CHAPTER I

$\boxed{1\text{-}1}$ $c = \lambda f$

$\boxed{1\text{-}2}$ Given $0\ \text{dBm} = 10^{-3}\ \text{W}$

$$10^{-3} \times 10^{4} = 10\ \text{W}$$
$$= 0\ \text{dBm} + 40\ \text{dB}$$
$$= 40\ \text{dBm} = 10\ \text{W}$$

1.3 Paging systems are designed to provide ultra-reliable coverage, even inside buildings. Since buildings can attenuate radio signals by 20 or 30 dB, to maximize the signal-to-noise ratio at each paging receiver, we need to reduce the noise level. This can be achieved by reducing the RF bandwidth to which the noise level is proportional. The small RF bandwidth thus result in low data rate.

In a paging system, the signal level in a receiver degrades when the distance between the receiver and the base station becomes large. If the coverage of a paging system is defined by the coverage area at which the signal-to-noise ratio is above a certain threshold, for a lower data rate, the noise level in the receiver will be smaller, thus for a fixed threshold, the coverge will be larger.

1.4 Since the coverage area of mobile cellular phone is generally larger than that of portable cellular phones, the power supply requirement of the mobile cellular phone is higher than that of the portable cellular phone. The coverage range of packet pagers is larger than that of the cordless phones. however, pagers are receive-only, whereas cordless phones use both a transmitter and receiver. The power supply requirement of packet pagers is much smaller than that of cordless phones. A pager typically has a battery life of a week or more, whereas a cordless phone may have a battery life of a few days.

1.4 Cont'd

In a mobile radio system, large coverage range requires high handset power and thus increases battery drain and reduces battery life. Small cell size, on the other hand, needs low handset power and therefore decreases battery drain and increases battery life.

1-5 In a simulcasting paging system, the paging receiver receives the dominant signal arriving from the transmitter closest to the paging receiver. If the predetection SNR at the receiver is greater than the FM threshold, the weaker signals are totally eliminated and the receiver is said to be "captured" by the desired signal. The capture effect in FM improves the output SNR.

In cellular radio systems, the receiver will also be affected by the co-channel interference coming from neighboring cells. Unless the signal-to-interference ratio at the receiver is greater than the threshold, the capture effect cannot help the cellular radio systems. See also section 6.1.

1-6 A walkie-talkie is a half duplex system. It allows two-way communication by using the same radio channel for both transmission and reception. At any given time, the user can only either transmit or receive information. The coverage range of walkie-talkie is high, and its required

1-6 Cont'd

infrastructure, complexity, hardware cost are all low.

A cordless telephone, on the other hand, is a full duplex system. It allows simultaneous two-way communication. Transmission and reception is on two different channels (FDD) although new cordless system are using TDD. The coverage range, required infrastructure, hardware cost of a cordless phone system are low and the complexity is moderate. User expectations are greater for a cordless telephone.

$\boxed{1\text{-}7}$ Pager - only receives, doesn't transmit

$\boxed{1\text{-}8}$ Cell phone - transmits over longer distances than cordless phone

$\boxed{1\text{-}9}$ If the user has <u>one 3-minute call every day</u>

$$\text{the battery life} = \frac{60\times1000 \text{ (mA-minute)}}{(60\times24-3)\times35 + 3\times250 \text{ (mA-minute)}}$$

$$\doteq 1.175 \text{ days} \doteq \underline{\underline{28.2 \text{ hours}}}$$

If the user has <u>one 3-minute call every 6 hours</u>

$$\text{the battery life} = \frac{60\times1000}{(60\times6-3)\times35 + 3\times250} \times 6 \doteq \underline{\underline{27.18 \text{ hours}}}$$

If the user has one <u>3-minute call every hour</u>.

$$\text{the battery life} = \frac{60\times1000}{(60-3)\times35 + 3\times250} \doteq \underline{\underline{21.86 \text{ hours}}}$$

$$\text{The maximum talk time} = \frac{60\times1000}{250} = 240 \text{ minutes} = \underline{\underline{4 \text{ hours}}}$$

1.10

1. Battery = 1000 mA/hr

Call = 250 mA
Receiver = 35 mA
Call Duration = 3 min = 0.05 hr

a) If the user makes one 3-minute call every day...
Average battery life =

during call: $r_c = 250 \text{ mA} \cdot (0.05 \text{ hr}) = 12.5 \text{ mA·hr}$

during rec: $r_w = 35 \text{ mA} \cdot \dfrac{(1440 - 3\,\text{min})}{60} = 838.25 \text{ mA·hr}$

total for 1 day = 850.75 mA·hr

Average life = $\dfrac{1000\,\text{mA} \cdot \text{hr}}{850.75\,\text{mA} \cdot \text{hr}} \cdot 24\,\text{hr}$ = **28.21 hours**

b) If the user makes 1 call every 6 hours...

Average battery life =

during call: $r_c = 12.5 \text{ mA·hr}$

during rec: $r_w = \dfrac{357}{60} \cdot 35 = 208.75 \text{ mA·hr}$

avg. For 1 call/6 hrs = 220.75 mA·hr

Average life = $\dfrac{1000\,\text{ma} \cdot \text{hr}}{220.75\,\text{ma} \cdot \text{hr}} \cdot 6\text{hr}$ = **27.18 hr**

If the user makes 1 call every hour...
Average battery life =

during call: $r_c = 12.5 \text{ mA·hr}$

during rec: $r_w = \dfrac{57}{60} \cdot 35 = 33.25 \text{ mA·hr}$

avg. For 1 call/hr = 45.75 mA·hr

1.10 Cont'd

$$\text{Average life} = \frac{\frac{1000\,ma \cdot hr}{45.75\,ma \cdot hr} \cdot 1hr}{} = \mathbf{21.86\ hr}$$

$$\text{Maximum talk time} = \frac{1000\,ma \cdot hr}{250\,mA} = \mathbf{4\ hours}$$

3 battery states

idle = 1mA
wake-up = 5 mA
transceiver = mA
Average battery life =

In order to verify the influence of the duration of these periods (idle, wake-up, and transceiver), let us write the expression for 1 hour:

1 hour:

1 Tc-hour call
1 Tw-hour wake-up mode
1 T-hour idle mode

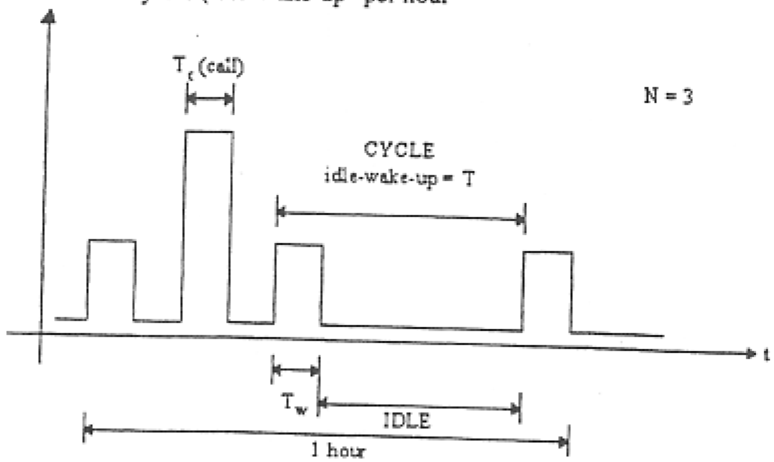
So we can write:

1 A·hr =

$$u\{T_c \times 0.25A + N_x t_w \times 0.035A + [N(T - T_w) - T_c]0.001\}$$

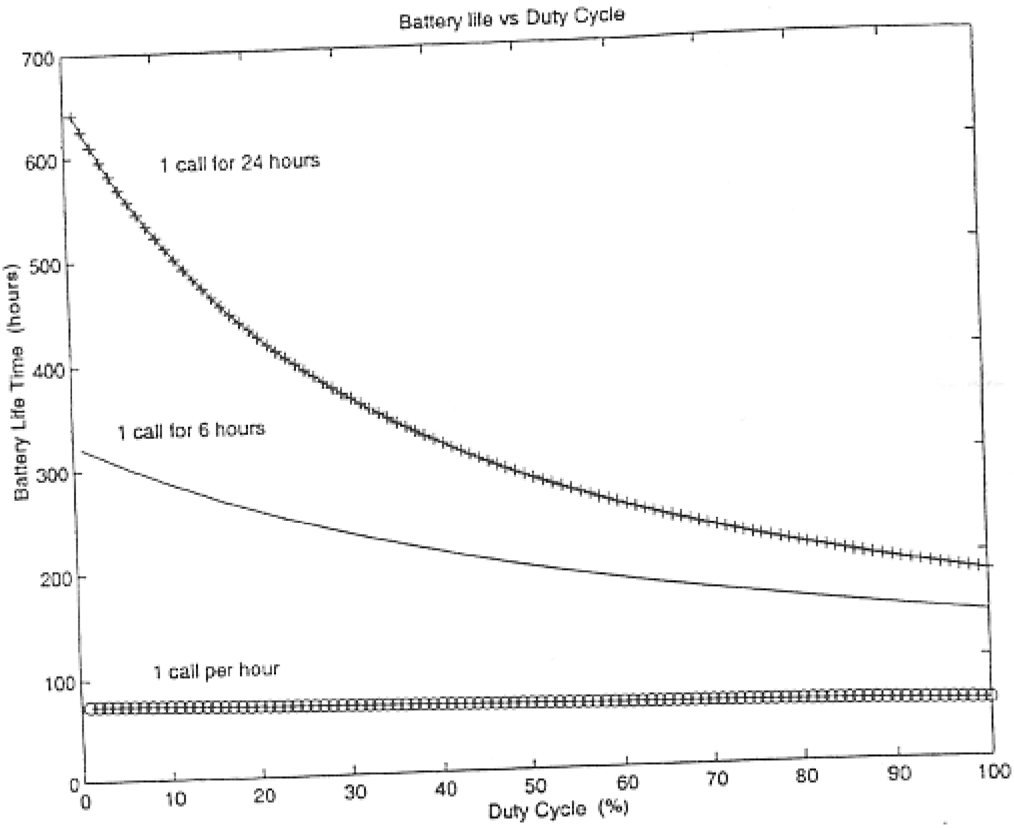
where u = # of hours of battery life

N = # of cycles (idle-wake-up" per hour



Observe that we consider that the call occurs during idle mode.

1.10 Cont'd



Battery life vs Duty Cycle
700
600
500
400
300
200
100
0
Battery Life Time (hours)
1 call for 24 hours
1 call for 6 hours
1 call per hour
0
10
20
30
40
50
60
70
80
90
100
Duty Cycle (%)

1.10 Cont'd

```
% Text 1.10

% Battery life for one 3 minute call every hour

for i=1:100
    x(i)=1/(((i/100)*(57/60)*0.005)+(((100-i)/100)*(57/60)*0.001)+(0.25*3/60)
);
end
plot(x,'o');

hold on;

% Battery life for one 3 minute call 6 hours

for j=1:100
    y(j)=(1/(((j/100)*(357/60)*0.005)+(((100-j)/100)*(357/60)*0.001)+(0.25*3/
60)))*6;
end;
plot(y);

hold on;

% Battery life for one 3 minute call every day

for k=1:100
    z(k)=(1/(((k/100)*(1437/60)*0.005)+(((100-k)/100)*(1437/60)*0.001)+(0.25*
3/60)))*24;
end;
plot(z,'+');

xlabel('Duty Cycle');
ylabel('Battery Life Time');
title('Battery life vs Duty Cycle');
```

1.10 Cont'd

Now, defining he duty cycle by,

$$D = \frac{T_w}{T} \rightarrow T_w = DT$$

Also, since N is the # of cycles during 1 hour, 1=NT

So, we can rewrite the expression for the battery life as:

$$1 = u\{T_c\, 0.25A + D0.035A + [1 - D - T_c]0.001\}$$

or

$$u = \{T_c\, 0.25A + D0.035A + [1 - D - T_c]0.001\}^{-1}$$

hours

In the figure on page 7a, we can see the curve for battery life x duty cycle for one 3 min. call/day, four 3 min. calls/day, and 24 3 min. calls/day.

We observe that since the power required by the phone during a call is much higher (250 mA) than during idle and wake-up states, the battery life is reduced dramatically.

1-11 For 3-minute call / day

$$\text{battery life} = \frac{60 \times 1000 \text{ (mA-minute)}}{(60 \times 24 - 3) \times 5 + 3 \times 80} \doteq 8.08 \text{ days} = \underline{\underline{193.94 \text{ hours}}}$$

For 3 minute-call / 6 hours.

$$\text{battery life} = \frac{60 \times 1000}{(60 \times 6 - 3) \times 5 + 3 \times 80} \times 6 \doteq \underline{\underline{177.78 \text{ hours}}}$$

For 3 minute-call / hour.

$$\text{battery life} = \frac{60 \times 1000}{(60 - 3) \times 5 + 3 \times 80} \doteq \underline{\underline{114.29 \text{ hours}}}$$

$$\text{The maximum talk time} = \frac{60 \times 1000}{80} = 750 \text{ minutes} = \underline{\underline{12.5 \text{ hours}}}$$

[1.12] Since the coverage range of the CT-2 system is lower than that of the cellular radio system, to obtain the same signal-to-noise ratio in the coverage area, a CT-2 handset requires less transmitted power than a cellular telephone, and thus a smaller battery drain.

[1.13] FM has several advantages over AM. The most important advantage is FM's superior noise suppression characteristics. With conventional AM, the modulating signal is impressed onto the carrier in the form of amplitude variations. However, noise introduced into the system also produces changes in the amplitude of the envelope. Therefore, the noise cannot be removed from the composite waveform without also removing a portion of the information signal. With FM, the information is impressed onto the carrier in the form of frequency variations. Therefore, with FM receivers,

1.13 Cont'd

amplitude variations caused by noise can be removed from the composite waveform simply by limiting the peaks of the envelope prior to detection. With FM, an improvement in the SNR is achieved during the demodulation process; thus, system performance in the presence of noise can be improved by limiting. See also section 6.1.

Another advantage that FM has over AM is the fact that low-level modulation can be used with subsequent highly efficient class C power amplifiers. Since the FM waveform does not vary in amplitude, the information is not lost by class C amplification as it is for AM, which requires class A amplifiers which are less power efficient.

$\boxed{1.14}$ (a) Factors led to the development of the GSM system for Europe:

① To solve the fragmentation problem of the first cellular Systems in Europe, enable the customer to use a single subscribe unit throughout Europe (Pan-European roaming).

② To provide extra ISDN-type facilities such as calling line identification, a high bit rate data bearer and Smart Card enabling and billing.

③ To provide a lower power budget and a better talk-time-to-weight-to-size ratio than analog.

1.14 Cont'd

④ To provide lower cost-per-subscriber with a unified European technology.

⑤ To reduce the effect of adjacent channel and co-channel interference, improve power control and hand-off and reduce the effect of multipath signal phase cancellation.

(b) Factors led to the development of the U.S. digital cellular system

① To offer large improvement in capacity in large cities.

② To reduce the cost, weight and size of the mobiles.

③ To improve the up-link and down-link quality of service to and from the mobile — improved access (less blocking) and fewer mid-conversation dropped calls.

In Europe, a new radio band was established (935-960 MHz down-link and 890-915 MHz uplink) for the GSM system. In North America, there was no new allocated band for the U.S. digital cellular system. The digital cellular system has to share the same allocated band with the analog System (AMPS). Therefore the digital and the analog systems have to be coexistent. For this reason, a dual-mode mobile unit was decided on; i.e., the unit can work on both analog and digital systems in the U.S..

$\boxed{1.15}$ Since the channel bandwidths for USDC, GSM and IS-95 are 30KHZ, 200KHZ and 1.25 MHz, respectively, we have, 15 Cont'd

$$\frac{N_{GSM}}{N_{USDC}} = \frac{200}{30} \doteq 6.67 \doteq 8.24\,dB, \quad \frac{N_{IS95}}{N_{USDC}} = \frac{1.25\times10^{3}}{30} \doteq 41.67 \doteq 16.2\,dB$$

Where N_{GSM}, N_{USDC}, and N_{IS95} are the noise level for the above systems, respectively. For the same transmitted power and distance, USDC will provide the best SNR at a receiver. The improvement is 8.24dB compared to GSM and 16.2dB compared to IS-95 system.

6. In the space-based cellular radio system, the satellite operates as a base station of the conventional cellular radio system. Thus the space-based cellular radio system can provide large coverage area, for instance, over mountains or rural areas where it is expensive to service the user community with a conventional system. But the space-based system will experience more time delay, require more transmit power and larger size of antenna due to the long link between the satellite and the mobile. The conventional cellular radio system could support a larger number of users for a given frequency allocation due to its smaller cell size. This can reduce the cost of service for each subscriber. However, space-based system offer tremendous promise for paging, data collection, and emergency communication as well as for global roaming.

1.17 Examples of paging standards culled from homework turned in by several EE6644 students, Spring'97.

Paging Standard	Multiple Access Techniques	Continent of Operation	Frequency Band	Modulation	Chanel Bandwidth	References
ReFLEX (Motorola)	TDMA	North America	pager receives on 940-941 MHz Pager transmits on 901-902 MHz	4-FSK	25 kHz	http://www.motorola.com http://www.pcia.com/flex.htm
pACT (Personal Air Communications Technology)	TDMA	North America	N-PCS Band 901-902 MHz 930-931 MHz 940-941 MHz	GMSK	versions will be available in 50/50 kHz and 50/12.5 kHz	http://www.pact-forum.org http://www.pcsi.com/html/tech1.sh http://www.ericsson.se/US/npcs/

Examples of cellular standards culled from homework turned in by several EE6644 students, Spring'97.

Cellular Standard	Multiple Access Techniques	Continent of Operation	Frequency Band	Modulation	Chanel Bandwidth	References
IS-136	TDMA	North America	800 MHz (cellular) 1900 MHz (PCS)	pi/4 DQPSK	30 kHz	http://www.isotel.com/is136.htm

Examples of PCS standards culled from homework turned in by several EE6644 students, Spring'97.

Cellular Standard	Multiple Access Techniques	Continent of Operation	Frequency Band	Modulation	Chanel Bandwidth	References
IS-665	W-CDMA	North America	1.85-1.99 GHz	requires coherent detection	5 MHz (basic) 10 & 15 MHz (future options)	Fukasawa *et al*, "Wideband CDMA System for Communications," *IEEE Communications Magaz.* October 1996, pp.116-123.
TETRA (Trans European Trunked Radio)	TDMA	Europe	400 MHz		25 kHz (6.25 kHz per voice channel)	http://www.cicd.com/tetra/tetr http://www.cicd.com/tetra/tetraf http://www.smithsys.co.uk/smithsys/techp http://www.smithsys.co.uk/smithsys/techp

1-18 Group 1: High power, wide area systems

Group 2: Low power, local area systems

Group 3: Low speed, wide area systems.

Group 4: High speed, local area systems.

Wireless Communication Systems	Group 1	Group 2	Group 3	Group 4
TV Remote Control		X		
Garage Door Opener		X		
Paging System	X		X	
Cordless Phone		X		
Cellular Phone	X		X	
AMPS	X		X	
IS-95	X		X	
GSM	X		X	X
IMT-2000	X		X	X
PACS		X		

1.19 It is the task of these organizations to develop regional and international standards. The competitive advantage in using different wireless standards in different parts of the world is that we can know the advantage and disadvantage of each standard by comparing them and then the improve-

1.19 Cont'd

ment of these standards is possible. Also, regional standards insure certain companies will have an advantage in manufacturing equipment. But there is a problem when different standards and different frequencies are used in different parts of the world. A subscriber will not work anywhere in the world. The problem is very complicated and is hard to solve unless a universal standard is adopted.

$\boxed{1\text{-}20}$ The overall objectives of IMT-2000 are to provide all services generally available through the fixed network (e.g., voice, fax and data) to mobile systems. It is intended to provide these services over a wide range of user densities and geographic coverage areas. Calls within the mobile system are routed to and from the intelligent network, either fixed or mobile via terrestrial or satellite links using at least four kinds of radio interface (R_1, R_2, R_3, R_4). The R_1 interface is used by mobile stations. The R_2 interface is used by indoor and outdoor personal stations (handsets). The R_3 interface is used by mobile stations communicating through a satellite, and

1.20 Cont'd

the R_4 interface is used by pagers. All mobile calls can be connected either directly to PSTN (public service telephone network) or via a mobile switch. The mobile systems can be either a narrowband or wideband.

CHAPTER 2

Solutions will vary over time and location.

CHAPTER 3

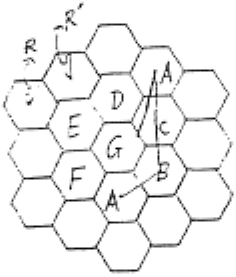
3.1 Generally, for $N = i^2 + i\cdot j + j^2$, we can do the following to find the nearest co-channel neighbors of a particular cell:

(1) move i cells along any chain of hexagons and then

(2) turn 60 degree counter-clockwise and move j cells

From the following figure, using the cosine law, we have

$$D^2 = [i\cdot(2R')]^2 + [j\cdot(2R')]^2 - 2i\cdot(2R')\cdot j\cdot(2R')\cdot\cos 120^\circ$$



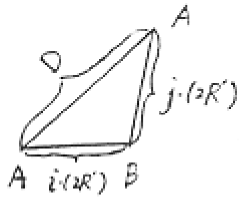
where $R' = \frac{\sqrt{3}}{2}R$, therefore

$$D = \sqrt{3i^2 R^2 + 3j^2\cdot R^2 + i\cdot j\cdot 3R^2}$$

$$= \sqrt{3(i^2 + i\cdot j + j^2)}\cdot R$$

$$= \sqrt{3N}\cdot R$$

Hence, $Q = \frac{D}{R} = \sqrt{3N}$



3.2

Example 1:

In general, the average power of $v_1(t)$ is $P_1 = \lim_{T\to\infty}\int_{-T/2}^{T/2}|v_1(t)|^2 dt = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} v_1(t)v_1{}^*(t)dt$

or, $P_1 = \langle |v_1(t)|^2\rangle = \|v_1(t)\|^2 = \underbrace{\underbrace{\langle v_1(t), v_1(t)\rangle}_{\text{scalar product}}}_{\text{mean value of the scalar product}}$

if the above is periodic over the interval T_0, then

$$P_1 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2}|v_1(t)|^2 dt = \frac{1}{T_0}\int_{T_0/2}^{T_0/2} v_1(t)v_1{}^*(t)dt$$

now,

$$P_{1+2} = \|v_1(t)+v_2(t)\|^2 = \|v_1(t)\|^2 + \langle v_1(t), v_2(t)\rangle + \langle v_2(t), v_1(t)\rangle + \|v_2(t)\|^2$$

and

$$\langle v_2(t), v_1(t)\rangle = \langle v_1(t), v_2(t)\rangle^*$$

and

$$\langle v_1(t), v_2(t)\rangle = \lim_{T\to\infty}\frac{1}{T}\int_{-T/2}^{T/2} v_1(t)v_2{}^*(t)dt$$

Similarly, if we can assume $v_1(t)$ and $v_2(t)$ are both periodic with period T_0, then

$$\langle v_1(t), v_2(t)\rangle = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} v_1(t)v_2{}^*(t)dt$$

If the signals are uncorrelated, then $v_1(t) + v_2(t)$ has a correlation of

$$R_{total} = R_1(\tau) + R_{1,2}(\tau) + R_{2,1}(\tau) + R_2(\tau)$$

If uncorrelated for all τ, then

$$R_{1,2}(\tau) = E[v_1(t)v_2(t+\tau)] = E[v_1(t)]E[v_2(t+\tau)]$$

If the signals are uncorrelated then $P_{total} = P_1 + P_2$ and at least one signal must have a zero mean. There are no other special conditions, since it was stated in the problem that the signals are statistically independent. A gaussian process is also assumed.

This homework problem submitted by Mark Glasgow, Northern Virginia Site, Commonwealth Graduate Engineering Program, Spring 1997.

3-2 Cont'd

Example 2

Given two independent voltages $v_1(t)$ and $V_2(t)$ that are added together, determine the normalized average power

$$P_{AV} = \overline{(v_1(t) + v_2(t))^2} = E[v_1^2] + E[v_2^2] + 2E[v_1]E[v_2]$$

If $E[v_1] = 0$ or $E[v_2] = 0$ or $E[v_1] = E[v_2] = 0$

then the resulting power is

$$P_{AV} = \overline{v_1^2} + \overline{v_2^2}$$

which is equal to the sum of the individual powers.

If $v_1(t)$ and $V_2(t)$ are uncorrelated (but not necessarily independent) the normalized average power of the voltage sum is

$$P_{AV} = \overline{(v_1(t) + v_2(t))^2} = E[v_1^2] + E[v_2^2] + 2E[v_1 v_2]$$

From the correlation property, if the two signals are uncorrelated

$$E[v_1 v_2] = E[v_1]E[v_2]$$

so two signals that are uncorrelated will have a combined average power equal to the sum of the individual powers if either signal is mean zero.

It should be noted that statistically independentr.v.s are uncorrelated, but uncorrelated r.v.s may or may not be statistically independent.

Also, if the signals are orthogonal,

$$E[v_1 v_2] = 0$$

This homework problem submitted by John B. Call, Commonwealth Graduate Engineering Program, Spring 1997.

$\boxed{3\text{-}3}$ Since $S = kN$, where N is the cluster size, we have

$$N = \frac{S}{k}.$$

By the definition of frequency reuse factor, we have

$$\text{frequency reuse factor} = \frac{1}{N} = \frac{k}{S}.$$

3-4 (a) 20 MHz / [25 kHz × 2] = 400 channels

(b) 400 / 4 = 100

3-5 (a) Let i_0 be the number of co-channel interfering cells, for omni-derectional antennas, $i_0 = 6$. Assume $n = 4$,

we have $\frac{S}{I} = \frac{(\sqrt{3N})^n}{i_0} > 15\,dB = 31.623 \Rightarrow N > 4.59$

$\Rightarrow \underline{\underline{N = 7}}$

(b) For 120° sectoring, $i_0 = 2$.

$\frac{S}{I} = \frac{(\sqrt{3N})^n}{i_0} > 31.623 \Rightarrow N > 2.65 \Rightarrow \underline{\underline{N = 3}}$.

(c) For 60° sectoring, $i_0 = 1$.

$\frac{S}{I} = \frac{(\sqrt{3N})^n}{i_0} > 31.623 \Rightarrow N > 1.87 \Rightarrow \underline{\underline{N = 3}}$

From (a), (b) and (c) we can see that using 120° sectoring can increase the capacity by a factor of 7/3, or 2.333. Although using 60° sectoring can also increase the capacity by the same factor, it will decrease the trunking efficiency. therefore we choose the 120° sectoring.

3.6 solution not available

3-7

a) Calls are not lost due to weak signal condition during handoff if:

$$\frac{\text{distance traveled during handoff}}{\text{mobile speed}} = \frac{d_{min} - d_{HO}}{v} \geq 4.5 \text{ seconds} \quad (2)$$

* $d_{min} \Rightarrow$ received power at BS_1 reaches $P_{r,min}$

$$P_{r,min} = -29\log_{10}(d_{min}) \Rightarrow d_{min} = 10^{-P_{r,min}/29} = 1083 \text{ m} \quad (3)$$

* $d_{HO} \Rightarrow$ received power at BS_1 reaches $P_{r,HO}$

$$P_{r,HO} = -29\log_{10}(d_{HO}) \Rightarrow d_{HO} = 10^{-P_{HO}/29} \quad (4)$$

Using (2),

$$\frac{1083 - 10^{-P_{r,HO}/29}}{22.22(m/s)} \geq 4.5 \text{ seconds} \quad (5)$$

$$P_{HO} \geq -86.8 \text{ dBm} \quad (6)$$

Thus,

$$\Delta = P_{r,HO} - P_{r,min} \Rightarrow \Delta \geq 1.2\text{dB}. \quad (7)$$

b) If we set Δ *too large*, several unnecessary handoffs will be requested and performed, increasing the signaling traffic between the base stations and mobile switching center (MSC). On the other hand, if Δ is *too small*, that is, $P_{r,HO}$ is only slightly greater than $P_{r,min}$, there will not be enough time to complete the handoff (especially for high speed mobiles), and calls may be lost due to weak signal condition.

3-8 For $n=3$

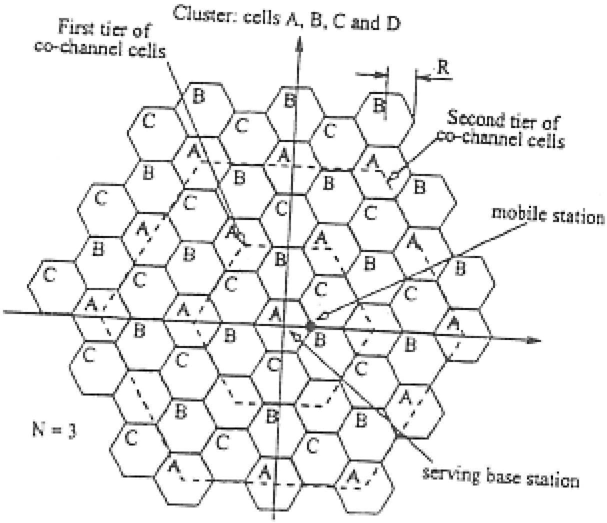
(a) $i_0 = 6$, $\frac{S}{I} = \frac{(\sqrt{3N})^n}{i_0} > 31.623 \Rightarrow N > 11 \Rightarrow \underline{\underline{N = 12}}$

(b) $i_0 = 2$, $\frac{S}{I} = \frac{(\sqrt{3N})^n}{i_0} > 31.623 \Rightarrow N > 5.29 \Rightarrow \underline{\underline{N = 7}}$

(c) $i_0 = 1$, $\frac{S}{I} = \frac{(\sqrt{3N})^n}{i_0} > 31.623 \Rightarrow N > 3.33 \Rightarrow \underline{\underline{N = 4}}$

From (a), (b) and (c), we can see that using 60° sectoring can increase the capacity by a factor of 12/3, or 4. For 120° sectoring, this factor is only 12/7, or 1.714. Therefore, we choose the 60° sectoring.

3-9



Cells A share the same channels

Figure 1: Cellular system using cluster size $N = 3$.

a) **Contribution of co-channel base stations in each tier to the total co-channel interference received at the mobile, and *SIR* at the mobile**

The contribution of co-channel base stations in the t-th tier to the total co-channel interference is the sum of the interference signals from those base stations, received at the mobile

$$I^{(t)} = \sum_{k=1}^{6t} P_{r,k}^{(t)}, \tag{2}$$

where $P_{r,k}^{(t)}$ is the received power at the mobile, from the k-th base station in the t-th tier, given by

$$P_{r,k}^{(t)} = P_t \left(\frac{1}{d_k^{(t)}} \right)^n . \tag{3}$$

$d_k^{(t)}$ is the distance between the mobile and the k-th base station in the t-th tier. In this problem, we assume that the mobile is located at the cell boundary (worst case situation), where it receives the weakest desired signal from its serving base station. Figure 1 shows the location of base stations and the mobile, for cluster size $N = 3$. Tables 5 through 7 show the location of the co-channel base stations in the first three tiers for cluster sizes $N = 1, 3, 4$ and 7.

The total interference at the mobile is, therefore,

$$I = \sum_{t=1}^{3} \sum_{k=1}^{6t} P_{r,k}^{(t)}. \tag{4}$$

Table 1 shows $I^{(t)}$ normalized with P_t/R^n, for each tier, and path loss exponents $n = 2$, 3 and 4 and cluster sizes $N = 1, 3, 4$ and 7.

3-9 Cont'd

Table 1: Interference $I^{(t)}$ for cluster size $N = 1, 3, 4$ and 7, and path loss exponents $n = 2, 3$ and 4.

cluster size	tier t	$I^{(t)}$ (normalized with P_t/R^n) $n=2$	$n=3$	$n=4$
$N=1$	1	2.7857	2.3580	2.1658
	2	1.2969	4.6609×10^{-1}	1.7725×10^{-1}
	3	8.3080×10^{-1}	1.8569×10^{-1}	4.2571×10^{-2}
$N=3$	1	7.5206×10^{-1}	2.9128×10^{-1}	1.1906×10^{-1}
	2	4.0218×10^{-1}	7.6130×10^{-2}	1.4731×10^{-2}
	3	2.6855×10^{-1}	3.3340×10^{-2}	4.1833×10^{-3}
$N=4$	1	5.4482×10^{-1}	1.7481×10^{-1}	5.8191×10^{-2}
	2	2.9909×10^{-1}	4.8511×10^{-2}	8.0107×10^{-3}
	3	2.0065×10^{-1}	2.1471×10^{-2}	2.3178×10^{-3}
$N=7$	1	3.0003×10^{-1}	6.9577×10^{-2}	1.6510×10^{-2}
	2	1.6906×10^{-1}	2.0444×10^{-2}	2.5031×10^{-3}
	3	1.1411×10^{-1}	9.1740×10^{-3}	7.4223×10^{-4}

Table 2: Contribution $C^{(t)}$ of co-channel base stations in the t-tier to the total co-channel interference: cluster sizes $N = 1, 3, 4$ and 7 and path loss exponents $n = 2$, 3 and 4.

cluster size	tier t	$C^{(t)}$ (%) $n=2$	$n=3$	$n=4$
$N=1$	1	56.70	78.34	90.79
	2	26.39	15.49	7.43
	3	16.91	6.17	1.78
$N=3$	1	52.86	72.68	86.29
	2	28.27	19.00	10.68
	3	18.87	8.32	3.03
$N=4$	1	52.16	71.41	84.93
	2	28.63	19.82	11.69
	3	19.21	8.77	3.38
$N=7$	1	51.45	70.14	83.57
	2	28.99	20.61	12.67
	3	19.57	9.25	3.76

We can also express the contribution of co-channel base stations in each tier to the total co-channel interference using

$$C^{(t)} = \frac{I^{(t)}}{I} \times 100\%. \tag{5}$$

Table 2 and Figure 2 show $C^{(t)}$ for each tier, and path loss exponents $n = 2$, 3 and 4 and cluster sizes $N = 1, 3, 4$ and 7.

3-9 Cont'd

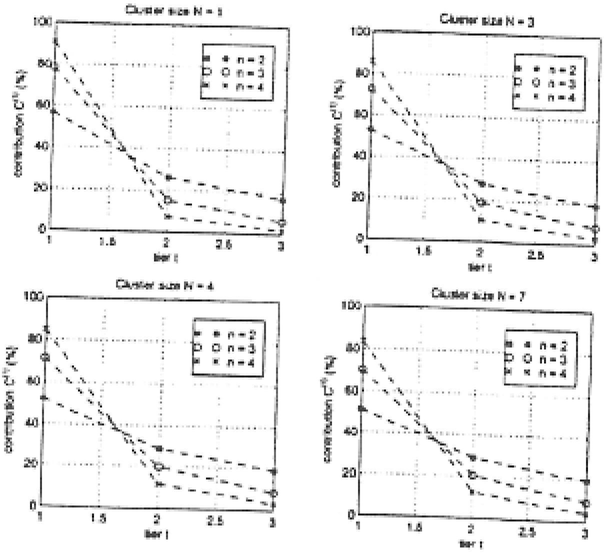


Figure 2: Contribution $C^{(t)}$ of co-channel base stations in the t-tier to the total co-channel interference.

The signal-to-interference ratio when only T tiers are assumed, denoted by $SIR^{(T)}$, is computed by

$$SIR^{(T)} = 10\log_{10}\frac{P_{des}}{\sum_{t=1}^{T} I^{(t)}} \tag{6}$$

where P_{des} is the power of the desired signal received at the mobile and given by $P_{des} = P_t\, R^{-n}$.

Table 3 and Figure 3 show $SIR^{(T)}$ for cluster sizes $N = 1, 3, 4$ and 7 and path loss exponents $n = 2, 3$ and 4.

b) **Co-channel interference analysis**

From the results presented in Figures 2 and 3, we conclude that the contribution of base stations in a given tier depends mainly on the path loss exponent.

The number of tiers we need to consider when computing *SIR* depends on the desired accuracy of the results. In this problem, we will assume that we can tolerate an error of 0.5 dB in the estimate of *SIR*. We have assumed that only base stations in the first three tiers produce considerable interference. Base stations in more distant tiers produce negligible interference. Therefore, the *true SIR* at the mobile is $SIR^{(3)}$

$$SIR^{(3)} = 10\log_{10}\frac{P_{des}}{I^{(1)} + I^{(2)} + I^{(3)}} \tag{7}$$

The error in the estimate of *SIR* when only the first T tiers are considered is

$$E_{SIR}^{T} = SIR^{(T)} - SIR^{(3)} \qquad T = 1, 2 \quad \text{(in dB)}. \tag{8}$$

3-9 Cont'd

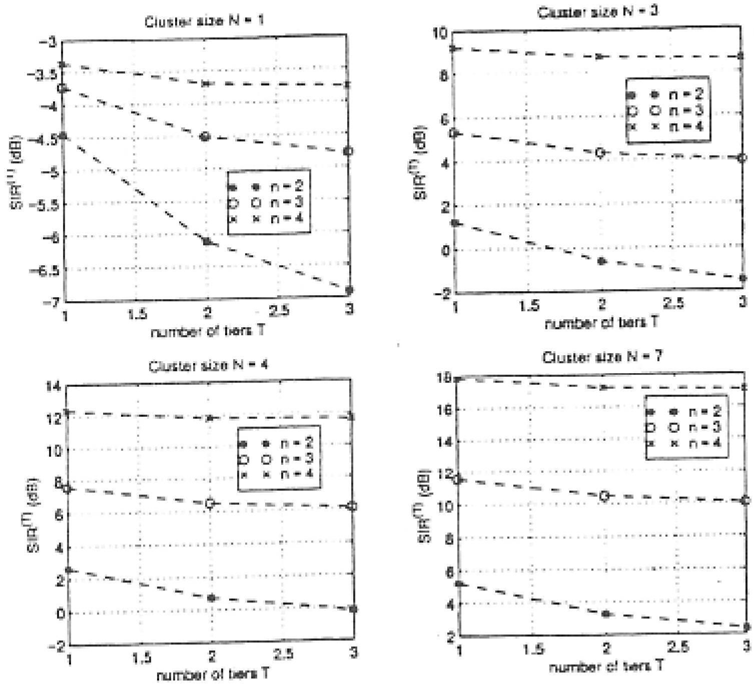


Figure 3: Signal-to-interference ratio at the mobile located at the cell boundary, when only the first T tiers of co-channel cells are considered.

Table 3: Signal-to-interference ratio at the mobile located at the cell boundary, when the first T tiers of co-channel cells are considered: cluster sizes $N = 1, 3, 4$ and 7 and path loss exponents $n = 2, 3$ and 4.

cluster size	T	$SIR^{(T)}$ (dB) $n = 2$	$n = 3$	$n = 4$
$N = 1$	1	-4.45	-3.73	-3.36
	2	-6.11	-4.51	-3.70
	3	**-6.91**	**-4.79**	**-3.78**
$N = 3$	1	1.24	5.36	9.24
	2	-0.62	4.35	8.74
	3	**-1.53**	**3.97**	**8.60**
$N = 4$	1	2.64	7.57	12.35
	2	0.74	6.51	11.79
	3	**-0.19**	**6.11**	**11.64**
$N = 7$	1	5.23	11.58	17.82
	2	3.29	10.46	17.21
	3	**2.34**	**10.04**	**17.04**

3-9 Cont'd

Using the results from Figure 3, we can compute E_{SIR}^T for cluster sizes $N = 1$, 3, 4 and 7 and path loss exponents $n = 2$, 3 and 4. Results are presented in Table 4.

We conclude: for error < 0.5 dB,

- **Path loss exponent $n = 2$:**
  We need to consider all three tiers, regardless of cluster size.
- **Path loss exponent $n = 3$:**
  We need to consider the first two tiers, regardless of cluster size.
- **Path loss exponent $n = 4$:**
  For cluster size $N = 1$, we may consider the first tier only; for cluster sizes $N = 3$, 4 and 7, we need to consider the first two tiers.

Table 4: Error in the estimate of *SIR* using only the first T tiers, for cluster sizes $N = 1$, 3, 4 and 7 and path loss exponents $n = 2$, 3 and 4.

cluster size	T	E_{SIR}^T (dB) $n=2$	$n=3$	$n=4$
$N=1$	1	2.46	1.06	0.42
	2	0.80	0.28	0.08
$N=3$	1	2.77	1.39	0.64
	2	0.91	0.38	0.13
$N=4$	1	2.83	1.46	0.71
	2	0.93	0.40	0.15
$N=7$	1	2.89	1.54	0.78
	2	0.95	0.42	0.17

Table 5: Location of co-channel base stations in the first tier. Serving cell is located at the origin of the coordinate system. Coordinates x and y are normalized with R.

	N = 1 x	N = 1 y	N = 3 x	N = 3 y	N = 4 x	N = 4 y	N = 7 x	N = 7 y
1	1.5000	0.8660	3.0000	0	3.0000	1.7321	4.5000	-0.8660
2	0.0000	1.7321	1.5000	2.5981	0.0000	3.4641	3.0000	3.4641
3	-1.5000	0.8660	-1.5000	2.5981	-3.0000	1.7321	-1.5000	4.3301
4	-1.5000	-0.8660	-3.0000	0.0000	-3.0000	-1.7321	-4.5000	0.8660
5	-0.0000	-1.7321	-1.5000	-2.5981	-0.0000	-3.4641	-3.0000	-3.4641
6	1.5000	-0.8660	1.5000	-2.5981	3.0000	-1.7321	1.5000	-4.3301

3-9 Cont'd

Table 6: Location of co-channel base stations in the **second tier**. Serving cell is located at the origin of the coordinate system. Coordinates x and y are normalized with R.

	N = 1		N = 3		N = 4		N = 7	
	x	y	x	y	x	y	x	y
7	3.0000	1.7321	6.0000	0	6.0000	3.4641	9.0000	-1.7321
8	1.5000	2.5981	4.5000	2.5981	3.0000	5.1962	7.5000	2.5981
9	0.0000	3.4641	3.0000	5.1962	0.0000	6.9282	6.0000	6.9282
10	-1.5000	2.5981	0.0000	5.1962	-3.0000	5.1962	1.5000	7.7942
11	-3.0000	1.7321	-3.0000	5.1962	-6.0000	3.4641	-3.0000	8.6603
12	-3.0000	0.0000	-4.5000	2.5981	-6.0000	0.0000	-6.0000	5.1962
13	-3.0000	-1.7321	-6.0000	0.0000	-6.0000	-3.4641	-9.0000	1.7321
14	-1.5000	-2.5981	-4.5000	-2.5981	-3.0000	-5.1962	-7.5000	-2.5981
15	-0.0000	-3.4641	-3.0000	-5.1962	-0.0000	-6.9282	-6.0000	-6.9282
16	1.5000	-2.5981	-0.0000	-5.1962	3.0000	-5.1962	-1.5000	-7.7942
17	3.0000	-1.7321	3.0000	-5.1962	6.0000	-3.4641	3.0000	-8.6603
18	3.0000	0	4.5000	-2.5981	6.0000	0	6.0000	-5.1962

Table 7: Location of co-channel base stations in the **third tier**. Serving cell is located at the origin of the coordinate system. Coordinates x and y are normalized with R.

	N = 1		N = 3		N = 4		N = 7	
	x	y	x	y	x	y	x	y
19	4.5000	2.5981	9.0000	0	9.0000	5.1962	13.5000	-2.5981
20	3.0000	3.4641	7.5000	2.5981	6.0000	6.9282	12.0000	1.7321
21	1.5000	4.3301	6.0000	5.1962	3.0000	8.6603	10.5000	6.0622
22	0.0000	5.1962	4.5000	7.7942	0.0000	10.3923	9.0000	10.3923
23	-1.5000	4.3301	1.5000	7.7942	-3.0000	8.6603	4.5000	11.2583
24	-3.0000	3.4641	-1.5000	7.7942	-6.0000	6.9282	0.0000	12.1244
25	-4.5000	2.5981	-4.5000	7.7942	-9.0000	5.1962	-4.5000	12.9904
26	-4.5000	0.8660	-6.0000	5.1962	-9.0000	1.7321	-7.5000	9.5263
27	-4.5000	-0.8660	-7.5000	2.5981	-9.0000	-1.7321	-10.5000	6.0622
28	-4.5000	-2.5981	-9.0000	0.0000	-9.0000	-5.1962	-13.5000	2.5981
29	-3.0000	-3.4641	-7.5000	-2.5981	-6.0000	-6.9282	-12.0000	-1.7321
30	-1.5000	-4.3301	-6.0000	-5.1962	-3.0000	-8.6603	-10.5000	-6.0622
31	-0.0000	-5.1962	-4.5000	-7.7942	-0.0000	-10.3923	-9.0000	-10.3923
32	1.5000	-4.3301	-1.5000	-7.7942	3.0000	-8.6603	-4.5000	-11.2583
33	3.0000	-3.4641	1.5000	-7.7942	6.0000	-6.9282	-0.0000	-12.1244
34	4.5000	-2.5981	4.5000	-7.7942	9.0000	-5.1962	4.5000	-12.9904
35	4.5000	-0.8660	6.0000	-5.1962	9.0000	-1.7321	7.5000	-9.5263
36	4.5000	0.8660	7.5000	-2.5981	9.0000	1.7321	10.5000	-6.0622

$\boxed{3\text{-}10}$

(a) $\dfrac{24\ \text{MHz}}{2 \cdot 30\ \text{kHz}} = 400$ channels

$\dfrac{400\ \text{channels}}{4\ \text{cells}} = 100$ channels/cell

(b) 90% of 100 Erlangs = 90 Erlangs

$90 = U A_u = U(0.1) \Rightarrow U = 900$ users

(c) offered: 90E ; $C = 100 \Rightarrow 0.03$ from graph (Fig. 3-6)

3% GOS

(d) Each sector has 33.3 channels ; GOS = 3%

from graph (Fig. 3-6) $\Rightarrow \approx 25$ Erlangs/sector

$25 = U A_u$ (per sector)

$\Rightarrow U = 250 \times 3$ sectors

$U = 750$ users

(e) $\dfrac{2500\ \text{km}^2}{5\ \text{km}^2} = 500$ cells $\Rightarrow 500 \times 900$ users/cell $= 450{,}000$ users

(f) 500 cells $\Rightarrow 500 \times 750$ user/cell $= 375{,}000$ users

3-11 By the same method used in example 3-9, when going from omni-directional antennas to 60° sectored antennas, the number of channels per sector $= \frac{57}{6} = 9.5$. Given $P_r[\text{blocking}] = 1\%$, from the Erlang B distribution we have the total offered traffic intensity per sector $A \doteq 4.1$ Erlangs.

For $\mu = 1$ call/hour, $H = 2$ minute/call, the number of calls that each seach sector can handle per hour is

$$U = \frac{A}{\mu H} = \frac{4.1}{\frac{1}{60} \cdot 2} = 123 \text{ users}$$

$\Rightarrow$ cell capacity $= 6 \times 123 = 738$ users, from example 2.9.

$\Rightarrow$ loss in trunking efficiency $= 1 - \frac{738}{1326} \doteq 0.44 = \underline{\underline{44\%}}$

3-12

(EIRP = 32 watts, cell radius = 10 km. GOS is 5%, blocked calls cleared. H = 2 minutes, and μ = 2 calls pr hour. Assume cell will be split into 4 cells.)

a) What is the current capacity of the "Radio Knob" cell?

