Chapter - 3
The Cellular Concept

Early design of mobile radio systems used high power TX over tall towers giving excellent coverage but cannot reuse the same frequency only at very distant areas.

The cellular concept was a major break through in solving the freq. reuse problem to increase system capacity.

Freq. Reuse

A large geographical area is divided into a no. of small areas (cells), each cell is allocated a subset of frequencies (channels), provided there is a sufficient separating distance between cells using the same frequency to minimize interference.

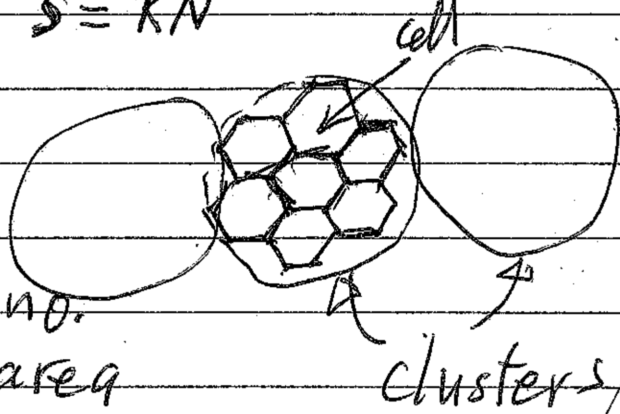
Base stations use lower power just sufficient to cover users in the cell and thus minimizing interference to neigboring cells having similar frequency.

Fig. (3.1)

Define:

- S: total no. of duplex channels available for use by the cellular system (all cells).
- k: No. of channels allocated to each cell.
- N: Cluster size (no. of cells in each cluster) (A cluster uses all S-channels)
- M: No. of replications of a cluster in a certain geographic area.

$\therefore \boxed{k = \frac{S}{N}}$ OR $S = kN$



The capacity C of a geographic area, i.e the no. of users served in the area

$$\boxed{C = MS} = \boxed{MkN}$$

$$M = \frac{\text{Area (city)}}{\text{Area (cluster)}}$$

$$\boxed{\text{Freq. Reuse factor} = 1/N}$$

$$\boxed{N - \text{typically} = 4, 7, 12}$$

ex) Let $N = 7$ cells/cluster
$k = 18$ channels/cell
$M = 3$ no. of cluster replicas in a geographical area.

Find capacity C of this area

$S = kN = 18 \times 7 = 126$ total no. of channel in syst

$C = MS = 3 \times 126 = 378$ user in this geographic area

Given a fixed geographic area with fixed no. of channels (S) and fixed cell size

If $N\uparrow$ then the cluster area increases proportionally
$\Rightarrow$ No. of replicas $M = \left(\dfrac{\text{total area}}{\text{cluster area}}\right) \Rightarrow$ decreases

$\therefore$ capacity $C (= MS)$ decreases

But SIR $\uparrow$ (SIR: signal-to-interference power ratio)

$$\boxed{\begin{array}{l} N\uparrow \Rightarrow SIR\uparrow \Rightarrow \text{But } C\downarrow \\ \hline N\downarrow \Rightarrow SIR\downarrow \Rightarrow \text{But } C\uparrow \end{array}}$$

ex) A cellular system has a total of (1001) channels for use, with a cell size of ($6\ Km^2$).
A geographical area of ($2100\ km^2$) is to be served by this system.
Find the no. of channels/cell and ~~the total~~ system capacity if
a) cluster size $N = 7$
b) " " $N = 4$

First we find M
No. of replicas

Solution
Given: $S = 1001$, $A_{cell} = 6\ Km^2$,
$A_{area} = 2100\ Km^2$, $A_{cluster} = N A_{cell}$

a) No. of channels / cell $K = S/N = \frac{1001}{7} = 143 \left(\frac{\text{chan.}}{\text{cell}}\right)$

$A_{cluster} = 7 \times A_{cell} = 7 \times 6 = 42\ Km^2$

$M\left(\begin{matrix}\text{No. of} \\ \text{replicas}\end{matrix}\right) = \frac{A_{area}}{A_{cluster}} = \frac{2100}{42} = 50$

∴ Capacity $C = MKN = 50 \times 143 \times 7$
$= 50050$ users

b) $N = 4$

$K = \frac{1001}{4} \simeq 250$ ch./cell

$A_{cluster} = N\,A_{cell} = 4 \times 6 = 24\ Km^2$

∴ $M = \frac{2100}{24} \simeq 87$

∴ $C = MKN = 87 \times 250 \times 4 = 87000$ users

∴ Reducing cluster size (N) caused an increase in overall capacity (C) but at the price of lower (SIR).

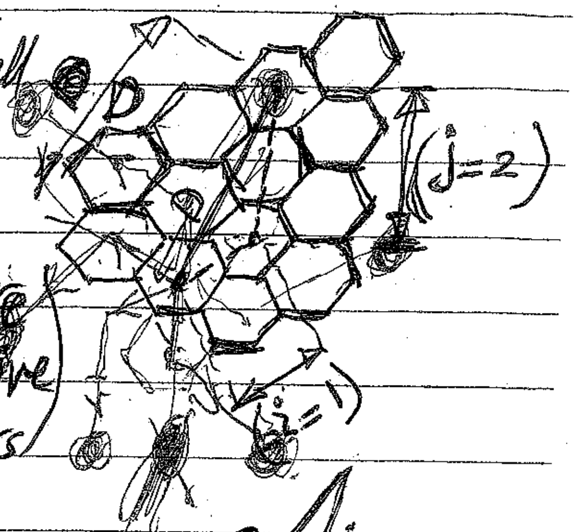
Geometry of Hexagonal Cells

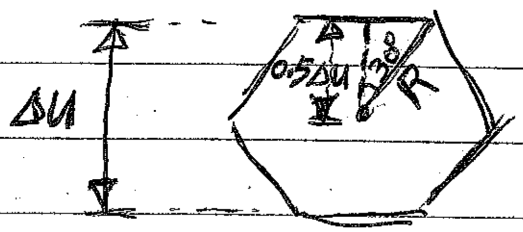
To find the nearest co-channel cell

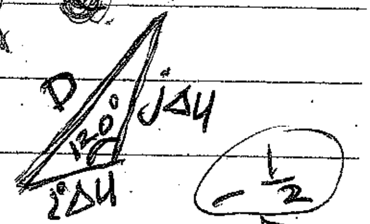
1- Move i-cells along any chain of hexagons.

2- Turn 60° counter clockwise and move j-cells.

(i, j: are non-negative integers)







$$\frac{\Delta u/2}{R} = \cos 30 = \frac{\sqrt{3}}{2}$$

$$\boxed{\Delta u = \sqrt{3}\, R}$$

$$D^2 = (i\Delta u)^2 + (j\Delta u)^2 - 2\, i\Delta u\, j\Delta u \cos 120$$

$\cos 120 = -\frac{1}{2}$

$$\boxed{D^2 = \left(i^2 + j^2 + ij\right)\Delta u^2}$$

To find the no. of cells in a cluster

A candidate cell has 6-nearest co-channel cells. Form a Large hexagon by joining the centers of the 6-nearest neighboring cochannel cells.

Area of Hexagon $= \underset{\text{a constant}}{K}(\text{radius})^2$

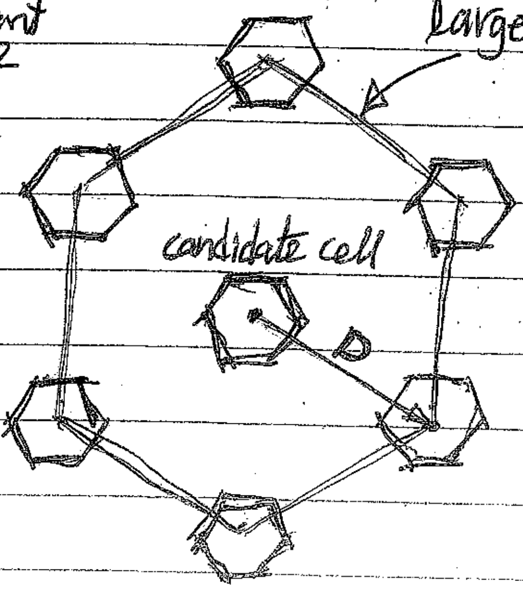
$A_{Large\text{-}Hex} = KD^2$

$= K(i^2 + j^2 + ij)\Delta u^2$

$A_{small\text{-}Hex} = KR^2$

No. of cells in Large Hex $= \dfrac{KD^2}{KR^2}$

$= D^2/R^2$



No. of cells in Large Hex. $= \dfrac{(i^2+j^2+ij)\,\Delta u^2}{R^2} = \dfrac{(i^2+j^2+ij)\,3R^2}{R^2}$

~~No. of~~ $= 3\,(i^2+j^2+ij)$

But geometrically, the large Hexagon encloses 3-candidate cells (sum of all portions of it) + 3-cells of every other different freq. cell $= 3N$

$\therefore\ 3\,(i^2+j^2+ij) = 3N$

$$\boxed{N = i^2+j^2+ij}$$

i, j : Non-negative integers

This is the cluster size if (i, j) represent the shifts specified for cochannel shifts

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Fig.(3.2)

ex) If a total BW = 33 MHz is allocated to a cellular system. System uses (25 KHz) simplex channels.

Find the number of channels/cell (= K) for
a) $N=4$ b) $N=7$

channel BW $= 2\times 25 = 50$ KHz

Total available channels $= S = \dfrac{33\times 10^3}{50} = 660$ channels

a) $K = S/N = 660/4 \simeq 165$ channels/cell

b) $K = 660/7 \simeq 95$ channels/cell

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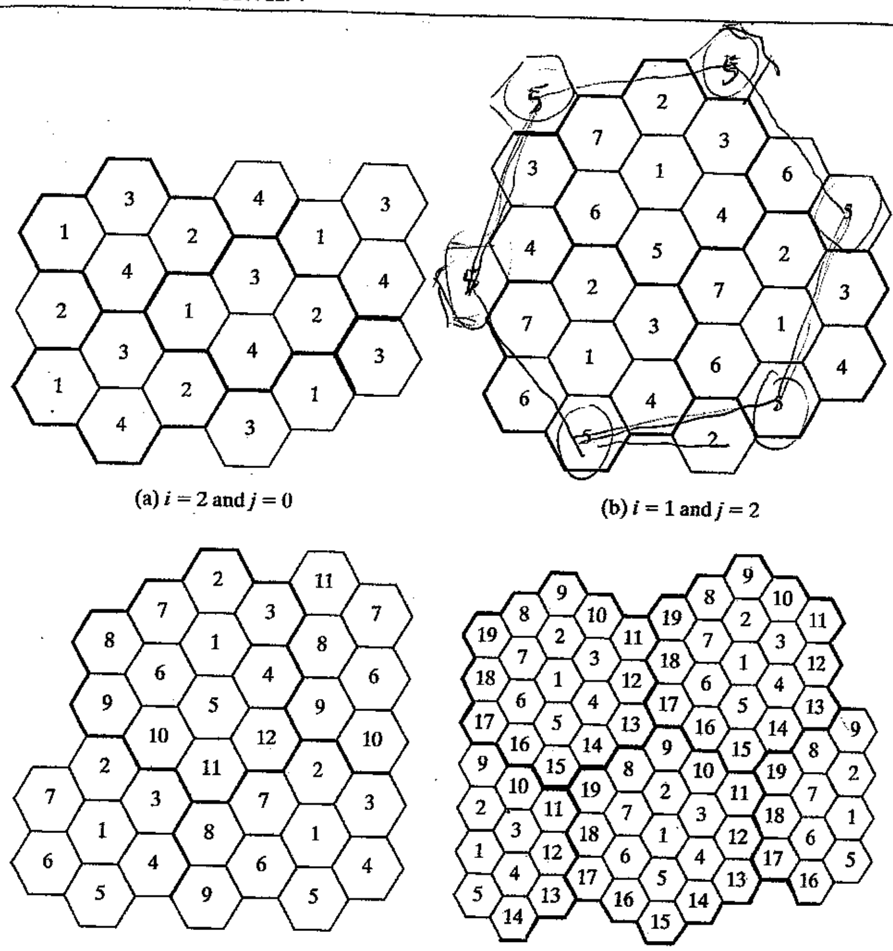

(a) $i = 2$ and $j = 0$

(b) $i = 1$ and $j = 2$

(c) $i = 2$ and $j = 2$

(d) $i = 2$ and $j = 3$

Figure 5.2 Cell clusters.

a. Calculate the total available channels,
b. Determine the number of control channels,
c. Determine the number of voice channels per cell, and
d. Determine an equitable distribution of control channels and voice channels in each cell.

Solution Given:

Total bandwidth = 30 MHz
Channel bandwidth = 25 kHz × 2 = 50 kHz/duplex channel

a. The total number of available channels = $\frac{30000}{50} = 600$.

Channel Assignment strategies

1- Fixed channel assignment

* Each cell is allocated a predetermined set of voice channels.
* If all cell channels are used, a new call request will be blocked (No service).
* In a "borrowing strategy" a cell is allowed to borrow unused channels from neighboring cells supervised by MSC.

2- Dynamic channel assignment

* No permenant channel allocation.
* when a call request is made, MSC assigns a channel to this cell, provided that it is not being used within min. reuse distance.
* Reduces Likelihood of call blocking.
* Increases complexity of MSC.

Handoff strategies

When mobile moves into a new cell during a conversation, MSC automatically transfers the call to a new channel of the new BS.

* Handoff is prioritized over call initiation.
* Must be imperceptible to users.

$\Delta = P_r(\text{handoff}) - P_r\left(\begin{matrix}\text{min.}\\ \text{acceptable}\end{matrix}\right)$

$\boxed{\text{Fig. (3.3)}}$

P_r : received power at BS

If Δ is too small $\Rightarrow$ P_r(handoff) is small $\rightarrow$ A call many be lost due to a weak signal

If Δ is too large $\Longrightarrow$ P_r(handoff) is large $\Rightarrow$ too many unnecessary handoff occur

Handoff for high speed users

Antennas of different heights and different powers are used to provide large and small colocated cells at a single location.

This umbrella cell approach provides large area coverage to high speed users and small area coverage to slow users, minimizing handoffs for high speed users.

$\boxed{\text{Fig. (3.3)}}$

Co-channel Interference (CCI) and system Capacity

Co-channel Interference (CCI): Is the interference which the mobile station (user) receives from the BS's of neighboring cells that operate at the same freq.

received signal = TX signal + CCI + Thermal noise

Thermal noise $\Rightarrow$ is overcome by increasing SNR (i.e signal power)

CCI $\Rightarrow$ cannot be combated by increasing TX power since as a system this means identical increase in TX power of all BS's, thus CCI of MS will increase by the same ratio.

To reduce CCI, cochannel cells must be separated by larger distances (however reducing capacity)

CCI depends on
- cell radius (R)
- distance (D) between nearest cochannel cells

Define Q: co-channel reuse ratio

$$\boxed{Q = \frac{D}{R}} = \frac{\sqrt{(i^2+j^2+ij)\Delta u^2}}{R} \qquad \Delta u = \sqrt{3}R$$

$$Q = \frac{\sqrt{N \times 3R^2}}{R} = \sqrt{3N}$$

$\therefore \boxed{Q = \frac{D}{R} = \sqrt{3N}}$ } co-channel reuse ratio

Define signal-to-Interference Ratio (SIR) as

$$SIR = \frac{S}{I} \begin{array}{l} \leftarrow \text{desired signal power} \\ \leftarrow \text{Interference power} \end{array}$$

$$SIR = \frac{S}{\sum_{i=1}^{i_0} I_i}$$

i_0 : No. of co-channel Interfering cells

I_i : Interference power from the BS of the i^{th} interfering co-channel cell

S : received power from the desired BS

But

$\boxed{\text{Average RX power} \propto d^{-n}}$

d : distance to source

n : path loss exponent $(2 \le n \le 6)$

Assuming: 1- TX power of all BS's are equal

2- (n) is the same within the area.

3- worst case interference for (MS) where (MS) is at cell edge $\Rightarrow$ distance $= R$

$\boxed{S = \text{const.}\ R^{-n}}$

desired RX power

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Fig.(3.5)

D: is the distance from the center of a cell to the center of its nearest cochannel cell.
Although MS is at the perimeter of a cell but we can approximate the distance of MS to cochannel centers as $\simeq D$

$$I_i = \text{const.}\, D^{-n}$$

$$\therefore\ SIR = \frac{S}{\sum_{i=1}^{i_0} I_i} = \frac{\text{const} \times R^{-n}}{\sum_{i=1}^{i_0} \text{const} \times D^{-n}} = \frac{R^{-n}}{i_0 D^{-n}}$$

$$\therefore\ \boxed{SIR = \frac{(D/R)^n}{i_0} = \frac{Q^n}{i_0} = \frac{(\sqrt{3N})^n}{i_0}}$$

$\therefore$ As $N\uparrow \Rightarrow Q\uparrow \Rightarrow SIR\uparrow \Rightarrow$ But $C\downarrow$

ex) If SIR of (15 dB) is needed for proper operation of a certain cellular system. Find the cluster size (N) that should be used to achieve maximum capacity if
a) $n=4$ b) $n=3$

$$SIR_{dB} = (\sqrt{3N})^n / i_0 \qquad i_0 = 6$$

~~Test for N = ...~~

~~... SIR_{dB} ... $\sqrt{3N}$~~

~~Let N≤?~~ We will test
$N = 3, 4, 7, 9, 12, 19 \ldots$

Let $N = 4$

$SIR_{dB} = 10 \log \frac{(\sqrt{3N})^{n}}{6 \leftarrow i_0} = 10 \log \frac{(\sqrt{3 \times 4})^{4}}{6} = 13.8$ dB

NOT good enough

Let $N = 7$

$SIR_{dB} = 10 \log \frac{(\sqrt{3 \times 7})^{4}}{6} = 18.66 \text{ dB} > 15 \text{ dB}$

$\therefore$ $\boxed{N = 7}$ is the desired choice

We select the smallest possible N to maximize capacity

b) $\boxed{n = 3}$ Let $N = 4$

$SIR_{dB} = 10 \log \frac{(\sqrt{3 \times 4})^{3}}{6} = 8.4 \text{ dB} < 15$

Let $N = 7$ $SIR_{dB} = 10 \log \frac{(\sqrt{3 \times 7})^{3}}{6} = 12.05 \text{ dB} < 15$

Let $N = 9$

$SIR_{dB} = 10 \log \frac{(\sqrt{3 \times 9})^{3}}{6} = 13.68 \text{ dB} < 15$

Let $N = 12$

$SIR_{dB} = 10 \log \frac{(\sqrt{3 \times 12})^{3}}{6} = 15.56 \text{ dB} > 15$

$\therefore$ The choice is $\boxed{N = 12}$

We select smallest possible N to maximize C

Adjacent channel Interference

Interference from signals which are adjacent in freq. to the desired signal is callent "adjacent channel Interference" (ACI)

To reduce ACI

1 - use modulation schemes with low out-of-band radiation (MSK is better than QPK or GMSK is better than MSK)

2 - Power Control ← the choice for GSM

3 - Careful design of BPF at RX front end.

4 - Proper channel interleaving by assigning adjacent channels to different cells.

5 - Avoid using adjacent channels in adjacent cells to further reduce ACI if N is large enough

6 - Separate the up-link and downlink by TDD or FDD.

Table. 3.2

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Trunking and Grade of Service (GOS)

Cellular Systems rely on "trunking" to accomodate a large no. of users in a Limited spectrum.

Trunking allows a large no. of users to share the relatively small no. of channels in a cell by providing acces to each user, on demand, from a pool of available channels. Trunking exploits the statistical behavior of users.

~~define~~

Grade of service GOS

- Blocked calls cleared (Erlang B) : GOS here is the probability of a call being blocked.

- Blocked calls Delayed (Erlang C) : GOS here is the probability of a call being delayed beyond a certain time.

Define: λ: average no. of calls / unit time for each user
H: Duration of a call (seconds) > Average

Each user generate a traffic intensity of (A_u) Erlangs given by $\boxed{A_u = \lambda H}$ traffic intensity / user in (Erlangs)

For U-users ~~there total tr~~

A : total traffic intensity for U-users

$$\boxed{A = U A_u}$$ Erlang

C : no. of channels in a trunked system

$$\boxed{A_c = U A_u / C}$$

A_c → traffic intensity per channel

Erlang B formula

Assumptions:- a) no. of calls/unit time has poisson dist.

b) Duration of a call follows exponential dist.

c) There is a finite no. of channels available in the pool.

$$\boxed{P_r[\text{blocking}] \triangleq GOS = \frac{A^C / C!}{\sum_{k=0}^{C} \frac{A^k}{k!}}}$$ Blocked calls cleared

C : No. of trunked channels of the system

A : Total offered traffic intensity (Erlangs)

Formula plotted in → Fig. (3.6)

ex) A certain urban area has 394 cells having a population of 0.1 million. Given:

$C = 19$ channel/cell

$\lambda = 2$ calls/hr for each user

$H = 3$ min/call average call time

Desired GOS $= 0.02$

Find a) No. of users that can be supported per cell

b) Total no. of users " " " in the urban area.

c) % market of populatn,

Given GOS $= 0.02$ and $C = 19$ ch/cell from curves we get

a) Total traffic intensity in cell $= 12$ Erlangs

Traffic intensity for a single user $A_u = \lambda H$

$$A_u = 2 \frac{\text{call}}{\text{hr}} \times \frac{3}{60} \text{hr} = 0.1 \text{ Erlangs}$$ for each user

Total no. of supportable $\frac{\text{users}}{\text{cell}} = \frac{A}{A_u} = \frac{12}{0.1} = 120$ user

b) Tot. No. of users/urban area $= 120 \frac{\text{user}}{\text{cell}} \times 394$ cell

$= 47280$ users

c) % market $= \frac{47280}{\text{population} \rightarrow 100000} \simeq 47.3\%$

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Erlang C formula

$$\boxed{P_r[\text{delay} > 0] = \frac{A^C}{A^C + C!\left(1-\frac{A}{C}\right)\sum_{k=0}^{C-1}\frac{A^k}{k!}}}$$ Blocked Calls delayed

Plotted in fig. Fig. (3.7)

$P_r[\text{delay} > t \mid \text{delay} > 0] = e^{-\left(\frac{C-A}{H}\right)t}$

$$P_r[\text{delay} > t] = P_r[\text{delay} > 0]\, P_r[\text{delay} > t \mid \text{delay} > 0]$$

$$\boxed{P_r[\text{delay} > t] = P_r[\text{delay} > 0]\, e^{-\frac{C-A}{H}t}}$$

Average delay for a call $= \boxed{D = P_r[\text{delay} > 0]\,\frac{H}{C-A}}$

in a $N=4$ cellular system

~~ex) A hexagonal area with $R = 1.387$ Km serviced by 60 channels. If the traffic intensity for each user is $A_u = 0.029$ Erlangs. Average no. of calls is $\lambda = 1$ call/hr.~~

ex) How many users can be supported for 0.5% blocking probability for following trunking systems with "blocked calls cleared". No. of channels are:

a) $C=1$ b) $C=5$ c) $C=10$ d) $C=20$ e) $C=100$

Given: Each user generates (0.1 Erlangs) of traffic.

$\Longrightarrow A_u = (0.1)$ Erlangs

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a) Given $GOS = 0.005$, $C = 1$ from fig.(3.6) we got the total traffic intensity of system $A = 0.005$ Erlangs

∴ Total no. of users $U = \frac{A}{A_u} = \frac{0.005}{0.1} = 0.05$ users

But actually one user could be supported on one channel. So $(U = 1)$.

b) with $C = 5$ and $GOS = 0.005$ we get the total traffic intensity $A = 1.13$ Erlangs.

∴ No. of supported users $U = \frac{A}{A_u} = \frac{1.13}{0.1}$

$U = 11$ users

c) $C = 10$, $GOS = 0.005 \Longrightarrow A = 3.96$

$U = A/A_u = 3.96/0.1 = 39$ users

d) $C = 20$, $GOS = 0.005 \Longrightarrow A = 11.1$

$U = A/A_u = 11.1/0.1 \simeq 110$ users

e) $C = 100$, $GOS = 0.005 \Longrightarrow A = 80.9$

$U = 80.9/0.1 = 809$ users

ex) An urban area with 2 million residents. There are three systems

System A: 394 cells with 19 channels each.

" B: 98 cells " 57 " "

" C: 49 cells " 100 " " . Given:

Required GOS (blocking) = 2%, $\lambda = 2$ calls/hr, $H = 3$ minutes

Find total no. of users supported by each

Sol System A

a) $GOS = 0.02$, $C = 19$

From fig. 3.6 ⟹ total traffic $A = 12$ Erlangs

Each user generates traffic

$$A_u = \lambda H = 2 \times (3/60) = 0.1 \text{ Erlangs}$$

∴ No. of users supported per cell $= U = \frac{A}{A_u} = \frac{12}{0.1} = 120$

System (A) can support total no. of users =

$= 120 \times 394 = 47280$

b) System (B)

$GOS = 0.02$, $C = 57$, ⟹ $A = 45$ Er

No. of supported users per cell $U = \frac{A}{A_u} = \frac{45}{0.1} = 450$

System (B) can support total no. of users =

$= 98 \times 450 = 44100$

c) System (C)

$GOS = 0.02$, $C = 100$ ⟹ $A = 88$ Er

$U = A/A_u = 88/0.1 = 880$

Total for system (C) $= 49 \times 880 = 43120$

percentage market penetration

Sys (A) $= 47280/2000000 = 2.36\%$

Sys (B) $= 44100/$ " $= 2.205\%$

(C) $= 43120/$ " $= 2.156\%$

ex) A city of 1300 $(mile)^2$ covered by a cellular system of seven-cell reuse pattern ($N = 7$). Cell radius = 4-miles. BW allocated to city = 40MHz with fullduplex channels of (60 KHz). The desired GOS = 0.02 Erlang B (Blocked calls cleared) and each user generates traffic of (0.03) Erlangs.

Find a) no. of channels per cell
b) traffic intensity of each cell.
c) max. carried traffic in the city
d) Total no. of users in the city with GOS = 0.02

a) Area of hexagonal cell $= 2.5981 R^2$ $(R = 4)$
$= 41.57$ $mile^2$

Total no. of channels $= \frac{40 MHz}{60 KHz} = 666$ channel

No. of channels per cell $= 666/7 = 95$ $\frac{chan.}{cell}$

b) $C = 95$, $GOS = 0.02 \Rightarrow$ traffic intensity $A = 84$ Erlangs/cell (Fig. 3.6)

c) Max. carried traffic in the city = No. of cells x $\frac{traffic}{cell}$

Total no. of cells $= \frac{\text{Area of city}}{A_{cell}} = \frac{1300}{41.57} = (31)$ cells

Max. traffic $= 31 \times 84$
$= 2604$ Erlangs

d) Total no. of users in city $= \frac{\text{Total traffic}}{\text{user traffic}}$
$= \frac{2604}{0.03} = 86800$ users

ex) A 4-cell ($N=4$) system with cell radius $= 1.387$ Km
Total no. of channels for system $= 60$ (ch).
Each user generates traffic intensity $A_u = 0.029$ (Erlang)
and $\lambda = 1$ call/hr. Assume Erlang C
system (Blocked calls delayed) with
GOS ($Pr[\text{delay} > 0]$) $= 5\%$.
a) How many users per Km^2 will the system support.
b) what is the prob. that a delayed call will have to wait for more than (10 s).
c) what is the prob. that a call will be delayed for more than (10) seconds.

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a) cell area $= 2.598(1.387)^2 = 5\ Km^2$
$N = 4$, Tot. no. of Ch $= 60$
No. of channels/cell $= 60/4 = 15$ ch/cell
using GOS $= 0.05$, $C = 15 \Rightarrow$ traffic/cell $A = 9$ Erlang
If each user generate $A_u = 0.029$
No. of users per cell $U = A/A_u = 9/0.029$
$= 310$ users/cell

$$\frac{\text{No. of user}}{Km^2} = \frac{310}{\text{cell area}} = \frac{310}{5} = 62\ \frac{\text{users}}{Km^2}$$

b) $Pr[\text{delay} > t \mid \text{delay}] = e^{-\frac{C-A}{H}t}$

$Pr[\text{dely} > 10 \mid \text{delay}] = e^{-\frac{(15-9)}{H} \times 10}$ ← second, ← second $= 0.5629$

$A_u = \lambda H$
$.029 = 1 \frac{\text{call}}{\text{hr}} \times H$
$H = 0.029$ hr $\quad H = 104.4$ sec

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c) $Pr[\text{dely} > 0] = 0.05$

$Pr[\text{delay} > 10s] = \cancel{Pr[\text{delay} > 0]}$

$= Pr[\text{dely} > 0]\, Pr[\text{dely} > 10 \mid \text{dely} > 0]$

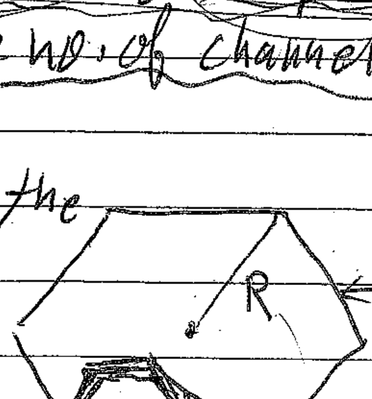
$= 0.05 \times 0.5627 = 0.0281$

$= 2.81\%$

Improving Coverage and Capacity of Cellular system

Can be done using

1. **Cell splitting**: Is the process of subdividing a congested cell into smaller cells ("microcells"), each with its own base station and a corresponding reduction in antenna height and TX power.
   * Microcells has the same no. of channels of the bigger cell.
   * Capacity increases since the no. of times channels are re-used increases.
   * The received signal power at the new & old cell boundary must be equal so that $\left(\frac{S}{I}\right)$ at boundary is approximately the same.



of microcell 239/3

P_{t2}: Tx power ~~P_r at old cell b~~, Fig. (3.9)

P_{t1}: TX power of the large cell

P_r [at old cell boundary] $\propto P_{t1} R^{-n}$

P_r [at new cell boundary] $\propto P_{t2} (R/2)^{-n}$

$$P_{r-new} = P_{r-old} \Rightarrow \frac{P_{t1} R^{-n}}{P_{t2} (R/2)^{-n}} = 1$$

Assume $n = 4$

$P_{t2} = \frac{P_{t1}}{16}$ ⇒ This is a reduction of 12 dB in power for microcells while maintaining S/I requirements.

Disadvantages of cell-splitting

1 - More BS's ⇒ cost ↑
2 - More handoff
3 - More complicated channel assignment

Notes

- The smaller cells Fig. 3.8 must be placed in such a way to preserve freq. reuse plan of the system. For example, microcell (G) is placed halfway between Larger stations of channels (G).

Splitting Process

In practice, not all cells are split at the same time. Different cell sizes will exist simultaneously. Hence, special care should be taken to keep the distance between co-channel cells at the required minimum.

For this reason, channels of the original (old) cells must be broken down into two groups:

1- those corresponding to smaller cell reuse requirements.
2- " " " Larger " " ".

(Larger cells dedicated to high speed traffic).

The size of the two groups depend on the stage of the splitting process:

1- At the beginning, there will be few channels in the small cell group.
2- As demand grows, more channels are required in small-cell group.
3- Splitting continues until all channels are used in the small-cell group ⇒ the entire system is now replaced by small cells.
4- Antenna downtilting (beaming towards ground rather than horizon) is often used to limit radio coverage of new small cells.

2- $\boxed{\text{Sectoring}}$

* Co-channel Interference (CCI) may be decreased by replacing the single omnidirectional antenna at the base station by several directional antennas, each radiating within a sector.
~~* Sectoring decreases the CCI received by MS.~~

* In Sectoring the channels used in a particular cell are ~~broken down~~ divided between sector of the cell.

f mobile located in the ight-most sector in the entral cell Labeled 5-V

Fig. (3.10)

* In 120° sectoring, ~~cells~~ co-channel cells affecting a MS are reduced from 6 to 2 (only the first tier has an effect)
* sectoring $\Rightarrow$ reduce CCI $\Rightarrow$ Increase capacity by reducing N (cluster size) $\Rightarrow C\uparrow$

Fig. (3.11)

ex) without sectoring $N = 7$, $n = 4$

$$\left(\frac{S}{I}\right)_{dB} = 10\log\frac{(\sqrt{3N})^n}{i_o} = 10\log\frac{(\sqrt{3\times7})^4}{6} = 18.6\text{ dB}$$

with 120° sectoring, $N=7$, $n=4$ but cochannel cells $= 2$

$$\left(\frac{S}{I}\right)_{dB} = 10\log\frac{(\sqrt{3\times7})^4}{2} = 23.4\text{ dB}$$

Since SIR$\uparrow$ we can use lower $N \Rightarrow M\uparrow \Rightarrow C\uparrow$

~~R~~ ENB 25a/3

Disadvantage of sectoring

1- Increasing no. of antennas, i.e. complexity

2- Increasing no. of hand-off's (sector to sector)

3- Decrease trunking efficiency

However, BS's can do sector → sector handoff without MSC intervention

ex) capacity Improvement

It is desired to have SIR ≥ 18 dB (minimum) for proper operation. What are the necessary cluster size, (N) a) without sectoring b) with 120° sectoring

Assume $\boxed{n = 4}$.

a) $SIR(dB) = 10 \log \frac{(\sqrt{3N})^n}{6}$ (without sectoring)

Let N = 4 ⇒ $SIR = 10 \log \frac{(\sqrt{3 \times 4})^4}{6}$ = 13.8 dB < 18

N = 7 $SIR = 10 \log \frac{(\sqrt{3 \times 7})^4}{6} = 18.66 > 18$

$\boxed{N = 7}$ satisfies requment.

b) with 120° sectoring (CCI comes from 2-cells)

let N = 3 $SIR = 10 \log \frac{(\sqrt{3 \times 3})^4}{2} = 16.07 < 18$

Let N = 4 $SIR = 10 \log \frac{(\sqrt{3 \times 4})^4}{2} = 18.57 > 18$

∴ $\boxed{N = 4}$ satisfies

∴ A capacity increase of $\boxed{\frac{7}{4}}$ by $\boxed{120°}$ sectoring

25 b / 3

sol.) $STR_{dB} = 10 \log \dfrac{(\sqrt{3N})^n}{i_0}$

a) without sectoring

$10 \log \dfrac{(\sqrt{3N})^4}{6} \geq 18$

$\log \left(\dfrac{9N^2}{6}\right) \geq 1.8 \quad 10^{\wedge}$

$\dfrac{9N^2}{6} \geq 10^{1.8} \Rightarrow N^2 \geq 42.06$

$\boxed{N \geq 6.48} \Rightarrow$

$N = [3, 4, 7, 9, 12, 13, 19, 21, 27 \ldots]$

$\therefore \boxed{N = 7}$ To satisfy $SIR \geq 18\,dB$

b) with 120° sectoring

$10 \log \dfrac{(\sqrt{3N})^4}{2} \geq 18 \Rightarrow \log \dfrac{9N^2}{2} \geq 1.8$

$\dfrac{9N^2}{2} \geq 10^{1.8} \Rightarrow N \geq 3.74$

$\therefore \boxed{N = 4}$ To satisfy $SIR \geq 18\,dB$

$\therefore$ ~~This~~ ~~give~~ 120°-sectoring gives a capacity increase of 7/4

1- Capacity first (determine N)
2 - GOS comes 2nd to look at.

26/3

ex) A cellular system has S = 210 full duplex channels. Assume (n = 4) and desired GOS = 1%. Find the SIR and trunking efficiency for
a) No sectoring N=7 b) 120° sectoring N=7 c) 120° sector N=4

---

a) $\boxed{N=7}$ so no. of channels/cell = 210/7 = $\boxed{30}$ ch/cell

(Fig. 3.6)
GOS = 0.01, C = 30 $\Longrightarrow$ $A_{cell} = 20.34$ Er

$SIR = 10\log\left(\frac{\sqrt{3N}}{6}\right)^n = 10\log\left(\frac{\sqrt{3\times7}}{6}\right)^4 = 18.66$ dB

b) N=7, 120° sect $i_0 = 2$
No. of channels/sector = 210/(7×3) = 10 ch/sector
GOS = 0.01, C = 10 $\Longrightarrow$ A = 4.46 Erlang/sector
$A_{cell} = 3\times4.46 = 13.38$

$SIR = 10\log\left(\frac{\sqrt{3\times7}}{2}\right)^4 = 23.43$ dB

Trunking efficiency
$\eta = \frac{A(1-GOS)}{C}$ ← ch/cell

c) N=4, 120° sect, $i_0 = 2$
No. of channels/sector = 210/(4×3) = 17 ch/sector
GOS = 0.01, C = 17, $\Longrightarrow$ A = 9.65 Erlang/sector
$A_{cell} = 9.65\times3 = 28.95$ Er/cell

$SIR = 10\log\left(\frac{\sqrt{3\times4}}{2}\right)^4 = 18.56$ dB

$\eta_a = 20.34(0.99)/30 = 67$ %
$\eta_b = 13.38(0.99)/30 = 44.1$ %
$\eta_c = 28.95(0.99)/(17\times3) = 56$ %

As we go from non-sector to 120°-sector
SIR ↑
But trunk eff
η ↓

3- Repeaters for range extension

- Often it is needed to provide dedicated coverage for hard-to-reach areas (eg. within buildings, valleys, tunnels) Radio retransmitters are used to provide such range extensions. Care must be taken in the positions and power setting for these repeaters and their antenna pattern.
- Distributed Antenna Systems (DAS) are sometime used with repeaters in buildings or tunnels.
- Some operators install microcells outside large buildings, and then install several repeaters with DAS inside buildings.
- Engineers can rapidly determine best placements for repeaters and DAS within buildings, by using special softwares such a (sitePlanner).

- Repeater increase SIR, and hence it is possible to use ~~larger~~ smaller (N) ~~of~~ ⇒ ~~Large~~ smaller cluster ⇒ Larger capacity.

Improves Coverage of strongly shadowed regions (inside buildings, tunnels)

4- Microcell Zone Concept   Fig. 3-13 + 3-14

The increased no. of handoffs in sectoring-approach causes a load on BSC or MSC. A solution is microcell concept. In this scheme:

- Each of the three (or possibly more) zone sites (having TX + RX) is connected to a single base station, sharing the same radio equipment.
- Zones are connected by (coaxial cable, fiberoptic cable or microwave link) to the base station.
- Multiple zones + one base station ⇒ make one cell
- At any time, a mobile is served by the zone of the strongest signal.
- As mobile moves from zone-to-zone it retains the same channel.
- No hand-off by BSC or MSC.
- BSC simply switches channel to a different zone

∴ A channel is active only in the particular zone in which mobile is travelling and hence BSC radiation is localized and interference is reduced.

- We need significantly less power to cover the same cell area (without sectoring)
- (N) can be smaller (cells are closer) while keeping same SIR (or effective D/R)

27c/3

Advantage of microcell zone [Fig. 3-14]

While the cell maintains a particular coverage radius, ~~cell~~ ~~CCI is reduced since~~ SIR is increased

since $SIR = \frac{(D/R)^n}{i_0}$

while effective D is slightly reduced, R is effectively reduced by a large factor $\Rightarrow$ SIR $\uparrow$

ex) Fig. 3-14 Assume SIR = 18 dB

A - No cell zone

For N = 7 ~~requires~~ $\frac{D}{R} = 4.6$ can achieve SIR = 18 dB

$\hookrightarrow$ corresponds to Δ

B - Cell zone

SIR of 18 dB can be achieved with $\frac{D_Z}{R_Z} = 4.6$ effective ratio

In Fig. 3-14 $\Rightarrow$ $\frac{D_Z}{R_Z} = 4.6 \xLeftrightarrow{\text{correspond to}} \left(\frac{D}{R} = 3\right)$ corresponding to $N = 3$

$\therefore$ cluster size is reduced from

$N = 7 \rightarrow$ to $N = 3$

$\Rightarrow$ 2.33 times capacity increase

27d / 3

More Examples

ex) A cellular system with desired SIR = 15 dB.
Find the optimal value of (N) for:
a) Omnidirectional antenna
b) 120° sectoring c) 60° sectoring
d) Should sectoring be used and if so what type?
(Assume n = 4, and consider trunking eff)

a) $SIR = 10\log\frac{(\sqrt{3N})^4}{6} = 10\log\frac{(\sqrt{3N})^4}{6} \geq 15$

$\log\left(\frac{9N^2}{6}\right) \geq \frac{15}{10} \Rightarrow \frac{9N^2}{6} \geq 10^{15/10}$

$\Rightarrow N \geq 4.59 \Rightarrow$ The choice is $\boxed{N=7}$

b) $SIR = 10\log\frac{(\sqrt{3N})^4}{2} = 10\log\frac{9N^2}{2} \geq 15$

$N \geq 2.65 \Rightarrow$ Choice is $\boxed{N=3}$

c) $SIR = 10\log\left(\frac{9N^2}{1}\right) \geq 15 \Rightarrow N \geq 1.87$

Choice is $\boxed{N=3}$

d) Sectoring ($120^\circ, 60^\circ$) achieves desired SIR = 15 dB with smaller (N) $\Rightarrow$ larger capacity (a factor 7/3 increase in capacity compared to N = 7 omnidirectional). We choose 120° since it has better trunking eff. than 60° sectoring, both giving similar capacity (N = 3 for both) ex) A total of 24 MHz Bandwidth is allocated to a cellular system which uses two 30 kHz simplex channels to form a duplex channel. Each cellphone user generates 0.1 Erlang of traffic (Erlang-B).

$\boxed{N=4}$

a) Find no. of channels in each cell
b) If each cell is to offer capacity that is 90% of perfect scheduling, find the max. no. of users that can be supported in a cell using Omnidirectional antenna.
c) What is the blocking probability of the system if the no. of users is the same as (b).
d) Using 120° sectoring what is the no. of users in a cell that result in the same blocking prob. of (c).

a) Total no. of channels $= \frac{24000}{2 \times 30} = 400$

No. of channels/cell $= \frac{400}{4} = \boxed{100}$

b) 1 Erlang $\Rightarrow$ ~~a call~~ time

If $\lambda = \frac{1 \text{ call}}{\text{unit time}}$ $H = 1$ unit time $A_u = 1$ Erlang
max. traffic a user can generate

(full - all time use of channel)

perfect scheduling $\Rightarrow$ Each user uses channel all the time

$\Rightarrow$ Traffic $A =$ No. of channels

$A = \frac{90}{100} \times$ No. of channel $= \frac{90}{100} \times 100 = \boxed{90}$ Erlang

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$$\text{No. of users} = \frac{A}{A_u} = \frac{90}{0.1} = \boxed{900} \text{ users}$$

c) If no. of users = 900 ~~and no. of channel~~
then total traffic intensity $A = 900 \times 0.1 = 90$ Erlg
~~then from fig. 3.6~~

No. of channels = 100

$\boxed{A = 90\,Er}$, $\boxed{C = 100}$ $\Longrightarrow$ Blocking prob. (GOS) $= \boxed{0.03}$
(Fig. 3.6)

d) Using 120° sector $C = 100/3 = 33.33$

$\text{GOS} = 0.03$, $\frac{C}{\text{sector}} = 33.33$ $\Longrightarrow$ $\frac{A}{\text{sector}} = 25$

$$\text{No. of } \frac{\text{users}}{\text{sector}} = \frac{A}{A_u} = \frac{25}{0.1} = 250$$

$$\text{Total no. of } \frac{\text{Users}}{\text{cell}} = 3 \times 250 = 750 \text{ users}$$

ex)

ex) Given a cell with 20-channels. Each user has $\lambda = 1$ call/Hr, $H = 105$ sec. If the ~~cell~~ no. of users in the cell is 480, and ~~determine~~ assuming a lost call delayed system (Erlang C), find the Prob.[delay > 20 sec].

$\underline{\text{Traffic}}_{\text{user}} = A_u = \lambda H = \frac{1}{3600} \times 105 = 0.02916$ Er

Total traffic $= u A_u = 480 \times 0.02916 = \boxed{14 \text{ Er}}$

$$\text{Prob.}[\text{delay} > 20\,s] = \text{Prob.}[\text{delay} > 0] \times \text{Prob}[\text{delay} > 20 \mid \text{delay} > 0]$$

$$= \text{Prob.}[\text{delay} > 0]\, e^{-\frac{(C-A)t}{H}}$$

From Erlang-C chart

$A = 14$ Er, $C = 20 \Rightarrow$ GOS[Prob(delay > 0)] $= \boxed{0.06}$

$$\text{Prob.}[\text{delay} > 20] = 0.06 \times e^{-(20-14)20/105}$$

$$= \boxed{0.019}$$

Suggested Problems

3.7, 3.8, 3.11, 3.12, 3.13, 3.15, 3.16
3.17, 3.18, 3.26

Chapter-4
Mobile Radio Propagation: Large-Scale Path Loss

The mobile radio channel places fundamental limitations on the performance of wireless communication systems.

EM wave propagation mechanisms

- reflection
- diffraction
- scattering

The interaction between waves coming from different paths cause fading at specific locations and the average strength of these waves decrease as the T-R separating distance is increased.

Large-Scale propagation model: This model predicts the mean signal strength for an arbitrary T-R separation

Small-Scale propagation model: This model characterizes the rapid fluctuations of the received signal strength over very short distances (few wavelengths) or short time durations.

* As mobile moves very small ~~[illegible]~~ distances the signal strength varies rapidly. amplitude

(Sum of large no. of paths with random phases)

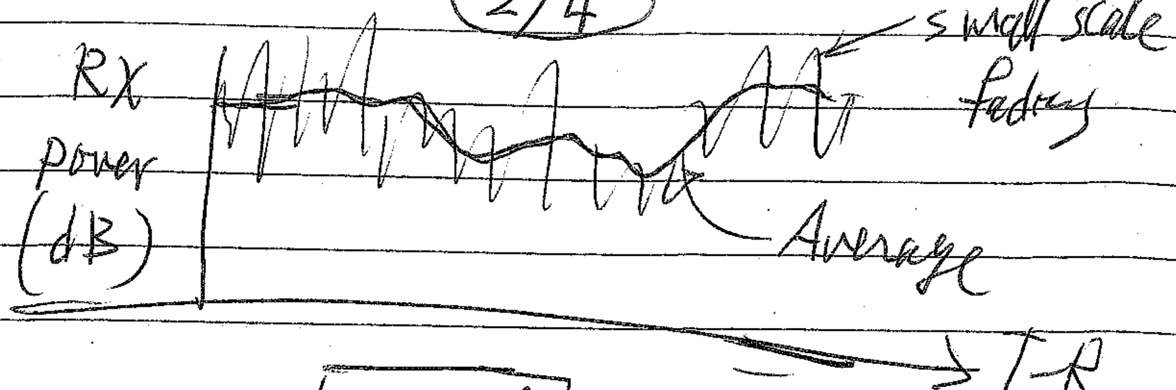


Fig. (4.1)

Free Space propagation Model

This model is used when the TX and RX have clear unobstructed Line of sight between them.

Power received by RX at a distance (d) is

$$\boxed{P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}} \quad \text{Friis free-Space equation} \quad (1)$$

P_t: transmitted power $\quad$ G_t: TX antenna gain
λ: wavelength (in meters) $\quad$ G_r: RX antenna gain
d: T-R separating distance (in meters).
L: system Loss factor ($L \geq 1$) due to transmission Line attenuation, filter and antenna Losses. If the system has no Losses then ($L = 1$).

Antenna gain: $G = \frac{4\pi A_e}{\lambda^2}$ (due to antenna directivity) $\quad$ A_e: effective aperture related to antenna size

$\lambda = \frac{c}{f_c} = \frac{2\pi c}{\omega_c}$ $\quad$ c ← speed of Light (meter/sec), ω_c ← carrier freq.

Note: $\boxed{P_r(d) \propto \frac{1}{d^2}} \Longrightarrow P_r(d) = K/d^2$

$$\Rightarrow P_r(d)_{dB} = 10\log K - 10\log d^2$$
$$= 10\log K - \underline{20\log d}$$

∴ $P_r(d)_{dB}$ decays with distance as 20 dB/decade

$$P_r(10d_0)_{dB} - P_r(d_0)_{dB} = 10\log K - 20\log(10d_0) - [10\log K - 20\log(d_0)]$$
$$= 20\log\frac{d_0}{10d_0} = 20\log 10^{-1} = -20\text{ dB}$$

∴ $P_r(d)_{dB}$ decreases by (20dB) as distance is increased by a factor of (10)

Effective Isotropic Radiated Power (EIRP)

$$\boxed{EIRP = P_t G_t}$$

Is the max. radiated power in the direction of max. antenna gain as compared to an isotropic radiator.

Effective Radiated Power (ERP)

Is the max. radiated Power compared to $\lambda/2$-dipole antenna (having a gain of 1.64 (2.15dB) above an isotrope)

$$\boxed{ERP = EIRP - 2.15dB}$$

In practice antenna gains are given in

1- dBi (dB gain with respect to isotropic antenna)
2- dB_d (" " " " " $\lambda/2$ dipole)

Path Loss (P_L(dB))

1- When antenna gains are included

$$P_L(dB) = 10\log\frac{P_t}{P_r} = -10\log\left[\frac{G_t G_r \lambda^2}{(4\pi)^2 d^2}\right]$$

2- When antenna gains are excluded

$$P_L(dB) = 10\log\frac{P_t}{P_r} = -10\log\left[\frac{\lambda^2}{(4\pi)^2 d^2}\right]$$

Far-Field (Fraunhofer region)

Friis model is only valid for distance (d) in the far field region, ~~For (d) to be in the far field regions~~ i.e. $(d > d_f)$ where $\boxed{d_f = \frac{2D^2}{\lambda}}$ D: Largest physical dimension of TX-antenna

and d_f should satisfy

1- $d_f \gg D$
2- $d_f \gg \lambda$

Free Space received power using close-in distances

Equation (1) ~~holds~~ does not hold for $(d=0)$.

Let (d_o) be close-in distance that is in the far-field region and is smaller than any distance in the communication system

$P_r(d_o)$: power received at (d_o) calculated by either (1) prediction from eqn (1)
OR (2) measured practically by taking average received power at several points with $(d=d_o)$.

$$\frac{P_r(d)}{P_r(d_o)} = \frac{d_o^2}{d^2}$$

$$P_r(d) = P_r(d_o)\left(\frac{d_o}{d}\right)^2 \quad (d_f \le d_o \le d)$$

Because of the large dynamic range of $P_r(\ )$ we use units of dBm, dBW

$P_r(d)(dB_m) = 10 \log P_r(d)$ where $P_r(d)$ is in (mW)

$$\boxed{P_r(d)(dBm) = 10 \log \left[\frac{P_r(d_o)\ W}{0.001\ W}\right] + 20 \log\left(\frac{d_o}{d}\right)}$$

$$(d_f \le d_o < d)$$

choice of (d_o)

For low gain antennas at (1-2) GHz (for easy calculations) we choose ⇧

$d_o = 1m$ for indoor environments
$d_o = 100m$ or $1Km$ in outdoor env.

