An Introduction to Wireless Communication Systems:

- **Mobile Station**: a station in the cellular radio service intended for use while in motion at unspecified locations.
- **Base Station**: a fixed station in a mobile radio system used for radio communication with mobile stations. Base stations are located at the center or on the edge of a coverage area and consist of radio channels and transmitter and receiver antennas mounted on a tower.
- **Control Channel**: a radio channel used for control purposes such as call setup, call request, or call initiation and others.
- **Forward Channel**: a radio channel used for transmission from the base station to the mobile station.

- **Reverce Channel**: a radio channel used for transmission from the mobile station to the base station.
- **Subscriber**: a user who pays subscription charges for using a mobile communication system.
- **Transceiver**: a device capable of simultaneously transmitting and receiving radio signals.
- **Simplex Systems**: communication systems which provide only one-way communication (e.g. Paging Systems).
- **Full Duplex Systems**: communication systems which allow simultaneous two-way communication. Transmission and reception is on two different channels (FDD) or during adjacent time slots on a single channel (TDD).
- **Half Duplex Systems**: communication systems which allow two-way communication by using the same radio channel for both transmission and reception. However, at any time,

the user can either transmit or receive (e.g. Push to Talk).

- Handoff: the process of transferring a mobile satation from on channel to another or from one base station to another.

- Roamer: a mobile station which operates in a service area other than that from which it has subscribed to.

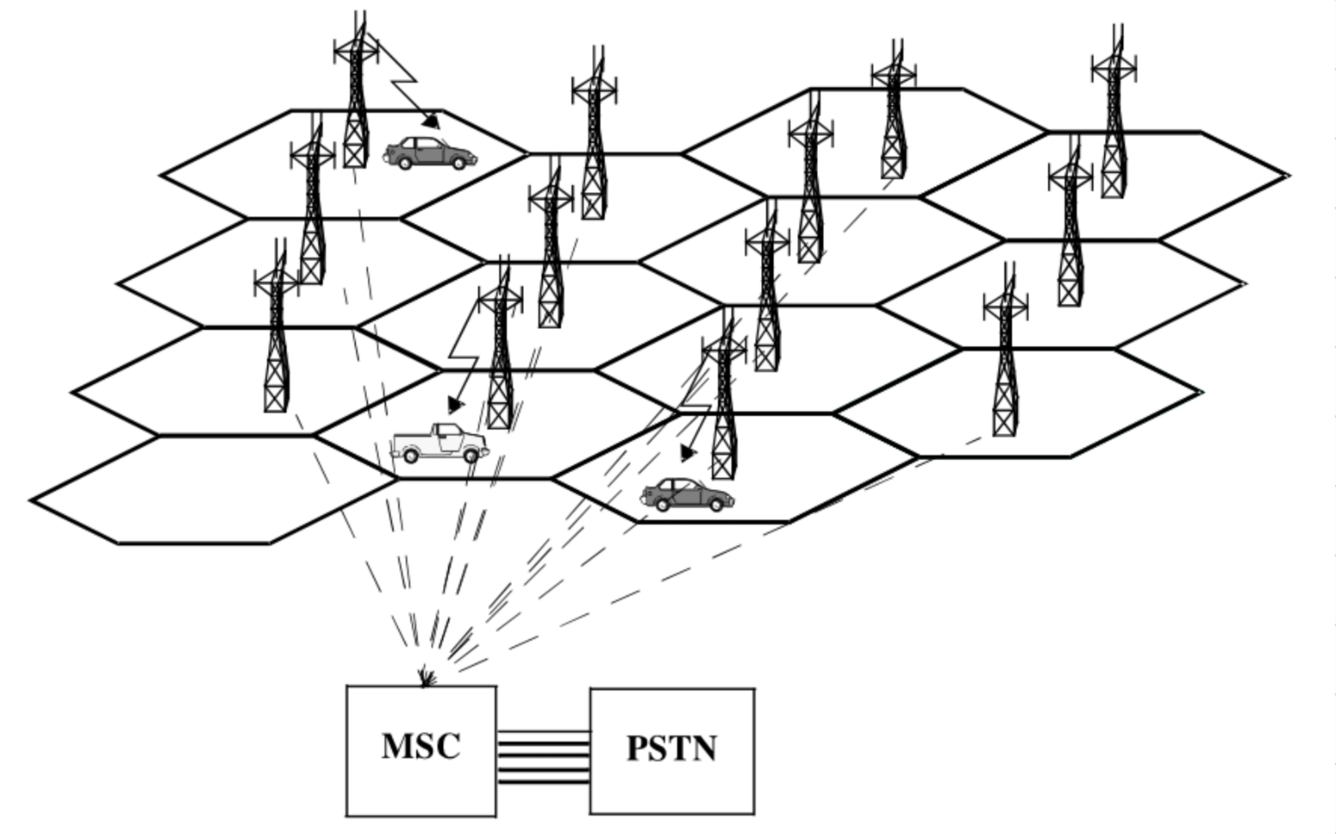
- Mobile Switching Center: in a cellular radio system, it connects the cellular base stations and the mobile stations to the PSTN (Public switched Telephone Network). MSC is sometimes called MTSO: Mobile Telephone Switching Office.

- FDD (Frequency Division Duplexing) provides two radio transmission channels one for the subscriber and the other of the base station so they may

transmit and receive signals at the same time.
At the base station, separate transmit and receive antennas are used to provide the two separate channels. At the subscriber unit, a single antenna is used for both transmitting and receiving the signals to and from the base station. While a device called a duplexer is used in the subscriber unit to enable the use of one antenna for simultaneous transmission and reception.
The channel for traffic from the base statio to the mobile user is the forward channel, while the channel for traffic from the mobile user to the base station is the reverse channel. FDD is used only in analog mobile radio systems.

- TDD (Time Division Duplexing) uses a single radio channel from the base station to the mobile user and from the mobile user to the base station by sharing the time. TDD is only possible in digital modulation in small area wireless systems to ensure low radio propagation time delay.

Cellular Telephone Systems:



Communication between a base station and a mobile user is defind by specified four different channels:

1. FVC (Forward Voice Channel): used for voice transmission from the base station to the mobiles.
2. RVC (Reverse Voice Channel): used for voice transmission from the mobiles to the base station

3. FCC (Forward Control Channel)

4. RCC (Reverse Control Channel)

- Both FCC & RCC are setup channels used in initiating calls and moving calls to unused voice channels. They are monitored by mobiles when they do not have a call in progress

MSC		Receives call from PSTN. Sends the requested MIN to all base stations.			Verifies that the mobile has a valid MIN, ESN pair.	Requests BS to move mobile to unused voice channel pair.		Connects the mobile with the calling party on the PSTN.
Base Station	FCC		Transmits page (MIN) for specified user.				Transmits data message for mobile to move to specific voice channel.	
	RCC			Receives MIN, ESN, Station Class Mark and passes to MSC.				
	FVC							Begin voice transmission.
	RVC							Begin voice reception.
Mobile	FCC		Receives page and matches the MIN with its own MIN.				Receives data messages to move to specified voice channel.	
	RCC			Acknowledges receipt of MIN and sends ESN and Station Class Mark.				
	FVC							Begin voice reception.
	RVC							Begin voice transmission.

time →

Diagram showing a call from a landline to a mobile is made.

- MIN : mobile identification number
- ESN : electronic serial number

MSC			Receives call initiation request from base station and verifies that the mobile has a valid MIN, ESN pair.	Instructs FCC of originating base station to move mobile to a pair of voice channels.		Connects the mobile with the called party on the PSTN.	
Base Station	FCC				Page for called mobile, instructing the mobile to move to voice channel.		
	RCC	Receives call initiation request and MIN, ESN, Station Class Mark.					
	FVC						Begin voice transmission.
	RVC						Begin voice reception.
Mobile	FCC				Receives page and matches the MIN with its own MIN. Receives instruction to move to voice channel.		
	RCC	Sends a call initiation request along with subscribe MIN and number of called party.					
	FVC						Begin voice reception.
	RVC						Begin voice transmission.

time →

Diagram showing a call from a mobile is made.

* Voice over IP (VoIP)
* Voice over LTE (VoLTE)

The Cellular Concept

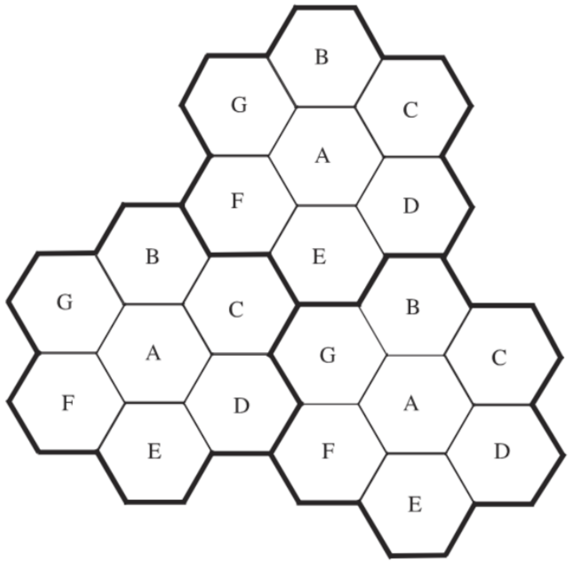
- The early mobile radio systems used a single high powered antenna mounted on a tall tower to provide a large coverage area.
- The cellular concept is to replace a single high powered antenna (large cell) with many low power antennas (small cells) to provide coverage to a small portion of the larger service area.
- Each base station is allocated a portion of the total number of channels available to the entire system.
- Neighboring base stations are assigned different groups of channels so that the interference between base stations and mobile users is minimized.
- As more channels are needed for new users, the number of base stations may be increased

along with decreasing the transmitter power to avoid interference (smaller cells), resulting in additional radio capacity with no additional radio spectrum.

- Each cellular base station covers a geographic area called a cell. The actual coverage of a cell is called the footprint.
- To cover an entire region without overlap and with equal area, we can use squares, equal-lateral triangles, or hexagons. For a given distance between the center of one of these shapes and its farthest perimeter points, the hexagon has the largest area. So fewer number of hexagon cells can cover a region compared to the other shapes.

- Frequency Reuse:

This figure illustrates the cellular concept of frequency reuse. Cells with the same letter use the same set of frequency. A cell cluster is outlined in bold line and replicated over the coverage area.



In this example, we have a cluster size $N = 7$.

The cluster is replicated $M = 3$ times.

The frequency reuse factor is given by $\frac{1}{N} = \frac{1}{7}$ here, since each cell within a cluster is assigned $\frac{1}{N}$ of the total available channels in the system.

So, for a total of S duplex channels available in a cellular system, each cell is allocated a group of K channels ($k < S$).

For N cells, S channels are divided into unique and disjoint channel groups which have the same number of channels, so the total number $S = kN$.

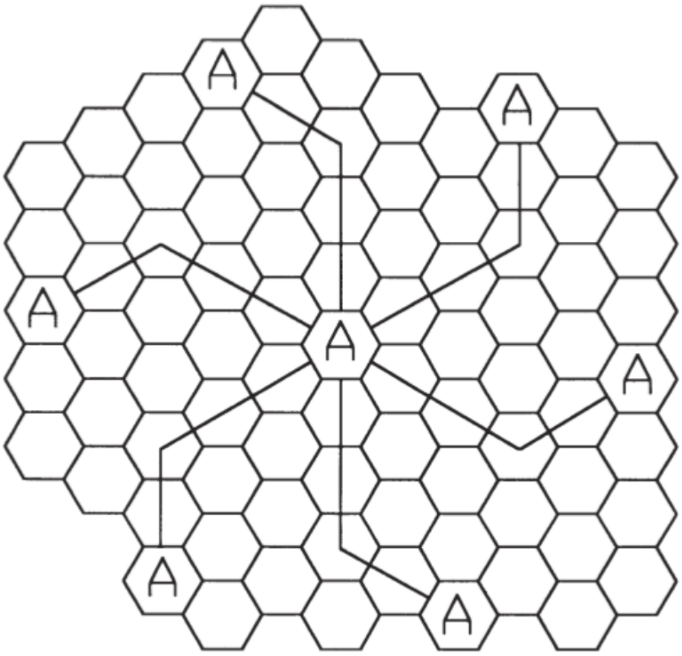
If the cluster is replicated M times within the system, the total number of duplex channels, C, that can be used, which is used as a measure of capacity $C = MS = MkN$.

- Since the hexagon has six neighbors, to cover an area without leaving gaps, the number of cells per cluster, i.e. the cluster size, N, can only have values that satisfy

$$N = i^2 + ij + j^2$$

where i and j are non-negative integers.

- The cells that reuse the same channels are called co-channel cells, here the cells with the letter A use the same channels.



To find the nearest co-channel neighbors of a particular cell :
first move i cells along any chain of hexagons, then turn $60°$ counter-clockwise and move j cells.
This illustration above is for $i=3$ and $j=2$,

$$N = i^2 + ij + j^2 = 3^2 + (3)(2) + 2^2 = 19.$$

Example: For a particular FDD cellular system, if a total of 33 MHz of bandwidth is allocated and uses two 25 kHz simplex channels to provide full duplex voice & control channels. If 1 MHz of the allocated spectrum is dedicated to control channels, compute the number of channels available per cell if a system uses:

(1) 4 cell reuse, (2) 7 cell reuse, (3) 12 cell reuse

Solution: $BW = 33\,MHz$

full duplex channel BW: $25\,kHz\,(2) = 50\,kHz$

Total full duplex channels: $\dfrac{33 \times 10^6}{50 \times 10^3} = 660$

Number of control channels: $\dfrac{1 \times 10^6}{50 \times 10^3} = 20$

Each cell needs only one control channel,

(1) $660 - 20 = 640$

$\frac{640}{4} = 160$

4 cells with 160 voice channels

4 control channels one per cell, 16 left.

(2) $\frac{640}{7} \simeq 91.43$, $(91)(7) = 637$, 3 left

3 cells with 92 voice channels

4 cells with 91 voice channels

7 control channels one per cell, 13 left

(3) $\frac{640}{12} \simeq 53.33$, $(53)(12) = 636$, 4 left

4 cells with 54 voice channels

8 cells with 53 voice channels

12 control channels one per cell, 8 left

Channel Assignment Strategies:

Channel assignment strategies can be classified as either fixed or dynamic.

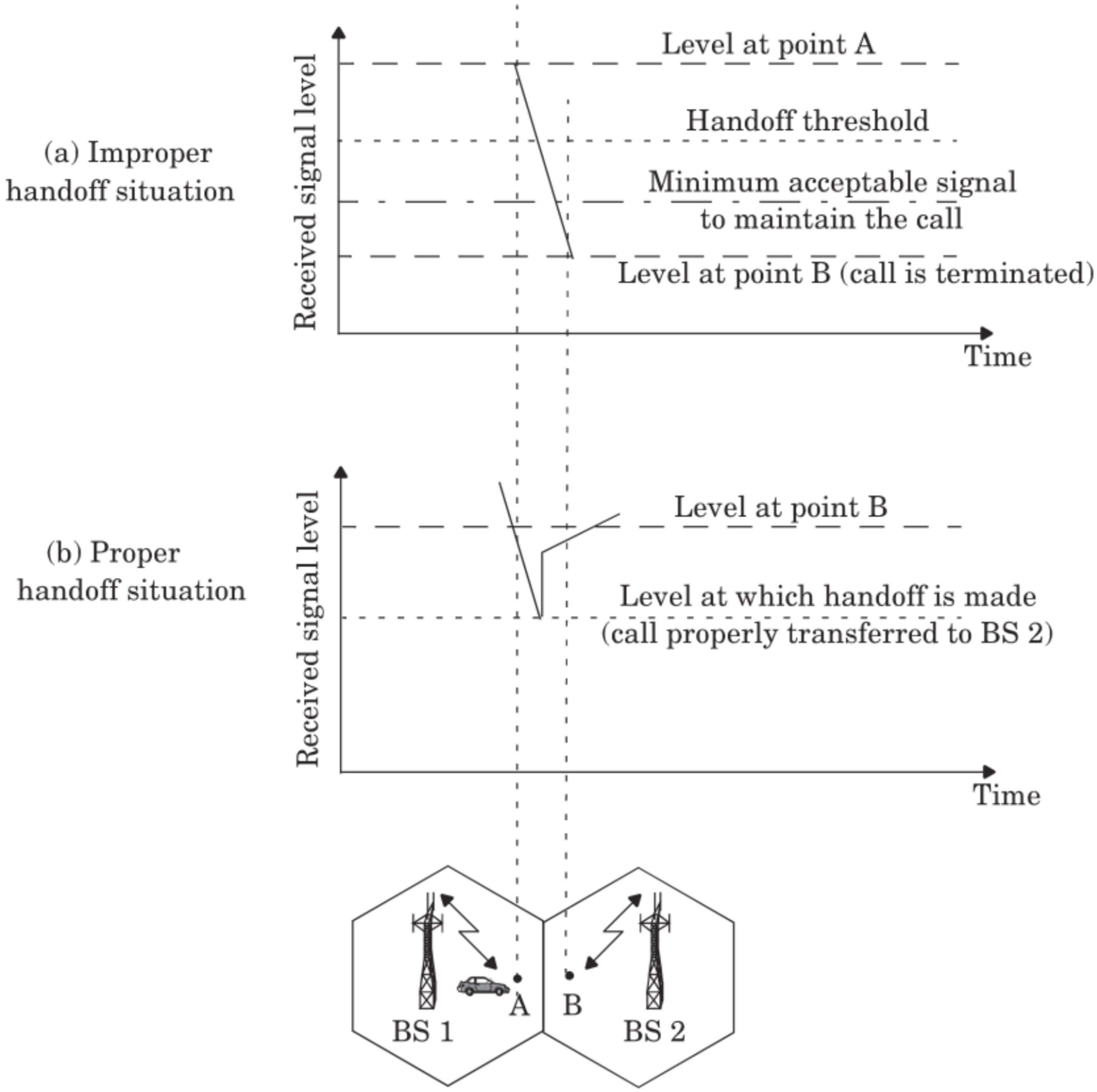
In a fixed channel assignment, a predetermined set of voice channels are allocated for each cell. If all channels in the cell are occupied, then the call is blocked and the user does not receive service. Some fixed strategies can allow the cell to borrow channels from neighboring cells if all its own channels are occupied, this strategy is called the borrowing strategy. Such borrowing is supervised by the mobile switching center (MSC) to ensure that this borrowing does not interfere with the calls in progress in the donor cell.

In dynamic channel assignment strategy, voice channels are not allocated to different cells permanently. However, channels are requested from the MSC by the serving base station each time a call request is made. The MSC allocates a frequency that is not in use in the cell or any other cell which falls within the minimum restricted distance of frequency reuse to avoid co-channel interference. Dynamic channel assignment strategies require the MSC to collect real-time data on channal occupancy, traffic distribution, and radio signal strength indications of all channels. This increases the storage and computational load on the system but provides the advantage of an increase in channel utilization and decrease the probability of call blocking.

- Handoff Strategies:

When a mobile user moves into a different cell while a call is in progress, the base station is changed and the voice and control channels are allocated to the new base station.

- Many handoff strategies prioritize handoff requests over call initiation requests.
- Handoffs must be performed successfully and as infrequently as possible, and without any notice from the user.



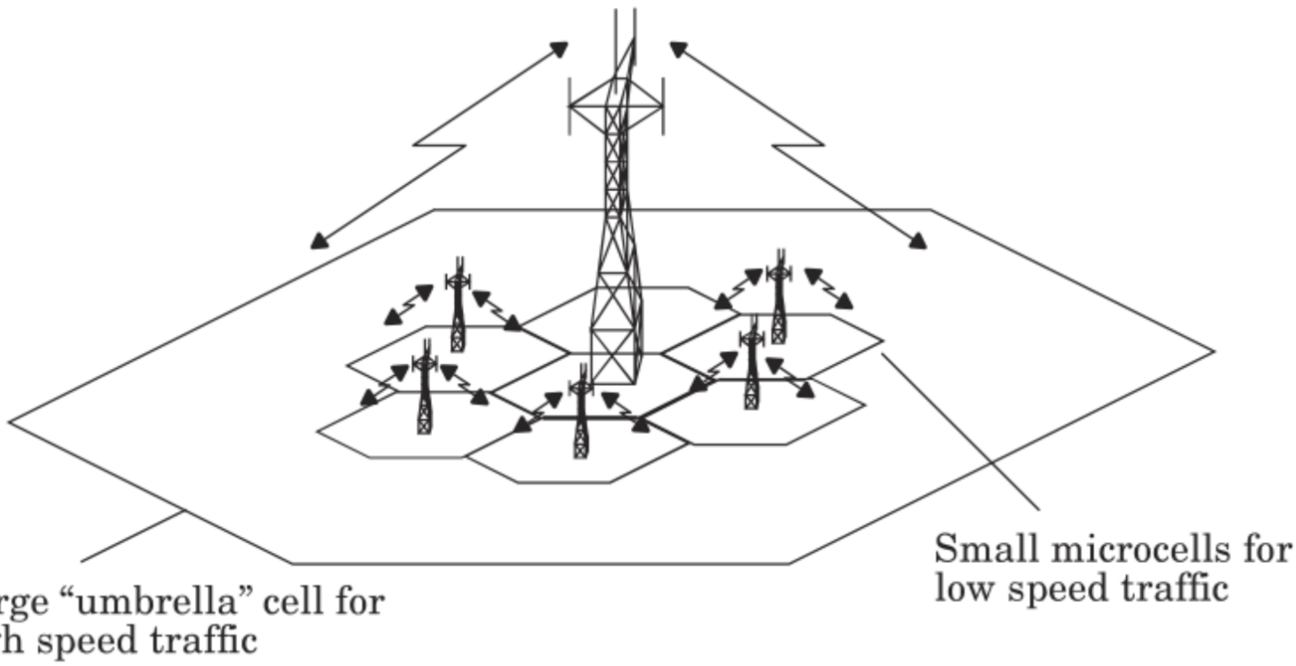
System designers specify a minimum usable signal for acceptable voice quality at the base station receiver, $P_{r,min.usable}$, and a slightly stronger signal level used as a threshold at which a handoff is made, $P_{r,handoff}$. This margin is given by:

$$\Delta = P_{r,handoff} - P_{r,min.usable}$$

Δ can't be too large or too small, if Δ is too large, unnecessary handoffs may occur causing burden on the MSC. If Δ is too small, there may be insufficient time to complete a handoff before the call is lost due to weak signal level.

Also, a call can be lost if the MSC caused an excessive delay in assigning a hand off channel, this can happen during high traffic due to either high computational loading at the MSC or no channels are available on any of the nearby base stations.

- High speed vehicles pass through a cell within seconds, while pedestrian users may never need a handoff during a call, so the use of the umbrella cell approach is used to provide for both cases.



This is done by using different antenna heights and different power levels at the same coverage area, and this is often done on the same building or tower. Larger cells will provide service to high speed moving vehicles while small cells service pedestrians.

- The speed of each user may be estimated by the base station or MSC by evaluating how fast the short-term average signal strength on the reverce voice channel (RVC) changes over time.

- When a pedestrian user provides a very strong signal to the base station while they walk away from the base station and have slow decay in the average signal strength. Even when the user has traveled well beyond the designed range of the cell. This happens when there is a line-of-sight (LOS) radio path between the user and the tower.

This may create interference and traffic management problems. This issue is called call dragging.

- One method for prioritizing handoffs is called guard channel concept, where a fraction of the total available channels in a cell is reserved only for handoff requests. In fixed channel assignment strategy, this method has the disadvatage of reducing the total channals for new call requests. However, for dynamic channel assignment strategies, guard channels offer efficient spectrum utilization since the channel allocations are demand based.

- Another method for prioritizing handoff is called queuing of handoff requests.

This method decreases the probability of forced termination of a call due to the lack of available channels. The queuing does not guarantee a handoff, since large delays will cause the received signal level to drop below the minimum required level to maintain communication and then leading to forced termination.

- In code division cellular systems or time division cellular systems, mobile users share the same channel in every cell, so the handoff term is for the change in different base stations that handle the radio communication. This handoff is done as multiple base stations receive the signal from a user, and allows the MSC to make a decision as to which signal to pass at any instance, this is called soft handoff. while the earlier handoff is called hard handoff.

- Interference and System Capacity:

Interference is the major limiting factor in the performance of cellular radio systems. Two major types of system generated cellular interference are co-channel interference and adjacent channel interference.

$\boxed{1}$ Co-channel Interference and System Capacity

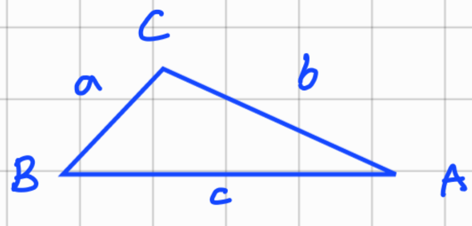
In a given coverage area, frequency reuse implies that there are several cells that use the same set of frequencies, these cells are called co-channel cells.

The interference between signals from these cells is called co-channel interference.

Thermal noise is overcome by increasing the signal-to-noise ratio (SNR) by increasing the transmitted power.

Co-channel interference can't be overcome by increasing the transmitted power, since the increase will result in an increase in interference to co-channel cells. To reduce co-channel interference, co-channel cells must be physically separated by a minimum distance to provide enough isolation due to propagation. When the size of each cell is approximately the same and the base stations transmit the same power, the co-channel signal to interference ratio becomes independent of the transmitted power and becomes a function of the radius of the cell (R) and the distance between centers of the nearest co-channel cells (D).

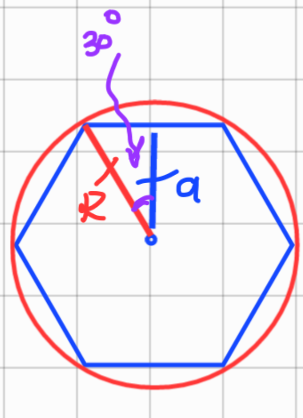
Recall:

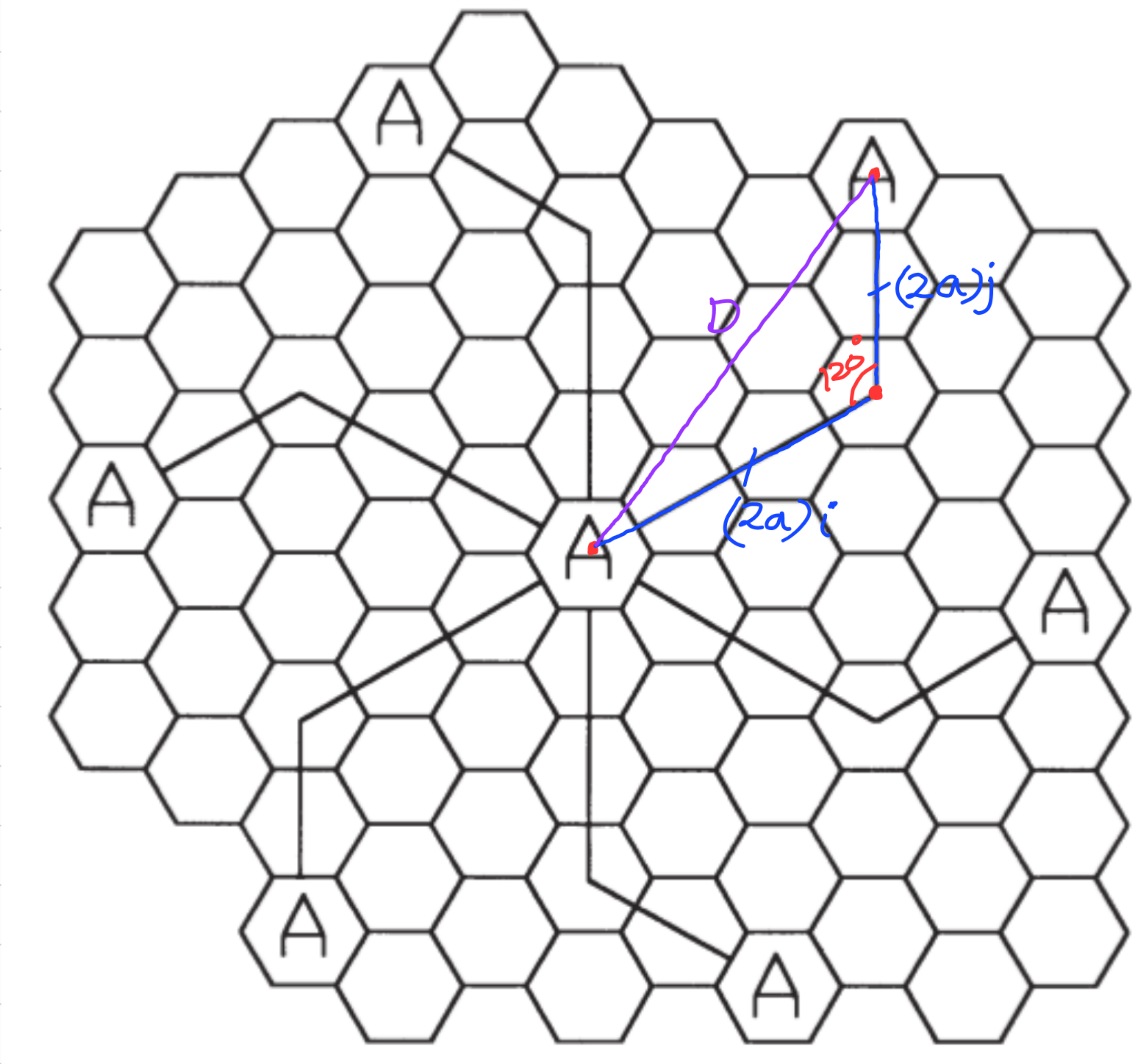


$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$R\cos 30 = a \Rightarrow \frac{a}{R} = \frac{\sqrt{3}}{2}$$

$$\frac{2a}{R} = \sqrt{3}$$





$$D^2 = (2ai)^2 + (2aj)^2 - 2(2ai)(2aj)\cos \cancel{120} \;\; -\tfrac{1}{2}$$

$$D^2 = (2a)^2\left[i^2 + j^2 + ij\right]$$

since $2a = \sqrt{3}R$, and $N = i^2 + j^2 + ij$

$$D^2 = 3R^2 N$$

$$\Rightarrow \frac{D}{R} = \sqrt{3N} =: Q$$

: a parameter called co-channel reuse ratio.

So by increasing Q, the spatial separation between co-channel cells (D) increases compared to the coverage distance of a cell (R)

While by decreasing Q, N is smaller, so to cover a same area we will need more cluster replication, larger M, leading to larger capacity C since $C = MKN$, leading to larger levels of co-channel interference. So in cellular design, a trad-off should be made for the suitable value of Q.

Recall: dB : decibel, B : bel $\Rightarrow 1\,dB = \frac{1}{10}\,B$

dB expresses the ratio of two values either power or root-power quantities (e.g. power is proportional to the square of valtage or current when impedance is constant).

$$1\,dB =: 10 \log 10^{\frac{1}{10}} = 10 \, \frac{1}{10} \log 10 = 1\,dB$$

This means that for a ratio say $\frac{P}{P_{ref}}$.

$10 \log \frac{P}{P_{ref}} = 1 \text{ dB}$ when $\frac{P}{P_{ref}} = 10^{\frac{1}{10}}$.

In a mobile radio channel, the propagation measurements show that the average received signal strength at any point decays as a power law of the distance of separation between a transmitter and a receiver.

Note: In statistics, a power law is a functional relationship between two quantities, when a relative change in one quantity results in a proportional relative change in the other quantity. It has its variable in its base, while the exponential function has its variable in its exponent.

Let d be the distance between the transmitter and receiver (T-R) separation distance. If P_o is the power received at a close-in (close to the transmitting antenna) reference point in the far field region of the antenna at a small distance d_o from the transmitting antenna, then the average received power P_r at a distance d from the transmitting antenna is approximated by

$$P_r = P_o \left(\frac{d}{d_o}\right)^{-n}$$

or

$$P_{r(dB)} = P_{o\,(dB)} - 10\, n \log\left(\frac{d}{d_o}\right)$$

n: is the path loss exponent, it depends on the environment as shown in the table.

Environment	Path Loss Exponent, *n*
Free space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

If D_i is the distance of the i^{th} co-channel cell interferer from a mobile user, then the received power at a given mobile due to the i^{th} interfering cell will be proportional to $(D_i)^{-n}$. Let i_0 be the number of co-channel interfering cells, then the signal-to-interference ratio (SIR or S/I) for a mobile receiver (on its forward channel)

$$\frac{S}{I} = \frac{S}{\sum_{i=1}^{i_0} I_i}$$

where S: is the desired power from the desired base station, I_i : is the interference power caused by the i^{th} interfering co-channel cell base station.

If I_i is known for all co-channel cells, then SIR can be found using the above equation.

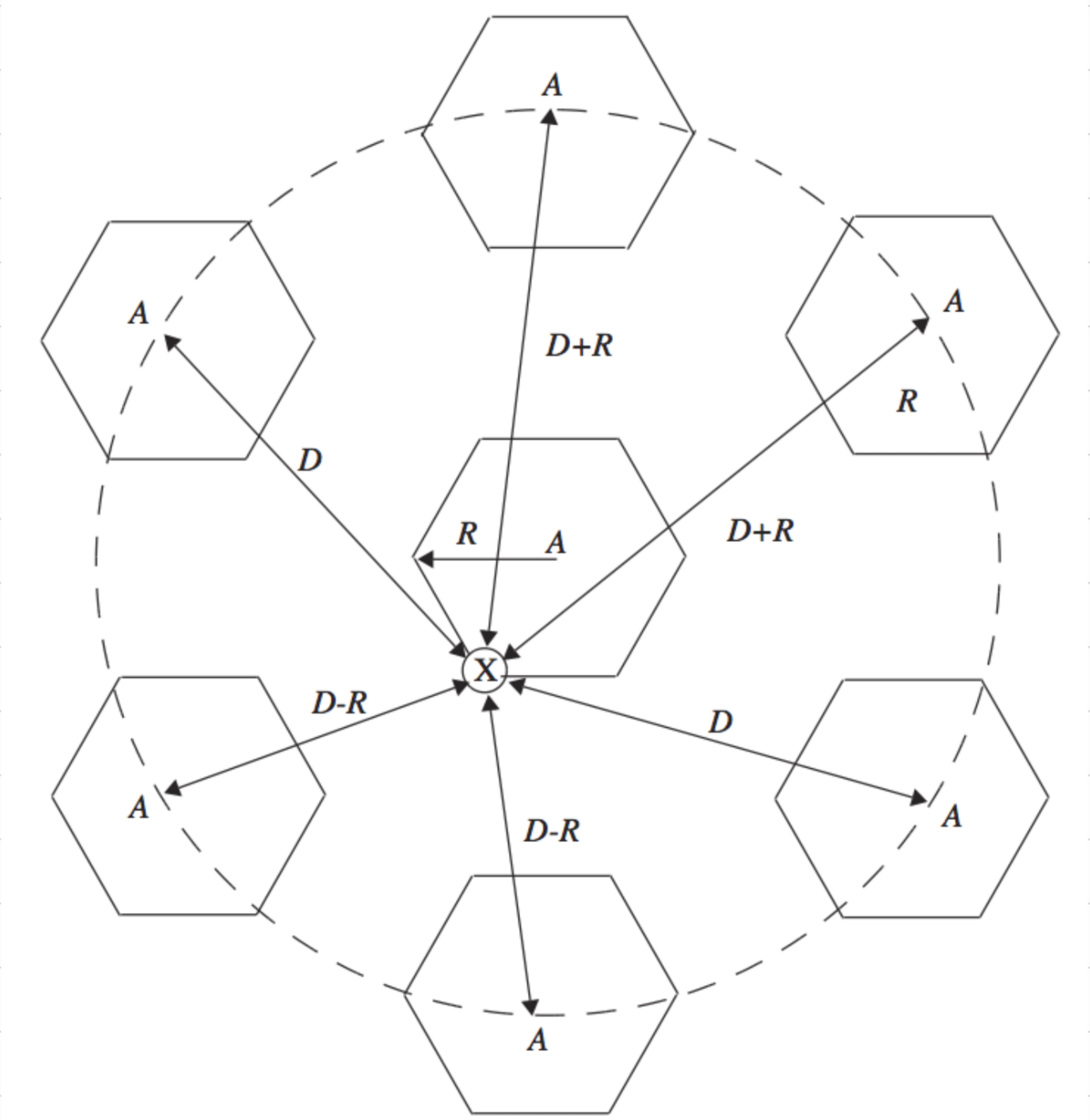
When the transmitted power of each base station is equal and the path loss exponent is the same throughout the coverage area, the SIR for a mobile is

$$\frac{S}{I} = \frac{R^{-n}}{\sum_{i=1}^{i_0} (D_i)^{-n}}$$

For the case of considering only the first layer of interfering cells and all the interfering base stations are equidistant from the desired base station with a distance D between cell centers,

$$\frac{S}{I} = \frac{R^{-n}}{i_0 D^{-n}} = \frac{1}{i_0}\left(\frac{D}{R}\right)^n = \frac{1}{i_0}\left(\sqrt{3N}\right)^n$$

For a more close approximation of SIR, consider an example where we have $N=7$, the first layer of interfering cells has 7 clusters, this is 7 co-channel cells interfering with the cell at the center as illustrated below.



Based on an appropriate approximation of the distances between co-channels to simplify the analyses, for the worst case, the mobile user is at the edge of its cell (at the cell boundary at point x). The mobile user is at a distance R from its base station, a distance (D-R) from the two closest interfering cells, a distance (D+R) from the two furthest interfering cells, and a distance D from the two left. Then

$$\frac{S}{I} = \frac{R^{-n}}{2(D-R)^{-n} + 2(D+R)^{-n} + 2D^{-n}}$$

if we multiply SIR by $\frac{R^n}{R^n}$ we can write the above result in terms of Q :

$$\frac{S}{I} = \frac{1}{2(Q-1)^{-n} + 2(Q-1)^{-n} + 2Q^{-n}}$$

for any path loss exponent n.

Example: If a SIR of 15 dB is required for satisfactory forward channel performance of a cellular system, what is the frequency reuse factor and cluster size that should be used for maximum capacity if the path loss is (a) $n=4$, (b) $n=3$.

Assume that there are six co-channel cells in the first tier, and all of them are at the same distance from the mobile. Use suitable approximations.

Solution:

(a) $n=4$, assume $N=7$, $Q=\sqrt{3N}=\sqrt{21}\simeq 4.583$

$$SIR=\frac{1}{6}Q^n=\frac{1}{6}(\sqrt{21})^4=73.5=18.66\,dB$$

Since SIR is greater than 15 dB, then $N=7$ can be used.

(b) $n=3$, assume $N=7$, $Q=\sqrt{3N}=\sqrt{21}\approx 4.583$

$SIR=\frac{1}{i_0}Q^n=\frac{1}{6}(\sqrt{21})^3=16.04=12.05\ dB$

since $12.05\ dB < 15\ dB$, then $N=7$

is not acceptable.

assume $N=12$, $Q=\sqrt{3N}=\sqrt{36}=6$

$SIR=\frac{1}{i_0}Q^n=\frac{1}{6}(6)^3=36=15.56\ dB$

since $15.56\ dB > 15\ dB$, then $N=12$

can be used.

- Aavailable mobile radio spectrum is divided into channels through an air interface standard that is used throughout a country or a continent.
- About 5% of the entire mobile spectrum is devoted to control channels, while the remaining 95% of the spectrum is dedicated to voice channels.
- Air interface standard ensures a distinction between voice and control channels.
- Control channels are important in initiating a call, so more conservative frequency reuse strategy is applied to control channels compared to voice channels, it is given a better SIR.

- Adjacent Channel Interference : is the interference resulting from signals which are adjacent in frequency to the desired signal.
- This interference results from imperfect receiver filters which allow nearby frequencies to leak into the passband.
- This will be serious when a close signal source with a strong transmitted signal is adjacent in frequency to a receiver that is trying to hear a weaker transmitted signal from a source far from the receiver. This is called near-far effect.
- Adjacent channel interference can be minimized through careful filtering and channel assignments.
- Frequency separation between each channel

in a given cell is kept as large as possible.

Many channel allocation schemes separate adjacent channels in a cell by $\underline{\underline{N}}$ channel bandwidths. Also, adjacent channels are not used in neighboring cells.

For example, if a mobile is close to a base station on the order of 20 times closer than another mobile and has energy spillout of its passband, then before receiver filtering, the SIR at the base station is approximately $SIR \simeq (20)^{-n}$ for an n path loss exponent.

For $n=4$, $SIR = 10 \log 20^{-4} = -52 \text{ dB}$

Example: This illustrates how channels are divided into subsets and allocated to different cells to minimize adjacent channel interference.

Table 3.2 AMPS Channel Allocation for A and B Side Carriers

1A	2A	3A	4A	5A	6A	7A	1B	2B	3B	4B	5B	6B	7B	1C	2C	3C	4C	5C	6C	7C	
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	
22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	41	42	
43	44	45	46	47	48	49	50	51	52	53	54	55	56	57	58	59	60	61	62	63	
64	65	66	67	68	69	70	71	72	73	74	75	76	77	78	79	80	81	82	83	84	
85	86	87	88	89	90	91	92	93	94	95	96	97	98	99	100	101	102	103	104	105	
106	107	108	109	110	111	112	113	114	115	116	117	118	119	120	121	122	123	124	125	126	
127	128	129	130	131	132	133	134	135	136	137	138	139	140	141	142	143	144	145	146	147	
148	149	150	151	152	153	154	155	156	157	158	159	160	161	162	163	164	165	166	167	168	
169	170	171	172	173	174	175	176	177	178	179	180	181	182	183	184	185	186	187	188	189	
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481	482	483	484	485	486	487	488	489	490	491	492	493	494	495	496	497	498	499	500	501	
502	503	504	505	506	507	508	509	510	511	512	513	514	515	516	517	518	519	520	521	522	
523	524	525	526	527	528	529	530	531	532	533	534	535	536	537	538	539	540	541	542	543	B
544	545	546	547	548	549	550	551	552	553	554	555	556	557	558	559	560	561	562	563	564	SIDE
565	566	567	568	569	570	571	572	573	574	575	576	577	578	579	580	581	582	583	584	585	
586	587	588	589	590	591	592	593	594	595	596	597	598	599	600	601	602	603	604	605	606	
607	608	609	610	611	6612	613	614	615	616	617	618	619	620	621	622	623	624	625	626	627	
628	629	630	631	632	633	634	635	636	637	638	639	640	641	642	643	644	645	646	647	648	
649	650	651	652	653	654	655	656	657	658	659	660	661	662	663	664	665	666	-	-	-	
-	-	-	-	-	717	718	719	720	721	722	723	724	725	726	727	728	729	730	731	732	
733	734	735	736	737	738	739	740	741	742	743	744	745	746	747	748	749	750	751	752	753	
754	755	756	757	758	759	760	761	762	763	764	765	766	767	768	769	770	771	772	773	774	
775	776	777	778	779	780	781	782	783	784	785	786	787	788	789	790	791	792	793	794	795	
796	797	798	799																		

voice channels

control channels

extended blocks

voice channels

extended blocks

This shows two operators in this service area, operator distinguished as block A and the other as block B.

Here we have 416 channels per operator, 395 voice channels and 21 control channels.

In this table, for a cluster size $N = 7$, each cell uses three subsets of channels, the three subsets are assigned to assure that every channel in the cell is separated from every other channel by at least seven channel spacings, each cell uses channels in the subsets iA, iB, iC where i is an integer from 1 to 7.

For control channels, each cell is assigned a single control channel, for an $N=7$, seven of these 21 control chanels will be used, then for two other neighboring clusters the remainding 14 control channels will be used. This means for an $N=7$, the voice channels will be reused for every seven cells, while the control channels will be reused for every 21 cells. Here every row has 21 channels & each row has about 19 voice channels, resulting in $(19)(3)=57$ voice channels, since the every channel in a cell is separated from every other channel by at least 7 channel spacings as mentioned above.

_ Power control for reducing interference:

the power level transmitted by any subscriber is under constant control by the serving base station to ensure the longest battery life for the subscriber and to reduce the SIR on the reverse channel.

- Trunking and Grade of Service (GOS):

- Trunking as a concept is to allow a large number of users to share the relatively small number of channels in a cell by providing access to each user, on demand, from a pool of available channels.

- Grade of Service (GOS): is a measure of the ability of a user to access a trunked system during the busiest hour during a week, month or a year.

- The two types of trunked systems that are common are:

(1) Blocked Calls Cleared (Erlang B)

(2) Blocked Calls Delayed (Erlang C)

Set-up Time: The time required to allocate a trunked radio channel to a requesting user.

Blocked Call: Call which cannot be completed at time of request, due to congestion. Also referred to as a *lost call*.

Holding Time: Average duration of a typical call. Denoted by H (in seconds).

Traffic Intensity: Measure of channel time utilization, which is the average channel occupancy measured in Erlangs. This is a dimensionless quantity and may be used to measure the time utilization of single or multiple channels. Denoted by A.

Load: Traffic intensity across the entire trunked radio system, measured in Erlangs.

Grade of Service (GOS): A measure of congestion which is specified as the probability of a call being blocked (for Erlang B), or the probability of a call being delayed beyond a certain amount of time (for Erlang C).

Request Rate: The average number of call requests per unit time. Denoted by λ seconds^{-1}.

- The traffic intensity per user (A_u) :

$$A_u = \lambda H$$

λ : is the average number of call requests per unit time for each user.

H : is the average duration of a call.

- The total offered traffic intensity (A) :

$$A = U A_u$$

U : number of users in a system with an unspecified number of channels.

- Note that the offered traffic is not necessarily the traffic carried by the trunked system.

- The traffic intensity per channel (A_c), if the traffic is equally distributed among the channels

$$A_c = A / C = U A_u / C$$

C : number of channels in a trunked system

- The maximum possible carried traffic is the total number of channels C (in Erlangs)

One Erlang represents the amount of traffic intensity carried by a channel that is completely occupied (i.e. one call-hour per hour or one call-minute per minute). For example, a channel that is occupied for thirty minutes during an hour carries 0.5 Erlang of traffic.

