Introduction to Mechanical Engineering Design

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Design concept

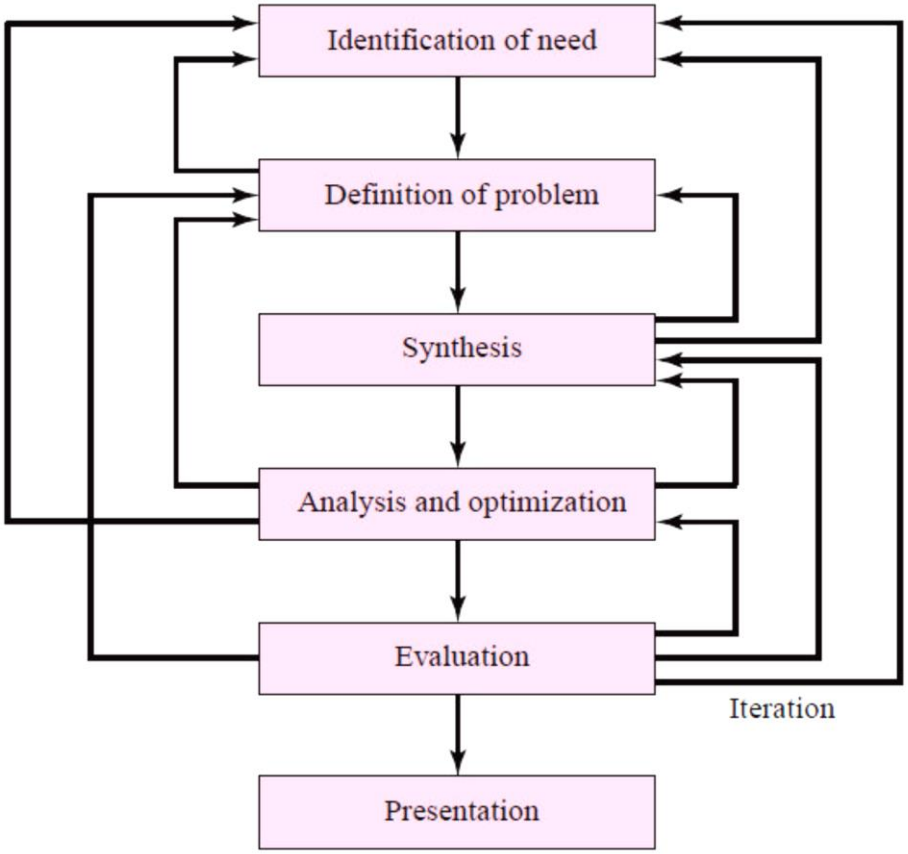
- *To **design** is, to meet a specific need, by formulating it into a problem and finding a solution to it.*
- *The result of the design process has to be, functional, usable, safe, reliable, but also manufactural, marketable and competitive.*
- ***Design** is an innovative, decision-making and highly iterative process.*

Other names for design

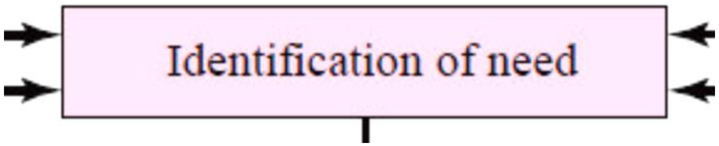
- *Mechanical Engineering Design*
- *Machine Design*
- *Machine-element Design*
- *Machine-component Design*
- *Systems Design*
- *Fluid-power Design*

Design phases

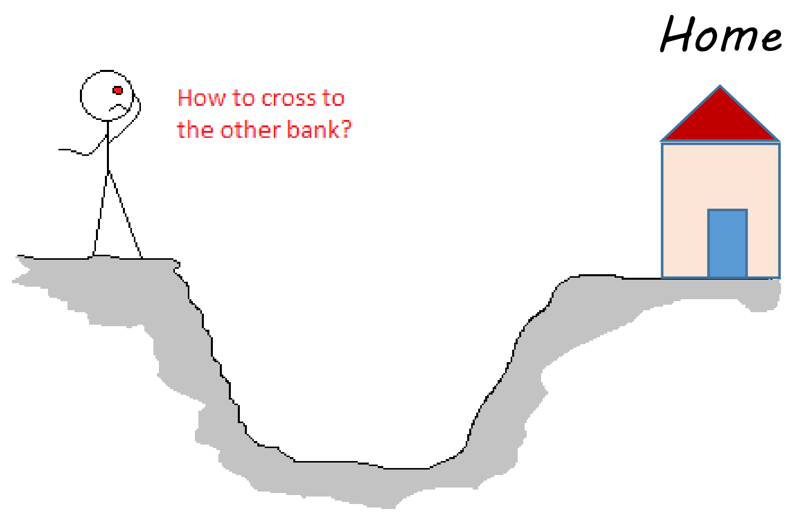
- Recognition, phrasing, vague, uneasiness, sense something not right
- More specific, characteristics, dimensions, limitations, constraints
- Concept, schemes
- Calculations, mathematical models, simulation, revise, improve, optimize, discard, iterate
- Prototype, testing, laboratory, proof, manufacturability, economics
- Communication, sales, marketing



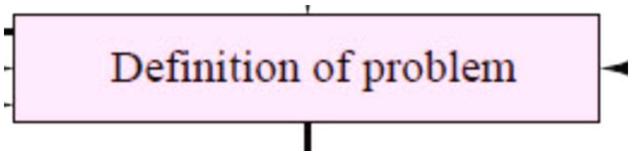
Design example



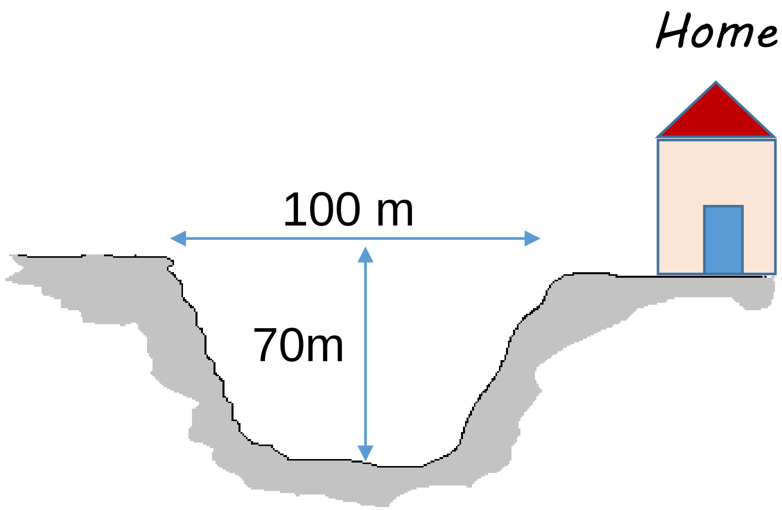
- *Recognition:* Deep valley, Difficult terrain?
- *Phrasing:* I need to cross to the other bank!!
- *Vague:* Do both banks on the same level?
- *Uneasiness:* time, effort, and money.

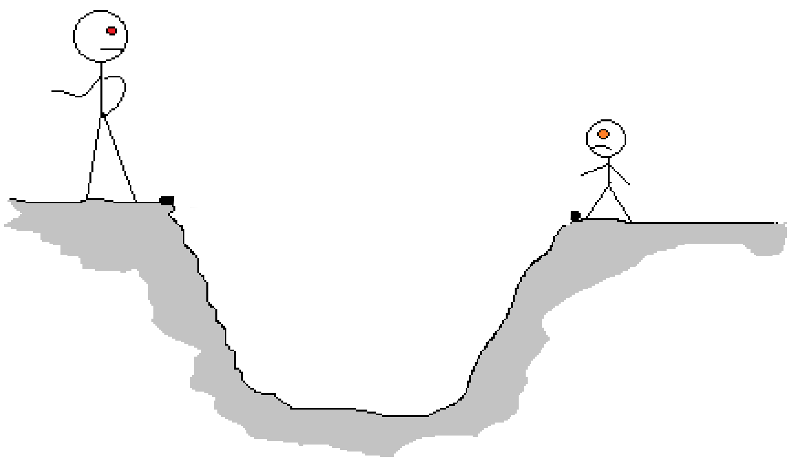


Design example

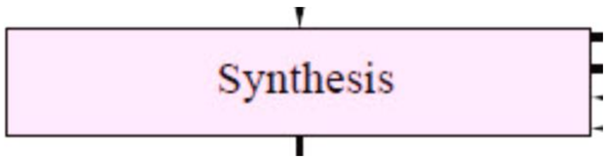


- *More specific: I need to do something to move to the other bank in short time and less effort.?*
- *Characteristics:*
- *Dimensions: 100 m long,1m wide. Thickness?*
- *Limitations: max. load 10000 kg.*
- *Constraints: Cost, safety, environmental...*

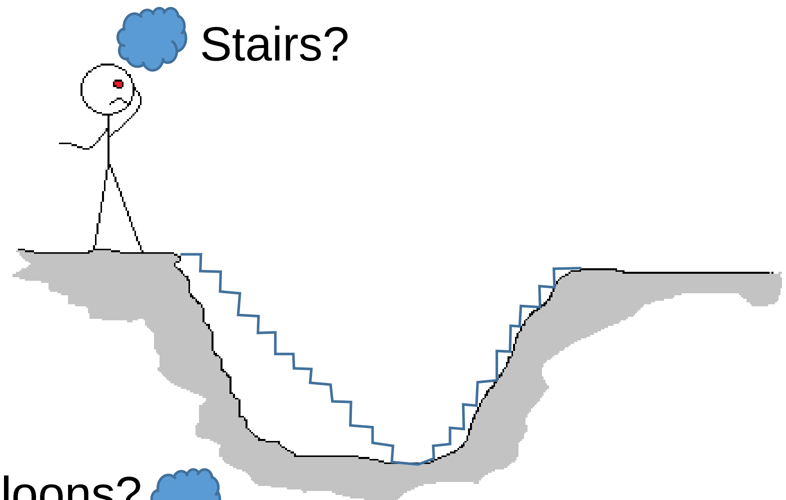


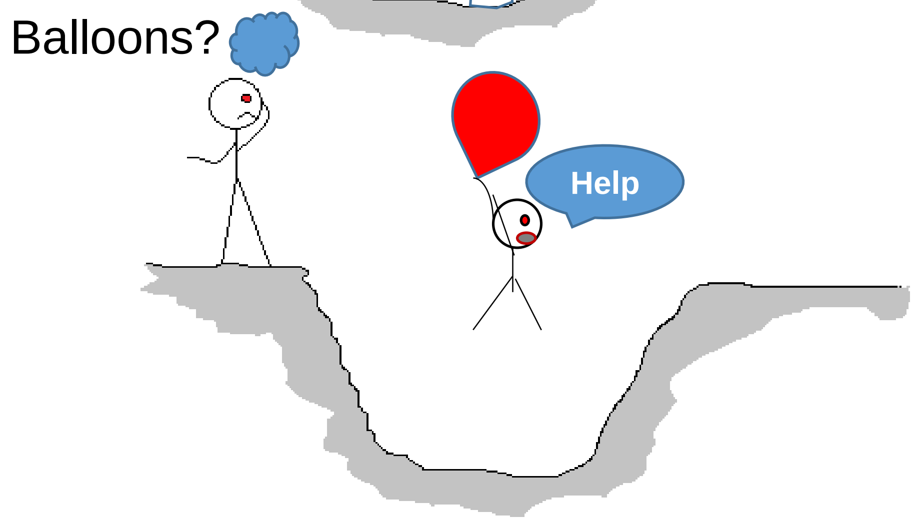


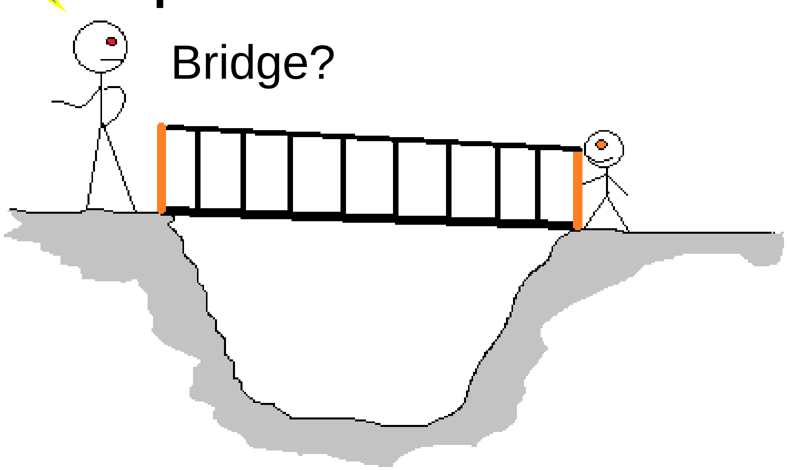
Design example



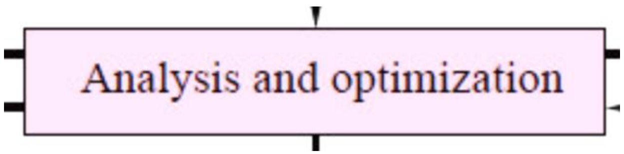
- *Concept: Bridge for people.*
- *Schemes:* timeline, financial support plan…



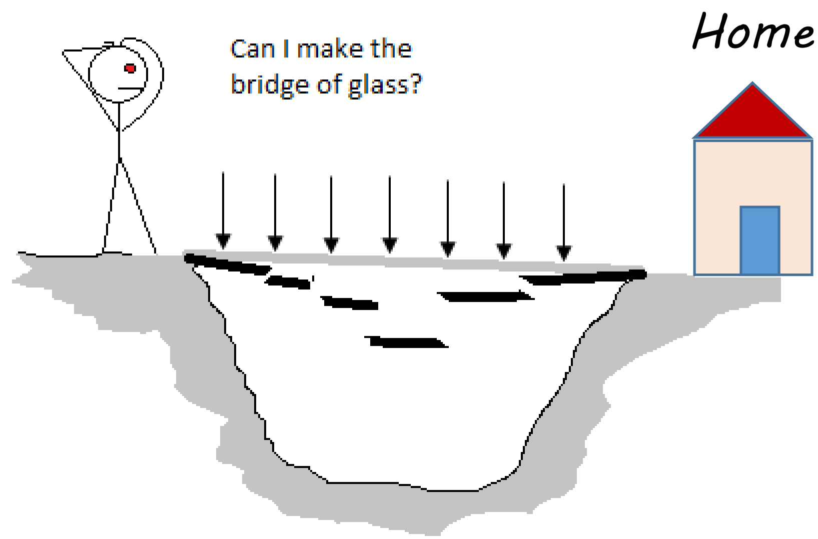




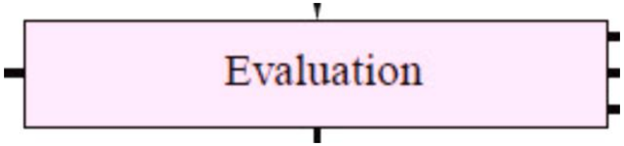
Design example



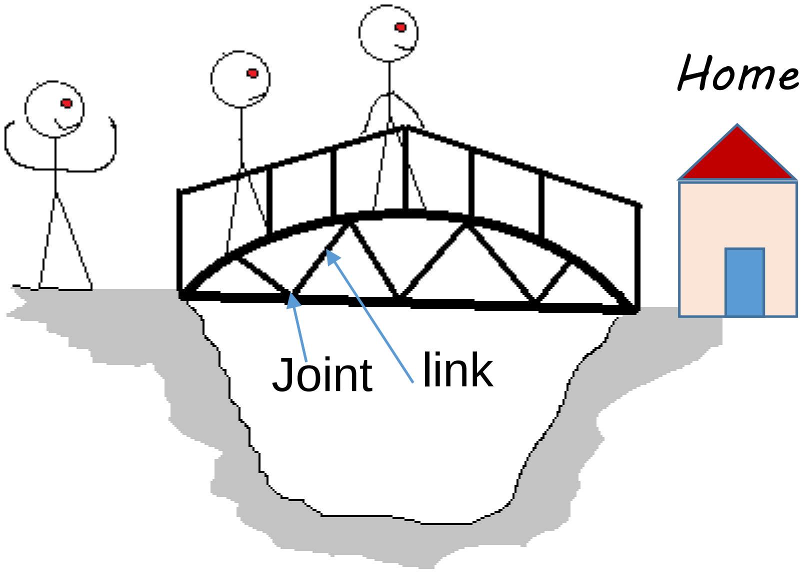
- *Calculations: Axial loading, Bending, Joints, Welding….*
- *Mathematical models,*
- *Simulation: Matlab, ANSYS*
- *Revise: Use another material, Welding or joints…*
- *Improve: Welding area, Number of joints,*
- *Optimize: Minimize used material…*
- *Discard*
- *Iterate*