6/4

ex) Find the far-field distance for an antenna with max. dimension of (1 meter) and operating freq = 900 MHz

$$d_f = \frac{2D^2}{\lambda} \Rightarrow D = 1$$

$$\lambda = c/f = \frac{3\times 10^8 \text{ m/s}}{900\times 10^6 \text{ Hz}} = \boxed{0.333 \text{ m}}$$

$$\boxed{d_f = \frac{2\times 1^2}{0.333} = \boxed{6 \text{ m}}}$$

ex) A TX produces 50W of power. Express the TX power in units of a) dBm b) dBW

c) If 50W is applied to a unity gain antenna with $f_{carrier}$ = 900 MHz, find the RX-power in (dBm) at a free space distance of (100m) from antenna.

d) what is $P_r(10\text{ Km})$. Assume unity gain for TX and RX antennas.

$$\lambda = c/f = 3\times 10^8 / 900\times 10^6 = \frac{1}{3} \text{ m}$$

a) $P_t(dB_m) = 10 \log \frac{P_t(W)}{0.001 W} = 10 \log \frac{50}{0.001} = 47 \text{ dBm}$

b) $P_t(dB_W) = 10 \log \frac{P_t(W)}{1W} = 10 \log \frac{50}{1} = 17 \text{ dBW}$

OR $\boxed{P(dB_m) = P(dBW) + 30}$ ← 10 log 1000

c) $P_r(d) = P_t G_t G_r \lambda^2 / [(4\pi)^2 d^2 L]$

$P_r(100m) = 50\times 1\times 1\times (1/3)^2 / [(4\pi)^2 \times 100^2 \times 1]$

$= 3.5\times 10^{-6}$ Watt $= 3.5\times 10^{-3}$ mW

$P_r(100m)\, dB_m = 10 \log P_r(mW) = 10 \log (3.5\times 10^{-3})$

$= -24.5$ dBm

7/4

d) To find $P_r(10Km)$ we select $d_o = 100m$

$$P_r(10Km)\,dBm = 10\log\left[\frac{P_r(\overset{100m}{d_o})\,W}{0.001\,W}\right] + 20\log\frac{d_o}{d}$$

$$= -24.5\,dBm + 20\log\frac{100}{10000}$$

$$= -24.5 + 20\log 10^{-2} = -24.5 - 40 = \boxed{-64.5\,dBm}$$

ex) Given $P_t = 50W$, $f = 900\,MHz$, $G_t = 1$, $G_t = 2$, Find $P_r(10Km)$ in dBm (let $L = 1$)

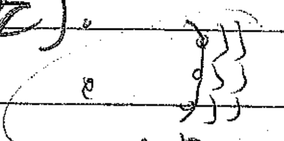
$$P_r(d) = 10\log\frac{P_tG_tG_r\lambda^2}{(4\pi)^2 d^2 L} = 10\log\frac{50\times1\times2\times(1/3)^2}{(4\pi)^2\times(10000)^2\times1}$$

$$= -91.5\,dBW \quad +30$$

$$= -61.5\,dBm$$

Propagation Mechanisms

① **Reflection:** Occurs when the wave hits an object which has very large dimensions compared to (λ), (ex. earth surface, buildings, ... etc).



② **Diffraction:** Occurs when the path between Tx and RX is obstructed by a surface with sharp irregularities (edges). It allows radio waves to propagate around curved surfaces and behind obstructions (Huygen's principle)

8/4

(3) **Scattering:** Waves are scattered (spread in all direction) when a wave passes through a medium with large no. of objects (obstacles) whose dimensions is small compared to (λ) (ex. rough surfaces, street signs, lamp posts ... etc).

rough: If we define critical height

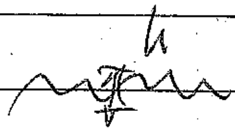
$$h_c = \frac{\lambda}{8 \sin \theta_i}$$

θ_i ← Angle of incidence

then a surface is considered to be

1- smooth if its min. to max. protuberance (h) is less than (h_c)

2- Rough if $h > h_c$



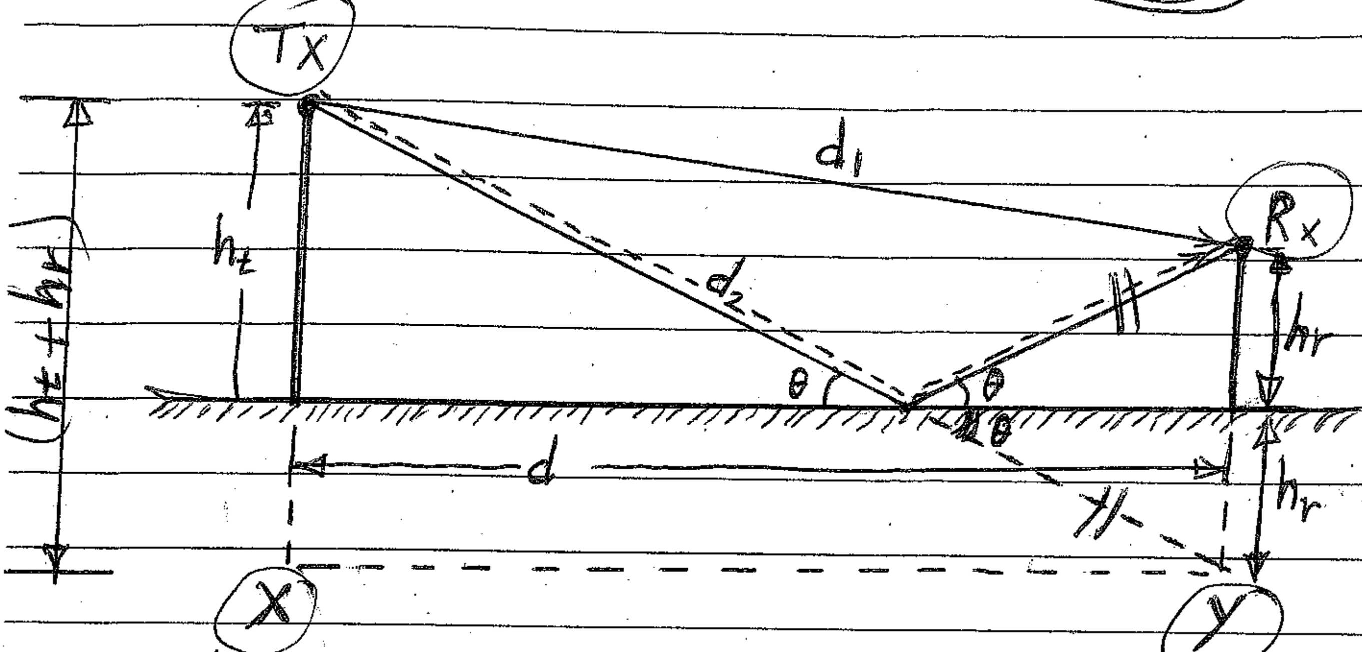
Ground Reflection (Two-ray) Model

A single path from TX to RX is very rare.
The two-ray ground reflection model is found a reasonably accurate model for:

1- A distance of several kilometers with tall towers (>50m).

2- Line of sight microcell channels in urban areas.

9/4



Assumptions

1- Over the distance of few kilometers used, earth is flat.

2- small (θ) due to the facts that

$$d \gg h_t \,, \; d \gg h_r$$

3- Hence at this small(θ), the reflection ($\rho = -1$) (for horizontal polarization) (see eqn. (4.25))

From figure $\Rightarrow$ $d_1^2 = (h_t - h_r)^2 + d^2$

OR $\quad d_1 = \sqrt{d^2 + (h_t - h_r)^2}$

FOR the triangle [TX − X − Y] we can write

$$[Tx - Y]^2 = [X - Y]^2 + [Tx - X]^2$$

$$d_2^2 = d^2 + (h_t + h_r)^2 \quad \text{OR} \quad d_2 = \sqrt{d^2 + (h_t + h_r)^2}$$

Let Δ = path difference = $d_2 - d_1$

$$\Delta = \sqrt{d^2 + (h_t + h_r)^2} - \sqrt{d^2 + (h_t - h_r)^2}$$

10/4

sing Taylor's Series

$$\sqrt{1+x} \simeq 1+\frac{x}{2} \quad (\text{for } x \ll 1)$$

$$\Delta = d\sqrt{1+\left(\frac{h_t+h_r}{d}\right)^2} - d\sqrt{1+\left(\frac{h_t-h_r}{d}\right)^2}$$

are very small since $d \gg h_t, h_r$
and because of squaring $(\,)^2$

$$= d\left[1+\frac{1}{2}\left(\frac{h_t^2+h_r^2+2h_th_r}{d^2}\right)\right] - d\left[1+\frac{1}{2}\left(\frac{h_t^2+h_r^2-2h_th_r}{d^2}\right)\right]$$

$$\boxed{\Delta = \frac{4h_th_r}{2d} = \frac{2h_th_r}{d}}$$

$\Delta\phi$ = carrier phase difference $= \frac{\Delta}{\lambda}2\pi = 2\pi\frac{2h_th_r}{d\lambda}$

$$\boxed{\Delta\phi = \frac{4\pi}{\lambda d}h_th_r}$$

e received electric field is the vector sum

$$|1+\alpha e^{-j(B)}e^{j\Delta\phi}|$$

carrier phase shift due to ground reflection $B=\pi$

attenuation factor $\simeq 1$

$$\to |1-e^{j\Delta\phi}|$$

ceived Power $= P_{LOS}|1-e^{j\Delta\phi}|^2$

$$P_r = P_tG_tG_r\left(\frac{\lambda}{4\pi d}\right)^2 |\underbrace{1-\cos\Delta\phi}_{Real} - j\underbrace{\sin\Delta\phi}_{Imag}|^2$$

$$= P_tG_tG_r\left(\frac{\lambda}{4\pi d}\right)^2[1+\cos^2\Delta\phi-2\cos\Delta\phi+\sin^2\Delta\phi]$$

$$[2-2\cos\Delta\phi]$$

$$2\left[1-\left(1-2\sin^2\frac{\Delta\phi}{2}\right)\right] \longrightarrow \left[4\sin^2\frac{\Delta\phi}{2}\right]$$

(11a/4)

$$P_r = P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 4\sin^2\left(\frac{4\pi}{\lambda d}\frac{h_t h_r}{2}\right) \leftarrow \frac{\Delta\phi}{2}$$

$$\therefore \boxed{P_r = 4 P_t G_t G_r \left(\frac{\lambda}{4\pi d}\right)^2 \sin^2\left(\frac{2\pi h_t h_r}{\lambda d}\right)}$$

For small $\Delta\phi$ (since $d >> h_t, h_r$) $\Rightarrow \sin\frac{\Delta\phi}{2} = \frac{\Delta\phi}{2}$

$$\therefore P_r = 4 P_t G_t G_r \frac{\lambda^2}{(4\pi)^2 d^2} \frac{(2\pi)^2 h_t^2 h_r^2}{\lambda^2 d^2}$$

$$\boxed{P_r = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4}}$$ Two-Ray reflection model

* $P_r \propto d^{-4}$ (P_r decays as d^{-4})
while for free space ($P_r \propto d^{-2}$)
* P_r falls at the rate of 40dB/decade with (d)
i.e P_r falls (40dB) when ($d_{new} = 10 d_{old}$) distance is increased 10-time.

ex) The distance between BS $\rightarrow$ mobile is $d = 5$ Km and $P_t = 2W$. freq $= 900$ MHz, $h_t = 1.5$ m, $h_r = 5$ m
Find received power at $d = 5$ Km where, $G_r = 2.55$ dB $= G_t$, using two-ray model.

$2.55 = 10 \log G \Rightarrow G = 10^{2.55/10} = 1.8$

$P_r(5\text{Km}) = P_t G_t G_r \left(\frac{h_t h_r}{d^2}\right)^2 = 5.9 \times 10^{-13}$ Watt -92.2 dBm

(dBW) $= 10 \log(5.9 \times 10^{-13}) = 10(\log 5.9 + \log 10^{-13}) = -122.29$ dBW

...ireless communications says that the received signal power is ... fourth power of the distance between TX and RX. This law is ... the received power for the case that only a direct (line-of-sight, ...ected wave, exists. For this specific case, the following equation is ... www.wiley.com/go/molisch):

$$P_{RX}(d) \approx P_{TX}G_{TX}G_{RX}\left(\frac{h_{TX}h_{RX}}{d^2}\right)^2 \tag{4.24}$$

... height of the transmit and the receive antenna, respectively; it is valid ...

$$d_{break} \gtrsim 4h_{TX}h_{RX}/\lambda \tag{4.25}$$

... the standard Friis' law, implies that the received power becomes ... Furthermore, it follows from Eq. (4.24) that the received power ... the height of both BS and MS.

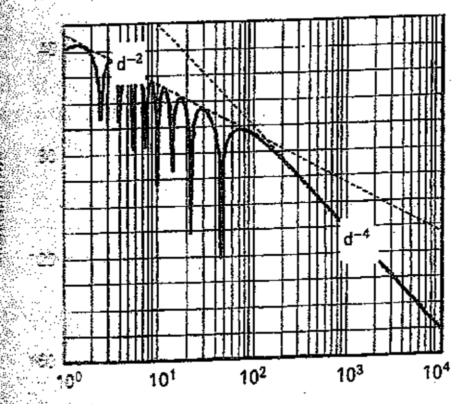
... useful to rewrite the power law on a logarithmic scale. Assuming that ... a breakpoint d_{break}, and from there with d^{-n}, then the received ...

$$... = P_{RX}(1\,\mathrm{m}) - 20\log(d_{break}|_m) - n10\log(d/d_{break}) \tag{4.26}$$

...ed power when there is a direct wave and a ground-reflected wave, ... (see Appendix 4.A) and Eq. (4.24). We find that the transition ...ent $n = 2$ and $n = 4$ is actually not a sharp breakpoint, but rather ...ble to strictly test statements about the onset of the d^{-4} law. ... the breakpoint is at $d = 90\,\mathrm{m}$; this seems to be approximated ...

... above (and in Appendix 4.A) are self-consistent, but it has to be ... a *universal* description of wireless channels. They do not agree ... realistic channels in several respects:

... valid decay exponent. $n = 2$ is fulfilled close to the transmit ... distances, values between $1.5 < n < 5.5$ have been measured, and



... ideally reflecting ground. Height of BS: 5 m. Height of MS: 1.5 m.

Diffraction – Fresnel Zone Geometry
(Knife edge diffraction model)

Diffraction is caused by propagation of secondary waves into shadowed region. It can be explained by "Huygen's principle":

"ALL points on a wavefront can be considered as point sources that produce secondary waves"

~~Fresnel zone Geometry (Knife edge diff. model)~~

* Estimation of signal attenuation caused by diffraction of waves over hills and buildings is essential in predicting the signal strength in a given service area.
* Diffraction losses is estimated by theoritical approximation modified by empirical corrections.
* The knife-edge model gives good insight into the order of magnitude of diffraction loss (This model treats hills, buildings as diffracting knife edge)

From Fig. (a), and assuming ($h \ll d_1, d_2$ and $h \gg \lambda$)

Δ: the difference between the direct path and the diffracted path

$$\Delta = (d_1' + d_2') - (d_1 + d_2)$$

$$= \left(\sqrt{d_1^2 + h^2} + \sqrt{d_2^2 + h^2}\right) - d_1 - d_2$$

$$= \left(d_1\sqrt{1 + (h/d_1)^2} + d_2\sqrt{1 + (h/d_2)^2}\right) - d_1 - d_2$$

using $\sqrt{1+x} \simeq 1 + \frac{x}{2}$ if $x \ll 1$

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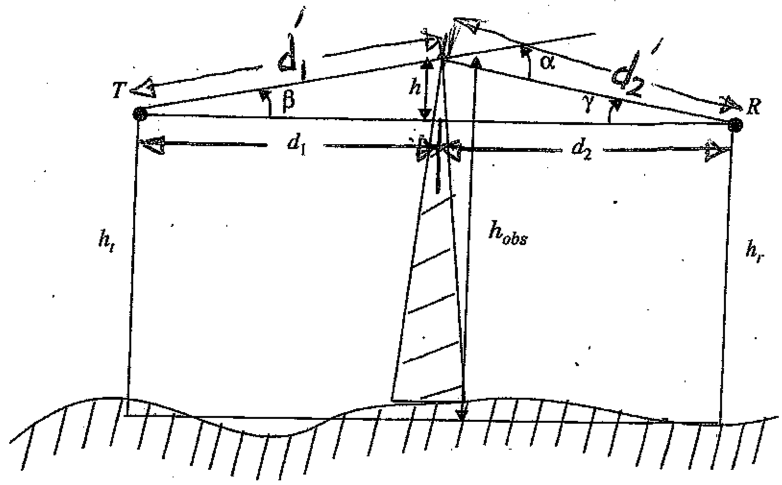
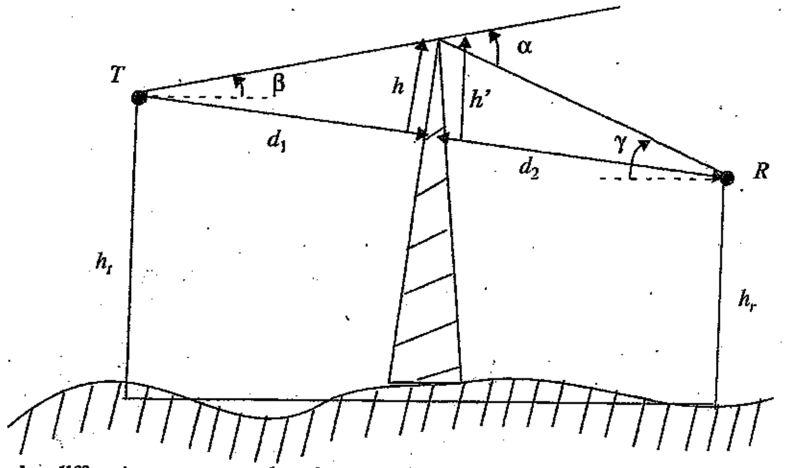
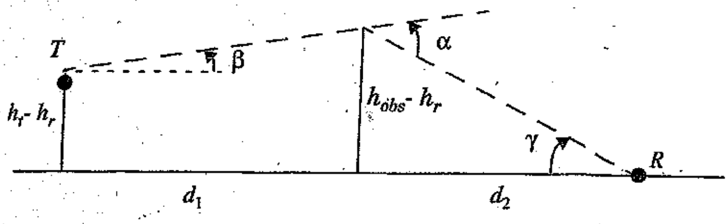


(a) Knife-edge diffraction geometry. The point T denotes the transmitter and R denotes the receiver, with an infinite knife-edge obstruction blocking the line-of-sight path.



(b) Knife-edge diffraction geometry when the transmitter and receiver are not at the same height. Note that if α and β are small and $h \ll d_1$ and d_2, then h and h' are virtually identical and the geometry may be redrawn as shown in Figure 4.10c.



(c) Equivalent knife-edge geometry where the smallest height (in this case h_r) is subtracted from all other heights.

Figure 4.10 Diagrams of knife-edge geometry.

$$\Delta \simeq d_1\left[\cancel{1}+\frac{1}{2}\left(\frac{h}{d_1}\right)^2\right]+d_2\left[\cancel{1}+\frac{1}{2}\left(\frac{h}{d_2}\right)^2\right]-\cancel{d_1}-\cancel{d_2}$$

$$\boxed{\Delta = \frac{h^2}{2}\left(\frac{1}{d_1}+\frac{1}{d_2}\right) = \frac{h^2}{2}\,\frac{d_1+d_2}{d_1 d_2}}$$

$\therefore$ The phase difference $\phi = \frac{2\pi}{\lambda}\Delta = \frac{\pi h^2}{\lambda}\left(\frac{d_1+d_2}{d_1 d_2}\right)$

14/4

Since $d \gg h_t, h_r$ all angles α, β, γ are small and hence we can use $(\tan x \simeq x)$

From Fig. (a)

$$\alpha = \beta + \gamma \approx \overset{\tan\beta}{\frac{h}{d_1}} + \overset{\tan\gamma}{\frac{h}{d_2}} = h\left(\frac{d_1+d_2}{d_1 d_2}\right) \quad \text{OR} \quad h = \alpha \frac{d_1 d_2}{d_1+d_2} \quad (A)$$

$$\therefore \cancel{\Phi = \frac{\pi h}{\lambda} \times h\left(\frac{d_1+d_2}{d_1 d_2}\right) = \frac{\pi h}{\lambda}\alpha}$$

Defining the Fresnel-Kirchoff diffraction parameter v

$\cancel{v = h\sqrt{\;}}$

$$\boxed{v = h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}}} \quad \text{For } h_t = h_r \text{ (Fig. a)}$$

$\rightarrow$ such that $\boxed{\Phi = \frac{\pi}{2} v^2}$

Using eqn (A) above to remove (h) $\Rightarrow$ $v = \alpha \frac{d_1 d_2}{d_1+d_2}\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}}$

$$\boxed{v = \alpha\sqrt{\frac{2 d_1 d_2}{\lambda(d_1+d_2)}}}$$

For Fig b. $\Rightarrow$ Fig. c where $h_t > h_r$ since we do not have (h).

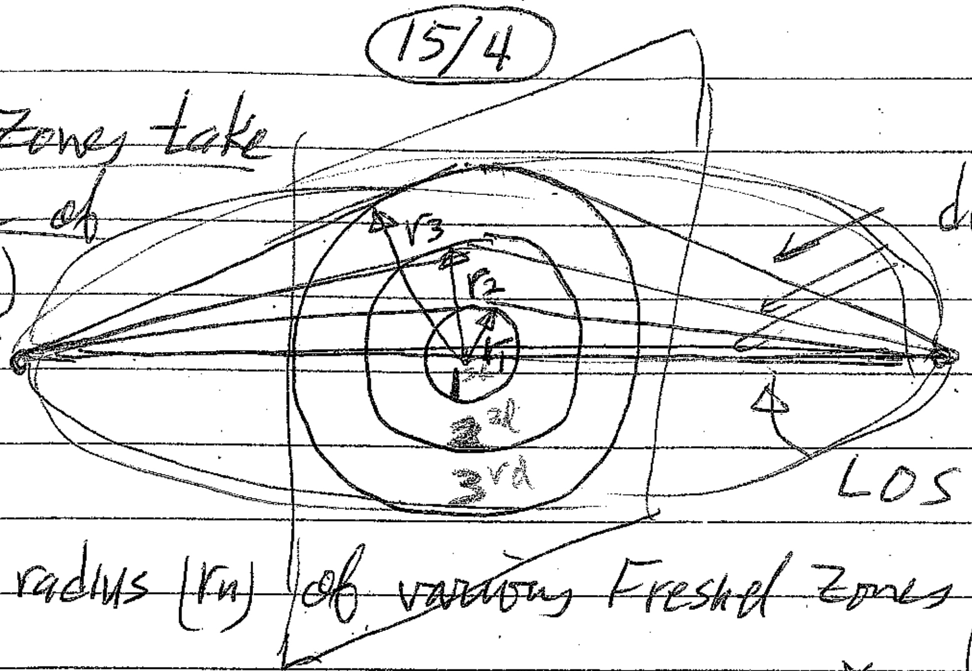
(since α, β, γ are small we can approximate d_1, d_2 in Fig. b) to be equal to d_1, d_2 in (Fig. a)).

and hence $\boxed{\Phi = \frac{\pi}{2} v^2}$

* Fresnel zones represent successive regions, that alternatively provide constructive and destructive interference to the total received signal, where diffracted path and LOS path have a difference of $(n\lambda/2)$.

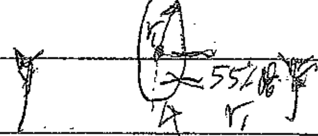
(15/4)

Fresnel Zones take the shape of ellipsoids)



To find radius (r_n) of various Fresnel Zones

$$\Delta = n\frac{\lambda}{2} = \frac{h^2}{2}\left(\frac{d_1+d_2}{d_1 d_2}\right)$$



$$\boxed{r_n = h(n) = \sqrt{\frac{n\lambda d_1 d_2}{d_1+d_2}}} \qquad \sqrt{\frac{n\lambda d_1 d_2}{d_1+d_2}}$$

The path passing at r_1 exceeds LOS path by $\lambda/2$
" " " " r_2 " " " " λ
" " " " r_3 " " " " $3\lambda/2$

not significantly alter diffrac Loss

∘ Paths passing through 1st Fresnel Zone add constructively to LOS path
" " " 2nd " " destructive
" " " 3rd " " constructive

Odd numbered Zones (1, 3, 5) add to signal.
Even " " (2, 4, 6 subtract from signal.
If an obstruction does not block the volume contained within the 1st Fresnel zone, diffraction loss may be neglected
Rule of thumb in the design of LOS Microwave Systems:-
As long as 55% of 1st zone is clear, further clearance does

Received Power $\doteq P_{LOS} + G_d(dB)$

16/4 $= 10\log\left[\frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}\right] + G_d(dB)$

Diffraction gain G_d (dB)

The diffraction gain due to the presence of a knife-edge, as compared to free space E-field

$F(v) = \frac{E_d}{E_0}$

$= 10\log|F(v)|^2 = 20\log|F(v)|$

$$\boxed{G_d(dB) = 20\log|F(v)|}$$

E_d: Elect. field of the knife-edge diffracted

E_0: Free space field in the absence of ground and knife-edge

Fig. page. 132

OR

$$G_d(dB) = \begin{cases} 0 & v \le -1 \\ 20\log(0.5 - 0.62v) & -1 \le v \le 0 \\ 20\log(0.5\exp(-0.95v)) & 0 \le v \le 1 \\ 20\log\left(0.4 - \sqrt{0.1184 - (0.38 - 0.1v)^2}\right) & 1 \le v \le 2.4 \\ 20\log\left(\frac{0.225}{v}\right) & v > 2.4 \end{cases}$$

→ Find the diffraction Loss

ex) For fig. (4.12), assume $\lambda = 1/3$, $d_1 = d_2 = 1$ km

a) $h = 25$ m b) $h = 0$ m c) $h = -25$ m

For each of a, b, c, identify the Fresnel zone within which the tip of the obstruction lies.

a) Equal $h_t = h_r \Rightarrow v = h\sqrt{\frac{2(d_1+d_2)}{\lambda d_1 d_2}} = 25\sqrt{\frac{2(1000+1000)}{(1/3)(1000\times1000)}}$

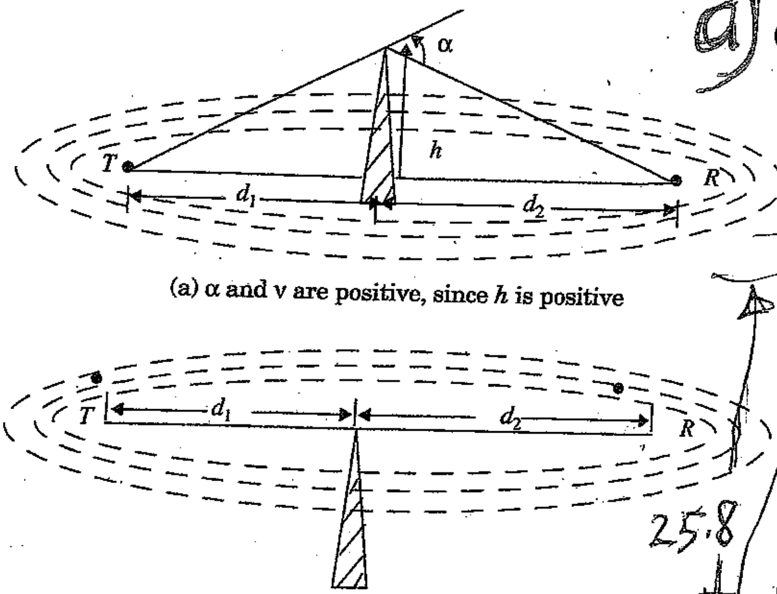
$v = 2.74$ From curves

Loss $G_d(dB) = 22$ dB

$r_n = h(n) = \sqrt{\frac{n\lambda d_1 d_2}{d_1+d_2}}$ we have $h = 25$ m

17/4

a) obstructi... within ... Zone (blocks ... Zones)

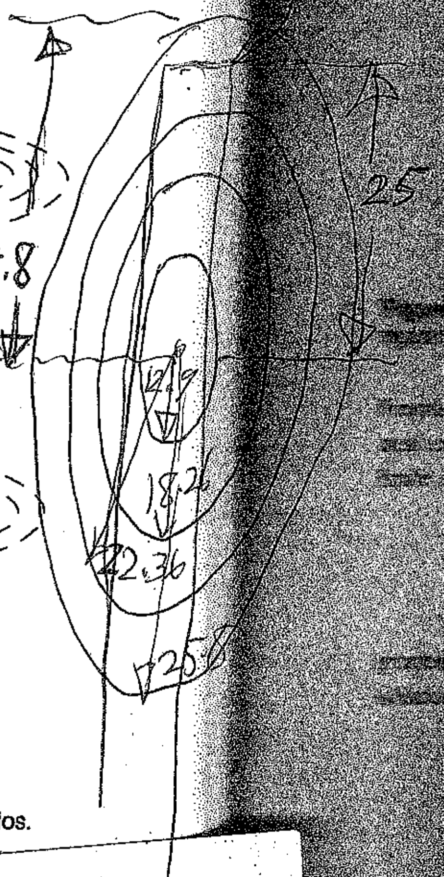


(a) α and ν are positive, since h is positive

(b) α and ν are equal to zero, since h is equal to zero

(c) α and ν are negative, since h is negative

Figure 4.12 Illustration of Fresnel zones for different knife-edge diffraction scenarios.



$r_1 = 12.91\,m$
$r_2 = 18.26\,m$
$r_3 = 22.36\,m$
$r_4 = 25.82\,m$

secondary Huygen's sources in the plane abo...
a knife-edge diff...

$$25 = \sqrt{\frac{n\,(1/3)\,(1000)^2}{2 \times 1000}} \Rightarrow \boxed{n = 3.75}$$

∴ the tip of obstruction blocks the 1st (3-zones).

b) $v = 0$ (since $h = 0$) $\Rightarrow$ Loss $= G_d(dB) = 6$ dB

also $n = 0$

tip of obstruction lies in the middle of the first zone.

c) $v = -2.74 \Rightarrow$ Loss $= 1$ dB

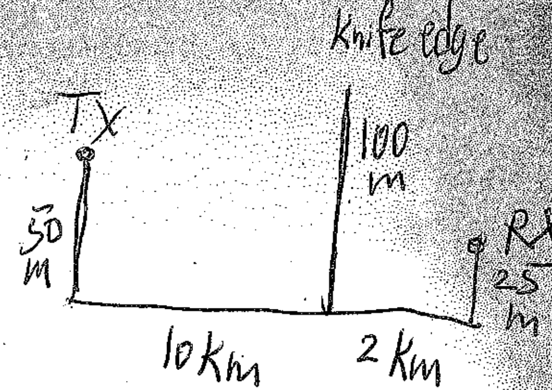
$$-25 = \sqrt{\frac{n(1/3)(1000)^2}{\ldots}} \longrightarrow \boxed{n = 3.75} \rightarrow$$ obstruction lies within 4th zone

18a/4

ex) $f = 900$ MHz

a) Find diffraction Loss.

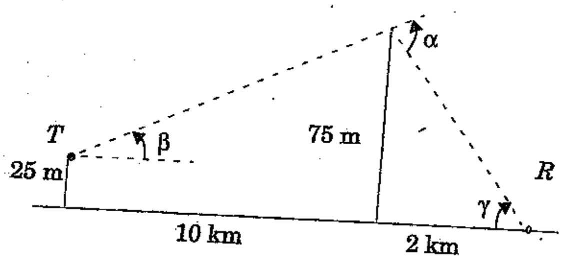
b) Find the height of obstacle required to induce 6 dB diffraction Loss.



Solution

(a) The wavelength $\lambda = \frac{c}{f} = \frac{3\times10^8}{900\times10^6} = \frac{1}{3}$ m.

Redraw the geometry by subtracting the height of the smallest structure.



$$\beta = \tan^{-1}\left(\frac{75-25}{10000}\right) = 0.2865°$$

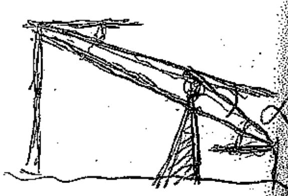
$$\gamma = \tan^{-1}\left(\frac{75}{2000}\right) = 2.15°$$

and

$$\alpha = \beta + \gamma = 2.434° = 0.0424 \text{ rad}$$

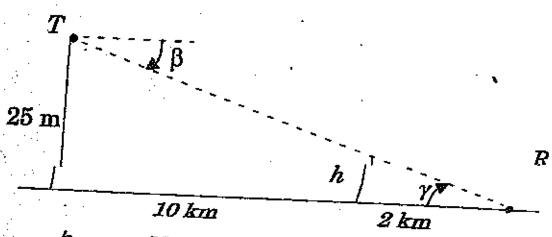
Then using Equation (4.56)

$$v = 0.0424\sqrt{\frac{2\times10000\times2000}{(1/3)\times(10000+2000)}} = 4.24.$$



From Figure 4.14 or (4.61.e), the diffraction loss is 25.5 dB.

(b) For 6 dB diffraction loss, $v = 0$. The obstruction height h may be found using similar triangles ($\beta = \gamma$), as shown below.

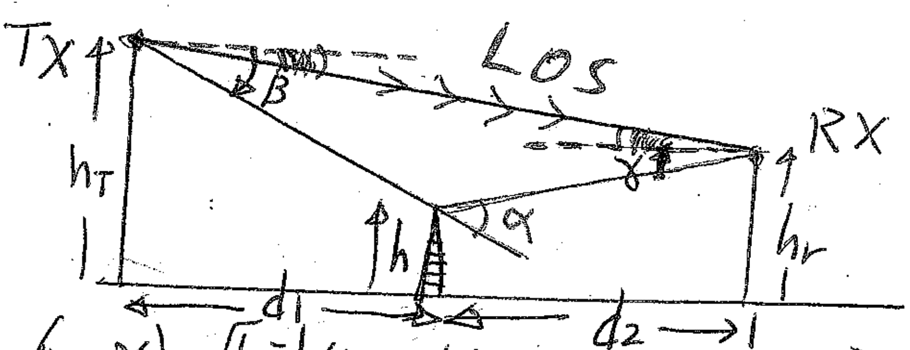


It follows that $\frac{h}{2000} = \frac{25}{12000}$, thus $h = 4.16$ m.

4.7.3 Multiple Knife-edge Diffraction

In many practical situations, especially in hilly terrain, the propagation path may consist of more than one obstruction, in which case the total diffraction loss due to all of the obstacles must be computed. Bullington [Bul47] suggested that the series of obstacles be replaced by a single

For $h_T \neq h_r$, the value of (α) can be (-ve)



$$\alpha = -(\beta + \gamma) = -\left[\tan^{-1}\left(\frac{h_T - h}{d_1}\right) + \tan^{-1}\left(\frac{h_r - h}{d_2}\right)\right]$$

We give it a (-ve) sign since obstruction is below LOS and hence (h) or (α) must be (-ve).

ex) It is desired to provide a good LOS link between two points 10-Km apart. A (15-m) hill is (2 Km) away from TX. Assuming $h_T = h_r$, what should the minimum antenna (h_r, h_T) height be? Frequency = 2.4 GHz.

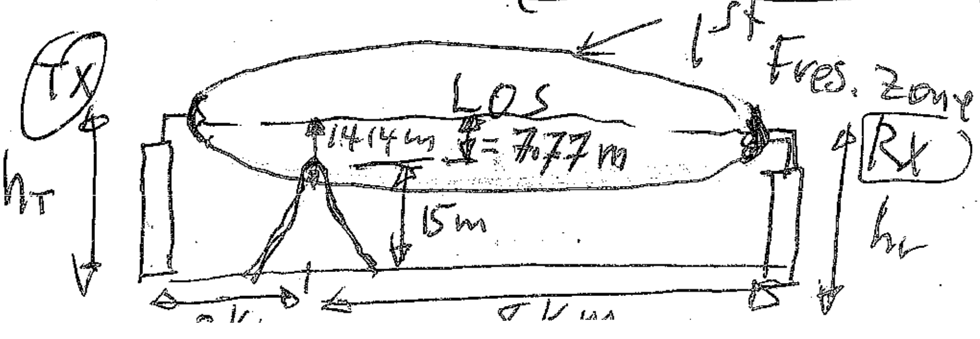
$\lambda = c/f = 3\times10^8 / 2.4\times10^9 = 1/8$ m

A good LOS requires at least 55% clearence of 1st zone.

$$r_1 = \sqrt{\frac{n\lambda d_1 d_2}{d_1 + d_2}} = \sqrt{\frac{1 \times \frac{1}{8} \times 2000 \times 8000}{2000 + 8000}} = 14.14 \text{ meters}$$

55% clearance $= 0.55 \times 14.14 = 7.77$ m } This should be the distance from LOS to top of hill

$\therefore h_T = h_r = 15 + 7.77 = \boxed{22.77 \text{ meters}}$



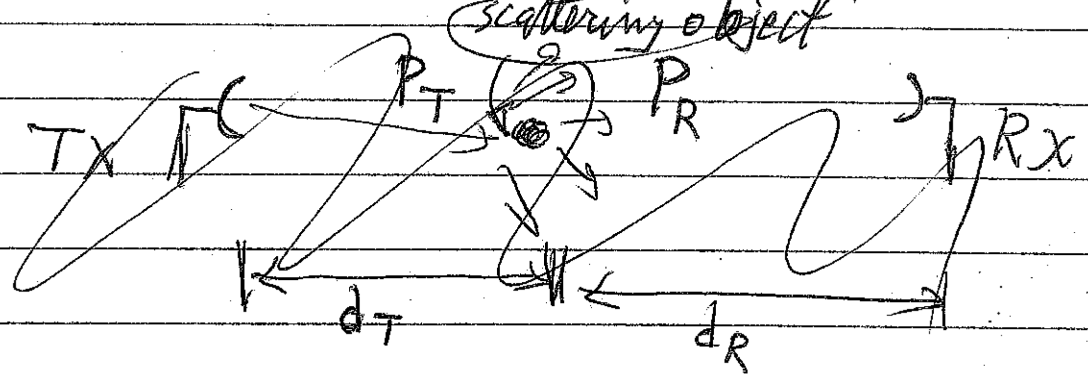
(19/4)

Scattering (Radar Cross section model)

(Lamps, posts, trees)

For urban mobile radio system, the radar cross section (RCS) model can be used to calculate the radiated power due to scattering in the direction of the RX as:-

$$P_R(dBm) = P_T(dBm) + G_T(dBi) + 20\log(\lambda) + RCS(dB.m^2) - 30\log(4\pi) - 20\log d_T - 20\log d_R$$



$$\underset{[dB.\,m^2]}{RCS} \simeq 10\log\left[\frac{\text{Object area}(m^2)}{1\,m^2}\right]$$

This equation can be applied to scatterers in the far-field of both TX & RX and it is useful for predicting received power which scatters of Large objects such as buildings

~~For medium and large size buildings located 5-10 Km away~~ $\cancel{RCS \simeq \underbrace{(14.1 - 55.7)}_{\text{range}} dB.m^2}$

(20/4)

Path Loss Models

Radio wave propagation models are derived to fit and recreate real measured data.

Log - distance path Loss Model

$P_{rec} \propto d^{-n}$

Practical measurements in different environment give the following logarithmic model

Average Large-scale path Loss $\overline{PL}(d) \propto \left(\frac{d}{d_0}\right)^n$

OR $\overline{PL}(d) = K\left(\frac{d}{d_0}\right)^n$ — (A)

$$\boxed{\overline{PL}(d)[dB] = \overline{PL}_{dB}(d_0) + 10n\log\left(\frac{d}{d_0}\right)}$$

avg path loss

$$\frac{\overline{PL}(d)}{\overline{PL}(d_0)} = \left(\frac{d}{d_0}\right)^n$$

where n: path loss exponent
d_0: close-in reference distance
d: distance between TX and RX

Env	n
Free space	2
Urban area cellular radio	2.7 - 3.5
Shadowed urban cell	3 - 5
In building Loss	1.6 - 1.8
obstructed in building	4 - 6
" " factories	2 - 3

At $d = d_0$

$\overline{PL}(d_0) = K\left(\frac{d_0}{d_0}\right)^n = K$

From (A)

$\overline{PL}(d)[dB] = 10\log K\left(\frac{d}{d_0}\right)^n$

$= 10\log K + 10n\log\left(\frac{d}{d_0}\right)$

$= \overline{PL}_{[dB]}(d_0) + 10n\log\left(\frac{d}{d_0}\right)$

Log-normal shadowing

The previous $\overline{PL(d)}$ represents the average value of path Loss for certain (d).

Practical measurements show that

$$\boxed{PL(d)[dB] = \overline{PL}(d) + X_\sigma}$$

Average

$$\boxed{= \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right) + X_\sigma}$$

Average

and

$$\boxed{P_r(d)[dBm] = P_t[dBm] - PL(d)[dB]}$$

Antenna gains are included in PL(d)

X_σ : zero-mean Gaussian distributed R.V. (in dB) with standard deviation σ (in dB).

(n, σ) are estimated practically by fitting the mathematical model to measured data.

Taking Average

$$\overline{P_r(d)}[dB] = \overline{P_r}(d_0)_{dB} - 10n\log\frac{d}{d_0}$$

Prob.

Fig. (4.17)

Rx Power at d

$$Pr[P_r(d) > \gamma] = Q\left(\frac{\gamma - \overline{P_r(d)}}{\sigma}\right)$$

where $Q(z) = \frac{1}{\sqrt{2\pi}} \int_z^\infty e^{-x/2}\, dx = \frac{1}{2}\left[1 - erf\left(\frac{z}{\sqrt{2}}\right)\right]$

$Q(z) = 1 - Q(-z)$ $\qquad erf(z) = \frac{2}{\sqrt{\pi}} \int_0^z e^{-x^2} dx$

(21b/4)

ex) A cellular System with $f = 900\,MHz$, Tx-power = 1 watt

Base station $G_t = 3\,dB$, $G_r = 0\,dB$

Region is characterized by $n = 4$, $\sigma = 8\,dB$

Close-in distance $d_0 = 1\,Km$. If a mobile Rx is placed at distance $d = 5\,Km$, Find

$Prob[P_r(5Km) > -90\,dBm]$

-90 ← Threshold for good detection

$$P_{r}(d_0)_{dB} = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2}$$

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{900\times10^6} = \boxed{\frac{1}{3}\,m}$$

$G_t = 10^{3/10} = 2$

$G_r = 10^{0} = 1$

$$P(1000m) = \frac{1\times2\times1\times(1/3)^2}{(4\pi)^2(1000)^2\times1} = 1.407\times10^{-9}\ \text{Watt}$$

$$P_r(d_0) = 10\log\frac{1.407\times10^{-9}}{0.001} = \boxed{-58.52}\ dBm$$

$$P_r(5\,Km) = P_r(1000m) - 10\,n\log\frac{5000}{1000}$$

$$= -58.52 - 10\times4\times\log\frac{5000}{1000} = \boxed{-86.48}\ dBm$$

$$Prob.[P_r(5Km) > -90] = Q\left(\frac{-90 - P_r(5Km)}{\sigma}\right)$$

$$= Q\left(\frac{-90-(-86.48)}{8}\right) = Q(-0.44)$$

$$= 1 - Q(0.44) = 1 - 0.32 = \boxed{0.68}$$

(21c/4)

ex) Tx power of 15 w with an antenna of gain $G_t(dB) = 12$ dB.
Rx $G_{r_{dB}} = 3$ dB, & receiver BW = 30 KHz.
$f_c = 1800$ MHz, Noise figure = 8 dB.
$n = 4$, $\sigma = 8$ dB, $d_0 = 1$ Km.
$\lambda = 3\times10^8/1800$
Find the max. T-R separation that will ensure that an SNR of at least 20 dB is provided 95% of the time

$$P_r(d_0) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d_0^2}$$

$G_{t_{dB}} = 12 = 10 \log G_t$

$\boxed{G_t = 10^{12/10} = \boxed{15.85}}$

$\boxed{G_r = 10^{3/10} = \boxed{2}}$

$c = f\lambda$

$\lambda = c/f = \frac{3\times10^8}{1800\times10^6} = \boxed{\frac{1}{6}}$ m

$$P_r(1Km) = \frac{15 \times 15.85 \times 2 \times (1/6)^2}{(4\pi)^2 \times (1000)^2}$$

$= 8.364\times10^{-8}$ watt

$= 10 \log \frac{8.364\times10^{-8}}{0.001} = -40.77$ dBm

0.001 ← mw

$F_{dB} = 8$ dB $= 10 \log F \Rightarrow F = 10^{8/10} = 6.3$ Noise Figure

Noise floor $= K \cdot BW \cdot F \cdot T_0$

$= (1.38\times10^{-23}) \times (30\times10^3) \times (6.3) \times (290)$

$= 7.56\times10^{-16} = \boxed{-121.2 \text{ dBm}}$

20 dB = RX signal power dBm − (Noise)

$20 = P_r - (-121.2)$ desired RX Power $P_r = -101.2$ dBm

prob.[SNR ≥ 20 dB] = 0.95 = Prob[$P_r(d) - (-121.2) \geq 20$]

= Prob.[$P_r(d) \geq -101.2$ dBm] = 0.95

$$Q\left(\frac{-101.2 - \overline{P_r(d)}}{\sigma}\right) = 0.95$$ ← NOT in Q-tables $\left(>\frac{1}{2}\right)$

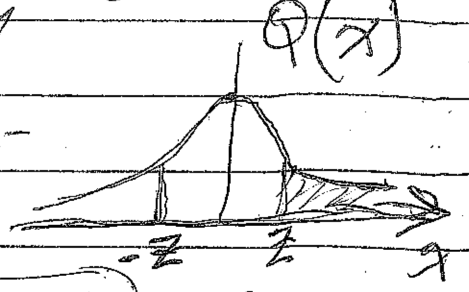
(21d/4)

~~End of Chap.4~~

~~-101.2 $\overline{P_r}$~~ & ~~1.645~~

If $0.95 > 0.5$

$$\boxed{Q(-z) = 1 - Q(z)}$$



$$Q\left(\frac{101.2+\overline{P_r}}{8}\right) = 1 - 0.95 = 0.05 \rightarrow \text{found in Tables}$$

$$\frac{101.2+\overline{P_r}}{8} = 1.64$$

$$\overline{P_r} = 8 \times 1.64 - 101.2 = \boxed{-88.04 \text{ dBm}}$$

$$\overline{P_r}(d) = -88.04 = \overline{P_r}(d_0) - 10n\log\left(\frac{d}{d_0}\right)$$

$$-88.04 = -40.77 - 10 \times 4 \log \frac{d}{d_0}$$

$$\log \frac{d}{d_0} = 1.182$$

$$\frac{d}{d_0} = 10^{1.182} = 15.2$$

1 Km

$$d = 15.2 \times d_0 = \boxed{15.2 \text{ Km}}$$

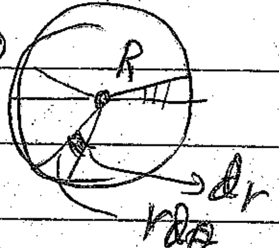
Suggested Problems Chap. 4

4.12, 15, 17, 19, 22, 23, 25, 28, 29

Determination of percentage of coverage area

It is useful to compute how the boundary coverage relates to the percent of area covered within the boundary.

