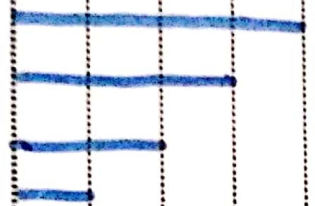


أساسيات الدوائر الكهربائية د. يحيى رواش

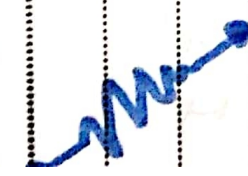
للطالبة المبدعة
حنين أبو العدس

إرادة - ثقة - تغيير

S
T
A
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Circuit

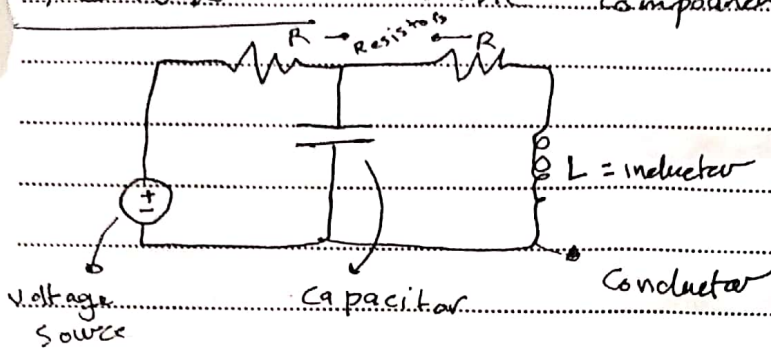


subject : circuit



Subject :

#Ch 2 Basic component and electrical circuit



Unit + scales

any measurable quantity can be described by number + unit
 like: 3 meter
 number ← unit

five something we must know

(*) charge :-

⇒ +ve → Proton → $+1.602 \times 10^{-19}$ C

⇒ -ve → electron → -1.602×10^{-19} C

⇒ unit → Coulomb

⇒ charge
 DC → $Q(t)$
 AC → $q(t)$

(*) current :-

is the time of flow of electrical charge through a conductor or circuit element

$$I = \frac{dq}{dt}$$

I ⇒ Amper

q ⇒ Coulomb

t ⇒ second

$$\left. \begin{matrix} I \\ q \\ t \end{matrix} \right\} \Rightarrow A = \frac{C}{s}$$

Subject: Complement ↕

Ex: let $q(t) = \cos t$ find $I(t)$ at $t = 3 \text{ ms}$?

$$I = \frac{dq}{dt}$$

3240

$$d q(t) = -\sin t \quad | \quad t = 3 \text{ ms}$$

$$= -\sin(3 \times 10^{-3})$$

$$= -2.999 \times 10^{-3} \text{ A}$$

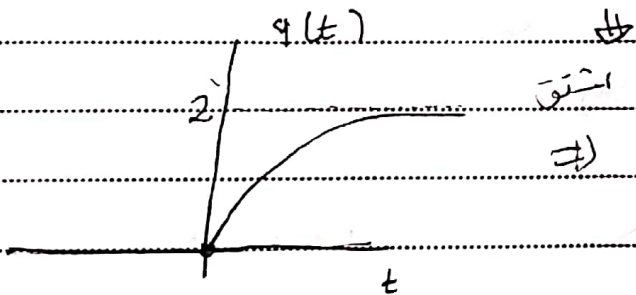
$$\rightarrow \frac{180}{\pi} \text{ degrees}$$

$$I(t) dt = dq(t)$$

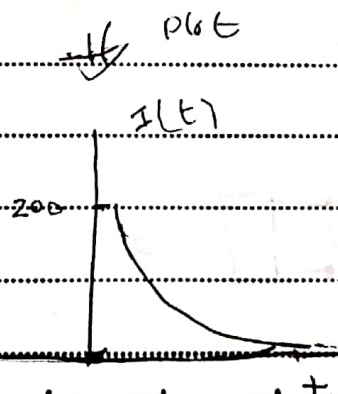
$$\int_{t_0}^t I(t) dt = \int_{t_0}^t dq(t)$$

$$\int_{t_0}^t I(t) dt = Q(t) - Q(t_0)$$

Ex: let $q(t) = \begin{cases} 0 & t < 0 \\ 2 - 2e^{-100t} & t > 0 \end{cases}$ plot $I(t), q(t)$?



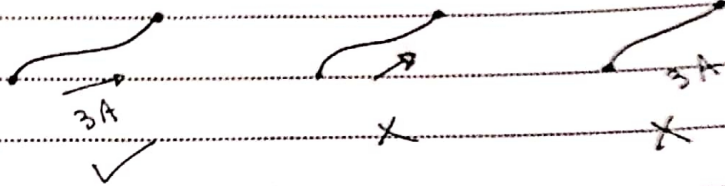
$$I(t) = \begin{cases} 0 & t < 0 \\ 200e^{-100t} & t > 0 \end{cases}$$



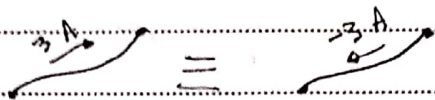
T A R S N O T E B O O K




(*) Current is a vector quantity?

- magnitude
- direction



also



type of current ⇒ 1) DC  2) AC  3) exp 

2) AC  3) damping 

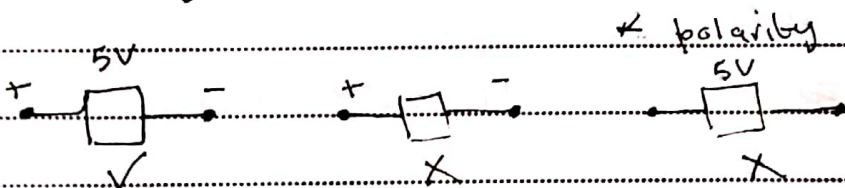
(*) Voltage :-

is a measure of the work required to move a charge through element.

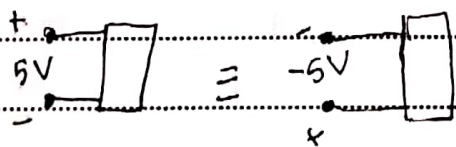
1] Units ⇒ Voltage = V, V

2] the voltage can exist between of electrical terminal whether the current is flowing or not

3] voltage determined by :- + Voltage mag.



also,

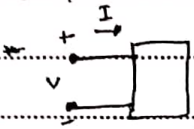


(*) power

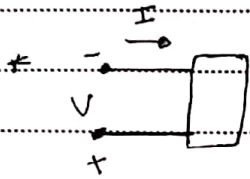
1] unit \Rightarrow watt = W

2] $P = VI$
 $W = V \times A$

3] passive sign convention:

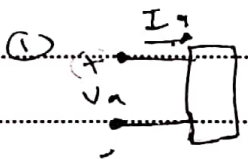


if the current entering ~~the~~ +ve terminal of element then the power is absorbed.

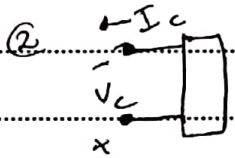


if the current is entering ~~the~~ -ve terminal of element then the power is supplied.

Find the power?



if $V_a = 12V$ $I_a = 2A$ $\Rightarrow P = VI = 12 \times 2 = 24W$ absorbed



$P = VI = 12 \times 3 = 36$ absorbed
36 supplied

$P_{\text{absorbed}} = -P_{\text{supplied}}$

Energy :-

1] unit \Rightarrow Joule

$w(t) = \int_{t_0}^t p(t) dt$

► Subject :

Ex find $w(t)$ if the voltage $v(t) = 12V$, $i(t) = 2e^{-t}A$

$0 \leq t \leq \infty$?

$$w(t) = \int_{t_0}^t p(t) dt$$

$$w(t) = \int_{t_0}^t p(t) dt \\ = \int_{t_0}^t 24e^{-t} dt$$

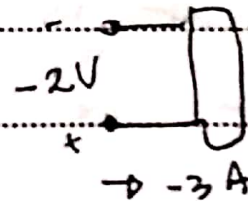
$$p(t) = v(t)i(t) \\ = (12)(2e^{-t}) \\ = 24e^{-t}$$

$$= 24 J$$

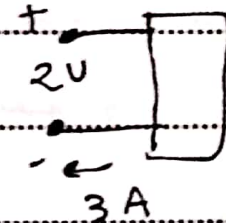
6. Find the power :

$$P = VI$$

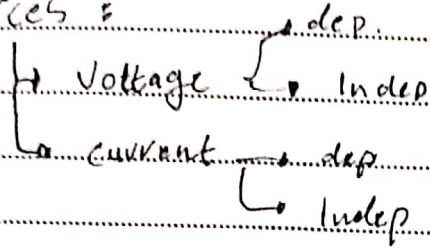
$$= 2 \times 3 = 6W \text{ absorbed}$$



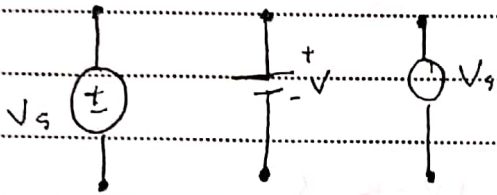
≡



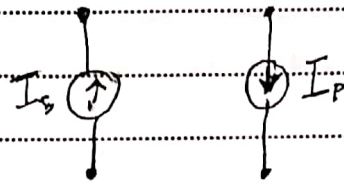
Sources :



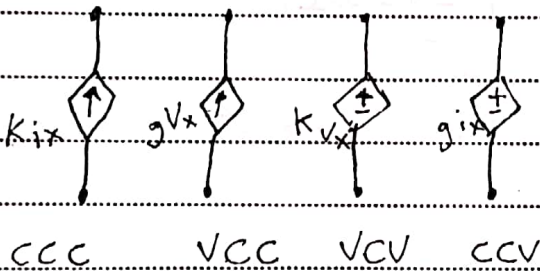
* Indep Voltage source :-



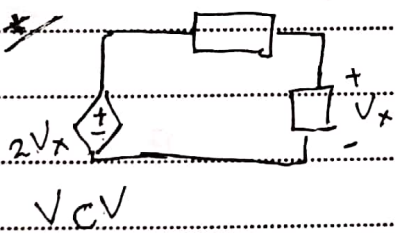
* Indep Current source



* dep Current and Voltage source :-

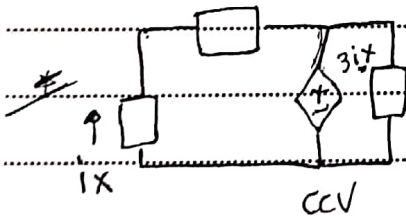


CCC VCC VCV CCV



if $V_x = 2$
then $2V_x = 4$

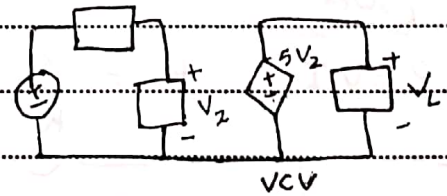
VCV



if $i_x = 3$
then $3i_x = 9$

CCV

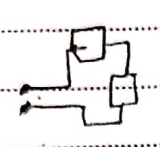
Ex let $V_2 = 3V$ find V_L



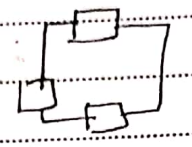
$5 \cdot V_2 = 5 \cdot 3 = 15$

so $V_L = 5V_2$ in Parallel

(*) network circuits:



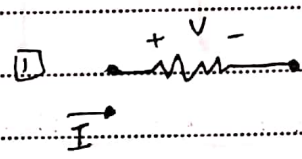
open loop
network



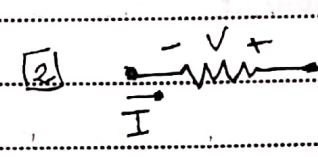
close loop
circuit

(*) Ohm's Law :-

$$V = RI$$



$$\{ +V = RI \}$$



$$\{ -V = RI \}$$

units :-

$$V \equiv \text{VOLT} \quad I \equiv \text{Amper} \quad R \equiv \text{ohm} \quad \Rightarrow \text{ohm} = \frac{V}{A}$$

(*) power

$$P = VI \quad P = \frac{V^2}{R} \quad P = I^2 R$$

* conductance :-

$$G = \frac{1}{R} \quad \text{unit: siemens or mho}$$

$$\therefore P = \frac{I^2}{G}$$

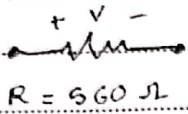
Subject: Complement to

VE

$\frac{V}{P}$

Find V, P ?

42.4 mA



$$\begin{aligned} +V &= RI \\ &= 560 \times 42.4 \times 10^{-3} \\ &= 23.7 V \end{aligned}$$

$$\begin{aligned} P &= I^2 R \\ &= 1.005 \text{ W absorbed} \end{aligned}$$

Ch. 3 : KVL + KCL

① node : a point at which two or more elements have common connection

② path : a set of nodes and elements that we pass through without passing through node more than once

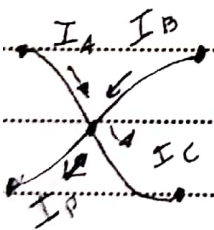
③ loop : if the node at which we started is the same as the node which we end then the path is loop

④ Branch : consist of one node and one element (implies path ↓)

* Kirchhoff's current law (KCL)

The algebraic sum of the current entering any node is zero.

$$\sum_{\text{node}} I = 0$$

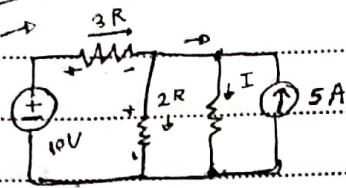


$$I_A + I_B - I_C - I_D = 0$$

T A R S N O T E B O O K

Subject :

Ex find I ?

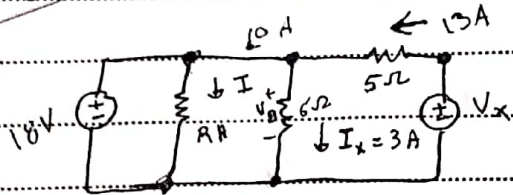


$$I = 6 A$$

$$+V = RI$$

$$I = \frac{10}{3}$$

Ex



find R_A ?

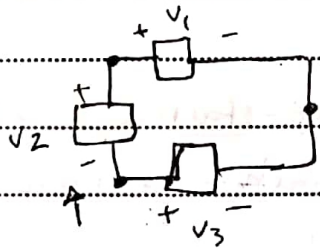
$$R_A = 1 \Omega$$

* Kirchhoff's Voltage Law (KVL)

The algebraic sum of the voltage around any loop is Zero.

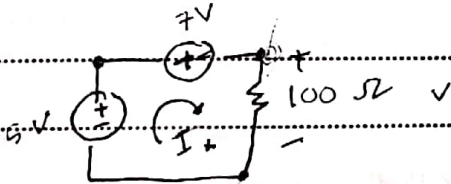
$$\sum V = 0$$

loop



$$-V_1 + V_2 - V_3 = 0$$

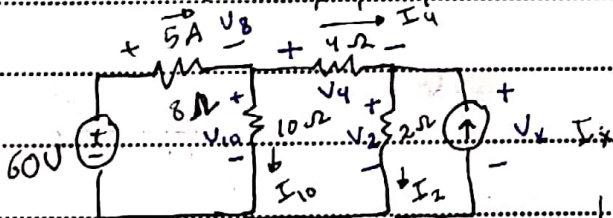
Ex find V_x , I_x ?



$$-5 - 7 + V_x = 0 \Rightarrow V_x = 12V$$

$$I_x = 120mA$$

Ex find V_x , I_x



$$+V = RI = 8 * 5 = 40V$$

$$-60 + 40 + V_{10} = 0 \Rightarrow V_{10} = 20V$$

$$+V_{10} = I_{10} R \Rightarrow I_{10} = 2A$$

$$I_4 = -5 + I_{10} \Rightarrow I_4 = 3A$$

$$+V_2 = I_2 * 2 \Rightarrow I_2 = 4A$$

$$+V_4 = I_4 * R = 3 * 4 = 12V$$

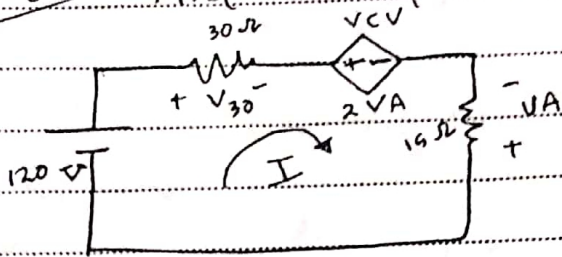
$$I_x = 1A$$

$$+12 + V_2 + 20 = 0 \Rightarrow V_2 = 8V$$

$$V_x = V_2 = 8V$$

Subject :

Ex Find the power?



$$-120 + V_{30} + 2VA - VA = 0$$

$$-120 + V_{30} + VA = 0$$

$$-120 + 30I + (-15I) = 0$$

$$-120 + 15I = 0 \Rightarrow I = 8A$$

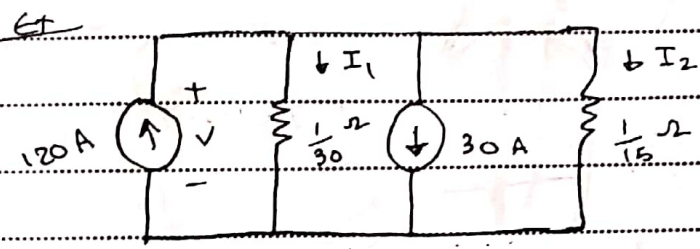
$$P_{120} = VI = 120 \times 8 = 960 \text{ w supplied}$$

$$P_{30} = (8 \times 30) \times 8 = 1920 \text{ w absorbed}$$

$$P_{2VA} = 2(15 \times 8) \times 8 = -1920 \text{ w absorbed}$$

$$P_{VA} = 15 \times 8 \times 8 = 960 \text{ w absorbed}$$

* single node pair circuit :-



$$120 = I_1 + 30 + I_2$$

$$I_1 = 30V = 30 \times 2 = 60A$$

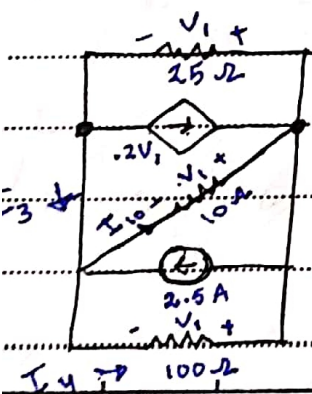
$$90 = I_1 + I_2$$

$$I_2 = 15V = 15 \times 2 = 30A$$

$$90 = 30V + 15V$$

$$90 = 45V \Rightarrow V = 2V$$

Ex Find I1, I2, I3, I4?



$$2.5 = I_1 + 2V_1 + I_3 + I_4$$

$$2.5 = \frac{-V_1}{25} + 2V_1 + \frac{-V_1}{10} + \frac{-V_1}{100} \Rightarrow V_1 = 50V$$

$$I_1 = -2A$$

$$I_2 = I_1 + I_3 + 2V_1 = 3A$$

$$I_4 = 10A$$

$$I_3 = 2.5 + 1.5 + 5 = 8A$$

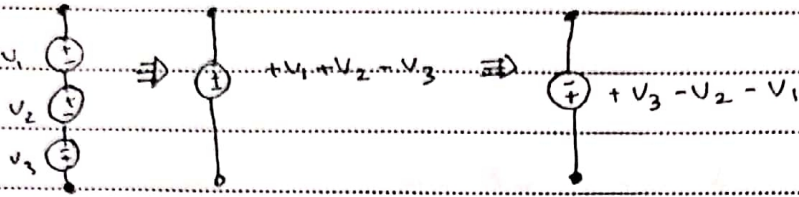
$$I_4 = -5A$$

$$I_4 = -0.5A$$

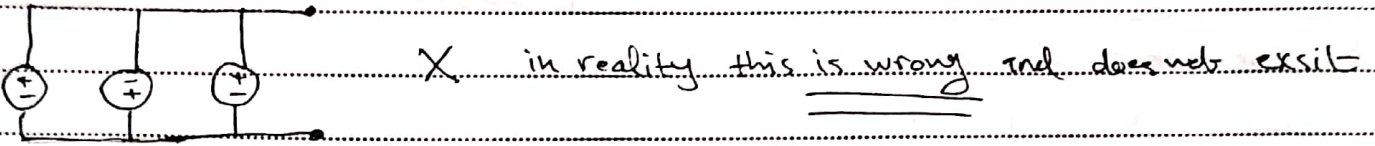
Subject : _____

* Sources \rightarrow series and parallel connected.

