POWER SYSTEM STABILITY

Introduction

Stability of a power system is its ability to return to normal or stable operating conditions after having been subjected to some form of disturbance. Conversely, instability means a condition denoting loss of synchronism or falling out of step.

Though stability of a power system is a single phenomenon, for the purpose of analysis, it is classified as Steady State Analysis and Transient Stability.

Increase in load is a kind of disturbance. If increase in loading takes place gradually and in small steps and the system withstands this change and performs satisfactorily, then the system is said to be in STADY STATE STABILITY. Thus the study of steady state stability is basically concerned with the determination of upper limit of machine's loading before losing synchronism, provided the loading is increased gradually at a slow rate.

In practice, load change may not be gradual. Further, there may be sudden disturbances due to

i) **Sudden change of load**
ii) **Switching operation**
iii) **Loss of generation**
iv) **Fault**

Following such sudden disturbances in the power system, rotor angular differences, rotor speeds, and power transfer undergo fast changes whose magnitudes are dependent upon the severity of disturbances. For a large disturbance, changes in angular differences may be so large as to cause the machine to fall out of step. This type of instability is known as TRANSIENT INSTABILITY. Transient stability is a fast phenomenon, usually occurring within one second for a generator close to the cause of disturbance.

Short circuit is a severe type of disturbance. During a fault, electrical powers from the nearby generators are reduced drastically, while powers from remote generators are scarily affected. In some cases, the system may be stable even with sustained fault; whereas in other cases system will be stable only if the fault is cleared with sufficient rapidity. Whether the system is stable on the occurrence of a fault depends not only on the system itself, but also on the type of fault, location of fault, clearing time and the method of clearing.

Transient stability limit is almost always lower than the steady state limit and hence it is much important. Transient stability limit depends on the type of disturbance, location and magnitude of disturbance.

Review of mechanics

Transient stability analysis involves some mechanical properties of the machines in the system. After every disturbance, the machines must adjust the relative angles of their rotors to meet the condition of the power transfer involved. The problem is mechanical as well as electrical.

The kinetic energy of an electric machine is given by

$$K.E. = \frac{1}{2} I \omega^2 \text{ Mega Joules} \qquad (1)$$

where I is the Moment of Inertia in Mega Joules sec.^{2} / elec. deg.^{2}

ω is the angular velocity in elec. deg. / sec.

Angular Momentum M = $I\,\omega$; Then from eqn. (1), K.E. can be written as

$$K.E. = \frac{1}{2} M \omega \text{ Mega Joules} \qquad (2)$$

The angular momentum M depends on the size of the machine as well as on its type.

The Inertia constant H is defined as the Mega Joules of stored energy of the machine at synchronous speed per MVA of the machine. When so defined, the relation between the Angular Momentum M and the Inertia constant H can be derived as follows.

Relationship between M and H

By definition $H = \dfrac{\text{Stored energy in MJ}}{\text{Machine's rating in MVA}}$

Let G be the rating of the machine in MVA. Then

$$\text{Stored energy} = G\,H \ \text{MJ} \qquad (3)$$

$$\text{Further, K.E.} = \frac{1}{2} M\,\omega \ \text{MJ} = \frac{1}{2} M\,(2\,\pi\,f) \ \text{MJ} = M \times \pi\,f \ \text{MJ} \qquad (4)$$

From eqns. (3) and (4), we get

$G\,H = M \times \pi\,f$; Thus

$$M = \frac{G\,H}{\pi\,f} \ \text{MJ sec. / elec. rad.} \qquad (5)$$

If the power is expressed in per unit, then G = 1.0 per unit and hence

$$M = \frac{H}{\pi\,f} \qquad (6)$$

While the angular momentum M depend on the size of the machine as well as on its type, inertia constant H does not vary very much with the size of the machine, The quantity H has a relatively narrow range of values for each class of machine.

Swing equation

The differential equation that relates the angular momentum M, the acceleration power P_a and the rotor angle δ is known as SWING EQUATION. Solution of swing equation will show how the rotor angle changes with respect to time following a disturbance. The plot of δ Vs t is called the SWING CURVE. Once the swing curve is known, the stability of the system can be assessed.

The flow of mechanical and electrical power in a generator and motor are shown in Fig. 1.

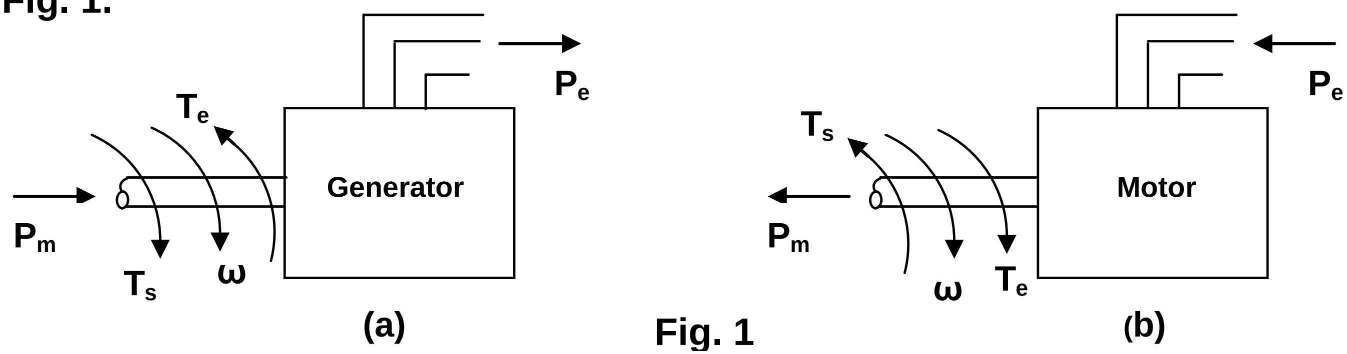


Fig. 1

Consider the generator shown in Fig. 1(a). It receives mechanical power P_m at the shaft torque T_s and the angular speed ω via. shaft from the prime-mover. It delivers electrical power P_e to the power system network via. the bus bars. The generator develops electromechanical torque T_e in opposition to the shaft torque T_s. At steady state, $T_s = T_e$.

Assuming that the windage and the friction torque are negligible, in a generator, accelerating torque acting on the rotor is given by

$$T_a = T_s - T_e \quad (7)$$

Multiplying by ω on both sides, we get

$$P_a = P_s - P_e \quad (8)$$

In case of motor

$$T_a = T_e - T_s \quad (9)$$

$$P_a = P_e - P_s \quad (10)$$

In general, the accelerating power is given by

$$P_a = \text{Input Power} - \text{Output Power} \quad (11)$$

$$P_a = T_a \omega = I \alpha \omega = M \alpha = M \frac{d^2\theta}{dt^2}$$

$$\text{Thus } M \frac{d^2\theta}{dt^2} = P_a \qquad (12)$$

Here θ = angular displacement (radians)

$$\omega = \frac{d\theta}{dt} = \text{angular velocity (rad. / sec.)}$$

$$\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \text{angular acceleration}$$

Now we can see how the angular displacement θ can be related to rotor angle δ.

Consider an object moving at a linear speed of $v_s \pm \Delta v$. It is required to find its displacement at any time t. For this purpose, introduce another object moving with a constant speed of vs. Then, at any time t, the displacement of the first object is given by

$$x = v_s t + d$$

where d is the displacement of the first object wrt the second as shown in Fig. 2.

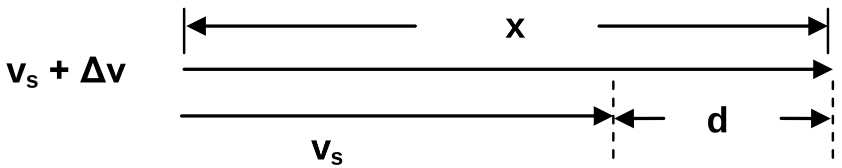


Fig. 2

Similarly in the case of angular movement, the angular displacement θ, at any time t is given by

$$\theta = \omega_s t + \delta \qquad (13)$$

where δ is the angular displacement of the rotor with respect to rotating reference axis which rotates at synchronous speed ω_s. The angle δ is also called as LOAD ANGLE or TORQUE ANGLE. In view of eqn.(13)

$$\frac{d\theta}{dt} = \omega_s + \frac{d\delta}{dt} \qquad (14)$$

$$\frac{d^2\theta}{dt^2} = \frac{d^2\delta}{dt^2} \qquad (15)$$

From equations (12) and (15), we get

$$M \frac{d^2\delta}{dt^2} = P_a \qquad (16)$$

The above equation is known as SWING EQUATION

In case damping power is to be included, then eqn.(16) gets modified as

$$M \frac{d^2\delta}{dt^2} + D \frac{d\delta}{dt} = P_a \qquad (17)$$

Swing curve, which is the plot of torque angle δ vs time t, can be obtained by solving the swing equation. Two typical swing curves are shown in Fig. 3.

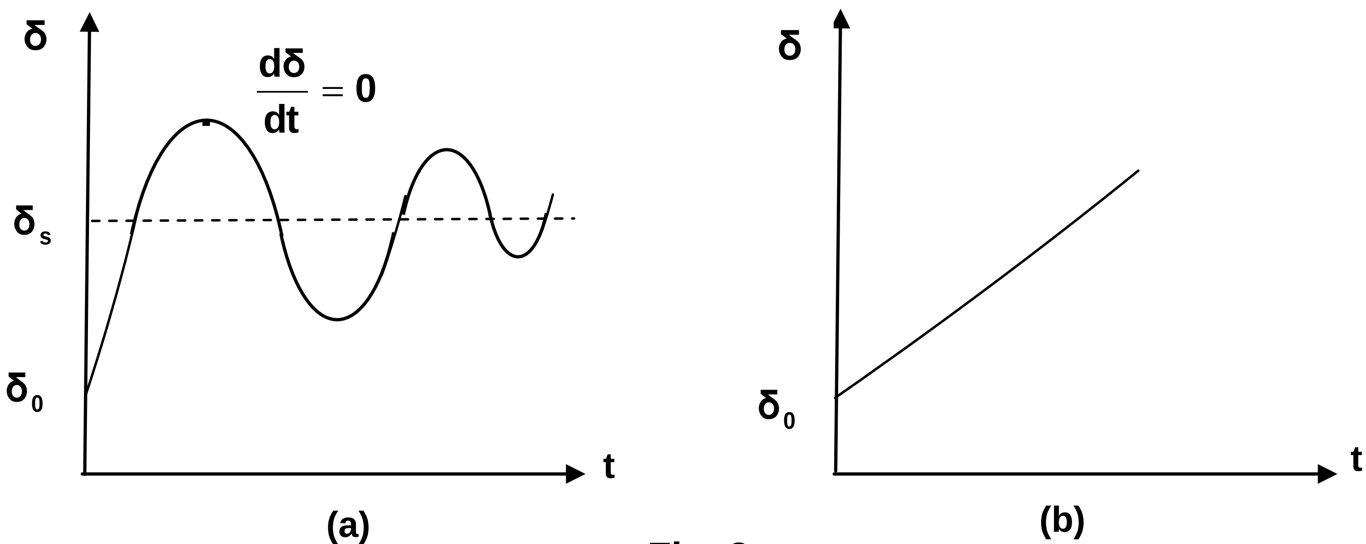


Fig. 3

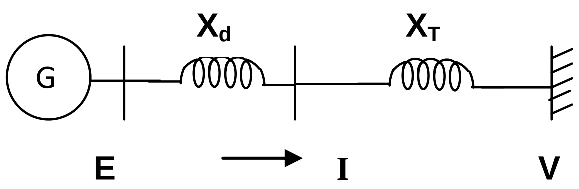
Swing curves are used to determine the stability of the system. If the rotor angle δ reaches a maximum and then decreases, then it shows that the system has transient stability. On the other hand if the rotor angle δ increases indefinitely, then it shows that the system is unstable.

We are going to study the stability of (1) a generator connected to infinite bus and (2) a synchronous motor drawing power from infinite bus.

We know that the complex power is given by

$P + j\,Q = V\,I^*$ i.e. $P - j\,Q = V^*\,I$ Thus real power $P = \mathrm{Re}\,\{V^*\,I\}$

Consider a generator connected to infinite bus.



V is the voltage at infinite bus.

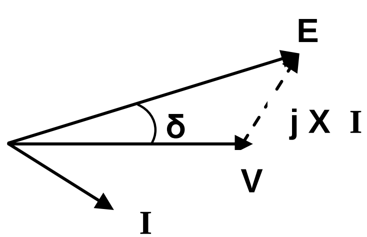
E is internal voltage of generator.

X is the total reactance

Taking this as ref. $V = |V| \angle 0^0$

$V + j\,X\,I = E$

phasor dia. can be obtained as



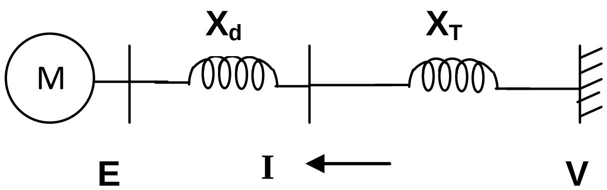
Internal voltage E leads V by angle δ.

Thus $E = |E| \angle\delta$

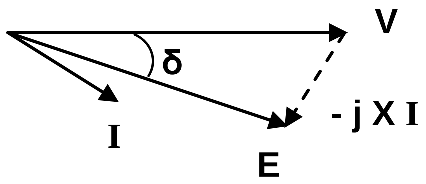
$$\text{Current } I = \frac{1}{jX}\left[\,|E|\cos\delta + j|E|\sin\delta - |V|\,\right]$$

Electric output power $P_e = \mathrm{Re}\left[\,|V|\;I\,\right] = \dfrac{|E|\,|V|}{X}\sin\delta = P_{max}\ \sin\delta$

Consider a synchronous motor drawing power from infinite bus.



$$V - jXI = E$$



Internal Voltage E lags the terminal voltage V by angle δ.

Thus $E = |E| \angle -\delta$ Current $I = \dfrac{1}{jX}\left[\,|V| - (|E|\cos\delta - j|E|\sin\delta)\,\right]$

Electric input power $P_e = \mathrm{Re}\,[\,|V|\, I\,] = \dfrac{|E||V|}{X}\sin\delta = P_{max}\sin\delta$

Thus Swing equation for alternator is

$$M\frac{d^2\delta}{dt^2} = P_m - P_{max}\sin\delta$$

Swing equation for motor is

$$M\frac{d^2\delta}{dt^2} = P_{max}\sin\delta - P_m$$

Notice that the swing equation is second order nonlinear differential equation

Equal area criterion

The accelerating power in swing equation will have sine term. Therefore the swing equation is non-linear differential equation and obtaining its solution is not simple. For two machine system and one machine connected to infinite bus bar, it is possible to say whether a system has transient stability or not, without solving the swing equation. Such criteria which decides the stability, makes use of equal area in power angle diagram and hence it is known as EQUAL AREA CRITERION. Thus the principle by which stability under transient conditions is determined without solving the swing equation, but makes use of areas in power angle diagram, is called the EQUAL AREA CRITERION.

From the Fig. 3, it is clear that if the rotor angle δ oscillates, then the system is stable. For δ to oscillate, it should reach a maximum value and then should decrease. At that point $\frac{d\delta}{dt} = 0$. Because of damping inherently present in the system, subsequence oscillations will be smaller and smaller. Thus while δ changes, if at one instant of time, $\frac{d\delta}{dt} = 0$, then the stability is ensured.

Let us find the condition for $\frac{d\delta}{dt}$ to become zero.

The swing equation for the alternator connected to the infinite bus bars is

$$M\frac{d^2\delta}{dt^2} = P_s - P_e \qquad (18)$$

Multiplying both sides by $\frac{d\delta}{dt}$, we get

$$M\frac{d^2\delta}{dt^2}\frac{d\delta}{dt} = (P_s - P_e)\frac{d\delta}{dt} \qquad \text{i.e.} \qquad \frac{1}{2}M\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2 = (P_s - P_e)\frac{d\delta}{dt} \qquad (19)$$

Thus

$$\frac{d}{dt}\left(\frac{d\delta}{dt}\right)^2\frac{dt}{d\delta} = \frac{2(P_s - P_e)}{M}\;; \qquad \text{i.e.} \qquad \frac{d}{d\delta}\left(\frac{d\delta}{dt}\right)^2 = \frac{2(P_s - P_e)}{M} \qquad \text{On integration}$$

$$\left(\frac{d\delta}{dt}\right)^2 = \int_{\delta_0}^{\delta}\frac{2(P_s - P_e)d\delta}{M} \qquad \text{i.e.} \qquad \frac{d\delta}{dt} = \sqrt{\int_{\delta_0}^{\delta}\frac{2(P_s - P_e)\,d\delta}{M}} \qquad (20)$$

Before the disturbance occurs, δ_0 was the torque angle. At that time $\frac{d\delta}{dt} = 0$. As soon as the disturbance occurs, $\frac{d\delta}{dt}$ is no longer zero and δ starts changing.

