

Exercises

Chapter 3

3.1 Classify the following random variables as discrete or continuous:

X: the number of automobile accidents per year in Virginia.

Y: the length of time to play 18 holes of golf.

M: the amount of milk produced yearly by a particular cow.

N: the number of eggs laid each month by a hen.

P: the number of building permits issued each month in a certain city.

Q: the weight of grain produced per acre.

3.2 An overseas shipment of 5 foreign automobiles contains 2 that have slight paint blemishes. If an agency receives 3 of these automobiles at random, list the elements of the sample space S using the letters B and Ar for blemished and nonblemished, respectively; then to each sample point assign a value x of the random variable X representing the number of automobiles purchased by the agency with paint blemishes.

3.3 Let W be a random variable giving the number of heads minus the number of tails in three tosses of a coin. List the elements of the sample space S for the three tosses of the coin and to each sample point assign a value w of W.

3.4 A coin is flipped until 3 heads in succession occur. List only those elements of the sample space that require 6 or less tosses. Is this a discrete sample space? Explain.

3.5 Determine the value c so that each of the following functions can serve as a probability distribution of the discrete random variable X: (a) $f(x) = c(x^2 + 4)$, for $x = 0, 1, 2, 3$; (b) $f(x) = c(1)(3^x)$, for $x = -1, 0, 1, 2$.

3.6 The shelf life, in days, for bottles of a certain prescribed medicine is a random variable having the density function

$$f(x) = \begin{cases} \frac{20,000}{(x+100)^3}, & x > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability that a bottle of this medicine will have a shelf life of (a) at least 200 days; (b) anywhere from 80 to 120 days.

3.7 The total number of hours, measured in units of 100 hours, that a family runs a vacuum cleaner over a period of one year is a continuous random variable X that has the density function

$$f(x) = \begin{cases} x, & 0 < x < 1, \\ 2 - x, & 1 \leq x < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

Find the probability that over a period of one year, a family runs their vacuum cleaner

- (a) less than 120 hours;
- (b) between 50 and 100 hours.

3.8 Find the probability distribution of the random variable W in Exercise 3.3. assuming that the coin is biased so that a head is twice as likely to occur as a tail.

3.9 The proportion of people who respond to a certain mail-order solicitation is a continuous random variable X that has the density function

$$f(x) = \begin{cases} \frac{2(x+2)}{5}, & 0 < x < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Show that $P(0 < X < 1) = 1$.
- (b) Find the probability that more than $1/4$ but fewer than $1/2$ of the people contacted will respond to this type of solicitation.

3.10 Find a formula for the probability distribution of the random variable X representing the outcome when a single die is rolled once.

3.11 A shipment of 7 television sets contains 2 defective sets. A hotel makes a random purchase of 3 of the sets. If x is the number of defective sets purchased by the hotel, find the probability distribution of X . Express the results graphically as a probability histogram.

3.12 An investment firm offers its customers municipal bonds that mature after varying numbers of years. Given that the cumulative distribution function of T , the number of years to maturity for a randomly selected bond, is,

$$F(t) = \begin{cases} 0, & t < 1, \\ \frac{1}{4}, & 1 \leq t < 3, \\ \frac{3}{4}, & 3 \leq t < 5, \\ \frac{5}{8}, & 5 \leq t < 7, \\ 1, & t \geq 7. \end{cases}$$

find

- (a) $P(T = 5)$;
- (b) $P(T > 3)$;
- (c) $P(1.4 < T < 6)$.

3.13 The probability distribution of A, the number of imperfections per 10 meters of a synthetic fabric in continuous rolls of uniform width, is given by

x	0	1	2	3	4
$f(x)$	0.41	0.37	0.16	0.05	0.01

Construct the cumulative distribution function of X.

3.14 The waiting time, in hours, between successive speeders spotted by a radar unit is a continuous random variable with cumulative distribution function

$$F(x) = \begin{cases} 0, & x < 0, \\ 1 - e^{-8x}, & x \geq 0. \end{cases}$$

Find the probability of waiting less than 12 minutes between successive speeders (a) using the cumulative distribution function of X; (b) using the probability density function of X.

