

Dr. Hadi Electronics II

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The following topics will be studied.

- 1) BJT Amplifiers (CH. 6)
- 2) MOSFET Amplifiers (CH. 4)
- 3) Operational Amplifier (CH. 9)
- 4) Amplifier Frequency Response (CH. 7)

BJT Amplifiers

Amplifier: It is a linear circuit used to magnify A.C input signal. Any amplifier contains:

D.C Sources: To bias the BJT in F.A.M with a certain operating point (V_{CEQ} , I_{CQ}).

A.C Source: to apply the required A.C input.

Resistors: for biasing and for voltage and current controlling.

Capacitors: D.C blocking, A.C Coupling.

Single-Stage BJT Amps.

There are three basic S.S. Amplifiers according to the common terminal.

a) Common-Emitter Amplifier

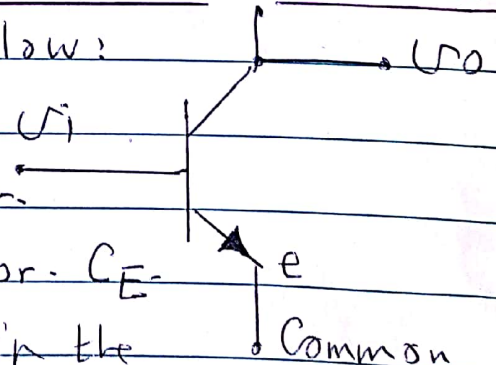
E → Comm. Terminal, input to base, V_{CC} from C. and can be classified as follow:

i- Basic C.E Amp.

ii- C.E with Emitter resistor.

iii- C.E with bypass capacitor. C_E

each ckt. will be discussed in the following:

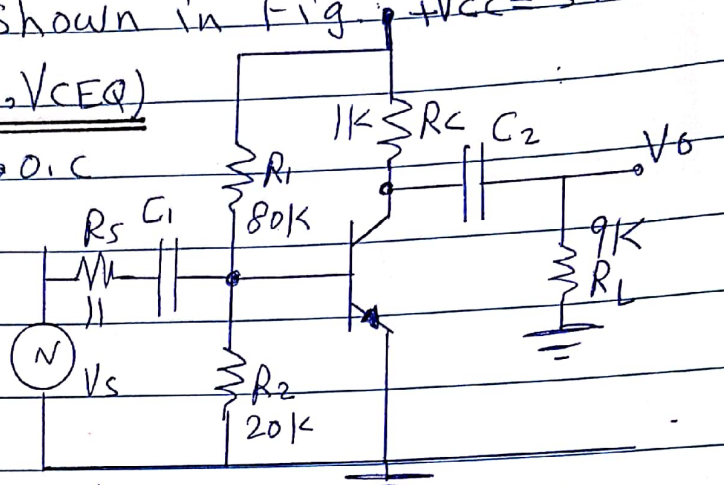


i) Basic Common-Emitter Amp

The BJT must be biased in F.A.M to be used as an Amp. So we must first perform D.C analysis and be sure that the BJT is in F.A.M, then start A.C analysis. Assume the ckt. shown in Fig. $+V_{CC} = 5V$

* Determine Q-pt (Find I_{CQ}, V_{CEQ})

* For D.C analysis all Caps \rightarrow o.c



$$R_{Th} = R_1 // R_2 = 16k\Omega$$

$$V_{Th} = \frac{V_{CC} R_2}{R_1 + R_2} = \frac{5 \times 20}{100} = 1V$$

$$-V_{Th} + I_B R_{Th} + V_{BE} = 0$$

$$I_B = \frac{(V_{Th} - V_{BE})}{R_{Th}} = \frac{(1 - 0.6)V}{16k} = 25\mu A \quad \beta = 100, V_{BE} = 0.6V$$

$$I_C = \beta I_B = 100 \times 25 = 2.5mA$$

$$-V_{CC} + I_C R_C + V_{CE} = 0$$

$$\therefore V_{CE} = 5 - 2.5 \times 1 = 2.5V$$

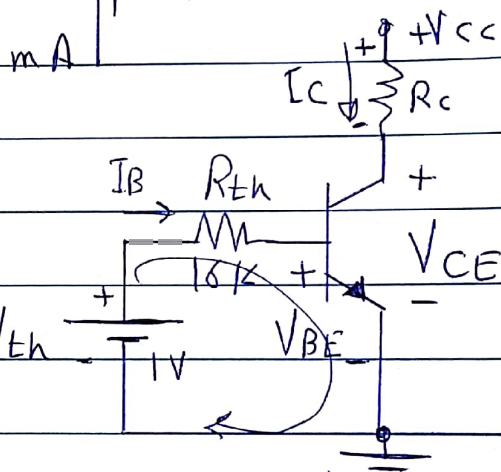
Since $I_B > 0$

\therefore B-E Jn. is Forward

and since $V_{CE} > V_{BE}$

\therefore B-C Jn. is reverse.

So the BJT is in F.A.M.



* Draw D.C.L.L indicating Q-pt and slope of D.C.L.L.

Write D.C.L.L eqn.

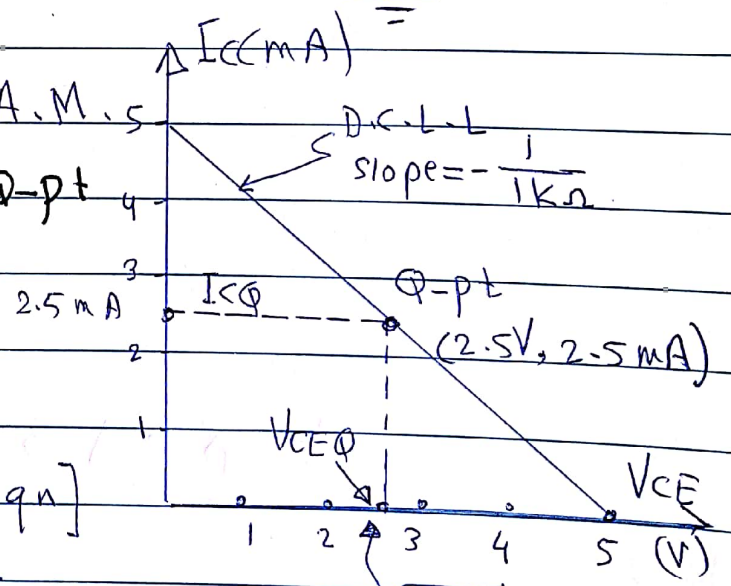
$$-V_{CC} + I_C R_C + V_{CE} = 0$$

$$V_{CE} = V_{CC} - I_C R_C \text{ [D.C.L.L eqn]}$$

$$\therefore \text{Slope of D.C.L.L} = -\frac{1}{R_C}$$

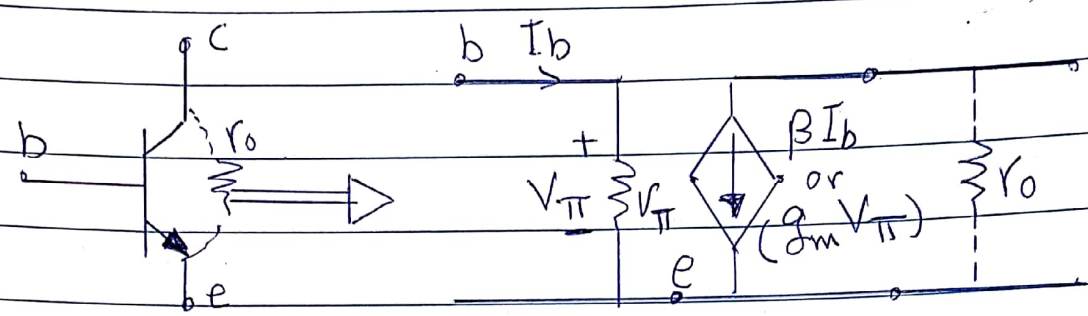
from eqn: i) for $I_C = 0, V_{CE} = V_{CC} \Rightarrow P_1 (5V, 0)$

for $V_{CE} = 0, I_C = \frac{V_{CC}}{R_C} = \frac{5V}{1k} = 5mA \Rightarrow P_2 (0, 5mA)$



A.C Analysis

To perform A.C Analysis, the BJT must be replaced by its model (equivalent cct.) called [hybrid- π] model



In this model: r_{π} : base-emitter resistance

and is given by: $r_{\pi} = \frac{\beta V_T}{I_{CQ}}$

β given, V_T = thermal voltage, $V_T = 0.026V = 26mV$
 I_{CQ} : Collector Current.

g_m : Transconductance (mA/V)

and given by: $g_m = \frac{I_{CQ}}{V_T}$

and if the BJT output resistance is considered (r_o), then it will be added as shown (between C & e)

and it is given by: $r_o = \frac{V_A}{I_{CQ}}$

where: V_A : is Early Voltage. It is given (50V \rightarrow 300V)

*NOTE: IF $V_A = \infty$ or NOT given, then $r_o = \infty$.

EXAMPLES: For the cct. shown calculate

A_v : Voltage gain $\rightarrow A_v = \frac{V_o}{V_s}$

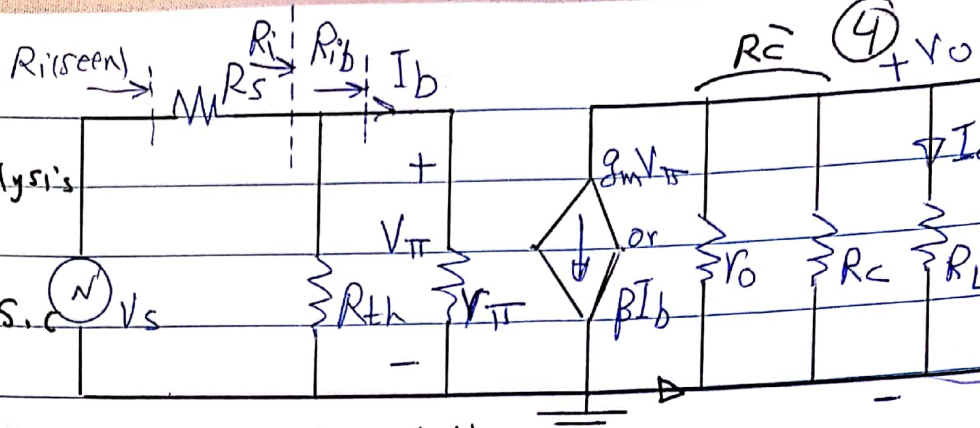
A_i : Current gain $\rightarrow A_i = \frac{I_o}{I_s}$

R_{in} : Input Resistance.

R_o : output \approx

* The A.C equivalent cct. is shown in the next page

* The A.C analysis is shown next page.



* For A.c analysis
 * all Caps \rightarrow S.c
 * all D.c Sources \rightarrow S.c
 * The BJT is replaced by its eqvt. cct (model).

① Small-Signal Voltage-gain: $A_v = \frac{V_o}{V_s} = \frac{V_o}{V_{\pi}} \times \frac{V_{\pi}}{V_s}$

$V_o = -g_m V_{\pi} R_L \Rightarrow \frac{V_o}{V_{\pi}} = -g_m R_L$ where $R_L = R_o \parallel R_c \parallel R_L$

$V_{\pi} = \frac{V_s \cdot R_i}{R_i + R_s} \Rightarrow \frac{V_{\pi}}{V_s} = \frac{R_i}{R_i + R_s}$ where $R_i = r_{\pi} \parallel R_{Th}$
 "using Voltage division Rule"

$\therefore A_v = -g_m R_L \frac{R_i}{R_i + R_s}$

⊖ means 180° phase-shift between V_o and V_s

$g_m = \frac{I_{CQ}}{V_T} = \frac{2.5 \text{ mA}}{26 \text{ mV}} = 96.154 \text{ mA/V}$

$r_{\pi} = \frac{\beta V_T}{I_{CQ}} = \frac{100 \times 26 \text{ mV}}{2.5 \text{ mA}} = 1.04 \text{ k}\Omega$, $R_o = \frac{V_A}{I_{CQ}} = \frac{100 \text{ V}}{2.5 \text{ mA}} = 40 \text{ k}\Omega$

$R_i = r_{\pi} \parallel R_{Th} = 1.04 \parallel 16 \text{ k} = 0.976 \text{ k}\Omega$

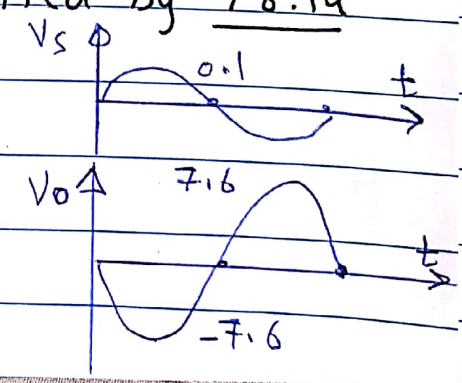
$\therefore \frac{V_o}{V_{\pi}} = -96.154 (40 \parallel 16 \parallel 9) = -96.154 \times 0.88 = -84.6$

$\frac{V_{\pi}}{V_s} = \frac{R_i}{R_i + R_s} = \frac{0.976}{0.976 + 0.1} = 0.9$

$\therefore A_v = -84.6 \times 0.9 = -76.14$

i.e. $V_o = -76.14 V_s \Rightarrow V_s$ is amplified by 76.14 and inverted by 180° .

[If (For exA: $V_s = 0.1 \sin \omega t \text{ V}$ then $V_o = -7.614 \sin \omega t \text{ V}$)]



② Small-Signal Current-gain $A_I = \frac{I_o}{I_s} = \frac{I_o}{I_b} \times \frac{I_b}{I_s}$

Using Current-division Rule "CDR" [From eqn. cct] →
 $I_o = \beta I_b \frac{R_c}{R_c + R_L} \rightarrow \frac{I_o}{I_b} = \frac{\beta R_c}{R_c + R_L}$ where $R_c = r_o \parallel R_c$

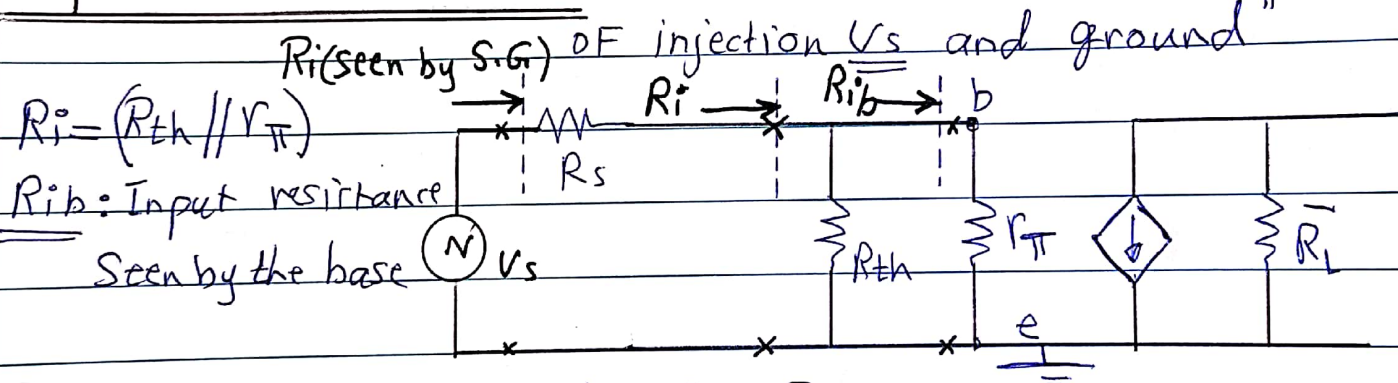
$I_b = \frac{I_s \cdot R_{th}}{R_{th} + r_{\pi}} \text{ (CDR)} \rightarrow \frac{I_b}{I_s} = \frac{R_{th}}{R_{th} + r_{\pi}}$

∴ $A_I = \frac{\beta R_c}{R_c + R_L} \frac{R_{th}}{R_{th} + r_{\pi}}$ $R_c = R_c \parallel r_o = 1 \parallel 40 = 0.975 K$

$A_I = \frac{100 \times 0.9756}{0.97549} \frac{16}{16 + 1.04} = \boxed{-9.183}$

this means that I_s is amplified by 9.183 and Inverted by 180° .

③ Input Resistance R_i : The resistance between the node



R_i (seen by Signal Generator) = $R_s + R_i$

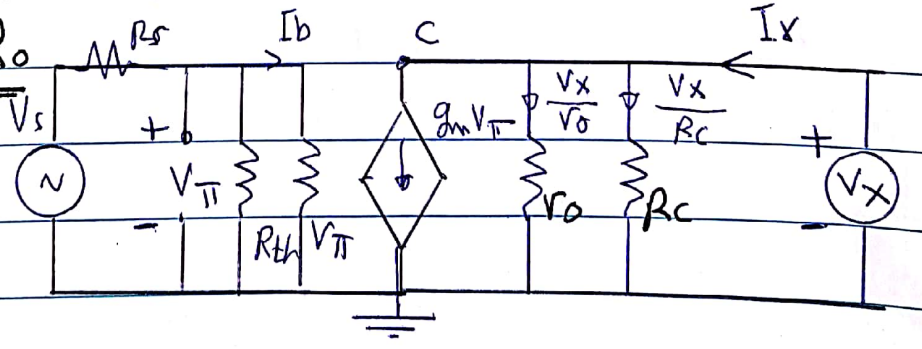
So $R_{ib} = r_{\pi} = 1.04 K\Omega$

$R_i = R_{th} \parallel r_{\pi} = 0.975 K\Omega$

R_i (seen by S.G) = $0.1 + 0.976 = 1.076 K\Omega$

④ output Resistance: R_o

$R_o = \frac{V_x}{I_x} \mid V_s = 0$



Write KCL at Node (c):

$$I_x = \frac{V_x}{R_o} + \frac{V_x}{r_o} + g_m V_{\pi}$$

but when $V_s = 0 \Rightarrow V_{\pi} = 0 \Rightarrow g_m V_{\pi} = 0$

$$\frac{I_x}{V_x} = \frac{1}{R_c} + \frac{1}{r_o} = \frac{1}{R_o}$$

$$R_o = R_c \parallel r_o$$

OR: We can say: when $V_s = 0, V_{\pi} = 0$ or $I_b = 0$, So the current source $g_m V_{\pi} = \beta I_b = 0$ i.e. it is open

So we can write $R_o = r_o \parallel R_c$

$$R_o = 1k \parallel 40k = 0.9756k\Omega$$

Av and AI Relation:

We can find Av in terms of AI and vice-versa

i) Av in terms of AI:

$$A_v = \frac{V_o}{V_s} = \frac{I_o R_L}{I_s (R_s + R_i)} = A_I \frac{R_L}{R_s + R_i}$$

ii) AI in terms of Av:

$$A_I = I_o / I_s = \frac{(V_o / R_L)}{(V_s / (R_s + R_i))} = \frac{V_o}{V_s} \frac{R_s + R_i}{R_L} = A_v \frac{R_s + R_i}{R_L}$$

*Be Careful!!! $V_o = I_o \cdot R_L$ and $I_o = V_o / R_L$ *

A.c. Load-line: a straight line drawn on transistor output characteristics relates a.c voltage and current (V_{ce}, I_c) with cct. elements. It intersects with D.C.L.L at Q-pt. and has a certain slope.

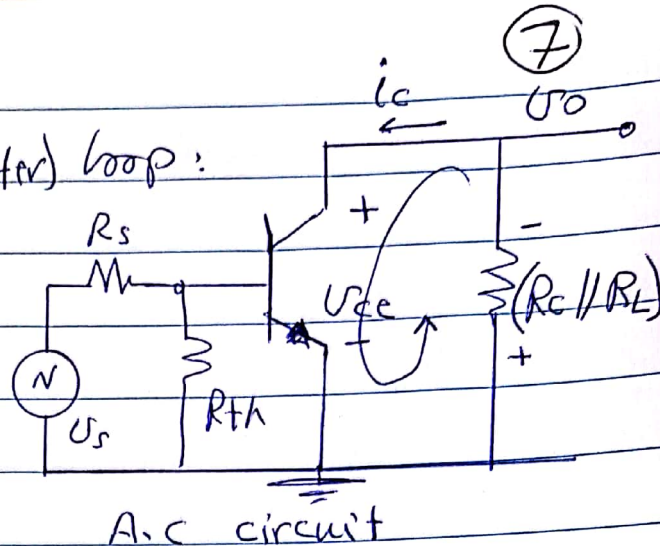
*The intersections with V_{ce} & I_c axis represent the max. values for I_c and V_{ce} for the cct.

Write KVL For (collector-emitter) loop:

$$V_{ce} + i_c (R_c // R_L) = 0$$

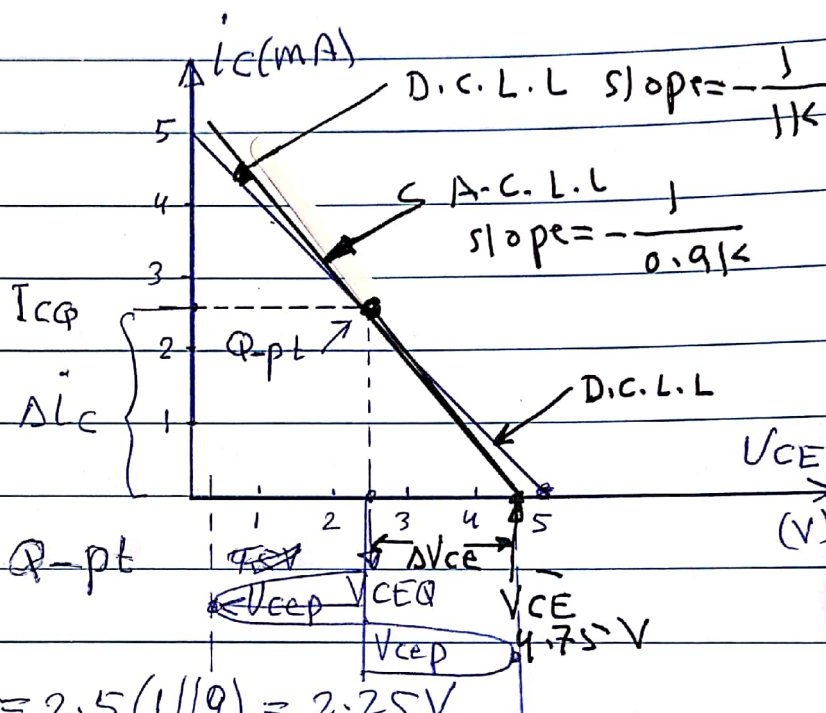
$$\boxed{V_{ce} = -i_c (R_c // R_L)} \text{ A.C.L.L. eqn.}$$

$$\text{Slope of A.C.L.L.} = -\frac{1}{(R_c // R_L)}$$



* It is a straight line passing through Q-pt. and has a slope = $-\frac{1}{R_c // R_L}$

* To Find another pt
 (Slope) = $\frac{1}{(R_c // R_L)} = \frac{\Delta i_c}{\Delta V_{ce}}$
 $\Delta V_{ce} = \Delta i_c (R_c // R_L)$



Since it passes through Q-pt
 So $\Delta i_c = I_{CQ}$

$$\Delta V_{ce} = I_{CQ} (R_c // R_L) = 2.5 (1 // 9) = 2.25V$$

$$V_{CE} = \Delta V_{ce} + V_{CEQ} = 2.5 + 2.25 = 4.75V$$

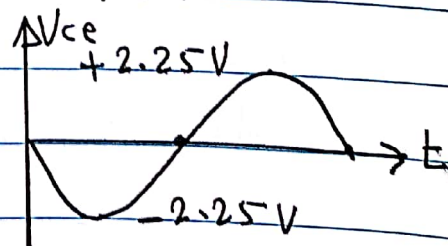
* We have two pts. and we can draw A.C.L.L.

* Maximum peak-to-peak Symmetrical Swings:

It is the Max. (p-to-p) Symmetrical V_{ce} Voltage without distortion. i.e. $V_{ce}(\text{peak}) - \Delta V_{ce} = I_{CQ} (R_c // R_L) = 2.25V$

So $V_{ce}(p-p)_{\text{max}} = 2 \Delta V_{ce} = 4.5V$ as shown.

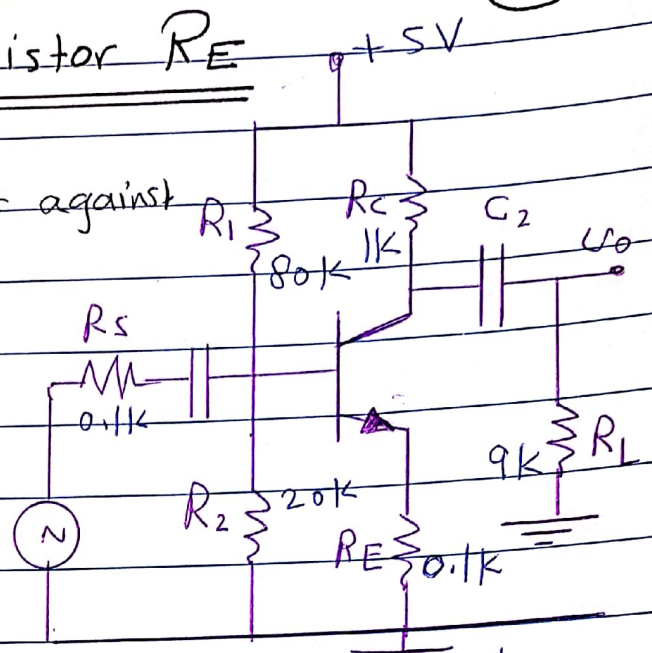
i.e. For this cct. the Max. peak-to-peak V_{ce} Voltage without distortion and Symmetrical is 4.5V



ii) C.E with Emitter Resistor R_E

" R_E is used to stabilize Q-pt against β Variation".

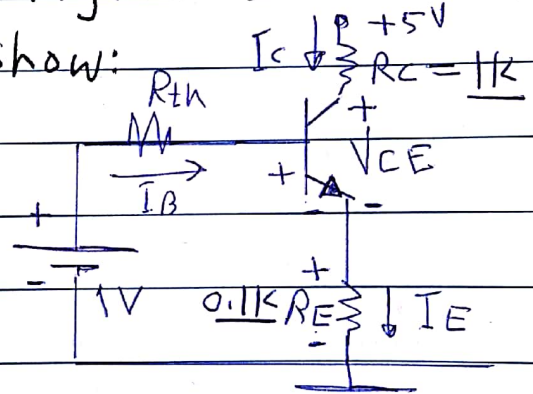
Consider the Basic-C-E cct. discussed before "without R_E " with $\beta = 100, I_{CQ} = 2.5 \text{ mA}, V_{CEQ} = 2.5 \text{ V}$. the BJT was in F.A.M.



* When $\beta = 200$, then $I_{CQ} = 5 \text{ mA}, V_{CE} = 0$, the BJT will be in Saturation mode and can't be used as an Amp.*

* IF R_E is connected as in this Fig. "C.E with R_E " the D.C analysis will be as show:

$R_{th} = 80 // 20 = 16 \text{ k}\Omega$
 $V_{th} = 5 \times \frac{20}{100} = 1 \text{ V}$



$V_{th} + I_B R_{th} + V_{BE} + I_E R_E = 0$

but $I_E = (\beta + 1) I_B$

$\therefore I_B = \frac{V_{th} - V_{BE}}{R_{th} + (\beta + 1) R_E}$

$\beta = 200, V_{BE} = 0.6 \text{ V}$

For $\beta = 200$

$\therefore I_B = \frac{(1 - 0.6) \text{ V}}{16 + 201 \times 0.1} = 0.011 \text{ mA}$

$I_{CQ} = \beta I_B = 2.216 \text{ mA}$

$I_{EQ} = (\beta + 1) I_B = 2.227 \text{ mA}$

$V_{CEQ} = 5 - I_C R_C - I_E R_E = 5 - 2.216 - 0.2227 = 2.561 \text{ V}$

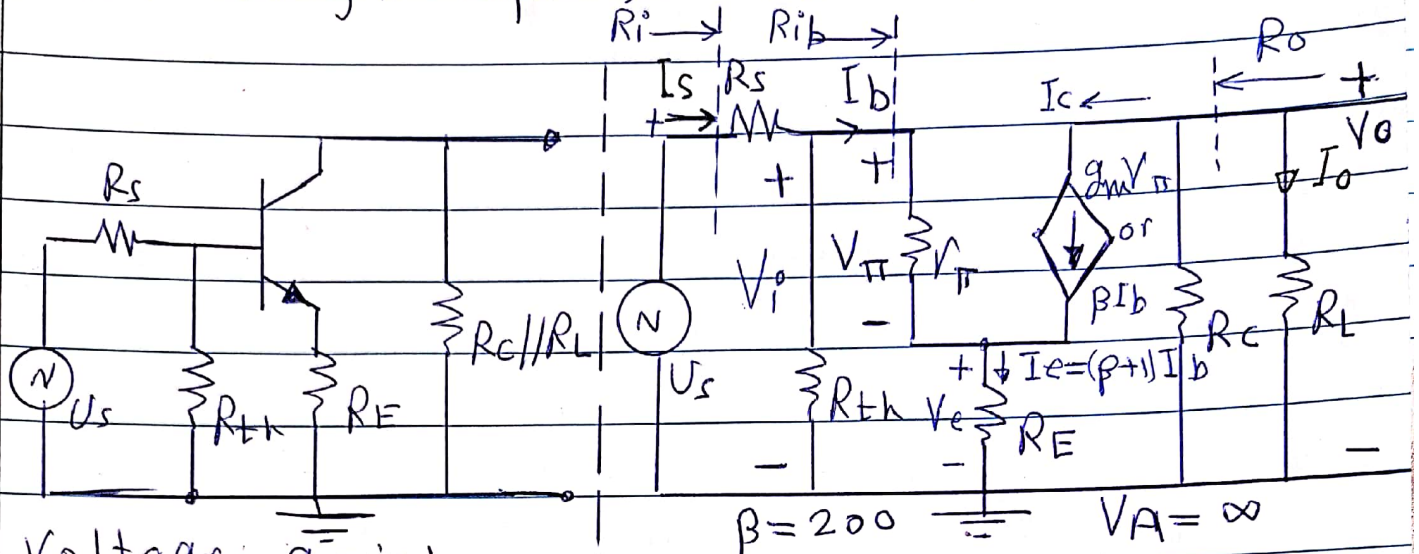
* The BJT is in F.A.M !!!

i.e R_E stabilize Q-pt and the BJT is in F.A.M even when $\beta = 200$.

A.c Analysis of C.E with RE

Calculate A_v, A_i, R_{ib}, R_i and R_o

* For A.c Analysis: Caps \rightarrow S.C, D.C Sources \rightarrow S.C



Voltage gain:

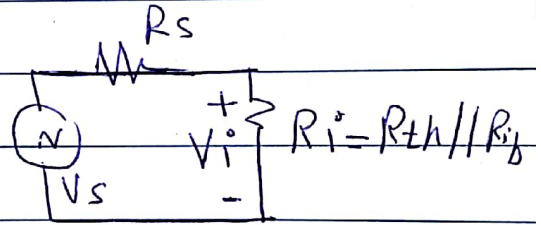
$$A_v = \frac{V_o}{V_s} = \frac{V_o}{V_i} \times \frac{V_i}{V_s}, \quad V_o = -\beta I_b (R_c // R_L)$$

$$-V_i + V_{\pi} + V_e = 0 \implies V_i = V_{\pi} + V_e = I_b (V_{\pi} + (\beta + 1) R_E)$$

$$\frac{V_o}{V_i} = \frac{-\beta (R_c // R_L)}{V_{\pi} + (\beta + 1) R_E}$$

$$V_i = \frac{(V_s \cdot R_i)}{R_i + R_s} \quad (\text{VDR})$$

$$\frac{V_i}{V_s} = \frac{R_i}{R_i + R_s} \quad \text{where } R_i = R_{th} // R_{ib}$$



$$A_v = \frac{-\beta (R_c // R_L)}{V_{\pi} + (\beta + 1) R_E} \cdot \frac{R_i}{R_i + R_s}$$

$$V_{\pi} = \frac{\beta V_T}{I_{CQ}}$$

this means that "RE reduces Av"

However: $R_i = R_{th} // R_{ib}$

$$R_{ib} = \frac{V_i}{I_b} = V_{\pi} + (\beta + 1) R_E$$

$$= \frac{200 \times 26}{2.216 \text{ mA}}$$

$$= 2.346 \text{ K}\Omega$$

$$R_{ib} = 2.346 + 201 \times 0.1$$

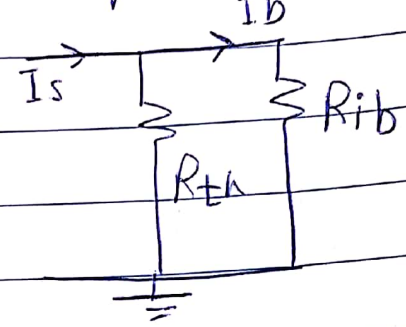
$$= 22.446 \text{ K}\Omega$$

$$R_i = R_{th} // R_{ib} = 16 // 22.446 = 9.341 \text{ K}\Omega$$

$$A_v = \frac{-200 \times 0.9}{2.346 + 20.1} \cdot \frac{9.341}{9.341 + 0.1} = \boxed{-7.934}$$

$R_o = \frac{V_x}{I_x} \Big|_{V_s=0}$ when $V_s=0, I_b=0, \beta I_b=0 \rightarrow 0.c$

$R_o = R_c = 1k\Omega$



$A_I = \frac{I_o}{I_s} = \frac{I_o}{I_b} \times \frac{I_b}{I_s}$

$I_o = -\beta I_b R_c \rightarrow \frac{I_o}{I_b} = \frac{-\beta R_c}{R_c + R_L}$

$I_b = \frac{I_s \cdot R_{th}}{R_{th} + R_{ib}} \text{ (CDR)} \rightarrow \frac{I_b}{I_s} = \frac{R_{th}}{R_{th} + R_{ib}}$

$A_I = \frac{-\beta R_c}{R_c + R_L} \cdot \frac{R_{th}}{R_{th} + R_{ib}} = \frac{-200 \times 1}{1+9} \cdot \frac{16}{16+22.4k\Omega} = -8.32$

OR: $A_I = \frac{I_o}{I_s} = \frac{(V_o/R_L)}{V_s/R_s + R_i} = A_v \cdot \frac{R_s + R_i}{R_L} = -8.32$

* Compare to Basic C.E Amp., C.E with R_E has the following ~~effects~~ characteristics:

- * Stabilize Q-pt. against β -variation (D.C) Advantages
- * Reduce A_v & A_I (Disadvantages)
- * Increase R_i (Advantages).
- * Stabilize A_v & A_I against β -variation (Advantages) as in the following.

Consider the eqn. For $A_v = \frac{-\beta(R_c // R_L)}{V_{\pi} + (\beta+1)R_E} \cdot \frac{R_{in}}{R_{in} + R_s}$

* IF $r_{\pi} \ll (\beta+1)R_E$ & $\beta \gg 1$ & $R_i \gg R_s$, then

$A_v \approx \frac{(R_c // R_L)}{R_E}$. This means A_v is approximately becomes independant on β .

* For this cct. $A_v = \frac{(9//1)}{0.1} = 9$

iii) C.E with bypass Cap. C_E:

To gain the advantages of R_E and Reduce its disadvantages of Reducing A_v & A_I.

* C_E is connected across R_E

1) * For D.C Analysis

All caps → o.c and the ckt. is analyzed as C.E with R_E and hence R_E stabilize Q-pt.

* From previous analysis:

$I_{BQ} = 0.011 \text{ mA}$

$I_{CQ} = 2.216 \text{ mA}$

$I_{EQ} = 2.227 \text{ mA}$

$V_{CEQ} = 2.561 \text{ V}$

* BJT in F.A.M.

2) For A.C Analysis, all caps → S.c the ckt. is analyzed as Basic C.E

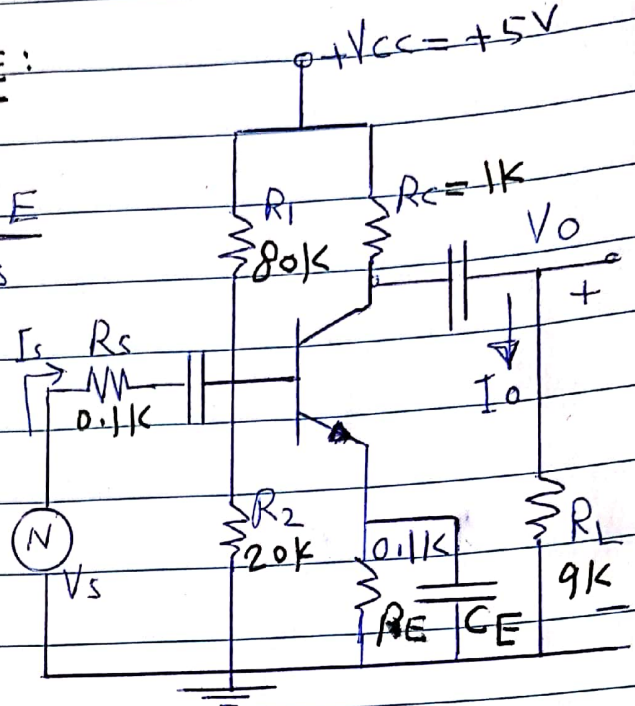
Amp.

$A_v = -g_m (R_c // R_L)$

$R_i = R_{\pi} // R_{th} = 2.046 \text{ k}\Omega$

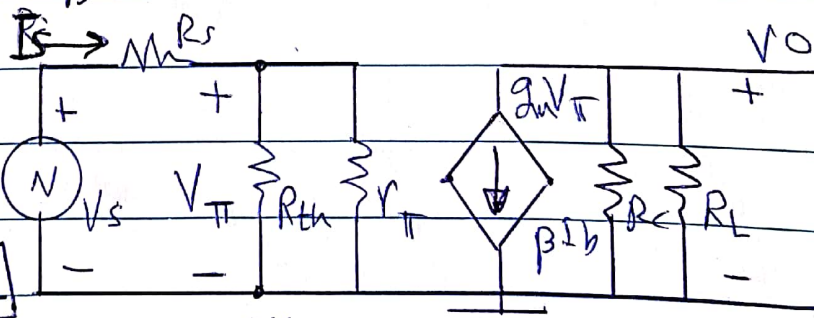
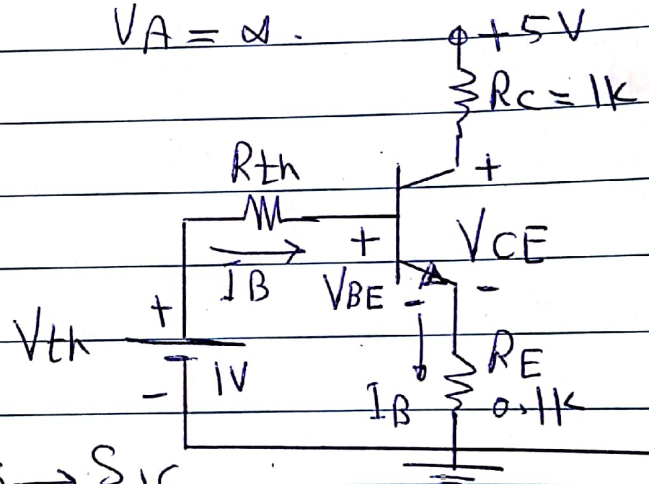
$A_v = -85.23 (1//9) \frac{2.046}{0.1+2.046} = -73.13$

$A_I = A_v \frac{R_i + R_s}{R_L} = 17.438$



$\beta = 200, V_{BE} = 0.6 \text{ V}$

$V_A = \infty$



$r_{\pi} = \frac{\beta V_T}{I_{CQ}} = 2.346 \text{ k}\Omega$

$R_i = 2.346 // 16 = 2.046 \text{ k}\Omega$

$g_m = I_{CQ} / V_T = 85.23 \text{ mA/V}$

Note: When C_E is connected, D.C values unchanged but $A_V \rightarrow$ increase, $A_I \rightarrow$ Increase but $R_i \rightarrow$ decrease !!!

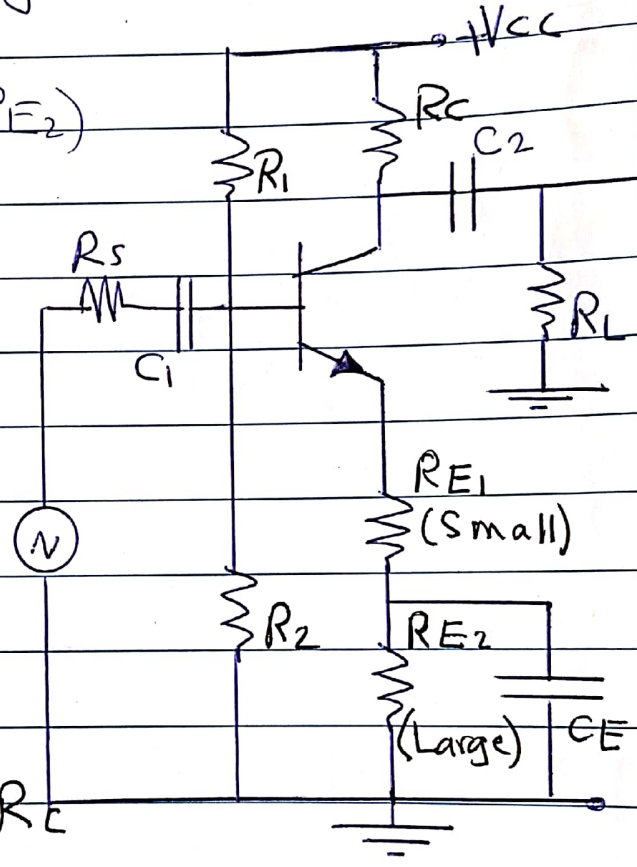
* In this case we ~~lose~~ have lost the advantages of high R_i and A_V & A_I stability. So some times R_E is made consisting of two parts R_{E1} & R_{E2} then connect C_E across the large R_E value, then we can gain all the advantages of R_E and minimize its disadvantages.

* For D.C analysis both $(R_{E1} + R_{E2})$ are included.

* For A.C Analysis, only R_{E1} is considered and since it is small, so it will NOT affect A_V , A_I , R_i too much.

* R_{E2} is short out for A.C Analysis and NOT affect A_V , A_I , R_i .

* For D.C-L.L, R_{E1} & R_{E2} , R_C both are present.