Torque angle δ will cease to change and the machine will again be operating at synchronous speed after a disturbance, when $\frac{d\delta}{dt} = 0$ or when

$$\int_{\delta_0}^{\delta} \frac{2(P_s - P_e)}{M} \, d\delta = 0 \quad \text{i.e.}$$

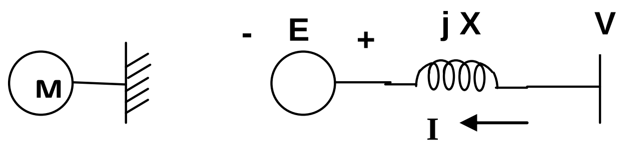
$$\int_{\delta_0}^{\delta} (P_s - P_e) \, d\delta = 0 \qquad (21)$$

If there exist a torque angle δ for which the above is satisfied, then the machine will attain a new operating point and hence it has transient stability.

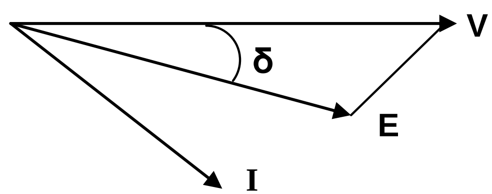
The machine will not remain at rest with respect to infinite bus at the first time when $\frac{d\delta}{dt} = 0$. But due to damping present in the system, during subsequent oscillation, maximum value of δ keeps on decreasing. Therefore, the fact that δ has momentarily stopped changing may be taken to indicate stability.

Sudden load increase on Synchronous motor

Let us consider a synchronous motor connected to an infinite bus bars.



$$E = V - I\,(jX)$$



$$P_e = \frac{|E|\,|V|}{X}\sin\delta = P_{max}\sin\delta$$

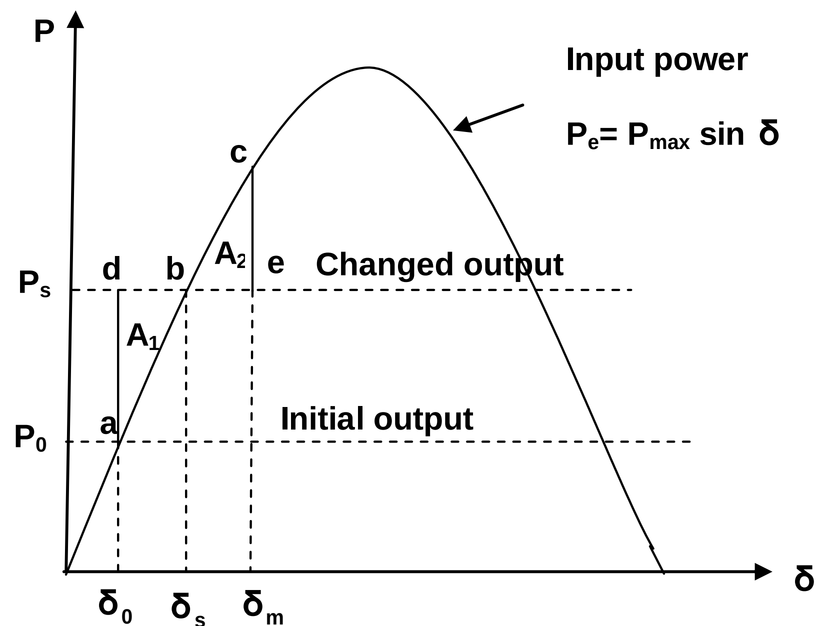
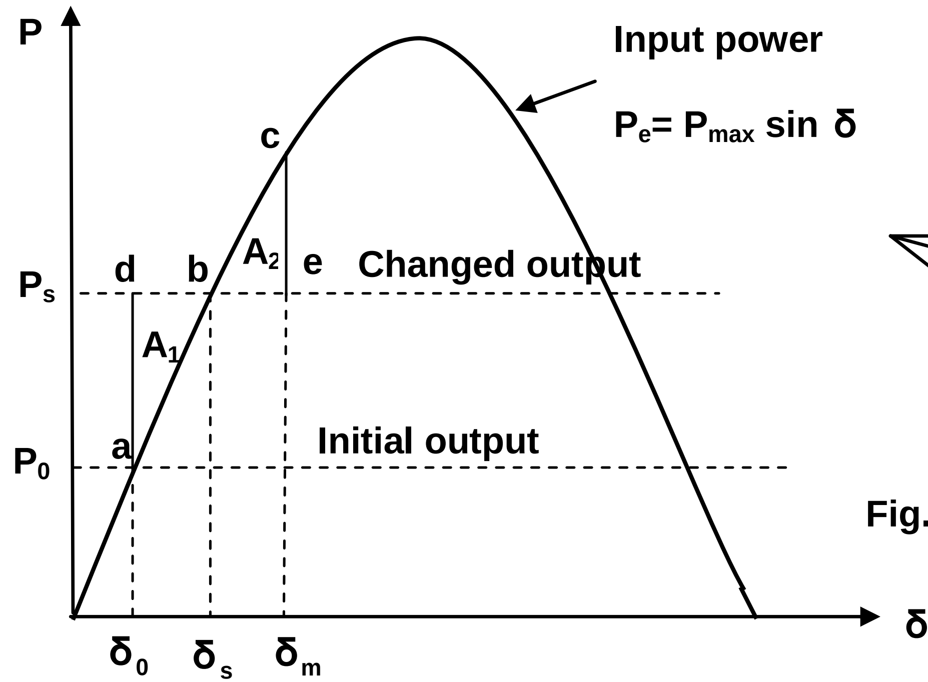


Fig. 4



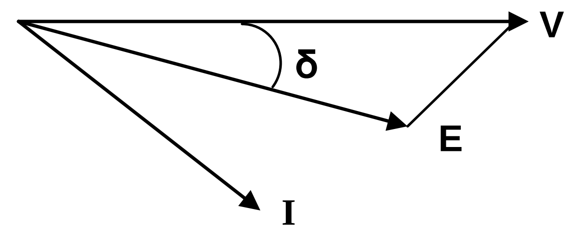


Fig. 4

The following changes occur when the load is increased suddenly.

Point a Initial condition; Input = output = P_0; $\omega = \omega_s$; $\delta = \delta_0$

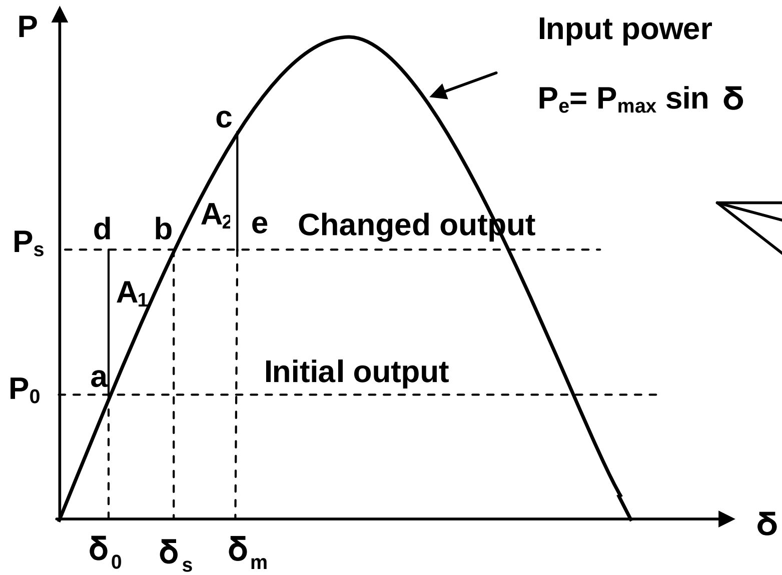
Due to sudden loading, output = P_s; output > Input;

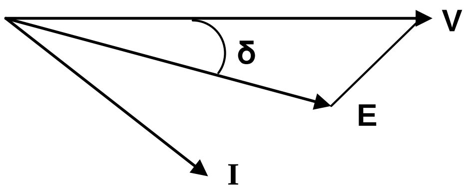
ω decreases from ω_s; δ increases from δ_0.

Between a-b Output > Input; Rotating mass starts loosing energy resulting deceleration; ω decreases; δ increases.

Point b Output = Input; $\omega = \omega_{min}$ which is less than ω_s; $\delta = \delta_s$

Since ω is less than ω_s, δ continues to increase.





Between b-c Input > output; Rotating masses start gaining energy; Acceleration; ω starts increasing from minimum value but still less than ω_s; δ continues to increase.

Point c Input > output; $\omega = \omega_s$; $\delta = \delta_m$; There is acceleration; ω is going to increase from ω_s; hence δ is going to decrease from δ_m.

Between c-b Input > output; Acceleration; ω increases and δ decreases.

Point b Input = output; $\omega = \omega_{max}$; $\delta = \delta_s$. Since ω is greater than ω_s, δ continues to decrease.

Between b-a Output > input; Deceleration; ω starts decreasing from ω_{max} ; but still greater than ω_s; δ continues to decrease.

Point a $\omega = \omega_s$; $\delta = \delta_0$; Output > Input; The cycle repeats.

Because of damping present in the system, subsequent oscillations become smaller and smaller and finally b will be the steady state operating point.

Interpretation of equal area

As discussed earlier (eqn. 21), the condition for stability is

$$\int_{\delta_0}^{\delta} (P_s - P_e)\, d\delta = 0 \quad \text{i.e.} \quad \int_{\delta_0}^{\delta} P_e\, d\delta = \int_{\delta_0}^{\delta} P_s\, d\delta$$

From Fig. 4, $\int_{\delta_0}^{\delta_m} P_e\, d\delta$ = area δ_0 a b c δ_m

and $\int_{\delta_0}^{\delta_m} P_s\, d\delta$ = area δ_0 a d e δ_m

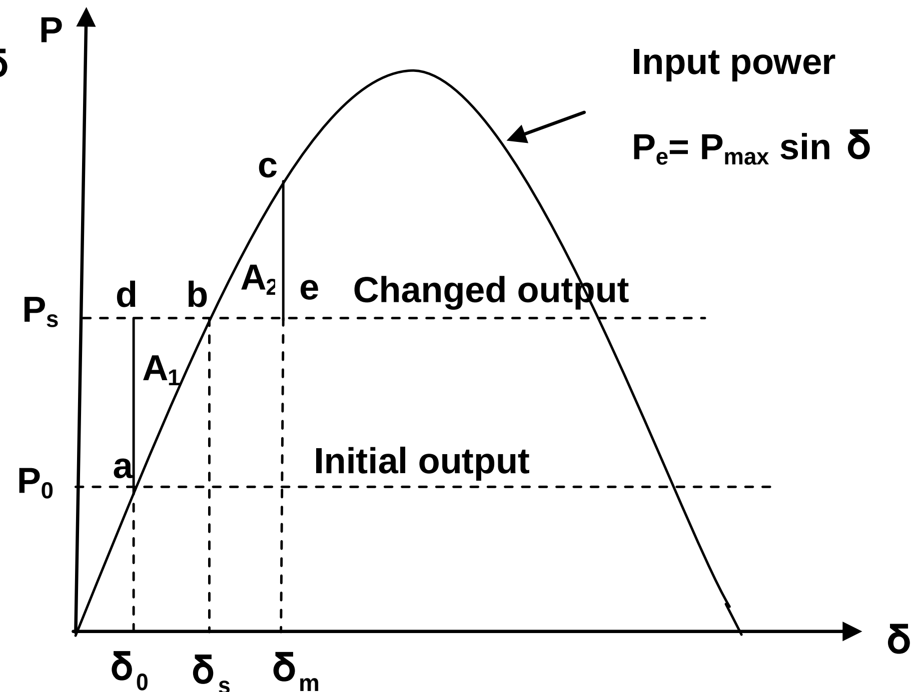


Fig. 4

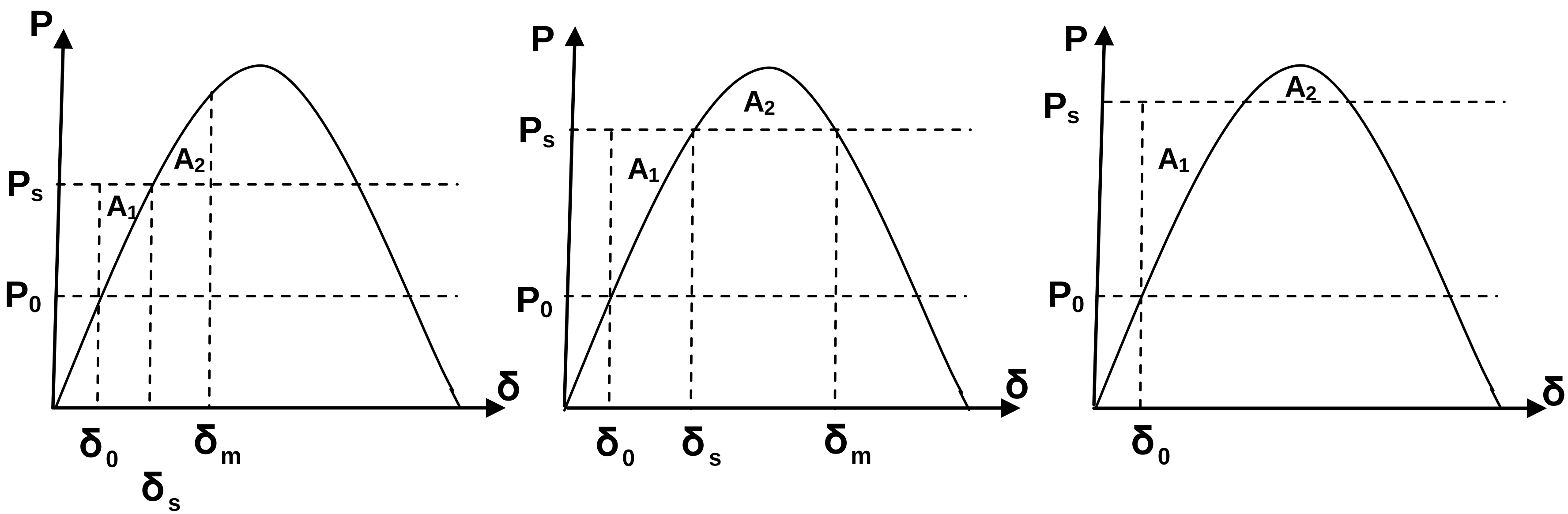
Thus for stability,

area δ_0 a b c δ_m = area δ_0 a d e δ_m

Subtracting area δ_0 a b e δ_m from both sides of above equation, we get $A_2 = A_1$.
Thus for stability,

$$A_2 = A_1 \tag{22}$$

Fig. 5 shows three different cases: The one shown in case a is STABLE. Case b indicates CRITICALLY STABLE while case c falls under UNSTABLE.



Case a **Case b** **Case c**

Fig. 5

Note that the areas A_1 and A_2 are obtained by finding the difference between INPUT and OUTPUT.

Example 1

A synchronous motor having a steady state stability limit of 200 MW is receiving 50 MW from the infinite bus bars. Find the maximum additional load that can be applied suddenly without causing instability.

Solution

Referring to Fig. 6,

for critical stability

$A_2 = A_1$

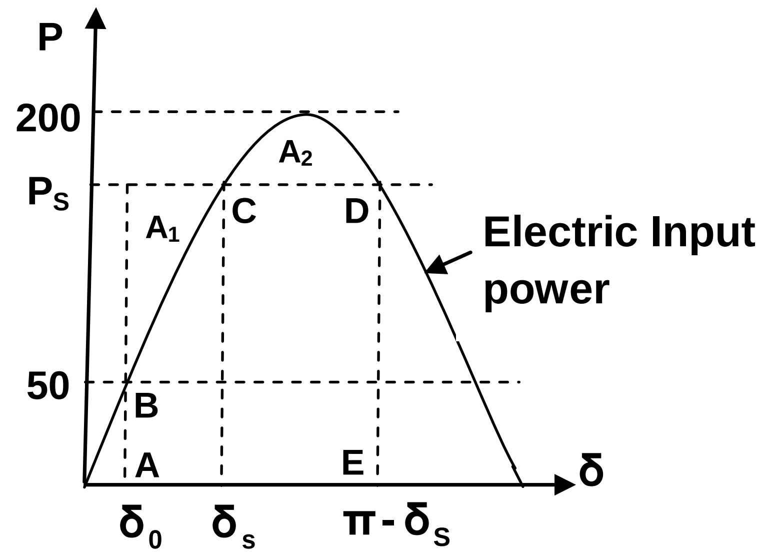


Fig. 6

$200 \sin \delta_0 = 50$ i.e.

$$\delta_0 = \sin^{-1} \frac{50}{200} = 0.25268 \text{ rad.}$$

Further $200 \sin \delta_S = P_S$

Adding area ABCDEA to both A_1 **and** A_2 **and equating the resulting areas**

$$200 \sin \delta_S \quad (\pi - \delta_S - \delta_0) \;=\; \int_{\delta_0}^{\Pi - \delta_S} 200 \sin \delta \, d\delta \quad \text{i.e.}$$

$$(\pi - \delta_S - \delta_0) \;\sin \delta_S \;=\; \cos \delta_0 - \cos (\pi - \delta_S) = \cos \delta_0 + \cos \delta_S \quad \text{i.e.}$$

$$(\pi - \delta_S - 0.25268) \sin \delta_S \;-\; \cos \delta_S \;=\; 0.9682458$$

The above equation can be solved by trial and error method.

