تقدم لجنة ElCoM الاكاديمية

دفتر لمادة:

استقرارية أنظمة القوى

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Sunday: 4-2-2018

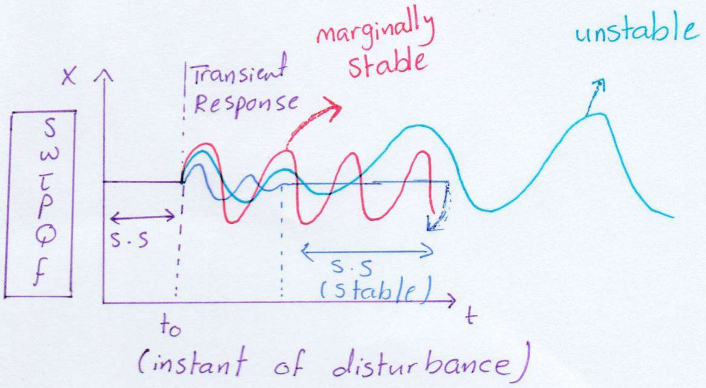
"Power System Stability"

- The aim of power system stability studies is to evaluate the impact of disturbances on the electromechanical systems in power systems.

↳ Synch. Gen

Large-Signal Stability Problem.

Small-Signal stability Problem.



(Transient)

Time domain Analysis

Non-Linear dynamical Model

* Large Signal Stability Problem: Studying the ability of the power system to maintain synchronisim after Large disturbance.

Large disturbance Examples: Symmetrical 3-Ø faults, Lightening
step change in AVR gain, step change in TG gain
step change in P demand, step change in Q demand

(Steady state)

Frequency domain Analysis

And / OR

Time domain Analysis

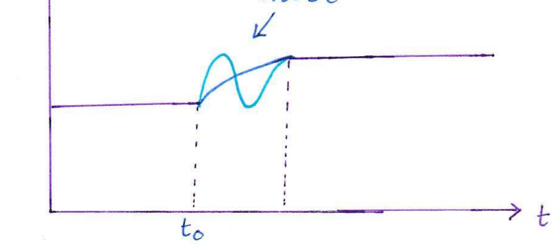
Linearization around the operating pt.

* Small Signal Stability Problem: studying the ability of the power system to maintain synchronisim after small disturbance.

Small disturbance Examples: Small variation in the parameters of the power system (P, Q, ω, I, V)

"Power System Stability"

Tuesday: 6-2-2018

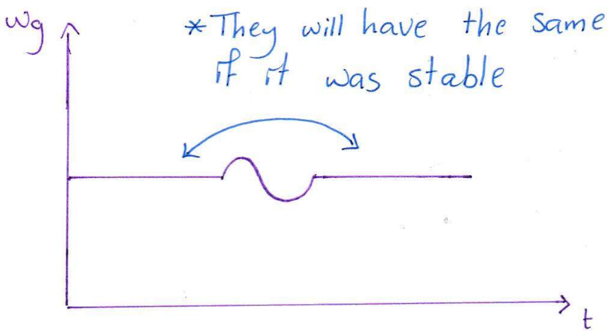


Could be one as these } defined by the constants of the equation (V, X, R)

t_o instant of disturbance

δ_g typically = 15°-60° , $w_g = w_s = 377$ rad/s $\Rightarrow$ $P = 2$
$f = 60$ Hz

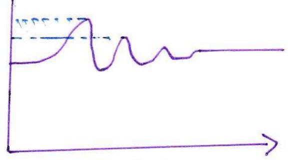
, $w_g = w_s = 157$ rad/s $\Rightarrow$ $P = 4$
$f = 50$ Hz



*They will have the same speed if it was stable

* First Swing Stable :

1st peak > 2nd peak



Stable

1^{st} peak < 2^{nd} peak

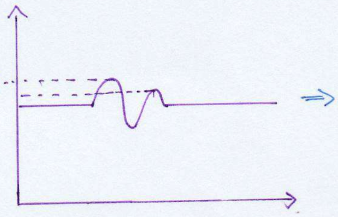
unstable

* If $\delta > 90°$ $\longrightarrow$ (unstable)

"Power System Stability"

Thursday: 8-2-2018

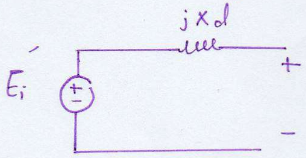
* Transient stability studies involve large-disturbances and therefore Linearization around the O.P is not permitted.



1st peak in the system response

→ Transiently stable system

* First swing transient stability: It uses a simple generator model which is E_i' (transient internal voltage) behind a transient reactance X_d'.



⇒ S.G eq. circuit during transient stability (1st swing).

-ve feedback electrical control system

* The dynamics of excitation system (AVR) and the dynamics of the Turbine Governor (TG) may or may not be included in the stability studies.

may not ⇒ For simplifying the analysis.

TG ↳ -ve feedback mechanical control system

* To facilitate computations, three fundemental assumptions are made in all stability studies:

[1] Only synchronous frequency currents & voltages are considered (.i.e Harmonics & DC offset values are neglected).

[2] Symmetrical components are used in the representations of unbalanced faults.

[3] Internal generated voltages are assumed unaffected by machine speed variations.

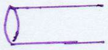
Thursday: 8-2-2018

*Damper windings effect on the stability studies are also neglected.

"Rotor Dynamics & Swing Equations"

$$J \frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e \Rightarrow \text{Newton's } 2^{nd} \text{ Law}$$

J: Total moment of inertia (Prime Mover + gen.)



$J = \frac{1}{2} m r^2$

θ_m: Angular displacement with respect to stationary axis

$\theta \rightarrow$ $\theta = \omega t$

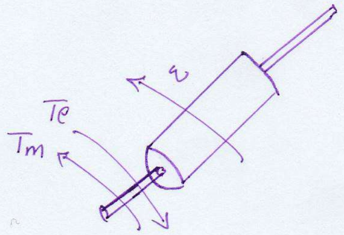
$v = \frac{d}{t}$

T_a: Accelerating Torque , $T_a = 0 \rightarrow$ in steady state ($T_m = T_e$)

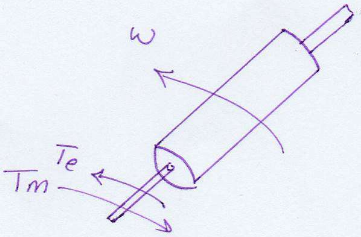
T_m: Mechanical input torque (Prime Mover)

T_e: Electromagnetic output torque $= \frac{P_{out}}{\omega_s}$

generator



Motor



"Rotor Dynamics & Swing Equations"

Sunday: 11-2-2018

* T_m is considered constant at any operating conditions and it is controlled by the governor which has relatively long time constant (1-2) s

* T_e → Corresponding to P_e → $T_e = \frac{P_e}{w_s}$

* P_e is the output electrical power from the synch. Gen.

$$P_e = \sqrt{3}\, V_L I_L \cos\phi \quad (W)$$

* During the stability studies, $\underline{\underline{3I^2R}}$ are neglected.

Copper Losses in the stator windings.

* θ_m is measured with respect to stationary reference axis on the stator, it's contineously increasing with time.

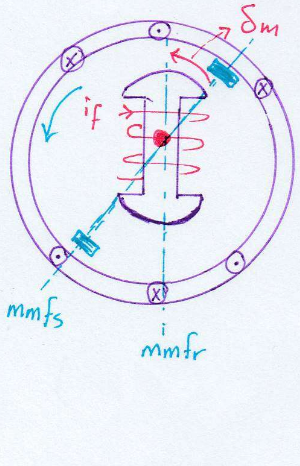
$\theta_m = \omega_{sm} t + \delta_m$, where δ_m: Load angle → increasing with the increase of the Load.

Rotor angle

Torque angle

Power angle

* δ_m = Zero at no Load



$$P_{out} = \frac{3 V_\phi E_A \sin\delta_m}{X_s}$$, where δ_m is:

The Phase difference between the induced and the terminal Voltages

$$\tilde{E}_A = \tilde{V}_\phi + \tilde{I}_A (R_A + jX_s)$$

$$E_A \angle\delta_m = V_\phi \angle 0° + \tilde{I}_A (R_A + jX_s)$$

"Rotor Dynamic & Swing Equations"

Sunday: 11-2-2018

$$\theta_m = \omega_m t + \delta_m \Rightarrow \frac{d\theta_m}{dt} = \omega_m + \frac{d\delta_m}{dt} \Rightarrow \frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

$$J\frac{d^2\delta_m}{dt^2} = T_a = T_m - T_e$$

$$\omega_m = \frac{d\theta_m}{dt} \Rightarrow \omega_m J\frac{d^2\delta_m}{dt^2} = \omega_m T_a = \omega_m T_m - \omega_m T_e$$

$$\underbrace{J\omega_m}_{\text{Angular moment of inertia }(M)}\frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e - \underset{3I^2R + P_{iron}}{\underline{\underline{P_{Losses}}}} \rightarrow \text{Neglected.}$$

$$M = J\omega_m = J\omega_s \Rightarrow \text{Constant inertia}$$

$$M\frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e$$

* In machines data sheet (manual), Another constant referred to inertia constant is used:

$$H = \frac{\text{Stored energy in Mega Joule at } \omega_s}{(\text{MVA})_{\text{rating}}} \quad (\text{Second})$$

$$H = \frac{\frac{1}{2}J\omega_{sm}^2}{S_{mach}} = \frac{\frac{1}{2}M\omega_{sm}}{S_{mach}} \Rightarrow M = \frac{2HS_{mach}}{\omega_{sm}}$$

$$\frac{2HS_{mach}}{\omega_{sm}}\cdot\frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e$$

Dividing by Smach: $\frac{2H}{\omega_s}\cdot\frac{d^2\delta_m}{dt^2} = \frac{P_a}{S_{mach}} = \frac{P_m}{S_{mach}} - \frac{P_e}{S_{mach}}$

Sunday: 11-2-2018

"Rotor Dynamic & Swing Equations"

$$\frac{2H}{w_s} \cdot \frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e \quad (PU) \Rightarrow$$ Swing equation of synch. Gen.

* δ_m could be in rad or in degree

$$\frac{2H}{2\pi f} \cdot \frac{d^2\overset{\rightarrow \text{ in rad}}{\delta_m}}{dt^2} = --$$

$$\frac{2H}{2(180)f} \cdot \frac{d^2\overset{\rightarrow \text{ in degree}}{\delta_m}}{dt^2} = --.$$

$$\boxed{\underbrace{\boxed{\frac{2H}{w_s}}}_{\text{Constant}} \cdot \frac{d^2\delta}{dt^2} = \underbrace{\boxed{P_m}}_{\text{Constant}} - \boxed{P_e}} \rightarrow P_e = \frac{V_\phi E_A}{X_s} \cdot \sin\delta$$

"2nd order non-linear differential equation"

* In state space representation:

$$\left.\begin{aligned} \frac{2H}{w_s} \cdot \frac{dw}{dt} &= P_m - P_e \\ \frac{d\delta}{dt} &= w - w_s \end{aligned}\right\} \qquad \begin{aligned} \frac{dw}{dt} &= \frac{d^2\delta}{dt^2} \\ w &= \frac{d\delta}{dt} \end{aligned}$$

"Further Considerations of Swing Equation"

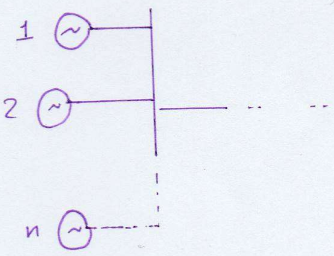
Tuesday: 13-2-2018

* In case of several synch. gen. only one MVA base common to all Parts is chosen to convert H from the machine base to system base:

$$H_{system} = H_{mach.} \frac{S_{mach}}{S_{system}}$$

* It is desired to minimize the total no. of swing equations.

1) Coherent Machines:



$$\frac{2H_1}{w_s} \cdot \frac{d^2\delta_1}{dt^2} = P_{M_1} - P_{e_1}$$

$$\frac{2H_2}{w_s} \cdot \frac{d^2\delta_2}{dt^2} = P_{m_2} - P_{e2}$$

* Equivalent swing equation for gen.1 & gen.2

$$\frac{2H}{w_s} \cdot \frac{d^2\delta}{dt^2} = P_m - P_e \quad , \quad H = H_1 + H_2$$

$P_m = P_{m1} + P_{m2}$

$P_e = P_{e1} + P_{e2}$

$\delta_1 = \delta_2 = \delta \Rightarrow$ Variation of δ

$w = w_1 = w_2 \Rightarrow$ Variation of w

* It is necessary to unify all the parameters of the machine on 1 base

Tuesday: 13-2-2018

"Further Considerations of Swing Equation"

Ex] 60 Hz power station has two units:

Unit #1: 500 MVA, 0.8 PF, 20 KV, 3600 rpm, $H_1 = 4.8\ S = 4.8\ MJ/MVA$

unit #2: 1333 MVA, 0.9 PF, 22 KV 11800 rpm, $H_2 = 3.27\ S = 3.27\ MJ/MVA$

calculate the equivalent $H_{constant}$ for the two machines on a 100 MVA base.

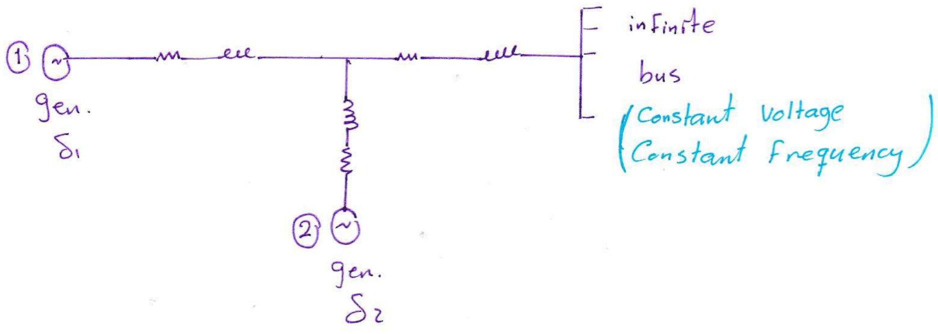
Kientic Energy = H×S

$$KE = (4.8 \times 500) + (3.27 \times 1333) = 6759\ MJ$$

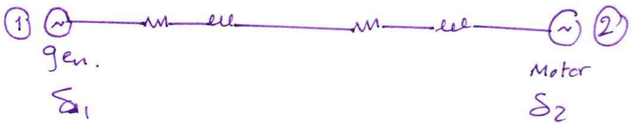
$$H_{eq} = \frac{K_E}{S_{base}} = \frac{6759}{100} = 67.59\ S$$

2) Non-Coherent Machines:

CASE A



CASE B



Equivalent Swing Equation:- for Case A

$$\frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2} = \frac{w_s}{2}\left[\frac{P_{m1} - P_{e1}}{H_1} - \frac{P_{m2} - P_{e2}}{H_2}\right]$$, Multiplying by $\frac{H_1 H_2}{H_1 + H_2}$

$$\frac{2H_{12}}{w_s} \cdot \frac{d^2\delta_{12}}{dt^2} = P_{m12} - P_{e12}$$, where:

$$H_{12} = \frac{H_1 H_2}{H_1 + H_2}, \quad \delta_{12} = \delta_1 - \delta_2$$

$$P_{m12} = \frac{P_{m1} H_2 - P_{m2} H_1}{H_1 + H_2}, \quad P_{e12} = \frac{P_{e1} H_2 - P_{e2} H_1}{H_1 + H_2}$$

Tuesday: 13-2-2018

"Further Considerations of Swing Equation"

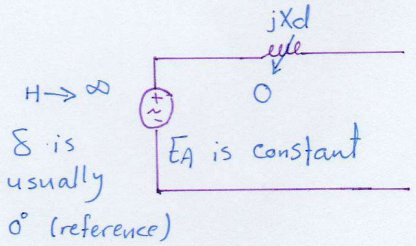
* For <u>case B</u> any change in the gen. output is absorbed by the motor.

$$\therefore P_{m1} = -P_{m2}$$

$$P_{e1} = -P_{e2}$$

* The stability studies are always relative, such that the oscillations are measured with respect to certain reference which is normally the infinite bus charectrized by constant internal voltage, zero impedance and <u>infinite inertia</u>.

* Equivalent cct. of the infinite bus



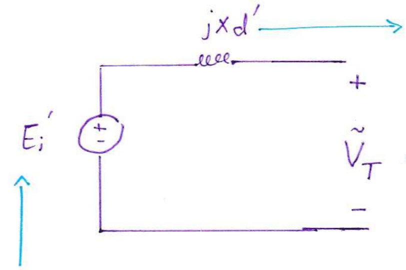
Thursday: 15.2.2018

"The Power Angle Equation"

* P_m is considered constant for all running conditions and therefore P_e determines whether the rotor accelerates, decelerates or remain at synch. speed.

* Changes on P_e are determined by conditions on the transmission & distribution networks and the loads.

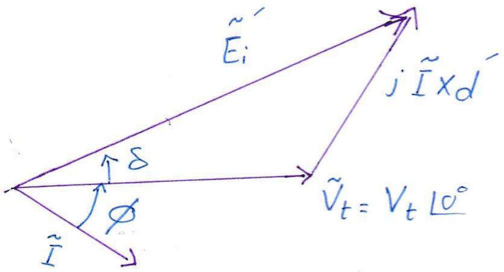
* Each generator is represented by:



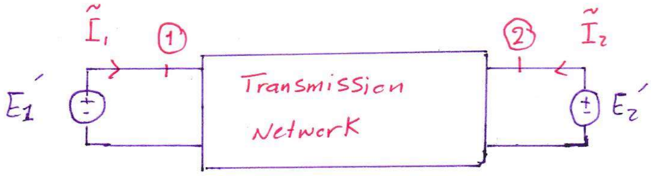
→ Transient reactance of the generator (given)

⇒ Armature winding resistance is neglected.

Transient internal generated voltage (constant)



$$\Rightarrow \tilde{E}_i' = \tilde{V}_t + j\tilde{I}X_d'$$



⇒ N = 2 (no. of buses)

$$\Rightarrow P_K + jQ_K = \tilde{V}_K \sum_{n=1}^{N} \Big(\underset{\text{Admittance}}{\underline{\underline{\tilde{Y}_{Kn}}}} \, \tilde{V}_n \Big)^*$$

⇒ Let K = 1, N = 2 :

$$P_1 + jQ_1 = \tilde{E}_1' \left(\tilde{Y}_{11} \tilde{E}_1' \right)^* + \tilde{E}_1' \left(\tilde{Y}_{12} \tilde{E}_2' \right)^*$$

"The Power Angle Equation"

Thursday: 15-2-2018

$\Rightarrow \tilde{E}_1' = |\tilde{E}_1| \angle \delta_1$, $\tilde{E}_2' = |\tilde{E}_2| \angle \delta_2$

$\Rightarrow \tilde{Y}_{11} = G_{11} + jB_{11} : |\tilde{Y}_{11}| \angle \theta_{11}$

$\Rightarrow \tilde{Y}_{12} = |\tilde{Y}_{12}| \angle \theta_{12}$

$$\Rightarrow \tilde{Y}_{bus} = \begin{bmatrix} \tilde{Y}_{11} & \tilde{Y}_{12} \\ \tilde{Y}_{21} & \tilde{Y}_{22} \end{bmatrix}$$

$\Rightarrow P_1 = |\tilde{E}_1'|^2 G_{11} + |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}| \cos(\delta_1 - \delta_2 - \theta_{12})$ → Power Angle Eq.

$\Rightarrow Q_1 = -|\tilde{E}_1'|^2 B_{11} + |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}| \sin(\delta_1 - \delta_2 - \theta_{12})$

⇒ let $\delta = \delta_1 - \delta_2$ & $\gamma = \theta_{12} - \frac{\pi}{2}$

$\rightarrow P_1 = |\tilde{E}_1'|^2 G_{11} + |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}| \sin(\delta - \gamma)$ → Power Angle Eq.

