



تقدم لجنة EiCoM الاكاديمية

دفتر لمادة:

أنظمة اتصالات لاسلكية

من شرح:

د. محمود حمادي

جزيل الشكر للطالب:

عدنان حورانبي



Introduction

EE 409433
Wireless Communication Systems
Mahmoud A. Smadi
Electrical Engineering Department
Hashemite University

Course Details

Instructor: Mahmoud A. Smadi
Email: smadi@hu.edu.jo

Class Hours:

S,T, Th: 9:00-10:00

Office Hours

Daily: 11:00-12:00

Classroom: 2006

Recommended Textbooks

- Theodore Rappaport, "**Wireless Communications: Principles and Practice**", 2nd edition, Prentice Hall, 2002.
 - Gordon Stuber, "**Principles of Mobile Communication**", 2nd edition, Kluwer Academic, 2001.
-
-
-

Grading

- There will be two midterms and one final exam
 - Attendance is important!
-
-

Outline

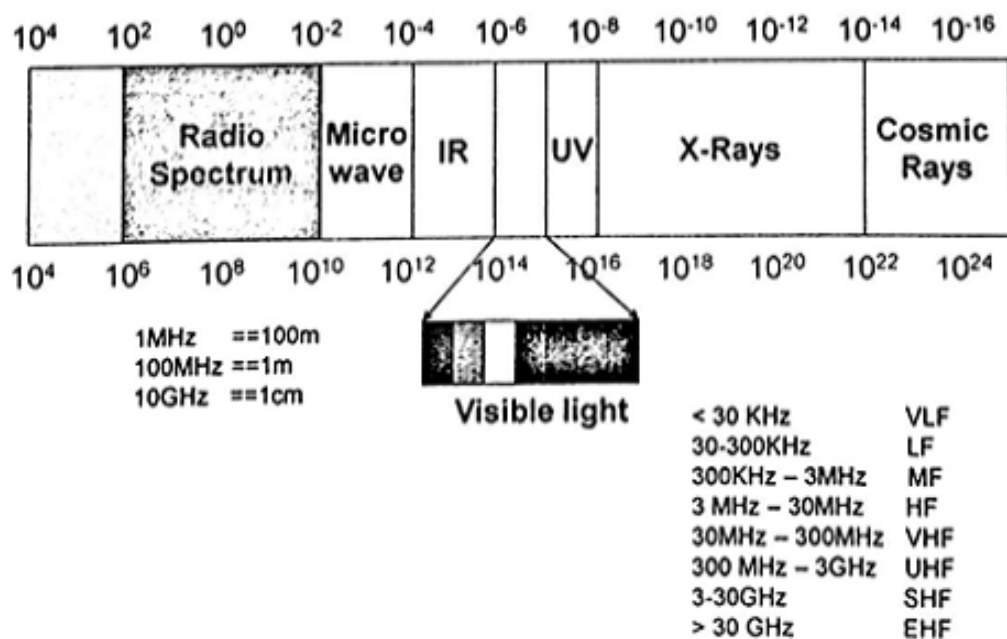
- **Wireless Link Characteristics**
 - Radio Propagation
 - Short and Long wave properties
 - Attenuation & Interference
 - Fading and Multi-path Fading
 - Transmit power and range
 - Bit Error Rate and Models
-

**What is Wireless and
Mobile Communication?**

Wireless Communication

- Transmitting voice and data using electromagnetic waves in open space
- Electromagnetic waves
 - Travel at speed of light ($c = 3 \times 10^8$ m/s)
 - Has a frequency (f) and wavelength (λ)
 - $c = f \times \lambda$
 - Higher frequency means higher energy photons
 - The higher the energy photon the more penetrating is the radiation

Electromagnetic Spectrum



Wavelength of Some Technologies

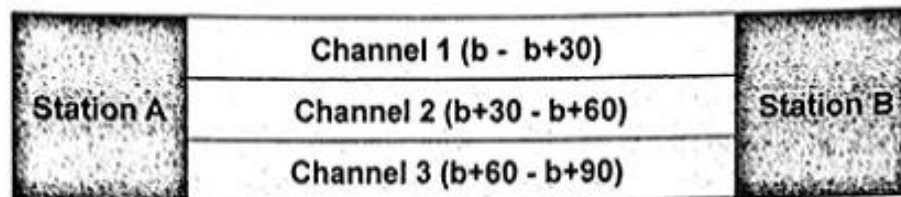
- GSM Phones:
 - frequency \approx 900 Mhz
 - wavelength \approx 33cm
 - PCS Phones
 - frequency \approx 1.8 Ghz
 - wavelength \approx 17.5 cm
 - Bluetooth:
 - frequency \approx 2.4Gz
 - wavelength \approx 12.5cm
-
-
-

Frequency Carriers/Channels

- The information from sender to receiver is carrier over a well defined frequency band.
 - This is called a channel
 - Each channel has a fixed frequency bandwidth (in KHz) and Capacity (bit-rate)
 - Different frequency bands (channels) can be used to transmit information in parallel and independently.
-

Example

- Assume a spectrum of 90KHz is allocated over a base frequency b for communication between stations A and B
- Assume each channel occupies 30KHz.
- There are 3 channels
- Each channel is simplex (Transmission occurs in one way)
- For full duplex communication:
 - Use two different channels (front and reverse channels)
 - Use time division in a channel



11

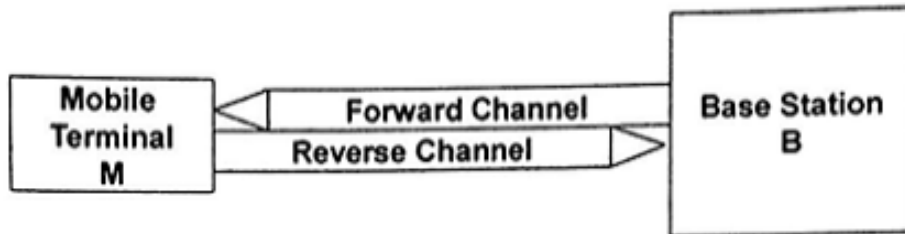
Simplex Communication

- Normally, on a channel, a station can transmit only in one way.
 - This is called simplex transmission
- To enable two-way communication (called full-duplex communication)
 - We can use Frequency Division Multiplexing
 - We can use Time Division Multiplexing

11

Duplex Communication - FDD

- FDD: Frequency Division Duplex

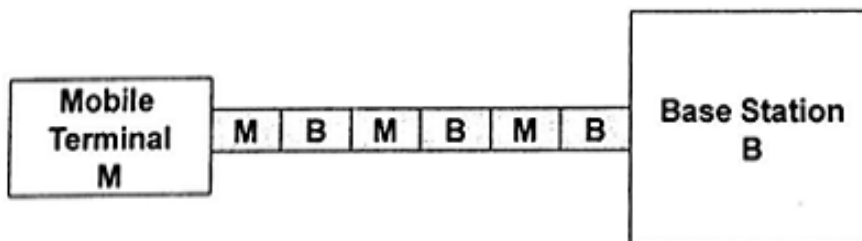


Forward Channel and Reverse Channel use different frequency bands

12

Duplex Communication - TDD

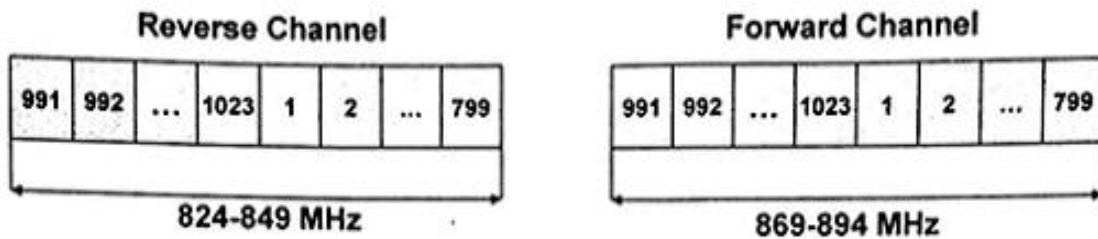
- TDD: Time Division Duplex



A single frequency channel is used. The channel is divided into time slots. Mobile station and base station transmit on the time slots alternately.

13

Example - Frequency Spectrum Allocation in U.S. Cellular Radio Service



Channel Number	Center Frequency (MHz)
Reverse Channel $1 \leq N \leq 799$	$0.030N + 825.0$
$991 \leq N \leq 1023$	$0.030(N-1023) + 825.0$
Forward Channel $1 \leq N \leq 799$	$0.030N + 870.0$
$991 \leq N \leq 1023$	$0.030(N-1023) + 870.0$
(Channels 800-990 are unused)	
Channel bandwidth is 45 MHz	

What is Mobility

- Initially Internet and Telephone Networks is designed assuming the user terminals are static
 - No change of location during a call/connection
 - A user terminals accesses the network always from a fixed location
- Mobility and portability
 - Portability means changing point of attachment to the network offline
 - Mobility means changing point of attachment to the network online

Degrees of Mobility

■ Walking Users

- Low speed
- Small roaming area
- Usually uses high-bandwidth/low-latency access

■ Vehicles

- High speeds
- Large roaming area
- Usually uses low-bandwidth/high-latency access
- Uses sophisticated terminal equipment (cell phones)

14

What is PCS?

■ Personal Communication Services

- A wide variety of network services that includes wireless access and personal mobility services
- Provided through a small terminal
- Enables communication at any time, at any place, and in any form.

■ The market for such services is tremendously big

- Think of cell-phone market

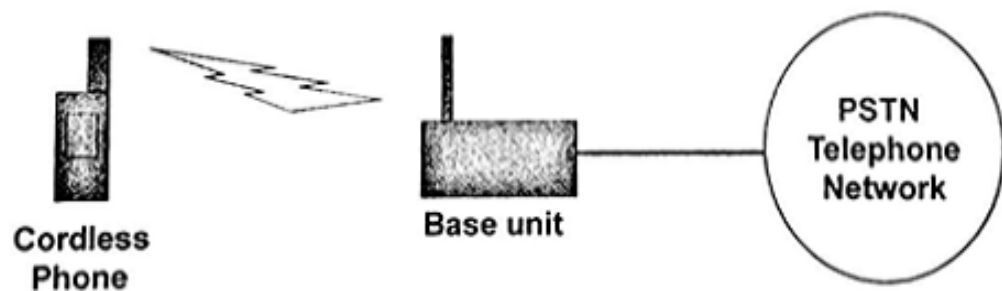
15

PCS Systems Classification

- Cordless Telephones
- Cellular Telephony (High-tier)
- Wide Area Wireless Data Systems (High-tier)
- High Speed Local and Personal Area Networks
- Paging Messaging Systems
- Satellite Based Mobile Systems
- 3G Systems

11

Examples: Cordless Telephones



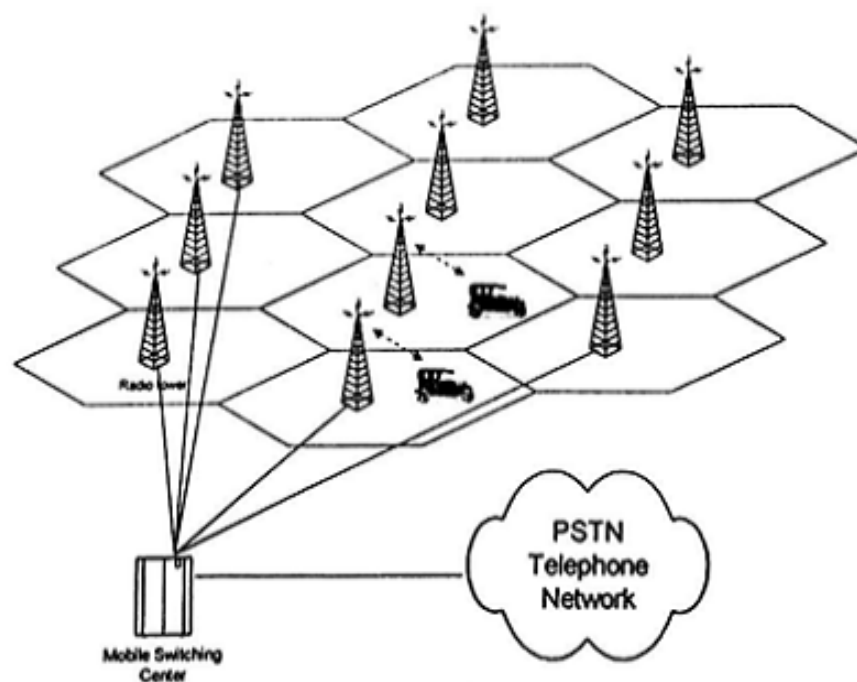
12

Cordless Telephones

- Characterized by
 - Low mobility (in terms of range and speed)
 - Low power consumption
 - Two-way voice communication
 - High circuit quality
 - Low cost equipment, small form factor and long talk-time
 - No handoffs between base units
- Appeared as analog devices
- Digital devices appeared later with CT2, DECT standards in Europe and ISM band technologies in USA

11

Cellular Telephony - Architecture



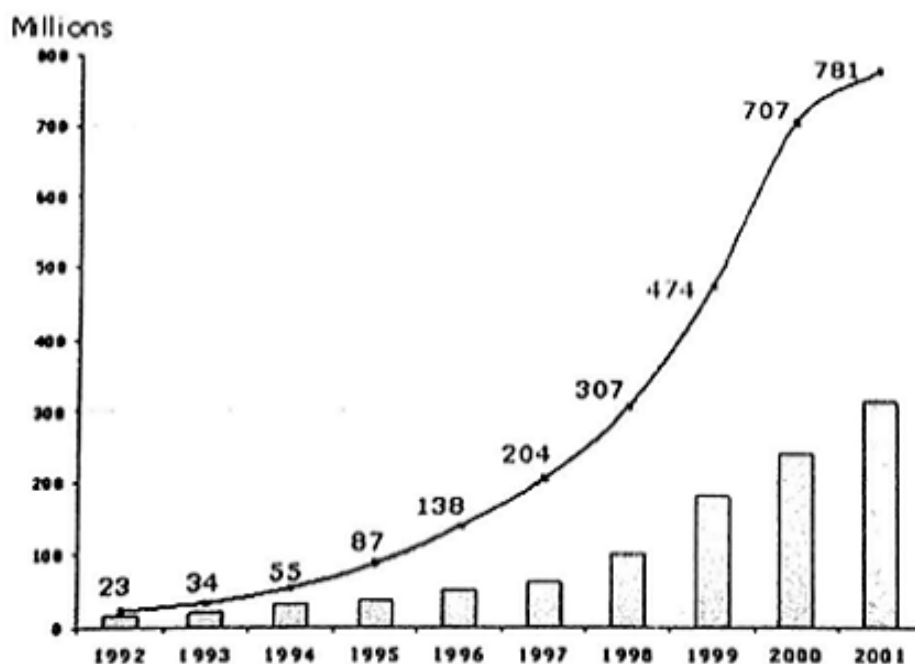
11

Cellular Telephony Systems

- Mobile users and handsets
 - Very complex circuitry and design
- Base stations
 - Provides gateway functionality between wireless and wireline links
 - ~1 million dollar
- Mobile switching centers
 - Connect cellular system to the terrestrial telephone network (PSTN)

11

World Cellular Subscriber Growth



11

Wireless System Definitions

□ **Mobile Station (MS)**

- A station in the cellular radio service intended for use while in motion at unspecified locations. They can be either hand-held personal units (portables) or installed on vehicles (mobiles)

□ **Base station (BS)**

- A fixed station in a mobile radio system used for radio communication with the mobile stations. Base stations are located at the center or edge of a coverage region. They consists of radio channels and transmitter and receiver antennas mounted on top of a tower.

10

Wireless System Definitions

□ **Mobile Switching Center (MSC)**

- Switching center which coordinates the routing of calls in a large service area. In a cellular radio system, the MSC connections the cellular base stations and the mobiles to the PSTN (telephone network). It is also called Mobile Telephone Switching Office (MTSO)

□ **Subscriber**

- A user who pays subscription charges for using a mobile communication system

□ **Transceiver (Txd)**

- A device capable of simultaneously transmitting and receiving radio signals

11

Wireless System Definitions

- **Control Channel**
 - Radio channel used for transmission of call setup, call request, call initiation and other beacon and control purposes.
- **Forward Channel**
 - Radio channel used for transmission of information from the base station to the mobile
- **Reverse Channel**
 - Radio channel used for transmission of information from mobile to base station

14

Wireless System Definitions

- **Simplex Systems**
 - Communication systems which provide only one-way communication
- **Half Duplex Systems**
 - Communication Systems which allow two-way communication by using the same radio channel for both transmission and reception. At any given time, the user can either transmit or receive information.
- **Full Duplex Systems**
 - Communication systems which allow simultaneous two-way communication. Transmission and reception is typically on two different channels (FDD).

15

Wireless System Definitions

□ Handoff

- The process of transferring a mobile station from one channel or base station to another.

□ Roamer

- A mobile station which operates in a service area (market) other than that from which service has been subscribed.

□ Page

- A brief message which is broadcast over the entire service area, usually in simulcast fashion by many base stations at the same time.

11

Major Mobile Radio Standards USA

Standard	Type	Year Intro	Multiple Access	Frequency Band (MHz)	Modulation	Channel BW (KHz)
AMPS	Cellular	1983	FDMA	824-894	FM	30
USDC	Cellular	1991	TDMA	824-894	DQPSK	30
CDPD	Cellular	1993	FH/Packet	824-894	GMSK	30
IS-95	Cellular/PCS	1993	CDMA	824-894 1800-2000	QPSK/BPSK	1250
FLEX	Paging	1993	Simplex	Several	4-FSK	15
DCS-1900 (GSM)	PCS	1994	TDMA	1850-1990	GMSK	200
PACS	Cordless/PCS	1994	TDMA/FDMA	1850-1990	DQPSK	300

12

Major Mobile Radio Standards - Europe

Standard	Type	Year Intro	Multiple Access	Frequency Band (MHz)	Modulation	Channel BW (KHz)
ETACS	Cellular	1985	FDMA	900	FM	25
NMT-900	Cellular	1986	FDMA	890-960	FM	12.5
GSM	Cellular/PCS	1990	TDMA	890-960	GMSK	200KHz
C-450	Cellular	1985	FDMA	450-465	FM	20-10
ERMES	Paging	1993	FDMA4	Several	4-FSK	25
CT2	Cordless	1989	FDMA	864-868	GFSK	100
DECT	Cordless	1993	TDMA	1880-1900	GFSK	1728
DCS-1800	Cordless/PCS	1993	TDMA	1710-1880	GMSK	200

11

Cellular Networks

- First Generation
 - Analog Systems
 - Analog Modulation, mostly FM
 - AMPS
 - Voice Traffic
 - FDMA/FDD multiple access
- Second Generation (2G)
 - Digital Systems
 - Digital Modulation
 - Voice Traffic
 - TDMA/FDD and CDMA/FDD multiple access
- 2.5G
 - Digital Systems
 - Voice + Low-datarate Data
- Third Generation
 - Digital
 - Voice + High-datarate Data
 - Multimedia Transmission also

11

2G Technologies

	cdmaOne (IS-95)	GSM, DCS-1900	IS-54/IS-136 PDC
Uplink Frequencies (MHz)	824-849 (Cellular) 1850-1910 (US PCS)	890-915 MHz (Europe) 1850-1910 (US PCS)	800 MHz, 1500 MHz (Japan) 1850-1910 (US PCS)
Downlink Frequencies	869-894 MHz (US Cellular) 1930-1990 MHz (US PCS)	935-960 (Europe) 1930-1990 (US PCS)	869-894 MHz (Cellular) 1930-1990 (US PCS) 800 MHz, 1500 MHz (Japan)
Deplexing	FDD	FDD	FDD
Multiple Access	CDMA	TDMA	TDMA
Modulation	BPSK with Quadrature Spreading	GMSK with BT=0.3	$\pi/4$ DQPSK
Carrier Separation	1.25 MHz	200 KHz	30 KHz (IS-136) (25 KHz PDC)
Channel Data Rate	1.2288 Mchips/sec	270.833 Kbps	48.6 Kbps (IS-136) 42 Kbps (PDC)
Voice Channels per carrier	64	8	3
Speech Coding	CELP at 13Kbps EVRC at 8Kbps	RPE-LTP at 13 Kbps	VSELP at 7.95 Kbps

rr

2.5 Technologies

■ Evolution of TDMA Systems

- HSCSD (high-speed circuit switched data) for 2.5G GSM
 - Dynamic TDMA Up to 57.6 Kbps data-rate
- GPRS (general packet radio service) for 2.5G GSM and IS-136
 - Up to 171.2 Kbps data-rate but supports more users than HSCSD
- EDGE (enhance data rate for global evolution) for 2.5G GSM and IS-136
 - Uses 8-PSK Up to 384 Kbps data-rate

■ Evolution of CDMA Systems

- IS-95B
 - Up to 64 Kbps

rr

3G Systems

■ Goals

- Voice and Data Transmission
 - Simultaneous voice and data access
- Multi-megabit Internet access
 - Interactive web sessions
- Voice-activated calls
- Multimedia Content
 - Live music

22

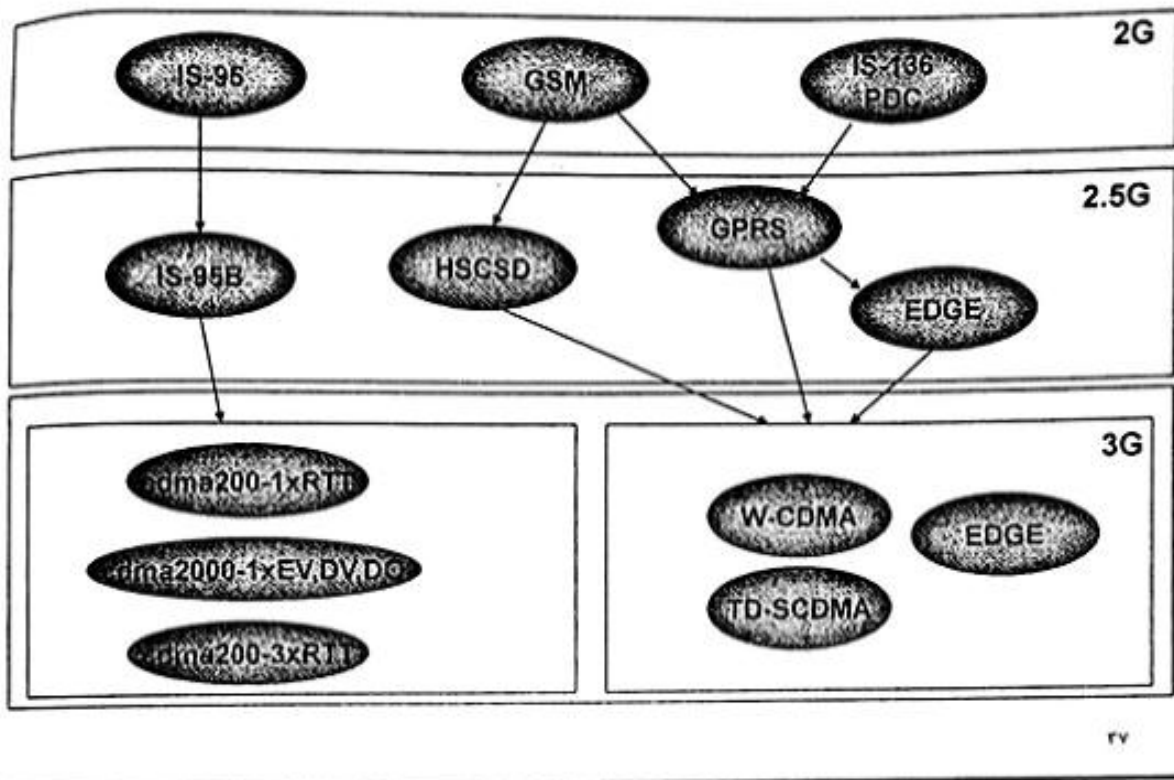
3G Systems

■ Evolution of Systems

- CDMA system evolved to CDMA2000
 - CDMA2000-1xRTT: Upto 307 Kbps
 - CDMA2000-1xEV:
 - CDMA2000-1xEVDO: upto 2.4 Mbps
 - CDMA2000-1xEVDV: 144 Kbps data rate
- GSM, IS-136 and PDC evolved to W-CDMA (Wideband CDMA) (also called UMTS: universal mobile)
 - Up to 2.048 Mbps data-rates
 - Future systems 8Mbps
 - Expected to be fully deployed by 2010-2015
- New spectrum is allocated for these technologies

23

Upgrade Paths for 2G Technologies



Objectives and Compromises in Designing Cellular Systems

- Objectives
 - Maximize users per MHz
 - Maximize users per cell site
 - Base stations are too costly
- Compromises made for these objectives
 - High power transmitters at base stations to increase the range
 - High power transmitters for user-sets
 - High user-set complexity
 - High power consumption due to complex digital signal processing
 - High network complexity

Features of Cellular Systems

- Low bit-rate speech coding: ≤ 13 Kb/s
 - Increases the number of users per MHz and cell-site
 - Decreases the voice quality
 - Some systems (like CDMA) make use of speech inactivity
- High Transmission Delay
 - 200 ms round-trip time
- High-complexity DSP (digital signal processing)
 - For speech coding and de-modulation

11

Features of Cellular Systems

- Fixed Channel Allocation
 - Channels are allocated to cell-sites statically
 - Dynamic allocation is complex to work also with handoffs
- Frequency Division Duplex
 - Network and system complexity for providing synchronization between network elements is relieved
 - TDD requires synchronized clock at both ends
- Mobile handset power control
 - Decreases the co-channel interference
 - Hence increases the system capacity
 - Implemented in CDMA systems

12

4G Wireless Technologies

4G wireless communications are developed for high speed broadband mobile capabilities.

Applications:-

- Wireless Broadband Internet Access
- Video Chat
- Mobile Television
- HDTV (High Definition TV)
- DVB (Digital Video Broadcasting)
- High Speed Data Transfer

4G Wireless Technologies

Main 4G

- WiMAX (Worldwide Interoperability for Microwave Access)
- 3GPP LTE (3rd Generation Partnership Project Long Term Evolution)

Focusing on mobility and broadband

UMB (Ultra Mobile Broadband)

Flash-OFDM (Fast Hopped OFDM)

WIMAX

- Provides up to 75 Mbps data rate
 - Uses OFDMA (orthogonal frequency division multiplexing)
 - High spectral efficiency 3-4 bits/sec/Hz
 - Techniques enabling high data rate
 - Uses adaptive modulation (higher power is assigned for weak channels according to waterfilling principle)
 - Use of smart antennas for beamforming
 - Use of multiple antennas for transmit diversity
- (MIMO) to
- reduce fading
 - Use of Error Correcting codes

LTE Long Term Evolution

Up Link: OFDM

Down Link: Single carrier OFDM (SC-OFDM) to reduce the high Peak – to-Average Ratio (PAR) of OFDM that causes problems to amplifiers due to the non-linearity region

Bandwidth: choice from 2-20 MHz.

Multi –Antennas: Up to 4x4 MIMO (At user and base station)

MIMO: 1- Spatial multiplexing (Increase data rate)(sending different data)

2- Transmit diversity (sending dependant data)

3- MIMO Precoding to maximize SINR

Coding: Uses Turbo Codes with interleaving.

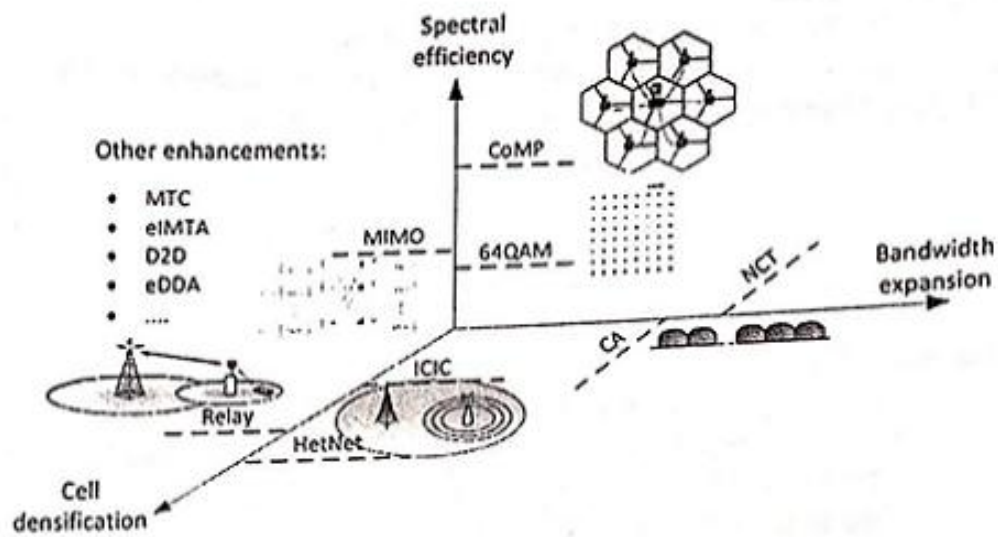
Downlink peak data rates (64QAM)

100 Mbps(SISO), 172.8 Mbps (2x2 MIMO) 326.4 Mbps (4x4 MIMO)

Uplink peak data rates (single antenna)

50Mbps (QPSK) 57.6Mbps(16QAM) 86.4Mbps(64QAM)

● The three dimensions for capacity improvement



5G enabling Technologies

Goals:

- 100 Mbps for mobile users
- 1Gbps for fixed users

using any or combinations of the following three approaches:

- Additional spectrum (bps),
 - Increase spectral efficiency (bps/Hz)
 - Dense deployments - femto cells (bps/Hz/Km).
- 20-60 MHz channel BW
 - Opportunistic OFDMA (Multiuser diversity)
 - cognitive radio,
 - cooperative methods,
 - distributed MIMO and
 - Massive MIMO

5G enabling Technologies

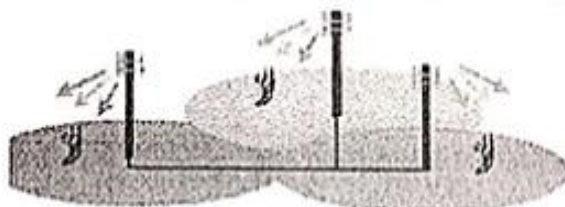
Cooperative Diversity

- . MIMO size is limited for portable devices.
- . An alternative for the MIMO spatial diversity, cooperation of in-cell users.
 - 1- One user may use another's user's resources to improve his transmission rate.
 - 2- A Relay node may be added to assist all users.
- . Requires channels knowledge.

Distributed antenna systems

- . Antenna elements are spatially distributed in the cell
- . Each distributed antenna element is connected to BS by fiber optics or LOS.
- . This acts as a large MIMO system.
- . Disadvantage requires channel knowledge

Where is MIMO Headed?



Coordinated MIMO



Massive MIMO



mmWave MIMO

Candidate architectures for 5G cellular

Wireless Local Area Networks (WLAN)

- Are low-power short range devices for providing private computer communications over a workplace

WLAN

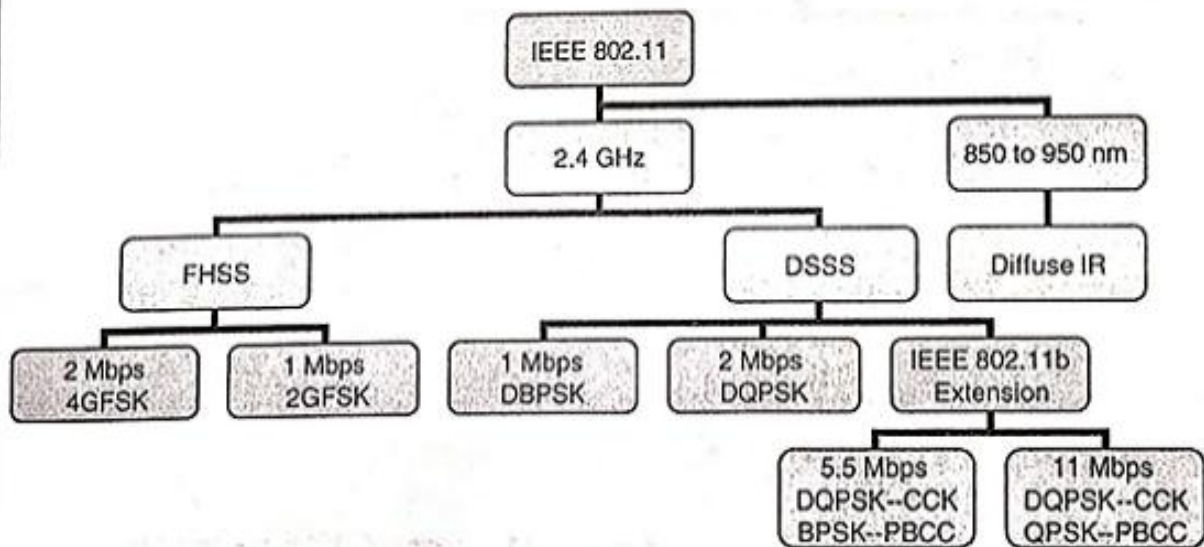


Figure 2.10 Overview of the IEEE 802.11 Wireless LAN standard.

Table 2.4 IEEE 802.11b Channels for Both DS-SS and FH-SS WLAN Standards

Country	Frequency Range Available	DSSS Channels Available	FHSS Channels Available
United States	2.4 to 2.4835 GHz	1 through 11	2 through 80
Canada	2.4 to 2.4835 GHz	1 through 11	2 through 80
Japan	2.4 to 2.497 GHz	1 through 14	2 through 95
France	2.4465 to 2.4835 GHz	10 through 13	48 through 82
Spain	2.445 to 2.4835 GHz	10 through 11	47 through 73
Remainder of Europe	2.4 to 2.4835	1 through 13	2 through 80

Bluetooth and Personal Area Networks (PAN)

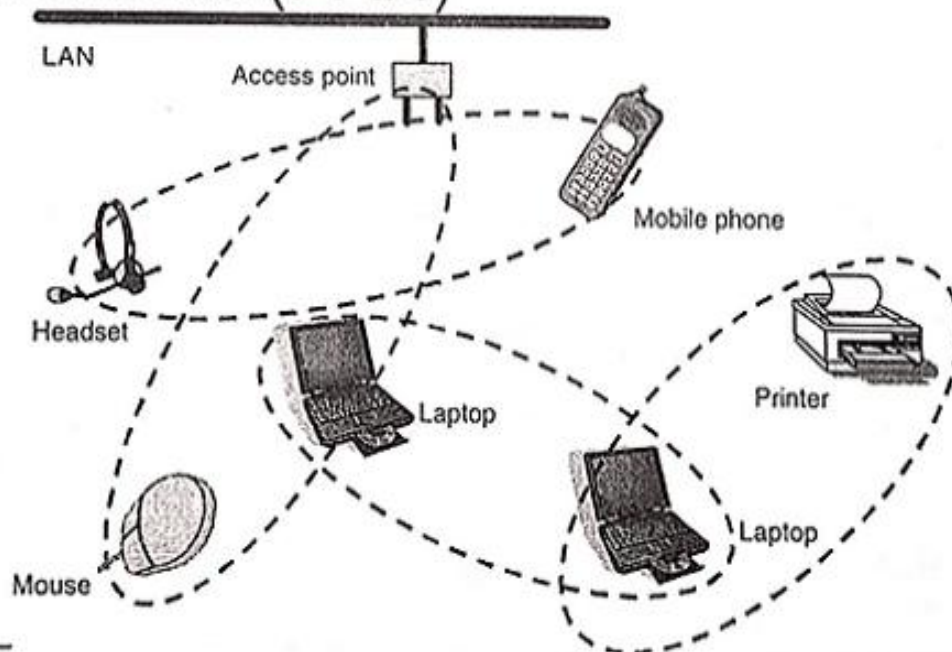


Figure 2.17 Example of a Personal Area Network (PAN) as provided by the Bluetooth standard.

Bluetooth and Personal Area Networks (PAN)

There is a great user appreciation of removing wires from various devices (printers, mouse, headphones,..etc).

- . Bluetooth operates in the (2400-2483.5 MHz) band.
- . Employs frequency Hopping 1600 Hop/sec.
- . One or more data packets over each slot.
- . Each channel has a 1MHz BW and 1 Mbps data rate
- . Uses GFSK modulation
- . Due to FH, it can stand very high interference levels.

