ElCoM
ELECTRICAL COMPUTER MECHATRONICS

تقدم لجنة ElCoM الاكاديمية

تلخيص لمادة

تصميم ميكانيكي

جزيل الشكر للطالبة:

سارة آبو سارة

Design Mechanics..

Sunday
20 / Jan / 2019

Ch.1 Concept of Stress

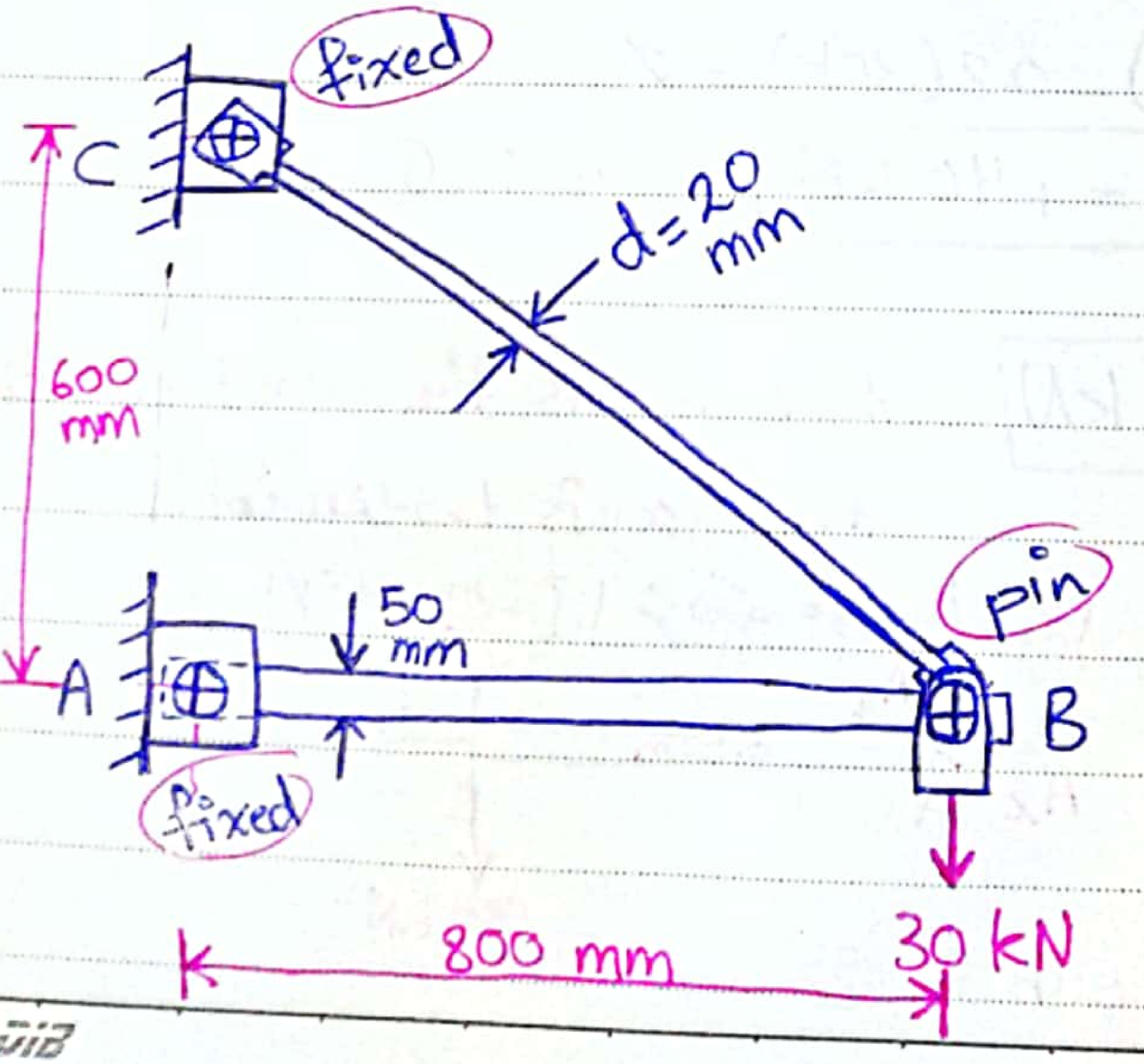
1.1 Introduction :

The main objective of the study of the mechanics is to provide the future engineer with the means of analysing and designing various machines and load-bearing structures.

Design & analysis of agiven structure → تحديد → stresses
→ deformation

1.2 A Short Review of the methods of the Static :

review the basic methods of statics while determining the forces in the members of a simple structure.

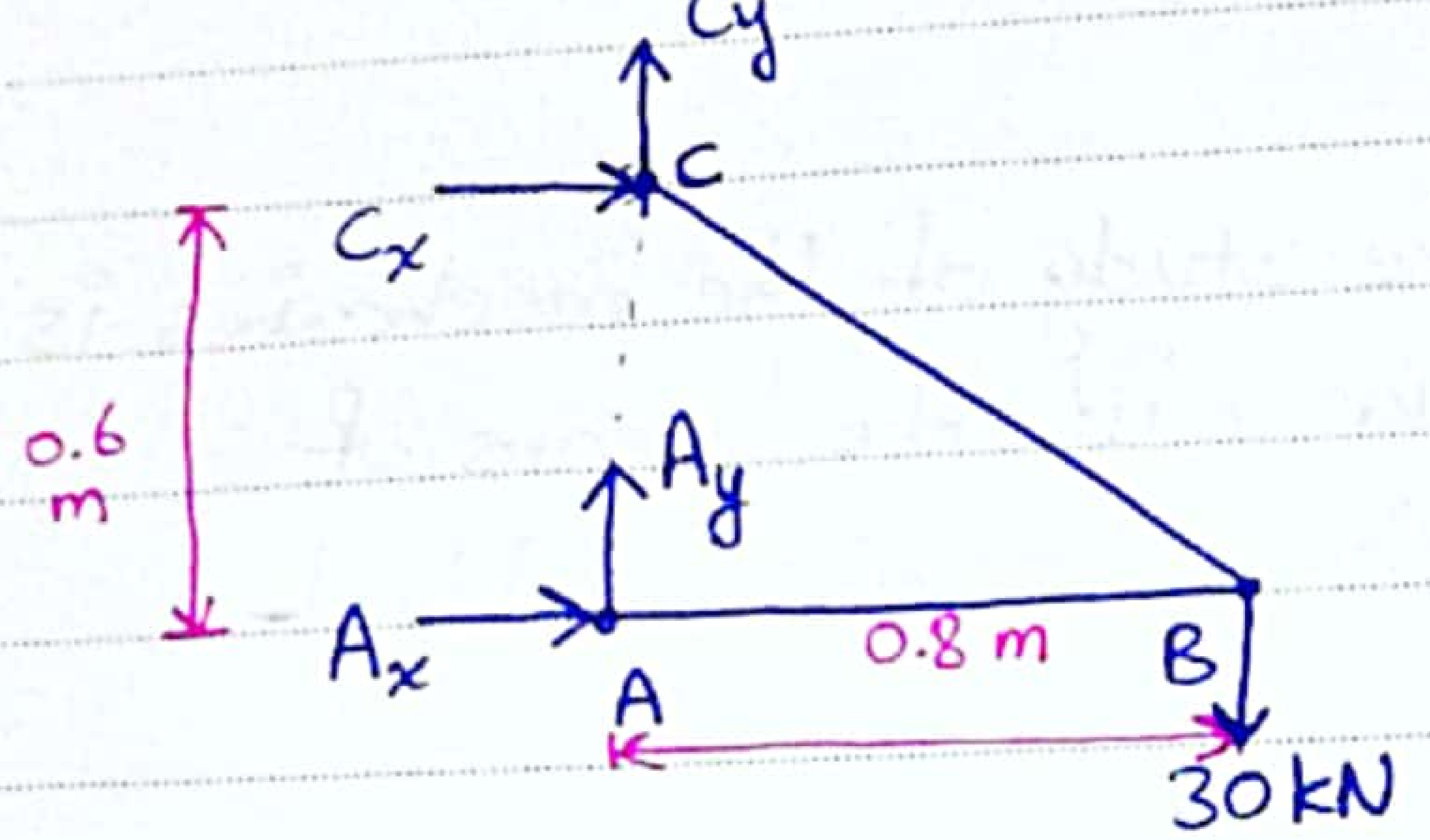


* Boom (AB) used to support a 30 kN load that has a rectangular cross sectional area.
* Rod (CB) has a circular cross sectional area..

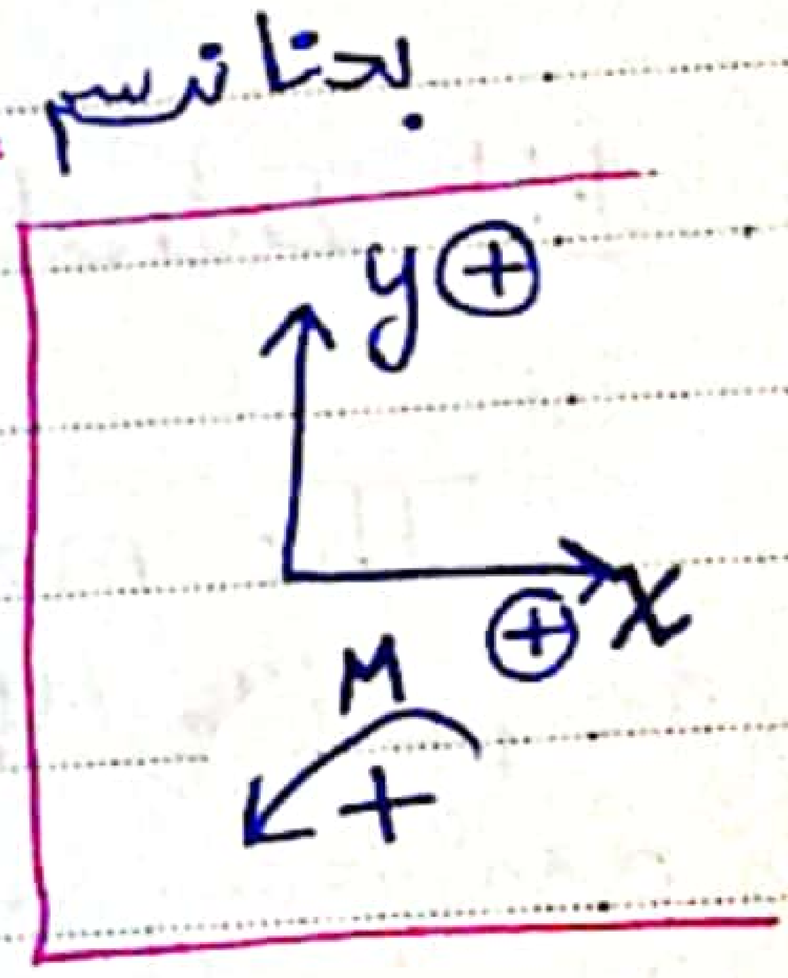
/ /

* حتى نعمل (analysis) للـ(structure) السابق

بدنا نرسم FBD توضح من خلاله الـ [Reaction Forces] ← static structure



4 unknowns



Using the equilibrium equations

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M = 0$

- $\Sigma F_x = 0 \Rightarrow C_x + A_x = 0 \quad \dots\dots (1)$

- $\Sigma F_y = 0 \Rightarrow C_y + A_y - 30k = 0 \quad \dots\dots (2)$

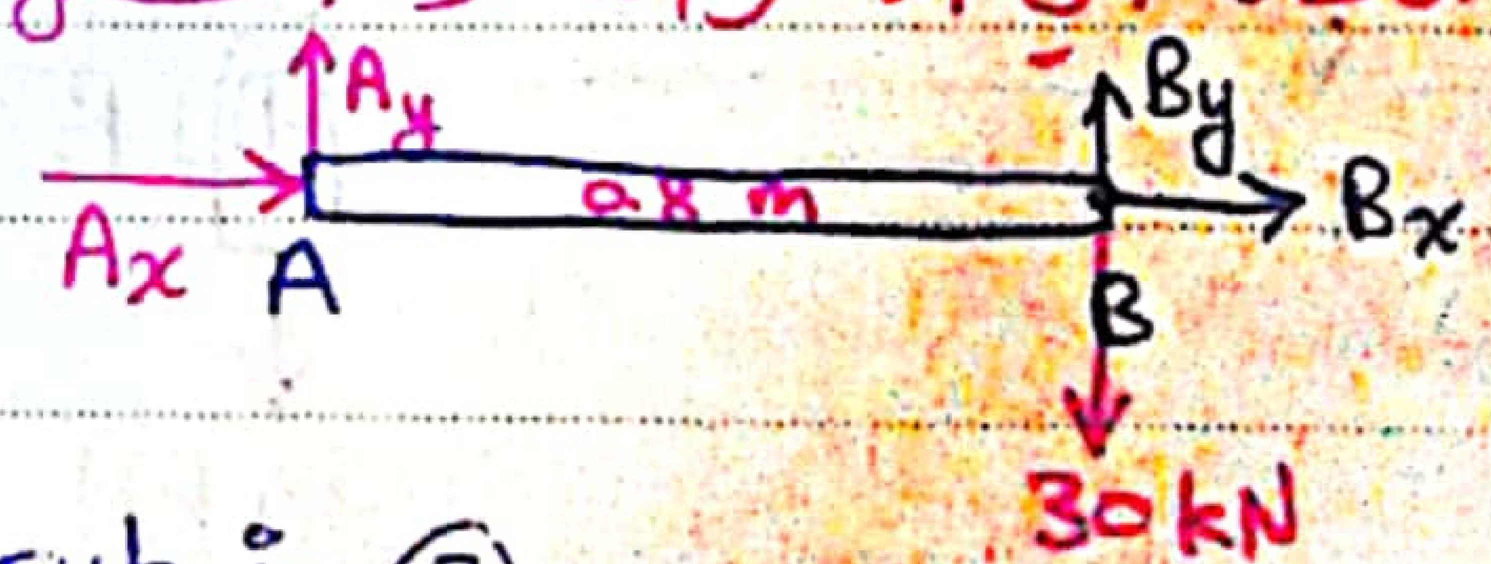
- $\Sigma M_c = 0 \Rightarrow A_x(0.6) - 0.8(30k) = 0$

$\boxed{A_x = +40 \text{ kN}}$ sub in (1)

أهملت العمودية عند حساب العزوم

$\boxed{C_x = -40 \text{ kN}}$, that means the correct direction for C_x is to the left

بعكس الاتجاه اللي أنا مفترضه فوق بالشكل



- $\Sigma M_B = 0$

$-A_y(0.8) = 0$

$\boxed{A_y = 0 \text{ kN}}$ sub in (2)

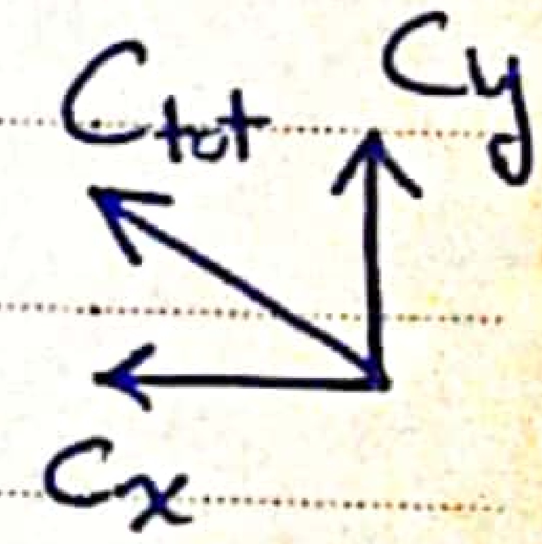
$\boxed{C_y = 30 \text{ kN}}$

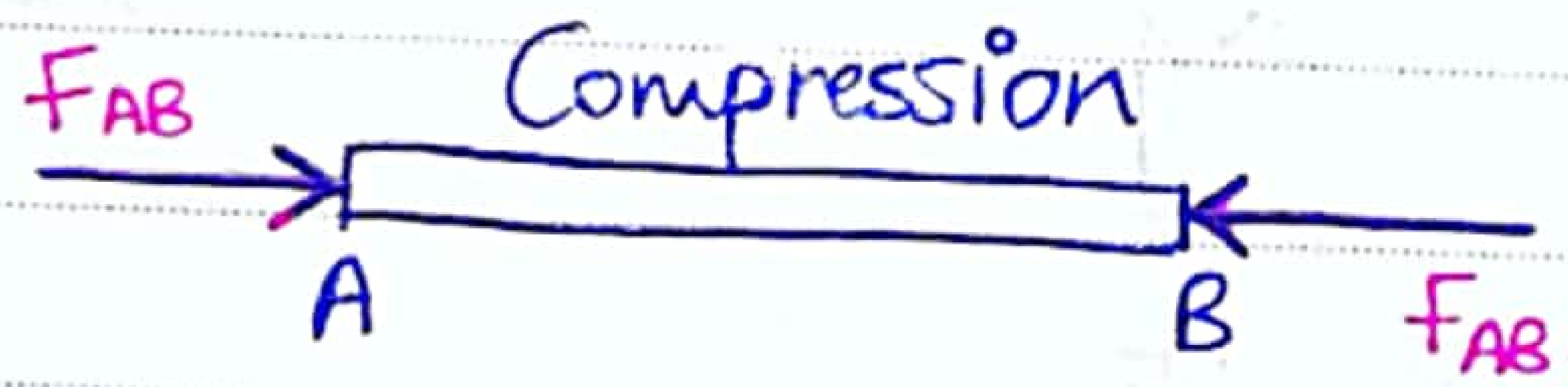
Remember ⇒ * النقطة اللي باخد عندها المومنت ما بيكون في مومنت للقوى المتمثلة فيها ، زي عند النقطة B
القوى على B ما ناخذهم بعين الاعتبار لما نحسب محصلة المومنت عند B

* نفصل كل member لحاله ونشتغل عليه لإيجاد المجاهيل والاتجاه اللي بفرضه على AB للقوى لازم تكون عكسها لما باخذ CB عند النقطة B

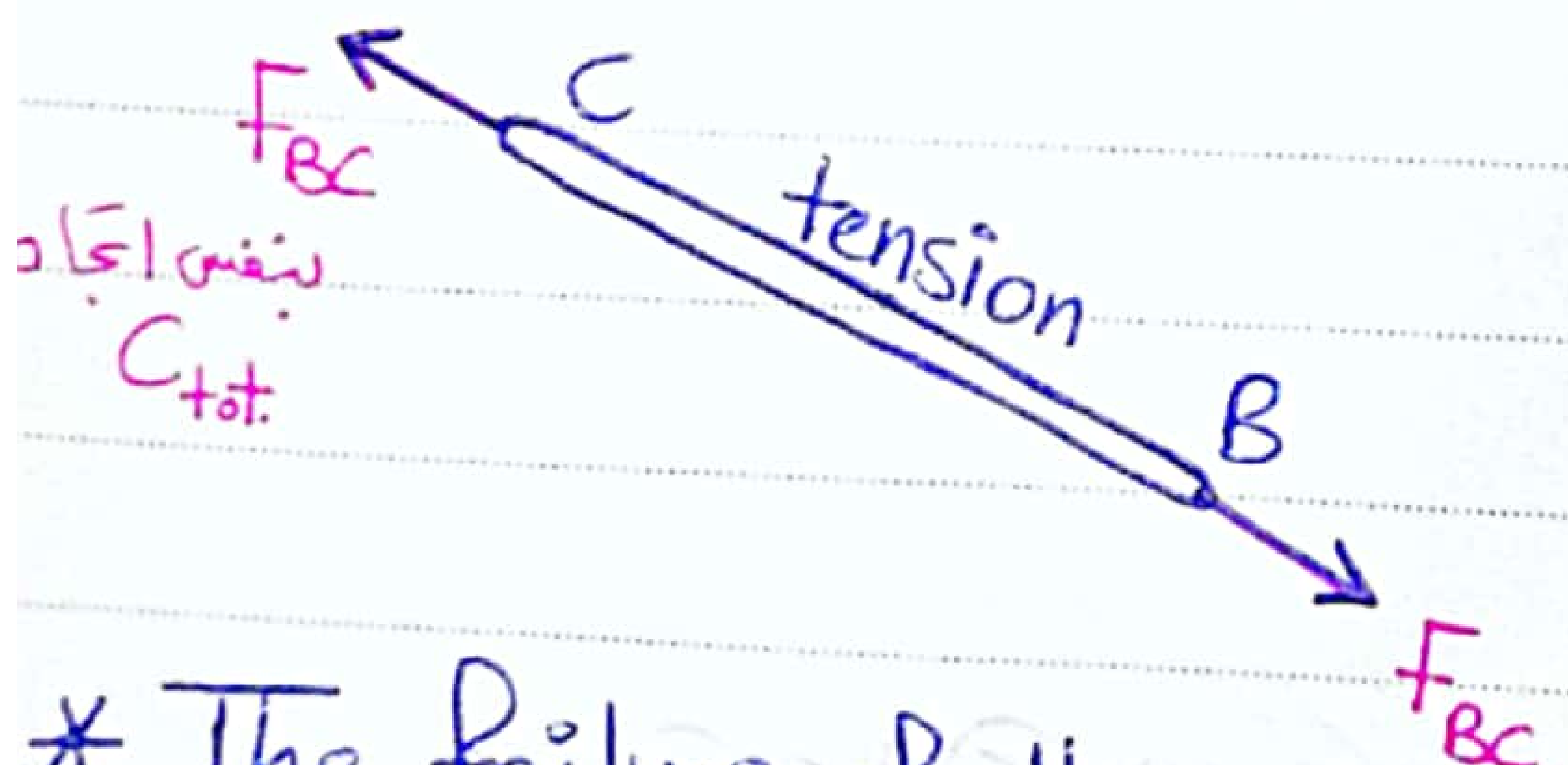
* النقطة B هي عبارة عن joint بتربط بين [2 members] فنفرض عندها pin

$$C_{tot} = \sqrt{C_x^2 + C_y^2} = \sqrt{(-40)^2 + (30)^2} = 50\,kN = F_{BC}$$





$F_{AB} = A_x = 40\,kN$



$F_{BC} = C_{tot} = 50\,kN$

* The failure of the system depends on
  → Load
  → Material
  → Cross Sectional Area

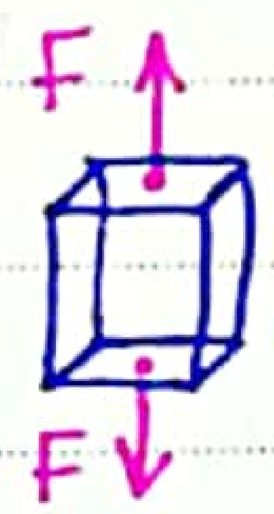
Tuesday
22/1/2019

1.3 Stress in the members of structure :

The structure can be safely supported depends on →
1- Internal Force
2- Cross Sectional Area
3- The material of which the rod is made from.

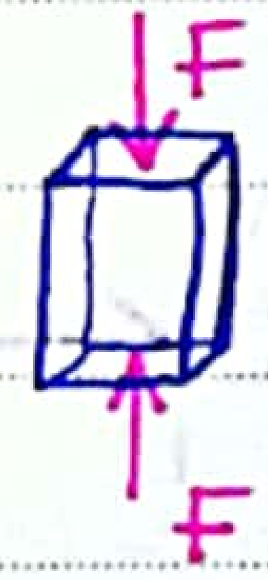
∴ Stress ≡ the force per unit area or intensity of the forces distributed over a given section.

$$\left[\sigma = \frac{F}{A}\right] \qquad [\text{Pascal}] = \frac{[N]}{[m^2]}$$



σ (+ve)

the force → tension
→ compression



σ (-ve)

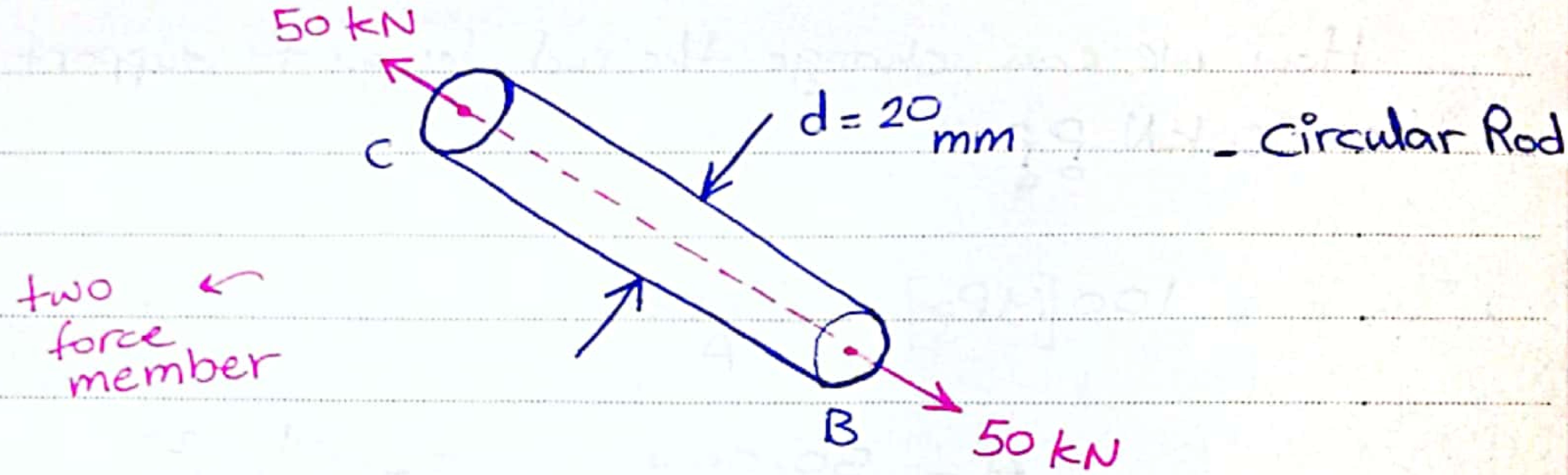
Recall) $1\,kPa = 10^3\,Pa$
$1\,MPa = 10^6\,pa$
$1\,GPa = 10^9\,pa$

قيمة ال σ اللي بوجدها (Normal stress)

$$[\sigma < \sigma_{allowed}]$$

1.4 Analysis and Design :

According to the previous simple structure, assume that rod (BC) is made of a steel with a max. allowable stress $\boxed{\sigma_{all} = 165\ \text{MPa}}$. Knowing that $\boxed{F_{BC} = 50\ \text{kN}}$



* أنا بعرف انه هاد الـ (rod) بتحمل (stress) لغاية 165 MPa
فلحتى أحدد اذا هو بيقدر يتحمل القوة 50 kN ولا لأ، بحتاج أحسب
قيمة الـ [stress] الناتجة عن هال 50 kN المؤثرة على الـ (BC Rod)!

$$\sigma = \frac{50\ \text{kN}}{\pi(0.1)^2} = (+)\ 159\ \text{MPa} < 165\ \text{MPa}$$

لأنّ
tension force

∴ اذن بقدر الـ rod يتحمل هاي القوة المؤثرة عليه [50 kN]

/ /

⇒ At the same previous value of the force (50 kN),
ازا غيّرنا الـ [material] الخاصة بالـ "BC rod"
من (steel) لألمنيوم بحيث أصبحت

$$\boxed{\sigma_{allow} = 100\ \text{MPa}}$$

How we can change the rod design to support the 50 kN ??

$$100\ [\text{MPa}] = \frac{50\ [\text{kN}]}{A}$$

$$A = \frac{50,000}{100 * 10^6} = 5*10^{-4}\ [m^2]$$

$$5*10^{-4} = \pi r^2 \Rightarrow r = 12.62\ \text{mm}$$

رح أشتري القطعة بناءً

$[d = 25.2\ \text{mm}]$ على طول 26 mm لأنه

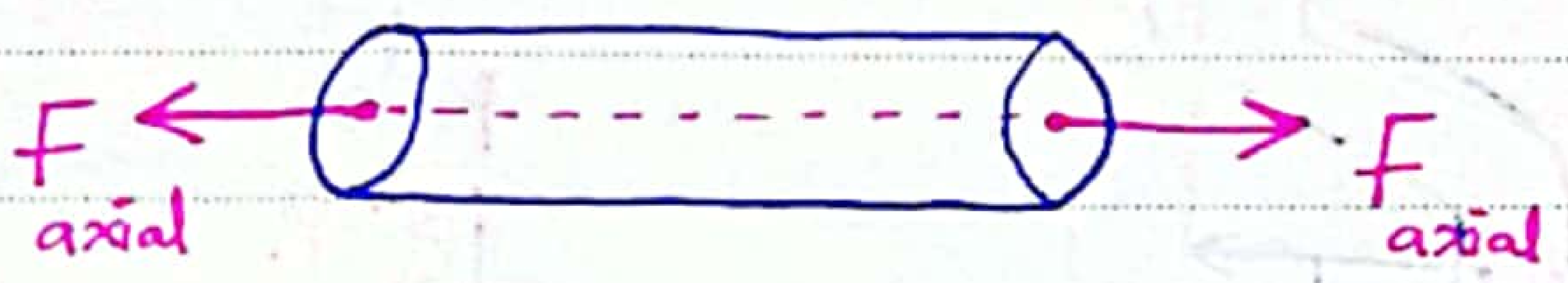
مارح أقدر ألاقيها بـ 25.2 وما بقدر أختار الـ 25 مبرداد σ عن المسموح به ..

∴ we need to increase the rod diameter 26 mm or more

لأنه اذا كان أقل من هيك معطيني (σ) أكثر من [100 MPa]

1.5 Axial loading ; Normal Stress :

Axial loading → When the force directed along the axis of the rod
(F_{axial})

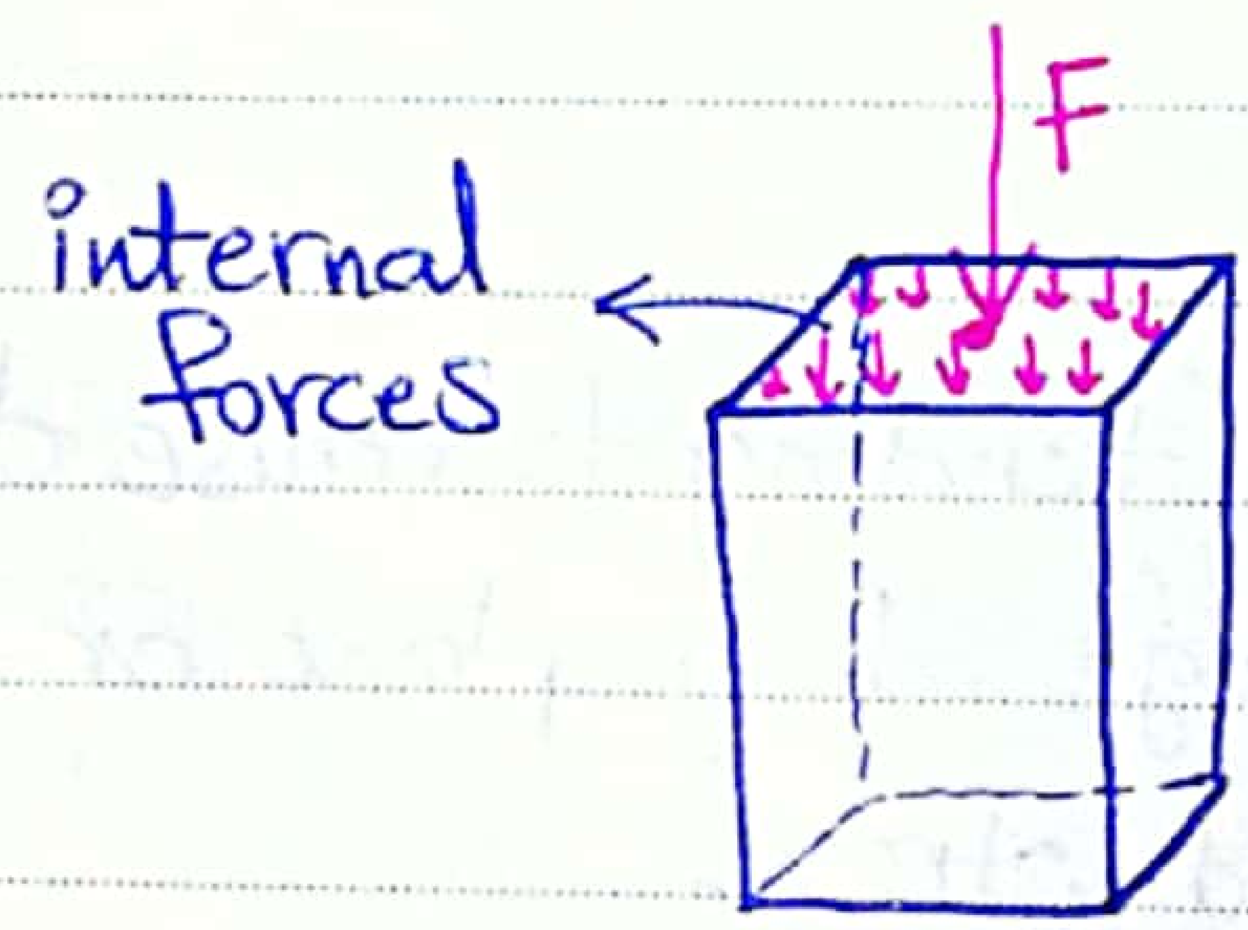


Normal stress ≡ occures when a member is loaded by an axial force

$$\boxed{\sigma_{normal} = \frac{F_{axial}}{A}}$$

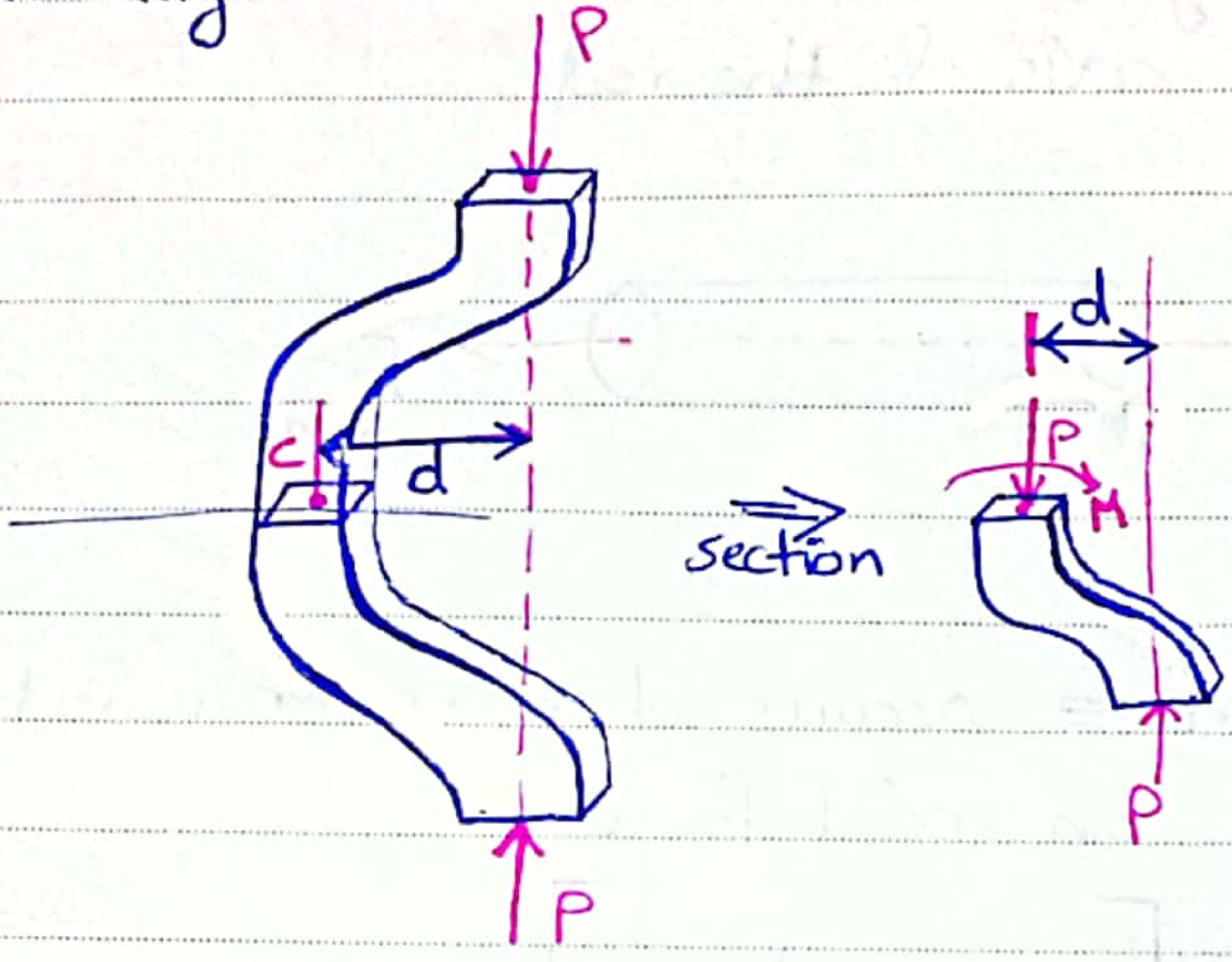
زي اللي على ال BC Rod

وبهالحالة بتكون القوة موزّعة بشكل منتظم على ال area



∴ a uniform distribution of stress is possible only if the line of action of the concentrated load passes through the centroid of the section considered (centric Loading).

However, if a two force member is loaded axially, but eccentrically

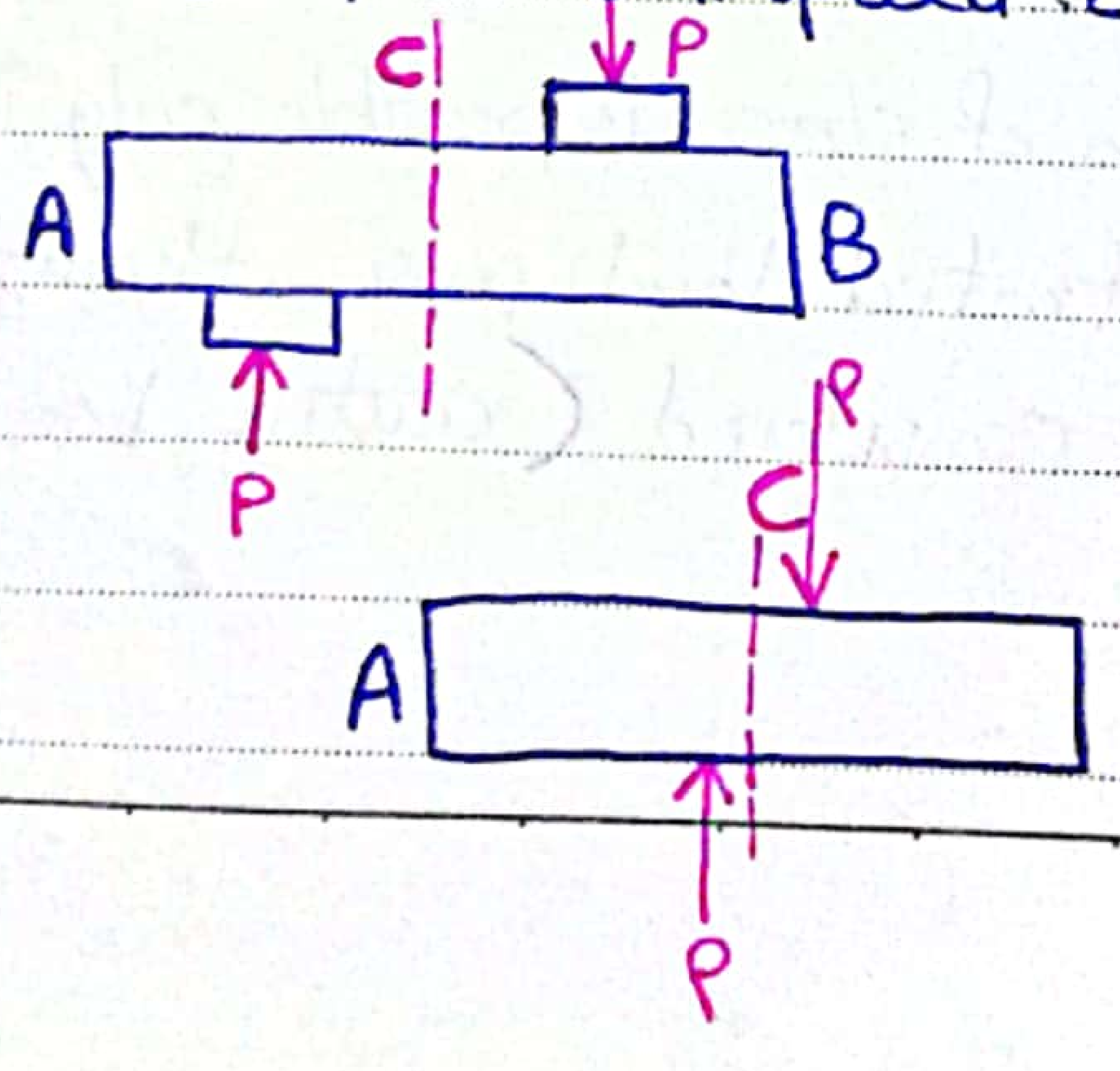


$M = Pd$

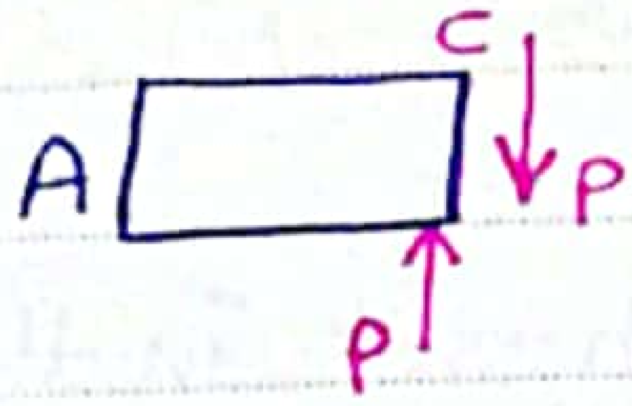
The distribution of forces, thus the corresponding distribution of stresses ⇒ [Non_uniform]

1.6 Shearing Stress :

Occured due to a force tending to cause deformation of a material by slippage along plane or planes parallel to the imposed stress.

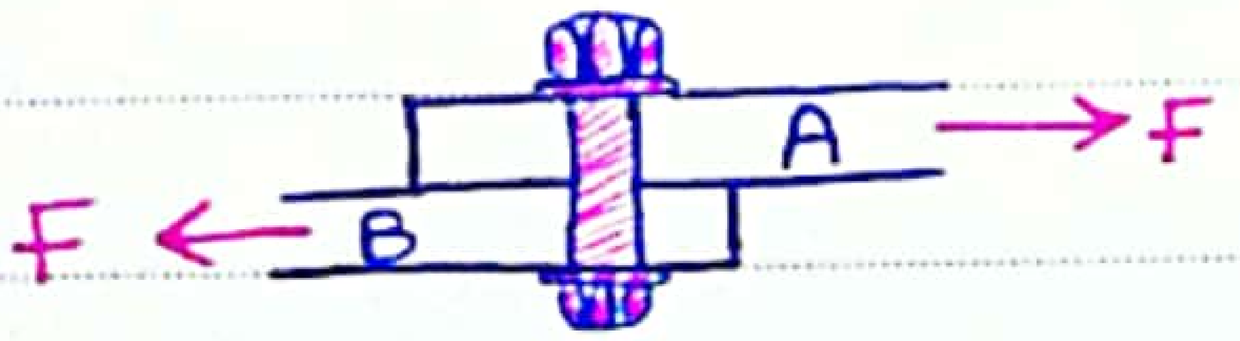


[يعتمد على ال
Contact Area
اللي تم تأثر عليها القوة
و القوة نفسها]



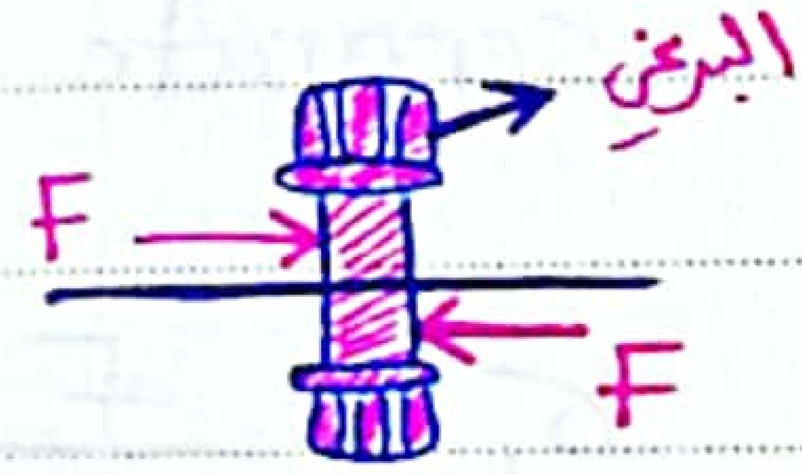
$$\boxed{\tau = \frac{F}{A}}$$

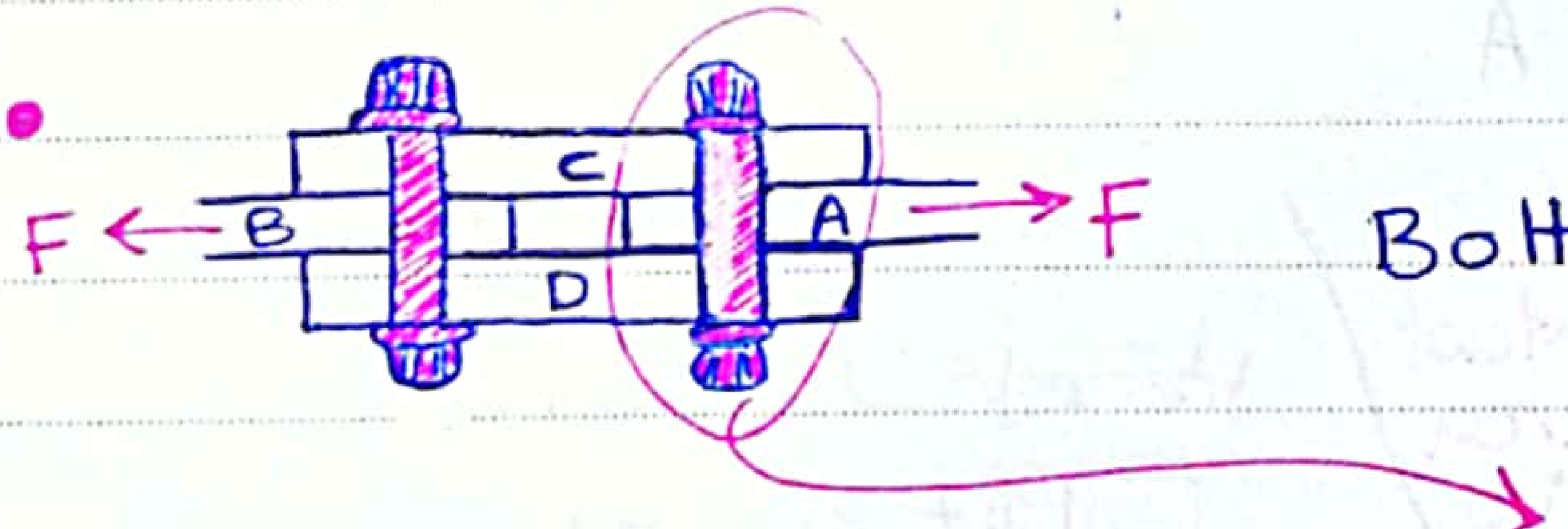
* Shearing stresses are commonly found in bolts, pins, and rivets used to connect various structural members and machine components.

- 

Bolt subject to single shear

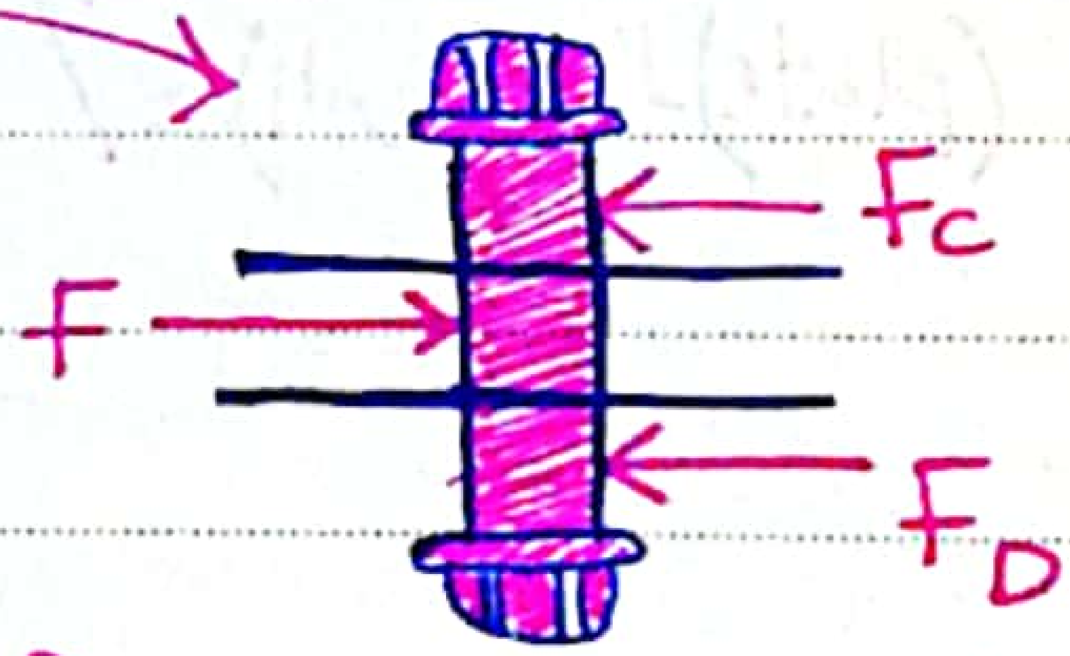
$$\boxed{\tau = \frac{F}{A}}$$



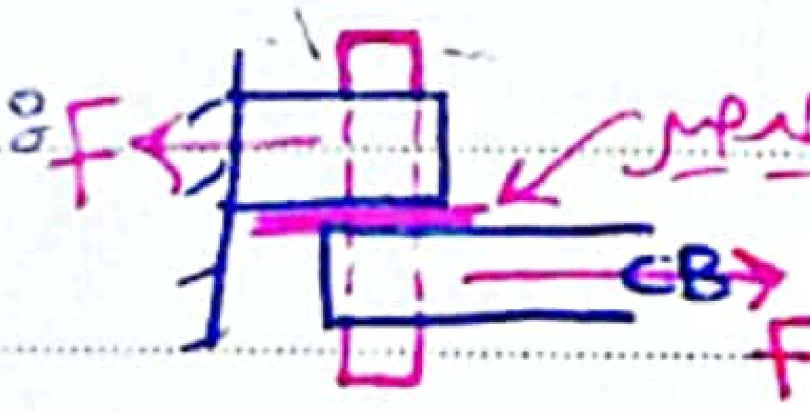
- 

Bolts subject to double shear ⇒ [سطحين قص]

$$\boxed{\tau = \frac{F/2}{A} = \frac{F}{2A}}$$

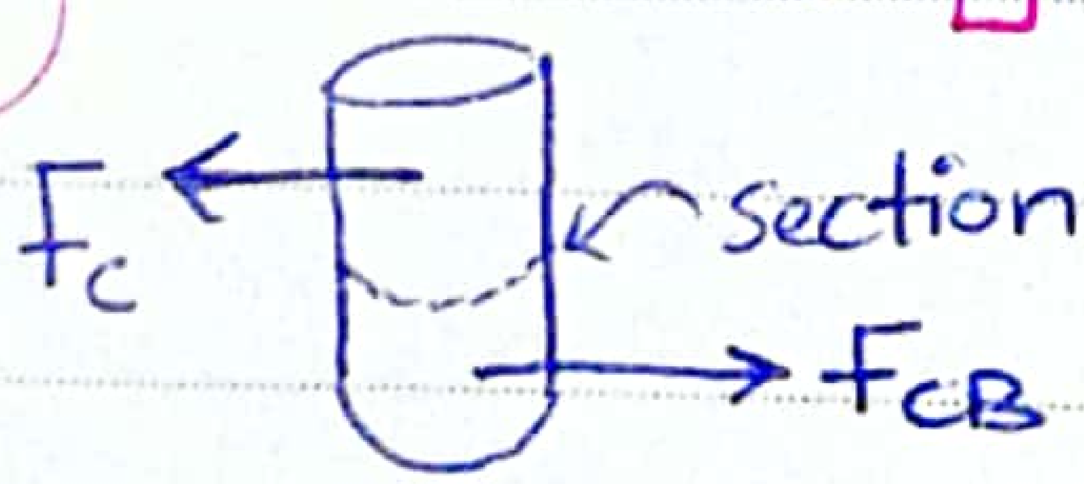


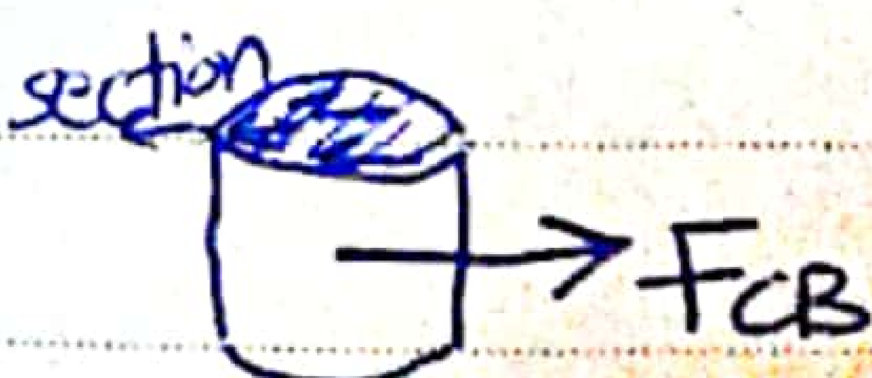
* at pin $\underline{C}$:



سطوح تعرض الكسر (Shearing Surface)

بالشكل من فوق





$\boxed{\text{قوى أفقية}}$

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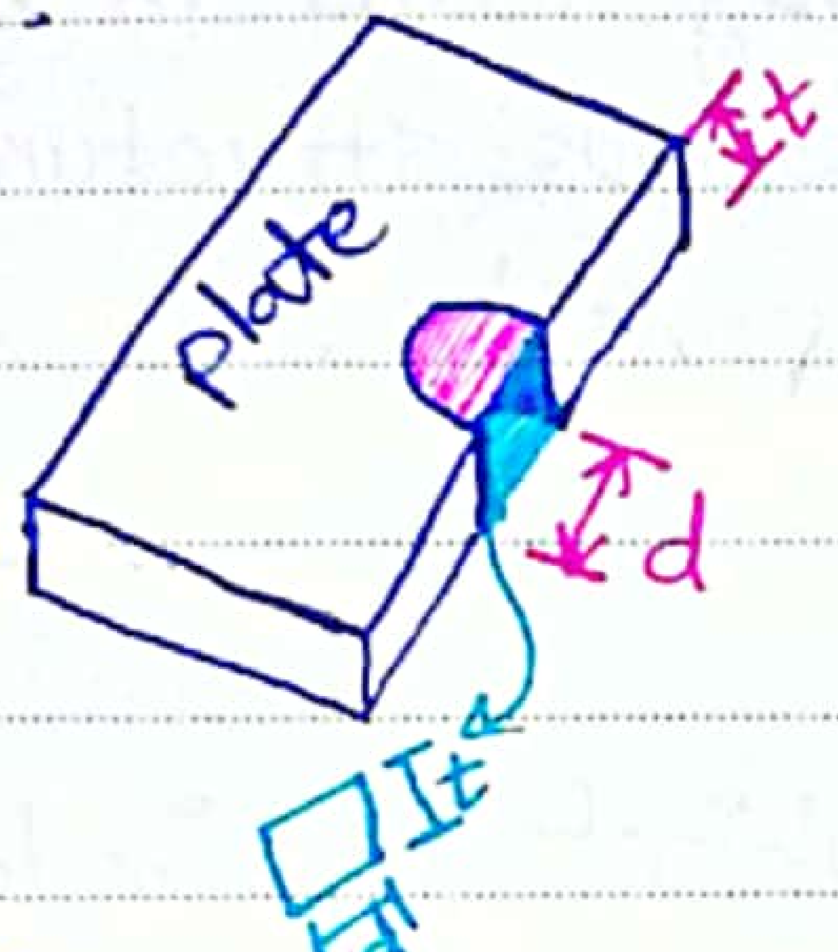
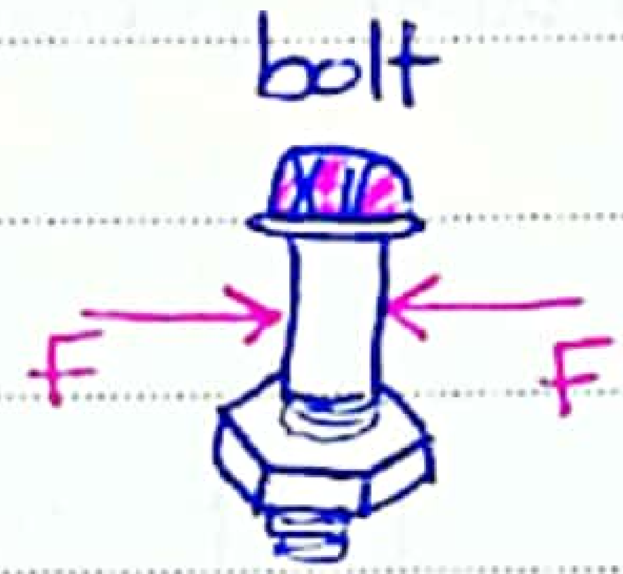
27/ 1 /2019

1.7 Bearing Stress in Connections :
"Contact"

Bolts, pins, and rivets create stresses in the members they connect, along the bearing surface, or surface of contact.





t : thickness of plate

d : bolt diameter

∴ bearing stress → is the contact pressure between the separate bodies.

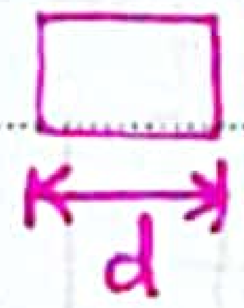
$$\sigma_b = \frac{F}{A} = \frac{F}{td}$$

contact area / بين (البرغي) و ال (plate)

لأنه لما تطلع من

قدام ملاقي الشكل مستطيل

أبعاده ⇐ t



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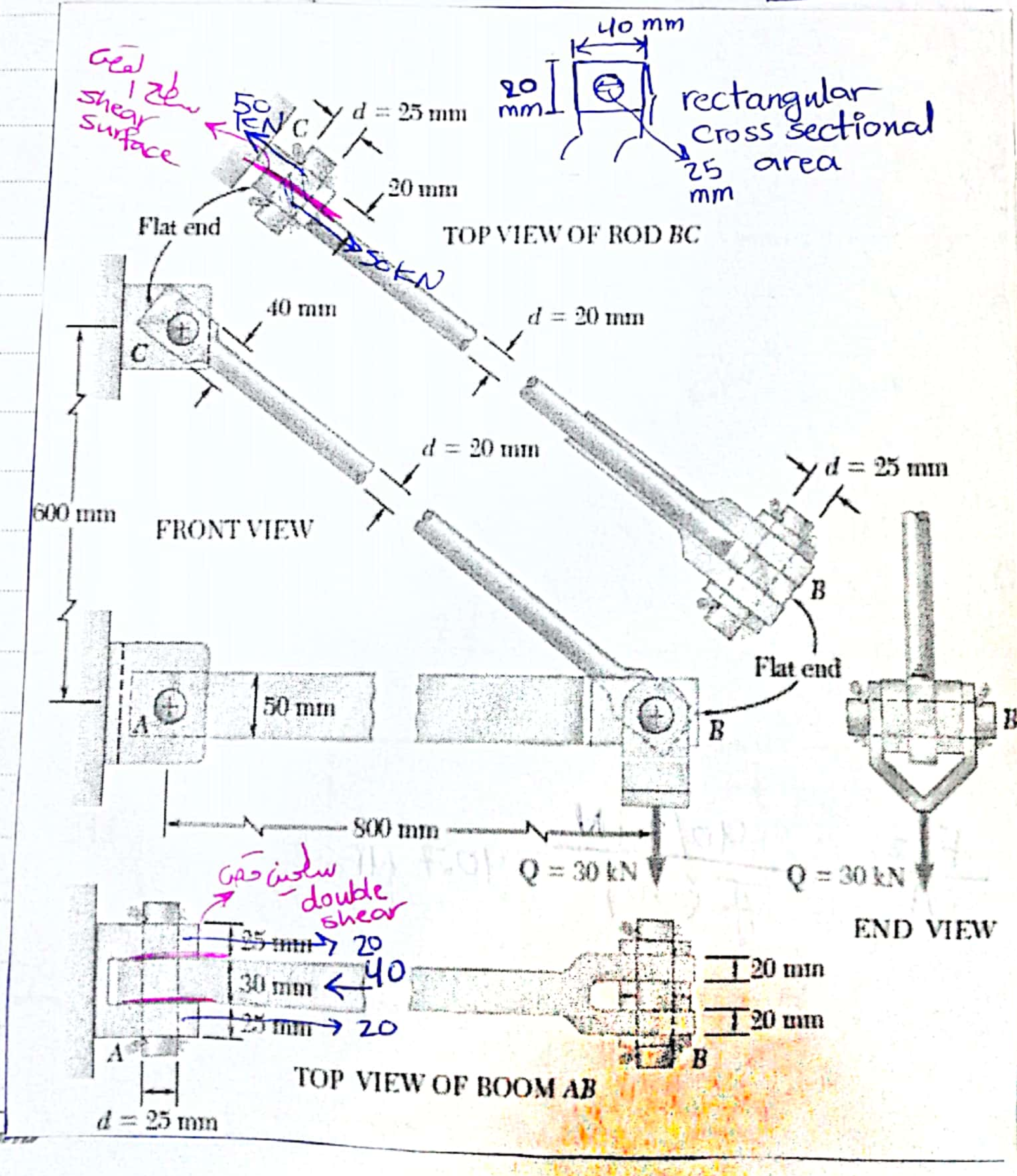
1.8 Application to the analysis and design of simple structures :

بسكشن 1.2 أوجدنا الـ
(Forces)
للـ [Structure] بالكامل
هلا بدنا نوجدله الـ (stresses)

Ex :



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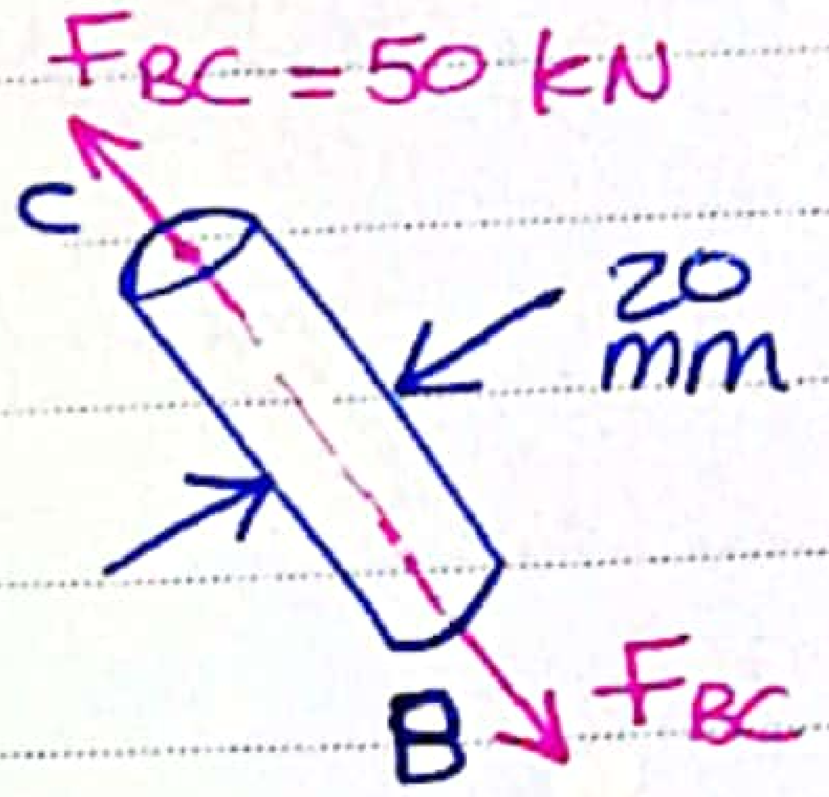
a. Determination of the normal stress in Boom AB and rod BC :

(BC)

$$\sigma_{\substack{normal\\BC}} = \frac{F}{A} = \frac{50\ kN}{\pi (0.1)^2}$$

$$= +159\ MPa$$



also, flat parts at the end of the Rod BC (at C) are under tension, where a hole is located

$\boxed{Area = \pi r^2}$

للدائرة

$$\sigma_{\substack{normally\\C}} = \frac{F}{A} = \frac{50{,}000\ N}{20 * (40-25) * 10^{-6}} = 167\ MPa$$

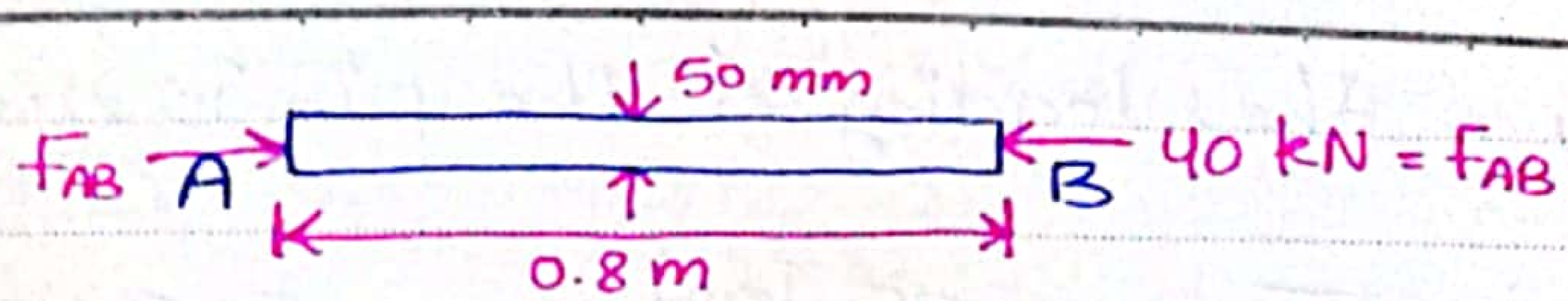
the stress will ⇐ أقوى (stress) بكون عند أضعف منطقة * reach a much larger value, close to the hole.

هاد معناه مع ازدياد اللود ، ال [rod] حيفشل على المنطقة القريبة من ال holes كونها الأضعف

So, in its design therefore, could be improved by increasing the width or the thickness of the flat ends of the rod.