Design example



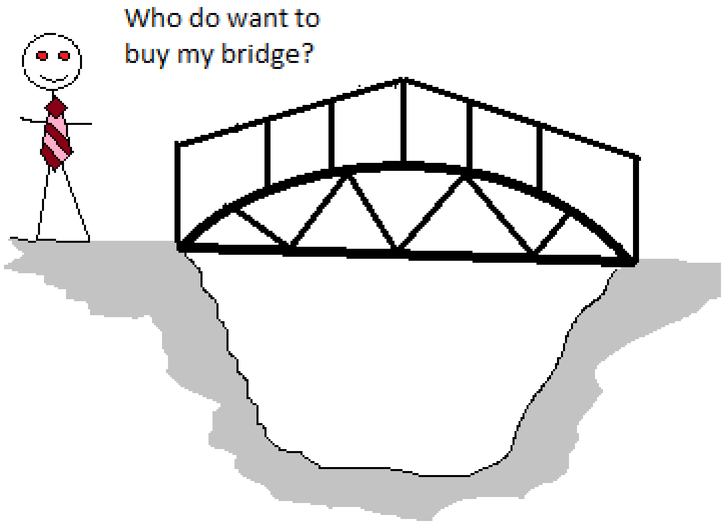
- Prototype: Create the bridge.
- Testing: Make many experiments.
- Laboratory: Check samples for more validations.
- Proof:
- Manufacturability: Available material, Tools, mass production.
- Economics: Total cost, cost estimate



Design example

Presentation

- Communication: companies, universities.
- Sales:
- Marketing:



Design considerations

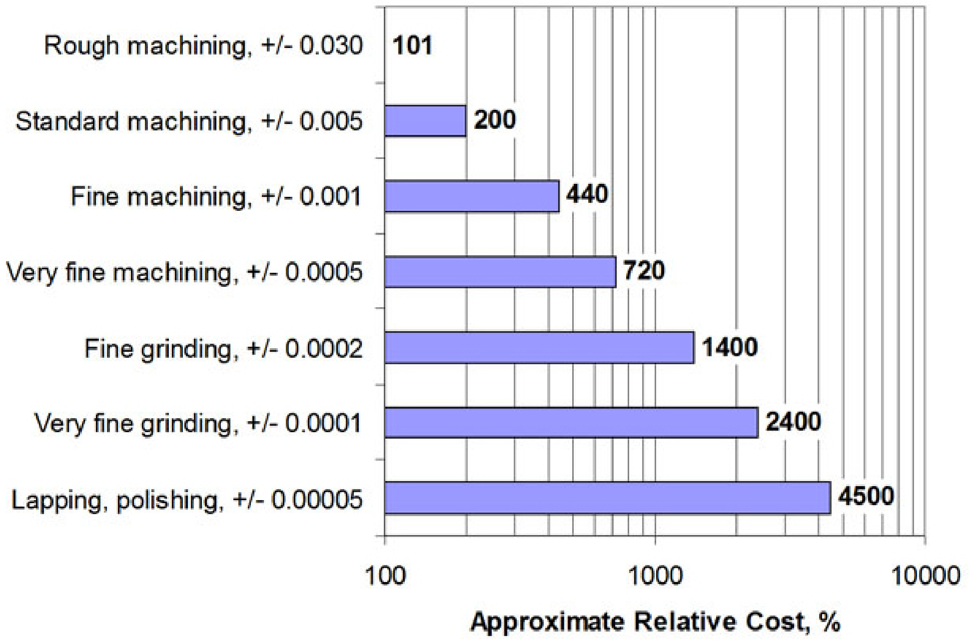
- *size*
- *weight*
- *volume*
- *surface*
- *manufacturability*
- *cost*
- *life*
- *wear*
- *corrosion*
- *functionality*
- *utility*
- *reliability*
- *maintenance*
- *strength / stress*
- *distortion / deflection / stiffness*
- *safety*
- *thermal properties*
- *friction*
- *lubrication*
- *noise*
- *control*
- *shape*
- *styling*
- *marketability*
- *liability*
- *remanufacturing / resource recovery*

Design tools and resources

- **Computer-Aided Design (CAD)**
- *Aries, AutoCAD, CadKey, I-Deas, Unigraphics, CATIA,*
- *SolidWorks, ProEngineer, SolidEdge*
- **Computer-Aided Engineering (CAE)**
- *Algor, ANSYS, ABAQUS, NASTRAN, FLUENT, CFD, FIDAP,*
- *ADAMS, Working Model, AutoDYN*
- **Technical Information**
- *internet, societies, government, libraries, vendors*

Economics

- Standard Sizes (standards and codes, ISO, DIN, ASME)
- Large Tolerances
- Breakeven Points
- Cost Estimates



Safety

- Design factor

the *design factor* is recalculated after rounding to standardized sizes, dimensions and materials and is then called *factor of safety*.

$$n_d = \frac{\text{loss-of-function parameter}}{\text{maximum allowable parameter}}$$

Factor of safety example

A solid circular rod of diameter d undergoes a bending moment $M = 1000$ lbf · in inducing a stress $\sigma = 16M/(\pi d^3)$. Using a material strength of 25 kpsi and a *design factor* of 2.5, determine the minimum diameter of the rod. Using Table A–17 select a preferred fractional diameter and determine the resulting *factor of safety*.

Fraction of Inches
$\frac{1}{64}, \frac{1}{32}, \frac{1}{16}, \frac{3}{32}, \frac{1}{8}, \frac{5}{32}, \frac{3}{16}, \frac{1}{4}, \frac{5}{16}, \frac{3}{8}, \frac{7}{16}, \frac{1}{2}, \frac{9}{16}, \frac{5}{8}, \frac{11}{16}, \frac{3}{4}, \frac{7}{8}, 1, 1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}, 2, 2\frac{1}{4}, 2\frac{1}{2}, 2\frac{3}{4}, 3, 3\frac{1}{4}, 3\frac{1}{2}, 3\frac{3}{4}, 4, 4\frac{1}{4}, 4\frac{1}{2}, 4\frac{3}{4}, 5, 5\frac{1}{4}, 5\frac{1}{2}, 5\frac{3}{4}, 6, 6\frac{1}{2}, 7, 7\frac{1}{2}, 8, 8\frac{1}{2}, 9, 9\frac{1}{2}, 10, 10\frac{1}{2}, 11, 11\frac{1}{2}, 12, 12\frac{1}{2}, 13, 13\frac{1}{2}, 14, 14\frac{1}{2}, 15, 15\frac{1}{2}, 16, 16\frac{1}{2}, 17, 17\frac{1}{2}, 18, 18\frac{1}{2}, 19, 19\frac{1}{2}, 20$

step 1: using as a *loss-of-function parameter*, *the material strength* **S** *and the given* *design factor* **nd**, *calculate the maximum allowable stress* **σ**
step 2: solve the stress equation for the diameter **d**
step 3: round the calculated diameter **d** *to the closest larger diameter from the standard*
step 4: calculated the stress **σ**, *based on the standardized diameter*
step 5: use the new calculated stress **σ** *and the given material strength* **S**, *to calculate thenew* *design factor*, *which is now called the* *factor of safety*

Reliability and liability

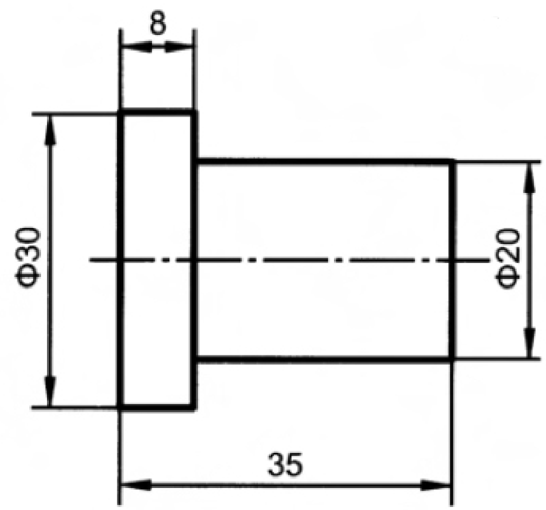
- *The reliability method of design is one in which we obtain the distribution of stresses and the distribution of strengths and the relate these two in order to achieve an acceptable success rate*

$$R = 1 - p_f$$

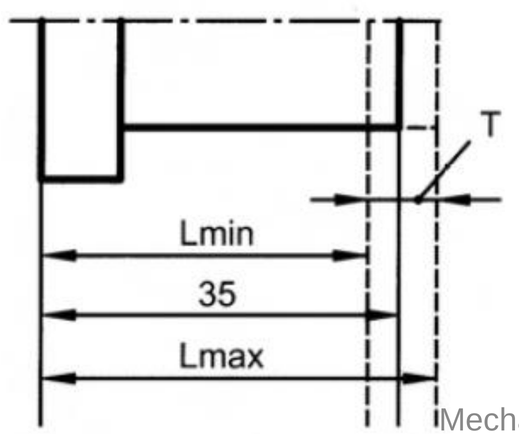
- *where* *pf* *is the probability of failure*

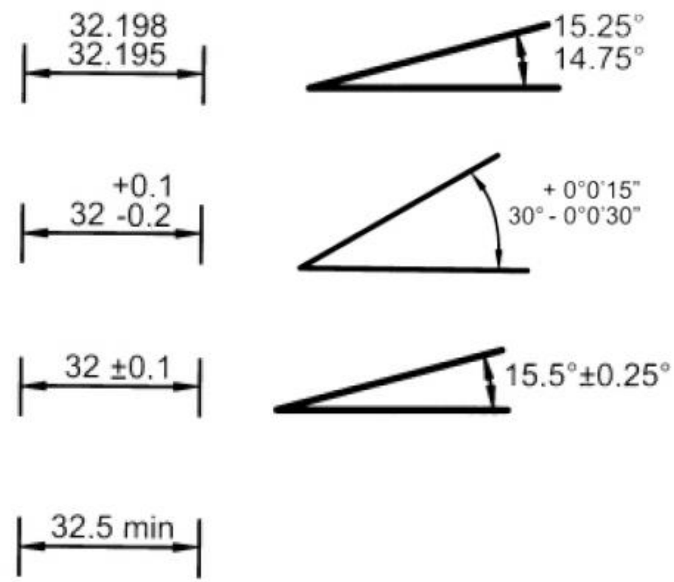
- *liability is the* *promise of no-failure* *(under certain conditions) by the manufacturer, frequently overstated by sales and marketing*

Dimensions and tolerances



T = Lmax-Lmin





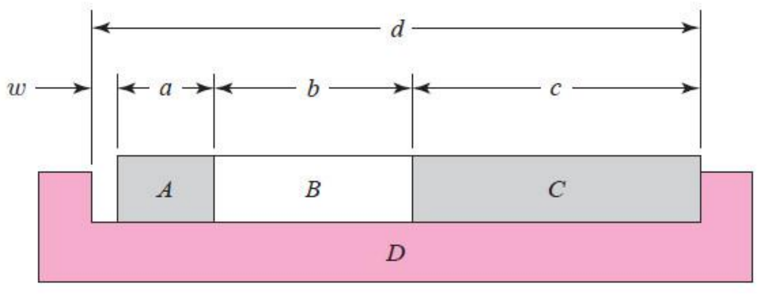
- *nominal size*
- *limits*
- *tolerance*
- *clearance*
- *interference*

Tolerance example

Three blocks A, B, and C and a grooved block D have dimensions a, b, c, and d as follows:

$$a = 1.500 \pm 0.001 \qquad b = 2.000 \pm 0.003$$

$$c = 3.000 \pm 0.004 \qquad d = 6.520 \pm 0.010$$



- *the nominal clearance* **w** *is, the difference between nominal* **d** *and the sum of nominal* **a, b** *and* **c**
- *the clearance* **w** *maximum limit is, the difference between the maximum limit of* **d** *and the sum of the minimum limits of* **a, b** *and* **c**
- *the clearance* **w** *minimum limit is, the minimum limit of* **d** *and the sum of the maximum limits of* **a, b** *and* **c**

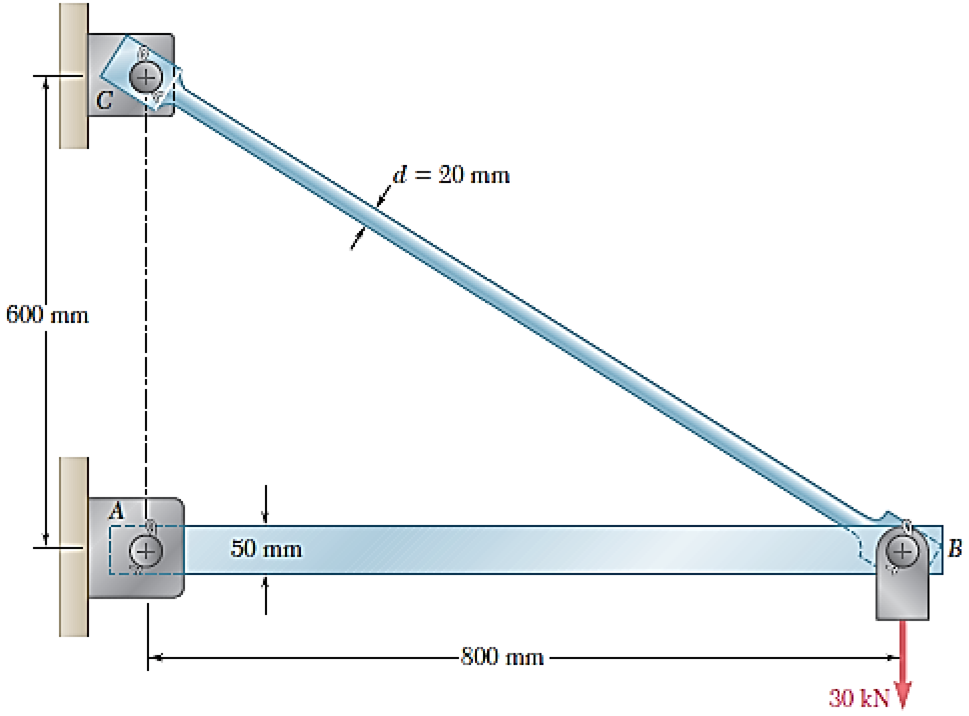
CH1: Concept of stress

Mechanical design
Mechatronics Engineering
Hashemite University
Dr. Mohammad Hayajneh

Structure under static load

- Consider the structure shown in figure which was designed to support a 30-kN load.
- Boom *AB* with a 30 x 50 mm rectangular cross section.
- Rod *BC* with a 20mm diameter circular cross section.
- The boom and the rod are connected by a pin at *B* and are supported by pins and brackets at *A* and *C*, respectively.

The goal: determining the forces on each member in the structure.