Sectoring improves S/I

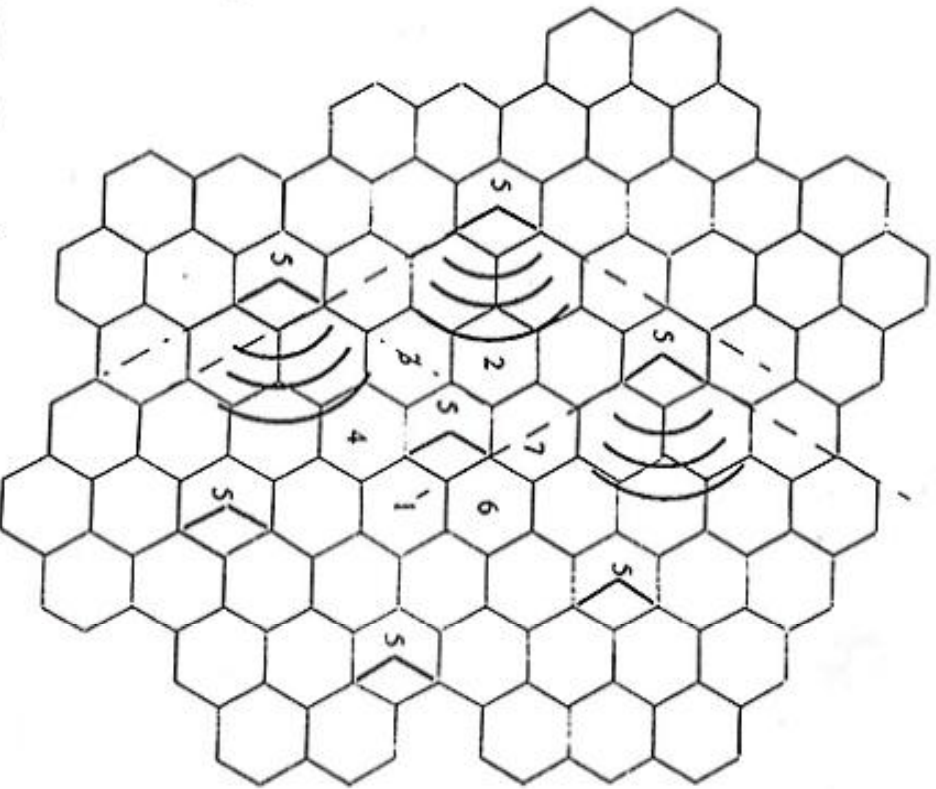


Figure 3.11 Illustration of how 120° sectoring reduces interference from co-channel cells. Out of the 6 co-channel cells in the first tier, only two of them interfere with the center cell. If omnidirectional antennas were used at each base station, all six co-channel cells would interfere with the center cell.

Cells are split to add channels with no new spectrum usage

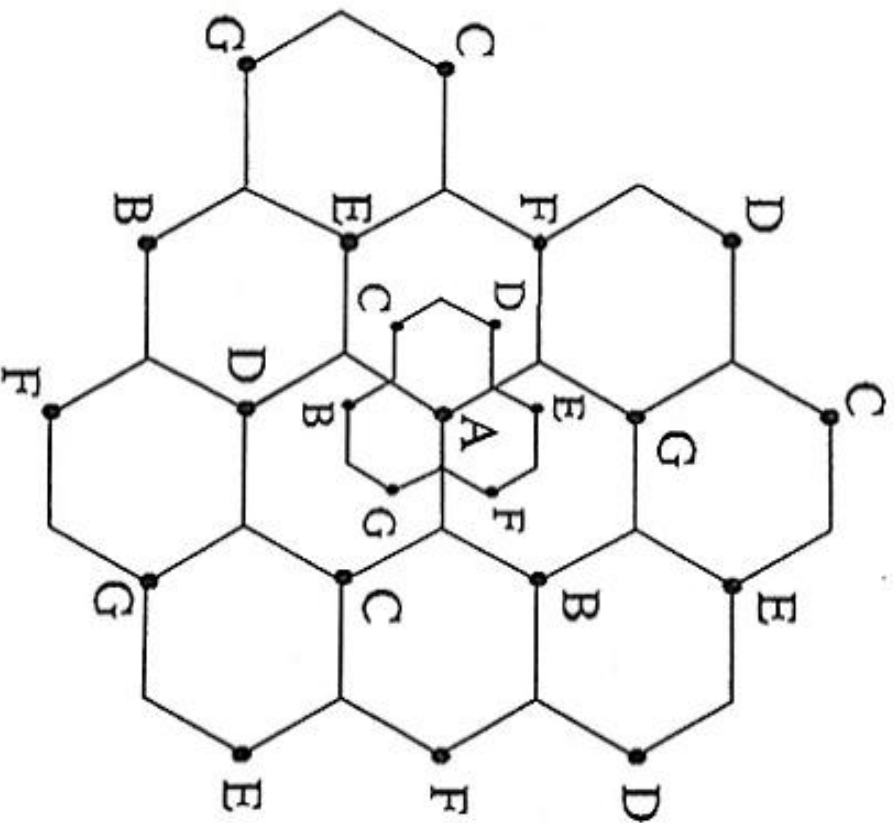


Figure 3.8 Illustration of cell splitting.

Sectoring improves S/I

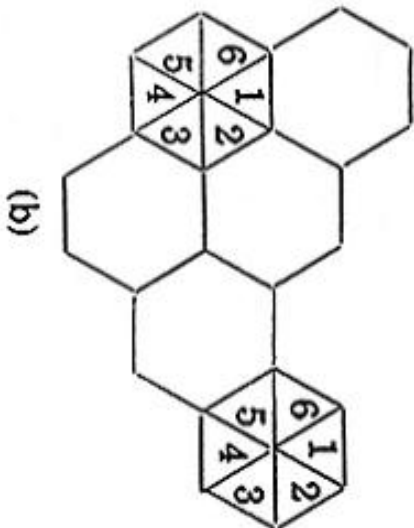
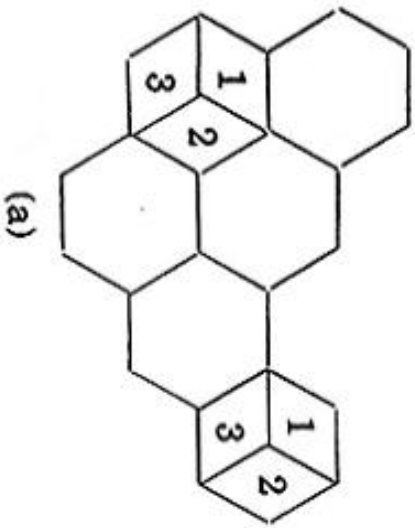


Figure 3.10 (a) 120° sectoring; (b) 60° sectoring.

19-cell reuse example ($N=19$)

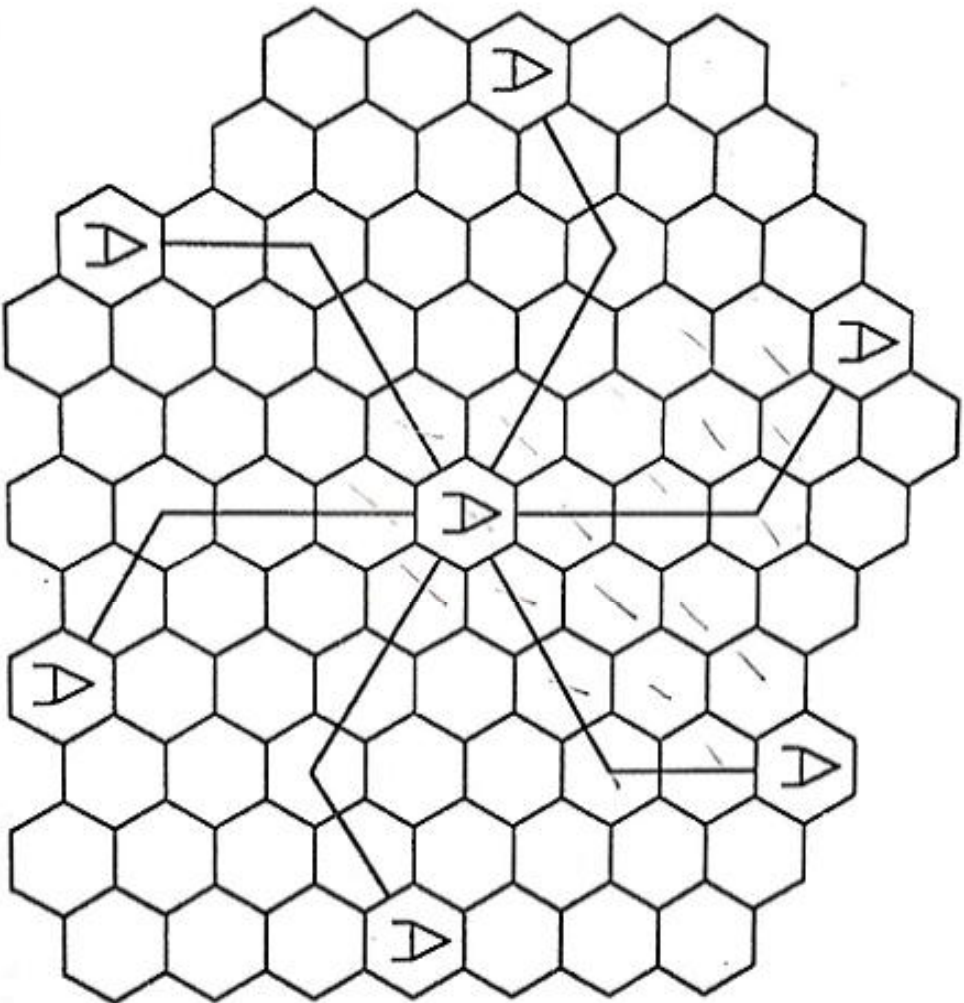


Figure 3.2 Method of locating co-channel cells in a cellular system. In this example, $N = 19$ (i.e., $I = 3$, $J = 2$). (Adapted from [Oet83] © IEEE.)

Co-channel cells for 7-cell reuse

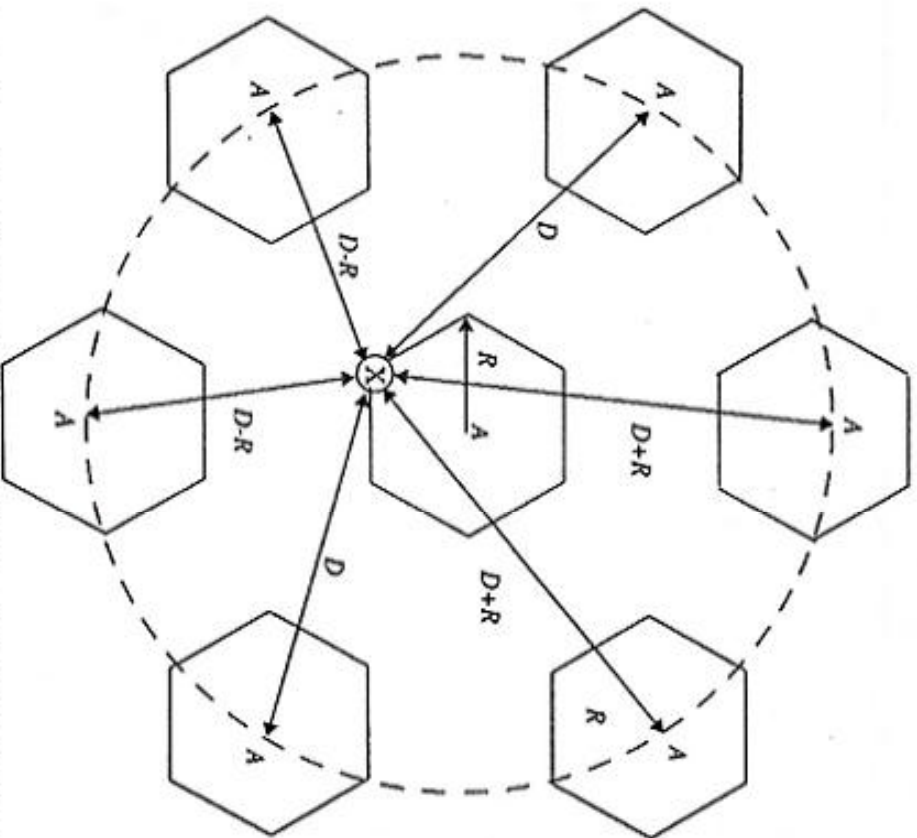


Figure 3.5 Illustration of the first tier of co-channel cells for a cluster size of $N = 7$. An approximation of the exact geometry is shown here, whereas the exact geometry is given in [Lee86]. When the mobile is at the cell boundary (point X), it experiences worst case co-channel interference on the forward channel. The marked distances between the mobile and different co-channel cells are based on approximations made for easy analysis.

Erlang B

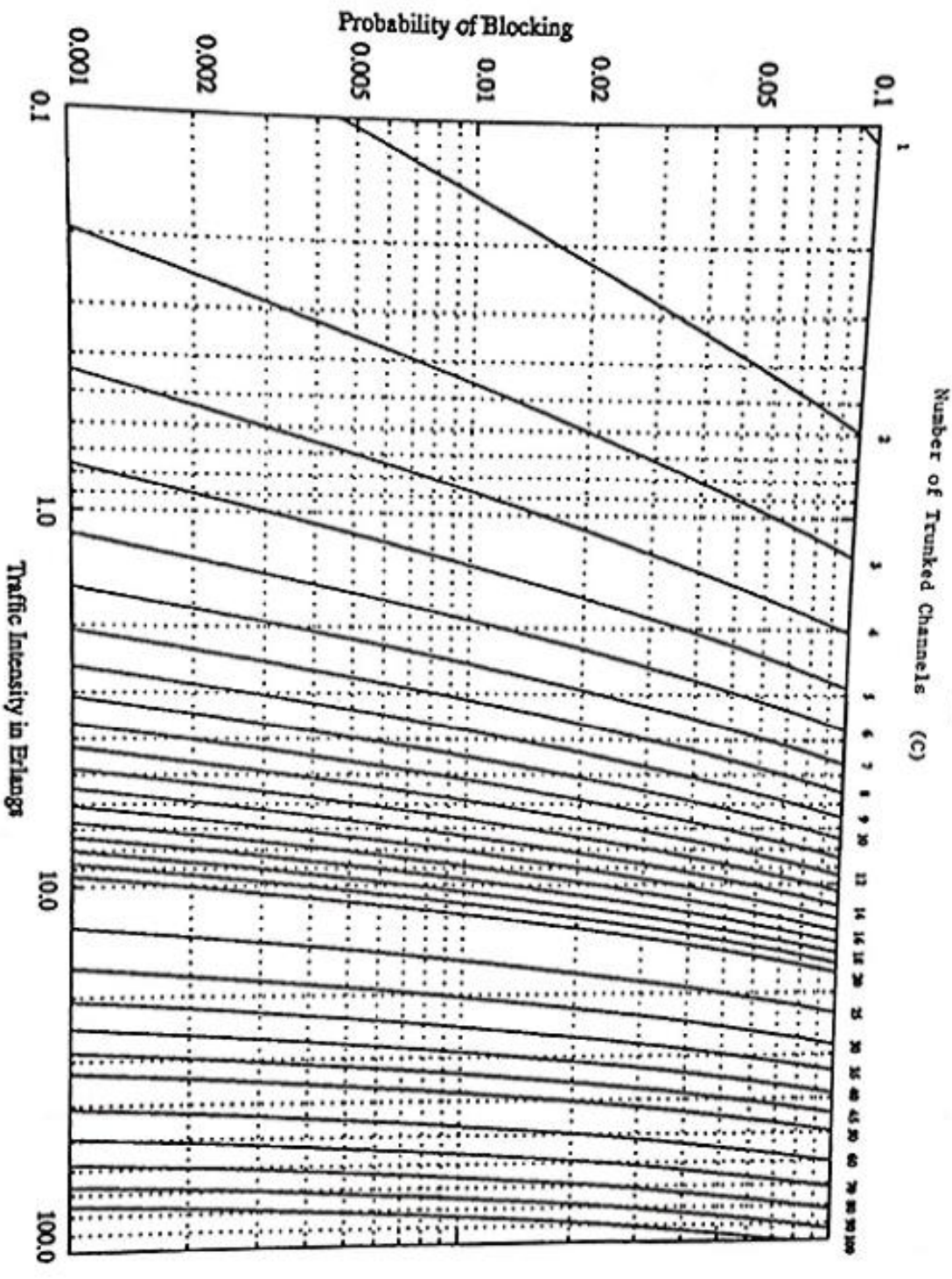


Figure 3.6 The Erlang B chart showing the probability of blocking as functions of the number of channels and traffic intensity in Erlangs.

Erlang C

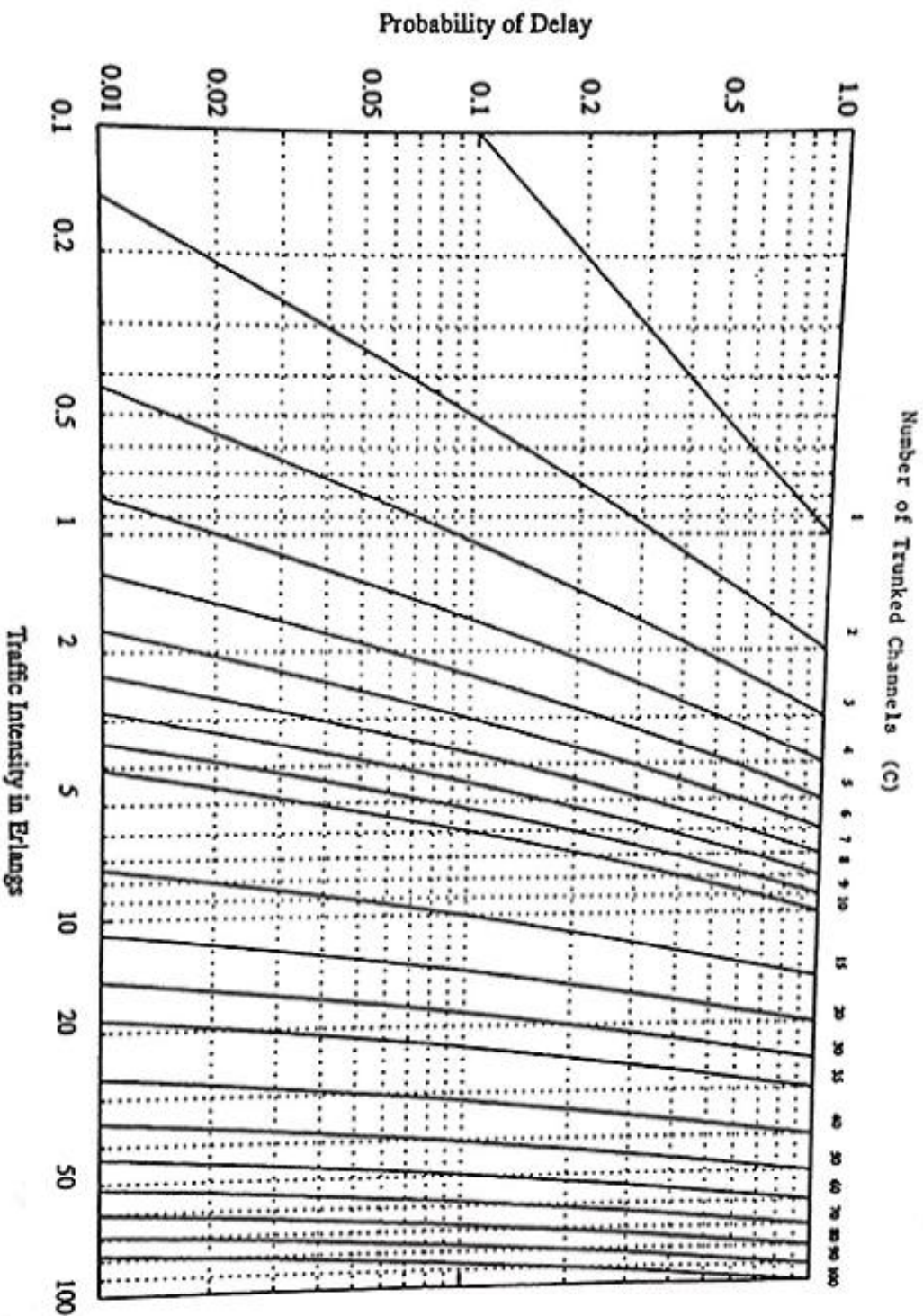
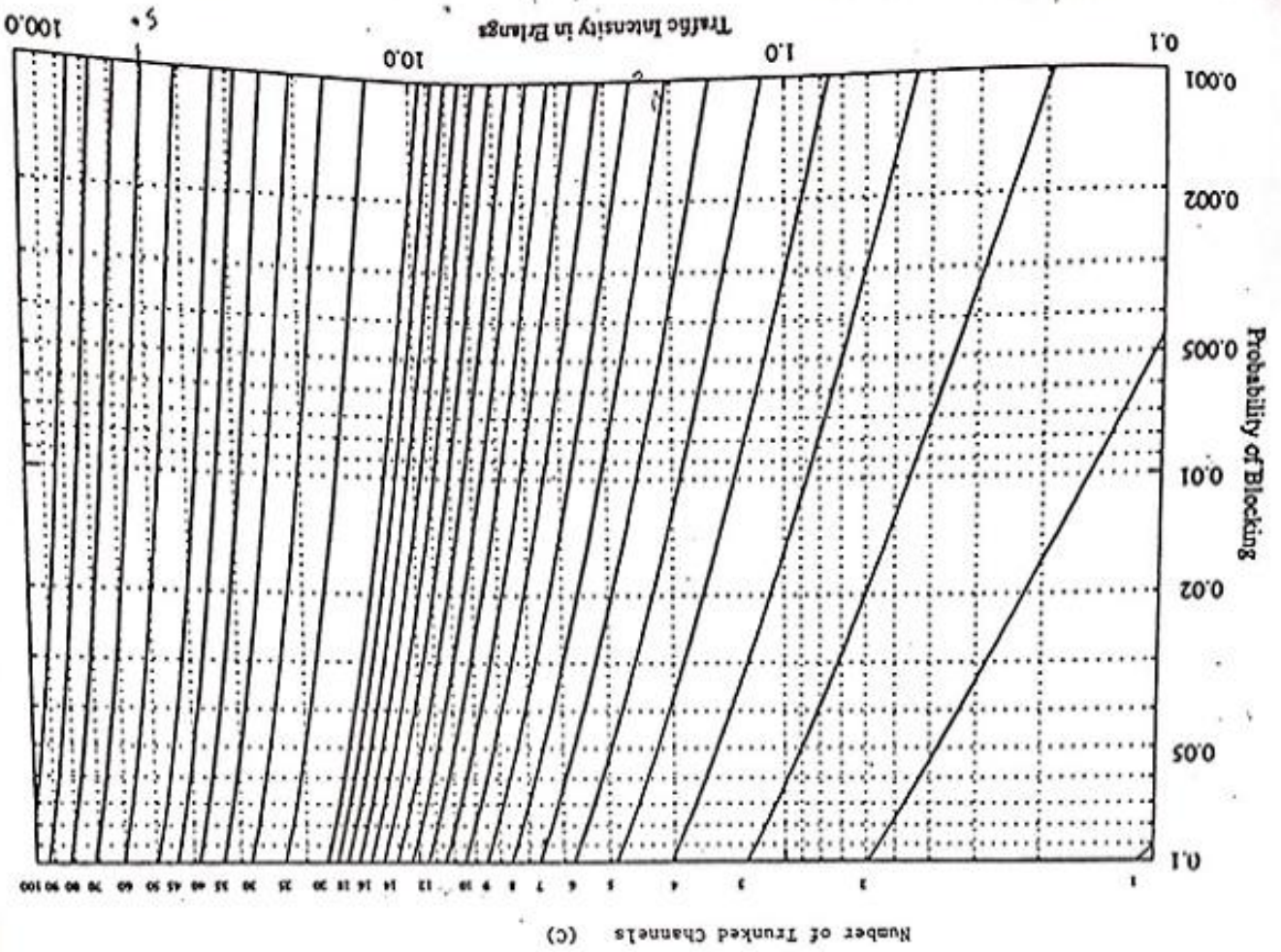
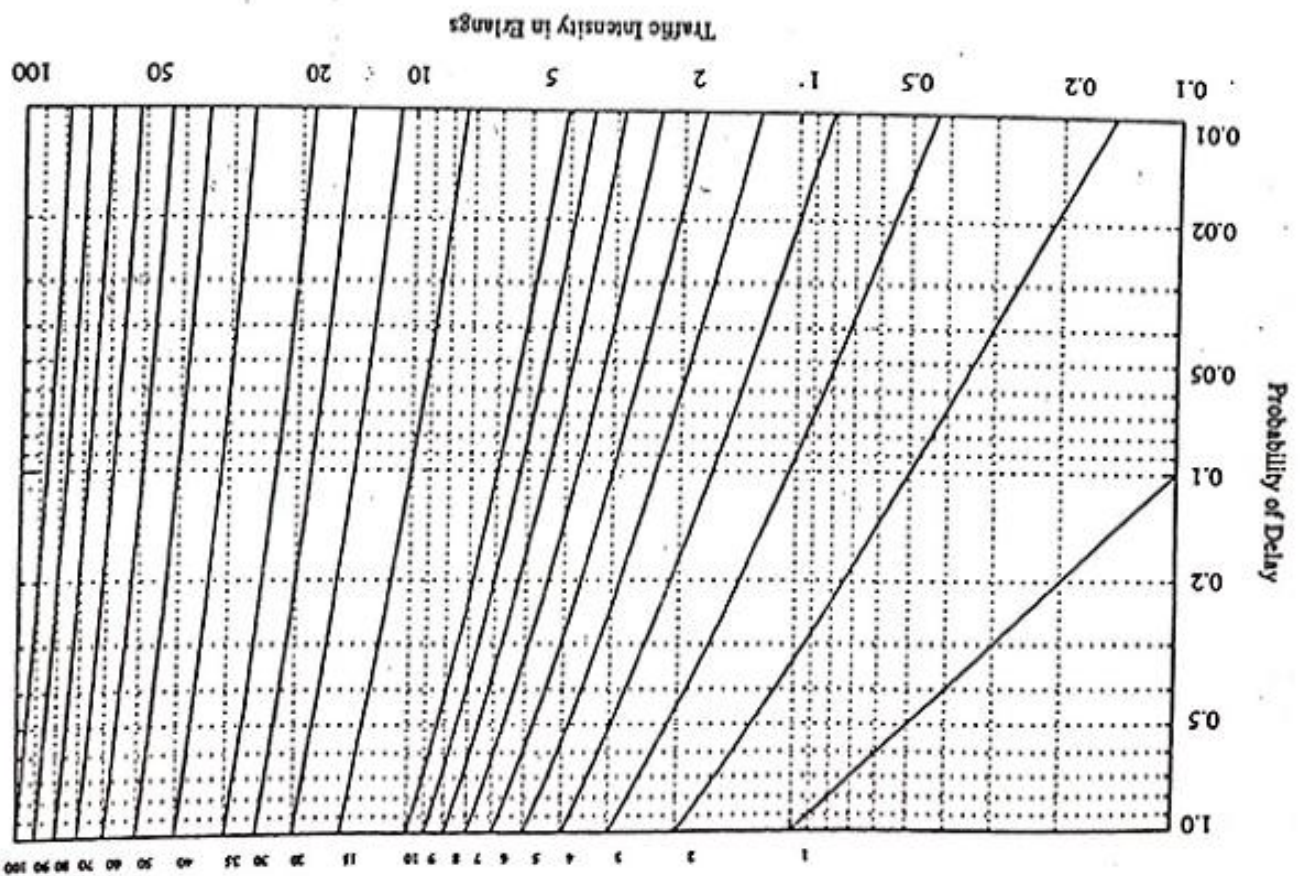


Figure 3.7 The Erlang C chart showing the probability of a call being delayed as a function of the number of channels and traffic intensity in Erlangs.



(C) Number of Trunked Channels



Chapter 3 :- 1

Cellular System Concepts :

* The bandwidth or Spectrum is Scarce & expensive. !! Use freq. reuse.

Frequency reuse : The total available radio channels (S) is distributed among a number of cellular base stations (BS), so that they can be reused again in a adjacent geographical area



Clustering.

Cell \rightarrow Cells \equiv Cluster \rightarrow Clusters \equiv Cellular Sys.

let N : Cluster Size (# of Cells / Cluster)

k : # of Channels / Cell.

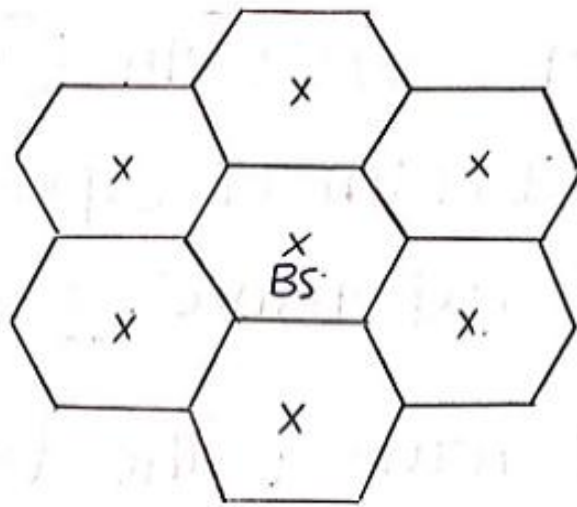
m : # of Clusters / System.

$\rightarrow N \times k$: # of Channels / Cluster.

* System Capacity (C) = kNm .

\rightarrow Target is to make (C) large as possible.

7 Cell / cluster :-



Example: $N=7$, $m=3$, & $k=18$

cells/cluster

clusters/system

channels/cell

• ہر آؤس میں

• 7 cells

$$C = 7 \times 3 \times 18 = 378 \text{ Channel/system}$$

or: user/system

@ instant of time.

If $N=3$, $m=7$, & $k=?$

for the same (S).

$$k = \frac{18 \times 7}{3} = 42 \text{ channel/cell.}$$

$$C_{\text{new}} = 42 \times 3 \times 7 = 882 \text{ Channel/system.}$$

AS $N \uparrow$, $C \downarrow$ & $CCI \downarrow$

AS $N \downarrow$, $C \uparrow$ & $CCI \uparrow$

cochannel interference. 2

* Cell-Shape usually used hexagon geometry.

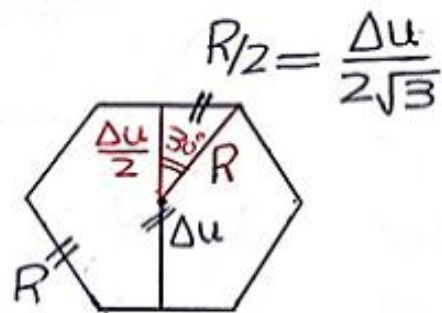
* Theorem:

$$N = i^2 + j^2 + ij, \quad (i, j : \text{integer} \geq 1)$$

آقل $N = 3$

اختیاره
لیس عشوار

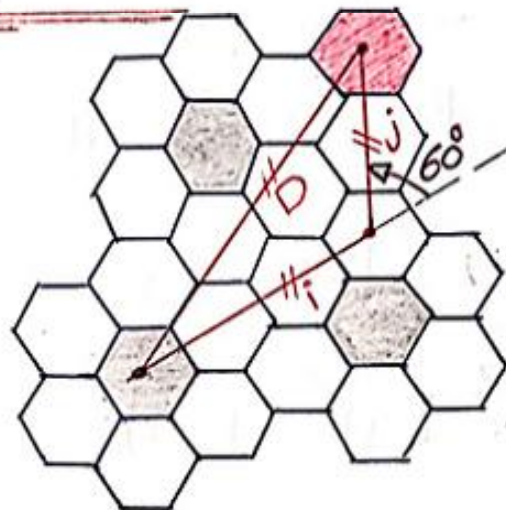
$$\Delta u = \sqrt{3} R$$



area of hexagon

= area of \triangle * 12 .

$$= 12 * \frac{1}{2} * \frac{\Delta u}{2\sqrt{3}} * \frac{\Delta u}{2}$$



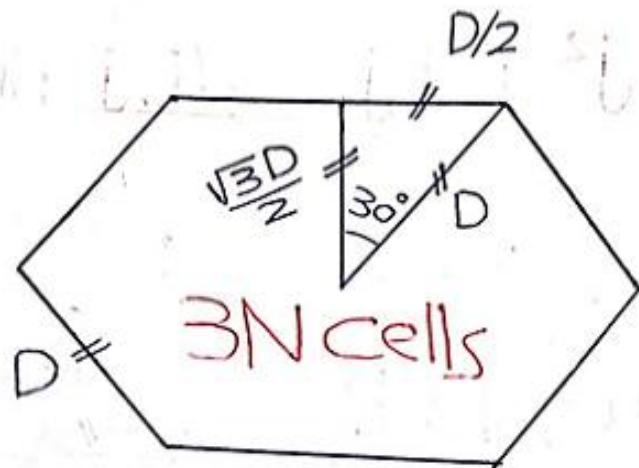
i - times
60° CCW
j - times
↓
CCI

From using cosine law:-

$$D^2 = (i \Delta u)^2 + (j \Delta u)^2 - 2 i \Delta u \cdot j \Delta u \cdot \cos 120^\circ$$

3

$$\rightarrow D^2 = (i^2 + j^2 + ij) \Delta u^2$$



$$\text{area} = \frac{1}{2} \cdot \frac{\sqrt{3}D}{2} \cdot \frac{D}{2} \times 12 = \frac{3\sqrt{3}}{2} D^2$$

$$= \underbrace{3 \cdot N}_{\text{\# of cells}} \cdot \frac{1}{2} \cdot \frac{\Delta u}{2} \cdot \frac{\Delta u}{2\sqrt{3}} \times 12$$

cell area.

$$D^2 = (\Delta u)^2 N$$

$$\therefore N = i^2 + j^2 + ij$$

physically: $i, j \rightarrow$ record.

19-cell reuse example ($N=19$)

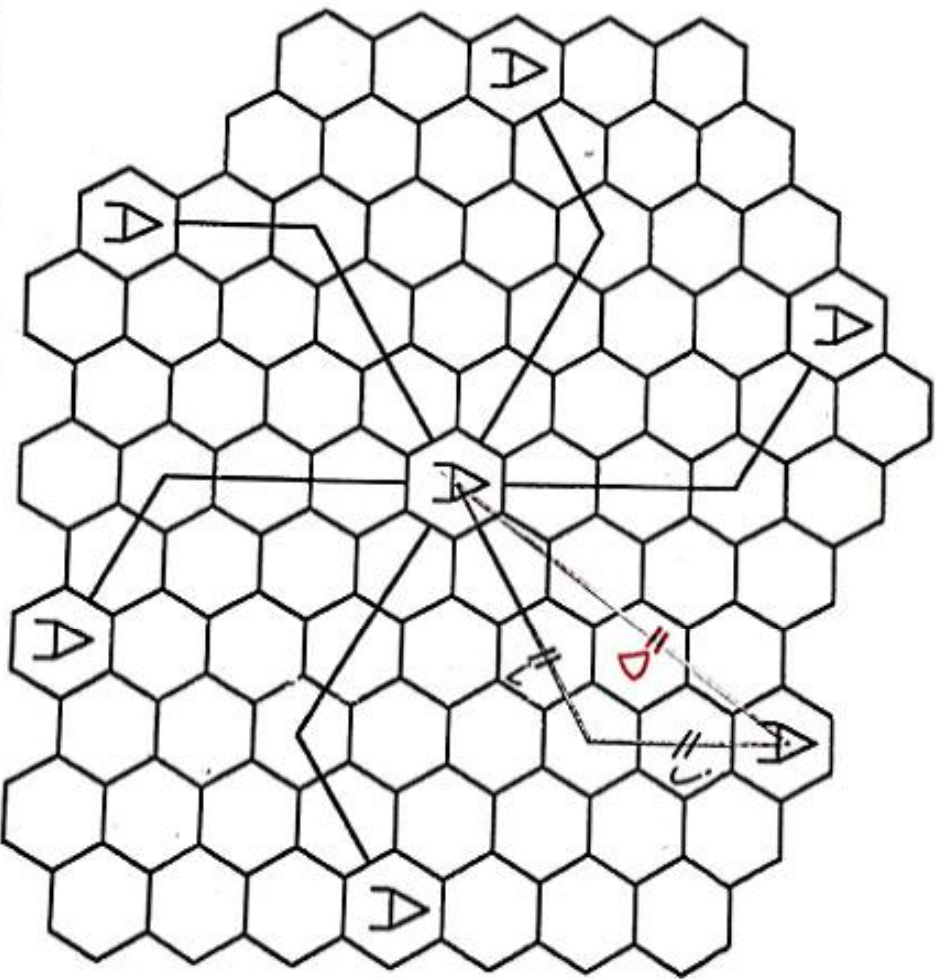


Figure 3.2 Method of locating co-channel cells in a cellular system. In this example, $N = 19$ (i.e., $l = 3$, $j = 2$). (Adapted from [Oet83] © IEEE.)

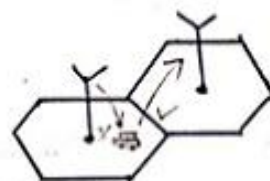
Channel Assignment Method :

Channels Assigned to Cells

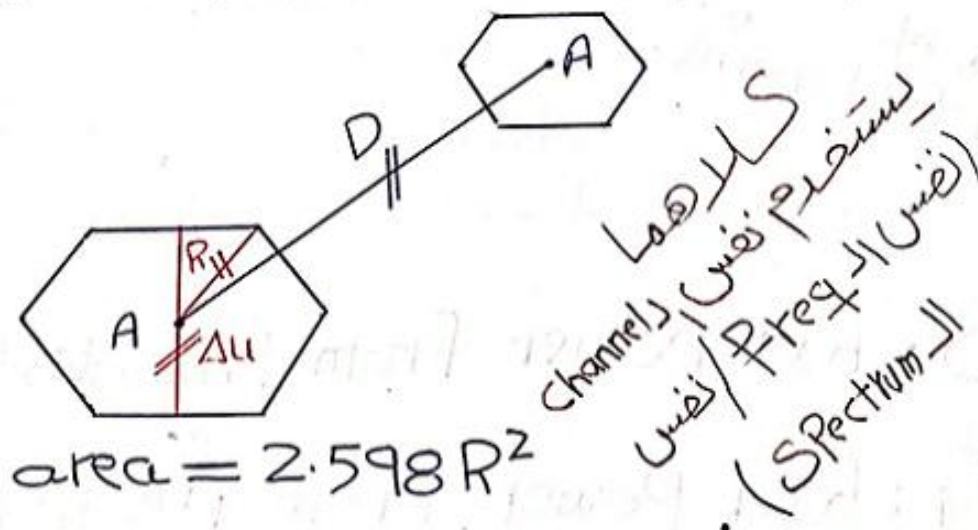
- a) Fixed Assignment (Static).
 - ↳ not change based on traffic.
 - ↳ **predetermined**, in each cell fixed no. of channels.
 - ↳ suffer from blocking problem.
- b) Dynamic Assignment (on demand)
 - ↳ change based on traffic.
 - ↳ better performance → complexity * *لكن عن حساب التعقيد*

Hand off (Hand over) :

When a mobile station moves from one cell to another cell, it should hand off the new base station.