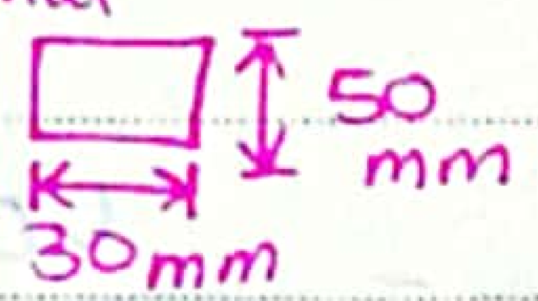
/ /

(AB)



$$\sigma_{\text{normally AB}} = \frac{F}{A} = \frac{-40{,}000\ N}{30 * 50 * 10^{-6}} = -26.7\ MPa \rightarrow \text{comp. force pushes on the pins}$$

A → rectangular cross sectional area



b. Determination of the shearing stress in various Connections :

→ such as bolts, pins, and rivets

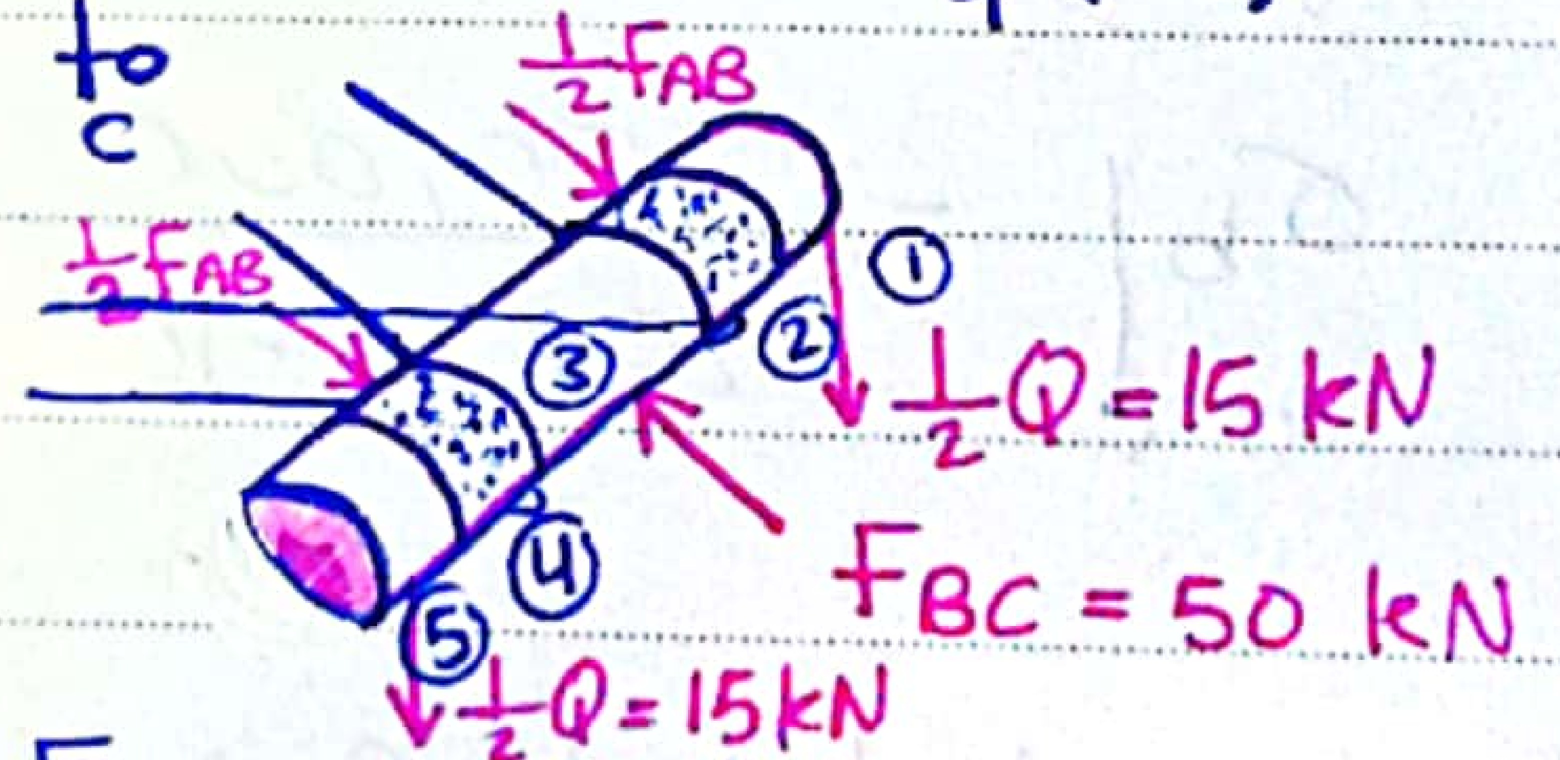
- at pin C → $\tau_c = \dfrac{F}{A} = \dfrac{50{,}000}{\frac{\pi}{4}(25*10^{-3})^2} = 102\ MPa$

  single shear

- at pin A → $\tau_a = \dfrac{F/2}{A} = \dfrac{\frac{40}{2}\ kN}{\frac{\pi}{4}(25)^2 * 10^{-6}} = 40.7\ MPa$

  double shear

- at pin B →



② & ④ → $\frac{1}{2} F_{AB} = 20\ kN$

أكبر [shear stress] بيصير في ال (pin B) هو عند المقطع في المنتصف ⇐ $\boxed{F = 25\ kN}$

Since the loading of the pin is symmetric

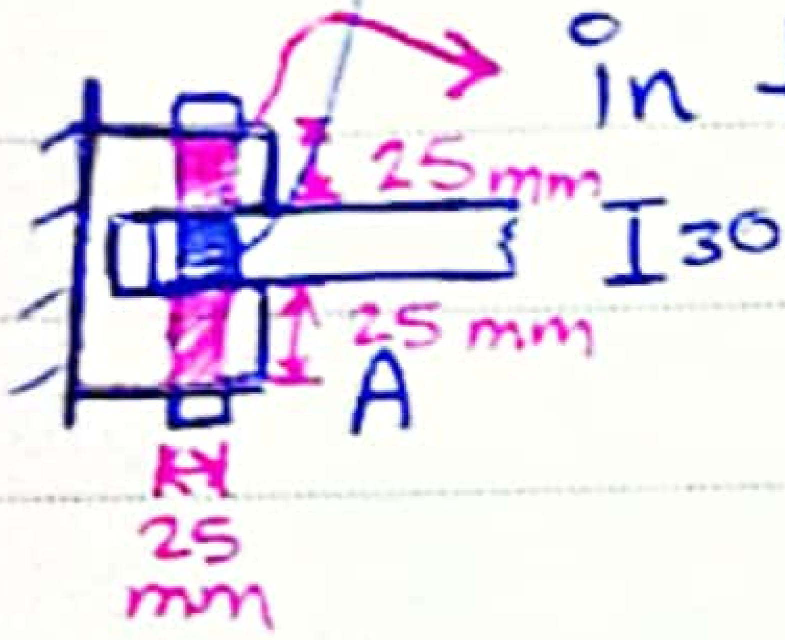
$$\tau_B = \frac{25\ kN}{\frac{\pi}{4}(25 * 10^{-3})^2} = 50.9\ MPa$$

C. Determination of the Bearing stresses :

• at A : on the rod

$$\sigma_b\Big|_{at\ A} = \frac{P}{td} = \frac{40{,}000\ N}{30 * 25 * 10^{-6}} = 53.3\ MPa$$

t d



in the bracket at A →

$$\sigma_b = \frac{40{,}000\ N}{50 * 25 * 10^{-6}} = 32\ MPa$$

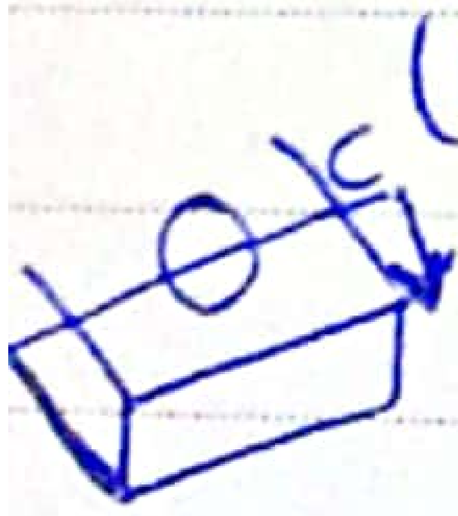
• at B : in the member (AB)

$$\sigma_b\Big|_B = \frac{40{,}000}{40 * 25 * 10^{-6}} = 40\ MPa$$

في منطقتين كل وحدة عليها 20 mm

• at C : in the member (BC)

$$\sigma_b\Big|_{at\ C} = \frac{50{,}000}{20 * 25 * 10^{-6}} = 100\ MPa$$

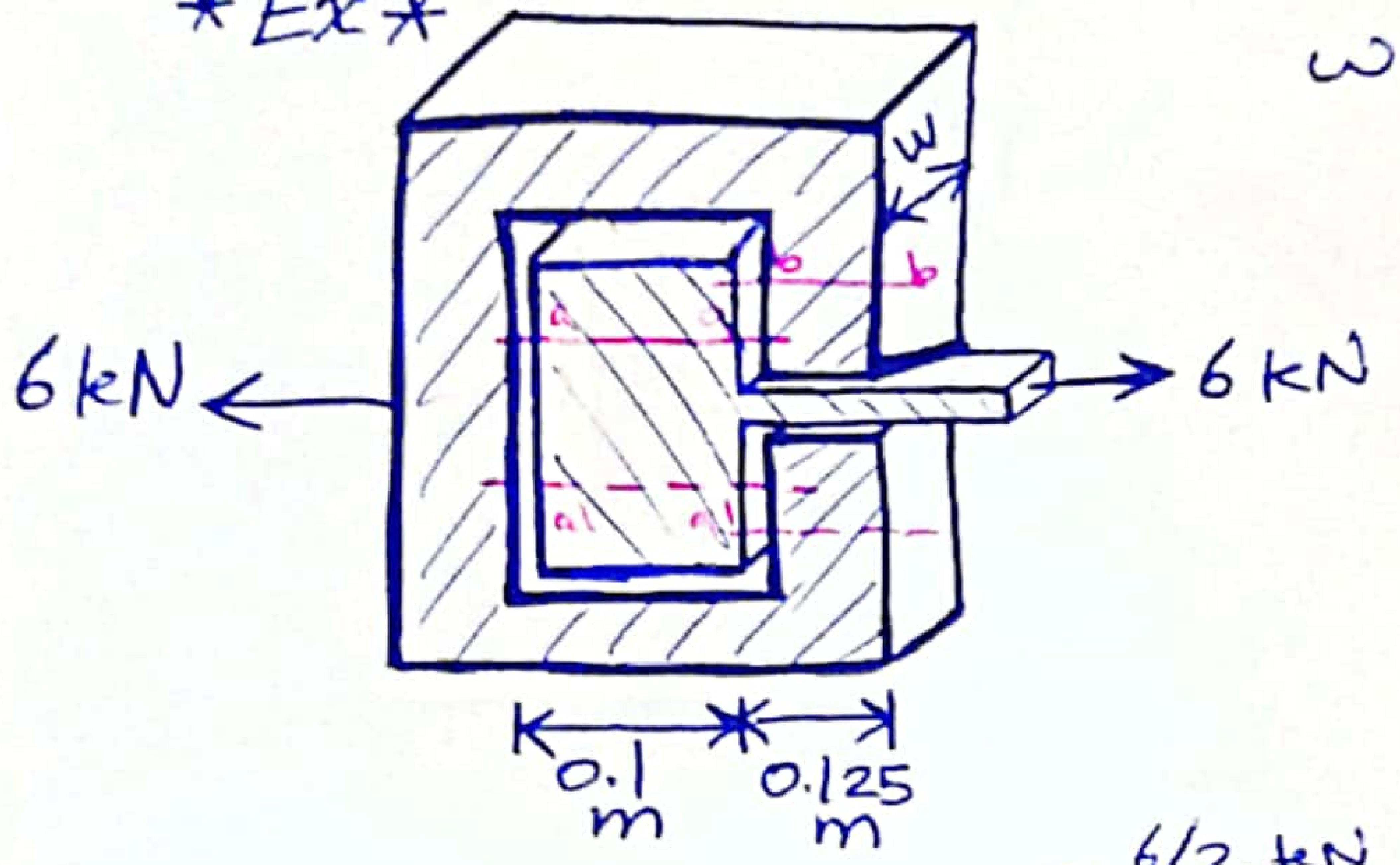


in the bracket at C →

$$\sigma_b = \frac{50{,}000}{40 * 25 * 10^{-6}} = 50\ MPa$$

Ex

$\omega = 150\ mm$



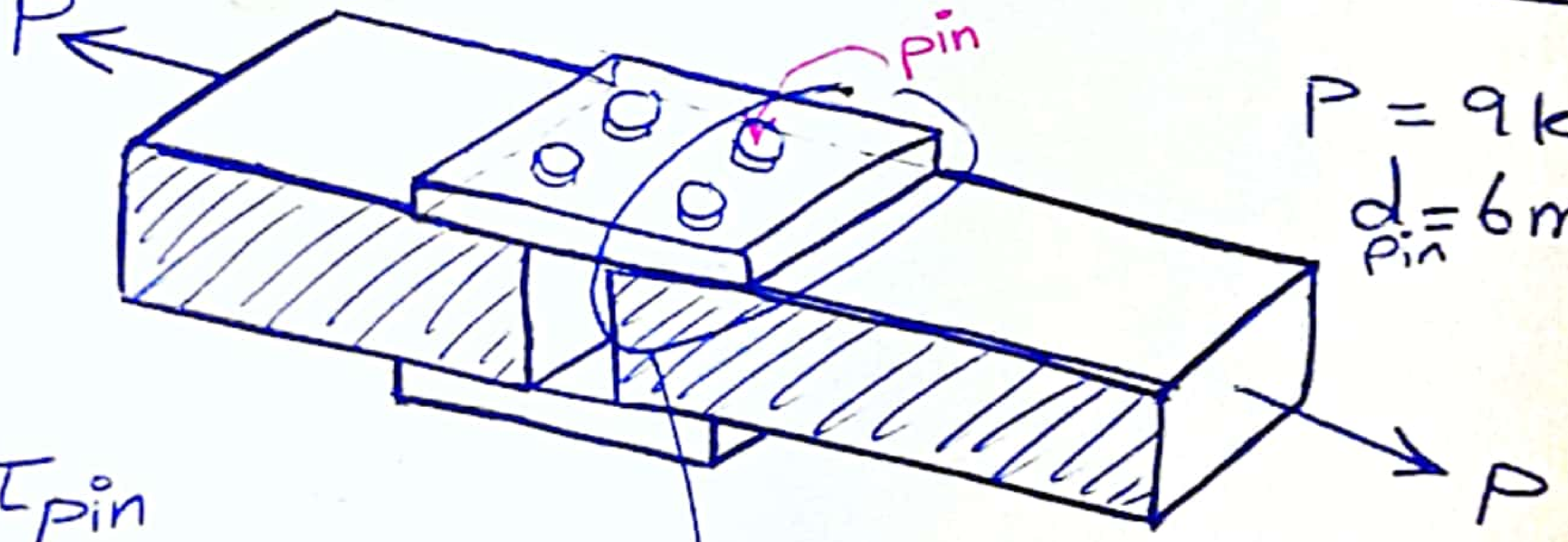
Find : $\tau_{a-a} = \dfrac{3 * 10^3}{(0.1)(0.15)} = 200\ kPa$ (6/2 kN)

$$\tau_{b-b} = \frac{6000/2}{(0.125)(0.15)} = 160\ kPa$$

Ex

$P = 9\ kN$

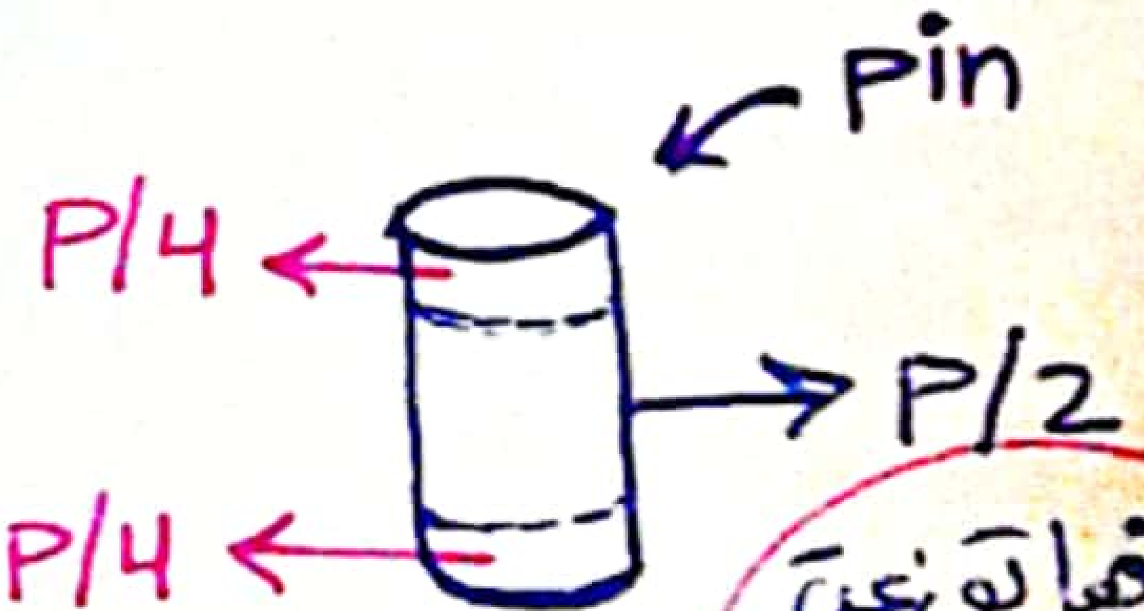
$d_{pin} = 6\ mm$



find τ_{pin}

$$\tau_{pin} = \frac{P/4}{A_o}$$

$$= \frac{9000/4}{\frac{\pi}{4}(6 * 10^{-3})^2} = 79.6\ MPa$$



لأنها توزعت على 2pins

Double shear

لو كانوا بدل 2pins ⇐ 3pins

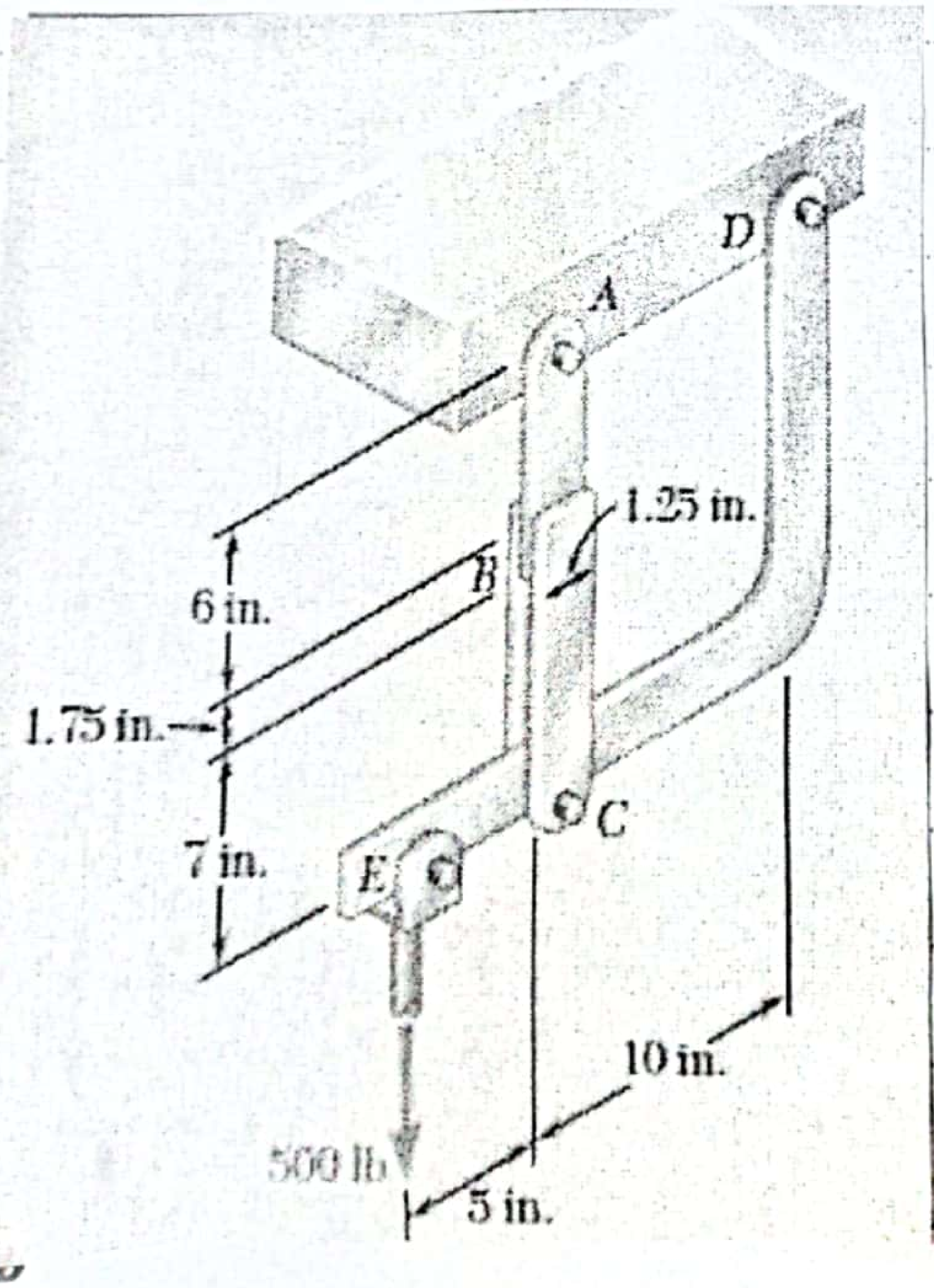
$$\tau_{pin} = \frac{P/6}{A_o}$$

Sample Problem (1.1) ⇒

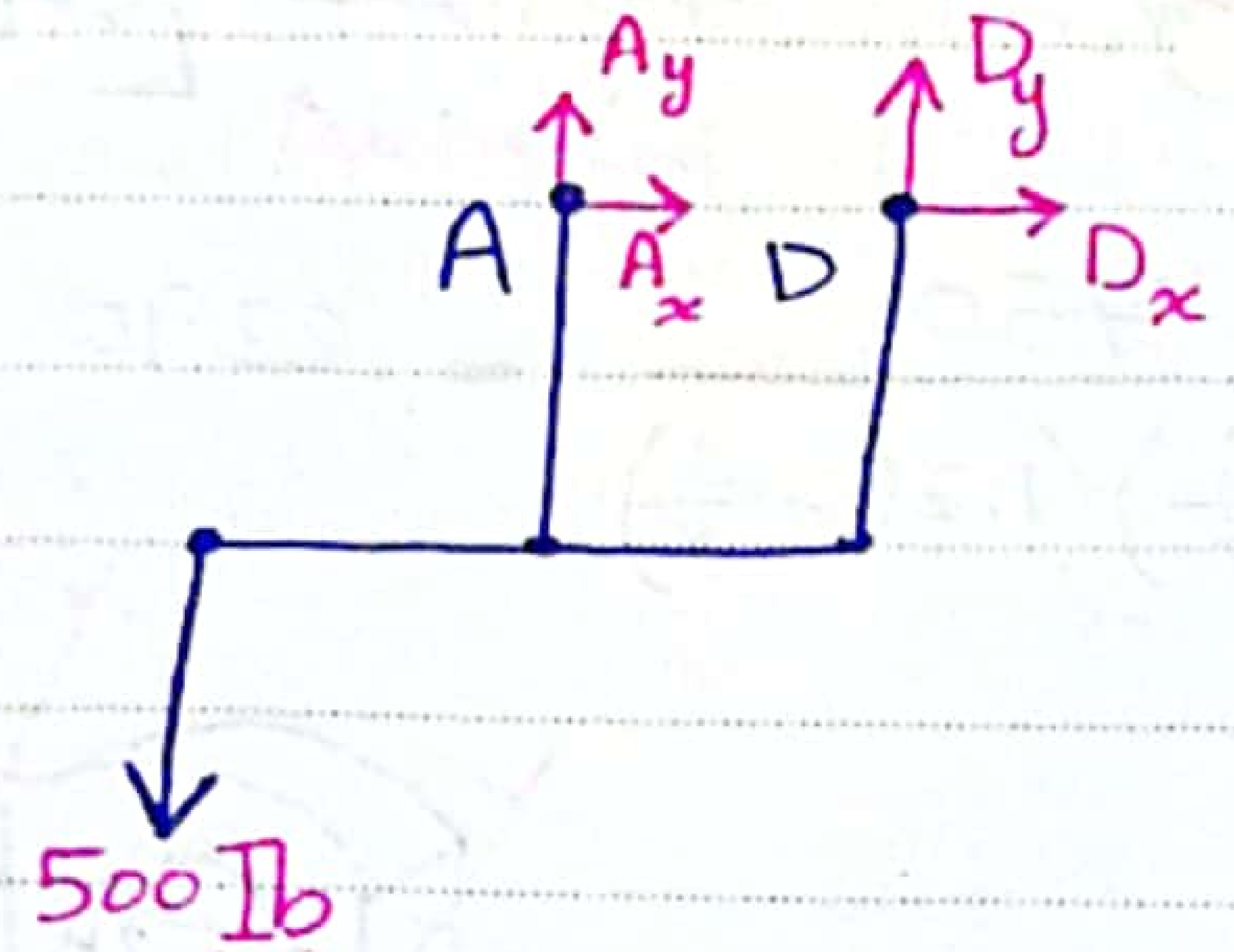
In the hanger shown, the upper portion of link ABC is $\frac{3}{8}$ inch thick and the lower portions are each $\frac{1}{4}$ inch thick.

Epoxy resin is used to bond the upper and lower portions together at B. The pin at A is of $\frac{3}{8}$ inch diameter while a $\frac{1}{4}$ inch diameter pin is used at C. Determine:

a) the shearing stress in pin A ?
b) 〃 〃 〃 〃 〃 C ?
c) 〃 largest normal stress in link ABC ?
d) 〃 average shearing stress on the bonded surfaces at B ?
e) the bearing stress in the link at C ?



→ free body diagram



(Fixed point) عند ال
Reaction مجهولات ال
Forces

- $\circlearrowright^{+}$ $\Sigma M_D = \emptyset$

$$-A_y(10) + 500(15) = \emptyset \Rightarrow \boxed{A_y = 750 \text{ Ib}}$$

- $\Sigma F_y = \emptyset$

$$A_y + D_y - 500 = \emptyset \Rightarrow \boxed{D_y = -250 \text{ Ib}}$$

a) $\tau_A = \frac{F_{AC}}{A} = \frac{750 \text{ Ib}}{\frac{\pi}{4}\left(\frac{3}{8}\right)^2} = 6790 \text{ psi}$

$\left[\frac{3}{8} \text{ inch}\right]$ → diameter pin which is in single shear...

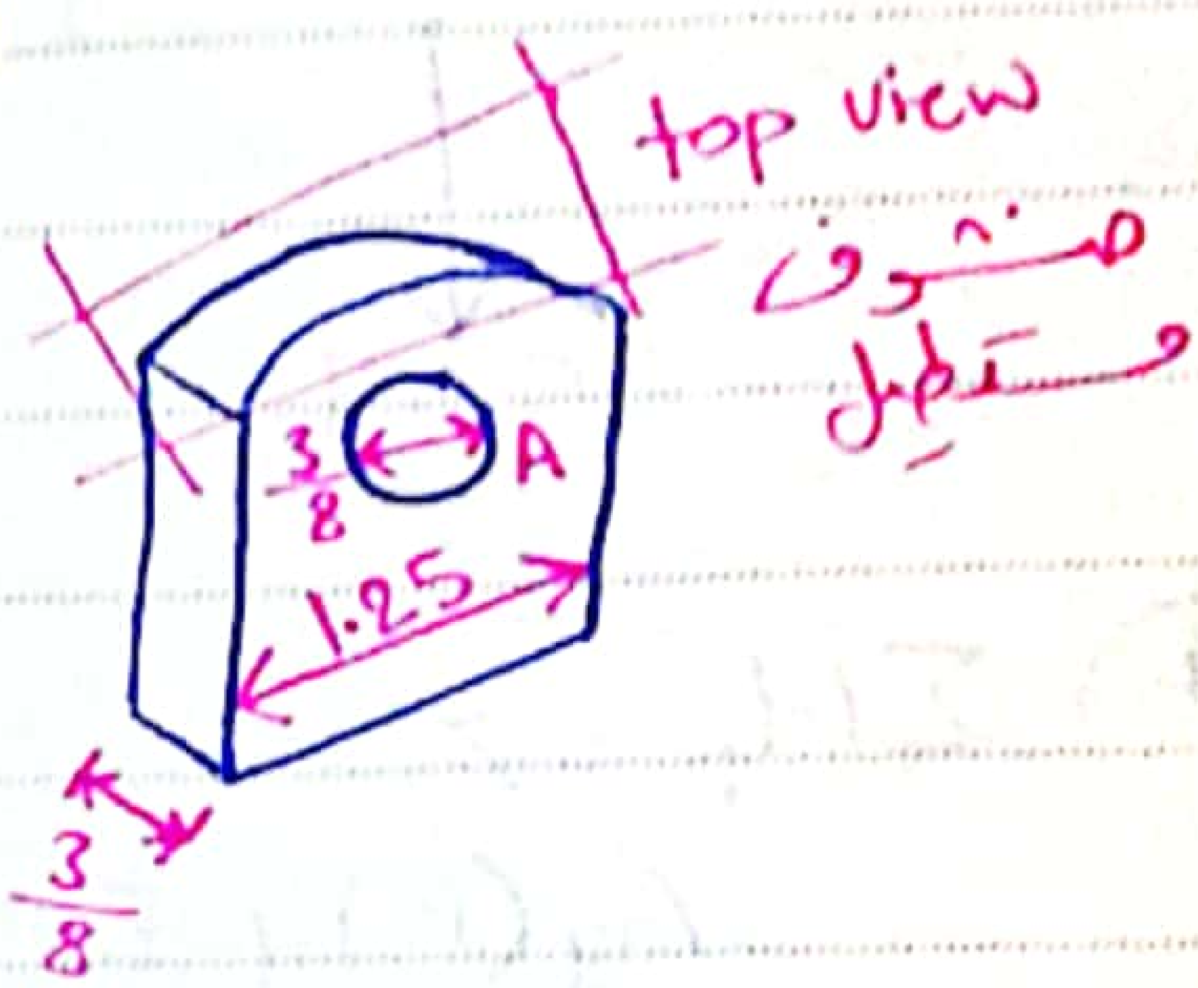
b) $\tau_C = \frac{\frac{1}{2} F_{AC}}{A} = \frac{750/}{\frac{\pi}{4}\left(\frac{1}{4}\right)^2} = 7640 \text{ psi}$

diameter pin which is in double shear...

c) largest normal stress in the link ABC occurs where the area is smallest [at A]

أصغر منطقة

$$\sigma_A \Big)_{\text{normal}} = \frac{F_{AC}}{A_{net}} = \frac{750}{\left(\frac{3}{8}\right)\left(1.25 - \frac{3}{8}\right)} = 2290 \text{ psi}$$



d) $\tau_B = \frac{F}{A} = \frac{750/2}{1.25 * 1.75}$

because there are 2 plates

at B : double shear

أضفت الـ $\boxed{\text{area}}$ هون كونه ما في برغي فرح تكون المساحة تاعة الـ

[Contact Surface]

e) $\sigma_b \Big|_{\text{at } C} = \frac{F}{A} = \frac{750/2}{\underset{t}{0.25} * \underset{d}{0.25}}$

$= 6000$ psi

for the two brackets → $f = 750$ Ib

Sample Problem (1.2) ⇒

The steel tie bar shown is to be designed to carry a tension force of magnitude $P = 120\ kN$ when bolted between double brackets at A and B. The bar will be fabricated from 20 mm thick plate stock. For the grade of steel to be used, the max. allowable stresses are : $\boxed{\sigma = 175\ MPa,\ \tau = 100\ MPa,\ \sigma_b = 350\ MPa}$.

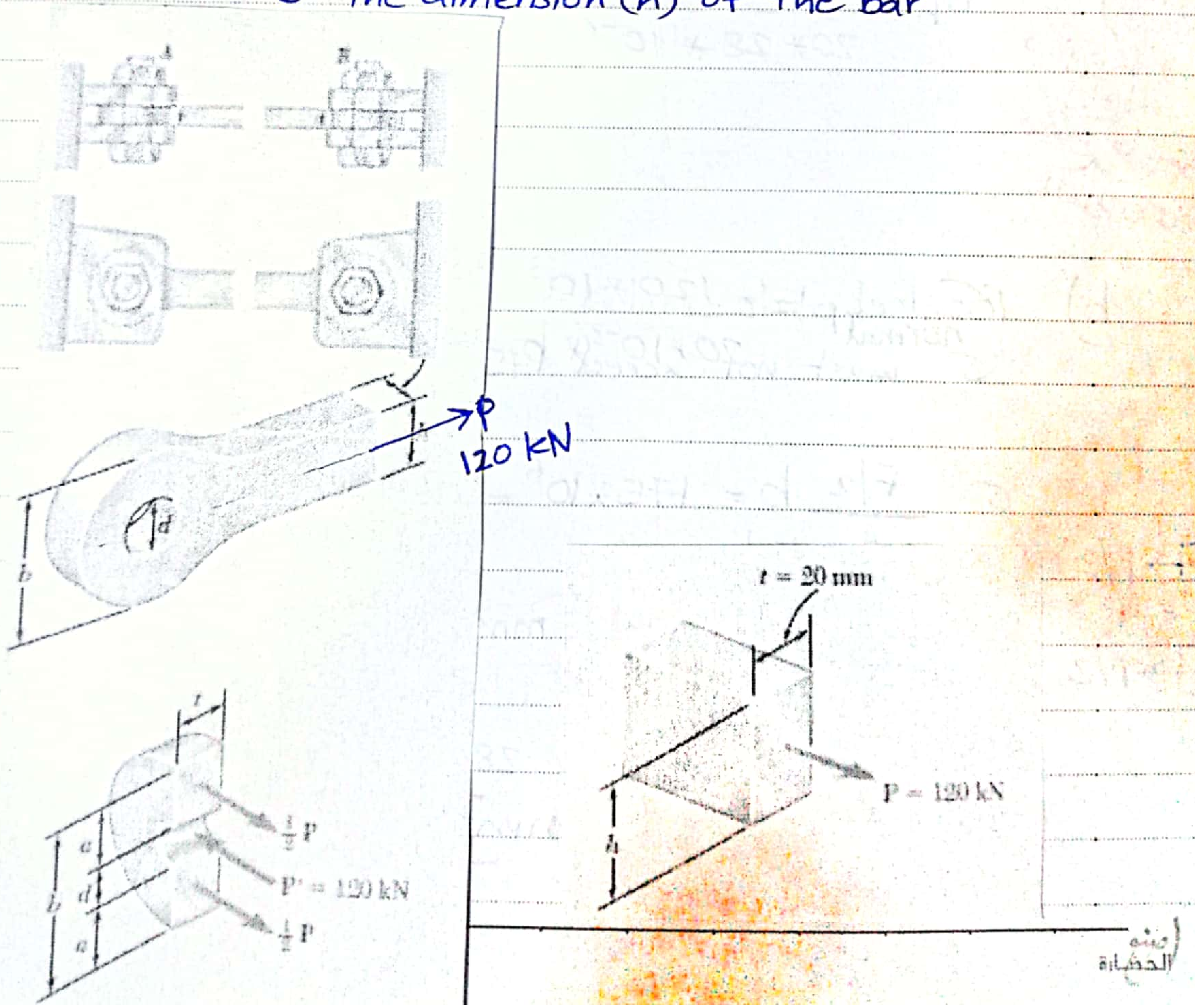
double shear

Design the tie bar by determining the required values of :

a- the diameter (d) of the bolt

b- the dimension (b) at each end of the bar

c- the dimension (h) of the bar



a) the bolt is in double shear

$$F_1 = \frac{1}{2} P = 60 \text{ kN}$$

120/2

$$\tau = \frac{F_1}{A} = \frac{60 \text{ kN}}{\frac{\pi}{4} d^2} = 100 \text{ MPa}$$

$$\boxed{d = 27.6 \text{ mm}}$$

منصحّحة على 28 mm

فنشيّك على الـ σ_b من خلال قيمة الـ d اللي أوجدناها

لأنه الـ (bearing stress) صايرة على الـ $\boxed{\text{hole}}$ مطرح ما بتضغط الفورس كاملة

$$\sigma_b = \frac{120 * 10^3 \text{ N}}{20 * 28 * 10^{-6}} = 214 \text{ MPa} < 350 \text{ MPa}$$

الديزاين صحيح ✓

b) the steel plate thickness = 20 mm

σ_{avg} ~~must~~ not exceed 175 MPa

$$\sigma = \frac{F/2}{A \rightarrow t\,d} \Rightarrow 175 * 10^6 = \frac{60 * 10^3 \text{ N}}{(0.02)\, a}$$

→ P/2

← P

→ P/2

$$a = 17.14 \text{ mm}$$

$$b = d + 2a = 28 \text{ mm} + 2(17.14) \text{ mm}$$

$$\boxed{b = 62.3 \text{ mm}}$$ #

c) Dimension h of the bar

$$\sigma = \frac{P}{t\,h} \Rightarrow 175\text{ MPa} = \frac{120\text{ kN}}{0.02 * h}$$

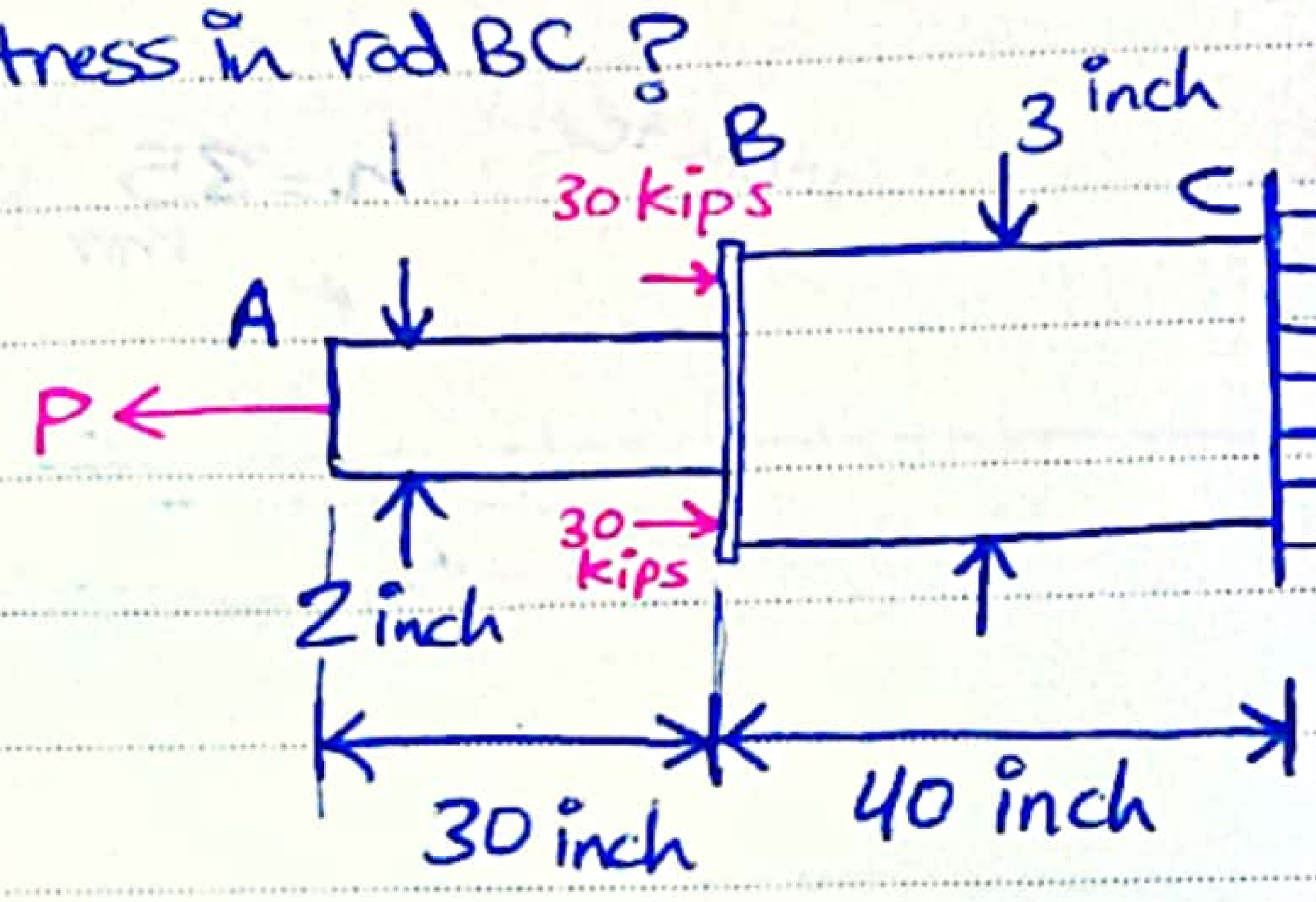
$$\boxed{h = 34.3\text{ mm}}$$ #

ultimate $h = 35$ mm فنستخدم

Problems / /

1.3 Two solid cylindrical rods AB and BC are welded together at B and loaded as shown. Determine the magnitude of the force P for which the tensile stress in rod AB has the same magnitude as the compressive stress in rod BC?

Ans : 18.2 Kips



- $A_{AB} = \frac{\pi}{4}(2)^2 = 3.14 \text{ in}^2$

$$\sigma_{AB} = \frac{P}{3.14} = 0.318\,P$$

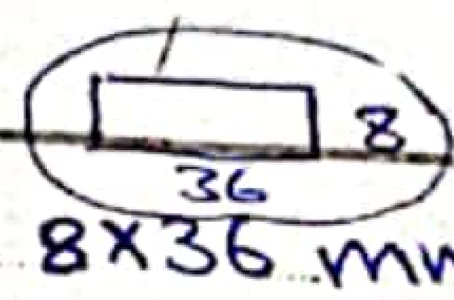
- $A_{BC} = \frac{\pi}{4}(3)^2 = 7.068 \text{ in}^2$

$$\sigma_{BC} = \frac{30(2) - P}{7.068} = 8.488 - 0.141\,P$$

$$\sigma_{AB} = \sigma_{BC}$$

$$0.318\,P = 8.488 - 0.141\,P$$

$$\boxed{P = 18.5 \text{ Kips}}$$



1.7 Each of the four vertical links has an 8×36 mm uniform rectangular cross section and each of the four pins has a 16 mm diameter. Determine the max. value of the average normal stress in the links connecting

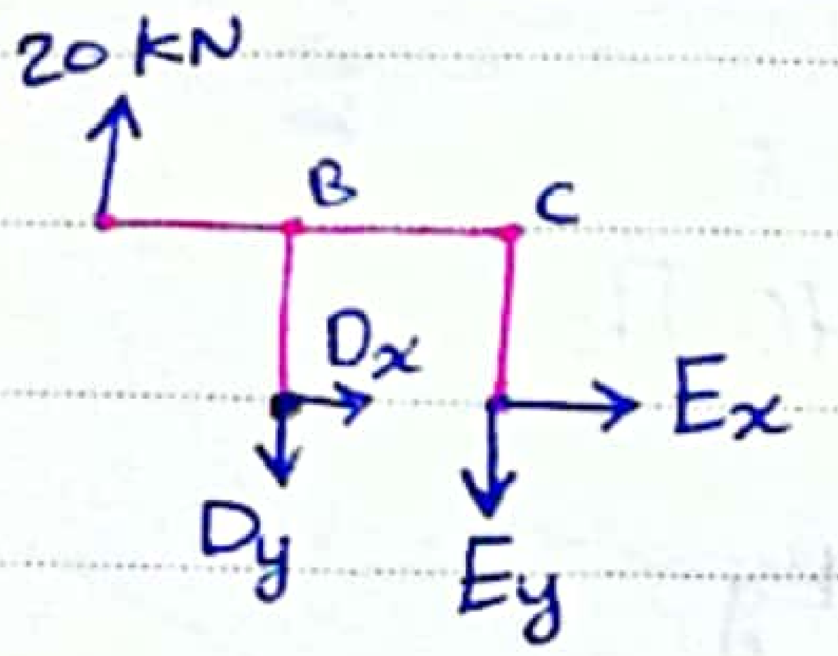
a) points B and D

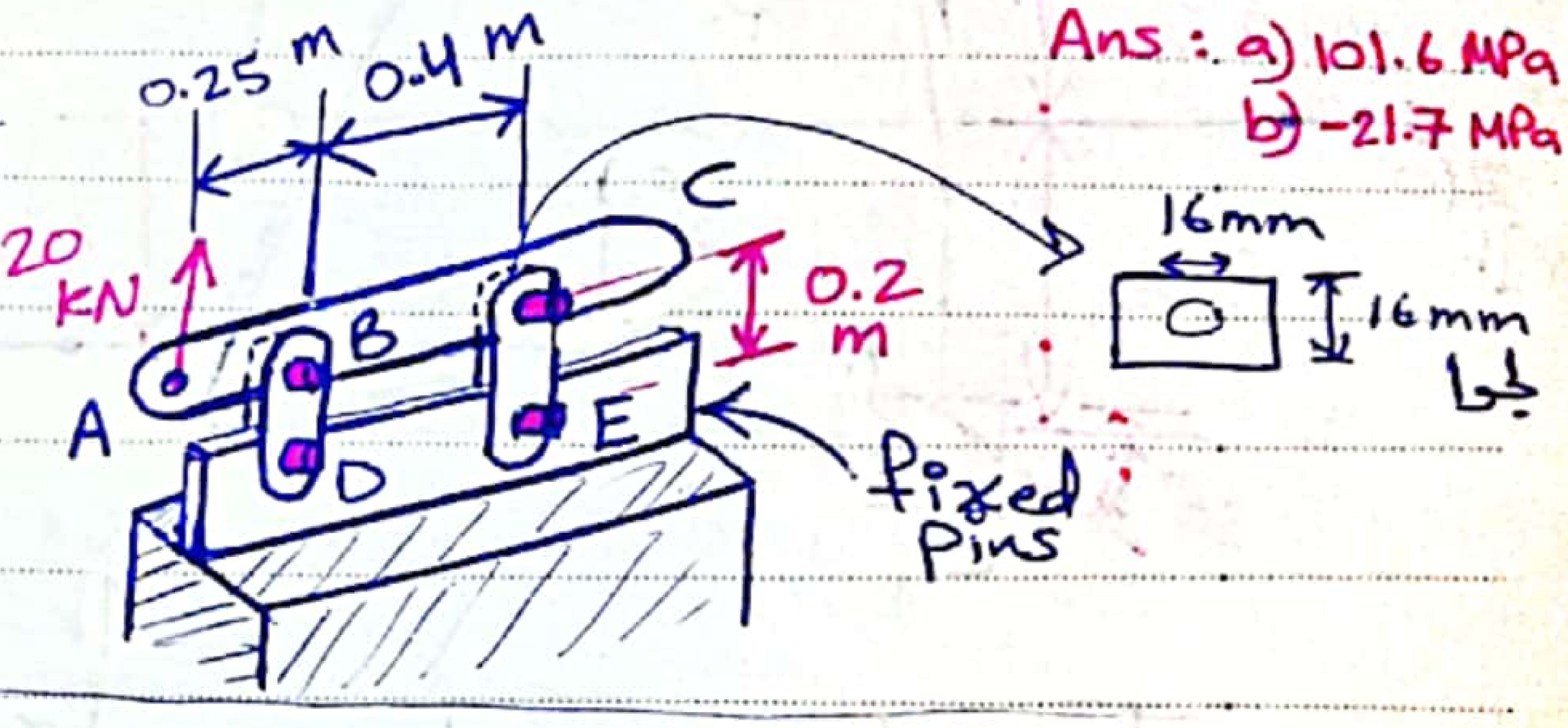
b) points C and E

Ans: a) 101.6 MPa
b) −21.7 MPa

Sol

FBD:





• $\Sigma M_E = \emptyset \rightarrow$ (+)

$$-D_y(0.4) + 20{,}000(0.4+0.25) = \emptyset \quad \boxed{D_y = 32{,}500\ N} \text{ tension}$$

• $\Sigma F_y = \emptyset \rightarrow \quad 20{,}000 - D_y - E_y = \emptyset \quad \boxed{E_y = -12{,}500\ N}$ Compression.

• $\Sigma F_x = \emptyset \rightarrow \quad E_x + D_x = \emptyset$

* Net area of one link for tension $= 0.008(0.036 - 0.016)$

$= 160 * 10^{-6}\ m^2$

For 2 parallel links ∴

$$\boxed{A_{net} = 320 * 10^{-6}\ m^2}$$

a) $\sigma_{BD} = \dfrac{F}{A}$

$$= \frac{32{,}500\ N}{(0.008)(0.036-0.016)2}$$

$= 101.6$ MPa

b) Area for one link in compression $= (0.008)(0.036) = 288 * 10^{-6}\ m^2$

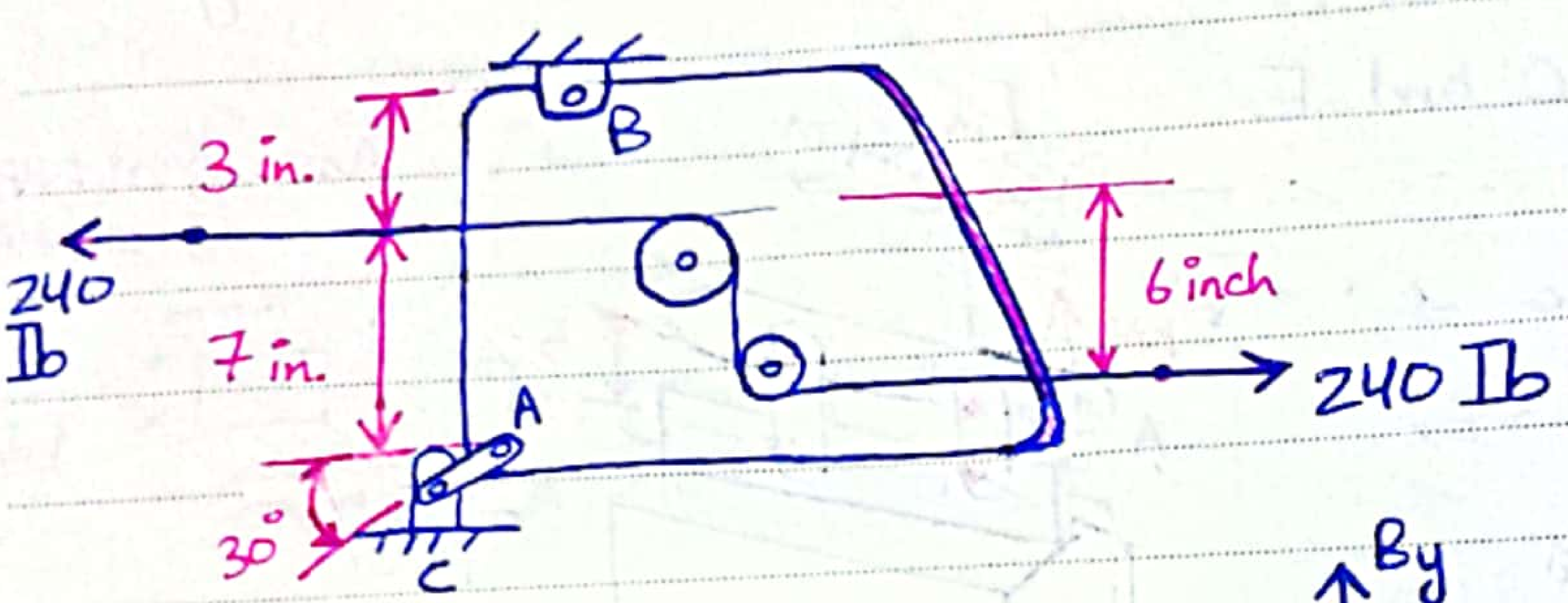
For 2 parallel links, $A = 576 * 10^{-6}\ m^2$

$$\sigma_{CE} = \frac{-12{,}500}{576 * 10^{-6}} = -21.7\ \text{MPa}$$

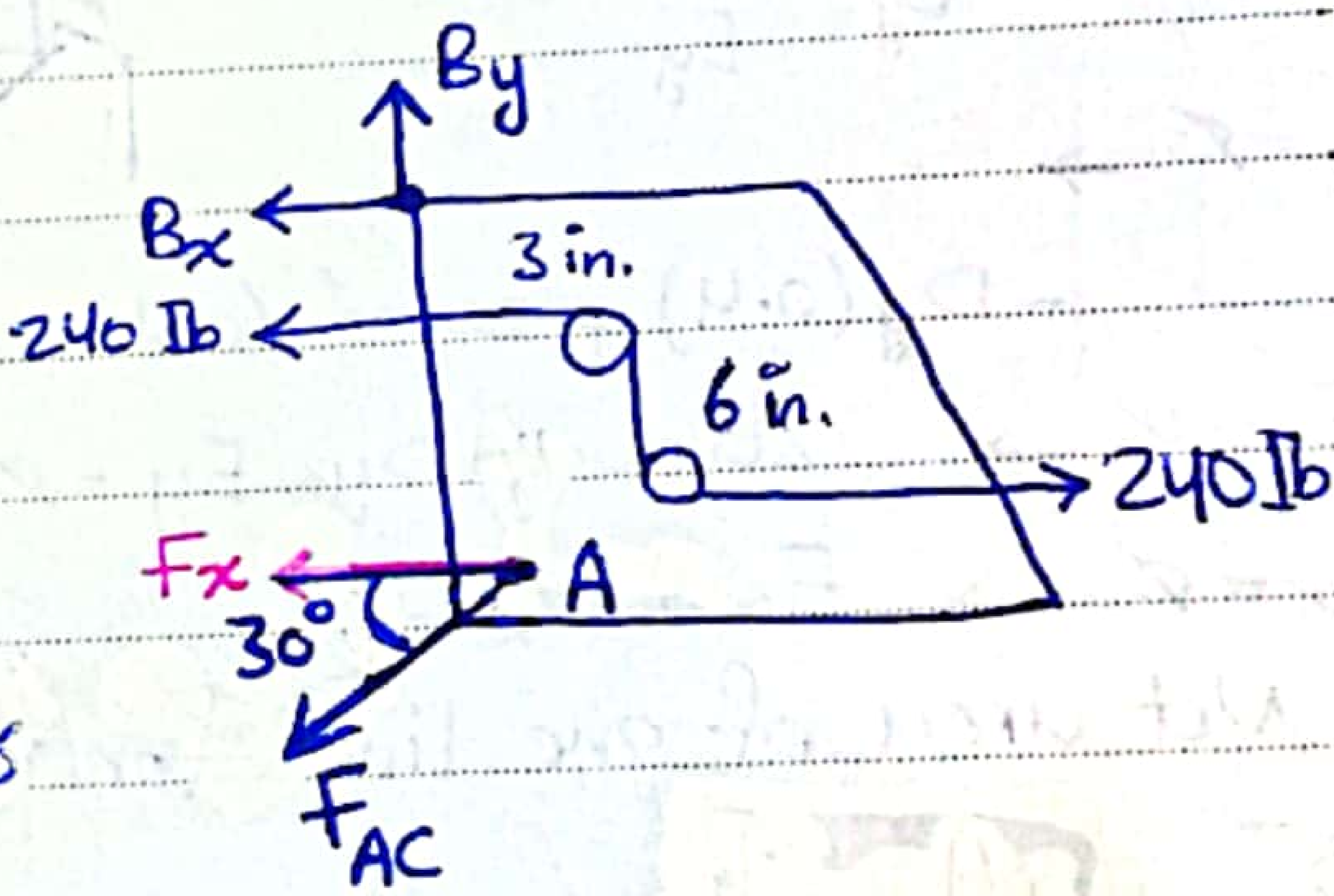
/ /

1.9 Link AC has a uniform rectangular cross section $\frac{1}{16}$ in. thick and $\frac{1}{4}$ in. wide. Determine the normal stress in the central portion of the link?

Ans : 10.64 ksi



$$\left[\sigma_{AC} = \frac{F_{AC}}{A}\right]$$



- $\Sigma M_B = \emptyset$ (+)

$$-240(3) - F_x(10) + 240(9) = \emptyset$$

$$F_x = 144 \text{ Ib}$$

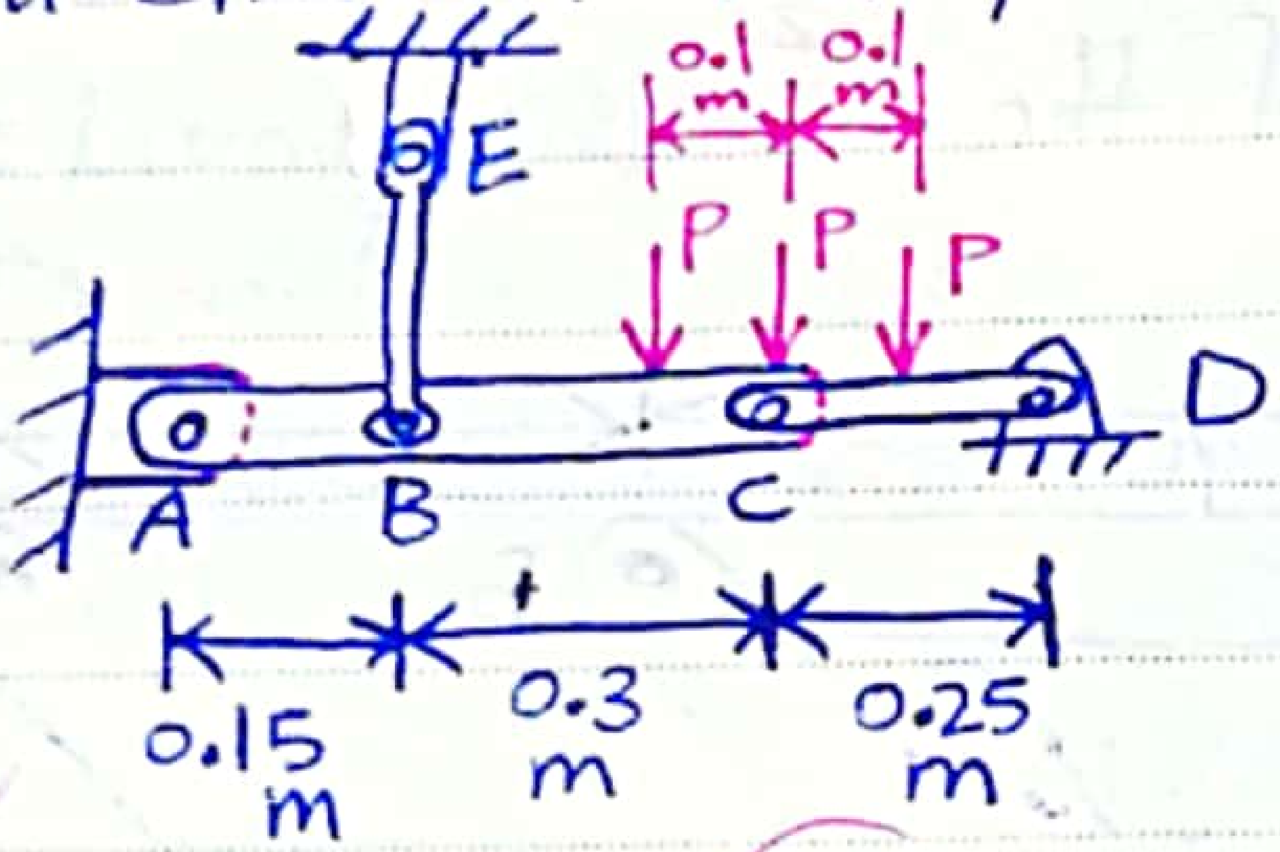
where, $F_x = F_{AC}\cos(30)$

$$F_{AC} = \frac{144}{\cos(30)} = 166.28 \text{ Ib}$$

$$\sigma_{AC} = \frac{166.28 \text{ Ib}}{\frac{1}{16} * \frac{1}{4}} = 10.64 \text{ ksi}$$

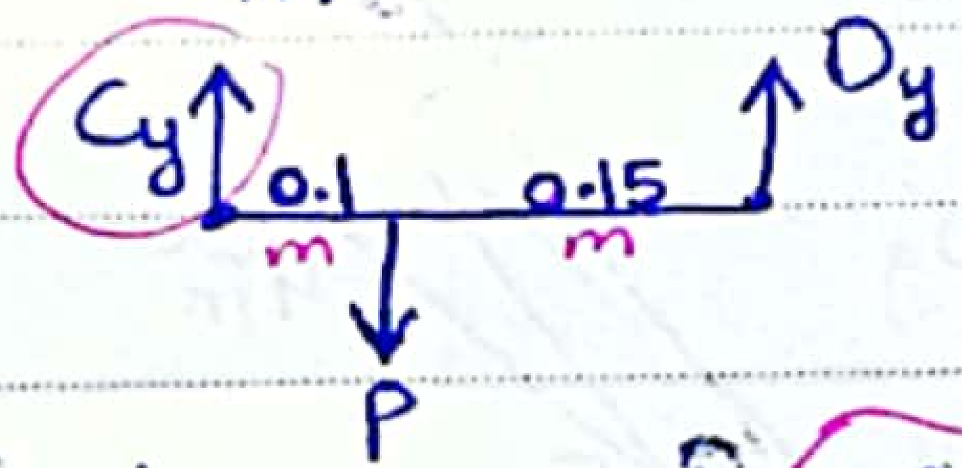
#

1.10 Three forces, each of magnitude $\boxed{P = 4\ kN}$, are applied to the mechanism shown. Determine the cross sectional area of the uniform portion of rod BE for which the normal stress in that portion is +100 MPa ?



Ans : 285 mm²

→ FBD for CD Link :

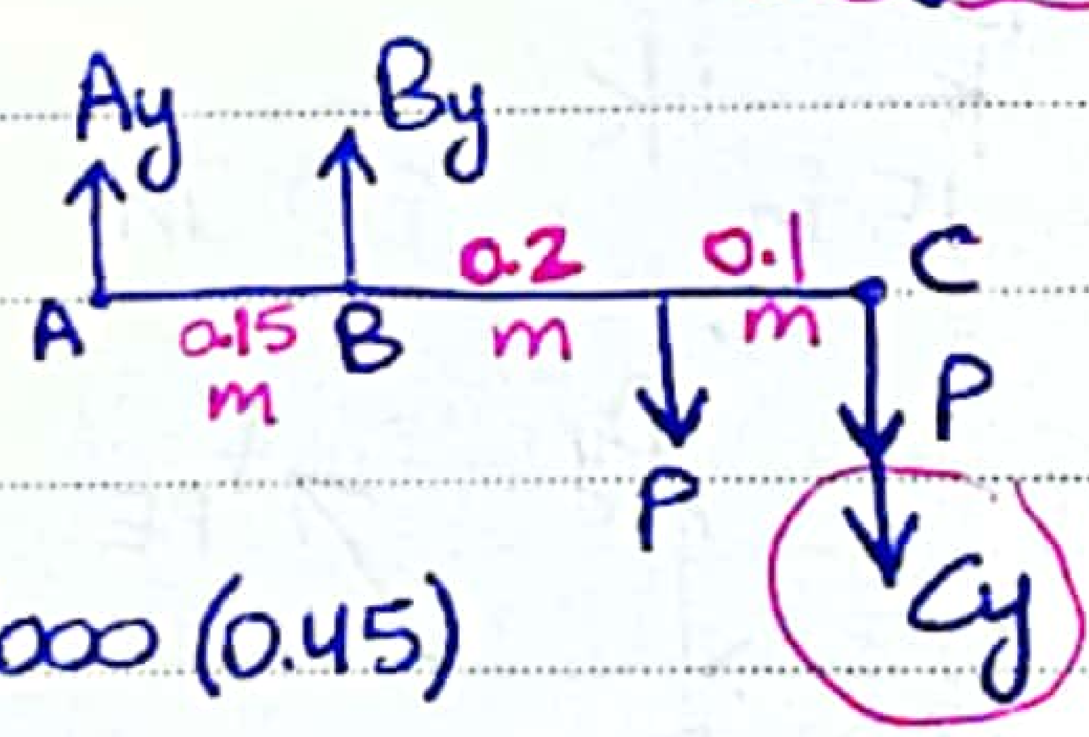


• $\Sigma M_D = \emptyset$ (+)

$C_y(0.1 + 0.15) - 4000(0.15) = \emptyset$

$C_y = 2400\ N$

→ FBD for link ABC :



• $\Sigma M_A = \emptyset$ (+)

$B_y(0.15) - 4000(0.35) - 4000(0.45) - 2400(0.45) = \emptyset$

$B_y = 28.53\ kN$

⇒ Remember

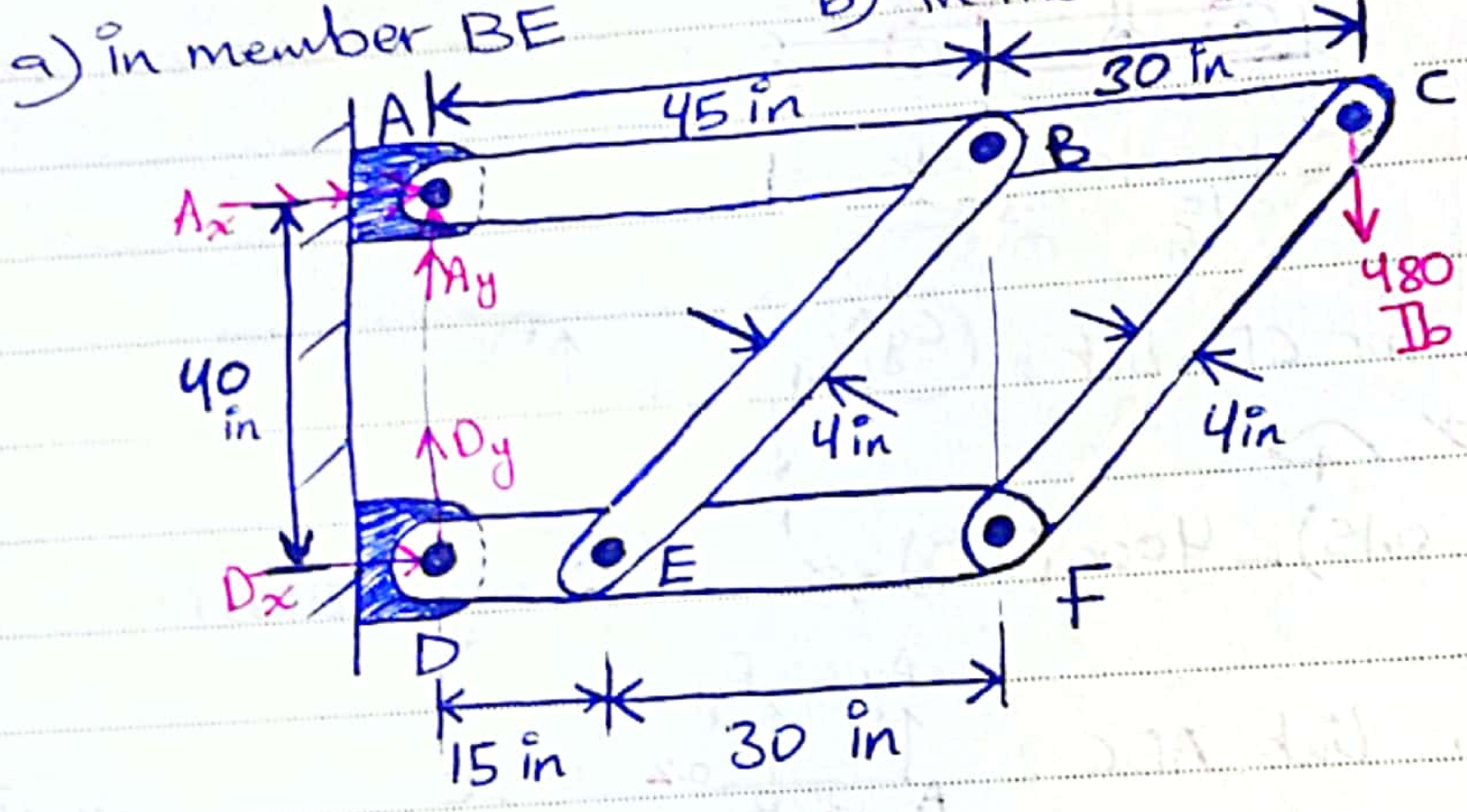
أي اتجاه بفرضه ع (link) معين ، لازم أعمل عكسه على الـ (link) الآخر التابع إله ..

$$100 * 10^6 = \frac{28.53 * 10^3}{A\ [m^2]}$$

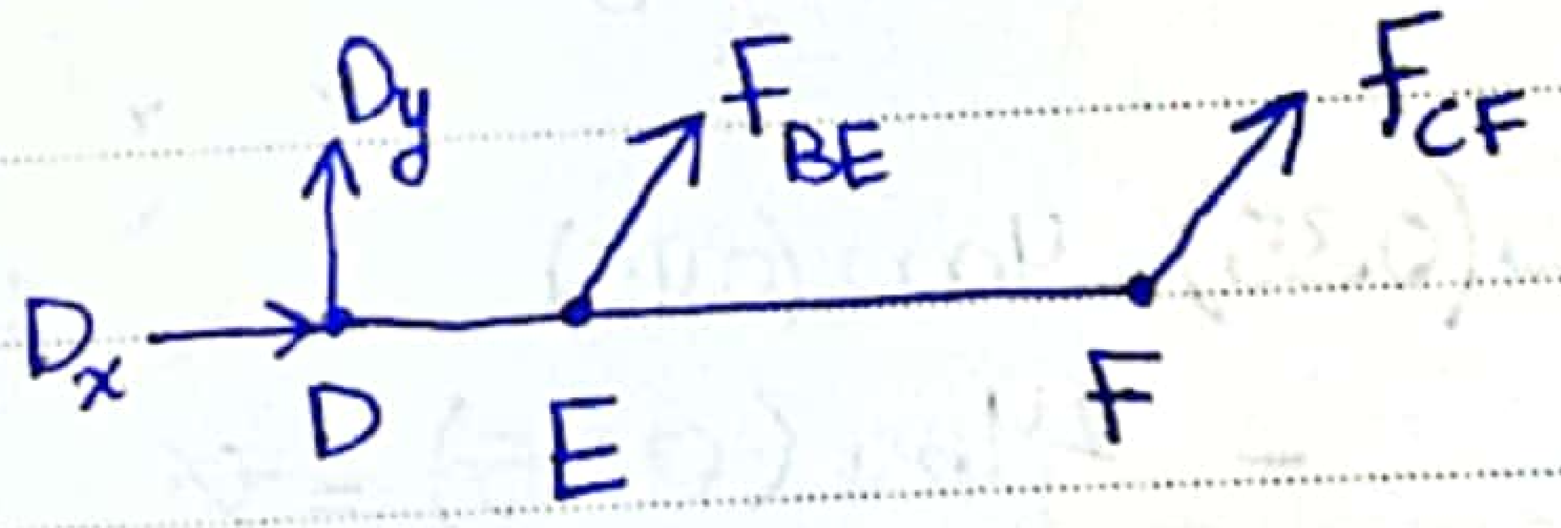
$$A = 2.85 * 10^{-4}\ \cancel{m^2} * \frac{1\ cm^2}{10^{-6}\ \cancel{m^2}}$$

$$= 285\ mm^2$$

1.11 The frame shown consists of four wooden members, ABC, DEF, BE and CF. Knowing that each member has a 2×4 in. rectangular cross section and that each pin has a $\frac{1}{2}$ in. diameter, determine the max. value of the average normal stress
a) in member BE b) in member CF



FBD :



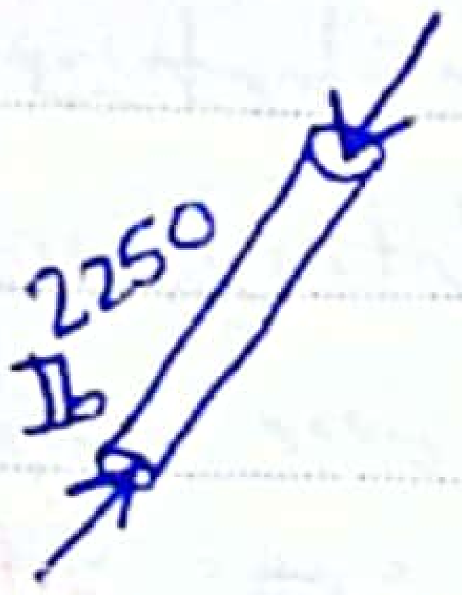
$\circlearrowleft + \ \Sigma M_A = \emptyset \Rightarrow 40 D_x - (30+45)\,480 = \emptyset$ (للشكل كامل قوة)

$* \quad D_x = 900 \text{ lb}$

using DEF member, the reaction at D must be Parallel to F_{BE} and F_{CF}

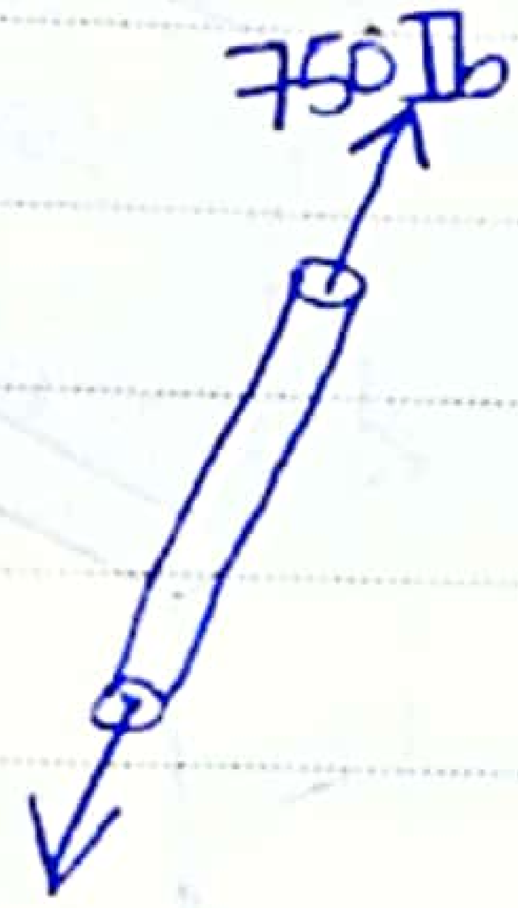
* $D_y = \frac{4}{3} D_x = 1200 \text{ lb}$

$+\sum M_F = 0 \Rightarrow -30\left(\frac{4}{5} F_{BE}\right) - (30+15) D_y = 0$



$F_{BE} = -2250 \text{ lb}$ compression force

$+\sum M_E = 0 \Rightarrow 30\left(\frac{4}{5} F_{CF}\right) - 15(D_y) = 0$



$F_{CF} = 750 \text{ lb}$ tension force

a) $\sigma_{BE} = \dfrac{-2250\,[\text{lb}]}{2 * 4\ [\text{in}^2]} = -281 \text{ psi}$

min. section area occurs at pin

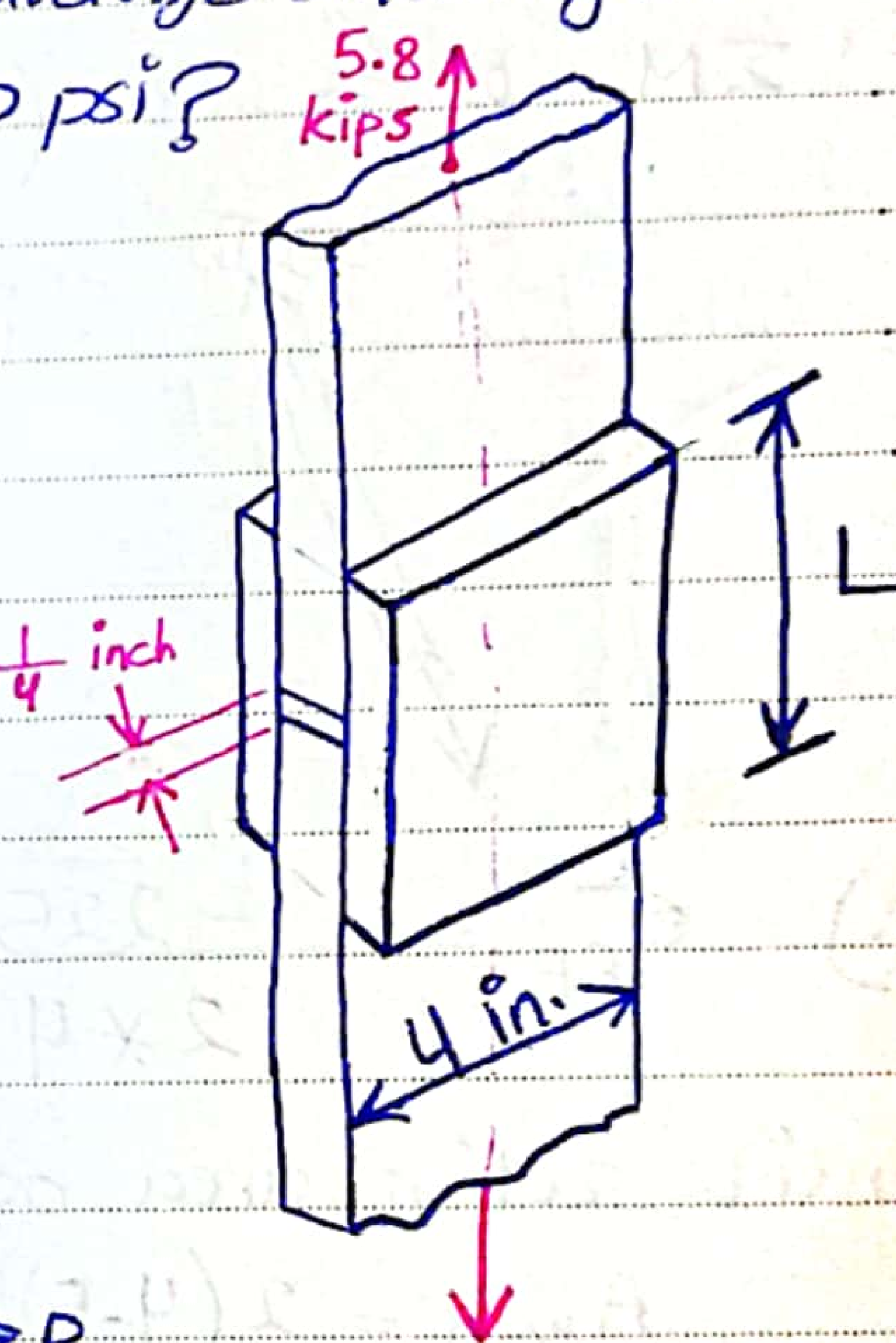
$A_{min} = 2(4 - 0.5) = 7 \text{ in}^2$

b) $\sigma_{CF} = \dfrac{750\ [\text{lb}]}{7\ [\text{in}^2]} = 107.1 \text{ psi}$

/ /

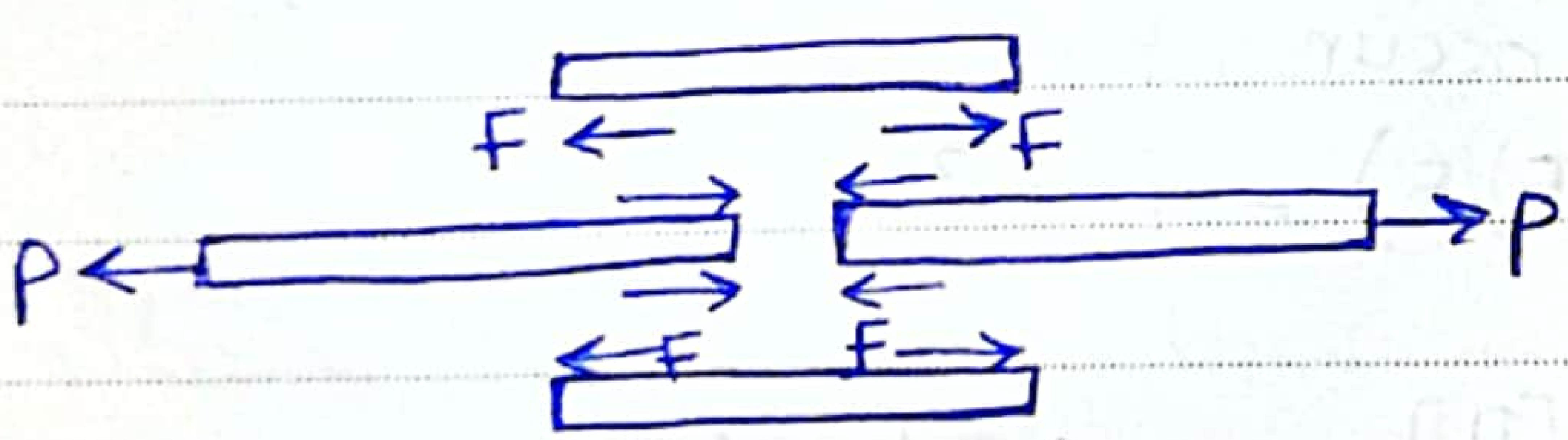
1.16 The wooden members A and B are to be joined by plywood splice plates that will be fully glued on the surfaces in contact. As part of the design of the joint, and knowing that the clearance between the ends of the members is to be $\frac{1}{4}$ inch, determine the smallest allowable length (L) if the average shearing stress in the glue isn't to exceed 120 psi?