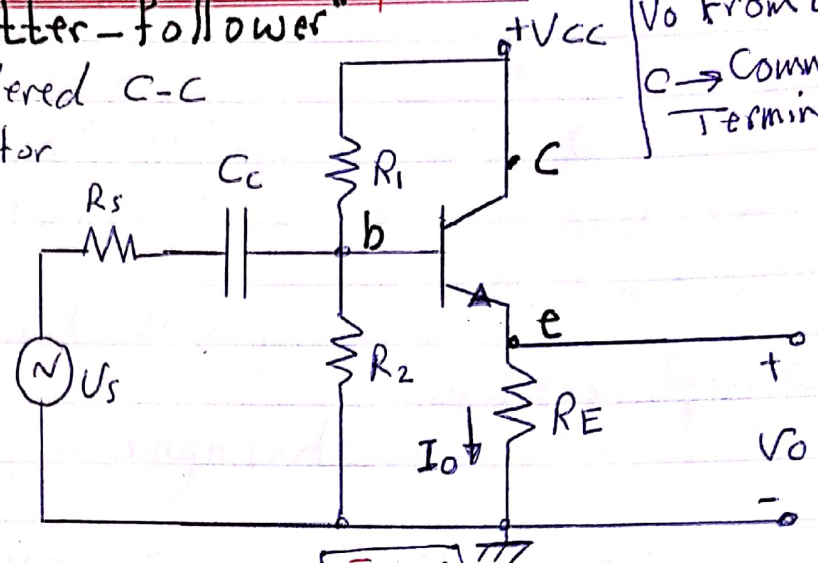
D.C.L.L slope =
$$-\frac{1}{R_C + (R_{E1} + R_{E2}) \frac{\beta + 1}{\beta}}$$

* A.C.L.L, R_C , R_L , R_{E1}

slope =
$$-\frac{1}{(R_C // R_L) + \frac{\beta + 1}{\beta} R_{E1}}$$

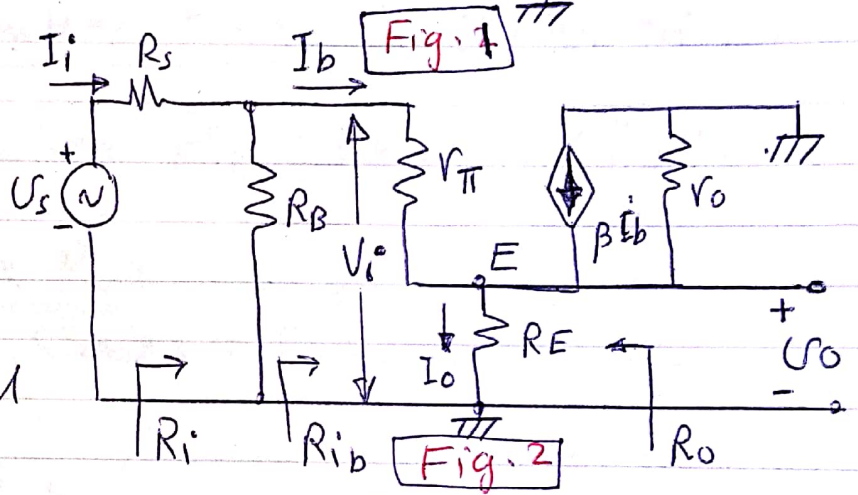
② Common-Collector Amplifier.
 * Emitter-follower

Fig. 1 shows the Standard C-C Amp. where the Collector is connected directly to V_{CC} which is grounded for a.c signals.

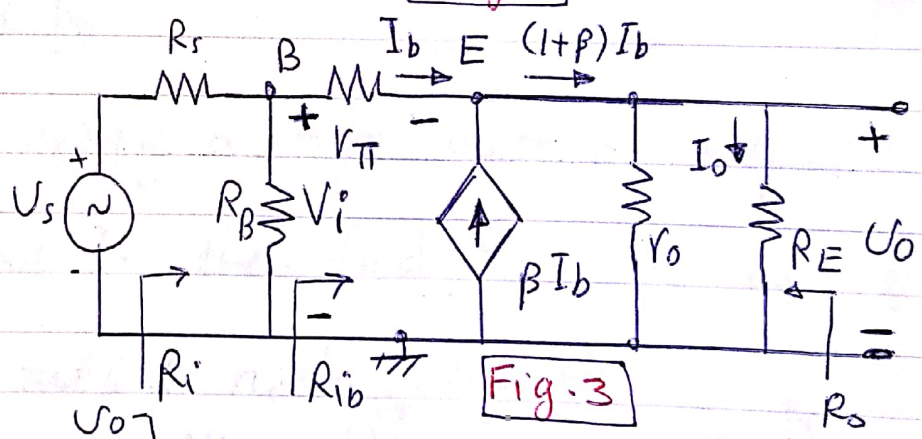


$V_B \rightarrow$ base
 V_o From E
 $C \rightarrow$ Common Terminal

* This cct. can be analysed as C-E with Emitter Resist. and taking the o/p from Emitter (Fig. 2) or can be redrawn



as in Fig. 3 and treated as (C-C) with all signal ground at the same point.



* However both Eqnt. ccts. will give the same Amplifier parameters expression.

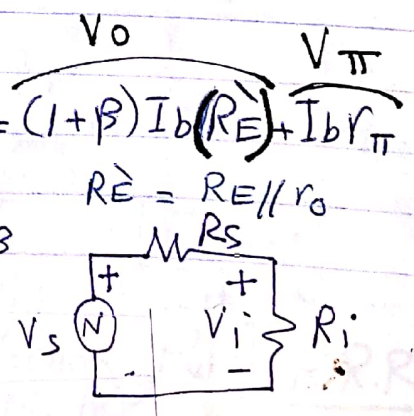
From Fig. 3

Voltage-gain: $[A_v = \frac{V_o}{V_s}]$

$$V_o = (1 + \beta) I_b (r_o \parallel R_E) \quad , \quad V_i = V_o + V_{\pi} = (1 + \beta) I_b (R_E) + I_b r_{\pi}$$

$$\frac{V_i}{V_s} = \frac{R_i}{R_i + R_s} \quad \text{where} \quad R_i = R_{ib} \parallel R_B$$

$$\frac{V_i}{I_b} = R_{ib} = r_{\pi} + (1 + \beta)(R_E \parallel r_o)$$



$$(V_o/V_i) = \frac{(1+\beta)(R_E \parallel r_o)}{r_{\pi} + (1+\beta)(R_E \parallel r_o)}$$

$$\frac{V_o}{V_s} = \frac{V_o}{V_i} \times \frac{V_i}{V_s} = \frac{(1+\beta)(R_E \parallel r_o)}{r_{\pi} + (1+\beta)(R_E \parallel r_o)} \times \frac{R_i}{R_i + R_s}$$

For $R_i \gg R_s$, $(1+\beta)(R_E \parallel r_o) \gg r_{\pi}$, then ① $A_v \approx 1$

$A_v \approx 1$ (Unity and zero phase). ② $\phi = 0^\circ$
 i.e. V_o follows V_s in magnitude and phase. *and taken from C*

It is called (Emitter-Follower). ③ high R_i

$$R_i = R_B \parallel R_{ib} \text{ where } R_{ib} = r_{\pi} + (1+\beta)(R_E \parallel r_o)$$

Output Resistance R_o :

Consider the eqnt. ckt. shown in Fig. 4

* R_o can be found by:

setting $V_s = 0$, applying a voltage source at output V_x which will cause I_x , then find $R_o = \frac{V_x}{I_x}$ by

writing KCL at output node (Node E)

$$g_m V_{\pi} + I_x = \frac{V_x}{R_E} + \frac{V_x}{(R_s \parallel R_B) + r_{\pi}}$$

then subti. $V_{\pi} = - \frac{V_x r_{\pi}}{r_{\pi} + (R_s \parallel R_B)}$

$$\text{and Find } \frac{V_x}{I_x} = R_o = \left(\frac{r_{\pi} + (R_s \parallel R_B)}{1+\beta} \right) \parallel R_E \parallel r_o$$

④ R_o Low R_o !!

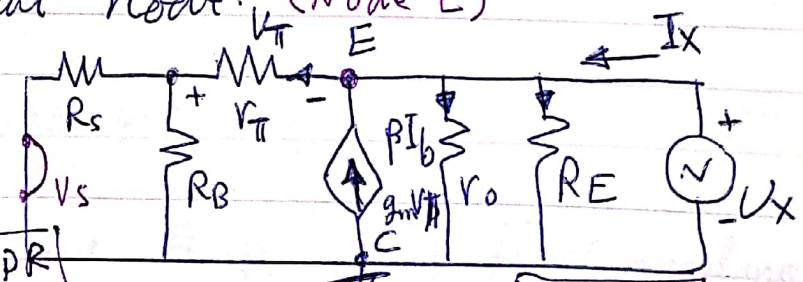
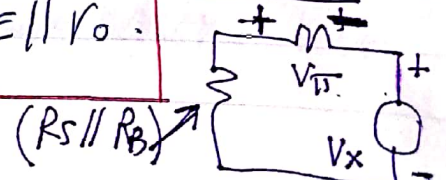


Fig. 4



A.c Current-gain A_i :

$A_i = \frac{I_o}{I_i} = \frac{I_o}{I_b} \times \frac{I_b}{I_i}$, From Eqnt. cct.

$I_o = \frac{(1+\beta) I_b r_o}{r_o + R_E} \Rightarrow (I_o/I_b) = (1+\beta) \frac{r_o}{r_o + R_E}$

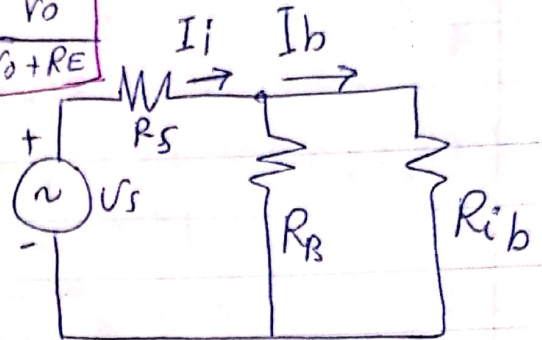
and $I_b = I_i \frac{R_B}{R_B + R_{i_b}}$

$\therefore \frac{I_b}{I_i} = \frac{R_B}{R_B + R_{i_b}}$

$A_i = \frac{I_o}{I_b} \times \frac{I_b}{I_i} = (1+\beta) \frac{R_B}{R_B + R_{i_b}} \frac{r_o}{r_o + R_E}$

For $r_o \gg R_E$, and $R_B \gg R_{i_b}$ then

$A_i \approx 1+\beta$ (high A_i)



Characteristics of C-C:

- 1) Approximately unity voltage-gain. ($A_v \approx 1$) ✓
- 2) Zero phase-shift. ✓
- 3) high Current gain $A_i \approx 1+\beta$. ✓
- 4) High input impedance ✓
- 5) Low output impedance ✓

So it is used as S-S power Amp. or Buffer (impedance transformer) to provide No loading connection effect.
OR: minimize loading effect.

Resistance Reflection Rule "RRR" and Its Inverse "IRRR"

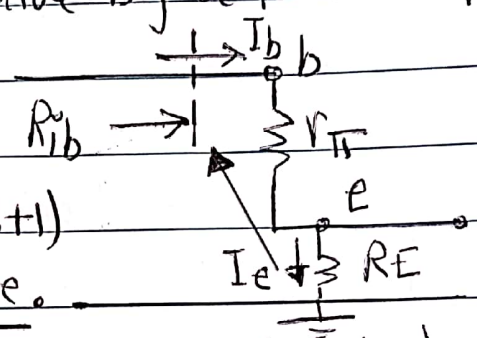
This rule used to calculate resistance looking from Emitter or base

R.R.R → reflection of resistance from E → B

It is based on that $I_e = (\beta + 1)I_b$ and since $I \propto \frac{1}{R}$ so any resistance in the emitter, is seen from base by multiplying its value by a factor of $(\beta + 1)$ as shown in Fig.

$$R_{ib} = r_{\pi} + (\beta + 1)R_E$$

i.e R_E is multiplied by $(\beta + 1)$ when it is seen from base.



∴ R.R.R → from e → to base multiply by $(\beta + 1)$

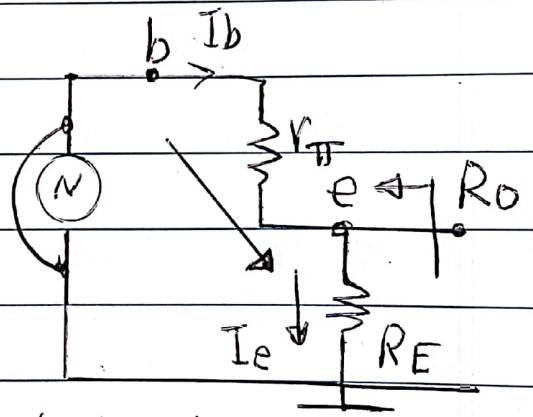
*I.R.R.R: Reflection resistance from base to emitter

To reflect a resistance from base to emitter divide its value

by $(\beta + 1)$. i.e

r_{π} is seen from emitter to be

$$\left(\frac{r_{\pi}}{\beta + 1}\right) \text{ So } R_o = \left(\frac{r_{\pi}}{\beta + 1}\right) \parallel R_E$$



* So we can use this rule to calculate input or output resistance for C.C. Amp. directly without using voltage, current eqns. For example for R_i

we can say: $R_i = R_{th} \parallel R_{ib}$ and

$R_{ib} = r_{\pi} + (\beta + 1)(R_o \parallel R_E)$ using R.R.R. (e → b)

$$\text{and: } R_o = \left[\frac{(\beta \parallel R_{th}) + r_{\pi}}{\beta + 1} \right] \parallel R_o \parallel R_E \text{ using IRRR (b → e)}$$

③ Common-Base Amp

17
 $V_s \rightarrow$ emitter
 V_o From Collector
 $B \rightarrow$ Common Ter.

Voltage-gain:

$$A_v = \frac{V_o}{V_s} = \frac{V_o}{V_{\pi}} \times \frac{V_{\pi}}{V_s}$$

$$V_o = -g_m V_{\pi} (R_c \parallel R_L)$$

$$V_{\pi} = V_i = \frac{-V_s \cdot R_i}{R_i + R_s}$$

$$\frac{V_o}{V_i} = \frac{R_i}{R_i + R_s} \quad (V_i = -V_{\pi})$$

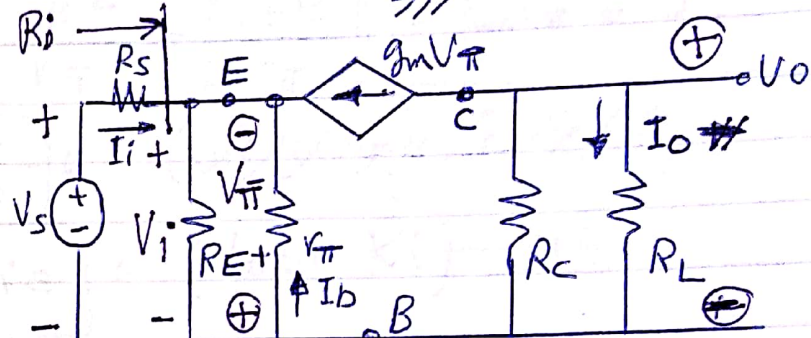
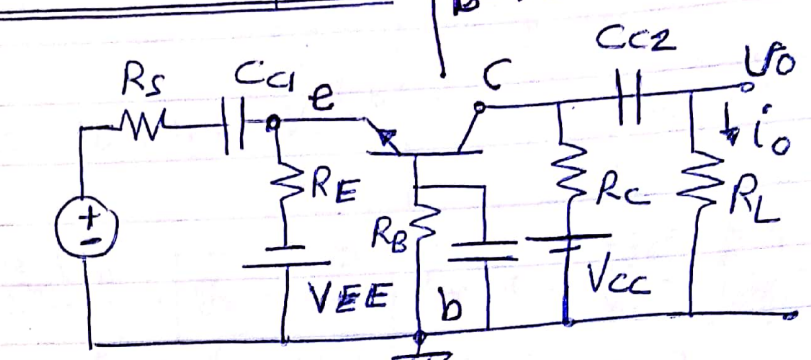
Where $R_i = \left(\frac{r_{\pi}}{1+\beta} \right) \parallel R_E$

① Low R_i

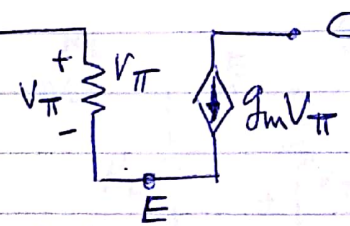
$$\frac{V_o}{V_s} = g_m (R_c \parallel R_L) \frac{R_i}{R_i + R_s}$$

But $\frac{r_{\pi}}{1+\beta} \ll R_E \Rightarrow R_i \approx \frac{r_{\pi}}{1+\beta}$

For $R_s = 0$, $A_v \approx g_m (R_c \parallel R_L)$



OR using RRR] A.C eqnt. cct-



hybrid- π model

Current-gain: $A_i = \frac{I_o}{I_i}$

$$I_o = -g_m V_{\pi} \frac{R_c}{R_c + R_L} \quad \text{[CDR]}$$

$$V_{\pi} = -I_i R_i = \left[\left(\frac{r_{\pi}}{1+\beta} \right) \parallel R_E \right] I_i$$

$$\therefore \frac{I_o}{I_i} = +g_m \frac{R_c}{R_c + R_L} \left[\left(\frac{r_{\pi}}{1+\beta} \right) \parallel R_E \right]$$

Since $\frac{r_{\pi}}{1+\beta} \ll R_E$, and For $R_L = \infty$, then

$$A_i \approx g_m \frac{r_{\pi}}{1+\beta} \approx \frac{\beta}{1+\beta} \approx 1$$

③ $A_i \approx 1$

KCL at Node E

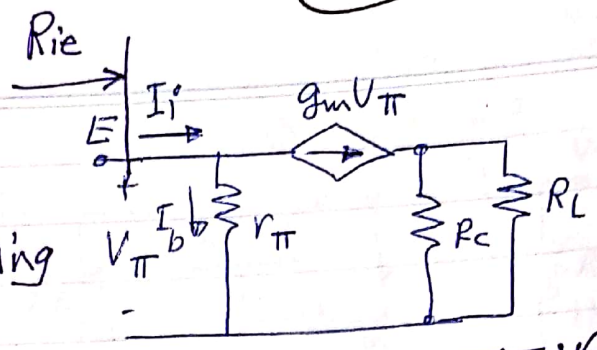
KCL at E node

$$I_i + \frac{V_{\pi}}{R_E} + \frac{V_{\pi}}{r_{\pi}} + g_m V_{\pi} = 0$$

this will give

$$V_{\pi} = -I_i \left[\left(\frac{r_{\pi}}{1+\beta} \right) \parallel R_E \right]$$

Input Resistance: R_i



$R_{ie} = \frac{V_{\pi}}{I_i}$ (Input Resistance looking to emitter).

$$I_i = I_b + g_m V_{\pi} = \frac{V_{\pi}}{r_{\pi}} + g_m V_{\pi}$$

$[I_b = v_{\pi}/r_{\pi}]$

$$\frac{I_i}{V_{\pi}} = \frac{1}{r_{\pi}} + g_m = \frac{1}{r_{\pi}} + \frac{\beta}{r_{\pi}}$$

$g_m = \frac{\beta}{r_{\pi}}$

$g_m r_{\pi} = \frac{I_{cQ}}{V_T} \approx \frac{\beta V_T}{I_{cQ}}$

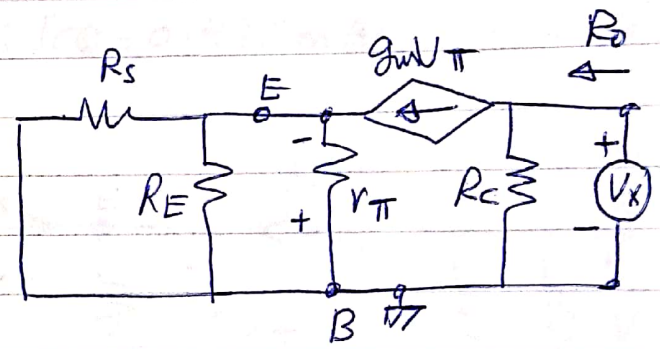
$\therefore g_m r_{\pi} = \beta$

$\therefore V_{\pi}/I_i = R_{ie} = \frac{r_{\pi}}{1+\beta}$ (OR using R/R/R).

$R_i = R_{ie} // R_E \approx R_{ie}$ (low R_i) \rightarrow (4)

Output Resistance: R_o

Setting $V_s = 0$ & Applying V_x at output terminal.



KCL at node E gives

$$g_m V_{\pi} + \frac{V_{\pi}}{r_{\pi}} + \frac{V_{\pi}}{R_E} + \frac{V_{\pi}}{R_s} = 0 \text{ which gives } V_{\pi} = 0$$

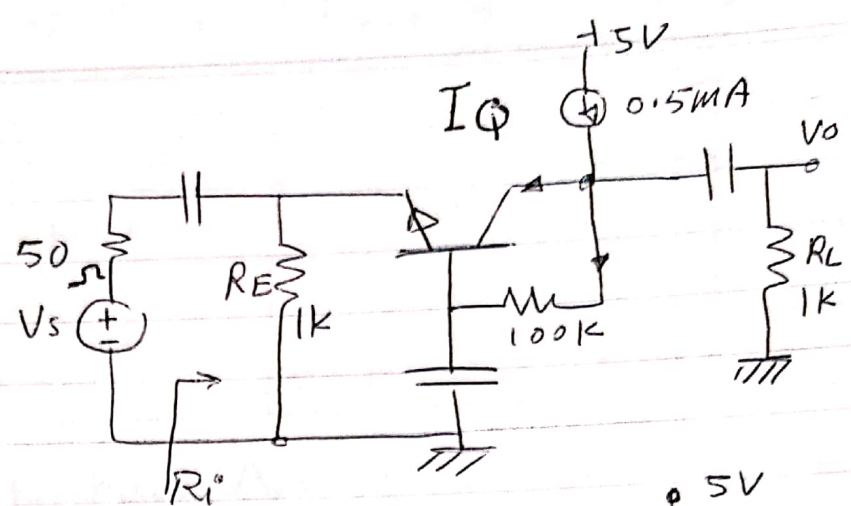
this means $g_m V_{\pi} = 0$ and consequently

$R_o = R_c$ (high since $r_o = \infty$)
main c/cs of C.B

- (1) High voltage gain ✓
- (3) low input Resistance ✓
- (2) low Current gain ($A_i \leq 1$) ✓
- (4) high output Resistance. ✓
- (5) Zero phase-shift ✓

P 4.47: $\beta = 100, V_A = \infty$

- 1) Find V_B, V_C, V_E
- 2) Find A_v, R_i



$$I_E = I_Q = 0.5 \text{ mA}$$

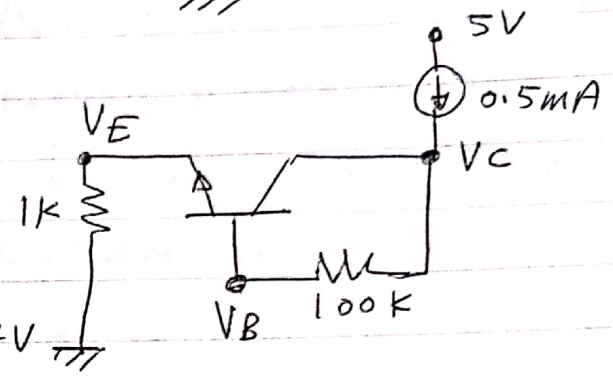
$$I_B = \frac{I_E}{1 + \beta} = \frac{0.5}{101} = 4.95 \mu\text{A}$$

$$I_{CQ} = \beta I_B = 0.495 \text{ mA}$$

$$V_E = I_E R_E = 0.5 \times 1 = 0.5 \text{ V}$$

$$V_B = V_E + V_{BE} = 0.5 + 0.7 = 1.2 \text{ V}$$

$$V_C = V_B + I_B R_B = 1.2 + 4.95 \times 100 \text{ k} = 1.7 \text{ V}$$



② $V_{\pi} = \frac{100 \times 26}{0.495} = 5.252 \text{ k}\Omega$ ($I_{CQ} = 0.495 \text{ mA}$)

$$g_m = \frac{0.495}{26} = 19 \text{ mA/V}$$

$$R_i = R_E \parallel \frac{r_{\pi}}{1 + \beta} \text{ [using RRR]}$$

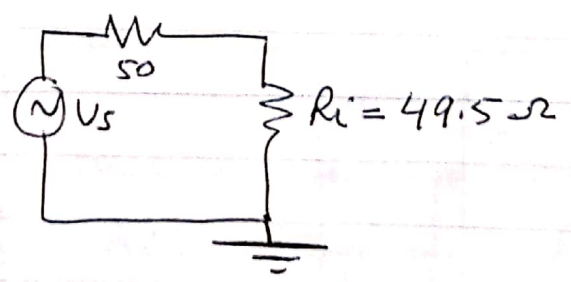
$$= 1 \text{ k} \parallel \frac{5.25 \text{ k}}{101} = 49.5 \Omega$$

$$R_o = R_B = 100 \text{ k}\Omega$$

$$R_B \parallel R_L = 1 \parallel 100 \approx 1 \text{ k}\Omega$$

$$V_o = -g_m V_{\pi} (R_B \parallel R_L) \Rightarrow \frac{V_o}{V_{\pi}} = -g_m (R_B \parallel R_L) = -19 \text{ mA/V} \times 1 \text{ k} = -19$$

$$\frac{V_{\pi}}{V_s} = -\frac{R_i}{R_i + R_s} = -0.497$$



$$\frac{V_o}{V_s} = 9.45 = A_v$$

Summary & Comparison of Single-Stage BJT Amps.

Amp	A_V	A_I	ϕ	Input Resistance	output Resistance
C.E	>1	>1	180°	Moderate	Moderate to high
C.C	≤ 1	>1	0	High	Low
C.B	>1	≤ 1	0	Low	Moderate to high

* In order to design or to have Amp. with certain specification, which can't be achieved using a single stage such as: " $A > 1$ & low R_o , low R_i & low R_o , very high A_V , very high A_I " we have to use multistage Amps.

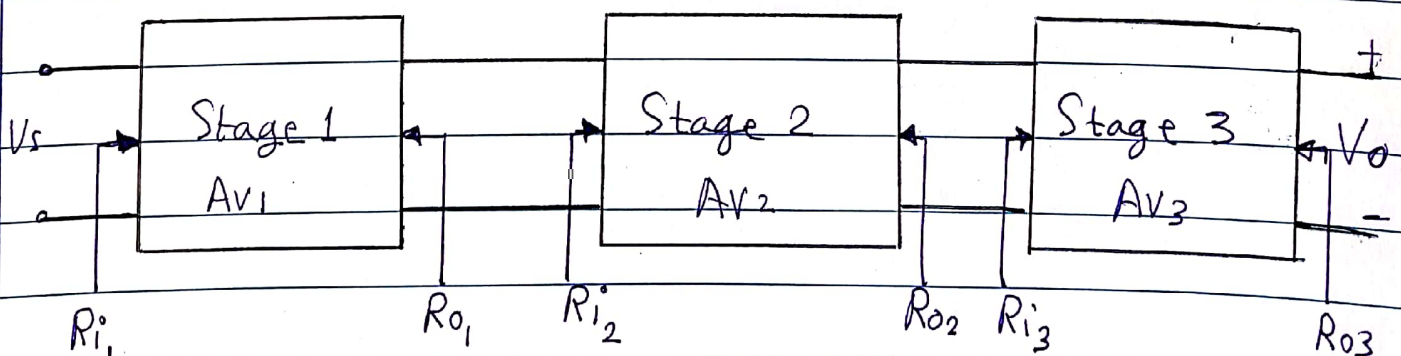
Multistages Amplifiers

In most applications "combined specifications" single-stage will not meet the required specifications so, multistage is used.

* Multistage Amp. contains more than one transistor at least two and used to achieve certain specifications which can't be achieved using single stage. They can be "Cascade Amp." or "Cascode Amp."

① Cascade Configuration:

The stages are connected in series as shown in Fig.

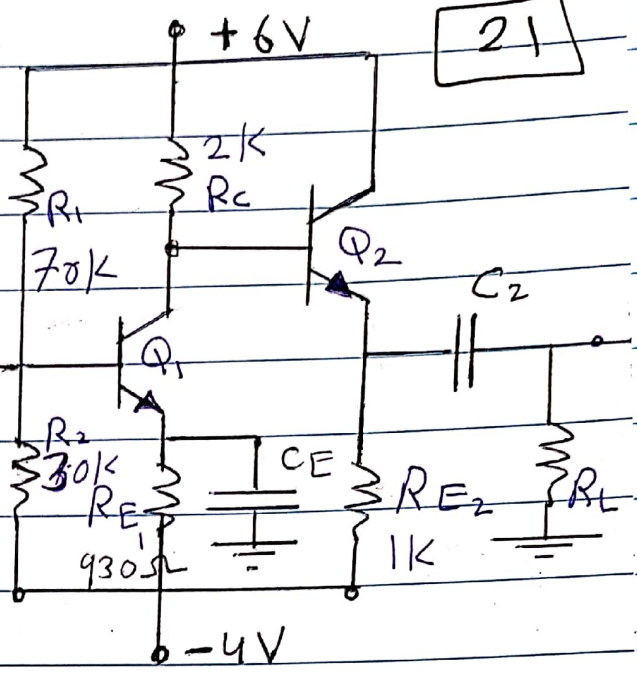


EXA: For the cct. shown

Q₁ & Q₂ are identical with
 $\beta = 100, V_{BE} = 0.7V, \mu_A = \infty$

1) Determine Q-pt For each transistor (I_{C1}, V_{CEQ1}, I_{C2} and V_{CEQ2}).

2) Draw s.s. A.c eqnt. cct. and calculate overall gain $A_v = \frac{V_o}{V_s}$



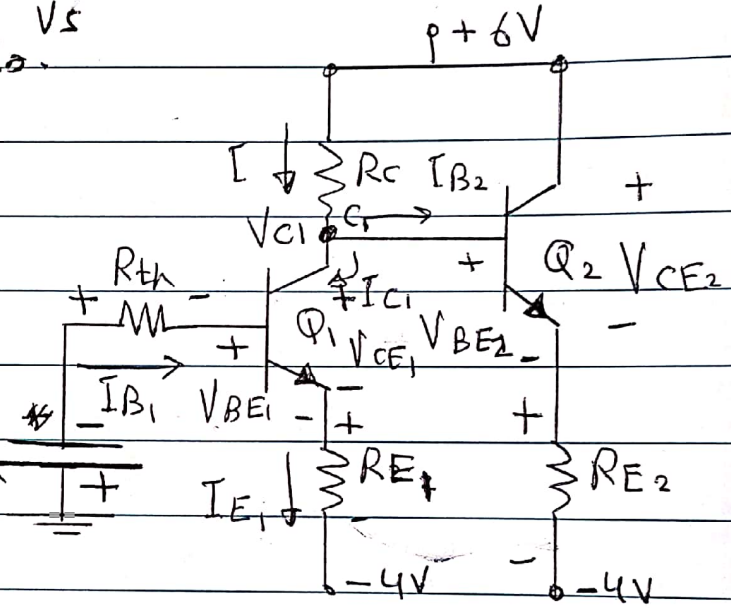
overall $A_i = \frac{I_o}{I_s}, R_i$ and R_o .

1) D.C Analysis:

All Caps. are open-cct. and A.c Source \rightarrow S.C

$$R_{th} = \frac{70 \times 30}{70 + 30} = 21k\Omega$$

$$V_{th} = \frac{6 \times 30}{70 + 30} - \frac{4 \times 70}{100} = -1V$$



$$V_{th} + I_{B1} R_{th} + V_{BE} + (\beta + 1) I_{B1} R_{E1} - 4 = 0$$

$$\therefore I_{B1} = \frac{(3 - 0.7)V}{21 + 101 \times 930} = \frac{2.3V}{(21 + 93.93)k} = 0.02mA$$

$$I_{C1} = \beta I_{B1} = 2mA, I_{E1} = (\beta + 1) I_{B1} = 2.02mA$$

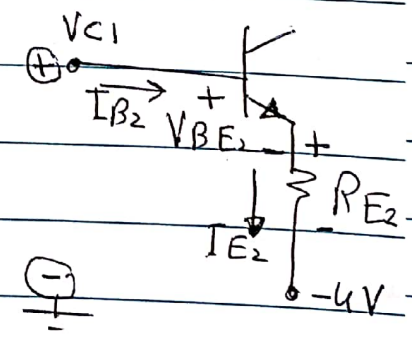
$$V_{CEQ1} = 6 - I_{RC} - I_{E1} R_{E1} = 10 - I_{RC} - I_{E1} R_{E1}$$

to Find I? write KCL at node C1

$$I = I_{C1} + I_{B2} = 2mA + I_{B2}$$

$$-V_{C1} + V_{BE2} + (\beta + 1) I_{B2} R_{E2} - 4 = 0$$

$$I_{B2} = \frac{V_{C1} - 0.7 + 4}{(\beta + 1) R_{E2}} = \frac{V_{C1} + 3.3}{101 R_{E2}} = \frac{V_{C1} + 3.3}{101}$$



$$\frac{6 - V_{C1}}{R_c} = 2 + \frac{V_{C1} + 3.3}{101} \rightarrow \frac{6 - V_{C1}}{2} = 2 + \frac{V_{C1} + 3.3}{101}$$

$$303 - 50.5 V_{C1} = 202 + V_{C1} + 3.3$$

$$V_{C1} = \frac{303 - 202 - 3.3}{49.5} = 1.897V$$

$$I = \frac{6 - 1.897}{51.5} = 2.0515mA$$

$$I_{B2} = \frac{1.897 + 3.3}{101} = 0.0515mA$$

$$V_{CE1} = 10 - 2.0515 \times 2 - 2.02 \times 0.93 = 4.02V$$

∴ Q₁ is in FAM. (I_{B1} > 0, V_{CE1} > V_{BE1})

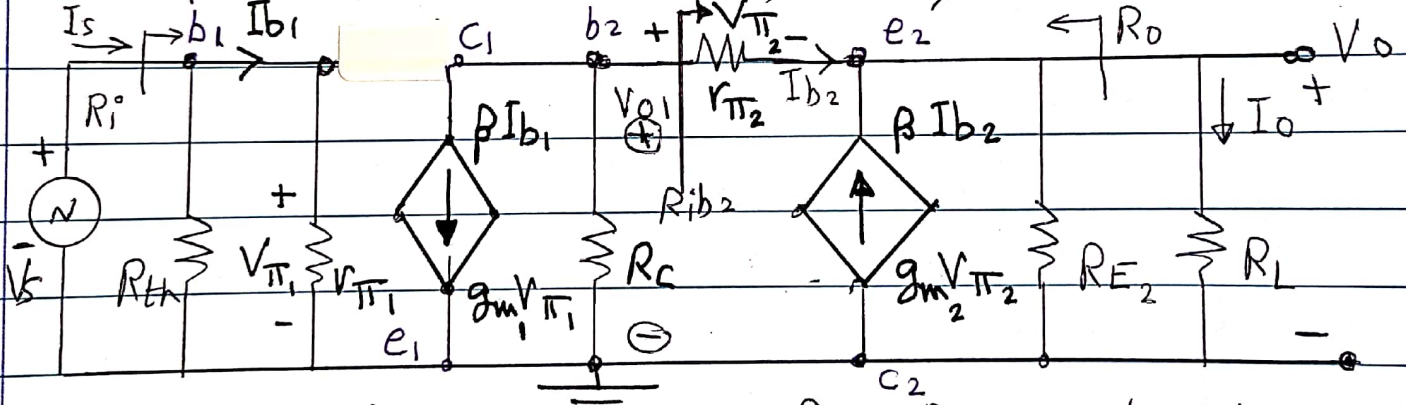
For Q₂: I_{B2} = 0.0515mA ⇒ I_{CQ2} = 100 × 0.0515 =

$$I_{CQ2} = 5.15mA, I_{E2} = (\beta + 1) I_{B2} = 5.2015mA$$

$$-V_{CC} + V_{CEQ2} + I_{E2} R_{E2} - 4 = 0$$

$$V_{CEQ2} = V_{CC} + 4 - I_{E2} R_{E2} = 10 - 5.201 \times 1 = 4.799V$$

Also Q₂ is in FAM "I_{B2} > 0, V_{CEQ2} > V_{BE2}"



⇒ In multistage Analysis's S.S. A.C eqnt. cct.

"Label the terminals of each transistor in the cct"

$$A_v = \frac{V_o}{V_s} = \frac{V_o}{V_{o1}} \times \frac{V_{o1}}{V_s}$$

$$\frac{V_o}{V_{o1}} = \frac{(\beta + 1)(R_{E2} \parallel R_L)}{r_{\pi_2} + (\beta + 1)(R_{E2} \parallel R_L)}$$

$$V_o = + (\beta + 1) I_{B2} (R_{E2} \parallel R_L)$$

$$V_{o1} = -g_{m1} V_{\pi_1} (R_c \parallel R_{iB2})$$

$$-V_{o1} + V_{\pi_2} + V_o = 0$$

and $V_{\pi_1} = V_s$

$$V_{o1} = V_{\pi_2} + V_o$$

$$V_{o1} = I_{B2} (r_{\pi_2} + (\beta + 1)(R_{E2} \parallel R_L))$$

$$\therefore \frac{V_{o1}}{V_s} = -g_{m1} (R_c \parallel R_{iB2})$$

$$A_v = \frac{V_o}{V_s} = \overbrace{-g_{m1}(R_c \parallel R_{i_{b2}})}^{A_{V1}} \cdot \overbrace{\frac{(\beta+1)(R_{E2} \parallel R_L)}{r_{\pi 2} + (\beta+1)(R_{E2} \parallel R_L)}}^{A_{V2}}$$

$$g_{m1} = \frac{I_{CQ1}}{V_T} = \frac{2\text{mA}}{26\text{mV}} = 76.9\text{mA}$$

$$r_{\pi 1} = \frac{\beta V_T}{I_{CQ1}} = \frac{100 \times 26}{2} = 1.3\text{K}\Omega$$

$$r_{\pi 2} = \frac{\beta V_T}{I_{CQ2}} = \frac{100 \times 26}{5.15\text{mA}} = 0.505$$

$$R_{i_{b2}} = (\beta+1)(R_{E2} \parallel R_L) + r_{\pi 2} = 101(1 \parallel 9) + 0.505 = 91.4\text{K}\Omega$$

$$A_v = -76.9(2 \parallel 91.4) \cdot \frac{90.9}{91.4} = -149.68$$

$$A_{V1} = -g_{m1}(R_c \parallel R_L) = -150.5 \text{ (C.E. Amp.)}$$

$$A_{V2} = \frac{(\beta+1)(R_{E2} \parallel R_L)}{r_{\pi 2} + (\beta+1)(R_{E2} \parallel R_L)} = 0.994 \text{ (C.C. Amp.)}$$

$$A_I = A_v \frac{R_i}{R_L} = -149.68 \frac{1.224\text{K}}{9\text{K}} = -20.356\text{K}\Omega$$

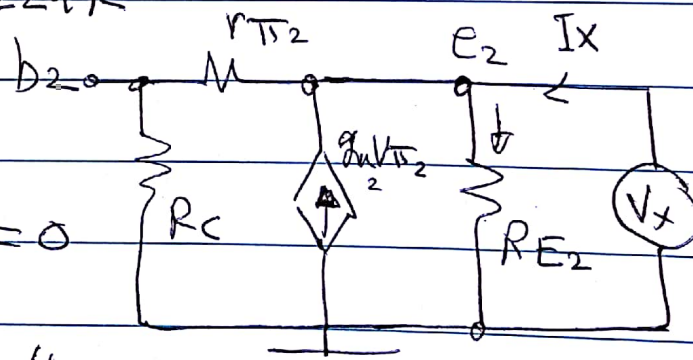
where $R_i = r_{\pi 1} \parallel R_{Th} = 1.3 \parallel 21 = 1.224\text{K}\Omega$

$$R_o = \frac{V_x}{I_x} \Big|_{V_s=0}$$

When $V_s=0$, $V_{\pi 1}=0$, $g_{m1}V_{\pi 1}=0$

Using IRRR:

$$R_o = \left(\frac{R_c + r_{\pi 2}}{\beta+1} \right) \parallel R_{E2} = 0.0248 \parallel 1 = 24.2 \Omega$$



Cascade Amp.

2

Darlington pair configuration:

$$I_o = g_{m2} V_{\pi 2} + g_{m1} V_{\pi 1}$$

$$V_{\pi 1} = r_{\pi 1} I_i$$

$$g_{m1} V_{\pi 1} = g_{m1} r_{\pi 1} I_i = \beta_1 I_i$$

$$\begin{aligned} g_{m2} V_{\pi 2} &= g_{m2} (g_{m1} V_{\pi 1} + I_i) r_{\pi 2} \\ &= g_{m2} (\beta_1 I_i + I_i) r_{\pi 2} \\ &= \beta_2 (1 + \beta_1) I_i \end{aligned}$$

$$\therefore I_o = I_i (\beta_2 (1 + \beta_1) + \beta_1)$$

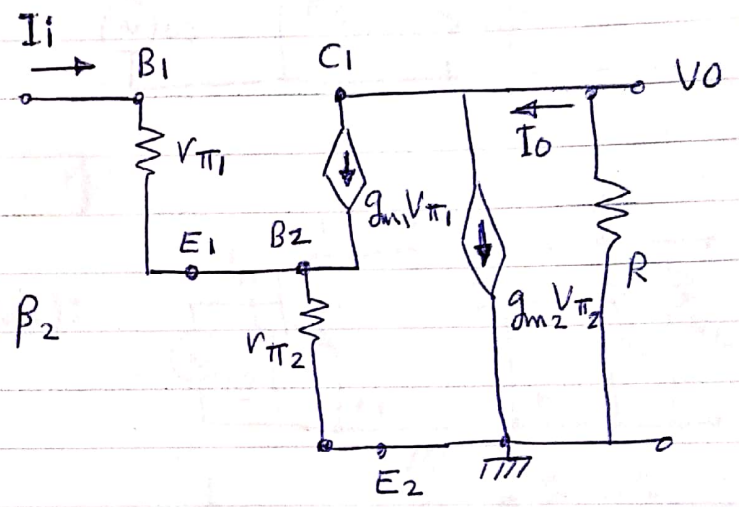
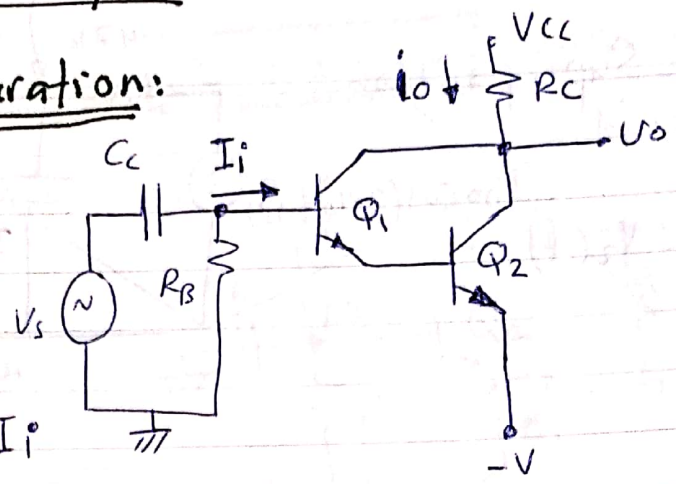
$$\therefore A_i = \beta_1 + \beta_2 (1 + \beta_1) \approx \beta_1 \beta_2$$

$$R_i = r_{\pi 1} + (1 + \beta_1) r_{\pi 2}$$

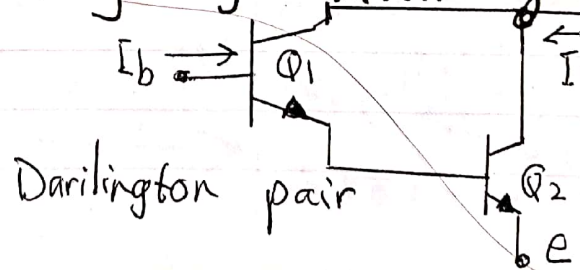
but $r_{\pi 1} = \frac{\beta_1 V_T}{I_{CQ1}}$ and $I_{CQ1} \approx I_{EQ1} = \frac{I_{CQ2}}{\beta_2}$

$$\therefore r_{\pi 1} = \frac{\beta_1 V_T}{I_{CQ2}} = \beta_1 \left[\beta_2 \frac{V_T}{I_{CQ2}} \right] = \beta_1 r_{\pi 2}$$

$$\therefore R_i = \beta_1 r_{\pi 2} + (1 + \beta_1) r_{\pi 2} \approx 2 \beta_1 r_{\pi 2}$$



- * Darlington pair is used to provide very high current-gain $\approx \beta_1 \beta_2$ or β^2 for identical transistors.
- * The input resistance is large due to β multiplication.
- * This ckt. provide very high current gain $\approx \beta^2$ and high R_i .