CoChannel interference (CCI) :



CCI Ratio (Φ) $\triangleq \frac{D}{R}$

Ratio \uparrow ; interference \downarrow .

$$D^2 = N \Delta u^2 = 3NR^2.$$

D: distance between two adjacent interference.

$$\Phi = \sqrt{3N}.$$

Note that : Signal - to - interference ratio (SIR) :-

$$SIR = \frac{S}{\sum_{i=1}^L I_i}$$

no. of interference.

$\rightarrow \text{max.} = 6$
 أسوأ الحالات
 ياؤن كل 6 الی و الی
 بقترو نفس ال freq
 أو ال Channel أو ال Spectrum الموجودة عندك.

S: Rxd power from the desired BS.

I_i : Rxd Power from i th interference.

$$S = \frac{P_t}{(R)^n}, \quad n : \text{path loss exponent} > 1.$$

$$I_i = \frac{P_{ti}}{(D_i)^n}$$

assume $P_t = P_{ti}$: transmitted Power.

$$D_i = D$$

$$SIR = \frac{(D/R)^n}{i_0} = \frac{Q^n}{i_0}$$

Example: $SIR \geq 18 \text{ dB} \geq 63.1 = 10^{1.8}$

$$i_0 = 6, \quad n = 4$$

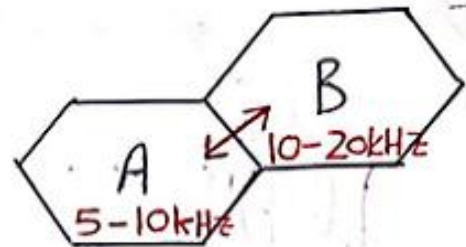
$$SIR = \frac{Q^n}{i_0} \rightarrow Q = (63.1 \times 6)^{\frac{1}{4}} = 4.411$$

$$N \geq 6.49 \therefore i^2 + j^2 + ij \quad \boxed{N=3, SIR > 18 \text{ dB}}$$

أقرب N مسوية 7 cells cluster
or 12, 19, ...

6 و 3 مابتحقق القانون

Adjacent Channel interference:



A	B	C	D	E	}	Bad.
1	2	3	4	5		
6	7	8	9	10		

A	B	C	D	E	}	Good.
1	3	5	4	6		
2	8	10	7	9		

Table (3.2)

power allocation :

Dynamic BS reduces Tx power as it moves toward MS, & Vice Versa.

3

Trunking & Grade of Service (GOS):
To allow a large no. of user to Share the relatively small no. of Channels in the Cell providing access to each user on demand; From a pool of available Channel, let:

λ : avg. no. of Calls / unit time.

H : avg. duration of the Call (S).

A_u : traffic intensity / user.

$$A_u = \lambda H \text{ (Erlang).}$$

A : total traffic intensity / System.

$$A = U A_u \text{ (Erlang).}$$

& U is no. of users / System (Cell/Cluster/or Total system).

Erlang B-Chart : (Probability of blocking)

Assumptions:

- 1- no. of Calls / unit has Poisson distribution
- 2- Call duration (H) is exponential R.V.
- 3- finite no. of Channels.

$$P_r [\text{blocking}] = \frac{A_c / C!}{\sum_{k=0}^C \frac{A^k}{k!}}$$

of truncated channels.

Example: we have 394 cells urban Area with $C = 19$ channels/cell,

$\lambda = 2$ calls/hr., $H = 3$ min/call, &

GOS = 2% :

a. $A_u = \lambda H$
 $= \frac{2}{60} \times 3 = 0.1$ Erlang.

b. traffic intensity in system ?

From Chart : $A = 12$ Erlang.

c. no. of users ?

1-Per Cell $\rightarrow A = A_u U \rightarrow U = \frac{A}{A_u} = 120.$

2-Per System $\rightarrow U * \# \text{ of cell}$
 $= 120 * 394$
 $= 47,280.$

2

Erlang B

Number of Trunked Channels (C)

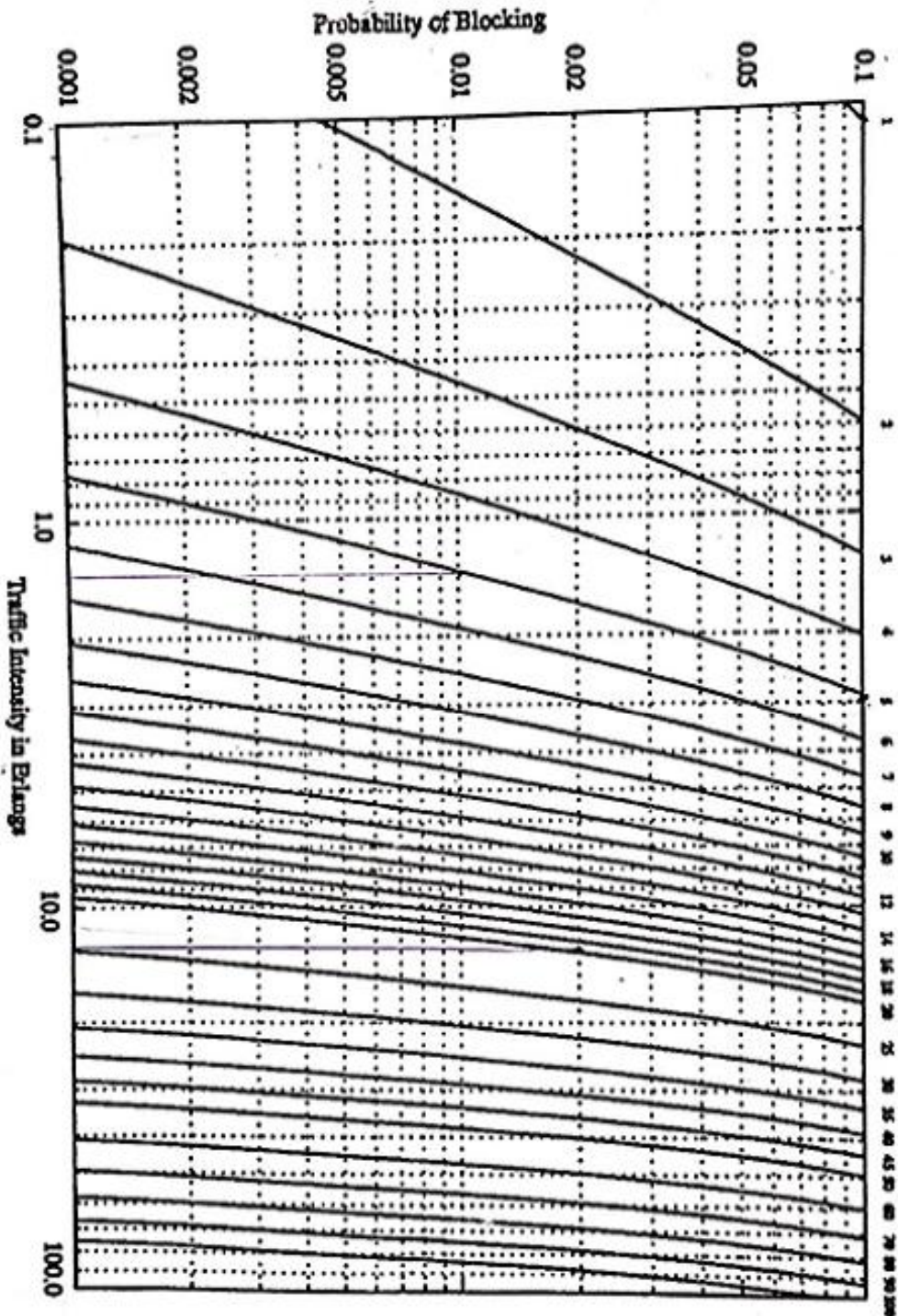


Figure 3.6 The Erlang B chart showing the probability of blocking as functions of the number of channels and traffic intensity in Erlangs.

Erlang C-Chart : (probability of delay)

$$P_r[\text{delay} > 0] = \frac{A^c}{A^c + C! \left(1 - \frac{A}{C}\right) \sum_{k=0}^{c-1} \frac{A^k}{k!}}$$

where :

A : traffic intensity (Erlang).

C : no. of truncated Channels.

$$P_r[\text{delay} > t] = \underbrace{P_r[\text{delay} > 0]}_{\text{From C-Chart}} * \underbrace{P_r[\text{delay} > t / \text{delay} > 0]}_{e^{-\frac{(C-A)t}{H}}}$$

Example: Cell hexagon, $R = 1.387$ km,
60 Channels/system,

$$A_u = \lambda * H = 0.029 \text{ Erlang.}$$

$$\lambda = 1 \text{ call/hr.}, N = 4 \text{ Cell.}$$

$$P_r[\text{delay} > 0] = 0.05 (5\%)$$

a) # of users/km² ?

$$C = \frac{60}{4} = 15 \text{ Channel/Cell.}$$

From C-chart, $A = 9$ Erlang.

$$\therefore \text{Total \# of user/Cell} = \frac{A}{A_u} = 310 \text{ users}$$

$$\text{A} \quad A = 2.598 R^2 \\ = (2.6)(1.387)^2 = 5 \text{ km}^2.$$

$$\# \text{ of users/km}^2 = \frac{310}{5} = 62 \text{ user/km}^2$$

b) $P_r[\text{delay} > 10\text{s} / \text{delay} > 0]$?

$$H = \frac{0.029}{1 \text{ hr.}} \times 3600 = 104.4 \text{ sec.}$$

$$P_r[\text{delay} > 10\text{s} / \text{delay} > 0] = e^{-\frac{(15-9)10}{104.4}} \\ = 0.563.$$

c) $P_r[\text{delay} > 10\text{s}]$

$$= 0.05 \times 0.563 = 0.028$$

$t \uparrow$, $P_r[\text{delay} > t / \text{delay} > 0] \downarrow$

4

Erlang C

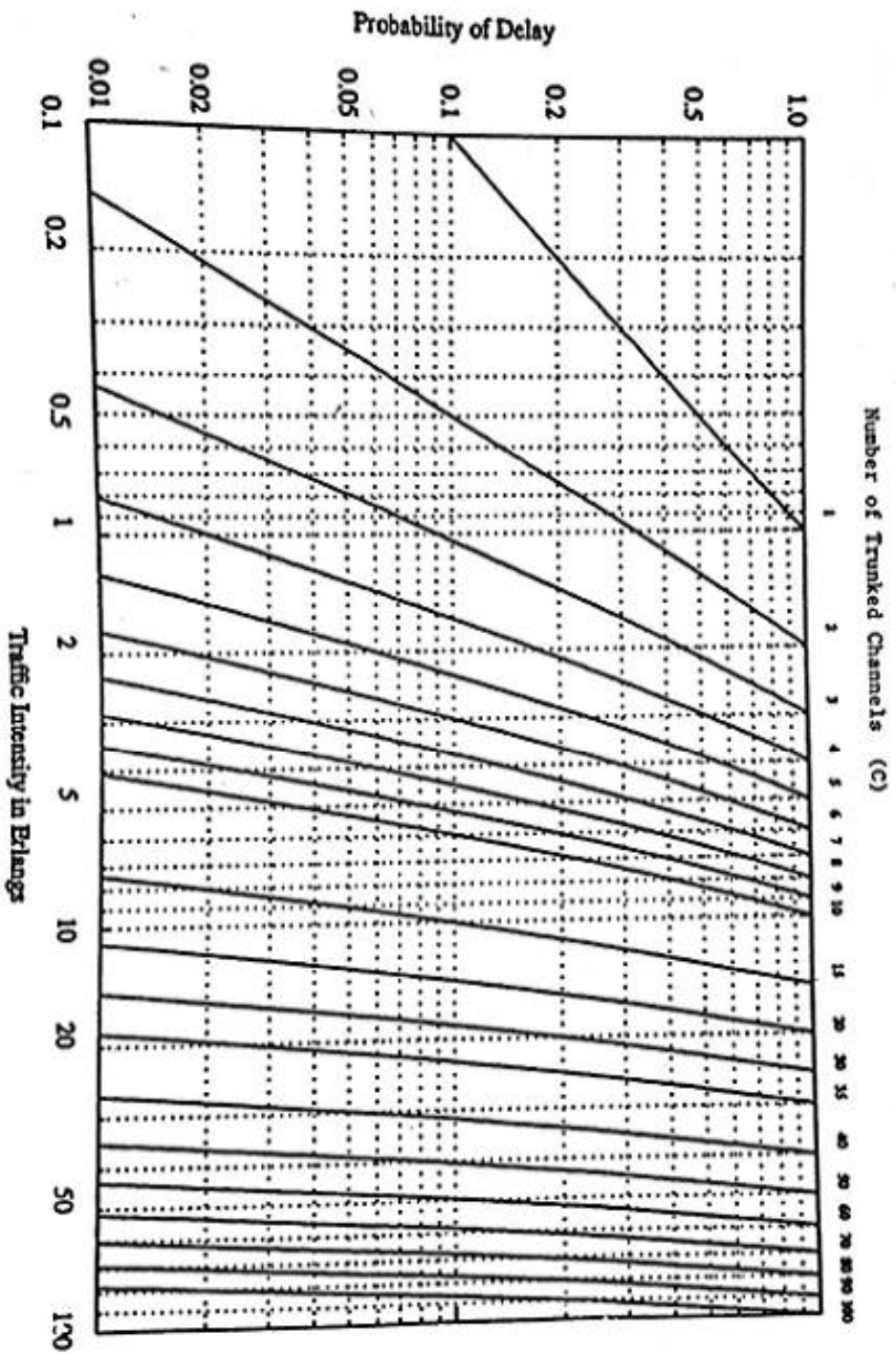


Figure 3.7 The Erlang C chart showing the probability of a call being delayed as a function of the number of channels and traffic intensity in Erlangs.

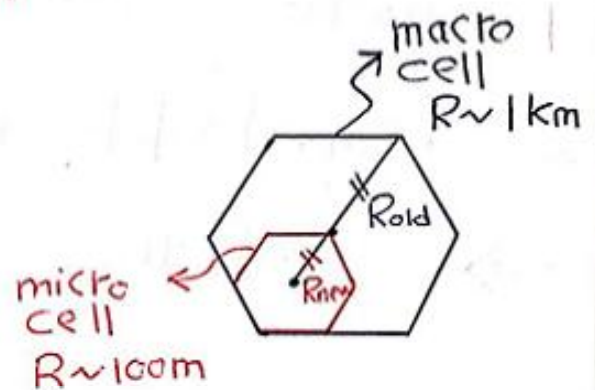
* Improving Capacity of Cellular Sys.:

□ Cell Splitting: (Fig. 3.8).

Decreasing the Cell radius (R) (size) & keep the CoChannel interference Ratio

$$\left(\Phi = \frac{D}{R} \right) \rightarrow R \text{ بتلايا بنفس نسبة } \Phi \text{ كما كان في السابق.}$$

$$= \sqrt{3N}$$



note that no. of Channels/micro cell = no. of Channels/macro cell.

$$(SIR)_{\text{new}} = (SIR)_{\text{old}}$$

$$\frac{P_{\text{trans,new}}}{R_{\text{new}}^n} = \frac{P_{\text{trans,old}}}{R_{\text{old}}^n}$$

$$\rightarrow \text{if } R_{\text{new}} = \frac{1}{2} R_{\text{old}}$$

$$n = 4$$

5!

$R \downarrow$, transmitter power \downarrow

$$\therefore \frac{P_{t,new}}{P_{t,old}} = \frac{1}{16}$$

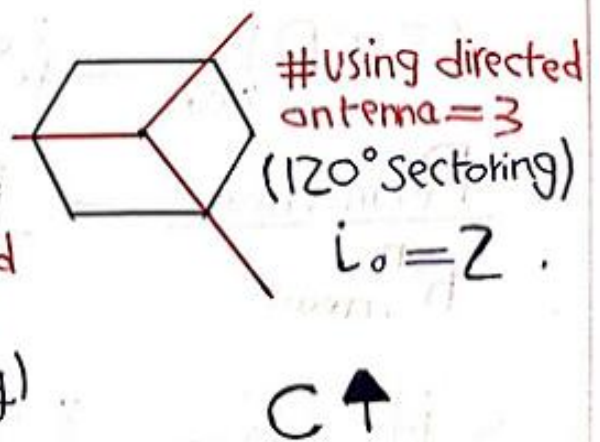
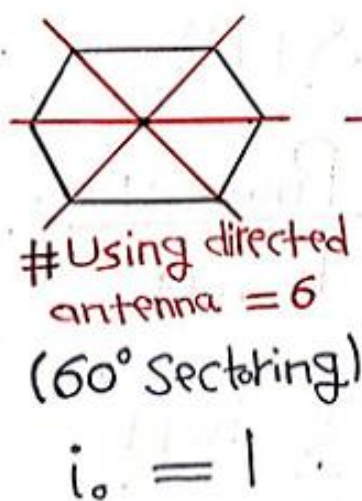
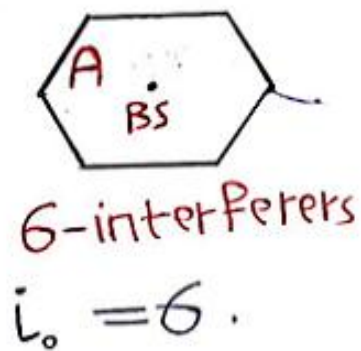
$\therefore C \uparrow$

* disadvantages :-

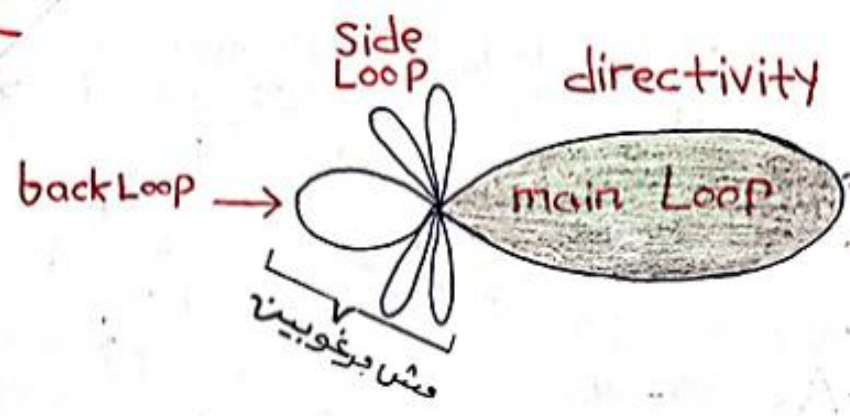
- 1 - more cost \$ of more base station.
 - 2 - hand off more complicated.
- \Rightarrow system complexity \uparrow .

• $P_{t,new} > P_{t,old} \rightarrow (SIR)_{new} > (SIR)_{old}$

2 Antenna sectoring :-



et / the



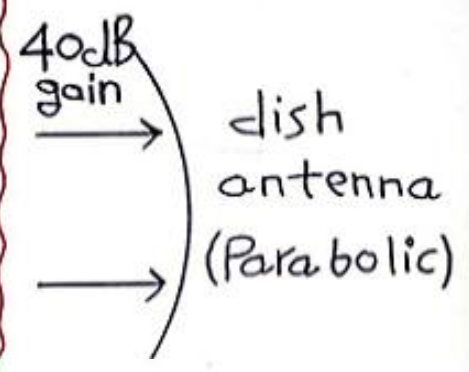
Isotropic
 بيت بكل الاتجاهات بنفس القوة.
 Ideal antenna
 لا بكل sectoring.

Example :- 120° sectoring,
 N = 7, n = 4 :

$$(SIR)_{no\ Sec.} = \frac{(\sqrt{3N})^n}{L_0} = \frac{(\sqrt{21})^4}{6} = 18\text{ dB.}$$

$$(SIR)_{120^\circ\ Sec.} = \frac{(\sqrt{21})^4}{2} = 24\text{ dB.}$$

↓
 6 dB
 advantage



أفضل الأنواع.

op. : N ↓ , (SIR)_{120°} = (SIR)_{no sec.} , C ↑ .

disadvantage :
 Complexity of the system.
 (design) → needs omni-antenna (multi-directional).

Chapter 4 :

Large-Scale Propagation models:

 P_t

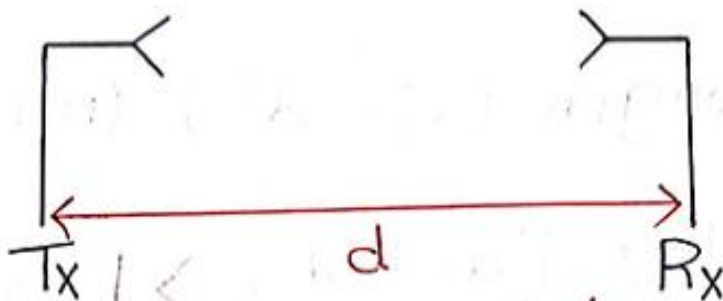
Reflection
diffraction
Scattering

 P_r

Radio waves are subject to several propagation mechanisms (phenomena);

- Reflection.
- diffraction.
- Scattering.

d) direct line of sight (LOS) ??
rare



$d \gg \lambda \rightarrow$ large scale (Chapter 4).

$d \sim \lambda \rightarrow$ Small scale (Chapter 5).

$$C = \lambda f$$

$$\lambda \propto \frac{1}{f}$$

if $f = 1 \text{ GHz}$, $\lambda = 30 \text{ cm}$.

□ Free Space model or (LOS) :
(ideal model)
"reference model".

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}, \quad G = \frac{4\pi A_e}{\lambda^2}$$

↑
Received
Power

A_e : effective
antenna
area.
($A_e > A_{\text{phy}}$)
مستویات
البرق

G_t, G_r : antenna gains. → linear scale
(not dB).

λ : wave length ($C = \lambda f$) (m).

L : Path loss factor, > 1

$L = 1 \rightarrow$ Free
space.

$$P_t > P_r$$

* Path loss :- $(P_t - P_r)$ dB.
(R)

$$\triangleq 10 \log \frac{P_t}{P_r} \text{ in } \underline{\text{dB}}.$$

$$= 20 \log(4\pi d) + 10 \log L - 10 \log(G_t G_r) - 20 \log \lambda$$

(dB).

$$\text{dBm} = 10 \log \left(\frac{P}{1 \text{ mW}} \right).$$

or : $\text{dBm} = \text{dB} + 30$.

Ex: $1 \text{ mW} = 0 \text{ dBm} = -30 \text{ dB}$.

$1 \text{ W} = 30 \text{ dBm} = 1 \text{ dB}$.

Example: $P_t = 50 \text{ W}$, $f = 900 \text{ MHz}$, $\lambda = \frac{1}{3} \text{ m}$

$G_t = 1$, $G_r = 2$, $L = 1$,

$P_r(10 \text{ km}) = -91.5 \text{ dB}$

or -61.5 dBm .



$$P_L = 10 \log(50) + 91.5$$

$$= 108.5 \text{ dB}.$$

* Propagation mechanisms :

a) Reflection :-

Occurs, when the wave length (λ) hits an object with dimensions $\gg \lambda$.

(buildings, cars, mountains, ...).



b) Diffraction :-

When the wave length λ hits a surface with sharp irregularities (edges).
 حواف



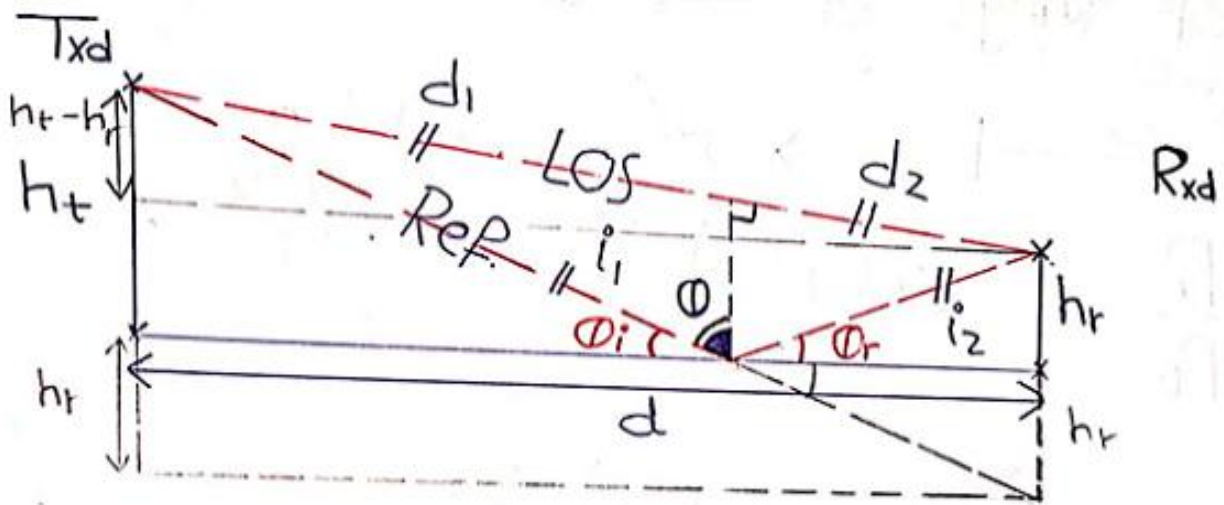
c) Scattering :-

When the wave length (λ) hits large no. of objects with dimensions $\sim \lambda$.

(Rain drops, fogs, ...).

2 Reflection (Ground wave or 2-Ray model) :-

↑
2 waves → LOS & another come from reflection.



$$i_1 + i_2 \gg d_1 + d_2$$

$$\phi = 90 - \phi_i$$

$$\frac{P_r}{P_t} = G_t G_r \left(\frac{\lambda}{4\pi d} \right)^2 \left| 1 + \rho e^{j\phi} \right|^2$$

↑
↑
 LOS reflection

ρ : reflection coefficient.

ϕ : phase shift reflected wave. (ref. LOS).

$$\rho = \frac{\cos \theta - \sqrt{\epsilon - \sin^2 \theta}}{\cos \theta + \sqrt{\epsilon - \sin^2 \theta}}$$

$d \uparrow, \theta \downarrow$

For large $d \rightarrow \theta \approx 90^\circ$.

$\rho \approx -1 \rightarrow$ كل الساقم ينعكس •

$$\frac{P_r}{P_t} = G_t G_r \left(\frac{\lambda}{4\pi d} \right)^2 |1 - e^{j\phi}|^2$$

$$\phi = \omega_c t = \frac{2\pi f_c \Delta d}{c = \lambda f}$$

$$\Delta d = (i_1 + i_2) - (d_1 + d_2)$$

$$\rightarrow \phi = \frac{2\pi f_c \Delta d}{\lambda f} = \frac{2\pi \Delta d}{\lambda}$$

$$*(d_1 + d_2)^2 = (h_t - h_r)^2 + d^2$$

$$*(i_1 + i_2)^2 = (h_t + h_r)^2 + d^2$$

$$\Delta d = d \left[\sqrt{\left(\frac{h_t + h_r}{d}\right)^2 + 1} - \sqrt{\left(\frac{h_t - h_r}{d}\right)^2 + 1} \right]$$

$$* h_t + h_r \ll d \rightarrow \sqrt{1+x} \approx 1 + \frac{x}{2}, x \ll 1$$

$$\begin{aligned} \therefore \phi &\approx \frac{2\pi}{\lambda} d \left[1 + \frac{1}{2} \left(\frac{h_t + h_r}{d}\right)^2 - 1 - \frac{1}{2} \left(\frac{h_t - h_r}{d}\right)^2 \right] \\ &= \frac{4\pi}{\lambda d} h_t h_r \end{aligned}$$

$$\frac{P_r}{P_t} = G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 \underbrace{|1 - e^{j\phi}|^2}_{\substack{2(1 - \cos \phi) \\ 2(2 \sin^2 \frac{\phi}{2}) \\ 4 \sin^2 \frac{\phi}{2}}}$$

$$|X|^2 = X X^*$$

$$\therefore 4 \sin^2 \frac{\phi}{2} \approx 4 \frac{\phi^2}{4} = \phi^2$$

$$\phi \ll \rightarrow \sin \frac{\phi}{2} \approx \frac{\phi}{2}$$

$$\therefore \frac{P_r}{P_t} = G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 \cdot \left(\frac{4\pi}{\lambda d}\right)^2 h_t^2 h_r^2$$

$$= G_t G_r \left(\frac{1}{d^2}\right) \cdot \left(\frac{1}{d^2}\right) h_t^2 h_r^2$$

$$= G_t G_r \left(\frac{1}{d^4}\right) h_t^2 h_r^2 \rightarrow$$

$$= G_t G_r \left(\frac{h_t h_r}{d^2}\right)^2$$

تضايفت المسافة
خسرت $\frac{1}{16}$ من P_r

$$\therefore P_r \propto \frac{1}{d^4} \rightarrow \text{أسوأ بكثير من LOS}$$

in ideal case (LOS) $\rightarrow P_r \propto \frac{1}{d^2}$.

$$10 \log \frac{1}{16} = -12 \text{ dB.}$$

if d is doubled, we lose 12 dB.
(in ideal, we lose 6 dB).

$$\begin{aligned} \therefore P_r(\text{dB}) &= 10 \log \frac{P_t}{P_r} \\ &= 40 \log d - 20 \log(h_t h_r) - 10 \log(G_t G_r) \end{aligned}$$

Example: BS & MS, $d = 5 \text{ km}$,

$G_t = G_r = 2.55 \text{ dB}$, $f = 900 \text{ MHz}$.

$h_t = 1.5 \text{ m}$, $h_r = 5 \text{ m}$, $P_t = 2 \text{ W}$,

$P_r(5 \text{ km})$? using 2-Ray model.

$$P_r(5 \text{ km}) = 5.9 \times 10^{-13} \text{ W.}$$

$$= -122.2 \text{ dB.}$$

$$= -92.2 \text{ dBm.}$$

$$G_t = G_r = 10^{\frac{2.55}{10}}$$

* Practical Propagation models :-

practical Radio propagation is derived using a combination of Analytical & Impirical models or methods.

1] Long distance model :-

$$\bar{P}_L(d) \propto \left(\frac{d}{d_0}\right)^n$$

n : Path loss exponent, $2 \leq n \leq 4$

↑ average Path Loss. (RV).
↓ reference distance. which \bar{P}_L is known.

* $\bar{P}_L \uparrow$, Power Recived \downarrow .

↓ empirically
↓ by certain model.

$$\bar{P}_{L,d}(\text{dB}) = \bar{P}_{L,d_0}(\text{dB}) + 10n \log\left(\frac{d}{d_0}\right).$$

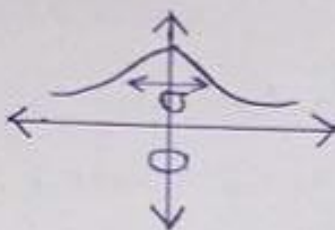
2] Log-normal Shadwing model :

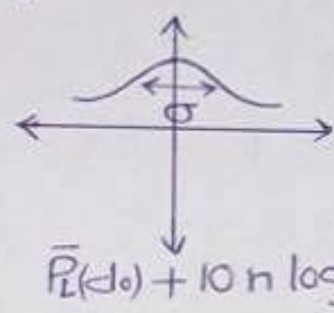
[Take into account the dynamic environment].

$$P_L(d) [\text{dB}] = \underbrace{\bar{P}_L(d_0)}_{\text{mean } \checkmark} + 10n \log\left(\frac{d}{d_0}\right) + \underbrace{X_\sigma}_{\substack{\text{log normal} \\ \text{mean} = 0 \\ \sigma \text{ const.}}}$$

↑ actual Path Loss.

X_σ : Zero mean Gaussian Variable
in (dB), with STD (σ) in (dB)
& Variance (σ^2) in (dB²).

X_σ :  $X_\sigma \approx (0, \sigma \text{ dB})$.

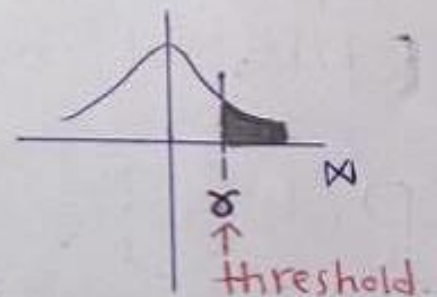
$\bar{P}_{L,d}$ (dB) :  $\bar{P}_{L,d} + 10n \log\left(\frac{d}{d_0}\right)$.

$$P_{rec}(d) = P_t - P_L(d)$$

$$= P_t - \bar{P}_L(d_0) - 10n \log\left(\frac{d}{d_0}\right) - X_\sigma$$

$$f_{P_{rec}}(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x - \bar{P}_{rec}(d))^2}{2\sigma^2}}$$

$\bar{P}_{rec}(d)$
↑
mean value



$$P_r[P_{rec}(d) > \gamma] = \int_{\gamma}^{\infty} f_{P_{rec}}(x) \cdot dx$$

$$= Q\left(\frac{\gamma - \bar{P}_{rec}}{\sigma}\right)$$

$$\begin{aligned}
 Q(x) &\triangleq \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-\frac{y^2}{2}} \cdot dy \\
 &= \frac{1}{2} \left[1 - \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right) \right] \\
 &= \frac{1}{2} \operatorname{erfc}\left(\frac{x}{\sqrt{2}}\right)
 \end{aligned}$$

∴ $P_r(d)$ is R.V has Pdf :

$$\frac{1}{\sqrt{2\pi} \sigma^2} e^{-\frac{(x - \bar{P}_r(d))^2}{2\sigma^2}}$$

Percentage of Coverage area $U(\delta)$

Determine the percentage of location within a circle of Radius R , in which the Avg. R_{xd} signal power from a certain base station exceeds a certain threshold (δ) .

$$U(\delta) = \frac{1}{\pi R^2} \int P_r[P_r(r) > \delta] dA$$

$$= \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R \left(Q\left(\frac{\delta - \bar{P}_{rec}(r)}{\sigma}\right) \right) \cdot r \, dr \, d\theta$$

$$\begin{aligned}
 x &= r \cos \theta \\
 y &= r \sin \theta
 \end{aligned}$$

$$dx dy = r \, dr \, d\theta$$

Polar.

$$U(\delta) = \frac{2}{R^2} \int_0^R \varphi(r) r dr$$

$$\therefore U(\delta) = \frac{2}{R^2} \int_0^R \frac{1}{2} \left[1 - \operatorname{erf} \left(\frac{\delta - \frac{P_t + \bar{P}_L(d_0) + 10n \log \frac{r}{d_0}}{\sqrt{2} \sigma}}{\sqrt{2} \sigma} \right) \right] r dr$$

Let:

$$a = \frac{\delta - P_t + \bar{P}_L(d_0) + 10n \log \left(\frac{R}{d_0} \right)}{\sqrt{2} \cdot \sigma}$$

$$b = \frac{10n \log e}{\sqrt{2} \cdot \sigma}$$

$$\log \frac{r}{R} = \frac{\log_e \left(\frac{r}{R} \right)}{\log_e 10} = \frac{\ln \left(\frac{r}{R} \right)}{\ln 10}$$

$$\rightarrow (\ln 10)^{-1} = \log_e$$

$$\frac{r}{d_0} = \frac{r}{R} \cdot \frac{R}{d_0}$$

$$\log \left(\frac{r}{d_0} \right) = \log \left(\frac{r}{R} \right) + \log \left(\frac{R}{d_0} \right)$$

$$U(\delta) = \frac{2}{R^2} \int \frac{1}{2} - \frac{1}{2} \operatorname{erf} \left[a + b \ln \frac{r}{R} \right] r dr$$

$$= \frac{1}{2} - \frac{1}{R^2} \int_0^R \operatorname{erf} \left(a + b \ln \frac{r}{R} \right) r dr$$

$$r = R e^{\frac{t-a}{b}} \quad ; \quad dr = \frac{R}{b} e^{\frac{t-a}{b}} dt$$

$$\therefore U(\delta) = \frac{1}{2} - \frac{1}{R^2} \int_{-\infty}^a R e^{\frac{t-a}{b}} \operatorname{erf}(t) \cdot \frac{R}{b} e^{\frac{t-a}{b}} dt$$

Outdoor Propagation Models

1] Okumura Model. (fig. 3.23)

2] Hata Model :-

$$\bar{P}_L(\text{urban}) [\text{dB}] = 69.55 + 26.16 \log f_c \overset{\text{MHz}}{\leftarrow} - 13.82 \log h_{te} - a(h_{re}) + (44.9 - 6.55 \log h_{te}) * \log d .$$

h_{te} : effective T_x height (in m).

h_{re} : effective R_x height (in m).

d : distance between R_x & T_x (in km).