δ_S	0.85	0.9	0.95
RHS	0.8718	0.9363	0.9954

Using linear interpolation between second and third points we get δ_S = 0.927 rad.

0.927 rad. = 53.11 deg.

Thus $P_S = 200 \sin 53.11^0 = 159.96$ MW

Maximum additional load possible = 159.96 – 50 = 109.96 MW

Opening of one of the parallel lines

When a generator is supplying power to an infinite bus over two parallel transmission lines, the opening of one of the lines will result in increase in the equivalent reactance and hence decrease in the maximum power transferred. Because of this, depending upon the initial operating power, the generator may loose synchronism even though the load could be supplied over the remaining line under steady state condition.

Consider the system shown in Fig. 7. The power angle diagrams corresponding to stable and unstable conditions are shown in Fig. 8.

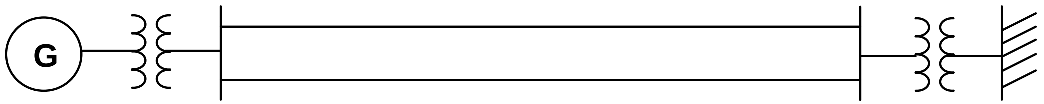


Fig. 7

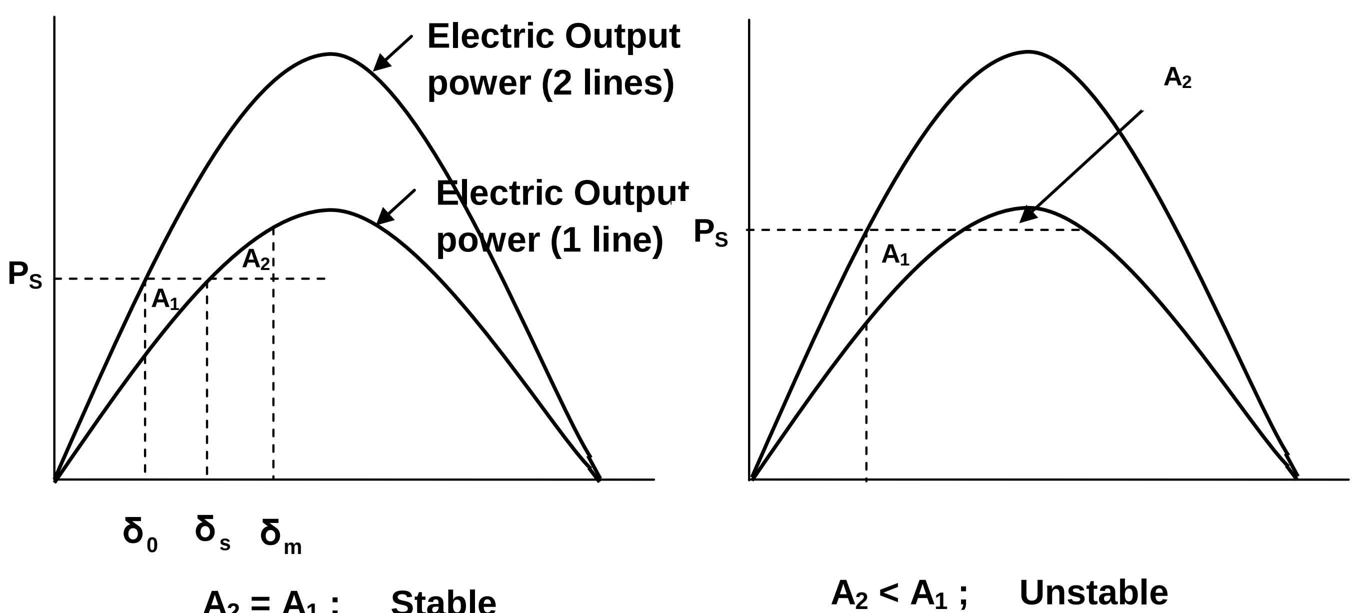


Fig. 8

Short circuit occurring in the system

Short circuit occurring in the system often causes loss of stability even though the fault may be removed by isolating it from the rest of the system in a relatively short time. A three phase fault at one end of a double circuit line is shown in Fig. 9(a) which can be reduced as shown in Fig. 9(b).

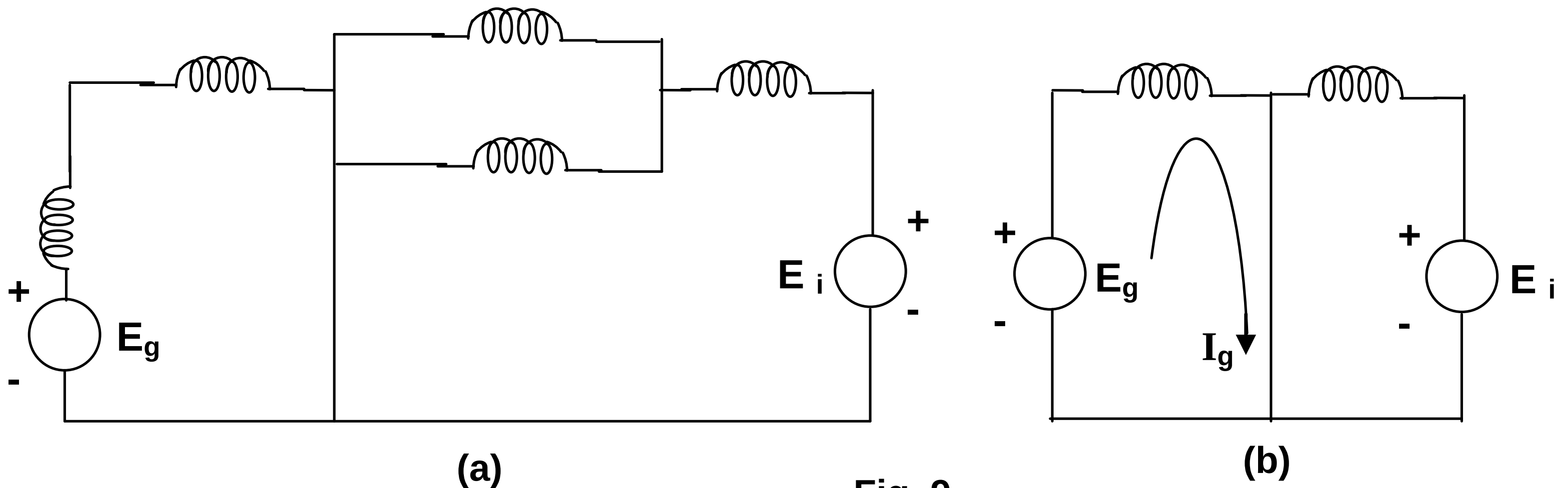


Fig. 9

It is to be noted that all the current from the generator flows through the fault and this current I_g lags the generator voltage by 90^0. Thus the real power output of the generator is zero. Normally the input power to the generator remains unaltered. Therefore, if the fault is sustained, the load angle δ will increase indefinitely because entire the input power will be used for acceleration. This may result in unstable condition.

When the three phase fault occurring at one end of a double circuit line is disconnected by opening the circuit breakers at both ends of the faulted line, power is again transmitted. If the fault is cleared before the rotor angle reaches a particular value, the system will remain stable; otherwise it will loose stability as shown in Fig. 10.

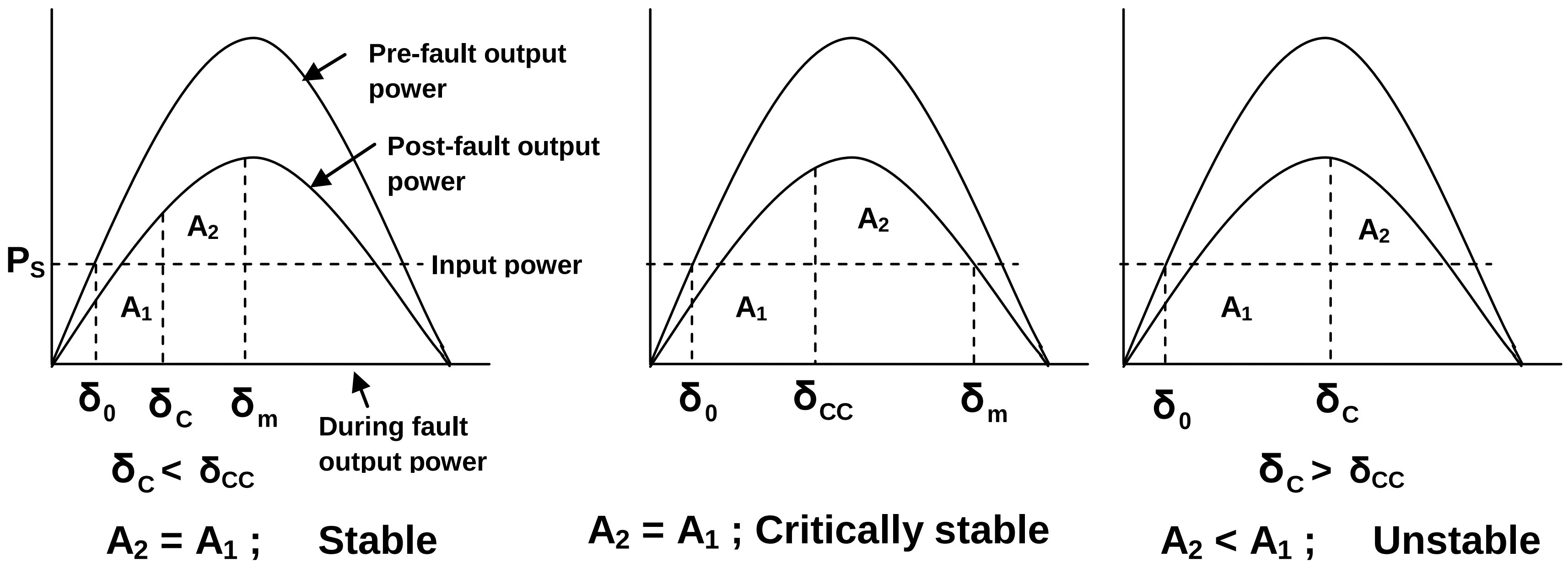


Fig. 10

Note that the areas A_1 and A_2 are obtained by finding difference between INPUT and OUTPUT.

When a three phase fault occurs at some point on a double circuit line, other than on the extreme ends, as shown in Fig. 11(a), there is some finite impedance between the paralleling buses and the fault. Therefore, some power is transmitted during the fault and it may be calculated after reducing the network to a delta connected circuit between the internal voltage of the generator and the infinite bus as shown in Fig. 11(b).

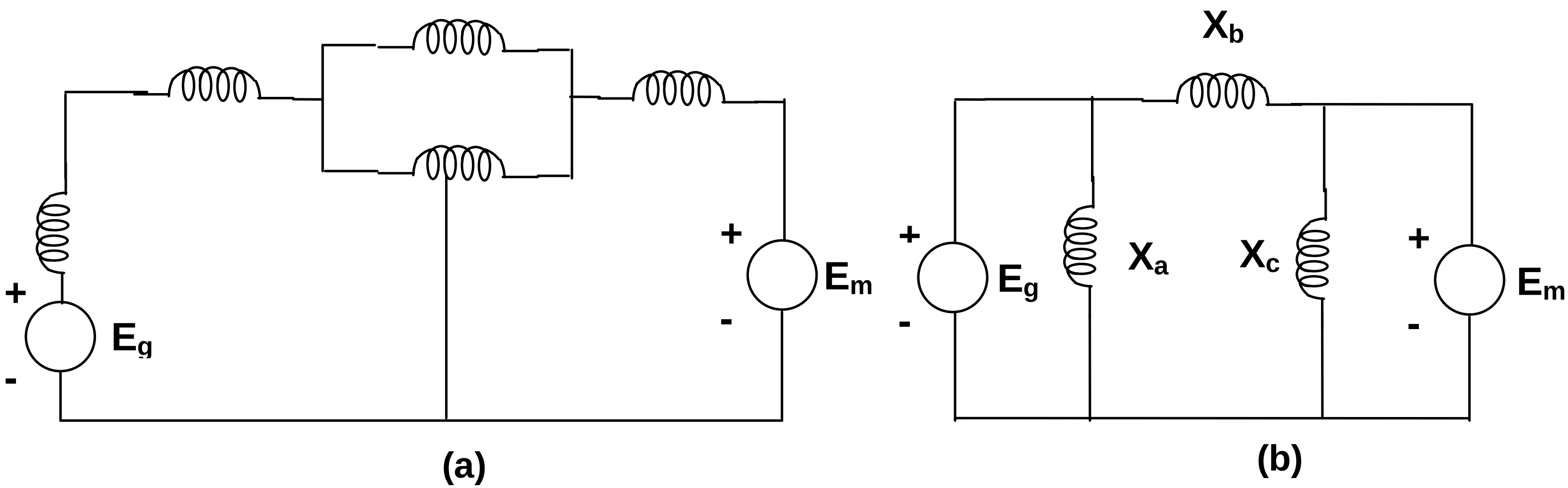


Fig. 11

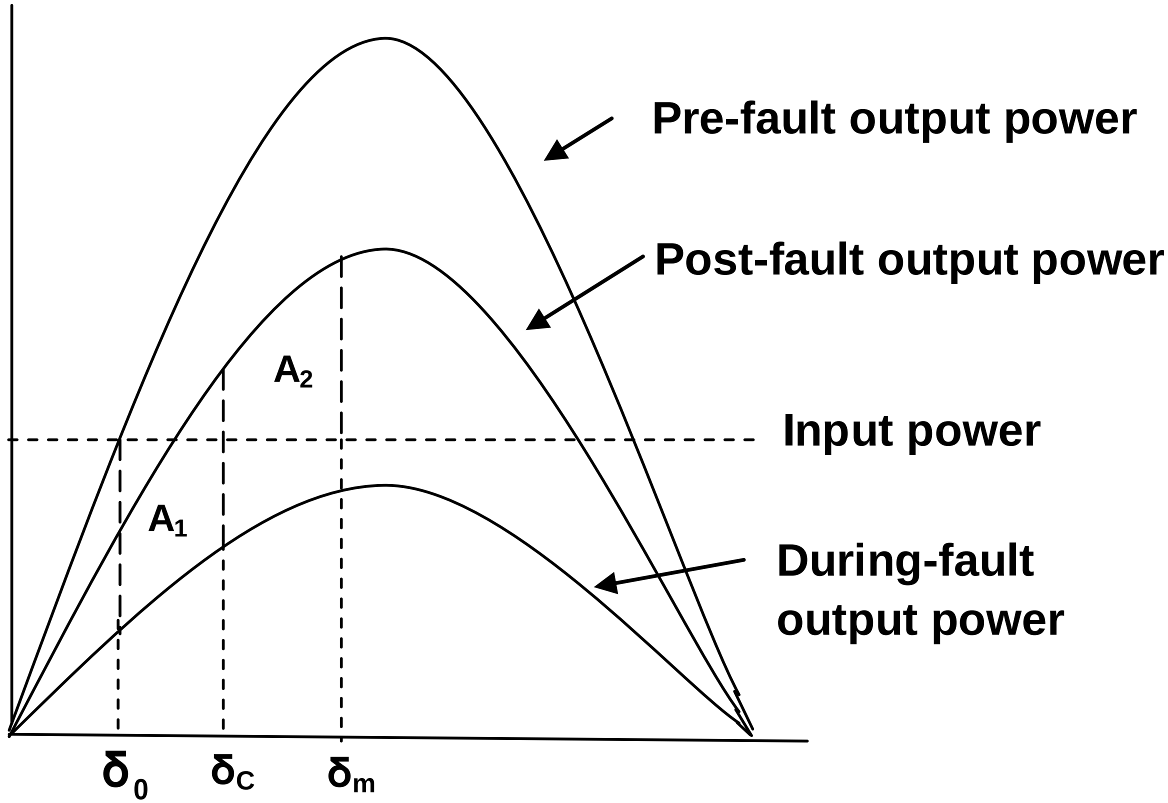
Power transmitted during the fault = $\frac{|E_g||E_m|}{X_b}\sin\delta$ (23)

Stable, critically stable and unstable conditions of such systems are shown:

$\delta_C < \delta_{CC}$;