$\rightarrow Q_1 = -|\tilde{E}_1'| B_{11} + |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}| \cos(\delta - \gamma)$

$$\boxed{P_1 = P_c + P_{max} \sin(\delta - \gamma)}$$

where: $P_c = |\tilde{E}_1'|^2 G_{11}$, $P_{max} = |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}|$

---

* Let $N = \underline{\underline{3}}$:-

$P_1 = |\tilde{E}_1'|^2 G_{11} + |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}| \sin(\delta_1 - \delta_2 - \gamma_{12}) + |\tilde{E}_1'||\tilde{E}_3'| \sin(\delta_1 - \delta_3 - \gamma_{13}) . |\tilde{Y}_{13}|$

$P_2 = |\tilde{E}_2'|^2 G_{22} + |\tilde{E}_2'||\tilde{E}_1'||\tilde{Y}_{21}| \sin(\delta_2 - \delta_1 - \gamma_{21}) + |\tilde{E}_2'||\tilde{E}_3'||\tilde{Y}_{23}| \sin(\delta_2 - \delta_3 - \gamma_{23})$

$P_3 = |\tilde{E}_3'|^2 G_{33} + |\tilde{E}_3'||\tilde{E}_1'||\tilde{Y}_{31}| \sin(\delta_3 - \delta_1 - \gamma_{31}) + |\tilde{E}_3'||\tilde{E}_2'||\tilde{Y}_{32}| \sin(\delta_3 - \delta_2 - \gamma_{32})$

---

* P_c , P_{max} & γ are constants for given network configuration.

* $\tilde{E}_1'$ & $\tilde{E}_2'$ are assumed to be constant.

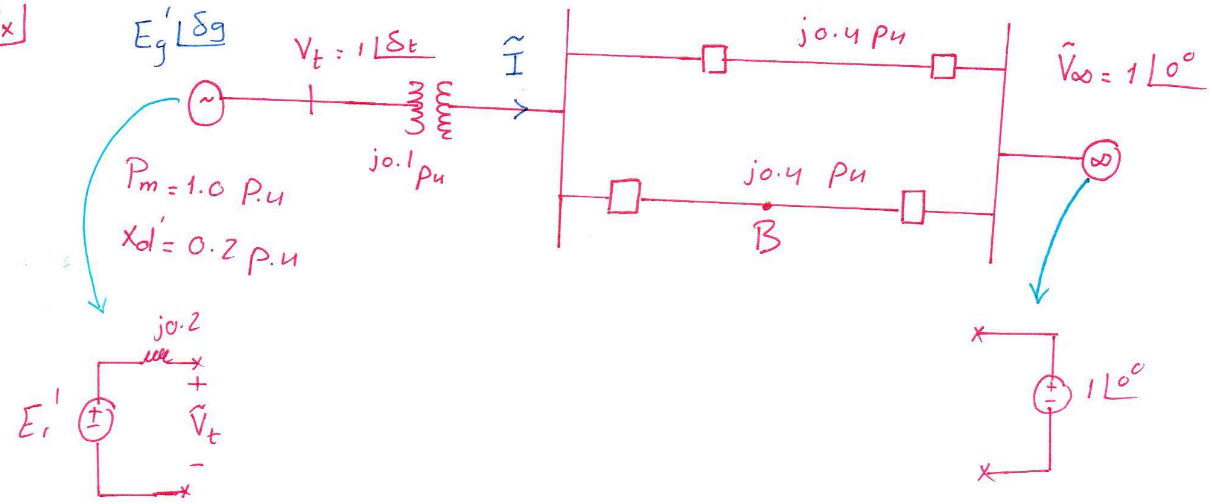
* If the network is considered without resistances ⇒ $G_{11} = 0$ ∴ $P_c = 0$ & $\gamma = 0$

$\rightarrow P_1 = P_{max} \sin\delta \Rightarrow$ where $P_{max} = \dfrac{|\tilde{E}_1'||\tilde{E}_2'|}{X_{12}}$ → Transfer reactance.

" The Power Angle Equation "

Thursday: 15-2-2018

Ex



Determine the power angle equation ?

1) By KVL :

$$\overset{?}{E_g'} = \tilde{V}_\infty + \overset{?}{\tilde{I}}(j0.2 + j0.1 + j0.2)$$

&

$$\tilde{I} = \frac{\tilde{V}_t - \tilde{V}_\infty}{j0.3} = \frac{1 \angle \overset{?}{\delta_t} - 1 \angle 0°}{j0.3}$$

∴ Can't be solve.

2) using :

$$P_m = P_e = \frac{|\tilde{E}'| \, |\tilde{V}_\infty| \cdot \sin\delta_g}{0.2 + 0.1 + 0.2}$$

$$1 = \frac{\overset{?}{|\tilde{E}'|} \, |1| \, \sin \overset{?}{\delta_g}}{0.5 \leftarrow x_T}$$

∴ Can't be solve.

3) using :

$$P_m = P_e = P_c + P_{max} \cdot \sin(\delta - \gamma)$$

if $R = 0 \Rightarrow P_c = 0$, $\gamma = 0$

∴ $P_e = P_{max} \cdot \sin\delta$

$$\Rightarrow P_e = \frac{|\tilde{V}_t| \, |\tilde{V}_\infty|}{0.2 + 0.1} \cdot \sin\delta_t \Rightarrow 1 = \frac{(1)(1)}{\underset{x_T \rightarrow}{0.2 + 0.1}} \cdot \sin\delta_t$$

∴ $\delta_t = 17.458°$

"The Power Angle Equation" Sunday: 18-2-2018

$$\Rightarrow \tilde{I} = \frac{1\angle 17.458^\circ - 1\angle 0^\circ}{j0.3} = 1.012\angle 8.729^\circ \text{ p.u}$$

$$\Rightarrow E_g' = 1\angle 0^\circ + (1.012\angle 8.729^\circ)(j0.5)$$

$$= 1.05\angle 28.44^\circ \text{ p.u} = |E_i'|\angle \delta g$$

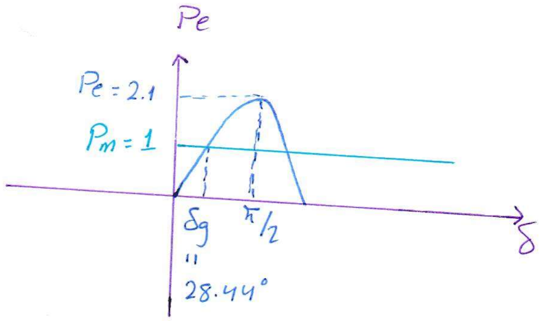
⇒ Power Angle Eq:-

$$P_e = P_c + P_{max} \cdot \sin(\delta - \gamma)$$

$$P_e = 0 + \frac{|\tilde{E}'||\tilde{V}_\infty|}{x_T} \sin(\delta g - 0)$$

$$P_e = \frac{(1.05)(1)}{0.5} \cdot \sin \delta g$$

$$P_e = 2.10 \sin \delta g$$



⇒ Swing Eq:

$$\frac{H}{180f} \cdot \frac{d^2\delta g}{dt^2} = P_m - P_e$$

$$\frac{H}{180f} \cdot \frac{d^2\delta g}{dt^2} = 1 - 2.1 \sin \delta g$$

⇒ other equations:-

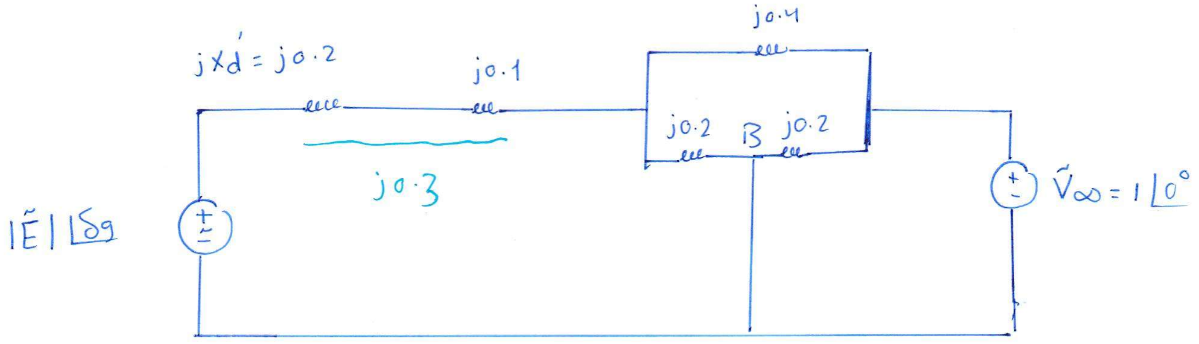
$$P_m = P_e = \frac{|\tilde{E}'||\tilde{V}_t|}{0.2} \cdot \sin(\delta g - \delta t) \quad \checkmark$$

$$P_e = |\tilde{E}_1'|^2 \underset{0}{\cancel{G_{11}}} + \frac{|\tilde{E}_1'||\tilde{E}_2'|}{|\tilde{Z}_{12}|} \sin(\delta_1 - \underset{0}{\cancel{\delta_2}} - \underset{0}{\cancel{\gamma_{12}}}) \leftarrow$$

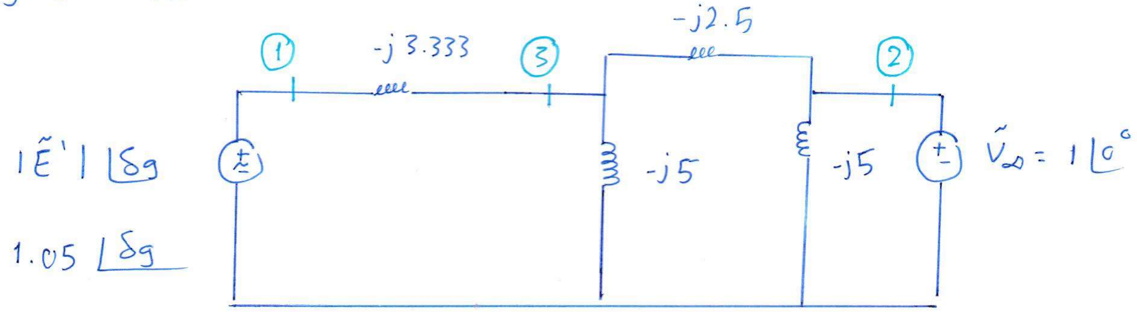
general Equation
using: Bus 1 @ Vt
Bus 2 @ ∞ bus

Ex] For the previous example a 3_ϕ symmetrical Fault occurs at pt. B of the transmission line, determine the power angle equation For the system with the fault and the swing equation, using H = 5 seconds

⇒ Equivalent cct :-



Redrawing:
using admittances



* $\tilde{Y}_{bus}$ matrix during Fault :-

$$Y_{bus} = \begin{bmatrix} -j3.333 & 0 & j3.333 \\ 0 & -j7.5 & j2.5 \\ j3.333 & j2.5 & -j10.833 \end{bmatrix}_{3\times 3}$$

* Bus #3 must be eliminated using Kron reduction as its Voltage is unknown by assumption.

"Power Angle Equation"

Tuesday: 20-2-2018

$$\tilde{Y}_{jK}(\text{new}) = \tilde{Y}_{jK}(\text{old}) - \frac{\tilde{Y}_{jP}\,\tilde{Y}_{PK}}{\tilde{Y}_{PP}} \quad , \quad P = \underline{\underline{3}}$$

$$\tilde{Y}_{11}(\text{new}) = -j3.333 - \frac{(j3.333)(j3.333)}{-j10.833} = -j2.308$$

$$\tilde{Y}_{12}(\text{new}) = 0 - \frac{(j3.333)(j2.5)}{-j10.833} = j0.769$$

$$\tilde{Y}_{21}(\text{new}) = j0.769$$

$$\tilde{Y}_{22}(\text{new}) = -j6.923$$

$$\Rightarrow \quad \tilde{Y}_{\text{bus, reduced}} = \begin{bmatrix} -j2.308 & j0.769 \\ j0.769 & -j6.923 \end{bmatrix}$$

* Power Angle Equation

$$\Rightarrow P_1 = |\tilde{E}_1'|^2 G_{11} + |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}| \cdot \sin(\delta_1 - \delta_2 - \gamma_{12})$$

$$P_1 = 0 + (1.05)(1)(0.769) \cdot \sin(\delta g - 0 - 0)$$

$$P_{e_1} = 0.808 \cdot \sin(\delta g)$$

* Swing Equation during Fault:

$$\frac{5}{180f} \cdot \frac{d^2 \delta g}{dt^2} = \overset{\text{Pm is Constant}}{1.0} - 0.808 \cdot \sin(\delta g)$$

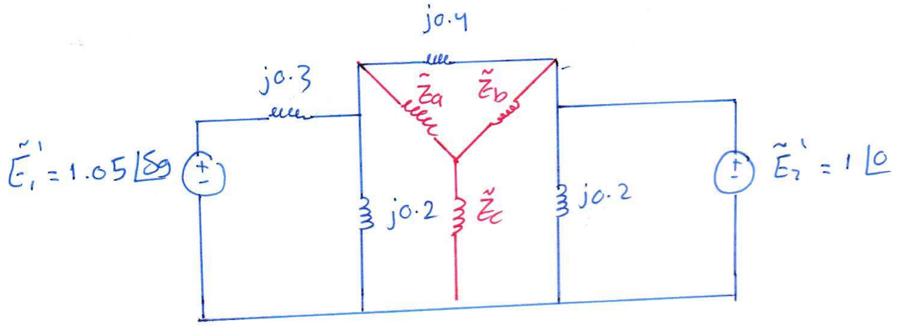
* The method of stability Analysis using the Ybus matrix is always Correct for any power system Configuration.

"Power Angle Equation"

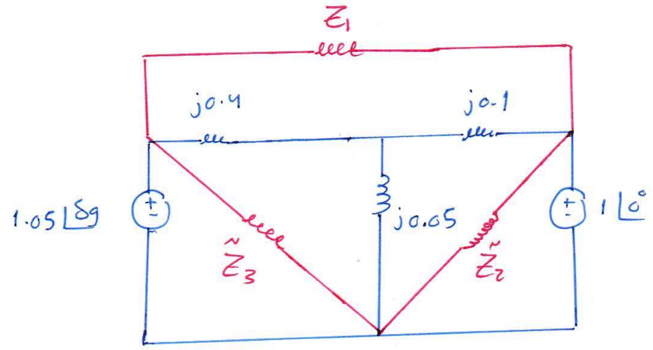
Tuesday: 20-2-2018

OR: If the configuration allows to have the transfer reactance, then:-

(Back to Example)



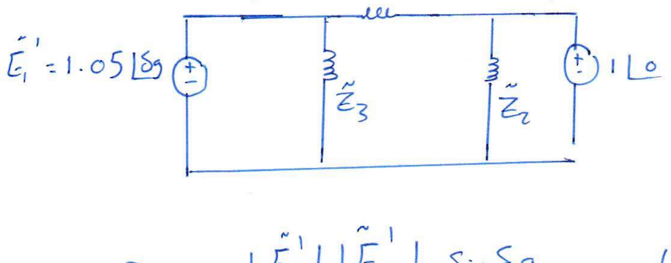
$\tilde{Z}_a = j0.1$, $\tilde{Z}_b = j0.1$, $\tilde{Z}_c = j0.05$



$\tilde{Z}_1 = j1.3$

$\tilde{Z}_2 =$ --

$\tilde{Z}_3 =$ --



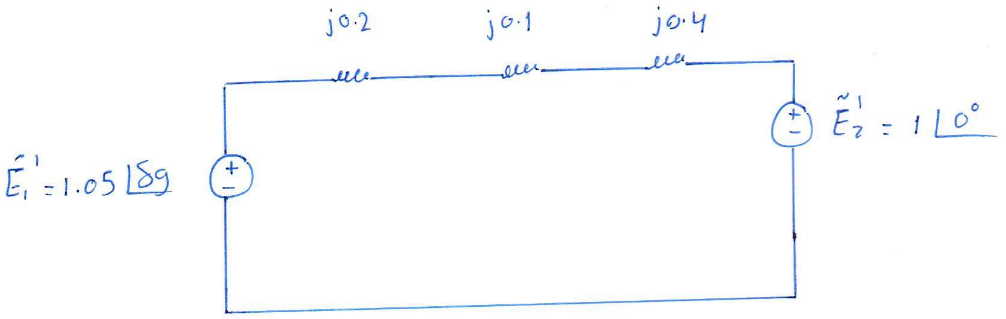
$$\Rightarrow P_{e_1} = \frac{|\tilde{E}_1'|\,|\tilde{E}_2'|\,.\sin\delta_g}{X_T} = \frac{(1.05)\,(1)}{1.3}\,.\sin\delta_g$$

$$P_{e_1} = 0.808 \sin\delta_g$$

"Power Angle Equation"

Tuesday: 20-2-2018

Ex) The faulted line of the previous example is opened from its two ends, calculate the power angle equation for the post fault period?



$$P_{e_1} = \frac{(1.05)(1)}{0.2+0.1+0.4} \cdot \sin \delta g$$

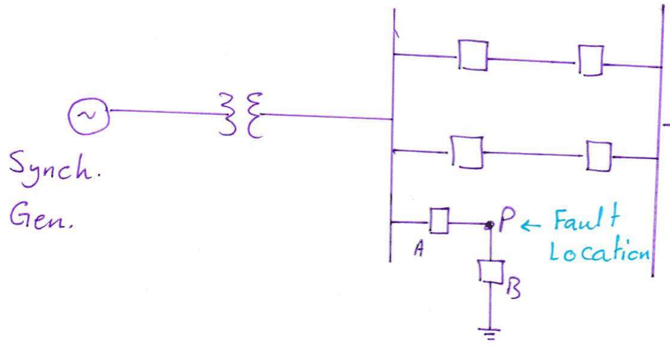
$$P_{e_1} = 1.5 \cdot \sin \delta g \leftarrow \text{Power Angle Equation}$$

$$\frac{5}{180f} \cdot \frac{d^2\delta g}{dt^2} = 1 - 1.5 \sin \delta g \leftarrow \text{Swing Equation}$$

"Equal Area Criterion of Stability"

Thursday: 22_2_2018

* A swing equation is a non_Linear 2^{nd} order differential equation.
* Analytical solution is not possible except in special cases
* Numerical solution is always possible.
* Equal area criterion is an analytical approach.



→ Initially, assume all circuit breakers are "on" except <u>B</u>

* Pre the Fault:

$$P_m = P_e = P_{max} \sin\delta$$

$$\delta_0 = \sin^{-1}\left[\frac{P_m}{P_{max}}\right]$$

* During Fault:

$$P_e = 0 \rightarrow \delta = 0$$

$$\frac{2H}{w_s} \cdot \frac{d^2\delta}{dt^2} = P_m - P_e \rightarrow P_m > P_e$$

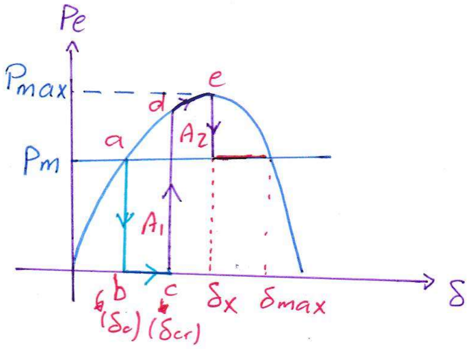
∴ Acceleration

* Post Fault:

$P_e > P_m$ ∴ descceleration

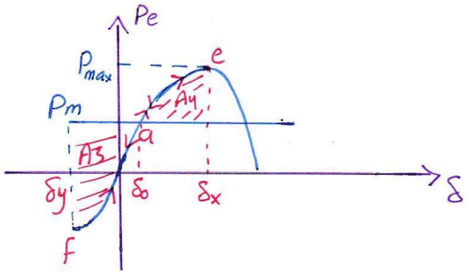
* New steady-state:

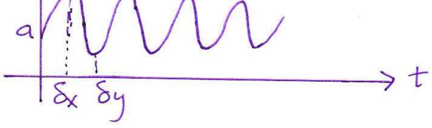
$$P_m = P_e$$



* Lossless System (no damping parameters)

$R = 0$ $K_D = 0$





"Equal Area Criterion of Stability"

Thursday:22-2-2018

* During Fault:

$$\frac{2H}{w_s} \cdot \frac{d^2\delta}{dt^2} = P_m$$

increase in speed.

$$\frac{d^2\delta}{dt^2} = \frac{w_s}{2H} \cdot P_m \rightarrow \quad \underline{\underline{w}} = \frac{d\delta}{dt} = \int_0^t \frac{d^2\delta}{dt^2} \cdot dt$$

$$\therefore \quad w = \int_0^t \frac{w_s}{2H} \cdot P_m \, dt = \frac{w_s}{2H} \cdot P_m \cdot t$$

$$\rightarrow \quad \delta = \int \frac{d\delta}{dt} \cdot dt = \int \frac{w_s}{2H} \cdot P_m \cdot \underline{\underline{t}} \, dt$$

$$\delta = \frac{w_s}{2H} \cdot P_m \cdot \frac{t^2}{\underline{\underline{2}}} + \delta_0$$

---

* In steady-state representation:

1) $w_r = \frac{d\delta}{dt} = w - w_s \rightarrow$ Speed.