③ Cascode configuration: **(C-E & C-B)** Configuration

The advantages of this Connection is:

- 1- high output Resistance
 - 2- good frequency Response.
- *Used as "Wideband" Amp.

$$V_o = -g_{m2} V_{\pi 2} (R_c // R_L)$$

$$V_{\pi 2} = \left(\frac{r_{\pi 2}}{1 + \beta_2} \right) g_{m1} V_{\pi 1} = R_{ie2} (g_{m1} V_{\pi 1})$$

and $V_{\pi 1} = V_s$ So:

$$V_{\pi 2} = \left(\frac{r_{\pi 2}}{1 + \beta_2} \right) g_{m1} V_s$$

or at node C_1 : (KCL yields):

$$g_{m2} V_{\pi 2} + \frac{V_{\pi 2}}{r_{\pi 2}} = g_{m1} V_{\pi 1}$$

$$\therefore V_{\pi 2} = \left(\frac{r_{\pi 2}}{1 + \beta_2} \right) g_{m1} V_{\pi 1}$$

$$\therefore (V_o / V_s) = -g_{m1} g_{m2} (R_c // R_L) \frac{r_{\pi 2}}{1 + \beta_2}$$

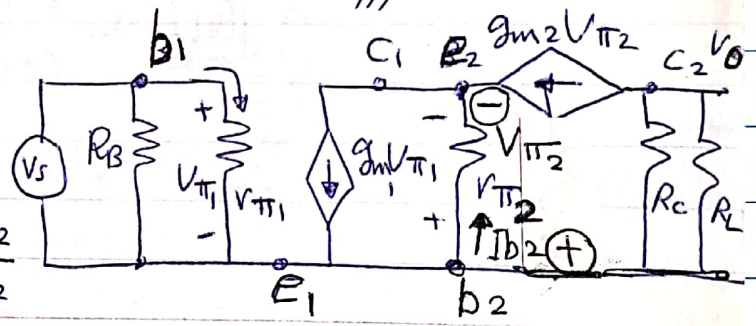
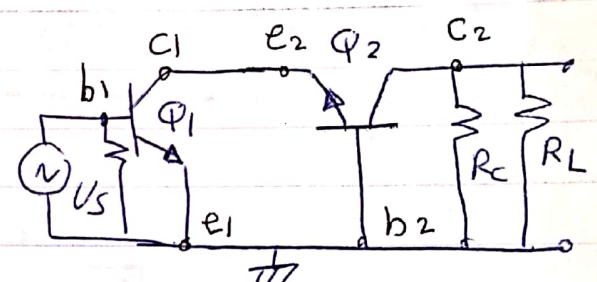
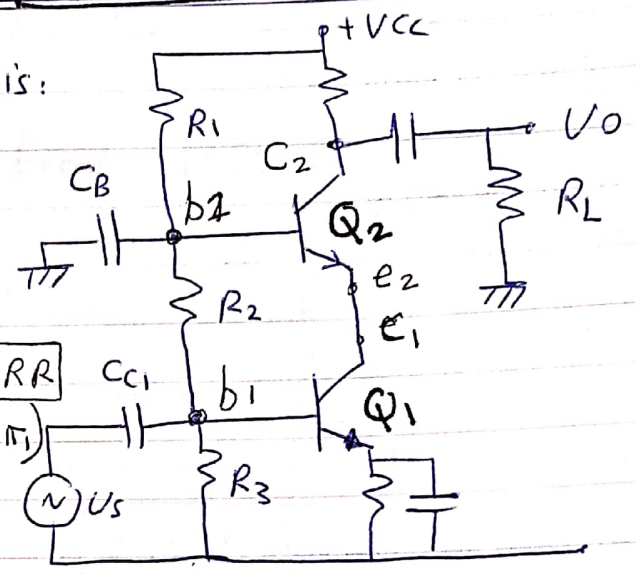
$$= -g_{m1} (R_c // R_L) \frac{\beta_2}{1 + \beta_2} \approx -g_{m1} (R_c // R_L)$$

$$R_i = R_B // r_{\pi 1}$$

$$R_o = R_c$$

NOTE: A_v of this Amp. is

A_v of the CE Amp.?? Why!!!



$$V_{\pi 2} \left(g_{m2} + \frac{1}{r_{\pi 2}} \right) = g_{m1} V_{\pi 1}$$

$$V_{\pi 2} \left(\frac{g_{m2} r_{\pi 2} + 1}{r_{\pi 2}} \right) = g_{m1} V_{\pi 1}$$

$$g_{m2} r_{\pi 2} = \beta$$

$$V_{\pi 2} \left(\frac{\beta + 1}{r_{\pi 2}} \right) = g_{m1} V_{\pi 1}$$

$$\therefore V_{\pi 2} = \left(\frac{r_{\pi 2}}{\beta + 1} \right) g_{m1} V_{\pi 1}$$

EXA1: Common Collector

25

EXA: For the cct. shown, the BJT has: $\beta = 75$,
 $V_{BE} = 0.7V$, $V_A = 60V$, $V_T = 26mV$

- 1) Determine I_{CQ} , V_{CEQ} .
- 2) Write D.C. L.L. eqn. and find its slope?
- 3) Draw S.S.A.C. eqn. cct. and Calculate A_v , A_i , R_i and R_o .

① * For D.C. Analysis

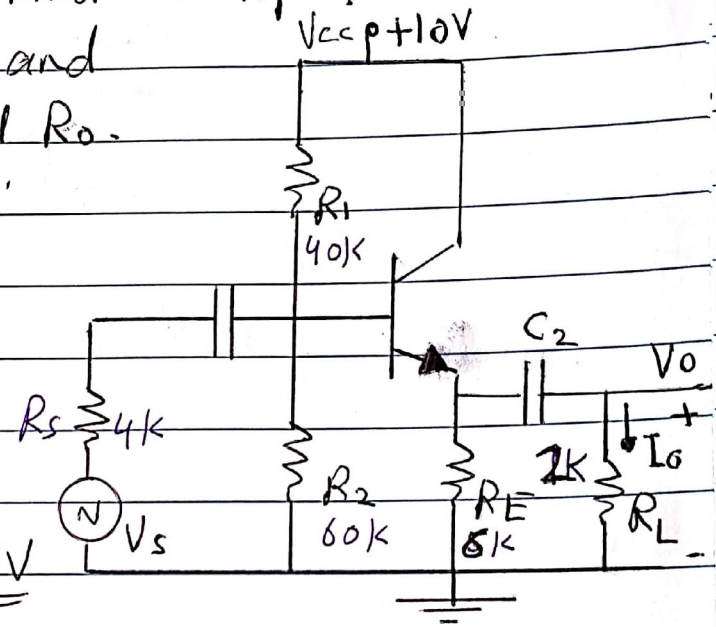
ALL Caps \rightarrow Open-cct.

A.C. Source \rightarrow Short-cct.

* Assume the BJT in FAM

$$R_{th} = R_1 \parallel R_2 = 24K\Omega$$

$$V_{th} = \frac{V_{cc} \cdot R_2}{R_1 + R_2} = \frac{10 \times 60}{100} = 6V$$



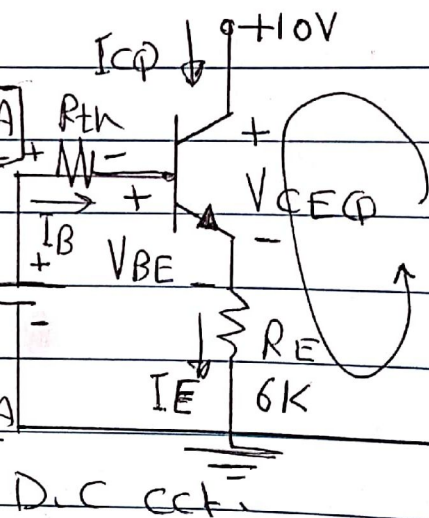
$$-V_{th} + I_B R_{th} + V_{BE} + (\beta + 1) I_B R_E = 0$$

$$\therefore I_B = \frac{V_{th} - V_{BE}}{R_{th} + (\beta + 1) R_E} = \frac{(6 - 0.7)V}{(24 + 76 \times 6)K} = 11\mu A$$

$$I_{CQ} = \beta I_{BQ} = 75 \times 0.011 = 0.825mA$$

$$-V_{cc} + V_{CE} + I_E R_E = 0, I_E = (\beta + 1) I_B$$

$$V_{CE} = 10 - 0.836 \times 6 = 4.984V = 0.836mA$$



D.C. cct.

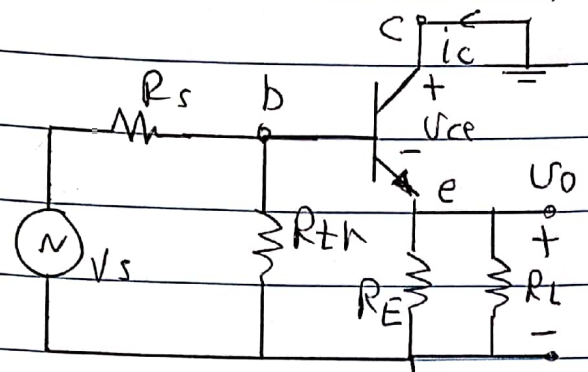
② D.C. L.L: KVL For C-E loop:

$$-10 + V_{CE} + I_E R_E = 0, \text{ but } I_E = \frac{\beta + 1}{\beta} I_C$$

$$V_{CE} = 10 - I_C \left(\frac{\beta + 1}{\beta} R_E \right)$$

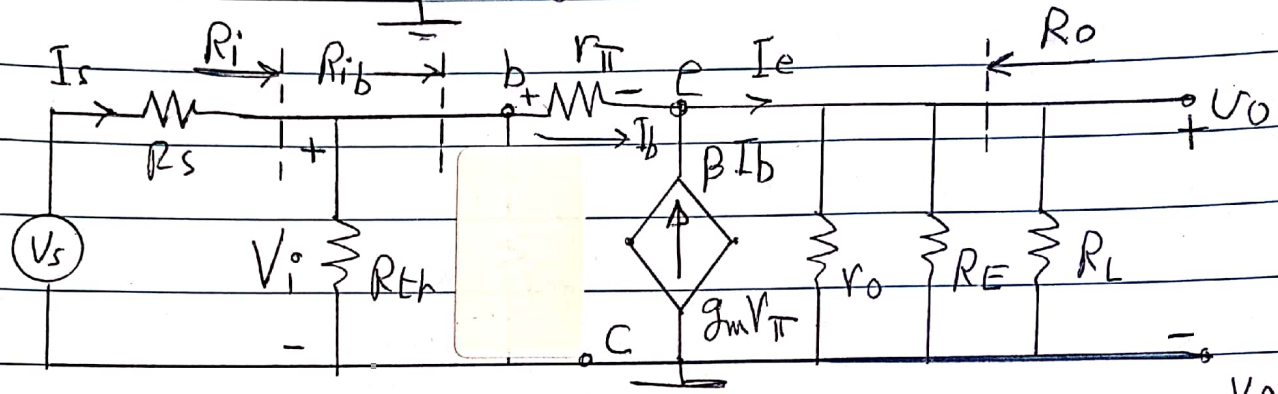
$$\therefore \text{slope of D.C. L.L} = \frac{-1}{\frac{\beta + 1}{\beta} R_E} = \frac{-\beta}{(\beta + 1) R_E}$$

3) * For A.c Analysis, all Caps \rightarrow S.c and D.c Sources \rightarrow S.c



A.c.cct:

A.c. L.L eqn.
 $V_{ce} + I_e(R_E \parallel R_L) = 0$
 $V_{ce} = -\frac{\beta + 1}{\beta} (R_E \parallel R_L) I_b$
 $\therefore \text{slope} = \frac{-\beta}{(\beta + 1)(R_E \parallel R_L)}$



S.S. A.c eqn. cct.

$$A_v = \frac{V_o}{V_s} = \frac{V_o}{V_i} \times \frac{V_i}{V_s}$$

$$V_o = (\beta + 1) I_b R_L, \quad V_i = V_{\pi} + V_o = I_b (r_{\pi} + (\beta + 1) R_L)$$

$$\therefore \frac{V_o}{V_i} = \frac{\beta + 1 R_L}{r_{\pi} + (\beta + 1) R_L} \quad \text{where } R_L = R_o \parallel R_L \parallel R_E$$

$$R_o = \frac{V_A}{I_{CQ}} = \frac{60V}{0.825mA} = 72.72K$$

$$r_{\pi} = \frac{\beta V_T}{I_{CQ}} = \frac{76 \times 26mV}{0.825mA} = 2.363K$$

$$V_i = \frac{V_s \cdot R_i}{R_i + R_s} \rightarrow \frac{V_o}{V_s} = \frac{R_i}{R_i + R_s}$$

$$R_i = R_{th} \parallel R_{i'b}, \quad R_{i'b} = (\beta + 1) (R_L) + r_{\pi}$$

$$\therefore A_v = \frac{(\beta + 1) R_L}{r_{\pi} + (\beta + 1) R_L} \cdot \frac{R_i}{R_i + R_s}$$

$$A_v = \frac{76 \times 1.47}{2.363 + 76 \times 1.47} \cdot \frac{19.829}{4 + 19.829} = \boxed{0.814}$$

$$R_o = \frac{(R_s \parallel R_{th}) + r_{\pi}}{\beta + 1} \parallel R_o \parallel R_E = 76 \Omega$$

$$* A_I = A_v \frac{R_i + R_s}{R_L} = 0.814 \frac{19.829 + 4}{2} = 9.7$$

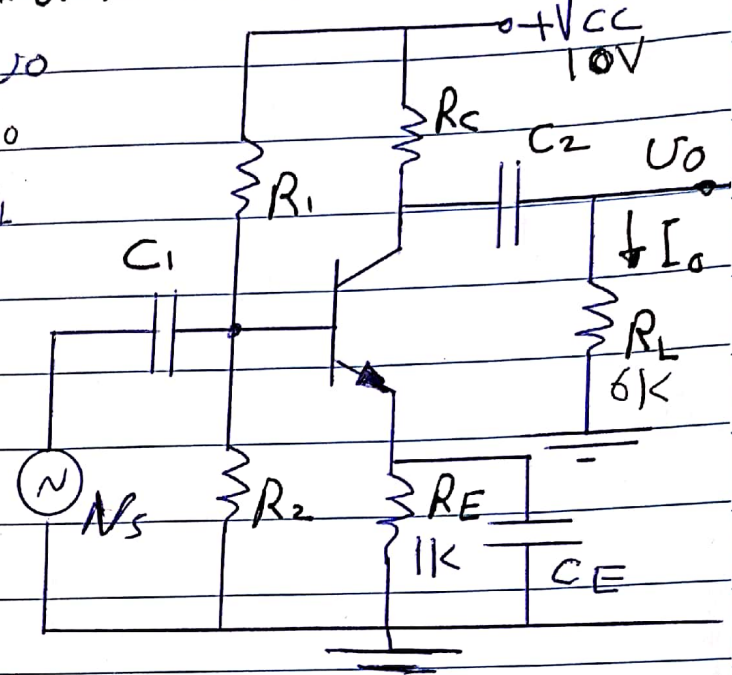
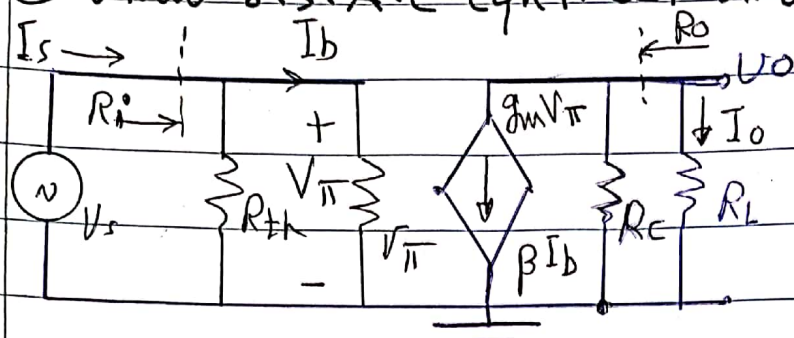
$$R_L = R_E \parallel R_o \parallel R_L = 6 \parallel 2 \parallel 72.72 = 1.47K \Omega$$

$$R_{i'b} = 76 \times 1.47 + r_{\pi} = 114.1K \Omega$$

$$R_i = R_{th} \parallel R_{i'b} = 24 \parallel 114.1 = 19.829K$$

EXA.: (Design) ① Design the cct. shown to have $A_v = -100$ and be bias-stable (Find R_c, R_1 & R_2).

② draw s.s.A.C eqnt. cct. and Find R_{in}, R_o & A_I ?



From A.C Analysis's

$$A_v = \frac{v_o}{v_s} = -g_m v_{\pi} (R_c \parallel R_L)$$

where $v_{\pi} = v_s$

$$\therefore A_v = -g_m (R_c \parallel R_L)$$

$$g_m = \frac{I_{CQ}}{V_T} = \frac{1.3 \text{ mA}}{26 \text{ mV}} = 50 \frac{\text{mA}}{\text{V}}$$

$$-100 = -50 (R_c \parallel R_L) = -50 \frac{R_c R_L}{R_c + R_L}$$

$$R_c \parallel R_L = 2 \text{ k}\Omega = \frac{R_c R_L}{R_c + R_L}$$

$$\therefore R_c = \frac{R_E \cdot R_L}{R_L - R_c} = \frac{6 \times 2}{6 - 2} = 3 \text{ k}\Omega$$

$\beta = 100, V_{BE} = 0.7 \text{ V}, V_A = \infty$
 $I_{CQ} = 1.3 \text{ mA}$ Given!!
 $V_T = 26 \text{ mV}$

$$R_o = \frac{v_x}{i_x} \Big|_{v_s = 0} \rightarrow R_o = R_c = 3 \text{ k}\Omega$$

$$R_i = R_{th} \parallel r_{\pi}, \quad r_{\pi} = \frac{\beta V_T}{I_{CQ}} = \frac{100 \times 26}{1.3 \text{ mA}} = 2 \text{ k}\Omega$$

$$R_{th} = 0.1 (\beta + 1) R_E \quad [\text{For bias-stable design}]$$

$$\therefore R_{th} = 0.1 (101) \times 1 = 10.1 \text{ k}\Omega$$

$$\therefore R_i = 10.1 \parallel 2 \text{ k} = 1.67 \text{ k}\Omega$$

$$A_I = A_v \frac{R_i}{R_L} = -100 \frac{1.67}{6} = -27.83$$

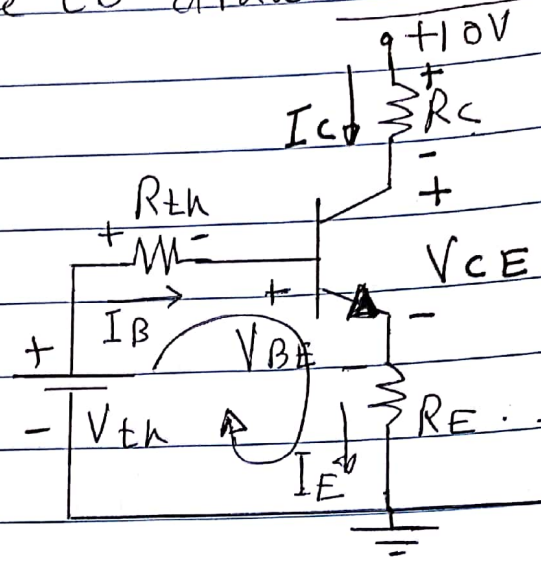
To Find R_1 & R_2 we have to draw D.C. ckt
 For a bias-stable design:

choose $R_{th} = 0.1(\beta + 1)R_E$
 $= 0.1(101) \times 1 = 10.1 \text{ k}\Omega$

but $R_{th} = R_1 \cdot R_2 = 10.1 \text{ k}\Omega$

and $V_{th} = \frac{V_{cc} R_2}{R_1 + R_2}$

$R_1 \cdot V_{th} = \frac{R_1 + R_2}{R_1 \cdot R_2} \cdot R_{th} \cdot V_{cc}$
 $V_{th} = \frac{R_{th} V_{cc}}{R_1 + R_2}$



$R_1 = \frac{V_{cc} R_{th}}{V_{th}}$

$V_{th} + I_B R_{th} + V_{BE} + (\beta + 1) I_B R_E = 0$

$V_{th} = V_{BE} + I_B (R_{th} + (\beta + 1) R_E)$

$I_B = \frac{I_C}{\beta} = \frac{1.3}{100} = 0.013 \text{ mA}$

$V_{th} = 0.7 + 0.013(10.1 + 101 \times 1) = 2.1443 \text{ V}$

$R_1 = \frac{10}{2.1443} (10.1) = 47.1 \text{ k}\Omega$

$R_2 = \frac{R_1 \cdot R_{th}}{R_1 - R_{th}} = \frac{47.1 \times 10.1}{47.1 - 10.1} = 12.857 \text{ k}\Omega$

D.C. L.L eqn: (IF it is required)

$-V_{cc} + I_C R_C + V_{CE} + I_E R_E = 0$, but $I_E = \frac{\beta + 1}{\beta} I_C$

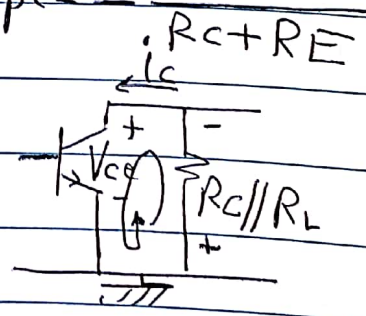
$V_{CE} = V_{cc} - I_C (R_C + \frac{\beta + 1}{\beta} R_E)$

Slope = $\frac{1}{R_C + \frac{\beta + 1}{\beta} R_E}$ * For $\beta \gg 1$, Slope = $\frac{1}{R_C + R_E}$

A.C. L.L eqn: From A.C ckt.

$V_{ce} + i_c (R_C // R_L) = 0 \Rightarrow V_{ce} = -i_c (R_C // R_L)$

slope = $\frac{1}{(R_C // R_L)}$



MOSFET Amplifiers CH. 4

For a MOSFET to be used as an Amplifier, it must be biased in Saturation Region.

To be sure that MOSFET in Sat. Regn., D.C analysis must be performed and check that $V_{DS} > V_{DS(sat)}$ where $V_{DS(sat)} = V_{GS} - V_{TN}$ and V_{TN} is given, V_{GS} is obtained from the circuit.

Single-Stage MOSFETs Amps.

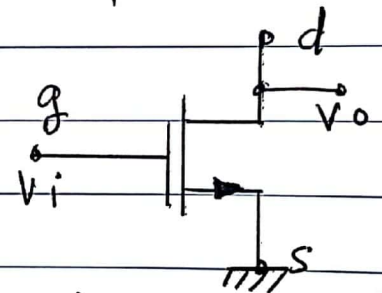
There are three configurations according to the common terminal.

i) Common-Source Amplifier (C.S. Amp).

V_i is given to gate

V_o is taken from drain

Source is Common terminal.

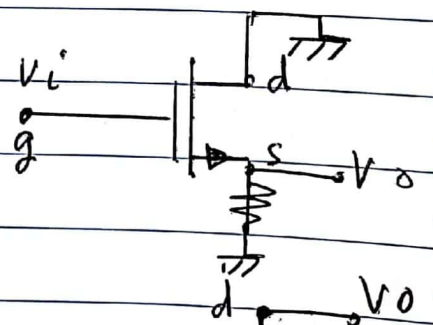


ii) Common-drain Amp. (Source-Follower).

$V_i \rightarrow$ to gate

$V_o \rightarrow$ from Source

Drain is Common-terminal.

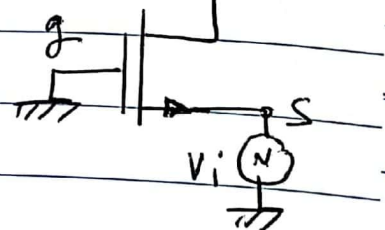


iii) Common-gate terminal

$V_i \rightarrow$ to Source

$V_o \rightarrow$ from drain

gate is Common terminal.



① Common-Source Amp.

i) Basic C.S. Amp.

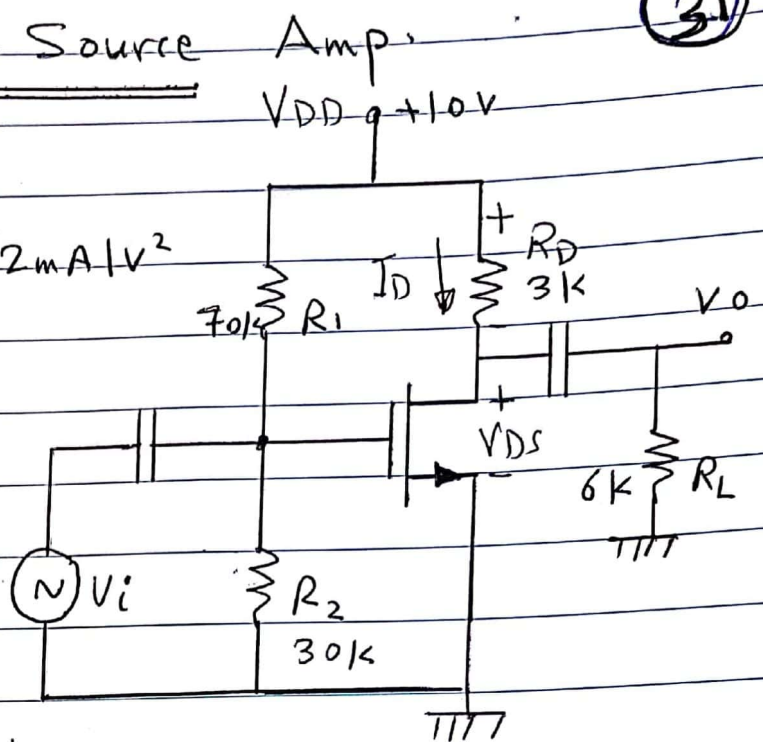
Given $V_{TN} = 2V$, $K_n = 2mA/V^2$

$\lambda = 0.01V^{-1}$

a) Calculate I_D , V_{DS}

b) Draw S.S.A.c eqn.

cct. & find A_v , A_i , R_i , R_o



A) For D.c Analysis:

assume MOSFET in Sat.

$$I_D = K_n (V_{GS} - V_{TN})^2$$

from the cct. $V_{GS} = V_G - V_S = \frac{10 \times 30}{100} - 0 = 3V$

$$I_D = 2(3 - 2)^2 = 2mA$$

$$V_{DS} = 10 - I_D R_D = 10 - 3 \times 2 = 4V$$

$$V_{DS(sat)} = V_{GS} - V_{TN} = 3 - 2 = 1V$$

Since $V_{DS} > V_{DS(sat)} \Rightarrow$ MOSFET is in Sat.

* Write D.c Load-line eqn, Find slope, Draw D.c.L.L?

$$-V_{DD} + I_D R_D + V_{DS} = 0$$

$$\therefore V_{DS} = 10 - I_D R_D \text{ (D.c.L.L eqn.)}$$

$$\text{slope} = - \frac{1}{R_D} = - \frac{1}{3k}$$

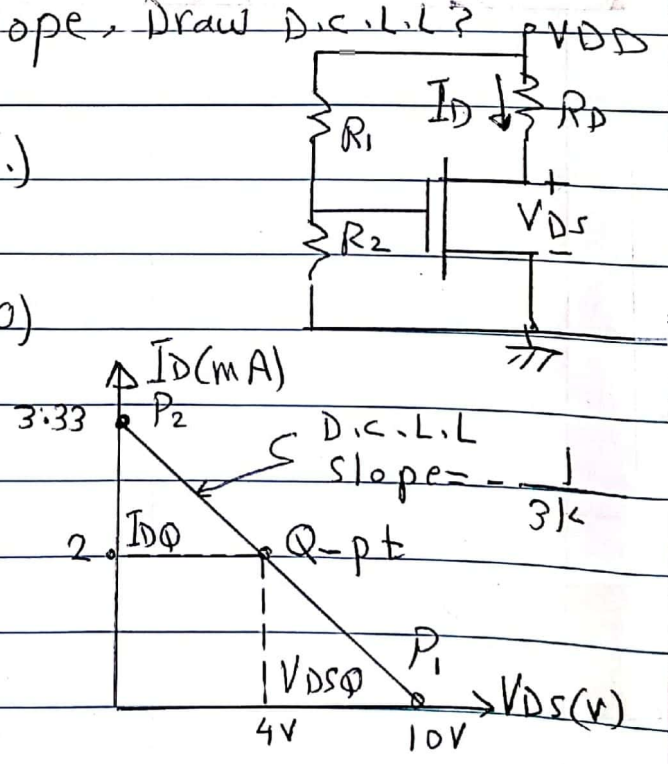
* For $I_D = 0$, $V_{DS} = 10V \rightarrow P_1(10V, 0)$

* For $V_{DS} = 0$, $I_D = 10/3k = 3.33mA$

$P_2(0V, 3.33mA)$

power dissipation: $P_D = I_D \cdot V_{DS}$

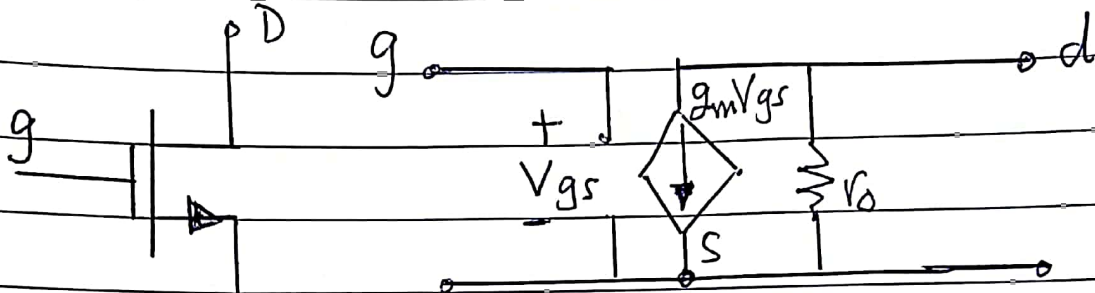
$$\therefore P_D = 2 \times 4 = 8mW$$



2) A-c Analysis:

To perform A-c Analysis, we have to draw A.c.cst. and replace MOSFET by its model.

MOSFET Model



where: g_m^s : Transconductance (mA/V)

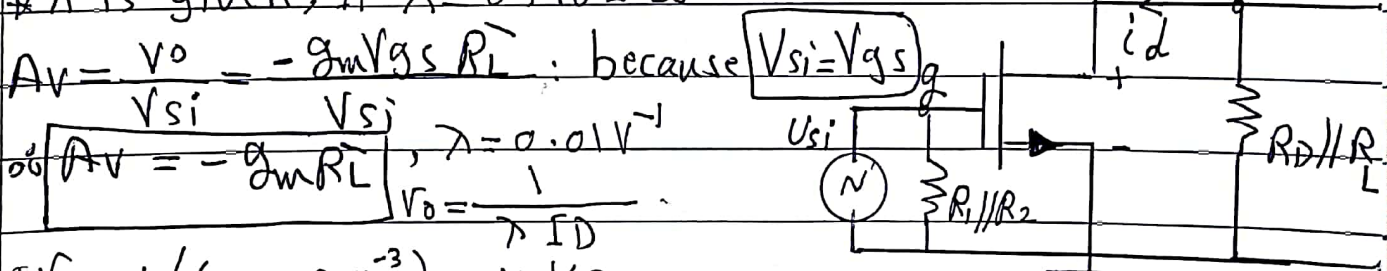
$$g_m = \frac{\partial I_D}{\partial V_{GS}} = 2K_n(V_{GS} - V_{TN})$$

OR $g_m = 2\sqrt{K_n I_D}$

r_o : MOSFET output resistance $\Rightarrow r_o = \frac{1}{\lambda I_D}$

where: λ : Channel length modulation parameters in (V^{-1}) .

* λ is given, If $\lambda = 0, r_o = \infty$



$A_v = \frac{V_o}{V_{si}} = -g_m V_{gs} R_L$: because $V_{si} = V_{gs}$

$A_v = -g_m R_L$, $\lambda = 0.01 V^{-1}$
 $r_o = \frac{1}{\lambda I_D}$

$r_o = 1 / (0.01 \times 2 \times 10^{-3}) = 50 k\Omega$

$g_m = 2\sqrt{K_n I_D} = 2\sqrt{2 \times 2} = 4 mA/V$

$A_v = -4(3 // 6 // 50) = -7.7$

$R_i = R_g = R_1 // R_2 = 21 k\Omega$

$R_o = \frac{V_x}{I_x} | V_{si} = 0$

When $V_{si} = 0, V_{gs} = 0, g_m V_{gs} = 0 \Rightarrow$ o.c $R_o = r_o // R_D$

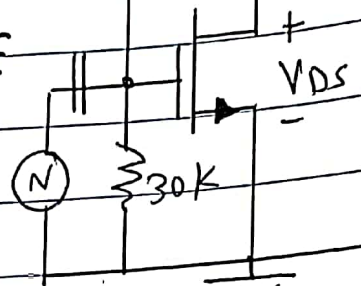
$R_o = 50 // 3 = 2.83 k\Omega$

ii) C.S with R_s :

R_s is used to stabilize Q-pt against K_n parameter variation which is an Advantages.

Basic

Fig.1



* Consider the basic cct. discussed in (i) When $K_n = 2 \text{ mA/V}^2$, $V_G = 3\text{V}$, $V_s = 0$, $I_D = 2 \text{ mA}$, $V_{DS} = 3\text{V}$ and $V_{DS}(\text{sat}) = V_{GS} - V_{TN} = 1\text{V}$ and MOSFET in Sat. Regn.

But $[K_n = 4 \text{ mA/V}^2]$, $I_D = 4(3-2)^2 = 4 \text{ mA}$

$V_{DS} = 10 - I_D R_D = 10 - 4 \times 3 = -2\text{V}$

∴ The MOSFET will be in Linear Regn and can't be used as Amplifier.

Now: If $R_s = 1\text{k}\Omega$ is connected in Fig. 10V

* Given: $V_{TN} = 2\text{V}$, $K_n = 4 \text{ mA/V}^2$, $\lambda = 0$

1) Find V_{GS} , I_D , V_{DS} .

2) Draw s.s. A.C eqn. cct. and determine A_v , R_i , R_o

D.c. Analysis:

all Caps. \rightarrow open-cct.

A.C Source \rightarrow S.C

Assume the MOSFET in Sat. Regn.

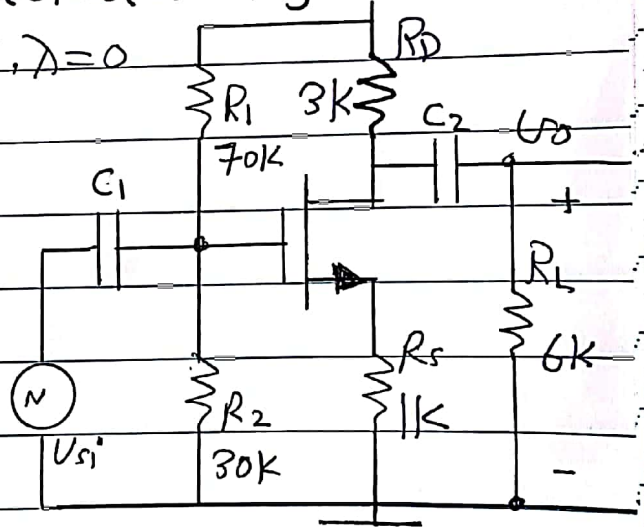
$I_D = K_n (V_{GS} - V_{TN})^2$

$V_{GS} = V_G - V_s$, $V_G = \frac{10 \times 30}{100} = 3\text{V}$

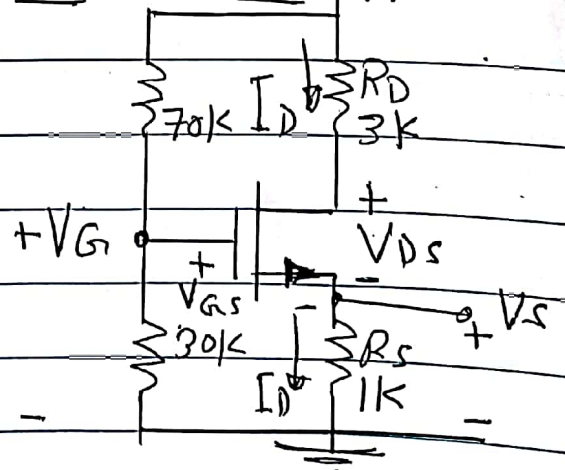
$V_s = I_D R_s = 1 \times I_D = I_D$

∴ $V_{GS} = 3 - I_D$ from this eqn.

$I_D = 3 - V_{GS}$



c.s with R_s



$$3 - V_{GS} = 4(V_{GS}^2 - 4V_{GS} + 4)$$

$$4V_{GS}^2 - 15V_{GS} + 13 = 0$$

$$V_{GS} = \frac{15 \pm \sqrt{(-15)^2 - 4 \times 4 \times 13}}{2 \times 4} = \frac{15 \pm 4.123}{8} = \boxed{2.39V} \text{ or } \boxed{1.36V}$$

$$I_D = 4 - 2.39 = 0.61 \text{ mA}$$

$$-10 + I_D R_D + V_{DS} + I_D R_S = 0$$

$$V_{DS} = 10 - 0.61(3H) = 7.56V$$

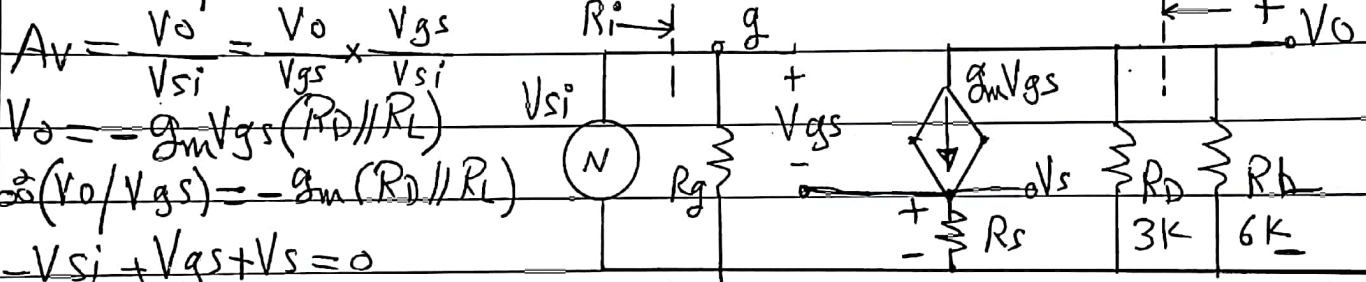
$$V_{DS(sat)} = V_{GS} - V_{TN} = 2.39 - 2 = 0.39V$$

Since $V_{DS} > V_{DS(sat)}$ MOSFET is in Sat. Regn.

R_S stabilize Q-pt against K_n parameter Variation!!

* A.C Analysis:

ALL caps. and D.C Sources → Short-cct.



$$A_v = \frac{V_o}{V_{si}} = \frac{V_o}{V_{gs}} \times \frac{V_{gs}}{V_{si}}$$

$$V_o = -g_m V_{gs} (R_D \parallel R_L)$$

$$\frac{V_o}{V_{gs}} = -g_m (R_D \parallel R_L)$$

$$-V_{si} + V_{gs} + V_s = 0$$

$$V_{si} = V_{gs} + g_m V_{gs} R_s \Rightarrow V_{si} = V_{gs} (1 + g_m R_s)$$

$$\frac{V_{gs}}{V_{si}} = \frac{1}{1 + g_m R_s}$$

$$A_v = \frac{-g_m (R_D \parallel R_L)}{1 + g_m R_s} \quad \boxed{\text{i.e } R_s \text{ reduces } A_v}$$

For this cct. $g_m = 2\sqrt{K_n I_D} = 2\sqrt{4 \times 0.61} = 3.124 \text{ mA/V}$

$$A_v = \frac{-3.124 \times (3 \parallel 6)}{1 + 3.124 \times 1} = -1.515$$

$$R_i = R_g = 70 \parallel 30 = 21 \text{ K}\Omega$$

$$R_o = \frac{V_x}{I_x} |_{V_{si}=0} \Rightarrow R_o = R_D = 3 \text{ K}\Omega$$

A.c & D.c. Load-line

1) D.c. L.L → KVL from D.c. ckt

$$-V_{DD} + I_D R_D + V_{DS} + I_D R_S = 0$$

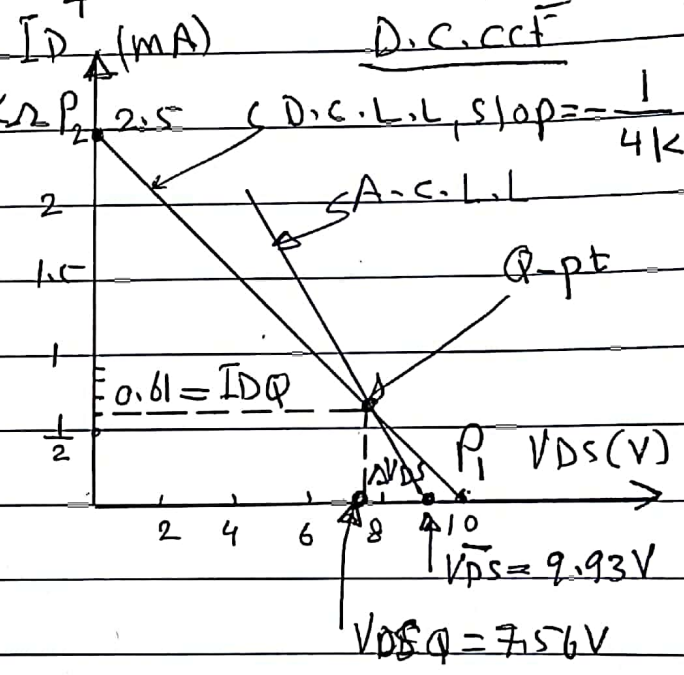
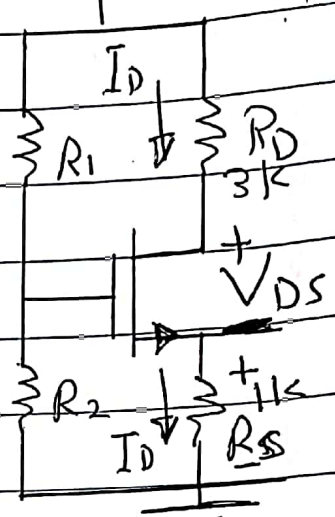
$$\therefore V_{DS} = V_{DD} - I_D (R_D + R_S) \quad \text{D.c. L.L eqn.}$$

i) For $I_D = 0$, $V_{DS} = V_{DD} = 10V \rightarrow P_1 (10V, 0mA)$

ii) For $V_{DS} = 0$, $I_D = V_{DD} / (R_D + R_S) = \frac{10}{4} = 2.5mA$

$P_2 (0V, 2.5mA)$

$$\text{Slope} = -1 / (R_D + R_S) = -1 / 4k\Omega \quad \text{D.c. L.L, slope} = -\frac{1}{4k}$$



2) A.c. L.L: Consider A.c. ckt.

