ElCoM
ELECTRICAL COMPUTER MECHATRONICS

تقدم لجنة ElCoM الاكاديمية

دفتر لمادة:

انظمة اتصالات لاسلكية

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"Cellcular Systems"

9-10-2017

S: Total no. of full-duplex channels available to cellcular system.

K: No. of channels/cell.

N: Cluster size (a cluster uses all S-channels)

$\Rightarrow K = \frac{S}{N}$

M: No. of replications of a cluster in a geographic area.

$\Rightarrow M = \frac{Area_{city}}{Area_{cluster}}$

C ⇒ capacity: No. of users in a geographic area.

$\Rightarrow C = MS.$

Ex] N = 7 cells/cluster, K = 18 ch/cell, M = 3
Find C?

$C = MS = MKN = 3 \times 18 \times 7 = 378$ users/city

⇒ Given fixed city Area, fixed cell Area, fixed S, then:

* If $N\uparrow \Rightarrow A_{cluster}\uparrow \Rightarrow M\downarrow \Rightarrow C\downarrow \Rightarrow SIR\uparrow$

Signal to interference Power ratio.

* If $N\downarrow \Rightarrow A_{cluster}\downarrow \Rightarrow M\uparrow \Rightarrow C\uparrow \Rightarrow SIR\downarrow$

"Cellcurar System" 9-10-2017

Ex) S = 1001 channels, cell Area = 6 Km^2
City Area = 2100 Km^2, Find capacity (C) for:

a) N = 7 b) N = 4

a) $M = \frac{2100}{6 \times 7} = 50$ $\Rightarrow$ C = MS = 50050 users/city

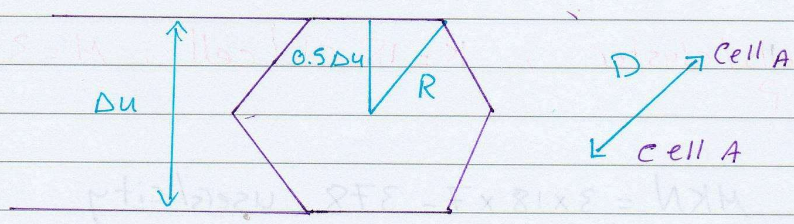
b) $M = \frac{2100}{6 \times 4} = 87$ $\Rightarrow$ C = MS = 87087 users/city

"Geometry of Hexagonal cells"

$\Rightarrow$ To find the nearest co-channel cell:

1) Move i-cells (integer) along any chain of hexagons.

2) Turn 60° counter clockwise and move j-cells.



$\frac{0.5\Delta U}{R} = \cos 30 = \frac{\sqrt{3}}{2}$

$\Rightarrow \Delta U = \sqrt{3}\, R$

$\Rightarrow D^2 = (i\Delta U)^2 + (j\Delta U)^2 - 2\,(i\Delta U)(j\Delta U)\cos 120°$

$\Rightarrow D^2 = (i^2 + j^2 + ij)\,\Delta U^2$

"Cellcular Systems" 11-10-2017

"Finding no. of cells in a cluster (N)"

$\Rightarrow$ Area of Large Hex $= KD^2$

$$= K(i^2 + j^2 + ij)\,\Delta u^2$$

$\Rightarrow$ Area of cell $= KR^2$

No. of cells in Large Hex $= \dfrac{KD^2}{KR^2} = \dfrac{(i^2 + j^2 + ij)\,\Delta u^2}{R^2}$

$$= 3(i^2 + j^2 + ij) = 3N$$

$\Rightarrow N = i^2 + j^2 + ij$

" Co-channel Interference (CCI)"

CCI: Is the interference seen by the mobile station from the B.S's of neighboring cells that operate at the same frequency.

Rx-Signal: Tx-Signal + CCI + thermal noise.

↳ overcome by increasing power

→ CCI: Can't be combated by increasing power.

$\Rightarrow$ To reduce CCI we need to increase D

" N

$\Rightarrow Q = \dfrac{D}{R}$

↓

Co-channel reuse ratio

$$Q = \frac{\sqrt{(i^2+j^2+ij)\,Du^2}}{R} \Rightarrow Du^2 = 3R^2$$

$$\Rightarrow Q = \frac{\sqrt{3N \cdot R^2}}{R} = \sqrt{3N}$$

Signal-to-Interference Ratio (SIR):

$$\Rightarrow SIR = \frac{S \leftarrow \text{desired signal power}}{I \leftarrow \text{Interference power}}$$

$$= \frac{S}{\sum_{i=1}^{i_o} I_i \leftarrow \text{CCI from B.S no. } i}$$

$\Rightarrow$ Rx power $\propto d^{-n} = \text{Const.} * d^{-n}$

d: distance from Tx to Rx

n: Path-Loss exponent $(2 \leq n \leq 6)$

$$SIR = \frac{\cancel{\text{Const}}.\, R^{-n}}{\sum_{i=1}^{i_o} \cancel{\text{Const}}.\, D^{-n}} = \frac{R^{-n}}{i_o D^{-n}} = \frac{(D/R)^n}{i_o}$$

$$\Rightarrow SIR = \frac{(\sqrt{3N})^n}{i_o \leftarrow \text{No. of interference B.S's}}$$

$\Rightarrow$ As $N \uparrow \rightarrow SIR \uparrow \rightarrow C \downarrow$

"Cellcular Systems"

16-10-2017

Ex] If the desired SIR is (15 dB) in a certain cellcular system, Find the cluster size (N) that should be used to achieve maximum capacity for:

a) $n=4$ b) $n=3$

$\Rightarrow SIR = \frac{(\sqrt{3N})^n}{6} \Rightarrow i_0 = 6$ if it's non-directional.

$SIR_{dB} = 10 \log \frac{(\sqrt{3N})}{6}$

a) $n=4$ $N = 3, 4, 7, 9, 12, 19$

↳ using $N=4$, $N = i^2+j^2+ij$

$\Rightarrow SIR_{dB} = 10 \log \frac{(\sqrt{3\times4})^4}{6} = 13.8\ dB < 15\ dB$ X

↳ using $N=7$

$SIR_{dB} = 10 \log \frac{(\sqrt{3\times7})^4}{6} = 18.66\ dB > 15\ dB$ ✓

∴ select $N=7$

b) $n=3$

↳ using $N=7$

$SIR_{dB} = 10 \log \frac{(\sqrt{3\times7})^3}{6} = 12.05\ dB$ X

↳ using $N=9 \Rightarrow SIR_{dB} = 13.68$ X

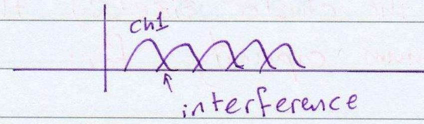
↳ using $N=12 \Rightarrow SIR_{dB} = 15.56\ dB$ ✓

∴ select $N=12$

"Cellcuar Systems"

16_10_2017

" Adjacent Channel Interference (ACI) "



⇒ Reducing ACI Methods:

1_ Power Control
2_ Some Modulation Types (i.e. GMSK)
3_ Good BPF
4_ Channel Interleaving

" Trunking and Grade of Service (GOS) "

1) Blocked Calls Cleared (Erlang_B)

GOS: Is the prob. of blocking a call.

2) Blocked Calls delayed (Erlang_C)

GOS: Is the prob. of delaying a call beyond a certain time.

λ: Average no. of calls/unit time for each user.

H: Average duration of a call (in sec)

⇒ Eeach user generates traffic intensity of:_

⇒ $A_u = \lambda H$ in (Er : Erlang)

⇒ For u_users , the total traffic is:

$A = u A_u$ (Er)

C: No. of channels in the trunked system

⇒ Erlang_B:

$$Pr\,[blocking] = GOS = \frac{A^C / C!}{\sum_{k=0}^{C} A^k / k!}$$

" Cellcular Systems" 16.10.2017

Ex] A certain city with 394 cells and population = 0.1 million with $C = 19$ ch/cell

$\lambda = 2$ calls/hour

$H = 3$ minutes

If the desired GOS_Er_B = Pr [Blocking] = 0.02

Find:

1) Number of users that can be supported by the cell
2) Total no. of users/city
3) % market of population.

1) $A_u = \lambda H = \frac{2}{60} \times 3 = 0.1$ Er

↳ using GOS = 0.02

$C = 19 \Rightarrow$ From Erlang_B

$A = 12$

& $u = \frac{A}{A_u} = \frac{12}{0.1} = 120$ users

2) Total no. of users = $394 \times 120 = 47,780$ users/city

3) $\% = \frac{47,780}{100,000} = 47.78\%$

Ex] GOS (Er_B) = 0.5 % = 0.005

$A_u = 0.1$ Er

Find no. of users/cell for:

a) $C = 5 \Rightarrow A = 1.13$, $u = 11.3$

b) $C = 20 \Rightarrow A = 11.1$, $u = 111$

c) $C = 100 \Rightarrow A = 80.4$, $u = 804$

Ex] City of 2 million population, we have 3 systems:

Sys_A : 394 cell with 19 ch/cell

Sys_B : 98 cell with 57 ch/cell

Sys_C : 49 cell with 100 ch/cell

GOS (Er_B) = 2% = 0.02 → Prob [Block] , $\lambda = 2$ calls/Hr , $H = 3$ min

"Cellcular Systems" 18-10-2017

a) Find total no. of users for each system?

⇒ for System _A :

$A_u = 0.1 = \lambda H$

$A = 12$ Er ⇒ $u = \frac{12}{0.1} = 120$ user/cell

Total no. = $394 \times 120 = 47280$ user/city

⇒ For system _B

$C = 57$, $GOS = 0.02$ ⇒ $A = 45$ Er

$u = \frac{45}{0.1} = 450$

Total no. = $450 \times 48 = 44100$

⇒ For system _C

$C = 100$, $GOS = 0.02$ ⇒ $A = 88$ Er

$u = \frac{88}{0.1} = 880$ user / cell

Total no. = $880 \times 49 = 43120$

b) Find the market population for each system?

⇒ System _A

market population % = $\frac{47280}{200\,000} = 2.36\,\%$

⇒ system _B

Market pop % = 2.206 %

⇒ system _C ⇒ Market pop % = 2.116 % 8

"Cellcular Systems"

18.10.2017

Ex] city of 1300 $mile^2$, $A_{cell} = 2.59\ R^2$ ← given
$N = 7$, $A_u = 0.03$ Er
Total B.W allocated = 40 MHz , cell radius = 4 miles
Full duplex Ch = 60 KHz , GOS (Er-B) = 0.02

a) no. of channels/cell
b) traffic intensity per cell
c) Max. traffic in the city
d) Total no. of users/city

Solution ⇒ $A_{cell} = 2.59\ R^2 = 2.59(4)^2 = 41.57\ mile^2$

a) no. of channels / cell $= \frac{S}{N}$ ⇒ $S = \frac{\text{Total BW}}{\text{ch BW}} = \frac{40 \times 10^3\ KHZ}{60\ KHZ}$

$= 666$ ch

∴ no. of channels / cell $= \frac{666}{7} = 95$ Ch/cell

b) $C = 95$, $GOS = 0.02$ } $A = 89$ Er

c) Max traffic of city = A * no. of cells

No. of cells $= \frac{A_{city}}{A_{cell}} = \frac{1300}{41.5} = 31$ cells.

∴ Max traffic $= 31 \times 84 = 2604$ Er

d) Total no. of users/city $= \frac{2604}{0.03} = 86800$ users.

⇒ "Erlang-C" ⇐

Prob_[Delay > 0] ⇒ Curve

$$\text{Prob}\_[\text{Delay} > t] = \text{prob}\_[\text{Delay} > 0]\, e^{-\left(\frac{C-A}{H}\right)t}$$

$$\text{prob}\_[\text{Delay} > t \mid \text{delay} > 0] = e^{-\left(\frac{C-A}{H}\right)t}$$

"Cellcular Systems" 18.10.2017

Ex) $N = 4$

$A_{cell} = 5\ Km^2$

Total no. of Channels $= 60 \leftarrow S$

$A_u = 0.029\ Er$, $\lambda = 1$ call / Hr

GOS (Er_C) $= p[\text{delay} > 0] = 5\% = 0.05$

a) Find no. of users supported / Km^2

b) Find p[a delayed call will wait more than 10 s]

c) prop. that a call is delayed >10 s

Solution:

a) $\text{No. of } \frac{Ch}{cell} = \frac{S}{N} = \frac{60}{4} = 15\ Ch/cell$

$GOS = 0.05$, $C = 15 \Rightarrow A = (9\ Er)$

$u = \frac{9}{0.029} = 310$ users/cell

$\text{No. of users}/Km^2 = \frac{310}{5} = 62\ user/Km^2$

b) $P[\text{delay} > 10, \text{delay} > 0] = e^{-\left(\frac{C-A}{H}\right)t}$

$= e^{-\left(\frac{15-9}{104.4\ sec}\right) \times 10\ sec}$

$A_u = \lambda H$

$= 0.563$

c) $p[\text{delay} \geq 10] = GOS\, e^{-\left(\frac{C-A}{H}\right)t}$

$= 0.05 \times 0.5629 = 2.81\%$

"Cellcular Systems" 18-10-2017

"Improving Coverage and Capacity of cellcular systems"

1) Cell Splitting:-

$\Rightarrow$ Power reduction for smaller cells.

$P_{rec.}$ [of older cell boundary] $\propto P_{t_1} R^{-n}$

(older = Larger)

$P_{rec.}$ [of new cell boundary] $\propto P_{t_2} \left(\frac{R}{2}\right)^{-n}$

(new = Small)

$\Rightarrow P_{t_1} (R)^{-n} = P_{t_2} \left(\frac{R}{2}\right)^{-n}$

Assume $n=4$

$\frac{P_{t2}}{P_{t1}} = \frac{1}{16}$

* Disadvantages:

1) More BS's cost
2) More Hand-offs
3) More complicated channel assignment.

$\Rightarrow$ Usually antenna down tilting is used to limit the power to neighboring cells.

2) Sectoring

$\Rightarrow$ CCI can be reduced by replacing omnidirectional antennas by several directional antennas.

$\Rightarrow$ Total no. of channels in a cell is divided between its sectors.

$\Rightarrow SIR = \frac{(\sqrt{3N})^n}{i_o}$ $\Rightarrow$ sectoring $\Rightarrow$ SIR $\uparrow$
if we reduce $N \Rightarrow$ Cap. $\uparrow$

* $120^\circ \Rightarrow i_o = 2$, $60^\circ \Rightarrow i_o = 1$

"Cellcular systems"

23.10.2017

Ex) $N=7$, $n=4$

⇒ without sectoring:-

$$SIR = 10 \log \left(\frac{(\sqrt{3N})^n}{i_o} \right)$$

$$= 10 \log \left(\frac{(\sqrt{3\times7})^4}{6} \right) = 18.6 \text{ dB}$$

⇒ with 120° sectoring

$$SIR = 10 \log \left(\frac{(\sqrt{3\times7})^4}{2} \right) = 23.4 \text{ dB}$$

∴ Sectoring → SIR ↑ → So we can reduce N ⇒ M ↑ ⇒ C ↑

Disadvantages:

1) Complexity (increasing no. of antennas)

2) Increasing no. of handoff → but it's base station Controlled

3) Decrease trunking efficiency

Ex) $n=4$ SIR (desired) = 18 dB

Find the capacity increase when using 120° sect?