$a(h_{re})$: R_x correction factor.

For Small & medium size city :

$$a(h_{re}) = (1.1 \log f_c - 0.7) h_{re} - (1.56 \log f_c - 0.8) \text{ dB}$$

For Large city :

$$a(h_{re}) = \begin{cases} 8.29 (\log 1.54 h_{re})^2 - 1.1 \text{ (dB)}, & f_c \ll 300 \text{ MHz} \\ 3.2 (\log 11.75 h_{re})^2 - 4.97 \text{ (dB)}, & f_c \gg 300 \text{ MHz} \end{cases}$$

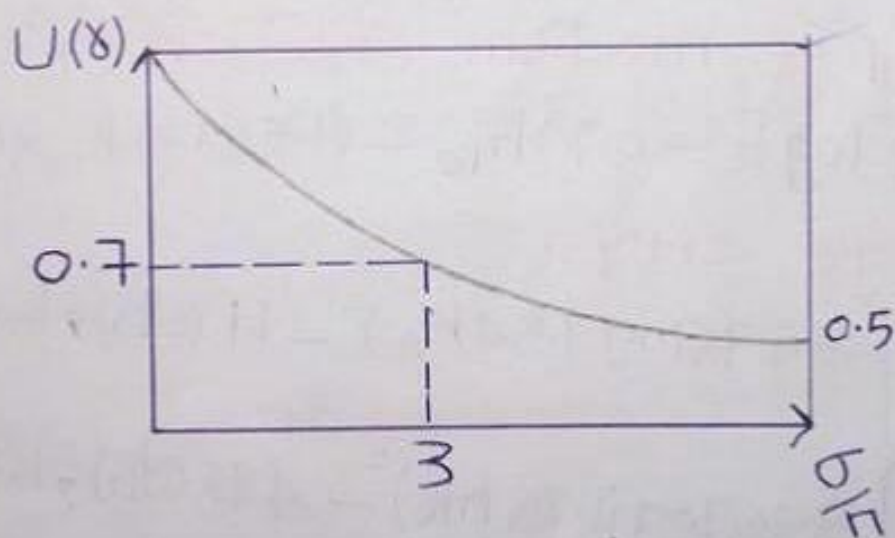
$$= \frac{1}{2} - \frac{e^{-\frac{2a}{b}}}{b} \int_{-\infty}^a e^{\frac{2t}{b}} \cdot \text{erf}(t) dt.$$

from integration Tables :

$$\int_{-\infty}^x \text{erfc}(Ax) e^{Bx} dx = \frac{1}{B} e^{Bx} \cdot \text{erf}(Ax) - \frac{1}{B} e^{-\frac{B^2}{4A^2}} \cdot \text{erf}\left(Ax - \frac{B}{2A}\right).$$

Example :- [fig (3.18)] :-

$$n=3, \sigma=9, P[P_r > 8] = 0.5 :$$



• 50% boundary coverage provides
70% area coverage.

* To Obtain path Loss in Suburban area, :

$$\bar{P}_L (\text{dB}) = \bar{P}_L (\text{urban}) - 2 \left(\log \frac{f_c}{28} \right)^2 - 5.4$$

Example:-

$P_t = 1\text{W}$, $G_t = 10\text{dB}$, $f_c = 600\text{MHz}$,
 $h_{te} = 30\text{m}$, $G_r = 0\text{dB}$, $d = 5\text{km}$,
 $\sigma = 8\text{dB}$, $h_{re} = 1\text{m}$.

find the $P_r [P_r > -125\text{dBm}]$ in Urban Small city using Hata Model. ?

$$\begin{aligned} \bar{P}_L &= 69.55 + 26.16 \log(600) - 13.82 \log(30) \\ &- a(h_{re}) + [44.9 - 6.55 \log(30)] \log(5) \\ &= 147.7 (\text{dB}). \end{aligned}$$

$$\bar{P}_r (\text{dBm}) = 30 + 10 + 0 - 147.6 = -107.6 \text{ dBm}.$$

$$P_r [P_r > -125 \text{ dBm}] = \Phi \left(\frac{-125 + 107.6}{8} \right).$$

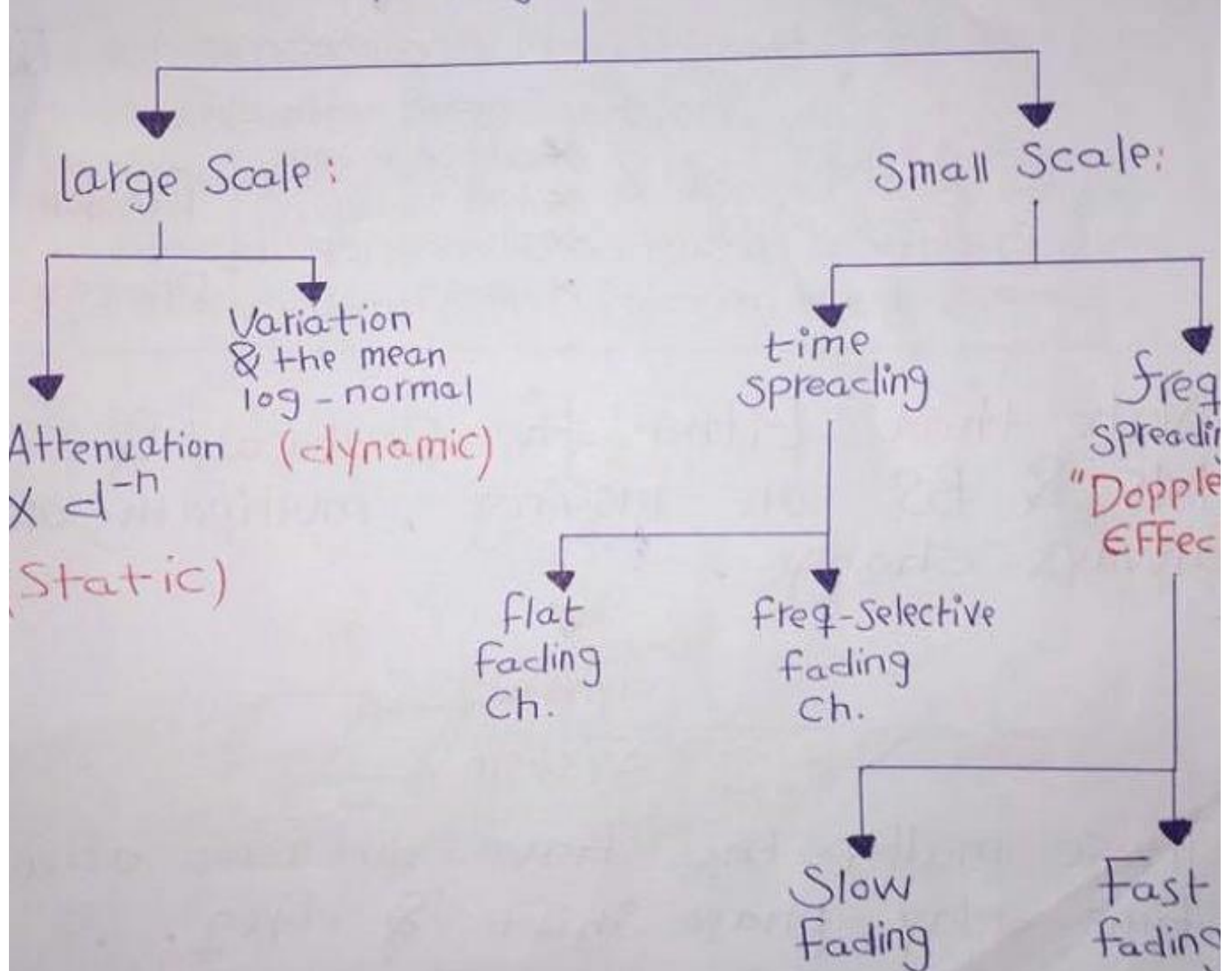
↑
 كاسازد تفرقی قیمة (P_r)

From table
 $= 98.5\%$

Chapter 5 : Small-Scale multi-path Fading: 2

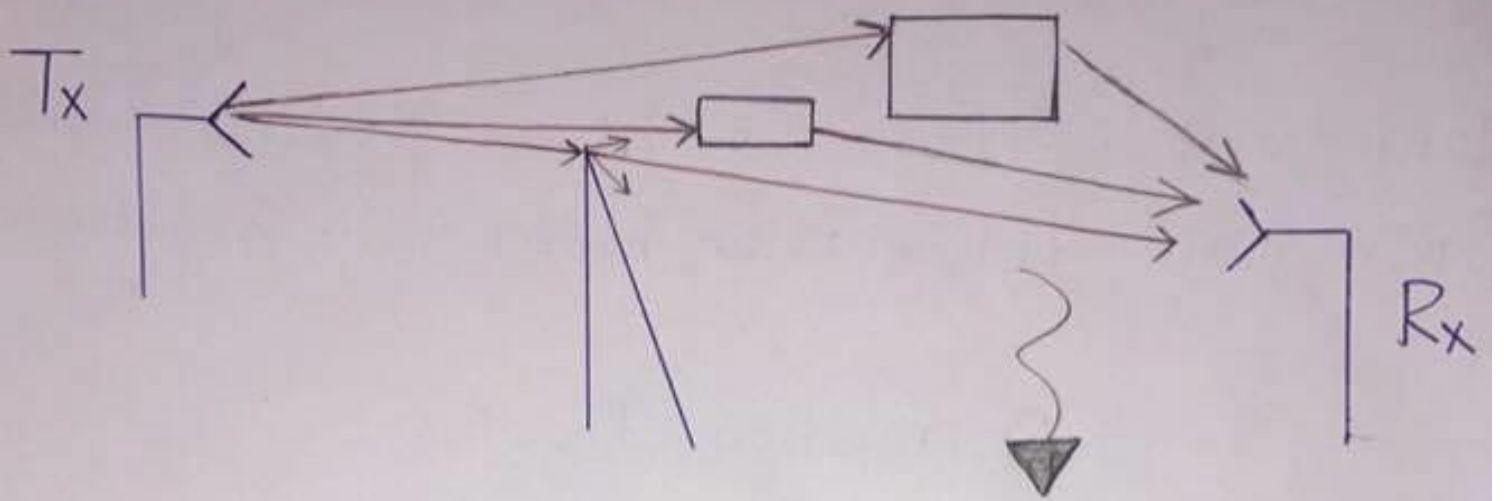
large Scale : Attenuation happen in large distance
Small Scale : Attenuation happen with λ distance

Propagation Effect :



As a designer, the target is to get Flat, Slow Fading.

Multipath Fading :-



multicopies of the T_x signal with diff.:

- *attenuation.
 - *time delay.
 - *phase shift.
 - *freq.
- } Random Process.

Note that: Either the Obstacles & MS & BS are moving, multipaths are always change.

dynamic System.

Those multipaths have different atten. time delay, phase shift & freq. .

Ex: Carrier freq. ($f_c = 1850 \text{ MHz}$) vehicle moving at const. Speed of 26.82 m/s

Find the Rx carrier freq. :

1- Toward Tx ($\phi = 0$).

2- Away from Tx ($\phi = 180^\circ$).

3- \perp to the direction ($\phi = 90^\circ$).

$$f_d = \frac{v}{\lambda} \cos \phi, \quad \lambda = \frac{3 \times 10^8}{18.5 \times 10^8} = 0.162 \text{ m}$$

$$1- f_d = \frac{26.8}{0.162} \cos 0 = 165.4 \text{ Hz}$$

$$f = f_c + f_d$$

$$2- f_d = -165.4 \text{ Hz}$$

$$f = f_c - f_d$$

$$3- f_d = 0$$

$$f = f_c$$

$$\Delta\phi = \frac{2\pi}{\lambda} \Delta L$$

$$= \frac{2\pi}{\lambda} \cdot v \cos\theta \Delta t$$

$$= \frac{2\pi}{\lambda} v \Delta t \cos\theta$$

Doppler Effect: $f_d \triangleq \frac{1}{2\pi} \Delta\phi'$ * مشتقة بالنسبة للزمن

"التغير بالـ freq."

$$= \frac{v}{\lambda} \cos\theta$$

λ known.

$$f_{d, \max} = \frac{v}{\lambda} \quad (\theta = 0)$$

\rightarrow between the radar & the car.

if $\theta = 90^\circ$, $f = \text{zero}$.

$$f_c \uparrow, \lambda \downarrow, f_d \uparrow$$

@ const. v

MS Rxd from:

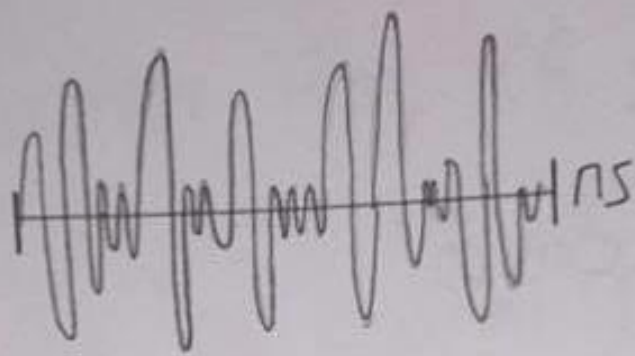
$$f = f_c \pm f_d$$

with known λ ,
he can measure $(v \cos\theta) \checkmark$

\downarrow
Rxd
Freq.

$\rightarrow (+)$: BS moves toward MS.

$\rightarrow (-)$: BS moves away from MS. Δ



maybe constructive.

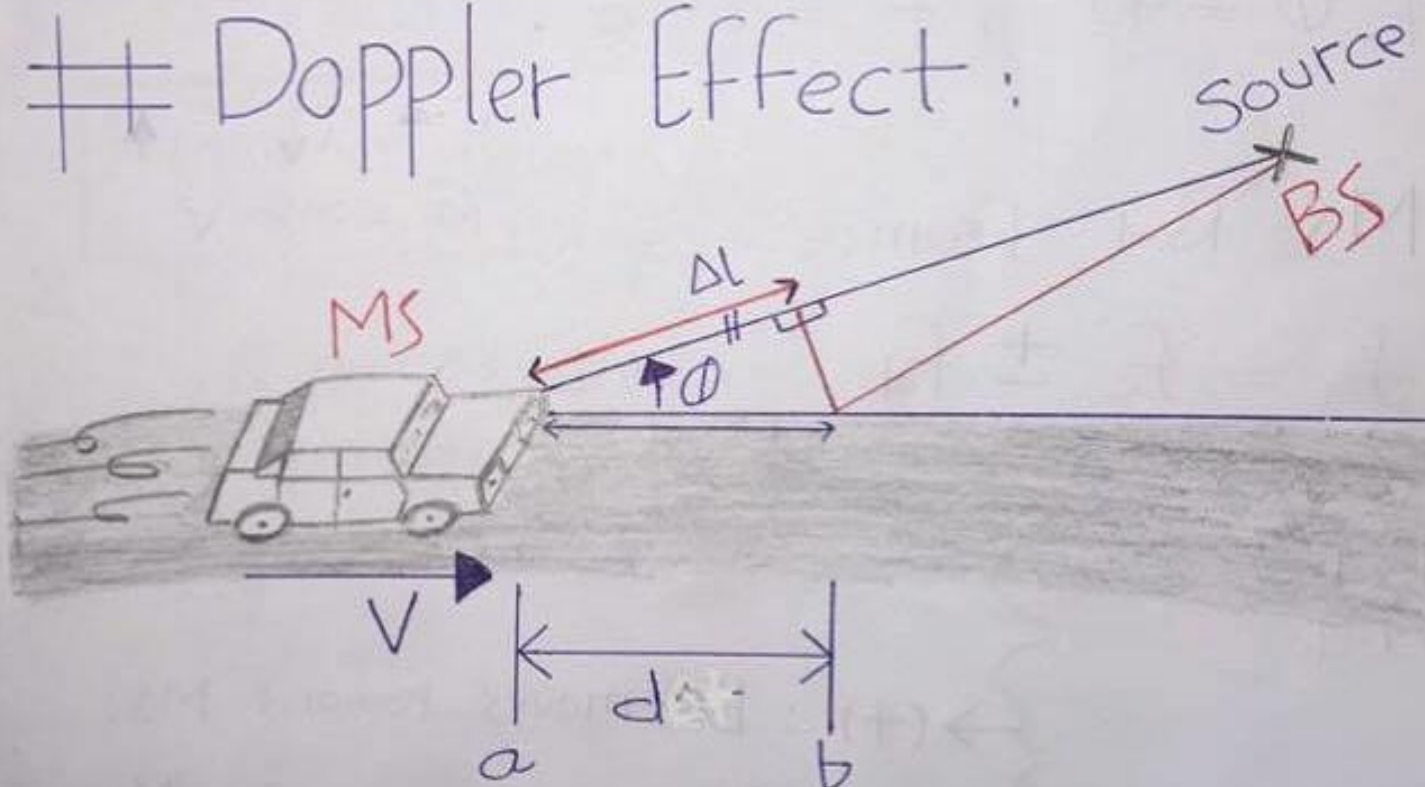
maybe destructive.

Small Scale.

parameters that effect on multipath fading
(Influences on multipath) :-

- 1- multipath propagation.
- 2- Speed of the T_x & R_x (MS & BS).
- 3- Speed of the obsticals Surrounding.
- 4- bandwidth of the transmitted signal.

Doppler Effect:



Multipath Fading :

Suppose that the T_{xd} base band signal has a complex envelope,

$g_s(t) \rightarrow$ The R_{xd} multipath signal can be written as : $g_r(t) = \sum_{k=1}^N \rho_k g_s(t - \tau_k) e^{j\phi_k} + n(t)$

$$0 \leq t \leq T_s$$

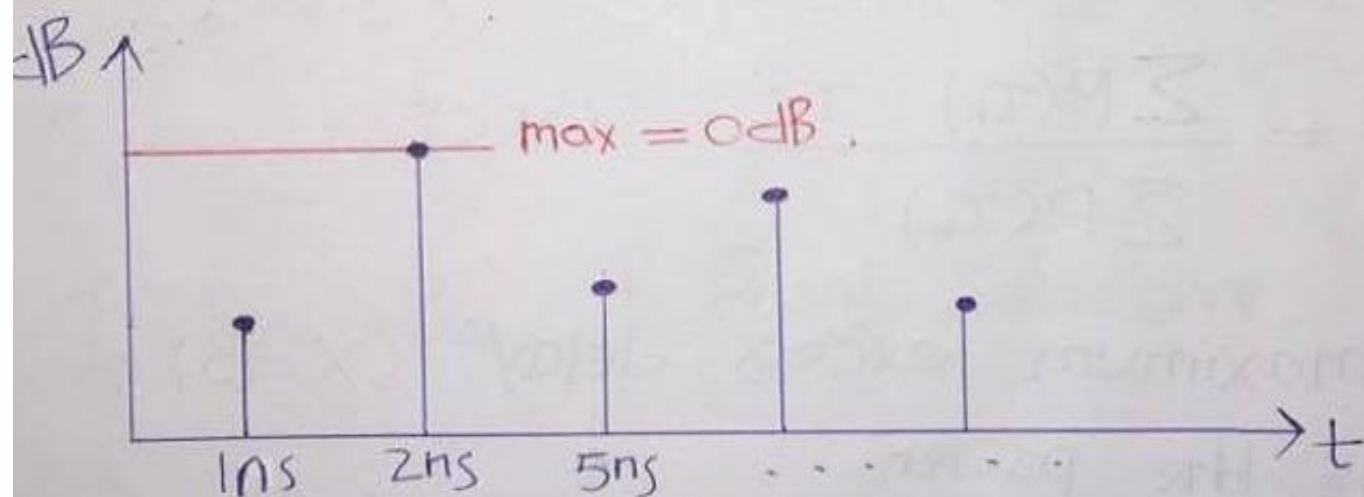
N : # of multipaths.

ρ_k : attenuation on k^{th} path.

ϕ_k : Phase Shift on the k^{th} path.

τ_k : time delay on the k^{th} path.

* Normalized R_{xd} power profile:



Time dispersion parameters :

[1] mean excess delay : (Average)

$$\bar{\tau} = \frac{\sum P(\tau_k) \cdot \tau_k}{\sum P(\tau_k)} \text{ (Sec)}$$

$P(\tau_k)$: Normalized R_{xd} Power of τ_k .

[2] RMS delay spread :

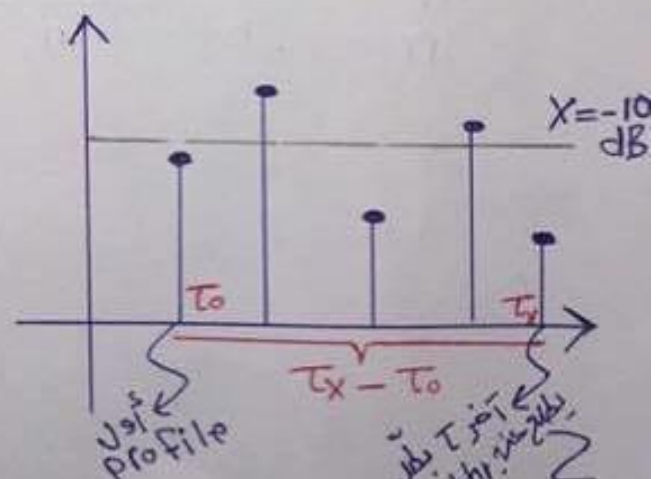
$$\sigma_{\tau} = \sqrt{\overline{\tau^2} - (\bar{\tau})^2} \text{ (Sec)}$$

STD

$$\overline{\tau^2} = \frac{\sum P(\tau_k) \cdot \tau_k^2}{\sum P(\tau_k)} \text{ (Sec)}^2$$

[3] maximum excess delay (X dB) :

Where the power falls to X dB below the first R_{xd} power path.



We define the Coherence Bandwidth (B_c) as the range of frequencies over which the Channel can be considered flat. (i.e. Passes all Spectral Components with same gain).

$$B_c = \begin{cases} \frac{1}{50 \sigma_\tau} & , \text{freq. corr.} > 90\% . \\ \frac{1}{5 \sigma_\tau} & , \text{freq. corr.} > 50\% . \end{cases}$$

(Hz)

If $BW < B_c$: Flat.
 ↑
 of Tx'd Signal

كل ال Spectral comp. التي جاي
 تنضرب بنفس ال gain.

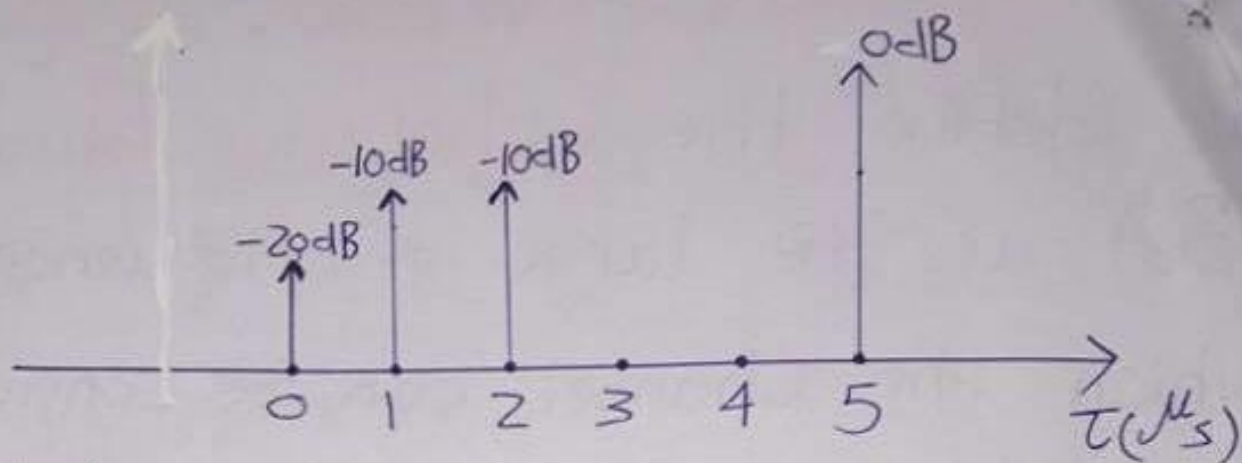
If $BW > B_c$: Freq. Selective.

OR:

If $T_s > \sigma_\tau$: Flat.

If $T_s < \sigma_\tau$: Freq. Selective.

Ex :



Calculate:

$$1. \bar{\tau} = \frac{\sum P(\tau_k) \tau_k}{\sum P(\tau_k)} = \frac{(0.01)(0) + (0.1)(1) + (0.1)(2) + (1)(5)}{0.01 + 0.1 + 0.1 + 1} = 4.38 \mu s. \#$$

$$P_1 = 10^{-2} = 0.01 \text{ W.}$$

$$P_2 = P_3 = 10^{-1} = 0.1 \text{ W.}$$

$$P_4 = 10^0 = 1 \text{ W.}$$

$$2. \overline{\tau^2} = \frac{\sum P(\tau_k) \tau_k^2}{\sum P(\tau_k)} = \frac{(0.01) + (0.1)^2 + (5)^2}{0.01 + 0.1 + 0.1 + 1} = 21.07 (\mu s)^2. \#$$

$$3. \sigma_{\tau} = \sqrt{21.07 - (4.38)^2} = 1.37 \mu s.$$

4. maximum Excess delay @

-10 dB : $5 \mu s$. max. excess delay < 10 dB

-7 dB : $5 \mu s$. " " " < 7 dB

-15 dB : $1 \mu s$. " " " < 15 dB

#

4

$$\begin{aligned} 5- 50\% \text{ Coh. BW} &= \frac{1}{5 \sigma_{\tau}} \\ &= \frac{1}{5(1.37\mu)} \\ &= 146 \text{ kHz} = B_s. \end{aligned}$$

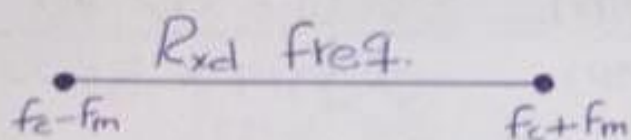
AMPS $\rightarrow B_c < 30\text{kHz} \rightarrow$ need equalizer.

GSM $\rightarrow B_c > 200\text{kHz} \rightarrow$ need equalizer.

Frequency Spreading or Dispersion & Coherence time T_c :

If the Tx'd freq. is f_c ;
due to doppler spreading, the freq.
will $f_c \pm f_m$

$$\rightarrow \text{max. doppler freq} = \frac{v}{\lambda} ; \theta = 0 \text{ or } \pi$$



We define coherence time $T_c \propto \frac{1}{f_m}$

In General,

$$T_c \text{ (50\% correlation)} = \frac{0.423}{f_m}$$

∞ a channel is slow faded iff
 $T_c > T_s$.

& a channel is fast faded iff
 $T_c < T_s$.

\rightarrow bit/symbol
Period.

* Our Target is to have flat
& slow channel.

Example : $V = 60 \text{ mile/hr}$, $f_c = 900 \text{ MHz}$

$$\lambda = \frac{C}{f_c} = \frac{3 \times 10^8}{900 \text{ M}} = 0.333 \text{ m.}$$

$$f_m = \frac{V}{\lambda} = \frac{60 (1.6 \text{ km/hr})}{0.33 \text{ m.}} = \frac{60 (1.6 \times 10^3 \text{ m})}{0.33 \text{ m} \times 3600 \text{ sec}}$$

$$T_c(50\%) = \frac{0.423}{f_m} = 6.67 \text{ msec.} = 80.8 \text{ Hz.}$$

as long as $T_s < 6.67 \text{ ms}$, or

as long as $R_s \geq 150 \text{ sym/sec}$
Sym. rate $\hookrightarrow (6.67 \text{ m})^{-1}$

The Channel is Slow.

* Our Target is to have flat
& slow channel.

Example : $V = 60 \text{ mile/hr}$, $f_c = 900 \text{ MHz}$

$$\lambda = \frac{C}{f_c} = \frac{3 \times 10^8}{900 \text{ M}} = 0.333 \text{ m.}$$

$$f_m = \frac{V}{\lambda} = \frac{60 (1.6 \text{ km/hr})}{0.33 \text{ m.}} = \frac{60 (1.6 \times 10^3 \text{ m})}{0.33 \text{ m} \times 3600 \text{ sec}}$$

$$T_c(50\%) = \frac{0.423}{f_m} = 6.67 \text{ msec.} = 80.8 \text{ Hz.}$$

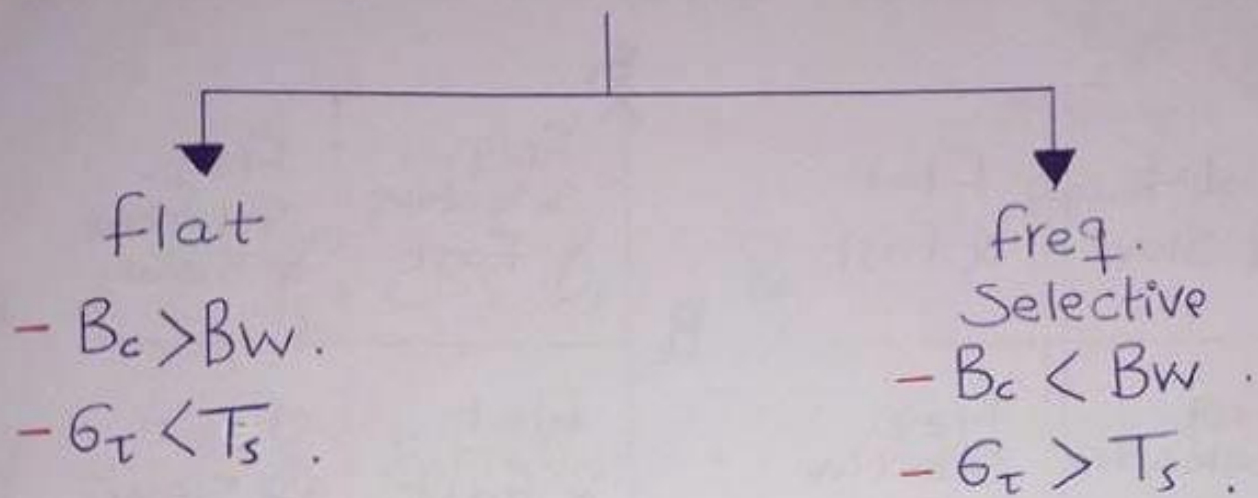
as long as $T_s < 6.67 \text{ ms}$, or

as long as $R_s \geq 150 \text{ sym/sec}$
Sym. rate $\hookrightarrow (6.67 \text{ m})^{-1}$

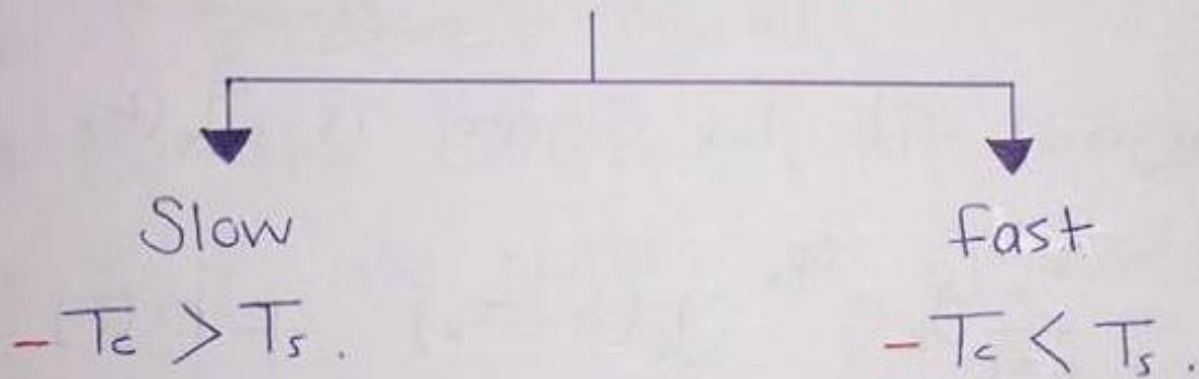
The Channel is Slow.

flat

Fading [Base on time spreading]



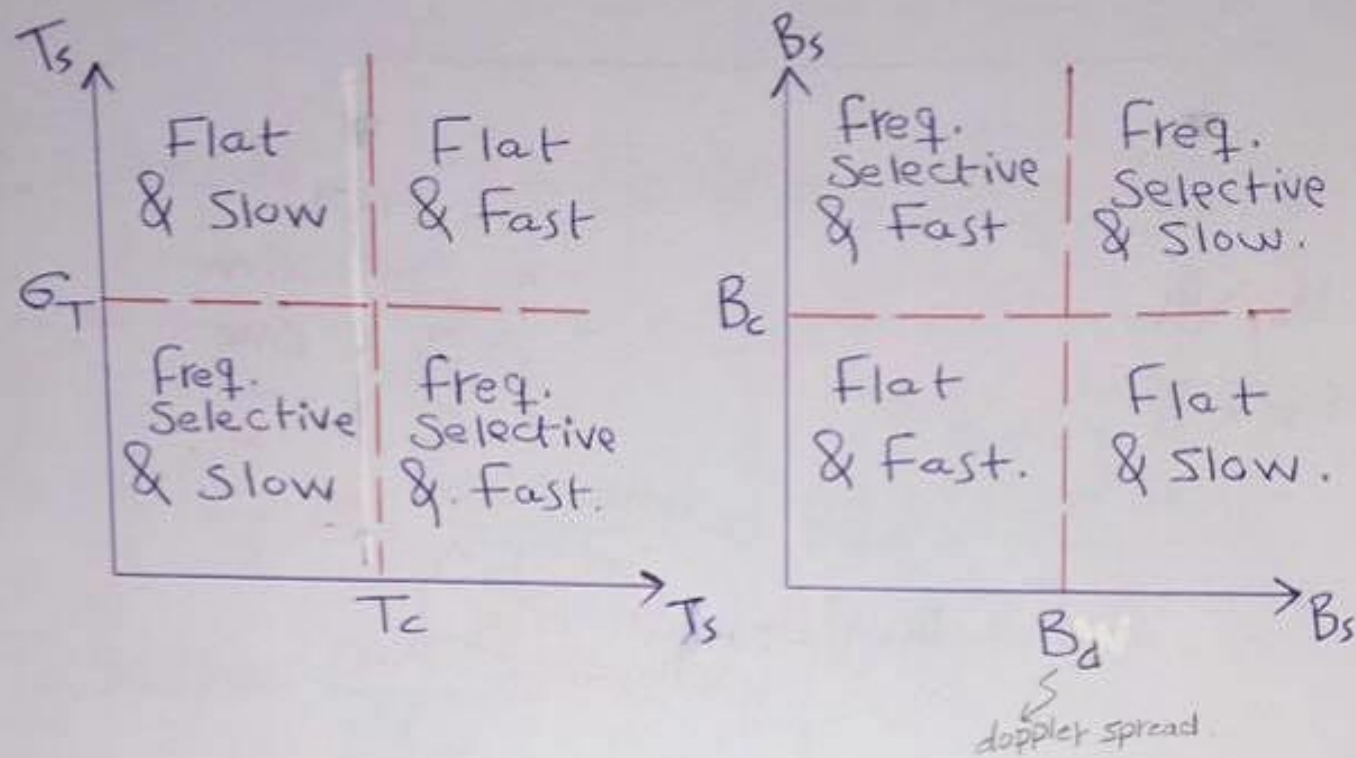
Fading [Base on Freq. spreading]



* $BW, T_s \rightarrow$ بقدر اتخام نيم / * $B_c, T_c \rightarrow$ ما بقدر اتخام نيم (من حضايش! Ch.)

$BW \propto R_s$
 \uparrow
 data Rate.