KVL For (d → s) loop:

$$v_{ds} + i_d R_S + i_d (R_D // R_L) = 0$$

$$\therefore v_{ds} = -i_d [R_S + (R_D // R_L)]$$

$$\therefore \text{slope} = -\frac{1}{R_S + (R_D // R_L)} = -\frac{1}{3k}$$

$$\overline{v_{DS}} = V_{DSQ} + \Delta V_{DS}$$

$$\text{slope} = \frac{1}{R_S + (R_D // R_L)} = \frac{\Delta I_D}{\Delta V_{DS}}$$

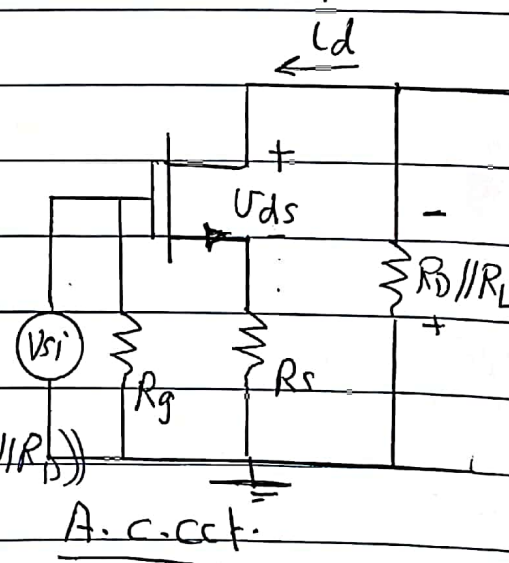
$$\therefore \Delta V_{DS} = \Delta I_D [R_S + (R_D // R_L)]$$

$$\Delta I_D = I_{DQ} = 0.61mA$$

$$\therefore \Delta V_{DS} = 0.61(1+2) = 1.83V$$

$$\overline{V_{DS}} = 7.56 + 1.83 = 9.39V$$

$$\text{Max. peak-to-peak } (V_{ds}) = 2 I_{DQ} (R_S + (R_D // R_L)) = 3.66V$$



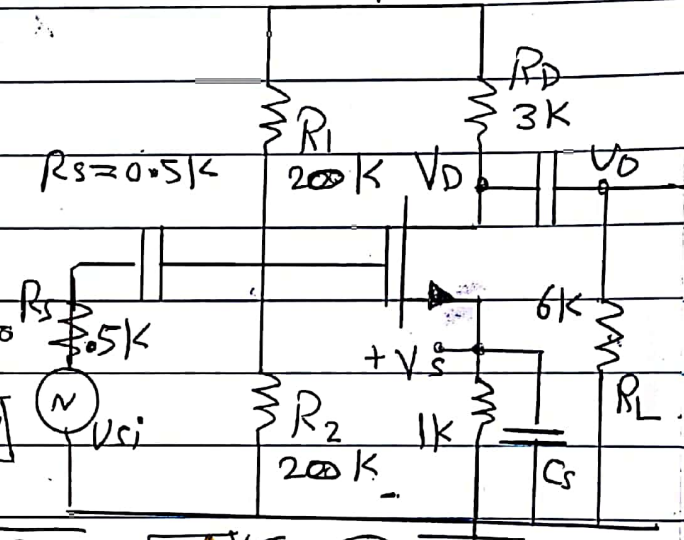
iii) C.S with bypass Capacitor "Cs"

- * To cancel the effect of R_s in reducing A_v , C_s is connected across R_s , where in D.C Analysis C_s is open, R_s is present and stabilize Q-pt.
- * For A.C Analysis C_s is short \rightarrow Shorting R_s and cancel its effect \rightarrow hence increase A_v .

EXA: For the ckt. shown, the MOSFET parameters are: $K_n = 2 \text{ mA/V}^2$, $V_{TN} = 2 \text{ V}$, $\lambda = 0.02 \text{ V}^{-1}$ and the ckt. is biased at $I_D = 2 \text{ mA}$. $V_{DD} = +10 \text{ V}$

* Determine: V_{DS}, V_D, V_S
 V_G, V_{GS}

* Draw s.s. A.C eqnt. ckt. and Find A_v, R_i, R_o



$$V_S = I_D R_S = 2 \times 1 = 2 \text{ V}$$

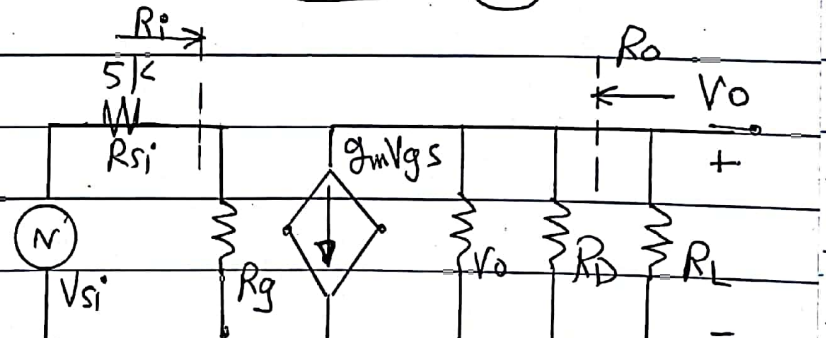
$$V_D = V_{DD} - I_D R_D = 10 - 2 \times 3 = 4 \text{ V}$$

$$V_{DS} = V_D - V_S = 4 - 2 = 2 \text{ V}$$

$$V_{GS} = V_{TN} \pm \sqrt{\frac{I_D}{K}} = 2 \pm \sqrt{\frac{2}{2}} = 3 \text{ V or } 1 \text{ V}$$

$$V_G = \frac{10 \times 200}{200 + 200} = 5 \text{ V}$$

$$A_v = \frac{V_o}{V_{si}} = \frac{V_o}{V_{GS}} \times \frac{V_{GS}}{V_{si}}$$



$$V_o = -g_m V_{GS} R_L$$

$$R_L = R_D \parallel R_L$$

$$r_o = (1/\lambda I_{DQ}) = 1/(0.02 \times 2 \times 10^{-3}) = 25 \text{ k}\Omega \quad R_o = R_D \parallel r_o = 2.676 \text{ k}\Omega$$

$$R_L = 25 \parallel 3 \parallel 6 = 1.852 \text{ k}\Omega, \quad g_m = 2\sqrt{K_n I_D} = 2\sqrt{2 \times 2} = 4 \text{ mA/V}$$

$$A_v(V_{GS}/V_{si}) = (R_L / (R_L + R_s)), \quad R_i = R_g = 20 \parallel 20 = 10 \text{ k}\Omega$$

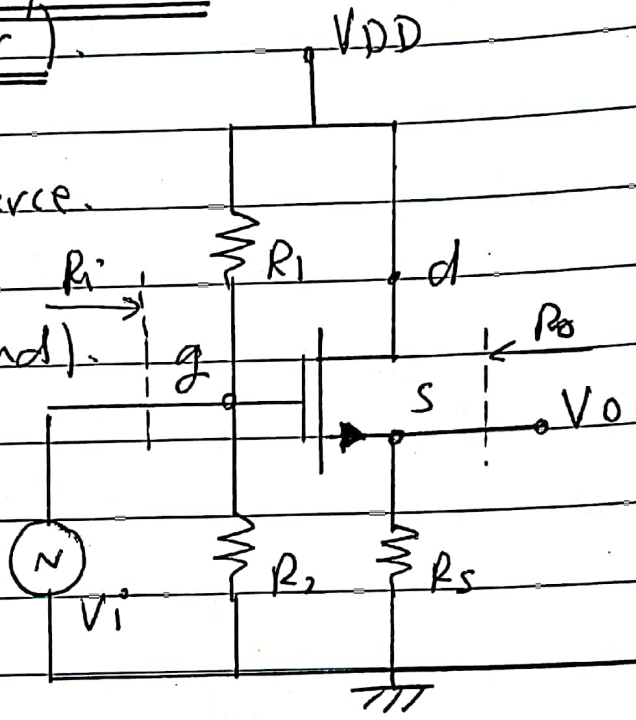
$$A_v = -g_m R_L \frac{R_i}{R_i + R_s} = -4 \times 1.852 \frac{10}{10 + 0.5} = -7$$

$$R_o = R_D \parallel r_o = 2.676 \text{ k}\Omega$$

② Common-Drain Amplifier
(Source-follower)

In this Amp.
 V_i to gate, V_o from Source.
 drain: Common-terminal
 (For A.c analysis d is at ground).

Draw S.S. A.c eqnt. cct.
 and determine expressions
 for A_v , R_i & R_o



$$V_o = g_m V_{gs} (r_o \parallel R_s)$$

$$-V_i + V_{gs} + V_o = 0$$

$$V_i = V_{gs} + g_m V_{gs} (r_o \parallel R_s)$$

$$V_i = V_{gs} (1 + g_m (r_o \parallel R_s))$$

$$\therefore V_{gs} = \frac{V_i}{1 + g_m (r_o \parallel R_s)}$$

$$V_o = \frac{g_m (r_o \parallel R_s)}{1 + g_m (r_o \parallel R_s)} V_i$$

$$A_v = \frac{V_o}{V_i} = \frac{g_m (r_o \parallel R_s)}{1 + g_m (r_o \parallel R_s)} \quad \therefore A_v < 1, \phi = 0^\circ$$

$$R_i = R_1 \parallel R_2$$

$$R_o = \frac{V_x}{I_x} \Big|_{V_i=0}; \quad I_x + g_m V_{gs} = \frac{V_x}{r_o} + \frac{V_x}{R_s}$$

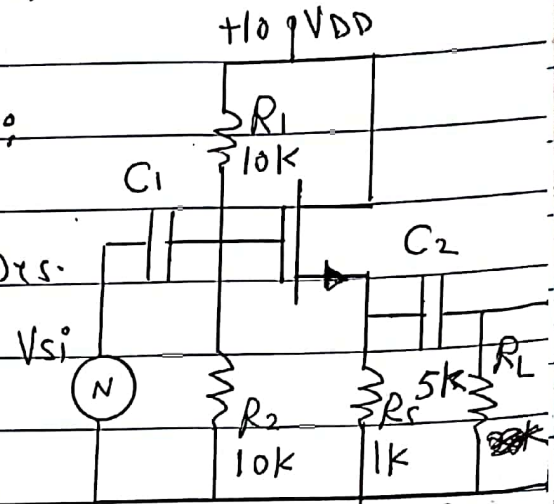
when $V_i = 0$, $V_{gs} = -V_x$

$$I_x = V_x \left(g_m + \frac{1}{r_o} + \frac{1}{R_s} \right) \Rightarrow R_o = \frac{V_x}{I_x} = \frac{1}{g_m \parallel r_o \parallel R_s}$$

EXA: For the ckt. shown in Fig. the MOSFET has, $k_n = 2 \text{ mA/V}^2$, $V_{TN} = 2 \text{ V}$, $\lambda = 0.025 \text{ V}^{-1}$. The ckt. is biased at $I_D = 2 \text{ mA}$.

1) Draw S.S. A.C eqnt. ckt. and Find:
 A_v , R_i , R_o

2) Write D.C. & A.C. L.L and Find slopes.



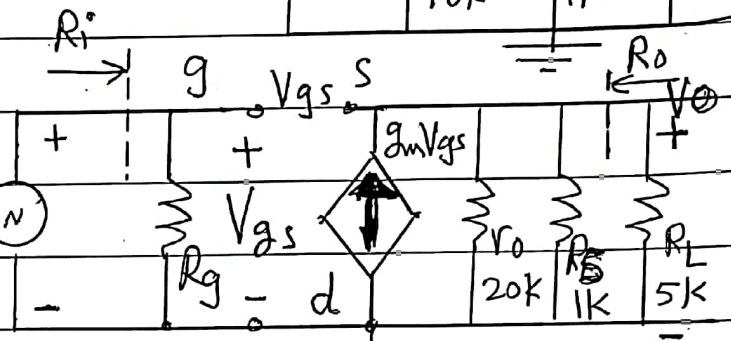
$$A_v = \frac{V_o}{V_{si}} = \frac{V_o}{V_{gs}} \times \frac{V_{gs}}{V_{si}}$$

$$V_o = g_m V_{gs} \bar{R}_L$$

$$(V_o/V_{gs}) = g_m \bar{R}_L$$

where $\bar{R}_L = r_o \parallel R_D \parallel R_L$

$$r_o = \frac{1}{\lambda I_D} = \frac{1}{0.025 \times 2 \times 10^{-3}} = 20 \text{ k}\Omega$$



$$g_m = 2\sqrt{k_n I_D} = 2\sqrt{2 \times 2} = 4 \frac{\text{mA}}{\text{V}} \quad \text{S.S. A.C eqnt. ckt.}$$

$$-V_{si} + V_{gs} + V_o = 0 \Rightarrow V_{si} = V_{gs} + g_m V_{gs} \bar{R}_L = V_{gs}(1 + g_m \bar{R}_L)$$

$$\therefore (V_{gs}/V_{si}) = \frac{1}{1 + g_m \bar{R}_L}$$

$$\therefore A_v = \frac{g_m \bar{R}_L}{1 + g_m \bar{R}_L} \quad \bar{R}_L = 20 \parallel 10 \parallel 1 = 0.8 \text{ k}\Omega$$

$$A_v = (4 \times 0.8) / (1 + 4 \times 0.8) = 0.762$$

$$R_i = R_g = 10 \parallel 10 = 5 \text{ k}\Omega$$

$$R_o = \left. \frac{V_x}{I_x} \right|_{V_{si}=0} = \frac{1}{g_m} \parallel r_o \parallel R_D = \frac{1}{4 \times 10^{-3}} \parallel 20 \parallel 10 = 198 \Omega$$

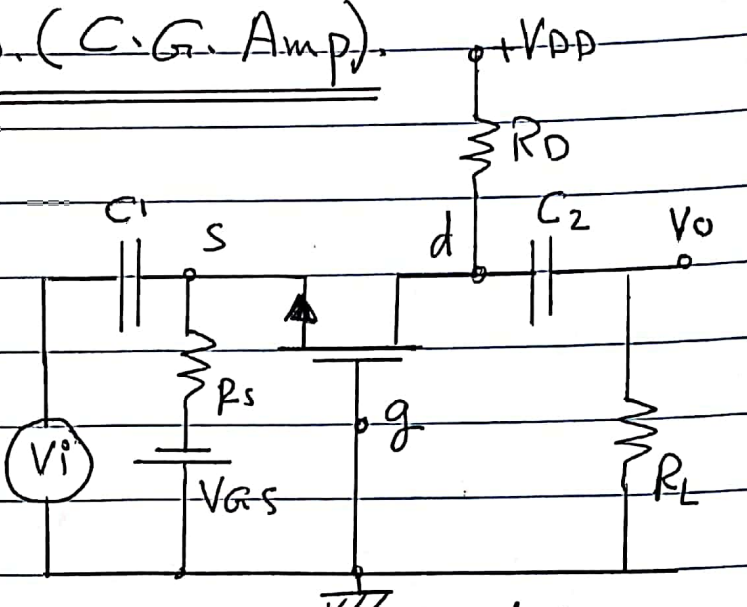
$$\therefore R_o = 0.25 \parallel 20 \parallel 10 = 198 \Omega$$

D.C. L.L : $V_{DS} = V_{DD} - I_D R_D \Rightarrow \text{slope} = -\frac{1}{R_D}$

A.C. L.L : $V_{ds} = -i_d (R_D \parallel R_L) \Rightarrow \text{slope} = -\frac{1}{(R_D \parallel R_L)}$

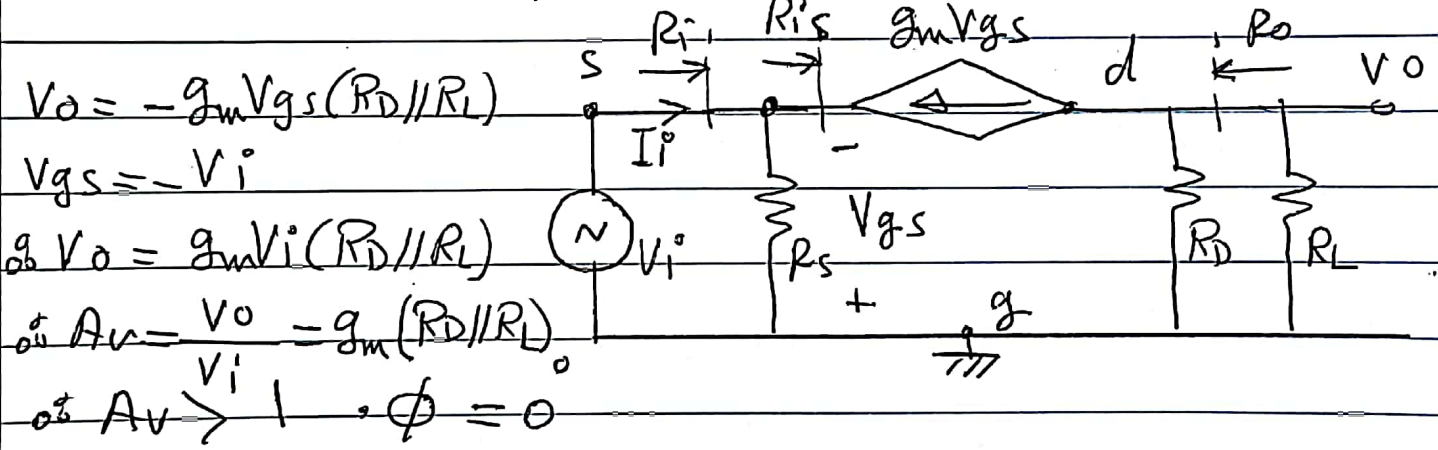
③ Common Gate Amp. (C.G. Amp.)

For this Amp:
 V_i to Source
 V_o from drain
 Gate: is Common-Terminal
 For the cct. shown
 assume certain values



for K_n, V_{TN} and certain resistance and D.C Source such that the MOSFET is in Saturation Regn.:

Draw S.S. A.c eqnt. cct. and find A_v, R_i & R_o .



$$R_o = \frac{V_x}{I_x} \Big|_{v_i=0} \Rightarrow R_D$$

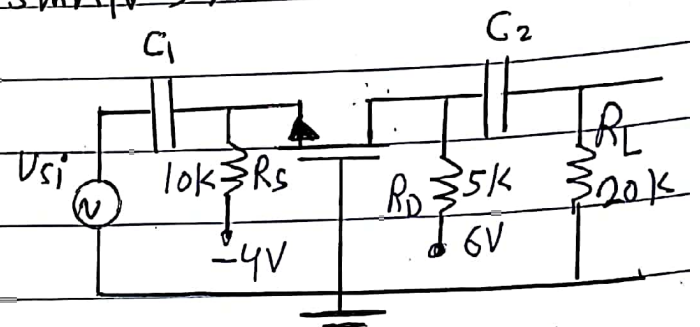
$$R_i = \frac{V_i}{I_i} = R_s // R_{is}$$

$$R_{is} = \frac{V_i - V_{gs}}{-g_m V_{gs}} = \frac{1}{g_m}$$

$$R_i = R_s // \frac{1}{g_m}$$

EXA: "C.G. Amp": For the cct. shown, the MOSFET has the parameters: $V_{TN} = 1V$, $K_n = 3mA/V^2$, $\lambda = 0$

- 1) Determine I_{DQ} , V_{DSQ}
- 2) Draw s.s.A.c eqnt. cct. and calculate A_v , R_i & R_o



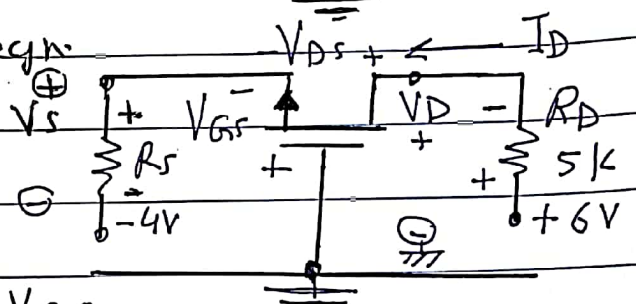
D.C Analysis:

Assume the MOSFET in Sat. Regn.

$$I_D = K_n (V_{GS} - V_{TN})^2$$

$$V_{GS} = V_G - V_S, \quad V_G = 0$$

$$V_{GS} + I_D R_S - 4 = 0$$



$$V_{GS} = 4 - I_D R_S \Rightarrow I_D = \frac{4 - V_{GS}}{10} = 3(V_{GS}^2 - 2V_{GS} + 1)$$

$$4 - V_{GS} = 30V_{GS}^2 - 60V_{GS} + 30 = 0$$

$$30V_{GS}^2 - 59V_{GS} + 26 = 0$$

$$V_{GS} = \frac{59 \pm \sqrt{(59)^2 - 4 \times 30 \times 26}}{60} = \frac{59 \pm 19}{60} = \boxed{1.3V} \text{ or } \boxed{0.667}$$

$$I_D = \frac{4 - 1.3}{10} = \frac{60}{60} = \boxed{0.27mA}$$

$$-6 + I_D R_S + V_{DS} + I_D R_S - 4 = 0$$

$$V_{DS} = 10 - I_D (R_D + R_S) = 10 - 0.27(5 + 10) = \boxed{5.95V}$$

$$V_D = 6 - I_D R_D = 4.65V, \quad V_S = I_D R_S - 4 = -1.3V$$

A.C Analysis: All caps. & D.C Sources \rightarrow S.C

$$A_v = \frac{V_o}{V_{gs}} \times \frac{V_{gs}}{V_{si}} = \frac{V_o}{V_{si}}$$

$$V_o = -g_m V_{gs} (R_D \parallel R_L)$$

$$\frac{V_o}{V_{gs}} = -g_m (R_D \parallel R_L)$$

$$\frac{V_{gs}}{V_{si}} = -V_{si} \rightarrow \frac{V_{gs}}{V_{si}} = 1$$

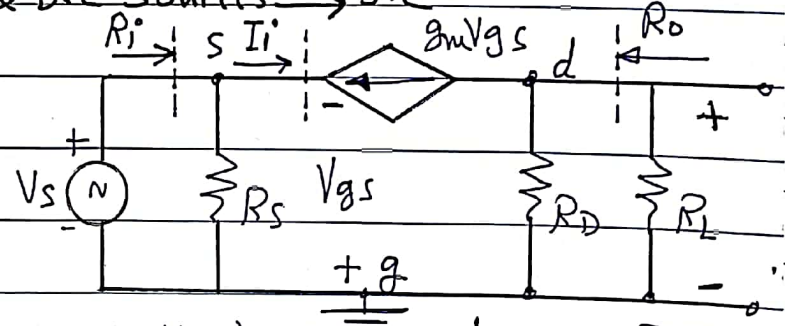
$$A_v = +g_m (R_D \parallel R_L) = 1.8(5 \parallel 20) = \boxed{7.2}$$

$$g_m = 2\sqrt{K_n I_D} = 1.8 \frac{mA}{V}$$

$$R_i = R_S \parallel R_{is}, \quad R_{is} = \frac{-V_{gs}}{-g_m V_{gs}} = \frac{1}{g_m} = 0.555k$$

$$R_i = 10 \parallel 0.555 = 0.526k \Omega$$

$$R_o = \frac{V_x}{I_x} \Big|_{V_{si}=0} \Rightarrow R_o = R_D = 5k \Omega$$



Amps. which contain more than one Transistor and used to achieve certain Combined specifications which can't be achieved using single stage.

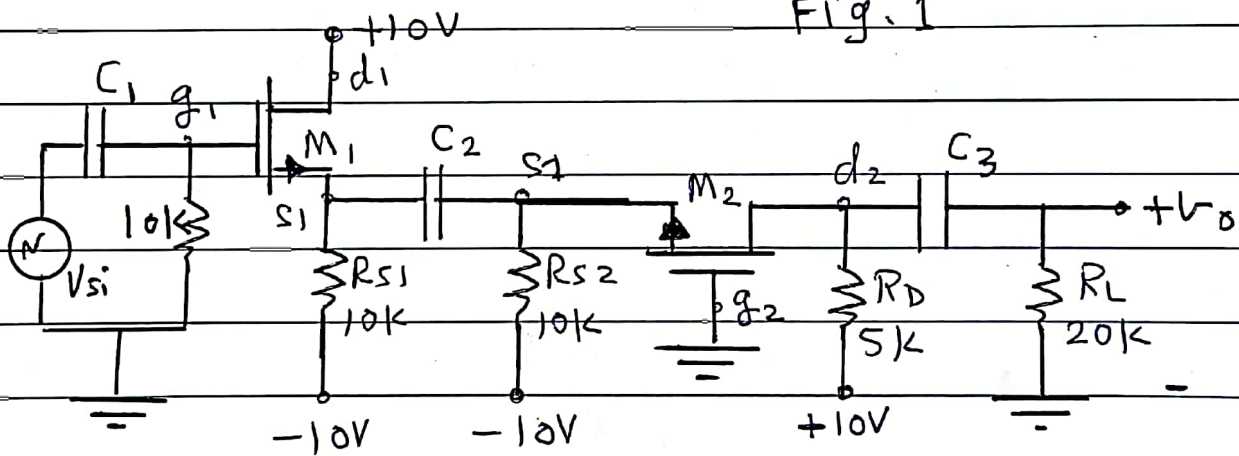
1) Cascade Multistage: These are connected in series and used to obtain high A_v or other characteristics.

EXA1: Consider the circuit shown in Fig-1 where M_1 & M_2 are identical MOSFETs

with $V_{TN} = 2V$ and $K_n = 4 \text{ mA/V}^2$, $\lambda = 0$.

- 1) Determine I_{D1} , I_{D2} , V_{DS1} and V_{DS2} .
- 2) Draw s.s. A.c eqnt. ckt. and calculate A_v , R_{in} and R_o ?

Fig. 1



This Amp. contain C.D Amp & C.G Amp.

D.C Analysis: For 1st Stage.

$$I_{D1} = K_n (V_{GS1} - V_{TN})^2$$

$$V_{GS1} = V_{G1} - V_{S1} = I_{D1} R_G - (I_{D1} R_{S1} - 10)$$

$$\text{or } V_{GS1} = 10 - 10 I_{D1}$$

$$I_{D1} = \frac{10 - V_{GS1}}{10} = 4 (V_{GS1} - 4)^2$$

$$10 - V_{GS1} = 40 V_{GS1}^2 - 160 V_{GS1} + 160$$

$$40 V_{GS1}^2 - 159 V_{GS1} + 150 = 0 \rightarrow V_{GS1} = 2.43V$$

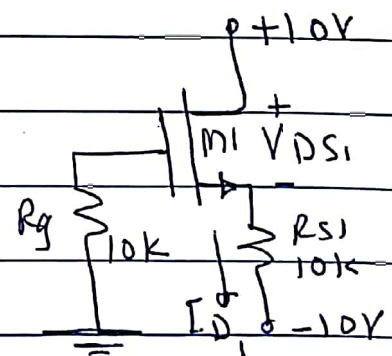


Fig. 2

$$I_{D1} = \frac{10 - 2.43}{10} = 0.756 \text{ mA}$$

$$V_{DS1} = 10 + 10 - 10 \times 0.756 = 12.4 \text{ V}$$

For M_2 :

$$I_{D2} = K_n (V_{GS2} - V_{TN2})^2$$

where $V_{GS2} = V_{G2} - V_{S2} = 0 - (10 I_{D2} - 10)$

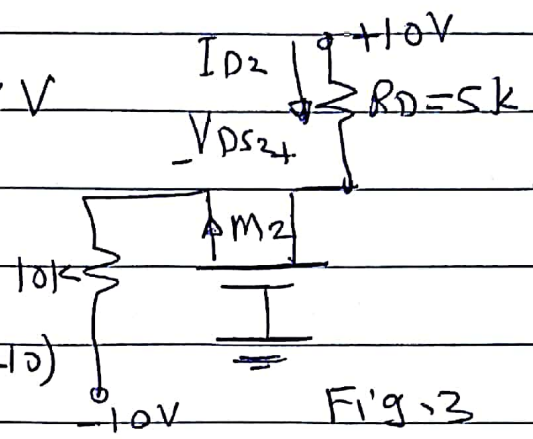


Fig. 3

$$V_{GS2} = 10 - 10 I_{D2}, I_{D2} = \frac{10 - V_{GS2}}{10}$$

$$\frac{10 - V_{GS2}}{10} = 4 (V_{GS2}^2 - 4 V_{GS2} + 4)$$

So $I_{D2} = 0.756 \text{ mA}$

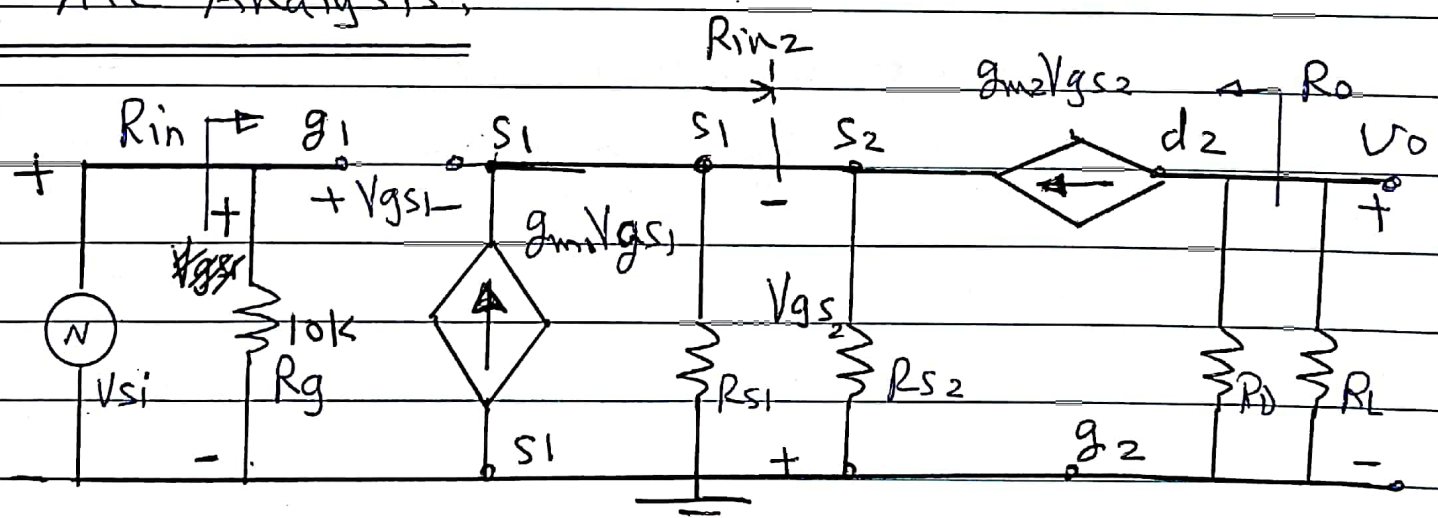
$$V_{DS2} = 10 + 10 - I_{D2} (R_D + R_{S2}) = 20 - 0.756 (5 + 10) \times 10^3$$

$\therefore V_{DS2} = 8.65 \text{ V}$

For both MOSFET $V_{DS(sat)} - V_{GS} - V_{TN} = 0.4 \text{ V}$
and $V_{DS1}, V_{DS2} > V_{DS(sat)}$

So both MOSFETs in Saturation.

A.c Analysis:



S.S. A.c eqnt. cct.

Fig. 4

$R_{in} = R_g = 10 \text{ k}\Omega$, $R_o = \text{RD} = 5 \text{ k}\Omega$

$$g_{m2} = 2 \sqrt{K_n I_D} = 3.47 \text{ mA/V}$$

$\therefore A_{v} = \frac{g_{m2} R_o}{1 + g_{m2} R_o}$

$$A_v = \frac{V_o}{V_{s_i}} = \frac{V_o}{V_{o_1}} \times \frac{V_{o_1}}{V_{s_i}} = A_{v_2} \times A_{v_1}$$

$$\frac{V_o}{V_{o_1}} = \frac{+g_{m_2}(R_D \parallel R_L)V_{gs_2}}{V_{gs_2}} = g_{m_2}(R_D \parallel R_L) \quad (\text{C.G. Amp.})$$

$$\frac{V_{o_1}}{V_{s_i}} = \frac{g_{m_1}V_{gs_1}(R_{s_1} \parallel R_{in_2})}{\frac{V_{s_i}}{1 + g_{m_1}V_{gs_1}(R_{s_1} \parallel R_{in_2})}} = \frac{g_{m_1}(R_{s_1} \parallel R_{in_2})}{1 + g_{m_1}(R_{s_1} \parallel R_{in_2})}$$

$$\therefore A_v = g_{m_2}(R_D \parallel R_L) \frac{g_{m_1}(R_{s_1} \parallel R_{in_2})}{1 + g_{m_1}(R_{s_1} \parallel R_{in_2})}$$

$$R_{in_2} = \frac{1}{g_{m_2}} \parallel R_{s_2} = 288 \parallel 10 \text{ k} = 0.28 \text{ k} \Omega \quad (\text{C.G. Amp.})$$

$$R_{s_1} \parallel R_{in_2} = 0.272 \text{ k} \Omega$$

$$A_v = 3.47(5 \parallel 20) \frac{3.47 \times 0.272}{3.47 \times 0.272 + 1} = 6.7$$

* The overall A_v is > 1

* The phase-shift is zero

* ~~The output resistance is "low".~~

Cascode Multistage

This ckt. is shown in Fig. in which M_1 is connected as C.S Amp. and M_2 is connected as C.G Amp. This Amp. is used for high-freq response Amp. "wide band Amp."

EXA: For the ckt. shown

M_1 & M_2 are identical

with $V_{TN} = 1.0V$

$K_n = 0.8 mA/V^2$

$\lambda = 0$

* D.C Analysis:

Find V_{DS1} , V_{DS2} , I_D

all Caps. are open.

$$I = \frac{15V}{100 + 50 + 150} = 0.05 mA$$

$$V_{G1} = I \times R_1 = 0.05 \times 50 = 2.5V$$

$$V_{S1} = I_D R_S = 5 I_D$$

$$V_{GS1} = V_{G1} - V_{S1} = 2.5 - 5 I_D$$

$$\therefore I_D = \frac{2.5 - V_{GS1}}{5} = 0.8 (V_{GS1}^2 - 2V_{GS1} + 1)$$

$$\therefore 4V_{GS1}^2 - 8V_{GS1} + 4 = 2.5 - V_{GS1}$$

$$4V_{GS1}^2 - 7V_{GS1} + 1.5 = 0$$

$$\therefore V_{GS1} = \frac{7 \pm \sqrt{49 - 4 \times 4 \times 1.5}}{8} = \frac{7.75}{8} = 1.5V$$

$$I_D = \frac{2.5 - 1.5}{5K} = 0.2 mA$$

$$V_{GS1} = V_{GS2} = 1.5V \quad (\text{Since } I_{D1} = I_{D2}, V_{TN1} = V_{TN2}, K_{n1} = K_{n2})$$

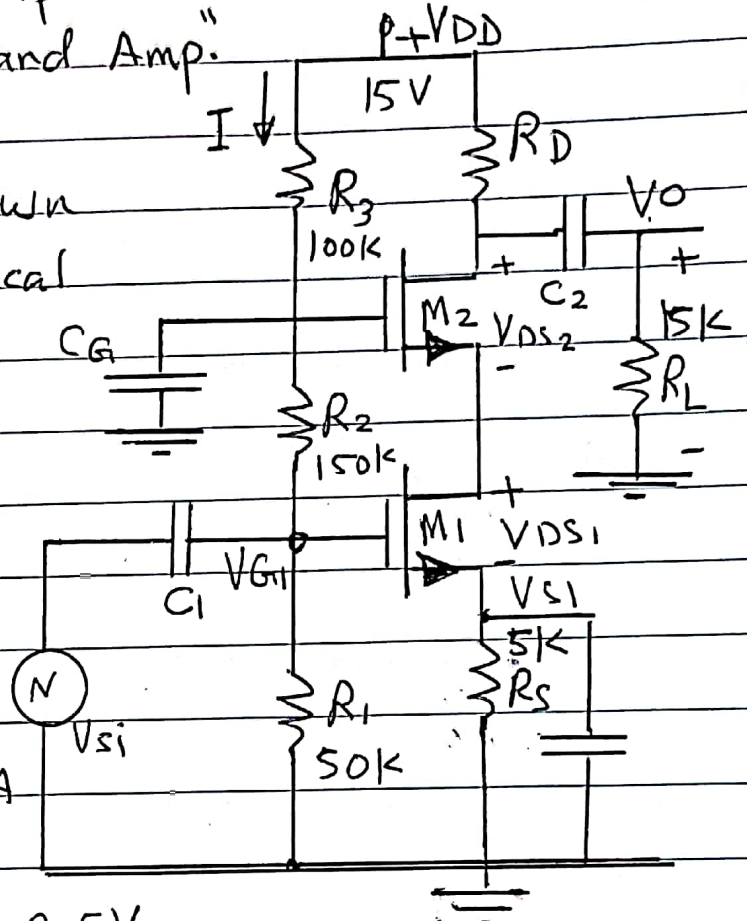


Fig. 5

$$-V_{DD} + I R_3 + V_{GS2} + V_{DS1} + I_D R_S = 0$$

$$V_{DS1} = 15 - 0.05 \times 100 - 1.5 - 0.2 \times 5 = 7.5 \text{ V}$$

$$-V_{DD} + I_D R_D + V_{DS2} + V_{DS1} + I_D R_S = 0$$

$$\therefore V_{DS2} = 15 - 7.5 - 10 \times 0.2 = 5.5 \text{ V}$$

$$V_{DS(\text{sat})} = V_{GS} - V_{TN} = 1.5 - 1 = 0.5 \text{ V}$$

\(\therefore\) Both MOSFETs are in Saturation Regn.

"A.C Analysis"

all Caps. & D.C Source are S.C.

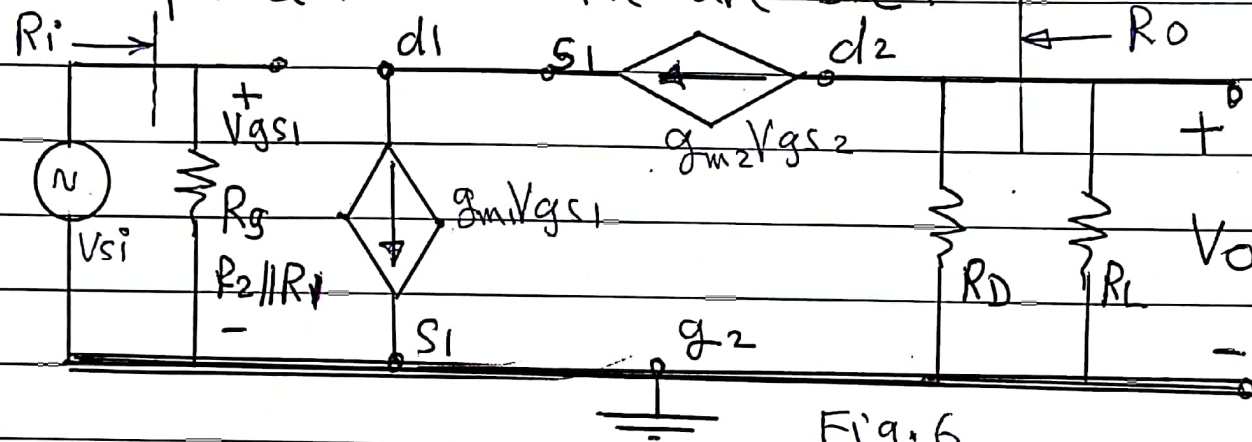


Fig. 6

$$A_v = \frac{V_o}{V_{si}} = \frac{-g_{m1} V_{gs1} (R_D \parallel R_L)}{V_{gs1}} = -g_{m1} (R_D \parallel R_L)$$

$$g_m = 2\sqrt{K_n I_D} = 2\sqrt{0.8 \times 0.2} = 0.8 \text{ mA/V}$$

$$\therefore A_v = -0.8 (10 \parallel 15) = -4.8$$

$$R_{in} = R_2 \parallel R_1 = 150 \parallel 50 = 37.5 \text{ k}\Omega$$

$$R_o = \left. \frac{V_x}{I_x} \right|_{V_{si}=0}$$

When $V_{si} = 0$, $V_{gs1} = 0$, $g_{m1} V_{gs1} = 0 \Rightarrow \text{O.C.}$

$$\therefore R_o = R_D = 10 \text{ k}\Omega$$

NOTE

THE AV OF this Amp. is (Av) of the C.S. Amp.??

Operational Amplifier 'Op-Amp' 1

Op-Amp. is an electronic device in IC form, contain multistages direct connected, with a very-high voltage gain. Compared to BJT and FET, It is Voltage-Controlled-Voltage-Source "VCVC".