Using the functions defined in problem 2.7

$\mu := 2$ $\quad H := \frac{2}{60}$ $\quad A_u := 0.067$ Erlangs

$P := .05$ Probability of blocked calls

$C := 57$ Assume N=7 cell, AMPS

$A := 40$ Initial guess

$A_T(P,C) := \text{root}(GOS(A,C)-P,A)$ Solve iteratively for total traffic

$A_T(P,C) = 51.528$ Erlangs

Number of users is $U := \frac{A_T(P,C)}{A_u}$ $\quad U = 772.921$ or 772 users

b) What is the radius and transmit power of the new cells?

Since the 4 new cells must cover the area of the old cell, the radius of the new cells must be R/2, where R is the radius of the old cell. Then the area covered by the new cells is

$$4\pi\left(\frac{R}{2}\right)^2 := 4\pi\left(\frac{R^2}{4}\right) = \pi R^2$$ which equals the area of the original cell

3-12 Cont'd

To maintain the same SNR, the power at the edge of the new cells must equal the power at the edge of the original cell or

$P_{ori}:=P_{new}$ $\quad P_1R^{-4}:=P_2\left(\frac{R}{2}\right)^{-4}$ $\quad$ and $\quad P_2:=\frac{P_1}{16}$

where P_1 and P_2 are the powers of the base station in the old and new cells respectively.

If P_1=32 watts, then P_2=2 watts.

c) How many channels are needed in the new cells to maintain frequency reuse stability in the system?

C:=57 **Each new cell gets the number of channels of the original cell once the cell splitting process is complete.**

d) If traffic is uniformly distributed, what is the new traffic carried by each new cell? Will the probability of blocking in these new cells be below 0.1% after the split?

$U:=\frac{772}{4}$ **U=193 users per new cell**

$A:=U\cdot A_u$ A=12.867 Erlangs

GOS(12.87,57)=0 **The probability of blocking is less than .1%**

3-13 Since users are uniformly distributed over the area, each cell in the cluster is assigned the same number of channels:

$$N_C = \frac{M}{N}, \tag{8}$$

where

N_C = number of channels per cell

M = number of channels available in the system (300 channels)

N = cluster size (9)

Given the number of channels per cell and the designed blocking probability $P_b = 1\%$, we can compute the maximum carried traffic per cell in Erlang (C_C) using the Erlang B formula

$$C_C = Erlang(N_C, P_b), \tag{10}$$

and the maximum carried traffic in the system C:

$$C = C_C \times 84 \tag{11}$$

Since each user offers a traffic of 0.04 Erlangs, the maximum number of users supported by the system is

$$N_U = \frac{C}{0.04} \tag{12}$$

Table 2: Number of channels per cell (N_C), carried traffic per cell (C_C), total carried traffic in the system (C), and maximum number of users in the system (N_U), for cluster sizes N = 4, 7 and 12. Blocking probability 1%.

Cluster size N	channels per cell (N_C)	carried traffic per cell (N_C)	total carried traffic C	number of users N_U
4	75	60.73 Erl	5101.09 Erl	127527
7	42	30.77 Erl	2584.81 Erl	64620
12	25	16.12 Erl	1354.49 Erl	33862

3-14

1) Since mobiles, with ongoing calls, are crossing cell boundaries (leaving a cell and entering another), the average call duration, from a particular base station perspective, decreases.
2) Some ongoing calls entering a cell will be forced to terminate due to lack of available channels in their new cells, reducing the average call duration
3) When mobiles with ongoing calls are entering a particular cell, the call request rate of that cell is composed by *new* call request rate (call requests from mobiles already within the cell), and requests made by mobiles entering the cell (with ongoing calls).

3-15 (a) Given GOS = 2%

For C = 4 channels, from the Erlang B chart

$A_{total} \doteq 1.1$ Erlangs $\Rightarrow A_{perchannel} = \frac{A_{tot}}{C} = 0.275$ Erlangs

For C = 20 channels.

$A_{total} \doteq 14$ Erlangs $\Rightarrow A_{perchannel} = \frac{14}{20} = 0.7$ Erlangs

For C = 40 channels

$A_{total} \doteq 31$ Erlangs $\Rightarrow A_{perchannel} = \frac{31}{40} = 0.775$ Erlangs

(b) $U = \frac{A_{total}}{\mu \cdot H} = \frac{31}{\frac{1}{3600} \times 105} \doteq 1063$ users

(c) For C = 4 channels, $A_{total} = 1.1$ Erlangs, $H = 105$ seconds/call

from the Erlang C chart, we have

$Pr[delay > 0] \doteq 0.03$

$\Rightarrow Pr[delay > 20\,sec] = Pr[delay > 0] \cdot \exp[-(C - A_{total}) \cdot 20sec/H]$

$= 0.03 \times \exp[-(4-1.1) \times 20/105] \doteq 0.017$

3.15 Cont'd

For $C = 20$ channels. $A_{total} = 14$ Erlangs, we have

$Pr[\text{delay} > 0] \doteq 0.06$

$\Rightarrow Pr[\text{delay} > 20\text{sec}] = Pr[\text{delay} > 0] \cdot \exp[-(C - A_{total}) \cdot 20\text{sec}/H]$

$= 0.06 \times \exp[-(20-14) \times 20/105] \doteq \underline{\underline{0.019}}$

For $C = 40$ channels. $A_{total} = 31$ Erlangs, we have

$Pr[\text{delay} > 0] \doteq 0.07$

$\Rightarrow Pr[\text{delay} > 20\text{sec}] = 0.07 \times \exp[-(40-31) \times 20/105] \doteq \underline{\underline{0.013}}$

(d) From (c) we can see that the probability that a call will be delayed for more than 20 seconds in a lost call delayed system is less than 2% for all the different channel numbers. Thus a lost call delayed system perform better than a system that drops blocked calls.

$\boxed{3.16}$ For 7 cell reuse pattern, the interference signal power from another transmitter is

$$P_I = P_t \cdot \left(\frac{D}{d_0}\right)^{-n} = P_t \cdot \left(\frac{\sqrt{3N} \cdot r}{d_0}\right)^{-n}$$

Where P_t is the transmit power in base station, D is the distance to the center of the nearest co-channel cells, r is the major radius. In this case, $P_t = 1\text{mW}$, $N = 7$, $d_0 = 1\text{m}$, $n = 3$, thus we have

3-16 Cont'd

$$10\log_{10}\frac{P_t\cdot\left(\frac{\sqrt{3N}\cdot r}{d_0}\right)^{-n}}{1mW} < -100\,dBm$$

$$\Rightarrow 10\log_{10}\frac{1mW\times\left(\frac{\sqrt{3\times7}\cdot r}{1m}\right)^{-3}}{1mW} < -100\,dBm \Rightarrow \underline{\underline{r > 470.1\,m}}$$

<u>For 4 cell reuse pattern</u>. $N=4$, we have

$$10\log_{10}\frac{1mW\times\left(\frac{\sqrt{3\times4}\cdot r}{1m}\right)^{-3}}{1mW} < -100\,dBm \Rightarrow \underline{\underline{r > 621.9\,m}}$$

$\boxed{3\text{-}17}$ area of a cell (hexagon) $= \frac{3\sqrt{3}}{2}\cdot r^2 = \frac{3\sqrt{3}}{2}\cdot(0.4701)^2 \doteq 0.574\,Km^2$

number of users in a cell $U=$ area of a cell $\times$ user density

$= 0.574\times9000 \doteq 5167$ users

$$\Rightarrow A = U\cdot\mu\cdot H = 5167\times\frac{1}{60}\times1 \doteq 86.1 \text{ Erlangs}.$$

Given $C=90$, from Erlang C chart, we have the probability that a call will be delayed

$$Pr[\text{delay}>0] \doteq 0.5$$

$$\Rightarrow Pr[\text{delay}>20\text{ sec}] = Pr[\text{delay}>0]\cdot Pr[\text{delay}>20\,|\,\text{delay}]$$

$$= 0.5\times\exp[-(90-86.1)\times20/60]$$

$$\underline{\underline{\doteq 0.136}}$$

$\boxed{3\text{-}18}$ See section 3.7.1.

If $n=3$. $P_{t2} = \frac{P_{t1}}{2^n} = \frac{P_{t1}}{8}$

3.19 For 4 cell reuse pattern and 3 sectors per cell, we can divide the 395 voice channels into 4x3=12 subsets, each containing about 33 channels. In each subset, the closest adjacent channel is 12 channels away. Each cell uses three subsets of channels; one subset for one sector. This channel assignment is illustrated in the chart below along with the control channel assigment. With respect to the chart, each cell uses channels in the subset iA+iB+iC. where i is an integer from 1 to 4 and A, B, and C represent 3 different sectors, respectively. The total number of voice channel in a cell is about 99. The underlined set of numbers correspond to the control channels.

1A	2A	3A	4A	1B	2B	3B	4B	1C	2C	3C	4C
334	335	336	337	338	339	340	341	342	343	344	345
346	347	348	349	350	351	352	353	354	355	356	357
358	359	360	361	362	363	364	365	366	367	368	369
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
658	659	660	661	662	663	664	665	666	717	718	719
—	—	—	—	—	—	—	—	720	721	722	723
724	725	726	727	728	729	730	731	732	733	734	735
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
796	797	798	799								

3.20 Similarly, for 4 cell reuse pattern and 6 sectors per cell, we can divided the 395 voice channels into $4 \times 6 = 24$ subsets. Each subset is labeled with an integer number i followed by a letter, i ranges from 1 to 4 and the letter from A to F. The integer number and the letter denote the cell number and sector name, respectively. Therefore each cell uses channels in the subset $iA + iB + iC + iD + iE + iF$.

3.21 Algorithm 1:

For a 7 cell reuse pattern with 3 sectors per cell, each cell uses about 57 channels, and each sector uses 19 channels. Since for the $120°$ sectoring, the number of interfers in the first level is reduced from six to two, we can divide the 19 channels of each sector into 3 groups, each with about 6 channels. For example, the 18 channels in sector A of cell 1 can be divided into 3 groups as follows.

group 1: {1, 22, 43, 64, 85, 106}
group 2: {127, 148, 169, 190, 211, 231}
group 3: {253, 274, 295, 670, 691, 712, 1003}

When one sector uses the channels in one of the three groups, the other two interfering sectors should first choose the

3.21 Cont'd

channels from the other two groups, and this should be continued until all the channels in one group have been occupied. When this happens, the sector can use the channels in other groups

Algorithm 2:

In this algorithm, the 19 channels are used as a whole group. When one sector uses the channels in this group, the other two interfering sector use the remaining channels. When all the 19 channels have been occupied by the three sectors, the MSC will borrow the channels that are not occupied from the other sectors in the co-channel cell.

$\boxed{3.22}$ (a) For AMPS. the channel bandwidth $B_W = 30KHz$

Given noise figure $F = 10dB = 10$. we have

noise floor $= K \cdot B_W \cdot F \cdot T_0$, where K is Boltzman constant.
$T_0 = 290°K$

$\Rightarrow$ noise floor $= 1.38 \times 10^{-23} \times 30 \times 10^3 \times 10 \times 290$
$\doteq 1.2 \times 10^{-15}\,(W) \doteq \underline{\underline{-119.2\,(dBm)}}$

(b) For GSM, $B_W = 200KHz$

$\Rightarrow$ noise floor $= K \cdot B_W \cdot F\, T_0 = 1.38 \times 10^{-23} \times 200 \times 10^3 \times 10 \times 290$
$\doteq 8 \times 10^{-15}\,(W) \doteq \underline{\underline{-111\,(dBm)}}$

(c) For USDC. $B_W = 30KHz$

$\Rightarrow$ noise floor $= \underline{\underline{-119.2\,(dBm)}}$

(d) For DECT, $B_W = 1.728\ MHZ$

$\Rightarrow$ noise floor $= \underline{\underline{-101.6\,(dBm)}}$

(e) For IS-95. $B_W = 1.2288\ MHZ$

$\Rightarrow$ noise floor $= \underline{\underline{-103.1\,(dBm)}}$

(f) For CT-2, $B_W = 100KHZ$

$\Rightarrow$ noise floor $= \underline{\underline{-114\,(dBm)}}$

$\boxed{3.23}$ (a) SNR = signal level (dBm) − noise floor (dBm)
$= -90 - (-119.2) = \underline{\underline{29.2\ dB}}$

3.23 Cont'd

(b) $SNR = -90 - (-111) = \underline{\underline{21\,dB}}$

(c) $SNR = -90 - (-119.2) = \underline{\underline{29.2\,dB}}$

(d) $SNR = -90 - (-101.6) = \underline{\underline{11.6\,dB}}$

(e) $SNR = -90 - (-103.1) = \underline{\underline{13.1\,dB}}$

(f) $SNR = -90 - (-114) = \underline{\underline{24\,dB}}$

3.24 See the appendix in the text book.

3.25 See the MATLAB program p3-25.m and Fig. p3-25(a),(b).

In Fig. p2-17(b), the ratio of the traffic intensity of the sectored system to that of the omni-directional antenna system, $\frac{A_{sector}}{A_{omni}}$, is shown for different number of channels. Also shown is the ratio of the SIR of the omni-directional antenna system to that of the sectored system, $\frac{SIR_{omni}}{SIR_{sector}}$. From the figure we can see that $\frac{A_{sector}}{A_{omni}} > \frac{SIR_{omni}}{SIR_{sector}}$, or $\frac{A_{sector}}{A_{omni}} \cdot \frac{SIR_{sector}}{SIR_{omni}} > 1$

that means trunking loss is always less than the SIR gain.

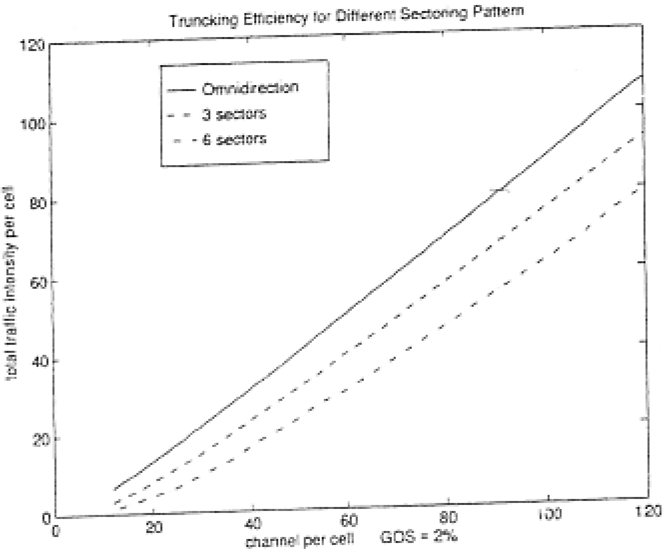


Fig. p3 - 25(a)

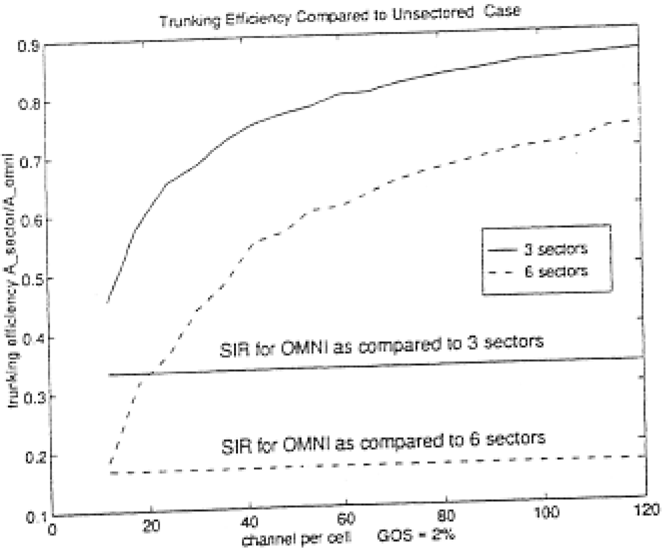


Fig. p3. 25(b)

3.26 (a) Given $u = 3$ calls/hour, $H = 5$ minutes/call.

$\Rightarrow A_u = u \cdot H = \frac{3}{60} \times 5 = \underline{\underline{0.25 \text{ Erlangs}}}$

(b) Given $C = 1$, $A_u = 0.25$ Erlangs, $GOS = 0.01$.

from the Erlang B chart, we have $A = 0.01$

$\Rightarrow$ number of users, $U = \frac{A}{A_u} = \frac{0.01}{0.25} = 0.04$ users.

But, actually one user could be supported on one channel.

So $U = \underline{\underline{1 \text{ user}}}$

(c) Given $C = 5$, $A_u = 0.25$ Erlangs, $GOS = 0.01$

$\Rightarrow A = 1.4$ Erlangs $\Rightarrow U = \frac{A}{A_u} = \frac{1.4}{0.25} = 5.6$ users

So $U = \underline{\underline{5 \text{ users}}}$

(d) $U = 5 \times 2 = 10$ users, $\Rightarrow A = U \cdot A_u = 10 \times 0.25 = 2.5$ Erlangs

For $C = 5$, $A = 2.5$ Erlangs $\Rightarrow GOS \doteq \underline{\underline{0.07}}$

This is not acceptable performance. That means 7 out of 100 calls will be blocked due to channel occupancy during the busiest hour. Generally, $GOS \leq 0.02$ is desired.

3.27 (a) The AMPS system is duplex.

Given total bandwidth $BW_{total} = 50$ MHz, total number of channel $N = 832$ channels. we have

the bandwidth for each channel $B_{rv} = \frac{BW_{total}}{N} = \frac{50 \times 10^6}{832} = \underline{\underline{60 \text{kHz}}}$

3.27 Cont'd.

This bandwidth of 60KHz for the duplex channel is split into two one-way channels, a forward channel (from the base-station to the subscriber) and a reverse channel (from the subscriber to the base station), each with bandwidth of 30kHz. The forward channel is exactly 45 MHz higher than the reverse channel.

(b) For $F_{fw} = 880.560\text{MHz} \Rightarrow F_{reverse} = F_{fw} - 45 = \underline{\underline{835.560\text{ MHz}}}$

(c) Given $N = 832$, total number of control channel $N_{con} = 42$, we have total number of voice channel

$N_{vo} = N - N_{con} = 832 - 42 = 790$.

$\Rightarrow$ number of voice channels for each carrier

$$N_{vo,A} = N_{vo,B} = \frac{N_{vo}}{2} = \frac{790}{2} = \underline{\underline{395 \text{ channels}}}$$

number of control channels for each carrier

$$N_{con,A} = N_{con,B} = \frac{N_{con}}{2} = \frac{42}{2} = \underline{\underline{21 \text{ channels}}}$$

(d) See example 3.3

(e) <u>For 7-cell reuse</u>; $N = 7$, $\Rightarrow Q = \frac{D}{R} = \sqrt{3N} = \sqrt{21} = 4.58$

$\Rightarrow D = \underline{\underline{4.58R}}$, where R is the radius of the cell.

For 4-cell reuse. $N = 4$, $\Rightarrow Q = \sqrt{3N} = \sqrt{12} \doteq 3.46$

$\Rightarrow D = \underline{\underline{3.46R}}$

a) *Minimum SIR*

In order to compute the minimun SIR at the mobile, we need to determine the number of interfering base stations in each possible configuration, which can be done by inspecting Figures 1 and 2. Table 1 shows the number of interfering base stations in the first tier, when 3 sectors ($BW = 120^o$) and 6 sectors ($BW = 60^o$) are used, for cluster sizes $N = 3$ and 4.

Using expression (1), we determine the minimum SIR (approximation) in each configuration (path loss exponent $n = 4$). Results are shown in Table 2.

Therefore, cluster size $N = 3$ cannot be used, since the minimum SIR achieved is below $SIR = 18.7$ dB. On the other hand, both configurations using cluster size $N = 4$ are feasible, regarding co-channel interference (assuming that a difference of 0.1 dB is negligible).

b) *Maximum carried traffic per cell*

Let us now computer the carried traffic per cell, when sectoring is used. As we know, each sector is assigned a subset of the set of channels assigned to the cell.

For example, for cluster size $N = 3$, each cell is assigned $300/3 = 100$ channels. If six sectors are employed, each sector is assigned $100/6 \approx 16$ channels. Using Erlang B formula, we find that each sector carries a maximum traffic of 9.83 Erlangs at a blocking probability of 0.02. Therefore, the maximum traffic carried by each cell is $9.83 \times 6 = 58.97$ Erlangs. Repeating this procedure, we can compute the maximum carried traffic per cell for other beamwidths and cluster sizes. Table 3 presents the results.

Table 1: Number of interfering base stations in the first tier (i_0) when 3 sectors ($BW = 120^o$) and 6 sectors ($BW = 60^o$) are used.

N	$BW = 60^o$	$BW = 120^o$
3	2	3
4	1	2

Table 2: Minimum SIR achieved when sectoring is used, for cluster sizes $N = 3$ and 4.

N	$BW = 60^o$	$BW = 120^o$
3	16.1 dB	14.3 dB
4	21.6 dB	18.6 dB

Table 3: Maximum carried traffic per cell (in Erlangs) when sectoring is used, for cluster sizes $N = 3$ and 4. 300 channels available in the system, $P_b = 2\%$

N	$BW = 60^o$	$BW = 120^o$
3	58.97	73.88
4	39.69	52.51

3.28 Cont'd

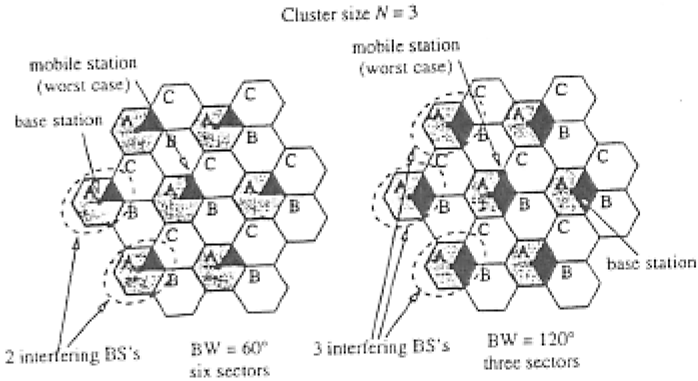


Figure 1 : Cluster size $N=3$, three & six sectors

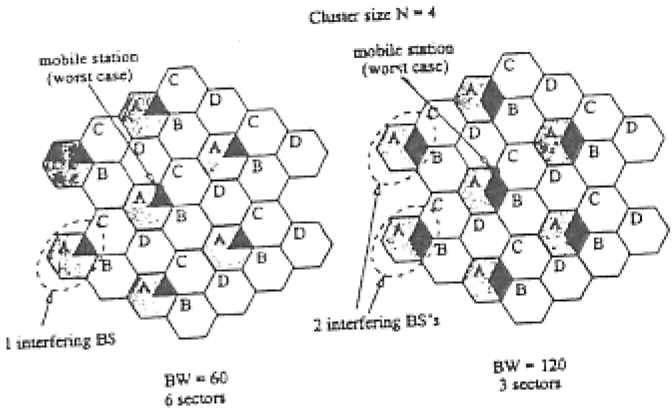


Figure 2: Cluster size $N=4$, three & six sectors

3.29 (a) Given loan $= \$6\times10^6$, Cost of MTSO. $C_{MTSO} = \$1.5\times10^6$. cost of a base station, $C_{bs} = \$5\times10^5$, Cost of advertisement. $C_{ad} = \$5\times10^5$, we have,

the number of the base stations we are able to install.

$$N = \frac{\text{loan} - C_{MTSO} - C_{ad}}{C_{bs}} = \frac{6\times10^6 - 1.5\times10^6 - 5\times10^5}{5\times10^5} = \underline{\underline{8}}$$

(b) Given $N = 8$ cells, total coverage area $A_{tot} = 140\,Km^2$.

$\Rightarrow$ coverage area of each cell $A_{each} = \frac{A_{tot}}{N} = \frac{140}{8} = \underline{\underline{17.5\,Km^2}}$

Since $A_{each} = 2.6R^2$, we have

$$R = \sqrt{\frac{A_{each}}{2.6}} = \sqrt{\frac{17.5}{2.6}} \doteq \underline{\underline{2.6\,Km}}$$

(c) For each year, each customer will pay $P = 50\times12 = \$600$,

Assume the number of customers on the first day of service is M, the gross billing revenus by the end of the fourth year of operation is

$$G = (M + 2M + 4M + 8M)\cdot P = 15M\cdot P$$

We need $G \geq \$10\times10^6 \Rightarrow 15MP \geq 10\times10^6$

$$\Rightarrow M \geq \frac{10\times10^6}{15\cdot P} = \frac{10\times10^6}{15\times600} \doteq 1111.1.$$

Hence the minimum number of customer on the first day of service is $\underline{\underline{1112}}$

(d) number of users per square Km $= \frac{M}{A_{tot}} = \frac{1112}{140} \doteq \underline{\underline{8\text{ users}/Km}}$

CHAPTER 4

4.1

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2} = \frac{(1/3)^2(10)(1)(1)}{(4\pi)^2(1000)^2} = 7.036^{-9}\ W$$

4.2 a)

$$P_r = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2} = \frac{(50)(1)(1)(1/20)^2}{(4\pi)^2(10^4)^2} = 7.910^{-12}\ W = -81\ dBm$$

b)

$$P_r = P_d \cdot A_e = \left[\frac{|E|^2}{120\pi}\right] \cdot A_e = \frac{|E|^2}{120\pi} \cdot \frac{G_r \lambda^2}{4\pi}$$

$$A_e = \frac{G\lambda^2}{4\pi} \qquad E = P_r \cdot (120\pi)(4\pi)/\lambda^2 G_r = 3.9 \times 10^{-3}\ V/m$$

c)

$$P_r = \frac{\left[\frac{V_{ant}}{2}\right]^2}{50\Omega} \Rightarrow \sqrt{7.9\, 10^{-12} \cdot 50 \cdot 4} = V_{ant} \quad \text{open circuit}$$

$$V_{ant} = 4 \cdot 10^{-5}\ \text{Volts rms} \quad \text{open circuit}$$

$$V_{rcvr} = \frac{V_{ant}}{2} = 2 \cdot 10^{-5}\ \text{Volts rms}$$

4.3 Fraunhofer Distance:

$$a = 4.6\ \text{cm} \qquad (2)$$

$$b = 3.5\ \text{cm} \qquad (3)$$

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{60 \times 10^9} = 0.005\ \text{m} \qquad (4)$$

In azimuth:

$$HPBW = \frac{51\lambda}{b} = \frac{51 \times 0.005}{0.035} = 7.3° \qquad (5)$$

In elevation:

$$HPBW = \frac{51\lambda}{a} = \frac{51 \times 0.005}{0.046} = 5.5° \qquad (6)$$

$$G = \frac{4\pi A_e}{\lambda^2} = \frac{4\pi ab}{\lambda^2} = 29\ \text{dB} \qquad (7)$$

$$D = \sqrt{a^2 + b^2} = 5.78\ \text{cm} \qquad (8)$$

$$D_f = \frac{2D^2}{\lambda} = \frac{2 \times 0.0578^2}{0.005} = 1.34\ \text{m} \qquad (9)$$

4.4

$$G_t = G_r = 29 \text{ dB} \tag{10}$$
$$P_t = 30 \text{ dBm} \tag{11}$$
$$\lambda = \frac{c}{f} = 0.005 \text{ m} \tag{12}$$
$$d_0 = 1 \text{ m} \tag{13}$$
$$d_1 = 100 \text{ m} \tag{14}$$
$$d_2 = 1000 \text{ m} \tag{15}$$
$$PL(d_0) = 20\log_{10}\frac{4\pi d_0}{\lambda} = 20\log_{10}\frac{4\pi}{0.005} = 68 \text{ dB} \tag{16}$$
$$PL(d_1) = PL(d_0) + 20\log_{10}\frac{d_1}{d_0} = 108 \text{ dB} \tag{17}$$
$$PL(d_2) = PL(d_0) + 20\log_{10}\frac{d_2}{d_0} = 128 \text{ dB} \tag{18}$$

(19)

$$P_r = P_t + G_t + G_r - PL = 30 + 29 + 29 - PL = 88 - PL \tag{20}$$
$$P_r(d_0) = 88 - 68 = 20 \text{ dBm} \tag{21}$$
$$P_r(d_1) = 88 - 108 = -20 \text{ dBm} \tag{22}$$
$$P_r(d_2) = 88 - 128 = -40 \text{ dBm} \tag{23}$$

(24)

$$V = \sqrt{4\ P_r\ R_{ant}} \tag{25}$$
$$V(d_1) = 0.0447 \text{ v} \tag{26}$$
$$V(d_2) = 0.0045 \text{ v} \tag{27}$$

4.5

$$\Gamma_{\parallel} = \frac{-\epsilon_2 \sin\theta_i + \sqrt{\epsilon_2 - \cos^2\theta_i}}{\epsilon_2 \sin\theta_i + \sqrt{\epsilon_2 - \cos^2\theta_i}}$$
$$\Gamma_{\perp} = \frac{\sin\theta_i - \sqrt{\epsilon_2 - \cos^2\theta_i}}{\sin\theta_i + \sqrt{\epsilon_2 - \cos^2\theta_i}}$$

At $\theta_i = 30^0$

	Ground	Brick	Limestone	Glass	Water
ϵ_r	15	4.44	7.51	4	81
$\Gamma_{\parallel}$	-0.33	-0.07	-0.18	-0.05	-0.64
$\Gamma_{\perp}$	-0.77	-0.59	-0.68	-0.57	-0.89

4.6

$$h_c = \frac{\lambda}{8\sin\theta_i}$$

(Note: θ_i in this equation refers to the angle between the direction of incident wave and the surface.)

Surface roughness depends on wavelength (carrier frequency) and incident angle. Surface appears "rougher" with the increase of frequency or decrease of incident angle (the angle between the direction of wave propagation and the surface normal).

4.7 When $\theta = \theta_i$, $\Gamma_{\parallel} = 0$

$$\Rightarrow \Gamma_{\parallel} = \frac{-\varepsilon_r \sin\theta_i + \sqrt{\varepsilon_r - \cos^2\theta_i}}{\varepsilon_r \sin\theta_i + \sqrt{\varepsilon_r - \cos^2\theta_i}} = 0$$

$$\Rightarrow -\varepsilon_r \cdot \sin\theta_i + \sqrt{\varepsilon_r - \cos^2\theta_i} = 0$$

$$\Rightarrow \sqrt{\varepsilon_r - \cos^2\theta_i} = \varepsilon_r \cdot \sin\theta_i$$

$$\Rightarrow (\varepsilon_r^2 - 1)\sin^2\theta_i = \varepsilon_r - 1$$

$$\Rightarrow \sin\theta_i = \frac{\sqrt{\varepsilon_r - 1}}{\sqrt{\varepsilon_r^2 - 1}}$$

4.8 (a) The advantages of the two-ray ground reflection model in the analysis of path loss is that it considers both the direct path and a ground reflected propagation path between transmitter and receiver. The disavantage is that this model is oversimplified in that it does not include important factors such as terrain profile, vegetation and buildings.

(b) Generally, when $d > 10(h_t + h_r)$, we can say that $d \gg h_t + h_r$, and thus may apply the two ray model.

<u>For $h_t = 35m$, $h_r = 3m$, $d = 250m$</u>

$d < 10(h_t + h_r) = 380\ m$

Hence <u>the two ray model could not be applied.</u>

<u>For $h_t = 30m$, $h_r = 1.5m$, $d = 450m$</u>

$d > 10(h_t + h_r) = 315\ m$

Hence <u>the two ray model could be applied.</u>

4.8 Cont'd

(c) Using the two ray model, we can see that at large distances, the received power falls off with distance raised to the fourth power or at a rate of 40dB/decade, and the received power and path loss are independent of frequency

4.9

$$\Delta = d'' - d'$$
$$= \sqrt{(h_t + h_r)^2 + d^2} - \sqrt{(h_t - h_r)^2 + d^2}$$
$$= d\left[1 + \left(\frac{h_t + h_r}{d}\right)^2\right]^{\frac{1}{2}} - d\left[1 + \left(\frac{h_t - h_r}{d}\right)^2\right]^{\frac{1}{2}}$$

For $d \gg h_t + h_r$, $\left(\frac{h_t + h_r}{d}\right)^2 \ll 1$, $\left(\frac{h_t - h_r}{d}\right)^2 \ll 1$.

Using Taylor series approximation, we have

$$\Delta \approx d\left[1 + \frac{1}{2}\left(\frac{h_t + h_r}{d}\right)^2\right] - d\left[1 + \frac{1}{2}\left(\frac{h_t - h_r}{d}\right)^2\right]$$
$$= d \cdot \frac{1}{2}\left[\left(\frac{h_t + h_r}{d}\right)^2 - \left(\frac{h_t - h_r}{d}\right)^2\right]$$
$$= d \cdot \frac{1}{2} \cdot \frac{4 h_t \cdot h_r}{d^2}$$
$$= \frac{2 h_t \cdot h_r}{d}$$

4.10 When $d \gg h_t + h_r$, we have $\theta_\Delta = \frac{2\pi}{\lambda} \cdot \frac{2 h_t \cdot h_r}{d}$

$$\Rightarrow d = \frac{4\pi}{\lambda} \cdot \frac{h_t \cdot h_r}{\theta_\Delta}$$

$$\tan\theta_i = \frac{h_t + h_r}{d} < \tan 5^\circ \Rightarrow \frac{h_t + h_r}{\frac{4\pi}{\lambda} \cdot \frac{h_t \cdot h_r}{\theta_\Delta}} < \tan 5^\circ$$

$$\Rightarrow \frac{1 + \frac{h_r}{h_t}}{\frac{4\pi}{\lambda} \cdot \frac{h_r}{\theta_\Delta}} < \tan 5^\circ \Rightarrow h_t > \frac{h_r}{\frac{4\pi \cdot h_r \tan 5^\circ}{\lambda \cdot \theta_\Delta} - 1}$$

For $h_r = 2m$, $\theta_\Delta = 6.261$, $\lambda = \frac{c}{f} = \frac{3 \times 10^8}{9 \times 10^2} = 0.333m$

$$\Rightarrow h_t > \frac{2}{\frac{4\pi \times 2 \times \tan 5^\circ}{\frac{1}{3} \times 6.261} - 1} \Rightarrow \underline{\underline{h_{t\,min} = 37.7m}}$$

4.10 Cont'd

$$\Rightarrow d_{min} = \frac{4\pi}{\lambda}\cdot\frac{h_{t\,min}\cdot h_r}{\theta_\Delta} = \frac{4\pi}{0.333}\times\frac{37.7\times 2}{6.261} \doteq \underline{\underline{453.77\,(m)}}$$

$\boxed{4.11}$ At the location of the signal nulls at the receiver.

$$\theta_\Delta = \frac{2\pi}{\lambda}\cdot\frac{2h_t\cdot h_r}{d} = 2i\pi,\quad i = 1, 2, \cdots$$

$$\Rightarrow \underline{\underline{d_{nulls} = \frac{2h_t\cdot h_r}{i\lambda}}}$$, where i is a positive integer such that $d_{nulls} > d_0$.

$\boxed{4.12}$ Approximate : $\underline{\underline{P_r = P_t\cdot G_t\cdot G_r\cdot\frac{h_t^2\cdot h_r^2}{d^4}}}$

Exact : $|E_{TOT}(d)| = \frac{E_0\cdot d_0}{d}\sqrt{2-2\cos\theta_\Delta}$

$$\Rightarrow |E_{TOT}(d)|^2 = \frac{E_0^2\cdot d_0^2}{d^2}\cdot(2-2\cos\theta_\Delta)$$

$$P_r(d) = \frac{|E_{TOT}(d)|^2}{120\pi}\cdot A_e = \frac{E_0^2\cdot d_0^2}{d^2}\cdot(2-2\cos\theta_\Delta)\cdot\frac{A_e}{120\pi}$$

$$E_0^2 = \frac{P_t\cdot G_t\cdot G_r\cdot\lambda^2}{(4\pi)^2 d_0^2}\cdot\frac{120\pi}{A_e}$$

$$\Rightarrow P_r(d) = \frac{P_t\cdot G_t\cdot G_r\cdot\lambda^2}{(4\pi)^2\cdot d_0^2}\cdot\frac{120\pi}{A_e}\cdot\frac{d_0^2}{d^2}\cdot(2-2\cos\theta_\Delta)\cdot\frac{A_e}{120\pi}$$

$$= \underline{\underline{\frac{P_t\cdot G_t\cdot G_r\cdot\lambda^2}{(4\pi)^2\cdot d^2}\cdot 4\cdot\sin^2\frac{\theta_\Delta}{2}}}$$

where $\theta_\Delta = \frac{2\pi}{\lambda}\cdot\frac{2h_t\cdot h_r}{d}$.

See problem 4.24 (pg. 58) for the plot.

$\boxed{4.13}$ See problem 4.24 (pg 58) for the plot

$$\Gamma = 1 \Rightarrow \underline{\underline{P_r(d) = \frac{P_t\cdot G_t\cdot G_r\cdot\lambda^2}{(4\pi)^2\cdot d^2}\cdot 4\cdot\cos^2\frac{\theta_\Delta}{2}}}$$

4.14

a)

$$\lambda = \frac{c}{f} = 0.1579\text{m} \quad (8)$$

$$P_r(d) = 10\log_{10}(\frac{P_t G_t G_r \lambda^2}{(4\pi d)^2}) \quad (9)$$

$$= 10\log_{10}\frac{50 \times 1 \times 2 \times 0.1597^2}{(4\pi \times 10^4)^2} \quad (10)$$

$$= 10\log_{10}(1.58 \times 10^{-10}) \quad (11)$$

$$= -98\text{dBw} \quad (12)$$

$$= -68\text{dBm} \quad (13)$$

b)

$$A_e = G_r\frac{\lambda^2}{4\pi} \quad (14)$$

$$P_r(d) = \frac{|E|^2}{120\pi}A_e \quad (15)$$

$$|E| = \sqrt{\frac{P_r(d)120\pi}{G_r\lambda^2/4\pi}} = 3.67 \text{ mV/m} \quad (16)$$

c)

$$V = \sqrt{P_r(d) \times 4R_{ant}} = \sqrt{1.58 \times 10^{10} \times 4 \times 50} = 0.178\text{mV} \quad (17)$$

d) In order to use the 2-ray model approximation, the following condition must be held

$$d > \frac{20h_t h_r}{\lambda} = 9500 \text{ m} \quad (18)$$

Since d=10000 m, we can use the following equations for the 2-ray ground reflection model,

$$P_r(W) = P_t G_t G_r \frac{h_t^2 h_r^2}{d^4} \quad (19)$$

$$= 50 * 1 * 2 * \frac{50^2 1.5^2}{10000^4} \quad (20)$$

$$= 5.625 \times 10^{-11} \text{ W} \quad (21)$$

$$P_r(dBm) = -72.5 \text{ dBm} \quad (22)$$

4.15 We need to find a d_f such that $\Delta = d'' - d = \frac{\lambda}{2}$.

$$\Delta = d'' - d = \sqrt{(h_t+h_r)^2 + d_f^2} - \sqrt{(h_t-h_r)^2 + d_f^2}$$

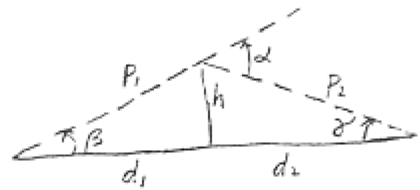
$$\Rightarrow \sqrt{(h_t+h_r)^2 + d_f^2} - \sqrt{(h_t-h_r)^2 + d_f^2} = \frac{\lambda}{2}$$

$$\Rightarrow (h_t+h_r)^2 + d_f^2 = (h_t-h_r)^2 + d_f^2 + \frac{\lambda^2}{4} + \lambda \cdot \sqrt{(h_t-h_r)^2 + d_f^2}$$

$$\Rightarrow d_f = \sqrt{\frac{16 h_t^2 \cdot h_r^2}{\lambda^2} - (h_t^2 + h_r^2) + \frac{\lambda^2}{16}}$$

4.16 (a) $P_1 = \sqrt{d_1^2 + h^2} = d_1 \sqrt{1 + \left(\frac{h}{d_1}\right)^2}$

$P_2 = \sqrt{d_2^2 + h^2} = d_2 \sqrt{1 + \left(\frac{h}{d_2}\right)^2}$



Since $d_1, d_2 \gg h \gg \lambda$, $\frac{h}{d_1}, \frac{h}{d_2} \ll 1$. using Taylor series approximation, we have

$$P_1 \doteq d_1 \left[1 + \frac{1}{2}\left(\frac{h}{d_1}\right)^2\right] = d_1 + \frac{1}{2}\frac{h^2}{d_1}$$

$$P_2 \doteq d_2 \cdot \left[1 + \frac{1}{2}\left(\frac{h}{d_2}\right)^2\right] = d_2 + \frac{1}{2}\frac{h^2}{d_2}$$

$$\Rightarrow \Delta = P_1 + P_2 - (d_1 + d_2)$$

$$\doteq \left(d_1 + \frac{1}{2}\frac{h^2}{d_1}\right) + \left(d_2 + \frac{1}{2}\frac{h^2}{d_2}\right) - (d_1 + d_2)$$

$$= \frac{h^2}{2}\left(\frac{d_1 + d_2}{d_1 d_2}\right)$$

and $\phi = \frac{2\pi\Delta}{\lambda} \doteq \frac{2\pi}{\lambda}\left[\frac{h^2}{2}\left(\frac{d_1+d_2}{d_1 \cdot d_2}\right)\right]$.

(b) From the definition of ν, $\frac{\nu^2 \pi}{2} = \phi$

$$\Rightarrow \nu = \sqrt{\phi \cdot \frac{2}{\pi}}$$

$$\Rightarrow \nu = \sqrt{\frac{2\pi}{\lambda}\left[\frac{h^2}{2}\left(\frac{d_1+d_2}{d_1 \cdot d_2}\right)\right] \cdot \frac{2}{\pi}} = h \cdot \sqrt{\frac{2}{\lambda} \cdot \frac{d_1+d_2}{d_1 d_2}}$$

Since $\tan\beta = \frac{h}{d_1} \ll 1$, $\tan\gamma = \frac{h}{d_2} \ll 1$, we have

$$\beta \approx \tan\beta = \frac{h}{d_1}, \quad \gamma \approx \tan\gamma = \frac{h}{d_2}$$

4.16 Cont'd

$$\Rightarrow \alpha = \beta + \gamma = \frac{h}{d_1} + \frac{h}{d_2} = h\left(\frac{d_1+d_2}{d_1 \cdot d_2}\right)$$

$$\Rightarrow h = \alpha \cdot \frac{d_1 \cdot d_2}{d_1+d_2}$$

$$\Rightarrow \nu = h \cdot \sqrt{\frac{2}{\lambda} \cdot \frac{d_1+d_2}{d_1 \cdot d_2}} = \alpha \cdot \frac{d_1 \cdot d_2}{d_1+d_2} \cdot \sqrt{\frac{2}{\lambda} \cdot \frac{d_1+d_2}{d_1 \cdot d_2}}$$

$$= \alpha \cdot \sqrt{\frac{2 d_1 d_2}{\lambda(d_1+d_2)}}$$

4.17 A general design rule for microwave links is 55% clearance of the first Fresnel zone. For a 1 km link at 2.5 GHz, what is the maximum first Fresnel zone radius? What clearance is required for this system ?

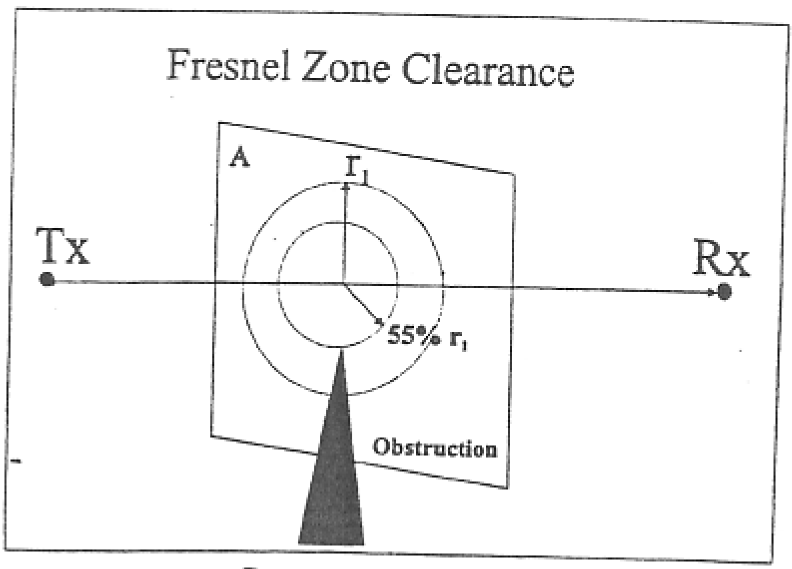
Solution

$$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{2.5 \times 10^9} = 0.12 \text{ m} \tag{23}$$

For the first Fresnel zone: n=1. The maximum Fresnel zone radius occurs for $d_1 = d_2 = 500m$. Using Equation (4.56), the Fresnel zone radius is found to be

$$r_n = \sqrt{\frac{n\lambda d_1 d_2}{d_1 + d_2}} = \sqrt{\frac{1 \times 0.12 \times 500 \times 500}{500 + 500}} = 5.48 \text{ m} \tag{24}$$

Thus, 55% first Fresnel zone clearance would require at least $5.48 \times 55\% = 3.01m$ above the obstruction to the LOS path as shown in the figure below.



Fresnel zone clearance.

$\boxed{4.18}$ Diffracted power decreases with the increase of the frequency as shown in Figure 1.

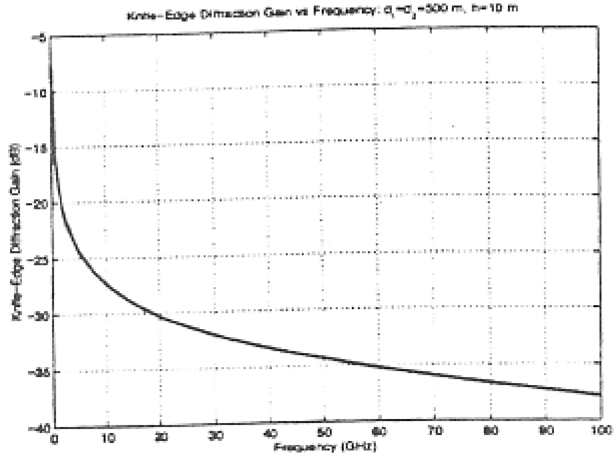


Figure 1: Diffracted power vs. frequency.

$\boxed{4.19}$ Given $P_t = 10W$. $G_t = 10dB = 10$, $L = 1dB \doteq 1.259$.