Ans : 12.33 inch



→ There are 4 separate areas that are glued.
Each of these areas transmits one half the 5.8 kips load. Thus

$$F = \frac{1}{2} * 5.8 = 2.9 \text{ kips}$$



let L = length of one glued area and $w = 4$ inch be its width.

For each glued area , $\boxed{A = LW}$

/ /

Avergae shearing stress : $\tau = \frac{F}{A} = \frac{F}{LW}$

$$\left[\tau_{allow} = 120\ psi\right]$$

$$L = \frac{F}{\tau W} = \frac{2900}{120 * 4\ in.} = 6.042\ in.$$

- Total length $(L_{tot}) = 6.042 + \frac{1}{4} + 6.042$

gap

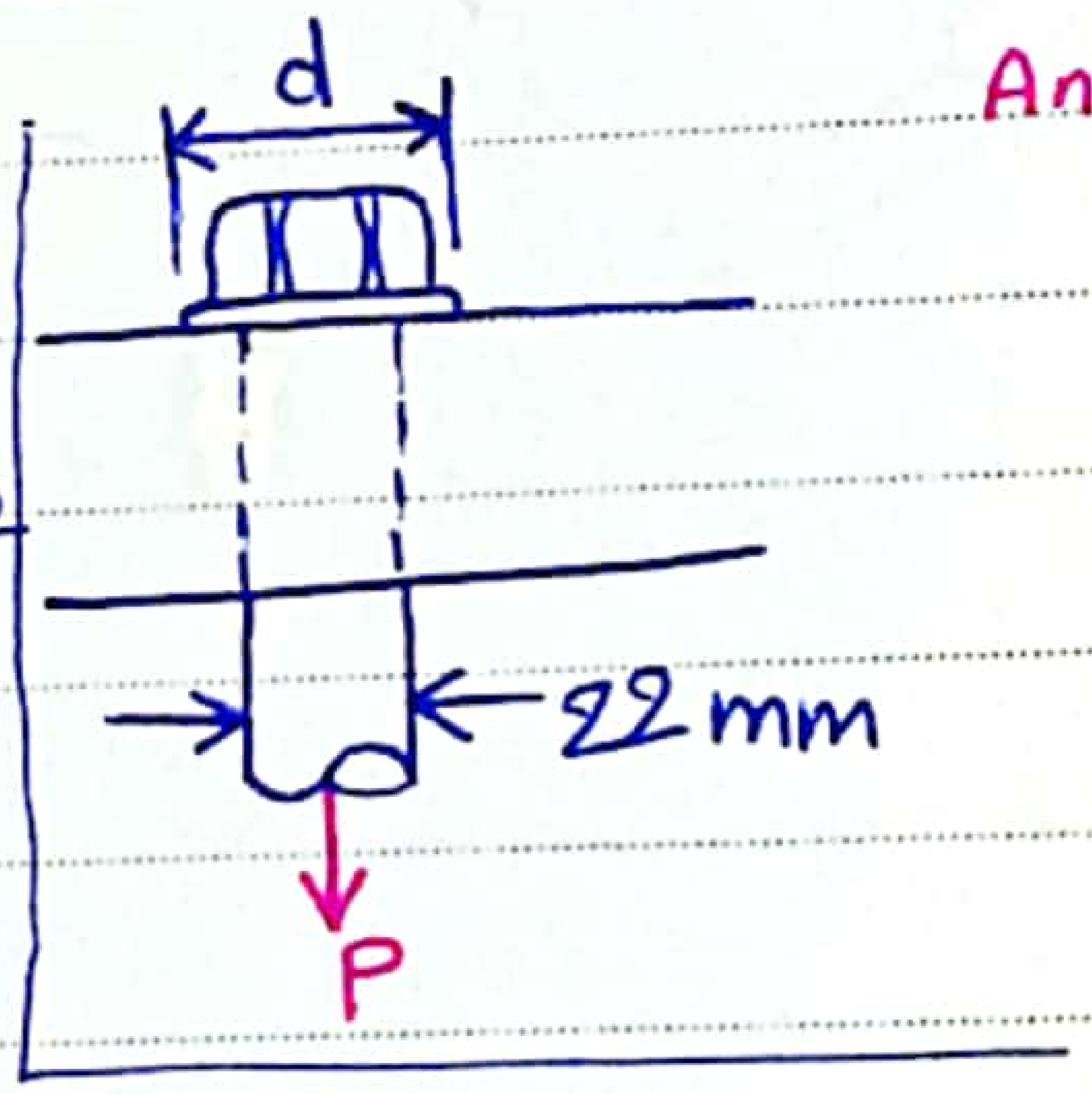
$= 12.33$ in.

#

/ /

1.19 The load P applied to a steel rod is distributed to a timber support by an annular washer. The diameter of the rod is 22 mm and the inner diameter of the washer is 25 mm, which is slightly larger than the diameter of the hole. Determine the smallest allowable outer diameter d of the washer knowing that the axial normal stress in the steel rod is 35 Mpa and that the average bearing stress between the washer & the timber must not exceed 5 MPa?

Ans : 63.3 mm



→ steel rod : $A = \frac{\pi}{4}(0.022)^2$

$= 380.13 * 10^{-6}\ m^2$

$\boxed{\sigma = 35 * 10^6\ pa}$ given

$P = (35 * 10^6)(380.13 * 10^{-6})$

$= 13.305 * 10^3\ N$

Washer : $\boxed{\sigma_b = 5 * 10^6\ Pa}$ given

required bearing area : $A_b = \frac{P}{\sigma_b} = \frac{13.305 * 10^3\ N}{5 * 10^6\ Pa}$

$= 2.6609 * 10^{-3}\ m^2$

But, $A_b = \frac{\pi}{4}(d^2 - d_i^2)$

$\sqrt{d^2 = d_i^2 + \frac{4A_b}{\pi}} = \sqrt{(0.025)^2 + \frac{4(2.6609 * 10^{-3})}{\pi}}$

$63.3 * 10^{-3}\ m$ → $d = 63.3\ mm$

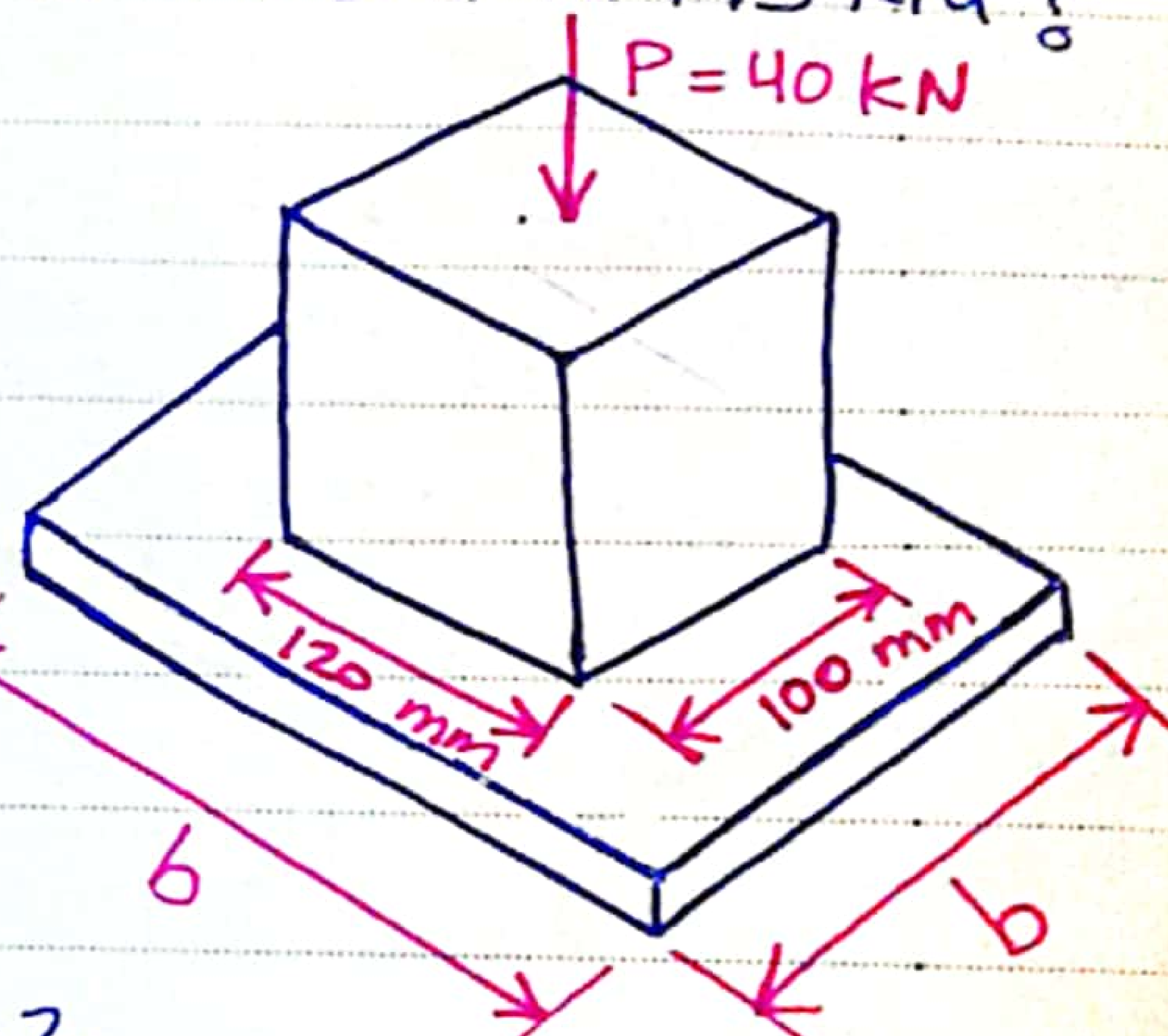
ALADIB

1.22 A 40 kN axial load is applied to a short wooden past that is supported by a concrete footing resting on undistributed soil. Determine: a) the max. bearing stress on the concrete footing

b) the size of the footing for which the average bearing stress in the soil in 145 kPa?

Ans: a) 3.33 MPa

b) 525 mm



a- Bearing stress on concrete footing

$P = 40\ kN$

$A = 100 * 120 = 12 * 10^{-3}\ m^2$

$$\sigma = \frac{40 * 10^3\ N}{12 * 10^{-3}\ m^2} = 3.33\ MPa \quad \#$$

b- footing area

$$A = \frac{P}{\sigma} = \frac{40 * 10^3\ N}{145 * 10^3\ Pa} = 0.27586\ m^2$$

Since the area is square, $\boxed{A = b^2}$ قانون مساحة المربع

$$b = \sqrt{A} = 0.525\ m = 525\ mm \quad \#$$

Ch.2 Stress & Strain – Axial Loading

يعني مارح نحكي عن ال bending/shear stresses

2.1 Introduction

In Chap1, we learned how to determine the stresses created in members and connections by the loads applied to a structure or machine.

But indeed, it's not always possible to determine the forces in the members of a structure by applying only the principles of statics.

Hence, in this chapter we will learn the analysis of deformation that will help us in the determination of stresses.

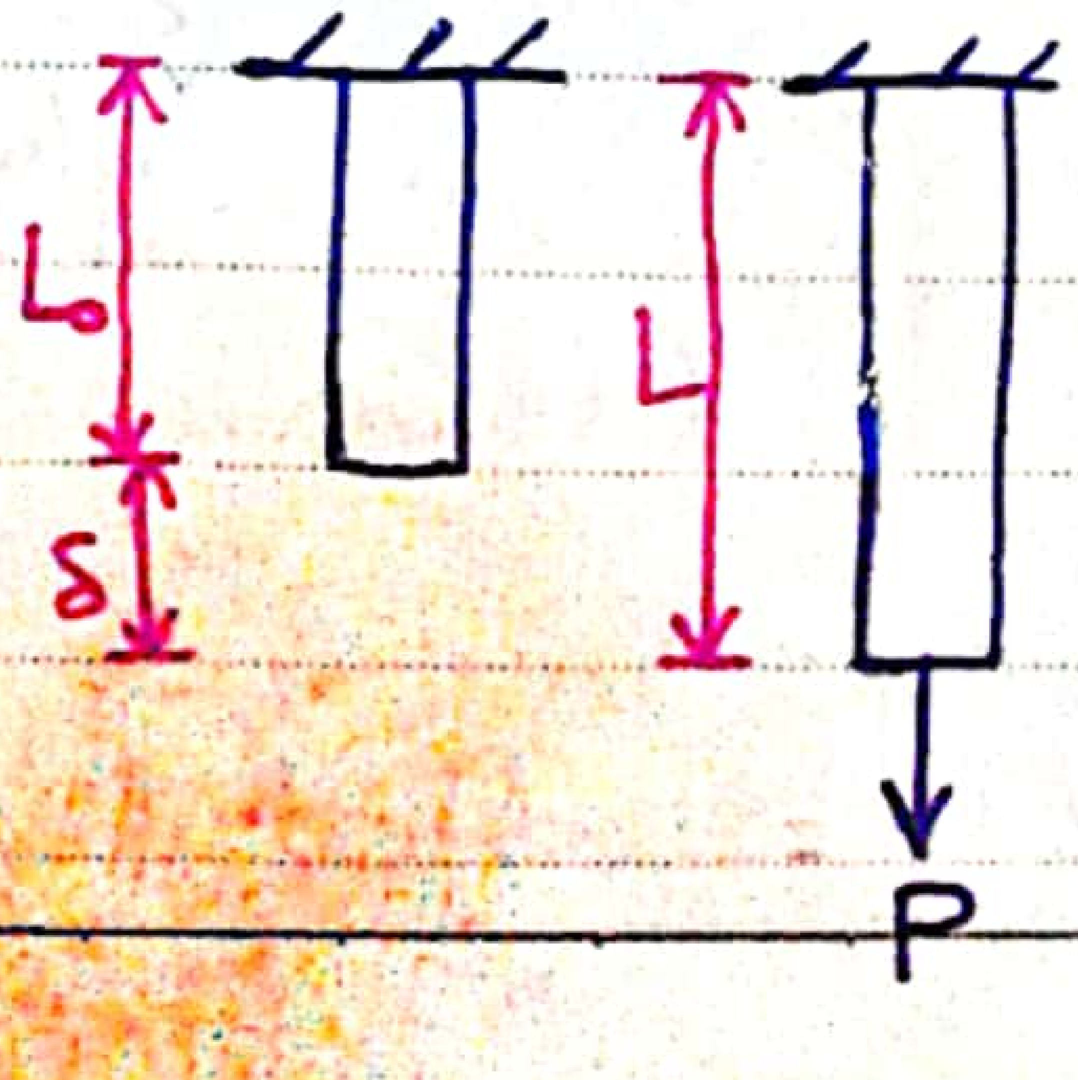
2.2 Normal Strain Under Axial Loading

the normal strain in a rod under axial Loading (ϵ) $\equiv$ the deformation per unit length of that rod

δ (deformation) $= L - L_0$

$$\epsilon = \frac{\delta}{L_0}$$

L_0 → initial length

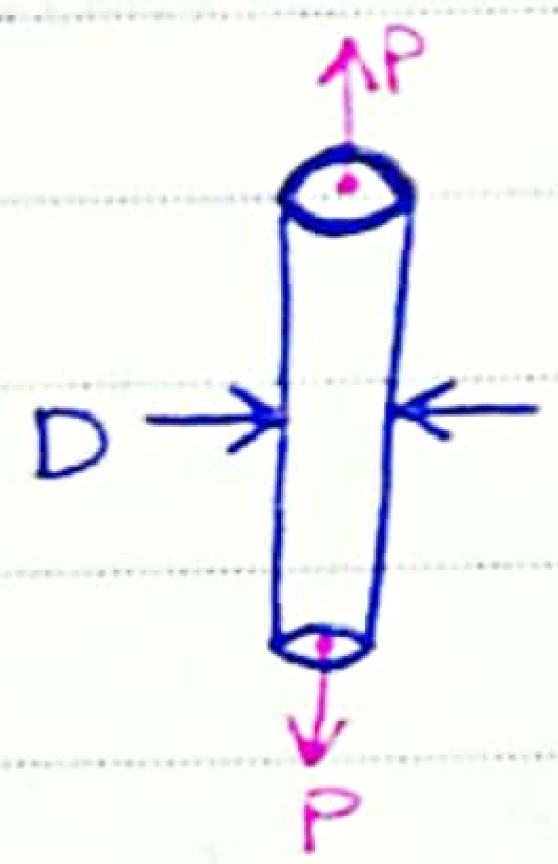


Consider, for instance, a bar of length $L = 0.6$ m and uniform cross section, which undergoes a deformation $\delta = 150 * 10^{-6}$ m. The corresponding strain is:

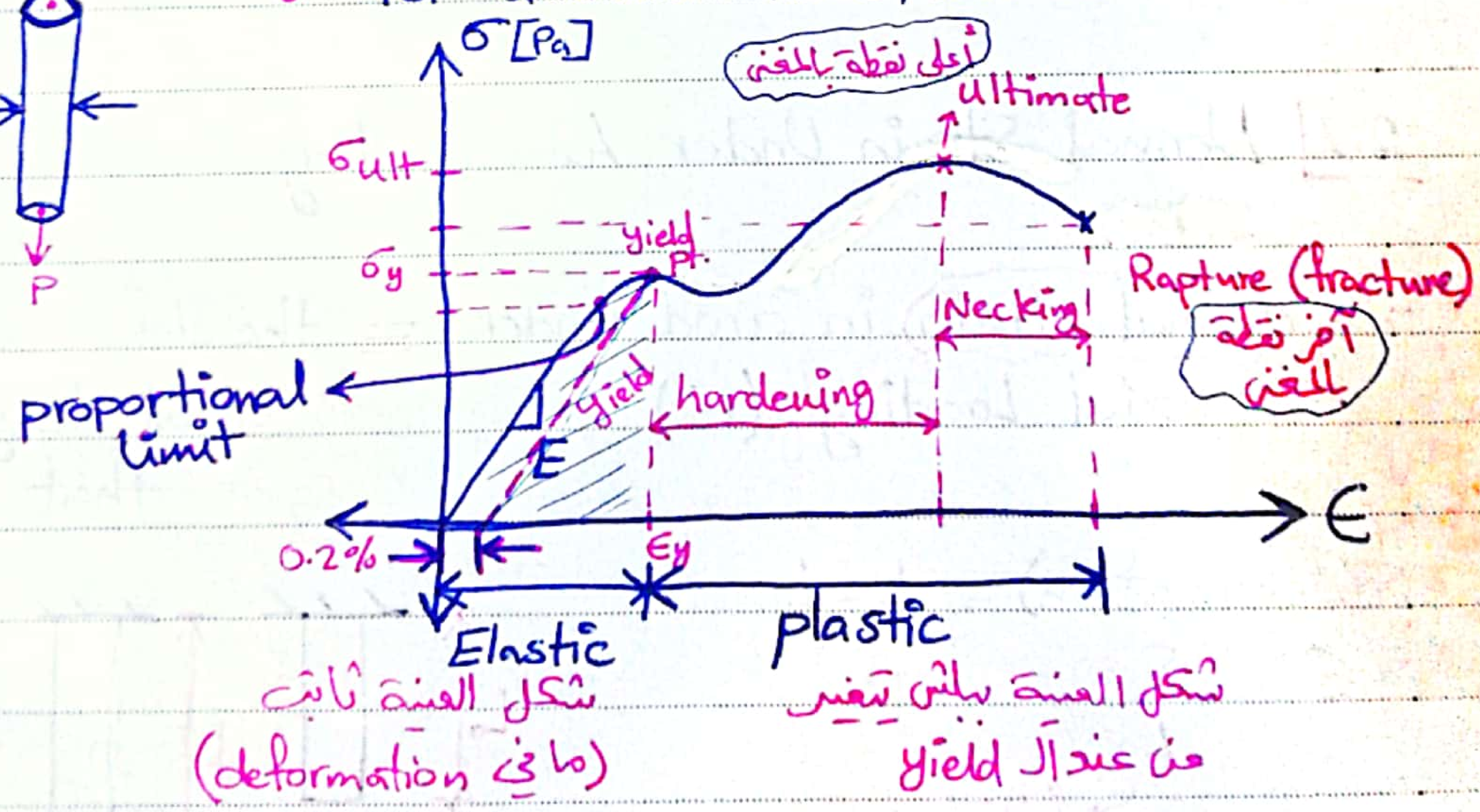
$$\epsilon = \frac{\delta}{L_o} = \frac{150 * 10^{-6}\ [\cancel{m}]}{0.6\ [\cancel{m}]} = 250 * 10^{-6}$$

2.3 | Stress-Strain Diagram

It's used to determine some important characteristics for the materials like
[E, material type. brittle or ductile., elastic or plastic deformation, ...]



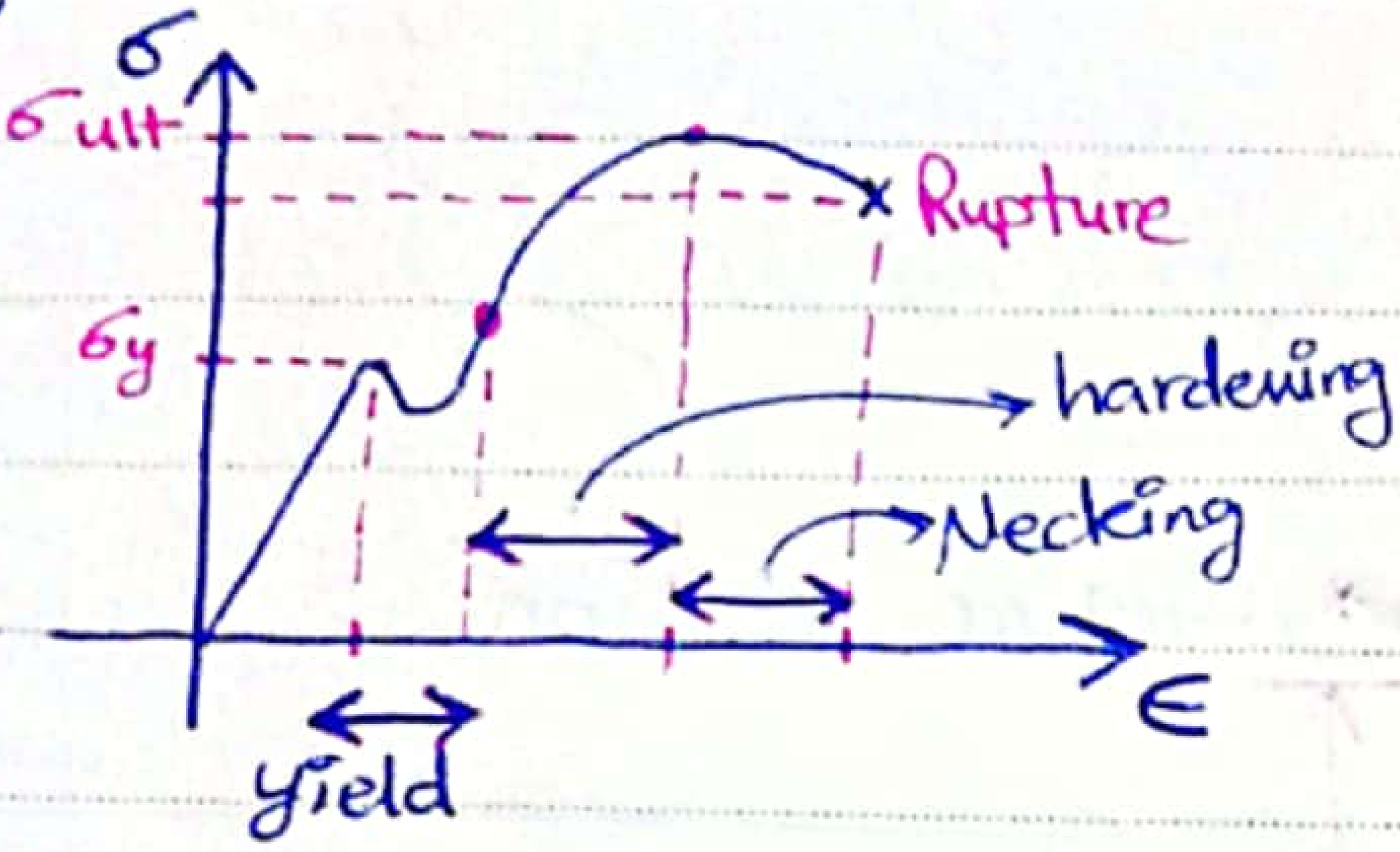
- For a tension force, ductile materials:



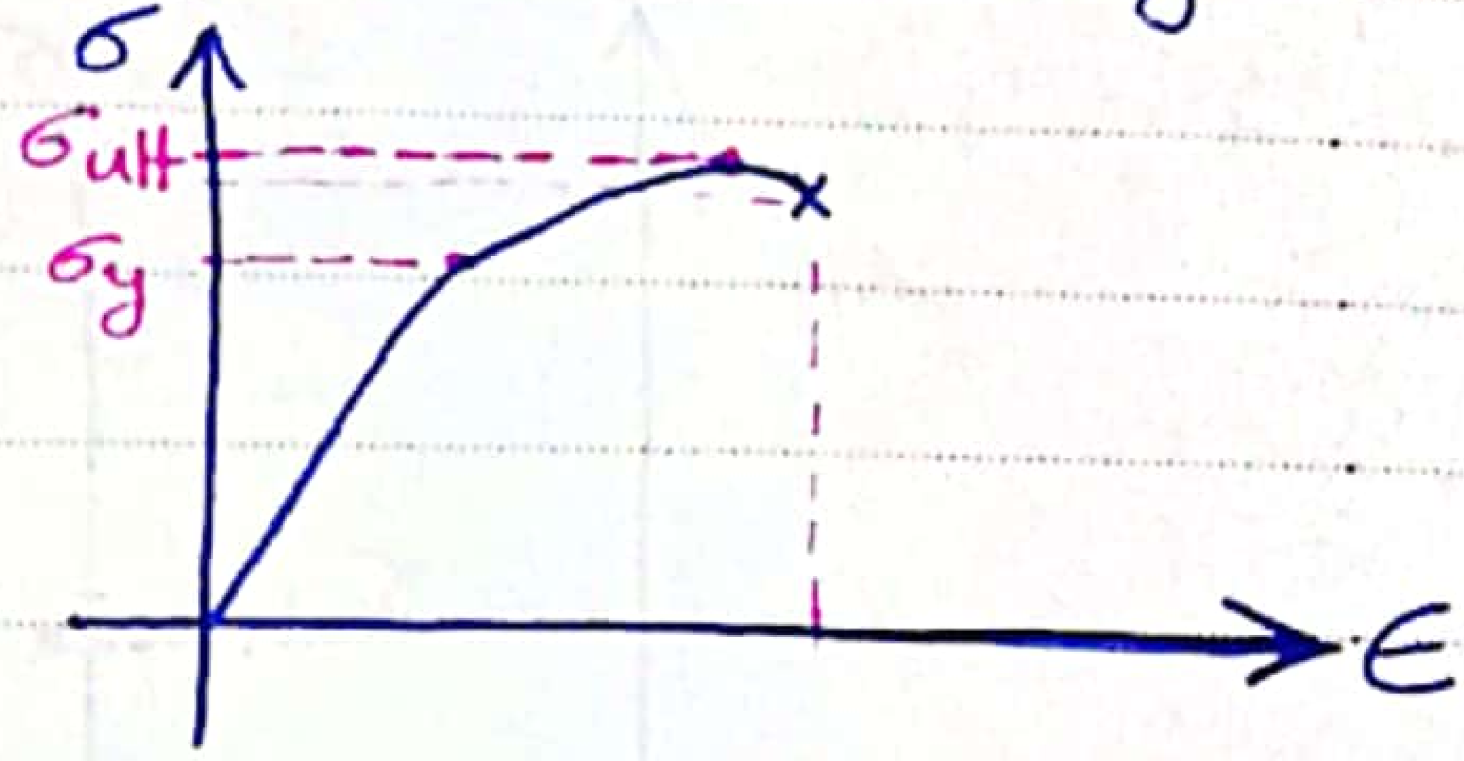
0.2 % ≡ an offset to determine the yield strength

- **Ductile Materials →**

a) Low Carbon Steel

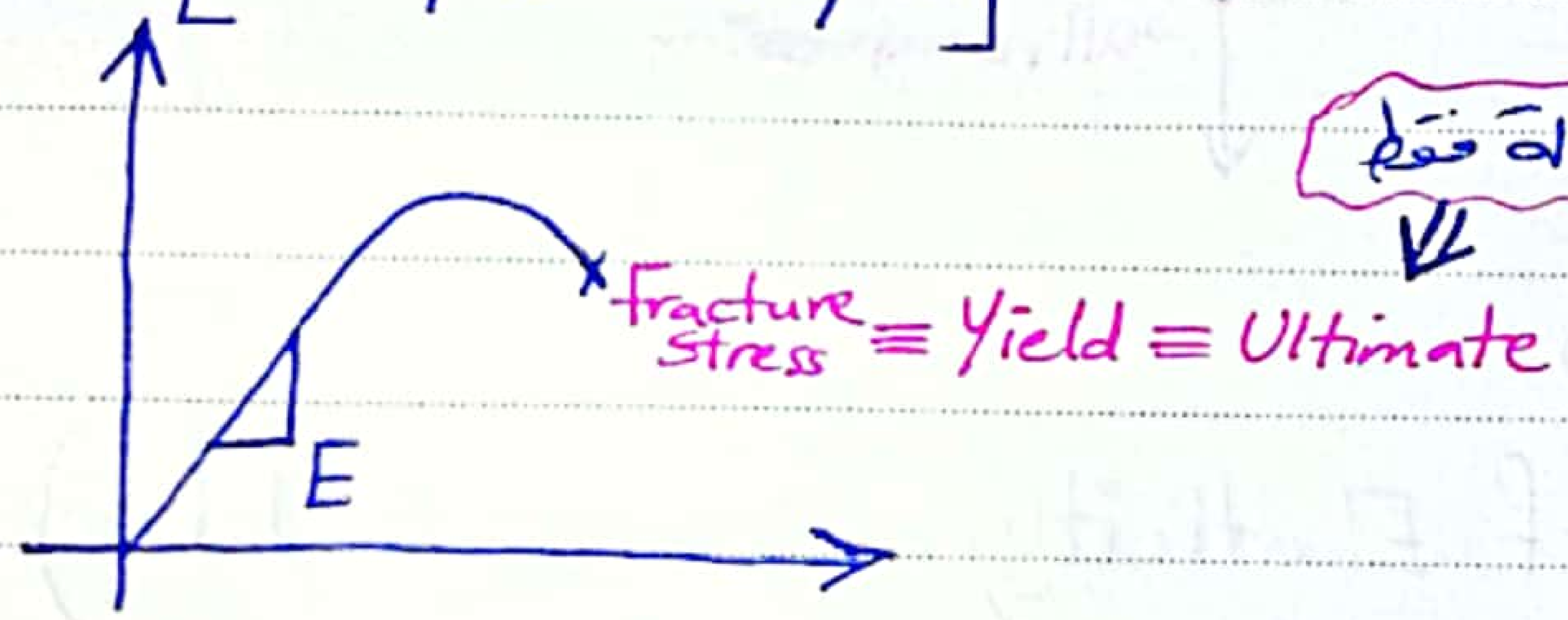


b) Aluminuim Alloy



- **Brittle Materials →**

Like [Brass / Concrete / ...]



⇒ σ_{ult} / compression > σ_{ult}) tension

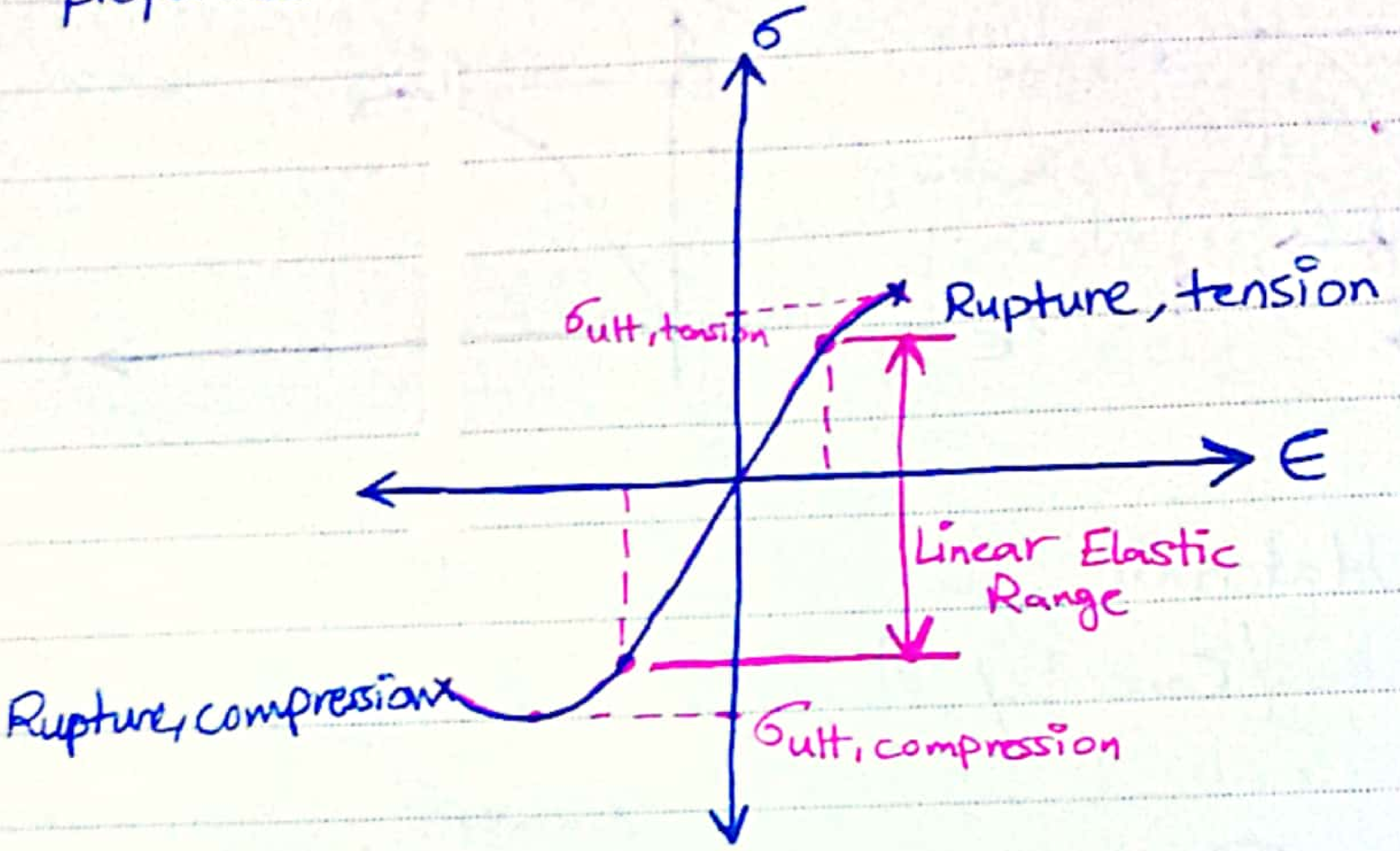
- due to the presence of flaws such as microscopic cracks or cavities, which tend to weaken the material in tension ...

شكل الكسر

° Fracture shape in the tension → ductile (Necking „ Cup & Cone „ 45°)
→ brittle (Flattening 90°)

due to the normal stress

an example of brittle material with different properties in tension & compression is provided by "concrete"



2.5 Hooke's Law

- E (Modulus of Elasticity) = (Young's Modulus) ⇒

$$\boxed{\sigma = E\,\epsilon}$$ Hooke's Law

(E) تمثل ال [slope] منحنى (σ-ϵ) في منطقة ال (elastic) فقط..

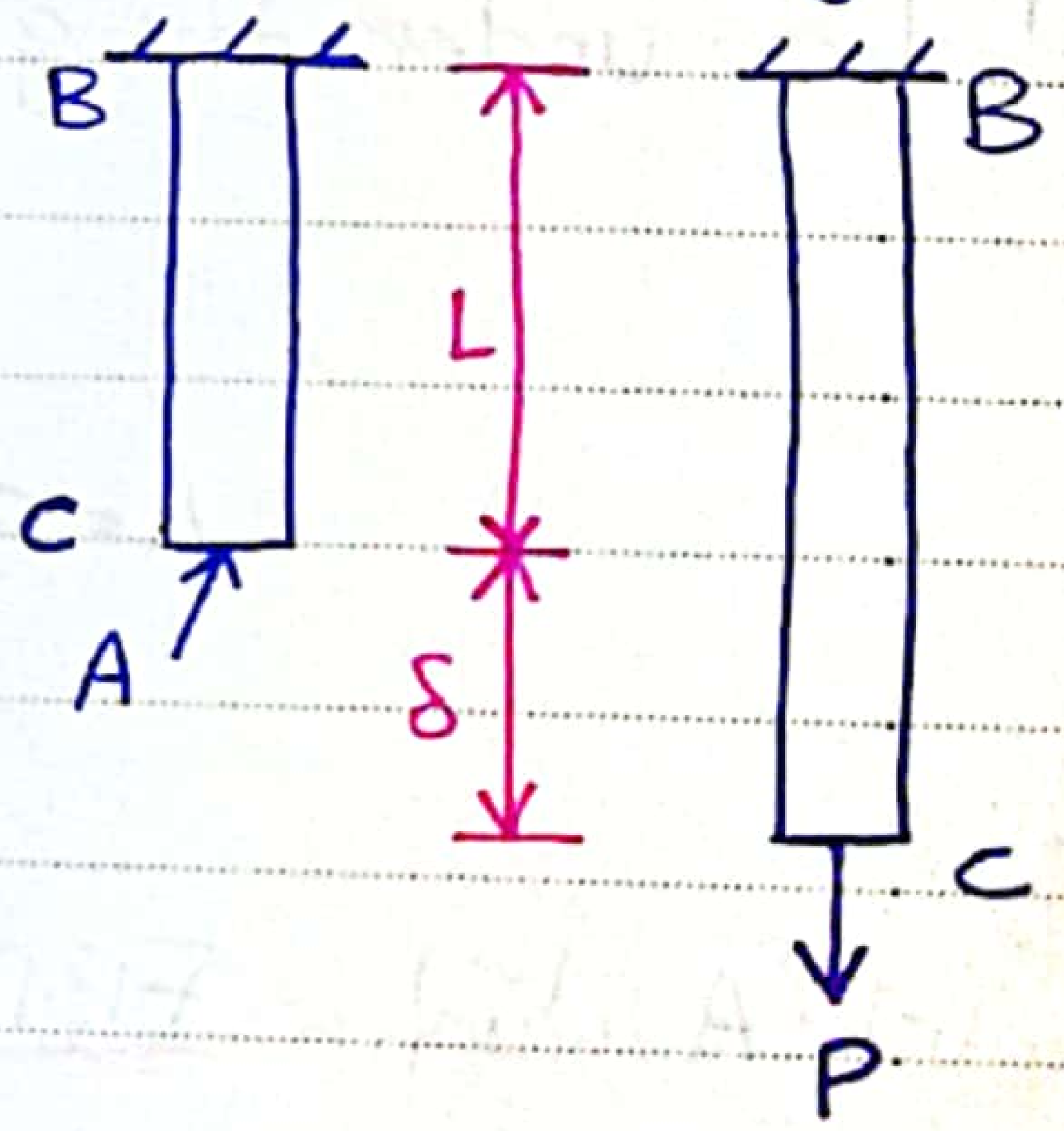
Linear Region

2.7 Repeated Loadings ; Fatigue

Fatigue ≡ loading are repeated thousands or millions of times on a specimen

In such cases, rupture will occur at a stress much lower than the static breaking strength.

2.8 Deformations of Members Under Axial Loading



If the resulting axial stress

$$\boxed{\sigma = \frac{P}{A}}$$

doesn't exceed the proportional limit [يعني لساتو بمنطقة ال elastic ال Linear]

$$\boxed{\sigma = E \epsilon}$$

$$\left[\epsilon = \frac{\sigma}{E} = \frac{P}{AE}\right] \quad , \quad \epsilon = \frac{\delta}{L}$$

$$\therefore \boxed{\delta = \frac{PL}{AE}}$$

⇒ used only if the rod has a uniform cross section of area A and is loaded at its end.

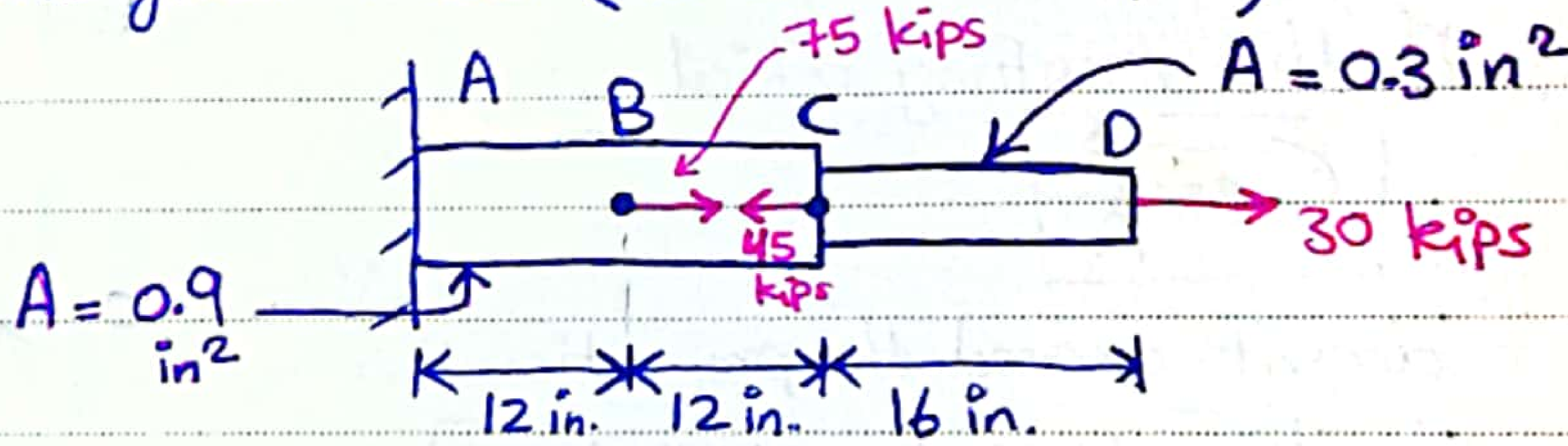
If the rod is loaded at other points, possible of different materials or it consists of various cross sections, we'll use ⇒

$$\left[\ \delta = \sum_i \frac{P_i L_i}{A_i E_i}\ \right]$$

Ex (2.01)

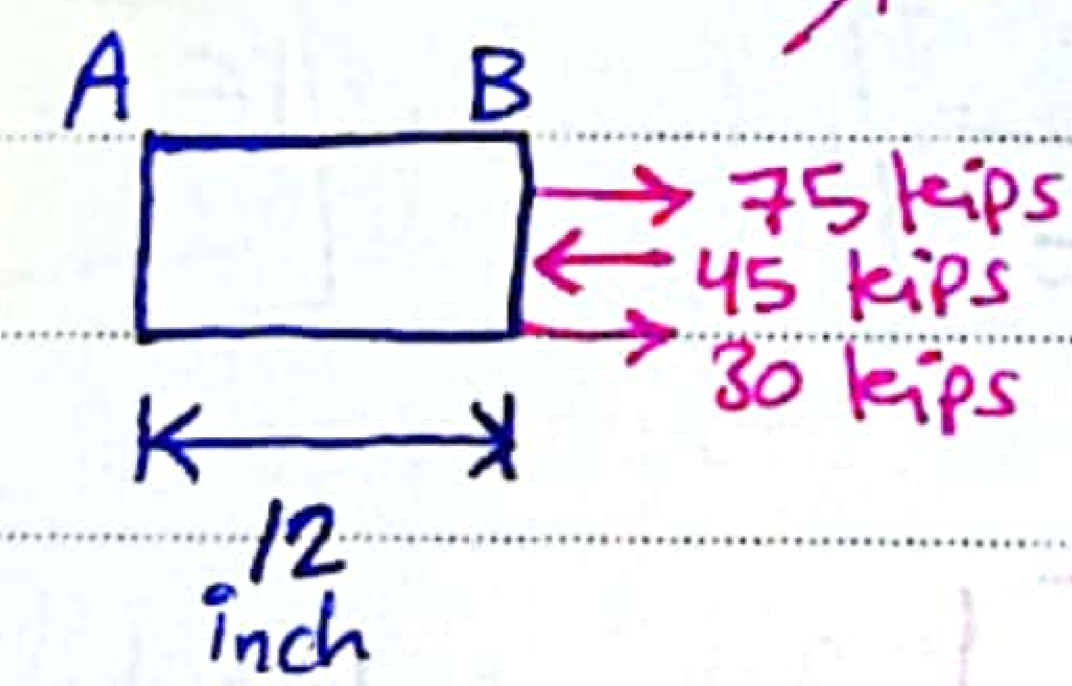
الـ (rod) كامل من نفس المادة

Determine the deformation of the steel rod shown below under the given loads ($E = 29 * 10^6$ psi)

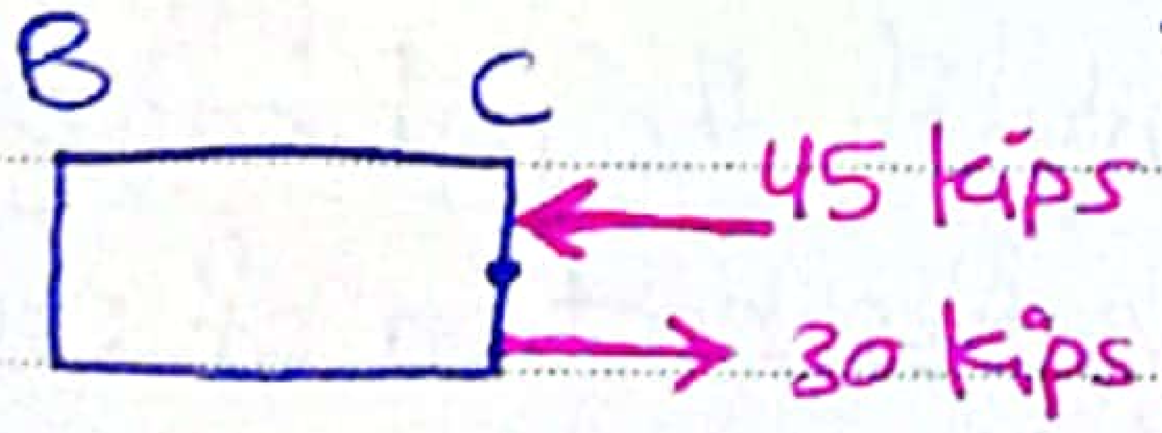


* باخد كل [Component Part] لحاله وبعمله FBD من النقطة (A) باتجاه اليمين
وباخد معها أي فورس عينيت برضو ..

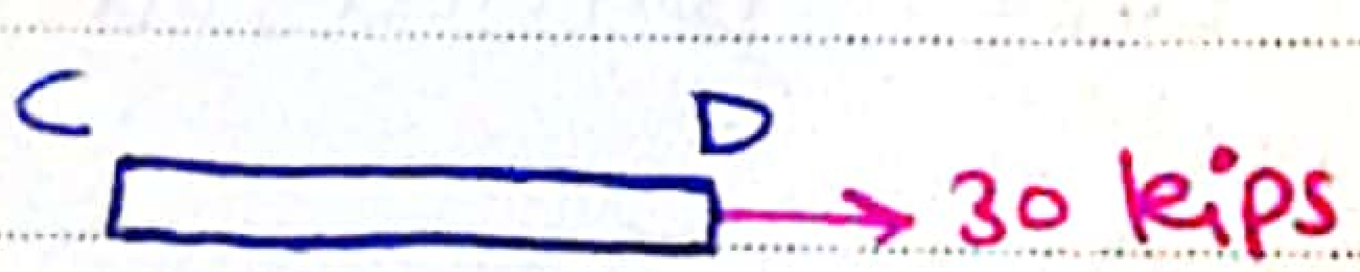
[→ +ve]



$P_{AB} = 75 - 45 + 30$ ①
$= 60$ kips



$P_{BC} = 30 - 45 = -15$ kips ②



$P_{CD} = 30$ kips ③

$$\therefore \quad \delta = \frac{P_1 L_1}{A_1 E} + \frac{P_2 L_2}{A_2 E} + \frac{P_3 L_3}{A_3 E}$$

$A_1 = A_2 = 0.9 \ in^2$

$L_1 = L_2 = 12 \ in.$

$L_3 = 16 \ in.$

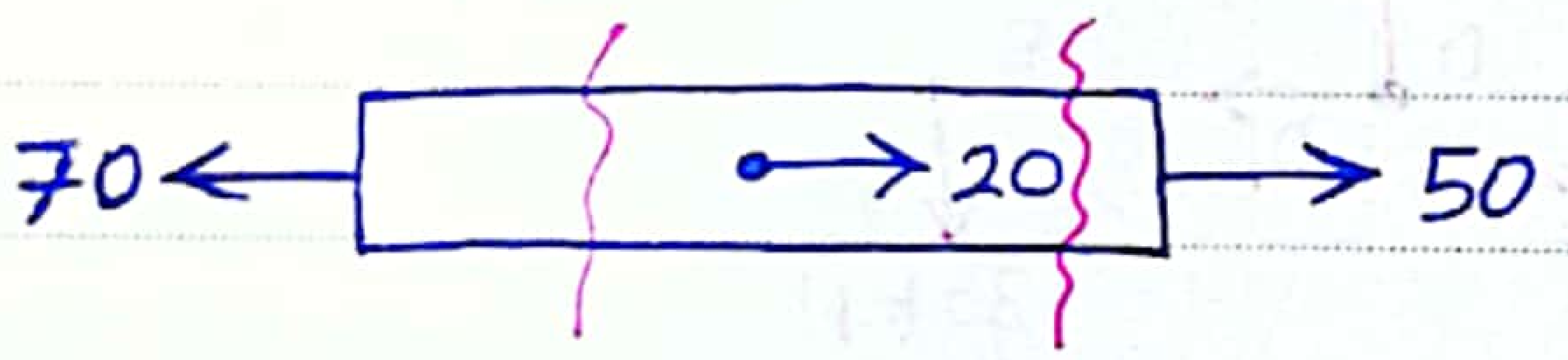
$A_3 = 0.3 \ in^2$

$$\Rightarrow \delta = \frac{60 * 10^3 * 12}{0.9 * 29 * 10^6} + \frac{-15 * 10^3 * 12}{0.9 * 29 * 10^6} + \frac{30 * 10^3 * 16}{0.3 * 29 * 10^6}$$

$$\delta = 75.9 * 10^{-3} \ in.$$

#

ex →



لو طلب مني أحسب الـ (σ_{normal})

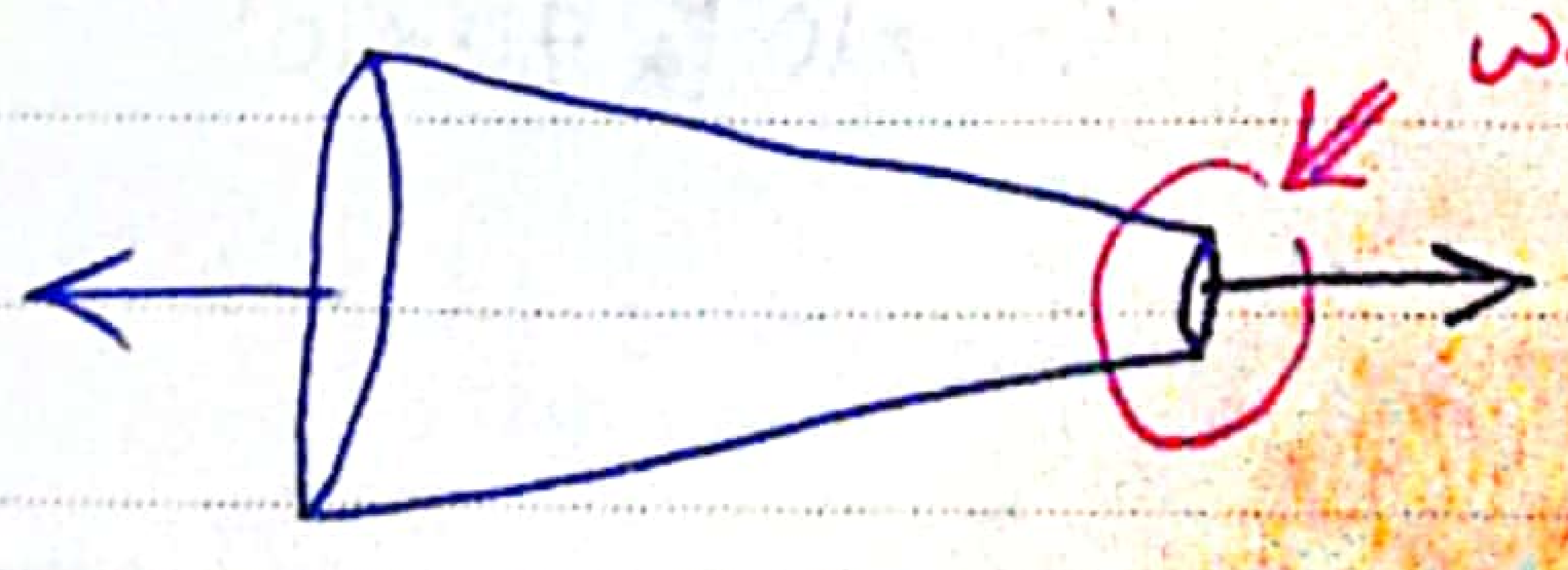
take a section at each pt. where the internal force or the cross section area has changed.

* كلما زادت الـ A بحصل على أقل stress ولذلك بيتم حساب الـ stress

في حالة الـ max. منه عشان أعرف أديه بتحمل

دائماً لاحظ الـ (A) عند أقل مكان/ قيمة لها ، مثلاً ∴

we will take (A) From this place

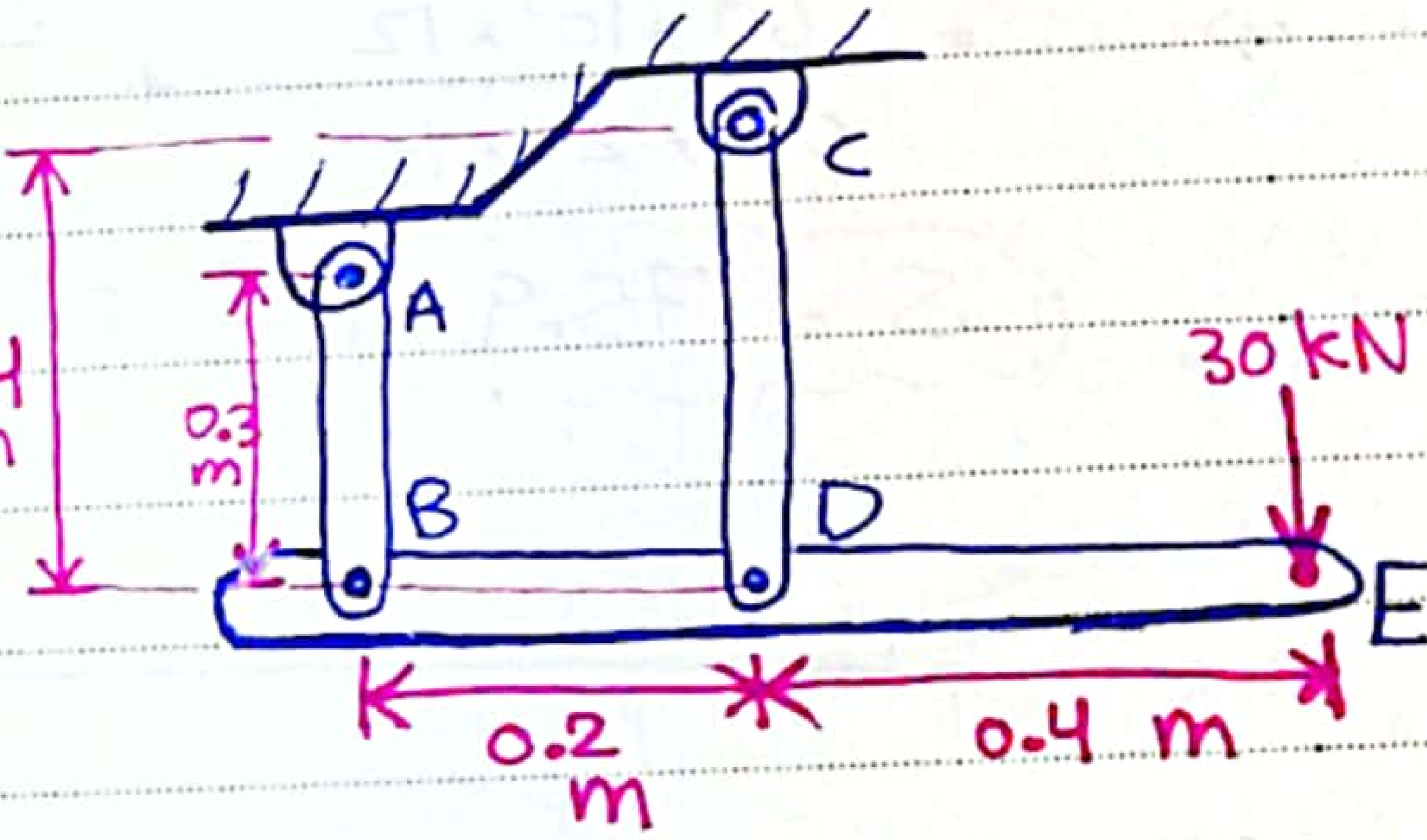


Link CD is made of steel ($E = 200 GPa$) and has across sectional area of $600 \ mm^2$.

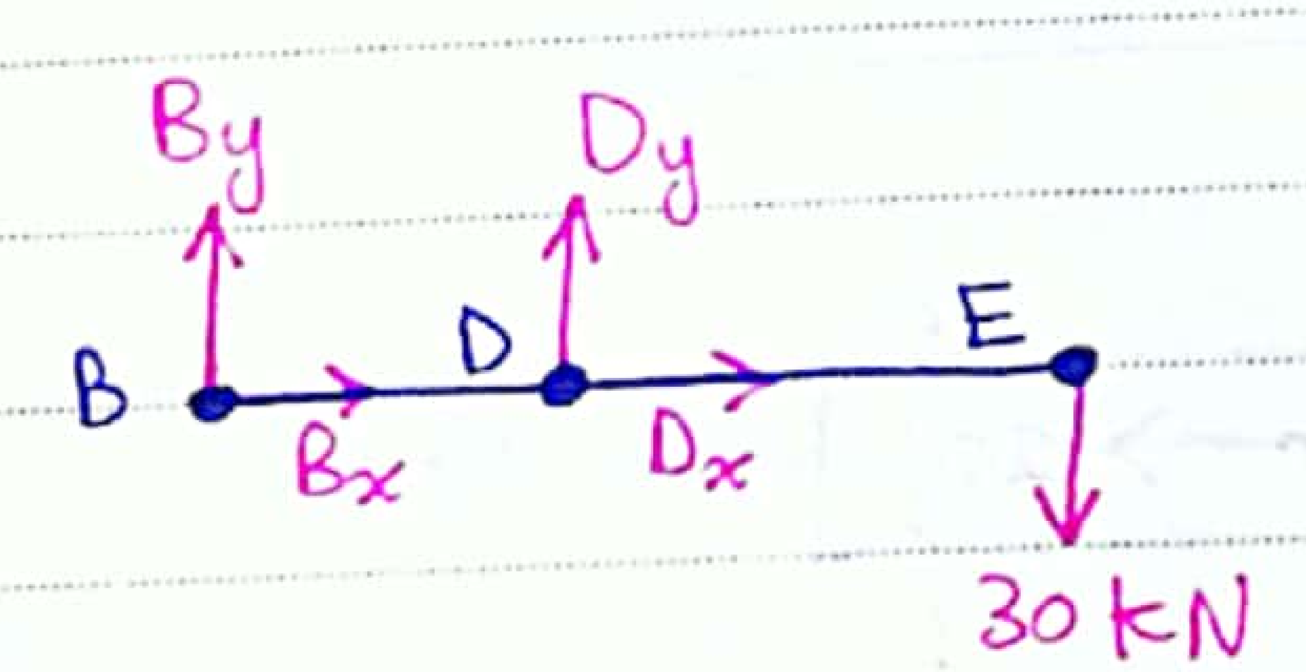
Sample Problem (2.1) ⇒

The rigid bar BDE is supported by two links AB and CD. Link AB is made of aluminum ($E = 70 GPa$) and has across sectional area of $500 \ mm^2$; For the 30 kN force shown, determine the deflection of :

a) B b) D c) E



Sol



$\overset{+}{\curvearrowright} \Sigma M_D = 0 , \quad 30{,}000(0.4) + B_y(0.2) = 0$

$B_y = F_{AB} = -60 \ kN$ compression

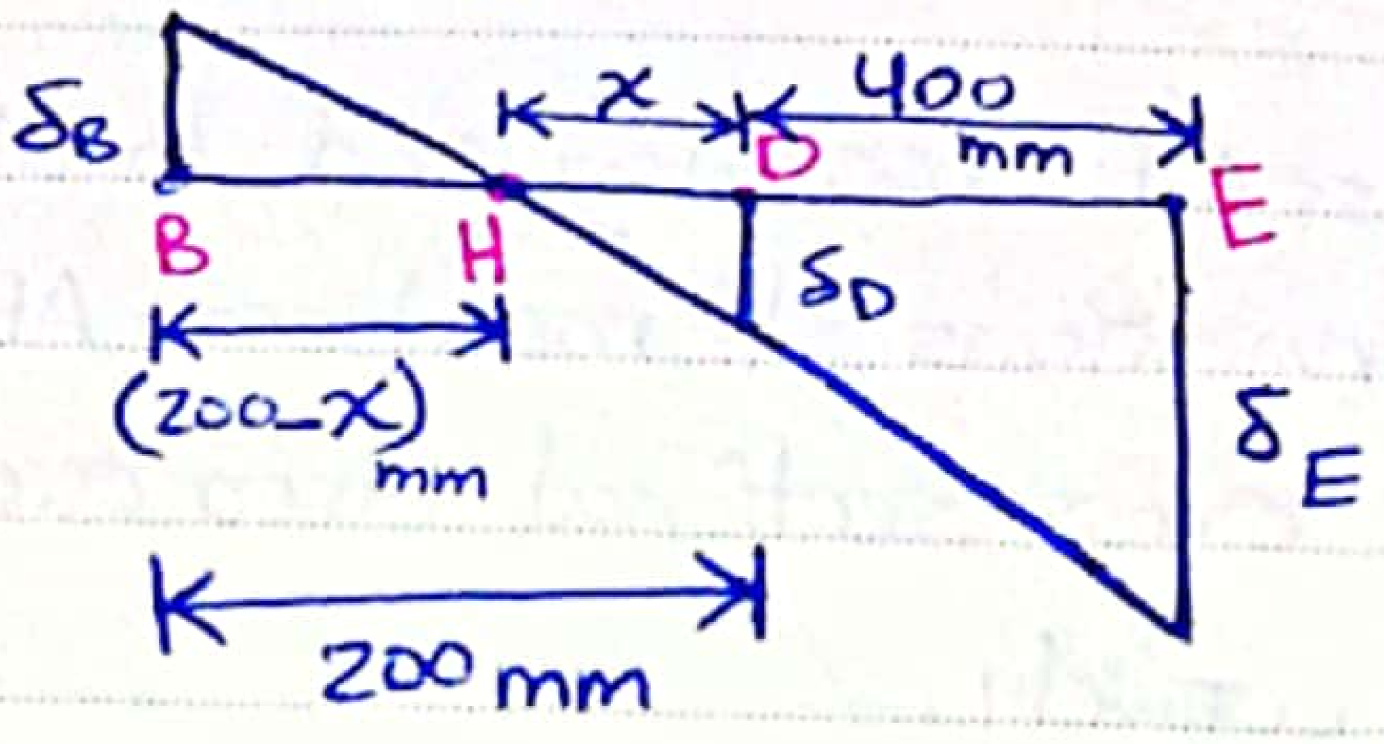
$\overset{+}{\curvearrowright} \Sigma M_B = 0 , \quad -D_y(0.2) + 30{,}000(0.6) = 0$

$D_y = F_{CD} = 90 \ kN$ tension

a) $\delta_B = \dfrac{P_{AB} L_{AB}}{A_{AB} E_{AB}} = \dfrac{-60 * 10^3 * 0.3}{500 * 10^{-6} * 70 * 10^9} = -5.14 * 10^{-4} \ m$

b) $\delta_D = \frac{P_{CD} L_{CD}}{A_{CD} E_{CD}} = \frac{90*10^3*0.4}{600*10^{-6}*200*10^9} = 3*10^{-4}$ m

c)



$\delta_D = 0.3$ mm

$\delta_B = 0.514$ mm

$$\frac{\delta_B}{\delta_D} = \frac{200-x}{x}$$

[من خلال قانون تشابه المثلثات]

$$0.514\,x = 0.3(200-x)$$

$$(0.514+0.3)\,x = 0.3*200$$

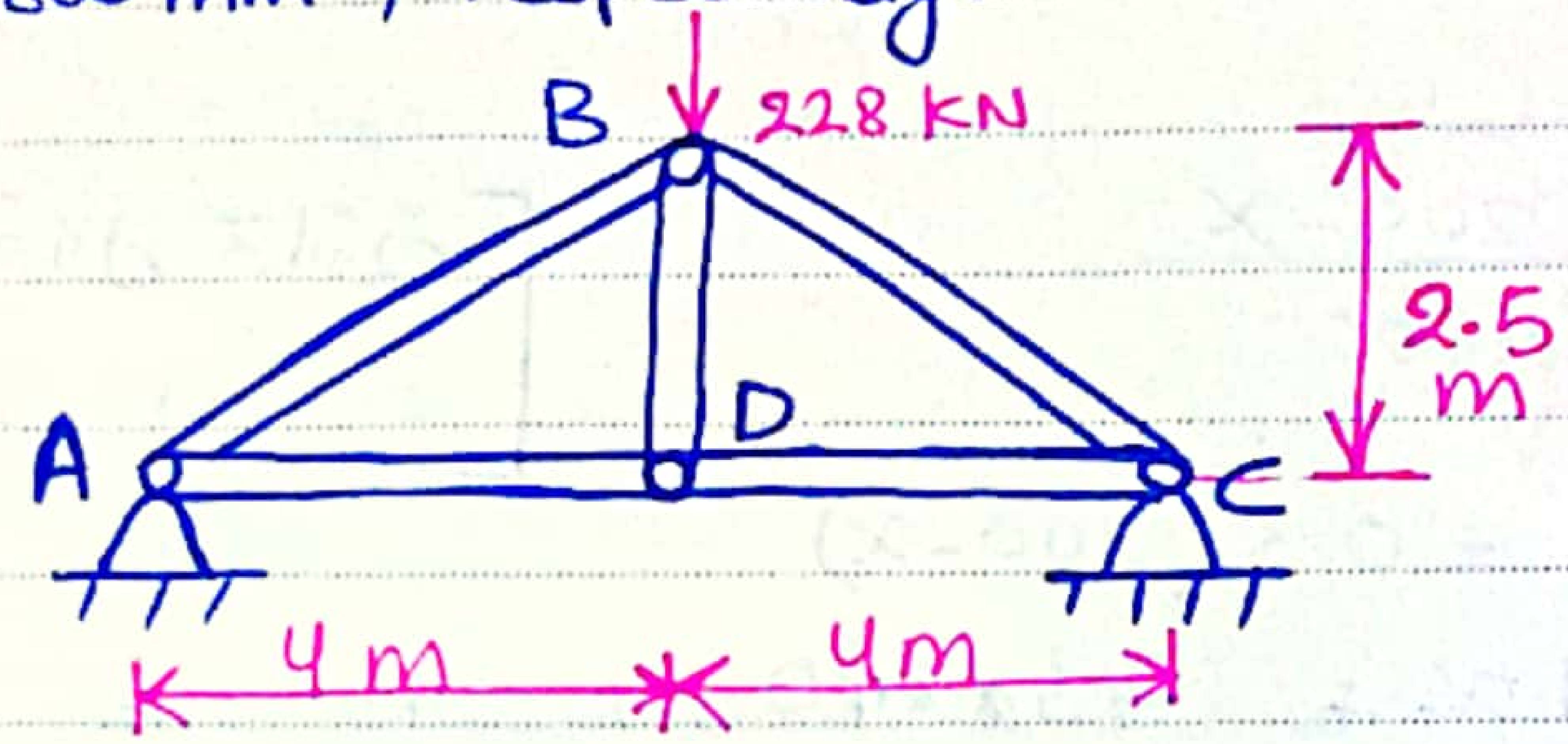
$$x = 73.7 \text{ mm}$$

$$\frac{\delta_E}{\delta_D} = \frac{400+73.7}{73.7} \Rightarrow \boxed{\delta_E = 1.928 \text{ mm}}$$

#

Problems

2.23) For the steel truss ($E = 200\ GPa$) and loading shown, determine the deformations of members AB and AD, knowing that their cross sectional area are $2400\ mm^2$ and $1800\ mm^2$, respectively.