Given a desired received signal strength threshold (γ), we will compute $\boxed{U(\gamma)}$, the percentage of area where the received signal $\geq \gamma$, given a known likelihood of coverage at the cell boundary (circular cell rad. $= R$).



$$U(\gamma) = \frac{1}{\pi R^2} \int_0^{2\pi}\int_0^{R} \text{Prob.}\left[P_r(r) > \gamma\right] r\,dr\,d\theta$$

Total area of circular area

RX power is Gaussian in (dB)

But (1) $\text{Prob.}\left[\overline{P_r(r)} > \gamma\right] = Q\left(\dfrac{\gamma - \overline{P_r(r)}}{\sigma}\right)$ $\overline{PL(r)}$

(2) $= \dfrac{1}{2} - \dfrac{1}{2}\,\text{erf}\left(\dfrac{\gamma - \overline{P_r(r)}}{\sigma\sqrt{2}}\right)$

(3) $= \dfrac{1}{2} - \dfrac{1}{2}\,\text{erf}\left[\dfrac{\gamma - \left[P_t - \left(\overline{PL}(d_0) + 10n\log(r/d_0)\right)\right]}{\sigma\sqrt{2}}\right]$

In order to determine the path loss as referenced to the cell boundary ($r = R$) it is clear that

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-x^2}\,dx$$

$$\overline{PL}(r) = \overline{PL}(d_0) + 10n\log\left(\frac{R}{d_0}\right) + 10n\log\left(\frac{r}{R}\right)$$

$\log x = (\log e) x (\ln x)$

23/4

$$\text{Prob}[P_r(r) > \gamma] = \frac{1}{2} - \frac{1}{2}\text{erf}\left[\frac{\gamma - [P_t - (\overline{PL}(d_0) + 10n\log(\frac{R}{d_0}) + 10n\log(\frac{r}{R}))]}{\sigma\sqrt{2}}\right]$$

Let $a = (\gamma - P_t + \overline{PL}(d_0) + 10n\log(R/d_0))/\sigma\sqrt{2}$

$b = 10n\log e/\sigma\sqrt{2}$

$a + b\ln\frac{r}{R}$

$$U(\gamma) = \frac{1}{\pi R^2}\int_0^{2\pi}\int_0^R \left(\frac{1}{2} - \frac{1}{2}\text{erf}\left[a + b\ln\frac{r}{R}\right]\right) r\,dr\,d\theta$$

$$= \frac{1}{\pi R^2}\left\{\int_0^{2\pi}\int_0^R \frac{1}{2} r\,dr\,d\theta - \int_0^{2\pi}\int_0^R \frac{1}{2}\text{erf}\left[a + b\ln\frac{r}{R}\right] r\,dr\,d\theta\right\}$$

$$= \frac{1}{\pi R^2}\left\{\int_0^{2\pi} \frac{r^2}{2\times 2}\Big|_0^R d\theta - \int_0^{2\pi}\int_0^R \frac{1}{2}\text{erf}\left[a + b\ln\frac{r}{R}\right] r\,dr\,d\theta\right\}$$

$$= \frac{1}{\pi R^2}\left\{\theta\Big|_0^{2\pi}\frac{R^2}{4} - \int_0^{2\pi}\int_0^R \frac{1}{2}\text{erf}\left[a + b\ln\frac{r}{R}\right] r\,dr\,d\theta\right\}$$

$$= \frac{2\pi R^2}{\pi R^2 \times 4} - \frac{1}{\pi R^2}\int_0^R \frac{1}{2}\text{erf}\left[a + b\ln\frac{r}{R}\right] r\,dr \int_0^{2\pi} d\theta$$

$$U(\gamma) = \frac{1}{2} - \frac{2\pi}{\pi R^2}\int_0^R \frac{1}{2} r\,\text{erf}\left(a + b\ln\frac{r}{R}\right) dr$$

$$= \frac{1}{2} - \frac{1}{R^2}\int_0^R r\,\text{erf}\left(a + b\ln\frac{r}{R}\right) dr$$

$\boxed{\log_a x = \frac{\ln x}{\ln a}}$

OR

$\boxed{\ln x = \frac{\log x}{\log e}}$

We substitute ~~$t = a + b\log(r/R)$~~

$t = a + b\ln(r/R)$

$\Rightarrow \frac{t-a}{b} = \ln\frac{r}{R} \Rightarrow r = Re^{\frac{t-a}{b}}$

$\Rightarrow dr = \frac{R}{b}e^{\frac{t-a}{b}}dt$

Integration Limits

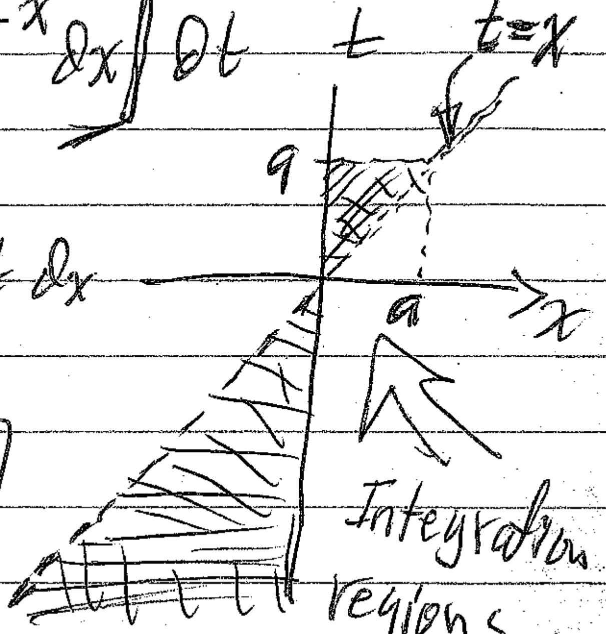
t	r
$-\infty$	0
a	R

24/4

$$U(\gamma) = \frac{1}{2} - \frac{1}{\cancel{R}^2}\int_{-\infty}^{a} \cancel{R}\, e^{\frac{t-a}{b}}\, \mathrm{erf}(t)\, \frac{R}{b}\, e^{\frac{t-a}{b}}\, dt$$

$$= \frac{1}{2} - \frac{1}{b}\int_{-\infty}^{a} e^{\frac{2(t-a)}{b}}\, \mathrm{erf}(t)\, dt \qquad \left\{ \mathrm{erf}(z) = \frac{2}{\sqrt{\pi}}\int_0^z e^{-x^2}\, dx \right.$$

$$= \frac{1}{2} - \frac{1}{b}\int_{-\infty}^{a} e^{\frac{2(t-a)}{b}} \left[\frac{2}{\sqrt{\pi}} \int_0^t e^{-x^2}\, dx \right] dt$$



$$= \frac{1}{2} - \frac{2e^{-2a/b}}{\sqrt{\pi}\, b} \left[\int_0^a e^{-x^2} \int_{t=x}^{a} e^{2t/b}\, dt\, dx - \int_{-\infty}^{0} e^{-x^2} \int_{t=-\infty}^{x} e^{2t/b}\, dt\, dx \right]$$

لا تكامل e^{x^2} موجبة لأنه تحول الى erf السالب ال x السالب

$$= \frac{1}{2} - \frac{2e^{-2a/b}}{b\sqrt{\pi}} \left\{ \int_0^a e^{-x^2} \left[\frac{e^{2t/b}}{2/b} \right]_x^a dx - \int_{-\infty}^{0} e^{-x^2} \left[\frac{e^{2t/b}}{2/b} \right]_{-\infty}^{x} dx \right\}$$

$$= \frac{1}{2} - \frac{2e^{-2a/b}}{b\sqrt{\pi}} \left\{ \frac{b}{2}\int_0^a e^{-x^2} \left[e^{2a/b} - e^{2x/b} \right] dx - \frac{b}{2}\int_{-\infty}^{0} e^{-x^2} \left[e^{2x/b} - e^{-\infty} \right] dx \right\}$$

$$= \frac{1}{2} - \frac{e^{-2a/b}}{\sqrt{\pi}} \left\{ \left[e^{2a/b}\int_0^a e^{-x^2} dx - \int_0^a e^{-x^2 + \frac{2}{b}x} dx \right] - \left[\int_{-\infty}^{0} e^{-x^2 + \frac{2}{b}x} dx \right] \right\}$$

$$= \frac{1}{2} - \frac{1}{2}\,\frac{2}{\sqrt{\pi}}\int_0^a e^{-x^2} dx + \frac{e^{-2a/b}}{\sqrt{\pi}}\int_0^a e^{\frac{1}{b^2}}\, e^{-\left(x - \frac{1}{b}\right)^2} dx + \frac{e^{-2a/b}}{\sqrt{\pi}}\int_{-\infty}^{0} e^{\frac{1}{b^2}}\, e^{-\left(x + \frac{1}{b}\right)^2} dx$$

Let $m = x - 1/b$ let $n = x + 1/b$

25/4

$$u(x) = \frac{1}{2} - \frac{1}{2}\,erf(a) + e^{(1-2ab)/b^2}\,\frac{1}{2}\times\frac{2}{\sqrt{\pi}}\int_{-1/b}^{a-\frac{1}{b}} e^{-m^2}\,dm + e^{(1-2ab)/b^2}\,\frac{1}{2}\,\frac{2}{\sqrt{\pi}}\int_{-\infty}^{1/b} e^{-n^2}\,dn$$

كتعريف erf هو بحيث سالبة لأنه
نعكس الحدود سالب لـ x

$$= \frac{1}{2} - \frac{1}{2}\,erf(a) + \frac{1}{2}\,e^{(1-2ab)/b^2}\,\frac{2}{\sqrt{\pi}}\left[\cancel{-\int_{-1/b}^{0} e^{-m^2}\,dm} + \int_{0}^{a-1/b} e^{-m^2}\,dm + \int_{-\infty}^{0} e^{-n^2}\,dn + \cancel{\int_{0}^{1/b} e^{-n^2}\,dn}\right]$$

$$= \frac{1}{2} - \frac{1}{2}\,erf(a) + \frac{1}{2}\,e^{(1-2ab)/b^2}\left[\frac{2}{\sqrt{\pi}}\int_{0}^{(ab-1)/b} e^{-m^2}\,dm + \frac{2}{\sqrt{\pi}}\int_{-\infty}^{0} e^{-n^2}\,dn\right]$$

$$= \frac{1}{2} - \frac{1}{2}\,erf(a) + \frac{1}{2}\,e^{\frac{1-2ab}{b^2}}\left[erf\left(\frac{ab-1}{b}\right) + 1\right]$$

erf is odd fn

$$= \frac{1}{2} - \frac{1}{2}\,erf(a) + \frac{1}{2}\,e^{\frac{1-2ab}{b^2}}\left[1 - erf\left(\frac{1-ab}{b}\right)\right]$$

$erf(x) = -erf(-x)$

26/4

$$U(\gamma) = \frac{1}{2} - \frac{1}{\cancel{R^2}} \int_{-\infty}^{a} \cancel{R} e^{\frac{t-a}{b}} \operatorname{erf}(t) \frac{\cancel{R}}{b} e^{\frac{t-a}{b}}\, dt$$

$$= \frac{1}{2} - \frac{e^{-\frac{2a}{b}}}{b} \int_{-\infty}^{a} e^{2t/b} \operatorname{erf}(t)\, dt$$

But (from tables)

$$\boxed{\int \operatorname{erf}(Ax) e^{Bx} dx = \frac{1}{B} e^{Bx} \operatorname{erf}(Ax) - \frac{1}{B} e^{\frac{B^2}{4A^2}} \operatorname{erf}\left(Ax - \frac{B}{2A}\right)}$$

$\boxed{A = 1}$ $\boxed{B = 2/b}$ $\therefore \frac{B^2}{4A^2} = \frac{4}{b^2 \times 4} = \frac{1}{b^2}$

$$\therefore U(\gamma) = \frac{1}{2} - \frac{e^{-2a/b}}{b} \left\{ \left[\frac{b}{2} e^{\frac{2}{b}a} \operatorname{erf}(a) - \frac{b}{2} e^{1/b^2} \operatorname{erf}\left(a - \frac{1}{b}\right) \right] - \left[\frac{b}{2} e^{\frac{2}{b}(-\infty)} \operatorname{erf}(-\infty) - \frac{b}{2} e^{\frac{1}{b^2}} \operatorname{erf}\left(-\infty - \frac{1}{b}\right) \right] \right\}$$

$e^{\frac{2}{b}(-\infty)} = 0$

we cannot plot curves (infinite no. of P_t, γ, R ...) for each

$$= \frac{1}{2} - \frac{1}{2} \operatorname{erf}(a) + \frac{1}{2} e^{\frac{1-2ab}{b^2}} \operatorname{erf}\left(\frac{ab-1}{b}\right) - \frac{1}{2} e^{\frac{1-2ab}{b^2}} \operatorname{erf}(-\infty)$$

$\operatorname{erf}(-\infty) = -1$

$$U(\gamma) = \frac{1}{2} - \frac{1}{2} \operatorname{erf}(a) + \frac{1}{2} e^{\frac{1-2ab}{b^2}} \left[1 - \operatorname{erf}\left(\frac{1-ab}{b}\right) \right]$$

a: depends on Prob[Power(R) > γ] b: depends on $\frac{\sigma}{n}$ [$\operatorname{erf}(x) = -\operatorname{erf}(-x)$]

By choosing the average received power at radius $= R$ as

$$\overline{P_r(R)} = \gamma$$

$$\therefore \Rightarrow a = \gamma - \underbrace{(P_t - \overline{PL}(d_0) + 10 n \log(R/d_0))}_{= \overline{P_r(R)} = \gamma} / \sigma\sqrt{2} = 0$$

$$\therefore U(\gamma) = \frac{1}{2}\left[1 - \operatorname{erf}(0) + e^{1/b^2}\left(1 - \operatorname{erf}\left(\frac{1}{b}\right)\right)\right] =$$

$$= \boxed{\frac{1}{2}\left[1 + e^{1/b^2}\left(1 - \operatorname{erf}(1/b)\right)\right]} \quad \text{for } \overline{P_r(R)} = \gamma$$

Fig. 4-18

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Notes on the curve of $U(\gamma)$ r4 (Fig. 4.18)

RX power at $d=R$

A – Given fixed (specified) values of σ and n, the curves of $U(\gamma)$ do not depend on (γ), but only on Prob.$[P_r(R)>\gamma]$. This is because curve is plotted as follows:–

1– Choose Prob.$[P_r(R)>\gamma]$ as a certain specified value (Do not specify γ-itself).

~~This will specify the value of~~

2– Since Prob.$[P_r(R)>\gamma] = Q\left(\dfrac{\gamma - \overline{P_r(\gamma)}}{\sigma}\right)$ then

specify Prob.$[P_r(R)>\gamma] \Rightarrow$ specifies the value of

$(\gamma - \overline{P_r(R)})/\sigma \Rightarrow$ specify $\underbrace{\dfrac{\gamma - [P_t - \overline{PL}(d_0) + 10n\log(R/d_0)]}{\sigma}}_{\sqrt{2}a}$

$\Rightarrow$ specify $\boxed{\sqrt{2}a} \Rightarrow$ specifies a.

3 – (b) is also specified since n, σ are specified.

4 – $U(\gamma)$ is specified since it is a function of (a, b), without specifying γ-value.

B – $U(\gamma)$ depends on $\boxed{\frac{\sigma}{n}}$ and not on (σ) or (n)

As seen above in (A), specifying Prob.$[P_r(R)>\gamma]$ leads to specifying the value of (a) and hence remains to specify the b-value.

But $b = \text{const} \times (n/\sigma)$ and does not depend on σ or n.

$\Rightarrow$ b depends only on the ratio (σ/n).

$\therefore$ $U(\gamma)$ is specified depending on $\begin{cases} \sigma/n \ \& \\ \text{Prob.}[P_r(R)>\gamma] \end{cases}$

28/4

ex) Given the following measurements with close-in distance $\boxed{d_0 = 100m}$ →

T-R distance	RX-power
100 m	0 dBm
200 m	-20 dBm
1000 m	-35 dBm
3000 m	-70 dBm

a) Find the minimum mean square error (MMSE) estimate for the path loss exponent (n).

b) Find (σ). c) Find the estimated (average) received power at d = 2Km).

d) predict the ~~likelihood~~ Prob.[P_r(2 Km) > -60 dBm]

e) Predict percentage of area within a (2 Km) radius cell that RX-signal > -60 dB.

---

a) First we estimate the received powers in the table above in terms of (n) using

$$P_r(d_i) = P_r(d_0) - 10n\log(d_i/d_0)$$

The random part X_σ is assumed = zero for MMSE

$$\hat{P}_r(200m) = 0\,dBm - 10n\log(200/100) = \boxed{-3n}$$

$$P_r(100m) = \boxed{0}\,dBm$$

$$\hat{P}_r(1000m) = 0\,dBm - 10n\log(1000/100) = \boxed{-10n}$$

$$\hat{P}_r(3000m) = 0 - 10n\log(3000/100) = \boxed{-14.77n}$$

The sum of the squared errors between the measured and estimated values of $P_r(d_i)$ is given by

$$J(n) = \sum_{i=1}^{K} (P_i - \hat{P}_i)^2$$

$$J(n) = (0-0)^2 + (-20-(-3n))^2 + (-35-(-10n))^2 + (-70-(-14.77n))^2$$

$$= 6525 - 2887.8n + 327.153n^2$$

To find (n) the minimizes the MMSE error

$$\frac{\partial J(n)}{\partial n} = 0 = -2887.8 + 2\times 327.153\,n \Rightarrow \boxed{\hat{n} = 4.4}$$

b) σ is the sample variance $(x-\bar{x})^2$

$$\sigma^2 = \frac{1}{4}\left[(0-0)^2 + (-20-(-3\times4.4))^2 + (-35-(-10\times4.4))^2 + (-70-(-14.77\times4.4))^2\right]$$

mean value (only d/d_0)

$$\sigma^2 = 38.09\ (dB)^2$$

$$\sigma = 6.17\ dB.$$

c) At $d = 2\,Km$

Average

$$P_r(d=2Km) = \overline{P_r(d_0)} - 10\,n\log\left(\frac{2Km}{d_0}\right)$$

$$= 0 - 10\times4.4\times\log(2000/100) = -57.24\ dBm$$

mean value

Note: To simulate random shadowing at ($d=2Km$), we add (X_σ) Gaussian zero mean (R.V.) to ↑ $\sigma = 6.17$

d) The mean value of the power received at $d = 2Km$ is $P_r(2Km) = -57.24$ dBm. As a Gaussian R.V. in dBm

$$Prob[P_r(2Km) > -60\,dBm] = Q\left[\frac{-60-(-57.24)}{6.17}\right]$$

$$= Q(-0.447) = 1 - Q(0.447)$$

$$= \boxed{67.4\%} \Leftarrow 1 - 0.326$$

e) To find percentage area within 2-km radius cell for $P_r > -60$dBm, we need to find the boundary coverage Prob.$[P_r(2Km) > -60dBm]$ and use (Fig. 4.18). This was found in (d) above a $\simeq 67.4\%$.

Given $(\sigma/n) = 6.17/4.4 = 1.402$.

$$\underbrace{(\sigma/n = 1.402) \ \&\ \text{boundary coverage} = 67.4\%}$$

$$\Rightarrow U(\gamma) = 88\%$$

percentage area coverage $= 88\%$

30+30b Okumura Curve

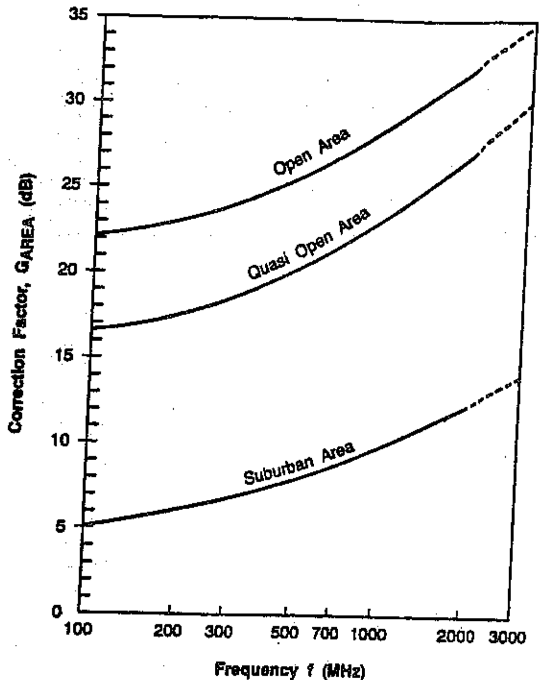


Figure 4.24 Correction factor, G_{AREA}, for different types of terrain [from [Oku68] © IEEE].

Okumura's model is wholly based on measured data and does not provide any analytical explanation. For many situations, extrapolations of the derived curves can be made to obtain values outside the measurement range, although the validity of such extrapolations depends on the circumstances and the smoothness of the curve in question.

Okumura's model is considered to be among the simplest and best in terms of accuracy in path loss prediction for mature cellular and land mobile radio systems in cluttered environments. It is very practical and has become a standard for system planning in modern land mobile radio systems in Japan. The major disadvantage with the model is its slow response to rapid changes in terrain, therefore the model is fairly good in urban and suburban areas, but not as good in rural areas. Common standard deviations between predicted and measured path loss values are around 10 dB to 14 dB.

Example 4.10

Find the median path loss using Okumura's model for d = 50 km, h_{te} = 100 m, h_{re} = 10 m in a suburban environment. If the base station transmitter radiates an EIRP of 1 kW at a carrier frequency of 900 MHz, find the power at the receiver (assume a unity gain receiving antenna).

ex) Find the median path loss using Okumura's model for $d = 50 Km$, $h_{te} = 100 m$, $h_{re} = 10 m$ in suburban environment. If BS radiates $\boxed{EIRP = 1 Kw}$ at $f = 900 MHz$, find the ~~Rec~~ received power assuming $G_r = 1$ (unity gain for receive antenna).

{ G_t is included in EIRP
{ G_r is considered in find equation

$$L_{median} = L_F + A(f,d) - G(h_{te}) - G(h_{re}) - G_{Area}$$

since EIRP is given we will use free-space path loss that does not include antenna gains.

$$L_F = 10 \log \left[\frac{\lambda^2}{(4\pi)^2 d^2} \right] = 10 \log \left[\frac{(3\times10^8 / 900\times10^6)^2}{(4\pi)^2 (50\times10^3)^2} \right] = 125.5 \text{ dB}$$

From Okumura Curves

$(f = 900 MHz,\ d = 50 Km) \Longrightarrow A(f,d) = 43 dB$

From Fig.(4.24)

$(f = 900 MHz,\ \text{suburban area}) \Longrightarrow G_{Area} = 9 dB$

$$G(h_{te}) = 20 \log \left(\frac{\overset{h_{te}}{100}}{200} \right) = -6 \text{ dB}$$

$$G(h_{re}) = 20 \log \left(\frac{\overset{h_{re}}{10}}{3} \right) = 10.46 \text{ dB}$$

$$\therefore L_{median} = 125.5 + 43 - (-6) - (10.46) - 9 = \boxed{155.04 \text{ dB}}$$

$$P_r(d = 50 Km) = EIRP(dBm) - L_{median} + G_r(dB)$$

$$= 10 \log \left(\frac{1000}{0.001} \right) - 155.04 + \underbrace{10 \log 1}_{0}$$

$$= 10 \log 10^6 - 155.04 = 60 - 155.04 = \boxed{-95.04 \text{ dBm}}$$

receiver caused by the second diffraction edge with the first diffraction edge as the source. The two attenuations sum to give the additional loss caused by the obstacles that is added to the free space loss or the plane earth loss, whichever is larger.

For three diffraction edges, the outer diffraction edges must contain a single diffraction edge in between. This is detected by calculating the line between the two outer diffraction edges. If an obstacle between the two outer edges passes through the line, then it is concluded that a third diffraction edge exists (see Figure 4.22). Again, the Epstein and Peterson method is used to calculate the shadow loss caused by the obstacles. For all other cases of more than three diffraction edges, the profile between the outer two obstacles is approximated by a single, virtual knife edge. After the approximation, the problem is that of a three edge calculation.

This method is very attractive because it can read in a digital elevation map and perform a site-specific propagation computation on the elevation data. It can produce a signal strength contour that has been reported to be good within a few dB. The disadvantages are that it cannot adequately predict propagation effects due to foliage, buildings, other man-made structures, and it does not account for multipath propagation other than ground reflection, so additional loss factors are often included. Propagation prediction algorithms which use terrain information are typically used for the design of modern wireless systems.

4.10.3 Okumura Model

Okumura's model is one of the most widely used models for signal prediction in urban areas. This model is applicable for frequencies in the range 150 MHz to 1920 MHz (although it is typically extrapolated up to 3000 MHz) and distances of 1 km to 100 km. It can be used for base station antenna heights ranging from 30 m to 1000 m.

Okumura developed a set of curves giving the median attenuation relative to free space (A_{mu}), in an urban area over a quasi-smooth terrain with a base station effective antenna height (h_{te}) of 200 m and a mobile antenna height (h_{re}) of 3 m. These curves were developed from extensive measurements using vertical omnidirectional antennas at both the base and mobile, and are plotted as a function of frequency in the range 100 MHz to 1920 MHz and as a function of distance from the base station in the range 1 km to 100 km. To determine path loss using Okumura's model, the free space path loss between the points of interest is first determined, and then the value of $A_{mu}(f, d)$ (as read from the curves) is added to it along with correction factors to account for the type of terrain. The model can be expressed as

$$L_{50}(\text{dB}) = L_F + A_{mu}(f, d) - G(h_{te}) - G(h_{re}) - G_{AREA} \tag{4.80}$$

where L_{50} is the 50th percentile (i.e., median) value of propagation path loss, L_F is the free space propagation loss, A_{mu} is the median attenuation relative to free space, $G(h_{te})$ is the base station antenna height gain factor, $G(h_{re})$ is the mobile antenna height gain factor, and G_{AREA} is the gain due to the type of environment. Note that the antenna height gains are strictly a function of height and have nothing to do with antenna patterns.

Hata Model

Is an empirical formulation of the graphical path loss provided by Okumura, valid for $(150 \rightarrow 1500)$ MHz

$$\boxed{L_{median}(urban)_{dB} = 69.55 + 26.16 \log f_c - 13.82 \log(h_{te}) - a(h_{re}) + (44.9 - 6.55 \log(h_{te})) \log d}$$

f_c: is the freq. in (MHz) for $150 \rightarrow 1500$ MHz

Acceptable range (meter)

$(30 \rightarrow 200)$ h_{te}: effective BS TX antenna height (in meters)

$(1 \rightarrow 10)$ h_{re}: " MS RX " " (" ")

d: is the T-R separation distance (in Km)

$a(h_{re})$: Correction factor for MS antenna

1 - For small to medium size city

$$\boxed{a(h_{re}) = (1.1 \log f_c - 0.7) h_{re} - (1.56 \log f_c - 0.8)} \quad \boxed{dB}$$

2 - For Large City

$$a(h_{re}) = 8.29 (\log 1.54 h_{re})^2 - 1.1 \quad \boxed{dB} \quad (\text{for } f_c \leq 300 \text{ MHz})$$

$$a(h_{re}) = 3.2 (\log 11.75 h_{re})^2 - 4.97 \quad \boxed{dB} \quad (\text{for } f_c \geq 300 \text{ MHz})$$

Area modifications

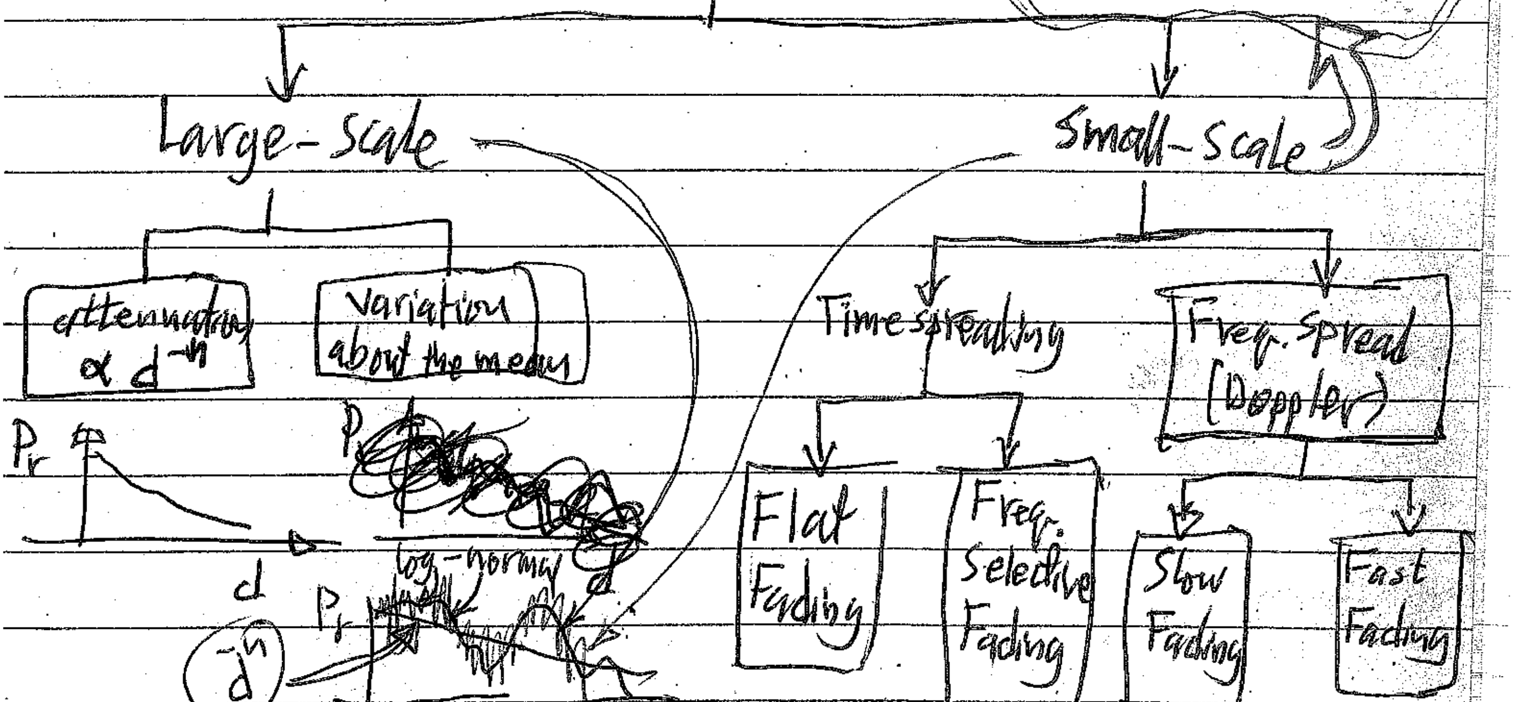
(Suburban area) $L_{median} = L_{median}(urban) - 2[\log(f_c/28)]^2 - 5.4$

(Open rural area) $L_{median} = L_{median}(urban) - 4.78 (\log f_c)^2 + 18.33 \log f_c - 40.94$

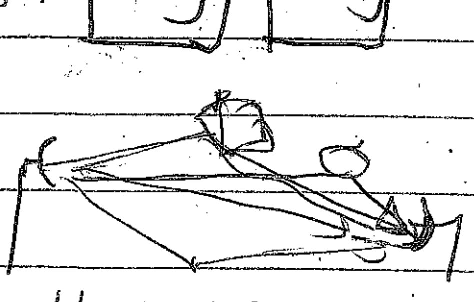
Chapter-5
Mobile Radio Propagation: Small-Scale Fading and Multipath

Small-scale fading describes rapid fluctuations in the received signal strength over short time durations or short travel distances so that large-scale path loss ~~may~~ effect may be ignored. These rapid fluctuations affect every aspect of ~~RX~~-TX design: ① Dynamic range ② Equalization ③ Diversity ④ modulation ⑤ EC codes

Propagation effects



Multipath fading



In addition to LOS path (if there is one) there are usually multiple delayed w/ attenuated paths (due to diffraction, reflection scattering)

Doppler effect

$\cos\omega_1 t + \cos\omega_2 t = 2\cos\left(\frac{\omega_1 t+\omega_2 t}{2}\right)\cos\left(\frac{\omega_1 t-\omega_2 t}{2}\right)$

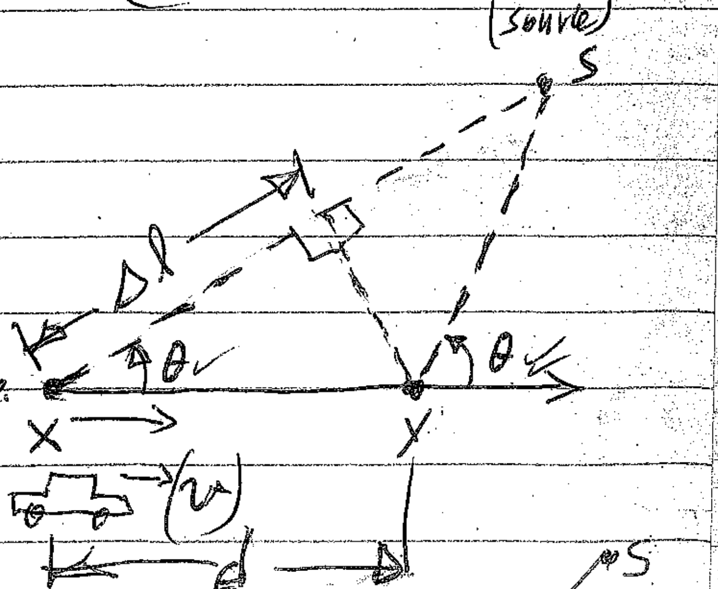
* Either reflectors or mobile or both may be moving $\Rightarrow$ multipath will be constantly changing.

* These paths have different attenuations, delays phase shifts and hence they may combine constructively or destructively (causing fading or weak signal)

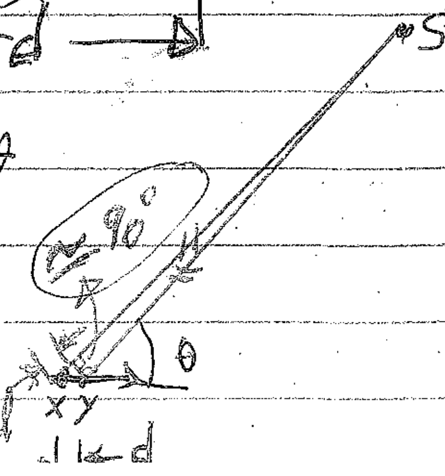
Factors influencing Fading

a) Multipath propagation
b) Speed of mobile $\Rightarrow$ Doppler shift $\Rightarrow$ (Random power fluctuations)
c) Speed of surrounding objects (also causing Doppler shift)
d) Transmission BW of signal (compared to coherence BW)

Doppler shift



Consider a mobile moving from (X → Y) along the st. Line.
Assuming source is far ($\theta \simeq \theta$) (see figure below)
Assuming velocity $= v$
Path difference $\Delta l = d\cos\theta = v\Delta t\cos\theta$
where Δt: time required for mobile from X → Y



Phase change due to motion $= \Delta\phi = \dfrac{2\pi\Delta l}{\lambda}$

$\therefore \Delta\phi = \frac{2\pi v \Delta t}{\lambda} \cos\theta$

$\therefore$ Apparent change in received freq. which is called (Doppler shift) is

$$\boxed{f_d = \frac{1}{2\pi}\frac{\Delta\phi}{\Delta t} = \boxed{\frac{v}{\lambda}\cos\theta}}$$ Doppler shift

$\frac{1}{2\pi}$ → to change $w \to f$

f_d is (+ve) if mobile is moving towards Source.
f_d is (−ve) if mobile is moving away from Source.

Multipath components from a ~~cw~~ continuous wave (CW) signal that arrive from different directions contribute to Doppler spreading for RX signal thus increasing signal (BW).

having approximately the same delay

received freq. $\boxed{f = f_c \pm f_d}$

ex) A transmitter radiates with $f_c = 1850$ MHz. A vehicle moving at 60 mpH, find the received carrier frequency ~~if~~ ~~the~~ the mobile is moving

a) directly towards TX

b) " away from TX.

c) In a direction ⊥ to the direction of arrival of TX signals

$f_c = 1850$ MHz

$$f_d = \frac{v}{\lambda}\cos\theta$$

$$\lambda = c/f_c = \frac{3\times10^8}{1850\times10^6} = 0.162\,\text{m}$$

vehicle speed v = 60 mph = 26.82 m/s

a) $f = f_c + f_d = 1850\times10^6 + \frac{26.82}{0.162}\cos 0^\circ$

$\theta = 0$

$= 1850.00016$ MHz

b) $f = f_c - f_d$ (because $\theta = 180^\circ$)

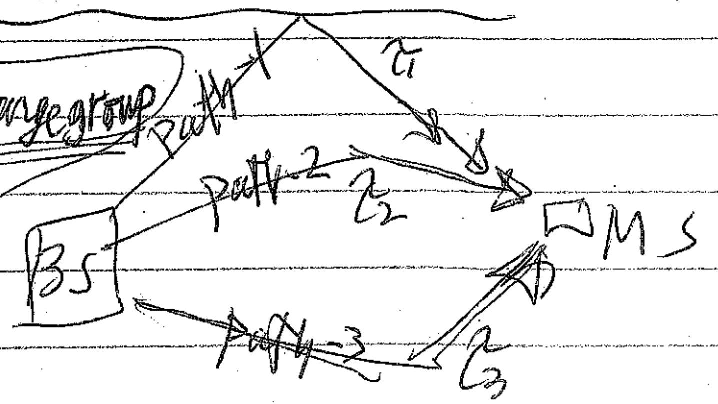
$= 1850\times10^6 - \frac{26.82}{0.162} = 1849.9998$ MHz

c) ~~H~~ $\theta = 90$ $\cos\theta = 0$

$f_d = 0$ No Doppler shift

$f = f_c = 1850$ MHz

Each path consists of ~~large group~~ no. of micropaths of nearly similar τ.



Model of the multipath channel

The multipath propagation with N-distinct paths leads to modelling the channel effect as a "Linear time-varying" system which can be modeled as an LTI system for short time durations.

Channel Impulse response (Complex envelope)

Baseband

The complex envelope model is equivalent to down-shifting the signal (at its f_c freq.) and the channel ~~resp~~ function $H(w)$ down by (f_c) to zero-frequency (Baseband) where it is easier to analyze, simulate and study without the carrier freq. while preserving all channel and signal characteristics.

The baseband (Complex envelope) channel impulse response

For an N-path time-varying channel, the impulse response

$$\boxed{h(\tau, t) = \sum_{n=1}^{N} a_n(t)\, e^{-j\theta_n(t)}\, \delta(\tau - \tau_n(t))}$$

where

$$\boxed{\theta_n(t) = w_c \tau_n(t) + \phi_i = 2\pi f_c \tau_n(t) + \phi_i}$$

ϕ_i ⟸ describes additional channel phase shift

$\theta_n(t)$: Carrier phase-shift of the n^{th} path at (time = t)

$a_n(t)$: Attenuation of the n^{th} path at (time = t).

$\tau_n(t)$: Propagation delay of the n^{th} path at (time = t).

We define the TX signal as $\boxed{x(t)}$ (Complex envelope signal Model)

The received signal is

$$r(t) = \int_{-\infty}^{\infty} h(\tau,t)\,x(t-\tau)\,d\tau$$

For the case of LTI channel (short time durations θ_n

$$\boxed{h(\tau) = \sum_{n=1}^{N} a_n e^{-j(\omega_c \tau_n + \phi_i)} \delta(\tau-\tau_n)}$$ LTI channel

The received signal is then (complex envelope)

$$r(t) = \int_{-\infty}^{\infty} h(\tau)\,x(t-\tau)\,d\tau = \int_{-\infty}^{\infty}\left[\sum_{n=1}^{N} a_n e^{-j(\omega_c\tau_n+\phi_i)}\delta(\tau-\tau_n)\right] x(t-\tau)\,d\tau$$

$$= \sum_{n=1}^{N} a_n e^{-j(\omega_c\tau_n+\phi_i)} \int_{-\infty}^{\infty} \delta(\tau-\tau_n)\,x(t-\tau)\,d\tau$$

$$\boxed{r(t) = \sum_{n=1}^{N} a_n e^{-j\theta_n} x(t-\tau_n)}$$ RX-signal for LTI channel

$\theta_n = \omega_c \tau + \phi_i$ ← additional phase effects of the channel on n^{th} path

Time Dispersion Parameters

The "power delay profile" of the channel is the spatial average of $|h(t;\tau)|^2$ by making several measurements in different locations of a local area and taking average

$$P(\tau) = K\,\overline{|h(t;\tau)|^2}$$ ← sample average

Several important channel parameters are derived from the "power delay profile" $P(\tau)$

a) Mean Excess delay
$$\boxed{\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\,\tau_k}{\sum_k P(\tau_k)}}$$

Note

$$\bar{\tau} = E[\tau] = \sum_k \text{prob.}(\tau_k)\,\tau_k \quad (\text{discrete R.V.})$$

$$\text{Prob.}(\tau_k) \simeq \frac{P(\tau_k)}{\sum_k P(\tau_k)} \Rightarrow \text{gives above eqn.}$$

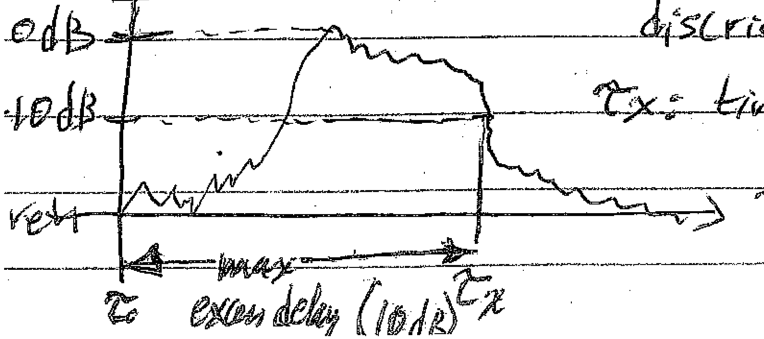
b) rms delay spread
$$\boxed{\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2}}$$

where
$$\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \boxed{\frac{\sum_k P(\tau_k)\,\tau_k^2}{\sum_k P(\tau_k)}}$$

c) max. excess delay (X dB) : Is the time delay during which multipath power falls to (X dB) below maximum $= \tau_x - \tau_0$

τ_0 : 1st arriving signal whose power is above a certain threshold so that it is discriminated from thermal noise

τ_x : time for (X dB) below max.



Coherence Bandwidth (B_c)

similar to the correlation

$$R_{xx}(\tau) \xleftrightarrow{F} S_{xx}(w) = H(w)H^*(w)$$

By Duality

$$\left.\begin{array}{l}\text{power delay}\\ \text{profile}\end{array}\right\} = |\underbrace{h(\tau)h^*(\tau)}_{\text{spectrum in } \tau\text{-domain}}| \xleftrightarrow{F} R_{HH}(v) = E[H(w)H(w+v)]$$

correlation in the frequency domain

Coherence BW (B_c) is the range of frequencies over which the channel $H(w)$ is considered "flat" i.e two freqs are highly correlated in the $H(w)$ if they are ($f_2 - f_1 < B_c$) which means that these two frequencies are affected nearly equally in gain and phase by the channel.

$$\therefore B_c \propto \frac{1}{\sigma_\tau} \simeq \begin{cases} \dfrac{1}{50\sigma_\tau} & \text{freq. correlation} > 90\% \\ \dfrac{1}{5\sigma_\tau} & \text{freq. correlation} > 50\% \end{cases}$$

~~ex)~~

~~Doppler Spread and Coherence Time~~

ex) Find the mean ~~excess~~ delay, rms delay spread and max. excess delay (10dB) for channel below. Estimate the (50%) B_c. Would this channel be suitable for AMPS or GSM systems without using an equalizer.

9/5 (6)

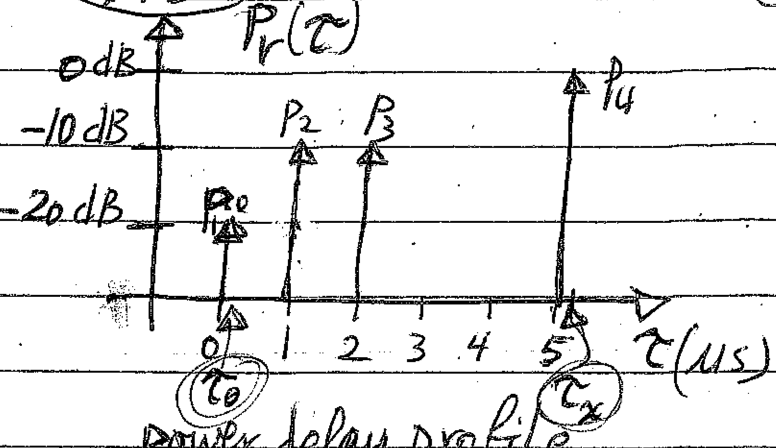
max. excess delay (10dB) =
$= 5 - 0 = \boxed{5 \mu s}$

$-20 = 10 \log P_1$

$P_1 = 10^{-20/10} = 0.01$

$P_2 = P_3 = 10^{-10/10} = 0.1$

$P_4 = 1$



power delay profile

$$\bar{\tau} = \frac{\sum P(\tau_k)\tau_k}{\sum P(\tau_k)} = \frac{(1)(5)+(0.1)(2)+(0.1)(1)+(0.01)(0)}{1+0.1+0.1+0.01}$$

$$\boxed{\bar{\tau} = 4.38 \mu s}$$

$$\overline{\tau^2} = \frac{\sum P(\tau_k)\tau_k^2}{\sum P(\tau_k)} = \frac{1\times 5^2 + 0.1\times 2^2 + 0.1\times 1^2 + 0.01\times 0^2}{1+0.1+0.1+0.01}$$

$$\boxed{\overline{\tau^2} = 21.07 \mu s^2}$$

$$\sigma_\tau = \sqrt{\overline{\tau^2} - \bar{\tau}^2}$$
$$= \sqrt{21.07 - 4.38^2}$$
$$= \boxed{1.37 \mu s}$$

$$B_c^{(50\%)} \cong 1/(5\sigma_\tau) = 1/(5\times 1.37) = 146 \text{ KHz}$$

Coherence BW

(30 kHz) AMPS: since $B_c > 30$ KHz AMPS work without equalizer

(200 KHz) GSM: since $B_c < 200$ KHz GSM needs an equalizer to work.

Freq. Spreading (10/5) (8)

Doppler Spread and Coherence Time

It describe the time-varying nature of the channel (small scale region).

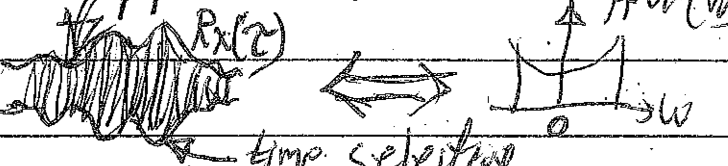
Mobile movement causes Doppler Spread (broadening of the signal spectrum)

$$f_c \longrightarrow f_c - f_d < f < f_c + f_d$$

$\Rightarrow$ Time variation in carrier amplitude

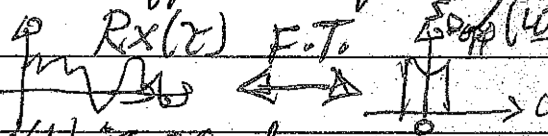
$\Rightarrow$ 〃 〃 in the channel

Let $\boxed{f_m = \dfrac{v}{\lambda}}$ (max. Doppler shift) only carrier



Coherence Time (T_c)

Is the time during which $h(t)$ is approximately invariant, i.e. it gives a measure of time separation of two signals such that they are affected equally by the channel



$$T_c \propto \frac{1}{f_m}$$ where $x(t)$ is a complex baseband WSS R.P. power spectral density

$$T_c \text{ (50\% time correlation)} = \boxed{\frac{9}{16\pi f_m}}$$

Or a more popular practical value

$$\boxed{T_c = \frac{0.423}{f_m}}$$

usually Gaussian R.P.

$R_x(\tau) = E[x(t)x(t+\tau)]$

Coherence time implies that two signals arriving with a time separation $> T_c$ are affected differently by the channel.

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10

ex) vehicle speed = 60 mph, $f_c = 900$ MHz $\left(\lambda = \frac{1}{3}\text{ m}\right)$

then $T_c = \frac{9}{16\pi f_m} = 2.22$ msec

$T_{sym} < T_c$ OR

As long as symbol rate $> \frac{1}{T_c}$ ($= 454$ bps)

$T_{symb} < T_c$

the channel will not cause distortion due to motion (However there could be distortion due to freq. selective fading)

Using the practical value $T_c = \frac{0.423}{f_m} = 6.77$ ms ($f_m = 80.4$)

∴ Symbol rate $> \frac{1}{6.77} > 150$ bps to avoid distortion

60 mph ⇒ 26.8 m/s

$f_m = \frac{v}{\lambda} = \frac{26.8}{1/3} = \boxed{80.4}$ Hz

Types of Small-Scale Fading

Small-Scale Fading (Based on time dispersion)

Time Dispersion ⇒ Freq. Selectivity

Flat Fading

1. $BW_{signal} < BW_{channel}$ $(\equiv B_c)$ coherence BW
2. $\sigma_\tau < T_s$ (symbol period)

rule of thumb: channel is flat fading if $\sigma_\tau \leq 0.1 T_s$

Freq. Selective Fading

1. $BW_{sig.} > BW_{ch}$ $(\equiv B_c)$
2. $\sigma_\tau > T_s$

Small-Scale Fading (Based on Doppler Spread)

Freq. Dispersion ⇒ Time Selectivity

Fast Fading

1. High Doppler Spread
2. $T_c < T_s$
3. Channel variation faster than baseband signal variations

Slow Fading

1. Low Doppler Spread
2. $T_c > T_s$
3. Channel variations slower than BB signal variations

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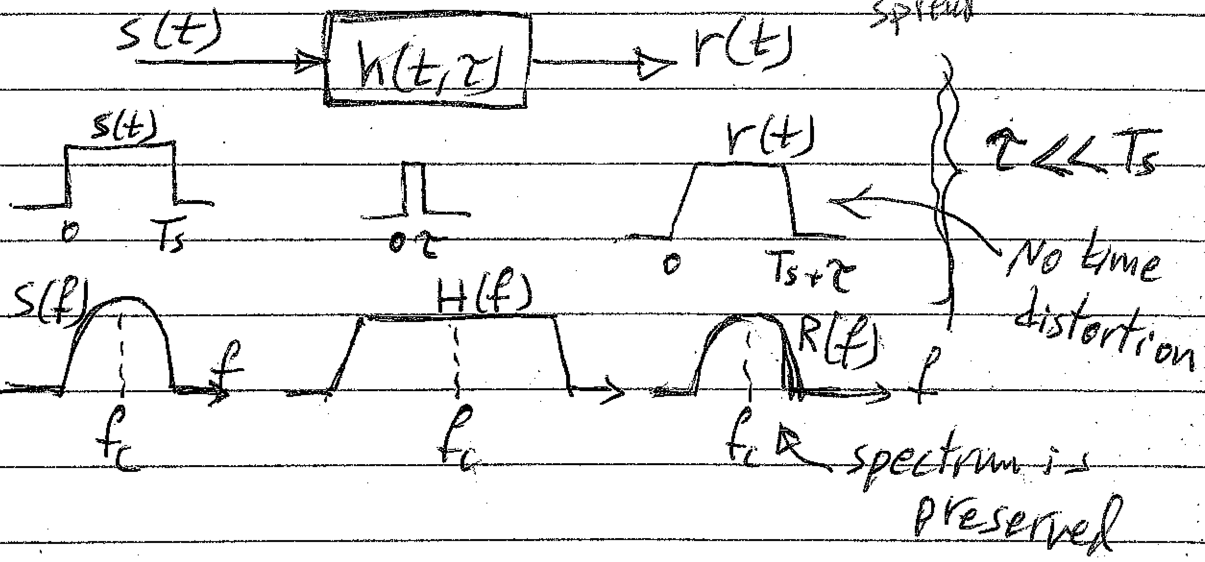
Fading effects due to multipath delay spread

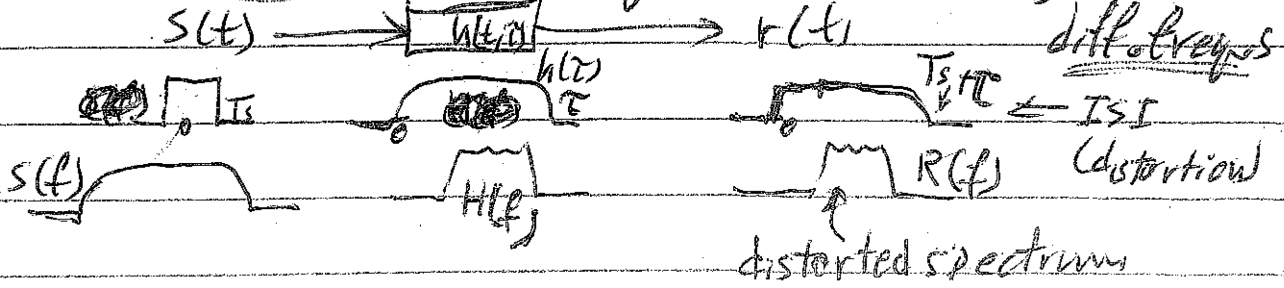
Flat Fading ($BW_{ch} > BW_{sig}$)

- Channel has a constant gain and Linear phase over BW that is greater than (BW_{sig}).
- Spectral characteristics of signal is preserved.
- Channel gain may change over time causing RX signal to vary in gain but its spectrum is preserved (undistorted).
- $h(\tau)$ is approximated by one $\delta(t)$ (if channel delay spread $\ll T_{symbol}$).



Freq. Selective Fading ($BW_{sig} > BW_{ch}$)

- Constant gain BW of channel $<$ BW signal
- $\sigma_\tau > T_s$ (RX signal includes multiple version of $x(t)$ delayed attenuated $\Rightarrow$ distortortion
- $h(\tau)$ = LTI with several taps.
- $\Rightarrow$ Causing ISI, viewed in Freq. domain as diff. gain for diff. freqs



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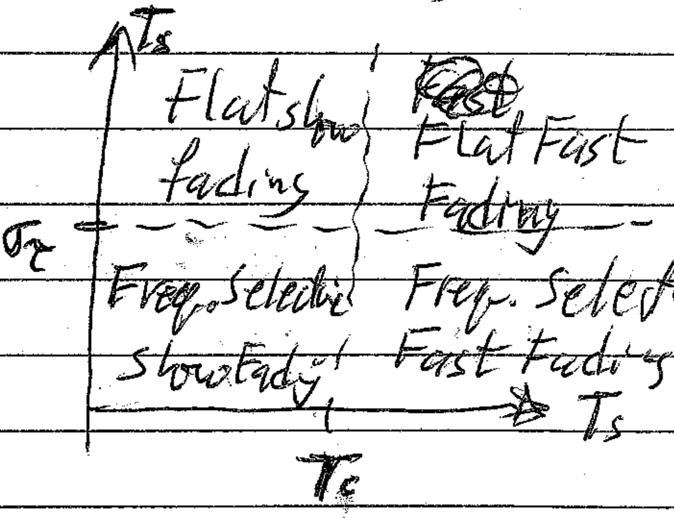
Fading Effects due to Doppler spread

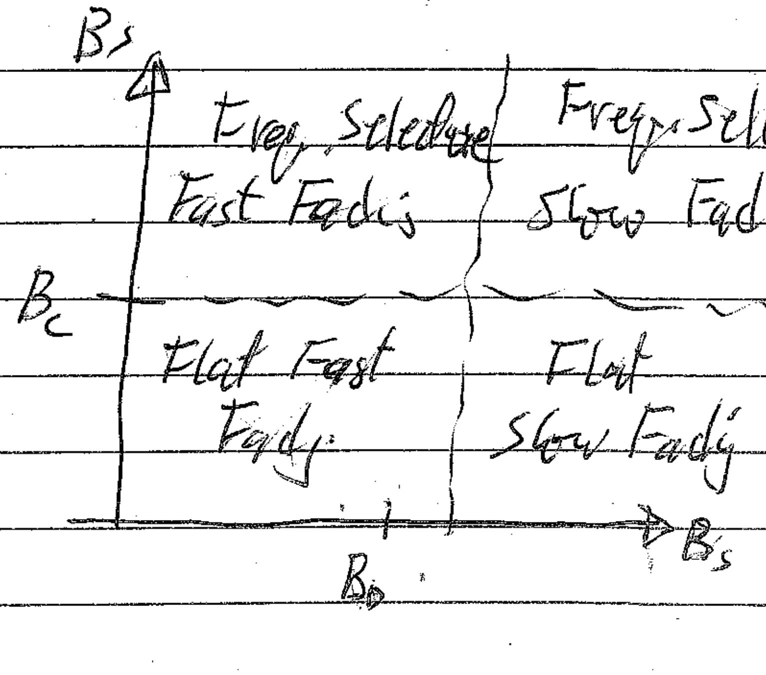
Fast Fading

- Channel impulse response changes rapidly within the symbol period ($T_c < T_s$) ($B_D > B_s$)
- In practice this type of fading occurs for very low data rates.
- Doppler spread causes freq. dispersion
- Is not related to whether channel is flat or freq. selective

Slow Fading

- $h(t)$ changes at a rate that is much slower than data rate.
- ($T_s \ll T_c$) ($B_s \gg B_D$)





(13b/5)

(x) ~~Effect~~ Determine Effect of Doppler Spread / Time dispersion on GSM frame.

Bit period = 3.7 μsec
Max. Vehicle speed = 250 Km/Hr
τ_{max} = 15 μsec (4500 m)
$f_c \simeq$ 900 MHz ($\lambda \simeq$ 1/3 m)

Data	training 26 bits	Data

0.546 msec
Frame length
156-bit

Coherence time

$$v = \frac{250 \times 10^3}{3600} = 69.4 \text{ m/s}$$

$$f_m = v/\lambda = 69.4/(1/3) = 208.3 \text{ Hz}$$

$$\boxed{T_c(50\%) = \frac{9}{16\pi f_m} = \boxed{0.86 \text{ msec}}}$$

$\therefore$ $\boxed{\text{Frame length} = \frac{0.546}{0.86} T_c = \boxed{0.63 T_c}}$

Moreover, training is in the middle
$\therefore$ channel in the middle differs by 0.3 T_c from both sides (nearly constant).