⇒ without sect.

$$SIR \geq 18 \text{ dB}$$

$$10 * \log \left(\frac{(\sqrt{3N})^4}{6} \right) \geq 18$$

$$\log \frac{(3N)^2}{6} \geq 1.8$$

$$\frac{(3N)^2}{6} \geq 10^{1.8} \Rightarrow N \geq 6.48$$

⇐ $N=7$

3, 4, 7, 9, 12, 13, 19 - -

⇒ with sectoring:

$$10 \log \frac{(\sqrt{3N})^4}{2} \geq 18$$

$$\frac{9N^2}{2} \geq 10^{1.8} \Rightarrow N \geq 3.74$$

$$N = 4$$

∴ Capacity increased by $\frac{7}{4}$

Ex) A cellcular system , S = 210 ch , n = 4
GOS (Er_B) = 1 %
⇒ Find SIR and trunking efficiency (η) for:
a) No_sect. , N = 7 b) 120°_sect , N = 7
c) 120°_sect , N = 4

a) No_sect. ⇒ N = 7

$$SIR = 10 \log \frac{(\sqrt{3 \times 7})^4}{6} = 18.66 \text{ dB}$$

$$\eta = \frac{A(1 - GOS)}{C}$$

$$C = \frac{210}{7} = 30 \text{ ch/cell}$$

$$GOS = 0.01 \Rightarrow A = 20.34 \text{ Er}$$

$$\eta_a = \frac{20.34(0.99)}{30} = 67\ \%$$

b) N = 7 , 120°_sect , $i_o = 2$

$$SIR = 10 \log \frac{(\sqrt{3 \times 7})^4}{2} = 23.43 \text{ dB}$$

$$GOS = 0.01 \Rightarrow C = \frac{210}{7 \times 3} = 10 \text{ ch/sect.} \Rightarrow A_{sect} = 4.46 \frac{Er}{sect.}$$

$$A_{cell} = 3 A_{sect} = 13.38 \text{ Er}$$

$$\eta_b = \frac{13.38(0.99)}{30} = 44.1\ \%$$

c) 120°_sect , $N = 4$

$$SIR = 10 \log\left(\frac{(\sqrt{3\times4})^4}{2}\right) = 18.56 \text{ dB}$$

$GOS = 0.01$

$$C = \frac{210}{4\times3} = 17 \text{ ch/sect.}$$

$$A_{sec} = 9.65 \frac{Er}{sec.} \Rightarrow A_{cell} = 3\times9.65 = 28.95 \frac{Er}{cell}$$

$$\eta_c = 56\%$$

3) Repeaters for range extension

To provide coverage for hard_to_reach areas (Factories, Large buildings, tunnels --).

$\Rightarrow$ SIR$\uparrow$, we can reduce $N \Rightarrow C\uparrow$

4) Microcell Zone concept.

$\Rightarrow$ Cell is divided into Zones (3_Zones).

$\Rightarrow$ Each Zone has its own Tx, Rx.

$\Rightarrow$ All Zones are connected to the same base station.

$\Rightarrow$ As a mobile moves from one Zone into another $\Rightarrow$ Switching occurs (Not hand-off)

$\Rightarrow$ We can use lower Tx power.

$\Rightarrow$ SIR $\Rightarrow$ C$\uparrow$

Ex) Cellcular Sys. , desired SIR = 15 dB, Find the optimal value of N for:

a) Omni_directional antenna.

b) 120°_sect.

c) 60°_sect.

use $n=4$

"Cellcular Systems"

23-10-2017

a) $SIR = 10 \log\left(\frac{(\sqrt{3N})^4}{6}\right) \geq 15$

$N \geq 4.59 \Rightarrow N = 7$

b) $10 \log\left(\frac{(\sqrt{3N})^4}{2}\right) \geq 15$ ← better than (c) in 7

$N \geq 2.65 \Rightarrow N = 3$

c) $10 \log\left(\frac{(\sqrt{3N})^4}{1}\right) \geq 15$

$N \geq 1.87 \Rightarrow N = 3$

Ex] Total BW = 24 MHZ , Full duplex ch = 60 KHZ
$A_u = 0.1$ Er $N = 4$.

a) If each cell offers capacity 90% of perfect scheduling, Find max. no. of users/cell.

b) Find pr[blocking] if no. of users is the same as in (a)

c) Using 120° sect. Find no. of users/cell to result in GOS of (b).

⇒ a) Total no. of ch $= \frac{24000}{60} = 400$ ch.

$K\left(\frac{\text{No. of ch}}{\text{cell}}\right) = \frac{400}{4} = 100$ ch/cell

$A = 0.9 \times 100 = 90$ Er

No. of users $= \frac{90}{0.1} = 900$ users.

b) $A = 90$ Er

$C = 100 \Rightarrow GOS\ (Er\_B) = 0.03$

c) $C = \frac{100}{3} = 33$, $GOS = 0.03 \Rightarrow A_{sect} = 25$ Er $\Rightarrow A_{cell} = 3 \times 25 = 75$

Total no. of users $= 75/0.1 = 750$ users.

"Cellular Systems"

23_10_2017

Ex] $C = 20$ ch/cell

$\lambda = 1$ call/hour , $H = 105$ sec. , No. of users $= 480$

⇒ Find prob. [delay > 20 sec.]

$$\text{prob. [delay} > 20] = \text{pr [delay} > 0] \, e^{-\left(\frac{C-A}{H}\right)t}$$

pr [delay > 0] ↙ GOS

$A_u = \lambda H = 0.029$ Er

$A = 480 \times A_u = 14$ Er

$A = 14$, $C = 20$ } → GOS (Er_C) = 0.06

$$\therefore \text{prob [delay} > 20] = 0.06 \, e^{-\left[\frac{20-14}{105}\right]} = 0.019$$

Problems: 3. (7_8_11_12_13_15_16_17_18_26)

Ex) A Tx produces 50W power, power is applied to a unity gain antenna ($f_c = 900$ MHZ, $G_r = 1$)

a) Find Rx-power in (dBm) at a free space distance (100 m)

b) Find Rx-power at 10 Km.

$$\Rightarrow P_r(100m) = \frac{P_t G_T G_r \lambda^2}{(4\pi)^2 d^2} \Rightarrow \lambda = \frac{3\times10^8}{900\times10^6} = \frac{1}{3} \text{ m}$$

c/f

$$P_r(100m) = \frac{50\times1\times1\times(\frac{1}{3})^2}{(4\pi^2)(100)^2} = 3.5\times10^{-6} \text{ watt.}$$

$$P_r(100m)_{dBm} = 10 \operatorname{Log}\left(\frac{3.5\times10^{-6}}{10^{-3}}\right) = -24.5 \text{ dBm}$$

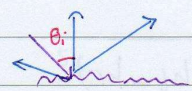
$$\Rightarrow P_{dBm}(10 \text{ Km}) = P_r(100m)_{dBm} + 20 \operatorname{Log}\left(\frac{d_o}{d}\right)$$

$$P_{dBm}(10Km) = -24.5 + 20 \operatorname{Log}\left(\frac{100}{10K}\right)$$

$$P_{dBm}(10Km) = -24.5 - 40 = -64.5 \text{ dBm.}$$

$\Rightarrow$ Scattering (spreading in all directions)

distance from bottom & peak $\leftarrow h$ $\updownarrow$ =



$\leftarrow$ rough surface.

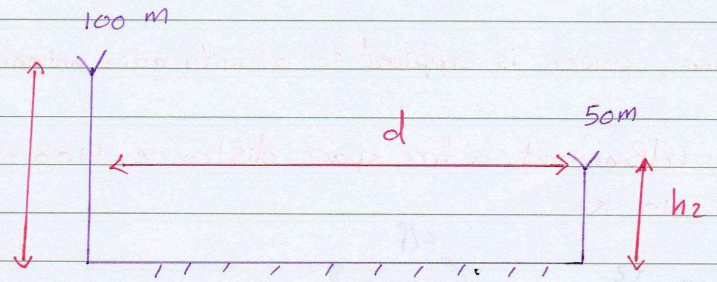
hc: Critical height. $\Rightarrow h_c = \frac{\lambda}{8 \sin\theta_i}$

The surface is:

1) rough if $h > h_c$

2) Smooth if $h < h_c$

"Ground Reflection (2-ray) Model". 30_10_2017



Assumptions:

1) Large seperating distance (several kilometers)
2) Ground is a perfect reflector
3) $d >> h_1$ & $d >> h_2$ $\Longrightarrow$ from these assumptions it becomes propotional to d^{-4} instead of d^{-2}

↳ From (figure 4.8): Method of Images.

$$d_1^2 = d^2 + (h_t - h_r)^2$$

$$d_1 = \sqrt{d^2 + (h_t - h_r)^2}$$

$$d_2 = \sqrt{d^2 + (h_t + h_r)^2}$$

Δ = path diff. $= d_2 - d_1$

$$\Delta = \sqrt{d^2 + (h_t + h_r)^2} - \sqrt{d^2 + (h_t - h_r)^2}$$

approximation: $\sqrt{1+x} = 1 + \frac{x}{2}$ if $x << 1$

$$\Delta = d\sqrt{1 + \left(\frac{h_t + h_r}{d}\right)^2} - d\sqrt{1 + \left(\frac{h_t - h_r}{d}\right)^2}$$

$$\Delta = d\left[1 + \frac{1}{2}\left(\frac{h_t^2 + h_r^2 + 2h_t h_r}{d^2}\right)\right] - d\left[1 + \frac{1}{2}\left(\frac{h_t^2 + h_r^2 - 2h_t h_r}{d^2}\right)\right]$$

$$\Delta = \frac{2h_t h_r}{d}$$

"The 2-Ray Model"

$$\Rightarrow \Delta\phi = \left(\frac{2h_t h_r}{d}\right) \cdot \frac{2\pi}{\lambda} = \frac{4\pi}{\lambda d} h_t h_r$$

$$P_{recieved} = P_{LOS} \left|1 - e^{j\Delta\phi}\right|^2 \Rightarrow \text{Two phases are added.}$$

Line of sight power

$$\left|(1 - \cos\Delta\phi - j\sin\Delta\phi)\right|^2$$
$$= (1 - 2\cos\Delta\phi + \cos^2\Delta\phi + \sin^2\Delta\phi)$$
$$= 2 - 2\cos\Delta\phi$$
$$= 2\left[1 - \left(1 - 2\sin^2\frac{\Delta\phi}{2}\right)\right]$$
$$= 4\sin^2\left(\frac{\Delta\phi}{2}\right)$$

$$P_{recieved} = P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 \cdot 4\sin^2\left(\frac{4\pi}{\lambda d} \cdot \frac{h_t h_r}{2}\right)$$

$$P_{recieved} = 4 P_t G_t G_r \left(\frac{\lambda^2}{(4\pi d)^2}\right) \cdot \sin^2\left(\frac{2\pi h_t h_r}{\lambda d}\right) \Rightarrow \text{Exact}$$

For small $\Delta\phi \Rightarrow \sin\left(\frac{\Delta\phi}{2}\right) \approx \frac{\Delta\phi}{2}$

$$P_{rec.} = P_t G_t G_r \left(\frac{h_t h_r}{d^2}\right)^2 \Rightarrow \text{The two-ray model (approx.)}$$
$$= P_t G_t G_r \left(\frac{h_t^2 h_r^2}{d^4}\right)$$

directly with d^{-4}

Ex) $f_0 = 900\,MHz$, $h_r = 5m$, $h_t = 1.5m$, $P_t = 2w$, $G_r = G_t = 2.55dB$
Find the recieved power at $d = 5km$, using the two ray model.

$$\Rightarrow 2.55 = 10\log G_r \Rightarrow G_r = 10^{2.55/10} = 1.8 = G_t$$

$$P_r(5km) = P_t G_t G_r \left(\frac{h_t h_r}{d^2}\right)^2 = 5.8 \times 10^{-13}\ \text{watt}$$

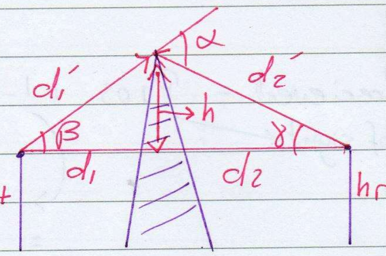
$$P_r(5km)_{dBm} = 10\log\left(\frac{5.8\times 10^{-13}}{10^{-3}}\right) = -92.3\,dBm.$$

"Knife-Edge Diffraction"

6-11-2017

$\Rightarrow$ In our derivations we will take the LOS as a reference.

Δ: Path difference.



$$\Delta = d_1' + d_2' - (d_1 + d_2)$$

$$\Delta = \sqrt{d_1^2 + h^2} + \sqrt{d_2^2 + h} - d_1 - d_2$$

$$\sqrt{1+x} \approx 1 + \frac{x}{2} \quad \text{if } x \ll 1$$

$$\Delta = d_1 \sqrt{1 + \left(\frac{h}{d_1}\right)^2} + d_2 \sqrt{1 + \left(\frac{h}{d_2}\right)^2} - d_1 - d_2$$

$$\Delta = d_1 \left[1 + \frac{1}{2} \left(\frac{h}{d_1}\right)^2 \right] + d_2 \left[1 + \frac{1}{2} \left(\frac{h}{d_2}\right)^2 \right] - d_1 - d_2$$

$$\Delta = \frac{h^2}{2} \cdot \left(\frac{d_1 + d_2}{d_1 d_2} \right)$$

$$\Delta\phi = \frac{\Delta . 2\pi}{\lambda} = \frac{\pi h^2}{\lambda} \left(\frac{d_1 + d_2}{d_1 d_2} \right)$$ $\Rightarrow$ The phase difference for the same height.

$\beta \rightarrow$ The angle between the horizontal line and the line from the Tx to the knife-edge.

$\gamma \rightarrow$ The angle between the horizontal line and the line from the Rx to the knife-edge.

$$\alpha = \beta + \gamma \qquad \Rightarrow h \approx h'$$

$$\alpha = \frac{h}{d_1} + \frac{h}{d_2} = h \left(\frac{d_1 + d_2}{d_1 d_2} \right)$$

$$h = \alpha \frac{d_1 d_2}{d_1 + d_2}$$ $\rightarrow$ relation between α & h.