■ Characterization of Flat & Slow faded wireless channel : "best Scenatio"



R_{xd} signal if T_{xd} signal is $g_s(t)$:

$$g_r(t) = \sum_{k=1}^N \rho_k e^{j\phi_k} \cdot g_s(t - \tau_k) + n(t)$$

$$0 \leq t < T_s$$

* Since the channel is flat :

$$T_{max} < T_s, \therefore g_s(t - \tau_k) = g_s(t)$$

$$\rightarrow g_r(t) = g_s(t) \sum_{k=1}^N \rho_k e^{j\phi_k} + n(t)$$

* Channel gain $\triangleq h$ Random

$$\rightarrow f_{x,y}(x,y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{2\sigma^2}}$$

$$X, Y \rightarrow N(0, \sigma^2)$$

$$h = X + jY = r e^{j\phi}$$

↑
distribution J
mag. ← r

The Channel gain envelope or Amplitude is :

↙
magnitude

$$\alpha = \sqrt{X^2 + Y^2}, \quad X = \alpha \cos \phi$$

$$Y = \alpha \sin \phi$$

& the Channel phase :

$$\phi = \tan^{-1}\left(\frac{Y}{X}\right)$$

↙
argument

X, Y are jointly gaussian,

By the Jacobian Transformation:

$$f_{\alpha\phi}(\alpha, \phi) = \frac{f_{x,y}(x^{-1}, y^{-1})}{J(x,y)}$$

↑
R.V. J

$h \triangleq X + jY \rightarrow$ effect on mag. & phase

↑
المهارة
لأنها بتأثر على
الpower

$$X = \sum_{k=1}^N P_k \cos \phi_k \quad , \quad \&$$

$$Y = \sum_{k=1}^N P_k \sin \phi_k \quad .$$

* Since the channel is slow & $N \gg 1$,
all P_k 's almost equal.
of paths is large enough

∴ By the Central Limit Theorem:
 X & Y are Normal or Gaussian RV's

$$P_k \approx P.$$

↓
CLT

“ N identically distributed
RV's (X_i 's),
then $X = \sum X_i$ as $N \rightarrow \infty$
 $\Rightarrow X$ is Gaussian or
Normal R.V. ”

$$\begin{aligned}
 (X, Y) &= \frac{1}{\begin{vmatrix} \frac{\partial x}{\partial \alpha} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial \alpha} & \frac{\partial y}{\partial \phi} \end{vmatrix}} \\
 &= \frac{1}{\begin{vmatrix} \cos \phi & -\alpha \sin \phi \\ \sin \phi & \alpha \cos \phi \end{vmatrix}} = \frac{1}{\alpha} .
 \end{aligned}$$

$$\rightarrow f_{\alpha\phi}(\alpha, \phi) = \frac{\frac{1}{2\pi\sigma^2} e^{\frac{-(\alpha^2 \cos^2 \phi + \alpha^2 \sin^2 \phi)}{2\sigma^2}}}{\frac{1}{\alpha}}$$

$$= \frac{\alpha}{2\pi\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}} \quad , \quad \alpha \rightarrow \text{always +ve}$$

∇
 $\frac{1}{2\pi}$: distribution of ϕ is Uniform $(-\pi, \pi)$.

$$\begin{aligned}
 f_{\phi}(\phi) &= \int f_{\alpha\phi}(\alpha, \phi) d\alpha \\
 &= \frac{1}{2\pi} \quad , \quad |\phi| < \pi .
 \end{aligned}$$

$$\begin{aligned} \therefore f_{\alpha}(\alpha) &= \int_{-\pi}^{\pi} f_{\alpha\phi}(\alpha, \phi) \cdot d\phi \\ &= \int_{-\pi}^{\pi} \frac{\alpha}{2\pi\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}} \cdot d\phi \\ &= \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}}, \quad \alpha > 0 \end{aligned}$$

$$\therefore f_{\alpha}(\alpha) = \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}}, \quad \alpha > 0.$$

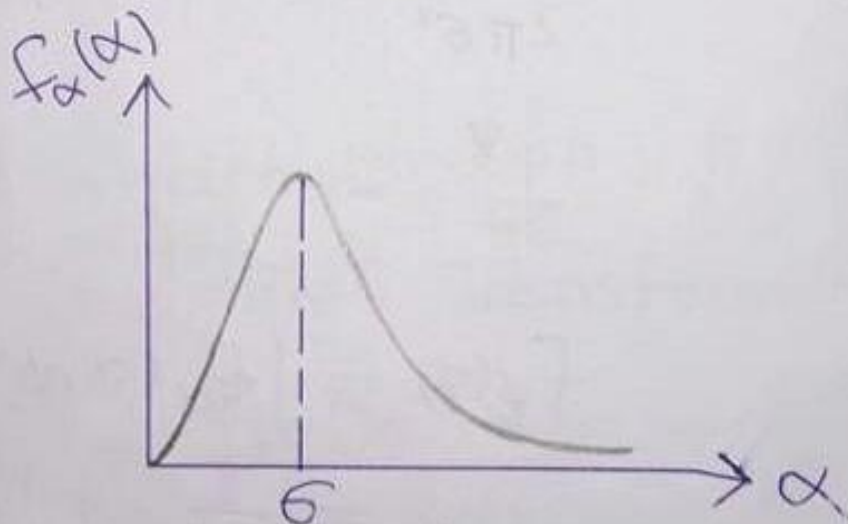
envelope for Z R.V.

α is called Rayleigh Fading R.V.

* Cdf of Rayleigh is exponential e^{-x} .

x & y \rightarrow independent & uncorrelated (gaussian).

α & ϕ \rightarrow independent.



* Variance of x & y .

$$E[\alpha] = \frac{1}{\sigma^2} \int_0^{\infty} \alpha^2 e^{-\frac{\alpha^2}{2\sigma^2}} d\alpha \quad \text{TRICKS}$$

$$u = \alpha \longrightarrow du = d\alpha$$

$$\frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}} = dv \longrightarrow v = -e^{-\frac{\alpha^2}{2\sigma^2}}$$

$$E[\alpha] = -\alpha e^{-\frac{\alpha^2}{2\sigma^2}} \Big|_0^{\infty} + \int_0^{\infty} e^{-\frac{\alpha^2}{2\sigma^2}} d\alpha \quad * \frac{\sqrt{2\pi}\sigma}{\sqrt{2\pi}\sigma}$$

$$= 0 + \frac{\sqrt{2\pi}\sigma}{2}$$

$$= \sigma \sqrt{\frac{\pi}{2}}$$

$$E[\alpha^2] = 2\sigma^2$$

$$\bullet \text{RMS}(\alpha) = \sqrt{2}\sigma$$

$$= \frac{1}{\sigma^2} \int_0^{\infty} \alpha^3 e^{-\frac{\alpha^2}{2\sigma^2}} d\alpha$$

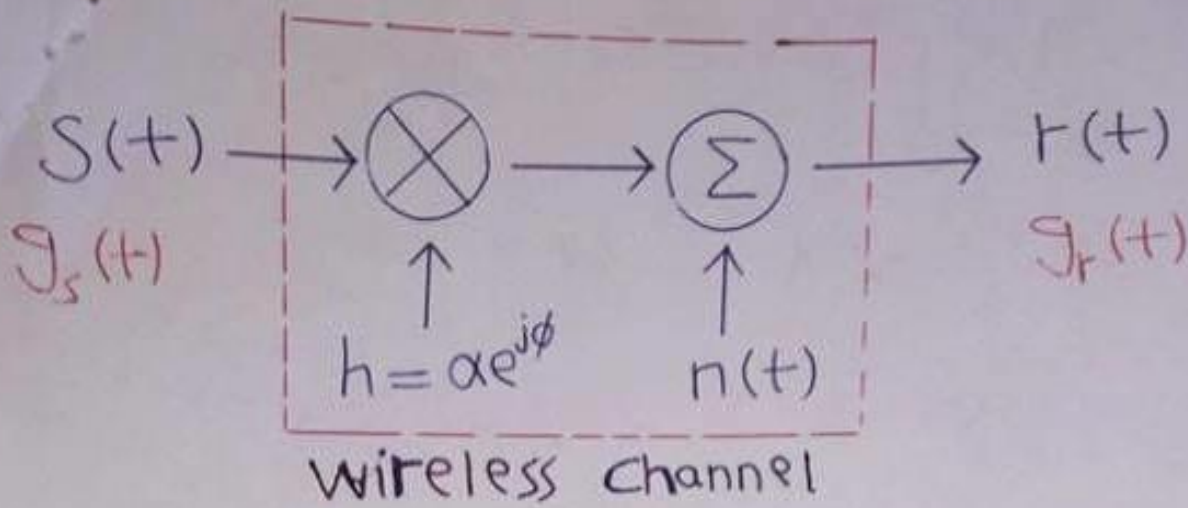
$$\sigma_{\alpha}^2 = 2\sigma^2 - \frac{\sigma^2}{2} \pi = \frac{(4-\pi)\sigma^2}{2}$$

$$f_{\alpha^2}(\alpha) = \frac{1}{2\sigma^2} e^{-\frac{\alpha}{2\sigma^2}} \quad \blacktriangleright E[\alpha^2]$$

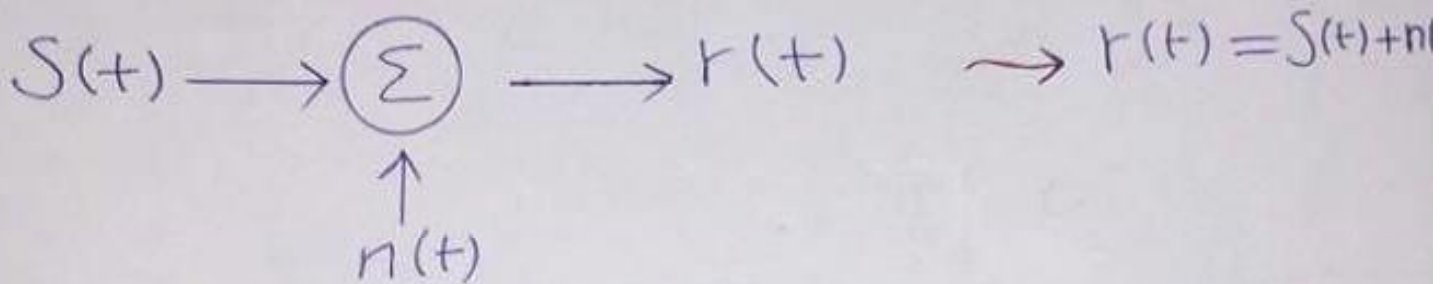
\hookrightarrow power.

$\alpha \rightarrow$ magnitude.

\blacktriangledown
exponential.



Rayleigh Faded Channel.
or Multiplicative Channel.



AWGN

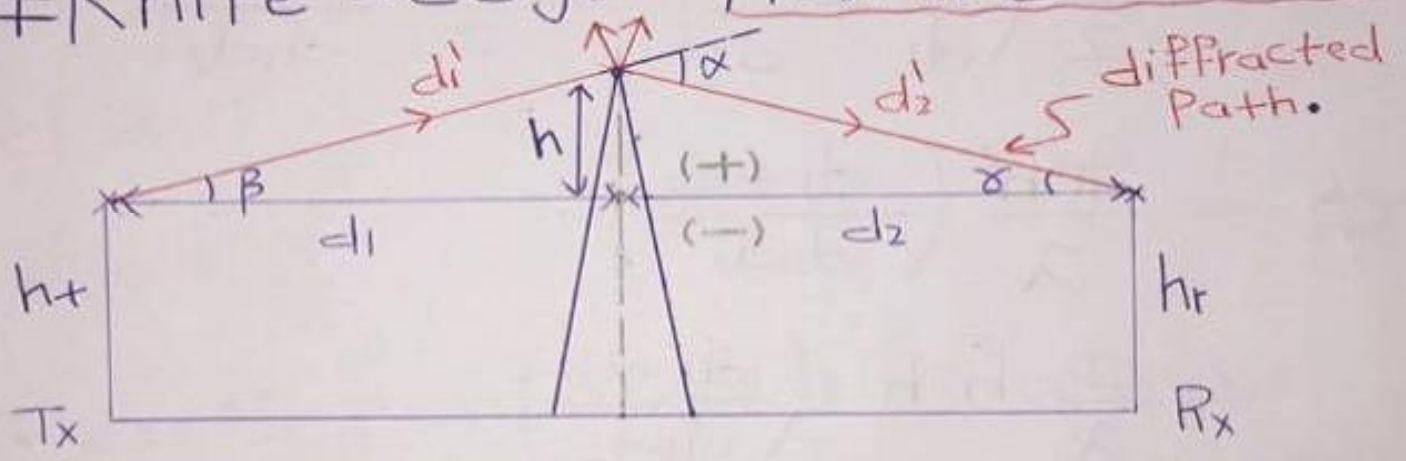
3] Diffraction :-

* Huggens principle ;

"All points on a wave form consider a point source for the production of Secondary wave forms"



Knife-edge "Fresnel zone model"



phy. $d_1 + d_2 = LOS$; but here there is no LOS path.

$$h_t = h_r$$

IF $h \ll d_1 \& d_2 \therefore h \gg \lambda$

$$\phi = \frac{2\pi}{\lambda} \Delta d$$

↑
Phase shift between 2 paths, the distance between them Δd .

$$\begin{aligned}
 \Delta d &= (d_1' + d_2') - (d_1 + d_2) \\
 &= \sqrt{d_1^2 + h^2} + \sqrt{d_2^2 + h^2} - d_1 - d_2 \\
 &= d_1 \sqrt{1 + \underbrace{\left(\frac{h}{d_1}\right)^2}_{\text{small}}} + d_2 \sqrt{1 + \underbrace{\left(\frac{h}{d_2}\right)^2}_{\text{small}}} - d_1 - d_2 \\
 &\approx d_1 \left[1 + \frac{1}{2} \left(\frac{h}{d_1}\right)^2\right] + d_2 \left[1 + \frac{1}{2} \left(\frac{h}{d_2}\right)^2\right] - d_1 - d_2 \\
 &= \frac{h^2}{2} \left(\frac{1}{d_1} + \frac{1}{d_2}\right) = \frac{h^2}{2} \frac{d_1 + d_2}{d_1 d_2}
 \end{aligned}$$

$$\phi = \frac{\pi h^2}{\lambda} \left(\frac{d_1 + d_2}{d_1 d_2} \right)$$

$$= \frac{\pi}{\lambda} \cdot h \cdot h \cdot \left(\frac{d_1 + d_2}{d_1 d_2} \right)$$

$$\alpha = \beta + \delta$$

$$\approx \frac{h}{d_1} + \frac{h}{d_2}$$

$$\text{; } \boxed{\tan x \approx x \text{ if } x \ll 1}$$

$$\therefore \phi = \frac{\pi h}{\lambda} \alpha$$

• الزاوية بين الـ Path والـ diff Path
 • الزاوية بين الـ Path والـ diff Path

$$\Delta d = \frac{h^2}{2} \left(\frac{d_1 + d_2}{d_1 d_2} \right) = n \frac{\lambda}{2}$$

$$\therefore r_n = h(n) = \sqrt{\frac{n \lambda d_1 d_2}{d_1 + d_2}}$$

radius

r_1 : exceeds LOS path by $\lambda/2$, π .
distance phase

r_2 : exceeds LOS path by λ , 2π .

r_3 : exceeds LOS path by $3\lambda/2$, 3π .

:

For : $n=1 \rightarrow \Delta d = \frac{\lambda}{2}$ & $\phi = \frac{2\pi \Delta d}{\lambda} = \pi$

* totally destructive.

$n=2 \rightarrow \Delta d = \lambda$ & $\phi = 2\pi$

* totally constructive

$$\therefore G_d(\text{dB}) = 20 \log |F(\nu)|$$

↑

diffraction

loss in (dB).

#($P_t - P_r$).

$\rightarrow \frac{P_r}{P_t}$ in linear scale.

↑

defined in

Eq. (4.61 a-e).

or in Fig. 4.14.

* Define Fresnel-Kirchoff parameter:

$$v \triangleq h \sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}}$$

$$= \alpha \sqrt{\frac{2 d_1 d_2}{\lambda (d_1 + d_2)}}$$

$h \rightarrow +ve \rightarrow$ only diff path.
 LOS X.
 $h \rightarrow -ve \rightarrow$ diff. &
 LOS ✓

Also,

$$\phi = \frac{\pi h}{\lambda} \alpha = \frac{\pi}{2} v^2$$

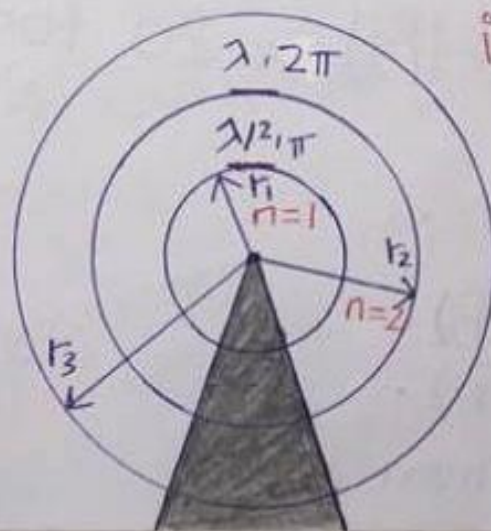
$$\Delta d = n \frac{\lambda}{2}$$

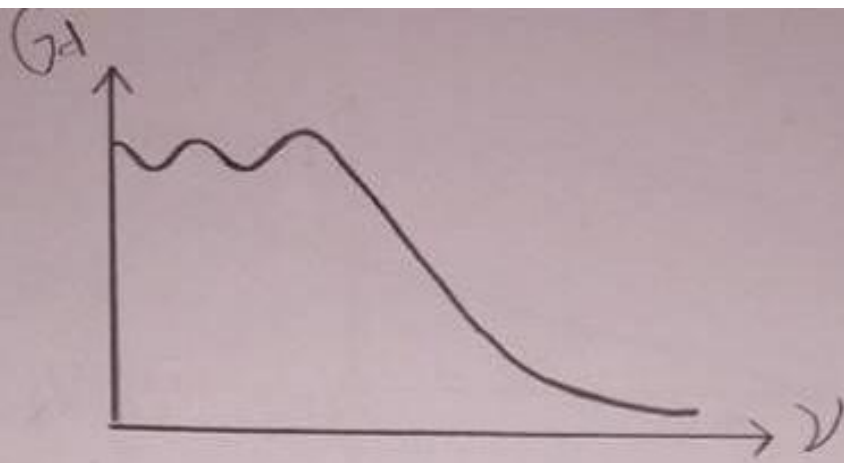
$$\therefore \phi = k \frac{\pi}{2}$$

* Fresnel zone represents successive regions where waves have a path length from T_x & R_x greater than $n \frac{\lambda}{2}$ of the LOS.

reference.

\uparrow
 $\frac{n \lambda}{2}$
 int. > 0





Example: Compute diffraction loss for $\lambda = 1/3$ m, $d_1 = 1$ km, $d_2 = 1$ km.

a) $h = 25$ m. b) $h = 0$. c) $h = -25$ m.

& identify the Fresnel zone within which the tip of obstacle lies.

a) $h = 25$ m:

$$v = h \sqrt{\frac{2(d_1 + d_2)}{\lambda d_1 d_2}}$$

$$= 2.74$$

From Fig. 4.14:

$$G_d(\text{dB}) = 22 \text{ dB.}$$

$$\Delta d = \frac{h^2}{2} \left(\frac{d_1 + d_2}{d_1 d_2} \right)$$

$$= h^2 \frac{\lambda}{2}$$

$$n = 3.75$$

∴ Zone # 3.

b) $h = 0$:

$$v = 0$$

$$G_d = 6 \text{ dB.}$$

∴ center of first zone

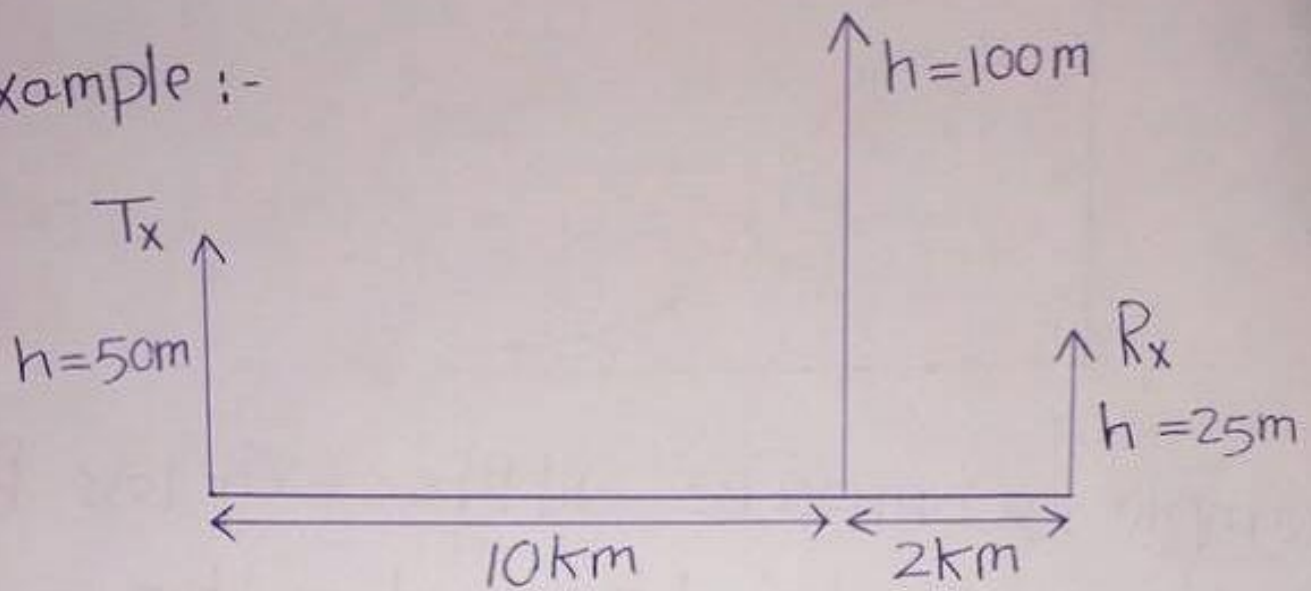
c) $h = -25$ m:

$$v = -2.74$$

$$G_d \approx 1 \text{ dB.}$$

∴ Zone # 3.

Example :-

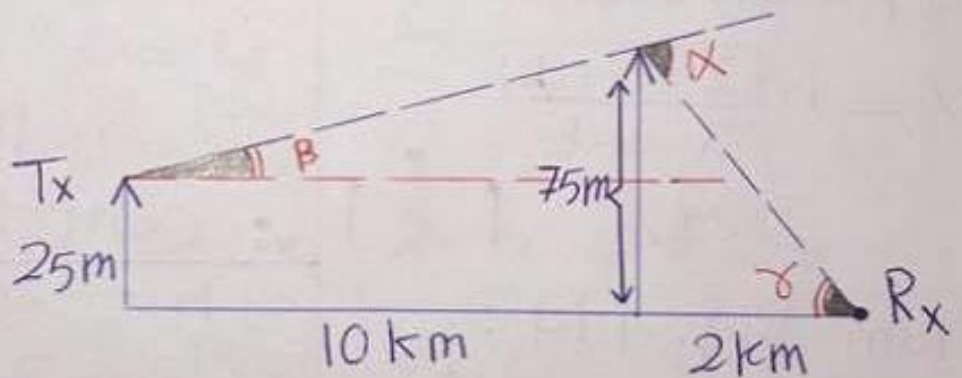


1] Find $G_d(\text{dB})$? [Analysis].

2] h of the obstacle that induced $G_d = -6\text{dB}$? [Design]. $\{\lambda = 1/3\text{m}\}$.

Solution :-

1)

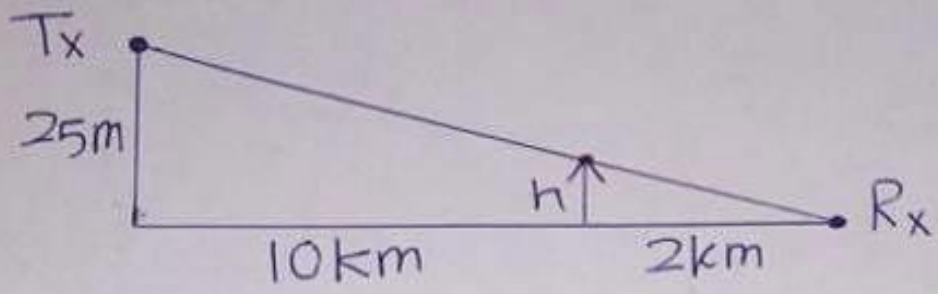


$$\alpha = \delta + \beta$$

$$= \frac{75}{2000} + \frac{50}{10,000} = 0.0424 \text{ rad.}$$

$$V = \alpha \sqrt{\frac{2d_1d_2}{\lambda(d_1+d_2)}} = 4.24$$

From fig 3.14 : $G_d(\text{dB}) = -25.5\text{dB}$.



$$\frac{h}{2k} = \frac{25}{12k} \rightarrow h = 4.16 \text{ m.}$$

4] Scattering : [Radar Cross Section model

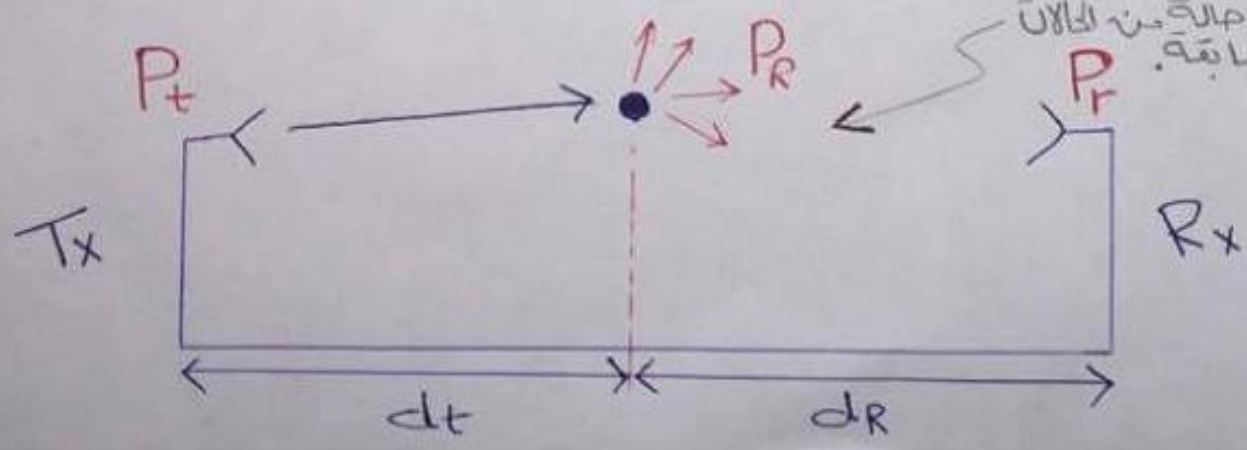
For urban system, the Radar Cross Section (RCS) model can be used to calculate the radiated power (P_R) due to scattering in the direction of the R_x

$$P_R \text{ (dBm)} = P_T \text{ (dBm)} + G_t \text{ (dB)} + 20 \log (\lambda) + RCS \text{ (dB} \cdot \text{m}^2) - 30 \log (4\pi) - 20 \log (d_T) - 20 \log (d_R).$$

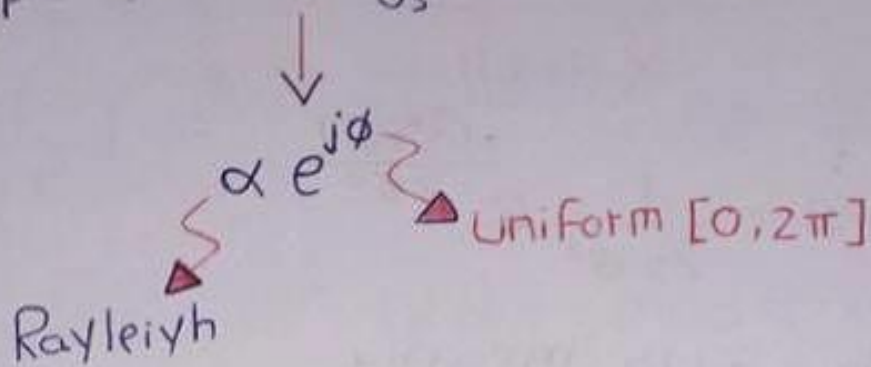
بيننا Tx و ال

For obstacles

• ممكن يصير غيرا أي حالة من الحالات السابقة.

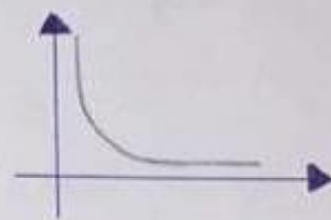


$$g_r(t) = h g_s(t) + n(t)$$



[NO LOS]

α^2 : exponential : $f_{\alpha^2}(\alpha) = \frac{1}{2\sigma^2} e^{-\frac{\alpha}{2\sigma^2}}$



Wireless Channel c/c in LOS :-

In this case :

$$h = \underbrace{p_1 e^{j\phi_1}}_{\text{LOS}} + \underbrace{\sum_{k=2}^N p_k e^{j\phi_k}}_{\text{Other multi-paths}}$$

LOS

Other multi-paths.

* will be dominant.

* Rxd power $\propto \frac{1}{d^2}$.

$$\rightarrow h = X_1 + jY_1 + \underbrace{\sum_{k=2}^N X_k + jY_k}_{\tilde{X} + j\tilde{Y}}$$

We know that:

$$\tilde{X} + j\tilde{Y} \rightarrow f_{\tilde{x}\tilde{y}}(\tilde{x}, \tilde{y}) = \frac{1}{2\pi\sigma^2} e^{\frac{-(\tilde{x}^2 + \tilde{y}^2)}{2\sigma^2}}$$

with zero mean.

OR:

$$f_{x,y}(x,y) = \frac{1}{2\pi\sigma^2} e^{\frac{-[(\tilde{x}-x_1)^2 + (\tilde{y}-y_1)^2]}{2\sigma^2}}$$

Where:

$$* x_1 = E[X_1]$$

$$* y_1 = E[Y_1]$$

* Convert to polar coordinate:

$$\tilde{x} = \alpha \cos \phi, \quad \tilde{y} = \alpha \sin \phi$$

$$\therefore f_{\alpha\phi}(\alpha, \phi) = \frac{f_{\tilde{x}\tilde{y}}(\tilde{x}^{-1}, \tilde{y}^{-1})}{J(\tilde{x}, \tilde{y})}$$

$$J(\tilde{x}, \tilde{y}) \rightarrow \frac{1}{\alpha}$$

$$= \frac{\alpha}{2\pi\sigma^2} e^{\frac{-[(\alpha \cos \phi - x_1)^2 + (\alpha \sin \phi - x_2)^2]}{2\sigma^2}}$$

$$f_{\alpha}(\alpha) = \int_{-\pi}^{\pi} f_{\alpha\phi}(\alpha, \phi) \cdot d\phi$$

$$= \frac{\alpha}{\sigma^2} e^{\frac{-(\alpha^2 + A^2)}{2\sigma^2}} \cdot \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{\frac{\alpha(x_1 \cos \phi + y_1 \sin \phi)}{\sigma^2}} \cdot d\phi$$

where:

$$A^2 = x_1^2 + y_1^2.$$

$$\frac{1}{2\pi} \int_{-\pi}^{\pi} e^{\frac{\alpha(x_1 \cos \phi + y_1 \sin \phi)}{\sigma^2}} \cdot d\phi \triangleq I_0\left(\frac{A\alpha}{\sigma^2}\right).$$

$I_n(x)$: n^{th} order modified Bessel function of the first kind.

$$I_n(x) = \frac{1}{\pi} \int_0^{\pi} e^{x \cos \phi} \cdot \cos(n\phi) \cdot d\phi.$$

$$\therefore f_{\alpha}(\alpha) = \frac{\alpha}{\sigma^2} e^{\frac{-(\alpha^2 + A^2)}{2\sigma^2}} \cdot I_0\left(\frac{A\alpha}{\sigma^2}\right).$$

Rician or Ricean

* Pdf of envelope wireless channel with LOS.

mag.

$A^2 = x_1^2 + y_1^2$; avg. power in LOS.

→ For Rician R.V, we define:

$$K = \frac{A^2}{2\sigma^2}$$



→ Variance.

Rician Parameter.
(SNR in LOS)

$$\ast \boxed{I_0(0) = 1}.$$

Note that:

IF $A=0 \rightarrow$ NO LOS

$f_{\alpha}(\alpha) : \text{Rayleigh} = \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}}$

$$* r(t) = \alpha e^{j\phi} s(t) + n(t)$$

BER : BPSK
BFSK
M-QAM
M-PAM

} $\propto \Phi(\text{SNR})$

$$\Phi\left(\sqrt{\frac{E}{N_0}}\right)$$

* Assume α const. (Actually α is RV)

$$\Phi\left(\sqrt{\frac{\alpha^2 E}{N_0}}\right) \equiv \Phi\left(\alpha \sqrt{\frac{E}{N_0}}\right)$$

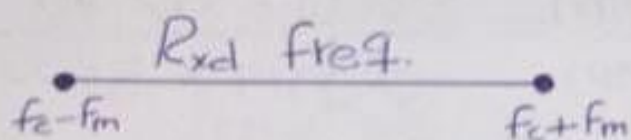
* α , RV:

$$\int_0^{\infty} \Phi\left(\alpha \sqrt{\frac{E}{N_0}}\right) f(\alpha) d\alpha \neq$$

Frequency Spreading or Dispersion & Coherence time T_c :

If the Tx'd freq. is f_c ;
due to doppler spreading, the freq.
will $f_c \pm f_m$

$$\rightarrow \text{max. doppler freq} = \frac{v}{\lambda} ; \theta = 0 \text{ or } \pi$$



We define coherence time $T_c \propto \frac{1}{f_m}$

In General,

$$T_c \text{ (50\% correlation)} = \frac{0.423}{f_m}$$

∞ a channel is slow faded iff
 $T_c > T_s$.

& a channel is fast faded iff
 $T_c < T_s$.

\rightarrow bit/symbol
Period.

* Avg. Fade duration : 7

Avg. period of time for the Rx'd Signal to be below a certain threshold R .

$$\bar{T}_{fd} \triangleq \frac{P_r[\alpha < R]}{N_R} = \frac{\int_0^R f(\alpha) \cdot d\alpha}{N_R} \rightarrow \text{cdf for Rayleigh}$$

$$\rightarrow \int_0^R f(\alpha) \cdot d\alpha = \int_0^R \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}} \cdot d\alpha \quad \text{--- Rayleigh}$$
$$= 1 - e^{-\rho^2}$$

$$* \rho = \frac{R}{R_{rms}} = \frac{R}{\sqrt{2\sigma^2}}$$

$$* R_{rms}^2 = 2\sigma^2 \rightarrow R_{rms} = \sqrt{2\sigma^2}$$

$$\therefore \bar{T}_{fd} = \frac{1 - e^{-\rho^2}}{\sqrt{2\pi} f_m \rho e^{-\rho^2}}$$
$$= \frac{e^{\rho^2} - 1}{\sqrt{2\pi} f_m \rho} \quad (S) \cdot \times \times$$

Example : $\rho = 0.707$, $f_m = 20$ kHz

a) $\bar{T}_{fd} = ?$

$$\bar{T}_{fd} = \frac{e^{\rho^2} - 1}{\sqrt{2\pi} f_m \rho} = 18.3 \text{ ms}$$

b) For binary digital system,

$$R_b = 50 \text{ bps} , T_b = \frac{1}{R_b} = 20 \text{ ms}$$

$\rightarrow T_b > \bar{T}_{fd}$: Fast channel.

c) Find bit ERROR Rate for $R_b = 50$ bps

if a bit ERROR occurs whenever a portion of a bit encounters a fade with $\rho = 0.1$?

$$\rho = 0.1 ; N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2} \\ = 4.96 \approx 5 \text{ crossing/sec}$$

$$\text{BER} = \frac{5}{50} = 10\%$$

Frequency Selective Channels

To examine the effect of freq. selective channel, we consider the simple two path model,

$$g_r(t) = \underline{g_s(t) P_1 e^{j\phi_1}} + \underline{g_s(t - \tau_m) P_2 e^{j\phi_2}},$$

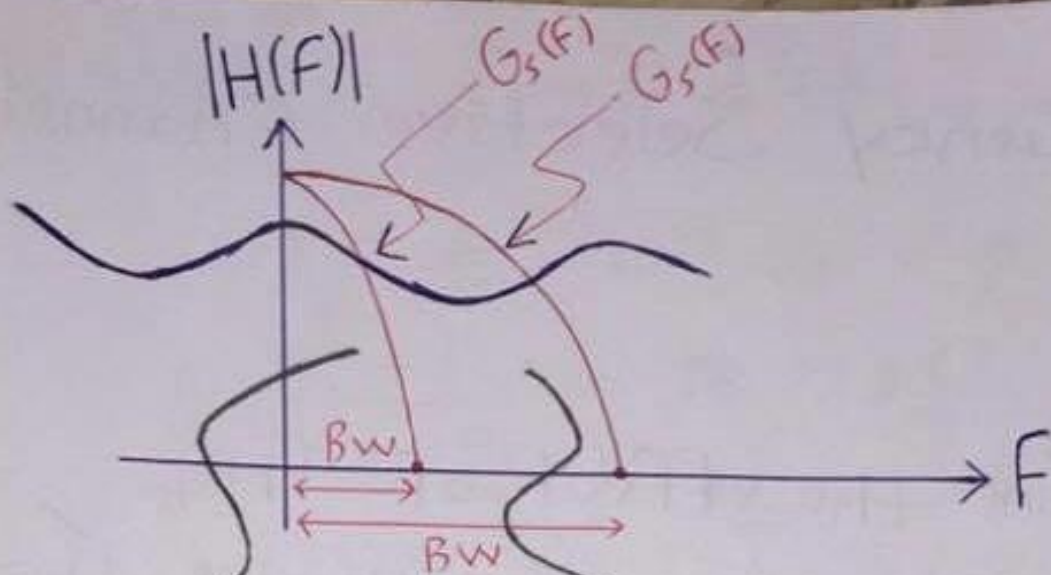
$$0 \leq t \leq T_s$$

$$T_s < \tau_m$$

$$h(t) = P_1 e^{j\phi_1} \delta(t) + P_2 e^{j\phi_2} \delta(t - \tau_m)$$

or

$$H(f) = P_1 e^{j\phi_1} \times (1) + P_2 e^{j[\phi_2 - 2\pi f \tau_m]}$$



flat faded
Channel

freq. selective
Channel.