3.15 Find the cumulative distribution function of the random variable X representing the number of defectives in Exercise 3.11. Then using $F(x)$, find

- (a) $P(X = 1)$;
- (b) $P(0 < X \leq 2)$.

3.16 Construct a graph of the cumulative distribution function of Exercise 3.15.

3.17 A continuous random variable X that can assume values between $x = 1$ and $x = 3$ has a density function given by $f(x) = 1/2$. (a) Show that the area under the curve is equal to 1. (b) Find $P(2 < X < 2.5)$. (c) Find $P(X < 1.6)$.

3.18 A continuous random variable X that can assume values between $x = 2$ and $x = 5$ has a density function given by $f(x) = 2(1 + a;)/27$. Find (a) $P(X < 4)$; (b) $P(3 < X < 4)$.

3.19 For the density function of Exercise 3.17, find $F(x)$. Use it to evaluate $P(2 < X < 2.5)$.

3.20 For the density function of Exercise 3.18, find $F(x)$, and use it to evaluate $P(3 < X < 4)$.

3.21 Consider the density function

$$f(x) = \begin{cases} k\sqrt{x}, & 0 < X < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Evaluate k .
- (b) Find $F(x)$ and use it to evaluate

$$P(0.3 < X < 0.6).$$

3.22 Three cards are drawn in succession from a deck without replacement. Find the probability distribution for the number of spades.

3.23 Find the cumulative distribution function of the random variable W in Exercise 3.8. Using $F(w)$, find (a) $P(W > 0)$; (b) $P(-1 < W < 3)$.

3.24 Find the probability distribution for the number of jazz CDs when 4 CDs are selected at random from a collection consisting of 5 jazz CDs, 2 classical CDs, and 3 rock CDs. Express your results by means of a formula.

3.25 From a box containing 4 dimes and 2 nickels, 3 coins are selected at random without replacement. Find the probability distribution for the total T of the 3 coins. Express the probability distribution graphically as a probability histogram.

3.26 From a box containing 4 black balls and 2 green balls, 3 balls are drawn in succession, each ball being replaced in the box before the next draw is made. Find the probability distribution for the number of green balls.

3.27 The time to failure in hours of an important piece of electronic equipment used in a manufactured DVD player has the density function

$$f(x) = \begin{cases} \frac{1}{2000} \exp(-x/2000), & x \geq 0, \\ 0, & x < 0. \end{cases}$$

- (a) Find $F(x)$.
- (b) Determine the probability that the component (and thus the DVD player) lasts more than 1000 hours before the component needs to be replaced.
- (c) Determine the probability that the component fails before 2000 hours.

3.28 A cereal manufacturer is aware that the weight of the product in the box varies slightly from box to box. In fact, considerable historical data has allowed the determination of the density function that describes the probability structure for the weight (in ounces). In fact, letting X be the random variable weight, in ounces, the density function can be described as

$$f(x) = \begin{cases} \frac{2}{5}, & 23.75 \leq x \leq 26.25, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Verify that this is a valid density function. (b) Determine the probability that the weight is smaller than 24 ounces. (c) The company desires that the weight exceeding 26 ounces is an

extremely rare occurrence. What is the probability that this "rare occurrence" does actually occur?

3.29 An important factor in solid missile fuel is the particle size distribution. Significant problems occur if the particle sizes are too large. From production data in the past, it has been determined that the particle size (in micrometers) distribution is characterized by

$$f(x) = \begin{cases} 3x^{-4}, & x > 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Verify that this is a valid density function. (b) Evaluate $F(x)$. (c) What is the probability that a random particle from the manufactured fuel exceeds 4 micrometers?