- For a system designd for a GOS of 2% blocking implies that the channel allocations are designed so that 2 out of 100 calls will be blocked due to channel occupancy during the busiest hour.

[1] The Blocked Call Cleared trunked system :

for every user who requests service, it is assumed there are no setup time and the user is given immediate access to a channel if one is available, if no channels are available, the requesting user is

blocked without access and is free to try again later.
This type offers no queuing for call requests.
This type assumes that calls arrive as Poisson distribution, it is assumed that there are infinite users, there are memoryless arrivals of requests, this means that all users including blocked users may request a channel at any time. Also the probability of a user occupying a channel is exponentially distributed, this means that the call length has an exponential distribution with longer calls less likely to happen. In addition, there are finite number of channels available to the trunking pool. This leads to the Erlang B formula that determines the probability that a call is blocked. It is a measure of the GOS for

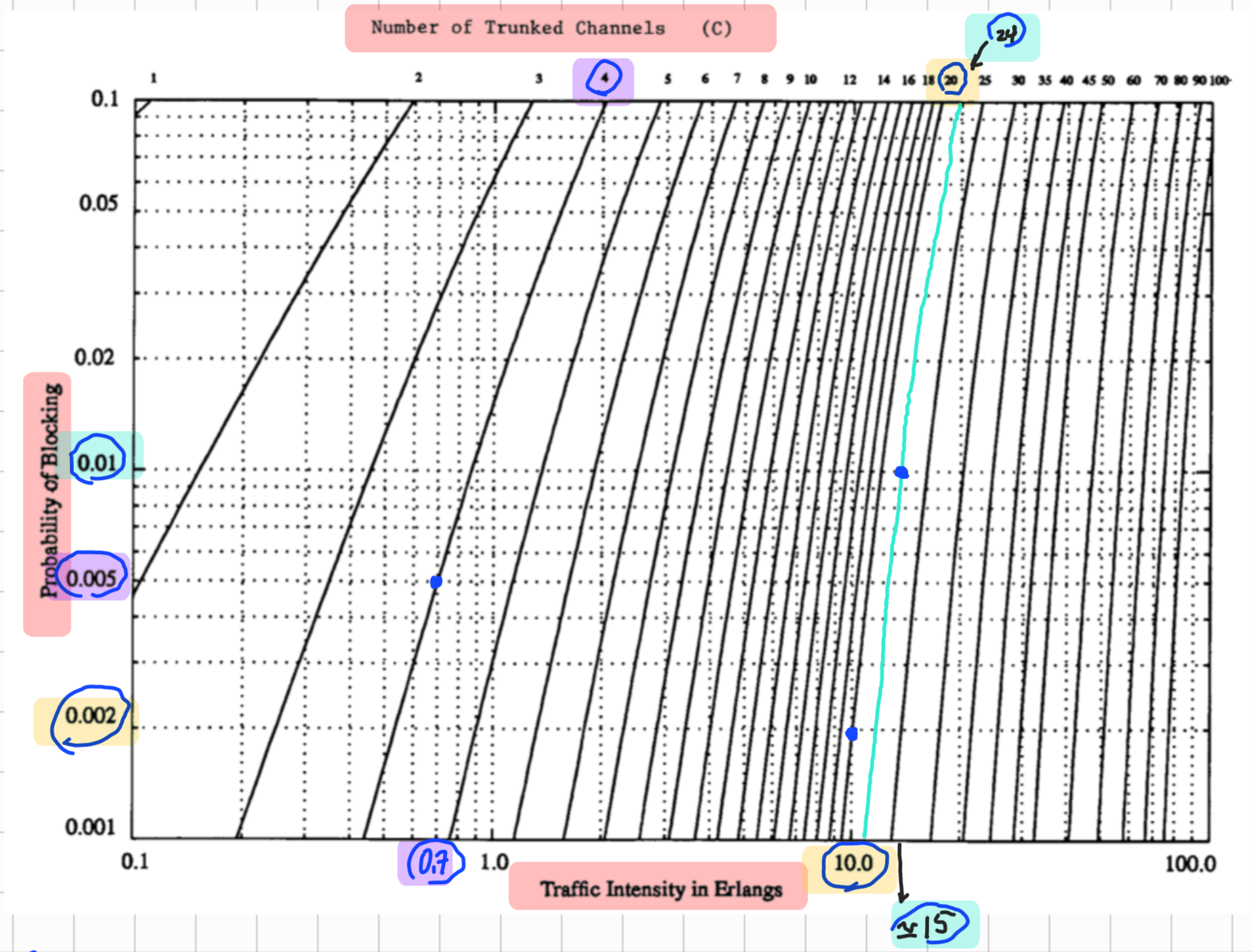
a blocked calls cleared trunking system.

$$P\{\text{Blocking}\} = \frac{\frac{A^C}{C!}}{\sum_{k=0}^{C} \frac{A^k}{k!}} =: \text{GOS}$$

C is the number of trunked channels offered by a trunked radio system, A is the total offered traffic.

The below table shows the capacity (in Erlangs) of a blocked call cleared trunked system for various values of P{Blocking} (i.e. GOS) and the number of channels

Number of Channels C	Capacity (Erlangs) for GOS			
	GOS = 0.01	GOS = 0.005	GOS = 0.002	GOS = 0.001
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2



The above figure is a plot of Erlang B formula.

The top x-axis has the number of trunked channels C.

The bottom x-axis has the traffic intensity A.

The y-axis has the probability of call blocking. All for the blocked calls cleared trunking system.

Example: for $C = 20$ and $GOS = 0.002$, $A \simeq 10$.

for $C = 4$ and $GOS = 0.002$, $A \simeq 0.7$

Also, for $GOS = 0.002$ and $A = 10$, $C = 20$

for $C = 24$ and $GOS = 0.01$, $A \simeq 15$.

[2] The Blocked Calls Delayed Trunked System:

In this system, if a channel is not available immediately, the call is hold in a queue until a channel is available. Here the GOS is the probability that a call is blocked after waiting a specific length of time in a queue. So, to find GOS here, the system denies access to the user at first, then the user waits in a queue, then it is blocked.

The probability of a call not having immediate access to a system is determined by the Erlang C formula: $P\{delay > 0\} = \dfrac{A^C}{A^C + C!\left(1 - \frac{A}{C}\right)\sum_{k=0}^{C-1} \frac{A^k}{k!}}$

So the probability that a delayed call is forced to wait more than t seconds is the GOS of such trunked system. It is measured as follows. If the probability that a call is delayed $t > t$ sec. given that the call had no immidiate access to a channel:

$$P\{delay > t \mid delay > 0\} = \exp\left\{ -\frac{(C-A)t}{H} \right\}$$

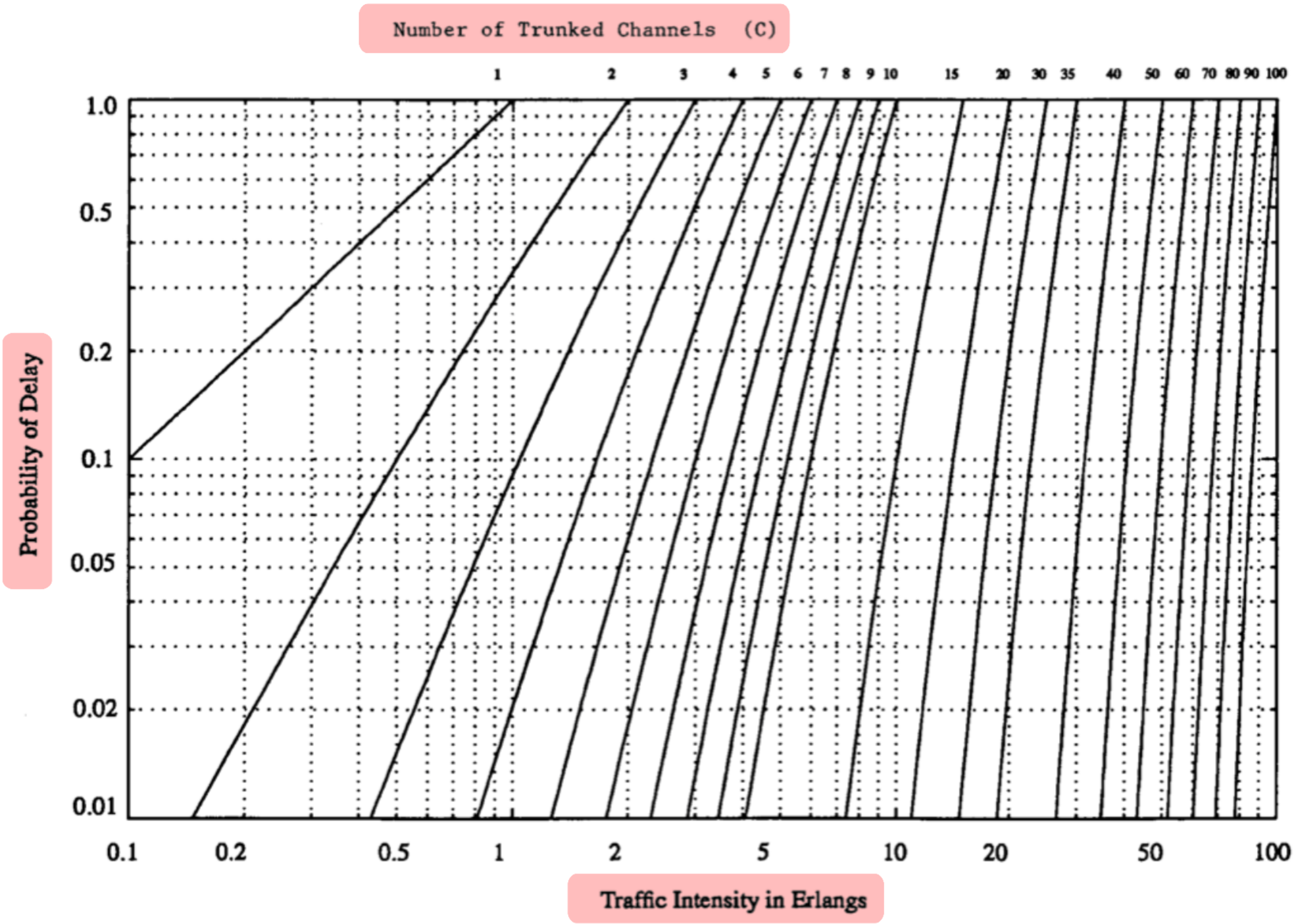
Then

$$P\{delay > t\} = P\{delay > t, delay > 0\}$$

$$= P\{delay > t \mid delay > 0\} P\{delay > 0\}$$

$$= \exp\left\{ -\frac{(C-A)t}{H} \right\} P\{delay > 0\}$$

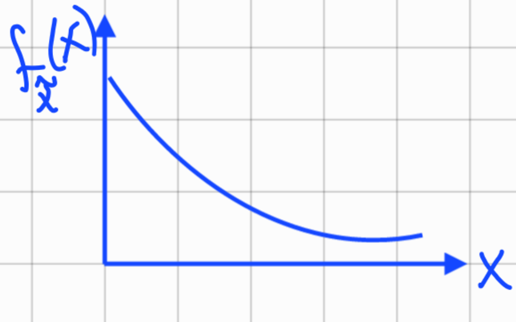
where $P\{delay > 0\}$ can be found from Erlang C formula or Erlang C chart.



Recall: For exponential distribution : $f_{\tilde{x}}(x) = \begin{cases} \lambda e^{-\lambda x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$

$E[\tilde{x}] = 1/\lambda$:= mean := expected value

$P\{\tilde{x} \geq x\} = e^{-\lambda x}$



$P\{\tilde{x} \leq x\} = \begin{cases} 1 - e^{-\lambda x} & , x \geq 0 \\ 0 & , x < 0 \end{cases}$

- The average delay of queued calls is $\dfrac{H}{C-A}$
- From Erlang B & Erlang C charts, from knowing two parameters, one can find the third parameter.

Example 3.4

How many users can be supported for 0.5% blocking probability for the following number of trunked channels in a blocked calls cleared system? (a) 1, (b) 5, (c) 10, (d) 20, (e) 100. Assume each user generates 0.1 Erlangs of traffic.

$GOS = 0.5\%$, $A_u = 0.1$

(b) $C=5$, $GOS = 0.5\%$, from Erlang B chart

$A \simeq 1$, $U = \frac{A}{A_u} = \frac{1}{0.1} = 10$ user

(c) $C=10$, $GOS = 0.5\%$, from Erlang B chart

$A \simeq 4$, $U = \frac{A}{A_u} = \frac{4}{0.1} = 40$ user

(d) $C=20$, $GOS = 0.5\%$, from Erlang B chart

$A \simeq 11$, $U = \frac{11}{0.1} = 110$ users

(e) $C = 100$, $GOS = 0.5\%$, from Erlang B chart

$A \simeq 80$, $U = \frac{80}{0.1} = 800$ users

Example 3.5

An urban area has a population of two million residents. Three competing trunked mobile networks (systems A, B, and C) provide cellular service in this area. System A has 394 cells with 19 channels each, system B has 98 cells with 57 channels each, and system C has 49 cells, each with 100 channels. Find the number of users that can be supported at 2% blocking if each user averages two calls per hour at an average call duration of three minutes. Assuming that all three trunked systems are operated at maximum capacity, compute the percentage market penetration of each cellular provider.

System A: GOS = 2% , C = 19 per cell , 394 cells

$A_u = \lambda H = \frac{2}{60}(3) = 0.1$ Erlang

For GOS = 2% , C = 19 $\Rightarrow$ from chart Erlang B

$A \simeq 12$, $U = \frac{A}{A_u} = \frac{12}{0.1} = 120$ user per cell

for 394 cells $\Rightarrow$ $(120)(394) = 47280$ users per sys.

percentage of market penetration $= \frac{47280}{2\times10^6} = 2.364\%$

System B: GOS = 2% , C = 57 per cell , 98 cells

$A_u = \lambda H = \frac{2}{60}(3) = 0.1$ Erlang

For GOS = 2% , C = 57 $\Rightarrow$ from chart Erlang B

$A \simeq 45$, $U = \frac{A}{A_u} = \frac{45}{0.1} = 450$ user per cell for 98 cells $\Rightarrow (450)(98) = 44100$ users per sys.

percentage of market penetration $= \frac{44100}{2\times10^6} = 2.205\%$

System C: $GOS = 2\%$, $C = 100$ per cell, 49 cells

$A_u = \lambda H = \frac{2}{60}(3) = 0.1$ Erlang

For $GOS = 2\%$, $C = 100 \Rightarrow$ from chart Erlang B

$A \simeq 90$, $U = \frac{90}{0.1} = 900$ user per cell

for 49 cells $\Rightarrow (900)(49) = 44100$ users per sys.

percentage of market penetration $= \frac{44100}{2\times10^6} = 2.205\%$

For the three systems the percentage of market penetration is $\left(\frac{47280 + 44100 + 44100}{2\times10^6}\right)$

$= \frac{135480}{2\times10^6} = 6.774\%$

or simply $2.364\% + 2.205\% + 2.205\% = 6.774\%$

Example 3.6

A certain city has an area of 1,300 square miles and is covered by a cellular system using a seven-cell reuse pattern. Each cell has a radius of four miles and the city is allocated 40 MHz of spectrum with a full duplex channel bandwidth of 60 kHz. Assume a GOS of 2% for an Erlang B system is specified. If the offered traffic per user is 0.03 Erlangs, compute (a) the number of cells in the service area, (b) the number of channels per cell, (c) traffic intensity of each cell, (d) the maximum carried traffic, (e) the total number of users that can be served for 2% GOS, (f) the number of mobiles per unique channel (where it is understood that channels are reused), and (g) the theoretical maximum number of users that could be served at one time by the system.

(a) area of a hex. $= \frac{3\sqrt{3}}{2} R^2 = \frac{3\sqrt{3}}{2}(4)^2 = 41.57$ mil.^{2}

\# of cells per whole area $= \frac{1300}{41.57} = 31.27 \Rightarrow 31$ cells.

(b) \# of channels per sys $= \frac{40 \times 10^6}{60 \times 10^3} = 666$ per sys.

\# of channels per cell $= \frac{666}{7} = 95$ per cell

(c) A per cell $\Rightarrow$ GOS $= 2\%$, $C = 95 \Rightarrow A = 85$

(d) A per sys. $\Rightarrow (31)(85) = 2635$

(e) $U = \frac{A}{A_u} = \frac{2635}{0.03} = 87833$ users in sys.

(f) users per channel $= \frac{87833}{666} = 131$ users / channel.

(g) at one time, max users = \# of channels in the sys.

$= (\# \text{of ch. per cell})(\# \text{of cells}) = (95)(31) = 2945$

Example 3.7

A hexagonal cell within a four-cell system has a radius of 1.387 km. A total of 60 channels are used within the entire system. If the load per user is 0.029 Erlangs, and $\lambda = 1$ call/hour, compute the following for an Erlang C system that has a 5% probability of a delayed call:

(a) How many users per square kilometer will this system support?

(b) What is the probability that a delayed call will have to wait for more than 10 s?

(c) What is the probability that a call will be delayed for more than 10 seconds?

(a) $N = 4$, 60 channels in sys., $A_u = 0.029$

$R = 1.387 \Rightarrow \text{area} = \frac{3\sqrt{3}}{2}(1.387)^2 \simeq 5 \text{ km}^2$

$\# \text{of channels per cell} = \frac{60}{4} = 15$

For $GOS = 0.05$, $C = 60 \Rightarrow A$ for sys. $\simeq 36$

For $GOS = 0.05$, $C = 15 \Rightarrow A$ per cell $\simeq 9$

$\Rightarrow A$ for sys. $= (4)(9) = 36$ (same as above).

$\# \text{of users per cell} = \frac{A \text{ per cell}}{A_u} = \frac{9}{0.029} \simeq 310$

$\# \text{of users per km} = \frac{\# \text{of users per cell}}{\text{cell area}} = \frac{310}{5} = 62$

(b) $C = 15$, $A = 9$, $H = \frac{A_u}{\lambda} = \frac{0.029}{1/hr.} = 0.029(60)60 = 104.4$ sec.

$$P\{\text{delay} > 10s \mid \text{delay} > 0\} = \exp\left\{-\frac{(C-A)t}{H}\right\}$$

$$= \exp\left\{-\frac{(15-9)\,10}{104.4}\right\} = 56.287\%$$

(c) $P\{\text{delay} > 10s\} = P\{\text{delay} > 10s \mid \text{delay} > 0\}\, P\{\text{delay} > 0\}$

$$= (0.56287)(0.05)$$

$$= 2.814\ \%$$

- Trunking efficiency: is a measure of the number of users (capacity C) that can be offered a particular GOS with a particular configuration of fixed channels. That is, the number of users handled by a trunked system is altered by the way channels ar grouped in a trunked system.

Number of Channels C	Capacity (Erlangs) for GOS = 0.01	= 0.005	= 0.002	= 0.001
2	0.153	0.105	0.065	0.046
4	0.869	0.701	0.535	0.439
5	1.36	1.13	0.900	0.762
10	4.46	3.96	3.43	3.09
20	12.0	11.1	10.1	9.41
24	15.3	14.2	13.0	12.2
40	29.0	27.3	25.7	24.5
70	56.1	53.7	51.0	49.2
100	84.1	80.9	77.4	75.2

10 trunked channels at GOS = 0.01 ⇒ 4.46 Erlangs

two 5 trunked channels at GOS = 0.01 ⇒ 2(1.36) = 2.72

100 trunked channels at GOS = 0.002 ⇒ 77.4 Erlangs

five 20 trunked channels at GOS = 0.002 ⇒ 5(10.1) = 50.5

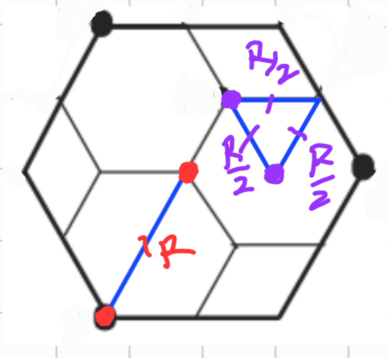
Improving Coverage and Capacity in Cellular Systems:

1. Cell Splitting
2. Cell Sectoring
3. Repeaters of Range Extension
4. Microcell Zone Concept

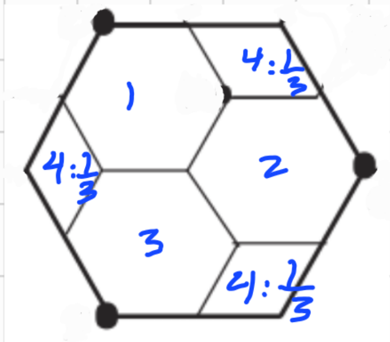
[1] Cell Splitting : is the process of subdividing a congested cell into smaller cells, each new cell with its own base station, and the antenna height is reduced as well as the transmitted power.

- This process of cell splitting will increase the capacity of the cellular system since it increases the number of times the channels are reused per unit area, the new smaller cells are called microcells

- If we reduce the radius of every cell into half, to cover the



same area with the smaller cells, we will use four times as many cells. It is shown geometrically to the right.

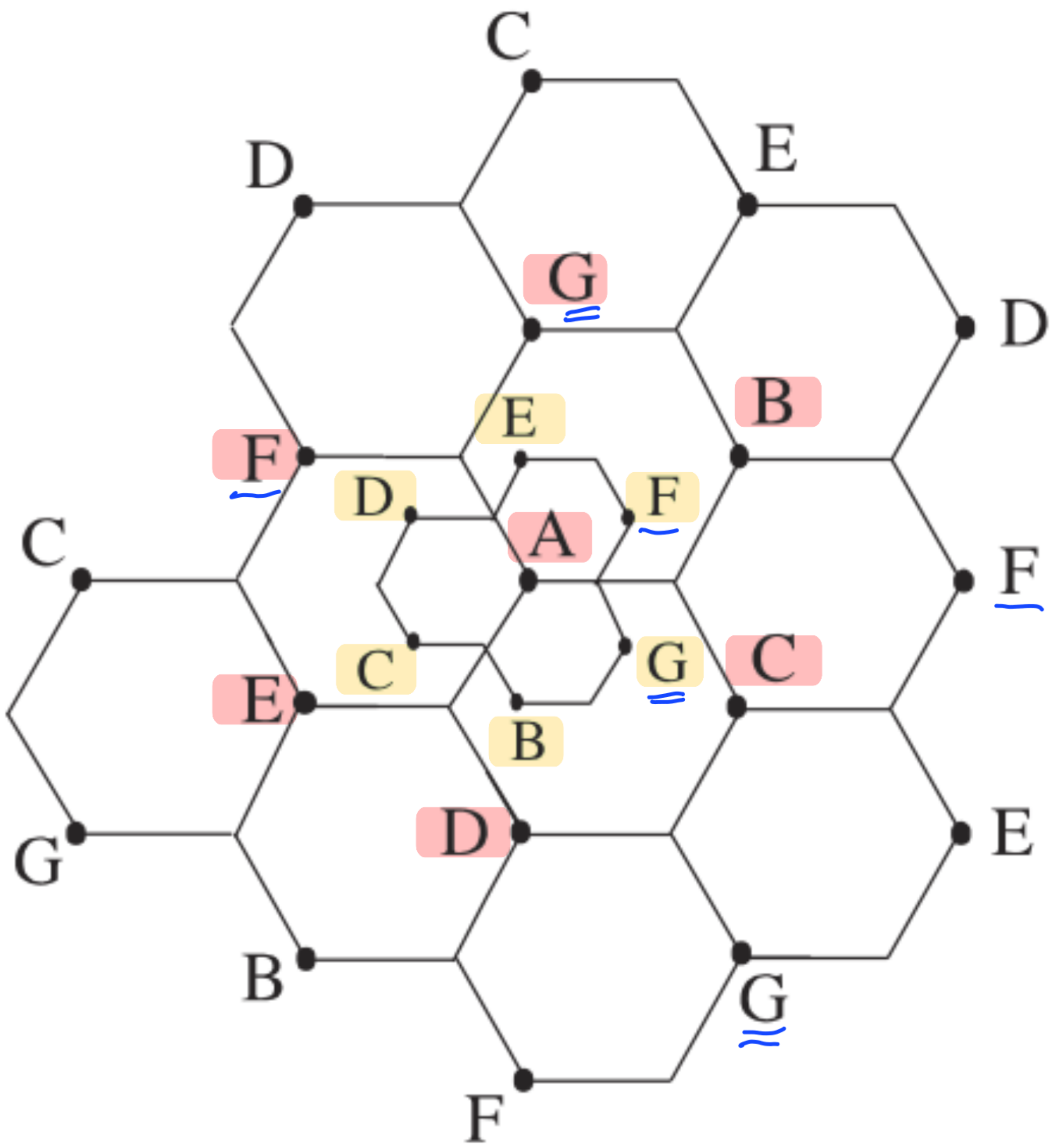


Also the ratio between both areas is

$$\text{ratio} : \frac{\text{area of hex}, R}{\text{area of hex}, \frac{R}{2}} = \frac{(3\sqrt{3}/2)\,R^2}{(3\sqrt{3}/2)\left(\frac{R}{2}\right)^2} = 4$$

- When the number of cells increase (N), the number of clusters over the same area increases (M), then the number of channels per system increases in the same area (system capacity C).
- In cell splitting, the system capacity grows by replacing large cells with small cells while maintaining the co-channel reuse ratio $Q = \frac{D}{R}$
- For example, if the base stations are placed at the corners of the cells as shown in the below illustration, and the cell A is

saturated with traffic, where the GOS becomes lower then accepted, that is when calls are blocked more than the accepted rate. A solution could be through cell splitting. The frequency reuse for the microcells is the same for the original large cells.



where the microcell base stations are placed half way between the two larger original base stations using the same channel sets.

- So cell splitting is simply scaling the geometry of the cluster by scaling the radius of each new microcell. Here, R is half the original cell.
- The transmit power of the microcells must be reduced. This is done to ensure the frequency reuse plan for the new cells behaves the same as the original cells. So, the transmit power at the boundaries of both original and new cells should be equal.

That is: $P_r\{\text{at original cell boundary}\} \propto P_{(t,\,Large)}\, R^{-n}$

and $P_r\{\text{at new cell boundary}\} \propto P_{(t,\,Small)}\, R^{-n}$

where $P_{(t,\,Large)}$ and $P_{(t,\,Small)}$ are the transmit powers of the large and small cell base stations, respectively, and n is the path loss exponent.

For example, if $n = 4$, and the new cells have a radius half the original cell. We set

P_r {at orginal cell boundary} = P_r {at new cell boundary}

then $$P_{(t,Small)} = \frac{1}{16} P_{(t,Large)}$$

or

$$P_{(t,Small)} = P_{(t,Large)} - 10\log 16$$

$$P_{(t,Small)} = P_{(t,Large)} - 12\,dB$$

this means that the transmit power must be reduced by 12 dB in the new smaller cells to maintaine the required SIR, for this example.

- In practice, not all cells are split at the same time, and sometimes providers can't find perfectly situated real estate. So, channel assignements will be more complecated. This leads to having both small cells and large cells simultaneously.

This will result in having a transmit power for small cells different than the transmit power for large cells. For this reason, channels in the original cell must be divided into two channel groups, one for the smaller cell reuse and the other for the larger cell reuse. The sizes of the two channel groups depend on the stage of the splitting process. At the early stages of splitting, fewer channels will be in the smaller cells. Then as the demand grows, more channels will be required, and then the smaller cells will require more channels. This splitting process continues until all the channels are used in the smaller cells, at which point all the original larger cells are split into smaller cells and the entire system is rescaled to have a smaller radius per cell.

- Antenna downtilting: here the antenna's radiated energy is focused from the base station toward the ground to limit the radio coverage of newly formed microcells rather than toward the horizon.

Example 3.8

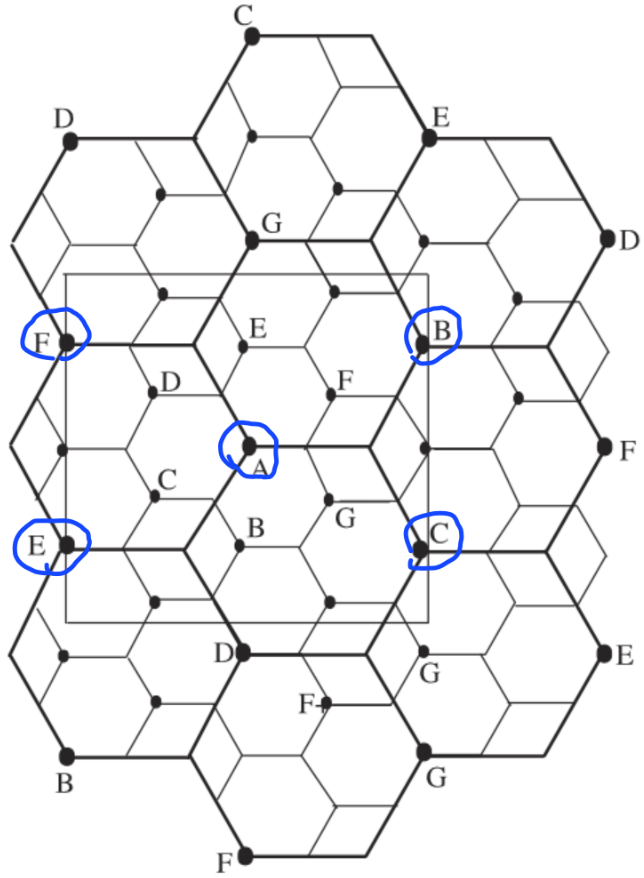
Consider Figure 3.9. Assume each base station uses 60 channels, regardless of cell size. If each original cell has a radius of 1 km and each microcell has a radius of 0.5 km, find the number of channels contained in a 3 km by 3 km square centered around A under the following conditions: (a) without the use of microcells; (b) when the lettered microcells as shown in Figure 3.9 are used; and (c) if all the original base stations are replaced by microcells. Assume cells on the edge of the square to be contained within the square.

(a) 5 base stations,

60 channels per base station:

# of channels in such area =

$(5)(60) = 300$ channel.



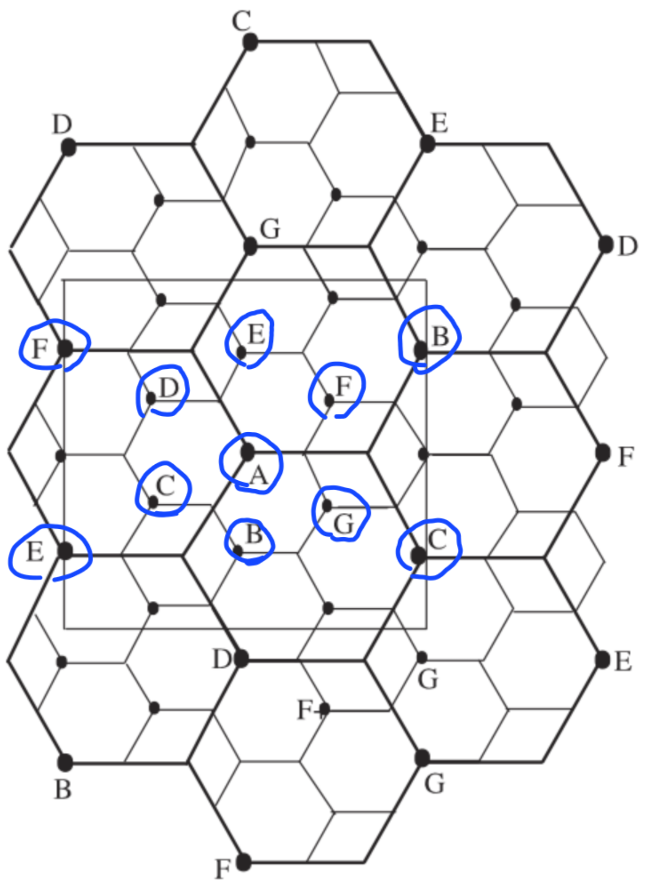
(b) 5 + 6 = 11 base stations,
60 channels / base station:

# of channels in such area =

(11)(60) = 660 channels

$\frac{660}{300}$ = 2.2 times more channels

compared to the case in (a)



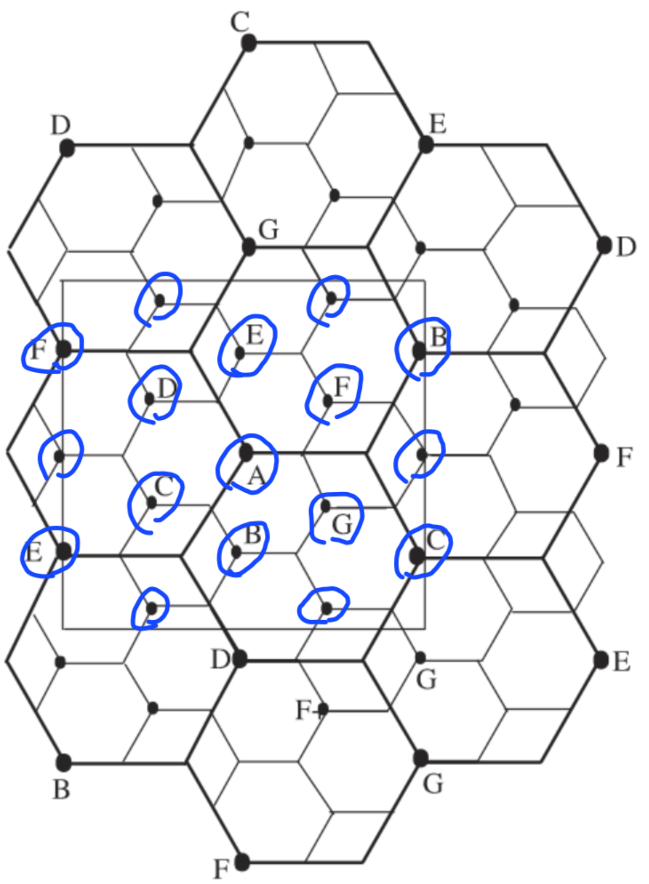
(c) 11 + 6 = 17 base stations,
60 channels / base station:

# of channels in such area =

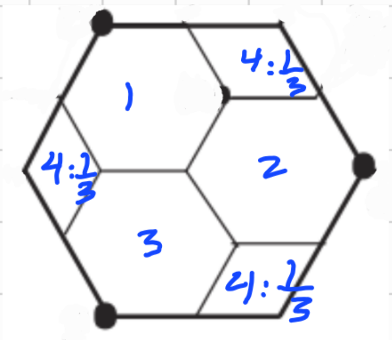
(17)(60) = 1020 channels

$\frac{1020}{300}$ = 3.4 times more channels

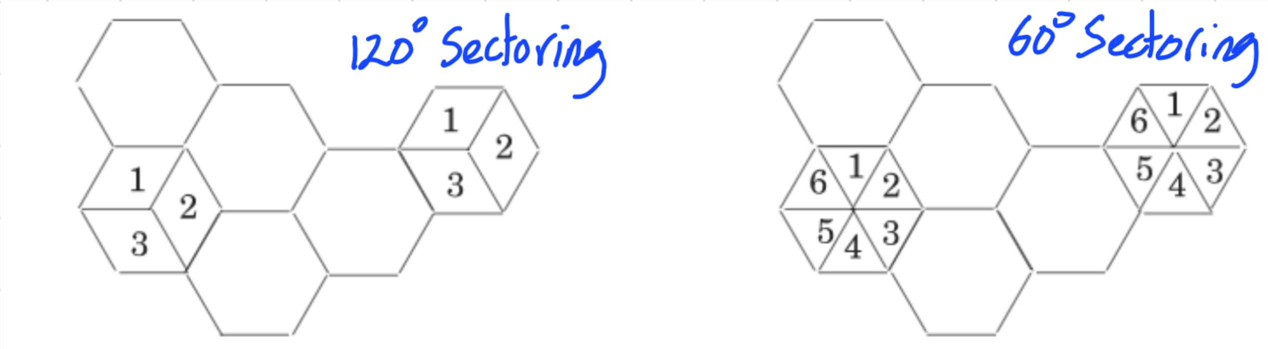
compared to the case in (a)



For this example, if all cells were split into microcells with half the radius of the original cell, then the number of channels will increase by four, since the ratio between the original cells area and the new microcells will be four. So, the number of cells in the covered area will increase by the factor of four, as illustrated earlier & below.

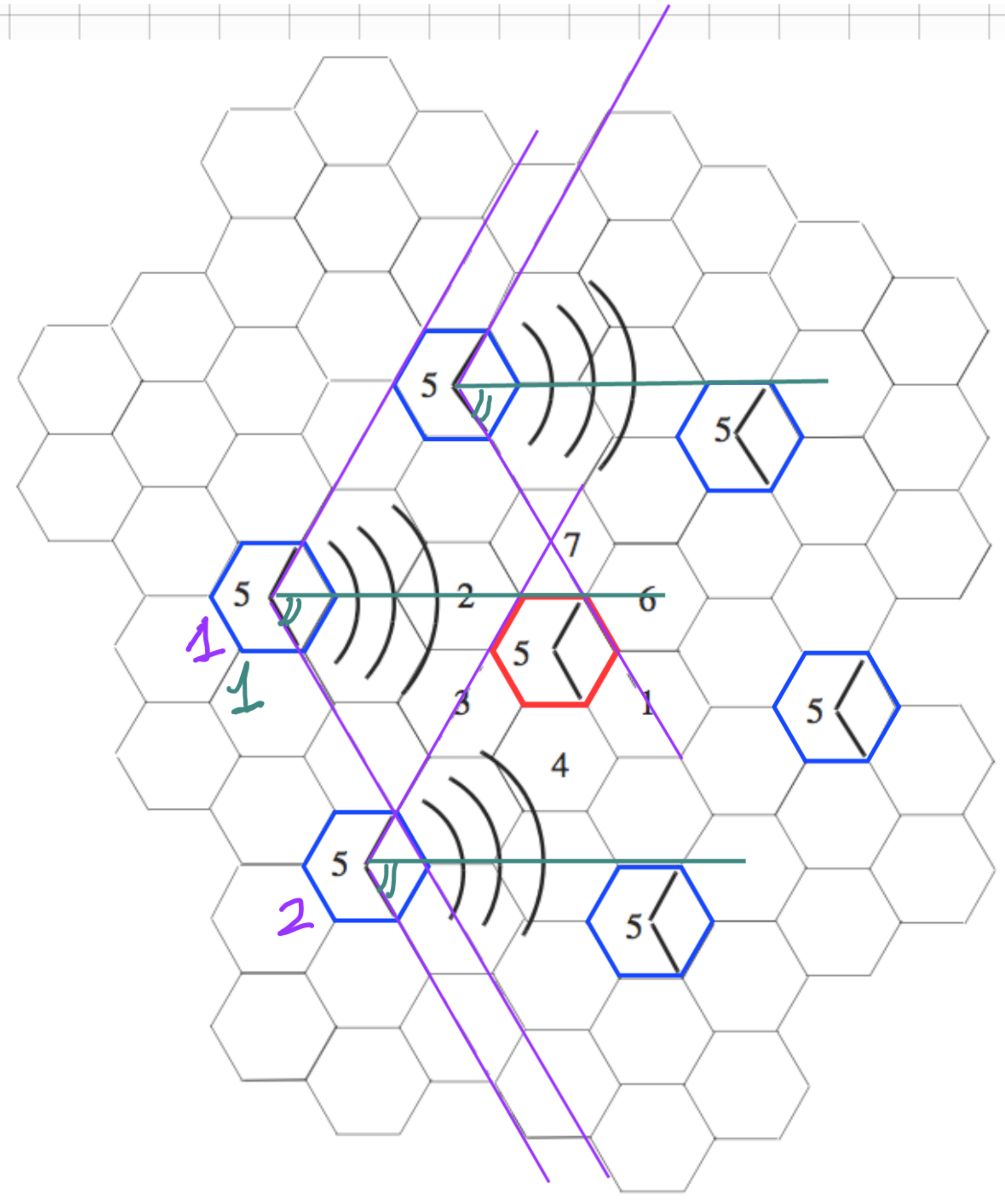


2 Cell Sectoring: is the process of replacing a single omnidirectional antenna at the base station by several directional antennas, each radiating within a specified sector.



for 120° sectoring: $i_o = 2$

for 60° sectoring $i_o = 1$



- By using directional antennas, a given cell will receive interference and transmit interference from and to only a fraction of the co-channel cells. The reduction in co-channel interference depends on the amount of sectoring.
- In cell sectoring, the channels used in a particular cell are broken into sectored groups, and they are used only within a particular sector.
- As illustrated above, when 120° sectoring is used, the number of interferers in the first tier is reduced from six to two, since only two cells have sectors with antenna patterns which radiate into a cell.
- So the resulting SIR for this case is reduced, leading to a significant improvement

compared to the omnidirectional case.

- This improvement allows the system designer to decrease the cluster size N $\left(Q = \frac{D}{R} = \sqrt{3N}\right)$ in order to improve the frequency reuse for a better system capacity.
- Antenna downtilting is used to improve the SIR as well
- However, sectoring increases the number of antennas at each base station, and decreases trunking efficiency as a result of channels being sectored into groups within a cell.
- Between sectors, handoff should be made, however, such handoff is usually handled at the base station not at the MSC.

Example 3.9

Consider a cellular system in which an average call lasts two minutes, and the probability of blocking is to be no more than 1%. Assume that every subscriber makes one call per hour, on average. If there are a total of 395 traffic channels for a seven-cell reuse system, there will be about 57 traffic channels per cell. Assume that blocked calls are cleared so the blocking is described by the Erlang B distribution.

$$A_u = H\lambda = 2\left(\frac{1}{60}\right) = 0.03$$

For the unsectored system : $GOS = 1\%$, $C = 57$:

# of supported users by the system at any time

is $U = \frac{A}{A_u} = \frac{44.2}{0.03} = 1326$

For $120°$ sectoring :

# of channels per sector $= \frac{57}{3} = 19$ channels

# of supported users per sector =

$$U = \frac{A}{A_u} = \frac{11.2}{0.03} = 336$$

# of supported users per cell =

$(336)(3) = 1008$

$\Rightarrow \frac{1008}{1326} = 76\%$ $\Rightarrow$ which is 24% less users

For 60° sectoring :

# of channels per sector $= \frac{57}{6} = 9$

# of supported users per sector =

$$U = \frac{A}{A_u} = \frac{3.7}{0.03} = 123$$

# of supported users per cell =

$$(123)(6) = 738$$

$\frac{738}{1326} \simeq 55\% \Rightarrow$ which is 45% less users.

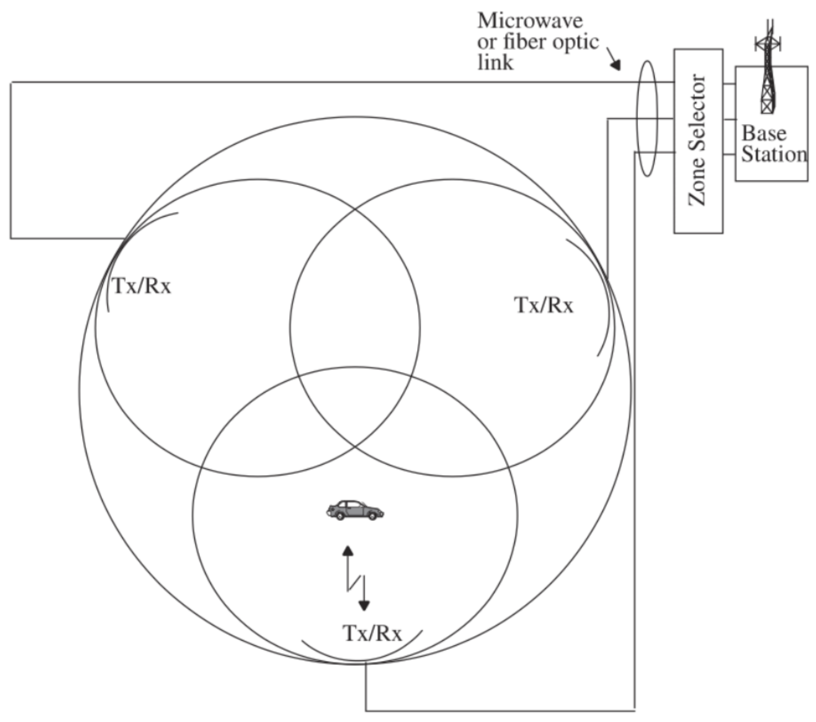
3 Repeaters for Range Extension :

Cellular providers cover even hard-to-reach areas such as within buildings or tunnels. They are usually used to provide range extension through radio retransmission. Repeaters are bidirectional, they simultaneously send and receive signals from and to a serving base station. Repeaters amplify and retransmit signals, so the noise and interference in the received signal is amplified with the retransmitted signal. So the provider must consider the proper place for the repeater and adjust the amplification levels in both forward and reverse links.

- A repeater only modifies the coverage area, it doesn't increase the capacity of the system, it just repeats.