"Knife_Edge Diffraction"

6.11.2017

assuming

$$\Delta\phi = \frac{\pi}{2} v^2 \rightarrow v = h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}} \Rightarrow \text{For Equal Heights } (h_t = h_r)$$

$$\rightarrow v = \alpha\sqrt{\frac{2 d_1 d_2}{\lambda (d_1+d_2)}} \Rightarrow \text{For } (h_t \neq h_r)$$

" Fresnel Screens or Fresnel Zones"

Zones $\Rightarrow$ path passing r_1 exceeds LOS by $\lambda/2$
path passing r_2 exceeds LOS by λ
path passing r_3 exceeds LOS by 1.5λ

* Odd Zones add to signal.
* Even Zones substract from signal

Constructive & distractive Zones for the signal

$$\Delta = n\frac{\lambda}{2} = \frac{h^2}{2}\left(\frac{d_1+d_2}{d_1 d_2}\right)$$

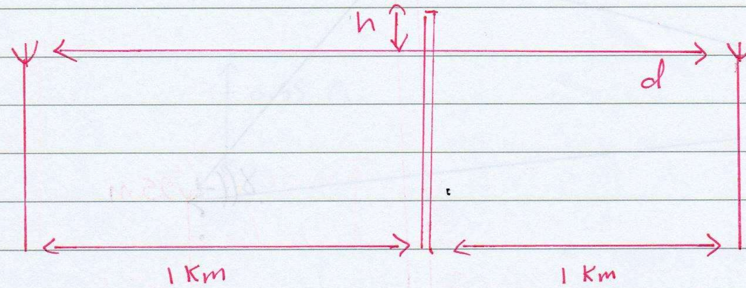
$$r_n = \sqrt{\frac{n\lambda d_1 d_2}{d_1+d_2}}$$

* Rule of thumb for LOS
  $\Rightarrow$ As long as 55% of the first Fresnel Zone is cleared further clearance is not required $\Rightarrow$ (we have good LOS)

. $P_{rec.} = P_{LOS} + G_{diffraction}$

"Examples" 8-11-2017

Ex]



$\lambda = \frac{1}{3}$

a) $h = 25\,m$

b) $h = 0$

c) $h = -25\,m$

Find Diffraction Loss, within which Fresnal Zone the obstruction lies?

a) $v = h\sqrt{\dfrac{2(d_1+d_2)}{\lambda d_1 d_2}} = 25\sqrt{\dfrac{2(1000+1000)}{\frac{1}{3}\times 1000\times 1000}} = 2.74$

Loss(dB) = 22 dB

b) $v = 0 \Rightarrow$ Loss(dB) = 6dB

c) $v = -2.74 \Rightarrow$ Loss (dB) = 1dB

$\Rightarrow r_n = \sqrt{\dfrac{n\lambda(d_1 d_2)}{d_1+d_2}}$ For a) $\Rightarrow 25 = \sqrt{\dfrac{n(\frac{1}{3})(1000)^2}{2000}}$

$n = 3.75 \Rightarrow$ within the 4th Zone above LOS.

For b) $\Rightarrow$ $h=0$, in the middle of 1st Zone

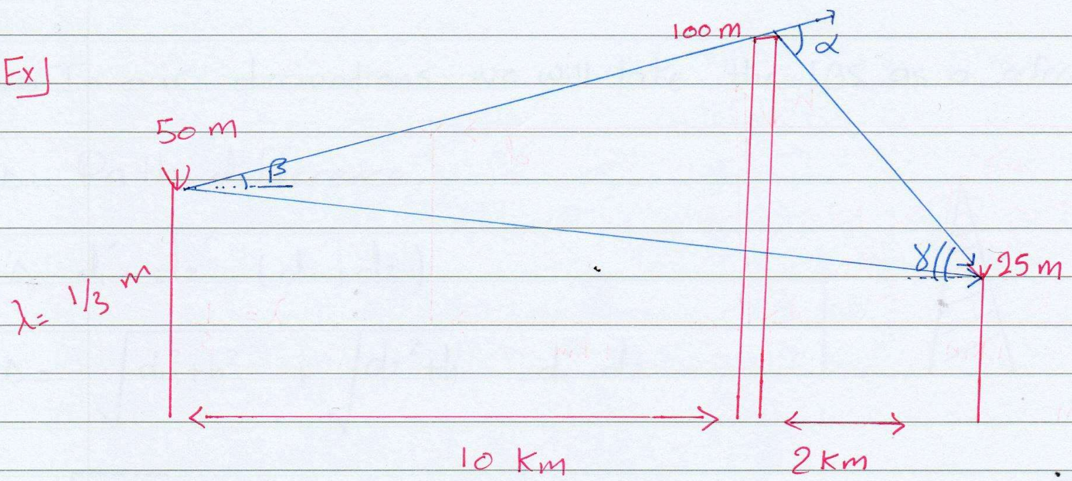
For c) $\Rightarrow$ $h = -25$, in the 4th Zone below LOS.

or by drawing 8-

$r_1 = 12.9$, $r_2 = 18.26$, $r_3 = 22.36$, $r_4 = 25.8$

"Examples" 8-11-2017

Ex]



Find:-

a) Diff. loss

b) height of obstacle to give 6 dB loss.

a)

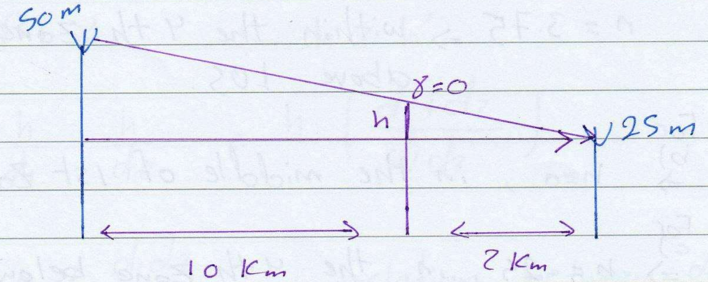
$\alpha = \beta + \gamma$

$\alpha = \tan^{-1}\left(\frac{100-50}{10000}\right) + \tan^{-1}\left(\frac{100-25}{2000}\right) = 2.434^{\circ} = 0.0424$

$v = 0.0424 \sqrt{\frac{2(10000)(2000)}{\frac{1}{3}(10000+2000)}} = 4.24$

loss = 25.5 dB

b) $6\ dB \Rightarrow v = 0 \rightarrow \left[\begin{array}{l} h = 0 \\ \alpha = 0 \end{array}\right.$



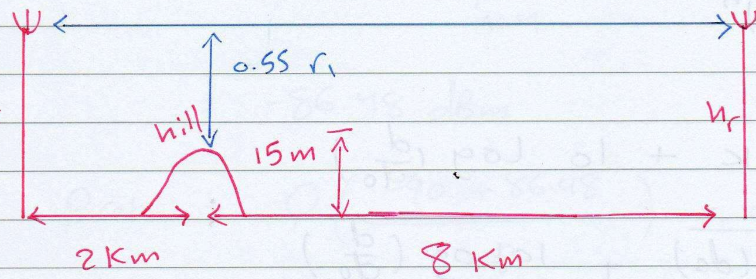
$\frac{h}{2000} = \frac{50-25}{12000} \Rightarrow h = 4.16\ m$

$\therefore$ height = 25 + 4.16 = 29.16 m

"Examples"

8-11-2017

Ex)



$\lambda = \frac{3\times10^8}{2.4\times10^9} = \frac{1}{8}$ m

$h_t = h_r$

Find $h_t (= h_r)$ to provide a good LOS. $f = 2.4$ GHz.

$$r_1 = \sqrt{\frac{1 \times \frac{1}{8} \times 2000 \times 8000}{2000 + 8000}} = 14.14 \text{ m}$$

$0.55\, r_1 = 7.77$ m

$\therefore h_t = h_r = 15 + 7.77 = 22.77$ m.

"Practical Link Budget Design using Path Loss Model" 8-11-2017

$$\Rightarrow \quad \overline{P_L}(d) = K\left(\frac{d}{d_0}\right)^n$$

and $\overline{P_L}(d_0) = K$

$$\therefore \quad \overline{P_L}(d) = 10 \log K + 10 \log\left(\frac{d}{d_0}\right)^n$$

$$= \overline{P_L}(d_0) + 10 n \log\left(\frac{d}{d_0}\right)$$

$$\Rightarrow \quad P_r(d)_{dBm} = P_{t\,dBm} - P_L(d)$$

$$P_r(d)_{dBm} = P_t - \left[\overline{P_L}(d_0) + 10 n \log\left(\frac{d}{d_0}\right) + \overline{X}_\sigma\right]$$

$$\overline{P_r}(d)_{dBm} = \overline{P_r}(d_0) - 10 n \log\left(\frac{d}{d_0}\right)$$

Ex] A cellular system with $f = 900$ MHz

Tx-power = 1 watt, $G_t = 3$dB, $G_r = 0$dB

$d_0 = 1$ Km, $d = 5$ Km

Find Prob. $[P_r(5\text{Km}) > -90 \text{ dBm}]$, where $\sigma = 8$dB, $n = 4$

given: $\lambda = \frac{3\times10^8}{9\times10^8} = \frac{1}{3}$, $G_t = 10^{3/10} = 2$ & $G_r = 10^0 = 1$

we know that $\text{prob}[P_r(5\text{Km}) > -90\text{dBm}] = Q\left(\frac{-90 - \overline{P_r}(5\text{Km})}{\sigma}\right)$

$$\Rightarrow \quad \overline{P_r}(d_0) = \overline{P_r}(1\text{Km}) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2} = \frac{(1)(2)(1)\left(\frac{1}{3}\right)^2}{(4\pi)^2 (1000)^2}$$

$$\overline{P_r}(1\text{ Km}) = 1.407\times10^{-9} \text{ watt} = 10 \log\left(\frac{1.407\times10^{-9}}{0.001}\right) \text{dBm}$$

$$= -58.8 \text{ dBm}$$

$$\overline{P_r}(5\text{Km}) = \overline{P_r}(1\text{Km}) - 10 n \log\left(\frac{d}{d_0}\right) = -58.8 - 10(4) \log\left(\frac{5000}{1000}\right)$$

$$= -86.48 \text{ dBm}$$

Finally $\Rightarrow$ $\text{Prob.}\left[\overline{P_r}(5\text{Km}) > -90\,\text{dBm}\right] = Q\left(\dfrac{-90+86.48}{8}\right)$

$= Q(-0.44) = 1 - Q(0.44) = 1 - 0.32 = 0.68$

Ex] Tx_power = 15 W , $G_t = 12\,\text{dB}$, $G_r = 3\,\text{dB}$, BW = 30 KHZ , $F_c = 1800$ MHZ , $d_0 = 1$ Km , Noise Figure = 8 dB , T = 290 kelvin
Find the max. Tx_Rx distance that will ensure SNR $\geq$ 20 dB , n=4 with prob. = 95% and $\sigma = 8$ dB

$G_t = 10^{G_t\text{dB}/10} = 10^{12/10} = 15.85$

$G_r = 10^{3/10} = 2$

$F = 10^{8/10} = 6.3$

$\Rightarrow$ given Noise power = $K * BW * F * T$

$= 1.38 \times 10^{-23} \times 30 \times 10^{3} \times 6.3 \times 290$

$= 7.56 \times 10^{-16} = -121.2$ dBm

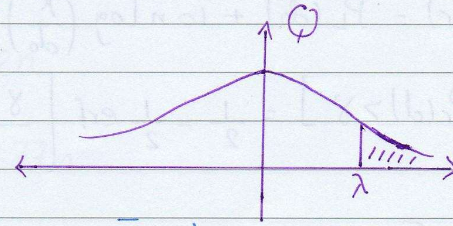
$\text{Prob.}\left[\text{SNR} \geq 20\,\text{dB}\right] = \text{prob.}\left[\left\{P_r(d) - \text{Noise Power}\right\} \geq 20\,\text{dB}\right] = 0.95$

$= \text{prob.}\left[P_r(d) \geq 20 + \text{Noise Power}\right] = 0.95$

$= \text{Prob.}\left[P_r(d) \geq -101.2\,\text{dBm}\right] = 0.95$

$\Rightarrow 0.95 = Q\left(\dfrac{-101.2 - \overline{P_r}(d)}{\sigma}\right)$

$1 - 0.95 = Q\left(\dfrac{101.2 + \overline{P_r}(d)}{8}\right)$



$0.05 = Q\left(\dfrac{101.2 + \overline{P_r}(d)}{8}\right) \Rightarrow \dfrac{101.2 + \overline{P_r}(d)}{8} = 1.64$

$\Rightarrow \overline{P_r}(d) = -88.04$ dBm

$\overline{P_r}(d_0 = 1\text{Km}) = \dfrac{P_t G_t G_r \lambda^2}{(4\pi d)^2} = \dfrac{15 \times 15.85 \times 2 \left(\frac{1}{6}\right)^2}{(4\pi \times 1000)^2} = 8.364 \times 10^{-8}$ watt

$P_r(d_0 = 1\text{Km}) = 10 \log\left(\dfrac{8.364 \times 10^{-8}}{0.001}\right) = -40.77$ dBm

$$\bar{P}_r(d) = \bar{P}_r(1\,Km) - 10n \operatorname{Log}\left(\frac{d}{1000}\right)$$

$$-88.04 = -40.77 - 40 \operatorname{Log}\left(\frac{d}{1000}\right)$$

$$\operatorname{Log}\frac{d}{1000} = 1.182 \Rightarrow \frac{d}{1000} = 10^{1.182} = 15.2$$

$$d = 15.2\ Km$$

" Percentage of Coverage Area"

$\Rightarrow$ Given a signal power threshold (γ), we need to find $u(\gamma)$, the percentage of cell area where the Rx_Power $\geq \gamma$

$$u = \frac{1}{\pi R^2} \int_{\theta=0}^{2\pi} \int_{r=0}^{R} \text{prob.}\left[P_r(r) > \gamma\right] r\,dr\,d\theta$$

$$\text{Prob.}\left[P_r(r) > \gamma\right] = Q\left(\frac{\gamma - \bar{P}_r(r)}{\sigma}\right) = \frac{1}{2} - \frac{1}{2}\operatorname{erf}\left(\frac{\gamma - \bar{P}_r(r)}{\sigma\sqrt{2}}\right)$$

$$= \frac{1}{2} - \frac{1}{2}\operatorname{erf}\left(\frac{\gamma - [P_t - (\overbrace{\bar{P}_L(d_0) + 10n \operatorname{Log}(\frac{r}{d_0})}^{\bar{P}_L(r)})]}{\sigma\sqrt{2}}\right)$$

$$\bar{P}_L(r) = \bar{P}_L(d_0) + 10n \operatorname{Log}\left(\frac{R}{d_0}\right) + 10n \operatorname{Log}\left(\frac{r}{R}\right)$$

$$\text{Prob.}\left[P_r(d) > \gamma\right] = \frac{1}{2} - \frac{1}{2}\operatorname{erf}\left[\frac{\gamma - [P_t - (\bar{P}_L(d_0)) + 10n \log(\frac{R}{d_0}) + 10n \operatorname{Log}(\frac{r}{R})]}{\sigma\sqrt{2}}\right]$$