3.30 Measurements of scientific systems are always subject to variation, some more than others. There are many structures for measurement error and statisticians spend a great deal of time modeling these errors. Suppose the measurement error X of a certain physical quantity is decided by the density function

$$f(x) = \begin{cases} k(3 - x^2), & -1 \leq x \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Determine k that renders $f(x)$ a valid density function. (b) Find the probability that a random error in measurement is less than $1/2$. (c) For this particular measurement, it is undesirable if the magnitude of the error (i.e., $|a|$), exceeds 0.8 . What is the probability that this occurs?

3.31 Based on extensive testing, it is determined by the manufacturer of a washing machine that the time Y (in years) before a major repair is required is characterized by the probability density function

$$f(x) = \begin{cases} \frac{1}{4}e^{-y/4}, & y \geq 0, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Critics would certainly consider the product a bargain if it is unlikely to require a major repair before the sixth year. Comment on this by determining $P(Y > 6)$. (b) What is the probability that a major repair occurs in the first year?

3.32 The proportion of the budgets for a certain type of industrial company that is allotted to environmental and pollution control is coming under scrutiny. A data collection project determines that the distribution of these proportions is given by

$$f(y) = \begin{cases} 5(1-y)^4, & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Verify that the above is a valid density. (b) What is the probability that a company chosen at random expends less than 10% of its budget on environmental and pollution controls? (c) What is the probability that a company selected at random spends more than 50% on environmental and pollution control?

3.33 Suppose a special type of small data processing firm is so specialized that some have difficulty making a profit in their first year of operation. The pdf that characterizes the proportion Y that make a profit is given by

$$f(x) = \begin{cases} ky^4(1-y)^3, & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) What is the value of k that renders the above a valid density function? (b) Find the probability that at most 50% of the firms make a profit in the first year. (c) Find the probability that at least 80% of the firms make a profit in the first year.

3.34 Magnetron tubejs are produced from an automated assembly line. A sampling plan is used periodically to assess quality on the lengths of the tubes. This measurement is subject to uncertainty. It is thought that the probability that a random tube meets length specification is 0.99. A sampling plan is used in which the lengths of 5 random tubes are measured. (a) Show that the probability function of Y , the number out of 5 that meet length specification, is given by the following discrete probability function

$$f(y) = \frac{5!}{y!(5-y)!} (0.99)^y (0.01)^{5-y},$$

for $y = 0, 1, 2, 3, 4, 5$.

- (b) Suppose random selections are made off the line and 3 are outside specifications. Use $f(y)$ above either to support or refute the conjecture that the probability is 0.99 that a single tube meets specifications.

3.35 Suppose it is known from large amounts of historical data that X , the number of cars that arrive at a specific intersection during a 20 second time period, is characterized by the following discrete probability function

$$f(x) = \frac{e^{-6} 6^x}{x!}, \quad x = 0, 1, 2, \dots$$

- (a) Find the probability that in a specific 20-second time period, more than 8 cars arrive at the intersection.
 (b) Find the probability that only 2 cars arrive.

3.36 On a laboratory assignment, if the equipment is working, the density function of the observed outcome, X is

$$f(x) = \begin{cases} 2(1-x), & 0 < x < 1, \\ 0, & \text{otherwise.} \end{cases}$$

- (a) Calculate $P(X < 1/3)$.
 (b) What is the probability that X will exceed 0.5?
 (c) Given that $X > 0.5$, what is the probability that X will be less than 0.75?

3.37 Determine the values of c so that the following functions represent joint **probability** distributions of the random variables X and Y :

(a) $f(x, y) = cxy$, for $x = 1, 2, 3$; $y = 1, 2, 3$;

(b) $f(x, y) = c|x - y|$, for $x = -2, 0, 2$; $y = -2, 3$.

3.38 If the joint probability distribution of X and Y is given by

$$f(x, y) = \frac{x + y}{30}, \quad \text{for } x = 0, 1, 2, 3; \quad y = (1, 1, 2,$$

find

(a) $P(X \leq 2, Y = 1)$;

(b) $P(X > 2, Y \leq 1)$;

(c) $P(X > Y)$;

(d) $P(X + Y = 4)$.