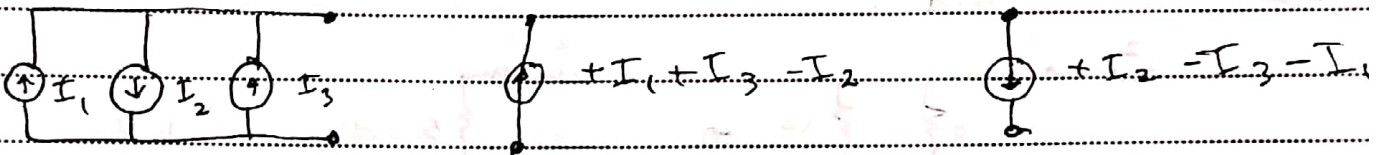
1] Voltage in series:



2] Voltage in parallel:

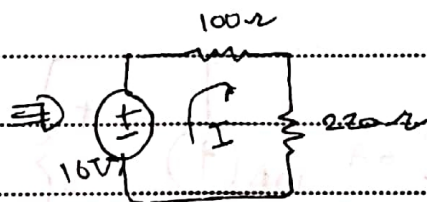
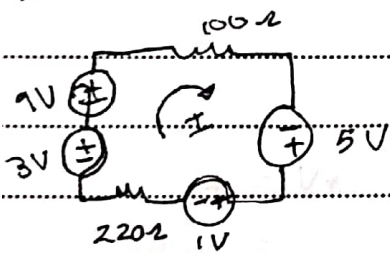


* 1] Current in Parallel



2] Current in series :- does not exist in real life

find I ?



$$-16 + 100I + 220I = 0$$

$$-16 + 320I = 0$$

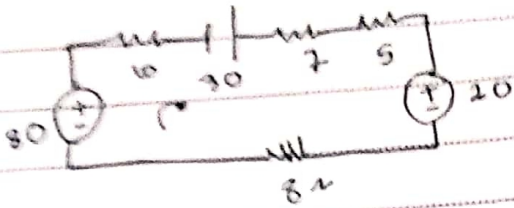
$$320I = 16$$

3] Resistors in series: $R_{eq} = R_1 + R_2 + R_3$

$$I = 50 \text{ mA}$$

Subject:

Ex: find I?



$$-50 + 10I + 30 - 7I - 5I + 20 - I \cdot 8 = 0$$

$$-80 - 30I - 30 + 20 = 0$$

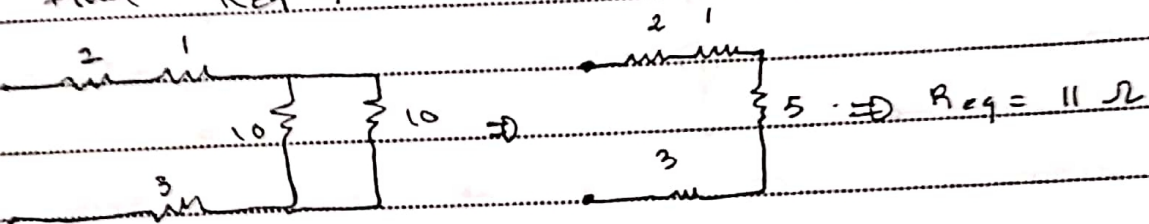
$$-30I - 90 = 0$$

$$I = -3A$$

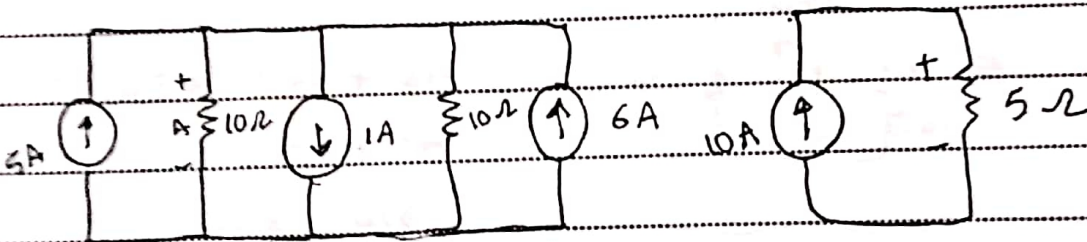
Ex] Resistor in parallel:-

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

Ex find Req?

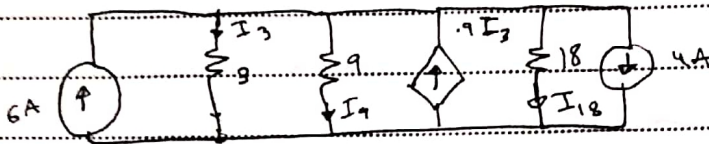
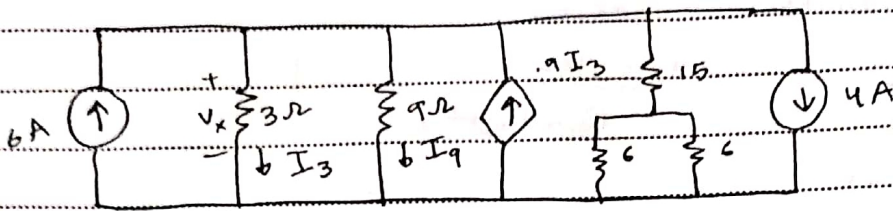


Ex: find V?



$$\begin{aligned} +V &= RI \\ &= 5 \times 10 \\ &= 50V \end{aligned}$$

Find the power in the dep. source



$$P = I U$$

$$= V_x + 0.1 I_3$$

$$= V_x + 0.1 \left(\frac{V_x}{3} \right)$$

$$= 3 (V_x)^2 \rightarrow = 30 \text{ w supplied}$$

$$\frac{0.1}{0.9} = 0.1$$

$$6 + 0.1 I_3 = I_3 + I_9 + I_{18} + 4$$

$$2 = 0.1 I_3 + I_9 + I_{18}$$

$$2 = 0.1 \left(\frac{V_x}{3} \right) + \frac{V_x}{9} + \frac{V_x}{18}$$

$$2 = \frac{0.1}{3} V_x + \frac{1}{9} V_x + \frac{1}{18} V_x$$

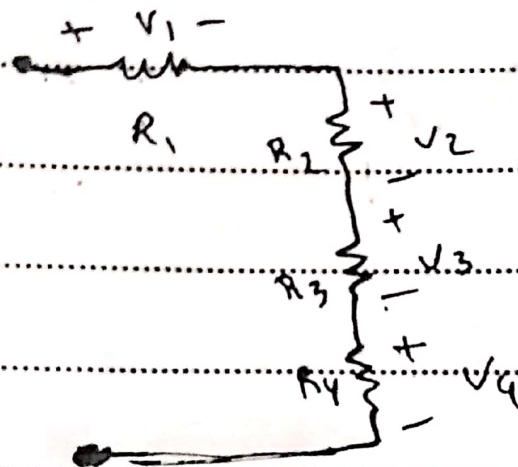
$$2 = 0.2 V_x \Rightarrow V_x = 10$$

► Subject : Circuit

Jul 1 29th 1 2022

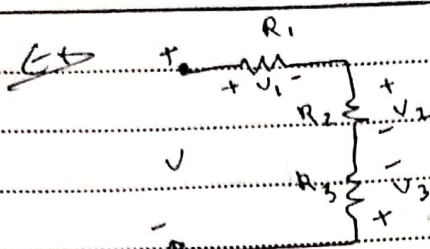
⊛ Voltage and current division :-

▢ Voltage for resistor in series :-



$$V_n = \frac{R_n}{R_1 + R_2 + R_3} V_1 \quad [V]$$

Subject :

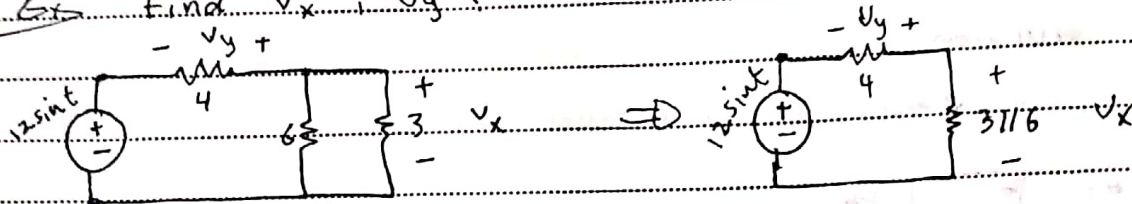


$$V_1 = \frac{R_1}{R_1 + R_2 + R_3} [V]$$

$$V_3 = \frac{R_3}{R_1 + R_2 + R_3} [-V]$$

$$V_2 = \frac{R_2}{R_1 + R_2 + R_3} [V]$$

Ex Find V_x , V_y ?



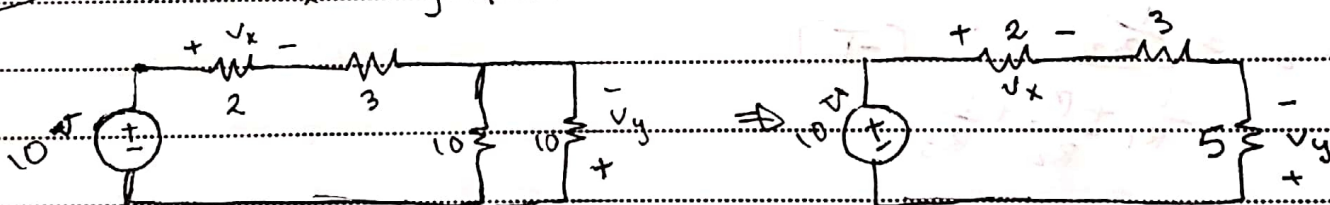
$$R_{eq} = 3 || 6 = \left[\frac{1}{3} + \frac{1}{6} \right]^{-1}$$

$$= 2$$

then $V_x = \frac{2}{2+4} [12 \sin t] = 4 \sin t \text{ V}$

$$V_y = \frac{4}{2+4} [-12 \sin t] = -8 \sin t \text{ V}$$

Ex find V_x , V_y ?



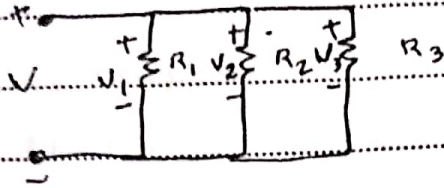
then $V_x = \frac{2}{2+3+5} [10] = 2 \text{ V}$

$$V_y = \frac{5}{2+3+5} [10] = -5 \text{ V}$$

S T A R S N O T E B O O K

Subject :

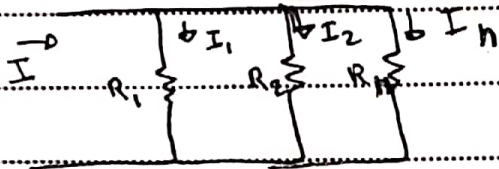
* Voltage for resistor in parallel



$$V = V_1 = V_2 = V_3$$

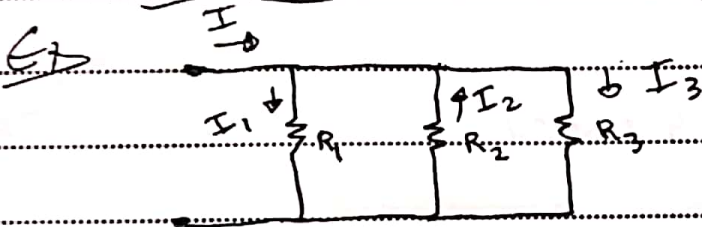
* Current division

(1) Current for resistor in parallel



$$I_n = \frac{I}{R_n} \quad [I]$$

$$\frac{I}{\frac{1}{R_1} + \frac{1}{R_2} + \dots + \frac{1}{R_n}}$$



$$I_1 = \frac{I}{R_1} \quad [I]$$

$$\frac{I}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

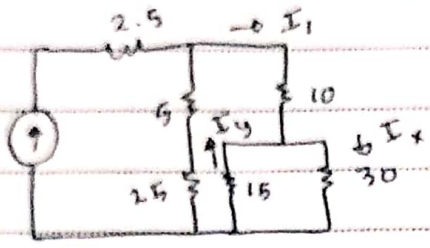
$$I_2 = \frac{I}{R_2} \quad [I]$$

$$\frac{I}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

$$I_3 = \frac{I}{R_3} \quad [I]$$

$$\frac{I}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}}$$

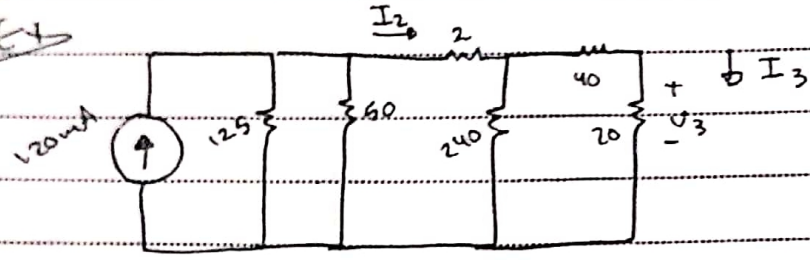
Ex 1. Let $I_1 = 100 \text{ mA}$ find I_x, I_y ?



$$I_x = \frac{\frac{1}{30}}{\frac{1}{30} + \frac{1}{15}} [100 \text{ m}] = 33.33 \text{ mA}$$

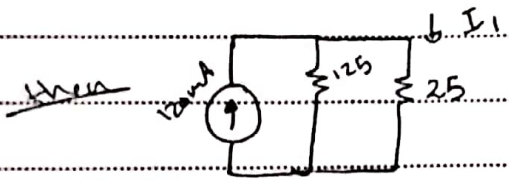
$$I_y = \frac{\frac{1}{15}}{\frac{1}{30} + \frac{1}{15}} [100 \text{ m}] = 76.66 \text{ mA}$$

Ex 2



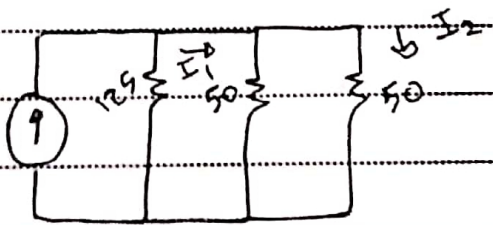
$$R_{eq1} = \left[\left[\left[40 + 20 \right] \parallel 240 \right] + 2 \right] \parallel 50$$

$$= 25$$



$$I_1 = \frac{\frac{1}{25}}{\frac{1}{125} + \frac{1}{25}} [120 \text{ m}] = 100 \text{ mA}$$

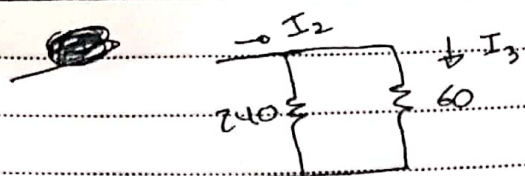
$$R_{eq2} = \left[\left[40 + 20 \right] \parallel 240 \right] + 2 = 50$$



$$I_2 = \frac{\frac{1}{50}}{\frac{1}{50} + \frac{1}{50}} [I_1] = 50 \text{ mA}$$

OR

$$I_2 = \frac{\frac{1}{50}}{\frac{1}{50} + \frac{1}{50} + \frac{1}{125}} [120 \text{ m}] = 50 \text{ mA}$$

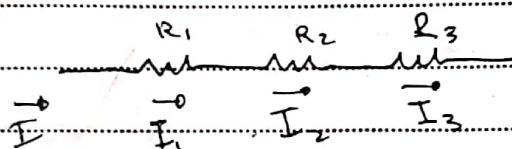


$$+V_3 = 20 I_3$$

$$I_3 = \frac{1}{\frac{1}{60} + \frac{1}{240}} \quad (50 \text{ mA})$$

$$\Delta +V_3 = 20 + I_3 = 0.8 \text{ V}$$

* Current for Resistor in Series :-



$$I = I_1 = I_2 = I_3$$

CH 4 : nodal and mesh analysis :

solving linear system of equation :

1] $a_1 x_1 = b_1$

2] $a_1 x_1 + a_2 x_2 = b_1$

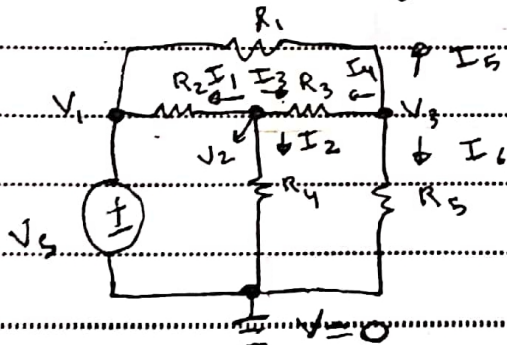
$$a_3 x_1 + a_4 x_2 = b_2$$

3] $a_1 x_1 + a_2 x_2 = b_1$

$$a_4 x_1 + a_5 x_2 + a_6 x_3 = b_2$$

$$a_7 x_1 + a_8 x_2 + a_9 x_3 = b_3$$

① nodal analysis :



① assume reference voltage a below

② assume voltage on each node

③ assume current entering each Resistor

④ Apply KCL

at nodal 1 :

$$+V_1 - 0 = V_s \Rightarrow V_1 = V_s$$

at nodal 2 :

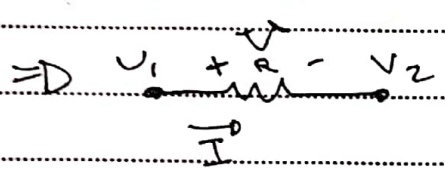
$$I_1 + I_2 + I_3 = 0$$

$$\frac{V_2 - V_1}{R_2} + \frac{V_2 - 0}{R_4} + \frac{V_2 - V_3}{R_3} = 0 \quad \text{--- (2)}$$

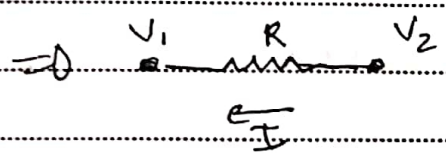
at nodal 3 :

$$I_4 + I_5 + I_6 = 0$$

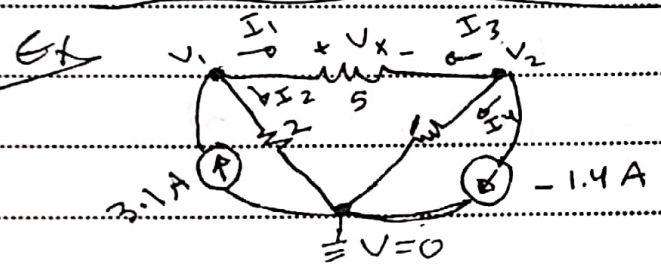
$$\frac{V_3 - V_2}{R_3} + \frac{V_3 - V_1}{R_1} + \frac{V_3 - 0}{R_3} = 0 \quad \text{--- (3)}$$



$$I = \frac{V}{R} = \frac{+V_1 - V_2}{R}$$



$$I = \frac{V_2 - V_1}{R}$$



at nodal 2 :

$$I_3 + I_4 + (-1.4) = 0$$

$$\frac{V_2 - V_1}{5} + \frac{V_2 - 0}{1} + (-1.4) = 0 \quad \text{--- (1)}$$

at nodal 1 :

$$V_1 = 5V \quad V_2 = 2V$$

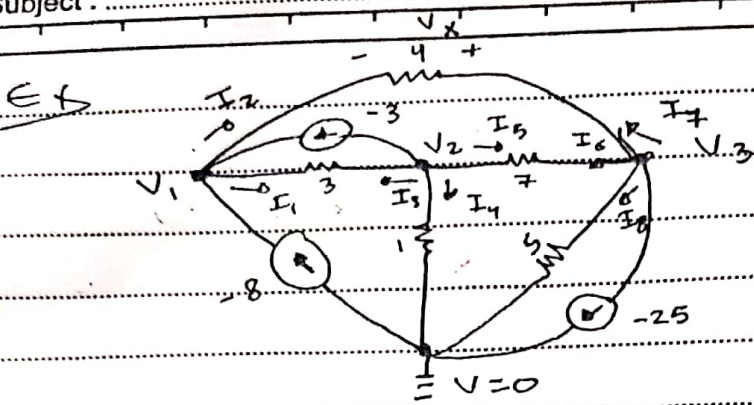
$$3.1 = I_2 + I_3$$

$$I_1 = \frac{V_1 - V_2}{5} = \frac{5 - 2}{5} = \frac{3}{5} \text{ A}$$

$$3.1 = \frac{V_1 - V_2}{5} + \frac{V_1 - 0}{2} \quad \text{--- (1)}$$

$$V_x = +V_1 - V_2 = 5 - 2 = 3V$$

Subject: _____



at nodal 1:

$$-8 + (-3) = I_1 + I_2$$

$$-11 = \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{4} \quad \text{--- (1)}$$

at nodal 2:

$$-3 + I_3 + I_4 + I_6 = 0$$

$$-3 + \frac{V_2 - V_1}{3} + \frac{V_2 - 0}{1} + \frac{V_2 - V_3}{7} = 0 \quad \text{--- (2)}$$

at nodal 3:

$$-25 + I_6 + I_7 + I_8 = 0$$

$$-25 + \frac{V_3 - V_2}{7} + \frac{V_2 - V_1}{4} + \frac{V_3 - 0}{5} = 0 \quad \text{--- (3)}$$

$$V_1 = 5.412 \text{ V}$$

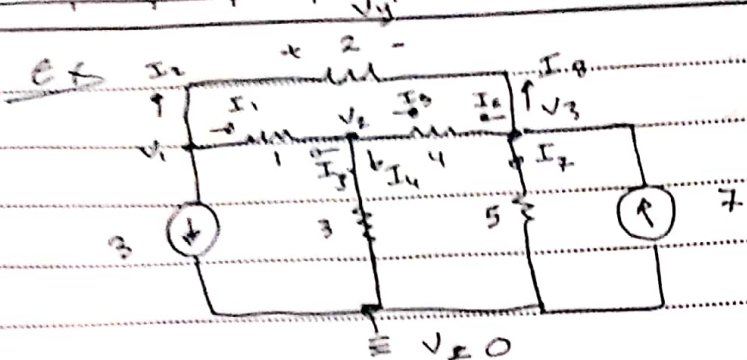
$$V_2 = 7.736 \text{ V}$$

$$V_3 = 46.32 \text{ V}$$

$$I_5 = \frac{V_2 - V_3}{7} = \boxed{} \text{ A}$$

$$V_x = -V_1 + V_3 = \boxed{} \text{ V}$$

Subject:



at nodal 1:

$$3 + I_1 + I_2 = 0$$

$$3 + \frac{V_1 - V_2}{1} + \frac{V_1 - V_3}{2} = 0 \quad \text{--- (1)}$$

at nodal 2:

$$I_3 + I_4 + I_5 = 0$$

$$\frac{V_2 - V_1}{3} + \frac{V_2 - 0}{5} + \frac{V_2 - V_3}{4} = 0 \quad \text{--- (2)}$$

at nodal 3:

$$7 = I_6 + I_7 + I_8$$

$$7 = \frac{V_3 - V_2}{4} + \frac{V_2 - 0}{5} + \frac{V_3 - V_1}{2} \quad \text{--- (3)}$$