Let $a = \left[\gamma - P_t + \bar{P}_L(d_0) + 10n \operatorname{Log}\left(\frac{R}{d_0}\right)\right] / \sigma\sqrt{2}$

$b = [10n \operatorname{Log} e] / \sigma\sqrt{2}$

$$u(\gamma) = \frac{1}{\pi R^2} \int_0^{2\pi}\int_0^{R} \left(\frac{1}{2} - \frac{1}{2}\operatorname{erf}\left[a + b \ln\frac{r}{R}\right]\right) r\,dr\,d\theta$$

$$u(\gamma) = \frac{1}{2} - \frac{1}{2}\operatorname{erf}(a) + \frac{1}{2} e^{\frac{1-2ab}{b^2}} \left[1 - \operatorname{erf}\left(\frac{1-ab}{b}\right)\right]$$

"Percentage of Coverage Area" 20-11-2017

depends on :-

1) $\frac{\sigma}{n}$

2) $Pr\left(Pow(R) \geq \gamma\right) = Q\left(\frac{\gamma - \overline{Pow}(R)}{\sigma}\right)$

Ex| Given the following power measurements :-

	dist (m)	Rx- Power
$d_0 =$	100 m	0 dBm
	200 m	-20 dBm
	1000 m	-35 dBm
	3000 m	-70 dBm

a) Find the minimum mean square of error estimate of σ, n

b) Find percentage of Area coverage for the cell with radius of 2 Km ($R = 2$ Km) for $\gamma = -60$ dBm.

a) $\hat{P}r(d) = \overline{Pr}(d_0) - 10\,n\,Log\left(\frac{d}{d_0}\right) \Rightarrow$ where $d_0 = 100$ m

$\hat{P}r(200) = 0 - 10\,n\,Log\left(\frac{200}{100}\right) = -3\,n$

$\hat{P}r(1000) = 0 - 10n\,log\left(\frac{1000}{100}\right) = -10n$

$\hat{P}r(3000) = 0 - 10\,n\,Log\left(\frac{3000}{100}\right) = -14.77\,n$

$\Rightarrow$ Assume $J(n) = \sum_{i=1}^{K} \left(\bar{P}_i - \hat{P}_i\right)^2$

$J(n) = (0-0)^2 + (-20-(-3n))^2 + (-35-(-10n))^2 + (-70-(-14.77n))^2$

$J(n) = 6525 - 2887.8\,n + 327.153\,n^2$

$\frac{dJ}{dn} = 0 = -2887.8 + 2\,(327.153)n \Longrightarrow \hat{n} = 4.4$

"Percentage of Coverage Area" 20-11-2017

$$\sigma^2 = \frac{1}{4}\left[(0-0)^2 + (-20-(-3\times4.4))^2 + (-35-(-10\times4.4))^2 + (-70-(-14.77\times4.4))^2\right]$$

$$\sigma^2 = 38.09$$

$$\hat{\sigma} = 6.17 \text{ dB}$$

b) $u(\gamma = -60 \text{ dBm})$

$$Prob(P_{ow}(2km) > -60dBm) = Q\left(\frac{-60 - \bar{P}_{ow}(2km)}{\sigma}\right)$$

$$\bar{P}_{ow}(2km) = \bar{P}_{ow}(d_o) - 10\,n\,Log\left(\frac{d}{d_o}\right)$$

$$= 0 - 10(4.4)\log\left(\frac{2000}{100}\right) = -57.24 \text{ dBm}$$

$$Prob.(P_{ow}(2km) > -60dBm) = Q\left(\frac{-60+57.24}{6.17}\right) = Q(-0.477)$$

$$= 1 - Q(0.477)$$

$\therefore$ $Prob.(P_{ow}(2km) > -60 \text{ dBm}) = 1 - 0.326 = 67.4\%$

& $\frac{\sigma}{n} = \frac{6.17}{4.4} = 1.402$

$\Rightarrow$ From the curve :- $u(\gamma) = 88\%$

"Okumura Model"

Ex] Find the average path loss using okumura model.

$d = 50$ Km
$h_t = 100$ m
$h_r = 10$ m
EIRP = 1 KW
↳ $P_t G_t$

Sub-Urban Area
$f = 900$ MHz
$G_r = 1$

$$L_f = 10 \log \frac{\lambda^2}{(4\pi d)^2} = -10 \log \frac{(1/3)^2}{(4\pi * 50000)^2} = 125.5 \text{ dB}$$

$A(f,d) = A(900 \text{ MHz}, 50 \text{ Km}) = 43$ dB ⟵ From the Curve

$G_{Area}(900 \text{ MHz, Sub-urban}) = 9$ dB ⟵ From the Curve!

$$G(h_t) = 20 \log\left(\frac{100}{200}\right) = -6 \text{ dB}$$

$$G(h_r) = 20 \log\left(\frac{10}{3}\right) = 10.46 \text{ dB}$$

$$\bar{L} = 125.5 + 43 - (-6) - 10.46 - 9 = 155.04 \text{ dB}$$

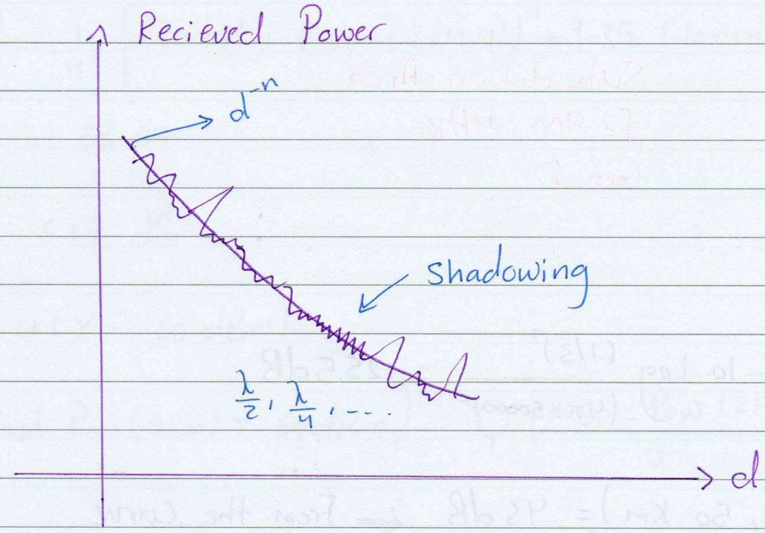
$$\bar{P_r}(50 \text{ km}) = \text{EIRP} - \bar{L} + G_r(\text{dB})$$

$$= 10 \log\left(\frac{1000}{0.001}\right) - 155.04 + 0$$

$$= -95.04 \text{ dBm}$$

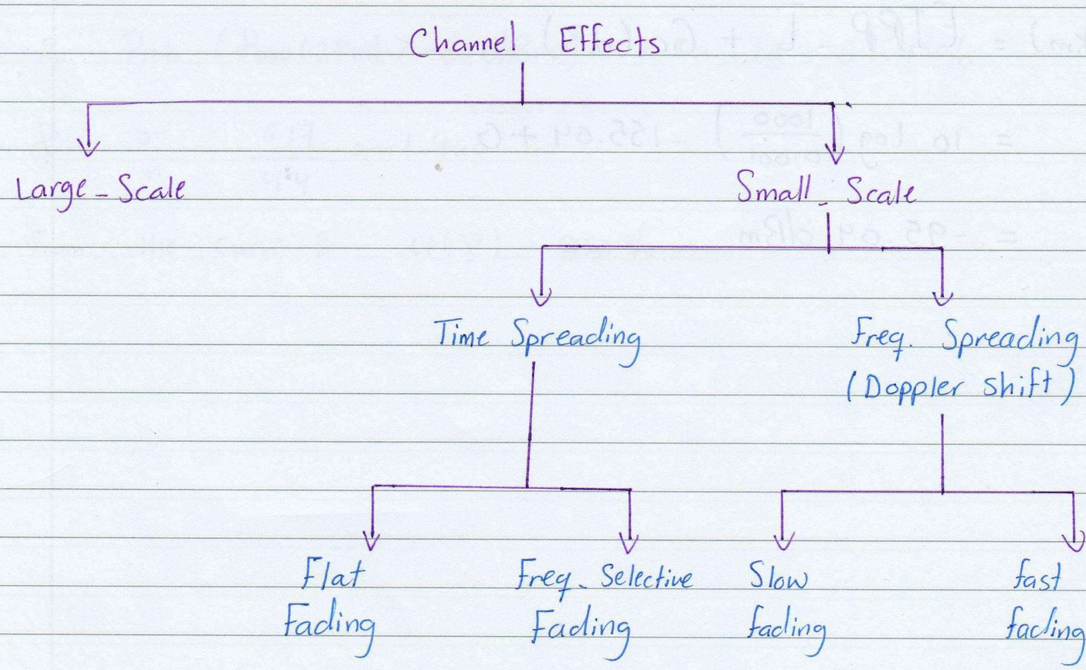
Chp 5: "Small_ Scale Fading and Multipath"

20_11_2017



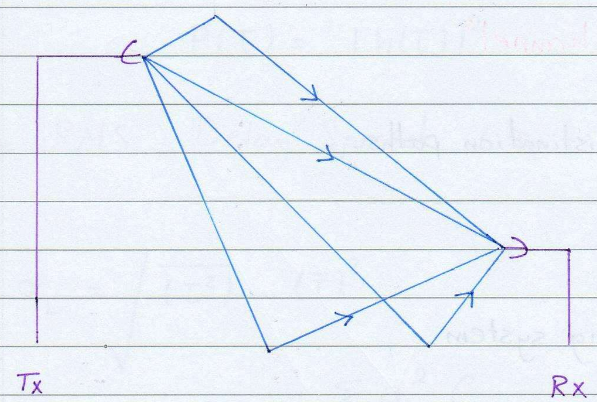
⇒ Small_ Scale fading affects the design of:

1_ Dynamic Range
2_ Modulation + Coding
3_ Diversity
4_ Equalization.

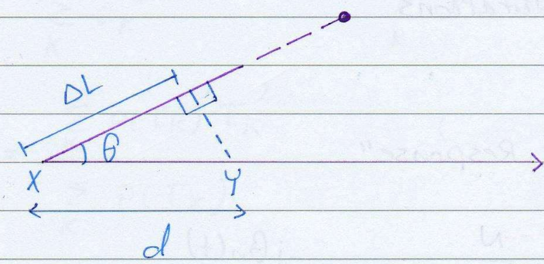


"Small Scale Fading"

⇒ Rapid fluctuations in the Rx_ Signal power over Short distances (time_intervals)



"Doppler Shift"



⇒ $\Delta L = d \cos\theta = v \Delta t \cos\theta$

⇒ Phase change due to motion → $\Delta\phi = 2\pi\left(\frac{\Delta L}{\lambda}\right)$

$$\Delta\phi = \frac{2\pi v \Delta t \cos\theta}{\lambda} \Rightarrow f_d = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \frac{v\cos\theta}{\lambda}$$

Ex) $f_c = 1850$ MHZ , $v = 60$ miles/hour $= 26.82$ m/s

Find the recieved frequency if the motion is :_

a) Toward Tx b) Away from Tx c) Perpendicular to Tx

$\longrightarrow$ Tx $\longleftarrow$ Tx $\downarrow$ Tx

$$\lambda = \frac{3\times10^8}{1850\times10^6} = 0.162 \text{ m}$$

a) $f_{rx} = f_c + f_d = 1850\times10^6 + \frac{26.82}{0.162}\cos\theta = 1850.00016$ MHZ.

b) $f_{rx} = f_c + \frac{v}{\lambda}\cos 180^\circ = f_c - \frac{v}{\lambda} = 1849.99984$ MHZ.

c) $f_{rx} = f_c + \frac{v}{\lambda}\cos\left(\frac{\pi}{2}\right) = f_c = 1850$ MHZ.

$\Rightarrow f_m = \frac{v}{\lambda}$ is the max. Doppler shift.

"Model Of the Multi_path Channel"

* The signal propagates in N_distination paths.

* The channel is Linear

* The system is a time_varying system.

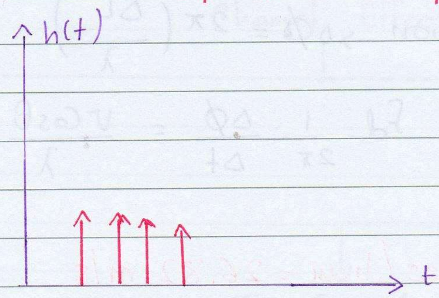
→ Channel can be assumed to be an LTI system over Short time_durations

"Channel Impulse Response"

No. of paths

$$\Rightarrow h(\tau, t) = \sum_{n=1}^{N} a_n(t) e^{-j\theta_n(t)} \delta(\tau - \tau_n(t))$$

Amplitude of n^{th} path   Delay of n^{th} path.



⇒ over short time durations :_

$$h(\tau) = \sum_{n=1}^{N} a_n e^{-j\theta_n} \delta(\tau - \tau_n) \Leftarrow \text{LTI}$$

$$r(t) = x(t) * h(t) = \sum_{n=1}^{N} a_n e^{-j\theta_n} x(t - \tau_n)$$

"Power Delay Profile (PDP)"

PDP is a spatial average of $|h(t)|^2$ by making several measurements in different locations.