3.39 From a sack of fruit containing 3 oranges, 2 apples, and 3 bananas, a random sample of 4 pieces of fruit is selected. If X is the number of oranges and Y is the number of apples in the sample, find (a) the joint probability distribution of X and Y ; (b) $P[(X, Y) \in A]$, where A is the region that is given by $\{(x, y) | x + y < 2\}$.

3.40 A privately owned liquor store operates both a drive-in facility and a walk-in facility. On a randomly selected day, let X and Y , respectively, be the proportions of the time that the drive-in and walk-in facilities are in use, and suppose that the joint density function of these random variables is

$$f(x, y) = \begin{cases} \frac{2}{3}(x + 2y), & 0 \leq x \leq 1, \quad 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Find the marginal density of X.
- (b) Find the marginal density of Y.
- (c) Find the probability that the drive-in facility is busy less than one-half of the time.

3.41 A candy company distributes boxes of chocolates with a mixture of creams, toffees, and cordials. Suppose that the weight of each box is 1 kilogram, but the individual weights of the creams, toffees, and cordials vary from box to box. For a randomly selected box, let X and Y represent the weights of the creams and the toffees, respectively, and suppose that, the joint density function of these variables is

$$f(x, y) = \begin{cases} 24xy, & 0 \leq x \leq 1, \quad 0 \leq y \leq 1, \\ & x + y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Find the probability that in a given box the cordials account for more than 1/2 of the weight.
- (b) Find the marginal density for the weight of the creams.
- (c) Find the probability that the weight of the toffees in a box is less than 1/8 of a kilogram if it is known that creams constitute 3/4 of the weight.

3.42 Let X and Y denote the lengths of life, in years, of two components in an electronic system. If the joint density function of these variables is

$$f(x, y) = \begin{cases} e^{-(x+y)}, & x > 0, \quad y > 0, \\ 0, & \text{elsewhere,} \end{cases}$$

$$\text{find } P(0 < X < 1 \mid Y = 2).$$

3.43 Let X denote the reaction time, in seconds, to a certain stimulus and Y denote the temperature (°F) at which a certain reaction starts to take place. Suppose that two random variables X and Y have the joint density

$$f(x, y) = \begin{cases} 4xy, & 0 < x < 1, 0 < y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find

- (a) $P(0 \leq X \leq \frac{1}{2} \text{ and } \frac{1}{4} \leq Y \leq \frac{1}{2})$;
- (b) $P(X < Y)$.

3.44 Each rear tire on an experimental airplane is supposed to be filled to a pressure of 40 pound per square inch (psi). Let X denote the actual air pressure for the right tire and Y denote the actual air pressure for the left tire. Suppose that X and Y are random variables with the joint density

$$f(x, y) = \begin{cases} k(x^2 + y^2), & 30 \leq x < 50; \\ & 30 \leq y < 50, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Find k .
- (b) Find $P(30 < X < 40 \text{ and } 40 < Y < 50)$.
- (c) Find the probability that both tires are underfilled.

3.45 Let X denote the diameter of an armored electric cable and Y denote the diameter of the ceramic mold that makes the cable. Both X and Y are scaled so that they range between 0 and 1. Suppose that X and Y have the joint density

$$f(x, y) = \begin{cases} \frac{1}{y}, & 0 < x < y < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

Find $P(X + Y > 1/2)$.

3.46 Referring to Exercise 3.38, find

- (a) the marginal distribution of X;
- (b) the marginal distribution of Y.

3.47 The amount of kerosene, in thousands of liters, in a tank at the beginning of any day is a random amount Y from which a random amount X is sold during that day. Suppose that the tank is not resupplied during the day so that $x < y$, and assume that the joint density function of these variables is

$$f_{X,Y}(x,y) = \begin{cases} 2, & 0 < x < y, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Determine if X and Y are independent.
- (b) Find $P(1/4 < X < 1/2 \mid Y = 3/4)$.