$A_2 = A_1$;

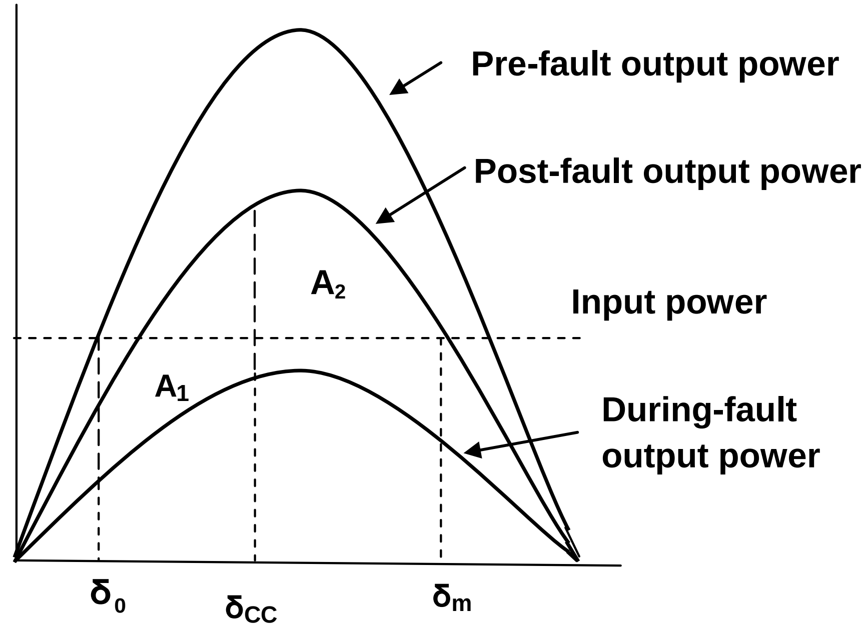
STABLE



$\delta_C = \delta_{CC}$;

$A_2 = A_1$;

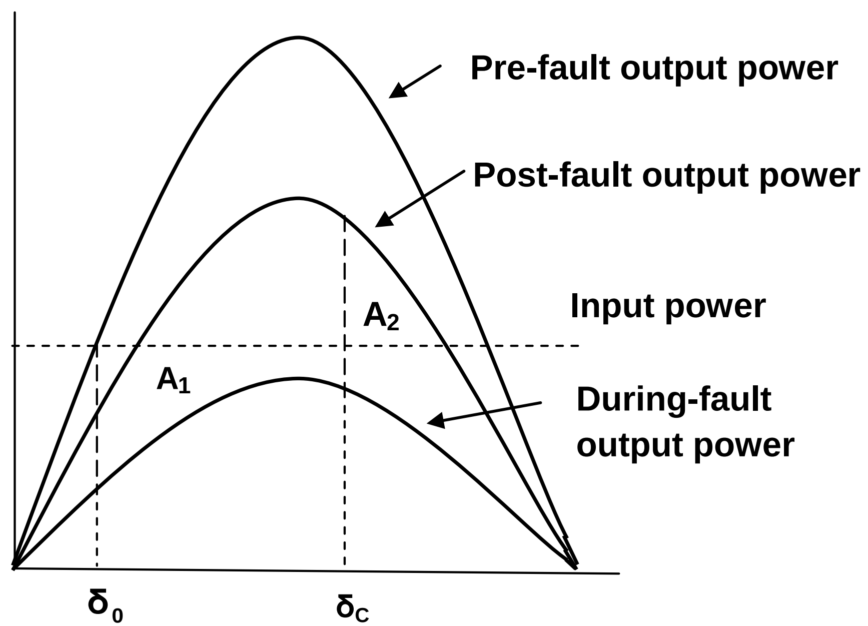
CRITICALLY STABLE



$\delta_C > \delta_{CC}$;

$A_2 < A_1$;

UNSTABLE



Example 2

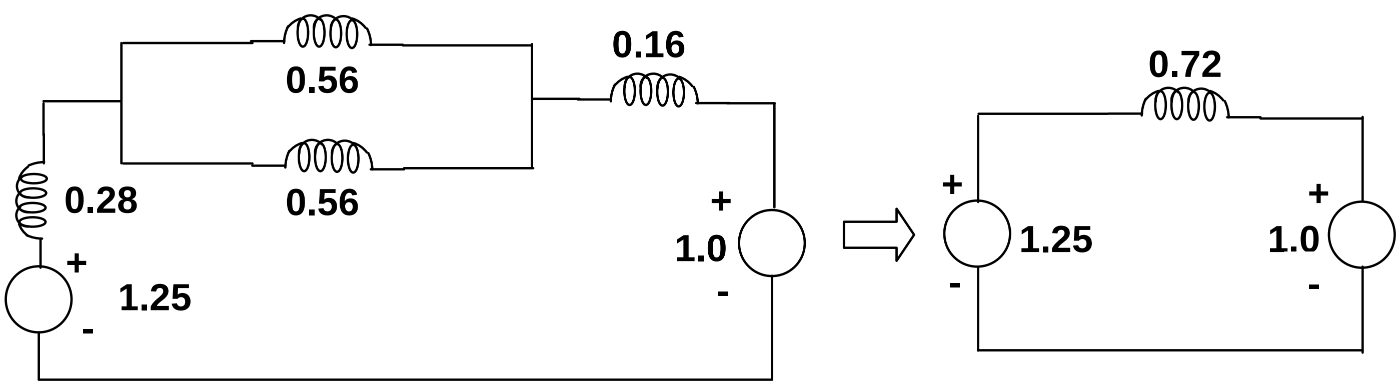
In the power system shown in Fig. 12, three phase fault occurs at P and the faulty line was opened a little later. Find the power output equations for the pre-fault, during fault and post-fault conditions.

Values marked are p.u. reactances

Fig. 12

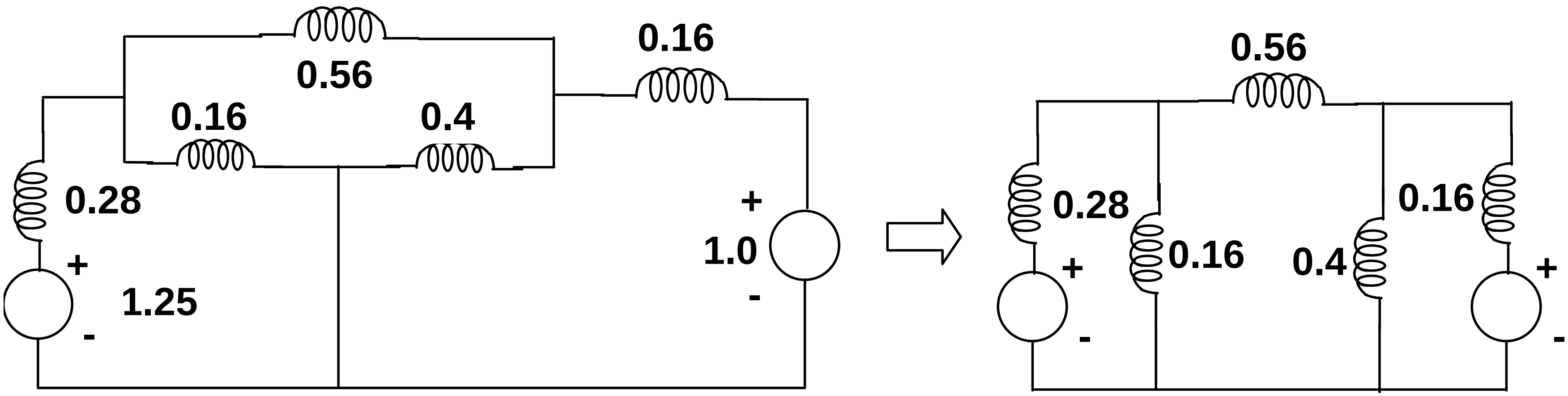
Solution

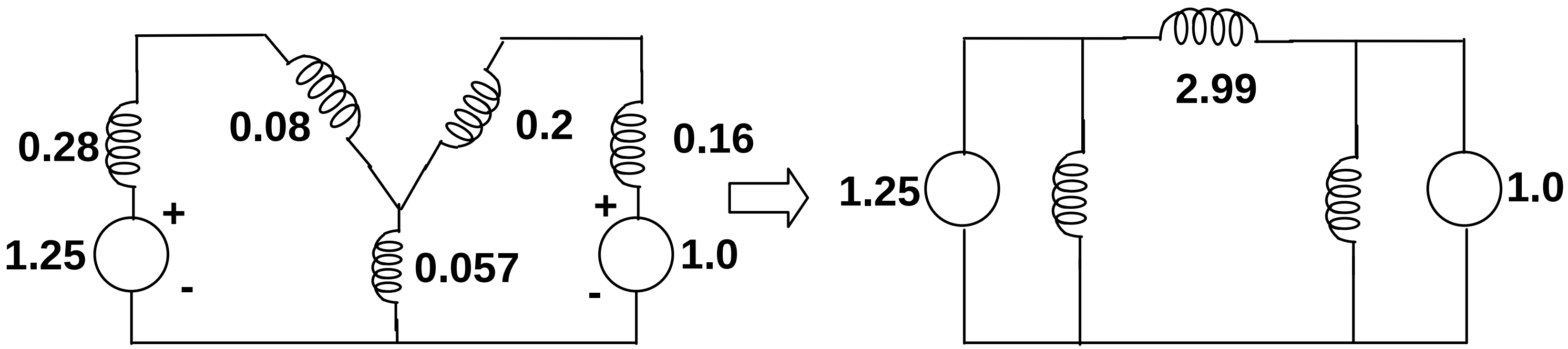
Pre-fault condition



Power output $P_e = \dfrac{1.25 \times 1.0}{0.72} \sin\delta = 1.736 \sin\delta$

During fault condition:

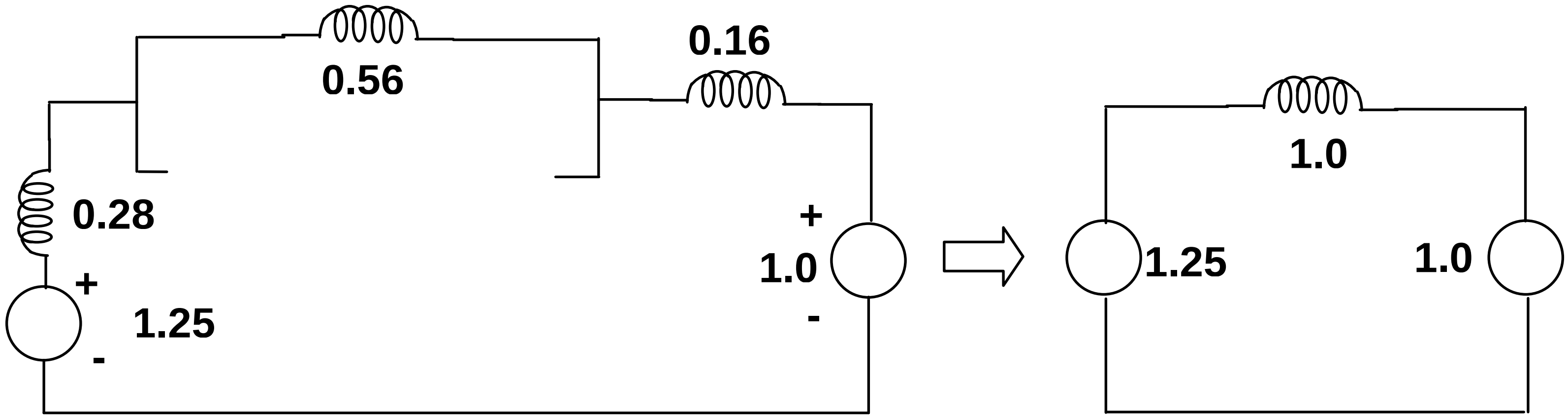




(0.36x0.36 + 0.36x0.057 + 0.057x0.36) / 0.057 = 2.99

Power output $P_e = \dfrac{1.25 \times 1.0}{2.99} \sin\delta = 0.418 \sin\delta$

Post-fault condition:



Power output $P_e = \dfrac{1.25 \times 1.0}{1.0} \sin\delta = 1.25 \sin\delta$

Thus power output equations are:

Pre-fault $P_e = P_{m1} \sin\delta = 1.736 \sin\delta$

During fault $P_e = P_{m2} \sin\delta = 0.418 \sin\delta$

Post fault $P_e = P_{m3} \sin\delta = 1.25 \sin\delta$

Here

$P_{m1} = 1.736$; $P_{m2} = 0.418$; $P_{m3} = 1.25$;

Expression for critical clearing angle δ_{CC}

Consider the power angle diagrams shown in Fig. 13

$$P_{m1} \sin \delta_0 = P_s$$

$$P_{m3} \sin \delta_s = P_s$$

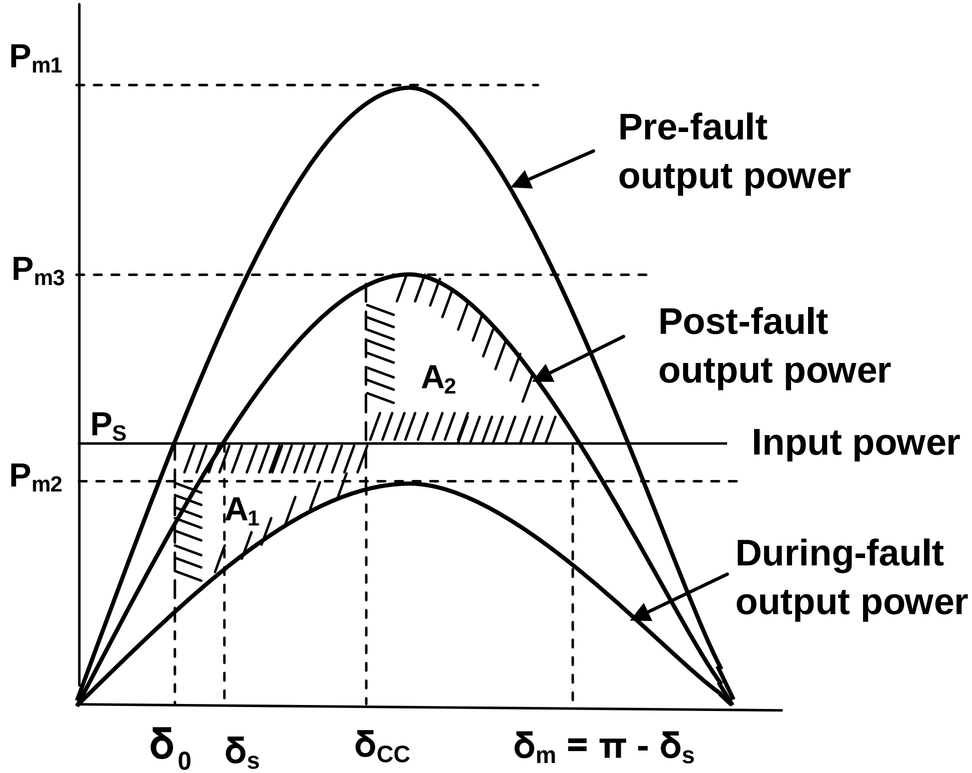


Fig. 13

$$\text{Area } A_1 = P_S(\delta_{CC} - \delta_0) - \int_{\delta_0}^{\delta_{CC}} P_{m2} \sin\delta \, d\delta$$

$$= P_S \delta_{CC} - P_S \delta_0 + P_{m2} \cos \delta_{CC} - P_{m2} \cos \delta_0 \qquad (24)$$

$$\text{Area } A_2 = \int_{\delta_{CC}}^{\delta_m} P_{m3} \sin\delta \, d\delta - P_S(\delta_m - \delta_{CC})$$

$$= P_{m3} \cos \delta_{CC} - P_{m3} \cos \delta_m - P_S \delta_m + P_S \delta_{CC} \qquad (25)$$

$$A_1 = P_S\,\delta_{CC} - P_S\,\delta_0 + P_{m2}\cos\delta_{CC} - P_{m2}\cos\delta_0 \qquad (24)$$

$$A_2 = P_{m3}\cos\delta_{CC} - P_{m3}\cos\delta_m - P_S\,\delta_m + P_S\,\delta_{CC} \qquad (25)$$

Area A_2 = Area A_1

$$P_{m3}\cos\delta_{CC} - P_{m3}\cos\delta_m - P_S\,\delta_m + \cancel{P_S\,\delta_{CC}} = \cancel{P_S\,\delta_{CC}} - P_S\delta_0 + P_{m2}\cos\delta_{CC} - P_{m2}\cos\delta_0$$

$$(P_{m3} - P_{m2})\cos\delta_{CC} = P_S(\delta_m - \delta_0) + P_{m3}\cos\delta_m - P_{m2}\cos\delta_0$$

$$\cos\delta_{CC} = \frac{P_S(\delta_m - \delta_0) + P_{m3}\cos\delta_m - P_{m2}\cos\delta_0}{P_{m3} - P_{m2}}$$

Thus CRITICAL CLEARING ANGLE is given by

$$\delta_{CC} = \cos^{-1}\left[\frac{P_S(\delta_m - \delta_0) + P_{m3}\cos\delta_m - P_{m2}\cos\delta_0}{P_{m3} - P_{m2}}\right] \qquad (26)$$

Here the angles are in radian. Further, since

$P_{m1}\sin\delta_0 = P_s$, $P_{m3}\sin\delta_s = P_s$ and $\delta_m = \pi - \delta_s$ angles δ_0 and δ_m are given by

$$\delta_0 = \sin^{-1}\left(\frac{P_S}{P_{m1}}\right) \qquad \delta_m = \pi - \sin^{-1}\left(\frac{P_S}{P_{m3}}\right) \qquad (27)$$

Example 3

In the power system described in the previous example, if the generator was delivering 1.0 p.u. just before the fault occurs, calculate δ_{cc}.