2) $\frac{2H}{w_s} \cdot \frac{dw_r}{dt} = P_m - P_e \rightarrow$ Rotor angle.

$\rightarrow$ [Multiplying $\boxed{2}$ by $w_r = \frac{d\delta}{dt}$]

$$\frac{2H}{w_s} \cdot w_r \cdot \frac{dw_r}{dt} = (P_m - P_e) \cdot \frac{d\delta}{dt} \Rightarrow \frac{2H}{w_s} \cdot \frac{1}{2} \frac{d(w_r^2)}{dt} = (P_m - P_e) \frac{d\delta}{dt}$$

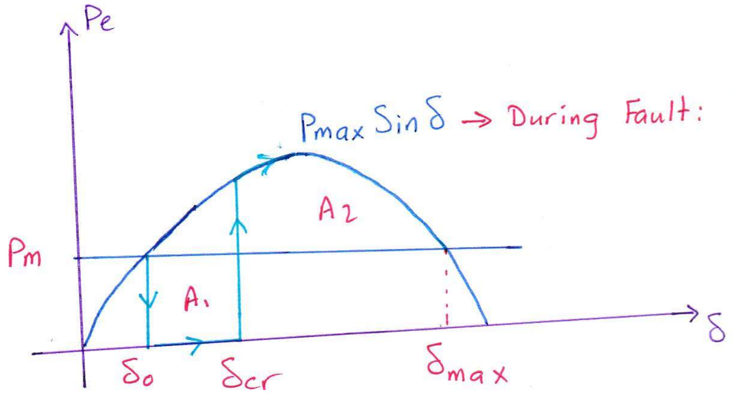
$\rightarrow$ [Multiplying by dt]

$$\frac{H}{w_s} \cdot \int_{w_{r_1}}^{w_{r_2}} d(w_r^2) = \int_{\delta_1}^{\delta_2} (P_m - P_e) \, d\delta$$

$$\Rightarrow \frac{H}{w_s} \left[w_{r_2}^2 - w_{r_1}^2 \right] = \int_{\delta_1}^{\delta_2} (P_m - P_e) \, d\delta$$

"Equal Area Criterion of Stability"

Sunday: 25_2_2018



$\rightarrow$ During Fault: $P_m = \frac{E'V}{X} = \underline{\underline{0}}$

$\rightarrow A_1 = A_2$

$$\int_{\delta_o}^{\delta_{cr}} P_m \cdot d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_{max} . \sin\delta - P_m) d\delta$$

$$P_m(\delta_{cr} - \delta_o) = P_{max} \cdot \cos\delta \Big]_{\delta_{max}}^{\delta_{cr}} - P_m \Big]_{\delta_{cr}}^{\delta_{max}}$$

$$\cos\delta_{cr} = \frac{P_m}{P_{max}} (\delta_{max} - \delta_o) + \cos\delta_{max} \Rightarrow \delta_{cr} = \_\_\_$$

$\vdots$

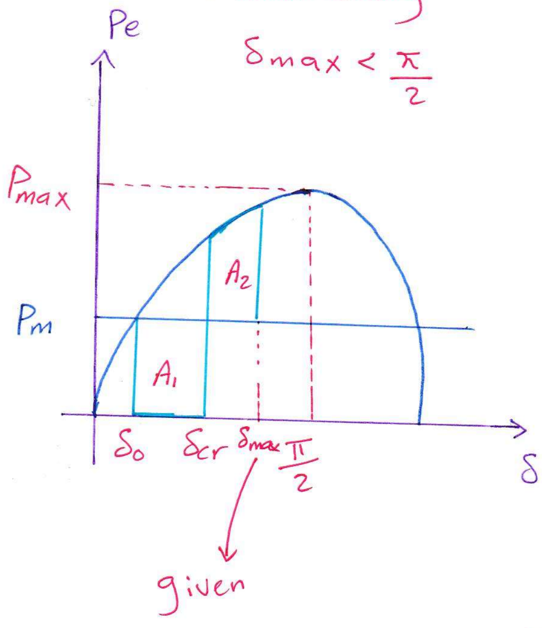
$$\delta(t) = \frac{w_s P_m}{4H} . t^2 + \delta_o$$

$$\delta_{cr} = \frac{w_s . P_m}{4H} . t_{cr}^2 + \delta_o$$

$$t_{cr} = \sqrt{\frac{4H(\delta_{cr} - \delta_o)}{w_s . P_m}}$$

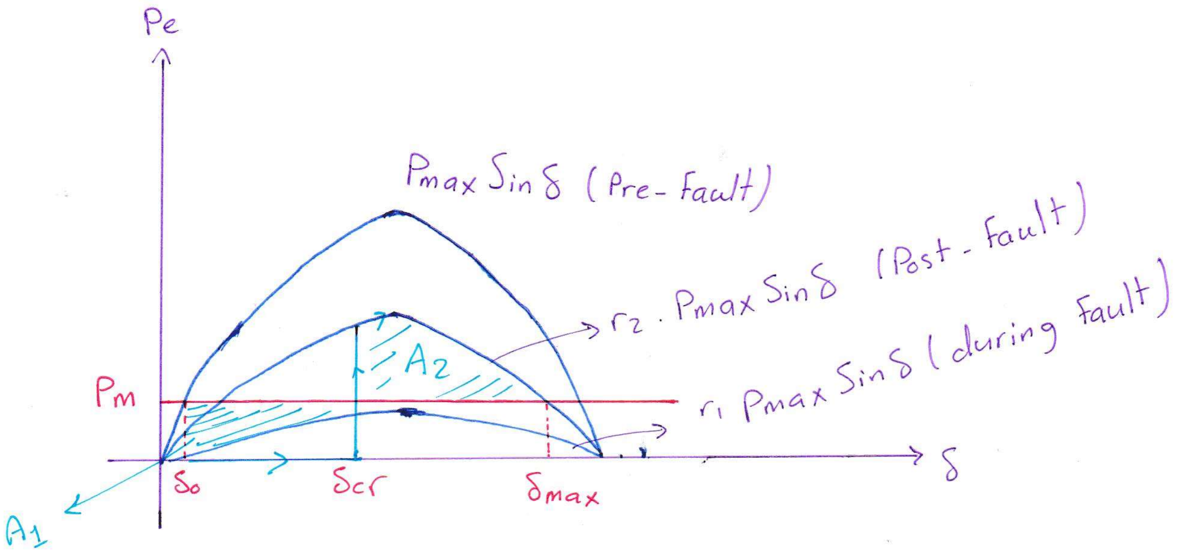
* Practically

$\delta_{max} < \frac{\pi}{2}$



"Further Applications of The Equal Area Criterion"

Tuesday: 27-2-2018



$$P_e = P_{max} . \sin\delta = P_m$$

$$\Rightarrow \delta_o = \sin^{-1}\left[\frac{P_m}{P_{max}}\right]$$

$$\Rightarrow \delta_{max} = \sin^{-1}\left[\frac{P_m}{r_2 P_{max}}\right]$$

$$\Rightarrow A_1 = A_2$$

$$\int_{\delta_o}^{\delta_{cr}} (P_m - r_1 P_{max} \sin\delta)\, d\delta = \int_{\delta_{cr}}^{\delta_{max}} (r_2 P_{max} \sin\delta - P_m)\, d\delta$$

$$\Rightarrow \cos\delta_{cr} = \frac{\left(\frac{P_m}{P_{max}}\right)(\delta_{max} - \delta_o) + r_2 \cos\delta_{max} - r_1 \cos\delta_o}{r_2 - r_1}$$

"Stability Study Approach of Multi-Machines"

Thursday: 1-3-2018

* Multi: 3 machines or more

* To simplify the analysis of multi-machines stability studies:

1) P, Q, V, I at each bus is calculated before the fault using power flow studies.

2) $\bar{E}'$ For each generating bus is calculated using:

$$\bar{E}' = \bar{V}_t + j\bar{I}X_d'$$

3) Each Load is converted into constant admittance to ground:

$$\bar{Y}_L = \frac{1}{\bar{Z}_L} \quad \text{or} \quad \bar{Y}_L = \frac{(P_L + jQ_L)^*}{|V_L|^2} = \frac{P_L - jQ_L}{|V_L|^2}$$

* X_d' and $\bar{Y}_L$ must be included with the Y_{bus} matrix. Also the series impedance of T.L and transformers plus the shunt admittance of the T.L if given.

* The Y_{bus} matrix must be modified for the during and post fault networks.

* Power angle calculations should be done at each generating bus.

⇒ Assumptions:

1) The mechanical input power for the pre, during and post fault is constant.

2) Damping power is neglected. (mechanical damping).

3) Each machine is represented by constant transient reactance in series with constant transient internal Voltage. (jX_d' in series with $|\bar{E}'|\angle\delta$)

" Stability Study Approach of Multi-Machines"

Thursday: 1-3-2018

4) The mechanical rotor angle equals the electrical phase angle "delta".
(The angle of the internal voltage).

5) All loads are considered as shunt admittances to the ground with constant values at all conditions. (Pre / during / Post).

6) The configuration of the power system at all conditions must be known.

7) Steady-state values before the fault are calculated using load Flow study.

---

* Consider a 3 buses network:

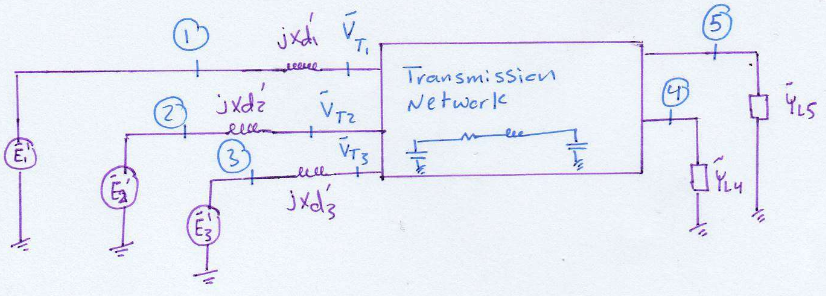
$$P_{e1} = |\bar{E}_1'|^2 . G_{11} + |\bar{E}_1'||\bar{E}_2'||\bar{Y}_{12}| \cdot \sin(\delta_1 - \delta_2 - \gamma_{12}) + |\bar{E}_1'||\bar{E}_3'||\bar{Y}_{13}| \cdot \sin(\delta_1 - \delta_3 - \gamma_{13})$$

$$P_{e2} = |\bar{E}_2'|^2 G_{22} + |\bar{E}_2'||\bar{E}_1'||\bar{Y}_{21}| \sin(\delta_2 - \delta_1 - \gamma_{21}) + |\bar{E}_2'||\bar{E}_3'||\bar{Y}_{23}| \sin(\delta_2 - \delta_3 - \gamma_{23})$$

$$\Rightarrow \gamma_{23} = \underline{\underline{\theta_{23}}} - \pi/2$$

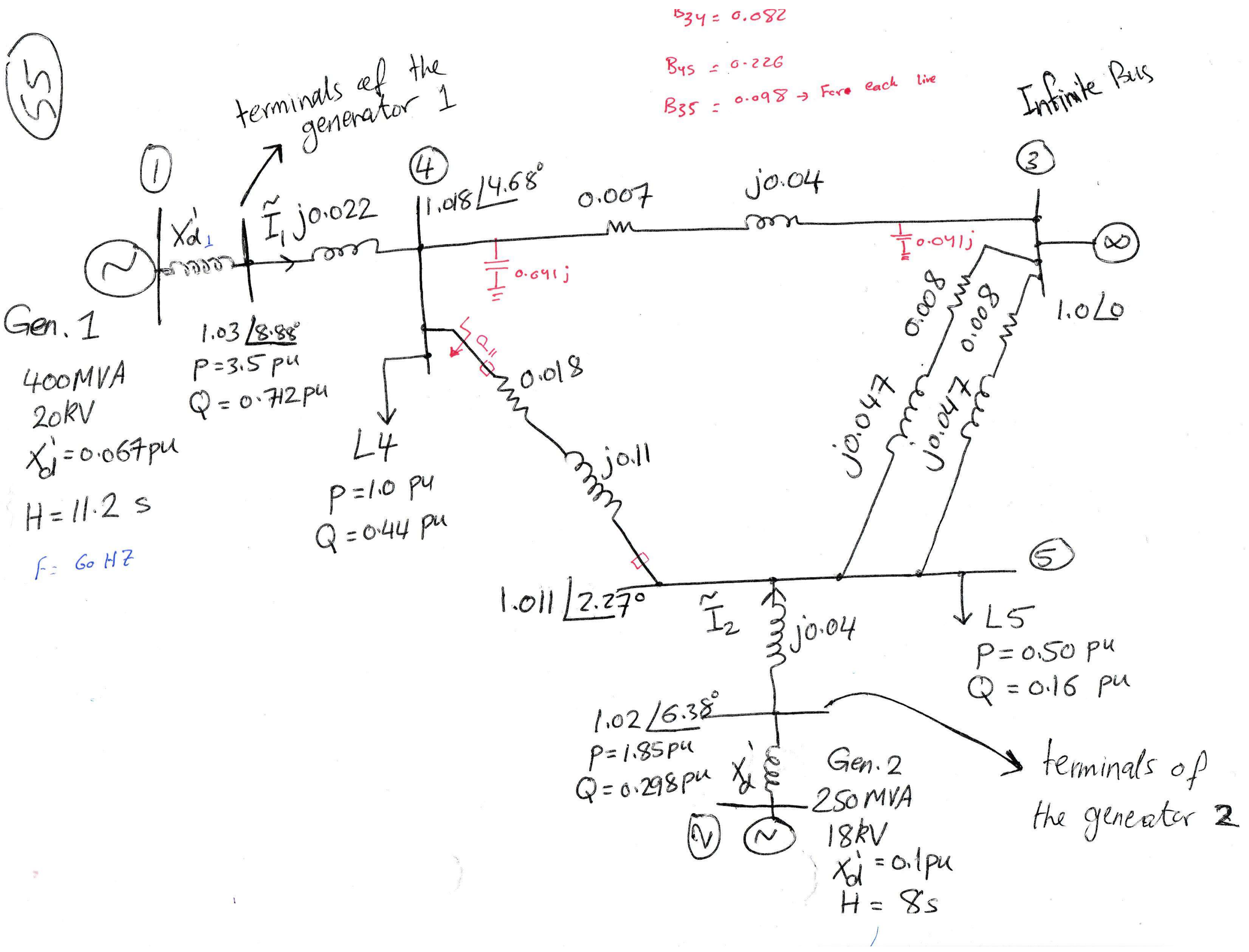
↳ The angle of $\bar{Y}_{13}$

* The swing equation for each generator bus is calculated.



\# Note: Kron reduction is used to eliminate all buses having unknown voltage by assumption. (during & Post).

\# of swing equations = # of Machines − 1


55
B34 = 0.082
B45 = 0.226
B35 = 0.098 → Fore each line
terminals of the generator 1
Infinite Bus
①
④
③
⑤
②
∞
X'_{d_1}
$\tilde{I}_1$ j0.022
1.018∠4.68°
0.007
j0.04
0.041j
0.041j
1.0∠0
Gen. 1
400MVA
20KV
X'_d = 0.067pu
H = 11.2 s
f = 60 HZ
1.03∠8.88°
P = 3.5 pu
Q = 0.712pu
L4
P = 1.0 pu
Q = 0.44 pu
$P_{\parallel}$
0.018
j0.11
0.008
0.008
j0.047
j0.047
1.011∠2.27°
$\tilde{I}_2$
j0.04
L5
P = 0.50 pu
Q = 0.16 pu
1.02∠6.38°
P = 1.85pu
Q = 0.298pu
X'_d
Gen. 2
250MVA
18KV
X'_d = 0.1pu
H = 8s
terminals of the generator 2

"Stability Study Approach of Multi- Machines"

Tuesday: 6-3-2018

* Pre - Fault Analysis :

$$\Rightarrow \tilde{E}_1' = \tilde{V}_{t1} + j I_1 X_{d1}'$$

$$\text{where , } I_1 = \frac{\tilde{V}_{t1} - \tilde{V}_4}{j0.022} = \frac{1.03 \angle 8.88^\circ - 1.018 \angle 4.68^\circ}{j0.022} = 3.468 \angle -2.619 \text{ pu}$$

$$\underline{\text{OR}} , \quad I_1 = \frac{S_1^*}{V_{t1}^*} = \frac{(P_1 + jQ_1)^*}{V_{t1}^*} = \frac{3.5 - j0.712}{1.03 \angle -8.88^\circ} = 3.468 \angle -2.619 \text{ pu}$$

$$\therefore \quad \tilde{E}_1' = 1.03 \angle 8.88^\circ + j\left[(3.468)\angle -2.619\right] \cdot \left[0.067\right] = 1.1 \angle 20.82^\circ \text{ pu}$$

$\Rightarrow \delta_1 = 20.82 \leftarrow$ Pre- Fault

$\Rightarrow \quad \tilde{E}_2' = 1.065 \angle 16.19^\circ$ pu $\leftarrow$ Calculated like $\tilde{E}_1'$

$\Rightarrow \delta_2 = 16.19^\circ \leftarrow$ Pre - Fault

$\Rightarrow \quad \tilde{E}_3' = 1 \angle 0^\circ \quad \leftarrow$ Infinite Bus

$\Rightarrow \delta_3 = 0^\circ$

$$\Rightarrow \delta_{13} = \delta_1 - \delta_3 = \delta_1$$

$$\Rightarrow \delta_{23} = \delta_2 - \delta_3 = \delta_2$$

$$\Rightarrow \tilde{y}_{L4} = \frac{P_{L4} - jQ_{L4}}{|V_4|^2} = \frac{1 - j0.44}{(1.018)^2} = 0.965 - j0.425 \text{ pu}$$

$$\Rightarrow \tilde{y}_{L5} = \frac{P_{L5} - jQ_{L5}}{|V_5|^2} = \frac{0.5 - j0.16}{(1.011)^2} = 0.489 - j0.157 \text{ pu}$$

"Stability Study Approach of Multi-Machines"