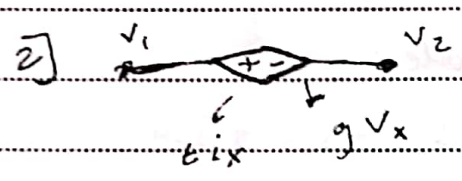
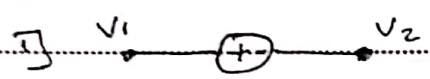
$$V_1 = 5.235 \text{ V}$$

$$V_2 = 5.12 \text{ V}$$

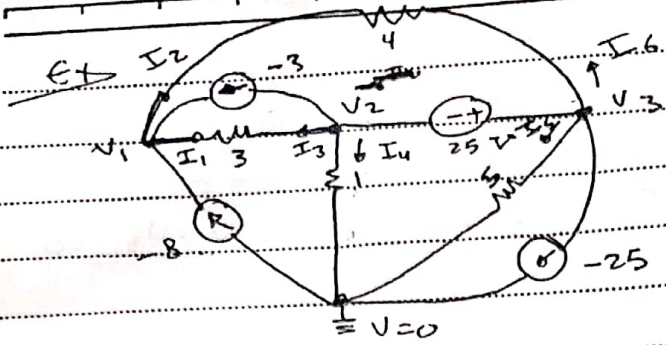
$$V_3 = 11.47 \text{ V}$$

$$I_6 = \frac{V_3 - V_2}{4} \quad V_y = +V_1 - V_3$$

⊕ super nodal



Subject :



at nodal 1 :-

$$-8 - 3 = I_1 + I_2$$

$$-11 = \frac{V_1 - V_2}{3} + \frac{V_1 - V_3}{4} \quad \text{--- (1)}$$

at super node :-

~~$$-3 + I_3 + I_4 + I_5 + I_6 + (-25) = 0$$~~

$$-3 + \frac{V_2 - V_1}{3} + \frac{V_2 - 0}{1} + \frac{V_3 - 0}{5} + \frac{V_3 - V_1}{4} - 25 = 0$$

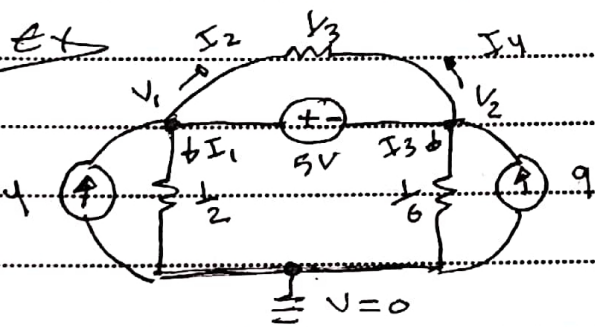
from super node

$$-V_2 + V_3 = 22 \quad \text{--- (2)}$$

$$V_1 = 1.071 \text{ V}$$

$$V_2 =$$

$$V_3 =$$



at super node :-

$$4 + 9 = I_1 + I_2 + I_3 + I_4$$

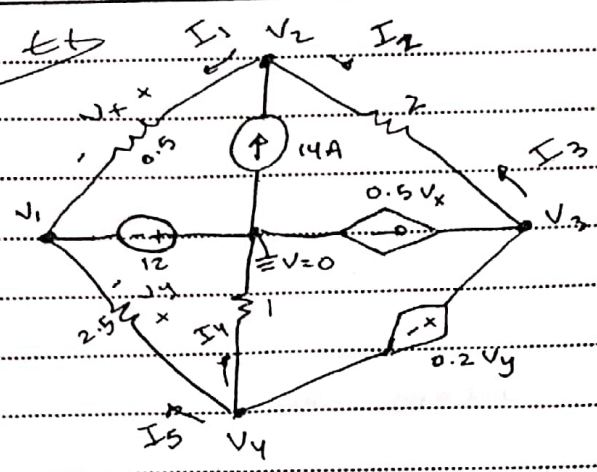
$$13 = \frac{V_1 - 0}{1/2} + \frac{V_1 - V_2}{1/3} + \frac{V_2 - 0}{1/6} + \frac{V_2 - V_1}{1/3}$$

$$+ V_1 - V_2 = 5 \quad \text{--- (2)}$$

$$V_1 = 5.375 \text{ V}$$

$$V_2 = 375 \text{ mV}$$

S T A R S N O T E B O O K



at super node 1

$$-V_1 + 0 = 12$$

$$V_1 = -12 \text{ V} \quad \dots (1)$$

at node 2:

$$14 = I_1 + I_2$$

$$14 = \frac{V_2 - V_1}{0.5} + \frac{V_2 - V_3}{2} \quad \dots (2)$$

at super node 2:

$$I_3 + I_4 + I_5 = 0.5 V_x$$

$$\frac{V_3 - V_2}{2} + \frac{V_4 - 0}{1} + \frac{V_4 - V_1}{2.5} = 0.5 [-V_1 + V_2] \quad \dots (3)$$

$$\Rightarrow -V_4 + V_3 = 0.2 V_y$$

$$-V_4 + V_3 = 0.2 [-V_1 + V_4] \quad \dots (4)$$

$$V_1 = -12 \text{ V}$$

$$V_3 = 0 \text{ V}$$

$$V_x = -V_1 + V_2 = 12 - 4 = 8 \text{ V}$$

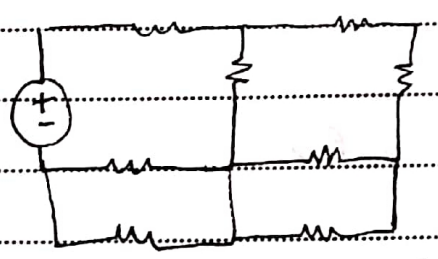
$$V_2 = -4 \text{ V}$$

$$V_4 = -2 \text{ V}$$

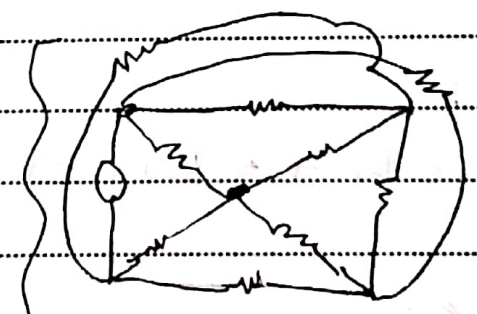
$$V_y = -V_1 + V_4 = 12 - 2 = 10 \text{ V}$$

mesh analysis:

* mesh analysis is applicable only to planar circuit:

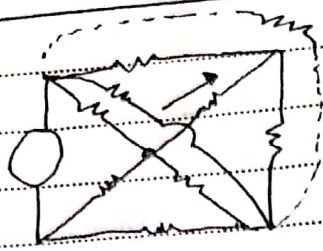


Planar circuit



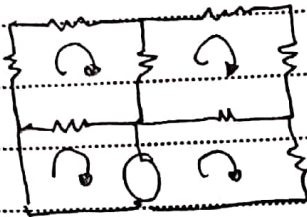
non-planar circuit

Subject: _____



Planar

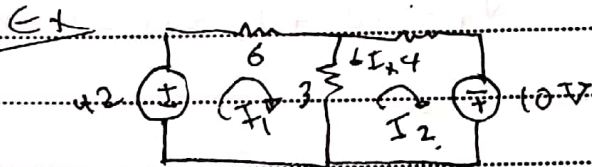
* mesh = loop that does not contain any other loop



* mesh analysis

1) assume current on each loop

2) apply KVL



$$I_x = I_1 - I_2$$

at loop 1:

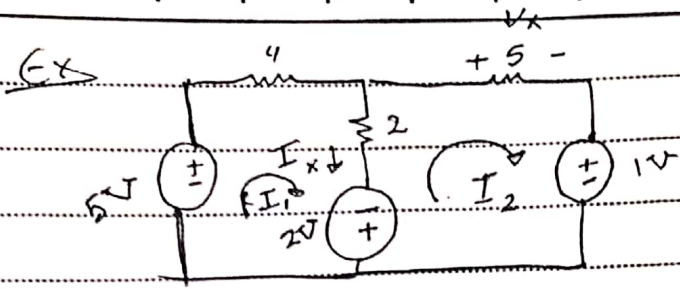
$$-42 + 6I_1 + 3[I_1 - I_2] = 0 \quad \text{--- (1)}$$

at loop 2:

$$3[I_2 - I_1] + 4I_2 - 10 = 0 \quad \text{--- (2)}$$

$$I_1 = 6A \quad I_2 = 4A$$

Subject :



at loop 1 :-

$$-5 + 4I_1 + 2[I_1 - I_2] - 2 = 0 \quad \text{--- (1)}$$

at loop 2 :-

$$2 + 2[I_2 - I_1] + 5I_2 + 1 = 0 \quad \text{--- (2)}$$

$$I_1 = 1.132 \text{ A}$$

$$I_2 = -0.1053 \text{ A}$$

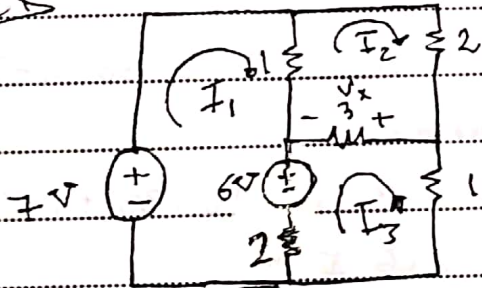
$$+V_x = 5I_2 = \underline{\quad} \text{ V}$$

$$P_{2V} = VI = 2(I_x)$$

$$= 2[+I_1 - I_2]$$

$$= \underline{\quad} \text{ W supplied}$$

Ex



at loop 1:-

$$-7 + 1[I_1 - I_2] + 6 + 2[I_1 - I_3] = 0$$

at loop 2:-

$$1[I_2 - I_1] + 2I_2 + 3[I_2 - I_3] = 0$$

at loop 3:-

$$2[I_3 - I_1] - 6 + 3[I_3 - I_2] + 1I_3 = 0$$

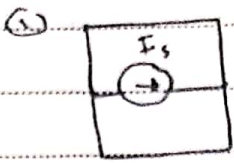
$$I_1 = 3 \text{ A} \quad I_2 = 2 \text{ A} \quad I_3 = 3 \text{ A}$$

$$I_x = I_3 - I_2 = 3 - 2 = 1 \text{ A}$$

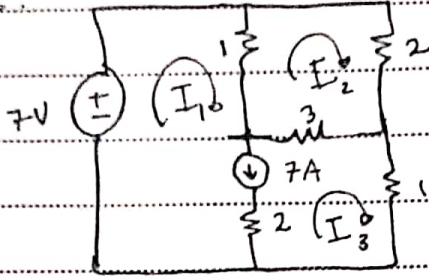
$$-V_x = 3I_x = 3(1) \Rightarrow V_x = -3 \text{ V}$$

► Subject: _____

⊗ Super mesh:



Ex:



at super mesh 1

$$-7 + 1[I_1 - I_2] + 3[I_3 - I_2] + 1I_3 = 0$$

at loop 2:

$$1[I_2 - I_1] + 2[I_2] + 3[I_2 - I_3] = 0 \quad \dots (2)$$

⇓

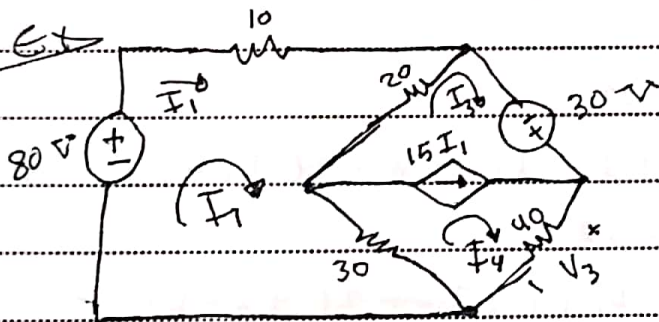
$$+I_1 - I_3 = 7 \quad \dots (3)$$

$$I_1 = 9A \quad I_2 = 2.5A \quad I_3 = 2A$$

$$I_y = +I_1 - I_2 = 9 - 2.5 = 6.5A$$

$$+V_y = 1[I_y] = 6.5V$$

Ex:



at loop 1:

$$-80 + 10I_1 + 20[I_1 - I_3] + 30[I_1 - I_4] = 0$$

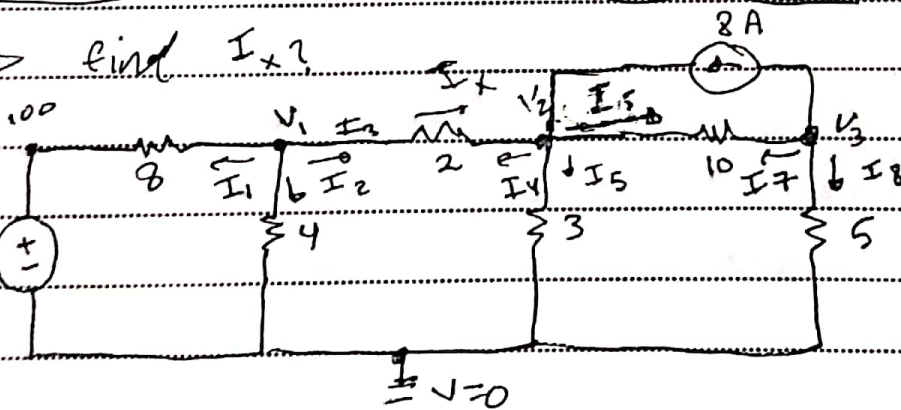
at super mesh 1

$$20[I_3 - I_1] - 30 + 40I_4 + 30[I_4 - I_1] = 0$$

$$+I_4 - I_3 = 15I_1$$

$$I_1 = 0.584A \quad I_3 = -6.15A \quad I_4 = 2.605A \quad V_3 = 104.2V$$

Ex find I_x ?



using nodal analysis:

at nodal 1

$$I_1 + I_2 + I_3 = 0$$

$$= \frac{V_1 - 100}{8} + \frac{V_1 - 0}{4} + \frac{V_1 - V_2}{2}$$

$$= 0$$

at node 2:

$$8 = I_4 + I_5 + I_6$$

$$8 = \frac{V_2 - V_1}{2} + \frac{V_2 - 0}{3} + \frac{V_2 - V_3}{10} \quad \dots (2)$$

at node 3:

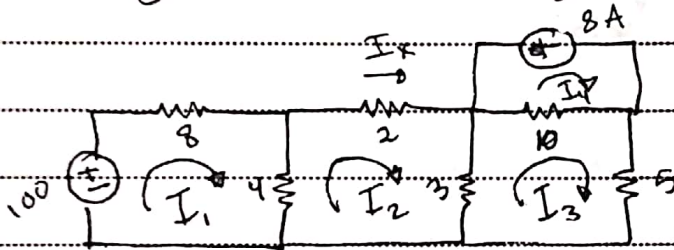
$$8 + I_7 + I_8 = 0$$

$$8 + \frac{V_3 - V_2}{10} + \frac{V_3 - 0}{5} = 0 \quad \dots (3)$$

$$V_1 = 25.89 \text{ V} \quad V_2 = 20.31 \text{ V} \quad V_3 = \text{---} \text{ V}$$

$$I_x = \frac{V_1 - V_2}{2} = 2.79 \text{ A}$$

Using mesh analysis:



$$I_4 = -8 \text{ A}$$

at loop 1:

$$-100 + 8I_1 + 4(I_1 - I_2) = 0$$

at loop 2:

$$4(I_2 - I_1) + 2I_2 + 3(I_2 - I_3) = 0$$

at loop 3:

$$3(I_3 - I_2) + 10(I_3 + 8) + 5I_3 = 0$$

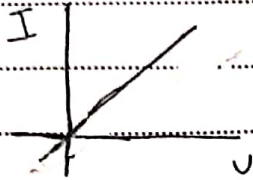
So $I_2 = 2.79 \text{ A}$ $I_1 = \text{---} \text{ A}$ $I_3 = \text{---} \text{ A}$

$$I_x = I_2 = 2.79 \text{ A}$$

CH 5:

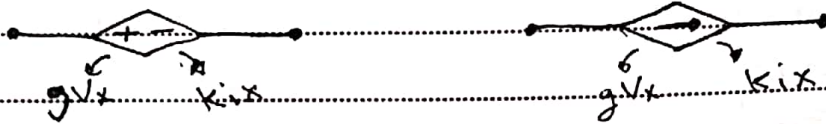
linear element and linear circuit

1) linear element: is passive element that has linear voltage-current relationship.



2) linear dep. source:

is dep. current or voltage whose output current or voltage is proportional to the first power of voltage or current.



3) linear circuit: composed

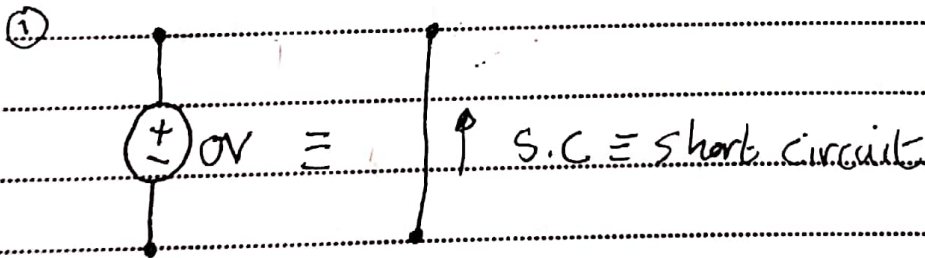
1) indep. source

2) linear dep. source

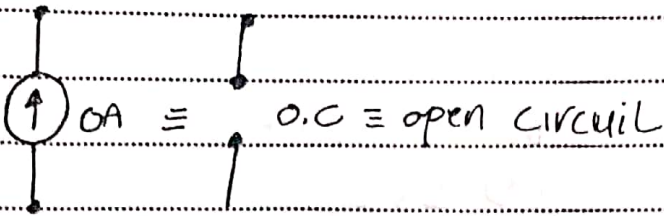
3) linear element

*) super position:

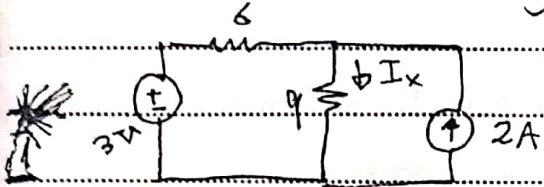
the response in a linear circuit having more than one indep. source can be obtained by adding the responses caused by the separate indep. source acting alone.



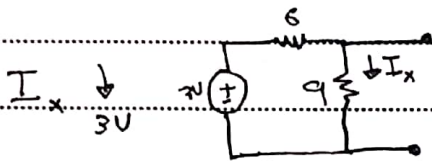
②



Find I_x using super position



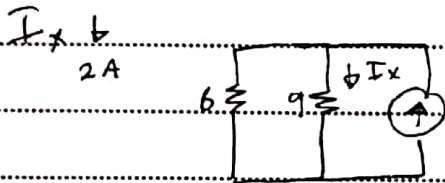
$$I_x = I_x \downarrow_{3V} + I_x \downarrow_{2A}$$



using KVL:

$$-3 + 6I_x + 9I_x = 0$$

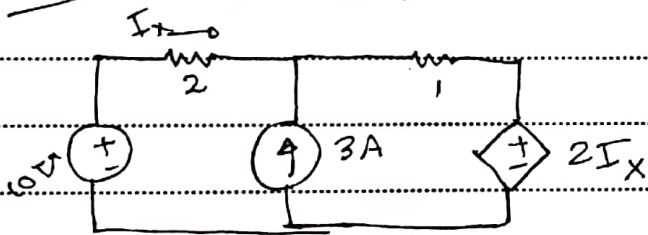
$$15I_x = 3 \Rightarrow I_x = .2A$$



$$I_x = \frac{1}{\frac{1}{9} + \frac{1}{6}} (2) = .8A$$

$$I_{x \text{ total}} = 2 + .8 = 2.8A$$

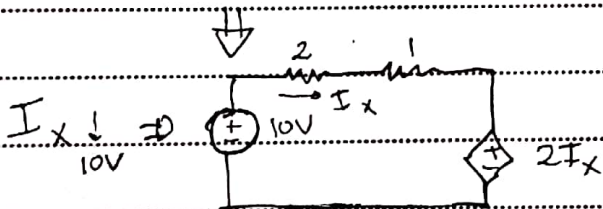
Find I_x ?



$$I_x = I_x \downarrow_{10V} + I_x \downarrow_{3A}$$

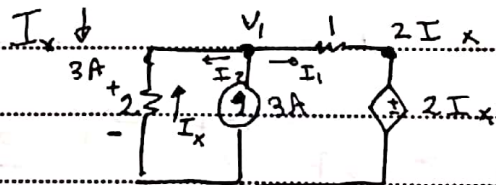
$$= 2 + (-.6)$$

$$\Rightarrow I_x = 1.4A$$



Using KVL

$$-10 + 2I_x + 1I_x + 2I_x = 0 \Rightarrow I_x = 2A$$

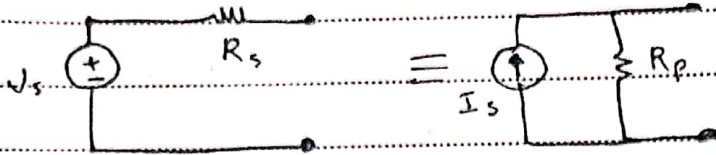


using nodal analysis:

$$3 = I_1 + I_2 \Rightarrow 3 = \frac{V_1 - 2I_x}{1} + \frac{V_1 - 0}{2}$$

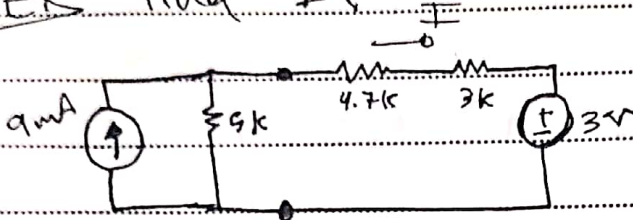
$$-V_1 = I_x(2) \Rightarrow V_1 = -2I_x \Rightarrow I_x = -.6$$

⊕ Source Transformation :



- ① $R_s = R_p$
- ② $V_s = I_s R_p$
- ③ $I_s = \frac{V_s}{R_s}$

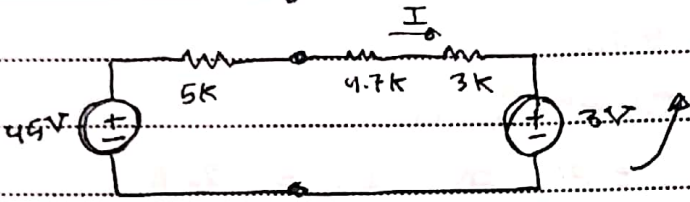
Ex find I?