* $Bw \ll \frac{1}{T_m} = B_c$

* $Bw \gg \frac{1}{T_m}$

* $T_s \gg T_m$

* $T_s \ll T_m$

* Time domain:

* freq. domain:

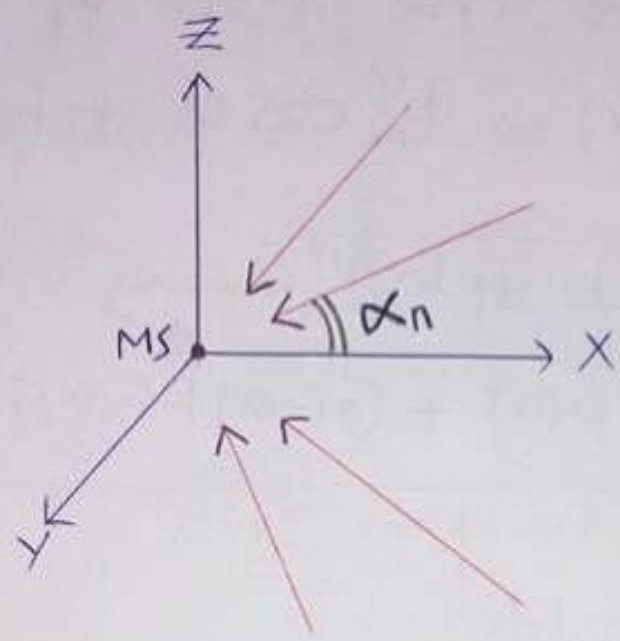
$Bw \checkmark$

$T_s \checkmark$

$R_s \uparrow, Bw \uparrow, T_s \downarrow.$

Statistical Model for multi-path flat faded Channels :

* [Clark's model] :



$$0 \leq \alpha_n \leq 2\pi$$

n^{th} arriving wave with α_n has doppler

Shift $f_n = \frac{v}{\lambda} \cos \alpha_n$
 $= f_m \cos \alpha_n$

$f_c \rightarrow f_c \pm f_m$

\uparrow
max.

let $P(\alpha) d\alpha$: fraction of total incoming power with $d\alpha$ of the arriving angle α .

$G(\alpha)$: MS antenna gain.

∴ Total R_{xd} Power by the MS will be:

$$P_r = \int_0^{2\pi} G(\alpha) P(\alpha) \cdot d\alpha$$

another method to calculate Power:

$$R_x(\tau) \rightarrow \int_{-\infty}^{\infty} G_x(f) \cdot df \quad (\text{W/Hz})$$

PSD

or let $S(f)$ be the PSD of the R_{xd} signal: $f(\alpha) = f_m \cos \alpha + f_c$.

$$|df| = f_m \sin(\alpha) \cdot (d\alpha)$$

even Fun.

$$\infty \frac{S(f)|df|}{\text{fraction power}} = \frac{(G(\alpha)P(\alpha) + G(-\alpha)P(-\alpha))^2 |d\alpha|}{\text{fraction power}}$$

$$S(f) = 2 G(\alpha) P(\alpha) \frac{|d\alpha|}{|df|}$$

$$= \frac{2 G(\alpha) P(\alpha)}{f_m \sin \alpha}$$

$$f(\alpha) = f_m \cos \alpha + f_c \rightarrow \alpha = \cos^{-1} \left(\frac{f - f_c}{f_m} \right)$$

$$\sin \alpha = \sqrt{1 - \left(\frac{f - f_c}{f_m} \right)^2}$$

Car. Freq

max doppler

$$= f_m \sqrt{1 - \left(\frac{f - f_c}{f_m} \right)^2}$$

$$S(f) = \frac{2 G(\alpha) P(\alpha)}{f_m \sqrt{1 - \left[\frac{f - f_c}{f_m} \right]^2}}$$

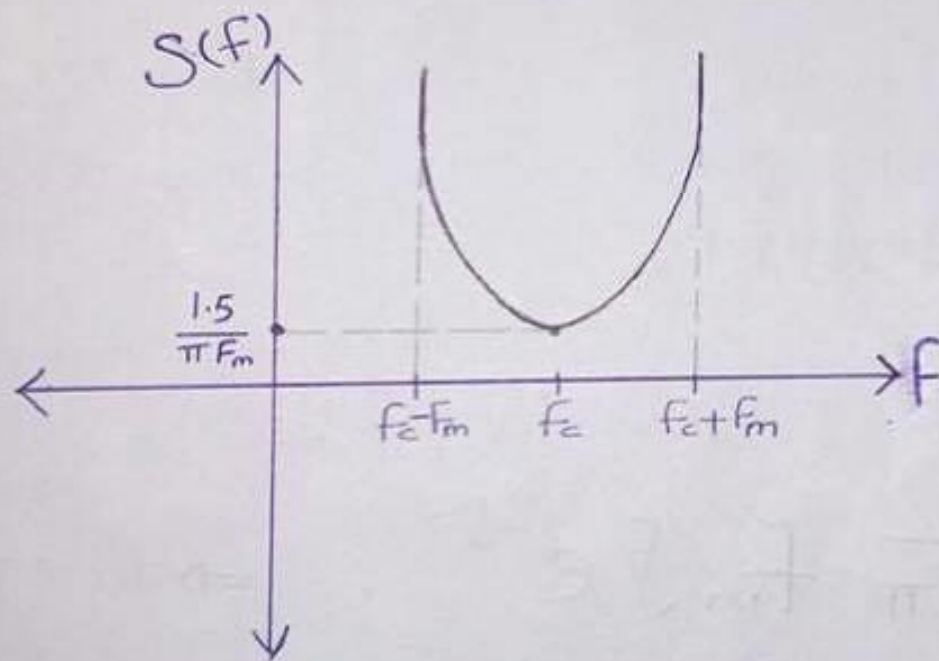
Clark's
model.

Total PSD
as a Fun. of f .

for $\frac{\lambda}{4}$ vertical Antenna,

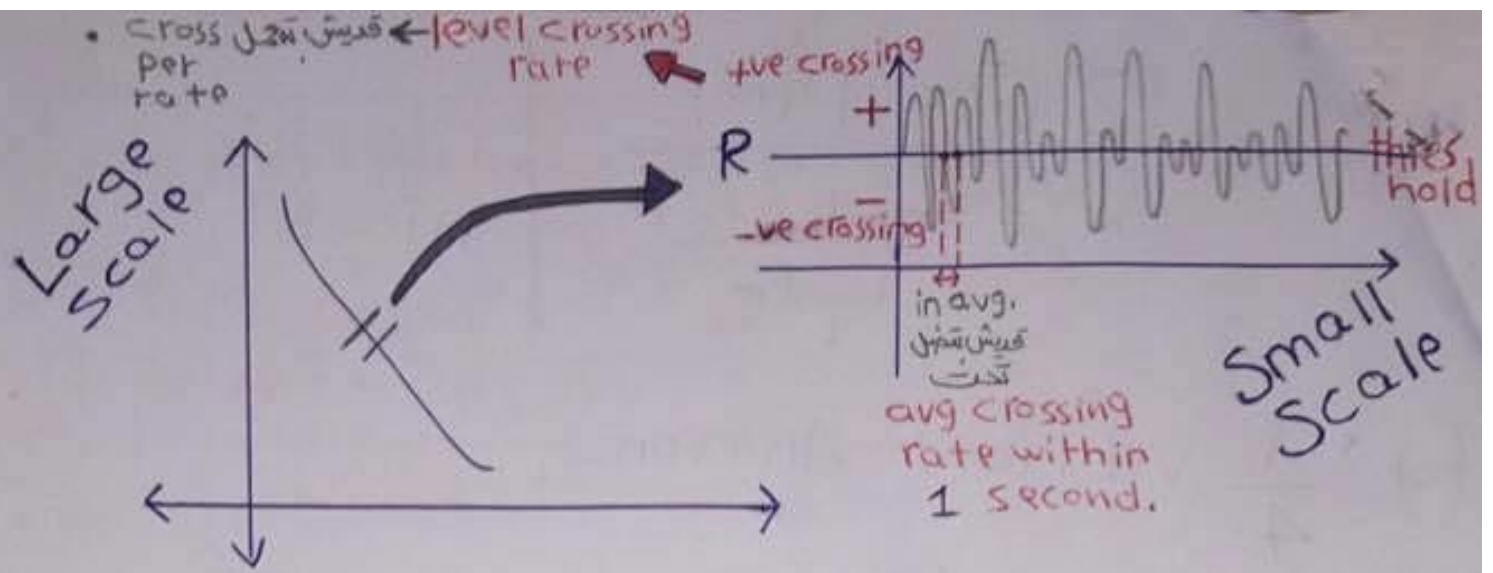
$$G(\alpha) P(\alpha) = \frac{1.5}{2\pi}$$

$$S(f) = \frac{1.5}{\pi f_m \sqrt{1 - \left[\frac{f - f_c}{f_m} \right]^2}}$$



Total Power :

$$\int_{-\infty}^{\infty} S(f) df \rightarrow \int_{f_c - f_m}^{f_c + f_m} S(f) df \cdot$$



* Level crossing Rate:
 (+ve) level crossing Rate (N_R), &
 average fade duration (\bar{T}_{fd}) of a
 Rayleigh fading signal are very
 important parameters for wireless
 system designers.

(+) LCR (crossing / sec)

$$= \sqrt{2\pi} f_m \rho e^{-\rho^2} \Rightarrow \text{integer.}$$

where $\rho \triangleq \frac{R}{R_{rms}}$

$$R_{rms}^2 \triangleq E[\alpha^2] = 2\sigma^2 = \int_0^{\infty} \frac{\alpha^3}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}} d\alpha$$

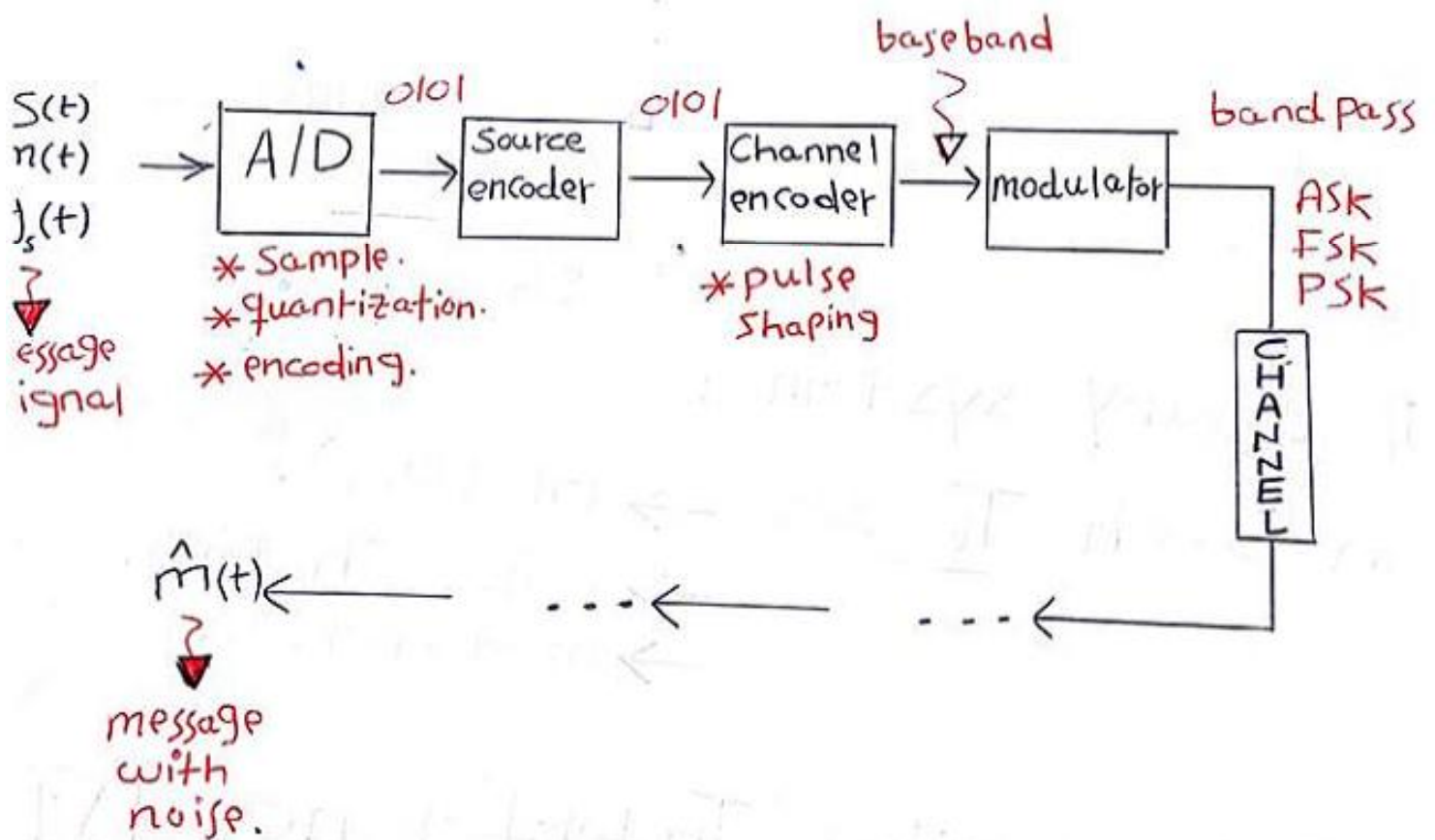
$$\alpha = \sqrt{x^2 + y^2}$$

(σ_x^2) (σ_y^2)

Chapter 6: Digital modulation Techniques in Wireless Channel (Fading Channel)

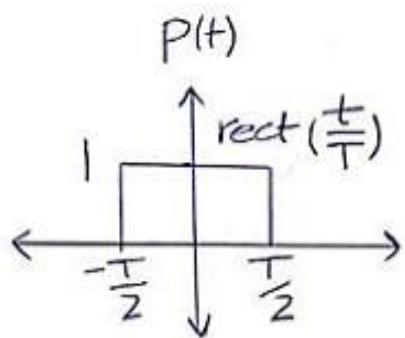
$$h = \alpha e^{j\theta}$$

In General, The generic block diagram of any digital comm. system :-



* pulse Shaping $[P(t)]$:-

0, 1 \rightarrow pulses.



* time domain.

$\rightarrow T \text{ sinc}(fT)$

* freq. domain.



- No channels use the rect. as pulse Shaping.

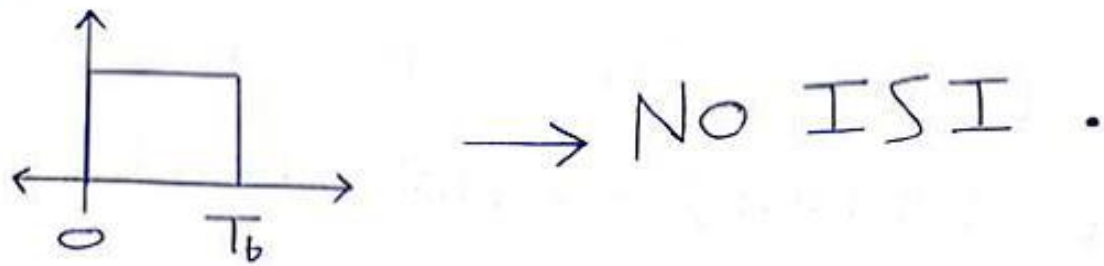
if binary system :-

in each T_b sec. \rightarrow bit (0, 1). Source of bit

\rightarrow Pulse Shaping.

\rightarrow modulation.

* Inter Symbol Interference **ISI**
any pulse Shaping signal must satisfies No ISI.



* advantages of using rect. as pulse shaping :-

* band limited in time.

* Not suffer from ISI.

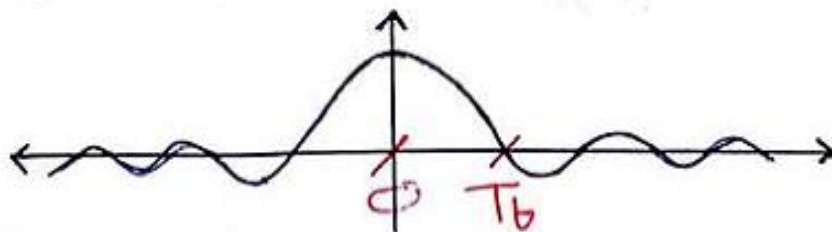
* disadvantages :

* in freq. domain → sinc

↓
need infinite BW.

* any pulse with T_b & multiple integers of T_b has a zero crossing

⇒ No ISI.



Any pulse with zero crossing @ nT_b ($n: \text{int.}$) \rightarrow HAS NO ISI

\rightarrow Nyquist Pulse.

1) Rectangular pulse:

$\text{rect}\left(\frac{t}{T}\right) \rightarrow$ Nyquist pulse \checkmark

but Bw $\uparrow \infty$ \times .

2) Sinc pulse:

$\text{Sinc}\left(\frac{t}{T}\right)$

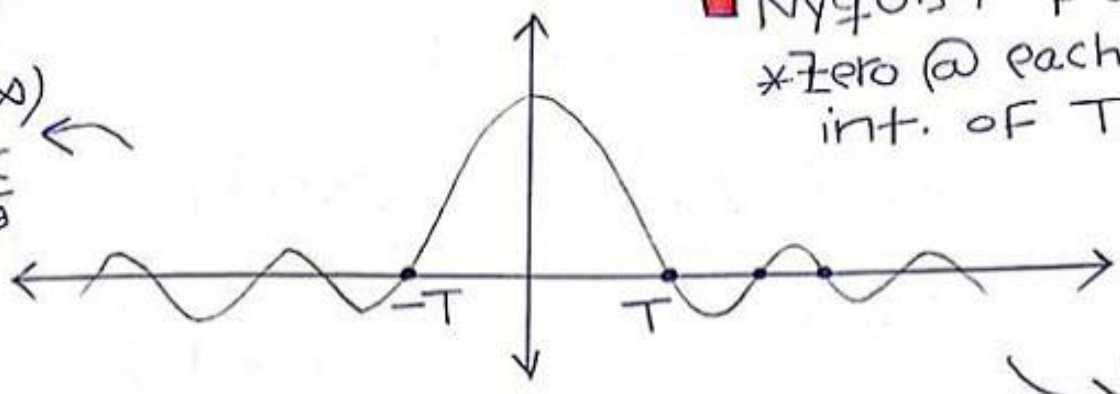
$\text{Sinc}(x) \triangleq \frac{\sin(x)}{x} \rightarrow$ zero crossing @ $\pi, 2\pi, 3\pi, \dots$

$\triangleq \frac{\sin(\pi x)}{\pi x} \rightarrow$ zero crossing @ $1, 2, 3, \dots$

FD

■ Nyquist pulse ✓
* zero @ each multiple int. of T.

$(-\infty, \infty)$
sinc
قناة



$(-\infty, \infty)$
sinc
بند

→ Source send $(-\infty, \infty)$ signal. ✓

* BW (finite) ✓

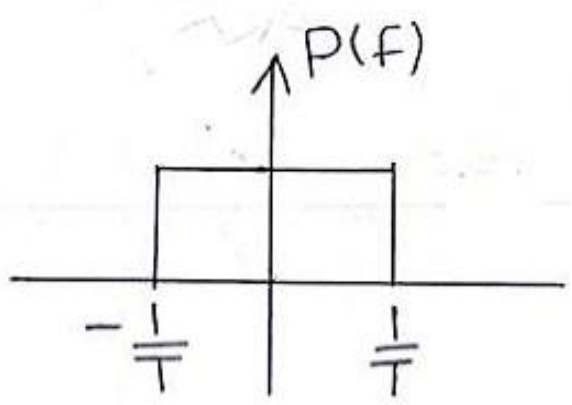
* $BW \propto \frac{1}{T}$

* one degree of freedom → T

بقدر التحكم فيها

و بتحكم بال BW من خلالها

in freq. domain :-



c) Raised cosine :-

$$P(t) = \frac{\sin(\pi t/T)}{\pi t} \frac{\cos(\alpha \pi t/T)}{1 - \left(\frac{4\alpha t}{2T}\right)^2}$$

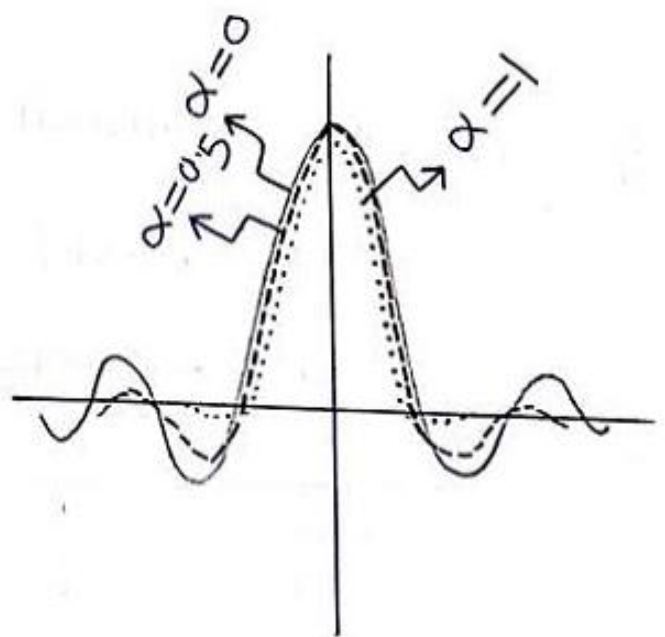
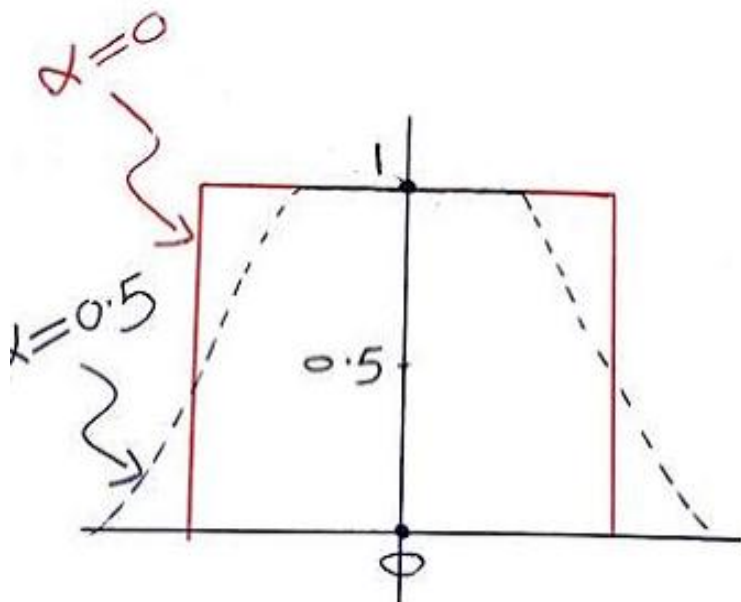
* at any multiple int. of $t \rightarrow P(t) = \text{zero}$

α : roll-off factor.

* HAS Two degree of freedom $\rightarrow T$ & α

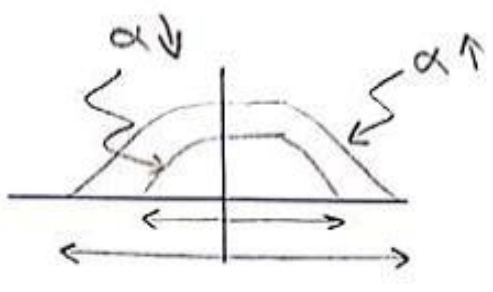
زیادتی اونقدره

• Bw eff. 1 Power eff. 1



$\alpha \uparrow$, BW \uparrow , Side loop \downarrow ,
 Total power \downarrow , Power eff. \uparrow .

As $\alpha \downarrow$, BW \downarrow , Side loop \uparrow ,
 Total power \uparrow , Power eff. \downarrow .

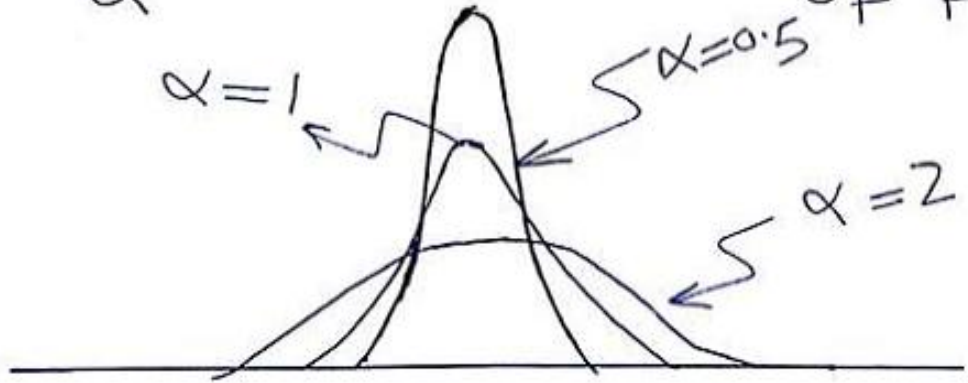


GSM \rightarrow using raised cosine pulse shaping signal with $\alpha = 0.5$

d) Gaussian pulse:

$$P(t) = \frac{\sqrt{\pi}}{\alpha} e^{-\frac{\pi^2 t^2}{\alpha^2}}$$

\rightarrow one degree of freedom



Target \rightarrow Pulse BW \downarrow , P \downarrow , efficient

Digital Systems :-

1] BPSK :-

T_{xd} :

$$S_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos(\omega_c t + \phi_i) , 0 \leq t \leq T_b .$$

E_b : bit Energy (J).

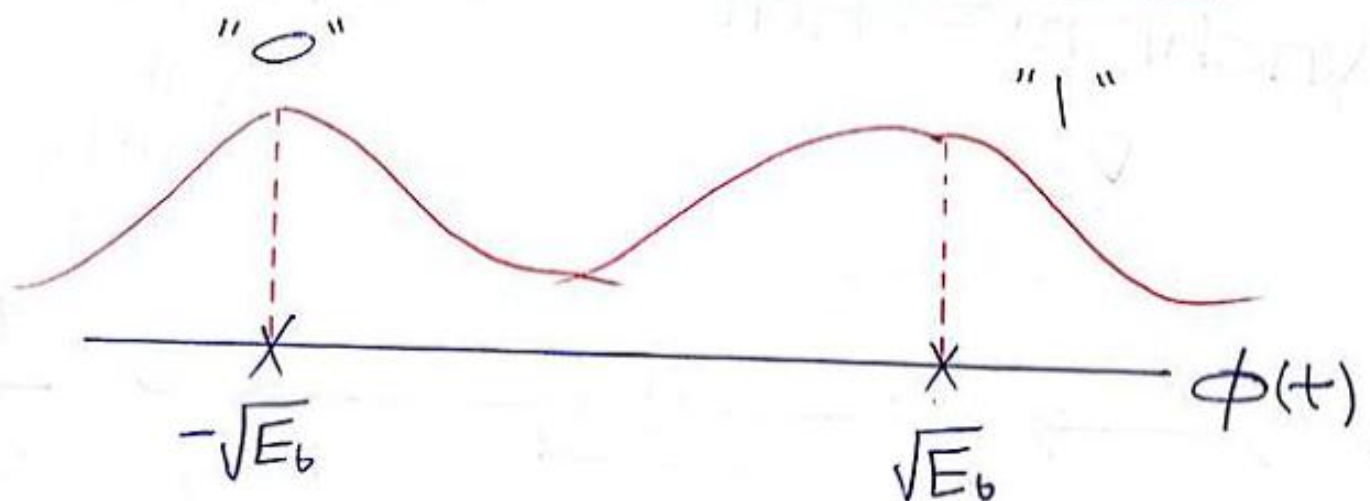
T_b : bit period (s).

ω_c : carrier freq.

ϕ_i : carrier phase shift = 0, π .

$$\Rightarrow S_i(t) = \pm \sqrt{\frac{2E_b}{T_b}} \cos(\omega_c t) .$$

$$= \pm \sqrt{E_b} \left[\sqrt{\frac{2}{T_b}} \cos(\omega_c t) \right] \phi(t)$$



Rxd :

$$r(t) = s_i(t) + n(t)$$

AWGN
(0, $\frac{N_0}{2}$)

$$P_b(e) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) = Q(\sqrt{2 \text{SNR}})$$

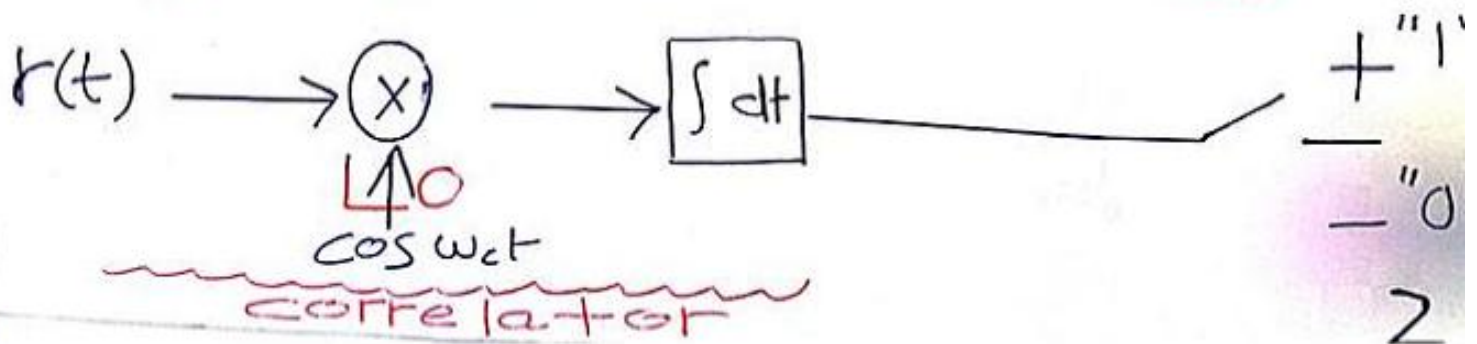
$$\frac{E_b}{N_0} = \text{SNR}$$

$\frac{N_0}{\omega/\text{Hz}}$ $\Delta \frac{J}{\omega} \cdot \text{Hz} = \frac{W}{\omega} \rightarrow \text{unitless.}$

* BPSK need perfect phase synchronization \rightarrow due to PLL.

between phase rec. sig. & lo. phase.

Voltage control osci.



Non-coherent PSK (Differential PSK) :-

DPSK

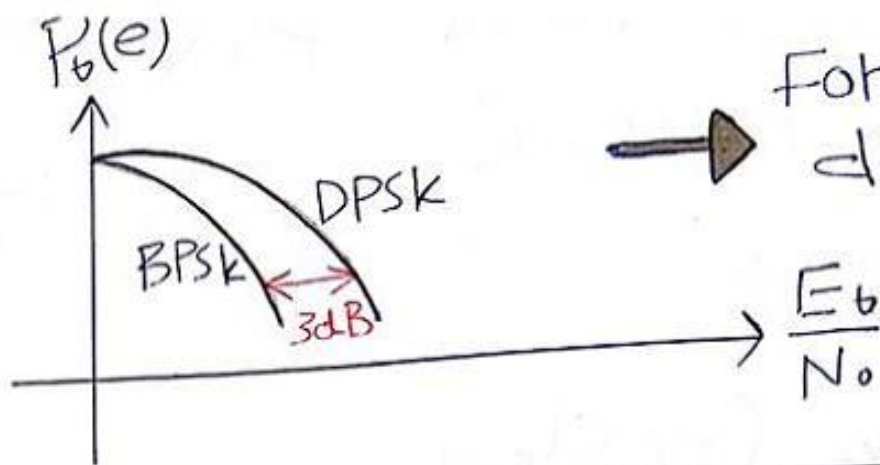
$$d_k = m_k \oplus d_{k-1}$$

m_k : current bit
 d_{k-1} : previous encoded bit.
 d_k : Differently encoded bit.

0	\oplus	0	\rightarrow	1	$\Phi_i = \pi$
0	\oplus	1	\rightarrow	0	} $\Phi_i = 0$
1	\oplus	0	\rightarrow	0	
1	\oplus	1	\rightarrow	1	$\Phi_i = \pi$

* DPSK To solve the phase synchronization in BPSK.

$$P_b(e) = \frac{1}{2} e^{-\frac{E_b}{N_0}} = \frac{1}{2} e^{-\text{SNR}}$$



③ Quadrature PSK (ϕ PSK) (4-PSK) :

$$S_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left[\omega_c t + (i-1) \frac{\pi}{2}\right]$$

$$i = 1, 2, 3, 4$$

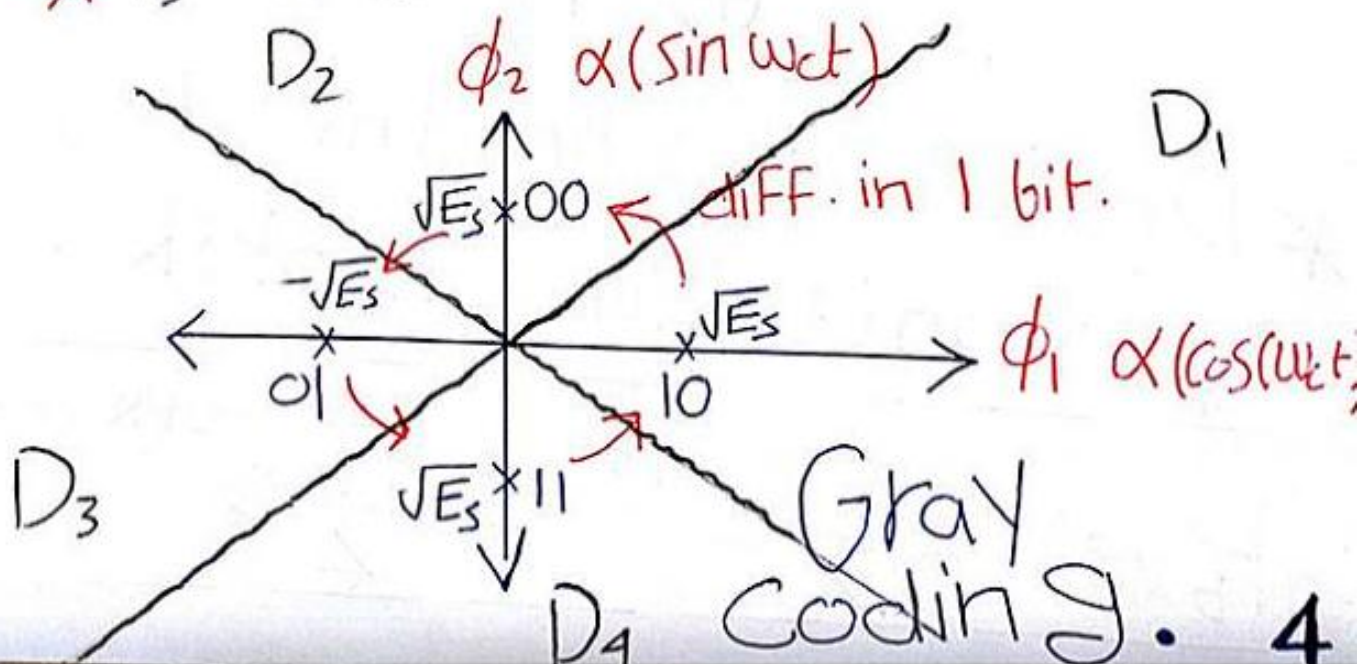
For 4-PSK:

$$* E_s = 2E_b$$

$$* T_s = 2T_b$$

$$0 \leq t \leq T_s$$

$$N = 2^b$$



$$\text{erfc}(x) = \frac{1}{2} \Phi\left(\frac{x}{\sqrt{2}}\right)$$

we can show that :

$$P_b(e) = 1 - \left(1 - \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right)\right)^2$$

$$= 1 - P_{cx} P_{cy}$$

$$= 1 - (1 - P_{ex})^2$$

$$= \text{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right) - \frac{1}{4} \text{erfc}^2\left(\sqrt{\frac{E_s}{2N_0}}\right)$$

for large SNR : $\frac{E_s}{N_0} \gg 1$, $\text{erfc} \ll 1$

$$\therefore P_b(e) \approx \text{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right)$$

$$P_b(e) = \frac{1}{2} P_b(e) \quad \dots \quad \Phi \text{PSK}$$

$$= \frac{1}{2} \text{erfc}\left(\sqrt{\frac{E_b}{N_0}}\right)$$

$$= \Phi\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$\Phi(x) = \frac{1}{2} \text{erfc}\left(\frac{x}{\sqrt{2}}\right)$$

* QPSK has the same power
(same performance) as BPSK, & S:
needs $\frac{1}{2}$ BW of BPSK.

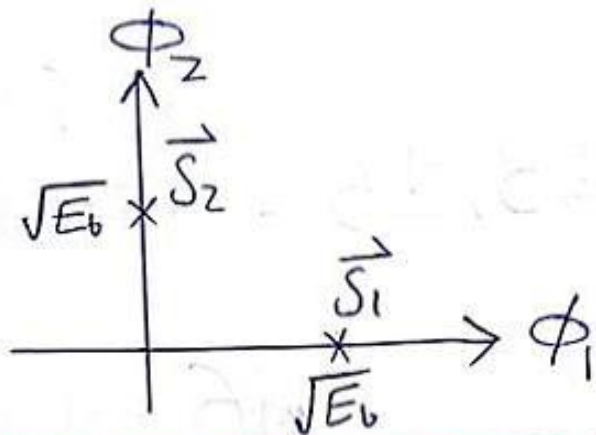
$$\boxed{\text{QPSK} \equiv 4\text{-QAM}}$$

→ QPSK needs more complicated
phase synchroniser.