Sunday: 11-3-2018

⇒ Pre-Fault Ybus Matrix:

$$\bar{Y}_{11} = \frac{1}{j0.067 + j0.022} = -j11.236$$

$\bar{Y}_{12} = 0 + j0$

$\bar{Y}_{13} = 0 + j0$

$$\bar{Y}_{14} = \frac{-1}{j0.067 + j0.022} = j11.236$$

$\bar{Y}_{15} = 0 + j0$

$\bar{Y}_{31} = \bar{Y}_{13} = 0 + j0$

$\bar{Y}_{32} = \bar{Y}_{23} = 0 + j0$

$$\bar{Y}_{33} = \frac{1}{0.007 + j0.04} + \frac{2}{0.008 + j0.047} + \frac{j0.082}{2}$$

$$+ \frac{j0.098}{2} + \frac{j0.098}{2} \leftarrow \text{Shunt admittance}$$

$$\bar{Y}_{33} = 11.284 - j65.473 \leftarrow \text{3 digits}$$

$$\bar{Y}_{34} = \frac{-1}{0.007 + j0.04}$$

$\bar{Y}_{34} = -4.245 + j24.257$

$$\bar{Y}_{35} = \frac{-2}{0.008 + j0.047}$$

$\bar{Y}_{35} = -7.039 + j41.355$

$\bar{Y}_{21} = \bar{Y}_{12} = 0 + j0$

$$\bar{Y}_{22} = \frac{1}{j0.1 + j0.04} = -j7.143$$

$\bar{Y}_{23} = 0 + j0$

$\bar{Y}_{24} = 0 + j0$

$$\bar{Y}_{25} = \frac{-1}{j0.1 + j0.04} = j7.143$$

$\bar{Y}_{41} = \bar{Y}_{14}$, $\bar{Y}_{42} = \bar{Y}_{24}$, $\bar{Y}_{43} = \bar{Y}_{34}$ ✓

$$\bar{Y}_{44} = \frac{1}{j0.067 + j0.022} + \frac{1}{0.007 + j0.04} + \frac{j0.082}{2}$$

$$+ \frac{1}{0.018 + j0.11} + \frac{j0.226}{2} + \underbrace{0.965 - j0.425}_{\bar{y}_{L4}}$$

$\bar{Y}_{44} = 6.659 - j44.618$

$$\bar{Y}_{45} = \frac{-1}{0.018 + j0.11} = -1.449 + j8.854$$

$\bar{Y}_{51} = \bar{Y}_{15}$, $\bar{Y}_{52} = \bar{Y}_{25}$, $\bar{Y}_{53} = \bar{Y}_{35}$, $\bar{Y}_{54} = \bar{Y}_{45}$ ✓

$$\bar{Y}_{55} = \frac{1}{j0.04 + j0.1} + \frac{2}{0.008 + j0.047} + \frac{2\,j0.098}{2}$$

$$+ \frac{1}{0.018 + j0.11} + \frac{j0.226}{2} + \underbrace{0.489 - j0.157}_{\bar{y}_{L5}}$$

$\bar{Y}_{55} = 8.977 - j57.297$

" Stability Study Approach of Multi_Machines "

Sunday: 11-3-2018

$$\Rightarrow P_{e1} = |\bar{E}_1'|^2 \cdot \cancelto{0}{G_{11}} + |\bar{E}_1'||\bar{E}_2'||\cancelto{0}{\bar{Y}_{12}}| \sin(\delta_1 - \delta_2 - \gamma_{12}) + |\bar{E}_1'||\bar{E}_3'||\cancelto{0}{\bar{Y}_{13}}| \sin(\delta_1 - \delta_3 - \gamma_{13})$$

$$+ |\bar{E}_1'||\bar{E}_4'||\cancelto{11.236}{\bar{Y}_{14}}| \sin(\delta_1 - \cancelto{4.68^\circ}{\delta_4} - \cancelto{90-90=0}{\gamma_{14}}) + |\bar{E}_1||\bar{E}_5||\cancelto{0}{\bar{Y}_{15}}| \sin(\delta_1 - \delta_5 - \gamma_{15})$$

$$\therefore P_{e1} = 0 + 0 + 0 + (1.1)(1.018)(11.236) \cdot \sin(\delta_1 - 4.68^\circ) + 0$$

$$P_{e1} = 12.582 \sin(\delta_1 - 4.68^\circ) \Leftarrow \text{Power Angle eq. For Gen. 1}$$

$$\Rightarrow P_{e2} = |\bar{E}_2'|^2 \cdot \cancelto{0}{G_{22}} + |\bar{E}_2'||\bar{E}_1||\cancelto{0}{\bar{Y}_{21}}| \sin(\delta_2 - \delta_1 - \gamma_{21}) + |\bar{E}_2'||\bar{E}_3'||\cancelto{0}{\bar{Y}_{23}}| \sin(\delta_2 - \delta_3 - \gamma_{23})$$

$$+ |\bar{E}_2'||\bar{E}_4||\cancelto{0}{\bar{Y}_{24}}| \sin(\delta_2 - \delta_4 - \gamma_{24}) + |\bar{E}_2'||\bar{E}_5'||\cancelto{7.143}{\bar{Y}_{25}}| \sin(\delta_2 - \cancelto{2.27^\circ}{\delta_5} - \cancelto{90-90=0}{\gamma_{25}})$$

$$\therefore P_{e2} = 0 + 0 + 0 + 0 + (1.065)(1.011)(7.143) \sin(\delta_2 - 2.27^\circ)$$

$$P_{e2} = 7.69 \sin(\delta_2 - 2.27^\circ) \Leftarrow \text{Power Angle eq. For Gen. 2}$$

* Swing equations for Gen. 1 & Gen. 2 in the pre-Fault:

1) $$\frac{2H_1}{2(180)f} \cdot \frac{d^2\delta_1}{dt^2} = P_{m1} - P_{e1}$$

$$\Rightarrow \frac{d^2\delta_1}{dt^2} = \frac{180f}{11.2}\left(3.5 - 12.582 \sin(\delta_1 - 4.68^\circ)\right)$$

2) $$\frac{2H_2}{2(180)f} \cdot \frac{d^2\delta_2}{dt^2} = P_{m2} - P_{e2}$$

$$\Rightarrow \frac{d^2\delta_2}{dt^2} = \frac{180f}{8}\left(1.85 - 7.69 \sin(\delta_2 - 2.27^\circ)\right)$$

"Stability Study Approach of Multi-Machines"

Tuesday: 13.3.2018

* During-Fault Analysis:

⇒ Assuming a 3-ϕ Symmetrical Fault occured at $\underline{\underline{P}}$, then:

- * $\breve{Y}_{14}, \breve{Y}_{24}, \breve{Y}_{34}, \breve{Y}_{44}, \breve{Y}_{45}, \breve{Y}_{41}, \breve{Y}_{42}, \breve{Y}_{43}, \breve{Y}_{54} = 0$
- * Bus # 1,2,3 have constant voltages. → generated & infinite buses.
- * Bus # 4 must be removed as it is short circuited to ground.
- * Bus # 5 has to be eliminated using Kron reduction, because it's Voltage is unknown.

⇒ During-Fault Y-bus Matrix:

$$\begin{array}{r} \\ \\ \\ \text{For Bus \# 5} \rightarrow \end{array} \begin{bmatrix} -j11.26 & 0 & 0 & 0 \\ 0 & -j7.143 & 0 & j7.143 \\ 0 & 0 & 11.2841-j65.473 & -7.039+j41.355 \\ 0 & j7.143 & -7.039+j41.355 & 8.977-j37.297 \end{bmatrix}_{4\times4}$$

⇒ After doing Kron reduction using:

$$\breve{Y}_{jk}(\text{new}) = \breve{Y}_{jk}(\text{old}) - \frac{\breve{Y}_{jp}\,\breve{Y}_{pk}}{\breve{Y}_{pp}}, \quad P=4, \ (\text{originally } \underline{\underline{5}})$$

⇒ The results are:

$$\begin{bmatrix} -j11.236 & 0+j0 & 0+j0 \\ 0+j0 & 0.136-j6.274 & 5.167\angle 90.755^\circ \\ 0+j0 & -0.0681+j5.166 & 5.798-j35.639 \end{bmatrix}_{3\times3}$$

"Stability Study Approach of Multi-Machines"

Tuesday: 13-3-2018

$\Rightarrow P_{e1} = 0$ ← From Figure

OR

$$P_{e1} = |\vec{E}_1'|^2 \cancelto{0}{G_{11}} + |\vec{E}_1'||\vec{E}_2'|\cancelto{0}{|\bar{Y}_{12}|} \cdot \sin(\delta_1 - \delta_2 - \gamma_{12}) + |\vec{E}_1||\vec{E}_3|\cancelto{0}{|\bar{Y}_{13}|} \sin(\delta_1 - \delta_3 - \gamma_{13})$$

$\therefore P_{e1} = 0$ ← Power Angle eq. for Gen. 1

$$\Rightarrow P_{e2} = |\vec{E}_2'|^2 \cdot G_{22} + |\vec{E}_2'||\vec{E}_1'|\cancelto{0}{|\bar{Y}_{21}|} \sin(\delta_2 - \delta_1 - \gamma_{21}) + |\vec{E}_2'||\vec{E}_3'||\bar{Y}_{23}| \sin(\delta_2 - \cancelto{0}{\delta_3} - \gamma_{23})$$

$$P_{e2} = (1.065)^2(0.136) + 0 + (1.065)(1)(5.167) \sin(\delta_2 - 0 - (90.755 - 90))$$

$\therefore P_{e2} = 0.155 + 5.5023 \sin(\delta_2 - 0.755^\circ)$ ← Power Angle eq. for Gen. 2

* Swing equations for Gen. 1 & Gen. 2 in the during fault:

1) $$\frac{2H_1}{\omega_s} \cdot \frac{d^2\delta_1}{dt^2} = 3.5 - 0$$

$$\Rightarrow \frac{d^2\delta_1}{dt^2} = \frac{180 f}{11.2}(3.5)$$

2) $$\frac{2H_1}{\omega_s} \cdot \frac{d^2\delta_2}{dt^2} = 1.85 - [0.155 + 5.5023 \sin(\delta_2 - 0.755^\circ)]$$

$$\Rightarrow \frac{d^2\delta_2}{dt^2} = \frac{180 f}{8}(1.695 - 5.5023 \sin(\delta_2 - 0.755^\circ))$$

"Stability Study Approach of Multi_Machines"

Thursday: 15-3-2018

* Post-Fault Analysis:

⇒ The Y_{bus} matrix of the post Fault must be reconstructed without the Line 4-5 and it's shunt admittances.

⇒ Bus #4 & 5 must be eliminated using Kron reduction.

⇒ L_4 & L_5 will not be changed.

⇒ Post-Fault Y-bus Matrix after eliminating bus #4 & 5:

$$\begin{bmatrix} 0.5005 - j7.790 & 0+j0 & 7.632\angle 91.664^\circ \\ 0+j0 & 0.159 - j6.117 & 6.098\angle 90.897^\circ \\ 7.632\angle 91.664^\circ & 6.098\angle 90.897^\circ & 1.393 - j13.873 \end{bmatrix}$$

⇒ $$Pe_1 = |\bar{E}_1'|^2 G_{11} + |\bar{E}_1'||\bar{E}_2'|\cancelto{0}{|\bar{Y}_{12}|} \sin(\delta_1 - \delta_2 - \gamma_{12}) + |\bar{E}_1'||\bar{E}_3'||\bar{Y}_{13}| \sin(\delta_1 - \delta_3 - \gamma_{13})$$

$$Pe_1 = (1.1)^2 (0.5005) + 0 + (1.1)(1)(7.632) . \sin(\delta_1 - 0 - (91.664 - 90))$$

∴ $Pe_1 = 0.606 + 8.396 \sin(\delta_1 - 1.664^\circ)$ ← Power Angle Eq. For Gen. 1

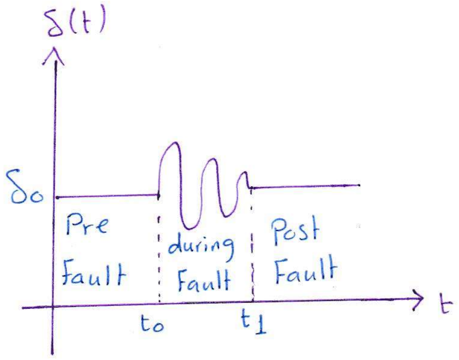
⇒ $Pe_2 = 0.1804 + 6.493 \sin(\delta_2 - 0.847^\circ)$ ← Power Angle Eq. For Gen. 2

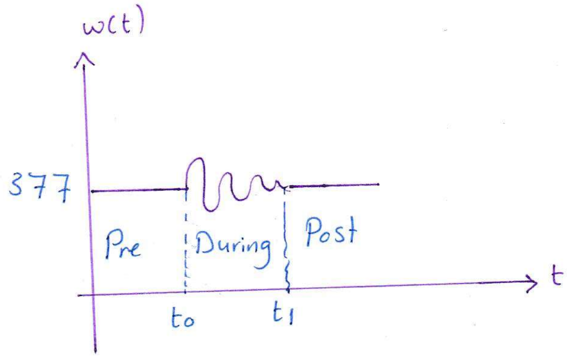
* Swing equations for Gen.1 & Gen.2 in the post-fault:

1) $$\frac{d^2\delta_1}{dt^2} = \frac{180f}{11.2}\left(2.8944 - 8.3955 \sin(\delta_1 - 1.664^\circ)\right)$$

2) $$\frac{d^2\delta_2}{dt^2} = \frac{180f}{8}\left(1.6696 - 6.4934 \sin(\delta_2 - 0.847^\circ)\right)$$

" Solving Swing equations using MATLAB " | Sunday: 18- 3- 2018





* For the 1st Gen. :-

*use F= 60 Hz

$$\overset{x_1}{\frac{d\omega_1}{dt}} = \frac{(180)(60)}{11.2}\left[3.5 - 12.582 \sin(\delta_1 - \overset{0.082\ rad}{\cancel{4.68^\circ}})\right]$$

$$\overset{x_2}{\frac{d\delta_1}{dt}} = \omega_1 - 377$$

} Pre - Fault state space Swing equations.

$$\frac{d\omega_1}{dt} = \frac{(180)(60)}{11.2}[3.5]$$

$$\frac{d\delta_1}{dt} = \omega_1 - 377$$

} During - Fault

$$\frac{d\omega_1}{dt} = \frac{(180)(60)}{11.2}\left[2.894 - 8.39 \sin(\delta_1 - \overset{0.029\ rad}{\cancel{1.664^\circ}})\right]$$

$$\frac{d\delta_1}{dt} = \omega_1 - 377$$

} Post - Fault

"Solving Swing Equations using MATLAB"

Sunday: 18-3-2018

* MATLAB Code:

1) Defining 3-functions:→

```
Function   Xprime = prefault1 (t, x)

Xprime = [(180*60/11.2)*(3.5-12.582 * Sin (X(2) - 0.082));

          x(1) -377];
```

Prefault1.m

```
function   Xprime = duringfault1 ( t , X)

Xprime =  [ (180*60 / 11.2) *3.5 ;

             X(1) - 377] ;
```

duringfault1.m

```
Function   Xprime = post fault1  (t, x)

Xprime =  [ (180* 60 / 11.2)* (2.894 -8.396 * Sin (X(2) - 0.029 ));

           X(1) - 377 ] ;
```

post fault1.m

2) Solving the equations:→

```
Tspan  =  [0   5] ;

  yo  =   [377; 0.3639] ;

[t1 , X] = ode 23s ( 'Prefault1' , Tspan , yo ) ;

     a = Length (t1) ;

  yo =  [ x (a,1) ; x (a,2)] ;
```

PreFault

"Solving Swing Equations using MATLAB"

Sunday: 18-3-2018

```
Tspan = [ 5   5.225 ] ;
[t2 , y] = ode23s ('duringfault1' , Tspan, yo) ;
    b = length (t2) ;
    yo = [ y(b,1) ; y(b,2)] ;
```

during fault

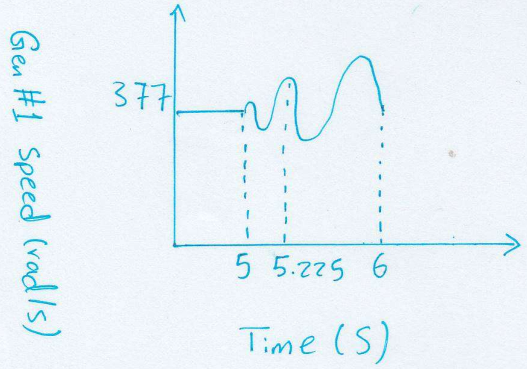
```
Tspan = [5.225   6]
[t3, Z] = ode23s ('postfault1' , Tspan , yo) ;
```

Post fault

3) Plotting Results →

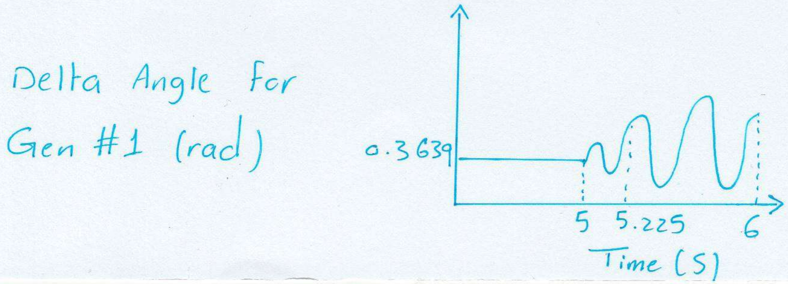
```
Plot (t1 , x(:,1) , t2 ,y(:,1) , t3 , Z(:,1))
xlabel ( 'Time (S)')   ;
ylabel ( 'Gen#1 speed (rad/s)') ;
```

Plotting speed



```
Plot (t1 , x(:,2) , t2 , y(:,2) , t3 , Z(:,2))
xlabel ('Time (S)') ;
ylabel ('Delta Angle for Gen #1 (rad)') ;
```

Plotting Delta Angle



"Factors Affecting Transient Stability"

Tuesday: 20-3-2018

* The higher the value of H constant, the better the stability of the power system. Because the higher weight withstands the change in speed.

* The lower the value of δ_0, the better the stability; as the low value is far away from 90°.

* The lower P_m the better the stability, because:

  1) δ_0 is lower (Far away from 90°), Since $\delta_0 = \sin^{-1}\left[\frac{P_m}{P_{max}}\right]$
  2) Accelerating power is lower, due to lower stored mechanical power.

* The higher P_{max}, the better the stability, because:
  1) δ_0 is lower.
  2) Accelerating power is lower.

$$\Rightarrow P_{max} = \frac{E_1 E_2}{X_{12}} \longleftarrow$$

* The higher power system reactances, the worst the stability.

∴ Adding more transmission lines in parallel improves the stability, for two reasons:

  1) lower equivalent reactance.
  2) Providing more paths for the electrical power during the fault.

* Adding shunt capacitances reduces the reactances, and thus improves the stability. Also it improves P.F & V.R.

  Note: Shunt capacitances are added in the transmission system.

"Factors Affecting Transient Stability"

Tuesday: 20-3-2018

* Adding series capacitances improve the stability

Condition: Providing that the subsynchronous resonance phonemon does not take place.

⇒ Subsynchronous resonance: Electromechanical interaction occurs if the electrical frequency ($f = \frac{1}{2\pi\sqrt{LC}}$) approaches the value of one of the mechanical natural frequencies.

$$X_L = X_C \Rightarrow wL = \frac{1}{wC} \Rightarrow w^2 = \frac{1}{LC} \Rightarrow f = \frac{1}{2\pi\sqrt{LC}}$$

* -Ve feedback controllers techniques improve the stability.

⇒ AVR has larger affect than TG.

* Damping ratio improves the stability.