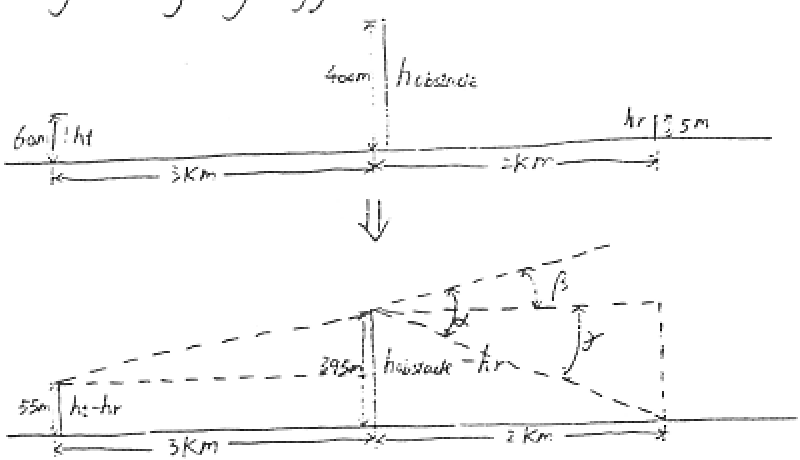
$G_r = 3dB = 2$, $f_c = 900MHz$, $d = 3000 + 2000 = 5000m$,

We have $\lambda_c = \frac{c}{f_c} \doteq 0.333(m)$ and free space received power

$$P_{freespace} = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda_c^2}{(4\pi)^2 d^2 \cdot L} = \frac{10 \times 10 \times 2 \times 0.333^2}{(4\pi)^2 \times (5000)^2 \times 1.259}$$

$$\doteq 4.48 \times 10^{-9}(w) \doteq \underline{\underline{-53.5\,dBm}}$$

For the geometry shown below, we can redraw it in another geometry by approximation.



From the figure above we have

4.19 Cont'd

$$\tan\beta = \frac{h_{obstacle} - h_t}{d_1} = \frac{400-60}{3000} \doteq 0.1133 \Rightarrow \beta \doteq 0.11285\ (rad)$$

$$\tan\gamma = \frac{h_{obstacle} - h_r}{d_2} = \frac{400-5}{2000} = 0.1975 \Rightarrow \gamma \doteq 0.1950\ (rad)$$

$$\Rightarrow \alpha = \beta + \gamma = 0.11285 + 0.195 \doteq 0.3078\ (rad)$$

and $v = \alpha \cdot \sqrt{\frac{2 d_1 d_2}{\lambda (d_1+d_2)}} = 0.3078 \times \sqrt{\frac{2 \times 3000 \times 2000}{0.333 \times (3000+2000)}} \doteq 26.12$

Using the approximation equation (3.59.e), we obtain

$$G_d (dB) = 20 \cdot \log_{10} \left(\frac{0.225}{v}\right) \qquad v > 2.4$$

$$= 20 \cdot \log_{10} \left(\frac{0.225}{26.12}\right)$$

$$\doteq -41.3\ dB.$$

$$\Rightarrow P_{received} = P_{freespace} (dBm) + G_d$$

$$= -53.5 - 41.3$$

$$= \underline{\underline{-94.8\ dBm}}$$

$\Rightarrow$ loss due to diffraction $L_d = P_{freespace} - P_{received} = \underline{\underline{41.3\ dB}}$

$\boxed{4.20}$ (a) $f_c = 50\ MHz \Rightarrow \lambda_c = \frac{c}{f} = 6\ m$

$$\Rightarrow P_{freespace} = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda_c^2}{(4\pi)^2 \cdot d^2 \cdot L} \doteq 1.45 \times 10^{-6} (W) \doteq -28.4\ (dBm)$$

$$v = \alpha \cdot \sqrt{\frac{2 d_1 d_2}{\lambda_c (d_1+d_2)}} = 0.3078 \times \sqrt{\frac{2 \times 3000 \times 2000}{6 \times (3000+2000)}} \doteq 6.156$$

$$\Rightarrow G_d = 20 \log_{10} \left(\frac{0.225}{v}\right) = 20 \log_{10} \left(\frac{0.225}{6.156}\right) \doteq -28.7\ dB$$

$$\Rightarrow P_{received} = P_{freespace} + G_d = -28.4 - 28.7 = \underline{\underline{-57.1\ dBm}}$$

$$L_d = -G_d = \underline{\underline{31.7\ dB}}$$

4.20 Cont'd

(b) $f_c = 1900\,MHZ \Rightarrow \lambda_c = \frac{c}{f_c} \doteq 0.158\,(m)$

$$\Rightarrow P_{freespace} = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda_c^2}{(4\pi)^2 \cdot d^2 \cdot L} \doteq 10^{-9}(W) = -60\,(dBm)$$

$$\nu = \alpha \cdot \sqrt{\frac{2 d_1 \cdot d_2}{\lambda_c (d_1 + d_2)}} = 0.3078 \cdot \sqrt{\frac{2 \times 3000 \times 2000}{0.158 \times 5000}} \doteq 37.9$$

$$\Rightarrow G_d = 20 \log\left(\frac{0.225}{\nu}\right) = 20 \cdot \log\left(\frac{0.225}{37.9}\right) = -44.5\,dB$$

$$\Rightarrow P_{received} = P_{freespace} + G_d = -60 - 44.5 = \underline{\underline{-104.5\,dBm}}$$

$$L_d = -G_d = \underline{\underline{44.5\,dB}}$$

$\boxed{4.21}$

$$PL(d) = PL_0 + 10n\log(d/d_0)$$
$$PL_i(d_i) = PL_0 + 10n\log(d_i/d_0) + x_i$$

Error between the measurements and the prediction:

$$x_i = PL_i(d_i) - PL_0 - 10n\log(\frac{d_i}{d_0})$$

MSE:

$$\frac{1}{N}\sum_{i=1}^{N} x_i^2 = \frac{1}{N}\sum_{i=1}^{N}[PL_i - PL_0 - 10n\log(\frac{d_i}{d_0})]^2 = F(n)$$

$$\frac{d\,F(n)}{d\,n} = \frac{1}{N}\sum_{i=1}^{N} 2[PL_i - PL_0 - 10n\log(\frac{d_i}{d_0})](-10\log\frac{d_i}{d_0}) = 0$$

$$n = \frac{\sum_{i=1}^{N}(PL_i - PL_0)\log\frac{d_i}{d_0}}{\sum_{i=1}^{N} 10(\log\frac{d_i}{d_0})^2}$$

$\boxed{4.22}$ $n = -3.5 \Rightarrow \overline{P_r(d)}\,(dBm) = -35\log_{10}\frac{d}{d_0} + P_r(d_0)(dBm)$

$$= -35 \cdot \log_{10}\frac{10}{1} + 0 = -35\,dBm$$

For $\gamma = -25\,dBm$, $Pr[P_r(d) > \gamma] = Q\left(\frac{\gamma - \overline{P_r(d)}}{\sigma}\right) = 10\%$

$$\Rightarrow Q\left(\frac{-25-(-35)}{\sigma}\right) = Q\left(\frac{10}{\sigma}\right) = 10\%$$

$$\Rightarrow \frac{10}{\sigma} \doteq 1.29 \Rightarrow \sigma \doteq \underline{\underline{7.75\,dB}}$$

4.23 (a) For free space, $P_r = P_0 \left(\frac{d_0}{d}\right)^2$

Given $P_0 = 10^{-6}(W) = -30\,dBm$, $d_0 = 1\,Km$.

For $d = 2\,Km$, $P_r = 10^{-6} \cdot \left(\frac{1}{2}\right)^2 = 2.5 \times 10^{-7}(W) \doteq -36\,dBm$

Similarly, For $d = 5\,Km$, $P_r \doteq -44\,dBm$

For $d = 10\,Km$, $P_r \doteq -50\,dBm$

For $d = 20\,Km$, $P_r = -56\,dBm$

(b) For $n = 3$, $P_r = P_0 \cdot \left(\frac{d_0}{d}\right)^3$

4.23 Cont'd

For $d = 2Km$. $P_r = 10^{-6} \cdot (\frac{1}{2})^3 = 1.27 \times 10^{-7} (W) = \underline{\underline{-39\,dBm}}$

For $d = 5Km$, $P_r = \underline{\underline{-51\,dBm}}$

For $d = 10Km$, $P_r = \underline{\underline{-60\,dBm}}$

For $d = 20Km$, $P_r = \underline{\underline{-69\,dBm}}$

(c) For $n = 4$, $P_r = P_0 \left(\frac{d_0}{d}\right)^4$

For $d = 2Km$, $P_r = \underline{\underline{-42\,dBm}}$ For $d = 5Km$, $P_r = \underline{\underline{-58\,dBm}}$

For $d = 10Km$, $P_r = \underline{\underline{-70\,dBm}}$ For $d = 20km$, $P_r = \underline{\underline{-82\,dBm}}$

(d) For two ray ground reflection model using the exact expression

$$P_r(d_0) = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 d_0^2} \Rightarrow P_t = \frac{P_r(d_0) \cdot (4\pi)^2 \cdot d_0^2}{G_t \cdot G_r \cdot \lambda^2}$$

Given $P_r = 10^{-6}\,W$, $d_0 = 1Km$, $G_t = G_r = 0dB = 1$. $\lambda = \frac{c}{f_c} \doteq 0.1667m$,

$$\Rightarrow P_t = \frac{10^{-6} \times (4\pi)^2 \times (1000)^2}{1 \times 1 \times 0.1667^2} \doteq 5.679 \times 10^3 (W)$$

From problem 4.12, the exact expression is

$$P_r(d) = \frac{P_t \cdot G_t\, G_r\, \lambda^2}{(4\pi)^2 \cdot d^2} \cdot 4 \cdot \sin^2\left(\frac{\theta_\Delta}{2}\right), \quad \text{where } \theta_\Delta = \frac{2\pi}{\lambda} \cdot \frac{2h_t \cdot h_r}{d}$$

For $d = 2Km$, $\theta_\Delta = \frac{2\pi}{0.1667} \times \frac{2 \times 40 \times 3}{2000} = 4.5216$ rads

$$\Rightarrow P_r = \frac{5.679 \times 10^3 \times 1 \times 1 \times (0.1667)^2}{(4\pi)^2 \times (2000)^2} \times 4 \times \sin^2\left(\frac{4.5216}{2}\right)$$

$$\doteq 5.97 \times 10^{-7} (W) \doteq \underline{\underline{-32.25\,dBm}}$$

Similarly. For $d = 5Km$, $\theta_\Delta \doteq 1.809$ rads

$\Rightarrow P_r \doteq 9.88 \times 10^{-8} (W) \doteq \underline{\underline{-40\,dBm}}$

For $d = 10km$, $\theta_\Delta = 0.904$ rads

$\Rightarrow P_r = 7.64 \times 10^{-9} (W) \doteq \underline{\underline{-51.17\,dBm}}$

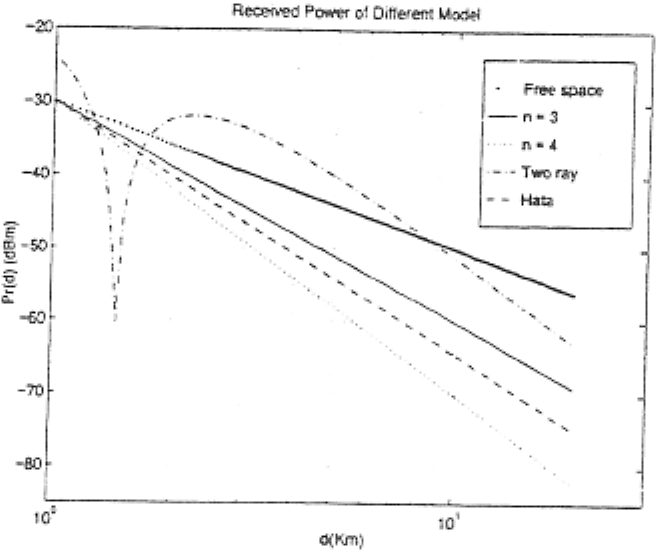


Fig. p4_23

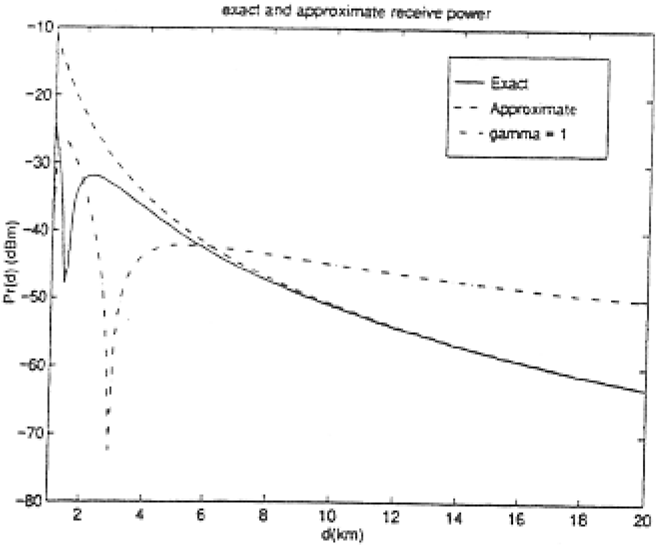


Fig. p4_24

4.23 Cont'd

For $d = 20$ Km. $\theta_\Delta \doteq 0.452$ rads

$\Rightarrow P_r = 5.02 \times 10^{-10} (W) \doteq \underline{\underline{-63 \text{ dBm}}}$

(e) For Extended Hata model.

$$L_{50}(\text{urban}) = 46.3 + 33.9 \log_{10} f_c - 13.82 \log_{10} h_t - a(h_r) + (44.9 - 6.55 \log_{10} h_t) \log_{10} d + C_M$$

Where $C_M = 3$ dB for large city

$a(h_r) = 3.2(\log_{10} 11.75 h_{re})^2 - 4.97$ dB for $f_c \geq 400$ MHz

$\Rightarrow a(h_r) = 3.2[\log_{10}(11.75 \times 3)]^2 - 4.97 \doteq 2.69$ dB

For $d = 2$ km, $L_{50}(2\text{Km}) = 46.3 + 33.9 \log_{10} 1800 - 13.82 \log_{10} 40 - 2.69 + (44.9 - 6.55 \log_{10} 40) \log_{10} 2 + 3$

$= 145.18$ dB

Since $L_{50}(1\text{Km}) = 134.8$ dB

$\Rightarrow P_r = P_0(\text{dBm}) - [L_{50}(20\text{Km}) - L_{50}(1\text{Km})]$

$= -30 - [145.18 - 134.8] = \underline{\underline{-40.38 \text{ dBm}}}$

Similarly, for $d = 5$ Km, $P_r = \underline{\underline{-54.07 \text{ dBm}}}$

for $d = 10$ Km, $P_r = \underline{\underline{-64.4 \text{ dBm}}}$

for $d = 20$ Km, $P_r = \underline{\underline{-74.8 \text{ dBm}}}$

See the MATLAB program p4_23.m and Fig. p4_23.

$\boxed{4.24}$ See the MATLAB program p4.24.m and Fig. p4_24.

It can be seen from the figure generated by the program that when $d > 5$ Km, these two models agree and when $d < 5$ Km, the models disagree. When $d > \frac{20\pi h_t h_r}{3\lambda}$,

4.24 Cont'd

We can use the approximation instead of the exact expression in cellular system design.

$\boxed{4.25}$

a) Handoff $\rightarrow$ two independent events: $P_{r,1} < P_{r,HO}$ and $P_{r,2} > P_{r,min}$. Therefore, the probability that a handoff occurs is given by

$$Pr[\text{ handoff }] = Pr[P_{r,1} < P_{r,HO}] \times Pr[P_{r,2} > P_{r,min}], \tag{2}$$

where $P_{r,1}$ and $P_{r,2}$ are the received signals at BS_1 and BS_2, respectively. $P_{r,1}$ is given by

$$\begin{aligned} P_{r,1} &= \underbrace{P_0 - 10n\log_{10}(d_1/d_0)}_{m_1} + \chi_1 \\ &= m_1 + \chi_1. \end{aligned} \tag{3}$$

Likewise for $P_{r,2}$

$$\begin{aligned} P_{r,2} &= \underbrace{P_0 - 10n\log_{10}[(D-d_1)/d_0]}_{m_2} + \chi_2 \\ &= m_2 + \chi_2. \end{aligned} \tag{4}$$

Therefore, for a given distance d_1, $P_{r,1}$ and $P_{r,2}$ are Gaussian variables with standard deviation σ and means m_1 and m_2, respectively.
Thus, the probability $Pr[P_{r,1} < P_{r,HO}]$ is

$$\begin{aligned} Pr[P_{r,1} < P_{r,HO}] &= \int_{-\infty}^{P_{r,HO}} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-(x-m_1)^2}{2\sigma^2}\right] dx \\ &= 1 - Q\left(\frac{P_{r,HO} - m_1}{\sigma}\right). \end{aligned} \tag{5}$$

where $Q(x)$ is the Q-function:

$$Q(x) = \int_x^{\infty} \frac{1}{\sqrt{2\pi}} \exp\left(\frac{-y^2}{2}\right) dy. \tag{6}$$

Likewise

$$\begin{aligned} Pr[P_{r,2} > P_{r,min}] &= \int_{P_{r,min}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[\frac{-(x-m_2)^2}{2\sigma^2}\right] dx \\ &= Q\left(\frac{P_{r,min} - m_2}{\sigma}\right). \end{aligned} \tag{7}$$

Substituting (3)-(7) into (2), we have the probability that a handoff occurs as a function of the distance d_1. Figure 2 shows the received area average power at both base stations (m_1 and m_2) (Not required in the homework!). Figure 3 shows the probability that a handoff occurs, along with the probabilities $Pr[P_{r,1} < P_{r,HO}]$ and $Pr[P_{r,2} > P_{r,min}]$.

b) From the plot in Figure 3, the distance d_{ho}, such that the probability that a handoff occurs is equal to 80%, is $d_{ho} \approx 1000$ m.

4.25 Cont'd.

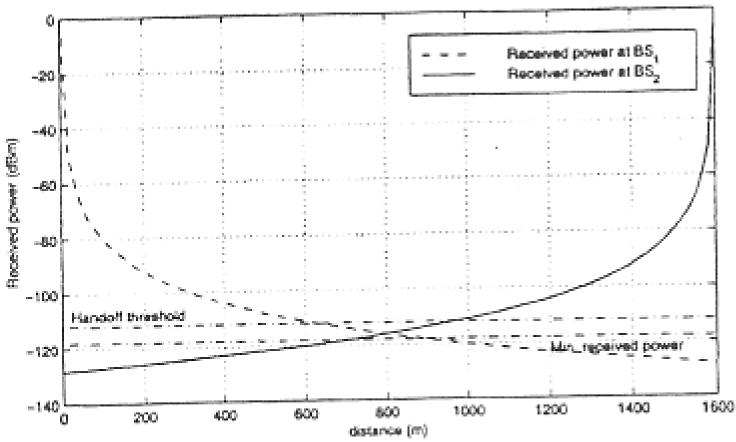


Figure 2: Received area average power levels at the mobile, from both base stations.

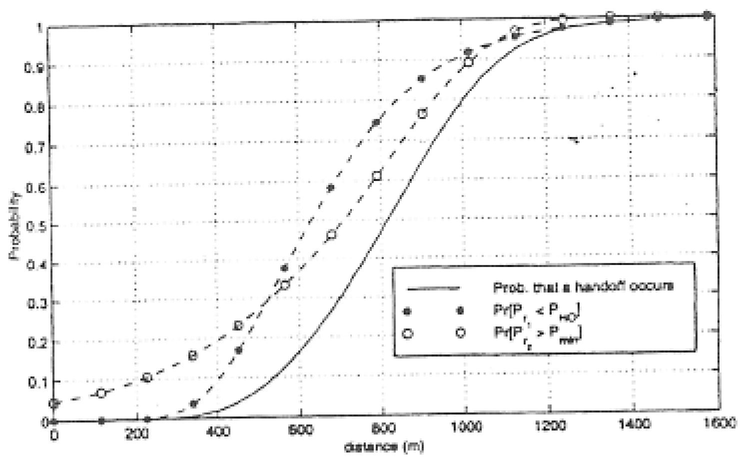


Figure 3: Probability that a handoff occurs, $Pr[P_{r,1} < P_{r,HO}]$ and $Pr[P_{r,2} > P_{r,min}]$

4.26 The area coverage level

$$U(\gamma) = \frac{1}{\pi R^2}\int \text{Prob}[P_r(r) > \gamma]\, dA$$

$$= \frac{2\pi}{\pi R^2}\int_0^R \text{Prob}[P_r(r) > \gamma]\cdot r \cdot dr$$

$$= \frac{2}{R^2}\int_0^R r\cdot Q\left[\frac{\gamma - \overline{P_r(r)}}{\sigma}\right]\cdot dr \qquad (1)$$

Where $\overline{P_r(r)} = \alpha - 10\cdot n\cdot \log_{10}\frac{r}{R}$, and α, expressed in dB, is a constant determined from the transmitter power, antenna heights and gains, and so on, then

$$Q\left[\frac{\gamma - \overline{P_r(r)}}{\sigma}\right] = \frac{1}{2} - \frac{1}{2}\,erf\left[\frac{\gamma - \alpha + 10\cdot r\cdot \log_{10}\left(\frac{r}{R}\right)}{\sigma\cdot\sqrt{2}}\right]$$

Then letting $a = \frac{\gamma-\alpha}{\sigma\sqrt{2}}$, $b = (10\cdot n\cdot \log_{10} e)/(\sigma\sqrt{2})$, we get

$$U(\gamma) = \frac{1}{2} - \frac{1}{R^2}\int_0^R r\cdot erf\left(a + b\ln\frac{r}{R}\right) dr \qquad (2)$$

Letting $t = a + b\cdot \ln\frac{r}{R}$, we have

$$r = R\cdot e^{\frac{t-a}{b}}, \quad dr = \frac{R}{b}\cdot e^{\frac{t-a}{b}}\cdot dt$$

Substituting $t = a + b\ln\frac{r}{R}$ into (2), we obtain

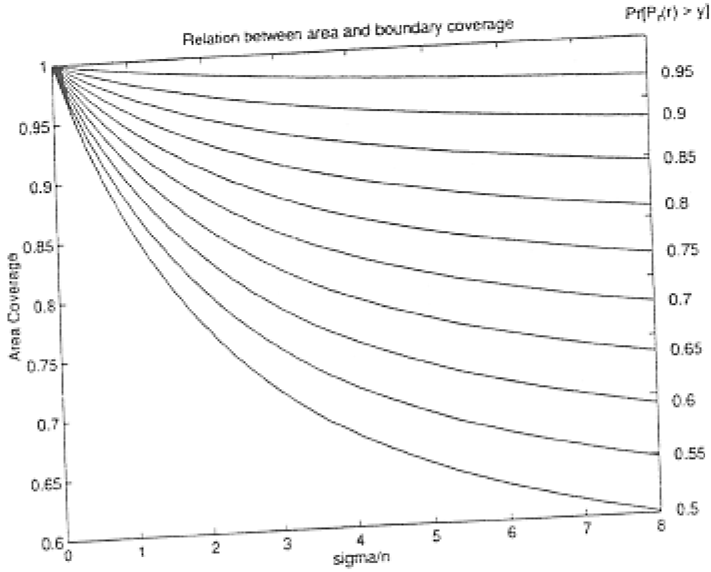
$$U(\gamma) = \frac{1}{2} - \frac{1}{R^2}\int_{-\infty}^{a} R\cdot e^{\frac{t-a}{b}}\cdot erf(t)\cdot \frac{R}{b}\cdot e^{\frac{t-a}{b}}\cdot dt$$

$$= \frac{1}{2} - \frac{e^{\frac{-2a}{b}}}{b}\int_{-\infty}^{a} e^{\frac{2t}{b}}\cdot erf(t)\, dt \qquad (3)$$

Substituting $t' = -t$ into (3), we obtain

$$U(\gamma) = \frac{1}{2} - \frac{e^{\frac{-2a}{b}}}{b}\cdot\int_{-a}^{\infty} e^{-\frac{2t}{b}}\cdot erf(-t)\, dt$$

$$= \frac{1}{2} + \frac{e^{-\frac{2a}{b}}}{b}\int_{-a}^{\infty} e^{-\frac{2t}{b}}\cdot erf(t)\, dt \qquad (4)$$



Relation between area and boundary coverage
Pr[P_r(r) > y]
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0.55
0.5
Area Coverage
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
0.6
0
1
2
3
4
5
6
7
8
sigma/n

Fig. pA-26

4.26 Cont'd

Using the integral formula

$$\int \mathrm{erf}(Ax)e^{Bx}dx = \frac{1}{B}e^{Bx}\mathrm{erf}(Ax) - \frac{1}{B}\exp\left(\frac{B^2}{4A^2}\right)\mathrm{erf}\left(Ax - \frac{B}{2A}\right)$$

from (4) we have

$$U(\gamma) = \frac{1}{2}\left\{1-\mathrm{erf}(a) + e^{\frac{1-2ab}{b^2}}\left[1-\mathrm{erf}\left(\frac{1-ab}{b}\right)\right]\right\}. \quad (5)$$

When $a=0$, (5) becomes

$$U(\gamma) = \frac{1}{2}\left\{1 + e^{\frac{1}{b^2}}\left[1-\mathrm{erf}\left(\frac{1}{b}\right)\right]\right\} \quad (6).$$

See the MATLAB program p426.m and Fig. p4.26.

$\boxed{4.27}$ Given noise figure $F = 8dB \doteq 6.3$, receiver bandwidth $B_W = 30KHz$,

$\Rightarrow$ noise floor $= K \cdot B_W \cdot F \cdot T_0$, where K is Boltzman constant, $T_0 = 290°K$

$\Rightarrow$ noise floor $= 1.38\times10^{-23}\times30\times10^3\times6.3\times290$
$\doteq 7.56\times10^{-16}(W) \doteq -121.2\ (dBm)$

$\Rightarrow$ threshold γ = noise floor (dBm) + SNR (dB)
$= -121.2 + 20 = -101.2\ (dBm)$

Since $Pr[P_r(d_{max}) > \gamma] = Q\left(\frac{\gamma - \overline{P_r(d_{max})}}{\sigma}\right) = 0.95$, we have

$$\frac{\gamma - \overline{P_r(d_{max})}}{\sigma} \doteq -1.645$$

$$\Rightarrow \overline{P_r(d_{max})} = \gamma + 1.645\sigma = -101.2 + 1.645\times8 = -88.04(dBm)$$

Given $P_t = 15W$, $\lambda = \frac{c}{f} \doteq 0.1667m$, $G_t = 12dB \doteq 15.85$.
$G_r = 3dB = 2$

$$\Rightarrow P_r(d_0) = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d_0^2}$$

4.27 Cont'd

$$= \frac{15 \times 15.85 \times 2 \times 0.1667^2}{(4\pi)^2 \times (1000)^2} \doteq 8.373 \times 10^{-8} (W) \doteq -40.77 \, dBm$$

Since $\overline{P_r(d_{max})} = P_r(d_0)\,(dBm) - 10 \cdot n \log_{10}\left(\frac{d_{max}}{d_0}\right)$, we have

$$10 \times 4 \log_{10}\left(\frac{d_{max}}{d_0}\right) = P_r(d_0) - \overline{P_r(d_{max})} = -40.77 - (-88.04)$$

$$\Rightarrow \log_{10} \frac{d_{max}}{d_0} \doteq 1.182$$

$$\Rightarrow d_{max} \doteq \underline{\underline{15.2 \, (Km)}}$$

$\boxed{4.28}$ noise floor $= K \cdot B_W \cdot F \cdot T_0$

$$= 1.38 \times 10^{-23} \times 30 \times 10^3 \times 10 \times 290 \doteq 1.2 \times 10^{-15} (W)$$

$$\doteq -119.2 \, (dBm)$$

$\Rightarrow$ threshold $\gamma =$ noise floor $(dBm) + SNR(dB)$

$$= -119.2 + 25 = -94.2 \, (dBm)$$

Given $EIRP = P_t \cdot G_t = 100W$, $G_r = 0\,dB = 1$, $d_0 = 1Km$, $\lambda = \frac{c}{f} = 0.333m$

$$P_r(d_0) = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d_0^2} = \frac{100 \times 1 \times 0.333^2}{(4\pi)^2 \times (1000)^2} \doteq 7.04 \times 10^{-8} (W)$$

$$\doteq -41.5 \, dBm$$

For $d = 10Km$, $n = 4$.

$$\overline{P_r(d)} = P_r(d_0) - 10 \cdot n \cdot \log_{10}\left(\frac{d}{d_0}\right) = -41.5 - 40 = -81.5 \, dBm$$

$$\Rightarrow Pr(P_r(d) > \gamma) = Q\left[\frac{\gamma - \overline{P_r(d)}}{\sigma}\right] = Q\left[\frac{-94.2 - (-81.5)}{8}\right]$$

$$= Q(-1.5875) \doteq \underline{\underline{0.944}}$$

4.29

(a) Find the minimum mean square error (MMSE) estimate for the path loss exponent, n.

First note that $P_r(100m) = 0\,dBm = P_r(d_o)$
$P_r(d) = P_r(d_o) - PL(d)$

d	P_r (dBm)
100	0
200	-25
1000	-35
2000	-38

$$J_n = \sum_{i=1}^{4}\left[P_i - \hat{P}_i\right]^2 = \underbrace{[0-0]^2}_{100m} + \underbrace{\left[-25-\left(0-10n\log\tfrac{200}{100}\right)\right]^2}_{200m}$$

$$+\left[-35-\left(0-10n\log\tfrac{1000}{100}\right)\right]^2 + \left[-38-\left(0-10n\log\tfrac{2000}{100}\right)\right]^2$$

$$= 625 - 150n + 9n^2 + 1225 - 700n + 100n^2 + 1444 - 988n + 169n^2$$

$$\therefore J_n = 278n^2 - 1838n + 3294$$

$$\frac{dJ_n}{dn} = 556n - 1838 \Rightarrow \boxed{n = 3.30}$$

(b) Calculate the standard deviation of shadowing about the mean value.

$$\sigma = \sqrt{\sigma^2} = \sqrt{\frac{J_n}{4}}\Bigg|_{n=3.30}$$

$$= \sqrt{278(3.3)^2 - 1838(3.3) + 3294}\,/\sqrt{4}$$

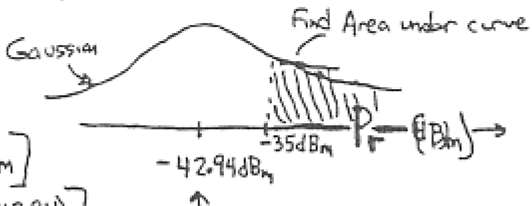
$$= \frac{\sqrt{3027 - 6065.4 + 3294}}{\sqrt{4}} = \frac{\sqrt{255.6}}{\sqrt{4}} = \boxed{8\,dB}$$

(c) Estimate the received power at $d = 2$ km using the resulting model.

$$P_r(d) = P_r(d_o) - PL(d) = 0\,dBm - 10[3.3]\log_{10}\left(\frac{2000}{100}\right)$$

$$= (0 - 42.94)\,dBm = \boxed{-42.94\,dBm}$$

(d) Predict the likelihood that the received signal level at 2 km will be greater than -35 dBm. Express your answer as a Q-function.



$$P_r\left[P_r(2km) \geq -35\,dBm\right]$$

$$= Q\left[\frac{-35-(-42.94)}{8}\right]$$

$$= Q\left(\frac{7.93}{8}\right) = Q(0.99) \approx Q(1)$$

4.30

a)

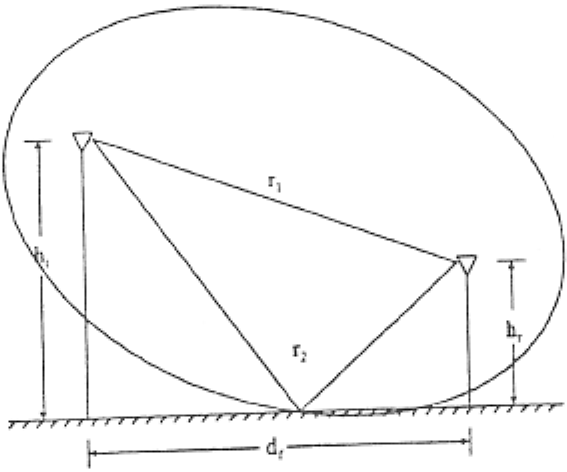


Figure 1

From the paper, "The first Fresnel zone is defined as an ellipsoid whose foci are the transmit and receive antennas. The distance from either antenna to a point on the ellipsoid and back to the other antenna is $\lambda/2$ greater tan the direct path distance, r_1, shown in Figure 1. The break point will be considered as the distance for which the ground begins to obstruct the first Fresnel zone."

We wish to show that the distance, d_f, at which the first Fresnel zone becomes obstructed, is given by

$$d_f = \frac{1}{\lambda}\sqrt{\left(\Sigma^2 - \Delta^2\right)^2 - 2\left(\Sigma^2 + \Delta^2\right)\left(\frac{\lambda}{2}\right)^2 + \left(\frac{\lambda}{2}\right)^4}\,. \tag{1}$$

So, we want the reflection point of r_2 to occur at the "break point," so we can assume that r_2 bounces off the ellipsoid and then goes to the receive antenna. As a result,

$$r_2 = r_1 + \frac{\lambda}{2}. \tag{2}$$

4.30 Cont'd

Now, we can use image theory and the geometry shown in Figure 2 to complete the derivation of the expression for d_f

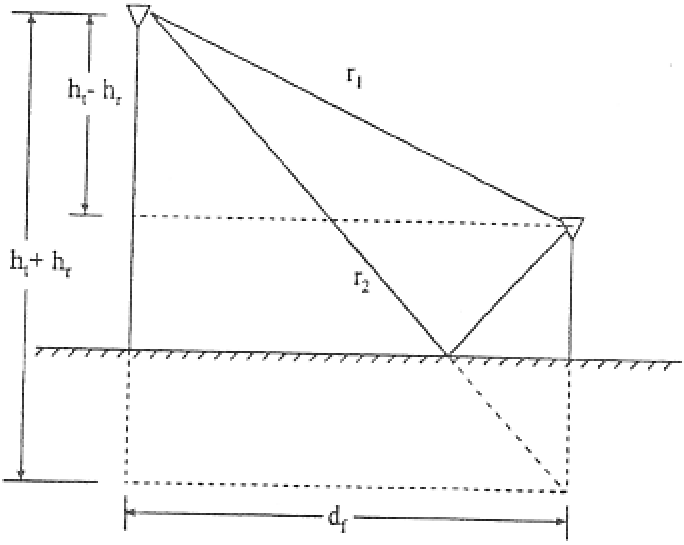


Figure 2

We see from the geometry that $d_f^2 + (h_t + h_r)^2 = r_1^2$ and $d_f^2 + (h_t - h_r)^2 = r_2^2$. Now, if we let $\Sigma = h_t + h_r$ and $\Delta = h_t - h_r$, this leads to $d_f^2 + \Sigma^2 = r_1^2$ and $d_f^2 + \Delta^2 = r_2^2$. Substituting into (2),

$$d_f^2 + \Sigma^2 = \left(r_1 + \frac{\lambda}{2}\right)^2 = r_1^2 + \lambda r_1 + \left(\frac{\lambda}{2}\right)^2$$

$$d_f^2 + \Sigma^2 = d_f^2 + \Delta^2 + \lambda\sqrt{d_f^2 + \Delta^2} + \left(\frac{\lambda}{2}\right)^2$$

$$\left\{(\Sigma^2 - \Delta^2) - \left(\frac{\lambda}{2}\right)^2 = \lambda\sqrt{d_f^2 + \Delta^2}\right\}^2$$

$$(\Sigma^2 - \Delta^2)^2 - 2(\Sigma^2 - \Delta^2)\left(\frac{\lambda}{2}\right)^2 + \left(\frac{\lambda}{2}\right)^4 = \lambda^2 d_f^2 + \lambda^2\Delta^2$$

$$(\Sigma^2 - \Delta^2)^2 - 2\left(\frac{\lambda}{2}\right)^2\Sigma^2 + 2\left(\frac{\lambda}{2}\right)^2\Delta^2 + \left(\frac{\lambda}{2}\right)^4 - \underbrace{\lambda^2\Delta^2}_{4\left(\frac{\lambda}{2}\right)^2\Delta^2} = \lambda^2 d_f^2$$

$$\lambda^2 d_f^2 = (\Sigma^2 - \Delta^2)^2 - 2\left(\frac{\lambda}{2}\right)^2\Sigma^2 - 2\left(\frac{\lambda}{2}\right)^2\Delta^2 + \left(\frac{\lambda}{2}\right)^4$$

$$\Rightarrow \quad d_f = \frac{1}{\lambda}\sqrt{(\Sigma^2 - \Delta^2)^2 - 2(\Sigma^2 + \Delta^2)\left(\frac{\lambda}{2}\right)^2 + \left(\frac{\lambda}{2}\right)^4}.$$

4.30 Cont'd

b.) (1) For obstructed environments, we use a single regression power law model relating path loss to T-R separation. From Table II in the paper, we may use the path loss exponent n=2.56 for transmit antenna height of 8.5 m.

$$PL(d) = (10n)\log_{10}(d) + p_1$$

where p_1 = PL(d_0) is the path loss in dB at the reference distance of d_0 = 1 m (at 1900 MHz, p_1 = 38.0 dB, as given in the paper. The received signal power is found from

$$P_R(dBm) = P_T(dBm) + G_T(dBi) + G_R(dBi) - PL(dB)$$

Since it was given that the transmit and receive antennas are unity gain, $G_T = G_R = 1 = 0dBi$; the transmitted power is $P_T = 1W = 0dBW = 30dBm$ so $P_R(dBm) = 30(dBm) - PL(dB)$. This leads to the following results:

T-R	PL(dB)	P_r(dBm)
50 m	81.49	-51.49
100 m	89.2	-59.2
1000 m	114.8	-84.8

(2) For LOS environments, we use the double regression model. There are two possible models, one in which the break point is fixed at the firstFresnel zone clearance distance, and one in which the break point is determined by a curve fit. Either is acceptable.

$$\text{Fresnel Best-Fit:} \quad PL_1(d) = \begin{cases} (10n_1)\log_{10}(d) + p_1 & \text{for } 1 < d < d_f \\ (10n_2)\log_{10}(d/d_f) & \\ \quad + (10n_1)\log_{10}(d_f) & \\ \quad + p_1 & \text{for } d > d_f \end{cases}$$

$$\text{MMSE Best Fit:} \quad PL_2(d) = \begin{cases} (10n_1^*)\log_{10}(d) + p_1 & \text{for } 1 < d < d_b \\ (10n_2^*)\log_{10}(d/d_b) & \\ \quad + (10n_1^*)\log_{10}(d_b) & \\ \quad + p_1 & \text{for } d > d_b \end{cases}$$

4.30 Cont'd.

Using the following values from Table I of the paper,

	Fresnel Best-Fit				MMSE Best Fit			
	n_1	n_2	σ(dB)	d_f(m)	n_1^*	n_2^*	σ(dB)	d_f(m)
Med (8.5 m)	2.17	3.36	7.88	366	2.14	6.87	7.23	884

leads to the following results:

	Fresnel Best-Fit		MMSE Best Fit	
T-R	PL(dB)	P_r(dBm)	PL(dB)	P_r(dBm)
50 m	74.87	-44.87	74.36	-44.36
100 m	81.4	-51.4	80.8	-50.8
1000 m	108.3	-78.3	104.73	-74.73

c.) Since we are using the log-normal shadowing model, the received signal powers in dBm should fit a Gaussian probability density function, with mean values equal to the path loss values calculated in part b. As a result, we may use the Q-function to determine the likelihood that the received signal will be greater than or equal to 70dBm. However, the Q-function must be normalized to have the desired mean and standard deviation.

From Appendix D of the text, the Gaussian pdf is given by:

$$\Pr(x \geq x_0) = \int_{x_0}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} e^{-(x-m)^2/(2\sigma^2)} dx$$

and may be rewritten through the use of the substitution $y = \frac{x-m}{\sigma}$, giving

$$\Pr\left(y > \frac{x_0 - m}{\sigma} \right) = \int_{\left(\frac{x_0-m}{\sigma}\right)}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-y^2/2} dy .$$

As a result of the definition of the Q-function, we find that

$$\Pr\left(y > \frac{x_0 - m}{\sigma} \right) = Q\left(\frac{x_0 - m}{\sigma} \right) = Q(z) .$$

4.30 Cont'd.

(a) In obstructed environments, for T-R = 50 m, using σ = 7.67 from Table II,

$$\Pr\left[P_r \geq -70dBm\right] = \Pr\left[y > \frac{-70+51.49}{7.67}\right] = Q(-2.41) = 1 - Q(2.41)$$

Using Table F.1 in the text, Q(2.41)≈0.008, so $\Pr\left[P_r \geq -70dBm\right] \approx 0.992$. Thus, there is a 99.2% likelihood that the received signal power will exceed -70dBm. The values for T-R = 100 m and T-R = 1000 m are calculated in a similar manner, leading to the following results:

T-R	$\Pr\left[P_r \geq -70dBm\right]$
50 m	0.992 → 99.2%
100 m	0.921 → 92.1%
1000 m	0.027 → 2.7%

(b) For LOS environments, the likelihood is calculated in the same way as for obstructed environments, except that σ=7.88 for the Fresnel best fit, and σ=7.23 for the MMSE best fit (from Table I of the paper). This leads to the following:

	Fresnel Best-Fit	MMSE Best-Fit
T-R	$\Pr\left[P_r \geq -70dBm\right]$	$\Pr\left[P_r \geq -70dBm\right]$
50 m	0.9993 → 99.93%	0.9998 → 99.98%
100 m	0.9908 → 99.08%	0.9961 → 99.61%
1000 m	0.1587 → 15.87%	0.2581 → 25.81%

d) The overbound model is given by $\sigma_d = e^{0.065PL(d)}$, where σ_d is the rms delay spread in nanoseconds, and $PL(d)$ is the path loss in dB. Using the values calculated for path loss in part 2 leads to the following results:

T-R	OBS σ_d (ns)	Fresnel Best-Fit σ_d (ns)	MMSE Best-Fit σ_d (ns)
50 m	199.71	129.87	125.64
100 m	329.64	198.54	190.95
1000 m	1740.6	1140.82	904.56

4.30 Cont'd.

e.) Let's assume that the noise figure of 6 dB includes noise contributions from both the antenna and the receiver. Then the noise power referred to the receiver input is $N = FkT_0B$, where $k=1.38\times10^{-23}$ J/K is Boltzmann's constant, B is the receiver bandwidth, T_0 is the ambient room temperature, and F is the noise figure. We'll assume that T_0 = 290 K.

For GSM, B = 200 kHz, so

$$N = FkT_0B = (10^{0.6})(1.38\times10^{-23})(290)(200000) = 3.187\times10^{-15}W = -114.97dBm$$

From class notes, GSM has C/I requirement of 13 dB, so the minimum received signal power for GSM is $\mathbf{P_{min}}$ **= -114.97 + 13 = -101.97 dBm.**

For IS-136, B = 30 kHz, so

$$N = FkT_0B = (10^{0.6})(1.38\times10^{-23})(290)(30000) = 4.780\times10^{-16}W = -123.21dBm$$

From class notes, IS-136 has C/I requirement of 14 dB, so the minimum received signal power for IS-136 is $\mathbf{P_{min}}$ **= -123.21 + 14 = -109.21 dBm.**

All of the received signal power values calculated in part 2 for both obstructed and LOS environments are substantially higher than these minimum required values, so the receivers will not be noise-limited, at least up to 1 km distance. Therefore, **if we assume that thermal noise is the only limiting factor, it will be possible to receive either GSM or IS-136 signals at these distances and in these environments.**

Now, let's consider the limitation caused by the time delay spread. The probler statement says that we should assume that an adaptive equalizer is not neede mobile receiver when the symbol duration is greater than 10 times therms d spread. So, given our *estimated* values for rms delay spread calculated i need to determine the minimum symbol duration, or maximum symbo so that we don't have to use an equalizer → **[symbol duration]$_{min}$ = 10** leads to the following requirements:

4.30 Cont'd.

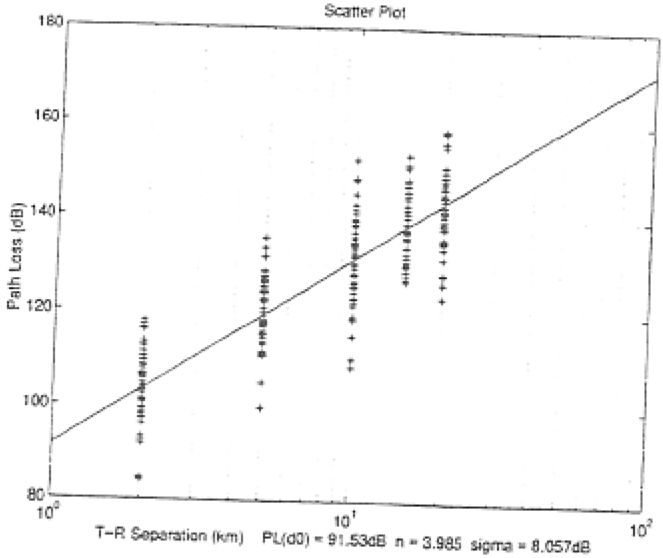
Environment	T-R	σ_τ (ns)	Min. signal period (ns)	Max. Symbol Rate (symbols/s)
OBS	50 m	199.71	1997.1	500,726
	100 m	329.64	3296.4	303,361
	1000 m	1740.63	17406.3	57,451
LOS Fresnel Best-Fit	50 m	129.87	1298.7	770,001
	100 m	198.54	1985.4	503,677
	1000 m	1140.82	11408.2	87,656
LOS MMSE Best-Fit	50 m	125.64	1256.4	795,925
	100 m	190.95	1909.5	523,697
	1000 m	904.56	9045.6	110,551

From Table 11.12 of the text, we find that the data rate for GSM is 270,833 bits/sec, and the data rate for IS-136 is 48,600 bits/sec. GSM uses one bit per symbol, so its symbol rate and data rate are the same. IS-136 uses $\pi/4$ DQPSK, so it has 2 bits per symbol, and its symbol rate is half the data rate, or 24,300 symbols/sec.

If we compare the IS-136 symbol rate against the values calculated for the maximum symbol rate allowable without an equalizer, we find that the symbol rate for GSM is less than the maximum symbol rate for all T-R separations in all environments. Therefore, it will be possible to receive IS-136 signals in all environments at all T-R separations without an equalizer.

Comparing the GSM symbol rate against the maximum symbol rate allowable without an equalizer, we find that the symbol rate for GSM is less than the maximum symbol rate for all T-R separations in all environments**EXCEPT** for a T-R separation of 1 km. We find that for a T-R separation of 1 km, the actual symbol rate used in GSM is higher than the maximum symbol rate allowable without an equalizer. Therefore, we must use an equalizer to receive GSM signals in all environments at a T-R separation of 1 km. For T-R separations of 50 m and 100 m, an equalizer is not required for reception of GSM signals.