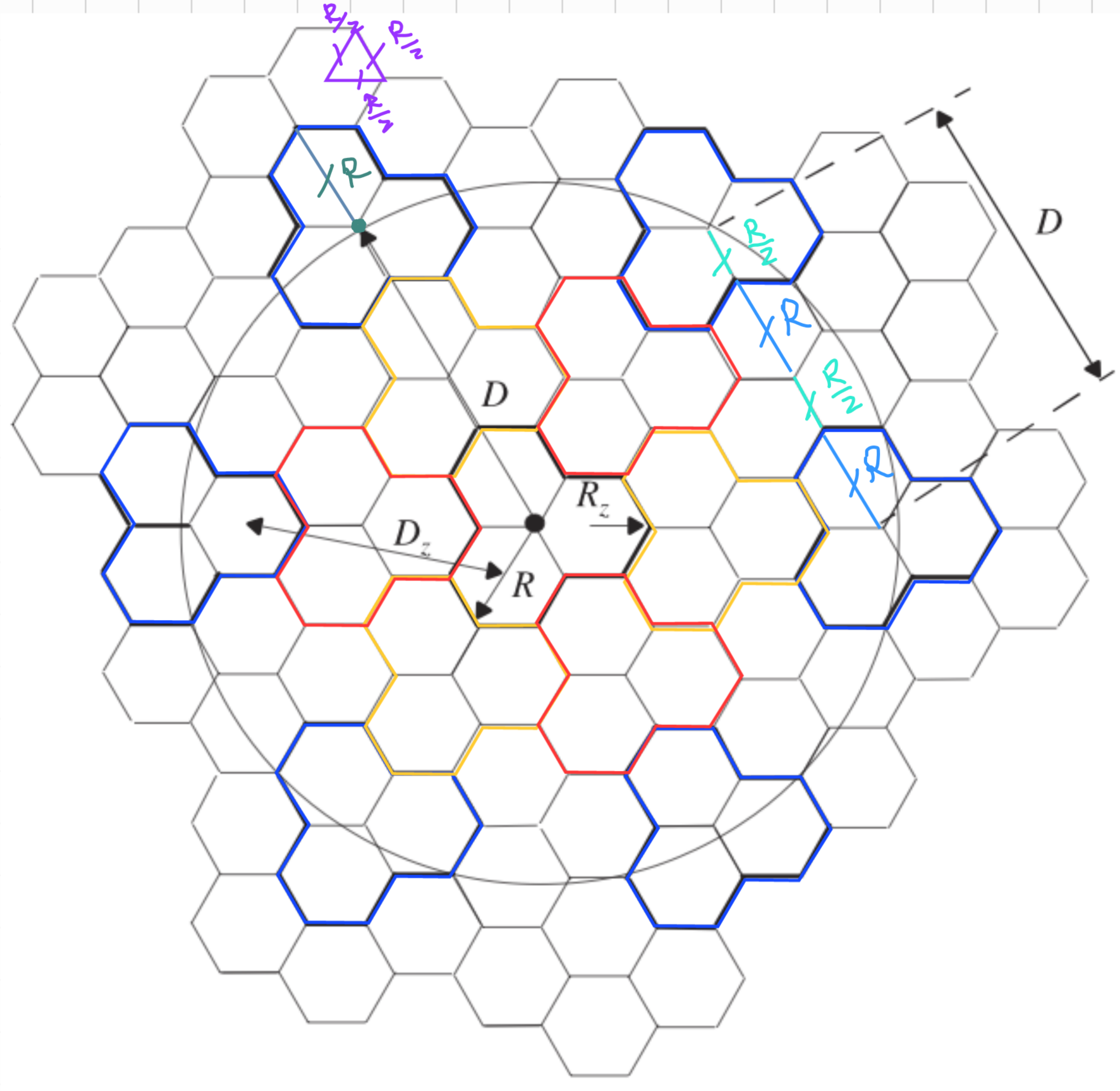
[4] Microcell Zone Concept:

In cell sectoring, an increase in load due to handoff as well as a decrease in trunking efficency effect the cellular service. To overcome these issues the microcell zone concept was proposed. The proposal was based on a microcell zone cocept for seven cell reuse. Each base station with multiple zone sites, three in this figure, form a cell. These zones are connected to their base station by coaxial cables, fiberoptic cables, or microwave links.



As a mobile travels within a cell, it is served by the zone with the strongest signal. Instead of handoff, here simply the base station switches the channel to a different zone site as the mobile travels within the cell, unlike in sectoring. Thus, a used channel is active only in a particular zone site, and since the transmit power is less compared to the case of one centered base station per cell, the interference between co-channel cells is reduced. Also, there is no trunking for the channels for different zone sites, so this will not degrade trunking efficiency as in sectoring.

Then, for a seven cell reuse system using this concept and a three zone site per cell, each cell can be seen as a group of three hexagons. Let each zone site have a radius R_z, then $R_z = \frac{1}{2} R$, where R is the cell radius. To find the capacity of the system we should find the appropiate Q



$R = 2R_z$ (R : cell radius, R_z : zone radius)

$D = 3R$ (from the figure)

$\Rightarrow \frac{D}{R} = 3 =: Q$

$Q = \sqrt{3N} \Rightarrow N = 3$

Thus, using the microcell concept reduced the cluster size from $N=7$ to $N=3$ resulting in an increase in the system's capacity. That is, for an $N=7$ conventional system and a microcell zone system with $N=3$, the SIR is the same.

3.3 Show that the frequency reuse factor for a cellular system is given by k/S, where k is the average number of channels per cell and S is the total number of channels available to the cellular service provider.

avg. # of channels/cell = $\dfrac{\text{total \# of channels}}{\text{cluster size}}$

$$\Rightarrow \quad K = \frac{S}{N}$$

$$\frac{K}{S} = \frac{1}{N} := \text{freq. reuse factor.}$$

3.5 A cellular service provider decides to use a digital TDMA scheme which can tolerate a signal-to-interference ratio of 15 dB in the worst case. Find the optimal value of N for (a) omnidirectional antennas, (b) 120° sectoring, and (c) 60° sectoring. Should sectoring be used? If so, which case (60° or 120°) should be used? (Assume a path loss exponent of $n = 4$ and consider trunking efficiency.)

(a) $i_0 = 6$, $SIR = \dfrac{R^{-n}}{\sum_{i_0} (D_i)^{-n}} = \dfrac{R^{-4}}{6D^{-4}} = \dfrac{1}{6}\left(\dfrac{D}{R}\right)^4$

$$= \frac{1}{6}\left(\sqrt{3N}\right)^4 = \frac{1}{6} 9N^2$$

we need $SIR = \dfrac{3}{2}N^2 > 15\,dB \Rightarrow 15\,dB = 10\log x$

$$x = 10^{15/10}$$

$$\frac{3}{2}N^2 > 31.62$$

$$N^2 > 21.08$$

$$N > 4.59$$

$$\Rightarrow N = 7$$

(b) $i_0 = 2$, $SIR = \frac{1}{2}(\sqrt{3N})^4 = \frac{1}{2} 9N^2$

we need $SIR = \frac{9}{2} N^2 > 15\,dB = 31.62$

$$\frac{9}{2} N^2 > 31.62$$

$$N^2 > 7.03$$

$$N > 2.65 \Rightarrow N = 3$$

(c) $i_0 = 1$, $SIR = (\sqrt{3N})^4 = 9N^2$

we need $SIR = 9N^2 > 31.62$

$$\Rightarrow N^2 > 3.51$$

$$N > 1.87 \Rightarrow N = 3$$

Since $N=3$ provides more capacity than $N=7$, and $120°$ sectoring has a better truking efficiency than $60°$ sectoring, then case of $120°$ sectoring is best here.

3.10 A total of 24 MHz of bandwidth is allocated to a particular FDD cellular telephone system that uses two 30 kHz simplex channels to provide full duplex voice and control channels. Assume each cell phone user generates 0.1 Erlangs of traffic. Assume Erlang B is used.

(a) Find the number of channels in each cell for a four-cell reuse system.

(b) If each cell is to offer capacity that is 90% of perfect scheduling, find the maximum number of users that can be supported per cell where omnidirectional antennas are used at each base station.

(c) What is the blocking probability of the system in (b) when the maximum number of users are available in the user pool?

(d) If each new cell now uses 120° sectoring instead of omnidirectional for each base station, what is the new total number of users that can be supported per cell for the same blocking probability as in (c)?

(e) If each cell covers five square kilometers, then how many subscribers could be supported in an urban market that is 50 km × 50 km for the case of omnidirectional base station antennas?

(f) If each cell covers five square kilometers, then how many subscribers could be supported in an urban market that is 50 km × 50 km for the case of 120° sectored antennas?

(a) $\dfrac{24 \times 10^6}{2(30) \times 10^3} = 400$ channels, $\dfrac{400}{4} = 100$ channels / cell

(b) Capacity 90% of perfect scheduling =: 90 Erlang as traffic intensity for the system (i.e. $A = 90$)

$\Rightarrow U = \dfrac{A}{A_u} = \dfrac{90}{0.1} = 900$ users can be serviced.

(c) $A = 90$, while $C = 100$,

$\Rightarrow$ From Erlang B chart, $GOS = 3\%$

(d) each sector will have $\dfrac{100}{3}$ channels

From Erlang B chart for $C = 33, GOS = 3\% \Rightarrow A \simeq 26$

$\Rightarrow$ for $A = 26$, $A_u = 0.1$

$\Rightarrow U = \frac{26}{0.1} = 260$ user/sector

$260(3) = 780$ user/cell.

(e) cell area $= 5\,km^2$, covered area $= (50)^2 = 2500\,km^2$

$\frac{2500}{5} = 500$ cells required.

$\Rightarrow$ from (b) we can support 900 users/cell

$\Rightarrow 500(900) = 450000$ users for the whole system.

(f) from (d) we can support 780 users/cell

$\Rightarrow 500(780) = 390000$ users for the whole system.

3.13 A certain area is covered by a cellular radio system with 84 cells and a cluster size N. 300 voice channels are available for the system. Users are uniformly distributed over the area covered by the cellular system, and the offered traffic per user is 0.04 Erlang. Assume that blocked calls are cleared and the designated blocking probability is $P_b = 1\%$.

(a) Determine the maximum carried traffic per cell if cluster size $N = 4$ is used. Repeat for cluster sizes $N = 7$ and 12.

(b) Determine the maximum number of users that can be served by the system for a blocking probability of 1% and cluster size $N = 4$. Repeat for cluster sizes $N = 7$ and 12.

for $N = 4$, $C = 300$, $GOS = 1\%$, $A_u = 0.04$

# of channels per cell $= \frac{300}{4} = 75$

A per cell from Erlang B chart $\Rightarrow A \simeq 60$ Erlang

$\Rightarrow U = \frac{A}{A_u} = \frac{60}{0.04} = 1500$ per cell

# of supported users for whole system $= (84)(1500)$
$= 126000$ users.

also traffic intensity for the whole system is
$(84)(60) = 5040$ Erlang

3.26 Assume each user of a single base station mobile radio system averages three calls per hour, each call lasting an average of 5 minutes.

(a) What is the traffic intensity for each user?

(b) Find the number of users that could use the system with 1% blocking if only one channel is available.

(c) Find the number of users that could use the system with 1% blocking if five trunked channels are available.

(d) If the number of users you found in (c) is suddenly doubled, what is the new blocking probability of the five channel trunked mobile radio system? Would this be acceptable performance? Justify why or why not.

(a) $A_u = H\lambda = (5)\frac{3}{60} = 0.25$ Erlang.

(c) $C = 5$, $GOS = 1\%$ $\Rightarrow A \simeq 1.4$

$U = \frac{A}{A_u} = 5$ users

(d) if $U = 5(2) = 10$ (more than what the system can afford].

$\Rightarrow$ the traffic intinsity $A = U A_u = 10\,(0.25) = 2.5$

(which is double than when the users were half)

$\Rightarrow$ From Erlang B chart for $A = 2.5$ & $C = 5$,

$\Rightarrow GOS \simeq 0.07 = 7\%$ which is not acceptable

Mobile Radio Wave Propagation

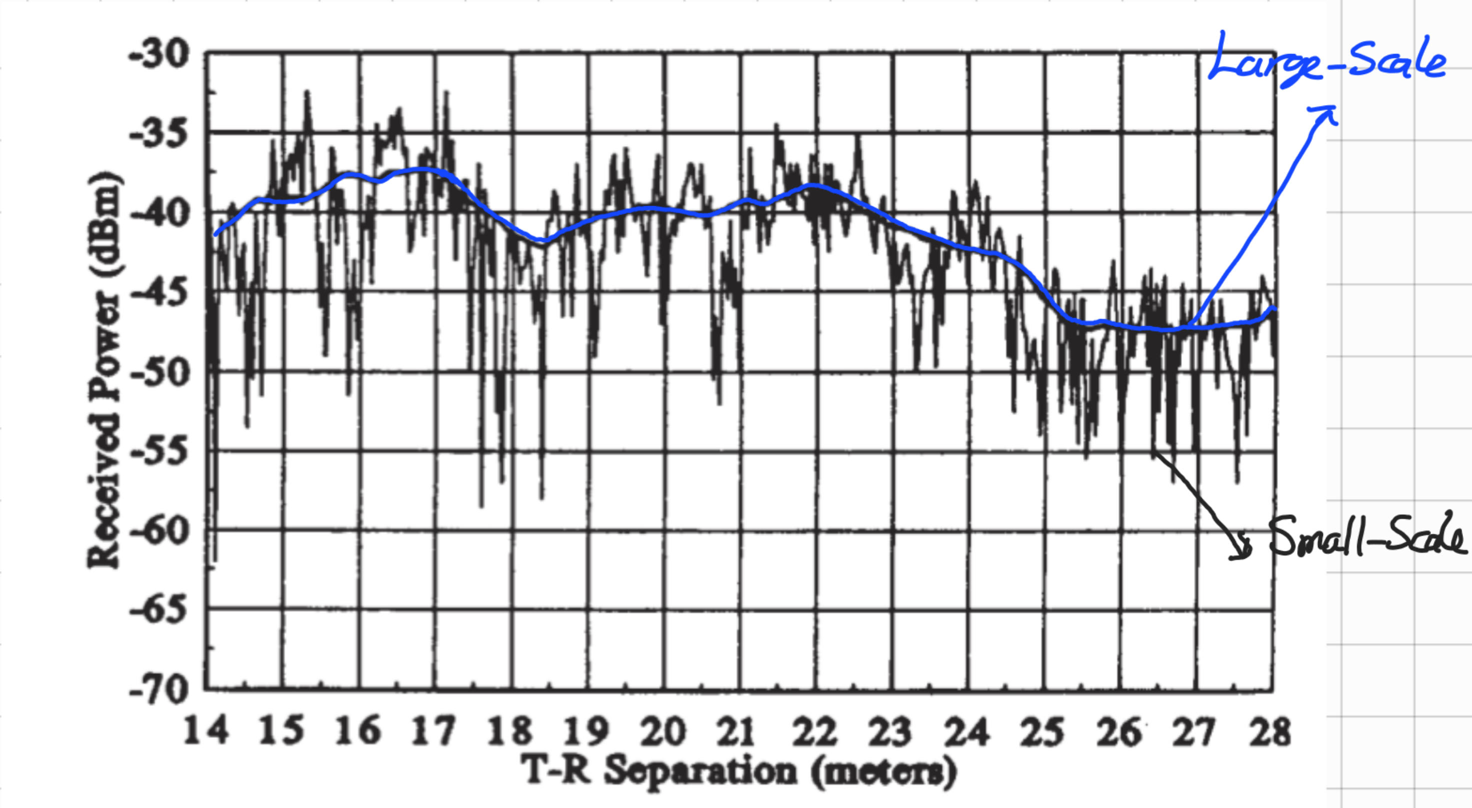
Radio channels unlike wired channels are not predicatable, they are extremely random and are not easily modeled. Modeling the radio channel is one of the most difficult parts of designing a mobile radio system.

[1] Large-Scale Propagation : Chapter 4

Large-scale propagation models predict the average received signal strength at a given distance from the transmitter, as well as the variability of the signal strength in a close proximity to a particular location. These models are used to estimate the coverage area of a transmitter.

[2] Small-Scale Propagation : Chapter 5

Small-scale propagation models characterize the rapid fluctuations of the received signal strength over a short time durations (on the order of seconds) or over very short travel distances (a few wavelengths).



Large-Scale Propagation :

- Free Space Propagation Model :

This model is used to predict received signal strength when the transmitter and receiver have a clear line-of-sight path between them.

e.g. Satellite communication systems and wireless systems with line-of-sight radio links use this model.

The free space powe received is given by Friis free space equation :

$$P_r = \frac{P_t\, G_t\, G_r\, \lambda^2}{(4\pi)^2\, d^2\, L}$$

where d : the distance separating the receiver antenna from the radiating transmitter antenna in meters ($d \neq 0$)

P_t : the transmitted power , P_r = the received power

G_t : the transmitter antenna gain, G_r : receiver antenna gain

L : the system hardware loss, not related to propagation ($L \geq 1$)
$L = 1$ indicates no loss in the system

λ : the wavelength in meters, Path loss exponent for free space $n = 2$.

The λ is related to the carrier frequency by

$$\lambda = \frac{c}{f_c} = \frac{2\pi c}{\omega_c}$$

f_c : carrier frequency in Hz, ω_c : carrier frequency in rad/sec.
c : speed of light in meters/sec. $\simeq 3 \times 10^8$ m/s

G_t and G_r are dimensionless quantities, they are related to the antennas effective aperture A_e (which is related to the physical size of the antenna) :

$$G = \frac{4\pi A_e}{\lambda^2}$$

P_t and P_r must be expressed in the same units

The path loss (PL) represents signal attenuation as a possitive quantity measured in dB, it is the difference between P_t and P_r in dB.

The path loss for free space model when antenna gains are included :

$$PL(dB) = 10 \log \frac{P_t}{P_r} = -10 \log \left[\frac{G_t G_r \lambda^2}{(4\pi)^2 d^2} \right]$$

The path loss when the antenna gains are excluded, the antennas are assumed to have unity gain, then

$$PL(dB) = 10 \log \frac{P_t}{P_r} = -10 \log \left[\frac{\lambda^2}{(4\pi)^2 d^2} \right]$$

This propagation model, Friss free space model, can predict P_r only if d is in the far-field of the transmitting antenna, also called Fraunhofer-region. This region is defined as the region beyond the far-field distane d_f, where

$$d_f = \frac{2D^2}{\lambda}$$

where D : is the largest physical linear dimension of the antenna

Also, $d_f \gg D$ and $d_f \gg \lambda$.

If the received power at a distance d_o is $P_r(d_o)$, then the received power at a distance $d \geq d_o \geq d_f$

$$P_r(d) = P_r(d_o) \left(\frac{d_o}{d} \right)^2 \quad \sim \text{equ (A)}$$

Path loss exponent for free space $n=2$.

The reference distance d_o is typically chosen to be 1 meter in indoor environments, 100 m or 1 km in outdoor environments, so the computulations for path loss are easy in dB units.

Equ. (A) can be in dBW or in dBm as:

$$P_r(d)\,dBW = 10\log\left(P_r(d_o)\right) + 20\log\left(\frac{d_o}{d}\right)$$

OR

$$P_r(d)\,dBm = 10\log\left[\frac{P_r(d_o)}{10^{-3}\,W}\right] + 20\log\left(\frac{d_o}{d}\right)$$

Example 4.1
Find the far-field distance for an antenna with maximum dimension of 1 m and operating frequency of 900 MHz.

$$d_f = \frac{2D^2}{\lambda} \quad , \quad \lambda = \frac{c}{f_c} = \frac{3\times10^8}{9\times10^8} = \frac{1}{3}$$

$$= \frac{2(1)^2}{1/3}$$

$$d_f = 6 \text{ m}$$

so $\quad d \geq d_f$

Example 4.2

If a transmitter produces 50 W of power, express the transmit power in units of (a) dBm, and (b) dBW. If 50 W is applied to a unity gain antenna with a 900 MHz carrier frequency, find the received power in dBm at a free space distance of 100 m from the antenna. What is $P_r(10\text{ km})$? Assume unity gain for the receiver antenna.

(a) $P_t = 10\log\left(\frac{50}{10^{-3}}\right) = 47\text{ dBm}$

(b) $P_t = 10\log 50 = 17\text{ dBW}$

$G_t = G_r = 1$, $f_c = 9\times10^8$, $P_r(d=100m)$, $P_r(d=10^4 m)$

$\lambda = \frac{c}{f_c} = \frac{3\times10^8}{9\times10^8} = \frac{1}{3}$

$$P_r\Big|_{d=100m} = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2} = \frac{50(1)(1)\left(\frac{1}{3}\right)^2}{(4\pi)^2(100)^2} = 3.5\times10^{-6}\text{ W}$$

$$P_r(dBm) = 10\log\left(\frac{3.5\times10^{-6}}{10^{-3}}\right) = -24.5\text{ dBm}$$

$$P_r(dBm)\Big|_{d=10^4 m} = P_r(dBm)\Big|_{d=100m} + 20\log\left(\frac{100}{10^4}\right)$$

$$= -24.5\text{ dBm} - 40\text{ dB} = -64.5\text{ dBm}$$

OR in dBW:

$$P_r(dBW) = 10\log(3.5\times10^{-6}) = -54.5\,dBW$$

$$P_r(dBW)\Big|_{d=10^4 m} = P_r(dBW)\Big|_{d=100m} + 20\log\left(\frac{100}{10^4}\right)$$

$$= -54.5\,dBW - 40\,dB = -94.5\,dBW$$

$$\Rightarrow -94.5\,dBW - 10\log\left(\frac{1}{10^{-3}}\right) =$$

$$-94.5\,dBW + 30\,dB =$$

$$-64.5\,dBm$$

The Three Basic Propagation Mechanisms :

1. **Reflection**: occurs when a propagating electromagnetic wave hits an object that has very large dimensions compared to the wavelength of the propagating wave.
(e.g. the surface of the earth, buildings, walls, ...)

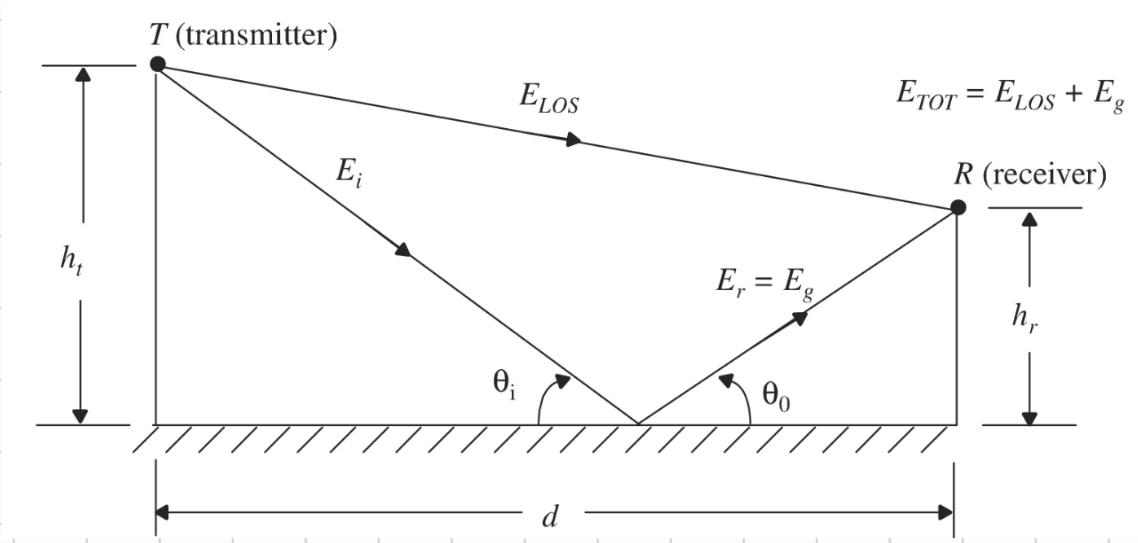
2. **Diffraction**: occurs when the radio path between the transmitter and receiver is obstructed by a surface that has sharp edges. This causes the propagating wave to bend around the obstacle. (e.g. a hill, a mountain, ...)

3. **Scattering**: occurs when the medium in which the wave propagates has objects with dimensions that are small compared to the wavelength, and the number of objects per unit volume is large.
(e.g. trees, street signs, lamp posts, ...)

1 Reflection

1.1 Ground Reflection (Two-Ray) Model

- In this model, the direct path and a ground reflected propagation path between transmitter and receiver are considered. This model is reasonably accurate for predicting large-scale signal strength over distances of several kilometers for mobile radio systems that use tall towers that exceed 50 m, as well as for line-of-sight microcells in urban environments.
- The maximum T-R separation distance is at most a few tens of kilometers, so the earth may be assumed to be flat.
- The total received electric field, E_{Tot}, is the result of the direct line-of-sight (LOS) component, E_{LOS}, and the ground reflection component, E_g.
- Assume perfect ground reflection

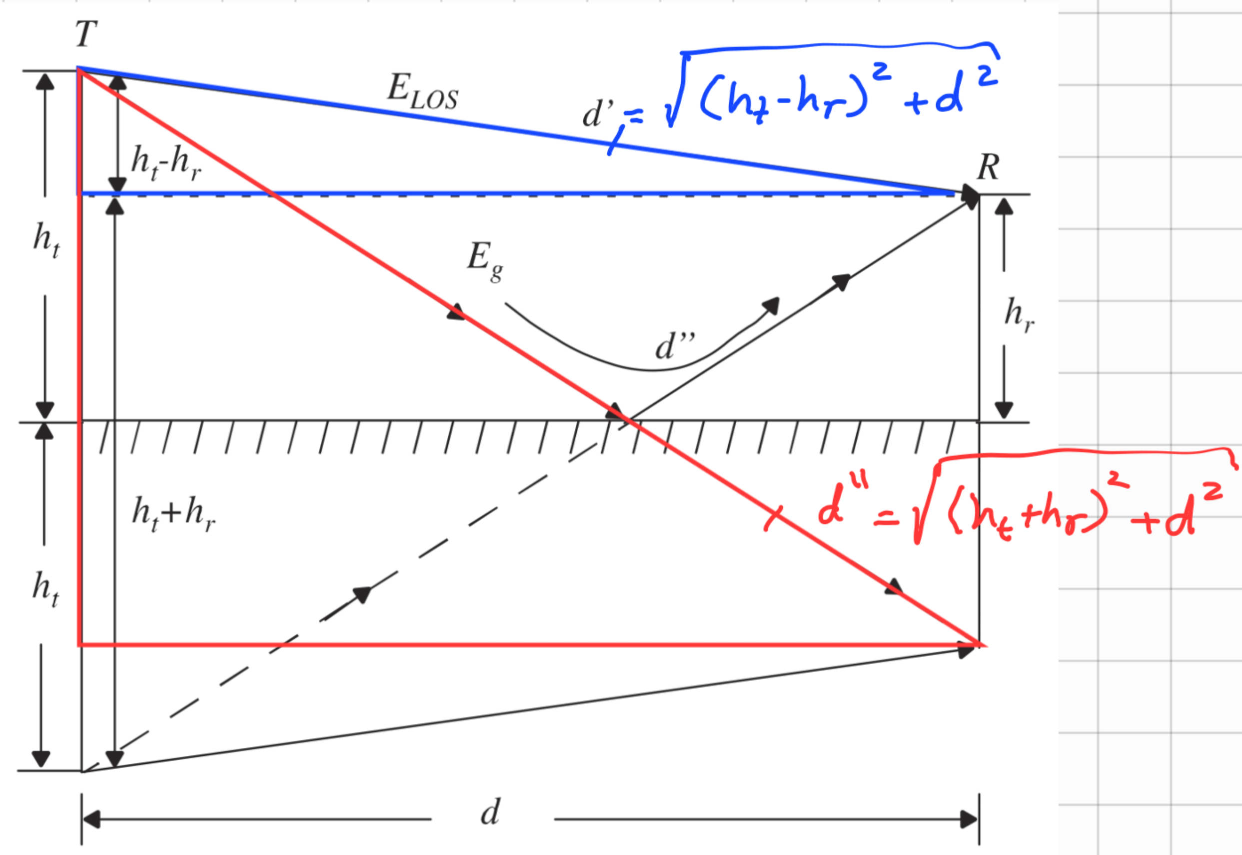


E_i : incident E-field, E_r : reflected E-field, $\theta_i = \theta_0$ for reflection in dielectrics.

h_t : height of transmitter, h_r : height of receiver, E_0 : free-space E-field at a reference distance d_0 from the transmitter. Then for $d > d_0$,

$$E(d,t) = \frac{E_0 d_0}{d} \cos\left[w_c\left(t - \frac{d}{c}\right)\right]$$

where the envelope of the E-field at d is $\left|\frac{E_0 d_0}{d}\right|$



Then the two propagating waves at the receiver will be

$$E_{Tot} = E_{LOS}(d', t) + E_g(d'', t)$$

$$E_{LOS}(d', t) = \frac{E_0 d_0}{d'} \cos\left[w_c\left(t - \frac{d'}{c}\right)\right]$$

$$E_g(d'', t) = \Gamma \frac{E_0 d_0}{d''} \cos\left[w_c\left(t - \frac{d''}{c}\right)\right]$$

where Gamma $\Gamma = -1$: is the reflection coefficient for ground.

- To find the phase difference, we find the path difference $\Delta = d'' - d'$, since the phase difference θ_Δ is

$$\theta_\Delta = \frac{w_c d''}{c} - \frac{w_c d'}{c} \Rightarrow \theta_\Delta = \frac{w_c}{c} \Delta = \frac{2\pi f_c}{c} \Delta = \frac{2\pi}{\lambda} \Delta$$

while the time delay τ_d is

$$\tau_d = \frac{d''}{c} - \frac{d'}{c} \Rightarrow \tau_d = \frac{\Delta}{c} = \frac{1}{c} \frac{\theta_\Delta \lambda}{2\pi} = \frac{\theta_\Delta}{2\pi f_c}$$

If $d \gg (h_t + h_r)$, then :

$$d'' = \sqrt{(h_t + h_r)^2 + d^2}$$

$$= d\sqrt{1 + \left(\frac{h_t + h_r}{d}\right)^2} \quad , \quad (1+x)^n \simeq 1 + nx \text{ if } x \ll 1$$

$$\simeq d\left(1+\frac{1}{2}\left(\frac{h_t+h_r}{d}\right)^2\right)$$

$$d'' = d + \frac{d}{2d^2}\left(h_t^2 + 2h_t h_r + h_r^2\right)$$

also, $$d' = \sqrt{(h_t-h_r)^2 + d^2}$$

$$= d\sqrt{1+\left(\frac{h_t-h_r}{d}\right)^2}$$

$$\simeq d\left(1+\frac{1}{2}\left(\frac{h_t-h_r}{d}\right)^2\right)$$

$$= d + \frac{d}{2d^2}\left(h_t^2 - 2h_t h_r + h_r^2\right)$$

$$\Rightarrow \Delta = d'' - d'$$

$$\simeq \frac{1}{2d}\left(2h_t h_r + 2h_t h_r\right)$$

$$\Delta \simeq \frac{2h_t h_r}{d} \quad \text{for} \quad d \gg h_t + h_r$$

As d becomes large, $\Delta = d'' - d'$ becomes very small, then the amplitudes of E_{LOS} and E_g become almost the same with only a phase difference.

That is, $\left|\frac{E_0 d_0}{d}\right| \simeq \left|\frac{E_0 d_0}{d'}\right| \simeq \left|\frac{E_0 d_0}{d''}\right|$.

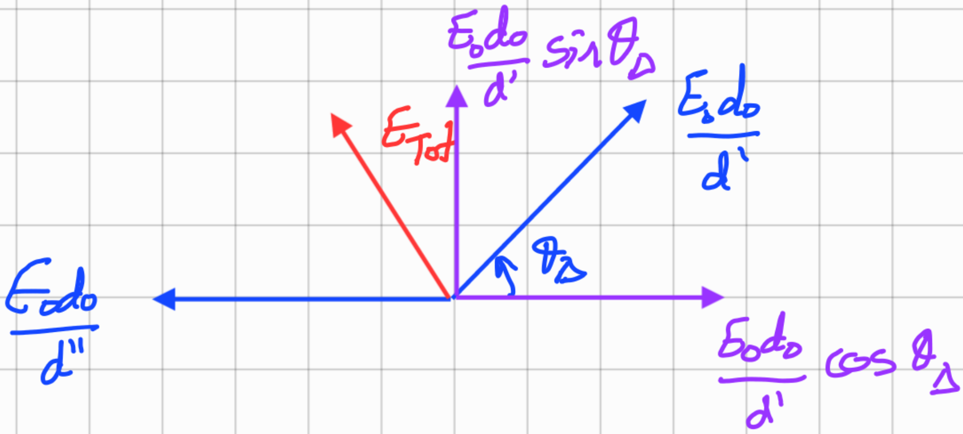
The received E-field at $t = \frac{d''}{c}$,

$$E_{Tot}\left(d, t=\frac{d''}{c}\right) = E_{LOS}\left(d', t=\frac{d''}{c}\right) + E_g\left(d'', t=\frac{d''}{c}\right)$$

$$= \frac{E_0 d_0}{d'} \cos\left[\omega_c\left(\frac{d''-d'}{c}\right)\right] - \frac{E_0 d_0}{d''} \cos\left[\omega_c\left(\frac{d''-d''}{c}\right)\right]$$

$$= \frac{E_0 d_0}{d'} \cos\left(\omega_c \frac{\Delta}{c}\right) - \frac{E_0 d_0}{d''}$$

$$\simeq \frac{E_0 d_0}{d} \underline{\left|(\theta_\Delta - \pi)\right.}$$



$$|E_{Tot}(d)| \simeq \sqrt{\left[\frac{E_0 d_0}{d}(\cos\theta_\Delta - 1)\right]^2 + \left[\frac{E_0 d_0}{d}\sin\theta_\Delta\right]^2}$$

$$= \frac{E_0 d_0}{d}\left[(\cos\theta_\Delta - 1)^2 + \sin^2\theta_\Delta\right]^{\frac{1}{2}}$$

simplify the use $\sin^2\theta = \frac{1}{2}\left[1-\cos 2\theta\right]$

$$\left|E_{Tot}(d)\right| = \frac{E_0 d_0}{d}\, 2 \sin\frac{\theta_\Delta}{2} \quad \sim (*)$$

if θ_Δ is not too small, $E_{Tot}(d)$ decays in an oscillating way.

if θ_Δ is too small, then $\sin\frac{\theta_\Delta}{2} \simeq \frac{\theta_\Delta}{2}$, this is when

$\frac{\theta_\Delta}{2} < 0.3$ rad, $(0.3 \text{ rad} \simeq 17.19^\circ)$

Since $\theta_\Delta = \frac{2\pi\Delta}{\lambda}$, and $\Delta \simeq \frac{2h_t h_r}{d}$

$$\Rightarrow \theta_\Delta \simeq \frac{4\pi h_t h_r}{\lambda d}$$

So $\frac{\theta_\Delta}{2} \simeq \frac{2\pi h_t h_r}{\lambda d}$

when $\frac{\theta_\Delta}{2} < 0.3$ rad

$$\frac{\theta_\Delta}{2} \simeq \frac{2\pi h_t h_r}{\lambda d} < 0.3 \quad \sim (**)$$

$$\Rightarrow \frac{\lambda d}{2\pi h_t h_r} > \frac{10}{3}$$

$$d > \frac{20\pi h_t h_r}{3\lambda} \simeq \frac{20 h_t h_r}{\lambda}$$

So $E_{Tot}(d) \simeq \dfrac{2E_o d_o}{d} \dfrac{2\pi h_t h_r}{\lambda d}$, from (*) & (**)

$E_{Tot}(d) \simeq \dfrac{k}{d^2}$, k : constant related to E_o, h_t, h_r, λ

From received E-field, E_{Tot}, to received power, P_r,

$$P_r = P_r\{\text{freespace}\}\left[2\sin\frac{\theta_\Delta}{2}\right]^2$$

$$= P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 [4]\left[\frac{2\pi h_t h_r}{\lambda d}\right]^2$$

$$= P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 \left[\frac{4\pi h_t h_r}{\lambda d}\right]^2$$

$$P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}$$

at large distances, $d \gg \sqrt{h_t h_r} \Rightarrow P_r \propto \dfrac{1}{d^4}$, which is a faster decay compared to free-space propagation model.

So the path loss (PL) in dB is :

$$PL(dB) = 40\log d - \left[10\log G_t + 10\log G_r + 20\log h_t + 20\log h_r\right]$$

Example :

If the distance between the mobile and the base station is 5km with $G_t = G_r = 2.55\,dB$, $P_t = 3\,W$, $h_t = 50\,m$, $h_r = 2\,m$, $f_c = 900\,MHz$. Find

a) the effective aperture of the receiver antenna

b) the received power at the mobile using two-ray ground reflection mode

(a) $G = \frac{4\pi A_e}{\lambda^2}$, $\lambda = \frac{c}{f} = \frac{3\times10^8}{9\times10^8} = 0.33\ m$

$G = 2.55\,dB$, $10\log x = 2.55 \Rightarrow x = 10^{0.255} = 1.79887 \approx 1.8$

$$A_e = \frac{G\lambda^2}{4\pi} = \frac{(1.8)(0.33)^2}{4\pi} = 0.016$$

(b) $$P_r(d = 5km) = P_t G_t G_r \left(\frac{h_t h_r}{d^2}\right)^2$$

$$= 3(1.8)^2 \left(\frac{(50)(2)}{(5\times10^3)^2}\right)^2$$

$$P_r = 1.04\times10^{-10}\ W$$

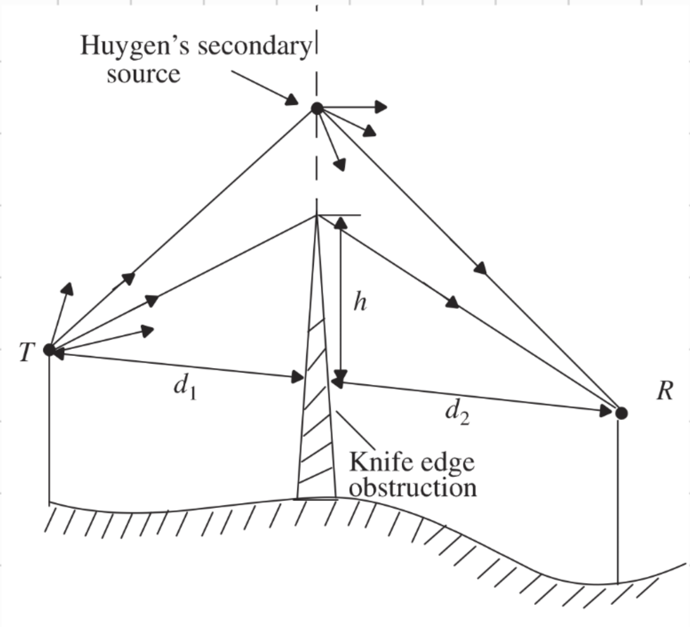
or $P_r = -99.84\ dBW$

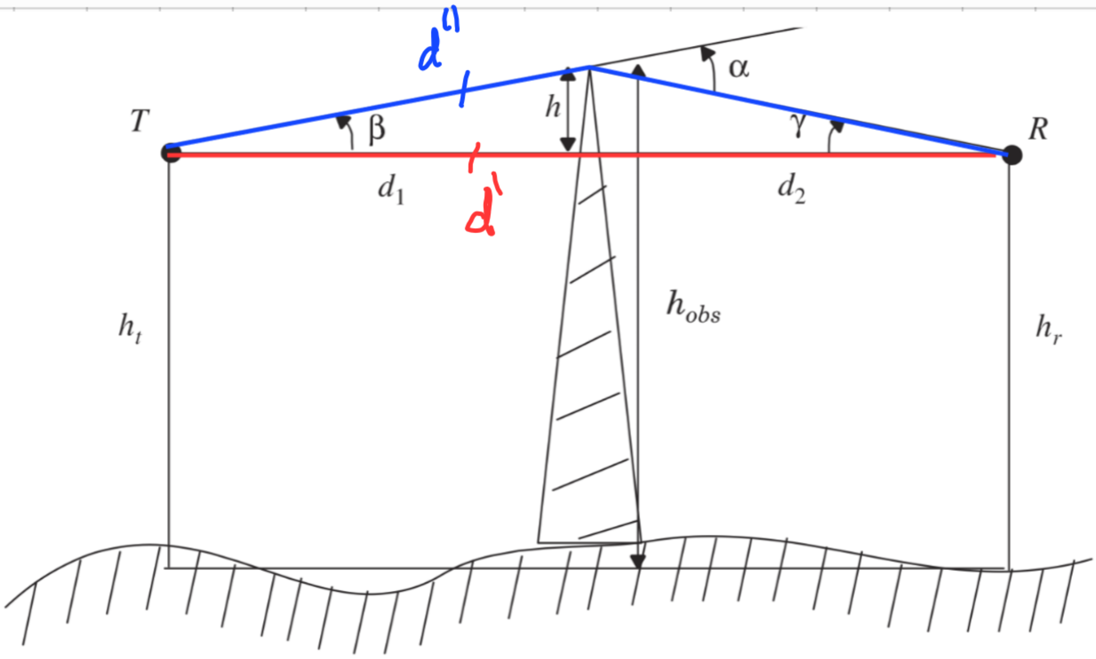
2. Diffraction

2.1 Knife-edge diffraction model

Diffraction allows radio signals to propagate behind obstacles into its shadowed region, this will cause the received signal strength to decrease, however, still with sufficient strength to produce a useful signal.

Diffraction can be explained by Huygen's principle. It explains wave propagation as if every point on the wave will make its own wave, with such point as its center. So the propagation of the wave into the shadowed region is the vector sum of the components of all the waves that reach the space behind the obstacle.



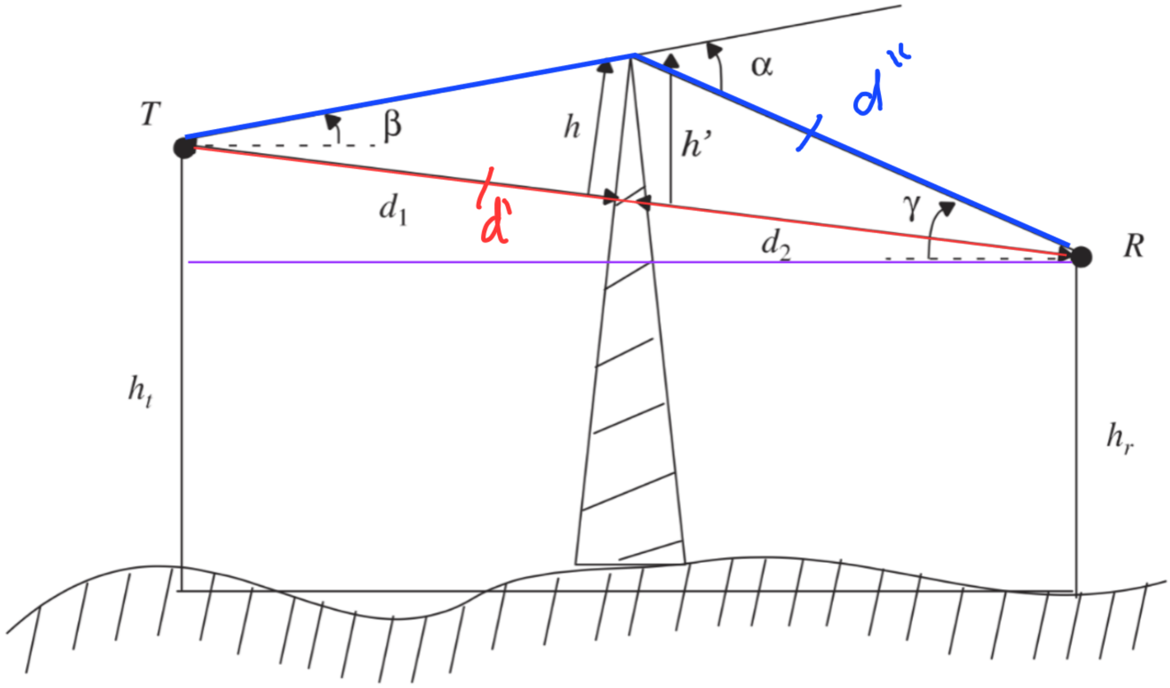


(a) Knife-edge diffraction geometry. The point T denotes the transmitter and R denotes the receiver, with an infinite knife-edge obstruction blocking the line-of-sight path.

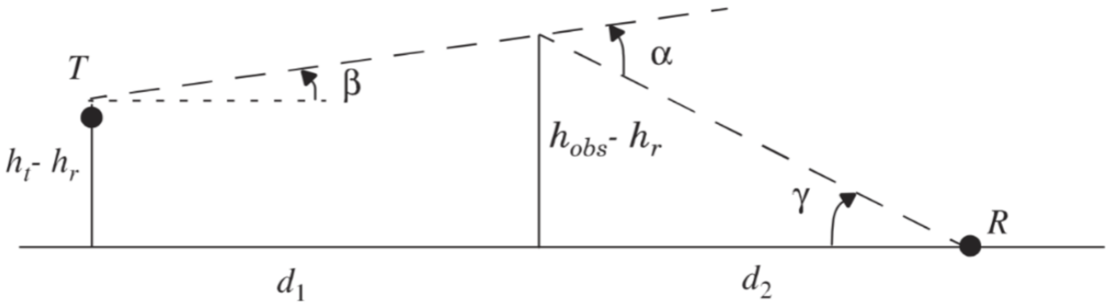
Consider an obstacle as shown above, assume that its width is infinite as if it is going into and out of the paper. Let its distance from the transmitter be denoted by d_1, its distance from the receiver d_2, and its height h. Assume $h \ll d_1$, $h \ll d_2$, and $h \gg \lambda$. Let the difference between the direct path and the diffracted path be denoted by Δ and is called excess path length. To find Δ:

$$\Delta = d'' - d'$$

$$= \left[\sqrt{d_1^2+h^2} + \sqrt{d_2^2+h^2}\right] - \left[d_1 + d_2\right]$$



(b) Knife-edge diffraction geometry when the transmitter and receiver are not at the same height. Note that if α and β are small and $h << d_1$ and d_2, then h and h' are virtually identical and the geometry may be redrawn as shown in Figure 4.10c.