Structure under static load: Forces

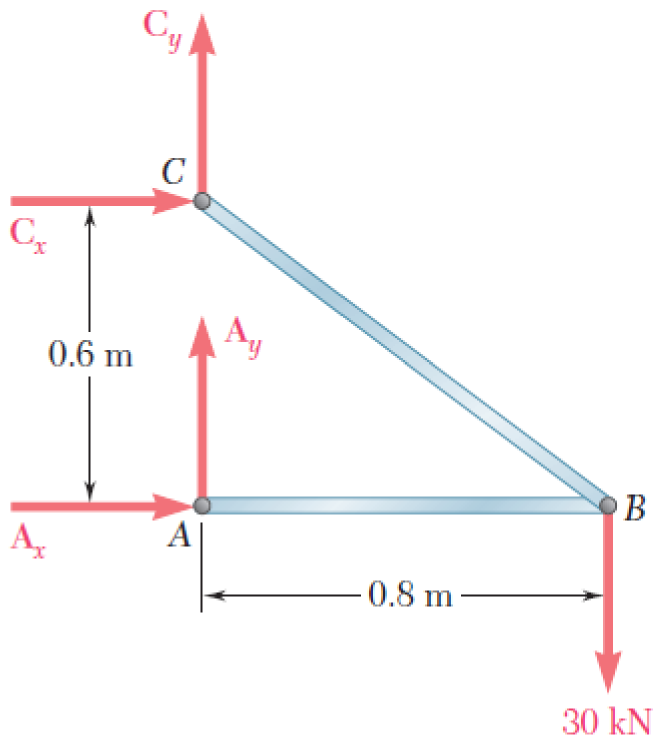
First step should be to draw a free-body diagram of the structure without its supports at A and B.

$$+\circlearrowleft \Sigma M_C = 0: \qquad A_x(0.6\text{ m}) - (30\text{ kN})(0.8\text{ m}) = 0$$
$$A_x = +40\text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0: \qquad A_x + C_x = 0$$
$$C_x = -A_x \qquad C_x = -40\text{ kN}$$

$$+\uparrow \Sigma F_y = 0: \qquad A_y + C_y - 30\text{ kN} = 0$$
$$A_y + C_y = +30\text{ kN}$$

We have found two of the four unknowns



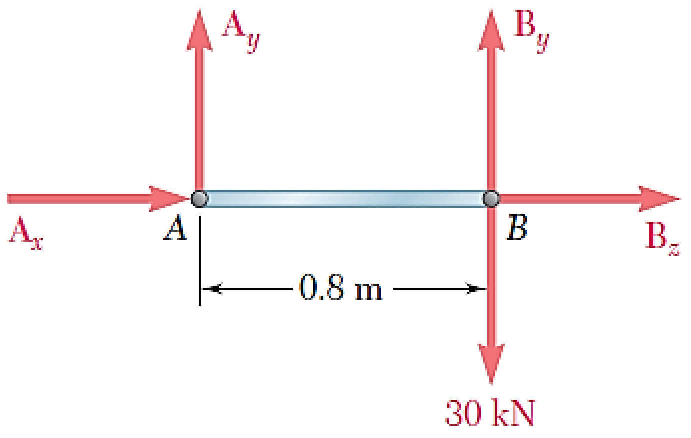
Structure under static load: Forces

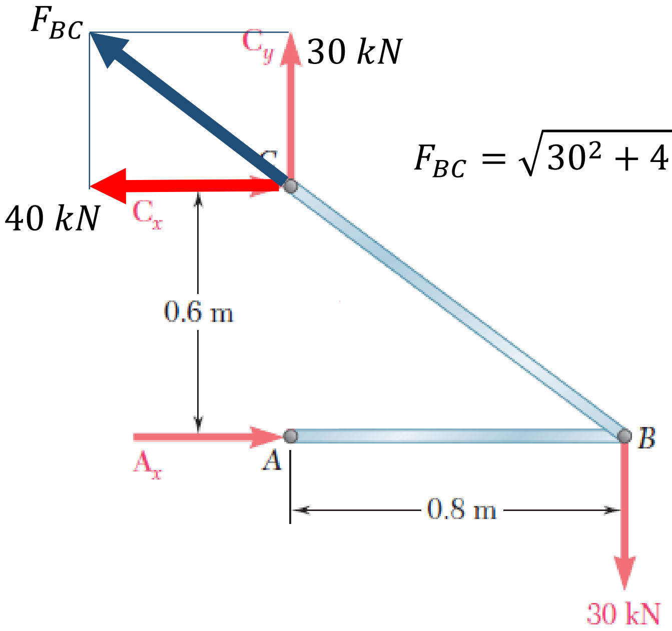
Considering the free body diagram of the boom AB, we write the following equilibrium equation:

$$+\circlearrowleft \Sigma M_B = 0: \qquad -A_y(0.8 \text{ m}) = 0 \qquad A_y = 0$$

Substituting $A_y = 0$ in $A_y + C_y = +30 \text{ kN}$ we have:

$$\mathbf{C}_y = 30 \text{ kN} \uparrow$$



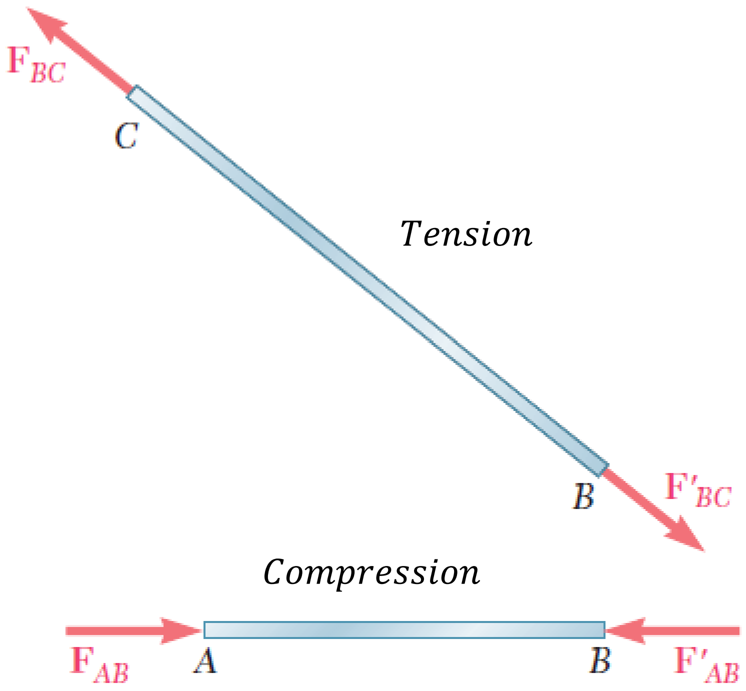


$$F_{BC} = \sqrt{30^2 + 40^2} = 50 \ kN$$

Structure under static load: Forces

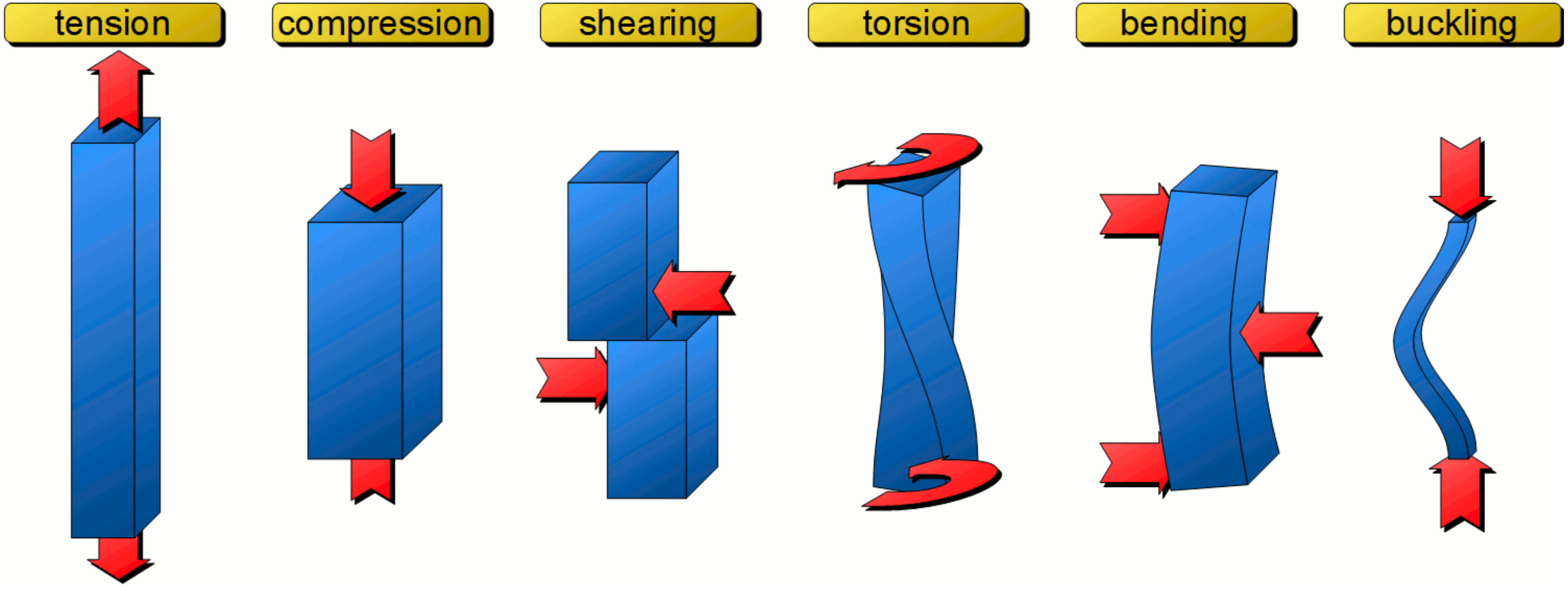
We call AB and BC are two-force members.

for a two-force member the lines of action of the resultants of the forces acting at each of the two points are equal and opposite and pass through both points.



Stresses

The force per unit area, or intensity of the forces distributed over a given section, is called the ***stress*** on that section



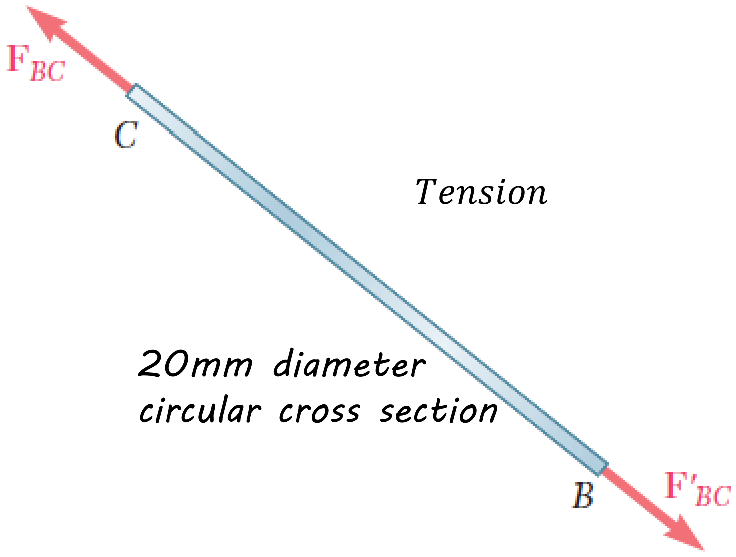
AXIAL LOADING; NORMAL STRESS

- We say that the rod BC is under *axial loading*
- Forces F_{BC} and F'_{BC} acting on its ends B and C are directed along the axis of the rod.
- The corresponding stress is described as a <u>normal stress</u>:

$$\sigma = \frac{P}{A}$$

P ← Axial force

A ← Cross sectional area



Example: For the rod BC calculate the <u>normal stress</u>:

$$F_{BC} = F'_{BC} = 50\ kN$$

Now, we calculate the area of circular cross section such that:

$$A = \frac{\pi}{4} d^2 = \frac{\pi}{4}(0.02)^2 = 0.314 \times 10^{-3} m^2$$

Tension

$$\sigma = \frac{50 \times 10^3\ N}{0.314 \times 10^{-3} m^2} = \boxed{159.24 \times 10^6\ Pa}$$

STRESS Units

the stress will be expressed in $\frac{N}{m^2}$. This unit is called a pascal (Pa). However, one finds that the pascal is an exceedingly small quantity and that, in practice, multiples of this unit must be used, namely, the kilopascal (kPa), the megapascal (MPa), and the gigapascal (GPa).
We have:

$$1 \text{ kPa} = 10^3 \text{ Pa} = 10^3 \text{ N/m}^2$$

$$1 \text{ MPa} = 10^6 \text{ Pa} = 10^6 \text{ N/m}^2$$

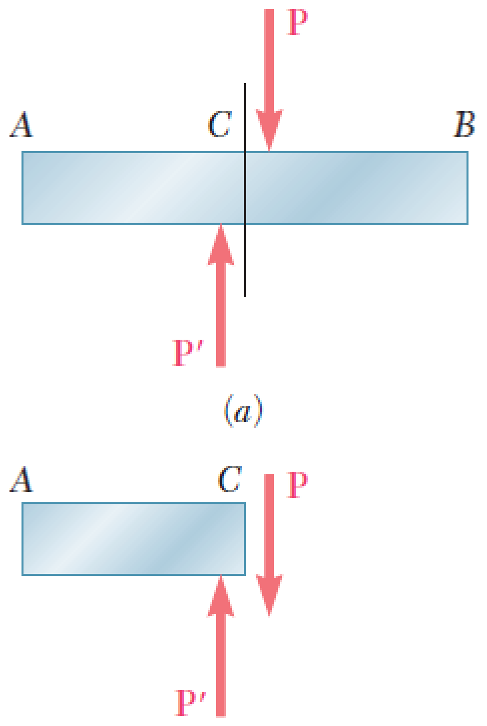
$$1 \text{ GPa} = 10^9 \text{ Pa} = 10^9 \text{ N/m}^2$$

When U.S. customary units are used, the force P is usually expressed in pounds (lb) or kilopounds (kip), and the cross-sectional area A in square inches (in^2). The stress will then be expressed in pounds per square inch (psi) or kilopounds per square inch (ksi).

SHEARING STRESS

A different type of stress is obtained when transverse forces P and P' are applied to a member AB. Passing a section at C between the points of application of the two forces we obtain the diagram of portion AC.

We conclude that internal forces must exist in the plane of the section, and that their resultant is equal to P. These elementary internal forces are called *shearing forces*, and the magnitude P of their resultant is the *shear* in the section. Dividing the shear P by the area A of the cross section, we obtain the *average shearing stress* in the section

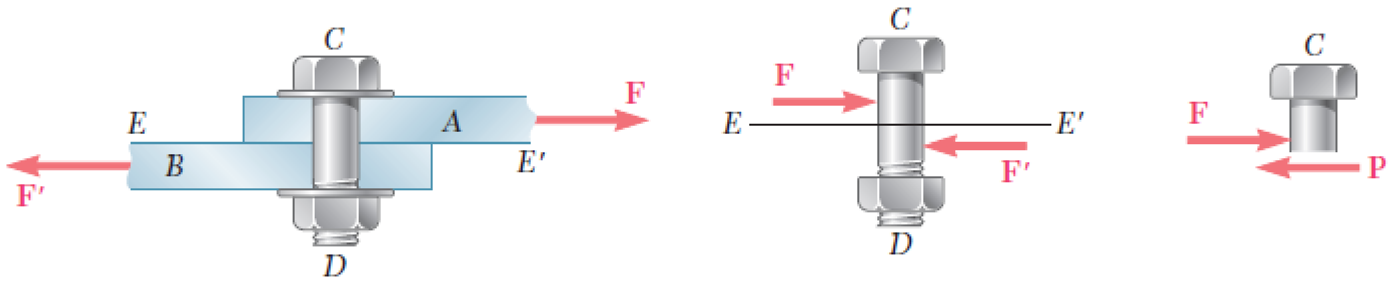


$$\tau = \frac{P}{A}$$

SHEARING STRESS

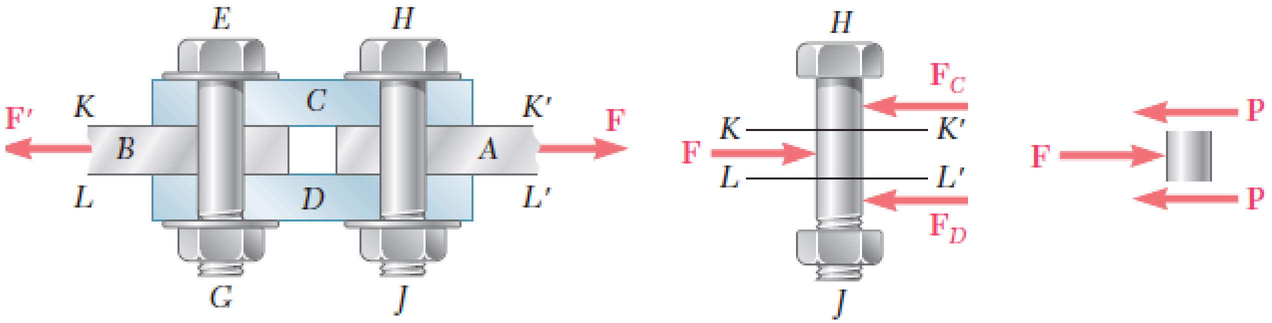
Shearing stresses are commonly found in bolts, pins, and rivets used to connect various structural members and machine components