4.31 See the MATLAB program p4_31.m and the following figure.



4.32 See the MATLAB program p4.32.m

4.33 a) Given total bandwidth $B_{W tot} = 30 MHz$ (one-way)

channel bandwidth $B_W = 200 KHZ$

$\Rightarrow$ total radio channel $S = \frac{B_{W tot}}{B_W} = \frac{30 \times 10^6}{200 \times 10^3} = \underline{\underline{150}}$

b) number of users that can be served by a base Station during fully loaded operation $= 8 \times 64 = \underline{\underline{512}}$

c) Given $n=4$, $\sigma = 8 dB$, $U(\gamma) = 0.9$, from Figure 4.18, we have $P_\gamma(R) \doteq 0.73$, where $\gamma = -90 dBm$.

$$P_\gamma(R) = Pr[P_r(R) > \gamma] = Q\left[\frac{\gamma - \overline{P_r(R)}}{\sigma}\right] = 0.73$$

$$\Rightarrow \frac{\overline{P_r(R)} - \gamma}{\sigma} \doteq 0.61 \Rightarrow \overline{P_r(R)} = 0.61\sigma + \gamma = 0.61 \times 8 + (-90)$$
$$= -85.12 dBm$$

$$\Rightarrow \overline{PL(R)} = P_t(dBm) - \overline{P_r(R)}(dBm)$$
$$= 10\log_{10}(20/10^{-3}) - (-85.12) \doteq 128.13 dB$$

Assume $d_0 = 1Km$, $G_r = 0dB = 1$, miscellaneous loss $L = 0dB = 1$, given $G_t = 10dB = 10$, $f_c = 1960 MHZ$ (corresponding to the largest $\overline{PL}(d_0)$), we have $\lambda = \frac{c}{f_c} \doteq 0.153 (m)$

$$\Rightarrow \overline{PL(d_0)} = -10\log_{10}\left[\frac{G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d_0^2}\right] = -10\log_{10}\left[\frac{10 \times 1 \times (0.153)^2}{(4\pi)^2 \times (1000)^2}\right]$$
$$\doteq 88.3 dB$$

Since $\overline{PL(R)} = \overline{PL(d_0)} + 10 \cdot n \log_{10}\frac{R}{d_0}$, for $n=4$, we have

$$R = 10^{\frac{\overline{PL(R)} - \overline{PL(d_0)}}{10n}} \cdot d_0$$
$$= 10^{\frac{128.13 - 88.3}{40}} \cdot 1 = 9.88 (Km)$$

$\boxed{4.33}$ Cont'd

$$\Rightarrow N = \frac{\text{Total Area}}{2.6R^2} = \frac{2500}{2.6 \times 9.88^2} \doteq \underline{\underline{10}}$$

(d) Let's first relate the channel number to it's center frequency

	Channel Number	Center Frequency (MHz)
Reverse Channel	$1 \le N \le 150$	$0.2N + 1849.9$
Forward Channel	$1 \le N \le 150$	$0.2N + 1929.9$

We can see that the forward and reverse channel in each pair are separated by 80 MHz

Since the cluster size is equal to 4, we divide the total 150 radio channel into $4 \times 3 = 12$ subsets. The following chart illustrates these 12 subsets.

1A	2A	3A	4A	1B	2B	3B	4B	1C	2C	3C	4C
1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
133	134	135	136	137	138	139	140	141	142	143	144
145	146	147	148	149	150						

It can be seen from the chart that each subset contains about 12 channels. In each subset, the closest adjacent channel is 12 channels away. In a four cell reuse system, each cell uses channel chosen from the subsets. $iA + iB + iC$. Where i is an integer from 1 to 4. Therefore each cell can use about 37 channels. The adjacent channel

4.33 Cont'd

distance is 4 channels. Now we can describe the channel reuse scheme for each cell in the city

Cell Number: 10

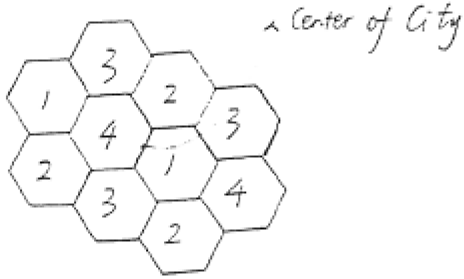
Number of channels used by each cell:

cell 1: 38 cell 2: 38 cell 3: 37 cell 4: 37

Cluster Size: $N=4$

Nearest reuse distance: $D=\sqrt{3N}\cdot R=\sqrt{12}\cdot R=3.46\times 9.88 \doteq 34.2\text{Km}$

The distribution of the 10 cells over the city is shown in the following figure. $(N=i^2+i\cdot j+j^2=4 \Rightarrow i=2,\ j=0)$



e) i) Ten cells are available throughout the entire city

ii) Total radio channels = cluster size $\times S = 4\times 150 = 600.$

iii) Total User Channels = total radio channels $\times$ 8users/channel
$= 600\times 8 = 4800$

f) Total cost of base station $=10\times 500{,}000 = \$5\times10^6$

Total cost of radio channel $= 600\times 50{,}000 = \$3\times 10^7$

Total cost of system $= 5\times10^6 + 3\times10^7 = \$3.5\times10^7$

4.33 Cont'd

g) Given Probability of Blocking = 5%

Number of user channels per cell: $C = 38 \times 8 = 304$

Assume traffic intensity per user $A_u = uH = 3 \times \frac{2}{60} = 0.1$ Erlangs.

$\Rightarrow$ number of users that can be supported per cell at start-up $= U = A/A_u = 300/0.1 = \underline{\underline{3000}}$

$\Rightarrow$ maximum number of subscribers that can be supported at start up $User_{number} = U \cdot Cell_{number} = 3000 \times 10 = \underline{\underline{3 \times 10^4}}$

h) cost per user $= \dfrac{3.5 \times 10^7 \times 10\%}{3 \times 10^4} \doteq \underline{\underline{\$116.67}}$

4.34 Given $P_t = 10W$, $G_t = 6dB = 4$, $G_r = 3dB = 2$,

$\lambda = \frac{c}{f_c} = \frac{300}{890} \doteq 0.337m$, $d_0 = 1Km$, we have

$$P_r(d_0) = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d_0^2} = \frac{10 \times 4 \times 2 \times 0.337^2}{(4\pi)^2 \times (1000)^2} = 5.76 \times 10^{-8} (W)$$

$$\doteq -42.4 \text{ dBm}$$

At the boundary of B_1,

$$P_r(R) = P_r(d_0) - 25\log_{10}\left(\frac{R}{d_0}\right) = -42.4 - 25\log_{10}\left(\frac{R}{d_0}\right)$$

$$d_2 = \sqrt{(3.3R)^2 + \left(\frac{\sqrt{3}}{2}R\right)^2} \doteq 3.6R$$

$$\Rightarrow \overline{P_r(d_2)} = -42.4 - 40\log_{10}\left(\frac{3.6R}{d_0}\right) = -42.4 - 40\log_{10}\left(\frac{R}{d_0}\right) - 22.25$$

$$\Rightarrow \overline{C/I} = P_r(R) - \overline{P_r(d_2)}$$

$$= -42.4 - 25\log_{10}\left(\frac{R}{d_0}\right) - \left[-42.4 - 40\log_{10}\left(\frac{R}{d_0}\right) - 22.25\right]$$

$$= 22.25 + 15\log_{10}\left(\frac{R}{d_0}\right)$$

4.34 Cont'd

The variance of C/I is equal to $\sigma = 7\,dB$

$$\Rightarrow P_r\left[\frac{C}{I} > 18dB\right] = Q\left[\frac{18-\overline{C/I}}{\sigma}\right]$$

$$= Q\left[\frac{18-(22.25+15\log_{10}(\frac{R}{d_0}))}{7}\right] > 99\%$$

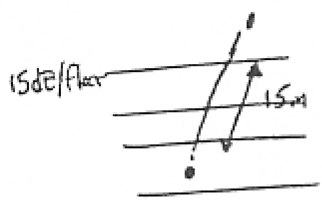
$$\Rightarrow \frac{18-[22.25+15\log_{10}(\frac{R}{d_0})]}{7} < -2.33$$

$$\Rightarrow -15\log_{10}\left(\frac{R}{d_0}\right) < -12.03$$

$$\Rightarrow \frac{R}{d_0} > 6.34 \quad \Rightarrow R > 6.34\,d_0 = 6.34\,Km$$

$$\Rightarrow R_{min} = \underline{\underline{6.34\,Km}}$$

$\boxed{4.35}$



$$PL(d)\,dB = 40 + 20\log(15) + \sum_{i=1}^{3} 15$$

$$PL(d)\,dB = 40 + 20\log 15 + 45$$

$$PL(d)\,dB = 85 + 20[1.2] = 85 + 24 = \underline{\underline{109\,dB}}$$

$$P_r = P_t - PL = 20dBm - 109\,dB$$

$$= \underline{\underline{-89\,dBm}}$$

CHAPTER 5

$\boxed{5.1}$ $c = \lambda f \Rightarrow \lambda = \frac{c}{f} = \frac{3\,10^8 m/s}{1.95\,10^9 Hz} = 0.154\,m$

$f_d = \frac{v}{\lambda}\cos\theta \qquad f_{d_{max}} = \frac{v}{\lambda}\ ;\ -f_{d_{max}} = -\frac{v}{\lambda}$

$v = 1\,km/hr \Rightarrow v = 0.278\,m/s \Rightarrow f_d = 1.8\,Hz$

$v = 5\,km/hr \Rightarrow v = 1.39\,m/s \Rightarrow f_d = 9.03\,Hz$

$v = 100\,km/hr \Rightarrow v = 27.8\,m/s \Rightarrow f_d = 180.5\,Hz$

$v = 1000\,km/hr \Rightarrow v = 278\,m/s \Rightarrow f_d = 1805\,Hz$

$\therefore$ @ 1 km/hr, spectral edges are 1949.9999982 mHz
and 1950.0000018 mHz

@ 5km/hr, spectral edges are 1949.99999097 mHz
and 1950.00000903 mHz

@ 100km/hr, edges are 1949.9998195 mHz
and 1950.0001805 mHz

@ 1000 km/hr, edges are 1949.998195 mHz
1950.001805 mHz

5.2 (a) A 0 Hz (No Doppler shift) condition occurs when either 1) the receiver, the channel and the transmitter are all not moving; or 2) when the receiver is moving along a line that is exactly perpendicular to a line drawn between the transmitter and receiver. Note that the perpendicular direction implied two different directions may be traveled by the receiver to obtain 0 Hz Doppler.

(b) Positive maximum Doppler occurs when the receiver is moving in a straight line towards the signal source (where the angle between the line drawn from the receiver to transmitter and the line of receiver travel is zero).

(c) Maximum negative Doppler occurs when the receiver is moving in a straight line away from the signal source (where the angle between the line drawn from the transmitter to receiver and the line of the receiver travel is pi).

(d) Positive half-maximum Doppler occurs for two different physical conditions, where cosine of the angle between transmitter and receiver is = 0.5 (when the angle made between the receiver's line of travel and the line drawn from the receiver to the transmitter is either +60 or -60 degrees). This shows that the Doppler frequency is not unique for a given direction of travel, except for the maximum and minimum Doppler shift cases.

5.3 Show that if x, h, and y are BP signals,

$$\begin{aligned} x(t) &= Re\left[c(t)\ e^{\jmath\omega_c t}\right] \\ h(t) &= Re\left[h_b(t)\ e^{\jmath\omega_c t}\right] \\ y(t) &= Re\left[r(t)\ e^{\jmath\omega_c t}\right] \end{aligned} \tag{11}$$

then,

$$r(t) = \frac{1}{2}c(t) \star h_b(t). \tag{12}$$

We know

$$y(t) = x(t) \star h(t) \Leftrightarrow Y(f) = X(f)H(f) \tag{13}$$

$$x(t) = Re\left[c(t)\ e^{\jmath\omega_c t}\right] = \frac{1}{2}c(t)\ e^{\jmath\omega_c t} + \frac{1}{2}c^*(t)\ e^{-\jmath\omega_c t} \tag{14}$$

$$\begin{aligned} X(f) &= \mathcal{F}[x(t)] \\ &= \frac{1}{2}\mathcal{F}[c(t)\ e^{\jmath\omega_c t}] + \frac{1}{2}\mathcal{F}[c^*(t)\ e^{-\jmath\omega_c t}] \end{aligned} \tag{15}$$

Note that

$$\mathcal{F}[c^*(t)] = C^*(-f) \text{ (from F.T Theory, use complex signal translation)} \tag{16}$$

$$X(f) = \frac{1}{2}[C(f - f_c) + C^*(-(f + f_c))] \tag{17}$$

and it follows

$$H(f) = \frac{1}{2}[H_b(f - f_c) + H_b^*(-f - f_c)] \tag{18}$$

5.3 Cont'd.

Now, look at $Y(f) = X(f)H(f)$ and multiply

$$Y(f) = \frac{1}{4}[C(f-f_c) + C^*(-f-f_c)] \times [H_b(f-f_c) + H_b^*(-f-f_c)] \tag{19}$$

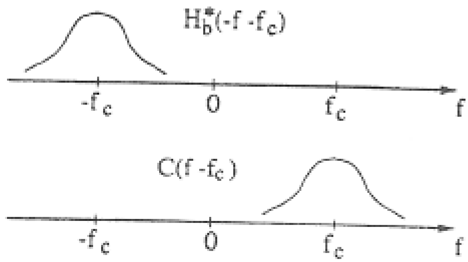


Figure 1: Spectra $H_b^*(-f-f_c)$ and $C(f-f_c)$ do not overlap.

Now note that spectra do not overlap when $C(f-f_c) \times H_b^*(-f-f_c) = 0$ (see Figure 1). Likewise, $C^*(-f-f_c) \times H_b(f-f_c) = 0$. Thus

$$\begin{aligned} Y(f) &= \frac{1}{4}[C(f-f_c)H_b(f-f_c) + C^*(-f-f_c) \times H_b^*(-f-f_c)] \\ &= \frac{1}{2}\left[\frac{1}{2}C(f-f_c)H_b(f-f_c) + \frac{1}{2}C^*(-f-f_c) \times H_b^*(-f-f_c)\right] \end{aligned} \tag{20}$$

Now note

$$\begin{aligned} y(t) &= Re\left[r(t)\, e^{\jmath\omega_c t}\right] \\ &= \frac{1}{2}r(t)\, e^{\jmath\omega_c t} + \frac{1}{2}r^*(t)\, e^{-\jmath\omega_c t} \end{aligned} \tag{21}$$

$$\mathcal{F}[y(t)] = \frac{1}{2}R(f-f_c) + \frac{1}{2}R^*(-f-f_c) \tag{22}$$

where $R(f) = \frac{1}{2}\, C(f)H(f)$. Thus

$$Y(f) = \frac{1}{2}\left[R(f-f_c) + R^*(-f-f_c)\right] \tag{23}$$

$$y(t) = \frac{1}{2}c(t) * h_b(t). \qquad \text{QED} \tag{24}$$

$\boxed{5.4}$ The block diagram of a binary spread spectrum sliding correlator multipath measurement system is shown in Fig. 5.7. Section 5.3.2 illustrates how it is used to measure power delay profiles.

(a) Given $T_c = 10ns$, $\ell = 1023$, $\alpha - \beta = 30kHz$, we have

$$\alpha = \frac{1}{T_c} = \frac{1}{10ns} = 10^8\,Hz$$

Slide factor $\gamma = \frac{\alpha}{\alpha-\beta} = \frac{10^8}{30\times10^3} \doteq 3333.3$

$\Rightarrow$ time between maximal correlation

$$\Delta T = \gamma \cdot \ell \cdot T_c = 3333.3 \times 1023 \times 10ns = \underline{\underline{34\ mS}}$$

(b) Sweep Setting $= \frac{\Delta T}{10} = 3.4\ ms/div$

$\therefore$ practical oscilloscope setting $= \underline{\underline{5ms/div}}$

(c) required IF passband bandwidth $Bw_s = 2(\alpha-\beta) = \underline{\underline{60\ KHZ}}$

The time resolution $\Delta\tau$ of this system is about

$$\Delta\tau = 2T_c = 20\,ns \quad (\text{or } 10ns \text{ rms width})$$

For a direct pulse system with similar time resolution, the passband bandwidth is $Bw_d = \frac{2}{\Delta\tau} = 100MHZ$.

Comparing the passband bandwidth of these two system, we can see $Bw_s \ll Bw_d$, that means using a spread Spectrum System, we can reject much of the passband noise and interference, thus improving the coverage range for a given transmitter power.

$\boxed{5.5}$ $B_c \approx \frac{1}{5\sigma_\tau} \approx 2\, B_{baseband} \geq \frac{2}{T_s}$

for flat fading

$$\therefore \frac{2}{T_s} \leq \frac{1}{5\sigma_\tau}$$

$$\therefore T_s \geq 10\,\sigma_\tau \text{ for flat fading}$$

5.6 For (a), $\bar{\tau} = \dfrac{1\times 0 + 1\times 50 + 0.1\times 75 + 0.01\times 100}{1+1+0.1+0.01} \doteq 27.725\,(ns)$

$$\overline{\tau^2} = \frac{1\times 0 + 1\times 50^2 + 0.1\times 75^2 + 0.01\times 100^2}{1+1+0.1+0.01} \doteq 1498.8\,(ns^2)$$

$\Rightarrow$ the rms delay spread $\sigma_\tau = \sqrt{1498.8 - 27.725^2} \doteq 27\,(ns)$

Since $\dfrac{\sigma_\tau}{T_s} \le 0.1$, $\quad T_s \ge 10\,\sigma_\tau = 270\,ns$

$\Rightarrow$ Smallest symbol period $T_{s\,min} = \underline{\underline{270\,ns}}$

greatest data rate $\quad R_{max} = \dfrac{1}{T_{s\,min}} \doteq \underline{\underline{3.7\,Mbps}}$

For (b), $\bar{\tau} = \dfrac{0.01\times 0 + 0.1\times 5 + 1\times 10}{0.01+0.1+1} \doteq 9.46\,(\mu s)$

$$\overline{\tau^2} = \frac{0.01\times 0 + 0.1\times 5^2 + 1\times 10^2}{0.01+0.1+1} = 92.34\,(\mu s^2)$$

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2} = \sqrt{92.34 - (9.46)^2} \doteq 1.688\,(\mu s)$$

$T_{s\,min} = 10\,\sigma_\tau = \underline{\underline{16.88\,(\mu s)}} \qquad R_{max} = \dfrac{1}{T_{s\,min}} \doteq \underline{\underline{59.25\,kbps}}$

5.7 (a) $T_s = \dfrac{1}{100\,kbps} = 10^{-5}\,s.$ $\qquad \boxed{0 \le \sigma_\tau \le 10^{-6}\,s}$

if $\sigma_\tau \le \frac{1}{10} T_s \Rightarrow$ flat fading

$$T_s \ge 10\,\sigma_\tau \qquad \sigma_\tau \le \frac{T_s}{10}$$

(b) $f_d = \dfrac{v}{\lambda}$

$c = \lambda f$

$\lambda = c/f = 3\,10^8 / 5.8\,10^9 \approx \underline{\underline{0.05\,m}}$

$$\text{velocity} = \frac{30\text{ miles}}{hr} \cdot \frac{1\,hr}{3600\,sec} \cdot \frac{5280'}{1\text{ mile}} \cdot \frac{12''}{1'} \cdot \frac{2.54\,cm}{1''} \cdot \frac{1\,m}{100\,cm}$$

$$= \frac{30 \cdot 5280 \cdot 12 \cdot 2.54}{3600 \cdot 100} = 13\ m/s$$

$$f_d = \frac{13}{0.05} = \boxed{260\,Hz}$$

5.7 Cont'd.

Coherence Time Define $\begin{bmatrix} 90\% \\ 50\% \end{bmatrix}$

$T_c \approx \frac{1}{f_m} = \boxed{0.004s}$

$T_c \approx \frac{9}{16\pi f_m} = \frac{1}{5 f_m} = \boxed{0.00125}$

(c) Here we have $f_d = 200Hz$; $T_s \simeq 10^{-5}s$; $T_c \approx 10^{-3}s$

slow fading $\Rightarrow T_s << T_c$ here $10^{-5} << 10^{-3}$

$\boxed{\text{slow fading}}$

d) pick your T_c, then

$\#$ bits sent $= R_b \cdot T_s = \frac{10^5 b}{s} \cdot 10^{-3} s \simeq \underline{\underline{100}}$

$\boxed{5.8}$ For (a), the 90% correlation coherence bandwidth is

$B_{c0.9} \doteq \frac{1}{50\sigma_\tau} = \frac{1}{50 \times 27(nS)} \doteq \underline{\underline{740\ KHZ}}$

the 50% correlation coherence bandwidth is

$B_{c0.5} \doteq \frac{1}{5\sigma_\tau} = \underline{\underline{7.4\ MHZ}}$

For (b), $B_{c0.9} = \frac{1}{50\sigma_\tau} = \frac{1}{50 \times 1.688(\mu S)} \doteq \underline{\underline{11.85\ KHZ}}$

$B_{c0.5} = \frac{1}{5\sigma_\tau} \doteq \underline{\underline{118.5\ KHz}}$

$\boxed{5.9}$ For a binary modulated signal,

Symbol period $T_s = \frac{1}{\text{bit rate}} = \frac{1}{R} \Rightarrow T_s = \frac{1}{25 \times 10^3} = 40(\mu s)$

$\Rightarrow \sigma_\tau \leq 0.1 T_s = 0.1 \times 40 = \underline{\underline{4\ (\mu s)}}$

5.9 Cont'd

For an 8-PSK system, Symbol period $T_s = \frac{3}{\text{bit rate}} = \frac{3}{R}$

$$\Rightarrow T_s = \frac{3}{75\times10^3} = 40\ (\mu s)$$

$$\Rightarrow \sigma \le 0.1\, T_s = 0.1\times 40 = \underline{\underline{4\ (\mu s)}}$$

$\boxed{5.10}$

$$\sqrt{2\pi}\, f_m \times e^{-\rho^2} + \left[\sqrt{2\pi}\, f_m \rho\right]\left[-2\rho e^{-\rho^2}\right] = 0$$

$$\sqrt{2\pi}\, f_m \left[e^{-\rho^2} + -2\rho^2 e^{-\rho^2}\right] = 0$$

$$1 = 2\rho^2$$

$$\rho = \sqrt{\tfrac{1}{2}} = 0.707$$

$\boxed{5.11}$ $P(r<R) = \int_{-\infty}^{R} p(r)dr$

$$= \int_0^R \frac{r}{\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2}\right) dr$$

$$= \frac{1}{2}\int_0^R \frac{1}{\sigma^2} \exp\left(\frac{-r^2}{2\sigma^2}\right) dr^2$$

$$\overset{t=r^2}{=} \frac{1}{2}\int_0^{R^2} \frac{1}{\sigma^2} \exp\left(\frac{-t}{2\sigma^2}\right) dt$$

$$= \frac{1}{2}\cdot\frac{1}{\sigma^2}(-2\sigma^2)\cdot \exp\left(\frac{-t}{2\sigma^2}\right)\Big|_0^{R^2}$$

$$= 1 - \exp\left(\frac{-R^2}{2\sigma^2}\right)$$

For $\rho = -10dB = 0.316$, the percentage of time that a signal is 10dB or more below the rms value for a Rayleigh fading signal is $P_{o,1} = 1-\exp(-\rho^2) = 1-\exp(-0.316^2) \doteq \underline{\underline{9.5\%}}$

$\boxed{5.12}$ (a) Since $N_R = \sqrt{2\pi}\, f_m \cdot \rho e^{-\rho^2}$, the ratio of the desired signal level to rms signal level that maximizes N_R is the solution of the equation

$$\frac{dN_R}{d\rho} = 0 \Rightarrow \sqrt{2\pi}\, f_m \cdot (1-2\rho^2)\cdot e^{-\rho^2} = 0$$

$$\Rightarrow 1-2\rho^2 = 0 \Rightarrow \underline{\underline{\rho = \frac{1}{\sqrt{2}} = -3dB}}$$

(b) $\lambda = \frac{c}{f_c} = \frac{3\times 10^8}{900\times 10^6} \doteq 0.33\,(m)$ $\quad V = \frac{50\times 10^3 m}{3600\, sec} \doteq 13.89\, m/s$

$\Rightarrow f_m = \frac{V}{\lambda} = \frac{13.89}{0.33} \doteq 41.67\,(HZ)$

5.12 Cont'd

From (a), we have $\rho = \frac{1}{\sqrt{2}}$

$\Rightarrow N_R = \sqrt{2\pi}\cdot f_m \cdot \rho \cdot e^{-\rho^2} = \sqrt{2\pi} \times 41.67 \times \frac{1}{\sqrt{2}} \cdot e^{-\frac{1}{2}} = 44.8$ (crossings/s)

$\Rightarrow$ the maximum number of times the signal envelop will fade below the level found in (a) during a 1 minute test $= N_R \cdot t = 44.8 \times 60 = \underline{\underline{2688}}$

(c) $\bar{\tau} = \dfrac{e^{\rho^2}-1}{\rho f_m \sqrt{2\pi}} = \dfrac{e^{\frac{1}{2}}-1}{\frac{1}{\sqrt{2}} \times 41.67 \times \sqrt{2\pi}} \doteq \underline{\underline{8.8\ (ms)}}$

$\boxed{5.13}$ For $\rho = -10dB \doteq 0.316$,

$\bar{\tau} = \dfrac{e^{\rho^2}-1}{\rho f_m \sqrt{2\pi}} \Rightarrow f_m = \dfrac{e^{\rho^2}-1}{\rho \cdot \bar{\tau} \sqrt{2\pi}} = \dfrac{e^{0.316^2}-1}{0.316 \times 10^{-3} \times \sqrt{2\pi}} \doteq 132.8\ (HZ)$

$f_m = \frac{v}{\lambda} \Rightarrow V = f_m \cdot \lambda = f_m \cdot \frac{c}{f_c} = 132.8 \times \frac{1}{3} \doteq 44.3\ (m/s)$

$\Rightarrow d = V \cdot t = 44.3 \times 10 = \underline{\underline{443\ m}}$

For $\rho = 1$, $N_R = \sqrt{2\pi} \cdot f_m \cdot \rho \cdot e^{-\rho^2} = \sqrt{2\pi} \times 132.8 \times 1 \times e^{-1} \doteq 122.4$ (crossings/s)

$\Rightarrow$ Total number of fade the signal undergo at the rms threshold level during a 10 second interval $= N_R \cdot t = \underline{\underline{408}}$

$\boxed{5.14}$ From Fig. P5.14, we have

$$v(t) = \begin{cases} 3t & 0 \le t < 10 \\ 30 & 10 \le t < 90 \\ 300-3t & 90 \le t < 100 \end{cases}$$

$\lambda = \frac{c}{f_c} = \frac{3 \times 10^8}{900 \times 10^6} \doteq 0.33\ (m)$

For $\rho = 0.1$, $T = 100$ Second, we have

$$N_R = \frac{1}{T}\int_0^T \sqrt{2\pi} f_m \cdot \rho \cdot e^{-\rho^2} dt$$

5.14 Cont'd

$$= \frac{1}{T}\int_0^T \sqrt{2\pi}\cdot\frac{V(t)}{\lambda}\cdot\rho\cdot e^{-\rho^2}dt$$

$$= \frac{1}{T}\cdot\frac{\sqrt{2\pi}}{\lambda}\cdot\rho\cdot e^{-\rho^2}\cdot\int_0^T V(t)\,dt$$

$$= \frac{1}{100}\times\frac{\sqrt{2\pi}}{0.33}\times 0.1\times e^{-0.1^2}\cdot\left[2\times\int_0^{10}3t\,dt+\int_{10}^{90}30\,dt\right]$$

$$= \frac{1}{100}\times\frac{\sqrt{2\pi}}{0.33}\times 0.1\times e^{-0.1^2}\times 2700$$

$$\doteq 20.1 \text{ (crossings/s)}$$

$$\bar{\tau} = \frac{1}{N_R}\cdot Pr[r\le R] = \frac{1-e^{-\rho^2}}{N_R} = \frac{1-e^{-0.01}}{20.1} \doteq 4.95\times10^{-4}(s)$$

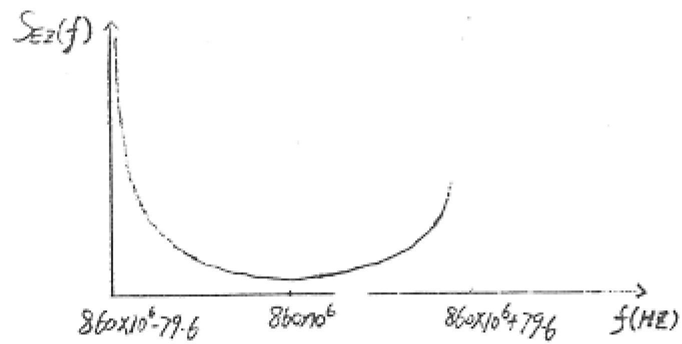
$$= 0.495\,(ms)$$

$\boxed{5.15}$ (a) $\lambda = \frac{c}{f_c} = \frac{3\times10^8 m/s}{860\times10^6 Hz} \doteq 0.349\,(m)$

$$V = \frac{100\times10^3 m}{3600\ sec} \doteq 27.78\,(m/s)$$

$$\Rightarrow f_m = \frac{V}{\lambda} = \frac{27.78}{0.349} \doteq 79.6\ HZ$$

The Doppler spectrum is shown as follows.



(b) For $\rho = -20dB = 0.1$,

$$N_R = \sqrt{2\pi}\cdot f_m\cdot\rho\cdot e^{-\rho^2} = \sqrt{2\pi}\times79.6\times0.1\times e^{-0.01} \doteq 19.7 \text{ (crossings/s)}$$

5.15 Cont'd

$$\bar{\tau} = \frac{e^{\rho^2}-1}{\rho f_m \sqrt{2\pi}} = \frac{e^{0.01}-1}{0.1 \times 79.6 \times \sqrt{2\pi}} \doteq \underline{\underline{0.5\,(ms)}}$$

$\boxed{5.16}$ When $\sigma/T_s \le 0.1$, no equalizer is required.

<u>For USDC</u> using $\frac{\pi}{4}$ DQPSK modulation.

Symbol period $T_s = \frac{2}{\text{RF Data Rate}} = \frac{2}{48.6 \times 10^3} \doteq 41.2\,(\mu s)$

$\Rightarrow \sigma_{max} = 0.1\,T_s = 0.1 \times 41.2 = \underline{\underline{4.12\,(\mu s)}}$

<u>For GSM</u> using 0.3 GMSK modulation,

$$T_s = \frac{1}{0.3 \times \text{RF Data Rate}} = \frac{1}{0.3 \times 270833 \times 10^3} \doteq 12.3\,(\mu s)$$

$\Rightarrow \sigma_{max} = 0.1\,T_s = 0.1 \times 12.3 = \underline{\underline{1.23\,(\mu s)}}$

<u>For DECT</u> using 0.3 GMSK modulation,

$$T_s = \frac{1}{0.3 \times \text{RF Data Rate}} = \frac{1}{0.3 \times 1152 \times 10^3} \doteq 2.89\,(\mu s)$$

$\Rightarrow \sigma_{max} = 0.1\,T_s = 0.1 \times 2.89 = \underline{\underline{0.289\,(\mu s)}}$

$\boxed{5.17}$ (a) $\cos(a+b) = \cos a \cos b - \sin a \sin b$,

Rewrite $E_z(t) = E_0 \sum_{n=1}^{\infty} C_n \cos(2\pi f_c t + \theta_n)$ (1)

$$= \underbrace{E_0 \sum_{n=1}^{\infty} C_n \cos(2\pi f_c t + \phi_n)}_{T_c(t)} \cos 2\pi f_c t - \underbrace{E_0 \sum_{n=1}^{\infty} C_n \sin(2\pi f_n t + \phi_n)}_{T_s(t)} \sin 2\pi f_c t$$

5.17 Cont'd.

(b) The ϕ_n are independent and identically distributed uniformly on $(0, 2\pi]$

$$p(\phi_n) = \frac{1}{2\pi}$$

$$0 \le \phi_n < 2\pi$$

(c) The C_n are random variables

(d) $\overline{|E_z(t)|^2} = \overline{E_0^2 \sum_{n=1}^{\infty} C_n^2 \cos^2(2\pi f_c t + \theta_n)}$ from (1)

$$= E_0^2 \sum_{n=1}^{\infty} \overline{C_n^2 [1/2 + 1/2 \cos 2[2\pi f_c t + \theta_n]]}$$

assume C_n are independent of θ_n (i.e. ϕ_n)

$$= E_0^2 \sum_{n=1}^{\infty} \overline{\cancel{C_n}^2}\left[\overline{1/2 + 1/2 \cos(2\cdot[2\pi f_c t + \theta_n])}\right]$$

$$\therefore \overline{|E_z(t)|^2} = \frac{E_0^2}{2} + \underbrace{\int_0^{2\pi} 1/2 \cos[4\pi f_c t + 4\pi f_n t + 2\phi_n]\cdot\left(\overset{p(\phi_n)}{\frac{1}{2\pi}}\right) d\phi_n}_{=0}$$

(e) Large sum of components causes $E_z(t)$ to have Gaussian distribution Envelope of a Gaussian yields a Rayleigh distribution

$$|E_z(t)| = r \Rightarrow p(r) = \frac{r}{\sigma^2} e^{-r^2/2\sigma^2} \quad r \ge 0;\ p(r) = 0;\ r < 0$$

(f) Rician distributed fading (also spelled Ricean)

(g) $K = \frac{A^2/2}{\sigma^2} = \frac{A^2}{2\sigma^2}$

where $\sigma^2 = \frac{E_0^2}{2} = \underbrace{\overline{|E_z|^2}}_{\text{from (d)}} - \overset{0}{\cancel{\overline{|E_z|}^2}}$

Here $A = 5E_0$ given $K = \frac{(5E_0)^2}{2\left(\frac{E_0^2}{2}\right)} = 25$

$\boxed{5.18}$ For a $\frac{5}{8}\lambda$ vertical monopole, $G(\alpha)=1.75$, thus

$$S_{Ez}(f)=\frac{1.75}{\pi f_m\sqrt{1-\left(\frac{f-f_c}{f_m}\right)^2}}$$

The RF Doppler spectrum and the corresponding baseband spectrum out of an envelop detector are shown in Fig. 5.20 and Fig. 5.21 respectively.

$\boxed{5.19}$ Consider two independent, zero-mean Gaussian random variables with equal variance σ^2. Their joint density function is given by:

$$f_{xy}(X,Y)=\left[\frac{1}{\sqrt{2\pi\sigma^2}}\exp\left(-\frac{X^2}{2\sigma^2}\right)\right]\cdot\left[\frac{1}{\sqrt{2\pi\sigma^2}}\cdot\exp\left(-\frac{Y^2}{2\sigma^2}\right)\right]$$

$$=\frac{1}{2\pi\sigma^2}\exp\left(-\frac{X^2+Y^2}{2\sigma^2}\right)$$

Let Z represent the magnitude (envelope) of the sum of two independent identically distributed complex (quadrature) Gaussian sources, we have $Z=\sqrt{X^2+Y^2}$. The cumulative distribution of Z is given by

$$F_z(Z)=P_r(z<Z)$$

$$=\iint_{\sqrt{X^2+Y^2}<Z} f_{xy}(X,Y)\,dX\,dY$$

$$\overset{\substack{x=r\cos\theta\\ y=r\sin\theta}}{=}\int_0^{2\pi}\int_0^{r} r\cdot\frac{1}{2\pi\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right)dr\,d\theta$$

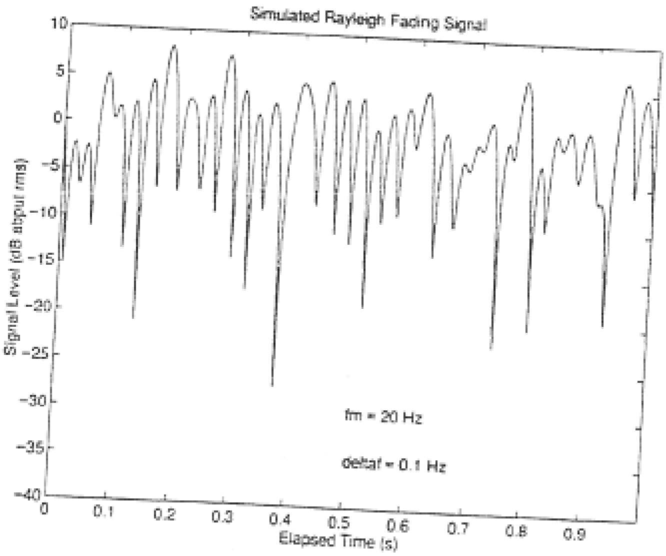
$$=1-\exp\left(-\frac{Z^2}{2\sigma^2}\right),\quad Z\geq 0$$

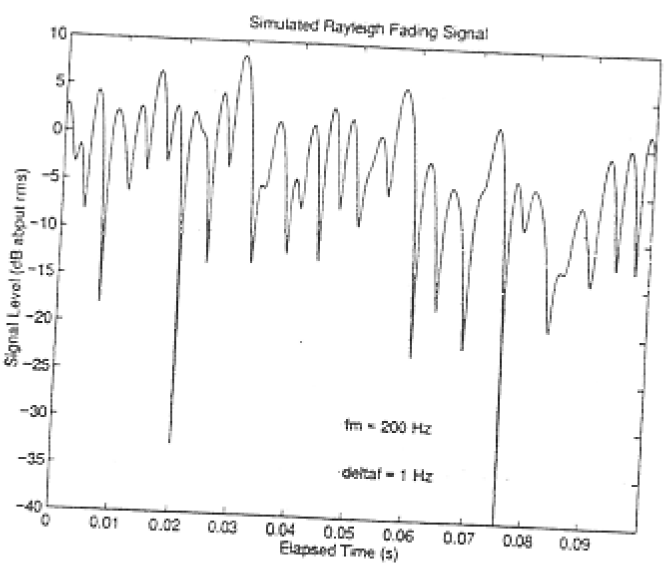
Hence $f_z(Z)=\frac{Z}{\sigma^2}\exp\left(-\frac{Z^2}{2\sigma^2}\right),\quad Z\geq 0.$

We can see that the random variable Z is Rayleigh distributed

5.21 To simulate Ricean fading, simply add a fixed, DC component at f=0 in the Doppler Spectrum filter, where the amplitude of the added DC component is weighted properly in proportion with the existing "noise" power in the other randomly generated spectral components in order to yield the desired K-factor. Note that by putting the single constant component at DC, you are assured of retaining the properties of a real signal in the time domain.

5.22 See MATLAB program P5_22.m and the figures below





5.23 Theoretical value

(a) $f_d = 20\,Hz$

for $\rho = \frac{R}{R_{RMS}} = 1$, we have

$$N_R = \sqrt{2\pi}\cdot f_d\cdot \rho \cdot e^{-\rho^2} = \sqrt{2\pi}\times 20\times 1\times e^{-1} \doteq \underline{\underline{18.44 \text{ crossings/sec}}}$$

$$\bar{\tau} = \frac{e^{\rho^2}-1}{\sqrt{2\pi}\cdot f_d\cdot \rho} = \frac{e-1}{\sqrt{2\pi}\times 20\times 1} \doteq 0.0343\ (sec) = \underline{\underline{34.3\ ms}}$$

for $\rho = 0.1$, we have

$$N_R = \sqrt{2\pi}\times 20\times 0.1\times e^{-0.1^2} \doteq \underline{\underline{4.96 \text{ crossings/sec}}}$$

$$\bar{\tau} = \frac{e^{0.1^2}-1}{\sqrt{2\pi}\times 20\times 0.1} \doteq 2\times 10^{-3}\ (s) = \underline{\underline{2\ ms}}$$

for $\rho = 0.01$, we have

$$N_R = \sqrt{2\pi}\times 20\times 0.01\times e^{-0.01^2} \doteq \underline{\underline{0.5 \text{ crossings/sec}}}$$

$$\bar{\tau} = \frac{e^{0.01^2}-1}{\sqrt{2\pi}\times 20\times 0.01} \doteq 2\times 10^{-4}\ (s) = \underline{\underline{0.2\ ms}}$$

(b) $f_d = 200\,Hz$

for $\rho = 1$, we have

$$N_R = \sqrt{2\pi}\cdot f_d\,\rho\cdot e^{-\rho^2} \doteq 18.44\times 10 = \underline{\underline{184.4 \text{ crossings/sec}}}$$

$$\bar{\tau} = \frac{e^{\rho^2}-1}{\sqrt{2\pi}\cdot f_m\cdot \rho} \doteq \frac{34.3}{10} = \underline{\underline{3.43\ (ms)}}$$

for $\rho = 0.1$, we have

$N_R \doteq \underline{\underline{49.6 \text{ crossings/sec}}}$

$\bar{\tau} \doteq \underline{\underline{0.2\ ms}}$

for $\rho = 0.01$, we have

$N_R \doteq \underline{\underline{5 \text{ crossings/sec}}}$ $\quad \bar{\tau} \doteq 0.02\ ms = \underline{\underline{20\ \mu s}}$

5.23 Cont'd

See the MATLAB program p5_23.m for simulation result. The simulation result is close to the theoretical value when ρ is large. When ρ becomes small (e.g., $\rho = 0.01$), the simulation result is different from the theoretical value. This is because when ρ is very small, N_R and $\bar{\tau}$ become small, if the number of points of the IFFT in the simulation is not large enough, the resolution in time domain, Δt, should be larger than $\bar{\tau}$, and thus we will miss some fading event. If we increase the IFFT number of points, we can obtain more accurate result. For example, for $f_m = 20\text{HZ}$, if we take 200 points in the frequency domain from 0 to f_d, we have $\Delta f = \frac{f_m}{200} = 0.1\text{ Hz}$. With $N = 8192$ point IFFT, the resolution in time domain is

$$\Delta t_{8192} = \frac{1}{f_s} = \frac{1}{N \cdot \Delta f} = \frac{1}{8192 \times 0.1} \doteq 1.22 \times 10^{-3}(S) = 1.22\,mS$$

we can see that $\Delta t_{8192} > \bar{\tau} = 0.2\,mS$ when $\rho = 0.01$, therefore we will miss almost all of the fading events. With $N = 8192 \times 8$, $\Delta t_{8192\times 8} = 0.153\,mS < \bar{\tau} = 0.2\,mS$, thus we can get more accurate results if 8 times as many points are used.

$\boxed{5.24}$ See the MATLAB program p5_24.m and Fig. p5-24.

5.24 Cont'd.

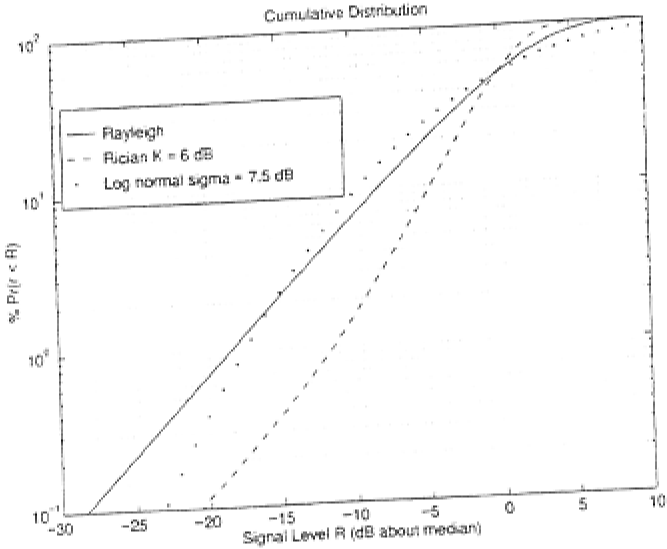


Fig. p5_24

5.25 See the MATLAB program p5.25.m and Fig p5_25 (a), (b).

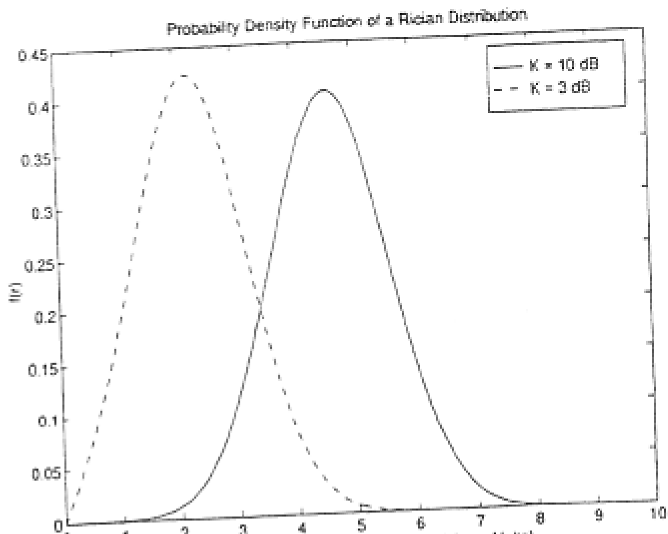


Fig. p5_25(a)

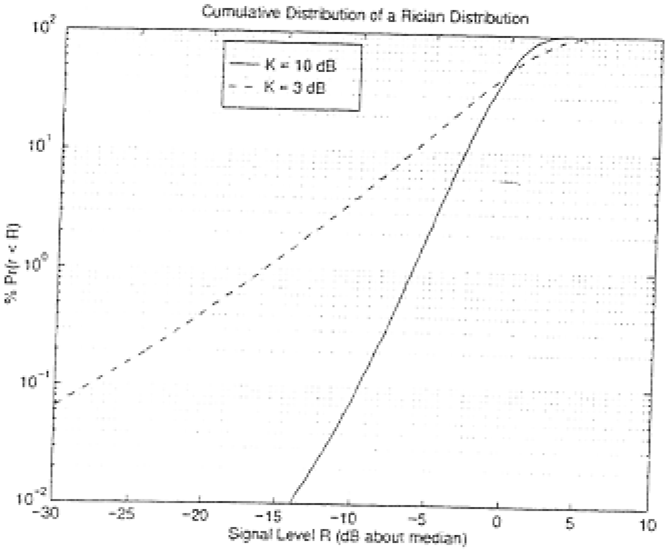


Fig. p5_25(b)

$\boxed{5.26}$ (a) $\underline{K = 10\ dB}$

Signal Level R (dB about median) $= -80 - (-70) = -10\ dB$

From the figure in problem 5.25, we have

$$Pr(r < -80\,dBm) = 0.08\%$$

$$\Rightarrow Pr(r > -80\,dBm) = 1 - Pr(r < -80\,dBm) = \underline{\underline{99.92\%}}$$

(b) $\underline{K = 3\ dB}$

Similarly, we have $Pr(r < -80\,dBm) = 4\%$

$$\Rightarrow Pr(r > -80\,dBm) = \underline{\underline{96\%}}$$

5.27 The local average power delay profile in a particular environment is found to be

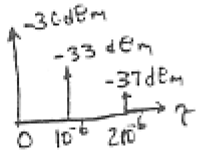
$$P(\tau) = \sum_{n=0}^{2} \frac{10^{-6}}{n^2+1}\delta(\tau - n10^{-6}) \qquad (5)$$

(a) Sketch the Power Delay Profile of the channel in dBm.

$\tau = 0 \Rightarrow P(\tau) = 10^{-6}\ W \qquad P_{dBm} = 10\log\left[\frac{x}{10^{-3}W}\right]$

$\tau = 10^{-6}s \Rightarrow P(\tau) = 5\,10^{-7}\,W$

$\tau = 2\,10^{-6}s \Rightarrow P(\tau) = 2\,10^{-7}\,W$



(b) What is the local average power in dBm?

$$P_{AVG} = \int_0^{\infty} P(\tau)\,d\tau = \sum_{n=0}^{2} P(\tau) = 10^{-6} + 5\,10^{-7} + 2\,10^{-7} = 1.7\,10^{-6}\,W$$

$$10\log\left[\frac{1.7\,10^{-6}}{10^{-3}}\right] = \boxed{-27.7\,dBm}$$

(c) What is the rms delay spread of the channel?