Statics : Reactions are 114 kN upward at A and C
Member BD is a zero force member.

$$L_{AB} = \sqrt{4^2 + 2.5^2} = 4.717\ m$$

Use joint A as a free body

$$+\uparrow \Sigma F_y = 0 : \quad 114 + \frac{2.5}{4.717} F_{AB} = 0$$

$$F_{AB} = -215.10\ kN$$

$$\xrightarrow{+} \Sigma F_x = 0 : \quad F_{AD} + \frac{4}{4.717} F_{AB} = 0$$

$$F_{AD} = 182.4\ kN$$

Member AB :

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{E A_{AB}}$$

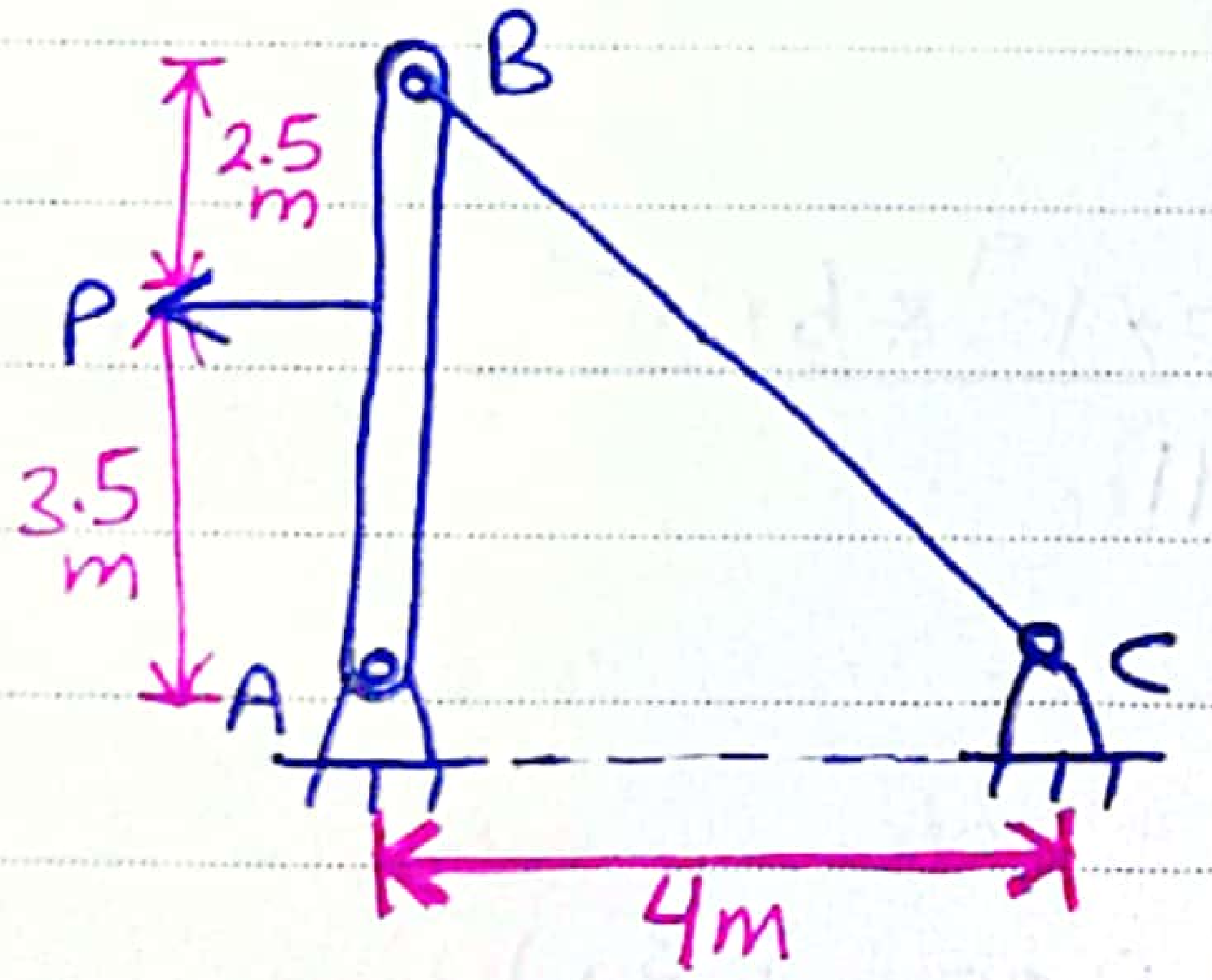
$$= \frac{-215.10 * 10^3 * 4.717}{200 * 10^9 * 2400 * 10^{-6}} = -2.11 * 10^{-3} \ m$$

Member AD :

$$\delta_{AD} = \frac{F_{AD} L_{AD}}{E A_{AD}}$$

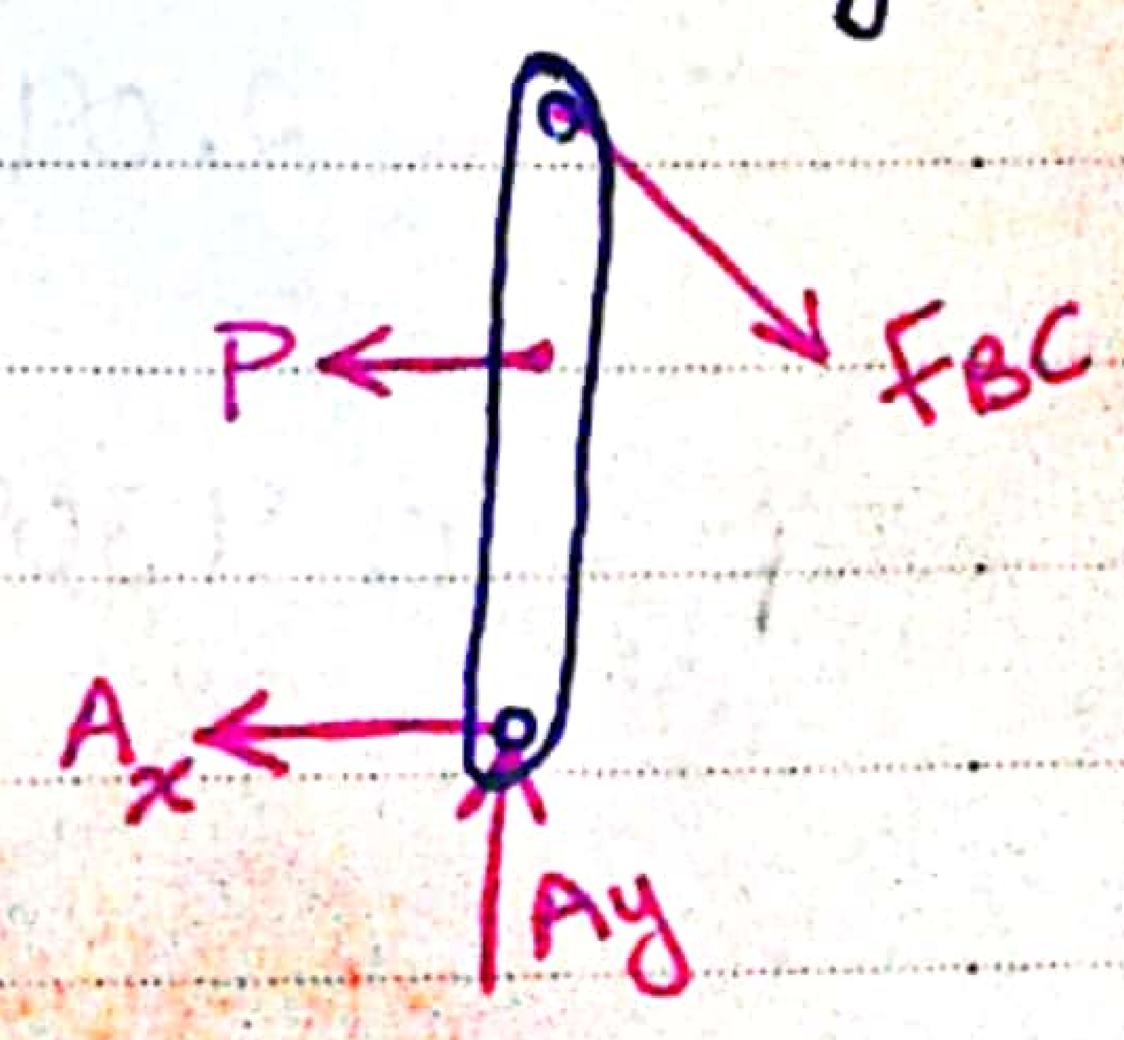
$$= \frac{182.4 * 10^3 * 4}{200 * 10^9 * 1800 * 10^{-6}} = 2.03 * 10^{-3} \ m$$

2.13) The 4mm diameter cable BC is made of a steel with E = 200 GPa. Knowing that the max. stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the max. load P that can be applied as shown :



$$L_{BC} = \sqrt{6^2 + 4^2} = 7.211 \, m$$

use AB as a Free body :



$\circlearrowleft + \sum M_A = 0$

$$3.5P - 6 * \frac{4}{7.2111} F_{BC} = 0$$

$$P = 0.9509 F_{BC}$$

* Considering allowable stress: $\sigma = 190 * 10^6$ pa

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4} (0.004)^2 = 12.566 * 10^{-6} m^2$$

$$\sigma = \frac{F_{BC}}{A}$$

$$\therefore F_{BC} = \sigma A = 190 * 10^6 * 12.566 * 10^{-6}$$
$$= 2.388 * 10^3 N$$

* Considering allowable elongation: $\delta = 6 * 10^{-3} m$

$$\delta = \frac{F_{BC} L_{BC}}{E A_{BC}}$$

$$F_{BC} = \frac{12.566 * 10^{-6} * 200 * 10^9 * 6 * 10^{-3}}{7.2111}$$
$$= 2.091 * 10^3 N$$

$$P = 0.9509 F_{BC} = 1.988 * 10^3 N$$

#

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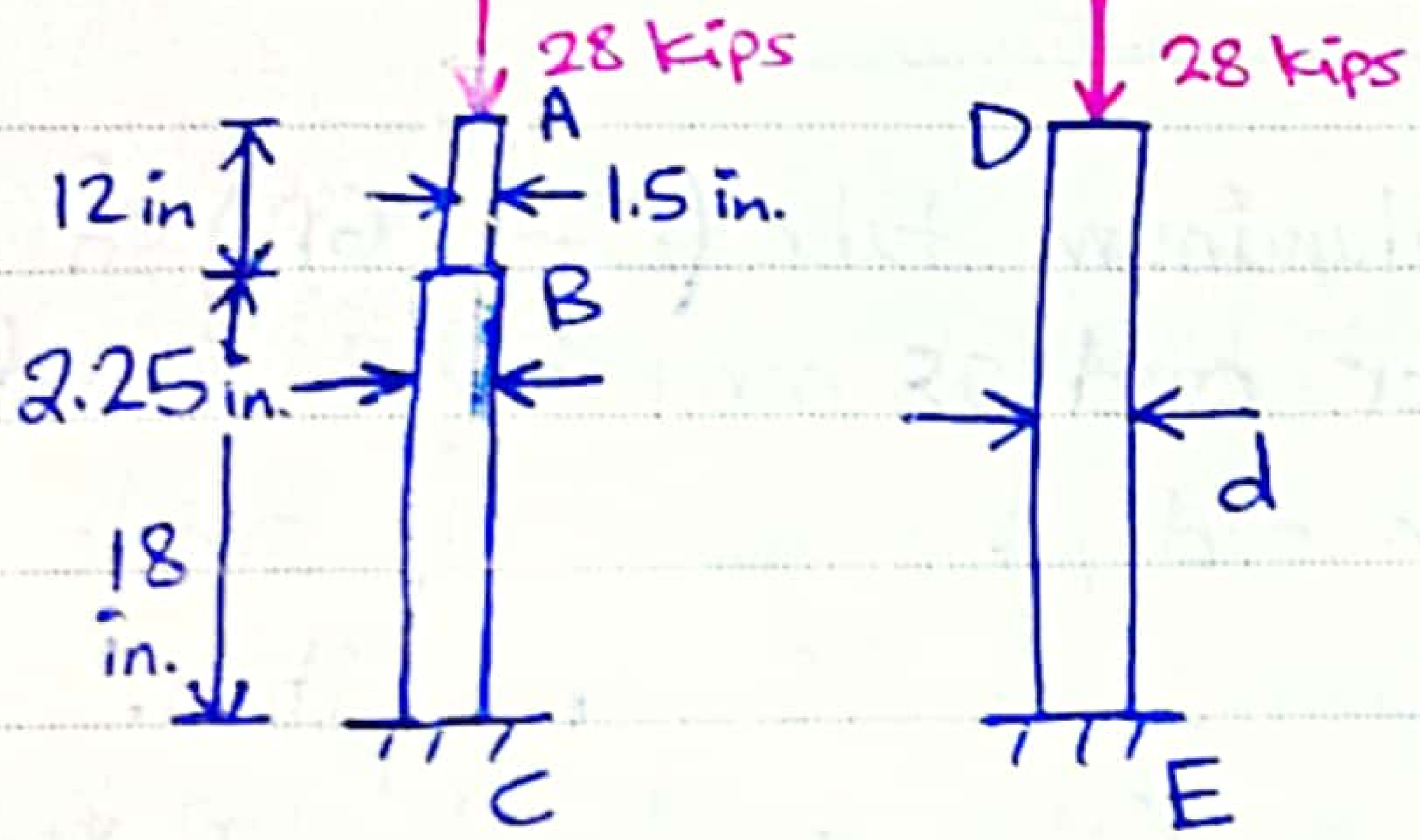
2.14) The aluminum rod ABC ($E = 10.1 * 10^6$ psi), which consists of two cylindrical portions AB and BC, is to be replaced with a cylindrical steel rod DE ($E = 29 * 10^6$ psi) of the same overall length. Determine the min. required diameter d of the steel rod if its vertical deformation isn't to exceed the deformation of the aluminium rod under the same load and if the allowable stress in the steel rod isn't to exceed 24 ksi.



→ Deformation of aluminum rod

$$\delta_A = \frac{PL_{AB}}{A_{AB}E} + \frac{PL_{BC}}{A_{BC}E}$$

$$= \frac{28 * 10^3}{10.1 * 10^6}\left[\frac{12}{\frac{\pi}{4}(1.5)^2} + \frac{18}{\frac{\pi}{4}(2.25)^2}\right]$$

$$= 0.031376 \text{ in.}$$

Steel rod, $\left[\delta = 0.031376 \text{ inch}\right]$

$$\delta = \frac{PL}{EA} \Rightarrow A = \frac{28 * 10^3 * 30}{29 * 10^6 * 0.031376} = 0.92317 \text{ in}^2$$

/ /

$$\sigma = \frac{P}{A} \Rightarrow A = \frac{28 * 10^3}{24 * 10^3} = 1.1667 \text{ in}^2$$

Required area is the larger value

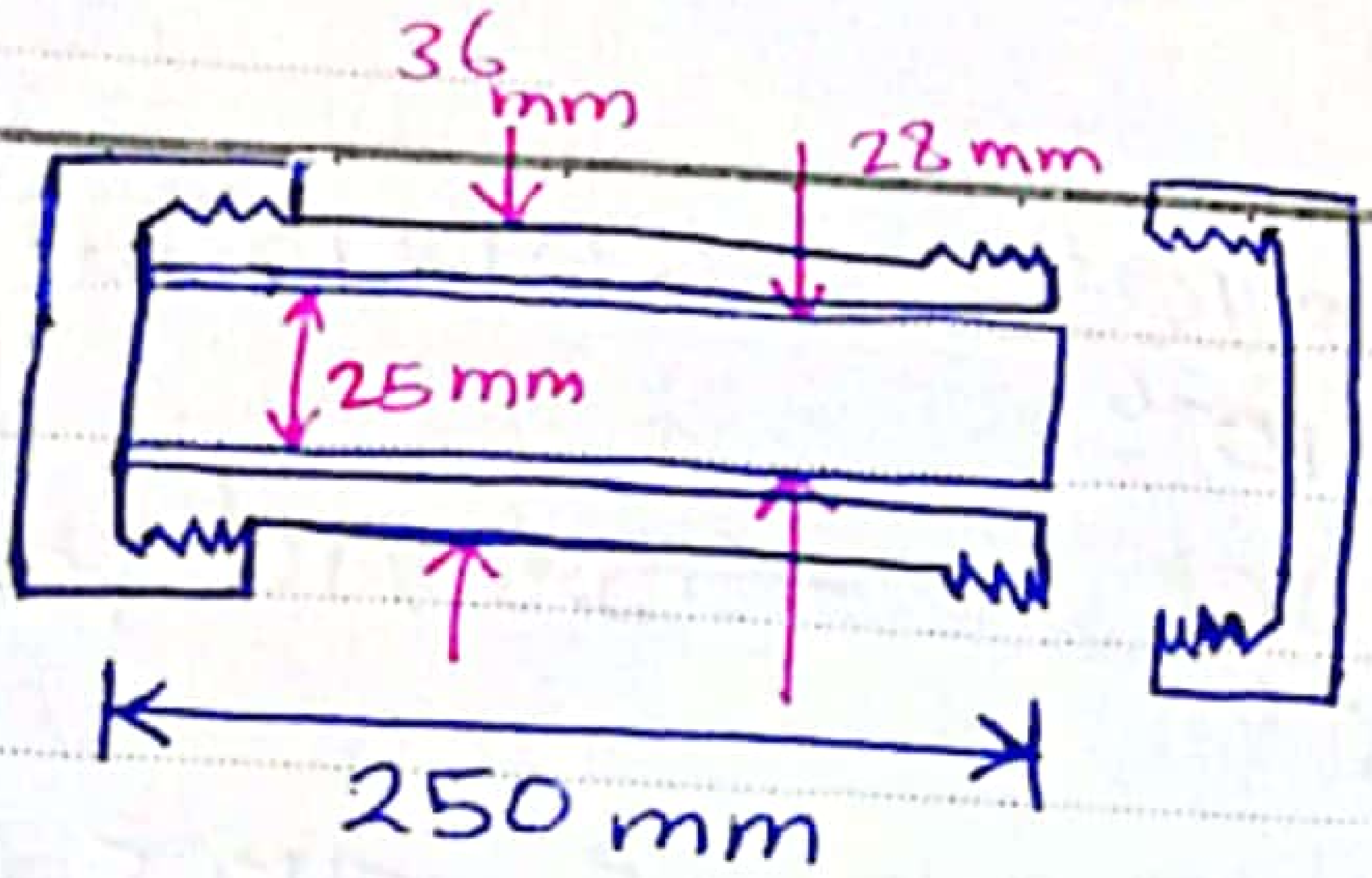
Diameter $\Rightarrow$ $d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(1.6667)}{\pi}}$

$d = 1.219$ in.

2.17) A 250 mm long aluminum tube ($E = 70$ GPa) of 36 mm outer diameter and 28 mm inner diameter can be closed at both ends by means of single threaded screw-on covers of 1.5 mm pitch. With one cover screwed on tight, a solid brass rod ($E = 105$ GPa) of 25 mm diameter is placed inside the tube and the second cover is screwed on. Since the rod is slightly longer than the tube, it's observed that the cover must be forced against the rod by rotating it one quarter of a turn before it can be tightly closed. Determine:

a) the average normal stress in the tube and in the rod?

b) the deformations of the tube and of the rod?



$$\rightarrow \quad A_{tube} = \frac{\pi}{4}(d_o^2 - d_i^2) = \frac{\pi}{4}(36^2 - 28^2) = 402.12\,mm^2$$

$$A_{rod} = \frac{\pi}{4}d^2 = \frac{\pi}{4}(25^2) = 490.87\,mm^2$$

$$\delta_{tube} = \frac{P\,L}{E_{tube}\,A_{tube}} = \frac{P(0.25)}{70*10^9*402.12*10^{-6}} = 8.8815*10^{-9}P$$

$$\delta_{rod} = \frac{-\,PL}{E_{rod}\,A_{rod}} = \frac{P(0.25)}{105*10^6*490.87*10^{-6}} = -4.8505*10^{-9}P$$

$$* \quad \delta^* = \frac{1}{4}\,\text{turn} * 1.5\,mm = 0.375\,mm = 375*10^{-6}\,m$$

$$[\delta_{tube} = \delta^* + \delta_{rod}] \quad \underline{\text{or}} \quad [\delta_{tube} - \delta_{rod} = \delta^*]$$

$$8.8815*10^{-9}P + 4.8505*10^{-9}P = 375*10^{-6}$$

$$P = \frac{0.375*10^{-3}}{(8.8815+4.8505)\,10^{-9}} = 27.308*10^{3}\,N$$

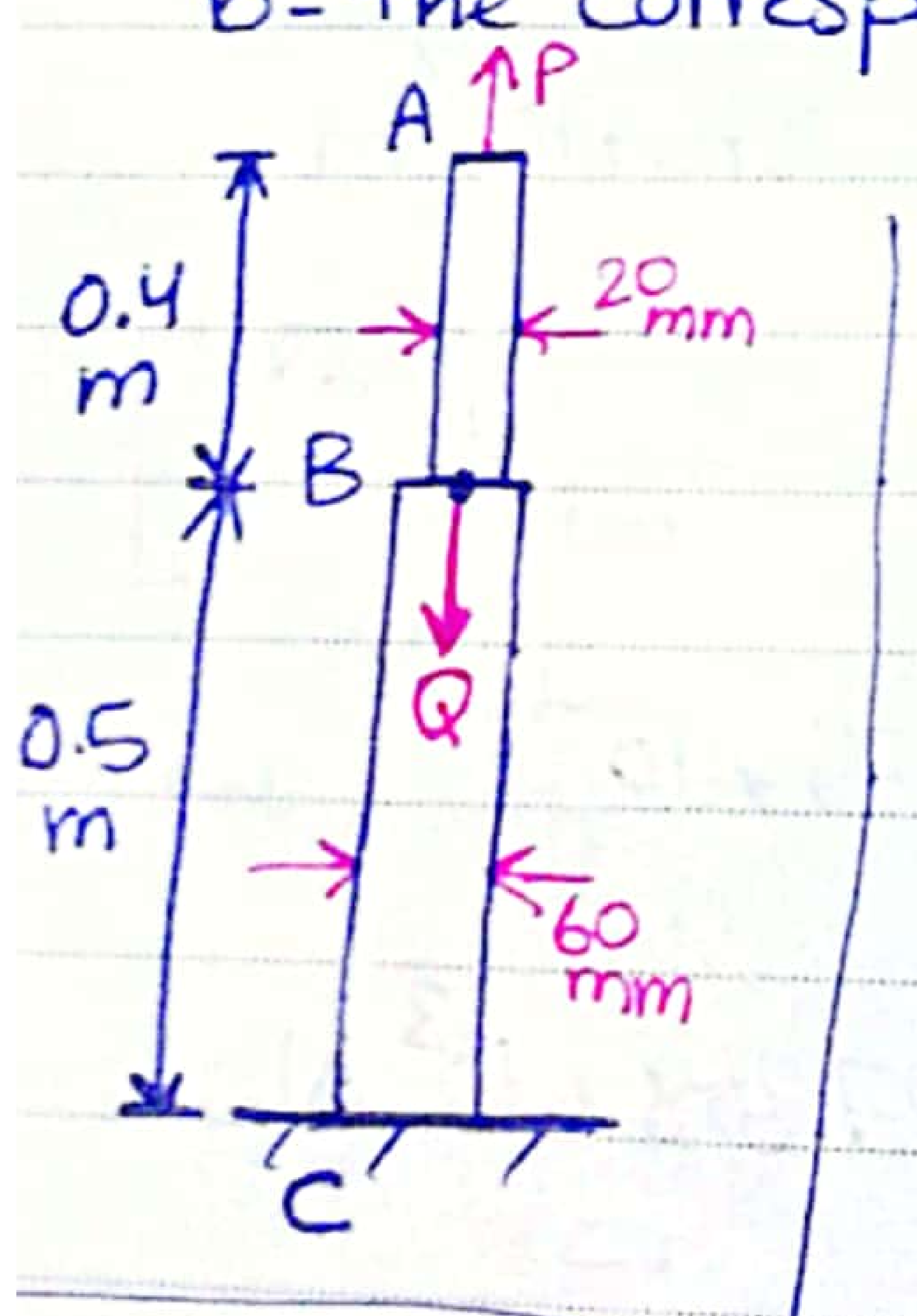
a) $\sigma_{tube} = \frac{27.308 * 10^3}{402.12 * 10^{-6}} = 67.9 * 10^6$ pa

$\sigma_{rod} = \frac{-27.308 * 10^3}{490.87 * 10^{-6}} = -55.6 * 10^6$ pa

b) $\delta_{tube} = 8.8815 * 10^{-9} * 27.308 * 10^3 = 242.5 * 10^{-6}$ m

$\delta_{rod} = -(4.8505 * 10^{-9})(27.308 * 10^3)$
$= -132.5 * 10^{-6}$ m

2.19) Both portions of the rod ABC are made of an aluminum for which E = 70 GPa. Knowing that the magnitude of P is 4 kN, determine:
a- the value of Q so that the deflection at A is zero
b- the corresponding deflection of B



a-) $A_{AB} = \frac{\pi}{4} d_{AB}^2 = \frac{\pi}{4}(0.02)^2$
$= 314.16 * 10^{-6}$ m^2

$A_{BC} = \frac{\pi}{4} d_{BC}^2 = \frac{\pi}{4}(0.06)^2$
$= 2.8274 * 10^{-3}$ m^2

/ /

- force in member $\underline{AB}$ is tension (P)

Elongation →

$$\delta_{AB} = \frac{P L_{AB}}{E A_{AB}} = \frac{4*10^3*0.4}{70*10^9*314.16*10^{-6}} = 72.756*10^{-6}\ m$$

- force in member $\underline{BC}$ is (Q−P) compression

Shortening →

$$\delta_{BC} = \frac{(Q-P) L_{BC}}{E A_{BC}} = \frac{(Q+P)(0.5)}{70*10^9*2.8274*10^{-3}}$$

$$= 2.5263*10^{-9}(Q-P)$$

⇒ for zero deflection at A, $\delta_{BC} = \delta_{AB}$

$$2.5263*10^{-9}(Q-P) = 72.756*10^{-6}$$

$$\therefore \quad Q-P = 28.8*10^3 N$$

$$Q = 28.8*10^3 + 4*10^3 = 32.8*10^3 N$$

b) $\delta_{AB} = \delta_{BC} = \delta_B = 72.756*10^{-6}\ m$

$$\delta_{AB} = 0.0728\ mm \downarrow$$

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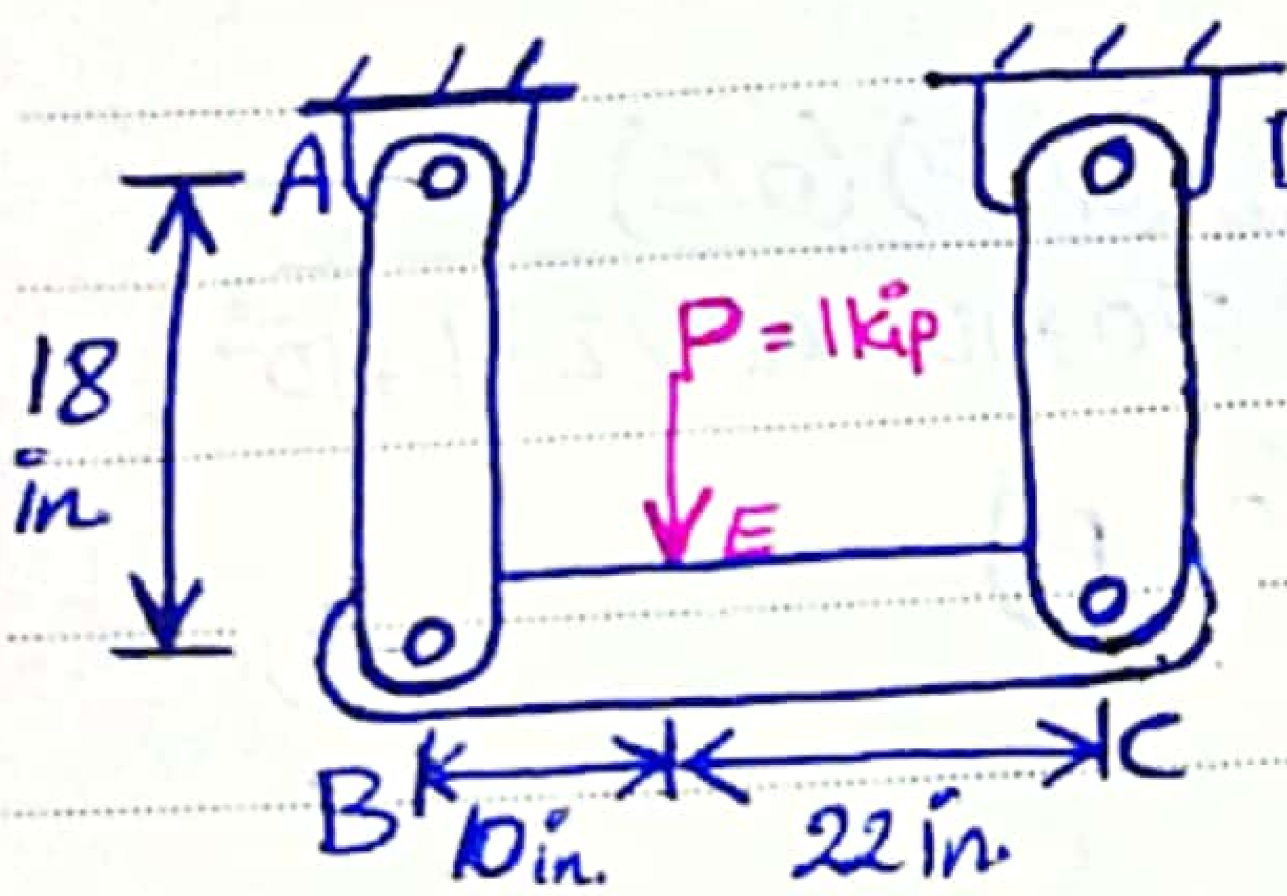
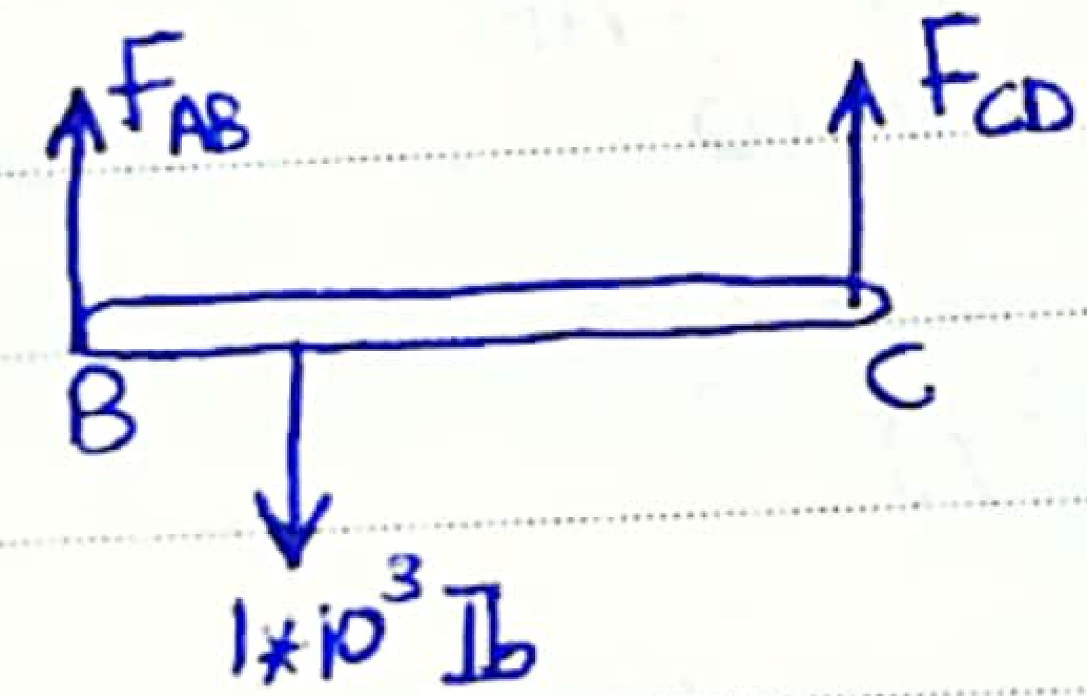
2.25) Each of the links AB and CD is made of aluminum ($E = 10.9 * 10^6$ psi) and has a cross-sectional area of $0.2\ in^2$. Knowing that they support the rigid member BC, determine the deflection of point E?





$\circlearrowleft + \Sigma M_c = \emptyset$:

$-(32) F_{AB} + 22(1*10^3) = \emptyset$

$\boxed{F_{AB} = 687.5 \text{ Ib}}$

$+\uparrow \Sigma F_y = \emptyset$: $687.5 - 1*10^3 + F_{CD} = \emptyset$

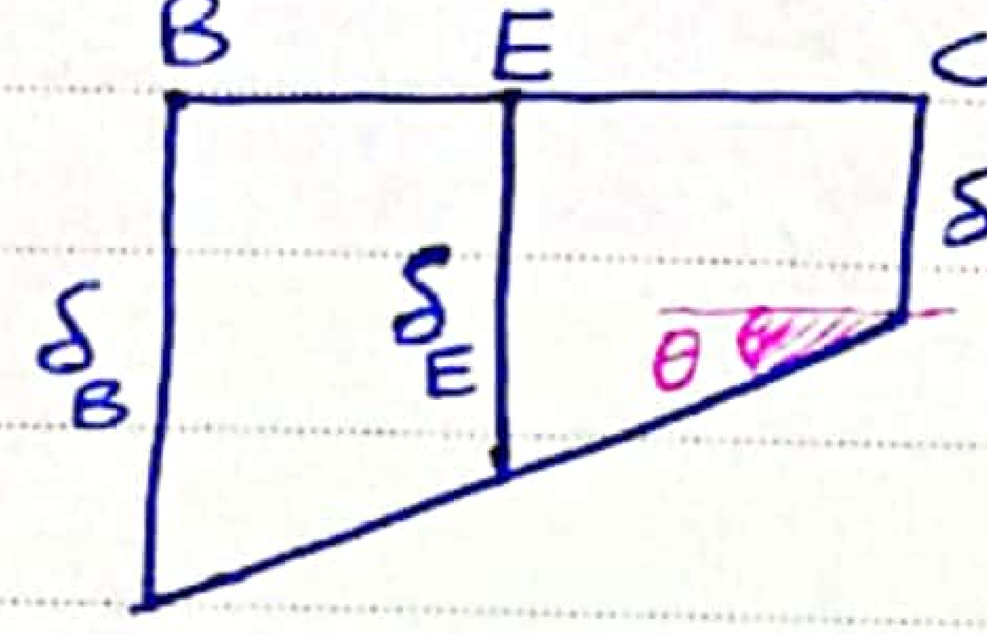
$\boxed{F_{CD} = 312.5 \text{ Ib}}$

$$\delta_{AB} = \frac{F_{AB} L_{AB}}{EA} = \frac{687.5 * 18}{10.9*10^6 * 0.2} = 5.6766 * 10^{-3} \text{ in.}$$
$$= \delta_B$$

$$\delta_{CD} = \frac{F_{CD} L_{CD}}{EA} = \frac{312.5 * 18}{10.9 * 10^6 * 0.2} = 2.5803 * 10^{-3} \text{ in.}$$
$$= \delta_c$$

/ /

Deformation diagram:



$$\underset{\theta}{\text{Slope}} = \frac{\delta_B - \delta_C}{L_{BC}} = \frac{3.0963 * 10^{-3}}{32}$$

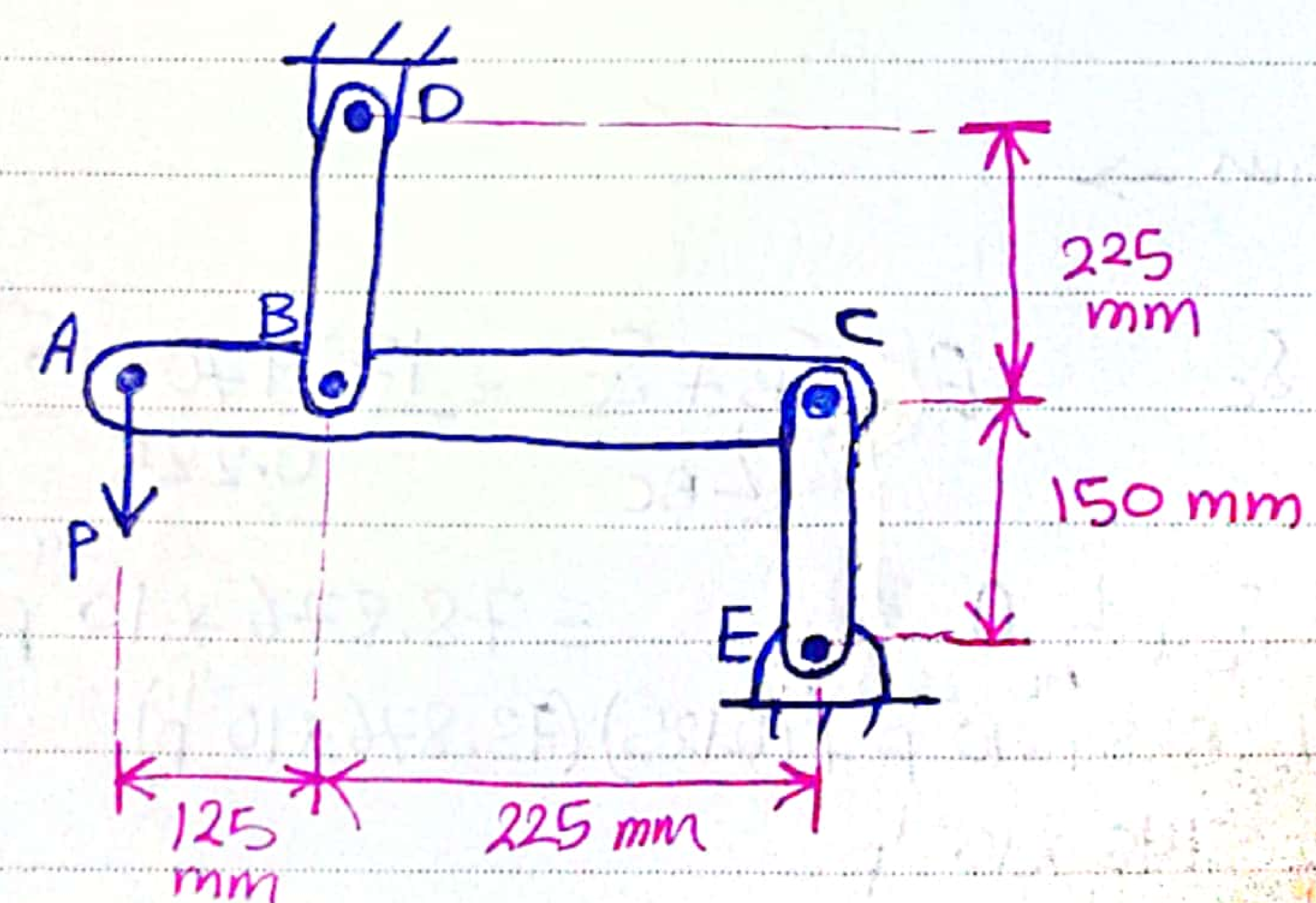
$$= 96.759 * 10^{-6} \text{ rad}$$

$$\delta_E = \delta_c + L_{EC}\,\theta$$

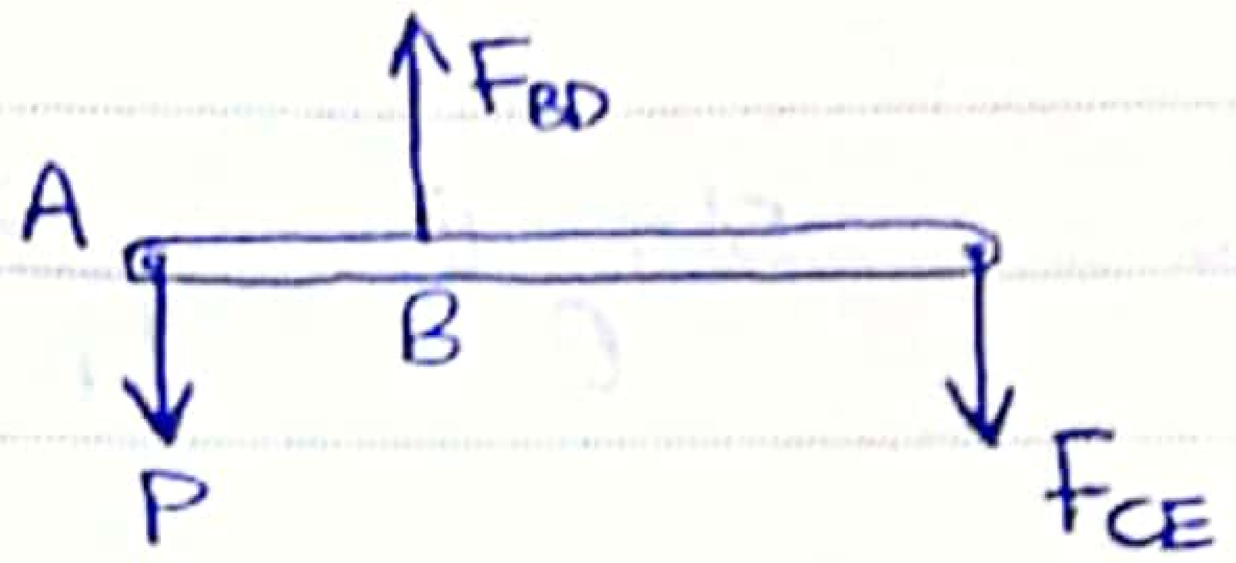
$$= 2.5803 * 10^{-3} + (22)(96.759 * 10^{-6})$$

$$= 4.7090 * 10^{-3} \text{ in.} \downarrow$$

2.27) Link BD is made of brass ($E = 105$ GPa) and has a cross sectional area of 240 mm². Link CE is made of aluminum ($E = 72$ GPa) and has a cross sectional area of 300 mm². Knowing that they support rigid member ABC, determine the max. force P that can be applied vertically at point A if the deflection of A isn't to exceed 0.35 mm?



FBD for member AC:



$\circlearrowleft + \Sigma M_C = 0$: $0.35P - 0.225F_{BD} = 0$

$F_{BD} = 1.555P$

$\circlearrowleft + \Sigma M_B = 0$: $0.125P - 0.225F_{CE} = 0$

$F_{CE} = 0.55556P$

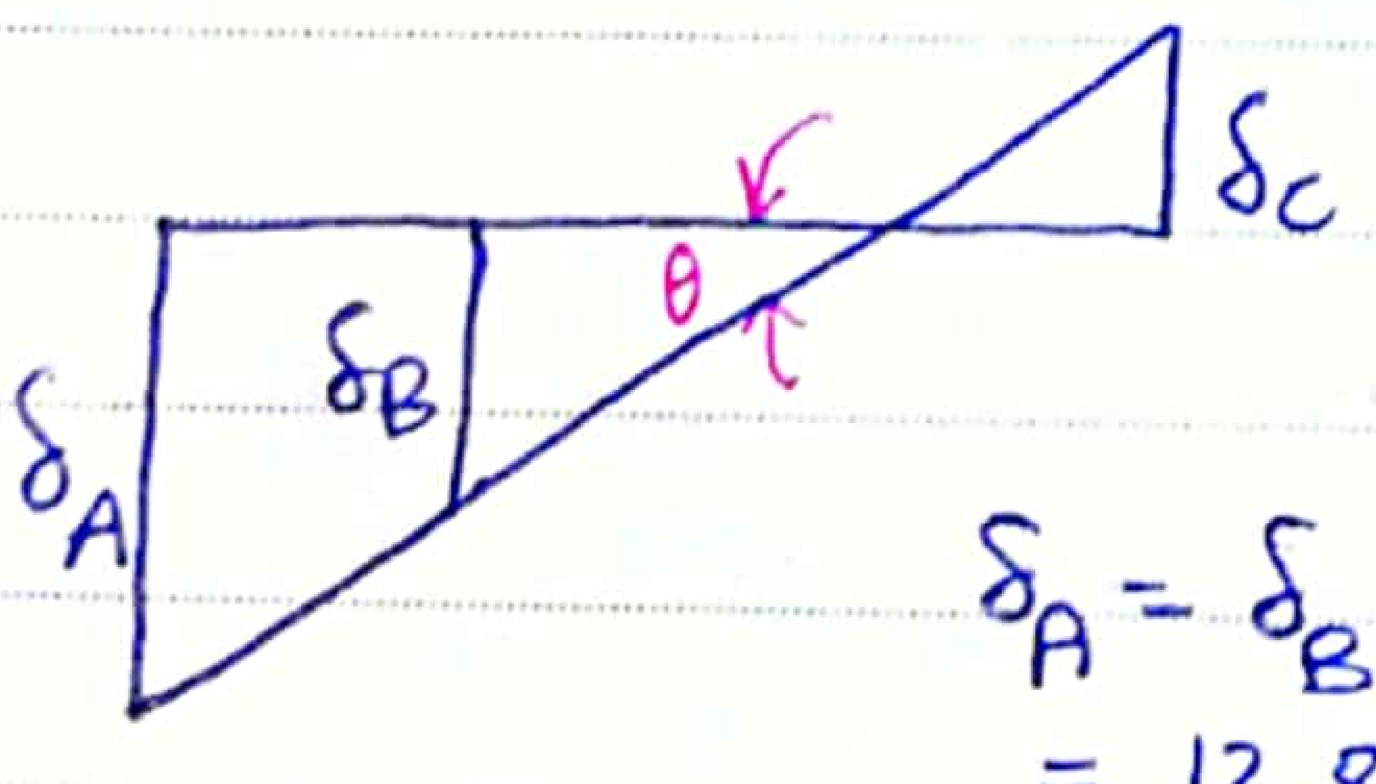
$$\delta_B = \delta_{BD} = \frac{F_{BD}L_{BD}}{E_{BD}A_{BD}} = \frac{1.55556P * 0.225}{105*10^9 * 240*10^{-6}}$$

$$= 13.88 * 10^{-9} P$$

$$\delta_C = \delta_{CE} = \frac{F_{CE}L_{CE}}{E_{CE}A_{CE}} = \frac{0.55556P * 0.150}{72*10^9 * 300*10^{-6}}$$

$$= 3.8581 * 10^{-9} P$$

Deformation Diagram →



$$\theta = \frac{\delta_B + \delta_C}{L_{BC}} = \frac{17.7470 * 10^{-9} P}{0.225}$$

$$= 78.876 * 10^{-9} P$$

$$\delta_A = \delta_B + L_{AB}\theta$$

$$= 13.8889 * 10^{-9} P + (0.125)(78.876 * 10^{-9} P)$$

$$= 23.748 * 10^{-9} P$$

apply displacement limit: $\delta_A = 0.35 * 10^{-3} m = 23.748 * 10^{-9} P$

$P = 14.7381 * 10^3 N$

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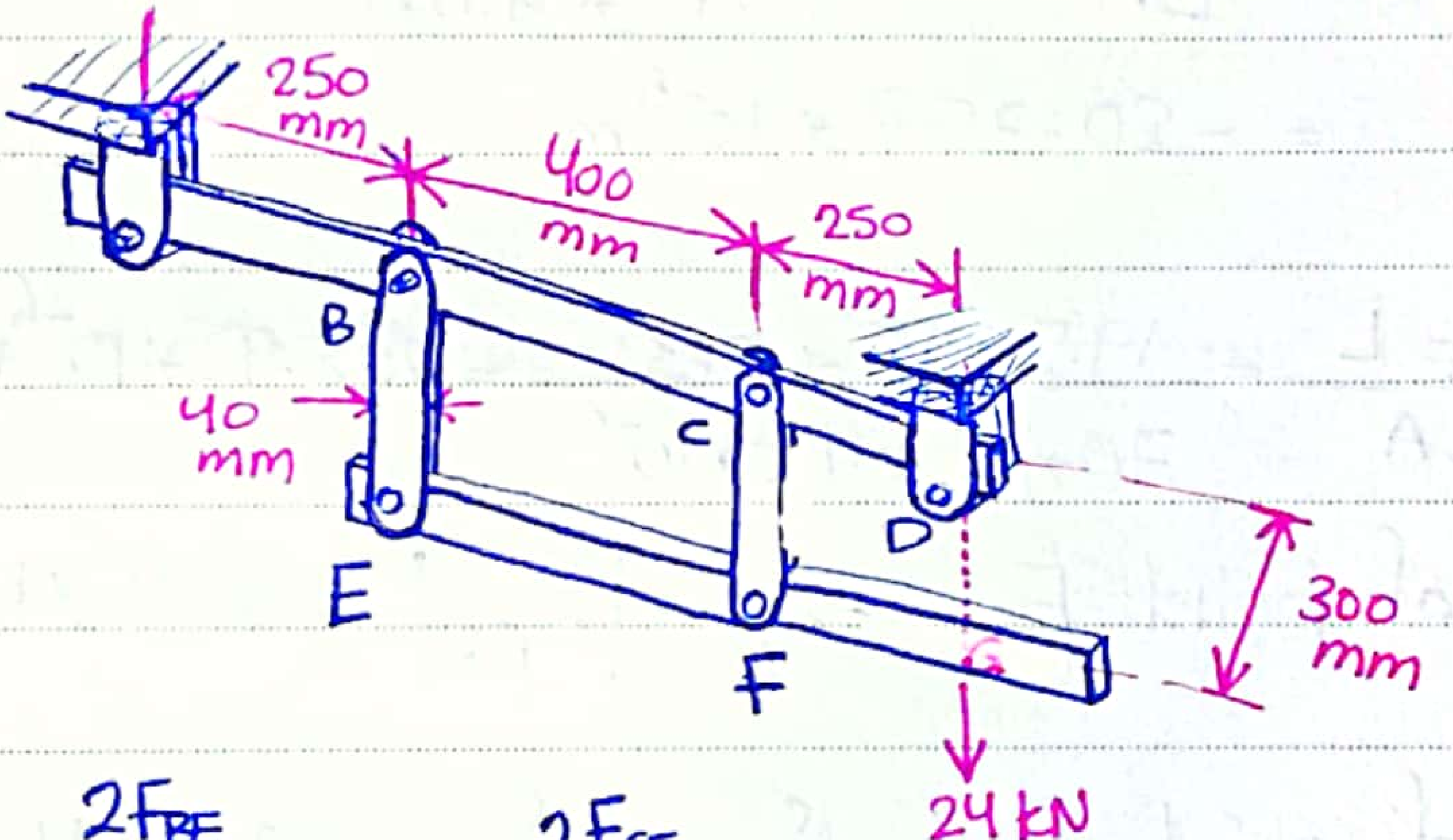
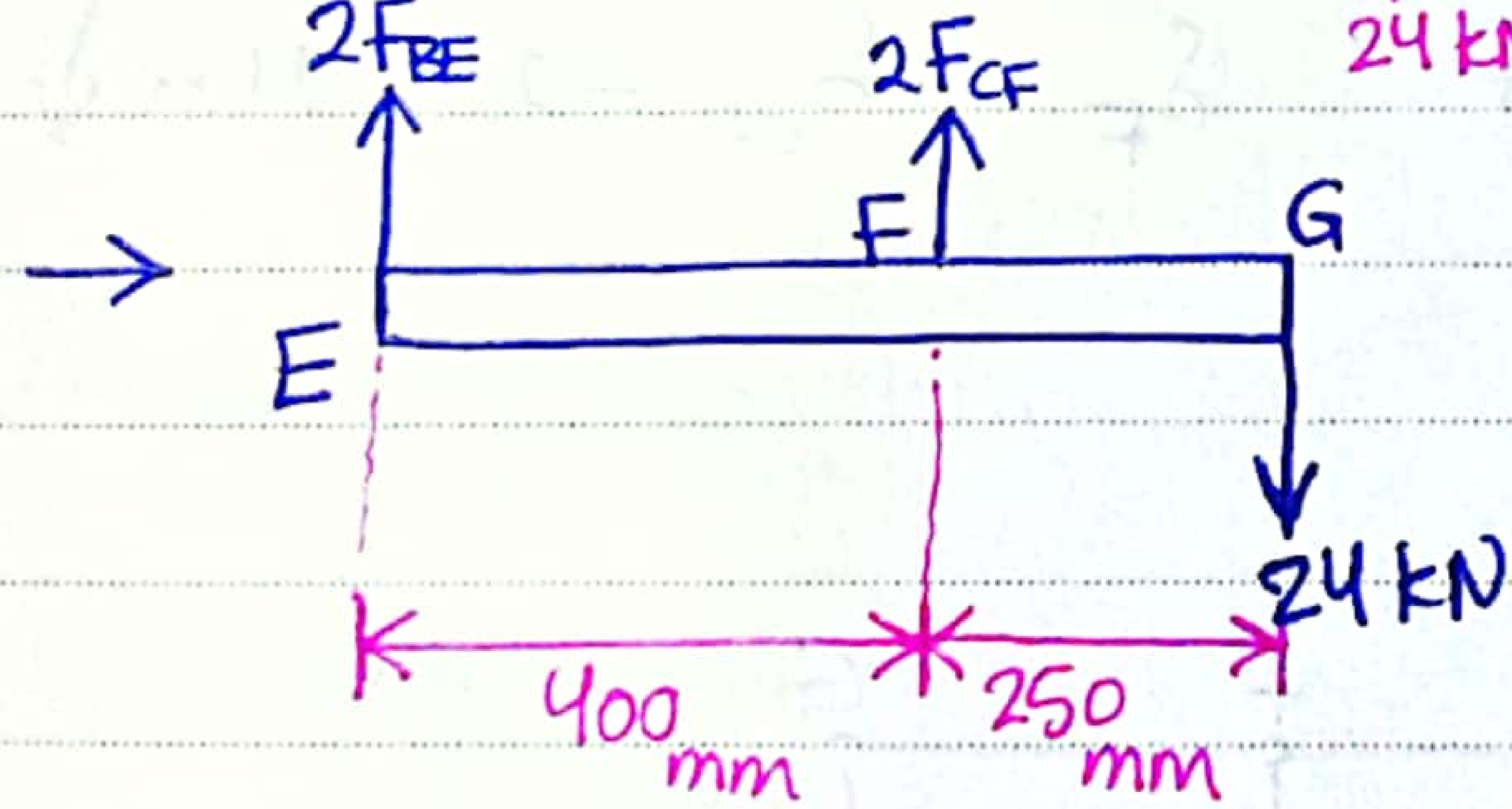
228) Each of the four vertical links connecting the two rigid horizontal members is made of aluminum $E = 70\ GPa$ and has a uniform rectangular cross section of 10×40 mm. For the loading shown, determine the deflection of: a) point A

b) point F

c) point G





$\circlearrowleft + \Sigma M_F = \emptyset$: $-(400)(2F_{BE}) - (250)(24) = \emptyset$

$F_{BE} = -7.5$ kN

$\circlearrowleft + \Sigma M_E = \emptyset$: $400(2F_{CF}) - (650)(24) = \emptyset$

$F_{CF} = 19.5$ kN

Area of one link :

$$A = 10 \times 40 = 400\ mm^2$$
$$= 400 \times 10^{-6}\ m^2$$

Length : $L = 300\ mm$

Deformations

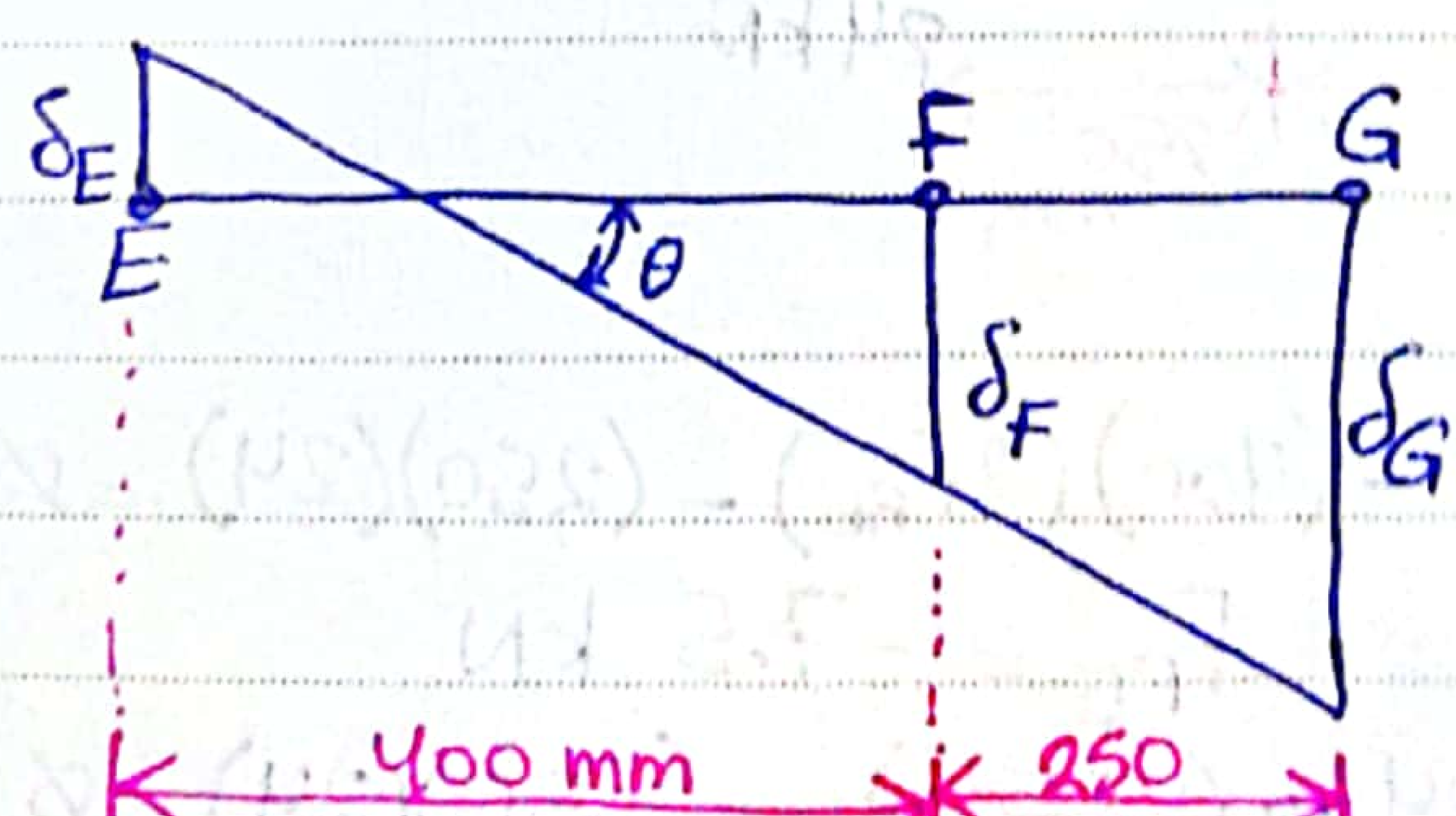
$$\delta_{BE} = \frac{F_{BE}L}{EA} = \frac{-7.5 \times 10^3 \times 0.3}{70 \times 10^9 \times 400 \times 10^{-6}}$$
$$= -80.357 \times 10^{-6}\ m$$

$$\delta_{CF} = \frac{F_{CF}L}{EA} = \frac{19.5 \times 10^3 \times 0.3}{70 \times 10^9 \times 400 \times 10^{-6}} = 208.93 \times 10^{-6}\ m$$

a) Deflection of point E $\quad \delta_E = |\delta_{BF}| = 80.4\ \mu m \uparrow$

b) Deflection of point F $\quad \delta_F = \delta_{CF} = 209\ \mu m \downarrow$

Geometry Change ⇒



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Let θ be the small change in slope angle

$$\theta = \frac{\delta_E + \delta_F}{L_{EF}} = \frac{80.357 * 10^{-6} + 208.93 * 10^{-6}}{0.4} = 723.22 * 10^{-6} \text{ rad}$$

c) Deflection of point G

$$\delta_G = \delta_F + L_{FG}\,\theta$$

$$= 208.93 * 10^{-6} + (0.25)(723.22 * 10^{-6})$$

$$= 389.73 * 10^{-6} \text{ m}$$

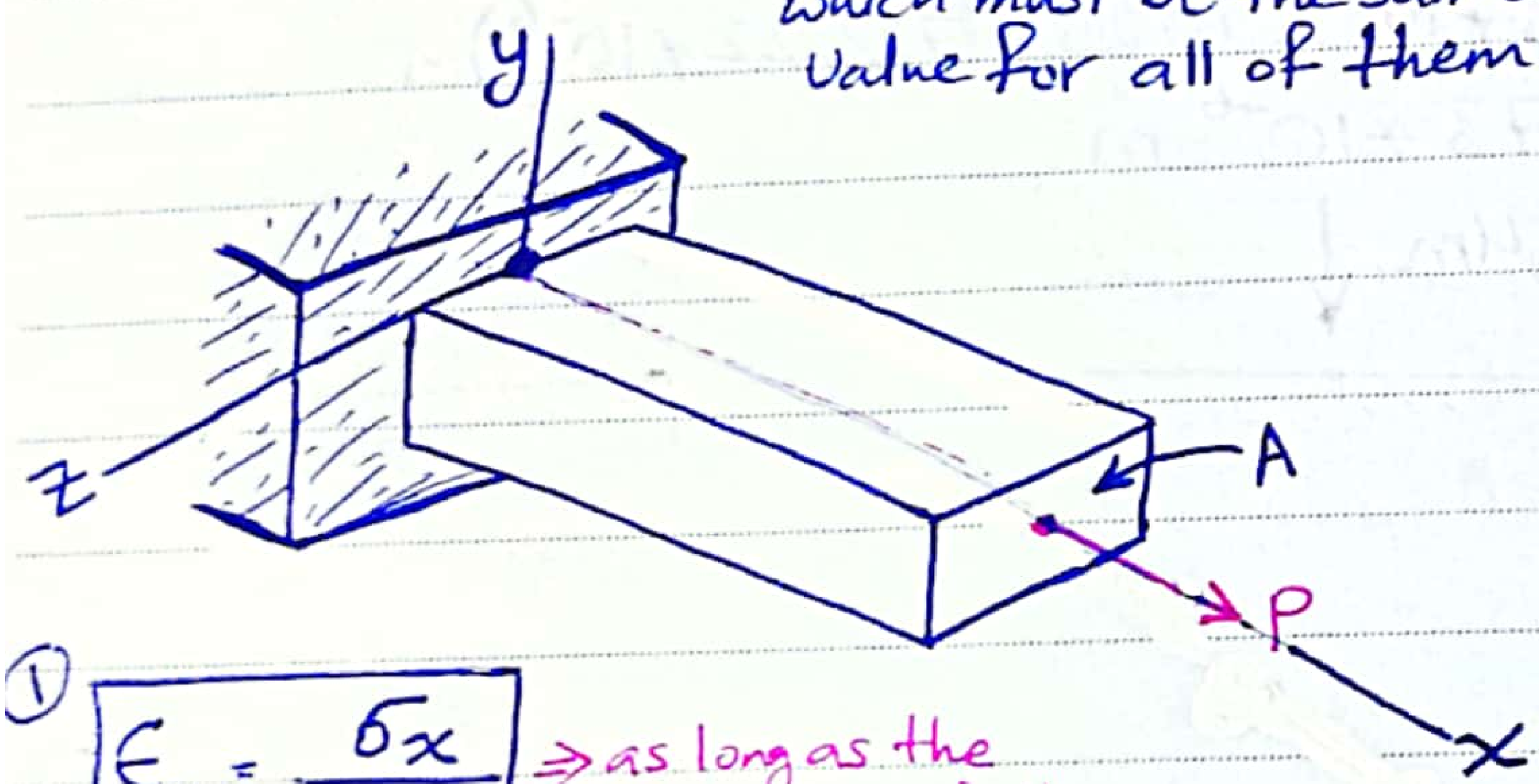
$$\approx 390\ \mu m \downarrow$$

2.11 Poisson's Ratio

When an axial load P is applied to a homogeneous, slender bar, it causes a strain.

strain → along the axis of the bar $[\epsilon_x]$ ← axial strain

strain → along any transverse direction $[\epsilon_y]$, $[\epsilon_z]$ ← lateral strain which must be the same value for all of them → $\epsilon_y = \epsilon_z$



$$\left[\begin{array}{c}\sigma_y = \sigma_z = \emptyset \\ \text{due to zero force}\end{array}\right]$$

① $\boxed{\epsilon_x = \frac{\sigma_x}{E}}$ ⇒ as long as the elastic limit of the material isn't exceeded.

$$\epsilon_y \neq \epsilon_z \neq \epsilon_x \neq \emptyset$$

$$\left[\nu = -\frac{\text{lateral strain}}{\text{axial strain}}\right] \rightarrow \text{poisson's ratio}$$

② $\nu = \frac{-\epsilon_y}{\epsilon_x} = \frac{-\epsilon_z}{\epsilon_x}$

used to get a (+ve) value for ν

/ /

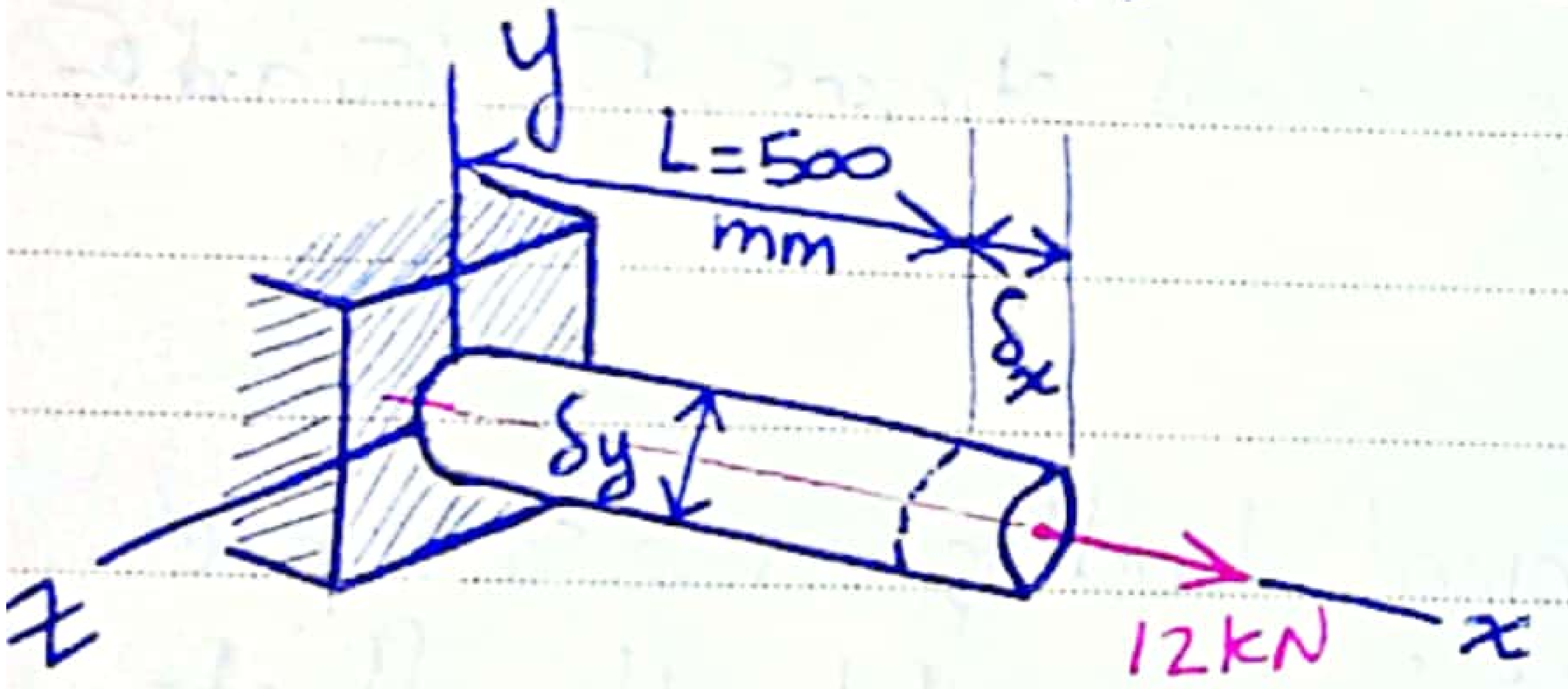
From ① & ② we get →

$$\left[\epsilon_y = \epsilon_z = -\vartheta\epsilon_x = \frac{-\vartheta\sigma_x}{E}\right]$$

→ describes the condition of strain under an axial load applied in a direction parallel to the x axis.

Ex (2.07)

A 500 mm long, 16 mm diameter rod made of a homogenous, isotropic material is observed to increase in length by 300 μm, and to decrease in diameter by 2.4 μm when subjected to an axial $\boxed{12\,kN}$ load. Determine the modulus of elasticity (E) & poisson's ratio (ϑ) of the material ?



$d_{rod} = 16\,mm$

$\delta_y = -2.4\,\mu m$

$\delta_x = 300\,\mu m$

→ $A_{rod} = \pi r^2 = \pi(8 * 10^{-3}\,m)^2 = 201 * 10^{-6}\,m^2$

$$\sigma_x = \frac{P}{A} = \frac{12 * 10^3\,N}{201 * 10^{-6}\,m^2} = 59.7\,MPa$$

$$\epsilon_x = \frac{\delta x}{L} = \frac{300 * 10^{-6}}{500 * 10^{-3}} = 600 * 10^{-6}$$

$$\epsilon_y = \frac{\delta y}{d} = \frac{-2.4 * 10^{-6}}{16 * 10^{-3}} = -150 * 10^{-6}$$

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From hooke's law, $\boxed{\sigma_x = E\epsilon_x}$ we obtain:

$$E = \frac{\sigma_x}{\epsilon_x} = \frac{59.7*10^6 \text{ Pa}}{600*10^{-6}} = 99.5 \text{ GPa}$$

$$\vartheta = \frac{-\epsilon_y}{\epsilon_x} = \frac{-150*10^{-6}}{600*10^{-6}} = 0.25$$

2.12 Multiaxial Loading ; Generlized Hooke's Law

In this section, we'll consider structural elements subjected to loads acting in the directions of the three coordinate axes and producing normal stresses, σ_x, σ_y and σ_z that they aren't zeros ..

The effect of a given combined loading on a structure can be obtained by determining separately the effects of the various loads and combining the results obtained, provided that the following conditions are satisfied:

1. Each effect is linearly related to the load that produces it
2. The deformation resulting from any given load is small & doesn't affect the conditions of application of the other loads.

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In the multiaxial loading, the 1st condition will be satisfied if the stresses don't exceed the proportional limit of the material.
the 2nd condition will be satisfied if the stress on any given face doesn't cause deformations of the other faces that are large enough to affect the computation of the stresses on those faces ...

$$\epsilon_x = \frac{\sigma_x}{E} - \frac{\nu \sigma_y}{E} - \frac{\nu \sigma_z}{E}$$

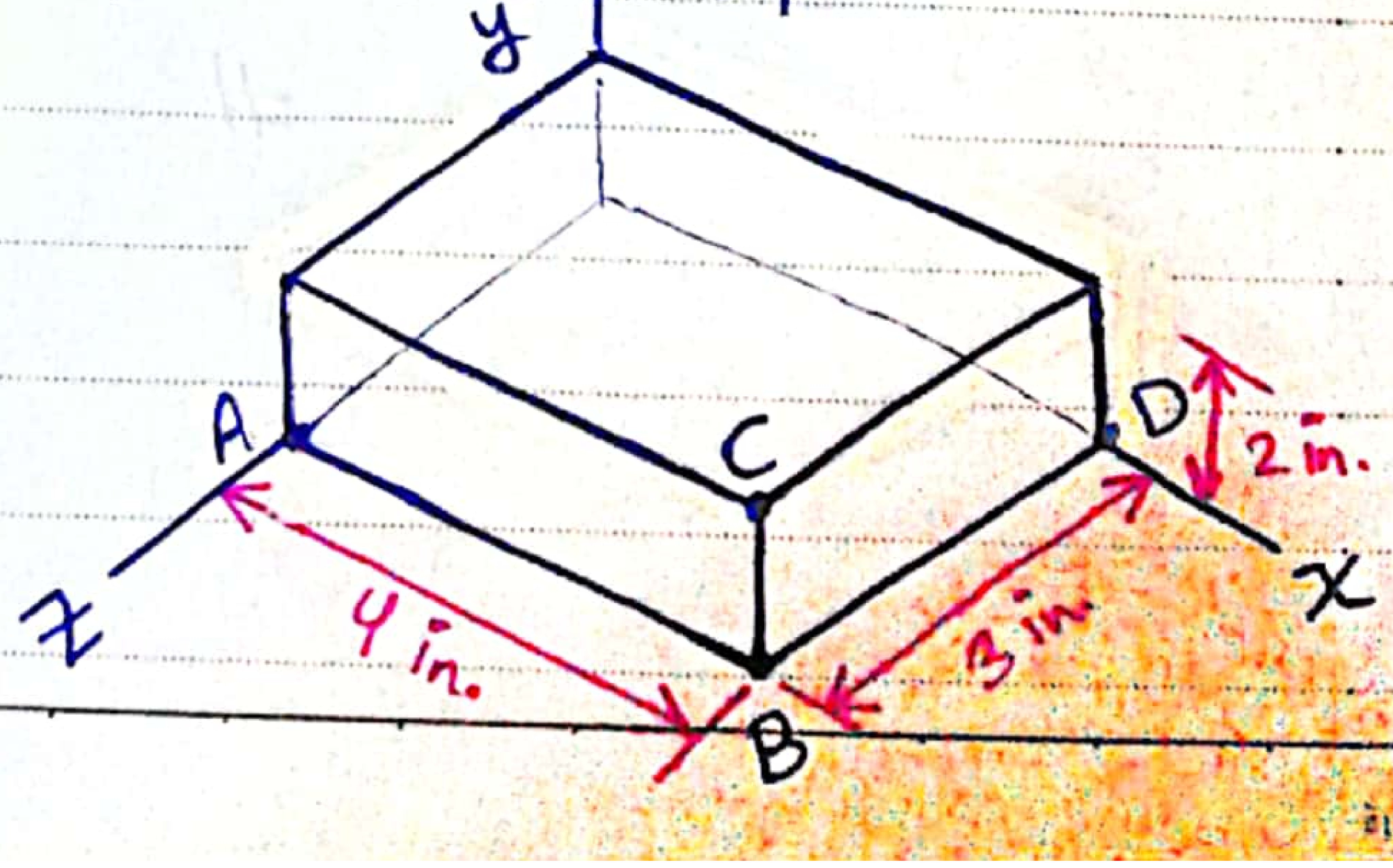
$$\epsilon_y = -\frac{\nu \sigma_x}{E} + \frac{\sigma_y}{E} - \frac{\nu \sigma_z}{E}$$

$$\epsilon_z = -\frac{\nu \sigma_x}{E} - \frac{\nu \sigma_y}{E} + \frac{\sigma_z}{E}$$

→ Referred to as the generalized Hooke's law for (the multiaxial loading) of a homogenous isotropic material.

Ex (2-08)

The steel block shown below is subjected to a uniform pressure on all its faces. Knowing that the change in length of edge AB is $-1.2 * 10^{-3}$ inch, determine :
a- the change in length of the other two edges
b- the pressure p applied to the faces of the block.
Assume $E = 29 * 10^6$ psi and $\nu = 0.29$



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a) Change in length of other edges :

$$\epsilon_x = \epsilon_y = \epsilon_z = \frac{-P}{E}(1-2\nu)$$

Since $\Rightarrow$ $\epsilon_x = \frac{\delta x}{AB} = \frac{-1.2*10^{-3} \text{ in.}}{4 \text{ in.}}$

$= -300*10^{-6}$

we get // $\epsilon_y = \epsilon_z = \epsilon_x = -300*10^{-6}$

from which it follows that

$$\delta_y = \epsilon_y (BC) = (-300*10^{-6})(2) = -600*10^{-6} \text{ in.}$$
$$\delta_z = \epsilon_z (BD) = (-300*10^{-6})(3) = -900*10^{-6} \text{ in.}$$

b) Pressure :

$$P = -\frac{E\epsilon_x}{1-2\nu}$$

$$= \frac{-(29*10^6 \text{ psi})(-300*10^{-6})}{1-0.58}$$

$= 20.7$ ksi

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Second

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Ch.3 Torsion

3.1 Introduction

Torsion is twisting of an object due to an applied torque.