The Symbol for op-Amp. is shown in Fig. 1 where:

$$V_o = A_o(V_2 - V_1) \quad \text{Inverting Terminal } V_1$$

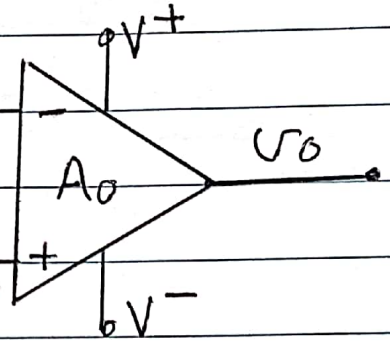
A_o : open-loop gain

V_2, V_1

NonInverting T. V_2

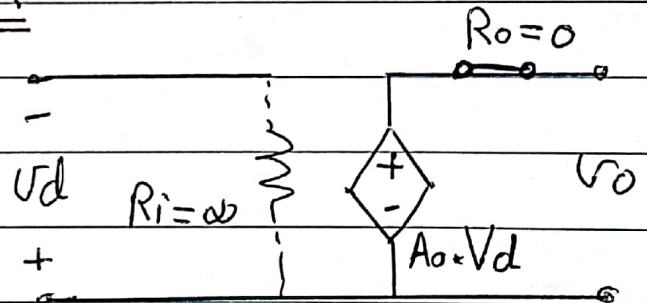
(Signal on nonInverting & Inverting Terminal)

V^+ and V^- are positive and negative biasing voltage.



Ideal op-Amp, parameters:

- 1) $R_{in} = \infty$
- 2) $R_o = 0$
- 3) open-loop gain = ∞
- 4) Bandwidth $B.W = \infty$



Ideal Op-Amp eqnt. cct.

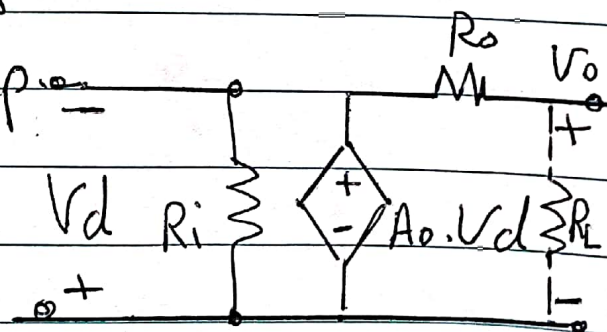
nonideal op-Amp parameters

The real op-Amps, parameters depend on the type of op-Amp.

(Technology: either Bipolar, MOSFET or BiFET)

where $R_{in} \neq \infty, R_o \neq 0$

$B.W \neq \infty, A_o \neq \infty$



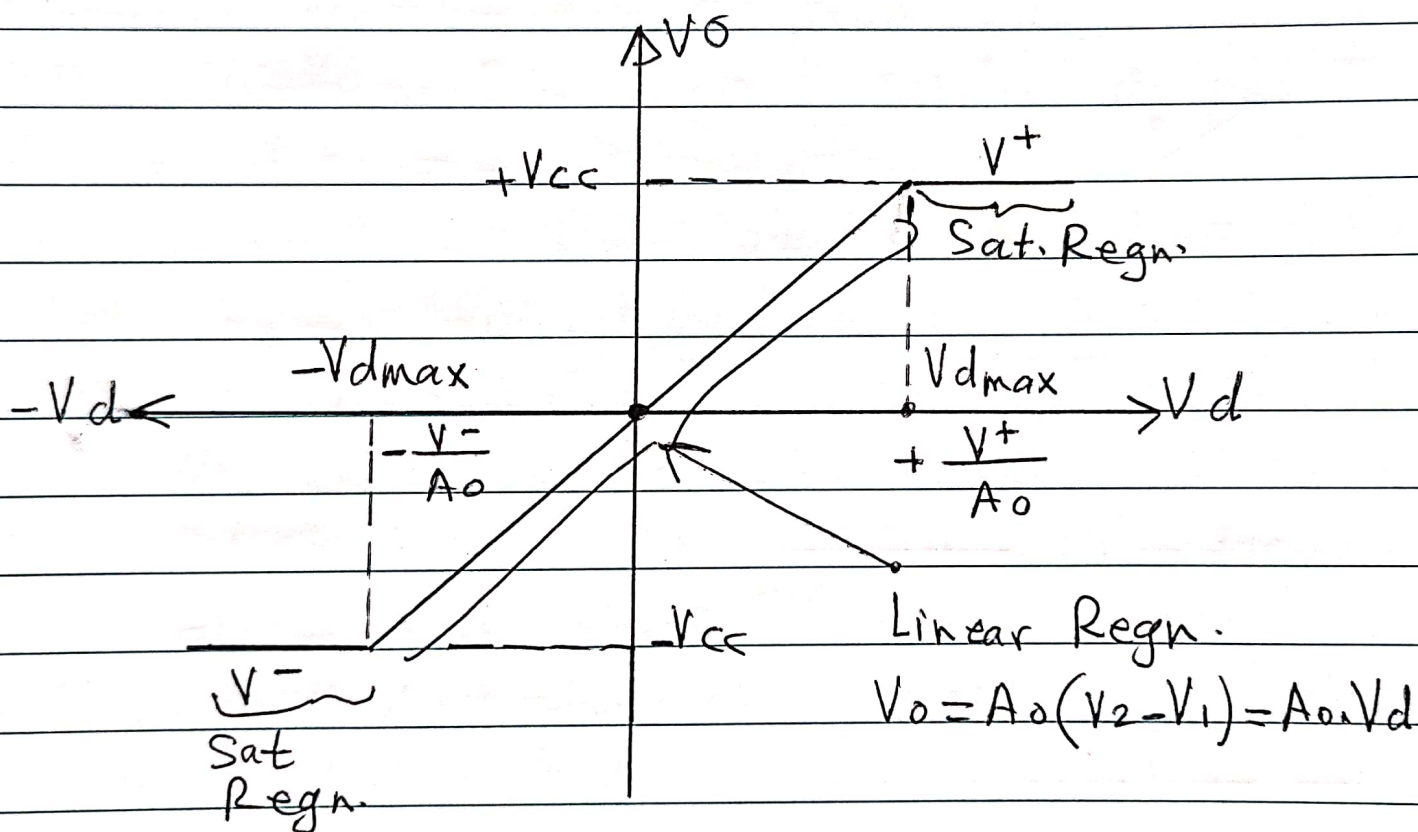
If R_L is connected, then:

In ideal Op-Amp: $V_o = A_o \cdot V_d$ (Independent on R_L)

In nonideal op-Amp: $V_o = \frac{A_o \cdot V_d \cdot R_L}{R_L + R_o}$ (depends on R_L).

Transfer characteristics: "a plot of V_o versus V_i "

This c/c is obtained from this eqn. $V_o = A_o (V_2 - V_1)$
but it is not exceed $\pm V$ i.e. $[V^- < V_o < V^+]$



Operating Regns:

Non-

- ① Saturation Regn: when $[-V_{dmax} > V_d < V_{dmax}]$
in this Regn: $V_o = A_o (V_2 - V_1)$ } Linear Regn. }
- ② Saturation Regn, when $[-V_{dmax} < V_d > V_{dmax}]$
In this Regn: $V_o = \pm V$ (either V^+ or V^-)

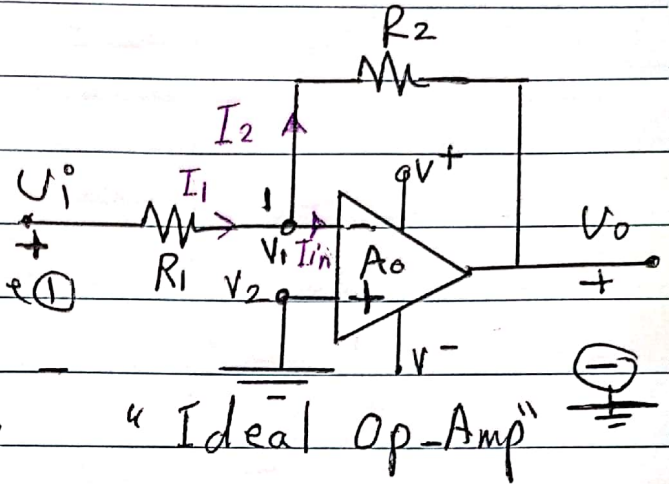
Op-Amp. Applications

3

① Linear Applications:

a) Inverting Amplifier

To determine the voltage gain of this Amp. follow the procedure: KCL at Node ①



$$I_1 = I_2 + I_{in}$$

$$\frac{V_i - V_1}{R_1} = \frac{V_1 - V_o}{R_2} + I_{in} \quad \text{①}$$

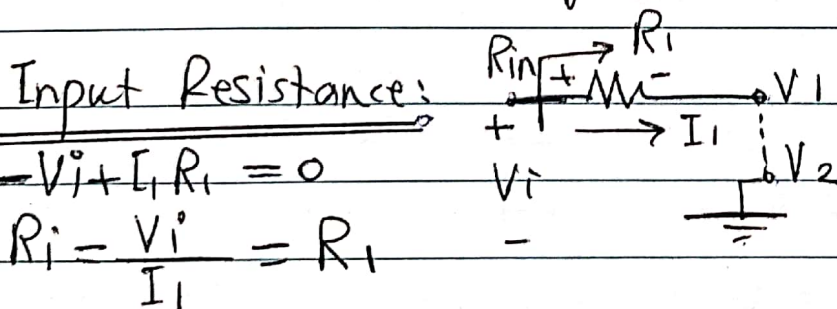
but $I_{in} = 0$ (since $R_{in} = \infty$)
and $V_1 = V_2 = 0$ (Virtual ground)

So: eqn. ① becomes:

$$\frac{V_i}{R_1} = \frac{-V_o}{R_2} \Rightarrow \frac{V_o}{V_i} = \frac{-R_2}{R_1} = A_v$$

A_v is closed-loop gain.

⊖ Inverting (180° phase-shift)



Virtual short & Virtual ground

$V_1 = V_2$ (V. short)
means $V_1 = V_2$ but Node ① and ② are not connected.

* If V_1 or V_2 is connected to ground then $V_1 = V_2 = 0$ (Virtual ground) i.e. $V_1 = 0$ but node ① is NOT connected to ground.

notice: $A_o \rightarrow$ open-loop gain

$A_v \rightarrow$ closed-loop

EXA: design an Inverting Amp. to have $A_v = -50$ and $R_{in} = 10k\Omega$. draw $v_o(t)$ for $v_i = 0.1 \sin \omega t$

Solution:

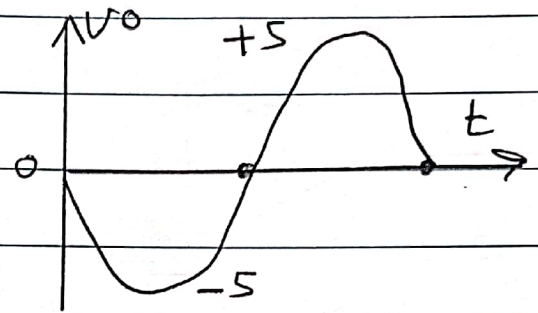
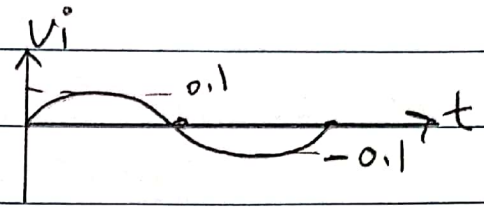
Since $R_{in} = R_1$ (For Inv. Amp).

So R_1 must be $10k\Omega$

$$A_v = -\frac{R_2}{R_1} = -50$$

$$\text{So } R_2 = 50R_1 = 500k\Omega$$

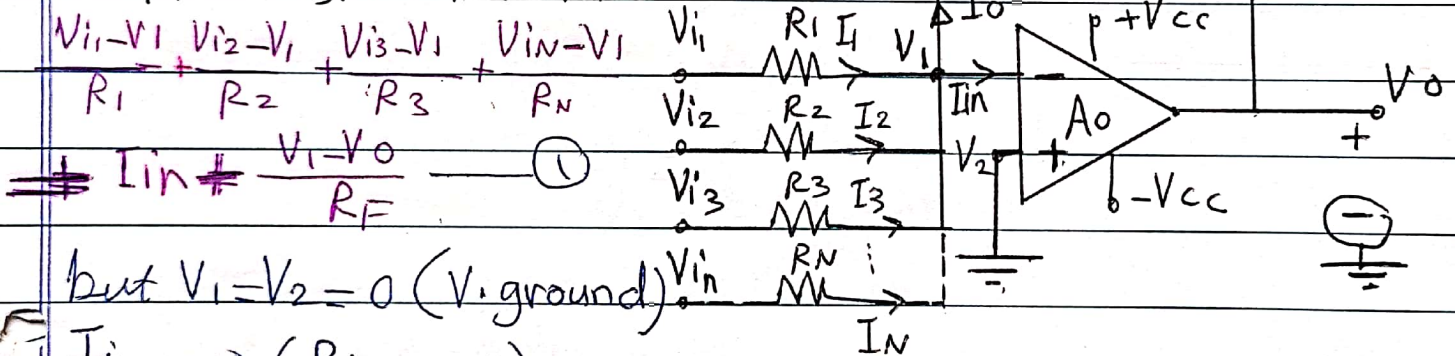
$$v_o = A_v \cdot v_i = -50 \times 0.1 \sin \omega t = -5 \sin \omega t \text{ V}$$



(b) Summing Amplifier:

KCL at Node ① gives:

$$I_1 + I_2 + I_3 + \dots + I_N - I_{in} + I_o = 0$$



but $v_1 = v_2 = 0$ (V. ground)

$$I_{in} = 0 \quad (R_{in} = \infty)$$

So eqn ① becomes:
$$\frac{v_{i1}}{R_1} + \frac{v_{i2}}{R_2} + \frac{v_{i3}}{R_3} + \dots + \frac{v_{iN}}{R_N} = -\frac{v_o}{R_F}$$

$$v_o = -\left(\frac{R_F v_{i1}}{R_1} + \frac{R_F v_{i2}}{R_2} + \frac{R_F v_{i3}}{R_3} + \dots + \frac{v_{iN} R_F}{R_N} \right)$$

each v_i will be multiplied (amplified) by a certain factor, Summed, and Inverted.

If all signals are required to be amplified by same gain, then choose $R_1 = R_2 = R_3 = \dots = R_N = R_F$

and $V_o = -\frac{R_F}{R} (V_{i_1} + V_{i_2} + V_{i_3} + \dots + V_{i_N})$
 V_i can be a.c or d.c signal"

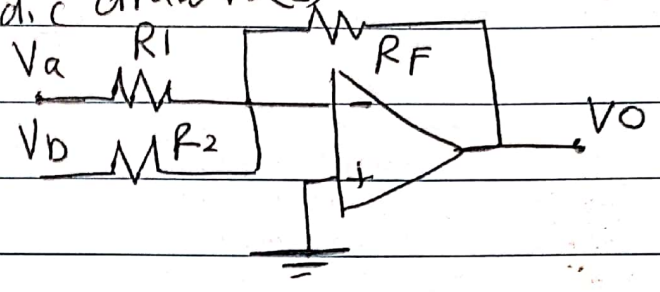
EXA:

- a) Design an Amp to produce $V_o = -(5V_a + 10V_b)$.
- b) For $V_a = 2 \sin \omega t$ V, $V_b = 0.3$ V d.c draw $V_o(t)$

Compared to:

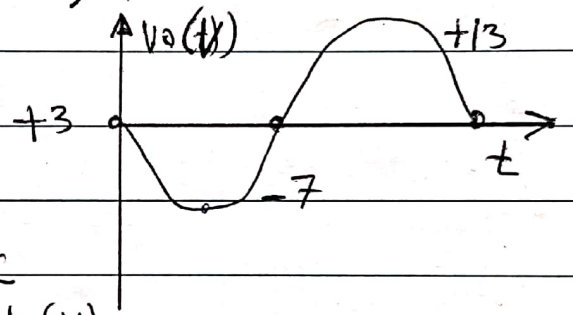
$$V_o = -\left(\frac{R_F}{R_1} V_a + \frac{R_F}{R_2} V_b\right)$$

$$\frac{R_F}{R_1} = 5, \quad \frac{R_F}{R_2} = 10$$



let $R_F = 20 \text{ k}\Omega$, so $R_1 = 4 \text{ k}\Omega$, $R_2 = 2 \text{ k}\Omega$

b) $V_o = -(5 \times 2 \sin \omega t + (10)(-0.3)) = 3 + 10 \sin \omega t$ V



EXA: Design a cct. to produce $V_o = -10(V_a + V_b)$.
 draw $V_o(t)$ when $V_a = 0.4$ V d.c
 $V_b = 0.2 \sin \omega t$ (v)

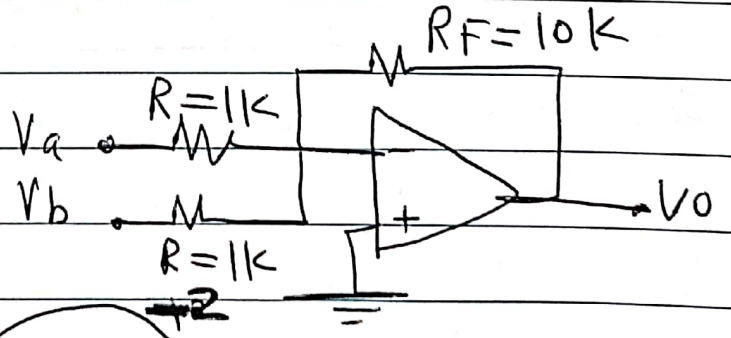
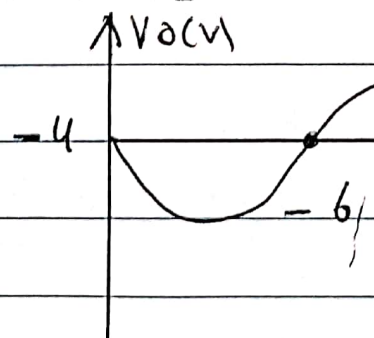
Solution: Compared V_o eqn. with

$$V_o = -\frac{R_F}{R} (V_{i_1} + V_{i_2})$$

$$\frac{R_F}{R} = 10, \text{ let } R_F = 10 \text{ k}, R = 1 \text{ k}\Omega$$

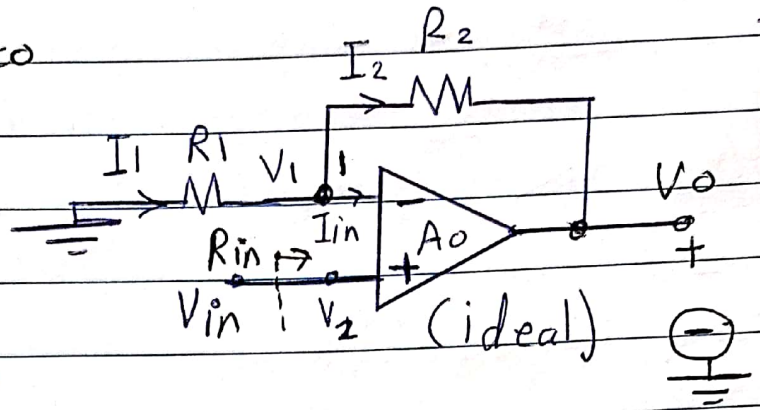
$$V_o = -10(0.4 + 0.2 \sin \omega t)$$

$$= -4 - 2 \sin \omega t$$



③ Non Inverting Amp.

The input is given to noninverting terminal



KCL at Node ①

$$I_1 = I_2 + I_{in}$$

$$\frac{0 - V_1}{R_1} = \frac{V_1 - V_0}{R_2} + I_{in}$$

but $I_{in} = 0$, $V_1 = V_2 = V_i$ (Virtual short)

$$\frac{-V_{in}}{R_1} = \frac{V_{in} - V_0}{R_2} \Rightarrow \frac{V_0}{R_2} = \frac{V_{in}}{R_1} + \frac{V_{in}}{R_2}$$

$$\text{So } V_0 = \left(1 + \frac{R_2}{R_1}\right) V_{in}$$

$$\frac{V_0}{V_{in}} = A_v = \left(1 + \frac{R_2}{R_1}\right)$$

Vin will be amplified by $\left[1 + \frac{R_2}{R_1}\right]$ and NOT inverted

Input resistance $R_{in} = \infty$.

④ Voltage-Follower "Buffer"

It is a special case of noninverting in which $R_1 = \infty$ and $R_2 = 0$

$V_1 = V_2 = V_{in}$ (Virtual short)

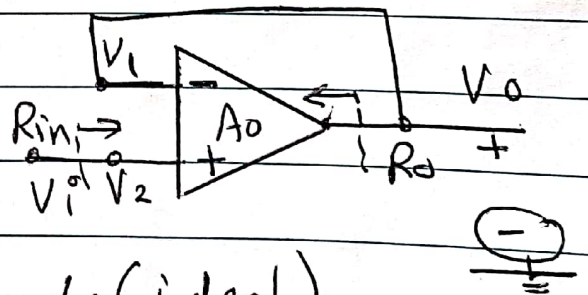
and since $V_1 = V_0$

$$\text{or } V_0 = V_{in} \Rightarrow A_v = \frac{V_0}{V_{in}} = 1$$

i.e V_0 follows V_{in} in magnitude (ideal)

and sign, so it is called voltage-follower

$R_{in} = \infty$, $R_o = 0$, $A_v = 1$, $\phi = 0$ characteristics of ideal Voltage-Follower.



* It is used to minimize loading effects *

EXA: For the ckt. shown in Fig. assume ideal Op-Amp. Calculate and draw $V_o(t)$ for the indicated input.

$(V_i = 8 \sin \omega t \text{ V})$

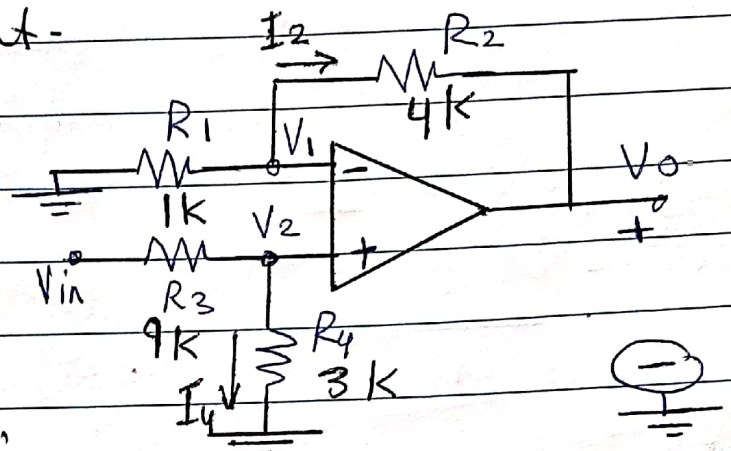
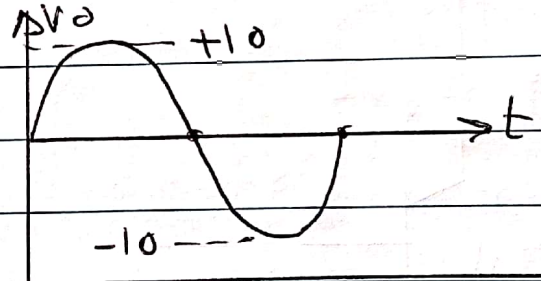
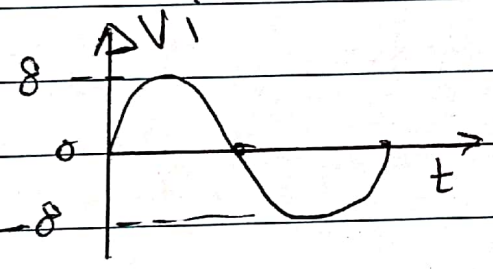
$$V_o = \left(1 + \frac{R_2}{R_1}\right) V_2$$

$$V_2 = \frac{V_{in} \cdot R_4}{R_3 + R_4}$$

$$V_o = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{R_4}{R_3 + R_4}\right) V_{in}$$

$$\therefore \frac{V_o}{V_{in}} = \left(1 + \frac{R_2}{R_1}\right) \left(\frac{R_4}{R_3 + R_4}\right) = \left(1 + \frac{4}{1}\right) \left(\frac{3}{9 + 3}\right) = 1.25$$

$$\therefore V_o = A_v \cdot V_{in} = 1.25 \times 8 \sin \omega t = 10 \sin \omega t$$



EXA: For the above ckt. Calculate I_u, I_2, V_1 For the same input.

$$V_2 = \frac{R_4}{R_3 + R_4} V_i = \frac{3}{9 + 3} (8 \sin \omega t) = 2 \sin \omega t \text{ V}$$

$$I_u = \frac{V_2}{R_4} = \frac{2 \sin \omega t}{3 \text{ K}} = 0.66 \sin \omega t \text{ mA}$$

$$V_1 = V_2 \text{ (Virtual short)} = 2 \sin \omega t \text{ (V)}$$

$$I_2 = \frac{V_1 - V_o}{R_2} = \frac{2 \sin \omega t - 10 \sin \omega t}{4 \text{ K}} = -2 \sin \omega t \text{ mA}$$

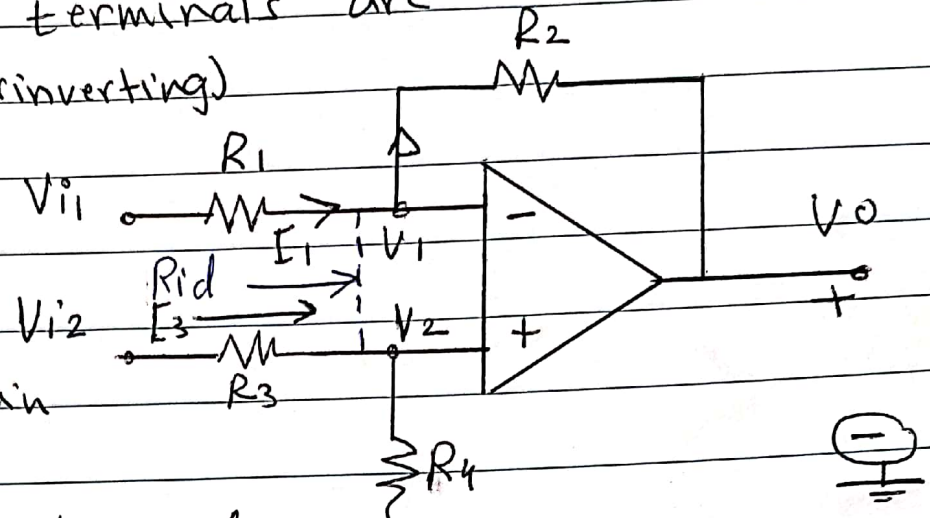
5) Difference Amp.

In this ckt. both terminals are used (Inverting & noninverting)

the o/p will be in the form

$$V_o = A_d (V_{i2} - V_{i1})$$

where A_d : the gain of the diff. Amp.

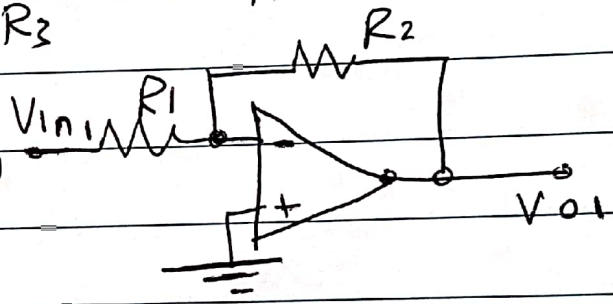


* Superposition will be used

to find the expression for A_d

1) For V_{i1} ($V_{i2} = 0$), $V_2 = \frac{V_{i2} \times R_4}{R_4 + R_3} = 0$, so the ckt. will be as shown.

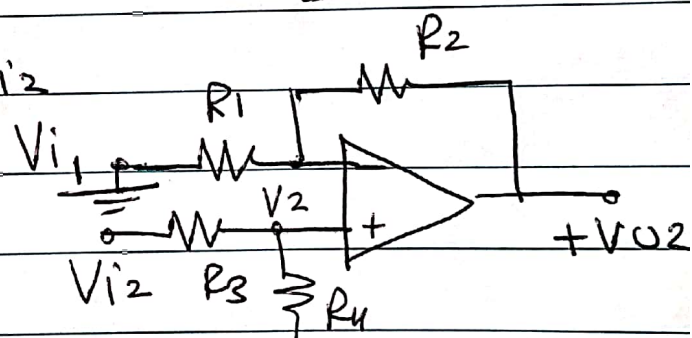
$$V_{o1} = -\frac{R_2}{R_1} V_{i1} \text{ (Inverting Amp)}$$



2) For V_{i2} ($V_{i1} = 0$)

$$V_{o2} = \left(\frac{R_4}{R_4 + R_3} \right) \left(1 + \frac{R_2}{R_1} \right) V_{i2}$$

(Non Inverting)



$$V_o = V_{o1} + V_{o2}$$

$$V_o = \left\{ \left(1 + \frac{R_2}{R_1} \right) \left(\frac{R_4/R_3}{1 + \frac{R_4}{R_3}} \right) V_{i2} - \frac{R_2}{R_1} V_{i1} \right\}$$

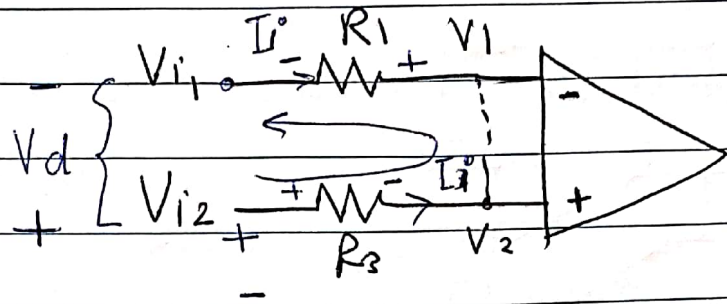
If $\left(\frac{R_4}{R_3} = \frac{R_2}{R_1} \right)$ then, $V_o = \frac{R_4}{R_3} (V_{i2} - V_{i1})$

$$A_d = \frac{V_o}{V_{i2} - V_{i1}} = \left[\frac{R_4}{R_3} = \frac{R_2}{R_1} \right]$$

Input Resistance R_{id} :

(9)

$$R_{id} = \frac{V_d}{I}$$



$$V_d + I_i R_3 + I_i R_1 = 0$$

$$V_d = I_i (R_1 + R_3)$$

$$\therefore R_{id} = \frac{V_d}{I_i} = R_1 + R_3$$

EXA: Design a difference Amp. to have $R_{id} = 6k$ and $A_d = 50$? Calculate V_o for $V_{i1} = 2V$ and $V_{i2} = 2.1V$

Solution:

For this ckt. $A_d = \frac{R_2}{R_1}$ (with assumption $\frac{R_2}{R_1} = \frac{R_4}{R_3}$)

also $R_{id} = R_1 + R_3$

"If we choose $R_1 = R_3$ & $R_2 = R_4$, we satisfy the above constraint. (assumption)

So $(R_1 + R_3) = 6k = R_{id}$ then $R_1 = R_3 = 3k$

$$A_d = \frac{R_2}{R_1} = 50 \Rightarrow R_2 = R_4 = 50 \times 3 = 150k\Omega$$

$$V_o = A_d (V_{i2} - V_{i1})$$

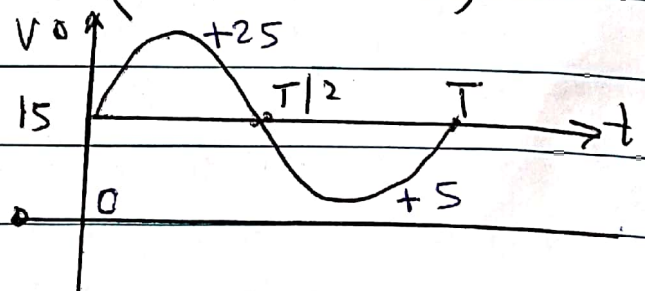
$$= 50(2.1 - 2) = 5V$$

* For $V_{i1} = -0.2 \sin \omega t$

$$V_{i2} = 0.3 V_{d.c}$$

$$V_o = 50(0.3 - (-0.2 \sin \omega t))$$

$$= 50(0.3 + 0.2 \sin \omega t) = (15 + 10 \sin \omega t) V$$



EXA 2: Design a diff. Amp. to have $A_d = 100$, $R_i = 20k$ using an ideal op-Amp.

To satisfy $\frac{R_2}{R_1} = \frac{R_4}{R_3}$

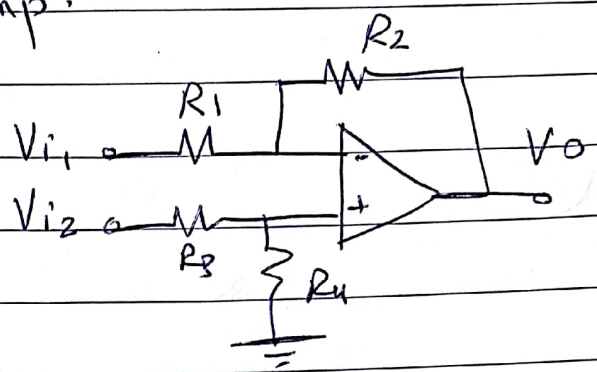
let $R_1 = R_3$, $R_2 = R_4$

then $R_{in} = 2R_1 = 2R_3 = 20k\Omega$

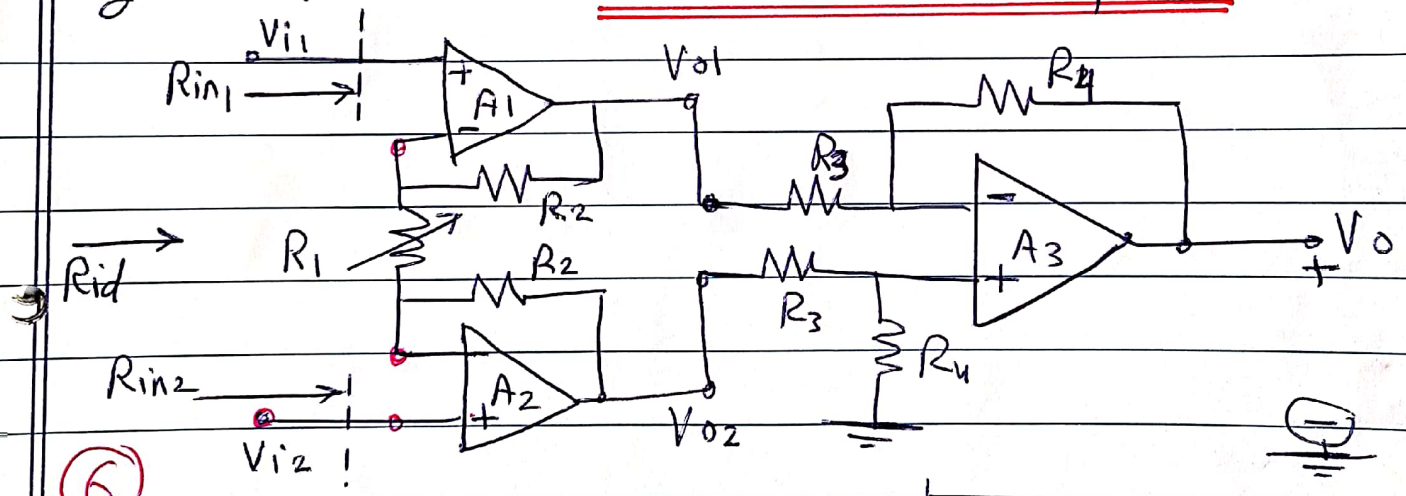
$\therefore R_1 = R_3 = 10k\Omega$

Since $A_d = \frac{R_2}{R_1} = \frac{R_4}{R_3} = 100$

$\therefore R_2 = R_4 = 1000k\Omega = 1M\Omega$



\therefore To increase R_{in} or A_d we must accept high value for R_2 & R_4 which is NOT desired in design!!!
So to solve this problem and achieve high A_d & high R_{in} we use an Instrumentation Amplifier



6

Instrumentation Amp.

It contains three Op-Amps.
 A_1 & A_2 \rightarrow Noninverting Amp.
 A_3 \rightarrow Difference Amps.

$R_{in1} = \infty$, $R_{in2} = \infty$

$R_{id} = \infty$.

This Amp achieves:
high, adjustable, single-element dependant gain,
 and very high input resistance. Using proper values for resistance.

To derive an expression for the differential voltage-gain (A_d), superposition theorem is used.

① Effect of V_{i1} ($V_{i2} = 0$) A_1 is Noninverting

$$\text{with } V_{o1} = \left(1 + \frac{R_2}{R_1}\right) V_{i1}$$

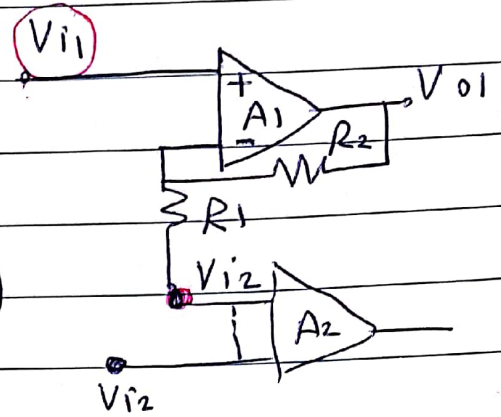
② Effect of V_{i2} ($V_{i1} = 0$)

A_1 is inverting

$$V_{o1} = -\frac{R_2}{R_1} V_{i2}$$

but $V_{o1} = V_{o1} + V_{o1}$ (Superposition)

$$\text{So } V_{o1} = \left(1 + \frac{R_2}{R_1}\right) V_{i1} - \frac{R_2}{R_1} V_{i2}$$



① The Same For A_2 :

Noninverting for V_{i2} & Inverting For V_{i1} , So

$$V_{o2} = \left(1 + \frac{R_2}{R_1}\right) V_{i2} - \frac{R_2}{R_1} V_{i1}$$

But $V_o = (V_{o2} - V_{o1}) \frac{R_4}{R_3}$ (Difference Amp.)

$$\text{So } V_o = \frac{R_4}{R_3} \left[\left\{ \left(1 + \frac{R_2}{R_1}\right) V_{i2} - \frac{R_2}{R_1} V_{i1} \right\} - \left\{ \left(1 + \frac{R_2}{R_1}\right) V_{i1} - \frac{R_2}{R_1} V_{i2} \right\} \right]$$

$$\text{So } V_o = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1}\right) (V_{i2} - V_{i1})$$

$$\text{So } A_d = \frac{V_o}{V_{i2} - V_{i1}} = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1}\right)$$

and since there are two resistances of R_2, R_3, R_4 and only single resistance named R_1 , so A_d is single-element dependant, which is R_1 i.e. to change A_d , we can change R_1 only NOT all resistances, which is desirable in design.

EXA: Design an I.A to have a gain ranging from (5 → 500) Using a Max. resistance 100kΩ assuming ideal Op-Amps.

Solution: to have adjustable gain, choose R_1 to be a variable resistance consisting of Fixed resistance R_{1F} and a Variable resistance R_{1V}

R_{1F} is to limit the gain at Max. value even when R_{1V} is set to zero and prevent Op-Amp. Saturation.

let R_{1V} is 100kΩ potentiometer

① $A_d = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_1} \right)$
when R_1 is min, $A_d \rightarrow$ Max

$R_{1min} = R_{1F} (R_{1V} = 0)$

when R_1 is Max, $A_d \rightarrow$ Min

$R_{1max} = 100k + R_{1F}$

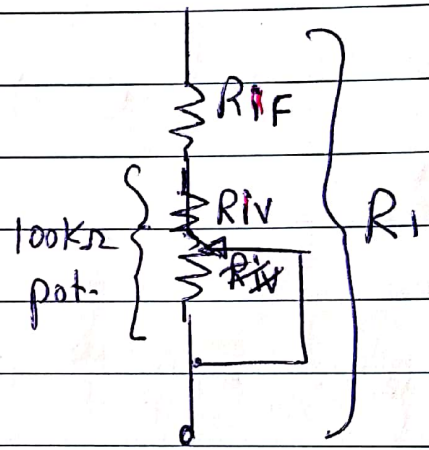
∴ $500 = \frac{R_4}{R_3} \left(1 + \frac{2R_2}{R_{1F}} \right)$

let $\frac{R_4}{R_3} = 2$

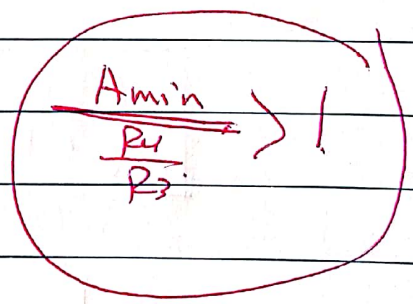
∴ $2R_2 = 249 R_{1F}$ — ①

⑤ $= \frac{R_4}{R_3} \left(1 + \frac{2R_2}{100 + R_{1F}} \right)$

∴ $2R_2 = 150 + 1.5 R_{1F}$ — ②



$R_1 = R_{1F} + R_{1V}$



How to choose $\frac{R_4}{R_3}$

equate eqn. ① & ② gives: $R_{1F} = 0.606 k\Omega$

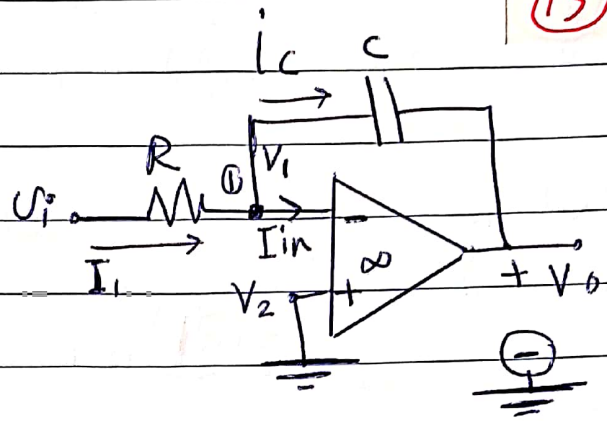
$R_2 = 75.5 k\Omega$, let $R_3 = 1 k\Omega$ so $R_4 = 2 k\Omega$

So we achieve high, adjustable, single-element dependant gain with reasonable resistance values "In desired range".

(7)

(13)

Integrator:



KCL at Node ①

$$I_i = I_{in} + I_c$$

$$\frac{V_i - V_1}{R} = I_{in} + C \frac{dV_c}{dt}$$

Where $V_c = V_1 - V_o$

but $I_{in} = 0$, ($R_{in} = \infty$) and $V_2 = V_1 = 0$ (Virtual short ground)

$$\frac{V_i}{R} = 0 + -C \frac{dV_o}{dt}$$

$$\therefore dV_o = -\frac{V_i}{RC} dt \Rightarrow \int dV_o = -\frac{1}{RC} \int V_i dt$$

$$\therefore V_o = -\frac{1}{RC} \int V_i dt$$

EXA: Calculate & Draw $V_o(t)$ for the indicated input.