$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$

$$\bar{\tau} = \frac{\sum P_n \tau_n}{P} = \frac{10^{-6}\cdot 0 + \left[\frac{1}{2}10^{-6}\right]10^{-6} + \left[\frac{1}{5}10^{-6}\right]10^{-6}}{1.7\ 10^{-6}} = 5.3\,10^{-7}s$$

Note $P = \sum P_n = \sum a_n^2$

$$\overline{\tau^2} = \frac{\sum P_n \tau_n^2}{P} = \frac{10^{-6}\cdot 0^2 + \left[\frac{1}{2}10^{-6}\right][10^{-6}]^2 + \left[\frac{1}{5}10^{-6}\right](2\,10^{-6})^2}{1.7\ 10^{-6}} = 7.65\,10^{-13}\,s^2$$

$\sigma_\tau = \sqrt{7.65\,10^{-13} - (5.29\,10^{-7})^2} = \boxed{0.696\,\mu s}$

(d) If 256 QAM modulation having a bit rate of 2 Megabits per second is applied to the channel, will the modulation undergo flat or frequency selective fading? Explain your answer.

256 QAM implies $R_b = [\log_2 256]R_s \therefore R_b = 8R_s \therefore R_s = \frac{2\cdot10^6}{8} = 250\,ksps$

$\sigma_\tau \doteq 0.7\mu s \quad ; \quad T_s = 4\,\mu s$, we do not know the BW of the signal, since pulse shaping is not specified $\quad T_s = 1/R_s = 4\,\mu s$

Since $T_s > \sigma_\tau \Rightarrow$ FLAT FADING

(e) Over what bandwidth will the channel appear to have constant gain?

Define Coherence Bandwidth to solve this:

90% Correlation of Amplitudes occurs over $B_c = \frac{1}{50\sigma_\tau}$

50% Correlation of Amplitudes occurs over $B_c = \frac{1}{5\sigma_\tau}$

For 90% Correlation: $B_c = \frac{1}{50[0.7\mu s]} = 28.571\,kHz$

For 50% Correlation: $B_c = \frac{1}{5[0.7\mu s]} = 285.7\,kHz$

5.28 (a) mean excess delay $\bar{\tau} = \dfrac{1\times 0 + 0.1\times 1 + 1\times 2}{1+0.1+1} = 1\,(\mu s)$

$$\overline{\tau^2} = \frac{1\times 0 + 0.1\times 1 + 1\times 2^2}{1+0.1+1} \doteq 1.95\,(\mu s^2)$$

rms delay spread $\sigma_\tau = \sqrt{1.95 - 1^2} \doteq \underline{\underline{0.976\,(\mu s)}}$

(b) maximum excess delay (20 dB) $= \underline{\underline{2\,\mu s}}$

(c) $T_{min} = 10\,\sigma_\tau = 10\times 0.976 = 9.76\,(\mu s)$

$\Rightarrow$ maximum RF symbol Rate $= \dfrac{1}{T_{min}} = \dfrac{1}{9.76\times 10^{-6}} \doteq \underline{\underline{102\text{ Kbp}}}$

(d) Let $f_c = 900$ MHZ $\Rightarrow \lambda = \dfrac{c}{f_c} \doteq 0.33\,(m)$

$$V = \frac{30\times 10^3 m}{3600 S} \doteq 8.33\,(m/s)$$

$$\Rightarrow f_m = \frac{V}{\lambda} = \frac{8.33}{0.33} = 25\,(HZ)$$

$\Rightarrow$ Coherence time $T_C = \dfrac{0.423}{f_m} = \dfrac{0.423}{25} \doteq \underline{\underline{0.017\,(S)}}$

5.29 (a) $\lambda = \dfrac{c}{f_c} = \dfrac{3\times 10^8 m/s}{6\times 10^9 Hz} = 0.05\,(m)$

$$V = \frac{80\times 10^3 m}{3600 S} \doteq 22.22\,(m/s)$$

$$\Rightarrow f_m = \frac{V}{\lambda} = \frac{22.22}{0.05} \doteq 444.4\,(HZ)$$

For $\rho = 1$, $N_R = \sqrt{2\pi}\cdot f_m \cdot \rho e^{-\rho^2} = \sqrt{2\pi}\times 444.4\times 1\times e^{-1}$
$\doteq 409.7$ (crossings/sec)

$\Rightarrow$ Number of positive-going zero crossings about the rms value that occur over a 5 second interval
$= N_R\cdot t = 409.7\times 5 \doteq \underline{\underline{2048}}.$

5.29 Cont'd

(b) $\bar{\tau} = \frac{e^{\rho^2}-1}{\rho \cdot f_m\sqrt{2\pi}} = \frac{e^{1}-1}{1\times 444.4\times\sqrt{2\pi}} \doteq 1.54\times 10^{-3}(s) = 1.54\,(ms)$

(c) For $\rho = -20dB = 0.1$, we have

$$\bar{\tau} = \frac{e^{\rho^2}-1}{\rho \cdot f_m\cdot\sqrt{2\pi}} = \frac{e^{0.01}-1}{0.1\times 444.4\times\sqrt{2\pi}} \doteq 9.02\times 10^{-5}(s) = \underline{\underline{90.2\,(\mu s)}}$$

$\boxed{5.30}$

$$\text{Fast Fading} \;:\; f_M \gg \frac{1}{T_s}$$
$$\text{Slow Fading} \;:\; f_M \ll \frac{1}{T_s}$$

$$\text{Flat Fading} \;:\; T_s \gg$$
$$\text{Freq. Selective Fading} \;:\; T_s \ll$$

(a)

$$f_m \text{ Doppler} \approx 200 \text{ Hz (highway speed)}$$

$$\sigma_\tau(\text{urban}) \approx 2\mu s \quad \text{(typical)}$$

Given

$$1/T_s = 500 \text{ kbps} \Rightarrow T_s \approx 2\mu s.$$

$$\text{Slow fading} \;:\; \text{since } f_M \ll \frac{1}{T_s}$$
$$\text{Freq. Selective Fading} \;:\; \text{since } T_s < \sigma_\tau$$

(b)

$$\text{Flat Fading} \;:\; \text{since } T_s \gg \sigma_\tau$$
$$\text{Slow Fading} \;:\; \text{since } f_M \ll \frac{1}{T_s}$$

(b)

$$\text{Fast Fading} \;:\; \text{since } f_M \gg \frac{1}{T_s}$$
$$\text{Flat Fading} \;:\; \text{since } T_s \gg \sigma_\tau$$

5.31 See the MATLAB program p5-31.m and Fig. p5-31.

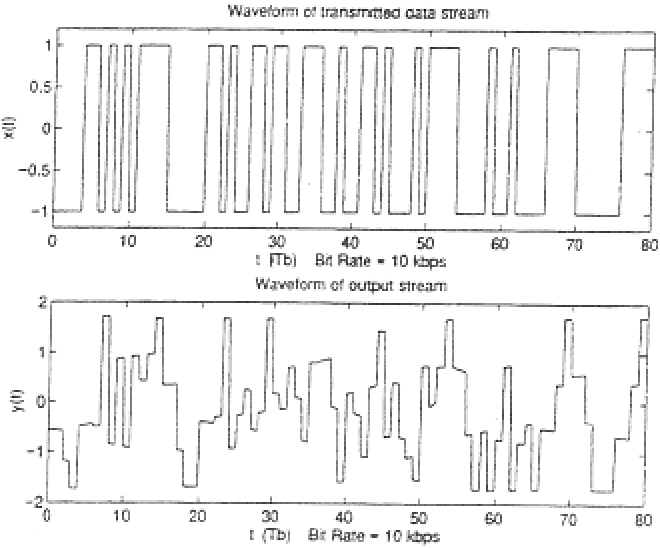


Fig. p5_31

5.32 Since the maximum Doppler shift f_m and the vehicular velocity V are related by the equation $f_m = \frac{V}{\lambda}$, for a given λ, if we can detect f_m, we know the velocity V. There are several ways to detect f_m.

1. Detect the bandwidth B_W of the unmodulated CW carrier, then $f_m = \frac{B_W}{2}$.

2. Detect the level crossing rate N_R of a Rayleigh fading signal, then $f_m = \frac{1}{\sqrt{2\pi}\rho} \cdot e^{\rho^2} \cdot N_R$, where ρ is the value of the specified level R, normalized to the local rms amplitude of the fading envelope.

CHAPTER 6

6.1 Given $f_m = \frac{5}{2\pi} \doteq 0.796$ Hz, $\beta_f = 10$.

$\Rightarrow$ bandwidth of the FM signal $B_T = 2(\beta_f + 1)\cdot f_m$

$B_T = 2\times(10+1)\times 0.796 \doteq \underline{\underline{17.5 \text{ Hz}}}$

For $f_c = \frac{\omega_c}{2\pi} \doteq 796.18$ Hz, the upper sideband frequency

$= f_c$ to $f_c + \frac{B_T}{2} = \underline{\underline{796.18 \text{ to } 804.93 \text{ Hz}}}$

the lower sideband frequency $= f_c - \frac{B_T}{2}$ to f_c

$= \underline{\underline{787.43 \text{ to } 796.18 \text{ Hz}}}$

6.2 For $m(t) = \sin(1000\pi t)$, given $\Delta f = 1$ KHz, $A_m = 2$ volts,

$f_m = \frac{1000\pi}{2\pi} = 500$ Hz, we have

$k_f = \frac{\Delta f}{A_m} = \frac{1000}{2} = 500$ Hz/volt

Thus, for $A_m = 8$ V, $f_m = 2000$ Hz, with $f_c = 2\times 10^6$ Hz,

$A_c = 4$ V, we have

$$S_{FM}(t) = A_c \cdot \cos\left[2\pi f_c t + 2\pi k_f \cdot \int_{-\infty}^{t} m(\eta)\,d\eta\right]$$

$$= 4\cdot\cos\left[2\pi\times 2\times 10^6 t - 2\pi\times 500\times\frac{8}{2000}\cdot\cos(2\pi\times 2000)\right]$$

$$= 4\cdot\cos\left[4\times 10^6\pi t - 4\pi\cos(2\pi\times 2000)\right]$$

$$= \underline{\underline{4\cdot\cos\left[4\times 10^6\pi t - 4\pi\cos(4000\pi)\right]}}$$

6.3 (a) See the MATLAB program p6_03.m and Fig. p6_03

(b) See Fig. p6_03.

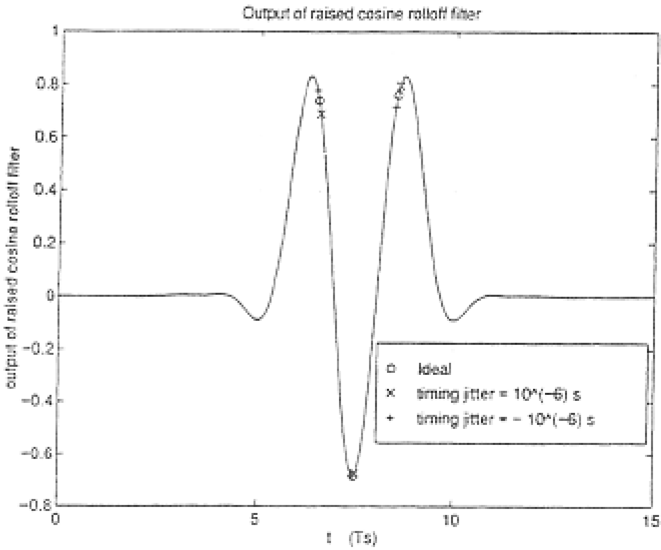


Fig. p6_03

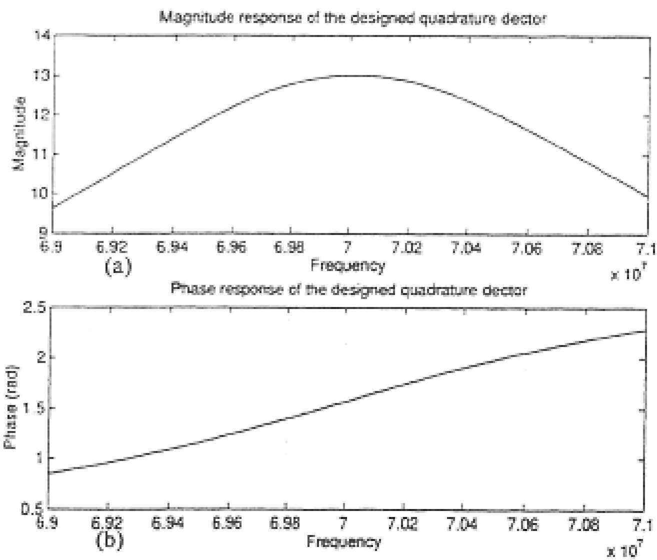


Fig. p6_08 (a), (b)

6.3 Cont'd

(c) The table below illustrates the voltage at each sample point for ideal and timing jitter of $\pm 10^{-6}$ case.

	1	0	1
Ideal	0.7395	−0.6834	0.7609
Timing jitter of 10^{-6} s	0.6884	−0.6727	0.7954
Timing jitter of -10^{-6} s	0.7795	−0.6799	0.7154

$\boxed{6.4}$ Maximum Doppler spread $f_m = \frac{V}{\lambda} = \frac{80 \times 1.6 \times 10^3 / 3600}{\frac{300}{440}}$

$\doteq 52.1 \text{ Hz}$

$\Rightarrow$ notch width $= 2 f_m = 2 \times 52.1 = \underline{\underline{104.2 \text{ Hz}}}$

$\boxed{6.5}$ $\beta_f = \frac{f_d}{W} = \frac{12}{4} = \underline{\underline{3}}$

$\boxed{6.6}$ For AMPS FM transmissions, $\beta_f = 3$.

$\Rightarrow$ output SNR improvement factor $\doteq 3\beta_f^2(\beta_f + 1)$

$= 3 \times 3^2 \times (3+1) = 108 \doteq 20.33 \text{ dB}$

$\Rightarrow (SNR)_{out} = (SNR)_{in} + 20.33 \text{ dB}$

$= 10 + 20.33 = \underline{\underline{30.33 \text{ dB}}}$

If $(SNR)_{in}$ is increased by 10dB, the corresponding increase out of the detector is also $\underline{\underline{10 \text{ dB}}}$

$\boxed{6.7}$ See section 6.3.3.

$\boxed{6.8}$ As shown in Example 6.4, for $f_c = 70\,MHz$, $B = 200\,KHz$, at the largest IF frequency $f_i = f_c + \frac{B}{2} = 7.01\times10^7\,Hz$, we require

$$Q\cdot\left(\frac{f_i}{f_c} - \frac{f_c}{f_i}\right) = \tan 5^\circ$$

$$\Rightarrow\ Q\cdot\left(\frac{7.01\times10^7}{7\times10^7} - \frac{7\times10^7}{7.01\times10^7}\right) \doteq \tan 5^\circ$$

$$\Rightarrow\ Q = 30.64$$

Since $Q = \frac{R}{\omega_c L}$, choose $L = \underline{\underline{0.1\,\mu H}}$, we have

$$R = Q\cdot\omega_c\cdot L = 30.64\times2\pi\times70\times10^6\times0.1\times10^{-6}$$

$$= 1.347\times10^3\,\Omega = \underline{\underline{1.347\,K\Omega}}$$

Since $Q = R\cdot\omega_c(C_1+C)$, we have

$$C_1+C = \frac{Q}{R\cdot\omega_c} = \frac{30.64}{1.347\times10^3\times2\pi\times70\times10^6} = 51.74\,PF$$

Assuming $C_1 = 21.74\,PF \doteq \underline{\underline{22\,pF}}$, we get $C = \underline{\underline{30\,pF}}$

$$\Rightarrow\ |H(f)| = \frac{2\pi f R C_1}{\sqrt{1+Q^2\cdot\left(\frac{f}{f_c} - \frac{f_c}{f}\right)^2}} = \underline{\underline{\frac{1.86\times10^{-7} f}{\sqrt{1+938.8\cdot\left(\frac{f}{70\times10^6} - \frac{70\times10^6}{f}\right)^2}}}}$$

$$\angle H(f) = \frac{\pi}{2} + \tan^{-1}\left[Q\left(\frac{f}{f_c} - \frac{f_c}{f}\right)\right]$$

$$= \underline{\underline{\frac{\pi}{2} + \tan^{-1}\left[30.64\cdot\left(\frac{f}{70\times10^6} - \frac{70\times10^6}{f}\right)\right]}}$$

6.8 Cont'd

The MATLAB program p6_08.m plots the amplitude and phase response of this quadrature detector which are shown in Fig. p6_08 (a) and (b), respectively (pg. 102).

$\boxed{6.9}$ See the MATLAB program p6_09.m and Fig. 6_09a (a), (b), (c), (d) and (e) and Fig. 6_09b (a), (b), (c), (d) and (e).

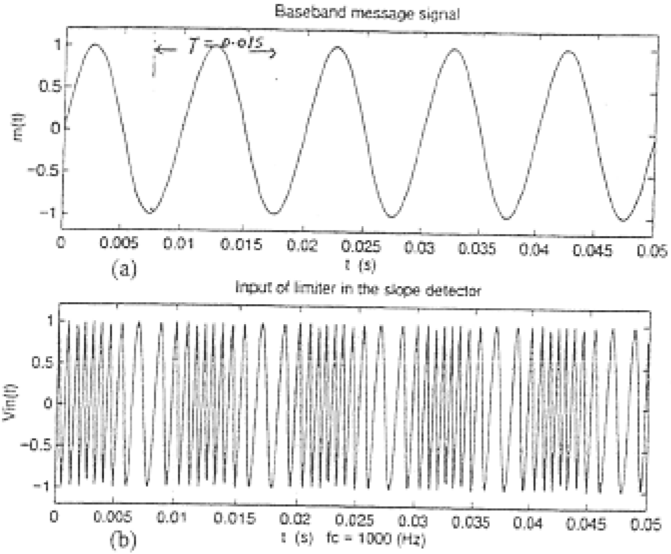


Fig. p6_09a (a), (b)

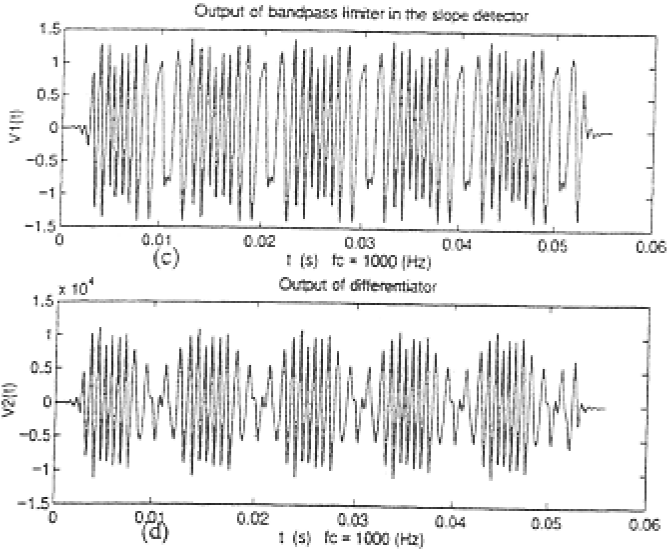


Fig. p6_09a (c), (d)

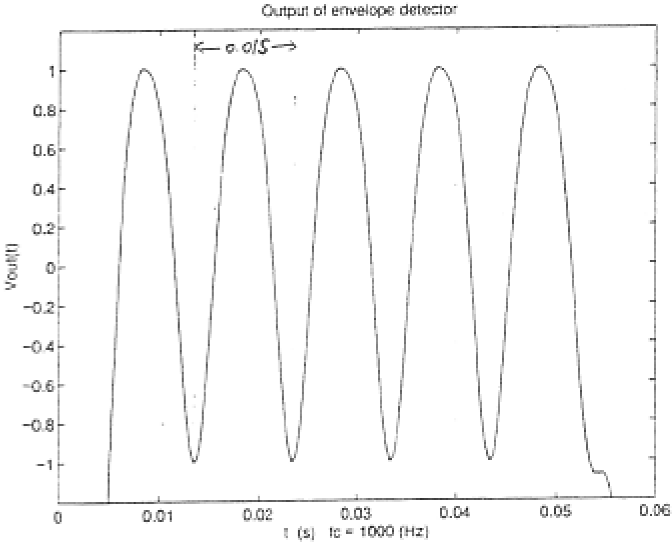


Fig. p6_09a (e)

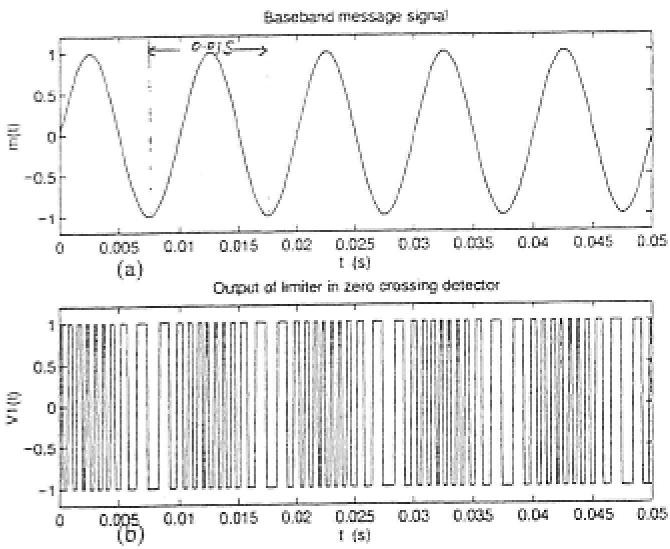


Fig. p6_09b (a), (b)

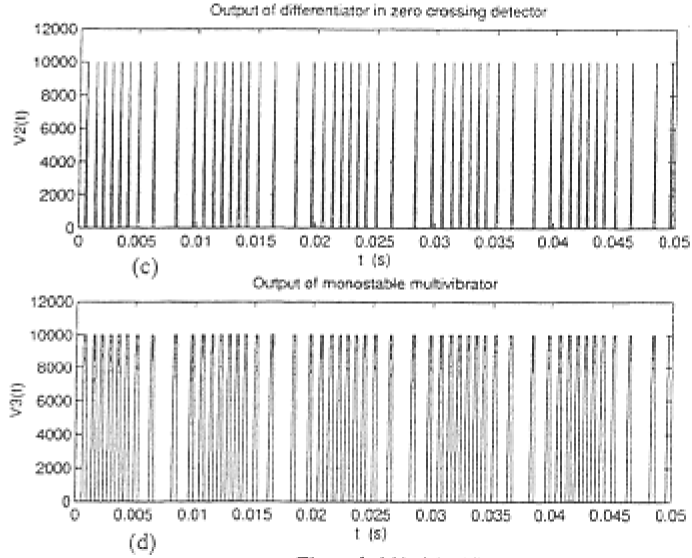


Fig. p6_09b (c), (d)

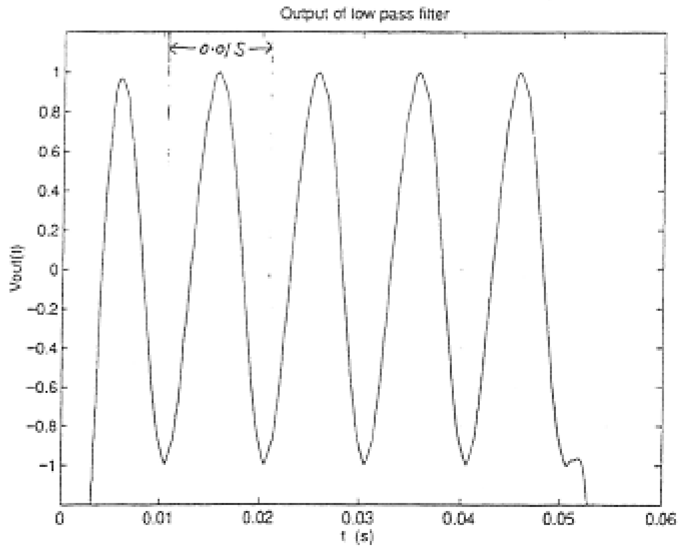


Fig. p6_09b (e)

6.8 Cont'd

The MATLAB program p6_08.m plots the amplitude and phase response of this quadrature detector which are shown in Fig. p6_08 (a) and (b), respectively (pg. 102).

$\boxed{6.9}$ See the MATLAB program p6_09.m and Fig. 6_09a (a), (b), (c), (d) and (e) and Fig. 6_09b (a), (b), (c), (d) and (e).

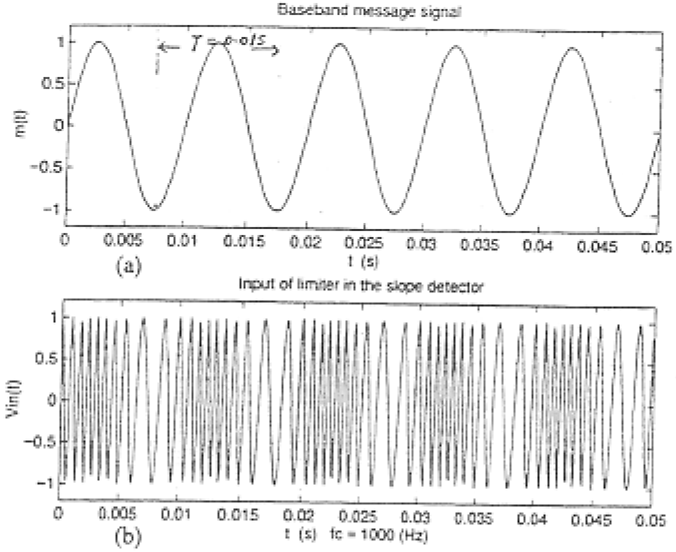


Fig. p6_09a (a), (b)

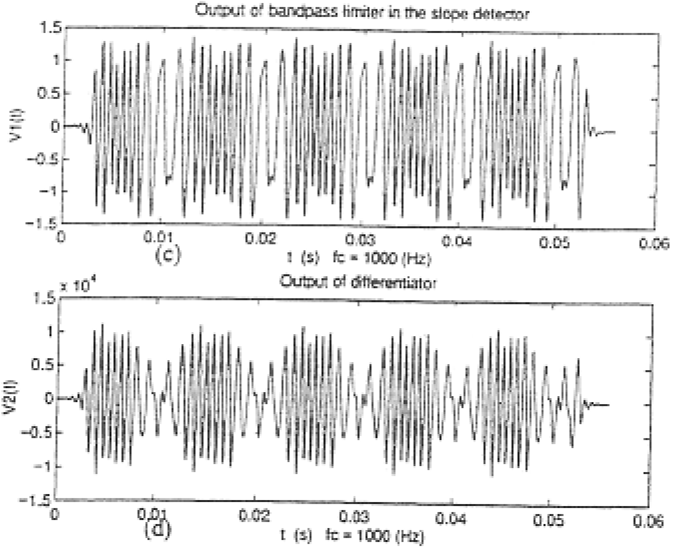


Fig. p6_09a (c), (d)

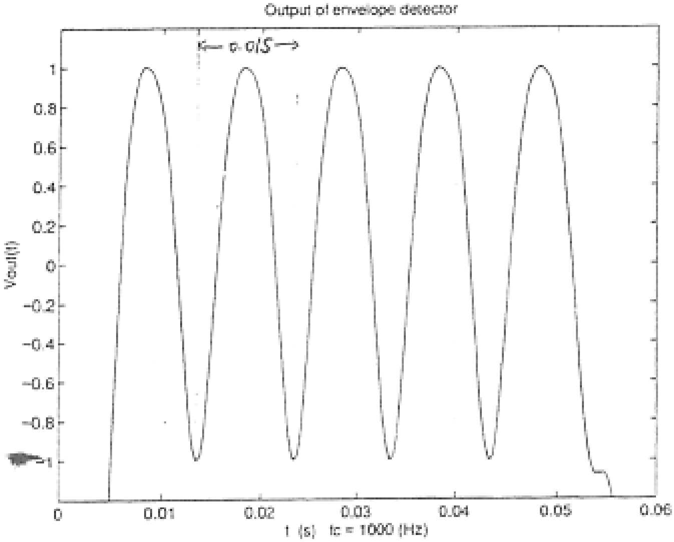


Fig. p6_09a (e)

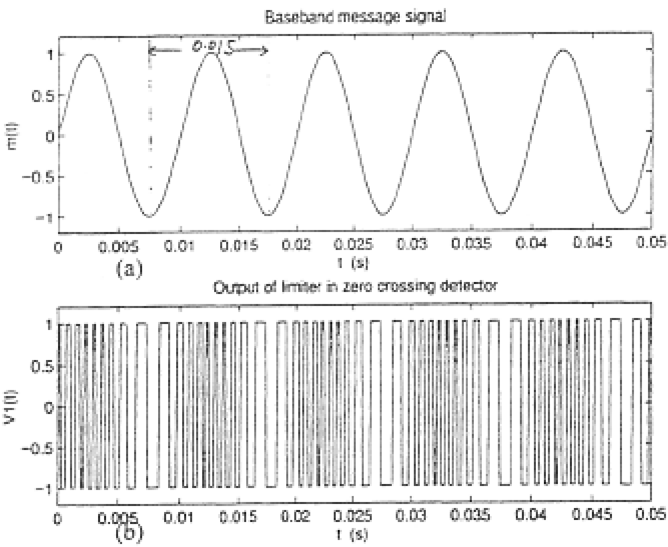


Fig. p6_09b (a), (b)

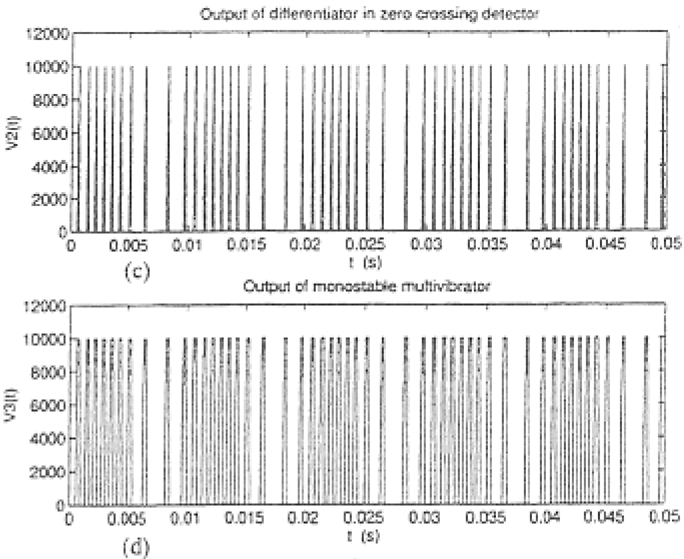


Fig. p6_09b (c), (d)

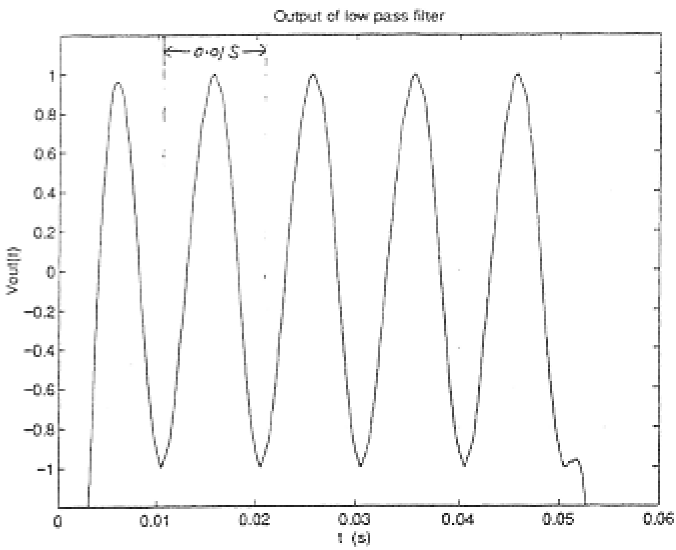


Fig. p6_09b (e)

6.10 See BPSK receiver in section 6.8.1

6.11 See the MATLAB program p6_11.m and Fig. p6_11

$$P_{e,BPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \quad P_{e,DPSK} = \frac{1}{2}\exp\left(-\frac{E_b}{N_0}\right)$$

$$P_{e,QPSK} = Q\left(\sqrt{\frac{2E_b}{N_0}}\right), \quad P_{e,FSK,NC} = \frac{1}{2}\exp\left(-\frac{E_b}{2N_0}\right)$$

BPSK, DPSK and QPSK are all linear constant envelope modulation techniques. They can save bandwidth but are poor in power efficiency. Pulse shaping can make the modulation techniques non-constant envelope and even more bandwidth efficient. BPSK and QPSK all need coherent detection which is more complicated than the non-coherent detection. FSK is a nonlinear constant envelope modulation. Using class C amplifier, it is power efficient but occupies a larger bandwidth than linear modulation schemes, even when pulse shaping is used, FSK techniques are not as bandwidth efficient as linear techniques. FSK can use noncoherent detection.

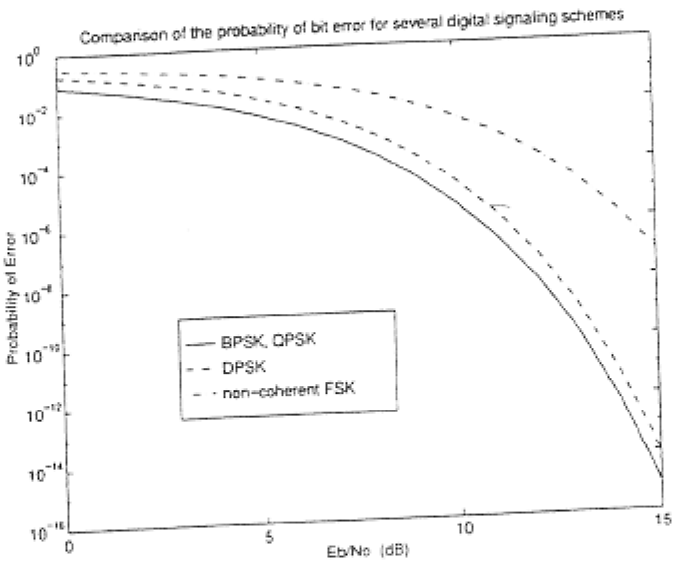


Fig. p6_11

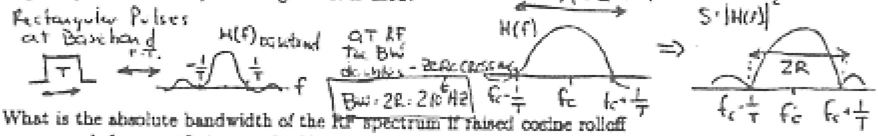
6.12

Assume a binary bit stream is to be modulated on an RF carrier. If the baseband bit stream has a data rate of 1 Megabit per second, then:

$T = 10^{-6}$ s

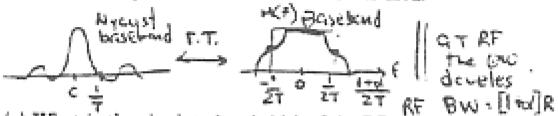
$R = 10^{6}$ s

(a) What is the first-zero crossing bandwidth of the RF spectrum if simple rectangular pulses are used, assuming BPSK is used?



$\boxed{BW = 2R = 2\,10^6\ Hz}$

(b) What is the absolute bandwidth of the RF spectrum if raised cosine rolloff pulses are used, for $\alpha = 1$? Assume BPSK is used.



$\alpha = 1$

$BW = (1+1)\,10^6 = \boxed{2\,10^6\ Hz}$

(c) What is the absolute bandwidth of the RF spectrum if raised cosine rolloff pulses are used, for $\alpha = \frac{1}{3}$? Assume BPSK is used.

$$RF\ BW = \left[1 + \tfrac{1}{3}\right] 10^6 = \boxed{1.333\ 10^6\ Hz}$$

(d) If a timing jitter of 10^{-6} seconds exists at the receiver and raised cosine rolloff pulses are used, will the detector experience intersymbol interference from the adjacent symbols? Explain.

Nyquist pulses produce no ISI for all k, t such that $t = kT_s$ except for $k=0$ (the desired pulse). If the jitter is exactly 10^{-6} s, then the timing offset is exactly one symbol. Thus, there will be no ISI except for the next symbol, which will be perfectly received. i.e. the receiver will be off sync by T.

(e) If GMSK modulation is to be used and a 3 dB bandwidth of 500 kHz is used for the Gaussian low pass filter, what is the proper choice for the FM peak frequency deviation, ΔF?

$$\Delta F = \frac{1}{4T_b} = \frac{R_b}{4} = \frac{10^6}{4} = \boxed{250\ kHz}$$

(NOT A function of 3dB BW!)

(f) For GMSK modulation using $BT \le 0.5$, how many spectral sidelobes occur?

No sidelobes occur at $BT < \frac{1}{2}$

I will accept two sidelobes (one on each side of carrier)

6.13

For $SNR = 30dB = 1000$, $B = 200KHZ$, the maximum possible data rate, $C = B \cdot \log_2(1 + \frac{S}{N}) = 200 \times 10^3 \times \log_2(1+1000) \doteq 1.99\ Mbps$

The GSM data rate is 270.833 bps. which is only about $0.136C$

6.14 For IS-54, $R = 48.6$ Kbps, $B = 30$ KHz.

$$\Rightarrow \eta_B = \frac{R}{B} = \frac{48.6}{30} = 1.62 \text{ bps/Hz}$$

For GSM, $R = 270.833$ Kbps. $B = 200$ KHz

$$\Rightarrow \eta_B = \frac{270.833}{200} = 1.35 \text{ bps/Hz}$$

For PDC, $R = 42$ kbps, $B = 25$ KHz $\Rightarrow \eta_B = \frac{42}{25} = 1.68$ bps/Hz

For IS-95, the bandwidth efficiency depends on the number of users K. For $R = 9.6$ Kbps/S, $B = 1.2288$ MHz.

$$\Rightarrow \eta_B = \frac{K \cdot R}{B} = \frac{K \cdot 9.6 \times 10^3}{1.2288 \times 10^6} \doteq K \cdot 7.8 \times 10^{-3} \text{ bps/Hz}$$

If $SNR = 20dB = 100$, the theoretical spectral efficiency

$$\eta_{B\max} = \log_2(1+SNR) = \log_2(1+100) \doteq 6.66 \text{ bps/Hz}$$

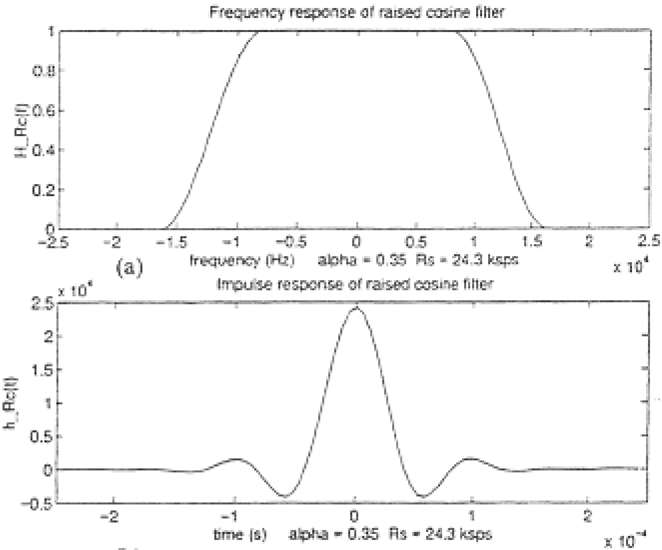
6.15 See the MATLAB program p6_15.m and Fig. p6_15 (a) and (b)

$$H_{RC}(f) = \begin{cases} 1 & 0 \le |f| \le \frac{1-\alpha}{2T_s} \\ \frac{1}{2}\left\{1 + \cos\left[\frac{\pi \cdot (|f| \cdot 2T_s - 1 + \alpha)}{2\alpha}\right]\right\} & \frac{(1-\alpha)}{2T_s} < |f| \le \frac{(1+\alpha)}{2T_s} \\ 0 & |f| > \frac{(1+\alpha)}{2T_s} \end{cases}$$

$$h_{RC}(t) = \frac{\sin\left(\frac{\pi t}{T_s}\right)}{\pi t} \cdot \frac{\cos\left(\frac{\pi \alpha t}{T_s}\right)}{1 - \left(\frac{4\alpha t}{2T_s}\right)^2}$$

Fraction of the total radiated energy that will fall out-of-band

$$= 1 - \frac{\int_{-15K}^{15K} H_{RC}^2(f) \cdot \left(\frac{\sin(\pi f T_s)}{\pi f T_s}\right)^2 df}{\int_{-\frac{(1+\alpha)R_s}{2}}^{\frac{(1+\alpha)R_s}{2}} H_{RC}^2(f) \cdot \left(\frac{\sin(\pi f T_s)}{\pi f T_s}\right)^2 df} \doteq 3 \times 10^{-5} = 0.003\%$$


Frequency response of raised cosine filter
H_Rc(f)
frequency (Hz) alpha = 0.35 Rs = 24.3 ksps
x 10^4
(a)
Impulse response of raised cosine filter
x 10^4
h_Rc(t)
time (s) alpha = 0.35 Rs = 24.3 ksps
x 10^-4

(b)

Fig. p 6_15(a), (b)

$\boxed{6.16}$ $\underline{BT_s = 0.5}$, $T_s = \dfrac{1}{19.2\,\text{KSPS}}$ (See also problem 6.27)

$\Rightarrow B = \dfrac{0.5}{T_s} = 0.5 \times 19.2 \times 10^3 = 9.6\ \text{KHz}$

$\Rightarrow \alpha = \dfrac{1.1774}{2 \cdot B} = \dfrac{1.1774}{2 \times 9.6 \times 10^3} \doteq 6.13 \times 10^{-5}$

$\Rightarrow H_G(f) = \exp(-\alpha^2 f^2) = \underline{\underline{\exp(-3.75 \times 10^{-9} f^2)}}$

$h_G(t) = \dfrac{\sqrt{\pi}}{\alpha} \exp\left(-\dfrac{\pi^2}{\alpha^2} t^2\right) \doteq \underline{\underline{28907.08 \exp(-2.62 \times 10^9 t^2)}}$

$\underline{BT_s = 0.2}$, $\Rightarrow B = \dfrac{0.2}{T_s} = 0.2 \times 19.2 \times 10^3 = 3.84\ \text{KHz}$

$\Rightarrow \alpha = \dfrac{1.1774}{2 \cdot B} = \dfrac{1.1774}{2 \times 3.84 \times 10^3} \doteq 1.533 \times 10^{-4}$

$\Rightarrow H_G(f) = \exp(-\alpha^2 f^2) = \underline{\underline{\exp(-2.35 \times 10^{-8} f^2)}}$

$h_G(t) = \dfrac{\sqrt{\pi}}{\alpha} \exp\left(-\dfrac{\pi^2}{\alpha^2} \cdot t^2\right) \doteq \underline{\underline{11559\ \exp(-4.195 \times 10^8 t^2)}}$

$\underline{BT_s = 0.75}$, $\Rightarrow B = \dfrac{0.75}{T_s} = 0.75 \times 19.2 \times 10^3 = 14.4\ \text{KHz}$

$\Rightarrow \alpha = \dfrac{1.1774}{2 \cdot B} = \dfrac{1.1774}{2 \times 14.4 \times 10^3} \doteq 4.088 \times 10^{-5}$

$\Rightarrow H_G(f) = \exp(-\alpha^2 f^2) = \underline{\underline{\exp(-1.67 \times 10^{-9} \cdot f^2)}}$

$h_G(t) = \dfrac{\sqrt{\pi}}{\alpha} \exp\left(-\dfrac{\pi^2}{\alpha^2} \cdot t^2\right) \doteq \underline{\underline{4.334 \times 10^4 \cdot \exp(-5.8998 \times 10^9 t^2)}}$

The impulse response and frequency response are shown in Fig. p6_16 (a) and (b), respectively. Using the MATLAB program p6_16.M, We can calculate the fraction, F_{out}, of the total radiated energy that would fall out-of-band. $\underline{\text{For } BT_s = 0.5}$, $F_{out} \doteq \underline{\underline{2.32 \times 10^{-3}}}$

$\underline{\text{For } BT_s = 0.2}$, $F_{out} \doteq \underline{\underline{2.21 \times 10^{-7}}}$

$\underline{\text{For } BT_s = 0.75}$. $F_{out} \doteq \underline{\underline{9.91 \times 10^{-3}}}$

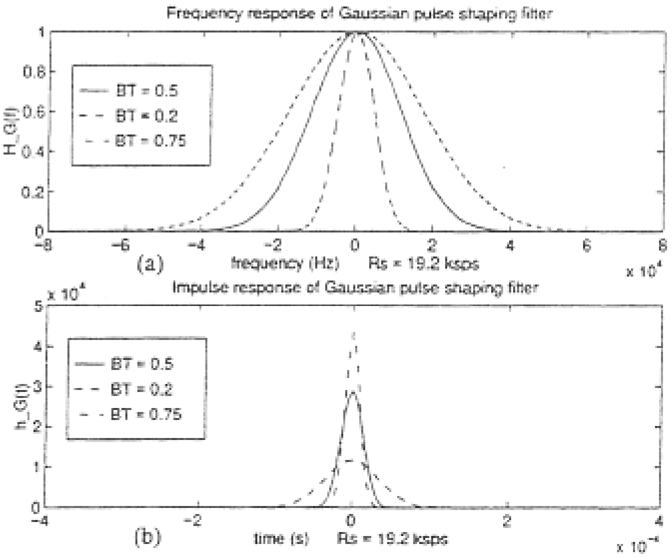

Frequency response of Gaussian pulse shaping filter
H_G(f)
BT = 0.5
BT = 0.2
BT = 0.75
frequency (Hz)
Rs = 19.2 ksps
x 10^4
(a)
Impulse response of Gaussian pulse shaping filter
x 10^4
h_G(t)
BT = 0.5
BT = 0.2
BT = 0.75
time (s)
Rs = 19.2 ksps
(b)

Fig. p6-16 (a), (b)

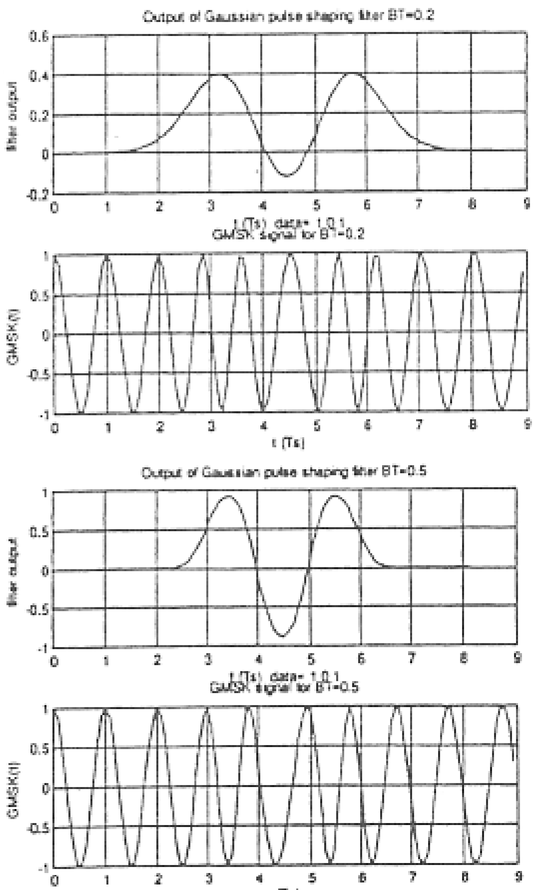
$\boxed{6.17}$

Please see the MATLAB program attached in this solution. To be comparable, we use the input NRZ bit stream is [1 0 1]. Applying different BT values, we can see the output of the bi-polar NRZ waveform from the Gaussian pulse-shaping filter with impulse response:

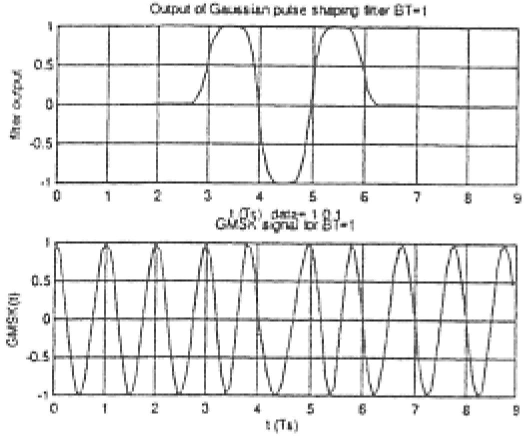
$$h_G(t) = \frac{\sqrt{\pi}}{\alpha} \exp\left[\frac{-\pi^2}{\alpha^2} t^2\right]$$

with α=0.5587/B (B is 3dB baseband bandwidth). The filter can be achieved in MATLAB by time-domain convolution or frequency domain multiplication (by means of DFT /IDFT).

