~~طلب~~ شابلي

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Power System ①

Stability

تأثير

* Stability studies which evaluate the impact of disturbance on the electromechanical dynamic behavior of the system are of two types:

إضطراب

(large signal stability)

① transient stability : studying the ability of the synchronous generator to maintain synchronism with the power system after a large distarbance like three-phase fault, step change in AVR gain, step change in active power demand, step change in reactive power demand and lightening.

القدرة

الحفاظ

عبارة عن فولتج عالي

(small signal stability)

② steady-state stability : studying the stability of the system under small incremental variations in parameters or operating conditions about steady-state equilibrium point.

distarbance

تدريجي

النقطة التي نحصل على أقل انحراف

تشابك كبير

(2)

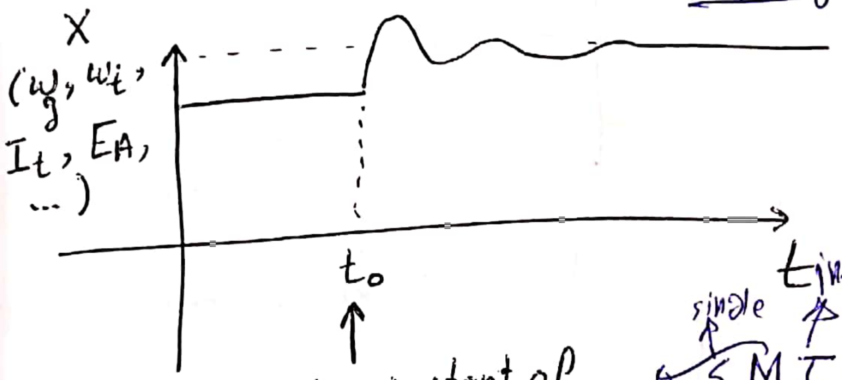
* In transient stability studies, the system models used are extensive (واسعة الانتشار) as the present-day power systems are vast (ضخمة), heavily interconnected (مترابطة) systems with hundreds of machines which can interact (تتفاعل) through the medium of their extra-high-voltage and ultra-high-voltage networks. These machines have associated (مرتبطة) excitation systems and turbine-governing (مسيطر) control systems. If the resultant nonlinear differential and algebraic equations of the overall system are to be solved, then either direct methods or iterative step-by-step procedures must be used.

* A power system is in a steady-state operating condition if all the measured physical quantities describing the operating condition of the system can be considered (يعتبر) constant for purposes (هدف) of analysis. When operating in a steady-state condition if a sudden (مفاجئ) change or sequence of changes occurs in one or more of the parameters of the system, or in one or more of its operating quantities, we say that the system has

dergone a disturbance from its steady-state operating condition. ③

W_t: rotational speed of generator

بارامتر واحد بحدد إذ الكل بتايل أولا



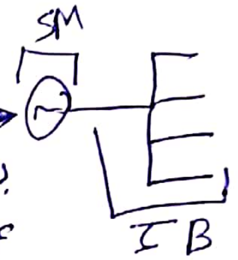
↑
the instant of disturbance

IB: constant Voltage frequency

SMIB — Single Machine Infinity Bus

عدد الجنيراتر الواصلة مع بعض بنفس المحطة

بعبر عن كل PS



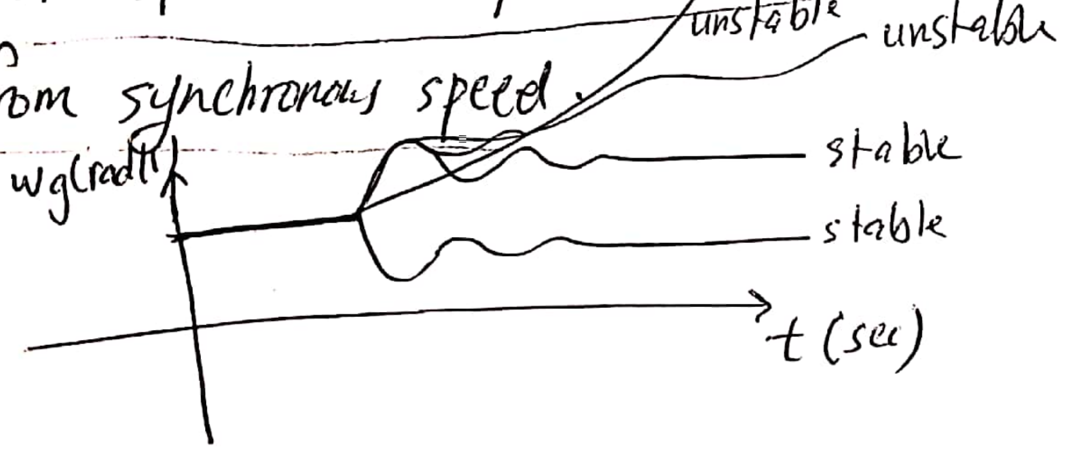
* steady-state stability studies are usually less extensive in scope than transient stability studies and often involve a single machine operating into an infinite bus or just a few machines undergoing one or more small disturbances. Thus, steady-state stability studies examine the stability of the system under small incremental variations in parameters or operating conditions about a steady-state equilibrium point. The nonlinear differential and algebraic equations of the system are replaced by a set of linear equations

④

which are then solved by methods of linear analysis to determine if the system is steady-state stable.

Since transient stability studies involve large disturbances, linearization of the system equations is not permitted. Transient stability is sometimes studied on a first-swing rather than a multiswing basis. First-swing transient stability studies use a reasonably simple generator model consisting of the transient internal voltage E_i' behind transient reactance X_d'. In such studies the excitation systems and turbine-governing control systems of the generating units are not represented. Usually, the time periode under study is the first second following a system fault or other large disturbance. If the machines of the system are found to remain essentially in synchronism within the first second, the system is regarded as being transiently stable. Multiswing stability studies extend over a longer

study period, and therefore the effects of (5)
the generating units control system must be considered
since they can affect the dynamic performance of
the units during the extended period. Machine models
of greater sophistication are then needed to properly
reflect the behavior of the system. Thus, excitation
systems and turbine-governing control systems
may or may not be represented in steady-state
and transient stability studies depending on the
objectives. In all stability studies, the objective
is to determine whether or not the rotors of the
machines being perturbed return to constant
speed operation. Obviously, this means that the
rotor speeds have departed at least temporarily
from synchronous speed.



تسهيل الحسابات

To facilitate computation, three fundamental assumptions are made in all stability studies:

① Only synchronous frequency currents and voltages are considered. Consequently, dc offset currents and harmonic components are neglected.

+ve seq & −ve seq & zero seq

② Symmetrical components are used in the representation of unbalanced faults. (+ve se

③ Generated voltage is considered unaffected by machine speed variations. بالحقيقة بتتأثر بس أنا بفرض إنه لا

$\rightarrow E_a = 4.44\,\phi f \rightarrow \frac{NP}{120}$

Sometimes (In large/small signal stability)

* ..., the effect of damper windings, AVR and TG are neglected and three-phase balanced faults are generally considered.

Damper winding: copper burse s.c from the two ends place in the pole faces in the rotor. they are used to damp oscillation of error

Rotor Dynamics and the Swing Equation ⑦

* Basically, the accelerating torque is the product of the moment of inertia of the rotor times its angular acceleration.

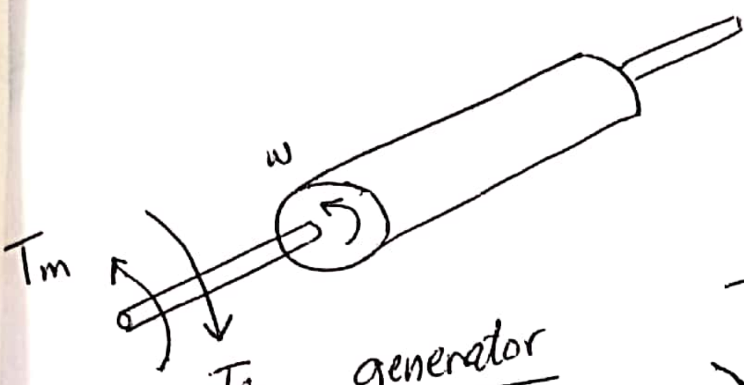
mass ↑ → J ↑

$$J \frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e \quad N.m$$

- θ_m: angular displacement of the rotor with respect to a stationary axis (rad. mechanical)
- J: total moment of inertia (kgm^2)
- t: time (sec)
- T_a: accelerating torque
- T_m: mechanical or shaft torque (input mechanical torque)
- T_e: electromagnetic torque (due to electrical loads (current))

No. load $T_e = \text{Zero}$ so $T_m = \text{Zero}$ because no O/P Power

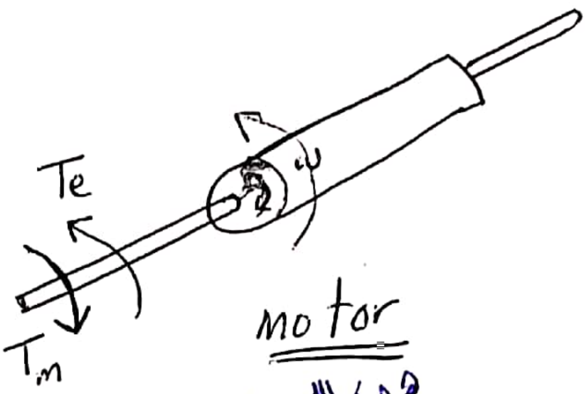
* T_m and T_e are considered positive for the synchronous generator. Under steady state condition $T_m = T_e$ and therefore $T_a = 0$. The generator in this case is running at synchronous speed.



generator

قوة الآلة بتحركه طاقة ميكانيكية

Tm



motor

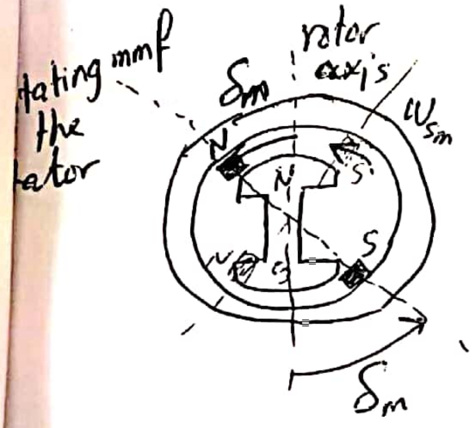
قوة اللي بحرك ال rotor طاقة كهربائية

* Tm is considered constant at any given operating condition. This assumption is a fair one for generators even though input from the prime mover is controlled by governors. Governors do not act until after a change in speed is sensed, and so they are not considered effective during the time period in which rotor dynamics are of interest in our stability studies here.

* Te corresponds to the air-gap power in the machine and thus accounts for the total output power of the generator plus $|I|^2R$ losses in the armature winding.

* θ_m is measured with respect to a (9)
Stationary reference axis on the stator. It is an absolute measure of rotor angle. Consequently, it continuously increases with time even at constant synchronous speed. Since the rotor speed relative to synchronous speed is of interest, it is more convenient to measure the rotor angular position with respect to a reference axis which rotates at synchronous speed. Therefore,

$$\theta_m = \omega_{sm} t + \delta_m$$



angular displacement of the rotor in mech. radians from the synchronously rotating reference axis.

at no-load $\delta_m = 0$. $\Rightarrow$ $\theta_m = \omega_{sm} t$

$$\frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt}$$

$\frac{d\delta_m}{dt} = 0$ if the load is constant and/or at no-load conditions $\Rightarrow \frac{d\theta_m}{dt} = \omega_{sm}$

$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2} \Rightarrow$ the rotor acceleration measured in mechanical radians per second squared.

$$\therefore \quad J\frac{d^2\delta_m}{dt^2} = T_a = T_m - T_e \quad Nm.$$

For convenient notational purposes let

$$\omega_m = \frac{d\theta_m}{dt}$$

multiplying the above equation by ω_m

$$J\omega_m\frac{d^2\delta_m}{dt^2} = \omega_m T_a = \omega_m T_m - \omega_m T_e$$

$$J\omega_m\frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e$$

P_e ← electrical power crossing air gap

P_a: accelerating power

P_m: mechanical power

* Usually, $|I^2|R$ is neglected and therefore P_e is considered as the output power of the generator.

* $J\omega_m$ is the angular momentum of the rotor at synchronous speed ω_{sm} and denoted by M and called the inertia constant of the machine.

$$M\frac{d^2\delta_M}{dt^2} = P_a = P_m - P_e \quad W$$

$\Rightarrow$ normally used as power is more convenient for calculations.

$M = J\omega_m$

* In machine data, another constant refered to inertia is often encountered. It is called "H" constant which is defined by:

$$H = \frac{\text{stored kinetic energy in MJ at } \omega_s}{MVA_{rating}}$$

$$= \frac{\frac{1}{2}J\omega_{sm}^2}{S_{mach}} = \frac{\frac{1}{2}M\omega_{sm}}{S_{mach}} \quad MJ/MVA$$

solving for M, we obtain

$$M = \frac{2H}{\omega_{sm}} S_{mach} \quad MJ/\text{mech rad.}$$

$$\underset{\text{mech. rad/s}}{\frac{2H}{\omega_{sm}}} \frac{d^2\delta_M}{dt^2} = \frac{P_a}{S_{mach}} = \frac{P_m - P_e}{S_{mach}}$$

mech. rad.

could be mech. or elec.

$$\frac{2H}{\omega_s} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad pu$$

swing equation

2nd order differential equation (nonlinear)

could be mech. or elec. per second

ω_s & δ must be consistent.

elec. rad.

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad pu$$

or

elec. degrees

$$\frac{H}{180 f} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad pu$$

* Swing equation is a 2^{nd} order nonlinear differential equation. It can be written as two first order differential equations as: 13

$$\left.\begin{aligned} \frac{2H}{\omega_s}\frac{d\omega}{dt} &= P_m - P_e \\ \frac{d\delta}{dt} &= \omega - \omega_s \end{aligned}\right] \text{ normally solved numerically}$$

Further Considerations of the Swing Equation

* In a stability study of a power system with many synchronous machines only one MVA base common to all parts of the system can be chosen.

To convert H from the machine base to system base:

$$H_{system} = H_{mach} \frac{S_{mach}}{S_{system}}$$

Machine manufacturers use the symbol WR^2 to specify for the rotating parts of a generating unit (including the prime mover). It is the weight in pounds multiplied by the square of the radius in feet. Table 16.1 PP. 703 provides the inertia constant H for different generators.

(14)

Ex. Develop formula to calculate the H constant from WR^2 and then evaluate H for a nuclear generating unit at 1333 MVA, 1800 r/min with $WR^2 = 5820000$ Ib-ft^2.

Hint: 550 ft-Ib/s $= 746$ W

$$H = \frac{\text{stored kinetic energy in MJ at } \omega_s}{\text{machine rating in MVA}}$$

$$= \frac{\left(\frac{746}{550} \times 10^{-6}\right) \frac{1}{2} \frac{WR^2}{32.2} \left(\frac{2\pi \, r/min}{60}\right)^2}{S_{mach}}$$

$$\boxed{H = \frac{2.31 \times 10^{-10} \, WR^2 \, (r/min)^2}{S_{mach}}}$$

Inserting the given machine data,

$$H = \frac{(2.31 \times 10^{-10})(5820000)(1800)^2}{1333}$$

$$= 3.27 \ MJ/MVA$$

Converting to the base of 100 MVA gives:

$$H = 3.27 \ \frac{1333}{100} = 43.56 \ MJ/MVA$$

* In stability study, it is desirable to minimize the number of swing equations to be solved. This can be done if the transmission line fault or other disturbance on the system affects the machines within a power plant so that their rotors swing together. In such cases the machines within the plants can be combined into a single equivalent machine just as if their rotors were mechanically coupled and only one swing equation must be written for them. Consider a power plant with

two generators connected to the same bus which is electrically remote from the network disturbances. The swing equations on the common system base are:

$$\frac{2H_1}{\omega_s}\frac{d^2\delta_1}{dt^2} = P_{m_1} - P_{e_1} \quad pu$$

$$\frac{2H_2}{\omega_s}\frac{d^2\delta_2}{dt^2} = P_{m_2} - P_{e_2} \quad pu$$

two generators at the same power plant

Adding the two equations and denoting δ_1 & δ_2 by δ as the two rotors swing together, we obtain

$\delta_1 = \delta_2$

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e \quad pu$$

$P_m = P_{m_1} + P_{m_2}$ $\quad$ $P_e = P_{e_1} + P_{e_2}$

$H = H_1 + H_2$

Ex. Two 60 Hz generating units operate in parallel within the same power plant and have the following ratings:

Unit #1: 500 MVA, 0.85 PF, 20 kV, 3600 r/min

$$H_1 = 4.8 \text{ MJ/MVA}$$

Unit #2: 1333 MVA, 0.9 PF, 22 kV, 1800 r/min

$$H_2 = 3.27 \text{ MJ/MVA}$$

Calculate the equivalent H constant for the two units on a 100 MVA base.

$$KE = (4.8 * 500) + (3.27 * 1333) = 6759 \text{ MJ}$$

Therefore, the H constant for the equivalent machine on 100 MVA base is:

$$H = 67.59 \text{ MJ/MVA}.$$

This value can be used in a single swing equation provided the machines swing together.

* Machines which swing together (connected at the same bus) are called "coherent" machines.

$\delta = \delta_1 = \delta_2$

* For any pair of noncoherent machines, the system swing equations can be written as:

$$\frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2} = \frac{\omega_s}{2}\left(\frac{P_{m_1} - P_{e_1}}{H_1} - \frac{P_{m_2} - P_{e_2}}{H_2}\right)$$

multiplying each side by $\frac{H_1 H_2}{H_1 + H_2}$ and rearranging we find that:

$$\frac{H_1 H_2}{H_1 + H_2}\,\frac{d^2(\delta_1 - \delta_2)}{dt^2} = \frac{\omega_s}{2}\left[\frac{P_{m_1} H_2 - P_{m_2} H_1}{H_1 + H_2} - \frac{P_{e_1} H_2 - P_{e_2} H_1}{H_1 + H_2}\right]$$

$$\frac{2}{\omega_s} H_{12} \frac{d^2\delta_{12}}{dt^2} = P_{m_{12}} - P_{e_{12}} \Rightarrow$$ two noncoherent machines

$$\delta_{12} = \delta_1 - \delta_2$$

$$H_{12} = \frac{H_1 H_2}{H_1 + H_2}$$

$$P_{m12} = \frac{P_{m_1} H_2 - P_{m_2} H_1}{H_1 + H_2}$$

$$P_{e12} = \frac{P_{e_1} H_2 - P_{e_2} H_1}{H_1 + H_2}$$

An important

A ~~noteworthy~~ application of these equations (noncoherent) concerns a two-machine system having only one generator (machine one) and a synchronous motor (machine two) connected by a network of pure reactances. Whatever change occurs in the generator output is thus absorbed by the motor and we can write:

$$P_{m_1} = -P_{m_2} = P_m$$

$$P_{e_1} = -P_{e_2} = P_e$$

Under these conditions, $P_{m12} = P_m$ & $P_{e12} = P_e$

nd the result is:

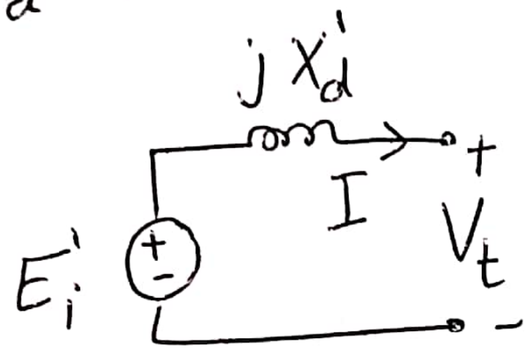
$$\frac{2H_{12}}{\omega_s}\frac{d^2\delta_{12}}{dt^2} = P_m - P_e$$

* The previous discussion emphasizes the relative nature of the system stability property and shows that the essential features of stability are revealed by consideration of two machine problems. Such problems are of two types: those having one machine of finite inertia swinging with respect to an infinite bus and those having two finite inertia machines swinging with respect to each other. An infinite bus may be considered for stability purposes as a bus at which there is located a machine of constant internal voltage, having zero impedance and infinite inertia.

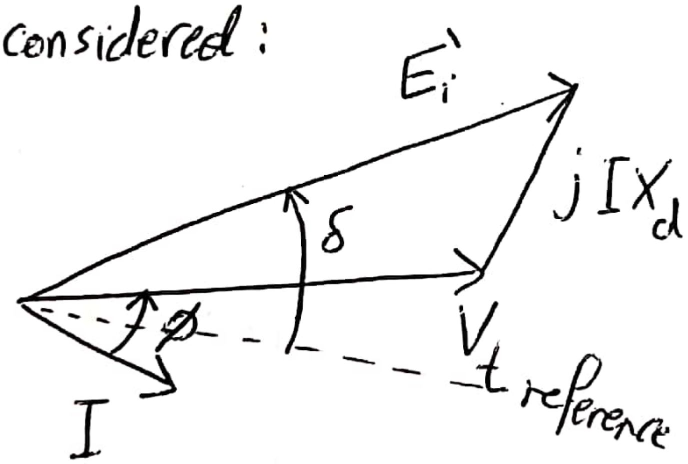
The power Angle Equation

In the swing equation of the generator, the input mechanical power from the prime mover P_m is considered constant. This is a reasonable assumption because conditions in the electrical network can be expected to change before the control governor can cause the turbine to react. ~~Stne~~ Since P_m is constant, P_e determines whether the rotor accelerates, decelerates or remain at synchronous speed. Changes in P_e are determined by conditions on the transmission and distribution networks and the loads on the system to which the generator supplies power. Electrical network disturbances resulting from severe load changes, network faults, or circuit breaker operations may cause the generator output P_e to change rapidly in which case electromechanical transients exist.

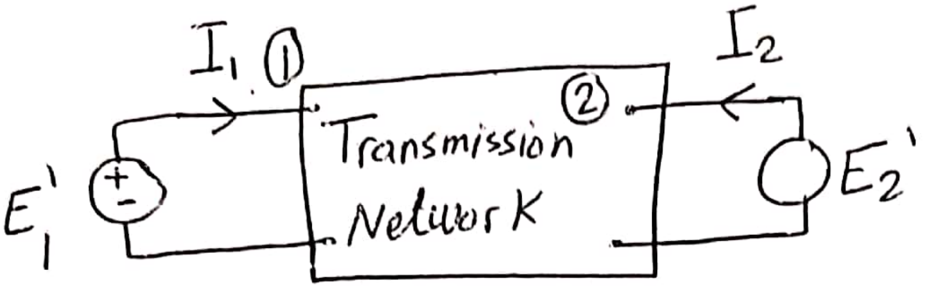
Each synchronous machine is represented for transient stability studies by its transient internal Voltage E_i' in series with the transient reactance X_d' :



* Armature (stator) resistance is neglected in most cases and therefore the following phasor diagram is considered:



* The following figure represents a generator supplying power through a transmission line to a receiving-end system at bus 1.



$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix}$$

but : (from chapter 9 of the book)

$$P_K + jQ_K = V_K \sum_{n=1}^{N} (Y_{kn} V_n)^*$$

Let $K = 1$ & $N = 2$ and replacing

V with E', we obtain

$$P_1 + jQ_1 = E_1'(Y_{11} E_1')^* + E_1'(Y_{12} E_2')^*$$

if we define $\tilde{E}_1' = |E_1'| \angle \delta_1$ $\tilde{E}_2' = |E_2'| \angle \delta_2$

$\tilde{Y}_{11} = G_{11} + jB_{11}$ $\tilde{Y}_{12} = |Y_{12}| \angle \theta_{12}$

then

$$P_1 = |E_1'|^2 G_{11} + |E_1'||E_2'||Y_{12}| \cos(\delta_1 - \delta_2 - \theta_{12})$$

$$Q_1 = -|E_1'|^2 B_{11} + |E_1'||E_2'||Y_{12}| \sin(\delta_1 - \delta_2 - \theta_{12})$$

if we let

$$\delta = \delta_1 - \delta_2$$

and

$$\gamma = \theta_{12} - \frac{\pi}{2}$$

then

power angle equation

$$P_1 = |E_1'|^2 G_{11} + |E_1'||E_2'||Y_{12}| \sin(\delta - \gamma)$$

$$Q_1 = -|E_1'|^2 B_{11} - |E_1'||E_2'||Y_{12}| \cos(\delta - \gamma)$$

The equation of P_1 can be rewritten as:

$$P_e = P_c + P_{max} \sin(\delta - \gamma)$$

$$P_c = |E_1'|^2 G_{11} \quad \& \quad P_{max} = |E_1'||E_2'||Y_{12}|$$

The parameters P_c, P_{max} and γ are constants for a given network configuration and constant voltage magnitudes $|E_1'|$ and $|E_2'|$. →

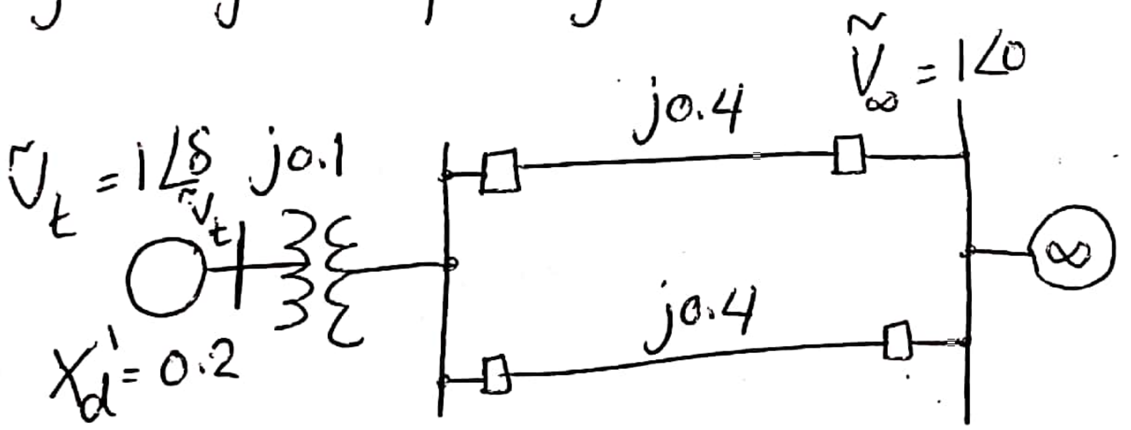
* If the network is considered without resistance, all the elements of $\tilde{Y}_{bus}$ are susceptances and then G_{11} and γ are both zero. The power angle equation will then be :

$$P_e = P_{max} \sin \delta$$

where

$$P_{max} = \frac{|E_1'||E_2'|}{X}$$ and X is the transfer reactance between E_1' and E_2'.