3.48 Referring to Exercise 3.39, find (a) $f_Y(y)$ for all values of y; (b) $P(Y = 0 \mid X = 2)$.

3.49 Let X denote the number of times a certain numerical control machine will malfunction: 1, 2, or 3 times on any given day. Let Y denote the number of times a technician is called on an emergency call. Their joint probability distribution is given as

$f(x, y)$		x		
		1	2	3
y	1	0.05	0.05	0.1
	2	0.05	0.1	0.35
	3	0	0.2	0.1

- (a) Evaluate the marginal distribution of X.
- (b) Evaluate the marginal distribution of Y.
- (c) Find $P(Y = 3 \mid X = 2)$.

3.50 Suppose that X and Y have the following joint probability distribution:

$f(x,y)$		x	
		2	4
y	1	0.10	0.15
	3	0.20	0.30
	5	0.10	0.15

3.51 Consider an experiment that consists of 2 rolls of a balanced die. If X is the number of 4s and Y is the number of os obtained in the 2 rolls of the die, find

- (a) the joint probability distribution of A and V;
 (b) $P[(X, Y) \in A]$, where A is the region $\{(x,y) \mid 2x + y < 3\}$.

3.52 Let X denote the: number of heads and Y the number of heads minus the number of tails when 3 coins are tossed. Find the joint probability distribution of X and Y.

3.53 Three cards are drawn without replacement from the 12 face cards (jacks, queens, and kings) of an ordinary deck of 52 playing cards. Let X be the number of kings selected and Y the number of jacks. Find (a) the joint probability distribution of X and Y; (b) $P[(X,Y) \in A]$, where A is the region given by $\{(x,y) \mid x + y > 2\}$.

3.54 A coin is tossed twice. Let Z denote the number of heads on the first toss and W the total number of heads on the 2 tosses. If the coin is unbalanced and a head has a 40% chance of occurring, find (a) the joint probability distribution of W and Z; (b) the marginal distribution of W\ (c) the marginal distribution of Z; (d) the probability that at least 1 head occurs.

3.55 Given the joint density function

$$f(x, y) = \begin{cases} \frac{6-x-y}{8}; & 0 < x < 2, \ 2 < y < 4, \\ 0, & \text{elsewhere,} \end{cases}$$

find $P(1 < Y < 3 \mid X = 1)$.

3.56 Determine whether the two random variables of Exercise 3.49 are dependent or independent.

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3.58 The joint density function of the random variables X and Y is

$$f(x, y) \equiv \begin{cases} 6x, & 0 < x < 1, \ 0 < y < 1 - x, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Show that X and Y are not independent.

(b) Find $P(X > 0.3 \mid Y = 0.5)$.

3.59 Let X , Y , and Z have the joint probability density function

$$f(x, y, z) = \begin{cases} kxy^2z, & 0 < x, y \leq 1; \ 0 < z < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

(a) Find k .

(b) Find $P(X < \frac{1}{4}, Y > \frac{1}{2}, 1 < Z < 2)$.

3.60 Determine whether the two random variables of Exercise 3.43 are dependent or independent.

3.61 Determine whether the two random variables of Exercise 3.44 are dependent or independent.

3.62 The joint probability density function of the random variables X , Y , and Z is

$$f(x, y, z) = \begin{cases} \frac{4xyz^2}{9}, & 0 < x, y \leq 1; \ 0 < z < 3, \\ 0, & \text{elsewhere.} \end{cases}$$

3.62 The joint probability density function of the random variables X , Y , and Z is

$$f(x, y, z) = \begin{cases} \frac{4xyz^2}{9}, & 0 < x, y < 1; \quad 0 < z < 3, \\ 0, & \text{elsewhere.} \end{cases}$$

Find

- (a) the joint marginal density function of Y and Z ;
- (b) the marginal density of Y ;
- (c) $P(\frac{1}{4} < X < \frac{1}{2}, Y > \frac{1}{3}, KZ < 2)$;
- (d) $P(0 < X < \frac{1}{2} \mid Y = \frac{1}{2}, Z = 2)$.

