ELECTRICAL COMPUTER MECHATRONICS

تقدم لجنة ElCoM الاكاديمية

دفتر لمادة:

استقراريـة أنظمة القوى

جزيل الشكر للطالب:

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Power System Stability

CHAPTER 16

POWER SYSTEM STABILITY

Ch.16 :- power system stability :-

* Stability studies which evaluate the impact of disturbances on the electromechanical dynamic behavior of the power system.
* power system stability is of two types :-

1] large signal (transient) stability :- studying the ability of the power system to maintain synchronism in case the power system is subjected to large disturbance, large signal stability is normally studied by solving the swing equation in time domain. (2nd order non-linear differential equation).

- Example of large disturbances :-

→ Three-phase Fault.
→ Step change in the power demand.
→ Step disconnecting of large power.
→ Step change in the Automatic Voltage Regulating gain (AVR gain).
→ Step change in the Turbine Governor gain (TG gain).
→ Lightning (صواعق)

بتعمل
① over voltage
② تؤدي إلى انهيار العوازل وانهيارها يؤدي الى وجود الـ Fault

2] Small-Signal (Steady-State stability) :- Studying the ability of the power system to maintain synchronism in case it is subjected to Small disturbance, Small signal stability is normally studied by linearizing the Swing equation around the operating point and obtaining the eigenvalues of the linearize model.

poles of the characteristics equation.

⇒ Example of Small disturbance :-

⟶ Small incremental change in the power or Small change in the system parameters (voltage, current, Frequency, PF, ... etc).

⇘ ومن الأمثلة عليه أيضاً تغيير Automatic voltage Regulator gain

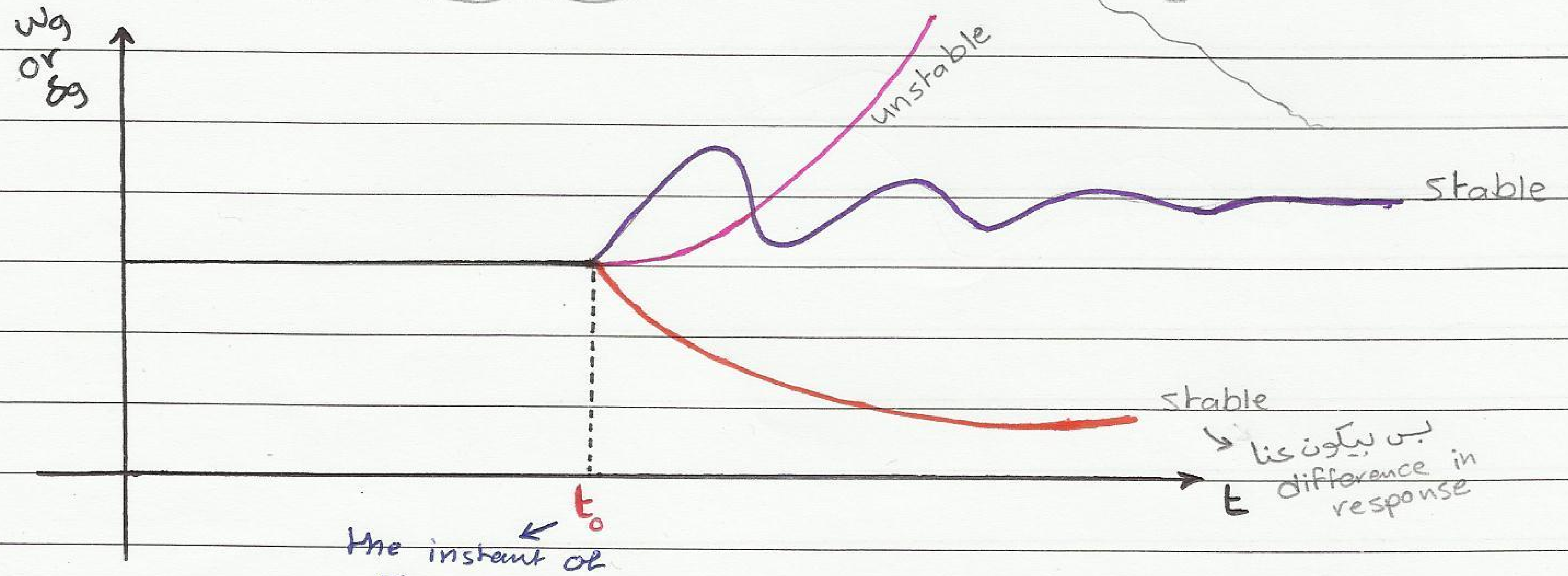
* ملاحظات بسيطة وهامة *

* إذا اردنا تغيير قيمة المقاومة، فإننا نقوم بتغيير الـ Temp لأن درجة الحرارة تؤثر بشكل مباشر على قيمة الـ Resistors.

* إذا أردنا تغيير قيم المكثفات والملفات (C, L) فإننا نقوم بتغيير الـ Frequency (f)، ويجب الانتباه الى ان تغيير قيمة الـ f تؤثر على قيمة R بسبب وجود الـ Skin effect لكن لا نغيرها الاهتمام لأننا نتعامل مع انظمة ثابتة constant power system.

* من الممكن ان نحصل على Pout = صفر وتكون الـ Pin = قيمة ثابتة، عندها تكون السرعة عالية جداً بالاضافة الى high acceleration.

اللي بيحدد شكل هاي الحركات هي الـ Poles for characteristics equation



the instant of Fault (3-Ø Fault Symmetrical)

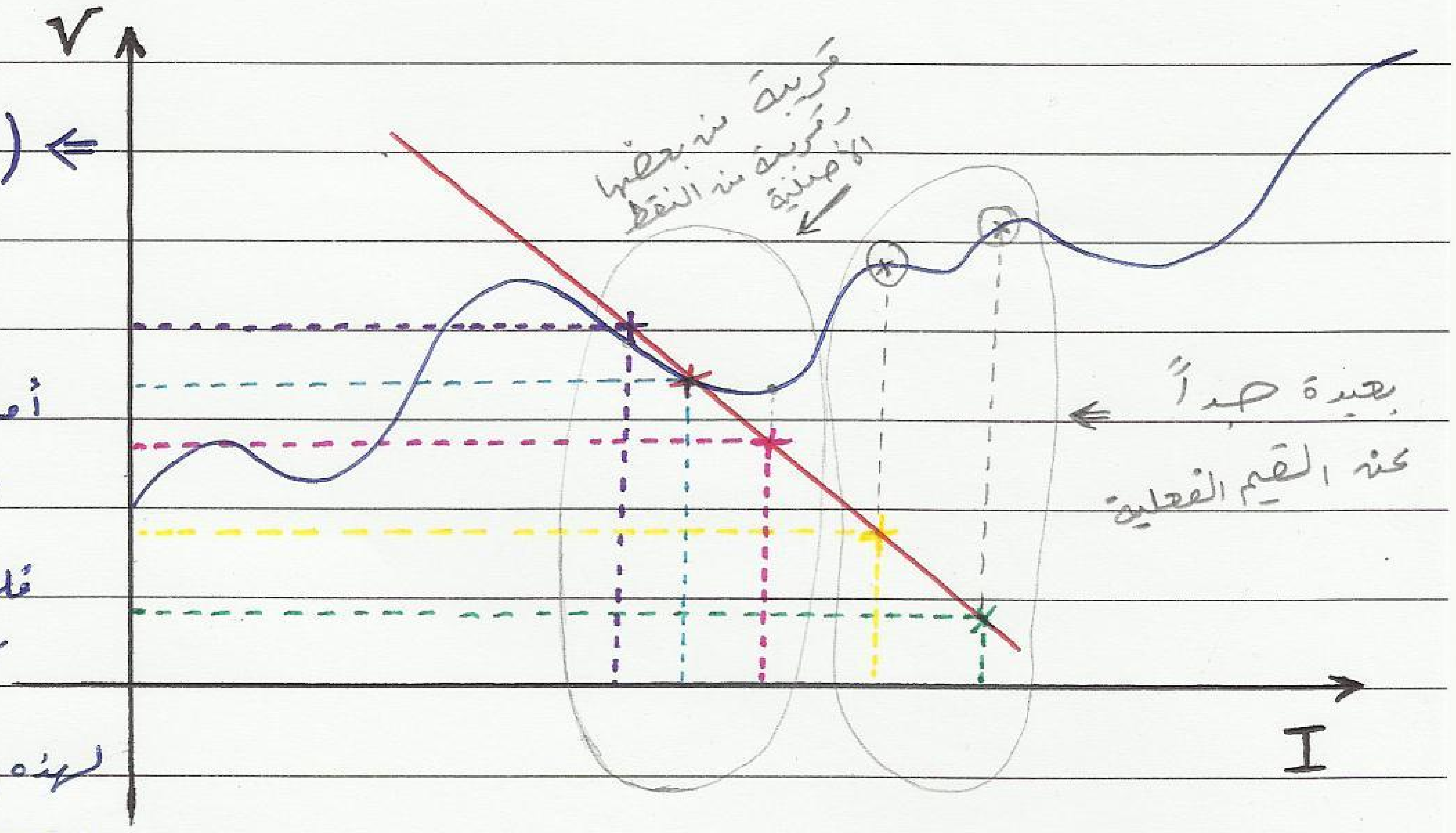
أو ممكن يكون أي disturbance من المذكورة بالصفحة السابقة أو أي step change ممكن يحدث للـ system

" general response of the power system due to large disturbance "

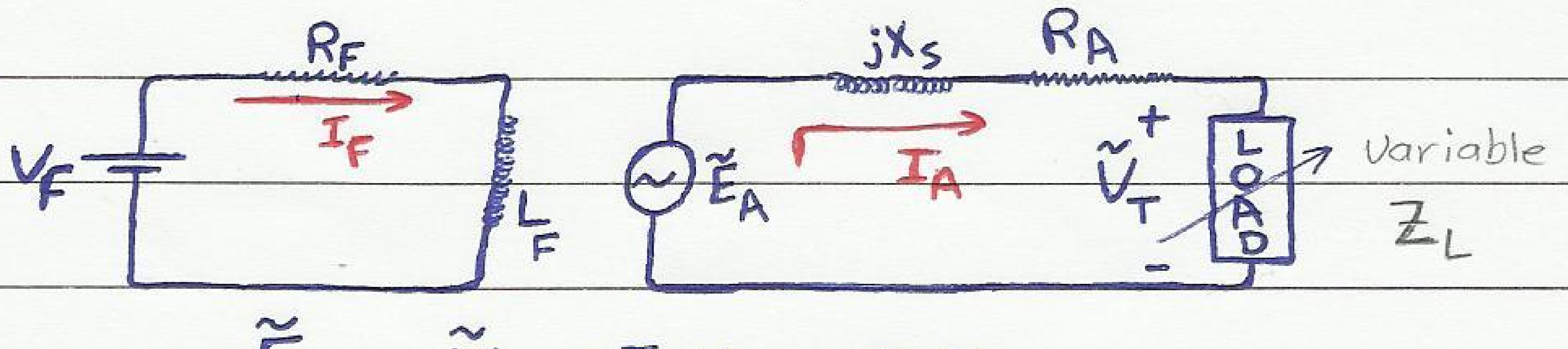
* Steady - State stability is less extensive than transient stability and the normally involves a single generators operating in to a infinite Bus.

* Since transient stability involves large disturbances, linearization is not permitted.

(بمعنى آخر أنه الـ linearization نستطيع الاعتماد عليه فقط في الـ Small disturbances.
أما في الـ large disturbances الموجودة في الـ transient stability فلا نستطيع استخدامها لأننا نبتعد كثيراً عن الـ curve الأصلي. والحل لهذه المشكلة أنه نعمل linearization مرة أخرى عند النقطة التي وصلنا لها. والصعوبة تكمن في معرفة مكان هذه النقطة.)



* AVR :- Automatic Voltage Regulator, which is a negative Feedback control, where the Field current (I_F) is modified automatically to adjust ($\tilde{E}_A$) to keep constant (V_T).



$$\tilde{E}_A = \tilde{V}_T + jI_A X_S + I_A R_A$$

as the load increase $\Rightarrow Z_L \downarrow$, $I_A \uparrow$, Voltage drop on Resistor $R\ (V_R) \uparrow$

إذا كانت E_A ثابتة (constant) فإن قيمة V_T سوف تقل وتتناقص تدريجياً مع زيادة الحمل ، ونستطيع زيادتها عن طريق زيادة قيمة I_F ، وبالتالي زيادة قيمة E_A فتزداد قيمة V_T .

Note : if the load increase , $Z_L \downarrow$?? why ?!! جواب :- لأن الـ load يتم وصلها على التوازي (in parallel) .

* TG :- Turbine Governor.

if the load increase, the speed ↓ ⇒ it's contreal and measure the Frequency of the system.

but we need constant speed to obtain constant frequency.

$$F = \frac{nP}{120}$$

and we control the Frequency of the system by governer set point (increase or decrease the input fuel)

* notes :-

- AVR ⇒ Q , TG ⇒ P
- in AVR the $T_{AVR} = 0.001$ s to reaction.
  in TG the $T_{TG} = 0.2$ second to reaction.
- في ال AVR بيكون تأثيره على ال Fault اسرع بكثير من TG
- ال AVR اسرع من TG لأنه pure electrical
  اما TG فيحتوي على mechanical system

* to Facilitate (simplify) computation three Fundamental assumption are made in all stability studies :-

1] Only Synchronous Frequency currents and voltages are considered.

- DC offset (voltage and current) are neglected (لا تدخل في الحسابات).
- harmonics neglected (does not exist).
- Variation on speed does not effect on frequency (تغيير س ⇐ لا يغير ال f).

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2] Symmetrical components are used in case of an balanced Faults.

3] generated voltage is considered constant even the machine (generator) speed varies during stability study.

* **note :-**

* نلاحظ مما سبقه أن هناك ترابط ما بين النقطة الثالثة والنقطة الأولى كما سبق ، ذلك لأن تغيير السرعة لا يؤدي فقط الى تغيير الـ f بل يؤدي أيضاً الى تغيير الـ voltage لأن الـ f يؤثر على الفولتية بالعلاقة التالية :-

$$E_A = 4.44\,\phi f N$$

* In large signal stability the effect of damper windings, AVR and TG are sometimes neglected, and 3-ϕ balanced Faults are considered.

أسوأهم لأن $P_{out} = Zero$

symmetrical Faults

* **16.2 :- Rotor Dynamics and the Swing equation :-**

$$J\frac{d^2\theta_m}{dt^2} = T_a = T_m - T_e$$

total moment of inertia $Kg \cdot m^2$ of the rotor mass

acceleration torque

applied mechanical torque (input From the prime mover)

Electrical (Anti) torque $\Rightarrow K\phi I_A$ where I_A the current in stator.

θ_m : angular displacement of the rotor w.r.t reference axis.

t : time, in seconds (s)

T_a : net acceleration torque, in N.m

T_m : the mechanical or shaft torque supplied by the prime mover less retarding (تأخير) torque due to rotational losses, in N.m

T_e : net electrical or electromagnetic torque, in N.m

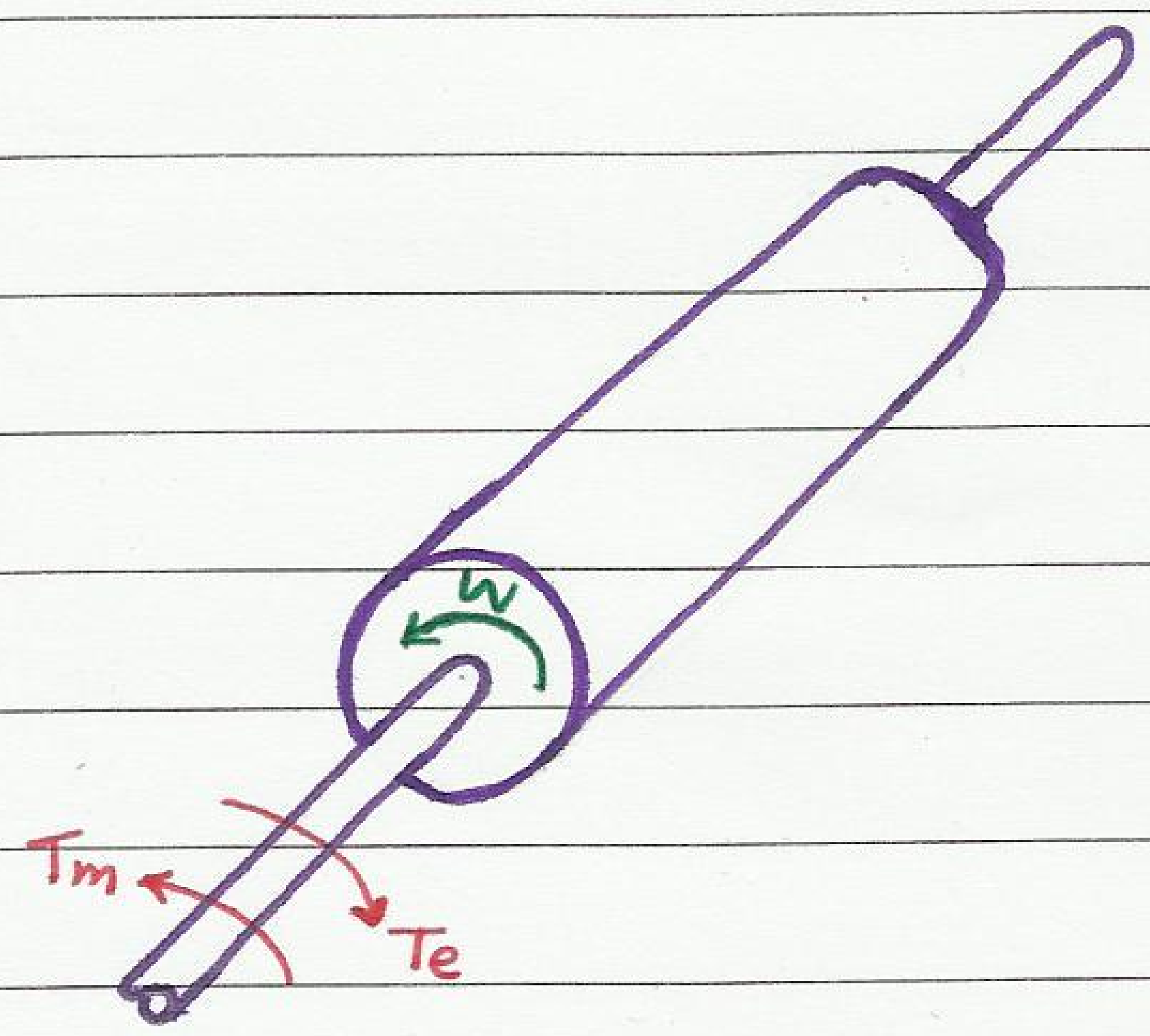
$\Rightarrow T_m$ & T_e are considered positive $+ve$ in case of generators.

* In steady-state conditions, $T_a = 0 \Rightarrow T_m = T_e$

  and the time derivative term $= 0$ $\left(\frac{d^2\theta_m}{dt^2} = 0 \right)$.

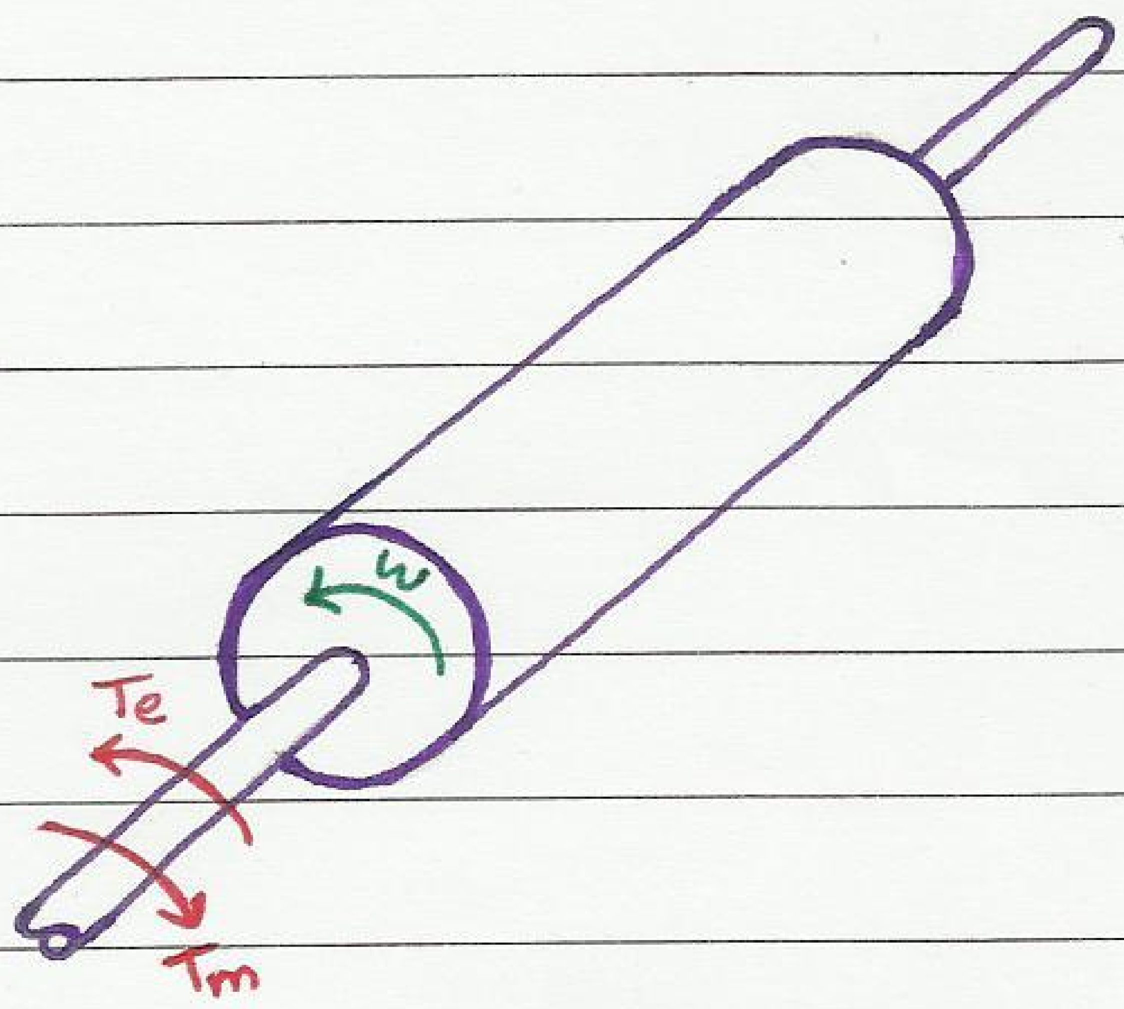
* at no-load $T_e = 0 \Rightarrow T_m = 0$ (Ideally). $\Rightarrow$ فعلياً لا تساوي صفر بسبب وجود الـ losses

* if we applied this equation on motor, then the direction of power flow is change $(T_a = T_e - T_m)$.



* This is the generator

لأن T_m بأتجاه (w)



* This is motor

لأن T_m بعكس اتجاه (w)

* T_m is considered constant at any given operating conditions.

  لـ يعني لو صار عنا Fault $\Leftarrow$ فإن $(P_{out} = 0)$ وبالتالي $(T_e = 0)$ لكن T_m تبقى ثابتة (constant)

* T_e (Anti or electrical torque) depends on the value of the output of electrical power from the generator.

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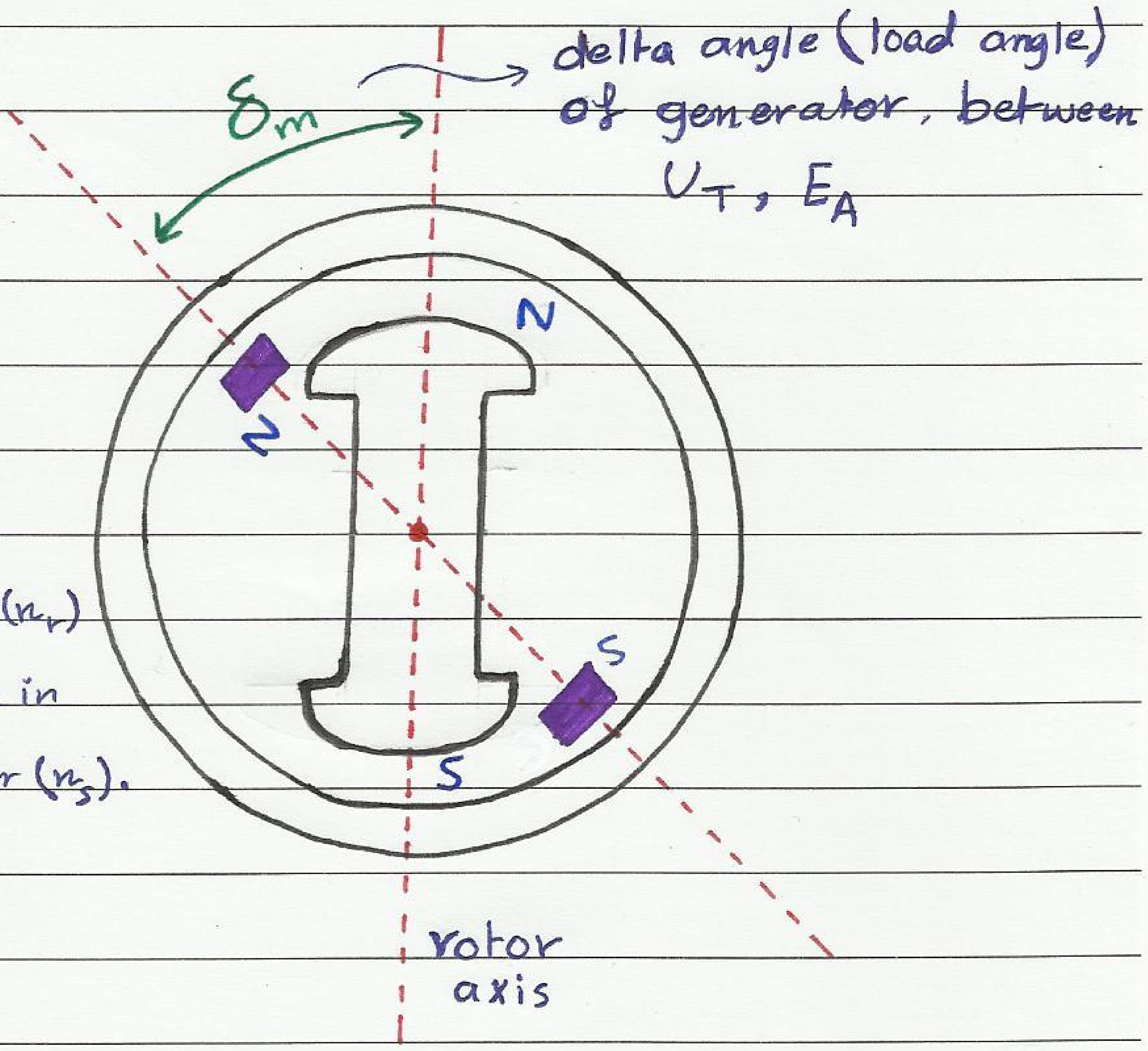
* T_m is controlled by governor, governor do not act until a change in speed is sensed and normally its time constant is relatively long (around 0.2 second).

* θ_m is measured with respect to a stationary reference axis on the stator.

$$\theta_m = \underset{\text{synchronous speed}}{\omega_{s_m}} t + \delta_m$$ $\Rightarrow$ increasing with time (periodic).

* If the load $\uparrow$, $\delta_m \uparrow$



* there are two north poles and two South poles.

N, S of the rotor $\Rightarrow$ due to excitation current $\Rightarrow$ rotating mmf for rotor (n_r)

N, S of stator $\Rightarrow$ due to 3-ø current in the stator $\Rightarrow$ rotating mmf for stator (n_s).

* $\delta_m \leq 90°$ $\Rightarrow$ stable.

$\delta_m = 90°$ $\Rightarrow$ marginally stable.

$\delta_m > 90°$ $\Rightarrow$ Unstable.

* At no-load $\delta_m = 0$ $\Rightarrow$ $V_T = E_A$, $P_e = T_e = 0$

* $P = \dfrac{3 E_A V_T}{X_s} \sin\delta$ (non-linearity)

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$$\theta_m = \omega_{sm} t + \delta_m$$

$\Rightarrow$

$$\frac{d\theta_m}{dt} = \omega_{sm} + \frac{d\delta_m}{dt} \Longrightarrow \frac{d\delta_m}{dt} = 0 \quad \text{if}$$

→ the load is constant (no change)
and / or
→ at no-load conditions

$$\frac{d^2\theta_m}{dt^2} = \frac{d^2\delta_m}{dt^2}$$

$$J\frac{d^2\delta_m}{dt^2} = T_a = T_m - T_e \quad N.m$$

Let $\omega_m = \frac{d\theta_m}{dt}$

$$J\omega_m \frac{d^2\delta_m}{dt^2} = \omega_m T_a = \omega_m T_m - \omega_m T_e$$

$\Rightarrow P = T.\omega$

$$J\omega_m \frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e$$

where,

P_a : accelerating power.

P_m : mechanical power developed by the prime mover.

P_e : electrical power output from the generator (air gap electrical power).

* Assumption :- $I^2 . R_A = 0$ $\Rightarrow$ copper losses in the generator (and in transformer $P_e = P_{out}$)

* $J.\omega_m$ is the angular momentum of the rotor denoted by **M** called "inertia constant".

$$M\frac{d^2\delta_m}{dt^2} = P_a = P_m - P_e$$

* In machine data, another constant referred to inertia is often encountered called "H" constant and defined as :-

$$H = \frac{\text{stored kinetic energy in MJ at } \omega_s}{MVA_{rating}}$$

$$H = \frac{\frac{1}{2} J \omega_{sm}^2}{S_{mach.}} = \frac{\frac{1}{2} M \omega_{sm}}{S_{mach.}} \quad MJ/MVA$$

$$M = \frac{2H}{\omega_{sm}} S_{mach.} \quad MJ/mech. rad$$

$$\frac{2H}{\omega_{sm}} \frac{d^2\delta_m}{dt^2} = \frac{P_a}{S_{mach.}} = \frac{P_m - P_e}{S_{mach.}}$$

$$\frac{2H}{\omega_s} \cdot \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \ pu$$ ⇒ In per unit } swing equation

ω_s could be mechanical or electrical per second

* ω_s & δ must be consistent. (both electrical or both mechanical)

$$\frac{H}{\pi f} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad pu$$

when δ is in electrical radians (elec. rad)

or

$$\frac{H}{180 f} \frac{d^2\delta}{dt^2} = P_a = P_m - P_e \quad pu$$

when δ is in electrical degrees (elec. deg)

} 2nd order non-linear D.E

* note :-

معنى كلمة consistent ⇐ إذا كانت δ بال (degrees) بنستخدم بالقانون 180

اما إذا كانت δ بال (radians) بنستخدم بالقانون π

* the swing equation is a 2nd order non-linear differential equation it can be written in a state-space representation.