Ex. The single line diagram of the following figure shows a generator connected through parallel transmission lines to a large metropolitan system considered as an infinite bus. The machine is delivering 1.0 pu power and both the terminal voltage and the infinite bus voltage are 1.0 pu. The reactances of the lines are on common system base. Determine the power-angle equation for the given system operating conditions.



$$X = 0.1 + 0.4 // 0.4 = 0.3 \text{ pu}$$

$$P_e = P_{max} \sin\delta = \frac{(1)(1)}{0.3} \sin\delta = 3.333 \sin\delta$$

Solving for δ :

$$1 = 3.3333 \sin\delta \Rightarrow \sin\delta = \frac{1}{3.3333}$$

$$\delta = 17.458^\circ \Rightarrow \tilde{V}_t = 1.0\angle 17.458^\circ$$

The output current from the generator can be calculated as :

$$\tilde{I} = \frac{1\angle 17.458^\circ - 1\angle 0}{j0.3} = 1.012\angle 8.729^\circ \text{ pu}$$

The transient internal voltage E_1' is :

$$E_1' = \tilde{V}_t + \tilde{I}\, j0.2$$

$$= 1\angle 17.458^\circ + (1.012)\angle 8.729^\circ)(j0.2)$$

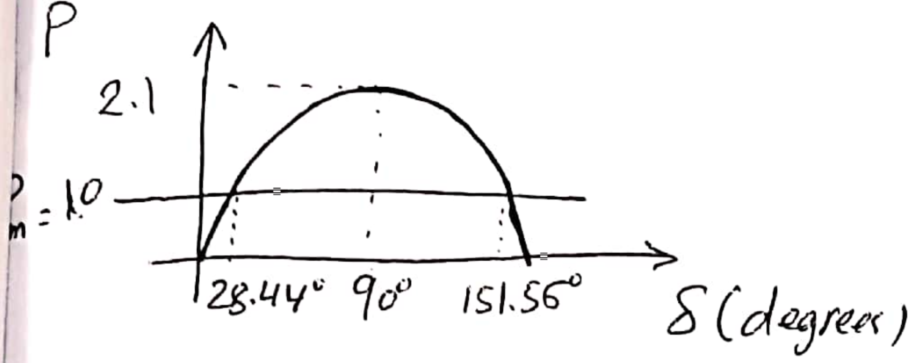
$$= 1.05\angle 28.44^\circ \text{ pu}$$

* The power-angle equation relating the transient voltage E_i' and the infinite bus will be :

$$X = 0.2 + 0.1 + 0.4 // 0.4 = 0.5 \text{ pu}$$

$$P_e = \frac{(1.05)(1)}{0.5} \sin\delta = 2.10 \sin\delta \text{ pu}$$

where δ is the machine rotor angle w.r.t the infinite bus.

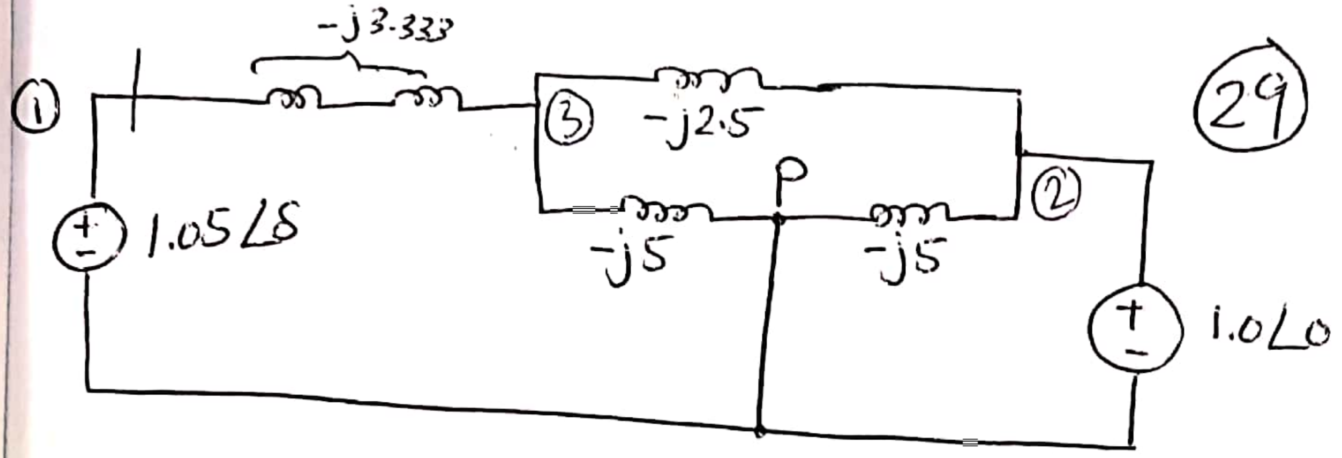


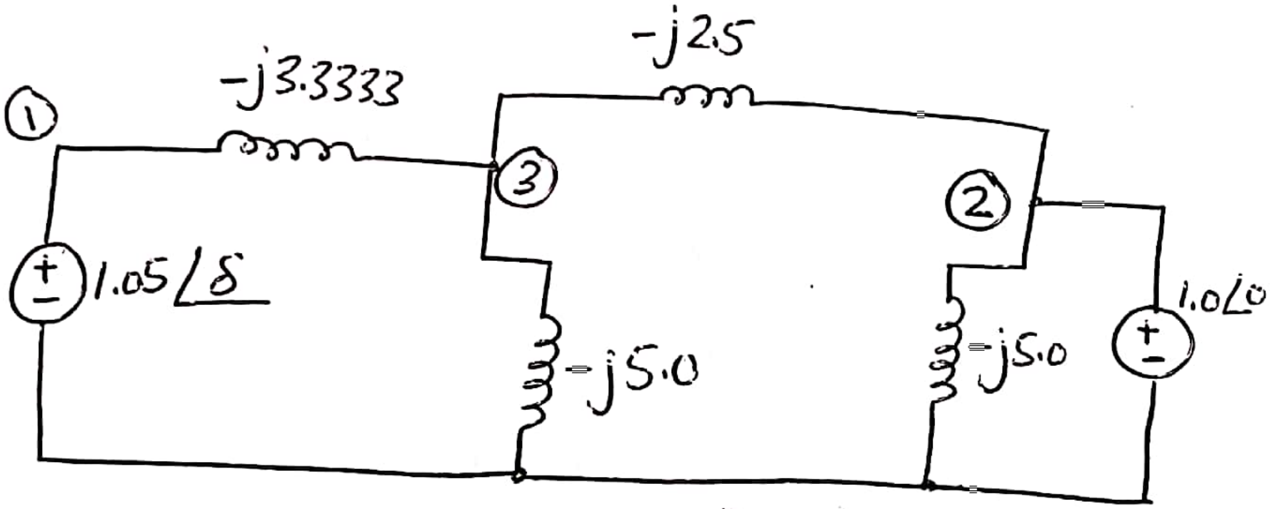
The swing equation for the machine may be written as:

$$\frac{H}{180f}\frac{d^2\delta}{dt^2} = 1.0 - 2.10 \sin\delta \quad pu$$

EX. For the previous example, a three-phase fault occurs at point P of the transmission line. Determine the power-angle equation for the system with the fault on and the corresponding swing equation. Take $H = 5\ MJ/MVA$.

The network with the fault at point P is:





$$Y_{bus} = \begin{bmatrix} -j3.3333 & 0 & j3.3333 \\ 0 & -j7.5 & j2.5 \\ j3.3333 & j2.5 & -j10.8333 \end{bmatrix}$$

Note: The voltage at bus 3 is unknow and therefore can be removed by "Kron Elimination"

Bus 3 has no external source connection and it may be removed by the node elimination procedure of <u>Sec. 7.4</u> to yield the reduced bus admittance matrix:

Bus #3 is eliminated using Krom reduction as it is Voltage is unknown :

$\underline{P=3}$

$$\tilde{Y}_{jK}(new) = \tilde{Y}_{jK} - \frac{\tilde{Y}_{jp}\tilde{Y}_{pK}}{\tilde{Y}_{pp}}$$

$$\tilde{Y}_{11}(new) = -j3.3333 - \frac{(j3.3333)(j3.3333)}{-j10.8333}$$

$$= -j2.3077 \text{ pu}$$

$$\tilde{Y}_{12}(new) = 0 - \frac{(j3.3333)(j2.5)}{-j10.8333}$$

$$= j0.7692 \text{ pu}$$

$$\tilde{Y}_{21}(new) = \tilde{Y}_{12}(new) = j0.7692 \text{ pu}$$

$$\tilde{Y}_{22}(new) = -j7.5 - \frac{(j2.5)(j2.5)}{-j10.83333}$$

$$= -j6.9231$$

$$\tilde{Y}_{bus_{reduced}} = \begin{bmatrix} \tilde{Y}_{11} & \tilde{Y}_{12} \\ \tilde{Y}_{21} & \tilde{Y}_{22} \end{bmatrix}$$

$$= \begin{bmatrix} -j2.308 & j0.769 \\ j0.769 & -j6.923 \end{bmatrix}$$

$$P_{max} = \underbrace{(1.05)(1)(0.769)}_{|E_i'||E_2'||Y_{12}|} = 0.808 \text{ pu}$$

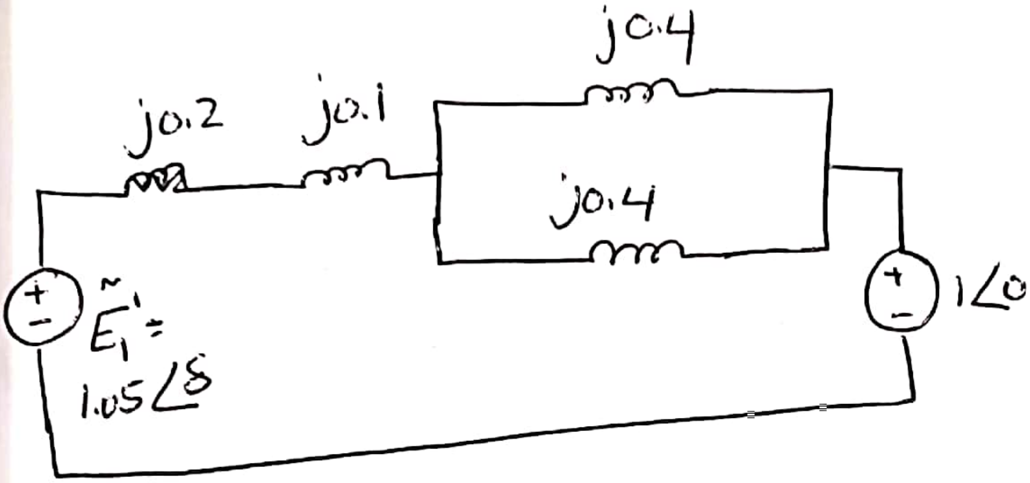
and therefore,

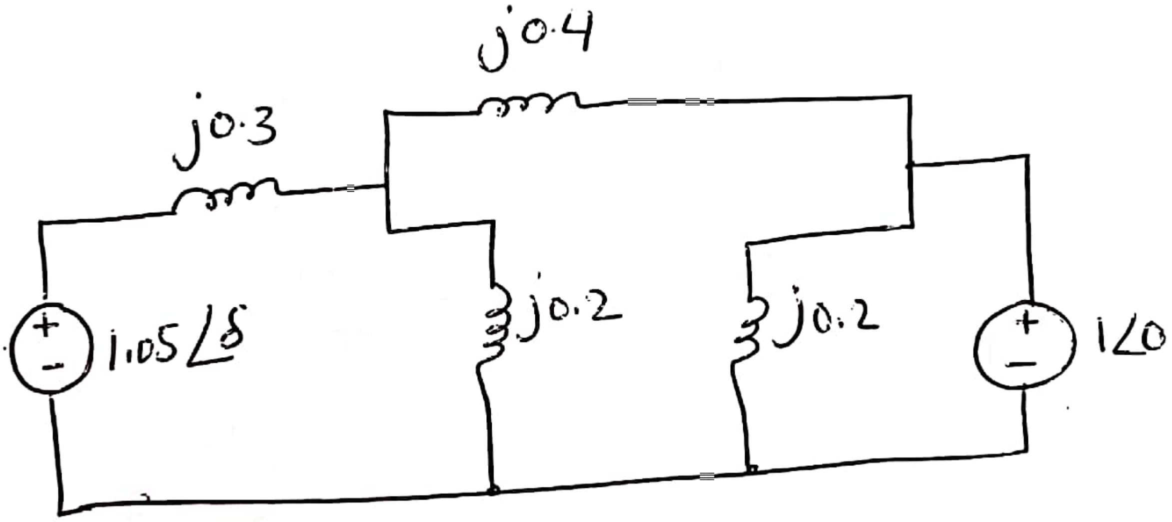
$$P_e = 0.808 \sin \delta \text{ pu}$$

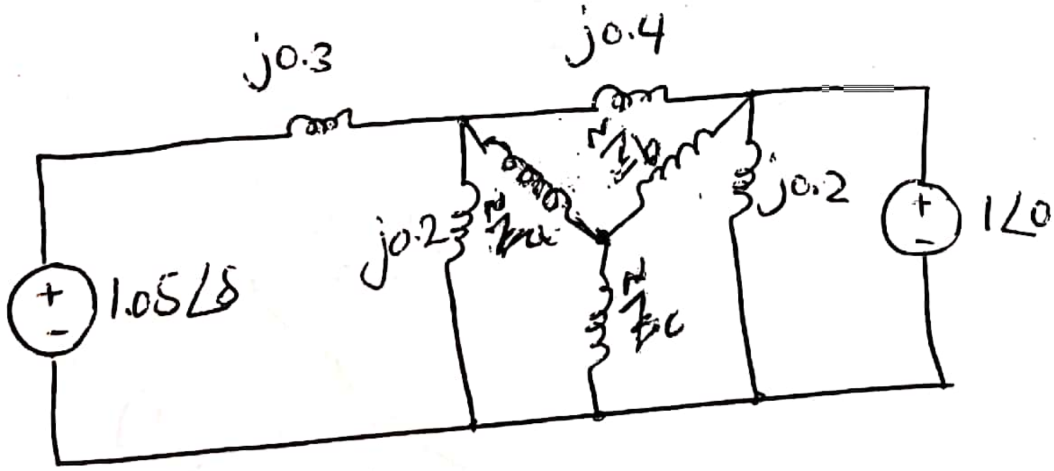
The corresponding swing equation is:

$$\frac{5}{180f} \frac{d^2\delta}{dt^2} = 1.0 - 0.808 \sin \delta \text{ pu}$$

OR


j0.4
j0.2
j0.1
j0.4
$\tilde{E}_1' =$
1.05∠δ
1∠0


j0.4
j0.3
1.05∠δ
j0.2
j0.2
1∠0

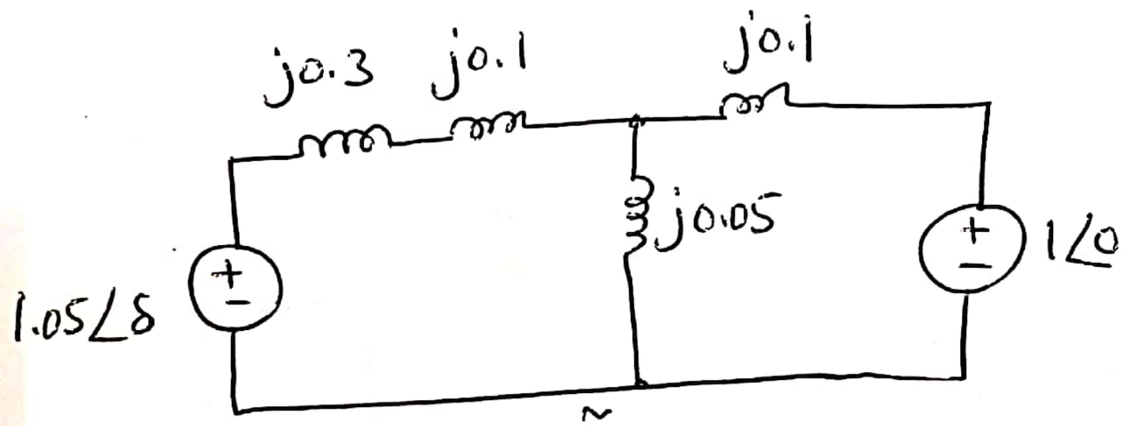

j0.3
j0.4
$\tilde{Z}_b$
j0.2
j0.2
$\tilde{Z}_a$
1.05∠δ
$\tilde{Z}_c$
1∠0

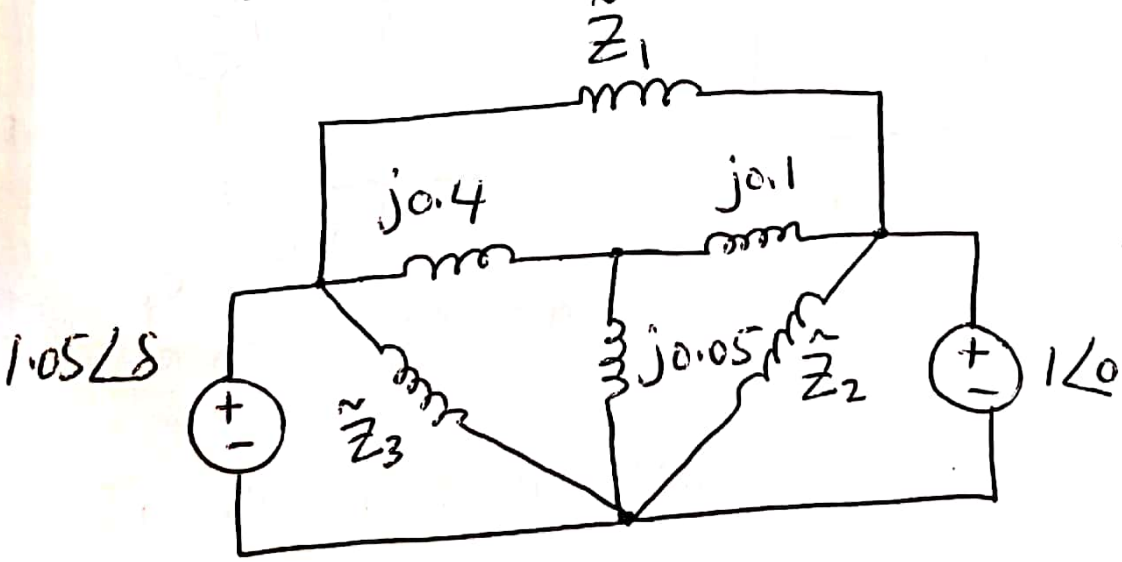
$$\tilde{Z}_a = \frac{(j0.2)(j0.4)}{j0.2+j0.4+j0.2} = \frac{-0.08}{j0.8}$$

$$= j0.1$$

$$\tilde{Z}_b = \frac{(j0.4)(j0.2)}{j0.8} = j0.1$$

$$\tilde{Z}_c = \frac{(j0.2)(j0.2)}{j0.8} = \frac{-0.04}{j0.8} = j0.05$$





33

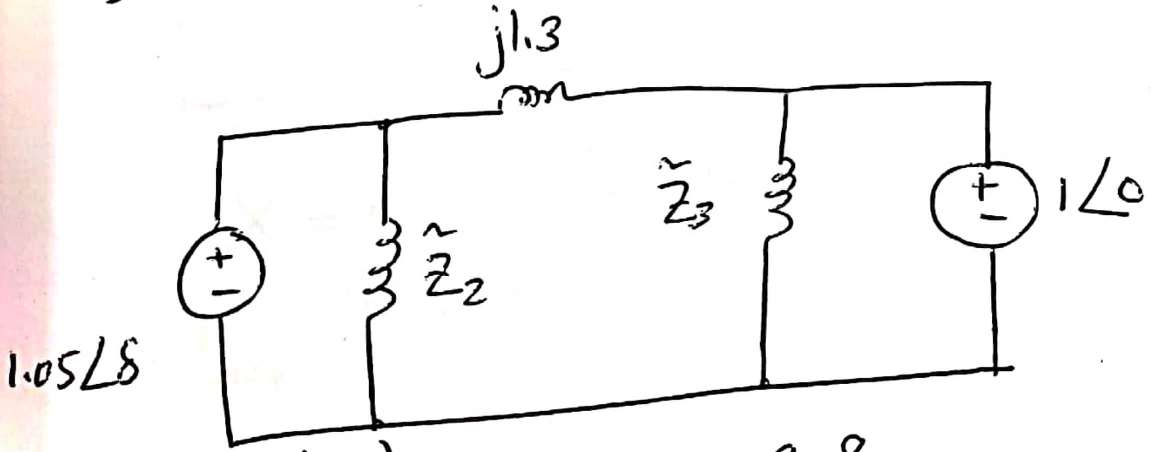
$$\tilde{Z}_1 = \frac{(j0.4)(j0.05) + (j0.4)(j0.1) + (j0.1)(j0.05)}{j0.05}$$

$$= \frac{-0.02 - 0.04 - 0.005}{j0.05} = \frac{-0.065}{j0.05}$$

$$= j1.3$$

$$\tilde{Z}_2 =$$

$$\tilde{Z}_3 =$$



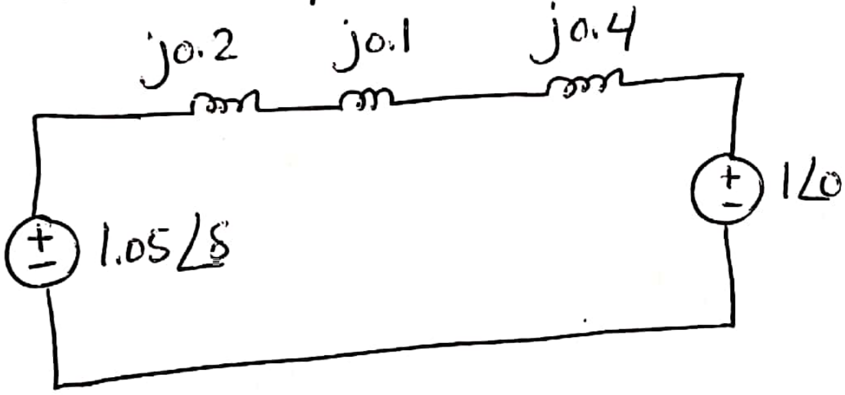
$$P_{max} = \frac{(1.05)(1)}{1.3}\sin\delta = 0.808 \sin\delta$$

The corresponding swing equation is:

$$\frac{5}{180 f}\frac{d^2\delta}{dt^2} = 1.0 - 0.808 \sin\delta \text{ pu}$$

Ex. The fault on the system is cleared by simultaneous opening of the circuit breakers at each end of the affected line. Determine the power-angle equation and the swing equation of the postfault period.