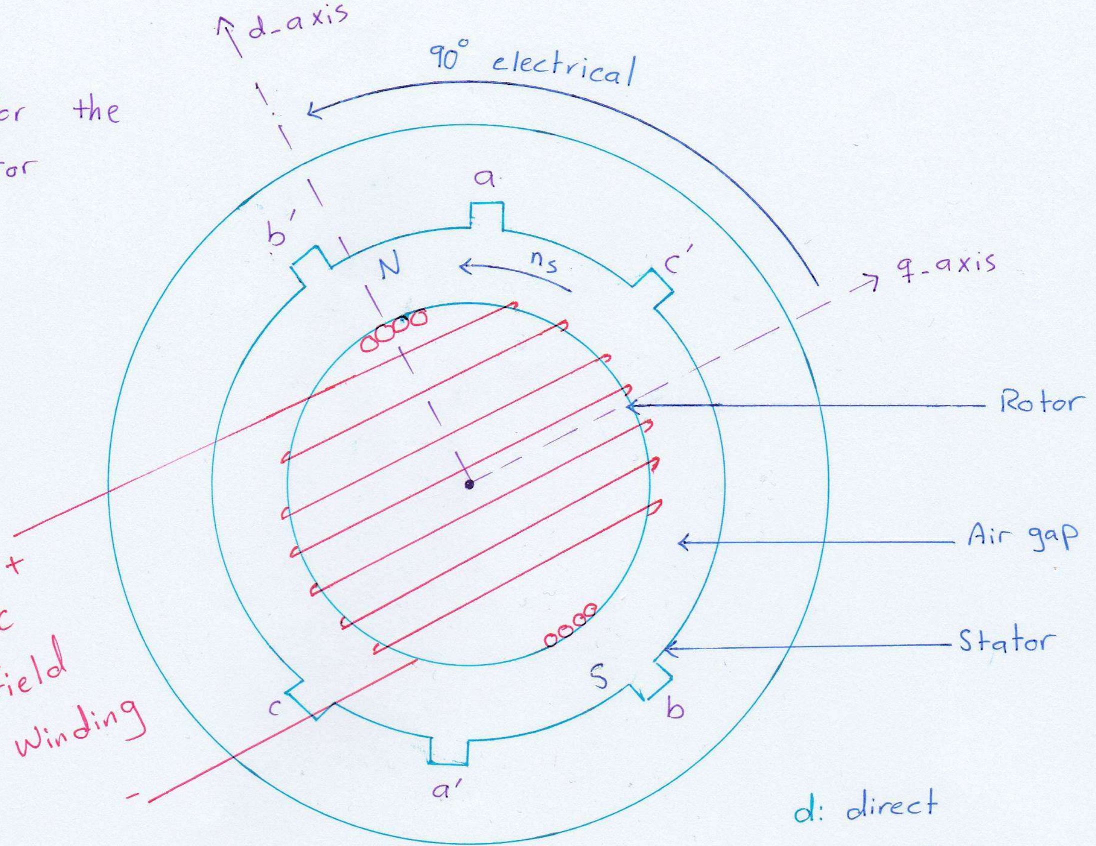
⇒ 1) Damping for oscillations

2) Starting Synch. motor as induction motor.

"Dynamical Model for the Synch. Generator"

Thursday: 22-3-2018

1) Construction for the cylindrical rotor



d: direct

q: quadrature

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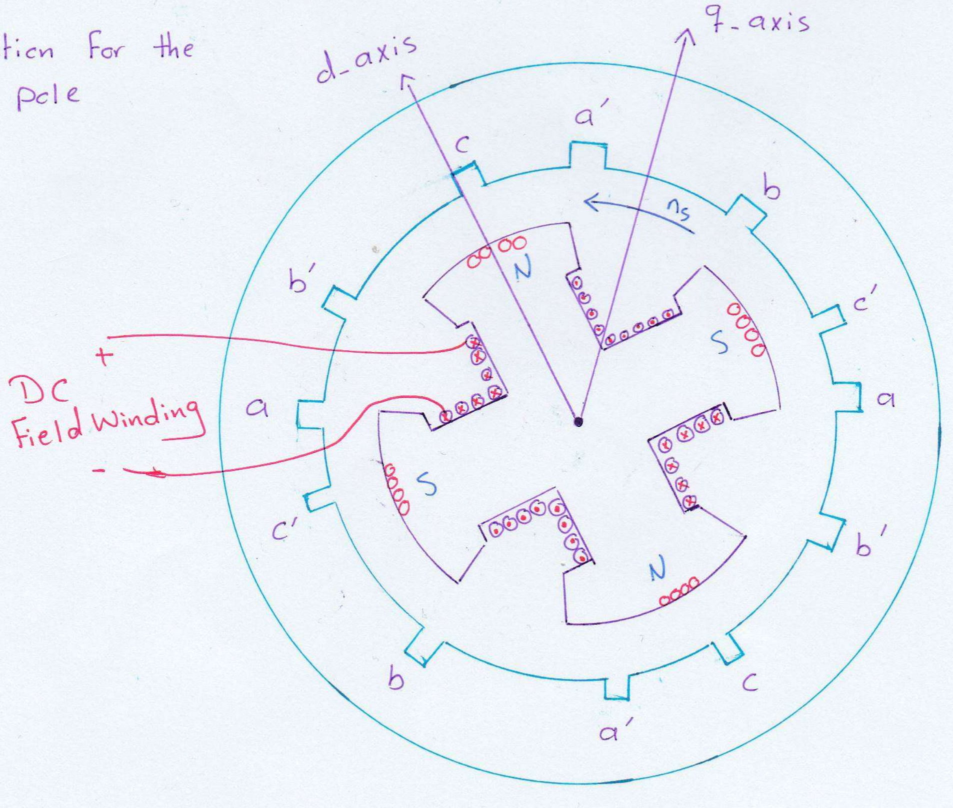
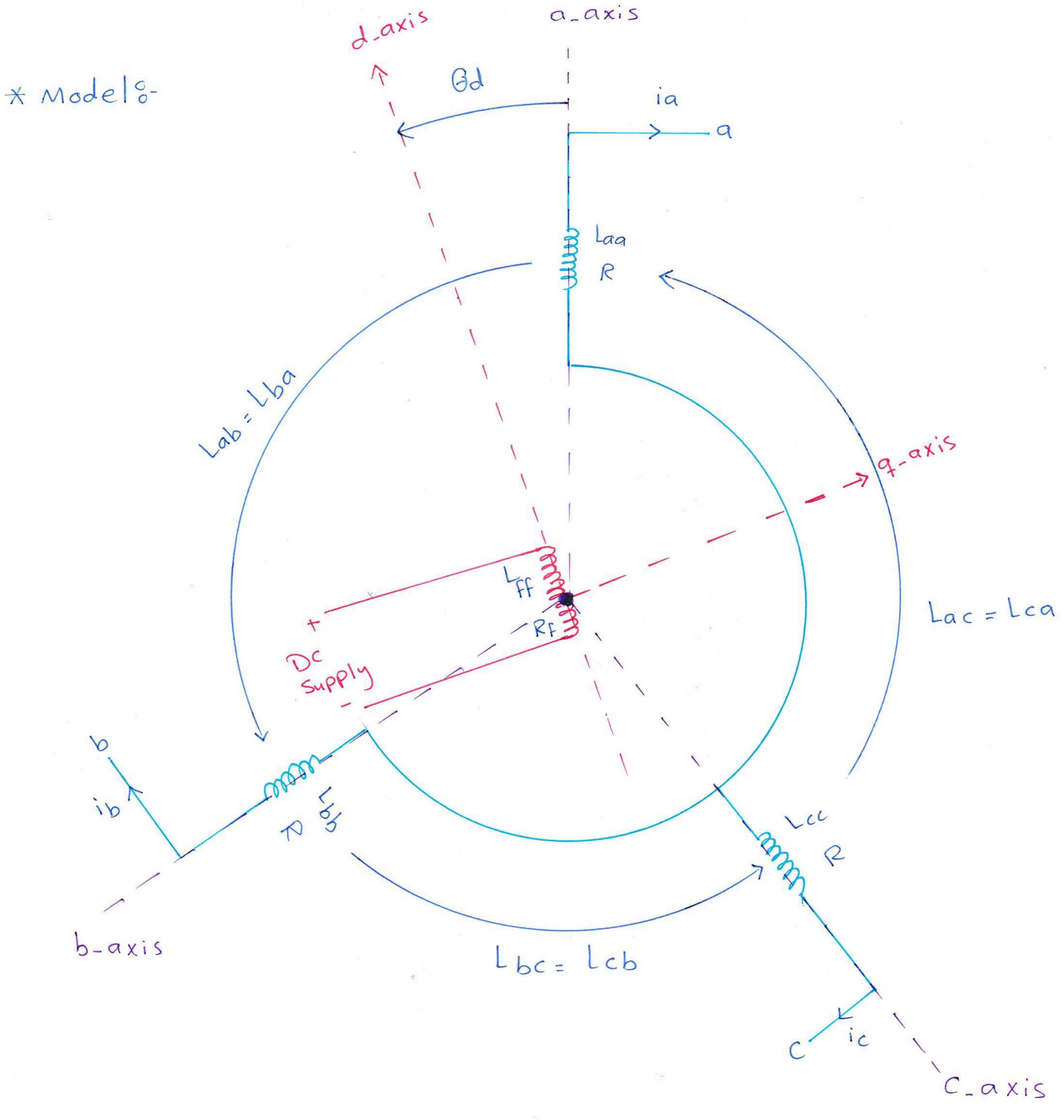
2) Construction for the salient pole



"Dynamical Model for the Synch. Generator"

Thursday: 22-3-2018

* Model:-



"Dynamical Model of the Synch. Generator"

Sunday: 25-3-2018

* Each phase winding has a self inductance:

$$L_s = L_{aa} = L_{bb} = L_{cc}$$

* Each two phase windings have mutual inductance.

$$-M_s = L_{ab} = L_{bc} = L_{ca}$$

* The mutual inductance between the field winding "f" and each of the stator coils volves with the rotor position θ_d as:

$$L_{af} = M_f \cos\theta_d$$

$$L_{bf} = M_f \cos(\theta_d - 120^\circ)$$

$$L_{cf} = M_f \cos(\theta_d - 240^\circ)$$

* The field coil has a constant self inductance L_{ff}. I_f is constant because in both types of machines the field winding on the d-axis Produces magnetic flux through similar magnetic path in the stator for all positions of the rotor. (neglecting the effect of stator slots).

* Flux Linkages for each phase:

$$\lambda_a = L_{aa} \cdot i_a + L_{ab} \cdot i_b + L_{ac} \cdot i_c + L_{af} \cdot i_f$$

$$\lambda_a = L_s \cdot i_a - M_s(i_b + i_c) + L_{af} \cdot i_f$$

$$\lambda_a = L_s \cdot i_a - M_s(-i_a) + L_{af} \cdot i_f$$

$$\Rightarrow \lambda_a = (L_s + M_s) \cdot i_a + L_{af} \cdot i_f \Leftarrow$$

⇒ Similarly :

⇒ $\lambda_b = (l_s + M_s).i_b + l_{ab}.i_f \Leftarrow$

⇒ $\lambda_c = (l_s + M_s).i_c + l_{ac}.i_f \Leftarrow$

⇒ $\theta_d = \omega t + \theta_{do}$

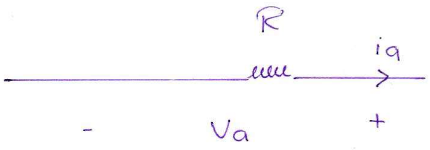
$$\frac{d\theta_d}{dt} = \omega$$

⇒ At steady-state : $i_f = I_f$

$$\lambda_a = (l_s + M_s).i_a + M_f.I_f.\cos(\omega t + \theta_{do})$$

$$\lambda_b = (l_s + M_s).i_b + M_f.I_f.\cos(\omega t + \theta_{do} - 120^\circ)$$

$$\lambda_c = (l_s + M_s).i_c + M_f.I_f.\cos(\omega t + \theta_{do} - 240^\circ)$$



$$V_a = -R.i_a - \frac{d\lambda_a}{dt}$$

$$V_a = -R.i_a - \frac{d}{dt}\left[(l_s + M_s)i_a + M_f.I_f.\cos(\omega t + \theta_{do})\right]$$

$$V_a = \underbrace{-R.i_a}_{\text{Voltage drop across the resistance}} - \underbrace{(l_s + M_s).\frac{di_a}{dt}}_{\text{Voltage drop across the reactance}} + \underbrace{\omega.M_f.I_f.\sin(\omega t + \theta_{do})}_{\text{induced voltage } (e_a')}$$

i.e, internal generated voltage

"Dynamical Model of the Synch. Generator"

Sunday: 25-3-2018

$\Rightarrow e_a' = \sqrt{2} \cdot |E_i| \cdot \sin(wt - \theta_{do})$

where, $|E_i| = \dfrac{w \cdot M_F \cdot I_F}{\sqrt{2}}$

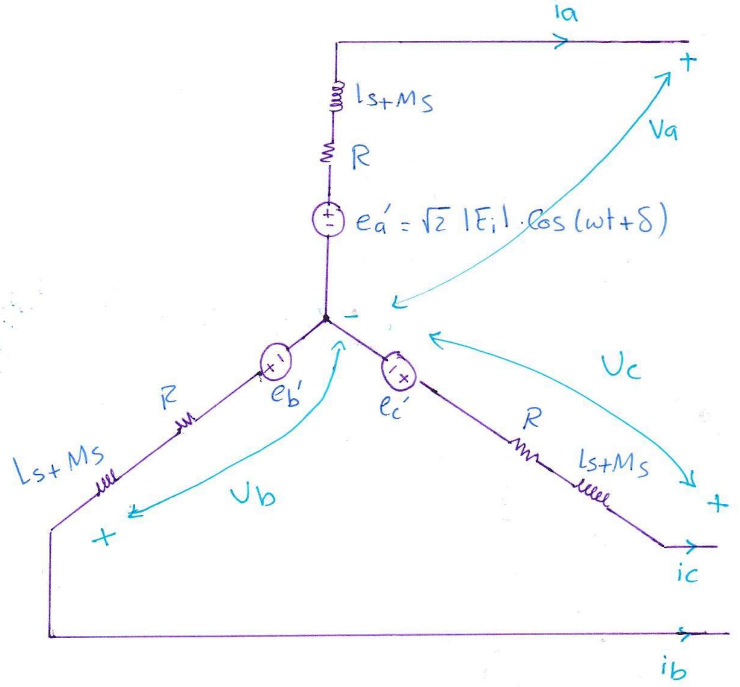
* θ_{do} indicates the position of the field winding with respect to the a-axis at $t=0$

$\Rightarrow$ let $\delta \triangleq \theta_{do} - 90^\circ$ $\Rightarrow$ indicates the position of q-axis

$\theta_d = wt + \theta_{do} = wt + \delta + 90^\circ$

$\Rightarrow e_a' = \sqrt{2}\, |E_i| \cdot \cos(wt + \delta)$

$\Rightarrow V_a = -R \cdot i_a - (l_s + M_s) \cdot \dfrac{di_a}{dt} + \sqrt{2}\, |E_i| \cdot \cos(wt + \delta)$

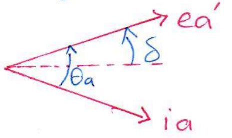


$\Rightarrow i_a = \sqrt{2}\, |I_a| \cos(wt + \delta - \theta_a)$

$i_b = \sqrt{2}\, |I_a| \cos(wt + \delta - \theta_a - 120^\circ)$

$i_c = \sqrt{2}\, |I_a| \cos(wt + \delta - \theta_a - 240^\circ)$

θ_a: Phase angle of the current w.r.t e_a'



$\Rightarrow |I_a| = |I_b| = |I_c|$ = rms value of the phase current

"Dynamical Model of the Synch. Generator"

Tuesday: 27-3-2018

⇒ The flux linkage of the field winding:

$$\lambda_F = L_{ff} \cdot I_F + M_F \left[i_a \cos\theta_d + i_b \cdot \cos(\theta_d - 120^\circ) + i_c \cdot \cos(\theta_d - 240^\circ) \right]$$

where,

$$i_a \cos\theta_d = \sqrt{2}\,|I_a| \cdot \cos(\omega t + \delta - \theta_a) \cdot \cos(\omega t + \delta + 90^\circ)$$

$$i_b \cos(\theta_d - 120^\circ) = \sqrt{2}\,|I_b| \cdot \cos(\omega t + \delta - \theta_a - 120^\circ) \cdot \cos(\omega t + \delta + 90^\circ - 120^\circ)$$

$$i_c \cos(\theta_d - 240^\circ) = \sqrt{2}\,|I_c| \cdot \cos(\omega t + \delta - \theta_a - 240^\circ) \cdot \cos(\omega t + \delta + 90^\circ - 240^\circ)$$

Also, $2\cos\alpha \cdot \cos\beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$

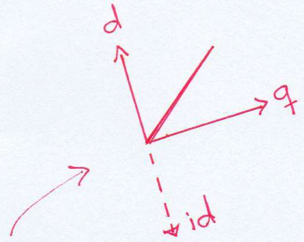
$$\Rightarrow \quad i_a \cdot \cos\theta_d = \frac{|I_a|}{\sqrt{2}} \left[-\sin\theta_a - \sin[2(\omega t + \delta) - \theta_a] \right]$$

$$i_b \cdot \cos(\theta_d - 120^\circ) = \frac{|I_a|}{\sqrt{2}} \left[-\sin\theta_a - \sin[2(\omega t + \delta) - \theta_a - 120^\circ] \right]$$

$$i_c \cdot \cos(\theta_d - 240^\circ) = \frac{|I_a|}{\sqrt{2}} \left[-\sin\theta_a - \sin[2(\omega t + \delta) - \theta_a - 240^\circ] \right]$$

$$\Rightarrow \quad \boxed{\lambda_{ff} = L_{FF} I_F - \frac{3M_F}{\sqrt{2}} |I_a| \cdot \sin(\theta_a)} \Leftarrow$$

$$\Rightarrow \quad \lambda_{ff} = L_{ff} \cdot I_f + \sqrt{\tfrac{3}{2}} \cdot M_f \cdot i_d$$



where, i_d = d-axis armature (stator) current.

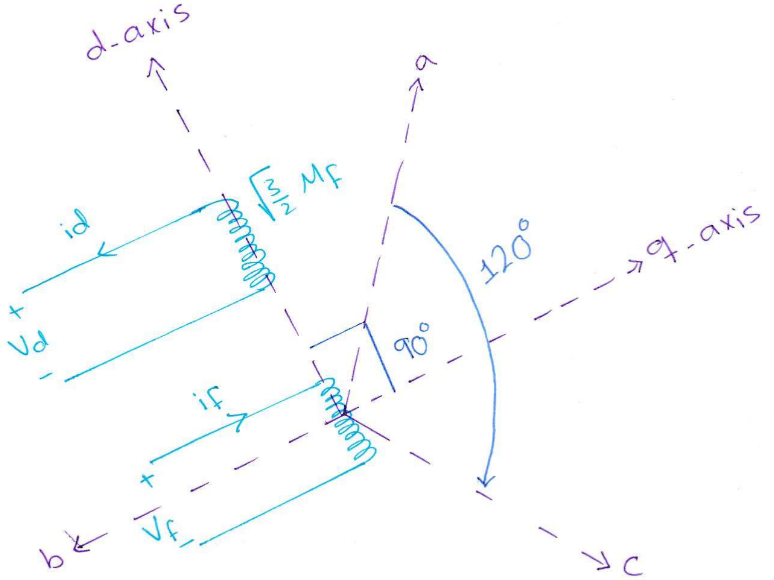
$i_d = -\sqrt{3}\,|I_a| \cdot \sin\theta_a$ ⇒ No time, ∴ Acts like DC

Adv: Acts like 1 current instead of 3.

"Dynamical Model of the Synch. Generator"

Tuesday: 27-3-2018

* The flux Linkages with the field winding due to the Combination of ia, ib, and ic don't vary with time, they can be treated as coming from dc steady current in a dc circuit concident with the d-axis and therefor they are staticnary with respect to the field current.