بمعنى آخر ⇒ (to solve swing equation it must be represented in state-Space).

$$\frac{2H}{\omega_s}\frac{d\omega}{dt} = P_m - P_e \quad pu$$

$$\frac{d\delta}{dt} = \omega - \omega_s$$

let $X_1 = \omega$

$X_2 = \delta$

$$\frac{2H}{\omega_s}\dot{X_1} = P_m - P_e$$

$$\dot{X_2} = X_1 - \omega_s$$

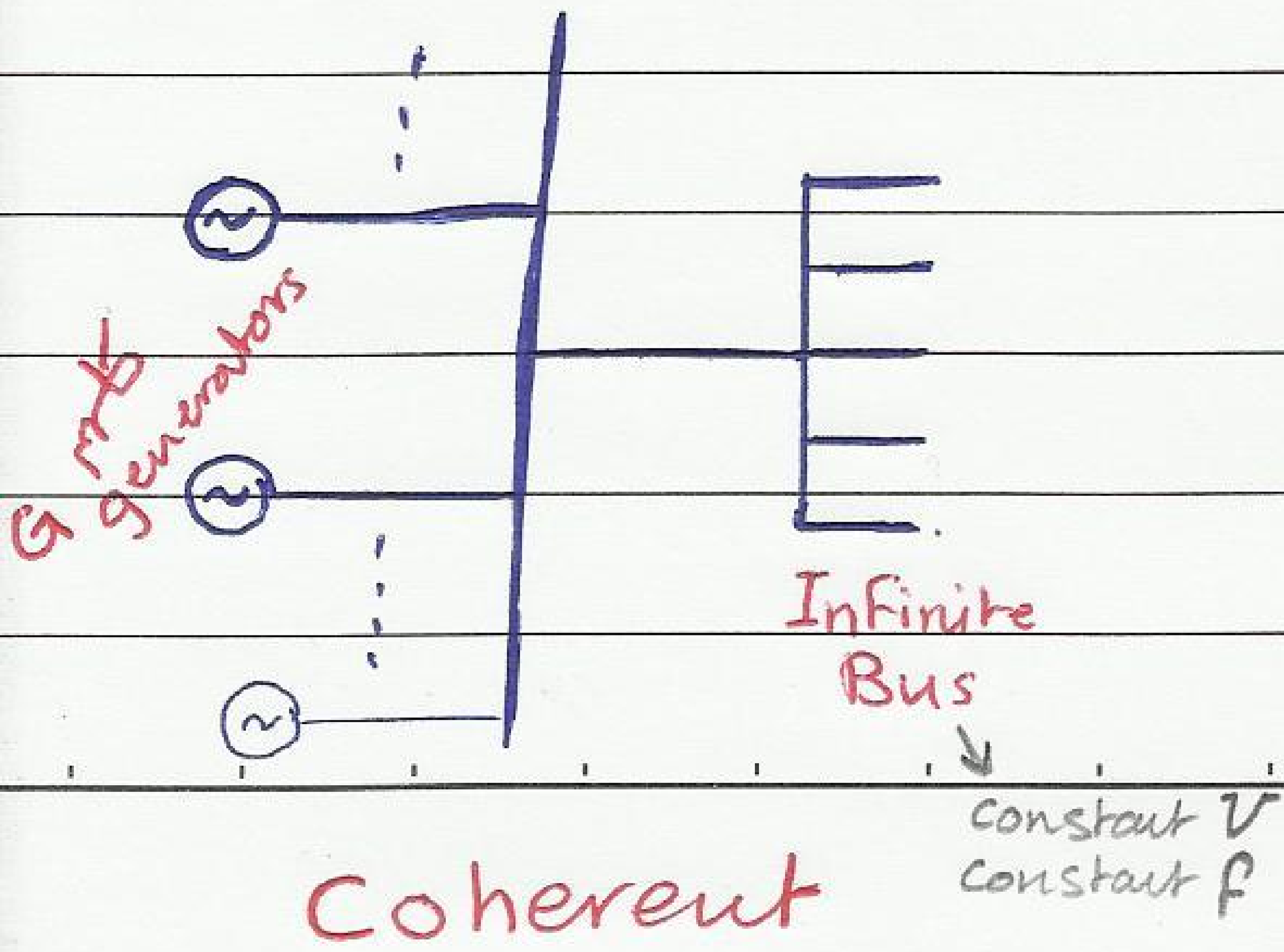
* 16.3 :- Further considerations of the swing equation :-

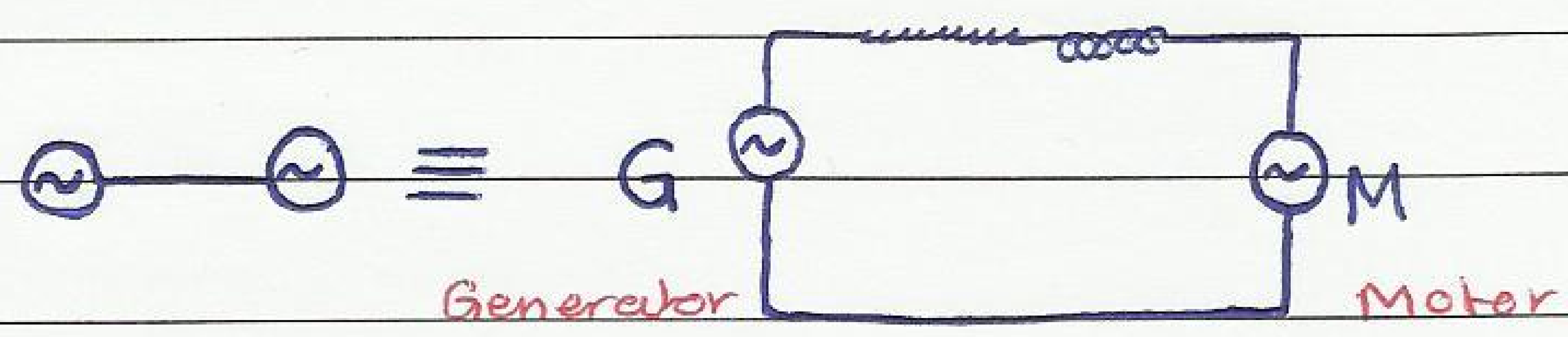
- In stability study of more than one machine, only one MVA base common to all parts of the system are chosen.

بمعنى آخر ⇒ In stability studies which many synchronous generator, only one MVA base is chosen to connect H from one machine base to another.

* To convert H from one machine base to system base :-

$$H_{system} = H_{mach.}\frac{S_{mach.}}{S_{system}}$$





Coherent

non-coherent

* In stability study it's desired to minimize the number of swing equation, in such cases the machine within the plantes (power station) are compined into a single equivalent one just as if the rotor are mechanically coupled and only one swing equation is written.

* For the one case #1 (coherent system) :-

$$\frac{2H_1}{\omega_s}\frac{d^2\delta_1}{dt^2} = P_{m_1} - P_{e_1} \quad \text{.........} (1)$$

$$\frac{2H_2}{\omega_s}\frac{d^2\delta_2}{dt^2} = P_{m_2} - P_{e_2} \quad \text{.........} (2)$$

combining the two generators togother (1) + (2) =

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e \qquad pu$$

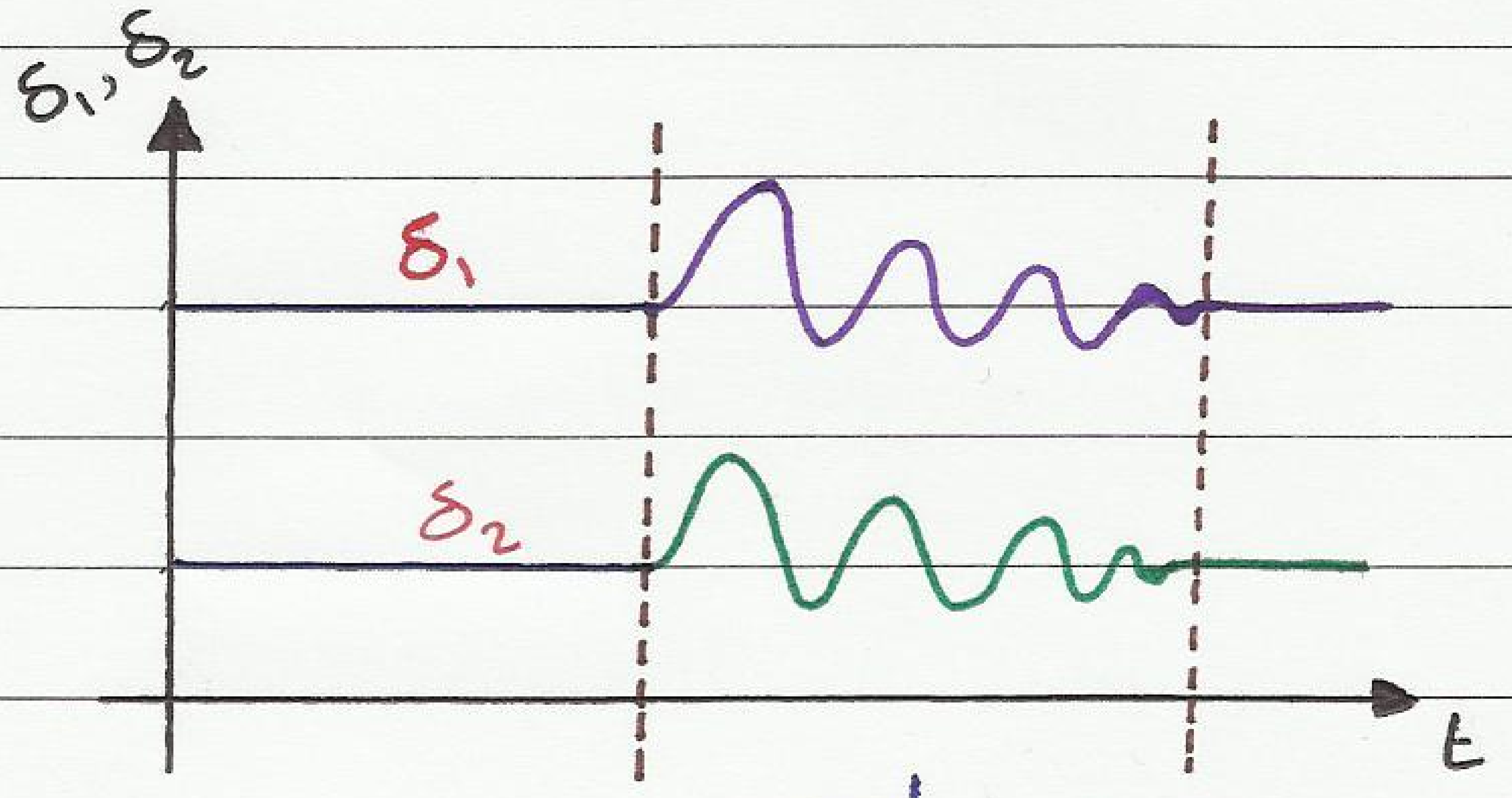
where,

$H = H_1 + H_2$

$P_m = P_{m_1} + P_{m_2}$

$P_e = P_{e_1} + P_{e_2}$

$\delta = \delta_1 = \delta_2 \Rightarrow$ Synchronized

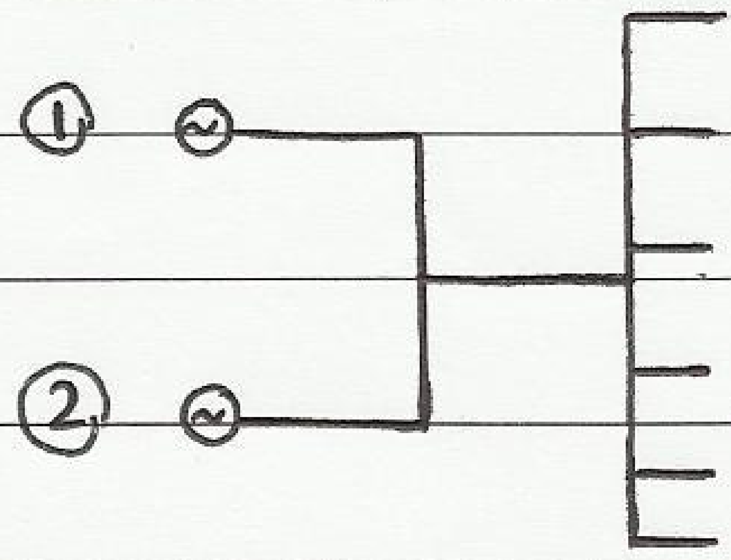


نفس الشكل لأنهم موصولين على نفس Bus ، وبالتالي

حيكون الهم نفس الموجة التذبذبية (same oscillation).

* H proportional with moment of inertia.

Ex:- two 60 Hz generating units operate in parallel within the same power plant and have the following ratings :-



Unit # 1 :- 500 MVA , 0.8 pF , 20 kV , 3600 rpm.

$H_1 = 4.8$ MJ/MVA

Unit # 2 :- 1333 MVA , 0.9 pF , 22 kV , 3600 rpm.

$H_2 = 3.27$ MJ/MVA

Find the $H_{eq.}$ for the two units on 100 MVA base ??

Sol. $KE = (4.8 * 500) + (3.27 * 1333) = 6759$ MJ

$$H_{eq.} = \frac{6759}{100} = 67.59 \text{ MJ/MVA}$$

notes: ① الوحدة الأولى قيمة الفولتية 20kV والوحدة الثانية 22kV
وهذا يرجع لـ الشركة المصنعة وال rated قيمة للgenerator.
② نلاحظ أن السرعة متساوية في الوحدتين
وهذا يدل على أنهما لديها نفس عدد ال poles $\left(p = \frac{120 f}{n}\right)$

* machines which swing together (connected at the same Bus) are called coherent machine. (δ angle Oscillate together).

* For any pair for non-coherent machine, the system swing equation :-

أي أن كل mach ، تعمل oscillate with respect to gother.

$$\frac{d^2\delta_1}{dt^2} - \frac{d^2\delta_2}{dt^2} = \frac{\omega_s}{2}\left[\frac{P_{m1} - P_{e1}}{H_1} - \frac{P_{m2} - P_{e2}}{H_2}\right]$$

$$\frac{2H_{12}}{\omega_s}\frac{d^2\delta_{12}}{dt^2} = P_{m_{12}} - P_{e_{12}}$$

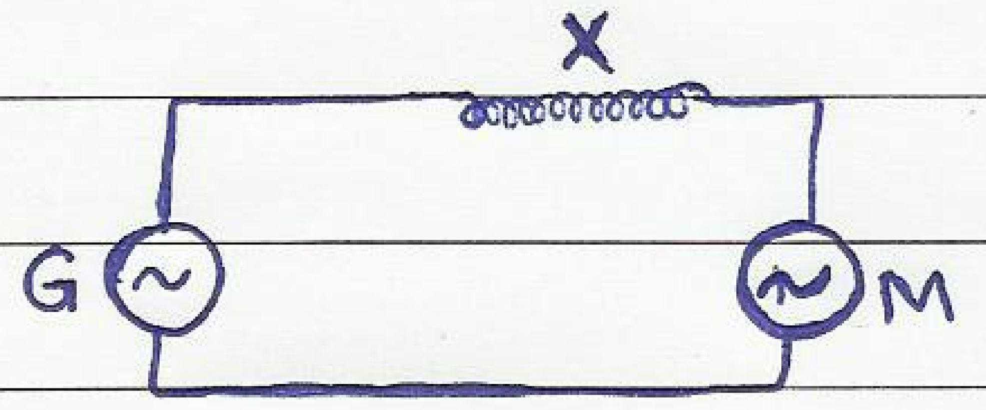
where,

$$H_{12} = \frac{H_1 H_2}{H_1 + H_2}$$

$$P_{m_{12}} = (P_{m_1} H_2 - P_{m_2} H_1) / (H_1 + H_2)$$

$$P_{e_{12}} = (P_{e_1} H_2 - P_{e_2} H_1) / (H_1 + H_2)$$

$$\delta_{12} = \delta_1 - \delta_2$$



non-coherent machine

$$P_{m_1} = -P_{m_2} = P_m$$

$$P_{e_1} = -P_{e_2} = P_e$$

Then,

$$P_{m_{12}} = P_m$$

$$P_{e_{12}} = P_e$$

لأنه أي power تنتج من الـ generator
لازم يصير لها consumed من الـ Motor

* the previous discussion shows the relative Nature of the system stability :-
one machine swinging with respect to ① an infinite Bus, and two finite ② inertia swinging with respect to each other.
⇓
non-coherent system case

infinite Bus = Infinite inertia *

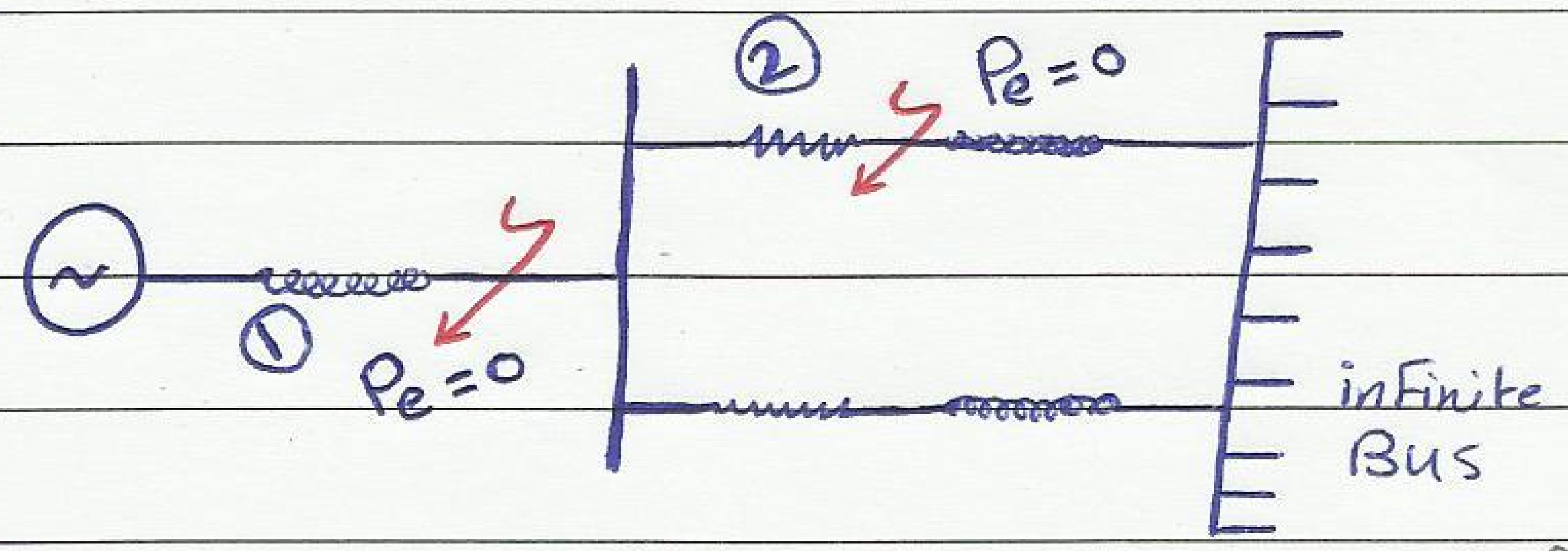
* $H \uparrow \Rightarrow$ more stable.

* An Infinite Bus may be considered for stability purposes as a bus at which there is a machine of constant voltage having zero impedance and infinite inertia.

* 16.4 :- The power angle equation :-

- In swing equation P_m is considered constant. This is a reasonable assumption because conditions in electrical network change before the control governor can cause the Turbine To react. changes in P_e are determined by conditions on the transmission and distribution networks and the load in the system.
- Since P_m is constant, P_e determines whether the rotor accelerate or decelerate or remain at synchronous speed.

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e$$

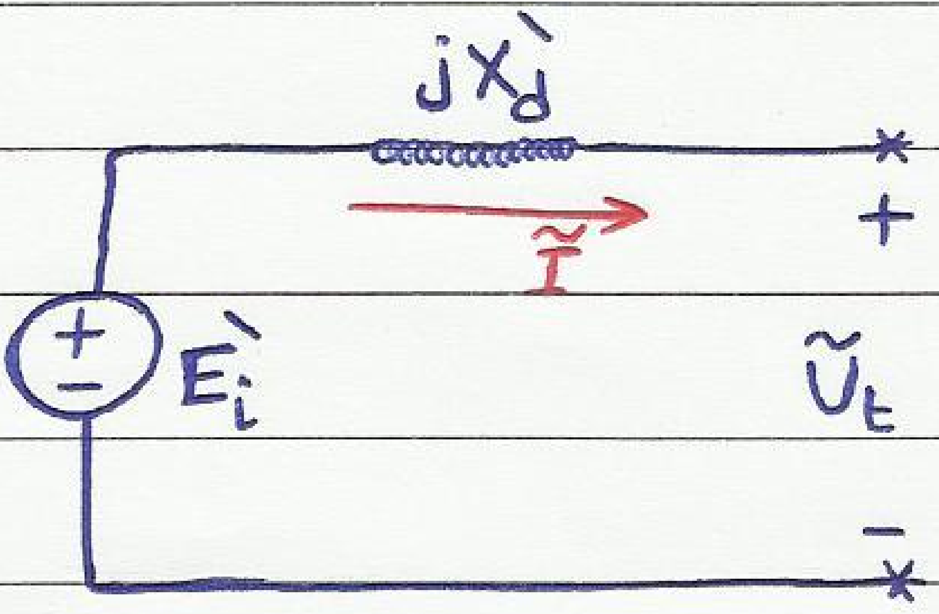


$$\hookrightarrow \frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m \Longrightarrow \boxed{\delta = \alpha_1 t^2 + \alpha_2 t + \alpha_3}$$

$\delta > 90 \Rightarrow$ unstable system

* For transient stability studies the equivalent generator is :

- Each synchronous machine is represented by its transient internal voltage E_i' in series with the transient reactance X_d'.



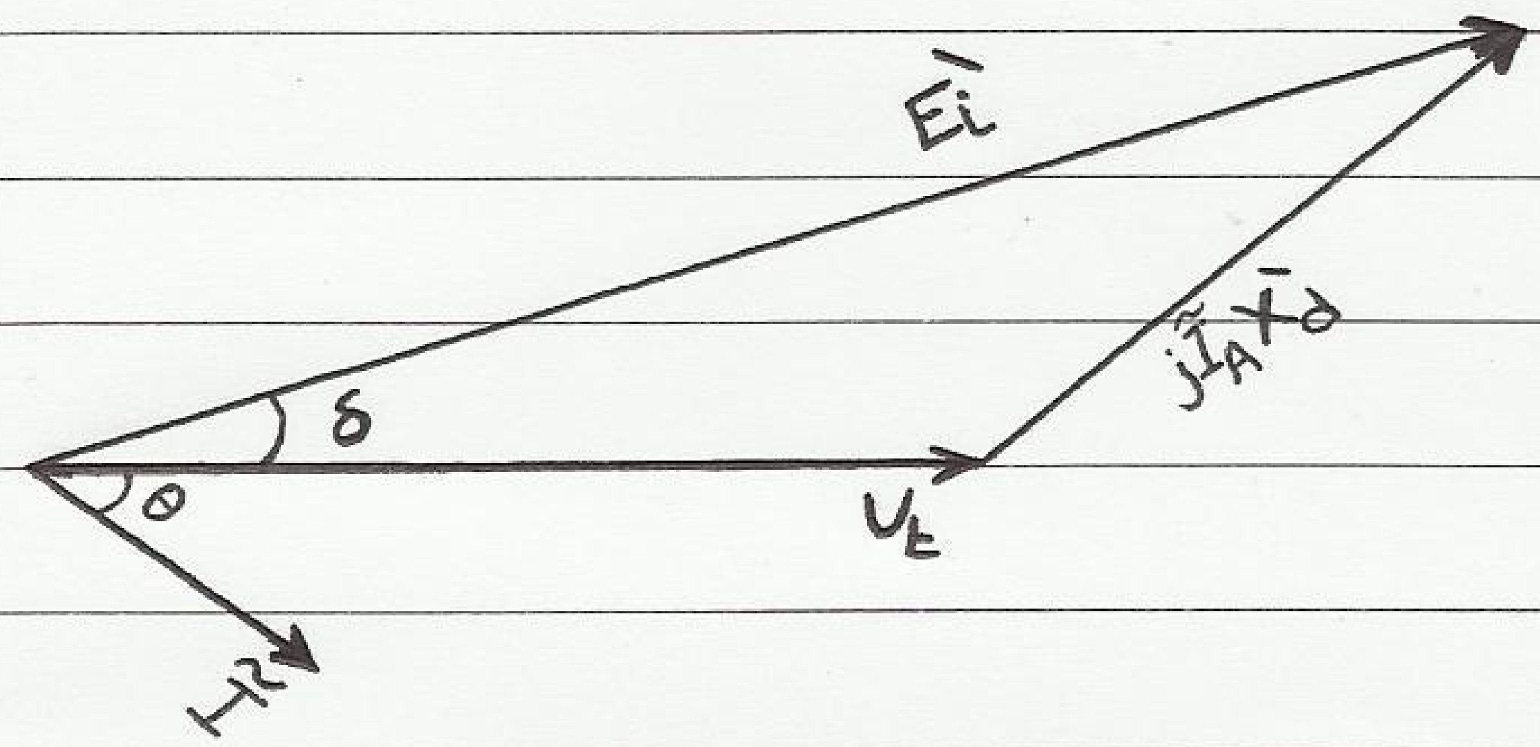
X_d' : transient reactance.

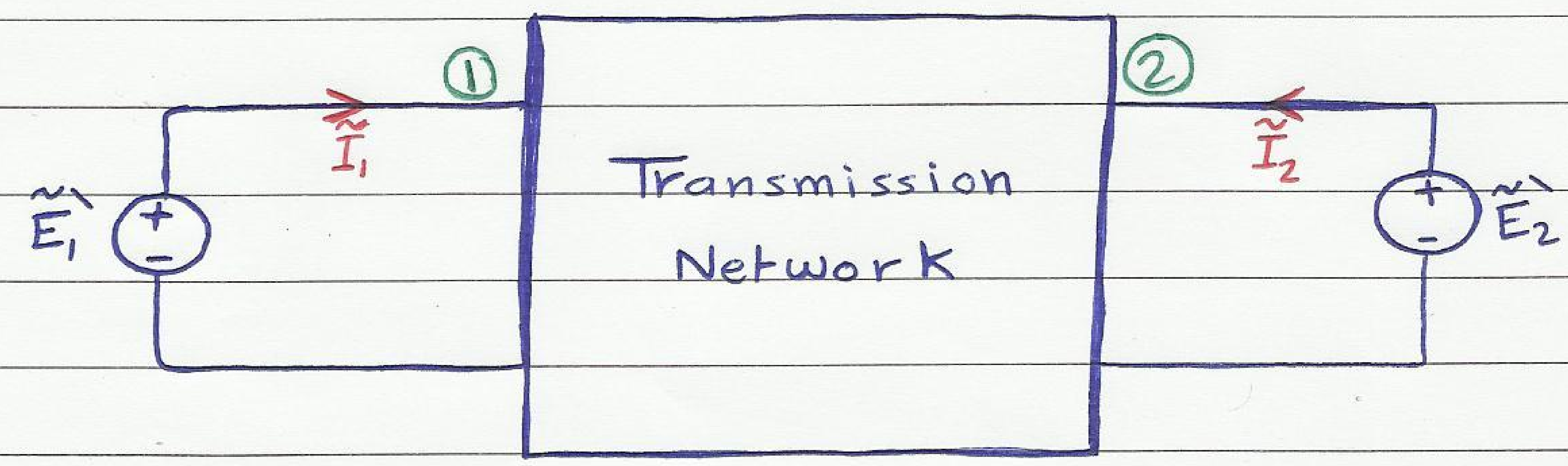
E_i' : transient induced voltage.

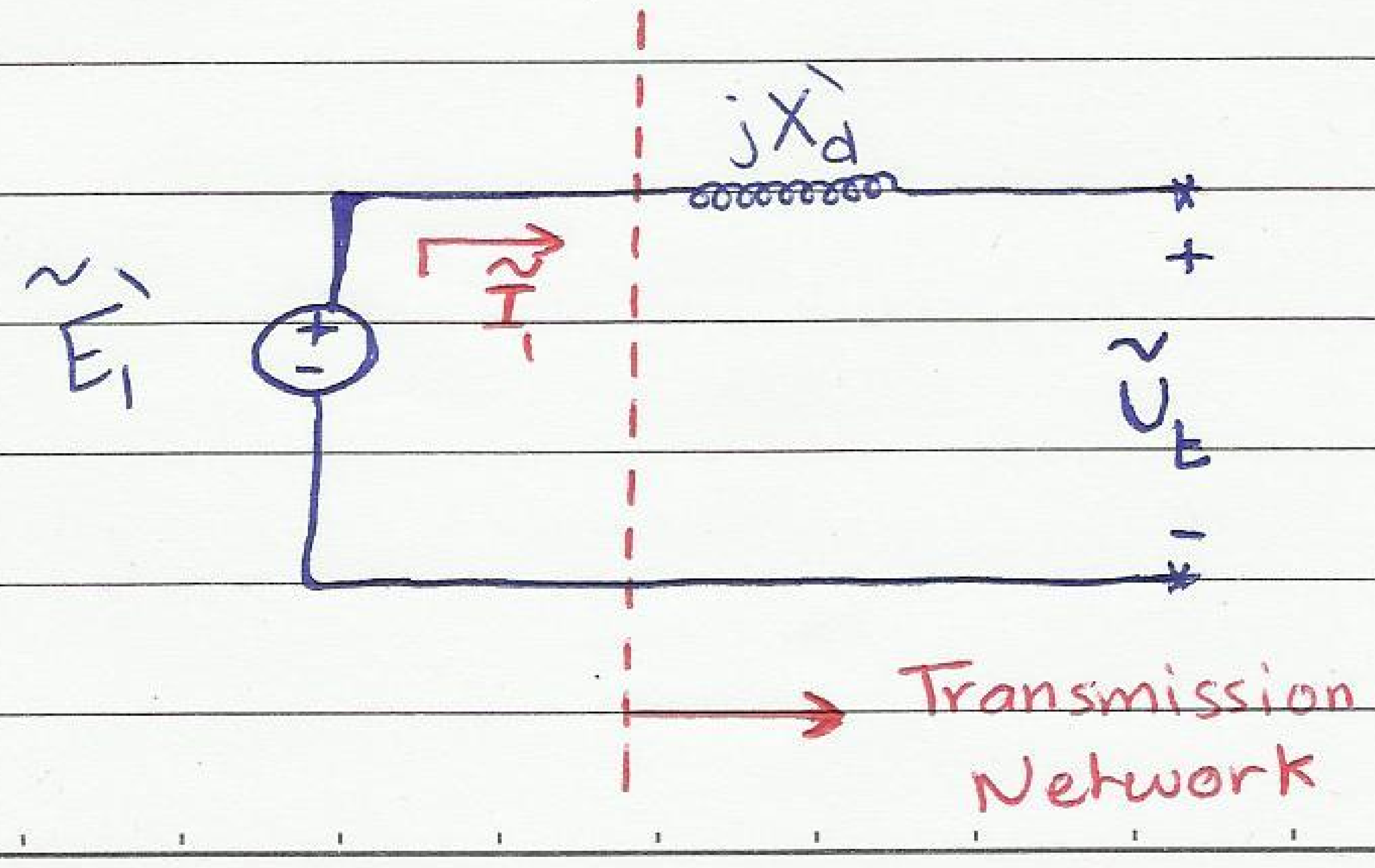
R_A neglected.

* phasor diagram :-

$$\boxed{P_e = \frac{3\, E_i' \, U_t}{X_d'} \sin\delta}$$







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$$\Rightarrow \quad \tilde{Y}_{bus} = \begin{bmatrix} \tilde{Y}_{11} & \tilde{Y}_{12} \\ \tilde{Y}_{21} & \tilde{Y}_{22} \end{bmatrix} \Rightarrow$$ Bus Admittance Matrix

ch. 9 $$P_k + jQ_k = \tilde{V}_k \sum_{n=1}^{N} (\tilde{Y}_{kn} V_n)^* \Rightarrow$$ power at bus "k".