Time Spread

σ_τ ≤ 15 μsec (4500 meter) will be σ_τ = 10 μs

$T_s < \sigma_\tau$ $\therefore$ Freq. selective fading
needs equalizer 15 μs / 3.7 μs = $\boxed{4 \text{ bit ISI}}$
needs at least (4-5) time, training ≃ 26 bit

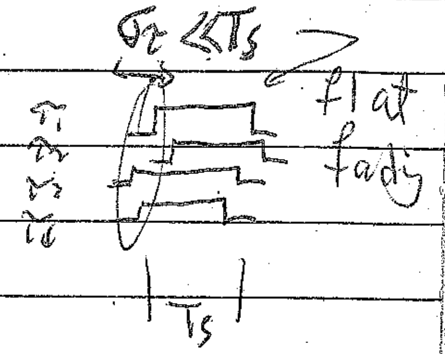
Rayleigh and Ricean Distributions

These distributions describe the envelope of a flat fading signal assuming that delay spread $\ll T_{symbol}$.
(LOS or NO-LOS)

~~Rayleigh~~

Rayleigh Fading (Non-LOS)

The received signal is



$$r(t) = \sum_{n=1}^{N} a_n(t)\, e^{-j\omega_c \tau_n(t)}\, x(t-\tau_n(t))$$

$$\simeq \underbrace{\boxed{\sum_{n=1}^{N} a_n(t)\, e^{-j\omega_c \tau_n(t)}}}\, x(t-\tau) \quad \text{Assuming delay spread} \ll T_{symbol}$$

∴ The complex channel gain is

$$Z(t) = \sum_{n=1}^{N} a_n(t)\, e^{-j\overbrace{\omega_c \tau_n(t)}^{\theta_n(t)}} = X + jY$$

$$X = \sum_{n=1}^{N} a_n(t)\cos\theta_n(t) \qquad Y = \sum_{n=1}^{N} a_n(t)\sin\theta_n(t)$$

Due to the large no. of path component that contribute to X, Y
Using Central Limit Theorem X and Y are Gaussian R.V.s with zero-means and equal variances σ^2.

$$f_{X,Y}(x,y) = \frac{1}{2\pi\sigma^2} e^{-(x^2+y^2)/2\sigma^2}$$

The envelope $r = \sqrt{X^2+Y^2}$, phase $\phi = \tan^{-1}\frac{Y}{X}$

(15~~4~~5)

To find the pdf of r, ϕ

$$f_{r,\phi}(r,\phi) = f_{x,y}(x_1, y_1)\,|J_1|$$

single root

$$\boxed{x_1 = r\cos\phi \;,\; y_1 = r\sin\phi}$$

$$J_1 = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \phi} \end{vmatrix} = \begin{vmatrix} \cos\phi & -r\sin\phi \\ \sin\phi & r\cos\phi \end{vmatrix}$$

$$= r\cos^2\phi + r\sin^2\phi = r$$

$$f_{r,\phi}(r,\phi) = \frac{r}{2\pi\sigma^2} e^{-(r^2\cos^2\phi + r^2\sin^2\phi)/2\sigma^2}$$

$$f_{R,\Phi}(r,\phi) = \frac{r}{2\pi\sigma^2} e^{-r^2/2\sigma^2} \begin{cases} r > 0 \\ -\pi < \phi < \pi \end{cases}$$

The envelope, with NO-LOS is the marginal pdf

$$f_R(r) = \int_{-\pi}^{\pi} \frac{r}{2\pi\sigma^2} e^{-r^2/2\sigma^2}\, d\phi = \boxed{\frac{r}{\sigma^2} e^{-\frac{r^2}{2\sigma^2}} \quad r > 0}$$

$$f_\phi(\phi) = \int_0^\infty \frac{r}{2\pi\sigma^2} e^{-r^2/2\sigma^2}\, dr$$

$$= \frac{1}{2\pi}\int_0^\infty \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} = -\frac{1}{2\pi}\left[e^{-r^2/2\sigma^2}\right]_0^\infty = \frac{1}{2\pi}\left[e^0 - e^{-\infty}\right]$$

$$f_\phi(\phi) = \boxed{\frac{1}{2\pi}} \text{ (uniform in } 0, 2\pi)$$

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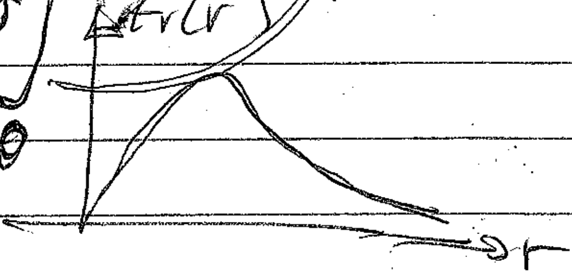
Note that

a) $E(r) = \sqrt{\frac{\pi}{2}}\,\sigma$

b) $E(r^2) = 2\sigma^2 \Longrightarrow$ $r^2 = x^2 + y^2$, $\overline{r^2} = \overline{x^2} + \overline{y^2} = \sigma^2 + \sigma^2 = 2\sigma^2$ (zero mean)

c) $\sigma_r^2 = \overline{r^2} - \overline{r}^2 = 0.4292\,\sigma^2$

d) $P[r \le A] = \int_0^A \frac{r}{\sigma^2} e^{-r^2/2\sigma^2}\,dr = \left[-e^{-r^2/2\sigma^2}\right]_0^A = 1 - e^{-A^2/2\sigma^2}$



~~proof of a)~~

$E(r) = \int_{r=0}^{\infty} \frac{r^2}{\sigma^2} e^{-r^2/2\sigma^2}\,dr$

let $u = r \Rightarrow du = dr$

$dV = \frac{r e^{-r^2/2\sigma^2}}{\sigma^2}\,dr \Longrightarrow V = -e^{-r^2/2\sigma^2}$

$E[r] = uV - \int v\,du = -re^{-r^2/2\sigma^2}\Big|_0^\infty - \int_0^\infty -e^{-r^2/2\sigma^2}\,dr \;\times\frac{\sqrt{2\pi\sigma^2}}{\sqrt{2\pi\sigma^2}}$

$= (0 - 0) + \sqrt{2\pi\sigma^2}\int_0^\infty \frac{e^{-r^2/2\sigma^2}}{\sqrt{2\pi\sigma^2}}\,dr$

$= \frac{\sqrt{2\pi}\,\sigma^2}{\sigma} \times \frac{1}{2} = \boxed{\sigma\sqrt{\frac{\pi}{2}}}$

√

$E(r) = \frac{\sqrt{2\pi}}{\sigma}\int_0^\infty \frac{r^2 e^{-r^2/2\sigma^2}}{\sqrt{2\pi}\sigma}\,dr = \frac{\sqrt{2\pi}}{\sigma}\int_0^\infty \frac{r^2}{\sqrt{2\pi\sigma^2}} e^{-r^2/2\sigma^2}\,dr$

$= \frac{\sqrt{2\pi}}{\sigma}\left[\frac{1}{2}\text{variance}\right] = \frac{\sqrt{2\pi}}{\sigma}\,\frac{\sigma^2}{2}$ zero mean Gaussian

$= \sigma\sqrt{\frac{\pi}{2}}$

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Ricean Fading (LOS)

If there exists a Line-of-sight LOS path then the channel is

$$z(t) = \sum_{n=1}^{N} a_n(t) e^{-j w_c \tau_n(t)} + A_0(t) e^{-j w_c \tau_0(t)}$$

$w_c \tau_0(t) = \theta$

where $A_0(t)$ is the deterministic part that's strong

$$\therefore \quad f_{X,Y}(x,y) = \frac{1}{2\pi\sigma^2} e^{-(x - A\cos\theta)^2/2\sigma^2} e^{-(y - A\sin\theta)^2/2\sigma^2}$$

$$\boxed{x = r\cos\psi, \quad y = r\sin\psi \qquad r = \sqrt{x^2+y^2} \qquad \psi = \tan^{-1}(y/x)}$$

As for Rayleigh $\Rightarrow |J| = r$

$$f_{R,\Psi}(r,\psi) = f_{X,Y}(x^{-1}, y^{-1})\,|J|$$

$$= \frac{r}{2\pi\sigma^2} e^{-[(r\cos\psi - A\cos\theta)^2 + (r\sin\psi - A\sin\theta)^2]/2\sigma^2}$$

$$= \frac{r}{2\pi\sigma^2} e^{-[r^2\cos^2\psi + A^2\cos^2\theta - 2Ar\cos\theta\cos\psi + r^2\sin^2\psi + A^2\sin^2\theta - 2rA\sin\psi\sin\theta]/2\sigma^2}$$

$$= \frac{r}{2\pi\sigma^2} e^{-[r^2 + A^2]/2\sigma^2} e^{+2Ar(\cos\theta\cos\psi + \sin\theta\sin\psi)/2\sigma^2}$$

$$f_{R,\Psi}(r,\psi) = \frac{r}{\sigma^2} e^{-(r^2+A^2)/2\sigma^2} \frac{1}{2\pi} e^{\frac{Ar}{\sigma^2}\cos(\psi-\theta)} \quad \begin{cases} r \geq 0 \\ 0 \leq \psi \leq 2\pi \end{cases}$$

Marginal Pdf

$$f_R(r) = \int_0^{2\pi} f_{R,\Psi}(r,\psi)\, d\psi$$

$$= \frac{r}{\sigma^2} e^{-(r^2+A^2)/2\sigma^2} \frac{1}{2\pi} \int_0^{2\pi} e^{\frac{Ar}{\sigma^2}\cos(\psi-\theta)}\, d\psi$$

let $\phi = \psi - \theta$ (change of variables)

$$f_R(r) = \frac{r}{\sigma^2} e^{-(r^2+A^2)/2\sigma^2} \frac{1}{2\pi} \int_{-\theta}^{2\pi-\theta} e^{\frac{Ar}{\sigma^2}\cos\phi}\, d\phi$$

identical

$$= \frac{r}{\sigma^2} e^{-(r^2+A^2)/2\sigma^2} \frac{1}{2\pi} \int_0^{2\pi} e^{\frac{Ar}{\sigma^2}\cos\phi}\, d\phi$$

$$\boxed{f_R(r) = \frac{r}{\sigma^2} e^{-(r^2+A^2)/2\sigma^2} I_0\left(\frac{Ar}{\sigma^2}\right) \quad \text{for } r \geq 0}$$

Ricean distribution

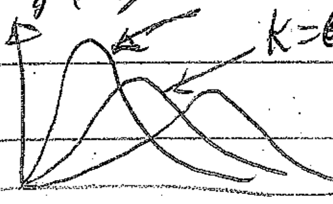
where I_0 : zeroth order modified Bessel fn. of 1st Kind

$$I_0(x) = \frac{1}{2\pi} \int_0^{2\pi} e^{x\cos\theta}\, d\theta$$

Note: Ricean distribution is often described in terms of

$$K = \frac{\text{power of LOS component}}{\text{Total power of scattered paths}}$$

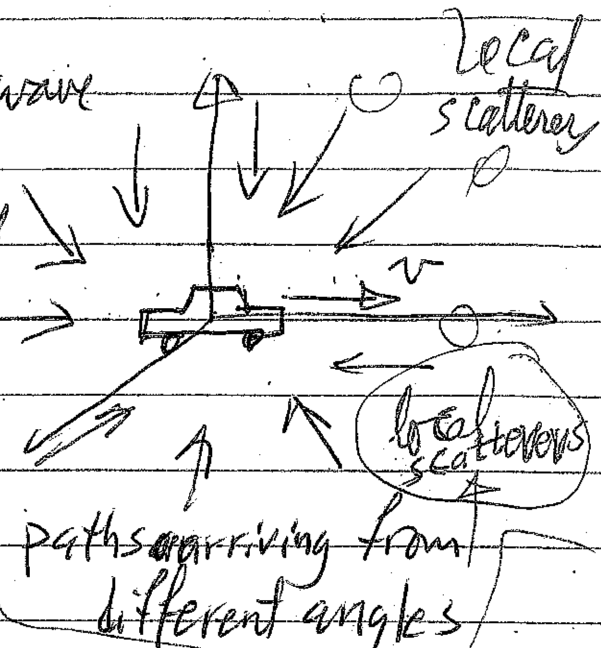
$$\boxed{K_{dB} = 10 \log \frac{A^2}{2\sigma^2}}$$



Statistics of signal received by a moving MS

Spectral shape due to Doppler spread (Clarke's Model)

- Assume TX signal is a continuous wave (CW) sinusoidal signal.
- If the MS is moving then different paths coming from different directions give rise to different freq. shifts (NO LOS)
- This leads to broadening of the received spectrum.



- Due to the large no. of paths, the RX envelope is a complex Gaussian RP (Rayleigh fading).

The n^{th} wave arriving at angle (α_n) gives Doppler shift

$$f_n = \frac{v}{\lambda} \cos\alpha_n$$

Let $P(\alpha)\,d\alpha$: the fraction of the total incoming power within angle $|d\alpha|$ at (α)

A : Average RX power with respect to an isotropic antenna.

$G(\alpha)$: Antenna gain as a fn. of (α).

The total RX power

$$P_r = \int_0^{2\pi} A\,G(\alpha)\,P(\alpha)\,d\alpha$$

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where $AG(\alpha)p(\alpha)\,d\alpha$: the differential variation of RX power with angle.

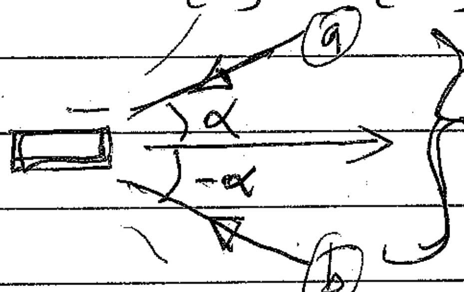
Let the CW signal has freq. $= f_c$

The freq. of RX signal arriving at angle (α) is

$$f = f(\alpha) = \frac{v}{\lambda}\cos\alpha + f_c = f_m\cos\alpha + f_c \quad (1)$$

where f_m : max. Doppler shift $= \frac{v}{\lambda}$

Here we have $f(\alpha) = f(-\alpha)$



Both paths (a), (b) yield the same freq. shift

Let $S(f)$: power spectrum of RX signal.

The differential variation of RX power with freq. is $S(f)|df|$

differential power at freq. (f) = differential power at the corresponding angle (α)

$$S(f)|df| = A[P(\alpha)G(\alpha) + P(-\alpha)G(-\alpha)]|d\alpha|$$

~~$S(f)|df| = 2A[P(\alpha)G(\alpha)]|d\alpha|$ — (2)~~

~~example $S(f=10\,Hz)|\Delta f| = 2A[P(\alpha(f_{10Hz}))G(\alpha(10Hz))]|\Delta\alpha|$~~

~~$df/d\alpha\ (\Delta f/\Delta\alpha) = -f_m \sin\alpha$~~

But from (1) $df = -f_m \sin\alpha\, d\alpha$

$|df| = |d\alpha||-\sin\alpha|\, f_m \Longrightarrow$ Substitute in (2)

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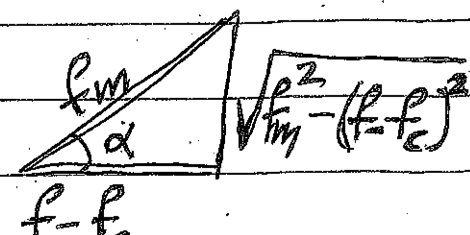
$$S(f)\,|\cancel{d\alpha}|\,|-\sin\alpha|\,f_m = A\left[P(\alpha)G(\alpha)+P(-\alpha)G(-\alpha)\right]|\cancel{d\alpha}|$$

$$S(f) = \frac{A\left[P(\alpha)G(\alpha)+P(-\alpha)G(-\alpha)\right]}{f_m\,|-\sin\alpha|}$$

From (1) $\Rightarrow$



$$\cos\alpha = (f-f_c)/f_m$$

$$\sin\alpha = \frac{\sqrt{f_m^2-(f-f_c)^2}}{f_m} = \sqrt{1-\left(\frac{f-f_c}{f_m}\right)^2}$$

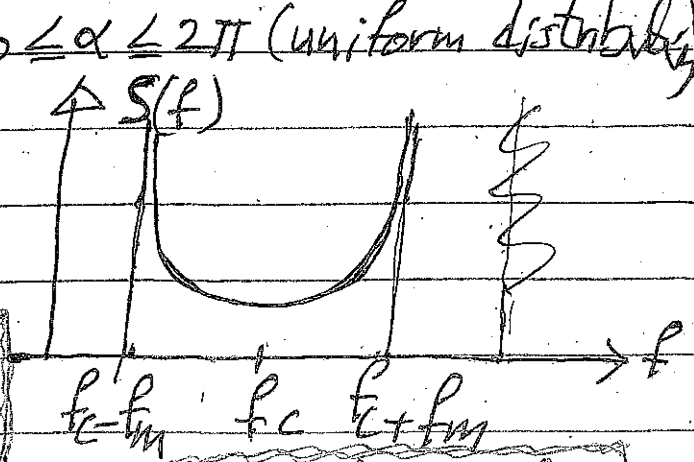
$$S(f) = \frac{A\left[P(\alpha)G(\alpha)+P(-\alpha)G(-\alpha)\right]}{f_m\sqrt{1-\left(\frac{f-f_c}{f_m}\right)^2}}$$

But $\boxed{\alpha = \cos^{-1}\left(\frac{f-f_c}{f_m}\right)}$

$$\therefore\ \boxed{S(f) = \left.\frac{A\left[P(\alpha)G(\alpha)+P(-\alpha)G(-\alpha)\right]}{f_m\sqrt{1-\left(\frac{f-f_c}{f_m}\right)^2}}\right|_{\alpha=\cos^{-1}\left(\frac{f-f_c}{f_m}\right)}}$$

For $f_c-f_m \le f \le f_c+f_m$

Assuming
- $A=1$
- vertical $\lambda/4$ antenna $G(\alpha)=1.5$
- $P(\alpha) = 1/2\pi \quad 0\le\alpha\le 2\pi$ (uniform distribution)



$$S(f) = \frac{2\times 1\times 1.5\times(1/2\pi)}{f_m\sqrt{1-(f-f_c/f_m)^2}}$$

$$\boxed{S(f) = \frac{1.5}{\pi f_m\sqrt{1-\left(\frac{f-f_c}{f_m}\right)^2}}}$$

$\Rightarrow$ For $(f_c-f_m \le f \le f_c+f_m)$

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lation method uses the concept of in-phase and quadrature modulation paths to produce a simulated signal representing Equation (5.63) with spectral and temporal characteristics very close to measured data.

As shown in Figure 5.22(b), two independent Gaussian low pass noise sources are used to produce in-phase and quadrature fading branches. Each Gaussian source may be formed by summing two independent Gaussian random variables which are orthogonal (i.e., $g = a + jb$, where a and b are real Gaussian random variables and g is complex Gaussian). By using the spectral filter defined by Equation (5.78) to shape the random signals in the frequency domain, accurate time domain waveforms of Doppler fading can be produced by using an inverse fast Fourier transform (IFFT) at the last stage of the simulator.

Smith [Smi75] demonstrated a simple computer program that implements Figure 5.22(b). His method uses a complex Gaussian random number generator (noise source) to produce a baseband line spectrum with complex weights in the positive frequency band. The maximum

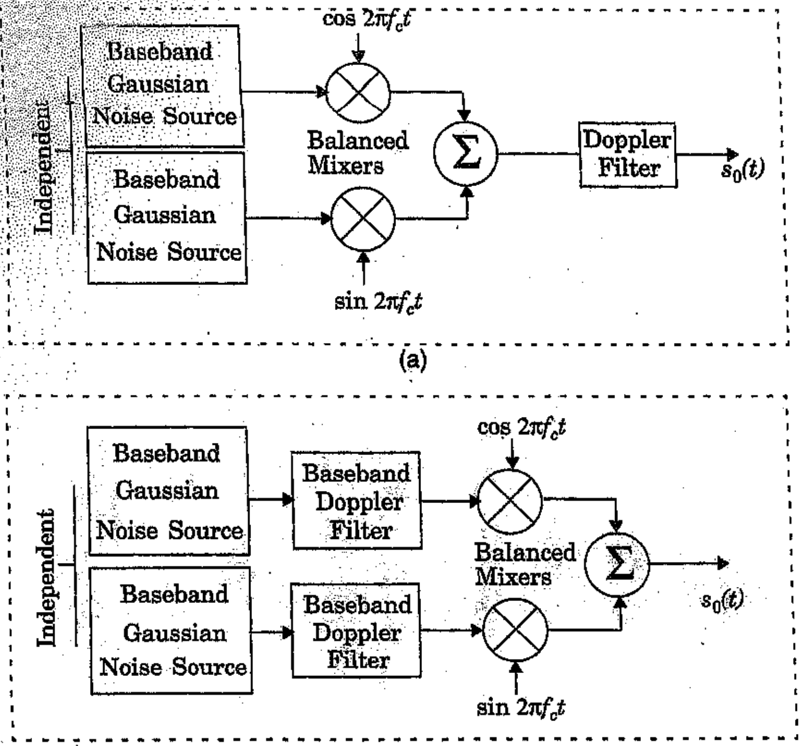

Figure 5.22 Simulator using quadrature amplitude modulation with (a) RF Doppler filter and (b) baseband Doppler filter.

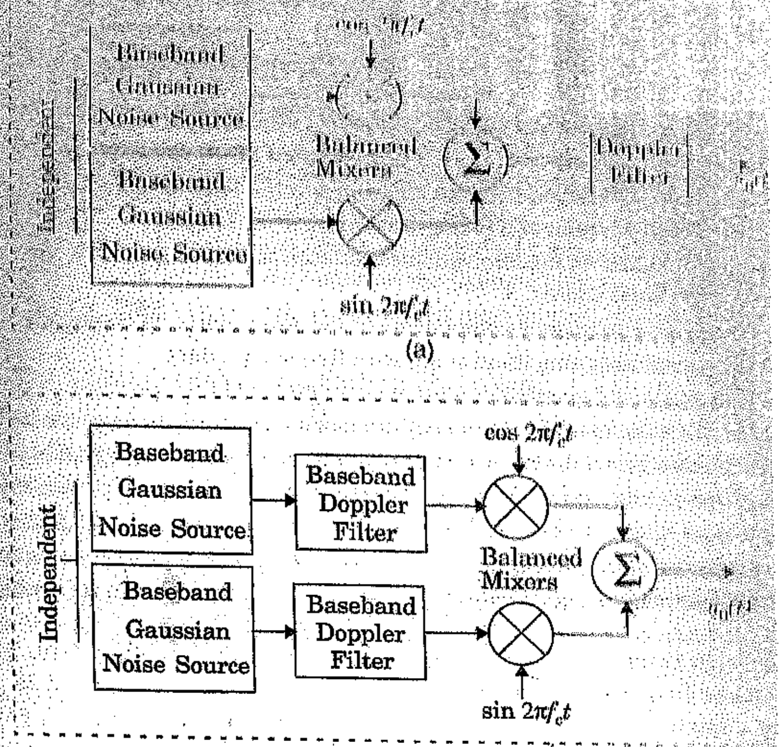

Figure 5.23 Simulator using quadrature amplitude modulation with (a) RF Do... (b) baseband Doppler filter.

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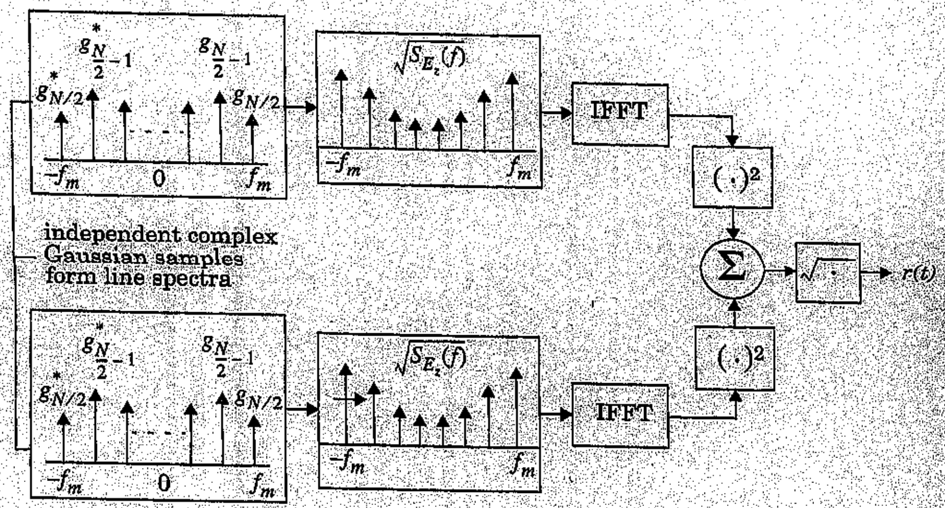
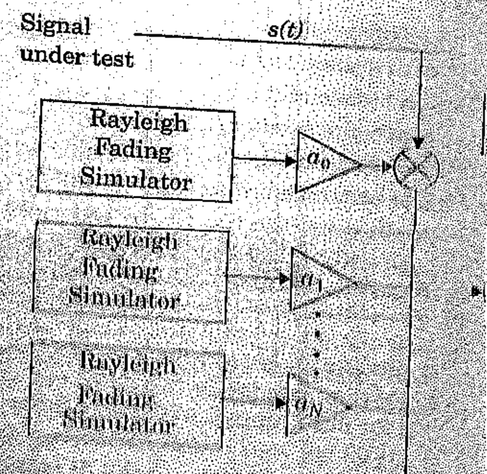


Figure 5.24 Frequency domain implementation of a Rayleigh fading simulator at baseband

To implement the simulator shown in Figure 5.24, the following steps are used:

1. Specify the number of frequency domain points (N) used to represent $\sqrt{S_{E_z}(f)}$ and the maximum Doppler frequency shift (f_m). The value used for N is usually a power of two.
2. Compute the frequency spacing between adjacent spectral lines as $\Delta f = 2f_m/(N-1)$. This defines the time duration of a fading waveform, $T = 1/\Delta f$.
3. Generate complex Gaussian random variables for each of the $N/2$ positive frequency components of the noise source.
4. Construct the negative frequency components of the noise source by conjugating positive frequency values and assigning these at negative frequency values.
5. Multiply the in-phase and quadrature noise sources by the fading spectrum $\sqrt{S_{E_z}(f)}$.
6. Perform an IFFT on the resulting frequency domain signals from the in-phase and quadrature arms to get two N-length time series, and add the squares of each signal point in time to create an N-point time series like under the radical of Equation (5.67). Note that each quadrature arm should be a real signal after the IFFT to model Equation (5.63).
7. Take the square root of the sum obtained in Step 6 to obtain an N-point time series of a simulated Rayleigh fading signal with the proper Doppler spread and time correlation.



B $r(t)$: is a R.P. represents the envelope of the received signal of a CW TX signal.

$$r(t) \triangleq |z(t)| = \sqrt{x^2(t)+y^2(t)}$$

when $z(t) = x(t) + j\,y(t)$

$x(t), y(t)$: are stat. ind. SSS zero-mean Gaussian R.P.'s, ~~each has~~ having identical ~~PSD~~ PSD & $R_{xx}(\tau)$.

$$S_{zz}(w) = \frac{1.5}{\pi \sqrt{f_m - f^2}}$$ (Due to Doppler spread)

since $x(t), y(t)$ have identical PSD we have
$$S_{xx}(w) = S_{yy}(w) = \frac{1}{2} S_{zz}(w)$$

$$= \frac{0.75 \times K}{\pi \sqrt{f_m - f^2}}$$ ← for power scaling.

$$R_{xx}(\tau) = \cancel{R_{xx}(\tau)} = F^{-1}[S_{xx}(w)] = R_{xx}(\tau)$$

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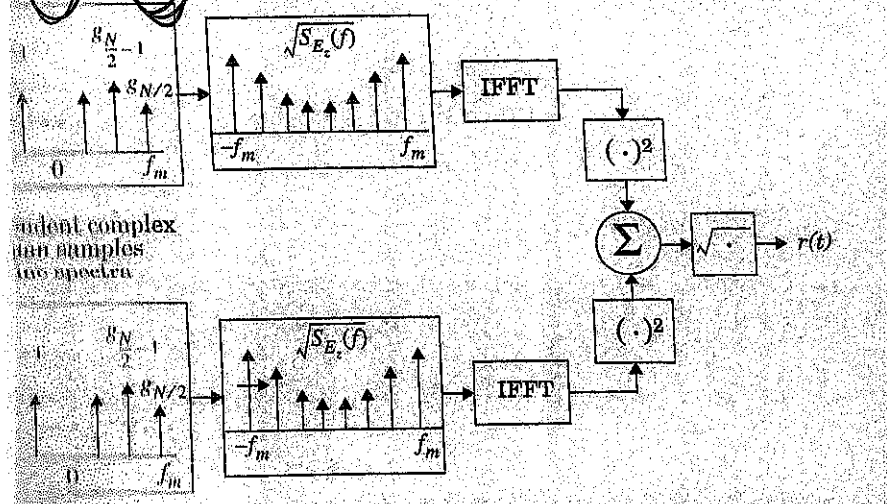

... Frequency domain implementation of a Rayleigh fading simulator at baseband

...plement the simulator shown in Figure 5.24, the following steps are used:

...ify the number of frequency domain points (N) used to represent $\sqrt{S_{E_z}(f)}$ and the ...imum Doppler frequency shift (f_m). The value used for N is usually a power of two. ...pute the frequency spacing between adjacent spectral lines as $\Delta f = 2f_m/(N-1)$. ...defines the time duration of a fading waveform, $T = 1/\Delta f$.

...rate complex Gaussian random variables for each of the $N/2$ positive frequency ...ponents of the noise source.

...struct the negative frequency components of the noise source by conjugating positive ...uency values and assigning these at negative frequency values.

...tiply the in-phase and quadrature noise sources by the fading spectrum $\sqrt{S_{E_z}(f)}$.

...form an IFFT on the resulting frequency domain signals from the in-phase and quadra... arms to get two N-length time series, and add the squares of each signal point in time ...eate an N-point time series like under the radical of Equation (5.67). Note that each ...drature arm should be a real signal after the IFFT to model Equation (5.63).

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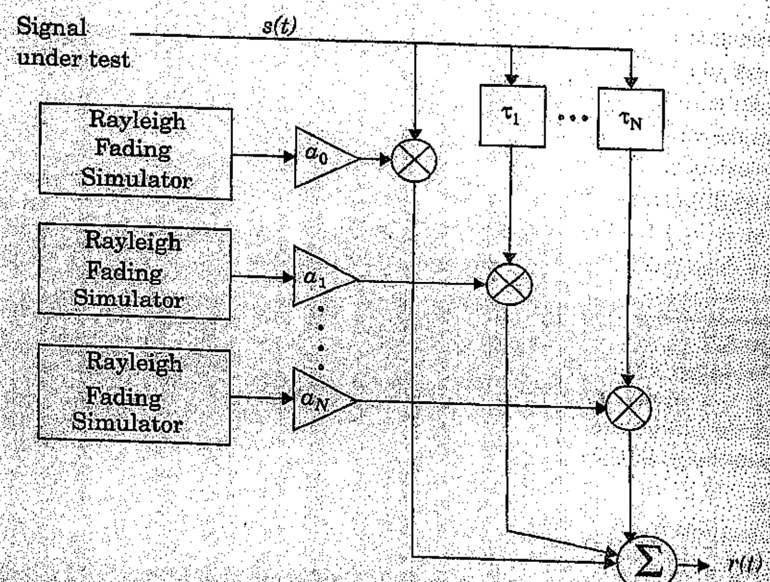


Figure 5.25 A signal may be applied to a Rayleigh fading simulator to determine performance i... a wide range of channel conditions. Both flat and frequency selective fading conditions may b... simulated, depending on gain and time delay settings.

...implement the IFFT such that each arm of Figure 5.24 produces a real time domain signal a... given by $T_c(t)$ and $T_s(t)$ in Equations (5.64) and (5.65).

To determine the impact of flat fading on an applied signal $s(t)$, one merely needs to mul...tiply the applied signal by $r(t)$, the output of the fading simulator. To determine the impact of ...more than one multipath component, a convolution must be performed as shown in Figure 5.25.

5.7.3 Level Crossing and Fading Statistics

Rice computed joint statistics for a mathematical problem which is similar to Clarke's fading mode... [Cla68], and thereby provided simple expressions for computing the average number of level cross...ing and the duration of fades. The *level crossing rate* (LCR) and *average fade duration* of a Ray...leigh fading signal are two important statistics which are useful for designing error control code... and diversity schemes to be used in mobile communication systems, since it becomes possible to ... relate the time rate of change of the received signal to the signal level and velocity of the mobile.

The *level crossing rate* (LCR) is defined as the expected rate at which the Rayleigh fading ...

~~Bgpex~~ 246/5

Level Crossing Rate and Averange Fade Duration

Assumptions of the Baseband Model

Level Crossing Rate (LCR) and Average Fade Duration

The Level crossing rate (LCR) and the Averag fade duration of a Rayleigh fading signal are two important statistics needed for the design of Error Control Codes and diversity schemes used in mobile communications.
→ Interleaving

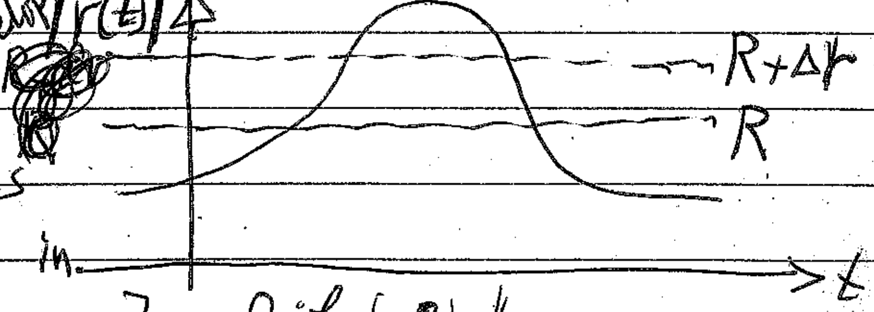
LCR

(No. of crossings / second)

Is the expected rate at which the Rayleigh fading envelope (normalized to rms signal level) crosses a specified level in the positive-going direction.

r: envelope

$\dot{r}$: time derivative of envelope



Given the joint statistics of $(r, \dot{r})$, if (r) is in the range of $[R, R+\Delta r]$ and if $(\dot{r})$ has a specific value (let it be $\dot{r}$ for generality) then the No. of crossings per second (N) is

depends on prob. of specific slope $\dot{r}$ at $r=R$

slope $= \dot{r}$

$$N = \frac{\dot{r}}{\Delta r} \begin{matrix} \leftarrow \text{volt/sec} \\ \leftarrow \text{volt} \end{matrix} \Rightarrow \frac{\text{cross}}{\text{sec}}$$

But N is a R.V. depending on $f(r, \dot{r})$, hence the average crossing rate is

$$N_R = E[N] = \int_{\dot{r}=0}^{\infty} \int_{r=R}^{R+\Delta r} \left(\frac{\dot{r}}{\Delta r}\right) f(r, \dot{r}) \, dr \, d\dot{r}$$

$$= \int_{\dot{r}=0}^{\infty} \frac{\dot{r}}{\Delta r} \left[\int_{r=R}^{R+\Delta r} f(r, \dot{r}) \, dr \right] d\dot{r} = \int_{\dot{r}=0}^{\infty} \frac{\dot{r}}{\Delta r} f(R, \dot{r}) \int_{r=R}^{R+\Delta r} dr \, d\dot{r}$$

As $\Delta r \to 0$

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$$N_R = \int_{r^o=0}^{\infty} \frac{r^o}{\Delta r} f(R, r^o) \Big[r \Big]_R^{R+\Delta r} \, dr^o = \int_{r^o=0}^{\infty} \frac{r^o}{\cancel{\Delta r}} f(R, r^o) \cancel{\Delta r} \, dr^o$$

$$\boxed{N_R = \int_{r^o=0}^{\infty} r^o f(R, r^o) \, dr^o}$$

Next we need to find the joint density $f(r, r^o)$.

To do this we will start by computing a joint pdf that will be useful to find $f(r, r^o)$.

$f(x, y, dx/dt, dy/dt)$

<u>Note</u>

Here $x = X(t)$, $y = Y(t)$, ----

For simplicity we drop (t) from $X(t)$ $Y(t)$ -- and use x, y based on WSS assumption and on the fact that we need joint pdf all at time $= (t)$ for all variables.

x, y are obviously Gaussian (zero-mean). dx/dt and dy/dt are also Gaussian (zero mean), since differentiation is an LTI system.

x and y are independent. x and dx/dt are also uncorrelated ($=$ independent) which can be shown from freq. domain relations. So is y & $\frac{dy}{dt}$

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The joint pdf of $x, y, \underbrace{\partial x/\partial t}_{x^o}, \underbrace{\partial y/\partial t}_{y^o}$ is jointly Gaussian (all are independent)

Let $\sigma_x^2 = \sigma_y^2 = \alpha^2$, $\sigma_{\frac{\partial x}{\partial t}}^2 = \sigma_{\frac{\partial y}{\partial t}}^2 = \beta^2$

$$f(x, y, x^o, y^o) = f_X(x) f_Y(y) f_{X^o}(x^o) f_Y(y^o)$$

$$= \frac{1}{(2\pi)^2 \alpha^2 \beta^2} e^{-\frac{1}{2}\left(\frac{x^2}{\alpha^2} + \frac{y^2}{\alpha^2} + \frac{x^{o2}}{\beta^2} + \frac{y^{o2}}{\beta^2}\right)}$$

$$\alpha^2 = E[X^2(t)] = R_{XX}(\tau = 0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(w) e^{jw\tau} \partial w \quad \tau = 0$$

$$\therefore \alpha^2 = \frac{1}{2\pi}\int_{-\infty}^{\infty} S_{XX}(w)\, 2\pi\, \partial f \qquad \text{But } w = 2\pi f, \quad \partial w = 2\pi\, \partial f$$

$$\alpha^2 = \int_{-\infty}^{\infty} S_{XX}(w)\, \partial f$$

Since x, y, x^o, y^o represent the baseband model or the I, Q components, from downshift (by f_c) of $\boxed{\text{eqn. 5.78}}$ in Rappaport we get

$$\underset{\text{Doppler}}{S_D(w)} = \frac{1.5K}{\pi\sqrt{f_m - f^2}} \quad K: \text{stands for power} \quad \Longleftrightarrow \left(-f_m \le f \le f_m\right)$$

$$S_{XX}(w) = \frac{1}{2} S_D(w) = \frac{3K/4}{\pi\sqrt{f_m - f^2}}$$

(identical Imag. & Real parts x, y)

$$\alpha^2 = \int_{-\infty}^{\infty} \frac{3K/4}{\pi\sqrt{f_m^2 - f^2}}\, \partial f = \frac{3K}{4\pi}\left[\sin^{-1}\frac{f}{f_m}\right]_{-f_m}^{f_m}$$

$$= \frac{3K}{4\pi}\left[\sin^{-1}(1) - \sin^{-1}(-1)\right] = \frac{3K}{4\pi}\left[\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)\right]$$

$$\boxed{\alpha^2 = 3K/4}$$

Next we find $\langle B^2 \rangle$

Since $x^o = dx/dt \Longrightarrow S_{x^o x^o}(w) = w^2 S_{xx}(w)$

$$B^2 = R_{x^o x^o}(\tau = 0) = \int_{-\infty}^{\infty} S_{x^o x^o}(w)\,df = \int_{-\infty}^{\infty} w^2 S_{xx}(w)\,df$$

$$B^2 = \int_{-\infty}^{\infty} (2\pi f)^2 S_{xx}(w)\,df = (2\pi)^2 \int_{-\infty}^{\infty} S_{xx}(w) f^2\,df$$

$$= (2\pi)^2 \int_{-f_m}^{f_m} \frac{f^2 \times 3K/4}{\pi\sqrt{f_m^2 - f^2}}\,df = 3K\pi \int_{-f_m}^{f_m} \frac{f^2}{\sqrt{f_m^2 - f^2}}\,df$$

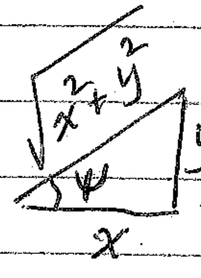
$$= 3K\pi \left[\underbrace{-\frac{f}{2}\sqrt{f_m^2 - f^2}}_{=0} + \frac{f_m^2}{2}\sin^{-1}\frac{f}{f_m} \right]_{-f_m}^{f_m}$$

$$\boxed{B^2 = 1.5 K\pi^2 f_m^2 = 2\pi^2 f_m^2 \alpha^2}$$

Before we can find $f(r, r^o)$ that we need we have to use auxillary variable ideas by finding a larger joint pdf and then compute $f(r, r^o)$ as a marginal density

Define $\boxed{f(r, r^o, \psi, \psi^o)}$

$$\psi = \tan^{-1}\frac{y}{x}, \quad \psi^o = \frac{d\psi}{dt}$$



$$r = \sqrt{x^2 + y^2} \quad \text{—— (1)}$$

$$r^o = \frac{dr}{dt} = \frac{d}{dt}(x^2+y^2)^{1/2} = \frac{1}{\cancel{2}}(x^2+y^2)^{-\frac{1}{2}}\left(\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}\right)$$

$$r^o = \frac{x x^o + y y^o}{\sqrt{x^2+y^2}} = x^o\cos\psi + y^o\sin\psi \quad \text{—— (2)}$$

$$\psi^o = \frac{d}{dt}\left[\tan^{-1}\left(\frac{y}{x}\right)\right] = \frac{1 \times \frac{x\frac{dy}{dt} - y\frac{dx}{dt}}{x^2}}{1 + \left(\frac{y}{x}\right)^2} = \frac{x y^o - y x^o}{x^2 + y^2}$$

$$\psi^o = \frac{1}{\sqrt{x^2+y^2}}\,\frac{x y^o - y x^o}{\sqrt{x^2+y^2}} = \frac{1}{r}\left[y^o\cos\psi - x^o\sin\psi\right]$$

$$\psi^\circ r = y^\circ \cos\psi - x^\circ \sin\psi \quad \text{——— } (3)$$

(2) $\times \sin\psi \Rightarrow r^\circ \sin\psi = x^\circ \cos\psi \sin\psi + y^\circ \sin^2\psi$

(3) $\times \cos\psi \Rightarrow \psi^\circ r \cos\psi = y^\circ \cos^2\psi - x^\circ \sin\psi \cos\psi$

(+)

$$\boxed{r^\circ \sin\psi + \psi^\circ r \cos\psi = y^\circ} \quad \text{——— } (A)$$

(2) $\times \cos\psi \Rightarrow r^\circ \cos\psi = x^\circ \cos^2\psi + y^\circ \sin\psi \cos\psi$

(3) $\times \sin\psi \Rightarrow \psi^\circ r \sin\psi = y^\circ \cos\psi \sin\psi - x^\circ \sin^2\psi$

(−)

$$\boxed{r^\circ \cos\psi - \psi^\circ r \sin\psi = x^\circ} \quad \text{——— } (B)$$

$$\boxed{\begin{array}{l} x = r\cos\psi \quad \text{——— } (C) \\ y = r\sin\psi \quad \text{——— } (D) \end{array}}$$

Equations A, B, C, D give $T_1^{-1}, T_2^{-1}, T_3^{-1}, T_4^{-1}$ needed to find Jacobian for transformation from

$$f(x, y, x^\circ, y^\circ) \Longrightarrow f(r, r^\circ, \psi, \psi^\circ)$$

$$J = \det \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial r^\circ} & \frac{\partial x}{\partial \psi} & \frac{\partial x}{\partial \psi^\circ} \\ \partial y/\partial r & \partial y/\partial r^\circ & \partial y/\partial \psi & \partial y/\partial \psi^\circ \\ \partial x^\circ/\partial r & \partial x^\circ/\partial r^\circ & \partial x^\circ/\partial \psi & \partial x^\circ/\partial \psi^\circ \\ \partial y^\circ/\partial r & \partial y^\circ/\partial r^\circ & \partial y^\circ/\partial \psi & \partial y^\circ/\partial \psi^\circ \end{vmatrix}$$

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$$J = \det \begin{bmatrix} \cos\psi & 0 & -r\sin\psi & 0 \\ \sin\psi & 0 & r\cos\psi & 0 \\ -\dot\psi\sin\psi & \cos\psi & -\dot r\sin\psi - \dot\psi r\cos\psi & -r\sin\psi \\ \dot\psi\cos\psi & \sin\psi & \dot r\cos\psi - \dot\psi r\sin\psi & r\cos\psi \end{bmatrix}$$

$$= \cos\psi\left[-r\cos\psi\left(r\cos^2\psi + r\sin^2\psi\right)\right]$$
$$- r\sin\psi\left[\sin\psi\left(r\cos^2\psi + r\sin^2\psi\right)\right]$$
$$= -r^2\cos^2\psi - r^2\sin^2\psi = -r^2$$

$$|J| = |-r^2| = r^2$$

$$\therefore f(r,\dot r,\psi,\dot\psi) = f_{x,y,\dot x,\dot y}\left(T_1^{-1}, T_2^{-1}, T_3^{-1}, T_4^{-1}\right)|J|$$

$$= \frac{r^2}{(2\pi)^2\alpha^2\beta^2} e^{-\frac{1}{2}\left[\frac{r^2\cos^2\psi}{\alpha^2} + \frac{r^2\sin^2\psi}{\alpha^2} + \frac{(\dot r\cos\psi - \dot\psi r\sin\psi)^2}{\beta^2} + (\dot r\sin\psi + \dot\psi r\cos\psi)^2/\beta^2\right]}$$

$$= \frac{r^2}{(2\pi)^2\alpha^2\beta^2} e^{-\frac{1}{2}\left[\frac{r^2}{\alpha^2} + \frac{\dot r^2\cos^2\psi + \dot\psi^2 r^2\sin^2\psi - \cancel{2\dot r\dot\psi r\sin\psi\cos\psi}}{\beta^2} + \frac{\dot r^2\sin^2\psi + \dot\psi^2 r^2\cos^2\psi + \cancel{2\dot r\dot\psi r\sin\psi\cos\psi}}{\beta^2}\right]}$$

$$\boxed{f(r,\dot r,\psi,\dot\psi) = \frac{r^2}{(2\pi)^2\alpha^2\beta^2} e^{-\frac{1}{2}\left[\frac{r^2}{\alpha^2} + \frac{\dot r^2 + \dot\psi^2 r^2}{\beta^2}\right]}}$$

$$f(r,\dot r) = \int_{\psi=-\pi}^{\pi}\int_{\dot\psi=-\infty}^{\infty} f(r,\dot r,\psi,\dot\psi)\, d\psi\, d\dot\psi$$

$$= \frac{r^2}{4\pi^2\alpha^2\beta^2}\int_{\dot\psi=-\infty}^{\infty} e^{-\frac{1}{2}\left[\frac{r^2}{\alpha^2} + \frac{\dot r^2 + \dot\psi^2 r^2}{\beta^2}\right]} d\dot\psi \underbrace{\int_{-\pi}^{\pi} d\psi}_{2\pi}$$

(31/5) 7

$$f(r,r^0) = \frac{r^2}{2\pi\alpha^2\beta^2} e^{-\frac{1}{2}\left(\frac{r^2}{\alpha^2}+\frac{r_0^2}{\beta^2}\right)} \int_{\psi^0=-\infty}^{\infty} e^{-\frac{1}{2}\frac{\psi_0^2}{\beta^2/r^2}}\, d\psi^0$$

$$= \frac{r}{\sqrt{2\pi}\,\alpha^2\beta} e^{-\frac{1}{2}\left[\frac{r^2}{\alpha^2}+\frac{r_0^2}{\beta^2}\right]} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\beta^2/r^2}} e^{-\frac{1}{2}\frac{\psi_0^2}{\beta^2/r^2}} = 1$$

$$\sigma^2 = \frac{\beta^2}{r^2}$$

$$\boxed{f(r,r^0) = \frac{r}{\alpha^2\sqrt{2\pi\beta^2}} e^{-\frac{r^2}{2\alpha^2}} e^{-r_0^2/2\beta^2}}$$

$$\therefore N_R = (-\beta^2)\int_{r^0=0}^{\infty}\left(-\frac{1}{\beta^2}\right) r^0 \frac{R}{\alpha^2\sqrt{2\pi\beta^2}} e^{-\frac{R^2}{2\alpha^2}} e^{-r_0^2/2\beta^2}\, dr^0$$

$$= \frac{R}{\alpha^2\sqrt{2\pi\beta^2}} e^{-R^2/2\alpha^2} (-\beta^2)\left[e^{-r_0^2/2\beta^2}\right]_0^{\infty} \rightarrow \left(e^{-\infty} - e^0\right) = (-1)$$

$$= \frac{\beta R e^{-R^2/2\alpha^2}}{\sqrt{2\pi}\,\alpha^2} = \frac{\overset{\beta}{\sqrt{2}\,\pi f_m \alpha}\, R e^{-R^2/2\alpha^2}}{\sqrt{2\pi}\,\alpha^2}$$

$$= \sqrt{\pi} f_m \frac{R}{\alpha} e^{-R^2/2\alpha^2} \times \frac{\sqrt{2}}{\sqrt{2}} = \sqrt{2\pi} f_m \frac{R}{\sqrt{2}\alpha} e^{-(R/\sqrt{2}\alpha)^2}$$

$$\boxed{N_R = \sqrt{2\pi} f_m \rho e^{-\rho^2}}$$

$$\boxed{\rho = \frac{R}{\sqrt{2}\alpha} = \frac{R}{R_{rms}}}$$

$$(\sqrt{2}\alpha)^2 + (\sqrt{2}\alpha)^2 =$$

$r_{rms} = \sqrt{E[|r|^2]}$ But $r = x + jy$

$$= \sqrt{E[x^2+y^2]}$$

$$= \sqrt{\overline{x^2+y^2}} = \sqrt{\alpha^2+\alpha^2} = \sqrt{2}\alpha$$

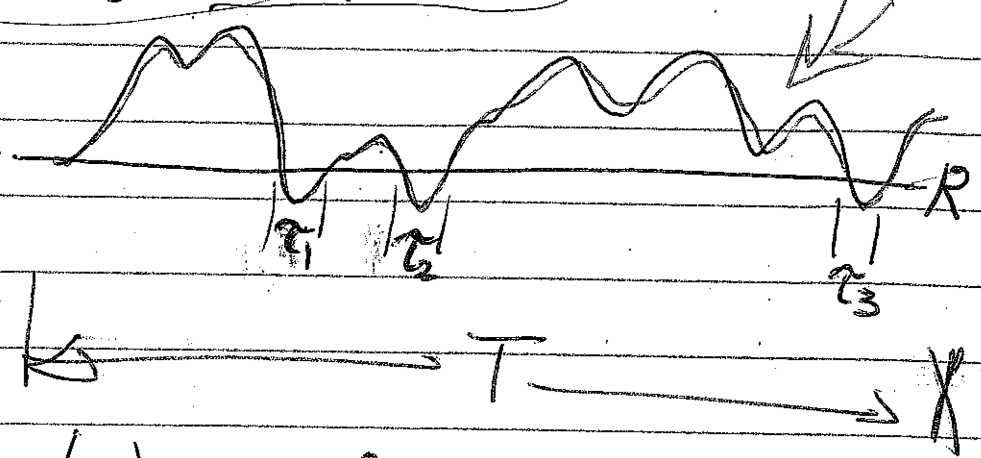
Average Fade Duration ($\bar{\tau}$)

Is the average period of time for which the received signal remains below a specified level (R).

duration of i^{th} fade

$$\bar{\tau} = \frac{\sum \tau_i \text{ (over period } = T)}{\text{No. of crossings in the period } (T)} \qquad \left(= \frac{\tau_1 + \tau_2 + \tau_3}{3} \right)$$

No. of fades during period (T)



$$\text{No. of crossings in } (T) = \frac{\text{No. of crossings}}{\text{second}} \times T \text{ (second)}$$

$$= N_R \, T$$

$$\therefore \bar{\tau} = \frac{\sum \tau_i}{N_R \, T} \quad \Big| \;\Rightarrow \text{But } \left.\frac{\sum \tau_i}{T}\right|_{\text{As } T \to \infty} = P_r[r \leq R]$$

As $T \to \infty$

$$\therefore \bar{\tau} = \frac{P_r[r \leq R]}{N_R}$$

$$P_r[r \leq R] = \int_0^R f_r(r)\,dr = 1 - e^{-\rho^2} \quad \text{p.16}$$

$$\bar{\tau} = \frac{1 - e^{-\rho^2}}{\sqrt{2\pi}\, f_m \, \rho \, e^{-\rho^2}}$$

$$\boxed{\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho \, f_m \sqrt{2\pi}}}$$

Average fade duration

ex) Compute the +ve-going level crossing rate for $\rho = 1$ for Rayleigh fading, when f_m (max. Doppler freq.) = 20 Hz. ~~What is the max. velocity of the mobile for this Doppler freq. if f_c = 900 MHz.~~

$$N_R = \sqrt{2\pi}\, f_m\, \rho\, e^{-\rho^2} = \sqrt{2\pi} \times 20 \times 1\, e^{-1} = 18.44$$

crossings/second

$$f_{d_{max}} = \frac{v}{\lambda}$$

$$v = f_{max}\,\lambda = 20 \times \frac{1}{3}\text{m} = 6.66\ \frac{\text{m}}{\text{s}} = 24\ \frac{\text{km}}{\text{hr}}$$

ex) Find the average fade duration for threshold levels $\rho = 0.01$, $\rho = 0.1$, $\rho = 1$, if Doppler ~~freq~~ $f_m = 200$ Hz.

$$\bar{\tau} = \frac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}}$$

For $\rho = 0.01$ $\quad \bar{\tau} = \dfrac{e^{(0.01)^2} - 1}{(0.01)200\sqrt{2\pi}} = \boxed{19.9\,\mu s}$

$\rho = 0.1$ $\quad \bar{\tau} = \dfrac{e^{0.1^2} - 1}{(0.1)200\sqrt{2\pi}} = \boxed{200\,\mu s}$

$\rho = 1$ $\quad \bar{\tau} = \boxed{3.43\,ms}$

Note on: 8-PSK, 256-QAM (3 4/5) ← No. of bits/symbol

ex) ~~Find the~~ Given that Doppler freq is 20 Hz.

a) Find $\bar{\tau}$ for threshold $\rho = 0.707$ with regards to $\bar{\tau}$

b) For 50 bps dig. modulation, is Rayleigh fading fast or slow?

c) Find average no. of bit errors per second.

d) (Assume that a bit error occurs whenever any portion of a bit encounters a fade for which ($\rho < 0.1$)).