1) Single shear

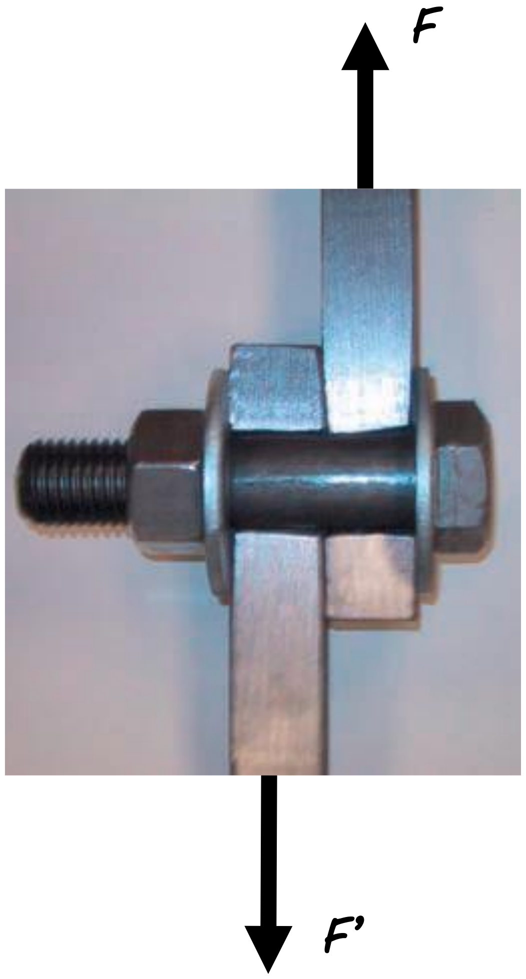


$$\tau_{ave} = \frac{P}{A} = \frac{F}{A}$$

2) Double shear



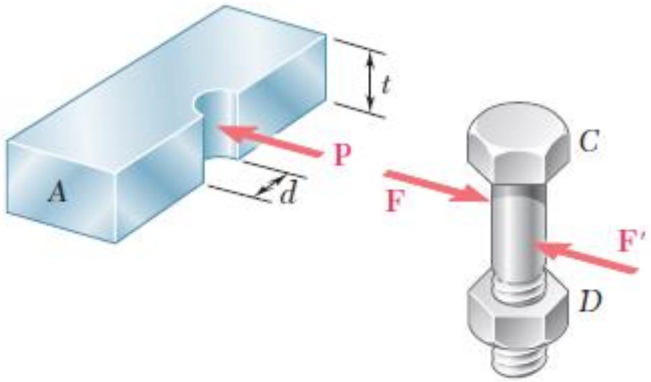
$$\tau_{ave} = \frac{P}{A} = \frac{F/2}{A} = \frac{F}{2A}$$



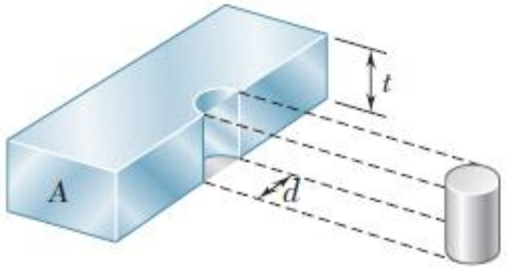
Bearing Stress

Bolts, pins, and rivets create stresses in the members they connect, along the *bearing surface*, or surface of contact

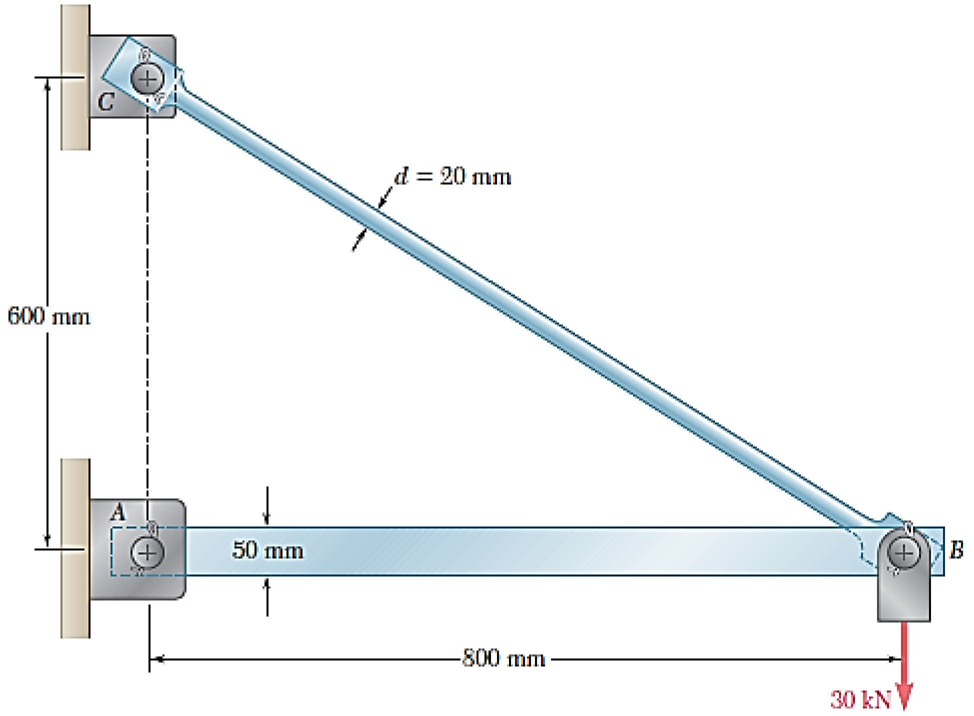
$$\sigma_b = \frac{P}{A} = \frac{p}{td}$$

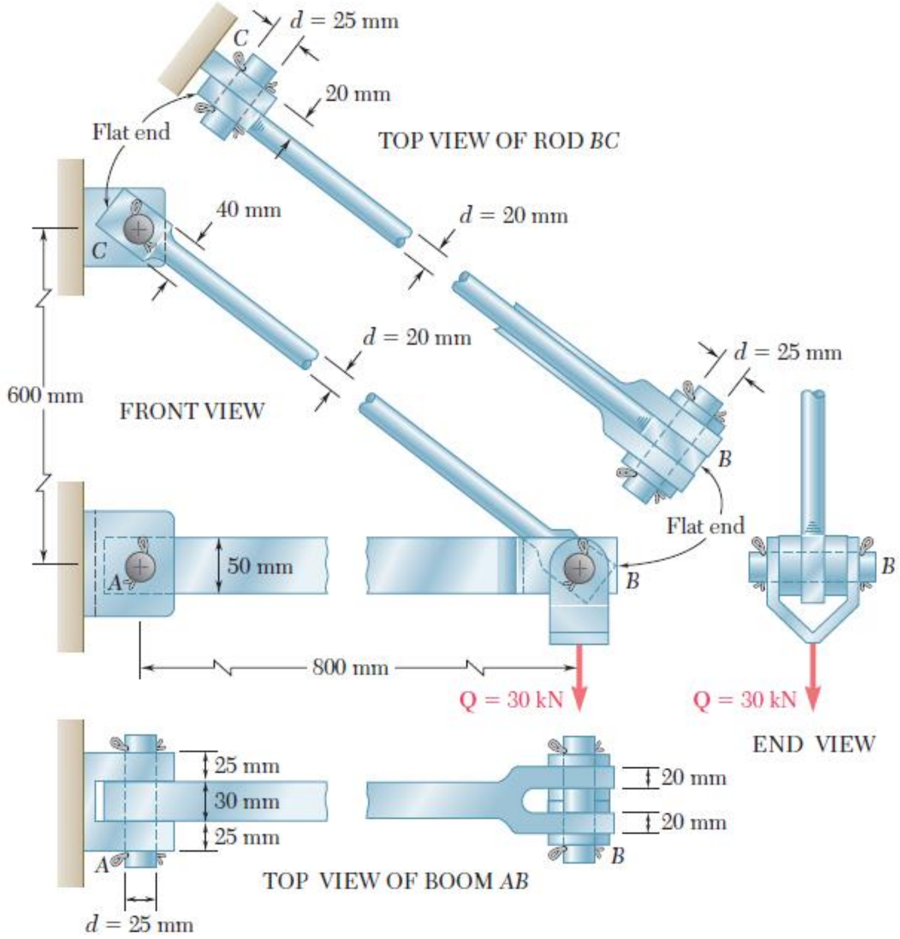


Exercise: If P= 30 KN, plate thickness equals to 20 mm, and bolt diameter equals to 10 mm
Calculate the bearing stress?



Application to the analysis and design of simple structures



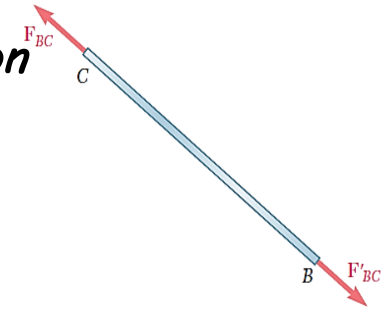


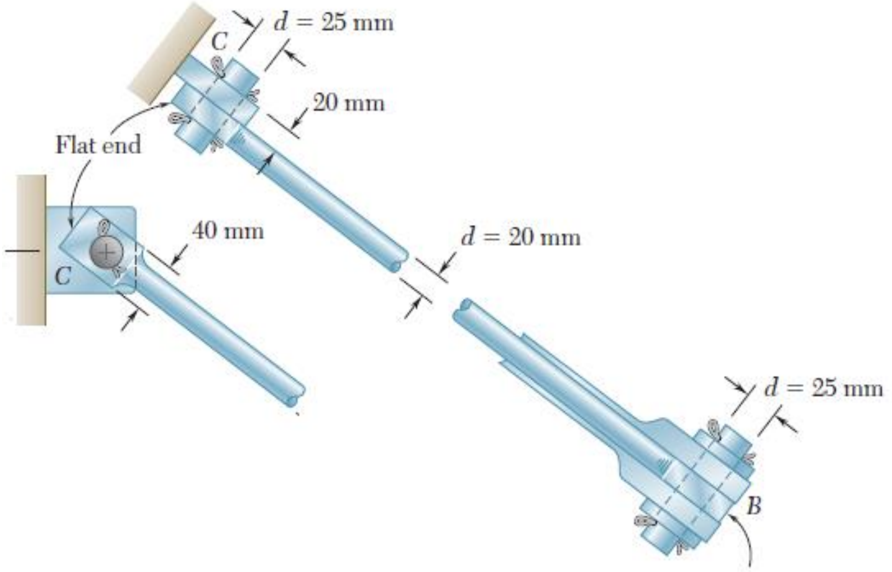
Application to the analysis and design of simple structures

1) Stresses in Rod BC

As we calculated previously, the force in rod BC is F_{BC} =50 kN (tension).

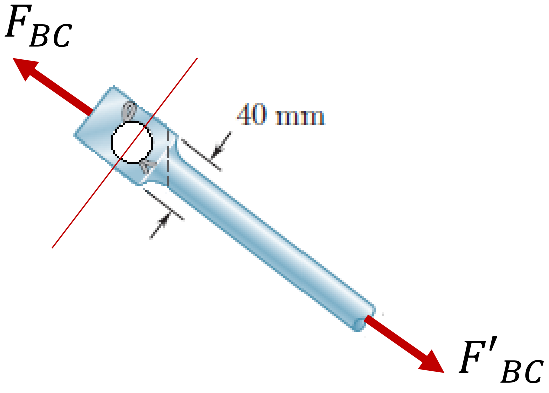
a) Normal stress in the circular cross section





$$\sigma = \frac{50 \times 10^{3}\ N}{\mathbf{0.314 \times 10^{-3} m^{2}}} = 159.24 \times 10^{6}\ Pa$$

b) Normal stress in the flat parts of the rod



$$(\sigma_{BC})_{end} = \frac{F}{A} = \frac{50\ kN}{(40-25)mm \times 20mm} =$$

$$\frac{50 \times 10^{3}\ N}{\mathbf{300 \times 10^{-6} m^{2}}} = 167\ MPa$$

Application to the analysis and design of simple structures

1) Stresses in Rod BC

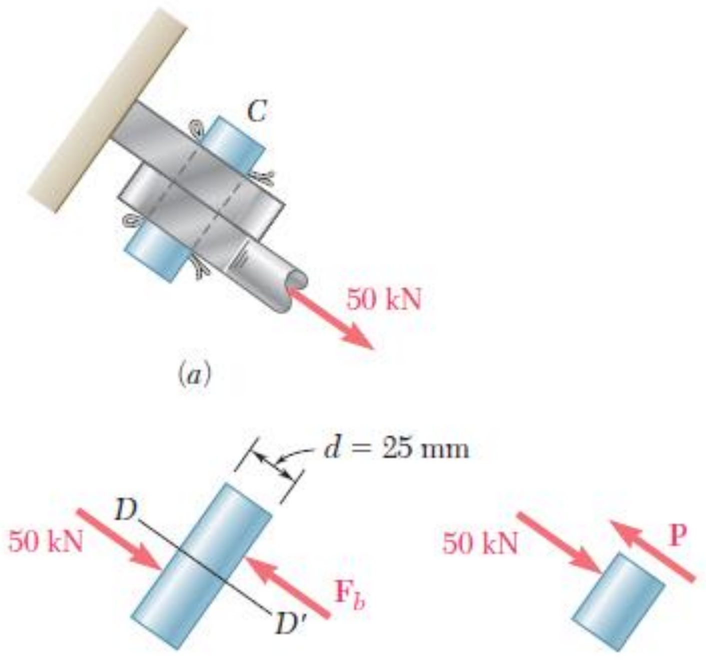
c) Shear stress on pin in joint C, Single Shear

$$A = \pi r^2 = \pi\left(\frac{25\text{ mm}}{2}\right)^2 = \pi(12.5 \times 10^{-3}\text{ m})^2 = 491 \times 10^{-6}\text{ m}^2$$

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{50 \times 10^3\text{ N}}{491 \times 10^{-6}\text{ m}^2} = 102\text{ MPa}$$

d) Bearing stress on link

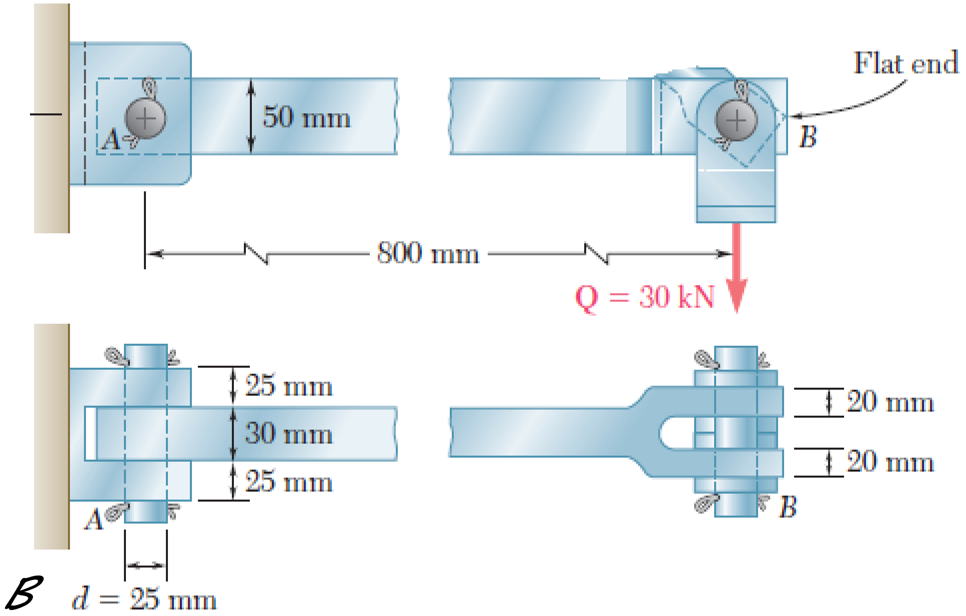
$$\sigma_b = \frac{P}{A} = \frac{50\ kN}{(20\ mm) \times (25 mm)} = ??$$



Application to the analysis and design of simple structures

2) Stresses in Boom AB

As we calculated previously, the force in AB is F_{AB} =40 kN (compression).



a) Normal stress in middle section

$$A = 30\,mm \times 50\,mm = 1.5 \times 10^{-3} m^2$$

$$\sigma_{AB} = -\frac{40 \times 10^3\ \text{N}}{1.5 \times 10^{-3}\ \text{m}^2} = -26.7 \times 10^6\ \text{Pa} = -26.7\ \text{MPa}$$

b) Normal stress at the end

Note that the sections of minimum area at A and B are not under stress, since the boom is in compression, and, therefore, *pushes* on the pins (instead of *pulling* on the pins as rod BC does).