Solution

$P_{m1} = 1.736$; $P_{m2} = 0.418$; $P_{m3} = 1.25$; $P_S = 1.0$

$1.736 \sin\delta_0 = 1.0$; $\sin\delta_0 = 0.576$; $\delta_0 = 0.6139$ rad.

$1.25 \sin\delta_s = 1.0$; $\sin\delta_s = 0.8$; $\delta_s = 0.9273$ rad.; $\delta_m = \pi - \delta_s = 2.2143$ rad.

$$\cos\delta_{cc} = \frac{P_S(\delta_m - \delta_0) + P_{m3}\cos\delta_m - P_{m2}\cos\delta_0}{P_{m3} - P_{m2}}$$

$$= \frac{1.0(2.2143 - 0.6139) + 1.25\cos 2.2143 - 0.418\cos 0.6139}{1.25 - 0.418} = 0.6114$$

Critical clearing angle $\delta_{cc} = 52.31^0$

STEP BY STEP SOLUTION OF OBTAINING SWING CURVE

The equal area criterion of stability is useful in determining whether or not a system will remain stable and in determining the angle through which the machine may be permitted to swing before a fault is cleared. It does not determine directly the length of time permitted before clearing a fault if stability is to be maintained.

In order to specify a circuit breaker of proper speed, the engineer must know the CRITICAL CLEARING TIME, which is the time taken by the machine to swing from its initial position to its critical clearing angle. If the Critical Clearing Angle (CCA) is determined by the equal area criterion, then to determine corresponding Critical Clearing Time (CCT), the swing curve for the sustained faulted condition is required.

The step by step method of obtaining swing curve, using hand calculation is necessarily simpler than some of the methods recommended for digital computer. In the method suitable for hand calculation,the period of interest is divided into several short intervals. The change in the angular position of the rotor during a short interval of time is computed by making the following assumptions.

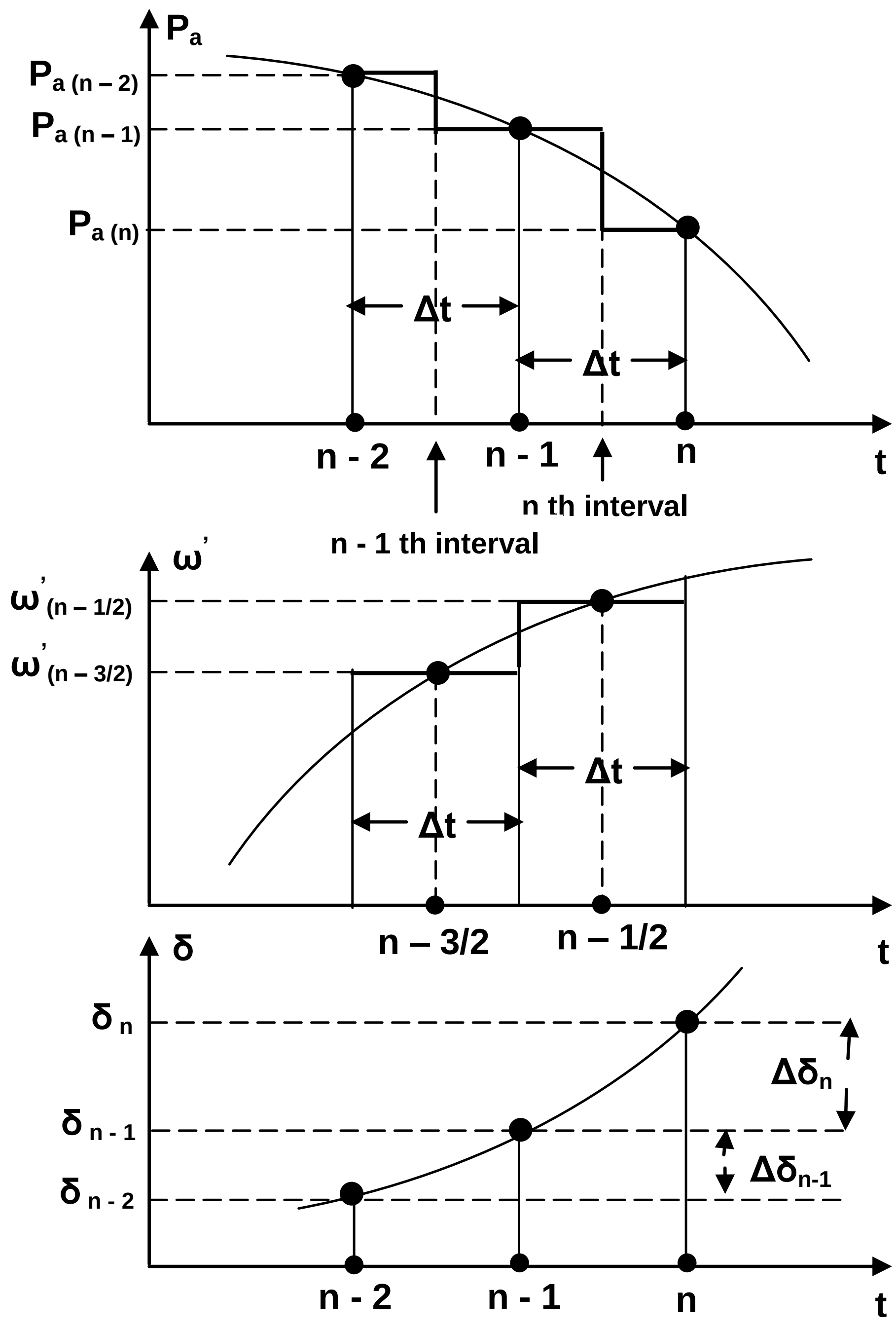
1. **The accelerating power P_a computed at the beginning of an interval is constant from the middle of the proceeding interval to the middle of the interval considered.**

2. $\frac{d\delta}{dt}$ **is constant throughout any interval at the value computed at the middle of the interval.**

Above assumptions are made to approximate continuously varying P_a and $\frac{d\delta}{dt}$ as stepped curve. Fig. 14 will help in visualizing the assumptions. The accelerating power is computed for the points enclosed in circles, at the beginning of n-1, n and n+1 th intervals. The step of P_a in the figure results from assumption 1.

Similarly ω' ($\frac{d\delta}{dt} = \frac{d\theta}{dt} - \omega_s$), the excess of angular velocity over the synchronous angular velocity is shown as a step curve that is constant throughout the interval, at the value computed at the midpoint.

Between the ordinates $n - \frac{3}{2}$ and $n - \frac{1}{2}$, there is a change in angular speed ω' caused by constant angular acceleration (caused by constant P_a).



Pa
Pa (n – 2)
Pa (n – 1)
Pa (n)
Δt
Δt
n - 2
n - 1
n
t
n th interval
n - 1 th interval
ω'
ω'(n – 1/2)
ω'(n – 3/2)
Δt
Δt
n – 3/2
n – 1/2
t
δ
δ n
δ n - 1
δ n - 2
Δδn
Δδn-1
n - 2
n - 1
n
t

Fig. 14

The change in angular speed ω' is

$$\omega'\left(n - \frac{1}{2}\right) - \omega'\left(n - \frac{3}{2}\right) = \text{Constant angular acceleration x time duration}$$

$$= \frac{P_{a(n-1)}}{M}\,\Delta t \qquad \left(\text{Because } \frac{d^2\delta}{dt^2} = \frac{1}{M}\,P_a\right) \qquad (28)$$

Similarly, change in δ over any interval = constant angular speed ω' x time duration. Thus

$$\Delta\delta_{(n)} = \omega'\left(n - \frac{1}{2}\right)\Delta t \qquad (29)$$

$$\Delta\delta_{(n-1)} = \omega'\left(n - \frac{3}{2}\right)\Delta t \qquad \text{and} \qquad (30)$$

Therefore $\Delta\delta_{(n)} - \Delta\delta_{(n-1)} = \left[\,\omega'\left(n - \frac{1}{2}\right) - \omega'\left(n - \frac{3}{2}\right)\right]\Delta t = \frac{P_{a(n-1)}}{M}\,(\Delta t)^2$

Thus $$\Delta\delta_{(n)} = \Delta\delta_{(n-1)} + \frac{P_{a(n-1)}}{M}\,(\Delta t)^2 \qquad (31)$$

Thus $$\Delta\delta_{(n)} = \Delta\delta_{(n-1)} + \frac{P_{a(n-1)}}{M}(\Delta t)^2 \quad (31)$$

Equation (31) shows that the change in torque angle during a given interval is equal to the change in torque angle during the proceeding interval plus the accelerating power at the beginning of the interval X $\frac{(\Delta t)^2}{M}$.

Torque angle δ at the end of n[th] interval can be computed as

$$\delta_{(n)} = \delta_{(n-1)} + \Delta\delta_{(n)} \quad (32)$$

where $$\Delta\delta_{(n)} = \Delta\delta_{(n-1)} + \frac{P_{a(n-1)}}{M}(\Delta t)^2 \quad (33)$$

The above two equations are known as Recursive equations using which approximate swing curve can be obtained.

The process of computation is now repeated to obtain $P_{a(n)}$, $\Delta\delta_{(n+1)}$ and $\delta_{(n+1)}$. The solution in discrete form is thus carried out over the desired length of time normally 0.05 sec. Greater accuracy of solution can be achieved by reducing the time duration of interval.

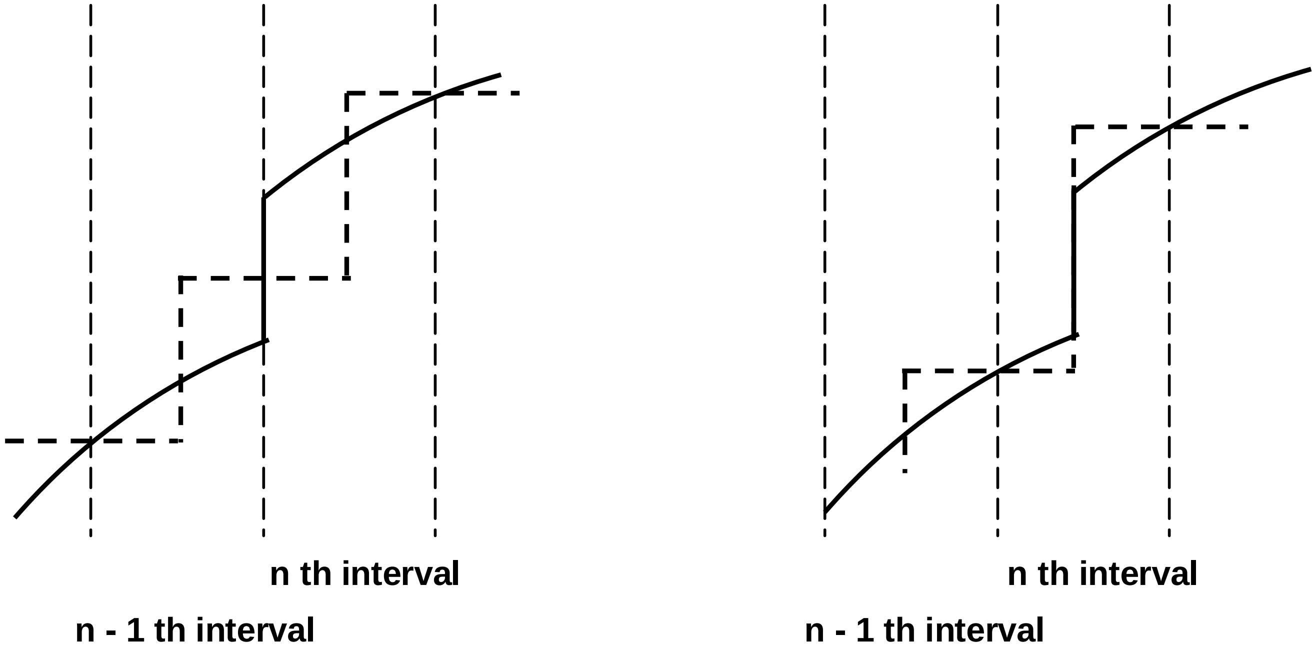
Any switching event such as occurrence of a fault or clearing of the fault causes discontinuity in the accelerating power P_a. If such a discontinuity occurs at the beginning of an interval then the average of the values of P_a just before and just after the discontinuity must be used.

Thus in computing the increment of angle occurring during the first interval after a fault is applied at time t = 0, becomes

$$\Delta\delta_1 = 0 + \frac{1}{2}\left(P_{a\,0^-} + P_{a\,0^+}\right) \frac{(\Delta t)^2}{M}$$

$$= \frac{1}{2} P_{a\,0^+} \frac{(\Delta t)^2}{M} \quad \text{(Because } P_{a\,0^-} = 0\text{)}$$

If the discontinuity occurs at the middle of an interval, no special procedure is needed. The correctness of this can be seen from Fig. 15.



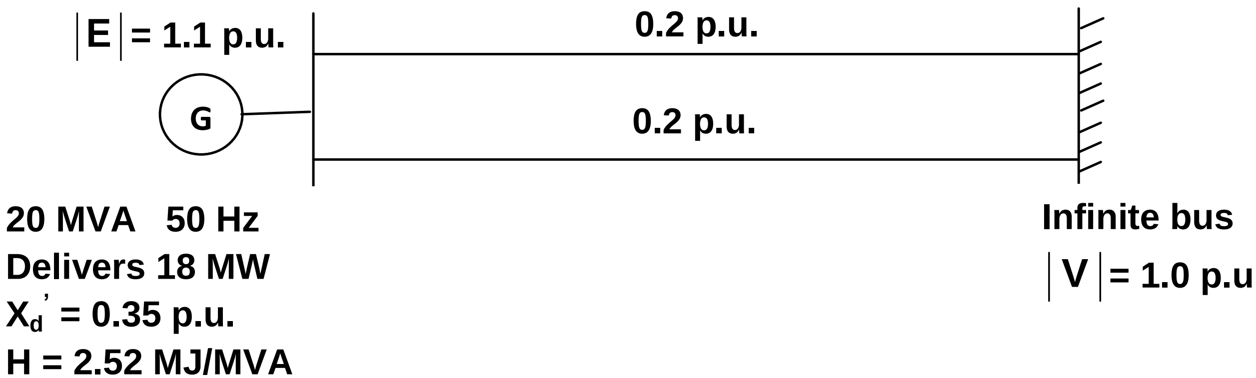
Discontinuity at the beginning of an interval

Discontinuity at the middle of an interval

Fig. 15

Example 4

A 20 MVA, 50 Hz generator delivers 18 MW over a double circuit line to an infinite bus. The generator has KE of 2.52 MJ / MVA at rated speed and its transient reactance is $X_d' = 0.35$ p.u. Each transmission line has a reactance of 0.2 p.u. on a 20 MVA base. $|E| = 1.1$ p.u. and infinite bus voltage $|V| = 1.0$ p.u. A three phase fault occurs at the mid point of one of the transmission lines. Obtain the swing curve over a period of 0.5 sec. if the fault is sustained.



G = 20 MVA = 1.0 p.u.