$\rightarrow P(\tau) = |h(\tau)|^2$

$\Rightarrow$ RMS Delay Spread (σ_τ)

$$\sigma_\tau = \sqrt{\overline{(\tau^2)} - (\bar{\tau})^2}$$

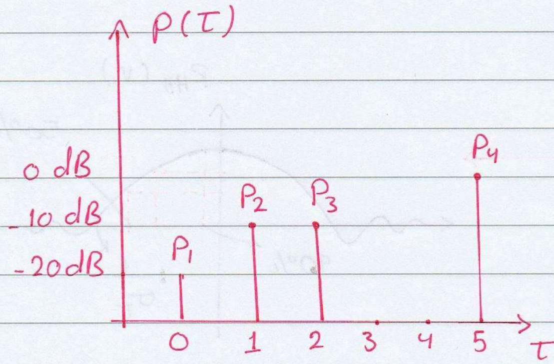
where :-

$$\bar{\tau} = \frac{\sum_K a_K^2 \tau_K}{\sum_K a_K^2} = \frac{\sum_K P(\tau_K)\tau_K}{\sum_K P(\tau_K)}$$

and

$$\overline{(\tau^2)} = \frac{\sum_K P(\tau_K)\tau_K^2}{\sum_K P(\tau_K)}$$

Ex)



$P_1 = 10^{-20/10} = 0.01$, $P_2 = 10^{-10/10} = 0.1 = P_3$, $P_4 = 10^0 = 1$

$$\bar{\tau} = \frac{\sum_K P(\tau_K)\tau_K}{\sum_K P(\tau_K)} = \frac{1*5 + 0.1*2 + 0.1*1 + 0.01*0}{1+0.1+0.1+0.01} = 4.38\ \mu\text{sec}$$

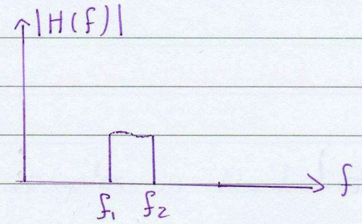
$$\overline{(\tau^2)} = \frac{1*5^2 + 0.1*2^2 + 0.1*1^2 + 0.01*0^2}{1+0.1+0.1+0.01} = 21.07\ (\mu s)^2$$

$$\sigma_\tau = \sqrt{21.07 - (4.38)^2} = 1.37\ \mu s$$

" Coherence Band-width (B_c) "

22-11-2017

The range of frequencies over which the channel $H(\omega)$ is considered "flat"

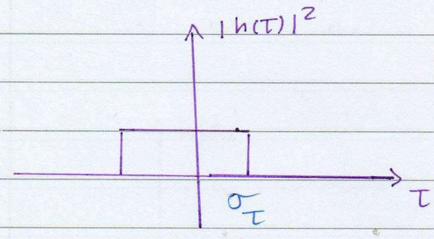


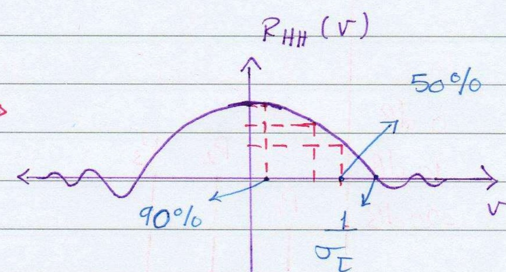
$\Rightarrow$ Two frequencies f_1 & f_2 are affected by the channel nearly equally if

$$f_2 - f_1 < B_c$$

where $B_c = \begin{cases} \dfrac{1}{50\sigma_\tau}, & \text{using 50 \% Correlation} \\ \dfrac{1}{500\sigma_\tau}, & \text{using 90 \% Correlation} \end{cases}$

* $|h(\tau)|^2 \longleftrightarrow R_{HH}(\nu)$





Ex] Given the previous PDP for a given channel, Is the channel suitable for:

a) GSM (200 KHZ)

b) AMPS (30 KHZ)

without needing an equalizor

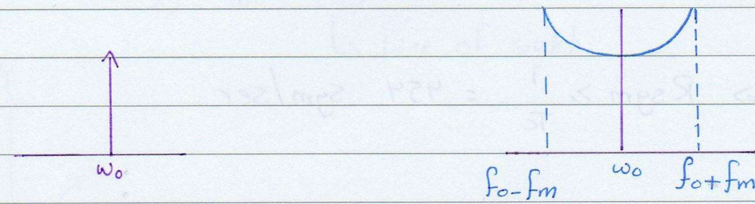
$\sigma_\tau = 1.37\ \mu s \Rightarrow B_c = \dfrac{1}{50\sigma_\tau} = 146\ KHZ$

$\therefore$ a) $BW > B_c \rightarrow$ needs an equalizor

b) $BW < B_c \rightarrow$ does not need an equalizor.

"Coherence Time T_c" 29.11.2017

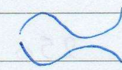
"Doppler Spread and Coherence Time (T_c)"

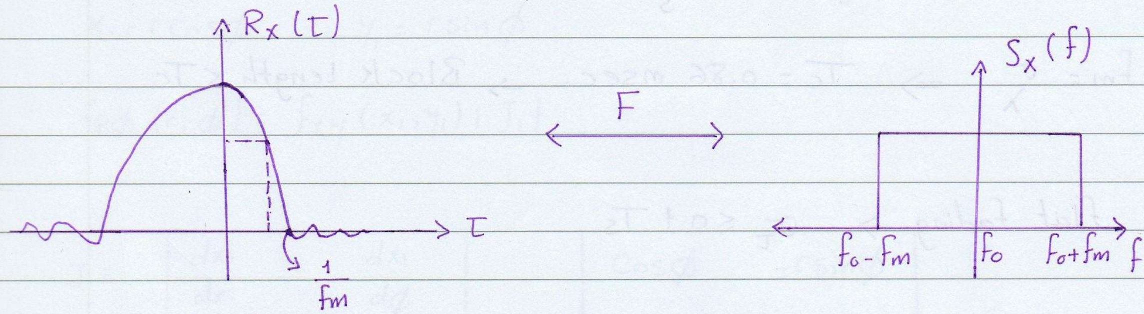


where $f_m = \frac{v}{\lambda}$

i.e) for two signals.

$$\rightarrow \cos\omega_1 t + \cos\omega_2 t = 2\cos\left(\frac{\omega_1-\omega_2}{2}t\right)\cos\left(\frac{\omega_1+\omega_2}{2}t\right)$$

 $\Leftarrow \omega_1 - \omega_2$



$$\underset{50\%}{T_c} = \frac{9}{16\pi f_m} = \frac{0.423}{f_m} \leftarrow \frac{v}{\lambda}$$

T_c: is the time over which $h(t)$ is approximately invariant.

Ex] A vehicle speed = 26.8 m/s = 60 mph.
f_c = 900 MHZ, find the minimum bit rate that will not cause distortion due to motion.

if: $T_{symbol} < T_c \rightarrow$ No distortion due to motion
$T_{symbol} > T_c \rightarrow$ Distortion due to motion.

$\therefore$ $T_c > T_{sym} \Rightarrow \frac{1}{T_c} < R_{sym}$ (symbol rate)

"Coherence Time"

29.11.2017

$$T_c = \frac{9}{16\pi f_m} \quad , \quad f_m = \frac{v}{\lambda} = \frac{26.8}{1/3} = 80.4 \text{ Hz}$$

$$T_c = 2.22 \text{ ms} \Rightarrow R_{sym} \geq \frac{1}{T_c} = 454 \text{ sym/sec.}$$

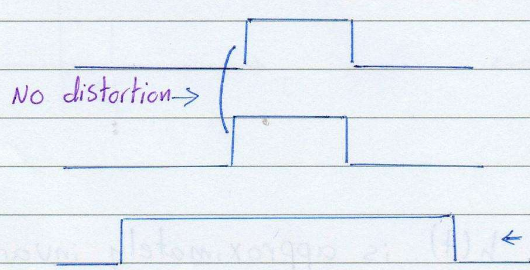
* If there is a distortion:

$$x(t) \longrightarrow \boxed{h(t)} \longrightarrow \boxed{h_I(t)}$$

* GSM is designed for $250 \text{ Km/Hr} = 69.4 \text{ m/sec.}$
& $\lambda = \frac{1}{3} \text{ m}$

$$f_m = \frac{v}{\lambda} \Rightarrow T_c = 0.86 \text{ msec.} \rightarrow \text{Block Length} < T_c$$

* For flat fading $\Rightarrow \sigma_\tau \leq 0.1\, T_s$



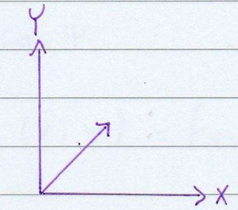
"Rayleigh Distortion"

29.11.2017

Flat Fading $\Rightarrow$ 1) $\sigma_\tau \ll T_{sym}$

2) NO _ LOS

$\hookrightarrow$ Line of sight.



X, Y: Gaussian R.V with Zero mean σ^2

$$r = \sqrt{X^2 + Y^2} \quad , \quad \phi = \tan^{-1}\left(\frac{Y}{X}\right)$$

$$f_{R,\phi}(r,\phi) \quad , \quad X, Y \rightarrow R, \phi$$

$$f_{XY}(x,y) = \frac{1}{2\pi\sigma^2} e^{-\left(\frac{x^2+y^2}{2\sigma^2}\right)}$$

$$x_1 = r\cos\phi \quad , \quad y_1 = r\sin\phi$$

$$f_{R\phi}(r,\phi) = f_{XY}(x_1, y_1)\,|J_1|$$

$$J_1 = \begin{vmatrix} \frac{dx_1}{dr} & \frac{dx_1}{d\phi} \\ \frac{dy_1}{dr} & \frac{dy_1}{d\phi} \end{vmatrix} = \begin{vmatrix} \cos\phi & -r\sin\phi \\ \sin\phi & r\cos\phi \end{vmatrix} = r$$

$$f_{R\phi}(r,\phi) = \frac{r}{2\pi\sigma^2} e^{-(r^2\cos^2\phi + r^2\sin^2\phi)/2\sigma^2}$$

$$f_{R\phi}(r,\phi) = \frac{r}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

$$f_r(r) = \int_{-\pi}^{\pi} f_{R\phi}(r,\phi)\, d\phi = 2\pi\left(\frac{r}{2\pi\sigma^2} e^{-\frac{r^2}{2\sigma^2}}\right) = \frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}}$$

$$= Q\left(\sqrt{\frac{2E_b r^2}{N_b}}\right)$$

"Rayleigh Distribution"

4-12-2017

⇒ For Rayleigh distribution:-

1) $E(r) = \sqrt{\frac{\pi}{2}}\ \sigma$

2) $E(r^2) = 2\sigma^2$

3) $\sigma_r^2 = \overline{r^2} - \overline{r}^2 = 0.4292\,\sigma^2$

4) $P(r \leq A)$ ⇐ discussed later.

Proof:-

1) $E(r) = \int_{r=0}^{\infty} r f_R(r)\, dr = \int_{r=0}^{\infty} \frac{r^2}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \cdot \frac{\sqrt{2\pi}}{\sqrt{2\pi}}$

$r \geq 0$

$= \frac{\sqrt{2\pi}}{\sigma} \int_{r=0}^{\infty} \frac{r^2 e^{-r^2/2\sigma}}{\sqrt{2\pi\sigma^2}} = \frac{\sqrt{2\pi}}{\sigma} \cdot \frac{\sigma^2}{2} = \sqrt{\frac{\pi}{2}}\ \sigma$

2) $E(r^2) = E[x^2 + y^2] = \overline{x^2} + \overline{y^2} = \sigma^2 + \sigma^2 = 2\sigma^2$

3) $P(r \leq A) = (-1) \int_0^A \left(\frac{-r}{\sigma^2}\right) e^{-r^2/2\sigma^2}\, dr = \left[-e^{-r^2/2\sigma^2}\right]_0^A$

$= 1 - e^{-A^2/2\sigma^2}$

⇒ Rician Fading

If there exist Line of sight (LOS) then:

$f_{XY}(x, y) = \frac{1}{2\pi\sigma^2}\, e^{\frac{-(x - A\cos\theta)^2}{2\sigma^2}} \cdot e^{\frac{-(y - A\sin\theta)^2}{2\sigma^2}}$

"Ricean Fading"

4-12-2017

$$r = \sqrt{x^2+y^2} \quad , \quad \psi = \tan^{-1}\frac{y}{x}$$

$$x_1 = r\cos\psi \quad , \quad y_1 = r\sin\psi \quad , \quad |J_1| = r$$

$$f_{R,\psi}(r,\psi) = f_{XY}(x_1,y_1)\,|J_1|$$

$$= \frac{r}{2\pi\sigma^2}\, e^{-\left[\frac{(r\cos\psi - A\cos\theta)^2 + (r\sin\psi - A\sin\theta)^2}{2\sigma^2}\right]}$$

$$= \frac{r}{2\pi\sigma^2}\left[e^{-\left(\frac{r^2+A^2}{2\sigma^2}\right)} \cdot e^{\frac{2Ar(\cos\theta\cos\psi + \sin\theta\sin\psi)}{2\sigma^2}}\right.$$

$$f_{R,\psi}(r,\psi) = \frac{r}{\sigma^2}\, e^{-\frac{(r^2+A^2)}{2\sigma^2}} \cdot \frac{1}{2\pi}\, e^{\frac{Ar}{\sigma^2}\cos(\psi-\theta)}$$

$$f_R(r) = \int_0^{2\pi} f_{R,\psi}(r,\psi)\, d\psi$$

$$= \frac{r}{\sigma^2}\, e^{\frac{-(r^2+A^2)}{2\sigma^2}} \cdot \frac{1}{2\pi}\int_0^{2\pi} e^{\frac{Ar}{\sigma^2}\cos(\psi-\theta)}\, d\psi$$

$$\Rightarrow \phi = \psi - \theta$$

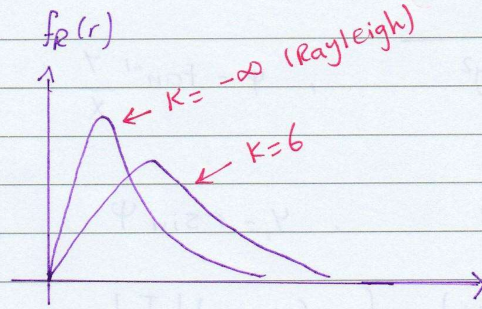
$$f_R(r) = \frac{r}{\sigma^2}\, e^{\frac{-(r^2+A^2)}{2\sigma^2}} \cdot \frac{1}{2\pi}\int_{-\theta}^{2\pi-\theta} e^{\frac{Ar}{\sigma^2}\cos\phi}\, d\phi$$

$$f_R(r) = \frac{r}{\sigma^2}\, e^{\frac{-(r^2+A^2)}{2\sigma^2}} \cdot I_0\!\left(\frac{Ar}{\sigma^2}\right)$$

where $I_0(x) = \frac{1}{2\pi}\int_0^{2\pi} e^{x\cos\theta}\, d\theta$ ← Zero-order modified Bessel fn. of 1st kind.

"Ricean Fading"   4-12-2017

$$K\,dB = 10 \log \frac{A^2}{2\sigma^2}$$



* for $K >> 1$: gaussian around the mean

" Spectral Shape due to Doppler Spread (Clarks Model)

$f_m = \frac{v}{\lambda}$, A: Average Rx Power

$f_d = f_m \cos\alpha$

$G(\alpha)$: Antenna gain as a fn. of angle.