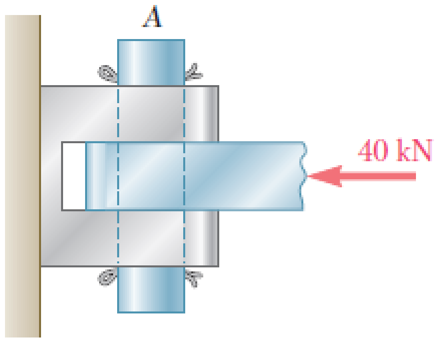
Application to the analysis and design of simple structures

1) Stresses in Boom AB

c) Shear stress on pin in joint A, Double Shear

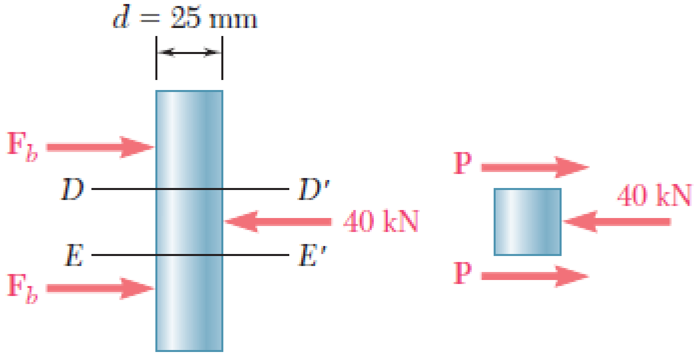
$$A = \pi r^2 = \pi\left(\frac{25\ \text{mm}}{2}\right)^2 = \pi(12.5 \times 10^{-3}\ \text{m})^2 = 491 \times 10^{-6}\ \text{m}^2$$

$$\tau_{\text{ave}} = \frac{P}{A} = \frac{20\ \text{kN}}{491 \times 10^{-6}\ \text{m}^2} = 40.7\ \text{MPa}$$

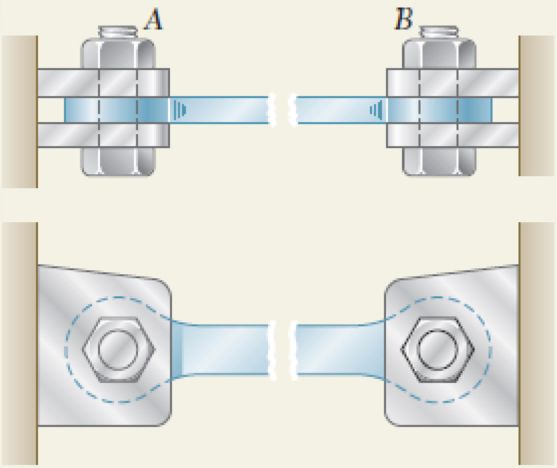


d) Bearing stress in joint A

$$\sigma_b = \frac{P}{A} = \frac{40\ kN}{(50\ mm) \times (25mm)} = 32\ MPa$$



SAMPLE PROBLEM 1.2



The steel tie bar shown is to be designed to carry a tension force of magnitude $P = 120$ kN when bolted between double brackets at A and B. The bar will be fabricated from 20-mm-thick plate stock. For the grade of steel to be used, the maximum allowable stresses are: $\sigma = 175$ MPa, $\tau = 100$ MPa, $\sigma_b = 350$ MPa. Design the tie bar by determining the required values of (*a*) the diameter d of the bolt, (*b*) the dimension b at each end of the bar, (*c*) the dimension h of the bar.

$$\tau = \frac{F}{A} = \frac{\frac{120}{2}kN}{\frac{\pi}{4}d^2} = 100\ MPa$$

$$\boxed{d = 27.6\ mm \approx 28\ mm}$$

For check

$$\sigma_b = \frac{120kN}{20mm \times 28mm} = 214\ MPa < 350MPa$$

$$\sigma_{end} = \frac{120kN}{(b-28)mm \times 20mm} = 175\text{MPa}$$

$$\boxed{b = 62.3\ mm}$$

$$\sigma_{middle} = \frac{120kN}{(h)mm \times 20mm} = 175\text{MPa}$$

$$\boxed{h = 34.3\ mm}$$

CH2: Stress and strain, Axial loading

Mechanical design

Mechatronics Engineering

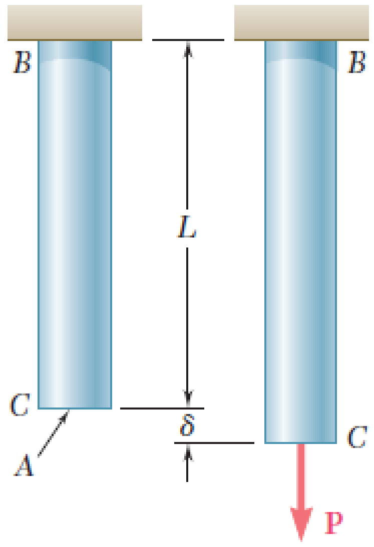
Hashemite University

Dr. Mohammad Hayajneh

Structure under static load

- Let us consider a rod BC, of length L and uniform cross-sectional area A, which is suspended from B. If we apply a load P to end C, the rod elongates.
- *normal strain* in a rod under axial loading is defined as the deformation per unit length of that rod.

$$\epsilon = \frac{\delta}{L}$$

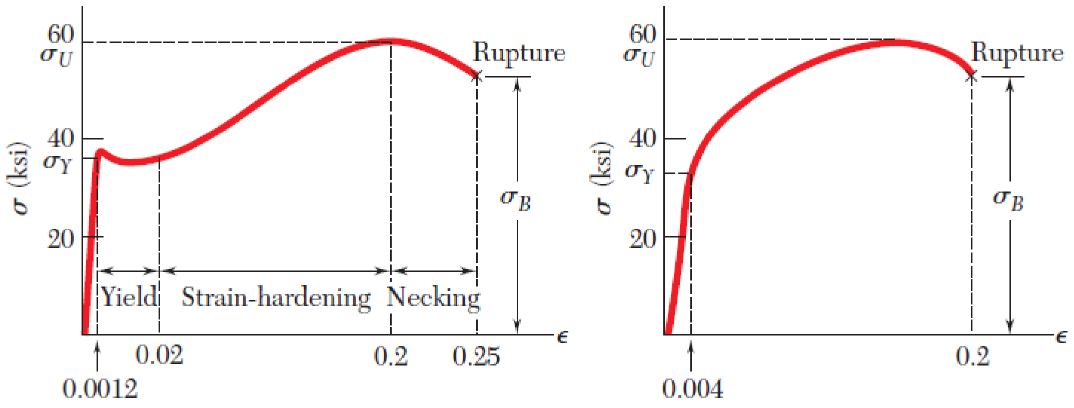


Example : Consider, for instance, a bar of length $L = 0.600$ m and uniform cross section, which undergoes a deformation $\delta = 150 \times 10^{-6}$ m. The corresponding strain is?

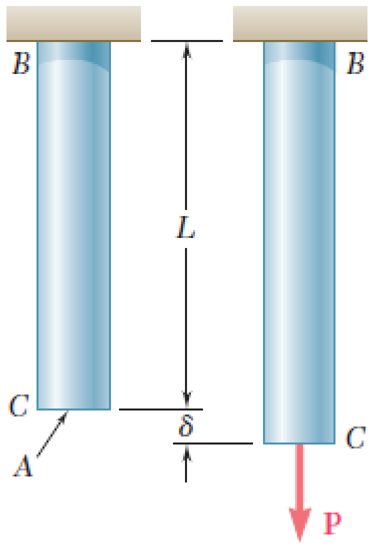
STRESS-STRAIN DIAGRAM

- To obtain the stress-strain diagram of a material, one usually conducts a *tensile test* on a specimen of the material.

- Ductile materials, which comprise structural steel, as well as many alloys of other metals, are characterized by their ability to *yield* at normal temperatures.



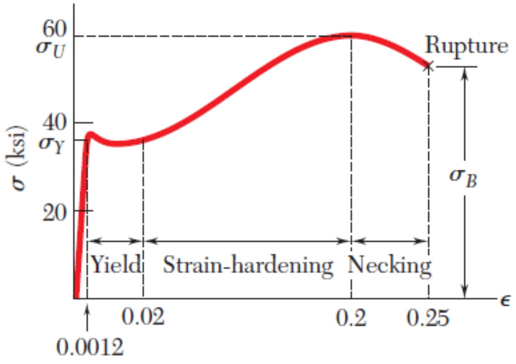
(a) Low-carbon steel (b) Aluminum alloy



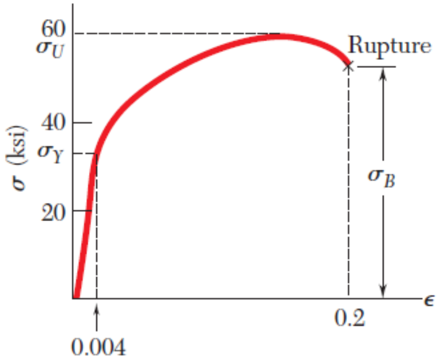
Force (p)	$\epsilon = \frac{\delta}{L}$	$\sigma = \frac{P}{A}$
10 KN		
20 KN		
30 KN		
….		

STRESS-STRAIN DIAGRAM

- As the specimen is subjected to an increasing load, its length first increases linearly with the load and at a very slow rate. Linear portion
- after a critical value σ_Y of the stress has been reached, the specimen undergoes a large deformation with a relatively small increase in the applied load. Yield portion
- As we can note from the stress-strain diagrams of two typical ductile materials, the elongation of the specimen after it has started to yield can be 200 times as large as its deformation before yield. Strain Hardening
- After a certain maximum value of the load has been reached, the diameter of a portion of the specimen begins to decrease, because of local instability This phenomenon is known as *necking*.



(a) Low-carbon steel



(b) Aluminum alloy

STRESS-STRAIN DIAGRAM

- Stresses
  - Yield stress (σ_Y)
  - Ultimate stress (σ_U)
  - Breaking stress (σ_B)
- The rupture occurs along a cone-shaped surface that forms an angle of approximately 45^o with the original surface of the specimen.

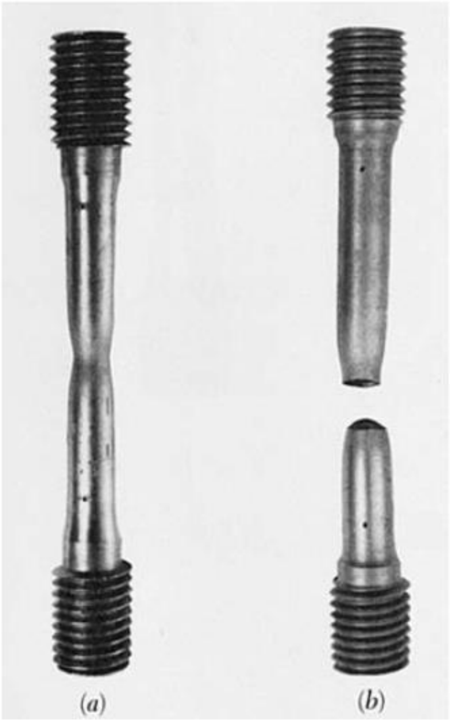
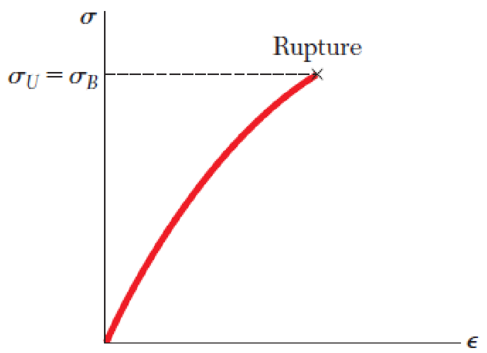


Photo 2.4 Tested specimen of a ductile material.

STRESS-STRAIN DIAGRAM

- Brittle materials, which comprise cast iron, glass, and stone, are characterized by the fact that rupture occurs without any noticeable prior change in the rate of elongation
- There is no difference between the ultimate strength and the breaking strength.
- The strain at the time of rupture is much smaller for brittle than for ductile materials.
- No necking in the specimen in the case of a brittle material.
- Rupture occurs along a surface perpendicular to the load



Hooke's law; modulus of elasticity

Most engineering structures are designed to undergo relatively small deformations, involving only the straight-line portion of the corresponding stress-strain diagram.

$$\sigma = E\epsilon \qquad \text{Hooke's law,}$$

E is called the *modulus of elasticity* of the material involved, or also *Young's modulus*.

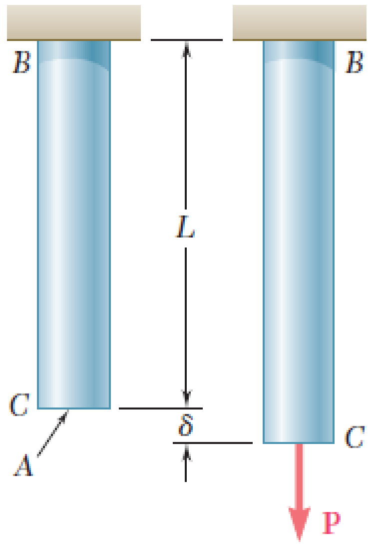
Deformations of members under axial loading

$$\sigma = P/A$$

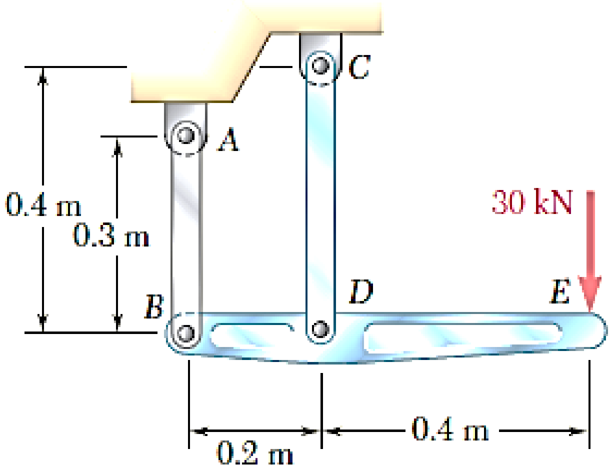
$$\sigma = E\epsilon \rightarrow \epsilon = \frac{\sigma}{E} = \frac{\delta}{L}$$

$$\epsilon = \frac{P/A}{E} = \frac{\delta}{L}$$

$$\delta = \frac{PL}{AE}$$



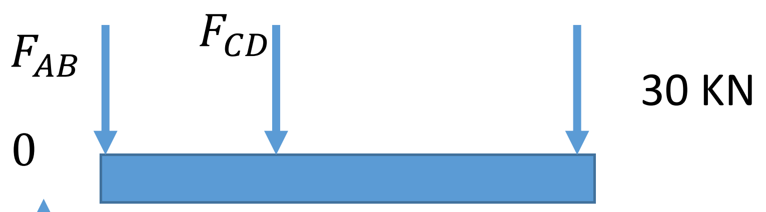
Deformations of members under axial loading



SAMPLE PROBLEM 2.1

The rigid bar *BDE* is supported by two links *AB* and *CD*. Link *AB* is made of aluminum (E = 70 GPa) and has a cross-sectional area of 500 mm^2; link *CD* is made of steel (E = 200 GPa) and has a cross-sectional area of 600 mm^2. For the 30-kN force shown, determine the deflection (*a*) of *B*, (*b*) of *D*, (*c*) of *E*.