(c) Equivalent knife-edge geometry where the smallest height (in this case h_r) is subtracted from all other heights.

$$\Rightarrow \Delta = d_1\sqrt{1+\left(\frac{h}{d_1}\right)^2} + d_2\sqrt{1+\left(\frac{h}{d_2}\right)^2} - d_1 - d_2$$

since $h \ll d_1$ & $h \ll d_2$, and $(1+x)^n \simeq 1+nx$ if $x \ll 1$

then $$\Delta \simeq d_1\left[1+\frac{1}{2}\left(\frac{h}{d_1}\right)^2\right] + d_2\left[1+\frac{1}{2}\left(\frac{h}{d_2}\right)^2\right] - d_1 - d_2$$

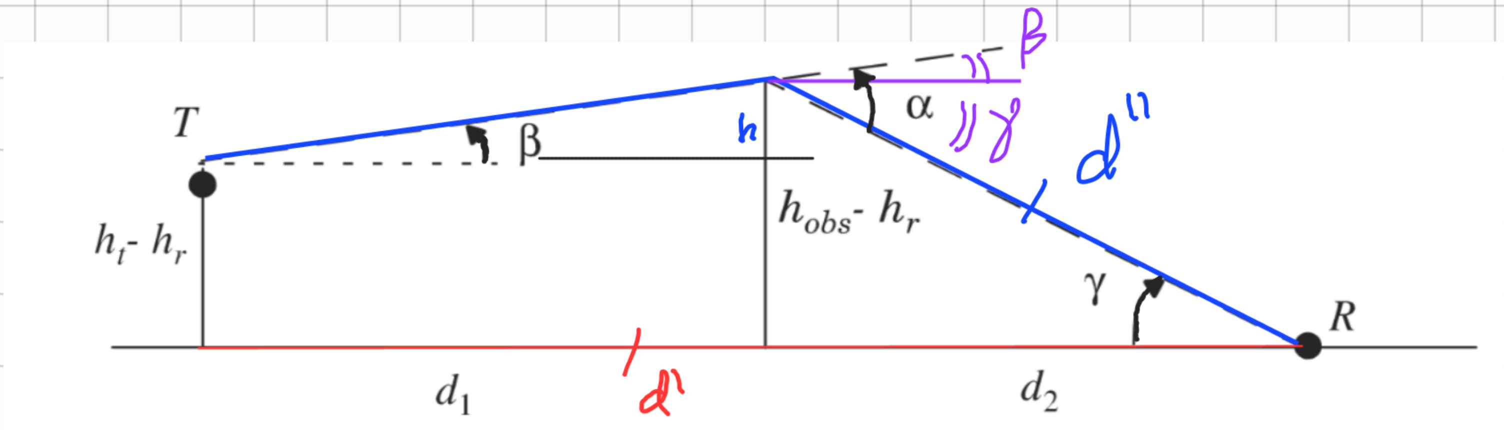
$$= \frac{h^2}{2d_1} + \frac{h^2}{2d_2} = \frac{h^2}{2}\left[\frac{1}{d_1}+\frac{1}{d_2}\right]$$

$$\Delta \simeq \frac{h^2}{2}\frac{(d_1+d_2)}{d_1 d_2}$$

With such excess path length Δ, the phase difference is:

$$\phi = \frac{2\pi\Delta}{\lambda} \approx \frac{2\pi}{\lambda}\frac{h^2}{2}\frac{(d_1+d_2)}{d_1 d_2}$$

Notice: ϕ is a function of height h, d_1, d_2, λ.



Assume small angles, for small angles $\tan x \simeq x$,

$$\alpha = \beta + \gamma \simeq \frac{h}{d_1} + \frac{h}{d_2} = h\left(\frac{1}{d_1}+\frac{1}{d_2}\right)$$

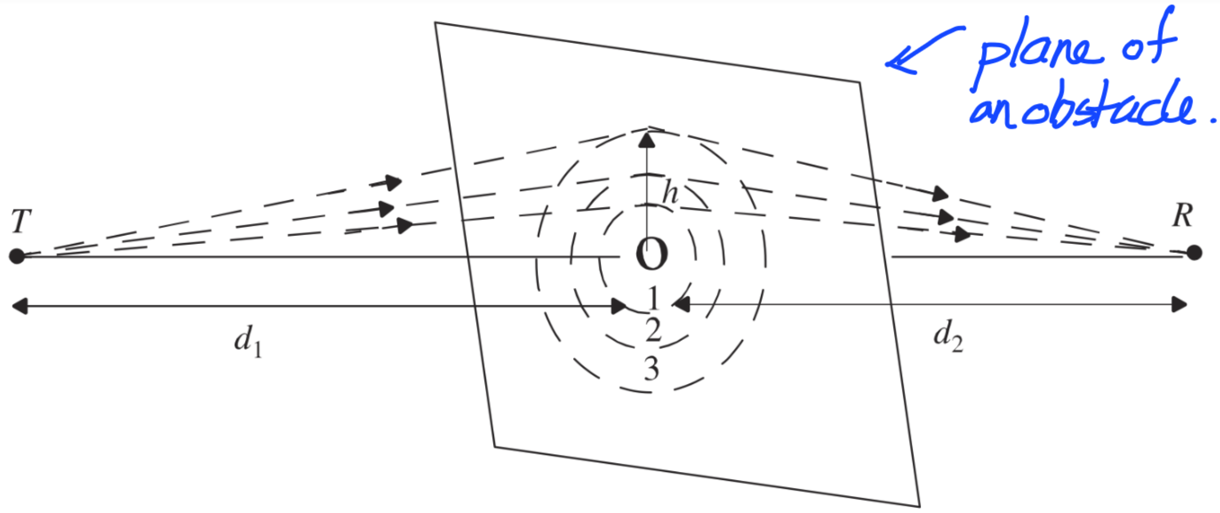
$$\alpha \simeq h\frac{(d_1+d_2)}{d_1 d_2}$$

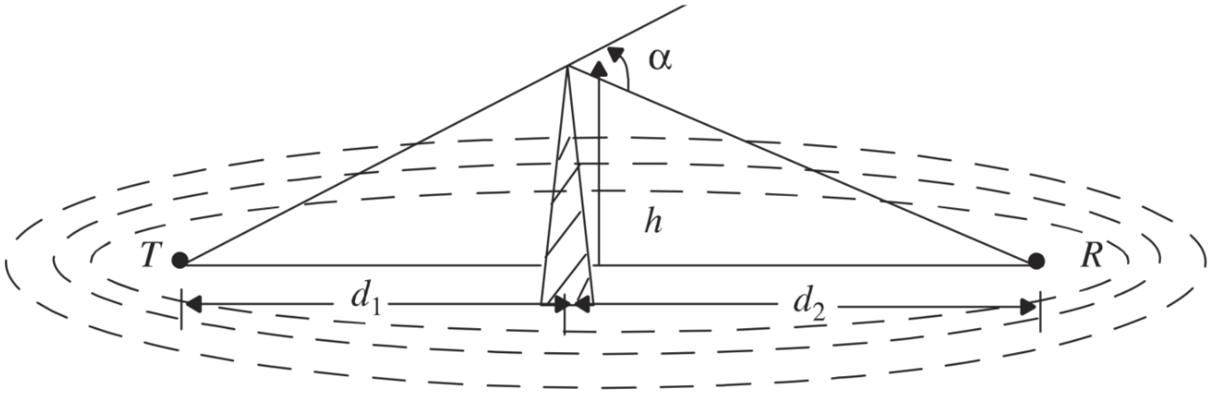
Fresnel-Kirchoff diffraction parameter ν (/nu/):

$$\nu = h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}} \simeq \alpha\sqrt{\frac{2 d_1 d_2}{\lambda(d_1+d_2)}}$$

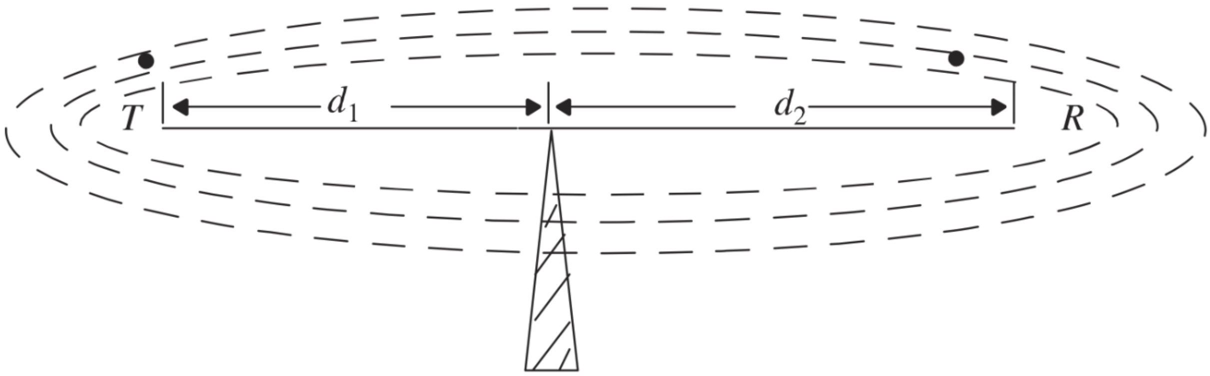
α: has units in radians.

Using ν parameter: $\phi = \frac{\pi}{2}\nu^2$

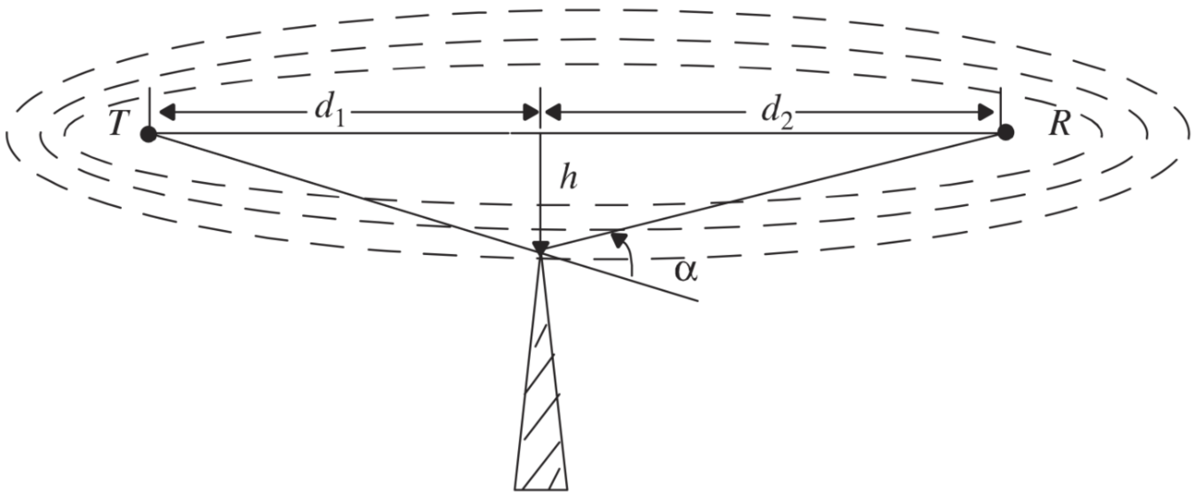




(a) α and ν are positive, since h is positive



(b) α and ν are equal to zero, since h is equal to zero



(c) α and ν are negative, since h is negative

- The ellipsoids represent Fresnel zones with the transmitter and receiver antenna at their foci.
- The family of ellipsoids are constructed by joining all the points for which the excess path length Δ is an integer multiple of half wavelengths, that is

$$\Delta = n\frac{\lambda}{2} \quad , n: \text{is an integer.}$$

e.g. the path traveling within the smallest circle corresponds to $n=1$, it will have an axcess path length of $\Delta = \frac{\lambda}{2}$ as compared to a line-of-sight path.
- The radii of the circles on the plane of an obstacle depend on the location of the plane. The radii become smaller when the plane of an obstacle moves closer to either the transmitter or the receiver. The radii become maximum when the plane is midway between the transmitter and receiver.

- The radius of the n^{th} Fresnel zone circle, r_n, can be expressed as follows: $r_n = \sqrt{\dfrac{n\lambda d_1 d_2}{d_1+d_2}}$

this approximation is valid for $d_1 \gg r_n$ & $d_2 \gg r_n$.

- The received energy will be a vector sum of the energy contributions from all unobstructed Fresnel zones.
- The successive Fresnel zones alternately provide construction and destruction interference to the total received signal.
- In general, if the obstacle does not block the volume contained within the first Fresnel zone, then the diffraction loss is minimal and the diffraction effects can be neglected.
- For line-of-sight links, if the first Fresnel zone is 55% clear, then other Fresnel zones do not significaly effect the diffraction loss.

- It is impossible to make precise estimations for diffraction losses. In practice, prediction is through theoretical approximation modified by necessary experiential corrections.
- When diffraction is caused by one obstacle, diffraction can be estimated by the knife-edge model.
- The electric field strength of a knife-edge diffracted wave, E_d, with respect to the free space field strength in the absence of both the ground and the knife edge, E_o, is :

$$\frac{E_d}{E_o} = F(\nu) = \frac{1+j}{2} \int_{\nu}^{\infty} \exp\left\{ \frac{-j\pi t^2}{2} \right\} dt$$

$F(\nu)$ is a function of the parameter ν.

$F(\nu)$ is usually evaluated from a graphical representation of $G_d(dB)$ or an approximation :

$G_d(dB)$ is the diffraction gain due to the knife edge as compared to the free space E-field,

$$G_d(dB) = 20\log|F(\nu)| := 20\log\left|\frac{E_d}{E_o}\right|$$

Then

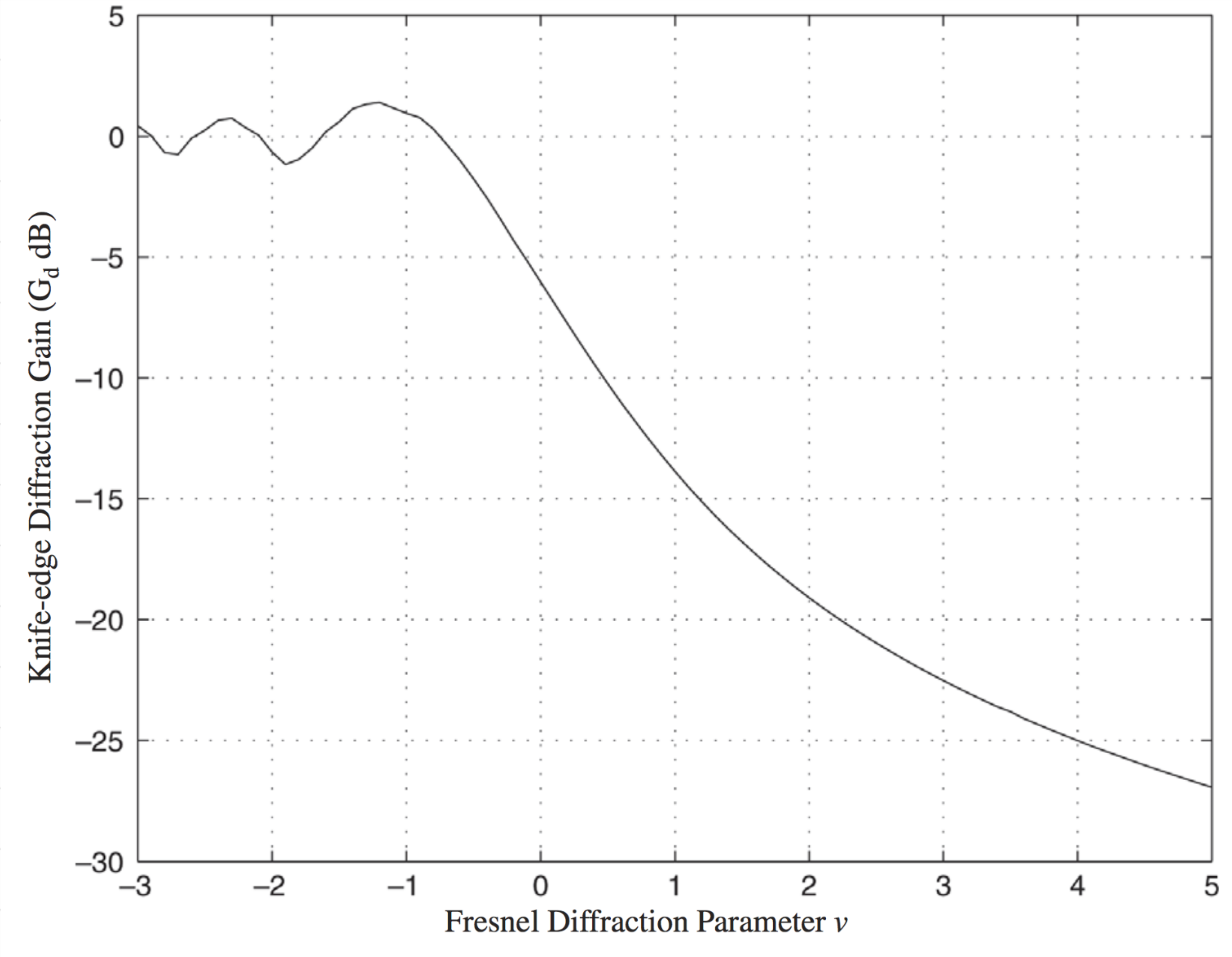
$$P_r(dB) = P_{r,\text{free-space}}(dB) + G_d(dB)$$

$$P_r(dB) = 10\log\left[\frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}\right] + G_d(dB)$$

The approximation of $G_d(dB)$:

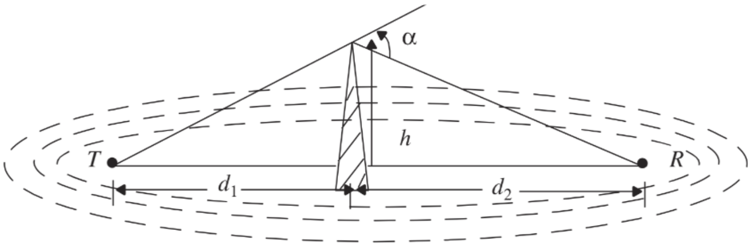
$$G_d(dB) = \begin{cases} 0 & , \nu \leq -1 \\ 20\log(0.5 - 0.62\nu) & , -1 \leq \nu \leq 0 \\ 20\log(0.5\, e^{-0.95\nu}) & , 0 \leq \nu \leq 1 \\ 20\log\left(0.4 - \sqrt{0.1184 - [0.38 - 0.1\nu]^2}\right), & 1 \leq \nu \leq 2.4 \\ 20\log\left(\frac{0.225}{\nu}\right) & , \nu > 2.4 \end{cases}$$

Below is the graphical representation of G_d(dB) as a function of ν.



Example 4.7

Compute the diffraction loss for the three cases shown in Figure 4.12. Assume $\lambda = 1/3$ m, $d_1 = 1$ km, $d_2 = 1$ km, and (a) $h = 25$ m, (b) $h = 0$, (c) $h = -25$ m. Compare your answers using values from Figure 4.14, as well as the approximate solution given by Equation (4.61.a)–(4.61.e). For each of these cases, identify the Fresnel zone within which the tip of the obstruction lies.



(a) α and v are positive, since h is positive

(a) $\nu = h\sqrt{\dfrac{2(d_1+d_2)}{\lambda d_1 d_2}}$

$= 25\sqrt{\dfrac{2(10^3+10^3)}{(1/3)(10^3)^2}} = 2.74$

- From the graphical representation, for $\nu = 2.74$

$G_d(dB) \simeq -22$ dB, so the diffraction loss $= 22$ dB

- Using the approximation, for $\nu = 2.74$

$G_d(dB) = 20\log\left(\dfrac{0.225}{\nu}\right) = -21.7$ dB

so the loss is 21.7 dB.

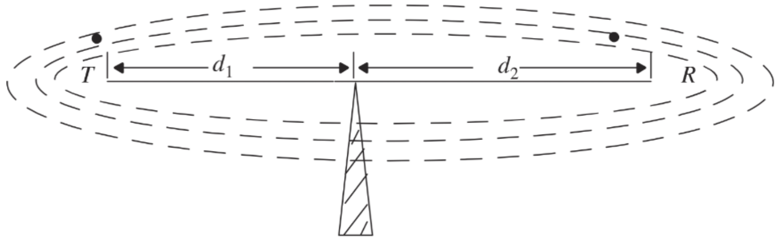
Since $\Delta = \dfrac{n\lambda}{2} \Rightarrow n = \dfrac{2\Delta}{\lambda}$ and

$\Delta \simeq \dfrac{h^2(d_1+d_2)}{2d_1d_2} = \dfrac{(25)^2(10^3+10^3)}{2\,10^3\,10^3} = 0.625$ m

so $n = \dfrac{2\Delta}{\lambda} = \dfrac{2(0.625)}{1/3} = 3.75$

so the tip of the obstacle completely blocks the first three Fresnel zones.

(b) $h = 0$



(b) α and ν are equal to zero, since h is equal to zero

$$\nu = h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}} = 0$$

From the graphical representation of $G_d(dB)$:

$G_d(dB) \simeq -6\ dB$, so the diffraction loss $= 6\ dB$.

From the approximation: $G_d(dB) = 20\log(0.5 - 0.62\nu)$

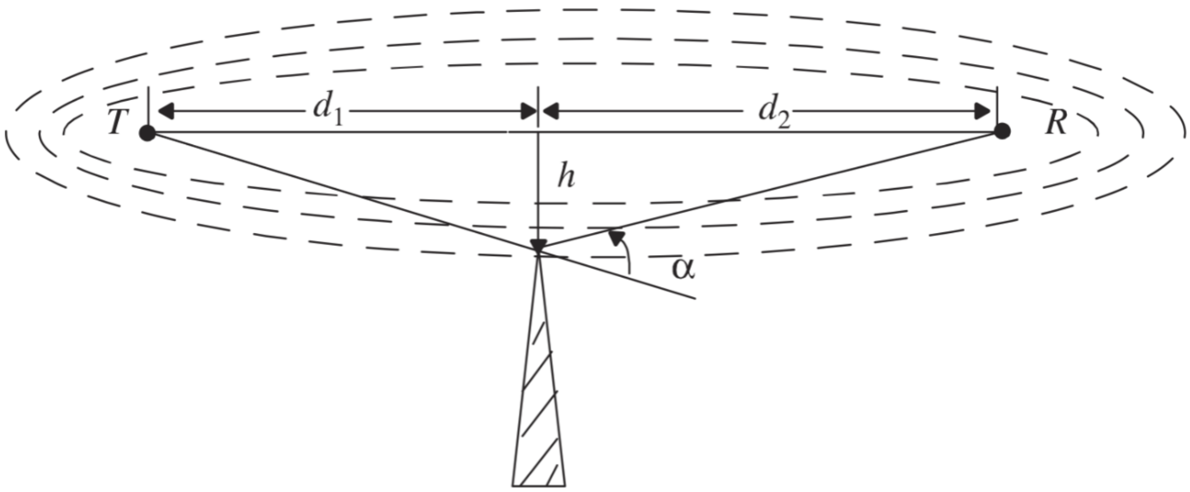
$G_d(dB) = -6.02\ dB$

So the diffraction loss $= 6.02\ dB$

$$\Rightarrow \Delta = \frac{h^2(d_1+d_2)}{2d_1d_2} = 0 \quad \Rightarrow \quad n = \frac{2\Delta}{\lambda} = 0$$

so the obstacle is in the first Fresnel zone.

(c)



(c) α and v are negative, since h is negative

$h = -25m$.

$$\nu = h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}} = -25\sqrt{\frac{2(2\,10^3)}{\frac{1}{3}10^6}} = -2.74$$

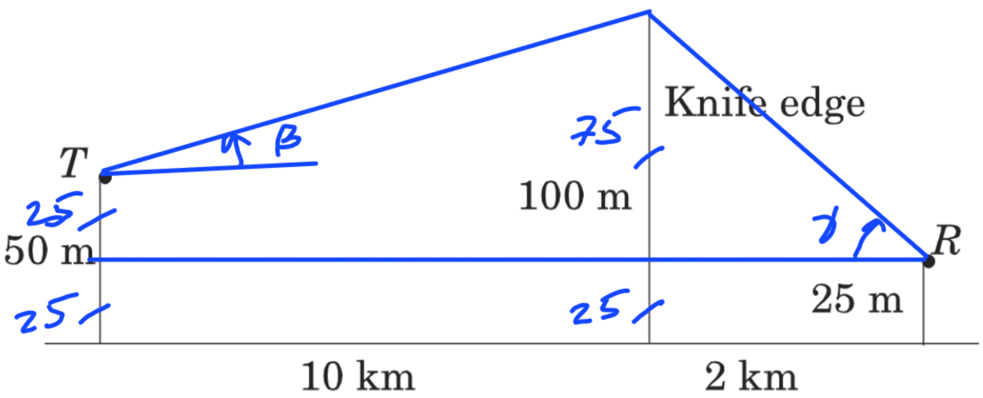
$G_d(dB) = 0$ dB from the approximation

also from the graph $G_d(dB) \simeq -1dB$, the loss $\simeq 1dB$

$$\Delta = \frac{h^2(d_1+d_2)}{2d_1d_2} = 0.625, \quad n = \frac{2\Delta}{\lambda} = 3.75,$$

however, since there is a line-of-sight path where the obstacle does not block it, then the diffraction loss is negligible even though it obstructs the first three Fresnel zones.

Example 4.8
Given the following geometry, determine (a) the loss due to knife-edge diffraction, and (b) the height of the obstacle required to induce 6 dB diffraction loss. Assume f = 900 MHz.



To find $G_d(dB)$ we should find ν.

$$\nu = h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}} = 75\sqrt{\frac{2(10\times10^3+2\times10^3)}{\frac{3\times10^8}{9\times10^8}(20\times10^6)}} = 4.5$$

$G_d(dB) = 20\log\left(\frac{0.225}{\nu}\right) = -28.52$ dB, loss $= 28.52$ dB

or if we use $\nu \simeq \alpha\sqrt{\frac{2d_1d_2}{\lambda(d_1+d_2)}}$

$$\alpha = \beta + \gamma = \tan^{-1}\left(\frac{50}{10\times10^3}\right) + \tan^{-1}\left(\frac{75}{2\times10^3}\right) = 0.04248\,\text{rad}$$

$$\nu \simeq 0.04248\sqrt{\frac{2(20\times10^6)}{\frac{3\times10^8}{9\times10^8}(12\times10^3)}} = 4.248$$

$G_d(dB) \simeq 20\log\left(\frac{0.225}{\nu}\right) = -25.52$ dB, loss $= 25.52$ dB

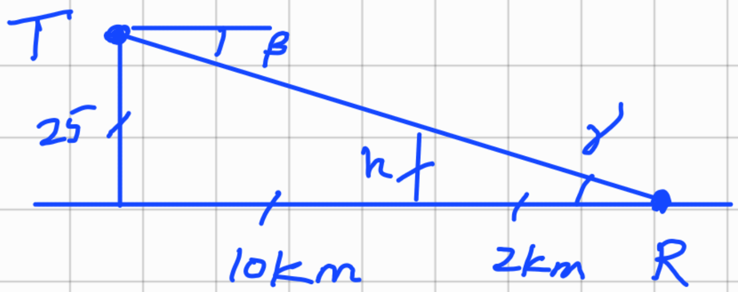
(b) 6 dB loss means $G_d(dB) = -6$ dB,

From the graph, $\nu = 0$.

from $\nu = h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}} \simeq \alpha\sqrt{\frac{2 d_1 d_2}{\lambda(d_1+d_2)}}$

to find $h \neq 0$ with $\nu = 0$, so $\alpha = 0$

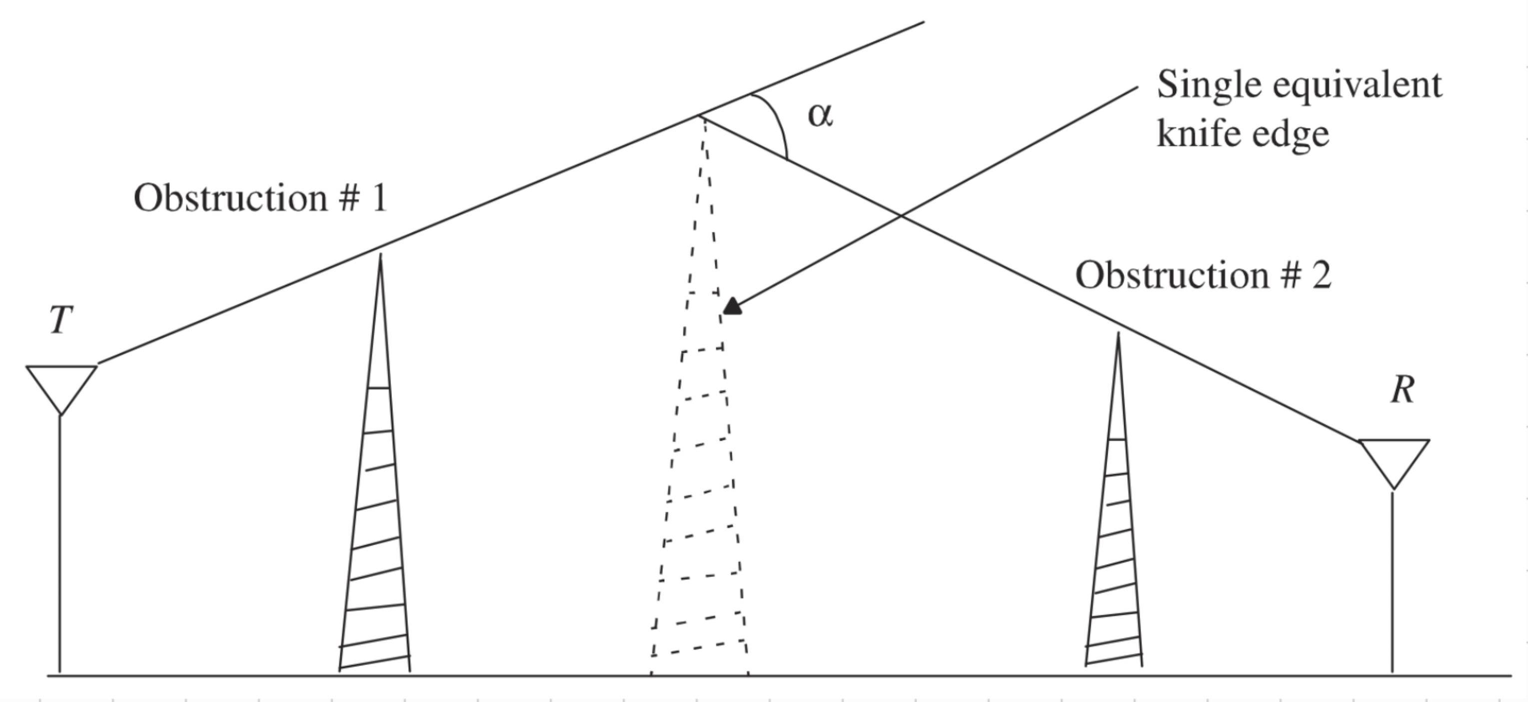
and $\beta = -\gamma$, then



$$\frac{h}{25} = \frac{2\times10^3}{12\times10^3}$$

$$\Rightarrow h = 4.167 \text{ m}$$

- For the case of multiple obstacles in the propagation path, Bullington suggested that the obstacles can be replaced by one equivalent obstacle so the path loss can be predicted using single knife-edge diffraction model as illustrated below.



3 Scattering:

3.1 Radar Cross Section (RCS) Model:

The received signal in a mobile environment is usually stronger than the prediction of reflection and diffraction models. This is caused by scattering due to the reflected energy off small objects (lamp posts) or off rough surfaces. If a large flat surface with dimentions much larger than the wavelength, it will reflect the signal, however, if the same large surface has a roughness, then it could cause scattering for the signal along its path. The roughness of a surface is tested using Rayleigh criterion, it studies the diffrence between reflected waves from the minimum protuberance (نتوء) to the maximum protuberance of such surface, if the phase difference is less than $\frac{\pi}{2}$, then the surface is smooth, that is if the heigh difference between the minimum protuberance and the maximum protuberance (h) is less than a critical height (h_c) for a given angle of incidence (θ_i), where $h_c = \dfrac{\lambda}{8 \sin \theta_i}$.

So the surface is smooth if $h < h_c$

and the surface is rough if $h > h_c$.

- The radar cross section (RCS) of a scattering object is defined as the ratio of the power density of the signal scattered in the direction of the receiver to the power density of the radio wave incident onto the scattering object (unit in m^2).

- When the scattering object is assumed to be in the far field of both the transmitter and receiver antennas, the received power for an urban environment is:

$$P_r(dB) = P_t(dB) + 10\log G_t + 20\log\lambda + RCS(dB\,m^2)$$
$$- 30\log(4\pi) - 20\log d_t - 20\log d_r$$

where d_t is the distance from the scattering object to the transmitter, d_r is the distance from the scattering object to the receiver.

- Practical Link Designs:

1. Log-distance Path Loss Model:

The average large-scale path loss for an arbitrary T-R separation is known theoretically and through measurements that it is a function of distance (d) and a path loss exponent (n).

$$\overline{PL}(d) = \overline{PL}(d_0) + 10n \log \frac{d}{d_0}$$

$\overline{PL}$: the bar denotes the ensemble average of all possible path loss values for a given distance d.

This function when plotted on a log-log scale, the path loss model is a straight line with a slope of 10n dB/decade.

Where n depends on the environment:

Environment	Path Loss Exponent, *n*
Free space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

while the reference path loss is either calculated using the free-space path loss: $PL(d_0) = 10\log \frac{P_t}{P_r} = -10\log\left[\frac{G_t G_r \lambda^2}{(4\pi)^2 d^2}\right]$

or through measurements at distance d_0.

- This model does not consider that for the same TR separation in an environment, $PL(d)$ could be very different.

2. Log-normal Shadowing:

- If we consider that $PL(d)$ could be very different for the same T-R separation, then this model could be used.

- Through measurements, the path loss at a distance d, $PL(d)$, is found to be random with a log-normal distribution with a mean that depends on the distance. That is,

$$PL(d) = \overline{PL}(d) + X_\sigma$$

$$= \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right) + X_\sigma$$

and $P_r(d) = P_t - (10\log G_t + 10\log G_r) - PL(d)$

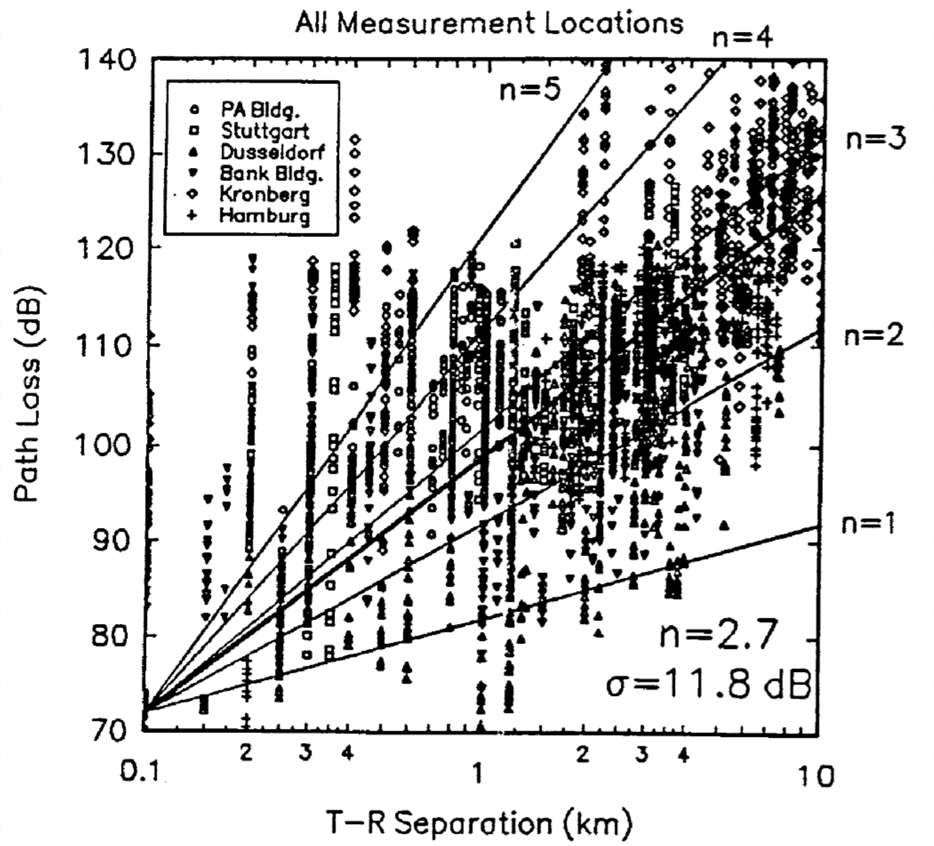
where X_σ is normal distributed random variable in dB, with zero mean, that is, log-normal with zero mean. This distribution describes a large number of measurements at locations with the same T-R separation due to the different level of cluttering on the propagation path. This case is called log-normal shadowing. Its standard deviation is σ and variance σ^2.

$\overline{PL}(d_0)$ is found either through measurements at distance d_0 or calculated using the free-space path loss:

$$PL(d_0) = 10\log\frac{P_t}{P_r} = -10\log\left[\frac{G_t G_r \lambda^2}{(4\pi)^2 d^2}\right]$$

The values of n and σ are found through data fitting by comparing the measurements to a mathmatical model.

- This plot shows data fitting for measured data for a range of T-R separations for multiple cities



- To find the received power at a distance d, $P_r(d)$, we should find the $PL(d)$, which depends on the random variable (RV) X_σ. Where $X_\sigma : \mathcal{N}(0, \sigma^2)$ i.e normal distributed with zero mean and variance σ^2.
- The probability that the received signal at a T-R separation has a power more than γ is denoted by $P\{P_r(d) > \gamma\}$ and the probability of such received signal has a power less than γ is denoted by $P\{P_r(d) < \gamma\}$. Since $X_\sigma : \mathcal{N}(0, \sigma^2)$, then $P_r(d)$ has a distribution $P_r(d) : \mathcal{N}\left(\overline{P_r(d)}, \sigma^2\right)$, and

$$P\{P_r(d) > \gamma\} = Q\left(\frac{\gamma - \overline{P_r(d)}}{\sigma}\right)$$

$$P\{P_r(d) < \gamma\} = Q\left(\frac{\overline{P_r(d)} - \gamma}{\sigma}\right)$$

where $Q(z) = 1 - Q(-z)$, $Q(0) = \frac{1}{2}$

with $Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty \exp\left(-\frac{x^2}{2}\right) dx$

for error function (erf) : $\operatorname{erf}(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-x^2} dx$

and the complementary error function (cerf) :

$$\operatorname{cerf}(z) = \frac{2}{\sqrt{\pi}} \int_z^\infty e^{-x^2} dx$$

$$\operatorname{erfc}(z) = 1 - \operatorname{erf}(z)$$

$$\Rightarrow \quad Q(z) = \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{z}{\sqrt{2}}\right)\right] = \frac{1}{2}\operatorname{erfc}\left(\frac{z}{\sqrt{2}}\right)$$

$$\operatorname{erfc}(z) = 2Q(\sqrt{2}\,z)$$

$$\operatorname{erf}(z) = 1 - 2Q(\sqrt{2}\,z)$$

Percentage of Coverage Area:

Using the probability of a received signal to fall below a desired threshold at a distance as found above, the percentage of coverage area can be found as follows.

For a circular coverage area of radius R from a base station and a desired received signal threshold γ, let $U(\gamma)$ denote the percentage of area with received signal strength equal or greater than the threshold γ. If $P\{P_r(r) > \gamma\}$ is the probability that the random received signal at $d=r$ is greater than the threshold γ, then

$$U(\gamma) = \frac{1}{R^2\pi} \int P\{P_r(r) > \gamma\} \, dA$$

$$= \frac{1}{R^2\pi} \int_{\phi=0}^{2\pi} \int_{r=0}^{R} P\{P_r(r) > \gamma\} \, r \, dr \, d\phi$$

$$= \frac{1}{R^2\pi} \int_{r=0}^{R} Q\left(\frac{\gamma - \overline{P_r(r)}}{\sigma}\right) r\,dr \int_{\phi=0}^{2\pi} d\phi$$

$$U(\gamma) = \frac{2}{R^2} \int_{r=0}^{R} Q\left(\frac{\gamma - \overline{P_r(r)}}{\sigma}\right) r\,dr$$

Since $\overline{P_r(r)} = P_t - \overline{PL(r)} = P_t - \left[\overline{PL}(d_0) + 10n\log\frac{r}{d_0}\right]$

to find PL with refrence to the cell boundary,

$$\overline{P_r(r)} = P_t - \left[\overline{PL}(d_0) + 10n\log\frac{r}{R} + 10n\log\frac{R}{d_0}\right]$$

$$\Rightarrow \frac{\gamma - \overline{P_r(r)}}{\sigma} = \left[\gamma - P_t + \overline{PL}(d_0) + 10n\log\frac{r}{R} + 10n\log\frac{R}{d_0}\right] / \sigma$$

let $a = \left[\gamma - P_t + \overline{PL}(d_0) + 10n\log\frac{R}{d_0}\right] / (\sqrt{2}\,\sigma)$

since $\ln x = \frac{\log x}{\log e}$ $\left(x = e^{\ln x} \Rightarrow \log x = \log e^{\ln x} \Rightarrow \log x = \ln x \log e\right)$

$$\Rightarrow \left(10n\log\frac{r}{R}\right) / (\sqrt{2}\,\sigma) = \left(10n\log e \ln\frac{r}{R}\right) / (\sqrt{2}\,\sigma) = b\ln\frac{r}{R}$$

let $b = (10n\log e)/(\sqrt{2}\,\sigma)$

$$\Rightarrow \frac{\gamma - \overline{P_r(r)}}{\sigma} = a + b\ln\frac{r}{R}$$

$$\Rightarrow U(\gamma) = \frac{2}{R^2}\int_{r=0}^{R} Q\left(\frac{\gamma - \overline{P_r(r)}}{\sigma}\right) r\,dr$$

$$= \frac{2}{R^2}\int_{r=0}^{R} Q\left(a + b\ln\frac{r}{R}\right) r\,dr$$

$$= \frac{2}{R^2}\int_{r=0}^{R} \frac{1}{2}\left[1 - \mathrm{erf}\left(a + b\ln\frac{r}{R}\right)\right] r\,dr$$

$$= \frac{1}{R^2}\left\{\left.\frac{r^2}{2}\right|_{r=0}^{R} - \int_{r=0}^{R} r\,\mathrm{erf}\left(a + b\ln\frac{r}{R}\right) dr\right\}$$

let $t = a + b\ln\frac{r}{R} \Rightarrow t - a = b\ln\frac{r}{R} \Rightarrow \frac{t-a}{b} = \ln\frac{r}{R}$

$$\Rightarrow \frac{r}{R} = e^{\frac{t-a}{b}} \Rightarrow r = R e^{\frac{t-a}{b}}$$

$$dr = \frac{R}{b} e^{\frac{t-a}{b}} dt$$

for $r = 0$, $t = -\infty$

$\ln 0 = -\infty$

$r = R$, $t = a + b\cancel{\ln\frac{R}{R}}^{0} = a$

$\ln 1 = 0$

$$\Rightarrow U(\gamma) = \frac{1}{R^2}\left[\frac{R^2}{2} - \int_{t=-\infty}^{a} R e^{\frac{t-a}{b}} \operatorname{erf}(t) \frac{R}{b} e^{\frac{t-a}{b}} dt\right]$$

$$= \frac{1}{2} - \frac{e^{-2a/b}}{b} \int_{t=-\infty}^{a} e^{\frac{2t}{b}} \operatorname{erf}(t)\, dt$$

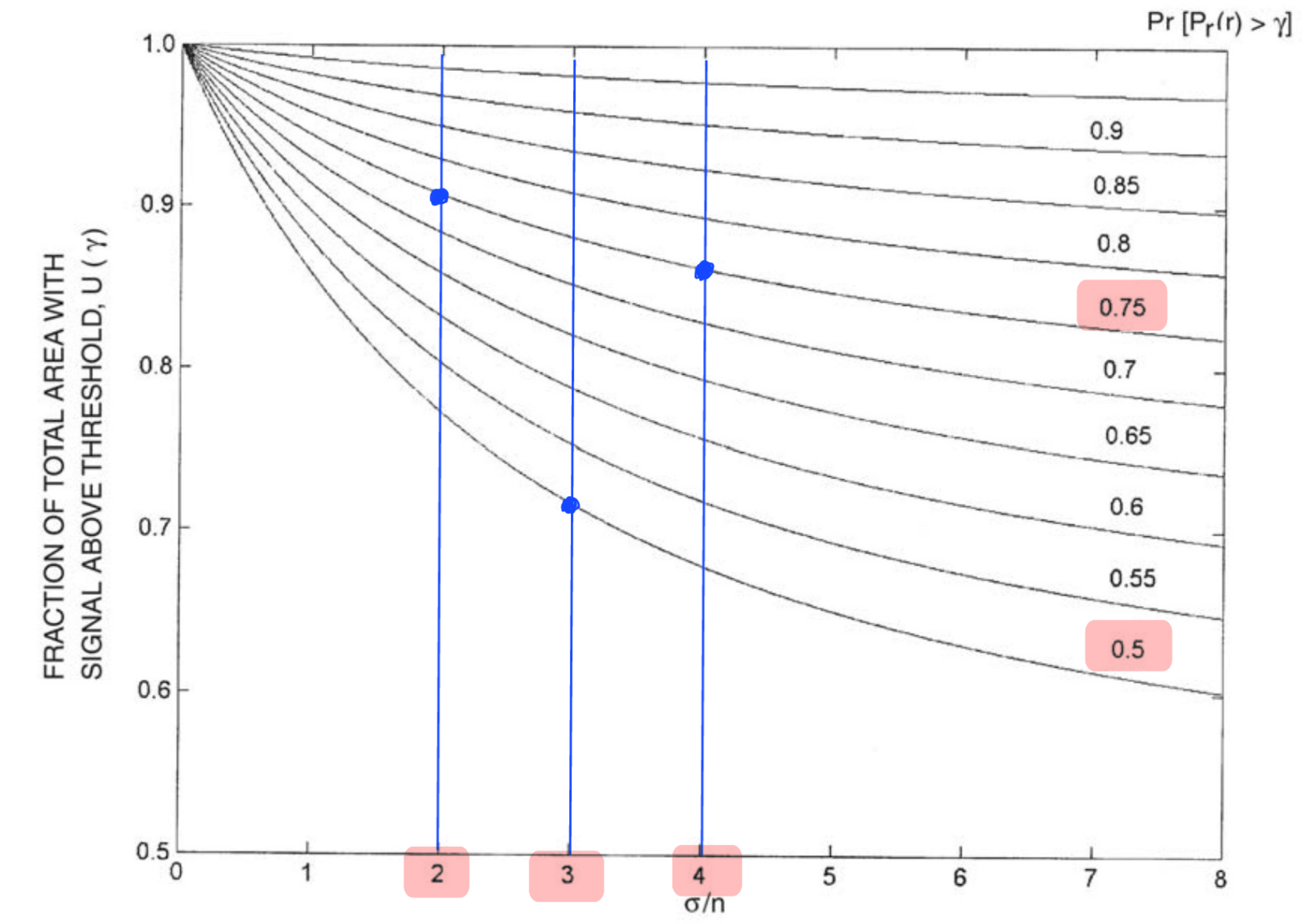
$$= \frac{1}{2} - \frac{e^{-2a/b}}{b} \int_{t=-\infty}^{a} e^{2t/b} \left[\frac{2}{\sqrt{\pi}} \int_{x=0}^{t} e^{-x^2} dx\right] dt$$

$$\vdots$$

$$U(\gamma) = \frac{1}{2}\left\{1 - \operatorname{erf}(a) + \exp\left(\frac{1-2ab}{b^2}\right)\left[1 - \operatorname{erf}\left(\frac{1-ab}{b}\right)\right]\right\}$$

If we choose $a = 0$ (i.e. $\overline{P_\gamma}(R) = \gamma$), then

$$U(\gamma) = \frac{1}{2}\left\{1 + \exp\left(\frac{1}{b^2}\right)\left[1 - \operatorname{erf}\left(\frac{1}{b}\right)\right]\right\}$$



These curves relate $U(\gamma)$ with the ratio of $\frac{\sigma}{n}$ and $P\{P_r(r) > \gamma\}$.