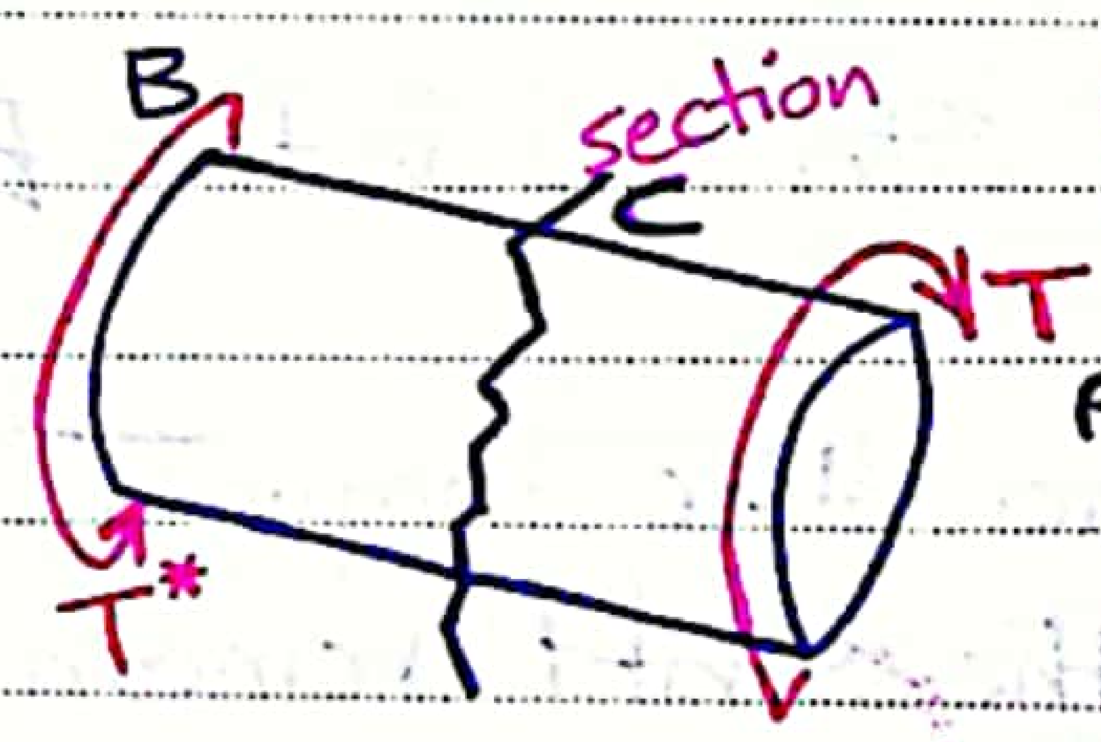
3.2 Discussion of the stresses in a shaft

ρ : distance

T : torque

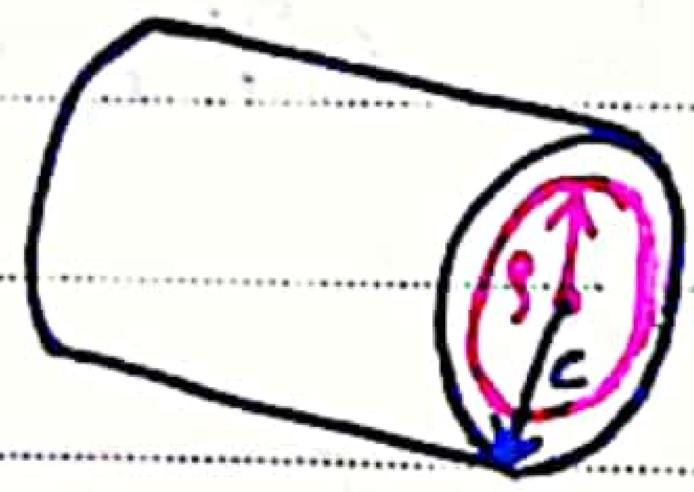
dA : area

τ : the shearing stress on the element



$$\boxed{T = \int \rho . dF} \quad \text{— (1)}$$

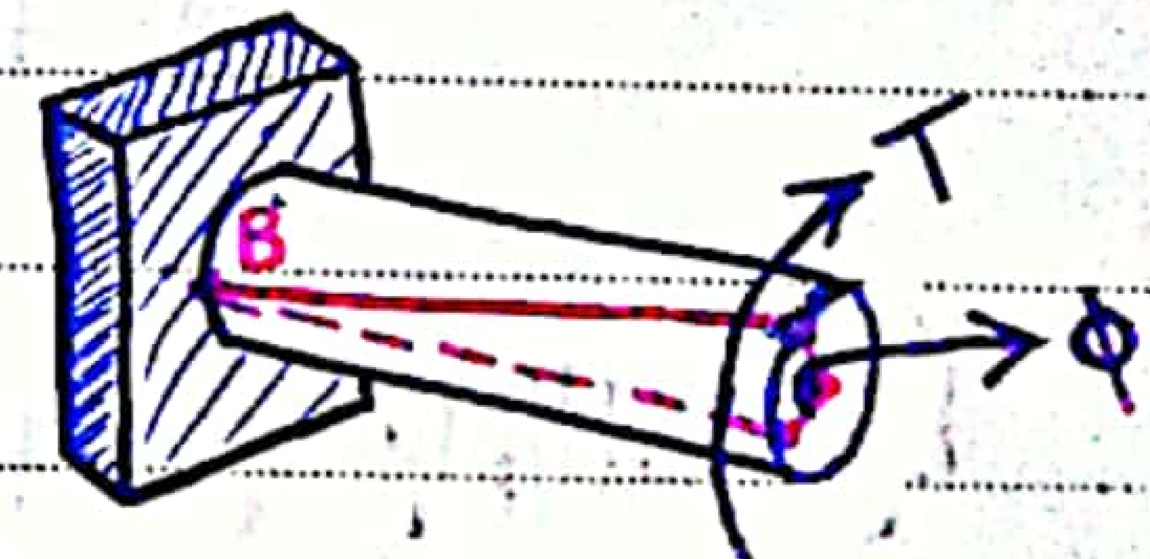
$dF = \tau \, dA$



c : shaft radius

3.3 Deformations in a circular shaft

$$\gamma \text{ (shear strain) [rad]} = \frac{\rho \phi}{L} \quad \text{— (2)}$$



the shearing strain in a circular shaft varies linearly with the distance from the axis of the shaft.

where : ϕ , twisting angle

L , shaft length

- the shearing strain (γ) is max. on the surface of the shaft at $\rho = c$

$$\gamma_{max} = \frac{c\,\phi}{L} \quad \text{— (3)}$$

$$\therefore \boxed{\gamma = \frac{\rho}{c}\,\gamma_{max}} \quad \text{— (4)}$$

3.4 Stresses in the Elastic Range

When the torque T is such that all shearing stresses in the shaft remain below the yield strength τ_y. Hooke's law will apply and there will be no permanent deformation.

$$\boxed{\tau = G\,\gamma}$$

where G: the modulus of rigidity or shear modulus of the material

- by multiplying equ. (4) by G, we get:

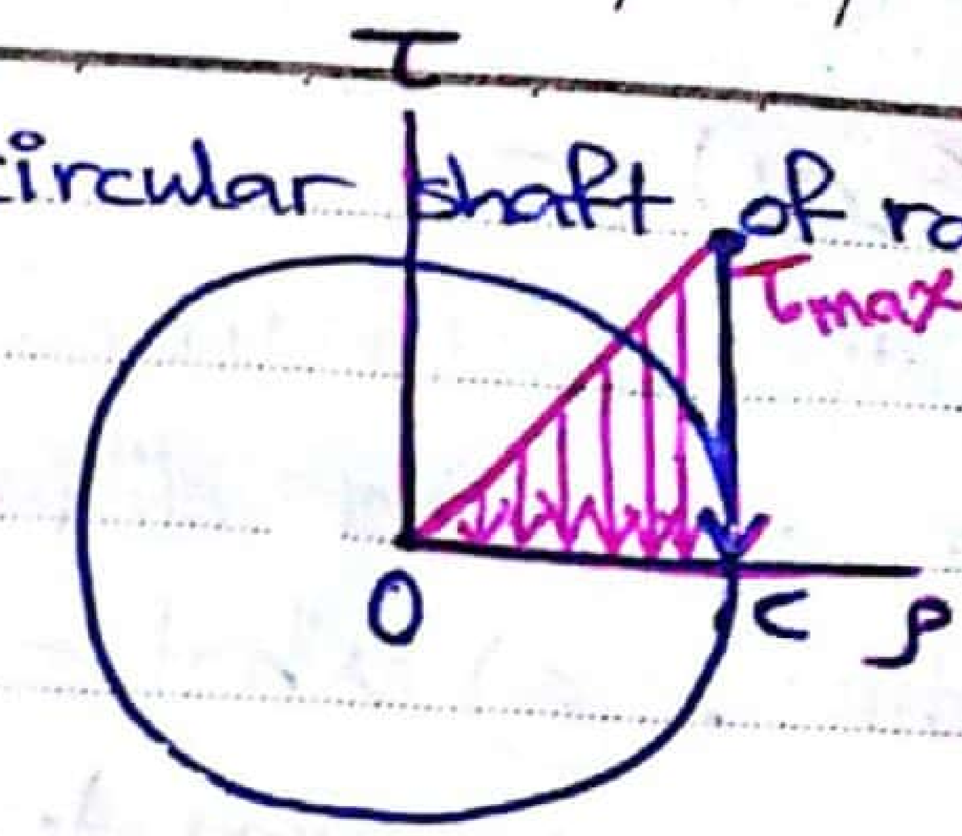
$$G\gamma = \frac{\rho}{c}\,G\gamma_{max}$$

$$\tau = \frac{\rho}{c}\,\tau_{max}$$

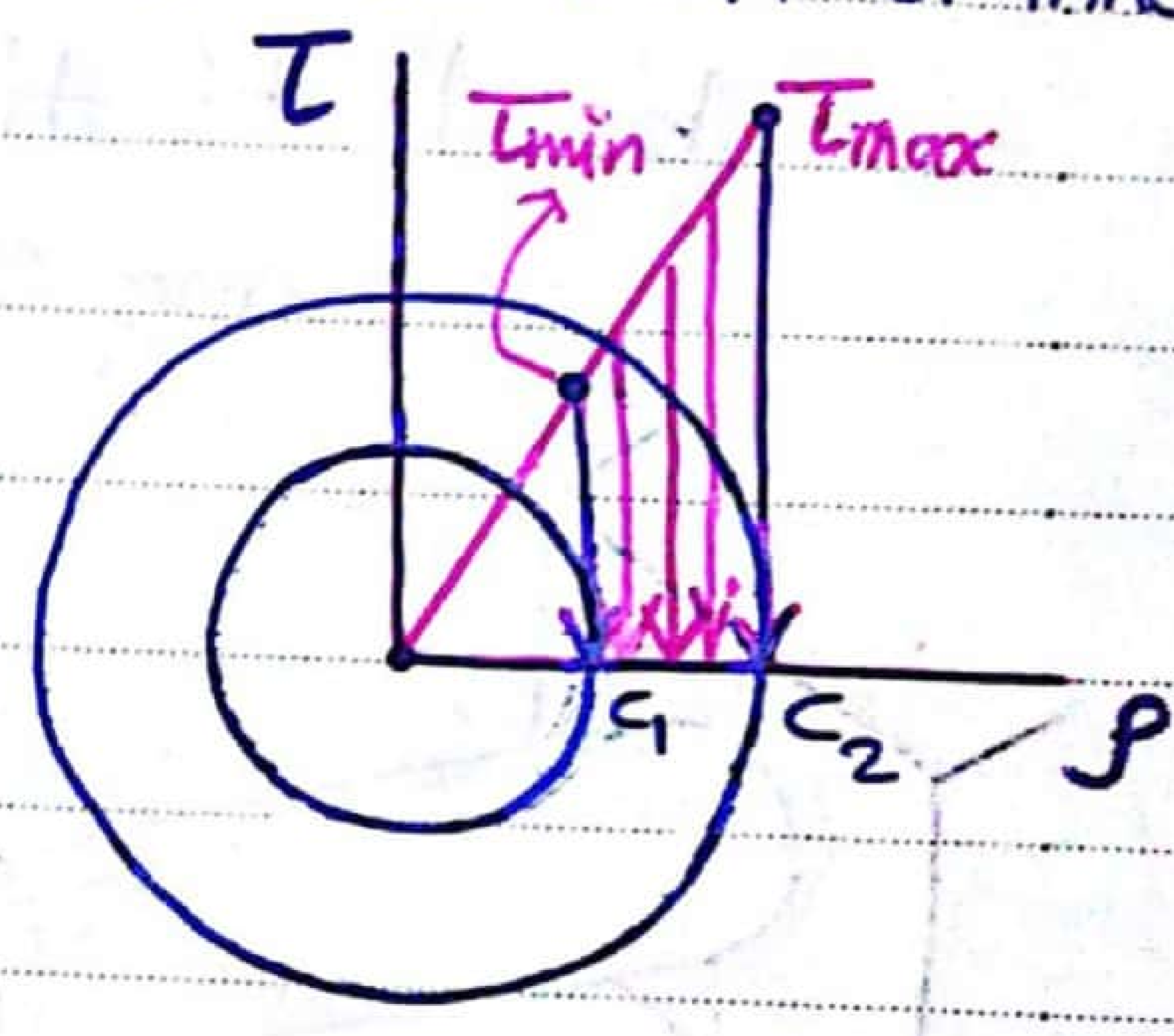
* هاي العلاقة معناتها إنه ال (shearing stress τ) على ال shaft بتتغير بشكل خطي مع المسافة (ρ) from the axis of the shaft... (من المحور)

within the elastic region ⇐

* the stress distribution in a solid circular shaft of radius $\underline{C}$.



* the stress distribution in a hollow circular shaft of inner radius $\underline{c_1}$ and outer radius $\underline{c_2}$.



$$\left[\tau_{min} = \frac{c_1}{c_2}\tau_{max}\right]$$

Remark) • τ_{max} occur in center

τ_{min} " " $R = C$

• polar moment of inertia (J) of a circle of radius (C) is : $\boxed{J = \frac{\pi}{2} C^4}$

For a hollow circular shaft c_1 & c_2 : (inner radius c_1, outer radius c_2)

$$\boxed{J = \frac{\pi}{2}\left(c_2^4 - c_1^4\right)}$$

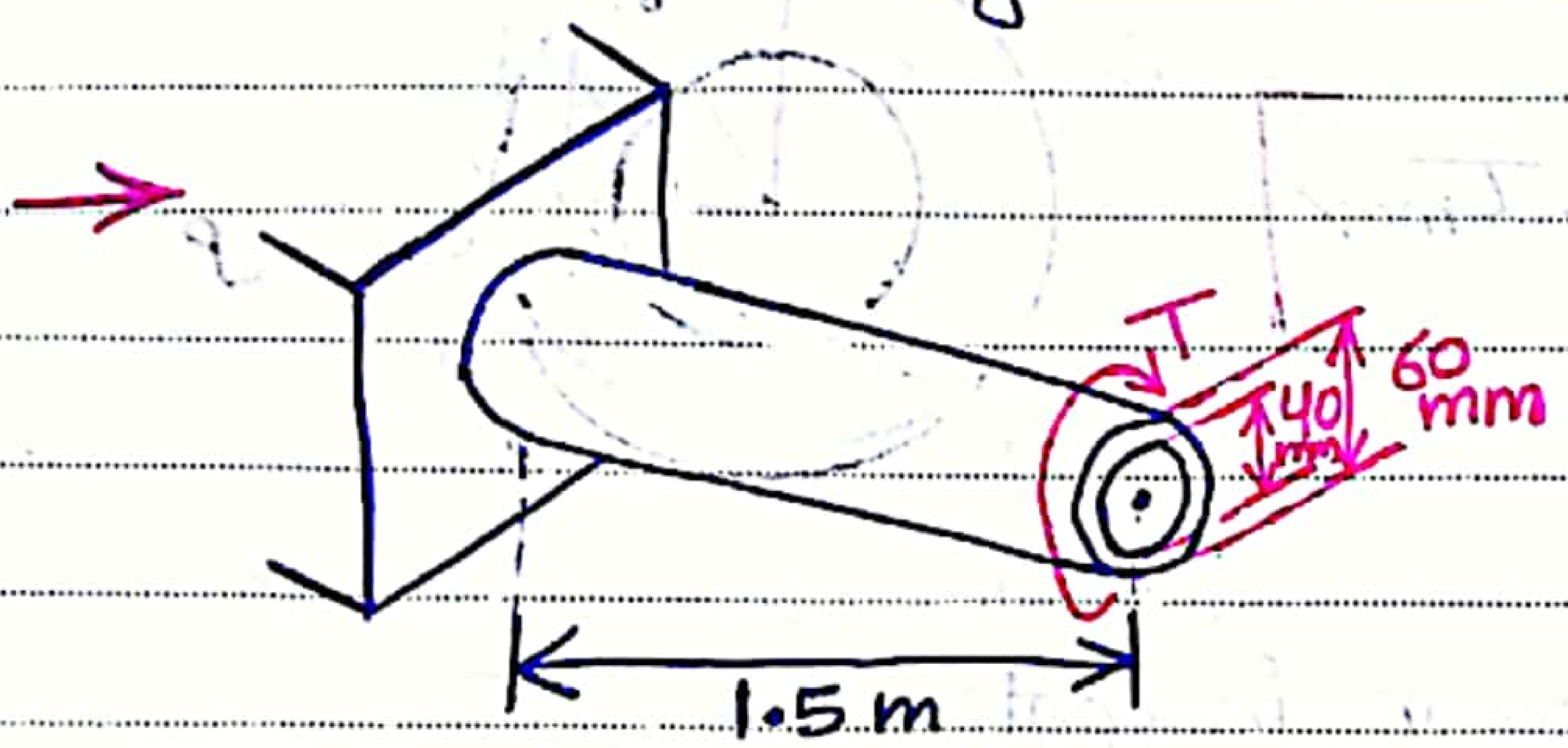
→ $T = \dfrac{\tau J}{\rho}$, Elastic Torsion Formula to express the shearing stress at any distance ρ from the axis of the shaft...

/ /

Ex(3.01) ::

A hollow cylindrical steel shaft is 1.5 m long and has inner and outer diameters respectively equal to 40 and 60 mm. a) What's the largest torque that can be applied to the shaft if the shearing stress isn't to exceed 120 MPa?

b) What's the corresponding min. value of the shearing stress in the shaft?



$c_1 = 0.02\ m$

$c_2 = 0.03\ m$

a-

$$T = \frac{J\,\tau_{max}}{c}$$

$$= \frac{\frac{\pi}{2}(c_2^4 - c_1^4) * 120 * 10^6}{c_2}$$

$$= \frac{\frac{\pi}{2}(0.03^4 - 0.02^4) * 120(10^6)}{0.03\ m}$$

$$= 4.08\ kN.m$$

b- the min. value of the shearing stress occurs on the inner surface of the shaft.

$$\tau_{min} = \frac{0.02}{0.03}(120 * 10^6) = 80\ MPa$$

τ_{min} for solid shaft = zero — NOTE

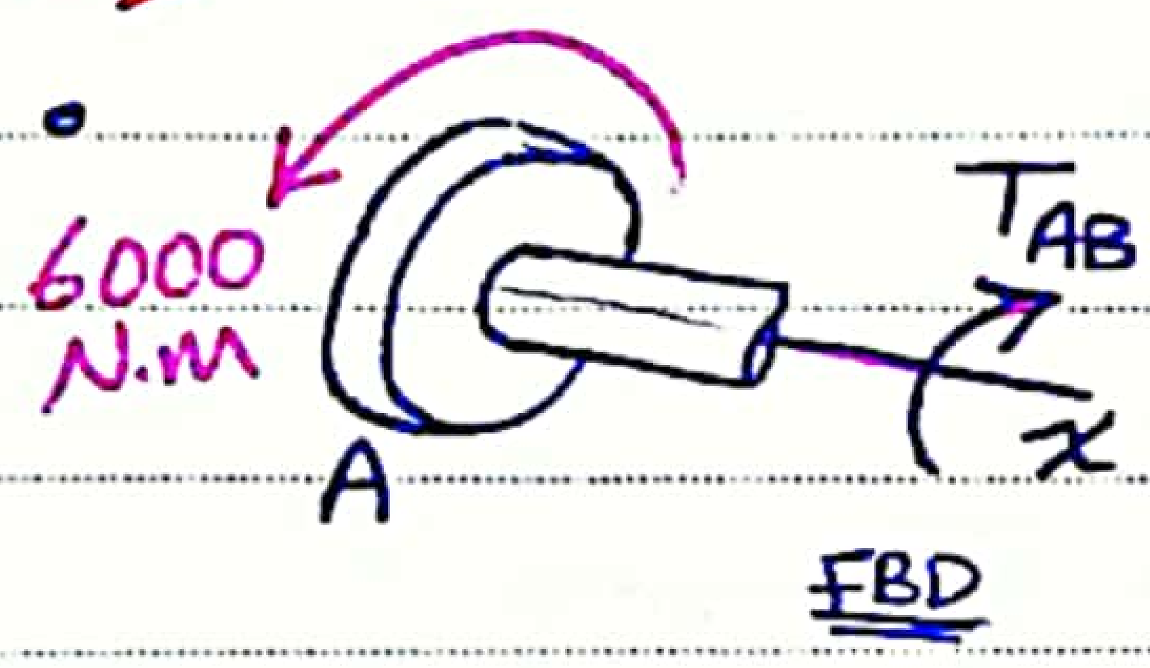
Sample Problem (3.1)

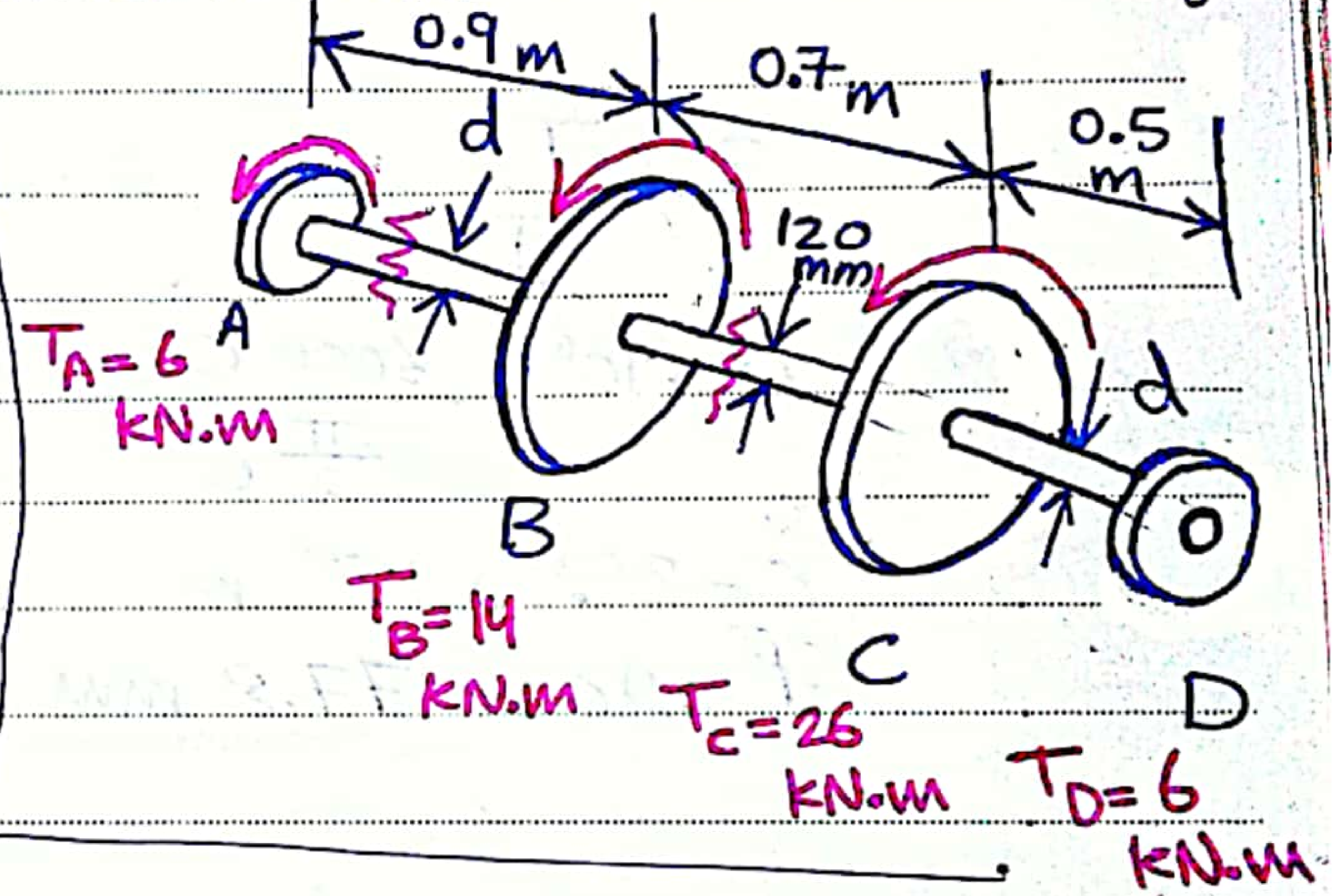
Shaft BC is hollow with inner & outer diameters of 90 mm and 120 mm, respectively. Shafts AB and CD are solid and of diameter d. For the loading shown, determine:

a) the max. and min. shearing stress in shaft BC?

b) the required diameter d of shafts AB and CD if the allowable shearing stress in these shafts is 65 MPa?

→ Solu.

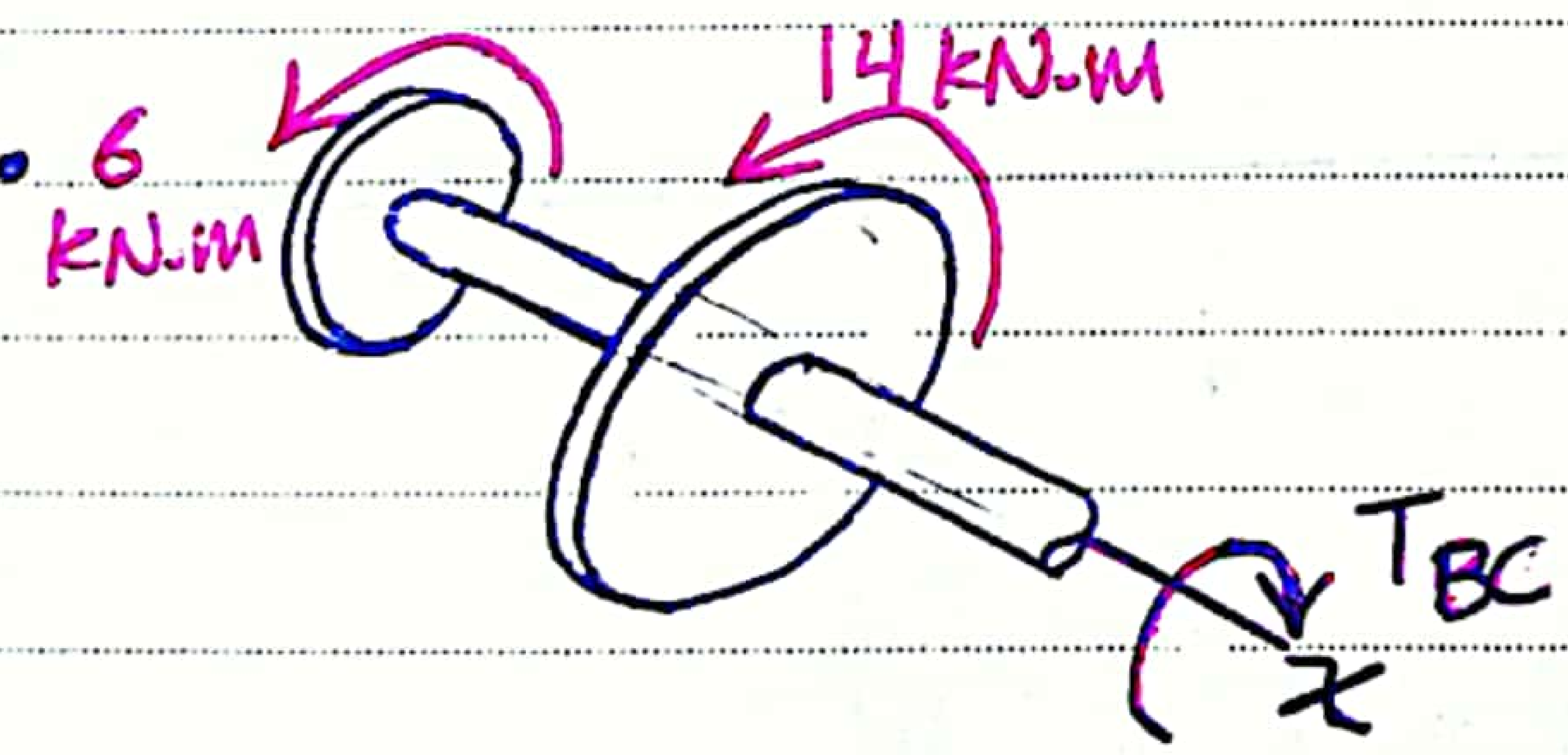




$\Sigma M_x = 0$

$6000 - T_{AB} = 0$

$T_{AB} = 6 \text{ kN.m}$



$\Sigma M_x = 0$

$6000 + 14{,}000 - T_{BC} = 0$

$T_{BC} = 20 \text{ kN.m}$ #

a- Shaft BC ⇒

$$\tau_{max} = \frac{T_{BC}\, C_2}{J} = \frac{20{,}000 * 0.06}{\frac{\pi}{2}(0.06^4 - 0.045^4)}$$

$$= 86.2 * 10^6 \, [Pa]$$

$$\tau_{min} = \frac{c_1}{c_2} \tau_{max}$$

$$= \frac{0.045}{0.06} (86.2 * 10^6)$$

$$= 64.7 \text{ MPa}$$

b- Shafts AB and CD ⇒

$$\tau = \frac{Tc}{J}$$

$$65 * 10^6 = \frac{6000\, c}{\frac{\pi}{2} c^4}$$

$$c = 38.9 * 10^{-3} \text{ m}$$

$$d = 2c = 77.8 \text{ mm}$$

/ /

Sample Problem (3.2)

The preliminary design of a large shaft connecting a motor to a generator calls for the use of a hollow shaft with inner and outer diameters of 4 in. and 6 in., respectively. Knowing that the allowable shearing stress is 12 ksi, determine the max. torque that can be transmitted

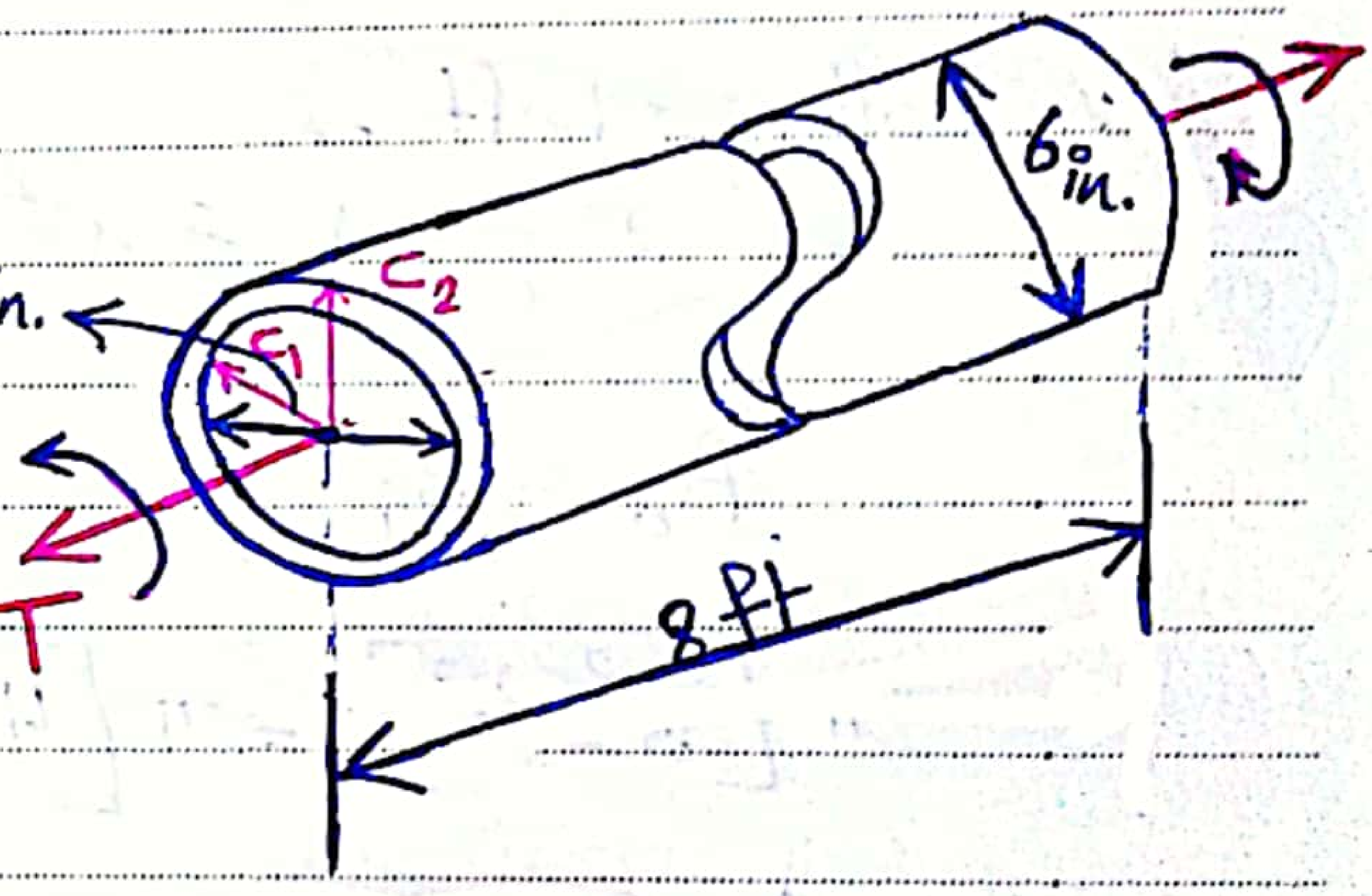
a) by the shaft as designed ?

b) by a solid shaft of the same weight ?

c) by a hollow shaft of the same weight and of 8 in. outer diameter ?

$C_2 = 3$ in.

$C_1 = 2$ in.



a- as designed...

$$\tau_{max} = \frac{T C_2}{J} \rightarrow 12\text{ksi} = \frac{T(3\text{ in.})}{\frac{\pi}{2}(3^4 - 2^4)}$$

$$T = 408 \text{ [kip.in]}$$

b- Solid shaft ...

their cross sectional area must be equal

$$A_a = A_b$$

$$\pi[3^2 - 2^2] = \pi C_3^2 \Rightarrow C_3 = 2.24 \text{ in.}$$

since $\tau_{all} = 12$ ksi , $\tau_{max} = \frac{T C_3}{J}$

$$12 \text{ ksi} = \frac{T(2.24)}{\frac{\pi}{2}(2.24)^4}$$

$$T = 211 [\text{kip.in}]$$

c. hollow shaft ...

برضو هتكون المساحة متساوية قبل الفرع اللي قبل

$$A_a = A_b$$

$$\pi[3^2 - 2^2] = \pi[4^2 - C_5^2]$$

$$C_5 = 3.317 \text{ in.}$$

$$\therefore J = \frac{\pi}{2}[4^4 - 3.317^4] = 212 [\text{in}^4]$$

$$\Rightarrow 12 \text{ ksi} = \frac{T(4)}{212}$$

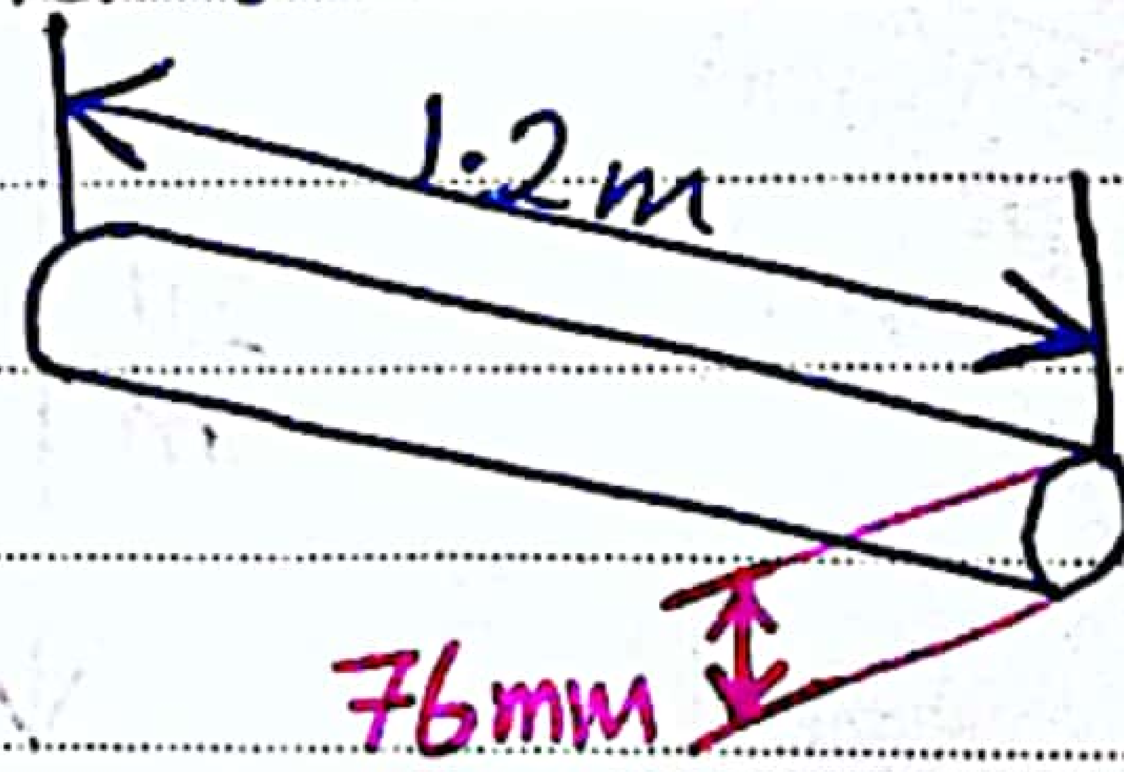
$$T = 636 [\text{kip.in}] \quad \#$$

/ /

Suggested Problems

3.1) a) Determine the max. shearing stress caused by a 4.6 kN.m torque T in the 76 mm diameter solid aluminum shaft shown.

b) Solve part a, assuming that the solid shaft has been replaced by a hollow shaft of the same outer diameter and of 24 mm inner diameter.



→ a-

$$\tau = \frac{Tc}{J}$$

$$= \frac{4.6 * 10^3 * 0.038}{3.2753 * 10^{-6}} = 53.4 * 10^6 [pa]$$

b- $$\tau = \frac{Tc}{J}$$

$$= \frac{4.6 * 10^3 * 0.038}{\frac{\pi}{2}(0.038^4 - 0.012^4)} = 53.9 * 10^6 [pa]$$

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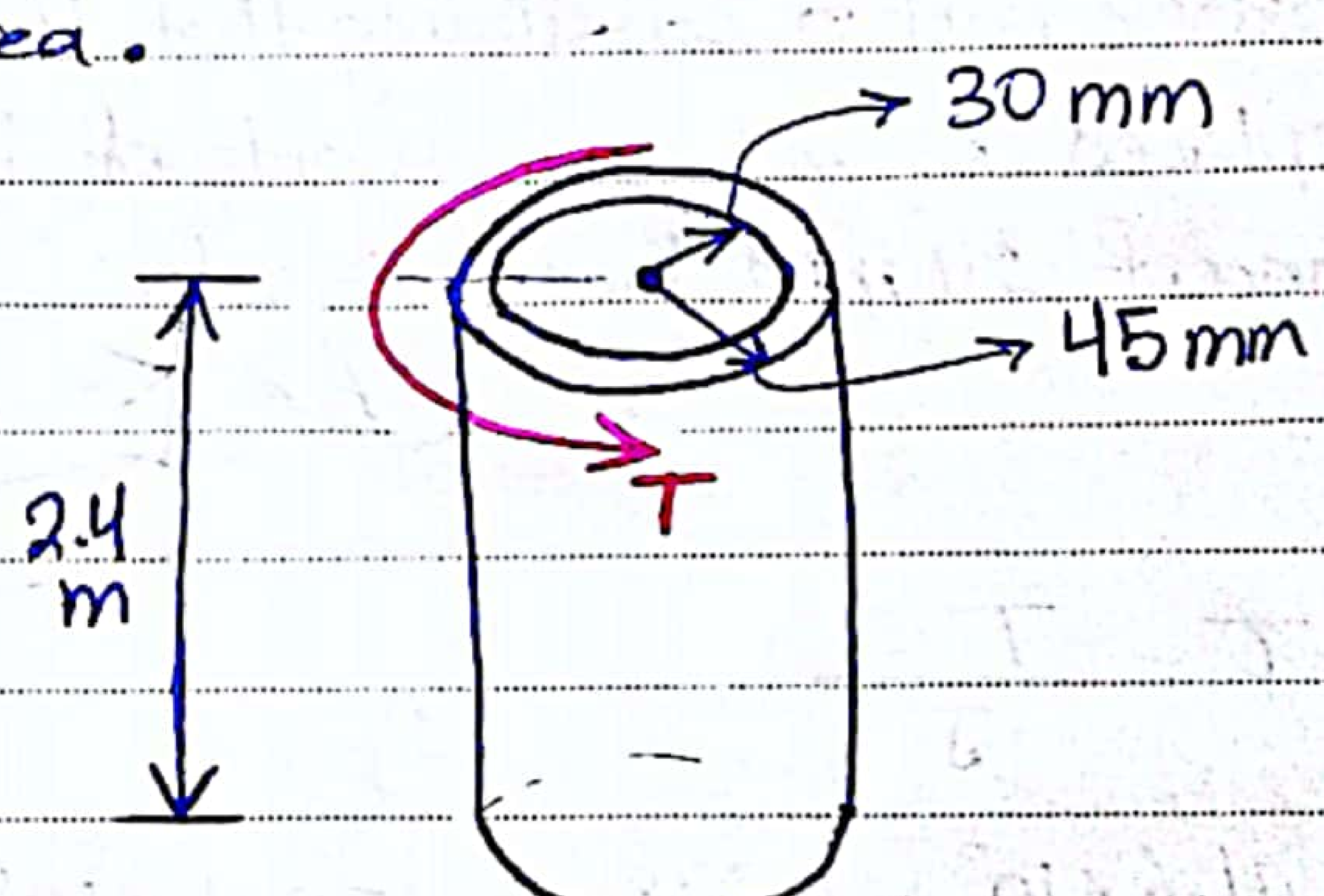
3.2) a) Determine the torque T that causes a max. shearing stress of 45 MPa in the hollow cylindrical steel shaft shown.

b) Determine the max. shearing stress caused by the same torque T in a solid cylindrical shaft of the same cross sectional area.



a-

$$T = \frac{\tau J}{c}$$

$$= \frac{45 * 10^{6} * \frac{\pi}{2}(45^4 - 30^4) * 10^{-6}}{45 * 10^{-3}}$$

$$= 5.17 \text{ kN.m}$$

b-

$$\tau = \frac{2T}{\pi c^3} = \frac{Tc}{J}$$

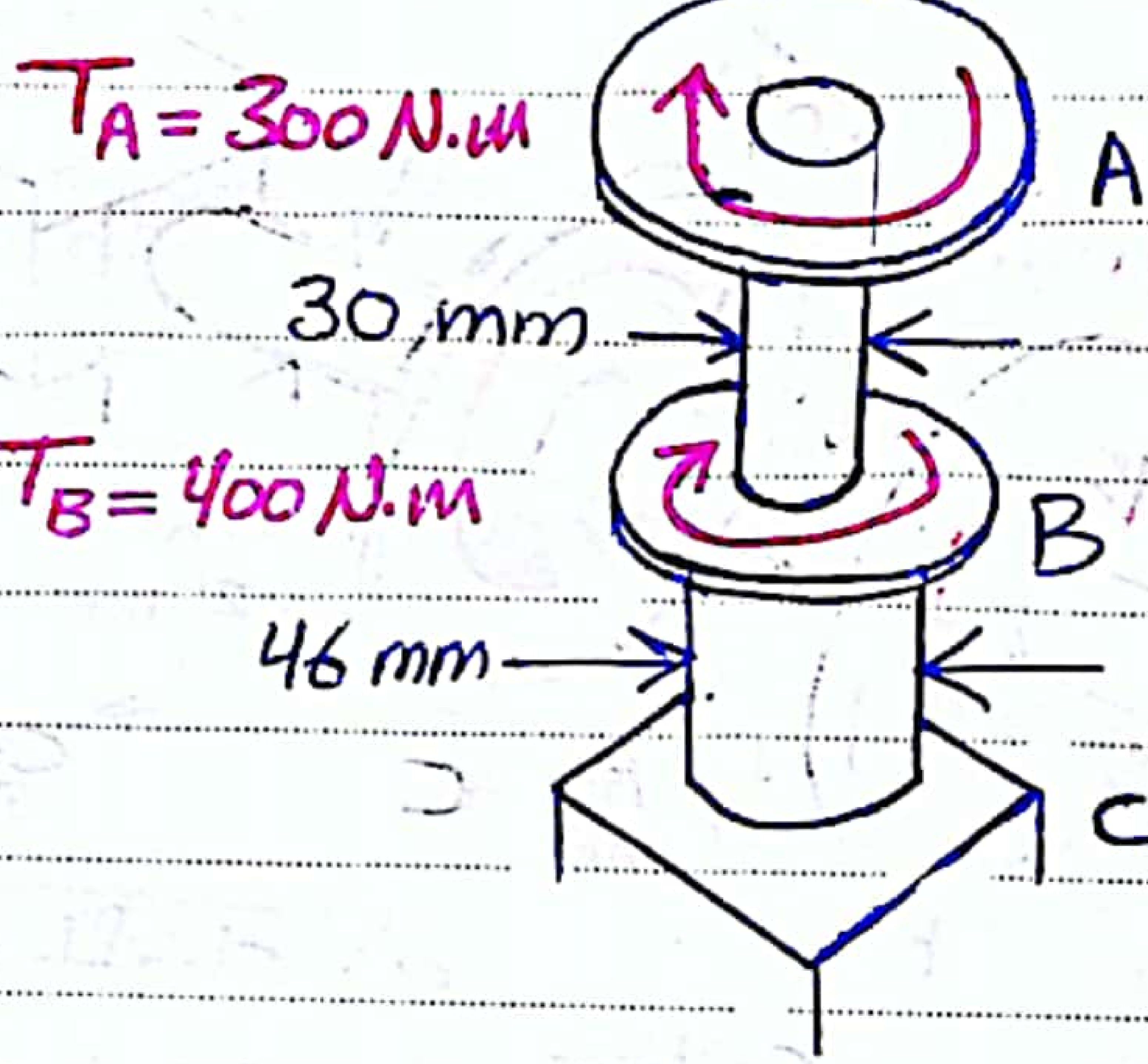
$$A = \pi(c_2^2 - c_1^2)$$ (with $c_2 = 45$, $c_1 = 30$)

$$= 3.5343 * 10^3 \text{ mm}^2$$

$$c = \sqrt{\frac{A}{\pi}} = 33.541 \text{ mm}$$

$$= \frac{2 * 5.1689 * 10^3}{\pi (0.033541)^3} = 87.2 (10^6) \text{ pa}$$

3.9) The torques shown are exerted on pulleys A and B. Knowing that both shafts are solid, determine the max. shearing stresses in a) shaft AB b) shaft BC



a- $T_{AB} = 300$ N.m

$d = 0.03$ m

$C = 0.015$ m

$$\tau_{max} = \frac{TC}{J} = \frac{2T}{\pi c^3}$$

$$= \frac{2(300)}{\pi (0.015)^3}$$

$$= 56.588 * 10^6 \text{ pa}$$

#

b- $T_{BC} = 300 + 400$

$= 700$ N.m

$d = 0.046$ m , $C = 0.023$ m

$$\tau_{max} = \frac{TC}{J} = \frac{2(T)}{\pi C^3} = \frac{2(700)}{\pi (0.023)^3} = 36.6 \text{ MPa}$$

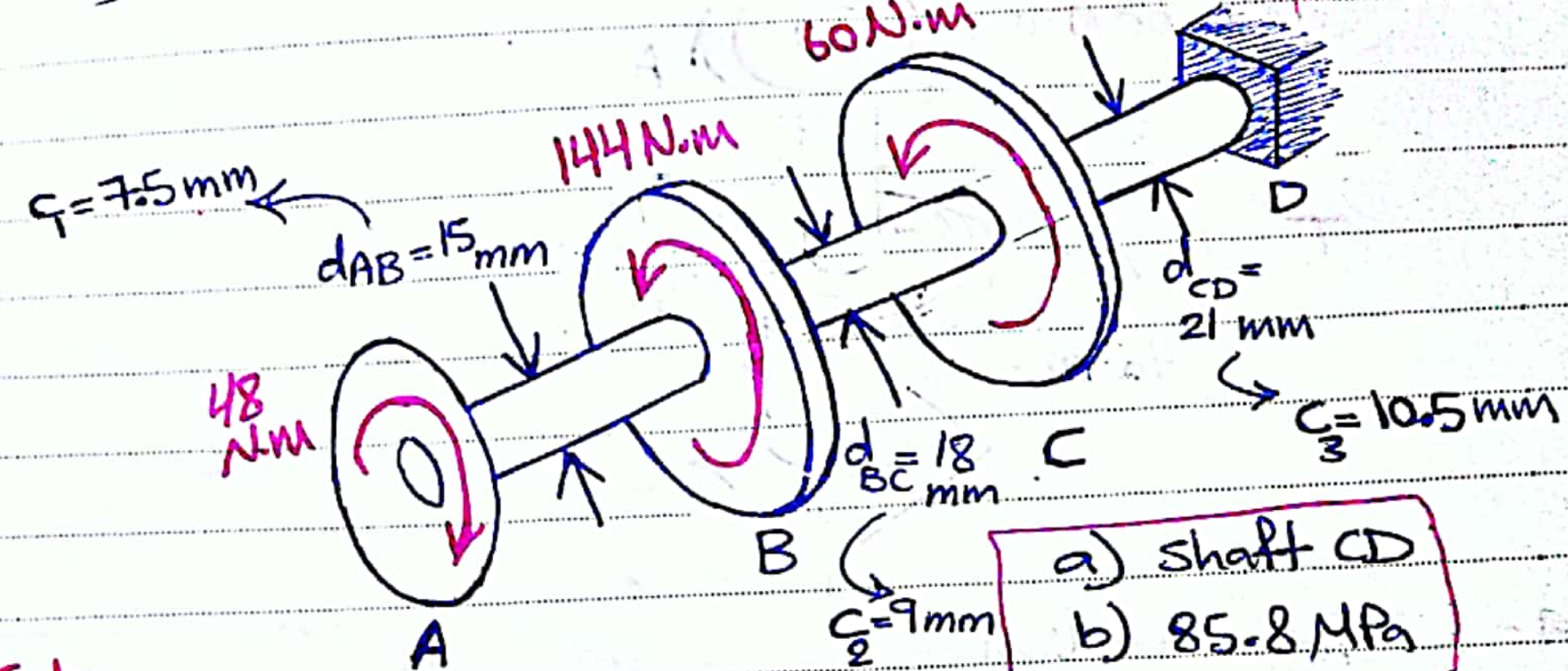
#

3.11) Knowing that each of the shafts AB, BC, and CD consists of a solid circular rod, determine:

a) the shaft in which the τ_{max} occurs

b) the magnitude of that stress

بدنا نعرف أكبر τ
نجيب ج أي shaft وجيبة
عند كل واحد منهم ..



a) shaft CD

b) 85.8 MPa

Solu.

- Shaft AB →

$$T_{AB} = 48 \text{ N.m}$$

$$\tau_{max} = \frac{TC}{J} = \frac{2T}{\pi c_1^3} = \frac{2(48)}{\pi(0.0075)^3}$$

$$= 72.433 \text{ MPa}$$

- Shaft BC →

$$T_{BC} = -48 + 144 = 96 \text{ N.m}$$

$$\tau_{max} = \frac{2(96)}{\pi(0.009)^3} = 83.835 \text{ MPa}$$

c_2

- Shaft CD → $T_{CD} = -48 + 144 + 60 = 156$ N.m

$$\tau_{max} = \frac{2(156)}{\pi(0.0105)^3} = \boxed{85.79 \text{ MPa}}$$

CD

c_3

ans. for (a)

/ /

3.12) Knowing that an 8 mm diameter hole has been drilled through each of the shafts AB, BC and CD, determine:
a- the shaft in which the max. τ occurs?
b- the magnitude of that stress?

حل نفس المشكل
بيقال !!

solu.

Hole : $C_h = 4\,mm$

Shaft AB : $T_{AB} = 48\ N.m$

$$\tau_{max} = \frac{TC}{J} = \frac{(48)(0.0075)}{\frac{\pi}{2}(0.0075^2 - 0.004^2)} = 78.81\ MPa$$

Shaft BC : $T_{BC} = 96\ N.m$

$$\tau_{max} = \frac{TC_2}{J} = \frac{96(0.009)}{\frac{\pi}{2}(0.009^4 - 0.004^4)} = 87.239\ MPa$$

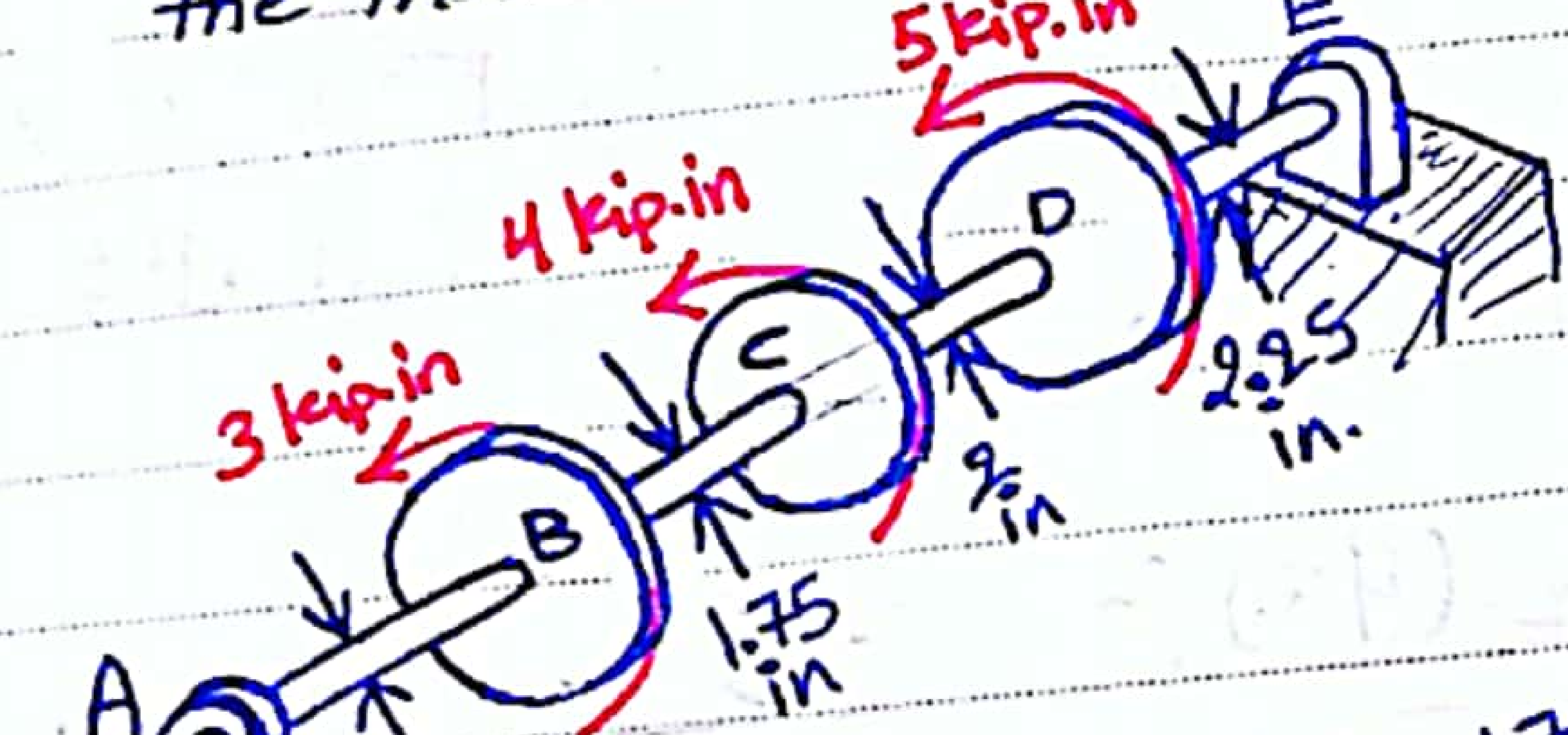
Shaft CD : $T_{CD} = 156\ N.m$

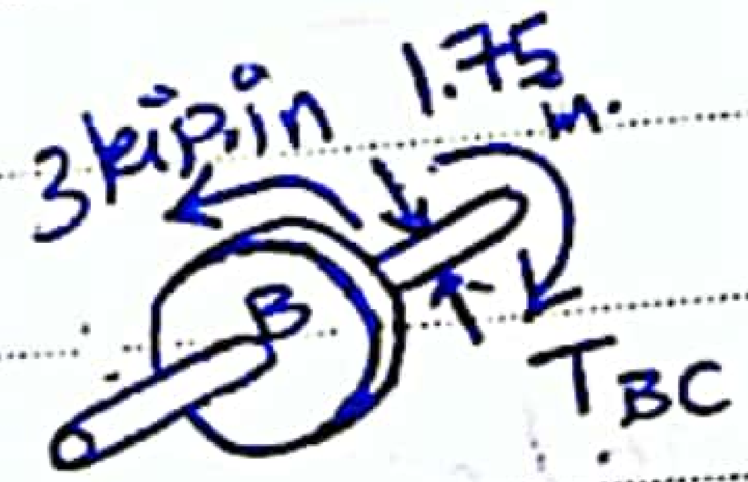
$$\tau_{max} = \frac{TC_3}{J} = \frac{156(0.0105)}{\frac{\pi}{2}(0.0105^4 - 0.004^4)}$$

$$= 87.636\ MPa$$

a- Shaft CD
b- 87.6 MPa

3.13) Under normal operating conditions, the electric motor exerts a 12 kip.in torque at E. Knowing that each shaft is solid, determine: the max. shearing stress in a) shaft BC b) shaft CD c) shaft DE





FBD

→ a) shaft BC :

$$T_{BC} = 3 \text{ kip.in}$$

$$\tau = \frac{Tc}{J} = \frac{Tc}{\frac{\pi}{2}c^4} = \frac{2T}{\pi c^3} = \frac{2(3)\text{ kip.in}}{\pi\left(\frac{1.75}{2}\right)^3}$$

$$\tau_{BC} = 2.85 \text{ ksi}$$

b) Shaft CD : $T_{CD} = 3 + 4 = 7$ kips.in

$$\tau = \frac{2(7)}{\pi\left(\frac{2}{2}\right)^3} = 4.46 \text{ ksi}$$

c) Shaft DE : $T_{DE} = 3 + 4 + 5 = 12$ kips.in

$$\tau = \frac{2(12)}{\pi\left(\frac{2.25}{2}\right)^3} = 5.37 \text{ ksi}$$

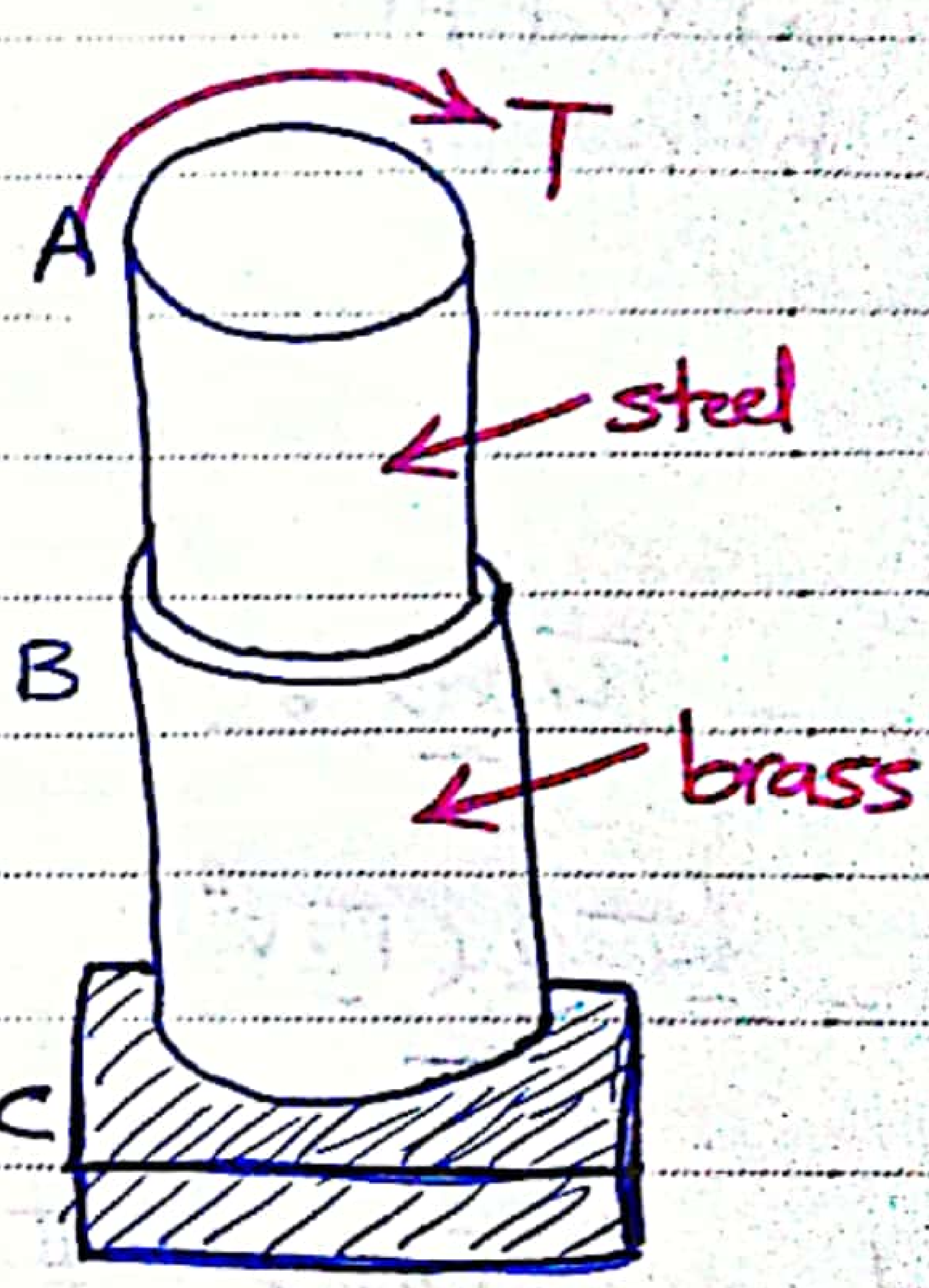
#

3.15) The allowable τ is 15 ksi in the 1.5 in. diameter steel rod AB and 8 ksi in the 1.8 in. diameter brass rod BC. Neglecting the effect of stress concentrations, determine the largest torque that can be applied at A?

$$\rightarrow T_{AB} = \left(\frac{\pi}{2} c^3 \tau_{max}\right) \leftarrow \frac{J \tau_{max}}{c}$$

$$= \frac{\pi}{2} (0.75^3)(15)$$

$$= 9.94 \ [\text{kip.in}]$$

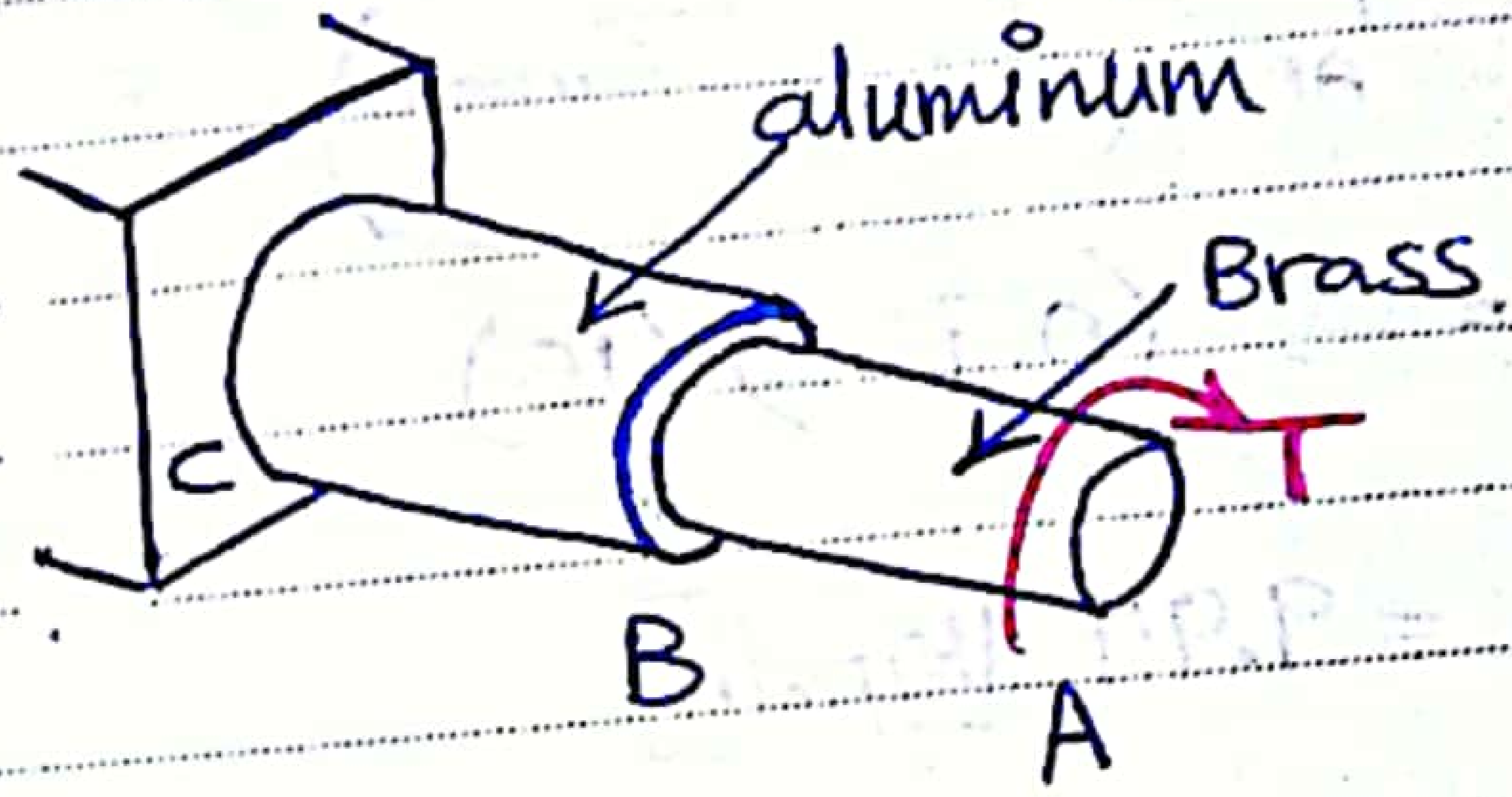


$$T_{BC} = \frac{\pi}{2} (0.9)^3 (8)$$

$$= 9.16 \ [\text{kip.in}]$$

the allowable torque is the smaller value...

3.17 The allowable stress is 50 MPa in the brass rod AB and 25 MPa in the aluminium rod BC. Knowing that a torque of magnitude T = 1250 N.m is applied at A, determine the required diameter of
a) rod AB
b) rod BC



$$\rightarrow \quad C = \frac{\tau_{max} J}{T}$$

$$C = \frac{(\tau_{max})(\frac{\pi}{2} C^4)}{T}$$

$$1 = \frac{\tau_{max}(\frac{\pi}{2} C^3)}{T} \Rightarrow C^3 = \frac{2T}{\pi \tau_{max}}$$

- $C_{AB}^3 = \frac{2(1250)}{\pi(50 \times 10^6)} = 15.9\,(10^{-6})\,m^3$

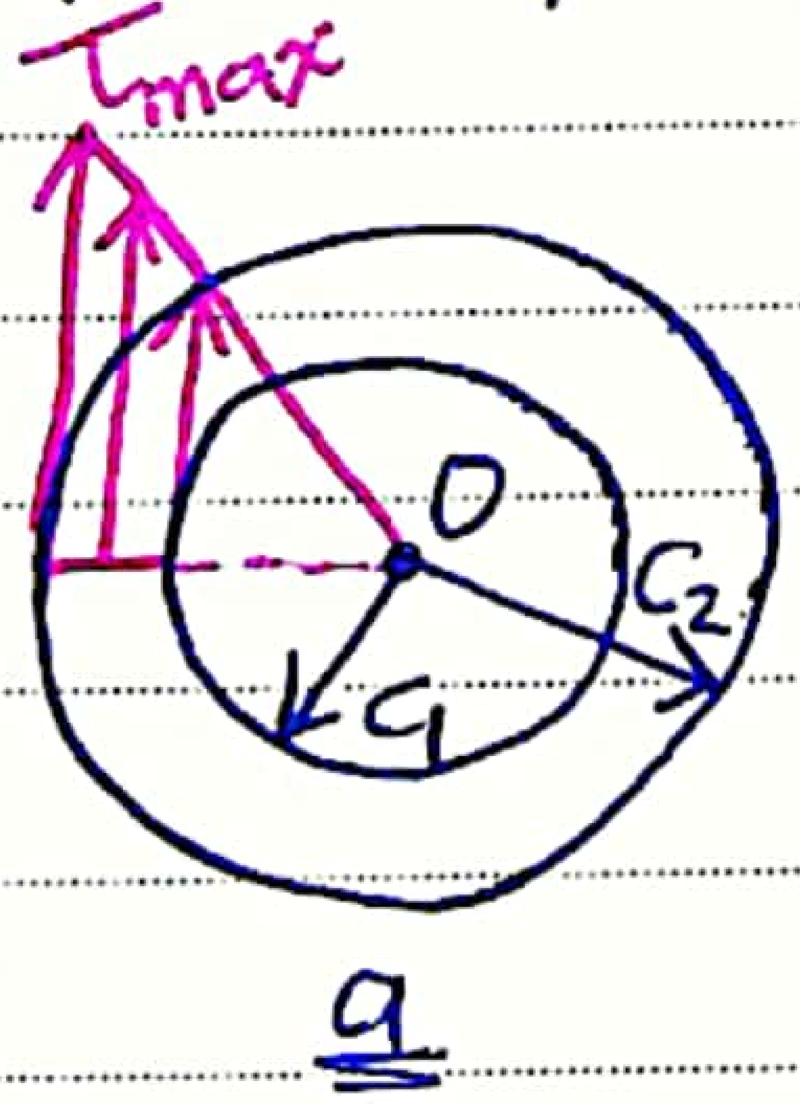
$$C_{AB} = 25.15\ mm \rightarrow d_{AB} = 50.3\ mm$$

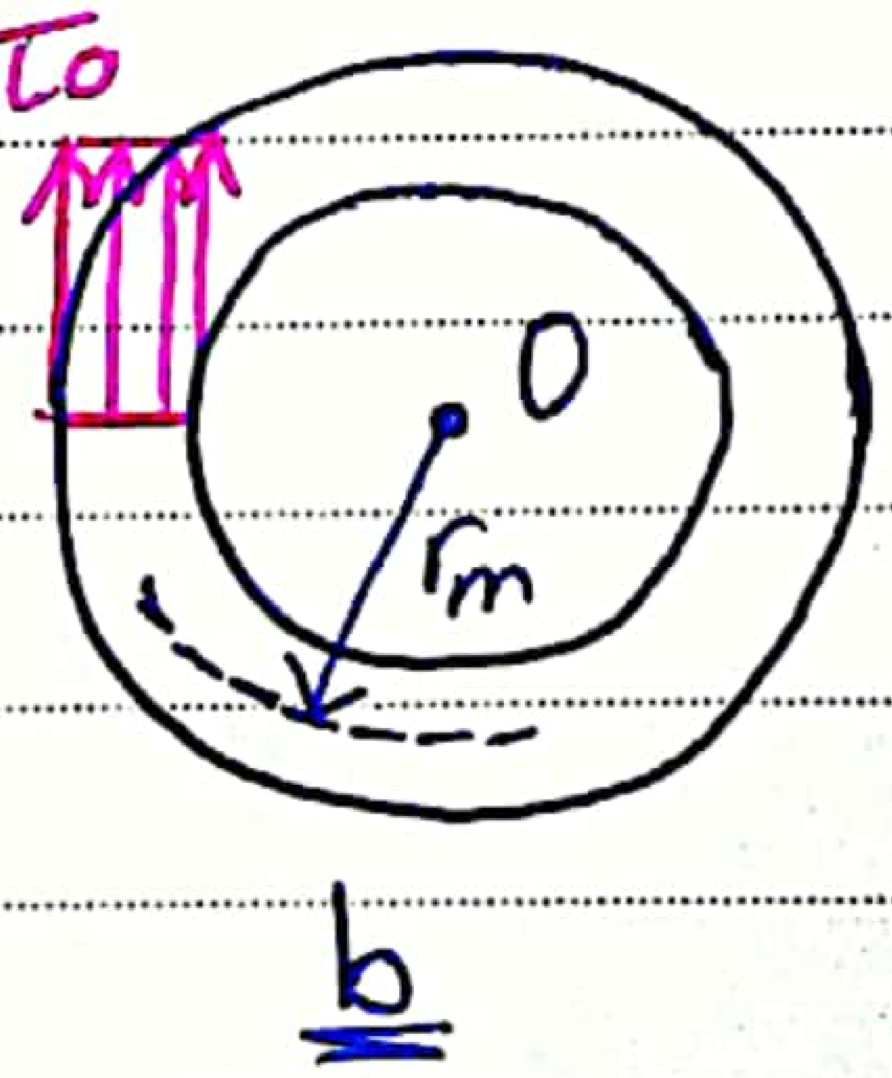
- $C_{BC}^3 = \frac{2(1250)}{\pi(25 \times 10^6)} = 31.831 \times 10^{-6}\ m^3$

$$C_{BC} = 31.69\ mm \rightarrow d_{BC} = 63.4\ mm$$

3.30) While the exact distribution of the shearing stresses in a holbw cylindrical shaft is as shown below, an approximate value can be obtained for τ_{max} by assuming that the stresses are uniformly distributed over the area A of the cross section, as shown in b below, and then furthur assuming that all of the elemantry shearing forces act at a distance from O equal to the mean radius $\frac{1}{2}(C_1+C_2)$ of the cross section. This approx. value $\tau_o = T/Ar_m$, where T : applied torque.