N: no. of buses

let $k=1$ & $N=2$

$$P_1 + jQ_1 = \tilde{V}_1 (\tilde{Y}_{11} \tilde{V}_1)^* + \tilde{V}_1 (\tilde{Y}_{12} \tilde{V}_2)^*$$

$$P_1 + jQ_1 = \tilde{E}_1' (\tilde{Y}_{11} \tilde{E}_1')^* + \tilde{E}_1' (\tilde{Y}_{12} \tilde{E}_2')^*$$

let $\tilde{E}_1' = |\tilde{E}_1'| \angle \delta_1$

$\tilde{E}_2' = |\tilde{E}_2'| \angle \delta_2$

$\tilde{Y}_{11} = |\tilde{Y}_{11}| \angle \theta_{11} = G_{11} + jB_{11}$

$\tilde{Y}_{12} = |\tilde{Y}_{12}| \angle \theta_{12}$

$$P_1 + jQ_1 = |\tilde{E}_1'| \angle \delta_1 \left(|\tilde{Y}_{11}| \angle \theta_{11} * |\tilde{E}_1'| \angle \delta_1\right)^* + |\tilde{E}_1'| \angle \delta_1 \left(|\tilde{Y}_{12}| \angle \theta_{12} * |\tilde{E}_2'| \angle \delta_2\right)^*$$

$\hookrightarrow$ But $|\tilde{Y}_{11}| \angle \theta_{11} * |\tilde{E}_1'| \angle \delta_1 = |\tilde{Y}_{11}||\tilde{E}_1'| \angle \theta_{11} + \delta_1$

and $|\tilde{Y}_{12}| \angle \theta_{12} * |\tilde{E}_2'| \angle \delta_2 = |\tilde{Y}_{12}||\tilde{E}_2'| \angle \theta_{12} + \delta_2$

$$\Rightarrow P_1 = |\tilde{E}_1'||\tilde{Y}_{11}||\tilde{E}_1'| \cos(-\theta_{11} - \delta_1 + \delta_1) + |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}| \cos(\delta_1 - \delta_2 - \theta_{12})$$

$$P_1 = |\tilde{E}_1'|^2 G_{11} + |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}| \cos(\delta_1 - \delta_2 - \theta_{12})$$

$$Q_1 = -|\tilde{E}_1'|^2 B_{11} + |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}| \sin(\delta_1 - \delta_2 - \theta_{12})$$

Let $\delta = \delta_1 - \delta_2$ $\Rightarrow$ non-coherent system

$\gamma = \theta_{12} - \frac{\pi}{2}$

$$P_1 = |\tilde{E}_1'|^2 G_{11} + |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}| \sin(\delta - \gamma)$$

$$Q_1 = -|\tilde{E}_1'|^2 B_{11} + |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}| \cos(\delta - \gamma)$$

* if $\tilde{E}_1$ is generator and $\tilde{E}_2$ is a Motor. then,

$$\boxed{P_e = P_c + P_{max.} \sin(\delta - \gamma)}$$ where :-

$P_c = |\tilde{E}_1'|^2 G_{11}$, $P_{max.} = |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}|$

certain active Power losses
(losses in Resistance)

* Notes :-

(1) The parameters P_c , P_{max} , γ are constants during stability study.
(كلهم constant ما عدا δ).

(2) the P_e equation is non-linearity cause :-

a- sin & cos.

b- Voltage :- الفولتية ما تضيف "non-linearity term" لأنه E_A اعتبرناها constant

(3) If the network is considered without resistances, all of the elements of the (Y) bus are susceptances and (G_{11}) will be zero and (γ) will be zero and the power angle equation will be $P_e = P_{max} \cdot \sin\delta$

$$P_{max} = \frac{|\tilde{E}_1| \cdot |\tilde{E}_2|}{X_{12}} \Rightarrow P_{max} = |\tilde{E}_1||\tilde{E}_2| \cdot Y_{12}$$

$$\hookrightarrow B_{12} = \frac{1}{X_{12}}$$

(4) $|\tilde{E}_i|$ are considered constants before, during and after the Faults.

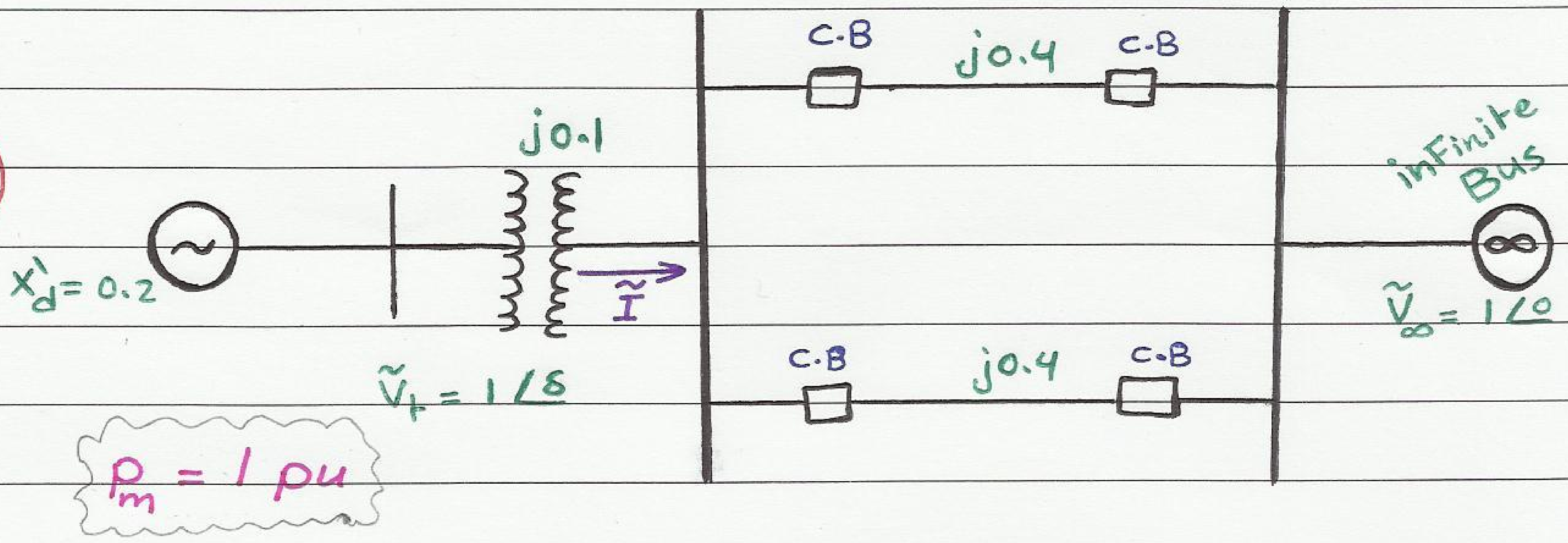
(5) γ will be zero since $\gamma = \theta_{12} - \frac{\pi}{2}$

$Y_{12} = |Y_{12}| \angle\theta_{12} = G_{12} + jB_{12} \Rightarrow \theta_{12} = \tan^{-1}\left(\frac{B}{G}\right)$ if there is no resistance then $\boxed{G_{12} = 0} \Rightarrow \theta_{12} = \tan^{-1}\left(\frac{B}{0}\right) = \tan^{-1}(\infty) = 90^\circ$

$\gamma = \theta_{12} - \frac{\pi}{2} = 90^\circ - 90^\circ = 0$

Ex:-

كل الـ system عبارة عن Per Unit



* Determine the power-angle equation for the given system?

Sol.:

ملاحظات هامة قبل الحل

1) من الملاحظ أنه لا يوجد فرق في القيمة القولية magnitude لـ $(\tilde{V}_\infty , \tilde{V}_t)$ ومع ذلك فإنه يوجد تيار للـ sys. $(\tilde{I})$ ؟! ذلك يرجع إلى أنه هناك فرق الـ phase بينهما مما يسمح بنشوء تيار ، ويجب أن يكون هذا الفرق موجباً وفي هذا المثال يجب أن تكون (δ) + موجبة لأن قيمة الـ phase في $\tilde{V}_\infty$ = صفر ، أما إذا كان الفرق (−) فيصبح التيار بالاتجاه المعاكس $\overleftarrow{\tilde{I}}$

2] at steady state in time domain $\Rightarrow \frac{d}{dt} = 0$ (no variation in time).
and the steady state in frequency domain $S = 0$.
that's mean at S.S $P_m = P_e = 1.0$ pu .

3] $X_{t\infty} = 0.1 + (0.4 // 0.4) = 0.1 + 0.2 = 0.3$ pu .

$X_{tot} = X_{t\infty} + X_d' = 0.3 + 0.2 = 0.5$ pu .

اذا :

$$P_e = |\tilde{E}_1'|^2 G_{11}^2 + |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}| \sin(\delta - \gamma)$$

For lossless lines $\Rightarrow R_{line} + R_{tran.} = 0$, that's mean
$G_{11} = 0$, $\gamma = 0$

$$P_e = |\tilde{E}_1'||\tilde{E}_2'||Y_{12}| \sin(\delta - \gamma)$$

$P_e = P_{max} \sin\delta$, $P_{max} = \frac{|E_1||E_2|}{X_{12}}$

$$P_e = \frac{E_1 E_2}{X_{12}} \sin\delta = \frac{|\tilde{V}_t||\tilde{V}_\infty|}{X_{t\infty}} \sin\delta = \frac{(1)(1)}{0.3} \sin\delta$$

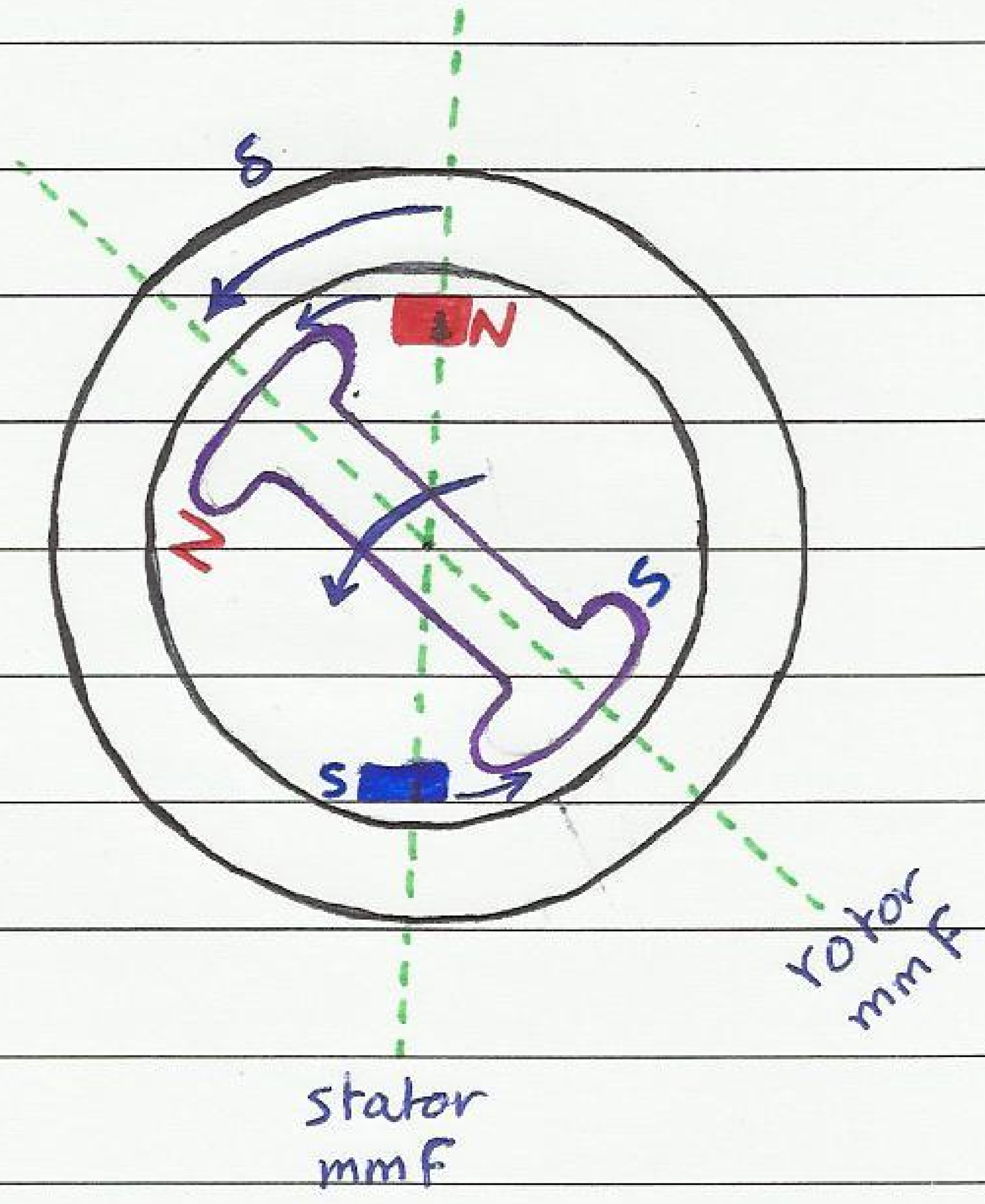
$$P_e = 3.3333 \sin\delta$$

At S.S $P_m = P_e$

$1.0 = 3.3333 \sin\delta$

$\delta = \sin^{-1} \frac{1.0}{3.3333}$

$$\boxed{\delta = 17.5^\circ}$$



$$* \quad \frac{H}{180f}\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\frac{H}{180f}\frac{d^2\delta}{dt^2} = 1 - 3.333 \sin\delta \quad \Rightarrow \text{Swing equation.}$$

$$- \quad \tilde{E}_1' = \tilde{V}_t + j\tilde{I}x$$

$$\tilde{I} = \frac{\tilde{V}_t \angle\delta - \tilde{V}_\infty \angle 0}{jX_{t\infty}} = \frac{1\angle 17.5^\circ - 1\angle 0^\circ}{j0.3} = 1.012 \angle 8.7^\circ \ \text{pu}$$

$$E_1' = \tilde{V}_t + j\tilde{I}X_d' \quad \Rightarrow \text{internal generator voltage (transient voltage)}$$

$$E_1' = 1\angle 17.5^\circ + j(1.012\angle 8.7^\circ)(0.2) = 1.05 \angle 28.4^\circ \ \text{pu}$$

للتأكد فقط

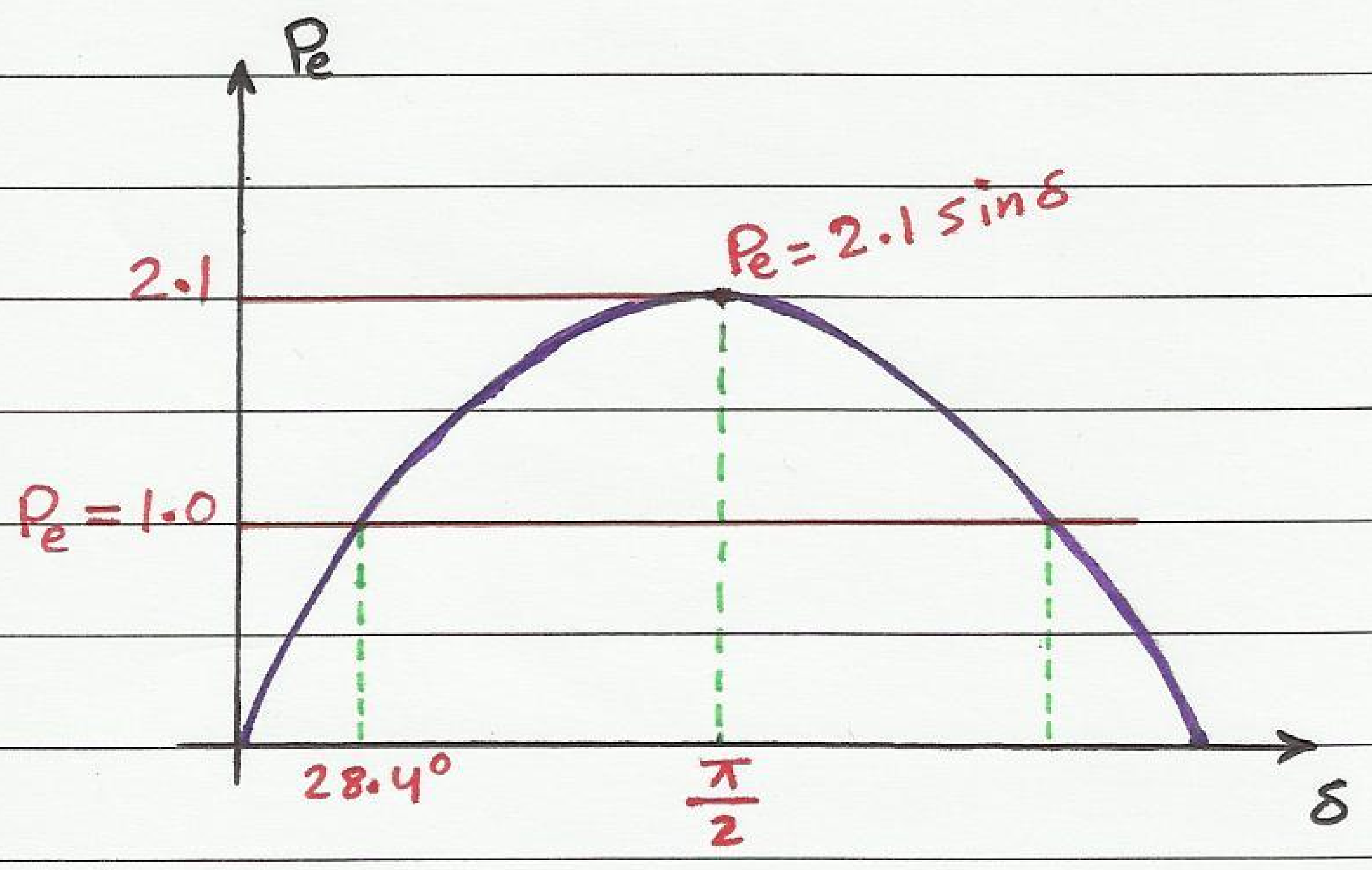
$$\begin{cases} P_e = \dfrac{E' \cdot V_\infty}{X_{tot}} \sin\delta = \dfrac{(1.05)(1)}{0.5} \sin(28.4) \approx 1.0 \\ P_e = \dfrac{E' \cdot V_t}{X_d'} \sin\delta = \dfrac{(1.05)(1)}{0.2} \sin(28.4 - 17.5) \approx 1.0 \end{cases}$$

* the power-angle equation relating the transient voltage E_1' and the infinite Bus (where δ is the machine torque-angle with respect to infinite Bus).

$$P_e = \frac{|\tilde{E}_1'| \cdot |\tilde{V}_\infty|}{X_{tot}} \sin\delta_g$$

$$= \frac{(1.05)(1)}{0.5} \sin\delta$$

$$\boxed{P_e = 2.1 \sin\delta}$$



* the swing equation of the machine :-

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\boxed{\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = 1.0 - 2.1 \sin\delta}$$

or, $$\frac{H}{180f}\frac{d^2\delta}{dt^2} = 1.0 - 2.1 \sin\delta_g$$

تتغير هذه المعادلة إذا تغيرت P_m ،
أو تغيير في الـ configuration of system
يعني إذا صار تغيير في خطوط النقل وحدث
في أي واحد فيها (Fault) تتغير المعادلة ويصير عنا
ثلاث حالات ومراحل لحل السؤال وهي :-
pre → during → post (after)
أي أننا يجب أن نجد ٣ معادلات (3 swing)
قبل الـ Fault وخلاله وبعده . وحساب الـ
initial conditions for each equation
لأن كل معادلة فيها مجهولين δ ، ω مثلاً في معادلة الـ pre
بتكون قيمة δ هي نفسها إلي حسبناها بالسؤال وبتكون initial للمعادلة إلي بعدها (during) لأنه في عنا بينه
مرحلة الـ pre والـ during ما بيسمى بالـ Boundary .

مدة الـ pre بتكون تقريباً
0.1 → 0.3 s

- The steady state value of (δ) is obtained by dropping out the time derivative terms and solving the resulting algabric equation.
- This value of (δ) the initial condition for the system if a Fault happen.

* Bus admittance matrix :-

admittance ← $y_{11} = \frac{1}{j0.2} + \frac{1}{j0.8} + \frac{1}{0.1 + j0.2}$

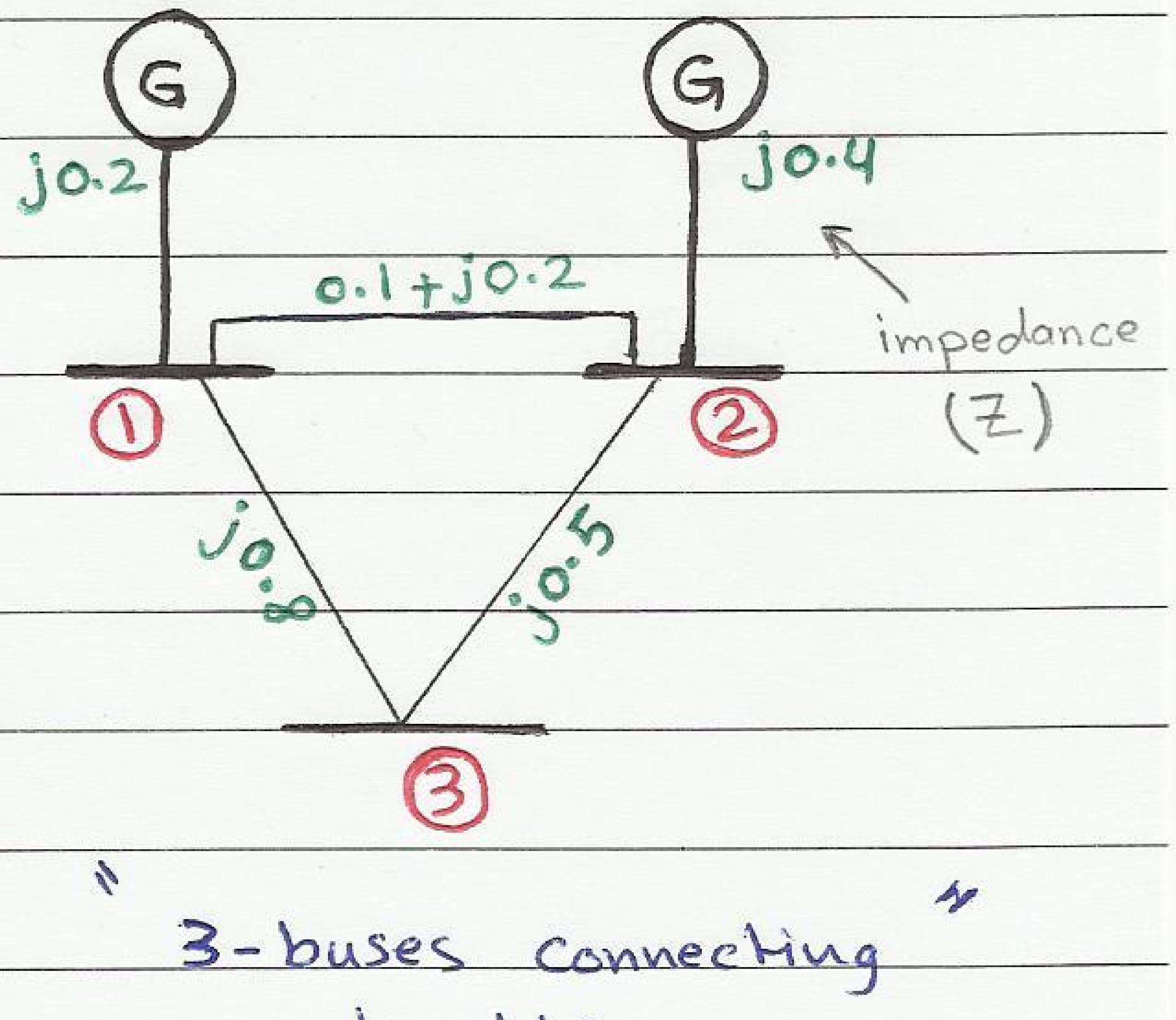
$y_{12} = -\frac{1}{0.1 + j0.2} = y_{21}$

$y_{13} = -\frac{1}{j0.8} = y_{31}$

$y_{22} = \frac{1}{j0.4} + \frac{1}{j0.5} + \frac{1}{0.1 + j0.2}$

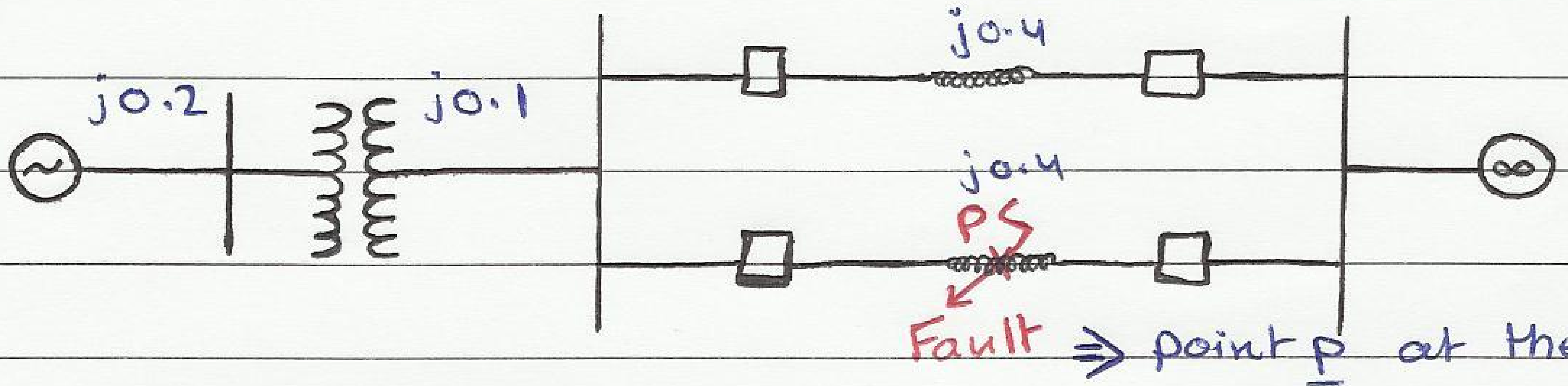
$y_{32} = -\frac{1}{j0.5} = y_{23}$

$y_{33} = \frac{1}{j0.8} + \frac{1}{j0.5}$



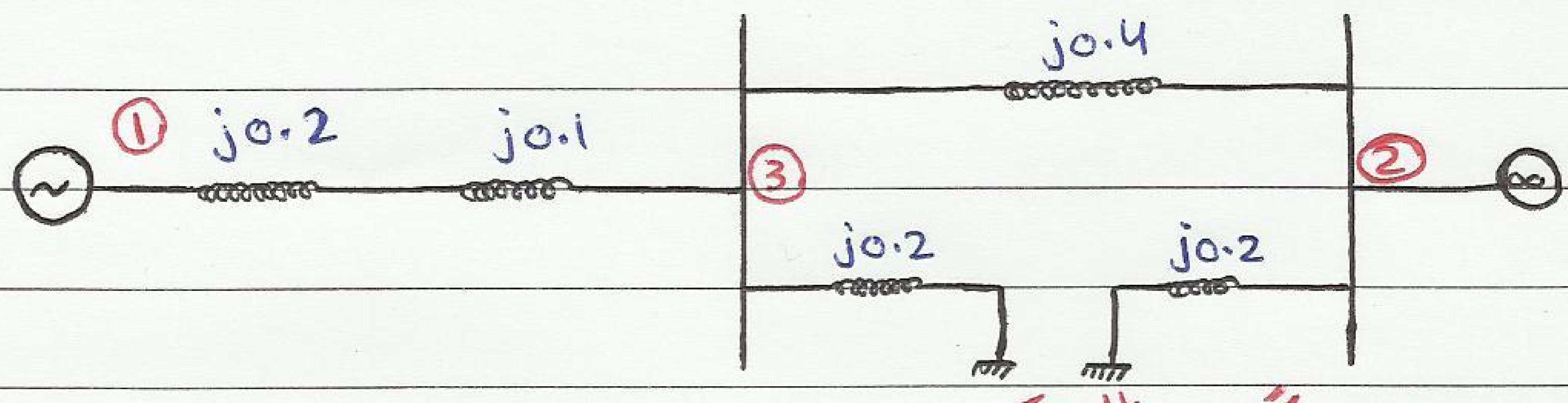
" 3-buses connecting together "

<u>Ex:-</u> For the previous example, a 3-ϕ Fault (symmetrical) at point p of the transmission line occurs as shown in the Following Figures, determine the power angle equation for the system with the Fault (during Fault) and corresponding the swing equation ($H = 5\,MJ/MVA$). ?



⇒ point p at the middle of the second transmission line.

رسم توضيحي يبين خطوات حل السؤال .



"reactance depends on length"

Figure (1)

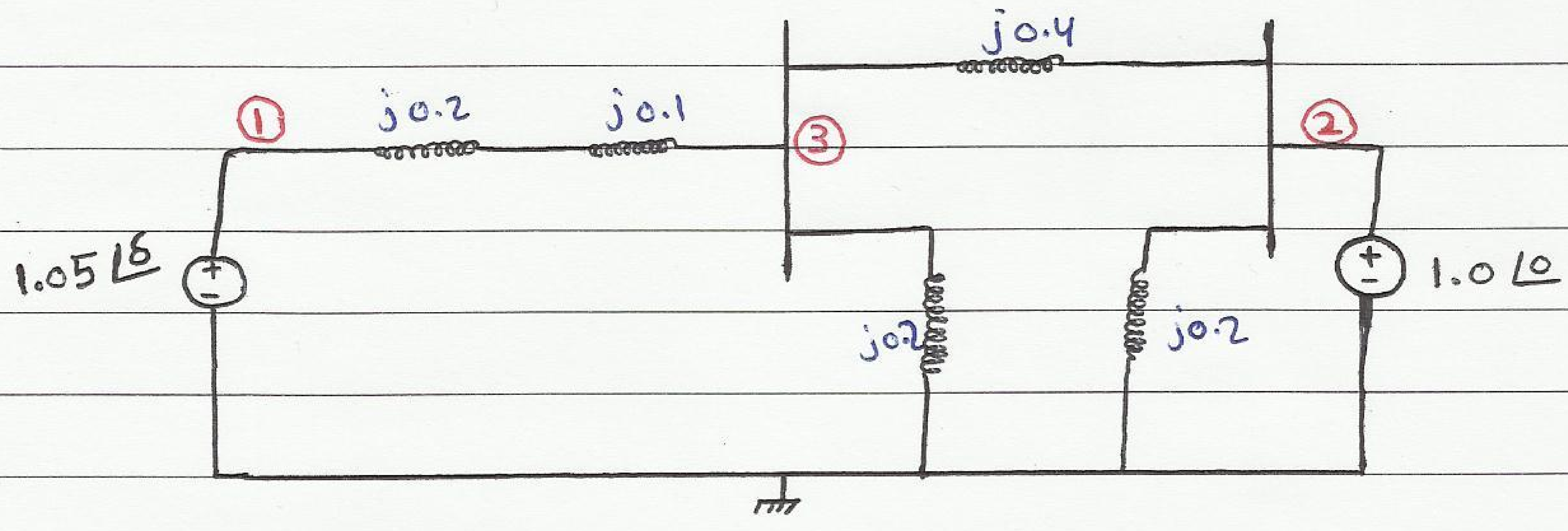


Figure (2)

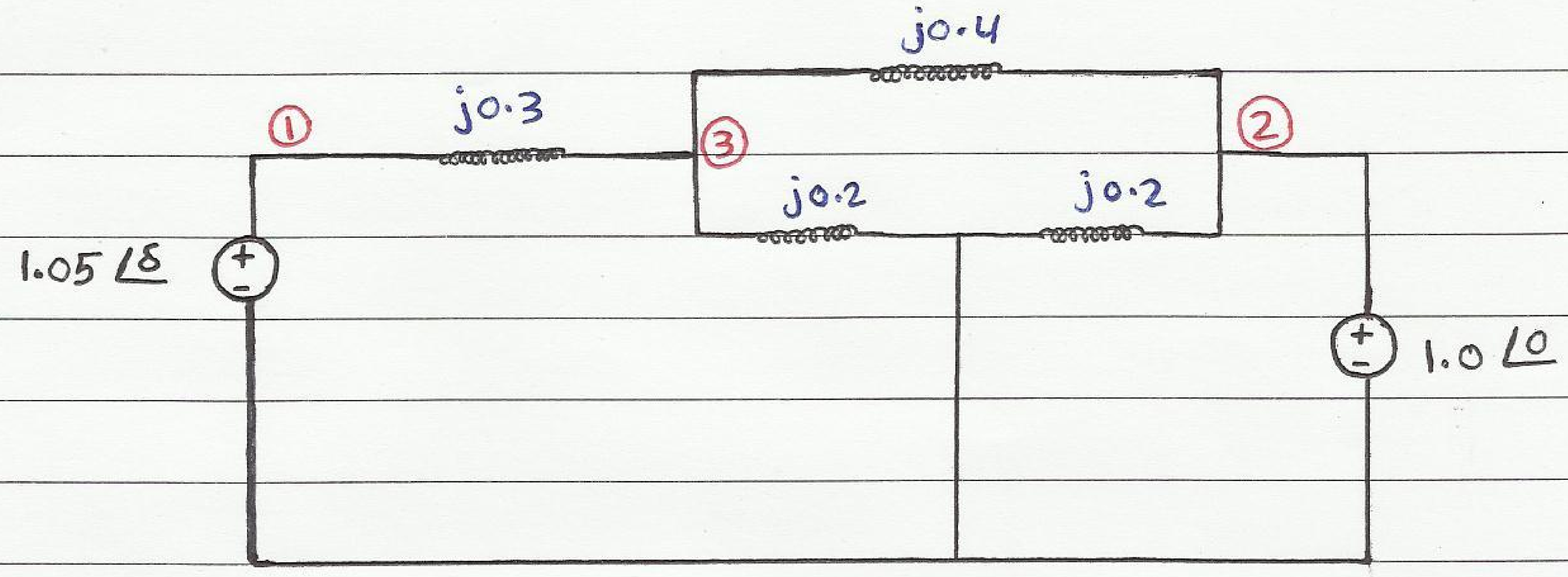


Figure (3)