(more accurate to make synchronise)
& should be much more sensitive.

BFSK (Orthogonal System):

$$s_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t), \quad i = 1, 2, 3, \dots$$



$$P_b(e) = Q\left(\sqrt{\frac{E_b}{N_0}}\right) = \frac{1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_b}{2N_0}}\right)$$

Similarity for Non-coherent (DFSK):

$$P_b(e) = \frac{1}{2} e^{-\frac{E_b}{2N_0}} = \frac{1}{2} e^{-\frac{\gamma}{2}}, \quad \frac{E_b}{N_0} = \text{SNR} = \gamma$$

& for DPSK:

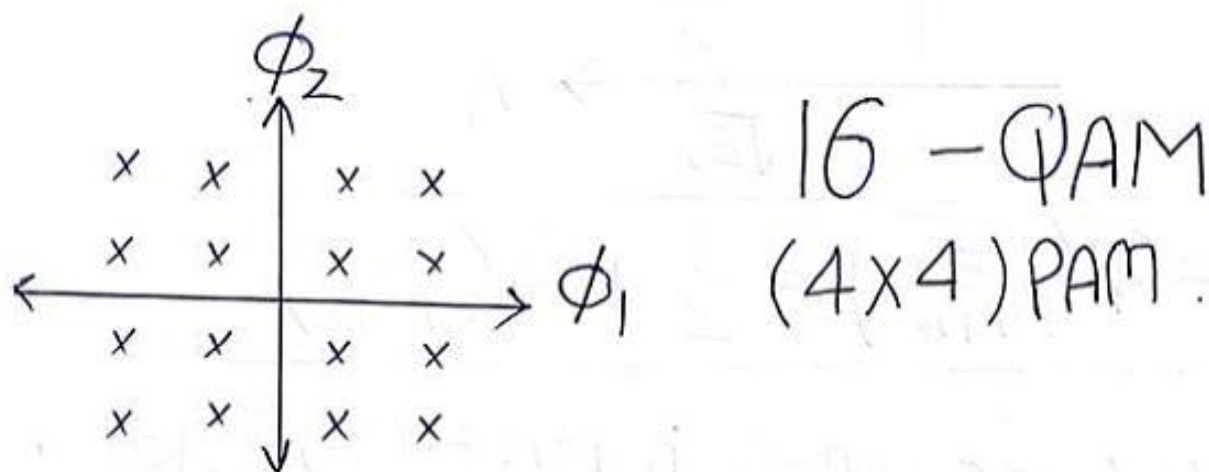
$$P_b(e) = \frac{1}{2} e^{-\gamma}$$

5 M-QAM (Square QAM):

$$S_i(t) = \sqrt{\frac{2E}{T_s}} a_i \cos(\omega_c t) + \sqrt{\frac{2E}{T_s}} b_i \sin(\omega_c t)$$

$i = 1, \dots, M$

$$a_i, b_i = \pm 1, \pm 3, \pm 5 \dots \pm \sqrt{M} - 1$$



$$P_B(e) = 2 \left(1 - \frac{1}{\sqrt{M}}\right) \operatorname{erfc} \left(\sqrt{\frac{3}{2} \frac{P_{\text{avg}}}{(M-1) N_0}} \right)$$

SNR $\equiv \gamma$

M-PSK :

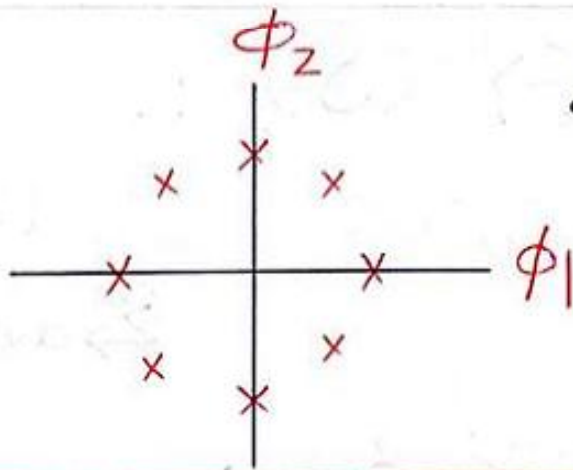
(N=2)

BPSK

QPSK

MPSK

$$S_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(\omega_c t + \frac{2\pi}{M}(i-1)\right)$$



8-PSK

$$P_s(e) \cong \text{erfc}\left(\sqrt{\frac{E_s}{N}} \sin \frac{\pi}{M}\right)$$

7] M-FSK : $N = M$

$$S_i(t) = \sqrt{\frac{2E_s}{T_s}} \cos\left(\frac{\pi}{T_s}(i+n_c)F\right)$$

$$i = 1, \dots, M.$$

$$f_c = \frac{n_c}{2T_s} ; n_c \rightarrow \text{integer } 1, 2, \dots$$

$$P_s(e) \leq \frac{M-1}{2} \operatorname{erfc}\left(\sqrt{\frac{E_s}{2N_0}}\right).$$

* BW efficiency (\mathcal{S}) :

$$\mathcal{S} = \frac{R_b}{\text{BW (Hz)}} \quad \begin{array}{l} \text{bit rate} = \frac{1}{T_b} \text{ (Hz)} \\ \text{duration} \end{array}$$

Transmission BW.

OR MPSK :

$$BW = \frac{2}{T_s} = \frac{2}{T_b \log_2 M} = \frac{2R_b}{\log_2 M} \cdot$$

$$\therefore \rho = \frac{\log_2 M}{2}$$

as $M \uparrow$, $\rho \uparrow$, $BW \downarrow$

MPSK is BW eff. system.

FOR MFSK :

$$BW = \frac{M}{2T_s} = \frac{M}{2T_b \log_2 M} = \frac{MR_b}{2 \log_2 M} \cdot$$

$$\therefore \rho = \frac{2 \log_2 M}{M}$$

as $M \uparrow$, $\rho \downarrow$, $BW \uparrow$

MFSK is power eff. system.

* Performance for Digital Systems in Multipath Fading Channels (wireless channels) :-

$$r(t) = \alpha e^{j\phi} S_i(t) + n(t)$$

gain, amp., voltage, or envelop of the channel.

channel phase $(0, 2\pi]$

AWGN
($0, N_0$)

not the power,
Power $\rightarrow \alpha^2$.

α : Rayleigh (Non LOS)
or Rician (LOS).

☑ BPSK :

$$r(t) = \alpha S + n$$

$\alpha^2 \leftrightarrow E_b / N_0$

Without fading: $P_b(e) = Q(\sqrt{2\gamma})$
 $= \frac{1}{2} \operatorname{erfc}(\sqrt{\gamma})$.

with Fading: (given α const.)

$$P_b(\alpha) = Q(\sqrt{2\gamma_b \alpha^2}), \quad \gamma_b = \frac{E_b}{N_0}$$

$$P_b(e) = \int_0^{\infty} P_b(\alpha) \cdot f(\alpha) \cdot d\alpha$$

avg. BER.

* Rayleigh Fading: $f(\alpha) = \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}}$

$$\Rightarrow P_b(\alpha) = Q\left(\sqrt{2 \frac{E_b}{N_0} \alpha^2}\right) = Q(\sqrt{2\gamma})$$

$\gamma = \frac{E_b}{N_0} \alpha^2$

exp. \uparrow

$$P_b(e) = \int_0^{\infty} Q(\sqrt{2\gamma}) \cdot f(\gamma) \cdot d\gamma$$

instantaneous SNR
with multipath
Fading (Rayleigh)

$$\gamma = \frac{E_b}{N_0} \alpha^2 \rightarrow \alpha = \sqrt{\frac{N_0}{E_b}} \gamma \quad f_\alpha(\alpha)$$

$$J = 2 \frac{E_b}{N_0} \cdot \sqrt{\frac{N_0}{E_b}} \gamma = 2 \sqrt{\frac{E_b \gamma}{N_0}} \quad \rightarrow P_b(e)$$

$$f(\alpha) = \frac{\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}}$$

$$Y = X^2$$

$$f_Y(Y) = \frac{f_X(X)}{J}$$

$$f_\gamma(\gamma) = \frac{\sqrt{\frac{N_0}{E_b}} \gamma e^{-\frac{N_0 \gamma}{E_b} / 2\sigma^2}}{2\sigma^2 \sqrt{\frac{E_b \gamma}{N_0}}}$$

$$= \frac{1}{2 \frac{E_b}{N_0} \sigma^2} e^{-\frac{\gamma}{2 \cdot \frac{E_b}{N_0} \cdot \sigma^2}}, \quad \gamma \geq 0.$$

$$* E[\alpha^2] = 2\sigma^2 \rightarrow * \gamma = \frac{E_b}{N_0} \alpha^2$$

$$* E[\gamma] = \frac{E_b}{N_0} 2\sigma^2$$

but: $2 \frac{E_b}{N_0} \sigma^2 = E[\gamma] \triangleq \bar{\gamma}_b \rightarrow \text{avg SNR} *$

avg SNR
Faded channel.

binary system.

$$f_x(x) = \frac{1}{\sqrt{x_b}} e^{-x/\sqrt{x_b}}$$

$$\rightarrow P_b(e) = \int_0^{\infty} \varphi(\sqrt{2x}) \cdot \frac{1}{\sqrt{x_b}} e^{-x/\sqrt{x_b}} \cdot dx$$

$$\varphi(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-t^2/2} \cdot dt$$

\Rightarrow By parts :

$$u = \varphi(\sqrt{2x}) \rightarrow du = \frac{-1}{\sqrt{2\pi}} e^{-x} \cdot \frac{\sqrt{2}}{2} x^{-1/2} dx$$

$$= \frac{-1}{2\sqrt{\pi}} \frac{e^{-x}}{\sqrt{x}}$$

$$dv = \frac{1}{\sqrt{x_b}} e^{-x/\sqrt{x_b}} \rightarrow v = -e^{-x/\sqrt{x_b}}$$

$$\int u dv = uv - \int v \cdot du$$

$$= -e^{-x/\sqrt{x_b}} \varphi(\sqrt{2x}) \Big|_0^{\infty} - \frac{1}{2\sqrt{\pi}} \int_0^{\infty} \frac{1}{\sqrt{x}} e^{-x(1+\frac{1}{\sqrt{x_b}})} \cdot dx$$

$$f_2 = 0 + \frac{1}{2} - \frac{1}{2\sqrt{\pi}} \int_0^{\infty} \frac{1}{\sqrt{x}} e^{-\frac{x}{2a^2}} \cdot dx$$

تویین

$$2a^2 = \frac{1}{1 + \frac{1}{\gamma_6}}$$

$$\rightarrow \gamma = x^2$$

$$d\gamma = 2x \cdot dx$$

$$\frac{1}{2} - \frac{1}{2\sqrt{\pi}} \int_0^{\infty} \frac{1}{x} e^{-\frac{x^2}{2a^2}} \cdot 2x \cdot dx$$

$$= \frac{1}{2} - \frac{1}{\sqrt{\pi}} \int_0^{\infty} e^{-\frac{x^2}{2a^2}} \cdot dx$$

$$= \frac{1}{2} - \left[\frac{1}{\sqrt{2\pi} a} \sqrt{2a} \int_0^{\infty} e^{-\frac{x^2}{2a^2}} \cdot dx \right] = \frac{1}{2}$$

$$= \frac{1}{2} - \frac{\sqrt{2} a}{2}$$

$$= \frac{1}{2} - \frac{a}{\sqrt{2}}$$

$$, a = \sqrt{\frac{1}{2(1 + \frac{1}{\gamma_6})}}$$

$$= \frac{1}{2} - \frac{1}{2} \sqrt{\frac{1}{1 + \frac{1}{\gamma_6}}}$$

$$P_b(e) = \frac{1}{2} \left(1 - \sqrt{\frac{\gamma_6}{1 + \gamma_6}} \right)$$

نصفاً طبقاً
تحت،
Gaussian

$$\gamma_6 = \frac{E_b}{N_0} \frac{E[\alpha^2]}{2\sigma^2}$$

4

in non-fading :-

$$P_b(e) = \frac{1}{2} \exp(-\gamma_b)$$

BER \uparrow in BPSK in Rayleigh Fading

wireless channel

استود من ال

non fading

من ناحية

Power eff.



wireless channel
unguided
medium.

BPSK :

$$P_b(e) = \frac{1}{2} \left(1 - \sqrt{\frac{\bar{\gamma}_b}{1 + \bar{\gamma}_b}} \right)$$

$$\begin{aligned} \bar{\gamma}_b &= E[\gamma] \\ &= \frac{E_b}{N_0} \cdot 2\sigma^2 \end{aligned}$$

for $\bar{\gamma}_b \gg 1$

$$\sqrt{\frac{x}{1+x}} \approx 1 - \frac{1}{2x}$$

Variance for gaussian distribution for x & y .

$$P_b(e) \approx \frac{1}{2} \left(1 - \left\{ 1 - \frac{1}{2\bar{\gamma}_b} \right\} \right)$$

$$P_b(e) = \frac{1}{4\bar{\gamma}_b}$$

loss

~ 12 dB

without fading: $P_b(e) = Q(\sqrt{2\gamma})$.

* Similar: (BFSK) :-

without fading :- $P_b(e) = Q(\sqrt{\gamma_b})$.

with fading :-

$$P_b(e) = \frac{1}{2} \left(1 - \sqrt{\frac{\bar{\gamma}_b/2}{1 + \bar{\gamma}_b/2}} \right)$$

$$P_b(e) = \frac{1}{2} \left(1 - \sqrt{\frac{\bar{\gamma}_b}{2 + \bar{\gamma}_b}} \right)$$

for $\bar{\gamma}_b \gg 1$:

$$P_b(e) \approx \frac{1}{2\bar{\gamma}_b}$$

DPSK :- \longrightarrow saved \longleftarrow data
 on phase difference.
 Non coherent BPSK

without Fading: $P_b(e) = \frac{1}{2} e^{-\bar{\gamma}_b}$, $\bar{\gamma}_b = \frac{E_b}{N_0}$

with Fading:

$$P_b(x) = \frac{1}{2} e^{-\bar{\gamma}_b \alpha^2}$$

$$P_b(e) = \int_0^{\infty} \frac{1}{2} e^{-\gamma} \cdot \frac{1}{\bar{\gamma}_b} e^{-\gamma/\bar{\gamma}_b} \cdot d\gamma$$

$$P_b(e) = \int_0^{\infty} \frac{1}{2\bar{\gamma}_b} e^{-\gamma(1 + \frac{1}{\bar{\gamma}_b})} \cdot d\gamma$$

$$P_b(e) = \frac{1}{2(1 + \bar{\gamma}_b)}$$

DFSK : Non Coherent BFSK :

without fading: $P_b(e) = \frac{1}{2} e^{-\frac{\gamma_b}{2}}$

with fading:

$$P_b(e) = \frac{1}{2(1 + \frac{\gamma_b}{2})} = \frac{1}{2 + \gamma_b}$$

H.W : QAM, QPSK, PAM, ... $P_b(e)$?

* Performance of digital systems in Rician fading:

$$f(\alpha) = \frac{\alpha}{\sigma^2} e^{-\frac{(\alpha^2 + A^2)}{2\sigma^2}} \cdot I_0\left(\frac{\alpha A}{\sigma^2}\right)$$

By transformation of RV's :

$$A^2 = x_1^2 + y_1^2$$

$$\gamma = \frac{E_b}{N_0} \alpha^2 = \gamma_b \alpha^2$$

$$f_\gamma(\gamma) = \frac{1+k}{\gamma_b} e^{-\left[\frac{\gamma(1+k) + k\gamma_b}{\gamma_b}\right]} \cdot I_0\left(\sqrt{\frac{4(1+k)k}{\gamma_b}}\right)$$

$$k = A^2 / \sigma^2$$

$$I_0(x) = \frac{1}{\pi} \int_0^{\pi} e^{x \cos \theta} \cdot d\theta$$

For BPSK, No closed Form Solution for $P_b(e)$.

$$\varphi(\cdot) * e \xrightarrow{\text{بوسه}} \text{بوسه}$$

For non-coherent BFSK:

$$P_b(\gamma) = \frac{1}{2} e^{-\frac{\gamma}{2}}$$

$$P_b(e) = \int_0^{\infty} \frac{1}{2} e^{-\frac{\gamma}{2}} \cdot f(\gamma) \cdot d\gamma$$

$$= \frac{1+k}{\bar{\gamma}_b + 2 + 2k} e^{-\left(\frac{k\bar{\gamma}_b}{\bar{\gamma}_b + 2 + 2k}\right)}$$

For DPSK:

Similar DPSK in Rician Fading:

$$P_b(e) = \frac{1+k}{2\bar{\gamma}_b + 2 + 2k} e^{-\frac{2k\bar{\gamma}_b}{2\bar{\gamma}_b + 2 + 2k}}$$

WIRELESS

(S S)
Spread Spectrum Communication Technique

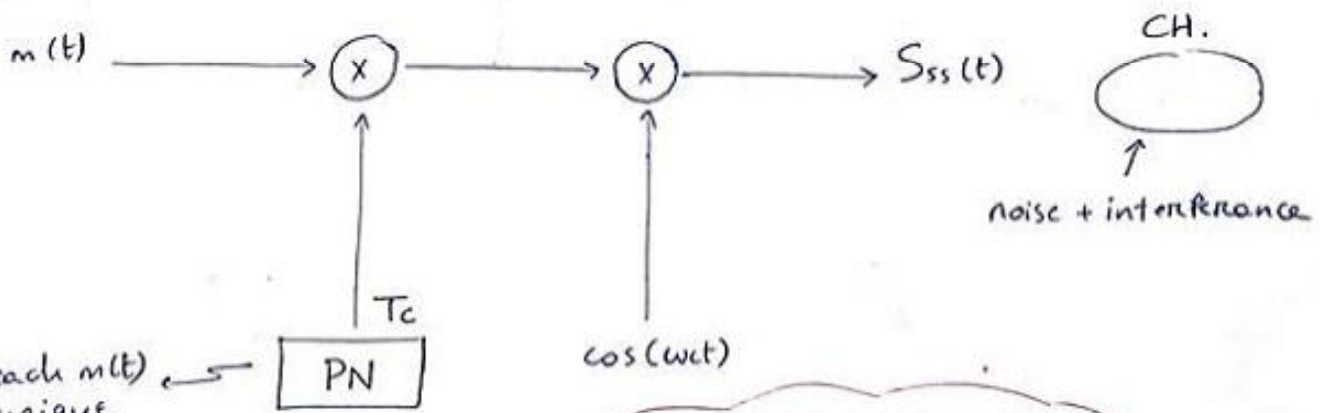
① Direct Sequence (DS-SS)

In DS-SS, a pseudo noise (PN) sequence with chip rate $R_c = \frac{1}{T_c}$ is used to spread the spectrum of the bandpass modulated signal (T_x) with data rate $R_s = \frac{1}{T_s}$ ($R_c \gg R_s$ or $T_c \ll T_s$)

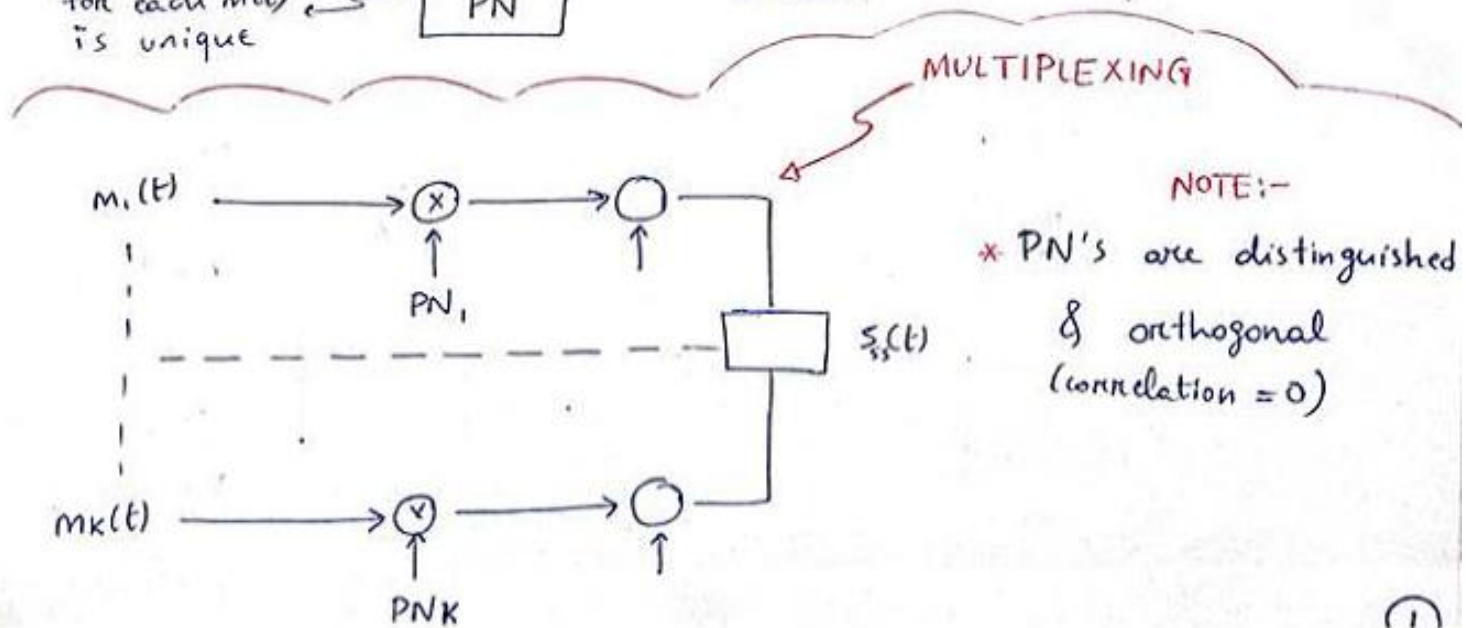
$\frac{R_c}{R_s} = N \rightarrow$ spreading gain
 (N is an integer)

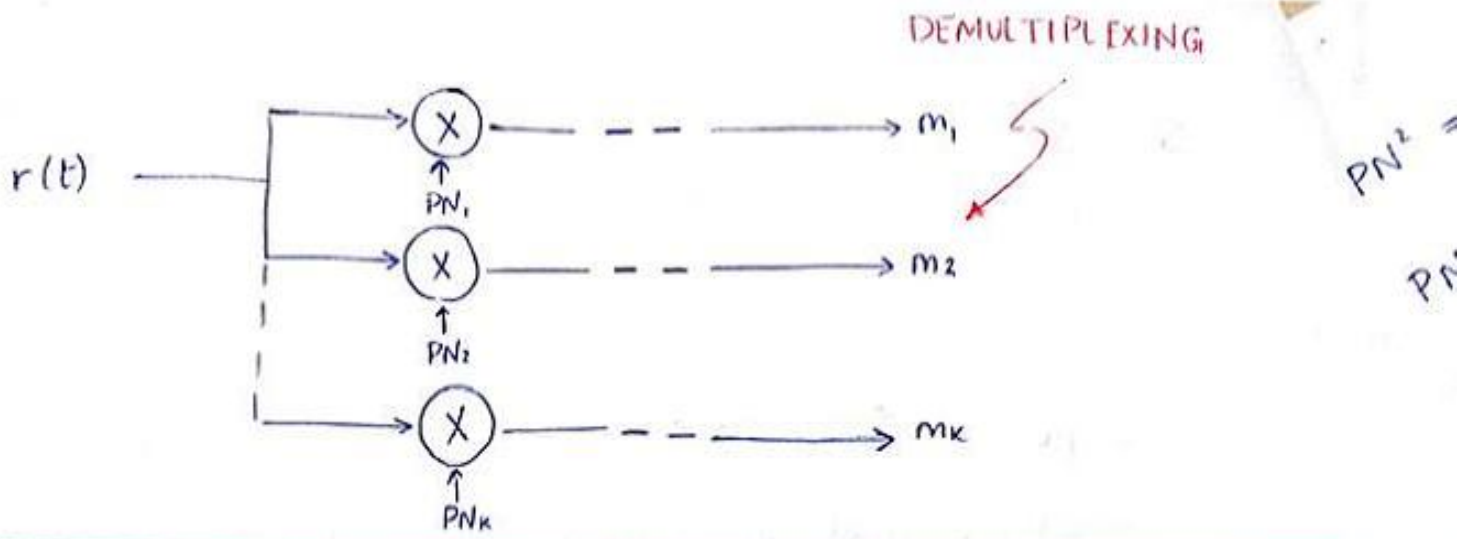
$\therefore \frac{T_s}{T_c} = N \rightarrow T_s = N T_c$

Tx

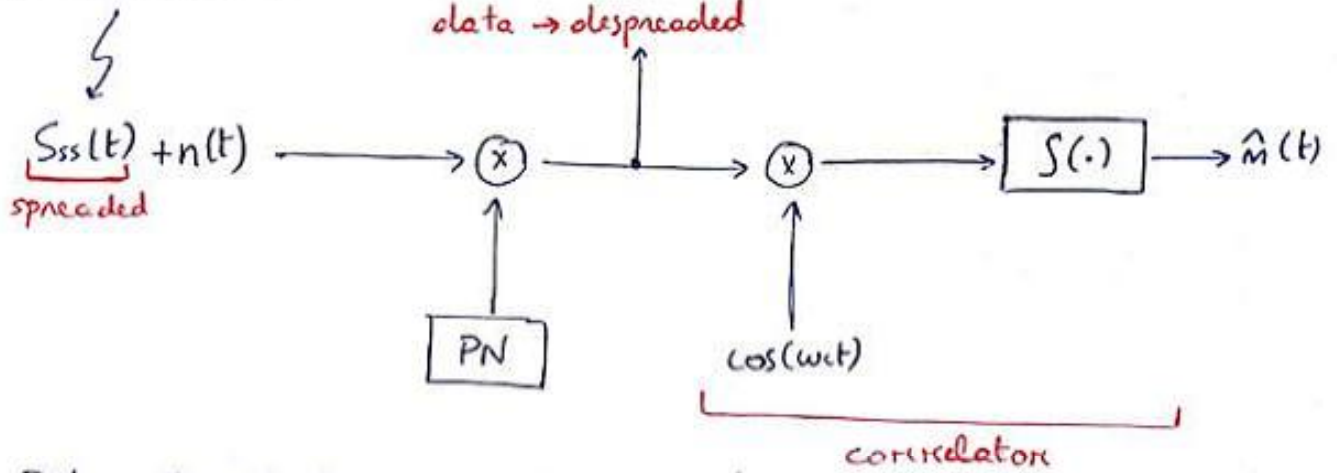


for each $m_i(t)$ is unique

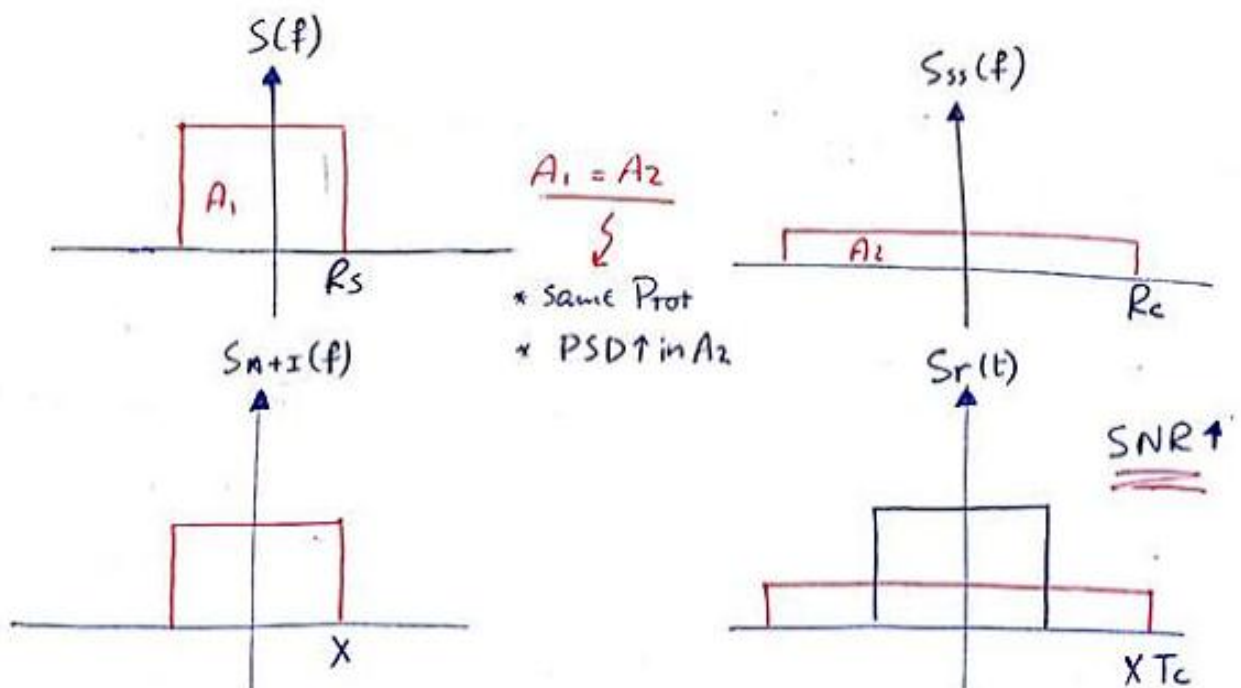




$$r(t) = S_{SS}(t) + n(t)$$



PN codes ± 1



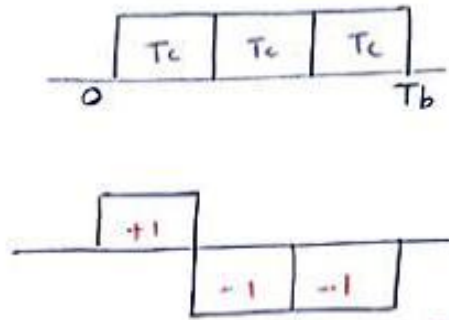
X كل ما قلت عرضها بار time domain بتوسع بال spectrum

$$PN^2 = 1$$

$$PN \Rightarrow +1 \quad -1 \quad -1$$

$$T_b = 3T_c$$

$$R_c = 3R_b$$



اذا رجعتنا
نضربنا بنفس PN
بترجع ال data
الاصليه

code division multiple axis

CDMA { Based on SS comm. technique }

total BW used CDMA
أكبر من sys عادي

K users \rightarrow K PN sequence all are orthogonal

\Rightarrow all users use all the spectrum all the time & each has unique code

ex CDMA 2-users ($N=6$) $\rightarrow T_s = 6T_c$

$$PN_1 = +1 \quad -1 \quad -1 \quad +1 \quad -1 \quad +1$$

$$PN_2 = +1 \quad +1 \quad -1 \quad -1 \quad +1 \quad +1$$

orthogonal
cross correlation = 0

BPSK "0" $\rightarrow -A$

"1" $\rightarrow +A$

user 1 sent "0" , user 2 sent "1"

$$d_1 : -1 \quad +1 \quad +1 \quad -1 \quad +1 \quad -1$$

$$d_2 : +1 \quad +1 \quad -1 \quad -1 \quad +1 \quad +1$$

$$d_1 \oplus d_2 = r$$

بتجمعهم CH.

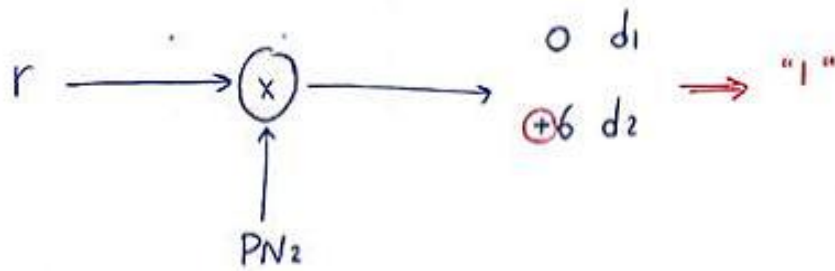
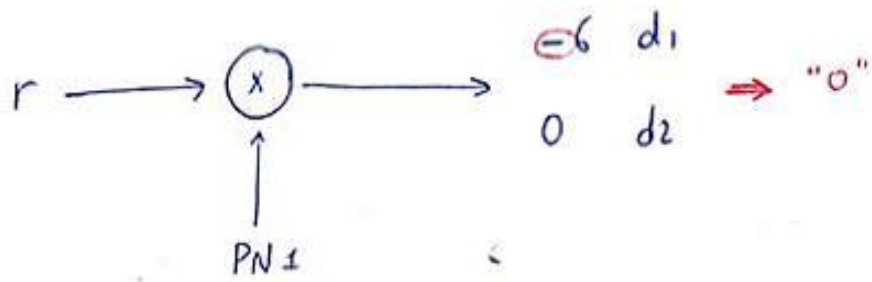
ستستخدموا اجزاء من
all the time
FDMA

a part of the
spectrum is used
all the time

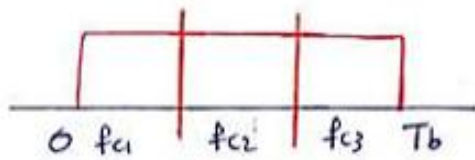
TDMA

all the spectrum
is used at different
times

ستستخدموا كل
على اجزاء من الوقت



② Frequency Hopping \rightarrow use multi-carriers



ال bit الوحدة يحملها
 على multi-carriers وال
 carriers بختلفر من bit
 ال bit

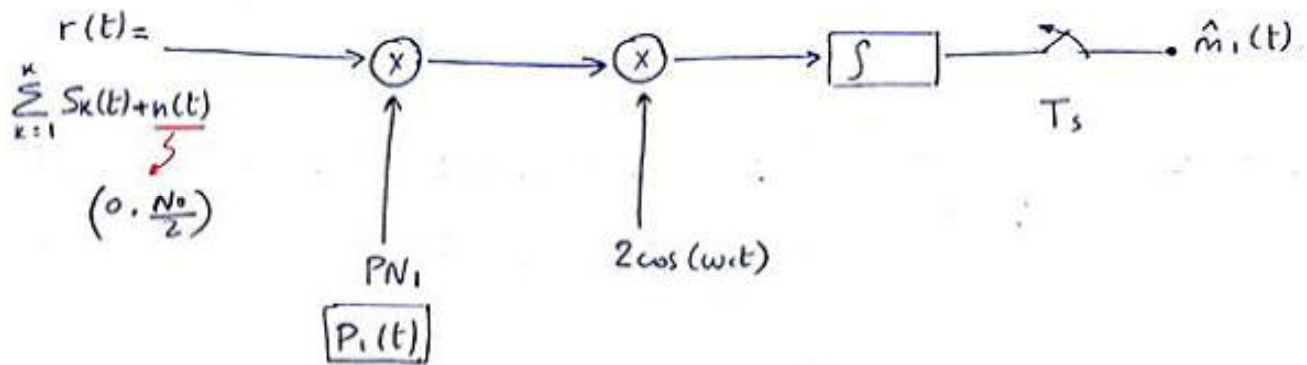
* Performance of BPSK in DS-SS *

Suppose we have K users in DS-SS system with processing gain $N = \frac{T_b}{T_c} = \frac{P_c}{P_b}$

Trans signal $S_k(t) = \sqrt{\frac{2E_b}{T_b}} \underbrace{m_k(t)}_{\substack{\text{k-user} \\ \text{bits } (\pm 1)}} \underbrace{P_k(t)}_{\substack{\text{PN seq} \\ (\pm 1)}} \cos(\omega_c t) \quad k = 1, \dots, K$

$P_k(t)$ are all orthogonal

Suppose that we are receiving $S_1(t)$ (recovering)



$$\hat{m}_1(t) = \underbrace{I_1}_{S_1(t)} + \sum_{k=2}^K \underbrace{I_k}_{\text{Interference } S_k(t) \rightarrow S_k(t)} + \hat{n}(t)$$

$$I_1 = \sqrt{\frac{E_b T_b}{2}} = \int_0^{T_b} S_1(t) P_1(t) \cos(\omega_c t) dt$$

$\hat{n}(t)$ is Gaussian with $E[\hat{n}^2] = \frac{N_0}{2} \cdot \frac{T_b}{2}$

By CLT $\rightarrow \sum I_k$ is Gaussian R.V with variance $\sigma^2 = \frac{N T_c^2}{6} \sum_{k=2}^K (P_k)$

{ The proof is shown in appendix E }

If all $P_k = P_0 \Rightarrow \sigma^2 = \frac{N(K-1) T_c^2 P_0}{6}$

avg power in the k th users (interference)

$$\sigma_N^2 = \underbrace{\frac{N_0 T_b}{4}}_{\text{thermal}} + \underbrace{\frac{N(K-1)T_c^2 P_0}{6}}_{\text{interference}}$$

$$P_b(e) = Q(\sqrt{\gamma}) = Q\left(\sqrt{\frac{P_0 T_b^2}{2\left[\underbrace{\frac{N(K-1)T_c^2 P_0}{6}}_{\text{SIR}} + \underbrace{\frac{N_0 T_b}{4}}_{\text{SNR}}\right]}}\right) \quad \text{where } T_b = NT_c$$

$$= Q\left(\sqrt{\frac{1}{\frac{K-1}{3N} + \frac{N_0}{2T_b P_0}}}\right)$$

* without DSS $\rightarrow K=0$

In interference limited system

$$P_b(e) \sim Q\left(\sqrt{\frac{3N}{K-1}}\right); \text{ As } N \uparrow P_b(e) \downarrow \text{ BW } \uparrow \text{ performance } \uparrow$$

$$\text{As } K \uparrow P_b(e) \uparrow \text{ BW } \downarrow \text{ performance } \downarrow$$

o° CDMA (SS) improve power efficiency & reduces BW efficiency

The challenge is to get all the K-users orthogonally.