Determine the ratio τ_{max}/τ_o of the true value of the max. τ and its approximate value τ_o for values of C_1/C_2 respectively equal to 1, 0.95, 0.75, 0.5 and 0





→ For a hollow shaft : $\tau_{max} = \frac{TC_2}{J} = \frac{2TC_2}{\pi(C_2^4-C_1^4)}$

$$= \frac{2TC_2}{\pi(C_2^2-C_1^2)(C_2^2+C_1^2)} = \frac{2TC_2}{A(C_2^2+C_1^2)}$$

By definition ⇒ $\tau_o = \frac{T}{Ar_m} = \frac{2T}{A(C_2+C_1)}$

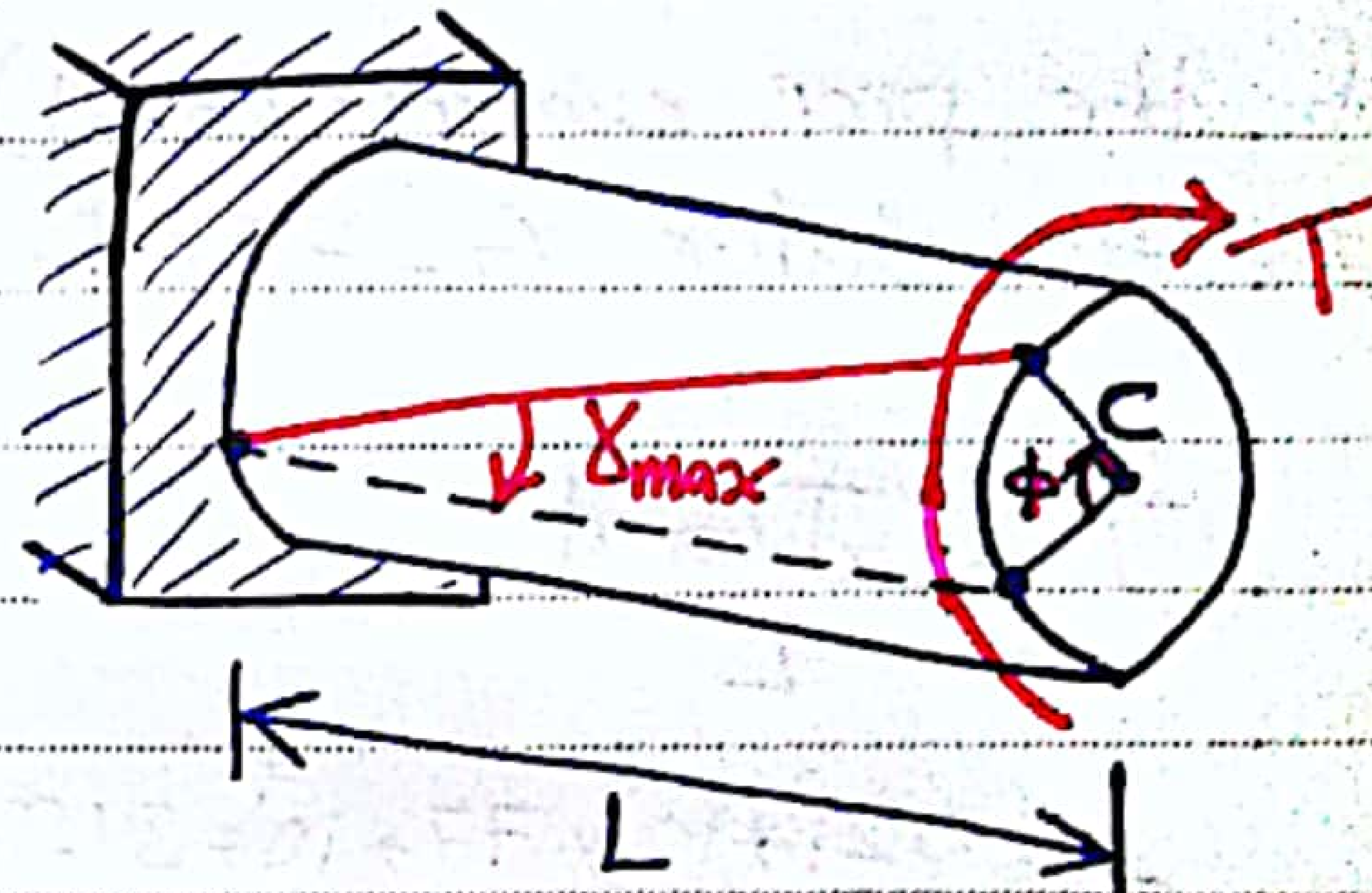
Dividing ....

$$\frac{\tau_{max}}{\tau_0} = \frac{C_2(C_2+C_1)}{C_2^2+C_1^2} = \frac{1+(C_1/C_2)}{1+(C_1/C_2)^2}$$

C_1/C_2	1	0.95	0.75	0.5	0
τ_{max}/τ_0	1	1.025	1.120	1.20	1

3.5 Angle of Twist in the Elastic Range

The entire shaft will be assumed to remain elastic.



Considering the case of the shaft of length (l) and of uniform cross section of radius (C) subjected to a torque (T) at its free end..

$$\gamma_{max} = \frac{\tau_{max}}{G} = \frac{TC}{JG}$$

(elastic) في منطقة الـ where we can apply hooke's law

$$\gamma_{max} = \frac{C\phi}{L}$$

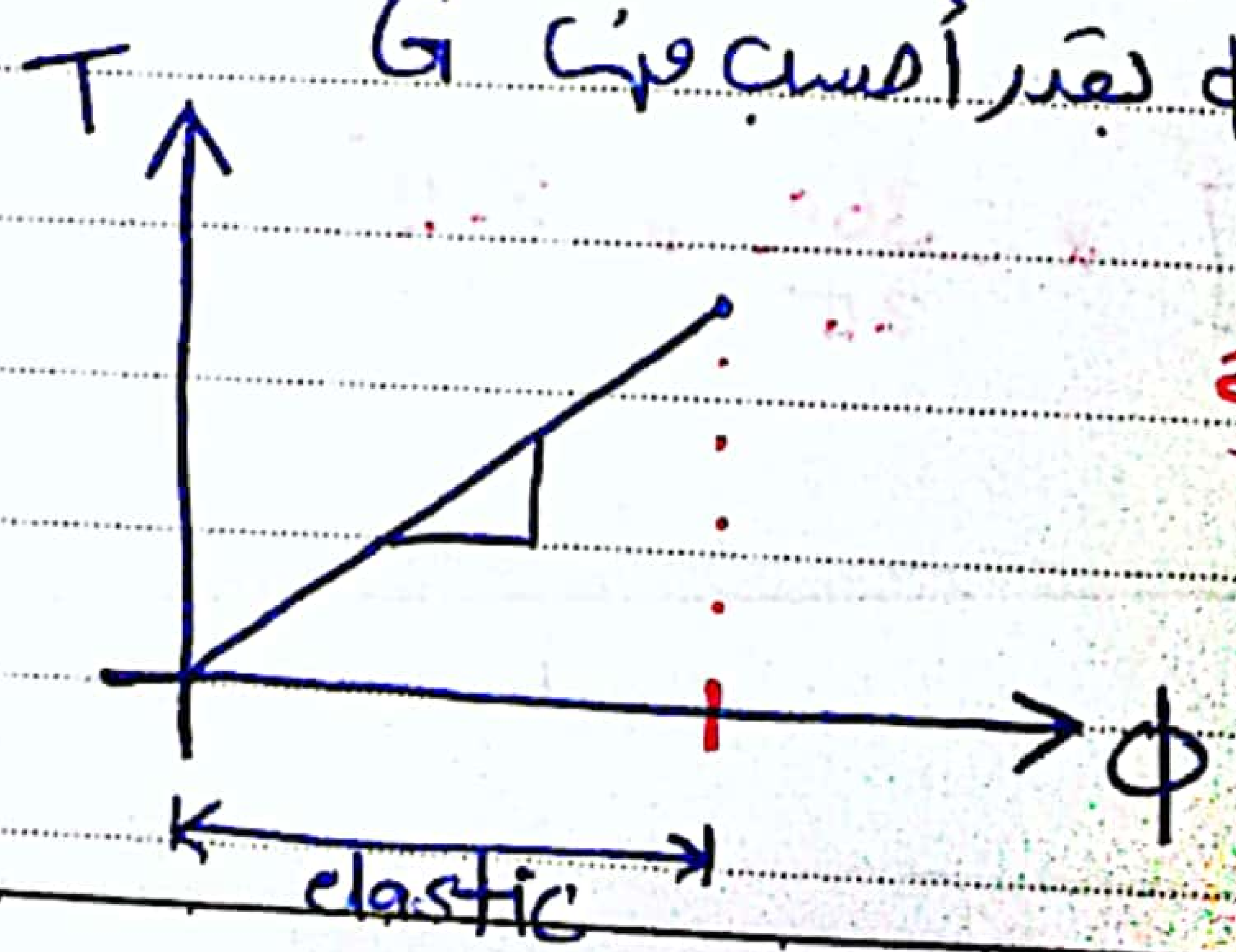
$$\Rightarrow \frac{\cancel{C}\phi}{L} = \frac{T\cancel{C}}{JG}$$

$$\boxed{\underset{[rad]}{\phi} = \frac{TL}{JG}}$$

→ for a homogenous shaft, and uniform cross section with loading at its end..

* كلما ازداد التورك بتزيد ϕ

* من خلال معرفتي لقيمة ϕ بقدر أحسب منه G



منطقة الـ elastic العلاقة طردية خطية بين ϕ والـ T

$$\left(\text{slope} = \frac{JG}{L}\right)$$

Ex (3.02)

What torque should be applied to the end of the shaft - of the prev. example (3.01) - to produce a twist of 2°? Use the value G = 77 GPa for the steel shaft.

$$T = \frac{JG\phi}{L}$$

$L = 1.5\ m$

$G = 77 * 10^{9}\ pa$

$$\left[\phi = 2° * \frac{2\pi}{360} = 34.9 * 10^{-3}\ \text{rad}\right]$$

$$= \frac{1.021 * 10^{-6} * 77 * 10^{9} * 34.9 (10^{-3})}{1.5}$$

$= 1.829\ kN.m$ #

Ex (3.03)

بعتبره ال min.

What angle of twist will create a shearing stress of 70 MPa on the inner surface of the hollow steel shaft of examples (3.01) (3.02)

$$\gamma_{min} = \frac{\tau_{min}}{G} = \frac{70 (10^{6})}{77 (10^{9})}$$

$= 909 * 10^{-6}$

$$\phi = \frac{L \gamma_{min}}{c_1} = \frac{1500\ [mm]\ 909 (10^{-6})}{20\ [mm]}$$

$$= 68.2 * 10^{-3}\ [rad] * \frac{360}{2\pi} = 3.91°$$

بحسب أول شي التورك
اللي بيطلعلي هاي قيمة
ال τ ومن التورك
نطلع φ

If the shaft is subjected to torques at locations other than its end, or if it consists of several portions with varions cross sections of different materials ...

$$\phi = \sum_i \frac{T_i L_i}{J_i G_i}$$

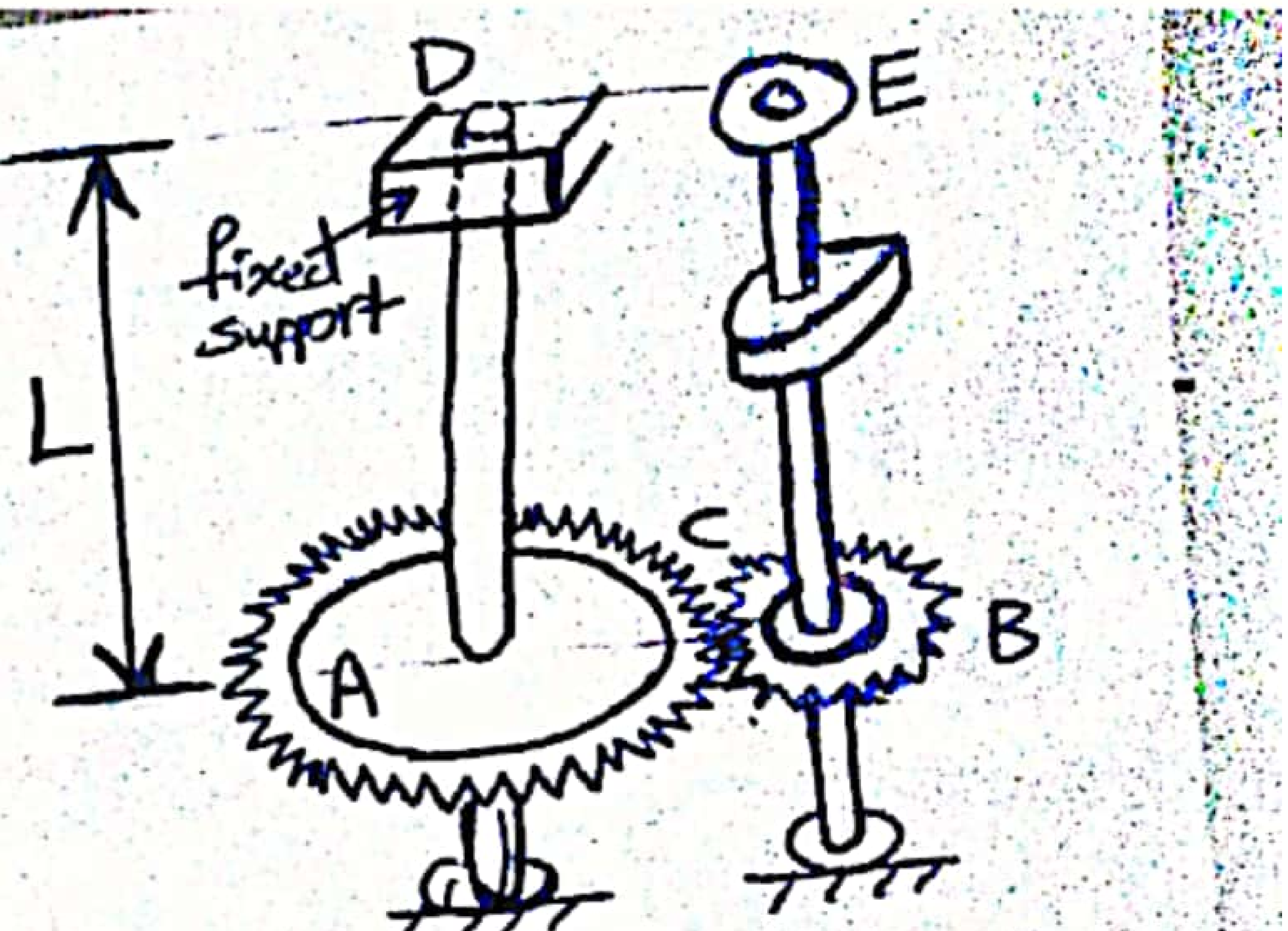
اشارة ϕ بتعتمد على
الـ internal torque

3.6 Statically Indeterminate Shafts

Because statics isn't sufficient in some cases to determine the external and internal torques, the shafts are said to be statically indeterminant.

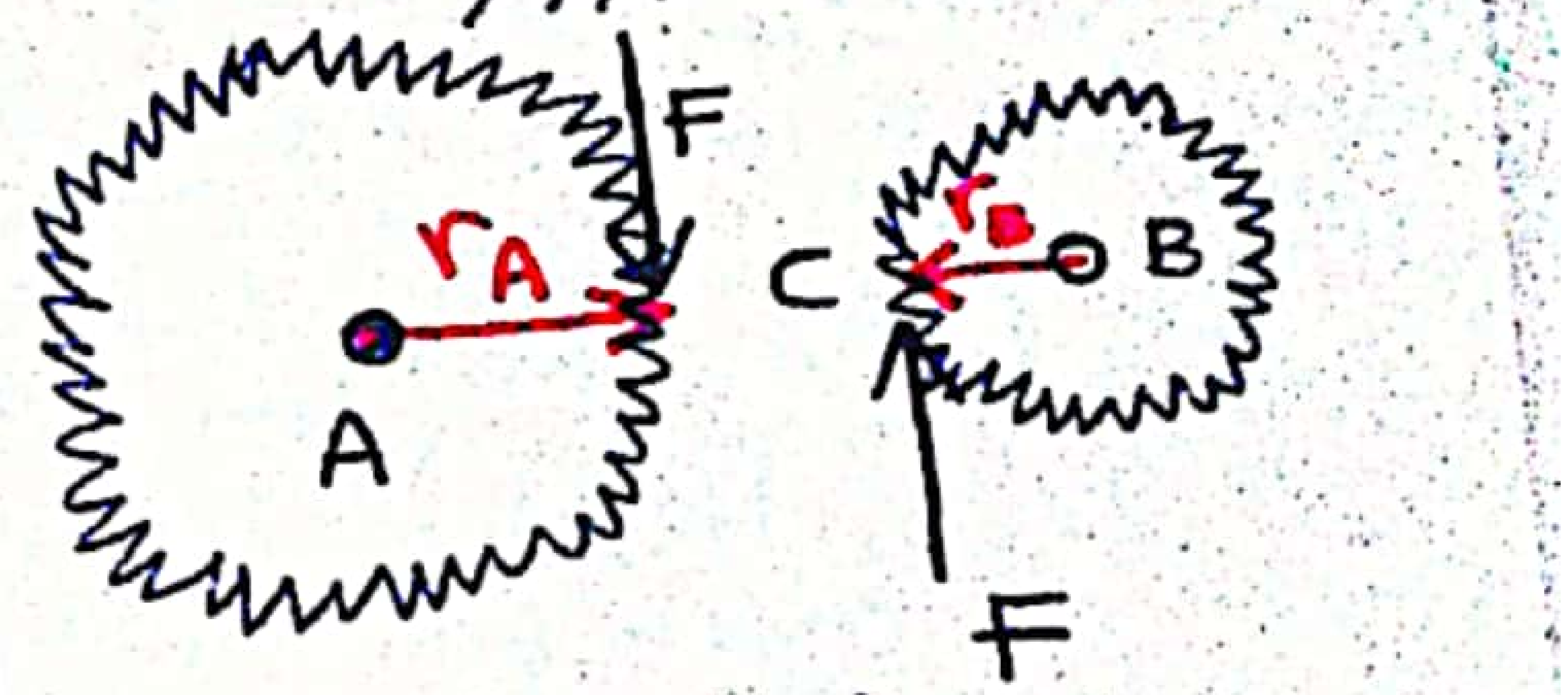
Ex(3.04)

Knowing that $\boxed{r_A = 2r_B}$, determine the angle of rotation of end E of shaft BE when the torque T is applied at E?



→ بما انه المقطع D ثبتة فإن ϕ_A
للجير A هيكون مساوي لل Shaft كاملة

$$\left[\phi_A = \frac{T_{AD} L}{JG} = \frac{2TL}{JG}\right]$$ due to $r_A = 2r_B$



$$r_A \phi_A = r_B \phi_B \Rightarrow \phi_B = \frac{r_A}{r_B}\phi_A = 2\phi_A$$

$$\left[\phi_B = \frac{4TL}{JG}\right]$$

$$* \quad \phi_{E/B} = \frac{T_{BE} L}{JG}$$

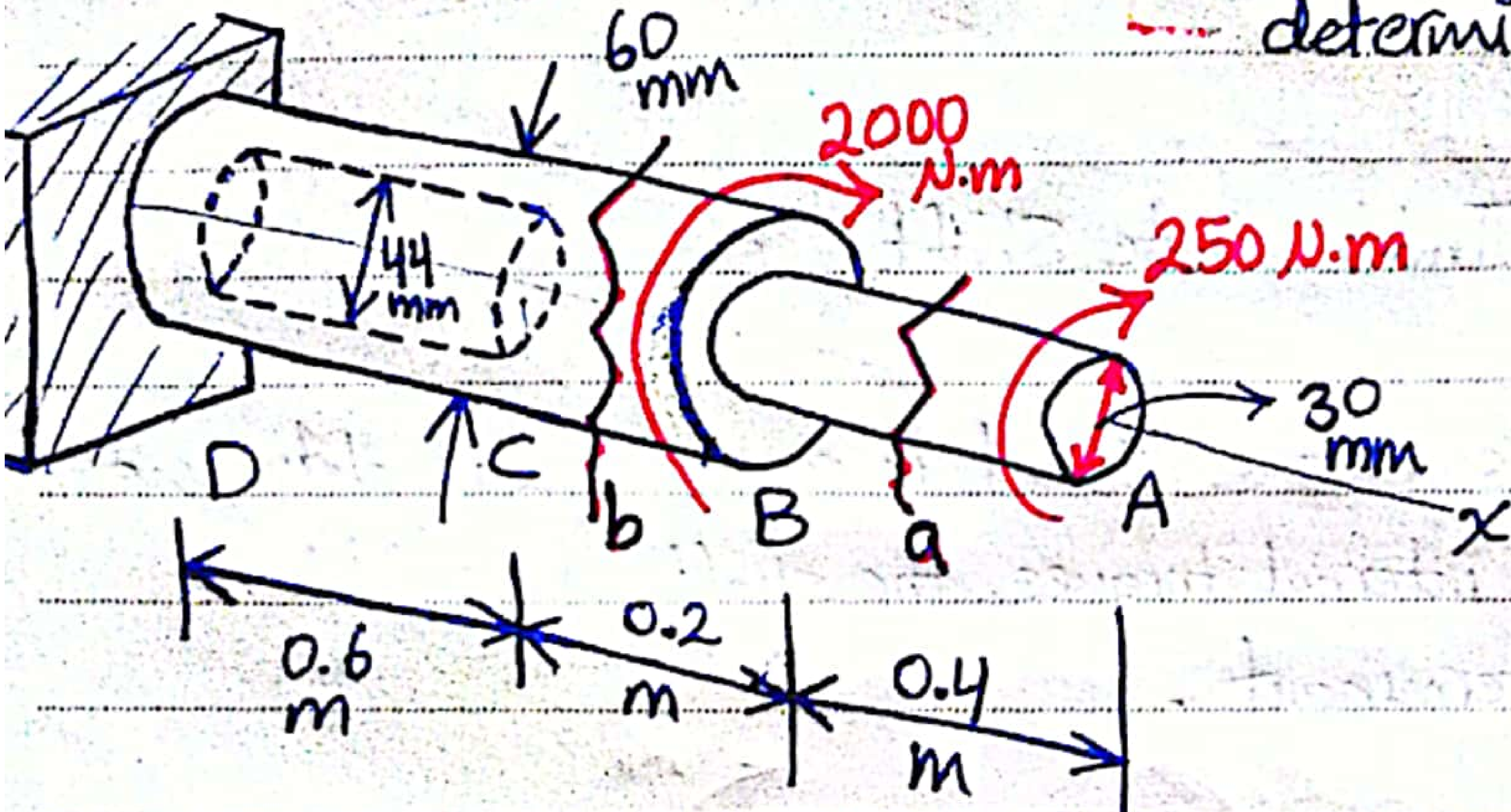
∴ the angle of rotation of end E is obtained by writing :-

$$\phi_E = \phi_B + \phi_{E/B}$$

$$= \frac{4TL}{JG} + \frac{TL}{JG} = \frac{5TL}{JG} \quad \#$$

Sample Problem (3.3)

The horizontal shaft AD is attached to a fixed base at D and is subjected to the torques shown. A 44 mm diameter hole has been drilled into portion CD of the shaft. Knowing that the entire shaft is made of steel for which $G = 77\,GPa$, determine ϕ at end A?



→ this shaft consists of 3 portions (AB, BC, CD)

by taking sec.(a) ⇒ $250 - T_{AB} = 0$ $\quad T_{AB} = 250\ N.m$

sec.(b) ⇒ $250 + 2000 - T_{BC} = 0$ $\quad T_{BC} = 2250\ N.m$

$\boxed{T_{BC} = T_{CD}}$ لأنه ما في تورك عند النقطة C

من كل سكشن أخذته فوق باخذ كل شي أمامه من اليمين

$$\therefore\ \phi_A = \sum \frac{T_i L_i}{J_i G} = \frac{1}{G}\left(\frac{T_{AB} L_{AB}}{J_{AB}} + \frac{T_{BC} L_{BC}}{J_{BC}} + \frac{T_{CD} L_{CD}}{J_{CD}}\right)$$

$$\phi_A = \frac{1}{77*10^9}\left[\frac{250*0.4}{\frac{\pi}{2}(0.015)^4} + \frac{2250*0.2}{\frac{\pi}{2}(0.03)^4} + \frac{2250*0.6}{\frac{\pi}{2}(0.03^4 - 0.022^4)}\right]$$

$$\phi_A = 0.0403 \text{ rad}$$

$$= 2.31^\circ \quad * \frac{360^\circ}{2\pi}$$

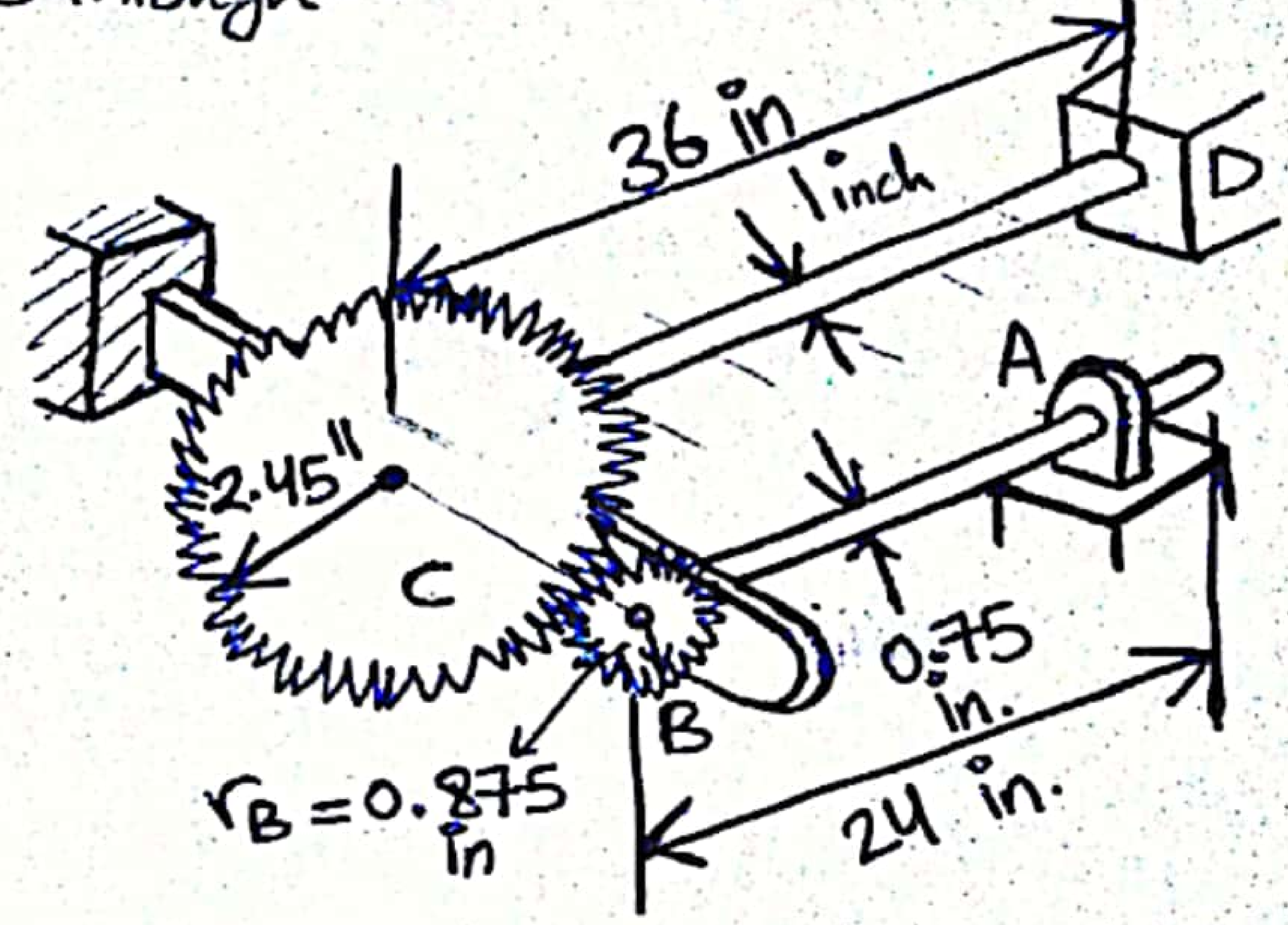
Scanned with CamScanner

Sample Problem (3.4)

Two solid steel shafts are connected by the gears shown. Knowing that for each shaft $G = 11.2 * 10^6$ psi and that the allowable shearing stress is 8 ksi. Determine:

a) the largest torque T_o that may be applied to end A of shaft AB

b) the corresponding angle through which end A of shaft AB rotates?



Gear B: $\Sigma M_B = 0$

$$F(0.875) - T_o = 0$$

Gear C: $\Sigma M_C = 0$

$$F(2.45) - T_{CD} = 0$$

$$T_{CD} = 2.8\, T_o$$

$$r_B \phi_B = r_C \phi_C \Rightarrow \phi_B = \phi_C \frac{2.45}{0.875} = 2.8\, \phi_C$$

a. $$8000 \text{ psi} = \frac{T_o (0.375)}{\frac{\pi}{2}(0.375)^4} \Rightarrow T_o = 663 \text{ lb.in}$$ / shaft AB

$$\boxed{T_{AB} = T_o}$$

$$8000 = \frac{2.8\, T_o (0.5)}{\frac{\pi}{2}(0.5)^4} \Rightarrow T_o = 561 \text{ lb.in}$$ / shaft CD

$$\boxed{T_o (2.8) = T_{CD}}$$

باخد أقل قيمة للاتنين

Max. permissible Torque: $T_o = 561$ lb.in

b. $$\phi_A = \phi_B + \phi_{A/B} = 2.8\, \phi_C + \phi_{A/B}$$

$$\phi_A = \frac{2.8\, (\overset{T_{CD}}{2.8 * 561} * 36)}{\frac{\pi}{2}(0.5)^4 (11.2 * 10^6)} + \frac{(561 * 24)}{\frac{\pi}{2}(0.375)^4 (11.2 * 10^6)}$$

$$= 8.26^\circ + 2.22^\circ = 10.48^\circ$$

#

Moment → Torsion (ch.3) ، المومنت اللي موازي الـ axial axis
→ Bending (ch.4) ، المومنت اللي موازي أي محور مو axial / (Lateral) بالـ

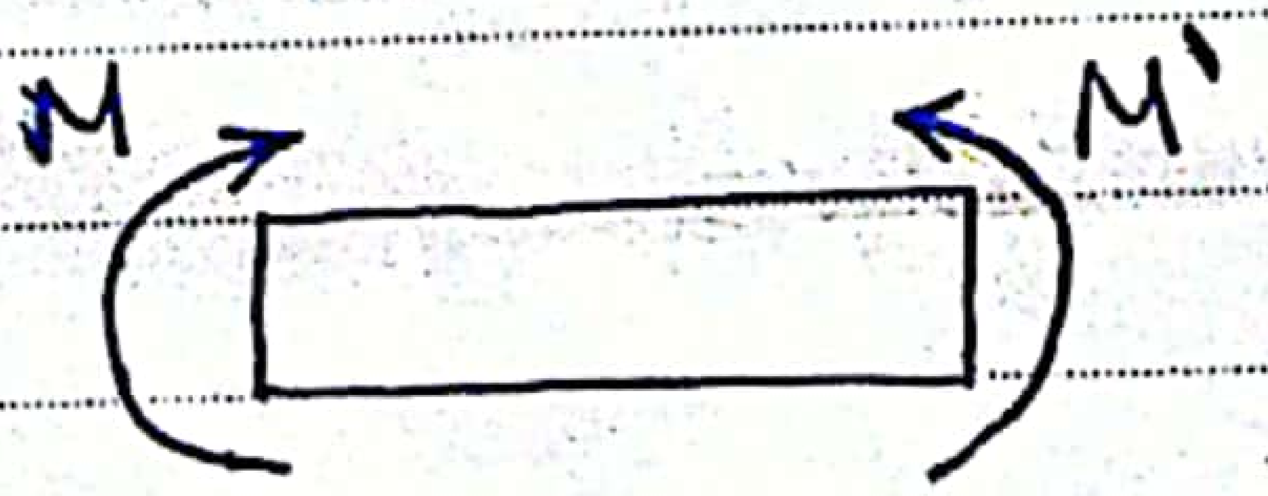
Ch.4 Pure Bending

4.1 Introduction

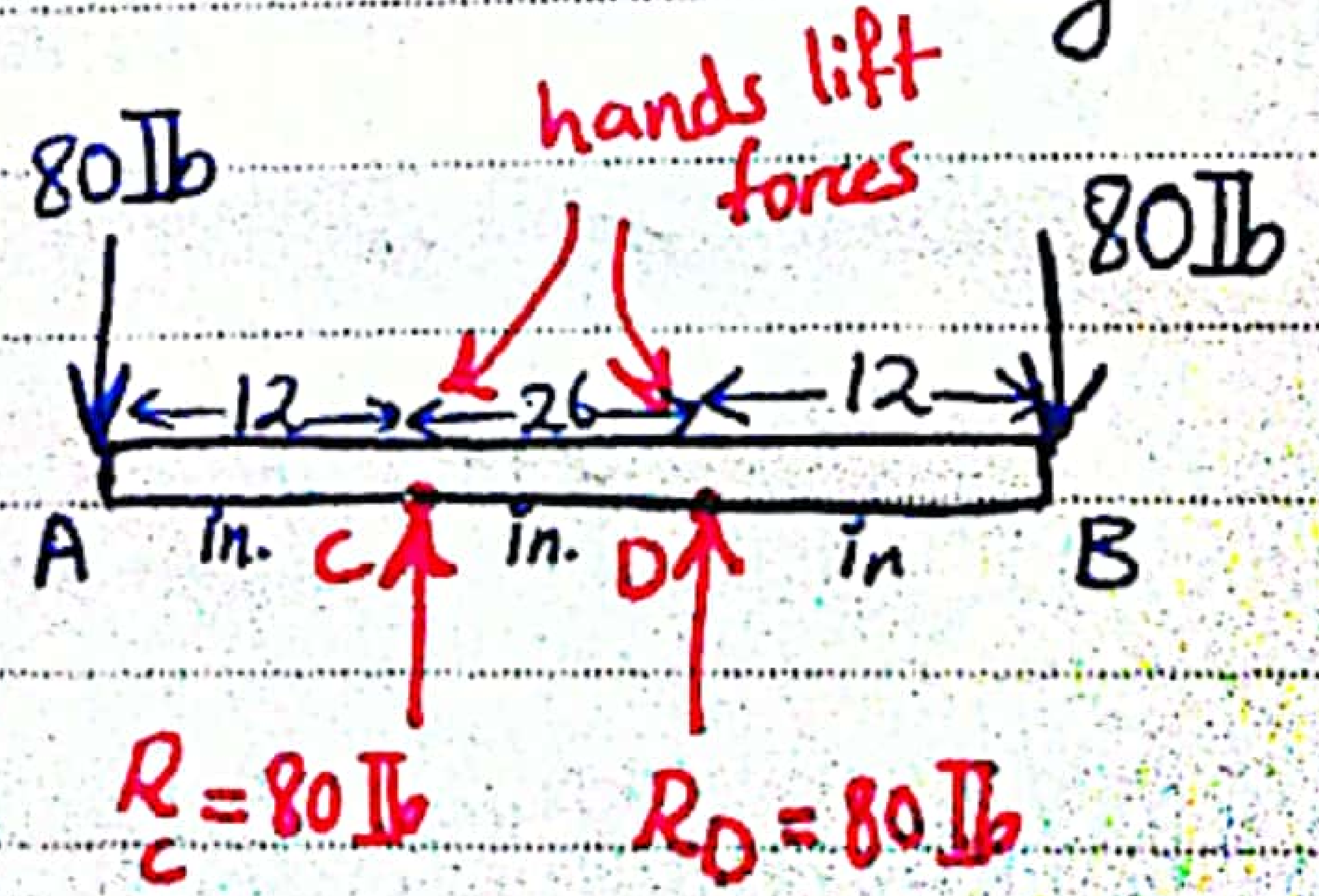
In this chapter, we'll analyze the stresses and strains in prismatic members subjected to **bending**.

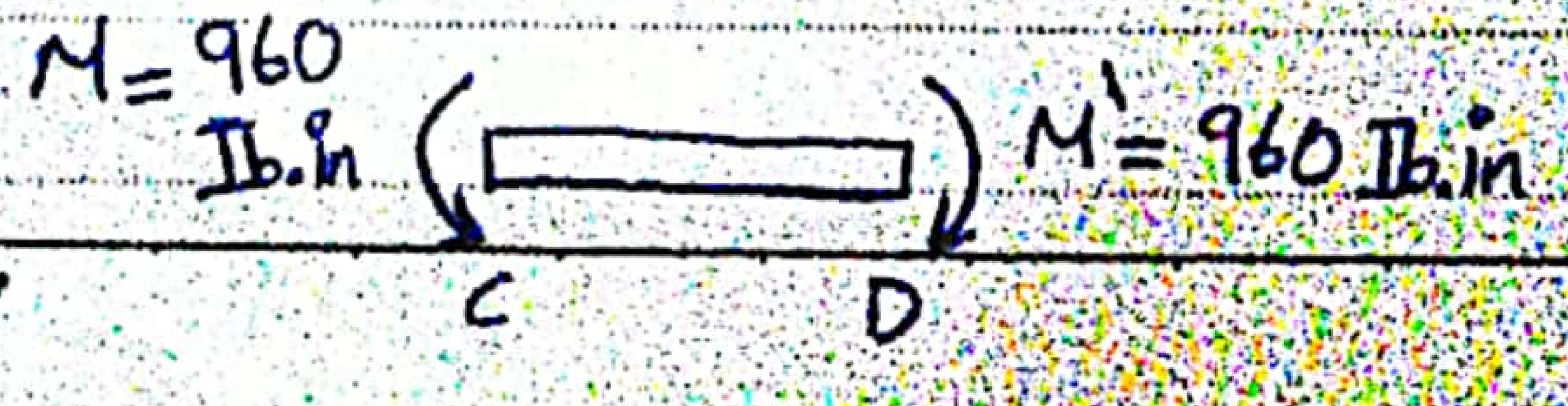
In addition, it will be devoted to the analysis of prismatic members subjected to equal & opposite couples M and M' acting in the same longitudinal plane. بحالة

∴ Members said to be in PURE bending.



* An example of pure bending : the bar as it's held overhead by a weight lifter. الأثقال



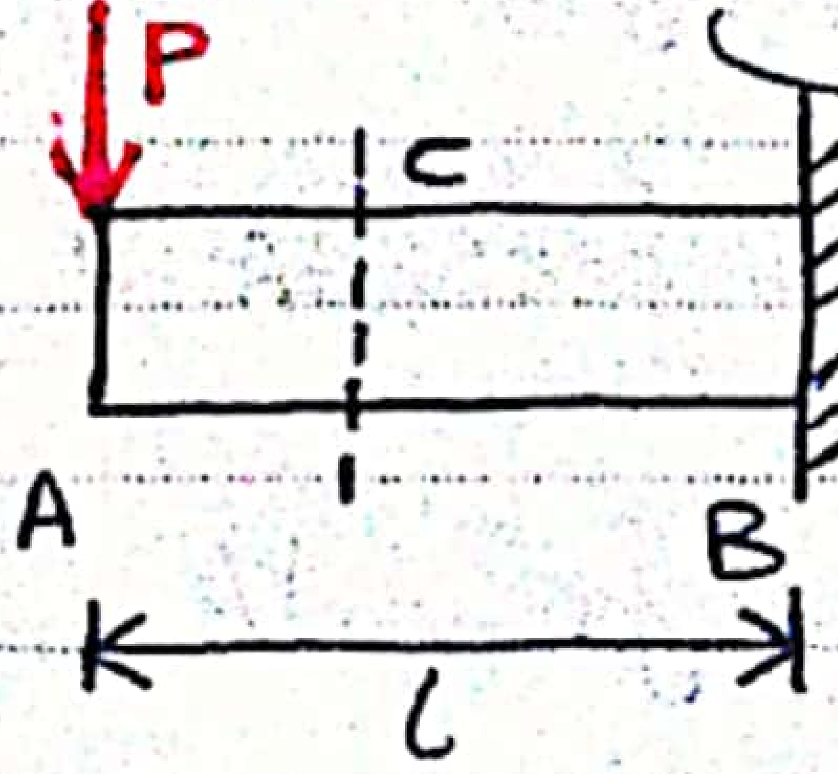


⇒ CD is in pure bending

/ /

* study of pure bending ≡ study of beams (prismatic members subjected to various types of transverse loads)

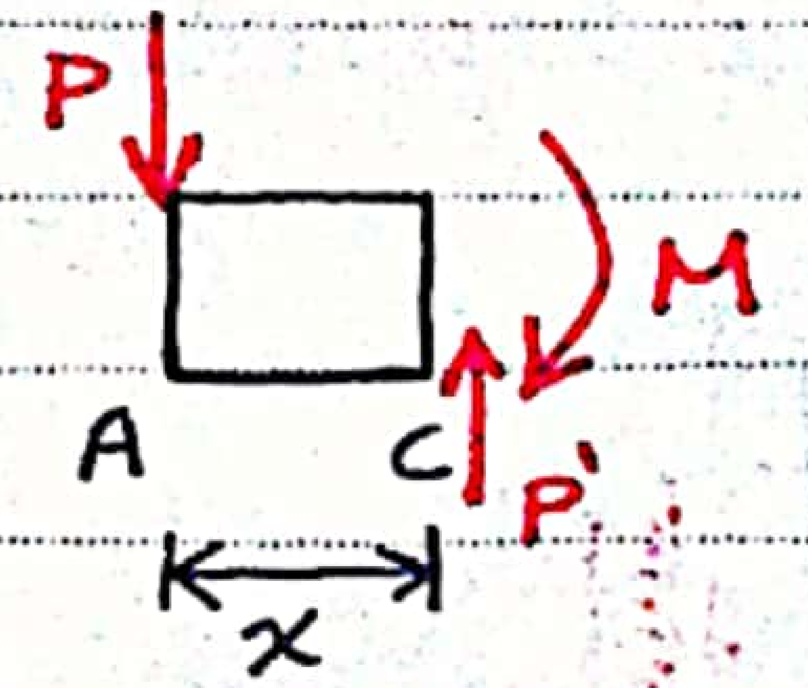
القوى اللي ما بتكون على الـ member axis



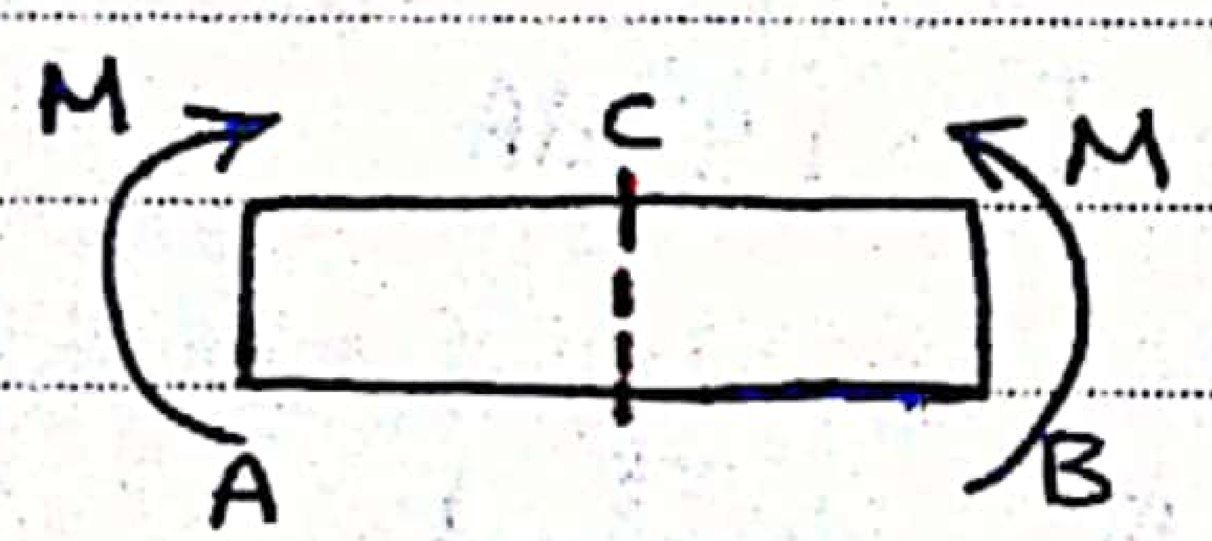
لو أخدنا سكشن من النقطة C
فلاحظ من الـ FBD للسكشن هاد
انه عندي فيه (internal forces)

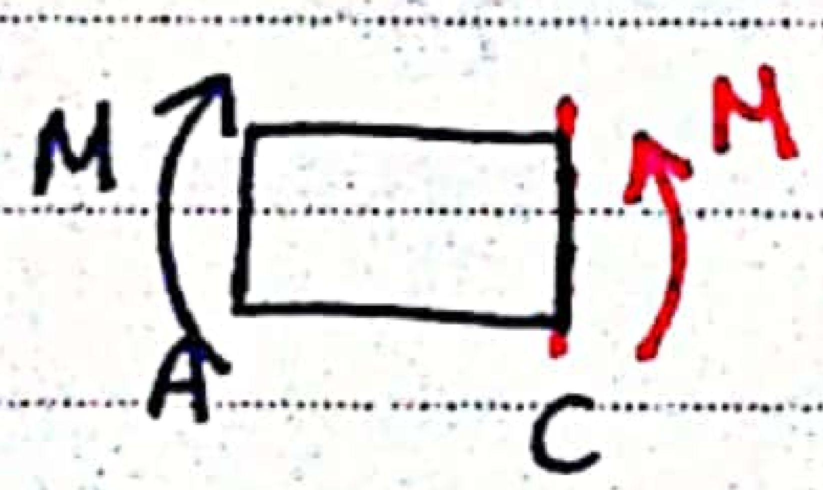
P' equal & opposite to P

M of magnitude $M = Px$



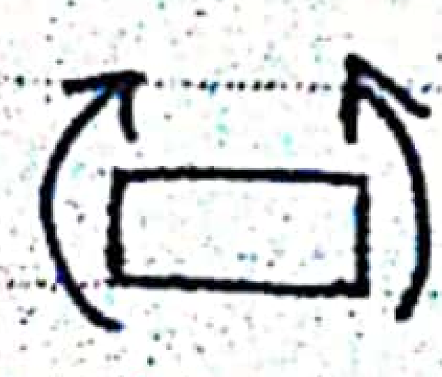
4.2 Symmetric Member in Pure Bending



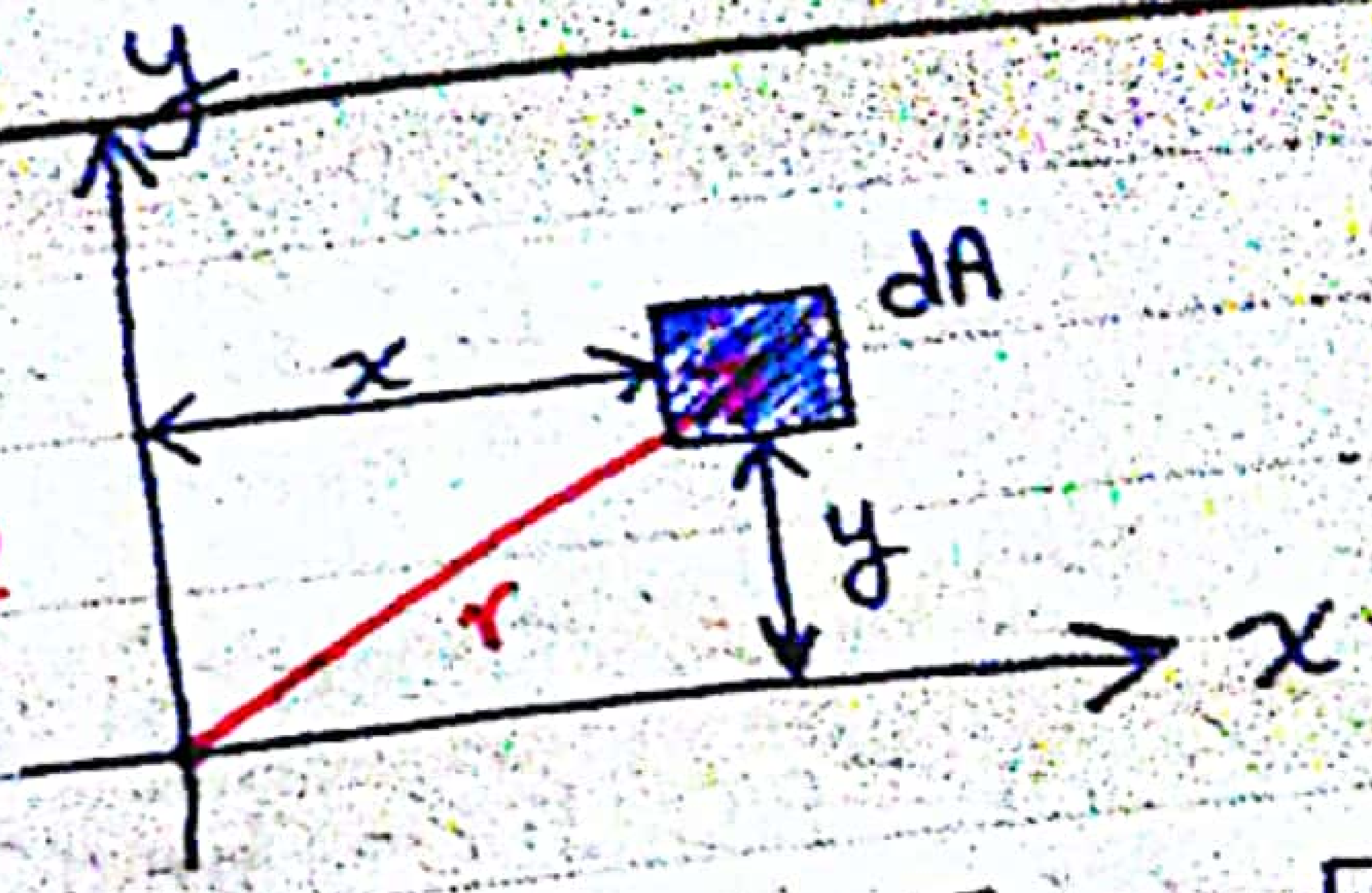


the internal forces in any cross section of a symmetric member in pure bending are equivalent to a couple.

The moment M of that couple is referred to as ⇒ bending moment


$+M$

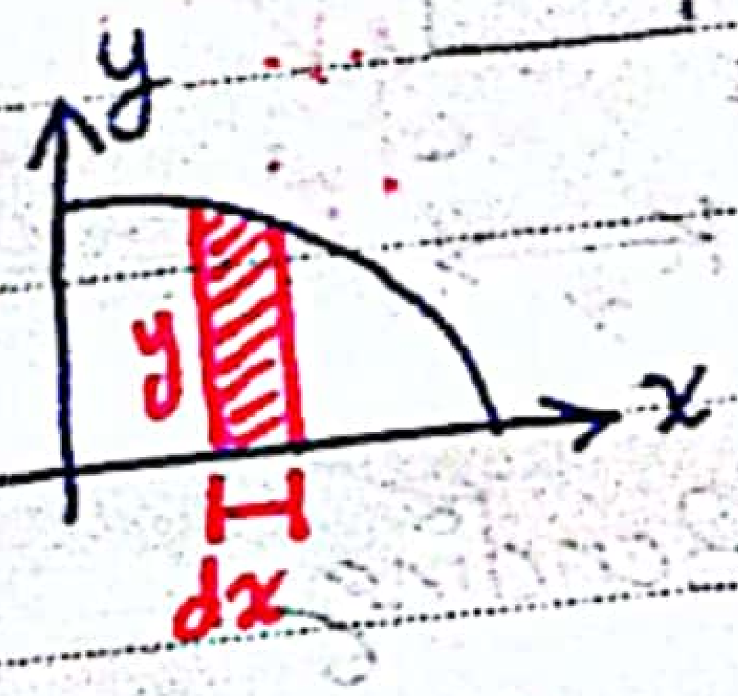

$-M$



$$\left.\begin{array}{l} I_x = \int y^2 . dA \\ I_y = \int x^2 . dA \end{array}\right\} \text{Moment of inertia}$$

polar [moment of inertia] $J = \int r^2 . dA = \int (x^2 + y^2)\, dA = I_x + I_y \quad [m^4]$

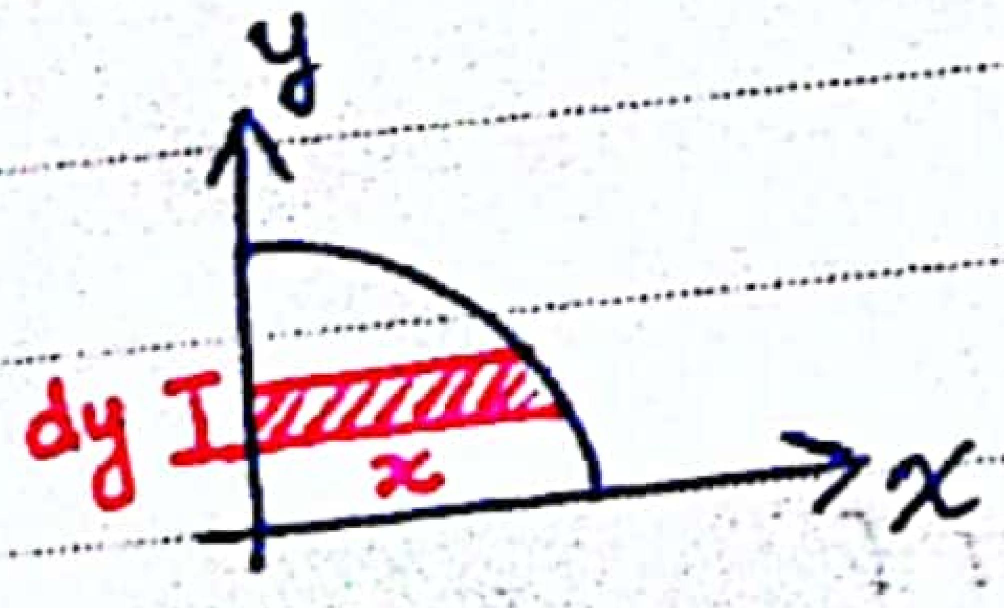
[Remark]



$y = ax + B$ ، مقطع عمودي

$\int dA = \int y . dx$

- $I_y = \int x^2 . dA$
- $I_x = \int y^2 . dA$

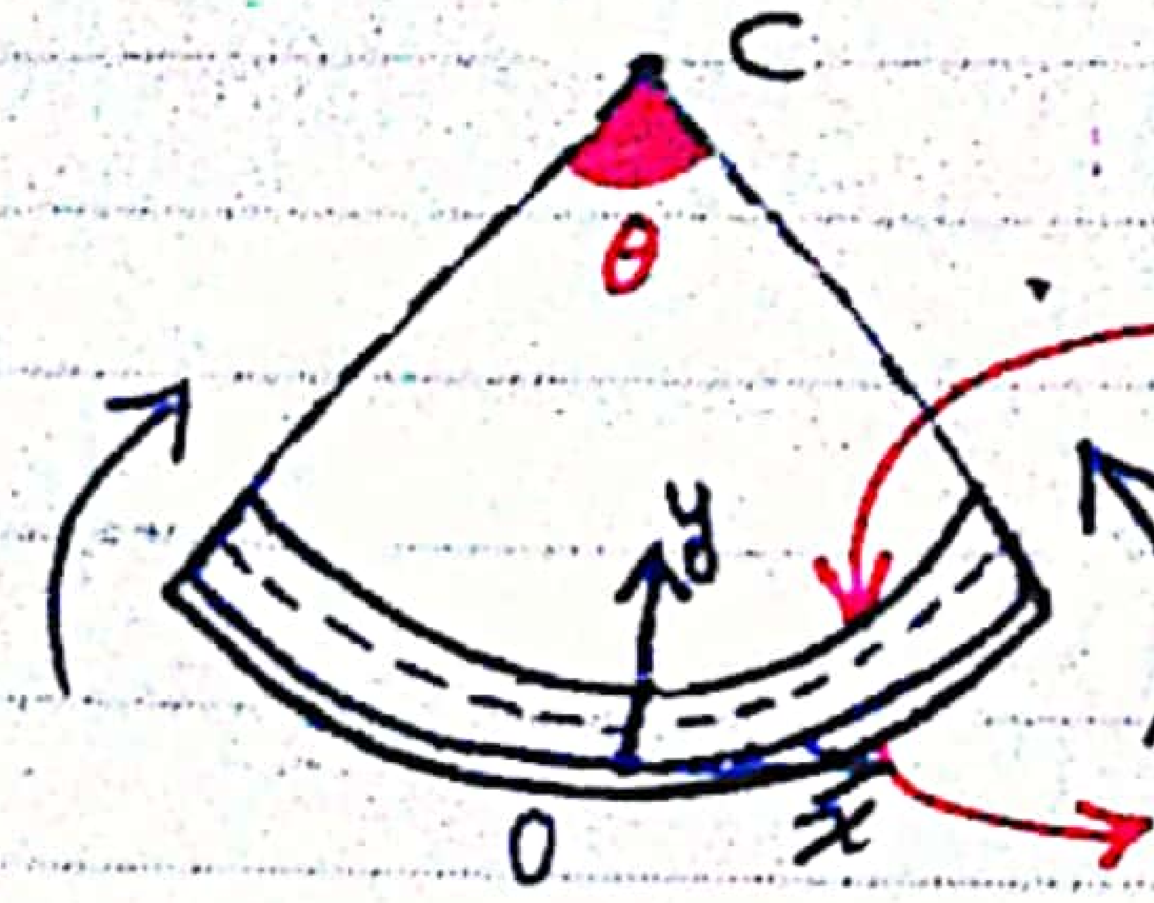


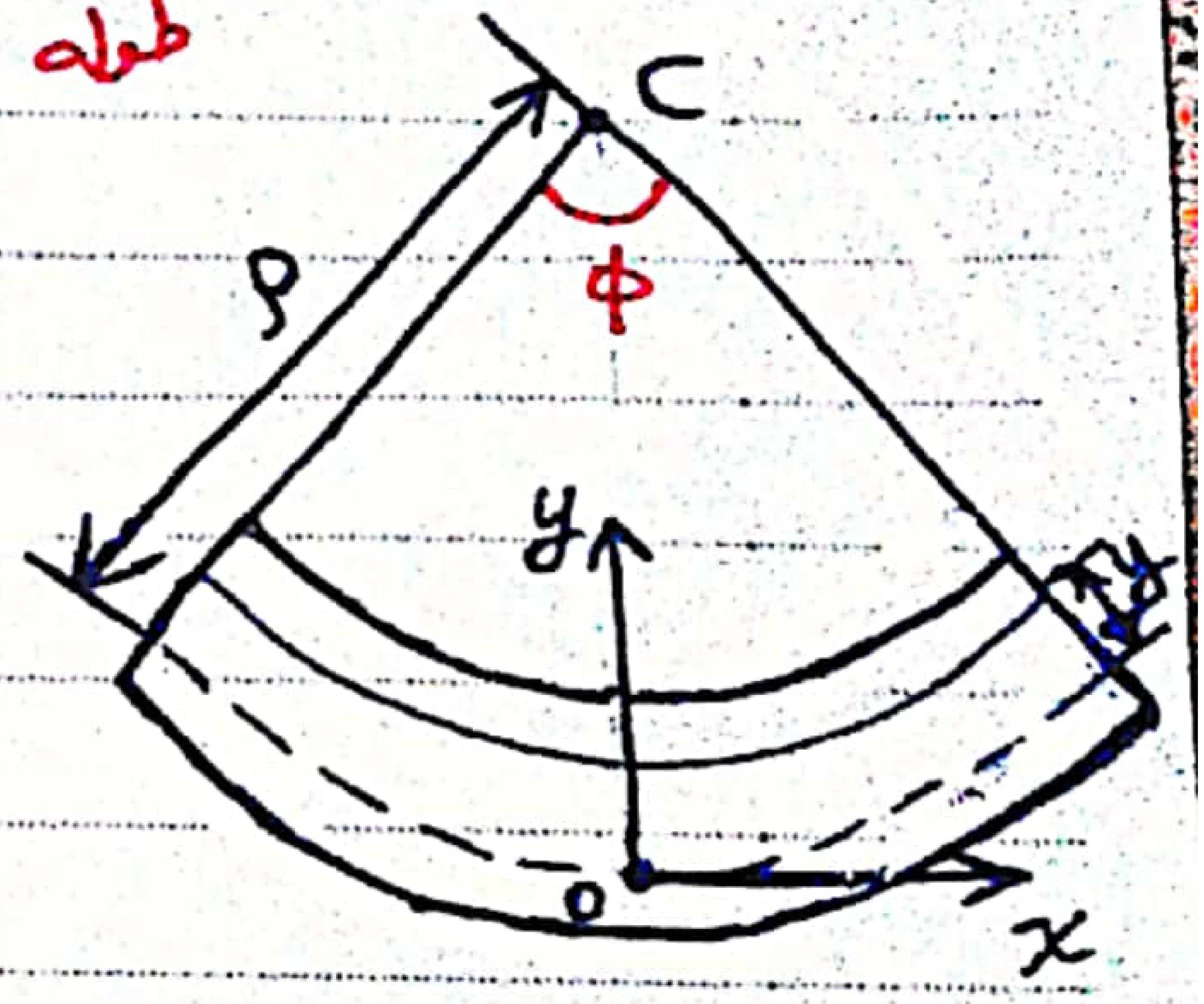
$x = ay + B$ ، مقطع أفقي

$\int dA = \int x . dy$

- $I_x = \int y^2 . dA$
- $I_y = \int x^2 . dA$

4.3 Deformations in a Symmetric Member in Pure Bending





$L = \rho\theta$

$L' = (\rho - y)\theta$

$\delta = L' - L \rightarrow \delta = (\rho - y)\theta - \rho\theta$

$\delta = -y\theta$

$$\epsilon_x = \frac{\delta}{L} = \frac{-y\theta}{\rho\theta}$$

$$\therefore \boxed{\epsilon_x = -\frac{y}{\rho}}$$

لأني شتغل ع +M

concave UP...

∴ the Longitudinal normal strain ϵ_x varies linearly with the distance y from the neutral surface...

4.4 | Stresses and deformations in the elastic range

Assuming the material to be homogenous, and denoting by E its modulus of elasticity, we have in the longitudinal x direction :

$$(\sigma_x = E\epsilon_x)$$

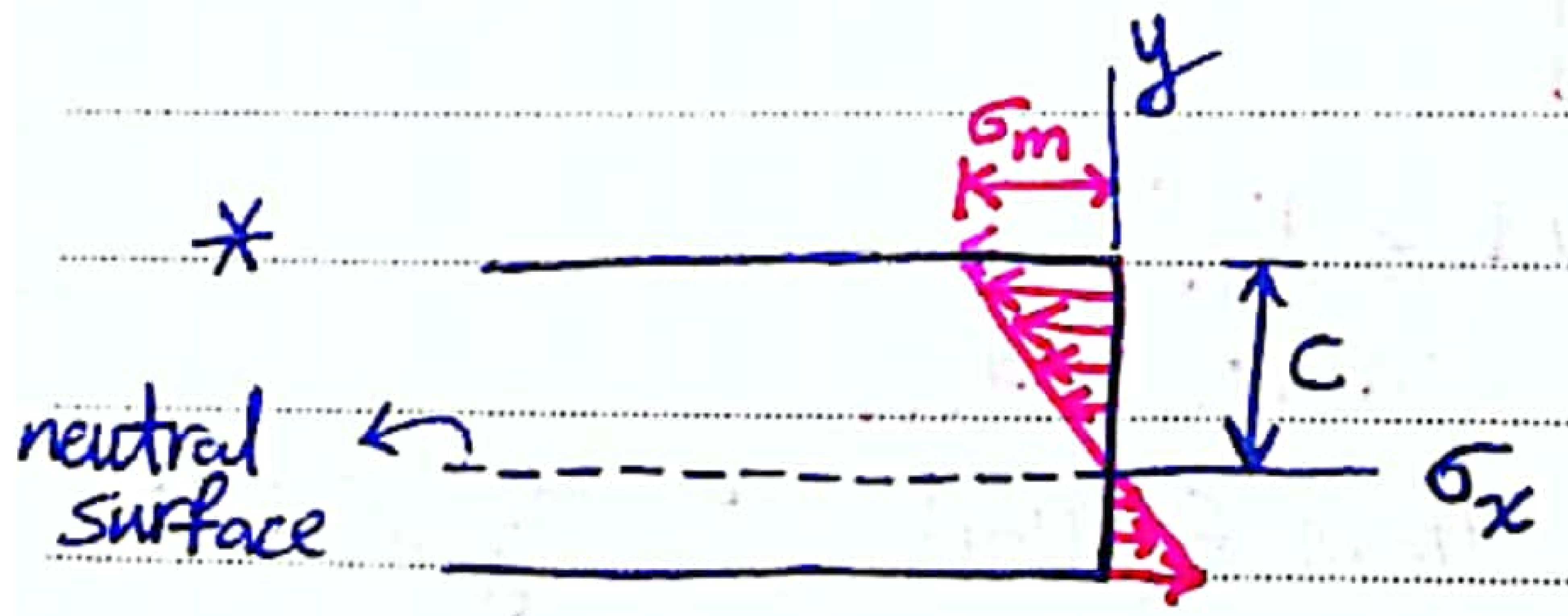
$$\Rightarrow \left[\epsilon_x = \frac{-y}{c}\epsilon_m\right] * E$$

$$\boxed{\sigma_x = \frac{-y}{c}\sigma_m}$$

$\sigma_m \to$ the max absolute value of the stress

$\therefore$ This result shows that, in the elastic range, the normal stress varies linearly with the distance from the neutral surface.

*



$\Rightarrow$ Bending Stresses

$$\int \sigma_x \, dA = \int \frac{-y}{c}\sigma_m \, dA = 0$$

shows that the first moment of the cross section about its neutral axis must be zero. $\Leftarrow \therefore \int y \cdot dA = 0$

for a member subjected to pure bending, and $\therefore$ يعني خاص

as long as the stresses remain in the elastic range, the [illegible] neutral axis passes through the centroid of the section.

$$M = \int (-y\sigma_x)\, dA$$

$$M = \int \left(-y * \frac{-y}{c} \sigma_m\right) dA = \frac{\sigma_m}{c} \int y^2 \cdot dA$$

$\therefore$ $\boxed{\sigma_m = \frac{MC}{I}}$ $\Rightarrow$ in the case of pure bending the neutral axis passes through the centroid of the cross section.

$\boxed{\sigma_x = \frac{-My}{I}}$ $\Rightarrow$ the normal stress σ_x at any distance y from the neutral axis

(Elastic flexure formulas) : بسمّي المعادلتين هدول

and σ_x caused by the bending or "flexing" of the member $\Rightarrow$ flexural stress

$\sigma_x < 0$, compressive above the neutral axis ($y > 0$) when the bending moment is +ve
(M)

$\sigma_x > 0$, tensile when M is -ve

Elastic section modulus $= S = \dfrac{I}{c}$ → depends only upon the geometry of the cross section.

$$\therefore \quad \boxed{\sigma_m = \frac{M}{S}}$$

The deformation of the member caused by M is measured by the curvature of the neutral surface.

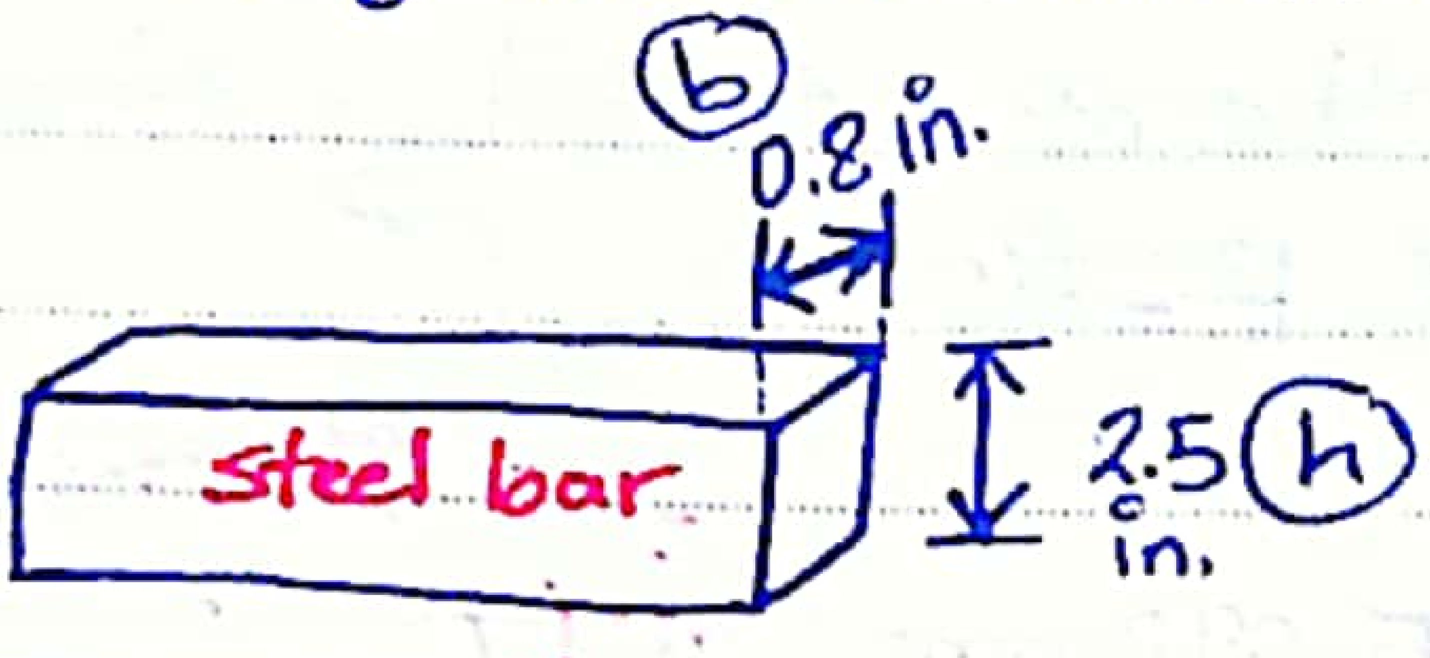
$$\boxed{\frac{1}{\rho} = \frac{\epsilon_m}{c}}$$

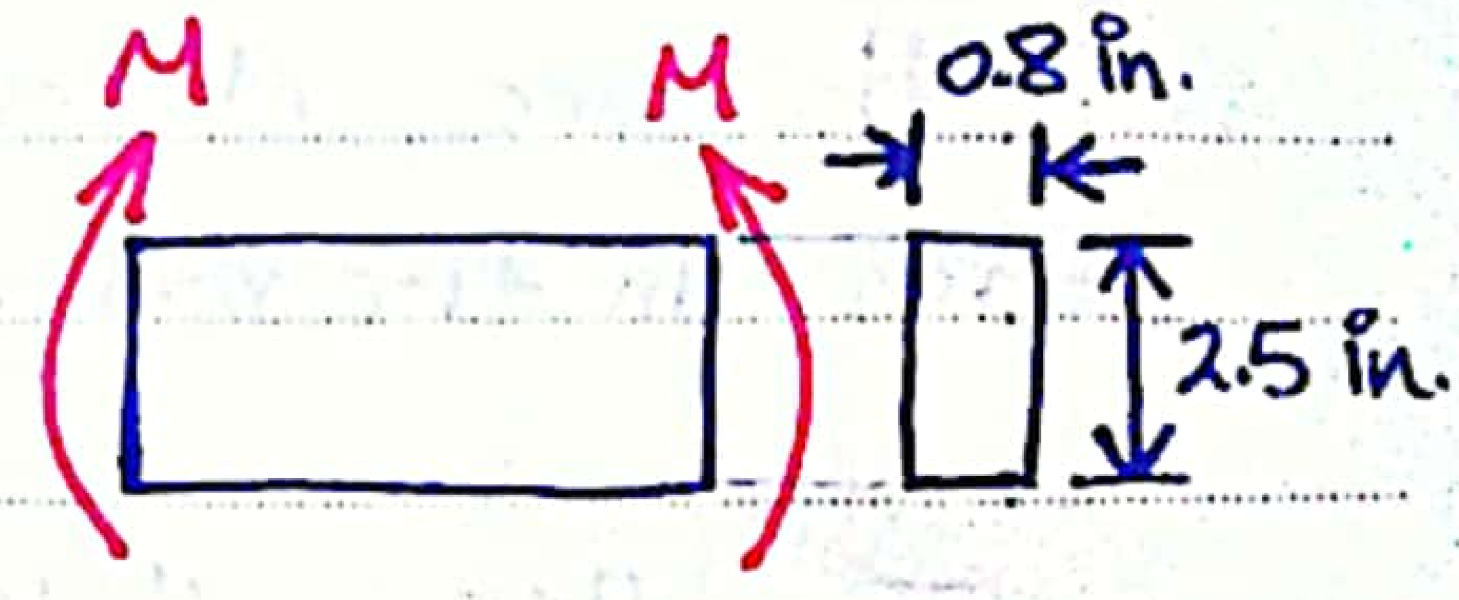
$$\boxed{\epsilon_m = \frac{\sigma_m}{E}}$$ $\Leftarrow$ منطقة ال elastic

$$\therefore \quad \frac{1}{\rho} = \frac{\sigma_m}{Ec} = \frac{1}{Ec}\left(\frac{Mc}{I}\right)$$

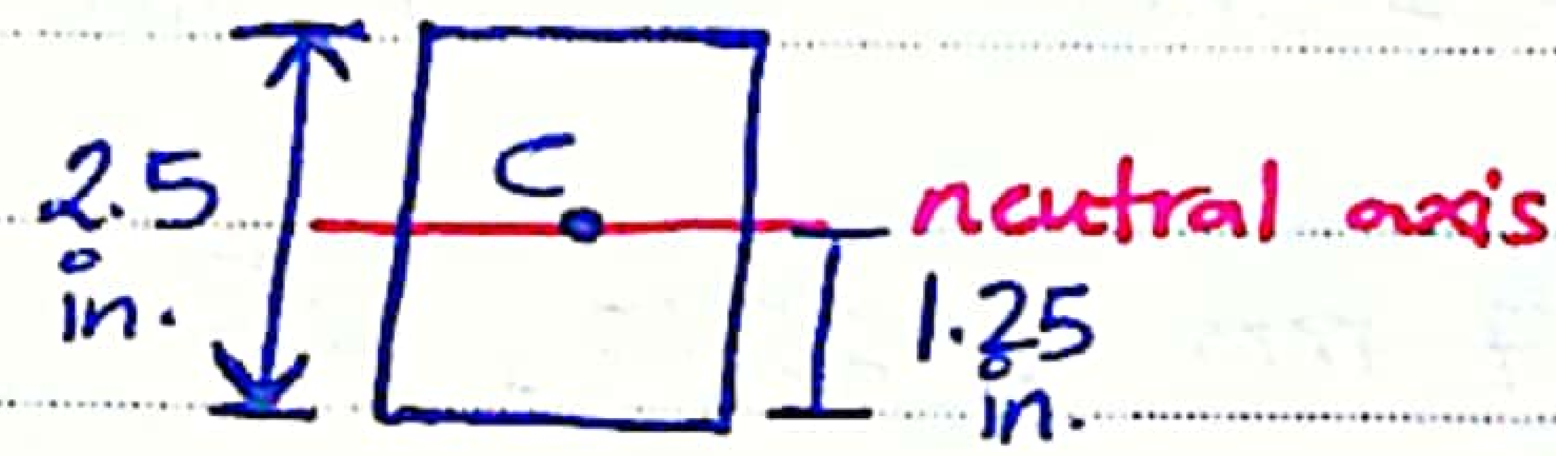
$$\boxed{\frac{1}{\rho} = \frac{M}{EI}}$$

Ex(4.01) A Steel bar of 0.8 X 2.5 in. rectangular cross sec. is subjected to two equal and opposite couples acting in the vertical plane of symmetry of the bar. Determine the value of the bending moment M that causes the bar to yield. Assume $\sigma_y = 36$ ksi





الـ (neutral axis) لازم تمر من خلال السنترويد بالـ cross sec.