* نلاحظ أن الرسمة تحتوي على قيم الـ impedance (Z) ، يتم تحويلها الى admittance (Y)

$$Y = \frac{1}{Z}$$

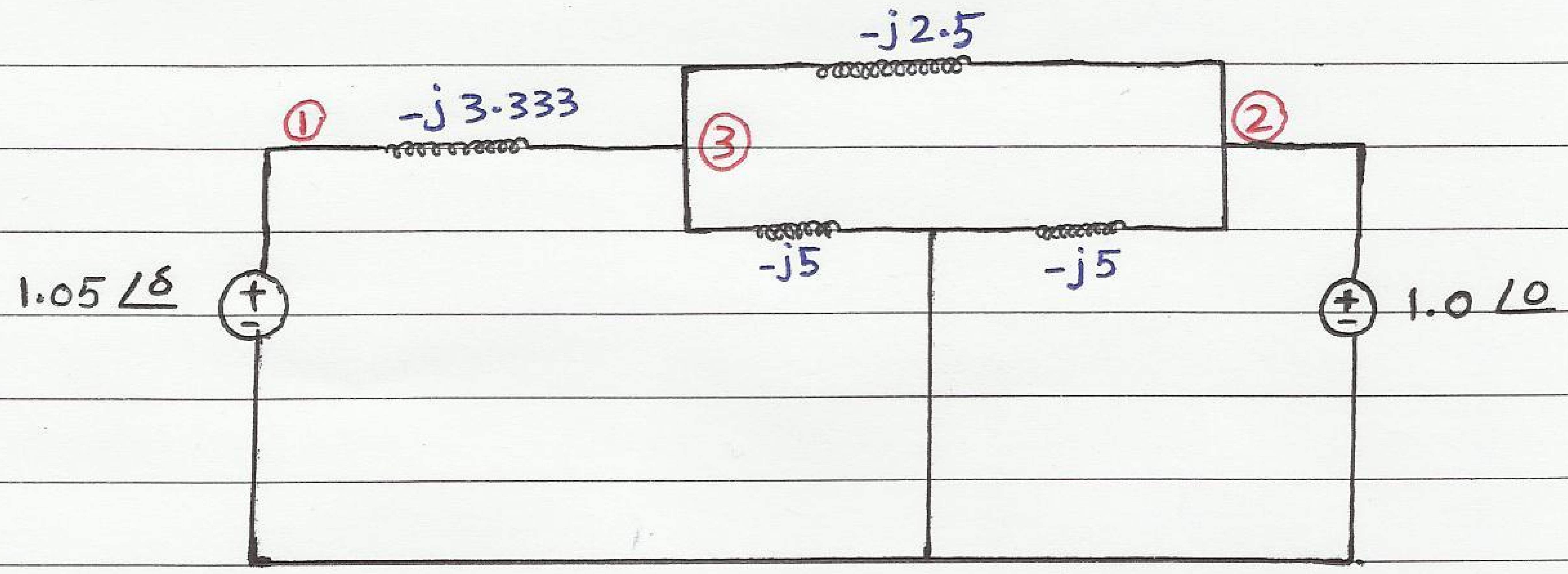


Figure (4)

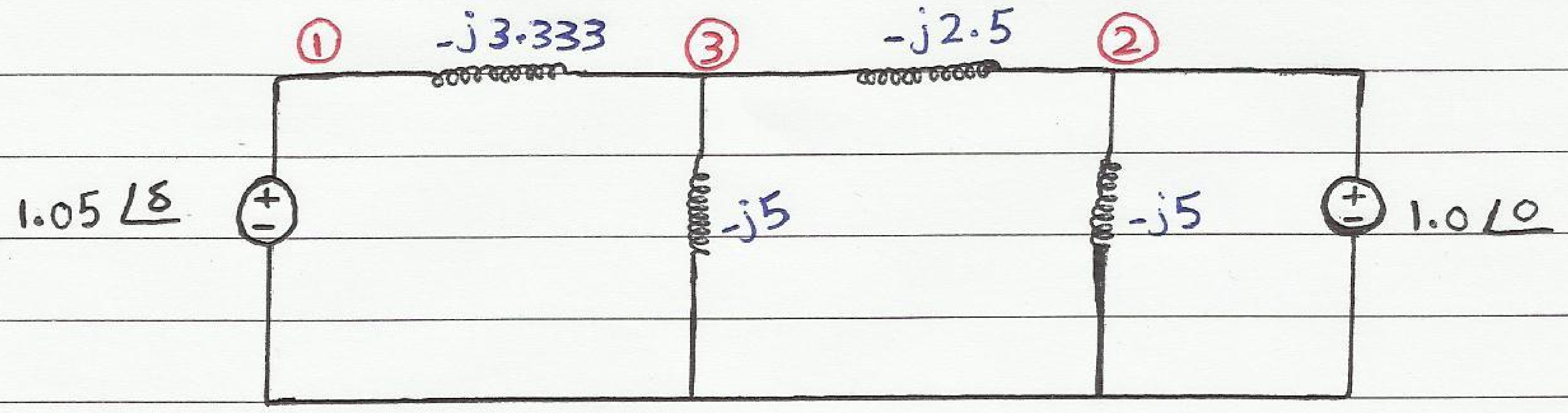


Figure (5)

During Fault equation circuit

$$\tilde{Y}_{bus} = \begin{bmatrix} \tilde{y}_{11} & \tilde{y}_{12} & \tilde{y}_{13} \\ \tilde{y}_{21} & \tilde{y}_{22} & \tilde{y}_{23} \\ \tilde{y}_{31} & \tilde{y}_{32} & \tilde{y}_{33} \end{bmatrix} \Rightarrow y_{bus} \text{ matrix during Fault}$$

$$\tilde{Y}_{bus} = \begin{bmatrix} -j3.333 & 0 & j3.333 \\ 0 & -j7.5 & j2.5 \\ j3.333 & j2.5 & -j10.833 \end{bmatrix}$$

- as Figure (5), bus number 3 must be eliminated by "Kron reduction technique" because it's voltage is Unknown.

$$\boxed{\tilde{Y}_{jk}(new) = \tilde{Y}_{jk}(old) - \frac{\tilde{Y}_{jP}\,\tilde{Y}_{PK}}{\tilde{Y}_{PP}}}$$

P: number of bus to be eliminated (removed).
j, k : the indecies of the new bus matrix.

- This equation is a general way of power system stability study applied at any power system.

$$\tilde{Y}_{11}(new) = \tilde{Y}_{11}(old) - \frac{\tilde{Y}_{13}\,\tilde{Y}_{31}}{\tilde{Y}_{33}} = -j3.333 - \frac{(j3.333)(j3.333)}{-j10.8333} = \underline{\underline{-j2.3077 \text{ pu}}}$$

$$\tilde{Y}_{12}(new) = \tilde{Y}_{12} - \frac{\tilde{Y}_{13}\,\tilde{Y}_{32}}{\tilde{Y}_{33}} = 0 - \frac{(j3.333)(j2.5)}{-j10.8333} = \underline{\underline{j0.7692 \text{ pu}}}$$

$$\tilde{Y}_{21}(new) = \tilde{Y}_{12}(new) = \underline{\underline{j0.7692 \text{ pu}}}$$

$$\tilde{Y}_{22}(new) = -j7.5 - \frac{(j2.5)(j2.5)}{-j10.8333} = \underline{\underline{-j6.9231}}$$

Sub. Date: / /

$$\tilde{Y}_{bus}(\text{reduced}) = \begin{bmatrix} -j2.3077 & j0.7692 \\ j0.7692 & -j6.9231 \end{bmatrix}$$

للتذكير $\tilde{Y}_{11} = G_{11} + jB_{11}$ $\qquad \tilde{Y}_{12} = G_{12} + jB_{12}$

$$P_e = P_c + P_{max} \sin(\delta - \gamma)$$

$$P_e = |\tilde{E}_1'|^2 G_{11} + |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}| \sin(\delta - \gamma) + \cdots \cancel{|Y_{13}|} \cdots + \cdots$$

Zero

$$G_{11} = 0 \Rightarrow P_c = 0 \Rightarrow |\tilde{E}_1'|^2 G_{11} = 0$$

$$\tilde{Y}_{11} = G_{11} + jB_{11} = B_{11} \angle 90^\circ \Rightarrow$$

$\sqrt{B^2+G^2} = |Y_{11}|$

$\theta = \tan^{-1} \frac{B}{G}$

إرجع إلى صفحة ١٨ بالملزمة رقم 5

$\hookrightarrow \gamma = \frac{\pi}{2} - \theta_{12}$

$\gamma = \frac{\pi}{2} - \frac{\pi}{2} = 0$

thus,

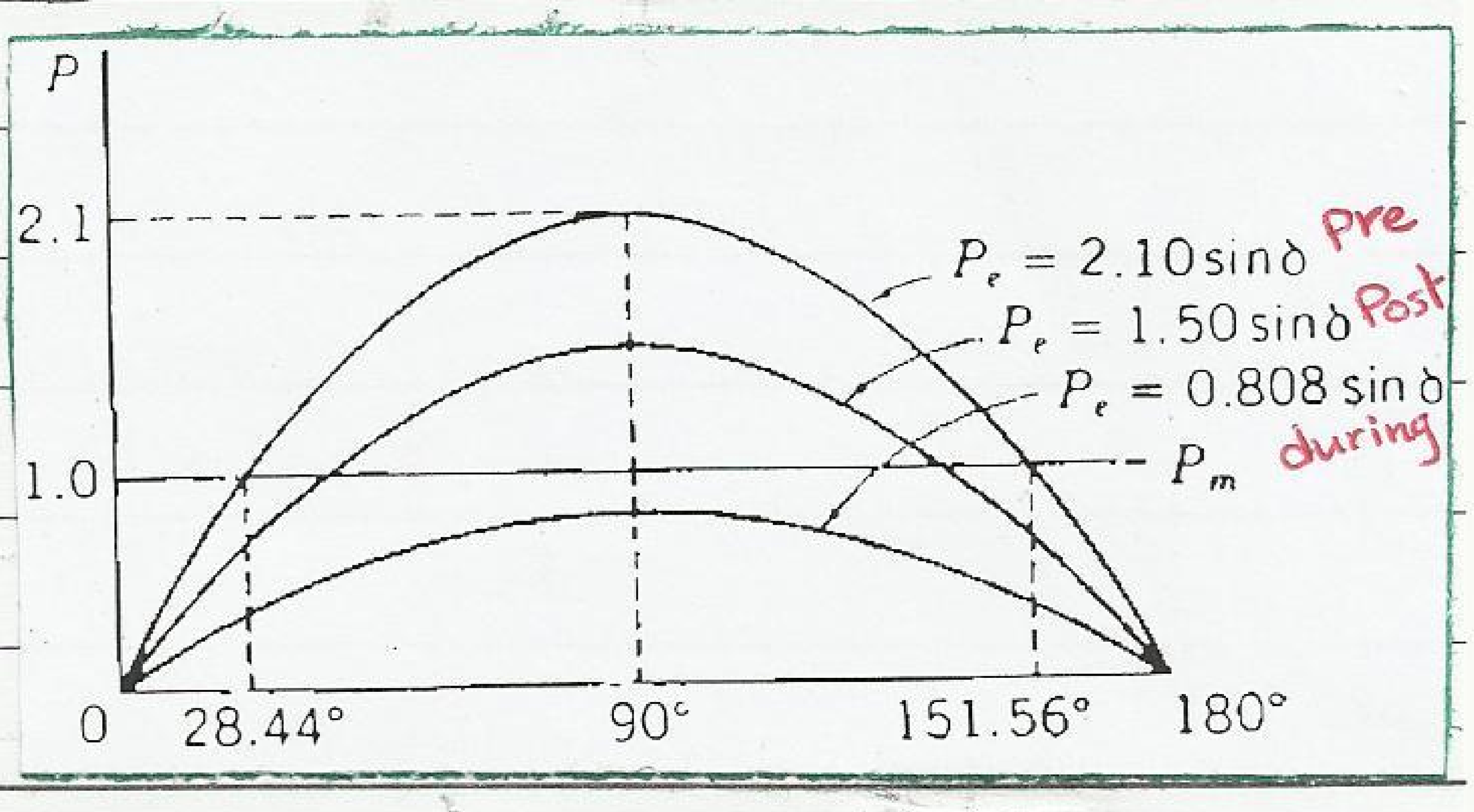
$$P_e = |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}| \sin\delta$$

$$P_e = (1.05)(1)(0.7692) \sin\delta = 0.808 \sin\delta$$

$$\frac{H}{180f} \frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\boxed{\frac{(5)}{(180)(60)} \frac{d^2\delta}{dt^2} = 1.0 - 0.808 \sin\delta}$$

during Fault swing equation



Another Solution :-

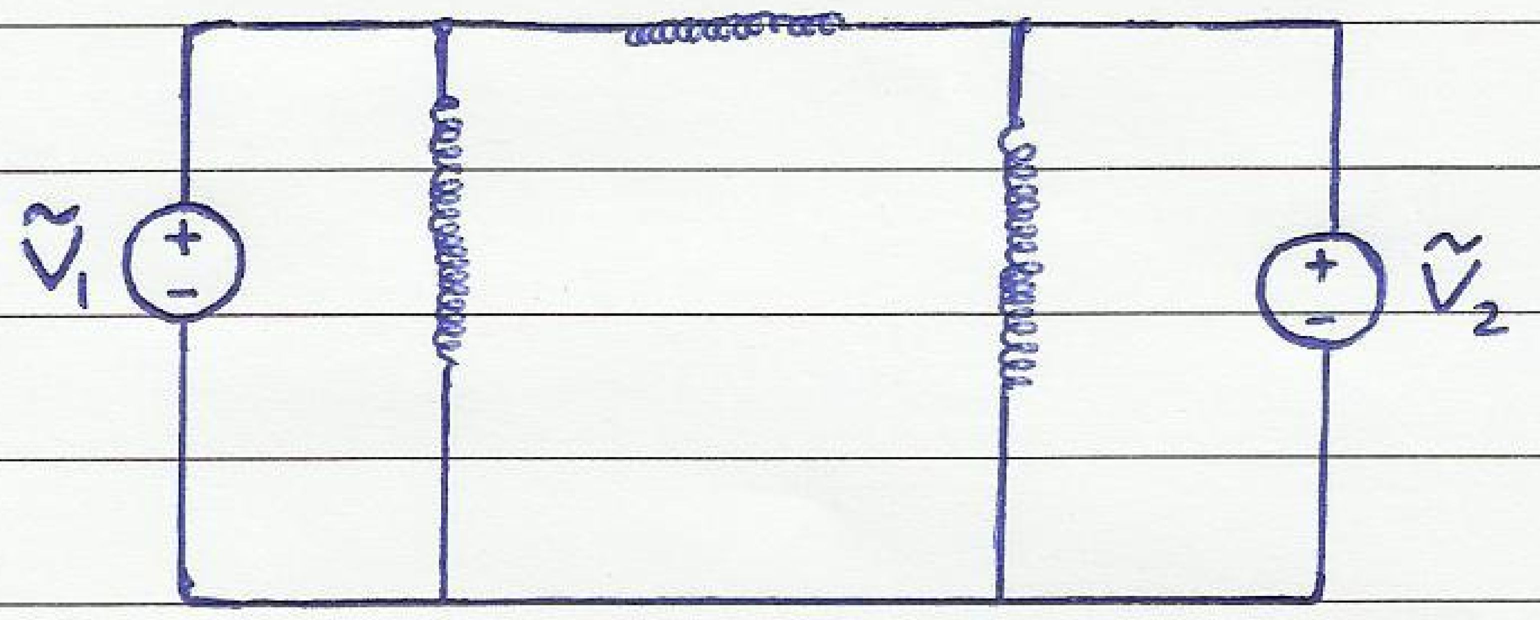
بس مو دايماً بنستخدم هاي الطريقة، بنستخدمها

إذا كان الـ sys شبيه بالـ sys الموجود

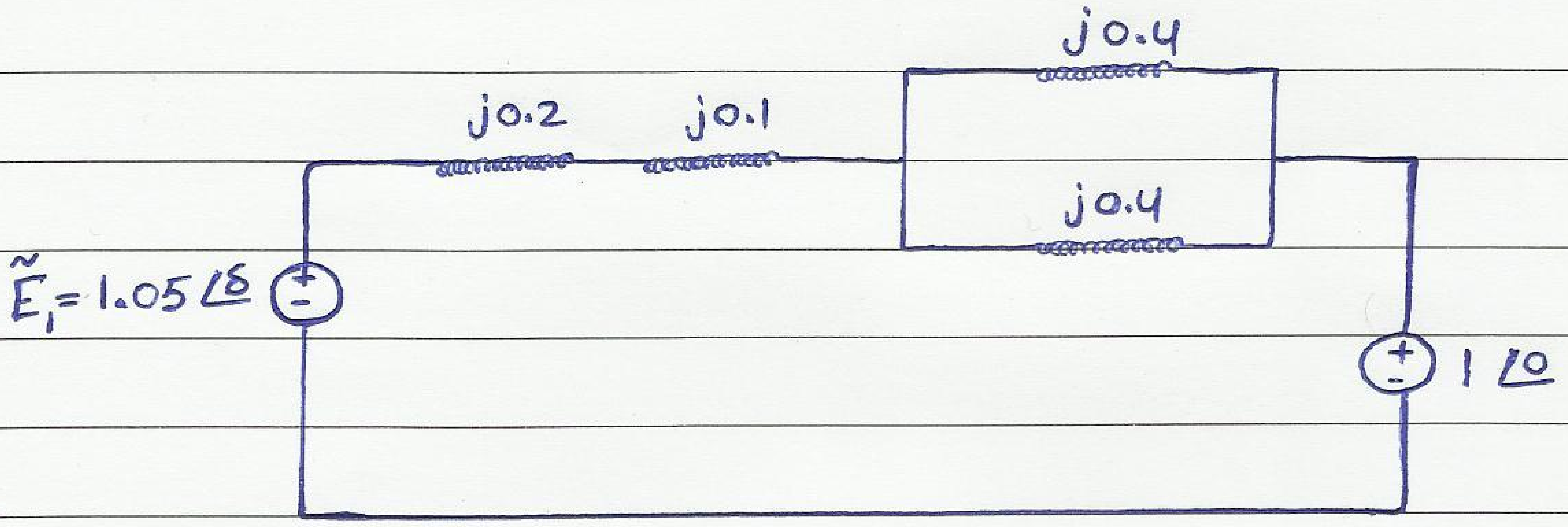
هنا حالياً (special case)

- Simply the circuit to get the following model :-

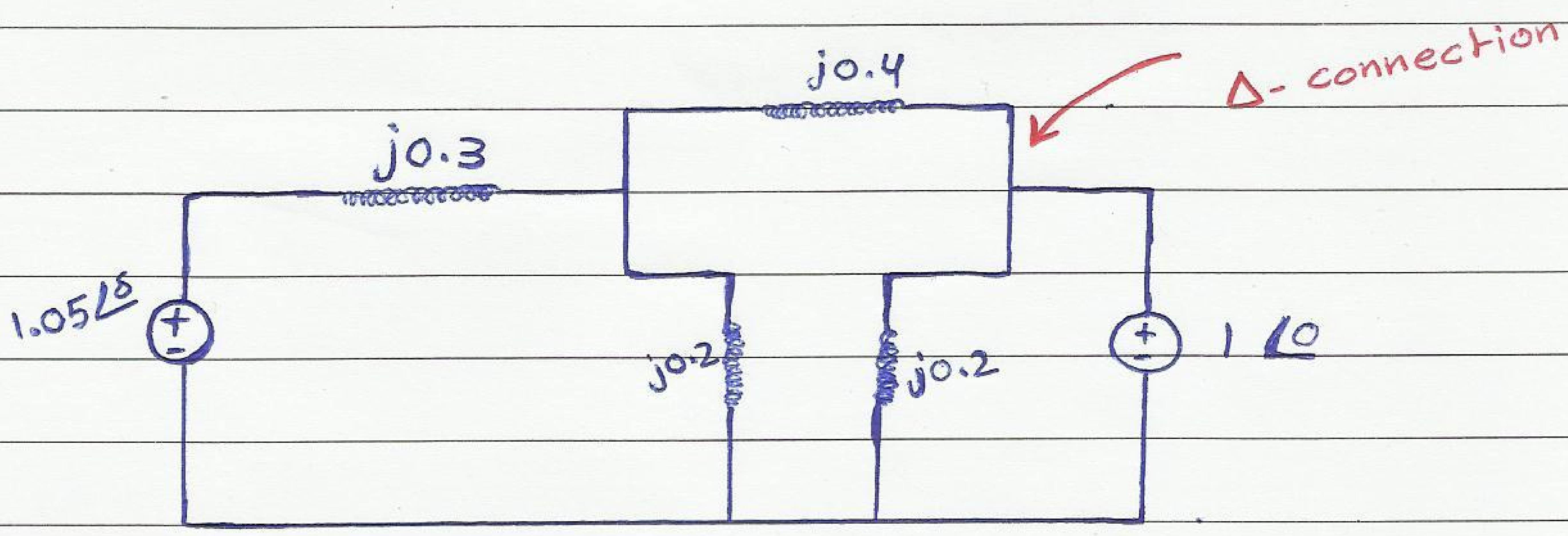
$$P_{12} = \frac{V_1 V_2}{X_{12}} \sin \delta$$