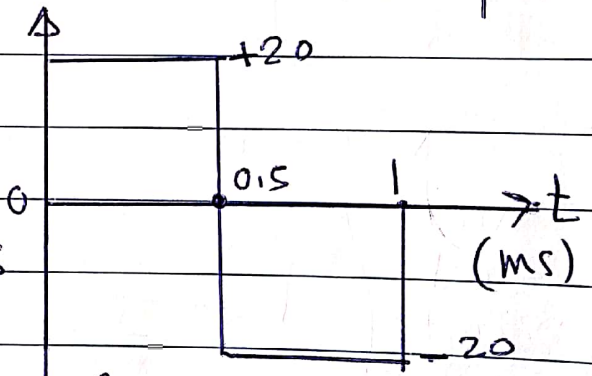
When $R = 1k\Omega$, $C = 1\mu F$

1) For $0 < t < 0.5ms$

$$V_o = -\frac{1}{10^3 \times 10^{-6}} \int_0^{0.5} 20 dt$$

$$= -1000 \times 20 t = -1000 \times 20 (0.5 - 0)ms$$

$$= -10V$$

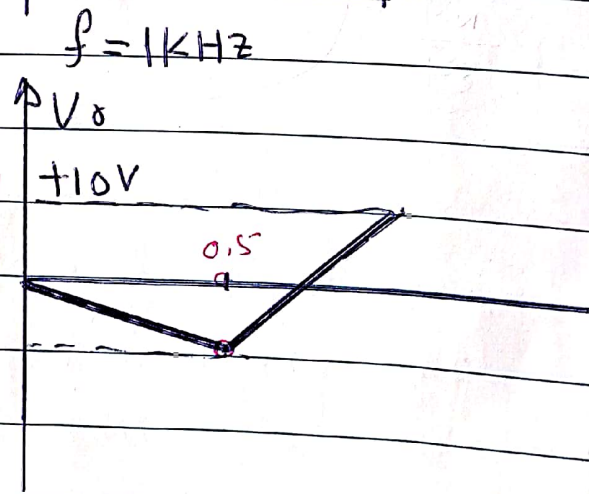


2) For $(0.5 < t < 1)ms$

$$V_o = -\frac{1}{RC} \int -20 dt$$

$$= +1000 \times 20 t = 2 \times 10^4 (1 - 0.5)ms$$

$$= +10V$$



i.e when V_i is Square-Wave -10 and V_o will be Inverted Triangular wave.

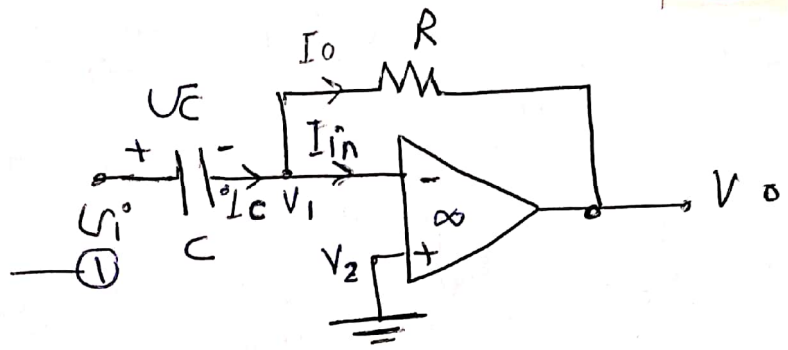
8

14

Differentiator

$$I_{in} + I_o = I_c$$

$$I_{in} + \frac{V_1 - V_o}{R} = C \frac{dV_c}{dt}$$



but $V_1 = V_2 = 0$ (Virtual ground) $\therefore I_{in} = 0$ ($R_{in} = \infty$)

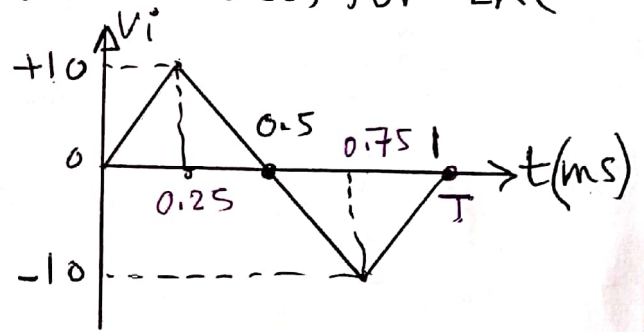
$$V_c = V_i - V_1 = V_i$$

\therefore eqn. ① will be: $-\frac{V_o}{R} = \frac{C dV_i}{dt}$

$$\therefore \boxed{V_o = -RC \frac{dV_i}{dt}} \quad \text{i.e. } V_o \propto \frac{dV_i}{dt}$$

EXAM: For the diff. shown, draw $V_o(t)$ for the indicated input.

When $R = 500 \Omega$, $C = 1 \mu F$.



i) For $(0 < t < 0.25) \text{ ms}$

$$\left(\frac{\Delta V_i}{\Delta t}\right) = \frac{(10 - 0)}{(0.25 - 0) \text{ ms}} = 40 \times 10^3 \text{ (V/s)}$$

$$\therefore V_o = -RC \frac{\Delta V_i}{\Delta t} = -0.5 \times 10 \times 10^{-6} \times 40 \times 10^3 = \boxed{-20 \text{ V}}$$

ii) For $(0.25 < t < 0.75) \text{ ms}$

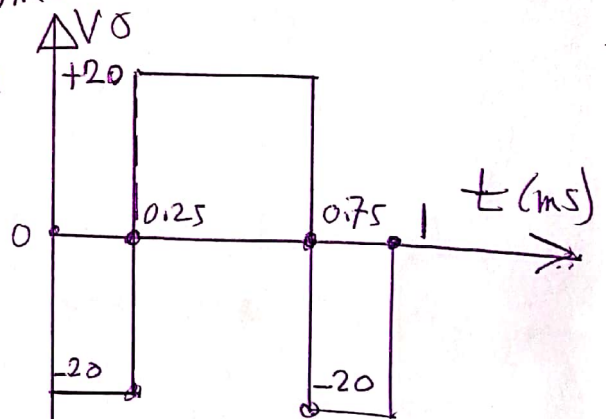
$$\left(\frac{\Delta V_i}{\Delta t}\right) = \frac{[-10 - (+10)]}{(0.75 - 0.25) \text{ ms}}$$

$$= -4 \times 10^4 \text{ V/s. } \boxed{V_o = +20 \text{ V}}$$

iii) For $(0.75 < t < 1) \text{ ms}$

$$\left(\frac{\Delta V_i}{\Delta t}\right) = \frac{0 - (-10)}{(1 - 0.75) \text{ ms}} = 4 \times 10^4 \text{ V/s}$$

$$\boxed{V_o = -20 \text{ V}}$$



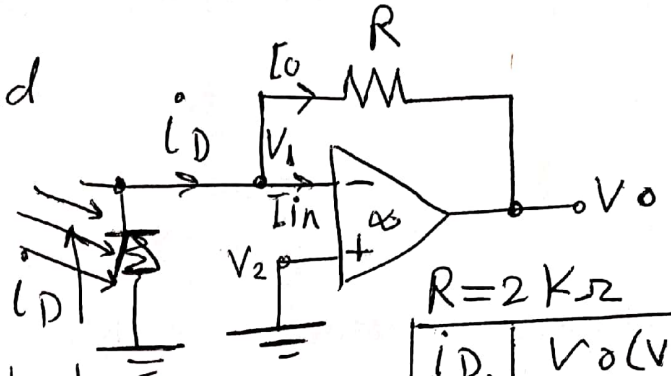
i.e. the o/p is Inverted Square wave.

9

Current-to-Voltage Converter (I-to-V)

15

Some time, it is required to convert the o/p Current (such as photodiode) into an output voltage.



The ckt. shown can do that:

$$i_D = I_{in} + I_0 \Rightarrow i_D = I_{in} + \frac{V_1 - V_0}{R}$$

but $I_{in} = 0$, $V_1 = V_2 = 0$ (ideal op-Amp)

$$\therefore V_0 = -I_D \cdot R \Rightarrow V_0 \propto I_D$$

$R = 2 \text{ K}\Omega$

i_D (mA)	V_0 (V)
1	-2V
3	-6V
5	-10V
7	-14V
10	-20V

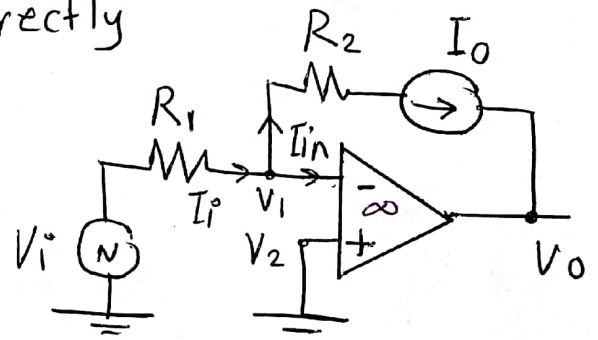
EXA: Calculate V_0 corresponding to i_D For $R = 2 \text{ K}$

10 Voltage-to-Current Converter: (V-to-I)

In this ckt. the o/p Current is directly proportional to input voltage.

$$I_i = I_0 + I_{in}$$

$$\frac{V_i - V_1}{R_1} = I_0 + I_{in}$$



but $V_1 = V_2 = 0$ (V.G) and $I_{in} = 0$ ($R_{in} = \infty$).

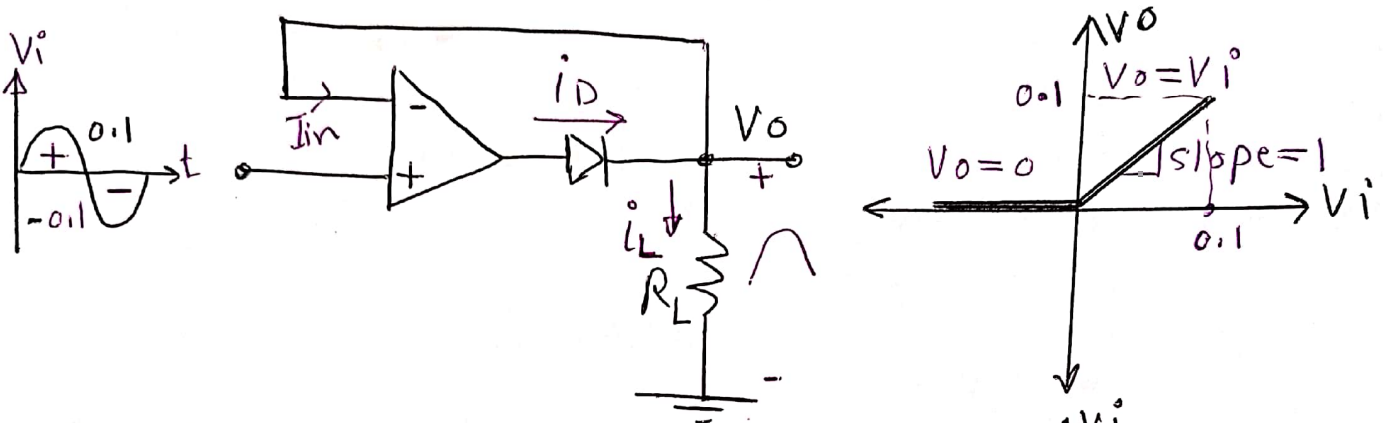
$$\therefore I_0 = \frac{-V_i}{R_1} \text{ i.e. } I_0 \propto V_i$$

a More complex ckt. can be used, but this is a simple ckt. Remember that the op-Amp. works in Linear Regn.

NonLinear Applications

① Precision Half-Wave Rectifier

To rectify signals with peak values $< V_D$ of diode, we use precision rectifier, which is an op-Amp with Diode. Such as shown in Fig.

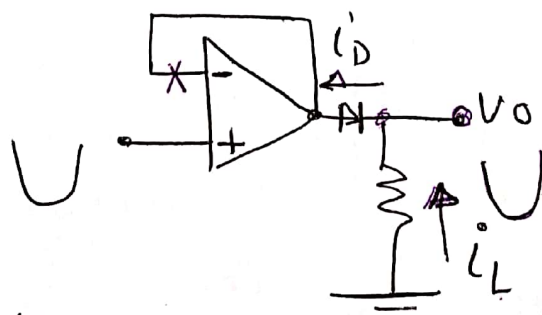


1) During +ve H.c of V_i ($V_i > 0$)
 $V_o = V_i$ (The ckt. is voltage-follower)
 So i_L & i_D are positive. This means the loop is closed and $V_o = V_i$

2) During -ve H.c of V_i ($V_i < 0$)
 the load and diode current (i_D & $i_L < 0$)
 which mean the diode is OFF

and the loop is broken, so $i_D = 0$
 and V_o is zero

∴ V_o is half-wave
 with average value
 $V_{o \text{ avg}} = \frac{V_{ip}}{\pi}$



* If we change the diode direction, V_o will be -ve HWR

(2)

Voltage-Comparator

(17)

In this ckt. the op-Amp. is used in open-loop mode (i.e No Connection between V_o and \ominus terminal, the op-Amp. works in Saturation mode with $V_o = A_o(V_2 - V_1) = V^+$ or $V^- (V_{cc}, -V_{cc})$

1) If $V_2 > V_1$ then $V_o = V^+$
($V_o = A_o \cdot V_d = \infty \rightarrow (V_o = V^+)$)

2) If $V_2 < V_1$, then $V_o = V^-$

$V_o = -A_o \cdot V_d = -\infty \rightarrow V_o = V^-$

ONE OF V_2 or V_1 will be a reference voltage the other will be the input voltage.

EXA: For the ckt. shown

Draw $V_o(t)$ for the indicated V_i

$$V_{ref} = V_1 = \frac{V^+ \cdot 2k}{(2+3)k} = 2V$$

① For $[0 < t < t_1], V_1 > V_2 \Rightarrow V_o = V^- = -5V$

② For $[t_1 < t < t_2], V_2 > V_1 \Rightarrow V_o = V^+ = +5V$

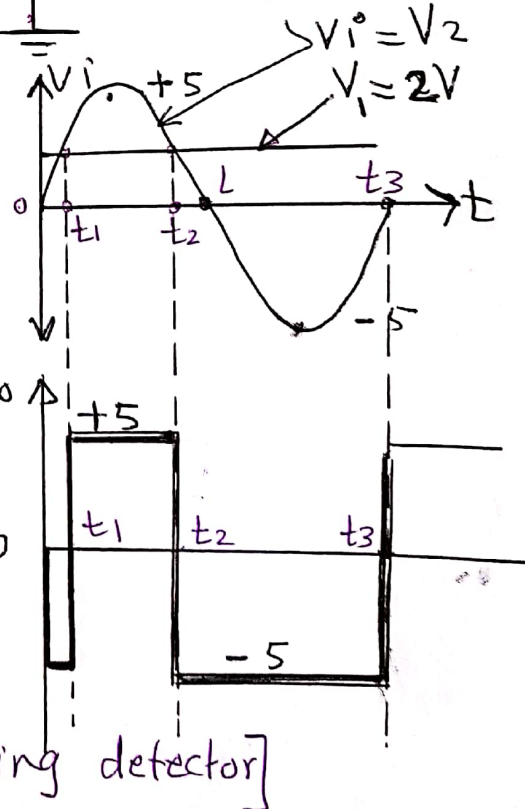
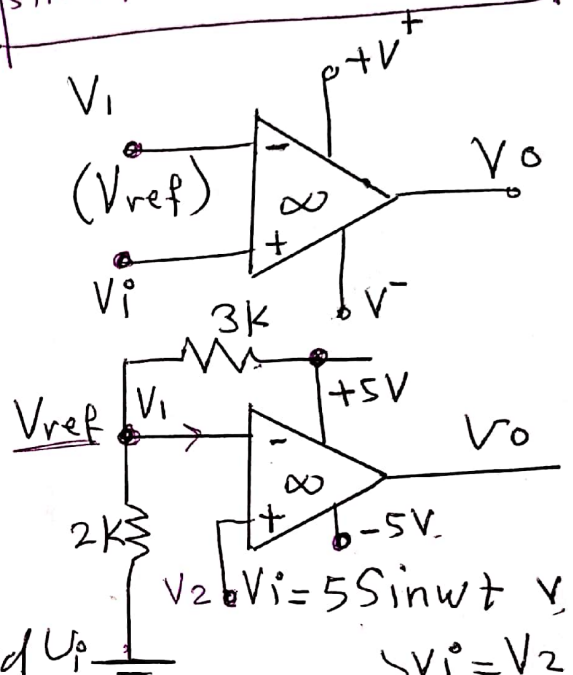
③ For $[t_2 < t < t_3], V_2 < V_1 \Rightarrow V_o = V^- = -5V$

So $V_o(t)$ is shown, Square-wave between V^+ and V^-

* IF $V_{ref} = 0$, then the o/p is Symmetrical Square-wave

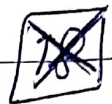
and the ckt. is called [Zero-Crossing detector]

Since $A_o = \infty, V_2 - V_1 = V_d$



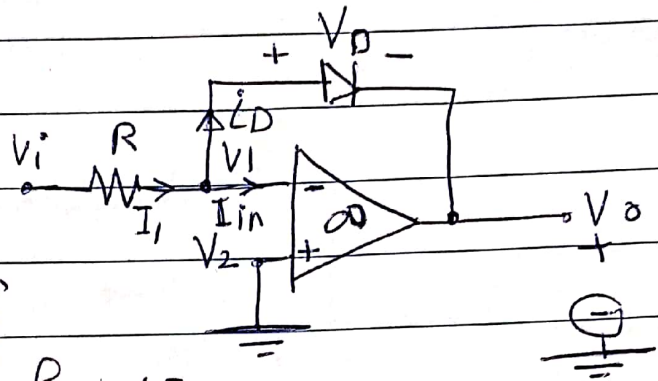
3

18



Log Amplifier:

In this ckt. the output is proportional to the log of input voltage



to derive an expression for V_o :

$$I_1 = I_{in} + I_D \text{ (KCL at Node 1)}$$

$$\frac{V_i - V_1}{R} = I_{in} + I_s e^{\frac{V_D}{nV_T}} \quad \text{--- (1)}$$

but $I_{in} = 0, V_1 = V_2 = 0$ (ideal op-Amp)

$$\text{also } V_D = V_1 - V_o = -V_o$$

So eqn. (1) will be

$$\frac{V_i}{R} = I_s e^{-\frac{V_o}{nV_T}} \rightarrow \frac{V_i}{R I_s} = e^{-\frac{V_o}{nV_T}}$$

$$\ln \left[\frac{V_i}{R \cdot I_s} \right] = -\frac{V_o}{nV_T} \Rightarrow \ln \left[\frac{V_i}{R \cdot I_s} \right] = -V_T \ln \left[\frac{V_i}{R \cdot I_s} \right] \text{ (For } n=1)$$

i.e. $V_o \propto \ln V_i$

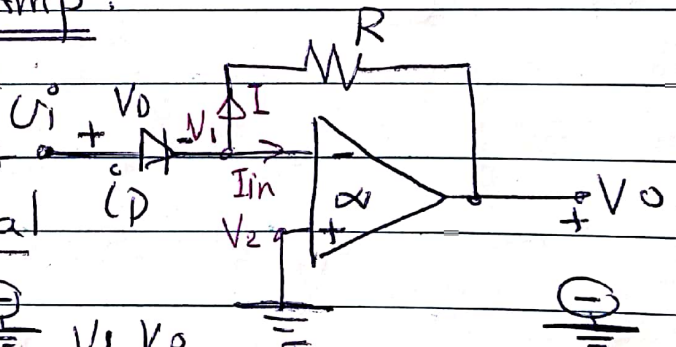
Antilog or Exponential Amp.

The Complement or Inverse

function of Log. Amp. is

Antilog or exponential

Amp.



$$I_D - I + I_{in} \rightarrow I_s e^{\frac{V_D}{nV_T}} = \frac{V_1 - V_o}{R} + I_{in} \quad \text{--- (1)}$$

but $I_{in} = 0, V_1 = V_2 = 0, V_D = V_1 - V_o = -V_o$ (ideal op-Amp)

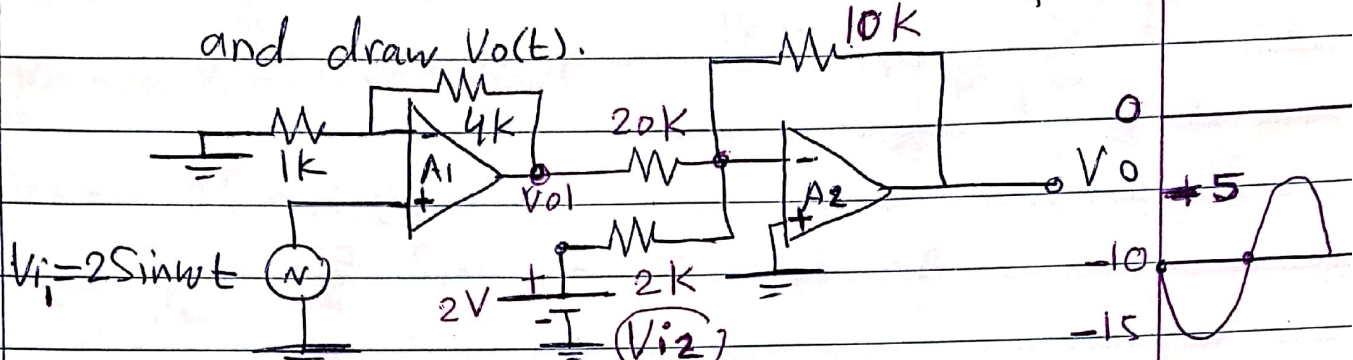
$$\text{eqn (1) will be: } I_s e^{\frac{V_i}{nV_T}} = \frac{-V_o}{R}$$

$$\therefore V_o = -I_s R e^{\frac{V_i}{nV_T}}$$

$$\text{i.e. } V_o \propto e^{\frac{V_i}{nV_T}}$$

EXAMPLES:

EXA 1: For the cct. shown, Assume ideal Op-Amp. Calculate V_o for the indicated inputs and draw $V_o(t)$.



$$V_o = - \left(\frac{10}{20} V_{o1} + \frac{10}{2} V_{i2} \right)$$

$$V_{o1} = \left(1 + \frac{4}{1} \right) V_i = 5 \times 2 \sin wt = 10 \sin wt \text{ (V)}$$

$$V_o = - \left(0.5 (10 \sin wt) + 5(2) \right) = -10 - 5 \sin wt \text{ (V)}$$

EXA 2: For the cct. shown, Assume ideal Op. Amp. Calculate V_o, I_Z, I_1 & I_2

Assume Z-D is "ON"

$$-10 + I R + V_Z = 0$$

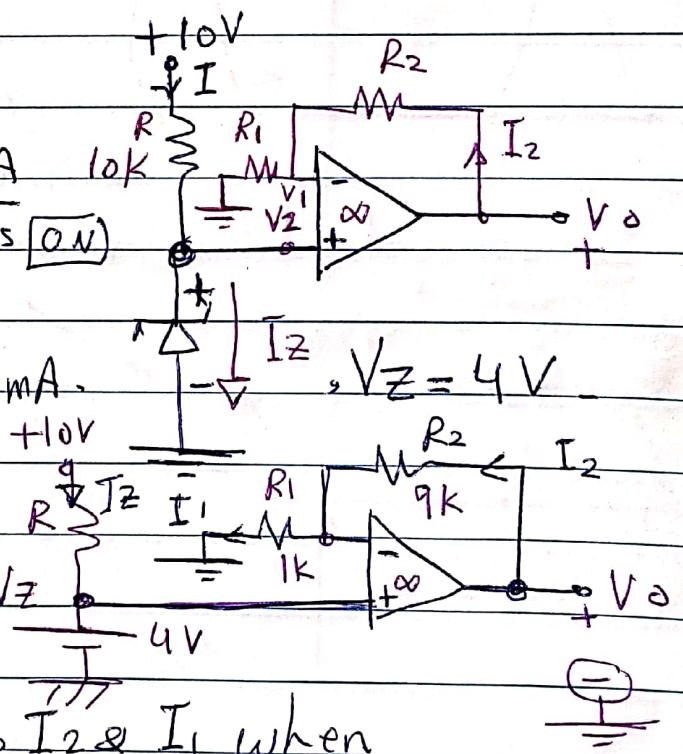
$$I R = I_Z = \frac{10 - 4}{10k} = 0.6 \text{ mA}$$

Since $I_Z > 0$, Z-D is ON

$$V_o = \left(1 + \frac{9}{1} \right) V_Z = 40 \text{ V}$$

$$I_1 = I_2 = \frac{V_o - 0}{9k + 1k} = \frac{40}{10} = 4 \text{ mA}$$

* This cct. is called Voltage Reference cct.

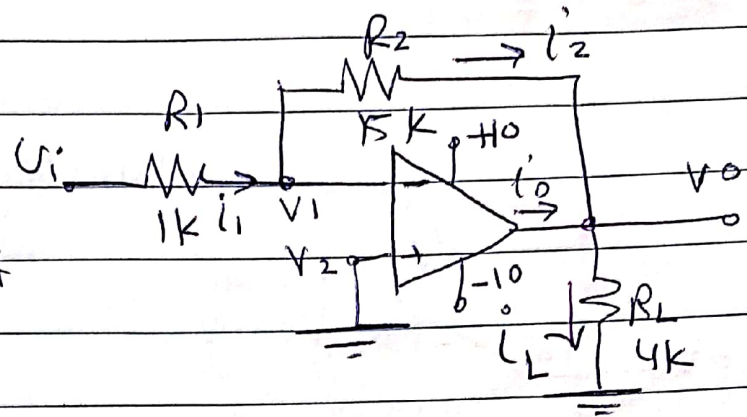


* Recalculate V_o, I_Z, I_2 & I_1 when $V_Z = 12 \text{ V} ???$

iii) $V_i = 0.8 \sin \omega t$ (V)

$$A_v = -\frac{R_2}{R_1} = -15$$

$$V_o = A_v \cdot V_i = -15 \times 0.8 \sin \omega t$$



$$V_o = -12 \sin \omega t$$
 (V)

$$i_L = \frac{V_o}{R_L} = \frac{-12}{4} \sin \omega t = -3 \sin \omega t$$
 mA

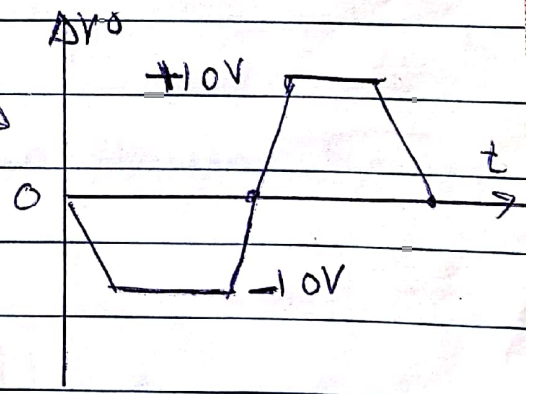
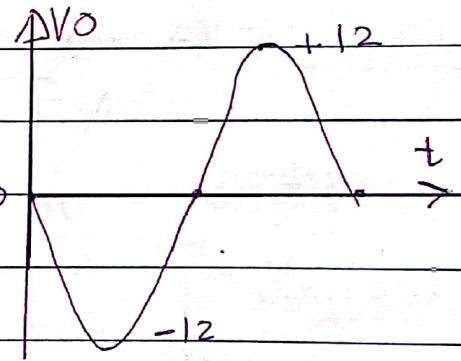
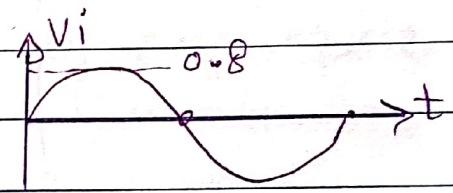
$$i_2 = \frac{V_1 - V_o}{R_2} = \frac{0 - (-12 \sin \omega t)}{15} = 0.8 \sin \omega t$$
 mA = i_1

$$i_o = i_L - i_2 = -3 \sin \omega t - 0.8 \sin \omega t = -3.8 \sin \omega t$$
 mA

$$R_{in} = R_1 = 1k\Omega$$

Draw $V_i(t)$ and $V_o(t)$

* Draw $V_o(t)$ if V^+ and V^- are (+10V) and (-10V)
 = If $V^+ = +10V$ & $V^- = -10V$
 then V_o will be saturated at +10V & -10V as shown

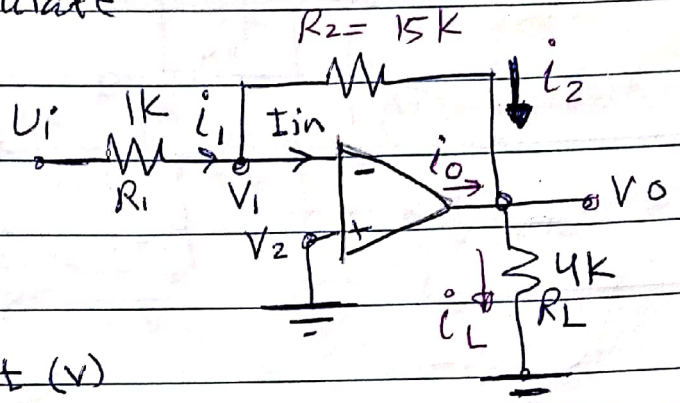


EXA 3:

Assume ideal Op-Amp. Calculate

$A_v, R_i, V_o, i_1, i_2, i_L, i_o$

For $V_i =$



i) $= 0.20V$ (d.c)

ii) $0.08V$ (d.c)

iii) $0.8 \sin \omega t$ (V) $\Rightarrow 0.8 \sin \omega t$ (V)

i) $A_v = -\frac{R_2}{R_1} = -15, V_o = A_v \cdot V_i = -15(-0.2) = 3V$

$i_L = \frac{V_o}{R_L} = \frac{3V}{4K} = 0.75 \text{ mA}$

$i_2 = \frac{V_1 - V_o}{R_2} = \frac{0 - 3}{15} = -0.2 \text{ mA} = i_1$ (because $I_{in} = 0$)

$R_{in} = R_1 = 1K\Omega$
Op-Amp. Source Current I_o

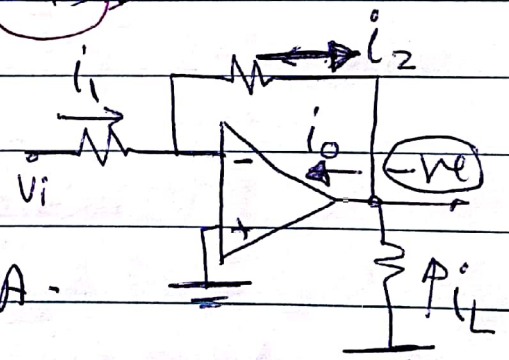
$i_o + i_2 = i_L \Rightarrow i_o = i_L - i_2 = 0.75 - (-0.2) = 0.95 \text{ mA}$

ii) For $V_i = 0.08V, A_v = -\frac{R_2}{R_1} = -15$

$V_o = A_v \cdot V_i = -15 \times 0.08 = -1.2V$

$i_L = \frac{V_o}{R_L} = \frac{-1.2}{4} = -0.3 \text{ mA}$

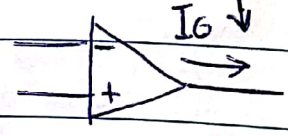
$i_2 = \frac{V_1 - V_o}{R_2} = \frac{0 - (-1.2)}{15K} = 0.08 \text{ mA}$



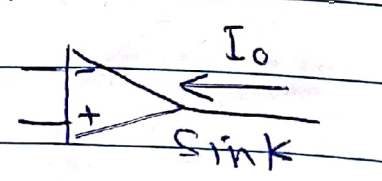
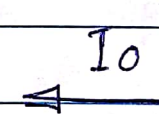
$i_o = i_L - i_2 = -0.3 - 0.08 = -0.38 \text{ mA}$

Sources I_o

$i_1 = \frac{V_i - V_1}{R_1} = \frac{0.08V}{1K} = 0.08 \text{ mA}$



"The Op Amp Sink Current" !!!



EXA 4: (A)

Assume ideal Op-Amp.

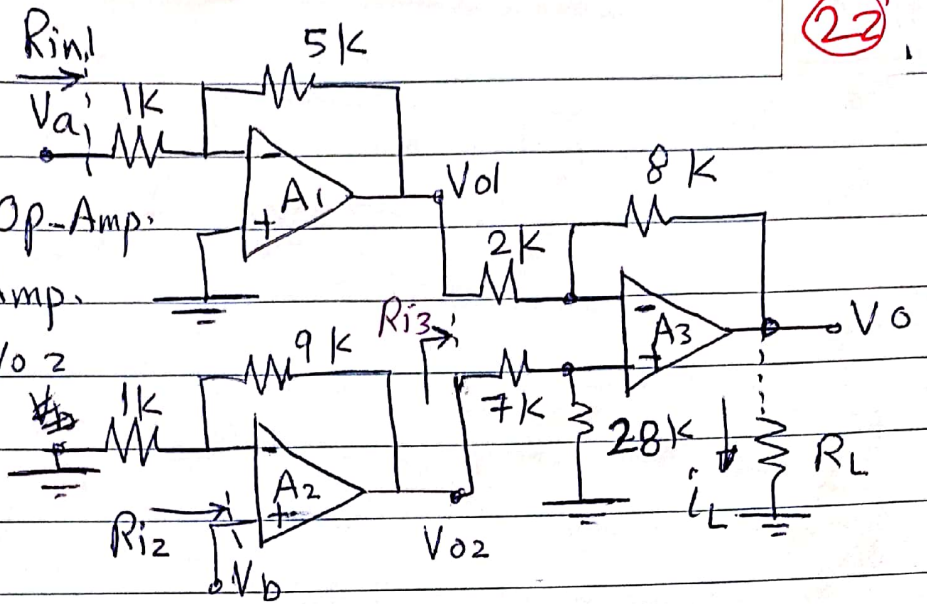
i) Name each Amp.

ii) Calculate V_{o1} , V_{o2}

& V_o in terms of V_a, V_b

iii) Calculate

$R_{in1}, R_{in2}, R_{in3}$?



A1: Inverting Amp, A2: Noninverting Amp.

A3: Difference Amp.

$$V_{o1} = -\frac{5}{1} V_a = -5V_a, \quad V_{o2} = \left(1 + \frac{9}{1}\right) V_b = 10V_b$$

$$V_o = \frac{8}{2} = \frac{28}{7} (V_{o2} - V_{o1}) = 4(10V_b - (-5V_a))$$

$$V_o = 40V_b + 20V_a$$

iii) $R_{in1} = R_1 = 1k\Omega$, $R_{in2} = \infty$, $R_{in3} = 2 + 7 = 9k\Omega$

EXA 4: (B)

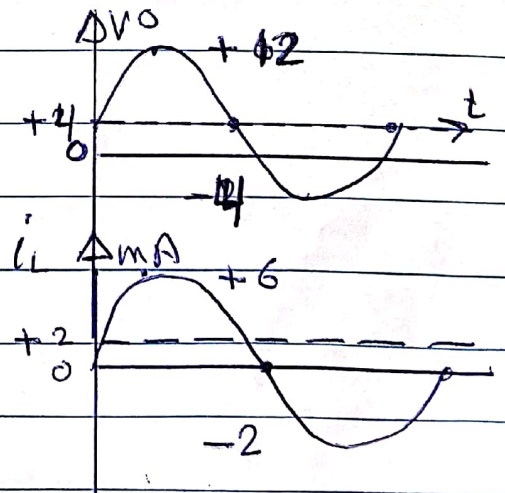
IF $R_L = 2k\Omega$ is connected, $V_a = 0.2V$ d.c

$V_b = 0.2 \sin \omega t$ (V), Calculate V_o , i_L & Draw $V_o(t)$, $i_L(t)$

* Since $V_o = 20V_a + 40V_b = 20 \times 0.2 + 40 \times 0.2 \sin \omega t$

$$\therefore V_o = 4 + 8 \sin \omega t \text{ (V)}$$

$$i_L = \frac{V_o}{R_L} = 2 + 4 \sin \omega t \text{ mA}$$



Frequency Response

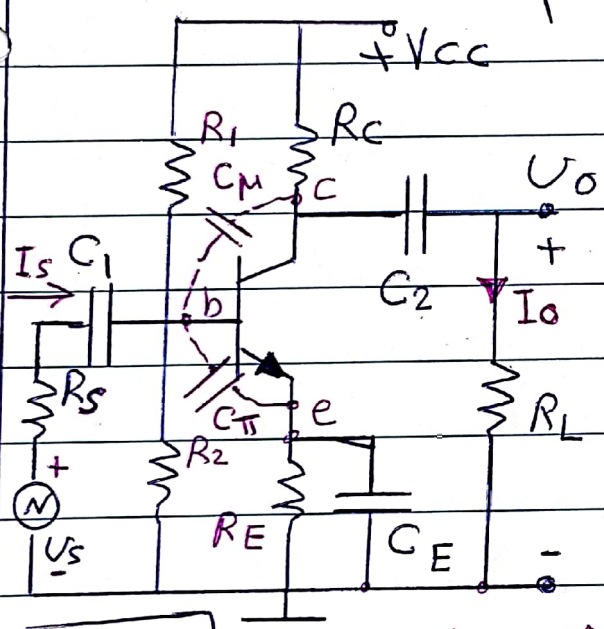
It is a plot of Amp. gain versus frequency

The gain can be Av or AI and can be in:
dB " $Av(dB) = 20 \log \frac{V_o}{V_s}$ and $AI(dB) = 20 \log \frac{I_o}{I_s}$

or (unitless) $Av = \frac{V_o}{V_s}$, $AI = \frac{I_o}{I_s}$

The freq. can be in (Hz) or ω (rad/sec).

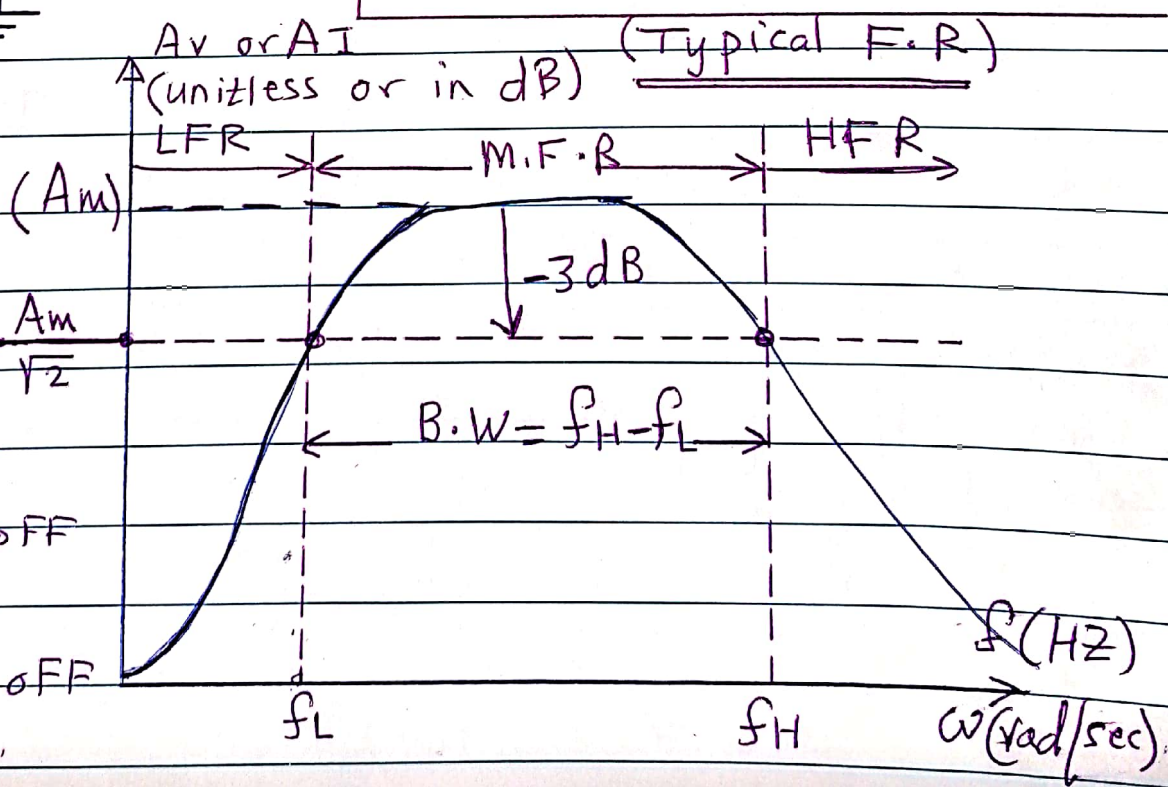
Consider the Amp. shown in Fig. 1 which is C.E BJT Amp. (C_M & C_{π} \rightarrow high-freq. caps).



C_1 & $C_2 \rightarrow$ Coupling Capacitor
 $C_E \rightarrow$ bypass Cap.

Any Amp. has ONE (at least) Coupling Cap (C_1, C_2) or bypass Cap C_E OR two or three has the F.R shown.

Fig. 1



$0.707 A_m = \frac{A_m}{\sqrt{2}}$

Fig. 2

f_L : low-cut off freq.
 f_H : high-cut off freq.

The typical freq. Resp. shown in Fig. 2 has three main regions:

① Low Freq. Regn. (LFR)

This Regn. extends from $(0 \rightarrow f_L)$, in which the gain is frequency dependant such that as freq. increases the gain increases. due to the effect of Coupling Cap. (C_1, C_2) and bypass Cap C_E . These capacitors has high reactances ($X_C = \frac{1}{2\pi fC}$) at low freq. So they effective at low-freq. Regn. This is "why the gain at LFR is frequency dependant"

⊗ * But "why as freq. ↑, the gain ↑" because these Caps. are in the series path of the input signal, so as $f \uparrow, X_C \downarrow$, more signal reaches base, more signal reaches collector and more signal reaches R_L So: as $f \uparrow, X_C \downarrow, V_b \uparrow, V_c \uparrow, V_o \uparrow, A_v \uparrow$. because since $(V_s = V_s \sin 2\pi f t)$ in freq. response we can increase $f \uparrow$ NOT V_s .

The value of " $f_L \rightarrow$ low freq depends on C_1, C_2, C_E . [In LFR: $C_T \& C_M \Rightarrow 0 \cdot C$] $X_{C_T} = X_{C_M} = \infty$]

So we can not neglect the effect of these capacitors.

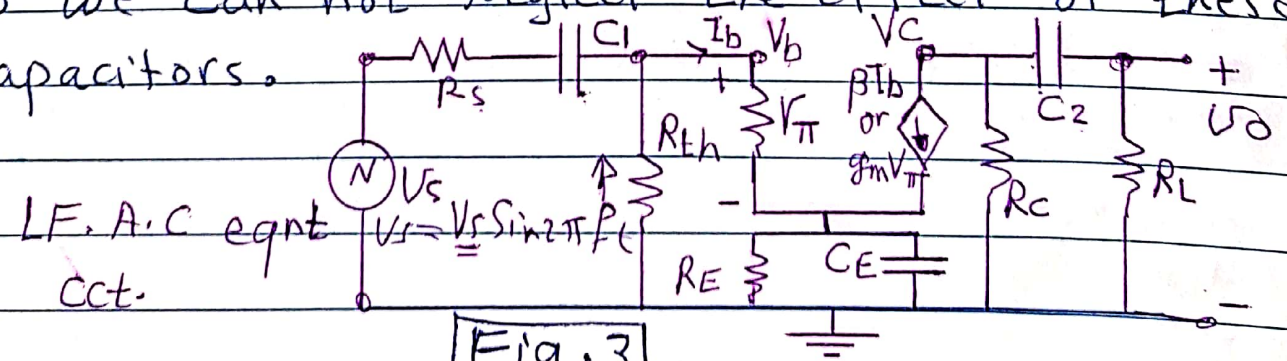


Fig. 3

② Medium freq. Regn "MFR"

This Regn. extends from $(f_L \rightarrow f_H)$ over a bandwidth $(B.W = f_H - f_L)$. This is a useful Regn of Amp. operation. $X_C = \frac{1}{2\pi f C}$

- In this Regn, all Coupling and bypass Caps. are considered short ~~circuits~~ ccts so the Amp. will behave as an pure resistive Amp. with a certain gain A_m . $C_{\pi} \& C_{\mu} \Rightarrow$ are open cct, $X_{C_{\pi}} = X_{C_{\mu}} = \infty$

- In this regn, the gain is freq. independant. "actually all A.C analysis's we did it in this term (BJT Amp. & MOSFET Amp) were in MFR.