---

a) $\bar{\tau} = \dfrac{e^{\rho^2} - 1}{\rho f_m \sqrt{2\pi}} = \dfrac{e^{0.707^2} - 1}{0.707 \times 20 \times \sqrt{2\pi}} = 18.3$ msec

b) Bit period $= \frac{1}{50} = 0.02$ sec $= 20$ msec

Since Bit period (20 msec) $> \bar{\tau}$ (18.3) msec average fade duration, the signal undergoes fast fading.

c) ~~Since a bit error occurs for $\rho < 0.1$~~

~~For $\rho < 0.1$~~ $\cancel{\bar{\tau} = \dfrac{e^{0.1^2} - 1}{0.1 \times 20 \times \sqrt{2\pi}} = 0.002 \text{ sec}}$

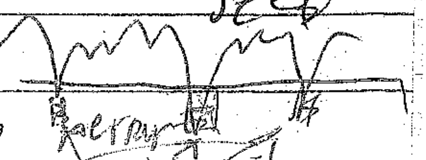
∴ Every fade hits one bit (on average).

$N_R \left(\frac{\text{cross}}{\text{sec}}\right)$ for $\rho = 0.1$

$= \sqrt{2\pi} \times 20 \times 0.1 e^{-0.1^2} = 4.96 \ \frac{\text{cross}}{\text{sec}}$

∴ $\frac{\text{No. of errors}}{\text{sec}} = \frac{\text{No. of fades}}{\text{sec}}$

$= 5$ errors/sec



Bit freq ⇒ 50 bits/sec

Prob.(Bit error) $= \frac{5}{50} = 10\%$

ex) A mobile receiver operates at $f_c = 860\,MHz$ and moves at 100 Km/hr.

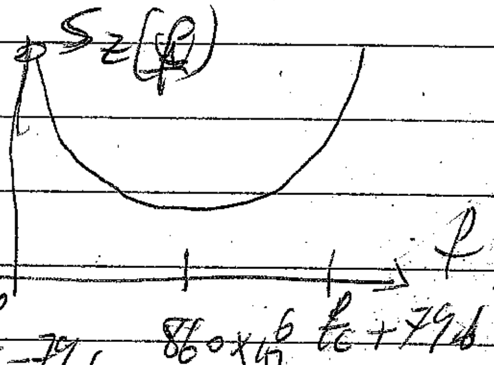
a) sketch the Doppler spectrum.

b) Calculate the level crossing rate & average fade duration for a signal level 20 dB below the r.m.s. level (i.e 20 dB below the local mean power).

a) $\lambda = c/f_c = 3\times10^8 / 860\times10^6 = 0.349\ m$

$v = \frac{100\times10^3}{3600}\frac{m}{s} = 27.78\ m/sec$

$f_m = \frac{v}{\lambda} = \frac{27.78}{0.349} = 79.6\ Hz$



b) $\rho = \frac{R}{R_{rms}}$ where R_{rms}^2 → local mean power predicted by the large scale path-loss model ($\propto d^{-n}$) + shadowing (log-normal)

$\rho_{dB} = 20\log\frac{R}{R_{rms}} \Longrightarrow -20\,dB = 20\log\frac{R}{R_{rms}}$

OR $\boxed{\rho_{dB} = 20\log\rho} \Longrightarrow -20 = 20\log\rho$

$\log\rho = -1 \Longrightarrow \boxed{\rho = 10^{-1} = 0.1}$

$N_R = \sqrt{2\pi}\, f_m\, \rho\, e^{-\rho^2} = \sqrt{2\pi}\,(79.6)\times 0.1\, e^{-(0.1)^2} = \boxed{19.7\ \frac{\text{cross}}{\text{sec}}}$

$\bar{\tau} = \frac{e^{\rho^2}-1}{\rho f_m\sqrt{2\pi}} = \frac{e^{0.01}-1}{0.1\times79.6\sqrt{2\pi}} = \boxed{0.5\ msec}$

36/5 end of chap 5

Coherence Distance (D_c)

- Is the separation distance in space over which a fading channel appears to be unchanged.
- D_c is important in the design of wireless receivers that employ spatial diversity to combat spatial selectivity.
- For static channel we have the relation

$$T_c = \frac{D_c}{v} \quad \text{OR} \quad D_c = v\,\frac{m}{sec} \times T_c(sec) = vT_c$$

- Definition of D_c may be based on the autocovariance fn. of the envelope.
- A convenient definition satisfying $\rho(D_c) = 0.5$ is given by

$$D_c = v\frac{9}{16\pi f_m} \qquad f_m = v/\lambda \qquad D_c = v\frac{9}{16\pi(v/\lambda)}$$

$$\boxed{D_c \simeq \frac{9\lambda}{16\pi}} \simeq 0.2\lambda$$

For Omnidirectional Rayleigh Channel (static channel)

Suggested Problems (Chap. 5)

5.1, 5.6, 5.7, 5.8, 5.9, 5.10, 5.11, 5.12, 5.13, 5.14, 5.15, 5.16, 5.19, 5.27, 5.28 5.29, 5.30, 5.32

Nyquist Pulses for zero-ISI

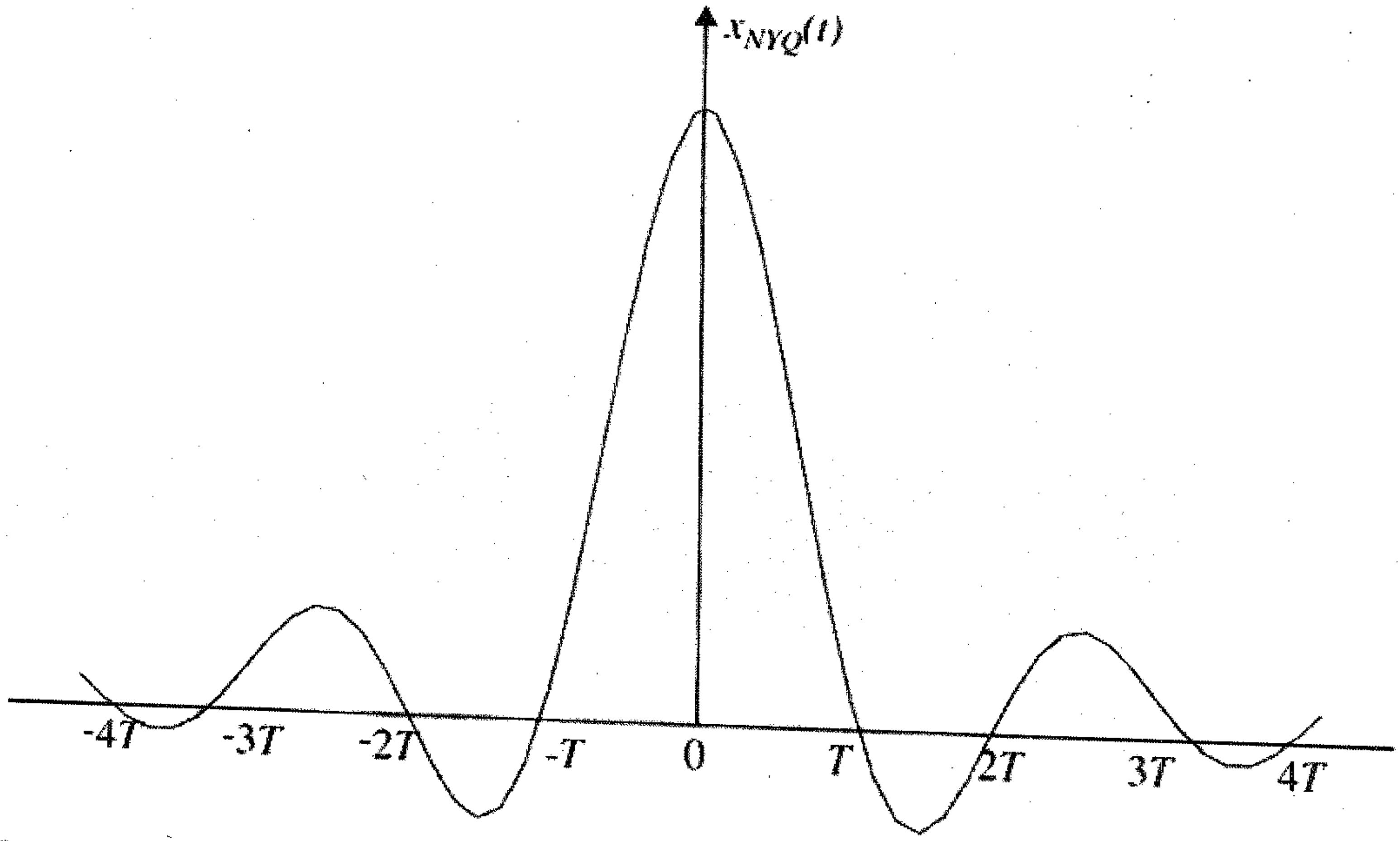


Figure 6.15 Nyquist ideal pulse shape for zero intersymbol interference.

Raised Cosine Spectrum

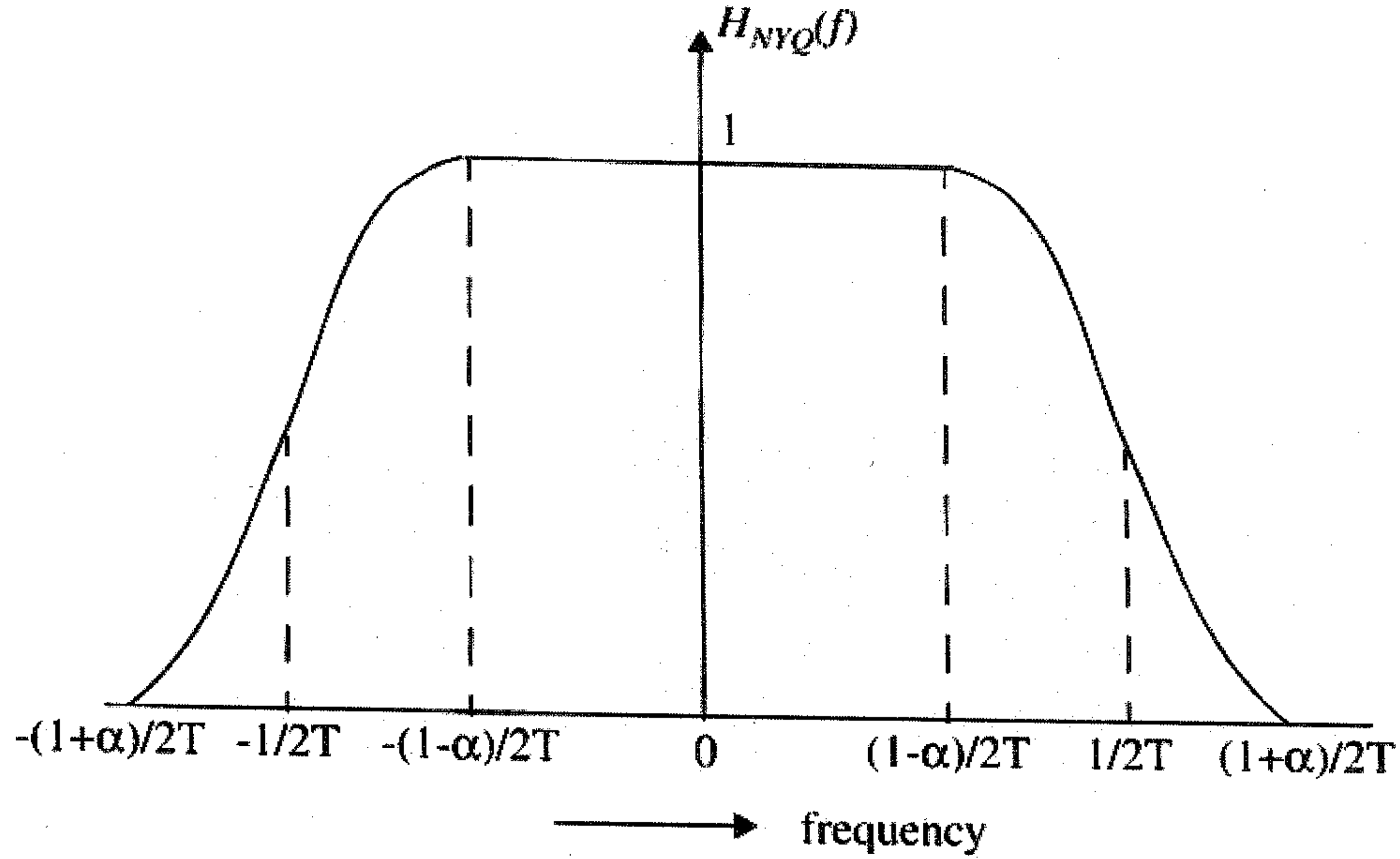


Figure 6.16 Transfer function of a Nyquist pulse-shaping filter at baseband.

Spectrum of Raised Cosine pulse

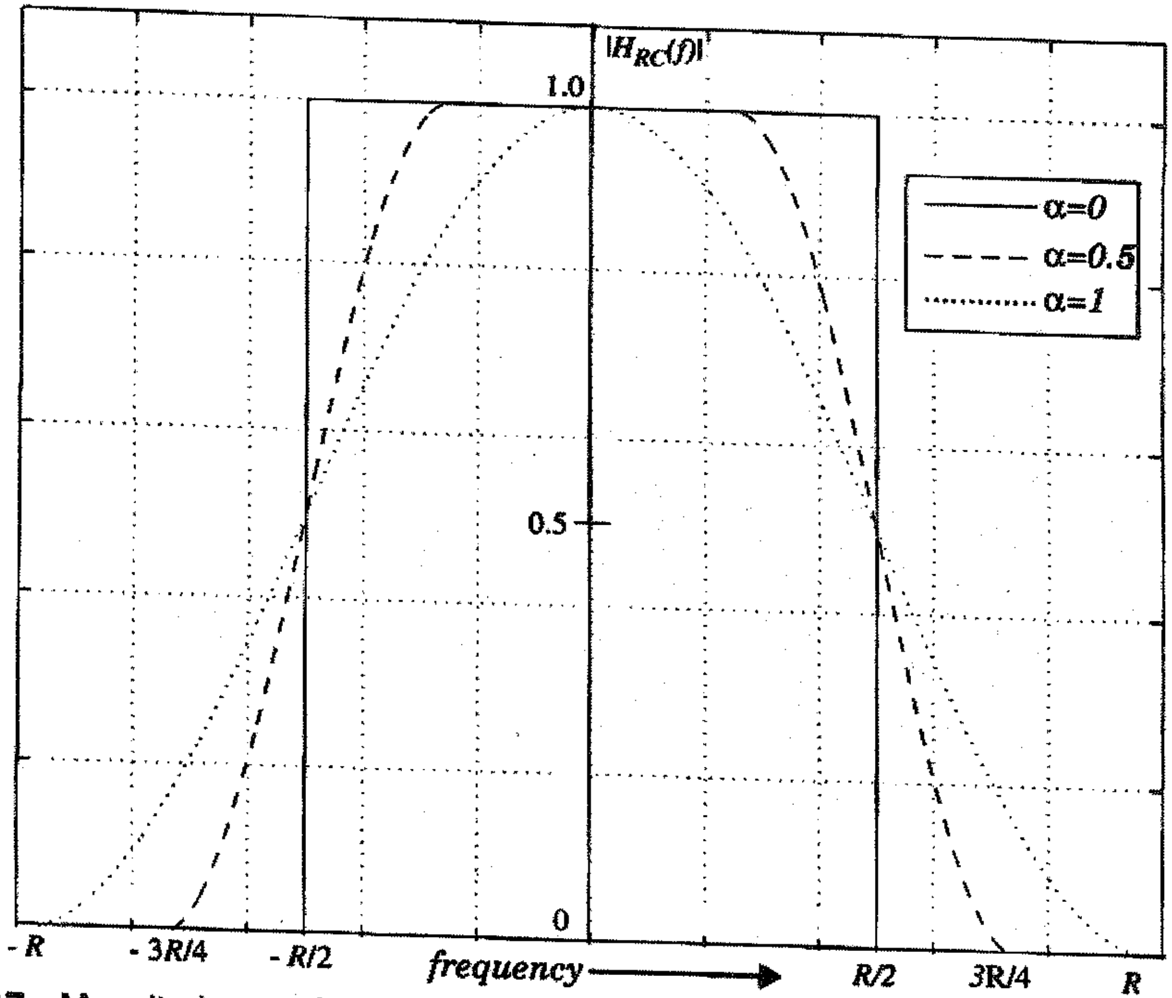


Figure 6.17 Magnitude transfer function of a raised cosine filter at baseband.

Raised Cosine pulses

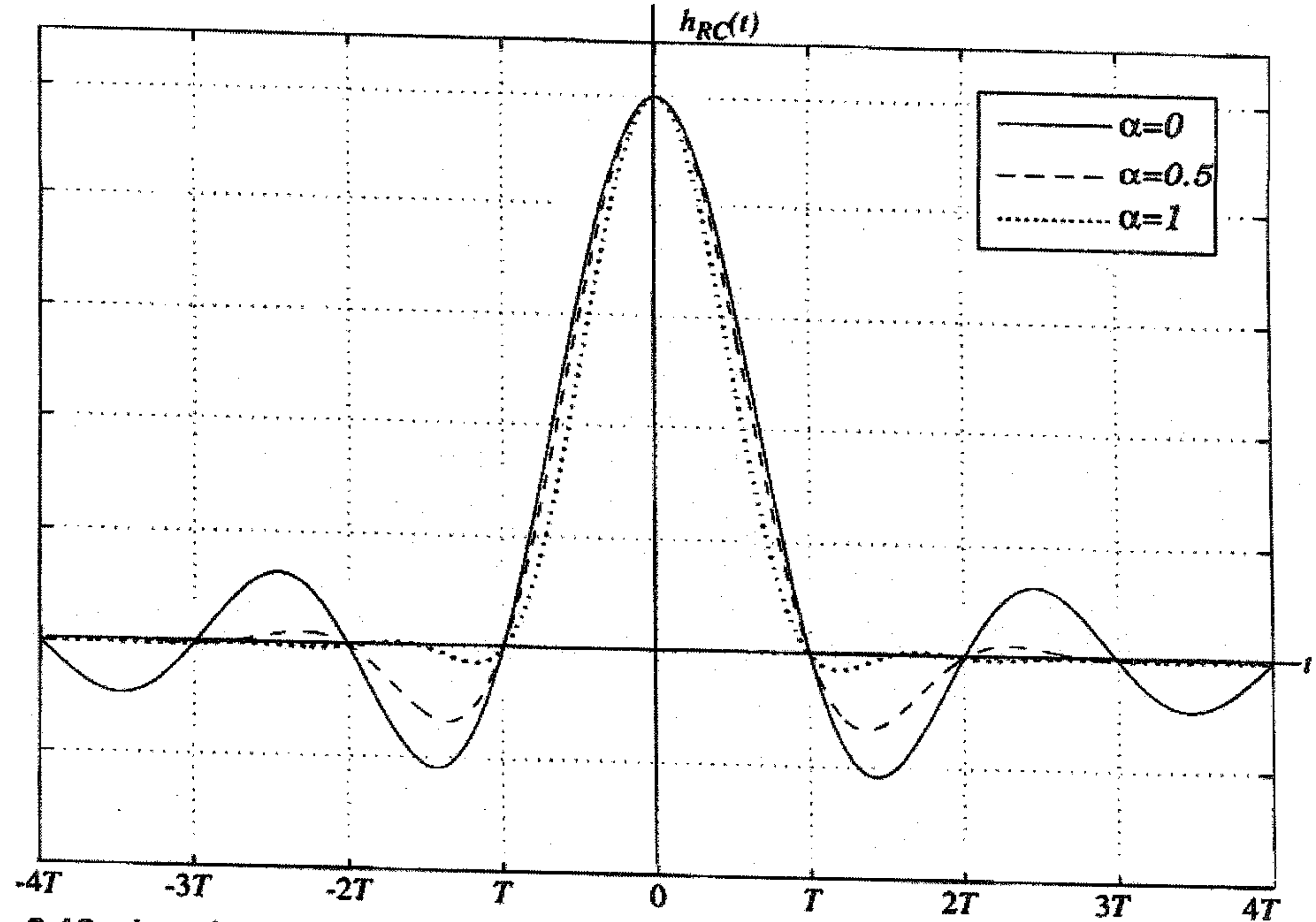


Figure 6.18 Impulse response of a raised cosine rolloff filter at baseband.

RF signal usig Raised Cosine

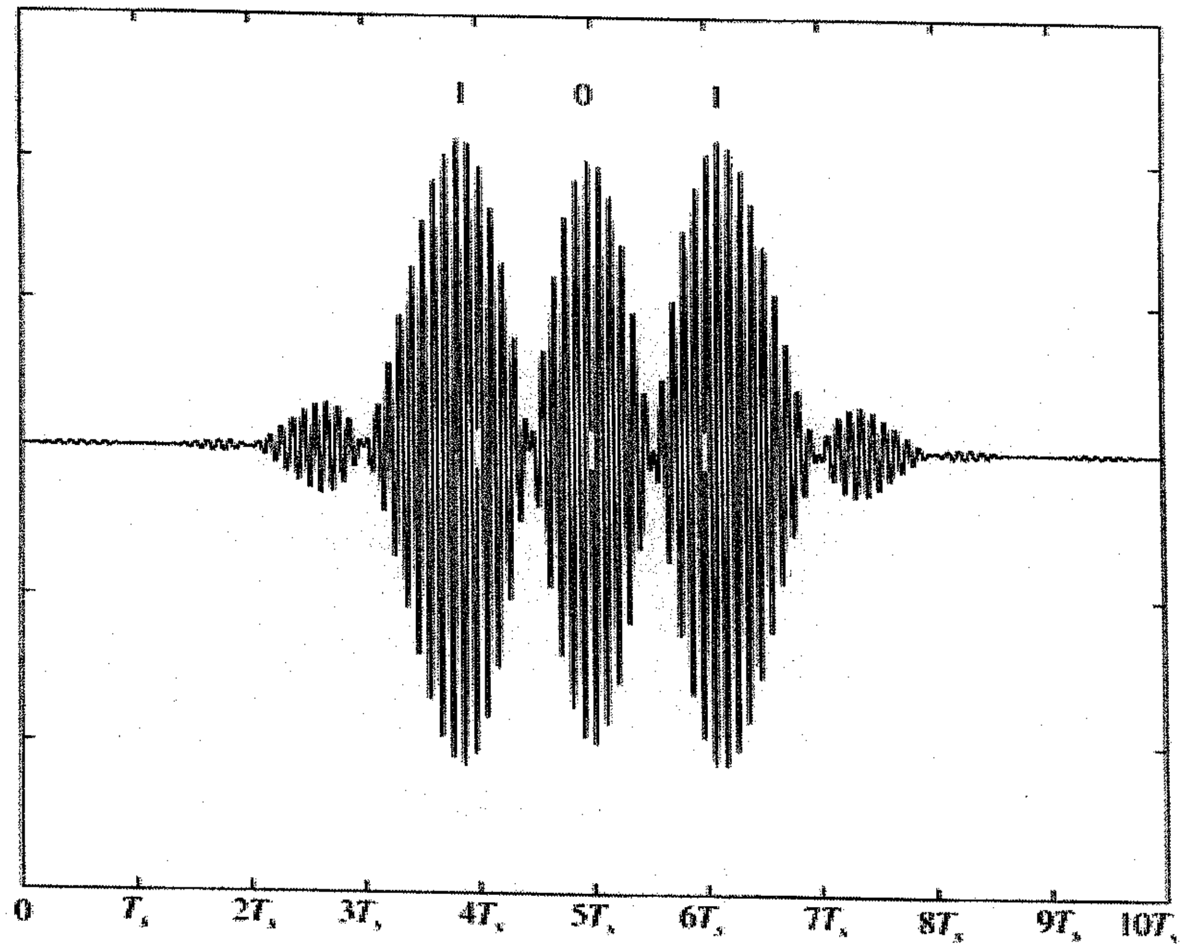


Figure 6.19 Raised cosine filtered ($\alpha = 0.5$) pulses corresponding to 1, 0, 1 data stream for a BPSK signal. Notice that the decision points (at $4T_s$, $5T_s$, $6T_s$) do not always correspond to the maximum values of the RF waveform.

Gaussian pulse-shapes

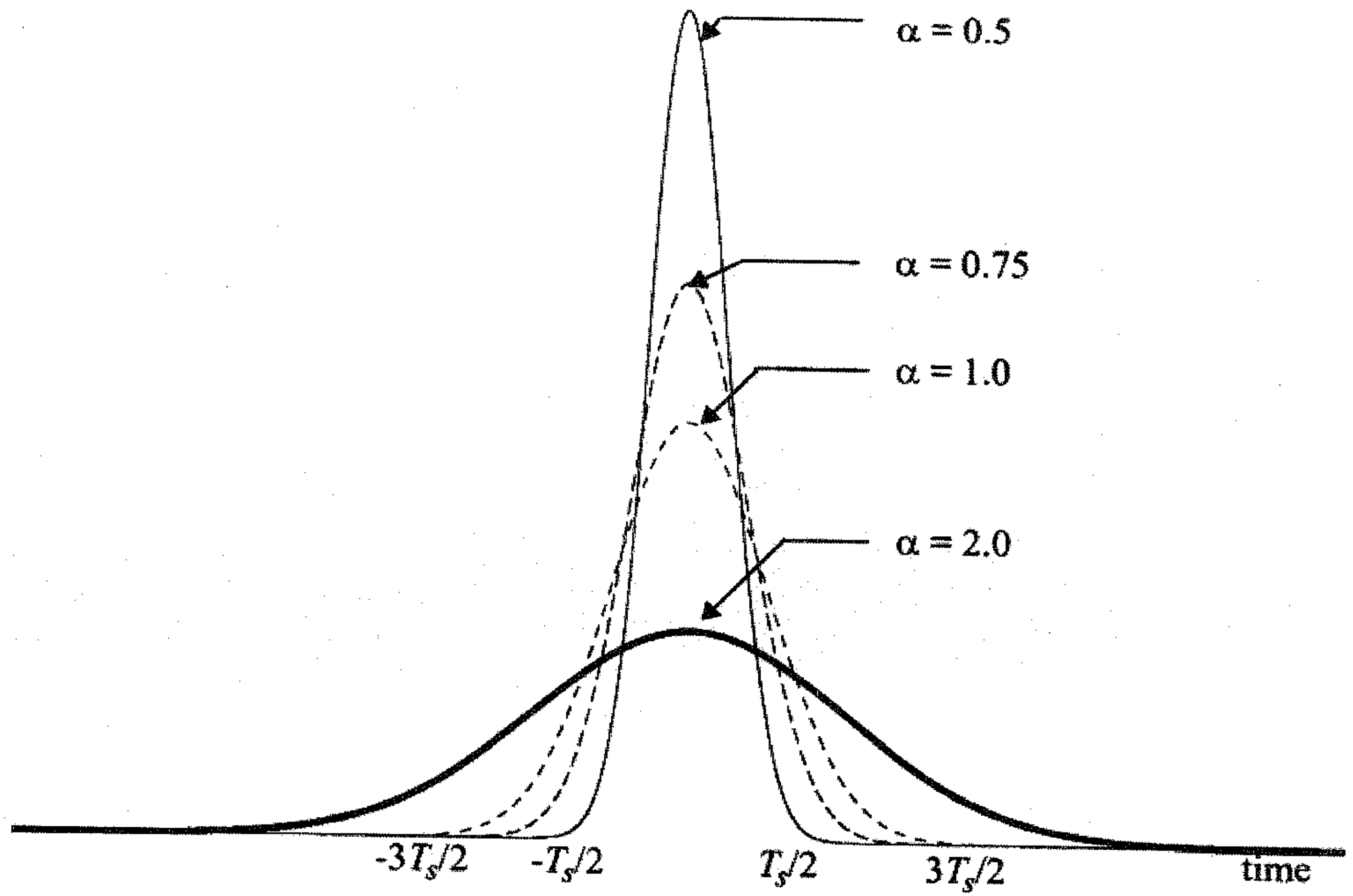


Figure 6.20 Impulse response of a Gaussian pulse-shaping filter.

BPSK constellation

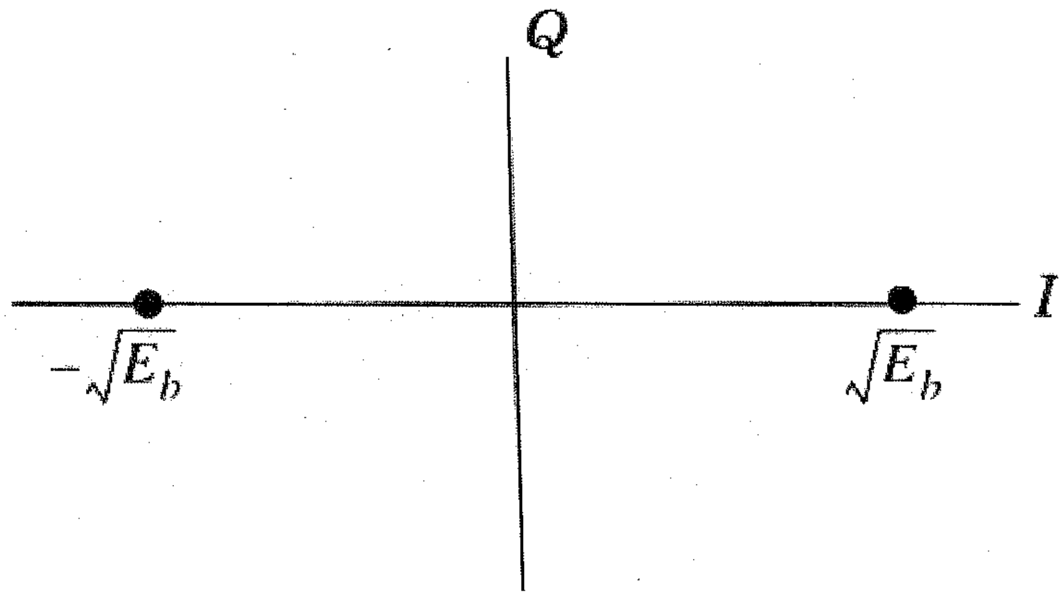
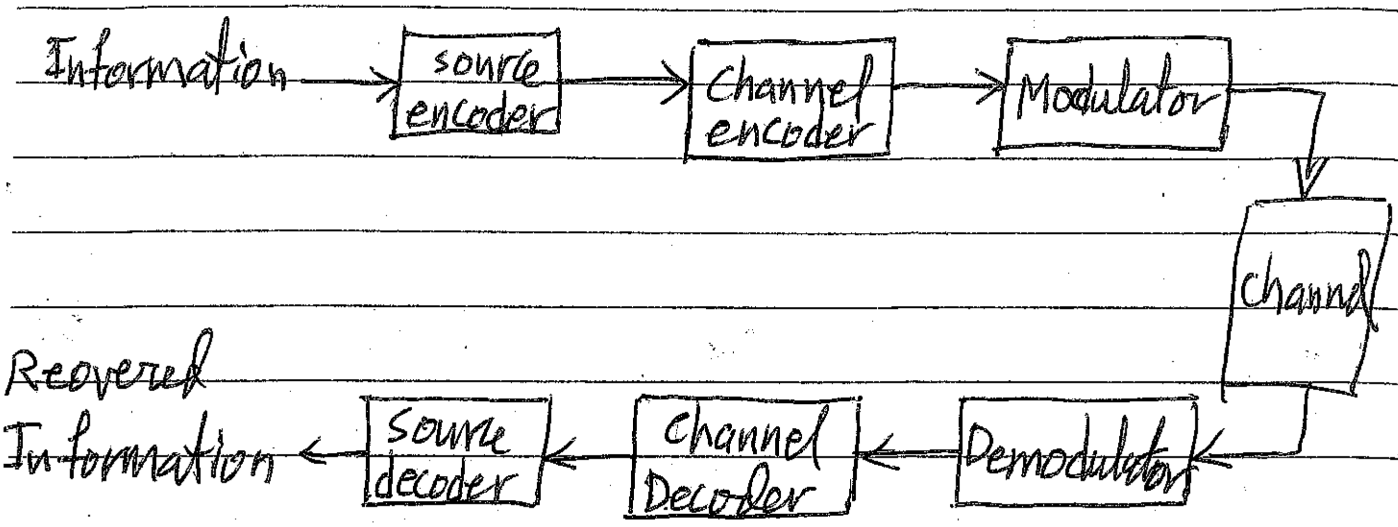


Figure 6.21 BPSK constellation diagram.

Chap. 6

Digital Modulation Techniques



Block Diagram of Digital Communication system

Bandwidth (BW) and Power Spectral density of dig. signals

A Baseband digitally modulated signal has the form

$$x(t) = \sum_{n=-\infty}^{\infty} I_n\, g(t-nT)$$

I_n: Information sequence, $g(t)$: pulse shape, T: symbol period

If $\{I_n\}$ is uncorrelated, zero-mean, data sequence (as it is often) then the power spectral density is

$$S_{xx}(w) = \frac{\sigma_I^2}{T} |G(w)|^2$$

σ_I^2: variance of $\{I_n\}$, $G(w) = F[g(t)]$

∴ The pulse shape $g(t)$ is the basic parameter that controlls BW.

BW Definitions

1- Null-to-Null BW (width of main lobe).
2- Half-power BW (3dB).
3- Federal Communications Commission (FCC) defines the BW as that Band containing (99%) of signal power Leaving 0.5% on each side.
4- A commonly used definition is that BW is the band outside of which PSD is below a certain threshold (45 dB or 60 dB).

Pulse Shaping Techniques

The choice of the pulse shape $g(t)$ is decided by:
1- BW requirement
2- Minimizing ISI

The 1^{st} stage of an optimal demodulator is a matched filter $g(T-t)$ (equivalent to a correlator demodulator), so that the output SNR is maximized. Moreover, BER becomes a fn. of only the bit energy and not the pulse shape.
The effective impulse response is

channel impulse response

$$h_{eff}(t) = \delta(t) * g(t) * h_c(t) * g(T-t)$$

Assuming using an equalizer filter to perfectly null the channel effect we get

$$h_{eff}(t) = \delta(t) * g(t) * \underbrace{h_c(t) * \overbrace{h_{eqz}(t)}^{\text{equalizer}}}_{\simeq\, \delta(t)} * g(T-t)$$

$$= g(t) * g(T-t)$$

$\therefore$ Equivalently the pulse shape that is effective for the overall (TX – RX system) (with equalization and matched filter) is $h_{eff}(t)$.

In practice, the effect of $\{h_{eff}(t)\}$ is often divided between TX-pulse shaping filter and RX-matched filter so that their convolution gives $h_{eff}(t)$

i.e., $$h_{eff}(t) = \underset{\text{TX}}{h_{tr}(t)} * \underset{\text{RX}}{h_{rec}(t)}$$

and each of these two filters has a transfer fn. $\sqrt{H_{eff}(w)}$.

$\therefore$ Effective pulse shape that is required to satisfy the (BW, ISI) requirements is $h_{eff}(t)$. Once selected, $h_{eff}(t)$ is divided between TX and RX.

ALL mobile users following the same system will see the same end-to-end pulse shape $\{h_{eff}(t)\}$ and not $g(t)$.

Choice of $\{h_{eff}(t)\}$

Nyquist criteria for ISI cancellation

Assuming a perfectly equalized system, ISI can be completely nulled if the overall pulse shape (h_{eff}) satisfies:

$$h_{eff}(t = nT_s) = \begin{cases} K \leftarrow \text{constant} & n = 0 \\ 0 & n \neq 0 \end{cases}$$

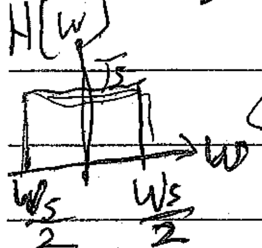
examples

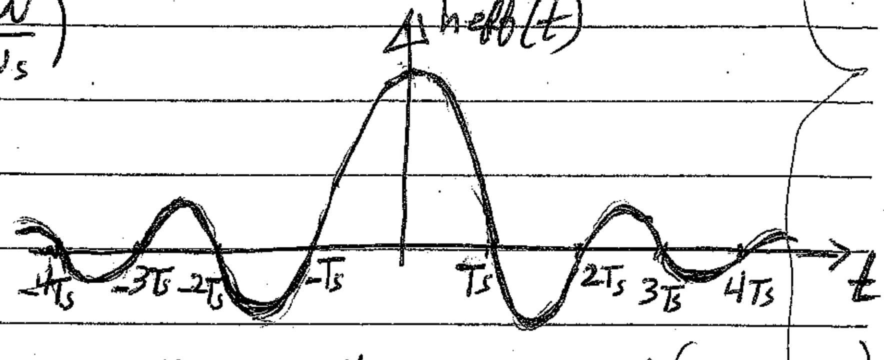
1- ~~Nyquist pulses~~ ~~For zero-ISI~~

$$\boxed{h_{eff}(t) = \frac{\sin(\pi t/T_s)}{(\pi t/T_s)}}$$

$\therefore$ $H(w) = T_s \text{rect}\left(\frac{w}{w_s}\right)$

$w_s = 2\pi/T_s$





$\therefore$ This pulse satisfies zero-ISI with min. BW of $w_s = \frac{2\pi}{T_s}$

Disadvantage: ~~Difficult to truncate~~ especially due to ~~....~~ $1/t$ i.e. slow decay

1- ~~Anticausal signal~~ (difficult to approximate)

2- $(\sin t/t)$ has a slope of $1/t$ at each zero-crossing thus any error in the sampling time (at nT_s) will cause significant ISI.

(Slope of $1/t^2$ or $1/t^3$ is more desirable to minimize ISI due to time jitter).

2- Rectangular $g(t)$

$g(t) = \text{rect}\left(\frac{t - 0.5T}{T}\right)$

with a matched filter $g(T-t)$

then overall effective pulse shape

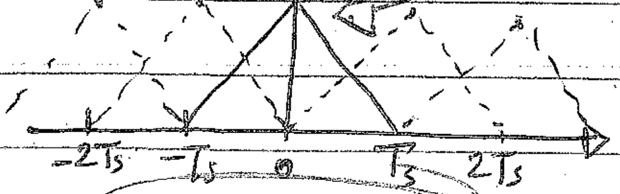
satisfies Nyquist criteria for zero-ISI

$$h_{eff} = g(t) * g(T-t) = \text{triangular}$$

$$H_{eff}(w) = \left[T\,\text{sinc}(wT/2)\right]^2$$

Disadvantages

- Time jitter causes large (ISI).
- PSD is large out of the main lobe (unacceptable).



zero-ISI

3- Raised cosine pulse shape

This is the most popular pulse shape used in mobile communications

$$H_{RC}(f) = \begin{cases} 1 & 0 \le |f| \le \frac{1-\alpha}{2T_s} \\ \frac{1}{2}\left[1 + \cos\left[\frac{\pi(|f| \cdot 2T_s - 1 + \alpha)}{2\alpha}\right]\right] & \frac{1-\alpha}{2T_s} \le |f| \le \frac{1+\alpha}{2T_s} \\ 0 & |f| > \frac{1+\alpha}{2T_s} \end{cases}$$

α : rolloff factor ($0 \to 1$)

$$h_{eff}(t) = h_{RC}(t) = \frac{\sin\left(\frac{\pi t}{T_s}\right)}{\pi t} \cdot \frac{\cos\left(\frac{\pi \alpha t}{T_s}\right)}{1 - \left(\frac{4\alpha t}{2T_s}\right)^2} \quad \Big\} \text{ satisfies zero-ISI}$$

Advantage

1- $h_{eff}(t)$ decays fast as $\simeq 1/t^3$ for $t \gg T_s$ at zero crossings

2- Increase of (α) increases (BW) but decreases sensetivity to timing jitter.

Hence $h_{eff}(t)$ can be truncated in time with little deviation in performance from Theory

~~can be truncated easily~~ two

timing jitter causes very small ISI

Notes

- Raised cosine is implemented by Tx-RX identical $\sqrt{H_{RC}(w)}$ filters so that we can acheive overall $h_{eff}(t) = h_{RC}(t)$ and at the same time provide a matched filter at RX for opt. perfmance.
- Implemented at baseband by truncating the infinite $h_{eff}(t)$ to ($\pm 6T_s$) about ($t=0$) for each symbol.
- Digital comm. systems with pulse shaping often store several symbols at a time inside the modulator and then clock out a group of symbols by using a look-up table which represent the waveform of stored symbols.

Relation of symbol rate (R_s) to filter BW ($=B$)

Baseband case

$$R_s = \frac{1}{T_s} = \frac{2B}{1+\alpha} \Rightarrow \boxed{BW_{BB} = \frac{1+\alpha}{2T_s}}$$

Fig. 6.17

RF passband

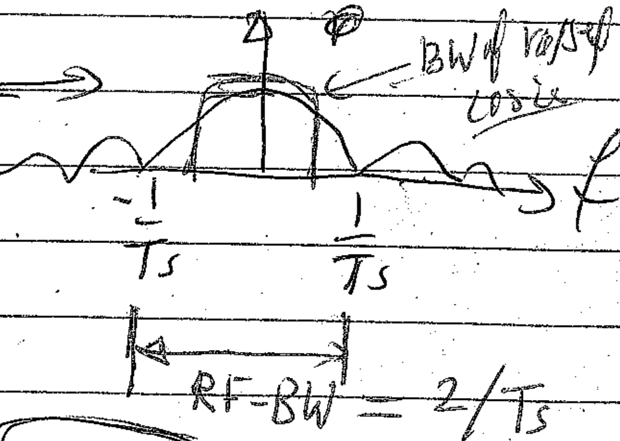
$$R_s = \frac{B}{1+\alpha} = \frac{1}{T_s} \Rightarrow \boxed{BW_{RF} = \frac{1+\alpha}{T_s}}$$

+ Fig. 6.18

ex) Find the 1st zero-crossings RF-BW of a rectangular pulse with $T_s = 41.06\ \mu sec$. Compare to BW of raised cosine pulse shape with $T_s = 41.06\ \mu s$, $\alpha = 0.35$.

Rectangular

$$\text{rect}\,\frac{t}{T_s} \longleftrightarrow T_s \,\text{sinc}\,\frac{wT_s}{2} \longleftrightarrow$$



1st null) $\frac{wT_s}{2} = \pi \Rightarrow w = \frac{2\pi}{T_s}$

or $f = \frac{1}{T_s}$

$$BW\text{-}RF = 2/T_s = \frac{2}{41.06\mu s} = 48.7\ KHz$$

Raised cosine ($\alpha = 0.35$)

$$BW\text{-}RF = (1+\alpha)\frac{1}{T_s} = \frac{1.35}{T_s \leftarrow 41.06\mu s} = 32.88\ KHz$$

6 b/16

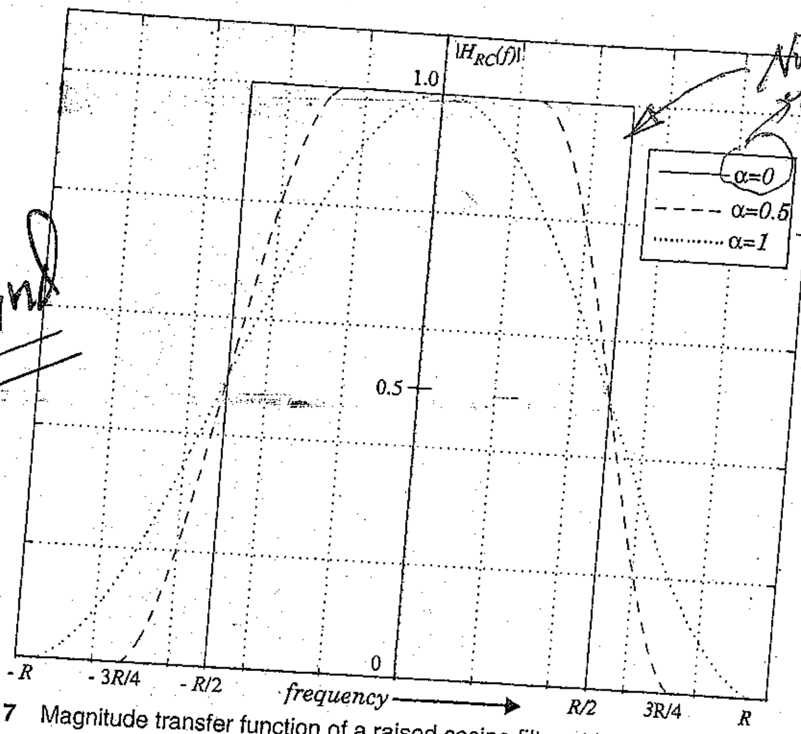


Figure 6.17 Magnitude transfer function of a raised cosine filter at baseband.

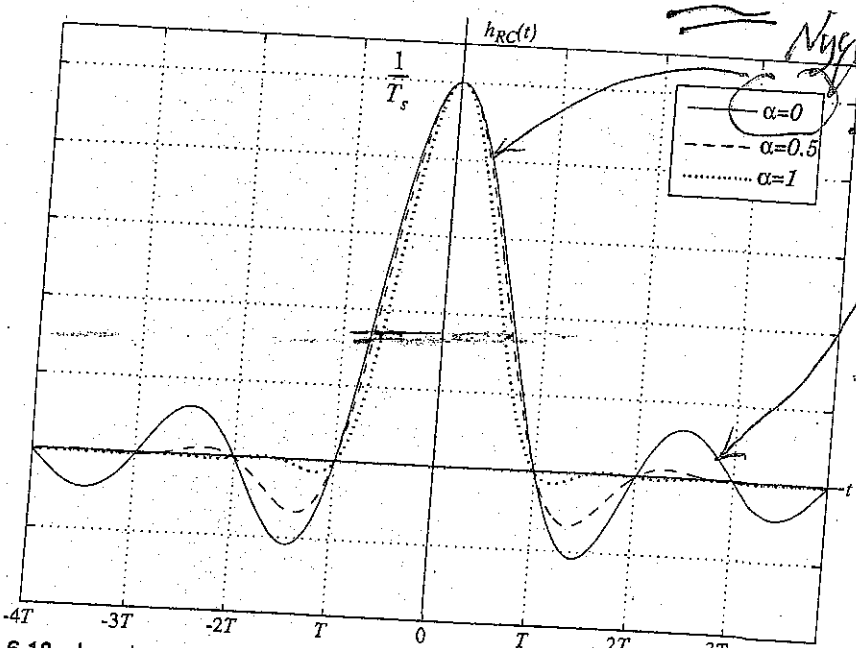


Figure 6.18 Impulse response of a raised cosine rolloff filter at baseband.

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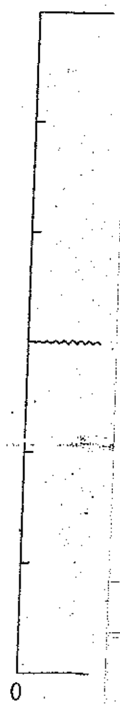


Figure 6.19 R
BPSK signal. N
maximum value

Digital Modulation Techniques

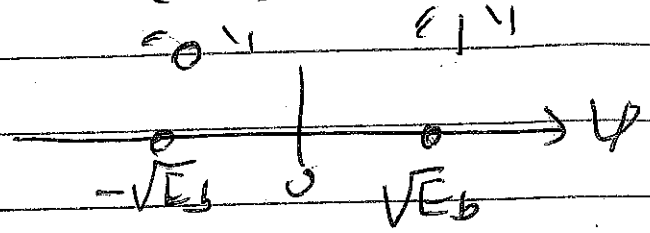
Binary Phase shift Keying (BPSK)

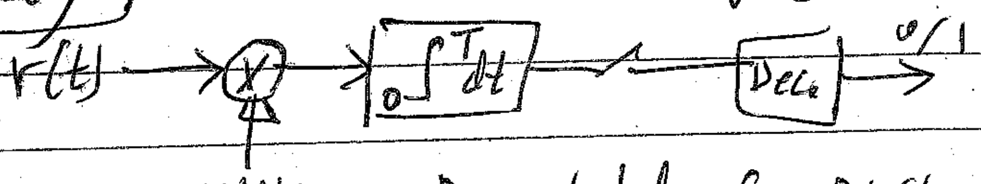
$$s_i(t) = g(t)\sqrt{\frac{2E_b}{T_b}}\cos(w_c t + \theta_i) \qquad 0 \le t \le T_b$$

overall pulse shape

$\theta_i = 0$ ("1"), $\theta_i = \pi$ ("0")

$$\boxed{P_{e,BPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)}$$





Demodulator for BPSK

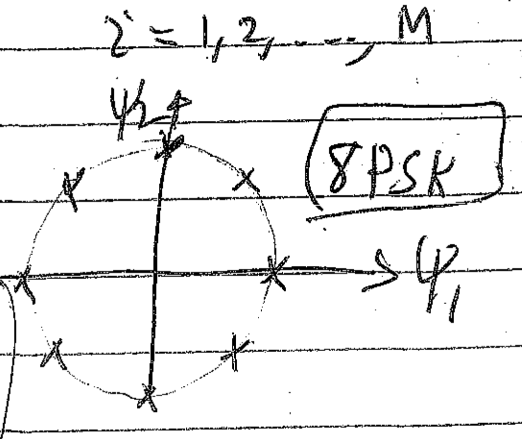
M-ary Phase shift Keying (MPSK)

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}}\cos\left[2\pi f_c t + \frac{2\pi}{M}(i-1)\right] \qquad 0 \le t \le T_s$$

$i = 1, 2, \ldots, M$

$$\varphi_1 = \sqrt{\frac{2}{T_s}}\cos(w_c t)$$

$$\varphi_2 = \sqrt{\frac{2}{T_s}}\sin(w_c t)$$



$$\boxed{P_e \le 2Q\left(\sqrt{\frac{2E_b\log_2 M}{N_0}\sin^2\left(\frac{\pi}{M}\right)}\right)}$$

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The power spectral density (PSD) of the complex envelope can be shown to be

$$P_{g_{\text{BPSK}}}(f) = 2E_b\left(\frac{\sin \pi f T_b}{\pi f T_b}\right)^2 \tag{6.70}$$

The PSD for the BPSK signal at RF can be evaluated by translating the baseband spectrum to the carrier frequency using the relation given in Equation (6.41).