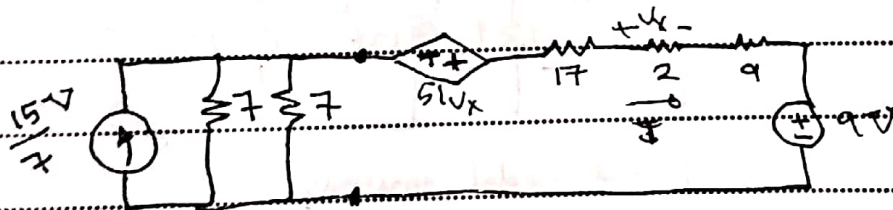
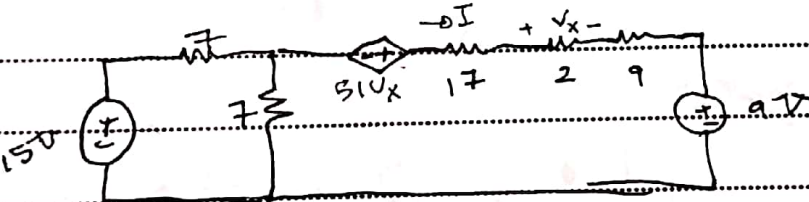
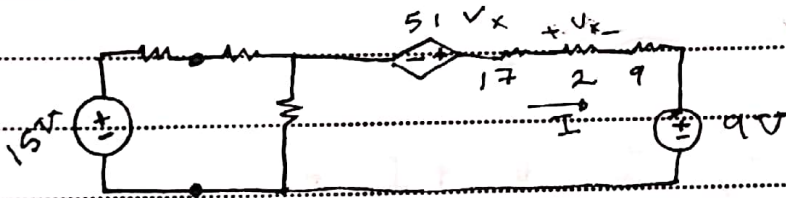
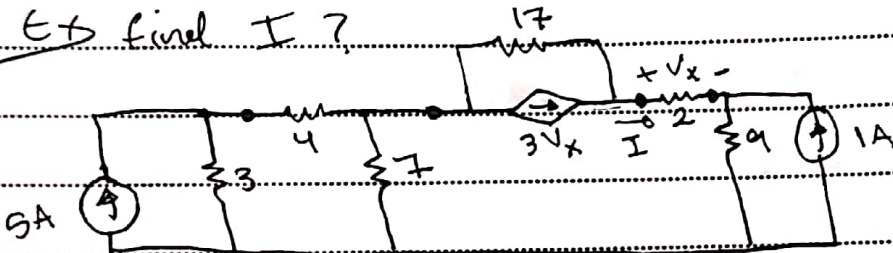
Using KVL :

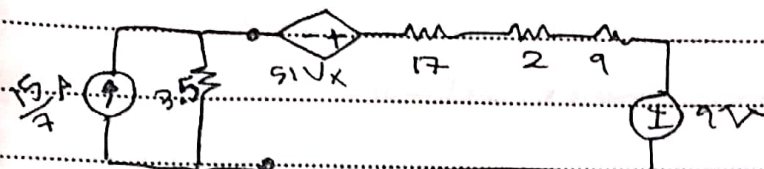
$$-45 + 5kI + 4.7kI + 3kI + 3 = 0$$

$$I = 3.307 \text{ mA}$$



Ex find I?





Now using KVL:

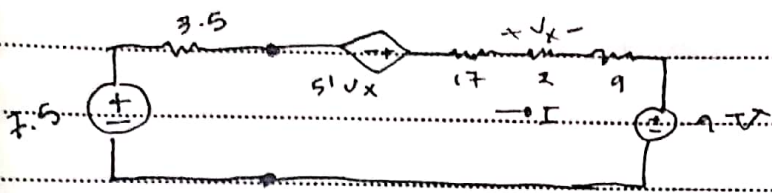
$$-7.5 + 3.5I - 51V_x + 17I + 2I + 9I + 9I = 0$$

$$17V_x = 2I$$

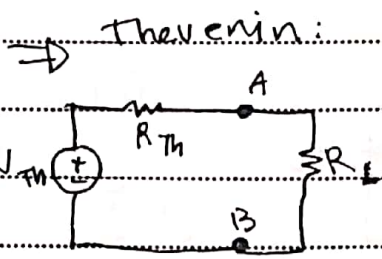
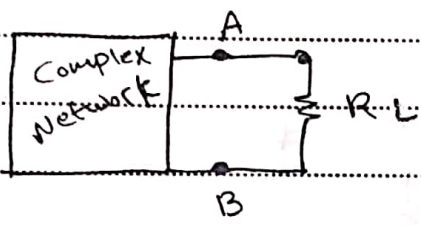
then

$$I = 21.28 \text{ mA}$$

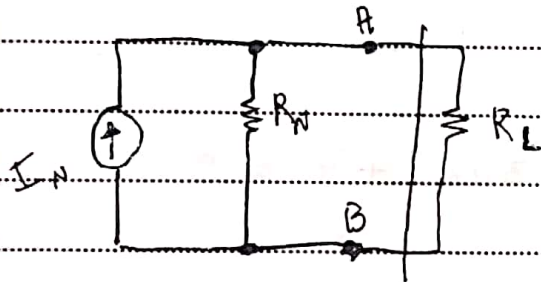
$$V_x = 42.4 \text{ mV}$$



Thevenin + norton equivalent:



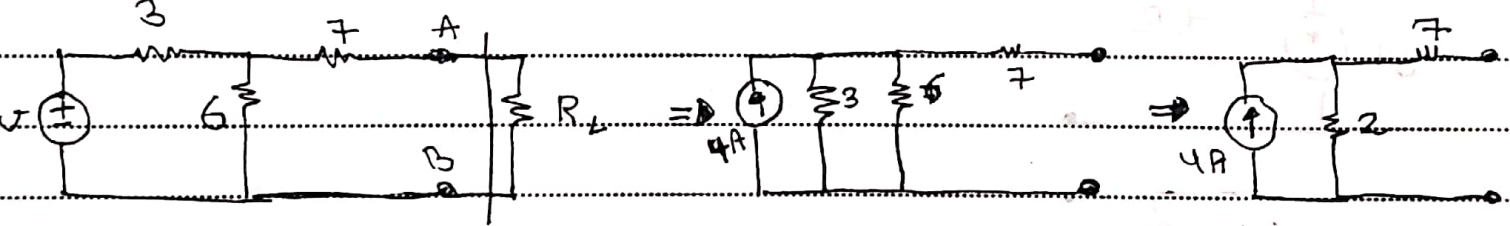
norton equivalent:



using source transformation:

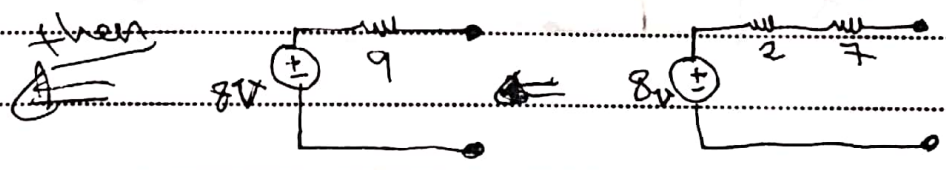
- ① $R_{th} = R_N$
- ② $V_{th} = I_N R_N$
- ③ $I_N = \frac{V_{th}}{R_{th}}$

Find Thevenin :-



$$R_{th} = 9 \Omega$$

$$V_{th} = 8 \text{ V}$$



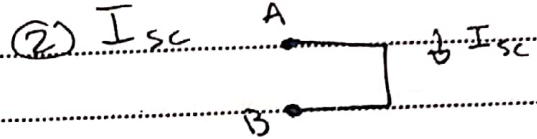
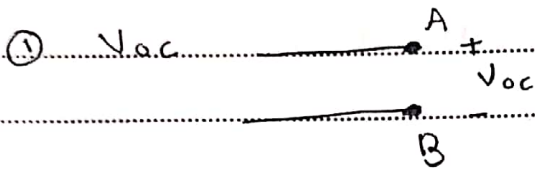
* Case one &

only Indep. source in the complex network

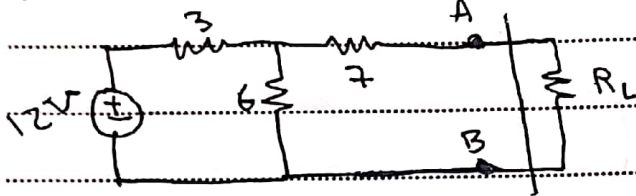
① $V_{th} = V_{oc}$

② $R_{th} = R_{eq}$ after killing all source

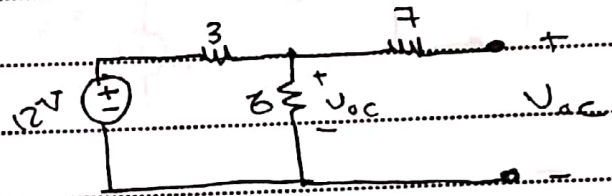
③ $I_N = I_{sc}$



Ex Find Thevenin:

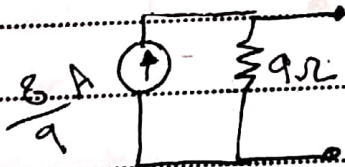
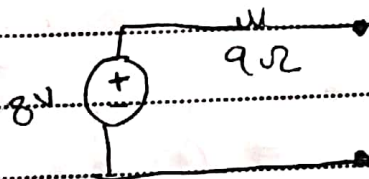
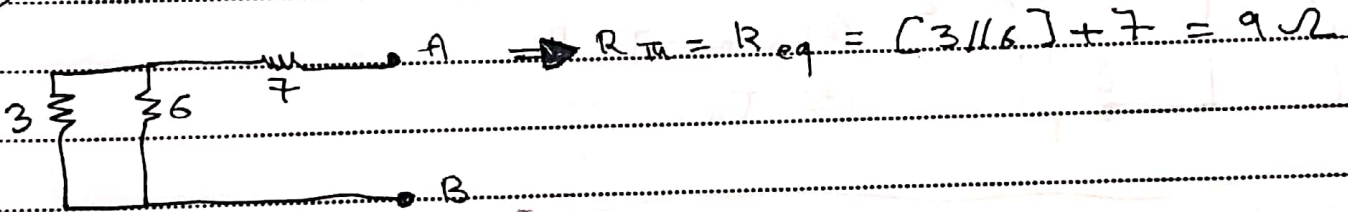


① $V_{th} = V_{oc}$

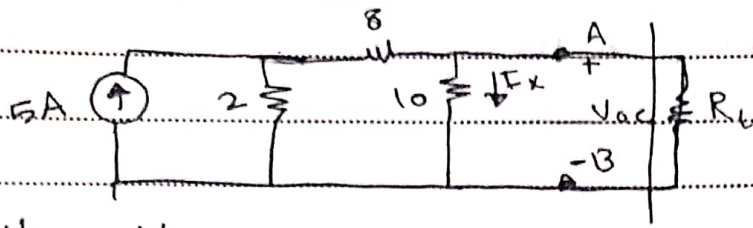


then $V_{oc} = \frac{6}{6+3} [12] = 8V$

② $R_{th} =$



Ex: find thevenin + norton:

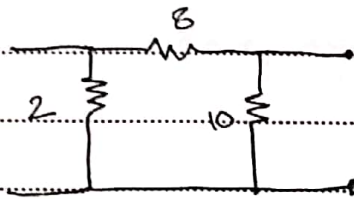


① $V_{Th} = V_{oc}$

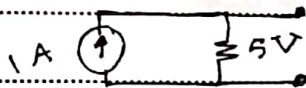
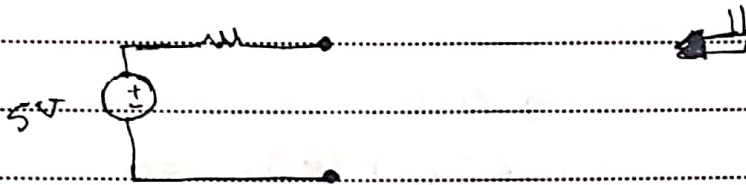
$V_{oc} = 10 I_x$

$= 10 \left[\frac{\frac{1}{10}}{\frac{1}{10} + \frac{1}{2}} \right] [5] = 5V$

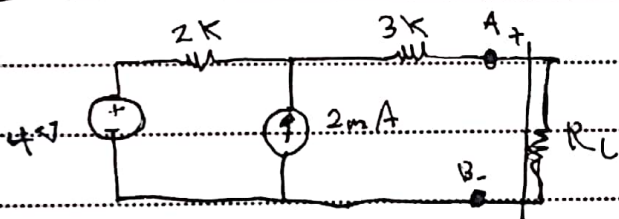
② $R_{Th} = R_{eq}$ After killing all sources



$R_{eq} = (2+8) \parallel 10 = 5 \Omega$



Ex: find thevenin:

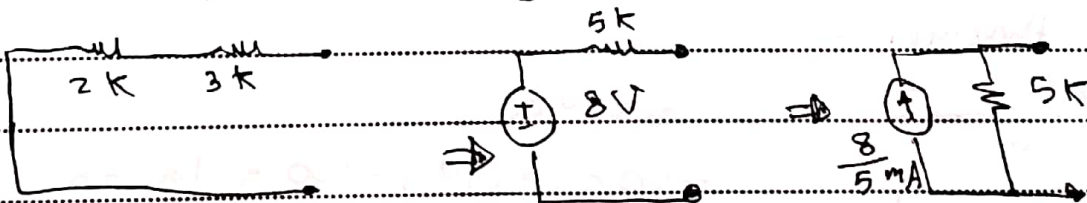


1] $V_{Th} = V_{oc}$

$-4 + 2K(-2m) + 3K(0) + V_{oc} = 0$

$V_{oc} = 8V$

2] $R_{Th} = R_{eq}$ after killing all sources



$R_{eq} = 2K + 3K = 5K$

9) Case two:

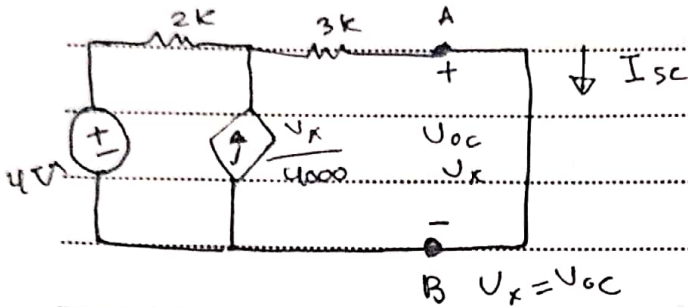
only dep + Indep sources in the complex network

1] $V_{th} = V_{oc}$

2] $I_{th} = I_{sc}$

3] $R_{th} = \frac{V_{oc}}{I_{sc}}$

Ex find thvenin:



1] $V_{th} = V_{oc}$

$$-4 + 2k \left(\frac{-V_x}{4000} \right) + 3k(0) + V_{oc} = 0$$

$V_{oc} = 8V$

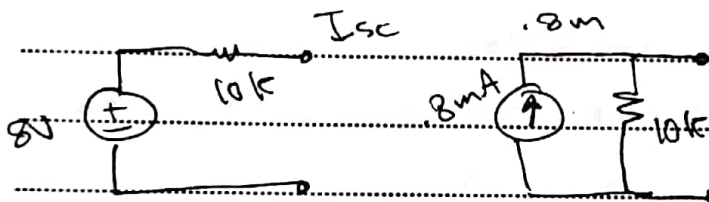
2] $I_{th} = I_{sc}$

using KVL:

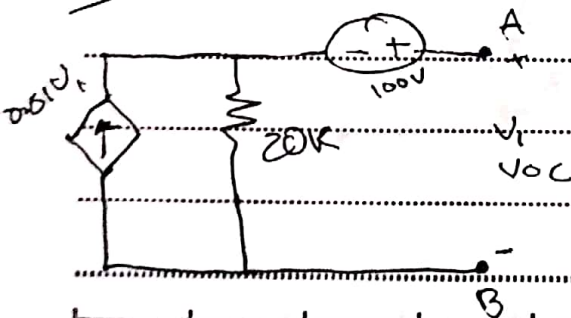
$$-4 + 2k I_{sc} + 3k I_{sc} = 0$$

$I_{sc} = 0.8 \text{ mA}$

3] $R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{8}{0.8} = 10 \text{ k}\Omega$



Ex find thvenin r-



$V_{th} = V_{oc}$

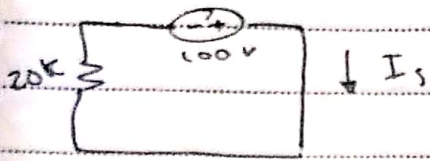
$$-20k(0.01V_1) - 100 + V_{oc} = 0$$

$V_{oc} = V_1$

$\therefore V_{oc} = -0.503 \text{ V}$

S T A R S N O T E B O O K

2] $I_w = I_{sc}$

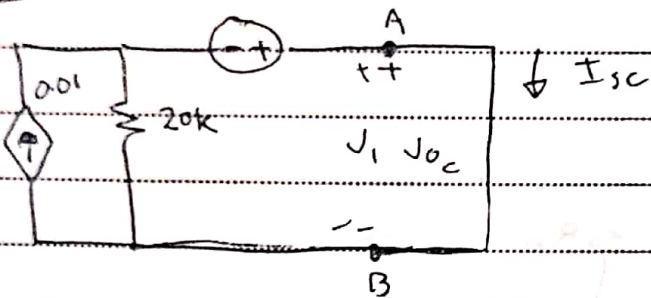


$$20k I_{sc} - 100 = 0$$

$$I_{sc} = 5mA$$

3] $R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{-0.503}{5m} = -100.6 \Omega$ unstable circuit

→ find Thevenin :



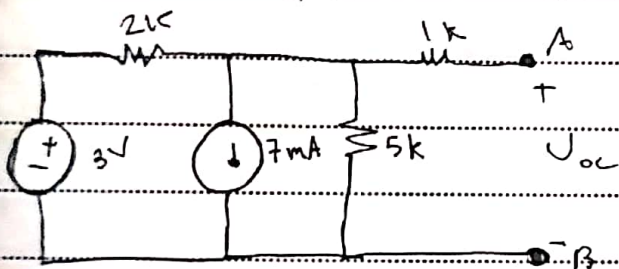
① $V_{th} = V_{oc}$

$$-20k(0.01V_1) - 100 + V_{oc} = 0$$

$$V_{oc} = V_1$$

$$\therefore V_{oc} = -0.503V$$

→ find thevenin:



① $V_{th} = V_{oc}$

V_{oc} ⇒ using super position

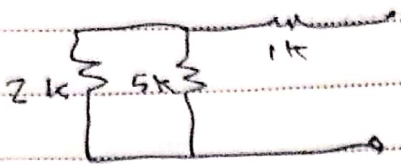
$$V_{oc} = V_{oc} \downarrow_{3V} + V_{oc} \downarrow_{7mA}$$

$$= \left[\frac{5k}{5k+2k} \right] (3) + \frac{1}{\frac{1}{5k} + \frac{1}{2k}} (-7m)(5k)$$

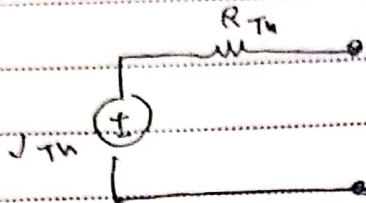
$$= \underline{\hspace{2cm}} V$$

► Subject :

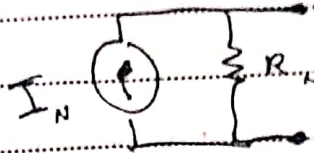
② $R_{Th} = R_{eq}$ after killing all sources



$$R_{eq} = (5k \parallel 2k) + 1k = R$$



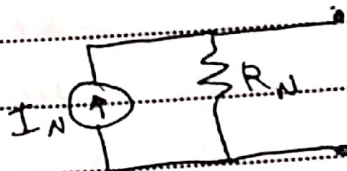
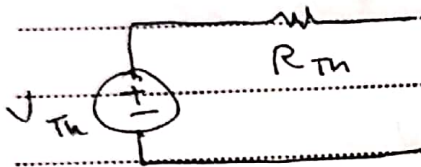
\Rightarrow



Thevenin + norton :

Thevenin :

norton :



Using source transformation

1] $R_{Th} = R_N$

2] $V_{Th} = I_N R_N$

3] $I_N = \frac{V_{Th}}{R_{Th}}$

① case one :

only 1 indep. source in a complex network

1] $V_{Th} = V_{oc}$

2] $R_{Th} = R_{eq}$ After killing all sources

3] $I_N = I_{sc}$

2) case two :

dep. + indep. source in complex network

1] $V_{th} = V_{oc}$

2] $I_{N} = I_{sc}$

3] $R_{th} = R_{N} = \frac{V_{oc}}{I_{sc}}$

3) case three :