- The equivalent cct. in M.F.R is shown below by assuming C_1, C_2 and C_E are short cct.

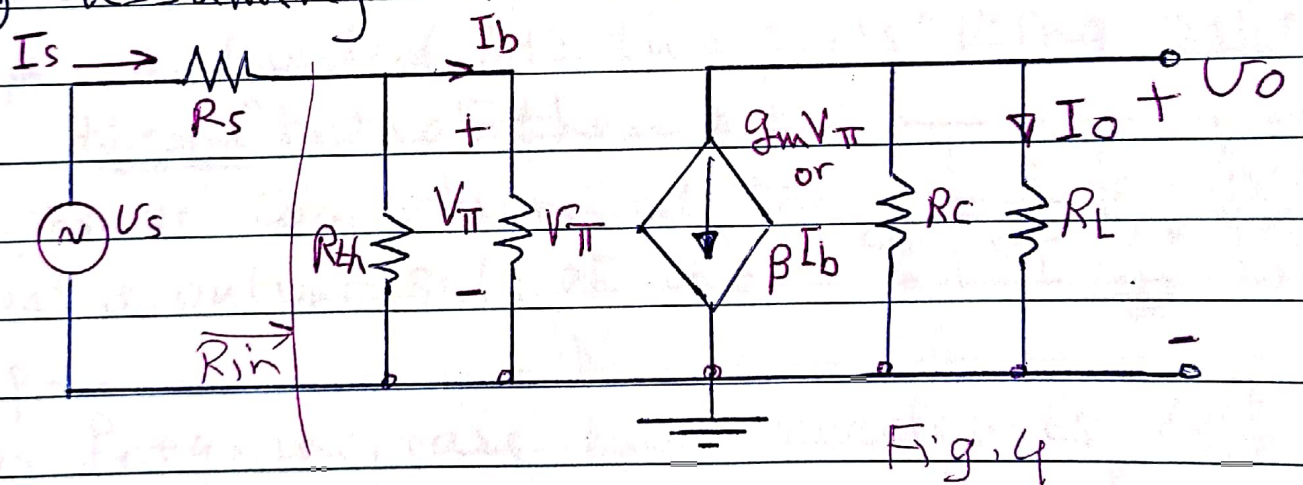


Fig. 4

MF A.C eqnt cct.

$$A_{vm} = \frac{-g_m (R_C \parallel R_L) R_{in}}{R_{in} + R_s}$$

$$R_{in} = R_{\pi} \parallel R_{B1}$$

③ High-Freq. Regn.

- This Regn. extends from $f_H \rightarrow \infty$.
- The gain is frequency dependant such that as $f \uparrow$ the gain decrease.
- In this Regn. C_1, C_2 & C_E are considered short circuit. but new stray capacitances (C_{π} & C_{μ}) appear effectivily (No physical existance) due to physical structure of BJT where:

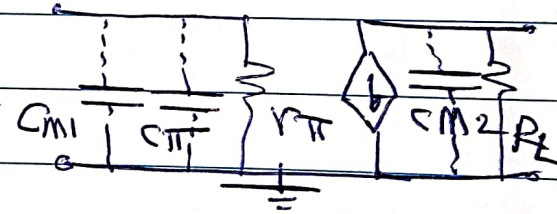
C_{π} : B-E Jn diffusion capacitance (F.w biased Jn) capacitance

C_{μ} : Reversed-biased Jn capacitance.

- These capacitances appear in high-freq. Regn. So the gain is freq. dependant.

- C_{μ} is divided into two parts "Using Miller theorem" both of them are

in parallel connection at the input & output side of the Amp.



- As freq. increase their reactances $X_C \downarrow$ So $(R_L // X_{C_{M2}})$ decrease and $|A_v| = \downarrow g_m (R_L // X_{C_{M2}})$ that is "why as $f \uparrow, A_v \downarrow$ in HFR".

However in this Regn. " C_1, C_2 & $C_E \rightarrow S.C$ " but C_{π} & $C_{\mu} \rightarrow$ effective.

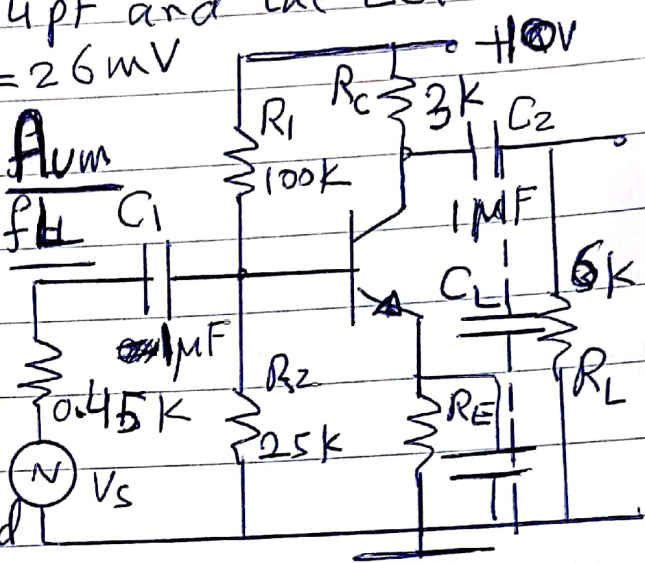
C_{π} & C_{μ} are given in data sheet of BJT

* Sometime C_L is connected in parallel with R_L to control f_H and make it NOT depend on C_{μ} & C_{π} .

Frequency Response Analysis:

EXA: For the cct. shown in Fig.1, the BJT has:
 $\beta = 100$, $C_T = 30pF$, $C_M = 4pF$ and the cct is:
 biased at $I_{CQ} = 2.6mA$, $V_T = 26mV$

- 1) Draw MF eqnt. cct. & Find A_{vm}
- 2) " HF " " " " "
- 3) " HF " " " " "
- 4) Sketch Freq. Response of the Amp. (A_v versus f)
- 5) If $C_L = 0.2nF$ is connected in parallel with R_L recalculate f_H and Comment??

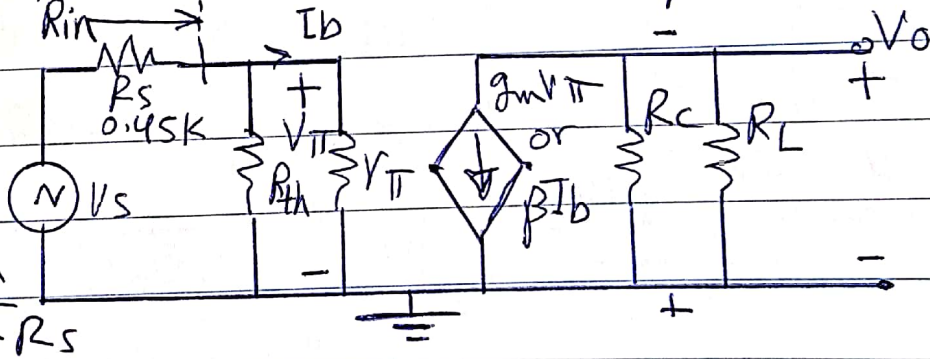


① MF eqnt. cct: $C_1, C_2, C_E \rightarrow S.C$, $C_T, C_M, C_L \rightarrow O.C$

$$A_v = \frac{V_o}{V_s}$$

$$= \frac{V_o}{V_{\pi}} \times \frac{V_{\pi}}{V_s}$$

$$= -g_m(R_C || R_L) \frac{R_{in}}{R_{in} + R_s}$$



$$g_m = \frac{I_{CQ}}{V_T} = \frac{2.6mA}{26mV} = 100 \text{ mA/V}$$

$$r_{\pi} = \frac{\beta V_T}{I_{CQ}} = 1k\Omega$$

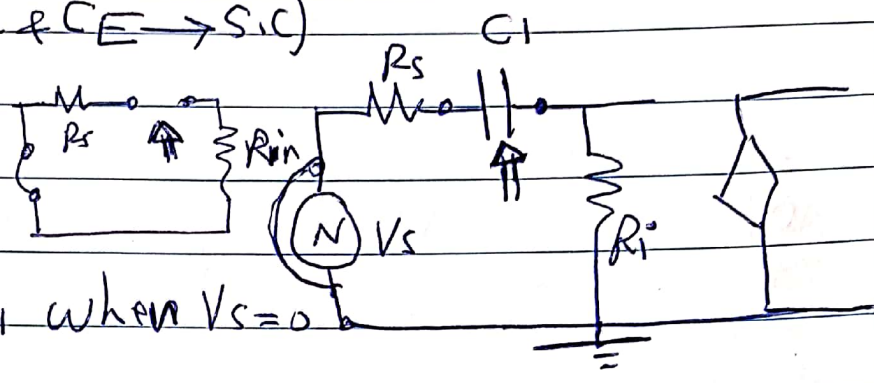
$$R_{in} = r_{\pi} || R_{th} = 1 || (100 || 25) = 0.952k\Omega$$

$$A_v = -100(3 || 6) \frac{0.952k}{(0.45 + 0.952)k} = -136$$

② L.F.R ($C_{\pi}, C_u, C_L \rightarrow$ o.c), C_1, C_2 & $C_E \rightarrow$ exist

(i) Effect of C_1 (C_2 & $C_E \rightarrow$ S.C)

$$f_{L1} = \frac{1}{2\pi C_1 R_{eq1}}$$



R_{eq1} : Rth seen by C_1 when $V_s = 0$

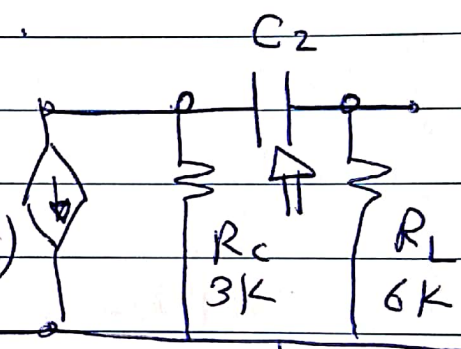
$$R_{eq} = R_{in} + R_s$$

$$\therefore f_{L1} = \frac{1}{2\pi C_1 (R_{in} + R_s)} = \frac{1}{2\pi \times 1 \times 10^{-6} \times (0.45 + 0.952) \text{K}}$$

$$f_{L1} = \frac{10^3}{2\pi \times 1.402} = 113.5 \text{ Hz}$$

(ii) Effect of C_2 (C_1 & $C_E \rightarrow$ S.C)

$$f_{L2} = \frac{1}{2\pi C_2 R_{eq2}}$$



R_{eq2} - Rth seen by C_2 (C_1 & $C_E \rightarrow$ o.c)

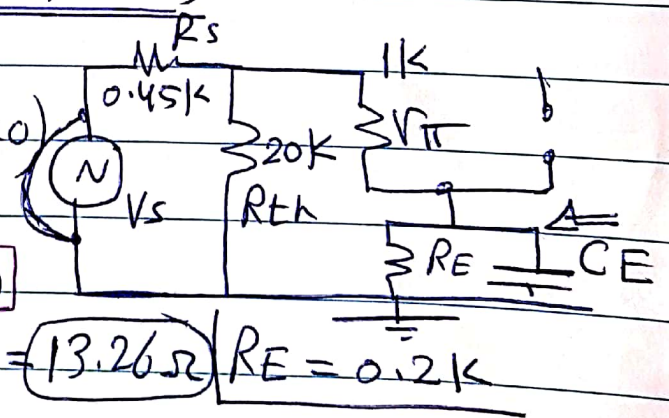
$$R_{eq2} = R_c + R_L = 9 \text{ K}\Omega$$

$$\therefore f_{L2} = \frac{1}{2\pi \times 10^{-6} (9) \times 10^3} = \frac{10^3}{18\pi} = 17.68 \text{ Hz}$$

(iii) Effect of C_E (C_1 & $C_2 \rightarrow$ S.C)

$$f_{L3} = \frac{1}{2\pi C_E R_{eq3}}$$

R_{eq3} = Rth seen by C_3 (when $V_s = 0$)



$$R_{eq} = \left(\frac{R_s \parallel R_{th} + r_{\pi}}{\beta + 1} \right) \parallel R_E$$

$$= \left(\frac{(0.45 \parallel 20) + 1}{101} \right) \parallel 0.2 = 13.26 \Omega \quad R_E = 0.2 \text{ K}$$

$$\therefore f_{L3} = \frac{1}{2\pi \times 2 \times 10^{-5} \times 13.26} = 600 \text{ Hz}$$

The effective f_L is the highest one (600 Hz)

③ High-Freq. Analysis (HFR).

29 ~~34~~

$C_1, C_2, C_E \rightarrow S.C, C_{\mu}, C_{\pi} \& C_L$ are effected.
 * Using Miller Theorem

C_{μ} is divided into two parts:

C_{M1} : between base and ground.

$$C_{M1} = C_{\mu}(1 - k)$$

C_{M2} : between collector & ground

$$C_{M2} = C_{\mu}\left(1 - \frac{1}{k}\right)$$

where $k = \frac{V_c}{V_b} = \frac{V_o}{V_{\pi}} = -g_m R_L$

$$k = -100(3/6) = -200$$

$$\therefore C_{M1} = 4(1 - (-200)) = 804 \text{ pF}$$

$$C_{M2} = 4\left(1 - \frac{1}{(-200)}\right) \approx C_{\mu} = 4 \text{ pF}$$

$$f_{Hi} = \frac{1}{2\pi C_i R_{eqi}}, \quad C_i = C_{\pi} + C_{M1} = 834 \text{ pF}$$

R_{eqi} : Rth seen by C_i (when $V_s = 0$)

$$R_{eqi} = R_s // R_{th} // r_{\pi} = 0.45 // 20 // 1 = 0.305 \text{ k}\Omega$$

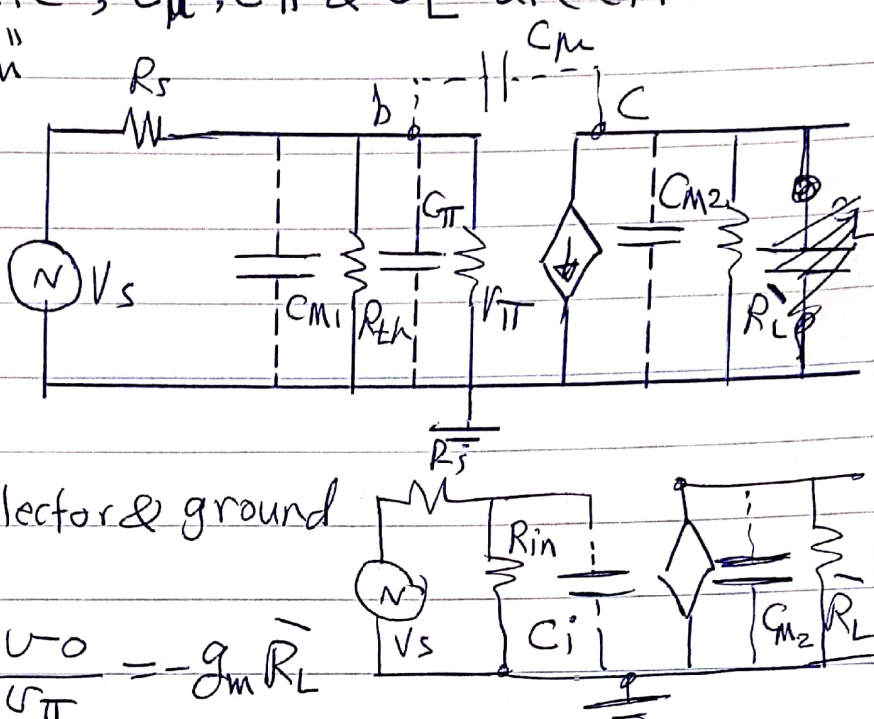
$$\therefore f_{Hi} = \frac{1}{2\pi \times 834 \times 10^{-12} \times 0.305 \times 10^3} = \frac{10^9}{2\pi \times 834 \times 0.305} = 625.7 \text{ K}$$

$$f_{Ho} = \frac{1}{2\pi C_o R_{eqo}}, \quad R_{eqo} = R_L = 2 \text{ k}\Omega, \quad C_o = C_{M2}$$

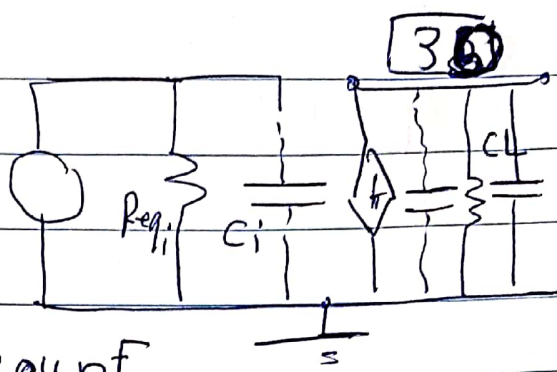
$$\therefore f_{Ho} = \frac{1}{2\pi \times 2 \times 10^3 \times 4 \times 10^{-12}} = \frac{10^9}{16\pi} = 19.9 \text{ MHz}$$

The effective f_H is the lowest value

$$\therefore f_H = 625.7 \text{ KHz}$$



⑤ If $C_L = 0.2 \mu F$ is connected in parallel with R_L , then $C_0 = C_{M2} + C_L$



$$= (4 + 200) \mu F = 204 \mu F$$

$$f_{H0} = \frac{1}{2\pi R_L C_0}$$

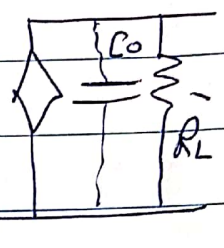
$$= \frac{1}{2\pi \times 10^3 \times 204 \times 10^{-6}}$$

$$= \frac{1}{2\pi \times 10^3 \times 2 \times 204 \times 10^{-6}}$$

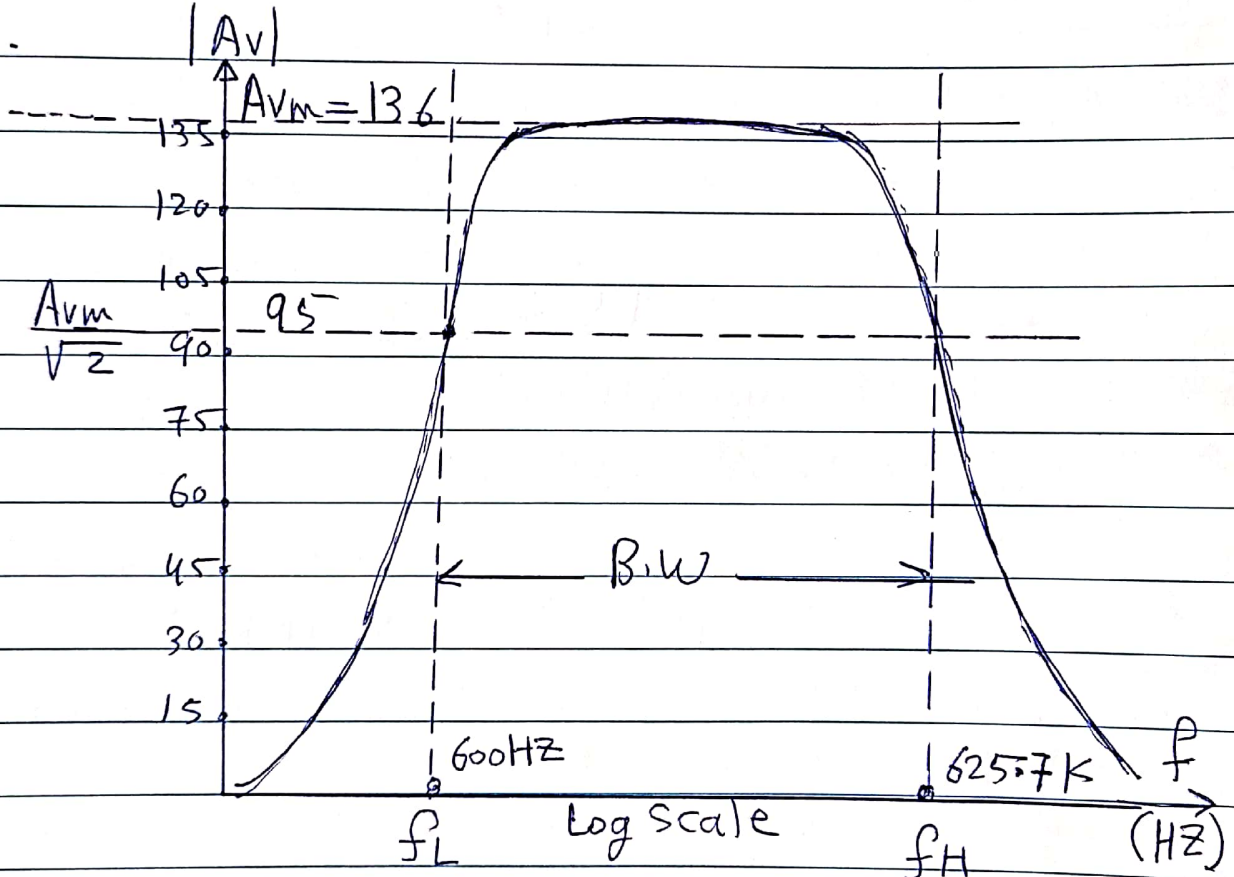
$$f_{H0} = \frac{10^9}{4\pi \times 204} = \frac{10^9}{816\pi}$$

$$= \frac{10^9}{816\pi}$$

$$= 396 \text{ KHz}$$

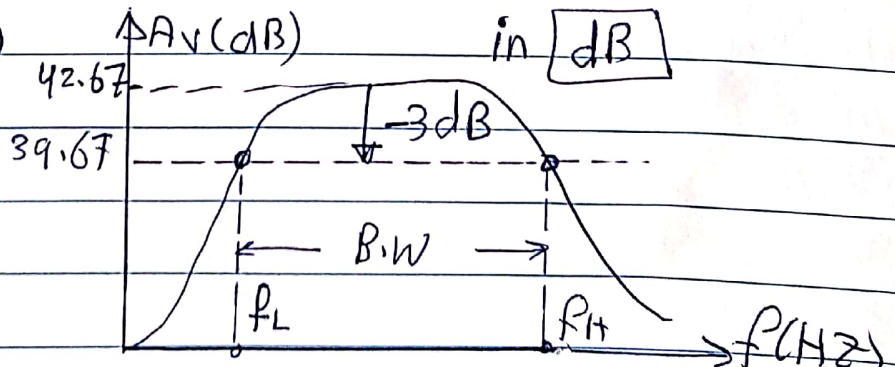


i.e the Value of C_L will control the Value of f_H .



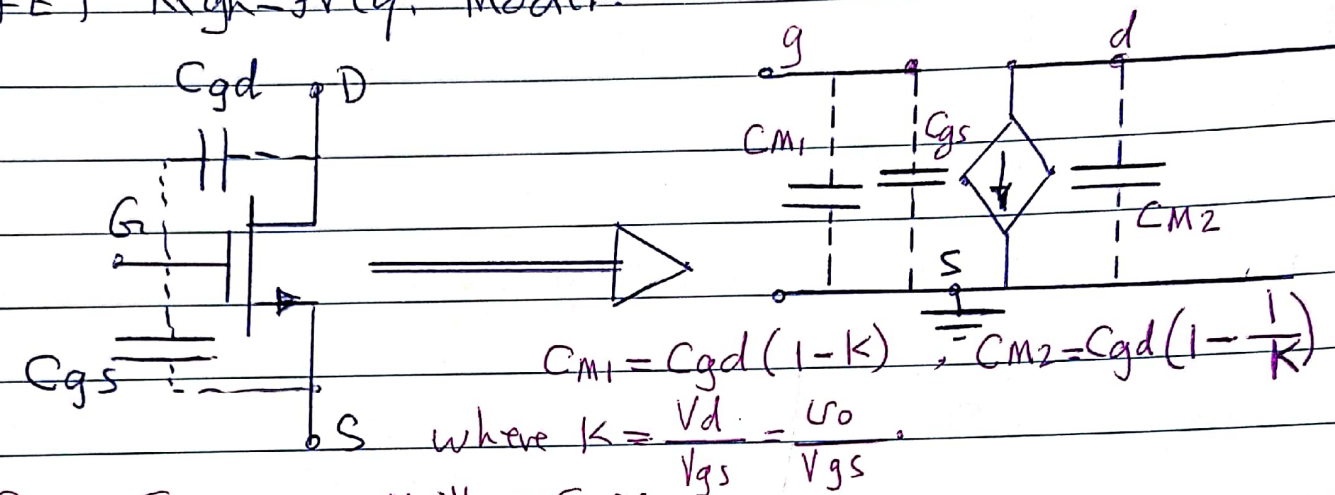
$$B.W = f_H - f_L = 625.7 - 0.6 = 625.1 \text{ KHz}$$

$$A_v(\text{dB}) = 20 \log(13.6) = 42.67 \text{ dB}$$



Frequency Response of FET Amp.

The low-cut-off frequencies are determined by coupling (C_1, C_2) and bypass cap (C_s). While the high-freq. (cut-off) freq. is determined by FET high-freq. cap. and load capacitor (if any).
 FET high-freq. Model:

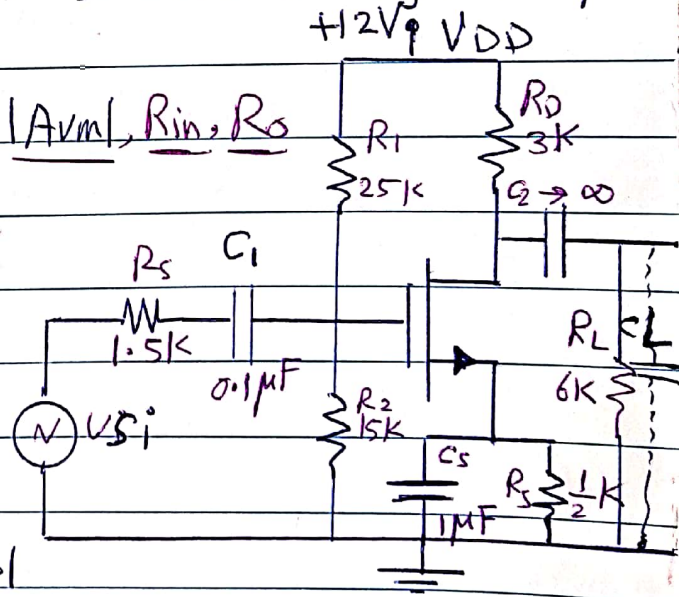


C_{M1} & C_{M2} are Miller Caps.

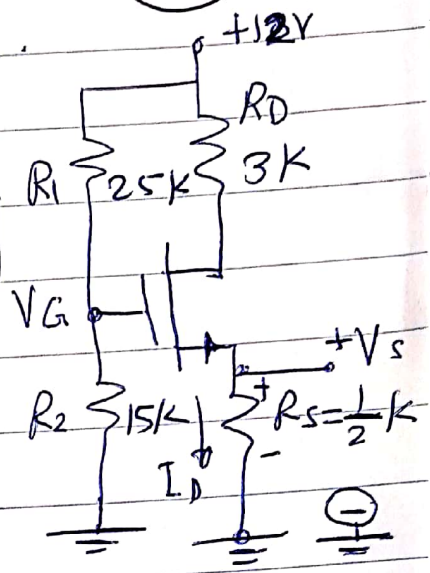
EXAMPLES:

For the cct. shown: (C.S) Amplifier, the MOSFET parameters: $K_n = 1 \text{ mA/V}^2$, $V_{TN} = 2 \text{ V}$, $\lambda = 0$, $C_{gd} = 0.2 \text{ pF}$ and $C_{gs} = 1 \text{ pF}$.

- 1) Draw MF eqnt. cct. and Find $|A_{v_{mid}}|$, R_{in} , R_o
- 2) Draw LF $\leq \leq \leq f_L$
- 3) \leq HF $\leq \leq \leq f_H$
- 4) Sketch freq. Response of Amp (A_v (dB) versus f).
- 5) what must be the value of C_L connected in parallel with R_L required to make $f_H = 500 \text{ KHz}$.



1) We have to find I_D , to find g_m .
 Assume the trans. in Sat. Regn.



$$I_D = K_n (V_{GS} - V_{TN})^2$$

$$V_{GS} = V_G - V_S = \frac{12 \times R_2}{R_1 + R_2} - I_D R_S$$

all cap are 0.c
 $V_S = I_D R_S$

$$V_{GS} = \frac{12 \times 15}{40} - 0.5 I_D = 4.5 - 0.5 I_D$$

$$I_D = \frac{(4.5 - V_{GS})^2}{0.5}$$

$$\frac{4.5 - V_{GS}}{0.5} = 1 (V_{GS} - 2) = V_{GS} - 4 V_{GS} + 4$$

$$0.5 V_{GS}^2 - 2 V_{GS} + 2 - 4.5 + V_{GS} = 0$$

$$0.5 V_{GS}^2 - V_{GS} - 2.5 = 0$$

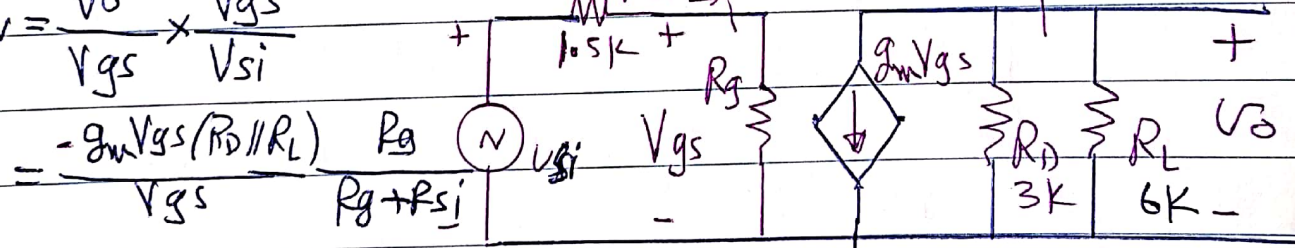
$$V_{GS} = \frac{1 \pm \sqrt{1+5}}{2 \times 0.5} = \frac{1 \pm \sqrt{6}}{1} = \boxed{3.45V} \text{ or } \boxed{-1.45}$$

$$I_D = \frac{(4.5 - 3.45)^2}{0.5} = \boxed{2.1 \text{ mA}}$$

$$g_m = 2 \sqrt{K_n I_D} = 2 \sqrt{1 \times 2.1} = \boxed{2.9 \text{ mA/V}}$$

* In MF Regn: $C_1, C_2, C_S \rightarrow$ S.C, $C_{GS} \& C_{GD} \rightarrow$ 0.c

$$A_v = \frac{V_o}{V_{gs}} \times \frac{V_{gs}}{V_{si}}$$



$$= \frac{-g_m V_{gs} (R_D \parallel R_L)}{V_{gs}} \frac{R_g}{R_g + R_{si}}$$

$$R_g = R_1 \parallel R_2 = 9.375 \text{ k} = R_{in}, \quad R_o = R_D = 3 \text{ k}$$

$$A_v = -g_m (R_D \parallel R_L) \frac{R_g}{R_g + R_{si}} = -2.9(2) \frac{9.375 \text{ k}}{(1.5 + 9.375) \text{ k}}$$

$$A_{vm} = -5 \Rightarrow |A_{vm}| = 5$$

$$|A_{vm}|(\text{dB}) = 20 \log 5 = 13.98 \text{ dB}$$

② Low-Freq. Regn.: $C_1, C_s \rightarrow$ exist, $C_2 \rightarrow$ S.C because

C_2 is ∞ in this example. $C_{gd}, C_{gs} \rightarrow 0.C$
 Effect of C_1 ($C_s \rightarrow$ S.C)

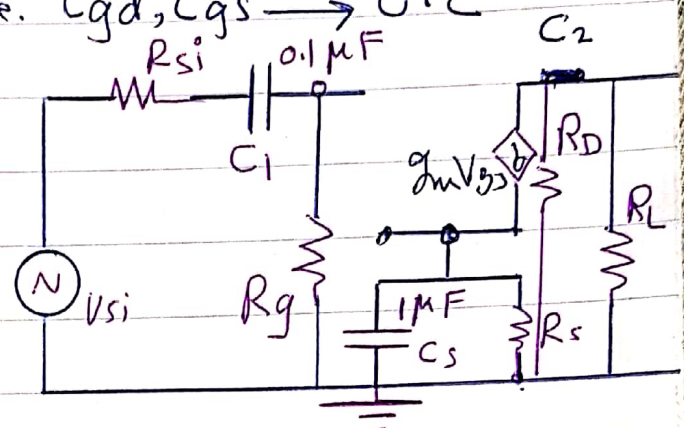
$$f_{L1} = \frac{1}{2\pi C_1 (R_{si} + R_g)}$$

$$= \frac{1}{2\pi \times 1 \times 10^{-7} \times 10.875 \times 10^3}$$

$$f_{L1} = \boxed{146 \text{ Hz}}$$

$$f_{L2} = \frac{1}{2\pi R_s C_s} = \frac{1}{2\pi \times 1 \times 10^{-6} \times 0.5 \times 10^3} = \boxed{318 \text{ Hz}}$$

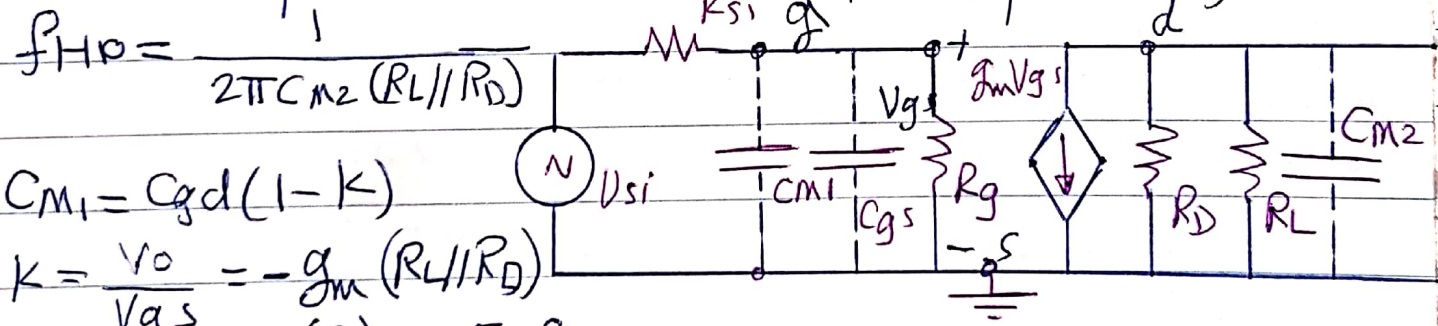
$$f_{L3} = \frac{1}{2\pi C_2 (R_L + R_D)} = \frac{1}{2\pi \times \infty \times 9 \times 10^3} = \boxed{0 \text{ Hz}}$$



∴ f_L (effective) is the highest value = 318 Hz.

③ High-Freq. Regn.: ($C_1, C_2, C_s \rightarrow$ S.C) and C_{gd} & C_{gs} are effective. and C_L (if any)

The HF eqn. cct. is shown (H.F. eqnt. cct).



$$C_{M1} = C_{gd}(1 - k)$$

$$k = \frac{V_o}{V_{gs}} = -g_m (R_L // R_D)$$

$$= -2.9(2) = -5.8$$

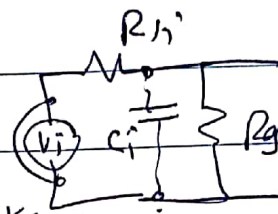
$$C_{M2} = C_{gd} \left(1 - \frac{1}{(-5.8)}\right) = \boxed{0.2345 \text{ pF}}$$

$$f_{H0} = \frac{1}{2\pi \times 0.2345 \times 10^{-12} \times 2 \times 10^3} = \boxed{339.349 \text{ MHz}}$$

$$f_{Hi} = \frac{1}{2\pi C_i R_{eq}}, \quad R_{eq} = R_{si} // R_g = 1.355 \text{ K}\Omega$$

$$C_i = C_{M1} + C_{gs}, \quad C_{M1} = C_{gd}(1 - k) = 0.2(1 + 5.8) = 1.36 \text{ pF}$$

$$C_i = 1 + 1.36 = 2.36 \text{ pF}$$



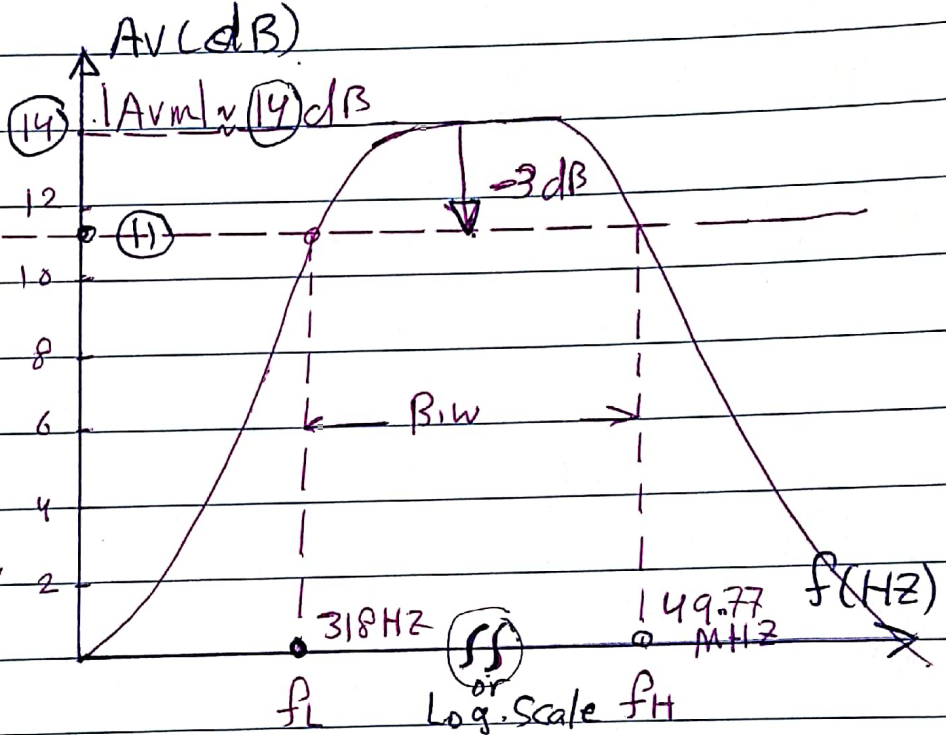
$$\frac{(R_s // R_L)}{(R_{eq i})} = 1.355 \times 10^3$$

$$f_{Hi} = \frac{1}{2\pi \times 2.36 \times 10^{-12} \times 1.355 \times 10^3} = 49.770 \text{ MHz}$$

f_H (effective) is the lowest Value = 49.77 MHz

(4) Freq. Resp.

$$|A_v| = [|A_{vm}| - 3] \text{ dB}$$



$$B.W = f_H - f_L$$

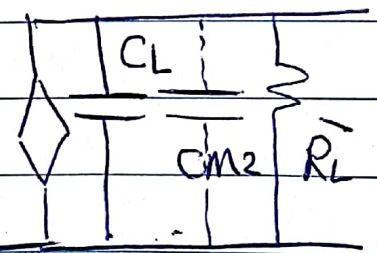
For $f_H \gg f_L$

$$B.W \approx f_H$$

$$\approx 49.77 \text{ MHz}$$

5) If $C_L = ?$ required to make $f_H = 500 \text{ KHz}$

$$f_H = \frac{1}{2\pi R_L (C_L + C_{M2})}$$



$$C_L + C_{M2} = \frac{1}{2\pi f_H R_L}$$

$$C_L + C_{M2} = \frac{1}{2\pi \times 500 \times 10^3 \times 2 \times 10^3} = \frac{10^9}{2\pi} = 159.155 \text{ pF}$$

$$C_L = 159.155 - 0.2345 = 158.920 \text{ pF}$$

Frequency Resp. of C.B. Amp.

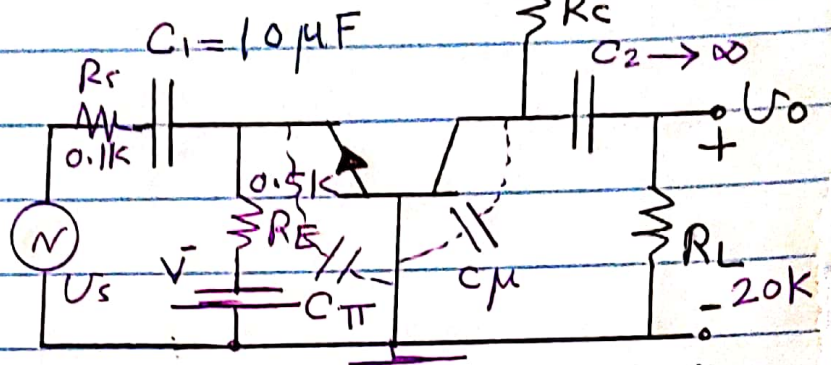
40

1) Draw MF eqnt cct and Find $|A_{vm}|$.

2) Draw LF eqnt. cct. and Find f_L

3) Draw HF eqnt. cct. & Find f_H .

4) Sketch (Bode Plot of the Amp.)



$C_{\pi} = 30\text{pF}, C_{\mu} = 5\text{pF}$
 $\beta = 130, I_{CQ} = 1\text{mA}$
 $V_T = 26\text{mV}$

1) MFR (all $C_1, C_2 \rightarrow \text{S.C.}$) & ($C_{\pi}, C_{\mu} \rightarrow \infty$).