Sol. بالرجوع الى الرسمة الأولى

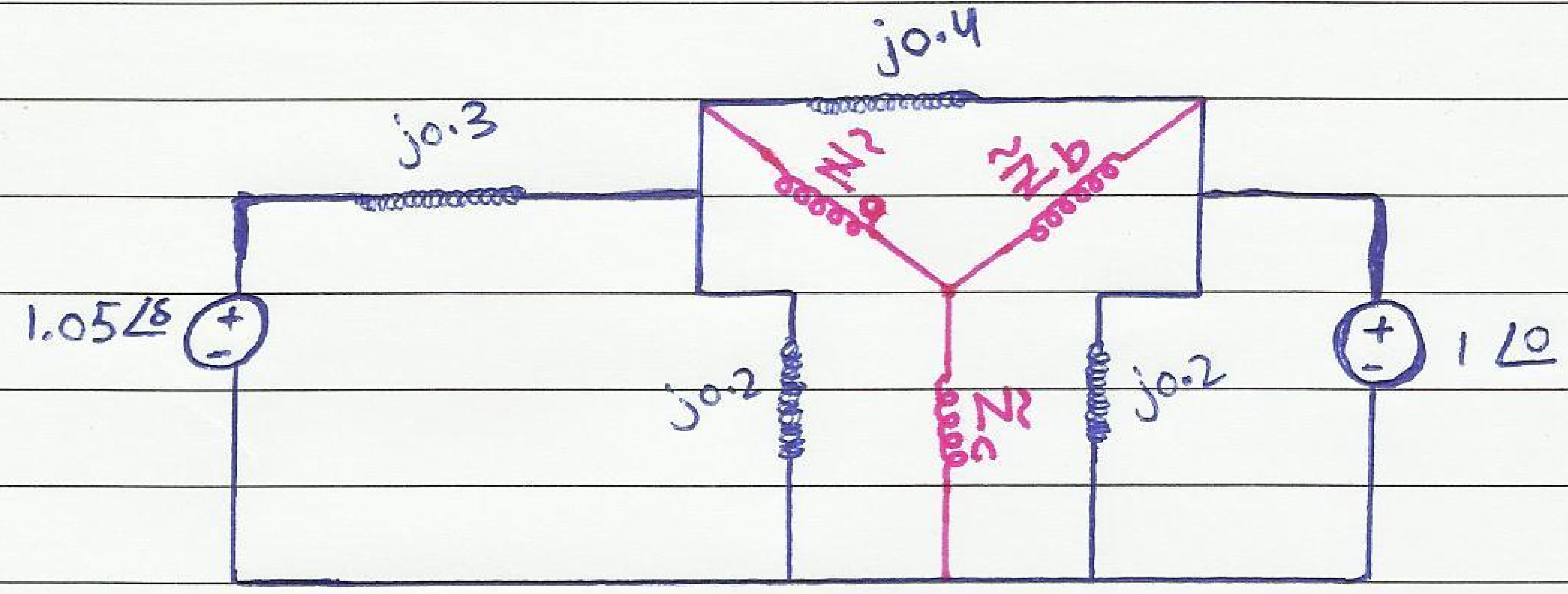


pre Fault



when the Fault is occur

⇒ Transforming the Δ-connection to Y-connection :-

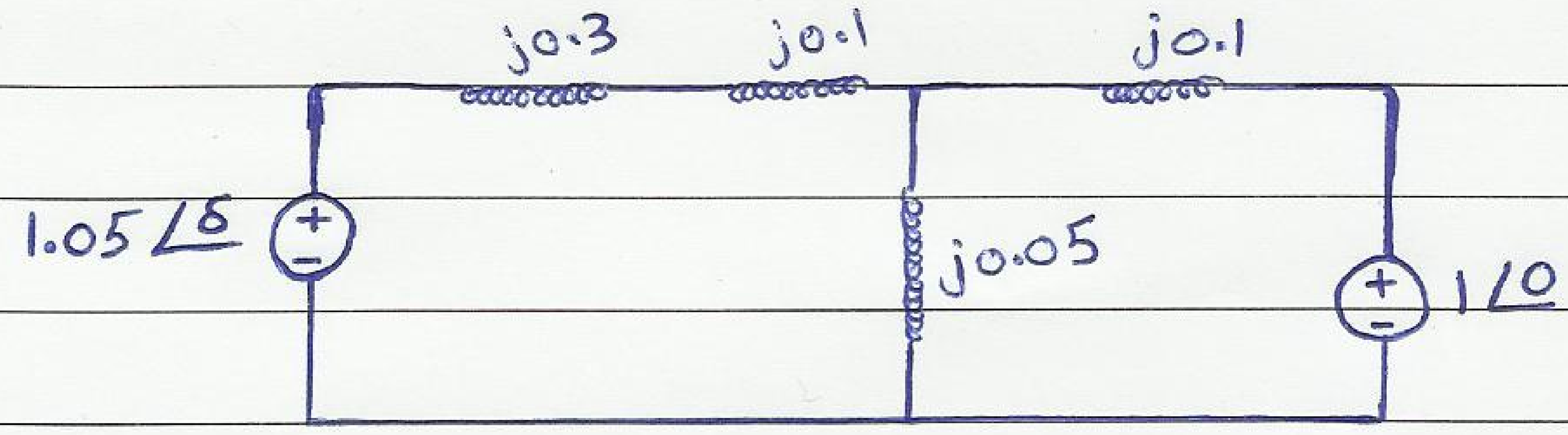


where $\tilde{Z}_a = \dfrac{(0.2)(0.4)}{0.2 + 0.4 + 0.2} = j0.1$

$\tilde{Z}_b = j0.1$

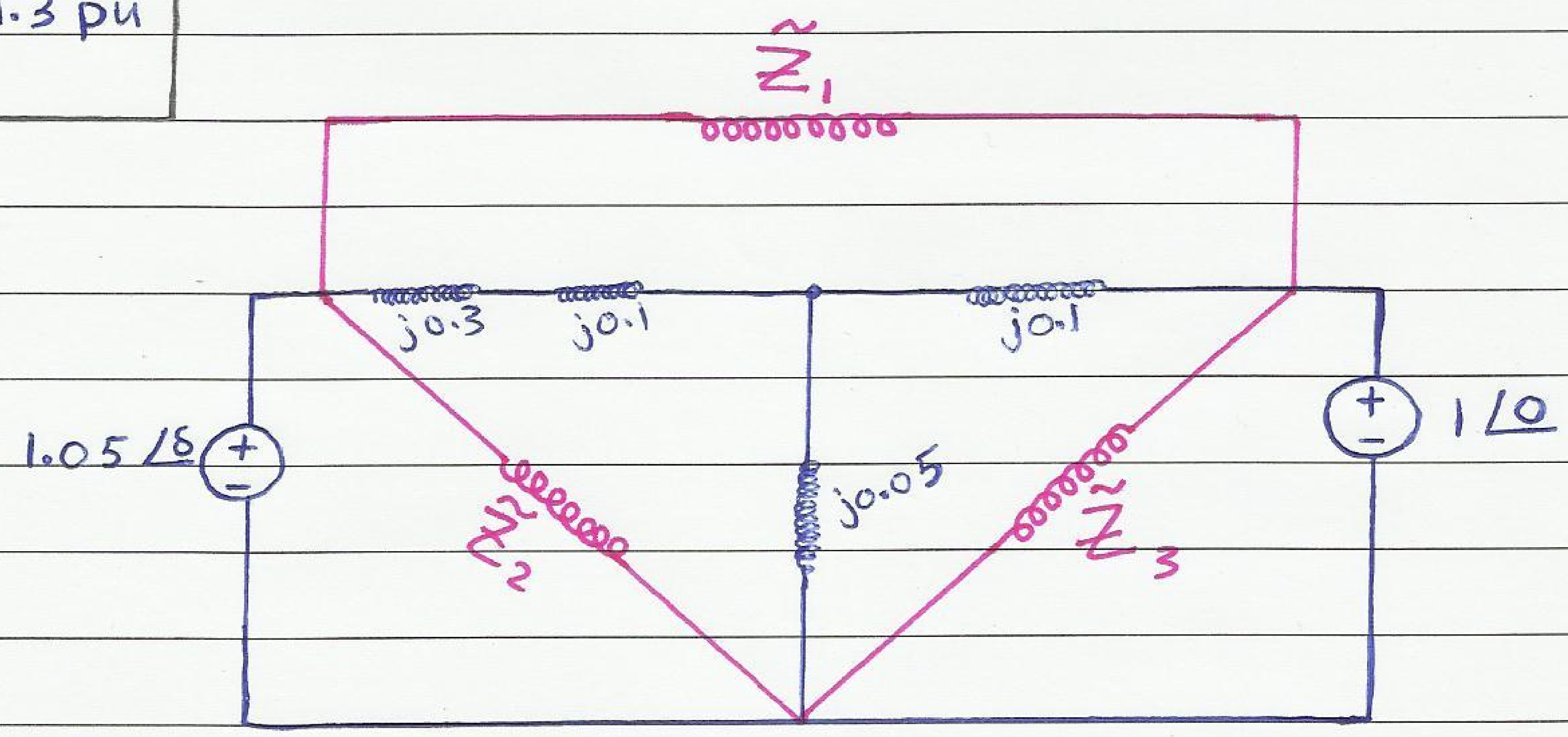
$\tilde{Z}_c = j0.05$

Y- connection

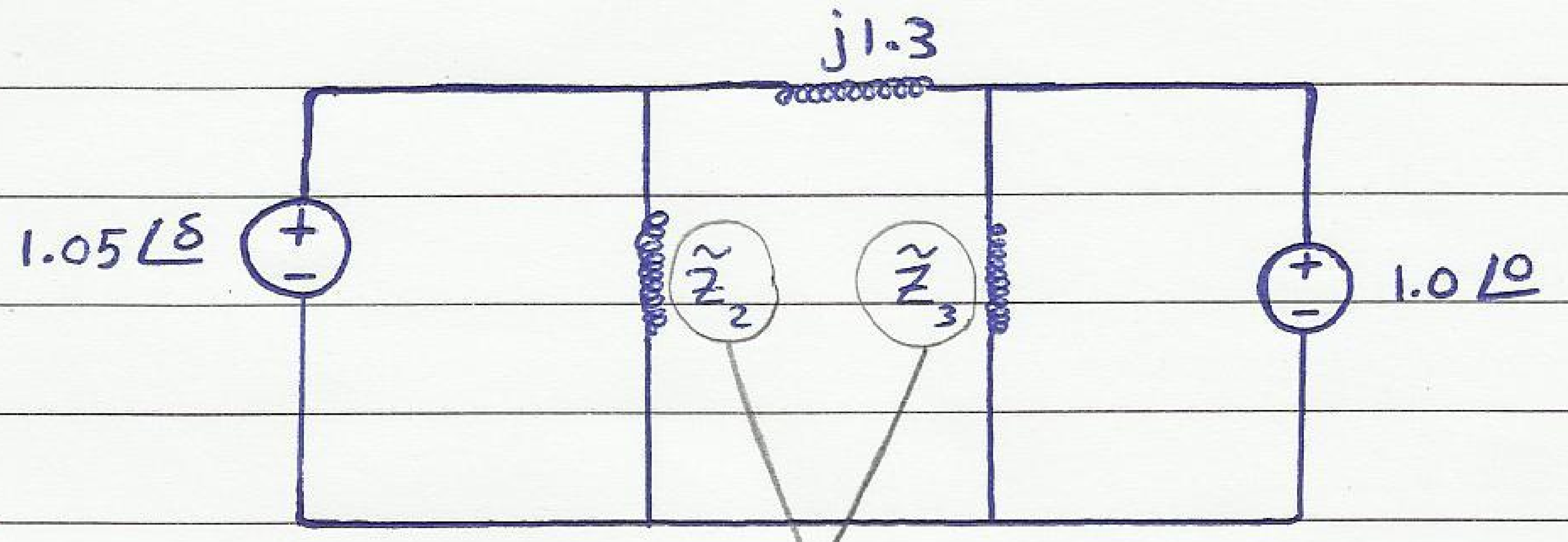


⇒ Transforming again the Y connection to Δ-connection :-

where $\tilde{Z}_1 = j1.3$ pu



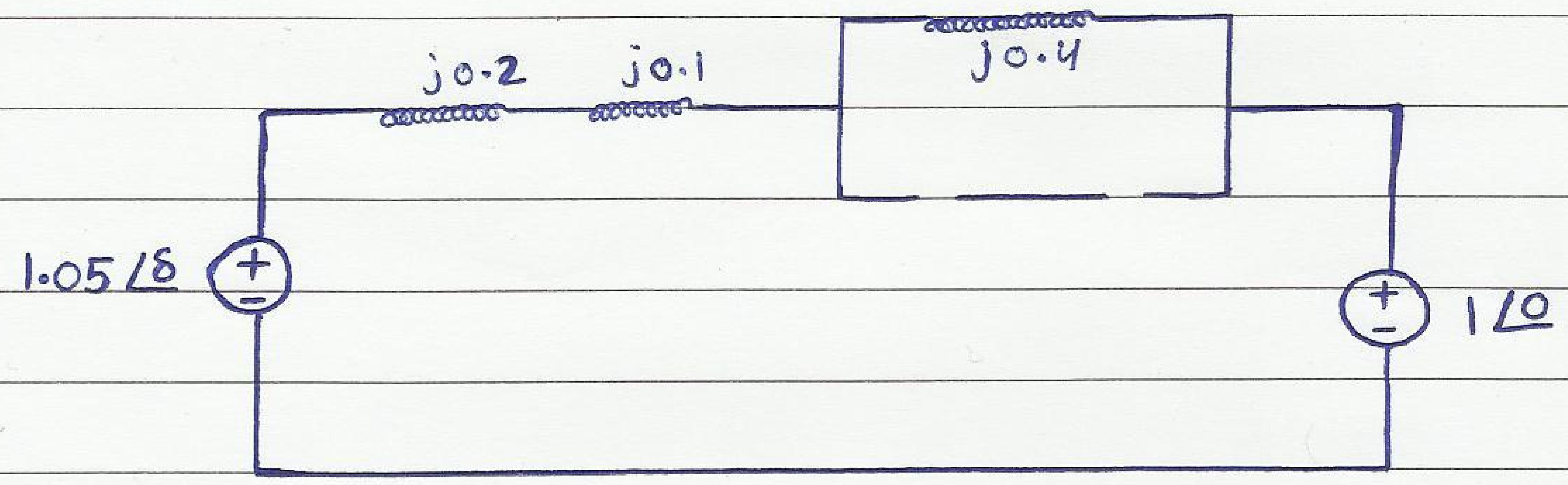
⇒



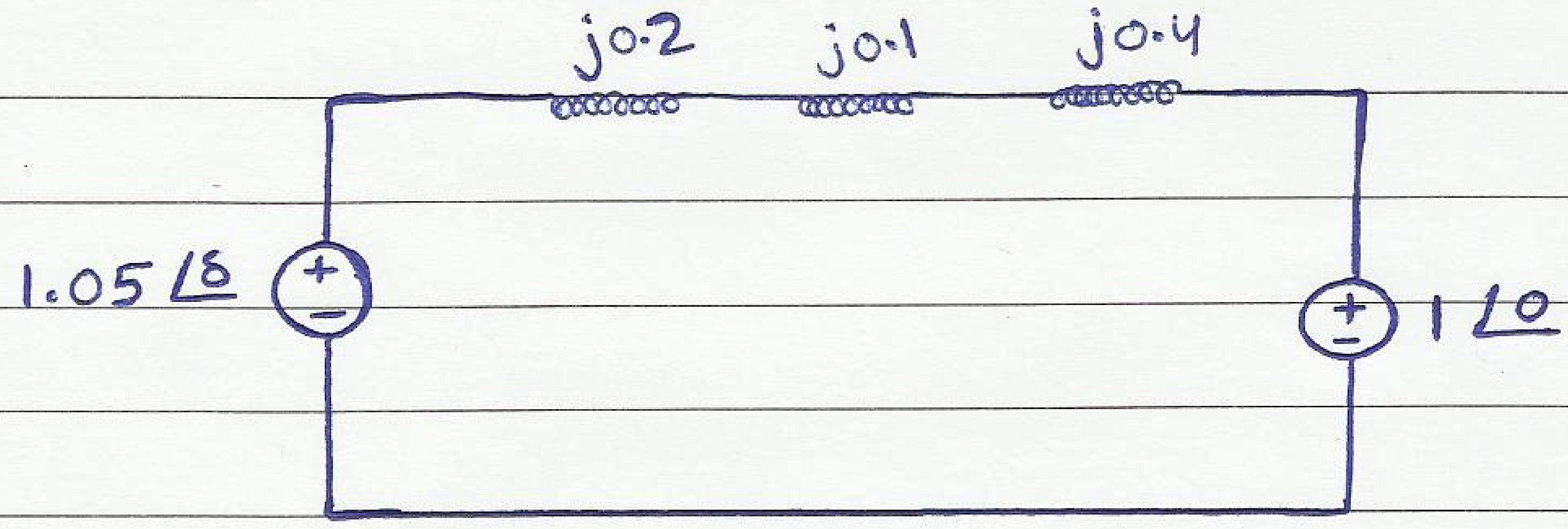
مابهتم لقيمهم لأنهم توازي مع $\tilde{E}_1$, $\tilde{V}_\infty$

$$P_e = \frac{(1.05)(1)}{1.3} \sin\delta = 0.808 \sin\delta$$

Ex: The Fault in the previous example is cleared by simultanuous opening the line From it's two ends, write the post Fault swing equation. (opening the circuit breaker at the begining and the end).



⇓

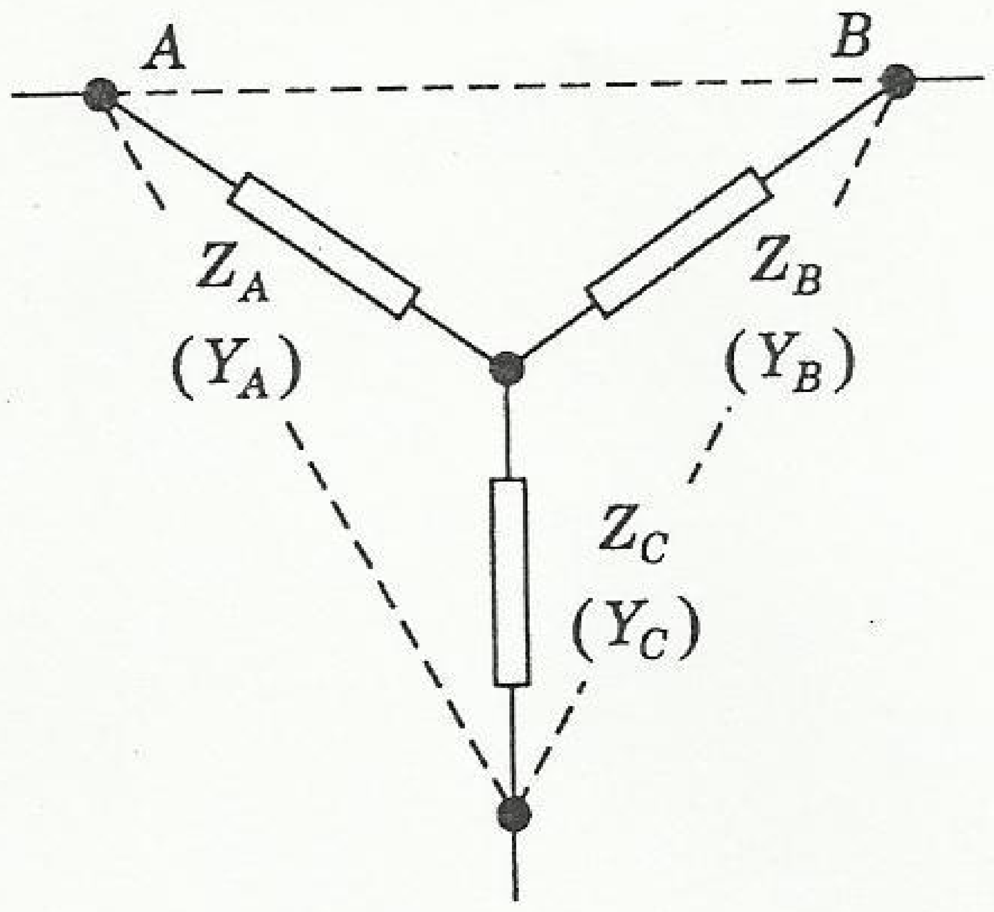
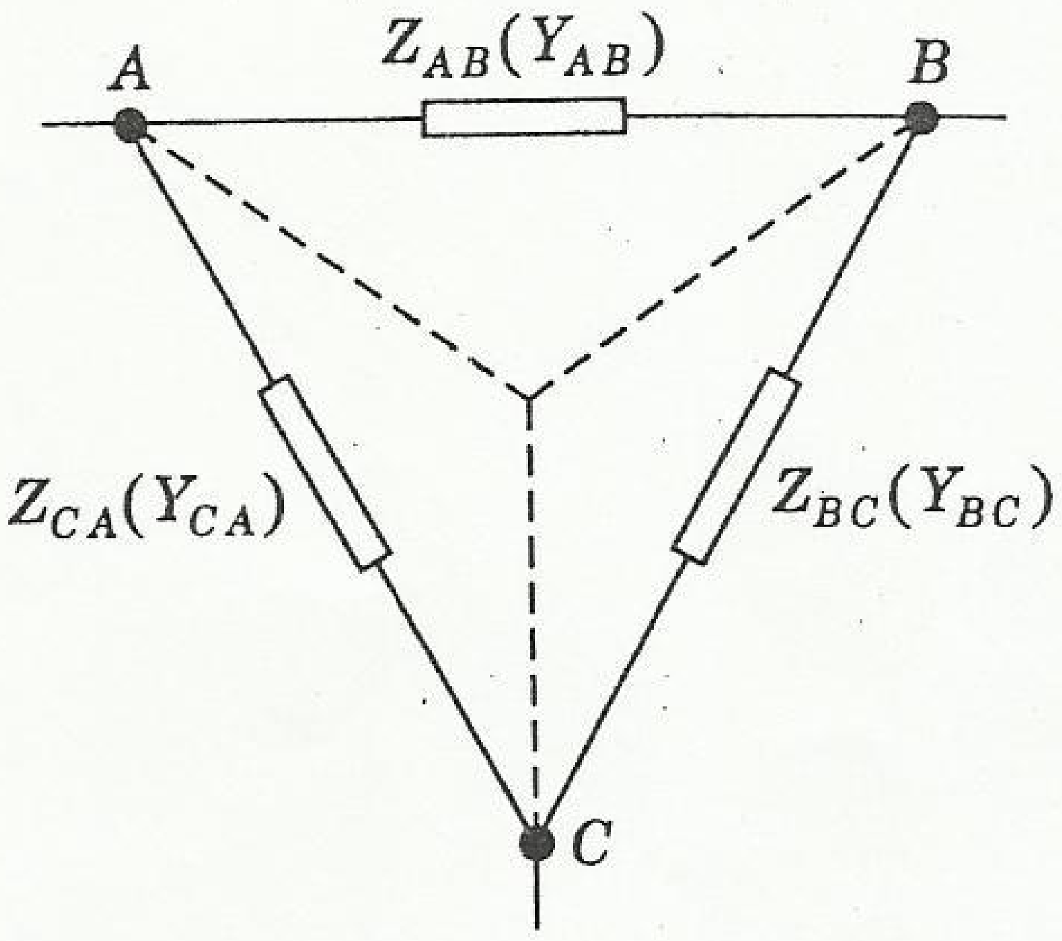


$$X_{tot} = j0.2 + j0.1 + j0.4 = j0.7$$

$$P_e = \frac{(1.05)(1)}{0.7} \sin\delta = 1.5 \sin\delta$$

$$\frac{2H}{\omega_s} \frac{d^2\delta}{dt^2} = 1.0 - 1.5 \sin\delta$$

TABLE 1.2
Y-Δ and Δ-Y transformations†

	
Δ → Y	Y → Δ
$Z_A = \dfrac{Z_{AB}Z_{CA}}{Z_{AB} + Z_{BC} + Z_{CA}}$	$Z_{AB} = \dfrac{Z_AZ_B + Z_BZ_C + Z_CZ_A}{Z_C}$
$Z_B = \dfrac{Z_{BC}Z_{AB}}{Z_{AB} + Z_{BC} + Z_{CA}}$	$Z_{BC} = \dfrac{Z_AZ_B + Z_BZ_C + Z_CZ_A}{Z_A}$
$Z_C = \dfrac{Z_{CA}Z_{BC}}{Z_{AB} + Z_{BC} + Z_{CA}}$	$Z_{CA} = \dfrac{Z_AZ_B + Z_BZ_C + Z_CZ_A}{Z_B}$
Δ → Y	Y → Δ
$Y_A = \dfrac{Y_{AB}Y_{CA} + Y_{BC}Y_{AB} + Y_{CA}Y_{BC}}{Y_{BC}}$	$Y_{AB} = \dfrac{Y_AY_B}{Y_A + Y_B + Y_C}$
$Y_B = \dfrac{Y_{AB}Y_{CA} + Y_{BC}Y_{AB} + Y_{CA}Y_{BC}}{Y_{CA}}$	$Y_{BC} = \dfrac{Y_BY_C}{Y_A + Y_B + Y_C}$
$Y_C = \dfrac{Y_{AB}Y_{CA} + Y_{BC}Y_{AB} + Y_{CA}Y_{BC}}{Y_{AB}}$	$Y_{CA} = \dfrac{Y_CY_A}{Y_A + Y_B + Y_C}$

† Admittances and impedances with the same subscripts are reciprocals of one another.

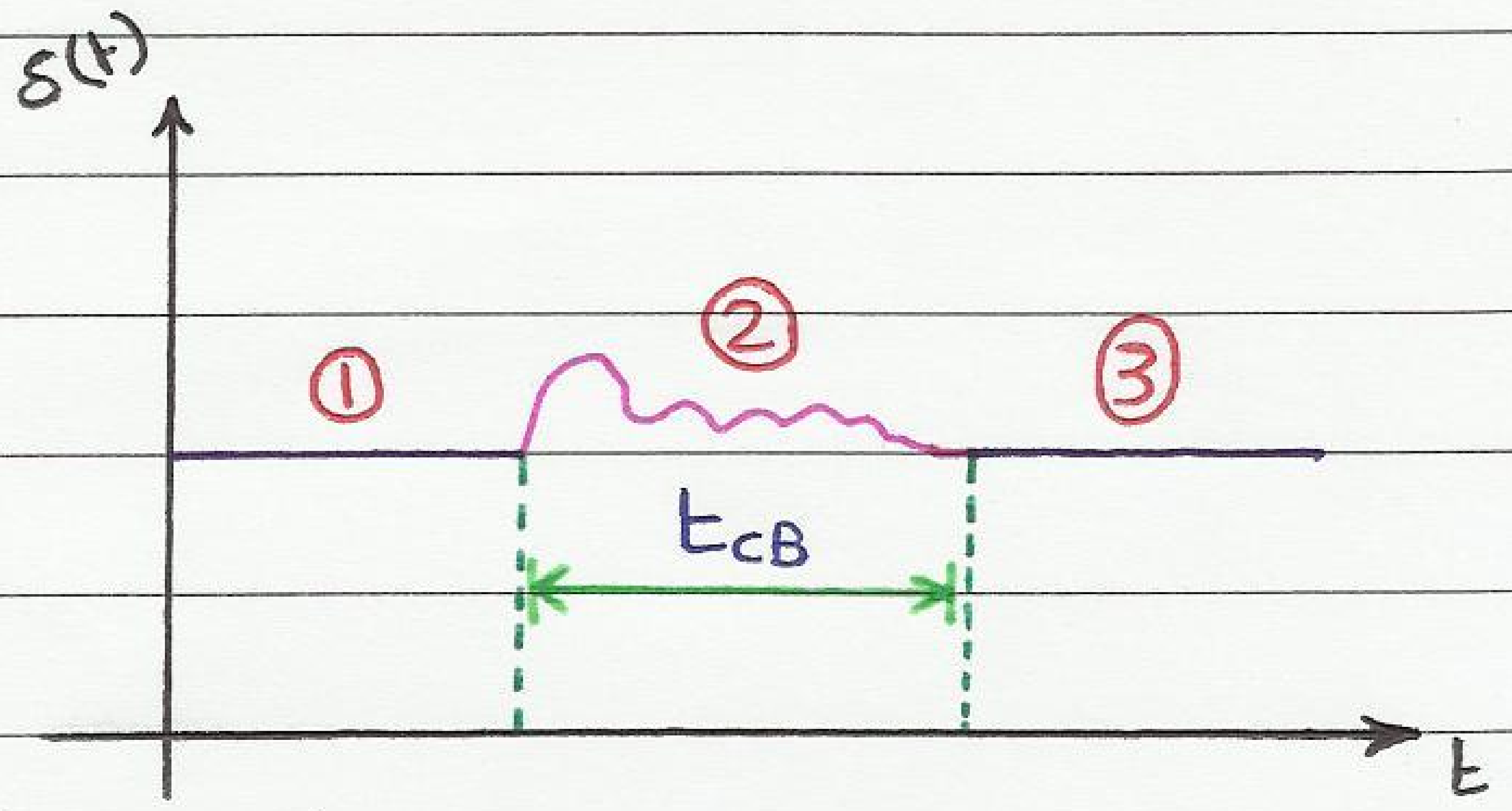
Z_Y in terms of the delta impedances Z_Δ's is

$$Z_Y = \frac{\text{product of adjacent } Z_\Delta\text{'s}}{\text{sum of } Z_\Delta\text{'s}} \tag{1.31}$$

So, when all the impedances in the Δ are equal (that is, balanced Z_Δ's), the impedance Z_Y of each phase of the equivalent Y is one-third the impedance of each phase of the Δ which it replaces. Likewise, in transforming from Z_Y's to

16.6 : Equal Area Criterion OF Stability :-

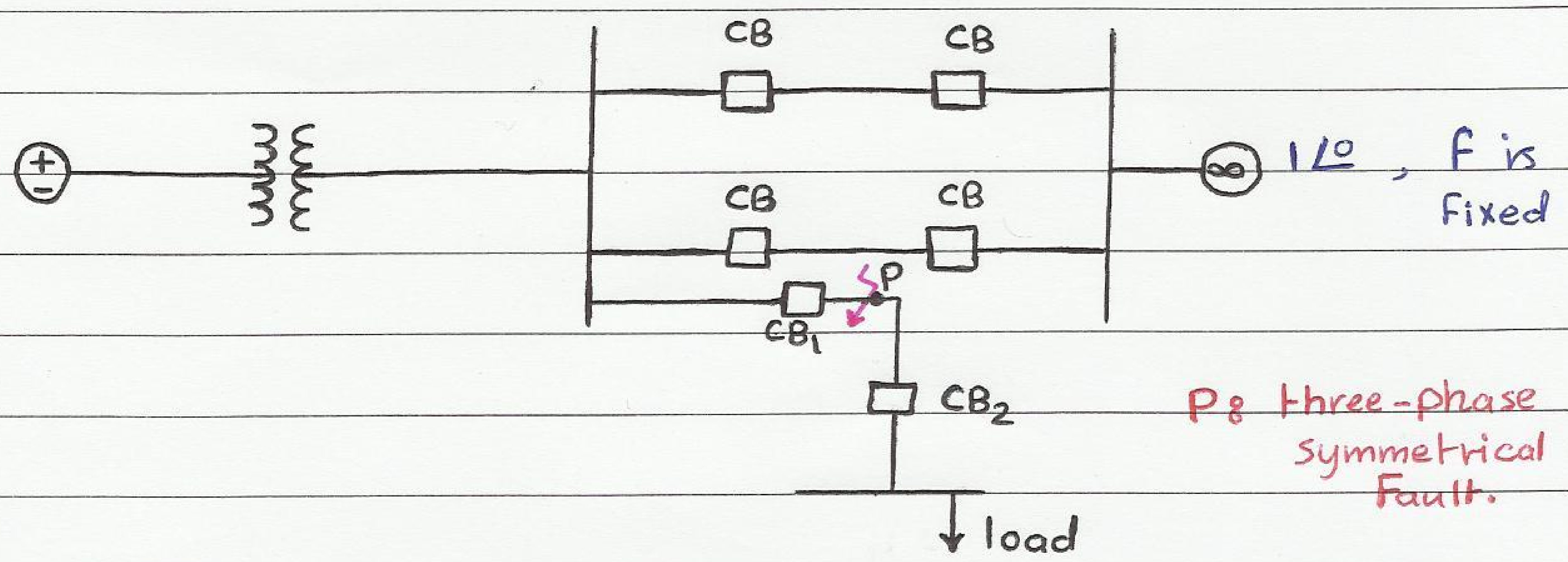
- swing equation is a 2nd order non-linear differential equation, explicit solution is not possible and therefore it solved numerically by using Matlab.



① → pre Fault
② → during Fault
③ → post Fault

t_{CB} : time For circuit breaker to disconnect.

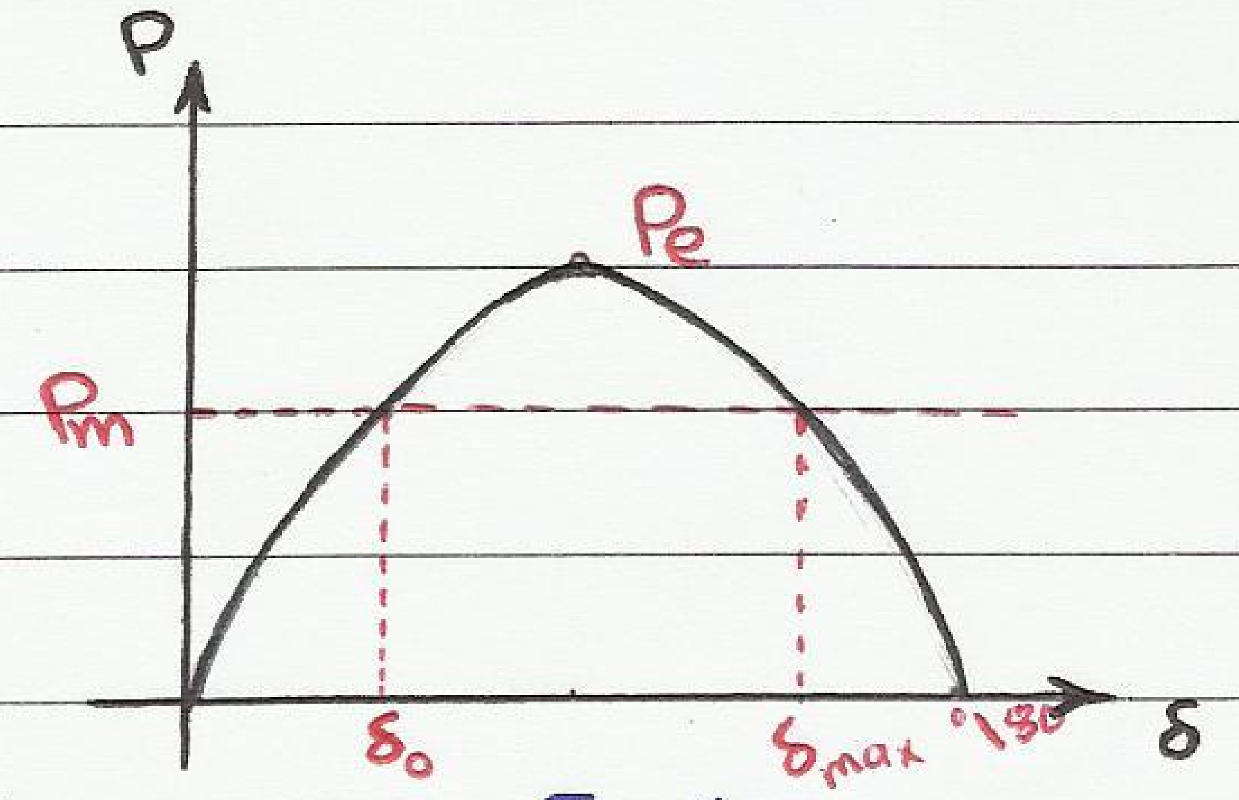
- In stability problem (large signal) three differential equation are solved successively ⇒ boundary condition (Final) of the First stage is the initial condition For the second, Same for the second and third.



$P_e = P_{max} \sin \delta$

$P_m = P_{max} \sin \delta_o$

$$\boxed{\delta_o = \sin^{-1}\left[\frac{P_m}{P_{max}}\right]}$$



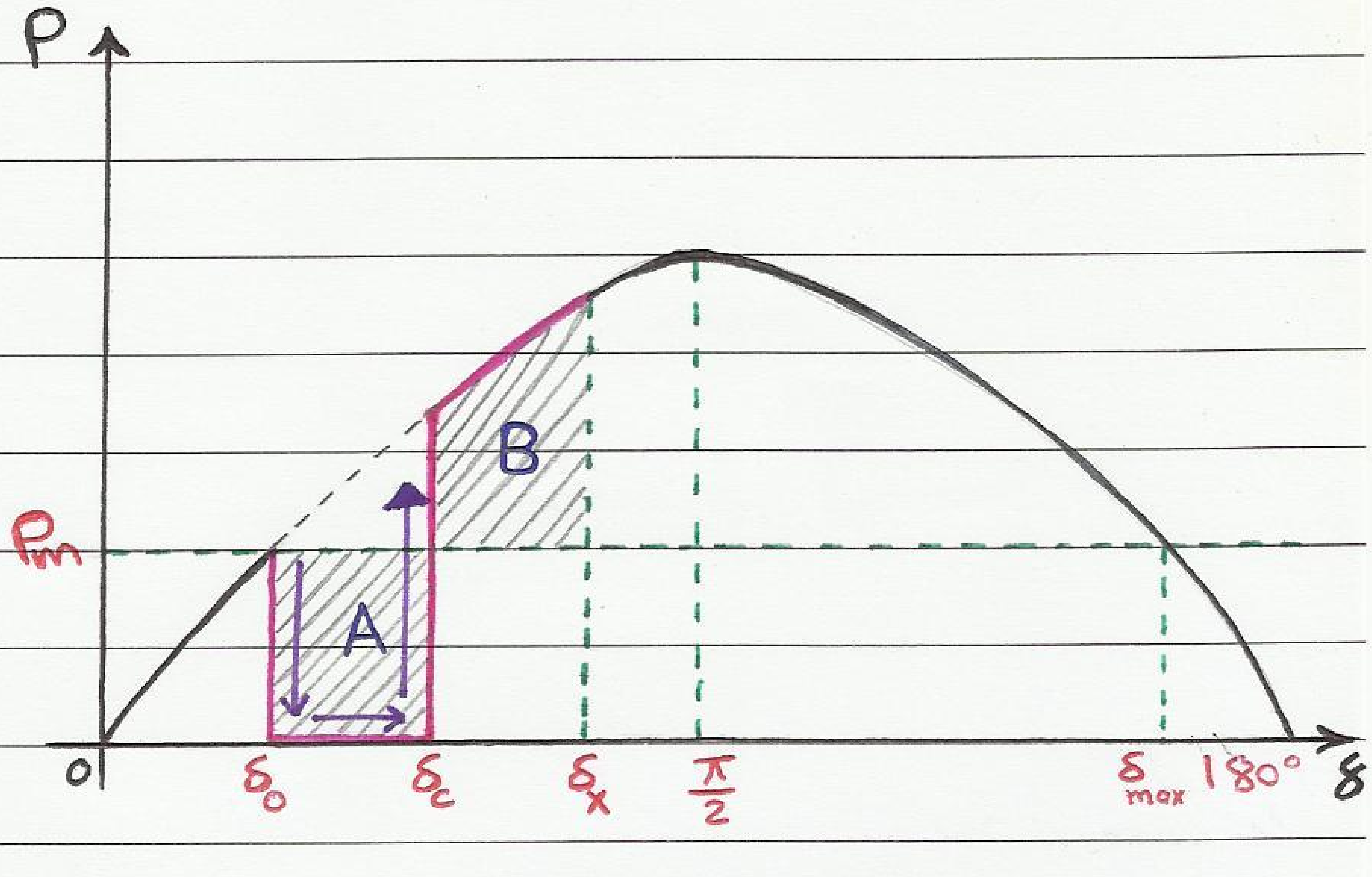
pre-Fault

* when the Fault occurs, the electrical power output is suddenly zero while the input mechanical power is unaltered.

where,

δ_c : The instant of cleaning the Fault.

δ_x : The angle that make the area A & B are equal.