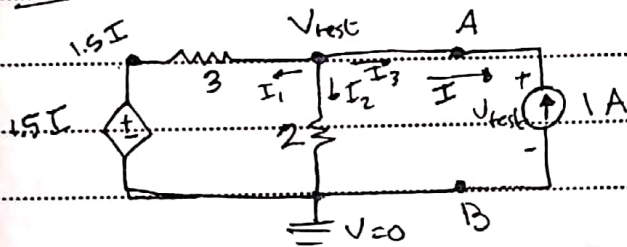
dep. source only in complex network

1) assume $I_{test} = 1A$ or $V_{test} = 1V$

2) find V_{test} or I_{test}

$R_{th} = \frac{V_{test}}{I_{test}}$ or $\frac{V_{test}}{I_{test}}$

Ex : find thevenin :



assume current 1A

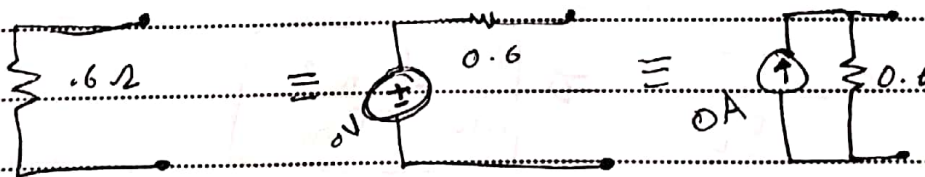
$I = -1A$

using nodal analysis :

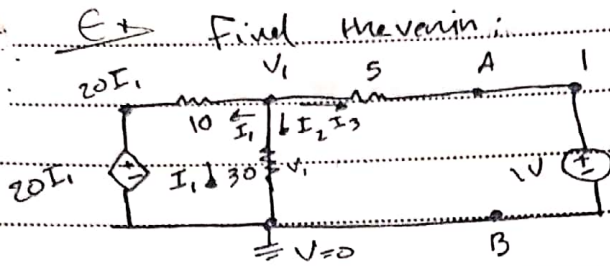
$I_1 + I_2 + I_3 = 0$

$\frac{V_{test} - 1.5I}{3} + \frac{V_{test} - 0}{2} + (-1) = 0 \Rightarrow V_{test} = 0.6V$

$R_{th} = \frac{V_{test}}{I_{test}} = \frac{0.6}{1} = 0.6 \Omega$



Subject :



$$R_{Th} = \frac{V_{test}}{I_{test}} = \frac{1}{I_{test}}$$

apa itu I_{test} ✓

using nodal analysis :

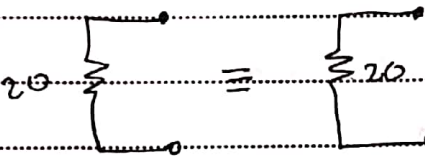
$$I_1 + I_2 + I_3 = 0$$

$$V_1 - 20I_1 + \frac{V_1 - 0}{30} + \frac{V_1 - 1}{5} = 0 \quad \text{--- (1)} \quad +V_1 = 30I_1 \quad \text{--- (2)}$$

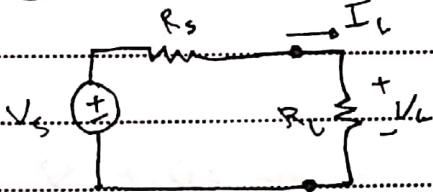
$$\therefore V_1 = 0.75 \text{ V}$$

$$I_{test} = -I_3 = -\left(\frac{V_1 - 1}{5}\right) = 50 \text{ mA}$$

$$R_{Th} = \frac{1}{50 \text{ mA}} = 20 \Omega$$



⊛ Max Power transfer to load :-



$$P_L = I_L^2 R_L = \left[\frac{V_s}{R_s + R_L}\right]^2 R_L$$

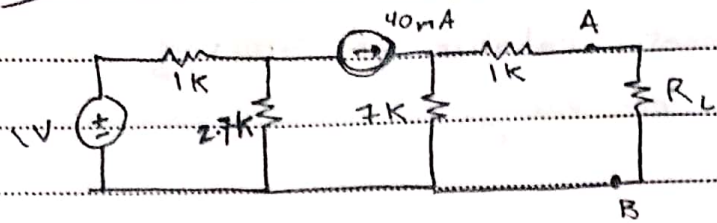
$$\frac{dP_L}{dR_L} = 0 \Rightarrow R_s = R_L$$

$$\Rightarrow P_{RL} = \left[\frac{V_s}{2R_L}\right]^2 R_L$$

$$= \frac{V_s^2}{4R_L^2} * R_L = \frac{V_s^2}{4R_L}$$

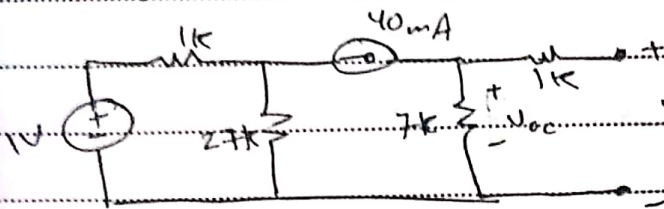
$$\Rightarrow P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

Ex find Max power transfer to load.



$$P_{max} = \frac{V_{th}^2}{4R_{th}}$$

$$V_{th} = V_{oc}$$



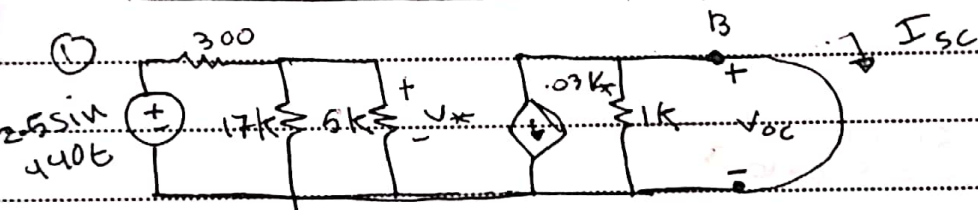
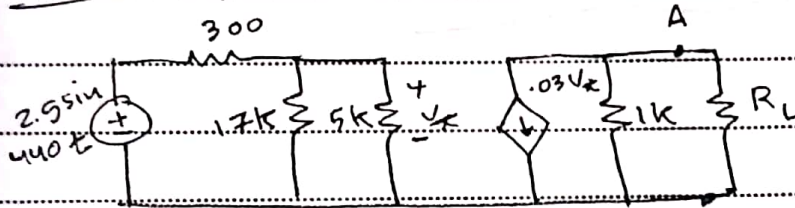
$$+V_{oc} = [7k][40mA] = 280V$$

$R_{th} = R_{eq}$ After killing All --

$$R_{eq} = 1k + 7k = 8k \Omega$$

$$\therefore P_{max} = \frac{(280)^2}{4 \times [8k]} = 2.45W$$

Ex: find Power max transfer to load.



$$-V_{oc} = 1k[0.03V_x]$$

$$V_{oc} = -30V_x$$

$$V_x = \frac{5k \parallel 17k}{300 + 5k \parallel 17k} [2.5 \sin 440t \text{ mV}]$$

$$\textcircled{2} I_N = I_{sc} = -0.03V_x$$

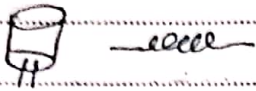
$$\textcircled{3} R_{th} = \frac{V_{oc}}{I_{sc}} = \frac{-30V_x}{-0.03V_x} = 1k$$

$$P_{max} = 1.211 \sin^2(440t) \text{ MW}$$

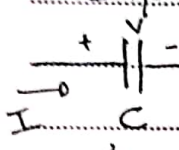
$$P_{max} = \frac{V_{th}^2}{4R_{th}} = \frac{[-30V_x]^2}{4[1k]}$$

CH 7 : Capacitors and Inductors

Both are passive element that capable of storing and delivering finite amount of energy.



Capacitor :



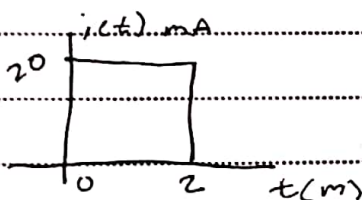
Unit of capacitor = Farad 'f'

$$i(t) = C \frac{dv(t)}{dt}$$

$$\int_{t_0}^t i(t) dt = \int_{t_0}^t C dv(t)$$

$$\frac{1}{C} \int_{t_0}^t i(t) dt = v(t) - v(t_0)$$

Ex find $v(t)$ on the capacitor



$$C = 5 \mu\text{f}$$

$$v(-\infty) = 0$$

$$i(t) = \begin{cases} 20 \times 10^{-3} & -\infty \leq t \leq 0 \\ 20 \times 10^{-3} & 0 \leq t \leq 2 \times 10^{-3} \\ 0 & 2 \times 10^{-3} \leq t \leq \infty \end{cases}$$

$$1) v(t) - v(-\infty) = \frac{1}{5 \times 10^{-6}} \int_{-\infty}^t 0 dt \Rightarrow v(t) = 0$$

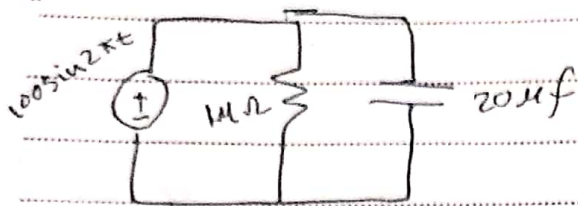
$$2) v(t) - v(0) = \frac{1}{5 \mu\text{f}} \int_0^t 20 \times 10^{-3} dt \Rightarrow v(t) = 4000t$$

$$3) v(t) - v(2 \times 10^{-3}) = \frac{1}{5 \mu\text{f}} \int_{2 \times 10^{-3}}^t 0 dt \Rightarrow v(t) = 8V$$

Subject: _____

Ex find $w_c(t)$ | $t = \frac{1}{4}$ s

$w_R(t)$ | $0 \leq t \leq 0.5$



$$\textcircled{1} w_c(t) = \frac{1}{2} C V^2(t)$$

$$= \frac{1}{2} [20 \times 10^{-6}] (100 \sin 2\pi(\frac{1}{4}))^2$$

$$= 100 \text{ mJ}$$

$$\textcircled{2} w_R(t) = \int_{t_0}^t P(t) dt$$

$$P(t) = \frac{V^2(t)}{R} = \frac{[100 \sin 2\pi t]^2}{1 \times 10^6}$$

$$w_R(t) = \int_0^t \frac{(100 \sin(2\pi t))^2}{1 \times 10^6} dt$$

$$= 2.5 \text{ mJ}$$

Ex find $w_c(t) = ?$, $C = 1000 \mu\text{f}$, $t = 50 \mu\text{s}$, $V(t) = 15 \cos 10^5 t$

Solu:

$$w_c(t) = \frac{1}{2} C V^2$$

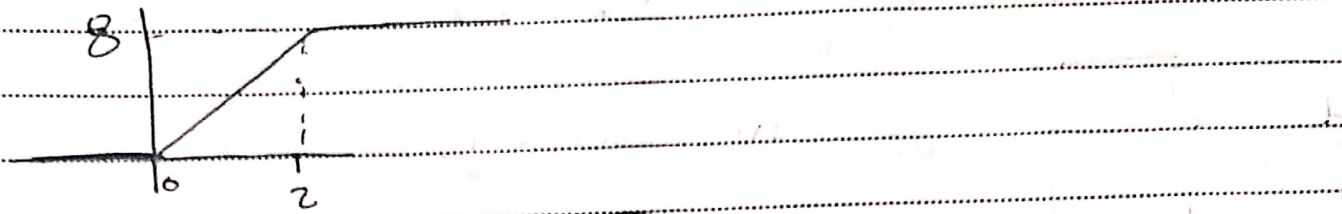
$$= \frac{1}{2} (1000 \times 10^{-6}) (1.5 \cos 10^5 (50 \mu))^2$$

$$= \dots \text{ J}$$

* characteristic of Ideal capacitor

1) There is no current through the capacitor if the voltage across it is not changing with time

$$i(t) = C \frac{dV(t)}{dt}$$



Ex: let $C = 5 \text{ f}$ and $V(t) = \sin t$, find $i(t)$ at $t = 10^{-3} \text{ s}$

$$i(t) = C \frac{dV(t)}{dt}$$

$$= 5 [\cos t]$$

$$= 5 [\cos(10^{-3} + \frac{180}{\pi})]$$

$$= \underline{\quad \quad \quad} \text{ A}$$

(*) Power :

$$P = V I$$

$$= V \left[C \frac{dV(t)}{dt} \right]$$

(*) Energy

$$W(t) = \int_{t_0}^t P(t) dt$$

$$= \int_{t_0}^t V C \frac{dV(t)}{dt} dt$$

$$= C \int_{t_0}^t V dV(t)$$

$$= C \left[\frac{V^2}{2} \right] \Big|_{t_0}^t$$

$$= \frac{1}{2} C [V^2(t) - V^2(t_0)]$$

Now let $V(t_0) = 0 \Rightarrow$

$$W_C(t) = \frac{1}{2} C V^2$$

② a finite amount of energy can be stored in a capacitor even if the current through capacitor is zero, such as the voltage across it is constant

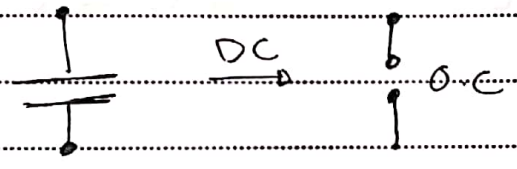
$$W_c(t) = \frac{1}{2} C V^2$$

③ it is impossible to change the voltage across capacitor by finite amount in zero time

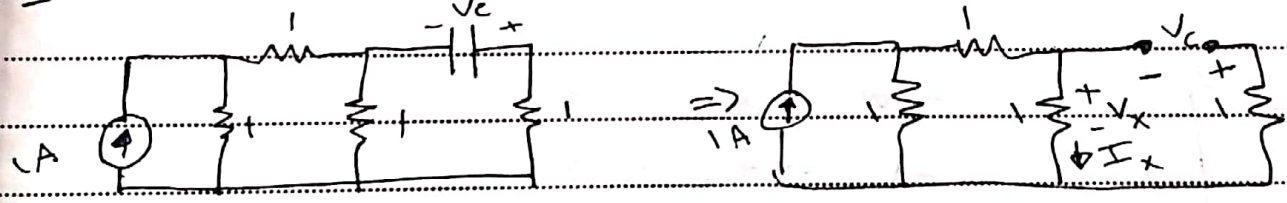
$$V_c(0^+) = V_c(0^-)$$

④ a capacitor never dissipate energy, but only store it although this is true for math. model it is not true for a physical model due to finite resistance

⑤ Capacitor in DC-analysis is open circuit



Ex find V_c ?



$$-V_x = V_c + 1(0) = 0$$

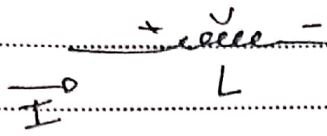
$$V_c = -V_x$$

$$\Rightarrow V_x = 1 I_x = 1 \left[\frac{\frac{1}{2}}{\frac{1}{2} + \frac{1}{1}} \right] [1] = \frac{1}{3}$$

$$\therefore V_c = -V_x = -\frac{1}{3} \text{ V}$$

Inductor :

Write in generic

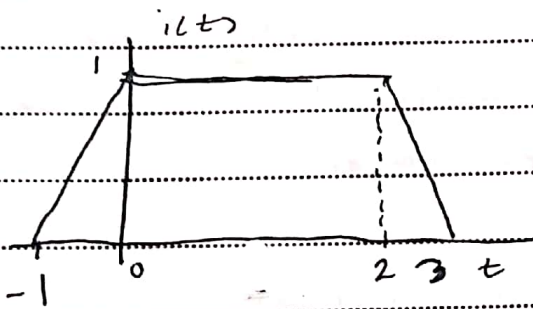


$$V(t) = L \frac{di(t)}{dt}$$

$$\int_{t_0}^t V(t) dt = \int_{t_0}^t L di(t)$$

$$\Rightarrow I(t) - I(t_0) = \frac{1}{L} \int_{t_0}^t V(t) dt$$

Ex let \$L = 3\$ H find \$V(t)\$



$$i(t) = \begin{cases} 0 & -\infty \leq t \leq -1 \\ t+1 & -1 \leq t \leq 0 \\ 1 & 0 \leq t \leq 2 \\ -t+3 & 2 \leq t \leq 3 \\ 0 & 3 \leq t \leq \infty \end{cases}$$

$$y - y_1 = m(x - x_1)$$

$(0, 1)$
 $(-1, 0)$

$$\begin{matrix} t & I \\ (2, 1) \\ (3, 0) \end{matrix}$$

$$y - 0 = \frac{1}{1} [t + 1]$$

$$I = t + 1$$

$$I - 0 = -1(t - 3)$$

$$I = -t + 3$$

$$\frac{di(t)}{dt} = \begin{cases} 0 & -\infty \leq t \leq -1 \\ 1 & -1 \leq t \leq 0 \\ 0 & 0 \leq t \leq 2 \\ -1 & 2 \leq t \leq 3 \\ 0 & 3 \leq t \leq \infty \end{cases}$$

$$V(t) = \begin{cases} 0 & " \\ 3 & " \\ 0 & " \\ -3 & " \\ 0 & " \end{cases}$$

Subject :

Ex: $L = 2H$ $v(t) = 6 \cos 5t$ v find $I(t)$, $I(-\frac{\pi}{2}) = 1A$

$$I(t) - I(-\frac{\pi}{2}) = \frac{1}{2} \int_{-\frac{\pi}{2}}^t 6 \cos 5t \, dt$$

$$I(t) = 6 \sin 5t + 1.6 \text{ A}$$

* Power :

$$P = VI$$

$$= L \frac{dI(t)}{dt} I$$

* Energy :

$$W_L(t) = \int_{t_0}^t v(t) \, dt$$

$$= \int_{t_0}^t L \frac{dI(t)}{dt} I \, dt$$

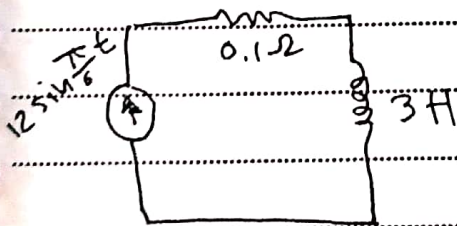
$$= L \int_{t_0}^t I \, dI(t)$$

$$= \frac{1}{2} L [I(t)^2 - I(t_0)^2] \quad \text{let } I(t_0) = 0$$

then

$$W_L(t) = \frac{1}{2} L I^2(t)$$

Ex: find $w_L(t)$ $t=3s$, $w_R(t)$ $0 \leq t \leq 6$



$$1) w_L(t) = \frac{1}{2} L I^2(t)$$

$$= \frac{1}{2} [3] [12 \sin \frac{\pi}{6} t]^2$$

$$w_L(3) = 216 \text{ J}$$

► Subject :

$$2) \omega_R(t) = \int_0^t p_R(t) dt$$

$$= \int_0^{6.28} [42 \sin \frac{\pi}{8} t]^2 (0.17) dt$$

$$= 43.2 \text{ J}$$

(*) characteristic of Ideal Inductor:

1] There is no voltage across inductor if the current through it is not changing with time.

$$v(t) = L \frac{dI(t)}{dt}$$

2] a finite amount of energy can be stored in an inductor even if the voltage across inductor is zero.

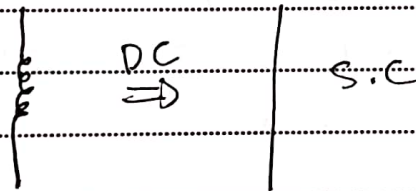
$$w_L(t) = \frac{1}{2} L I^2(t)$$

3] It is impossible to change the current through inductor by finite amount in zero time.

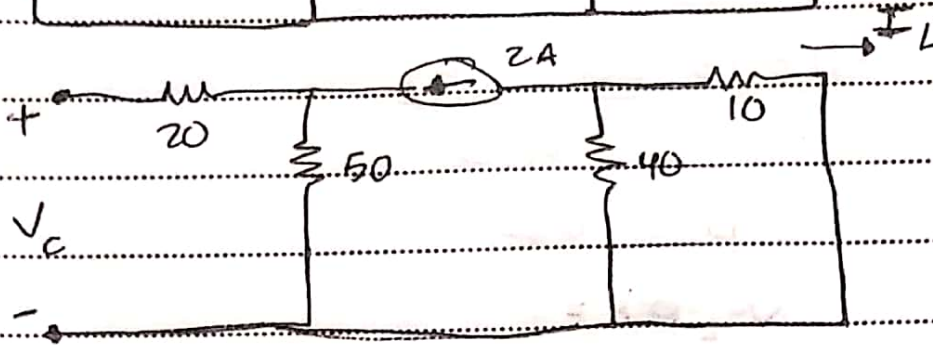
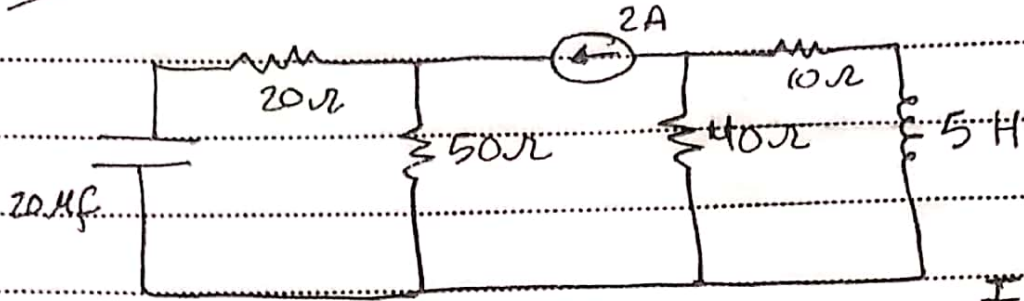
$$I_L(0^+) = I_L(0^-)$$

4] the inductor never dissipate energy but only store it even this is true for math model it is not true for physical model due to finite resistance.