Example: Find $U(\gamma)$ using the above curves,

1. for $n = 4$, $\sigma = 8$ dB, if $P\{P_r(r) > \gamma\} = 75\%$ (that is, the boundary has 75% boundary coverage, this means that 75% of the time the signal at the boundary is larger than the threshold). Then $U(\gamma) \simeq 90\%$.

2. For $n = 2$, $\sigma = 8$ dB, if $P\{P_r(r) > \gamma\} = 75\%$,

then $U(\gamma) \simeq 87\%$

3. For $n = 3$, $\sigma = 9$ dB, $P\{P_r(r) > \gamma\} = 50\%$,

then $U(\gamma) \simeq 71\%$.

Example 4.9

Four received power measurements were taken at distances of 100 m, 200 m, 1 km, and 3 km from a transmitter. These measured values are given in the following table. It is assumed that the path loss for these measurements follows the model in Equation (4.69.a), where d_0 = 100 m: (a) find the minimum mean square error (MMSE) estimate for the path loss exponent, n; (b) calculate the standard deviation about the mean value; (c) estimate the received power at d = 2 km using the resulting model; (d) predict the likelihood that the received signal level at 2 km will be greater than -60 dBm; and (e) predict the percentage of area within a 2 km radius cell that receives signals greater than –60 dBm, given the result in (d).

Distance from Transmitter	Received Power
100 m	0 dBm
200 m	–20 dBm
1000 m	–35 dBm
3000 m	–70 dBm

(a) The power estimations for the above locations for an unknown n are as follows:

$$P_r(d) = P_r(d_0) - 10n \log \frac{d}{d_0} \quad , \quad d_0 = 100\,m \quad , \quad P_r(d_0) = 0\,dBm$$

$$P_r(200) = 0\,dBm - 10n \log \frac{200}{100} = -3n\ dBm$$

$$P_r(1000) = 0\,dBm - 10n \log \frac{1000}{100} = -10n\ dBm$$

$$P_r(3000) = 0\,dBm - 10n \log \frac{3000}{100} = -14.77n\ dBm$$

The square error between the measured and the estimated:

$$SE = (0-0)^2 + (-3n-(-20))^2 + (-10n-(-35))^2 + (-14.77n-(-70))^2$$

$$= 9n^2 + 400 - 120n + 100n^2 + 1225 - 700n + 218.15n^2 + 4900 - 2067.8n$$

$SE = 327.15n^2 + 6525 - 2887.8n$

$MSE = \frac{SE}{4}$: mean square error

MMSE : min. mean square error, by minimizing MSE :

$(327.15n^2 + 6525 - 2887.8n)' = 0$

$2(327.15)n - 2887.8 = 0$

$n = \frac{2887.8}{654.3} = 4.41$

(b) Standard deviation σ, from the variance σ^2.

$$\sigma^2 = \frac{SE}{4} = \frac{1}{4}\left[0 + (-3(4.41)+20)^2 + (-10(4.41)+35)^2 + (-14.77(4.41)+70)^2\right]$$

$$= \frac{1}{4}\left[6.77^2 + 9.1^2 + 4.8643^2\right]$$

$\sigma^2 = 38.08$ dB

$\sigma = 6.17$ dB

(c) $P_r(d=2km) = P_r(100m) - 10n\log\frac{2000}{100}$

$= 0\ dBm - 10(4.41)\log 20$

$= -57.38\ dBm$

also if we consider X_σ with $\sigma = 6.17\ dB$

$P_r(d=2km) = -57.38 + X_\sigma$

where X_σ has zero mean & σ^2 variance.

(d) $P\{P_r(2km) > -60\ dBm\} = Q\left(\frac{\gamma - \overline{P_r}(d)}{\sigma}\right)$

$= Q\left(\frac{-60-(-57.38)}{6.17}\right) = Q(-0.4246)$

$= 1 - Q(0.4246) \simeq 1 - 0.34458 = 0.655$

$P\{P_r(2km) > -60\ dBm\} = 65.5\%$

(e) From the curves at the end of last lecture,

for $P\{P_r(d) > \gamma\} = 65.5\%$ and $\frac{\sigma}{n} = \frac{6.17}{4.41} \simeq 1.4$

$\Rightarrow U(\gamma) \simeq 90\%$

3. Okumura Model

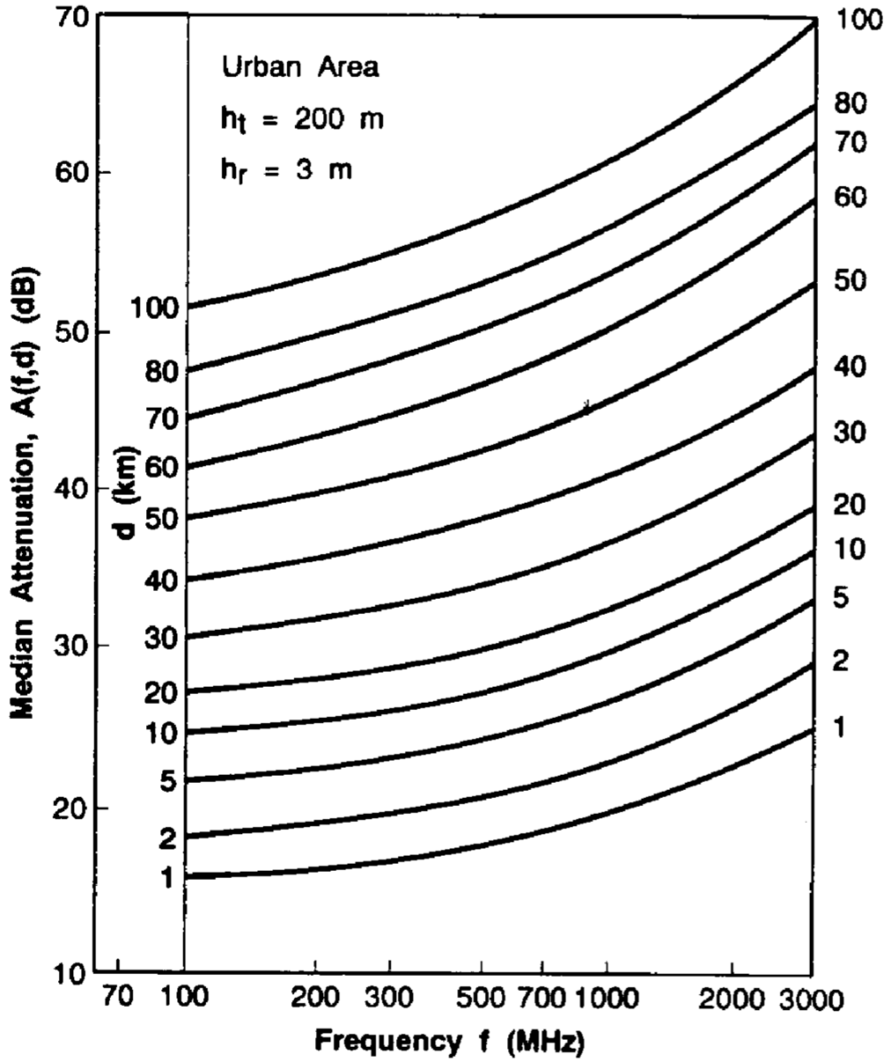
This model is used in urban areas, and for distances in the range of 1 km to 100 km and base station heights ranging from 30 m to 1000 m. Okumura developed a set of curves to give the median attenuation relative to free-space A_{mu} in an urban area with a base station effective height h_{te} of 200 m and a mobile antenna height h_{re} of 3 m, these curves were developed from extensive measurements at both the base station and the mobile, and plotted as a function of frequency in the range 100 MHz to 1920 MHz and extrapolated up to 3000 MHz, and as a function of distance from the base station and mobile in the range of 1 km to 100 km. To determine the path loss using Okumura's model, the free-space path loss is determined (L_F), then the value of the median attenuation $A_{mu}(f,d)$ is found from the curves, then the correction factors are added to find the median (50th percentile) value of propagation path loss is found (L_{50}), all as follows:

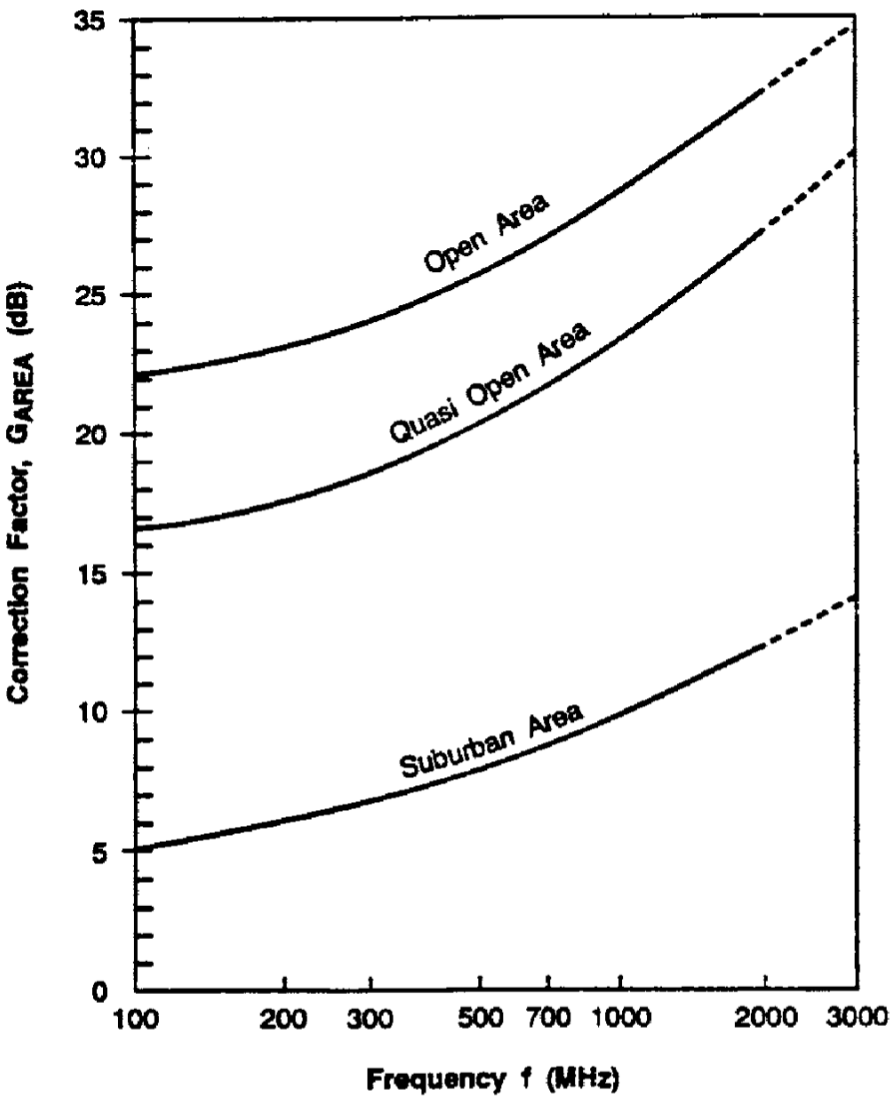
$$L_{50}(dB) = L_F + A_{mu}(f,d) - G(h_{te}) - G(h_{re}) - G_{AREA}$$

where $G(h_{te})$ and $G(h_{re})$ are the base station's and mobile's antenna height gain factors. G_{AREA} is the gain due to the type of environment, it is found from a set of curves for a range of frequencies as shown below. Also

$$G(h_{te}) = 20 \log\left(\frac{h_{te}}{200}\right), \quad 1000\,m > h_{te} > 30\,m$$

$$G(h_{re}) = \begin{cases} 10 \log\left(\frac{h_{re}}{3}\right), & h_{re} \leq 3\,m \\ 20 \log\left(\frac{h_{re}}{3}\right), & 10\,m > h_{re} > 3\,m \end{cases}$$


Urban Area
h_t = 200 m
h_r = 3 m
Median Attenuation, A(f,d) (dB)
d (km)
Frequency f (MHz)


Open Area
Quasi Open Area
Suburban Area
Correction Factor, G_{AREA} (dB)
Frequency f (MHz)

Example: For a median path loss using Okumura's model for $d = 50\,km$, $h_{te} = 100\,m$, $h_{re} = 10\,m$ in a suburban environment. If the base station transmits a power of 10^3 W using a carrier frequency of 900 MHz, find the received power at the receiver if $G_t = G_r = 1$.

$$P_r(d = 50\,km) = P_t - L_{50}$$

$$L_{50} = L_F + A_{mu}(f, d) - G(h_{te}) - G(h_{re}) - G_{AREA}$$

$$L_F = 10 \log\left[\frac{G_t G_r \lambda^2}{(4\pi)^2 d^2}\right] = 10 \log\left[\frac{(1)(1)\left(\frac{3\times10^8}{9\times10^8}\right)^2}{(4\pi)^2 (5\times10^4)^2}\right] = 125.5\,dB$$

$A_{mu}(900\,MHz, 50\,km) = 43\,dB$ (from the curves)

$G_{AREA}(f = 900\,MHz, \text{suburban}) = 9\,dB$ (from the curves)

$$G(h_{te}) = 20 \log \frac{100}{200} = -6\,dB$$

$$G(h_{re}) = 20 \log \frac{10}{3} = 10.46\,dB$$

$$P_r(d = 50\,km) = 10 \log 10^3 - (125.5 + 43 - (-6) - 10.46 - 9)$$

$$= 30\,dB - 155.04\,dB = -125.04\,dB$$

4. Hata Model

This model is a formulation of Okumura's graphs, it is valid for the frequency range of 150 MHz to 1500 MHz. It is applicable for situations other than urban through correction factors.

The formula for median path loss in urban areas:

$$L_{50}(\text{Urban}) = 69.55 + 26.16 \log f_c - 13.82 \log h_{te} - a(h_{re}) + (44.9 - 6.55 \log h_{te}) \log d .$$

where f_c is in MHz in the range from 150 MHz to 1500 MHz, h_{te} : effective base station antenna height in meters in the range of 30 m to 200 m, h_{re} : effective received antenna height in meters in the range of 1 m to 10 m, d : T-R separation in km, $a(h_{re})$ is the correction factor for effective mobile antenna height, it is a function of the size of the coverage area as follows:

For small to medium sized cities:

$$a(h_{re}) = (1.1 \log f_c - 0.7) h_{re} - (1.56 \log f_c - 0.8) \ \text{dB}$$

For large cities:

$$a(h_{re}) = \begin{cases} 8.29 (\log 1.54 h_{re})^2 - 1.1 \ \text{dB} , & f_c \leq 300 \text{ MHz} \\ 3.2 (\log 11.75 h_{re})^2 - 4.97 \ \text{dB} , & f_c \geq 300 \text{ MHz} \end{cases}$$

For suburban areas:

$$L_{50}(\text{Suburban}) = L_{50}(\text{Urban}) - 2 \left(\log \frac{f_c}{28} \right)^2 - 5.4$$

For open rural areas:

$$L_{50}(\text{open rural}) = L_{50}(\text{Urban}) - 4.78 (\log f_c)^2 + 18.33 \log f_c - 40.94$$

Hata model is suited for large cells, for cells of radius larger than 1 km.

4.19 If $P_t = 10$ W, $G_t = 10$ dB, $G_r = 3$ dB and $L = 1$ dB at 900 MHz, compute the received power for the knife-edge geometry shown in Figure P4.19. Compare this value with the theoretical free space received power if an obstruction did not exist. What is the path loss due to diffraction for this case?

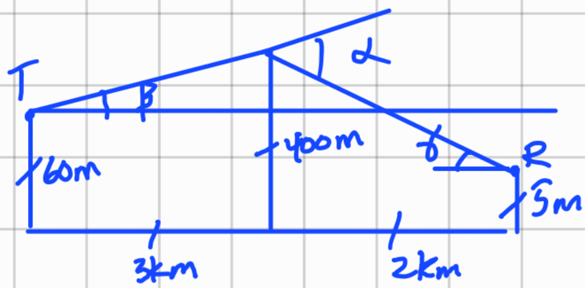
P_r : free-space
PL : diffraction
P_r : diffraction

$$P_r(\text{free-space}) = 10\log\frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}$$

$$= 10\log 10 + 10 + 3 + 20\log\frac{3\times10^8}{9\times10^8} - 20\log 4\pi - 20\log 5\times10^3 - 1$$

$$= 22 - 9.54 - 21.98 - 73.98 = -83.5 \text{ dB}$$

$$PL = -G_d(\text{dB}) \Rightarrow \nu = \alpha\sqrt{\frac{2d_1 d_2}{\lambda(d_1+d_2)}}$$



$$\alpha = \beta + \gamma = \tan^{-1}\left(\frac{400-60}{3\times10^3}\right) + \tan^{-1}\left(\frac{400-5}{2\times10^3}\right)$$

$$= 0.11285 + 0.1950$$
$$= 0.30785 \text{ rad}$$

$$\nu = (0.30785)\sqrt{\frac{2(3\times10^3)(2\times10^3)}{\frac{3\times10^8}{9\times10^8}(5\times10^3)}} = 26.122$$

for $\nu > 2.4$, $G_d = 20\log\left(\frac{0.225}{\nu}\right) = -41.30$ dB

$$PL = -G_d = 41.30 \text{ dB}$$

$$P_r(\text{diffraction}) = P_r(\text{free-space}) - PL = -83.5 - 41.30 = -124.8 \text{ dB}$$

4.23 If the received power at a reference distance $d_0 = 1$ km is equal to 1 microwatt, find the received powers at distances of 2 km, ~~5 km, 10 km, and 20 km~~ from the same transmitter for the following path loss models: (a) Free space; (b) $n = 3$; (c) $n = 4$; (d) two-ray ground reflection using the exact expression; and (e) extended Hata model for a large city environment. Assume $f = 1800$ MHz, $h_t = 40$ m, $h_r = 3$ m, $G_t = G_r = 0$ dB. ~~Plot each of these models on the same graph over the range of 1 km to 20 km. Comment on the differences between these five models~~.

(a) $P_r(2km) = P_r(d_0) - 10n \log \frac{d}{d_0}$

$= 10 \log 10^{-6} - 20 \log \frac{2 \times 10^3}{1 \times 10^3} =$

$= -60 - 6.021 = -66.021$ dB

(b) $n=3$, $P_r(2km) = P_r(d_0) - 30 \log 2$

$= -60 - 9.030 = -69.030$ dB

(c) $n=4$, $P_r(2km) = -60 - 40 \log 2 = -72.04$ dB

(d) The approximation:

$P_r(2km) = P_r(d_0)\Big|_{\text{free-space}} - PL$

$PL = 40 \log d - [10 \log G_t + 10 \log G_r + 20 \log h_t + 20 \log h_r]$

$= 40 \log 2 \times 10^3 - [0 + 0 + 20 \log 40 + 20 \log 3] = 90.46$

$P_r(2km) = 10 \log 10^{-6} - 90.46 = -60 - 90.46 = -150.46$ dB

4.27 A transmitter provides 15 W to an antenna having 12 dB gain. The receiver antenna has a gain of 3 dB and the receiver bandwidth is 30 kHz. ~~If the receiver system noise figure is 8 dB and~~ the carrier frequency is 1800 MHz, find the maximum T–R separation that will ensure that a ~~SNR of 20 dB~~ is provided for 95% of the time. Assume $n = 4$, $\sigma = 8$ dB, and $d_0 = 1$ km.

threshold $\gamma = -71.2$ dB

$$P\{P_r(d_{max}) > \gamma\} = Q\left(\frac{\gamma - \overline{P_r(d_{max})}}{\sigma}\right) = 95\%$$

$\Rightarrow$ this is when $\dfrac{\gamma - \overline{P_r(d_{max})}}{\sigma} \simeq -1.64$

$$\Rightarrow \frac{-71.2 - \overline{P_r(d_{max})}}{8} = -1.64$$

$$\overline{P_r(d_{max})} = -71.2 + 13.12 = -58.08 \text{ dB}$$

$$\overline{P_r(d_{max})} = \overline{P_r(d_0)}\Big|_{\text{free-space}} - 10n \log \frac{d_{max}}{d_0}$$

$$= 10\log\left(\frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d_0^2}\right) - 10(4)\log \frac{d_{max}}{d_0}$$

$$= 10\log 15 + 12 + 3 + 20\log \frac{3\times10^8}{18\times10^8} - 20\log 4\pi$$

$$- 20\log 10^3 - 40\log d_{max} + 40\log 10^3$$

$$-58.08 = 109.22 - 40\log d_{max}$$

$$40\log d_{max} = 167.3$$

$$d_{max} = 10^{\frac{167.3}{40}} = 10^{4.1825} = 15.22\times10^3 \text{ m}$$

$\gamma = -64.2$ dB

4.28 Assume ~~a SNR of 25 dB~~ is desired at the receiver. If a 900 MHz cellular transmitter has an ~~EIRP~~ of 100 W, and the AMPS receiver uses a 0 dB gain antenna ~~and has a 10 dB noise figure~~, find the percentage of time that the ~~desired SNR~~ γ is achieved at a distance of 10 km from the transmitter. Assume $n = 4$, $\sigma = 8$ dB, and $d_0 = 1$ km.

$P_tG_t = 100W$

$$\gamma = -64.2 \text{ dB}, \quad f_c = 9\times10^{8} \text{ Hz}, \quad n = 4, \quad \sigma = 8, \quad d_o = 1 \text{ km}$$

$$P_tG_t = 100 \text{ W}, \quad G_r = 0 \text{ dB}, \quad \text{find } P\{P_r(d=10\text{km}) > \gamma\}$$

$$\overline{P_r(d=10\text{km})} = \overline{P_r(d_o)} - 10n\log\frac{d}{d_o}$$

$$\overline{P_r(d_o)} = 10\log\frac{P_tG_tG_r\lambda^2}{(4\pi)^2 d_o^2} = 10\log P_tG_t + 10\log G_r + 20\log\lambda - 20\log 4\pi - 20\log d_o$$

$$= 10\log 10^2 + 0 + 20\log\frac{3\times10^8}{9\times10^8} - 20\log 4\pi - 20\log 10^3$$

$$= 20 - 9.54 - 21.98 - 60 = -71.52 \text{ dB}$$

$$\overline{P_r(d=10\text{km})} = -71.52 - 10(4)\log\frac{10^4}{10^3} = -111.52 \text{ dB}$$

4.29 Four received power measurements were taken at distances of 100 m, 200 m, 1 km, and 2 km from a transmitter. The measured values at these distances are –0 dBm, –25 dBm, –35 dBm, and –38 dBm, respectively. It is assumed that the path loss for these measurements follows the model

$$PL(d)[\mathrm{dB}] = \overline{PL}(d) + X_\sigma = \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right) + X_\sigma$$

where $d_0 = 100$ m.

(a) Find the minimum mean square error (MMSE) estimate for the path loss exponent, n.

(b) Calculate the standard deviation of shadowing about the mean value.

(c) Estimate the received power at $d = 2$ km using the resulting model.

(d) Predict the likelihood that the received signal level at 2 km will be greater than –35 dBm. Express your answer as a Q-function.

(a) $SE = (0-0)^2 + \left(-25 + 10n\log\frac{200}{100}\right)^2 + \left(-35 + 10n\log\frac{10^3}{10^2}\right)^2$

$+ \left(-38 + 10n\log\frac{2\times10^3}{10^2}\right)^2$

$= (625 + 9n^2 - 150n) + (1225 + 100n^2 - 700n)$

$+ (1444 + 169n^2 - 988n)$

$= 278n^2 + 3294 - 1838n$

$\left(\frac{SE}{4}\right)' = 2(278)n - 1838 = 0$

$n = 3.3$

(b) $\sigma \Rightarrow \sigma^2 = \frac{SE}{4} = \frac{278(3.3)^2 + 3294 - 1838(3.3)}{4}$

$= 64.005$

$\sigma = 8$

(c) $P_r(d = 2km) = P_r(d_0) - PL(d)$

$$= 0\,dBm - 10\,n\,\log\frac{2\times10^3}{100}$$

$$= -42.94\,dBm$$

(d) $P\{P_r(2km) > -35\,dBm\} = Q\left(\frac{\gamma - \overline{PL(2km)}}{\sigma}\right)$

$$= Q\left(\frac{-35\,dBm + 42.94\,dBm}{8}\right) \simeq Q\left(\frac{8}{8}\right)$$

$$\simeq Q(1)$$

Mobile Radio Wave Propagation:

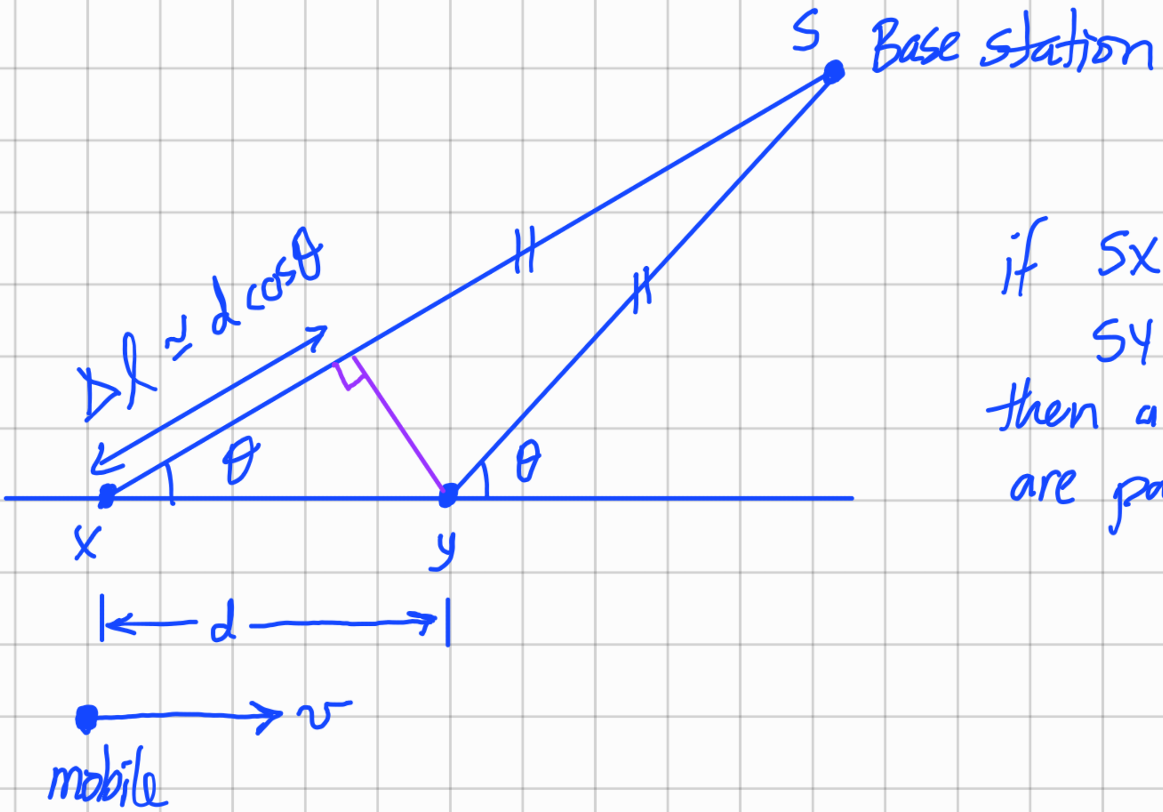
Small-Scale Multipath Propagation or Small-Scale Fading:

This describes the rapid fluctuations of the amplitudes, phases, or multipath delays of a radio signal over a short period of time or over a short travel distance, so that large-scale propagation effects may be ignored.

- Factors influencing small-scale fading:

1. Multipath propagation
2. Speed of the mobile
3. Speed of surrounding objects
4. The transmission bandwidth of the signal

Doppler shift and Doppler spread :



if $SX \gg d$ &
$SY \gg d$,
then assume SX & SY are parallel.

For a mobile moving in a constant velocity v from x to y a distance d, and a base station at location S, the difference in path lengths SX & SY is $\Delta l \simeq d\cos\theta = v\Delta t\cos\theta$ when assuming that SX & SY to be parallel, that is when $SX \gg d$ and $SY \gg d$,

and Δt is the time duration for the mobile to move from location x to y.

The phase difference in the received signal caused by the difference in path length Δl is

$$\Delta\phi = \frac{2\pi \Delta l}{\lambda} \simeq \frac{2\pi v \Delta t \cos\theta}{\lambda}$$

$$\Rightarrow \quad \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cos\theta =: f_d$$

$\Rightarrow$ The Doppler shift or the apparent change in frequency: $f_d = \frac{v}{\lambda}\cos\theta$

f_d : is positive when $\theta < 90$, that is when the mobile is moving toward the direction of the arrival of the wave, so the apparent received frequency increases.

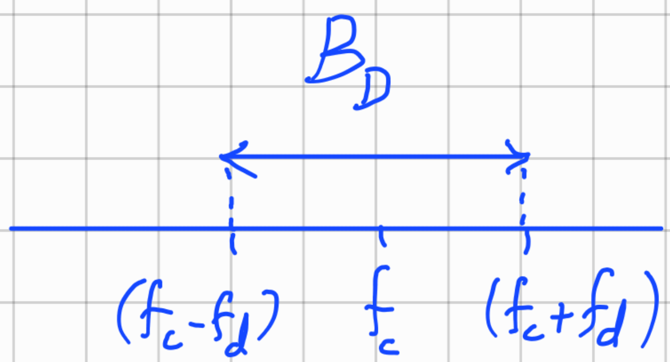
f_d : is negative when $\theta > 90$, that is when the mobile is moving away from the direction of the arrival of the wave, so the apparent received frequency decreases.

- The maximum Doppler shift denoted by f_m is when $\theta = 0$,

$$f_m = \frac{v}{\lambda} \cos 0 = \frac{v}{\lambda}$$

- When a pure sinusoidal tone of frequency f_c is transmitted, the received signal spectrum, called the Doppler spectrum, will have components in the range $(f_c - f_d)$ to $(f_c + f_d)$, f_d: Doppler shift

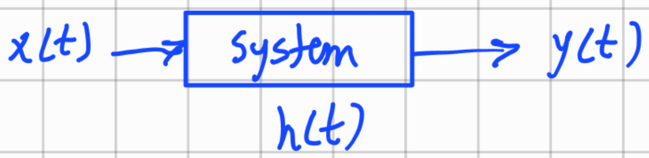
- The Doppler spread B_D is a measure of the spectral broadening and is defined as the range of frequencies over which the received Doppler spectrum is non-zero.



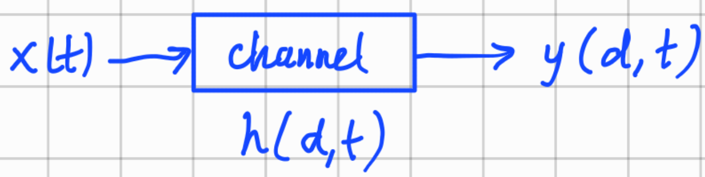
- The multipath components from a continuous signal that arrive from different directions contribute to Doppler spreading of the received signal, thus increasing the signal bandwidth.

- **Impulse Response Model of a Multipath Channel:**

For a system, to find the impulse response $h(t)$, we let $x(t) = \delta(t) :=$ impulse (i.e small pulse), then the output $y(t) = h(t)$.



To model a multipath channel, the mobile radio channel is modeled as a linear filter with a time varying impulse response, where the channel changes with time and distance.



- If we assume the mobile is moving in a constant velocity v over a short time, then $d = vt$, so

$$x(t) \longrightarrow \boxed{\text{channel}} \longrightarrow y(t)$$

$$h(vt, t)$$

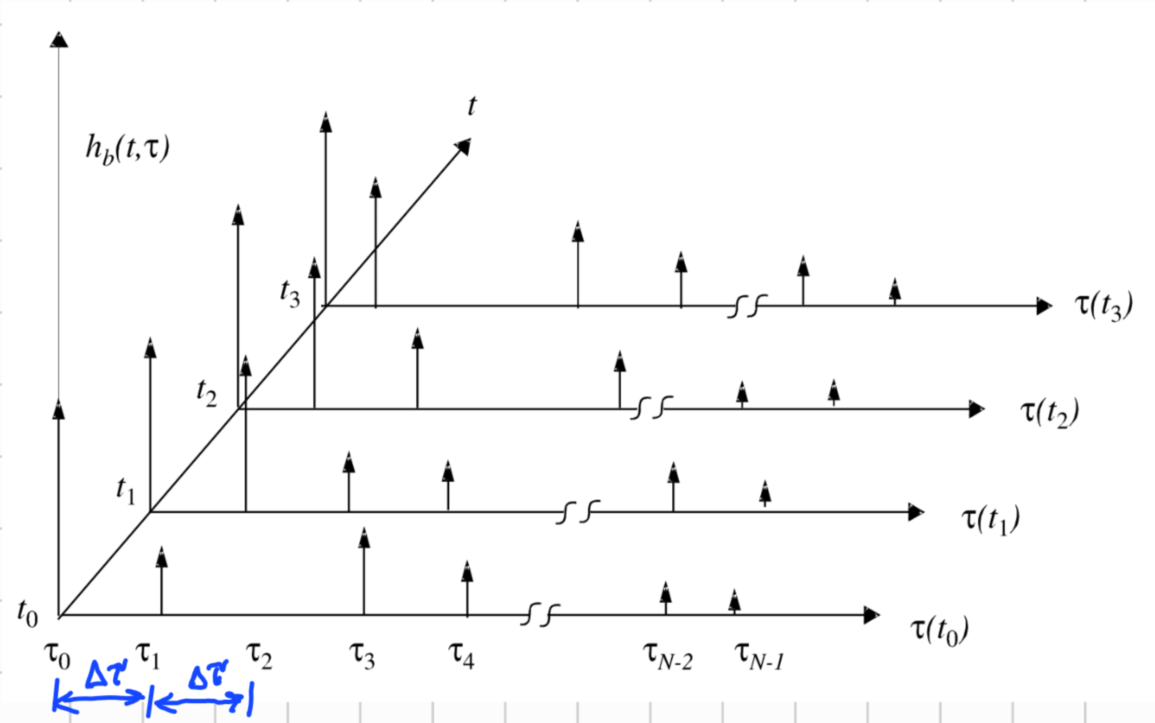
and $y(t)$ is a function of time

- If we have a multipath channel, then the received signal consists of a series of attenuated, time delayed, phase shifted replicas of the transmitted signal as follows

$$x(t) \longrightarrow \boxed{\text{multipath channel}} \longrightarrow y(t)$$

$$h(t, \tau)$$

where τ : is the time delay, it is quantized to widths of $\Delta\tau$



$$\text{and } h(t,\tau) = \sum_{i=0}^{N-1} a_i(t,\tau)\, e^{j\theta_i(t,\tau)}\, \delta\left(\tau - \tau_i(t)\right)$$

with $a_i(t,\tau)$: the channel attenuation of the i^{th} path at time t and time delay τ,

$\theta_i(t,\tau)$: the phase shift of the i^{th} path at time t and time delay τ

$\tau_i(t)$: the excess delay of the i^{th} path at time t

From the figure above, the delay axis τ is quatized into equal time delay segments called excess delay bins, each bin has a time delay width

$\Delta\tau = \tau_{i+1} - \tau_i$, $\tau_0 = 0$ and represents the first arriving signal at the receiver. For convention, $\tau_0 = 0$, then $\tau_1 - \tau_0 = \tau_1 = \Delta\tau$, and

$\tau_i - \tau_0 = \tau_i = i\Delta\tau$, for $i = \{0, 1, \cdots, N-1\}$

if N represents the total number of possible equally spaced multipath components including the first arriving component, then the maximum excess delay of the channel is $N\Delta\tau$.

- The received signal is then found from knowing the multipath channel impulse response $h(t,\tau)$

$$x(t) \longrightarrow \boxed{h(t,\tau)} \longrightarrow y(t)$$

- The plot representing the received power as a function of excess delay with respect to a fixed time delay reference is the power delay profile, $P(\tau)$, it is found by taking the spatial average of $|h(t,\tau)|^2$ over a local area (small-scale area), to provide a time invariant multipath power delay,

$$P(\tau) \simeq k\,\overline{|h(t,\tau)|^2}$$

and k is a gain

From the power delay profile, the following can be found:

1. Mean excess delay $\bar{\tau}$

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$$

$\bar{\tau}$: the first moment of the power delay profile.

2. The rms delay spread σ_τ

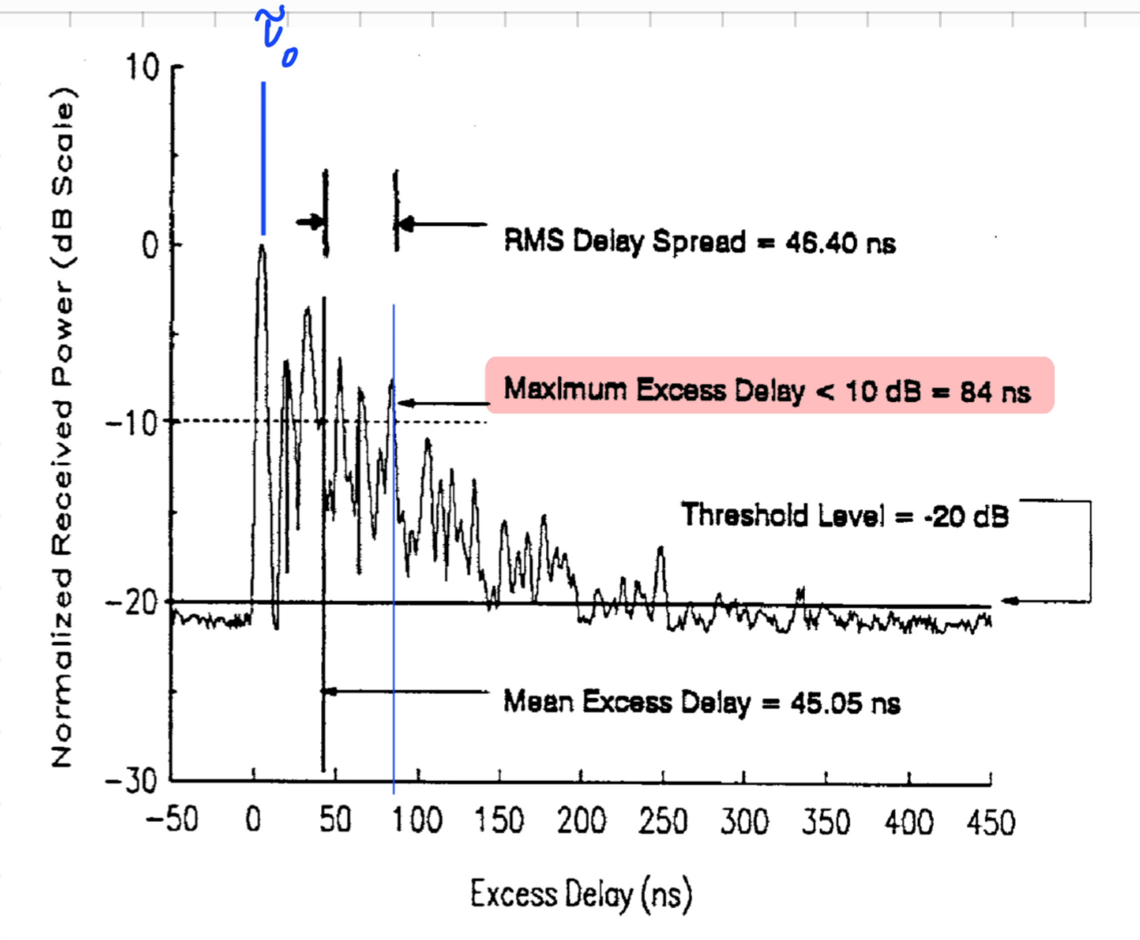
$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2} \quad , \text{ where}$$

$$\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}$$

$\overline{\tau^2}$: the second moment of the power delay profile

3. Maximum excess delay (X dB) for the power delay profile is the time delay during which multipath energy falls to X dB below the maximum, that is, $\tau_x - \tau_0$ where τ_0 is the first arriving signal and τ_x is the maximum delay at which a multipath component is within X dB of the strongest arriving multipath signal. The strongest arriving multipath signal does not necessarily arrive at τ_0. The value τ_x is called excess delay spread of a power delay profile.

- The below figure shows an example of a power delay profile, it shows the maximum excess delay 10 dB, it is from τ_0 to τ_x where τ_x is the time at which $P(\tau)$ is still within 10 dB of the max of $P(\tau) = 0$ dB, so $\tau_x \simeq 84$ ns,



then, the maximum excess delay $= \tau_x - \tau_0 = 84\text{ns} - 0$
$= 84\text{ns}$

- $\bar{\tau}$, $\overline{\tau^2}$, and σ_τ depend on the noise threshold, where the noise threshold is used to differentiate between the received multipath components and the thermal noise, if the noise threshold is set low, then the noise will be processed as multipath, then this will increase the values of $\bar{\tau}$, $\overline{\tau^2}$, and σ_τ

- Coherence time T_c :

- If two signals arrive with time separation greater than T_c, then they are affected differently by the channel.
- It is a measure of the time duration over which the channel impulse response is invariant.
  So, for time separation less than T_c, the channel is assumed to have a time invariant impulse response $h(t)$.
- It measures the similarity of the channel response at different times.
- It is the time duration over which two received signals have a strong posibility for amplitude correlation.
- For a max. doppler shift f_m,

$$T_c \simeq \frac{1}{f_m} \quad \sim \quad (*)$$

- If the coherence time is defined as the time over which the time correlation function is above 0.5, then the coherence time is approximately

$$T_c \simeq \frac{9}{16\pi f_m} \quad \text{---} \quad (**)$$

- The coherence time is usually defined as the geometric mean of (equ *) & (equ **), that is

$$T_c = \left[\frac{1}{f_m}\,\frac{9}{16\pi f_m}\right]^{\frac{1}{2}}$$

$$T_c = \sqrt{\frac{9}{16\pi f_m^2}} = \frac{0.423}{f_m}$$

- Coherence bandwidth B_c:

- Two sinusoids with frequency separation greater than B_c are affected differently by the channel.

- It is a measure of the range of frequencies over which the channel can be considered flat, that is, the channel will pass all spectral components with approximately equal gain and linear phase.

- It is the range of frequencies over which two frequency components have a strong posibility for amplitude correlation.

- If the coherence bandwidth is defined as the bandwidth over which the frequency correlation function is above 0.9, then

$$B_c \simeq \frac{1}{50\,\sigma_\tau}$$

where σ_τ is the rms delay spread.

- If the coherence bandwidth is defined as the bandwidth over which the frequency correlation function is above 0.5, then

$$B_c \simeq \frac{1}{5\,\sigma_\tau}$$

Types of Small-Scale Fading:

(A) Fading Due to Multipath Time Delay Spread :

1. Flat Fading

2. Frequency Selective Fading

(B) Fading Due to Doppler Spread :

1. Fast Fading

2. Slow Fading

Types of Small-Scale Fading:

(A) Fading Due to Multipath Time Delay Spread:

- Multipath propagation of the transmitted signal will cause the received signal to arrive spreaded over time, this is measured as rms delay spread σ_τ.
- The resulting fading due to this delay spread will depend on:

(1) the signal period T_s compared to the rms delay spread σ_τ

note: since the coherence bandwidth B_c is derived from the rms delay spread σ_τ, and the signal bandwidth B_s is the reciprocal of the signal period T_s (i.e $B_s = \frac{1}{T_s}$), then this fading will also depend on

(2) the signal bandwidth B_s compared to the coherence bandwidth B_c

1. Flat fading : in this type of fading, the spectral characteristics of the transmitted signal is preserved at the receiver, however, the strength of the signal can change if the channel gain changes over time, that would be due to the multipath.