Summation Moment at B=0



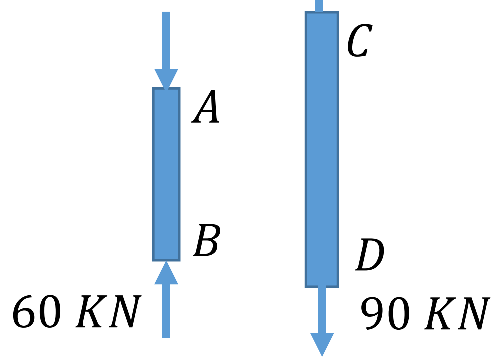
$$-F_{CD}(0.2) - 30(0.6) = 0$$

$$F_{CD} = -90\ KN = 90\ KN$$

Summation of forces =0

$$-30\ KN + 90KN - F_{AB} = 0$$

$$F_{AB} = 60\ KN$$

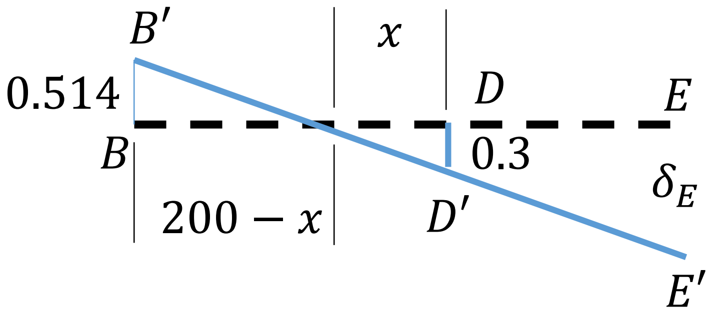


Deformations of members under axial loading

$$\delta_B = \frac{PL}{AE} = \frac{-60 \times 10^3(0.3)}{500 \times 10^{-6}(70 \times 10^9)} = -514 \times 10^{-6} m = -0.514 mm$$

$$\delta_D = \frac{PL}{AE} = \frac{90 \times 10^3(0.4)}{600 \times 10^{-6}(200 \times 10^9)} = 300 \times 10^{-6} m = 0.3\ mm$$

$$\delta_E =$$

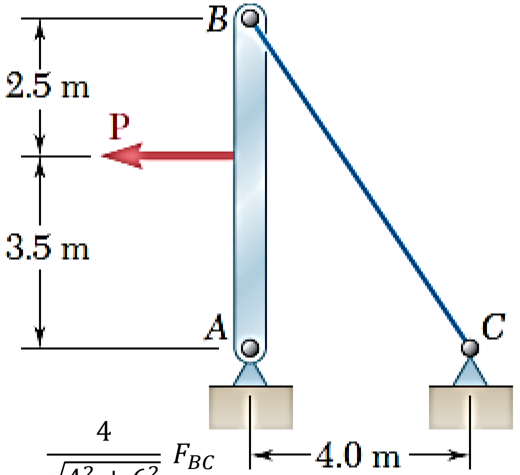


$\frac{0.3}{0.514} = \frac{x}{200-x}$ → $x = 73.7\ mm$

$\frac{0.3}{\delta_E} = \frac{73.7}{400+73.7}$ → $\delta_E = 1.93\ mm$

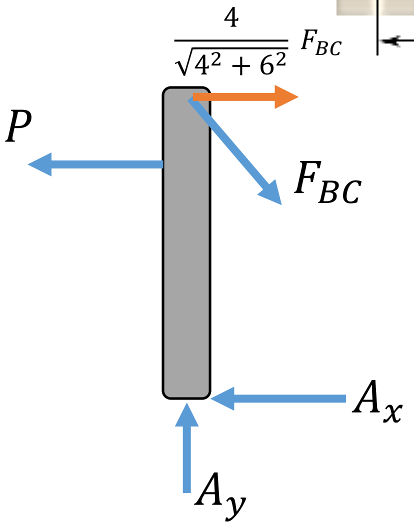
Deformations of members under axial loading

2.13 The 4-mm-diameter cable BC is made of a steel with $E = 200$ GPa. Knowing that the maximum stress in the cable must not exceed 190 MPa and that the elongation of the cable must not exceed 6 mm, find the maximum load **P** that can be applied as shown.



$$\sum M_A = 0 = 3.5P - 6 \times \frac{4}{\sqrt{4^2+6^2}} F_{BC}$$

$$P = 0.951 F_{BC} \rightarrow F_{BC} = \frac{P}{0.951}$$



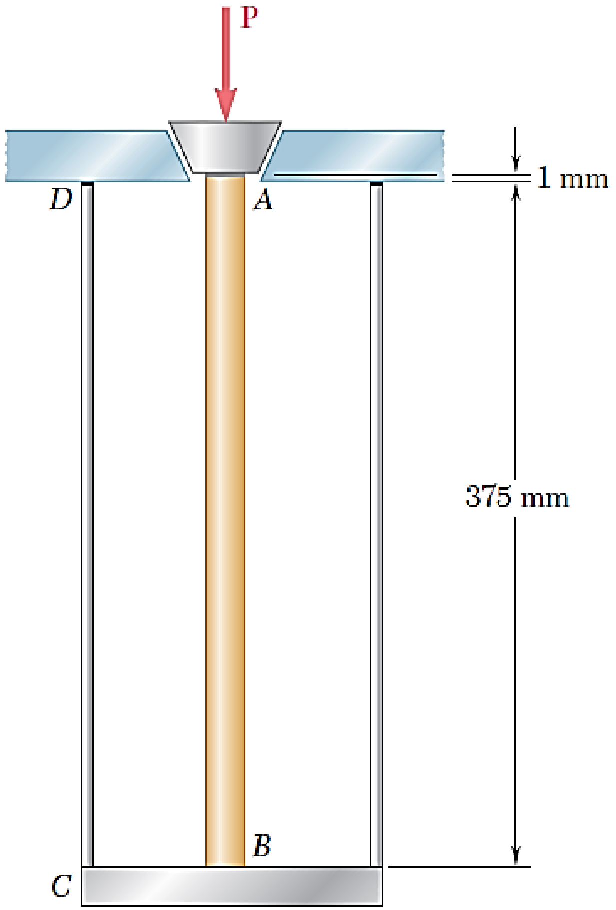
Condition 1 $$\delta_{cable} = \frac{PL}{AE} = \frac{\frac{P}{0.951}(7.21)}{\frac{\pi}{4}(0.004^2)(200\ GPa)} = 0.006m$$

$P_{deformation} = 1.989KN$ We choose this value

Condition 2 $$\sigma = \frac{F_{BC}}{A} = \frac{\frac{P}{0.951}}{\frac{\pi}{4}(0.004^2)} = 190 \times 10^6 \rightarrow P_{stress} = 2.27KN$$

Deformations of members under axial loading

2.16 The brass tube *AB* (E = 105 GPa) has a cross-sectional area of 140 mm^2 and is fitted with a plug at *A*. The tube is attached at *B* to a rigid plate that is itself attached at *C* to the bottom of an aluminum cylinder (E = 72 GPa) with a cross-sectional area of 250 mm^2. The cylinder is then hung from a support at *D*. In order to close the cylinder, the plug must move down through 1 mm. Determine the force **P** that must be applied to the cylinder.



$$\delta_{total} = 1mm =$$

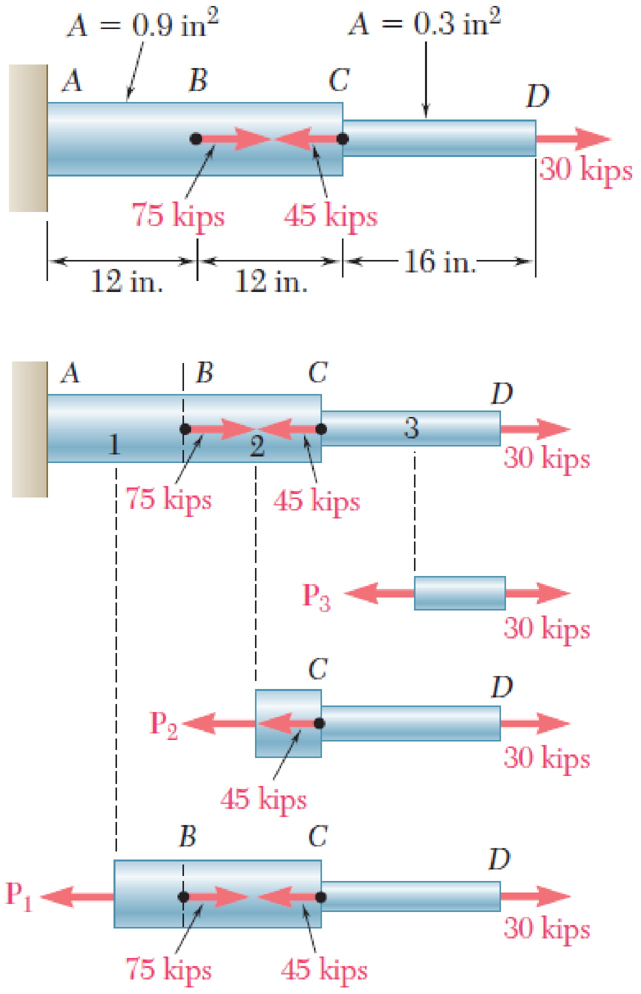
Deformations of members under axial loading

If the rod is loaded at other points, or if it consists of several portions of various cross sections and possibly of different materials, we must divide it into component parts that satisfy individually the required conditions. The deformation is given by:

$$\delta = \sum_i \frac{P_i L_i}{A_i E_i}$$

Deformations of members under axial loading

Determine the deformation of the steel rod shown in figure under the given loads ($E = 29 \times 10^6$psi).

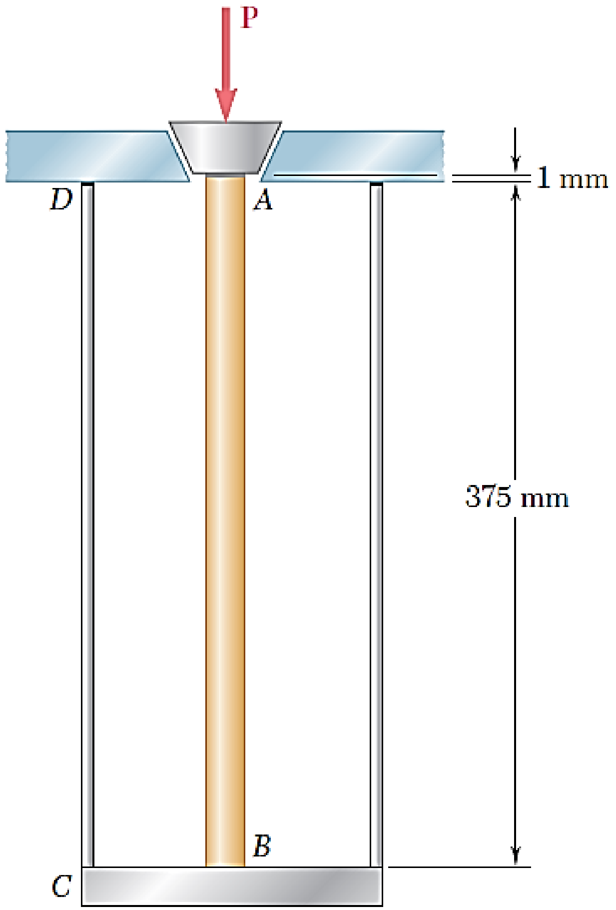


CH2: Poisson's ratio

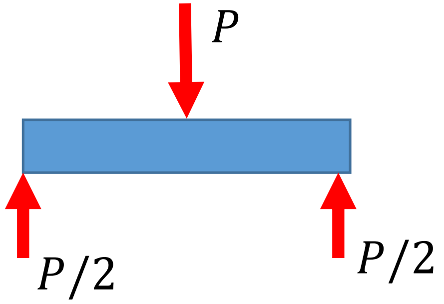
Mechanical design
Mechatronics Engineering
Hashemite University
Dr. Mohammad Hayajneh

Deformations of members under axial loading

2.16 The brass tube AB ($E = 105$ GPa) has a cross-sectional area of 140 mm^2 and is fitted with a plug at A. The tube is attached at B to a rigid plate that is itself attached at C to the bottom of an aluminum cylinder ($E = 72$ GPa) with a cross-sectional area of 250 mm^2. The cylinder is then hung from a support at D. In order to close the cylinder, the plug must move down through 1 mm. Determine the force **P** that must be applied to the cylinder.



$$\delta_{total} = 1mm = \frac{(P/2)(0.375)}{250 \times 10^{-6}(72 \times 10^{9})} - \frac{(P)(0.375)}{140 \times 10^{-6}(105 \times 10^{9})}$$



Deformations of members under axial loading

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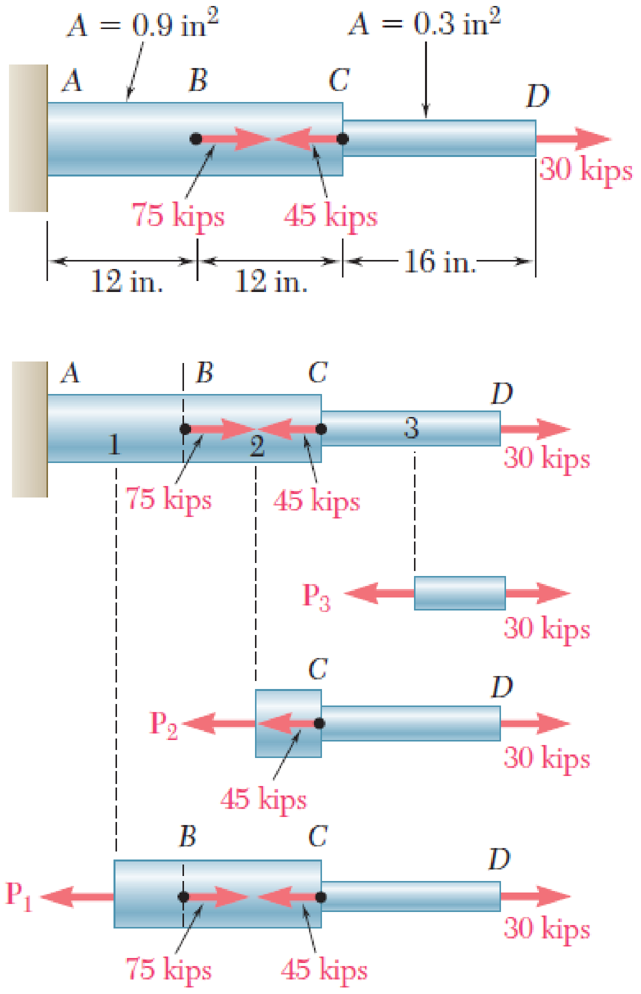
Deformations of members under axial loading

Determine the deformation of the steel rod shown in figure under the given loads ($E = 29 \times 10^6$psi).

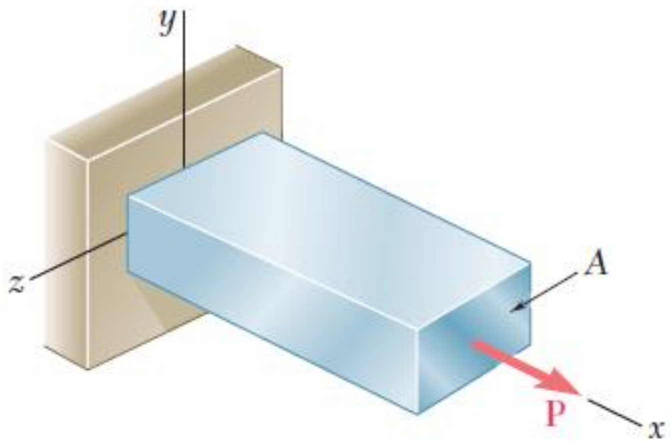


Poisson's ratio

When a homogeneous slender bar is axially loaded, the resulting stress and strain satisfy Hooke's law, as long as the elastic limit of the material is not exceeded.

$$\sigma = \epsilon E$$

$$\epsilon_x = \frac{\sigma_x}{E}$$

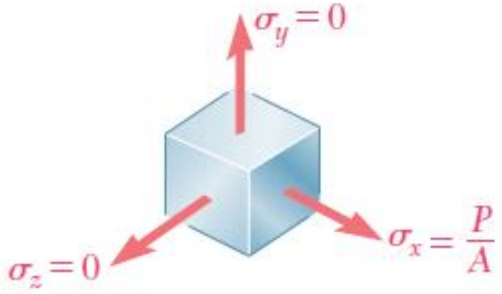


We also note that the normal stresses on faces respectively perpendicular to the y and z axes are zero ($\sigma_y = \sigma_z = 0$). This yields:

$$\epsilon_y = \frac{\sigma_y}{E} = \frac{0}{E} = 0$$

$$\epsilon_z = \frac{\sigma_z}{E} = \frac{0}{E} = 0$$

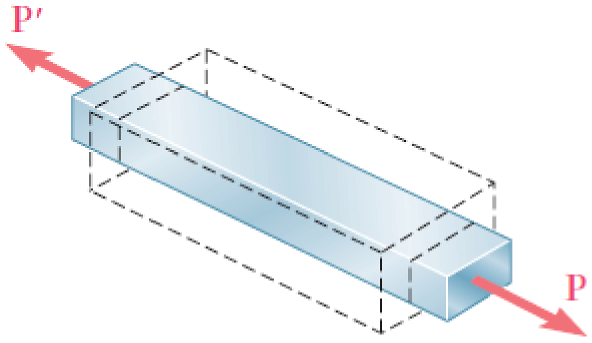
In practice, we notice that there are deflections occurred in other axes