Hence, the PSD of a BPSK signal at RF is given by

$$P_{\text{BPSK}}(f) = \frac{E_b}{2}\left[\left(\frac{\sin \pi(f-f_c)T_b}{\pi(f-f_c)T_b}\right)^2 + \left(\frac{\sin \pi(-f-f_c)T_b}{\pi(-f-f_c)T_b}\right)^2\right] \tag{6.71}$$

The PSD of the BPSK signal for both rectangular and raised cosine rolloff pulse shapes is plotted in Figure 6.22. The null-to-null bandwidth is found to be equal to twice the bit rate ($BW = 2R_b = 2/T_b$). From the plot, it can also be shown that 90% of the BPSK signal energy is contained within a bandwidth approximately equal to $1.6R_b$ for rectangular pulses, and all of the energy is within $1.5R_b$ for pulses with $\alpha = 0.5$ raised cosine filtering.

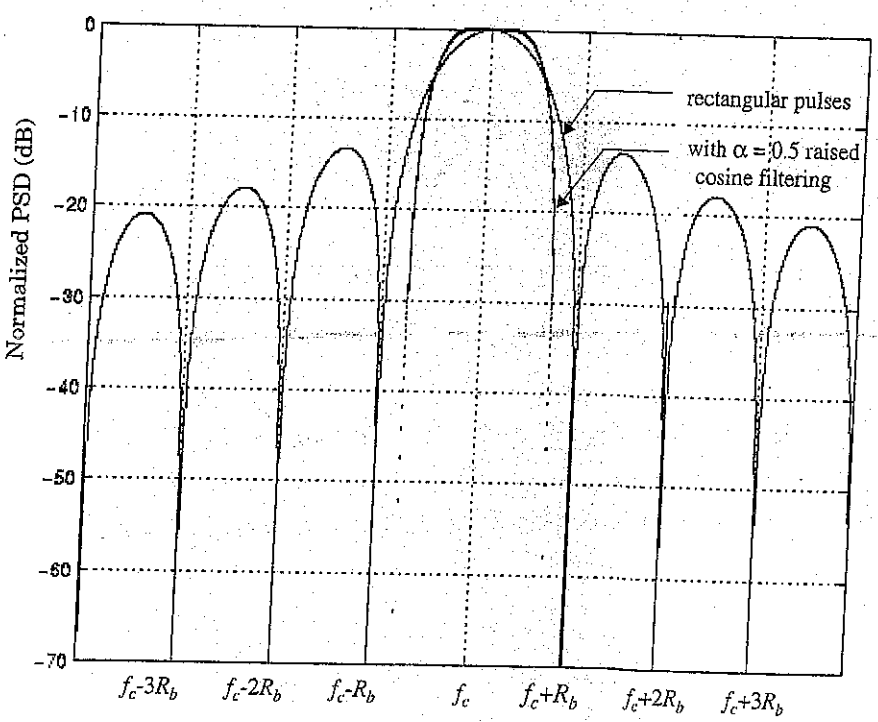

Figure 6.22 Power spectral density (PSD) of a BPSK signal.

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Differential phase shift Keying (DPSK)

- A form of PSK that avoids the need for coherent RX { cheaper and widely used
- The differentially encoded sequence $\{d_K\}$ is generated as

$$d_K = m_K \oplus d_{K-1}$$

← ~~invert~~ XOR

- At RX we can get $\{m_K\}$ as

$$\boxed{m_K = d_K \oplus d_{K-1}}$$

Table 6.1 Illustration of the Differential Encoding Process

$\{m_k\}$		1	0	0	1	0	1	1	0
$\{d_{k-1}\}$		1	1	0	1	1	0	0	0
$\{d_k\}$	1	1	0	1	1	0	0	0	1

obtain the DPSK signal. At the receiver, the original sequence is recovered from the demodulated differentially encoded signal through a complementary process, as shown in Figure 6.25.

While DPSK signaling has the advantage of reduced receiver complexity, its energy efficiency is inferior to that of coherent PSK by about 3 dB. The average probability of error for DPSK in additive white Gaussian noise is given by

$$\boxed{P_{e,\,\mathrm{DPSK}} = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)} \qquad (6.75)$$

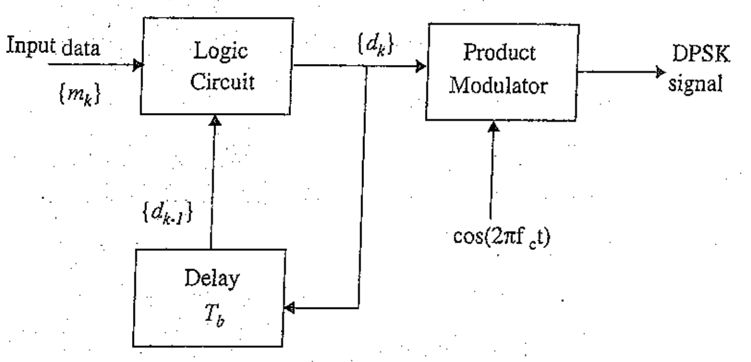


Figure 6.24 Block diagram of a DPSK transmitter.

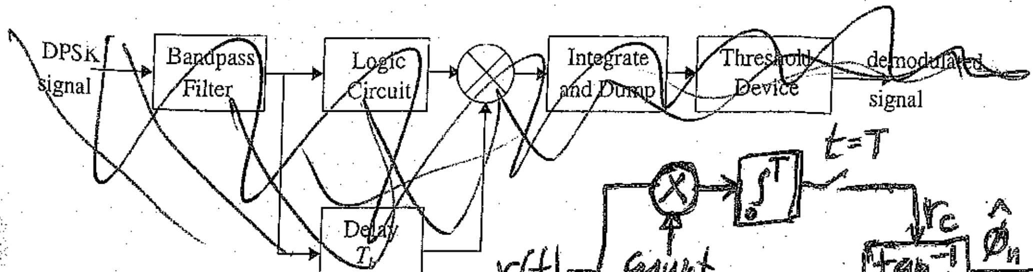
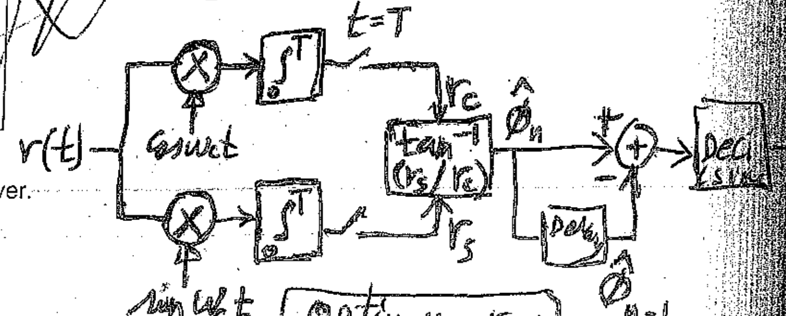


Figure 6.25 Block diagram of DPSK receiver.



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6.8.3 Quadrature Phase Shift Keying (QPSK)

Quadrature phase shift keying (QPSK) has twice the bandwidth efficiency of BPSK, since two bits are transmitted in a single modulation symbol. The phase of the carrier takes on one of four equally spaced values, such as 0, $\pi/2$, π, and $3\pi/2$, where each value of phase corresponds to a unique pair of message bits. The QPSK signal for this set of symbol states may be defined as

$$s_{\mathrm{QPSK}}(t) = \sqrt{\frac{2E_s}{T_s}}\cos\left[2\pi f_c t + (i-1)\frac{\pi}{2}\right] \qquad 0 \le t \le T_s \quad i = 1, 2, 3, 4 \tag{6.76}$$

where T_s is the symbol duration and is equal to twice the bit period.

Using trigonometric identities, the above equations can be rewritten for the interval $0 \le t \le T_s$ as

$$\begin{aligned} s_{\mathrm{QPSK}}(t) = {} & \sqrt{\frac{2E_s}{T_s}}\cos\left[(i-1)\frac{\pi}{2}\right]\cos(2\pi f_c t) \\ & - \sqrt{\frac{2E_s}{T_s}}\sin\left[(i-1)\frac{\pi}{2}\right]\sin(2\pi f_c t) \end{aligned} \tag{6.77}$$

If basis functions $\phi_1(t) = \sqrt{2/T_s}\cos(2\pi f_c t)$, $\phi_2(t) = \sqrt{2/T_s}\sin(2\pi f_c t)$ are defined over the interval $0 \le t \le T_s$ for the QPSK signal set, then the four signals in the set can be expressed in terms of the basis signals as

$$s_{\mathrm{QPSK}}(t) = \left\{\sqrt{E_s}\cos\left[(i-1)\frac{\pi}{2}\right]\phi_1(t) - \sqrt{E_s}\sin\left[(i-1)\frac{\pi}{2}\right]\phi_2(t)\right\} \quad i = 1, 2, 3, 4 \tag{6.78}$$

Based on this representation, a QPSK signal can be depicted using a two-dimensional constellation diagram with four points as shown in Figure 6.26(a). It should be noted that different QPSK signal sets can be derived by simply rotating the constellation. As an example, Figure 6.26(b) shows another QPSK signal set where the phase values are $\pi/4$, $3\pi/4$, $3\pi/4$, and $7\pi/4$.

From the constellation diagram of a QPSK signal, it can be seen that the distance between adjacent points in the constellation is $\sqrt{2E_s}$. Since each symbol corresponds to two bits, then $E_s = 2E_b$, thus the distance between two neighboring points in the QPSK constellation is equal to $2\sqrt{E_b}$. Substituting this in Equation (6.62), the average probability of bit error in the additive white Gaussian noise (AWGN) channel is obtained as

$$P_{e,\,\mathrm{QPSK}} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right) \tag{6.79}$$

BER

A striking result is that the bit error probability of QPSK is identical to BPSK, but twice as much data can be sent in the same bandwidth. Thus when compared to BPSK, QPSK provides twice the spectral efficiency with exactly the same energy efficiency.

Similar to BPSK, QPSK can also be differentially encoded to allow noncoherent detection.

Offset QPSK, π/4 QPSK) Avoid zero-crossings of envelope to avoid non-linear amplifier effect.

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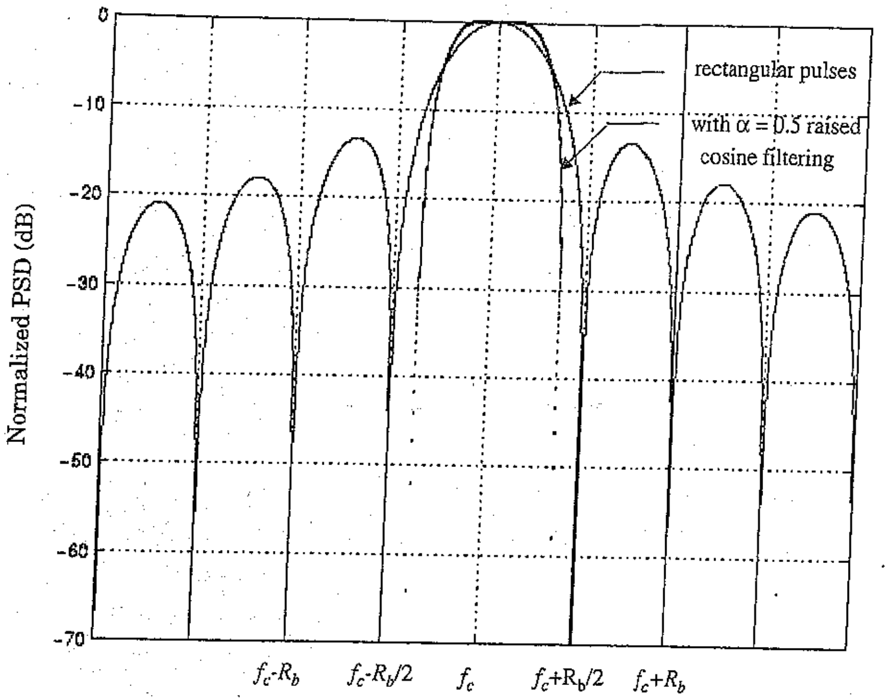


Figure 6.27 Power spectral density of a QPSK signal.

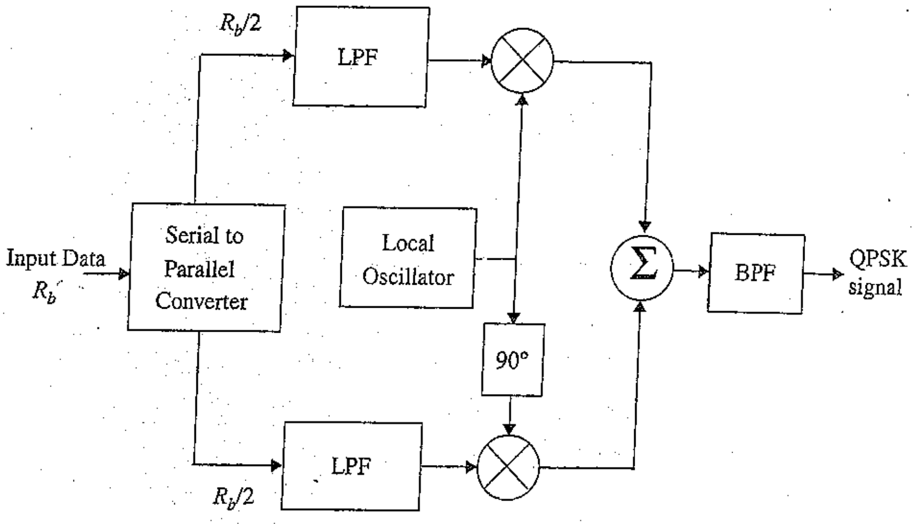


Figure 6.28 Block diagram of a QPSK transmitter.

10ㄷ/6

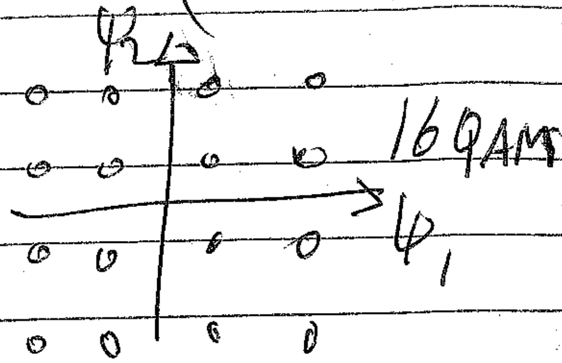
Mary QAM

$$s_i(t) = \sqrt{\frac{2E_0}{T_s}}\, a_i \cos(w_c t) + \sqrt{\frac{2E_0}{T_s}}\, b_i \sin(w_c t) \quad (i = 1, 2, \dots, M)$$

E_0 : energy of signal with lowest amplitude

a_i, b_i takes $\pm 1, \pm 3, \dots, \pm(\sqrt{M} - 1)$

$$\boxed{P_e \simeq 4\left(1 - \frac{1}{\sqrt{M}}\right) Q\left(\sqrt{\frac{2E_0}{N_0}}\right)}$$



$$\boxed{E_0 = \frac{3}{2} \frac{E_{av}}{M-1}}$$

Constant envelope modulation

Used by many practical mobile radio system.

Advantages: (Power Saving)

1- The efficient Class-C power amplifier can be used without degrading the signal spectrum (adding out-of-band radiation).

2 - Low out-of-band radiation of order -60dB, -70dB can be achieved.

3 - Limiter discriminator detection can be used providing high immunity against signal fluctuations due to Rayleigh fading.

Disadvantages

1- They occupy larger BW than Linear modulation schemes (using efficient pulse shapes like raised cos)

2- If BW efficiency is more important than power efficiency, constant envelope are not well-suited.

Binary Frequency Shift Keying (BFSK)

$$s_i(t) = \sqrt{\frac{2E_b}{T_b}} \cos(2\pi f_i t) \qquad i = 1,2$$

$$0 \le t \le T_b$$

$$\psi_1(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_1 t)$$

$$\vec{s}_1 = [\sqrt{E_b}, 0]$$

$$\psi_2(t) = \sqrt{\frac{2}{T_b}} \cos(2\pi f_2 t)$$

$$\vec{s}_2 = [0, \sqrt{E_b}]$$

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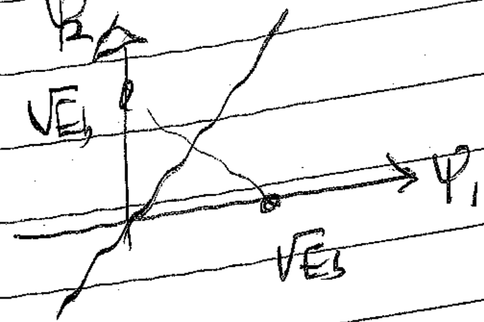
$$\boxed{P_{e,FSK} = Q\left(\sqrt{\frac{E_b}{N_0}}\right)}$$



Coherent Detection

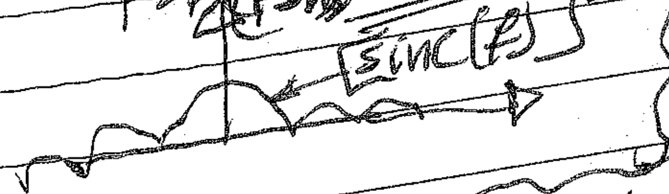
→ worse than BPSK and QPSK by (3dB)

Non-coherent FSK detection

→ $$\boxed{P_{e,FSK,NC} = \frac{1}{2} e^{-\frac{E_b}{2N_0}}}$$

worse than DPSK by 3dB

PSD of (FSK) $\propto \frac{1}{f^2}$



M-ary FSK

$$s_i(t) = \sqrt{\frac{2E_s}{T_s}}\cos\left[\frac{\pi}{T_s}(n_c+i)t\right] \quad 0\le t\le T_s$$

$i = 1, 2, \dots, M$

n_c: some fixed integer

$f_c = n_c/2T_s$

Coherent detection: $P_e \le (M-1)\,Q\left(\sqrt{\frac{E_b \log_2 M}{N_0}}\right)$

Non-coherent (envelope) Detection:

$$P_e = \sum_{k=1}^{M-1}\frac{(-1)^{k+1}}{k+1}\binom{M-1}{k}\exp\left(\frac{-kE_s}{(k+1)N_0}\right)$$

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Minimum Shift Keying (MSK)

Is a continuous-phase FSK (CPFSK) signal defined as

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos[2\pi f_c t + \theta(t)] \quad \text{--- (1)}$$

where $\theta(t)$ is a continuous function of time as follows

$$\theta(t) = \theta(0) \pm \frac{\pi h}{T_b} t \qquad 0 \le t \le T_b \quad \text{--- (2)}$$

(+) : send "1" , (−) : send "0"

$$s(t) = A\cos[2\pi f_c t + \theta(0) \pm \frac{\pi}{2T_b} t]$$

$h \triangleq$ deviation ratio $= \frac{1}{2}$ for MSK

$h = 0.5 \Rightarrow$ minimum frequency separation with orthogonality

$$\omega(t) = \frac{d\theta(t)}{dt} = \frac{d}{dt}\left[2\pi f_c t + \theta(0) \pm \frac{\pi}{2T_b} t\right]$$

$$= \boxed{2\pi f_c \pm \frac{\pi}{2T_b}} \Rightarrow \boxed{f(t) = \frac{\omega(t)}{2\pi}} = \boxed{f_c \pm \frac{1}{4T_b}}$$

$\boxed{f_1 = f_c + \frac{1}{4T_b}}$, $\boxed{f_2 = f_c - \frac{1}{4T_b}}$ (... separation ... orth.

$\therefore$ MSK is a form of FSK with

$$h = \frac{f_1 - f_2}{1/T_b} = 1/2$$

From (1) using Trig. Identity

$$s(t) = \sqrt{\frac{2E_b}{T_b}} \cos[\theta(t)] \ldots$$

(14/6)

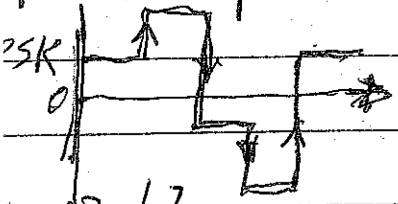
The in-phase component $s_I(t)$

$$s_I(t) = \sqrt{\frac{2E_b}{T_b}} \cos[\theta(t)]$$

where $\theta(t) = \theta(0) \pm \frac{\pi}{2T_b} t \qquad 0 \le t \le T_b$

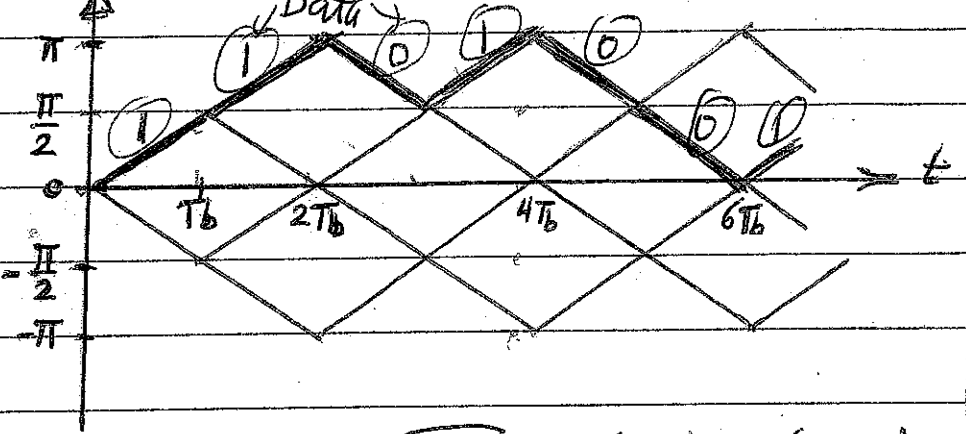
where (+) corresponds to (data = 1) & (−) to (data = 0).

∴ the in-phase component $s_I(t)$ consist of half-cycle cosine pulse shapes



3.17

p.85



$$s_I(t) = \sqrt{\frac{2E_b}{T_b}} \cos\left[\theta(0) \pm \frac{\pi}{2T_b} t\right] = \sqrt{\frac{2E_b}{T_b}} \cos(\theta(0)) \cos\left(\pm\frac{\pi}{2T_b} t\right)$$

But $\theta(0) = 0$ or π or $n\pi$

$$- \sqrt{\frac{2E_b}{T_b}} \sin(\theta(0)) \sin\left(\pm\frac{\pi}{2T_b} t\right)$$

0

$$\therefore s_I(t) = \sqrt{\frac{2E_b}{T_b}} \cos[\theta(0)] \cos\left(\frac{\pi}{2T_b} t\right) \qquad 0 \le t \le T_b$$

The same expression ~~of~~ $s_I(t)$ can be extended to $-T_b \le t \le 0$ since $\theta(t)$ is a linear fn. of time and hence can be written as $\theta(t) = \theta(0) \pm \frac{\pi}{2T_b} t \quad (-T_b \le t \le 0)$ where ($\pm$) depends on whether $\theta(-T_b)$ was ($\pi/2$) or ($-\pi/2$) and on the previous data (= "1" or "0").

In all cases {since $\cos(B) = \cos(-B)$} we have

$$s_I(t) = \sqrt{\frac{2E_b}{T_b}} \cos[\theta(0)] \cos\left(\frac{\pi}{2T_b} t\right) \quad (-T_b \le t \le 0)$$

(15/6)

$\therefore \quad s_I(t) = \pm\sqrt{\frac{2E_b}{T_b}} \cos\left(\frac{\pi}{2T_b} t\right) \quad \left[-T_b \le t \le T_b\right]$

where $(+)$ correspond to $\theta(0) = 0 \Rightarrow \cos 0 = 1$

" $(-)$ " " $= \pi \Rightarrow \cos \pi = -1$

$\therefore$ Polarity of pulse shape depends only on $\theta(0)$ regardless of the sequence of "1"s and "0"s transmitted before or after $(t=0)$.

The quadrature component $s_Q(t)$

Similarly we can show that

$$s_Q(t) = \pm\sqrt{\frac{2E_b}{T_b}} \sin\left(\frac{\pi}{2T_b} t\right) \quad \left[0 \le t \le 2T_b\right]$$

$(+)$ corresponds to $\theta(T_b) = \pi/2$

$(-)$ " " $\theta(T_b) = -\pi/2$

$$\left.\begin{array}{l} \theta(0) = 0, \ \theta(T_b) = \pi/2 \\ \theta(0) = \pi, \ \theta(T_b) = -\pi/2 \left(\text{equivalently } 3\pi/2\right) \end{array}\right\} \text{Transmission of Symbol "1"}$$

$$\left.\begin{array}{l} \theta(0) = \pi, \ \theta(T_b) = \pi/2 \\ \theta(0) = 0, \ \theta(T_b) = -\pi/2 \end{array}\right\} \text{Transmitted symbol "0"}$$

Orthonormal Basis for MSK

$$\varphi_1(t) = \sqrt{\frac{2}{T_b}} \cos\left(\frac{\pi}{2T_b} t\right) \cos(2\pi f_c t) \quad -T_b \le t \le T_b$$

$$\varphi_2(t) = \sqrt{\frac{2}{T_b}} \sin\left(\frac{\pi}{2T_b} t\right) \sin(2\pi f_c t) \quad 0 \le t \le 2T_b$$

(16/6)

$s(t) = s_1 \psi_1(t) + s_2 \psi_2(t)$

$$s_1 = \int_{-T_b}^{T_b} s(t)\,\psi_1(t)\,dt$$

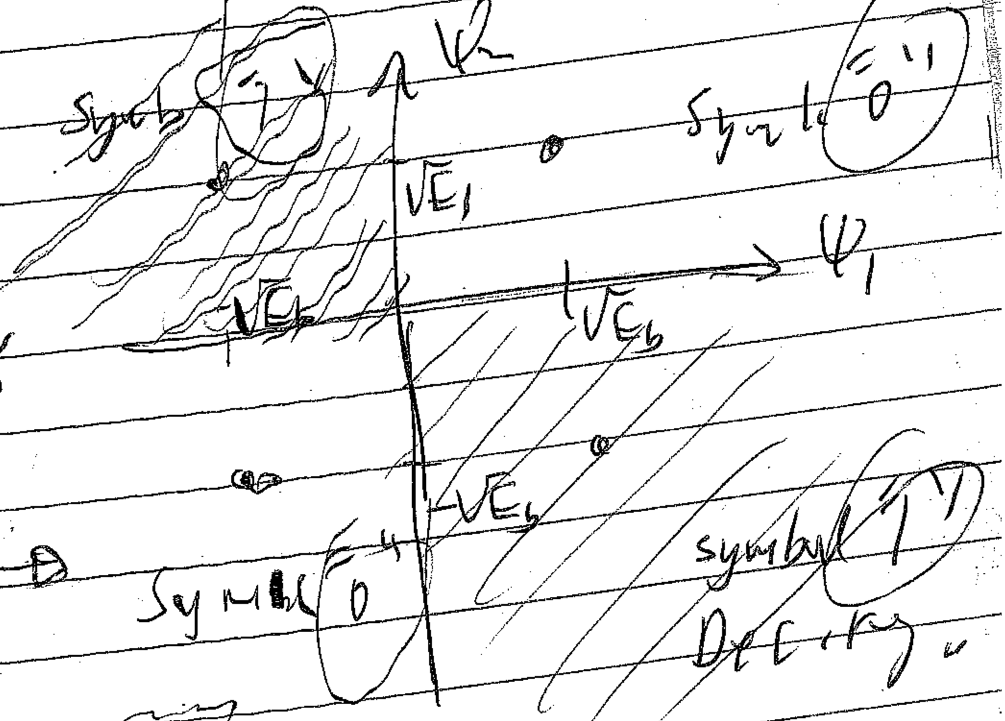
$$= \sqrt{E_b}\cos[\theta(0)]$$

$$s_1 = \begin{cases} +\sqrt{E_b} & \text{if } \theta(0) = 0 \\ -\sqrt{E_b} & \theta(0) = \pi \end{cases}$$

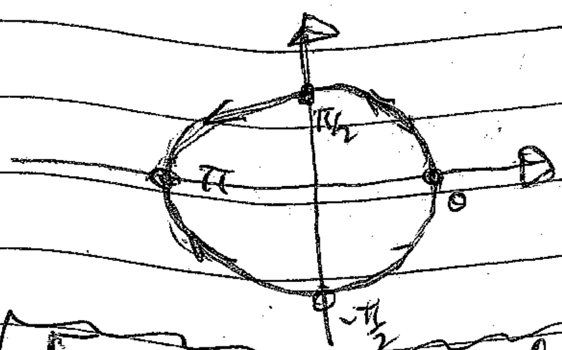
$$s_2 = \int_0^{2T_b} s(t)\,\psi_2(t)\,dt$$

$$= -\sqrt{E_b}\sin[\theta(T_b)]$$

$$s_2 = \begin{cases} +\sqrt{E_b} & \text{if } \theta(T_b) = -\pi/2 \\ -\sqrt{E_b} & \theta(T_b) = \pi/2 \end{cases}$$



performan is similar to BPSK



(For sin/cos basis functions)

Power Spectral density

The pulse shape for in-phase component is

$$g_1(t) = \begin{cases} \sqrt{\dfrac{2E_b}{T_b}}\cos\left(\dfrac{\pi t}{2T_b}\right) & \left(-T_b \le t \le T_b\right) \\ 0 & \end{cases}$$

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and for the quadrature component $0 \le t \le 2T_b$

$$g_2(t) = \sqrt{\frac{2E_b}{T_b}} \sin\left(\frac{\pi t}{2T_b}\right)$$

We find $G_1(w)$

$$\text{rect}\frac{t}{2T_b} \longleftrightarrow 2T_b \operatorname{sinc} \frac{w\,2T_b}{2}$$

$$\longleftrightarrow 2T_b \operatorname{sinc} wT_b$$

$$\cos(w_0 t)x(t) \longleftrightarrow 0.5\left[X(w-w_0) + X(w+w_0)\right]$$

$$\cos\left(\frac{\pi}{2T_b}t\right) \times \text{rect}\left(\frac{t}{2T_b}\right) \longleftrightarrow \frac{1}{2}\,2T_b\left[\operatorname{sinc}\left(w-\frac{\pi}{2T_b}\right)T_b + \operatorname{sinc}\left(w+\frac{\pi}{2T_b}\right)T_b\right]$$

$$-\sin(\pi/2 - wT_b)$$

$$\longleftrightarrow T_b\left[\frac{\sin(wT_b - \pi/2)}{wT_b - \pi/2} + \frac{\sin(wT_b + \pi/2)}{wT_b + \pi/2}\right]$$

$$\longrightarrow T_b\left[\frac{-\cos wT_b}{wT_b - \pi/2} + \frac{\cos(wT_b)}{wT_b + \pi/2}\right]$$

$$\longleftrightarrow T_b \frac{\cancel{-wT_b\cos wT_b} - (\pi/2)\cos wT_b + \cancel{wT_b\cos wT_b} - 0.5\pi\cos wT_b}{w^2T_b^2 - \pi^2/4}$$

$$\longleftrightarrow T_b \frac{-\pi\cos wT_b}{w^2T_b^2 - \pi^2/4} \times \sqrt{\frac{2E_b}{T_b}} \times \frac{4}{4}$$

$$(2\pi f)^2$$

$$G_1(f) = \frac{4\sqrt{2T_bE_b}}{\pi}\,\frac{-\cos 2\pi f T_b}{16f^2T_b^2 - 1}$$

$$|G_1(f)|^2 = \frac{32E_bT_b}{\pi^2}\left[\frac{\cos 2\pi f T_b}{16T_b^2f^2 - 1}\right]^2$$

$$\text{PSD of MSK} = \frac{|G_1(f)|^2 + |G_2(f)|^2}{2T_b} \rightarrow \text{similar to } G_1(f)$$

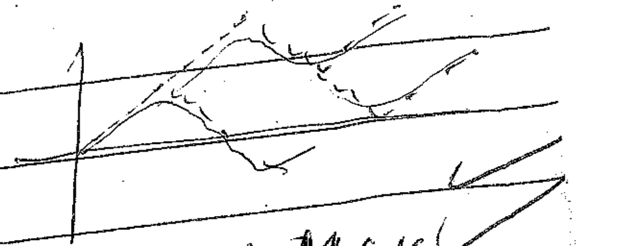
$$\boxed{\text{PSD of MSK} = \frac{32E_b}{\pi^2}\left[\frac{\cos 2\pi f T_b}{16T_b^2f^2 - 1}\right]^2} \propto \frac{1}{f^4}$$

faster than QPSK

For ~~slower~~ than QPSK (rectangular $g(t)$) ... $\propto \frac{1}{f^2}$

... envelope ...

(1816)



Gaussian MSK (GMSK)

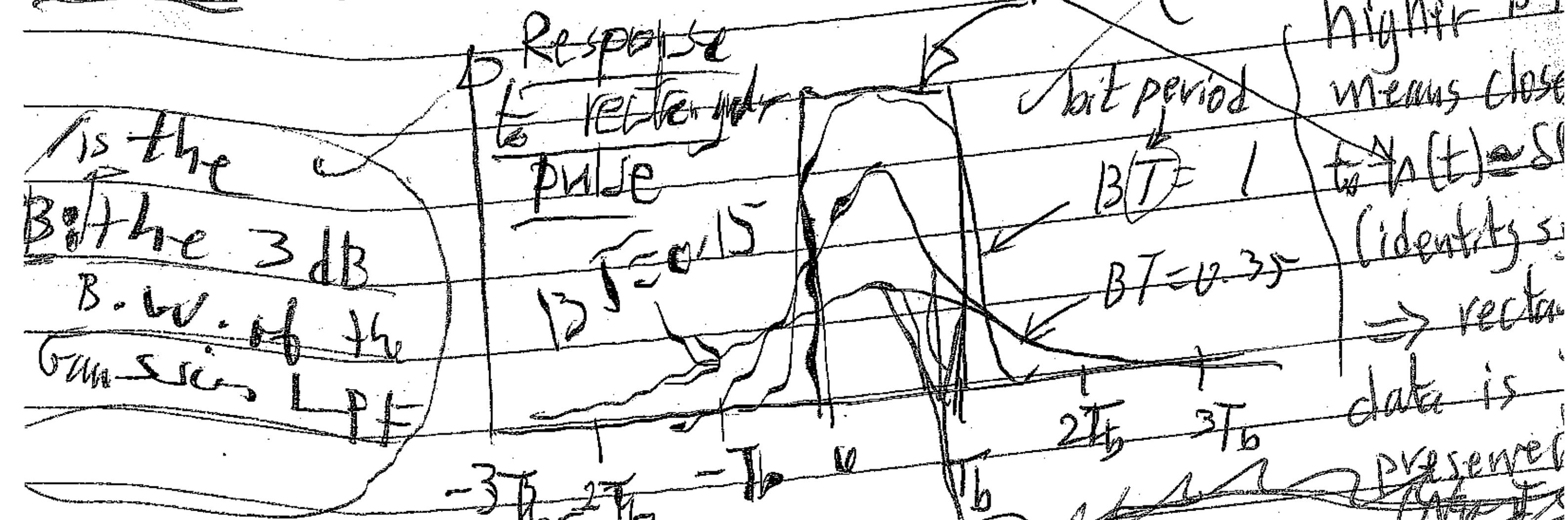
The out-of-band spectral characteristics of MSK, as good as they are, still do not satisfy the stringent requirements of certain applications in wireless communication.

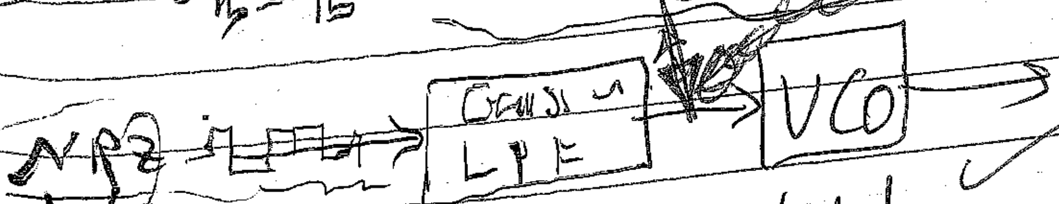
GMSK (modification of MSK) reduces the sidelobes by passing the nonreturn-to-zero (NRZ) binary data stream through a pulse-shaping filter with impulse response a Gaussian fn. (so is its freq. response).

{BT = Bandwidth × Bit duration}

This filtering smooths phase trajectories of MSK hence reducing sidelobes.

The response of the Gaussian filter to a rectangular pulse is shown





GMSK Modulator

If without this LPF it becomes MSK

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The parameter α is related to B, the 3 dB baseband bandwidth of $H_G(f)$, by

$$\alpha = \frac{\sqrt{\ln 2}}{\sqrt{2}B} = \frac{0.5887}{B} \tag{6.111}$$

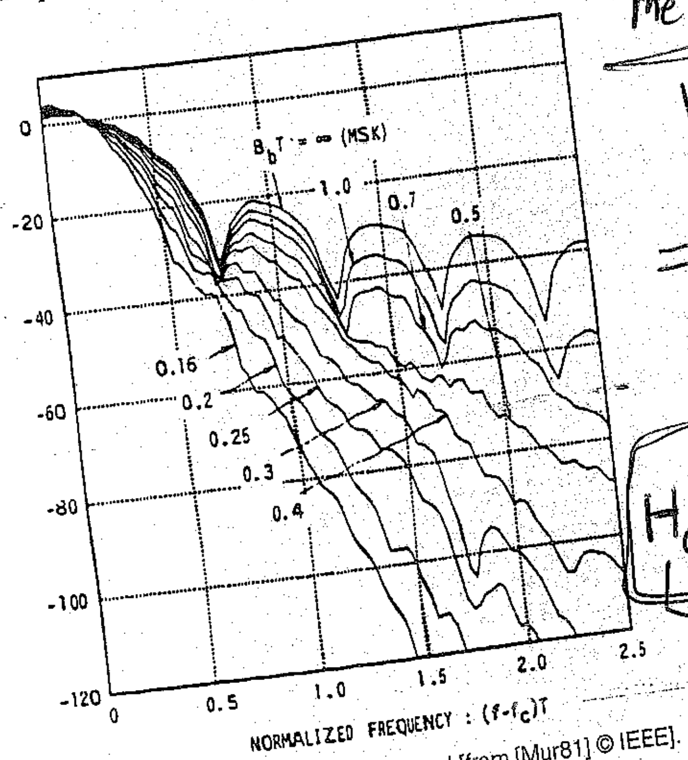
and the GMSK filter may be completely defined from B and the baseband symbol duration T. It is therefore customary to define GMSK by its BT product.

Figure 6.41 shows the simulated RF power spectrum of the GMSK signal for various values of BT. The power spectrum of MSK, which is equivalent to GMSK with a BT product of infinity, is also shown for comparison purposes. It is clearly seen from the graph that as the BT product decreases, the sidelobe levels fall off very rapidly. For example, for a $BT = 0.5$, the peak of the second lobe is more than 30 dB below the main lobe, whereas for simple MSK, the second lobe is only 20 dB below the main lobe. However, reducing BT increases the irreducible error rate produced by the low pass filter due to ISI. As shown in Section 6.12, mobile radio channels induce an irreducible error rate due to mobile velocity, so *as long as the GMSK irreducible error rate is less than that produced by the mobile channel, there is no penalty in using GMSK.* Table 6.3 shows occupied bandwidth containing a given percentage of power in a GMSK signal as a function of the BT product [Mur81].

The Gaussian LPF

$h_G(t)$ = impulse response

$$= \boxed{\frac{\sqrt{\pi}}{\alpha} \exp\left(-\frac{\pi^2}{\alpha^2} t^2\right)}$$



$$\boxed{H_G(f) = e^{-\alpha^2 f^2}}$$

has a 3-dB BW

$$\boxed{B = \frac{0.5887}{\alpha}}$$

Figure 6.41 Power spectral density of a GMSK signal [from [Mur81] © IEEE].

APPENDIX B

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GAUSSIAN PULSE-SHAPING FILTER

The Gaussian low-pass filter has a transfer function given by

$$H(f) = \exp(-\alpha^2 f^2) \tag{B.1}$$

The parameter α is related to B, the 3-dB bandwidth of the baseband Gaussian shaping filter. It is commonly expressed in terms of a normalized 3-dB bandwidth-symbol time product (BT_s):

B decreases

$$\alpha = \frac{\sqrt{\ln(2)}}{\sqrt{2}} \frac{T_s}{BT_s} \tag{B.2}$$

As α increases, the spectral occupancy of the Gaussian filter decreases and the impulse response spreads over adjacent symbols, leading to increased ISI at the receiver. The impulse response of the Gaussian filter in the continuous-time domain is given by h(t)

$$h(t) = \frac{\sqrt{\pi}}{\alpha} \exp\left[-\left(\frac{\pi}{\alpha} t\right)^2\right] \tag{B.3}$$

which could easily be rearranged (Eq. B.4) to reveal its fit with the canonical form of a zero-mean Gaussian random variable with standard deviation $\sigma_h = \alpha / \sqrt{2}\pi$:

$$h(t) = \frac{1}{\sqrt{2\pi}(\alpha/\sqrt{2}\pi)} \exp\left[-\frac{t^2}{2(\alpha/\sqrt{2}\pi)^2}\right] \tag{B.4}$$

Its integral from $-\infty$ to ∞ is, of course, 1.

All-Digital Frequency Synthesizer in Deep-Submicron CMOS, by Robert Bogdan Staszewski and Poras T. Balsara

Let us now express the Gaussian filter in the discrete-time domain. Let $t_0 = T_s/\mathrm{OSR}$ be an integer oversample of the symbol duration and $t = kt_0$, k being the sample index. The discrete-time impulse response becomes

$$h(kt_0) = \frac{\sqrt{\pi}}{\alpha}\exp\left[-\left(\frac{\pi}{\alpha}kt_0\right)^2\right] \tag{B.5}$$

Substituting Eq. B.2 and dropping explicit dependence on t_0 results in

$$h[k] = \underbrace{\frac{\sqrt{2\pi}}{\sqrt{\ln(2)}}\frac{BT_s}{T_s}}_{h_{\max}}\exp\left[-\left(\frac{\sqrt{2}\pi}{\sqrt{\ln(2)}}BT_s\frac{k}{\mathrm{OSR}}\right)^2\right] \tag{B.6}$$

The first factor in Eq. B.6 is the peak of the impulse frequency response:

$$h_{\max} = \frac{\sqrt{\pi}}{\alpha} = \frac{\sqrt{2\pi}}{\sqrt{\ln(2)}}\frac{BT_s}{T_s} \tag{B.7}$$

For Bluetooth, with $BT_s = 0.5$ and $T_s = 1\ \mu\text{s}$, we obtain $h_{\max} = 1.5054$ MHz. For GSM, with $BT_s = 0.3$ and $T_s = 3.692\ \mu\text{s}$, we obtain $h_{\max} = 244.62$ kHz.

For reasons described in Chapter 5, it is more efficient to operate on the *cumulative coefficients*

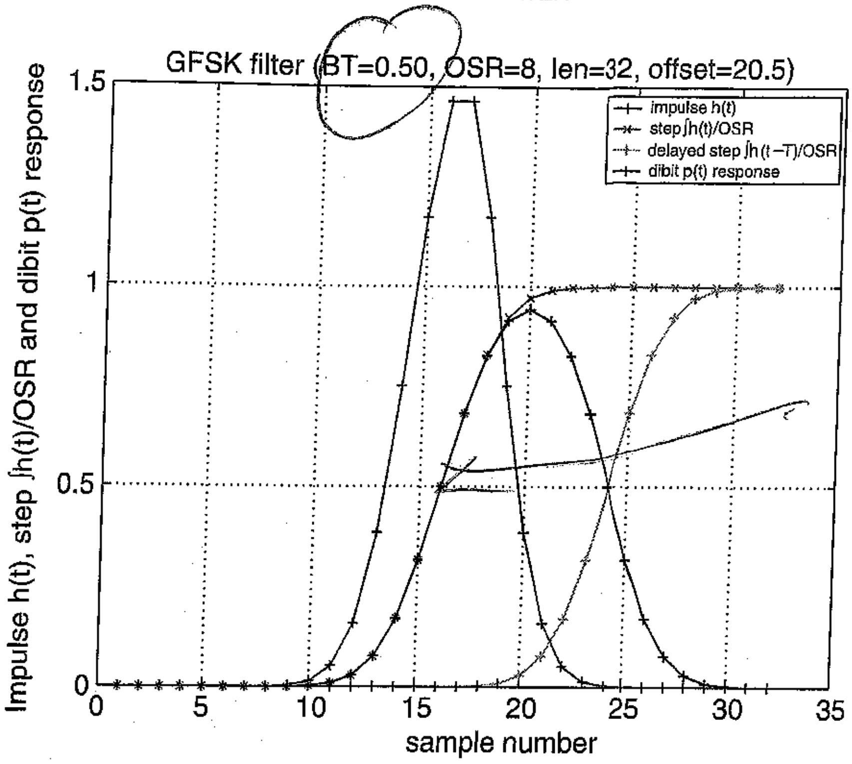
$$C[k] = \sum_{l=0}^{k-1} h[l] \tag{B.8}$$

which could be precalculated and stored in a look-up table, with $k = 0 \cdots \mathrm{OSR} - 1$ being the index. The minimum value of $C[k]$ is approximately zero and the maximum value is approximately 1, since the integral of Eq. B.4 is unity.

Figure B.1 shows the impulse $h[k]$, step $C[k]$, and di-bit responses (difference between step and symbol-delayed step responses) of the Bluetooth GFSK filter ($BT_s = 0.5$) with a length of three symbols, each symbol oversampled by 8.

Similarly, Fig. B.2 shows the impulse, step, and di-bit responses of the GSM GMSK filter ($BT_s = 0.3$) with a length of four symbols, each symbol oversampled by 8. It reveals much more intersymbol interference (ISI) than in the case of Bluetooth.

Figures B.3 and B.4 show the frequency responses of the Bluetooth and GSM filters with varying filter lengths of three, four, and five symbols. A filter length of three symbols is completely adequate for precise containment of the



unit step response

Figure B.1 Time response of a BLUETOOTH GFSK filter of four-symbol length ($BT_s = 0.3$, OSR $= 8$).

unit step response of Gauss. filter

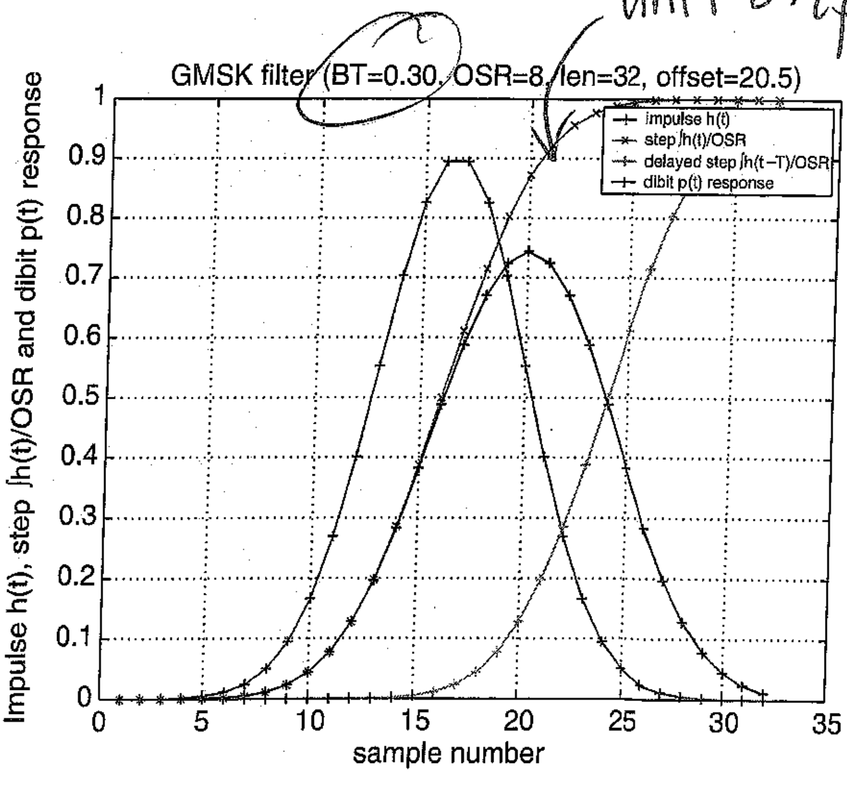


Figure B.2 Time response of a GSM GMSK filter of four-symbol length ($BT_s = 0.3$, OSR $= 8$).

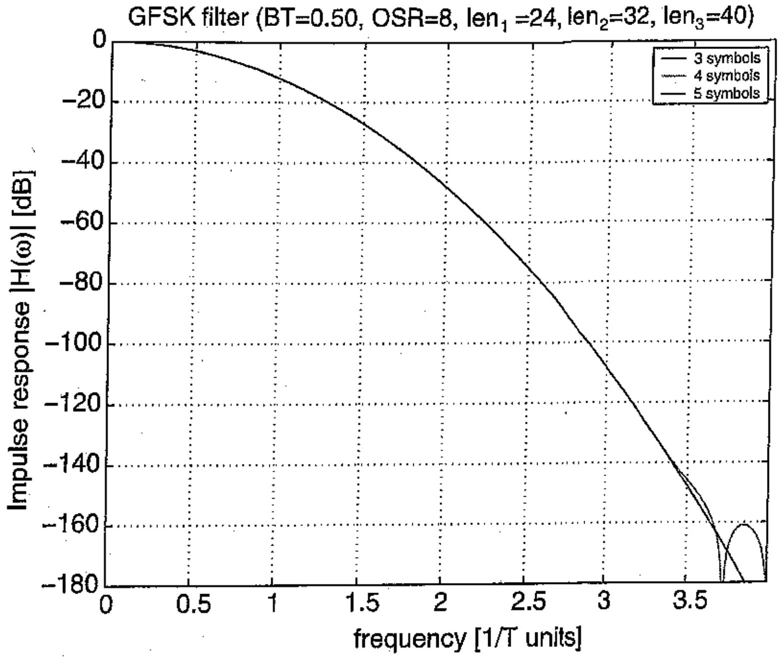


Figure B.3 Frequency response of a BLUETOOTH GFSK filter for filter lengths of three, four, and five symbols ($BT_s = 0.5$, OSR = 8).

modulated output spectrum and sufficient attenuation of frequency components in adjacent channels. However, due to the higher amount of ISI and much tougher requirements for the modulated output spectrum, the GSM-standard filter would require a filter length of at least four symbols.

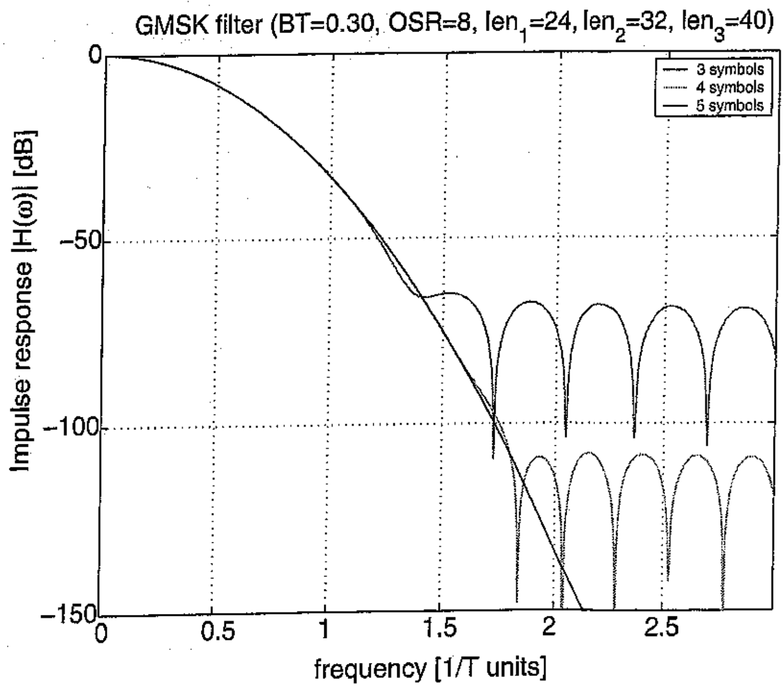


Figure B.4 Frequency response of a GSM GMSK filter for filter lengths of three, four, and five symbols ($BT_s = 0.3$, OSR = 8).