Angular momentum $M = \dfrac{GH}{180f} = \dfrac{1.0 \times 2.52}{180 \times 50} = 2.8 \times 10^{-4}\ \text{sec}^2/\text{elec. deg.}$

Let us choose Δt = 0.05 sec. **Then** $\dfrac{(\Delta t)^2}{M} = \dfrac{(0.05)^2 \times 10^4}{2.8} = 8.929$

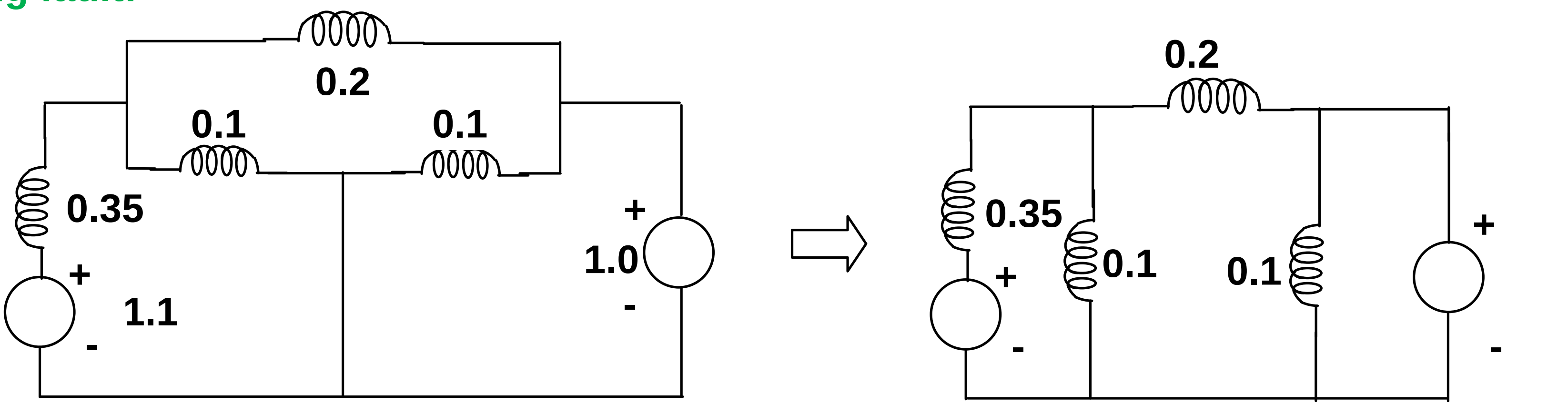
Recursive equations are

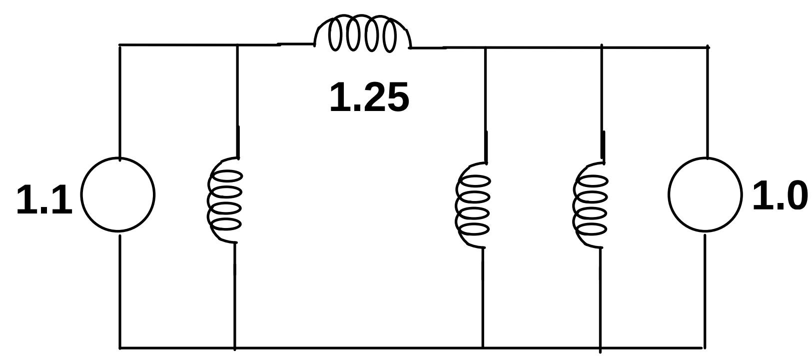
$$\delta_{(n)} = \delta_{(n-1)} + \Delta\delta_{(n)}$$

where $\Delta\delta_{(n)} = \Delta\delta_{(n-1)} + 8.929\, P_{a\,(n-1)}$

Pre fault: $X = 0.45$ p.u.; $P_e = \dfrac{1.1 \times 1.0}{0.45} \sin\delta = 2.44 \sin\delta$

During fault:





Converting the star 0.35, 0.1 and 0.2 as delta

$$P_e = \frac{1.1 \times 1.0}{1.25} \sin\delta = 0.88 \sin\delta$$

Initial calculations:

Before the occurrence of fault, there will not be acceleration i.e.

Input power is equal to output power. Therefore

Input power P_s = 18 MW = 0.9 p.u.

Initial power angle is given by

$$2.44 \sin \delta_0 = 0.9; \quad \text{Thus} \quad \delta_0 = 21.64$$

$$P_{a\,0^-} = 0; \quad P_{a\,0^+} = 0.9 - 0.88 \sin 21.64^0 = 0.576 \text{ p.u.}$$

$$P_{a\ average} = (\,0 + 0.576\,) / 2 = 0.288 \text{ p.u.}$$

First interval: $\Delta\delta_1 = 0 + P_{a\ average} \times \dfrac{(\Delta t)^2}{M} = 0.288 \times 8.929 = 2.57^0$

$$\text{Thus} \quad \delta(0.05) = 21.64 + 2.57 = 24.21^0$$

Subsequent calculations are shown below.

t sec.	δ deg.	P_{max}	P_e	$P_a = 0.9 - P_e$	8.929 P_a	Δδ
0^-	21.64	2.44	0.9	0		
0^+	21.64	0.88	0.324	0.576		
$0_{average}$	21.64			0.288	2.57	2.57
0.05	24.21	0.88	0.361	0.539	4.81	7.38
0.10	31.59	0.88	0.461	0.439	3.92	11.30
0.15	42.89	0.88	0.598	0.301	2.68	13.98
0.20	56.87	0.88	0.736	0.163	1.45	15.43
0.25	72.30	0.88	0.838	0.062	0.55	15.98
0.30	88.28	0.88	0.879	0.021	0.18	16.16
0.35	104.44	0.88	0.852	0.048	0.426	16.58
0.40	121.02	0.88	0.754	0.145	1.30	17.88
0.45	138.90	0.88	0.578	0.321	2.87	20.75
0.50	159.65					

Swing curve, rotor angle δ with respect to time, for sustained fault is plotted and shown in Fig. 16.

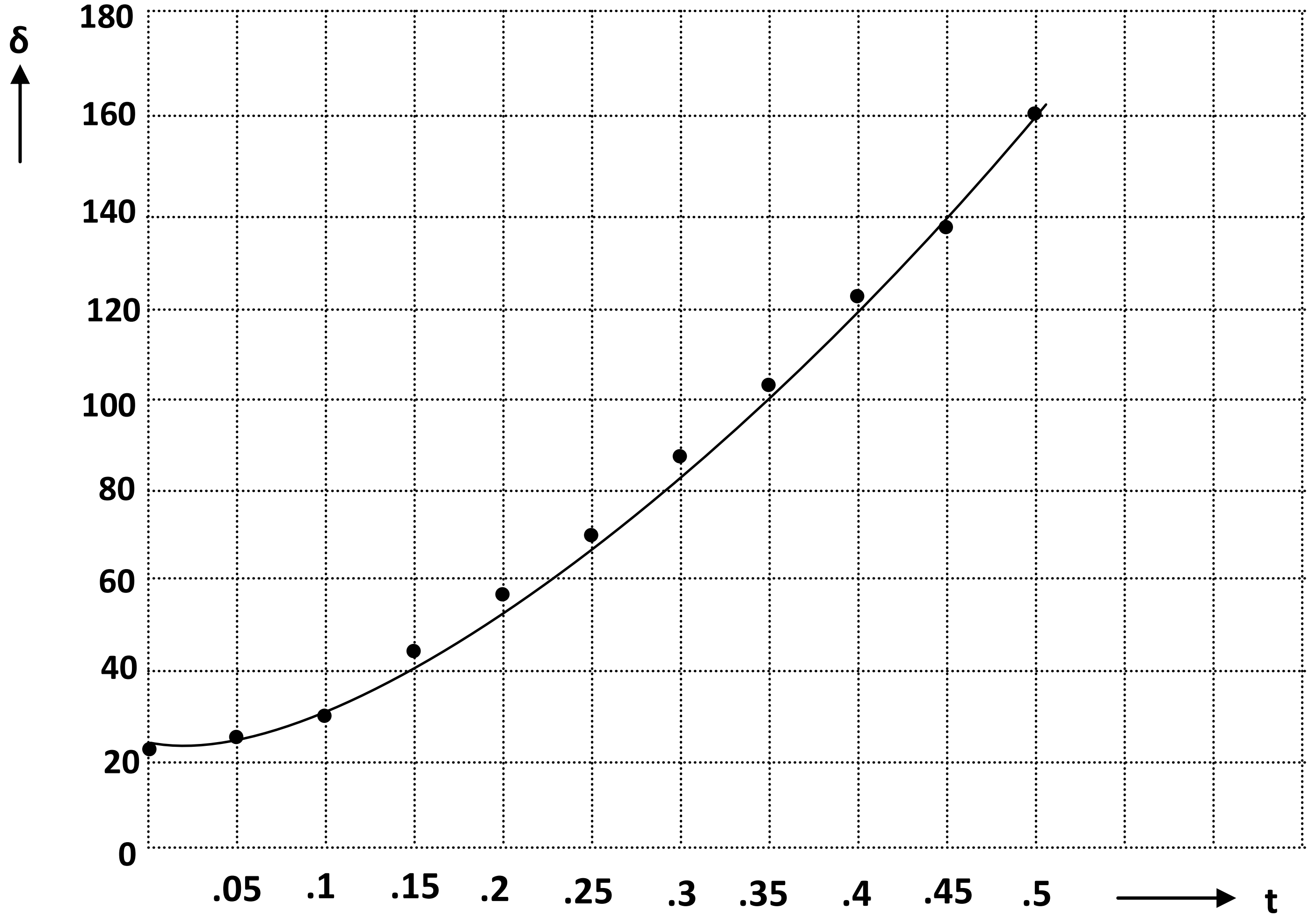


Fig. 16

Example 5

In the power system considered in the previous example, fault is cleared by opening the circuit breakers at both ends of the faulty line. Calculate the CCA and hence find CCT.

Solution

From the previous example: $P_s = 0.9$; $P_{m1} = 2.44$ **and** $P_{m2} = 0.88$

For the Post fault condition:

$X = 0.55$ p.u; $\quad P_e = \dfrac{1.1 \times 1.0}{0.55} \sin\delta = 2.0 \sin\delta$

Thus $P_s = 0.9$; $\quad P_{m1} = 2.44$; $\quad P_{m2} = 0.88$; $\quad P_{m3} = 2.0$

$$\cos\delta_{CC} = \frac{P_S(\delta_m - \delta_0) + P_{m3}\cos\delta_m - P_{m2}\cos\delta_0}{P_{m3} - P_{m2}}$$

$2.44 \sin\delta_0 = 0.9$; $\quad$ **Therefore** $\delta_0 = 0.3778$ **rad.**

$2.0 \sin\delta_s = 0.9$; **Thus** $\delta_s = 0.4668$ $\quad$ **Therefore** $\delta_m = \pi - \delta_s = 2.6748$ **rad**

$$\cos\delta_{CC} = \frac{0.9(2.6748 - 0.3778) + 2\cos(2.6748) - 0.88\cos(0.3778)}{2 - 0.88} = -0.47915$$

Thus CCA, $\delta_{CC} = 118.63^0$

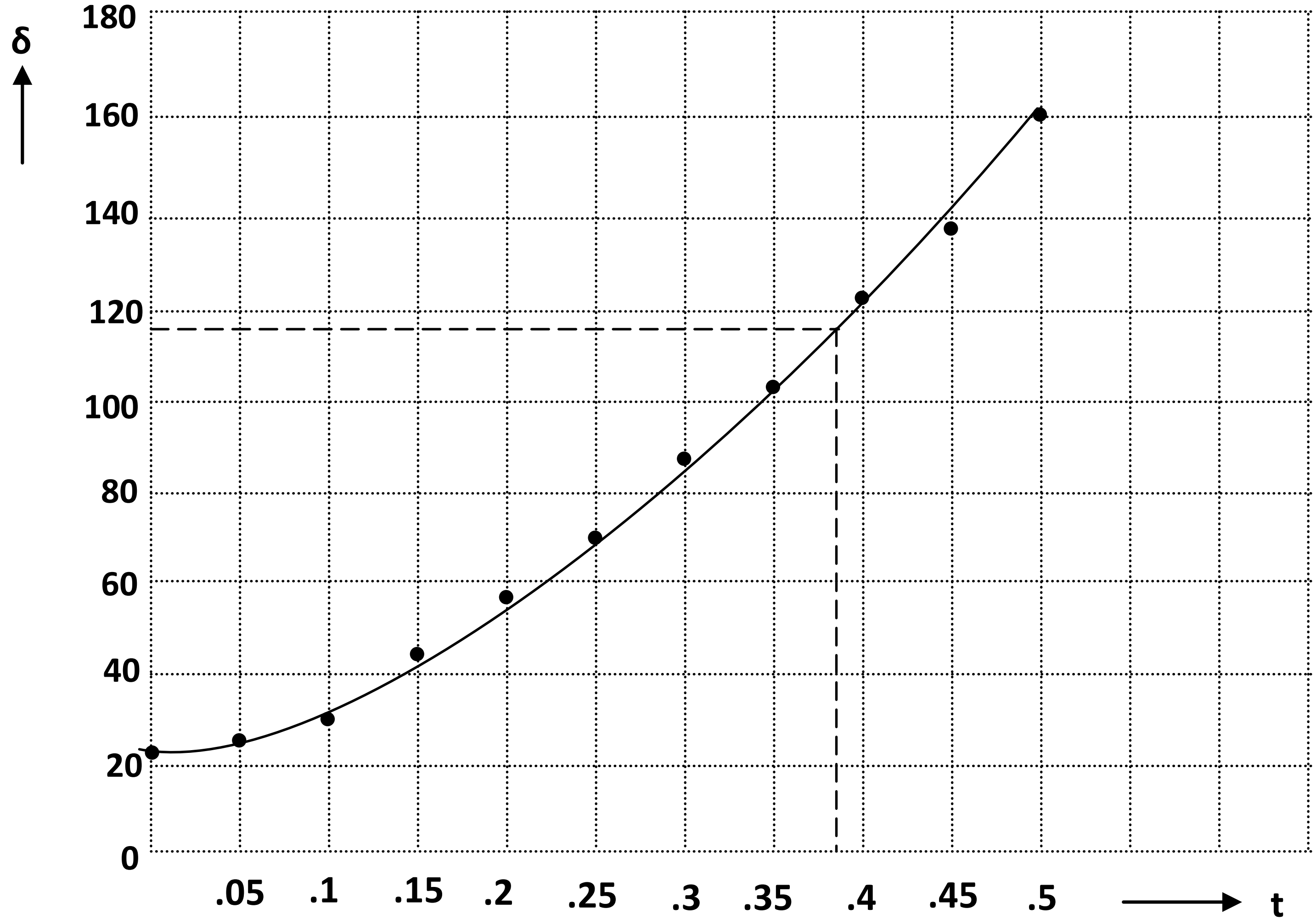


Fig. 17

Referring to the swing curve obtained for sustained fault condition, corresponding to CCA of 118.63^0, CCT can be obtained as 0.38 sec. as shown in Fig. 17.

SOLUTION OF SWING EQUATION BY MODIFIED EULER'S METHOD

Modified Euler's method is simple and efficient method of solving differential equations (DE)

Let us first consider solution of first order differential equation. Later we shall extend it for solving a set of first order DE. The swing equation is a second order DE which can be written as two first order DE and solution can be obtained using Modified Euler's method.

Let the given first order DE be

$$\frac{dx}{dt} = f(t, x) \qquad (34)$$

where t is the independent variable and x is the dependent variable. Let (t_0, x_0) be the initial solution and Δt is the increment in t. Then

$$t_1 = t_0 + \Delta t; \quad t_2 = t_1 + \Delta t; \quad t_n = t_{n-1} + \Delta t$$

First estimate of x_1 (value of x at time t_1) is denoted as $x_1^{(0)}$. Then

$$x_1^{(0)} = x_0 + \frac{dx}{dt}\Big|_0 \ \Delta t \qquad (35)$$

Thus $(t_1, x_1^{(0)})$ is the first estimated point of (t_1, x_1). Second and the final estimate of x_1 is calculated as

$$x_1 = x_0 + \frac{1}{2}\left(\frac{dx}{dt}\Big|_0 + \frac{dx}{dt}\Big|_1^{(0)}\right) \Delta t \qquad (36)$$

where $\frac{dx}{dt}\Big|_1^{(0)}$ is the value of $\frac{dx}{dt}$ computed at $(t_1, x_1^{(0)})$. Thus the next point

(t_1, x_1) is now known. Same procedure can be followed to get (t_2, x_2) and it can be repeated to obtain points (t_3, x_3), (t_4, x_4) ……..