Poisson's ratio

- In all engineering materials, the elongation produced by an axial tensile force P in the direction of the force is accompanied by a contraction in any transverse direction.
- In homogeneous and isotropic material , the strain must have the same value for any transverse direction. This is means

$$\epsilon_y = \epsilon_z \quad \text{(Lateral strain)}$$



Then we have

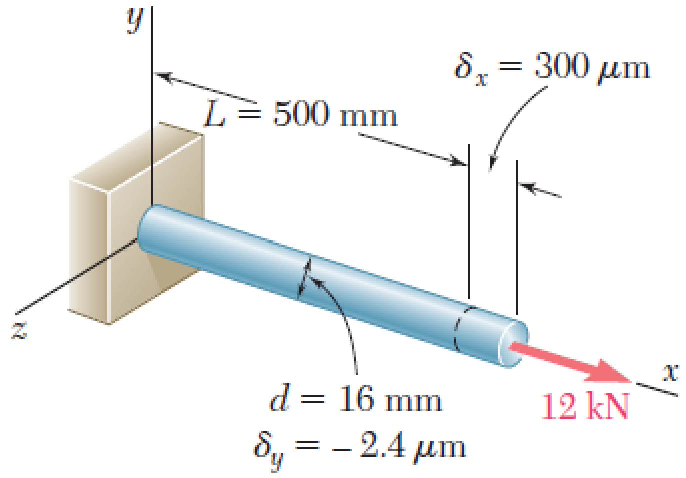
$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\epsilon_y}{\epsilon_x} = -\frac{\epsilon_z}{\epsilon_x}$$

Rearranging:

$$\epsilon_x = \frac{\sigma_x}{E}, \qquad \epsilon_y = \epsilon_z = -\nu\epsilon_x = -\frac{\nu\sigma_x}{E}$$

Poisson's ratio

A 500-mm-long, 16-mm-diameter rod made of a homogenous, isotropic material is observed to increase in length by 300 μm, and to decrease in diameter by 2.4 μm when subjected to an axial 12-kN load. Determine the modulus of elasticity and Poisson's ratio of the material.



$$\delta_x = \frac{PL}{AE} \qquad \frac{\delta_x}{L} = \frac{300 \times 10^{-6}}{0.5} = \epsilon_x = \frac{P}{AE} = 600 \times 10^{-6}$$

$$\frac{P}{AE} = 600 \times 10^{-6} \rightarrow E = \frac{12 \times 10^3}{\frac{\pi}{4} 0.016^2 (600 \times 10^{-6})} = 99.5\ GPa$$

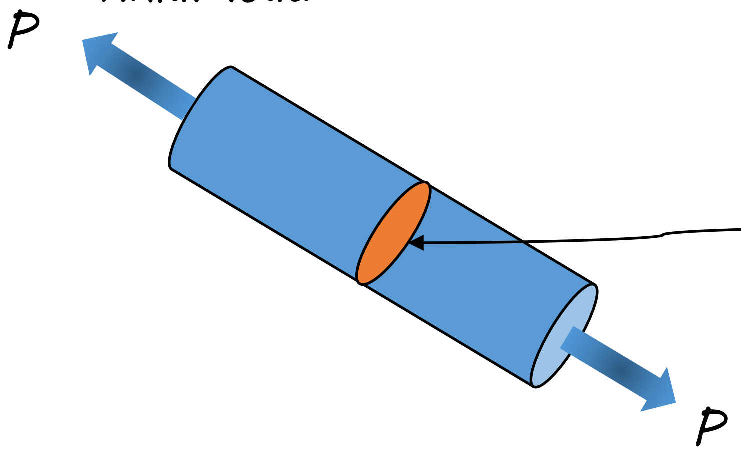
$$\epsilon_y = \frac{\delta_y}{d} = \frac{-2.4 \times 10^{-6}}{0.016} = -150 \times 10^{-6} \qquad \nu = -\frac{\epsilon_y}{\epsilon_x} = -\frac{-150}{600} = 0.25$$

Mechanical Deign (Torsion)

Mechatronics Engineering
Hashemite University
Dr. Mohammad Hayajneh

Remarks

Axial load

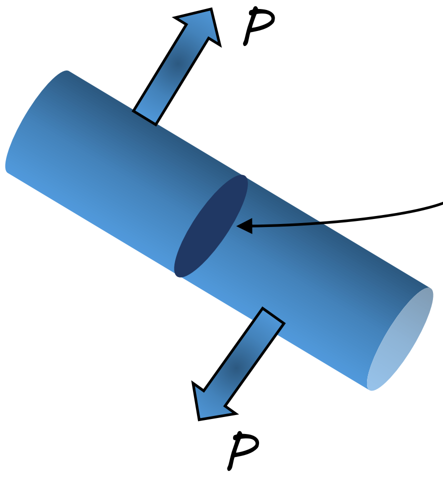


1) Axial stress or Normal stress

$$\sigma = \frac{P}{A}$$

Cross sectional area

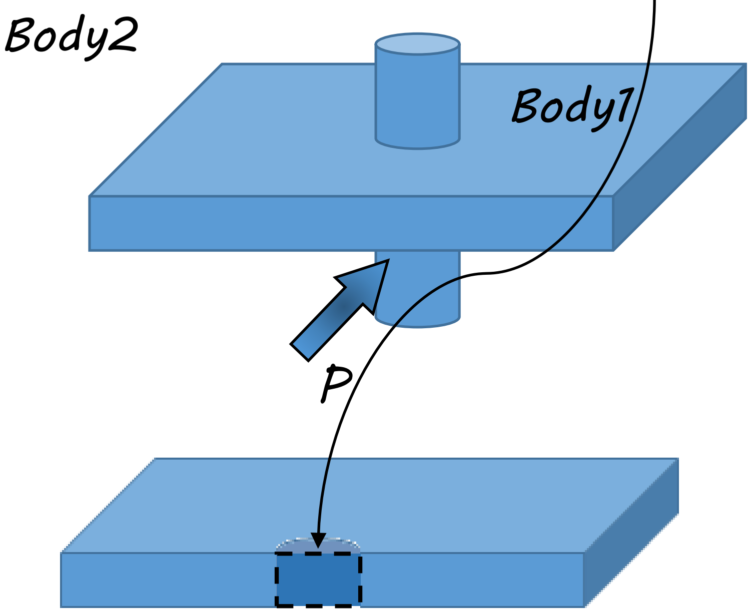
2) Shear stress



$$\tau = \frac{P}{A}$$

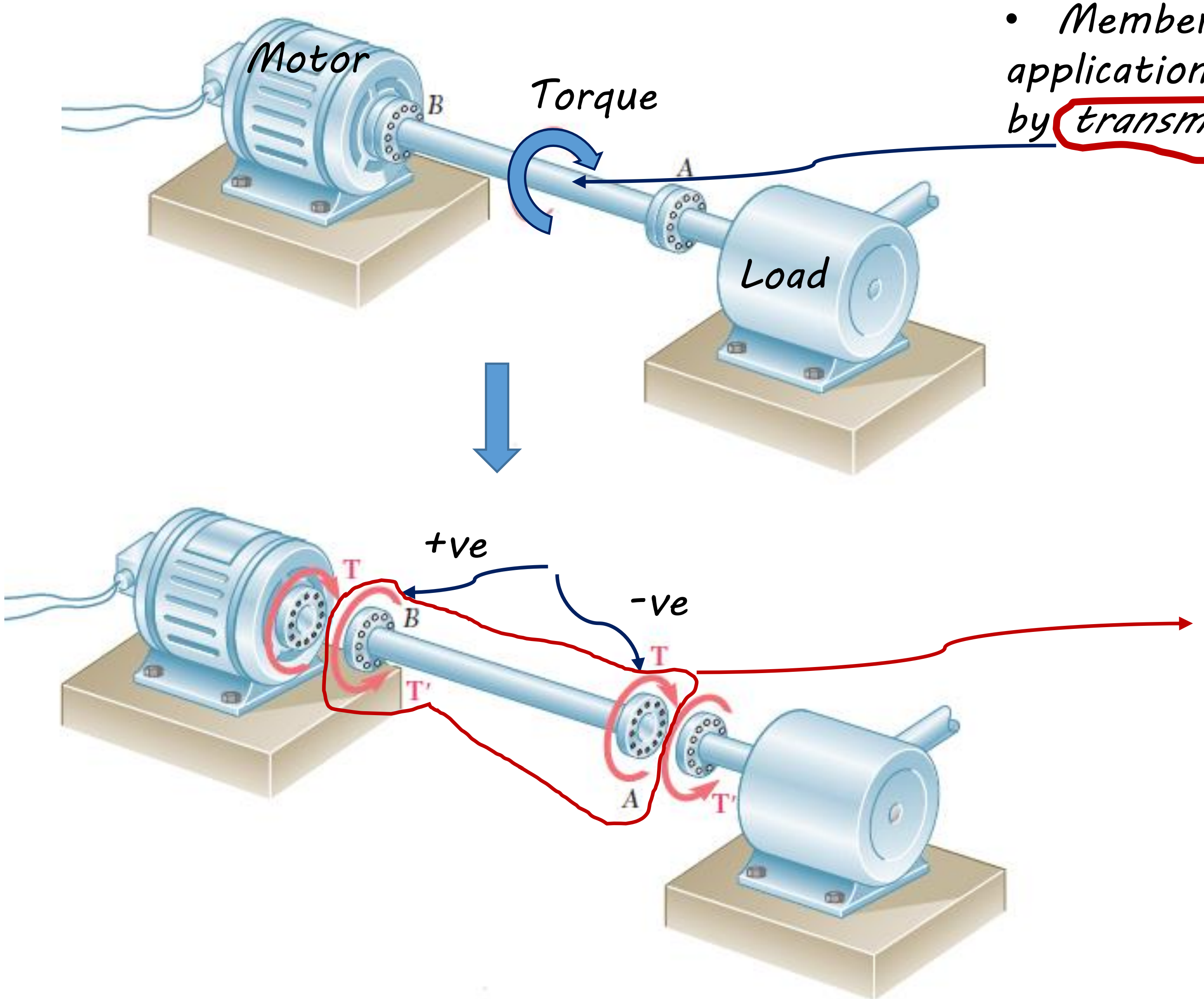
3) Bearing stress

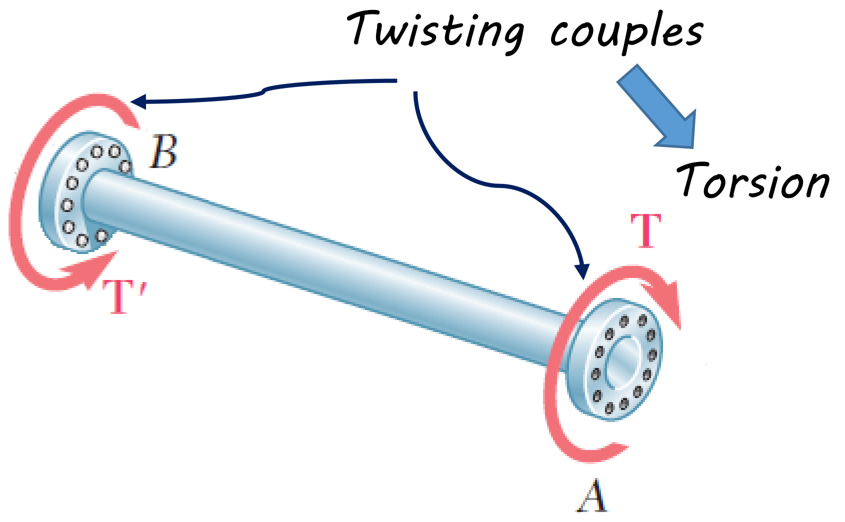
$$\sigma_b = \frac{P}{A_b}$$



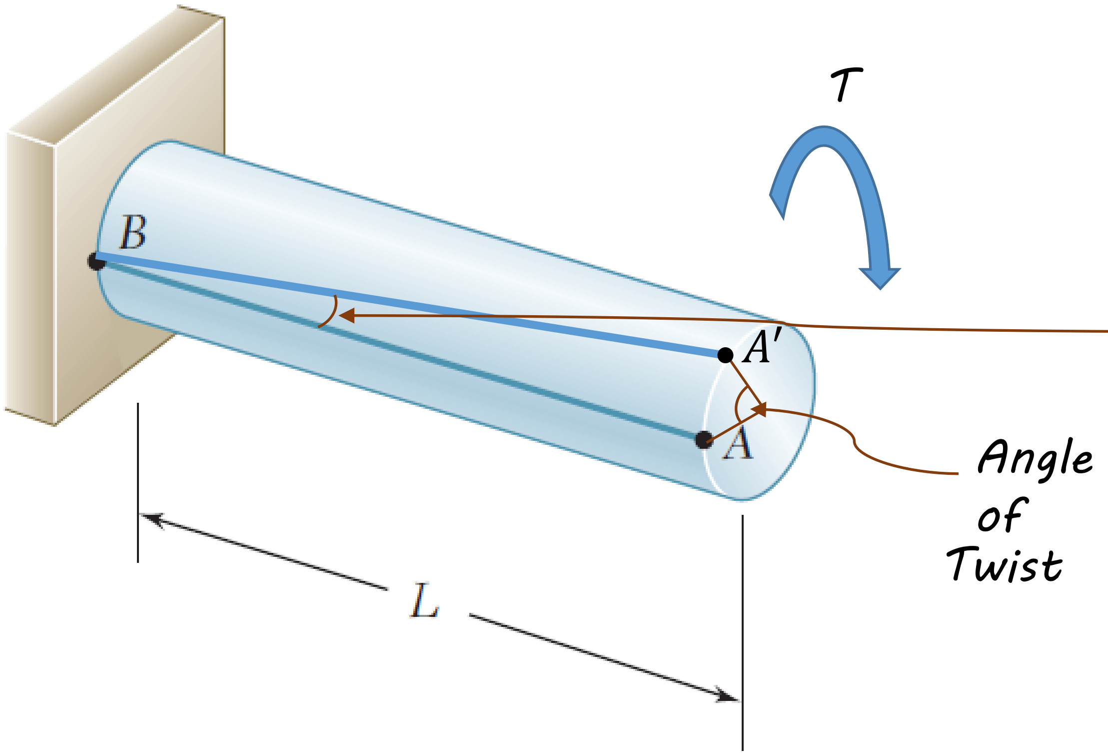
Torsion Definition

- *Members in torsion are existed in many engineering applications. The most common application is provided by transmission shafts.*