Vd : Armature (stator) equivalent winding rotating with the rotor

$$\Rightarrow V_{ff}' = R_f \cdot i_f + \frac{d\lambda_{ff}}{dt}$$

Ex

60 Hz, 3ϕ S.G , R=0

$l_{aa} = l_s = 2.7656$ mH , $M_f = 31.6950$ mH

$L_{ab} = M_s = 1.3828$ mH , $L_{ff} = 433.6569$ mH

$S_{rated} = 635$ MVA , 0.9 PF Lagging, 3600 rpm, 24 KV at rated

Conditions:

$V_a = 19596 \cos wt$ (V)

$i_a = 21603 \cos(wt - 25.8419°)$ (A)

Find: e_a', I_f, λ_f if the generator runs at rated conditions and a unity power factor

"Dynamical Model of the Synch. Generator"

Tuesday: 27-3-2018

A)

⇒ At rated conditions:

$$e_a' = \sqrt{2}\,|E_i|\,\cos(\omega t+\delta)$$

$$e_a' = V_a + (l_s+M_s)\cdot\frac{di_a}{dt}$$

$$e_a' = 19596\cos(\omega t) + (l_s+M_s)\cdot\frac{d}{dt}\left[21603\cos(\omega t-25.8419^\circ)\right]$$

⇒ After substituting parameters, with $\omega = 377$

then use:

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

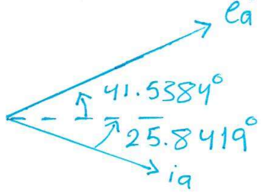
the result will be ⇒

$$e_a' = 45855\cos(\omega t+41.5384^\circ)$$

$$|E_i| = \frac{45855}{\sqrt{2}} \quad , \quad \delta = 41.5384^\circ$$

$$I_f = \frac{\sqrt{2}\,|E_i|}{\omega\cdot M_f} = \frac{\sqrt{2}\times\frac{45855}{\sqrt{2}}}{377\times 0.031695} = 3837.55\ A$$

$$\lambda_F = L_{ff}\cdot I_f - \frac{3M_f}{\sqrt{2}}\cdot|I_a|\cdot\sin\theta_a$$



$$\Rightarrow \theta_a = 41.5384+25.8419 = 67.3803^\circ$$

$$\Rightarrow |I_a| = \frac{21603}{\sqrt{2}}\ A$$

$$\lambda_f = 0.4336569\times 3837.55 - \frac{3}{\sqrt{2}}(0.031695)\left(\frac{21603}{\sqrt{2}}\right)\cdot\sin(67.3803^\circ)$$

$$\lambda_f = 716.123\ \text{wb-turns}$$

B)

⇒ At unity PF:

$$v_a = 19596 \cos(wt)$$

$$i_a = 21603 \cos(wt)$$

$$\Rightarrow \cos\phi = PF = 1$$

$$\phi = 0^\circ$$

$$\Rightarrow e_a' = \sqrt{2}\,|E_i| \cos(wt+\delta) = v_a + (L_s + M_s)\cdot\frac{di_a}{dt}$$

$$e_a' = 19596 \cos(wt) + (4.1484 \times 10^{-3})\cdot\frac{d}{dt}\left(21603 \cos(wt)\right)$$

$$e_a' = 19596 \cos(wt) - (4.1484 \times 10^{-3})(377)\left(21603 \sin(wt)\right)$$

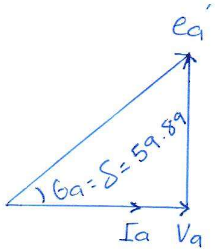
$$e_a' = 39057 \cos(wt + 59.8854)$$

$$\Rightarrow I_f = \frac{\sqrt{2}\,|E_i|}{w \cdot M_f}$$

$$I_f = \frac{19596 \times \sqrt{2}}{31.695 \times 10^{-3} \times 377} = 3269\ A$$

$$\Rightarrow \lambda_f = L_{ff} \cdot I_f - \frac{w \cdot M_f}{\sqrt{2}} \cdot |I_a| \cdot \sin\theta_a$$

$$\rightarrow I_a = \frac{21603}{\sqrt{2}} = 15275.6$$



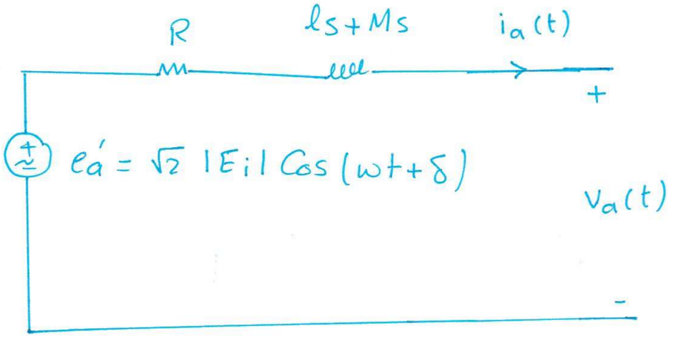
$$\therefore \lambda_f = (433.6569 \times 10^{-3})(3269) - \frac{(31.695 \times 10^{-3}) \times 377}{\sqrt{2}} \times (15275.6) \cdot \sin(59.89)$$

$$\lambda_f = 529.19 \text{ wb\_turns}$$

"Dynamical Model of the Synch. Generator"

Thursday: 29-3-2018

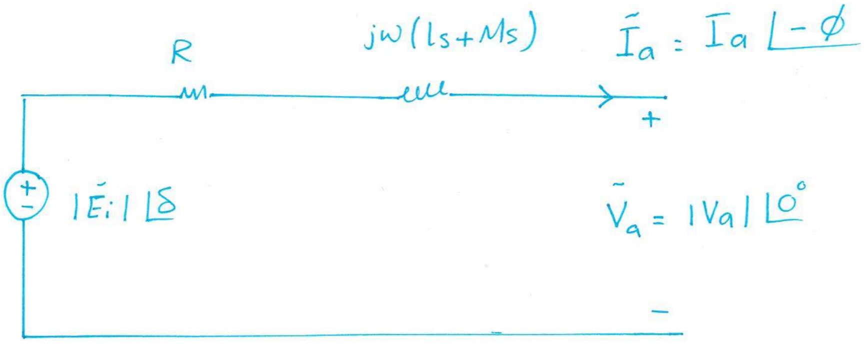
* Equivalent Circuit of the Synch. Generator ⇒



$$\Rightarrow i_a(t) = \sqrt{2}\,|I_a| \cdot \cos(\omega t - \phi)$$

$$v_a(t) = \sqrt{2}\,|V_a| \cdot \cos(\omega t)$$

⇒ In phasors:



$$\Rightarrow \bar{V}_a = \underbrace{\bar{E}_i}_{\text{Generated Voltage}} - \underbrace{R\,\bar{I}_a}_{\text{Voltage drop across R}} - \underbrace{j\omega L_s\,\bar{I}_a}_{\text{VD across self inductance}} - \underbrace{j\omega M_s \cdot \bar{I}_a}_{\text{VD across Mutual inductance}}$$

$$\Rightarrow X_d = \omega(L_s + M_s)$$

$$\bar{V}_a = \bar{E}_i - (R + jX_d)\,\bar{I}_a$$

"The Two-Axis Model"

Sunday: 1-4-2018

- In transient analysis the two-axis model is used for accurate results.

- In salient pole machine, the air gap is much narrower along the d-axis than the q-axis $(Lq \neq Ld) \Rightarrow (Xq \neq Xd)$

- For Cylindrical type. $(Xq = Xd)$

- In both types, the rotor sees the same airgap and magnetic circuit regardless of it's position $\Rightarrow$ The field winding has constant self inductance L_{ff} and both types have the same cosinusoidal mutual inductances L_{af}, L_{bf}, and L_{cf}.

- Through each revolution of the rotor (L_{aa}, L_{bb}, and L_{cc}) and (L_{ab}, L_{bc}, and L_{ca}) are not constants in the salient pole machines (varies as a function of the rotor position θ_d).

- $$\lambda_a = L_{aa} \cdot i_a + L_{ab} \cdot i_b + L_{ac} \cdot i_c + L_{af} \cdot i_f$$
  $$\lambda_b = L_{bb} \cdot i_b + L_{ba} \cdot i_a + L_{bc} \cdot i_c + L_{bf} \cdot i_f$$
  $$\lambda_c = L_{cc} \cdot i_c + L_{ca} \cdot i_a + L_{cb} \cdot i_b + L_{cf} \cdot i_f$$

- For salient pole machine, all coefficients are variable (time varying periodically).

- For cylindrical rotor all are constants.

- For salient pole machines, the time varying coefficients can be varried to constants using Park's transformation Matrix (P)

"The Two-Axis Model"

Sunday: 1-4-2018

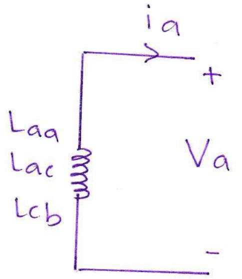
* The three phase currents:

$$\begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} = P \cdot \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

Where,

i_d: Equivalent d-axis stator phase current.
i_q: Equivalent q-axis stator phase current.
] Projected on the rotor ⇒ DC quantities.
i_0: Zero sequence Component (equals 0 in case of balanced condition)

$$\Rightarrow \quad i_a = \sqrt{2}\,|I_a| \cos(\omega t - \phi)$$
$$i_b = \sqrt{2}\,|I_a| \cos(\omega t - \phi - 120^\circ)$$
$$i_c = \sqrt{2}\,|I_a| \cos(\omega t - \phi - 240^\circ)$$



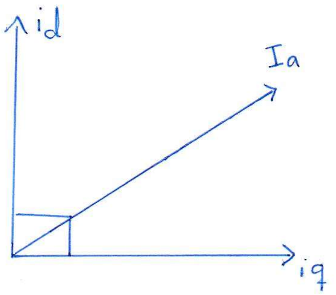
$$\Rightarrow \begin{bmatrix} v_d \\ v_q \\ v_o \end{bmatrix} = P \cdot \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad , \Rightarrow \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \end{bmatrix} = P \cdot \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix}$$

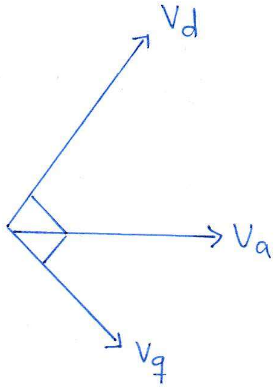
$$P = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos \theta_d & \cos(\theta_d - 120^\circ) & \cos(\theta_d - 240^\circ) \\ \sin \theta_d & \sin(\theta_d - 120^\circ) & \sin(\theta_d - 240^\circ) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

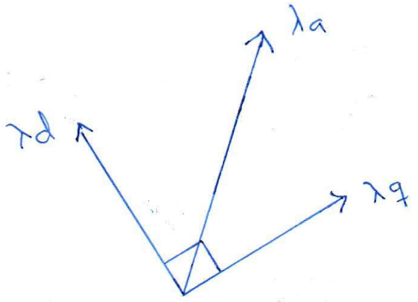
"The Two-Axis Model"

Sunday: 1-4-2018

$$I_a = \sqrt{i_d^2 + i_q^2}$$







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* Introductions P transformation for the flux Linkages:

$$\lambda_d = L_d \cdot i_d + \sqrt{\frac{3}{2}} M_f \cdot i_f$$

$$\lambda_q = L_q \cdot i_q$$

Where,

$$L_d = l_s + M_s + \frac{3}{2} l_m$$

$$L_q = l_s + M_s - \frac{3}{2} l_m$$

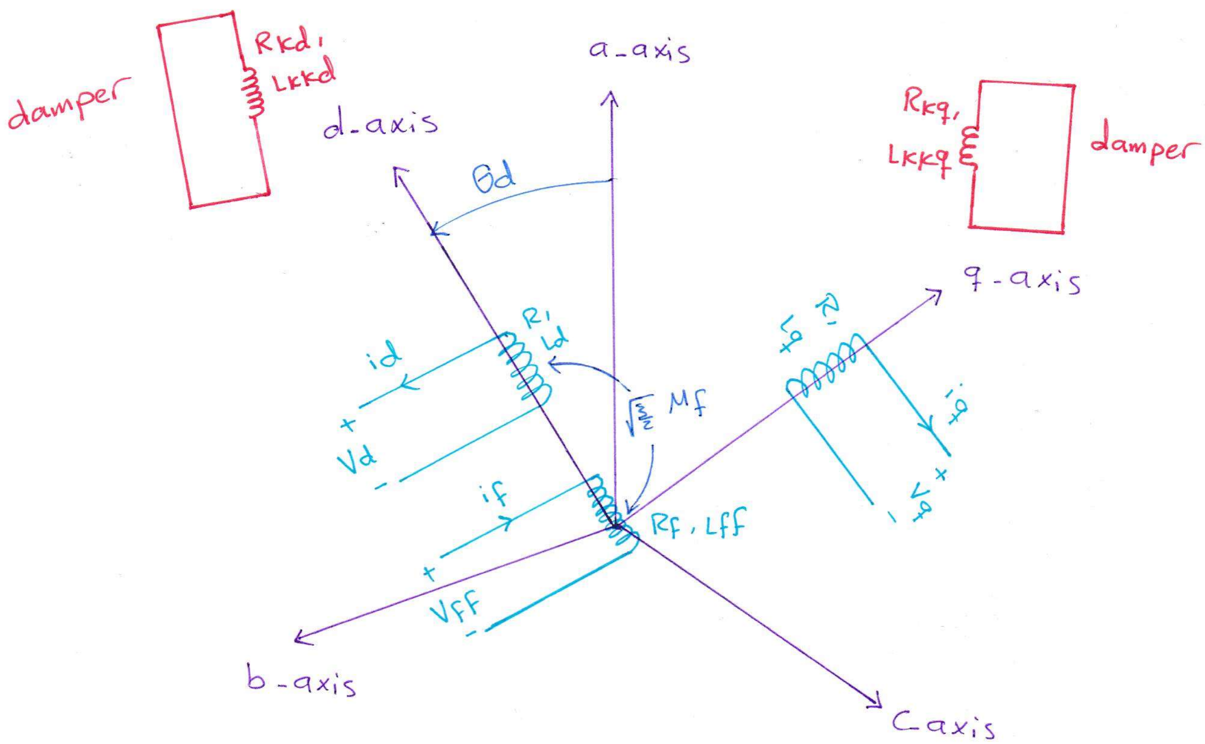
$$L_0 = l_s - 2 M_s$$

$$\lambda_f = \sqrt{\frac{3}{2}} M_f \cdot i_d + L_{FF} \cdot i_f$$

L_d: Self inductance of an equivalent d-axis armature winding rotating at the same speed as the Field carrying i_d.

"The Two - Axis Model"

Tuesday: 3-4-2018



$$\Rightarrow \quad V\phi = \sqrt{Vd^2 + Vq^2} \quad , \quad I\phi = \sqrt{id^2 + iq^2}$$

* There is no magnetic coupling between the windings on the d-axis with the windings on the q- axis

* Zero sequence components are neglected in case of balanced conditions.

* if $\Rightarrow$

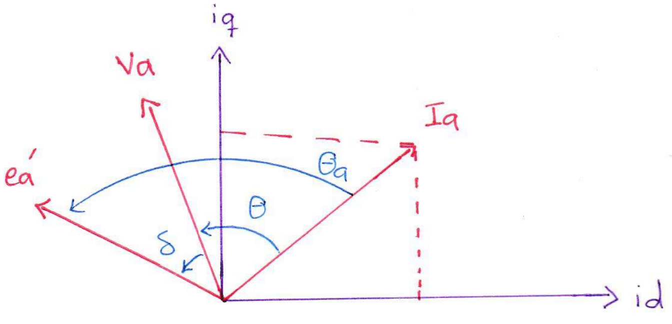
$$i_a = \sqrt{2}\ |I_a| \sin(\theta d - \theta a)$$

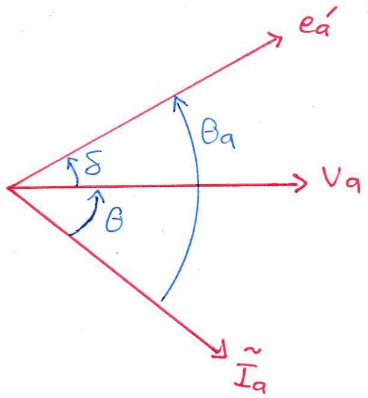
$$i_b = \sqrt{2}\ |I_a| \sin(\theta d - 120^\circ - \theta a)$$

$$i_c = \sqrt{2}\ |I_a| \sin(\theta d - 240^\circ - \theta a)$$

then ⇒ $i_d = -\sqrt{3}\,|I_a| \sin\theta_a = -\sqrt{3}\,|I_a| \sin(\theta+\delta)$

$i_q = \sqrt{3}\,|I_a| \cos\theta_a = \sqrt{3}\,|I_a| \cos(\theta+\delta)$





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* Voltage Equations :

⇒ $V_a = -R.i_a - \dot{\lambda}_a$

$V_b = -R.i_b - \dot{\lambda}_b$

$V_c = -R.i_c - \dot{\lambda}_c$

⇒ $V_d = -R.i_d - \dot{\lambda}_d - \omega.\lambda_q$

$V_q = -R.i_q - \dot{\lambda}_q + \omega.\lambda_d$

$V_o = -R.i_o - \dot{\lambda}_o$

Where ,

$\lambda_d = L_d.i_d + K.M_f.i_f$ ⇒ $K = \sqrt{\frac{3}{2}}$

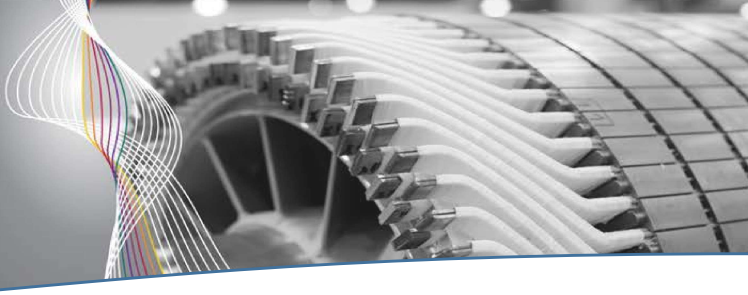
$\lambda_f = K.M_f.i_d + L_{ff}.i_f$

$\lambda_q = L_q.i_q$

$V_{ff} = R_f.i_f + \dot{\lambda}_f$

Non-Linear Dynamical Mathematical Model of S.G.

Tuesday: 3-4-2018



$-\lambda^{\cdot}{}_{d} = v_d + Ri_d + \omega\lambda_q$

$-L_d i^{\cdot}{}_{d} - k\,M_f i^{\cdot}{}_{f} = v_d + Ri_d + \omega\,L_q i_q$ [1] d-axis armature winding

$\searrow V_{\emptyset}sin\delta$

$-\lambda^{\cdot}{}_{q} = v_q + Ri_q - \omega\,\lambda_d$

$-L_q i^{\cdot}{}_{q} = v_q + Ri_q - \omega L_d i_d - \omega\,RM_f i_f$ [2] q-axis armature winding

$\searrow V_{\emptyset}cos\delta$

$\lambda^{\cdot}{}_{f} = v'_{ff} - Ri_f$

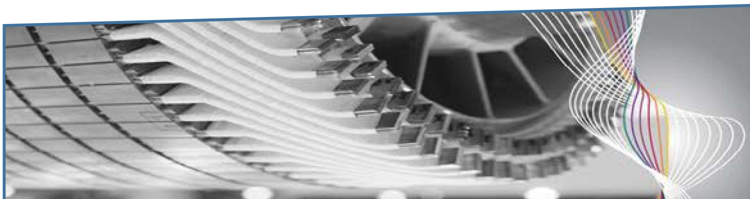
$k\,M_f i^{\cdot}{}_{d} + L_{ff} i^{\cdot}{}_{f} = v'_{ff} - Ri_f$ [3] d-axis field winding

$J\omega^{\cdot} = T_m - T_{out}$ [4]

$\delta^{\cdot} = \omega - \omega_s$ [5]

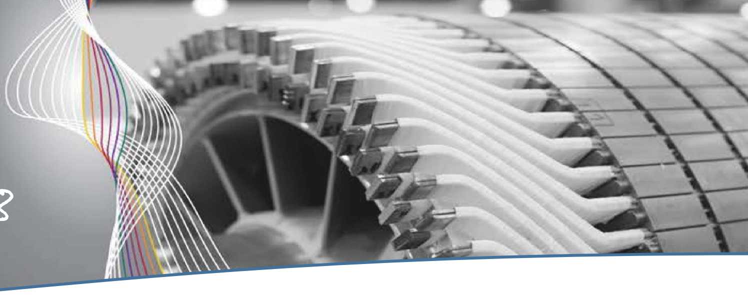
P_{out} is not a state variable and it can be expressed as function of constants and state variables as:

$P_{out} = v_a i_a + v_b i_b + v_c i_c$



Non-Linear Dynamical Mathematical Model of S.G.