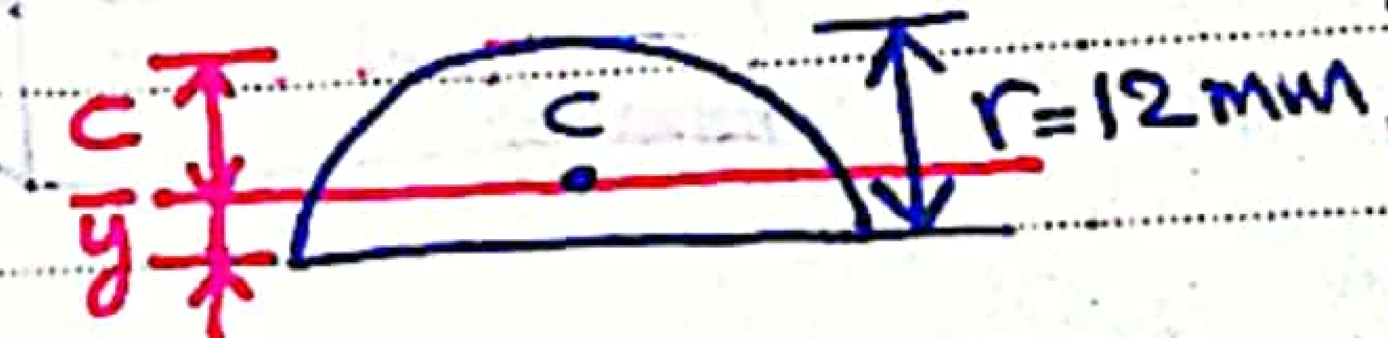
$$I = \frac{bh^3}{12} = \frac{(0.8)(2.5)^3}{12} = 1.042 \, [in^4]$$

the centroidal moment of inertia of the rec. cross sec.

$$M = \frac{I}{c}\sigma_m = \frac{1.042 * 36}{1.25} = 30 \, [kip.in]$$

Example (4.02)

An aluminum rod with a semicircular cross section of radius $r = 12$ mm is bent into the shape of a circular arc of mean radius $\rho = 2.5$ m. Knowing that the flat face of the rod is turned toward the center of curvature of the arc, determine the max. tensile and compressive stress in the rod. Use $E = 70$ GPa



$$\rightarrow \bar{y} = \frac{4r}{3\pi} = \frac{4(12)}{3\pi} = 5.093 \text{ mm}$$

$$c = r - \bar{y} = 12 - 5.093 = 6.907 \text{ mm}$$

$$\epsilon_m = \frac{c}{\rho} = \frac{6.907 \text{ mm}}{2.5 \text{ m}} = 2.763 * 10^{-3}$$

$$\sigma_m = E\,\epsilon_m = 70*10^9 * 2.763 * 10^{-3} = 193.4 \text{ MPa}$$

since this side of the rod faces away from the center of curvature, the stress obtained is a tensile stress.

the max. comp stress occurs on the flat side of the rod

$$\sigma_{comp} = -\frac{\bar{y}}{c}\sigma_m = -\frac{5.093}{6.907}(193.4*10^6) \text{ Pa}$$

$$= -142.6 \text{ MPa}$$

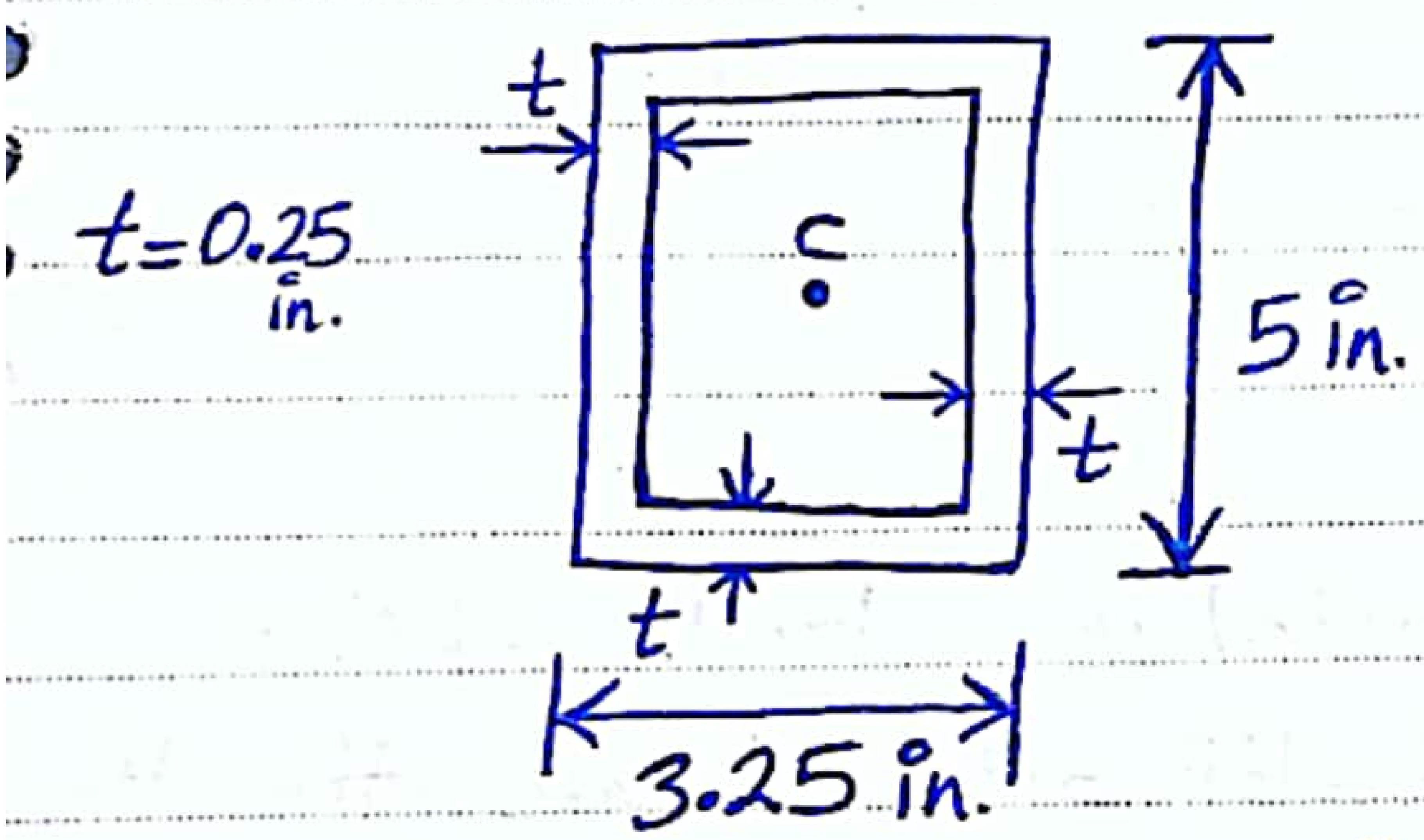
/ /

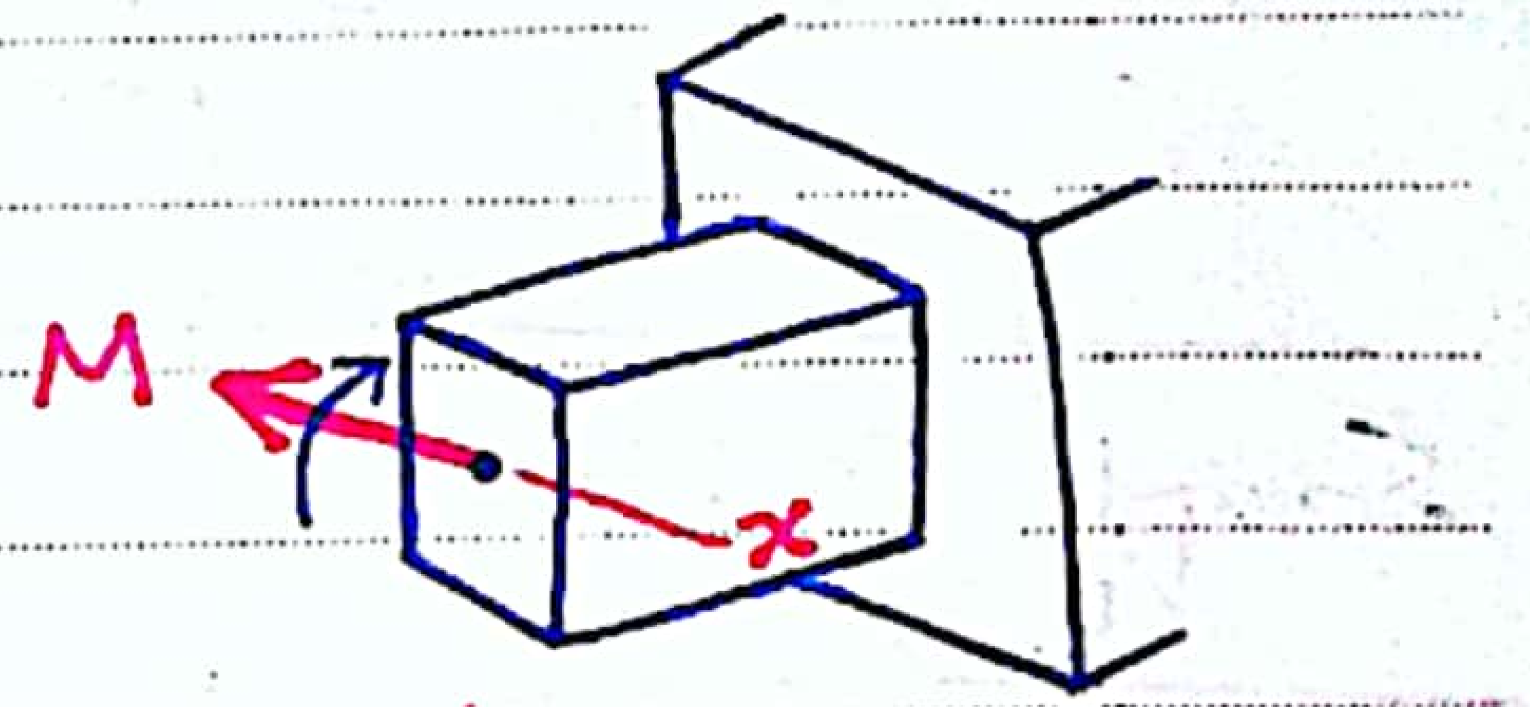
Sample Problem (4.1)

The rectangular tube shown is extruded from an aluminum alloy for which $\sigma_y = 40$ ksi, $\sigma_u = 60$ ksi, and $E = 10.6 * 10^6$ psi. Neglecting the fillets effect, determine :

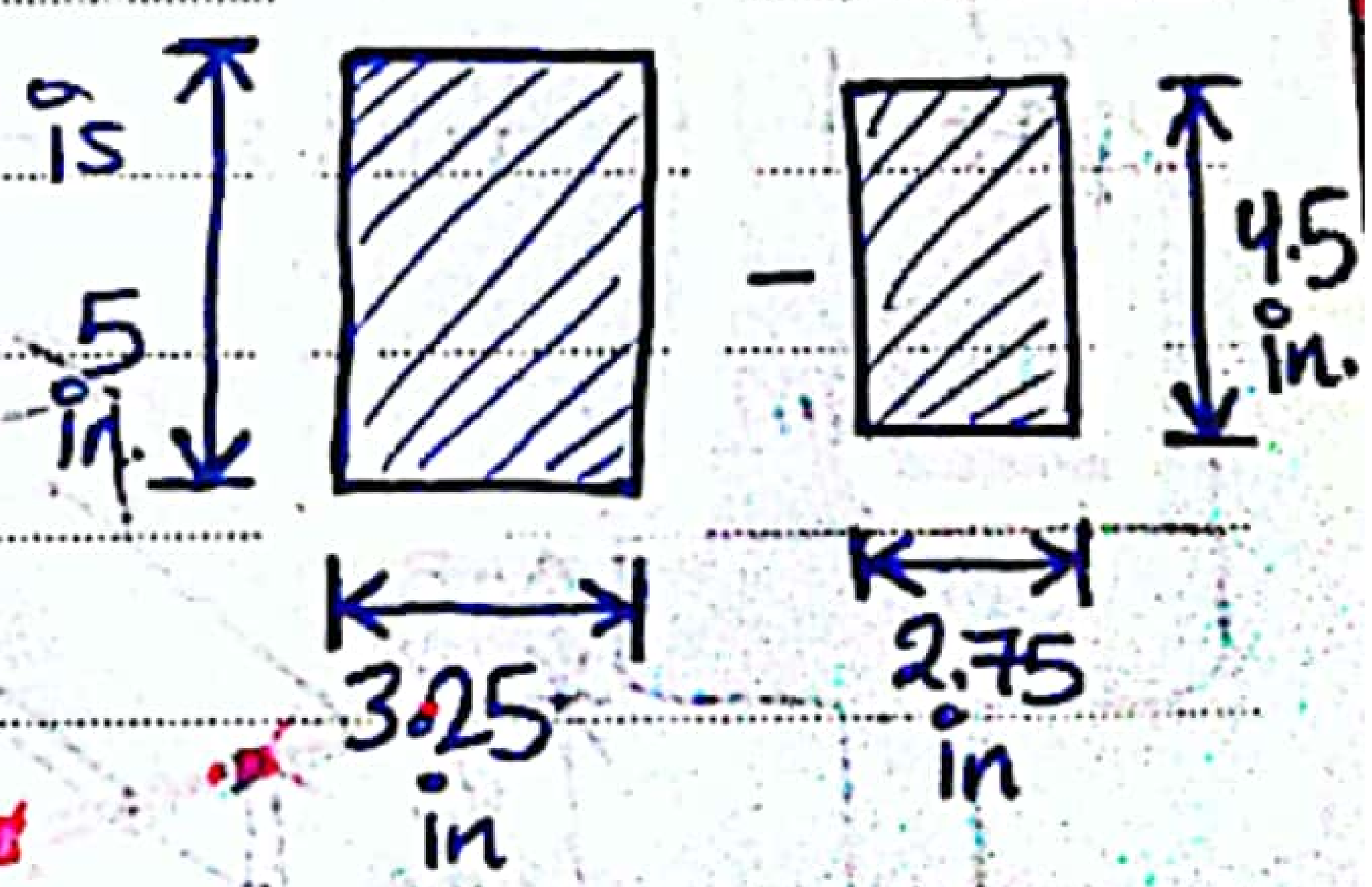
a) the bending moment M for which the factor of safety will be 3 ?

b) the corresponding radius of curvature of the tube ?





→ the cross sec. area of the tube is



$$I = \frac{3.25(5)^3}{12} - \frac{2.75(4.5)^3}{12}$$

$$= 12.97 \text{ in}^4$$

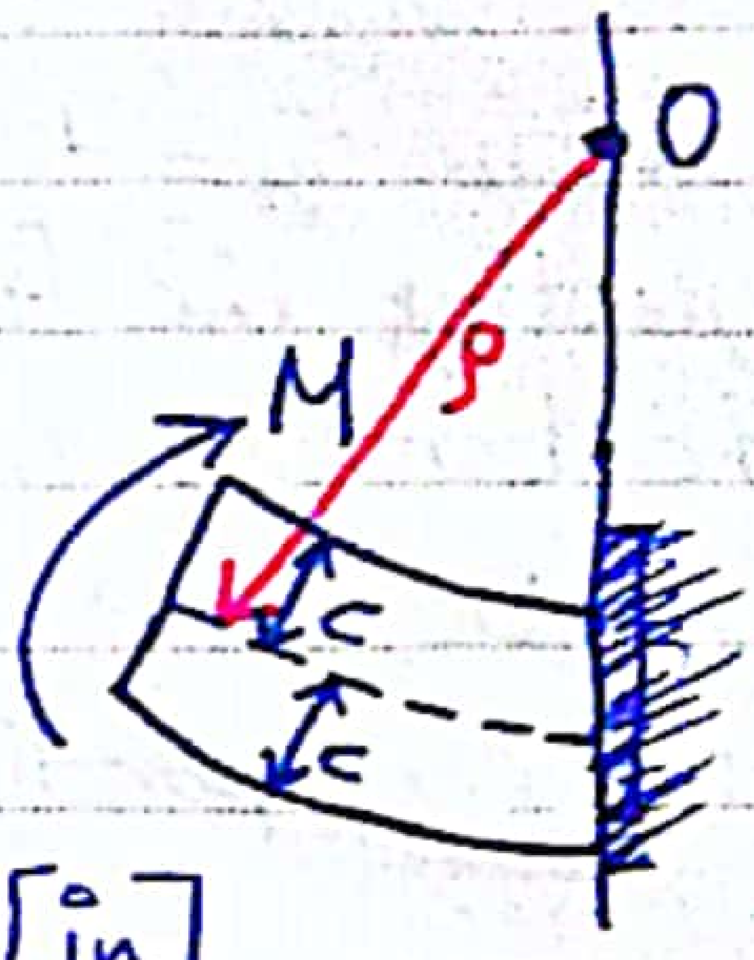
$$\sigma_{allowable} = \frac{\sigma_{ultimate}}{\text{factor of Safety (FS)}} = \frac{60 \text{ ksi}}{3} = 20 \text{ ksi} < \sigma_y$$

اذن ال tube لساته بمنطقة ال elastic

$$a.\quad M = \frac{I\sigma_{allow}}{c} = \frac{12.97 * 20 \text{ ksi}}{5 * \frac{1}{2}} = 103.8 \text{ kip.in}$$

b. $\dfrac{1}{\rho} = \dfrac{M}{EI} = \dfrac{103.8 \times 10^3\ [lb.in]}{(10.6 \times 10^6 psi)(12.97)} = 0.755 \times 10^{-3}$ in^{-1}

$\boxed{\rho = 1325\ in.}$



or $\epsilon_m = \dfrac{\sigma_{allow}}{E} = \dfrac{20\ ksi}{10.6 \times 10^6 psi}$

$= 1.887 \times 10^{-3}$

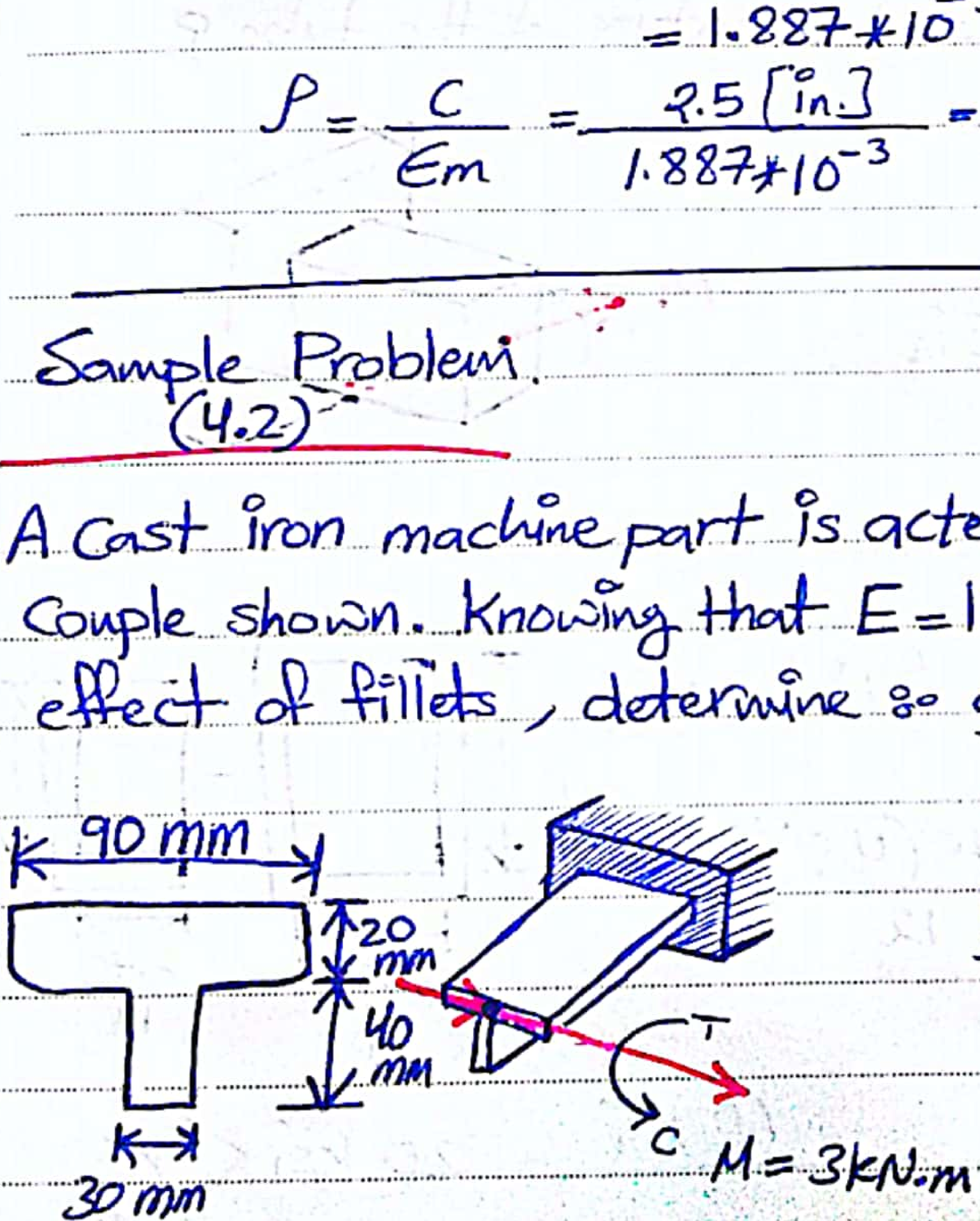
$\rho = \dfrac{c}{\epsilon_m} = \dfrac{2.5\ [in]}{1.887 \times 10^{-3}} = 1325 [in]$

#

Sample Problem (4.2)

A cast iron machine part is acted upon by the 3kN.m couple shown. Knowing that E = 165 GPa and neglecting the effect of fillets, determine :- a) the max. tensile and compressive stresses in the casting ?

b) the radius of curvature of the casting ?



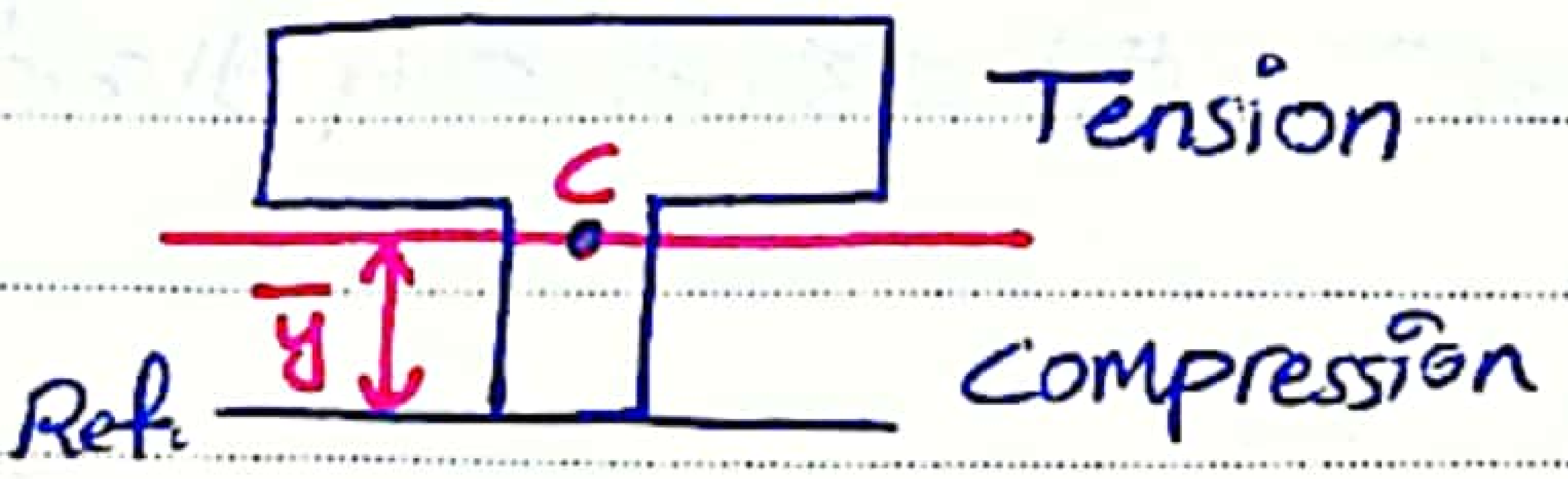
أول شي بحدد اتجاه ال M (منه بحدد اتجاه السنترويد ومنطقة ال tension وال compress)

tension

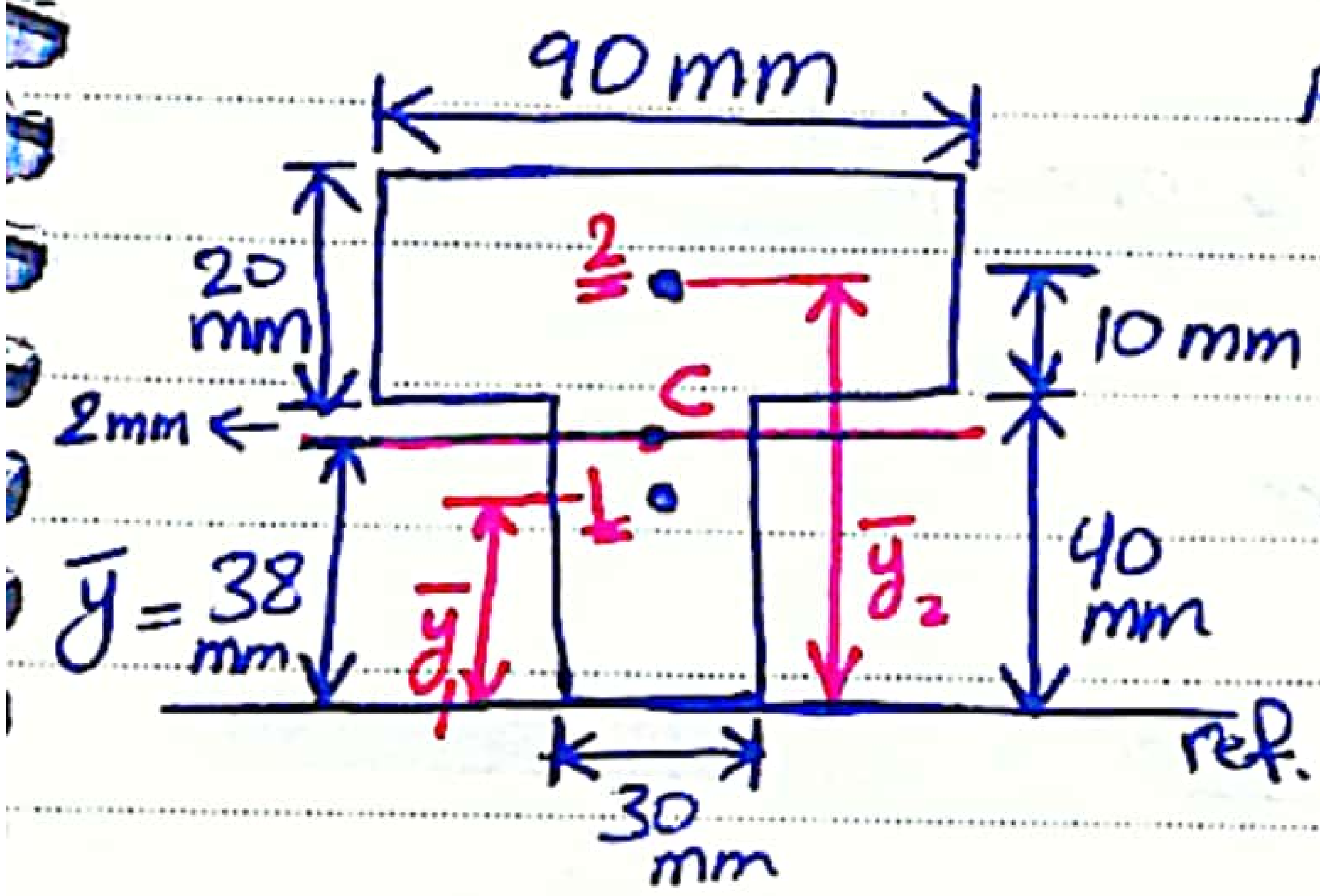
compression

الإبهام مع رأس السهم وحركة الأصابع هي اتجاه M والسنترويد بيكون على امتداد الإبهام

[المومنت (الأصابع) بتدور حوالين السنترويد (الإبهام)]



لحتى أطلع σ لازم أوجد $\bar{y}$ و I ←



$\bar{y}_1 = 20$ mm

$\bar{y}_2 = 50$ mm

$$\bar{y} = \frac{\Sigma A\, y_c}{\Sigma A} = \frac{(30*40)(20)+(90*20)(50)}{(30*40)+(90*20)} = 38 \text{ mm}$$

بما انه عندي أكثر من شكل منتظم

$$I = I_1 + I_2$$

$$\boxed{I_x = I_{\bar{x}} + A d^2}$$

المسافة بين سنتر كل شكل والسنترويد للشكل كاملاً

$$I_1 = \frac{30(40)^3}{12} + (30*40)(38-20)^2$$

$$I_2 = \frac{90(20)^3}{12} + (90*20)(12)^2$$

$$\therefore I = 868*10^{-9}\ m^4$$

من السنترويد لسطح ب

/ /

$$\sigma_{max}\Big|_{tension} = \frac{My}{I} = \frac{3000(22*10^{-3})}{868*10^{-9}} = 76\ MPa$$

تكون على سطح منطقة ال tension (من الأعلى للسنترويد)

$$\sigma_{max}\Big|_{comp} = \frac{My}{I} = \frac{-3000(38*10^{-3})}{868*10^{-9}} = -131.3\ MPa$$

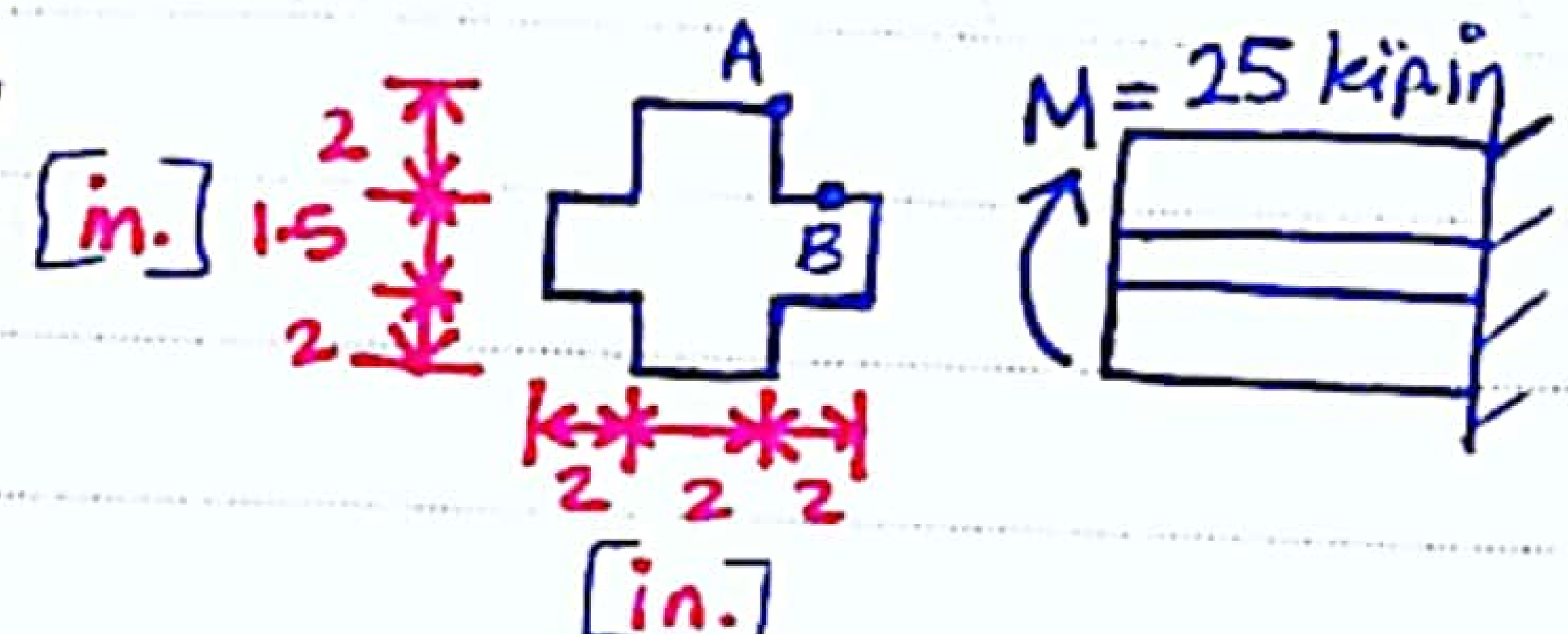
تكون على سطح منطقة ال comp. (من الأسفل للسنترويد)

b. $$\rho = \frac{EI}{M} = \frac{165*\cancel{10^{9}}*868*\cancel{10^{-9}}}{3000}$$

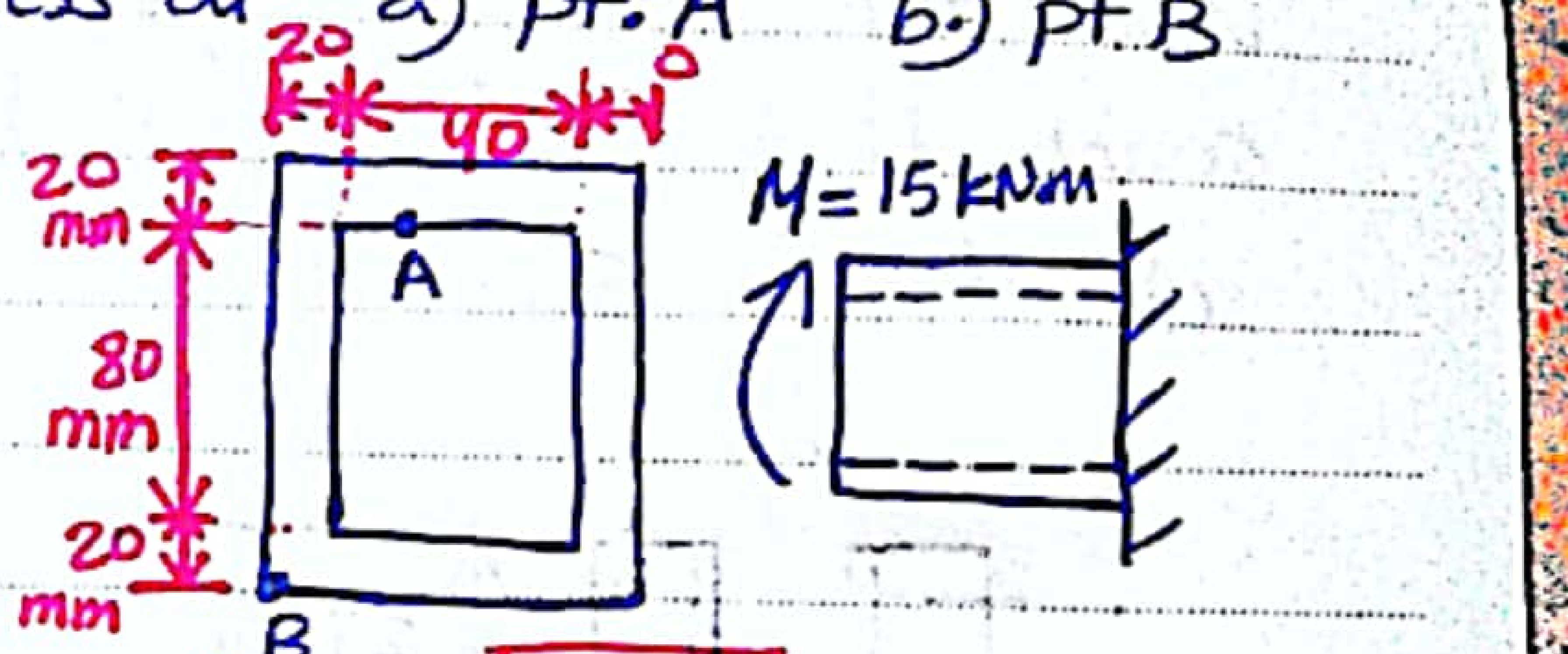
$$= 47.7\ m$$

Suggested Problems
Ch.4

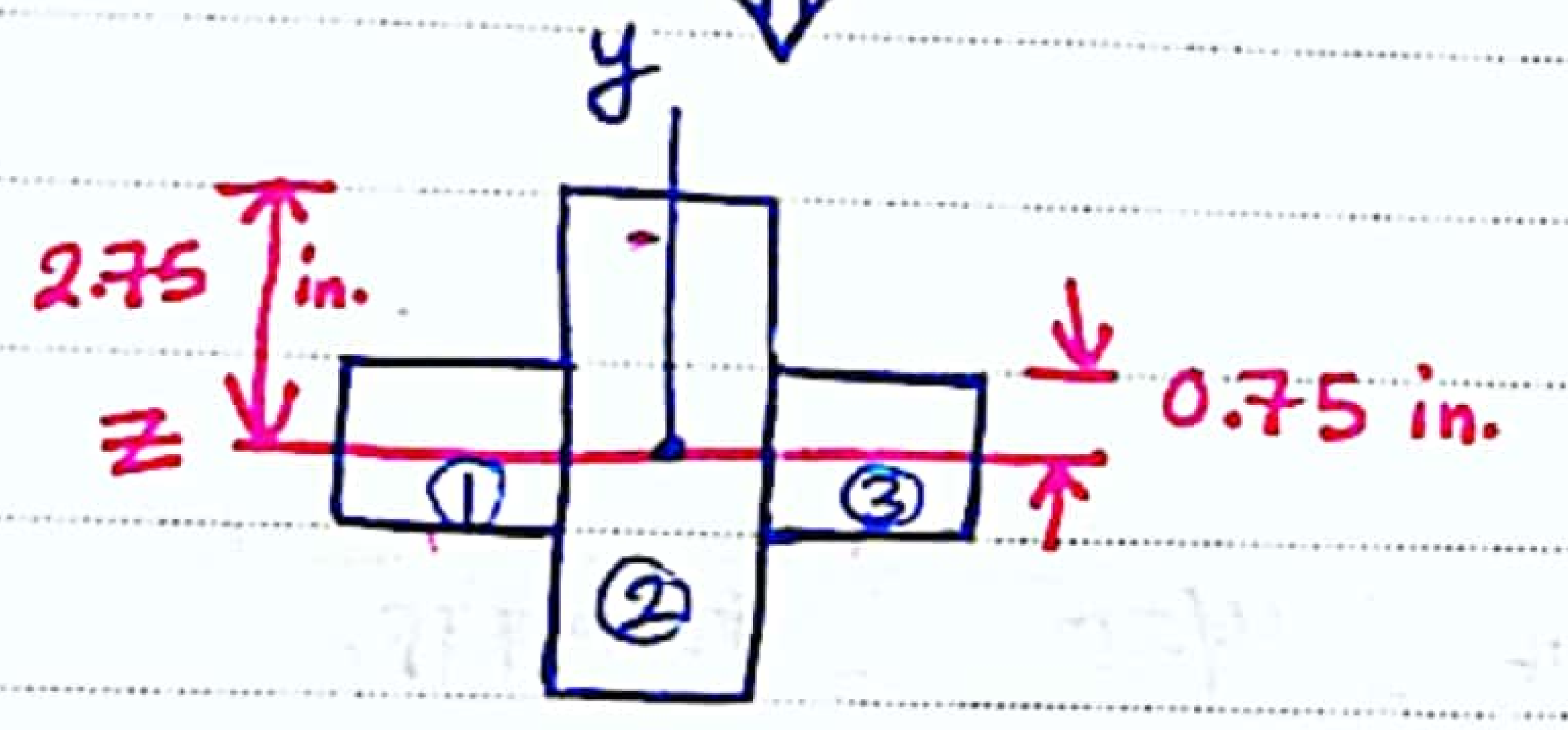
4.1 & 4.2) Knowing that the couple shown acts in a vertical plane, determine the stress at a) pt. A b.) pt. B



P4.1



P4.2



$$I = I_1 + I_2 + I_3$$

$$= \frac{2(1.5)^3}{12} + \frac{2(5.5)^3}{12} + \frac{2(1.5)^3}{12}$$

$$= 28.854 \text{ in}^4$$

a) $y_A = 2.75$ in. $\rightarrow \sigma_A = \frac{-M y_A}{I}$

$$= \frac{-25(2.75)}{28.854}$$

$$= -2.38 \text{ ksi}$$

b) $y_B = 0.75$ in. $\rightarrow \sigma_B = \frac{-M y_B}{I}$

$$= \frac{-(25)(0.75)}{28.854}$$

$$= -0.65 \text{ ksi}$$

For the section $I = I_1 - I_2$

$$I = \frac{80(120)^3}{12} - \frac{40(80)^3}{12} = 9.813 \times 10^6 \; [mm^4]$$

$$I = 9.8133 \times 10^{-6} \; [m^4]$$

a) $y_A = 0.04$ m $\rightarrow \sigma_A = \frac{-M y_A}{I}$

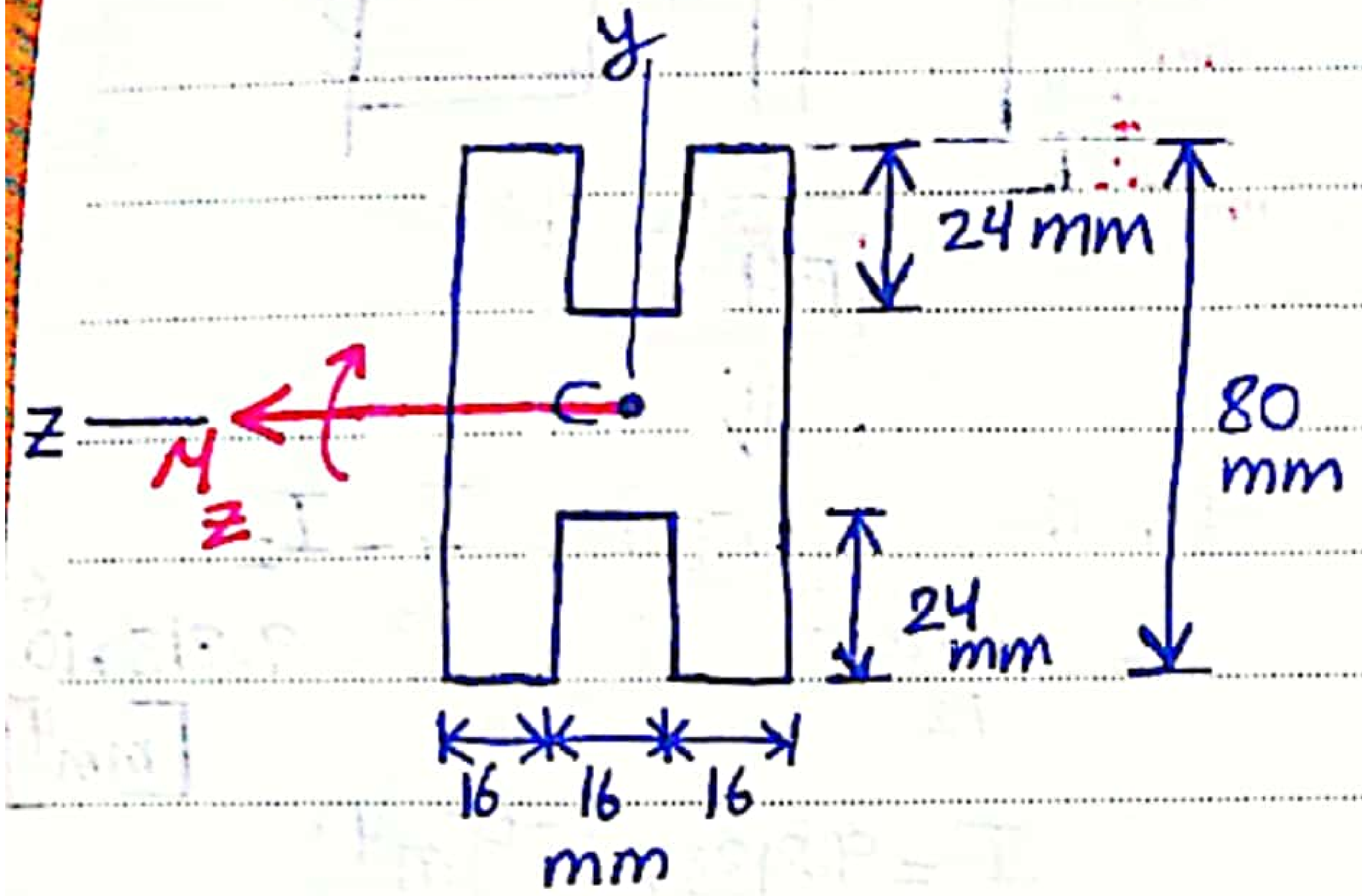
$$\sigma_A = \frac{-(15 \times 10^3)(0.04)}{9.8133 \times 10^{-6}} = -61.6 \text{ MPa}$$

b) $y_B = -0.06$ m $\rightarrow \sigma_B = \frac{-M y_B}{I}$

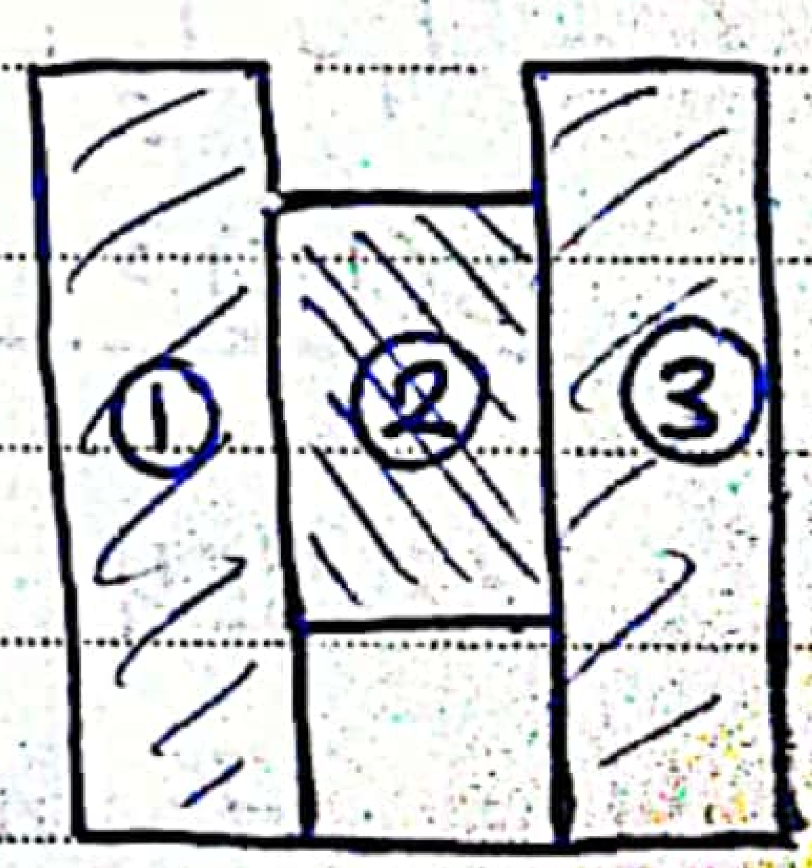
$$\sigma_B = \frac{-(15 \times 10^3)(-0.06)}{9.8133 \times 10^{-6}} = 91.7 \text{ MPa}$$

#

4.5) A beam of the cross section shown is extruded from an aluminum alloy for which $\sigma_y = 250\,MPa$ and $\sigma_u = 450\,MPa$. Using a factor of safety of 3, determine the largest couple that can be applied to the beam when it's bent about the z axis?



allowable stress $\Rightarrow = \dfrac{\sigma_{ult}}{F.S} = \dfrac{450}{3} = 150\,MPa$



Moment of inertia about z axis:

$$I_1 = \frac{16(80)^3}{12} = 682.67 * 10^3 [mm^4]$$

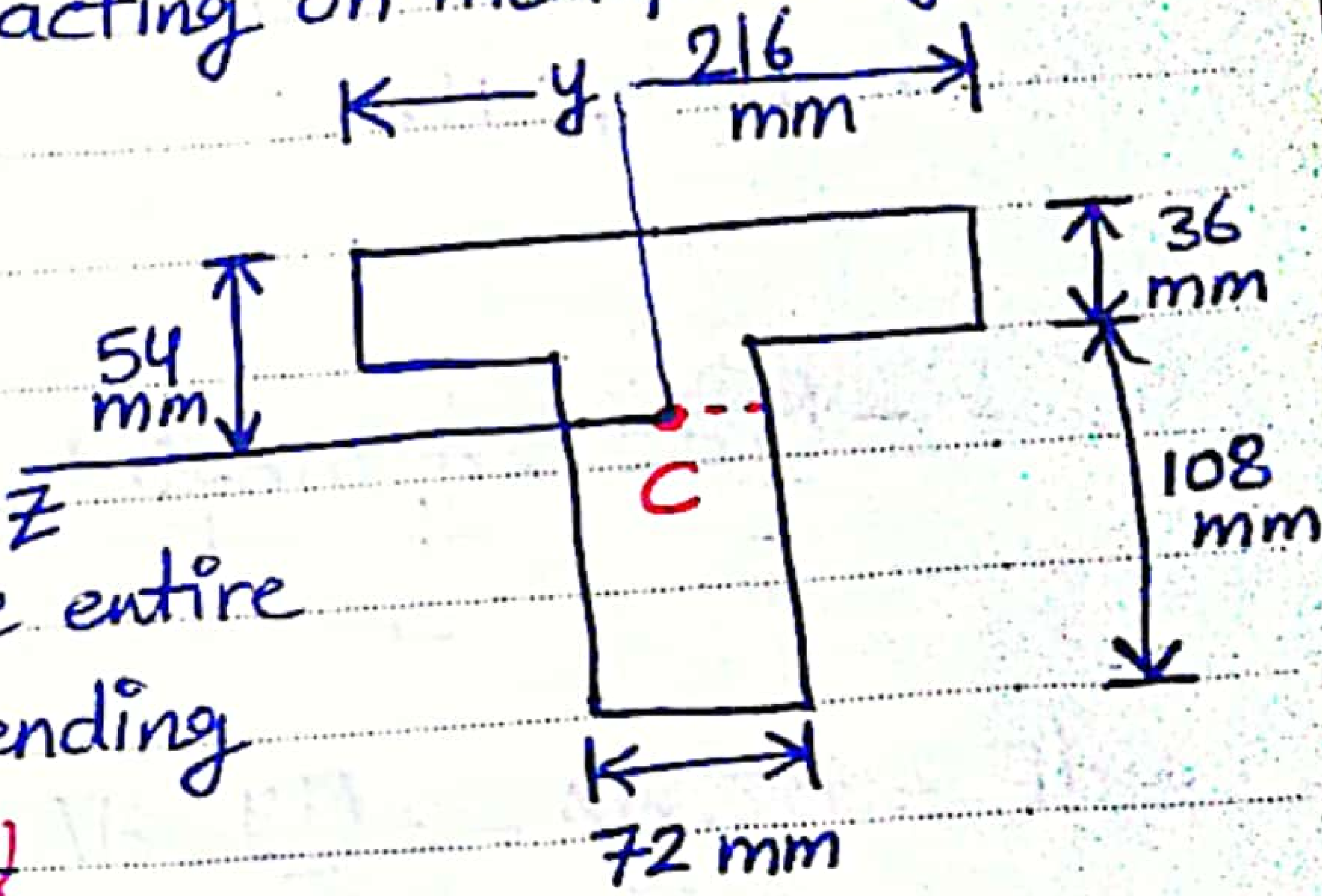
$$I_2 = \frac{16(32)^3}{12} = 43.69 * 10^3 [mm^4]$$

$$I_3 = I_1 = 682.67 * 10^3 [mm^4] \Rightarrow I = I_1 + I_2 + I_3$$
$$= 1.409 * 10^{-6}\, m^4$$

$$* \quad \sigma = \frac{MC}{I} \Rightarrow M = \frac{150 * 10^6 (Pa) * 1.409 * 10^{-6} [m^4]}{\frac{1}{2}(80) * 10^{-3} [m]}$$
$$= 5.28 * 10^3 [N.m]$$

#

4.12) Knowing that a beam of the cross sec. shown is bent about a horizontal axis and that the bending moment is 6 kN.m, determine the total force acting on the top flange?



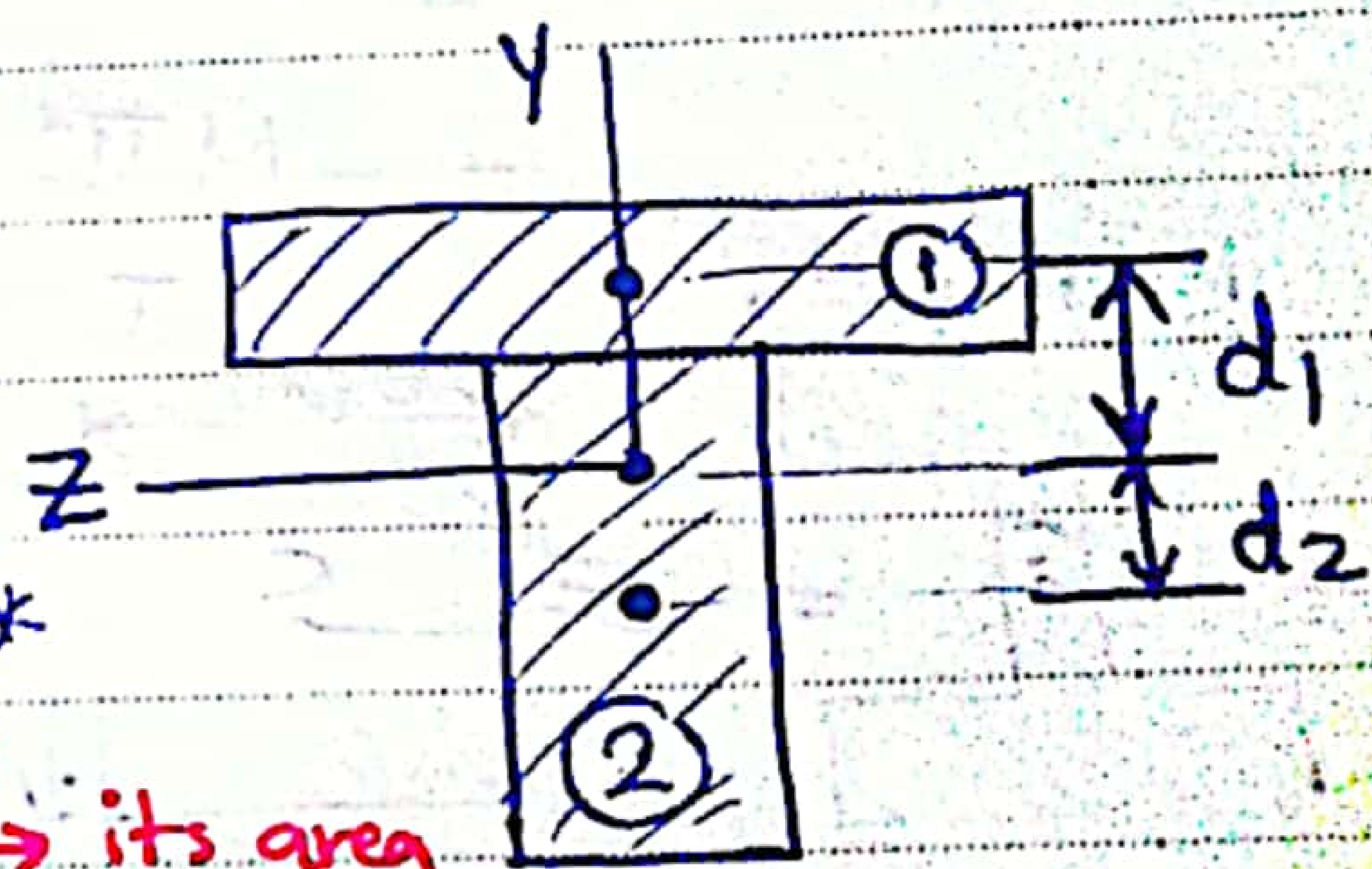
the stress distribution over the entire cross section is given by the bending stress formula :

$$\sigma_x = \frac{-My}{I}$$

y → coordinate with its origin on the neutral axis

I → moment لل of inertia لل (cross section)ل

the force on the shaded area :



$$dF = \sigma_x \, dA = \frac{-My}{I} dA$$

$$F = \int \frac{-My}{I} dA = \frac{-M}{I} \bar{y}^{*} A^{*}$$

A^{*} → its area

$\bar{y}^{*}$ → centroidal coordinate of the shaded portion

$d_1 = 54 - 18 = 36$ mm

$d_2 = 54 + 36 - 54 = 36$ mm

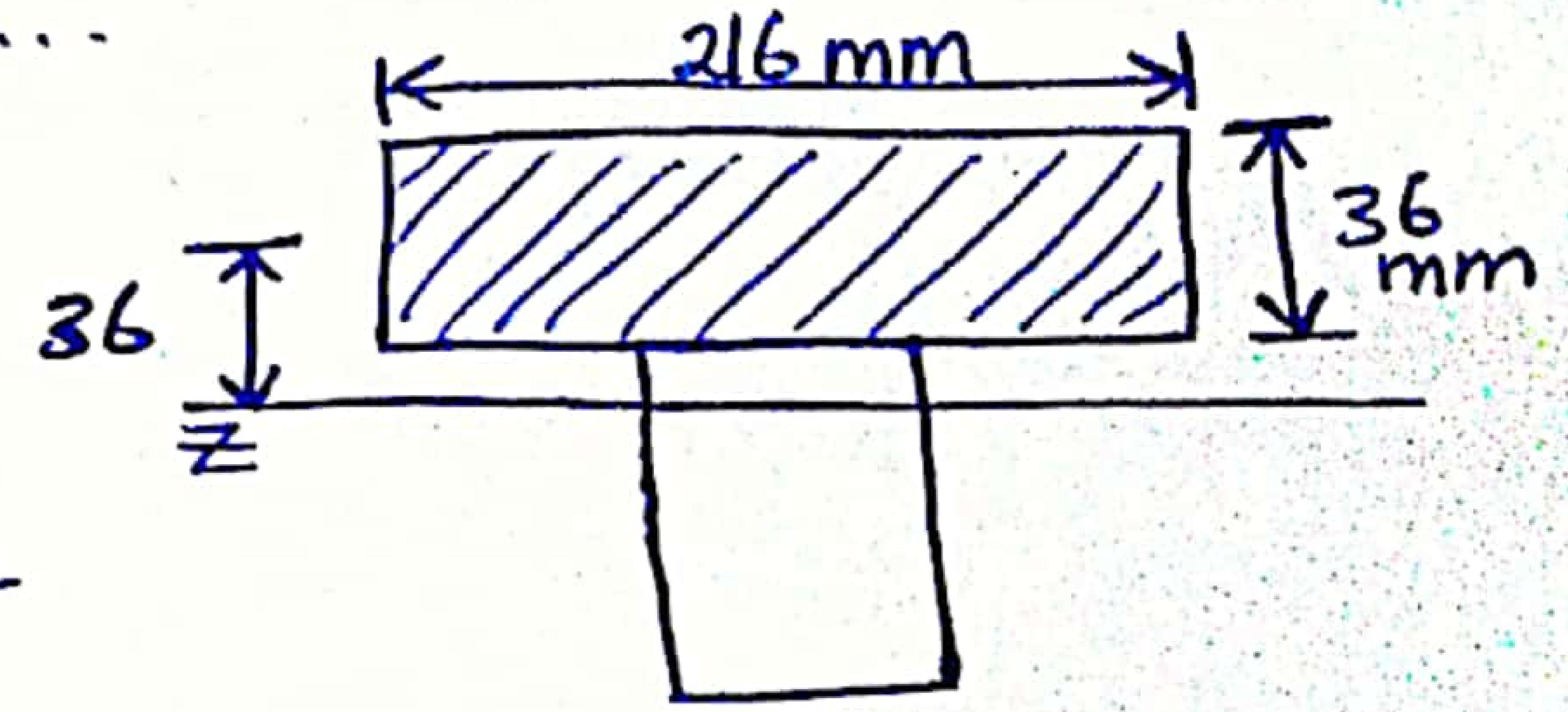
#

$$I_1 = \frac{216(36)^3}{12} + 216(36)(36)^2 = 10.9175 * 10^6 \text{ mm}^4$$

$$I_2 = \frac{72(108)^3}{12} + 72(108)(36)^2 = 17.636 * 10^6 \text{ mm}^4$$

$$I = I_1 + I_2 = 28.55 * 10^{-6} \text{ m}^4$$

for the shaded area...



$$A^* = 216(36) = 7776 \text{ mm}^2$$

$$\bar{y}^* = 36 \text{ mm}$$

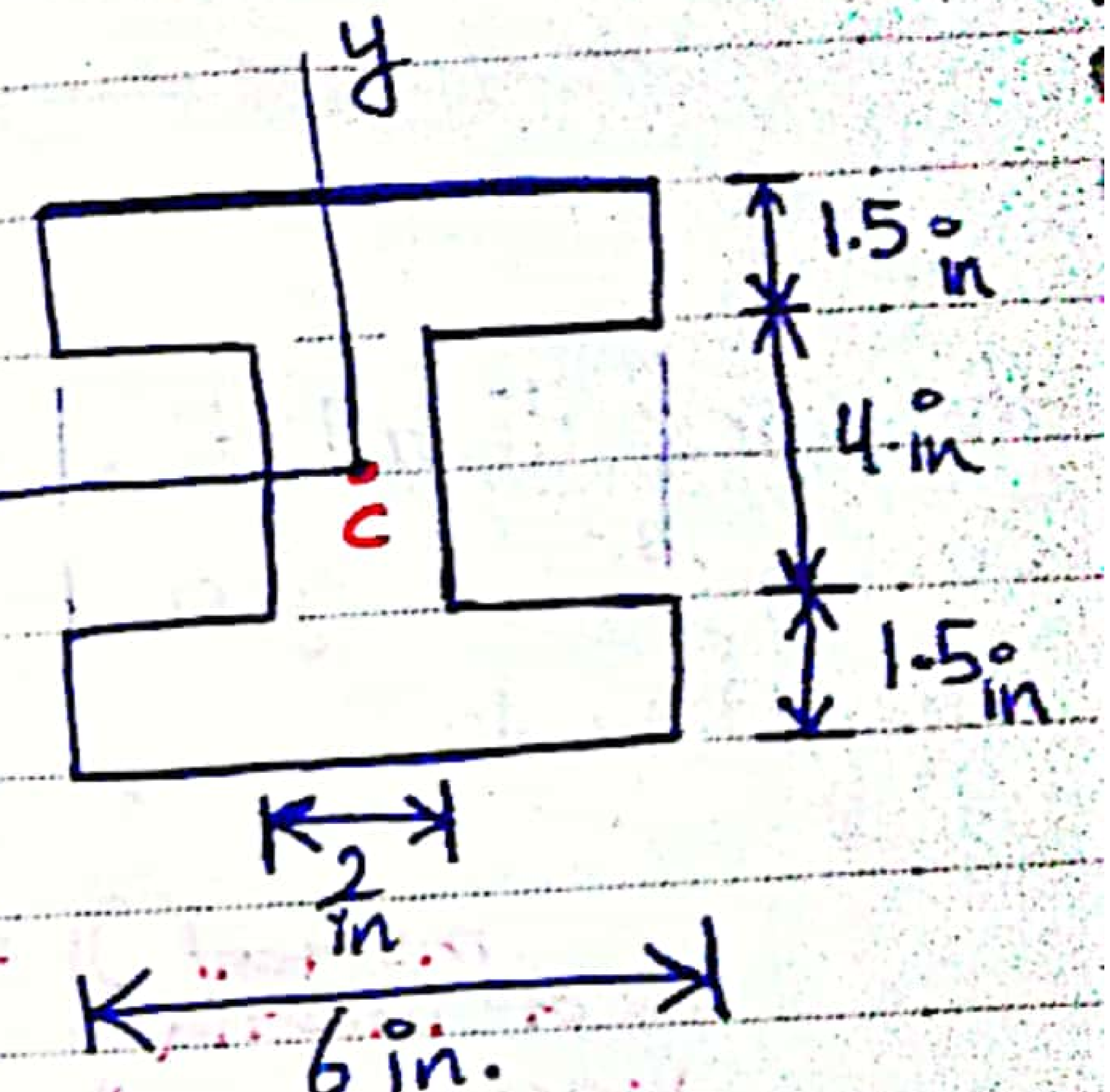
$$A^* \bar{y}^* = 279.936 * 10^{-6} \text{ m}^3$$

$$F = -\left| \frac{M A^* \bar{y}^*}{I} \right| = \frac{6*10^3 * 279.936 * 10^{-6}}{28.5535 * 10^{-6}}$$

$$\boxed{F = 58.8 \text{ kN}}$$

#

4.14) Knowing that a beam of the cross section shown is bent about a horizontal axis & that the bending moment is 50 kip.in, determine the total force acting
a) on the top flange   b) on the shaded portion of the web

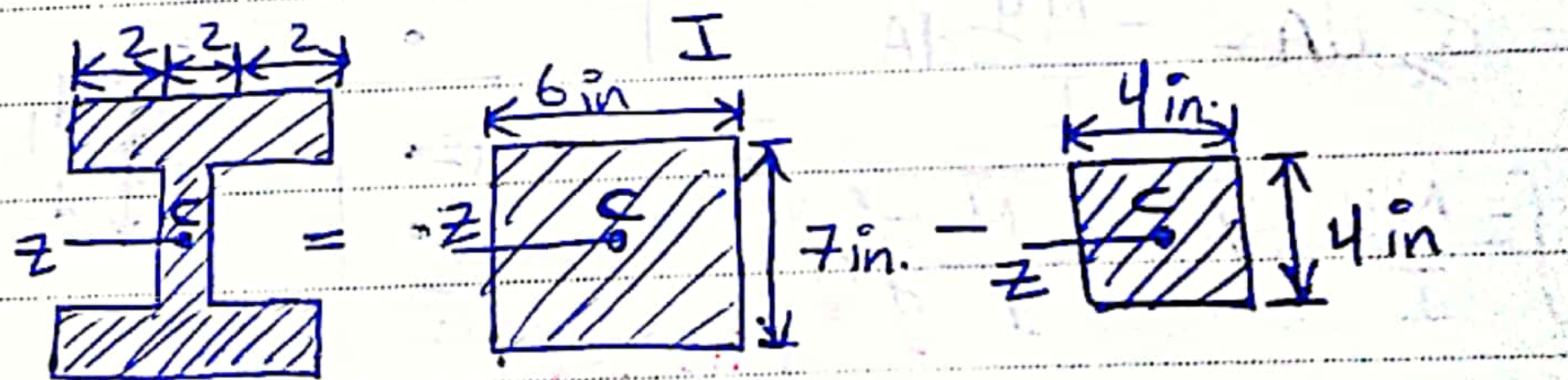


$\sigma_x = \frac{-My}{I}$, distributed over the entire cross section

$dF = \sigma_x \, dA = \frac{-My}{I} \, dA$

$F = \int dF = \frac{-M}{I} \int y \cdot dA$

$= \frac{-M \bar{y}^* A^*}{I}$



$I = \frac{6(7^3)}{12} - \frac{4(4)^3}{12} = 150.17 \, [in^4]$

$M = 50$ kip.in

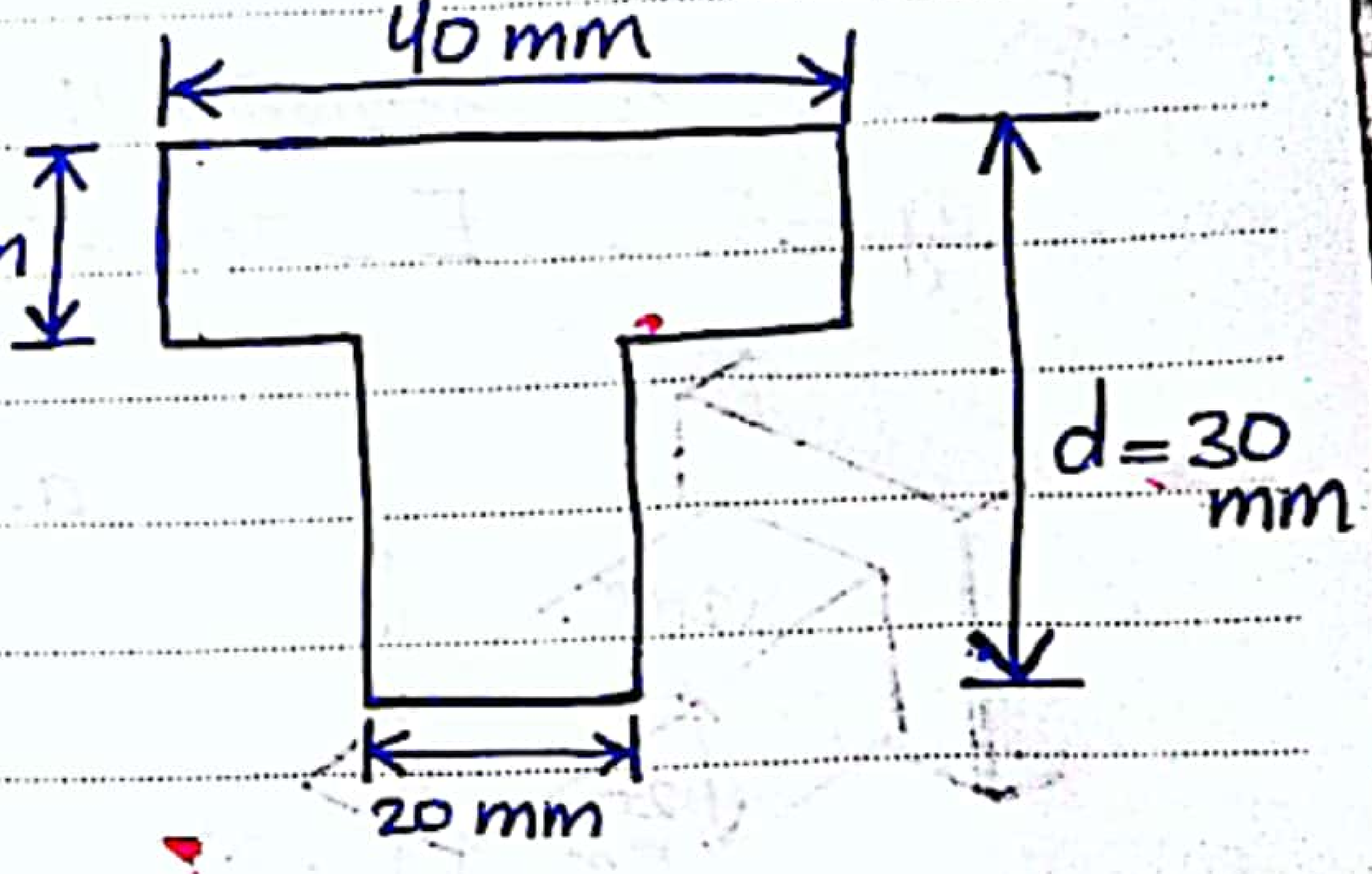
a) $A^* = 6(1.5) = 9 \, in^2$, $\bar{y}^* = 2 + 0.75 = 2.75$ in.