AWR BFSK :-

without fading :- $P_b(e) = Q(\sqrt{\gamma_b})$

$$\gamma_b = \frac{E_b}{N_0}$$

with fading :-

$$P_b(\alpha) = Q(\sqrt{\gamma_b \alpha^2})$$

$$\gamma = \frac{E_b}{N_0} \alpha^2$$

$$= Q(\sqrt{\gamma})$$

$$P_b(e) = \int_0^{\infty} Q(\sqrt{\gamma}) \cdot f_{\gamma}(\gamma) \cdot d\gamma$$

$$\gamma = \frac{E_b}{N_0} \alpha^2 \rightarrow \alpha = \sqrt{\frac{N_0}{E_b} \gamma}, \quad f_{\alpha}(\alpha) = \frac{2\alpha}{\sigma^2} e^{-\frac{\alpha^2}{2\sigma^2}}$$

$$J = 2 \cdot \frac{E_b}{N_0} \cdot \sqrt{\frac{N_0}{E_b} \gamma} = 2 \sqrt{\frac{E_b}{N_0} \gamma}$$

$$f_{\gamma}(\gamma) = \frac{\sqrt{\frac{N_0}{E_b} \gamma} \cdot e^{-\frac{N_0 \gamma}{E_b} / 2\sigma^2}}{2\sigma^2 \sqrt{\frac{E_b}{N_0} \gamma}} = \frac{1}{2\sigma^2 \frac{E_b}{N_0}} e^{-\frac{\gamma}{\frac{E_b}{N_0} \cdot 2\sigma^2}}$$

$$E[\gamma] = \gamma_b = 2\sigma^2 \frac{E_b}{N_0}$$

$$F_x(x) = \frac{1}{\delta_b} e^{-\frac{x}{\delta_b}}$$

$$P_b(e) = \int_0^{\infty} \varphi(\sqrt{x}) \cdot \frac{1}{\delta_b} e^{-\frac{x}{\delta_b}} \cdot dx$$

$$u = \varphi(\sqrt{x})$$

$$= \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-\frac{x^2}{2}} \cdot dx$$

$$= \frac{1}{\sqrt{2\pi}} \cdot \frac{e^{-\frac{x^2}{2}}}{-x} \Big|_0^{\infty}$$

$$= \frac{1}{\sqrt{2\pi}} \left[0 + \frac{e^{-\frac{x^2}{2}}}{\sqrt{x}} \right]$$

$$= \frac{1}{\sqrt{2\pi x}} e^{-\frac{x}{2}}$$

$$du = -\frac{1}{2\sqrt{2\pi x}} e^{-\frac{x}{2}}$$

$$\frac{x}{\sqrt{2\pi x}} \cdot \frac{x}{2} e^{-\frac{x}{2}}$$

$$dv = \frac{1}{\delta_b} e^{-\frac{x}{\delta_b}} \rightarrow v = -e^{-\frac{x}{\delta_b}}$$

$$P_b(e) = \varphi(\sqrt{x}) \cdot -e^{-\frac{x}{\delta_b}} \Big|_0^{\infty} - \int_0^{\infty} \frac{1}{2\sqrt{2\pi x}} e^{-x(\frac{1}{2} + \frac{1}{\delta_b})} \cdot dx$$

$$2a^2 = \frac{2\delta_b}{2 + \delta_b} = \frac{2}{1 + \frac{\delta_b}{2}}$$

$$0 + \frac{1}{2} - \frac{1}{2} \frac{1}{\sqrt{2\pi}} \int_0^{\infty} \frac{1}{\sqrt{\delta}} e^{-\delta/2a^2} \cdot d\delta$$

↖ ↗

$$\delta = x^2 \rightarrow d\delta = 2x dx$$

$$\rightarrow \frac{1}{2} - \frac{1 \cdot a}{\cancel{2\sqrt{2\pi}} \cdot a} \int_0^{\infty} \frac{1}{\sqrt{x^2}} e^{-\frac{x^2}{2a^2}} \cancel{2x} dx$$

$$= \frac{1}{2} - \frac{a}{2} = \frac{1}{2} - \frac{a}{2}$$

$$a = \sqrt{\frac{2}{2(1 + \frac{2}{\delta_b})}} ;$$

$$= \frac{1}{2} - \frac{\sqrt{\frac{2}{2(1 + \frac{2}{\delta_b})}}}{2} = \frac{1}{2} - \frac{1}{2} \sqrt{\frac{2}{2(1 + \frac{2}{\delta_b})}}$$

$$= \frac{1}{2} - \frac{1}{2} \sqrt{\frac{2}{\frac{2(\delta_b + 2)}{\delta_b}}} = \frac{1}{2} \left(1 - \sqrt{\frac{2\delta_b}{4 + 2\delta_b}} \right)$$

$$= \frac{1}{2} \left(1 - \sqrt{\frac{\delta_b}{2 + \delta_b}} \right)$$

OR DPSK :-

without fading: $P_b(e) = \frac{1}{2} e^{-\gamma_b}$

$$\gamma_b = \frac{E_b}{N_0}$$

with fading :-

$$P_b(\alpha) = \frac{1}{2} e^{-\gamma_b \cdot \alpha^2} = \frac{1}{2} e^{-\gamma}$$

$$P_b(e) = \int_0^{\infty} \frac{1}{2} e^{-\gamma} \cdot \frac{1}{\bar{\gamma}_b} e^{-\frac{\gamma}{\bar{\gamma}_b}} \cdot d\gamma$$

* نفس الطريقة استعمالها

$$= \frac{1}{2\bar{\gamma}_b} \left[\frac{e^{-\gamma(1+\frac{1}{\bar{\gamma}_b})}}{-(1+\frac{1}{\bar{\gamma}_b})} \right]_0^{\infty}$$

$$= \frac{1}{2\bar{\gamma}_b} \cdot \left[0 + \frac{1}{(1+\frac{1}{\bar{\gamma}_b})} \right]$$

$$= \frac{1}{2\bar{\gamma}_b + 2} = \frac{1}{2(1+\bar{\gamma}_b)} \quad \#$$

FOR :- DFSK :-

→ without :- $P_b(e) = \frac{1}{2} e^{-\frac{\gamma_b}{N}}$

$$\gamma_b = \frac{E_b}{N_0}$$

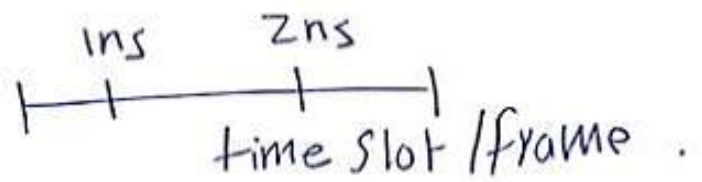
→ with fading :-

$$P_b(\alpha) = \frac{1}{2} e^{-\frac{\gamma_b}{2} \cdot \alpha^2} = \frac{1}{2} e^{-\frac{\alpha}{2}}$$

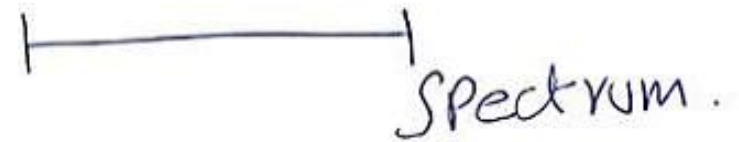
$$\begin{aligned} \therefore P_b(e) &= \int_0^{\infty} \frac{1}{2} e^{-\frac{\alpha}{2}} \cdot \frac{1}{\bar{\gamma}_b} e^{-\frac{\alpha}{\bar{\gamma}_b}} \cdot d\alpha \\ &= \int_0^{\infty} \frac{1}{2\bar{\gamma}_b} e^{-\alpha(\frac{1}{2} + \frac{1}{\bar{\gamma}_b})} \cdot d\alpha \\ &= \frac{1}{2\bar{\gamma}_b} \left[\frac{e^{-\alpha(\frac{1}{2} + \frac{1}{\bar{\gamma}_b})}}{-(\frac{1}{2} + \frac{1}{\bar{\gamma}_b})} \right]_0^{\infty} \\ &= \frac{1}{2\bar{\gamma}_b} \left[0 + \frac{1}{(\frac{1}{2} + \frac{1}{\bar{\gamma}_b})} \right] \\ &= \frac{1}{\bar{\gamma}_b + 2} \quad \neq \end{aligned}$$

CDMA (code digital multiple axis)

* TDMA
only with
digital source



* FDMA
with both
digital &
analog.



PN
pseudo noise

→ deterministic
cross correlation
 $r_{ij} = \delta_{ij}$
(orthogonal)

Spread Spectrum Com. System:

$X(t) \rightarrow$ Dig. $\frac{1}{T_b}$ ^{user bit/s}

$BW \propto \frac{1}{T_b}$

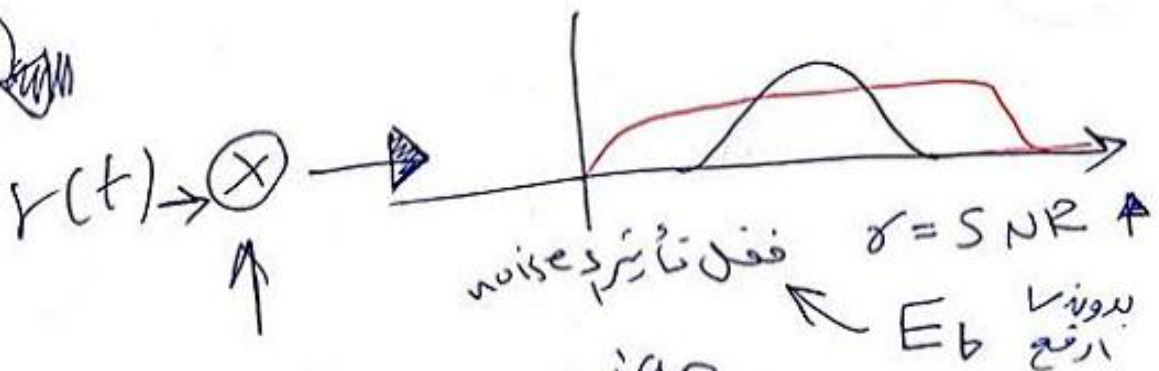
$BW \propto \frac{1}{T_b}$

Ex: $+1 \ -1 \ +1 \ -1$
 $T_p = \frac{1}{4} T_b$



PN (T_p) $T_p \ll T_b$

PSD \downarrow , P the same.



* PN #
 unique
 for all in network
 orthogonal

WIRELESS

CHAPTER 7 Diversity Techniques

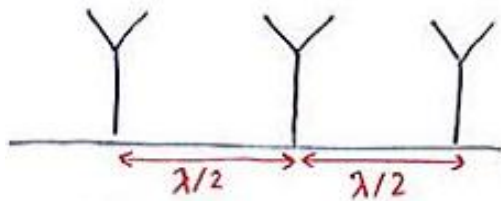
m -times bit \rightarrow bit بكثر من مرة

- * Diversity is a powerful technique to improve the performance of wireless faded-channels.
- * Diversity takes the advantage of the random^{nature} of mobile channel by finding independent (Tx) - at least highly uncorrelated - signal paths, for communication. If one path undergoes a deep fade, another independent path may have a strong signal which can be combined (Rx) with the weak one to improve the SNR (i.e. $\rightarrow P_b(\beta) \downarrow$) of the system.

Diversity Techniques (could be in Rx or Tx or in both - MIMO)

① Space Diversity (Spatial)

SISO
MISO
SIMO



minimum separation between any two antennas = $\frac{\lambda}{2}$

\rightarrow multi-antennas with distance $\geq \frac{\lambda}{2}$

\rightarrow it uses the same BW (spread spectrum) ^{NO} \rightarrow NO SPREADING spectrum

DISADVANTAGE: hard to be implemented in MS

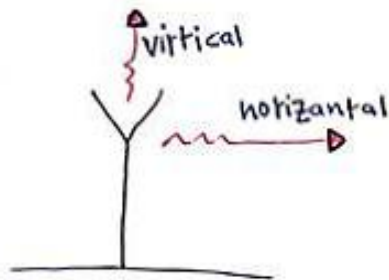
\rightarrow No limitation on # of copies.

ث. ٤٢

② Polarization Diversity

We transmit the signal in one antenna with two directions (horizontal & vertical - orthogonal) (90° phase shift).

DISADVANTAGE: limitation in the no. of copies (@ most two)



③ Frequency Diversity

We send the same signal with multi-carriers with frequency separation $\geq B_c$ (coherence BW)

DISADVANTAGE: BW inefficient.

uncorrelated / غير مترابطة

④ Time Diversity

We are sending the same signal with time interval separation

$\geq T_c$ (coherence time of the channel)

DISADVANTAGE: Power inefficient

uncorrelated / غير مترابطة

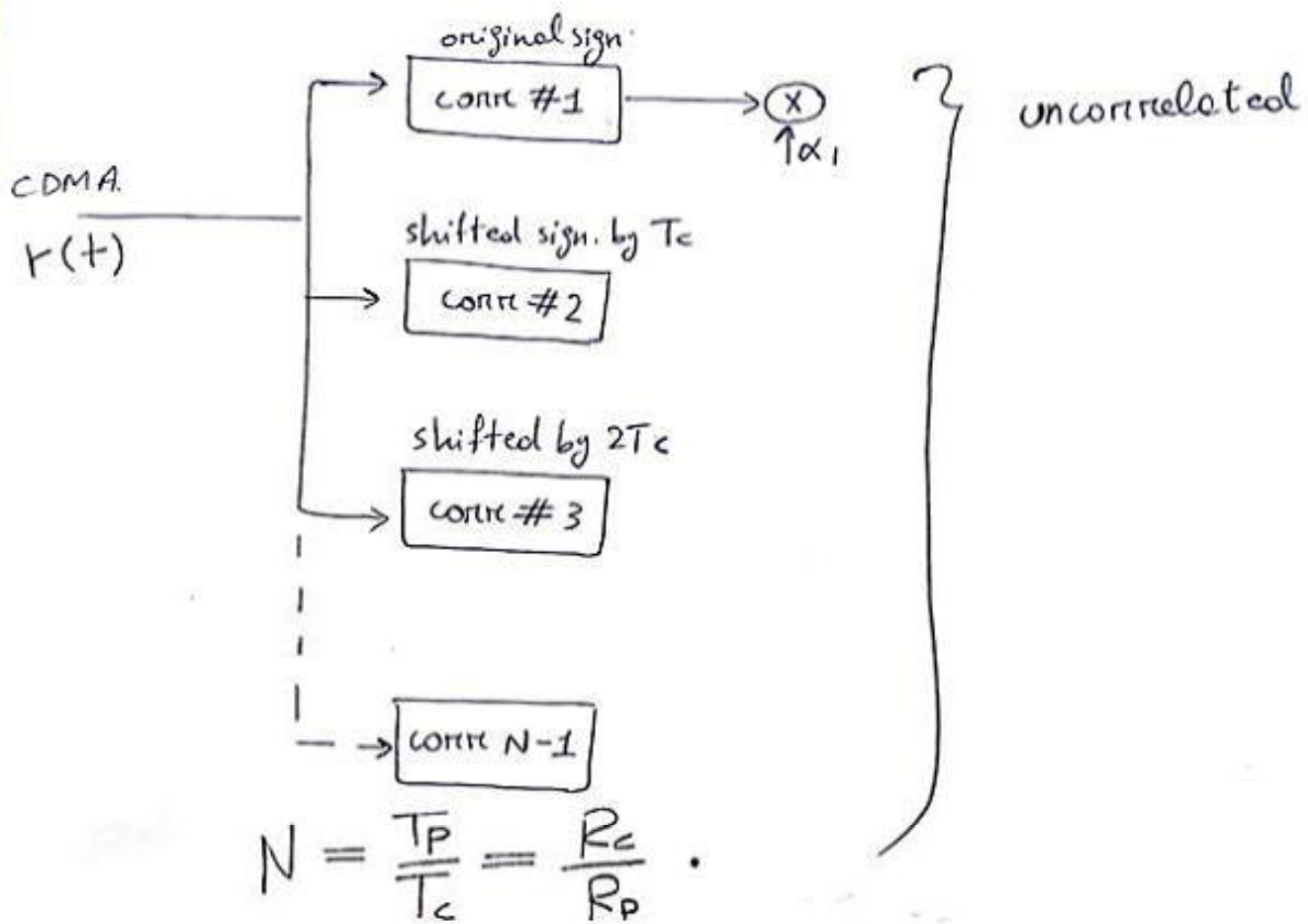
⑤ RAKE Rx (CDMA)



$$T_c = \frac{1}{6Z} T_b \rightarrow 6\text{-chips} \rightarrow \text{BW} \uparrow \text{ by 6 times.}$$

②

ensure that a delayed versions of the same CDMA signal by more than one chip are highly uncorrelated.



the same PN Seq. \rightarrow delayed version
 منسرا بقدر T_c

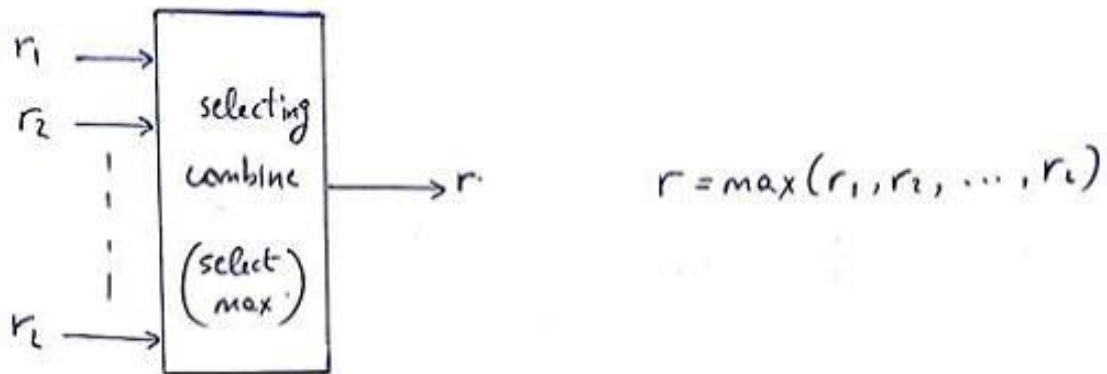


Auto. correlation

||
 # منسرا

Diversity combining techniques

① Selective combining (SC):-



For Rayleigh fading

$$\gamma_k = \frac{E_b}{N_0} \alpha_k^2$$

$$f_{\gamma}(\gamma_k) = \frac{1}{\bar{\gamma}_b} e^{-\gamma_k / \bar{\gamma}_b}, \quad \bar{\gamma}_b = \frac{E_b}{N_0} E[\alpha^2]$$

$$\gamma_{sc} = \max[\gamma_1, \gamma_2, \dots, \gamma_L]$$

$$* F_{\gamma_k} = \prod_{k=1}^L (1 - e^{-\gamma_k / \bar{\gamma}_b}) \rightarrow \text{for independent}$$

$$= (1 - e^{-\gamma / \bar{\gamma}_b})^L \rightarrow \text{for the maximum}$$

$$* \bar{\gamma}_{sc} = \int_0^{\infty} \gamma f_{\gamma_{sc}}(\gamma) d\gamma = E[\gamma_{sc}] \therefore f_{\gamma_{sc}} = \frac{L}{\bar{\gamma}_b} (1 - e^{-\gamma / \bar{\gamma}_b})^{L-1} e^{-\gamma / \bar{\gamma}_b}$$

$$= \frac{L}{\bar{\gamma}_b} \int_0^{\infty} \gamma (1 - e^{-\gamma / \bar{\gamma}_b})^{L-1} e^{-\gamma / \bar{\gamma}_b} d\gamma$$

By parts L-times :-

$$\bar{\gamma}_{sc} = \bar{\gamma}_b \sum_{k=1}^L \frac{1}{k}$$

* Improvement for SNR is :-

$$\sum_{k=1}^L \frac{1}{k} \rightarrow \bar{\gamma}_{sc} = \bar{\gamma}_b \sum_{k=1}^L \frac{1}{k} = \bar{\gamma}_b \left[1 + \frac{1}{2} + \frac{1}{3} + \dots + \frac{1}{L} \right]$$

SNR > 1;
diversity gain

For DPSK $P_e(\gamma) = \frac{1}{2} e^{-\gamma}$

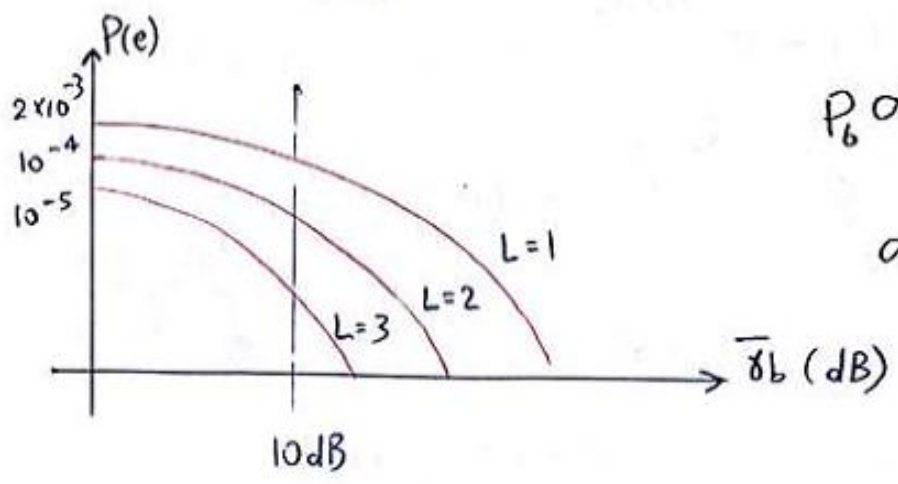
$$\rightarrow P(e) = \frac{1}{2} \cdot \frac{L}{\bar{\gamma}_b} \int_0^{\infty} e^{-\gamma(1 + \frac{1}{\bar{\gamma}_b})} \cdot (1 - e^{-\gamma/\bar{\gamma}_b})^{L-1} d\gamma$$

By parts L-times

$$\rightarrow P(e) = \frac{L}{2} \sum_{k=0}^{L-1} \frac{\binom{L-1}{k} (-1)^k}{1+k+\bar{\gamma}_b}; \quad \binom{x}{y} = \frac{x!}{y!(x-y)!}$$

→ for L=1 (no div.)

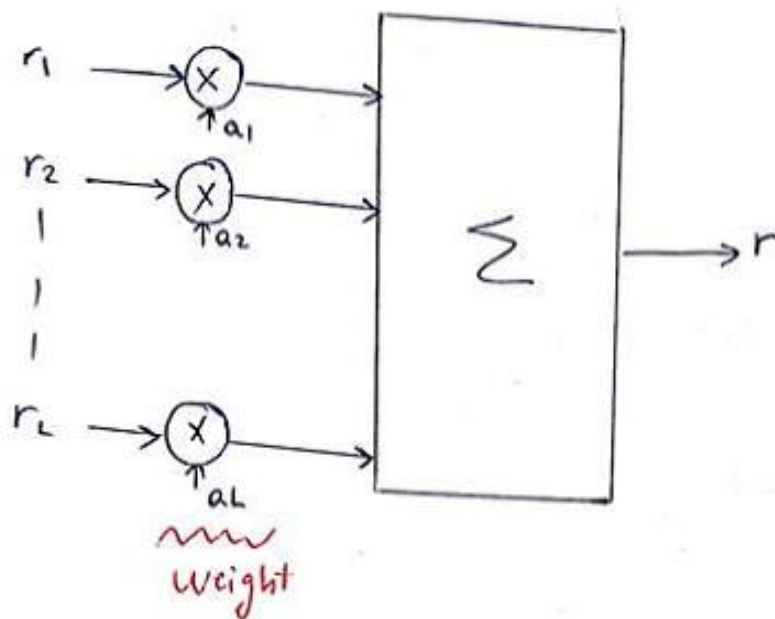
$$P(e) = \frac{1}{2} \cdot \frac{1}{1+\bar{\gamma}_b}$$



$$P_b \propto \frac{1}{SNR}$$

$$\propto \frac{1}{L}$$

Maximum Ratio Combining (Optimum)



$$r = \sum_{i=1}^L a_i r_i + \sum_{i=1}^L a_i n_i \quad \leftarrow N_0$$

\therefore SNR at the MRC is :-

$$\gamma_{\text{MRC}} = \frac{\left(\sum_{i=1}^L a_i r_i \right)^2}{N_0 \left(\sum a_i \right)^2}$$

By Schwartz inequality

$$\begin{aligned} \left(\sum a_i r_i \right)^2 &= \left(\sum a_i \sqrt{N_0} \frac{r_i}{\sqrt{N_0}} \right)^2 \\ &\leq \sum a_i^2 N_0 \sum \frac{r_i^2}{N_0} \end{aligned}$$

equality holds in $a_i = Kr_i$

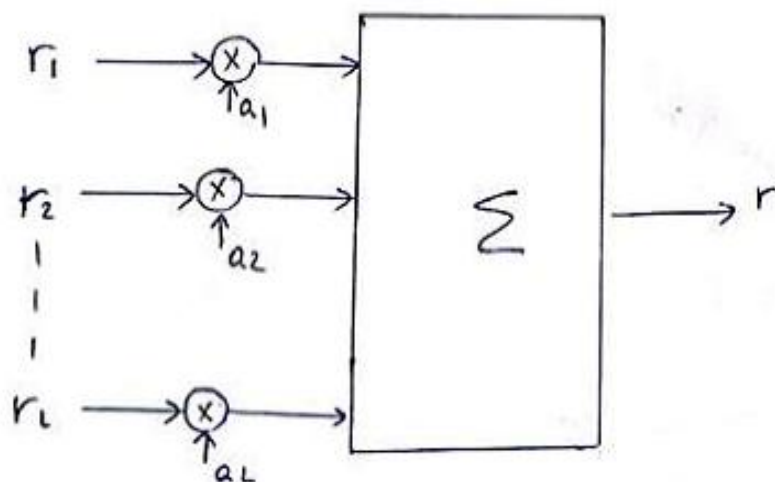
$$\gamma_{MRC} = \frac{\sum a_i^2 N_0 \sum \frac{r_i^2}{N_0}}{\sum a_i^2 \cdot N_0} = \sum \frac{r_i^2}{N_0} = \sum_{i=1}^L \gamma_i$$

If all paths are the same γ_i

$$\Rightarrow \gamma_{MRC} = L\gamma$$

* Price: - we need channel estimation

★ MRC (Max Ratio Combining) \Rightarrow OPTIMUM



channel estimation
on detection

$$a_i = Kr_i$$

$$\gamma_{MRC} = \sum_{i=1}^L \gamma_i = L\gamma$$

If all branches or paths are independent & by transform =
 nation of RVs

$$y = x + z$$

$$\rightarrow \text{pdf}_y = \text{pdf}_x \otimes \text{pdf}_z$$

$$f_{\gamma_{\text{MRC}}}(\gamma) = \frac{\gamma^{L-1}}{(L-1)! (\bar{\gamma}_b)^L} e^{-\gamma/\bar{\gamma}_b}$$

$$\text{since } \bar{\gamma}_b = \frac{E_b}{N_0} E[\alpha^2]$$

For BPSK system :-

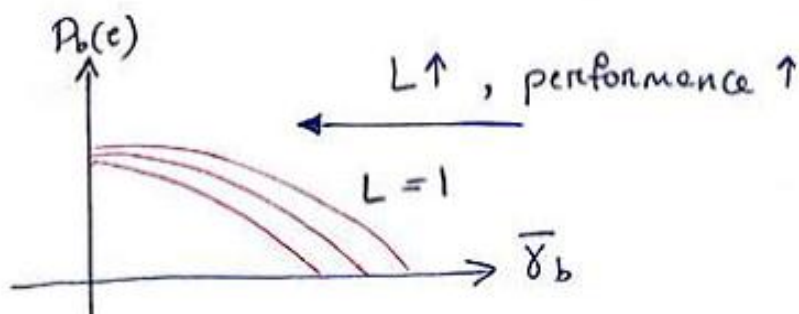
$$P_b(\cdot | e) = Q(\sqrt{2\gamma})$$

$$P_b(e) = \int_0^{\infty} Q(\sqrt{2\gamma}) f_{\gamma_{\text{MRC}}}(\gamma) d\gamma$$

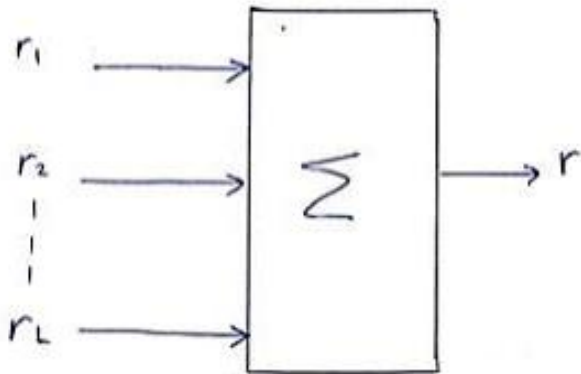
by parts for L-times :-

$$P_b(e) = \left(\frac{1 - \mu}{2} \right) \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left(\frac{1 + \mu}{2} \right)^k$$

$$\mu \triangleq \sqrt{\frac{\bar{\gamma}_b}{1 + \bar{\gamma}_b}}$$



③ Equal Gain Combining :-



$$r = \sum_{i=1}^L r_i$$

$\gamma_{EGC} =$ has no closed form solution

EGC ~~≠~~ مالة فامة من MRC
و هو اسوا منه

EGC performance is worst than MRC but no channel estimation is needed or required.

①

Q1: SIR = 15 dB, $n = 3$, $i_0 = 1$ (5 marks)

- 1- Find "N" For 60° sectoring
- 2- which is better for this system 60° or 120° sectoring

1- $SIR = \frac{(\sqrt{3}N)^n}{i_0}$, but convert dB \rightarrow w
 $SIR = 10^{\frac{15}{10}} = 31.6$

$$31.6 = \frac{(\sqrt{3}N)^3}{i_0=1}$$

$\lambda = 3$ calls/day $H = 6$ min/call (12 marks)
Urban Area \rightarrow 2 million (but has only 15% users)
 $N = 12$ & total spectrum = 75.6 MHz.
full-duplex channel = 140 kHz & Pr (delay) = 0.007 spectrum.

1- * of channels/cell

$$\text{* of channels} = \frac{75.6 \text{ M}}{140 \text{ k}} = 540 \text{ channels/system}$$

So, * of channels = $\frac{540}{12} = 45$ chan/cell.
* of cells $\leftarrow 12$

2- Traffic Intensity in cell (A)

From Erlang-C chart $\begin{cases} \rightarrow \text{Pr}(\text{delay}) = 0.007 \\ \rightarrow C = 45 \\ \rightarrow A = 38 \text{ Erlangs} \end{cases}$

3- Traffic Intensity per system

$$A \times \# \text{ of cells} = 38 \times 12 = 336 \text{ Erlangs}$$

4- $\text{Pr}(5\text{sec} < \text{delay} < 10\text{sec})$??

5- Area of system = 5000 km^2 Find Radius of R !

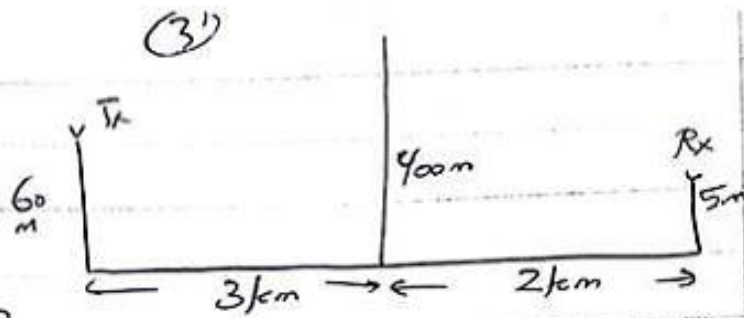
$$A = 5000 \text{ km}^2 / 12 = 416.6 \text{ km}^2 / \text{cell}$$

$$A_{\text{cell}} = 2.865 R^2 \rightarrow R = 12 \text{ km}$$

* من رگید *

6- Average $\text{Pr}(\text{delay})$ → sheet جانفہ بار
تعمیرہ مباشر

3: $h_t = 60\text{m}$
 $h_r = 5\text{m}$



$P_t = 10\text{w}$, $G_t = 10\text{dB}$
 $G_r = 3\text{dB}$
 $L = 1\text{dB}$

$F = 900\text{MHz}$ (8marks)

جواب في dB في 1 ج'

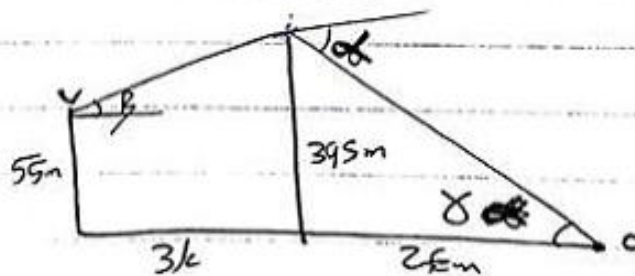
- 1- Find the Power at Receiver
- 2- ??

$$P_{\text{model}} = P_{\text{free space}} + \text{Ed}$$

$$P_r = 10 \frac{(10)(2)(0.33)^2}{(4\pi)^2 (5\text{k})^2 (1.26)} = 6.7 \times 10^{-9}$$

$$P_r = -51.7 \text{ dBm}$$

Ed??



$$\alpha = \arctan \left(\frac{2d_1 d_2}{\pi(d_1 + d_2)} \right)$$

$$\alpha = \beta + \gamma$$

$$= \frac{395 - 55}{3\text{k}} + \frac{395}{2\text{k}}$$

$$= \dots$$

بعد ما تطلع α بتسوف على انفاق

(4)

$\sqrt{\nu}$ $\left\{ \begin{array}{l} \rightarrow \nu < 3 \quad \text{from Culvert or equation} \\ \rightarrow \nu > 3 \quad \text{only equation} \end{array} \right.$

$$e_d(\text{dB}) = 20 \log \left(\frac{0.225}{\sqrt{\nu}} \right)$$

$$\therefore P_r = P_{\text{free space}} + e_d$$

①

$\lambda = 0.333 \text{ m}$

$U = 60 \text{ mile/hr}$, $f_c = 900 \text{ MHz}$. (6 marks)

- 1 - # of crossing at Zero level.
- 2 - " " " " -20dB level
- 3 - Avg fade duration from part (2).

1 - $N_R = \sqrt{2\pi} f_m \int_0^{\infty} e^{-\rho^2} d\rho$ (Zero level $\rightarrow \rho = 0 \text{ dB}$)
 $\rho = 0 \text{ dB} \rightarrow 1$

$\therefore N_R = \sqrt{2\pi} (80.8)(1)e^{-1}$
 $= \dots$

$f_m = \frac{U}{\lambda} = \frac{60 \times 1.6 \text{ km}}{0.333 \times \frac{3600}{3600} \text{ sec}}$
 $= 80.8 \text{ Hz}$

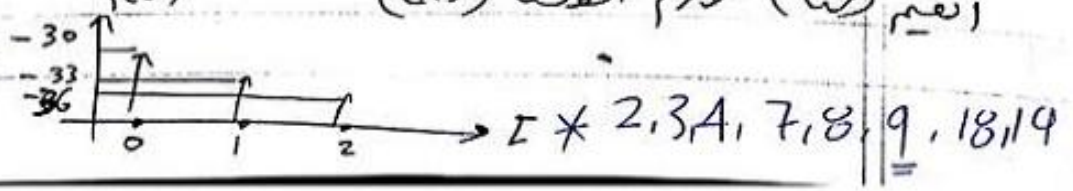
2 - $\rho = -20 \text{ dB} \rightarrow 0.02$

3 - $Tfd = \frac{e^{\rho^2} - 1}{\sqrt{2\pi} f_m \rho}$

2: $P(\epsilon) = \sum_{i=0}^2 \dots \delta(\tau - \epsilon_i)$ (6 marks) Power Delay Profile

1 - sketch power delay Profile. (dBm)

$\tau = 0$ $\bar{\tau} = 1$ $\bar{\tau} = 2$ $P(\tau)$ (dBm) (ω) $\bar{\tau}$



2

2- Find delay spread.

$$6\tau = \sqrt{\overline{t^2} - \bar{t}^2}$$

کے الصاف فون کے الصاف فون
 کے الصاف فون کے الصاف فون

تعريف هون (w)

3- system ~~slow~~ ^{slow} fading

$$T_c = \frac{0.423}{f_m} < T_s \quad ??$$

نی کی کہ مطاب ~~مطاب~~ نسبتہ ...
بہ حد بین ال فریک کواڈریٹ

Q3 :

Scattering & practical models (8 marks)
معم علی

مر اس حجتہ ال کتاب (نصفہ الاقدم)