Since we study $T_s \leftrightarrow \sigma_\tau$ and $B_s \leftrightarrow B_c$;

If $\sigma_\tau \ll T_s$ and $B_c \gg B_s$, then the channel will not change the transmitted signal at the receiver in frequency domain (flat fading channel), however, the channel might change the power of the received signal due to the traveling of the signal in multipaths if the channel is a time varying channel (amplitude varying channel).

Time domain

Frequency domain

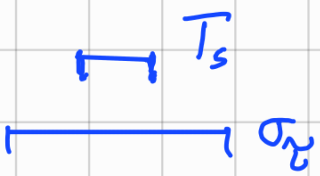
- A common rule, if $\sigma_\tau \leq \frac{T_s}{10}$, flat fading

2. Frequency Selective Fading : in this type of fading, the spectral characteristics of the transmitted signal is not preserved at the receiver, the channel will effect the transmitted signal with different gains for some frequencies, and the strength of the signal can change if the channel gain changes over time due to the multipaths.

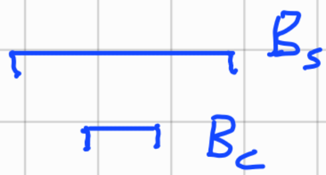
Since we study $T_s \leftrightarrow \sigma_\tau$ and $B_s \leftrightarrow B_c$;

If $\sigma_\tau > T_s$ and $B_c < B_s$, then the channel will change the transmitted signal at the receiver in frequency domain since it did not get transmitted within an rms delay spread σ_τ (intersymbol interference ISI), where this σ_τ gives a coherence bandwidth B_c that treats some frequencies differently (frequency selective fading).

Time domain



Frequency domain



- A common rule, if $\sigma_\tau > \frac{T_s}{10}$, frequency selective fading.

(B) Fading Due to Doppler Spread:

Fast and slow fading depend on the relationship between the time rate of change in the channel and the change in the transmitted baseband signal.

So we study the coherence time T_c of the channel compared to the signal period T_s of the transmitted signal, this coherence time T_c is related to the max. Doppler shift which is related to the Doppler spread of the channel B_D.

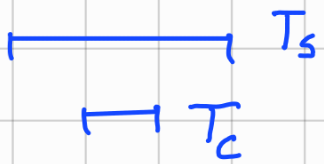
The resulting fading is due to the rate of change in the channel due to motion, the velocity of the mobile or the velocity of the objects in the channel. So we compare this rate of change in the channel with the change in the transmitted baseband signal through T_s, B_s & T_c, B_D

1. Fast Fading: in this type of fading, the coherence time T_c is less than the signal period leading to a change in the amplitude of the impulse response within a signal period T_s more than one value, this happens when we have low data rate, so this will lead to a fast change in the amplitude of the distortion caused by the channel (fast fading).

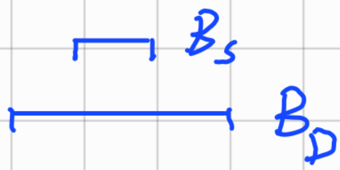
Or in frequency domain, with increasing Doppler spread of the channel B_D, the bandwidth of the transmitted baseband signal will be smaller compared to B_D, then fast fading will happen to the signal.

So, $T_c < T_s$ and $B_D > B_s$

Time domain



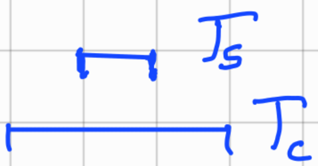
Frequency domain



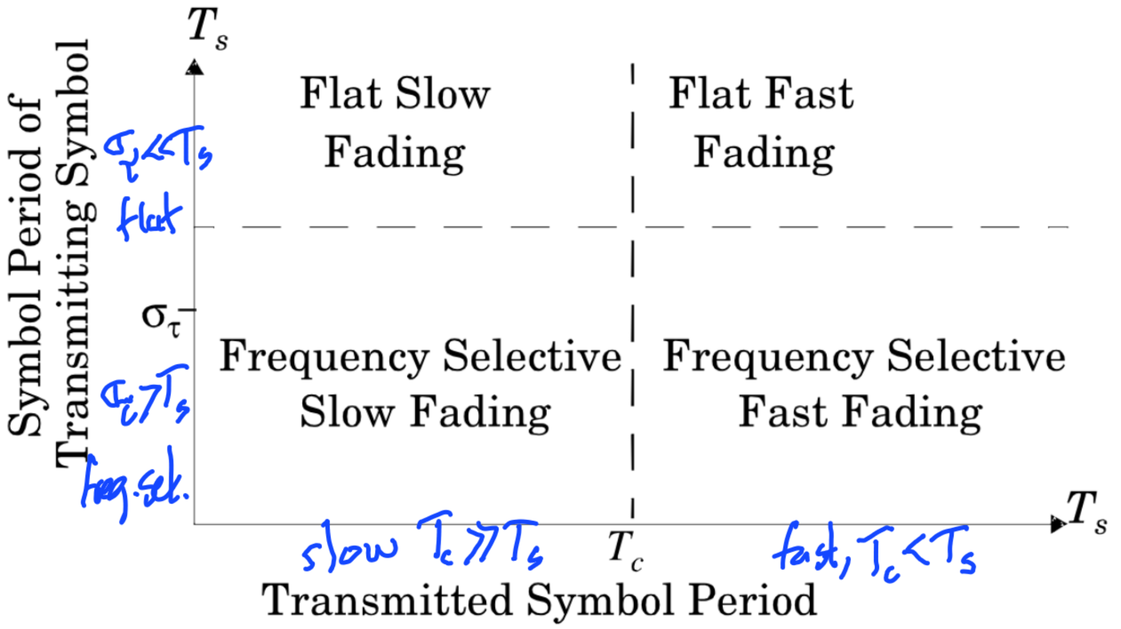
2. Slow Fading: in this type of fading, the channel impulse response changes much slower than the transmitted baseband signal, so the fading is slow fading, that is, the coherence time T_c is much larger than the signal period T_s. It follows that the Doppler spread B_D of the channel is much less than the signal bandwidth B_s.

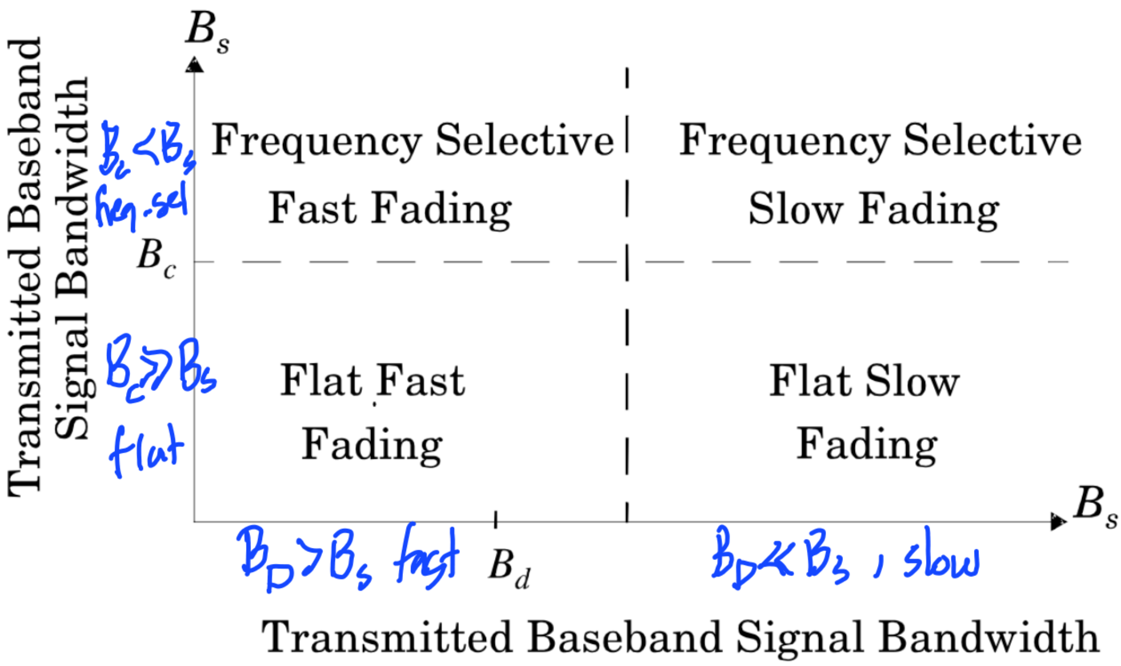
So, $T_c \gg T_s$ and $B_D \ll B_s$

Time domain



Frequency domain





flat fading : $\sigma_\tau \ll T_s$ & $B_c \gg B_s$

freq. selective fading: $\sigma_\tau > T_s$ & $B_c < B_s$

fast fading : $T_c < T_s$ & $B_D > B_s$

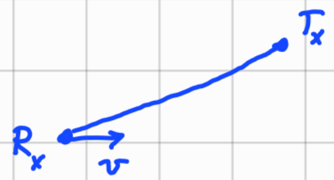
slow fading : $T_c \gg T_s$ & $B_D \ll B_s$

Example 5.1

Consider a transmitter which radiates a sinusoidal carrier frequency of 1850 MHz. For a vehicle moving 60 mph, compute the received carrier frequency if the mobile is moving (a) directly toward the transmitter, (b) directly away from the transmitter, and (c) in a direction which is perpendicular to the direction of arrival of the transmitted signal.

$v = 60$ miles/hr $\Rightarrow$ $v = \dfrac{60 \times 1.609}{60 \times 60} = 26.82$ Km/s, $v = 26.82$ m/s

$$f_d = \frac{v}{\lambda}\cos\theta = \frac{26.82}{3\times 10^8}(1850\times 10^6) = 165.39\cos\theta$$



(a) $\theta = 0$, $f_d = 165.39\cos 0 = 165.39$

$$f = f_c + f_d = 1850\times 10^6 + 165.39 = 1850.000165 \times 10^6 \text{ Hz}$$

(b) $\theta = 180$, $f_d = 165.39\cos 180 = -165.39$

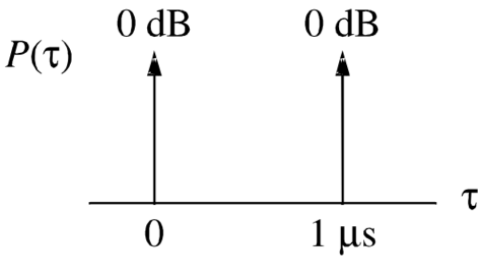
$$f = 1850\times 10^6 - 165.39 = 1849.999835 \times 10^6 \text{ Hz}$$

(c) $\theta = 90$, $f_d = 165.39\cos 90 = 0$

$f = 1850\times 10^6$ Hz, the received freq. did not change since $f_d = 0$

Example 5.4

Compute the RMS delay spread for the following power delay profile:



Since $10\log_{10} x = 0\,dB \Rightarrow x = 10^0 = 1\,W$

$$\bar{\tau} = \frac{\sum_k P(\tau_k)\,\tau_k}{\sum_k P(\tau_k)} = \frac{(1W)(0\mu s) + (1W)(1\mu s)}{1W + 1W} = 0.5 \times 10^{-6}\,s$$

$$\overline{\tau^2} = \frac{\sum_k P(\tau_k)\,\tau_k^2}{\sum_k P(\tau_k)} = \frac{(1)(0) + (1)(1\mu)^2}{1+1} = 0.5 \times 10^{-12}\,s^2$$

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2} = \left[0.5 \times 10^{-12} - (0.5 \times 10^{-6})^2\right]^{\frac{1}{2}}$$

$$= \sqrt{0.25} \times 10^{-6}$$

$$\sigma_\tau = 0.5\,\mu s$$

Example 5.5

Calculate the mean excess delay, rms delay spread, and the maximum excess delay (10 dB) for the multipath profile given in the figure below. Estimate the 50% coherence bandwidth of the channel.

1W := 0 dB

0.1 W := -10 dB

0.01 W := -20 dB

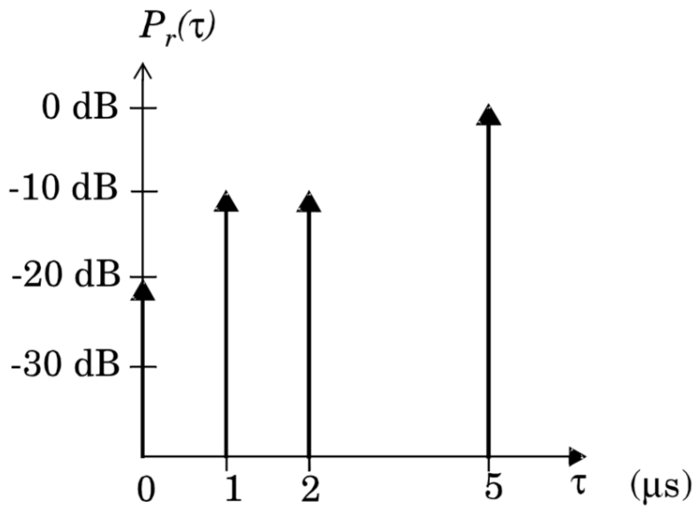


Figure E5.5

$$\bar{\tau} = \frac{\sum_k P(\tau_k)\,\tau_k}{\sum_k P(\tau_k)} = \frac{(0.01)(0) + (0.1)(1\mu) + (0.1)(2\mu) + (1)(5\mu)}{0.01 + 0.1 + 0.1 + 1}$$

$$= \frac{5.3 \times 10^{-6}}{1.21} = 4.38 \times 10^{-6}\ s$$

$$\overline{\tau^2} = \frac{\sum_k P(\tau_k)\,\tau_k^2}{\sum_k P(\tau_k)} = \frac{(0.01)(0) + (0.1)(1\mu)^2 + (0.1)(2\mu)^2 + (1)(5\mu)^2}{0.01 + 0.1 + 0.1 + 1}$$

$$= \frac{25.5 \times 10^{-12}}{1.21} = 21.674 \times 10^{-12}\ s^2$$

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2} = \left[21.074 \times 10^{-12} - (4.38 \times 10^{-6})^2 \right]^{\frac{1}{2}}$$

$$= 1.375\ \mu s$$

The max. excess delay (10 dB) : here $\tau_0 = 0$, the τ_x will be up to 10 dB less than the max $P_r(\tau)$ which is here $0\,dB - 10\,dB = -10\,dB$, this is from τ_0 to $\tau = 5\mu s$ since $P_r(\tau = 5\mu s)$ is still more than $-10\,dB$.

So max ecess delay (10 dB) here is

$$5\mu s - \tau_0 = 5\mu s .$$

The 50% coherence bandwidth is :

$$B_c \simeq \frac{1}{5\sigma_\tau} = \frac{1}{5(1.375\mu)} = 145.5\ KHz$$

Multipath Fading Channels:

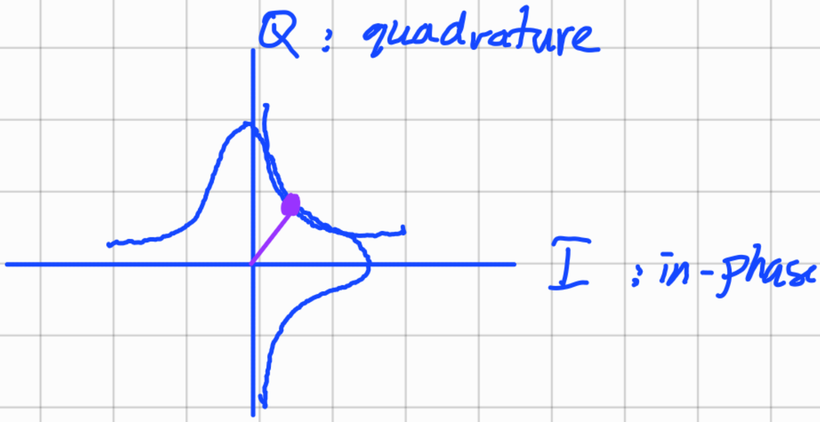
Since the channel can be as follows:

$$h(t,\tau) = \sum_{i=0}^{N-1} a_i(t,\tau)\, e^{j\theta_i(t,\tau)}\, \delta\left(\tau - \tau_i(t)\right)$$

then the received signal due to multipath components will have an energy:

$$E(t) = E_0 \sum_{i=0}^{N-1} E_i\, e^{j\phi_i} \quad , \quad E_0 : \text{local avg. energy}.$$

$$= E_0 \left[\underbrace{\sum_{i=0}^{N-1} E_i \cos\phi_i}_{\substack{\text{the sum is} \\ \text{a R.V. Gaussian} \\ \text{zero mean} \\ \sigma^2 \text{ var.}}} + j \underbrace{\sum_{i=0}^{N-1} E_i \sin\phi_i}_{\substack{\text{the sum is} \\ \text{R.V. Gaussian} \\ \text{zero mean} \\ \text{same var } \sigma^2}} \right]$$



then the magnitude of the energy $E(t)$

$$E(t) = E_c(t) + j E_s(t)$$

$$|E(t)| = \sqrt{E_c^2(t) + E_s^2(t)} := r(t)$$

and the magnitude of the energy is also the envelope of the received signal's energy.

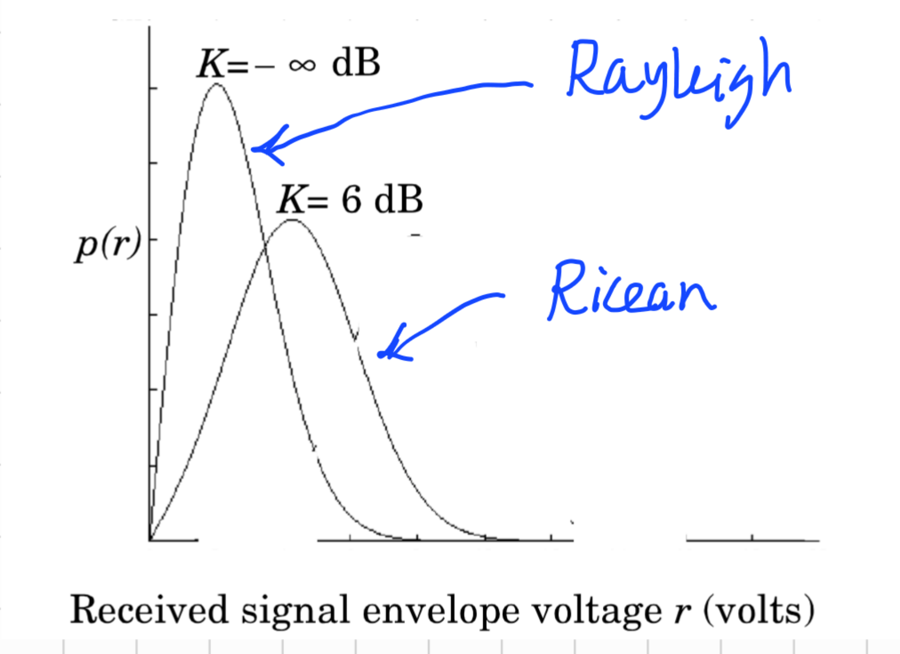
The envelope or magnitude of two quadrature Gaussian distributed signals both with zero mean and same variance follow a random distribution called Rayleigh distribution. This distribution describes the envelope of a flat fading received signal. However, for such distribution, if we had a signal component that is dominant (non-fading), where it propagated in a line-of-sight path, then the received signal will have a distribution that is superimposed on the dominant component. In this case, the distribution of the envelope of the received signal will no longer follow Rayleigh distribution, it will become Ricean distribution. That is,

$$E(t) = \sum_{i=0}^{N-1} E_i \, e^{j\phi_i}$$

$$= E_{dom.} + E_0 \sum_{i=1}^{N-1} E_i \, e^{j\phi_i}$$

$$= E_{dom.} + E_0 \left[\sum_{i=1}^{N-1} E_i \cos\phi_i + j \sum_{i=1}^{N-1} E_i \sin\phi_i \right]$$

So $E_{dominant}$ will act as a dc component to $E(t)$, and as $E_{dominant}$ becomes too weak, then the Ricean distribution becomes Rayleigh.



Rayleigh fading distribution: In multipath fading the random received signal envelope $r(t)$ has a Rayleigh distribution in the absence of a LOS component.

- The probability density function (PDF) is :

$$P\{\tilde{r}=r\} = \begin{cases} \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} , & r \geq 0 \\ 0 , & r < 0 \end{cases}$$

- The cumulative distribution function (CDF) is :

$$P\{\tilde{r} \leq r\} = \int_0^r P\{\tilde{r}=r\} \, dr$$

$$P\{\tilde{r} \leq r\} = 1 - e^{-\frac{r^2}{2\sigma^2}}$$

- The mean value, denoted by r_{mean} :

$$r_{mean} = E[\tilde{r}] = \int_0^{\infty} r \, P\{\tilde{r}=r\} \, dr$$

$$E[\tilde{r}] = \sigma \sqrt{\frac{\pi}{2}} = 1.2533 \, \sigma$$

- The variance, denoted by σ_r^2 :

$$\sigma_r^2 = E[\tilde{r}^2] - E^2[\tilde{r}]$$

$$= \int_0^\infty r^2 P\{\tilde{r}=r\}dr - \left(\sigma\sqrt{\frac{\pi}{2}}\right)^2$$

$$= 2\sigma^2 - \sigma^2\frac{\pi}{2}$$

$$\sigma_r^2 = \sigma^2\left(2-\frac{\pi}{2}\right) = 0.4292\,\sigma^2$$

where σ^2 is the variance of the original Gaussian random variables in

$$E(t) = E_0\left[\underbrace{\sum_N E_i\cos\phi_i}_{\mathcal{N}(0,\sigma^2)} + j\underbrace{\sum_N E_i\sin\phi_i}_{\mathcal{N}(0,\sigma^2)}\right]$$

σ: standard deviation of the received signal before envelope detection.

- The median, denoted by r_{median} :

$$P\{\tilde{r} \le r\} = 0.5 = \int_0^{r_{median}} P\{\tilde{r}=r\}dr$$

$$r_{median} = 1.177\,\sigma$$

however, the median in Gaussian is the mean.

The difference between r_{mean} and r_{median} in dB

is : $20\log\dfrac{r_{mean}}{r_{median}} = 20\log\dfrac{1.2533\sigma}{1.177\sigma} \simeq 0.55\,dB.$

For $E(t) = E_c(t) + j\,E_s(t)$, the variance

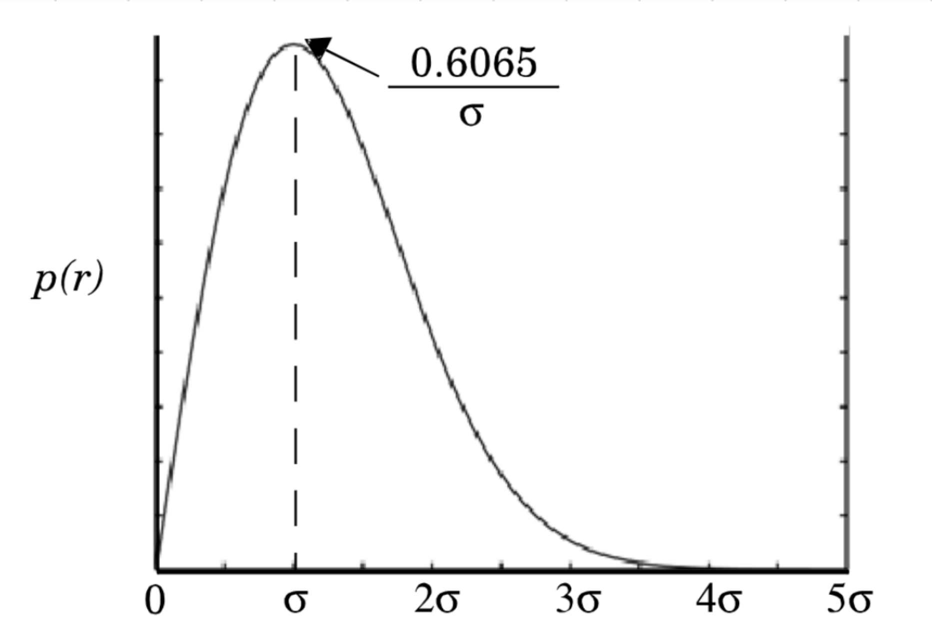
of the original Gaussian R.V.(s) is $\sigma^2 = \dfrac{E_0^2}{2}$.

_ The mode (i.e. the value that appears most often

in a set of random data set) is σ, and

it has a PDF value of $\dfrac{0.6065}{\sigma}$, as illustrated

below :



Rayleigh PDF.

Ricean fading distribution: In multipath fading the random received signal envelope $r(t)$ has a Ricean distribution in the presence of a LOS component.

- The Ricean PDF is

$$P\{\tilde{r}=r\} = \begin{cases} \dfrac{r}{\sigma^2} \exp\left\{-\dfrac{(r^2+A^2)}{2\sigma^2}\right\} I_0\left(\dfrac{Ar}{\sigma^2}\right), & A \geq 0 \;\&\; r \geq 0 \\ 0, & r < 0 \end{cases}$$

where A : is the peak amplitude of the dominant signal.

$I_0(\cdot)$: is the modified Bessel function of the first kind and zero order.

K : is the ratio between the dominant signal power A, and the variance of the multipath as

$$K = \frac{A^2}{2\sigma^2}, \quad \text{or in dB as:} \quad K(dB) = 10 \log \frac{A^2}{2\sigma^2} \; dB$$

where $2\sigma^2$ is the power received from all the other multipath components

Notice : when $K=0$, $K=-\infty$ (in dB), PDF is Rayligh.

- The envelope of the received signal and also the magnitude of the energy of the received signal is :

$$r(t) := |E(t)| = \sqrt{E_c^2(t) + E_s^2(t)},$$

$$E(t) = E_c(t) + j E_s(t)$$

Let dP_r denote the differential received power, for a differential angle $d\alpha$ of the angle α :

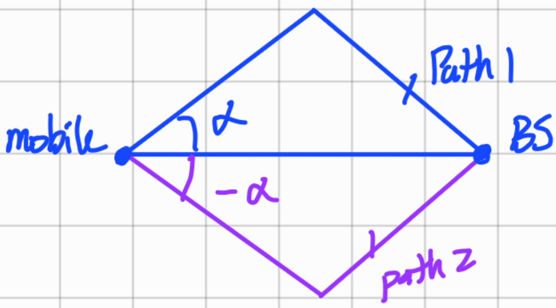
$$dP_r = A\, G(\alpha)\, P\{\alpha\}\, d\alpha \qquad \sim (1)$$

where A denotes the average received power.

$G(\alpha)$: gain of the mobile antenna as a function of the angle of arrival α

$P\{\alpha\}$: the PDF of the angle of arrival α over the interval $[0, 2\pi]$

- If the signal is a continuous signal of frequency f_c, then the instantaneous frequency of the received signal component arriving at an angle α is :



$$f(\alpha) = \frac{v}{\lambda}\cos\alpha + f_c = f_m \cos\alpha + f_c$$

$f(\alpha)$ is an even function of α, $f(\alpha) = f(-\alpha)$

- For the received signal, let $S(f)$ denote its power spectrum, then the differential received power with frequency f is:

$$dP_r = S(f)\,|df| \quad \sim (2)$$

By equating the differential received power of angle α with the differential received power with frequency f [equ (1) & equ (2)]:

$$S(f)\,|df| = 2\,A\,G(\alpha)\,P\{\alpha\}\,|d\alpha|$$

since $f(\alpha) = f_m \cos\alpha + f_c \Rightarrow df = f_m(-\sin\alpha)\,d\alpha$

$$\text{so } |df| = f_m\,|\sin\alpha|\,|d\alpha|$$

also $f(\alpha) - f_c = f_m \cos\alpha \Rightarrow \cos\alpha = \dfrac{f(\alpha) - f_c}{f_m}$

$$\alpha = \cos^{-1}\left[\frac{f(\alpha) - f_c}{f_m}\right]$$

also since $\sin^2\alpha + \cos^2\alpha = 1 \Rightarrow \sin\alpha = \sqrt{1 - \left(\dfrac{f(\alpha) - f_c}{f_m}\right)^2}$

since $|df| = f_m\,|\sin\alpha|\,|d\alpha| \Rightarrow \dfrac{|d\alpha|}{|df|} = \dfrac{1}{f_m\,|\sin\alpha|}$, and

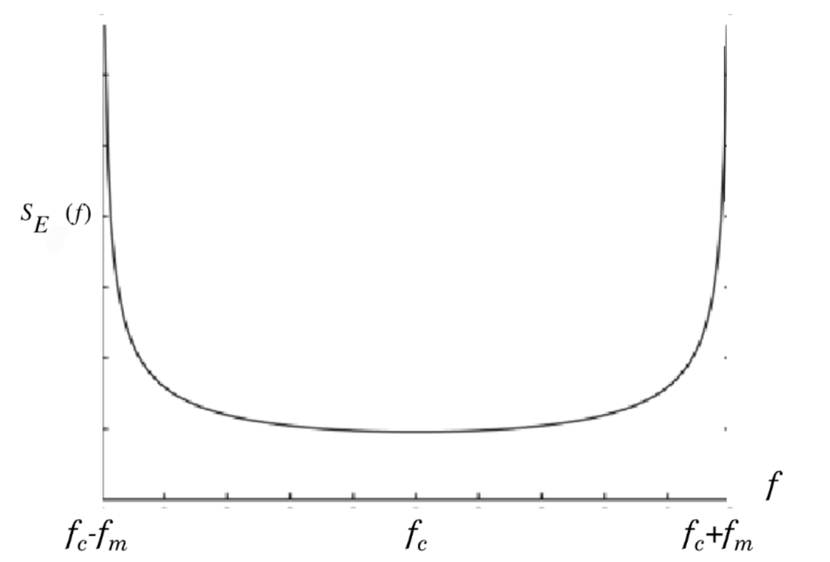
$$S(f) = \begin{cases} \dfrac{2\,A\,G(\alpha)\,P\{\alpha\}}{f_m\sqrt{1 - \left(\dfrac{f(\alpha) - f_c}{f_m}\right)^2}}, & |f - f_c| < f_m \\ 0, & |f - f_c| > f_m \end{cases} \qquad (\text{equ. } *)$$

and $S(f)$ at $|f-f_c| = f_m$ (i.e. $f = f_c \pm f_m$) is infinite,

this is when $\alpha = 0$ or 180, there is no problem with this

since the PDF of α is continuous and $P\{\alpha\}$ for exactly one

value is zero (i.e. $P\{\alpha=0\} = P\{\alpha=\infty\} = P\{\alpha=\alpha_i\} = 0$)



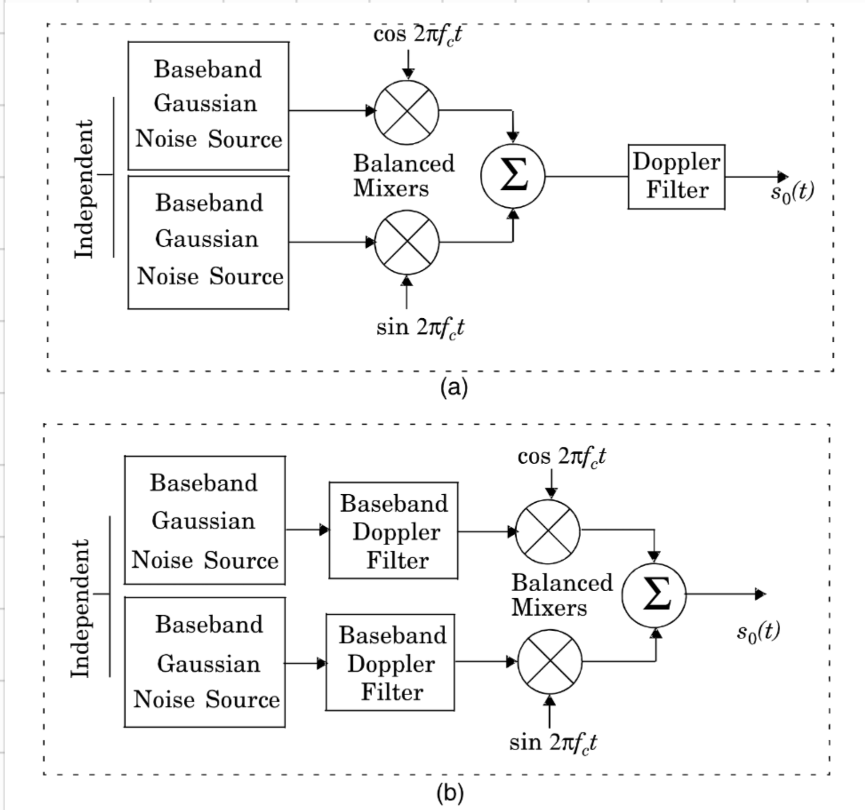
Example: For a mobile antenna of $G(\alpha) = 1.5$ and a uniform probability distribution function $P\{\alpha\} = \frac{1}{2\pi}$ over $[0, 2\pi]$, and average received power $A = 1$.

The output spectrum is:

$$S(f) = \frac{2\,A\,G(\alpha)\,P\{\alpha\}}{f_m\sqrt{1-\left(\frac{f(\alpha)-f_c}{f_m}\right)^2}} = \frac{2(1)(1.5)\frac{1}{2\pi}}{f_m\sqrt{1-\left(\frac{f-f_c}{f_m}\right)^2}}$$

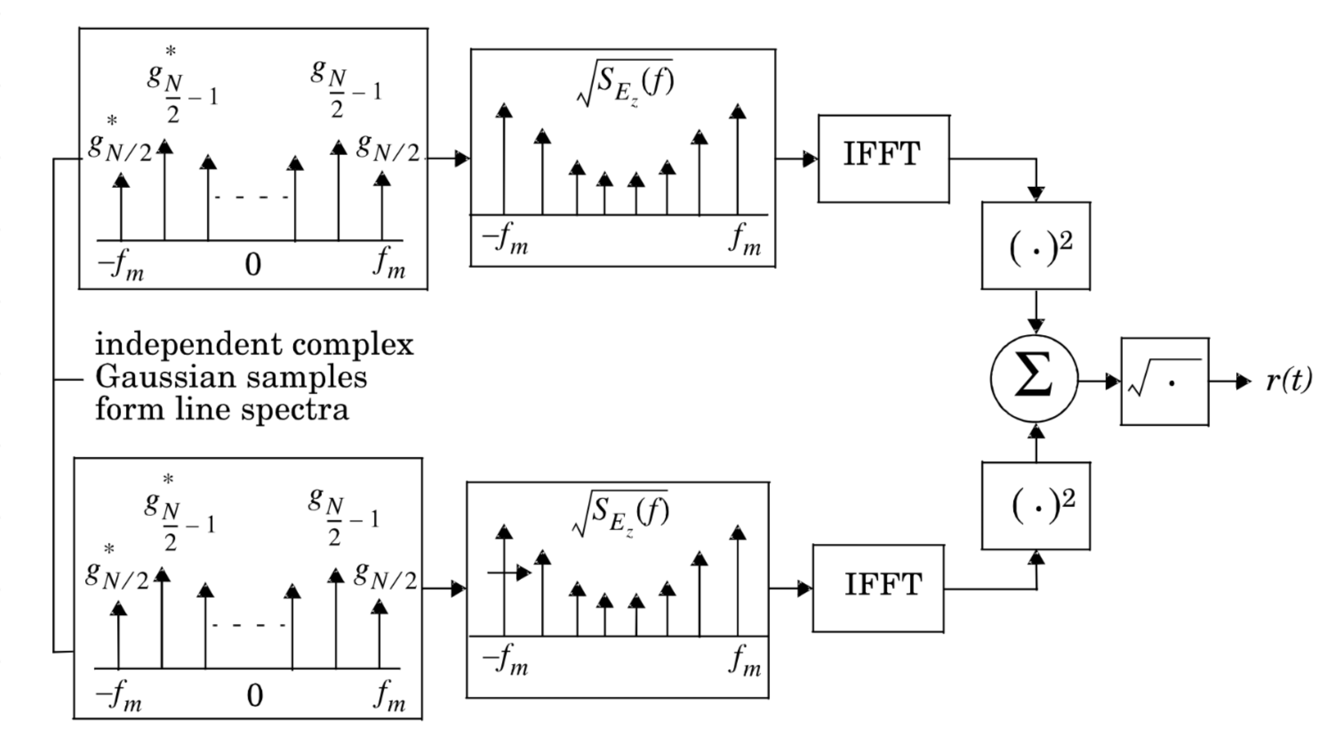
$$S(f) = \begin{cases} \dfrac{1.5}{\pi f_m\sqrt{1-\left(\frac{f-f_c}{f_m}\right)^2}}, & |f-f_c| < f_m \\ 0 & |f-f_c| > f_m \end{cases}$$

To simulate multipath fading close to measured data, the equation $E(t) = E_c(t) + jE_s(t)$, is simulated as follows:



Two Gaussian R.V.s values are obtained, then the values are shifted into the RF range by $\cos 2\pi f_c t$ and $\sin 2\pi f_c t$ to give two R.V.s that are in-phase and quadrature, then are summed. To shape the random signal in frequency domain according to the Doppler fading, an IFFT of the power spectral density $S(f)$ (as defined in equ.*) is transformed into a time domain of the Doppler fading to filter the signal, either after the summation as in Fig. (a) or before the modulation as in Fig. (b).

To simulate Rayleigh fading in frequency domain, the below steps are used:



1. The number of frequency domain points (N) is specified, usually a power of 2. Also, the max. Doppler frequency shift f_m.

2. Compute the frequency spacing between adjacent spectral points, $\Delta f = \frac{2f_m}{N-1}$

3. Generate $\frac{N}{2}$ complex Gaussian R.V.s for the $\frac{N}{2}$ positive frequency components as noise for each branche, where a complex Gaussian $g = a + jb$, where a & b are real Gaussian R.V.s.

4. To construct the negative frequency components of the noise, conjugate the positive frequency noise values and assign these at the negative frequency range.

5. Multiply the in-phase and quadrature noise values by the fading spectrum $\sqrt{S_E(f)}$

6. To convert to time domain, an IFFT is performed on the frequency domain signal for the in-phase signal and quadrature. The result is an N-point time series real signal on both branches.

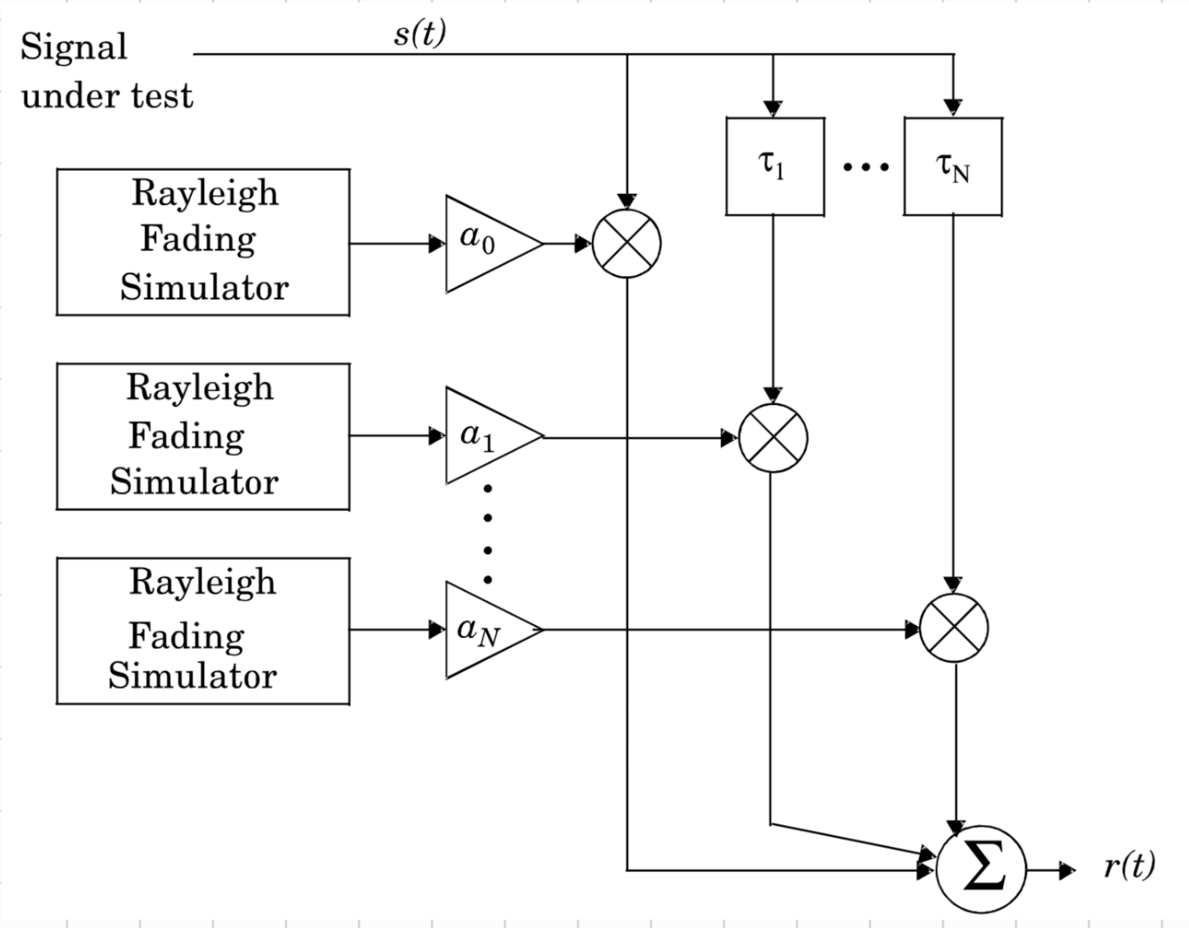
7. For each point in time, add the square of the amplitudes of the in-phase and quadrature then take the square root, this will result in N-point time series signal, each with the value of the envelope at such instant.

This N-point time series signal is Rayleigh distributed, as in $E(t) = E_o \sum_{i=0}^{N-1} E_i \, e^{j\phi_i}$, E_o : local avg. energy

$$E(t) = E_o \left[\sum_{i=0}^{N-1} E_i \cos\phi_i + j \sum_{i=0}^{N-1} E_i \sin\phi_i \right]$$

So to study the effect of flat fading on a signal, the transmitted signal is multiplied by the signal simulated through the Rayleigh simulator.

For more than one propagation path, several Rayleigh fading simulators may be used with varying gains and different time delays as below. Flat fading or frequency selective fading channels may be simulated by setting the appropriate gains and time delays (i.e. flat $B_c \gg B_s \Rightarrow \sigma_\tau \ll T_s$, freq. selective $B_c < B_s \Rightarrow \sigma_\tau > T_s$).



- **Level crossing rate (LCR)**: is the average rate at which the normalized Rayleigh fading envelope crosses a specified level in a positive-going (up-crossing) direction. The Rayleigh envelope is normalized to the local rms signal level.

For a specified level R and an rms R_{rms} for the amplitude of the envelope (signal value), the threshold level is $\rho = \frac{R}{R_{rms}}$. Then the average number of level crossings (up-crossing) per second is

$$N_R = \sqrt{2\pi}\, f_m\, \rho\, e^{-\rho^2} .$$

The max. rate of crossing occurs at threshold $\rho = \frac{1}{\sqrt{2}}$. In dB, this is $20\log\frac{1}{\sqrt{2}} = -3\,dB$, which is $3\,dB$ below the rms value, $\rho = \frac{R}{R_{rms}}$.

- Shallow fades happen frequently, deep fades only happen occasionally.

Example 5.7

For a Rayleigh fading signal, compute the positive-going level crossing rate for $\rho = 1$, when the maximum Doppler frequency (f_m) is 20 Hz. What is the maximum velocity of the mobile for this Doppler frequency if the carrier frequency is 900 MHz?

$$N_R = \sqrt{2\pi}\, f_m \rho e^{-\rho^2} = \sqrt{2\pi}\,(20)(1)\,e^{-1}$$

$$N_R = 18.44 \text{ crossings/sec.}$$

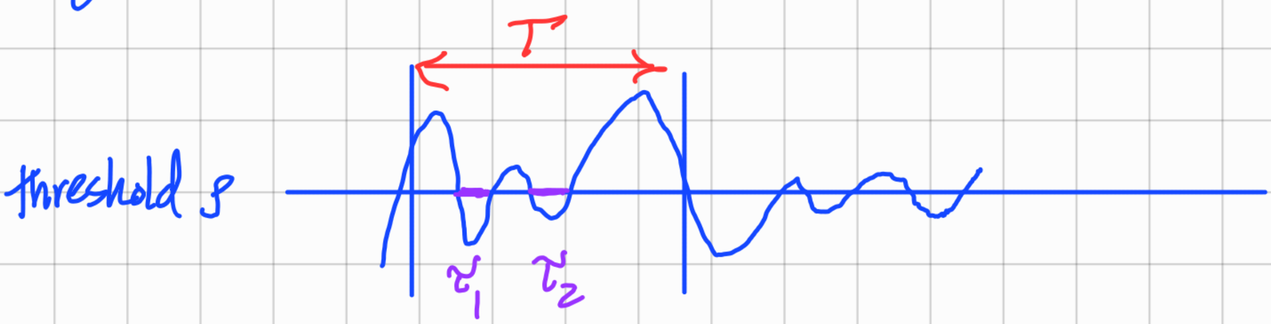
$$f_m = 20 = \frac{v}{\lambda} = \frac{v}{3\times10^8}\,900\times10^6$$

$$v = 6.66 \text{ m/s.}$$

- **Average fade duration $\bar{\tau}$**: is the average period of time for which the received signal is below a specified level R.

For Rayleigh fading, $\bar{\tau} = \frac{1}{N_R} P\{r \leq R\}$

where $P\{r \leq R\}$ is the probability that the received signal r is less than R.



$$P\{r \leq R\} = \frac{1}{T} \sum_i \tau_i$$

where τ_i : duration of the i^{th} fade.

T: the observation interval.