3.63 A tobacco company produces blends of tobacco with each blend containing various proportions of Turkish, domestic, and other tobaccos. The proportions of Turkish and domestic in a blend are random variables with joint density function (X = Turkish and Y = domestic)

$$f(x, y) = \begin{cases} 24xy, & 0 \leq x, y \leq 1: x + y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Find the probability that in a given box the Turkish tobacco accounts for over half the blend.
- (b) Find the marginal density function for the proportion of the domestic tobacco.
- (c) Find the probability that the proportion of Turkish tobacco is less than $1/8$ if it is known that the blend contains $3/4$ domestic tobacco.

3.64 An insurance company offers its policyholders a number of different premium payment options. For a randomly selected policyholder, let X be the number of months between successive payments. The cumulative distribution function of X is

$$F(x) = \begin{cases} 0, & \text{if } x < 1, \\ 0.4, & \text{if } 1 \leq x < 3, \\ 0.6, & \text{if } 3 \leq x < 5, \\ 0.8, & \text{if } 5 \leq x < 7, \\ 1.0, & \text{if } x \geq 7. \end{cases}$$

- (a) What is the probability mass function of X ?
- (b) Compute $P(4 < X < 7)$.

3.65 Two electronic components of a missile system work in harmony for the success of the total system. Let X and Y denote the life in hours of the two components. The joint density of X and Y is

$$f(x, y) = \begin{cases} ye^{-y(1+x)}, & x, y \geq 0, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Give the marginal density functions for both random variables.
- (b) What is the probability that both components will exceed 2 hours?

3.66 A service facility operates with two service lines. On a randomly selected day, let X be the proportion of time that the first line is in use whereas Y is the proportion of time that the second line is in use. Suppose that the joint probability density function for (A, V) is

$$f(x, y) = \begin{cases} \frac{3}{2}(x^2 + y^2), & 0 \leq x, y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- Compute the probability that neither line is busy more than half the time.
- Find the probability that the first line is busy more than 75% of the time.

3.67 Let the number of phone calls received by a switchboard during a 5-minute interval be a random variable X with probability function

$$f(x) = \frac{e^{-2}2^x}{x!}, \quad \text{for } x = 0, 1, 2, \dots$$

- Determine the probability that X equals 0, 1, 2, 3, 4, 5, and 6.
- Graph the probability mass function for these values of x .
- Determine the cumulative distribution function for these values of X .

3.68 Consider the random variables X and Y with joint density function

$$f(x, y) = \begin{cases} x + y, & 0 \leq x, y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Find the marginal distributions of X and Y .
- (b) Find $P(X > 0.5, Y > 0.5)$.

3.69 An industrial process manufactures items that can be classified as either defective or not defective. The probability that an item is defective is 0.1. An experiment is conducted in which 5 items are drawn randomly from the process. Let the random variable X be the number of defectives in this sample of 5. What is the probability mass function of X ?

3.70 Consider the following joint probability density function of the random variables X and Y :

$$f(x, y) = \begin{cases} \frac{3x-y}{9}, & 1 < x < 3, 1 < y < 2, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Find the marginal density functions of X and Y .
- (b) Are X and Y independent?
- (c) Find $P(X > 2)$.

3.71 The life span in hours of an electrical component is a random variable with cumulative distribution function

$$F(x) = \begin{cases} 1 - e^{-x/50}, & x > 0, \\ 0, & \text{elsewhere.} \end{cases}$$

- Determine its probability density function.
- Determine the probability that the life span of such a component will exceed 70 hours.

3.72 Pairs of pants are being produced by a particular outlet facility. The pants are “checked” by a group of 10 workers. The workers inspect pairs of pants taken randomly from the production line. Each inspector is assigned a number from 1 through 10. A buyer selects a pair of pants for purchase. Let the random variable X be the inspector number.

- Give a reasonable probability mass function for X .
- Plot the cumulative distribution function for X .

3.73 The shelf life of a product is a random variable that is related to consumer acceptance. It turns out that the shelf life Y in days of a certain type of bakery product has a density function

$$f(y) = \begin{cases} \frac{1}{2} e^{-y/2}, & 0 \leq y < \infty, \\ 0, & \text{elsewhere.} \end{cases}$$

What fraction of the loaves of this product, stocked today would you expect to be sellable 3 days from now?