Thursday: 5-4-2018



Using P-Matrix:

$$P_{out} = 0.75\, P\omega\, [\lambda_f i_q + (L_d - L_q) i_d i_q]$$

$$P_{out} = 0.75\, P\omega\, [R\, M_f i_d i_q + L_f i_q + L_d i_d i_q - L_q i_d i_q]$$

P: Number of poles

$$\tau_{out} = \frac{P_{out}}{\omega} = 0.75\, P\, [R\, M_f i_d i_q + L_f i_q + L_d i_d i_q - L_q i_d i_q]$$

In some special cases:

$$P_{out} = 3 \frac{V_{\emptyset} E_A}{X}\, sin\delta$$

$$\tau_{out} = 3 \frac{V_{\emptyset} E_A}{X\omega} sin\delta$$

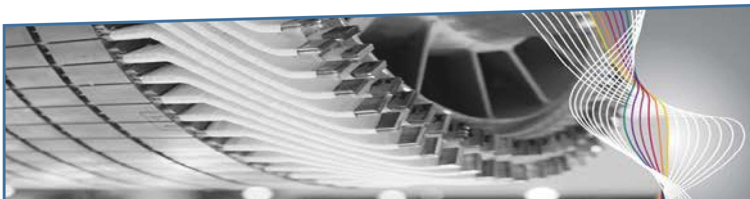
$\downarrow$

We can use this equation when these conditions are satisfied:

1) Lossless

2) $L_d = L_q = L_s$

$X_d = X_q = X_s$ (Cylindrical rotor)



Steady-State Performance

Thursday: 5-4-2018

In state space:

$x_1 = i_f \quad , \; x_2 = i_d \quad , \; x_3 = i_q \quad , \; x_4 = \omega \quad , \; x_5 = \delta$

Steady-State Performance ⟾ Operating Point , Equilibrium Solution

To obtain the steady-state model, drop out all time derivation terms:

$x^{\cdot}_1 = x^{\cdot}_2 = x^{\cdot}_3 = x^{\cdot}_4 = x^{\cdot}_5 = 0$

Then solve the resulting non-linear algebraic equations:

$$x_1 = i_f = \frac{V'_{ff}}{R_f}$$

$$x_4 = \omega = \omega_s$$

$$V_{\emptyset} sin\, \overset{\checkmark}{x_5} = -R\, \overset{\checkmark}{x_3} + \omega_s L_d \overset{\checkmark}{x_2} + \omega_s R M_f \boxed{i_f} \rightarrow \frac{V'_{ff}}{R_f}$$

$$V_{\emptyset} \cos x_5 = -R\, \overset{\checkmark}{x_2} - \omega_s L_q \overset{\checkmark}{x_3}$$

$$P_m = P_{out} = 0.75\, P \; [**]$$

$x_2, x_3\ \&\ x_5$ can be calculated numerically as the system is non-linear

Small Signal Stability of Power Systems

The dynamics of the power system can be described by set of first order non-linear ordinary differential equations

$$x^{\cdot}_i = f_i(\underbrace{x_1, x_2, \dots, x_n}_{\text{State variables}}; \underbrace{u_1, u_2, \dots, u_r}_{\text{Inputs}}; \underbrace{t}_{\text{Time}})$$

Order of the system

$$i = 5 \rightarrow \begin{array}{l} x^{\cdot}_1 = \\ x^{\cdot}_2 = \\ x^{\cdot}_3 = \\ x^{\cdot}_4 = \\ x^{\cdot}_5 = \end{array}$$

$$\underbrace{\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{pmatrix}}_{\text{State Vector}} \qquad \underbrace{\mathbf{u} = \begin{pmatrix} u_1 \\ u_2 \\ . \\ . \\ u_r \end{pmatrix}}_{\text{Input Vector}} \qquad \underbrace{\mathbf{f} = \begin{pmatrix} f_1 \\ f_2 \\ . \\ . \\ f_n \end{pmatrix}}_{\text{function Vector}}$$

$x^{\cdot} = f(x, u) \Longmapsto$ autonomous system , $x^{\cdot} = f(x, u, t) \Longmapsto$ non-autonomous system

The output variables:

$y = g(x, u)$

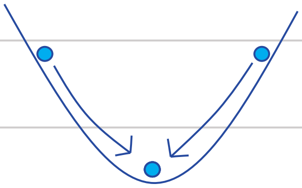
$$\mathbf{y} = \begin{pmatrix} y_1 \\ y_2 \\ . \\ . \\ y_n \end{pmatrix} \qquad \mathbf{g} = \begin{pmatrix} g_1 \\ g_2 \\ . \\ . \\ g_m \end{pmatrix}$$

Equilibrium points are those obtained when the time derivative terms equal zero.

$x^{\cdot}_1 = x^{\cdot}_2 = \cdots = x^{\cdot}_n = 0$

$f(x_0) = 0$

At equilibrium point, the system has the lowest energy (at rest).



A linear system has only one equilibrium solution.

Non-linear systems may have more than one equilibrium solutions, only one is the operating point.

Stability of a Dynamic System

The stability of a non-linear system (the steady-state stability) is classified into:

1) Local Stability

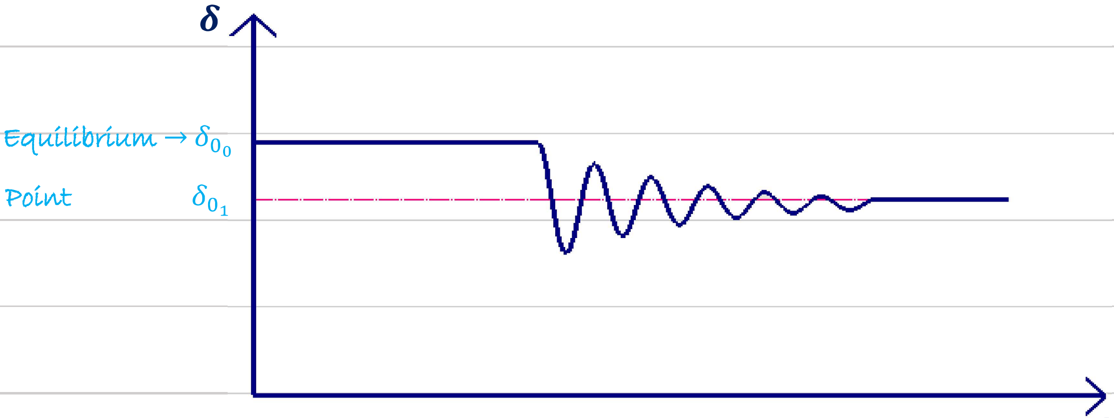
2) Finite Stability

3) Global Stability

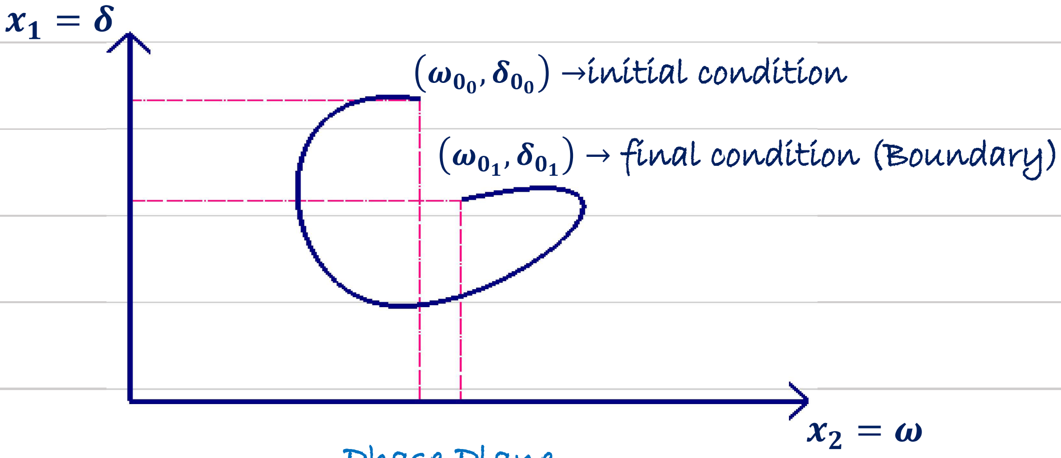
➢ Local Stability

The system is said to be locally stable around an operating point if when subjected to small perturbation, it remains within a small region surrounding the equilibrium point

disturbance



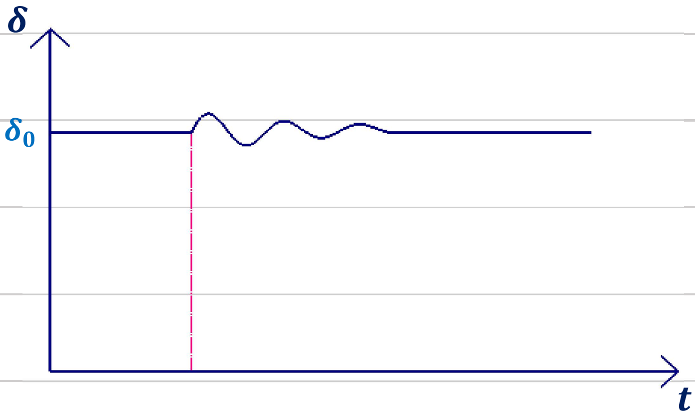
Time Domain Simulation

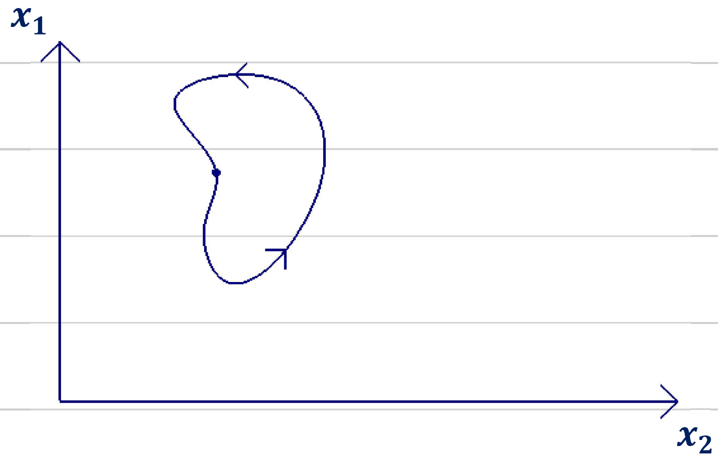


Phase Plane

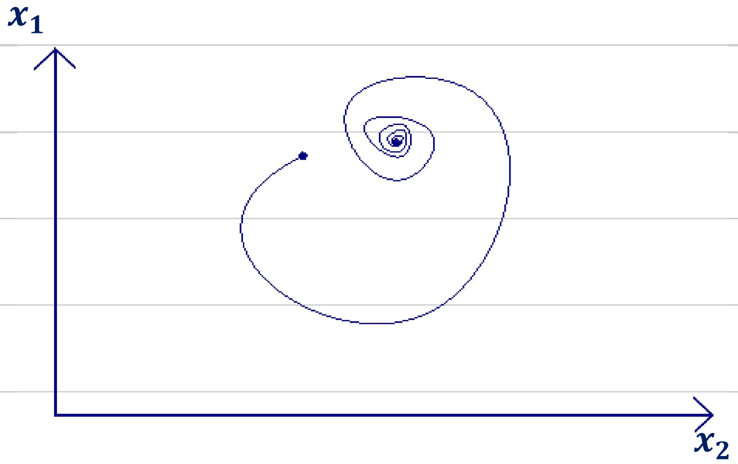
Sunday: 8-4-2018

If the equilibrium point returns to the same equilibrium, then the system is called asymptotically stable operating point



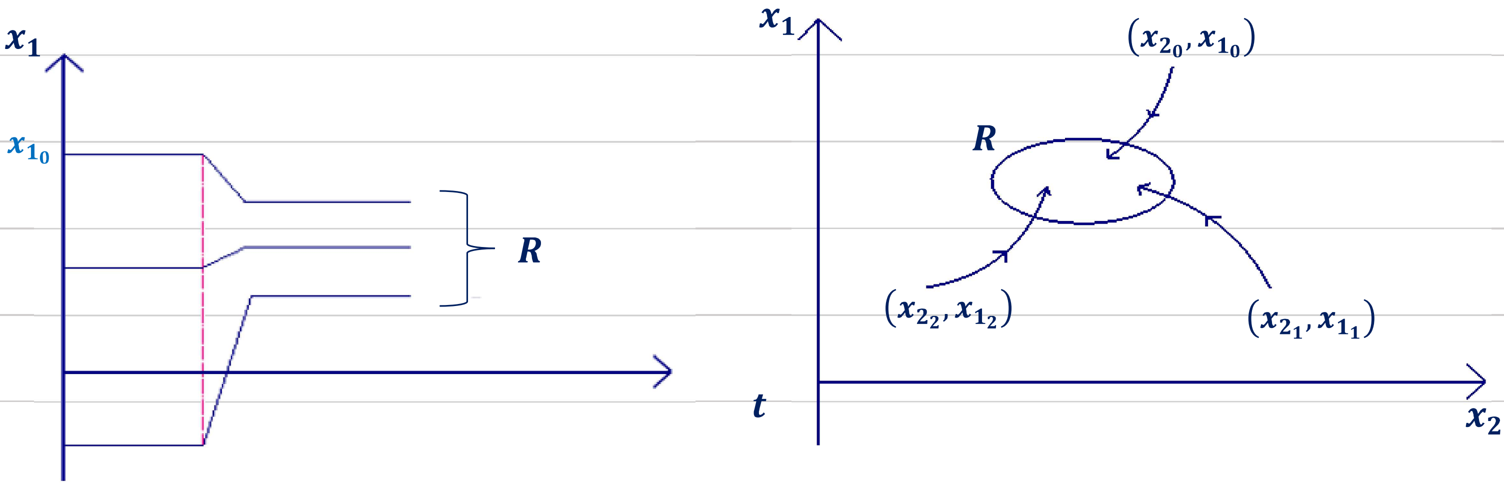


If the equilibrium point oscillates at a fixed frequency after the perturbation up to infinity, then the system has a limit cycle



Finite Stability

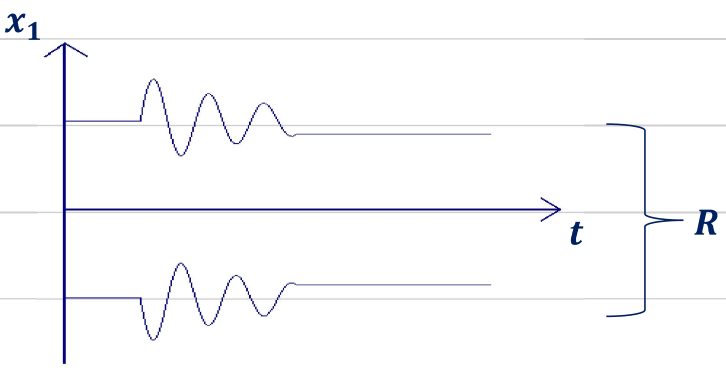
If the state of the system returns always to a certain finite region, then it is called finite stable operating point



Global Stability

The system is said globally stable if $\boldsymbol{R}$ includes the entire finite space

huge region



Linearization

Tuesday: 10-4-2018

The operating point stability of a non-linear dynamical system is studied using the eigen values of the linearized model around it

Let x_0 be the initial state vector and u_0 the input vector of the following:

$$x^{\cdot} = f(x, u)$$

Then , $x^{\cdot}_0 = f(x_0, u_0)$

Let us perturb the system from the above state:

$$x = x_0 + \underset{\text{small deviation}}{\underline{\underline{\Delta x}}}$$

$$x^{\cdot} = x^{\cdot}_0 + \Delta x^{\cdot}$$

$$x^{\cdot} = f[(x_0 + \Delta x), (u_0 + \Delta u)]$$

As the perturbation is relatively small, $f(x, u)$ can be expressed in terms of Taylor series expansion

$$x^{\cdot}_i = \cancelto{\text{Zero}}{x^{\cdot}_{i_0}} + \Delta x^{\cdot}_i = f_i[(x_0 + \Delta x), (u_0 + \Delta u)]$$

$$x^{\cdot}_i = \underset{\text{Zero}}{\cancel{f_i(x_0, u_0)}} + \frac{df_i}{dx_1}\Delta x_1 + \cdots + \frac{df_i}{dx_n}\Delta x_n + \frac{df_i}{du_1}\Delta u_1 + \cdots + \frac{df_i}{du_r}\Delta u_r$$

Tuesday: 10-4-2018

$$\Delta x^{\cdot}{}_i = \frac{df_i}{dx_1}\Delta x_1 + \cdots + \frac{df_i}{dx_n}\Delta x_n + \frac{df_i}{du_1}\Delta u_1 + \cdots + \frac{df_i}{du_r}\Delta u_r$$

As for the output variables:

$$\Delta y_i = \frac{dg_i}{d_{x1}}\Delta x_1 + \cdots + \frac{dg_i}{dx_n}\Delta x_n + \frac{dg_i}{du_1}\Delta u_1 + \cdots + \frac{dg_i}{du_r}\Delta u_r$$

- $\Delta x^{\cdot} = A.\Delta x + B.\Delta u$
- $\Delta y = C.\Delta x + D.\Delta u$

Δx: state vector (n)

Δy: output vector $(m: number\ of\ output\ variables)$

Δu: input vector (r)

$$A = \begin{Bmatrix} \frac{df_1}{dx_1} & \cdot & \cdot & \cdot & \frac{df_1}{dx_n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{df_n}{dx_1} & \cdot & \cdot & \cdot & \frac{df_n}{dx_n} \end{Bmatrix}_{n \times n}$$

State Matrix

$$B = \begin{Bmatrix} \frac{df_1}{du_1} & \cdot & \cdot & \cdot & \frac{df_1}{du_r} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{df_n}{du_1} & \cdot & \cdot & \cdot & \frac{df_n}{du_r} \end{Bmatrix}_{n \times r}$$

Input Matrix

$$C = \begin{Bmatrix} \frac{dg_1}{dx_1} & \cdot & \cdot & \cdot & \frac{dg_1}{dx_n} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{dg_m}{dx_1} & \cdot & \cdot & \cdot & \frac{dg_m}{dx_n} \end{Bmatrix}_{m \times n}$$

Output Matrix

Mostly Zero

$$D = \begin{Bmatrix} \frac{dg_1}{du_1} & \cdot & \cdot & \cdot & \frac{dg_1}{du_r} \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \frac{dg_m}{du_1} & \cdot & \cdot & \cdot & \frac{dg_m}{du_r} \end{Bmatrix}_{m \times r}$$

Feedforward Matrix

Operating Point Stability

Tuesday: 10-4-2018

- Eigen-values of the state matrix:

Roots of the characteristic equation

1) When the eigen-values is –ve real part → asymptotically stable O.P.

2) When at least one of the eigen-values has +ve real part → unstable O.P.