5] Inductor in DC-analysis is short circuit.



Ex: find $w_C(t)$, $w_C(\infty)$



$$V_C = 50(2) = 100 \text{ V}$$

$$w_C(t) = \frac{1}{2} C V^2 = \frac{1}{2} [20 \times 10^{-6}] [100]^2 = 0.1 \text{ J}$$

$$w_L(t) = \frac{1}{2} L I^2(t)$$

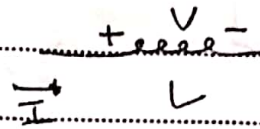
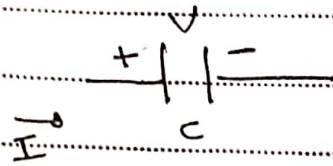
$$I_L = \frac{\frac{1}{10} [-2]}{\frac{1}{60} + \frac{1}{40}} = -1.6 \text{ A}$$

$$= \frac{1}{2} [5] [-1.6]^2$$

$$= 6.4 \text{ J}$$

Capacitor

Inductor



$$I(t) = C \frac{dV(t)}{dt}$$

$$V(t) = L \frac{dI(t)}{dt}$$

$$P = VI$$

$$P = VI$$

$$W_C = \frac{1}{2} CV^2$$

$$W_L(t) = \frac{1}{2} LI^2$$

$$V_C(0^+) = V_C(0^-)$$

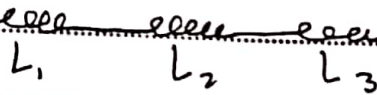
$$I_L(0^+) = I_L(0^-)$$

DC \Rightarrow O.C

DC \Rightarrow S.C

* Inductors:

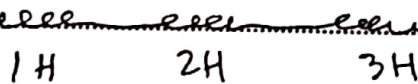
A) Inductors In Series:



$$L_{eq} = L_1 + L_2 + L_3$$

L_{eq}

\Rightarrow find L_{eq}

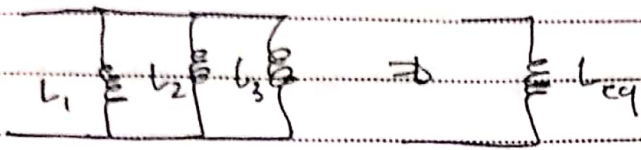


$$L_{eq} = 6H$$

S T A R S N O T E B O O K

S T A

B] Inductors In Parallel :



$$L_{eq} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3}}$$

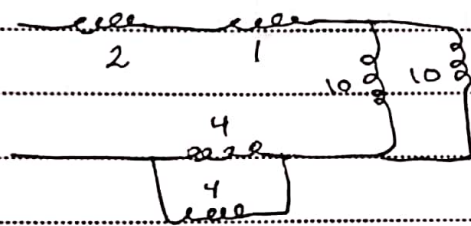
Ex) find \$L_{eq}\$?



$$\Rightarrow \frac{1}{L_{eq}} = \frac{1}{10} + \frac{1}{10}$$

$$L_{eq} = 5H$$

Ex) find \$L_{eq}\$?

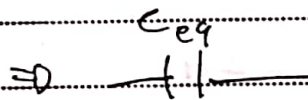
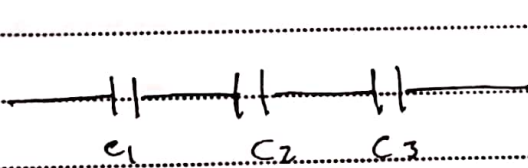


$$L_{eq} = 1 + 2 + [10 \parallel 10] + [4 \parallel 4]$$

$$= 10H$$

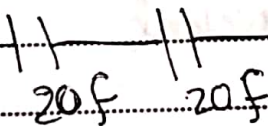
*] Capacitors :

A) Capacitors - in Series :



$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

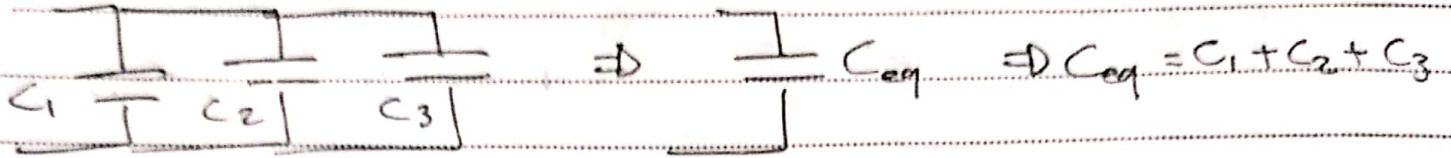
Ex) find \$C_{eq}\$?



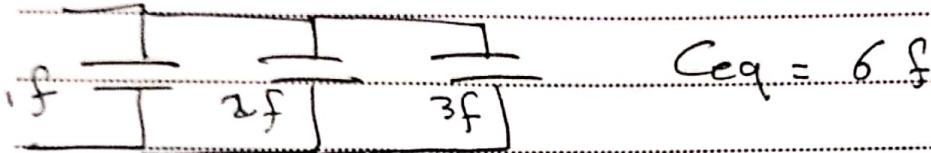
$$\frac{1}{C_{eq}} = \frac{1}{20} + \frac{1}{20}$$

$$C_{eq} = 10F$$

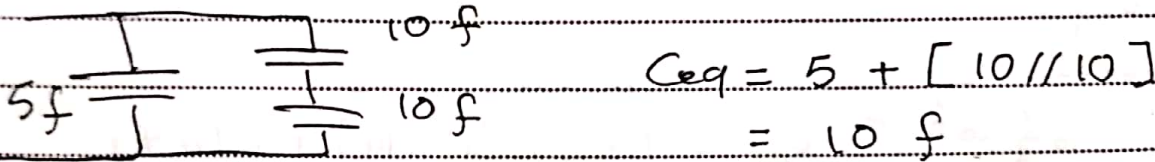
B) Capacitors in parallel:



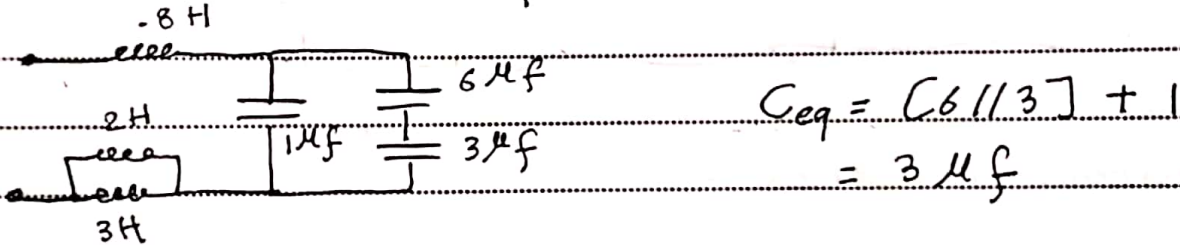
Ex: find C_{eq} ?



Ex: find C_{eq} ?

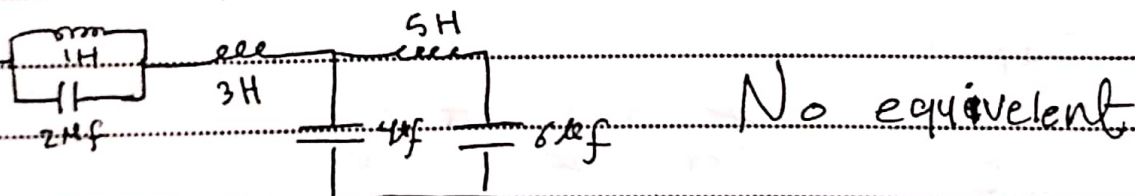


Ex: find the equi. circuit

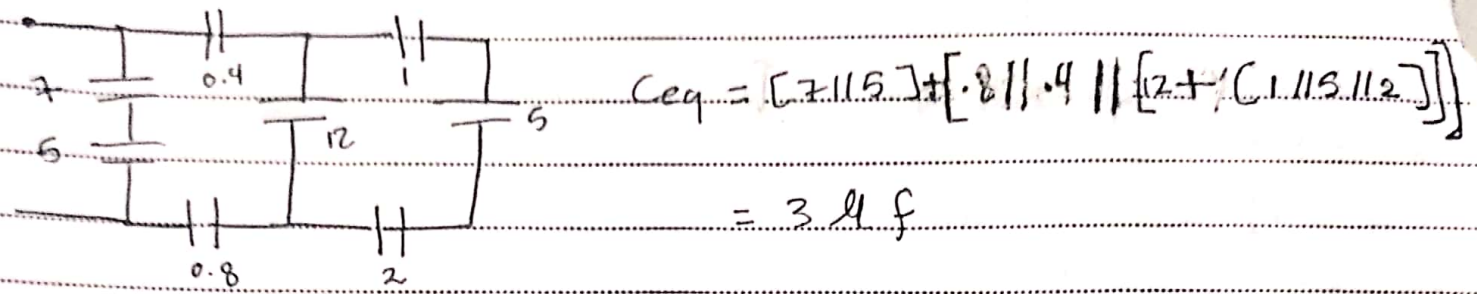


$L_{eq} = [2 || 3] + .8 = 2H$

Ex: find equ. circuit



Ex: Find C_{eq} ? All values in μf



CH 8: RL + RC - Circuit

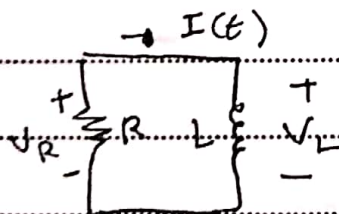
- ① Free source RL and RC circuit
- ② general RL and RC circuit
- ③ Driven RL and RC circuit

Ⓐ natural response: depend on general nature of circuit, types, size, numbers

Ⓑ transient response: any real circuit we construct can not store energy for ever the interval resistance will convert to heat and the signal will die out

Ⓒ forced response: depend on indep. source acting on a circuit

Ⓐ source-free RL-circuit ::



using KVL:

$$-V_R + V_L = 0$$

$$-V_R = R I(t) \quad \dots (1)$$

$$V_L(t) = L \frac{dI(t)}{dt} \quad \dots (2)$$

$$R I(t) + L \frac{dI(t)}{dt} = 0$$

$$\frac{dI(t)}{I(t)} + \frac{R}{L} dt = 0$$

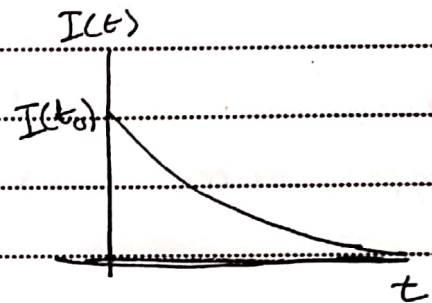
$$\int_{t_0}^t \frac{dI(t)}{I(t)} = \int_0^t -\frac{R}{L} dt$$

$$\ln [I(t)] \Big|_{t_0}^t = -\frac{R}{L} [t - 0]$$

$$\ln I(t) - \ln I(t_0) = -\frac{R}{L} t$$

$$\frac{\ln I(t)}{I(t_0)} = -\frac{R}{L} t$$

$$\therefore I(t) = I(t_0) e^{-R/L t}$$

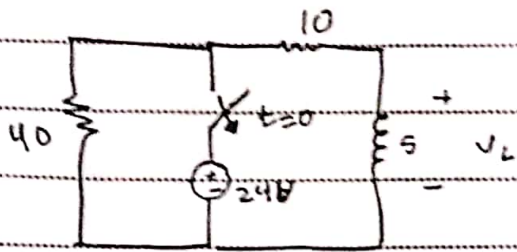


$$\textcircled{*} P_R = I^2 R = [I(t_0) e^{-R/L t}]^2 R$$

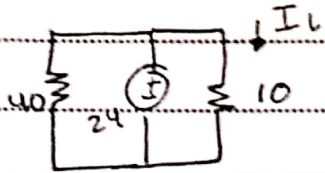
$$\textcircled{*} W_R(t) = \int_0^{\infty} P_R dt = \int_0^{\infty} [I(t_0) e^{-R/L t}]^2 R dt$$

$$= \frac{1}{2} L I^2(t_0)$$

Q: find $I_L(t)$, $V_L(t)$ $L = 5H$ $t = 200ms$



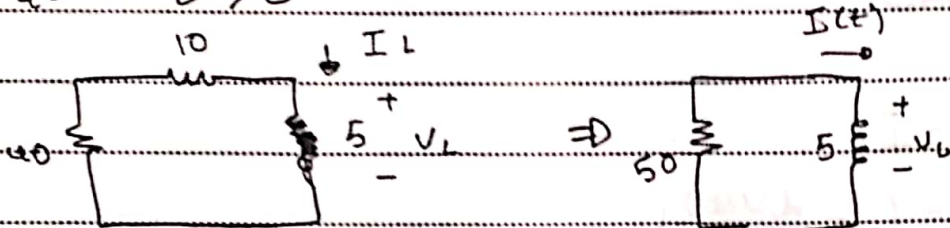
at $t < 0$



$$I_L(0^-) = \frac{24}{10} = 2.4 \text{ A}$$

$$I_L(0^+) = I_L(0^-) = 2.4 \text{ A}$$

at $t > 0$



$$I(t) = I(t_0) e^{-R/L t}$$

$$= 2.4 e^{-50/5 t}$$

$$= 2.4 e^{-10t}$$

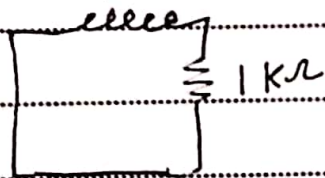
Now $V_L(t) = L \frac{dI(t)}{dt}$

$$V_L(t) = 5 \left[\frac{d[2.4 e^{-10t}]}{dt} \right]$$

$$I(200m) = 324.8 \text{ mA}$$

$$V_L(200m) = -16.24 \text{ V}$$

Q: find $W_L(t)$! $t = 2ms$ $W_L(0) = 7 \mu J$



$$I(t) = I(t_0) e^{-R/L t}$$

$$= I(t_0) e^{-1k/500n t}$$

$$\Rightarrow W_L(0) = \frac{1}{2} L I^2(0)$$

$$7 \mu J = \frac{1}{2} [500n] [I(0)]^2$$

$$I(0) = 5.292 \text{ A}$$

$$I(2n) = 5.292 e^{-1k/600n(2n)}$$

$$= 96.93 \text{ mA}$$

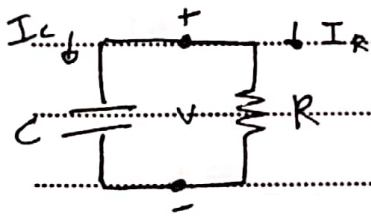
$$\text{Now } W_L(2n) = \frac{1}{2} L I^2(2n) \\ = 2.349 \text{ nJ}$$

⇒ Time constant of RL-circuit

$$\tau = \frac{L}{R}$$

$$\therefore I(t) = I(t_0) e^{-R/L t} = I(t_0) e^{-t/\tau}$$

⊛ Source free RC-circuit :

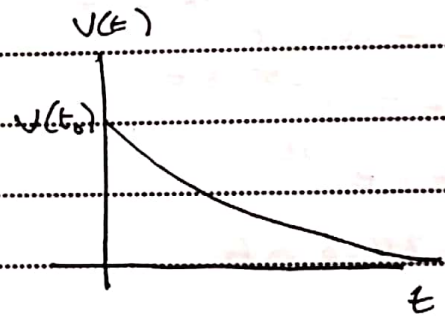


$$I_R = -I_C$$

$$\frac{V}{R} = -\left[C \frac{dV(t)}{dt} \right]$$

$$C \frac{dV(t)}{dt} + \frac{V}{R} = 0$$

$$\therefore V(t) = V(t_0) e^{-\frac{t}{RC}}$$



$$V_C(0^+) = V_C(0^-)$$

⊛ Time constant RC-circuit

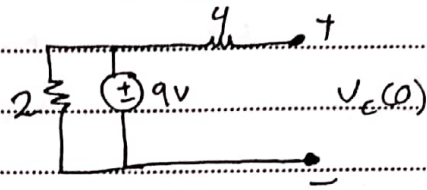
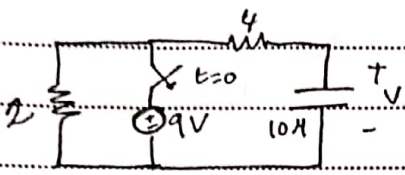
$$\tau = RC$$

$$\therefore V(t) = V(t_0) e^{-t/\tau}$$

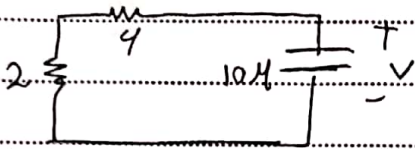
Ex: find $V(t)$

$t = 200 \mu s$

at $t < 0$



at $t > 0$



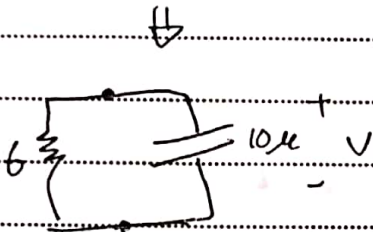
$$V_c(0^-) = 9V$$

$$V_c(0^+) = V_c(0^-) = 9V$$

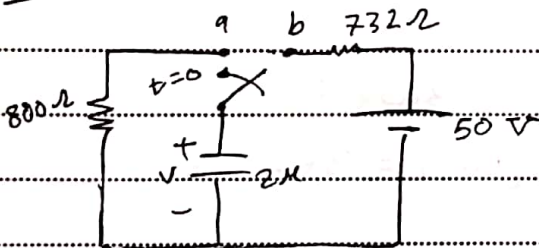
$$\Rightarrow V(t) = V(t_0) e^{-t/RC}$$

$$= 9 e^{-t/6 \times 10^{-6}}$$

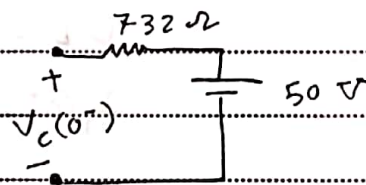
$$V(200 \mu) = 321.1 \text{ mV}$$



Ex: find $V(0)$, $V(2ms)$



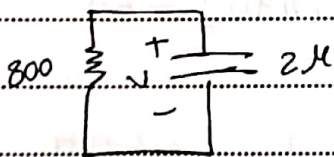
at $t < 0 = 0$



$$V_c(0^-) = 50V$$

$$V_c(0^+) = V_c(0^-) = 50V$$

at $t > 0$



$$V(t) = V(t_0) e^{-t/RC}$$

$$= 50 e^{-t/800 \times 2 \mu}$$

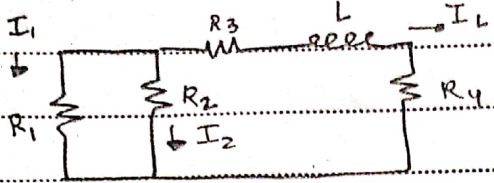
$$V(2m) = 50 e^{-\frac{2 \times 10^{-3}}{800 \times 2 \times 10^{-6}}}$$

$$= 50 e^{-\frac{10^3}{800}}$$

$$= 14.32 V$$

S T A R B N O T E B O O K

* General RL - Circuit



$L_{eq} = L$

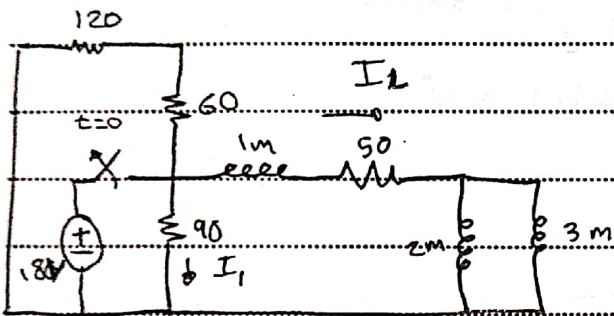
$R_{eq} = (R_1 || R_2) + R_3 + R_4$

$I_L(t) = I_L(t_0) e^{-R_{eq}/L_{eq} t}$

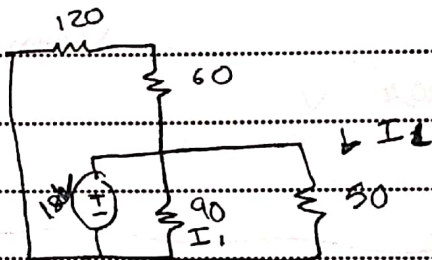
$I_1 = \frac{I_L}{\frac{1}{R_1} + \frac{1}{R_2}} [-I_L]$

$I_2 = \frac{I_L}{\frac{1}{R_2} + \frac{1}{R_3}} [-I_L]$

Ex: find I_1, I_2 ?