3.74 Passenger congestion is a service problem in airports. Trains are installed within the airport to reduce the congestion. With the use of the train, the time X that it takes in minutes to travel from the main terminal to a particular concourse has density function

$$f(x) = \begin{cases} \frac{1}{10}, & 0 \leq x \leq 10, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Show that the pdf above is a valid density function.
- (b) Find the probability that the time it takes a passenger to travel from the main terminal to the concourse will not exceed 7 minutes.

3.75 Impurities in the batch of final product of a chemical process often reflect a serious problem. From considerable plant data gathered, it is known that the proportion Y of impurities in a batch has a density function given by

$$f(y) = \begin{cases} 10(1 - y)^9, & 0 \leq y \leq 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- (a) Verify that the above is a valid density function.
- (b) A batch is **considered** not sellable and then not-acceptable if the percentage of impurities exceeds 60%. With the current quality of the process, what is the percentage of batches that are not acceptable?

3.76 The time Z in minutes between calls to an electrical supply system has the probability density function

$$f(z) = \begin{cases} \frac{1}{10} e^{-z/10}, & 0 < z < \infty, \\ 0, & \text{elsewhere.} \end{cases}$$

- What is the probability that there are no calls within a 20-minute time interval?
- What is the probability that the first call comes within 10 minutes of opening?

3.77 A chemical system that results from a chemical reaction has two important components among others in a blend. The joint distribution describing the proportion X_1 and X_2 of these two components is given by

$$f(x_1, x_2) = \begin{cases} 2, & 0 < x_1 < x_2 < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- Give the marginal distribution of X_1 .
- Give the marginal distribution of X_2 .
- What** is the probability that component proportions produce the results $X_1 < 0.2$ and $X_2 > 0.5$?
- Give the **conditional** distribution $f_{X_1|X_2}(x_1|x_2)$.

3.78 Consider the situation of Review Exercise 3.77. But suppose the joint distribution of the two proportions is given by

$$f(x_1, x_2) = \begin{cases} 6x_2, & 0 < x_2 < x_1 < 1, \\ 0, & \text{elsewhere.} \end{cases}$$

- Give the marginal distribution $f_{X_1}(x_1)$ of the proportion X_1 and verify that it is a valid density function.
- What is the probability that proportion X_2 is less than 0.5 given that X_1 is 0.7?

3.79 Consider the random variables X and Y that represent the number of vehicles that arrive at 2 separate street corners during a certain **2-minute** period. These street corners are fairly close together so it is important that traffic engineers deal with them jointly if necessary. The joint distribution of X and Y is known to be

$$f(x, y) = \frac{9}{16} \cdot \frac{1}{2^{x+y+1}},$$

for $x = 0, 1, 2, \dots$, and $y = 0, 1, 2, \dots$.

- Are the two random variables X and Y independent? Explain why or why not.
- What is the probability that during the time period in question less than 4 vehicles arrive at the two street corners?

3.80 The behavior of series of components play a huge role in scientific and engineering reliability problems. The reliability of the entire system is certainly no better than the weakest component in the series. In a series system, the components operate independently of each other. In a particular system containing three components the probability of meeting specification for components 1, 2, and 3, respectively, are 0.95, 0.99, and 0.92. What is the probability that the entire system works?

3.81 Another type of system that is employed in engineering work is a group of parallel components or a parallel system. In this more conservative approach, the probability that the system operates is larger than the probability that any component operates. The system fails only when all systems fail. Consider a situation in which there are 4 independent components in a parallel system with probability of operation given by

Component 1: 0.95; Component 2: 0.94

Component 3: 0.90; Component 4: 0.97.

What is the probability that the system does not fail?

3.82 Consider a system of components in which there are five independent components, each of which possesses an operational probability of 0.92. The system does have a redundancy built in such that it does not fail if 3 out of the 5 components are operational. What is the probability that the total system is operational?