The circuit of the postfault period is:



$$X = 0.2 + 0.1 + 0.4 = 0.7 \text{ pu}$$

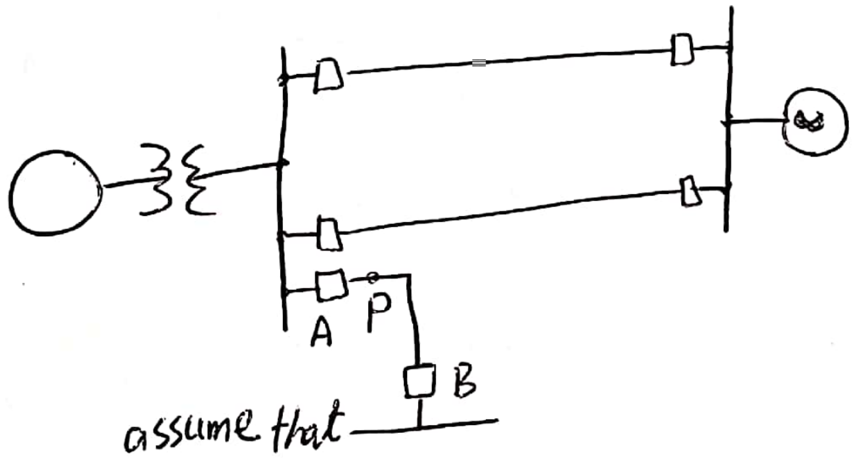
$$P_{max} = \frac{(1.05)(1)}{0.7} = 1.5 \text{ pu}$$

$$\frac{5}{180f}\frac{d^2\delta}{dt^2} = 1.0 - 1.5\sin\delta \quad \text{pu}$$

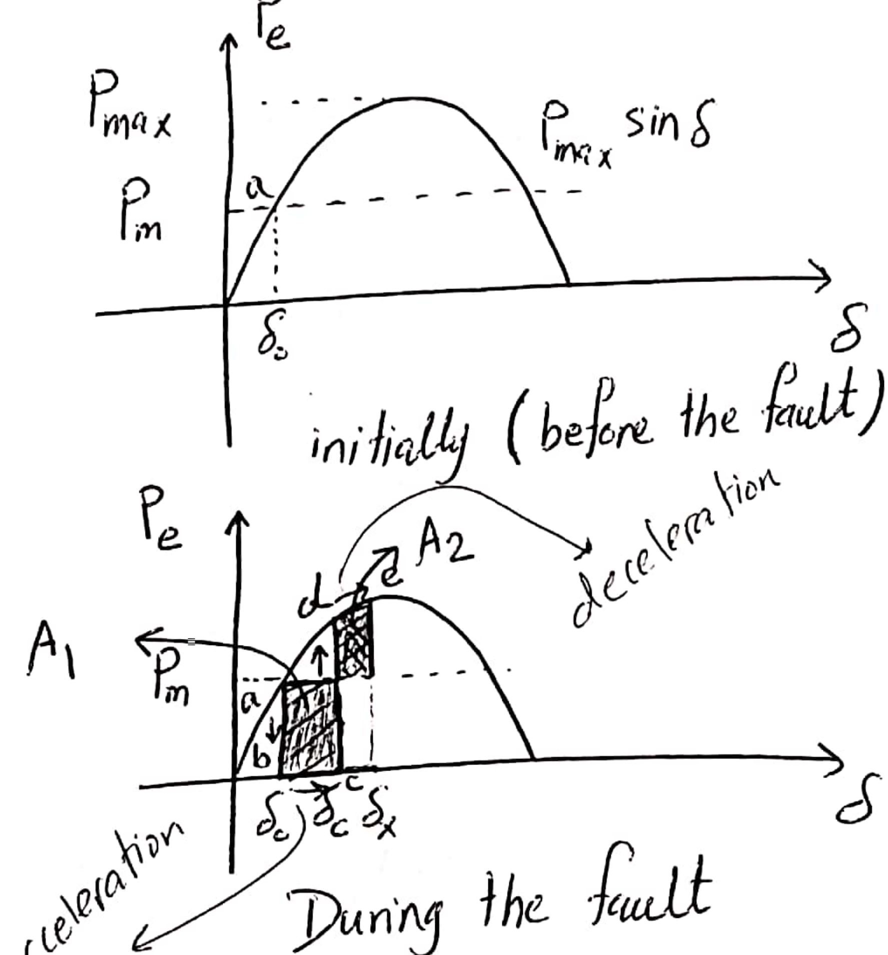
Equal-Area Criterion of Stability

* Swing equation is a second order ordinary nonlinear differential equation. Explicit solution is not possible. Numerically solutions using computer methods are therefore needed.

* Consider the following system:



assume that Initially circuit breaker A is closed and circuit breaker B is open. At point P a three-phase fault occurs and is cleared by circuit breaker A after a short period of time. The short circuit caused by the fault is effectively at the bus and therefore the electrical power output of from the generator during the fault is zero.



initially (before the fault)

During the fault

* During the fault the output electrical power equals zero and the rotor is subjected to acceleration Power P_a equals P_m.

If we denote the time to clear the fault by t_c, then the acceleration is constant for time t less than t_c and is given by:

$$\frac{H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m \Longrightarrow \text{During fault}$$

while the fault is on (during fault), the increase in velocity above ω_s is found by:

$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H} P_m$$

$$\underline{\omega} = \frac{d\delta}{dt} = \int_0^t \frac{d^2\delta}{dt^2} dt = \int_0^t \frac{\omega_s}{2H} P_m dt = \frac{\omega_s}{2H} P_m t$$

the increase in velocity

and δ will be:

$$\delta = \int \frac{\omega_s}{2H} P_m t + \delta_0$$

$$\delta = \frac{\omega_s P_m}{4H} t^2 + \delta_0 \leftarrow \text{initial value.}$$

$\therefore$ ω increase linearly and δ increases up to δ_c that is from b to c. At the instant of clearing the increase in rotor speed and the angle separation between the generator and the infinite bus are given by:

rease $\uparrow$ rotor speed $\omega\Big|_{t=t_c} = \dfrac{d\delta}{dt}\Big|_{t=t_c} = \dfrac{\omega_s P_M}{2H} t_c$

and $\delta(t)\Big|_{t=t_c} = \dfrac{\omega_s P_M}{4H} t_c^2 + \delta_0$

When the fault is cleared at the angle δ_c, the electrical power output suddenly increases to a value corresponding to point d on the power angle curve. At d the electrical power output exceeds the mechanical power input and thus the accelerating power is negative (deceleration). $P_m < P_e$

As a consequence, the rotor slows down as P_e goes from d to e. At e the rotor speed is again ω_s but the rotor angle increases to δ_x.

The value of δ_x is determined by the fact that A_1 and A_2 must be equal. (will be explained later why).

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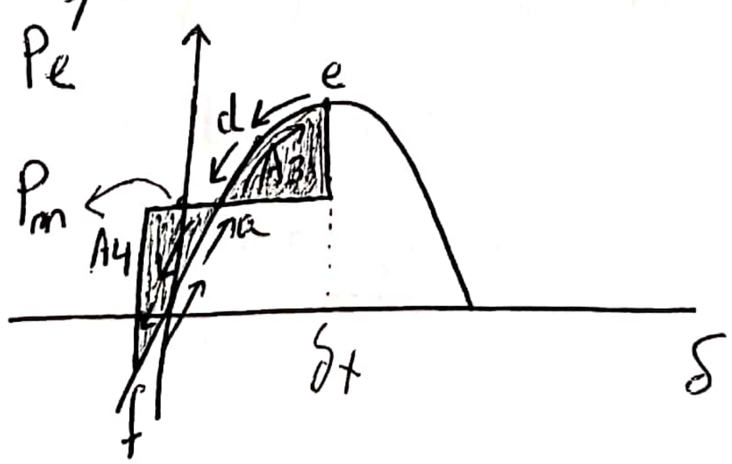
The accelerating power at e is still negative and so the rotor cannot remain at synchronous speed but continue to slow down.

and therefore the rotor angle moves back from δ_x at e along the power-angle curve to point a at which the rotor speed is less than ω_s. From a to f the mechanical power exceeds the electrical power and the rotor increases speed again until it reaches ω_s at f. Point f is located so that A_3 & A_4 are equal.



* In the absence of damping the rotor would continue to oscillate in the sequence f-a-e e-a-f and so with synchronous speed occuring at e and f.

* In a system where one machine is swinging with respect to an infinite bus we may use this principle of equality of areas, called the equal-area cirterion to determine the stability of the system under transient conditions without solving the swing equation.

* The swing equation for the machine connected to infinite bus is:

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e$$

Defining the angular velocity of the rotor relative to ω_s by:

$$\omega_r = \frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{2H}{\omega_s}\frac{d\omega_r}{dt} = P_m - P_e$$

multiplying both sides by $\omega_r = \frac{d\delta}{dt}$ gives:

$$\frac{2H}{\omega_s}\,\omega_r\frac{d\omega_r}{dt} = (P_m - P_e)\overbrace{\frac{d\delta}{dt}}^{\omega_r}$$

$$\boxed{\frac{d\omega_r^2}{dt} = 2\omega_r\frac{d\omega_r}{dt}}$$

$$\frac{2H}{\omega_s}\frac{1}{2}\frac{d(\omega_r^2)}{dt} = (P_m - P_e)\frac{d\delta}{dt}$$

multiplying both sides by dt yields

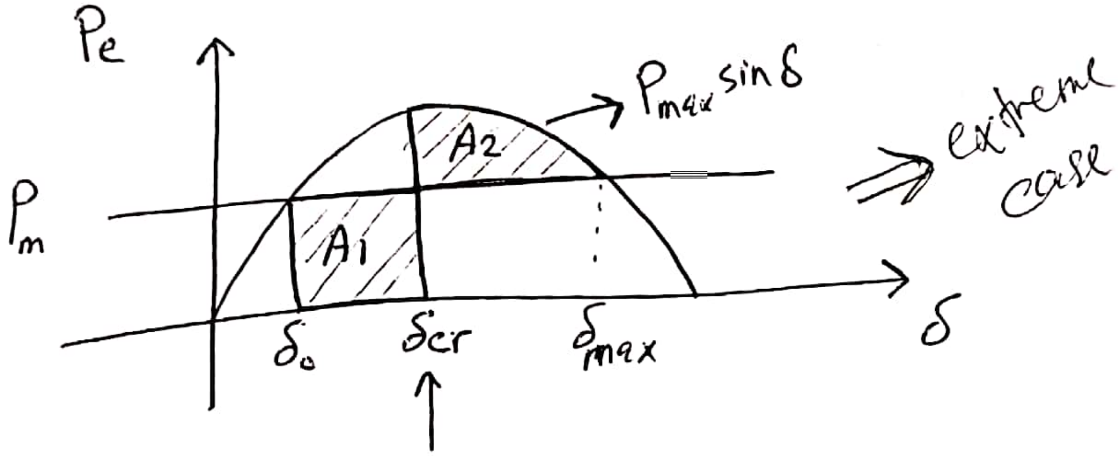
$$\frac{H}{\omega_s}\,d(\omega_r^2) = (P_m - P_e)\,d\delta$$

$$\int_{\omega_{r_1}}^{\omega_{r_2}} \frac{H}{\omega_s}\,d(\omega_r^2) = \int_{\delta_1}^{\delta_2} (P_m - P_e)\,d\delta$$

$$\frac{H}{\omega_s}\left(\omega_{r_2}^2 - \omega_{r_1}^2\right) = \int_{\delta_1}^{\delta_2} (P_m - P_e)\,d\delta$$

δ_1 corresponds to ωr_1 & δ_2 corresponds to ωr_2. (42)

* Coming back to the two equal areas:



$$A_1 = A_2$$

$$\int_{\delta_o}^{\delta_{cr}} P_m \, d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_{max} \sin\delta - P_m) \, d\delta$$

$$P_m(\delta_{cr} - \delta_o) = P_{max}(\cos\delta_{cr} - \cos\delta_{max}) - P_m(\delta_{max} - \delta_{cr})$$

$$P_m(\delta_{cr} - \delta_o) + P_m(\delta_{max} - \delta_{cr}) = P_{max}(\cos\delta_{cr} - \cos\delta_{max})$$

(43)

$$P_m\delta_{cr} - P_m\delta_o + P_m\delta_{max} - P_m\delta_{cr} = P_{max}(\cos\delta_{cr} - \cos\delta_{max})$$

$$P_m(\delta_{max} - \delta_o) = P_{max}(\cos\delta_{cr} - \cos\delta_{max})$$

$$\boxed{\cos\delta_{cr} = \frac{P_m}{P_{max}}(\delta_{max} - \delta_o) + \cos\delta_{max}}$$

but $\delta_{max} = \pi - \delta_o$

and $P_m = P_{max}\sin\delta_o$

$$\therefore \quad \delta_{cr} = \cos^{-1}\left[(\pi - 2\delta_o)\sin\delta_o - \cos\delta_o\right]$$

previously.

$$\delta_{cr} = \frac{\omega_s P_m}{4H} t_{cr}^2 + \delta_o$$

$$\boxed{t_{cr} = \sqrt{\frac{4H(\delta_{cr} - \delta_o)}{\omega_s P_m}}}$$

44

Ex. Calculate the critical clearing angle and the critical clearing time for the considered system (shown below) when the system is subjected to a three-phase fault at point P on the short transmission line. The initial conditions are the same as those earlier.

$H = 5$ MJ/MVA.

Sol. $P_e = P_{max} \sin\delta = 2.1 \sin\delta$

$\delta_0 = 28.44^\circ = 0.496$ rad.

$P_m = 1$

$$\delta_{cr} = \cos^{-1}\left[(\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0\right]$$

$$= \cos^{-1}\left[(\pi - (2)(0.496))\sin 28.44^\circ - \cos 28.44^\circ\right]$$

$$= 81.597^\circ = 1.425 \text{ rad.}$$

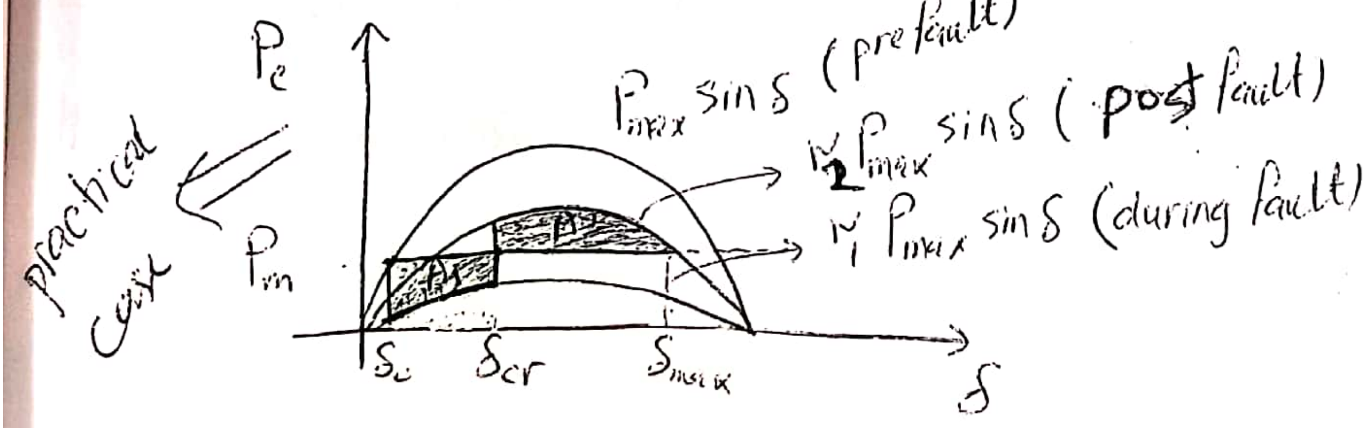
$$t_{cr} = \sqrt{\frac{4H(\delta_{cr} - \delta_0)}{\omega_s P_m}} = \sqrt{\frac{(4)(5)(1.425 - 0.496)}{377 \times 1}}$$

$$= 0.222 \text{ s}$$

Further Applications of The Equal-Area Criterion

* Equal-area criterion is a very useful mean for analyzing stability of a system of two machines or of a single machine suppling an infinite bus. For multimachine power system computer simulation (numerical techniques) are the only solution.

* In case of a single machine infinite bus power system described previously, if a three-phase fault occurs at one line which is then isolated after certain time, the following power-angle equations will be obtained:



* By evaluating A_1 & A_2 using the procedure steps of the previous section:

$$\cos\delta_{cr} = \frac{\left(\frac{P_m}{P_{max}}\right)(\delta_{max} - \delta_o) + r_2 \cos\delta_{max} - r_1 \cos\delta_o}{r_2 - r_1}$$

* A literal-form solution for the critical clearing time t_{cr} is not possible in this case.

* In all of the previous studies, three-phase faults are examined. The single line-to-ground fault occurs most frequently and the three-phase fault is the least frequent. For complete reliability a system should be designed for transient stability for three-phase faults at the worst locations.

(47)

Ex. Determine the critical clearing angle for three-phase fault described before when the initial system configuration and prefault operating conditions are as described before i.e.

Before the fault: $P_{max} \sin\delta = 2.1 \sin\delta$

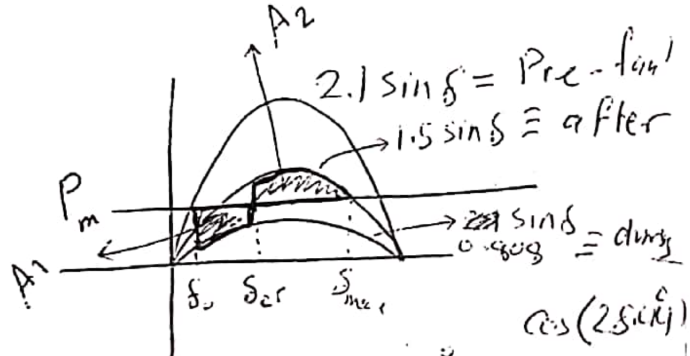
During the fault: $r_1 P_{max} \sin\delta = 0.808 \sin\delta$

After the fault: $r_2 P_{max} \sin\delta = 1.5 \sin\delta$

Sol. $r_1 = \dfrac{0.808}{2.1} = 0.385$ $\quad r_2 = \dfrac{1.5}{2.1} = 0.714$

$\delta_0 = 28.44^\circ = 0.496$ rad.

$\therefore \delta_{max} = 180^\circ - \sin^{-1}\left[\dfrac{1}{1.5}\right] = 138.2^\circ = 2.412$ rad.



Hence:

$$\delta_{cr} = \cos^{-1}\left[\dfrac{\left(\frac{1}{2.1}\right)(2.412 - 0.496) + 0.714\cos(138.19^\circ) - 0.385\cos(28.44^\circ)}{0.714 - 0.385}\right]$$

$= 82.725^\circ$.

Multimachine Stability Studies : Classical (48)

Representation

1.03.2017
3.03.2019

* In multimachine power systems (three machines or more) equal area criterion cannot be used directly. When a multimachine system operates under electromechanical transient conditions, intermachine oscillations occur through the medium of the transmission system connecting the machines. A typical frequency of such an oscillation is of the order 1-2 Hz, and this is superimposed upon the nominal 50 Hz frequency of the system. To ease the complexity of system modeling and thereby the computational burden, the following additional assumptions are commonly made in transient stability studies :

(1) The mechanical power input to each machine remains constant during the entire period of the swing curve computation.

2. Damping power is negligible.

3) Each machine is represented by a constant transient reactance in series with a constant transient internal voltage.

4) The mechanical rotor angle of each machine coincides with δ, the electrical phase angle of the transient internal voltage.

5) All loads are considered as shunt impedances to ground with values determined by conditions immediately prior to the transient conditions.

* The system stability model based on these assumptions is called the "classical stability model" and studies which use this model are called "classical stability studies".

In any transient stability studies, the system conditions before the fault and the network configuration both during and after it must be known. Consequently, in the multimachine case two preliminary steps are required:

1. The steady-state prefault conditions for the system are calculated using power-flow program.

2. The prefault network representation is determined and then modified to account for the fault and for the postfault conditions.

For multimachine stability problem, the following steps are used:

1. The values of power, reactive power and voltage at each generator terminal and load bus with all angles measured with respect to the slack bus must be known using the power-flow studies.

(2) The transient internal voltage of each generator is then calculated as:

$$E' = V_t + j X_d' I$$

(3) Each load is converted into a constant admittance to ground at its bus using the equation:

$$\tilde{Y}_L = \frac{P_L - j Q_L}{|V_L|^2}$$

(4) The bus admittance matrix which is used for the prefault power-flow calculation should be augmented to include the transient reactance of each generator and the shunt admittance of each load.

(Note: the injected current is zero at all buses except the internal buses of the generators).

(5) The bus admittance matrix is modified to correspond to the faulted and postfault conditions. Since only the generator internal buses have injections, all other buses

can be eliminated by Kron reduction such that the dimension of the modified matrix is equal to the the number of generators.

(6) During and after the fault, the power-angle equation can be written as:

$$P_{e_1} = |E_1'|^2 G_{11} + |E_1'||E_2'||Y_{12}| \cos(\delta_{12} - \theta_{12}) +$$
$$|E_1'||E_3'||Y_{13}| \cos(\delta_{13} - \theta_{13})$$

where $\delta_{12} = \delta_1 - \delta_2$.
$\delta_{13} = \delta_1 - \delta_3$

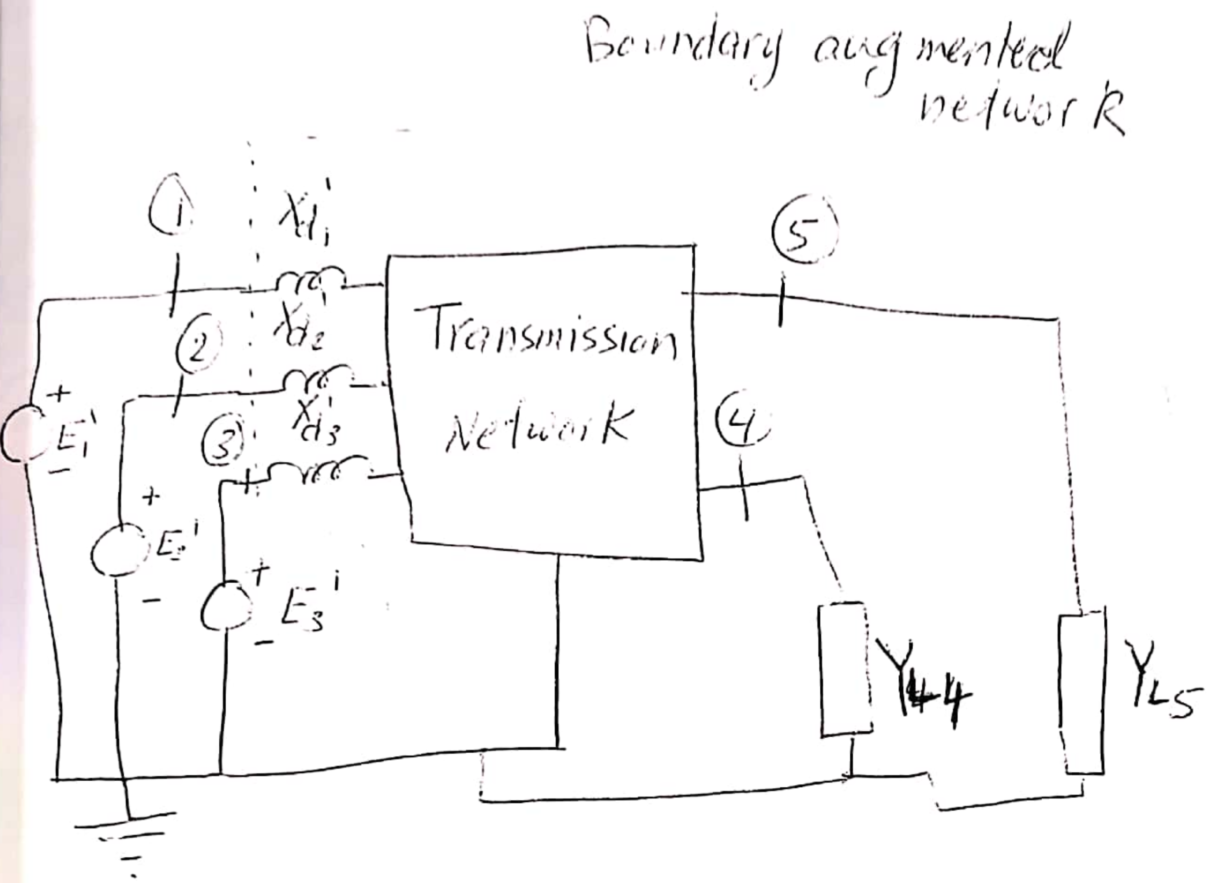
(7) Similar equations are written for P_{e_2} and P_{e_3}

(8) The swing equation is then written as:

$$\frac{2H_i}{\omega_s} \frac{d^2\delta_i}{dt^2} = P_{m_i} - P_{e_i} \qquad i = 1, 2 \,\&\, 3$$

for the three generators of

the following circuit:



EX A 60Hz, 230kV transmission system shown below has two generators of finite inertia and an infinite bus. The transformer and line data are given in the following Table. A three-phase fault occurs on line (4)-(5) near bus (4). Using the prefault power-flow solution given in the Table, determine the swing equation for each machine during the fault period. The generators have reactances and H values expressed on a 100MVA base as follows:

Gen. 1 : 400MVA, 20kV, $X_d' = 0.067$ pu $H = 11.2$ MJ/MVA

Gen. 2 : 250MVA, 18kV, $X_d' = 0.1$ pu $H = 8$ MJ/MVA

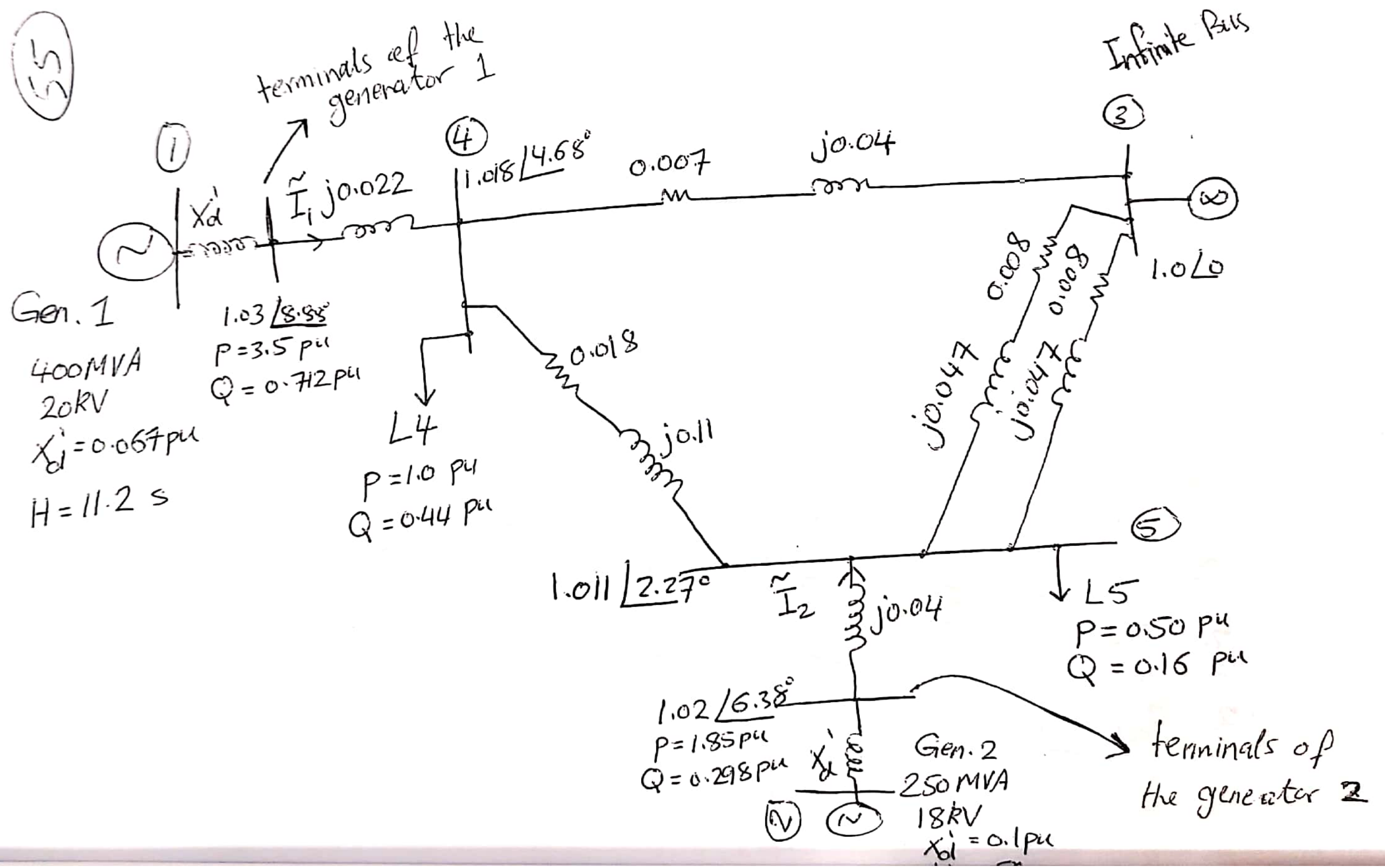
Bus to Bus	Series Z: R	Series Z: X	Shunt Y: B
① - ④ Transformer	—	0.022	—
② - ⑤ Transformer	—	0.04	—
Line ③ - ④	0.007	0.04	0.082
Line ③ - ⑤ (1)	0.008	0.047	0.098
Line ③ - ⑤ (2)	0.008	0.047	0.098
Line ④ - ⑤	0.018	0.110	0.226

All values are in pu on 230kV 100MVA base

Bus data and prefault load-flow values

Bus	Voltage	Generation P	Generation Q	Load P	Load Q
①	$1.03\angle 8.88°$	3.5	0.712	—	—
②	$1.02\angle 6.38°$	1.85	0.298		
③	$1.0\angle 0$	—	—	1.0	0.44
④	$1.018\angle 4.68°$	—	—	0.5	0.16
⑤	$1.011\angle 2.47°$	—			

values are in pu on 230kV 100MVA base


55
terminals of the generator 1
1
4
3
Infinite Bus
X_d'
$\tilde{I}_1$ j0.022
1.018∠4.68°
0.007
j0.04
1.0∠0
Gen. 1
400MVA
20KV
X_d'=0.067pu
H=11.2 s
1.03∠8.88°
P=3.5 pu
Q=0.712pu
L4
P=1.0 pu
Q=0.44 pu
0.018
j0.11
0.008
0.008
j0.047
j0.047
5
1.011∠2.27°
$\tilde{I}_2$
j0.04
L5
P=0.50 pu
Q=0.16 pu
1.02∠6.38°
P=1.85pu
Q=0.298pu
X_d'
Gen. 2
250 MVA
18KV
X_d' =0.1pu
terminals of the generator 2

Sol.

$$\tilde{E}_1' = 1.03\angle 8.88^\circ + j0.067\ \tilde{I}_1$$

$$\tilde{I}_1 = \frac{(P_1 + jQ_1)^*}{V_1^*} = \frac{3.5 - j0.712}{1.03\angle -8.88^\circ} = 3.468\angle -2.619^\circ \text{ pu}$$

$$\tilde{E}_1' = 1.03\angle 8.88^\circ + (j0.067)(3.468\angle -2.619^\circ) = 1.1\angle 20.82^\circ \text{ pu}$$

Hint:

$$\tilde{S} = \tilde{V}\,\tilde{I}^*$$

$$\tilde{I}^* = \frac{\tilde{S}}{\tilde{V}}$$

$$\tilde{I} = \frac{\tilde{S}^*}{V^*}$$

$$\tilde{E}_2' = 1.02\angle 6.38^\circ + j0.1\ \tilde{I}_2$$

$$\tilde{I}_2 = \frac{(P_2 + jQ_2)^*}{V_2^*} = \frac{1.85 - j0.298}{1.02\angle -6.38^\circ}$$

$$= 1.837\angle -2.77^\circ$$

$$\tilde{E}_2' = 1.02\angle 6.38^\circ + (j0.1)(1.837\angle -2.77^\circ)$$

$$= 1.065\angle 16.19^\circ \text{ pu}$$

The infinite bus voltage is:

$$E_3' = E_3 = 1.0\angle 0 \Rightarrow \delta_3 = 0$$

$$\delta_{13} = \delta_1 - \delta_3 = \delta_1$$

$$\delta_{23} = \delta_2 - \delta_3 = \delta_2$$

$$\tilde{Y}_{L4} = \frac{1.0 - j0.44}{(1.018)^2} = 0.9649 - j0.4246 \text{ pu}$$

$$\tilde{Y}_{L5} = \frac{0.5 - j0.16}{(1.011)^2} = 0.4892 - j0.1565 \text{ pu}$$

Hint:

$$\tilde{S} = \tilde{V}\tilde{I}^*$$

$$\tilde{S} = \tilde{V}\frac{\tilde{V}^*}{\tilde{Z}^*}$$

$$\tilde{S} = \frac{|V|^2}{\tilde{Z}^*}$$

$$\tilde{S} = |V|^2\tilde{Y}^*$$

$$\tilde{Y} = \frac{\tilde{S}^*}{|V|^2}$$

$$\tilde{Y}_{11} = \frac{1}{j0.067 + j0.022} = -j11.236 \text{ pu}$$

$$\tilde{Y}_{12} = 0$$

$$\tilde{Y}_{13} = 0$$

$$\tilde{Y}_{14} = \frac{-1}{j0.067 + j0.022} = j11.236 \text{ pu} \quad ?$$

$$\tilde{Y}_{15} = 0$$

$$\tilde{Y}_{21} = \tilde{Y}_{12} = 0$$

$$\tilde{Y}_{22} = \frac{1}{j0.1 + j0.04} = -j7.1429 \text{ pu}$$

$$\tilde{Y}_{23} = 0$$

$$\tilde{Y}_{24} = 0$$

$$\tilde{Y}_{25} = \frac{-1}{j0.04 + j0.1} = j7.1429 \quad ?$$

$Y_{31} = 0$

$\tilde{Y}_{32} = 0$

$$\tilde{Y}_{33} = \frac{1}{(j0.047 + 0.008)/2} + \frac{1}{0.007 + j0.04} + \frac{j0.082}{2} + \frac{j0.098}{2} + \frac{j0.098}{2}$$

$$= 11.2841 - j65.4732 \text{ pu}$$

$$\tilde{Y}_{34} = \frac{-1}{0.007 + j0.04} = -4.245 + j24.2571 \text{ pu}$$

$$\tilde{Y}_{35} = \frac{-1}{(0.008 + j0.047)/2} = -7.0392 + j41.355 \text{ pu}$$

$\tilde{Y}_{41} = \tilde{Y}_{14} = j11.2360$ pu

$\tilde{Y}_{42} = 0$

$\tilde{Y}_{43} = \tilde{Y}_{34} = -4.245 + j24.2571$ pu

$$\tilde{Y}_{44} = -j11.236 + \frac{1}{0.007 + j0.04} + \frac{1}{0.018 + j0.11} + j\frac{0.082}{2} + j\frac{0.226}{2} + 0.9649 - j0.4246$$

$$= 6.6587 - j44.6175 \text{ pu}$$

$$\tilde{Y}_{45} = \frac{-1}{0.018 + j0.11} = -1.4488 + j8.8538 \text{ pu}$$

$$\tilde{Y}_{51} = \tilde{Y}_{15} = 0$$

$$\tilde{Y}_{52} = \tilde{Y}_{25} = j7.1429 \text{ pu}$$

$$\tilde{Y}_{53} = \tilde{Y}_{35} = -7.0392 + j41.355 \text{ pu}$$

$$\tilde{Y}_{54} = \tilde{Y}_{45} = -1.4488 + j8.8538 \text{ pu}$$

$$\tilde{Y}_{55} = \frac{1}{j0.04 + j0.1} + \frac{1}{(0.008 + j0.047)/2} + \frac{j0.098}{2} + \frac{j0.098}{2} + \frac{j0.226}{2} + 0.4892 - j0.1565 + \frac{1}{0.018 + j0.11}$$

$$\tilde{Y}_{55} = -j7.1429 + 7.0392 - j41.355 + j0.049 + j0.049 + j0.113 + 0.4892 - j0.4246 + 1.4488 - j8.8538$$

$$\tilde{Y}_{55} = 8.9772 - j57.2972 \text{ pu}$$

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* The three-phase fault is at node (4) and therefore $Y_{14}, Y_{24}, Y_{34}, Y_{44}, Y_{54}, Y_{41}, Y_{42}, Y_{43}, Y_{44}, Y_{45}$ must be ~~eliminated~~ removed from the prefault Y_{bus} matrix. Now, in the faulted network:

Bus #1 has constant voltage. E_1' is assumed as it unchanged during stability studies.

Bus # 2 has constant Voltage E_2' as it is assumed unchanged during stability studies.

Bus #3 has constant Voltage E_3' as it is an infinite bus.

Bus #4 has been removed as it is shorted to ground

Bus #5 has to be eliminated by Kron reduction because it's voltage is unknown as:

$$Y_{jk}(\text{new}) = Y_{jk} - \frac{Y_{jp}Y_{pk}}{Y_{pp}}$$

where j and K take on all the integer values from 1 to N (total number of buses) except P as row P and column P are to be eliminated.

For our case, the prefault matrix after removing the row 4 and column 4 (short circuited bus) is:

$-j11.2360$	0	0	0
0	$-j7.1429$	0	$j7.1429$
0	0	11.2841 $-j65.4731$	-7.0392 $+j41.3550$
0	$j7.1429$	-7.0392 $+j41.3550$	8.9772 $-j57.2972$

Bus #5 4

$\underbrace{\text{The row 4 and column}}_{}$ are to eliminated using the Kron reduction.

$$\tilde{Y}_{11}(new) = Y_{11} - \frac{Y_{14}Y_{41}}{Y_{44}}$$

$$= -j11.2360 - \frac{(0)(0)}{8.9772 - j57.2972}$$

$$= -j11.2360 \text{ pu}$$

$$\tilde{Y}_{12}(new) = Y_{12} - \frac{Y_{14}Y_{42}}{Y_{44}}$$

$$= 0 - \frac{(0)(0)}{} = 0$$

$$\tilde{Y}_{13} = 0 - \frac{(0)(-7.0392 + j41.3550)}{} = 0$$

$$\tilde{Y}_{21} = 0$$

$$\tilde{Y}_{22} = -j7.1429 - \frac{(+j7.1429)(j7.1429)}{8.9772 - j57.2972}$$

$$= +0.1362 - j6.2738 \text{ pu}$$

$$\tilde{Y}_{23} = 0 - \frac{(j7.1429)(-7.0392 + j41.3550)}{8.9772 - j57.2972}$$

$$= -0.0681 + j5.1661 \text{ pu} = 5.1665 \angle 90.7552^\circ$$

$$\tilde{Y}_{31} = \hat{Y}_{13} = 0$$

$$\tilde{Y}_{32} = \hat{Y}_{23} = -0.0681 + j5.1661 \text{ pu}$$

$$\tilde{Y}_{33} = (11.2841 - j65.4731) - \frac{(-7.0392 + j41.3550)^2}{8.9772 - j57.2972}$$

$$= 5.7986 - j35.6299 \text{ pu}$$

$$P_{e_1} = 0$$

$$P_{e_2} = |E_2'|^2 G_{22} + |E_2'||E_1'| Y_{21} \cos(\delta_{21} - \theta_{21}) + |E_2'|\,|E_3|\, Y_{23} \cos(\delta_{23} - \theta_{23})$$

$$= (1.065)^2 (0.1362) + \underbrace{(1.065)(1.1)(0)\cos(\delta_{21} - 0)}_{\text{Zero}}$$

$$+ (1.065)(1)(5.1665)\cos(\delta_2 - \delta_3 - 90.7552^\circ + 90^\circ)$$

$$P_{e_2} = 0.1545 + 5.5023 \sin(\delta_2 - 0.7552^\circ) \text{ pu}$$

Therefore, the swing equations during the fault are:

$$\frac{d^2\delta_1}{dt^2} = \frac{180f}{H_1}(P_{m_1} - P_{e_1})$$

$$= \frac{180f}{11.2} 3.5$$

$$\frac{d^2\delta_2}{dt^2} = \frac{180f}{8}\left(1.85 - 0.1545 - 5.5023\sin(\delta_2 - 0.7552^\circ)\right)$$

$$= \frac{180f}{8}\left[1.6955 - 5.5023\sin(\delta_2 - 0.7552^\circ)\right]$$

Ex.

The three-phase fault of the previous example is cleared by simultaneously opening the circuit breakers at the ends of the faulted line. Determine the swing equations for the postfault period.

* Since the fault is cleared by removing the line 4 - 5, the prefault $\tilde{Y}_{bus}$ must be modified again. $Y_{45} = Y_{54} = 0$. Y_{44} & Y_{55} must also be modified by subtracting the series admittance of line 4 - 5 and the capacitive susceptance of one-half the line from them. Then reduce this matrix using Kron reduction for Bus #4 & Bus #5. Doing so one will end with:

$$\tilde{Y}_{11} = \underbrace{0.5005}_{G_{11}} - j7.7897 \text{ pu}$$

$$\tilde{Y}_{12} = 0$$

$$\tilde{Y}_{13} = -0.2216 + j7.6291 \text{ pu}$$

$$\tilde{Y}_{21} = \tilde{Y}_{12} = 0$$

$$\tilde{Y}_{22} = \underbrace{0.1591}_{G_{22}} - j6.1168 \text{ pu}$$

$$\tilde{Y}_{23} = -0.0901 + j6.0975 \text{ pu} = 6.0982\angle 90.8466^\circ$$

$\tilde{Y}_{31} = \tilde{Y}_{13} = -0.2216 + j7.6291$ pu $= 7.6323 \angle 91.6638^\circ$ (66)

$\tilde{Y}_{32} = \tilde{Y}_{23} = -0.0901 + j\,6.0975$ pu $= 6.098 \angle 90.85^\circ$

$\tilde{Y}_{33} = 1.3927 - j13.8728$

The power-angle equation of the first gen. is:

$$P_{e_1} = |\tilde{E}_1'|^2 G_{11} + |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{13}| \cos(\delta_{13} - \theta_{13})$$

$$= (1.1)^2(0.5005) + (1.1)(1)(7.6323)\cos(\delta_1 - 91.664^\circ)$$

$$= 0.6056 + 8.3955 \sin(\delta_1 - 1.664^\circ)$$

and

$$P_{e_2} = |\tilde{E}_2'|^2 G_{22} + |\tilde{E}_2'||\tilde{E}_3||\tilde{Y}_{23}| \cos(\delta_{23} - \theta_{23})$$

$$= (1.065)^2(0.1591) + (1.065)(1)\underset{(6.098)}{(\ )}(\delta_2 - 90.8466^\circ)$$

$$= 0.1804 + 6.4934 \sin(\delta_2 - 0.8466^\circ)$$

and therefore the swing equations are: (67)

$$\frac{d^2\delta_1}{dt^2} = \frac{180f}{11.2}\left[3.5 - (0.6056 + 8.3955\sin(\delta_1 - 1.664^\circ))\right]$$

$$= \frac{180f}{11.2}\left[2.8944 - 8.3955\sin(\delta_1 - 1.664^\circ)\right]$$

$$\frac{d^2\delta_2}{dt^2} = \frac{180f}{8}\left[1.85 - [0.1804 + 6.4934\sin(\delta_2 - 0.8466^\circ)]\right]$$

$$= \frac{180f}{8}\left[1.6696 - 6.4934\sin(\delta_2 - 0.8466^\circ)\right]$$

Step-By-Step Solution of The Swing Curve

Instead of this, will teach how to solve them using MATLAB via Ode45 subroutine ...!

Numerical Solution of Nonlinear Differential Equations using MATLAB:

(1) Operating point: This can be done by dropping out all the time derivative terms and solving the resulting nonlinear algebraic equations using the MATLAB instruction "solve".

EX

$$\frac{dX_1}{dt} = 2X_1 - X_2^2$$

$$\frac{dX_2}{dt} = X_1 - 2$$

$X1 = 2, 2$
$X2 = 2, -2$
equilibrium solution

MATLAB File

```
syms X1 X2

F1 = 2*X1 - X2^2;
F2 = X1-2;
es=solve(F1, F2);
es.X1 ⇒ This gives all values of X1
es.X2 ⇒ "       "            "      X2
```

(2) Time domain simulations of the nonlinear differential equations using "ode45" instruction.

MATLAB File example.m

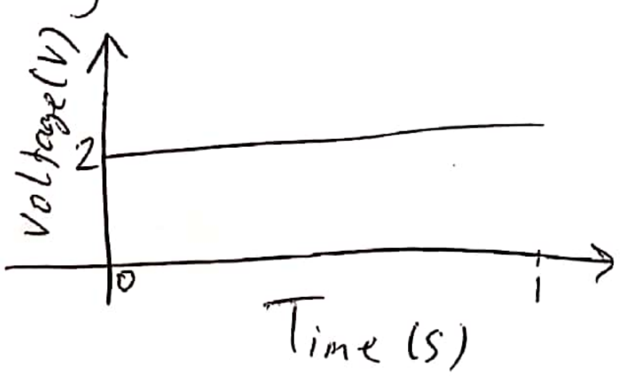
```
function xprime= example(t,x)
  xprime = [ 2 * x(1) - x(2)^2;
             x(1) - 2];
```

MATLAB File

```
Tspan = [0 1];
yo = [2; 2];

[t,x] = ode45('example', Tspan, yo)
plot(t,x(:,1));
xlablel(' Time (s)');
ylabel(' Voltage (V)');
```

Doing so gives



Coming back to our power system: 70

For the first generator

pre fault:

$$\frac{d^2\delta_1}{dt^2} = \frac{180f}{H_1}(P_{m_1} - P_{e_1})$$

$$P_{e_1} = |E_1'|^2 G_{11} + \underbrace{|E_1'||E_2'||Y_{12}|\cos(\delta_{12}-\theta_{12})}_{\text{Zero}}$$
$$+ \underbrace{|E_1'||E_3'||Y_{13}|\cos(\delta_{13}-\theta_{13})}_{\text{Zero}} + |E_1'||E_4'||Y_{14}|$$
$$\cos(\delta_{14}-\theta_{14}) + \underbrace{|E_1'||E_5'||Y_{15}|\cos(\delta_{15}-\theta_{15})}_{\text{Zero}}$$

$$P_{e_1} = (1.1)^2(0) + (1.1)(1.018)(11.2360)\cos(\delta_1 - \delta_4 - 90^\circ)$$

$$P_{e_1} = 12.582\sin\left(\delta_1 - 4.68\frac{\pi}{180}\right)$$

$$P_{e_1} = 12.582\sin(\delta_1 - 0.082)$$

$$\boxed{\frac{d^2\delta_1}{dt^2} = \frac{(180)(60)}{11.2}(3.5 - 12.582\sin(\delta_1 - 0.082))}$$

$$\frac{d\omega_1}{dt} = \frac{(180)(60)}{11.2}(3.5 - 12.582\sin(\delta_1 - 0.082))$$

$$\frac{d\delta_1}{dt} = \omega_1 - 377$$

prefault for G1

$$\frac{d^2\delta_2}{dt^2} = \frac{180f}{H_2}\left(P_{m_2} - P_{e_2}\right)$$

71

$$P_{e_2} = \overset{\text{Zero}}{\underbrace{|E_2'|^2 G_{22}}} + \overset{\text{Zero}}{\underbrace{|E_2'||E_1'||Y_{21}|\cos(\delta_{21} - \theta_{21})}}$$

$$+ \overset{\text{Zero}}{\underbrace{|E_2'||E_3'||Y_{23}|\cos(\delta_{23} - \theta_{23})}} +$$

$$\overset{\text{Zero}}{\underbrace{|E_2'||E_4'||Y_{24}|\cos(\delta_{24} - \theta_{24})}} + |E_2'||E_5'||Y_{25}|\cos(\delta_{25} - \theta_{25})$$

$$P_{m_2} = 1.85$$

$$P_{e_2} = (1.065)(1.011)(7.1429)\cos(\delta_2 - \delta_5 - 90^\circ)$$

$$P_{e_2} = 7.6909\sin(\delta_2 - 2.27^\circ)$$

$$P_{e_2} = 7.6909\sin(\delta_2 - 0.03962)$$

$$\boxed{\frac{d^2\delta_2}{dt^2} = \frac{(180)(60)}{8}\left(1.85 - 7.6909\sin(\delta_2 - 0.03962)\right)}$$

$$\frac{d\omega_2}{dt} = \frac{(180)(60)}{8}\left(1.85 - 7.6909\sin(\delta_2 - 0.03962)\right)$$

$$\frac{d\delta_2}{dt} = \omega_2 - 377$$

pre fault for G2

During fault

$$\frac{d^2\delta_1}{dt^2} = \frac{(180)(60)}{11.2}(3.5-0)$$

$$\frac{d^2\delta_1}{dt^2} = \frac{(180)(60)}{11.2}(3.5)$$

$$\frac{d\omega_1}{dt} = \frac{(180)(60)}{11.2}(3.5)$$

$$\frac{d\delta_1}{dt} = \omega_1 - 377$$

During Fault for G1

$$\frac{d^2\delta_2}{dt^2} = \frac{(180)(60)}{8}\left[1.6955 - 5.5023\sin(\delta_2 - 0.755^\circ)\right]$$

$$\frac{d\omega_2}{dt} = \frac{(180)(60)}{8}\left(1.6955 - 5.5023\sin(\delta_2 - 0.01318)\right)$$

$$\frac{d\delta_2}{dt} = \omega_2 - 377$$

During Fault for G2

Post fault (73)

$$\frac{d^2\delta_1}{dt^2} = \frac{(180)(60)}{11.2}\left(2.8944 - 8.3955\sin(\delta_1 - 1.664^\circ)\right)$$

$$\frac{d\omega_1}{dt} = \frac{(180)(60)}{11.2}\left(2.8944 - 8.3955\sin(\delta_1 - 0.029)\right)$$

$$\frac{d\delta_1}{dt} = \omega_1 - 377$$

post fault for G1

$$\frac{d^2\delta_2}{dt^2} = \frac{(180)(60)}{8}\left(1.6696 - 6.4934\sin(\delta_2 - 0.847^\circ)\right)$$

$$\frac{d\omega_2}{dt} = \frac{(180)(60)}{8}\left(1.6696 - 6.4934\sin(\delta_2 - 0.01478)\right)$$

$$\frac{d\delta_2}{dt} = \omega_2 - 377$$

post fault for G2

Operating point (prefault steady-state solution)

```
syms X1 X2
F1 = 3.5-12.582 sin(X2-0.082);
F2 = X1-377;
es = solve(F1, F2)
es.X1
es.X2
```

For G1

$X1 = 377, \quad X2 = 0.3639 \text{ rad} = 20.85^\circ$

prefault1

```
function Xprime = prefault1(t,x)
    Xprime = [(180*60/11.2)*(3.5-12.582*sin(X(2)-0.082));
              X(1)-377];
```

during fault1

```
function Xprime = duringfault1(t,x)
    Xprime = [(180*60*3.5)/11.2;
              X(1)-377];
```

```
function xprime = Postfault1(t,x)
xprime = [(180*60/11.2)*(2.8944- 8.3955*sin
                                    (x(2)-0.029));
          x(1)-377];
```

postfault1

```
Tspan=[0 10];
y0 = [377; 0.3639];
[t1, X] =ode23('prefault1', Tspan, y0)
a= length(t1)
y0 = [X(a,1); X(a,2)];
Tspan = [10 10.225];
[t2, y] =ode23('duringfault1', Tspan, y0)
a = length(t2)
y0 = [y(a,1); y(a,2)];
Tspan = [10.225 10.3];
[t3, Z] =ode23('postfault1', Tspan, y0)
plot(t1, X(:,2), t2, y(:,2), t3, Z(:,2))
```

Operating Point (prefault steady-State Solution)

```
syms X1 X2
  F1 = 1.85-7.6909 sin (X2-0.03962);
  F2 = X1-377;
  es = solve(F1, F2);
   es.X1 => This provides all values of X1
   es.X2 =>  "    "    "    "    "   X2
```

For G2

$X1 = 377$ $\qquad$ $X2 = 0.2825 \text{ rad} = 16.18^\circ$

prefault2

```
function xprime = prefault2(t,x)
   xprime=[(180*60/8)*(1.85-7.6909*sin(x(2)-0.03962));
           x(1)-377];
```

```
function xprime = duringfault2(t,x)
    xprime = [(180*60/8)*(1.6955-5.5023*sin(x(2)-
                                                  0.01318);
              x(1)-377];
```

during fault2

post fault2

```
function xprime = postfault2(t,x)
    xprime = [(180*60/8)*(1.6696-6.4934 sin(
                                            x(2)-0.01478);
              x(1)-377];
```

```
Tspan = [0 10];
y0 = [377; 0.2825];
[H,x] = ode23s('prefault2', Tspan, y0)
a = length(H)
y0 = [x(a,1), x(a,2)];   Tspan = [10 10.225];
[t2,y] = ode23s('duringfault2', Tspan, y0)
a = length(t2)
```

```
yo=[y(a,1);y(a,2)];
Tspan=[10.225 15];
[t3,Z]=ode235('postfault2', Tspan,yo)
plot(H,x(:,2);t2,y(:,2),t3,Z(:,2))
```

Factors Affecting Transient Stability

$P = P_{max} \sin\delta$

$\downarrow \sin\delta = \frac{P}{P_{max}\uparrow}$

① The smaller the H constant, the larger the angular swing during any time internal.