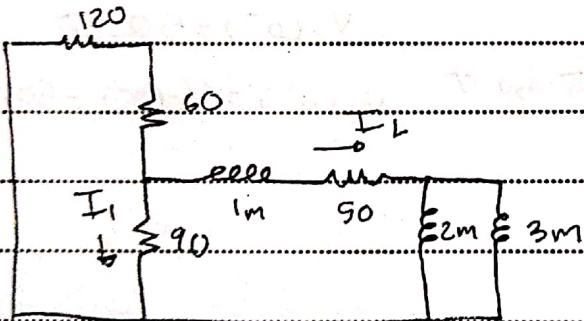


at $t < 0$



$I_1 = \frac{18}{90} = 200 \text{ mA}$

at $t > 0$



$I_2 = \frac{18}{50} = 360 \text{ mA}$

$I_L(t^+) = I_L(t^-) = 360 \text{ mA}$

$I_L(t) = I_L(t_0) e^{-R_{eq}/L_{eq} t}$
 $= 360 e^{-t/20 \mu} \text{ mA}$
 $= 360 e^{-50000 t} \text{ mA}$

$R_{eq} = [(60+120) || 90] + 50$
 $= 110 \Omega$

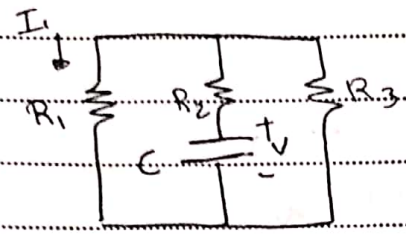
$L_{eq} = [2 || 3] + 1 = 2.2 \text{ mH}$

$I_1(t) = \frac{1}{\frac{1}{90} + \frac{1}{180}} [-360 e^{-50000 t}] \text{ mA}$

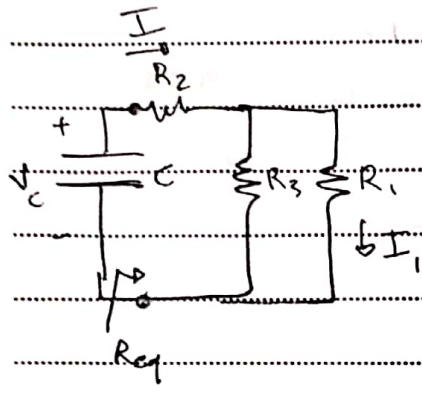
$\tau = \frac{L_{eq}}{R_{eq}} = 20 \mu\text{s}$

$= -240 e^{-50000 t} \text{ mA}$

*** General RC-circuit :**

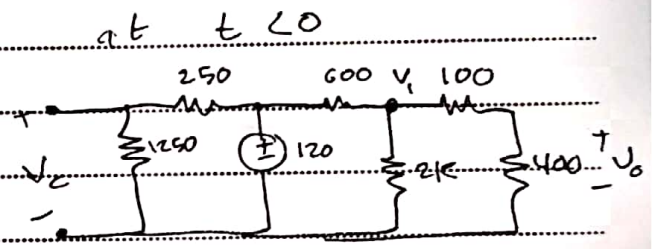
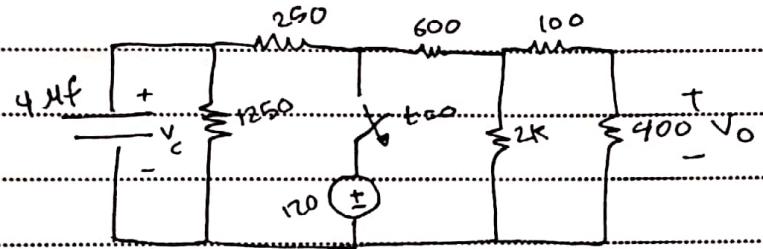


find $V(t)$, $I(t)$?
 $V_c(t) = V_c(t_0) e^{-t/R_{eq}C_{eq}}$
 $C_{eq} = C$
 $R_{eq} = R_1 || R_3 + R_2$

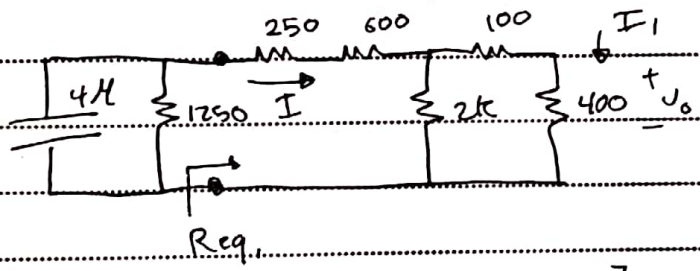


$I = \frac{V_c(t)}{R_{eq}}$ $R_{eq} = R_2 + R_3 || R_1$
 $I_1 = \frac{I}{\frac{1}{R_1} + \frac{1}{R_3}}$

Ex: find V_c , V_o at $t=0^+$, 0^- , 1.3 ms :



at $t > 0$



$V_c = \frac{1250}{1250 + 250} [120] = 100$ V

$V_c(0^+) = V_c(0^-) = 100$ V

$V_o = \frac{400}{100 + 400} [V_1]$

$R_{eq} = [(100 + 400) || 2k] + 600 + 250 = 1250 \Omega$

$V_1 = \frac{500 || 2k}{500 || 2k + 600} [120]$

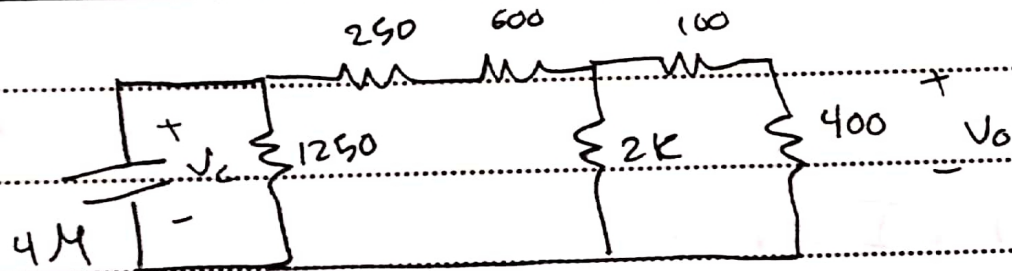
$I = \frac{100}{R_{eq}} = \frac{100}{1250} = .08$ A

$I_1 = \frac{1}{\frac{1}{500} + \frac{1}{2k}} [I] = 0.064$ A

then $V_o = 38.4$ V

$V_o = 400 I_1 = 25.6$ V

Subject :



at $t = 1.3 \text{ ms}$

$$V_c(t) = V_c(t_0) e^{-t/RC}$$

$$C_{eq} = 4 \text{ Mf}$$

$$R_{eq} = 1250 \parallel R_{eq1} = 625$$

$$\tau = RC = .0025$$

$$V_c(t) = 100 e^{-t/0.0025} \text{ V}$$

$$= 100 e^{-400t} \text{ V}$$

$$V_c(1.3 \text{ m}) = 59.45 \text{ V}$$

$$V_o(t) = V_o(t_0) e^{-t/R_{eq}C_{eq}}$$

$$= 25.6 e^{-400t} \text{ V}$$

$$V_o(1.3 \text{ m}) = 15.22 \text{ V}$$

Forced RL - circuit

1) with all indep. source zeroed out simplify the circuit to find R_{eq} , L_{eq}

$$I = \frac{L_{eq}}{R_{eq}}$$

2) as L_{eq} as short circuit use DC-analysis to find $I_L(0^-)$

3) with I_{eq} as short circuit use DC-analysis to find $I_L(\infty)$

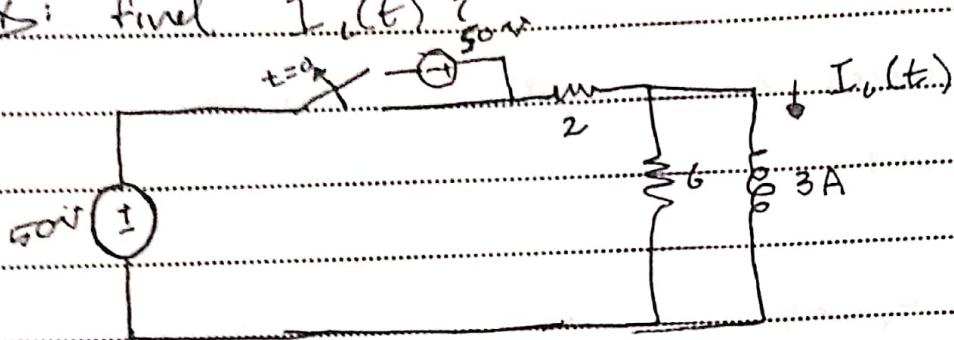
$$4) I_L(0^+) = I_L(0^-)$$

$$5) I_L(t) = I_L(\infty) + [I_L(0^+) - I_L(\infty)] e^{-t/\tau}$$

S T A R S N O T E B O O K

Subject:

Ex: find $I_L(t)$?



at $t < 0$

$$I_L(0^-) = \frac{90}{2} = 25 \text{ A}$$

$$-90 - 50 + 2I(t) = 0$$

$$I_L(0^+) = I_L(0^-) = 25 \text{ A}$$

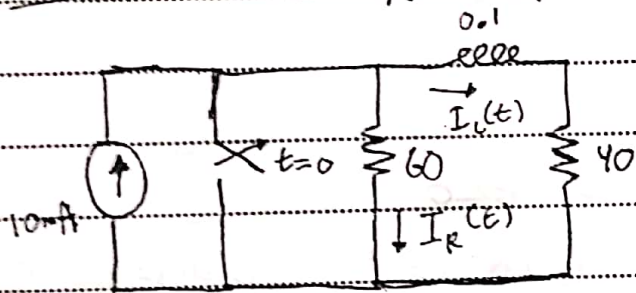
$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{3}{216} = 25$$

at $t > 0$

$$I_L(\infty) = \frac{100}{2} = 50 \text{ A}$$

$$I_L(t) = 50 + [25 - 50]e^{-t/2} = 50 - 25e^{-t/2} \text{ A}$$

Ex: find $I_R(t)$?



at $t < 0$

$$I_R(0^-) = 0 \text{ A}$$

$$I_L(0^-) = 0 \text{ A}$$

at $t > 0$

$$I_R(0^+) = 10 \text{ mA}$$

$$\tau = \frac{L_{eq}}{R_{eq}} = \frac{0.1}{60 + 40} = 1 \text{ ms}$$

$$I_R(\infty) = \frac{10}{\frac{1}{60} + \frac{1}{40}} [10 \text{ m}] = 4 \text{ mA}$$

$$I_R(t) = 4 + [10 - 4]e^{-t/1 \text{ ms}} \text{ mA}$$

$$I_R(1.9 \text{ ms}) = 5.339 \text{ mA}$$

Focred RC Circuit :

1] with all indep. sources Zeroed out simplify the circuit to find Req, Ceq.

$$\tau = R_{eq} C_{eq}$$

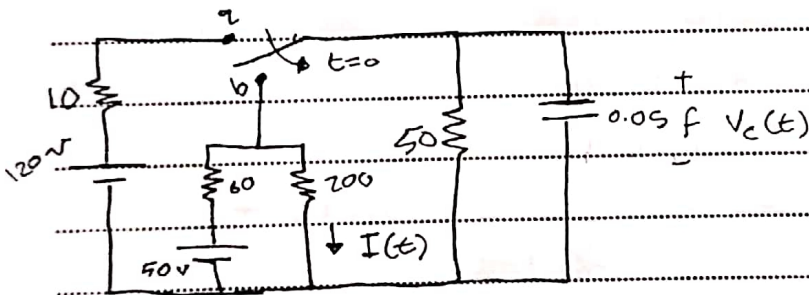
2] as Ceq as open circuit use DC-analysis to find $V_c(0^-)$

3] with Ceq as open circuit use DC-analysis to find $V_c(\infty)$

$$4) V_c(0^+) = V_c(0^-)$$

$$5) V_c(t) = V_c(\infty) + [V_c(0^+) - V_c(\infty)] e^{-t/\tau}$$

Ex: find $V_c(t)$, $I(t)$?



at $t < 0$

$$V_c(0^-) = \frac{50}{50+10} [120] = 100 \text{ V}$$

$$I(0^-) = \frac{50}{260} = 192.3 \text{ mA}$$

at $t > 0$

$$V_c(\infty) = \frac{50 \parallel 200}{50 \parallel 200 + 60} [50] = 20 \text{ V}$$

$$V_c(0^+) = V_c(0^-) = 100 \text{ V}$$

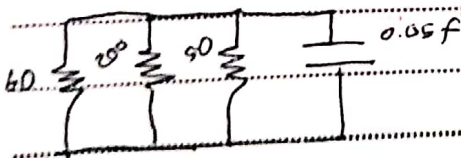
$$I(t) = \frac{V_c(t)}{200}$$

$$= 0.1 + 0.4 e^{-t/1.2} \text{ A}$$

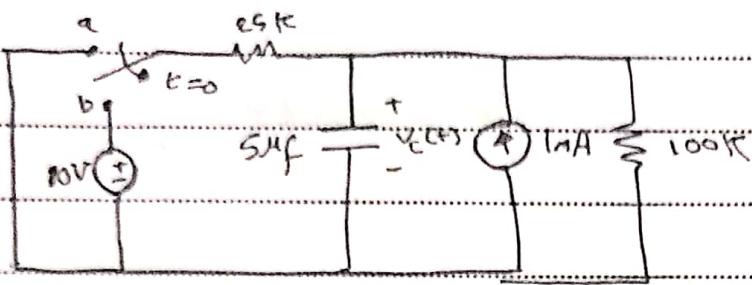
$$\tau = R_{eq} C_{eq}$$

$$= [60 \parallel 200 \parallel 50] [0.05] = 1.25$$

$$V_c(t) = 20 + [100 - 20] e^{-t/1.2}$$



Ex: find $V_c(t)$?

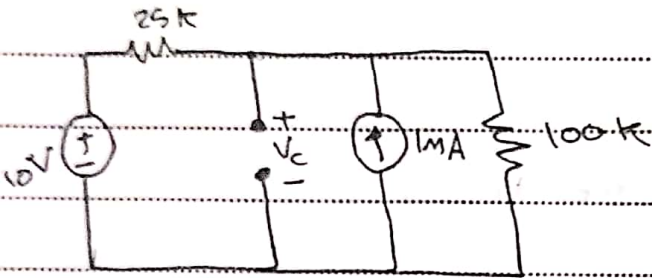


at $t < 0$

$$V_c(0^-) = (100k \parallel 25k)(1m) = 20V$$

$$V_c(0^+) = V_c(0^-) = 20V$$

at $t > 0$

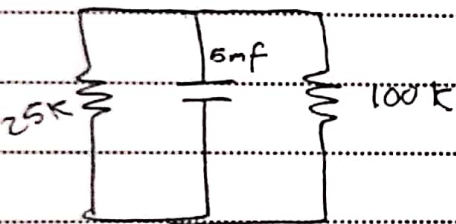


$$V_c(\infty) = V_{c \uparrow} + V_{c \downarrow}$$

$$= \frac{100k}{125k} [10] + (100k \parallel 25k)(1m)$$

$$= 28V$$

$$\tau = R_{eq} C_{eq}$$



$$= [25k \parallel 100k] [5\mu f]$$

$$= 100ms$$

$$V_c(t) = 28 + [20 - 28] e^{-t/1m}$$

$$V_c(80m) = 24.41V$$

CH. 10 :

Sinusoidal steady state analysis

$$V(t) = V_m \sin(\omega t + \theta)$$

V_m = amplitude

ωt = argument

θ = phase

ω = angular freq.

$$Ex: V(t) = 100 \sin(2\pi t - 30)$$

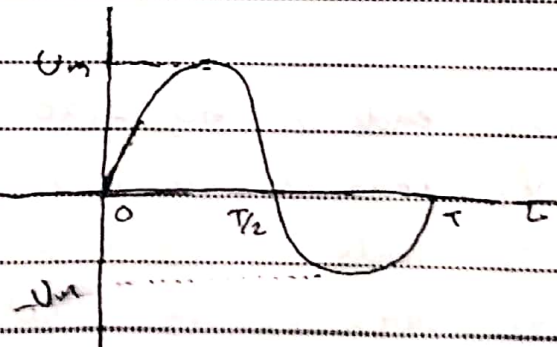
$$V_m = 100$$

$$f = \frac{1}{T}$$

$$\omega = 2\pi$$

$$\theta = -30$$

$$\omega = 2\pi f$$



$$V(t) = 100 \sin(2\pi 1000t - \pi/6)$$

$$\theta_{\text{rad}} = \theta_{\text{deg}} + \frac{\pi}{180}$$

$$V(t) = 100 \sin(2\pi 1000t - 30)$$

$$t = 1 \times 10^{-4}$$

$$\theta_{\text{deg}} = \theta_{\text{rad}} + \frac{180}{\pi}$$

$$V(t) = 100 \sin(2\pi 1000 \times 1 \times 10^{-4} - 30)$$

$$= 100 \sin\left(2\pi + \frac{180}{\pi} - 30\right)$$

$$= 10.45 \text{ V}$$

7) Lagging and leading signal:

Two sinusoidal waves whose phases are to be compared must:

- 1] Both be written as sine or cosine waves.
- 2] Both be written with positive amplitude
- 3] each have same freq.
- 4] \ominus front - \ominus back

$$1] -\sin(\omega t) = \sin(\omega t + 180)$$

$$2] -\cos(\omega t) = \cos(\omega t + 180)$$

$$3] \mp \sin(\omega t) = \cos(\omega t \pm 90)$$

$$4] \pm \cos(\omega t) = \sin(\omega t \pm 90)$$

Ex: Let $V_1(t) = U_{m1} \cos(\omega t + 10)$

$$V_2(t) = U_{m2} \sin(\omega t - 30)$$

$$V_1(t) = U_{m1} \sin(\omega t + 100)$$

1] V_2 leads V_1 by -130

2] V_2 lags V_1 by 130

3] V_1 leads V_2 by 130

4] V_1 lags V_2 by -130

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Ex: find by which I_1 lags V_1 :-

$$V_1(t) = 120 \cos(120\pi t - 40)$$

a) $I_1(t) = 25 \cos(120\pi t + 20)$

I_1 lags V_1 by -60

b) $I_1(t) = 1.4 \sin(120\pi t - 70)$

$$= 1.4 \cos(120\pi t - 160)$$

I_1 lags V_1 by 120

c) $I_1(t) = -0.8 \cos(120\pi t - 110)$

$$= 0.8 \cos(120\pi t + 70)$$

I_1 lags V_1 by -110

d) $I_1(t) = 2 \cos(40\pi t - 30)$

(Can not be compared)

Euler formula :-

$$e^{i\theta} = \cos \theta + i \sin \theta$$

any complex number Z

$$Z = x + iy$$

$$x = \text{Re}[Z] \quad y = \text{Im}[Z]$$

1] Rectangular form $\Rightarrow Z = x + iy$

2] Polar form $\Rightarrow Z = r \angle \theta$

3] exp. form $\Rightarrow Z = r e^{i\theta}$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1} \left[\frac{y}{x} \right]$$

$$x = r \cos \theta$$

$$y = r \sin \theta$$

► Subject :

$$z_1 = x_1 + iy_1 = r_1 \angle \theta_1$$

$$z_2 = x_2 + iy_2 = r_2 \angle \theta_2$$

$$1] z_1 + z_2 = (x_1 + x_2) + i(y_1 + y_2)$$

$$2] z_1 - z_2 = (x_1 - x_2) + i(y_1 - y_2)$$

$$3] z_1 z_2 = r_1 r_2 \angle (\theta_1 + \theta_2)$$

$$4] \frac{z_1}{z_2} = \frac{r_1}{r_2} \angle (\theta_1 - \theta_2)$$

$$5] \frac{1}{z_1} = \frac{1}{r_1} \angle -\theta_1$$

6] Conjugate of complex number

$$\overline{z_1} = x_1 - iy_1$$

Ex :

$$1] (2 \angle 30^\circ) (5 \angle -100^\circ) (1 + 2j) =$$

$$= (10 \angle -80^\circ) \left(\sqrt{1^2 + 2^2} \angle \tan^{-1}\left(\frac{2}{1}\right) \right)$$

$$= 22.36 \angle -16.57^\circ$$

$$= 22.36 \cos(-16.57^\circ) + i 22.36 \sin(-16.57^\circ)$$

$$= 21.43 - i 6.377$$

(b) $5 \angle -200 + 4 \angle 20 =$
 $= (5 \cos(-200) + i 5 \sin(-200)) + (4 \cos(20) + i 4 \sin(20))$
 $= -0.939 + i 3.078$

(c) $\frac{2-j7}{3-j} = \frac{\sqrt{2^2+7^2} \angle \tan^{-1}(-7/3)}{\sqrt{1^2+3^2} \angle \tan^{-1}(-1/3)} = 2.302 \angle -55.62^\circ$

~~XXXXXXXXXX~~

phasor transformation :-

1) $V(t) = V_m \cos(\omega t + \theta_1)$
 $V = V_m \angle \theta_1$

2) $I(t) = I_m \cos(\omega t + \theta_2)$
 $I = I_m \angle \theta_2$

Ex: time → phasor

1) $-5 \sin(580t - 110)$
 $\Rightarrow 5 \cos(580t - 20)$
 $\therefore 5 \angle -20$

2) $3 \cos(600t) - 5 \sin(600t + 110)$
 $= 3 \angle 0 + 5 \angle (600t + 200)$
 $= 3 \angle 0 + 5 \angle 200$

$3 \cos(0) + i 3 \sin(0) + 5 \cos(200) + i 5 \sin(200)$
 $= 2.41 \angle -134.8$

Ex 1 phasor \rightarrow time

$$\omega = 2000 \text{ rad/s}$$

$$t = 1 \text{ ms}$$

$$\boxed{1} \quad j10 \Rightarrow 0 + j10 = \sqrt{0^2 + 10^2} \angle \tan^{-1}(10/0) \\ = 10 \angle 90$$

$$\Rightarrow 10 \cos(2000t + 90) = 10 \cos(2000(1\text{m}) + 90) \\ = 10 \cos\left(2 * \frac{180}{\pi} + 90\right) \\ = -9.093 \text{ A}$$

$$\boxed{2} \quad 20 + j[10 \angle 20] = \\ (0 + j1) = 1 \angle 90$$

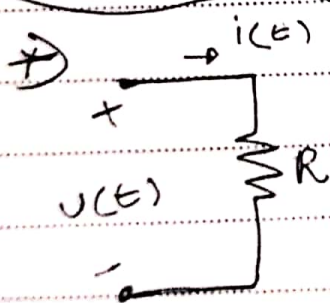
$$= 20 + [1 \angle 90][10 \angle 20]$$

$$= 20 + 10 \angle 110$$

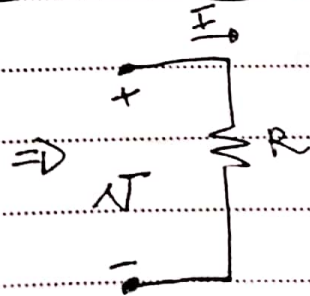
$$= 20 + 10 \cos(110) + j 10 \sin(110)$$

$$= 19.06 \angle 29.54 \text{ A}$$

$$\Rightarrow 19.06 \cos((29.54) + 2000t) \Big|_{t=1\text{ms}} \\ = 19.06 \cos\left(2 * \frac{180}{\pi} + 29.54\right) \\ = -15.45 \text{ A}$$



$$v(t) = R i(t)$$



$$V = R I$$

$$V_m \angle \theta_1 = R I_m \angle \theta_2$$

(2)