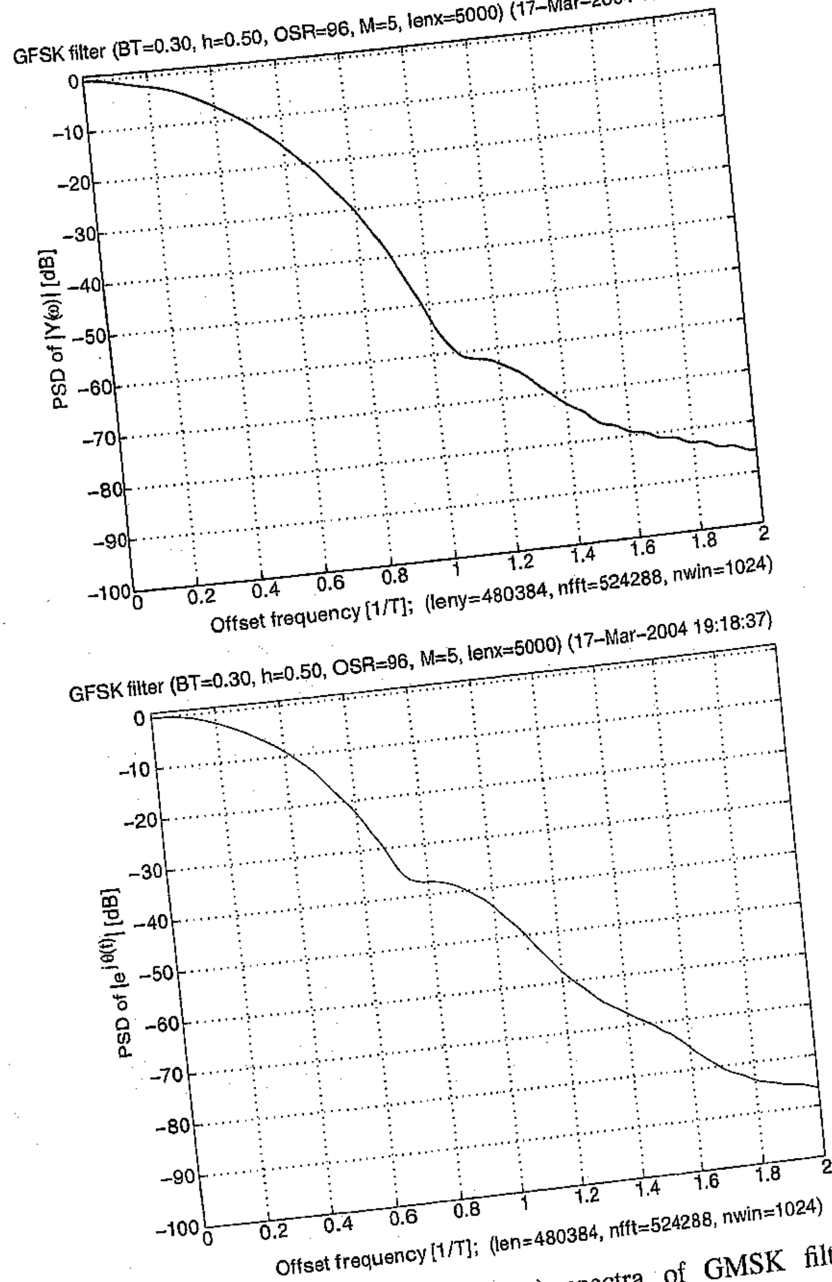


Figure B.5 Baseband (top) and RF (bottom) spectra of GMSK filter output with pseudorandom input (five-symbol length, $BT_s = 0.3$, OSR = 96).

Figure B.5 shows the spectrum of the baseband GMSK filter output FCW and RF port $\Re\{e^{j\theta}\}$ with pseudorandom input data, in which

$$\Delta f[k] = \mathrm{FCW}[k]\frac{f_R}{2^{W_F}} \tag{B.9}$$

and

$$\theta[k] = \frac{2\pi}{\mathrm{OSR}}\sum_{l=0}^{k-1}\Delta f[k] \tag{B.10}$$

- This Gaussian filtering introduces ISI.
- If $BT > 0.5$ there is little ISI.
- GMSK exchanges very good spectral efficiency + constant envelope with small ISI (+) with small degradation in error performance
- As long as this added error (due to ISI) is less than error caused by mobile channel then there is no penalty in using GMSK.

Filter parameters

$$\alpha = \frac{0.5887}{B}$$ B: 3dB BW of Gaussian filter.

Table 6.3 RF BW as a fraction of R_b

BT	90%	99%	99.9%	99.99%
0.2 GMSK	0.52	0.79	0.99	1.22
0.25 GMSK	0.57	0.86	1.09	1.37
0.5 GMSK	0.69	1.04	1.33	2.08
MSK	0.78	1.20	2.76	6.00

$$P_e = Q\left(\sqrt{\frac{2\gamma E_b}{N_o}}\right)$$

$$\gamma = \begin{cases} 0.68 & \text{for } BT = 0.25 \\ 0.85 & \text{for MSK } (BT = \infty) \end{cases}$$

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ex) Find the 3dB-BW of a Gaussian LPF for 0.25 GMSK with data rate $R_b = 270$ kbps. what is the 90% BW of the RF channel. Find α

$BT = 0.25$, $T = \frac{1}{R_b} = \frac{1}{270\times10^3} = 3.7\,\mu s$

$\therefore B = \frac{0.25}{T} = \frac{0.25}{3.7\times10^{-6}} = 67.567$ KHz (Gaussian LPF)

From table 90% BW of 0.25 GMSK $= 0.57 R_b$
$= 0.57 \times 270 = 153.9$ KHz

~~$\alpha = 0.5887 / B$~~

Performance (Prob. of error) of digital modulations in slow flat fading channels

Received signal

$r(t) = \alpha e^{-j\theta} s(t) + n(t)$

$e^{-j\theta}$ ← Phase shift due to channel

$n(t)$ ← noise

α ← Gain of channel (a Rayleigh R.V.)

BPSK performance

If (θ) is estimated then coherent detection is possible. For a constant value of (α) then the received signal energy is ($\alpha^2 E_b$)

$$P_{e,BPSK}(\text{Gaussian}) = Q\left(\sqrt{\frac{2E_b}{N_0}}\right)$$

$$P_{e,BPSK}\left[\text{for specific value of } (\alpha)\right] = Q\left(\sqrt{\frac{2\alpha^2 E_b}{N_0}}\right)$$

Instantaneous SNR

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Let $\gamma = \alpha^2 \frac{E_b}{N_0} \Rightarrow P_e = Q(\sqrt{2\gamma})$

$P_{e,BPSK}(\text{Rayleigh}) \equiv E[Q(\sqrt{2\gamma})]$ over γ

$\therefore$ we need $\boxed{f_\gamma(\gamma)}$ with transformation $\gamma = \alpha^2 E_b/N_0$

one root $\alpha_1 = \sqrt{\frac{\gamma N_0}{E_b}}$ $\quad \gamma' = 2\alpha \frac{E_b}{N_0}$

$f_\gamma(\gamma) = \frac{f_\alpha(\alpha_1)}{|T'(\alpha_1)|}$ $\quad\quad f_\alpha(\alpha) = \frac{\alpha}{\sigma^2} e^{-\alpha^2/2\sigma^2} \quad \alpha \geq 0$

$= \frac{\frac{\cancel{\alpha_1}}{\sigma^2} e^{-\alpha_1^2/2\sigma^2}}{2\cancel{\alpha_1}\frac{E_b}{N_0}} = \frac{1}{2\sigma^2 \frac{E_b}{N_0}} e^{-\frac{\gamma N_0}{E_b 2\sigma^2}}$ $\quad 2\sigma^2 \frac{E_b}{N_0} \rightarrow \bar{\gamma}$

$\alpha = \sqrt{x^2+y^2}$
$\alpha^2 = x^2+y^2$
$\overline{\alpha^2} = \sigma^2+\sigma^2$
$= 2\sigma^2$

We have defined $\gamma = \alpha^2 \frac{E_b}{N_0} \xRightarrow{\text{expectation}} \bar{\gamma} = \overline{\alpha^2}\, E_b/N_0$

(previous result) $\overline{\alpha^2} = 2\sigma^2$

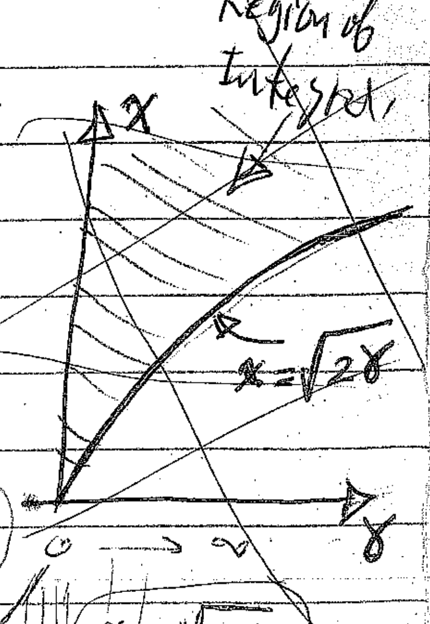
$\bar{\gamma} = 2\sigma^2 E_b/N_0 \Rightarrow$ subs. in $f_\gamma(\gamma)$

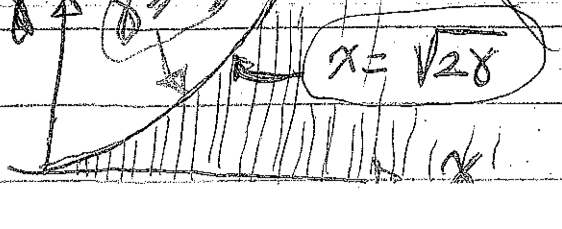
$$\boxed{f_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}}} \quad \gamma \geq 0$$

$$P_e(\text{Rayleigh}) = \int_0^\infty Q(\sqrt{2\gamma}) \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}}\, d\gamma$$

$$= \int_{\gamma=0}^{\infty} \left[\int_{\sqrt{2\gamma}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx\right] \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}} d\gamma$$

$$= \int_{x=0}^{\infty} \left[\int_{\gamma=0}^{\gamma = x^2/2} \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}} d\gamma\right] \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$





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$$= \int_{x=0}^{\infty} \left[-e^{-\gamma/\bar{\gamma}}\right]_0^{x^2/2} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$$

$$= \int_{x=0}^{\infty} \left[e^{0} - e^{-x^2/2\bar{\gamma}}\right] \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx$$

$$\frac{1+\bar{\gamma}}{\bar{\gamma}}$$

$$= \int_0^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2}\, dx - \int_0^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}\left(\frac{1}{\bar{\gamma}}+1\right)}\, dx$$

$$= \frac{1}{2} - \int_0^{\infty} \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2[\bar{\gamma}/(1+\bar{\gamma})]}}\, dx \times \frac{\sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}}{\sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}}$$

$$= \frac{1}{2} - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}} \int_0^{\infty} \frac{1}{\sqrt{2\pi \frac{\bar{\gamma}}{1+\bar{\gamma}}}} e^{-x^2/2[\bar{\gamma}/(1+\bar{\gamma})]}\, dx$$

$$P_{e_{BPSK}} = \frac{1}{2} - \frac{1}{2}\sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}} = \boxed{\frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{1+\bar{\gamma}}}\right)}$$

For $\bar{\gamma} \gg 1 \Rightarrow \sqrt{\frac{x}{1+x}} \simeq 1 - \frac{1}{2x}$ $\gg 1$

$$P_{e,BPSK} = \frac{1}{2}\left[1 - \left(1 - \frac{1}{2\bar{\gamma}}\right)\right] = \boxed{\frac{1}{4\bar{\gamma}}}$$

Similarly $P_{e,FSK} = \frac{1}{2}\left(1 - \sqrt{\frac{\bar{\gamma}}{2+\bar{\gamma}}}\right)$

$\bar{\gamma} \gg 1$

$$P_{e,FSK} \simeq \boxed{\frac{1}{2\bar{\gamma}}}$$

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DPSK performance

$$P_e(\text{Gaussian}) = \frac{1}{2} e^{-\frac{E_b}{N_0}}$$

$$P_e(\text{for specific}(\alpha)) = \frac{1}{2} e^{-\frac{\alpha^2 E_b}{N_0}} = \frac{1}{2} e^{-\gamma}$$

But $f_\gamma(\gamma) = \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}}$

$$P_e(\text{Rayleigh}) = E\left[\frac{1}{2} e^{-\gamma}\right] \text{ over } \gamma$$

$$= \int_0^\infty \frac{1}{2} e^{-\gamma} \frac{1}{\bar{\gamma}} e^{-\gamma/\bar{\gamma}} \, d\gamma$$

$$= \frac{1}{2\bar{\gamma}} \int_0^\infty e^{-\gamma\left(1+\frac{1}{\bar{\gamma}}\right)} d\gamma = \frac{1}{2\bar{\gamma}} \left. \frac{e^{-\gamma\left(1+\frac{1}{\bar{\gamma}}\right)}}{-\left(\frac{\bar{\gamma}+1}{\bar{\gamma}}\right)} \right|_0^\infty$$

$$= \frac{1}{2\bar{\gamma}} \frac{[e^{-\infty} - \overset{1}{e^{0}}]}{-\left(\frac{\bar{\gamma}+1}{\bar{\gamma}}\right)} = \boxed{\frac{1}{2+2\bar{\gamma}}} \simeq \boxed{\frac{1}{2\bar{\gamma}}} \text{ for } \bar{\gamma} \gg 1$$

Non-Coherent FSK

$$P_{e,NCFSK}(\text{Rayleigh}) = \boxed{\frac{1}{2+\bar{\gamma}}} \simeq \boxed{\frac{1}{\bar{\gamma}}} \text{ for } \bar{\gamma} \gg 1$$

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Performance in Rician flat fading

$$P_{e,NCFSK} = \frac{1+K}{\bar{\gamma}+2+2K} \exp\left(\frac{-K\bar{\gamma}}{\bar{\gamma}+2+2K}\right)$$

$$P_{e,DPSK} = \frac{1+K}{2(\bar{\gamma}+1+K)} \exp\left(\frac{-K\bar{\gamma}}{\bar{\gamma}+1+K}\right)$$

$$K = A^2/2\sigma^2$$

Bandwidth Efficiency of digital modulations

The choice of a specific modulation method depends on many factors; some of these are:

1- BER performance
2- BW efficiency (no. of bits/sec that can be sent over a limited BW.
3- Ease of implementation and cost.

BW efficiency $\rho = \frac{R_b}{BW}$ — R_b: Bit rate; BW: Channel BW

(M-ary PSK) and QAM — T_s: Symbol period

Assuming a pulse shape $g(t)$ with BW $\simeq 1/T_s$

(This assumption is based on assuming an approximation (extends $-\infty, \infty$) to Nyquist pulse which has zero ISI and $BW = RF = 1/T_s$. One approximation is (raised cosine).

$R_b =$ Symbol rate $\times \log_2 M = \frac{1}{T_s}\log_2 M$

$BW = 1/T_s$

$\rho = \frac{R_b}{BW} = \frac{(1/T_s)\log_2 M}{1/T_s} = \log_2 M$

$\therefore \boxed{\rho \geq 1}$

$\therefore$ As $M\uparrow$ $\rho = \frac{R_b}{BW}\uparrow$ we can send more bits/sec for the same BW but of course at the cost of BER or compensate by increasing power.

M-ary orthogonal FSK

Min. freq. separation for orthogonality is $\left(\frac{1}{2T_s}\right)$

$\therefore BW \simeq M \times \frac{1}{2T_s}$

(The required SNR per bit to achieve a target BER decreases as $M\uparrow$. $\therefore$ It is called power efficient) At the cost of large BW

Again

$R_b = \frac{1}{T_s} \times \log_2 M$

$\rho = \frac{R_b}{BW} = \frac{(1/T_s)\log_2 M}{(1/2T_s)M} = \frac{2\log_2 M}{M}$

$\rho = \frac{2\log_2 M}{M}$ $\therefore$ As $M\uparrow$ $\rho\downarrow$

$\therefore$ As $M\uparrow$, $\rho\downarrow \Rightarrow$ we can send smaller bits/sec for the same BW or increase BW.

MPSK-QAM): BW efficient, useful for BW limited channels with high SNR (ex. cable modems, LOS microwave)

M-FSK): Power efficient; useful for power-limited channels but with sufficiently large BW (ex. Deep space communications)

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Spread Spectrum Modulation Techniques

They are modulation techniques with BW's of several orders of magnitude larger than information BW.

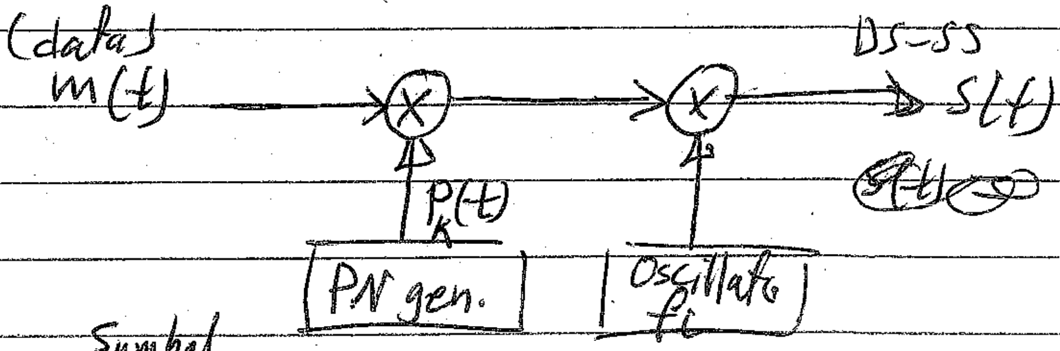
Applications: Antijamming in military comm, GPS, distance measurement, CDMA mobile comm.

The main types are

1- Direct Sequence Spread spectrum (DS-SS)
   (used in CDMA mobile comm.)
2- Frequency Hopping spread spectrum (FH-SS)
   (used in Bluetooth to mitigate various types of Interference)

Direct Sequence Spread Spectrum (DS-SS)

Spectrum spreading is achieved by multiplying the data sequence by a pseudo-noise (PN) sequence with rate much higher than data.



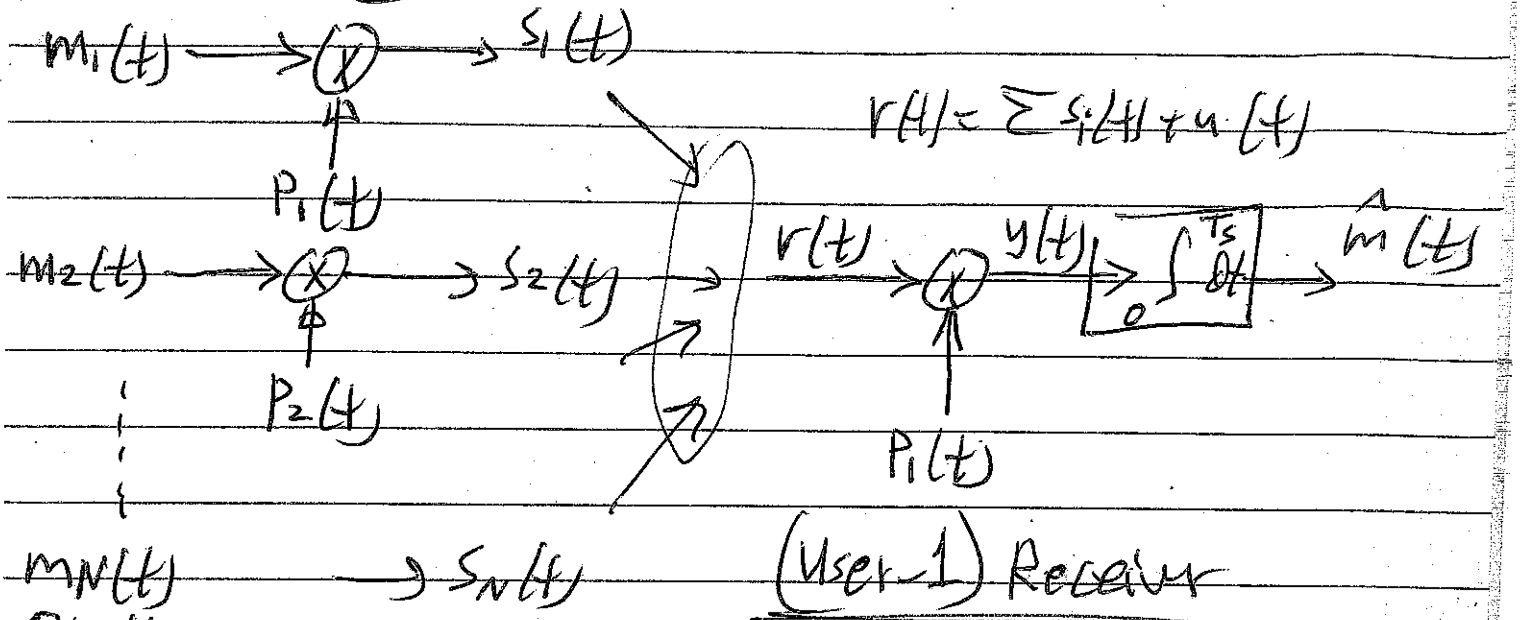
T_s : Symbol period T_c : Chip period

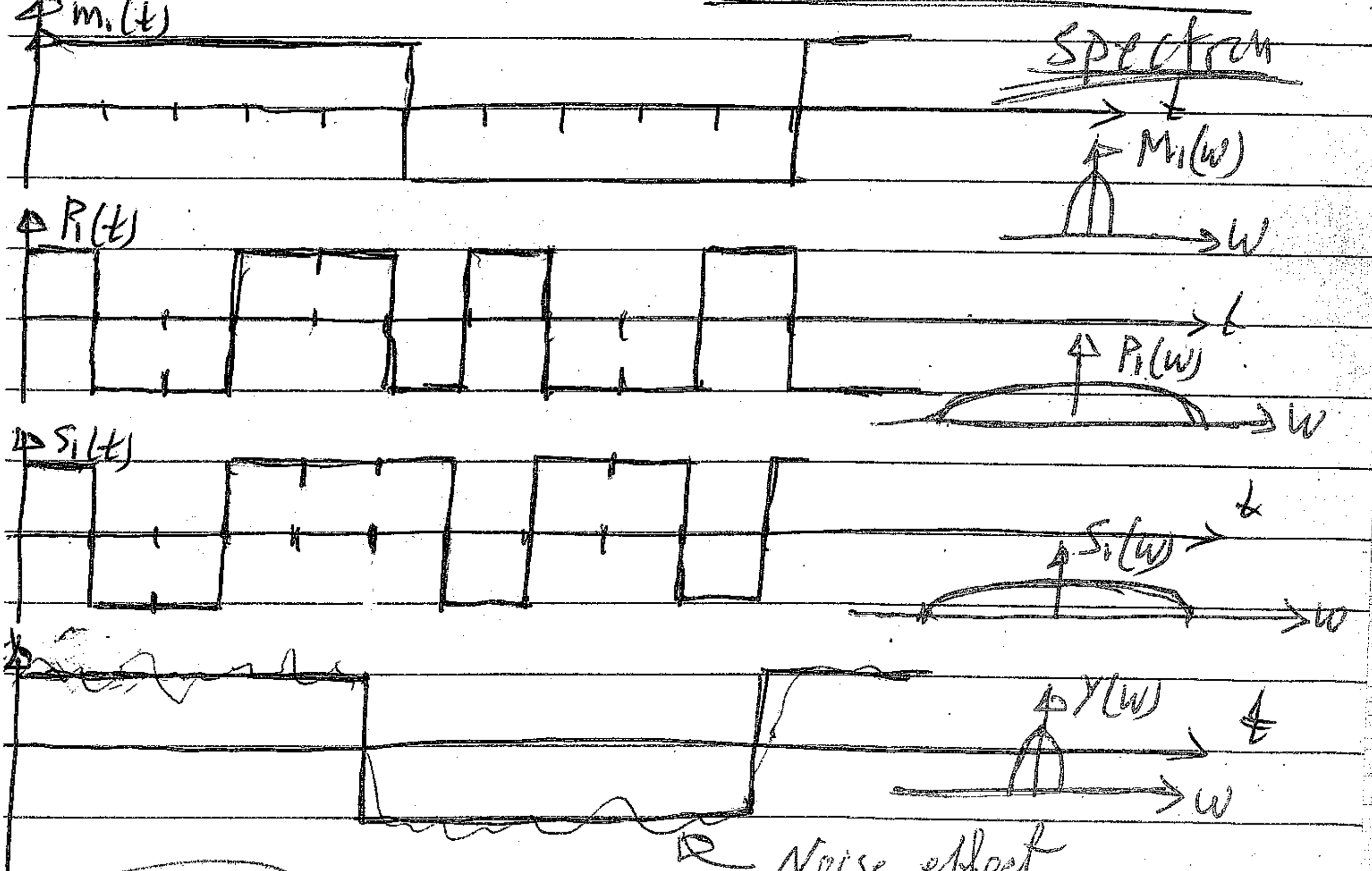
N (process gain) $= \frac{T_b}{T_c}$ (No. of chips / symbol)

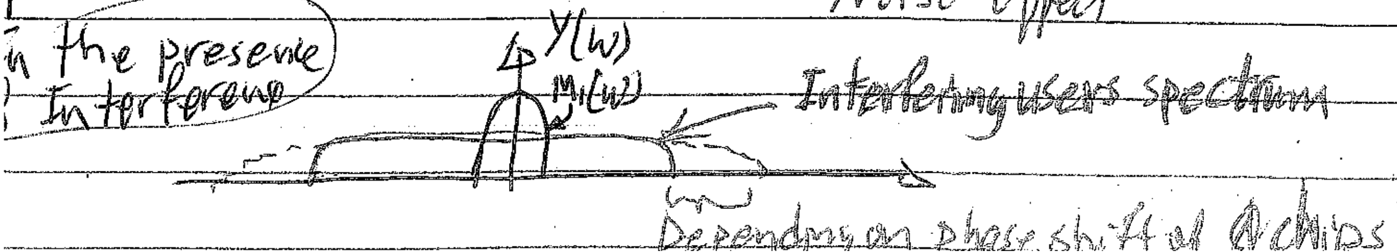
$$s(t) = \sqrt{\frac{2E_s}{T_s}}\, m(t)\, P_k(t) \cos(w_c t + \theta)$$

$m_k(t)$: (± 1) data sequence (for BPSK) for user k
$P_k(t)$: PN sequence [(± 1) random] chips for user-k.

Baseband Model







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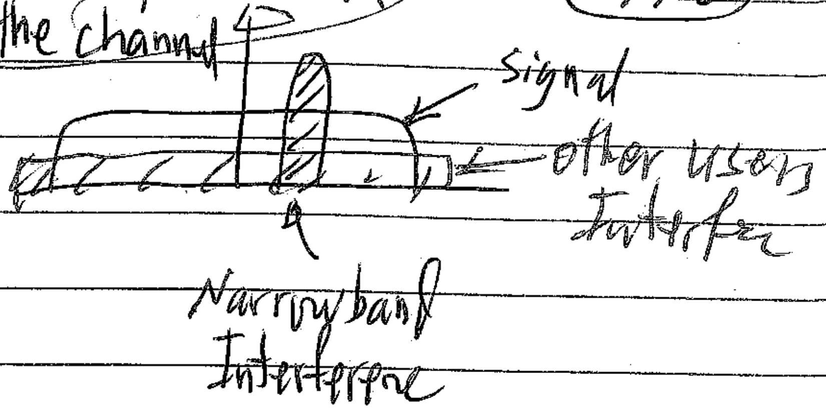
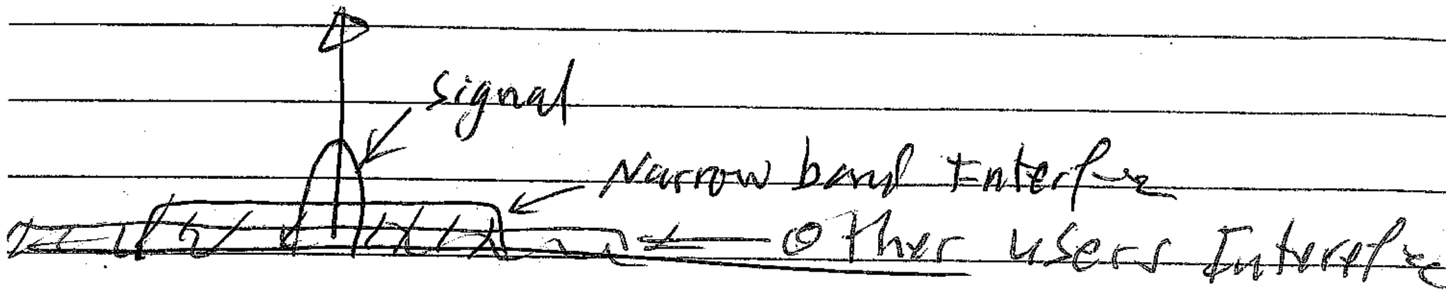
Suggested Problems
6.20, 6.21, 6.22
6.24, 6.31, 6.32, 6.34

Spectrum in the channel



Spectrum after De-spreading



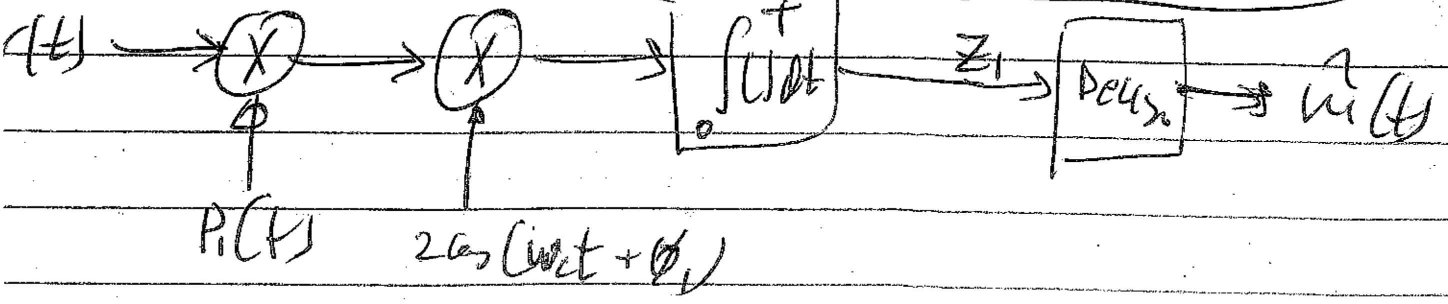
An approximate measure of interference rejection capability is given by $\dfrac{BW(\text{signal})}{BW(SS)}$

$\therefore$ Process Gain $= \dfrac{R_{chip}}{R_{symbol}} = \dfrac{T_{symbol}}{T_{chip}}$
PG

Performance of (DSSS)

User-1 receiver

Fig. 6.52



For a single user, $K = 1$, this expression reduces to the BER expression for BPSK modulation. For the interference limited case where thermal noise is not a factor, E_b/N_0 tends to infinity, and the BER expression has a value equal to

$$P_e = Q\left(\sqrt{\frac{3N}{K-1}}\right) \tag{6.145}$$

This is the irreducible error floor due to multiple access interference and assumes that all interferers provide equal power, the same as the desired user, at the DS–SS receiver. In practice, the *near–far problem* presents difficulty for DS–SS systems. Without careful power control of each mobile user, one close-in user may dominate the received signal energy at a base station, making the Gaussian assumption inaccurate [Pic91]. For a large number of users, the bit error rate is limited more by the multiple access interference than by thermal noise [Lib99]. Appendix E provides a detailed analysis of how to compute the BER for DS–SS systems. Chapter 9 illustrates how capacity of a DS–SS system changes with propagation and with multiple access interference.

6.11.5 Performance of Frequency Hopping Spread Spectrum

In FH–SS systems, several users independently hop their carrier frequencies while using BFSK modulation. If two users are not simultaneously utilizing the same frequency band, the probability of error for BFSK can be given by

$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right) \tag{6.146}$$

However, if two users transmit simultaneously in the same frequency band, a collision, or "hit", occurs. In this case, it is reasonable to assume that the probability of error is 0.5. Thus, the overall probability of bit error can be modeled as

$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)(1-p_h) + \frac{1}{2}p_h \tag{6.147}$$

where p_h is the probability of a hit, which must be determined. If there are M possible hopping channels (called slots), there is a $1/M$ probability that a given interferer will be present in the desired user's slot. If there are $K-1$ interfering users, the probability that at least one is present in the desired frequency slot is equal to one mm vs. the probability of no hits, given as

$$p_h = 1-\left(1-\frac{1}{M}\right)^{K-1} \approx \frac{K-1}{M} \tag{6.148}$$

assuming M is large. Substituting this in Equation (6.147) gives

$$P_e = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)\left(1-\frac{K-1}{M}\right) + \frac{1}{2}\left[\frac{K-1}{M}\right] \tag{6.149}$$

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$$Z_1 = \int_{(i-1)T+\tau_1}^{iT+\tau_1} r(t)\,p_1(t-\tau_1)\cos(\omega_c(t-\tau_1)+\theta_1)\,dt$$

desired — Interference from other users

$$Z_1 = I_1 + \sum_{K=2}^{K} I_K + \xi$$

$$I_1 = \int_0^T s_1(t)\,p_1(t)\cos(\omega_c t)\,dt = \sqrt{\frac{E_s T}{2}}$$

$$\xi = \int_0^T n(t)\,p_1(t)\cos(\omega_c t)\,dt \Rightarrow E[\xi^2] = \frac{N_0 T}{4}$$

$$I_K = \int_0^T s_K(t-\tau_K)\,p_1(t)\cos(\omega_c t)\,dt$$

Interfering users $\Rightarrow$ Gauss — (Central Limit Theorem)

It can be shown that

$$P_e = Q\left(\frac{1}{\sqrt{\frac{K-1}{3N} + \frac{N_0}{2E_b}}}\right)$$

For Interference Limited (neglecting Noise)

$$P_e = Q\left(\sqrt{\frac{3N}{K-1}}\right)$$

(Assuming power control with users of equal powers)

Intuitive

desired power $= 1$

K-Interferers $= (K-1) \times \frac{1}{N}$ reduction factor

$$\frac{S}{N} = \frac{1}{(K-1)/N} = \frac{N}{K-1} \times 3 \text{ for unsynchronized}$$

End of chp. 6

31/6

ex) In IS-95 CDMA, assume $K=20$ users sharing the same channel. The chip rate is 1.2288 MChip/sec and each user has data rate of (13) kbit/sec. If E_b/N_0 of 7.8 dB is provided for each user, find the Pe for each user. Assume BPSK.

$$P_e = Q\left(\frac{1}{\sqrt{\frac{K-1}{3N} + \frac{N_0}{2E_b}}}\right)$$

K: No. of users
N: process gain $= \frac{T_b}{T_c}$

$T_b = \frac{1}{R_b} = 1/13000$

$T_c = 1/R_{chip} = 1/(1.2288 \times 10^6)$

$N = \frac{T_b}{T_c} = \frac{1228800}{13000} = 94$

$7.8 = \frac{E_b}{N_0}$ dB

$7.8 = \frac{E_b}{N_0}$ dB $= 10 \log \frac{E_b}{N_0}$

$\frac{E_b}{N_0} = 10^{7.8/10} = 6.02$

Ans

$$P_e = Q\left(\sqrt{\frac{1}{\frac{20-1}{3 \times 94} + \frac{1}{2 \times 6}}}\right)$$

$= Q(2.58) = \boxed{0.0049}$

Chapter – 7
Diversity Techniques

- Diversity is a powerful communications technique to improve the performance of wireless faded channels.

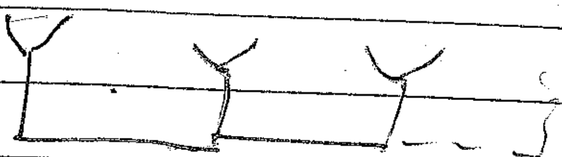
- Diversity exploits the random nature of a mobile channel by finding independent (or at least highly uncorrelated) signal paths for communicat...

- If one path undergoes a deep fade, another independent path may have a strong signal which can be combined with the weak one to improve the SNR (approximately 20-30 dB improvement) and hence the BER of the system.

Diversity techniques

① Space or antenna diversity

multiantenna, with spacing $> \lambda/2$ will receive independent paths.



isadvantage: Difficult to implement on MS due to size limitations

2 Polarization Diversity

Send the same signal on horizontal and vertical polarizations and can be combined at RX.
Disadvantage: Only two diversity branches.

3 Frequency Diversity

Sending the same signal on multicarrier freqs greater than coherence BW (B_c) so that they fade independently.

ex) Suburban area $\sigma_\tau = 0.5\,\mu s$

$$B_C = \frac{1}{5\sigma_\tau} = 400\ KHz \text{ (50\% correl)}$$

For 6-order diversity we need $6B_C$

$$BW = 6 \times B_C = 6 \times 400 = 2.4\ MHz$$

Disadvantage: BW inefficient

4 Time Diversity

Sending the same signal at different time instants that are separated by > coherence time (T_C) so that channel gain is independent

$$T_C = \frac{0.423}{f_m}$$

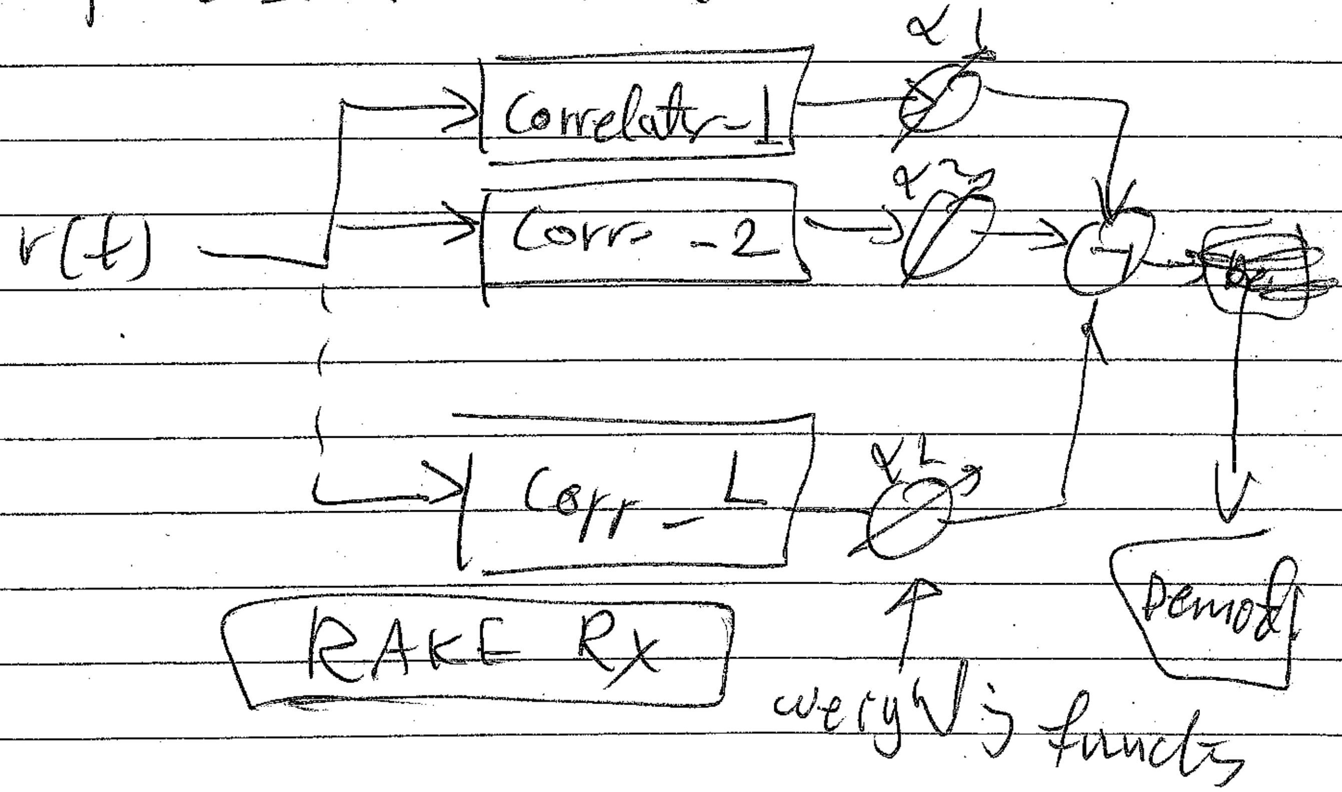
Disadvantage: Power inefficient.

3/7

5 RAKE Receiver diversity
Used in CDMA

- If PN codes are designed carefully then we can ensure that a delayed version of the same CDMA signal by more than one chip duration are highly uncorrelated.

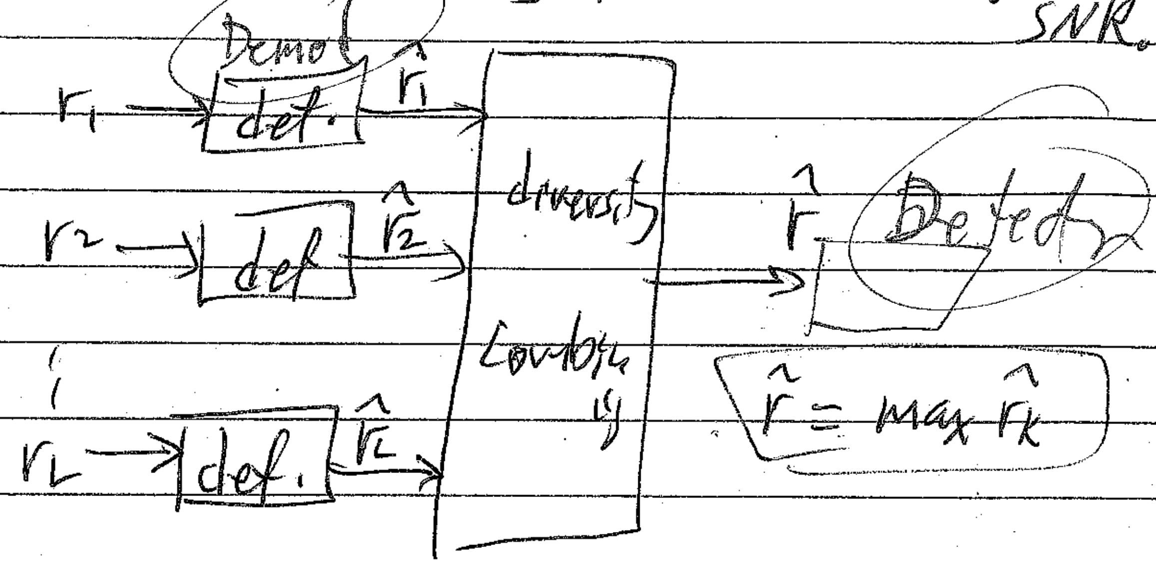
- RAKE receiver combines the time delayed versions of the CDMA signal, with weights proportional to each fingers SNR, to improve SNR and BER.



Diversity Combining Techniques

(1) Selective Combining (SC)

The Rx selects the diversity path with the highest SNR.



- Each channel (branch) has the same average SNR.
- Assume Rayleigh flat fading.

$$\text{Average SNR} \triangleq \bar{\gamma} = \frac{E_b}{N_0}\overline{\alpha^2} \qquad \left(\underline{\text{previously}}\quad \gamma = \frac{E_b}{N_0}\alpha^2\right)$$

Each branch has (instantaneous SNR) $= \gamma_i$

γ_i has PDF [from page (22/6)]

$$f_{\gamma_i}(\gamma) = \frac{1}{\bar{\gamma}_i} e^{-\gamma/\bar{\gamma}_i} \qquad \gamma \geq 0$$

Prob. that a single branch has instantaneous SNR < a threshold γ

$$\text{Prob}[\gamma_1 < \gamma] = \int_0^{\gamma} \frac{1}{\bar{\gamma}_i} e^{-\gamma_i/\bar{\gamma}_i} d\gamma_i = 1 - e^{-\gamma/\bar{\gamma}_i}$$

The distribution fn. of the ALL-Branch Combiner SNR is defined as

$(F_{\gamma_1}(\gamma) \times F_{\gamma_2}(\gamma) \times \cdots \times F_{\gamma_L}(\gamma))$ (5/7) $\bar{\gamma}_1, \bar{\gamma}_2 \cdots$ Average SNRs for channels 1, 2, --- 2

$F_{\gamma_{SC}}(\gamma)$ selective combine

$\gamma_{SC} = \max\{\gamma_1, \gamma_2, \ldots, \gamma_L\}$

$F_{SC}(\gamma) = \text{prob.}[\gamma_{SC} \le \gamma]$

$= \text{prob.}[\text{ALL } \gamma_1, \gamma_2, \ldots, \gamma_M \le \gamma]$ But all branches are stat. independent

$= P[\gamma_1 \le \gamma] P[\gamma_2 \le \gamma] \ldots P[\gamma_M \le \gamma] = (1-e^{-\gamma/\bar{\gamma}_1})(1-e^{-\gamma/\bar{\gamma}_2})(1-e^{-\gamma/\bar{\gamma}_3}) \ldots (1-e^{-\gamma/\bar{\gamma}_M})$

If Identical channels

$$F_{\gamma_{SC}}(\gamma) = \left(1-e^{-\gamma/\bar{\gamma}}\right)^L \qquad f(x) = \frac{\partial F(x)}{\partial x}$$

∴ the pdf is

$$f_{\gamma_{SC}}(\gamma) = L\left(1-e^{-\gamma/\bar{\gamma}}\right)^{L-1}\left(-e^{-\gamma/\bar{\gamma}}\right)\left(-\frac{1}{\bar{\gamma}}\right)$$

$$\boxed{f_{\gamma_{SC}}(\gamma) = \frac{L}{\bar{\gamma}}\left(1-e^{-\gamma/\bar{\gamma}}\right)^{L-1} e^{-\gamma/\bar{\gamma}}}$$

pdf of instantaneous SNR of the SC combiner

We can find the mean SNR (i.e $\bar{\gamma}_{SC}$) for selective combining

$$\bar{\gamma}_{SC} = \int_0^\infty \gamma f_{\gamma_{SC}}(\gamma)\, \partial\gamma = \int_0^\infty \frac{\gamma L}{\bar{\gamma}}\left(1-e^{-\gamma/\bar{\gamma}}\right)^{L-1} e^{-\gamma/\bar{\gamma}}\, \partial\gamma$$

Let $x = \frac{\gamma}{\bar{\gamma}} \Rightarrow \partial\gamma = \bar{\gamma}\, \partial x$

$$\bar{\gamma}_{SC} = \int_0^\infty L x \left(1-e^{-x}\right)^{L-1} e^{-x}\, \bar{\gamma}\, dx$$

$$= \bar{\gamma} \int_0^\infty L x \left(1-e^{-x}\right)^{L-1} e^{-x}\, dx$$

$(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^{n-k} b^k$

But from binomial expansion

$$\left(1-e^{-x}\right)^{L-1} = \sum_{l=0}^{L-1} \left(-e^{-x}\right)^l \binom{L-1}{l} = \sum_{l=0}^{L-1} (-1)^l \binom{L-1}{l} e^{-xl}$$

6a/7

$$\frac{\bar{\gamma}_{sc}}{\bar{\gamma}} = \int_0^\infty Lx \left[\sum_{l=0}^{L-1} (-1)^l \binom{L-1}{l} e^{-xl}\right] e^{-x}\,dx$$

$$= \sum_{l=0}^{L-1} L (-1)^l \binom{L-1}{l} \underbrace{\int_0^\infty \underset{u}{x}\, \underset{dv}{e^{-x(l+1)}\,dx}}_{\text{by parts}}$$

$$\frac{-xe^{-x(1+l)}}{(1+l)} - \frac{e^{-x(1+l)}}{(1+l)^2}\Bigg|_0^\infty = \frac{1}{(l+1)^2}$$

$$\frac{\bar{\gamma}_{sc}}{\bar{\gamma}} = \sum_{l=0}^{L-1} (-1)^l \binom{L-1}{l} \frac{L}{(l+1)^2}$$ can be shown to be

$$\frac{\bar{\gamma}_{sc}}{\bar{\gamma}} = \sum_{K=1}^{L} \frac{1}{K}$$

$$\boxed{\bar{\gamma}_{sc} = \bar{\gamma} \sum_{K=1}^{L} \frac{1}{K}}$$ Average SNR for selection combining

ex)

6b/7

ex) A four-branch selection diversity ($L=4$) is used where each branch receives an independent Rayleigh faded signal. The average SNR for each branch is 20 dB. Determine the probability that the SNR will drop below 10 dB for a) with the $L=4$ selective diversity b) without diversity
c) What the average SNR with $L=4$ selective diversity.