$F = \frac{50 * 9 * 2.75}{150.17} = 8.24$ kips

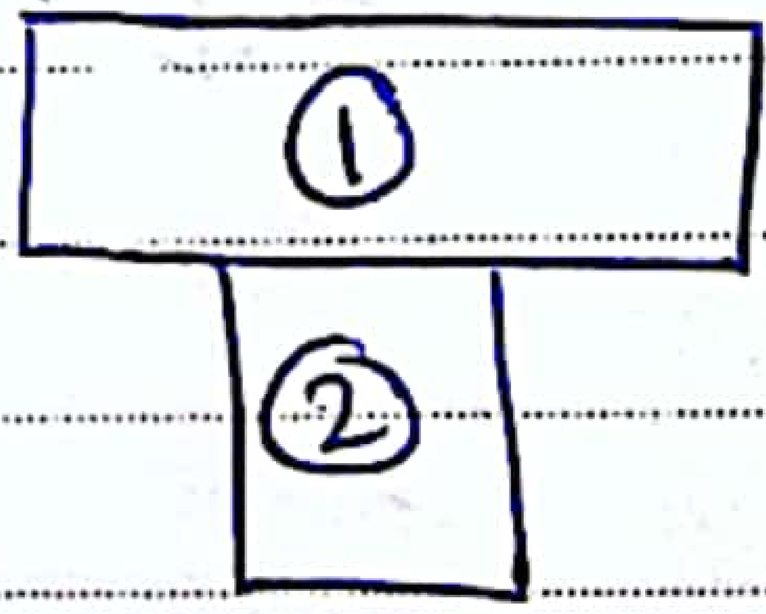
b) $F = \frac{50 * (2*2)(1)}{150.17} = 1.332$ kips

ALADIB

4.15) The beam shown is made of a nylon for which the allowable stress is 24 MPa in tension and 30 MPa in comp.. Determine the largest couple M that can be applied to the beam



	$A [mm^2]$	$\bar{Y}_o [mm]$	$A\bar{Y}_o [mm^3]$
1	600	22.5	$13.5 * 10^3$
2	300	7.5	$2.25 * 10^3$
Σ	900		$15.75 * 10^3$



$* \ \bar{Y}_o = \frac{15.5 * 10^3}{900} = 17.5 mm$

* the neutral axis lies 17.5 mm above the bottom

$y_{top} = 30 - 17.5 = 12.5 mm$

$y_{bot} = -17.5 mm$

$I_1 = \frac{b_1 h_1^3}{12} + A_1 d_1^3 = \frac{40(15)^3}{12} + 600(5)^2 = 26.25 * 10^3 [mm^4]$

$I_2 = \frac{20(15)^3}{12} + 300(10)^2 = 35.625 * 10^3 mm^4$

$I = I_1 + I_2 = 61.875 * 10^3 [mm^4]$

Top :

$M = \frac{24 * 10^6 * 61.875 * 10^{-9}}{0.0125} = 118.8 N.m$ — tension side

Bottom :

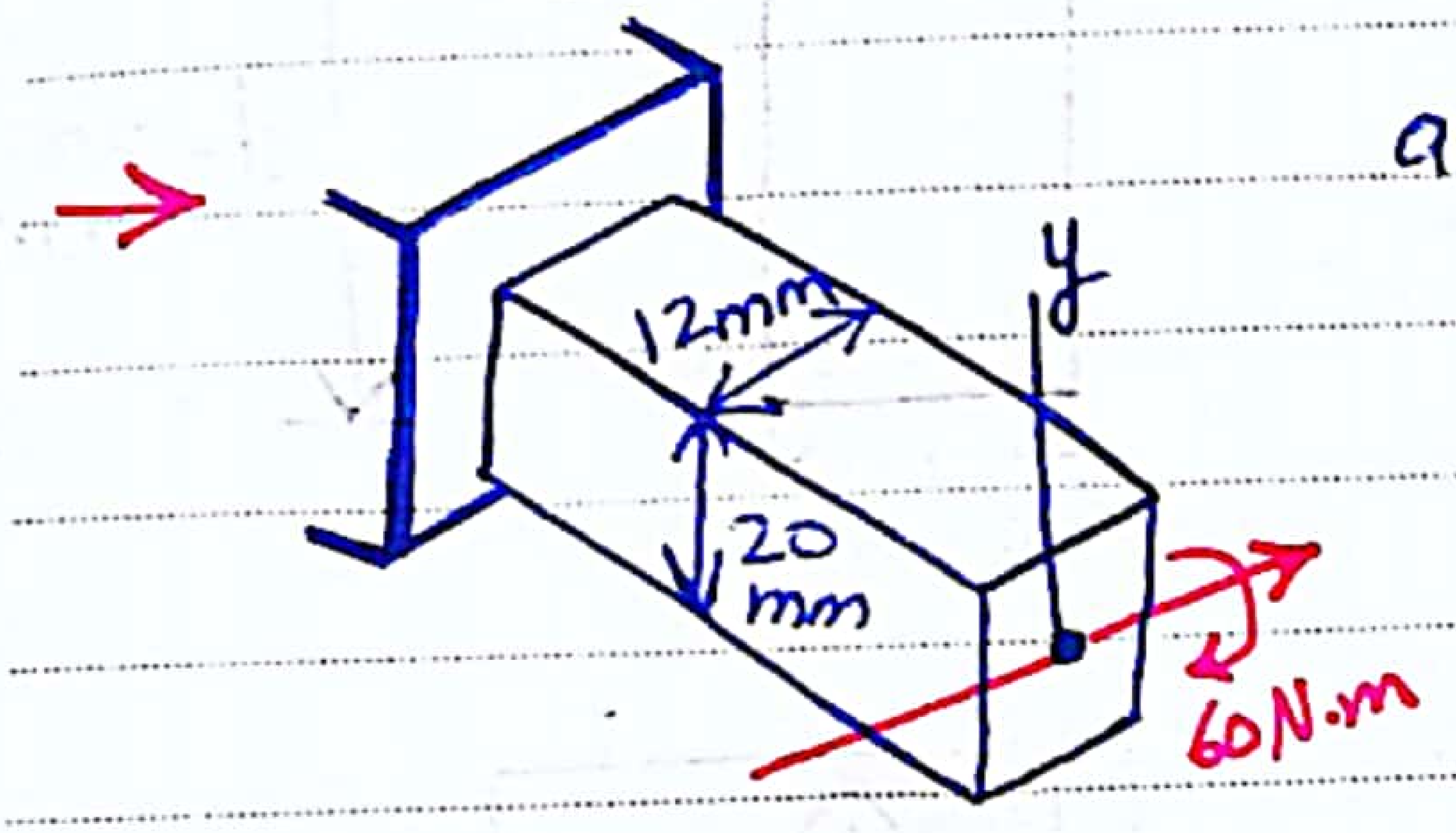
$M = \frac{30 * 10^6 * 61.875 * 10^{-9}}{0.0175} = 106.1 N.m$ compression side

$M = 106.1 N.m$ → the smallest value

4.24 A 60 N.m couple is applied to the steel bar shown.

a) Assuming that the couple is applied about the z axis as shown determine the max. stress and the radius of curvature of the bar?

b) solve a, assuming that the couple is applied about the y axis. E = 200 GPa



a- Bending about z axis →

$$\sigma = \frac{\overset{M}{60} * \left(\frac{20}{2} * 10^{-3}\right)^{c}}{\underset{I}{\left[\frac{12 (20)^3}{12} * 10^{-12}\right]}} = 75 \text{MPa}$$

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{60}{200*10^{9} * 8*10^{-9}}$$

$$= 37.5 * 10^{-3} \ m^{-1}$$

$$\rho = 26.7 \ m$$

b- Bending about y axis →

$$\sigma = \frac{60 * \left(12 * \frac{1}{2} * 10^{-3}\right)}{\frac{20 (12)^3}{12} * 10^{-12}} = 125 \text{ MPa}$$

$$\frac{1}{\rho} = \frac{60}{200*10^{9} * 2.88*10^{-9}} = 104.17 * 10^{-3} \ m^{-1}$$

$$\rho = 9.6 \ m$$

final o.o

17 / 3 / 2019
Sunday

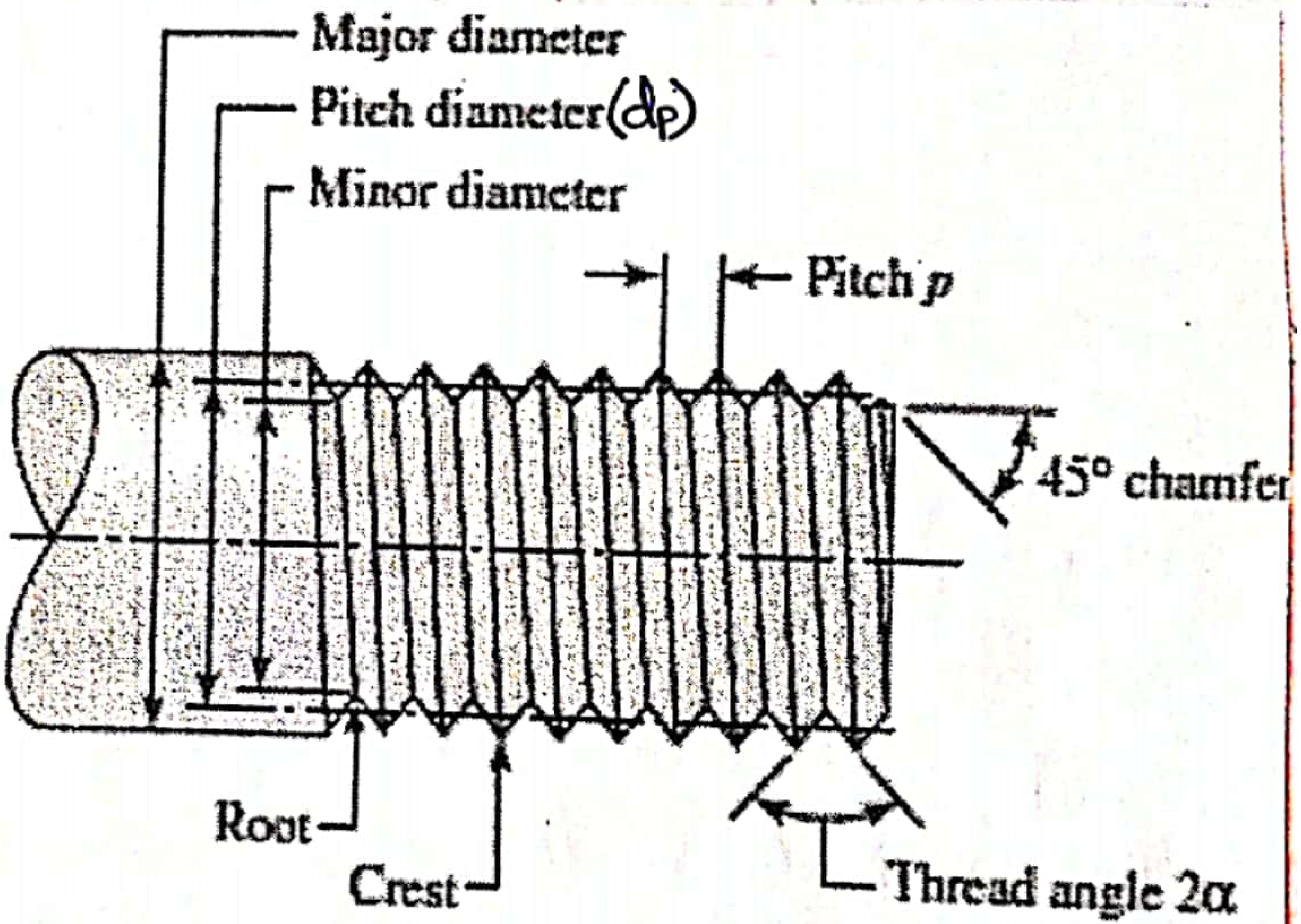
Ch. 8 Screws, Fasteners, and the Design of Nonpermanent Joints

Type of joints → permanent "welding"
→ Non permanent "Screws"

8.1 Thread Standards and Definitions

- The terminology of screw threads, illustrated in the figures below :

Sharp vee threads ⇒ shown for clarity; the crests and roots are actually flattened or rounded during the forming operation.



pitch (P) ≡ distance between adjacent thread forms measured parallel to the thread axis.

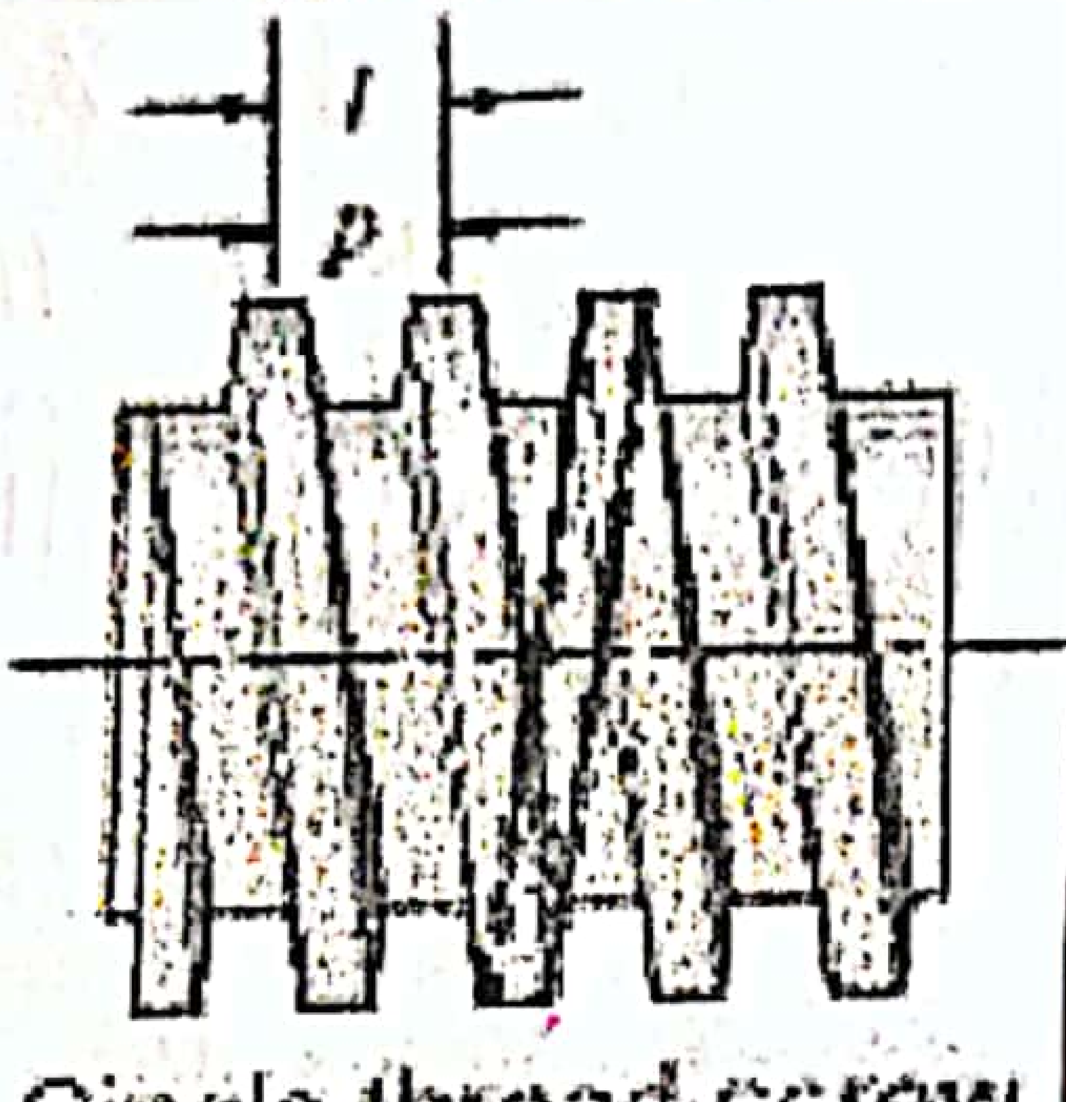
(d) major diameter ≡ the largest diameter of a screw thread.

(dr) minor diameter ≡ the smallest diameter of a screw thread.

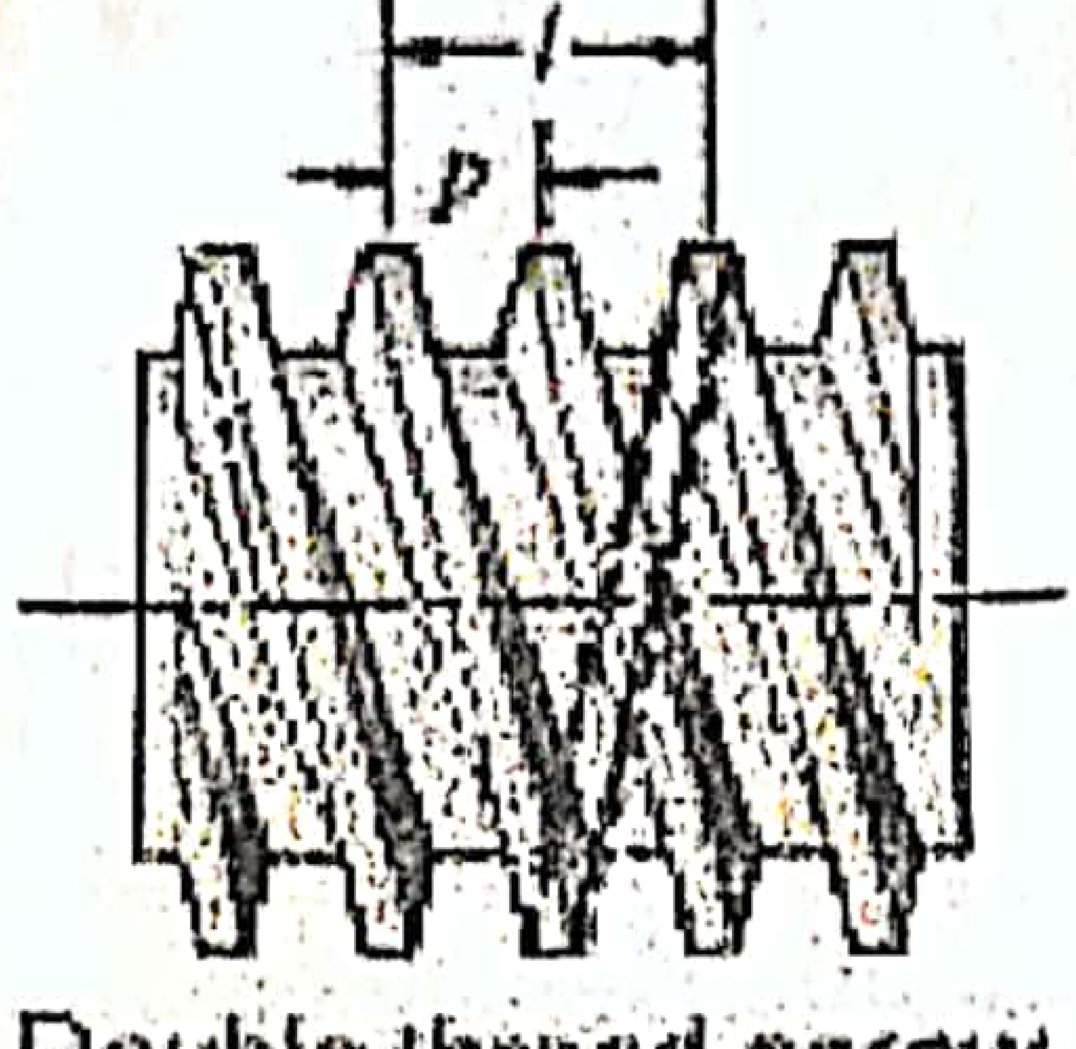
(or root diameter)

pitch diameter (d_p) ≡ is a theoretical diameter between the major and minor diameters.

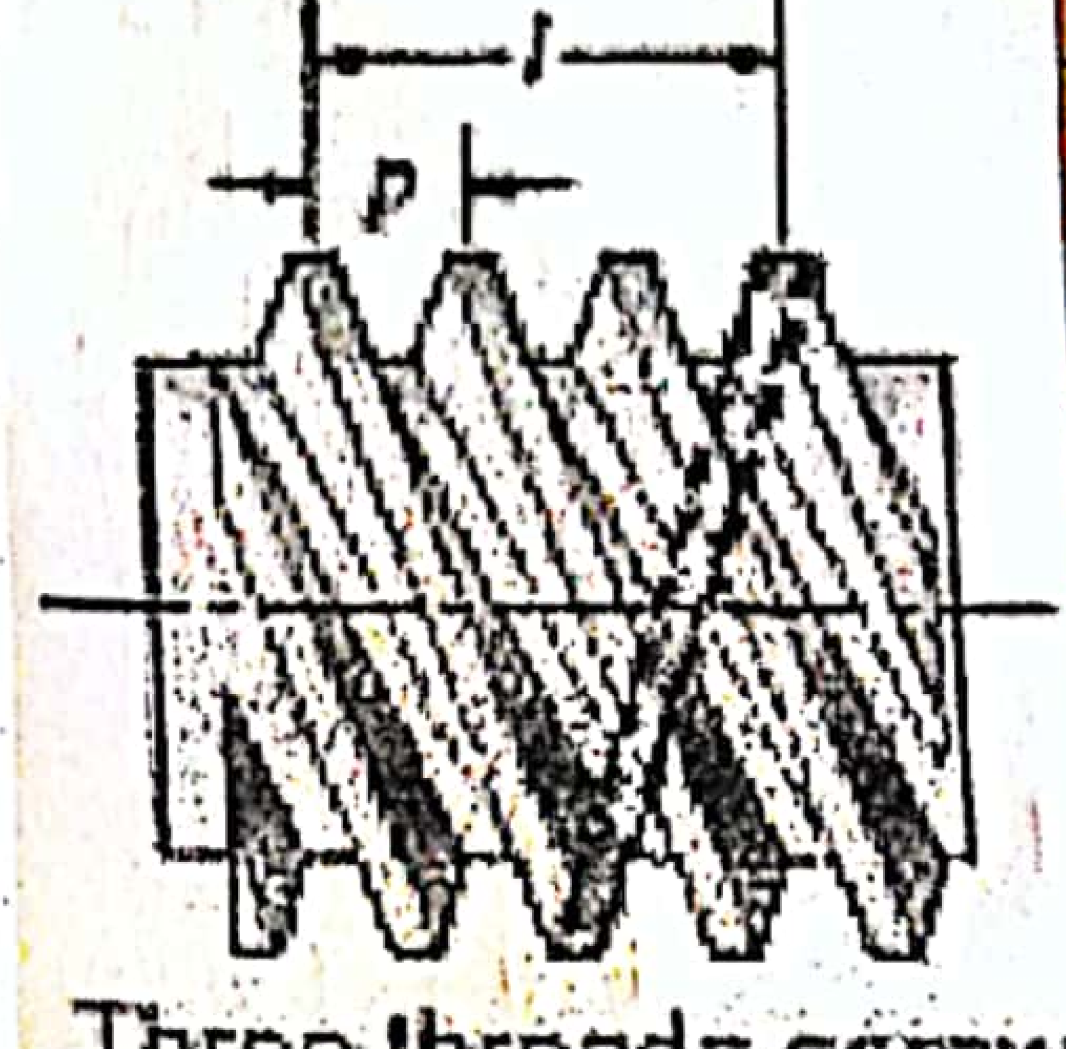
$l = P$ $l = 2P$ $l = 3P$



Single thread screw



Double thread screw



Three threads screw

Lead (L) ≡ the distance the nut moves parallel to the screw axis when the nut is given one turn.

المرفقين جداول

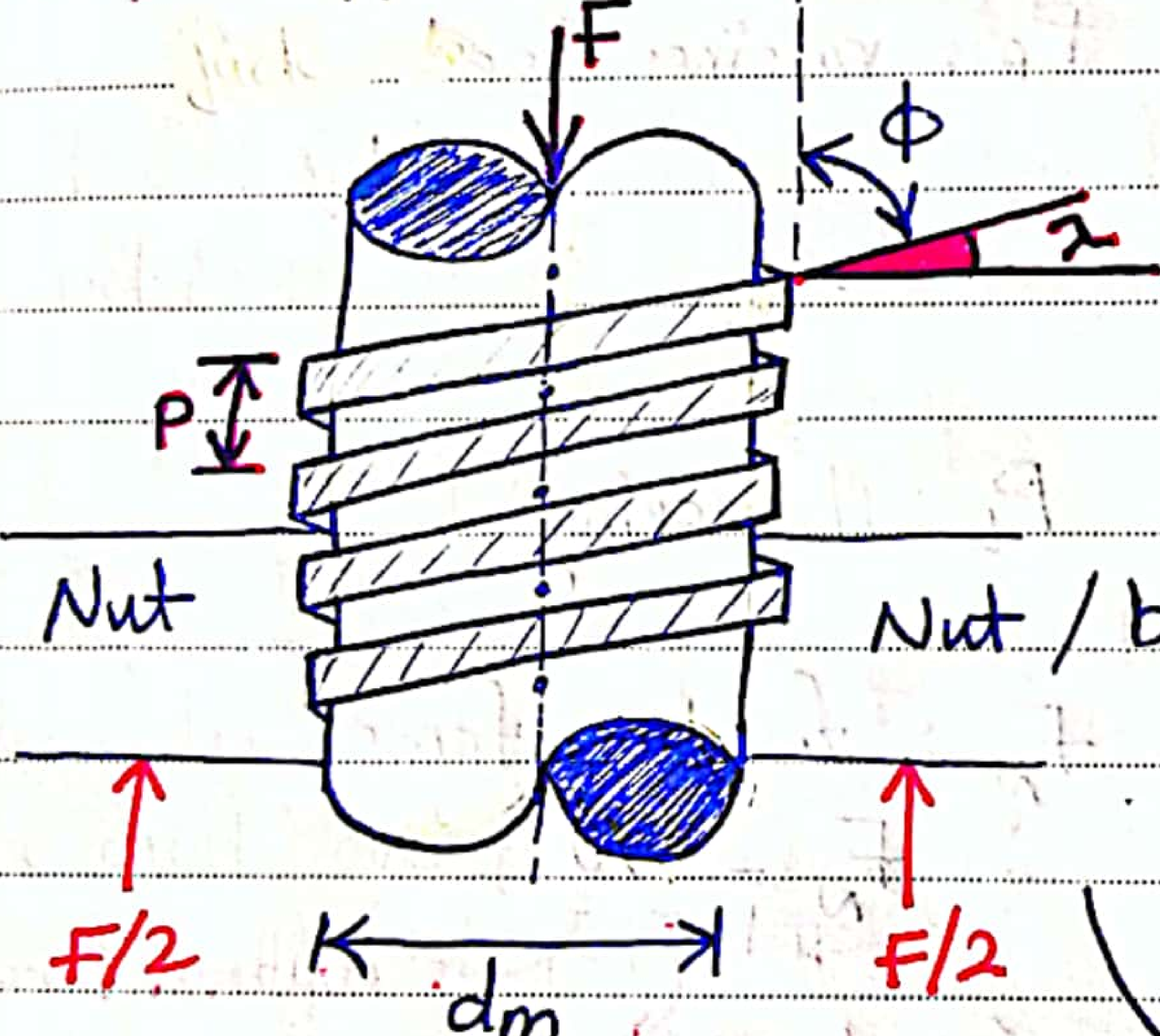
Tables (8-1) & (8-2) for specifying and desiguing threaded parts.

the thread size is specified by giving the pitch P for metric sizes and by giving the no. of threads per inch N. The screw sizes in (8-2) with diameter under 1/4 in are numbered or gauge sizes. The second column in (8-2) shows that a No.8 screw has a nominal major diameter of 0.1640 [in.]

8.2 The Mechanics of Power Screws

- Power Screw ≡ is a device used in machinery to change angular motion into linear motion, and, usually, to transmit power.

- An application of power screws to a power-driven jack



F : Comp. axial force

λ : Lead angle

ϕ : helix angle

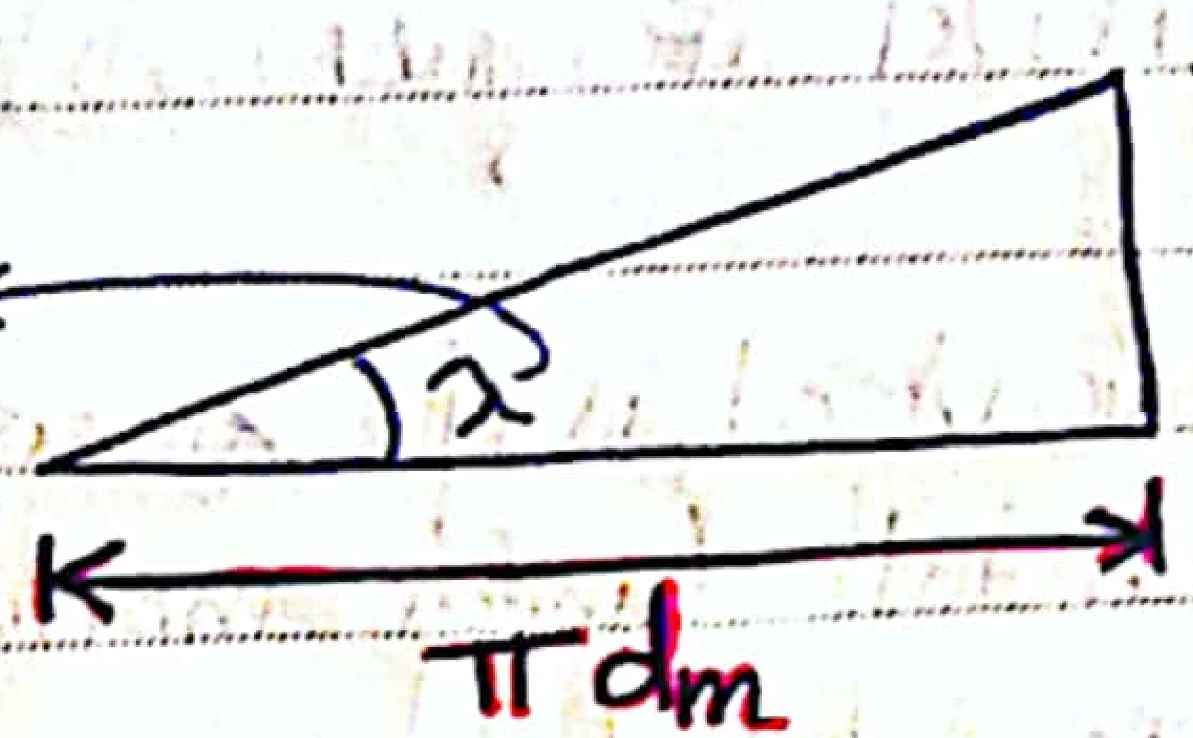
dm : mean diameter

Nut / body جزئ من ال Jack

المسنّن مربع
square threaded power screw with single thre[ad]

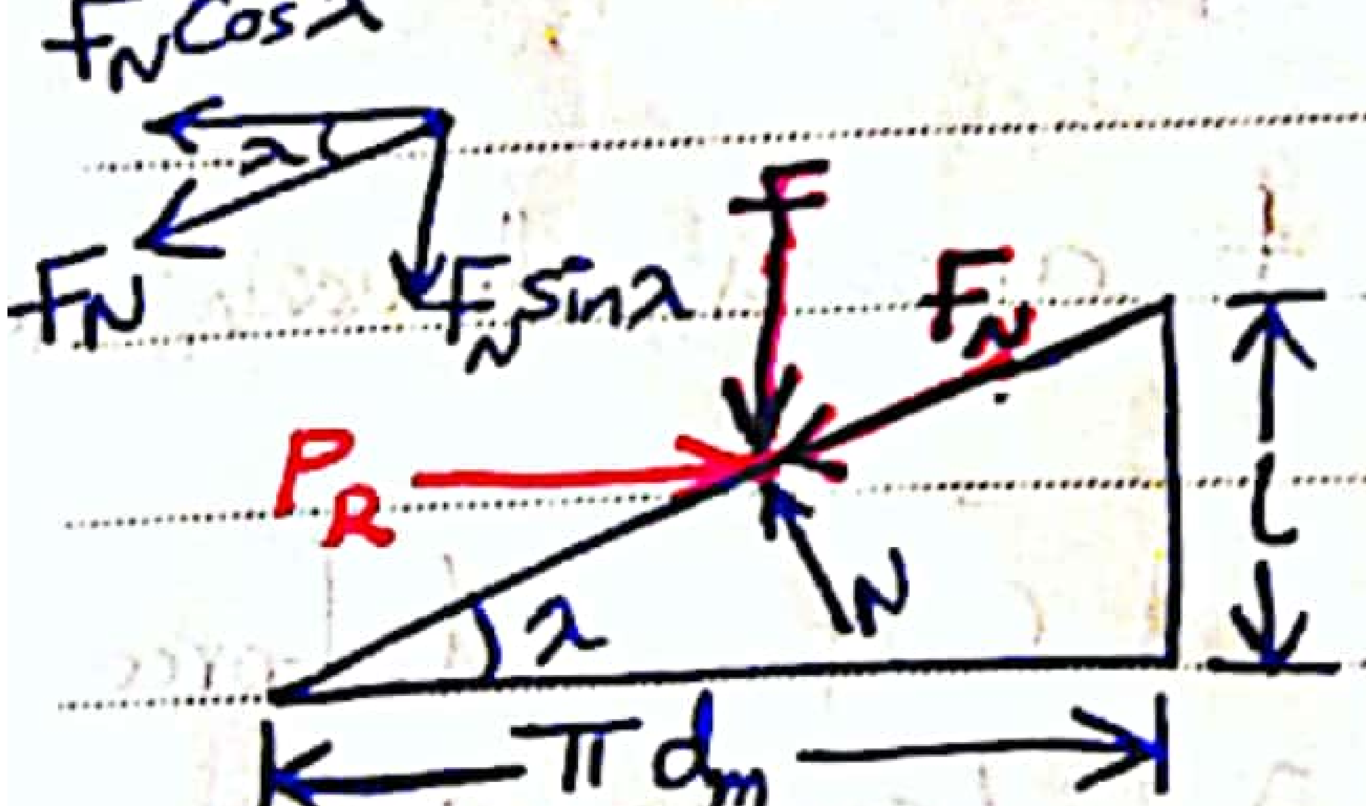
* بينا نوجد التورك اللي رح نحتاجه لحتى نرفع هاد اللود و التورك لحتى ننزله برضو ...

the lead angle of the thread

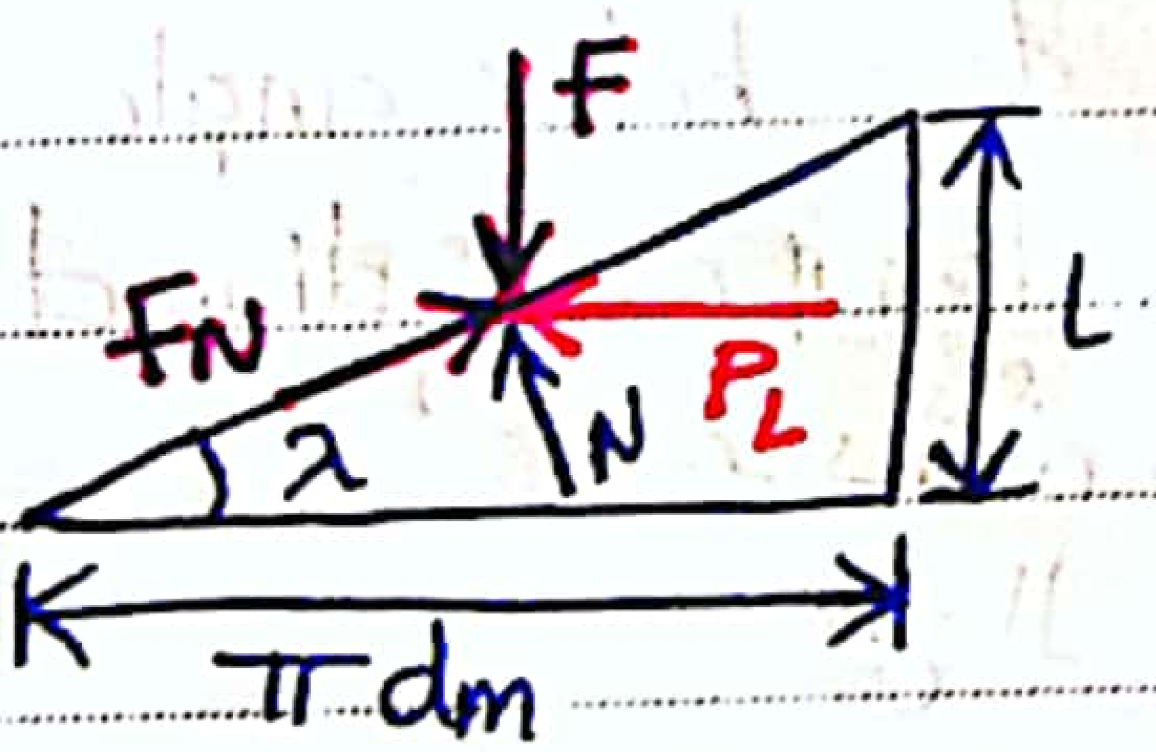


l → thread height (lead)

πd_m → the circumference of the mean thread diameter circle



⇒ P_R : raising load رفع



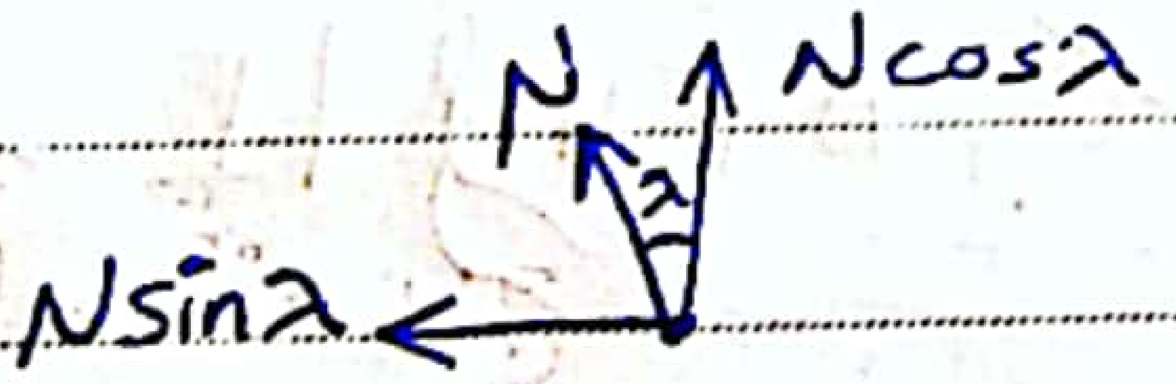
⇒ P_L : lowering load خفض

F_N : friction force

$F_N = fN$

f: معامل الاحتكاك friction coefficient

N: normal force



* For raising the load :

$$\sum F_x = 0 \Rightarrow \left[P_R - N \sin\lambda - fN\cos\lambda = 0 \right]$$

$$\sum F_y = 0 \Rightarrow \left[-F - fN\sin\lambda + N\cos\lambda = 0 \right]$$

/ /

dividing P_R by F :

$$\frac{P_R}{F} = \frac{\cancel{N}\sin\lambda + f\cancel{N}\cos\lambda}{-f\cancel{N}\sin\lambda + \cancel{N}\cos\lambda} \Rightarrow P_R = F\frac{(\sin\lambda + f\cos\lambda)}{(\cos\lambda - f\sin\lambda)}$$

بقسمة الحدود على $\cos\lambda$

Let $\tan\lambda = \frac{l}{\pi d_m}$

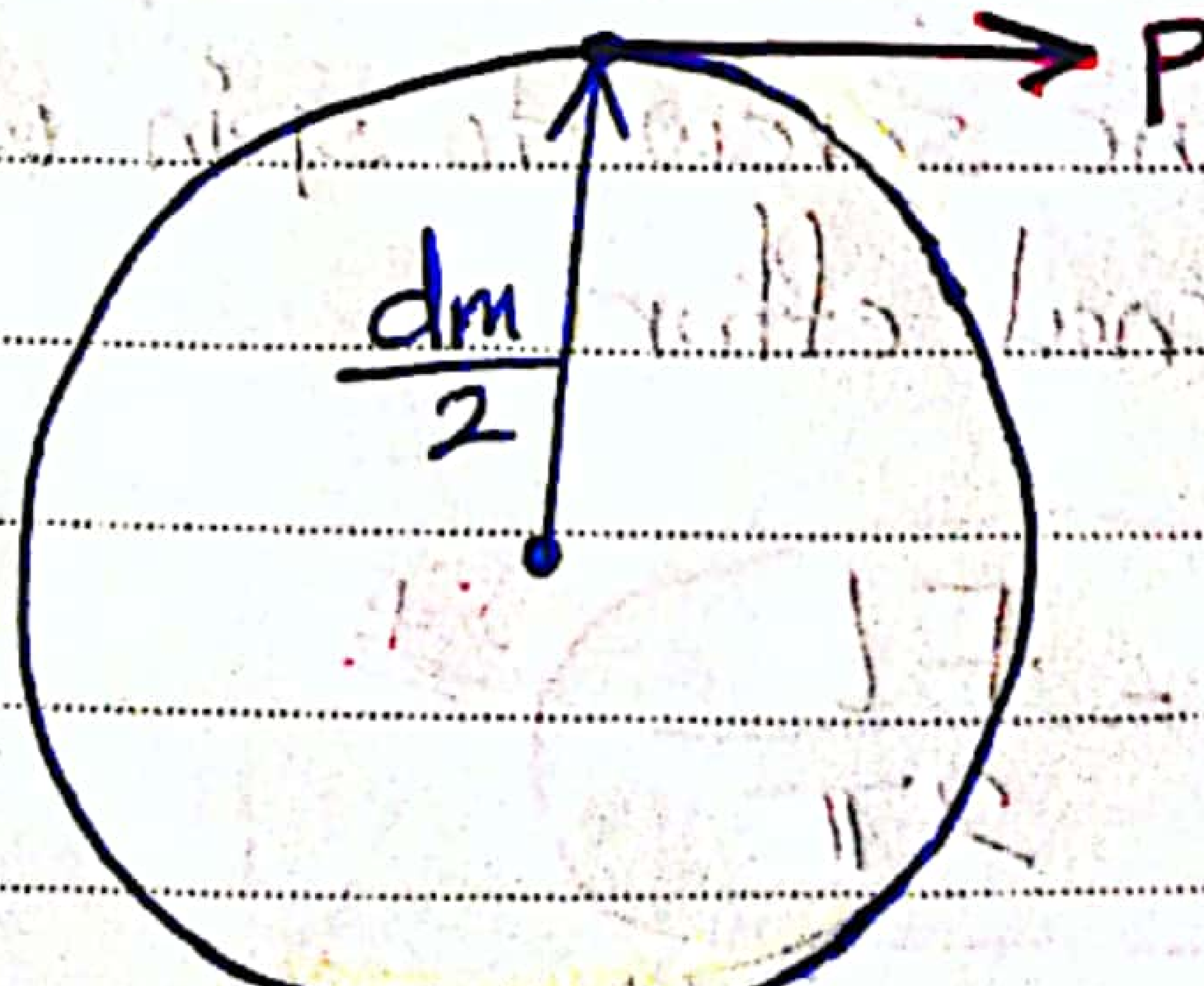
$$\boxed{P_R = F\frac{\left(\frac{l}{\pi d_m} + f\right)}{\left(1 - \frac{f\,l}{\pi d_m}\right)}}$$

حفظ

f : معامل الاحتكاك

- The torque for raising the load : $\boxed{T_R = P_R\left(\frac{d_m}{2}\right)}$

$$T_R = F\frac{d_m}{2}\left(\frac{\pi f d_m + l}{\pi d_m - f l}\right)$$



* For Lowering the Load :

$$\Sigma F_x = \emptyset \Rightarrow \left[-P_L + N\sin\lambda + fN\cos\lambda = \emptyset \right]$$

$$\Sigma F_y = \emptyset \Rightarrow \left[-F + fN\sin\lambda + N\cos\lambda = \emptyset \right]$$

$$\boxed{P_L = F\frac{\left(f - \frac{L}{\pi d_m}\right)}{\left(1 + f\frac{L}{\pi d_m}\right)}}$$

مهم

the torque required to overcome a part of the friction in lowering the load.

$$T_L = F\frac{d_m}{2}\frac{\left(f - \frac{L}{\pi d_m}\right)}{\left(1 + f\frac{L}{\pi d_m}\right)} = F\frac{d_m}{2}\left(\frac{\pi f d_m - L}{\pi d_m + fL}\right)$$

$\Rightarrow$ If $\pi f d_m > l \rightarrow f > \frac{l}{\pi d_m}$

$f > \tan\lambda$

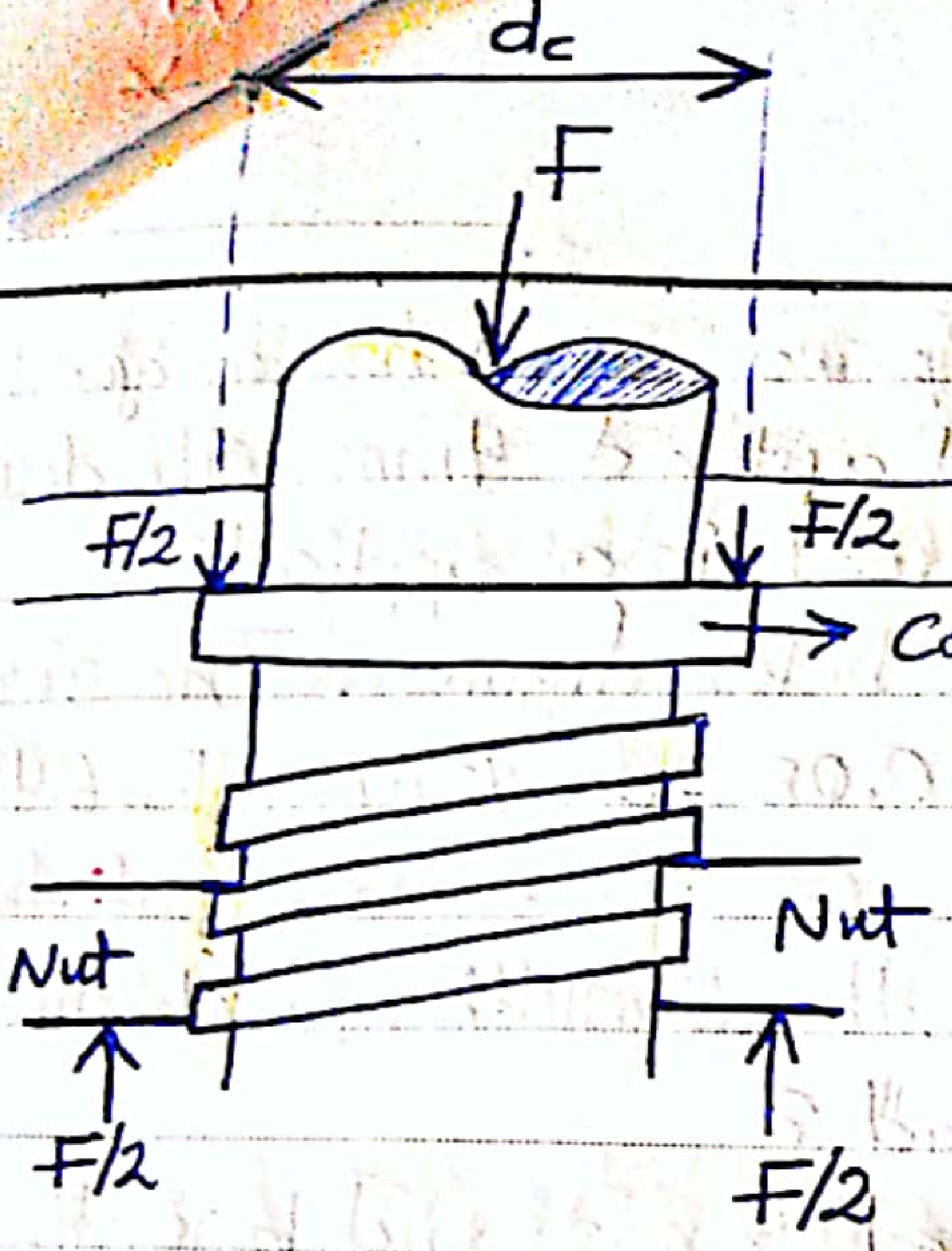
, when a +ve torque obtained from the above equation

حكي عن ال screw بهالحالة

Self locking : the Load will lower itself by causing the screw to spin without any external effort

if $f = \emptyset \rightarrow \boxed{T_o = \frac{FL}{2\pi}}$ مهم

$$\eta = \frac{T_o}{T_R} = \frac{FL}{2\pi T_R}$$



* When the screw is loaded axially, a thrust or collar bearing must be employed between the rotating and stationary members to carry the axial component..

f_c : the coefficient of collar friction

d_c : the mean collar diameter

$$f_c , d_c \Rightarrow T_c = \frac{F f_c d_c}{2}$$

then, $T_{total} = T_R + T_C$

or $T_{total} = T_L + T_C$

to calculate stresses for the above structure :

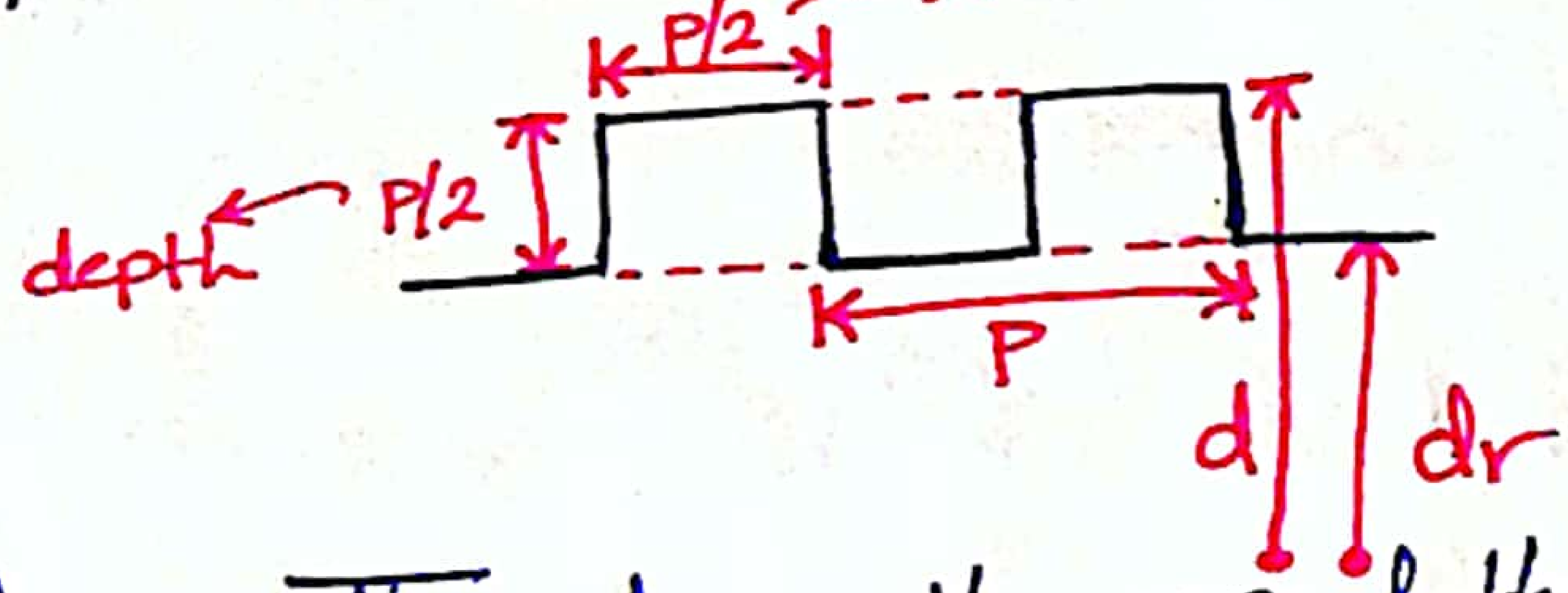
$$\sigma_{normal} = \frac{F}{A_1} = \frac{4F}{\pi (d_r)^2}$$

$$\tau_{max} = \frac{16T}{\pi (d_r)^3}$$ → for a maximum shear stress in torsion of the screw body

→ τ_{max} ال

تكون عند أكبر شي

* Square Thread → width

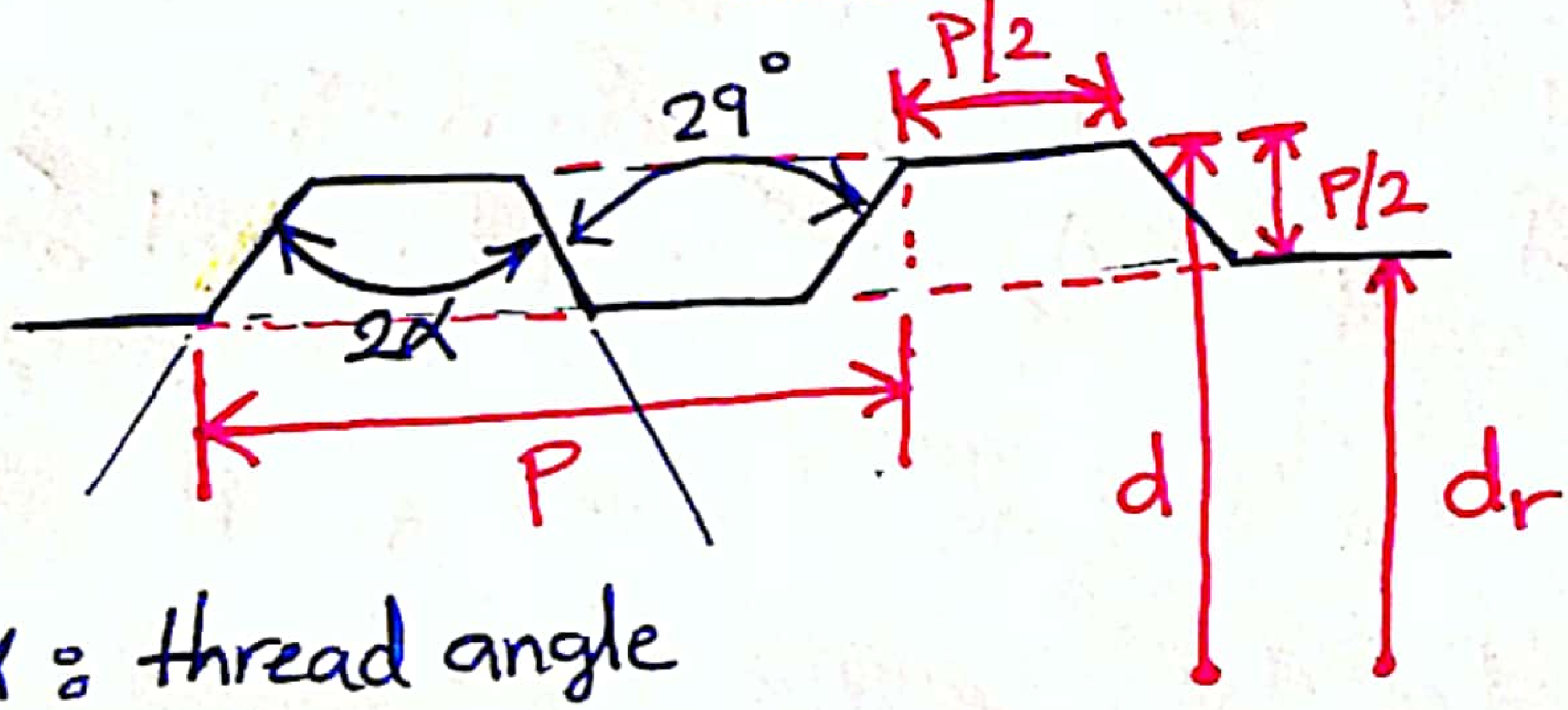


P : pitch

P/2 : width (depth)

* Acme Thread → the normal thread load is inclined to the axis because of the thread angle 2α and the lead angle λ

- For power screws, square better than Acme



2α : thread angle

Ex(8.1) A square thread power screw has a major diameter of 32 mm and a pitch of 4mm with double threads, and it's to be used in an application similar to that in the below figure. The given data include $f = f_c = 0.08$, $d_c = 40\,mm$, $F = 6.4\,kN$ per screw.

d_c → mean collar diameter

a) Find the thread depth, thread width, pitch diameter, minor diameter, and lead?

b) Find the torque required to raise and lower the load?

c) Find the efficiency during lifting the load?

d) Find the body stresses, torsional and compressive?

→ From the fig. shown previous for the square thread

a- mean diameter $(d_m) = d - (P/2) = 32 - \frac{4}{2} = 30\,mm$ (d: major)

thread depth = thread width $= \frac{\text{Pitch}}{2} = \frac{4}{2} = 2\,mm$

minor diameter $(d_r) = d - p = 32 - 4 = 28\,mm$

Lead $(l) = np = 2(4) = 8\,mm$

/ /

b- the torque required to turn the screw against the load is :

$$T_R = \frac{Fd_m}{2}\left(\frac{L+\pi f d_m}{\pi d_m - fL}\right) + \frac{F f_c d_c}{2} \quad T_c$$

$$T_R = \frac{6.4(30)}{2}\left(\frac{8+\pi(0.08)(30)}{\pi(30)-0.08(8)}\right) + \frac{6.4(0.08)(40)}{2}$$

$$= 15.94 + 10.24 = 26.18 \text{ N}$$

the load-lowering load torque is :

$$T_L = \frac{Fd_m}{2}\left(\frac{\pi f d_m - L}{\pi d_m + fL}\right) + \frac{F f_c d_c}{2}$$

$$= \frac{6.4(30)}{2}\left(\frac{\pi(0.08)(30)-8}{30\pi+0.08(8)}\right) + \frac{6.4(0.08)(40)}{2}$$

$$= -0.466 + 10.24$$

$$= 9.77 \text{ N.m}$$

indicates that the screw alone isn't self locking and would rotate under the action of the load except for the fact that the collar friction is present and must be overcome, too. Thus, the torque required to rotate the screw "with" the load < torque required to overcome collar friction alone.

c- The overall efficiency in raising the load is:

$$e = \frac{FL}{2\pi T_R} = \frac{6.4(8)}{2\pi(26.18)} = 0.311$$

d- The body shear stress (τ) due to torsional moment T_R at the outside of the screw body is:

$$= \frac{16 T_R}{\pi d_r^3} = \frac{16 * 26.18 * 10^3}{\pi (28)^3} = 6.07 \text{ MPa}$$

$$\sigma_{\substack{axial\\normal}} = \frac{-4F}{\pi d_r^2} = \frac{-4(6.4)(10^3)}{\pi(28^2)} = -10.39 \text{ MPa}$$

Bearing stress Calculation :-

Bearing, $\sigma_B = \frac{-2F}{\pi d_m n_t P}$, n_t : no. of engaged threads

bending, $\sigma_b = \frac{M}{Z} = \frac{6F}{\pi d_r n_t P}$ → the bending stress at the root of the thread

$$M = \frac{FP}{4}$$

$$Z = \frac{\pi}{24} d_r n_t P^2$$

The transverse τ at the centre of the root of the thread due to load F is :-

$$\tau = \frac{3V}{2A} = \frac{3F}{2\left(\frac{\pi d_r n_t P}{2}\right)}$$

$$\boxed{\tau = \frac{3F}{\pi d_r n_t P}}$$

and at the top of the root it's zero.

* The screw thread form is complicated from an analysis viewpoint.
A power screw lifting a load is in compression and its thread pitch is shortened by elastic deformation. Its engaging nut is in tension and its thread pitch is lengthened.
The engaged threads can't share the load equally

* at $F \Rightarrow 0.38F$ (the first engaged thread)
$n_t = 1$
will give the largest level of stresses in the threa nut combination.

Ex(8.1) تكملة

e) Find the bearing stress?

→ σ_B with one thread carrying 0.38F

$$\sigma_B = \frac{-2(0.38F)}{\pi d_m (1) P} = \frac{-2(0.38)(6.4 \times 10^3)}{\pi (30)(1)(4)}$$

$$\sigma_B = -12.9 \text{ MPa}$$

f) Find the thread bending stress at the root of the thread?

$$\sigma_b = \frac{6(0.38F)}{\pi d_r (1) P} = \frac{6(0.38)(6.4 \times 10^3)}{\pi (28)(1)(4)}$$

$$\sigma_b = 41.5 \text{ MPa}$$

F for $n_t = 1 \rightarrow 0.38F$

$n_t = 2 \rightarrow 0.25F$

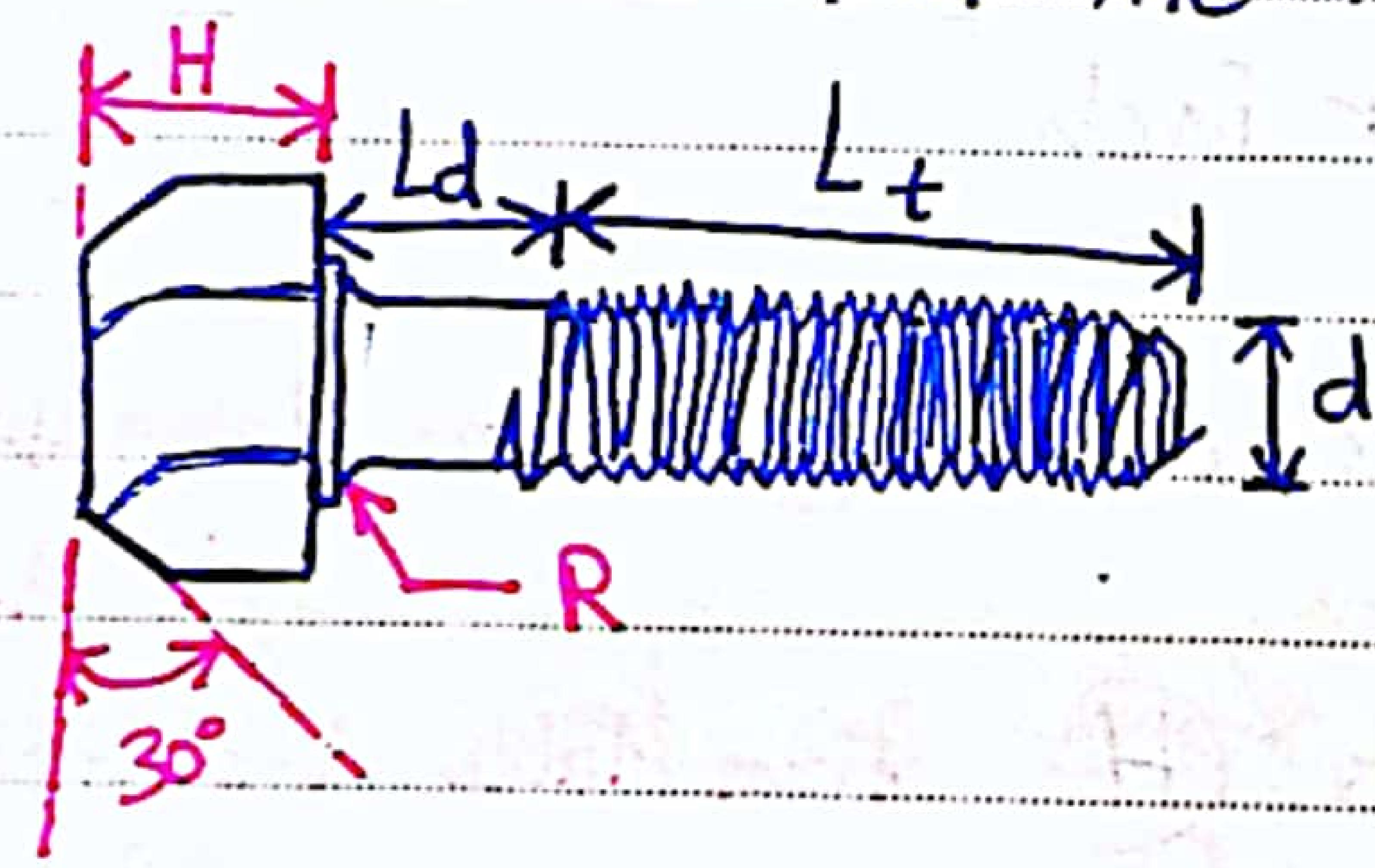
$n_t = 3 \rightarrow 0.18F$

8.3 Threaded Fasteners

The purpose of a bolt is to clamp 2 or more parts together. The clamping load stretches or elongates the bolt; the load is obtained by twisting the nut until the bolt has elongated almost to the elastic limit.

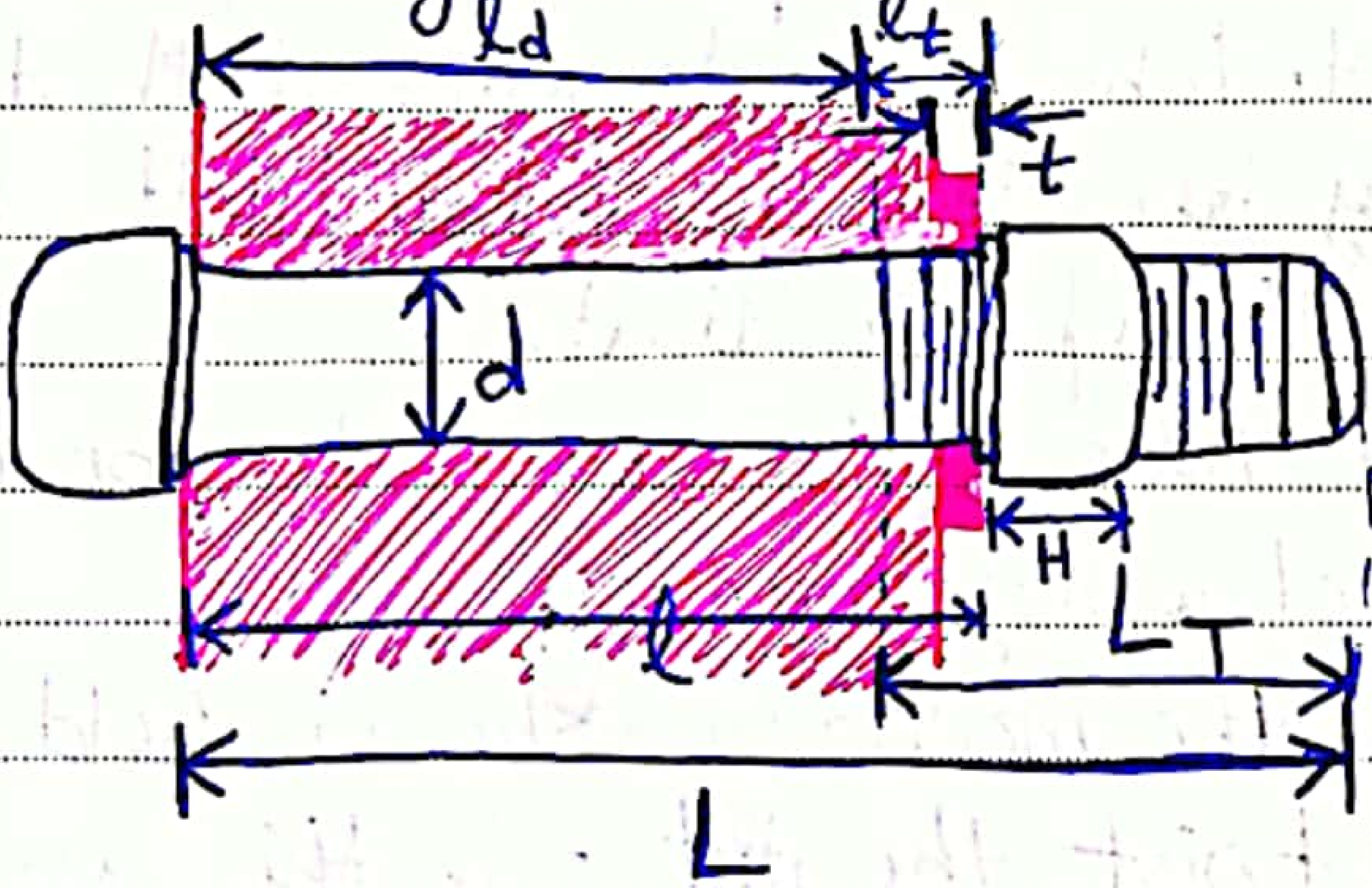
If the nut doesn't loosen, this bolt tension remains as the preload or clamping force.

When tightening, the mechanic should hold the bolt head stationary and twist the nut; in this way the bolt shank will not feel the thread-friction torque.



8.4 Joints _ Fastener Stiffness

- For finding fastener stiffness →



[from table 8-7 P.426]

⇒ Fig(a)

- Given fastener diameter d and pitch p in [mm] or no. of threads per inch

  - Washer Thickness : t from table A-32 (P.1056) or A-33 (P.1057)
  - Nut Thickness : H From table A-31 (P.1055)
  - Fastener stiffness : $k_b = \dfrac{A_d A_t E}{A_d l_t + A_t l_d}$

where A_t : area of threaded portion → from table 8-1, 8-2

A_d : area of unthreaded portion

$$\boxed{A_d = \frac{\pi}{4} d^2}$$

موجودين بـ table(8-7)

- Length of unthreaded portion in grip (l_d)

$$l_d = L - L_T$$

- Length of threaded portion in grip (l_t)

$$l_t = l - l_d$$

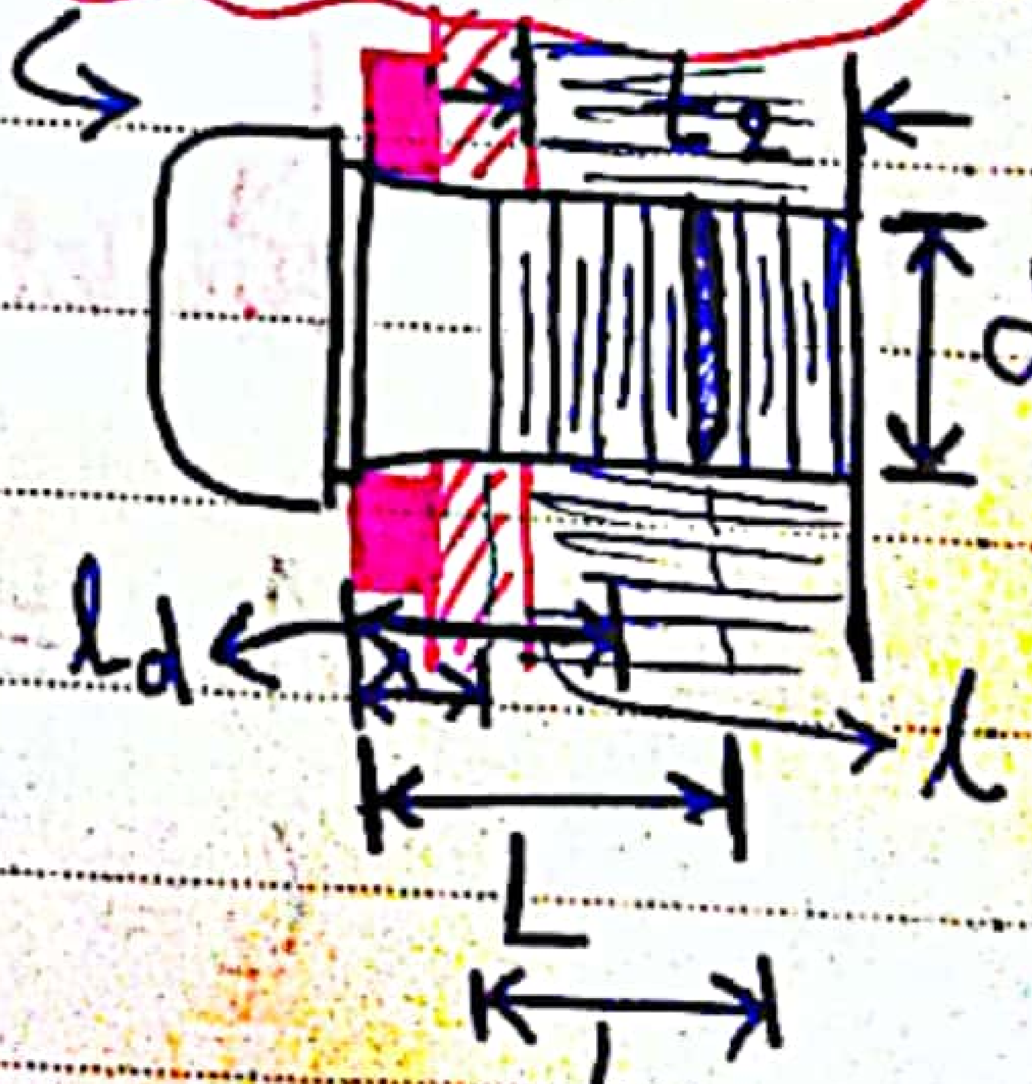
- Threaded Length (L_T)

$$L_T = \begin{cases} 2d + 6\,mm & ; L \leq 125\,mm,\ d \leq 48\,mm \\ 2d + 12\,mm & ; 125 < L \leq 200\,mm \\ 2d + 25\,mm & ; L > 200\,mm \end{cases}$$

- Fastener length (L)

Fig(a) $L > l + H$ or Fig(b) $L > h + 1.5d$

fig.(b) ⇒



- Grip Length (l)

fig(a) ⇒ $l \equiv$ thickness of all material squeezed between face of bolt and face of nut.

fig(b) ⇒ $l = \begin{cases} h + t_2/2 & ; t_2 < d \\ h + d/2 & ; t_2 \geq d \end{cases}$

/ /

- The stiffness of the portion of a bolt or screw within the clamped zone will generally consist of 2 parts, that of the unthreaded shank portion and that of the threaded portion.

Thus, the stiffness constant of the bolt is equivalent to the stiffnesses of 2 springs in series.

$$\frac{1}{k} = \frac{1}{k_1} + \frac{1}{k_2}$$

$$\boxed{k = \frac{k_1 k_2}{k_1 + k_2}}$$

- The spring rates of the threaded and unthreaded portions of the bolt in the clamped zone are:

$$\left[k_t = \frac{A_t E}{l_t}\right] \quad \& \quad \left[k_d = \frac{A_d E}{l_d}\right]$$

threaded unthreaded

where; $A_t \equiv$ tensile stress Area

$l_t \equiv$ length of threaded portion of grip

$A_d \equiv$ major diameter area of fastener

$l_d \equiv$ length of unthreaded portion in grip so, $k_b = \dfrac{A_d A_t E}{A_d l_t + A_t l_d}$

$k_b \equiv$ estimated effective stiffness of the bolt or cap screw in the clamped zone.

Problems

8.11) An M14 x 2 hex-head bolt with a nut is used to clamp together two 15 mm steel plates.
diameter

a- determine a suitable length for the bolt, rounded up to the nearest 5mm

b- determine the bolt stiffness

→ (a) from table A-31 → $H = 12.8$

$$L = l + H$$

$$= 2(15) + 12.8 = 42.8 \text{ mm} \approx 45 \text{ mm}$$

لأنه السؤال ماكلي

مقرب لعند أقرب 5mm

(b) $k_b = \dfrac{A_d A_t E}{A_d l_t + A_t l_d}$

$$l_d = L - L_T = 45 - 34 = 11 \text{ mm}$$

$$l_t = l - l_d = 2(15) - 11 = 19 \text{ mm}$$

$$A_d = \frac{\pi}{4} d^2 = \frac{\pi}{4}(14^2) = 153.9 \text{ mm}^2$$

/ /

$A_t = 115\ mm^2 \rightarrow$ From table 8-1

$E = 207\ GPa \rightarrow$ From table 8-8

$$k_b = \frac{(153.9)(115)(207)}{(153.9)(19) + (115)(11)}$$

$$= 874.6\ MN/m$$

8.12) Repeat the previous question with the addition of one 14R metric plain washer under the nut?

$\rightarrow$ From table A-31 $\rightarrow H = 12.8\ mm$

$$L = l + H + h \quad \leftarrow \text{nut thickness}$$
$$= 2(15) + 12.8 + 3.5$$
$$= 46.3\ mm$$
$$\approx 50\ mm$$

$L_T = 2(14) + 6 = 34\ mm$, $L \leq 125$

$50 \leq 125$

$2d + 5mm$

$l_d = L - L_T = 50 - 34$
$= 16\ mm$

$l_t = l - l_d = 30 - 16 = 14\ mm$

ALADIB

$50 \Rightarrow \boxed{k_b = 808.2\ MN/m}$