Ex: let $V(t) = 8 \cos(100t - 50) \text{ V}$, $R = 4 \Omega$, find $I(t)$

$$I(t) = \frac{V(t)}{R} = \frac{8 \cos(100t - 50)}{4} \text{ A}$$

max: $I = 2 \angle -50 \text{ A}$

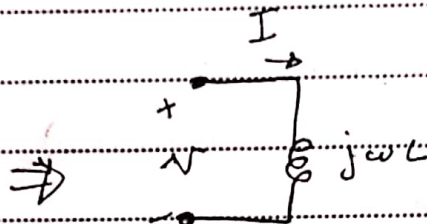
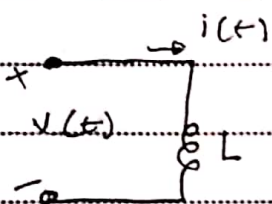
$$V = RI$$

$$\Rightarrow I = \frac{V}{R} = \frac{8 \angle -50}{4} = 2 \angle -50 \text{ A}$$

$$\therefore i(t) = 2 \cos(100t - 50) \text{ A}$$

⊛ Inductor :-

①



$$V(t) = L \frac{di(t)}{dt}$$

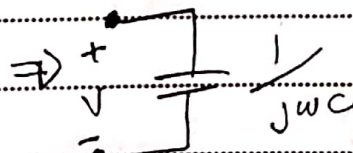
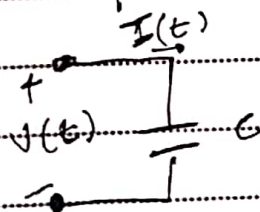
$$V = [j\omega L] I$$

Ex: $V = 8 \angle -50 \text{ V}$, $\omega = 100 \text{ rad/s}$, $L = 4 \text{ H}$ find I and $i(t)$

$$I = \frac{V}{j\omega L} = \frac{8 \angle -50}{j[100][4]} = 0.02 \angle -140^\circ \text{ A}$$

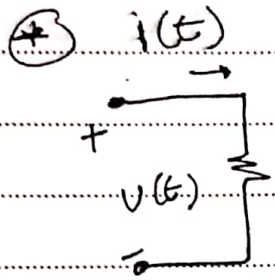
$$i(t) = 0.02 \cos(100t - 140) \text{ A}$$

⊛ Capacitor :-

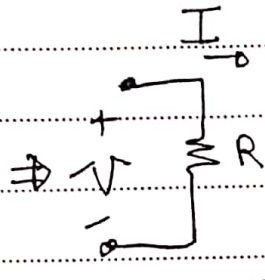


$$i(t) = C \frac{dV(t)}{dt}$$

$$V = \frac{1}{j\omega C} I$$



$$v(t) = RI(t)$$



$$V = RI$$

$$V_m \angle \theta_1 = R I_m \angle \theta_2$$

Ex: Let $v(t) = 8 \cos(100t - 50^\circ) \text{ V}$, $R = 4 \Omega$
Find $I(t)$?

$$I(t) = \frac{v(t)}{R} = 2 \cos(100t - 50^\circ) \text{ A}$$

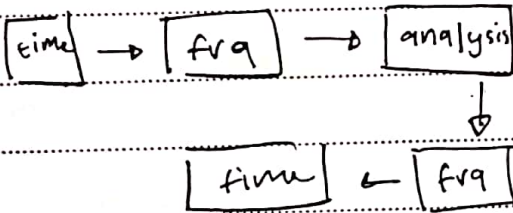
$$I = 2 \angle -50^\circ \text{ A}$$

General ohm law:

$$V = Z I, \quad Z_R = R$$

$$Z_L = j\omega L$$

$$Z_C = \frac{1}{j\omega C}$$



Ex: $V = 8 \angle -50^\circ \text{ V}$, $\omega = 100 \text{ rad/s}$, $L = 4 \text{ H}$ find I , $i(t)$

$$I = \frac{V}{j\omega L} = \frac{8 \angle -50^\circ}{j[100][4]} = 0.02 \angle -140^\circ \text{ A}$$

$$i(t) = 0.02 \cos(100t - 140^\circ) \text{ A}$$

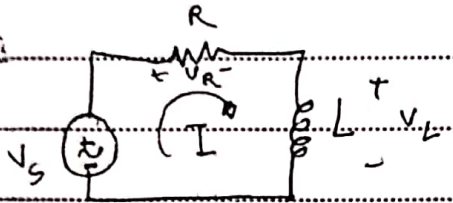
* KVL :-

$$\sum_{loop} V(t) = 0 \quad \sum_{loop} V = 0$$

* KCL :-

$$\sum_{node} I(t) = 0 \quad \sum_{node} I = 0$$

EX:



using KVL:

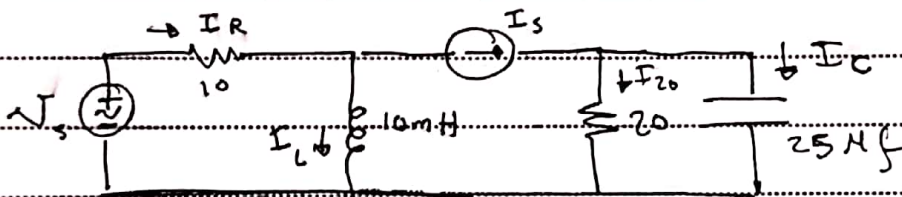
$$-V_s + V_R + V_L = 0$$

$$-V_s + RI + j\omega LI = 0$$

$$I = \frac{V_s}{R + j\omega L}$$

EX: $\omega = 1200 \text{ rad/s}$, $I_c = 1.2 \angle 28^\circ \text{ A}$, $I_L = 3 \angle 53^\circ \text{ A}$

$I_s \neq V_s$, $i_R(t) \rightarrow$ find



$10 \rightarrow 10$

$20 \rightarrow 20$

$10 \text{ mH} \rightarrow j\omega L \rightarrow 12j$

$25 \mu\text{F} \rightarrow \frac{1}{j\omega C} \rightarrow -33.33j$

$$V_c = Z_c I_c = [-33.33j][1.2 \angle 28^\circ]$$

$$= 40 \angle -62^\circ \text{ V}$$

$$I_{20} = \frac{40 \angle -62^\circ}{20} = 2 \angle -62^\circ \text{ A}$$

$$I_s = I_{20} + I_c = 2.33 \angle -31.04^\circ \text{ A}$$

$$I_R = I_s + I_L = 3.98 \angle 17.42^\circ \text{ A}$$

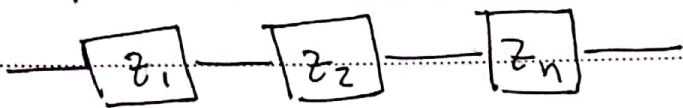
using KVL:-

$$-V_s + 10I_R + 12j[3 \angle 53^\circ] = 0$$

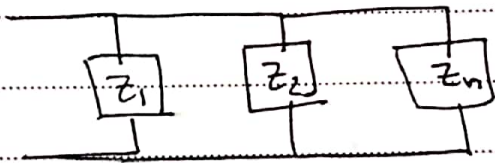
$$V_s = 34.86 \angle 74.91^\circ \text{ V}$$

$$i_R(t) = 3.98 \cos(1200t + 17.42^\circ) \text{ A}$$

Impedance:

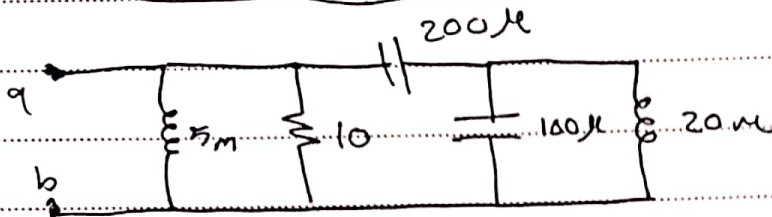


$$Z_{eq} = Z_1 + Z_2 + \dots + Z_n$$



$$\frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2} + \dots + \frac{1}{Z_n}$$

Ex 1



$$\omega = 1000 \text{ rad/s}$$

$$5 \text{ mH} \rightarrow j5$$

$$100 \mu\text{F} \rightarrow -j10$$

$$20 \text{ mH} \rightarrow j20$$

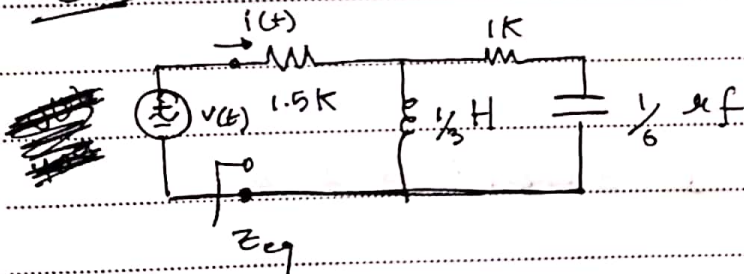
$$10 \rightarrow 10$$

$$200 \mu\text{F} \rightarrow -j5$$

$$\text{then: } Z_a = \left[\left[\left[j20 \parallel (-j10) \right] + [-j5] \right] \parallel 10 \parallel j5 \right]$$

$$= 2.809 + j4.49 \Omega$$

Ex 1 Find $i(t)$?



$$v(t) = 40 \sin 3000t$$

$$\Rightarrow 40 \cos(3000t - 90^\circ)$$

$$40 \angle -90$$

$$1.5 \text{ k} \rightarrow 1.5 \text{ k}$$

$$\frac{1}{3} \text{ H} \rightarrow 1 \text{ k} j \Omega$$

$$1 \text{ k} \rightarrow 1 \text{ k}$$

$$\frac{1}{6} \mu\text{F} \rightarrow -j2 \Omega$$

$$\therefore Z_{eq} = \left[\left[-j2 + 1 \text{ k} \right] \parallel 1 \text{ k} j \right] + 1.5 \text{ k} = 2.5 \angle 36.87 \text{ k} \Omega$$

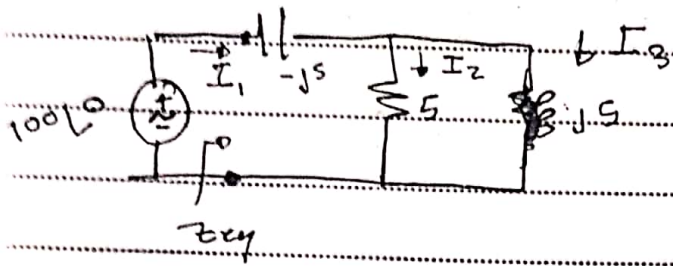
$$I = \frac{V_s}{Z_{eq}} = \frac{40 \angle -90}{2.5 \angle 36.87}$$

$$= 16 \angle -126.9 \text{ mA}$$

$$i(t) = 16 \cos(3000t - 126.9) \text{ mA}$$

S T A R S N O T E B O O K

Ex 1 find I_1, I_2, I_3



$$Z_{eq} = [5 \parallel j5] + [-j5]$$

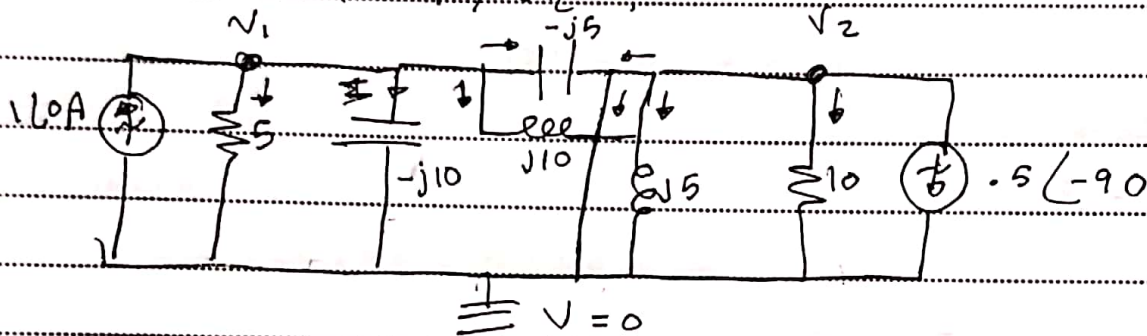
$$I_1 = \frac{100 \angle 0}{Z_{eq}} = 28.28 \angle 45^\circ \text{ A}$$

$$I_2 = \frac{1/5}{1/5 + 1/j5} [28.28 \angle 45^\circ]$$

$$= 20 \angle 90^\circ \text{ A}$$

$$I_3 = 20 \angle 0^\circ \text{ A}$$

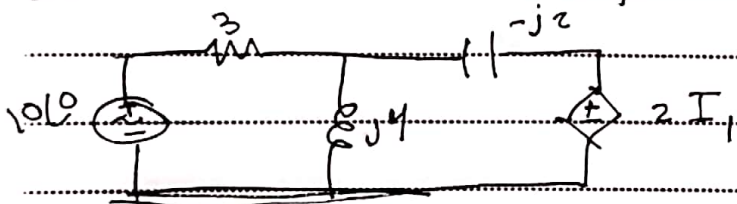
Ex 1 find V_1, V_2 ?



$$V_1 = 2.24 \angle -53.4^\circ \text{ V}$$

$$V_2 = 4.47 \angle 116.6^\circ \text{ V}$$

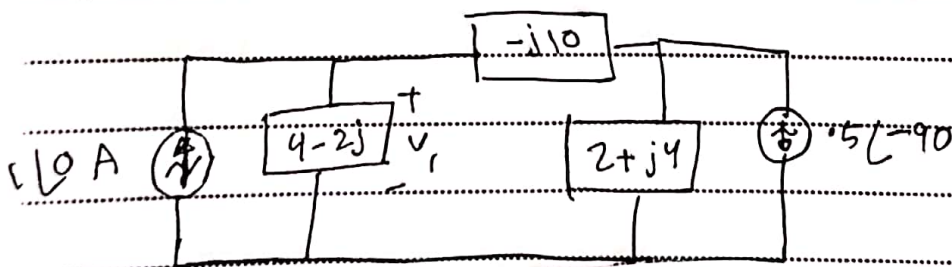
Ex: find I_1, I_2 ?



$$I_1 = 1.24 \angle 29.7^\circ \text{ A}$$

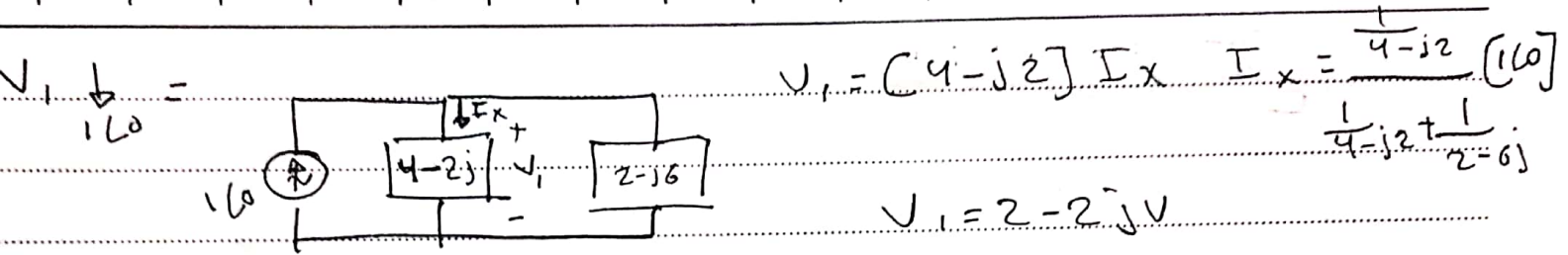
$$I_2 = 2.77 \angle 56.3^\circ \text{ A}$$

Ex: find V_1

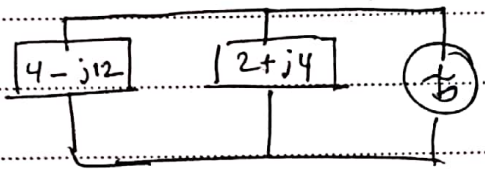


$$V_1 = V_{10} + V_{5\angle-90}$$

B T A R S N O T E B O O K



$V_1 \downarrow 0.5 \angle -90 :$



$V_1 = [4-j2] I_y$

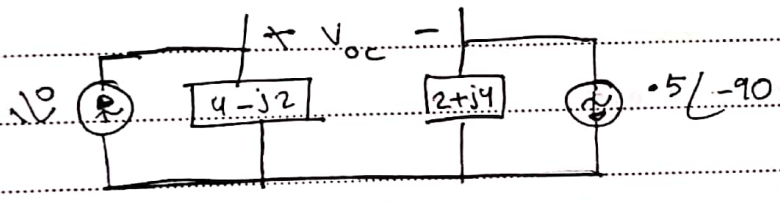
$I_y = \frac{1\angle -90}{(1/4-j2) + (1/2+j4)}$

$= -1 \text{ A}$

$V_1 = 2 - 2j + -1$

$= 1 - 2j \text{ V}$

EX: find thevenin



① $V_{Th} = V_{oc}$

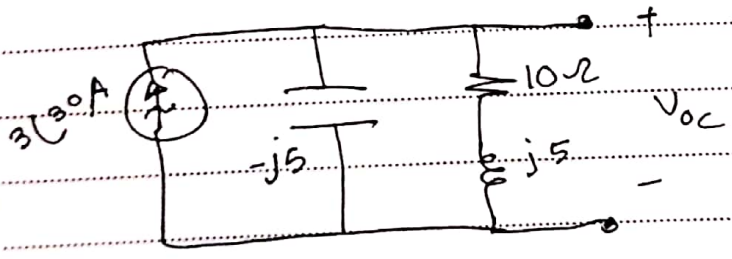
$V_{oc} = 1\angle 0 [4-j2] + 0.5\angle -90 [2+j4]$

$= 6 - j3 \text{ V}$

② $Z_{Th} = 4-j2 + 2+j4 = 6+j2 \Omega$

③ $I_{Norton} = \frac{V_{Th}}{Z_{Th}} = \frac{6-j3}{6+j2} \text{ A}$

EX: find thevenin:



$V_{Th} = V_{oc}$

$V_{oc} = [(10+j5) \parallel -j5] 3\angle 30$

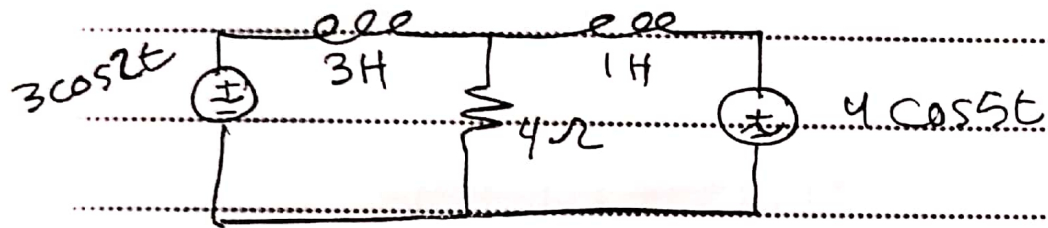
$= 16.77 \angle -33.43 \text{ V}$

$Z_{Th} = [10+j5] \parallel -j5$

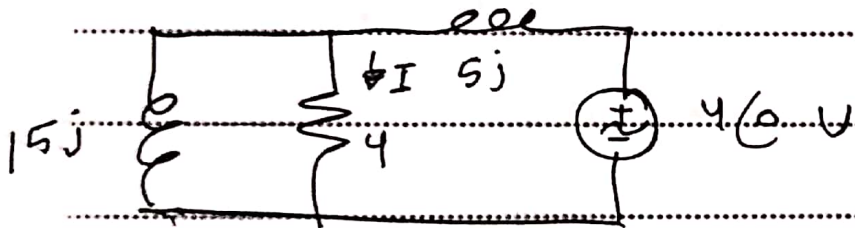
$= 5.59 \angle -63.43 \Omega$

$I_N = I_{sc} = 3\angle 30 \text{ A}$

Ex: find I ?



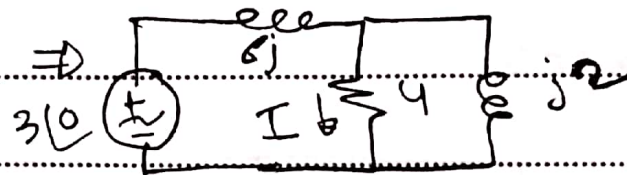
⇓



$$I = 547.1 \angle -43.15 \text{ mA}$$

$$I = I_{\downarrow} + I_{\downarrow}$$

$3 \cos 2t$ $4 \cos 5t$



$$I = \frac{V}{Y} = \frac{1}{4} \left[\frac{j2 \parallel 4}{4 \parallel j2 + 6j} \right] [3 \angle 0]$$

$$= 175.6 \angle -20.56 \text{ mA}$$

$$i(t) = 175.6 \cos(2t - 20.56)$$

$$+ 547.1 \cos(5t - 43.15) \text{ mA}$$