$\Rightarrow \quad f = f_c + f_m \cos\alpha$

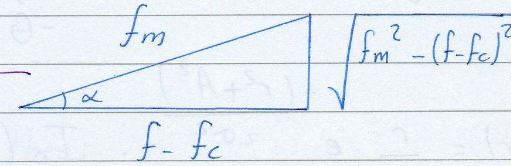
$$S(f)\,|df| = A\left[P(\alpha)G(\alpha) + P(-\alpha)G(-\alpha)\right]|d\alpha|$$

$$\frac{df}{d\alpha} = -f_m \sin\alpha$$

$$S(f)(-f_m \sin\alpha)\,|\cancel{d\alpha}| = A\left[P(\alpha)G(\alpha) + P(-\alpha)G(-\alpha)\right]|\cancel{d\alpha}|$$

$$S(f) = \frac{A\left[P(\alpha)G(\alpha) + P(-\alpha)G(-\alpha)\right]}{f_m\,|\sin\alpha|}$$

$$\sin\alpha = \frac{\sqrt{f_m^2 - (f-f_c)^2}}{f_m} \longleftarrow$$



$$S(f) = \frac{A\left[P(\alpha)G(\alpha) + P(-\alpha)G(-\alpha)\right]}{f_m\sqrt{1 - \left(\frac{f-f_c}{f_m}\right)^2}}, \quad f_c - f_m \le f \le f_c + f_m$$

"Clarks Model" 4.12.2017

$$\Rightarrow \alpha = \cos^{-1}\left(\frac{f-f_c}{f_m}\right)$$

Assume: $A=1$, $\frac{\lambda}{4}$ antenna, $G(\alpha)=1.5$, $P(\alpha)=\frac{1}{2\pi}$

$$S(f) = \frac{1.5}{\pi f_m \sqrt{1-\left(\frac{f-f_c}{f_m}\right)^2}}$$

σ_τ $\quad$ $1/50\sigma_\tau$

* Time Spreading $\rightarrow$ Frequency Selectivity , $\sigma_\tau \leq 0.1\, T_s$

* Frequency Spreading (Doppler) $\rightarrow$ Time Selectivity (Random AM)

"Level Crossing Rate (LCR)"

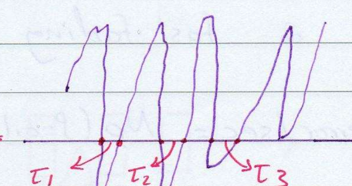
$$N_R = \sqrt{2\pi}\, f_m\, \rho\, e^{-\rho^2}$$

$f_m = \frac{v}{\lambda}$, $\rho = \frac{R}{R_{rms}}$ $R \leftarrow$ Amplitude, $R_{rms} \leftarrow$ rms value of Rayleigh.

"Average fade duration" ($\bar{\tau}$ or AFD)

is the average time for which the Rx signal remains below a specified level R.

$$\bar{\tau} = \frac{\Sigma \tau_i \text{ (over a period-}T\text{)}}{\text{No. of crossings in period-}T}$$

For this figure: 

$$\bar{\tau} = \frac{\tau_1 + \tau_2 + \tau_3}{3}$$

* No. of crossing in $(T) = N_R T$

$$\bar{\tau} = \frac{\sum \tau_i}{N_R T} \rightarrow P_r(r \leq R) = 1 - e^{\frac{-r^2}{2\sigma^2}}$$

$$\Rightarrow \bar{\tau} = \frac{P_r(r \leq R)}{N_R} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}$$

Ex] Find the LCR for $\rho = 1$ for Rayleigh fading when $f_m = 20\,hz$

$$N_R = \sqrt{2\pi}\, f_m \rho e^{-\rho^2} = \sqrt{2\pi}(20)(1)e^{-1} = 18.44 \text{ Cross/sec.}$$

Ex] Find the average fade duration ($\bar{\tau}$ or AFD), $f_m = 200\,hz$ for $\rho = 0.01, 0.1, 1$.

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}$$

$\Rightarrow \rho = 0.01 \rightarrow \bar{\tau} = 19.9\ \mu sec$

$\rho = 0.1 \rightarrow \bar{\tau} = 200\ \mu sec.$

$\rho = 1 \rightarrow \bar{\tau} = 3.43\ msec.$

Ex] $f_m = 20\,Hz$

a) Find $\bar{\tau}$ for $\rho = 0.707$

b) For 50 bit/sec is Rayleigh fading fast or slow.

c) Find the bit error rate assuming that each drop below R causes an error. $(\rho = 0.1)$

a) $\bar{\tau} = \dfrac{e^{0.707^2} - 1}{0.707 * 20\sqrt{2\pi}} = 18.3\ msec.$

b) $T_s = \dfrac{1}{50} = 20\ msec.$

* in general $T_s < T_c\ (50\%)$ ∴ slow

$T_s > \bar{\tau}$ ∴ fast fading

c) No. of error/sec $= N_R(\rho = 0.1) = \sqrt{2\pi}(20)(0.1)e^{-(0.1)^2}$

$= 4.96 \dfrac{\text{Cross}}{\text{sec}} = 4.96 \dfrac{\text{error}}{\text{sec}}$

$P_{err} = \dfrac{4.96}{50} \times 100\% \approx 10\%$

"LCR & AFD" 6-12-2017

Ex] $f_c = 860$ MHZ , $v = 100$ Km / hour
Find the level crossing rate and AFD for a signal to be 20 dB below RMS.

$$\rho_{dB} = 20 \log \rho = -20$$

$$\rho = 10^{-20/20} = 10^{-1} = 0.1 \quad , \quad \lambda = \frac{3\times10^8}{860\times10^6} = 0.349 \text{ m.}$$

$$v = \frac{100\times10^3}{3600} = 27.78 \text{ m/s}$$

$$\Rightarrow f_m = \frac{v}{\lambda} = 79.6 \text{ HZ}$$

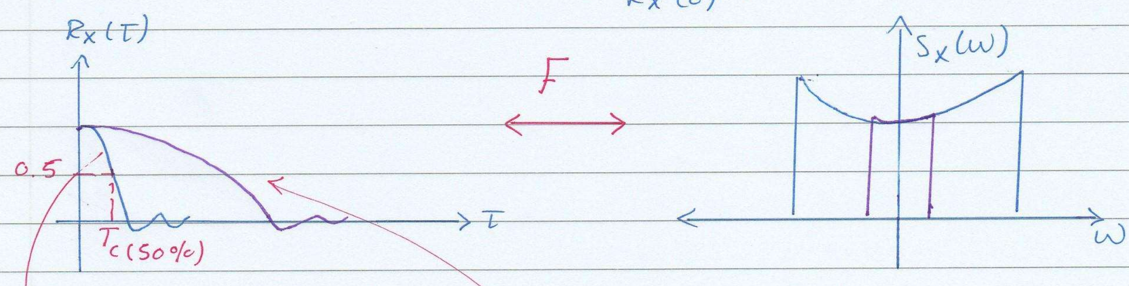
$$N_R = \sqrt{2\pi}\, f_m\, \rho\, e^{-\rho^2} = 19.7 \text{ cross/sec.}$$

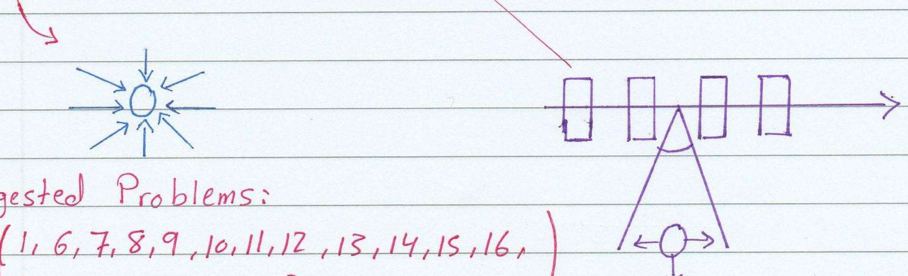
$$\bar{\tau} = 0.5 \text{ msec.}$$

"Coherence Distance (D_c)"
is the separating distance in space over which channel appears unchanged.

$$D_c = v * T_c = v * \left(\frac{9}{16\pi f_m}\right) = v\left(\frac{9}{16\pi \frac{v}{\lambda}}\right) = \frac{9\lambda}{16\pi} \approx 0.2\lambda$$

$$T_c(50\%) = \tau \text{ for which } \left|\frac{R_x(\tau)}{R_x(0)}\right| = 50\%$$





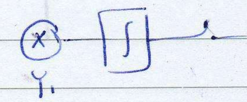
Suggested Problems:
5.(1, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 27, 28, 29, 30, 32)

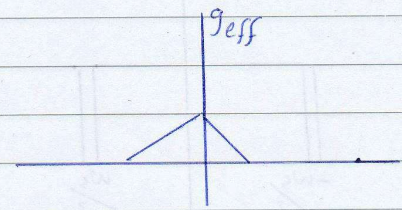
"Pulse Shaping Techniques" 6_12_2017

$$S(t) = g(t) \cdot \cos\left(w_c t + \frac{2\pi}{M} K\right) \quad , \quad K = 1, 2, \ldots M$$

* Choice of $g(t)$:

1) Bandwidth requirement
2) Minimize ISI.

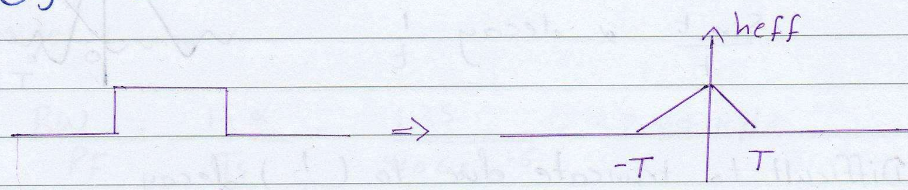


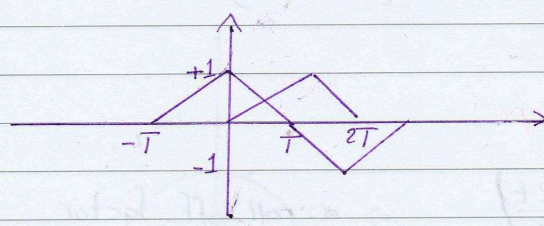


11_12_2017

$x(t) = \sum I_n g(t - nT)$, $g(t)$: pulse shape.

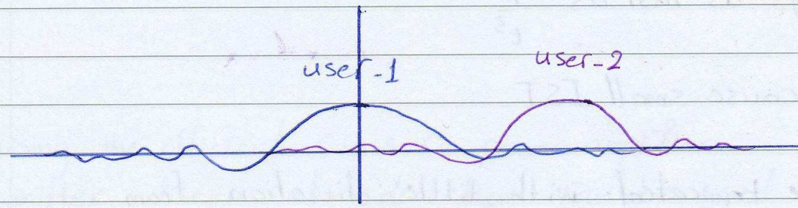
$h_{eff} = g(t) \circledast g(T - t)$





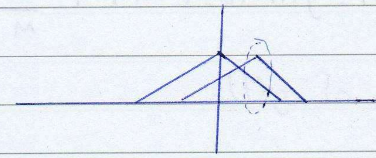
* Disadvantages of rectangular pulse shape:

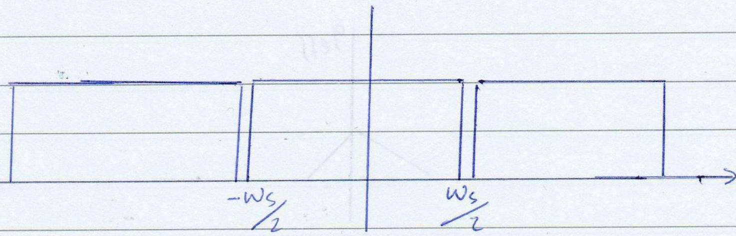
1) Small timing error causes large ISI
2) PSD has large out - of - main lobe power.



"Nyquist Criteria for ISI Cancellation" 11-12-2017

$$h_{eff}(t = nT_s) = \begin{cases} K & , n = 0 \\ 0 & , n \neq 0 \end{cases}$$

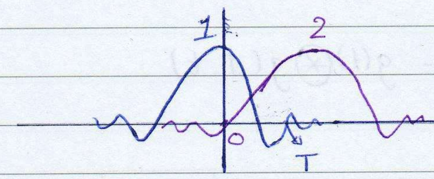




Disadvantages:

1) High ISI if there is timing error:

$\frac{\sin t}{t} \propto$ decay $\frac{1}{t}$



2) Difficult to truncate due to $\left(\frac{1}{t}\right)$ decay.

"Raised Cosine Pulse Shape"

$$h_{RC}(t) = \frac{\sin\left(\frac{\pi t}{T_s}\right)}{\pi t} \cdot \frac{\cos\left(\frac{\pi \alpha t}{T_s}\right)}{1 - \left(\frac{4\alpha t}{2T_s}\right)^2}$$ $\Rightarrow \alpha$: roll-off factor

Advantages:

1) $h_{eff}(t)$ decays as fast as $\frac{1}{t^3}$

2) Timing jitter cause small ISI

3) $h_{eff}(t)$ can be truncated with little diviation from ideal performance.

" Raised Cosine Pulse Shape" 11-12-2017.

$$BW_{B.B} = \frac{1+\alpha}{2\,T_s}$$ ← symbol period.

B.B: base band

$$BW_{RF} = \frac{1+\alpha}{T_s}$$

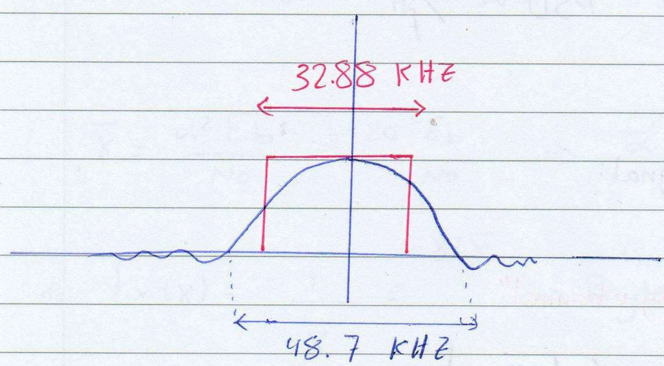
Ex) Find the zero-crossing RF. BW for a rectangular pulse shape compared to Raised-Cosine pulse shape with $T_s = 41.06\ \mu S$, $\alpha = 0.35$

Rectangular:

$$BW_{R.F}\ (\text{null-to-null}) = \frac{2}{T_s} = \frac{2}{41.06 \times 10^{-6}} = 48.7\ KHZ$$

Raised Cosine:

$$BW_{RF} = \frac{1+\alpha}{T_s} = \frac{1.35}{41.06 \times 10^{-6}} = 32.88\ KHZ.$$



" Constant Envelope Modulations"

Advantages:

1) using the efficient Class-C power amplifier
2) Limitter discriminator detection can be used with high immunity against fading.