② For a given shaft power P_m, if the initial rotor angle δ_0 is increased, δ_{max} is decreased and a smaller difference between δ_0 and δ_{cr} exists for a smaller P_{max} i.e. The higher P_{max} is, the higher the probability of maintaining stability under transient conditions tends to be.

3) Stability Control Techniques and transmission system designs have also been evolving to increase overall system stability. The control schemes include:

(a) Excitation systems.
(b) Turbine valve control.
(c) Faster fault clearing time.
(d) Single-pole operation of circuit breakers.

(4) P_{max} can be increased by:

(a) minimum transformer reactance.
(b) series capacitor compensation of lines.
(c) Additional transmission lines.

(5) Damper windings are also very effective mean for damping the oscillations of the rotor.

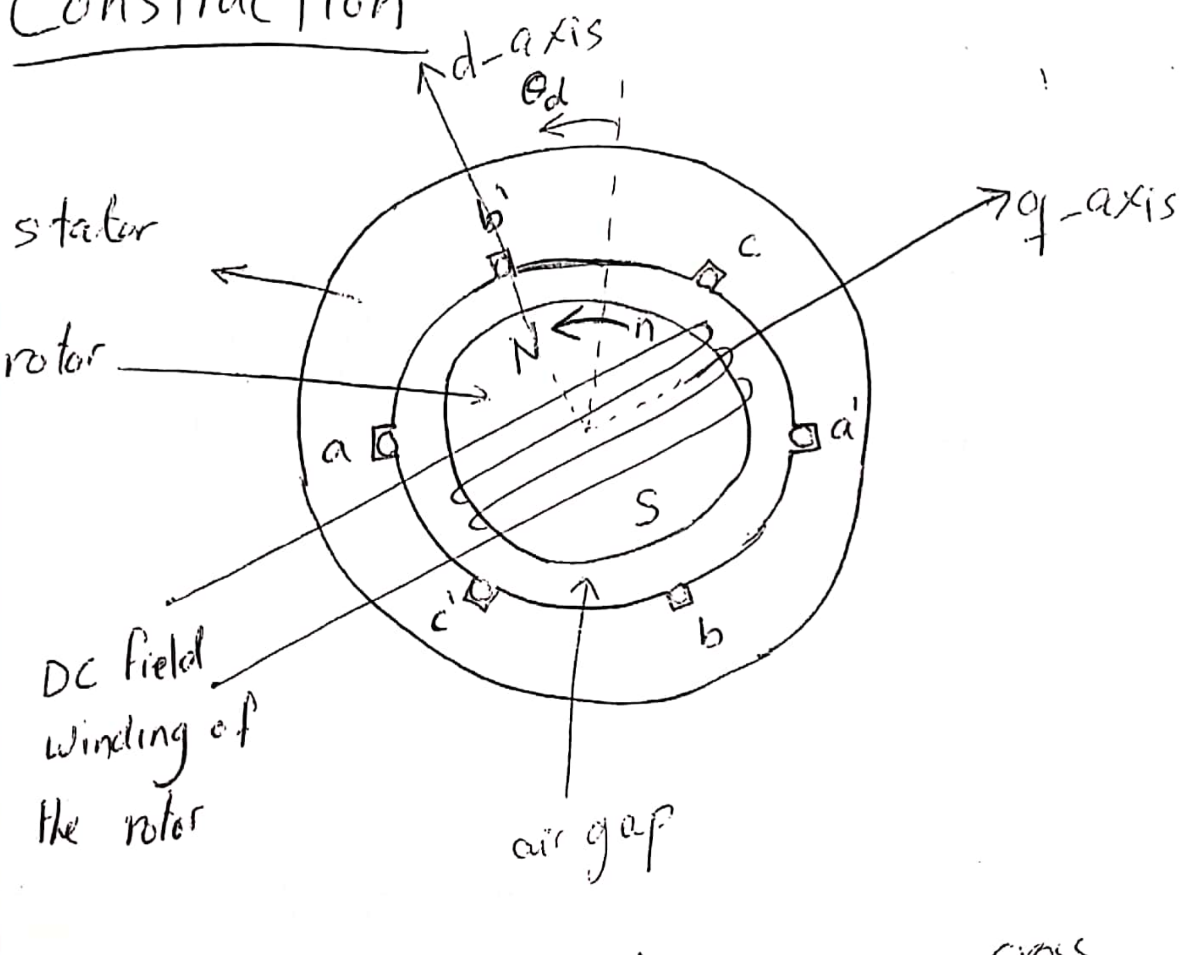
6) Increasing the number of parallel lines between two points is not only a common means of reducing reactance and therefore increasing P_{max} but also some power is transferred over the remaining line even during a three-phase fault on one of the lines.

7) The more power is transferred into the system during a fault, the lower the acceleration of the machine rotor and the greater the degree of stability.

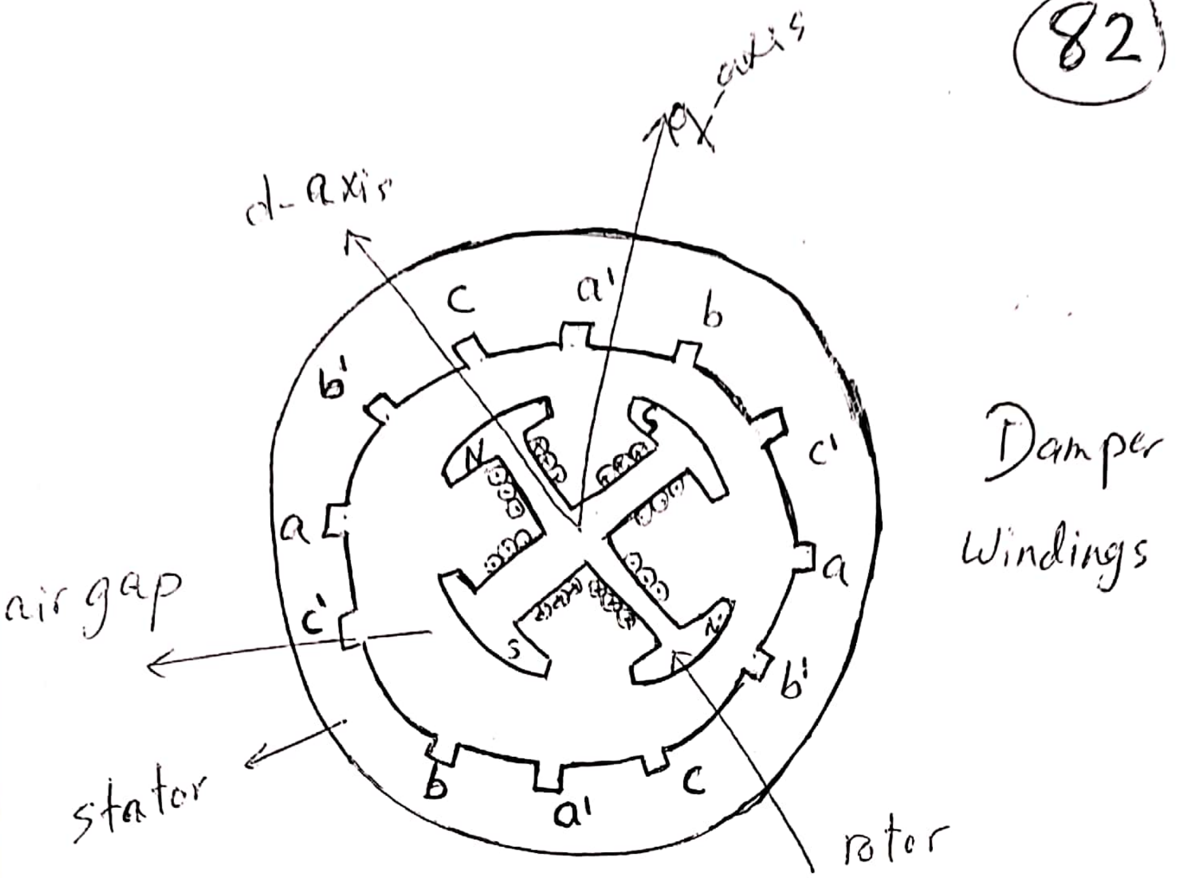
Ch. 3

The Synchronous Generator (Dynamical Model)

Construction



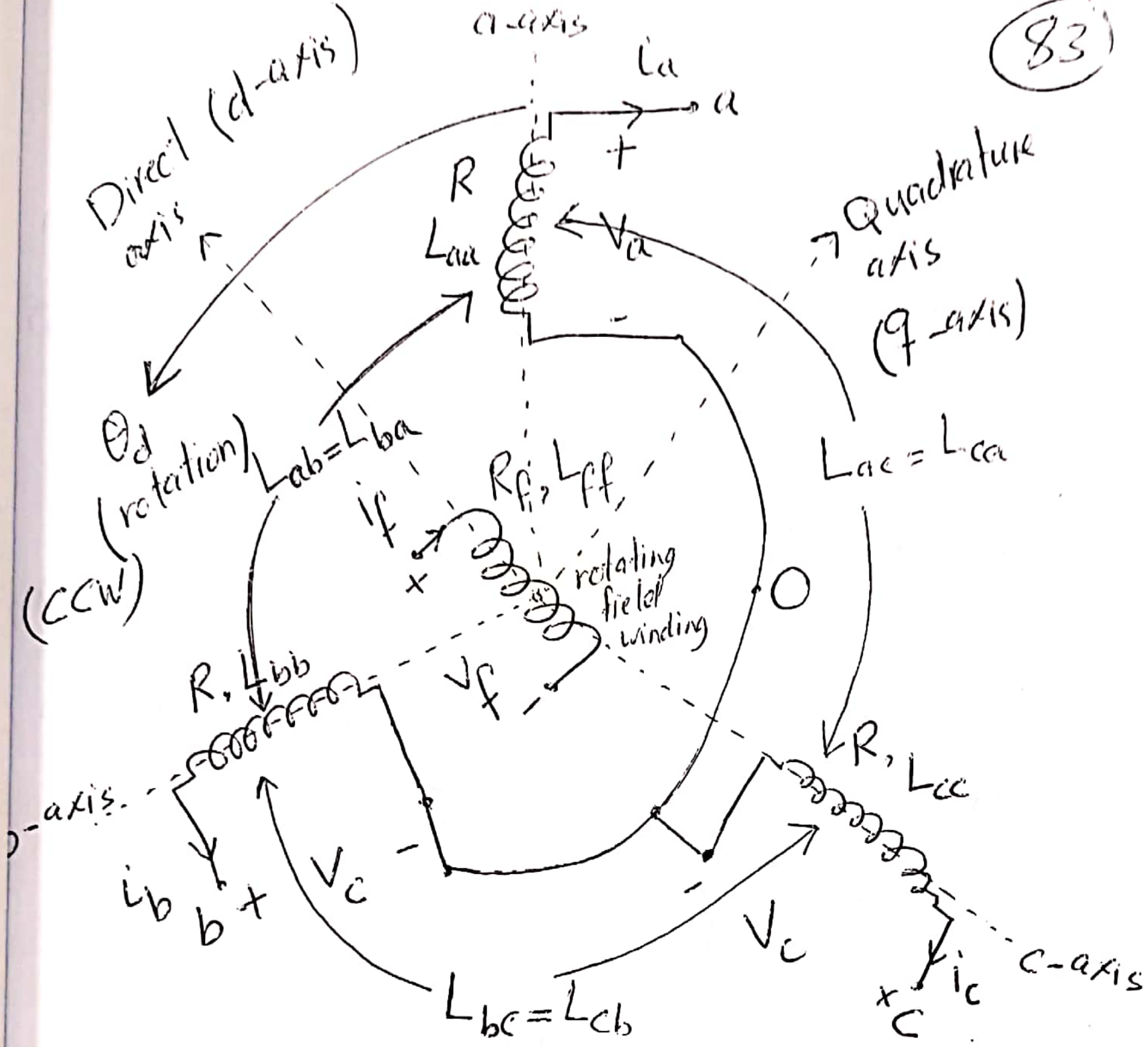
Two-pole cylinderical rotor cross section of three-phase synchronous generator



Four pole salient pole cross section of three-phase synchronous generator

$$f = \frac{np}{120}$$

* The field and armature windings of the synchronous machine are distributed in slots around the periphery of the air gap. The following figure shows them:



a, b and c coils, field coil, d-axis & q-axis of synchronous generator

* Each of concentrated stator (phase) winding has self inductance $\boxed{L_s = L_{aa} = L_{bb} = L_{cc}}$.

* The mutual inductances L_{ab}, L_{bc} and L_{ca} between each adjacent pair of stator concentrated coils are <u>negative</u> (convention) constant $-M_s$:

$$-M_s = L_{ab} = L_{bc} = L_{ca}$$

* The mutal inductance between the field coil f and each of the stator coils varies with the rotor position θ_d as a cosinusoidal function with maximum value of M_f as:

$$L_{af} = M_f \cos \theta_d$$

$$L_{bf} = M_f \cos(\theta_d - 120^\circ)$$

$$L_{cf} = M_f \cos(\theta_d - 240^\circ)$$

θ_d: rotor rotational angle

* The field coil has a <u>constant</u> self-inductance L_{ff}.

It is constant because in the round-rotor machine and in salient-pole machine the field winding on the d-axis produces flux through a similar magnetic path in the stator for all positions of the rotor (neglecting the small effect of armature slots). (When the rotor rotates, its flux rotates at the same speed and direction)

* Flux linkages with each of the coils a, b, c & f are due to its own current and the currents in the other three coils as:

Armature:

$$\lambda_a = \overset{L_s}{\overset{\uparrow}{L_{aa}}}\, i_a + \overset{-M_s}{\overset{\uparrow}{L_{ab}}}\, i_b + \overset{-M_s}{\overset{\uparrow}{L_{ac}}}\, i_c + L_{af}\, i_f$$

$$= L_s\, i_a - M_s\,(i_b + i_c) + L_{af}\, i_f$$

$$= L_s\, i_a - M_s\,(-i_a) + L_{af}\, i_f$$

$$\boxed{\lambda_a = (L_s + M_s)\, i_a + L_{af}\, i_f}$$

Similarly:

$$\lambda_b = (L_s + M_s)\, i_b + L_{bf}\, i_f$$

$$\lambda_c = (L_s + M_s)\, i_c + L_{cf}\, i_f$$

* For two pole machine,

$$\frac{d\theta_d}{dt} = \omega \quad \text{and} \quad \theta_d = \omega t + \theta_{do}$$

θ_{do} ← initial position of the field winding

* At steady-state, let $i_f = I_f$ then

$$\lambda_a = (L_s + M_f)\, i_a + M_f I_f \cos(\omega t + \theta_{do})$$

$$\lambda_b = (L_s + M_f)\, i_b + M_f I_f \cos(\omega t + \theta_{do} - 120^\circ)$$

$$\lambda_c = (L_s + M_f)\, i_c + M_f I_f \cos(\omega t + \theta_{do} - 240^\circ)$$

* If the coil a has resistance R, then the voltage drop V_a across the coil a to terminal O is:

$$V_a = -R i_a - \frac{d\lambda_a}{dt} = -R i_a - (L_s + M_s)\frac{di_a}{dt} + \underbrace{\omega M_f I_f \sin(\omega t + \theta_{do})}_{e_a'}$$

$$e_a' = \sqrt{2}\,|E_i| \sin(\omega t + \theta_{do}) \Rightarrow |E_i| = \frac{\omega M_f I_f}{\sqrt{2}}$$

no load voltage ← generated emf ← induced emf ← internal emf

* The angle θ_{do} indicates the position of the field winding (d-axis) relative to the a-axis at $t=0$.

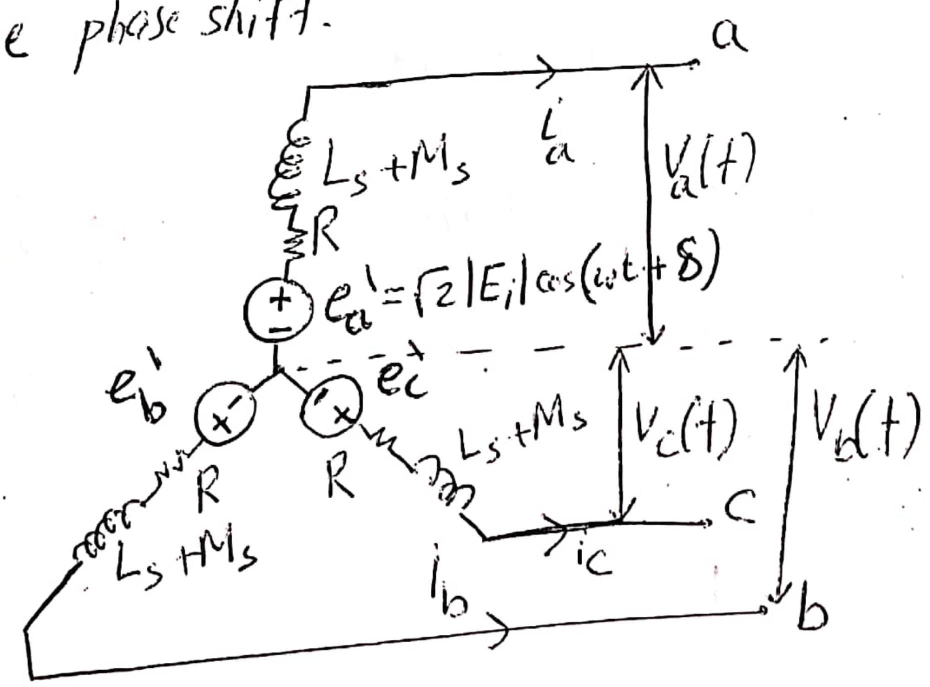
Hence $\delta \triangleq \theta_{do} - 90^\circ$ indicates the position of the q-axis. For later convenience set $\theta_{do} = \delta + 90^\circ$

$$\theta_d = \omega t + \theta_{do} = \omega t + \delta + 90$$

$$e_{a'} = \sqrt{2}|E_i| \cos(\omega t + \delta)$$

$$V_a = -Ri_a - (L_s + M_s)\frac{di_a}{dt} + \underbrace{\sqrt{2}|E_i|\cos(\omega t + \delta)}_{e_{a'}}$$

* $e_{b'}$ and $e_{c'}$ can be concluded from e_a' by introducing the phase shift.