Knowing (t_{n-1}, x_{n-1}), next point (t_n, x_n) can be computed as follows:

$$t_n = t_{n-1} + \Delta t \qquad (37)$$

$$x_n^{(0)} = x_{n-1} + \frac{dx}{dt}\Big|_{n-1} \Delta t \qquad (38)$$

Compute $\frac{dx}{dt}\Big|_n^{(0)}$ which is $\frac{dx}{dt}$ computed at $(t_n, x_n^{(0)})$. (39)

Then $$x_n = x_{n-1} + \frac{1}{2}\left(\frac{dx}{dt}\Big|_{n-1} + \frac{dx}{dt}\Big|_n^{(0)}\right) \Delta t \qquad (40)$$

Same procedure can be extended to solve a set of two first order DE given by

$$\frac{dx}{dt} = f_1(t, x, y) \quad \text{and} \quad \frac{dy}{dt} = f_2(t, x, y)$$

Knowing $(t_{n-1}, x_{n-1}, y_{n-1})$, next point (t_n, x_n, y_n) can be computed as follows:

$$t_n = t_{n-1} + \Delta t$$

$$x_n^{(0)} = x_{n-1} + \frac{dx}{dt}\Big|_{n-1} \Delta t$$

$$y_n^{(0)} = y_{n-1} + \frac{dy}{dt}\Big|_{n-1} \Delta t$$

Compute $\frac{dx}{dt}\Big|_n^{(0)}$ which is $\frac{dx}{dt}$ computed at $(t_n, x_n^{(0)}, y_n^{(0)})$ and

$\frac{dy}{dt}\Big|_n^{(0)}$ which is $\frac{dy}{dt}$ computed at $(t_n, x_n^{(0)}, y_n^{(0)})$

Then $x_n = x_{n-1} + \frac{1}{2}\left(\frac{dx}{dt}\Big|_{n-1} + \frac{dx}{dt}\Big|_n^{(0)}\right) \Delta t$ and

$$y_n = y_{n-1} + \frac{1}{2}\left(\frac{dy}{dt}\Big|_{n-1} + \frac{dy}{dt}\Big|_n^{(0)}\right) \Delta t$$

We know that the swing equation is

$$M \frac{d^2\delta}{dt^2} = P_a$$

When per unit values are used and the machine's rating is taken as base

$$M = \frac{H}{\pi f}$$

Therefore for a generator

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H} P_a = \frac{\pi f}{H}(P_s - P_e) = K(P_s - P_e) \text{ where } K = \frac{\pi f}{H}$$

The second order DE $\frac{d^2\delta}{dt^2} = K(P_s - P_e)$

can be written as two first order DE's given by

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{d\omega}{dt} = K(P_s - P_e)$$

Note that $\frac{d\delta}{dt}$ generally of the form $\frac{d\delta}{dt} = f_1(t, \delta, \omega)$. However, now it a function of ω alone. Similarly, $\frac{d\omega}{dt}$ generally of the form $\frac{d\omega}{dt} = f_2(t, \delta, \omega)$. However, now it a function of δ alone.

Just prior to the occurrence of the disturbance, $P_s - P_e = 0$ and $\omega = \omega_s$. The rotor angle can be computed as $\delta(0)$ and the corresponding angular velocity is $\omega(0)$. Thus the initial point is $(0, \delta(0), \omega(0))$.

As soon as disturbance occurs, electric network changes and the expression for electric power P_e in terms of rotor angle δ can be obtained. During fault condition, P_e shall be computed by the said expression.

Using Modified Euler's method δ_1 and ω_1 can be computed. Thus we get the next solution point as $(t_1, \delta_1, \omega_1)$. The procedure can be repeated to get subsequent solution points until next change in electric network, such as removal of faulted line occurs. As soon as electric network changes, corresponding expression for electric power need to be obtained and used in subsequent calculation.

The whole procedure can be carried out until t reaches the time upto which transient stability analysis is required.

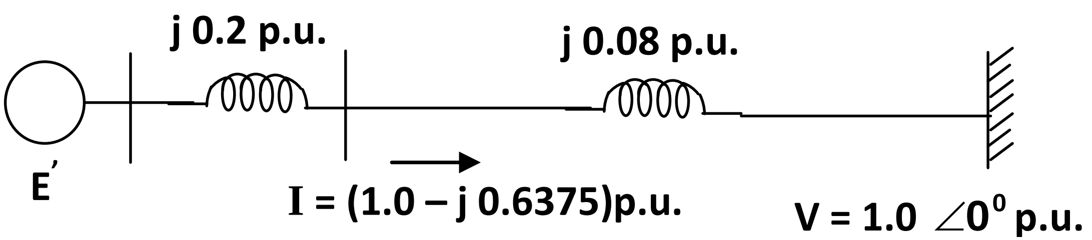
Example 6

An alternator rated for 100 MVA supplies 100 MW to an infinite bus through a line of reactance 0.08 p.u. on 100 MVA base. The machine has a transient reactance of 0.2 p.u. and its inertia constant is 4.0 p.u. on 100 MVA base. Taking the infinite bus voltage as reference, current supplied by the alternator is (1.0 – j 0.6375) p.u.

Calculate the torque angle and speed of the alternator for a period of 0.14 sec. when there is a three phase fault at the machine terminals and the fault is cleared in 0.1 sec. Use Modified Euler's method with a time increment of 0.02 sec.

Solution

100 MVA
P_s = 100 MW
H = 4



$E' = (1.0 + j0) + j\ 0.28\ (1.0 – j\ 0.6375) = 1.1785 + j\ 0.28 = 1.2113\ \angle 13.3651^0$ p.u.

Initial rotor angle $\delta = 13.3651^0 = 0.2333$ rad.

Shaft power P_s = 100 MW = 1.0 p.u. This remains same throughout the calculations.

Just before the fault, $P_e = P_s$ = 1.0 p.u.; Swing equation is:

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}(P_s - P_e) = \frac{50\pi}{4}(1 - P_e) = 39.2699(1 - P_e)$$

$\omega_s = 2\pi \times 50$ = 314.1593 rad. / sec.

The two first order DEs are:

$$\frac{d\delta}{dt} = \omega - 314.1593$$

$$\frac{d\omega}{dt} = 39.2699\,(1 - P_e)$$

Initial point is:
$\delta(0)$ = 0.2333 rad.
$\omega(0)$ = 314.1593 rad. / sec.

Since the fault is at the generator terminals, during fault $P_e = 0$

$$\frac{d\delta}{dt} = \omega - 314.1593 \qquad \frac{d\omega}{dt} = 39.2699\,(1 - P_e)$$

<u>To calculate δ(0.02) and ω(0.02)</u>

Initial point is:
δ(0) = 0.2333 rad.
ω(0) = 314.1593 rad. / sec.

First estimate:

$$\frac{d\delta}{dt} = 314.1593 - 314.1593 = 0$$

$$\frac{d\omega}{dt} = 39.2699\,(1 - 0) = 39.2699$$

δ = 0.2333 + (0 x 0.02) = 0.2333 rad.

ω = 314.1593 + (39.2699 x 0.02) = 314.9447 rad. / sec.

First estimated point is:
δ = 0.2333 rad.
ω = 314.9447 rad. / sec.

Second estimate:

$$\frac{d\delta}{dt} = 314.9447 - 314.1593 = 0.7854$$

$$\frac{d\omega}{dt} = 39.2699\,(1 - 0) = 39.2699; \text{ Thus}$$

$$\delta(0.02) = 0.2333 + \frac{1}{2}(0 + 0.7854) \times 0.02 = 0.24115 \text{ rad.}$$

$$\omega(0.02) = 314.1593 + \frac{1}{2}(39.2699 + 39.2699) \times 0.02 = 314.9447 \text{ rad. / sec.}$$

$$\frac{d\delta}{dt} = \omega - 314.1593 \qquad \frac{d\omega}{dt} = 39.2699\,(1 - P_e)$$

To calculate δ(0.04) and ω(0.04)

Latest point is:
δ(0.02) = 0.24115 rad.
ω(0.02) = 314.9447 rad. / sec.

First estimate

$$\frac{d\delta}{dt} = 314.9447 - 314.1593 = 0.7854$$

$$\frac{d\omega}{dt} = 39.2699\,(1 - 0) = 39.2699$$

$$\delta = 0.24115 + (0.7854 \times 0.02) = 0.2569 \text{ rad.}$$

$$\omega = 314.9447 + (39.2699 \times 0.02) = 315.7301 \text{ rad. / sec.}$$

Second estimate:

First estimated point is:
δ = 0.2569 rad.
ω = 315.7301 rad. / sec.

$$\frac{d\delta}{dt} = 315.7301 - 314.1593 = 1.5708$$

$$\frac{d\omega}{dt} = 39.2699\,(1 - 0) = 39.2699\text{; Thus}$$

$$\delta(0.04) = 0.24115 + \frac{1}{2}(0.7854 + 1.5708) \times 0.02 = 0.2647 \text{ rad.}$$

$$\omega(0.04) = 314.9447 + \frac{1}{2}(39.2699 + 39.2699) \times 0.02 = 315.7301 \text{ rad. / sec.}$$

Calculations can be repeated until the fault is cleared i.e. t = 0.1. The results are tabulated. Thus

$\delta(0.1)$ = 0.4297 rad.; $\omega(0.1)$ = 318.0869 rad. / sec.

Once the fault is cleared, reactance between internal voltage and the infinite bus is 0.28 and thus generator out put is;

$$P_e = \frac{1.2113 x 1.0}{0.28} \sin\delta = 4.3261 \sin\delta$$

In the subsequent calculation P_e must be obtained from the above equation.

<u>To calculate $\delta(0.12)$ and $\omega(0.12)$</u>

Latest point is:
$\delta(0.1)$ = 0.4297 rad.
$\omega(0.1)$ = 318.0869 rad. / sec.

$$\frac{d\delta}{dt} = \omega - 314.1593 \qquad \frac{d\omega}{dt} = 39.2699\,(1 - P_e)$$

<u>To calculate δ(0.12) and ω(0.12)</u>

Latest point is:
δ(0.1) = 0.4297 rad.
ω(0.1) = 318.0869 rad. / sec.

First estimate

$$\frac{d\delta}{dt} = 318.0869 - 314.1593 = 3.9276$$

$$\frac{d\omega}{dt} = 39.2699\,(\,1 - 4.3261 \sin 0.4297 \text{ rad.}\,) = -\,31.5041$$

δ = 0.4297 + (3.9276 x 0.02) = 0.50825 rad.

ω = 318.0869 + (- 31.5041 x 0.02) = 317.4568 rad. / sec.

Second estimate:

First estimated point is:
δ = 0.50825 rad.
ω = 317.4568 rad. / sec.

$$\frac{d\delta}{dt} = 317.4568 - 314.1593 = 3.2975$$

$$\frac{d\omega}{dt} = 39.2699\,(\,1 - 4.3261 \sin 0.50825 \text{ rad.}\,) = -\,43.4047\text{; Thus}$$

$$\delta(0.12) = 0.4297 + \frac{1}{2}(\,3.9276 + 3.2975\,) \times 0.02 = 0.50195 \text{ rad.}$$

$$\omega(0.12) = 318.0869 + \frac{1}{2}(\,-\,31.5041 - 43.4047\,) \times 0.02 = 317.3378 \text{ rad. / sec.}$$

Complete calculations are shown in the Table:

$$\frac{d\delta}{dt} = \omega - 314.1593; \qquad \frac{d\omega}{dt} = 39.2699\,(1 - P_e); \quad P_e = 0 \text{ for } t < 0.1 \text{ and } P_e = 4.3261 \sin\delta$$

t sec.	δ rad.	ω rad/sec	First Estimate				Second Estimate			
			dδ/dt	dω/dt	δ rad.	ω rad/sec	dδ/dt	dω/dt	δ rad.	ω rad/sec
0^-	0.2333	314.1593								
0^+	0.2333	314.1593	0	39.2699	0.2333	314.9447	0.7854	39.2699	0.2412	314.9447
0.02	0.2412	314.9447	0.7854	39.2699	0.2569	315.7301	1.5708	39.2699	0.2647	315.7301
0.04	0.2647	315.7301	1.5708	39.2699	0.2961	316.5155	2.3562	39.2699	0.304	316.5155
0.06	0.304	316.5155	2.3562	39.2699	0.3511	317.3009	3.1416	39.2699	0.359	317.3009
0.08	0.359	317.3009	3.1416	39.2699	0.4218	318.0863	3.927	39.2699	0.4297	318.0869
0.10^-	0.4297	318.0869								
0.10^+	0.4297	318.0869	3.9276	-31.504	0.5083	317.4568	3.2975	-43.405	0.502	317.3378
0.12	0.502	317.3378	3.1785	-42.468	0.5655	316.4884	2.3291	-51.761	0.557	316.3955
0.14	0.557	316.3955								

t sec	0	0.02	0.04	0.06	0.08	0.1	0.12	0.14
δ rad	0.2333	0.24115	0.2647	0.304	0.359	0.4297	0.50195	0.5570
δ deg	13.37	13.82	15.17	17.42	20.57	24.62	28.76	31.91
ω rad/sec	314.1593	314.9447	315.7301	316.5155	317.3009	318.0869	317.3378	316.3955

SOLUTION OF SWING EQUATION BY RUNGE KUTTA METHOD

Fourth order Runge Kutta (RK) method is one of the most commonly used methods of solving differential equation.

Consider the first order DE

$$\frac{dx}{dt} = f(t, x)$$

Let (t_m, x_m) be the initial point and h be the increment in time. Then

$$t_{m+1} = t_m + h$$

Fourth order RK method can be defined by the following five equations.

$$x_{m+1} = x_m + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \quad \text{where}$$

$$k_1 = f(t_m, x_m)\, h \qquad k_2 = f\left(t_m + \frac{h}{2}, x_m + \frac{k_1}{2}\right) h$$

$$k_3 = f\left(t_m + \frac{h}{2}, x_m + \frac{k_2}{2}\right) h \qquad k_4 = f(t_m + h, x_m + k_3)$$

Note that in this method, the function has to be evaluated four times in each step.

Same procedure can be extended to solve a set of first order DE such as

$$\frac{dx}{dt} = f_1 (t , x , y) \quad \text{and} \quad \frac{dy}{dt} = f_2 (t , x , y)$$

Initial solution point is (t_{m}, x_m, y_m). Then

$$x_{m+1} = x_m + \frac{1}{6}(k_1 + 2 k_2 + 2 k_3 + k_4) \qquad y_{m+1} = y_m + \frac{1}{6}(\ell_1 + 2 \ell_2 + 2 \ell_3 + \ell_4)$$

where

$$k_1 = f_1 (t_m, x_m, y_m) h \qquad \ell_1 = f_2 (t_m, x_m, y_m) h$$

$$k_2 = f_1 (t_m + \frac{h}{2}, x_m + \frac{k_1}{2}, y_m + \frac{\ell_1}{2}) h \qquad \ell_2 = f_2 (t_m + \frac{h}{2}, x_m + \frac{k_1}{2}, y_m + \frac{\ell_1}{2}) h$$

$$k_3 = f_1 (t_m + \frac{h}{2}, x_m + \frac{k_2}{2}, y_m + \frac{\ell_2}{2}) h \qquad \ell_3 = f_2 (t_m + \frac{h}{2}, x_m + \frac{k_2}{2}, y_m + \frac{\ell_2}{2}) h$$

$$k_4 = f_1 (t_m + h, x_m + k_3, y_m + \ell_3) h \qquad \ell_4 = f_2 (t_m + h, x_m + k_3, y_m + \ell_3) h$$

We know that the swing equation can be written as

$$\frac{d\delta}{dt} = \omega - \omega_s$$

$$\frac{d\omega}{dt} = K\,(P_s - P_e) \quad \text{where} \quad K = \frac{\pi f}{H}$$

The initial solution point is (0, $\delta(0)$, $\omega(0)$). When 4th order RK method is used, k_1, ℓ_1, k_2, ℓ_2, k_3, ℓ_3, k_4, ℓ_4 are computed and then the next solution point is obtained as

(t_1, δ_1, ω_1). This procedure can be repeated to get subsequent solution points.

Example 7

Consider the problem given in previous example and solve it using 4th order RK method.

Solution

As seen in the previous example, two first order DEs are

$$\frac{d\delta}{dt} = \omega - 314.1593$$

$$\frac{d\omega}{dt} = 39.2699\ (1 - P_e)$$

Initial point is:
$\delta(0) = 0.2333$ **rad.**
$\omega(0) = 314.1593$ **rad. / sec.**

During the first switching interval $t = 0^+$ **to 0.1 sec. electric output power** $P_e = 0$.

$$\frac{d\delta}{dt} = \omega - 314.1593 \qquad \frac{d\omega}{dt} = 39.2699\,(1 - P_e)$$

Initial point is:
$\delta(0) = 0.2333$ rad.
$\omega(0) = 314.1593$ rad. / sec.