During the Fault

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m \quad \Rightarrow \text{during Fault } P_e = 0$$

$$\frac{d^2\delta}{dt^2} = \frac{\omega_s}{2H} P_m$$

$$\frac{d\delta}{dt} = \int_0^t \frac{\omega_s}{2H} P_m \cdot dt = \frac{\omega_s}{2H} P_m t$$

$$\delta = \frac{\omega_s P_m}{4H} t^2 + \delta_o$$ initial value before Fault

$$\delta(t)\Big|_{t=t_c} = \delta_c = \frac{\omega_s}{4H} P_m t_c^2 + \delta_o$$

where, t_c : Fault cleaning time.

note :-

عملية الـ Damping تقسم الى قسمين :-

(1) electrical Damping :- وتكون من خلال الـ Resistors

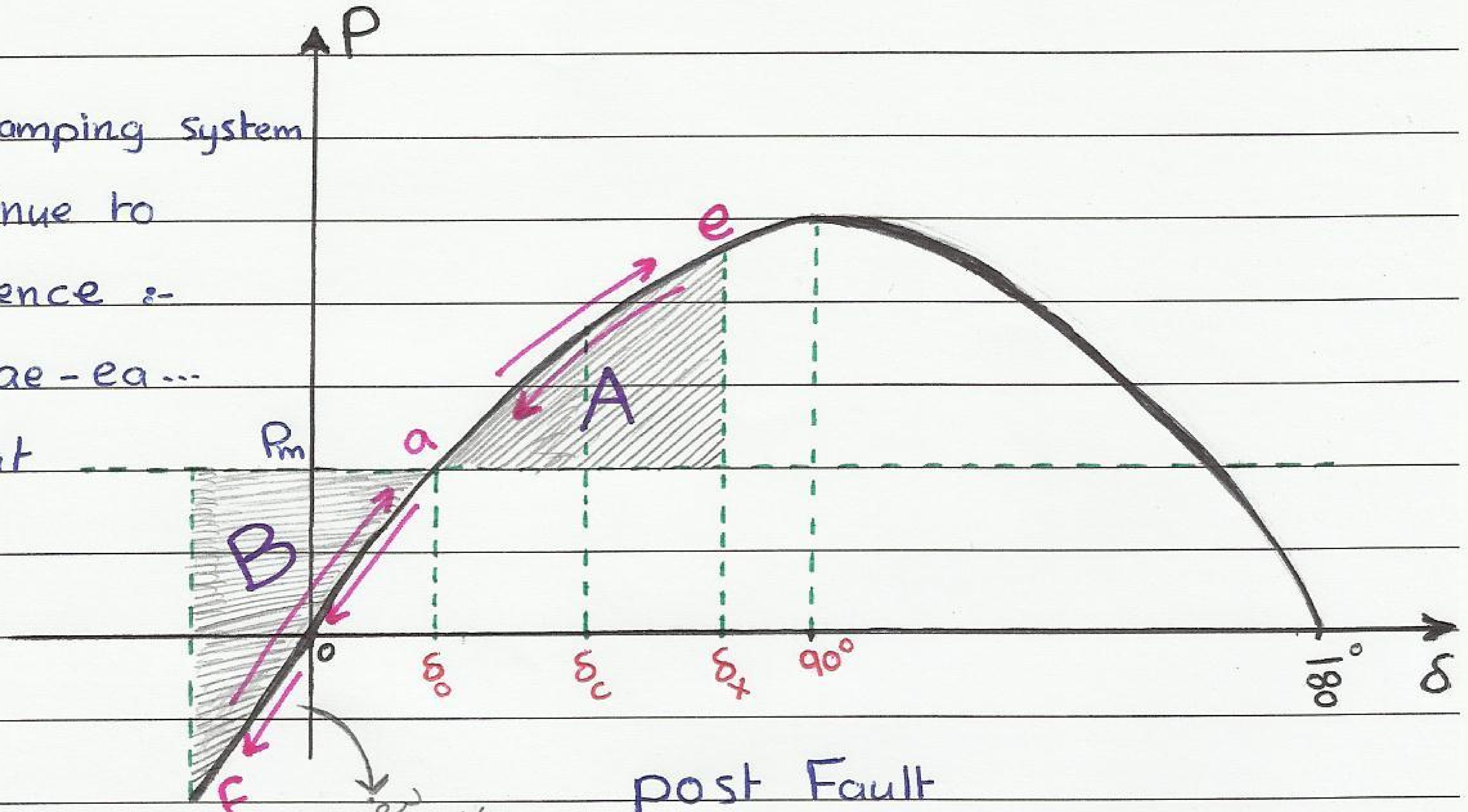
(2) mechanical Damping :- وتكون من خلال الـ Friction

* $P_a = P_m - P_e$

- in the absence of the damping system the rotor would continue to Oscillate in the sequence :-

ae - ea - af - fa - ae - ea ... .∞ , Such that

$A = B$



post Fault

بيضل كل طول
كي هالحال بسبب
الـ inertia
وبيضل عكس
لما يرجع

$$\frac{2H}{\omega_s}\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\omega_r = \frac{d\delta}{dt} = \omega - \omega_s$$

⇒ definition for the relative speed ω_r

$$\frac{2H}{\omega_s}\frac{d\omega_r}{dt} = P_m - P_e$$

$$\frac{2H}{\omega_s}\omega_r\frac{d\omega_r}{dt} = (P_m - P_e)\frac{d\delta}{dt}$$

$$\left(\frac{2H}{\omega_s}\frac{1}{2}\frac{d(\omega_r^2)}{dt} = (P_m - P_e)\frac{d\delta}{dt}\right).dt$$

$$\int_{\omega_{r_1}}^{\omega_{r_2}}\frac{H}{\omega_s}\,d(\omega_r^2) = \int_{\delta_1}^{\delta_2}(P_m - P_e)\,d\delta$$

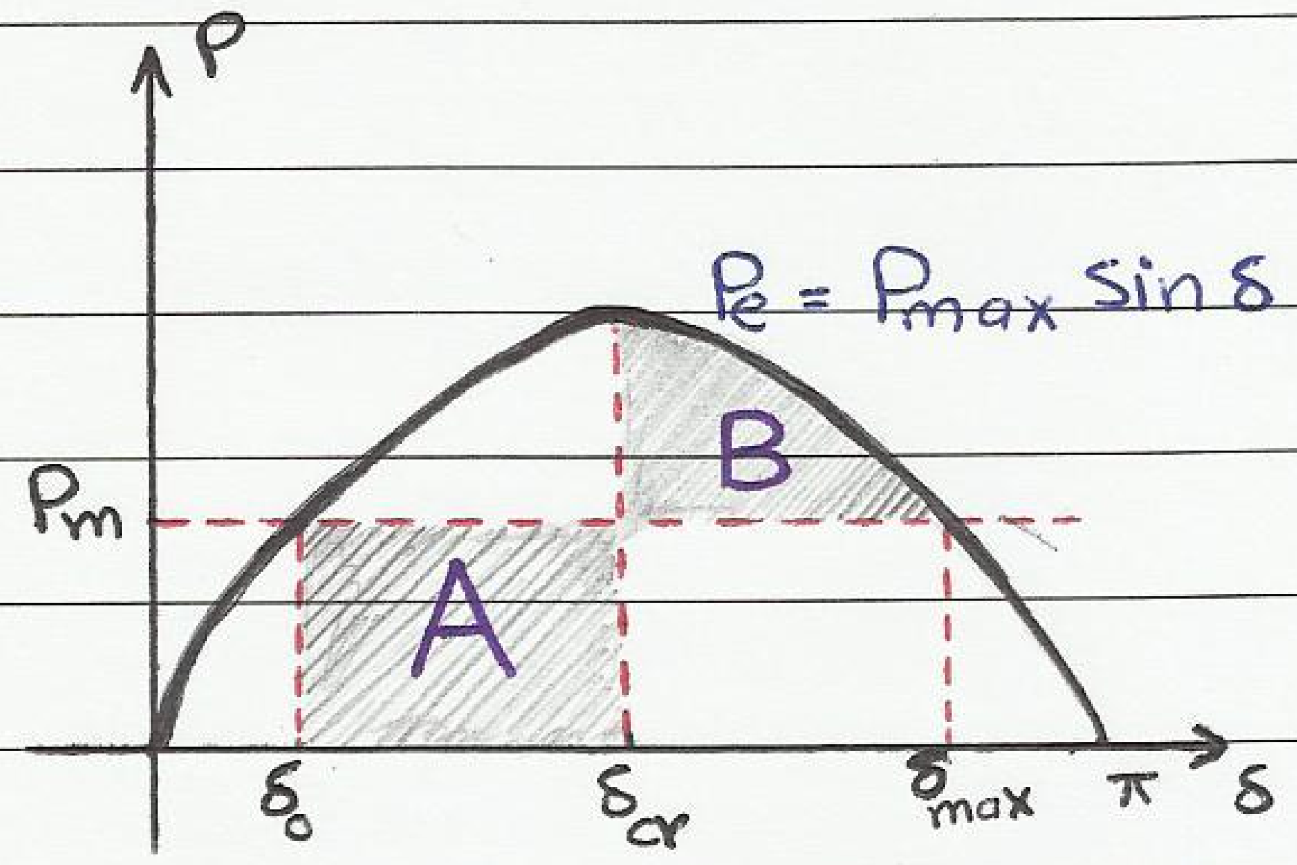
$$\frac{H}{\omega_s}(\omega_{r_2}^2 - \omega_{r_1}^2) = \int_{\delta_1}^{\delta_2}(P_m - P_e).d\delta$$

* For the Following case :-

$\boxed{A = B}$

$$A = \int_{\delta_0}^{\delta_{cr}} P_m . d\delta$$

$$B = \int_{\delta_{cr}}^{\delta_{max}} (P_{max}\sin\delta - P_m).d\delta$$



$$\Rightarrow \quad A = \int_{\delta_0}^{\delta_{cr}} P_m \, d\delta = P_m (\delta_{cr} - \delta_0)$$

$$B = \int_{\delta_{cr}}^{\delta_{max}} (P_{max} \sin\delta - P_m)\, d\delta = P_{max}(\cos\delta_{cr} - \cos\delta_{max}) - P_m(\delta_{max} - \delta_{cr})$$

$A = B$ then,

$$\int_{\delta_0}^{\delta_{cr}} P_m \, d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_{max} \sin\delta - P_m) \, d\delta$$

$$\hookrightarrow \quad \cos\delta_{cr} = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos\delta_{max}$$

$$\delta_{max} = \pi - \delta_0 \quad , \quad \delta_0 = \sin^{-1}\left[\frac{P_m}{P_{max}}\right]$$

elec. rad

$$P_m = P_{max} \sin\delta_0$$

$$\delta_{cr} = \cos^{-1}\left[(\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0\right]$$

critical clearing angle

$$\delta_{cr} = \frac{\omega_s P_m}{4H} t_{cr}^2 + \delta_0$$

$$t_{cr} = \sqrt{\frac{4H(\delta_{cr} - \delta_0)}{\omega_s P_m}}$$

critical clearing time

* with smaller critical clearing time (t_{cr}) the posibility of unstability is smaller.

Ex 16.7 :- Calculate the critical clearing angle and the critical clearing time For the system (Fig 16.8 page 718 on Book) when the system is subjected to a three phase Fault at point p on the short transmission line. The initial condition are the same as those in Example 16.3 and $H = 5\ MJ/MVA$?

Sol.

- the power angle equation is :

$$P_e = P_{max} \sin \delta = 2.1 \sin \delta$$

- the initial rotor angle is :

$$\delta_0 = 28.44^\circ = 0.496 \text{ elec. rad}$$

- mechanical input power P_m is 1.0 per unit

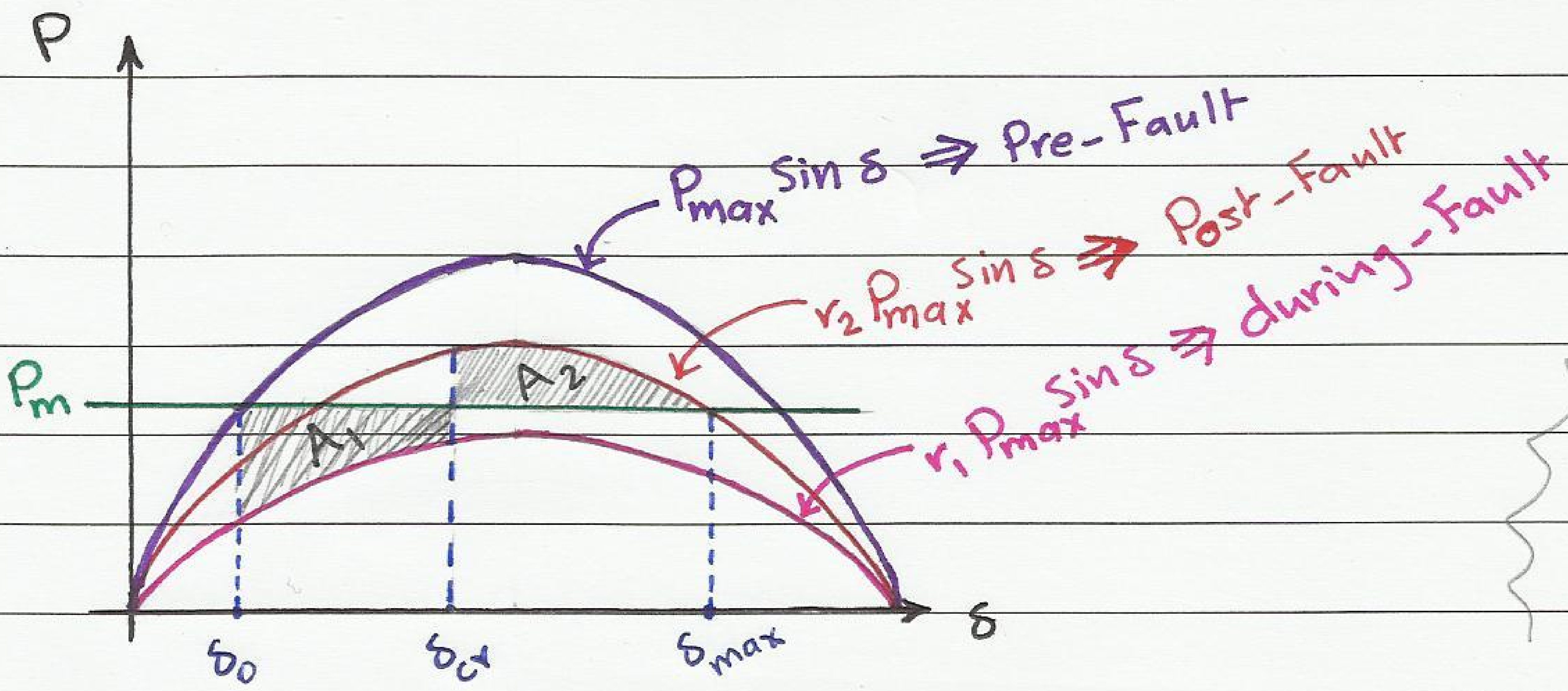
↳

$$\delta_{cr} = \cos^{-1}\left[(\pi - 2 * 0.496) \sin 28.44^\circ - \cos 28.44^\circ\right]$$

$$= 81.697^\circ = 1.426 \text{ elec rad}.$$

$$t_{cr} = \sqrt{\frac{4 * 5(1.426 - 0.496)}{377 * 1}} = 0.222\ s$$

* 16.7 :- Further Applications of the Equal Area Criterion :-



r_1, r_2 are constant

$A_1 = A_2$

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$$\Rightarrow \delta_0 = \sin^{-1}\left[\frac{P_m}{P_{max}}\right]$$

$$\cos\delta_{cr} = \frac{(P_m/P_{max})(\delta_{max} - \delta_0) + r_2\cos\delta_{max} - r_1\cos\delta_0}{(r_2 - r_1)}$$

Ex 16.8 p. 726 :- For pre, during, after Fault swing equation For the last example, calculate the critical clearing angle.

pre-Fault $\Rightarrow P_{max}\sin\delta = 2.1\ \sin\delta$

During-Fault $\Rightarrow r_1 P_{max}\sin\delta = 0.808\ \sin\delta$

post-Fault $\Rightarrow r_2 P_{max}\sin\delta = 1.5\ \sin\delta$

$$r_1 = \frac{0.808}{2.1} = 0.385$$

$$r_2 = \frac{1.5}{2.1} = 0.714$$

$$\delta_0 = \sin^{-1}\left[\frac{1}{2.1}\right] = 28.44^\circ = 0.496\ rad$$

$$\delta_{max} = 180 - \sin^{-1}\left[\frac{1}{1.5}\right] = 138.2^\circ = 2.412\ rad$$

$$\cos\delta_{cr} = \frac{(1/2.1)(2.412 - 0.496) + 0.714\cos(138.2^\circ) - 0.385\cos(28.44^\circ)}{(0.714 - 0.385)}$$

$$= 0.127$$

then,

$$\delta_{cr} = 82.726^\circ$$

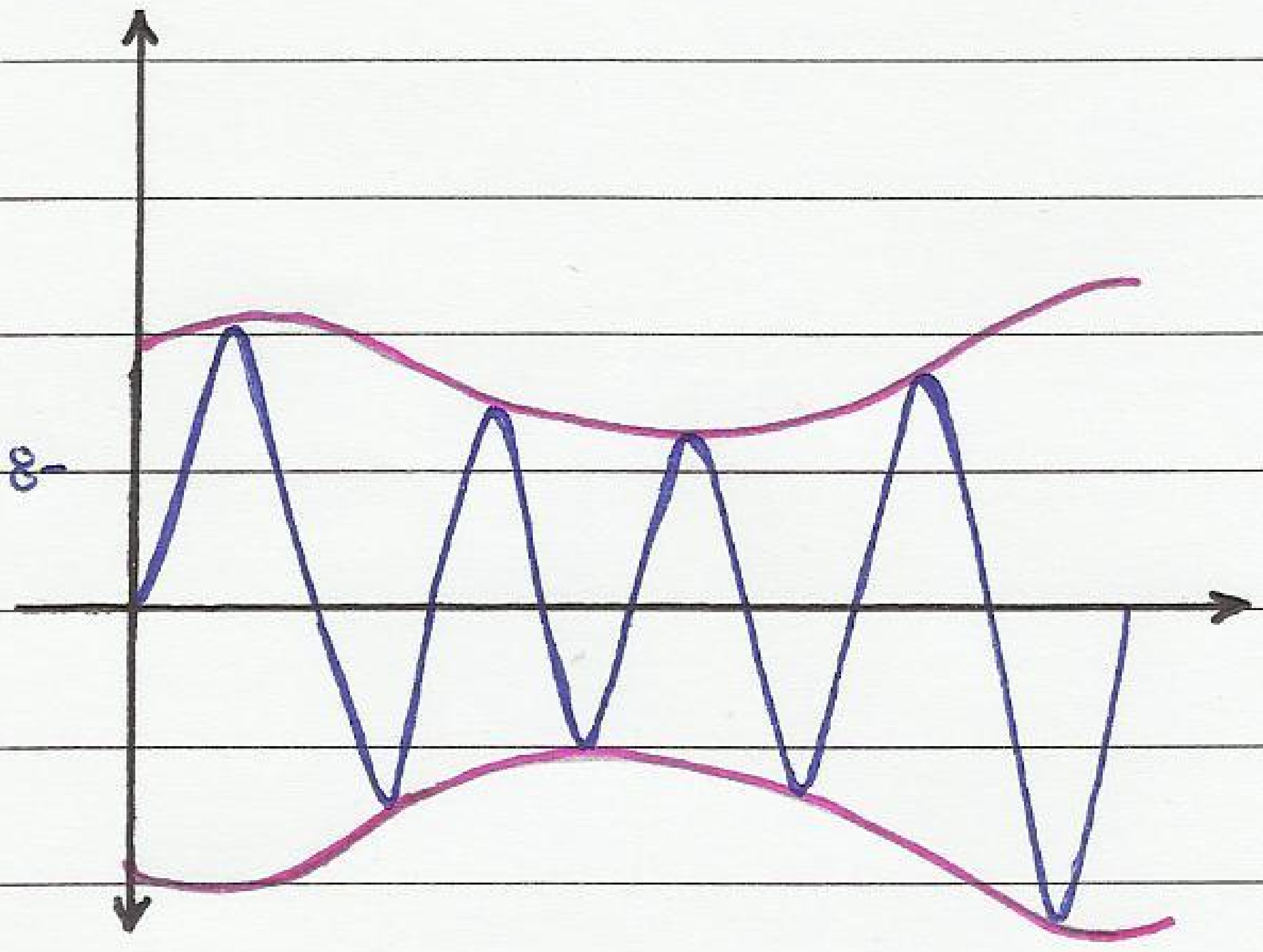
16.8 :- Multimachine Stability studies : Classical Representation :-

- Multimachine ⇒ 3 machines and more.

  SMIB ⇒ single machine infinite Bus. (multimachine يعتبر ✗)

- In Multimachine power system equal area Criterion can't be applied, and the only Solution is numerical solution. typical frequency of Oscillation of one machine to the other is 1-2 Hz superimposed on the 60 Hz.

- To Simplify the complexity of system modeling and therefore the computational load ← burden, the following assumption are made :-



1] P_m is constant for pre - during - after fault.

2] damping power is neglecting (mechanical friction, resistors of stator, damper winding).

3] each machine is represented by constant transient reactance in series with a constant transient internal voltage. (E' is constant)

4] the mechanical rotor angle δ_m is equal to the electrical phase angle δ_e of the transient internal voltage. (phase shift between two mmf equals the angle between E and V_t).

5] all loads are considered shunt impedances to ground and constant for pre - during - post fault condition.

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* Classical Stability model is the dynamical stability model obtain based on the previous mentioned conditions.

لـ عبارة عن تعريف الـ classical model ، يجب ذكر النقاط الثلاث السابقة إذا طلب التعريف بالامتحان

* For solving classical Stability studies :-

- the pre Fault steady state conditions are the initial condition for the during Fault.
- the boundary condition (Final-State) of the during Fault is initial condition For the post Fault.
- For each state (pre- during - Post) the configuration of the system might change

* For multimachine stability problem :-

(1) The values of power, reactive power, voltage and phase angle of each bus must be known by load-Flow study for the pre Fault state.

(2) The Transient internal voltage of each generator is then calculated as:

$$\tilde{E}' = \tilde{V}_t + jX_d' \tilde{I}_t$$

(3) each load is converted into constant admittance $\tilde{Y}$ connected to ground.

$$\tilde{Y}_L = \frac{P_L + jQ_L}{|V_L|^2}$$

(4) the bus admittance matrix (Y_{bus}) is used for the pre-Fault power Flow calculation should include the transient reactance of each generator and the shunt admittance of each load.

(5) the injected current is Zero at all buses except the generating buses.

(6) $\tilde{Y}_{bus}$ is modified to account for faulted and post fault condition. all buses can be eliminated by Kron reduction except the buses which have constant voltage, such that the dimention of modified matrix is equal the number of generators.

infinite Bus, slack Bus, generating Buses

(7) during and after fault the power angle equation is :-

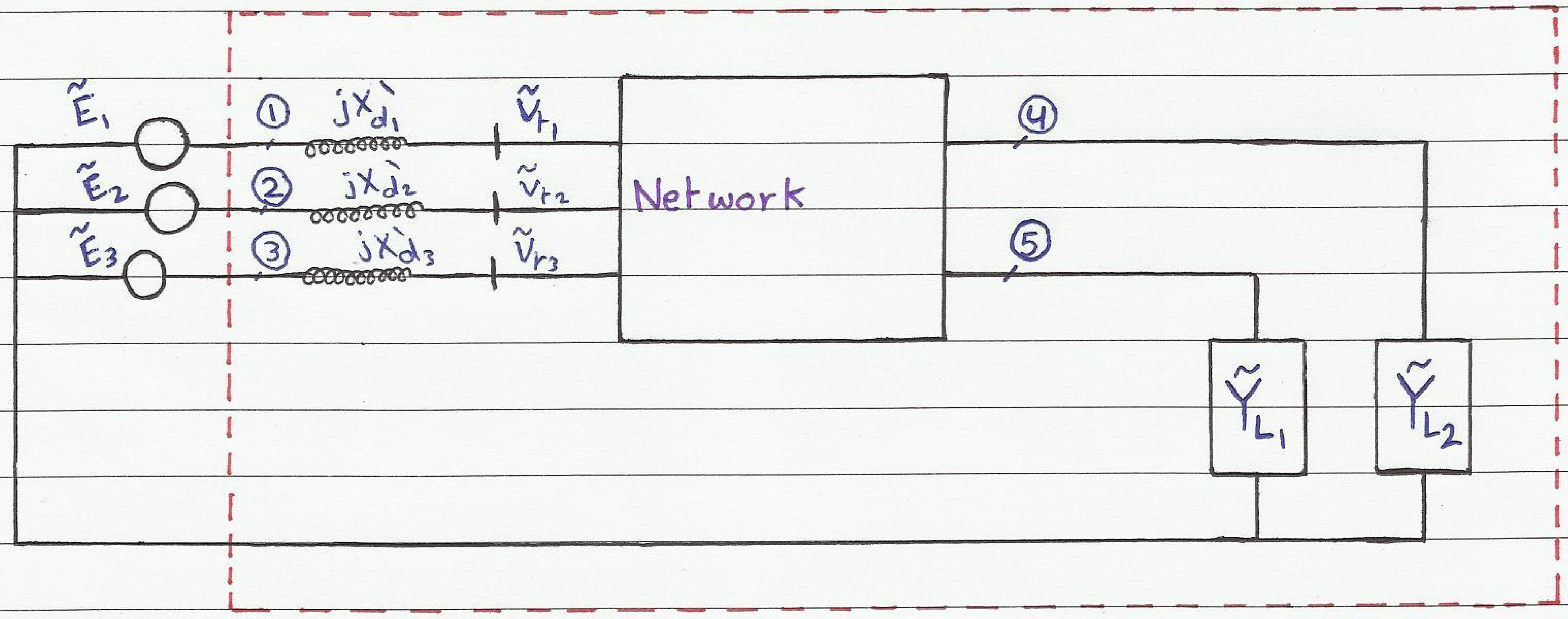
$$P_{e_i} = |\tilde{E}_1'|^2 G_{11} + |\tilde{E}_1'||\tilde{E}_2'||\tilde{Y}_{12}| \cos(\delta_{12} - \theta_{12}) + |\tilde{E}_1'||\tilde{E}_3'||\tilde{Y}_{13}| \cos(\delta_{13} - \theta_{13}) + \dots\dots$$

$$\delta_{12} = \delta_1 - \delta_2 \quad , \quad \delta_{13} = \delta_1 - \delta_3 \quad , \quad \dots$$

(8) Similar equations are written for all generating buses.

(9) the swing equation for each generator is written :-

$$\frac{2H_i}{\omega_s} \frac{d^2\delta_i}{dt^2} = P_{m_i} - P_{e_i}$$



the fault period. The generators have reactances and H values expressed on a 100MVA base as follows:

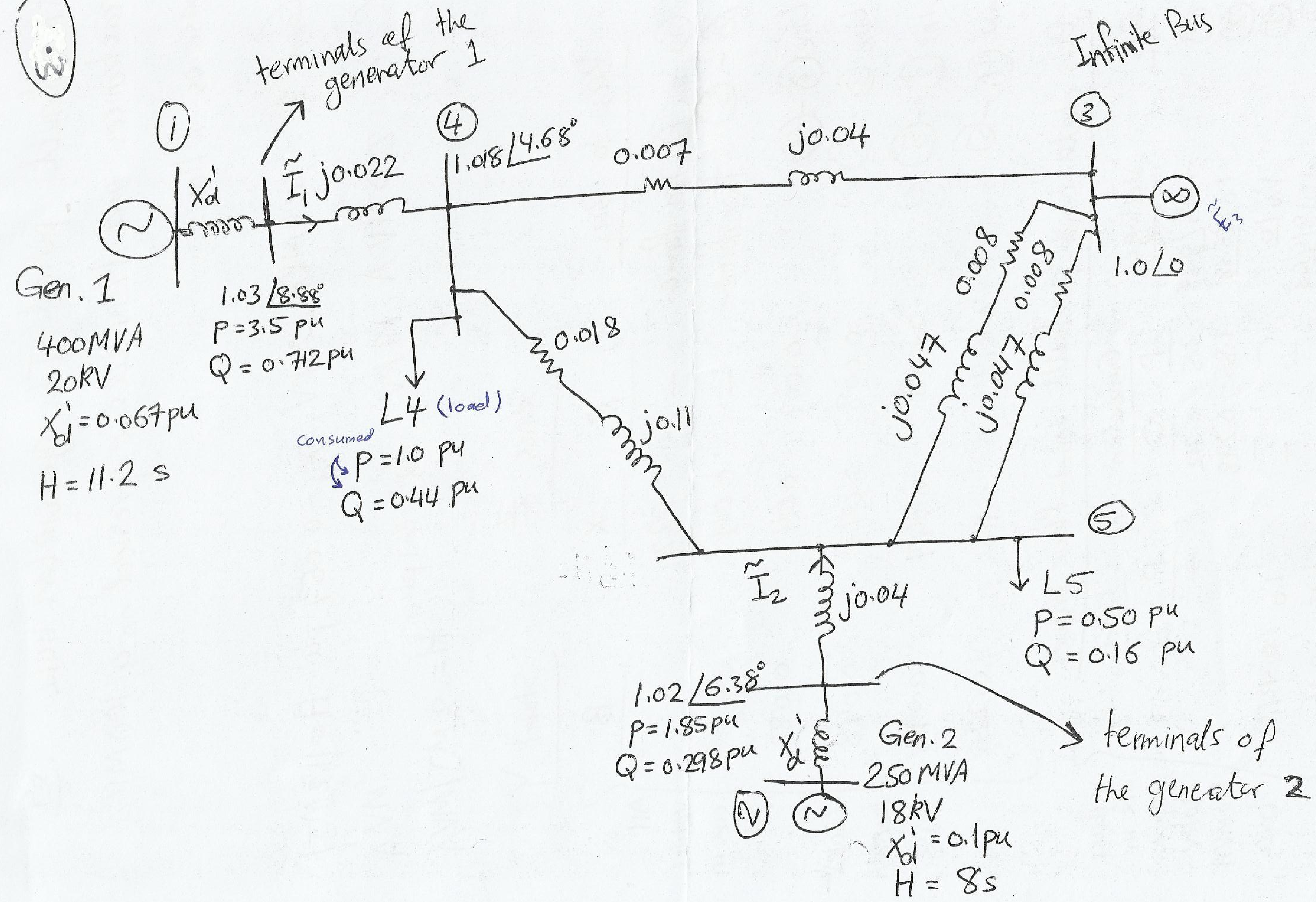
Gen. 1 : 400MVA, 20kV, $X_d' = 0.067$ pu $H = 11.2$ MJ/MVA

Gen. 2 : 250MVA, 18kV, $X_d' = 0.1$ pu $H = 8$ MJ/MVA

	Series Z		Shunt Y	
Bus to Bus	R	X	B	
① - ④ Transformer	—	0.022	—	All values are in pu on 230kV 100MVA base
② - ⑤ Transformer	—	0.04	—	
Line ③ - ④	0.007	0.04	0.082	
Line ③ - ⑤ (1)	0.008	0.047	0.098	
Line ③ - ⑤ (2)	0.008	0.047	0.098	
Line ④ - ⑤	0.018	0.110	0.226	

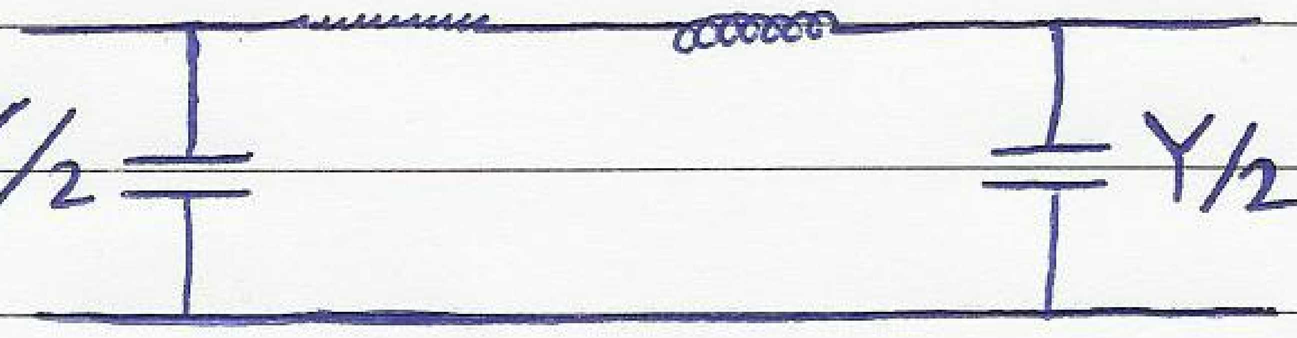
Bus data and prefault load-flow values

Bus	Voltage	Generation P	Generation Q	Load P	Load Q	
①	$1.03\angle 8.88°$	3.5	0.712	—	—	values are in pu on 230kV 100MVA base
②	$1.02\angle 6.38°$	1.85	0.298			
③	$1.0\angle 0$	—	—	1.0	0.44	
④	$1.018\angle 4.68°$	—	—	0.5	0.16	
⑤	$1.011\angle 2.27°$	—	—			


terminals of the generator 1
Infinite Bus
1
4
3
X_d'
$\tilde{I}_1$ j0.022
$1.018\angle 4.68°$
0.007
j0.04
∞
$\tilde{E}_3$
$1.0\angle 0$
Gen. 1
400MVA
20KV
$X_d' = 0.067$pu
H = 11.2 s
$1.03\angle 8.88°$
P = 3.5 pu
Q = 0.712 pu
0.018
j0.11
L4 (load)
consumed
P = 1.0 pu
Q = 0.44 pu
0.008
0.008
j0.047
j0.047
5
$\tilde{I}_2$
j0.04
L5
P = 0.50 pu
Q = 0.16 pu
$1.02\angle 6.38°$
P = 1.85 pu
Q = 0.298 pu
X_d'
Gen. 2
250 MVA
18KV
$X_d' = 0.1$pu
H = 8s
2
terminals of the generator 2

Ex 8- For the previous page, a system has 60 Hz, 230 kV, 2 generators and infinite Bus, Symmetrical 3-ϕ Fault occurs on line 4 & 5 near Bus 4 (electrically on Bus 4, practically beside Bus 4). determine the swing equation for each machine.

only on same indecies ⇐ shunt admittance on line and neglected if the line very short.



$$\tilde{S} = \tilde{V}\tilde{I}^* = \frac{\tilde{V}\tilde{V}^*}{Z^*}$$

$$\tilde{S} = \frac{|\tilde{V}|^2}{\tilde{Z}^*} \Rightarrow Z^* = \frac{|\tilde{V}|^2}{\tilde{S}}$$

$$\tilde{Y} = \frac{\tilde{S}^*}{|\tilde{V}|^2} = \frac{(P+jQ)^*}{|\tilde{V}|^2}$$

Sol.