"Minimum Shift Keying" 18.12.2017

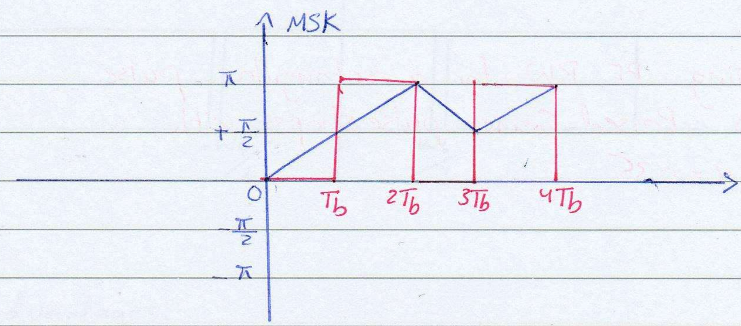
(MSK)

$$S(t) = \sqrt{\frac{2E_b}{T}} \cos\left(2\pi f_c t + \theta(0) \pm \frac{\pi}{2T_b} t\right)$$

data = 1

data = 0

$$0 \leq t \leq T_b$$



$$f_i(t) = \frac{1}{2\pi} \frac{d\theta}{dt}$$

$$= \frac{1}{2\pi}\left[2\pi f_c \pm \frac{2\pi}{4T_b}\right] = f_c + \frac{1}{4T_b} \rightarrow \frac{1}{4} \text{ bit rate}$$

$$PSD \propto 1/f^4$$

$\Rightarrow$ GMSK $\rightarrow$ $ISI < ISI_{signal}$

"Prob. of error in slow flat fading channel"

BPSK $\Rightarrow$ Pe_{BPSK} (Gaussian) $= Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$

For Rayleigh:

$\Rightarrow$ $r(t) = \alpha e^{-j\theta} S(t) + n(t)$

Rayleigh R.V.

$$P_{e_{BPSK}}\left(\underset{\alpha}{\text{specific}}\right) = Q\left(\sqrt{\frac{2\alpha^2 E_b}{N_0}}\right)$$

$$P_{e,BPSK}(\text{Rayleigh}) = E\left[Q\left(\sqrt{\frac{2\alpha^2 E_b}{N_0}}\right)\right]$$

$\Rightarrow$ assume $\gamma \triangleq \frac{\alpha^2 E_b}{N_0} \Rightarrow P_e(\text{Rayleigh}) = \overline{Q(\sqrt{2\gamma})}$

$$P_{e,BPSK} = \overline{Q(\sqrt{2\gamma})} = \int_{\gamma=0}^{\infty} Q(\sqrt{2\gamma})\, f_\gamma(\gamma)\, d\gamma$$

$$\Rightarrow f_\gamma(\gamma) = \frac{f_\alpha(\alpha_1)}{|T'(\alpha_1)|}, \quad \alpha_1 = \sqrt{\frac{\gamma N_0}{E_b}}$$

$$T'(\alpha) = \frac{2\alpha E_b}{N_0}, \quad f_\alpha(\alpha) = \frac{\alpha}{\sigma^2} e^{-\alpha^2/2\sigma^2}$$

$$f_\gamma(\gamma) = \frac{\cancel{\alpha_1}}{\sigma^2} \frac{e^{-\alpha_1^2/2\sigma^2}}{2\cancel{\alpha_1} E_b/N_0} = \frac{1}{2\sigma^2 \cdot \frac{E_b}{N_0}} e^{-\frac{\gamma N_0}{E_b \cdot 2\sigma^2}}$$

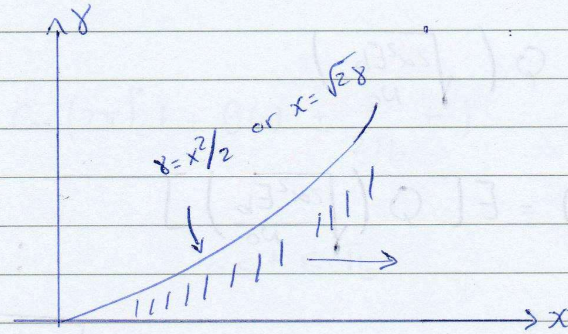
$$\overline{\gamma} = \frac{\overline{\alpha^2} E_b}{N_0} = \frac{2\sigma^2 E_b}{N_0} \Rightarrow \overline{\alpha^2} = 2\sigma^2$$

$$\therefore f_\gamma(\gamma) = \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}} \Rightarrow \text{Rayleigh.}$$

$$\Rightarrow P_e(\text{Ray}) = \int_{\gamma=0}^{\infty} Q(\sqrt{2\gamma})\, \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}}\, d\gamma$$

$$= \int_{\gamma=0}^{\infty} \left[\int_{\sqrt{2\gamma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx\right] \cdot \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}}\, d\gamma$$

"Prob. of error in a slow flat fading channel" 18.12.2017



$$P_e\,(\text{Rayleigh}) = \int_{x=0}^{\infty} \left[\int_{\gamma=0}^{x^2/2} \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}} \, d\gamma \right] \frac{e^{-x^2/2}}{\sqrt{2\pi}} \, dx$$

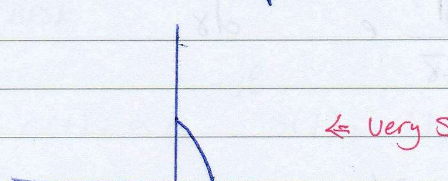
$$= \int_{x=0}^{\infty} \left[-e^{-\gamma/\bar{\gamma}} \right]_0^{x^2/2} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx$$

$$= \int_{x=0}^{\infty} \left[1 - e^{-x^2/2\bar{\gamma}} \right] \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx$$

$$= \int_{x=0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} \, dx - \int_{x=0}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2\left[\bar{\gamma}/(1+\bar{\gamma})\right]} \, dx$$

$$= \frac{1}{2} - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}} \int_{x=0}^{\infty} \frac{1}{\sqrt{2\pi \frac{\bar{\gamma}}{1+\bar{\gamma}}}} e^{-x^2/2\left[\frac{\bar{\gamma}}{1+\bar{\gamma}}\right]} \, dx$$

$$= \frac{1}{2} - \frac{1}{2} \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$$



⇐ very slow decay

"Prob. of error in a slow flat fading channel" 18.12.2017

DPSK Performance:-

$$P_{e,DPSK}(\text{Gaussian}) = \frac{1}{2} e^{-E_b/N_0}$$

$$P_{e,DPSK}(\text{Specific } \alpha) = \frac{1}{2} e^{-\alpha^2 E_b/N_0} = \frac{1}{2} e^{-\gamma}$$

$$P_{e,DPSK}(\text{Rayleigh}) = E\left[\frac{1}{2} e^{-\gamma}\right]$$

$$= \int_{\gamma=0}^{\infty} \frac{1}{2} e^{-\gamma} \cdot \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}} \, d\gamma$$

$$= \frac{1}{2\bar{\gamma}} \int_{\gamma=0}^{\infty} e^{-\gamma\left(1+\frac{1}{\bar{\gamma}}\right)} d\gamma$$

$$= \frac{1}{2\bar{\gamma}} \left[\frac{e^{-\gamma\left(1+\frac{1}{\bar{\gamma}}\right)}}{-\left(1+1/\bar{\gamma}\right)} \right]_0^{\infty}$$

$$= \frac{1}{2\cancel{\bar{\gamma}}} \left[\frac{\cancel{\bar{\gamma}}}{\bar{\gamma}+1} \right] = \frac{1}{2+2\bar{\gamma}}$$

$$\therefore \; P_{e,DPSK}(\text{Rayleigh}) = \frac{1}{2+2\bar{\gamma}}$$

"Spread Spectrum Modulation"

Modulation in which the modulated BW is much larger than the information BW.

1) Frequency Hopping SS
2) Direct sequence SS

} → Types for SS.

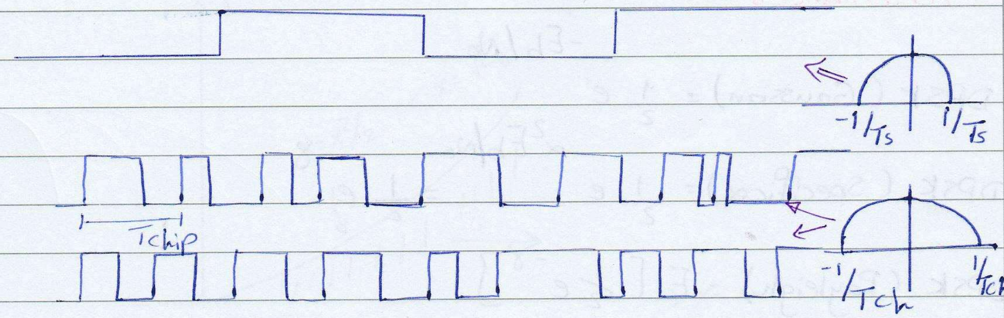
"S.S Modulation"

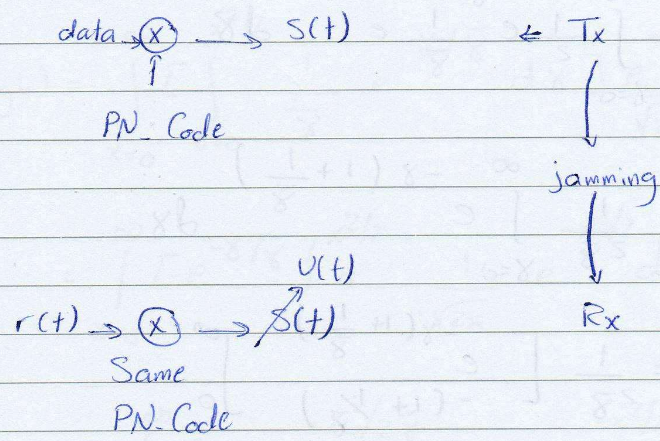
18.12.2017

→ data

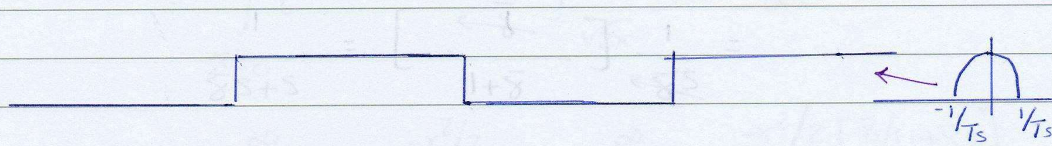
→ PN Code

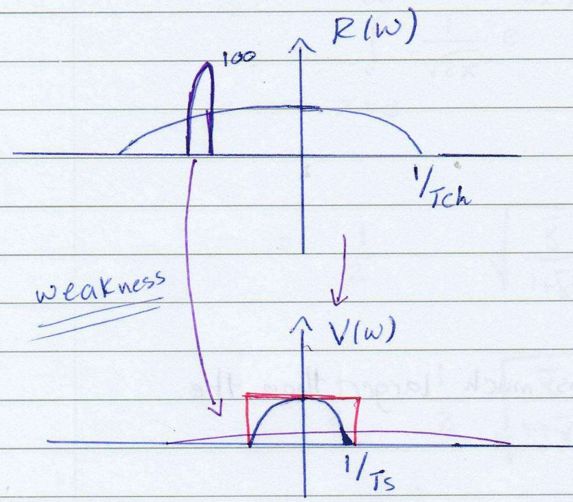
→ S(t)





→ U(t)





$$\Rightarrow \quad N = \left(\frac{T_{sym}}{T_{chip}}\right) = \frac{R_{chip}}{R_{sym}}$$

Process gain

KeskinColor

"RAKE Reciever"

20_12_2017

Resorvable Paths:

If $\tau_2 - \tau_1 > T_{chip}$

→ path 1 and path 2 are resolvable.

* 3G → 78 m , * 4G → 300 m

$$\rightarrow P_{e,\,CDMA,BPSK} = Q\left(\frac{1}{\sqrt{\frac{K-1}{3N} + \frac{N_o}{2E_b}}}\right)$$

K: No. of users

N: Process gain

No: Noise variance

Eb: Bit energy.

Ex) IS_95 , 2G , K = 20 users

Chip rate = 1.2288 $\frac{Mchip}{sec}$, Data rate = 13 Kbit/sec.

Eb/No = 7.8 dB , using BPSK Find Pe ?

$$E_b/N_o = 10^{7.8/10} = 6.02 \quad , \quad N = \frac{Chip\,Rate}{Data\,Rate} = \frac{T_{bit}}{T_{chip}} = \frac{1228800}{13000}$$

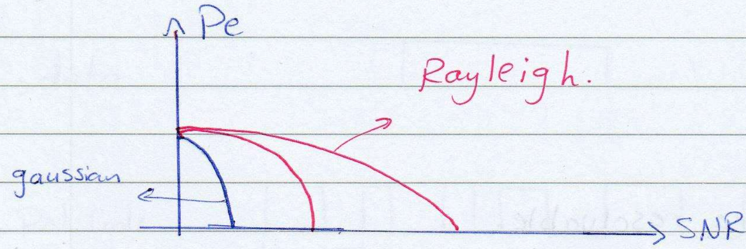
$N = 94$

$$P_e = Q\left(\frac{1}{\sqrt{\frac{20-1}{3N} + \frac{N_o}{2E_b}}}\right)$$

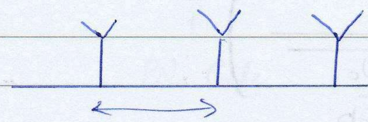
$$P_e = Q\left(\frac{1}{\sqrt{\frac{20-1}{3\times 94} + \frac{1}{2\times 6.02}}}\right) = 0.0049$$

"Diversity Techniques"

20.12.2017



1) Space or Antenna Diversity



Spacing > $\lambda/2$

Disadvantages:
Difficult to implement in mobile stations

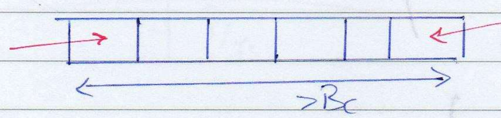
2) Polarization Diversity

H & V

Disadvantages:
Only Two branches.

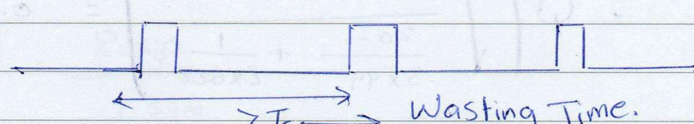
3) Frequency Diversity

Sending the same signal on multiple carriers.



Disadvantages: BW inefficient.

4) Time diversity



Wasting Time.