~~d) Find the prob that SNR > 30 dB?~~

Average SNR for each branch $= \bar{\gamma}_{dB} = 20\text{ dB} = 10\log\bar{\gamma}$

$$\therefore \bar{\gamma} = 10^{20/10} = 100$$

$$\gamma_{dB} = 10\text{ dB} \Rightarrow \gamma = 10^{10/10} = 10$$

a) $\text{Prob}[\gamma_{sc} \le 10] = \left(1-e^{-\gamma/\bar{\gamma}}\right)^L = \left(1-e^{-10/100}\right)^4 = \boxed{0.000082}$

b) without diversity

$$\text{prob}[\gamma \le 10] = \left(1-e^{-10/100}\right)^1 = \boxed{0.095}$$

$\therefore$ A very large improvement

c) $\bar{\gamma}_{sc} = \bar{\gamma}\sum_{k=1}^{L}\frac{1}{k} = 100\left[\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}\right] = 208.3$

$\bar{\gamma}_{sc\,dB} = 10\log\bar{\gamma}_{sc} = 23.18\text{dB}$ Approximately twice that without diversity

~~d) Prob. $[\gamma_{sc} > 30\text{dB}] = 1 - F(\gamma)$~~ $\gamma_{dB} = 30\text{dB} \Rightarrow \gamma = 1000$

Performance of DPSK with ~~SC~~

~~Pe~~ For Gaussian channel $\boxed{P_{e,DPSK} = \frac{1}{2} e^{-\gamma}}$ $\gamma = \frac{E_b}{N_0}$

$$P_{e,DPSK}(\text{with } SC) = E\left[\frac{1}{2} e^{-\gamma}\right] \text{ over } \gamma_{sc}$$

$$= \int_0^\infty \frac{1}{2} e^{-\gamma} f_{\gamma_{sc}}(\gamma)\, d\gamma \qquad \text{As before } \boxed{\bar{\gamma} = \frac{E_b}{N_0}\overline{\alpha^2}}$$

$$= \int_0^\infty \frac{1}{2} e^{-\gamma} \frac{L}{\bar{\gamma}} \left(1 - e^{-\gamma/\bar{\gamma}}\right)^{L-1} e^{-\gamma/\bar{\gamma}}\, d\gamma$$

$$= \frac{L}{2\bar{\gamma}} \int_0^\infty e^{-\gamma\left(1 + \frac{1}{\bar{\gamma}}\right)} \left(1 - e^{-\gamma/\bar{\gamma}}\right)^{L-1} d\gamma$$

Let $x = \gamma/\bar{\gamma} \Rightarrow d\gamma = \bar{\gamma}\, dx$

$$P_{e,DPSK}(SC) = \frac{L}{2\cancel{\bar{\gamma}}} \int_0^\infty e^{-x(\bar{\gamma}+1)} \left(1 - e^{-x}\right)^{L-1} \cancel{\bar{\gamma}}\, dx$$

$$= \frac{L}{2} \int_0^\infty e^{-x(\bar{\gamma}+1)} \boxed{\left(1 - e^{-x}\right)^{L-1}}\, dx \longrightarrow \text{Binomial Expansion}$$

$$= \frac{L}{2} \int_0^\infty e^{-x(\bar{\gamma}+1)} \left[\sum_{l=0}^{L-1} (-1)^l \binom{L-1}{l} e^{-xl}\right] dx$$

8/7

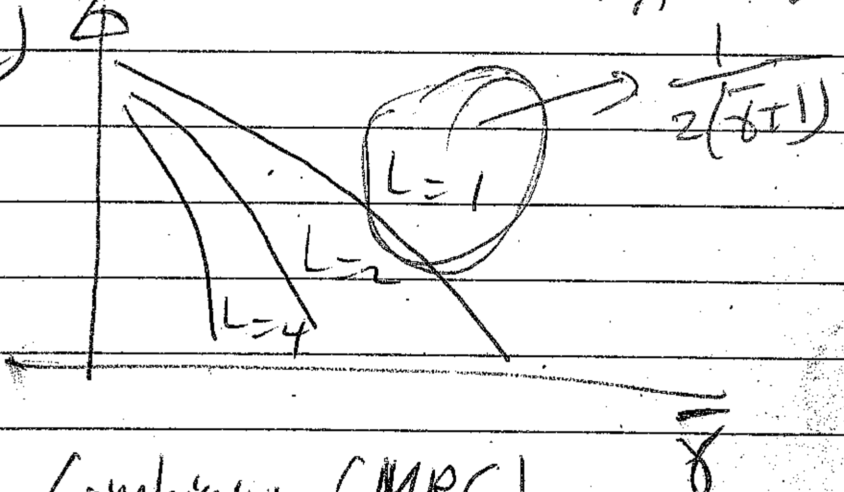
5

$$P_{e,DPSK}(SC) = \frac{L}{2}\sum_{l=0}^{L-1}(-1)^l\binom{L-1}{l}\int_0^\infty e^{-x(l+\bar{\gamma}+1)}\,dx$$

$$= \frac{L}{2}\sum_{l=0}^{L-1}(-1)^l\binom{L-1}{l}\left[\frac{e^{-x(l+\bar{\gamma}+1)}}{-(l+\bar{\gamma}+1)}\right]_0^\infty$$

$$= \boxed{\frac{L}{2}\sum_{l=0}^{L-1}(-1)^l\binom{L-1}{l}\left(\frac{1}{l+\bar{\gamma}+1}\right)}$$

As before



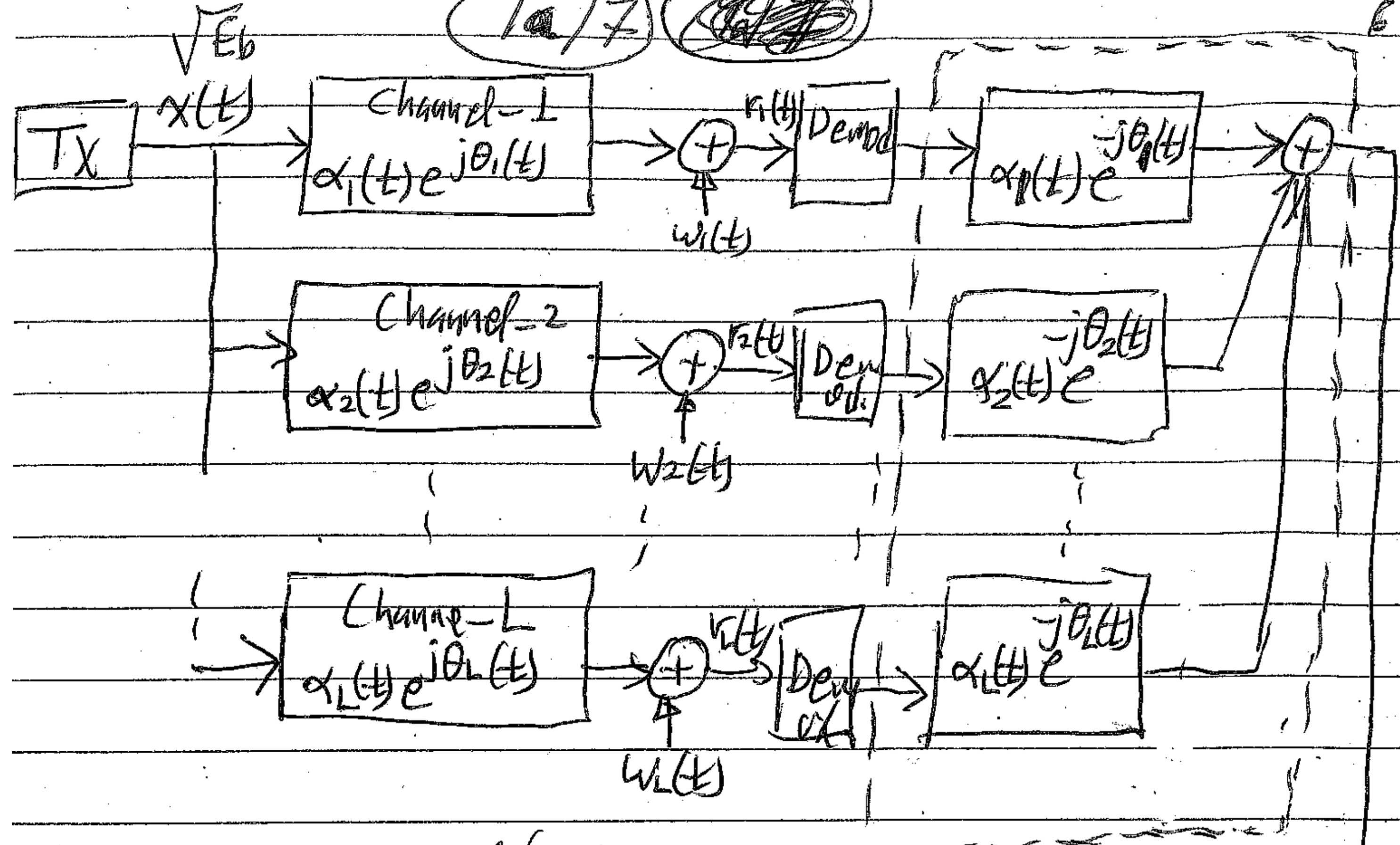
(2) Maximum Ratio Combining (MRC)

This combining technique assumes that the receiver is able to accurately estimate the amplitude fading $\alpha_l(t)$ and carrier phase $\theta_l(t)$ for each diversity branch of $l = 1, 2, \ldots, L$ branches. The receiver does:

1 - coherently demodulates the received signal of each branch.
2 - The l^{th} branch signal $r_l e^{j\theta_l(t)}$ is multiplied by $e^{-j\theta_l(t)}$ to remove phase offset (so that all branches add in phase).
3 - Each r_l is weighted by a gain $= \alpha_l$ of the same branch
4 - Branch outputs are added & output goes to decision device.

(9a/7)

6



It can be shown that (MRC) maximizes the output SNR and has the best performance (Shwartz inequality)   Maximal ratio Combiner

Dec. Device

Recovered Info

$$\vec{r} = \sum_{l=1}^{L} \left(\alpha_l e^{-j\theta_l}\right)\left(\alpha_l e^{j\theta_l} x + n_l\right)$$ → noise at demod output

$$= \left[\sum_{l=1}^{L} \alpha_l^2\right] x + \left[\sum_{l=1}^{L} \alpha_l e^{-j\theta_l} n_l\right]$$

$\sum_{l=1}^{L} \alpha_l^2 \sqrt{E_b}$ + noise

$$\vec{r} = g\vec{x} + \vec{n} \quad \text{where } g = \sum_{l=1}^{L} \alpha_l^2$$

$\frac{\alpha^2 E_b}{N_0}$ ← $\frac{2\sigma_n^2}{\alpha^2 E_b}$

$$\therefore \sigma_n^2 = \frac{N_0}{2} \sum_{l=1}^{L} \alpha_l^2 \qquad \vec{n} = \sum_{l=1}^{L} \alpha_l e^{-j\theta_l} n_l$$ Each (n_l) has a variance $\frac{N_0}{2}$

~~γ_{MRC} $x = \sqrt{E_b}$ for symbol "1" or $-\sqrt{E_b}$~~

output signal power $= (gx)^2 = \left(\sum_{l=1}^{L} \alpha_l^2 x\right)^2$

$$\gamma_{MRC} = \frac{E_b(MRC)}{N_0(MRC)} = \frac{\text{Signal power}}{\text{Noise power}} = \frac{\left[\sum_{l=1}^{L} \alpha_l^2 \sqrt{E_b}\right]^2}{2\frac{N_0}{2}\sum_{l=1}^{L} \alpha_l^2} = \frac{E_b}{N_0}\sum_{l=1}^{L} \alpha_l^2$$

$(2\sigma_n^2)$

9b/7

$$\gamma = \frac{\alpha^2 E_b}{N_0} \triangleq \frac{\overbrace{\text{instantaneous}}^{\text{specific}(\alpha)}\text{ bit energy}}{2\times\text{variance of } n(t)}$$

noise waveform

$N_0 \rightarrow 2\times\frac{N_0}{2}$

$$\gamma_{MRC} = \frac{\overbrace{\text{instantaneous}}^{\text{specific}(\alpha_1,\alpha_2,\ldots,\alpha_L)}\text{ bit energy after combining}}{2\times\underbrace{\text{variance of } n_{out}(t)}_{\frac{N_0(out)}{2}}}$$

$n_{out}(t)$ ← $n(t)$ after combining

$$\frac{N_0(out)}{2} = \frac{N_0}{2}\sum_{l=1}^{L}\alpha_l^2$$

Instantaneous bit energy of MRC output $= \left[\alpha_1^2\sqrt{E_b}+\alpha_2^2\sqrt{E_b}+\cdots+\alpha_L^2\sqrt{E_b}\right]^2$

$$= E_b\left(\sum_{l=1}^{L}\alpha_l^2\right)^2$$

$$\boxed{\gamma_{MRC} = \frac{E_b\left(\sum_{l=1}^{L}\alpha_l^2\right)^{\cancel{2}}}{\cancel{2}\times\frac{N_0}{\cancel{2}}\left(\sum_{l=1}^{L}\alpha_l^2\right)}} = \frac{E_b}{N_0}\sum_{l=1}^{L}\alpha_l^2 = \boxed{\sum_{l=1}^{L}\gamma_l}$$

~~Proof~~:

$\bar{a} = [\alpha_1 e^{j\theta_1}\ \alpha_2 e^{j\theta_2}\ \ldots\ \alpha_L e^{j\theta_L}]$

g = combining vector $\triangleq [g_1\ g_2\ \ldots\ g_L]^T$

$\bar{r}$ (combiner) $= \bar{a}\bar{g} \Rightarrow |r|^2 = r^* r = g^H a^H a g$

$= g^H A g$ where $A = a^H a$ (square matrix)

The maximizer of $g^H A g/|g|^2$ is the maximum eigenvector of $A(=a^H a)$. This choice maximizes SNR

Since A has one vector in its column span $v = a^H$ (scaling does not affect)

$Av = \lambda v \Rightarrow$ ... combiner

(10/7)

$\therefore\ \gamma_{MRC} = \sum_{l=1}^{L} \frac{E_b}{N_0}\alpha_l^2 = \sum_{l=1}^{L} \gamma_l$

$\therefore\ SNR_{MRC} = \sum$ SNR of each branch

Average SNR $\quad \overline{\gamma_{MRC}} = \sum_{l=1}^{L} \overline{\gamma_l} = L\,\overline{\gamma}$ (identical

$= L \frac{E_b}{N_0}\overline{\alpha^2}$

The pdf of γ_{MRC}

$$\boxed{f_{\gamma_{MRC}}(\gamma) = f_{\gamma_1}(\gamma) * f_{\gamma_2}(\gamma) * \cdots * f_{\gamma_L}(\gamma)}$$

$\cancel{\gamma_{MRC} = \gamma_1 * \gamma_2 * \cdots * \gamma_L}$ $\qquad$ $*$: convolution

ALL $\gamma_1, \gamma_2, \ldots \gamma_L$ are identical with

$$f_\gamma(\gamma) = \frac{1}{\overline{\gamma}} e^{-\gamma/\overline{\gamma}} \qquad \gamma \geq 0$$

$$\Phi_\gamma(\omega) = E\left[e^{j\omega\gamma}\right] = \int_0^\infty e^{j\omega\gamma}\frac{1}{\overline{\gamma}}e^{-\gamma/\overline{\gamma}}\,d\gamma$$

$$= \frac{1}{\overline{\gamma}}\int_0^\infty e^{\gamma(j\omega - \frac{1}{\overline{\gamma}})}\,d\gamma = \frac{1}{\overline{\gamma}} \left.\frac{e^{\gamma(j\omega-\frac{1}{\overline{\gamma}})}}{j\omega - \frac{1}{\overline{\gamma}}}\right|_0^\infty$$

$$= \frac{1}{\overline{\gamma}} \frac{e^{\infty(j\omega - \frac{1}{\overline{\gamma}})} - e^0}{j\omega - \frac{1}{\overline{\gamma}}} = \boxed{\frac{1}{1 - j\omega\overline{\gamma}}}$$

Since $\gamma_1, \gamma_2, \ldots, \gamma_L$ are statistically independent

$$\Phi_{\gamma_{MRC}}(\omega) = \boxed{\Phi_\gamma(\omega)}^L = \frac{1}{(1 - j\omega\overline{\gamma})^L}$$

$$f_{\gamma_{MRC}}(\gamma) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \Phi_{\gamma_{MRC}}(\omega)\, e^{-j\omega\gamma}\,d\omega$$

Borrowing from Fourier Transform pairs (diff. in freq. domain)

$$t^n e^{-at} u(t) \xleftrightarrow{F} \frac{n!}{(a+j\omega)^{n+1}}$$

Comparing to the definition of a characteristic fn we have $(-\omega)$ instead of (ω). Converting the above pair to γ, ω

$$\underbrace{\gamma^n e^{-a\gamma} u(\gamma)}_{f_\gamma(\gamma)} \longleftrightarrow \underbrace{\frac{n!}{(a-j\omega)^{n+1}}}_{\Phi_\gamma(\omega)}$$

$$\frac{\gamma^n e^{-a\gamma} u(\gamma)}{n!} \longleftrightarrow \frac{1}{(a-j\omega)^{n+1}}$$

But $\Phi_{\gamma MRC}(\omega) = \dfrac{1}{(1-j\omega\bar{\gamma})^L} = \dfrac{1}{\bar{\gamma}^L\left(\frac{1}{\bar{\gamma}}-j\omega\right)^L}$

letting $L = n+1 \Rightarrow n = L-1$ and $a = 1/\bar{\gamma}$

$$\gamma^{L-1} e^{-\frac{\gamma}{\bar{\gamma}}} u(\gamma) \xleftrightarrow{\text{scale fact}} \frac{1}{\left(\frac{1}{\bar{\gamma}}-j\omega\right)^L}$$

$$\frac{\gamma^{L-1} e^{-\frac{\gamma}{\bar{\gamma}}} u(\gamma)}{(L-1)!\,\bar{\gamma}^L} \longleftrightarrow \frac{1}{\bar{\gamma}^L\left(\frac{1}{\bar{\gamma}}-j\omega\right)^L}$$

$$\therefore \boxed{f_{\gamma MRC}(\gamma) = \frac{1}{(L-1)!\,(\bar{\gamma})^L}\,\gamma^{L-1} e^{-\gamma/\bar{\gamma}}} \quad \boxed{\gamma > 0}$$

$$\boxed{F_{\gamma MRC}(\gamma)} = \int_0^\gamma f_{\gamma MRC}(\gamma)\, d\gamma = \boxed{1 - e^{-\frac{\gamma}{\bar{\gamma}}} \sum_{k=1}^{L} \frac{(\gamma/\bar{\gamma})^{k-1}}{(k-1)!}}$$

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identical channel

Ex.) Consider a 6-branch diversity receiver:
a) Determine the improvement in average SNR for
i) selection diversity ii) Max ratio combining diversity
b) Assume independent Rayleigh channels
and if average SNR = 50 (Linear scale) for each branch, what is the prob. that SNR ≤ 5 for
i) MRC ii) selective diversity) No diversity

a) i) selection diversity

$$\bar{\gamma}_{SC} = \bar{\gamma} \sum_{K=1}^{6} \frac{1}{K} = \bar{\gamma}\left(\frac{1}{1}+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}\right)$$

$$\frac{\bar{\gamma}_{SC}}{\bar{\gamma}} = 2.45$$

ii) MRC $\bar{\gamma}_{MRC} = \sum_{i=1}^{L} \bar{\gamma}_i = L\bar{\gamma}$ (if identical channel)

$$\frac{\bar{\gamma}_{MRC}}{\bar{\gamma}} = L = 6$$ (better than s combining)

b) i) $\text{Prob}[\gamma_{MRC} < \gamma] = \text{prob}[\gamma_{MRC} < 5] = 1 - e^{-\frac{\gamma}{\bar{\gamma}}} \sum_{K=1}^{L} \frac{(\gamma/\bar{\gamma})^{K-1}}{(K-1)!}$

$$= 1 - e^{-0.1} \sum_{K=1}^{6} \frac{(0.1)^{K-1}}{(K-1)!} = \boxed{1.37 \times 10^{-15}}$$

(L = 6) $\quad \frac{\gamma}{\bar{\gamma}} = \frac{5}{50} = 0.1$

ii) selective Div.

$$\text{Prob}(\gamma_{SC} < \gamma) = \left(1 - e^{-\frac{\gamma}{\bar{\gamma}}}\right)^L = \left(1 - e^{-5/50}\right)^6 = \boxed{9.7 \times 1}$$

iii) No. Div. $\text{prob}(\gamma < 5) = 1 - e^{-5/50} = \boxed{9.95 \times}$

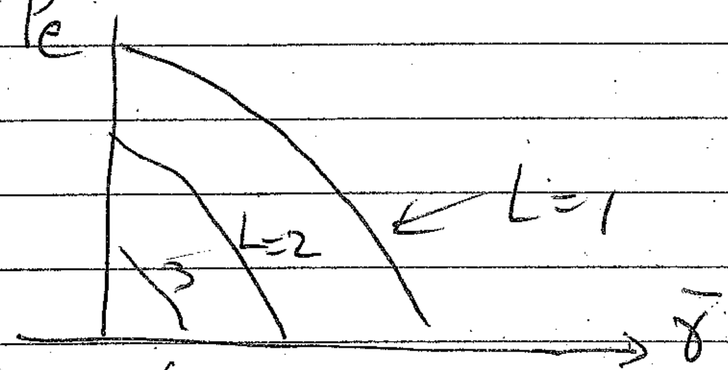
∴ Diversity provides very large improvement

The BER for BPSK with MRC

$$P_e = \int_0^{\infty} Q\left(\sqrt{2\gamma}\right) \frac{1}{(L-1)!\,(\bar{\gamma})^L} \gamma^{L-1} e^{-\gamma/\bar{\gamma}}\, d\gamma$$

$$= \left(\frac{1-\mu}{2}\right)^L \sum_{k=0}^{L-1} \binom{L-1+k}{k} \left(\frac{1+\mu}{2}\right)^k$$

where $\mu = \sqrt{\dfrac{\bar{\gamma}}{1+\bar{\gamma}}}$



(3) Equal Gain Combining (EGC)

No weighting is used

$$r = \sum_{l=1}^{L} r_l$$

$$\gamma_{EGC} = \frac{\left[\sum_{l=1}^{L} r_l\right]^2}{2N_0 L}$$

has no closed form PDF

7.7, 7.10, show ~~derive~~ $\sqrt{\gamma_{MRC}}$ (γ) 14/7 (End of chp. 7)

ex) A BPSK signal is transmitted to a receiver with (L=2) diversity with Maximal ratio combining. The channel of each diversity ~~channel~~ branch takes on a value of (1) with prob. = 0.9 and a value of (0.05) with prob. = 0.1. The two channels are independent. The noise in each branch is AWGN with $N_0/2$ PSD. Determine the Pe of the MRC receiver.

Given the PDF for $\alpha \Rightarrow f_\alpha(\alpha) = 0.1\,\delta(\alpha-0.05) + 0.9\,\delta(\alpha-1)$

But SNR is $\gamma = \frac{\alpha^2 E_b}{N_0}$ (Transformation) → γ has two value

1- $0.0025 \frac{E_b}{N_0}$ (p=0.1)

2- $\frac{E_b}{N_0}$ p(=0.9)

$\therefore \boxed{f_\gamma(\gamma) = 0.1\,\delta\left(\gamma - 0.0025\frac{E_b}{N_0}\right) + 0.9\,\delta\left(\gamma - \frac{E_b}{N_0}\right)}$ for each branch

$\gamma_{MRC} = \gamma_1 + \gamma_2 \Rightarrow f_{\gamma_{MRC}} = f_{\gamma_1}(\gamma) * f_{\gamma_2}(\gamma)$ } independent

$$f_{\gamma_{MRC}}(\gamma) = \left[0.1\,\delta\left(\gamma-0.0025\frac{E_b}{N_0}\right) + 0.9\,\delta\left(\gamma-\frac{E_b}{N_0}\right)\right] * \left[0.1\,\delta\left(\gamma-0.0025\frac{E_b}{N_0}\right) + 0.9\,\delta\left(\gamma-\frac{E_b}{N_0}\right)\right]$$

$$= \boxed{0.01\,\delta\left(\gamma-0.005\frac{E_b}{N_0}\right) + 0.81\,\delta\left(\gamma-\frac{2E_b}{N_0}\right) + 2\times0.09\,\delta\left(\gamma-1.0025\frac{E_b}{N_0}\right)}$$

$$P_e = E\left[Q\sqrt{2\gamma}\right] = \int_{-\infty}^{\infty} Q(\sqrt{2\gamma})\, f_{\gamma_{MRC}}(\gamma)\, d\gamma$$

$$= 0.01\,Q\left(\sqrt{\frac{0.01E_b}{N_0}}\right) + 0.81\,Q\left(\sqrt{\frac{4E_b}{N_0}}\right) + 0.18\,Q\left(\sqrt{\frac{2.005E_b}{N_0}}\right)$$

15/17

introduced by the fading channel as long as the phase distortion does not change much over a duration of two symbol intervals (similar to $\pi/4$-DQPSK discussed in Section 3.3). Therefor practice, the performance difference between DBPSK with selective diversity and coherent B with maximal ratio combining may not be as significant as that shown in Figures 4.6 and 4

Effect of ISI (freq-selective fading) on BER

4.3 CHANNEL EQUALIZATION

Consider the equivalent representation of a wireless communications system at baseband, wi channel encoding/decoding for simplicity. At the transmitter, the modulator maps the inform sequence to the data sequence $\{\vec{x}_n\}$ in the N-dimensional signal space (as discussed in Section depending on the modulation scheme used. For convenience, we can model the transmitter transmitter filter with an input signal $\vec{x}(t) = \sum_{n=-\infty}^{\infty} \vec{x}_n \delta(t - nT)$ such that the filter outpu the complex envelope of the signal actually transmitted, where T is the symbol interval o transmitted signal. As discussed in Chapter 2, a typical wireless channel introduces both fa and propagation delay dispersion to the transmitted signal, in addition to additive white Gau noise. From Chapter 3, we know that the optimum receiver for an AWGN channel is a ma filter receiver, consisting of a demodulator (a bank of matched filters) and a decision de The decision device makes the decision at the end of each symbol interval according to maximum-likelihood decision rule.

The transmitter filter, channel, and receiver filter together can be viewed as an effe channel for the input signal $\vec{x}(t)$, as shown in Figure 4.9. Let $c(t)$ denote the impulse resp of the effective channel. Then the output $\vec{r}(t)$ of the effective channel, sampled at the end of nth symbol interval, $t = nT$, can be represented as

$$
\begin{aligned}
\vec{r}(t)|_{t=nT} &= [\vec{x}(t) \star c(t) + \vec{n}(t)]|_{t=nT} \\
&= \left\{ \left[\sum_{l=-\infty}^{\infty} \vec{x}_l \delta(t - lT) \right] \star c(t) + \vec{n}(t) \right\} \Bigg|_{t=nT} \\
&= \left\{ \sum_{l=-\infty}^{\infty} \vec{x}_l c(t - lT) + \vec{n}(t) \right\} \Bigg|_{t=nT} \\
&= \sum_{l=-\infty}^{\infty} \vec{x}_l c(nT - lT) + \vec{n}(nT).
\end{aligned}
$$

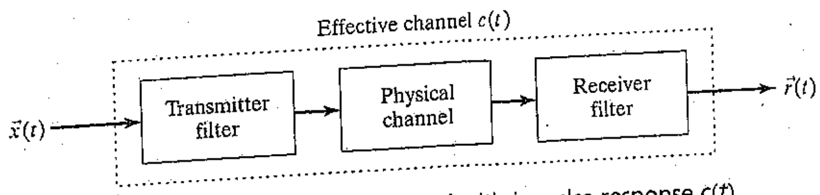


Figure 4.9 The effective channel with impulse response $c(t)$.

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[illegible] the filtered Gaussian noise vector due to the additive white Gaussian noise compo[illegible] by the physical channel. In discrete-time sequence representation with T as the [illegible] we have

$$\vec{r}_n = \sum_{l=-\infty}^{\infty} \vec{x}_l c_{n-l} + \vec{n}_n = \vec{x}_n \star c_n + \vec{n}_n, \tag{4.3.1}$$

[illegible] $c_n = c(nT)$, and $\vec{n}_n = \vec{n}(nT)$. Eq. (4.3.1) can be rewritten as

$r(n) = x(n) * h(n)$

Channel Impulse response

$$\vec{r}_n = c_0\vec{x}_n + \sum_{l=-\infty, l\neq n}^{\infty} \vec{x}_l c_{n-l} + \vec{n}_n. \tag{4.3.2}$$

[illegible] is the desired signal component (modified by the channel gain c_0) which contains [illegible] the nth transmitted symbol. As each entity in the effective channel can introduce [illegible] to the input signal, c_{n-l} at $n \neq l$ may not be zero. As a result, the second term [illegible] the ISI component due to other transmitted symbols, and the last term is the noise [illegible]. The ISI component makes it much more likely for the decision device in the receiver [illegible] error, as compared to the case without ISI.

[illegible] be shown that, to minimize the probability of transmission error, the optimum receiver [illegible] error sense) consists of a matched filter, an equalizer, and a maximum likelihood [illegible]. The matched filter is matched to the transmitter filter and the physical channel [illegible]. In other words, it is matched to the received signal waveforms and, therefore, is able [illegible] all the received signal energy. The equalizer is a transversal filter, and is needed to [illegible] for intersymbol interference as the matched filter introduces further time dispersion [illegible] signal.

[illegible] simplicity, in the following, we assume that the transmitted signal can be [illegible] a one-dimensional signal space. As a result, the vector representations such as $\vec{x}_n$, [illegible] can be simplified to the corresponding real-valued scalar representations.

[illegible] 4.5 Effect of ISI on Transmission Accuracy

[illegible] transmission system using BPSK, the matched filter receiver designed for an AWGN [illegible]. The physical channel introduces an additive Gaussian noise of zero mean and [illegible] $N_0/2$. In addition, it introduces ISI. At the end of the nth symbol interval, the [illegible] demodulator is

$$r_n = x_n + 0.5x_{n-1} + n_n,$$

$r(n) = x(n) + 0.5x(n-1) + n(n)$

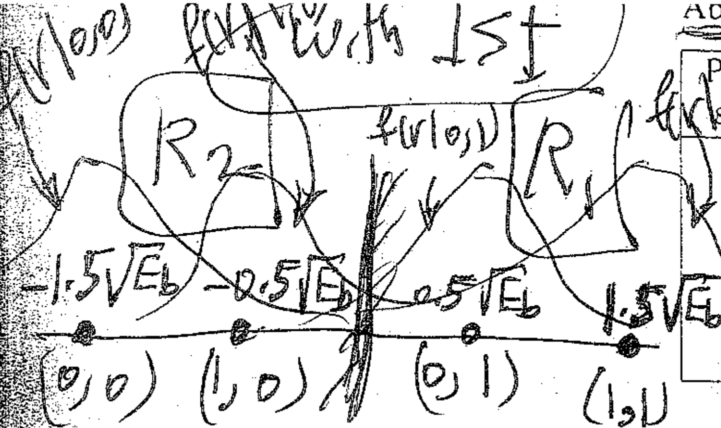
noise

[illegible] desired signal component $x_n = \sqrt{E_b}$ if symbol "1" was sent and $x_n = -\sqrt{E_b}$ if [illegible] was sent, E_b is the transmitted signal symbol energy, and the noise component n_n [illegible] random variable with zero mean and variance $N_0/2$. Determine the probability of [illegible] error.

[illegible] Without ISI, from Subsection 3.5.1, the probability of transmission error is

$$P_b = Q\left(\sqrt{\frac{2E_b}{N_0}}\right).$$

Absence of the Noise Component

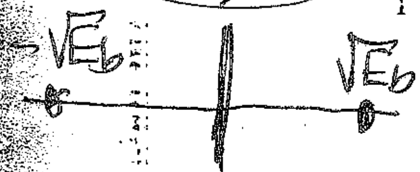


Previous symbol	Current symbol	Demodulator output without ISI	Demodulator output with ISI
"0"	"0"	$-\sqrt{E_b}$	$-1.5\sqrt{E_b}$
"0"	"1"	$\sqrt{E_b}$	$0.5\sqrt{E_b}$
"1"	"0"	$-\sqrt{E_b}$	$-0.5\sqrt{E_b}$
"1"	"1"	$\sqrt{E_b}$	$1.5\sqrt{E_b}$

new mean values

With ISI, the effect of ISI depends on whether or not the previous and the current transmitted symbols are the same. Table 4.1 lists the non-noise components in the demodulator output versus the transmitted symbols.

No-ISI



With the decision threshold setting at 0 for an AWGN channel and equally likely symbols "1" and "0", the probability of error is

$$P_b = \sum P(\text{error}|\text{previous symbol, current symbol})\,P(\text{previous symbol, current symbol})$$

$$= \frac{1}{4}[P(\text{error}|\text{"0","0"}) + P(\text{error}|\text{"0","1"}) + P(\text{error}|\text{"1","0"}) + P(\text{error}|\text{"1","1"})]$$

$\int_{R_1} f(r|0,0)\,dr \quad \int_{R_2} f(r|0,1)\,dr \quad \int_{R_1} f(r|1,0)\,dr \quad \int_{R_2} f(r|1,1)\,dr$

$$= \frac{1}{4}[P(-1.5\sqrt{E_b} + n_n > 0) + P(0.5\sqrt{E_b} + n_n < 0)$$

$$+ P(-0.5\sqrt{E_b} + n_n > 0) + P(1.5\sqrt{E_b} + n_n < 0)]$$

$$= \frac{1}{2}\left[Q\left(1.5\sqrt{\frac{2E_b}{N_0}}\right) + \underbrace{Q\left(0.5\sqrt{\frac{2E_b}{N_0}}\right)}_{\text{dominated term}}\right]$$

Figure 4.10 plots the probabilities of transmission error as a function of E_b/N_0 with and without ISI. It can be clearly observed that ISI severely degrades the transmission performance. At a $\boxed{\text{BER of } 10^{-3}}$, it requires an additional $\boxed{5.5\text{ dB}}$ in the transmitted power to $\boxed{\text{overcome the ISI.}}$ The required increase of the transmitted power increases as the required BER decreases

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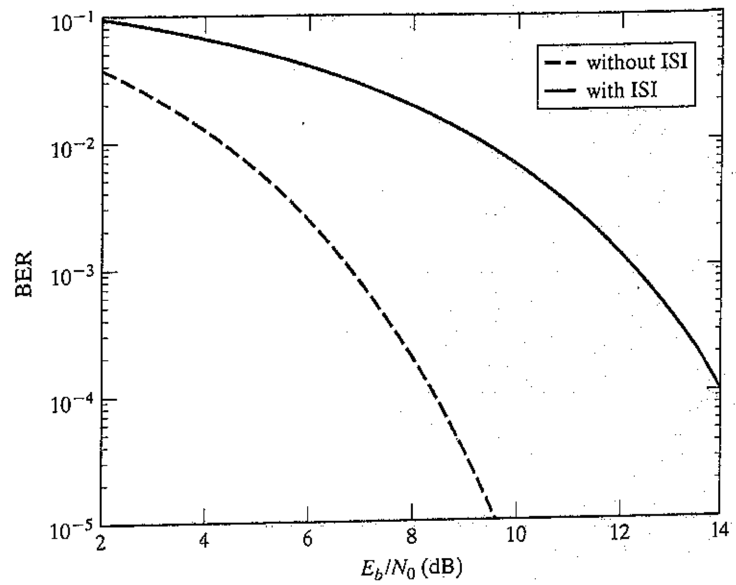

Figure 4.10 Comparison of the transmission accuracy with and without ISI.

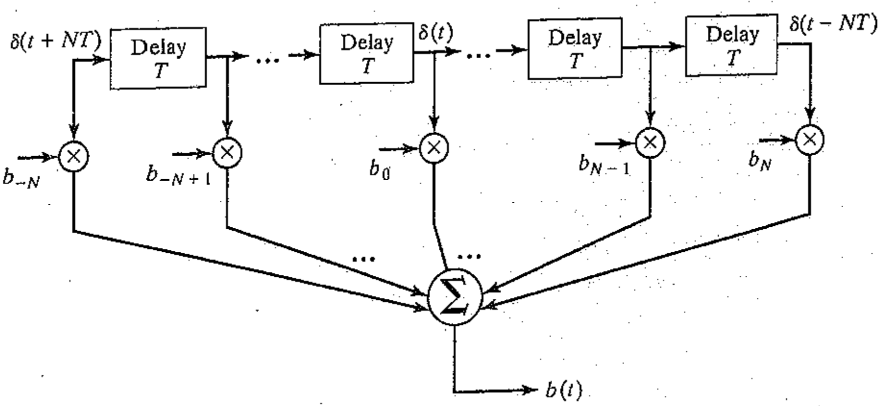

Figure 4.11 The tapped-delay-line linear equalizer.

The transfer function of the equalizer in the z domain is given by

$$B(z) = \sum_{k=-N}^{N} b_k z^{-k}.$$

Figure 4.12 shows the equalized system, where the effective channel and the equalizer are connected in tandem. Let $R(z) = \sum_{n=-\infty}^{\infty} r_n z^{-n}$ denote the z transform of the discrete time the (effective) channel such that the equalized system has a constant transfer function in the frequency region.

Example 4.6 Zero-Forcing Linear Equalizer with Infinite Taps

For the communication system in Example 4.5, design a linear equalizer to combat ISI, and determine the probability of transmission error with the channel equalization.

Solution The effective channel has an impulse response (in the absence of additive noise) given by

$$c(t) = \delta(t) + 0.5\delta(t - T).$$

Channel

The channel transfer function is then

$y(n) = x(n) + 0.5x(n-1)$

$$C(z) = \sum_{k=-\infty}^{\infty} c_k z^{-k} = 1 + 0.5z^{-1}.$$

$H(z) = \frac{Y(z)}{X(z)} = (1 + 0.5z^{-1})$

To completely equalize the channel, the transfer function of the equalizer should be

Equalizer ⇒

$$\begin{aligned} B(z) &= \frac{1}{C(z)} \\ &= \frac{1}{1 + 0.5z^{-1}} \\ &= 1 - 0.5z^{-1} + 0.5^2 z^{-2} - \ldots + (-0.5z^{-1})^k + \ldots, \quad |0.5z^{-1}| < 1. \end{aligned}$$

$\Rightarrow \frac{1}{1-(-0.5)z^{-1}} \xleftrightarrow{Z} b(n) = \left(-\frac{1}{2}\right)^n u(n)$

The equalization function can be implemented by a tapped-delay-line linear filter with a large number of taps (to approximate the infinite number of taps).

In the presence of additive noise n_n at the end of the nth symbol interval at the effective channel output, the noise component at the equalizer output at the same instant is

$$v_n = n_n \star b_n = \sum_{k=0}^{\infty} (-0.5)^k n_{n-k}.$$

Since n_n is a zero-mean Gaussian random variable with variance $N_0/2$ and is independent from sample to sample, v_n is also Gaussian with zero mean and variance equal to

$$\sigma_v^2 = \sum_{k=0}^{\infty} [(-0.5)^k]^2 (N_0/2) = 2N_0/3.$$

$\frac{N_0}{2}\left[1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \ldots\right]$

As a result, the probability of bit error with equalization is

$$P_b = Q\left(\sqrt{\frac{E_b}{2N_0/3}}\right) = Q\left(\sqrt{\frac{3E_b}{2N_0}}\right).$$

$= \frac{N_0/2}{1-\frac{1}{4}} = \frac{4}{3}\frac{N_0}{2} = \frac{2N_0}{3}$

Figure 4.15 shows the BER performance with and without the equalizer. The BER curve for the AWGN channel without ISI is also plotted for comparison. It is clear that equalization improves

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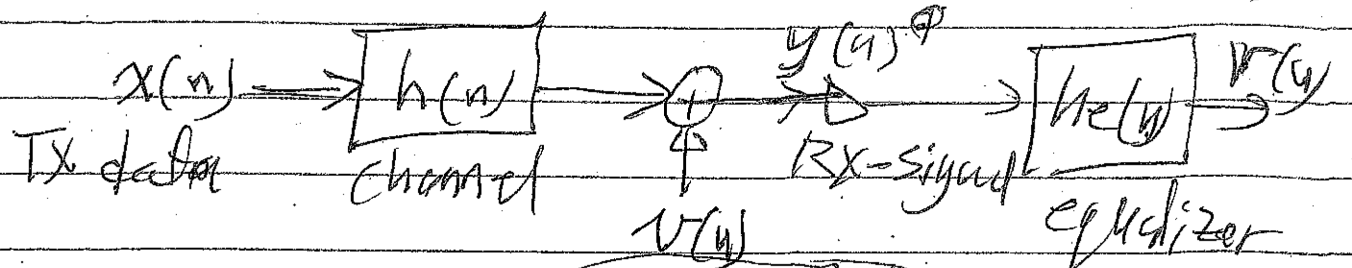
Zero-forcing (infinite length) equalizer



AWGN $\Rightarrow \sigma_v^2 = \frac{N_0}{2}$ zero mean

$y(n) = x(n) * h(n) + v(n)$

$Y(z) = X(z)H(z) + V(z)$

After the equalizer

$r(n) = h_e(n) * y(n) \Longrightarrow R(z) = Y(z)H_e(z)$

$R(z) = X(z)H(z)H_e(z) + V(z)H_e(z)$

For ZF equalizer $\Longrightarrow H_e(z) = 1/H(z)$

$\therefore R(z) = X(z) + \underbrace{V(z)H_e(z)}_{} = X(z) + W(z)$

$r(n) = x(n) + w(n)$

where $w(n) = v(n) * h_e(n)$

$\therefore w(n) = \sum_{k=0}^{\infty} h_e(k) v(n-k)$

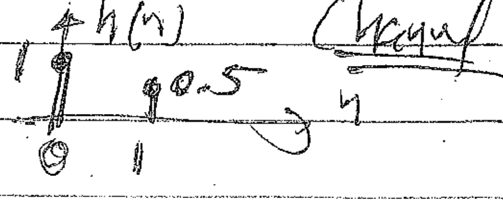
$\therefore \sigma_w^2 = \sigma_v^2 \sum_{k=0}^{\infty} |h_e(k)|^2$

ex) As in ex: 4.5 BPSK over the channel

$y(n) = x(n) + 0.5\,x(n-1) + v(n)$

$Y(z) = (1 + \frac{1}{2} z^{-1}) X(z) + V(z)$

$H_e(z) = 1/(1 + 0.5 z^{-1})$



$$H_e(z) = \frac{1}{1+0.5z^{-1}} = \frac{1}{1-(-0.5)z^{-1}}$$

After equalization $R(z) = X(z) + W(z)$

$r(n) = x(n) + w(n)$

where

$$\sigma_w^2 = \sigma_v^2 \sum_{k=0}^{\infty} |h_e(k)|^2$$

$$h_e(n) = (-0.5)^n u(n) = [1 \ -\tfrac{1}{2} \ \tfrac{1}{4} \ -\tfrac{1}{8} \ \dots$$

$$|h_e(k)|^2 = (0.25)^n u(n)$$

$$\sigma_w^2 = \sigma_v^2 \left(1 + \frac{1}{4} + \left(\frac{1}{4}\right)^2 + \left(\frac{1}{4}\right)^3 + \left(\frac{1}{4}\right)^4 \dots\right)$$

$$= \sigma_v^2 \frac{1}{1-\frac{1}{4}} = \frac{4}{3}\sigma_v^2$$

σ_v^2 BPSK det. $P_e = Q\sqrt{\frac{2E_b}{N_0}} = Q\left(\sqrt{\frac{E_b}{\sigma^2}}\right)$

$\frac{N_0}{2}$ $\sigma_w^2 = \frac{4}{3}\frac{N_0}{2} = \frac{2}{3}N_0$

$$P_{e,BPSK} = Q\left(\sqrt{\frac{E_b}{\sigma_w^2}}\right) = Q\left(\sqrt{\frac{E_b}{\frac{2}{3}N_0}}\right)$$

$$= Q\left(\sqrt{\frac{3E_b}{2N_0}}\right)$$

$r(n) = x(n) + w(n)$

$= \pm\sqrt{E_b} + w(n)$

$\sigma_w^2 = \frac{2}{3}N_0$

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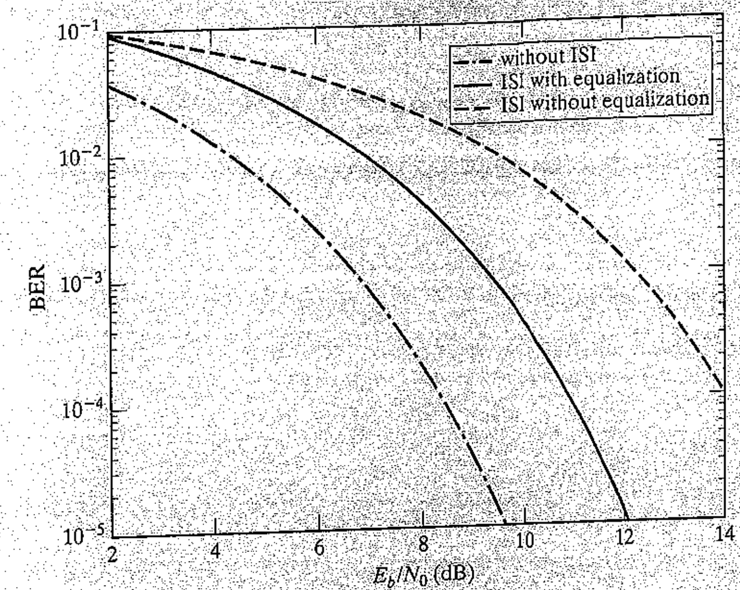

Figure 4.15 Comparison of the transmission accuracy with and without equalization.

the transmission accuracy. Also, because the equalizer increases the noise variance (power), the equalized system does not have a transmission accuracy as good as the system without ISI.

Example 4.7 Zero-Forcing Linear Equalizer with Finite Taps

The impulse response of a dispersive effective channel is

$$c(t) = \exp\left(-\frac{|t|}{3T}\right), \ -\infty < t < \infty,$$

where T is the transmitted symbol interval. Design a 3-tap zero-forcing linear equalizer for the channel.

Solution The discrete-time representation of the effective channel impulse response is $\{c_n\}$, where

$$c_n = c(t)|_{t=nT} = \exp\left(-\frac{|n|T}{3T}\right) = \exp\left(-\frac{|n|}{3}\right).$$

Let $\mathbf{b} = (b_{-1}, b_0, b_1)^T$ denote the tap coefficient vector of the 3-tap linear equalizer. The discrete-time representation of the equalized system impulse response is

$$h_n = c_n \star b_n = \sum_{k=-1}^{1} b_k c_{n-k}.$$

16.2 *Using the channel from Example 16.1, compute the noise enhancement for the DFE and ZF DFE.*

Example 16.1, remember that $\Xi(e^{j\omega T}) = 0.48\cos 2\omega T_S - 1.4\cos\omega T_S + 1.01$. Inserting Eq. (16.32), we obtain:

$$\xi(DFE - MMSE) = N_0 \exp\left(\frac{T_S}{2\pi}\int_{-\pi/T_S}^{\pi/T_S} \ln\left[\frac{1}{\Xi(e^{j\omega T}) + N_0}\right] d\omega\right) \tag{16.34}$$

$$= N_0 \exp\left(\frac{1}{2\pi}\int_{-\pi}^{\pi} \ln\left[\frac{1}{0.48\cos 2\omega - 1.4\cos\omega + 1.31}\right] d\omega\right) \tag{16.35}$$

$$\approx 0.33 \tag{16.36}$$

the output SNR is 2. Thus, the SNR deteriorates by 2 dB compared with the Additive Gaussian Noise (AWGN) case. For the ZF DFE, the noise variance at the output is so that the SNR deteriorates by 4.4 dB.

Maximum-likelihood sequence estimation – Viterbi detector

lizer structures considered up to now influence the decision about which *symbol* has nsmitted. For MLSE, on the other hand, we try to determine the *sequence of symbols* most likely been transmitted. This situation shows strong similarities to the decoding of ional codes. As a matter of fact, transmission through a delay-dispersive channel can be as convolutional encoding with a code rate $R_c = 1/1$. MLSE estimators give the best nce of all equalizers.

ber that the output signal of the time-discrete channel can be written as:

$$u_i = \sum_{n=0}^{L_c} f_n c_{i-n} + n_i \tag{16.37}$$

is Gaussian white noise with variance σ_n^2. For a sequence of N received values, the joint ity density function of the vector of received signals $\mathbf{u}$ (conditioned on the data vector $\mathbf{c}$ pulse response vector $\mathbf{f}$) is:[4]

$$pdf(\mathbf{u}|\mathbf{c};\mathbf{f}) = \frac{1}{\left(2\pi\sigma_n^2\right)^{N/2}} \exp\left(-\frac{1}{2\sigma_n^2}\sum_{i=1}^{N}\left|u_i - \sum_{n=0}^{L_c} f_n c_{i-n}\right|^2\right) \tag{16.38}$$

LSE of $\mathbf{c}$ (for a given $\mathbf{f}$) are the values of the vectors that maximize the joint probability function (pdf) $pdf(\mathbf{u}|\mathbf{c},\mathbf{f})$. As the variables only occur in the exponent, it is sufficient to ze:

$$\sum_{i=1}^{N}\left|u_i - \sum_{n=0}^{L_c} f_n c_{i-n}\right|^2 \tag{16.39}$$

and in the following, we assume that all transmit symbols are equally likely, such that MLSE and m-a-posteriori estimation are identical.

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As for convolutional decoding, various algorithms exist for determination of [illegible] sequence. The RX first generates all possible sequences that can result from convol[illegible] transmit sequences with the channel impulse response. We then try to find the seq[illegible] the smallest distance (best metric) from the received signal. The most straightfor[illegible] most computationally intensive) method is the *exhaustive search*. In practice, [illegible] algorithm is used instead.

MLSE as described above is only optimum if the additive noise at MLSE [illegible] Therefore, the sample values used at the detector have to be the output of a [illegible] filter. This filter has to be adapted to the current channel state, and each chann[illegible] requires spectral factorization. Due to these difficulties, the total input filter (matc[illegible] noise-whitening filter) is often replaced by a simple brickwall filter whose [illegible] approximately the inverse symbol duration. Note, however, that in this case [illegible] symbol no longer provides sufficient statistics.

Example 16.3 *Viterbi equalization.*

This example shows the working of the Viterbi detection of a symbol stre[illegible] through a channel with a discrete time impulse response:

$$\mathbf{f} = \begin{pmatrix} 1 \\ -0.5 \\ 0.3 \end{pmatrix}$$

The channel can be viewed as a tapped delay line (shift register) with weights [illegible] see the top part of Fig. 16.8 (the left top part shows the tapped delay line mo[illegible] part shows the "cell" model analogous to the convolutional codes discussed in [illegible] For ease of exposition, we chose a channel with a real impulse response, and [illegible]

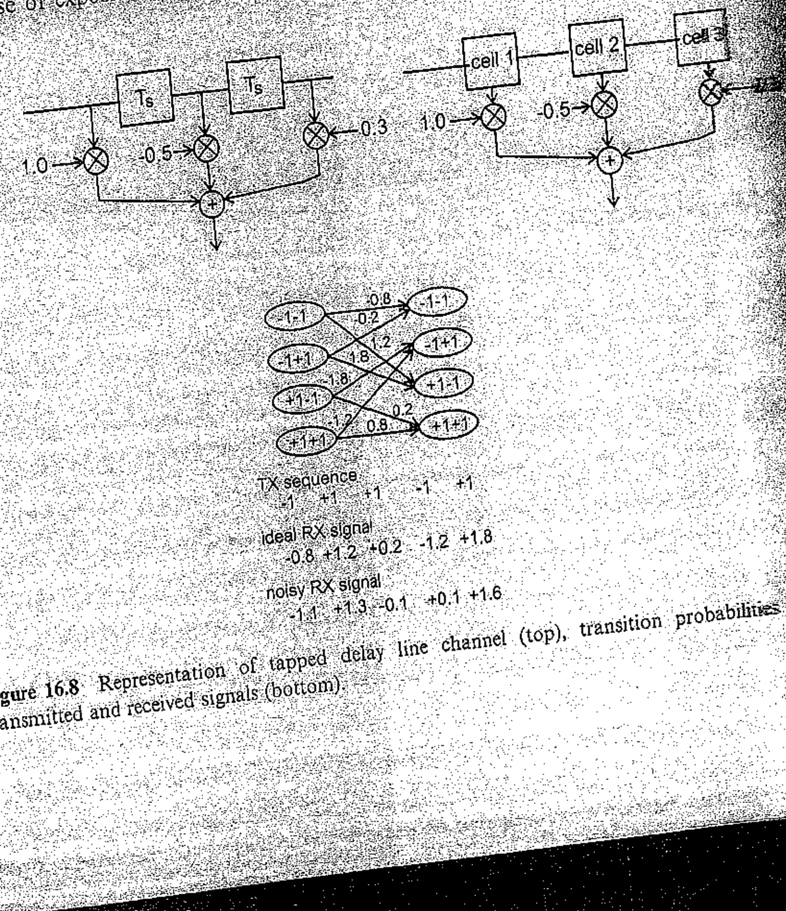

Figure 16.8 Representation of tapped delay line channel (top), transition probabilities [illegible] transmitted and received signals (bottom).

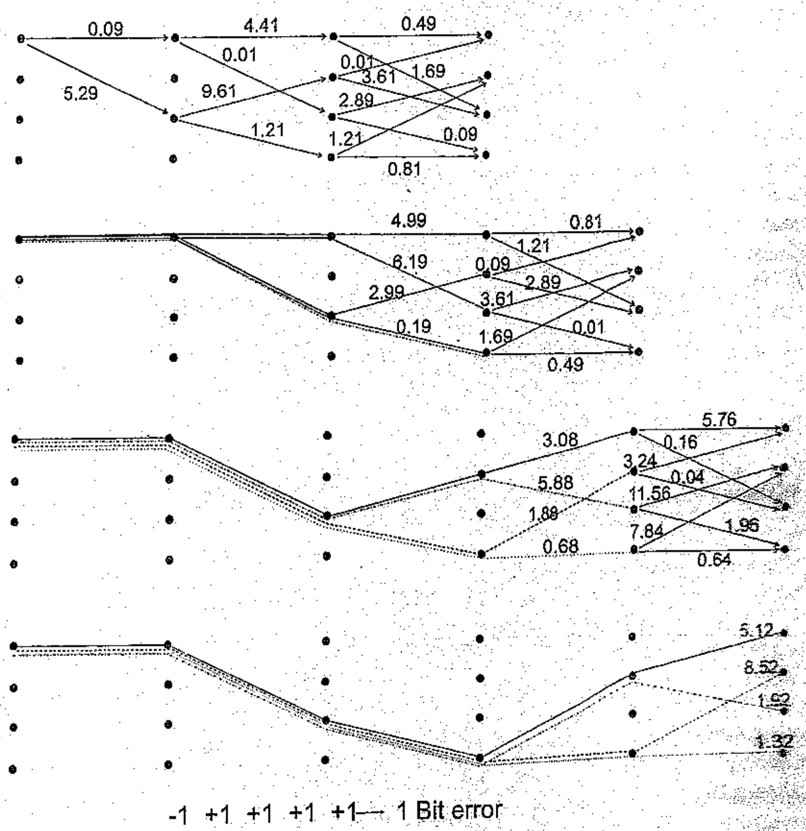

Figure 16.9 Viterbi algorithm for detection of the transmit sequence in a delay-dispersive channel.

Shift Keying (BPSK) as the modulation format. The lower part of Fig. 16.8 shows possible transitions in the trellis diagram. We have to consider four states in the trellis, as $L=2$ samples, and the number of possible states in a cell of the equivalent shift register is equal to the size of the modulation alphabet $M=2$. We assume furthermore that we know the starting state of the trellis – i.e., −1 −1 (e.g., because known bits have been transmitted before the start of our decoding). The bottom part of Fig. 16.9 shows the 'unfolding' of the trellis diagram. The numbers next to the transitions are metrics of the considered sequence.

16.5 Comparison of equalizer structures

Figure 16.10 shows a taxonomy of equalizer structures. When selecting an equalizer for a practical system, we have to consider the following criteria:

- *Minimization of the BER*: here MLSE is superior to all other structures. DFEs, though worse than MLSE estimators, are better than linear equalizers. The quantitative difference between the structures depends on the channel impulse response.