From Rayleigh distribution: $P\{r \leq R\} = 1 - e^{-\rho^2}$

Then, the average fade duration $\bar{\tau}$ is:

$$\bar{\tau} = \frac{1}{N_R} P\{r \leq R\} = \frac{1 - e^{-\rho^2}}{\sqrt{2\pi} f_m \rho e^{-\rho^2}}$$

$$\Rightarrow \bar{\tau} = \frac{e^{\rho^2} - 1}{\sqrt{2\pi} f_m \rho}$$

- The average fade duration $\bar{\tau}$ helps in determining the number of bits in error, since the error is determined by how often the signal is below a specified level on average, and how long it remains below such level on average.

Example 5.8
Find the average fade duration for threshold levels ρ = 0.01, ρ = 0.1, and ρ = 1, when the Doppler frequency is 200 Hz.

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\sqrt{2\pi} f_m \rho}$$

for $\rho = 0.01$, $\bar{\tau} = 19.948\,\mu s$

for $\rho = 0.1$, $\bar{\tau} = 200.471\,\mu s$

for $\rho = 1$, $\bar{\tau} = 3427.476\,\mu s$

Example 5.9

Find the average fade duration for a threshold level of $\rho = 0.707$ when the Doppler frequency is 20 Hz. For a binary digital modulation with bit duration of 50 bps, is the Rayleigh fading slow or fast? What is the average number of bit errors per second for the given data rate. Assume that a bit error occurs whenever any portion of a bit encounters a fade for which $\rho < 0.1$.

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\sqrt{2\pi} f_m \rho} = 18.30 \text{ ms}$$

For bit rate $B_R = 50$ bps, the bit period $T_B = \frac{1}{B_R} = 20$ ms

- Is it fast or slow fading, here $T_B > \bar{\tau}$

T_B ———— bit duration

$\bar{\tau}$ —— fade duration

the fading will be fast fading

- For $\rho = 0.1$, $\bar{\tau} = 2$ ms, then

$$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2} = \sqrt{2\pi}(20)(.1) e^{-0.1^2}$$

$$= 4.963 \text{ crossings/sec.} \simeq 5$$

We have about 5 errors per second, the bit duration is 20 ms, then we will have a BER = 0.1

5 errors $\rightarrow$ 1 sec.

? $\leftarrow$ 20 msec.

- Coherence distance D_c : is the separation distance in space over where a fading channel appears to be unchanged.

- The coherence distance is important in designing a wireless receiver that uses spatial diversity.

- In a static channel and an omnidirectional Rayleigh propagation, the coherence time and the coherence distance of a mobile receiver are related through the speed of the mobile :

$$D_c = v T_c$$

- If we use the approximation of T_c as

$T_c \simeq \dfrac{9}{16\pi f_m}$ & $D_c = v T_c$, then $D_c \simeq \dfrac{9\lambda}{16\pi}$

- A convenient coherence distance is the value of D_c is when the equation $\rho(D_c) = 0.5$, where $\rho = \dfrac{R}{R_{rms}}$

5.1 Determine the maximum and minimum spectral frequencies received from a stationary GSM transmitter that has a center frequency of exactly 1950.000000 MHz, assuming that the receiver is traveling at speeds of: (a) 1 km/hr; (b) 5 km/hr; (c) 100 km/hr; and (d) 1000 km/hr.

(a) $v = 1\,\text{km/hr} = \dfrac{10^3}{60 \times 60} = 0.278\,\text{m/s}$

$f_c = 1950 \times 10^6\,\text{Hz}$

$f_m = \dfrac{v}{\lambda} = \dfrac{0.278}{3 \times 10^8}\, 1950 \times 10^6 = 1.807\,\text{Hz}$

Doppler spread

$f_c - f_m \qquad f_c \qquad f_c + f_m$

$f_c - f_m = 1950 \times 10^6 - 1.807 = 1949999998.18\,\text{Hz}$

$f_c + f_m = 1950 \times 10^6 + 1.807 = 1950000001.807\,\text{Hz}$

5.6 If a particular modulation provides suitable BER performance whenever $\sigma_\tau / T_s \leq 0.1$ determine the smallest symbol period T_s (and thus the greatest symbol rate) that may be sent through RF channels shown in Figure P5.6, without using an equalizer.

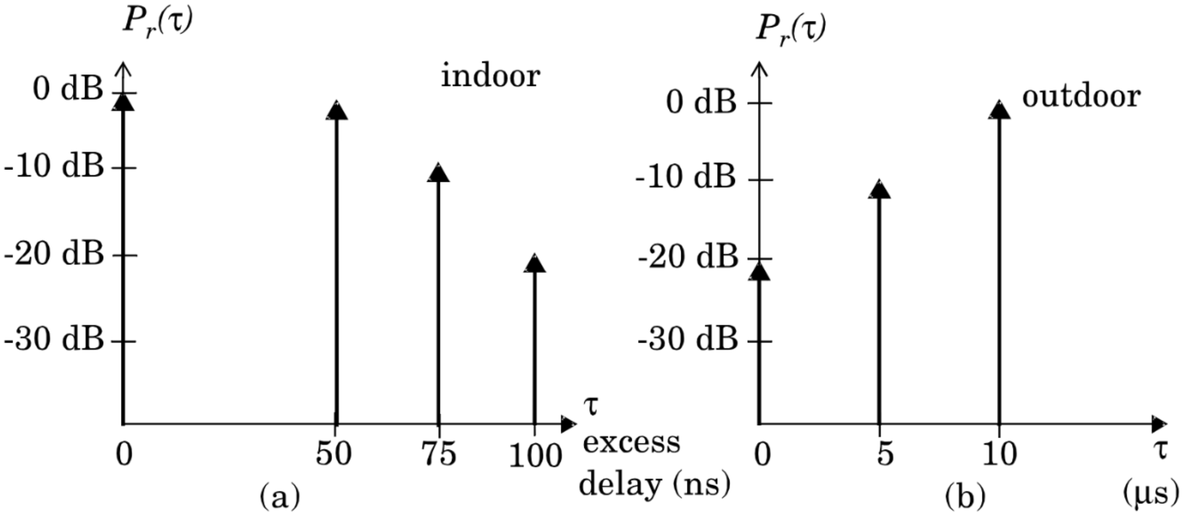


Figure P5.6 Two channel responses for Problem 5.6.

(a) $10 \log P_W = P_{dB} \Rightarrow P_W = 10^{P_{dB}/10}$,

$0\,dB \Rightarrow 10^0 = 1$, $-10\,dB \Rightarrow 10^{-1} = 0.1$, $-20\,dB \Rightarrow 10^{-2} = 0.01$

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2}$$

$$\bar{\tau} = \frac{(1)(0) + (1)(50\times10^{-9}) + (0.1)(75\times10^{-9}) + 0.01\,(100\times10^{-9})}{1+1+0.1+0.01} = 27.73\times10^{-9}\,s$$

$$\overline{\tau^2} = \frac{(1)(0) + (1)(50\times10^{-9})^2 + (0.1)(75\times10^{-9})^2 + 0.01(100\times10^{-9})^2}{1+1+0.1+0.01} = 1498.82\times10^{-18}\,s^2$$

$$\sigma_\tau = \sqrt{1498.82\times10^{-18} - (27.73\times10^{-9})^2} = 27.02\times10^{-9}\,s$$

Suitable BER when : $\sigma_\tau \leq 0.1\, T_s$, $T_s \geq 10\,\sigma_\tau$

$T_s \geq 270.2\times10^{-9}\,s$

So $270.2\times10^{-9}\,s$ is the smallest T_s .

5.7 If a baseband binary message with a bit rate $R_b = 100$ kbps is modulated by an RF carrier using BPSK, answer the following:

(a) Find the range of values required for the rms delay spread of the channel such that the received signal is a flat-fading signal.

(b) If the modulation carrier frequency is 5.8 GHz, what is the coherence time of the channel, assuming a vehicle speed of 30 miles per hour?

(c) For your answer in (b), is the channel "fast" or "slow" fading?

(d) Given your answer in (b), how many bits are sent while the channel appears "static"?

(a) $T_s = \frac{1}{R_b} = 10^{-5}$ s

σ_τ

if $\sigma_\tau \leq \frac{T_s}{10}$, then the channel is flat fading

so $\sigma_\tau \leq \frac{1}{10} 10^{-5} \Rightarrow \sigma_\tau \leq 10^{-4}$ sec.

(b) $f_c = 5.8 \times 10^9$ Hz, $v = 30$ mile/hr $= 13.41$ m/s

$$f_m = \frac{v}{\lambda} = \frac{v}{c} f_c = \frac{13.41 \times 5.8 \times 10^9}{3 \times 10^8} = 259.26 \text{ Hz}$$

$$T_c = 0.423 / f_m = 1.63 \text{ ms}$$

(c) $T_s = 10^{-5}$ s, $T_c = 1.63$ ms

T_c

T_s

$T_c > T_s \Rightarrow$ slow fading

(d) channel is constant within $T_c = 1.63$ ms, bitrate $R_b = 10^5$ bps

10^5 bits — 1 sec

? ← 1.63×10^{-3} sec $\Rightarrow$ Or simply $T_c R_b =$ 163 bits within a T_c

5.8 For the power delay profiles in Figure P5.6, estimate the 90% correlation and 50% correlation coherence bandwidths.

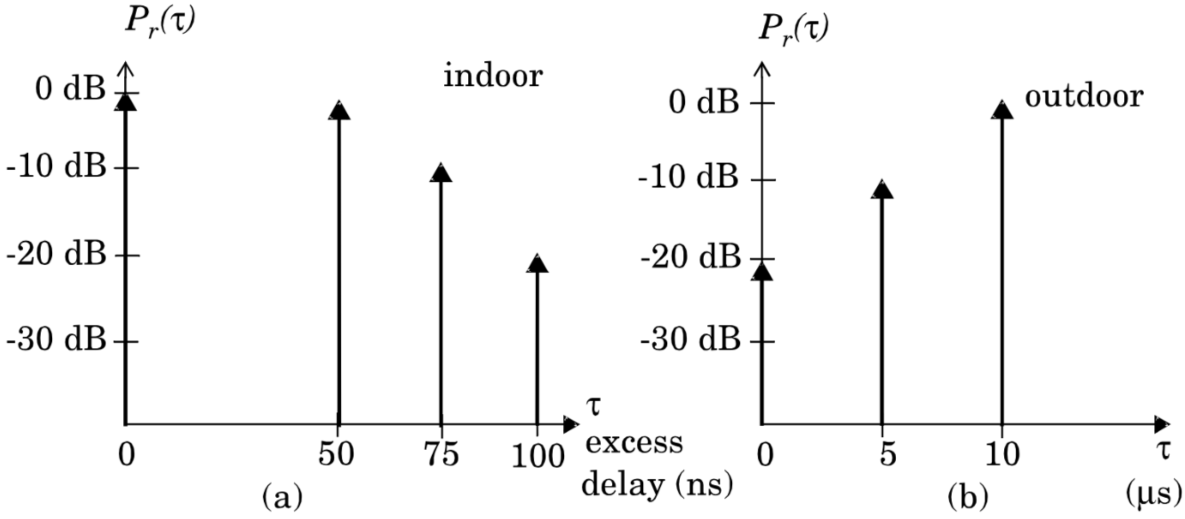


Figure P5.6 Two channel responses for Problem 5.6.

(a) From P5.6, $\sigma_\tau = 27.02 \times 10^{-9}$ s

For 90% correlation:

$$B_c = \frac{1}{50\sigma_\tau} = 740.2 \times 10^3 \text{ Hz}$$

For 50% correlation:

$$B_c = \frac{1}{5\sigma_\tau} = 7402 \times 10^3 \text{ Hz}$$

5.9 Approximately how large can the rms delay spread be in order for a binary modulated signal with a bit rate of 25 kbps to operate without an equalizer? What about an 8-PSK system with a bit rate of 75 kbps?

(a) $R_b = 25$ kbps $\Rightarrow T_s = \frac{1}{R_b} = 4 \times 10^{-5}$ sec

without an equalizer $\Rightarrow$ channel has flat fading

when $T_s \geq 10\sigma_\tau$, channel is flat fading

$\Rightarrow \sigma_\tau \leq 0.1\, T_s = 0.1\,(4 \times 10^{-5})$

$\sigma_\tau \leq 4 \times 10^{-6}$ sec.

(b) for 8-PSK with $R_b = 75$ kbps

8-PSK has 3 bits per symbol.

75k bits $\rightarrow$ 1sec
3 bits $\rightarrow T_s$

$\Rightarrow T_s = \frac{3}{75k} = 4 \times 10^{-5}$

OR simply $\Rightarrow T_b = \frac{1}{R_b} = \frac{1}{75k}$, $T_s = 3T_b = \frac{3}{75k} = 4 \times 10^{-5}$

so $\sigma_\tau \leq 0.1\, T_s \Rightarrow \sigma_\tau \leq 4 \times 10^{-6}$ sec (same as above)

5.10 Given that a Rayleigh-faded mobile radio signal has a level crossing rate of $N_r = \sqrt{2\pi} f_m \rho e^{-\rho^2}$, find the value of ρ for which N_r is a maximum.

We should find $\frac{dN_r}{d\rho} = 0$ to find at what value of ρ the function N_r is max.

$$\frac{dN_r}{d\rho} = \sqrt{2\pi} f_m (1) e^{-\rho^2} + \sqrt{2\pi} f_m \rho (-2\rho) e^{-\rho^2}$$

$$0 = \sqrt{2\pi} f_m e^{-\rho^2} - 2\sqrt{2\pi} f_m \rho^2 e^{-\rho^2}$$

$$2\rho^2 = 1$$

$$\Rightarrow \quad \rho^2 = \frac{1}{2}$$

$$\rho = \sqrt{\frac{1}{2}}$$

Recall, which is when, $20 \log \frac{1}{\sqrt{2}} = -3\,dB$, that is when R is 3dB less than R_{rms}

since $\rho = \frac{R}{R_{rms}}$

5.11 Given that the probability density function of a Rayleigh distributed envelope is given by $p(r) = \frac{r}{\sigma^2}\exp\left(\frac{-r^2}{2\sigma^2}\right)$, where σ^2 is the variance, show that the cumulative distribution function is given as $p(r<R) = 1 - \exp\left(\frac{-R^2}{2\sigma^2}\right)$.

$$P\{r<R\} = \int_0^R P\{r\}\,dr$$

$$= \int_0^R \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}\,dr$$

let $x = r^2$, if $r = 0 \Rightarrow x = 0$
if $r = R \Rightarrow x = R^2$

$\Rightarrow dx = 2r\,dr \Rightarrow r\,dr = dx/2$

$$P\{r<R\} = \frac{1}{2\sigma^2}\int_0^{R^2} e^{-x/2\sigma^2}\,dx$$

$$= \frac{1}{2\sigma^2}\,\frac{1}{-1/2\sigma^2}\, e^{-x/2\sigma^2}\Big|_0^{R^2}$$

$$= e^{-x/2\sigma^2}\Big|_{R^2}^{0}$$

$$P\{r<R\} = 1 - e^{-R^2/2\sigma^2}$$

5.12 The fading characteristics of a CW carrier in an urban area are to be measured. The following assumptions are made:

(1) The mobile receiver uses a simple vertical monopole.
(2) Large-scale fading due to path loss is ignored.
(3) The mobile has no line-of-sight path to the base station.
(4) The pdf of the received signal follows a Rayleigh distribution.

(a) Derive the ratio of the desired signal level to the rms signal level that maximizes the level crossing rate. Express your answer in dB.

(b) Assuming the maximum velocity of the mobile is 50 km/hr, and the carrier frequency is 900 MHz, determine the maximum number of times the signal envelope will fade below the level found in (a) during a 1 minute test.

(c) How long, on average, will each fade in (b) last?

(a) Same as in P5.10, $\rho = \frac{1}{\sqrt{2}}$

ρ in dB is $20 \log \frac{1}{\sqrt{2}} = -3\text{dB}$,

(b) $N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}$, $f_m = \frac{v}{\lambda}$, $c = \lambda f_c$

$$= \sqrt{2\pi}\ \frac{50 \times 10^3}{60 \times 60}\ \frac{9 \times 10^8}{3 \times 10^8}\ \frac{1}{\sqrt{2}}\ e^{-0.5}$$

$N_R = \sqrt{\pi}\ 41.67\ e^{-0.5} = 44.8$ per sec

$\Rightarrow$ # of crossings per minute $\Rightarrow 44.8 \times 60 = 2688$

(c) $$\bar{\tau} = \frac{e^{\rho^2} - 1}{\sqrt{2\pi} f_m \rho} = \frac{e^{0.5} - 1}{\sqrt{2\pi}\,(41.67)\,\frac{1}{\sqrt{2}}}$$

$\bar{\tau} = 8.78 \times 10^{3}$ sec

5.13 A vehicle receives a 900 MHz transmission while traveling at a constant velocity for 10 s. The average fade duration for a signal level 10 dB below the rms level is 1 ms. How far does the vehicle travel during the 10 s interval? How many fades does the signal undergo at the rms threshold level during a 10 s interval? Assume that the local mean remains constant during travel.

(1) $f_c = 9\times10^8$ Hz , $v =$ const. for 10 sec.

$\bar{\tau} = 10^{-3}$ sec , $\rho = \frac{R}{R_{rms}} =: -10dB \Rightarrow 20\log\rho = -10$

$\rho = 10^{-0.5}$

$= 0.316$

$d = vt = 10v \Rightarrow v = 0.1d$

$\bar{\tau} = 10^{-3} = \frac{e^{\rho^2} - 1}{\sqrt{2\pi} f_m \rho} \Rightarrow f_m = \frac{e^{\rho^2} - 1}{\sqrt{2\pi}\,\rho\, 10^{-3}} = 132.57$

$f_m = \frac{v}{\lambda} = \frac{0.1d}{3\times10^8}\, 9\times10^8 = 0.3d$

$132.57 = 0.3d \Rightarrow d = 441.92$ m

(2) at $R = R_{rms} \Rightarrow \rho = 1$

$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2} = \sqrt{2\pi}\,(132.57)(1)e^{-1}$

$= 122.25$ crossings per sec.

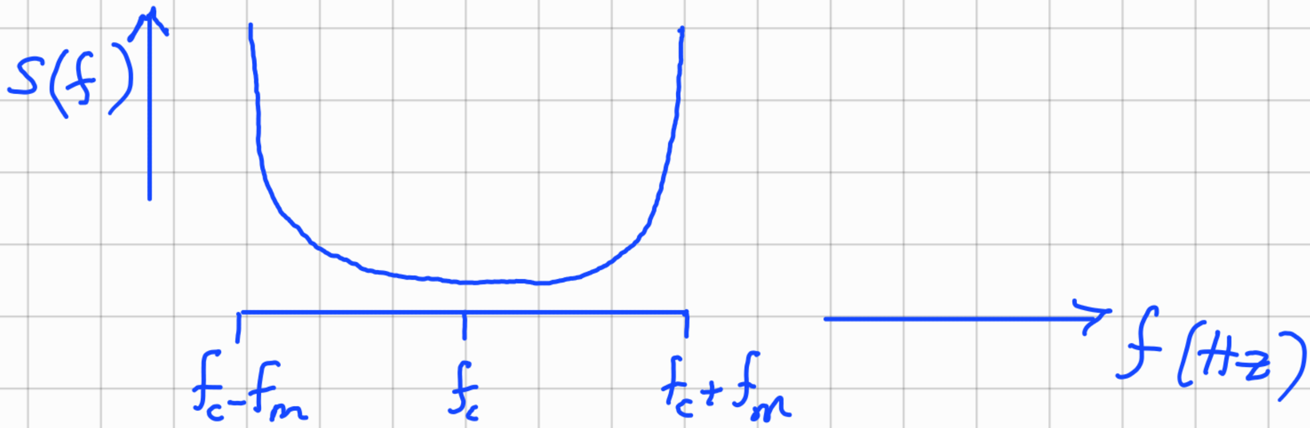
# of crossings per 10 sec $= 122.25(10) = 1222.5$

5.15 For a mobile receiver operating at frequency of 860 MHz and moving at 100 km/hr

(a) sketch the Doppler spectrum if a CW signal is transmitted and indicate the maximum and minimum frequencies

(b) calculate the level crossing rate and average fade duration if $\rho = -20$ dB.

(a) $$f_m = \frac{v}{\lambda} = \frac{v}{c} f_c = \frac{10^5}{60\times 60} \; \frac{860\times 10^6}{3\times 10^8} = 79.63 \text{ Hz}$$



860×10^6 Hz

859999920.4 Hz

860000079.6 Hz

(b) $\rho = -20\,dB \Rightarrow 20 \log \rho = -20$, $\rho = 10^{-1} = 0.1$

$$N_R = \sqrt{2\pi}\, f_m \,\rho\, e^{-\rho^2} = \sqrt{2\pi}\,(79.63)\,(0.1)\, e^{-0.01}$$

$= 19.76$ crossings per sec.

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\sqrt{2\pi}\, f_m\, \rho} = \frac{e^{0.01} - 1}{\sqrt{2\pi}\,(79.63)(0.1)}$$

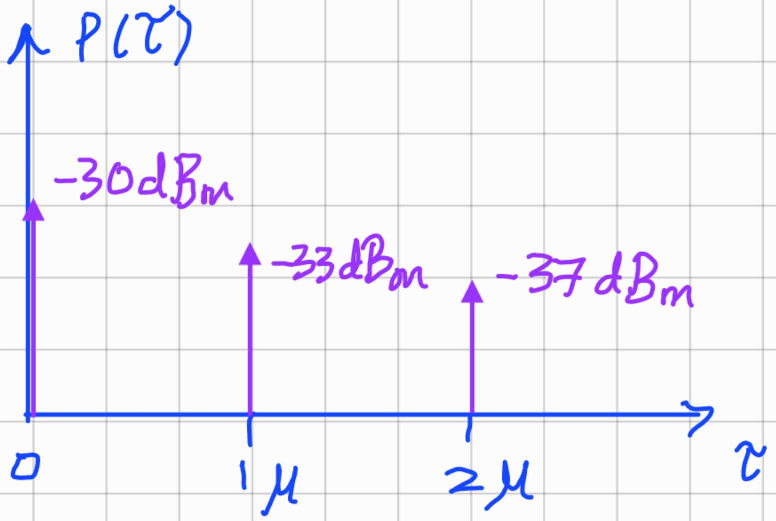
$= 0.5 \times 10^3$ sec

5.27 The local average power delay profile in a particular environment is found to be

$$P(\tau) = \sum_{n=0}^{2} \frac{10^{-6}}{n^2+1}\delta(\tau - n10^{-6})$$

(a) Sketch the Power Delay Profile of the channel in dBm.
(b) What is the local average power in dBm?
(c) What is the rms delay spread of the channel?
(d) If 256 QAM modulation having a bit rate of 2 Megabits per second is applied to the channel, will the modulation undergo flat or frequency selective fading? Explain your answer.
(e) Over what bandwidth will the channel appear to have constant gain?

(a)



$P(n=0) = \frac{10^{-6}}{1}\delta(\tau-0) = 10^{-6}\delta(\tau)$ W

$10\log 10^{-6} = -60$ dB $\Rightarrow -30$ dBm at say $\tau_0 = 0$

$P(n=1) = \frac{10^{-6}}{1+1}\delta(\tau-10^{-6}) = 0.5\times10^{-6}\delta(\tau-10^{-6})$ W

$10\log 0.5\times10^{-6} = -63$ dB $\Rightarrow -33$ dBm at $\tau_1 = 10^{-6}$

$P(n=2) = \frac{10^{-6}}{4+1}\delta(\tau-2\times10^{-6}) = 0.2\times10^{-6}\delta(\tau-2\times10^{-6})$ W

$10\log 0.2\times10^{-6} \simeq -67$ dB $\Rightarrow -37$ dBm at $\tau_2 = 2\times10^{-6}$

(b) The local avg. power $= 10^{-6} + 0.5\times10^{-6} + 0.2\times10^{-6}$

$= 1.7\times10^{-6}$ W $\Rightarrow -57.7$ dB $\Rightarrow -27.7$ dBm

(c) $$\bar{\tau} = \frac{10^{-6}(0) + 0.5\times10^{-6}(1\times10^{-6}) + 0.2\times10^{-6}(2\times10^{-6})}{10^{-6} + 0.5\times10^{-6} + 0.2\times10^{-6}}$$

$$= \frac{0.9\times10^{-12}}{1.7\times10^{-6}} = 0.53\times10^{-6}$$

$$\overline{\tau^2} = \frac{10^{-6}(0) + 0.5\times10^{-6}(1\times10^{-6})^2 + 0.2\times10^{-6}(2\times10^{-6})^2}{10^{-6} + 0.5\times10^{-6} + 0.2\times10^{-6}}$$

$$= \frac{1.3\times10^{-18}}{1.7\times10^{-6}} = 0.76\times10^{-12}$$

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2} = \sqrt{0.76\times10^{-12} - (0.53\times10^{-6})^2} \simeq 0.7\times10^{-6}\ \text{sec}$$

(d) For 256 QAM and $R_b = 2\times10^6$ bps

$256 =: 2^8$, so a symbol has 8 bits

Then $T_s = 8T_b = 8\frac{1}{R_b} = \frac{8}{2\times10^6} = 4\times10^{-6}$ sec

since $T_s > \sigma_\tau$

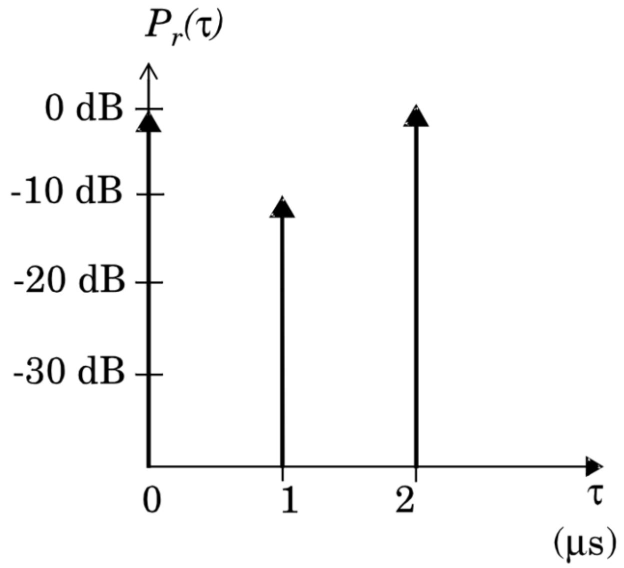
$\Rightarrow$ a flat fading channel.

(e) over a BW less or equal to coherence BW

for 90% coherence $B_c = \frac{1}{50\sigma_\tau} \simeq 28.57\times10^3$ Hz

for 50% coherence $B_c = \frac{1}{5\sigma_\tau} = 285.7\times10^3$ Hz

5.28 A local spatial average of a power delay profile measured at 900 MHz is shown in Figure P5.28.



(a) Determine the rms delay spread and mean excess delay for the channel.

(b) Determine the maximum excess delay (20 dB).

(c) If the channel is to be used with a modulation that requires an equalizer whenever the symbol duration T is less than 10 σ_τ, determine the maximum RF symbol rate that can be supported without requiring an equalizer.

(d) If a mobile traveling at 30 km/hr receives a signal through the channel, determine the time over which the channel appears stationary (or at least highly correlated).

(a) $10 \log P_W = P_{dB} \Rightarrow P_W = 10^{P_{dB}/10}$

$0\,dB \Rightarrow 10^0 = 1\,W, \quad -10\,dB \Rightarrow 10^{-1} = 0.1\,W$

$$\bar{\tau} = \frac{1(0) + 0.1(1\times10^{-6}) + 1(2\times10^{-6})}{1+0.1+1} = 1\times10^{-6}\ sec$$

$$\overline{\tau^2} = \frac{1(0) + 0.1(1\times10^{-6})^2 + 1(2\times10^{-6})^2}{1+0.1+1} = 1.95\times10^{-12}\ sec^2$$

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2} = \sqrt{1.95\times10^{-12} - (1\times10^{-6})^2} = 0.976\times10^{-6}\ sec$$

(b) max. excess delay 20dB $= 2\mu s - 0\mu s = 2\mu s$
i.e. the whole illustrated duration.

(c) $T_s \geq 10\sigma_\tau$ no equalizer, i.e. flat fading.

$$T_s \geq 10(.976\times10^{-6}) = 9.76\times10^{-6}$$

since $R_s = \dfrac{1}{T_s}$, then

$$R_s \leq \frac{1}{9.76\times10^{-6}} \simeq 102.5\times10^3 \text{ symbols per sec.}$$

so $R_s \leq 102.5\times10^3$ symbols per sec

will experience flat fading, and $R_s = 102.5\times10^3$ sym/sec

is the max. symbol rate that will experience flat fading and

won't require an equalizer.

(d) $v = 30\text{ km/hr} = \dfrac{30\times10^3}{60\times60} = 8.33\text{ m/s}$

$$T_c = \frac{0.423}{f_m} = \frac{0.423}{v/\lambda} = \frac{0.423}{8.33}\;\frac{3\times10^8}{9\times10^8} = 0.017\text{ sec}$$

during $T_c = 0.017$ sec the channel will appear stationary

5.29 A flat Rayleigh fading signal at 6 GHz is received by a mobile traveling at 80 km/hr.

(a) Determine the number of positive-going zero crossings about the rms value that occur over a 5 s interval.

(b) Determine the average duration of a fade below the rms level.

(c) Determine the average duration of a fade at a level of 20 dB below the rms value.

(a) $$f_m = \frac{v}{\lambda} = \frac{v}{c} f_c = \frac{80 \times 10^3}{60 \times 60} \frac{6 \times 10^9}{3 \times 10^8} \simeq 444.4 \text{ Hz}$$

for $R = R_{rms} \Rightarrow \rho = 1$

$$N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2} = 409.8 \text{ crossings/sec.}$$

in 5 sec $\Rightarrow$ # of crossings $= 5 N_R = 2049$

(b) $$\bar{\tau} = \frac{e^{\rho^2} - 1}{\sqrt{2\pi} f_m \rho} = \frac{e^1 - 1}{\sqrt{2\pi}(444.4)(1)} = 1.54 \times 10^{-3} \text{ sec}$$

(c) for $\rho = \frac{R}{R_{rms}}$, $\rho = -20$ dB, $20 \log \rho = -20 \Rightarrow \rho = 0.1$

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\sqrt{2\pi} f_m \rho} = \frac{e^{0.01} - 1}{\sqrt{2\pi}(444.4) 0.1} = 90.22 \times 10^{-6} \text{ sec}$$

5.30 For each of the three scenarios below, decide if the received signal is best described as undergoing fast fading, frequency selective fading, or flat fading.

(a) A binary modulation has a data rate of 500 kbps, $f_c = 1$ GHz and a typical urban radio channel is used. to provide communications to cars moving on a highway.

(b) A binary modulation has a data rate of 5 kbps, $f_c = 1$ GHz and a typical urban radio channel is used to provide communications to cars moving on a highway.

(c) A binary modulation has a data rate of 10 bps, $f_c = 1$ GHz and a typical urban radio channel is used to provide communications to cars moving on a highway.

For highways, $v = 110$ km/hr $= \dfrac{110 \times 10^3}{60 \times 60} = 30.56$ m/s

$$f_m = \frac{v}{\lambda} = \frac{v}{c} f_c = \frac{30.56}{3 \times 10^8} f_c = 10.187 \times 10^{-8} f_c$$

for $f_c = 10^9$ Hz $\Rightarrow f_m = 10.187 \times 10^{-8} \times 10^9 = 101.87$ Hz

Coherence time, $T_c = \dfrac{0.423}{f_m} = 4.15 \times 10^{-3}$ sec

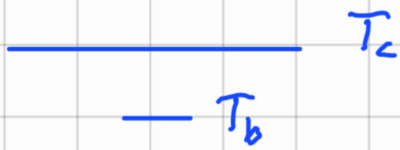
If for such urban environment $\sigma_\tau \simeq 10\,\mu s$

(a) $R_b = 5 \times 10^5$ bps, $T_b = \dfrac{1}{R_b} = 2 \times 10^{-6}$ sec

$10\mu > 2\mu \Rightarrow \sigma_\tau > T_b$ & $B_c < B_b$

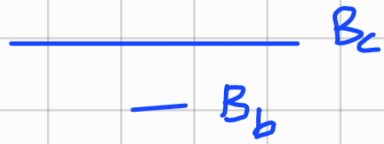
$\Rightarrow$ freq. selective fading

$2\mu < 4.15m \Rightarrow T_b < T_c \Rightarrow$ slow fading



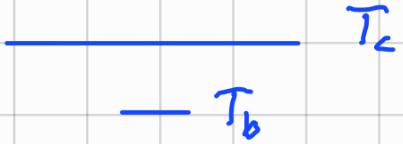
(b) $R_b = 5 \times 10^3$ bps , $T_b = \frac{1}{R_b} = 0.2 \times 10^{-3}$ sec

$10\mu < 0.2m \Rightarrow \sigma_\tau < T_b$ & $B_c > B_b$



$\Rightarrow$ flat fading

$0.2m < 4.15m \Rightarrow T_b < T_c \Rightarrow$ slow fading

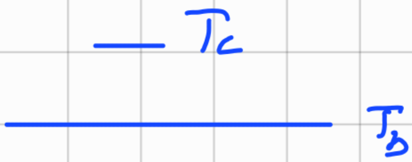


(c) $R_b = 10$ bps , $T_b = \frac{1}{R_b} = 0.1$ sec

$10\mu < 0.1 \Rightarrow \sigma_\tau < T_b$ & $B_c > B_b$

$\Rightarrow$ flat fading

$0.1 > 4.15m \Rightarrow T_b > T_c \Rightarrow$ fast fading



5.32 Based on concepts taught in this chapter, propose methods that could be used by a base station to determine the vehicular speed of a mobile user. Such methods are useful for handoff algorithms.

method 1 : The mobile velocity v is found in the Doppler shift and from the Doppler effect, the frequency will spread from $f_c - f_m$ to $f_c + f_m$ $\Rightarrow$ so the signal BW, B_s, will be $2f_m$ $\Rightarrow$ $B_s = 2f_m$ and $f_m = \frac{v}{\lambda}$

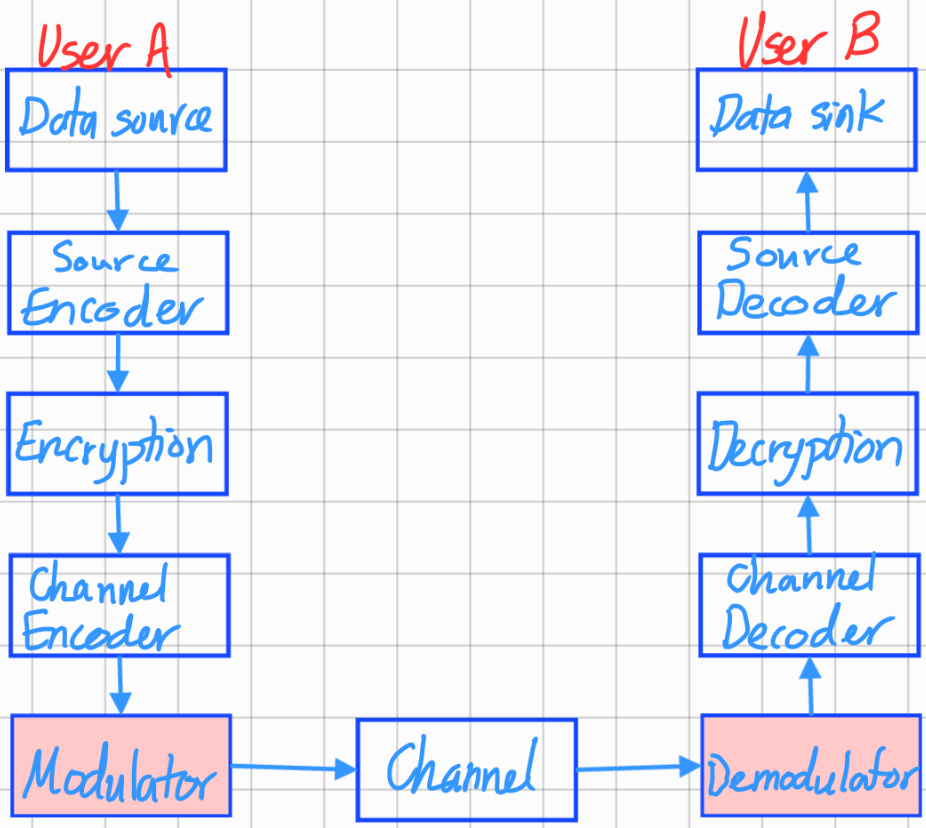
$$\Rightarrow v = \lambda f_m = \lambda \frac{B_s}{2}$$

method 2 : since mobile velocity v is found in the Doppler shift and the level crossing rate $N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}$, then by finding N_R for the received signal for a level R from the local rms value R_{rms} $\left(\rho = \frac{R}{R_{rms}}\right)$,

$$\text{as } N_R = \sqrt{2\pi} \frac{v}{\lambda} \rho e^{-\rho^2} \Rightarrow v = \frac{\lambda N_R}{\sqrt{2\pi}\, \rho} e^{\rho^2}$$

Modulation Techniques for Mobile Radio Systems

- Modulation is the process of encoding a message in a way suitable for transmission.
- Modulation translates a baseband signal (modulating signal) to a bandpass signal (modulated signal) at frequencies that are very high when compared to the baseband signal frequency.
- Modulation changes a high frequency carries signal in accordance with the change in the baseband signal.



- Demodulation is the process of extracting the baseband signal from the carrier signal at the receiver.
- Modulation: analog modulation, digital modulation.

Analog Modulation:

1. **Amplitude modulation (AM)**: the amplitude of a high frequency carrier signal is varied in accordance to the instantaneous amplitude of the modulating message signal.

For a carrier signal $A_c \cos 2\pi f_c t$, and modulating message signal $m(t)$, the AM signal $s_{AM}(t)$ can be as:

$$s_{AM}(t) = A_c\left[1 + m(t)\right] \cos 2\pi f_c t$$

for $m(t) = \frac{A_m}{A_c} \cos 2\pi f_m t$, the modulation index (k) is the ratio of the peak message signal amplitude to the peak carrier amplitude, expressed as percentage, as:

$$k = \frac{A_c \frac{A_m}{A_c}}{A_c} = \frac{A_m}{A_c}$$

If $k > 100\%$, the decoded signal will be distorted if the receiver uses envelope detection.

The spectrum of an AM signal is

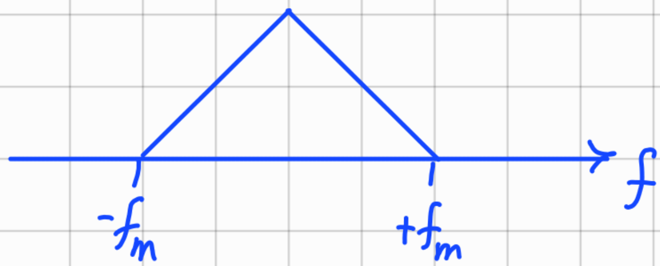
$$S_{AM}(f) = \frac{A_c}{2}\left[\delta(f-f_c) + M(f-f_c) + \delta(f+f_c) + M(f+f_c)\right]$$

where $M(f)$ is the message signal spectrum.

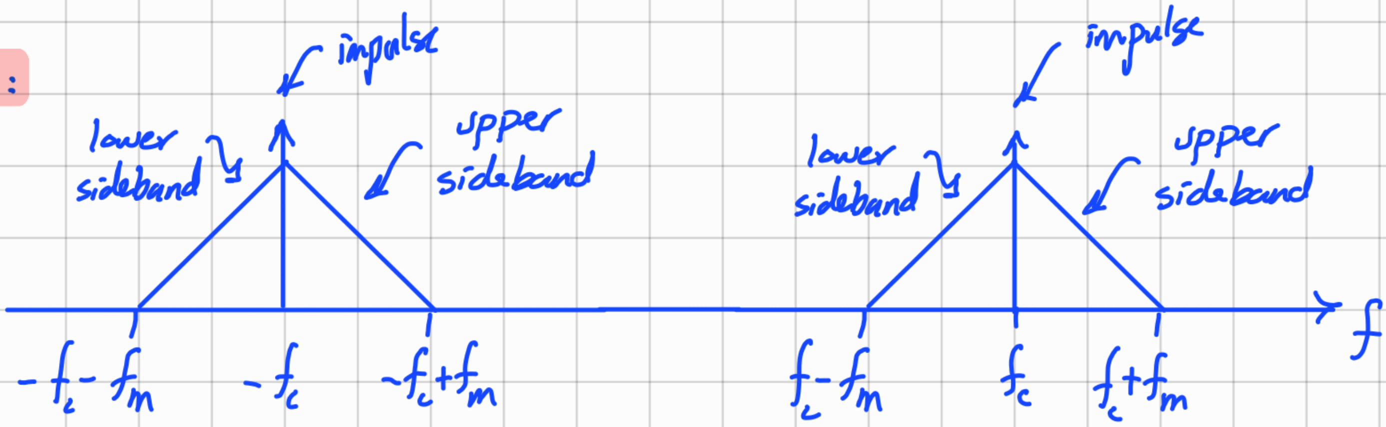
If f_m is the maximum frequency contained in the modulating message signal, then the RF bandwidth of an AM signal is

$$B_{AM} = 2f_m .$$

$|M(f)|$:



$|S_{AM}(f)|$:



Notice that the upper sideband and the lower sideband carry the same information.

2. Single sideband amplitude modulation (SSB-AM):

SSB-AM transmits only one of the two sidebands, either the upper or the lower, to redue the bandwidth into half the bandwith of the conventional AM.

$$S_{SSB} = A_c\left[m(t)\cos 2\pi f_c t \mp \hat{m}(t)\sin 2\pi f_c t\right]$$

the negative sign is for upper SSB and the positive sign is for lower, and $\hat{m}(t)$ is $-90°$ phase shifted from $m(t)$ as

$$\hat{m}(t) = m(t) \otimes h_{HT} \quad , \quad \text{with } H_{HT}(f) = \begin{cases} -j \ , & f>0 \\ j \ , & f<0 \end{cases}$$

3. Pilot tone SSB:

Doppler spread and Rayleigh fading shifts the signal spectrum causing frequency and amplitude variations in the received signal. In SSB-AM, this can be solved by transmitting a low level pilot tone along with the SSB signal as a reference for the phase and amplitude distortions in the received SSB caused by the fading and correct them.

Demodulation of AM signals :

- Coherent demodulator, product detector that converts a bandpass signal into a baseband signal.

If the input is an AM signal as :

$$R(t)\cos(2\pi f_c t + \theta_r)$$

then the output signal is

$$R(t)\cos(2\pi f_c t + \theta_r)A_o\cos(2\pi f_c t + \theta_o)$$

where θ_r : received signal phase, θ_o : oscillator phase.

The above signal can be written as :

$$\frac{1}{2}R(t)A_o\left[\cos(\theta_r - \theta_o) + \cos(2\pi f_c t + \theta_r + \theta_o)\right]$$

After the multiplication, the signal will enter a LPF to give the demodulated AM signal as :

$$\frac{1}{2}R(t)A_o\cos(\theta_r - \theta_o) = K R(t)$$

K : gain constant.

- Non coherent demodulator, envelope detector that has an output proportional to the envelope of the input signal.

If the input is an AM signal as:

$$R(t)\cos(2\pi f_c t + \theta_r)$$

then the output signal is $k|R(t)|$, where k is a gain constant.

- Envelope detectors can be used when the signal power is at least 10 dB greater than the noise power.
- Product detectors can be used with signal power to noise power well below 0 dB.

Angle Modulation :

1. Frequency modulation (FM): it is an angle modulation where the instantaneous frequency of the carrier signal is varied linearly with the baseband message signal $m(t)$.

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + \theta(t)\right]$$

$$= A_c \cos\left[2\pi f_c t + 2\pi k_f \int_{-\infty}^{t} m(t)\,dt\right]$$

where A_c : amplitude of the carrier , f_c : carrier frequency,

k_f : frequency deviation constant.

If the modulating signal is

$$m(t) = A_m \cos 2\pi f_m t$$

then the FM signal is

$$s_{FM}(t) = A_c \cos\left[2\pi f_c t + \frac{k_f A_m}{f_m} \sin 2\pi f_m t\right]$$

- Frequency modulation index β_f is the relation between the message amplitude and the bandwidth of the transmitted signal

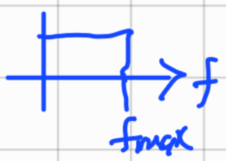
$$\beta_f = \frac{k_f A_m}{W} = \frac{\Delta f}{W}$$

where A_m: the peak value of the modulating signal,

Δf : the peak frequency deviation of the transmitter,

i.e. $\Delta f = k_f A_m$, W : maximum bandwidth of the modulating signal, for the case of low-pass signal

$W = f_{max}$,



$W = f_{max} - 0$

2. Phase modulation (PM) : it is an angle modulation where the angle $\theta(t)$ of the carrier signal varies linearly with the baseband message signal $m(t)$.

$$S_{PM}(t) = A_c \cos\left[2\pi f_c t + \theta(t)\right]$$

$$= A_c \cos\left[2\pi f_c t + k_\theta m(t)\right]$$

where k_θ : phase deviation constant.

- Phase modulation index β_p is the peak phase deviation of the transmitter $\Delta\theta$, $\beta_p = k_\theta A_m = \Delta\theta$

- Notice that an FM signal is a PM signal in which the modulating signal is integrated before modulation, or a PM signal is an FM signal in which the modulating signal is differentiated before modulation.