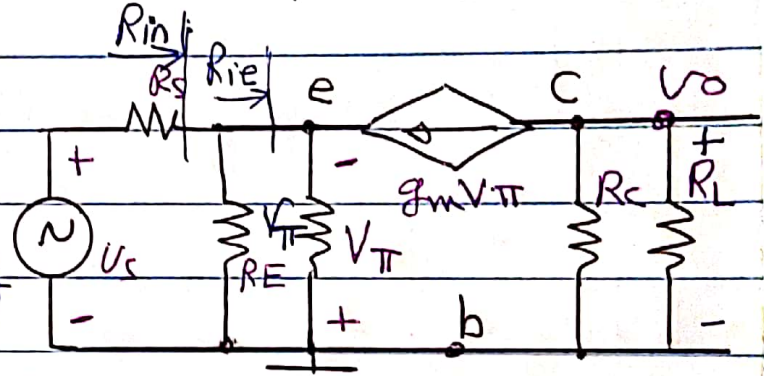
$$A_V = \frac{V_o}{V_s} \times \frac{V_{\pi}}{V_{\pi}}$$

$$V_o = -g_m V_{\pi} (R_c \parallel R_L)$$

$$\frac{V_o}{V_{\pi}} = -g_m (R_c \parallel R_L)$$

$$\frac{V_{\pi}}{V_s} = \frac{-R_{in}}{R_{in} + R_s}$$

$V_{\pi} = \frac{-V_s R_{in}}{R_{in} + R_s}$



$g_m = \frac{I_{CQ}}{V_T} = \frac{1\text{mA}}{26\text{mV}} = 38.5\text{mA/V}$ M.F. eqnt. cct.

$R_{in} = R_{ie} \parallel R_E, R_{ie} = \frac{V_{\pi}}{I_{CQ}}$
 $R_{in} = 0.5 \parallel \left(\frac{2.6}{130+1} \right) = 19.23 \Omega$

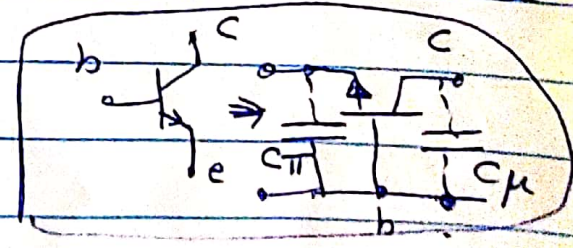
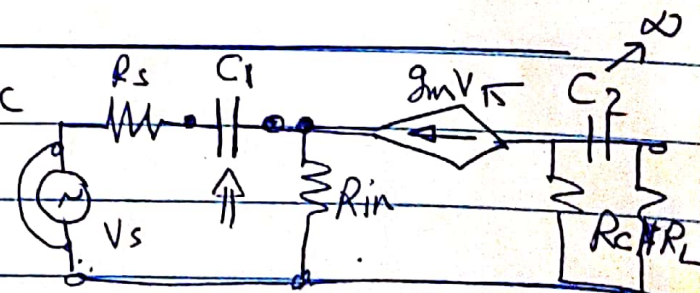
$A_{vm} = -g_m (R_c \parallel R_L) \frac{R_{in}}{R_{in} + R_s} = 38.5 \left(\frac{5 \parallel 20}{19.2 + 0.1} \right)$
 $A_{vm} = 24.8$

2) LFR: $C_{\pi}, C_{\mu} \rightarrow \text{O.C.}, C_2 \rightarrow \text{S.C.}$

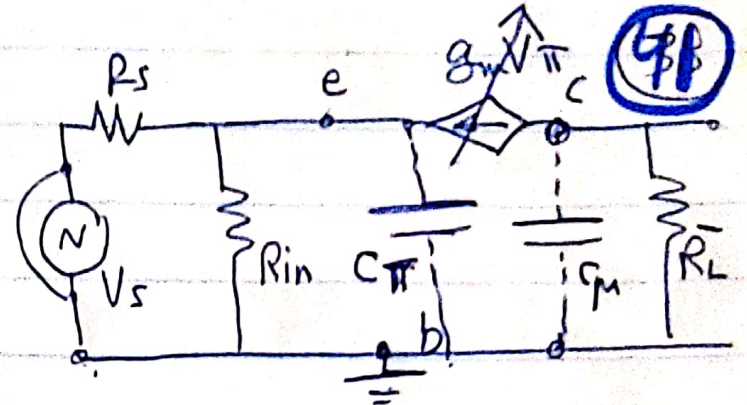
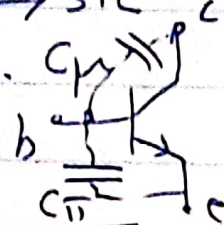
$$f_L = \frac{1}{2\pi C_1 (R_s + R_{in})}$$

$$= \frac{1}{2\pi (0.1 + 0.0192) \times 10 \times 10^{-6}}$$

$$= 133.5\text{Hz}$$



3) HFR: $C_1, C_2 \rightarrow S.C$
 C_{π}, C_{μ} exist.

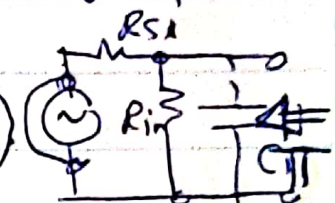


$$f_{Hi} = \frac{1}{2\pi C_{\pi} R_{eq}}$$

$R_{eq} = R_{th}$ seen by C_{π} when $V_s = 0$, $V_{\pi} = 0$, $g_m V_{\pi} \rightarrow 0$.

$$R_{eq} = R_s // R_{in} = 0.1 // 0.0192 = 16 \Omega$$

$$\therefore f_{Hi} = \frac{1}{2\pi \times 16 \times 30 \times 10^{-12}} = 331.572 \text{ MHz}$$



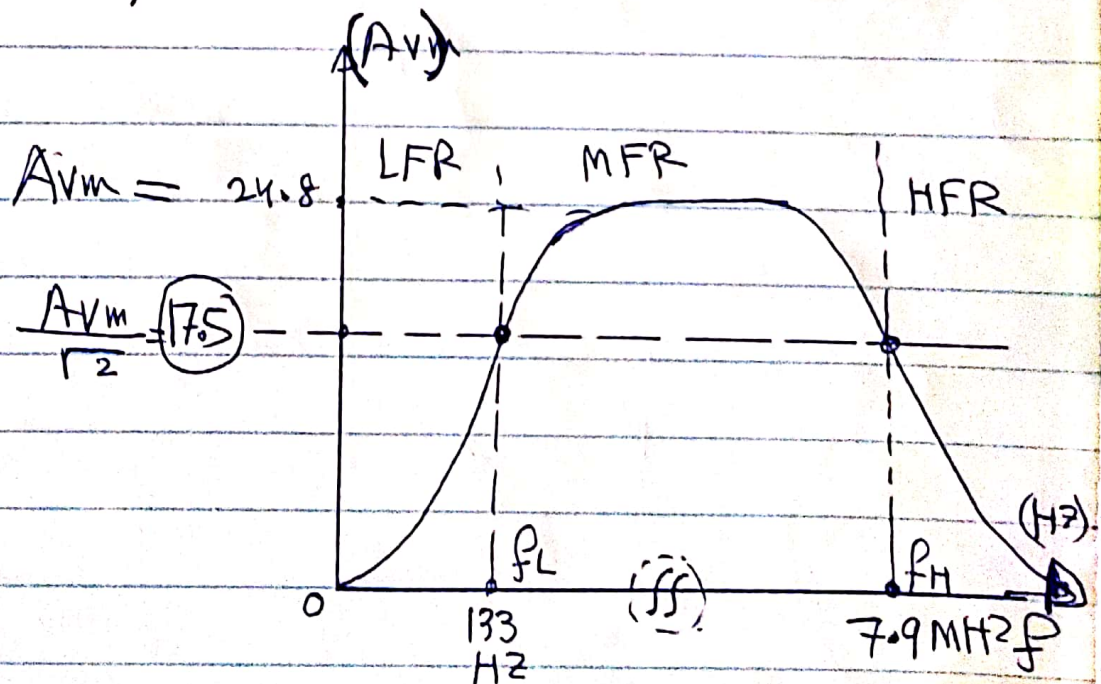
$$f_{Ho} = \frac{1}{2\pi C_{\mu} R_L}, \quad R_L = R_C // R_L = 5 // 20 = 4 \text{ k}\Omega$$

$$f_{Ho} = \frac{1}{2\pi \times 5 \times 4 \times 10^3 \times 10^{-12}} = 7.9577 \text{ MHz}$$



$\therefore f_H = 7.9577 \text{ MHz}$ which is due to C_{μ} ?

the B.W = $f_H - f_L = 7.9577 \text{ MHz} - 133 \text{ Hz} \approx 7.9577 \text{ MHz}$
 (\therefore B.W for C.B) B.W for C.E



Frequency Response of C.G. Amp.

Given: $V_{TN} = 1V, K_n = 1 \text{ mA/V}^2, \lambda = 0$

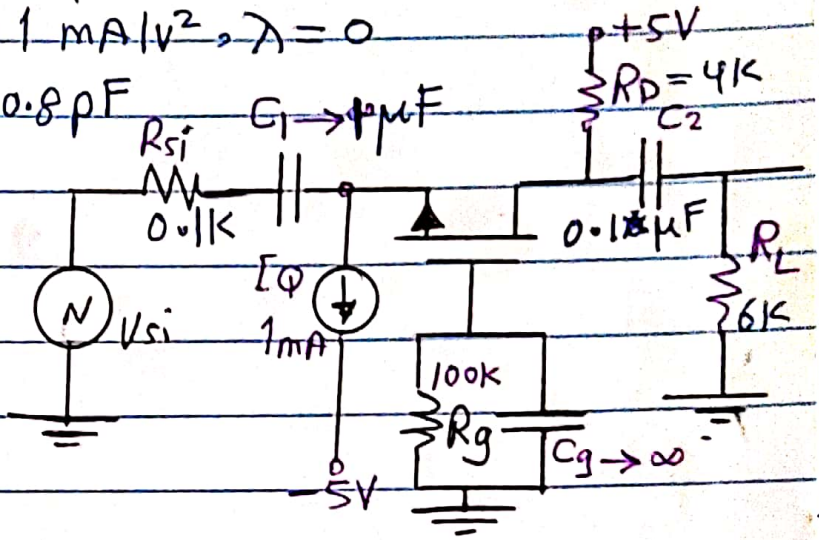
$C_{gs} = 5 \text{ pF}, C_{gd} = 0.8 \text{ pF}$

1) Draw MF eqnt. cct. & Find A_{vm} .

2) Draw LF eqnt. cct. and Determine f_L

3) Draw HF eqnt. cct. & Determine f_H .

4) Sketch Freq. Resp. of Amp. (A_v (dB) versus Freq)



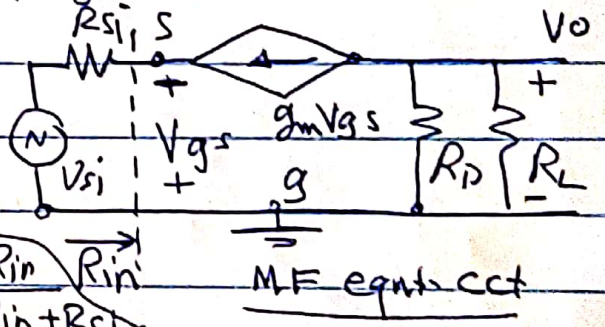
1) MFR: $C_1, C_2, C_g \rightarrow S.C, C_{gs} \text{ \& } C_{gd} \rightarrow O.C$

$I_D = I_S = 1 \text{ mA}$

$g_m = 2\sqrt{K_n I_D} = 2 \text{ mA/V}$

$R_{in} = R_{ig} = \frac{1}{g_m} = \frac{1}{2 \times 10^{-3}} = 0.5 \text{ k}\Omega$

$A_{vm} = \frac{V_o}{V_{gs}} \times \frac{V_{gs}}{V_{si}} = \frac{g_m V_{gs} (R_D || R_L) R_{in}}{V_{gs} (R_{in} + R_{si})}$



MF eqnt. cct

$\therefore A_{vm} = g_m (R_L || R_D) \frac{R_{in}}{R_{in} + R_{si}} = 2 (4 || 6) \frac{0.5}{0.5 + 0.1} = 4$

2) LFR: $C_{gs} \text{ \& } C_{gd} = O.C (C_1, C_2, C_g \rightarrow \text{EXIST})$

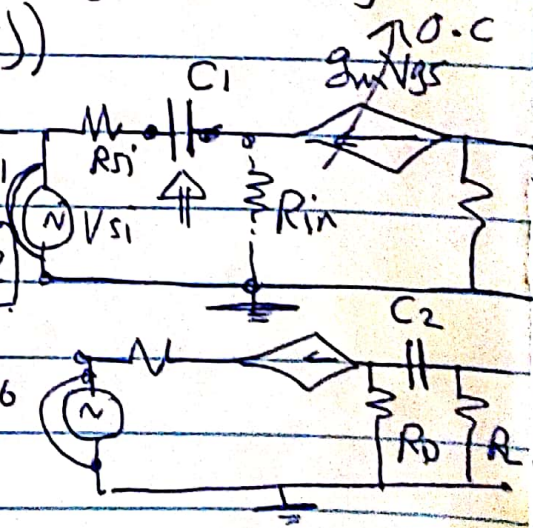
(i) Effect of (C_1) ($C_2 \text{ \& } C_g \rightarrow S.C$)

$f_{L1} = \frac{1}{2\pi C_1 (R_{si} + R_{in})} = \frac{1}{2\pi (0.1 + 0.5) \text{ k}\Omega \times 0.1 \mu\text{F}}$

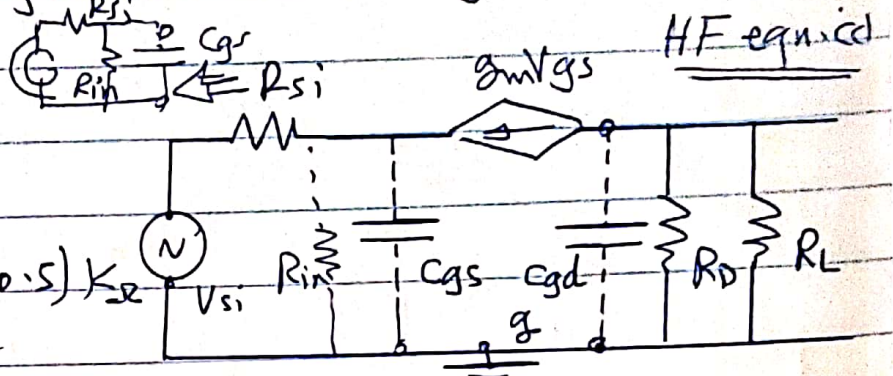
$f_{L1} = \frac{1}{2\pi \times 0.6 \times 10^3 \times 1 \times 10^{-6}} = 265 \text{ Hz}$

$f_{L2} = \frac{1}{2\pi C_2 (R_L + R_D)} = \frac{1}{2\pi \times 10 \times 10^3 \times 10 \times 10^{-6}}$

$f_{L2} = 159 \text{ Hz} \text{ \& } f_L = 265 \text{ Hz}$



3) HFR: $C_1, C_2, C_g \rightarrow S, C, C_{gs} \& C_{gd} \rightarrow$ exist.



$$f_{Hi} = \frac{1}{2\pi C_{gs} R_{eq}}$$

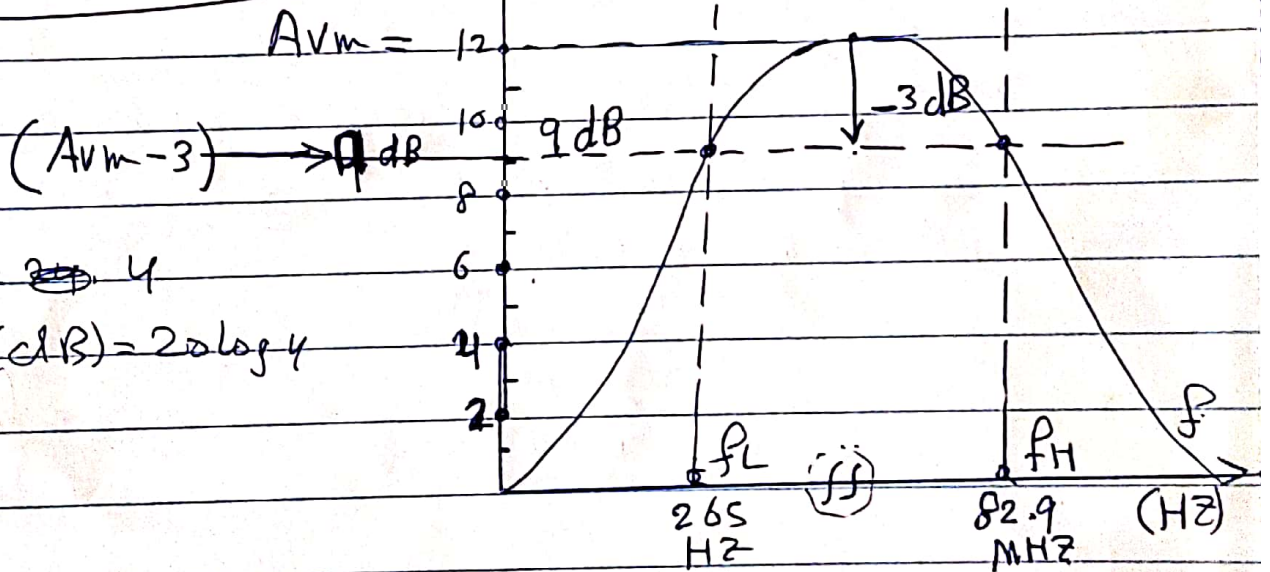
$$R_{eq} = R_{si} \parallel R_{in} = (0.1 \parallel 10.5) \text{ k}\Omega = 83.3 \Omega$$

$$f_{Hi} = \frac{1}{2\pi \times 5 \times 10^{-12} \times 83.3} = 382.124 \text{ MHz}$$

$$f_{Ho} = \frac{1}{2\pi C_{gd} (R_D \parallel R_L)} = \frac{1}{2\pi \times 0.8 \times 10^{-12} \times (4 \parallel 6) \times 10^3} = 10^9 \times 12.0637$$

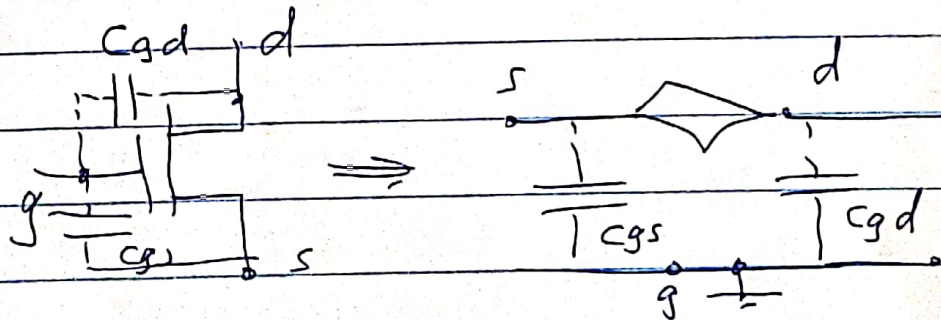
$$f_{Ho} = 82.9 \text{ MHz}$$

$$f_H = 82.9 \text{ MHz}$$



$$A_{vm} = 4$$

$$A_{vm}(\text{dB}) = 20 \log 4$$



Freq. Resp. For C.C & C.D Amp

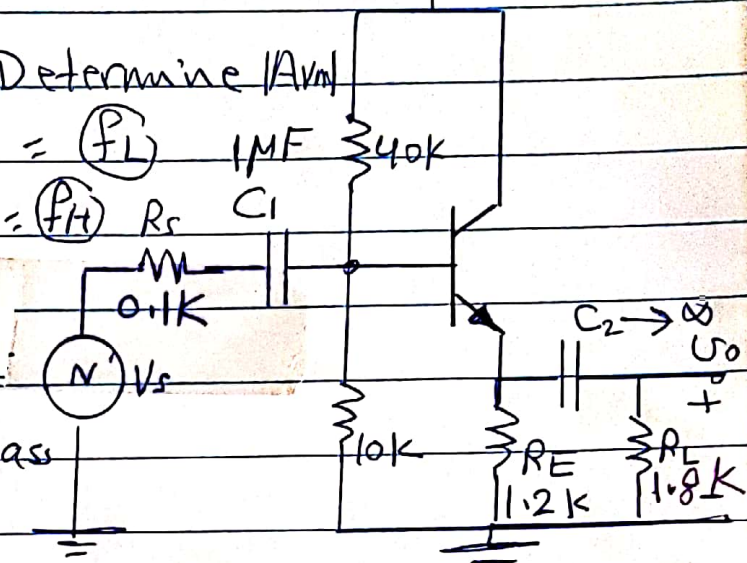
(Emitter follower & Source) 110V

EXA: For the cct. shown,

1) Draw MF eqnt. cct. & Determine $|A_{vm}|$

2) = LF = = = (FL) $1\mu F$ $40k$

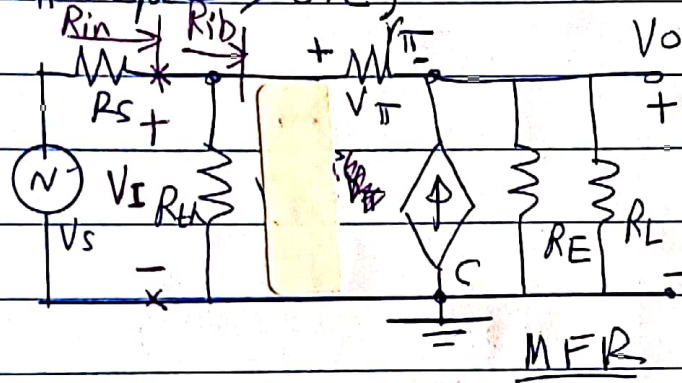
3) = HF = = = (FH) R_s C_1



① MFR: all coupling & bypass

$(C_1, C_2, C_E \rightarrow S.C)$

$(C_{\pi}, C_{\mu} \rightarrow O.C)$



$\beta = 100, V_{BE} = 0.7V, \lambda_A = \infty$
 $C_{\pi} = 40pF, C_{\mu} = 5pF$
 $I_{EQ} = 1mA, V_T = 26mV$

$R_L = 10k, R_E = 1.2k$

$$A_v = \frac{V_o}{V_s} = \frac{V_o}{V_{I}} \times \frac{V_{I}}{V_s}$$

$$\frac{V_o}{V_{I}} = \frac{(\beta+1)(R_L || R_E)}{V_T + (\beta+1)(R_L || R_E)}, \frac{V_{I}}{V_s} = \frac{R_{in}}{R_s + R_{in}}$$

$$R_{in} = R_{th} || R_{ib}, R_{ib} = (\beta+1)(R_L || R_E) + r_{\pi}$$

$$R_{ib} = 2.6 + 101(1.2 || 1.8) = 75.32 k\Omega$$

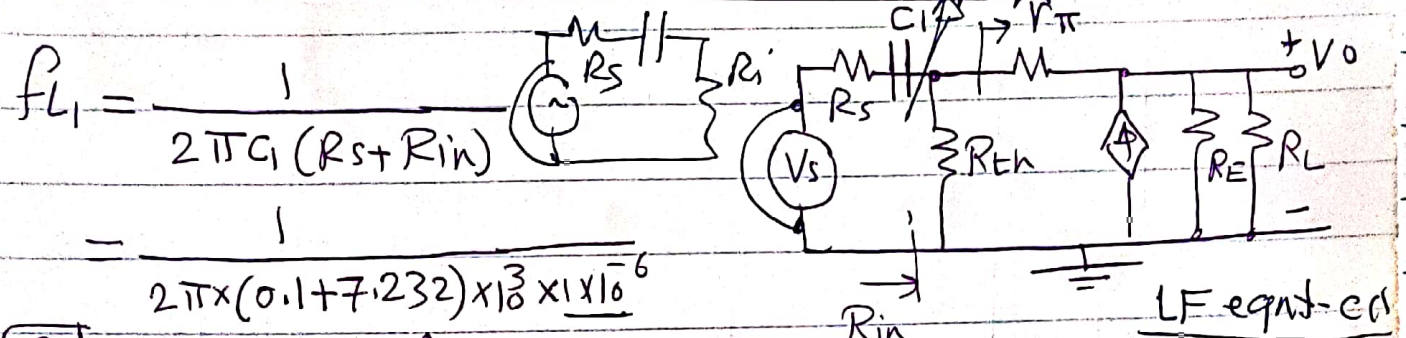
$$R_{th} = 40 || 110 = 8 k\Omega \therefore R_{in} = 8 || 75.32 k = 7.232 k\Omega$$

$$A_{vm} = \frac{(\beta+1)(R_L || R_E)}{V_T + (\beta+1)(R_L || R_E)} \times \frac{R_{in}}{R_{in} + R_s}$$

$$A_v = \frac{101 \times 0.72}{75.32} \times \frac{7.232}{0.1 + 7.232} = 0.952$$

$$r_{\pi} = \frac{\beta V_T}{I_{CQ}} = \frac{100 \times 26mV}{1mA} = 2.6 k\Omega$$

② LFR: $C_{\pi} \text{ \& } C_{\mu} \rightarrow 0, C_2 \rightarrow \text{Short}$



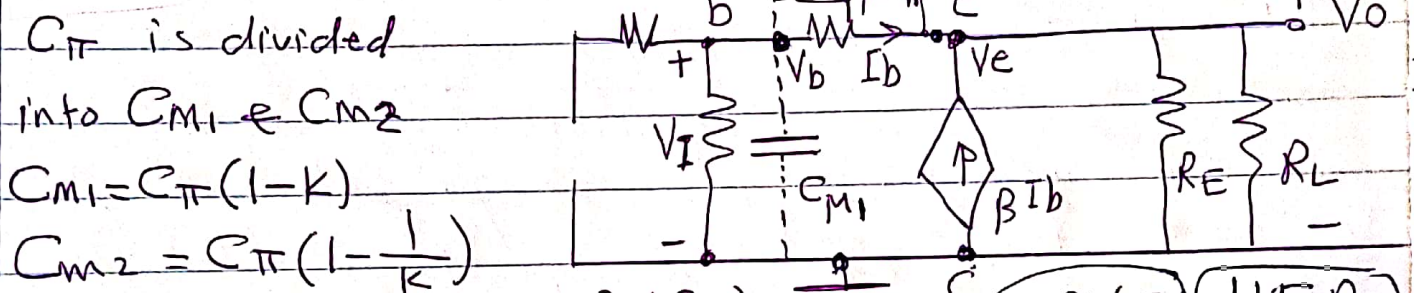
$$f_L = \frac{1}{2\pi C_1 (R_s + R_{in})}$$

$$= \frac{1}{2\pi \times (0.1 + 7.232) \times 10^3 \times 1 \times 10^{-6}}$$

$f_L = 21.7 \text{ Hz}$

$R_{in} = 7.232 \text{ K}\Omega$

③ HFR: $C_1, C_2 \rightarrow S, C_{\pi}, C_{\mu} \Rightarrow \text{effective}$



C_{π} is divided into $C_{M1} \text{ \& } C_{M2}$

$$C_{M1} = C_{\pi} (1 - k)$$

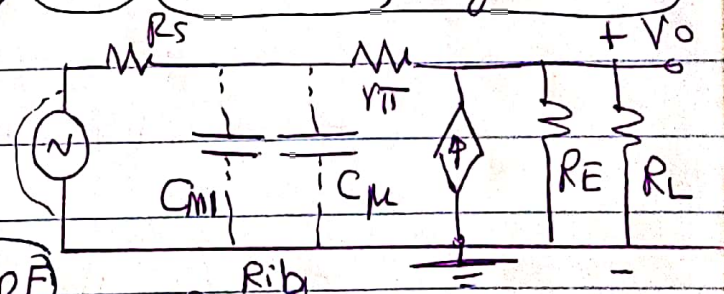
$$C_{M2} = C_{\pi} (1 - \frac{1}{k})$$

$$k = \frac{V_e}{V_b} = \frac{V_o}{V_i} = \frac{(\beta + 1)(R_L \parallel R_E)}{V_{\pi} + (\beta + 1)(R_E \parallel R_L)} = 0.965$$

$$C_{M1} = 40 (1 - 0.965) = 1.4 \text{ pF}$$

$$C_{M2} = 40 (1 - \frac{1}{0.965}) \Rightarrow \text{ve value} \Rightarrow \text{Neglected}$$

$$f_{Hi} = \frac{1}{2\pi C_i R_{eq}}$$



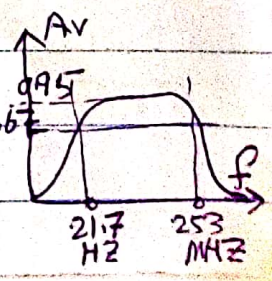
$$C_i = C_{\mu} + C_{M1} = 5 + 1.4 = 6.4 \text{ pF}$$

$$R_{eq} = (R_s \parallel R_{th}) \parallel \frac{R_{in}}{(\beta + 1)} \parallel (R_E \parallel R_L) + R_{\pi}$$

$$= (0.1 \parallel 8) \parallel \frac{101 \times 0.72}{2.6} + 2.6$$

$$= 98.7 \Omega \parallel 75.32 \text{ K} \approx 98 \Omega$$

$$f_H = \frac{1}{2\pi \times 98 \times 6.4 \times 10^{-12}} = 253.754 \text{ MHz}$$



Frequency Response of C.D. Amp.

43

Given: $K_n = 0.5 \text{ mA/V}^2$, $V_{TN} = 2 \text{ V}$, $\lambda = 0$

$C_{gs} = 5 \text{ pF}$, $C_{gd} = 1 \text{ pF}$

1) Calculate V_{GS} , I_D , V_{DS} .

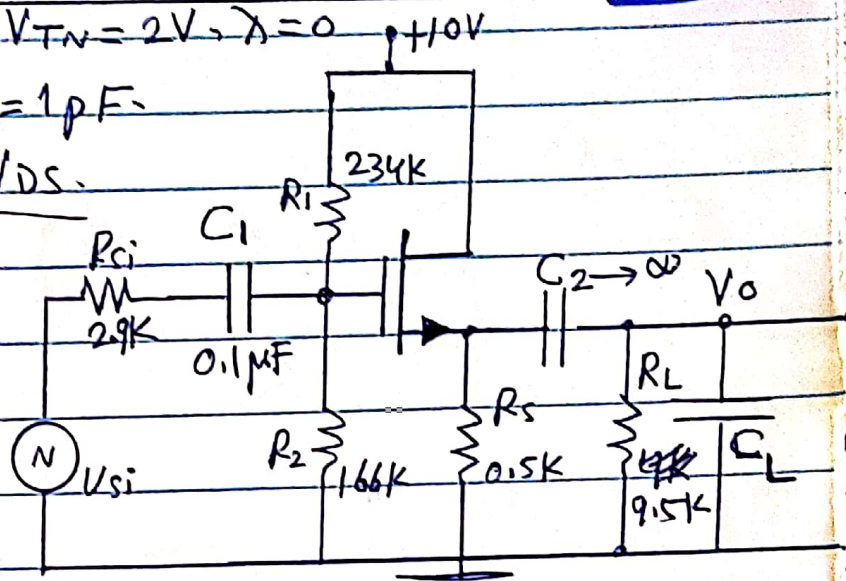
2) Draw MF eqnt. cct. and find A_{vm} ?

3) Determine f_L ?

4) Determine f_H ?

5) Sketch Freq. Resp.

6) Calculate C_L which makes $f_H = 1.5 \text{ MHz}$.



① For D.C Analysis, all Cap. \rightarrow O.C

Assume the MOSFET in Sat.

$$I_D = K_n (V_{GS} - V_{TN})^2 \quad \text{--- ①}$$

$$V_{GS} = V_G - V_S = 10 \times \frac{166}{400} - 0.5 I_D$$

$$4.15 - 0.5 I_D = V_{GS} \quad \text{--- ②}$$

$$I_D = \frac{4.15 - V_{GS}}{0.5} = 0.5 (V_{GS} - 4)^2$$

$$4.15 - V_{GS} = 0.25 V_{GS}^2 - V_{GS} + 4$$

$$0.25 V_{GS}^2 - V_{GS} + V_{GS} - 3.15 = 0$$

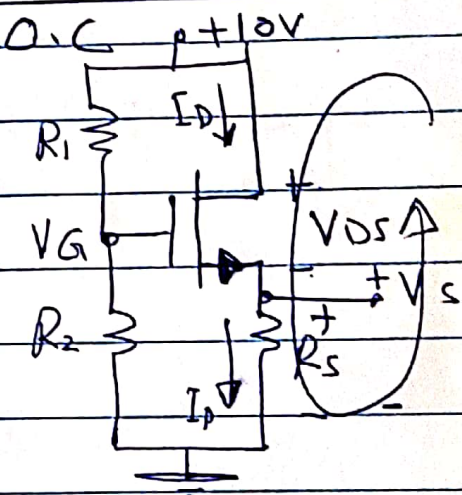
$$V_{GS} = \frac{1 \pm \sqrt{3.15}}{0.25} = 3.45 \text{ V} \quad \text{--- } V_{GS} = 3.55 \text{ V}$$

$$I_D = \frac{(4.15 - 3.55)^2}{0.5} = 1.2 \text{ mA}$$

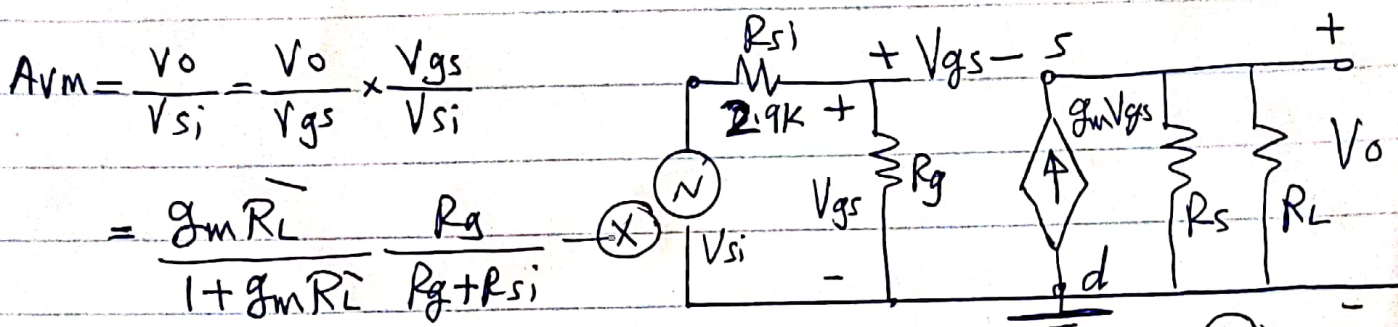
$$V_{DS} = 10 - I_D R_S = 10 - 1.2 \times 0.5 = 9.4 \text{ V}$$

$$V_{DS(sat)} = V_{GS} - V_{TN} = 3.45 - 2 = 1.45 \text{ V}$$

\therefore MOSFET in Saturation Regn.



② MFR: $C_1, C_2 \rightarrow S.C, C_{gd}, C_{gs} \Rightarrow O.C$



$$A_{vm} = \frac{V_o}{V_{si}} = \frac{V_o}{V_{gs}} \times \frac{V_{gs}}{V_{si}}$$

$$= \frac{g_m R_L}{1 + g_m R_L} \frac{R_g}{R_g + R_{si}}$$

$g_m = 2\sqrt{K_n I_D} = 1.55 \text{ mA/V}$ $R_g = R_1 \parallel R_2 = 97.1 \text{ k}\Omega$ ①

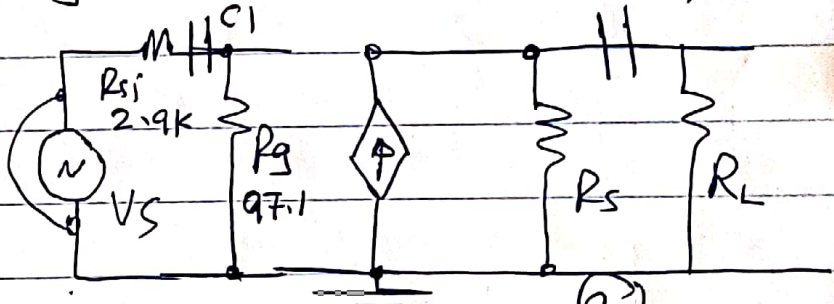
$R_L = R_s \parallel R_L = 0.5 \parallel 9.5 = 0.475 \text{ k}\Omega$

∴ $A_{vm} = \frac{1.55 \times 0.475}{1 + 1.55 \times 0.475} \frac{97.1}{2.9 + 97.1} = 0.411$

③ LFR: $C_{gs} \& C_{gd} \rightarrow O.C, C_1 \& C_2 \rightarrow \text{exist}$
 but $C_2 \rightarrow \infty$, so only C_1 is effective $C_2 \rightarrow \infty$

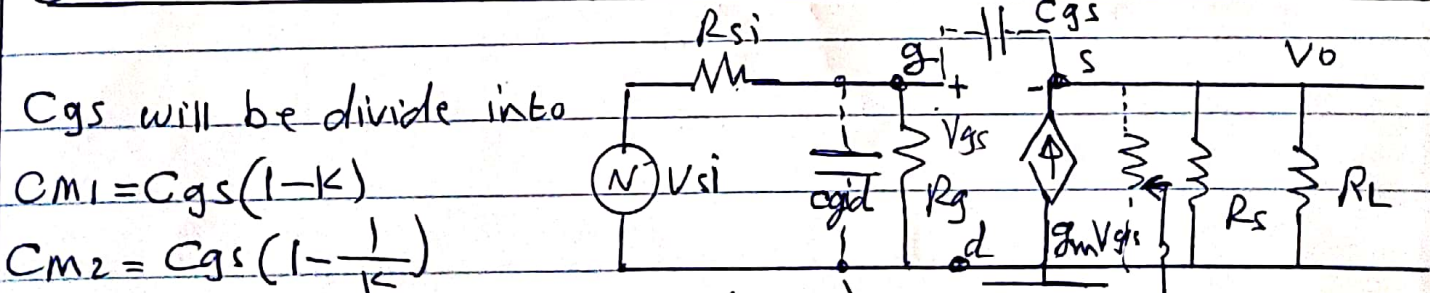
$$f_L = \frac{1}{2\pi(R_{si} + R_g)C_1}$$

$$f_L = \frac{1}{2\pi \times 100 \text{ k} \times 0.1 \times 10^{-6}}$$



$f_L \approx 16 \text{ Hz}$ $C_1 = 0.1 \mu\text{F}$ ②

④ HFR: $C_1 \rightarrow S.C, C_2 \rightarrow S.C, C_{gd}, C_{gs} \rightarrow \text{exists}$



C_{gs} will be divide into

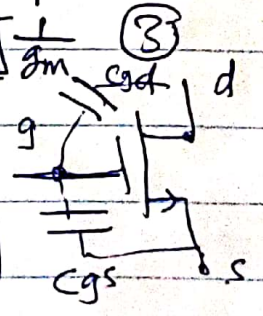
$$C_{M1} = C_{gs}(1 - k)$$

$$C_{M2} = C_{gs}(1 - \frac{1}{k})$$

$$k = \frac{V_o}{V_{gs}} = \frac{g_m(R_L \parallel R_s)}{1 + g_m(R_L \parallel R_s)} = \frac{1.55(0.475)}{1 + (1.55 \times 0.475)} = 0.424$$
 ③

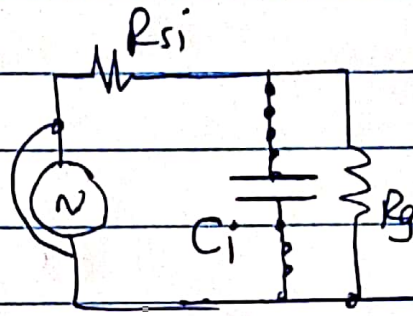
$C_{M1} = 5(1 - 0.424) = 2.88 \text{ pF}$

$C_{M2} = 5(1 - \frac{1}{0.424}) < 0 \text{ C - we neglected}$



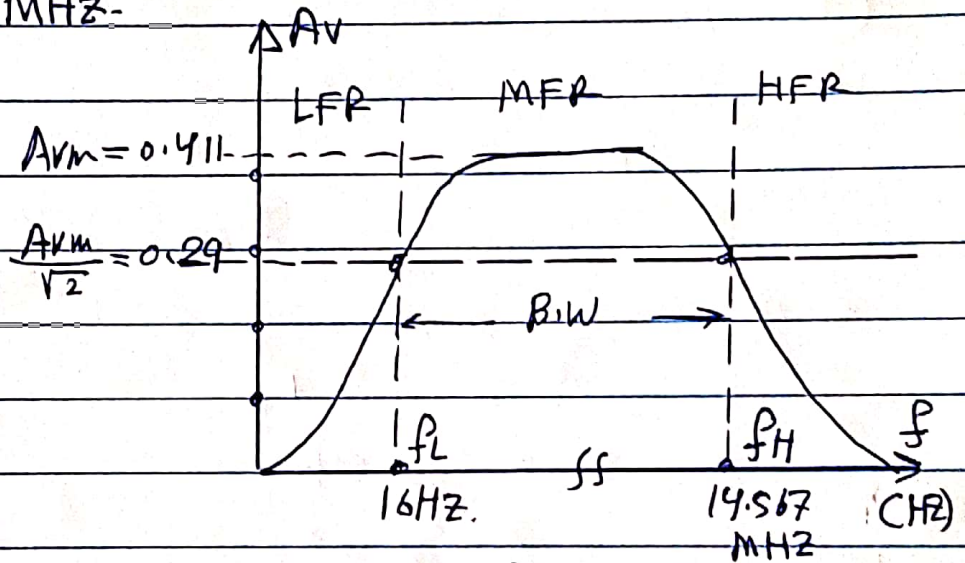
$$f_{Hi} = \frac{1}{2\pi C_i R_{eq}}$$

$$R_{eq} = R_{si} \parallel R_g = 2.9 \parallel 97.1 = 2.816 \text{ K}\Omega$$



$$f_{Hi} = \frac{1}{2\pi \times 2.816 \times 10^3 \times 3.88 \times 10^{-12}} = 14.567 \text{ MHz}$$

$$C_i = C_{M1} + C_{gd} = 3.88 \text{ pF}$$

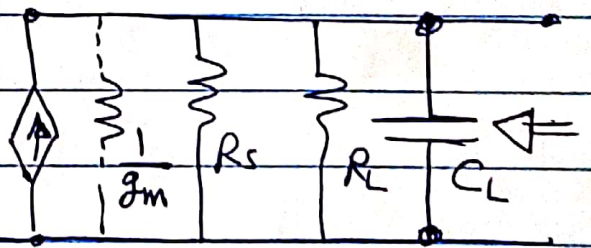


$$B.W = f_H - f_L \approx f_H$$

$$f_H = \frac{1}{2\pi C_L R_{eq}}$$

$$R_{eq} = \frac{1}{g_m \parallel R_s \parallel R_L}$$

$$g_m = \frac{1000}{1.55} = 0.645 \text{ K}\Omega$$



$$R_{eq} = 0.645 \parallel 0.5 \parallel 9.5 = 0.645 \parallel 0.475 = 0.273 \text{ K}\Omega$$

$$C_L = \frac{1}{2\pi f_H R_{eq}} = \frac{1}{2\pi \times 1.5 \times 10^6 \times 0.273 \times 10^3}$$

$$C_L = 388 \text{ pF}$$