Current i_a, i_b & i_c are:

$$i_a = \sqrt{2}\,|I_a| \cos(\omega t + \delta - \theta_a)$$

$$i_b = \sqrt{2}\,|I_b| \cos(\omega t + \delta - \theta_a - 120^\circ)$$

$$i_c = \sqrt{2}\,|I_c| \cos(\omega t + \delta - \theta_a - 240^\circ)$$

where $|I_a|$ is the rms value and θ_a is phase angle of the current w.r.t e_a' ~~(angle of power factor)~~,

* The expressions for L_{af}, L_{bf} & L_{cf} can be expressed as substituted into λ_f as:

$$\lambda_f = L_{ff} I_f + M_f \left[i_a \cos\theta_d + i_b \cos(\theta_d - 120^\circ) + i_c \cos(\theta_d - 240^\circ) \right]$$

Now, $i_a \cos\theta_d = \sqrt{2}\,|I_a| \cos(\omega t + \delta - \theta_a)\cos(\omega t + \delta + 90^\circ)$

but $2\cos\alpha\cos\beta = \cos(\alpha - \beta) + \cos(\alpha + \beta)$

Therefore,

$$i_a \cos\theta_d = \frac{|I_a|}{\sqrt{2}} \left\{ -\sin\theta_a - \sin(2(\omega t + \delta) - \theta_a) \right\}$$

similarly,

$$i_b \cos(\theta_d - 120^\circ) = \frac{|I_a|}{\sqrt{2}} \left\{ -\sin\theta_a - \sin(2(\omega t + \delta) - \theta_a - 120^\circ) \right\}$$

$$i_c \cos(\theta_d - 240^\circ) = \frac{|I_a|}{\sqrt{2}} \left\{ -\sin\theta_a - \sin(2(\omega t + \delta) - \theta_a - 240^\circ) \right\}.$$

$$\left[i_a \cos\theta_d + i_b \cos(\theta_d - 120^\circ) + i_c \cos(\theta_d - 240^\circ) \right] = -\frac{3|I_a|}{\sqrt{2}} \sin\theta_a$$

$\therefore$

$$\lambda_f = L_{ff} I_f - \frac{3M_f |I_a|}{\sqrt{2}} \sin\theta_a$$

$$= L_{ff} I_f + \sqrt{\frac{3}{2}} M_f i_d$$

where dc current $i_d = \sqrt{\frac{2}{3}} \left[i_a \cos\theta_d + i_b \cos(\theta_d - 120^\circ) + i_c \cos(\theta_d - 240^\circ) \right]$

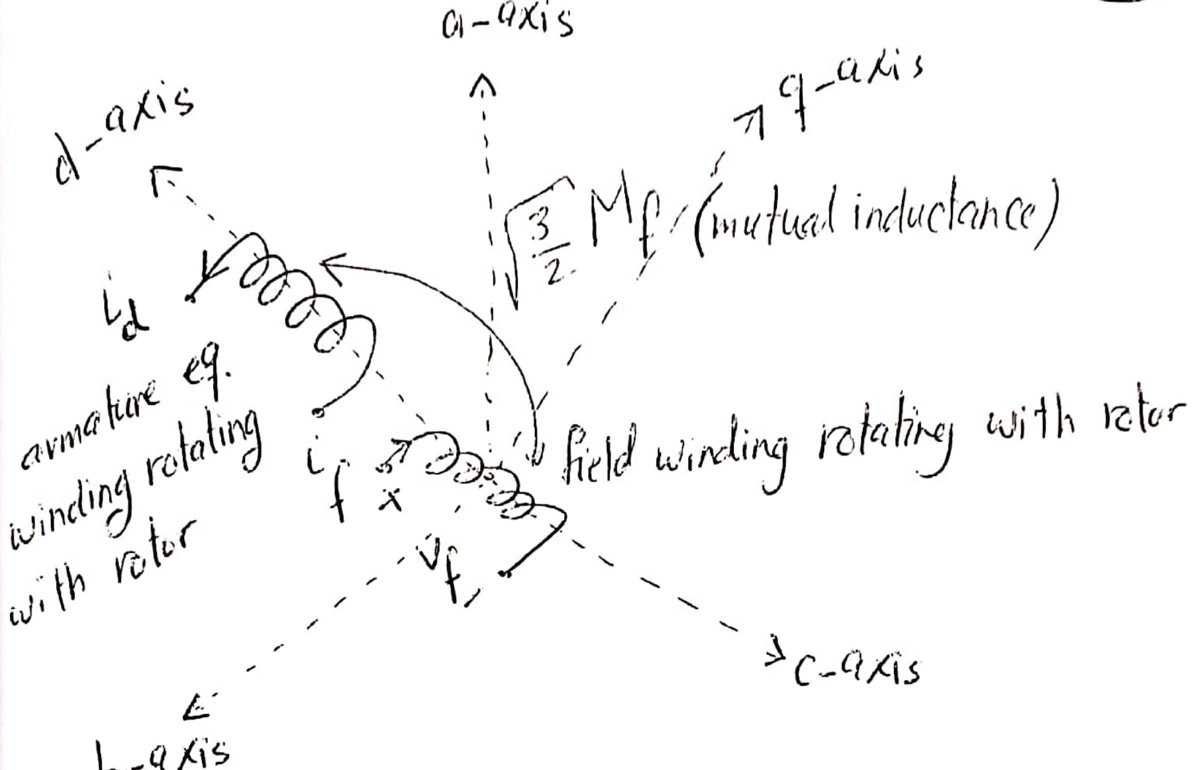
or $i_d = -\sqrt{3}\, |I_a| \sin\theta_a \rightarrow$ d-axis armature current

(90)

* Now, it can be concluded that the flux linkages with the field winding due to the combination of i_a, i_b and i_c do not vary with time. We can regard those flux linkages as coming from the steady dc current i_d in a fictitious dc circuit coincident with the d-axis and thus stationary w.r.t the field circuit. The two circuits rotate together in synchronism and have a mutual inductance $\sqrt{\frac{3}{2}}\, M_f$ between them. $V_{ff'}$ will be:

$$V_{ff'} = R_f i_f + \frac{d\lambda_f}{dt}$$

* The above conclusion can be drawn as:



Representing the armature of the synchronous machine by a direct-axis winding of mutual inductance $\sqrt{\frac{3}{2}} M_f$ with the field winding. Both windings rotate in synchronism

* In steady-state $V_{ff}' = R_f I_f$

EX. A 60Hz 3-ϕ synchronous generator with $R=0$ has the following inductance parameters:

$L_{aa} = L_s = 2.7656 \, mH$ $\quad M_f = 31.6950 \, mH$

$L_{ab} = M_s = 1.3828 \, mH$ $\quad L_{ff} = 433.6569 \, mH$

92

The machine is rated at 635MVA, 0.9 PF lagging, 3600 rpm, 24kV. At rated conditions:

$V_a = 19596 \cos\omega t$ V, $i_a = 21603 \cos(\omega t - 25.8419^\circ)$ A

Determine the magnitude of the synchronous internal voltage (e_a'), the field current I_f, and the flux linkages (λ_f) with the field winding. Calculate the values of these quantities when a load of 635MVA is served at rated voltage and unity PF. What is the field current for rated armature voltage on an open circuit.

Solution

At 0.9 PF lagging

$$e_a' = \sqrt{2}|E_i| \cos(\omega t + \delta)$$

$$= V_a + (L_s + M_s)\frac{di_a}{dt}$$

$$= 19596 \cos\omega t + (2.7656 + 1.3828) \times 10^{-3} \frac{d}{dt}\left[21603 \cos(\omega t - 25.8419^\circ)\right]$$

with $\omega = 2\pi f = (2\pi)(60) = 120\pi$

and $\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

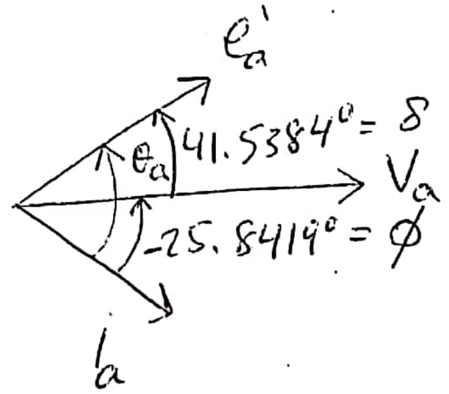
$$e_a' = 45855\cos(\omega t + 41.5384^\circ)\,V$$

Hence $\sqrt{2}|E_i| = 45855\,V$, $\delta = 41.5384^\circ$

$$I_f = \frac{\sqrt{2}|E_i|}{\omega M_f} = \frac{45855}{(120\pi)(31.6950\times10^{-3})} = 3838\,A$$

$$\lambda_f = L_{ff} I_f - \frac{3M_f}{\sqrt{2}}|I_a|\sin\theta_a$$

$$\theta_a = 25.8419^\circ + 41.5384^\circ = 67.3803^\circ$$



$$|I_a| = \frac{21603}{\sqrt{2}}$$

$$\lambda_f = (433.6569\times10^{-3})(3838) - \frac{(3)(31.6950\times10^{-3})}{\sqrt{2}}\,\frac{21603}{\sqrt{2}}\,\sin(67.3803^\circ)$$

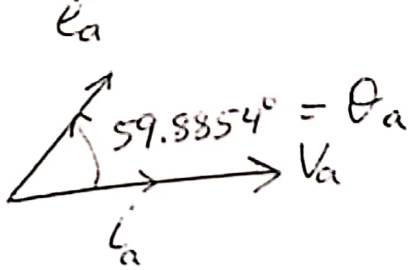
$= 716.32$ Wb-turns

At unity PF

$$e_a' = 19596\cos\omega t + (2.7656 + 1.3828)\times10^{-3}\frac{d}{dt}\left(21603\cos(\omega t)\right)$$

$$= 39057\cos(\omega t + 59.8854°)\ \text{V}$$

$$I_f = \frac{39057}{(120\pi)(31.6950\times10^{-3})} = 3269\ \text{A}$$



$$|I_a|\sin\theta_a = \frac{21603}{\sqrt{2}}\sin(59.8854°) = 13214\ \text{A}$$

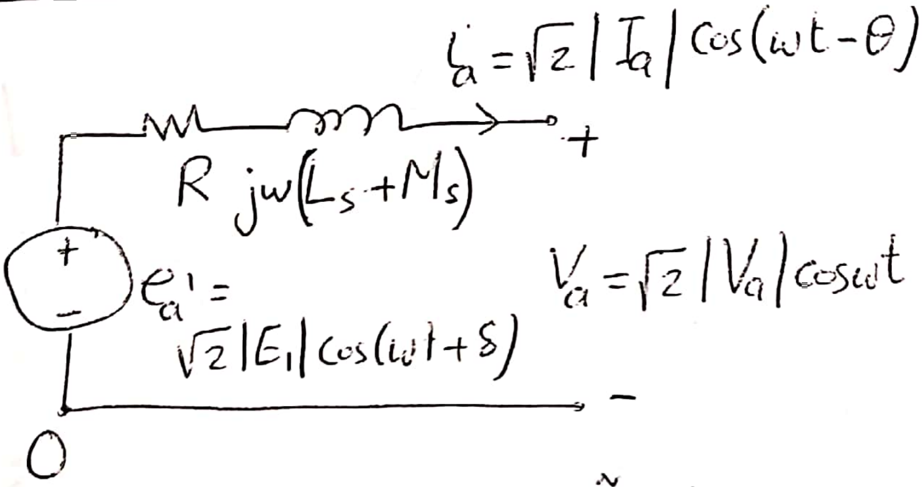
$$\lambda_f = (433.6569\times10^{-3})(3269) - \frac{(3)(31.6950\times10^{-3})}{\sqrt{2}}\,13214$$

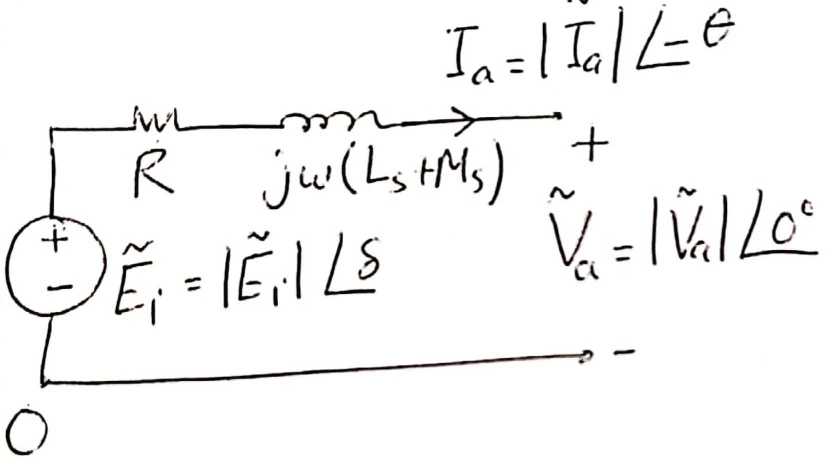
$$= 529.19\ \text{Wb-turns.}$$

Rated terminal voltage under open circuit conditions ($i_a = 0$) equals $19596 \Rightarrow I_f = \dfrac{\sqrt{2}\,|E_i|}{120\pi\, 31.695\times10^{-3}}$

$$= \frac{19596}{} = 1640\ \text{A}$$

Synchronous Reactance and Equivalent Circuits





$V_a(t) = \sqrt{2}|V_a|\cos\omega t \qquad e_a' = \sqrt{2}|E_i|\cos(\omega t + \delta)$

$i_a(t) = \sqrt{2}|I_a|\cos(\omega t - \theta)$

$$\tilde{V}_a = \underbrace{\tilde{E}_i}_{\text{Generated at no-load}} - \underbrace{R\tilde{I}_a}_{\text{Due to armature resistance}} - \underbrace{j\omega L_s \tilde{I}_a}_{\text{Due to armature self-reactance}} - \underbrace{j\omega M_s \tilde{I}_a}_{\text{Due to armature mutual reactance}}$$

let $\tilde{Z}_d = R + j X_d$

$= R + j\omega(L_s + M_s)$

$\tilde{V}_a = \tilde{E}_i - \tilde{I}_a \tilde{Z}_d$

$\tilde{V}_a = \tilde{E}_i - \tilde{I}_a R - j \tilde{I}_a X_d$

<u>Ex.</u> The 60 Hz synchronous generator described in the previous example is serving its rated load under steady-state operating conditions. Choosing the armature base equal to the rating of the machine, determine the value of the synchronous reactance and the phasor expressions for the stator quantities $\tilde{V}_a$, $\tilde{I}_a$ and $\tilde{E}_i$ in pu. If the base field current equals that value of I_f which produces rated terminal voltage under open-circuit conditions, determine the value of I_f under the specified operating conditions:

lution

Base kVA = 635, 000 kVA

Base kV_{LL} = 24 kV

$$\text{Base current} = \frac{635000}{\sqrt{3}\,24} = 15275.726\,A$$

$$\text{Base impedance} = \frac{24^2}{635} = 0.9071\,\Omega$$

$$X_d = \omega(L_s + M_s) = 120\pi\,(2.7656 + 1.3828) \times 10^{-3}$$

$$= 1.5639\,\Omega$$

$$X_d = \frac{1.5639}{0.9071} = 1.7241 \text{ pu}$$

$$\tilde{E}_i = \tilde{V}_a + jX_d\tilde{I}_a$$

$$= 1.0\angle 0 + j\,1.7241\ 1.0\angle -25.8419^\circ$$

$$\frac{\sqrt{2}|\tilde{E}_i|}{\omega M_f}$$

$$= 2.340\angle 41.5384^\circ \text{ pu}$$

From the previous example, the base field current required to produce 1.0 pu open circuit voltage is 1640A. Therefore, E_i is directly $\propto I_f$. Then 2.34 * 1640 = 3838 A under the above conditions.

The Two-Axis Machine Model

* For transient analysis, the two axis model is used for accurate results.

* In salient pole machine, the air gap is much narrower along the direct axis than along the quadrature axis ($X_d \neq X_q$) between poles.

* In synchronous machine (both types), the field sees the same air gap and magnetizing paths in the stator regardless of the rotor position. Consequently, the field winding has constant self inductance L_{ff} and both machine types have the same cosinusoidal mutual inductances L_{af}, L_{bf} and L_{cf}. Additionally, throughout each revolution of the rotor the self inductances L_{aa}, L_{bb} and L_{cc} of the stator winding and the mutual inductances

L_{ab}, L_{bc} and L_{ca} between them are (99)
not constant in the salient pole machine but
also vary as a function of the rotor angular
displacement θ_d. The flux linkages of phases
a, b and c are related to the currents by the
inductances so that:

$$\lambda_a = \underline{L_{aa}} i_a + \overset{\text{not constants}}{L_{ab}} i_b + L_{ac} i_c + \overset{\text{not constants}}{L_{af}} i_f$$

$$\lambda_b = L_{ba} i_a + \underline{L_{bb}} i_b + L_{bc} i_c + L_{bf} i_f$$

$$\lambda_c = L_{ca} i_a + L_{cb} i_b + \underline{L_{cc}} i_c + L_{cf} i_f$$

* For salient pole machine, all coefficients are
variable. For cylinderical rotor they are constant.

* For salient pole machine, the coefficients can be
changed to constant variable rotating in

synchronism with the rotor by means (Too)
of Park's transformation into d-axis, q-axis
and zero-sequence quantities. They are
distinguished by d, q and 0.

* The expressions of all coefficients is given in
Table 3.1 PP. 119.

* The three-phase currents i_a, i_b and i_c can be
transformed into d, q and o components using
the matrix P which was introduced by R. H.
Park.

$$\begin{bmatrix} i_d \\ i_q \\ i_o \end{bmatrix} = P \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix} \quad \begin{bmatrix} V_d \\ V_q \\ V_o \end{bmatrix} = P \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} \quad \begin{bmatrix} \lambda_d \\ \lambda_q \\ \lambda_o \end{bmatrix} = P \begin{bmatrix} \lambda_a \\ \lambda_b \\ \lambda_c \end{bmatrix}$$

* The matrix P is

$$\underbrace{P^{-1} = P^{T}}_{\text{orthogonality property}} = \sqrt{\frac{2}{3}} \begin{bmatrix} \cos\theta_d & \cos(\theta_d - 120^\circ) & \cos(\theta_d - 240^\circ) \\ \sin\theta_d & \sin(\theta_d - 120^\circ) & \sin(\theta_d - 240^\circ) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}$$

* Doing so provides :

$$\lambda_d = L_d i_d + \sqrt{\frac{3}{2}} M_f i_f$$

$$\lambda_q = L_q i_q$$

$$\lambda_o = L_o i_o \Longrightarrow \text{stationary appears in case of unbalanced conditions}$$

where :

$\boxed{L_d = L_s + M_s + \frac{3}{2} L_m}$. → d-axis inductance

$\boxed{L_q = L_s + M_s - \frac{3}{2} L_m}$ → q-axis inductance

$\boxed{L_o = L_s - 2M_s}$ → zero-sequence inductance

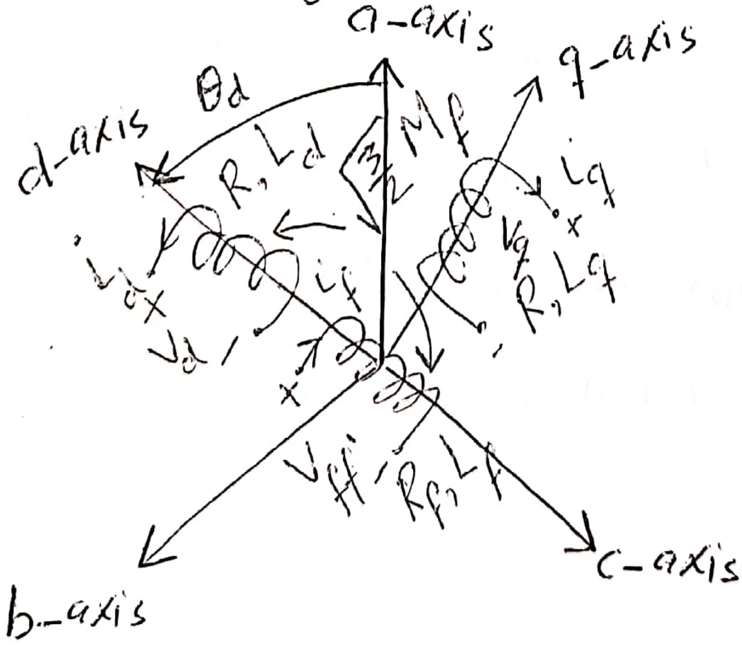
$\boxed{i_f \text{ is the actual field current}}$.

$\boxed{\lambda_f = \sqrt{\frac{3}{2}} M_f i_d + L_{ff} i_f}$

* L_d, L_q, i_d & i_q

* L_d is the self inductance of an equivalent d-axis armature winding which rotates at the same speed as the field and which carries current i_d to produce the same mmf on the d-axis as do the actual currents i_a, i_b & i_c. Similarly, L_q and i_q apply to the q-axis. i_d & i_q produce mmfs which are stationary w.r.t the rotor.

* The following figure shows this:



Armature equivalent d-axis & q-axis coils equivalent of salient pole synchronous generator.

* The fictitious d-axis winding and the f winding representing the physical field can be considered to act like two coupled coils which are stationary w.r.t each other as they rotate together sharing a mutual inductance $\sqrt{\frac{3}{2}}M_f$ between them. Furthermore, the field and the d-axis coils do not couple magnetically with the fictitious q winding on the q-axis.

* The zero-sequence inductance L_0 is associated with a stationary fictitious armature coil with no coupling to any other coils. Under balanced conditions this coil carries no current and therefore it is omitted from further discussions.

Ex Under steady-state operating conditions the armature of the salient pole synchronous generator carries symmetrical sinusoidal three-phase currents

$$i_a = \sqrt{2}\,|I_a|\sin(\theta_d - \theta_a)$$

$$i_b = \sqrt{2}\,|I_a|\sin(\theta_d - 120^\circ - \theta_a)$$

$$i_c = \sqrt{2}\,|I_a|\sin(\theta_d - 240^\circ - \theta_a)$$

where $\theta_d = \omega t + \delta + 90^\circ$. Using P-transformation matrix, find expressions for d-q-0 currents of the armature.

$$\begin{bmatrix} i_d \\ i_q \\ i_0 \end{bmatrix} = \sqrt{\frac{2}{3}} \underbrace{\begin{bmatrix} \cos\theta_d & \cos(\theta_d - 120^\circ) & \cos(\theta_d - 240^\circ) \\ \sin\theta_d & \sin(\theta_d - 120^\circ) & \sin(\theta_d - 240^\circ) \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{bmatrix}}_{[P]} \begin{bmatrix} i_a \\ i_b \\ i_c \end{bmatrix}$$

$$i_d = \sqrt{\frac{2}{3}}\left[i_a \cos\theta_d + i_b \cos(\theta_d - 120^\circ) + i_c \cos(\theta_d - 240^\circ)\right]$$

$$i_q = \sqrt{\frac{2}{3}}\left[i_a \sin\theta_d + i_b \sin(\theta_d - 120^\circ) + i_c \sin(\theta_d - 240^\circ)\right]$$

Zero under balanced conditions

$$i_0 = \sqrt{\frac{2}{3}}\left[\frac{1}{\sqrt{2}}(i_a + i_b + i_c)\right]$$

$$i_a \cos\theta_d = \sqrt{2}\,|I_a| \sin(\theta_d - \theta_a)\cos\theta_d$$

$$= \frac{|I_a|}{\sqrt{2}}\left[\sin(2\theta_d - \theta_a) - \sin\theta_a\right] \Rightarrow$$

using:

$$\boxed{2\sin\alpha\cos\beta = \sin(\alpha+\beta) + \sin(\alpha-\beta)}$$

Likewise,

$$i_b \cos(\theta_d - 120^\circ) =$$

forget this?

$$i_c \cos(\theta_d - 240^\circ) =$$

Hence,

$i_d = -\sqrt{3}\,|I_a|\sin\theta_{a} \Rightarrow$ constant value (not time varying quantity)

$= -\sqrt{3}\,|I_a|\sin(\theta + \delta)$

angle of the PF of the load

Similarly,

$i_q = \sqrt{3}\,|I_a|\cos(\theta + \delta)$.

Voltage equations : Salient pole Machine

* For voltage equations, P- transformation adds further simplifications as:

$$V_a = -R i_a - \frac{d\lambda_a}{dt}$$

$$V_b = -R i_b - \frac{d\lambda_b}{dt}$$

$$V_c = -R i_c - \frac{d\lambda_c}{dt}$$

V_a, V_b & V_c : line to neutral voltages.

Using P- matrix :

$$V_d = -R i_d - \frac{d\lambda_d}{dt} - \omega \lambda_q$$

$$V_q = -R i_q - \frac{d\lambda_q}{dt} + \omega \lambda_d$$

$$V_0 = -R i_0 - \frac{d\lambda_0}{dt}$$

$$\omega = \frac{d\theta_d}{dt}$$

where:

$$\lambda_d = L_d i_d + k M_f i_f$$

$$\lambda_f = k M_f i_d + L_{ff} i_f$$

$$V_d = -R i_d - \frac{d\lambda_d}{dt} - \omega \lambda_q$$

$$V_{ff}' = R_f i_f + \frac{d\lambda_f}{dt}$$

d-axis

$$k = \sqrt{\frac{3}{2}}$$

rotational emfs (speed voltages)

$$\lambda_q = L_q i_q$$

$$V_q = -R i_q - \frac{d\lambda_q}{dt} + \omega \lambda_d$$

q-axis

Note: The End of the "Book".

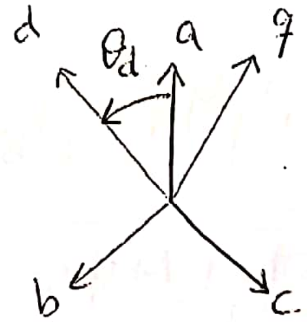
Nonlinear Dynamical Mathematical Model of Synchronous Generator

$$V_d = -R i_d - \frac{d\lambda_d}{dt} - \omega \lambda_q$$

$$\lambda_d = L_d i_d + kM_f i_f$$

$$\lambda_q = L_q i_q$$

$$-\frac{d\lambda_d}{dt} = V_d + R i_d + \omega \lambda_q$$



$$-\dot{\lambda}_d = +V_d + R i_d + \omega \lambda_q$$

$$-L_d \dot{i}_d - kM_f \dot{i}_f = V_d + R i_d + \omega (L_q i_q)$$

$$\boxed{-L_d \dot{i}_d - kM_f \dot{i}_f = V_a \cos\theta_d + R i_d + \omega L_q i_q} \quad \ldots (1)$$

d-axis armature equation

$$V'_{ff} = R_f i_f + \frac{d\lambda_f}{dt}$$

$$\frac{d\lambda_f}{dt} = V'_{ff} - R_f i_f$$

$$\boxed{kM_f \dot{i}_d + L_{ff} \dot{i}_f = V'_{ff} - R_f i_f} \quad \ldots (2)$$

d-axis field equation
(field winding equation)

$$V_q = -R i_q - \frac{d\lambda_q}{dt} + \omega\lambda_d$$

$$-\frac{d\lambda_q}{dt} = V_q + R i_q - \omega\lambda_d$$

$$-L_q \dot{i}_q = V_a \sin\theta_d + R i_q - \omega\left[L_d i_d + kM_f i_f\right]$$

$$\boxed{-L_q \dot{i}_q = V_a \sin\theta_d + R i_q - \omega L_d i_d - \omega kM_f i_f} \quad \ldots (3)$$

q-axis armature equation

* θ_d is the angle between the d-axis armature winding and the phase "a" terminal Voltage which the same angle between the induced Voltage (internal generated voltage) and the phase "a" terminal voltage. It is known as load angle or δ-angle of generator denoted as δ. For numerical simulations

$P_m = T_m \omega$

$$J \frac{d\omega}{dt} = T_m - T_{out}$$

J: moment of inertia of the rotating parts of the complete electromechanical system

$$\frac{d\delta}{dt} = \omega - \omega_s$$

, ω_s: synchronous mechanical rotational speed (Rad/s)

where, P_m: input mechanical power to the generator (Normally Constant)

P_{out}: output electrical power from the geerate

* P_{out} can be expressed as:

$$P_{out} = V_a i_a + V_b i_b + V_c i_c \text{ in a, b, c system}$$

Using P_matrix:

$$P_{out} = 0.75 P\omega \left[\lambda_f i_q + (L_d - L_q) i_d i_q \right]$$

$$= 0.75 P\omega \left[(kM_f i_d + L_{ff} i_f) i_q + L_d i_d i_q - L_q i_d i_q \right]$$

$$= 0.75 P\omega \left[kM_f i_d i_q + L_{ff} i_f i_q + L_d i_d i_q - L_q i_d i_q \right]$$

P: no. of poles

ω: rotational speed in mech. (Rad/s)

$$\boxed{J\dot{\omega} = T_m - 0.75P \left[L_{ff} i_f i_q + kM_f i_d i_q + L_d i_d i_q - L_q i_d i_q \right]} \quad (4)$$

$$\boxed{\dot{\delta} = \omega - \omega_s} \quad \cdots (5)$$

(4) & (5) are Know as mechanical (swing equation)

* Eqs. (1), (2), (3), (4) & (5) represent the nonlinear dynamical mathematical model of synchronous generator.