To calculate $\delta(0.02)$ and $\omega(0.02)$

$k_1 = (314.1593 - 314.1593) \times 0.02 = 0$ $\qquad \ell_1 = 39.2699\,(1 - 0) \times 0.02 = 0.7854$

$\delta(0) + k_1 / 2 = 0.2333;$ $\qquad \omega(0) + \ell_1 / 2 = 314.1593 + 0.3927 = 314.552$

$k_2 = (314.552 - 314.1593) \times 0.02 = 0.007854$ $\qquad \ell_2 = 39.2699\,(1 - 0) \times 0.02 = 0.7854$

$\delta(0) + k_2 / 2 = 0.2372;$ $\qquad \omega(0) + \ell_2 / 2 = 314.1593 + 0.3927 = 314.552$

$k_3 = (314.552 - 314.1593) \times 0.02 = 0.007854$ $\qquad \ell_3 = 39.2699\,(1 - 0) \times 0.02 = 0.7854$

$\delta(0) + k_3 = 0.2412;$ $\qquad \omega(0) + \ell_3 = 314.1593 + 0.7854 = 314.9447$

$k_4 = (314.9447 - 314.1593) \times 0.02 = 0.0157$ $\qquad \ell_4 = 39.2699\,(1 - 0) \times 0.02 = 0.7854$

$$\delta(0.02) = 0.2333 + \frac{1}{6}[\,0 + 2\,(0.007854) + 2\,(0.007854) + 0.0157\,] = 0.24115 \text{ rad.}$$

$$\omega(0.02) = 314.1593 + \frac{1}{6}[\,0.7854 + 2\,(0.7854) + 2\,(0.7854) + 0.7854\,]$$

$$= 314.9447 \text{ rad / sec.}$$

It is to be noted that up to 0.1 sec., since P_e remains at zero,

constants $\ell_1 = \ell_2 = \ell_3 = \ell_4 = 0.7854$

$$\frac{d\delta}{dt} = \omega - 314.1593 \qquad \frac{d\omega}{dt} = 39.2699\,(1 - P_e)$$

Latest point is:
$\delta(0.02) = 0.24115$ rad.
$\omega(0.02) = 314.9447$ rad. / sec.

<u>To calculate $\delta(0.04)$ and $\omega(0.04)$</u>

$k_1 = (314.9447 - 314.1593) \times 0.02 = 0.01571$

$\omega(0.02) + \ell_1 / 2 = 314.9447 + 0.3927 = 315.3374$

$k_2 = (315.3374 - 314.1593) \times 0.02 = 0.02356$

$\omega(0.02) + \ell_2 / 2 = 314.9447 + 0.3927 = 315.3374$

$k_3 = (315.3374 - 314.1593) \times 0.02 = 0.02356$

$\omega(0.02) + \ell_3 = 314.9947 + 0.7854 = 315.7301$

$k_4 = (315.7301 - 314.1593) \times 0.02 = 0.03142$

$$\delta(0.04) = 0.24115 + \frac{1}{6}[\,0.01571 + 2\,(0.02356) + 2\,(0.02356) + 0.03142\,] = 0.2647 \text{ rad.}$$

$\omega(0.04) = 314.9447 + 0.7854 = 315.7301$ rad / sec.

$$\frac{d\delta}{dt} = \omega - 314.1593 \qquad \frac{d\omega}{dt} = 39.2699\,(1 - P_e)$$

Latest point is:
$\delta(0.04) = 0.2647$ rad.
$\omega(0.04) = 315.7301$ rad. / sec.

To calculate δ(0.06) and ω(0.06)

$k_1 = (315.7301 – 314.1593) \times 0.02 = 0.03142$

$\omega(0.04) + \ell_1 / 2 = 315.7301 + 0.3927 = 316.1228$

$k_2 = (316.1228 – 314.1593) \times 0.02 = 0.03927$

$\omega(0.04) + \ell_2 / 2 = 315.7301 + 0.3927 = 316.1228$

$k_3 = (316.1228 – 314.1593) \times 0.02 = 0.03917$

$\omega(0.04) + \ell_3 = 315.7301 + 0.7854 = 316.5155$

$k_4 = (316.5155 – 314.1593) \times 0.02 = 0.04712$

$$\delta(0.06) = 0.2647 + \frac{1}{6}[\,0.03142 + 2\,(0.03927) + 2\,(0.03927) + 0.04712\,] = 0.304 \text{ rad.}$$

$\omega(0.06) = 315.7301 + 0.7854 = 316.5155$ rad / sec.

$$\frac{d\delta}{dt} = \omega - 314.1593 \qquad \frac{d\omega}{dt} = 39.2699\,(1 - P_e)$$

Latest point is:
$\delta(0.06)$ = 0.304 rad.
$\omega(0.06)$ = 316.5155 rad. / sec.

<u>To calculate $\delta(0.08)$ and $\omega(0.08)$</u>

$k_1 = (316.5155 - 314.1593) \times 0.02 = 0.04712$

$\omega(0.06) + \ell_1 / 2 = 316.5155 + 0.3927 = 316.9082$

$k_2 = (316.9082 - 314.1593) \times 0.02 = 0.05498$

$\omega(0.06) + \ell_2 / 2 = 316.5155 + 0.3927 = 316.9082$

$k_3 = (316.9082 - 314.1593) \times 0.02 = 0.05498$

$\omega(0.06) + \ell_3 = 316.5155 + 0.7854 = 317.3009$

$k_4 = (317.3009 - 314.1593) \times 0.02 = 0.06283$

$$\delta(0.08) = 0.2647 + \frac{1}{6}[\,0.04712 + 2\,(0.05498) + 2\,(0.05498) + 0.06283\,] = 0.359 \text{ rad.}$$

$\omega(0.08) = 316.5155 + 0.7854 = 317.3009$ rad / sec.

$$\frac{d\delta}{dt} = \omega - 314.1593 \qquad \frac{d\omega}{dt} = 39.2699\,(1 - P_e)$$

Latest point is:
$\delta(0.08)$ = 0.359 rad.
$\omega(0.08)$ = 317.3009 rad. / sec.

To calculate $\delta(0.1)$ and $\omega(0.1)$

k_1 = (317.3009 – 314.1593) x 0.02 = 0.06283

$\omega(0.08) + \ell_1 / 2$ = 317.3009 + 0.3927 = 317.6936

k_2 = (317.6936 – 314.1593) x 0.02 = 0.07069

$\omega(0.08) + \ell_2 / 2$ = 317.3009 + 0.3927 = 317.6936

k_3 = (317.6936 – 314.1593) x 0.02 = 0.07069

$\omega(0.08) + \ell_3$ = 317.3009 + 0.7854 = 318.0863

k_4 = (318.0863 – 314.1593) x 0.02 = 0.07854

$$\delta(0.1) = 0.359 + \frac{1}{6}[\,0.06283 + 2\,(0.07069) + 2\,(0.07069) + 0.07854\,] = 0.4297 \text{ rad.}$$

$\omega(0.1)$ = 317.1593 + 0.7854 = 318.0863 rad. / sec.

At t = 0.1 sec., the fault is cleared. As seen in the previous example, for t ≥ 0.1 sec., electric power output of the alternator is given by $P_e = 4.3261 \sin \delta$

$$\frac{d\delta}{dt} = \omega - 314.1593 \qquad \frac{d\omega}{dt} = 39.2699\,(1 - P_e)$$

Latest point is:

$\delta(0.1) = 0.4297$ rad.

$\omega(0.1) = 318.0863$ rad. / sec.

To calculate $\delta(0.12)$ and $\omega(0.12)$

$k_1 = (318.0863 – 314.1593) \times 0.02 = 0.07854$

$P_e = 4.3261 \sin (0.4297 \text{ rad.}) = 1.8022$

$\ell_1 = 39.2699 (1 – 1.8022) \times 0.02 = - 0.63$

$\delta(0.1) + k_1 / 2 = 0.4690$; $\omega(0.1) + \ell_1 / 2 = 318.0863 - 0.315 = 317.7713$

$k_2 = (317.7713 – 314.1593) \times 0.02 = 0.07224$

$P_e = 4.3261 \sin (0.469 \text{ rad.}) = 1.9554$

$\ell_2 = 39.2699 (1 – 1.9554) \times 0.02 = - 0.7504$

$\delta(0.1) + k_2 / 2 = 0.4658$; $\omega(0.1) + \ell_2 / 2 = 318.0863 + 0.3752 = 317.7111$

$k_3 = (317.7111 – 314.1593) \times 0.02 = 0.07104$

$P_e = 4.3261 \sin (0.4658 \text{ rad.}) = 1.9430$

$\ell_3 = 39.2699 (1 – 1.9430) \times 0.02 = - 0.7406$

$\delta(0.1) + k_3 = 0.5007;$ $\omega(0.1) + \ell_3 = 318.0863 - 0.7406 = 317.3457$

$k_4 = (317.3457 – 314.1593) \times 0.02 = 0.06373$

$P_e = 4.3261 \sin (0.5007 \text{ rad.}) = 2.0767$

$\ell_4 = 39.2699 (1 – 2.0767) \times 0.02 = - 0.8456$

$$\delta(0.12) = 0.4297 + \frac{1}{6}[0.07854 + 2 (0.07224) + 2 (0.07104) + 0.06373] = 0.5012 \text{ rad.}$$

$$\omega(0.12)= 318.0863 + \frac{1}{6}[- 0.63 - 2 (0.7504) - 2 (0.7406) - 0.8456]= 317.3434 \text{ rad. / sec.}$$

$$\frac{d\delta}{dt} = \omega - 314.1593 \qquad \frac{d\omega}{dt} = 39.2699\,(1 - P_e)$$

<u>To calculate δ(0.14) and ω(0.14)</u>

Latest point is:
$\delta(0.12)$ = 0.5012 rad.
$\omega(0.12)$ = 317.3434 rad. / sec.

k_1 = (317.3434 – 314.1593) x 0.02 = 0.06368

P_e = 4.3261 sin (0.5012 rad.) = 2.0786

ℓ_1 = 39.2699 (1 – 2.0786) x 0.02 = - 0.8471

$\delta(0.12) + k_1 / 2$ = 0.5330; $\omega(0.12) + \ell_1 / 2$ = 317.3434 - 0.42355 = 316.91985

k_2 = (316.91985 – 314.1593) x 0.02 = 0.05521

P_e = 4.3261 sin (0.533 rad.) = 2.1982

ℓ_2 = 39.2699 (1 – 2.1982) x 0.02 = - 0.9411

$\delta(0.12) + k_2 / 2$ = 0.5288; $\omega(0.12) + \ell_2 / 2$ = 317.3434 - 0.47055 = 316.87285

k_3 = (316.87285 – 314.1593) x 0.02 = 0.05427

P_e = 4.3261 sin (0.5288 rad.) = 2.1825

ℓ_3 = 39.2699 (1 – 2.1825) x 0.02 = - 0.9287

δ(0.12) + k_3 = 0.5555; ω(0.12) + ℓ_3 = 317.3434 - 0.9287 = 316.4147

k_4 = (316.4147 – 314.1593) x 0.02 = 0.04511

P_e = 4.3261 sin (0.5555 rad.) = 2.2814

ℓ_4 = 39.2699 (1 – 2.2814) x 0.02 = - 1.0064

$$\delta(0.14) = 0.5012 + \frac{1}{6}[\,0.06368 + 2\,(0.05521) + 2\,(0.05427) + 0.04511\,] = 0.5558 \text{ rad.}$$

$$\omega(0.14) = 317.3434 + \frac{1}{6}[-\,0.8471 - 2\,(0.9411) - 2\,(0.9287) - 1.0064\,] = 316.4112 \text{ rad. / sec.}$$

The results are tabulated.

t sec	0	0.02	0.04	0.06	0.08	0.1	0.12	0.14
δ rad	0.2333	0.24115	0.2647	0.304	0.359	0.4297	0.5012	0.5558
δ deg	13.37	13.82	15.17	17.42	20.57	24.62	28.72	31.84
ω rad/sec	314.1593	314.9447	315.7301	316.5155	317.3009	318.0863	317.3434	316.4112

Factors Affecting Transient Stability

The two factors mainly affecting the stability of a generator are

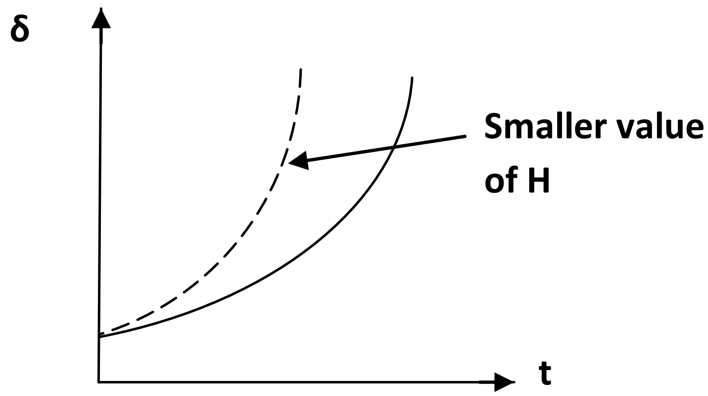
INERTIA CONSTANT H and TRANSIENT REACTANCE X_d'.

Smaller value of H:

Smaller the value of H means, value of M which is equal to H / π f is smaller. As seen in the step by step method

$$\Delta\delta_{(n)} = \Delta\delta_{(n-1)} + \frac{P_{a(n-1)}}{M} (\Delta t)^2$$

the angular swing of the machine in any interval is larger. This will result in lesser CCT and hence instability may result.



Larger value of X_d':

As the transient reactance of the machine increases, P_{max} decreases. This is so because the transient reactance forms part of over all series reactance of the system. All the three power output curves are lowered when P_{max} is decreased. Accordingly, for a given shaft power P_s, the initial rotor angle δ_0 is increased and maximum rotor angle δ_m is decreased. This results in smaller difference between δ_0 and δ_m as seen in Fig. 17.

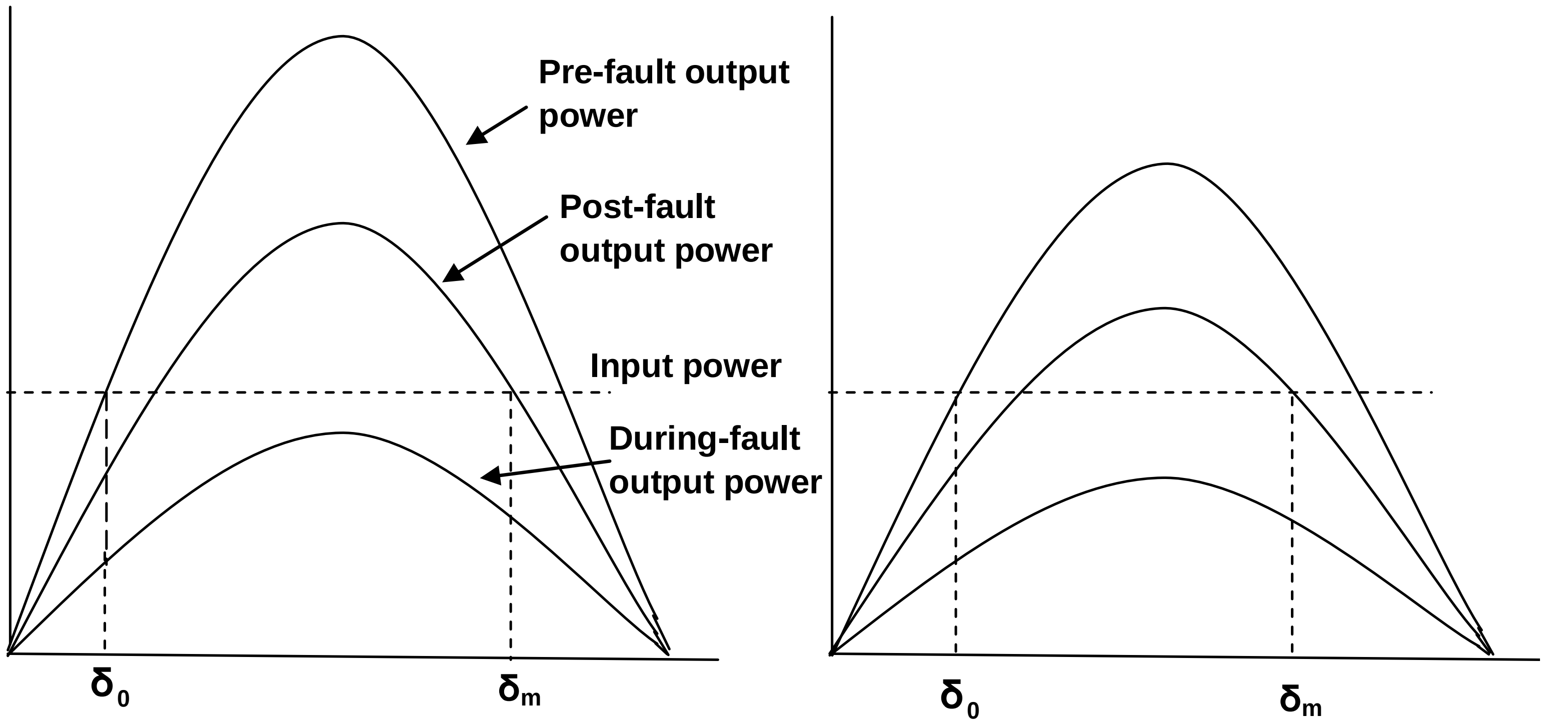


Fig. 17

The net result is that increased value of machine’s transient reactance constrains a machine to swing though a smaller angle from its original position before it reaches the critical clearing angle and the possibility of instability is more.

Thus any developments which lower the H constant and increase the transient reactance of the machine cause the CCT to decrease and lessen the possibility of maintaining the stability under transient conditions.