For G_1 :-

$$\tilde{E}_1' = 1.03\angle 8.88^\circ + j0.067\,\tilde{I}_1$$

$\hookrightarrow \tilde{E}_1' = \tilde{V}_t + jX_d'\tilde{I}$

$$\tilde{I}_1' = \frac{(P+jQ_1)^*}{V_1^*} = \frac{3.5 - j0.712}{1.03\angle -8.88} = 3.468\angle -2.62^\circ \text{ pu}$$

$$\tilde{E}_1' = 1.1\angle 20.82^\circ \text{ pu}$$

pre-Fault angle $\delta_1 = 20.82^\circ$ initial for during fault condition

For G_2 :-

$$\tilde{E}_2' = 1.02\angle 6.38^\circ + j0.1\,\tilde{I}_2$$

$$\tilde{I}_2 = \frac{(P+jQ_2)^*}{V_2^*} = \frac{1.85 - j0.298}{1.02\angle -6.38^\circ} = 1.84\angle -2.771^\circ \text{ pu}$$

$$\tilde{E}_2' = 1.065\angle 16.19^\circ \text{ pu}$$

$$\delta_2 = 16.19^\circ$$

The inFinite Bus :-

$\tilde{E}_3' = 1\angle 0$

$|\tilde{E}_3'| = 1 \quad , \quad \delta_3 = 0$

$\delta_{13} = \delta_1 - \delta_3 = \delta_1$

$\delta_{23} = \delta_2 - \delta_3 = \delta_2$

$$\tilde{Y}_{L4} = \frac{1.0 - j0.44}{(1.018)^2} = 0.9649 - j0.4246 \quad pu$$

$$\tilde{Y}_{L5} = \frac{0.5 - j0.16}{(1.011)^2} = 0.4892 - j0.1565 \quad pu$$

$$\tilde{Y}_{11} = \frac{1}{j0.067 + j0.022} = -j11.236 \quad pu$$

$\tilde{Y}_{12} = \tilde{Y}_{21} = 0$

$\tilde{Y}_{13} = \tilde{Y}_{31} = 0$

$$\tilde{Y}_{14} = \frac{-1}{j0.067 + j0.022} = j11.236 \quad pu = \tilde{Y}_{41}$$

$\tilde{Y}_{15} = \tilde{Y}_{51} = 0$

$$\tilde{Y}_{22} = \frac{1}{j0.1 + j0.04} = -j7.1429 \quad pu$$

$\tilde{Y}_{23} = \tilde{Y}_{32} = 0 \quad , \quad \tilde{Y}_{24} = \tilde{Y}_{42} = 0$

$$\tilde{Y}_{25} = \frac{-1}{j0.04 + j0.1} = j\,7.1429 = \tilde{Y}_{52}$$

$$\tilde{Y}_{33} = \frac{1}{(0.008 + j0.047)} + \frac{1}{(0.008 + j0.047)} + \frac{1}{0.007 + j0.04}$$

$$+ \frac{j0.082}{2} + \frac{j0.098}{2} + \frac{j0.098}{2} = 11.2841 - j65.4732 \text{ pu}$$

$$\tilde{Y}_{34} = \frac{-1}{0.007 + j0.04} = -4.245 + j\,24.2571 \text{ pu} = \tilde{Y}_{43}$$

$$\tilde{Y}_{35} = \frac{-1}{0.008 + j0.047} + \frac{-1}{0.008 + j0.047} = -7.0392 + j\,41.355 \text{ pu}$$

$$\tilde{Y}_{44} = \frac{1}{j0.022 + j0.067} + \frac{1}{0.007 + j0.04} + \frac{1}{0.018 + j0.11} + \underbrace{0.9649 - j0.4246}_{\tilde{Y}_{L4}}$$

$$+ \frac{j0.082}{2} + j\frac{0.226}{2}$$

$$\tilde{Y}_{45} = \tilde{Y}_{54} = \frac{-1}{0.018 + j0.11}$$

$$\tilde{Y}_{55} = \frac{1}{j0.04 + j0.1} + \frac{2}{(0.008 + j0.047)} + \frac{1}{0.018 + j0.11} + 0.4892 - j0.1565$$

$$+ \frac{j0.098}{2} + \frac{j0.098}{2} + \frac{j0.226}{2} = 8.977 - j57.297$$

$$\tilde{Y}_{Bus} = \begin{bmatrix} -j11.236 & 0 & 0 & j11.236 & 0 \\ 0 & -j7.1429 & 0 & 0 & j7.1429 \\ 0 & 0 & 11.28 - j65.4 & -4.2 + j24.2 & -7.03 + j41.3 \\ j11.236 & 0 & -4.2 + j24.2 & & \\ 0 & j7.1429 & -7.03 + j41.35 & & 8.977 - j57.29 \end{bmatrix}$$

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* pre - Fault equations :-

$$\frac{2H_1}{\omega_s} \cdot \frac{d^2\delta}{dt^2} = P_{m_1} - P_{e_1}$$

$$P_{e_1} = |E_1|^2 \underset{0}{G_{11}} + |E_1||E_2||\underset{0}{\tilde{Y}_{12}}|\cos(\delta_{12} - \theta_{12}) + |E_1||E_3||\underset{0}{\tilde{Y}_{13}}|\cos(\delta_{13} - \theta_{13}) + |E_1||E_4||Y_{14}|\cos(\delta_{14} - \theta_{14})$$

وطلعها

$$P_{e_1} = (1.1)(1.018)(11.236)\cos(\delta_1 - \delta_4 - \theta_{14})$$

$$= 12.58\cos(\delta_1 - 4.68 - 90)$$

$$= 12.58\sin(\delta_1 - 4.68^\circ) .$$

$$P_{e_2} = |E_2|^2 \underset{0}{G_{22}} + |E_2||E_1||\underset{0}{Y_{21}}|\cos(\delta_{21} - \theta_{21}) + |E_2||E_3||\underset{0}{Y_{23}}|\cos(\delta_{23} - \theta_{23}) + |E_2||E_4||\underset{0}{Y_{24}}|\cos(\delta_{24} - \theta_{24})$$

$$+ |E_2||E_5||Y_{25}|\cos(\delta_{25} - \theta_{25}) = (1.02)(1.011)(7.1429)\cos(\delta_2 - 2.27 - 90)$$

$$= 7.366\sin(\delta_2 - 2.27^\circ)$$

* the 3-ø Fault is at Bus 4 and therefore [Y_{14}, Y_{24}, Y_{34}, Y_{44}, Y_{54}, Y_{41}, Y_{42}, Y_{43}, Y_{44}, Y_{45}] must be completely eliminated.

$$Y_{Bus_1}^{(new)} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} & Y_{15} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} & Y_{25} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} & Y_{35} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} & Y_{45} \\ Y_{51} & Y_{52} & Y_{53} & Y_{54} & Y_{55} \end{bmatrix}$$

removed

⇒ Bus 1 :- has constant voltage, For three stages (pre, during, post) (remain same value).

Bus 2 :- has constant voltage (like Bus 1).

Bus 3 :- inFinite Bus (constant voltage).

Bus 4 :- is removed because the Fault is there.

Bus 5 :- has to be eliminated by Kron reduction because it's voltage during Fault is Unknown.

* AFter removing row 4 & coloumn 4 (short circuit), Y_{bus} will be 4X4 matrix :-

$$Y_{Bus}(new_1) = \begin{bmatrix} -j11.236 & 0 & 0 & 0 \\ 0 & -j7.1429 & 0 & j7.1429 \\ 0 & 0 & 11.281-j26.4731 & -7.0392+j41.3550 \\ 0 & j7.1429 & -7.0392+j41.3350 & 8.9772-j57.2972 \end{bmatrix}$$

* the row 4 & column 4 (Bus 5) in this matrix are to be eliminated by Kron reduction because the voltage across Bus 5 will change.

$$\tilde{Y}_{11}(new) = \tilde{Y}_{11} - \frac{\tilde{Y}_{14}\tilde{Y}_{41}}{\tilde{Y}_{44}} = -j11.2360 - \frac{(0)(0)}{8.9772-j57.2972}$$

$$= -j11.2360 \text{ pu}$$

$$\tilde{Y}_{12}(new) = \tilde{Y}_{21}(new) = 0$$

$$\tilde{Y}_{13}(new) = \tilde{Y}_{31}(new) = 0$$

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$$\Rightarrow \tilde{Y}_{22}(new) = 0.1362 - j6.2738 \text{ pu}$$

$$\tilde{Y}_{23}^{(new)} = -0.0681 + j5.1661 \text{ pu} = 5.1665 \angle 90.7552^\circ = \tilde{Y}_{32}(new)$$

$$\tilde{Y}_{33}(new) = 5.7986 - j35.6299 \text{ pu}$$

$$\underset{\underline{3\times3}}{\tilde{Y}_{Bus}(new_2)} = \begin{bmatrix} -j11.2360 & 0 & 0 \\ 0 & 0.1362-j6.2738 & -0.0681+j5.1661 \\ 0 & -0.0681+j5.1661 & 5.7986-j35.6299 \end{bmatrix}$$

$$P_{e_1} = |E_1|^2 \underbrace{G_{11}}_{0} + |E_1||E_2|\underbrace{|Y_{12}|}_{0}\cos(\delta_{12}-\theta_{12}) + |E_1||E_3|\underbrace{|Y_{13}|}_{0}\cos(\delta_{13}-\theta_{13}).$$

$$P_{e_1} = 0$$

$$P_{e_2} = |E_2|^2 G_{22} + |E_2||E_1|\underbrace{|Y_{21}|}_{0}\cos(\delta_{21}-\theta_{21}) + |E_2||E_3||Y_{23}|\cos(\delta_{23}-\theta_{23}).$$

$$P_{e_2} = (1.065)^2(0.1362) + (1.065)(1)(5.1665)\cos(\delta_2 - 0 - 90.7552^\circ).$$

$$P_{e_2} = 0.1545 + 5.5023 \sin(\delta_2 - 0.7552^\circ)$$

P_{e_3} = InFinite Bus.

ما في داعي أحسبه
لأنه ال Power
ما له swing equation
لأنه الطاقة معروفة
infinite Bus

* Swing equation (During Fault) :-

$$\frac{2H_1}{\omega_s}\frac{d^2\delta_1}{dt^2} = P_m - P_e$$

$$\frac{H}{180f}\frac{d^2\delta_1}{dt^2} = (3.5) - 0 \Rightarrow \frac{d^2\delta_1}{dt^2} = \frac{180f}{H}(3.5)$$

$$\delta_1(t) = \frac{180f}{H_1}\left[\frac{3.5}{2}t^2 + k_1 t + k_2\right]$$

$$\frac{d^2\delta_2}{dt^2} = \frac{180f}{H}\left[\underbrace{1.85 - 0.1545}_{1.6955} - 5.5023\sin(\delta_2 - 0.7552^\circ)\right]$$

numerical solution !!

* The Fault is cleared by Simultanuous opening of the circuit breaker at the two ends of the line determine the power angle and swing equation.
  - Because the Fault is cleared by removing the line 4-5 :-

    $\tilde{Y}_{45} = \tilde{Y}_{54} = 0$

    $\tilde{Y}_{44}$ & $\tilde{Y}_{55}$ must be modified by subtracting the series admittance of one half of the line.

  - Kron reduction For Buses 4 and 5 must be done :-

    $\tilde{Y}_{11}(new) = 0.5005 - j7.7897$ pu

    $\tilde{Y}_{12}(new) = \tilde{Y}_{21}(new) = 0$

    $\tilde{Y}_{13}(new) = -0.2216 + j7.6291$ pu $= \tilde{Y}_{31}(new) = 7.6323\angle 91.66^\circ$

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$\tilde{Y}_{22}(new) = 0.1591 - j6.1168$ pu

$\tilde{Y}_{23}(new) = -0.0901 + j6.097$ pu $= \tilde{Y}_{32}(new) = 6.098 \angle 90.85^\circ$ pu

$\tilde{Y}_{33}(new) = 1.3927 - j13.8728$ pu.

$$P_{e_1} = |\tilde{E}_1|^2 G_{11} + |\tilde{E}_1||\tilde{E}_2||\tilde{Y}_{12}| \cos(\delta_1 - \delta_2 - \theta_{12}) + |E_1||E_3||\tilde{Y}_{13}| \cos(\delta_1 - \delta_3 - \theta_{13})$$

$$= (1.1)^2(0.5005) + 0 + (1.1)(1)(7.6323)\cos(\delta_1 - 0 - 91.66^\circ)$$

$$P_{e_1} = 0.6056 + 8.3955 \sin(\delta_1 - 1.664^\circ)$$

$$P_{e_2} = |\tilde{E}_2|^2 G_{22} + |\tilde{E}_2||\tilde{E}_1||\tilde{Y}_{21}| \cos(\delta_2 - \delta_1 - \theta_{21}) + |\tilde{E}_2||\tilde{E}_3||\tilde{Y}_{23}| \cos(\delta_2 - \delta_3 - \theta_{23})$$

$$= (1.065)^2(0.1591) + 0 + (1.065)(1)(6.098)(\cos(\delta_2 - 0 - 90.85^\circ)$$

$$P_{e_2} = 0.1804 + 6.4934 \sin(\delta_2 - 0.85^\circ)$$

$$\frac{d^2\delta_1}{dt^2} = \frac{180f}{11.2}\left[3.5 - (0.6056 + 8.3955 \sin(\delta_1 - 1.664^\circ))\right]$$

$$\frac{d^2\delta_2}{dt^2} = \frac{180f}{8}\left[1.85 - (0.1804 + 6.4934 \sin(\delta_2 - 0.85^\circ))\right]$$

Ex:

$$\left.\begin{aligned} \frac{dx_1}{dt} &= 2x_1 - x_2^2 \\ \frac{dx_2}{dt} &= x_1 - 2 \end{aligned}\right\} \text{Steady - state solution} \quad \frac{dx_1}{dt} = \frac{dx_2}{dt} = 0$$

$$\left.\begin{aligned} 0 &= 2x_1 - x_2^2 \\ 0 &= x_1 - 2 \end{aligned}\right\} \left.\begin{aligned} x_1 &= 2 \\ x_2 &= \pm 2 \end{aligned}\right]$$

Pre-Fault solution
$\delta, \omega \Rightarrow$ initial condition for during Fault

الحل يوضع طريقتين ، كل بعدو "
يمكن في العادة نستخدم برامج كما في (Matlab
حل الـ الشكل العام & في الصفحة
اللي بعد

The code for initial conditions :

Editor - C:\Users\ALI AWAD\Documents\MATLAB\ssG1.m

```
syms x1 x2
F1=3.5-12.582*sin(x2-0.082);
F2=x1-377;
ss=solve(F1,F2);
a1=ss.x1
a2=ss.x2
```

MATLAB 7.10.0 (R2010a)

Command Window

```
a1 =

 377
 377


a2 =

      asin(1750/6291) + 41/500
 pi - asin(1750/6291) + 41/500

>>
```

الكـــود التالي بنستخدمه عشان نستدعي كل الـfunctionاللي موجودين عنا يعني بيستدعي الـ

Pre fault , during fault , post fault

Editor - C:\Users\ALI AWAD\Documents\MATLAB\G1_Solver.m

File Edit Text Go Cell Tools Debug Desktop Window Help

```
Tspan=[0 1];
yo=[377;0.3639];
[t1,x]=ode23tb('prefaultg1',Tspan,yo)


a=length(t1);
yo=[x(a,1);x(a,2)];
Tspan=[1 1.005];
[t2,y]=ode23tb('duringfaultg1',Tspan,yo)

aa=length(t2);
yo=[y(aa,1);y(aa,2)];
Tspan=[1.005 50];
[t3,z]=ode23tb('postfaultg1',Tspan,yo)

plot(t1,x(:,2),t2,y(:,2),t3,z(:,2))
xlabel('time (s)')
ylabel('Deltag Angle (rad)')
```

Editor - C:\Users\ALI AWAD\Documents\MATLAB\prefaultg1.m

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```
function xprime=prefaultg1(t,x)
xprime=[(180*60/11.2)*(3.5-12.582*sin(x(2)-0.082));
         x(1)-377];
```

01

Editor - C:\Users\ALI AWAD\Documents\MATLAB\duringfaultg1.m

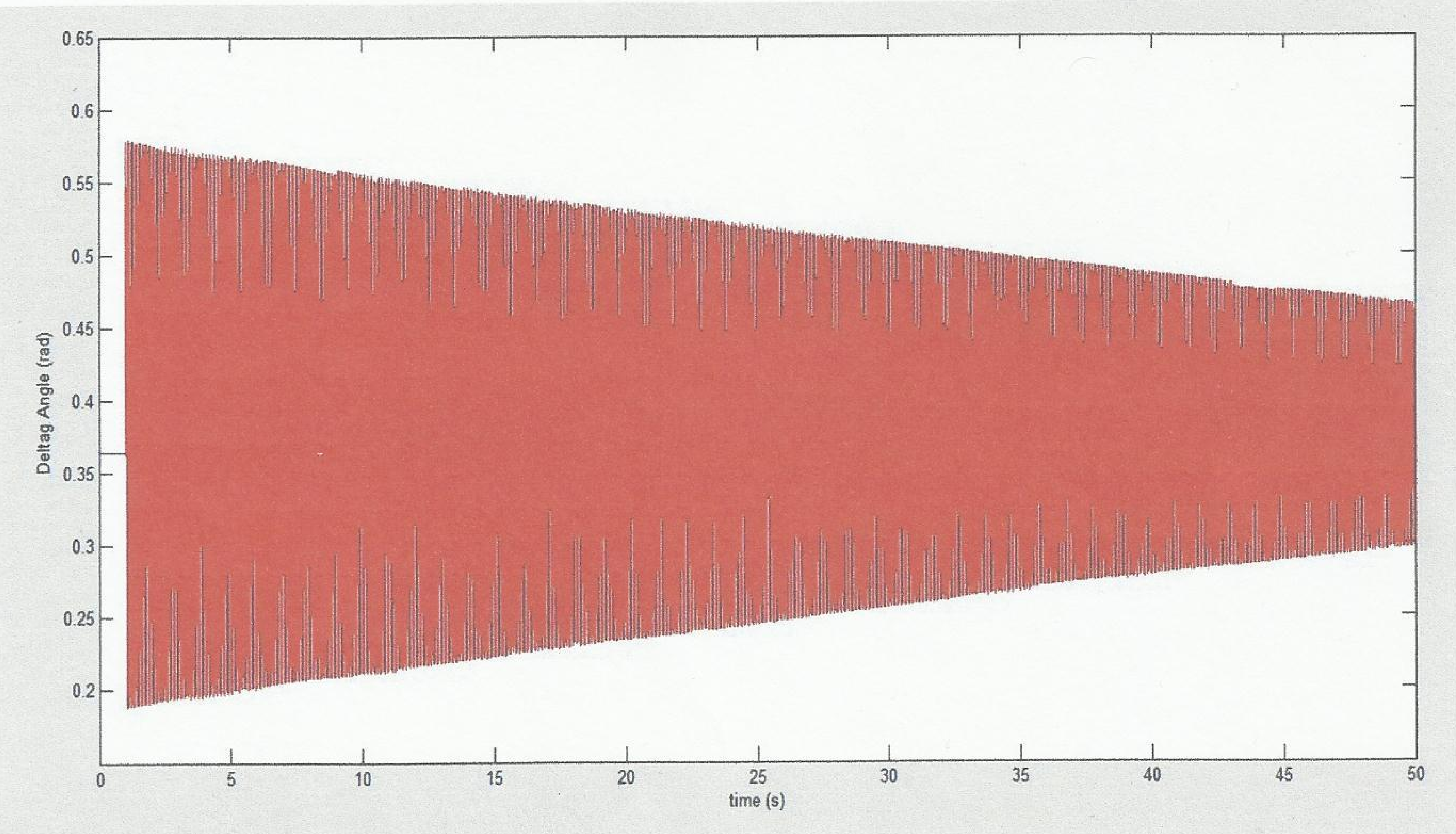
File Edit Text Go Cell Tools Debug Desktop Window Help

```
function xprime=duringfaultg1(t,x)
xprime=[(180*60/11.2)*3.5;
        x(1)-377];
```

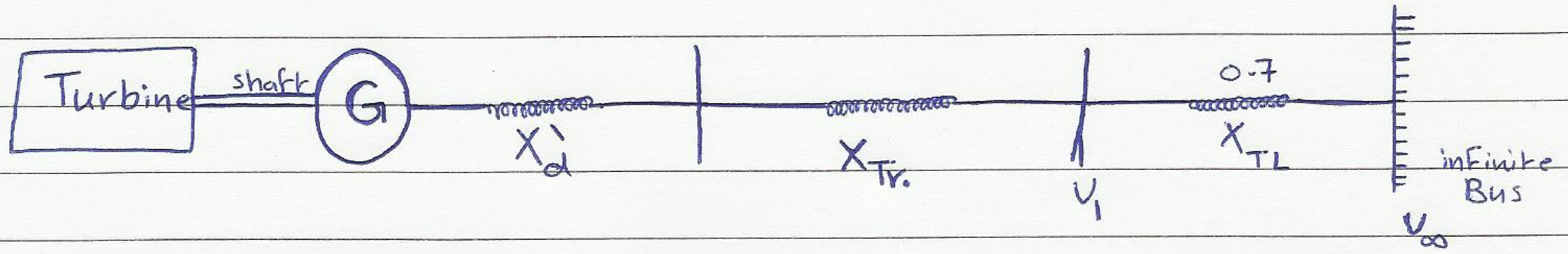
Editor - C:\Users\ALI AWAD\Documents\MATLAB\postfaultg1.m

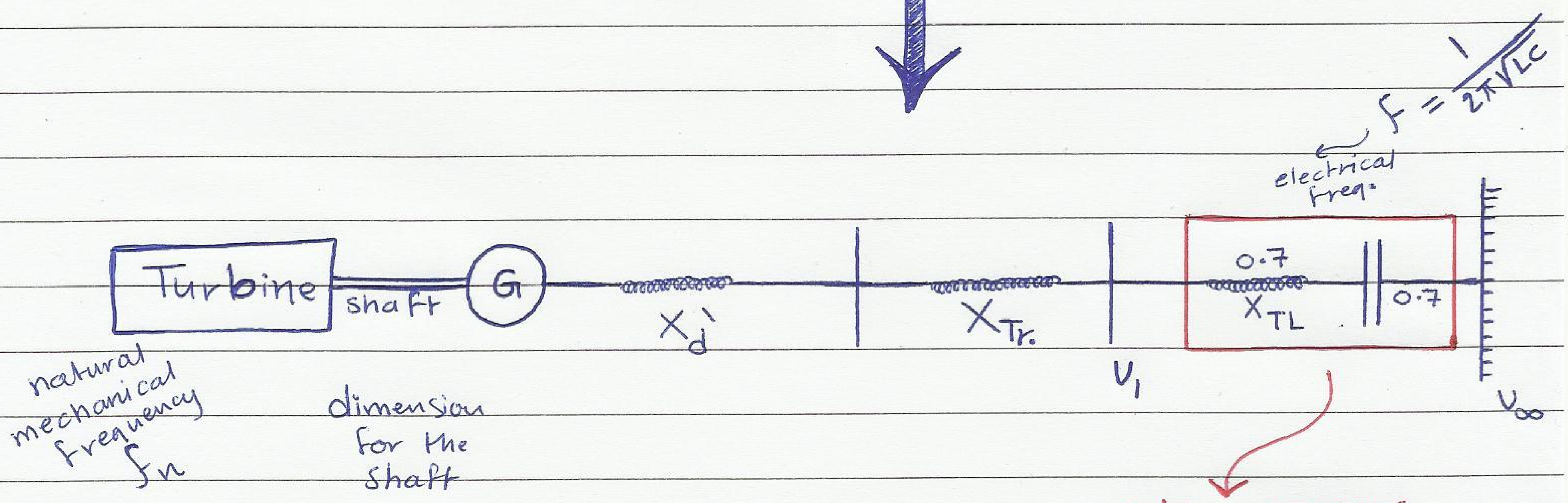
File Edit Text Go Cell Tools Debug Desktop Window Help