The signal is then sent to a FM modulator, in which the frequency deviation constant k_f can be calculated from the output values of the signal from the Gaussian pulse-shaping filter. The modulation index in the program is 0.5. The output plots of signal of the Gaussian pulse-shaping filter and FM modulator, with BT=0.2, 0.5 and 1 respectively, are shown in the following figures.

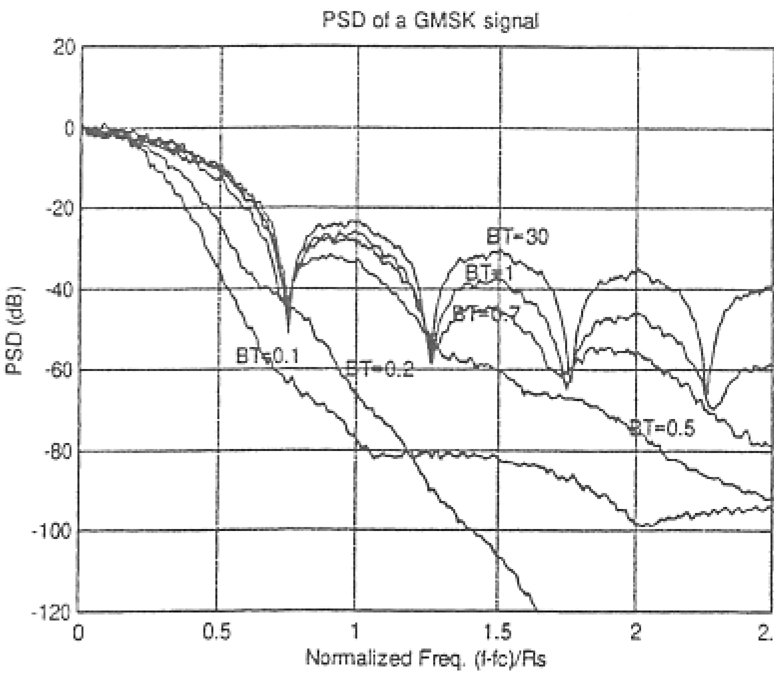


6.17 Cont'd.



We can see that ISI is introduced in the signals after the Gaussian pulse-shaping filter, especially for BT<0.5. This is the price paid for good spectral efficiency and constant envelope properties.

Similarly, we can recreate the simplified version of Figure 6.41 as below. The PSD can be calculated in MATLAB from its definition or by using existent function, such as *spectrum()* or *psd()*. The NRZ bit steam is randomly generated in this case. See MATLAB program p 6-17.M.
Note: There is 3dB difference compared with the graph in the textbook



6.17 Cont'd.

Table 6.3 in the textbook can be recreated by calculating the power density to reach the required percentages.

BT	90%	99%	99.9%	99.99%
0.1GMSK	0.4230	0.6470	0.8087	0.9455
0.2GMSK	0.5225	0.7962	0.9953	1.2068
0.5GMSK	0.6843	1.0326	1.2939	2.0404
0.7GMSK	0.7340	1.0824	1.8040	2.2021
1 GMSK	0.7465	1.1197	1.9533	2.8615
30 GMSK (MSK)	0.7838	1.1819	2.8118	6.8179

6-18 From equation (6.104), we have

$$S_{MSK}(t) = m_I(t)\cdot\cos\left(\frac{\pi t}{2T_b}\right)\cos(2\pi f_c t)$$
$$+ m_Q(t)\cdot\sin\left(\frac{\pi t}{2T_b}\right)\cdot\sin(2\pi f_c t) \qquad (p5\text{-}16)$$

There are four states that the pair $(m_I(t), m_Q(t))$ could be

1. When $m_I(t)=1$, $m_Q(t)=1$, equation p5-16 becomes

$$S_{MSK}(t) = \cos\left(\frac{\pi t}{2T_b}\right)\cdot\cos(2\pi f_c t) + \sin\left(\frac{\pi t}{2T_b}\right)\cdot\sin(2\pi f_c t)$$
$$= \cos\left[2\pi f_c t - \frac{\pi t}{2T_b}\right]$$
$$= \cos\left[2\pi f_c t - m_I(t)\cdot m_Q(t)\cdot\frac{\pi t}{2T_b} + 0\right]$$

2. when $m_I(t)=1$, $m_Q(t)=-1$, similarly, we have

$$S_{MSK}(t) = \cos\left[2\pi f_c t + \frac{\pi t}{2T_b}\right]$$
$$= \cos\left[2\pi f_c t - m_I(t)\cdot m_Q(t)\cdot\frac{\pi t}{2T_b} + 0\right]$$

3. When $m_I(t)=-1$, $m_Q(t)=1$. we have

$$S_{MSK}(t) = -\cos\left(\frac{\pi t}{2T_b}\right)\cos(2\pi f_c t) + \sin\left(\frac{\pi t}{2T_b}\right)\cdot\sin(2\pi f_c t)$$
$$= -\cos\left[2\pi f_c t + \frac{\pi t}{2T_b}\right]$$
$$= \cos\left[2\pi f_c t - m_I(t)\cdot m_Q(t)\cdot\frac{\pi t}{2T_b} + \pi\right]$$

4. When $m_I(t)=-1$, $m_Q(t)=-1$, we have

$$S_{MSK}(t) = -\cos\left[2\pi f_c t - \frac{\pi t}{2T_b}\right]$$
$$= \cos\left[2\pi f_c t - m_I(t)\cdot m_Q(t)\frac{\pi t}{2T_b} + \pi\right]$$

From the above, we can see that $S_{MSK}(t)$ can be expressed as

$$S_{MSK}(t) = \cos\left[2\pi f_c t - m_I(t)\cdot m_Q(t)\cdot\frac{\pi t}{2T_b} + \phi_K\right]$$

where $\phi_K = \begin{cases} 0 & \text{if } m_I(t)=1 \\ \pi & \text{if } m_I(t)=-1 \end{cases}$

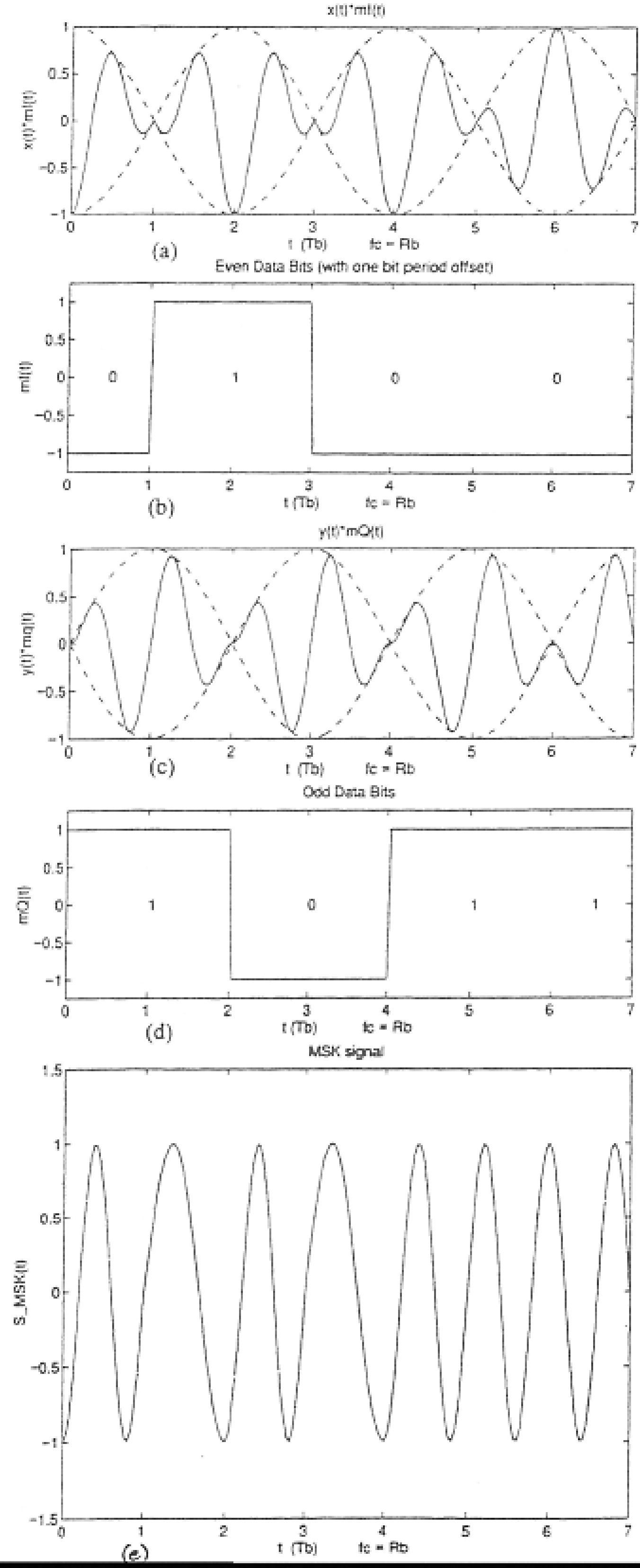
6.19 See the MATLAB program p6_19.m and Fig. p6_19(a), (b), (c), (d), (e), (f), (g), (h) and (i).

For a binary message stream 01100101, the serial data stream is converted to two parallel data streams, each with symbol rate as one half of the bit rate. The even data bits $m_I(t)$ (the first bit of data stream is labeled as bit 0) 0100 are first offset by one bit period and then multiplied by $x(t)$ (See Fig. 6.39 in Section 6.9.2), the odd data bit $m_Q(t)$ are multiplied by $y(t)$. The sum of these two multiplication results is the MSK signal. The signals $m_I(t)\cdot x(t)$, $m_I(t)$, $m_Q(t)\cdot y(t)$, $m_Q(t)$, and $S_{MSK}(t)$ are shown in Fig. p6-19 (a), (b), (c), (d), (e), respectively. In the receiver, the input of the integrator, $S_{MSK}(t)\cdot x(t)$ in the I channel is shown in Fig. p6-19(f) and the output of the integrator is shown in Fig. p6-19(g). For the Q channel, $S_{MSK}(t)\cdot y(t)$ is shown in Fig. p6-19(h) and the output of the integrator is shown in Fig. p6-19(i). In Fig. p6.19(g) and (i), the sampled signals as the input of the threshold detectors are also illustrated.

6.20 $P_b = Q\left(\sqrt{\frac{3N}{K-1}}\right) = Q\left(\sqrt{\frac{3\times 511}{63-1}}\right) = Q(4.9725) \doteq \underline{3.3\times 10^{-7}}$

In determine the above result, we assume that all interferers provide equal power, the same as the desired user. All users are considered orthogonal and independent, and the Gaussian approximation is assumed to be valid.

Fig. p6_19(a)-(e)


x(t)*mI(t)
x(t)*mI(t)
t (Tb) fc = Rb
(a)
Even Data Bits (with one bit period offset)
mI(t)
0
1
0
0
t (Tb) fc = Rb
(b)
y(t)*mQ(t)
y(t)*mq(t)
t (Tb) fc = Rb
(c)
Odd Data Bits
mQ(t)
1
0
1
1
t (Tb) fc = Rb
(d)
MSK signal
S_MSK(t)
t (Tb) fc = Rb
(e)

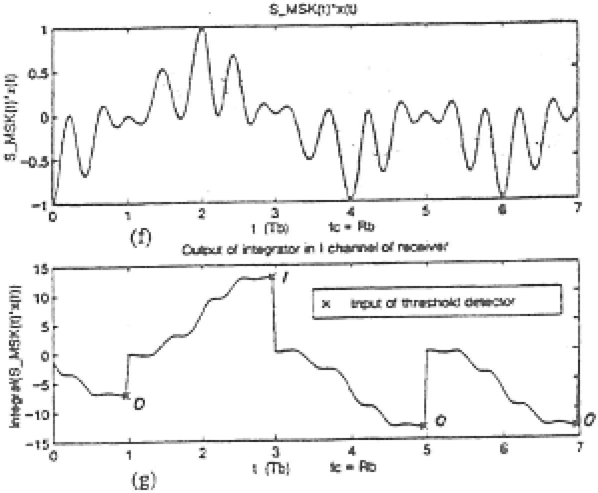


Fig. p6_19 (f), (g)

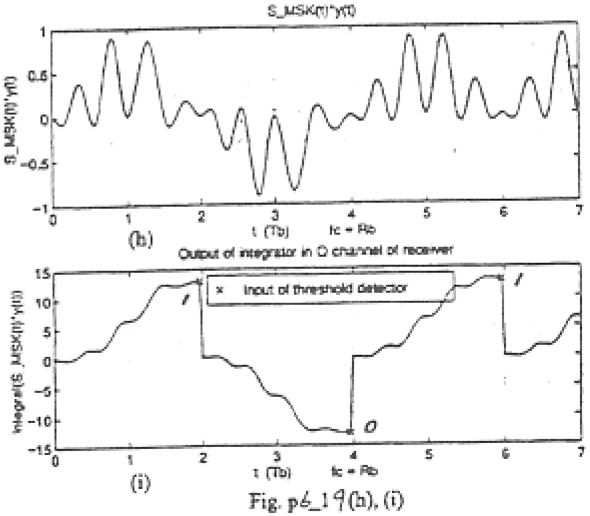


Fig. p6_19(h), (i)

$\boxed{6.21}$ $Q\left(\sqrt{\frac{3N}{K-1}}\right) = 10 \times 3.3 \times 10^{-7} = 3.3 \times 10^{-6}$

$\Rightarrow \sqrt{\frac{3N}{K-1}} = 4.5602 \Rightarrow \frac{3 \times 511}{K-1} = 20.3058 \Rightarrow \underline{K \doteq 76}$

$\boxed{6.22}$

T_c=1/1.2288Mcps, T_s=13kbps, 7.8dB ~ 6

so the processing gain is PG=T_s/T_c= 1.2288M/13k = 94

Assume BPSK,

$$BER = Q\left[\frac{1}{\sqrt{\frac{K-1}{3PG} + \frac{No}{2Eb}}}\right] = Q\left[\frac{1}{\sqrt{\frac{20-1}{3*94.52} + 1/12}}\right] = Q(2.58) = 0.0049$$

For **actual IS-95 system**, some coding overhead is added. So the bit rate is 19.2kbps, and since QPSK is used, the base band rate is thus 9600bps. This gives out PG=1.2288M/9600=**128.** The corresponding BER is:

$$BER = Q\left[\frac{1}{\sqrt{\frac{K-1}{3PG} + \frac{No}{2Eb}}}\right] = Q\left[\frac{1}{\sqrt{\frac{20-1}{3*128} + 1/12}}\right] = Q(2.74) = 0.0031$$

$\boxed{6.23}$

Assume P_k = Probability of collision, then $P_e = \frac{1}{2}e^{-\frac{E_b}{2N_o}}\left[1-P_k\right]+\frac{1}{2}P_k$.

P_k = Probability here is a collision with another user. There are M channels. The probability is equal to $1/M$. Assume the first user is on a certain channel. Probability of one user landing on one channel that is free from the first user is $\left[1-1/M\right]$. For $K-1$ users all landing on a free channel, the probability is $\left[1-1/M\right]^{K-1}$, therefore

$$P_k = 1-\left[1-1/M\right]^{K-1}$$

Assume M>>1.

Now $(1-x)^n = 1-x^n$ where x is an integer.

$$\text{Therefore } P_k = 1-\left[1-\frac{K-1}{M}\right] = \frac{K-1}{M}$$

$$\text{Therefore } P_e = \frac{1}{2}e^{-\frac{E_b}{2N_o}}\left[1-P_k\right]+\frac{1}{2}\left[\frac{K-1}{M}\right]$$

$$= \frac{1}{2}\left[\frac{K-1}{M}\right]$$

6.24

(a) In a DS-SS multiple user system, how many simultaneous users may be supported such that an average bit error rate of less than 10^{-3} is maintained for each user? Assume all users employ power control such that the received powers of each user are maintained at an average $E_b/N_o = 10\text{dB}$, and assume each user has a PN code that is produced from an 11-bit shift register.

here $\frac{E_b}{N_o} \neq \infty$ so must use

$$PE = Q\left[\frac{1}{\sqrt{\frac{k-1}{3N} + \frac{N_o}{2E_b}}}\right] \qquad \frac{E_b}{N_o} = 10$$

here $N = 2^M - 1 = 2^{11} - 1 = \underline{2047}$

Find K such that $PE = 10^{-3} \Rightarrow Q(3.1)$ from Table

$$\therefore 3.1 = \frac{1}{\sqrt{\frac{k-1}{3(2047)} + \frac{1}{20}}} \xrightarrow{(\,)^2} 9.6 = \frac{1}{\frac{k-1}{6141} + \frac{1}{20}} \Rightarrow 9.6 = \frac{20 \cdot 6141}{20(k-1) + 6141}$$

$$9.6[20(k-1) + 6141] = 20(6141) \Rightarrow 9.6[20(k-1)] = 10.4(6141) \Rightarrow 20(k-1) = (10.4)6141$$

$k-1 \approx 6700$

$\boxed{k \approx 330}$

(b) Using your answer in (a), what is the resulting average Probability of Error for a user if one more simultaneous user is added? Would this be noticeable to all the other users? Why or why not?

Not Noticeable

Not Noticeable, since one more user increments Q function by such a tiny bit. This is due to the PG of the DS-SS, which has an additional user provide only a very slight increase in MAI. This highlights the graceful degradation of DS-SS with additional users

6.25 (a) number of hops per second

$= 2\text{ hops/bit} \times R = 2 \times 25000 = \underline{\underline{5 \times 10^4 \text{ hops/sec}}}$

(b) For $\frac{E_b}{N_0} = 20dB = 100$ and a single user (assuming AWGN)

$P_b = \frac{1}{2} \cdot \exp(-\frac{E_b}{2N_0}) = \frac{1}{2} \cdot \exp(-\frac{100}{2}) \doteq \underline{\underline{9.64 \times 10^{-23} \doteq 0}}$

(c) number of possible hopping channels,

$M = \frac{20 \times 10^6}{50 \times 10^3} = 400 \Rightarrow P_b \doteq \frac{1}{2}\left[\frac{K-1}{M}\right] = \frac{1}{2} \times \frac{20}{400} = \underline{\underline{0.025}}$

(d) $P_b \approx \frac{1}{2} P_h = \frac{1}{2}\left[1 - (1 - \frac{1}{M})^{K-1}\right]$

$= \frac{1}{2}\left[1 - (1 - \frac{1}{400})^{200}\right] \doteq \underline{\underline{0.2}}$

6.26 Hints for solving Problem 6.26

To evaluate the probability of error, P_e, of a signal in flat Rayleigh fading, simply weight the P_e by the conditional likelihood of the signal being a particular value. That is,

$$P(\text{error}) = \int_0^\infty \underbrace{P(\text{error}\,|\text{specific } E_b/N_0)}_{\text{in text books - AWGN}} \cdot P(\text{specific } E_b/N_0)\, d\left(\frac{E_b}{N_0}\right) \quad (1)$$

where the probability density of the fading E_b/N_0 is given as the square of a Rayleigh distributed r.v., which is easily shown to be exponential, eqn. (6.155).

If we let X = random E_b/N_0 due to fading and let α^2 denote a chi-square (exponential) r.v. with the pdf of a squared Rayleigh distributed voltage, then

$$X = \alpha^2 \left(\frac{E_b}{N_0}\right) \quad (2)$$

Let's let

$$\Gamma = \overline{\alpha^2}\frac{E_b}{N_0}, \text{ the average value of } \frac{E_b}{N_0} \quad (3)$$

Then:

$$P_e(\Gamma) = \int_0^\infty P_e(X) \cdot \frac{1}{\Gamma} e^{-\frac{X}{\Gamma}}\, dX \text{ (Eqn. 1)} \quad (4)$$

is the value of P_e in flat slow Rayleigh fading.

6.26 Hints cont'd.

Hint: Derive the p.d.f. for

$$X = \alpha^2 \left(\frac{E_b}{N_0}\right) \tag{5}$$

where E_b/N_0 is a constant and α is Rayleigh, and you get (6.155), an exponential PDF.

Note a table of integrals can evaluate (Eqn. 1) where

$$P_e^{(1)}(x) = \frac{1}{2}e^{-\eta x} \tag{6}$$

$$\eta = \frac{1}{2} \text{ for noncoherent FSK} \tag{7}$$

$$\eta = 1 \text{ for DPSK} \tag{8}$$

and

$$P_e^{(2)}(x) = \frac{1}{2} erfc\sqrt{\beta x} \tag{9}$$

$$\beta = \frac{1}{2} \text{ : coherent FSK} \tag{10}$$

$$\beta = 1 \text{ coherent PSK} \tag{11}$$

A table of integrals can show:

$$P_b^{(1)} = \int_0^\infty P_b^{(1)}(X)\frac{1}{\Gamma}e^{-\frac{X}{\Gamma}}\,dX \tag{12}$$

$$= \frac{1}{2}\left[\frac{1}{1+\eta\Gamma}\right] \tag{13}$$

$$P_e^{(2)} = \int_0^\infty P_b^{(2)}(X)\frac{1}{\Gamma}e^{-\frac{X}{\Gamma}}\,dX \tag{14}$$

$$= \frac{1}{2}\left[\frac{1}{1-\sqrt{1+\frac{1}{\beta\Gamma}}}\right] \tag{15}$$

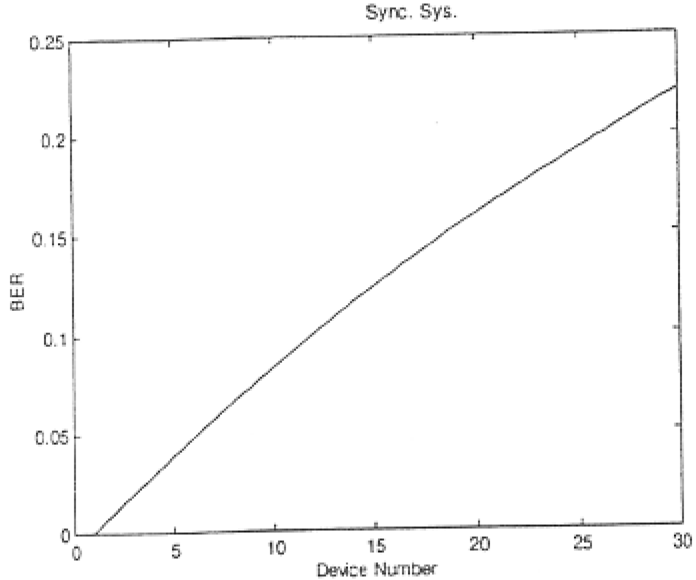
OR - if not closed form, use numerical integration for specific values of Γ.

6.26 Solution

We can treat the system as synchronized because they are in the nearby local area.

Using formula 6.147 and 6.149, we can plot the curve for BER vs. K. Please note that you should use the exact expression of 6.148, not the approximation expression, so that you can observe the change of the curve. A sample code in Matlab is enclosed.

When K=25, BER is above 0.2, which is redeemed as unacceptable because we need to apply significant coding scheme to reduce the end user BER. However, for the homework, any reasonable justification about number of acceptable users will be credited.



6.27 See the MATLAB program p6-27.m and Fig. p6-27a (a), (b). Fig. p6-27b(a), (b), Fig. p6-27c (a), (b) and Fig. p6-27d.

For a Gaussian lowpass filter with transfer function

$H_G(f) = \exp(-\alpha^2 f^2)$

the 3-dB bandwidth B is related with α by the following equation

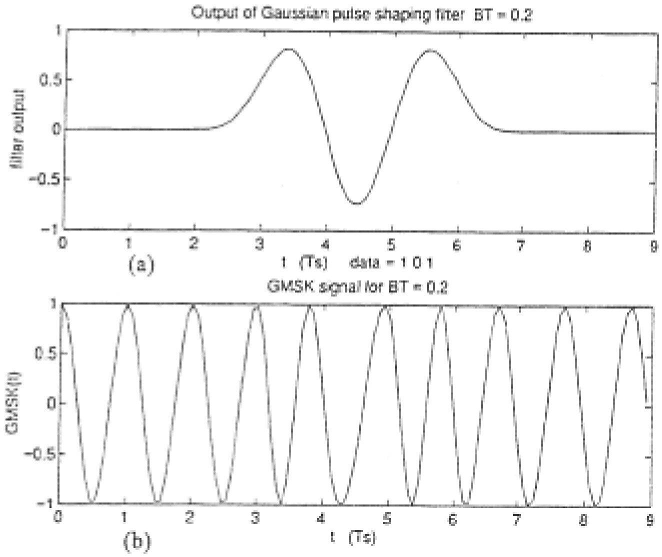


Fig. p6_27a (a), (b)

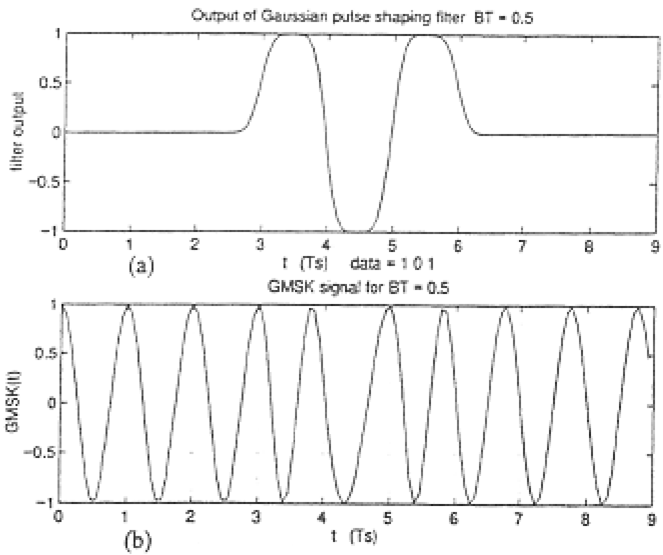


Fig. p6_27b (a), (b)

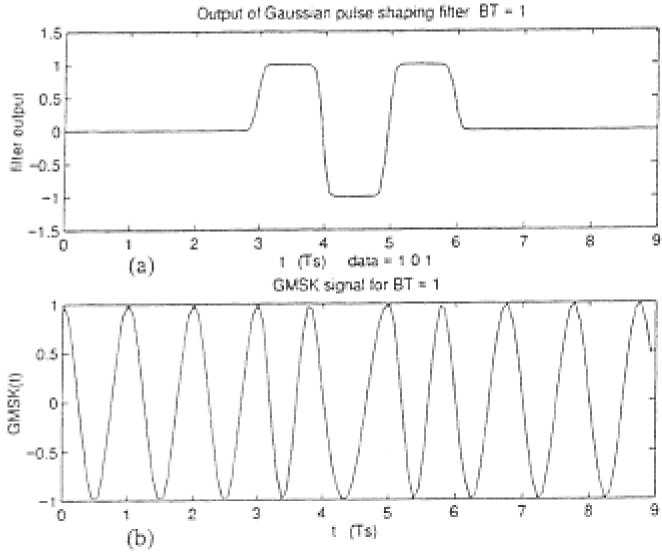


Fig. p6_27c (a), (b)

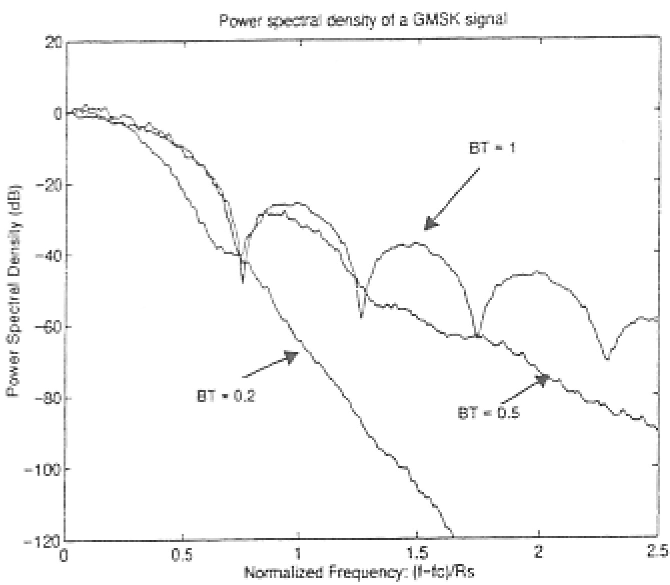


Fig. p6_27d

6.27 Cont'd

$$H_G(B) = \exp(-\alpha^2 B^2) = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad -\alpha^2 B^2 = -\frac{1}{2}\ln 2$$

$$\Rightarrow \quad \alpha = \sqrt{\frac{\ln 2}{2}} \cdot \frac{1}{B}$$

As shown in Fig. 6.42, the GMSK signal is generated by passing a NRZ message bit stream through a Gaussian baseband filter, followed by an FM modulator. with a modulation index of 0.5.

$\boxed{6.28}$ We define the RF bandwidth as the band of which everywhere outside, the power spectral density (PSD) is below -40 dB. From Fig. 6.41, we have

For BT = 0.25, $B_W = 2\times 0.83 R_s = 1.66 R_s$

For BT = 0.5, $B_W = 2\times 1.16 R_s = 2.32 R_s$

For BT = 1, $B_W = 2\times 1.6 R_s = 3.2 R_s$

For BT = 5, $B_W > 2\times 2.5 R_s = 5 R_s$

6.28 Cont'd

Since $P_e = Q\left(\sqrt{\frac{2\alpha E_b}{N_o}}\right)$, $\alpha \doteq \begin{cases} 0.68 & \text{for GMSK with } BT=0.25 \\ 0.85 & \text{for simple MSK } (BT=\infty) \end{cases}$

the E_b/N_o degradation for all these cases will be less than 1dB when compared to the optimum MSK, the larger the BT, the less the degradation.

From the above we can see that when BT decreases, the RF bandwidth becomes small. Although the BER increases, as long as the GMSK irreducible error rate is less than that produced by the mobile channel, there is no penalty in using GMSK.

$\boxed{6.29}$ The output $y(t)$ of the FM discriminator is the instantaneous frequency deviation of the input from the carrier frequency. For the input $x(t) = \cos(\omega_c t + \theta(t))$, $y(t) = \frac{d\theta(t)}{dt}$. Therefore, after integrated over one symbol period.

$$\phi = \int_{(k-1)T_s}^{kT_s} \frac{d\theta(t)}{dt} \cdot dt = \theta(kT_s) - \theta((k-1)T_s)$$

$$= \theta_k - \theta_{k-1}$$

From ϕ, using table 6.2, we can determine the transmitted data stream.

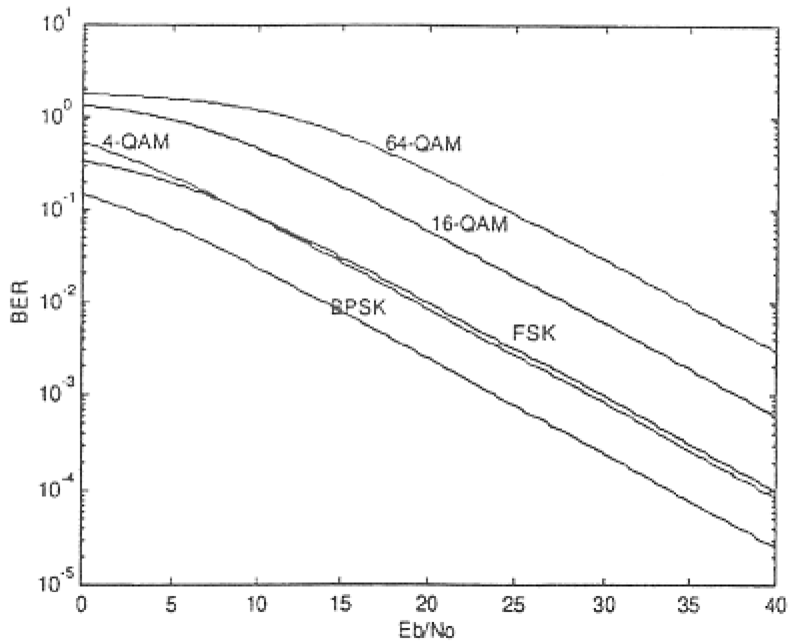
6.30

Using (6.156) and assume Rayleigh fading distribution model, we can evaluate the Bit Error Rate from :

$$Pe(\Gamma) = \int_0^\infty Pe(X) \bullet \frac{1}{\Gamma} e^{-\frac{X}{\Gamma}} dX$$

Substituting QAM Bit Error Rate formula (6.127) into Pe(X), where $X=\alpha^2(Eb/No)$, (α^2 can be normalized such that $\overline{\alpha^2} = 1$ and $\overline{X} = \Gamma$, the average Eb/No in fading channel) we can work out the curve by using **numerical integration** with your favorite math. software tool because closed form is hard to get.

Note that the QAM BER formula in (6.127) is derived from Union Bound (upper bound). So while we use it to plot the curves for QAM BER, at very low Eb/No, the BER is above 1. This is not true for the real world but is acceptable in the homework if you are using (6.127) to do the plots.



6.31

(a) Given that binary DPSK modulation has a bit error probability of

$$P_e = \frac{1}{2}e^{-\frac{E_b}{N_o}}$$

in AWGN, find the probability of error for DPSK in a Rayleigh, flat-fading channel.

$\Gamma = \text{Avg } \frac{E_b}{N_o}$

$\Gamma = \alpha^2 \frac{E_b}{N_o}$

$$P_E = \int_0^\infty P_e(x)\,P(x)\,dx \quad \text{where } X = \frac{\alpha^2 E_b}{N_o}$$

$\overline{\alpha^2} = 1$

$$\int_0^\infty \left[\frac{1}{2}e^{-x}\right]\left[\frac{1}{\Gamma}e^{-\frac{x}{\Gamma}}\right]dx \qquad \boxed{P(x) = \frac{1}{\Gamma}e^{-\frac{x}{\Gamma}} \quad x \geq 0}$$

$$= \frac{1}{2\Gamma}\int_0^\infty e^{-\left[\frac{\Gamma x + x}{\Gamma}\right]}dx$$

$$= \frac{1}{2\Gamma}\int_0^\infty e^{-x\left[\frac{\Gamma+1}{\Gamma}\right]}dx$$

$$= \left[\frac{-\Gamma}{(\Gamma+1)\,2\Gamma}\right]e^{-x}\Big|_0^\infty$$

$$= \frac{1}{2[\Gamma+1]} \quad [\text{eqn } 5.157]$$

(b) If the average SNR for a Rayleigh faded DPSK signal is 30dB, what is the probability of error at the receiver?

$\Gamma = 30\,\text{dB}$

if $\Gamma = 10^3$ $\quad P_E = \frac{1}{2[10^3+1]} \approx \frac{1}{2}10^{-3}$

$$P_E = \underline{5\;10^{-4}}$$

$\boxed{6.33}$ For the output $x(t) = \cos(w_c t + \theta_K)$, the output of the delayline is $x(t-T_s) = \cos(w_c t + \theta_{K-1})$

$\Rightarrow$ Output of LPF in the upper arm.

$W_K = \cos(\theta_K - \theta_{K-1})$

Output of LPF in the lower arm.

$Z_K = \sin(\theta_n - \theta_{K-1})$

$\Rightarrow$ output of decision device in the upper arm.

$S_I = 1$, if $W_K > 0$

$S_I = 0$, if $W_K < 0$

output of decision device in the lower arm.

$S_Q = 1$, if $Z_K > 0$

$S_Q = 0$, if $Z_K < 0$

Thus the transmitted data stream can be detected

$\boxed{6.34}$ For $P(x) = e^{-x}$, $x > 0$, from equation (6.155), we have $\Gamma = 1$.

$\Rightarrow P_{e,DPSK} = \frac{1}{2(1+\Gamma)} = \frac{1}{2(1+1)} = \underline{\underline{0.25}}$

$\boxed{6.35}$ (a) <u>For Rayleigh fading channel</u>, $P_{e,DPSK} = \frac{1}{2\Gamma}$. So the necessary average E_b/N_0 in order to detect DPSK with an average BER of 10^{-3} is

$$\frac{E_b}{N_0} = \frac{1}{2P_{e,DPSK}} = \frac{1}{2\times10^{-3}} = 500 \doteq 27\,dB.$$

(b) <u>For Ricean fading channel</u>, as shown in Example 6.11.

$$P_{e,DPSK} = \frac{1+K}{2(\Gamma+1+K)} \exp\left(\frac{-K\Gamma}{\Gamma+1+K}\right)$$

$\boxed{6.32}$ (a) Let $X = \frac{E_b}{N_0}\alpha^2$; the instantaneous SNR in fading channel

Let $\Gamma = \frac{E_b}{N_0}\overline{\alpha^2}$; the average SNR over the local area.

$$P_e(\Gamma) = \int_{-\infty}^{\infty} P_e(x)\, \underbrace{p(x)}_{} \, dX$$

given by exponential power distribution (the square of a Rayleigh voltage)

$$p(x) = \frac{1}{\Gamma} e^{-\frac{x}{\Gamma}} u(x)$$

$$\therefore P_e[\Gamma] = \int_0^{\infty} \left[\frac{1}{2} e^{-x}\right]\left[\frac{1}{\Gamma} e^{-\frac{x}{\Gamma}}\right] dx = \frac{1}{2\Gamma}\int_0^{\infty} e^{-x\left[1+\frac{1}{\Gamma}\right]} dX$$

$$= \frac{1}{2\Gamma}\frac{-1}{\left[1+\frac{1}{\Gamma}\right]}\left[e^{-x\left[1+\frac{1}{\Gamma}\right]}\right|_0^{\infty} = \frac{1}{2\Gamma}\frac{-1}{\left[\frac{\Gamma}{\Gamma}+\frac{1}{\Gamma}\right]}\left[0-1\right]$$

$$= \frac{1}{2[\Gamma+1]}$$

(b) in AWGN $10^{-3} = \frac{1}{2}e^{-x} \Rightarrow X = -\ln\left[2\, 10^{-3}\right] = 6.21$

$X_{dB} = 10\log_{10}(6.21) = \underline{7.9\,dB}$

in fading $10^{-3} = \frac{1}{2}\left[\frac{1}{\Gamma+1}\right] \overset{?}{\Rightarrow} \Gamma+1 = \frac{1}{2\,10^{-3}} \Rightarrow \Gamma = \frac{1}{2\,10^{-3}} - 1 = 499$

$10\log_{10}(499) = \underline{\underline{27\,dB}}$

Therefore, additional power needed is

$\Delta = 27 - 7.9 = \underline{\underline{19.1\,dB}}$ to overcome Rayleigh fading for comparable BER

6.35 Cont'd

Assume $K = 6\,dB$, for $P_e = 10^{-3}$, we have the average $\frac{E_b}{N_0}$, $\Gamma = 18\,dB$

If $K = 7\,dB$, $P_e = 10^{-3}$, we have $\Gamma = 16\,dB$

We can see that for Ricean fading channel, the necessary average $\frac{E_b}{N_0}$ for a particular P_e is less than that for Rayleigh fading channel. And when K increases, the necessary average $\frac{E_b}{N_0}$ decreases for fixed P_e.

$\boxed{6.36}$ (a) For Rayleigh fading channel. $P_{e,BPSK} = \frac{1}{4\Gamma}$

$$\Rightarrow \Gamma = \frac{1}{4 P_{e,BPSK}} = \frac{1}{4 \times 10^{-5}} \doteq 44\,dB$$

(b) $P_e = \int_0^\infty P_e(x) \cdot f(x)\,dx$. for Ricean fading channel. $f(x) = \frac{1+K}{\Gamma} \exp\left(-\frac{x(1+K)+K\Gamma}{\Gamma}\right) I_0\left(\sqrt{\frac{4(1+K)Kx}{\Gamma}}\right)$.

for BPSK, $P_e(x) = Q(\sqrt{2x})$. Therefore

$$P_e = \int_0^\infty Q(\sqrt{2x}) \cdot \frac{1+K}{\Gamma} \cdot \exp\left(-\frac{x(1+K)+K\Gamma}{\Gamma}\right) \cdot I_0\left(\sqrt{\frac{4(1+K)Kx}{\Gamma}}\right) dx.$$

This integral is calculated by the MATLAB program p6_36.m and the result is shown in Fig. P6-36

From Fig. P6-36, we obtain

if $K = 6\,dB$, for $P_e = 10^{-5}$, average $E_b/N_0 = 34\,dB$

if $K = 7\,dB$, for $P_e = 10^{-5}$, average $E_b/N_0 = 30.5\,dB$

$\boxed{6.37}$ Let $y = \alpha^2$, we have the pdf of y

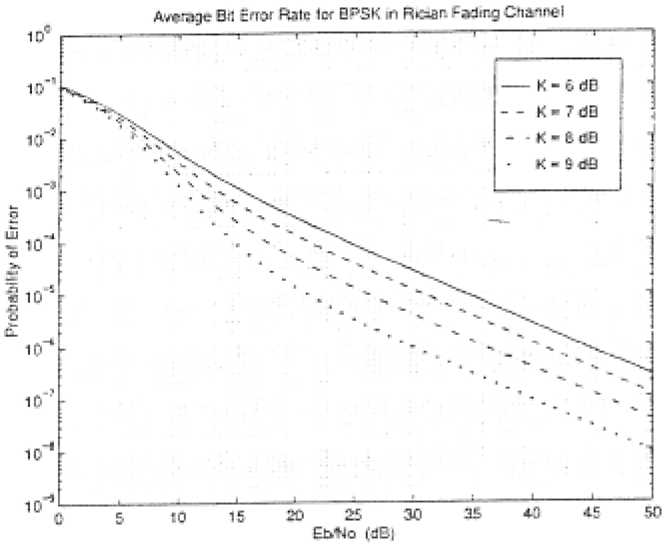

Average Bit Error Rate for BPSK in Rician Fading Channel
Probability of Error
Eb/No (dB)
K = 6 dB
K = 7 dB
K = 8 dB
K = 9 dB
0
5
10
15
20
25
30
35
40
45
50

Fig. p6_36

6.37 Cont'd

$$p_Y(y) = p_\alpha(\alpha) \cdot \frac{d\alpha}{dy}\Big|_{y=\alpha^2} \quad \text{where } p_\alpha(\alpha) = \frac{\alpha}{\sigma^2} \exp\left(-\frac{\alpha^2}{2\sigma^2}\right).$$

for $\alpha \geq 0$. Therefore,

$$p_Y(y) = \frac{\alpha}{\sigma^2} \cdot \exp\left(-\frac{\alpha^2}{2\sigma^2}\right) \cdot \frac{1}{\frac{dy}{d\alpha}}\Bigg|_{y=\alpha^2}$$

$$= \frac{\alpha}{\sigma^2} \cdot \exp\left(-\frac{\alpha^2}{2\sigma^2}\right) \cdot \frac{1}{2\alpha}\Big|_{y=\alpha^2}$$

$$= \frac{1}{2\sigma^2} \exp\left(-\frac{y}{2\sigma^2}\right) \quad \text{for } y \geq 0.$$

Since $\overline{\alpha^2} = 2\sigma^2$ (see chapter 5), we have

$$p_Y(y) = \frac{1}{\overline{\alpha^2}} \cdot \exp\left(\frac{-y}{\overline{\alpha^2}}\right).$$

$\boxed{6.38}$ $$P_{e,GMSK} = \frac{1}{2}\left(1 - \sqrt{\frac{\delta\Gamma}{\delta\Gamma + 1}}\right) = \frac{1}{2}\left(1 - \sqrt{\frac{1}{1 + \frac{1}{\delta\Gamma}}}\right)$$

$$= \frac{1}{2}\left[1 - \left(1 + \frac{1}{\delta\Gamma}\right)^{-\frac{1}{2}}\right]$$

Using Taylor's series approximation,

$$P_{e,GMSK} \doteq \frac{1}{2}\left[1 - \left(1 - \frac{1}{2\delta\Gamma}\right)\right] = \frac{1}{4\delta\Gamma}$$

$\boxed{6.39}$ See problem 6.38.