Example:

For $m(t) = 4\cos 2\pi\, 4\times10^{3} t$, in an FM modulator that has frequency deviation constant $k_f = 10^{4}$ Hz/V. Find the peak frequency deviation and the frequency modulation index.

peak freq. deviation $= \Delta f = k_f A_m = 4\times10^{4}$ Hz

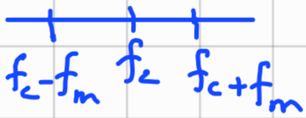
freq. modulation index $= \beta_f = \frac{\Delta f}{W} = \frac{4\times10^{4}}{f_m - 0} = \frac{4\times10^{4}}{4\times10^{3}} = 10$

An FM signal has 98% of the total transmitted power in a RF bandwidth B_T :

$$\left.\begin{aligned} B_T &= 2(\beta_f + 1) f_m && \text{(upper bound)} \\ B_T &= 2\Delta f && \text{(lower bound)} \end{aligned}\right\} \text{"Carson's rule"}$$

when $\beta_f \ll 1$,

$$B_T = 2(\beta_f + 1) f_m \simeq 2 f_m$$



when $\beta_f \gg 1$,

$$B_T = 2(\beta_f + 1) f_m \simeq 2\beta_f f_m = 2\frac{\Delta f}{f_m} f_m = 2\Delta f$$

Example: In a system with $\beta_f = 3$, $f_m = 4\,kHz$.

Use Carson's rule to find the upper and lower bounds of B_T.

Upper bound : $B_T = 2(\beta_f + 1) f_m = 2(3+1)\, 4 \times 10^3 = 32\,kHz$

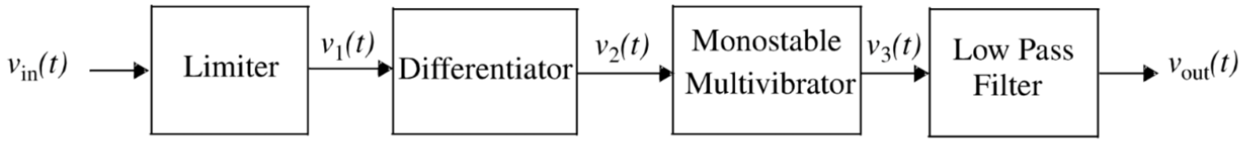
lower bound : $B_T = 2\Delta f = 2\beta_f W = 2(3)(4k - 0) = 24\,kHz$

FM modulation: one of the methods is called the direct method, where a voltage-controlled oscillators are used to vary the frequency of the carrier signal in accordance with the baseband signal amplitude.

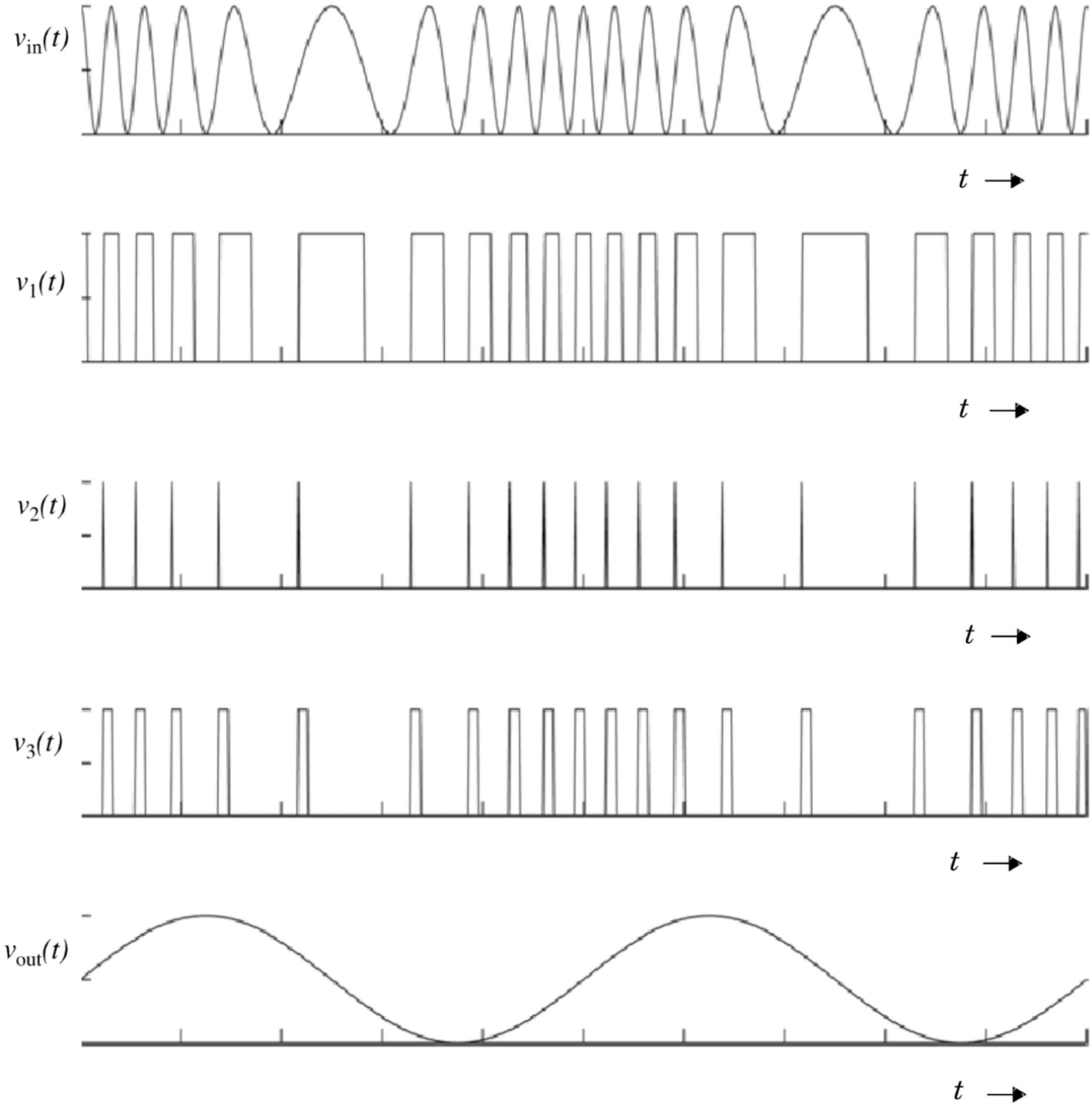
FM demodulation:

Slope detection: in this method, the demodulator takes the time derivative (slope) of the FM signal, then detects the resulting signal's envelope. The output is a signal with an amplitude that varies proportinonal to m(t).

Zero-crossing detection: in this method, the demodulator uses the zero-crossings to convert frequency to amplitude by counting zero-crossings, then generating pulses proportional to the signal above the threshold, then averaging the pulses' durations into a sequence of a unified pulse duration, then feading them into a LPF to generate the m(t) signal as its output.



Zero-crossing detector



Digital Modulation:

The performance of a modulation scheme is usually measured in terms of power efficiency and bandwidth efficiency.

- **Power efficiency** η_p \eta\ : is a measure of the tradeoff between the signal power and the noise power at the receiver input for a certain probability of error, usually measured as the ratio of the signal energy per bit to noise power spectral density E_b/N_o.

- **Bandwidth efficiency** η_B : is a measure of the tradeoff between the data rate and the bandwidth occupancy. It is the ratio of the throughput data rate in a given bandwidth, if R is the data rate in bits per second, and B is the bandwidth occupancy by the modulated RF signal, then $\eta_B = \frac{R}{B}$.

- The system capacity of a digital system is related to bandwidth efficiency η_B.

- In AWGN channels (non-fading channels), the channel capacity is upper bounded by Shannon's bound :

$$C = B\,\eta_{B,max} = B\log_2\left(1+\frac{S}{N}\right)$$

where C: channel capacity in bps, B: RF bandwidth, S/N: signal to noise ratio.

The therory states that for an arbitrarily small probability of error, the maximum possible bandwidth efficiency is limited by the noise in the channel.

- There is a tradeoff between bandwidth efficiency and power efficiency. For example, in error control coding, the coding decreases the bandwidth efficiency but reduces the required received power for a particular bit error rate, i.e. increases power efficiency.

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Example: In US Digital Cellular Standard, the data rate is 48.6kbps with channel conditions of 20dB SNR and RF bandwidth of 30kHz. However, the maximum theoretical data rate that can be transmitted is defined by Shannon's channal capacity as:

$$C = B\log_2(1+SNR)$$

$$= 30\times10^3 \log_2(1+10^{20/10})$$

$$= 30\times10^3 \frac{\log_{10} 101}{\log_{10} 2}$$

$$C = 199.746 \text{ kHz}$$

Recall: $\log_a b = \frac{\log_c b}{\log_c a}$

Only about 24% of the theoretical capacity is used in practice.

Example: In GSM, $R = 270.833$ kbps and $B = 200$ kHz in 10 dB SNR.

The theoretical capacity for such system is:

$$C = B\log_2(1+SNR) = 2\times10^5 \log_2(1+10^{10/10})$$

$$= 2\times10^5 \log_2 11 = 2\times10^5 \frac{\log_{10} 11}{\log_{10} 2}$$

$$C = 691.886 \text{ kbps}$$

Only about 40% of the theoretical capacity is used in practice.

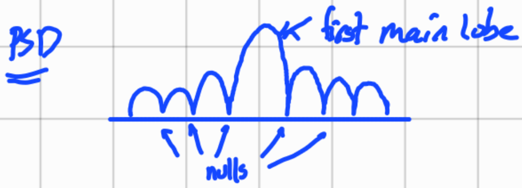
Signal Bandwidth: the definition of bandwidth depends on the context and no single definition suits all cases. However, all definitions are measured based on the power spectral density (PSD) of the signal.

For a signal $w(t)$, if we truncate it: $w_T(t) = \begin{cases} w(t) & , -T/2 < t < T/2 \\ 0 & , \text{else} \end{cases}$

The PSD of $w(t)$: $P_w(f) = \lim_{T \to \infty} \dfrac{\overline{|W_T(\omega)|^2}}{T}$, where $W_T(\omega) = \mathcal{F}\{w_T(t)\}$

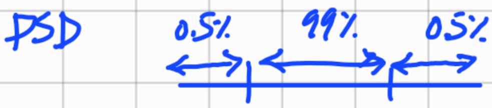
- **Absolute bandwidth:** the range of frequencies over which the signal has non-zero power spectral density.

- **First null-to-null bandwidth:** is the width of the first main spectral lobe.



- **Half-power bandwidth** or **3dB bandwidth:** the interval between frequencies at which the PSD has dropped to half-power or 3dB below peak power value.

- **Occupied bandwidth:** is the Federal Communication Commission (FCC) definition, it is the band that contains 99% of the signal power, it leaves 0.5% of the signal power above the upper band limit and 0.5% of the signal power below the lower band limit.



- A Bandwidth with the PSD is below a certain level outside it, for example, 45 dB attenuation outside the bandwidth, this means that the bandwidth is the band when the PSD is above 45 dB, and when the PSD is below 45 dB these frequencies are outside the band.

- Digital modulation represents a modulating signal that is digital data.

- This modulating signal can be represented as a time sequence of symbols or pulses. If each symbol has n bits of information with each having q possibilities (e.g. $q=2$ for binary), then each symbol has $m=q^n$ finite states, and $n=\log_q m$.

- In digital modulation, if there are m possible states, the modulator has a set S of modulation signals as: $S=\{s_1(t), s_2(t), \dots, s_m(t)\}$.

Then based on the information bits applied to the modulator, the modulator will choose a particular signal $s_i(t)$.

Example: (a) If $m=2$ & $n=1$, this is the case of binary and S will have only two signals and each signal represents one bit of information.

(b) If $m=4$, $q=2$, then $n=\log_2 4 = \frac{\log_{10} 4}{\log_{10} 2} = 2$, this has 4 states and each signal represents two bits of information, and each bit is either zero or one

(b) If the set S has m signals, then each symbol (or signal) will represent more than one bit, it can represent a maximum of $n = \log_2 m$ bits of information, and if $q = 2$, $n = \log_2 m$ bits per symbol.

- View the set of signals in S as a set of waveforms in a vector space.

Any signal in a vector space can be expressed as a linear combination of n orthonormal signals which form the basis of such vector space and n is the dimension. The basis can be viewed as a coordinate system for the vector space. That is, if the basis signals are:

$\{\phi_1(t), \phi_2(t), \ldots, \phi_n(t)\}$, then any signal in the set S can be represented as a linear combination of the basis as:

$$s_i(t) = \sum_{j=1}^{n} s_{ij}\,\phi_j(t)$$

The basis are orthonormal, that is, they are orthogonal to one another in time:

$$\int_{-\infty}^{\infty} \phi_i(t)\,\phi_j(t)\,dt = 0 \quad , \quad i \neq j$$

and each basis signal is normalized to have unit energy:

$$\int_{-\infty}^{\infty} \phi_i^2(t)\,dt = 1$$

- If the set of modulation signals S has signals equal to the number of basis signals, then all the signals in S are orthogonal to one another.

- If a modulation signal set S has two signals, $m=2$, two states. Then each modulated signal represents either one or zero, that is, it represents one bit. However, if the modulation signal set S has m signals, that is m states, then each modulated signal represents $n = \log_2 m$ bits per signal (or symbol). Then if we compare the two modulations, the one with $m=2$ states and the one with m states, we send per signal (or symbol) in the first only one bit of information, while in the other we send $n = \log_2 m$ bits per signal (or symbol). So if we compare bandwidths, the second has a bandwidth $\frac{1}{n}$ of the first

first $m=2$, $n=1$ bits per signal, $\overbrace{|\!\longrightarrow\!|}^{\text{signal}} T_s$, $R_b = R_s = \frac{1}{T_s} = \frac{1}{T_b}$

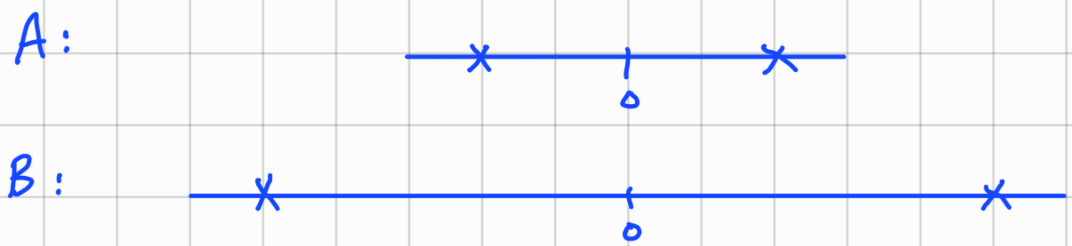
2nd m, $n = \log_2 m$ bits per signal, $\overbrace{|\!\longrightarrow\!|}^{\text{signal}} T_s$, $R_s = \frac{1}{T_s}$, $T_s = nT_b$

$$R_s = \frac{1}{nT_b}$$

- So the bandwidth occupied by a modulated signal decreases as the dimention increases.

- The probability of bit error is proportional to the distance between the closest points in the vector space. This distance is called the minimum distance.

- For a given dimension, if the modulation signals in set S are densely packed in the vector space, then the minimum distance is low, then the probability of bit error is high, i.e. high bit error rate (BER). In other words, increasing the separation means increasing the signal energy to increase the SNR.



$d_{min,A} < d_{min,B} \Rightarrow BER_A > BER_B$ because $SNR_A < SNR_B$

- The digital baseband signal can be coded with line coding to provide a particular spectral characteristics of a sequence of pulses before modulation.
- Also such baseband pulses can be passed through a filter in order to shape the sequence of pulses before modulation at the transmitter and also before demodulation at the receiver. This shaping is used to reduce the intersymbol interference (ISI) and to reduce the modulation bandwidth by spectral shaping. This technique is called Pulse shaping.

Line coding: some examples are :

1. Return-to-zero (RZ): the pulses return to zero within every bit period.
Improves time synchronization but causes spectral widening due to zero-return.

2. Non-return-to zero (NRZ): the pulses do not return to zero during a bit period, it stays at constant levels through a bit period.
Poor synchronization capabilities but more spectrally efficient.

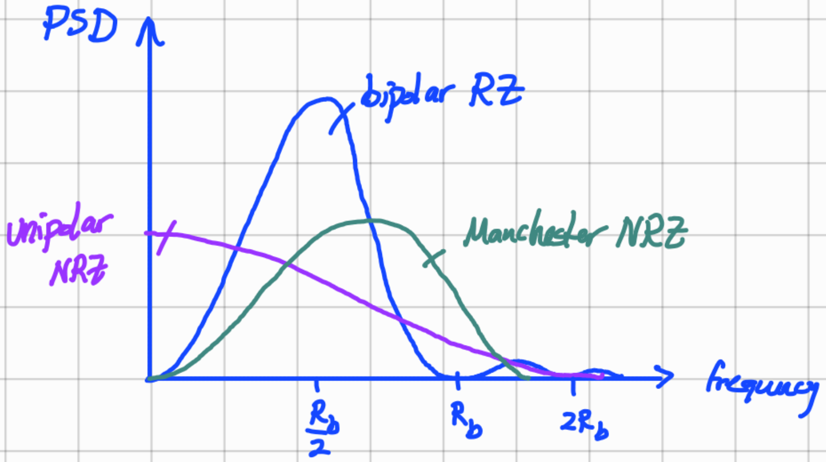
3. Manchester codes: a special type of NRZ, it uses two pulses per bit period, with a zero dc component. It provides synchronization without spectral widening as in RZ line coding.

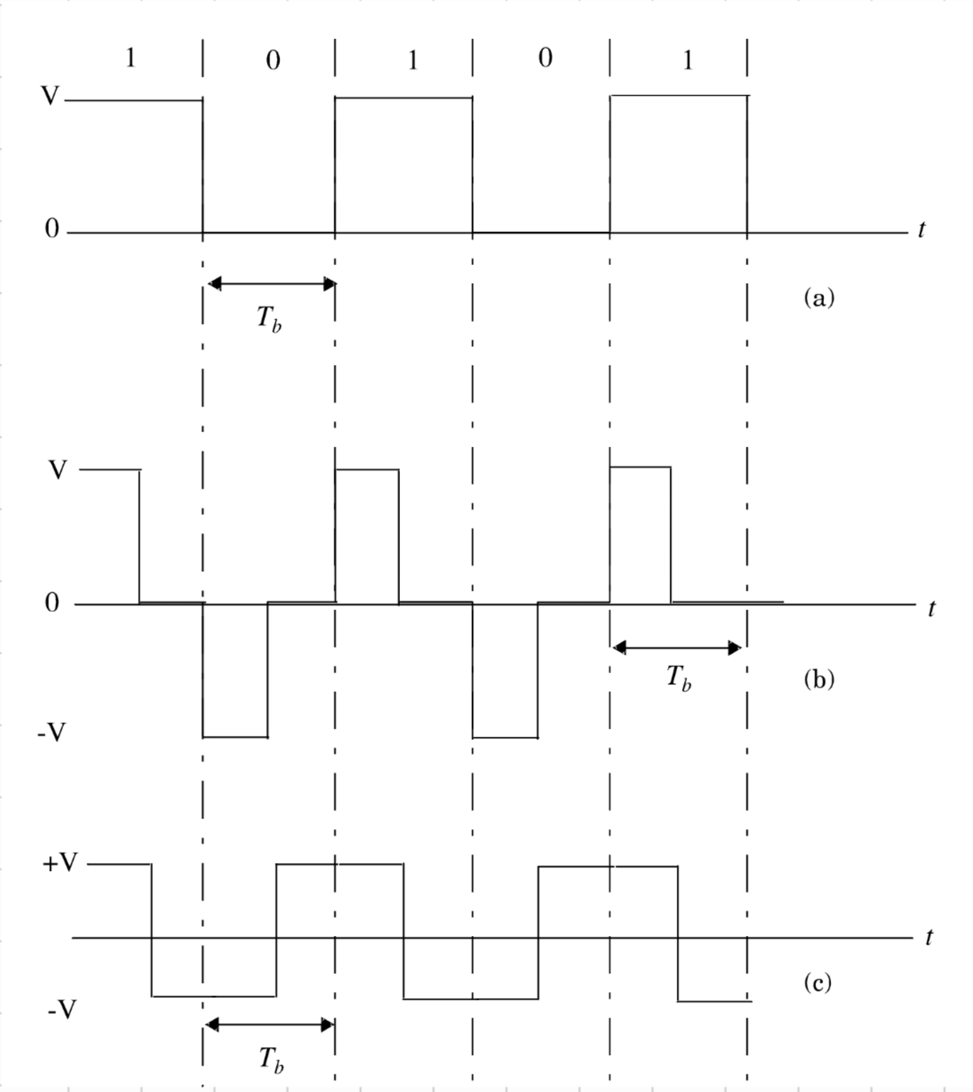
- These line codes can be either :

1. **Unipolar** : the amplitude levels are either 0 or A

2. **Bipolar** : the amplitude levels are either -A or A

Their Power spectral densities (PSDs) :





Unipolar NRZ

bipolar RZ

Manchester NRZ

Pulse shaping :

1. Nyquist pulse shaping filter

Nyquist was the first to solve the ISI problem, he cancels the ISI by having at every sampling instant at the receiver the response due to all symbols except the current symbol to all equal zero.

If $h_{eff}(t)$ is the impulse response of the overall communication system, then the Nyquist condition mathematically is :

$$h_{eff}(nT_s) = \begin{cases} K & n = 0 \\ 0 & n \neq 0 \end{cases}$$

where T_s : symbol period, n: integer, k: non-zero constant, and the effective transfer function of a system can be represented as :

$$h_{eff}(t) = \delta(t) * p(t) * h_c(t) * h_r(t)$$

where $p(t)$: pulse shape of a symbol, $h_c(t)$: channel impulse response, $h_r(t)$: receiver impulse response.

If we assume that the channel is ideal, $h_c(t) = \delta(t)$, and since the pulse shaping is a filter used at the transmitter to shape the sequence of pulses before modulation and also at the receiver before demodulation, then we will choose such shaping filters to both be $\sqrt{H_{eff}(f)}$. So, the impulse for the overall system will be $h_{eff}(t)$.

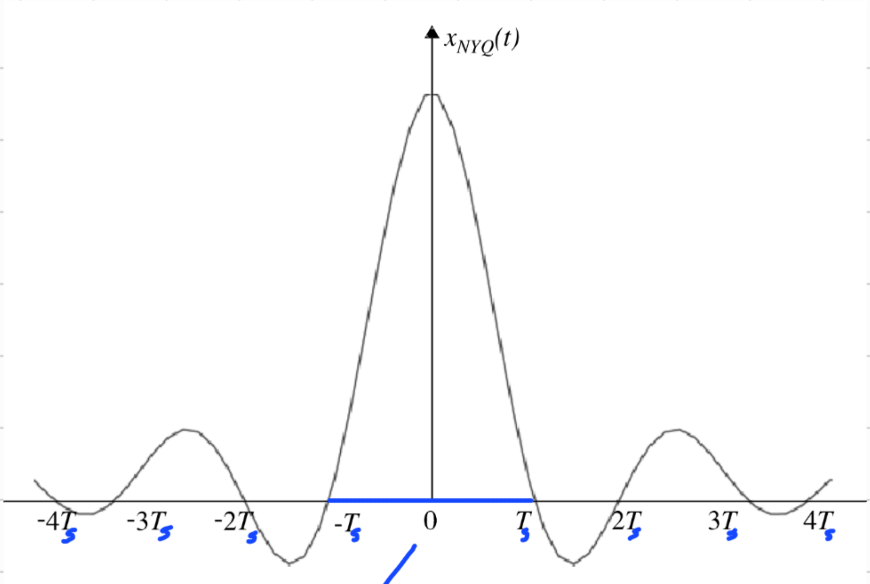
An ideal example for such $h_{eff}(t)$ is when

$$h_{eff}(t) = \frac{\sin\left(\frac{\pi t}{T_s}\right)}{\left(\frac{\pi t}{T_s}\right)} =: \operatorname{sinc}\left(\frac{\pi t}{T_s}\right)$$

Note: $\frac{\sin x}{x} =: (\sin x)\left(\frac{1}{x}\right)$

oscillation

decrease in amplitude



Nyquist ideal pulse shape for zero intersymbol interference.

So the signal duration (symbol duration) will be after the pulse shaping $2T_s$ where T_s is the symbol period

$$\left.\frac{\sin\left(\frac{\pi t}{T_s}\right)}{\frac{\pi t}{T_s}}\right|_{t=0} \Rightarrow \lim_{t\to 0} \frac{\sin\frac{\pi t}{T_s}}{\frac{\pi t}{T_s}}$$

$$= \lim_{t\to 0} \frac{\frac{\pi}{T_s}\cos\frac{\pi t}{T_s}}{\frac{\pi}{T_s}} = 1$$

$\operatorname{sinc}\frac{\pi t}{T_s} = 0$ is when

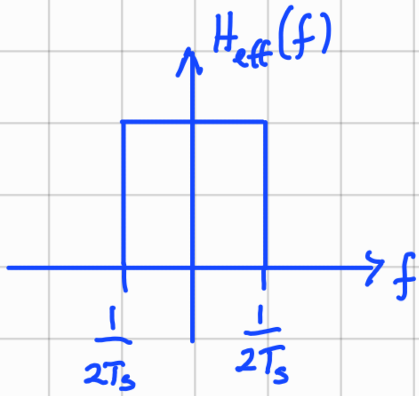
$\frac{\pi t}{T_s} = \pm n\pi$ except at $t=0$

$\Rightarrow t = \pm n T_s$

So such filter satisfies Nyquist's conditions

For such filter, its Fourier transform is :

$$H_{eff}(f) = \begin{cases} 1 & , -\frac{1}{2T_s} \le f \le \frac{1}{2T_s} \\ 0 & , f < -\frac{1}{2T_s}, \ f > \frac{1}{2T_s} \end{cases}$$



This filter is called rectangular filter.

So when the signal (or symbol) is passed through such filter before modulation, it will shape the signal to have a $BW = \frac{1}{2T_s}$ $\left(T_s = \frac{1}{R_s}\right)$.

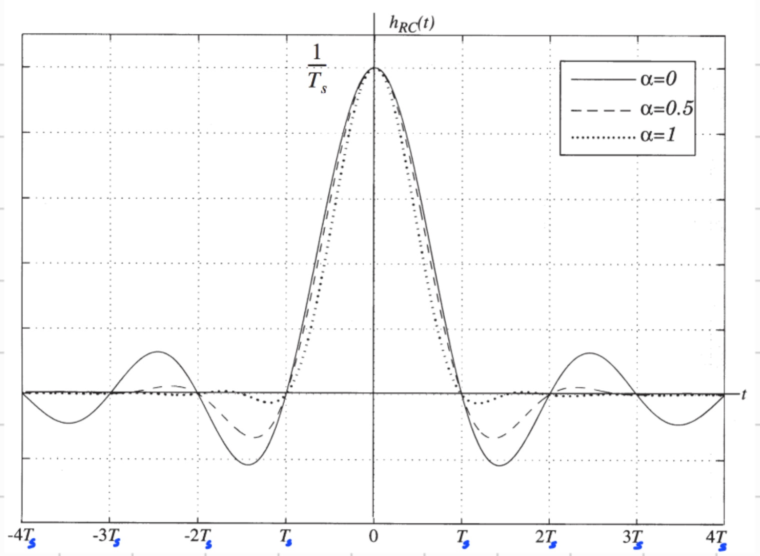
However, when such shaped signal (or symbol) is modulated, its passband bandwidth will double, so the RF passband bandwidth will be: $BW = \frac{1}{T_s}$ $\left(T_s = \frac{1}{R_s}\right)$.

2. Raised cosine pulse shaping filter

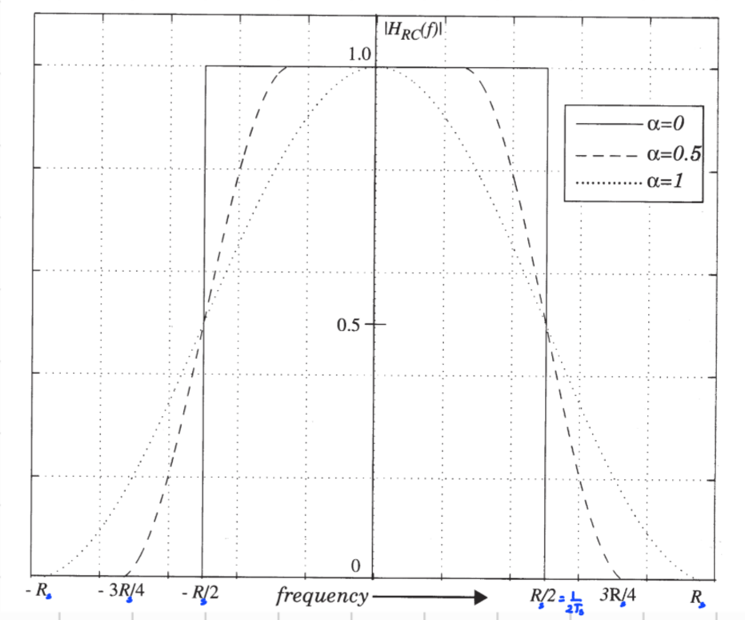
This type of filter satisfies Nyquist's condition. The transfer function is

$$h_{RC}(t) = \frac{\sin\left(\frac{\pi t}{T_s}\right)}{\pi t} \; \frac{\cos\left(\frac{\pi \alpha t}{T_s}\right)}{1-\left(\frac{4\alpha t}{2T_s}\right)^2}$$

where $0 \leq \alpha \leq 1$.



The rectangular filter is a RC filter with $\alpha = 0$.



$$H_{RC}(f) = \begin{cases} 1, & -\left(\frac{1-\alpha}{2T_s}\right) \leq f \leq \left(\frac{1-\alpha}{2T_s}\right) \\ \frac{1}{2} + \frac{1}{2}\cos\left[\frac{\pi\left(2T_s|f|-1+\alpha\right)}{2\alpha}\right], & \frac{1-\alpha}{2T_s} \leq |f| \leq \frac{1+\alpha}{2T_s} \\ 0, & |f| > \frac{1+\alpha}{2T_s} \end{cases}$$

So when the signal (or symbol) is passed through such filter before modulation, it will shape the signal to have a $BW = \frac{1+\alpha}{2T_s}$ $\left(T_s = \frac{1}{R_s}\right)$.

However, when such shaped signal (or symbol) is modulated, its passband bandwidth will double, so the RF passband bandwidth will be: $BW = \frac{1+\alpha}{T_s}$ $\left(T_s = \frac{1}{R_s}\right)$.

- **Digital modulation**: can be classified in general to:

(1) **Linear modulation**: the amplitude of the modulated signal varies linearly with the modulating digital signal $m(t)$, since the modulation signals in the set S vary in their amplitude.

(2) **Nonlinear modulation (constant envelope modulation)**: the amplitude of the modulated signal is constant with a varying phase (or frequency), since the modulation signals in set S vary in their phase (or frequency).

(3) **Combined linear and nonlinear modulation**: the amplitude and phase (or frequency) of the modulated signal both vary, since the modulation signals in set S vary in both amplitude and phase (or frequency).

Examples :

(1) Linear modulation :

(a) Binary Phase Shift Keying (BPSK):

Two modulation signals, $S=\{s_1(t), s_2(t)\}$, both with the same amplitude and a 180° phase separation. These two signals will have the same amplitude, however, one with a positive value and the other with its negative. So we need only one basis signal.

For example: $s_1(t) = A\cos(2\pi f_c t + \theta_c)$, $s_2(t) = -A\cos(2\pi f_c t + \theta_c)$

and if the pulses were shaped using a RC filter with α before BPSK modulation, then after modulation the

$$BW = \frac{1+\alpha}{T_s} = (1+\alpha)R_s = (1+\alpha)R_b \ , \quad \left(T_s = \frac{1}{R_s} \text{ and } R_s = R_b\right)$$

since each modulated signal represents one bit of information.

Where the BPSK constellation diagram can be as:

(b) Quadrature Phase Shift Keying (QPSK):

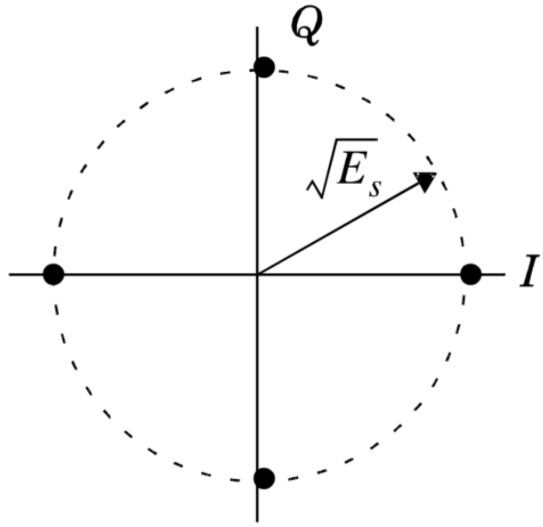
Four modulation signals, $S = \{s_1(t), s_2(t), s_3(t), s_4(t)\}$, $m=4$ and $q=2$, then $n = \log_2 4 = 2$, so we send two bits of information per signal (symbol). So we will need $n=2$ orthonormal basis signals to represent the four modulation signals.

For example: Basis: $\phi_1(t) = A\cos(2\pi f_c t)$, $\phi_2(t) = A\sin(2\pi f_c t)$

so we can have

$s_1(t) = B\,\phi_1(t)$, $s_2(t) = B\,\phi_2(t)$,

$s_3(t) = -B\,\phi_1(t)$, $s_4(t) = -B\,\phi_2(t)$.



If the pulses were shaped using a RC filter with α before QPSK modulation, then after modulation the BW:

$$BW = \frac{1+\alpha}{T_s} = \frac{1+\alpha}{2T_b} = (1+\alpha)R_s = (1+\alpha)\frac{R_b}{2} \quad \left(T_s = \frac{1}{R_s},\ T_s = 2T_b,\ R_s = \frac{R_b}{2}\right)$$

since each modulated signal (symbol) represents two bits of information.

(2) Nonlinear modulation (constant envelope modulation):

(a) Binary Frequency Shift keying (BFSK):

Two modulation signals with one basis, for example:

$$s_1(t) = A\cos\left(2\pi(f_c+\Delta f)t\right), \quad s_2(t) = A\cos\left(2\pi(f_c-\Delta f)t\right)$$

where Δf is a constant offset from the carrier frequency.

If the modulated signal was shaped using an RC filter with α, then the transmitted signal after BFSK using the two possible modulation signals will have a BW:

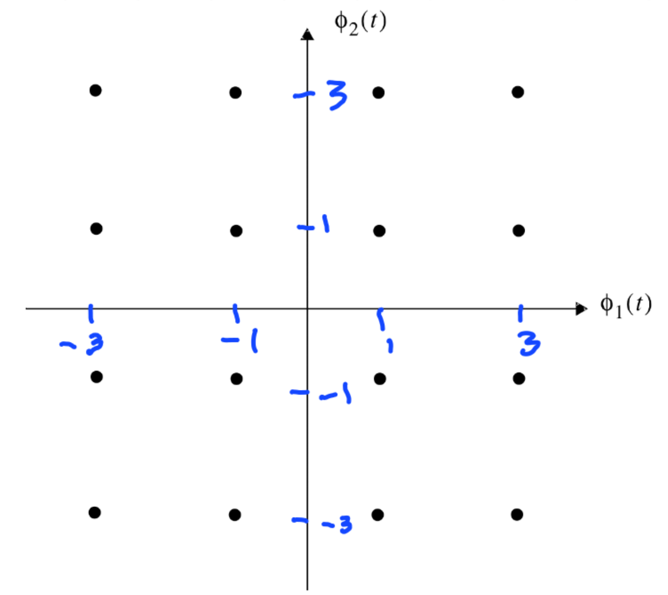
$$BW = \frac{1+\alpha}{T_s} + 2\Delta f = (1+\alpha)R_s + 2\Delta f = (1+\alpha)R_b + 2\Delta f$$

(3) Combined linear and nonlinear modulation:

(a) 16-ary Quadrature Amplitude Modulation (QAM):

here we have 16 states, $m = 16$. Both the amplitude and phase of the modulation signals will vary.

We will send 4 bits of information per signal (symbol), we need only two basis since both the amplitude and and phase carry information. For example,



$\phi_1(t) = A\cos(2\pi f_c t)$

$\phi_2(t) = A\sin(2\pi f_c t)$

and $s_i(t) = B a_i \phi_1(t) + B b_i \phi_2(t)$, $i = 1, 2, \dots, 16$

and (a_i, b_i) are integers chosen according to the location of the particular signal point on the constellation diagram above.

An example, $(a_i, b_i) = \begin{bmatrix} (-3,3) & (-1,3) & (1,3) & (3,3) \\ (-3,1) & (-1,1) & (1,1) & (3,1) \\ (-3,-1) & (-1,-1) & (1,-1) & (3,-1) \\ (-3,-3) & (-1,-3) & (1,-3) & (3,-3) \end{bmatrix}$

Spread spectrum modulation techniques:

These techniques use a transmission bandwidth that is several orders of magnitude greater than the minimum required signal bandwidth.

The system uses the same bandwidth for many users simultaneously without significantly interfering with one another.

(1) Direct sequence spread spectrum (DS-SS):

This system spreads the baseband data by directly multiplying the baseband data pulses with pseudo-noise (PN) sequence that is produced
/soo-dow/
by a pseudo-noise code generator before modulation.

A single pulse (or symbol) of the PN waveform is called a chip.

At the receiver, the spread spectrum signal is demodulated through cross-correlation with a locally-generated version of the PN sequence.

Cross-correlation with the correct PN sequence despreads the spread spectrum signal and restores the modulated signal and then it is demodulated. However, cross-correlating the signal from an undesired user with the incorrect PN sequence results in a very small amount of wideband noise at the receiver output.

(2) Frequency hopping spread spectrum (FH-SS):

This system changes the transmission frequency periodically. The frequecy hopping signal is a sequence of modulated data with time-varying, pseudo-random carrier frequencies. That is, the data is sent by hopping the carrier frequency to seemingly random channels which are known only to the desired receiver. On each channel, some data is sent before the channel hops again.

The set of possible carrier frequencies is called hopset. Hopping occurs over a frequency band that includes a number of channels. The bandwidth of the spectrum over which the hopping occurs is called the total hopping bandwidth. The bandwidth of a channel used in the hopset is called the instantaneous bandwidth. The time duration between hops is called hop duration or hopping period.

Frequency hopping is classified as fast or slow:

- Fast frequency hopping occurs if there is more than one frequency hope during each transmitted symbol.

- Slow frequency hopping occurs if one or more symbols are transmitted in a hop duration

- Diversity techniques:

These techniques use the randomness of radio propagation by finding independent or highly uncorrelated signal paths. Simply, if one radio path experiences deep fading then another independent path may have a strong signal. By having more than one path, both the instantaneous and average SNRs at the receiver may be improved.

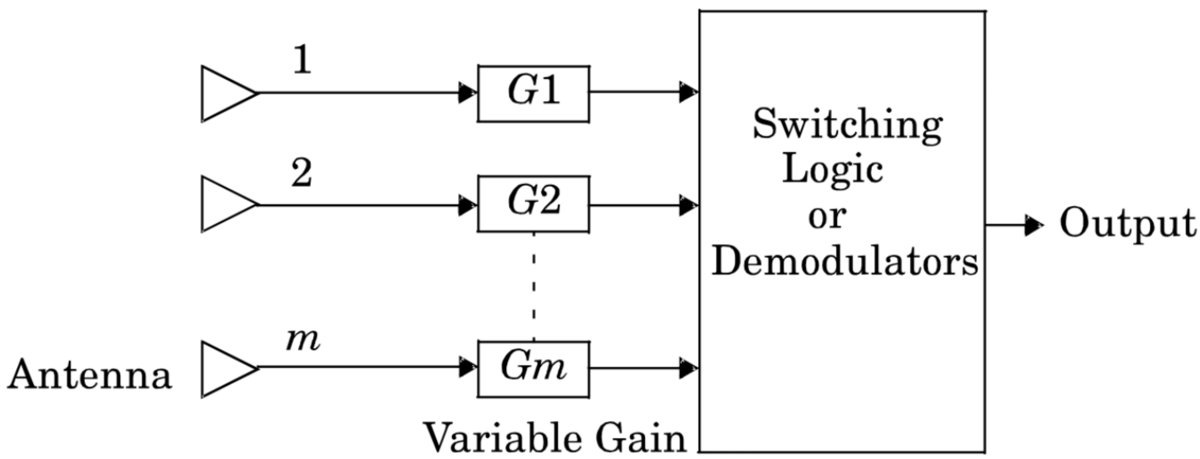
- Space diversity also called antenna diversity is one of the most used forms of diversity in wireless systems.

For small-scale and large-scale fading, microscopic diversity techniques and macroscopic diversity techniques can be used. The microscopic diversity techniques can be used at the mobile station where the signals are received from spatially separated antennas on the mobile with separations of one half wavelength or more. The macroscopic diversity techniques can be used at the basestation where the receiving antennas are separated on the order of several tens of wavelengths.

Space diversity reception methods can be classified as:

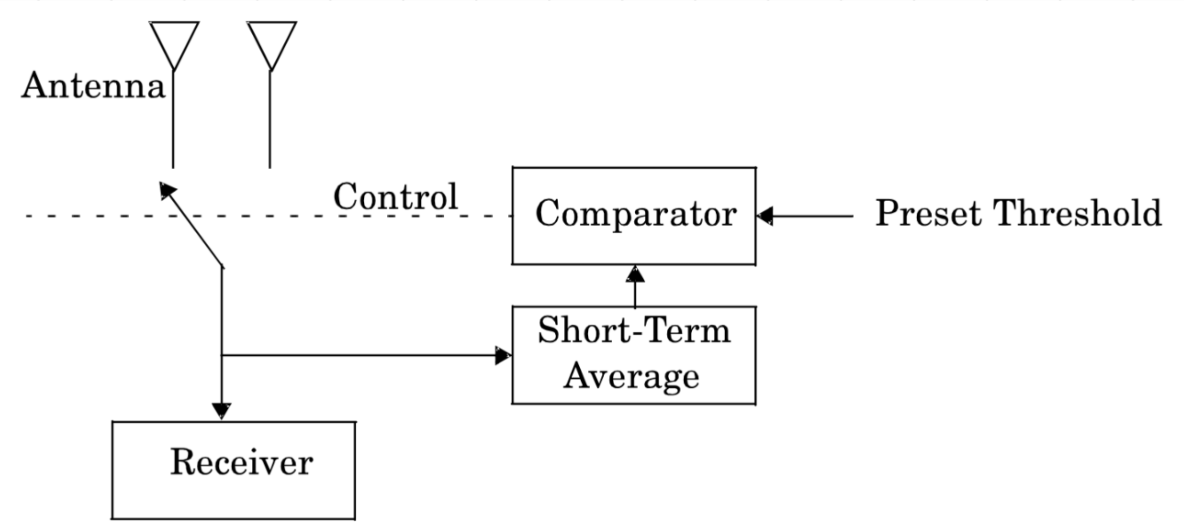
(1) Selection diversity:

This is the simplest technique, m demodulators are used to provide m diversity branches. The gain for these m branches is adjusted to provide the same average SNR for each branch. The branch with the highest intantaneous SNR is connected to the demodulator. The time of the selection circuitry is shorter than the reciprocal of the signal fading rate. This technique uses the best of the m received signals.



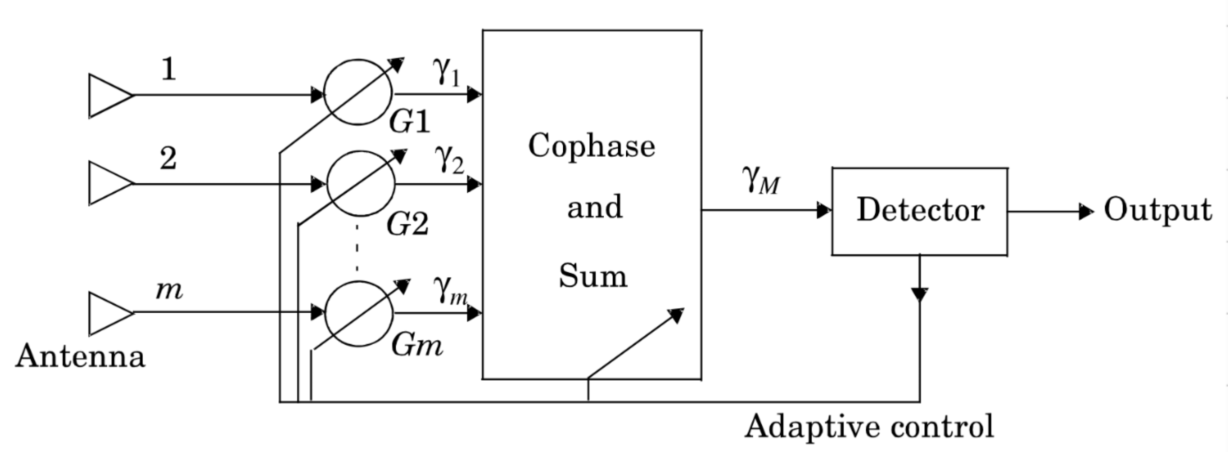
(2) Feedback diversity or scanning diversity:

This is similar to selection diversity except that instead of using the best of the m signals, the m signals are scanned in a fixed sequence until one is found to be above a predetermined threshold. This signal is then received until it falls below threshold and the scanning process is initiated again. This method is inferior to other methods, however, it is the simplest and it requires only one receiver.



(3) Maximal ratio combining:

In this method, the signals from all the m branches are weighted according to their individual SNR and then summed. The signals must be co-phased (have the same phase angle) before being summed. This combining method produces an output SNR equal to the sum of the individual SNRs. This will produce an output with acceptable SNR even when none of the individual signals have acceptable SNRs. This method gives the best reduction of fading.



(4) Equal gain diversity:

In this method, the branch weights are all set to unity and the signals from each branch are co-phased to provide equal gain combining diversity. This allows the receiver to use signals that are simultaneously received on each branch. This method will also produce an acceptable signal from a number of unacceptable signals. The performance is slightly inferior to maximal ratio combining and superior to selection diversity.