* P_{out} Can be approximated alternatively as:

$$P_{out} = \frac{3|E_i||V_a|}{X_s} \sin\delta \Rightarrow \text{if } X_d = X_q = X_s$$

(saliency is neglected)

OR

cylinderical rotor

and $R = 0$

State - Space Representation of Synchronous Generator

* In state space representation, the nonlinear dynamical Mathematical Model of synch. Gen. is:

assumption

$\boxed{X_1 = i_f}$ $\boxed{X_2 = i_d}$ $\boxed{X_3 = i_q}$ $\boxed{X_4 = \omega}$

$\boxed{X_5 = \delta}$

$$\boxed{L_{ff} \dot{X}_1 + kM_f \dot{X}_2 = V_{ff} - R_f X_1} \quad \cdots (1)$$

$$\boxed{-kM_f \dot{X}_1 - L_d \dot{X}_2 = V_a \cos X_5 + R X_2 + L_q X_4 X_3} \quad \cdots (2)$$

$$\boxed{-L_q \dot{X}_3 = V_a \sin X_5 + R X_3 - L_d X_4 X_2 - kM_f X_4 X_1} \quad \cdots (3)$$

$$\boxed{J \dot{X}_4 = T_m - 0.75P \left[L_{ff} X_1 X_3 + kM_f X_2 X_3 + L_d X_2 X_3 - L_q X_2 X_3 \right]} \quad \cdots (4)$$

$$\boxed{\dot{X}_5 = X_4 - W_s} \quad \cdots (5)$$

* In numerical simulations, V_a (terminal phase voltage) is normally treated as constant voltage.

Steady-State Performance

* It is well known that in steady-state conditions, the time derevative terms are zero. Doing so with the above 5 equations provide:

$$i_f = \frac{V'_{ff}}{R_f} \quad \cdots \quad \text{from } (1)$$

$$\omega = W_s \qquad \text{from } (5)$$

$$V_a \sin\delta = -R i_q + \underbrace{W_s L_d}_{X_d} i_d + \underbrace{\omega_s R M_f}_{X_{afd}} \frac{V'_{ff}}{R_f} \quad \text{from } (3)$$

$$V_a \cos\delta = -R i_d - \underbrace{\omega_s L_q}_{X_q} i_q \quad \text{from (2)}$$

$$P_m = 0.75 P \Big[\underbrace{\omega_s L_{ff}}_{X_{ffd}} \frac{V'_{ff}}{R_f} i_q + kM_f i_d i_q + L_d i_d i_q - L_q i_d i_q \Big]$$

Normally, i_f, i_d, i_q, δ & ω are solved numerically. (Could be by MATLAB™ using "solve").

for given V'_{ff}, P_m and V_a.

* The equilibrium solution is the solution of a set of nonlinear algebraic equations resulting from a nonlinear dynamical mathematical set of equations after dropping out all the time derivative terms. Normally, they are set of solutions. Only one represents the operating

point of the system.

Small-Signal stability of Power Systems

* Small signal stability is the ability of the power system to maintain synchronism when subjected to small disturbances.

* The dynamics of the power system can be described by set of first order nonlinear ordinary differential equations of the following form:

$$\dot{x}_i = f_i(x_1, x_2, \dots, \underset{\substack{\uparrow \\ \text{the order} \\ \text{of the system}}}{x_n}; u_1, u_2, \dots, \underset{\substack{\uparrow \\ \text{no. of inputs}}}{u_r}; \overset{\text{time}}{t})$$

* This can be written in the following form:

$$\dot{X} = f(X, u, t)$$

input variables

$$X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix} \quad u = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_r \end{bmatrix} \quad f = \begin{bmatrix} f_1 \\ f_2 \\ \vdots \\ f_n \end{bmatrix}$$

state variables

state vector

input vector

function vector

* if $\dot{X} = f(X, u) \Rightarrow$ autonomous system

$\dot{X} = f(X, u, t) \Rightarrow$ nonautonomous system (the time appears explicitly)

* The output variables can be observed in terms of the state variables and input variables as:

$$y = g(X, u) \quad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_m \end{bmatrix} \quad g = \begin{bmatrix} g_1 \\ g_2 \\ \vdots \\ g_m \end{bmatrix}$$

Equilibrium (singular) points

* The equilibrium points are those points where all the derivatives $\dot{x}_1, \dot{x}_2, \dots, \dot{x}_n$ are simultaneously zero. The system is accordingly at rest since all variables are constant and unvarying with time.

* The equilibrium point must satisfy the equation:

  $f(X_0) = 0$, X_0 : the state vector X at the equilibrium point.

* A linear system has only one equilibrium state. For a nonlinear system, there may be more than one. Only one is the operating point.

* In nonlinear systems like the power system, the equilibrium points are obtained by solving the resulting algebraic equations numerically. One should then think about the operating point.

Stability of a Dynamic System

* The stability of a linear system is entirely independent of the input and the state of a stable system. With zero input, the state always return to the origion of the state space, independent of the finite initial state. In contrast, the stability of a nonlinear system depends on the type and magnitude of input and the initial state.

* The stability of a nonlinear system is classified into the following categories :

- Local stability or stability in the small
- Finite stability
- Global stability or stability in the large

Local stability

* The system is said to be locally stable about an

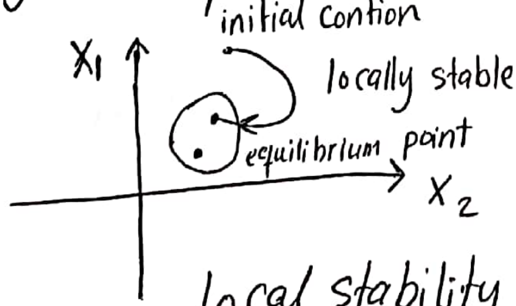
equilibrium point if, when subjected to small perturbation, it remains within a small region surrounding the equilibrium point.

* If, as t increases, the system returns to the origional state, it is said to be asymptotically stable.

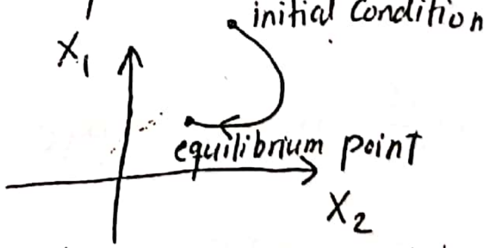
* It should be noted that the general definition of local stability does not require that the state return to the origional and therefore includes small limit cycles.

* Local stability (stability under small disturbance)

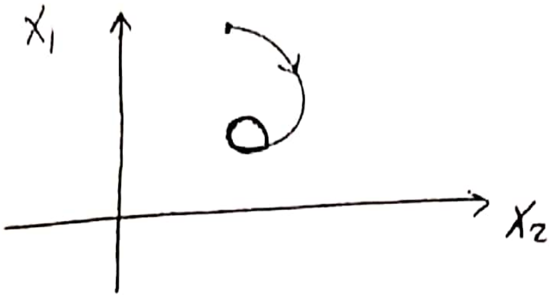
Can be studied by linearizing the nonlinear system equations about the equilibrium point.



local stability



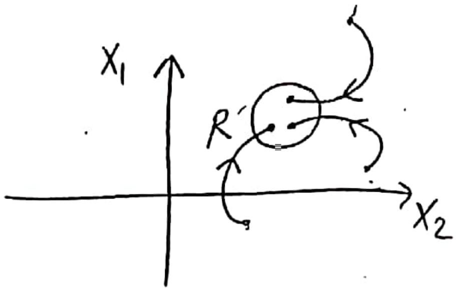
asymptotic stability



limit cycle

Finite stability

* If the state of the system remains within a finite region R, it is said to be stable with R.



stable within R

Global stability

* The system is said to be globally stable if R includes the entire finite space. (huge Region)

Linearization

Let X_0 be the initial state vector and u_0 the input vector corresponding to the equilibrium point about which the small-signal performance is to be investigated.

Since X_0 and u_0 satisfy

$$\dot{x} = f(x, u)$$

then

$$\dot{X}_0 = f(X_0, u_0) = 0$$

Let us perturb the system from the above state by letting

$$X = X_0 + \Delta X \qquad u = u_0 + \Delta u$$

where the prefex Δ denotes a small deviation.

Now,

$$\dot{X} = \dot{X}_0 + \Delta\dot{X}$$
$$= f[(X_0 + \Delta X), (u_0 + \Delta u)]$$

As the perturbations are assumed to be small, the nonlinear functions $f(X, u)$ can be expressed in terms of Taylor's series expansion. With terms involving second and higher order powers of ΔX and Δu neglected,

we may write:

$$\dot{X}_i = \dot{X}_{i0} + \Delta\dot{X}_i = f_i\left[(X_0+\Delta X), (u_0+\Delta u)\right]$$

$$= f_i(X_0, u_0) + \frac{\partial f_i}{\partial X_1}\Delta X_1 + \cdots + \frac{\partial f_i}{\partial X_n}\Delta X_n$$

$$+ \frac{\partial f_i}{\partial u_1}\Delta u_1 + \cdots + \frac{\partial f_i}{\partial u_r}\Delta u_r$$

since $\underbrace{\dot{X}_{i0}}_{\text{zero}} = \underbrace{f_i(X_0, u_0)}_{\text{zero}}$, we obtain

$$\Delta\dot{X}_i = \frac{\partial f_i}{\partial X_1}\Delta X_1 + \cdots + \frac{\partial f_i}{\partial X_n}\Delta X_n + \frac{\partial f_i}{\partial u_1}\Delta u_1 + \cdots$$

$$+ \frac{\partial f_i}{\partial u_r}\Delta u_r$$

Similarly, for the output variables:

$$\Delta y_j = \frac{\partial g_j}{\partial X_1}\Delta X_1 + \cdots + \frac{\partial g_j}{\partial X_n}\Delta X_n + \frac{\partial g_j}{\partial u_1}\Delta u_1 +$$

$$\cdots + \frac{\partial g_j}{\partial u_r}\Delta u_r$$

with $j = 1, 2, \ldots, m$. Therefore, the linearized forms are:

$$\Delta\dot{X} = A\,\Delta X + B\,\Delta u$$
$$\Delta y = C\,\Delta X + D\,\Delta u$$

where:

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial x_1} & \cdots & \frac{\partial f_n}{\partial x_n} \end{bmatrix} \quad B = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_r} \\ \cdots & \cdots & \cdots \\ \frac{\partial f_n}{\partial u_1} & \cdots & \frac{\partial f_n}{\partial u_r} \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{\partial g_1}{\partial x_1} & \cdots & \frac{\partial g_1}{\partial x_n} \\ \cdots & \cdots & \cdots \\ \frac{\partial g_m}{\partial x_1} & \cdots & \frac{\partial g_m}{\partial x_n} \end{bmatrix} \quad D = \begin{bmatrix} \frac{\partial g_1}{\partial u_1} & \cdots & \frac{\partial g_1}{\partial u_r} \\ \cdots & \cdots & \cdots \\ \frac{\partial g_m}{\partial u_1} & \cdots & \frac{\partial g_m}{\partial u_r} \end{bmatrix}$$

$D \Downarrow$ mostly zero

The above partial derivatives are evaluated at the equilibrium point about which the small perturbation is being analyzed.

Δx : state vector of dimension n

Δy : output vector of dimension m

Δu : input vector of dimension r

A : state matrix of size $n \times n$

B : input matrix of size $n \times r$

C : output matrix of size $m \times n$

D : feed forward matrix which defines the proportion of input which appears directly in the output of size $m \times r$

Analysis of Stability

* The stability of a [redacted] linear system is given by the roots of the characteristic equation of the system of first approximation i.e. by the eigenvalues of the matrix A as:

(i) When the eigenvalues have negative real parts, the original system is asymptotically stable.

(ii) When at least one of the eigenvalues has a positive real part, the origional system is unstable

(iii) When the eigenvalues have real parts equal to zero, it is not possible on the basis of the first approximation to say anything in the general.

* The Global stability may be studied by explicit solution of the nonlinear differential equations using digital Computers.

Eigenproperties of the State Matrix

Eigenvalues

* The eigenvalues of a matrix are given by the values of the scalar parameter λ for which there exist non-trivial solutions to the equation:

$$A\phi = \lambda\phi$$

where

A is an $n \times n$ matrix

ϕ is an $n \times 1$ vector

To find the eigenvalues:

$$(A - \lambda I)\phi = 0$$

For a non-trivial solution

$$\det(A - \lambda I) = 0$$

* The eigenvalues may be real or complex. If A is real, complex eigenvalues always occur in conjugate pairs.

* Similar matrices have identical eigenvalues.
* A matrix and its transpose have the same eigenvalues.

Eigenvectors

For any eigenvalues λ_i, the n-column vector ϕ_i which satisfy $A\phi = \lambda\phi$ is called the right eigenvector of A associated with eigenvalue λ_i.

Therefore:

$$A\phi_i = \lambda_i \phi_i$$

The eigenvector ϕ_i has the form:

$$\phi_i = \begin{bmatrix} \phi_{1i} \\ \phi_{2i} \\ \vdots \\ \phi_{ni} \end{bmatrix}$$

* Similarly, the n-row vector ψ_i which satisfies

$$\psi_i A = \lambda_i \psi_i$$

is called the left eigenvector associated with the eigenvalue λ_i.

* The left and right eigenvectors corresponding to different eigenvalues are orthogonal i.e.

$$\psi_j \phi_i = 0 \quad i \neq j$$

and in case of eigenvectors corresponding to the same eigenvalue.

$$\psi_i \phi_i = C_i$$

* It is common practice to normalize these vectors so that $\psi_i \phi_i = 1$

Eigenvalue and Stability

The stability of the system is determined by the eigenvalues as follows:

(a) A real eigenvalue corresponds to a non-oscillatory mode. A negative real represents a decaying mode. The larger its magnitude, the faster the decay.

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Complex eigenvalues occur in conjugate pairs and each pair corresponds to an oscillatory mode.

* The real component of the eigenvalues gives the damping and the imaginary component gives the frequency of oscillation. A negative real part represents a damped oscillation whereas a positive real part represents oscillation of increasing amplitude. Thus if

$$\lambda = \sigma \pm j\omega$$, then the frequency of oscillation in Hz is

$$f = \frac{\omega}{2\pi}$$. The damping ratio ζ is given by:

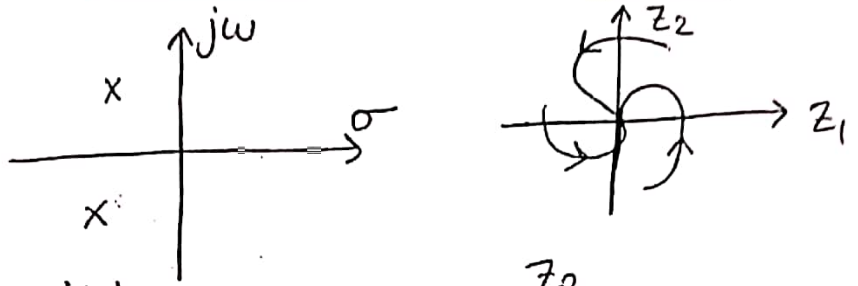
$$\zeta = \frac{-\sigma}{\sqrt{\sigma^2 + \omega^2}}$$

* The time response of the ith state variable is:

$$x_i(t) = \phi_{i1} c_1 e^{\lambda_1 t} + \phi_{i2} c_2 e^{\lambda_2 t} + \dots + \phi_{in} c_n e^{\lambda_n t} \rightarrow$$

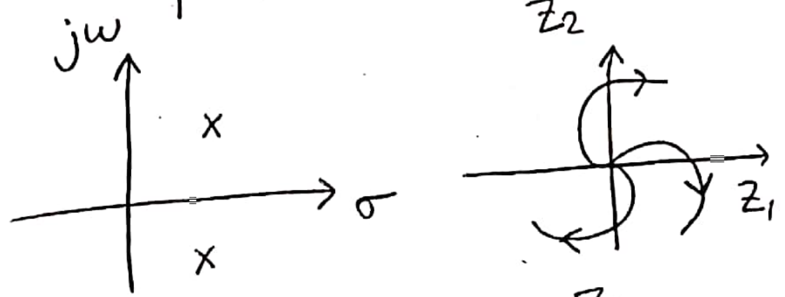
* The following figure shows the six different eigenvalue combinations and the corresponding trajectory behaviour around the singular points applicable to a two-dimensional case :

(1) eigenvalues — Trajectory — Type of singularity



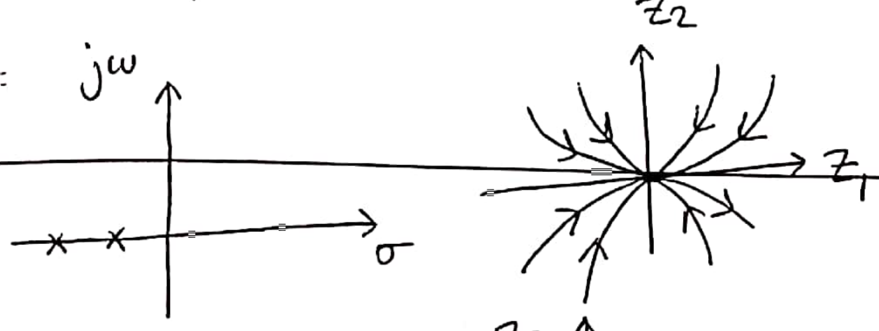
stable focus

(2)



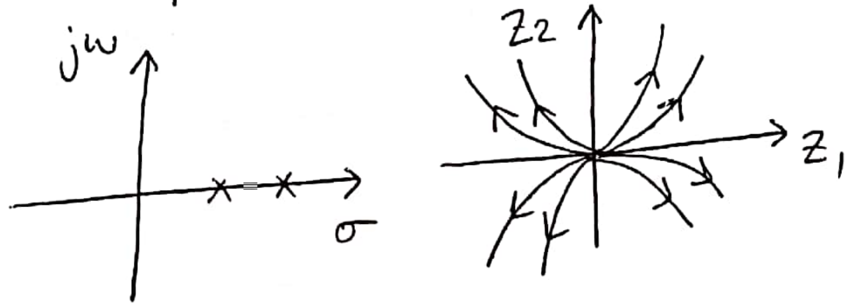
unstable focus

(3)

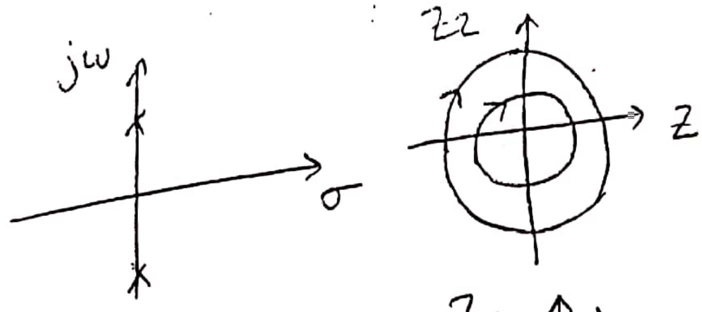


stable node

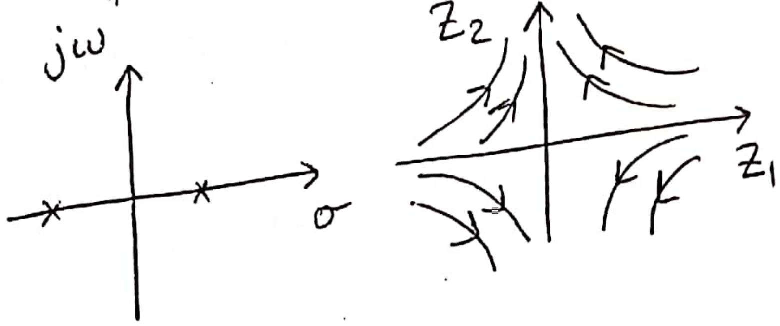
(4)



unstable node

(5) 

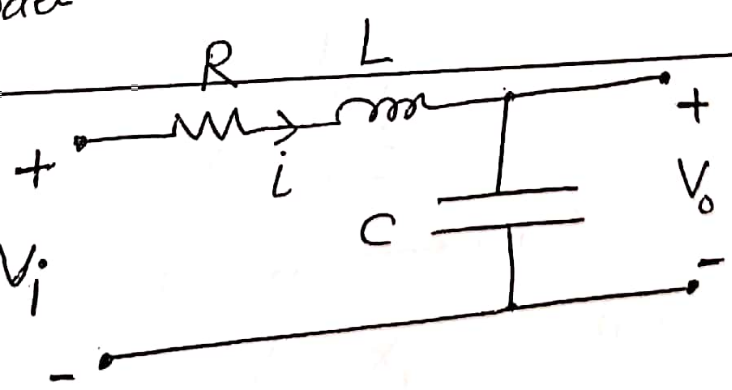
Vortex

(6) 

saddle

Singular points corresponding to six possible combinations of eigenvalue pairs

<u>EX.</u> The following is an RLC circuit. Study the eigen properties of the state matrix and examine its model characteristics



$$-V_i + Ri + L\frac{di}{dt} + V_o = 0$$

$$i = C\frac{dV_o}{dt}$$

$$RC\frac{dV_o}{dt} + LC\frac{d^2V_o}{dt^2} + V_o = V_i$$

In standard form:

$$\frac{d^2V_o}{dt^2} + (2\zeta\omega_n)\frac{dV_o}{dt} + \omega_n^2 V_o = \omega_n^2 V_i$$

where

$$\underbrace{\omega_n = \frac{1}{\sqrt{LC}}}_{\text{undamped natural frequency}} \quad \& \quad \underbrace{\zeta = \left(\frac{R}{2}\right) / \sqrt{L/C}}_{\text{damping ratio}}$$

In state-space representation:

$$x_1 = V_o$$

$$x_2 = \dot{x}_1 = \frac{dV_o}{dt}$$

$$\dot{x}_2 = \frac{d^2V_o}{dt^2}$$

$u = V_i \longrightarrow$ input

$y = V_o = x_1 \rightarrow$ output

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = -\omega_n^2 x_1 - (2\zeta\omega_n)x_2 + \omega_n^2 u$$

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In matrix form:

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 1\\ -\omega_n^2 & -2\zeta\omega_n\end{bmatrix}}_{A}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{\begin{bmatrix}0\\ \omega_n^2\end{bmatrix}}_{B} u$$

$$y = \underbrace{[1 \quad 0]}_{C}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \underbrace{0}_{D}u$$

$$|A - \lambda I| = 0$$

$$\left|\begin{bmatrix}0 & 1\\ -\omega_n^2 & -2\zeta\omega_n\end{bmatrix} - \begin{bmatrix}\lambda & 0\\ 0 & \lambda\end{bmatrix}\right| = 0$$

$$\begin{vmatrix}-\lambda & 1\\ -\omega_n^2 & -2\zeta\omega_n - \lambda\end{vmatrix} = 0$$

$$(-\lambda)(-2\zeta\omega_n - \lambda) + \omega_n^2 = 0$$

$$2\lambda\zeta\omega_n + \lambda^2 + \omega_n^2 = 0$$

$$\lambda^2 + 2\zeta\omega_n\lambda + \omega_n^2 = 0 \longrightarrow \text{characteristics}$$

$$\left.\begin{aligned}\lambda_1 &= -\zeta\omega_n + \omega_n\sqrt{\zeta^2 - 1}\\ \lambda_2 &= -\zeta\omega_n - \omega_n\sqrt{\zeta^2 - 1}\end{aligned}\right\}\ \text{eigen values}$$

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The right eigenvectors are:

$$A\phi_i = \lambda_i \phi_i$$

$$(A-\lambda_i I)\phi_i = 0$$

$$\begin{bmatrix} -\lambda_i & 1 \\ -\omega_n^2 & -2\zeta\omega_n-\lambda_i \end{bmatrix}\begin{bmatrix} \phi_{1i} \\ \phi_{2i} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$\left.\begin{aligned} -\lambda_i \phi_{1i} + \phi_{2i} &= 0 \\ -\omega_n^2 \phi_{1i} - (2\zeta\omega_n + \lambda_i)\phi_{2i} &= 0 \end{aligned}\right\}$$ not independent