CHAPTER 7

7.1 In this case $x_k = \sum_{n=0}^{N} w_{nk} \cdot y_{nk}$, let

$$\underline{Y}_k = [\, y_{0k} \;\; y_{1k} \;\; y_{2k} \; \cdots \; y_{Nk} \,]^T,$$

$$\underline{W}_k = [\, w_{0k} \;\; w_{1k} \;\; w_{2k} \; \cdots \; w_{Nk} \,]^T.$$

We have $x_k = \underline{Y}_k^T \cdot \underline{W}_k = \underline{W}_k^T \cdot \underline{Y}_k$, and

$$e_k = d_k - x_k = d_k - \underline{Y}_k^T \cdot \underline{W}_k = d_k - \underline{W}_k^T \cdot \underline{Y}_k.$$

We can see that the expression for x_k and e_k are the same as equation (7.11) and (7.12), thus the MSE are identical. Using the same method described in section 7.3, we have the optimum weight vector $\hat{w}$ for MMSE,

$$\underline{\hat{W}} = R^{-1} \cdot \underline{P}$$

where $R = E[\underline{Y}_k \cdot \underline{Y}_k^T] = E\begin{bmatrix} y_{0k}^2 & y_{0k} \cdot y_{1k} & \cdots & y_{0k} \cdot y_{Nk} \\ y_{1k}\, y_{0k} & y_{1k}^2 & \cdots & y_{1k} \cdot y_{Nk} \\ \cdots & \cdots & \cdots & \cdots \\ y_{Nk} \cdot y_{0k} & y_{Nk} \cdot y_{1k} & \cdots & y_{Nk}^2 \end{bmatrix}$

and $\underline{P} = E[d_k \cdot \underline{Y}_k] = E[\, d_k \cdot y_{0k} \;\; d_k\, y_{1k} \; \cdots \; d_k\, y_{Nk} \,]^T$

7.2 (a) Assume $N > 2$. We have

$$\underline{Y}_k = \left[\sin\frac{2\pi k}{N} \quad \sin\frac{2\pi(k-1)}{N} \right]^T. \qquad d_k = 2\cos\left(\frac{2\pi k}{N}\right).$$

$$\underline{W}_k = [\, w_0 \;\; w_1 \,]^T.$$

$$\Rightarrow R = E[\underline{Y}_k \cdot \underline{Y}_k^T] = E\begin{bmatrix} \sin^2\frac{2\pi k}{N} & \sin\frac{2\pi k}{N} \cdot \sin\frac{2\pi(k-1)}{N} \\ \sin\frac{2\pi(k-1)}{N} \cdot \sin\frac{2\pi k}{N} & \sin^2\frac{2\pi(k-1)}{N} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{1}{2}\cos\frac{2\pi}{N} \\ \frac{1}{2}\cos\frac{2\pi}{N} & \frac{1}{2} \end{bmatrix}$$

7.2 Cont'd

$$\underline{P} = E[d_k \cdot \underline{Y}_k] = E\left[2\cos\frac{2\pi k}{N}\cdot\sin\frac{2\pi k}{N} \quad 2\cos\left(\frac{2\pi k}{N}\right)\cdot\sin\frac{2\pi(k-1)}{N}\right]^T$$

$$= \left[0 \quad -\sin\frac{2\pi}{N}\right]^T$$

$$E[d_k^2] = E\left[4\cos^2\frac{2\pi k}{N}\right] = 2$$

$$\Rightarrow MSE = E[|e_k|^2] = E[d_k^2] + \underline{W}_k^T\cdot \mathbb{R}\cdot\underline{W} - 2\underline{P}^T\cdot\underline{W}$$

$$= 2 + [W_0 \quad W_1]\cdot\frac{1}{2}\begin{bmatrix} 1 & \cos\frac{2\pi}{N} \\ \cos\frac{2\pi}{N} & 1\end{bmatrix}\cdot\begin{bmatrix} W_0 \\ W_1\end{bmatrix} - 2\cdot\left[0 \quad -\sin\frac{2\pi}{N}\right]\begin{bmatrix} W_0 \\ W_1\end{bmatrix}$$

$$= \underline{\underline{2 + \frac{1}{2}\left(W_0^2 + W_1^2 + 2\cos\left(\frac{2\pi}{N}\right)\cdot W_0 W_1\right) + 2W_1\cdot\sin\frac{2\pi}{N}}}$$

(b) For $N>2$, we have

$$\underline{\hat{W}} = \mathbb{R}^{-1}\cdot\underline{P} = \frac{2\begin{bmatrix} 1 & -\cos\frac{2\pi}{N} \\ -\cos\frac{2\pi}{N} & 1\end{bmatrix}}{\sin^2\frac{2\pi}{N}}\cdot\left[0 \quad -\sin\frac{2\pi}{N}\right]^T$$

$$= 2\begin{bmatrix} \cos\frac{2\pi}{N} / \sin\frac{2\pi}{N} \\ -1/\sin\frac{2\pi}{N}\end{bmatrix}$$

$$\Rightarrow W_0 = \frac{2\cos\frac{2\pi}{N}}{\sin\frac{2\pi}{N}}, \quad W_1 = \frac{-2}{\sin\frac{2\pi}{N}}$$

$$\Rightarrow MMSE = 2 + \frac{1}{2}\left(\frac{4\cos^2\frac{2\pi}{N}}{\sin^2\frac{2\pi}{N}} - \frac{8\cos^2\frac{2\pi}{N}}{\sin^2\frac{2\pi}{N}} + \frac{4}{\sin^2\frac{2\pi}{N}}\right) - 2\cdot\frac{2}{\sin\frac{2\pi}{N}}\cdot\sin\frac{2\pi}{N}$$

$$= \underline{\underline{0}}$$

(c) $$MSE = 2 + \frac{1}{2}\left(W_0^2 + 2\cos\left(\frac{2\pi}{N}\right)\cdot W_0\cdot W_1 + W_1^2\right) + 2W_1\cdot\sin\frac{2\pi}{N}$$

$$= 2 + \frac{1}{2}(0+0+4) + 2\times(-2)\cdot\sin\frac{2\pi}{4}$$

$$= \underline{\underline{0}}$$

7.2 Cont'd

(d) $e_k = d_k - (W_0 \cdot Y_k + W_1 \cdot Y_{k-1})$

$$= 2\sin\left(\frac{2\pi k}{N}\right) - (-2)\cdot \sin\frac{2\pi(k-1)}{N}$$

$$= 2\cdot\left[\sin\left(\frac{\pi}{2}k\right) + \sin\left(\frac{\pi}{2}(k-1)\right)\right]$$

$$\Rightarrow E\left[|e_k|^2\right] = 2\cdot E\left[\sin^2\left(\frac{\pi}{2}k\right) + \sin^2\left(\frac{\pi}{2}(k-1)\right) + 2\sin\left(\frac{\pi}{2}k\right)\sin\left(\frac{\pi}{2}(k-1)\right)\right]$$

$$= 2\cdot\left(\frac{1}{2}+\frac{1}{2}+0\right) = \underline{\underline{2}}$$

$\boxed{7.3}$ $MSE = 2 + \frac{1}{2}\left[W_0^2 + 2\cos\left(\frac{2\pi}{N}\right)\cdot W_0 W_1 + W_1^2\right] + 2W_1\sin\frac{2\pi}{N} = 2^2 = 4$

$$\Rightarrow 2 + \frac{1}{2}\left[W_0^2 + 2\cos\left(\frac{2\pi}{5}\right)\cdot W_0 W_1 + W_1^2\right] + 2\cdot W_1 \sin\frac{2\pi}{5} = 4$$

$$\Rightarrow \underline{\underline{0.5W_0^2 + 0.5W_1^2 + 0.309W_0 W_1 + 1.902W_1 - 2 = 0}}$$

Any pair of W_0 and W_1 that can satisfy the above equation can have the rms value of $\varepsilon_k = 2$

$\boxed{7.4}$ Let N denote the number of coefficients in the equalizer, and M the time required between each iteration.

(a) For LMS, $M = (2N+1)\cdot 10^{-6}$ (s) $= \underline{\underline{2N+1\ \mu s}}$

(b) For Kalman RLS, $M = \underline{\underline{2.5N^2 + 4.5N\ \mu s}}$

(c) For square root RLS DFE, $M = \underline{\underline{1.5N^2 + 6.5N\ \mu s}}$

(d) For Gradient lattice DFE, $M = \underline{\underline{13N - 8\ \mu s}}$

7.5 For $f_d = 100\,Hz$, we have the coherence time

$$T_c = \sqrt{\frac{9}{16\pi f_d^2}} \doteq 4.23\ msec$$

Therefore the maximum time interval before retraining is 4.23 msec. Suppose $N=5$, for LMS algorithm, each updating of the equalizer needs time T_u, where

$$T_u = (2N+1) \times 10^{-6} \times \text{iteration numbers to converge}$$
$$= (2\times5+1)\times10^{-6}\times10^{3} = 11\times10^{-3}\,S = 11\ ms$$

Require $T_u < 10\%$ transmission overhead $= 10\% \times 4.23 = 0.423$ msec,

it is impossible to implement the LMS algorithm on such a low speed DSP chip. If the DSP chip can perform 27 Million multiplications per second. T_u becomes $0.41ms < 4.23\times10\% = 0.423ms$. Therefore for $f_d=100Hz$ and $N=5$, the LMS algorithm can be implemented on a 27 Million multiplications per second DSP chip.

Suppose each time slot of the signal contains 162 symbol, in 0.41ms time duration, there should be $162\times10\% \doteq 16$ symbols. Therefore the maximum symbol rate is

$$R_{smax} = \frac{16}{0.41} \doteq 39.02\ KSps$$ due to channel coherence.

Similarly, for RLS algorithm using the low speed DSP chip.

$$T_u = (2.5N^2 + 4.5N)\times10^{-6}\times 50 \text{ iterations}$$
$$= (2.5\times5^2 + 4.5\times5)\times10^{-6}\times50 = 4.25\times10^{-3}\,S$$
$$= 4.25\ ms > 4.23\times10\% = 0.423\ ms$$

7.5 Cont'd

Thus it is also impossible to implement the RLS algorithm on a DSP chip with 1 Million multiplications per second. The minimum speed of the DSP chip required is

$$\frac{4.25\,ms}{10\% \cdot T_c(ms)} = \frac{4.25}{0.1 \times 4.23} \doteq \underline{10.1 \text{ Million multiplications/sec}}.$$

$$\Rightarrow T_u = \frac{4.25ms}{10.1} \doteq 0.42ms, \quad R_{smax} = \frac{16}{0.42} \doteq \underline{\underline{38.02 \text{ ksps}}}.$$

If the DSP chip with <u>27 Million multiplications per second</u> is used, we have $T_u = \frac{4.25\,ms}{27} \doteq 0.157ms < 0.423ms$. and $R_{smax} = \frac{16}{0.157} = \underline{\underline{101.65 \text{ ksps}}}$. We can see that using the same speed DSP chip, RLS algorithm can handle higher data rate than LMS algorithm.

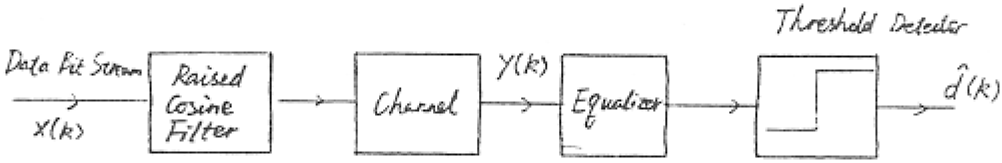
(b) For <u>$f_d = 1000\,Hz$</u>, $T_c = 0.423\,ms$, therefore the maximum time interval before retraining is $\underline{\underline{0.423\,ms}}$. We can see that even using a DSP chip that can perform 100 Million multiplication per second, since $T_u = \frac{11ms}{100} = 0.11ms > 10\% \cdot T_c = 0.0423\,ms$, <u>it is impossible to implement the LMS algorithm</u>. <u>For the RLS algorithm</u>, the minimum speed of the DSP chip required is $\frac{4.25ms}{10\% \cdot T_c(ms)} = \frac{4.25}{0.0423} \doteq 101$ million multiplications/sec, $\Rightarrow T_u = \frac{4.25\,ms}{101} \doteq 0.042ms$. $R_{smax} = \frac{16}{0.042} \doteq \underline{\underline{380.26\text{ksps}}}$

(c) For <u>$f_d = 10000\,Hz$</u>, $T_c = 0.0423\,ms$. It's <u>impossible</u> for both the LMS and RLS algorithms to implement using the DSP chip with current technology.

$\boxed{7.6}$ (a) See the MATLAB program p7-06.m

7.6 Cont'd

The block diagram of the system is shown in the figure below. The raised cosine filter is used to reduced the



bandwidth of the transmitted signal
Fig. p7_06a(a) shows the input $x(t)$ (in NRZ waveform).
Fig. p7-06a(b) shows the output of the two ray channel $y(t)$ ($x(t)$ has the rolloff factor of 1).
Fig. p7-06a(c) shows the output of the threshold detector $\hat{d}(t)$, (after the equalizer converges). Comparing Fig. p7-06a(a) and Fig. p7-06a(c), we can see that the data is recreated except for a time delay (use the bars in both figures for comparing)
Fig. p7-06a(d) shows the error signal used to update the coefficients of the equalizer.

(b) Fig. p7-06b shows the MMSE as a function of the number of iterations. The MMSE is calculated using the MATLAB program p7_06.m.

(c) From Fig. p7-06a(d) and Fig. p7_06b, we can see that about 400 iterations are required to obtain convergence

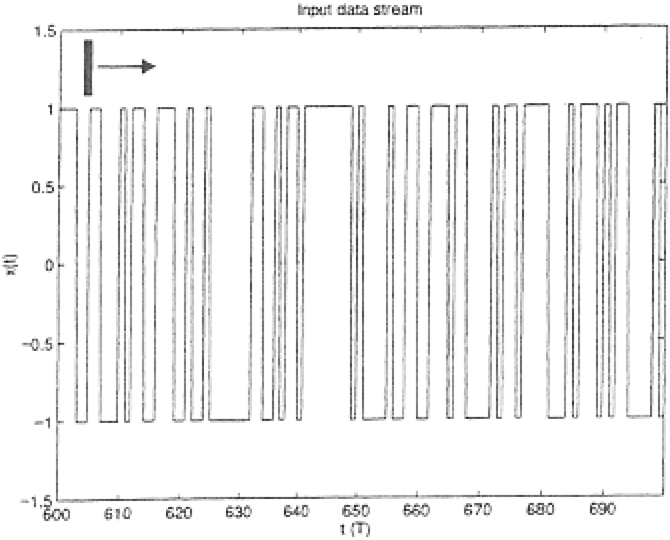


Fig. p7_06a (a)

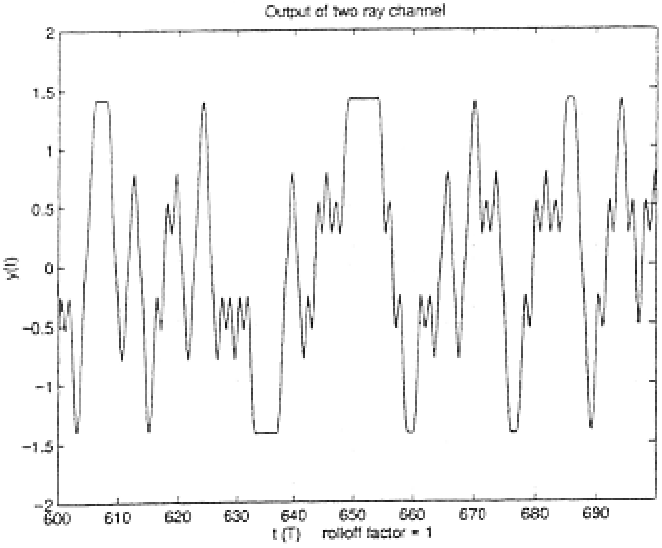


Fig. p7_06a (b)

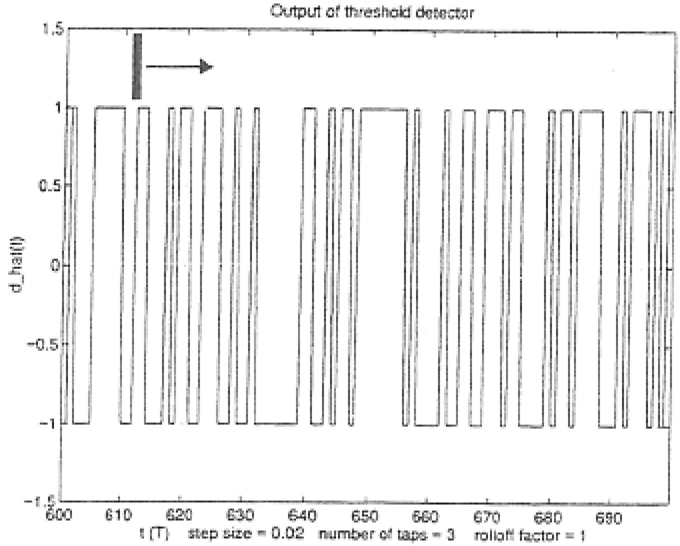


Fig. p7_06a (c)

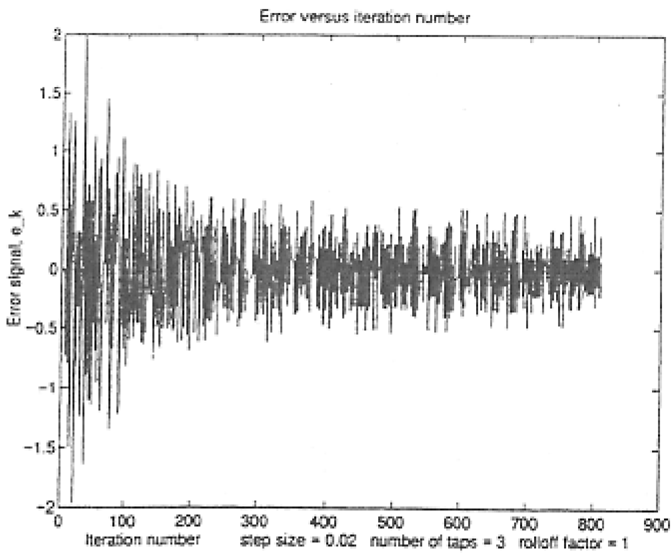


Fig. p7_06a (d)

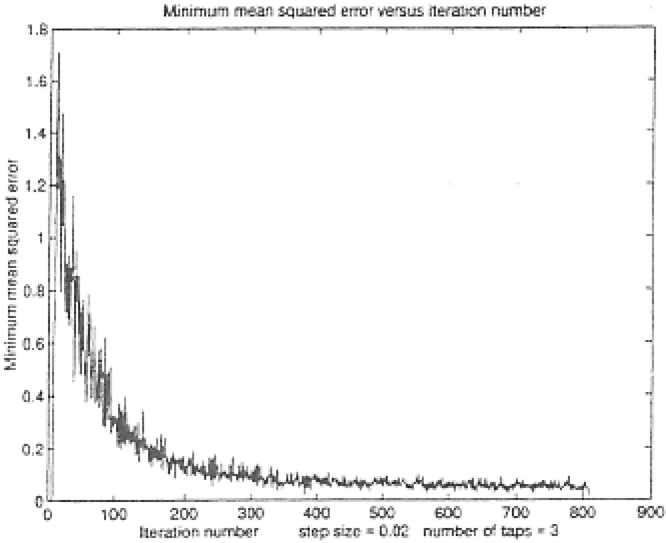


Fig. p7_06b

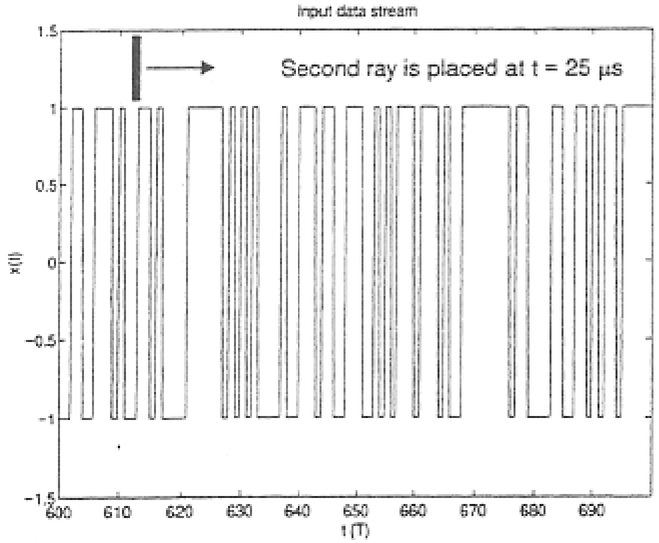


Fig. p7_06d (a)

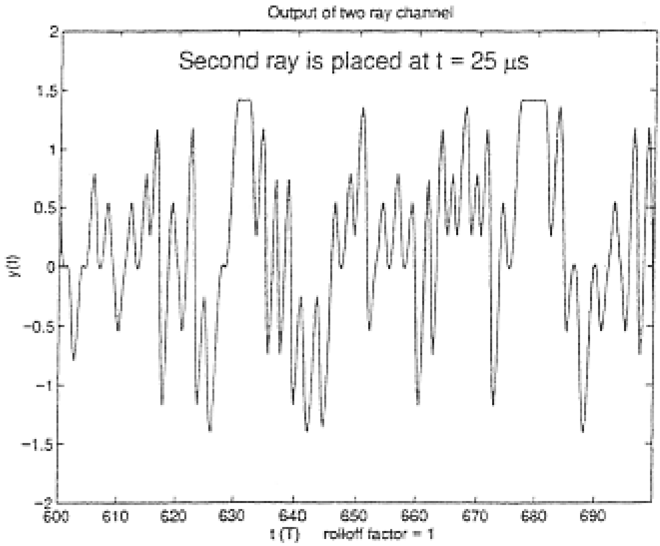


Fig. p7_06d (b)

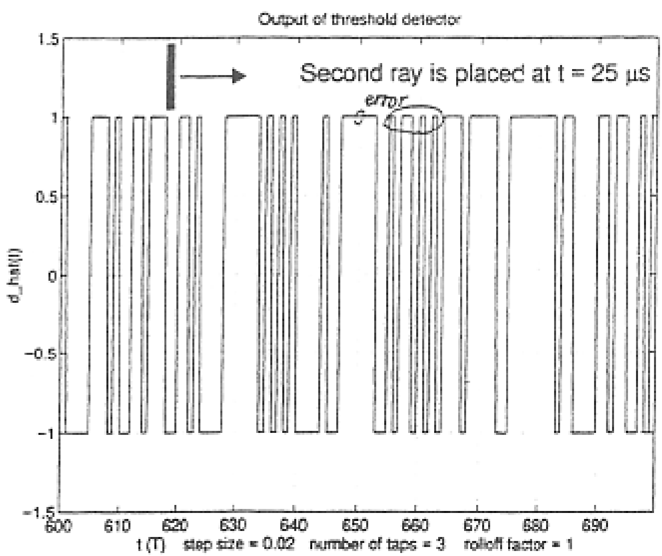


Fig. p7_06d (c)

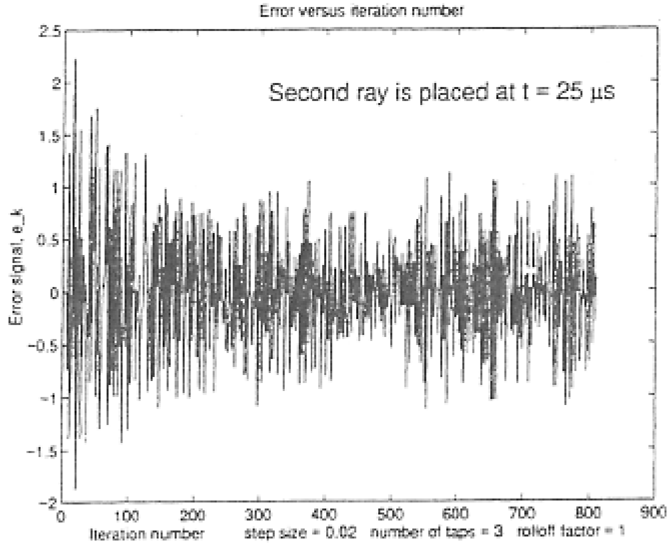


Fig. p7_06d (d)

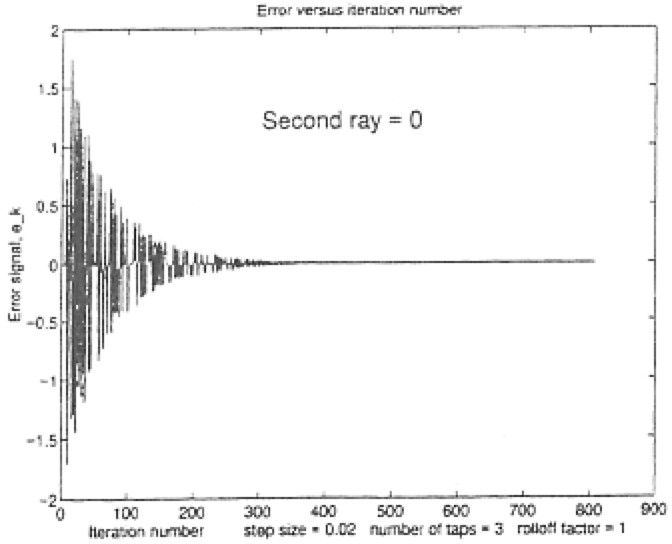


Fig. p7_06c

7.6 Cont'd

(d) If the second ray is placed at $t = 25\ \mu s$, the maximum delay spread is greater than the delay that the equalizer can offer, therefore <u>the data cannot be recreated correctly</u>. See Fig. p7-06d (a), (b), (c) and (d).

(e) When the second ray is set equal to zero, although the error after convergence becomes very small, it still exists. That's the equalizer noise. This is shown in Fig. p7-06e.

$\boxed{7.7}$ (a) Since $-6dB = \frac{1}{4}$, $Pr[\gamma_i \leq \frac{\gamma}{4}] = 1 - e^{-\frac{\gamma/4}{\Gamma}} = 0.2$, where γ is the SNR threshold, we have

$$\frac{\gamma}{4\Gamma} = -\ln 0.8 \Rightarrow \frac{\Gamma}{\gamma} = \frac{1}{-4\ln 0.8} \doteq 1.12 \doteq \underline{\underline{0.5\ dB}}$$

Therefore, the mean SNR of the Rayleigh fading signal is 0.5 dB above the SNR threshold. Using equation (7.59), we have

(b) P_2 (6dB below the mean SNR threshold) $= 0.2^2 = \underline{\underline{0.04}}$

(c) P_3 (6dB below the mean SNR threshold) $= 0.2^3 = \underline{\underline{0.008}}$

(d) P_4 (6dB below the mean SNR threshold) $= 0.2^4 = \underline{\underline{0.0016}}$

(e) From the above we can see that for a M branch selection diversity receiver, the probability that the

7.7 Cont'd

SNR will be 6dB below the mean SNR threshold is $\underline{\underline{0.2^M}}$.

$\boxed{7.8}$ See the MATLAB program p7_08.m and Fig. p7_08

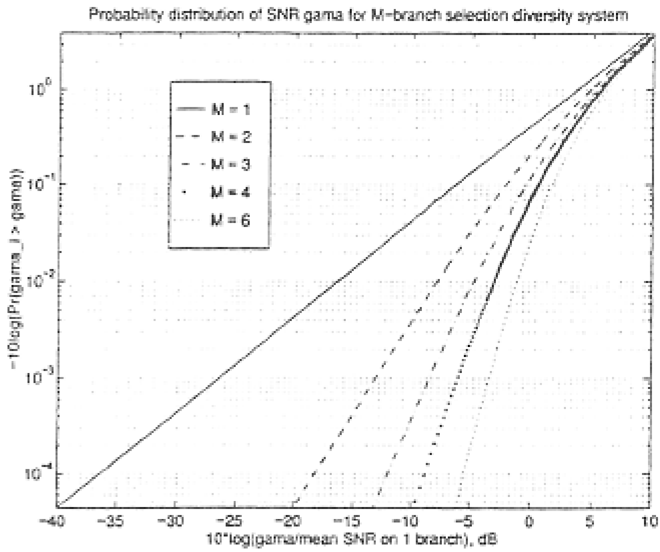
$\boxed{7.9}$ In the maximal ratio combiner, the signals in the M branches are cophased and added with appropriate branch weighting factors a_i. The resulting signal envelope is $r_m = \sum_{i=1}^{M} a_i r_i$, the total noise power is $N_T = N \cdot \sum_{i=1}^{M} a_i^2$, where N is the noise power per branch. Therefore, the resulting SNR is $\gamma_M = \frac{r_m^2}{2N_T} = \frac{\left(\sum_{i=1}^{M} a_i r_i\right)^2}{2 \cdot N \sum_{i=1}^{M} a_i^2}$. The weighting factors a_i, $i=1,2,\cdots M$, are the solutions for the set of equations $\frac{d\gamma_M}{da_i} = 0$, $i=1,2,\cdots M$.

It can be shown that if $a_i = r_i/N$, the γ_M will be maximized and $\gamma_M = \frac{1}{2}\frac{\sum_{i=1}^{M}(r_i^2/N)^2}{\sum_{i=1}^{M}(r_i^2/N^2)} = \frac{1}{2}\sum_{i=1}^{M}\frac{r_i^2}{N} = \sum_{i=1}^{M}\gamma_i$

where $\gamma_i = \frac{1}{2N}\cdot r_i^2$ is the branch SNR and can be represented as $\gamma_i = \frac{1}{2N}(x_i^2 + y_i^2)$, where x_i and y_i are independent Gaussian random variables of equal variance σ^2 and zero mean. Thus γ_M is a chi-square distribution of $2M$ Gaussian random variables with variance $\frac{\sigma^2}{2N} = \frac{1}{2}\Gamma$.

The density function is thus

$$p(\gamma_M) = \frac{\gamma_M^{M-1} e^{-\gamma_M/\Gamma}}{\Gamma^M (M-1)!}, \quad \gamma_M \geq 0$$



Probability distribution of SNR gama for M-branch selection diversity system
M = 1
M = 2
M = 3
M = 4
M = 6
-10log(Pr(gama_i > gama))
10*log(gama/mean SNR on 1 branch), dB
-40
-35
-30
-25
-20
-15
-10
-5
0
5
10

Fig. p7_08

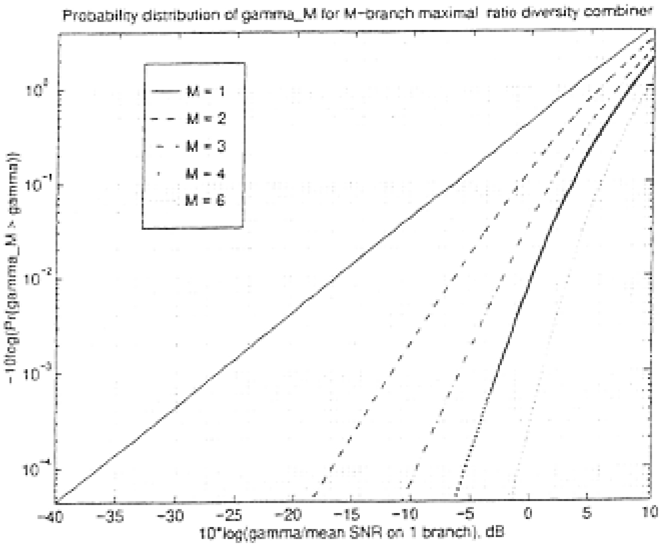


Fig. p7_09

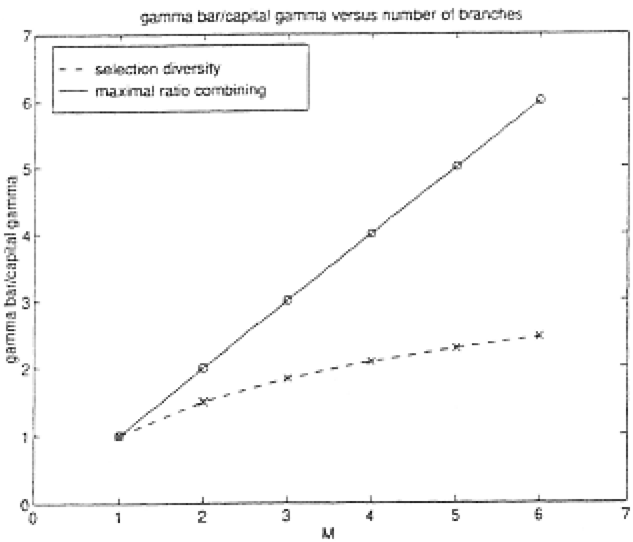


Fig. p7_10

7.9 Cont'd

and hence $P(\gamma_M \le \gamma) = \int_0^{\gamma} p(\gamma_M)\, d\gamma_M = 1 - e^{-\gamma/\Gamma} \sum_{k=1}^{M} \frac{(\gamma/\Gamma)^{k-1}}{(k-1)!}$

and $\overline{\gamma_M} = \sum_{i=1}^{M} \overline{\gamma_i} = \sum_{i=1}^{M} \Gamma = M\Gamma$.

The probability distribution of SNR $= \gamma_M$ as a function of γ/Γ of 1, 2, 3 and 4-branch diversity is shown in Fig. p7.09. (See MATLAB program p7-09.m)

$\boxed{7.10}$ See the MATLAB program p7-10.m and Fig. p7-10.

As shown in problem 7.9, <u>for the maximal ratio combiner</u>, the average SNR γ_M increases linearly with M, i.e.

$\underline{\underline{\frac{\overline{\gamma_M}}{\Gamma} = M}}$. <u>For selection diversity</u>,

$$p_M(\gamma) = \frac{d P_M(\gamma)}{d\gamma} = \frac{d}{d\gamma}\left[(1-e^{-\gamma/\Gamma})^M\right] = \frac{M}{\Gamma}\cdot(1-e^{-\gamma/\Gamma})^{M-1} e^{-\gamma/\Gamma}$$

Thus $\overline{\gamma} = \int_0^{\infty} \gamma \cdot p_M(\gamma)\, d\gamma = \Gamma \sum_{k=1}^{M} \frac{1}{k}$ and hence $\underline{\underline{\frac{\overline{\gamma}}{\Gamma} = \sum_{k=1}^{M} \frac{1}{k}}}$.

From Fig. p7-10, we can see that for the selection diversity, $\frac{\overline{\gamma}}{\Gamma}$ increases more slowly. Therefore, maximal ratio combining requires less branches than the selection diversity to achieve a specific average SNR improvement.

<u>For 6-branch maximal ratio combining</u>, the average SNR improvement is

7.10 Cont'd

$$10\log_{10}(\bar{\gamma}_M/\Gamma) = 10\log_{10}(6) \doteq \underline{\underline{7.78\ dB}}$$

For 6-branch selection diversity, the average SNR improvement is

$$10\log_{10}(\bar{\gamma}_M/\Gamma) = 10\log_{10}\left(\sum_{k=1}^{6}\frac{1}{k}\right) = 10\log_{10}(2.45) \doteq \underline{\underline{3.9\ dB}}$$

If $\frac{\gamma}{\Gamma} = 0.01$, for 6-branch maximal ratio combining,

$$P(\gamma_M \leq \gamma) = 1 - e^{-\gamma/\Gamma}\cdot\sum_{k=1}^{6}\frac{(\gamma/\Gamma)^{k-1}}{(k-1)!} \doteq 1 - e^{-0.01}\cdot\sum_{k=1}^{6}\frac{(0.01)^{k-1}}{(k-1)!}$$

$$\doteq 1 - e^{-0.01}\left[e^{0.01} - \frac{(0.01)^{7-1}}{(7-1)!}\right] \doteq \underline{\underline{1.37\times10^{-15}}}$$

For 6-branch selection diversity.

$$P(\gamma_M \leq \gamma) = (1 - e^{-\gamma/\Gamma})^M = (1 - e^{-0.01})^6 \doteq \underline{\underline{9.7\times10^{-13}}}$$

For a single Rayleigh fading channel.

$$P(\gamma_1 \leq \gamma) = 1 - e^{-\gamma/\Gamma} = 1 - e^{-0.01} \doteq \underline{\underline{9.95\times10^{-3}}}$$

$\boxed{7.11}$ (a) Based on the definition of y. (it should be more suitable to call y the compliment of the system reliability), we have

$$1 - y = \exp[-P^{-1}(x)/\gamma_0] \Longrightarrow \underline{\underline{\gamma_0 = \frac{-P^{-1}(x)}{\ln(1-y)}}}$$

(b) $\underline{\underline{y = \left[1 - e^{-\frac{P^{-1}(x)}{\gamma_0}}\right]^M}}$

7.11 Cont'd

(c) For BPSK, $P_e(\gamma) = Q(\sqrt{2\gamma})$. Given $X = 10^{-3}$, we have

$$\gamma_0 = \frac{-\frac{[Q^{-1}(X)]^2}{2}}{\ln(1-y)} \doteq \frac{-\frac{3.1^2}{2}}{\ln(1-10^{-3})} \doteq 4802.6 \doteq \underline{\underline{36.8\ dB}}$$

(d) In this case, $\gamma_0 = \dfrac{-P^{-1}(X)}{\ln(1-y^{\frac{1}{M}})}$, thus

$$\gamma_0 = \frac{-\frac{3.1^2}{2}}{\ln[1-(10^{-3})^{\frac{1}{4}}]} = 24.54 \doteq \underline{\underline{13.9\ dB}}$$

CHAPTER 8

8.1 For $n = 8$ bit, peak-to-peak amplitude = 2 volts.

we have, the stepsize Δ = peak-to-peak amplitude $\cdot 2^{-n}$

$$= 2 \times 2^{-8} = 2^{-7} \text{ volts}$$

Since the quantization error is uniformly distributed between $-\frac{1}{2}\Delta$ and $\frac{1}{2}\Delta$, the quantization noise power is $\sigma_e^2 = E[e^2] = \frac{1}{12}\cdot\Delta^2 = \frac{1}{12}\cdot(2^{-7})^2 = \frac{1}{12}\cdot 2^{-14}$ W.

For the sinusoid signal that spans the entire range of the quantizer, the signal power $S = \frac{1}{2}\times 1^2 = \frac{1}{2}$ W.

Therefore, the signal-to-quantization noise ratio (SQNR) is

$$SQNR = \frac{S}{\sigma_e^2} = \frac{\frac{1}{2}}{\frac{1}{12}\cdot 2^{-14}} = 12\times 2^{13} \doteq \underline{\underline{50 \text{ dB}}}$$

8.2 For an n bit uniform quantizer that spans the range $(-V, V)$, the step size of the quantizer is $\Delta = \frac{2V}{2^n} = \frac{V}{2^{n-1}}$.

Since the quantization error e is uniformly distributed between $-\frac{1}{2}\Delta$ and $\frac{1}{2}\Delta$, the quantization noise power is $\sigma_e^2 = E[e^2] = \frac{1}{12}\cdot\Delta^2 = \frac{1}{12}\cdot\frac{V^2}{2^{2(n-1)}}$

For an input signal $x(t)$, let $S(t)$ denotes the average signal power. Similar to equation 8.5, the signal-to-quantization noise ratio is

$$SQNR = \frac{S}{\sigma_e^2} = \frac{S}{\frac{1}{12}\cdot\frac{V^2}{2^{2(n-1)}}} = \frac{3\cdot 2^{2n}\cdot S}{V^2} = \underline{\underline{6.02n + 10\log_{10}\left(\frac{3S}{V^2}\right) \text{ d}}}$$

8.2 Cont'd

If $x(t)$ is uniformly distributed between $-V$ and V.

we have $S = \frac{1}{12} \cdot (2V)^2 = \frac{1}{3} V^2$. and

$$SQNR = 6.02n + 10\log_{10}\left(\frac{3 \cdot \frac{1}{3}V^2}{V^2}\right) = \underline{\underline{6.02n\,dB}}.$$

$\boxed{8.3}$ See the MATLAB program p803.m and Fig. p8-03.

For $\mu = 255$. $w(t) = 0.1$, $|v_o(t)| = \frac{\ln(1+255\times 0.1)}{\ln(1+255)} \doteq \underline{\underline{0.587\text{ volts}}}$

For $w(t) = 0.01$. $|v_o(t)| = \frac{\ln(1+255\times 0.01)}{\ln(1+255)} \doteq \underline{\underline{0.227\text{ volts}}}$

$\boxed{8.4}$ See the MATLAB program p804.m and Fig. P8-04.

For $A = 90$, $w(t) = 0.1$, Since $w(t) = 0.1 \geq \frac{1}{90}$, from equation (7.8), we have

$$|v_o(t)| = \frac{1+\ln(A|w(t)|)}{1+\ln A} = \frac{1+\ln(90\times 0.1)}{1+\ln 90} \doteq \underline{\underline{0.5813\text{ volts}}}$$

For $w(t) = 0.01$, Since $w(t) = 0.01 \leq \frac{1}{90}$, we have

$$|v_o(t)| = \frac{A\cdot|w(t)|}{1+\ln A} = \frac{90\times 0.01}{1+\ln 90} \doteq \underline{\underline{0.1636\text{ volts}}}$$

$\boxed{8.5}$ For μ law compander. from equation (7.7), we have $|v_o(t)| = \frac{\ln(1+\mu|w(t)|)}{\ln(1+\mu)}$

$$\Rightarrow |v_o(t)| \cdot \ln(1+\mu) = \ln(1+\mu|w(t)|)$$

$$\Rightarrow |w(t)| = \underline{\underline{\frac{(1+\mu)^{|v_o(t)|} - 1}{\mu}}}$$

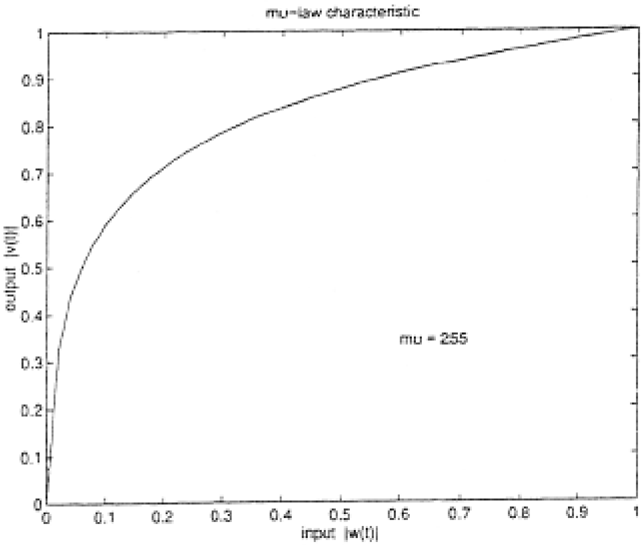


Fig. p8_03

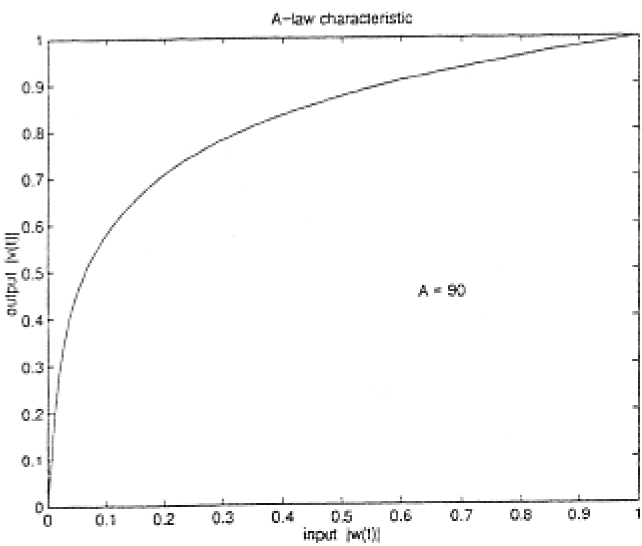


Fig. p8_04

8.5 Cont'd

For A law compander

$$|v_o(t)| = \begin{cases} \dfrac{A|w(t)|}{1+\ln A} & 0 \le |w(t)| \le \dfrac{1}{A} \\ \dfrac{1+\ln(A|w(t)|)}{1+\ln A} & \dfrac{1}{A} \le |w(t)| \le 1 \end{cases}$$

Thus when $|v_o(t)| \le \dfrac{1}{1+\ln A}$, $|v_o(t)| = \dfrac{A|w(t)|}{1+\ln A}$

$$\Rightarrow \quad |w(t)| = \frac{(1+\ln A)\cdot|v_o(t)|}{A}$$

when $\dfrac{1}{1+\ln A} \le |v_o(t)| \le 1$, $|v_o(t)| = \dfrac{1+\ln(A|w(t)|)}{1+\ln A}$

$$\Rightarrow \quad |v_o(t)|\cdot(1+\ln A) = 1+\ln(A|w(t)|)$$

$$\Rightarrow \quad |w(t)| = \frac{1}{A}\cdot\exp[\,|v_o(t)|(1+\ln A)-1]$$

Therefore $$|w(t)| = \begin{cases} \dfrac{(1+\ln A)\cdot|v_o(t)|}{A} & 0 \le |v_o(t)| \le \dfrac{1}{1+\ln A} \\ \dfrac{1}{A}\cdot\exp[\,|v_o(t)|(1+\ln A)-1] & \dfrac{1}{1+\ln A} \le |v_o(t)| \le 1 \end{cases}$$

$\boxed{8.6}$ For $n=4$, $\Delta = 0.25$ volts. the possible quantization levels are $(2i+1)\cdot\dfrac{\Delta}{2}$, $i=-8,-7,\cdots,0,1,\cdots 7$.

The relationship between the input x and the output $f_Q(x)$ of the quantizer can be expressed as

$$f_Q(x) = \begin{cases} -\dfrac{15}{2}\Delta\,, & x \le -7\Delta \\ (2i+1)\cdot\dfrac{\Delta}{2}\,, & i\Delta \le x \le (i+1)\Delta\,,\ i=-7,-6,\cdots,6 \\ \dfrac{15}{2}\Delta\,, & x \ge 7\Delta \end{cases}$$

The input-output characteristics of the uniform quantizer is shown in Fig.p8-6(a).

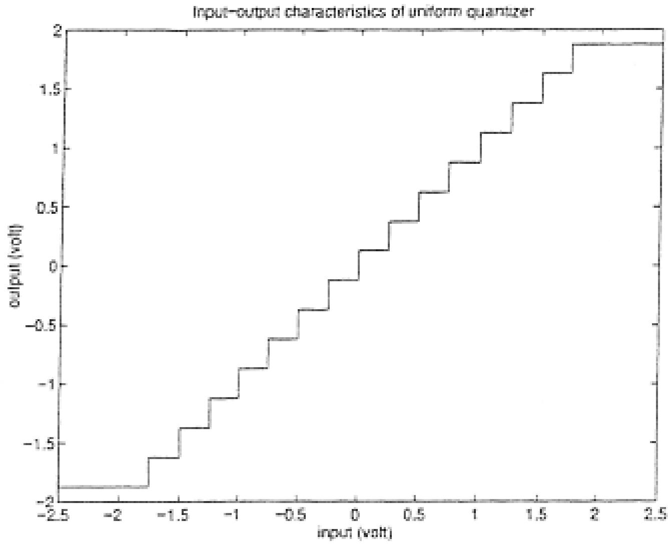


Fig. p8_06 (a)

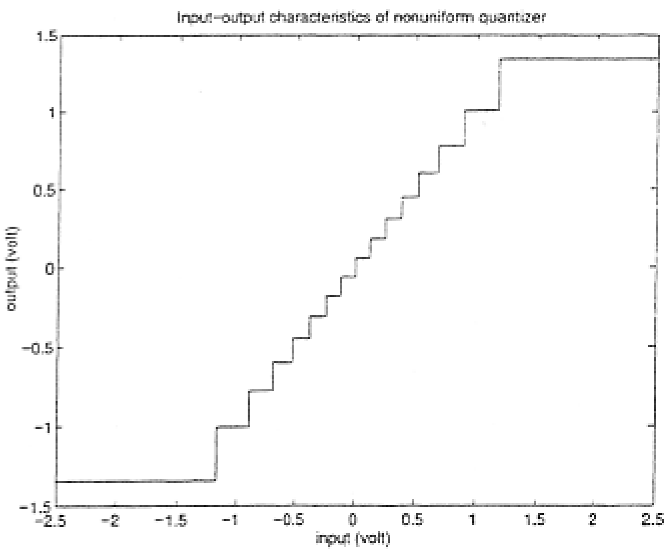


Fig. p8_06 (b)

8.6 Cont'd

From the problem statement, we have the pdf of x.

$$P_X(x) = \frac{1}{\sqrt{2\pi}\cdot 0.5} \exp\left(-\frac{x^2}{2\times 0.5^2}\right)$$

Therefore, the mean square error distortion

$$D = \int_{-\infty}^{\infty} (x - f_Q(x))^2 \cdot P_X(x)\,dx$$

$$= \int_{-\infty}^{-7\Delta} \left(x + \frac{15}{2}\Delta\right)^2 \cdot P_X(x)\,dx + \sum_{i=-7}^{7} \int_{i\Delta}^{(i+1)\Delta} \left[x - (2i+1)\frac{\Delta}{2}\right]^2 P_X(x)\,dx$$

$$+ \int_{7\Delta}^{\infty} \left(x - \frac{15}{2}\Delta\right)^2 \cdot P_X(x)\,dx.$$

Using the MATLAB program p8-06.m, we obtain

$$D \doteq \underline{\underline{0.0052}}$$

To minimize the distortion, we need to concentrate the quantization levels in regions of higher probability. Since the input signal has a greater probability around the zero amplitude, we need to place the quantization levels closer at amplitudes close to 0 and farther at amplitude close to $4 \times 0.5 = 2$. Using the Lloyd algorithm (see problem 7.10 for details), we can design a nonuniform quantizer that would minimize the mean square error distortion. From the MATLAB program P8-06.m, we obtain the quantization level and corresponding

8.6. Cont'd

boundaries as follows: (Since the quantization levels and corresponding boundaries are symmetrical about 0, we just list the positive part).