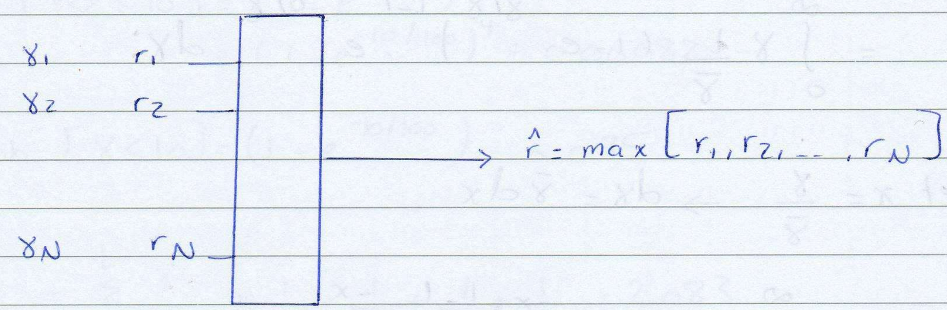
5) Rake Reciever

"Diversity Combining Techniques" 20-12-2017

1) Selective Combines :- (SC)

Rx select the diversity branch with high SNR,

$f_\gamma(\gamma)$



$$\gamma = \frac{\alpha^2 E_b}{N_0}$$

$$f_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}} \quad \text{(Rayleigh)}$$

$$\text{Prob.}[\gamma_{sc} \le \gamma] = F_{\gamma_{sc}}(\gamma)$$

$$\gamma_{sc} = \text{Max}[\gamma_1, \gamma_2, \ldots \gamma_L]$$

$$F_{\gamma_{sc}}(\gamma) = P[\gamma_1 \le \gamma] \cdot P[\gamma_2 \le \gamma] \ldots \cdot P[\gamma_L \le \gamma]$$

$$F_{\gamma_{sc}}(\gamma) = F_{\gamma_1}(\gamma) \cdot F_{\gamma_2}(\gamma) \ldots \cdot F_{\gamma_L}(\gamma)$$

For Rayleigh :

$$F_{\gamma_{sc}}(\gamma) = (1 - e^{-\gamma/\bar{\gamma}_1})(1 - e^{-\gamma/\bar{\gamma}_2}) \ldots (1 - e^{-\gamma/\bar{\gamma}_L})$$

Assuming identical L branches for Rayleigh:

$$F_{\gamma_{sc}}(\gamma) = (1 - e^{-\gamma/\bar{\gamma}})^L$$

$$f_{\gamma_{sc}}(\gamma) = L(1 - e^{-\gamma/\bar{\gamma}})^{L-1}(-e^{-\gamma/\bar{\gamma}})(^{-1}/_\gamma)$$

$$f_{\gamma_{sc}}(\gamma) = \frac{L}{\gamma}(1 - e^{-\gamma/\bar{\gamma}})^{L-1}(e^{-\gamma/\bar{\gamma}})$$ ← Ray. All identical.

Mean value : $\bar{\gamma}_{sc}$

$$\bar{\gamma}_{sc} = E(\gamma_{sc}) = \int_0^\infty \gamma f_{sc}(\gamma) d\gamma$$

$$= \int_0^\infty \gamma \frac{L}{\bar{\gamma}} (1-e^{-\gamma/\bar{\gamma}})^{L-1} . e^{-\gamma/\bar{\gamma}} d\gamma$$

Let $x = \frac{\gamma}{\bar{\gamma}} \rightarrow d\gamma = \bar{\gamma} dx$

$$\bar{\gamma}_{sc} = \int_0^\infty Lx (1-e^{-x})^{L-1} e^{-x} \bar{\gamma} dx$$

$$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$$

$$\frac{\bar{\gamma}_{sc}}{\bar{\gamma}} = \int_0^\infty Lx \left(\sum_{l=0}^{L-1} (-1)^l \binom{L-1}{l} e^{-xl} \right) e^{-x} dx$$

$$= \sum_{l=0}^{L-1} L (-1)^l \binom{L-1}{l} \int_0^\infty x e^{-x(l+1)} dx$$

$$\rightarrow \frac{1}{(1+l)^2}$$

$$\frac{\bar{\gamma}_{sc}}{\bar{\gamma}} = \sum_{l=0}^{L-1} (-1)^l \binom{L-1}{l} \frac{L}{(1+l)^2} = \sum_{k=1}^{L} \frac{1}{k}$$

Ex) 4_branches Rayleigh (identical) Average SNR = 20dB
Find Prob. [SNR ≤ 10 dB] for :_

a) with L = 4 selective Combining
b) without diversity
c) Find $\bar{\gamma}_{sc}$ for L = 4

"Selective Combines" 20-12-2017

a) $\bar{\gamma} = 10^{20/10} = 100$

$\bar{\gamma}_{th} = 10^{10/10} = 10$

$$\text{prob.}[\gamma_{sc} \leq 10] = (1 - e^{-\gamma/\bar{\gamma}})^L$$
$$= (1 - e^{-10/100})^4 = 0.00082$$

b) $$\text{Prob.}[\gamma \leq 10] = (1 - e^{-10/100}) = 0.095$$

c) $$\frac{\bar{\gamma}_{sc}}{\bar{\gamma}} = \sum_{k=1}^{L} \frac{1}{k} = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} = 2.083$$

$$\bar{\gamma}_{sc} = 2.083 * 100 = 208.3$$

" Prob. Of Error of DPSK with SC" :-

$$P_{e,DPSK}(\text{Gaussian}) = \frac{1}{2} e^{-\gamma}, \quad \gamma = E_b/N_o$$

$$P_{e,DPSK}(\text{with SC}) = \int_{\gamma=0}^{\infty} \frac{1}{2} e^{-\gamma} f_{\gamma_{sc}}(\gamma) d\gamma$$

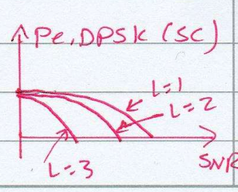
$$= \int_{\gamma=0}^{\infty} \frac{1}{2} e^{-\gamma} \left(\frac{L}{\bar{\gamma}}\right)(1 - e^{-\gamma/\bar{\gamma}})^{L-1} e^{-\gamma/\bar{\gamma}} d\gamma$$

Let $x = \gamma/\bar{\gamma} \rightarrow d\gamma = \bar{\gamma}\, dx$

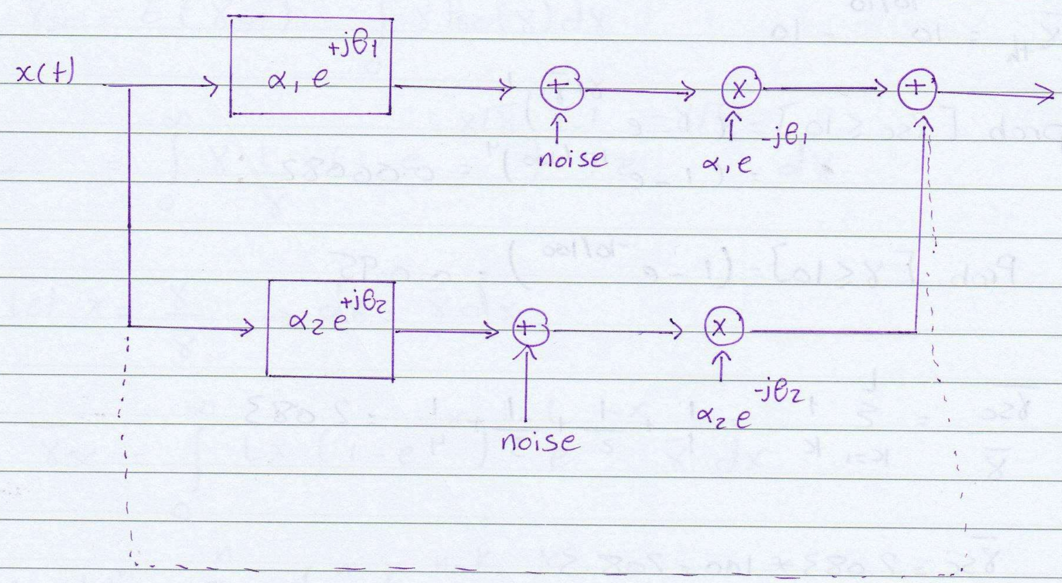
$$P_{e,DPSK}(\text{with SC}) = \frac{L}{2} \int_0^{\infty} e^{-x(\bar{\gamma}+1)} (1 - e^{-x})^{L-1} dx$$

$$P_e = \frac{L}{2} \sum_{l=0}^{L-1} (-1)^l \binom{L-1}{l} \left[\frac{e^{-x(l+\bar{\gamma}+1)}}{-(l+\bar{\gamma}+1)}\right]_{\gamma=0}^{\infty}$$

$$P_{e,DPSK}(SC) = \frac{L}{2} \sum_{l=0}^{L-1} (-1)^l \binom{L-1}{l} \left(\frac{1}{1+l+\bar{\gamma}}\right)$$



2) Maximum Ratio Combining (MRC).



$$\gamma_{MRC} = \gamma_1 + \gamma_2 + \dots + \gamma_L$$

$$\bar{\gamma}_{MRC} = \bar{\gamma}_1 + \bar{\gamma}_2 + \dots + \bar{\gamma}_L$$

$$\bar{\gamma}_{MRC} = L\bar{\gamma} \text{ (identical)}$$

$$f_{\gamma_{MRC}}(\gamma) = f_{\gamma_1}(\gamma) \circledast f_{\gamma_2}(\gamma) \dots \circledast f_{\gamma_L}(\gamma)$$

→ Assuming L. identical Rayleigh

$$f_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}}$$

$$\phi_\gamma(w) = E[e^{jw\gamma}] = \int_{\gamma=0}^{\infty} e^{jw\gamma} \cdot \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}} \, d\gamma$$

$$= \frac{1}{\bar{\gamma}} \int_0^{\infty} e^{\gamma(jw - 1/\bar{\gamma})} \, d\gamma$$

"Maximum Ratio Combining"

20-12-2017

$$\phi_\gamma(w) = \frac{1}{\bar{\gamma}} \cdot \frac{e^{\gamma(jw - \frac{1}{\bar{\gamma}})}}{jw - 1/\bar{\gamma}} \Bigg]_0^\infty = \frac{1}{1 - jw\bar{\gamma}}$$

$$\phi_{\gamma_{MRC}}(w) = [\phi_\gamma(w)]^L = \frac{1}{(1 - jw\bar{\gamma})^L}$$

$$t^n \cdot e^{-at} u(t) \overset{F}{\longleftrightarrow} \frac{n!}{(a+jw)^{n+1}}$$

$$\gamma^n \cdot e^{-a\gamma} u(\gamma) \overset{F}{\longleftrightarrow} \frac{n!}{(a+jw)^{n+1}}$$

$$\phi_{\gamma_{MRC}}(\gamma) = \frac{1}{\bar{\gamma}^L (\frac{1}{\bar{\gamma}} - jw)^L} \Bigg]$$

$a = 1/\bar{\gamma}$

$L = n+1$

$n = L - 1$

$$\Leftrightarrow \frac{\gamma^{L-1} e^{-\gamma/\bar{\gamma}} u(\gamma)}{(L-1)!\ \bar{\gamma}^L}$$

$$\rightarrow f_{\gamma_{MRC}}(\gamma) = \frac{1}{(L-1)!(\bar{\gamma})^L} \gamma^{L-1} e^{-\gamma/\bar{\gamma}}, \quad \gamma \geq 0$$

$$F_{\gamma_{MRC}}(\gamma) = 1 - e^{-\gamma/\bar{\gamma}} \sum_{k=1}^{L} \frac{(\gamma/\bar{\gamma})^{k-1}}{(k-1)!}$$

"Diversity Combining Techniques" 27-12-2017

Ex] $L=6$, Div. system (Rayleigh) → identical, $\bar{\gamma} = 15\,dB$

Find :

a) The improvement in SNR for:

i) SC ii) MRC

b) Find Prob. [SNR ≤ 5] for:

i) MRC ii) SC iii) No div.

Solution:-

a) , i) $\frac{\bar{\gamma}_{sc}}{\bar{\gamma}} = \sum_{K=1}^{6} \frac{1}{K} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6}$

$= 2.45$

ii) $\bar{\gamma}_{MRC} = \sum_{i=1}^{6} \bar{\gamma}_i = 6\bar{\gamma}$

$\therefore \frac{\bar{\gamma}_{MRC}}{\bar{\gamma}} = 6 \rightarrow$ Better

b) for $\gamma = 5dB$ & $\bar{\gamma} = 15\,dB \Rightarrow \frac{\gamma}{\bar{\gamma}} = \frac{10^{5/10}}{10^{15/10}} = 0.1$

i) $P[\gamma_{MRC} < 5] = 1 - e^{-\gamma/\bar{\gamma}} \sum_{K=1}^{6} \frac{(\frac{\gamma}{\bar{\gamma}})^{K-1}}{(K-1)!} = 1.27 \times 10^{-9}$

ii) $P[\gamma_{SC} < 5] = (1 - e^{-\gamma/\bar{\gamma}})^6 = 7.4267 \times 10^{-7}$

iii) $P[\gamma_{No.Div.} < 5] = 1 - e^{-\gamma/\bar{\gamma}} = 0.095$

" Pe, MRC (BPSK) "

$P_{e,MRC}\,(BPSK) = E(Q(\sqrt{2\gamma}))$

$$= \int_{\gamma=0}^{\infty} Q(\sqrt{2\gamma})\, \frac{1}{(L-1)!\,(\bar{\gamma})^L}\, .\gamma^{L-1}\, e^{-\gamma/\bar{\gamma}}\, d\gamma$$

"Prob. of Error in a MRC Div for a BPSK" 27-12-2017

$$P_{e,MRC}(BPSK) = \left(\frac{1-M}{2}\right)^L \cdot \sum_{K=0}^{L-1} \binom{L-1+K}{K} \left(\frac{1+M}{2}\right)^K$$

where $M = \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}$

Discrete R.V.

Ex| Find $P_{e,BPSK}(MRC)$ if $L=2$, $f_\alpha(\alpha) = 0.1\,\delta(\alpha-0.05) + 0.9\,\delta(\alpha-1)$

and $\gamma = \frac{\alpha^2 E_b}{N_0}$

Sol:-

$$P_{e,BPSK}(\text{Gaussian}) = Q(\sqrt{2\gamma})$$

$$f_\gamma(\gamma) = 0.1\,\delta\left(\alpha - 0.0025\frac{E_b}{N_0}\right) + 0.9\,\delta\left(\gamma - \frac{E_b}{N_0}\right)$$

$$f_{\gamma_{MRC}}(\gamma) = f_{\gamma_1}(\gamma) \otimes f_{\gamma_2}(\gamma)$$

$$= 0.01\,\delta\left(\gamma - 0.005\frac{E_b}{N_0}\right) + 0.81\,\delta\left(\gamma - \frac{E_b}{N_0}\right) + 0.18\,\delta\left(\gamma - 1.0025\frac{E_b}{N_0}\right)$$

$$P_{e,BPSK}(MRC) = E\left[Q(\sqrt{2\gamma})\right]$$

$$P_{e,BPSK}(MRC) = 0.01\,Q\left(\sqrt{\frac{0.01E_b}{N_0}}\right) + 0.81\,Q\left(\sqrt{\frac{2E_b}{N_0}}\right) + 0.18\,Q\left(\sqrt{\frac{2.005E_b}{N_0}}\right)$$