3) When at least one of the eigen-values has zero real part → it is not possible to say anything about the O.P. stability ⇨ Linearization cannot provide information ⇨ Go to non-linear mode

- Eigen properties of the state matrix eigen-values:

$A.\phi = \lambda.\phi$ $\phi: n \times 1$ eigen-vector

$(A - \lambda I).\phi = 0$ I: identity matrix

$|A - \lambda I| = 0$

- Eigen-vectors:

For any eigen-value $\boldsymbol{\lambda}_i$, the $\boldsymbol{n}$ – column vector $\boldsymbol{\phi}_i$ which satisfy $\boldsymbol{A}.\boldsymbol{\phi} = \boldsymbol{\lambda}.\boldsymbol{\phi}$ is called the right eigen-vector associated with the eigen –value $\boldsymbol{\lambda}_i$

$A.\phi_i = \lambda_i.\phi_i$, $\phi_i = \begin{bmatrix} \phi_{1i} \\ \phi_{2i} \\ . \\ . \\ \phi_{ni} \end{bmatrix}$

- Left eigen-vectors:

$\psi_i.A = \lambda_i.\psi_i$

Right and left eigen-vectors are orthogonal

$\psi_j.\phi_i = 0 \rightarrow i \neq j$

$\psi_i.\phi_i = C_i \;\rightarrow\; \psi_i.\phi_i = 1 \;\rightarrow$ if it is normalized

Eigen-Values and stability

a) Real eigen-value provides a non-oscillating mode; negative real represents a decaying mode, the larger it's magnitude the faster the decay.

b) Complex conjugates corresponding to oscillatory mode.

c) The real component of the eigen-values gives the damping and the imaginary part gives the frequency of oscillations

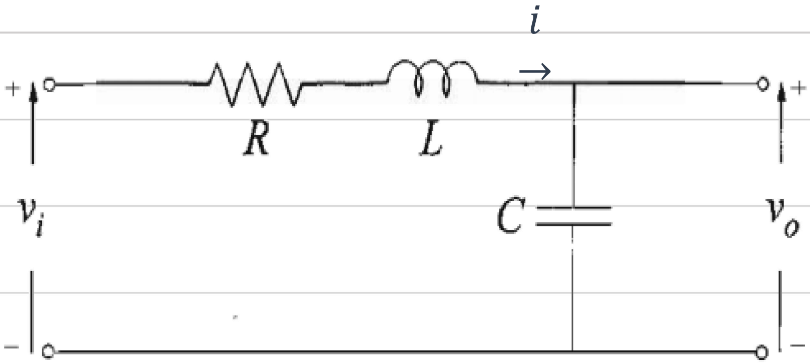
$\lambda = \sigma \pm j\omega$

$f = \frac{\omega_n}{2\pi}$, $\zeta = \frac{-\sigma}{\sqrt{\sigma^2+\omega^2}}$

The form of the time domain response:

$x_i(t) = \phi_{i1} C_1 e^{\lambda_1 t} + \phi_{i2} C_2 e^{\lambda_2 t} + \cdots + \phi_{in} C_n e^{\lambda_n t}$

Example:



Study the eigen-properties.

Solution:

$$-v_i + Ri + L\frac{di}{dt} + v_o = 0$$

$$i = c\frac{dv_o}{dt}$$

$$-v_i + RC\frac{dv_o}{dt} + LC\frac{d^2v_o}{dt^2} + vo = 0$$

Eigen-Values and Stability

Tuesday: 17-4-2018

In standard form:

$$\frac{d^2 v_o}{dt^2} + (2\zeta\omega_n)\frac{dv_o}{dt} + \omega_n^2\, v_o = \omega_n^2\, v_i$$

$\omega_n = \frac{1}{\sqrt{LC}}$, natural frequency

$\zeta = \frac{R/2}{\sqrt{L/C}}$, damping ratio

In state-space representation:

$x_1 = v_o$, $x_2 = x^{\cdot}{}_1 = \frac{dv_o}{dt}$

$x^{\cdot}{}_2 = \frac{d^2 v_o}{dt^2}$, $u = v_i$

$y = v_o = x_1$ → output , $x^{\cdot}{}_1 = x_2$

$x^{\cdot}{}_2 = -\omega_n^2\, x_1 - 2\,\zeta\omega_n\, x_2 + \omega_n^2\, u$

$$\begin{bmatrix} x^{\cdot}{}_1 \\ x^{\cdot}{}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \omega_n^2 \end{bmatrix} [u] \qquad \rightarrow x^{\cdot} = Ax + Bu$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 \qquad \rightarrow y = Cx + Du$$

$$|A - \lambda I| = 0$$

$$\left|\begin{bmatrix} 0 & 1 \\ -\omega_n^2 & -2\zeta\omega_n \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right| = 0$$

$$\left|\begin{bmatrix} -\lambda & 1 \\ -\omega_n^2 & -2\zeta\omega_n - \lambda \end{bmatrix}\right| = 0$$

$$-\lambda(-2\zeta\omega_n - \lambda) + \omega_n^2 = 0$$

$$\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0$$

$$\left.\begin{aligned} \lambda_1 &= -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1} \\ \lambda_2 &= -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1} \end{aligned}\right\} \text{Eigen-values}$$

The right eigen-vector:

$$A.\phi_i = \lambda_i.\phi_i$$

$$(A - \lambda_i I).\phi_i = 0$$

$$\begin{bmatrix} -\lambda_i & 1 \\ -\omega_n^2 & -2\zeta\omega_n - \lambda_i \end{bmatrix}\underbrace{\begin{bmatrix} \phi_{1i} \\ \phi_{2i} \end{bmatrix}}_{\text{right eigen-vectors}} = 0$$

$$\left.\begin{aligned} -\lambda_i\phi_{1i} + \phi_{2i} &= 0 \\ -\omega_n^2\phi_{1i} - (2\zeta\omega_n + \lambda_i)\phi_{2i} &= 0 \end{aligned}\right\} \text{ unknowns are } \phi_{1i} \,\&\, \phi_{2i}$$

Assumption:

Let $\phi_{1i} = 1 \rightarrow$ (normalized ; PU)

Then, $\phi_{2i} = \lambda_i$

$$\phi_1 = \begin{bmatrix} \phi_{11} \\ \phi_{21} \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1} \end{bmatrix}$$

$$\phi_2 = \begin{bmatrix} \phi_{12} \\ \phi_{22} \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda_2 \end{bmatrix} = \begin{bmatrix} 1 \\ -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1} \end{bmatrix}$$

$$\phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1} & -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1} \end{bmatrix}$$

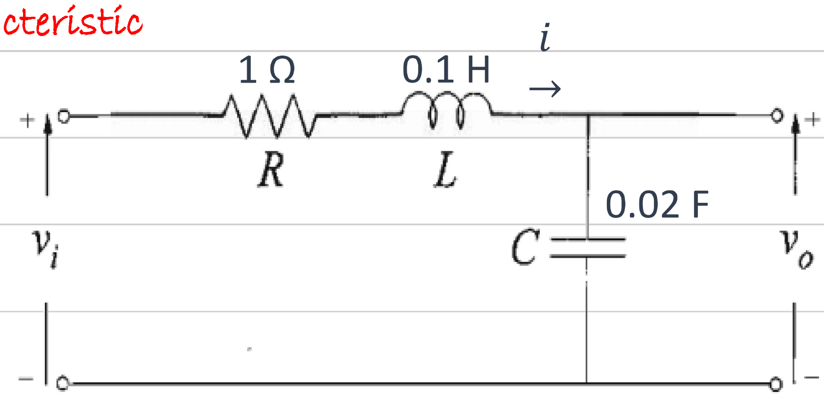
$$x_1(t) = \phi_{11}C_1e^{\lambda_1 t} + \phi_{12}C_2e^{\lambda_2 t}$$

$$x_1(t) = (1)C_1e^{\left(-\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}\,\right)t} + (1)C_2e^{\left(-\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}\,\right)t}$$

$$x_2(t) = \phi_{21}C_1e^{\lambda_1 t} + \phi_{22}C_2e^{\lambda_2 t}$$

$$x_2(t) = \left(-\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}\,\right)C_1e^{\left(-\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}\,\right)t} + \left(-\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}\,\right)C_2e^{\left(-\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}\,\right)t}$$

Example: for the following RLC circuit, study the eigen-properties of the static matrix and examine it's model characteristic



$$-v_i + Ri + L\frac{di}{dt} + v_o = 0$$

$$\frac{d^2 v_o}{dt^2} + 10\frac{dv_o}{dt} + 500\ v_o = 500\ v_i$$

Let:

$x_1 = v_o$ $\quad$ $, x_2 = x^{\cdot}{}_1 = v^{\cdot}{}_o$

$x^{\cdot}{}_2 = x^{\cdot\cdot}{}_1 = v^{\cdot\cdot}{}_o$ $\quad$ $, u = v_i$

$y = v_o = x_1$ $\quad$ $, x^{\cdot}{}_1 = x_2$

$x^{\cdot}{}_2 = -500\ x_1 - 10\ x_2 + 500\ u$

$$\begin{bmatrix} x^{\cdot}{}_1 \\ x^{\cdot}{}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -500 & -10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 500 \end{bmatrix} [u] \qquad \rightarrow x^{\cdot} = Ax + Bu$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0 \qquad \rightarrow y = Cx + Du$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 0 & 1 \\ -500 & -10 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = 0$$

$$\left| \begin{bmatrix} -\lambda & 1 \\ -500 & -10 - \lambda \end{bmatrix} \right| = 0$$

$\lambda^2 + 10\lambda + 500 = 0$ → Characteristic equation

$$\lambda_1 = -5 + j\,21.79$$

$$\lambda_2 = -5 - j\,21.79$$

The right eigen-vector:

$$A.\phi_i = \lambda_i.\phi_i$$

$$(A - \lambda_i I).\phi_i = 0$$

For $\lambda_1 = -5 + j\,21.79$:

$$\left[\begin{bmatrix} 0 & 1 \\ -500 & -10 \end{bmatrix} - \begin{bmatrix} -5 + j21.79 & 0 \\ 0 & -5 + j21.79 \end{bmatrix}\right] \begin{bmatrix} \phi_{11} \\ \phi_{21} \end{bmatrix} = 0$$

$$\left.\begin{aligned} &(5 - j21.79)\,\phi_{11} + \phi_{21} = 0 \\ &-500\,\phi_{11} + (-5 - j21.79)\,\phi_{21} = 0 \end{aligned}\right\} \text{Not independent}$$

Let $\phi_{11} = 1$ then, $\phi_{21} = -5 + j21.97$

$$\phi_1 = \begin{bmatrix} \phi_{11} \\ \phi_{21} \end{bmatrix} = \begin{bmatrix} 1 \\ -5 + j21.79 \end{bmatrix}$$

For $\lambda_2 = -5 - j\,21.79$:

$(A - \lambda_2 I).\phi_2 = 0$

$$\left.\begin{aligned} &(5 + j21.79)\,\phi_{12} + \phi_{22} = 0 \\ &-500\,\phi_{12} + (-5 + j21.79)\,\phi_{22} = 0 \end{aligned}\right\} \text{Not independent}$$

Let $\phi_{12} = 1$ then, $\phi_{22} = -5 - j21.97$

$$\phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ \lambda_1 & \lambda_2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -5 + j21.79 & -5 - j21.97 \end{bmatrix}$$

$$x_1(t) = \phi_{11} C_1 e^{\lambda_1 t} + \phi_{12} C_2 e^{\lambda_2 t}$$

$$x_1(t) = (1) C_1 e^{(-5+21.79)t} + (1) C_2 e^{(-5-21.79)t}$$

$$x_1(t) = (1) C_1 e^{-5t} e^{j\,21.79t} + (1) C_2 e^{-5t} e^{-j\,21.79t}$$, $C_1 \& C_2$ found from boundary or initial conditions

$$cos(21.79t) + j\,sin(21.79t) \qquad cos(21.79t) - j\,sin(21.79t)$$

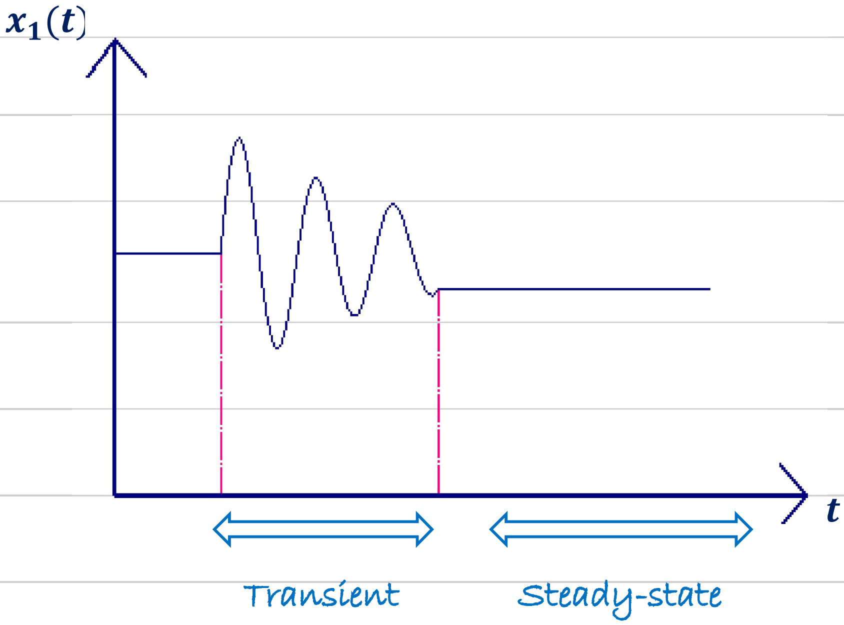
$$x_2(t) = \phi_{21} C_1 e^{\lambda_1 t} + \phi_{22} C_2 e^{\lambda_2 t}$$

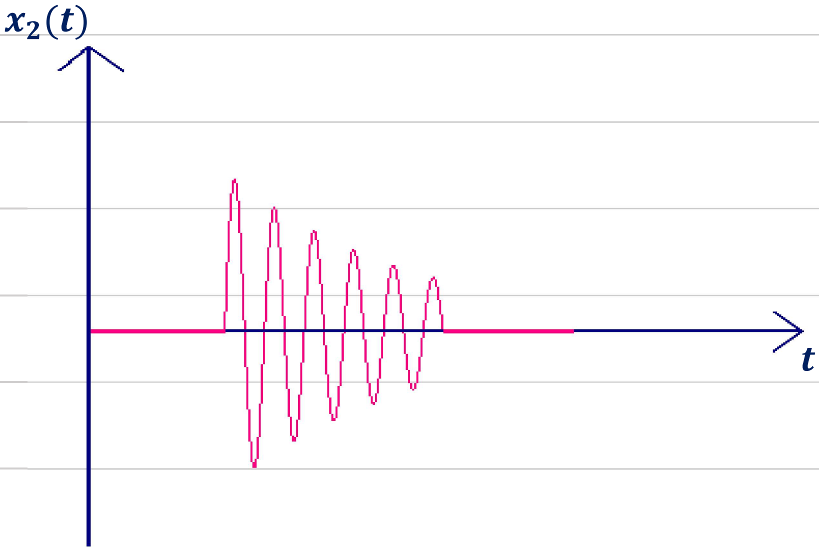
$$x_2(t) = (-5 + j21.79) C_1 e^{(-5+21.79)t} + (-5 - j21.79) C_2 e^{(-5-21.79)t}$$

$$x_2(t) = (-5 + j21.79) C_1 e^{-5t} e^{j\,21.79t} + (-5 - j21.79) C_2 e^{-5t} e^{-j\,21.79t}$$

$$cos(21.79t) + j\,sin(21.79t) \qquad cos(21.79t) - j\,sin(21.79t)$$

$\omega_n = 21.79 = 2\pi f \rightarrow f = \frac{21.79}{2\pi}$ → frequency of oscillation

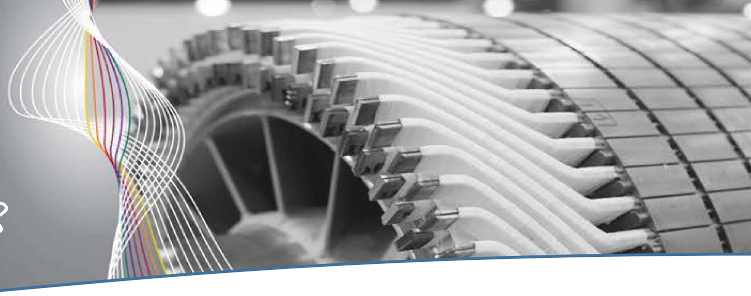




$$x^{\cdot}_{1} = x_2$$

$$x^{\cdot}_{2} = -500\, x_1 - 10\, x_2 + 500\, u$$

Eigen-Values and Stability



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Example: Operating point stability of a non-linear dynamical system

$\dot{x}_1 = 2\,x_1 + x_2^2 + 2$

$\dot{x}_2 = \sin x_1 - x_2$

Solution:

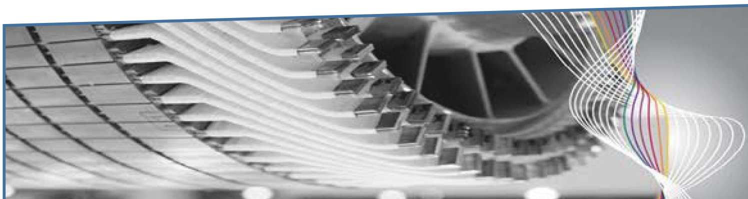
$$A = J = \begin{bmatrix} \frac{d\dot{x}_1}{dx_1} & \frac{d\dot{x}_1}{dx_2} \\ \frac{d\dot{x}_2}{dx_1} & \frac{d\dot{x}_2}{dx_2} \end{bmatrix} = \begin{bmatrix} 2 & 2\,x_2 \\ \cos x_1 & -1 \end{bmatrix}$$

Jacobian

$$\left.\begin{aligned} 2\,x_1 + x_2^2 + 2 &= 0 \\ \sin x_1 - x_2 &= 0 \end{aligned}\right\} \text{Non-linear algebraic equation}$$

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} -1.5 \\ -1 \end{bmatrix}$$

$$A|_{x_1\,\&\,x_2} = \begin{bmatrix} 2 & -2 \\ \cos(-1.5) & -1 \end{bmatrix} = \begin{bmatrix} 2 & 2\,x_2 \\ 0.0707 & -1 \end{bmatrix}$$



Eigen-Values and Stability

Tuesday: 24-4-2018

$$|A-\lambda I| = 0$$

$$\left|\begin{bmatrix}-2-\lambda & -2\\ 0.0707 & -1-\lambda\end{bmatrix}\right| = 0$$

$$\left.\begin{aligned}\lambda_1 &= 1.83\\ \lambda_2 &= 1.17\end{aligned}\right\} \text{Unstable O.P.}$$

$$\phi = \begin{bmatrix}\phi_{11} & \phi_{12}\\ \phi_{21} & \phi_{22}\end{bmatrix} = \begin{bmatrix}1 & 1\\ \lambda_1 & \lambda_2\end{bmatrix} = \begin{bmatrix}1 & 1\\ 1.83 & 1.17\end{bmatrix}$$

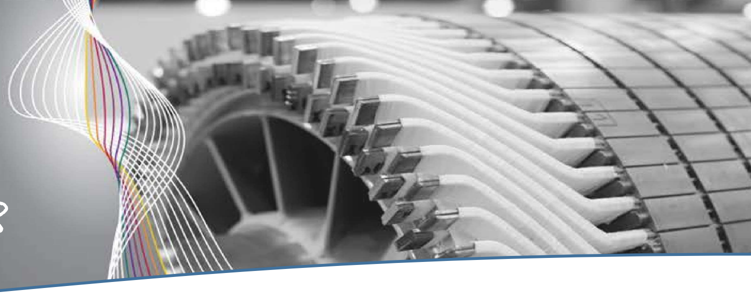
$$x_1(t) = \phi_{11}C_1e^{\lambda_1 t} + \phi_{12}C_2e^{\lambda_2 t}$$

$$x_1(t) = (1)C_1e^{1.83t} + (1)C_2e^{(1.17\,)t}$$

$$x_2(t) = \phi_{21}C_1e^{\lambda_1 t} + \phi_{22}C_2e^{\lambda_2 t}$$

$$x_2(t) = (1.83)C_1e^{1.83t} + (1.17)C_2e^{(1.17\,)t}$$

$\boldsymbol{C_1}$& $\boldsymbol{C_2}$ are obtained from additional information (initial or boundary conditions)



Eigen-Values and Stability

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$$A\dot{x}_1 + B\dot{x}_2 = f_1(x)$$

$$C\dot{x}_2 = f_2(x)$$

$$\begin{bmatrix} A & B \\ 0 & C \end{bmatrix} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} A & B \\ 0 & C \end{bmatrix}^{-1} \begin{bmatrix} f_1(x) \\ f_2(x) \end{bmatrix}$$

$$\dot{x}_1 = g_1(x)$$

$$\dot{x}_2 = g_2(x)$$

O.P.

$$0 = f_1(x)$$

$$0 = f_2(x)$$

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