Quantization level

= [0.0603 0.1826 0.3103 0.4479 0.6013 0.7807 1.0069 1.3419]

Boundaries

= [0 0.1215 0.2465 0.3791 0.5246 0.6910 0.8938 1.1744]

The input-output characteristics of the nonuniform quantizer is shown in Fig. P8_06(b).

The mean squared error distortion of this quantize is calculated by MATLAB program P8_06.m and is equal to 0.003

$\boxed{9.7}$ For perfect reconstruction of the bandpass signals, they need to be sampled at a Nyquist rate equal to twice the bandwidth of the signal. Therefore, the different sub-bands need to be sampled at the

8.7 Cont'd

following rates:

sub-band 1 $= 2\times(500-225) = 550$ samples/s

sub-band 2 $= 2\times(1200-500) = 1400$ samples/s

sub-band 3 $= 2\times(3000-1300) = 3400$ samples/s

$\Rightarrow$ data rate out of the sub-band coder

$= 550\times5 + 1400\times3 + 3400\times2 = 13750$ bits/s

$\Rightarrow$ data rate out of the channel coder

$= \frac{4}{3}\times 13750 \doteq 18333$ bits/s $= \underline{\underline{18.333 \text{ kbps}}}$

$\boxed{8.8}$ The significant factors which influence the choice of speech coders in mobile communication systems include:

1. The speech codec be robust to transmission errors
2. Implementation issues – how cost effective it is to implement
3. Power requirements – how much battery drain
4. Type of multiple access technique used
5. Type of modulation employed

See section 8.8 for details.

8.10 See the MATLAB program p8_10.m and Fig. p8_10.

Using training sequence, the quantization levels and corresponding boundaries are:

quantization levels:

-2.1653 -1.3728 -0.8564 -0.2539 0.2550 0.8393 1.3876 2.2484

boundaries:

-1.7691 -1.1146 -0.5552 0.0005 0.5472 1.1135 1.8180

Mean squared error distortion calculated from the test sequence is 0.0352.

The theoretical quantization levels and corresponding boundaries are:

quantization levels:

-2.1520 -1.3439 -0.7560 -0.2451 0.2451 0.7560 1.3439 2.1520

boundaries:

-1.7480 -1.0500 -0.5006 0 0.5006 1.0500 1.7480

Therotical lower bound on mean-squared error distortion is 0.0345

Mean squared error distortion for the therotical quantizer is (using the same test sequence) 0.0338.

The theoretical quantization levels and corresponding boundaries are calculated by using the Lloyd algorithm. Let q_i, $i=1,2,\cdots 8$ denote the quantization levels. X_i, $i=1,2,\cdots,7$ denote the corresponding boundaries. X and $Q(x)$ denote the input and output of the quantizer, respectively. the input and output characteristics of the quantizer can be represented as

$$Q(x)=\begin{cases} q_1, & -\infty < x \le x_1 \\ q_i, & x_{i-1} \le x \le x_i, \quad i=2,3,\cdots,7 \\ q_8, & x_7 < x < \infty \end{cases}$$

The Lloyd algorithm is as follows.

Step 1 Initialization: Set $m=0$, choose a set of initial boundary $X_i(0)$, $i=1,2,\cdots 7$.

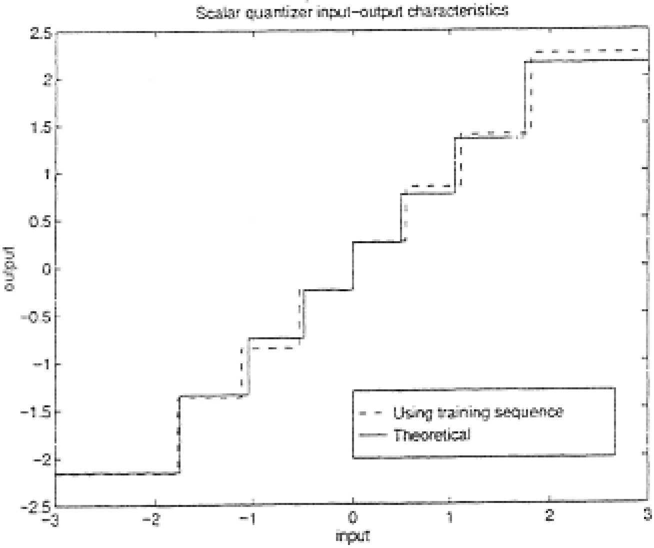


Fig. p8_10

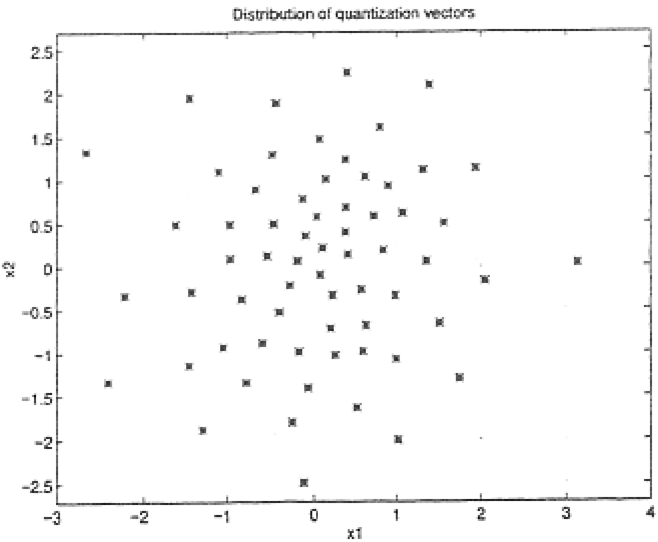


Fig. p8_11

8.10 Cont'd

Step 2: Quantization levels calculation:

$$q_i = \frac{\int_{Q_i} x f(x)\, dx}{\int_{Q_i} f(x)\, dx}, \quad i = 1, 2, \cdots, 8.$$

where $Q_1 = \{x: -\infty < x \le x_1(m)\}$, $Q_i = \{x: x_{i-1}(m) < x \le x_i(m),$
$i = 2, \cdots, 7\}$, $Q_8 = \{x: x_7(m) < x < \infty\}$.

Step 3: MSE distortion calculation: $D(m) = S - \sum_{i=1}^{8} q_i^2 \int_{Q_i} f(x)\,dx$

where $S = E\{s^2(t)\} = \int_{-\infty}^{\infty} x^2 f(x)\,dx$.

Step 4: Termination test: For $m \geq 2$, if the decrease in $D(m)$ at iteration m relative to $D(m-1)$ is below a certain threshold, stop; otherwise go to step 5

Step 5: Boundary updating: $m \leftarrow m+1$, $x_i(m) = \frac{1}{2}[q_i(m-1) + q_{i+1}(m-1)]$.
go to step 2.

From the above results we can see that due to the non-ideal random number generator, the data is not exactly normal distributed, therefore the MSE distortion calculated by using the test sequence for the ideal quantizer is less than the theoretical lower bound. For the ideal quantizer, the MSE distortion is always less than that for the quantizer derived by using the training sequence.

8.12 | See the MATLAB program p8_12.m and Fig. p8.12.

The 64 quantization vectors are (each column contains one vector)
Columns 1 through 7

0.0577	-1.8433	-0.6225	-1.1155	-2.0601	-0.1380	-0.0921
0.3076	-1.4062	-0.3863	-0.7736	-0.7074	-1:5222	-0.5689

Columns 8 through 14

0.2003	-1.5632	-0.2561	-1.9816	-0.9409	-0.8360	-0.9894
-0.2440	-1.9189	0.2848	-2.6935	-0.5288	-2.2484	-1.1809

Columns 15 through 21

1.2101	1.1565	2.0051	0.3935	0.4235	-0.0326	-0.6749
0.4419	1.8960	2.1411	-0.1250	1.7790	-0.1932	0.3061

Columns 22 through 28

-0.3221	0.1544	0.8663	-1.2890	0.4229	0.4424	2.4456
-0.3228	0.5541	-0.3847	-1.4615	1.3479	0.8415	1.3688

Columns 29 through 35

-0.2333	1.2679	-0.7651	0.1331	0.8701	0.3033	-0.7017
1.3738	-0.1312	0.8026	0.9053	0.1702	0.3488	-0.7243

Columns 36 through 42

0.4031	1.1372	0.0144	0.5464	-1.4392	-0.7975	-0.1514
-0.5522	0.8577	-0.9260	0.3412	-1.0051	-0.2046	0.0083

Columns 43 through 49

1.5928	1.4233	-0.4340	-0.1647	-2.5761	-0.7477	1.9103
1.4323	0.8812	-0.9211	0.7469	-1.8792	-1.0637	0.5321

Columns 50 through 56

0.5527	0.5871	-0.4250	-1.2357	0.7594	-0.4568	-0.4889
0.0519	-1.0419	-1.2822	-0.3979	2.4654	-0.5975	0.0831

Columns 57 through 63

-1.4924	-0.7798	0.7855	0.7272	-0.9640	-0.3679	0.1247
0.1370	-1.6209	0.6134	1.0414	0.1578	0.5105	0.0767

Column 64

1.0224
1.3258

The mean squared error distortion computed from the test sequence is 0.0249, which is less than that for the uncorrelation case in problem 7.11.

Comparing Fig. p8.12 and Fig. p8.11, we can see that the quantization vectors in this problem become more condense due to the correlation between the samples.

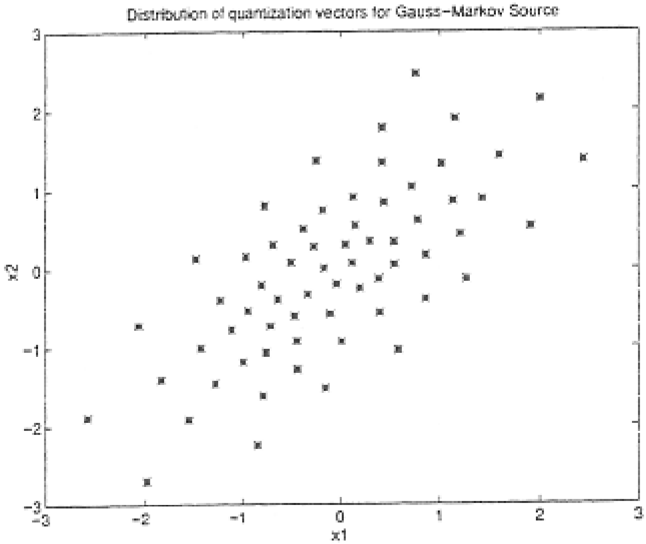

Distribution of quantization vectors for Gauss-Markov Source
x1
x2
-3
-2
-1
0
1
2
3

Fig. p8_12

8.12 Cont'd

For the length of the training sequence, the relative correlation, and the dimensions of the vector quantizer, increase one of the three factors and fix the other two, the MSE distortion will decrease.

CHAPTER 9

$\boxed{9.1}$ (a) raw data rate provided for each user

$$= \frac{270.833}{8} \doteq \underline{\underline{33.85 \text{ kbps}}}$$

(b) traffic efficiency for each user

$$= \left(1 - \frac{10.1}{33.85}\right) \times 100\% \doteq \underline{\underline{70\%}}$$

$\boxed{9.2}$ raw data rate provided for each user $= \frac{48.6}{3} = \underline{\underline{16.2 \text{ Kbps}}}$

$\boxed{9.3}$ (a) A frame has $6 \times 324 = 1944$ bits.

The number of overhead bits per frame is given by

$$b_{OH} = 6\times6 + 6\times6 + 6\times28 + 6\times12 + 6\times12 = 384 \text{ bits}$$

$$\Rightarrow \text{frame efficiency } \eta_f = \left[1 - \frac{384}{1944}\right] \times 100\% \doteq \underline{\underline{80.2\%}}$$

(b) For half rate speech coding.

raw data rate provided for each user $= \frac{48.6}{6} = \underline{\underline{8.1 \text{ Kbps}}}$

As shown in (a), frame efficiency $\eta_f = \underline{\underline{80.2\%}}$

$\boxed{9.4}$ (a) raw data rate provided for each user

$$= \frac{42.0}{3} = \underline{\underline{14 \text{ kbps}}}$$

(b) number of bit per frame

$= \text{frame duration} \times \text{data rate} = 6.667 \times 10^{-3} \times 42.0 \times 10^{3}$

$= 280$ bits/frame

9.4 Cont'd

$\Rightarrow$ number of information bit sent to each user per frame, $B_I = \dfrac{\text{number of bit/frame} \times \text{frame efficiency}}{\text{number of users per frame}}$

$$= \frac{280 \times 0.8}{3} \doteq \underline{\underline{74.67 \text{ bits}}}$$

(c) Similarly, $B_I = \dfrac{280 \times 0.8}{6} \doteq \underline{\underline{37.33 \text{ bits}}}$

(d) For half rate PDC, information data rate per user

$$= \frac{B_I}{\text{frame duration}} = \frac{37.33}{6.667 \times 10^{-3}} \doteq \underline{\underline{5.6 \text{ Kbps}}}$$

$\boxed{9.5}$ Intermodulation distortion products occur at frequencies $mf_1 + nf_2$ for all integer values of m and n. In this case, $f_1 = 0.03 \times 352 + 870$ MHz, $f_2 = 0.03 \times 360 + 870$ MHz. Therefore, the possible intermodulation frequencies are

$$mf_1 + nf_2 = m(0.03 \times 352 + 870) + n(0.03 \times 360 + 870)$$
$$= (m+n) \cdot 870 + 0.03 \times (352m + 360n)$$

Since the forward channels of U.S. AMPS occupy the frequency band from 869 MHz to 894 MHz, for those intermodulation frequencies that fall in the forward channel, m and n should satisfy

9.5 Cont'd

the condition such that $m+n=1$, otherwise, $mf_1+nf_2 > 894$ MHz or $mf_1+nf_2 < 869$ MHz. hence, we have

$$mf_1+nf_2 = 870 + 0.03 \times [352(m+n) - 352n + 360n]$$

$$= 870 + 0.03 \times [352+8n] \quad \text{or}$$

$$870 + 0.03 \times [(352+8n+1023) - 1023]$$

Thus the channels on the forward link that might carry interference due to intermodulation are

channel $352+8n$ for $-43 \le n \le 55$

channel $352+8n+1023$ for $-48 \le n \le -44$

$\boxed{9.6}$ In this case $f_1 = 0.03 \times 318 + 870 = 879.54$ MHz, $f_2 = 931.9375$ MHz. The possible intermodulation frequencies are

$$mf_1+nf_2 = m \cdot 879.54 + n \cdot 931.9375$$

$$= (m+n) \cdot 879.54 + n \cdot (931.9375 - 879.54)$$

$$= (m+n) \cdot 879.54 + 52.3975n$$

For $m+n=1$, $n=-1$, we have

$$mf_1+nf_2 = 879.54 - 52.3975 = \underline{\underline{827.1425 \text{ MHz}}}$$

This intermodulation frequency falls into the reverse channel band of U.S. AMPS.

9.7 (a) packet duration, $\tau = \frac{1000}{10\times10^6} = 10^{-4}$ S

$\Rightarrow$ traffic occupancy, $R = \lambda \cdot \tau = 10^3 \times 10^{-4} = 0.1$ Erlangs.

$\Rightarrow$ The normalized throughput of the system,

$$T = R \cdot e^{-2R} = 0.1 \times e^{-2\times0.1} \doteq \underline{\underline{0.082}}$$

(b) For unslotted ALOHA, when $R = \frac{1}{2}$, the throughput will be maximized, therefore,

$$\tau = \frac{R}{\lambda} = \frac{\frac{1}{2}}{10^3} = 5\times10^{-4} \text{ S}$$

$\Rightarrow$ number of bits per packet $= \tau \cdot$ bit rate

$= 5\times10^{-4} \times 10\times10^6 = \underline{\underline{5\times10^3 \text{ bits}}}$

9.8 (a) For Slotted ALOHA,

$$T = R \cdot e^{-R} = 0.1 \times e^{-0.1} \doteq \underline{\underline{0.09}}$$

(b) For slotted ALOHA, when $R=1$, the throughput will be maximized. Therefore, $\tau = \frac{R}{\lambda} = \frac{1}{10^3} = 10^{-3}$ S.

$\Rightarrow$ number of bits per packet $= \tau \cdot$ bit rate

$= 10^{-3} \times 10 \times 10^6 = \underline{\underline{10^4 \text{ bits}}}$

9.9 Propagation time $t_p = \frac{d}{c} = \frac{10\times10^3}{3\times10^8} \doteq 3.33\times10^{-5}$ S.

$\Rightarrow$ propagation delay $t_d = \frac{t_p \cdot R_b}{m} = \frac{3.33\times10^{-5}\times19.2\times10^3}{256}$

$= \underline{\underline{0.0025 \text{ packet transmission units}}}$

9.9 Cont'd

For slotted ALOHA, when $R=1$, the throughput will be maximized.

Therefore, $\tau_{optimum}$ should satisfy the condition $\lambda \cdot \tau_{optimum} = 1$

$\Rightarrow \tau_{optimum} = \frac{1}{\lambda}$. In this case the data rate $= 19.2$ Kbps,

$\Rightarrow$ bit period $= \frac{1}{19.2\,\text{Kbps}} \doteq 52.08\,\mu s$

$\Rightarrow \tau = 256$ bits/packet $\times$ $52.08\,\mu s$/bit $= \underline{\underline{13.33\,ms}}$

$\Rightarrow \lambda = \frac{R}{\tau} = \underline{\underline{75 \text{ packets/second}}}$.

$\boxed{9.10}$ For $n=3$, $\left(\frac{C}{I}\right)_{min} = 14\,dB \doteq 25.12$, we have the co-channel reuse factor, $Q \geq \left(6\left(\frac{C}{I}\right)_{min}\right)^{\frac{1}{n}}$

$\Rightarrow \sqrt{3N} \geq (6 \times 25.12)^{\frac{1}{3}} \doteq 5.32 \quad \Rightarrow N \geq 9.43$

$\Rightarrow N = 12$

$\Rightarrow$ number of analog channels per cell,

$$m = \frac{B_t}{B_c \cdot N} = \frac{20 \times 10^6}{30 \times 10^3 \times 12} \doteq \underline{\underline{55 \text{ channels/cell}}}$$

$\boxed{9.11}$ $\underline{\text{For } n=2}$, we need $Q \geq \left(6\left(\frac{C}{I}\right)_{min}\right)^{\frac{1}{2}} = (6 \times 25.12)^{\frac{1}{2}}$
$\doteq 12.28$

$\Rightarrow N \geq \frac{Q^2}{3} = \frac{12.28^2}{3} = 50.24 \Rightarrow N = 6^2 + 2^2 + 6 \times 2 = 52$

$$\Rightarrow m = \frac{B_T}{B_c \cdot N} = \frac{20 \times 10^6}{30 \times 10^3 \times 52} \doteq \underline{\underline{13 \text{ channels/cell}}}$$

9.11 Cont'd

For $n=4$, we need $Q \geq (6 \times 25.12)^{\frac{1}{4}} \doteq 3.5$

$\Rightarrow N \geq \frac{Q^2}{3} \doteq 4.09 \Rightarrow N = 7$

$\Rightarrow m = \frac{B_T}{B_C \cdot N} = \frac{20 \times 10^6}{30 \times 10^3 \times 7} \doteq \underline{\underline{95 \text{ channels/cell}}}$

$\boxed{9.12}$ $\frac{E_b}{N_0} = \frac{W/R}{N-1}$

$\Rightarrow W = \frac{E_b}{N_0} \cdot (N-1) \cdot R = 100 \times (100-1) \times 13 \times 10^3$

$= 1.287 \times 10^8$ chips/sec $= \underline{\underline{128.7 \text{ M chips/sec}}}$

$\boxed{9.13}$ For $\alpha = 0.4$, $\frac{E_b}{N_0} = \frac{W/R}{(N-1)\alpha}$

$\Rightarrow W = \frac{E_b}{N_0} (N-1) \cdot \alpha \cdot R = 1.287 \times 10^8 \times 0.4$

$\doteq \underline{\underline{51.46 \text{ M chips/sec}}}$

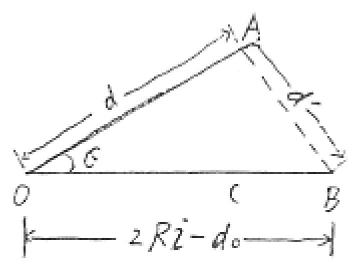
$\boxed{9.14}$ For tri-sectored CDMA system with voice activity $\alpha = 0.4$, $\frac{E_b}{N_0} = \frac{W/R}{(\frac{N}{3}-1) \cdot \alpha}$

$\Rightarrow W = \frac{E_b}{N_0} \cdot (\frac{N}{3}-1) \cdot \alpha \cdot R = 100 \times (\frac{100}{3}-1) \times 0.4 \times 13 \times 10^3$

$\doteq \underline{\underline{16.8 \text{ M chips/sec}}}$

$\boxed{9.15}$ For $(2i-1) \cdot R \leq d \leq (2i)R - d_0$,

from the figure left, we have



$|BC| = |OB| - |OC|$

$= 2Ri - d_0 - d\cos\theta$

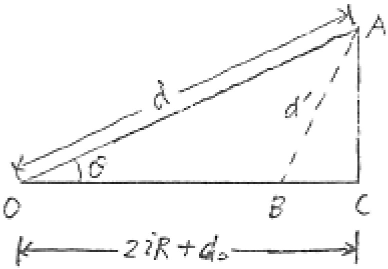
9.15 Cont'd

$$|AC| = d\cdot\sin\theta$$

$$\Rightarrow d' = \sqrt{|AC|^2 + |BC|^2}$$

$$= \sqrt{d^2\sin^2\theta + (2Ri - d_0 - d\cdot\cos\theta)^2}\,,\quad \text{for } (2i-1)R \le d \le (2i)R - d_0$$

For $(2i)R + d_0 \le d \le (2i+1)R$,



$$|BC| = |OC| - |OB|$$

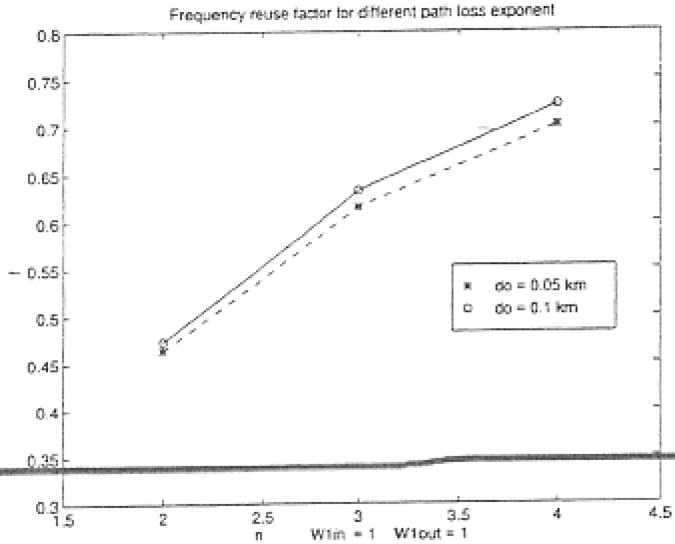
$$= d\cdot\cos\theta - (2iR + d_0)$$

$$|AC| = d\cdot\sin\theta$$

$$\Rightarrow d' = \sqrt{|AC|^2 + |BC|^2}$$

$$= \sqrt{d^2\sin^2\theta + (d\cos\theta - 2Ri - d_0)^2}\,,\quad \text{for } (2i)R + d_0 \le d \le (2i+1)R$$

$\boxed{9.16}$ See the MATLAB program p9-16.m and Fig. p9-16



Frequency reuse factor for different path loss exponent
0.8
0.75
0.7
0.65
0.6
0.55
0.5
0.45
0.4
0.35
0.3
1.5
2
2.5
3
3.5
4
4.5
n
W1in = 1
W1out = 1
do = 0.05 km
do = 0.1 km

Fig. p9_16

$\boxed{9.17}$ For a single cell CDMA system, $P_b = Q\left(\sqrt{\frac{3DN}{K-1}}\right)$.

$\Rightarrow \sqrt{\frac{3DN}{K-1}} = Q^{-1}(P_b)$. where $Q^{-1}(\cdot)$ is the inverse function of Q function

$$\Rightarrow K = \frac{3DN}{[Q^{-1}(P_b)]^2} + 1$$

Given $D = 10\,dB = 10$, $P_b = 10^{-3}$, $N = 511$, we have

$$K = \frac{3\times10\times511}{[Q^{-1}(10^{-3})]^2} + 1 \doteq \frac{3\times10\times511}{(3.1)^2} + 1 \doteq \underline{\underline{1596 \text{ users}}}$$

In actuality, only 511 users would likely be used in a single cell to ensure low cross-correlation between users.

$\boxed{9.18}$ In this case $P_b = Q\left(\sqrt{\frac{3DN}{(K-1)\cdot\alpha}}\right)$, where $\alpha = 0.4$ is the voice activity factor. Similarly, we have

$$K = \frac{3DN}{\alpha[Q^{-1}(P_b)]^2} + 1 = \frac{3\times10\times511}{0.4\times(3.1)^2} + 1 \doteq \underline{\underline{3989 \text{ users}}}$$

$\boxed{9.19}$ The frequency reuse factor f for reverse channel of CDMA cellular system, as a function of n, can be found from Table 9.4.

Assume the users are uniformly distributed, <u>for $n=2$</u>, from table 9.4, we get $f \doteq 0.46$. Given $D = 6\,dB = 4$, $N = 511$, we have

$$K = \frac{3fDN}{[Q^{-1}(P_b)]^2} + 1 = \frac{3\times0.46\times4\times511}{[Q^{-1}(0.01)]^2} + 1 \doteq \underline{\underline{520 \text{ users}}}$$

9.19 Cont'd

For $n=3$, $f \doteq 0.6$, $\Rightarrow K = \frac{3\times 0.6\times 4\times 511}{2\cdot 33^2} + 1 \doteq \underline{\underline{678 \text{ users}}}$

For $n=4$, $f \doteq 0.7$, $\Rightarrow K = \frac{3\times 0.7\times 4\times 511}{2\cdot 33^2} + 1 \doteq \underline{\underline{791 \text{ users}}}$

$\boxed{9.20}$ Using the concentric cellular geometry, for the ith surrounding layer, the inner and outer sectors of each cell have areas given by

$$A_{i\,in}/M_i = \left\{ \pi(2iR)^2 - \pi[(2i-1)R]^2 \right\}/8i$$
$$= \frac{4i-1}{8i}\cdot \pi R^2 = \frac{4i-1}{8i}\cdot A$$
$$A_{i\,out}/M_i = \left\{ \pi[(2i+1)\cdot R]^2 - \pi\cdot(2iR)^2 \right\}/8i$$
$$= \frac{4i+1}{8i}\cdot \pi R^2 = \frac{4i+1}{8i}\cdot A$$

Applying the weighting factors for the user density within the inner ($W_{i\,in}$) and outer ($W_{i\,out}$) sectors in the ith surrounding layer, we have,

$$U = KA = K\cdot\left[\frac{4i-1}{8i}\cdot W_{i\,in}\cdot A + \frac{4i+1}{8i} W_{i\,out}\cdot A\right].$$

For the equivalent hexagonal geometry, we have

$$\begin{cases} \frac{4i-1}{8i}\cdot W_{i\,in} = \frac{1}{2} \\ \frac{4i+1}{8i}\cdot W_{i\,out} = \frac{1}{2} \end{cases} \Longrightarrow \begin{cases} W_{i\,in} = \frac{4i}{4i-1} \\ W_{i\,out} = \frac{4i}{4i+1} \end{cases} \quad i=1,2,\cdots$$

For $i=2$, $W_{i\,in} = \underline{\underline{\frac{8}{7}}}$. $W_{i\,out} = \underline{\underline{\frac{8}{9}}}$

$\boxed{9.21}$ See the MATLAB program p9-21.m and Fig. p9-21.

(a) Assume the received power of each of the in-cell subscribers at the base receiver is $P_0 = 1W$. the received in-cell interference power

$$P_{I\text{-in-cell}} = (U-1)\cdot P_0 = 30-1 = \underline{\underline{29\,W}}$$

(b) Using MATLAB program p8-21.m, we have

$$P_{I\text{-out-of-cell}} \doteq \underline{\underline{11.27\,W}}$$

(c) Using MATLAB program p9-21.m, we have $f \doteq \underline{\underline{0.7209}}$

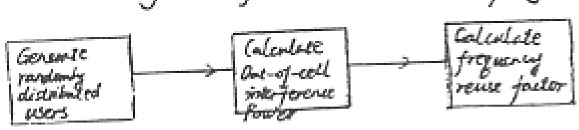
$\boxed{9.22}$ See the MATLAB program p9-21.m and Fig. p9-21

(a) $P_{I\text{-in-cell}} = \underline{\underline{29\,W}}$

(b) $P_{I\text{-out-of-cell}} \doteq \underline{\underline{17.51\,W}}$

(c) $f \doteq \underline{\underline{0.6244}}$.

The block diagram of the MATLAB program p9-21.m is shown below



To generate randomly distributed users in adjacent cells, we first generate randomly distributed users in the center cell, users in the adjacent cell can be generated by axis transforming. In the center cell, we first generate 30 users in one 60 degree sector, then rotate each 5 users of the 25 users to other sectors.

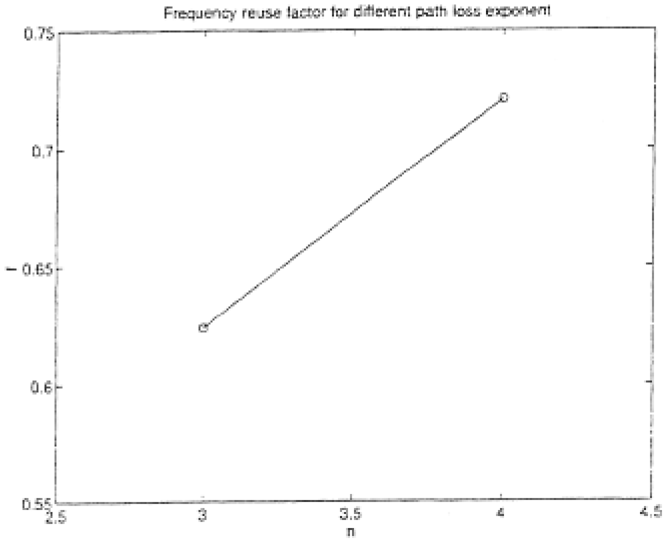

Frequency reuse factor for different path loss exponent
0.75
0.7
0.65
0.6
0.55
2.5
3
3.5
4
4.5
n

Fig. 9_21

CHAPTER 11

11.1

<u>Digital</u>
GSM, DECT, IS-136, IS-95

<u>Analog</u>
AMPS, ETACS

<u>TDMA</u>
GSM, DECT, IS-136

<u>CDMA</u>
IS-95

11.2 $\underline{\underline{B, C}}$

11.3 <u>AMPS, ETACS, N-AMPS</u>

11.4 Since the modulation scheme used by GSM is 0.3GMSK, the cut-off frequency of the baseband Gaussian pulse shaping filter, $B = R \cdot 0.3 = 270.83 \times 0.3$

$\doteq \underline{\underline{81.25 \text{ KHz}}}$

11.5 Let N denote the number of additional channels that can be accommodated in the 10MHz bandwidth.

For AMPS, $B = 30\text{KHz}$, $N = \dfrac{10 \times 10^6}{2B} = \underline{\underline{166 \text{ channels}}}$

For TACS, $B = 25\text{KHz}$, $N = \underline{\underline{200 \text{ channels}}}$

For ETACS, $B = 25\text{KHz}$, $N = \underline{\underline{200 \text{ channels}}}$

For NMT450, $B = 25\text{KHz}$, $N = \underline{\underline{200 \text{ channels}}}$

For NMT900, $B = 12.5\text{KHz}$, $N = \underline{\underline{400 \text{ channels}}}$

For C-450, $B = 10\text{KHz}$, $N = \underline{\underline{500 \text{ channels}}}$

For RTMS, $B = 25 \text{ KHz}$, $N = \underline{\underline{200 \text{ channels}}}$

For Radicom 2000, $B = 12.5 \text{ KHz}$, $N = \underline{\underline{400 \text{ channels}}}$

For NTT, $B = 6.25 \text{ KHz}$, $N = \underline{\underline{800 \text{ channels}}}$

or $B = 25 \text{ KHz}$, $N = \underline{\underline{200 \text{ channels}}}$

11.5 Cont'd

For JTAC/NTACS, B = 25 KHz, N = 200 channels

or B = 12.5 KHz, N = 400 channels

For IS-54, B = 30 KHz, N = 166 channels

For GSM, B = 200 KHz, N = 25 channels

For PDC, B = 25 KHz, N = 200 channels

For IS-95, B = 1.2288 MHz, N = 4 channels

11.6 The reasons are:

1. Reverse channel transmitted power is much less than the forward channel transmitted power
2. The mobile users are asynchronous with one another, which causes greater multiple access interference than on the synchronous link. Thus more coding is needed.

11.7 C

11.8 Different channels within the forward link in an IS-95 system are identified through their unique Walsh Code.

11.9 1.35 bps/Hz

11.10 B

$\boxed{11.11}$ IS-95

$\boxed{11.12}$ IS-54, PDC

$\boxed{11.13}$ See section 6.8.6

$\boxed{11.14}$ For a cluster size $N = 4$, the number of the full rate physical channels per cell that a GSM system can accommodate is

$$M = \frac{\text{Total Bandwidth allocated to the system}}{2 \times \text{one way channel bandwidth}} \times \begin{array}{l}\text{number of time slot}\\ \text{per frame}\end{array}$$

$$= \frac{50 \times 10^6}{2 \times 200 \times 10^3} \times 8 = \underline{\underline{1000 \text{ physical channels}}}$$

$\boxed{11.15}$ B

$\boxed{11.16}$ For AMPS system. $\beta = \frac{\Delta f}{W} = \frac{12 \times 10^3}{4 \times 10^3} = \underline{\underline{3}}$

$\boxed{11.17}$ C, D

$\boxed{11.18}$ B

$\boxed{11.19}$ 96 Kbps

$\boxed{11.20}$ 3

$\boxed{11.21}$ From section 11.4.2.5, we can see that the

11.21 Cont'd

transmitted power can be increased or decreased 1 dB every 1.25 ms, thus the maximum fade slope which can be compensated for by the reverse power control subchannel in the IS-95 CDMA system is

$$\frac{1\,dB}{1.25\,ms} = \underline{\underline{800\,dB/s}}$$

$\boxed{11.22}$ In GSM, in order to minimize the effect of sudden fades on the received data, the total of 456 encoded bits within each 20ms speech frame or control message frame are broken into eight 57 bit sub-blocks. These eight sub-blocks which make up a single speech frame are spread over 8 consecutive TCH time slots. (i.e., eight consecutive frames for a specific TS). Each TCH time slot carries two 57 bit blocks of data from two different 20 ms (456 bit) speech (or control) segments. Since the later 57 bits in one time slot are corresponding to the former speech frame, it is reasonable to place the 26 equalizer training bits before the block of the former speech frame such that the effect of

11.22 Cont'd

the time varying channel is minimized when the speech is reconstructed at the receiver.

The 8.25 bit guard period after the burst are utilized to prevent overlap of signal bursts arriving from different terminals at varying distances (i.e. different propagation times).

$\boxed{11.23}$ Assume the reference distance $d_0 = 1km$, the transmitted power $P_t = 1W$. $G_t = G_r = 1$, for GSM at 800 MHz. the received power at the reference distance

$$P_r(d_0)_{GSM} = \frac{P_t \cdot G_t \cdot G_r \cdot \lambda^2}{(4\pi)^2 \cdot d_0^2} = \frac{1 \times 1 \times 1 \times (\frac{300}{800})^2}{(4 \times 3.14)^2 \times 1000^2}$$

$$\doteq 8.9 \times 10^{-10} \, W \doteq -60.5 \, dBm$$

For DCS-1900 at 1900 MHz,

$$P_r(d_0)_{DCS\text{-}1900} = \frac{1 \times 1 \times 1 \times (\frac{300}{1900})^2}{(4 \times 3.14)^2 \times 1000^2} \doteq 1.58 \times 10^{-10} \, W \doteq -68 \, dBm$$

Assume the path loss exponent $n = 4$, given the sensitivity of the receiver $= -104$ dBm. we have

$$P_r(R) = P_r(d_0) + 10 \cdot n \log_{10}\left(\frac{d_0}{d}\right)$$

. where R is the radius of one cell.

11.23 Cont'd

$\Rightarrow$ For GSM at 800 MHz,

$$10 \times 4 \times \log_{10}\left(\frac{1000}{R_{GSM}}\right) = -104 - (-60.5)$$

$\Rightarrow$ $R_{GSM} \doteq 12.23$ Km

For DCS-1900 at 1900 MHz,

$$10 \times 4 \times \log_{10}\left(\frac{1000}{R_{DCS\text{-}1900}}\right) = -104 - (-68)$$

$\Rightarrow$ $R_{DCS\text{-}1900} \doteq 7.94$ Km

$\Rightarrow$ the area of one cell in GSM at 800 MHz.

$$A_{GSM} = 2.6 \cdot R_{GSM}^2 \doteq 388.89 \text{ Km}^2$$

the area of one cell in DCS-1900 at 1900 MHz.

$$A_{DCS\text{-}1900} = 2.6 R_{DCS}^2 = 163.91 \text{ Km}^2$$

$\Rightarrow$ number of ominidirectional cells required to cover a 1000 Km^2 area.

For GSM at 800 MHz, $N_{GSM} = \frac{1000}{388.89} \doteq 2.57 \doteq \underline{\underline{3 \text{ Cells}}}$

For DCS-1900 at 1900 MHz, $N_{DCS\text{-}1900} = \frac{1000}{163.91} = 6.1 \doteq \underline{\underline{7 \text{ Cells}}}$

$\boxed{11.24}$ Assume the carrier frequency for IS-95 is 900MHz, as shown in problem 11.23, we have

$$P_r(d_0)_{IS\text{-}95} = \frac{1 \times 1 \times 1 \times \left(\frac{300}{900}\right)^2}{(4 \times 3.14)^2 \times 1000^2} = 7.04 \times 10^{-10} \text{ W} \doteq -61.5 \text{ dBm}$$

$\Rightarrow$ $10 \times 4 \times \log_{10}\left(\frac{1000}{R_{IS\text{-}95}}\right) = -104 - (-61.5)$ $\Rightarrow$ $R_{IS\text{-}95} = 11.55\text{Km}$ $\Rightarrow$ $N_{IS\text{-}95} \doteq 2.88 \doteq \underline{\underline{3 \text{ Cells}}}$ at 900 MHz.

11.24 Cont'd

For unbaided IS-95 at 1900MHz, we have

$$P_r(d_0)_{IS\text{-}95} = \frac{1\times1\times1\times\left(\frac{300}{1900}\right)^2}{(4\times3.14)^2\times1000^2} \doteq 1.58\times10^{-10}W \doteq -68\,dBm.$$

$$\Rightarrow 10\times4\times\log_{10}\left(\frac{1000}{R_{IS\text{-}95}}\right) = -104-(-68) \Rightarrow R_{IS\text{-}95} \doteq 7.94\ Km$$

$$\Rightarrow N_{IS95} \doteq 6.1 \doteq \underline{\underline{7 \text{ cells at } 1900\,MHz}}$$

$\boxed{11.25}$ In AMPS, FDMA is used, the number of channels for each cell is fixed. If the system loading increases, as shown in chapter 2, the grade of service (GOS) decreases. Once the GOS falls below a threshold, the original cell should be split into smaller cells to accomodate more users. The cell splitting thus result in smaller cell coverage radius.

In IS-95 CDMA system, each user within a cell uses the same radio channel and users in adjacent cells also use the same radio channel. Increasing the system loading (number of users) in a CDMA system raises the noise floor in a linear manner. Once the noise floor reaches a threshold, the bit error rate becomes unacceptable, and the original cell needs to be divided into smaller cells such that the number of users in one cell becomes less.

11.25 Cont'd

In IS-54 TDMA, a single carrier frequency is shared with several users, where each user makes use of non-overlapping time slots. When the system loading increases, the grade of service decreases, and cell splitting is required such that more cells can be added to accomodate more users.

$\boxed{11.26}$ a) Imperfect fast power control at the mobile, which gives rise to an unacceptable level of multiple access interference on the reverse link due to fluctuations.

b) Deep shadowing of many mobiles, which causes the slow power control mechanism to fail in compensating mobile transmit powers beyond the maximum possible limit, thereby causing properly compensated users to draw out these deeply shadowed users.

$\boxed{11.27}$ The PACS version for the unlicensed PCS band is provided only 10MHz (1920~1930 MHz) bandwidth. To use FDD in such a narrow bandwidth (compared to the carrier frequency), expensive RF technology would be needed to separate the forward and reverse channel. TDD, on the other hand, enables each transceiver to operate as either a transmitter or receiver on the same frequency, and eliminates the need for separate forward and reverse frequency bands. So the PACS version for the unlicensed PCS band use TDD instead of FDD.

11.28 Since DECT system uses TDD technique with channel data rate of 1152 Kbps, it is very sensitive to multipath propagation and symbol timing. If it is deployed outdoors in an environment where significant multipath could occur, the large time delay spread will cause high bit error rates and timing jitter and will make the communication impossible.

11.29 See Fig. 11.2 for the allocation of bits in a USDC half-rate time slot

(a) channel data rate = 48.6 Kbps

(b) There are 260 user bits in each USDC time slot

(c) Time duration for each USDC frame $= \frac{40\text{ms}}{6} = 6.667\text{ms}$

(d) frame efficiency, $\eta_f = \frac{260}{324} \times 100\% \doteq 80.2\%$

11.30 (a) Possible SAT tones in AMPS: 5970, 6000 or 6030 Hz

(b) When a user terminates a call or turns the cellular phone off during a call, a ST tone is automatically sent by the subscriber unit. This allows the base station and the MSC to

11.30 Cont'd

know that the call was terminated deliberately by the user, as opposed to being dropping by the system.

(c) Severals ways a cellular phone call may be terminated.

1. User terminated the call deliberately. A ST tone is automatically sent by the subscriber unit.

2. Dropped calls due to interference. The SAT signal is interfered with or incorrectly detected at the subscriber unit or base station, causing the call to drop.

3. A user accidentally disconnects the power supply or battery to the subscriber unit, causing the call to drop without an ST tone.

4. Dropped call due to improper SAT or channel assignment by the base station or switch. The base station issues improper channel or SAT information, which the subscriber unit attempts to interpret but which conflicts with the switch programing.

5. Improper SAT tone regeneration by the subscriber unit. A SAT tone on the forward channel is received properly, but due to poor receiver design, drift, or interference, the reverse channel SAT tone is not properly transmitted.

11.30 Cont'd

6. Dropped calls due to inappropriate roaming. The MIN and ESN are not received correctly by the base station and the call is not allowed by the switch. Or, the call is passed between cellular systems which do not have roaming agreement.

11.31 (a) Gross RF data rate = $\underline{\underline{48.6 \text{ Kbps}}}$

(b) Gross RF data rate for the SACCH,

$$R_{SACCH} = \frac{6 \text{ time slots/frame} \times 12 \text{ bits/time slot}}{40 \text{ ms/frame}} = \underline{\underline{1.8 \text{ Kbps}}}$$

(c) Gross RF data rate for the CDVCC.

$$R_{CDVCC} = \frac{6 \times 12}{40} = \underline{\underline{1.8 \text{ Kbps}}}$$

11.31 Cont'd

(d) Gross data rate for synchronization, ramp-up, and guard time,

$$R_{SRG} = \frac{6 \times (28+6+6)}{40} = \underline{\underline{6\ \text{Kbps}}}$$

(e) $R_{SACCH} + R_{CDVCC} + R_{SRG} = 1.8 + 1.8 + 6 = 9.6$ Kbps

Gross RF data rate for user information data

$$R_{UI} = \frac{6 \times 260}{40} = 39\ \text{Kbps}$$

$\Longrightarrow$ $R_{UI} + R_{SACCH} + R_{CDVCC} + R_{SRG} = 39 + 9.6 = \underline{\underline{48.6\ \text{Kbps}}}$

(f) End user data rate provided in full rate USDC

$$= \frac{R_{UI}}{3} = \frac{39}{3} = \underline{\underline{13\ \text{Kbps}}}$$

11.32 (a) Data rate for the speech coder, $R_{SC} = \underline{\underline{13\ \text{Kbps}}}$

(b) From Fig. 10.12, data rate for speech error protection, $R_{SEP} = \dfrac{50 + 3 \times 2 + 132 + 4 \times 2}{20} = \underline{\underline{9.8\ \text{Kbps}}}$

(c) From Fig. 10.10, data rate for SACCH.

$$R_{SACCH} = \frac{2\ \text{frame/multiframe} \times 8\ \text{time slot/frame} \times 156.25\ \text{bits/time slot}}{120\ \text{ms/multiframe} \times 8\ \text{users}}$$

$$= \underline{\underline{2.604\ \text{Kbps}}}$$

11.32 Cont'd

(d) Data rate for guard time, ramp-up, synchronization, stealing flag, midample.

$$R_G = \frac{(3+1+1+26+3+8.25)\text{ bits/frame} \times 24 \text{ frame/multiframe}}{120 \text{ ms/multiframe}}$$

$$= \underline{\underline{8.45 \text{ Kbps}}}$$

$$\Rightarrow R_{SC} + R_{SEP} + R_{SACCH} + R_G = 13 + 9.8 + 2.604 + 8.45$$

$$= 33.854 \text{ Kbps}.$$

$\boxed{11.33}$ The frame number is sent with the base station identity code during the SCH burst. From Fig. 11.8, we can see that the interval between two consecutive SCH burst is 10 TDMA frames. Therefore,

$$T_{max} = \frac{235 \text{ ms/control multiframe}}{51 \text{ TDMA frames/control multiframe}} \times 10 \text{ frames}$$

$$\doteq \underline{\underline{46.08 \text{ ms}}}$$