```
function xprime=postfaultg1(t,x)
xprime=[(180*60/11.2)*(2.8944-8.3955*sin(x(2)-0.029));
        x(1)-377];
```



16.11 :- Factors Affecting Transient stability :-





series compansation ← S

$$P_{max} = \frac{V_1 \cdot V_\infty}{X_{eq}} \sin\delta \quad , \quad P_{max} \uparrow$$

* Subsynchronous Resonance :- (SSR) electromechanical interaction takesplace if the electrical frequency approaches one of the natural mechanical frequency of the turbine generator section.

II as constant moment of inertia (H) increased, system stability is increased.

Sub. Date: / /

2] P_m :- as P_m decrease, the system stability is increase.
the lower P_m more and more, the generator must be disconnecting from the system.

because $P_e = 0$

($P_m - P_e$ = very large value that's mean there is large difference between two values $\Rightarrow$ very high acceleration).

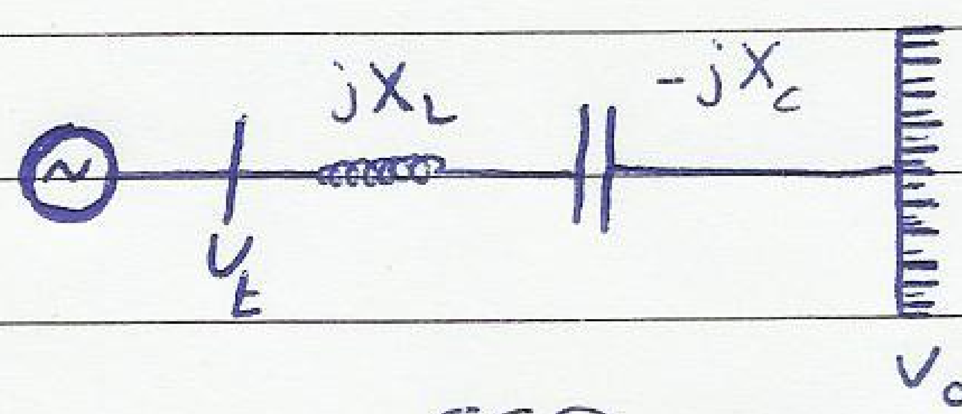
3] AVR :- Excitation control system or AVR tends, to increase the stability of power system.

4] Turbine Valve control :- Turbine governer, more stability system and it's operate when there is a difference between P_{in} and P_{out}.

5] t_{cr} : critical clearing time, the smaller t_{cr} and decrease it more and more is better the stability of the power system.

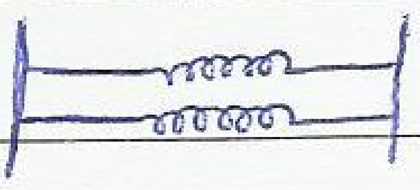
6] P_{max} :- static stability limit, higher P_{max} is more stable for the system because higher P_{max} is large difference between δ_e and $(\pi/2)$ and therefore more stable.

7] reduction in series reactance, $P_{max} \uparrow$, more stable.



SSR → reduce the value of X_{eq}

* Subsynchronous Resonance : SSR is interaction between the electrical system and mechanical system, connecting capacitors in series, it (X_c) is increase beyond a certain value then subsynchronous resonance might occure and cause unstability.



طريقة أخرى لتقليل قيمة الـ reactance
وهي توصيلها كالتوازي
مع الـ Bus

$$\uparrow P_{max} = \frac{V_1 \cdot V_\infty}{X_{eq} \downarrow}$$

OM

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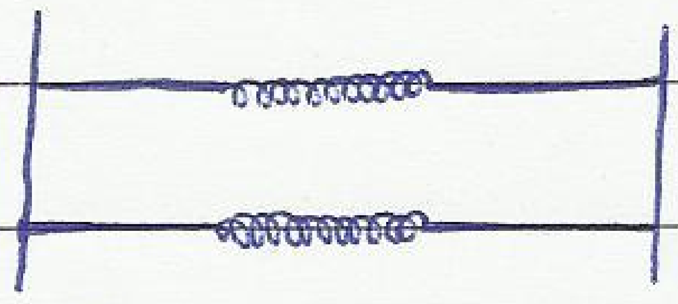
8] additional line : increase the number of parallel lines :-

(a) reducing the value of (X) and therefore increase P_{max}.



(b) Back up line :- some power is transffered over the remaining line during a 3-Ø Fault on one of them

↲ يعني إذا ضرب واحد من خطوط Fault الثاني بيظل شغال ← more stability.

* the higher the power transfer during Fault the more stable power system.

9] Damper winding :- improve the stability of power system.

10] power distributed : increase the number of power station.

↲ يعني آخر زيادة عدد الـ generator الموجودين في الـ system.

* the lower P_m ⇒ the lower load on the generator and lower angle δ ⇒ more stable system.

* if the initial angle δ_0 is increased, δ_{max} is decreased and a smaller difference between δ_0 and δ_{cr} exist. and therefore a smaller angular swing during any time interval.

11] photovoltaic generator.

12] Fuel cells.

END OF THE CHAPTER

Using $\mathbf{H}_{\mathbf{x}}^{(0)}$ obtained above, we have

$$\left[\mathbf{H}_{\mathbf{x}}^{(0)^T} \mathbf{R}^{-1} \mathbf{H}_{\mathbf{x}}^{(0)}\right]^{-1} =$$

$$\begin{bmatrix} 0.1884 \times 10^{-5} & 0.1355 \times 10^{-5} & 0.5578 \times 10^{-18} & 0.5571 \times 10^{-18} & 0.5583 \times 10^{-18} \\ & 0.1693 \times 10^{-5} & 0.4179 \times 10^{-18} & 0.4184 \times 10^{-18} & 0.4176 \times 10^{-18} \\ & & 0.4000 \times 10^{-3} & 0.4000 \times 10^{-3} & 0.4000 \times 10^{-3} \\ & & & 0.4019 \times 10^{-3} & 0.4014 \times 10^{-3} \\ & & & & 0.4017 \times 10^{-3} \end{bmatrix}$$

Finally, we have

$$\begin{bmatrix} \delta_2^{(1)} \\ \delta_3^{(1)} \\ |V_1|^{(1)} \\ |V_2|^{(1)} \\ |V_3|^{(1)} \end{bmatrix} = \begin{bmatrix} \delta_2^{(0)} \\ \delta_3^{(0)} \\ |V_1|^{(0)} \\ |V_2|^{(0)} \\ |V_3|^{(0)} \end{bmatrix} + \left(\mathbf{H}_{\mathbf{x}}^{(0)^T} \mathbf{R}^{-1} \mathbf{H}_{\mathbf{x}}^{(0)}\right)^{-1} \mathbf{H}_{\mathbf{x}}^{(0)^T} \mathbf{R}^{-1} \begin{bmatrix} e_1^{(0)} \\ e_2^{(0)} \\ e_3^{(0)} \\ e_4^{(0)} \\ e_5^{(0)} \\ e_6^{(0)} \\ e_7^{(0)} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.00948 \\ -0.03078 \\ 0.0 \\ 0.01003 \\ -0.00379 \end{bmatrix} = \begin{bmatrix} -0.00948 \text{ radian} \\ -0.03078 \text{ radian} \\ 1.0 \text{ per unit} \\ 1.01003 \text{ per unit} \\ 0.99621 \text{ per unit} \end{bmatrix}$$

Chapter 16 Problem Solutions

16.1 A 60-Hz four-pole turbogenerator rated 500 MVA, 22 kV has an inertia constant of $H = 7.5$ MJ/MVA. Find (*a*) the kinetic energy stored in the rotor at synchronous speed and (*b*) the angular acceleration if the electrical power developed is 400 MW when the input less the rotational losses is 740,000 hp.

Solution:

(*a*) Kinetic energy $= 500 \times 7.5 = 3750$ MJ

(*b*) Input power $= 740{,}000 \times 746 \times 10^{-6} = 552$ MW. By Eq. (16.14),

$$\text{Input power} - \text{rotational loss} = \frac{7.5}{180 \times 60}\frac{d^2\delta}{dt^2} = \frac{552 - 400}{500}$$

$$\frac{d^2\delta}{dt^2} = 437.8 \text{ elec. degrees/s}^2$$

For a four-pole machine,

$$\frac{d^2\delta}{dt^2} = \frac{437.8}{2} = 218.9 \text{ mech. degrees/s}^2$$

$$\text{or } 60 \times \frac{218.9}{360} = 36.5 \text{ rpm/s}^2$$

16.2 If the acceleration computed for the generator described in Prob. 16.1 is constant for a period of 15 cycles, find the change in δ in electrical degrees in that period and the speed in revolutions per minute at the end of 15 cycles. Assume that the generator is synchronized with a large system and has no accelerating torque before the 15-cycle period begins.

Solution:

$$\begin{aligned}
\text{duration of acceleration} &= \frac{15}{60} = 0.25 \text{ s} \\
\text{acceleration} &= 437.8 \text{ elec. degrees/s}^2 = 36.5 \text{ rpm/s} \\
\text{change in } \delta \text{ in 15 cycles} &= \frac{1}{2}(437.8)(0.25)^2 = 13.68 \text{ elec. degrees} \\
\text{synchronous speed} &= \frac{120 \times 60}{4} = 1800 \text{ rpm} \\
\text{After 15 cycles, speed} &= 1800 + 0.25 \times 36.5 = 1809.1 \text{ rpm}
\end{aligned}$$

16.3 The generator of Prob. 16.1 is delivering rated megavolt-amperes at 0.8 power factor lag when a fault reduces the electric power output by 40%. Determine the accelerating torque in newton-meters at the time the fault occurs. Neglect losses and assume constant power input to the shaft.

Solution:

$$\begin{aligned}
P_a &= \omega_m T_a = 0.8 \times 500 - 0.6 \times 0.8 \times 500 = 160 \text{ MW} \\
\omega_m &= \frac{2\pi f}{2} \text{ mech. radians/s} \\
T_a &= \frac{160 \times 10^6}{2\pi f/2} = 848,826 \text{ N·m}
\end{aligned}$$

16.4 Determine the WR^2 of the generator of Prob. 16.1.

Solution:

$$WR^2 = \frac{S_{mach} \times H \times 10^{10}}{2.31(\text{rpm})^2} = \frac{500 \times 7.5 \times 10^{10}}{2.31(1800)^2} = 5,010,422 \text{ lb-ft}^2$$

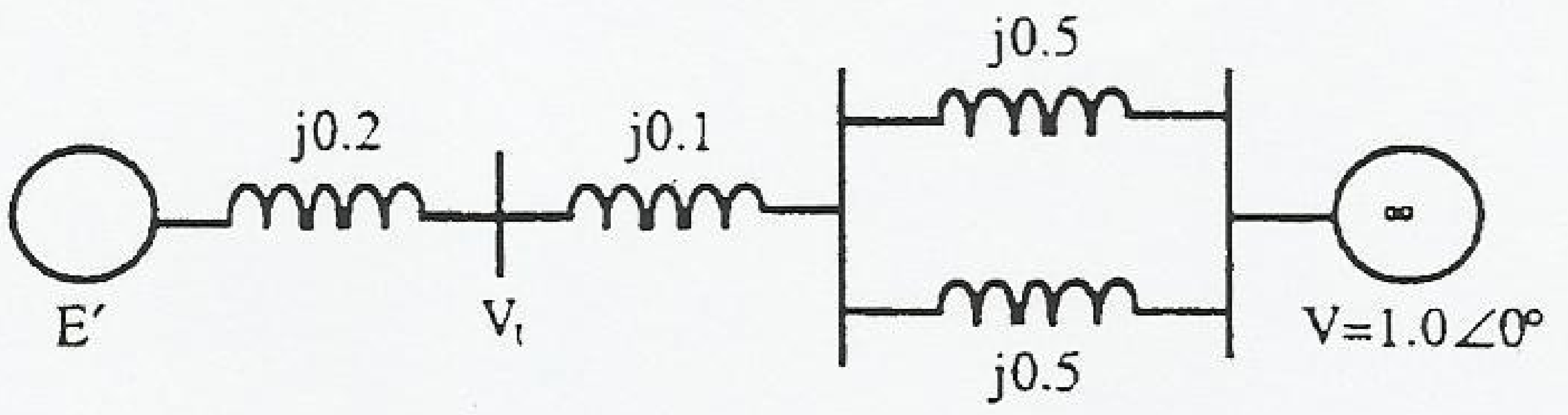
16.5 A generator having $H = 6$ MJ/MVA is connected to a synchronous motor having $H = 4$ MJ/MVA through a network of reactances. The generator is delivering power of 1.0 per unit to the motor when a fault occurs which reduces the delivered power. At the time when the reduced power delivered is 0.6 per unit determine the angular acceleration of the generator with respect to the motor.

Solution:

$$\begin{aligned}
\frac{6 \times 4}{6+4} \times \frac{1}{180f} \frac{d^2\delta_{12}}{dt^2} &= 1.0 - 0.6 \\
\frac{d^2\delta_{12}}{dt^2} &= 1800 \text{ elec. degrees/s}^2
\end{aligned}$$

16.6 A power system is identical to that of Example 16.3 except that the impedance of each of the parallel transmission lines is $j0.5$ and the delivered power is 0.8 per unit when both the terminal voltage of the machine and the voltage of the infinite bus are 1.0 per unit. Determine the power-angle equation for the system during the specified operating conditions.

Solution:



X between V_t and V is

$$j0.1 + \frac{j0.5}{2} = j0.35 \text{ per unit}$$

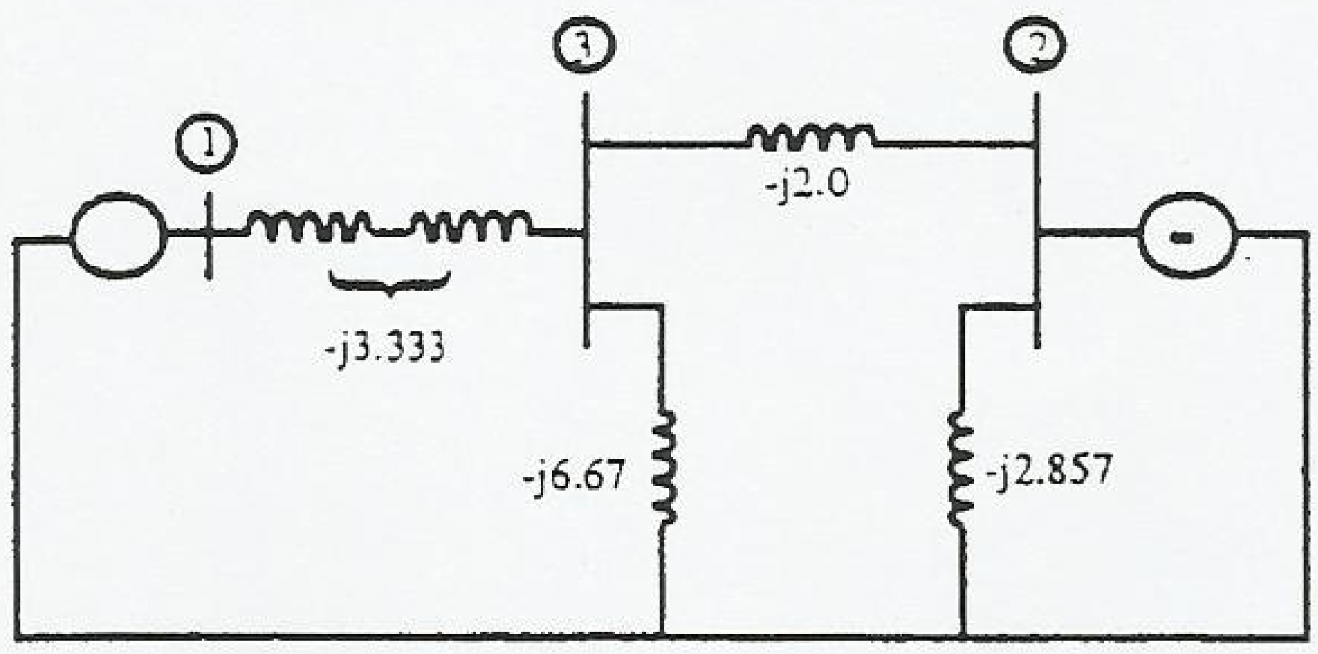
If $V_t = 1.0\angle\alpha$,

$$\begin{aligned}
\frac{1.0 \times 1.0}{j0.35}\sin\alpha &= 0.8, \quad \alpha = 16.26^\circ \\
I &= \frac{1.0\angle 16.26^\circ - 1.0\angle 0^\circ}{0.35\angle 90^\circ} = \frac{0.96 + j0.28 - 1.0}{j0.35} \\
&= 0.8 + j0.1143 = 0.8081\angle 8.13^\circ \\
E' &= 1.0\angle 16.26^\circ + 0.808\angle 8.13^\circ \times 0.2\angle 90^\circ \\
&= 0.96 + j0.28 - 0.023 + j0.16 = 1.0352\angle 25.15^\circ \\
P_e &= \frac{1.0352 \times 1.0}{0.35 + 0.20}\sin\delta = 1.882\sin\delta
\end{aligned}$$

16.7 If a three-phase fault occurs on the power system of Prob. 16.6 at a point on one of the transmission lines at a distance of 30% of the line length away from the sending-end terminal of the line, determine (a) the power-angle equation during the fault and (b) the swing equation. Assume the system is operating under the conditions specified in Prob. 16.6 when the fault occurs. Let $H = 5.0$ MJ/MVA as in Example 16.4.

Solution:

The circuit diagram with admittances marked in per unit and the fault as described is shown below:



$$
\mathbf{Y}_{\text{bus}} = \begin{bmatrix} -j3.333 & 0 & j3.333 \\ 0 & -j4.857 & j2.0 \\ j3.333 & j2.0 & -j12.0 \end{bmatrix}
$$

After elimination of node 3 by the usual method, in row 1, column 2 of the new $\mathbf{Y}_{\text{bus}}$ matrix,

$$
\begin{aligned}
Y_{12} &= \frac{j2.0 \times j3.333}{-j12} = j0.556 \\
P_e &= 1.0352 \times 1.0 \times 0.556 \sin\delta = 0.575 \sin\delta \\
\frac{5}{180f}\frac{d^2\delta}{dt^2} &= 0.8 - 0.575\sin\delta \\
\frac{d^2\delta}{dt^2} &= 36f(0.8 - 0.575\sin\delta)
\end{aligned}
$$

16.8 Series resistance in the transmission network results in positive values for P_c and γ in Eq. (16.80). For a given electrical power output, show the effects of resistance on the synchronizing coefficient S_p, the frequency of rotor oscillations and the damping of these oscillations.

Solution:

Equation (16.80) is $P_e = P_c + P_{max}\sin(\delta - \gamma)$ and Eq. (16.47) defines

$$
S_p = \left.\frac{dP_e}{d\delta}\right|_{\delta=\delta_0}
$$

So, if the network is resistive

$$
S_p = P_{max}\cos(\delta_0 - \gamma)
$$

This S_p is greater than that for a purely reactive network where $\gamma = 0$. Hence, by Eq. (16.50) which shows

$$
f_n = \sqrt{\frac{S_p\omega_s}{2H}}
$$

wherein f_n is correspondingly larger. We now define $\delta' = \delta - \gamma$ and $P'_m = P_m - P_c$ so that the swing equation becomes

$$
\frac{2H}{\omega_s} \times \frac{d^2\delta'}{dt^2} = P'_m - P_{max}\sin\delta'
$$

which must have a solution reflecting undamped oscillations (see footnote in Sec. 16.5) as in a purely reactive network. Consequently, series resistance cannot introduce damping of mechanical oscillations.

16.9 A generator having $H = 6.0$ MJ/MVA is delivering power of 1.0 per unit to an infinite bus through a purely reactive network when the occurrence of a fault reduces the generator output power to zero. The maximum power that could be delivered is 2.5 per unit. When the fault is cleared the original network conditions again exist. Determine the critical clearing angle and critical clearing time.

Solution:

$$\begin{aligned}
2.5 \sin \delta_0 &= 1.0 \\
\delta_0 &= 23.58^\circ \text{ or } 0.4115 \text{ rad} \\
\text{By Eq. (16.70), } \delta_{cr} &= \cos^{-1}\left[(\pi - 0.823)\sin 23.58^\circ - \cos 23.58^\circ\right] \\
&= \cos^{-1}(0.9275 - 0.9165) = 89.27^\circ = 1.560 \text{ rad} \\
\text{By Eq. (16.72), } t_{cr} &= \sqrt{\frac{4 \times 6\,(1.395 - 0.4115)}{2\pi 60 \times 1.0}} = 0.270 \text{ s}
\end{aligned}$$

16.10 A 60-Hz generator is supplying 60% of P_{max} to an infinite bus through a reactive network. A fault occurs which increases the reactance of the network between the generator internal voltage and the infinite bus by 400%. When the fault is cleared the maximum power that can be delivered is 80% of the original maximum value. Determine the critical clearing angle for the condition described.

Solution:

$$\begin{aligned}
P_{max} \sin \delta_0 &= 0.6 P_{max} \\
\delta_0 &= 36.87^\circ,\ 0.6435 \text{ rad} \\
r_1 &= 0.25 \qquad r_2 = 0.8 \\
r_2 P_{max} \sin \delta_{max} &= P_m \quad \text{(Fig. 16.11)} \\
\frac{P_m}{P_{max}} &= 0.6 \quad \text{(given)} \\
\sin \delta_{max} &= \frac{0.6}{0.8} = 0.75 \\
\delta_{max} &= 180^\circ - 48.59^\circ = 131.41^\circ = 2.294 \text{ rad} \\
\cos \delta_{cr} &= \frac{0.6(2.294 - 0.6435) + 0.8 \cos 131.4^\circ - 0.25 \cos 36.87^\circ}{0.8 - 0.25} = 0.475 \\
\delta_{cr} &= \cos^{-1} 0.475 = 61.64^\circ
\end{aligned}$$

16.11 If the generator of Prob. 16.10 has an inertia constant of $H = 6$ MJ/MVA and P_m (equal to 0.6 P_{max}) is 1.0 per-unit power, find the critical clearing time for the condition of Prob. 16.10. Use $\Delta t = 0.05$ to plot the necessary swing curve.

Solution:

From Prob. 16.10, $\delta_{cr} = 61.64°$ and t_{cr} can be read from the swing curve for a sustained fault

$$
\begin{aligned}
P_{max} &= \frac{1.0}{0.6} = 1.667 \text{ per unit} \\
P_e &= 1.667/4 = 0.4167 \text{ during fault} \\
k &= \frac{180 \times 60}{6}(0.05)^2 = 4.5 \\
\delta_0 &= 36.87° \qquad P_m = 1.0 \qquad P_c = 0 \qquad Y = 0
\end{aligned}
$$

Values in the table below were found by a computer program and rounded off only for tabulation.

t	P_e	P_a	kP_a	$\Delta\delta_n$	δ_n
0−	1.0	0			36.87°
0+	0.250	0.75			36.87°
0 av		0.375	1.688		36.87°
				1.688°	
0.05	0.260	0.740	3.331		38.56°
				5.019°	
0.10	0.287	0.713	3.207		43.58°
				8.226°	
0.15	0.328	0.673	3.026		51.81°
				11.252°	
0.20					63.05°

Problem 16.11 Solution Data

By linear interpolation,

$$
\begin{aligned}
t_c &\cong 0.15 + 0.05\left(\frac{61.64 - 51.81}{63.05 - 51.81}\right) \\
&\cong 0.15 + 0.044 = 0.194 \text{ s or } 11.6 \text{ cycles}
\end{aligned}
$$

16.12 For the system and fault conditions described in Probs. 16.6 and 16.7 determine the power-angle equation if the fault is cleared by the simultaneous opening of breakers at both ends of the faulted line at 4.5 cycles after the fault occurs. Then plot the swing curve of the generator through $t = 0.25$ s.

Solution:

From Prob. 16.6 and 16.7 $E' = 1.0352\angle 25.15°$ per unit and before the fault

$$P_e = 1.882 \sin\delta \qquad P_m = 0.8 \qquad \delta_0 = 25.15°$$

During the fault,

$$P_e = 0.575 \sin\delta$$

after clearing,

$$Y_{12} = \frac{1}{j0.3 + j0.5} = -j1.25 \text{ per unit}$$

and

$$\begin{aligned} P_e &= 1.0352 \times 1.0 \times 1.25 \sin\delta = 1.294 \sin\delta \\ k &= \frac{180 \times 60}{5}(0.05)^2 = 5.4 \\ 4.5 \text{ cycles} &= 0.075 \text{ s (middle of interval)} \end{aligned}$$

Values in the table below were found by a computer program and rounded off only for tabulation.

t	P_e	P_a	kP_a	$\Delta\delta_n$	δ_n
0−	0.8	0.0			25.15°
0+	0.244	0.556	3.000		25.15°
0 av			1.500		25.15°
				1.500°	
0.05	0.258	0.542	2.927		26.65°
				4.427°	
0.10	0.668	0.132	0.713		31.08°
				5.140°	
0.15	0.765	0.035	0.191		36.22°
				5.332°	
0.20	0.858	−0.058	−0.315		41.55°
				5.017°	
0.25					46.57°

Problem 16.12 Solution Data

Note: If the table is continued a maximum value of δ will be found equal to 56.20° at $t = 0.45$ s. At 0.55 s, $\delta = 52.56°$.

16.13 Extend Table 16.6 to find δ at $t = 1.00$ s.

Solution:

Continuing the computer program used to generate Table 16.6 and tabulating values only to the fourth decimal place we obtain:

t	$\delta_n - \gamma$	$P_{max}\sin(\delta_n - \gamma)$	P_a	kP_a	$\Delta\delta_n$	δ_n
0.85	16.9591	1.8940	−0.2244	−0.7575		17.8061°
					−3.2292°	
0.90	13.7299	1.5412	0.1284	0.4334		14.5769°
					−2.7957°	
0.95	10.9342	1.2317	0.4379	1.4780		11.7812°
					−1.3177°	
1.0						10.4634°

Problem 16.13 Solution Data

Note: At $t = 1.05$, $\delta = 11.1196°$.

Sample calculation (at $t = 0.85$ s):

$$\begin{aligned}
\delta_n - \gamma &= 17.8061 - 0.847 = 16.9591° \\
P_{max}\sin(\delta - \gamma) &= 6.4934 \sin 16.9591° = 1.8940 \\
P_a &= P_m - P_c - P_{max}\sin(\delta - \gamma) = 1.6696 - 1.8940 = -0.2244 \\
kP_a &= -0.7574 \\
\Delta\delta_n &= \Delta\delta_{n-1} - kP_a = -2.4716 - (-0.7574) = -3.2292°
\end{aligned}$$

16.14 Calculate the swing curve for machine 2 of Examples 16.9 – 16.11 for fault clearing at 0.05 s by the method described in Sec. 16.9. Compare the results with the values obtained by the production-type program and listed in Table 16.7.

Solution:

Using the computer programmed to obtain δ vs. t showing intermediate steps in the calculation and rounding off only for tabulation we have

t	$\delta_n - \gamma$	$P_{max} \sin(\delta_n - \gamma)$	P_a	kP_a	$\Delta\delta_n$	δ_n
0−			0.000			16.19°
0+	15.435	1.4644	0.2310			16.19°
0 av			0.1155	0.3898		16.19°
					0.3898°	
0.05−	15.8248	1.5005	0.1950			16.5798°
0.05+	15.7328	1.7607	−0.0911			
0.05 av			0.0520	0.1753		
					0.5653°	
0.10	16.2983	1.8223	−0.1527	−0.5153		17.1453°
					0.0500°	
0.15	16.3483	1.8227	−0.1581	−0.5337		17.1953°
					−0.4837°	
0.20	15.8685	1.7751	−0.1055	−0.3559		16.7155°
					−0.8396°	
0.25	15.0249	1.6833	−0.0137	−0.0464		15.8719°
					−0.8860°	
0.30	14.1389	1.5862	0.0834	0.2816		14.9859°
					−0.6044°	
0.35	13.5345	1.5197	0.1499	0.5061		14.3815°
					−0.0983°	
0.40	13.4361	1.5088	0.1608	0.5427		14.2831°
					0.4443°	
0.45	13.8804	1.5577	0.1119	0.3775		14.7274°
					0.8218°	
0.50						15.5493°
0.55						16.444°
0.60						17.0813°
0.65						17.2267°

Problem 16.14 Solution Data

Note: Collecting student prepared computer programs is suggested.

16.15 If the three-phase fault on the system of Example 16.9 occurs on line ④–⑤ at bus ⑤ and is cleared by simultaneous opening of breakers at both ends of the line at 4.5 cycles after the fault occurs prepare a table like that of Table 16.6 to plot the swing curve of machine 2 through $t = 0.30$ s.

Solution:

Before the fault and after clearing, the conditions are the same as in Examples 16.9 and 16.11. During the fault P_m is still 1.85 per unit for machine 2, but $P_e = 0$. So, $P_a = 1.85$ per unit. After clearing, $P_m - P_c = 1.6696$, $P_{max} = 6.4934$. $Y = 0.847°$. Clearing in 4.5 cycles, or $t = 0.075$ s. Values in the table below were obtained by a computer program and rounded off for tabulation only.

t	$P_{max}\sin(\delta_n - \gamma)$	P_a	kP_a	$\Delta\delta_n$	δ_n
0−	1.85	0	0	0°	16.19°
0+	0	1.850	6.244		16.19°
0 av		0.925	3.122		16.19°
				3.122°	
0.05	0	1.85	6.244		19.31°
				9.366°	
0.10	3.031	−1.362	−4.596		28.68°
				4.769°	
0.15	3.498	−1.829	−6.172		33.45°
				−1.403°	
0.20	3.363	−1.694	−5.717		32.04°
				−7.120°	
0.25	2.649	−0.979	−3.306		24.92°
				−10.425°	
0.30	1.533	0.137	0.463		14.50°
				−9.963°	
0.35	0.418	1.252	4.225		4.54°
				−5.738°	
0.40	−0.232	1.902	6.419		−1.20°
				0.681°	
0.45	−0.155	1.825	6.158		−0.52°
				6.839°	
0.50	0.619	1.051	3.546		6.32°
				10.385°	
0.55					16.70°

Problem 16.15 Solution Data

Note: Although the problem does not ask for values beyond $t = 0.30$ s, the table has been extended to show the extent of the variation of δ.

16.16 By applying the equal-area criterion to the swing curves obtained in Examples 16.9 and 16.10 for machine 1, (*a*) derive an equation for the critical clearing angle, (*b*) solve the equation by trial and error to evaluate δ_{cr} and (*c*) use Eq. (16.72) to find the critical clearing time.

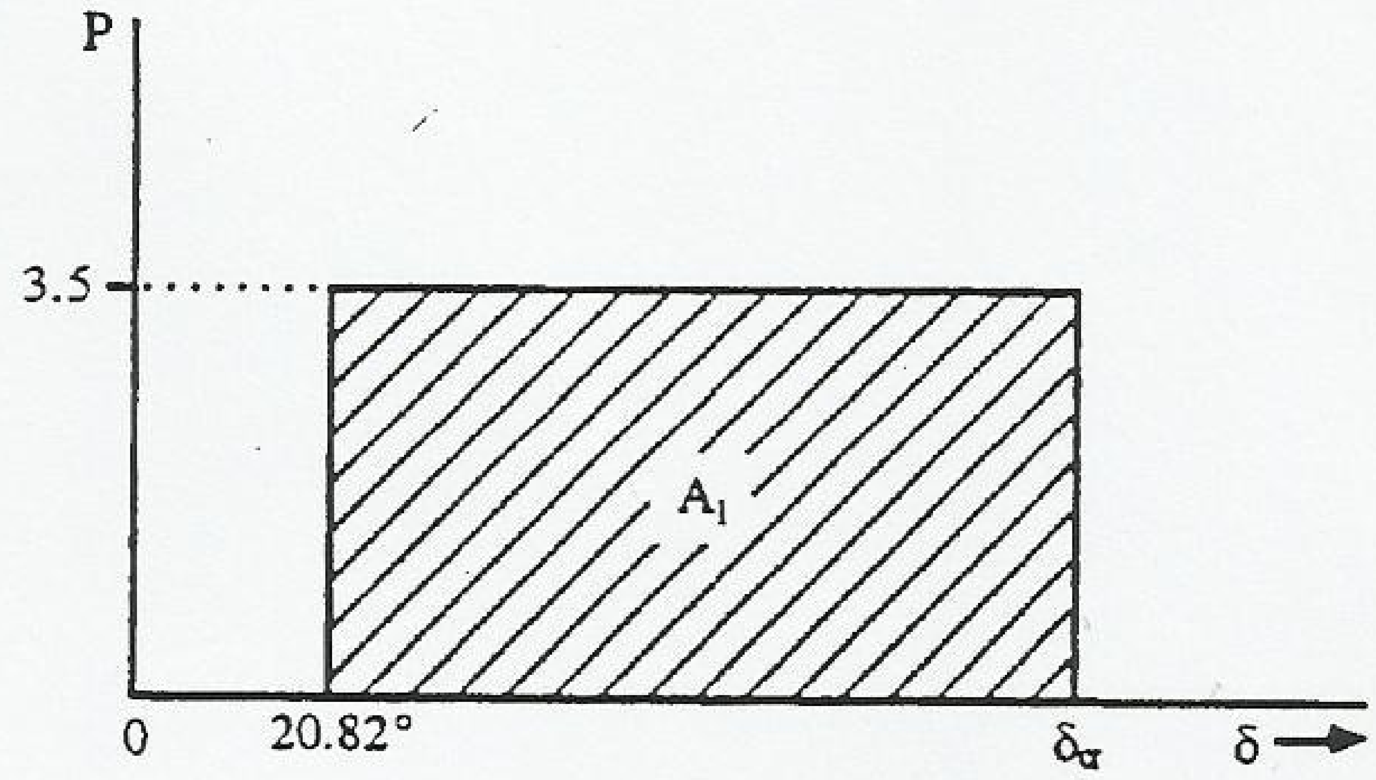
Solution:

Note: Students may need guidance in starting this problem which determines the critical clearing time for machine 1 for the fault specified in Example 16.9. This time must, of course, be less than 0.225 s as is evident from examination of Fig. 16.15 and Table 16.7.

(*a*) From Example 16.9 for machine 1:

$$P_m = 3.5 \text{ per unit} \quad \text{(Table 16.3)}$$
$$E_1' = 1.100\angle 20.82^\circ$$
$$\text{Thus, } \delta_0 = 20.82^\circ = 0.3634 \text{ rad}$$

Since the impedance between E_1' and the three-phase fault is pure inductive reactance, $P_e = 0$ during the fault and $P_a = P_m - P_e = 3.5$. The area A_1 for the equal-area criterion is shown below.

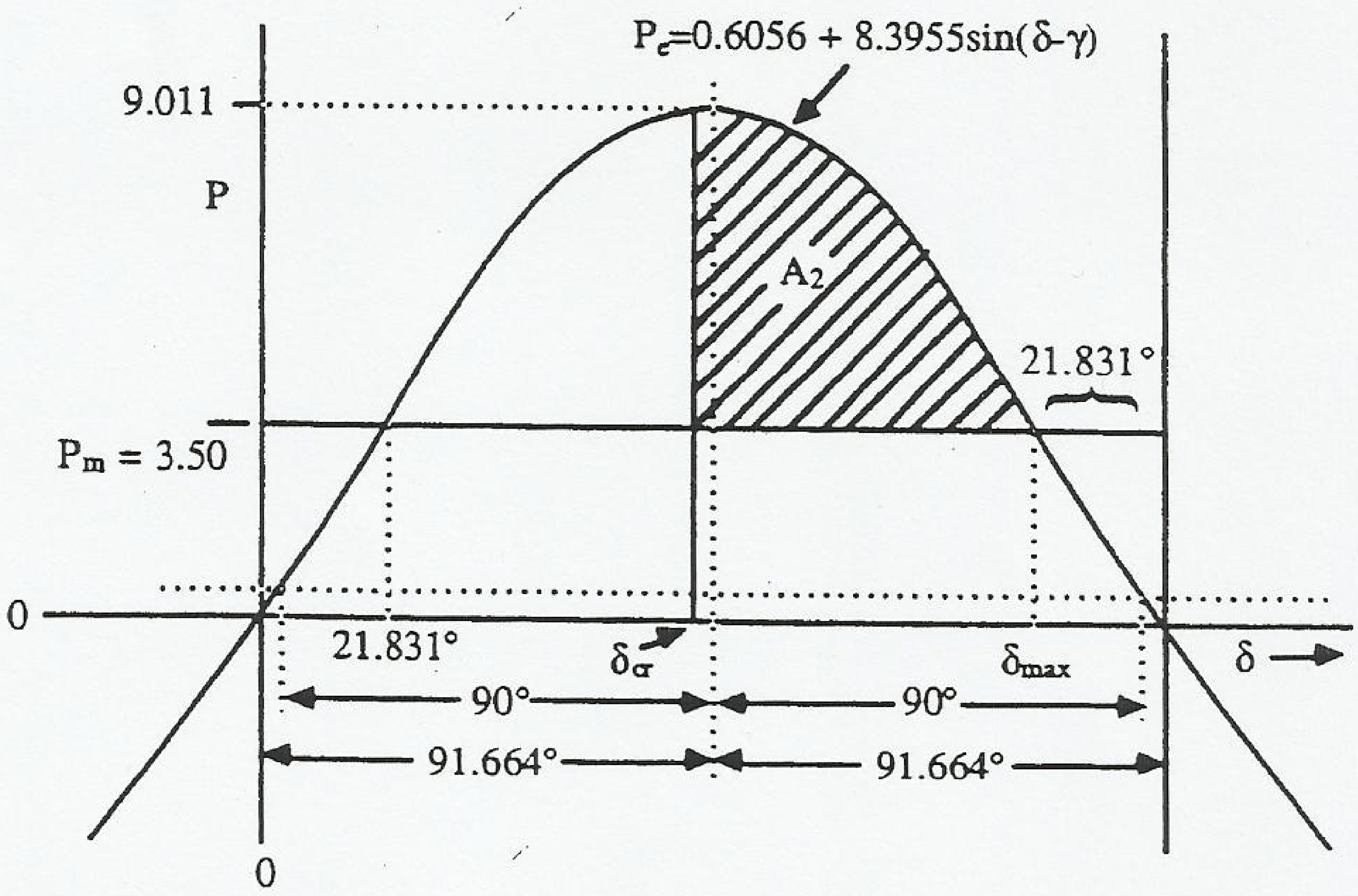


$$\text{where} \quad A_1 = 3.5(\delta_{cr} - 0.3634) = 3.5\delta_{cr} - 1.2719$$

From Example 16.10, the post-fault power-angle curve is given by

$$P_e = 0.6056 + 8.3955\sin(\delta - 1.664^\circ)$$

The curve, P_e vs. δ, is shown below:



Where P_m intercepts the fault curve,

$$3.5 = 0.6056 + 8.3955\sin(\delta - 1.664^\circ)$$

$$
\begin{aligned}
\delta &= 21.8309^\circ \\
P_{e,max} &= 0.6056 + 8.3955 \quad \text{where } \delta = 90^\circ + 1.664^\circ = 91.664^\circ \\
\delta_{max} &= 2 \times 91.664^\circ - 21.8309^\circ = 161.497^\circ = 2.8187 \text{ rad} \\
\text{Area } A_2 &= \int_{\delta_{cr}}^{\delta_{max}} \left[0.6056 + 8.3955 \sin(\delta - 1.664^\circ)\right] d\delta - 3.50\,(\delta_{max} - \delta_{cr}) \\
&= (0.6056 - 3.5)\,(\delta_{max} - \delta_{cr}) + 8.3955\left[\cos(\delta_{cr} - 1.664^\circ) - \cos(\delta_{max} - 1.664^\circ)\right] \\
&= -2.8944\,(2.8187 - \delta_{cr}) + 8.3955\left[\cos(\delta_{cr} - 1.664^\circ) - \cos(161.497^\circ - 1.664^\circ)\right] \\
&= -0.2776 + 2.8944\delta_{cr} + 8.3955\cos(\delta_{cr} - 1.664^\circ)
\end{aligned}
$$

Equating A_1 and A_2 yields

$$0.6056\delta_{cr} - 8.3955\cos(\delta_{cr} - 1.664^\circ) = 0.9943$$

(b) By trial and error we find

$$\delta_{cr} \cong 91.83^\circ = 1.6027 \text{ rad}$$

(c) The critical clearing time can be found from Eq. (16.72) since $P_e = 0$ during the fault:

$$t_{cr} = \sqrt{\frac{4 \times 11.2\,(1.6027 - 0.3644)}{377 \times 3.5}} = 0.205 \text{ s}$$