$\boxed{\text{always } n-1 \text{ independent equations}}$

* For the second order system, we can fix $\phi_{1i} = 1$

~~and determine ϕ_{2i} for each eigenvalue:~~

The eigenvector corresponding to λ_1 is:

$$\phi_1 = \begin{bmatrix} \phi_{11} \\ \phi_{21} \end{bmatrix} = \begin{bmatrix} 1 \\ \lambda_1 \end{bmatrix} = \begin{bmatrix} 1 \\ -\zeta\omega_n + \omega_n\sqrt{\zeta^2-1} \end{bmatrix}$$

The eigenvector corresponding to λ_2 is:

$$\phi_2 = \begin{bmatrix}\phi_{12}\\ \phi_{22}\end{bmatrix} = \begin{bmatrix}1\\ \lambda_2\end{bmatrix} = \begin{bmatrix}1\\ -\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}\end{bmatrix}$$

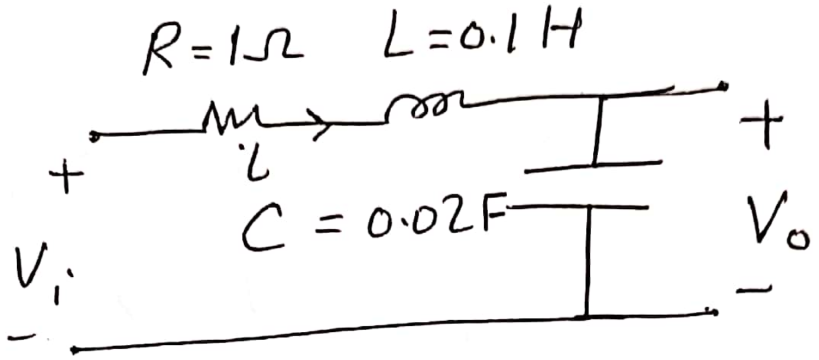
$$x_1(t) = \phi_{11} C_1 e^{\lambda_1 t} + \phi_{12} C_2 e^{\lambda_2 t}$$

$$\boxed{x_1(t) = c_1 e^{\left(-\zeta\omega_n + \omega_n\sqrt{\zeta^2-1}\right)t} + C_2 e^{\left(-\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}\right)t}}$$

$$x_2(t) = \phi_{21} c_1 e^{\left(-\zeta\omega_n + \omega_n\sqrt{\zeta^2-1}\right)t} + \phi_{22} C_2 e^{\left(-\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}\right)t}$$

$$\boxed{\begin{aligned} x_2(t) = {} & \left(-\zeta\omega_n + \omega_n\sqrt{\zeta^2-1}\right) c_1 e^{\left(-\zeta\omega_n + \omega_n\sqrt{\zeta^2-1}\right)t} \\ & + \left(-\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}\right) C_2 e^{\left(-\zeta\omega_n - \omega_n\sqrt{\zeta^2-1}\right)t} \end{aligned}}$$

Ex. For the following RLC circuit, study the eigen properties of the state matrix and examin its model characterstics.



$$-V_i + Ri + L\frac{di}{dt} + V_o = 0$$

$$i = C\frac{dV_o}{dt}$$

$$-V_i + RC\frac{dV_o}{dt} + LC\frac{d^2V_o}{dt^2} + V_o = 0$$

In standard form

$$\frac{d^2V_o}{dt^2} + (2\zeta\omega_n)\frac{dV_o}{dt} + \omega_n^2 V_o = \omega_n^2 V_i$$

$$\omega_n = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{(0.1)(0.02)}} = 22.36$$ undamped natural frequency

$$\zeta = \left(\frac{R}{2}\right)\Big/\sqrt{\frac{L}{C}} = \left(\frac{1}{2}\right)\Big/\sqrt{\frac{0.1}{0.02}} = 0.2236$$ damping ratio

~~In state representation:~~ ②

$\cancel{X_1 =}$

$$\frac{d^2V_o}{dt^2} + 10\frac{dV_o}{dt} + 500\,V_o = 500\,V_i$$

In state-space representation:

$$X_1 = V_o$$
$$X_2 = \dot{X}_1 = \frac{dV_o}{dt}$$
$$\dot{X}_2 = \frac{d^2V_o}{dt^2}$$
$$u = V_i \longrightarrow \text{input}$$
$$y = V_o = X_1 \longrightarrow \text{output}$$

$$\dot{X}_1 = X_2$$
$$\dot{X}_2 = -500X_1 - 10X_2 + 500u$$

In matrix form:

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & 1 \\ -500 & -10 \end{bmatrix}}_{A}\begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ 500 \end{bmatrix}}_{B} u$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 0u$$

$$|A - \lambda I| = 0$$

$$\left| \begin{bmatrix} 0 & 1 \\ -500 & -10 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right| = 0$$

$$\begin{vmatrix} -\lambda & 1 \\ -500 & -10-\lambda \end{vmatrix} = 0$$

$$(-\lambda)(-10-\lambda) + 500 = 0$$

$$10\lambda + \lambda^2 + 500 = 0$$

$\lambda^2 + 10\lambda + 500 = 0 \Rightarrow$ characteristic equation

$$\left.\begin{aligned} \lambda_1 &= -5 + j21.79 \\ \lambda_2 &= -5 - j21.79 \end{aligned}\right\} \text{eigenvalues}$$

The right eigenvectors are :

$$A\phi_i = \lambda_i \phi_i$$

$$(A - \lambda_i I)\,\phi_i = 0$$

For the first eigenvalue $i = 1$.

$$(A - \lambda_1 I)\phi_1 = 0$$

$$\left[\begin{bmatrix} 0 & 1 \\ -500 & -10 \end{bmatrix} - \begin{bmatrix} -5+j21.79 & 0 \\ 0 & -5+j21.79 \end{bmatrix}\right]\begin{bmatrix} \phi_{11} \\ \phi_{21} \end{bmatrix} = 0$$

$$\begin{bmatrix} 5-j21.79 & 1 \\ -500 & -10+5-j21.79 \end{bmatrix}\begin{bmatrix} \phi_{11} \\ \phi_{21} \end{bmatrix} = 0$$

$$\left.\begin{aligned} 5-j21.79\,\phi_{11} + \phi_{21} &= 0 \\ -500\,\phi_{11} + (-5-j21.79)\phi_{21} &= 0 \end{aligned}\right\} \text{not independent equations}$$

let $\phi_{11} = 1 \Rightarrow \phi_{21} = -5+j21.79$

$\therefore$ The right eigenvector for λ_1 is $\begin{bmatrix} 1 \\ -5+j21.79 \end{bmatrix}$

For the second eigenvalue $i=2$

$$(A-\lambda_2 I)\phi_2=0$$

$$\begin{bmatrix} 5+j21.79 & 1 \\ -500 & -10+5+j21.79 \end{bmatrix}\begin{bmatrix} \phi_{12} \\ \phi_{22} \end{bmatrix}=0$$

$$\left.\begin{aligned} 5+j21.79\,\phi_{12} + \phi_{22} &= 0 \\ -500\,\phi_{12} + (-5+j21.79)\,\phi_{22} &= 0 \end{aligned}\right\} \text{not independent}$$

let $\phi_{12}=1 \Rightarrow \phi_{22}=-5-j21.79$

$\therefore$ The right eigenvector for λ_2 is $\begin{bmatrix} 1 \\ -5-j21.79 \end{bmatrix}$

$$\therefore \phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ -5+j21.79 & -5-j21.79 \end{bmatrix}$$

Now

$$x_1(t) = C_1\,\phi_{11}\,e^{\lambda_1 t} + C_2\,\phi_{12}\,e^{\lambda_2 t}$$

$$x_1(t) = C_1\,e^{(-5+j21.79)t} + C_2\,e^{(-5-j21.79)t}$$

$$X_2(t) = C_1 \phi_{21} e^{\lambda_1 t} + C_2 \phi_{22} e^{\lambda_2 t}$$ (6)

$$= (-5+j21.79)\, C_1 e^{(-5+j21.79)t} + (-5-j21.79)\, C_2 e^{(-5-j21.79)t}$$

For C_1 & C_2, they can be calculated from the initial conditions.

Operating Point Stability of a Nonlinear Dynamical System

Ex. Study the stability of the operating point of the following dynamical system

$$\dot{x}_1 = 2x_1 + x_2^2 + 2$$

$$\dot{x}_2 = \sin x_1 - x_2$$

$$J = \begin{bmatrix} \dfrac{\partial \dot{x}_1}{\partial x_1} & \dfrac{\partial \dot{x}_1}{\partial x_2} \\ \dfrac{\partial \dot{x}_2}{\partial x_1} & \dfrac{\partial \dot{x}_2}{\partial x_2} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 2x_2 \\ \cos x_1 & -1 \end{bmatrix}$$

Operating point

$$0 = 2x_1 + x_2^2 + 2$$

$$0 = \sin x_1 - x_2$$

Solving gives $x_1 = -1.50$

$x_2 = -1.0$

Eigenvalues of the Jacobean Matrix at the operating Point

$$|J - \lambda I| =$$

$$\left| \begin{bmatrix} 2 & (2)(-1) \\ \cos(-1.5) & -1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right|$$

$$= \left| \begin{bmatrix} 2 & -2 \\ 0.0707 & -1 \end{bmatrix} - \begin{bmatrix} \lambda & 0 \\ 0 & \lambda \end{bmatrix} \right|$$

$$= \begin{vmatrix} 2-\lambda & -2 \\ 0.0707 & -1-\lambda \end{vmatrix}$$

$$= (2-\lambda)(-1-\lambda) + 0.1414$$

$$= \lambda^2 - 3\lambda + 2 + 0.1414 \Rightarrow$$ characteristic equation

$$= \lambda^2 - 3\lambda + 2.1414$$

$$= (\lambda - 1.83)(\lambda - 1.17)$$

Eigenvalues

$$\left.\begin{aligned} \lambda_1 &= 1.83 \\ \lambda_2 &= 1.17 \end{aligned}\right\}$$ unstable operating point

$$L_{ff}\dot{X}_1 + kM_f\dot{X}_2 = V'_{ff} - R_f X_1$$

$$-kM_f\dot{X}_1 - L_d\dot{X}_2 = V_a \sin X_5 + RX_2 + L_q X_4 X_3$$

$$-L_q\dot{X}_3 = V_a \cos X_5 + RX_3 - L_d X_4 X_2 - kM_f X_4 X_1$$

$$J\dot{X}_4 = T_m - 0.75\left[L_{ff} X_1 X_3 + kM_f X_2 X_3 + L_d X_2 X_3 - L_q X_2 X_3\right]$$

$$\dot{X}_5 = X_4 - \omega_s$$

$$A = \begin{bmatrix} L_{ff} & kM_f & 0 & 0 & 0 \\ -kM_f & -L_d & 0 & 0 & 0 \\ 0 & 0 & -L_q & 0 & 0 \\ 0 & 0 & 0 & J & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$F_1 = V'_{ff} - R_f X_1$$

$$F_2 = V_a \cos X_5 + R X_2 + L_q X_4 X_3$$

$$F_3 = V_a \sin X_5 + R X_3 - L_d X_4 X_2 - KM_f X_4 X_1$$

$$F_4 = T_m - 0.75\left[L_{ff} X_1 X_3 + KM_f X_2 X_3 + L_d X_2 X_3 - L_q X_2 X_3\right]$$

$$F_5 = X_4 - W_s$$

$$F = \begin{bmatrix} F_1 \\ F_2 \\ F_3 \\ F_4 \\ F_5 \end{bmatrix}$$

$$\dot{X} = A^{-1} F \Rightarrow \quad \begin{array}{l} \dot{X}_1 = \\ \dot{X}_2 = \\ \dot{X}_3 = \\ \dot{X}_4 = \\ \dot{X}_5 = \end{array}$$

solve $(F_1, F_2, F_3, F_4, F_5) \longrightarrow$ operating point

$$J = \begin{bmatrix} \frac{\partial \dot{x}_1}{\partial x_1} & \frac{\partial \dot{x}_1}{\partial x_2} & \frac{\partial \dot{x}_1}{\partial x_3} & \frac{\partial \dot{x}_1}{\partial x_4} & \frac{\partial \dot{x}_1}{\partial x_5} \\ \frac{\partial \dot{x}_2}{\partial x_1} & \frac{\partial \dot{x}_2}{\partial x_2} & \frac{\partial \dot{x}_2}{\partial x_3} & \frac{\partial \dot{x}_2}{\partial x_4} & \frac{\partial \dot{x}_2}{\partial x_5} \\ \frac{\partial \dot{x}_3}{\partial x_1} & - - - & & & \\ \frac{\partial \dot{x}_4}{\partial x_1} & & & & \\ \frac{\partial \dot{x}_5}{\partial x_1} & & & & \frac{\partial \dot{x}_5}{\partial x_5} \end{bmatrix}$$

J = Jacobean(

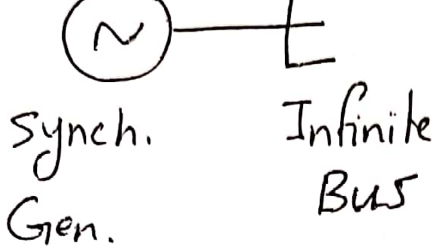
J = subs(J, X_1, a_1)

J = subs(J, X_2, a_2)

$\vdots$

E_i = eig(J)

Write the MATLAB code which finds the steady-state solution of the synch. gen. connected to infinite bus.



$$L_{ff}\,\dot{i}_f + kM_f\,\dot{i}_d = V_{ff} - R_f\, i_f$$

$$-kM_f\,\dot{i}_f - L_d\,\dot{i}_d = V_a \sin\delta + R\, i_d - \omega L_q\, i_q$$

$$-L_q\,\dot{i}_q = V_a \cos\delta + R\, i_q - \omega\, kM_f\, i_f + \omega L_d\, i_d$$

$$J\,\dot{\omega} = T_m - 0.75P\left[kM_f\, i_d i_q + L_{ff}\, i_f i_q + L_d\, i_d i_q - L_q\, i_d i_q\right]$$

$$\dot{\delta} = \omega - 377$$

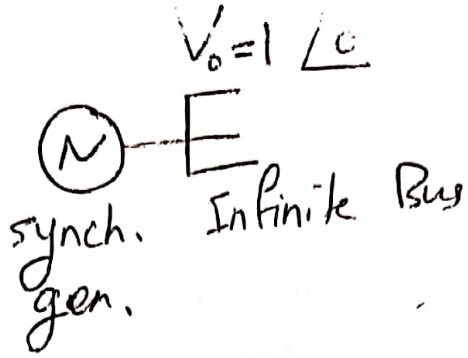
$L_{ff} = 0.1169$ H $\quad kM_f = 0.1083$ H

$L_d = 0.1184$ H $\quad V_a = 120$ V $\quad R = 0.1217\,\Omega$

$L_q = 0.1141$ H $\quad J = 0.5$ kg.m^2 $\quad T_m = 0.5$ N/m

$V_{ff} = 2$ V

* For the following power system, study the small signal stability :



① $X_{ffd}\,\dot{i}_{fd} - X_{afd}\,\dot{i}_d = \omega_0 \dfrac{R_{fd}}{X_{afd}} E_{fd} - \omega_0 R_{fd}\, i_{fd}$

② $X_{afd}\,\dot{i}_{fd} - X_d\,\dot{i}_d = \omega_0 V_0 \sin\delta_g + \omega_0 r_a X_2 - \omega_0 \omega_g X_q i_q$

③ $-X_q\,\dot{i}_q = \omega_0 V_0 \cos\delta_g - \omega_0 \omega_g X_{afd}\, i_{fd} + \omega_0 \omega_g X_d i_d$
$\qquad + \omega_0 r_a i_q$

④ $\dot{\delta}_g = \omega_0(\omega_g - 1)$

⑤ $J.\ \omega_g = T_m - X_{afd}\, i_q i_{fd} + X_d i_d i_q - X_q i_d i_q$

$\boxed{X_1 = i_{fd}}$ $\boxed{X_2 = i_d}$ $\boxed{X_3 = i_q}$ $\boxed{X_4 = \delta_g}$ $\boxed{X_5 = \omega_g}$

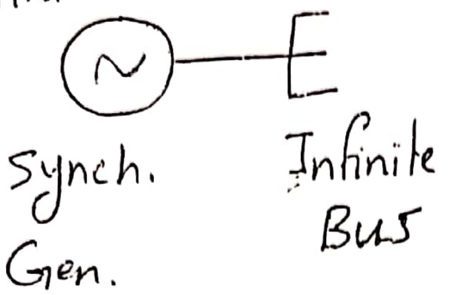
$X_{ffd} = 1.097$ $\quad X_{afd} = 0.847$ $\quad R_{fd} = 0.0003925$

$X_q = 0.66$ $\quad V_0 = 1.0$ $\quad r_a = 0.005$

$X_d = 1.0$

$J = 6.62$ $\quad \omega_0 = 377$ $\quad E_{fd} = 1.7$ $\quad T_m = 0.91$

Write the MATLAB code which finds the steady-state solution of the synch. gen. connected to infinite bus.



$$L_{ff}\dot{i}_f + kM_f \dot{i}_d = V_{ff} - R_f i_f$$

$$-kM_f \dot{i}_f - L_d \dot{i}_d = V_a \sin\delta + R i_d - w L_q i_q$$

$$-L_q \dot{i}_q = V_a \cos\delta + R i_q - w kM_f i_f + w L_d i_d$$

$$J\dot{w} = T_m - 0.75P\left[kM_f i_d i_q + L_{ff} i_f i_q + L_d i_d i_q - L_q i_d i_q\right]$$

$$\dot{\delta} = w - 377$$

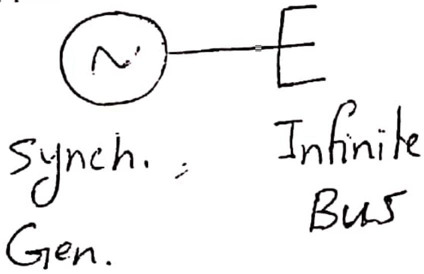
$L_{ff} = 0.1169\ H$ $\quad kM_f = 0.1083\ H$

$L_d = 0.1184\ H$ $\quad V_a = 120V$ $\quad R = 0.1217\ \Omega$

$L_q = 0.1141\ H$ $\quad J = 0.5\ kg.m^2$ $\quad T_m = 0.5\ Nm$

$V_{ff} = 2V$

Write the MATLAB code which finds the steady-state solution of the synch. gen. connected to infinite bus.



$$L_{ff}\dot{i}_f + kM_f \dot{i}_d = V_{ff} - R_f i_f$$

$$-kM_f \dot{i}_f - L_d \dot{i}_d = V_a \sin\delta + R i_d - \omega L_q i_q$$

$$-L_q \dot{i}_q = V_a \cos\delta + R i_q - \omega kM_f i_f + \omega L_d i_d$$

$$J\dot{\omega} = T_m - 0.75P\left[kM_f i_d i_q + L_{ff} i_f i_q + L_d i_d i_q - L_q i_d i_q\right]$$

$$\dot{\delta} = \omega - 377$$

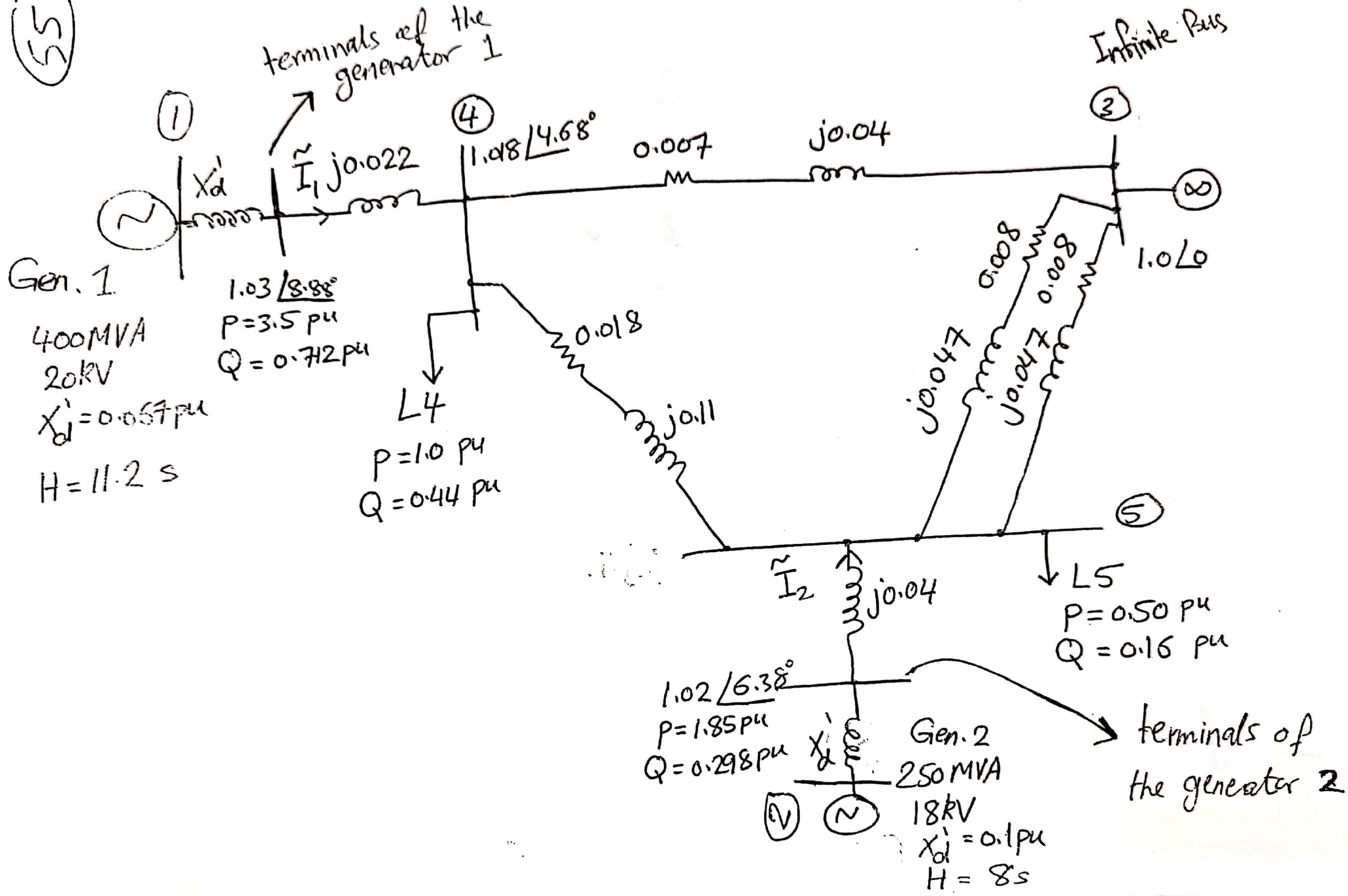
$L_{ff} = 0.1169$ H $\quad kM_f = 0.1083$ H

$L_d = 0.1184$ H $\quad V_a = 120$ V $\quad R = 0.1217\,\Omega$

$L_q = 0.1141$ H $\quad J = 0.5$ kg.m^2 $\quad T_m = 0.5$ Nm

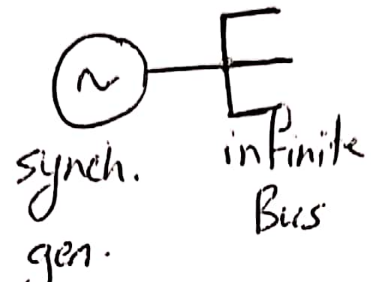
$V_{ff} = 2$ V

55



terminals of the generator 1
1
4
3
Infinite Bus
Xd'
$\tilde{I}_1$ j0.022
1.08∠4.68°
0.007
j0.04
1.0∠0
Gen. 1
400MVA
20kV
Xd' = 0.057pu
H = 11.2 s
1.03∠8.88°
P=3.5 pu
Q = 0.712pu
L4
P=1.0 pu
Q=0.44 pu
0.018
j0.11
0.008
0.008
j0.047
j0.047
5
$\tilde{I}_2$
j0.04
L5
P=0.50 pu
Q = 0.16 pu
1.02∠6.38°
P=1.85pu
Q=0.298pu
Xd'
Gen. 2
250MVA
18kV
Xd' = 0.1pu
H = 8s
terminals of the generator 2

Ex. For the following power system (synchronous generator connected to an infinite bus), study its small signal stability.



The numerical parameters of the generator are in per unit on the base of the generator itself:

$L_{ff} = 1.6286$ $\quad\quad$ $kM_f = 1.51$

$R_f = 0.00096$ $\quad\quad$ $L_d = 1.65$

$V_a = 1.0$ $\quad\quad$ $R = 0.0045$

$L_q = 1.59$ $\quad\quad$ $T_m = 1.0$

$J = 1.7581$ $\quad\quad$ $\omega_s = 1.0$

$P = 1$ (in per unit system) $\quad V'_{ff} = 1.7$