DEFORMATIONS IN A CIRCULAR SHAFT



ϕ (Phi) Angle of twist

L Rod length

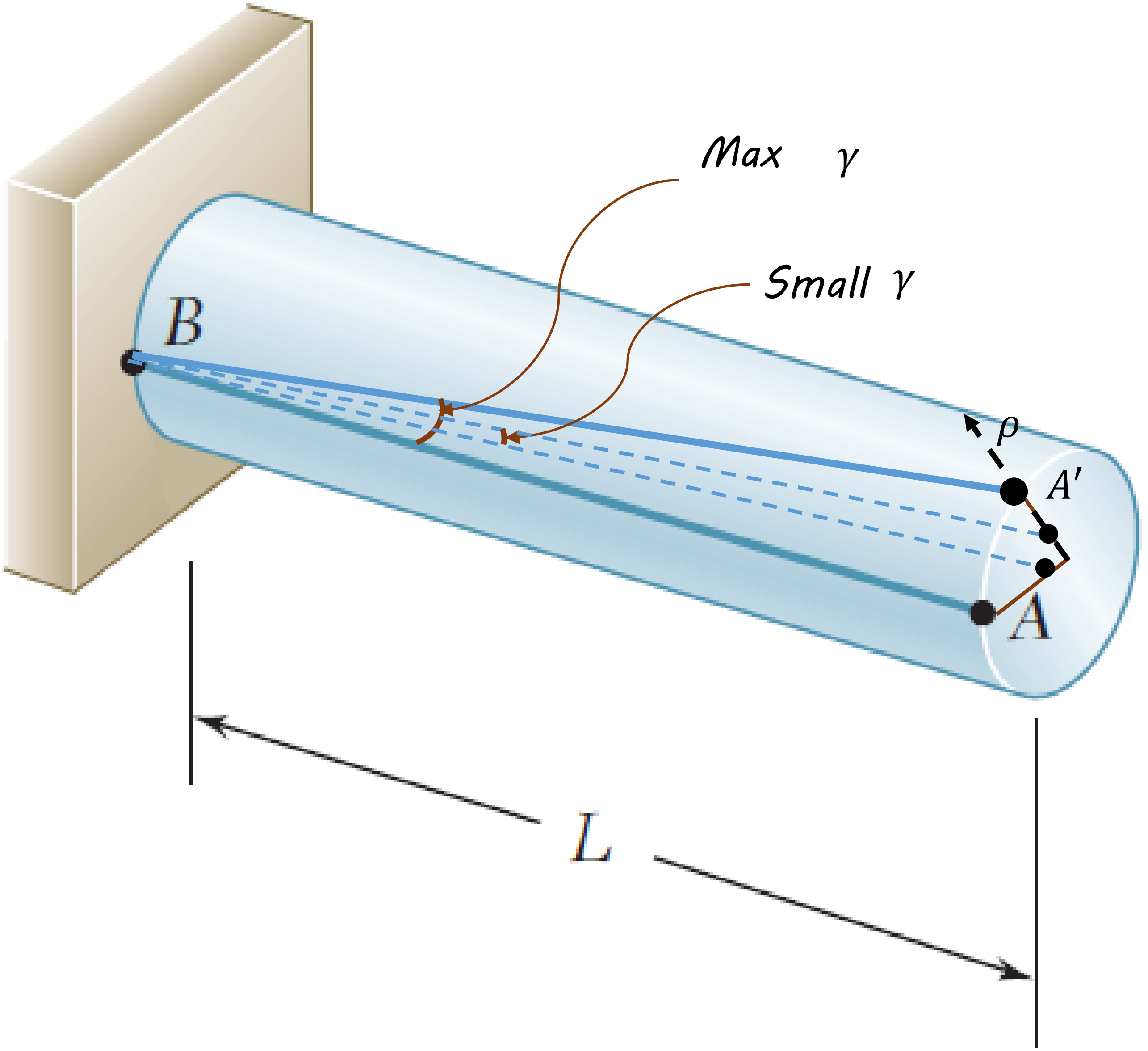
c Rod radius

γ (Gamma) Shearing strain

$$\gamma L = AA'$$

$$\phi c = AA'$$

$$\gamma L = \phi c \quad \Rightarrow \quad \gamma = \frac{c\phi}{L}$$



$$\gamma_{max} = \frac{c\phi}{L}$$

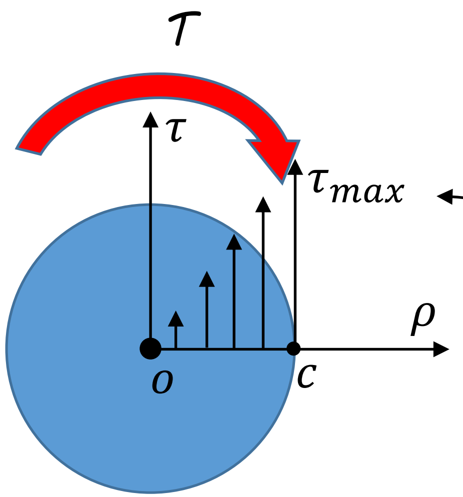
When $\rho = 0 \rightarrow \gamma = 0$

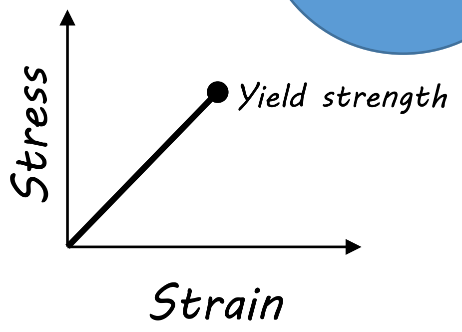
When $\rho = \mathrm{c} \rightarrow \gamma = Max$

$$\gamma = \frac{\rho\phi}{L}$$ General Equation

Thus, the shearing strain in a circular shaft varies linearly with the distance from the axis of the shaft.

STRESSES IN THE ELASTIC RANGE





Hook's law

$$\tau = G\gamma$$

G → *Modulus of rigidity*

τ → *Shearing stress*

$$\tau_{max} = G\gamma_{max}$$

$$G = \frac{\tau_{max}}{\gamma_{max}} = \frac{\tau}{\gamma}$$

$$\gamma = \frac{\rho\phi}{L}$$

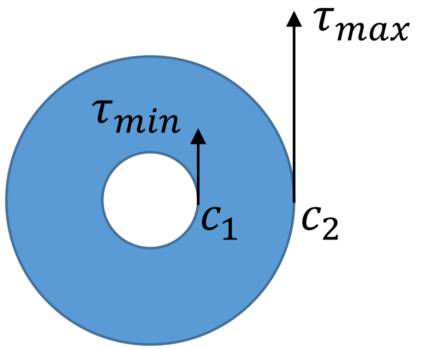
$$\gamma_{max} = \frac{c\phi}{L}$$

$$\tau = \frac{\rho}{c}\tau_{max}$$

$$\tau_o = \frac{0}{c}\tau_{max} = 0$$

$$\tau_c = \frac{c}{c}\tau_{max} = \tau_{max}$$

The shearing stress in the shaft varies linearly with the distance ρ from the axis of the shaft

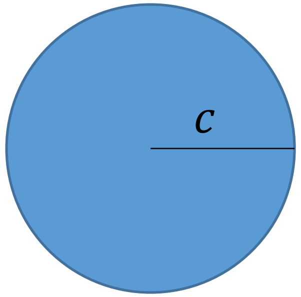


Torque and stress

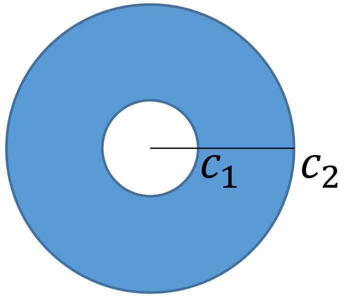
$$T = \frac{\tau_{max} J}{c}$$ → Polar moment of inertia

$$\tau_{max} = \frac{Tc}{J}$$

$$\tau = \frac{T\rho}{J}$$

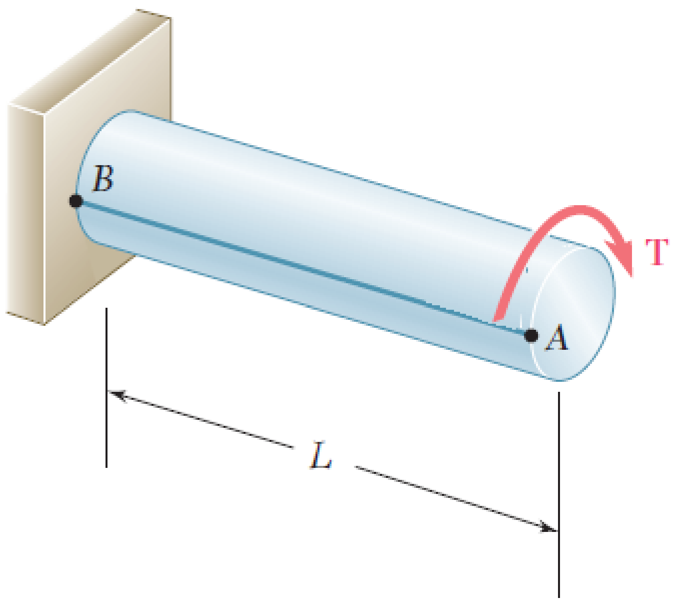


$$J = \frac{1}{2}\pi c^4$$



$$J = \frac{1}{2}\pi c_2^4 - \frac{1}{2}\pi c_1^4 = \frac{1}{2}(\pi c_2^4 - \pi c_1^4)$$

Example



A cylindrical Aluminum shaft is 1.5 m long and has a diameter of 60 mm. (a) What is the largest torque (T) that can be applied to the shaft if the shearing stress is not to exceed 70 MPa?

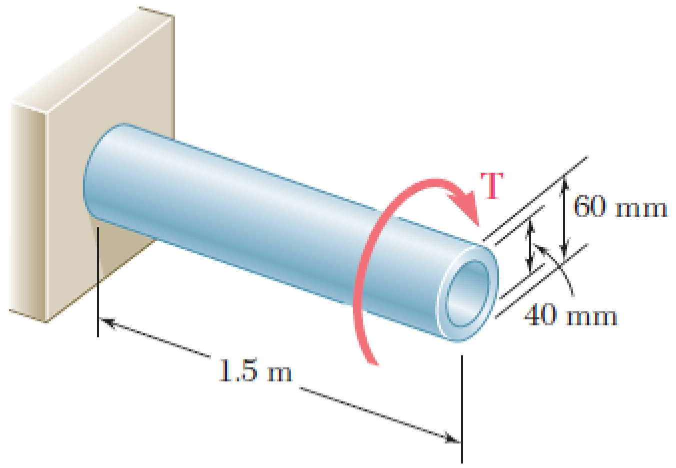
$$T = \frac{\tau_{max} J}{c} \qquad J = \frac{1}{2}\pi c^4$$

The solution is:

$$J = \frac{1}{2}\pi(0.03)^4 = 1.27 \times 10^{-6} m^4$$

$$T = \frac{70 \times 10^6 (1.27 \times 10^{-6})}{30 \times 10^{-3}} = 2.96\ KN.m$$

Example



A hollow cylindrical steel shaft is 1.5 m long and has inner and outer diameters respectively equal to 40 and 60 mm. (a) What is the largest torque that can be applied to the shaft if the shearing stress is not to exceed 120 MPa? (b) What is the corresponding minimum value of the shearing stress in the shaft?

$$T = \frac{\tau_{max} J}{c} \qquad J = \frac{1}{2}(\pi c_2^4 - \pi c_1^4)$$

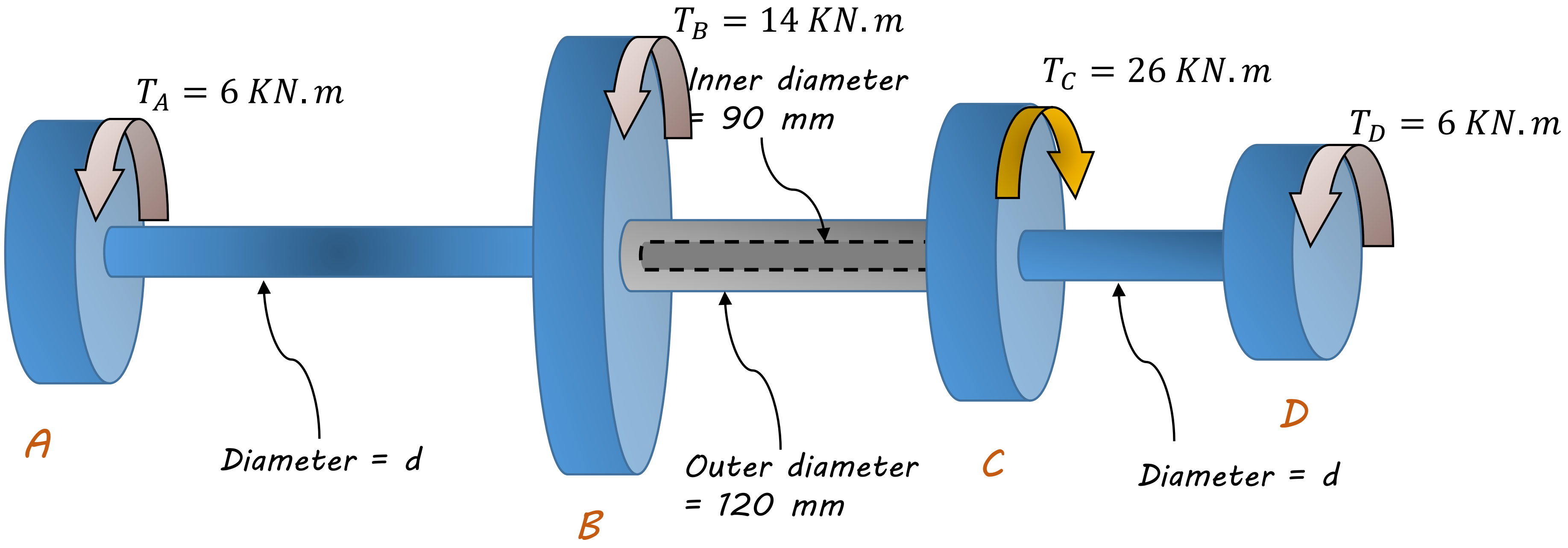
The solution is:

$$J = \frac{1}{2}(\pi(0.03^4) - \pi(0.02^2)) = 1.02 \times 10^{-6}\ m^4$$

$$T = \frac{120 \times 10^6 (1.02 \times 10^{-6})}{0.03} = 4.08\ KN.m$$

$$\tau_{min} = \frac{c_1}{c_2}\tau_{max} = \frac{20}{30} \times 120\ MPa = 80\ MPa$$

Example



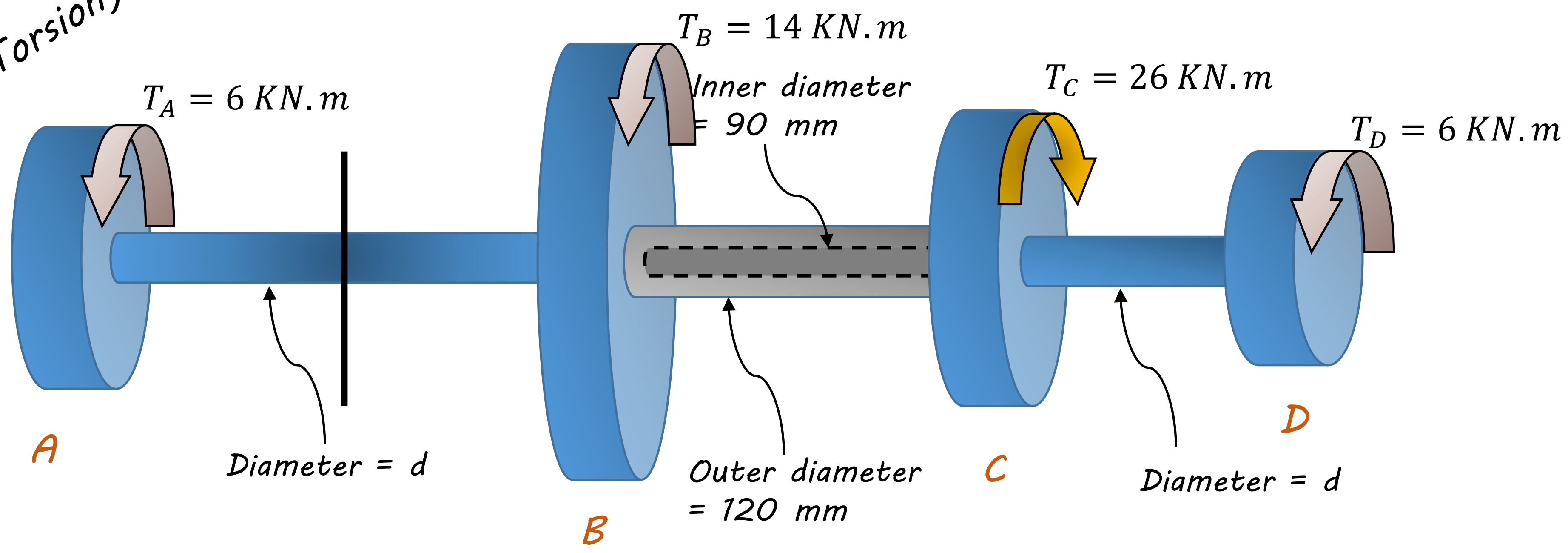
Determine:

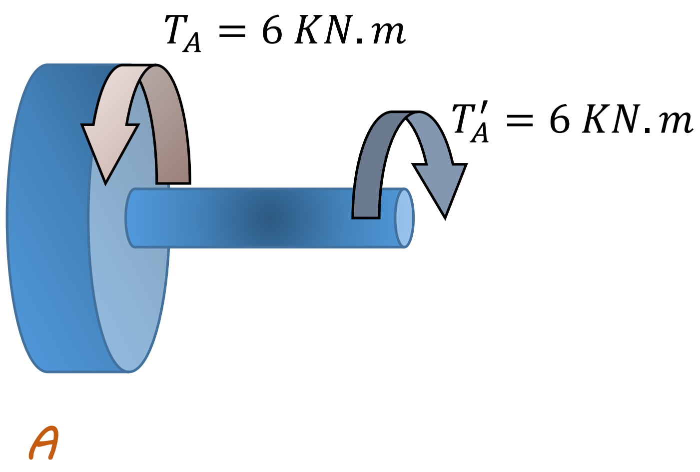
A) the maximum and minimum shearing stress in shaft BC.

B) the required diameter d of shafts AB and CD if the allowable shearing stress in these shafts is 65 MPa.

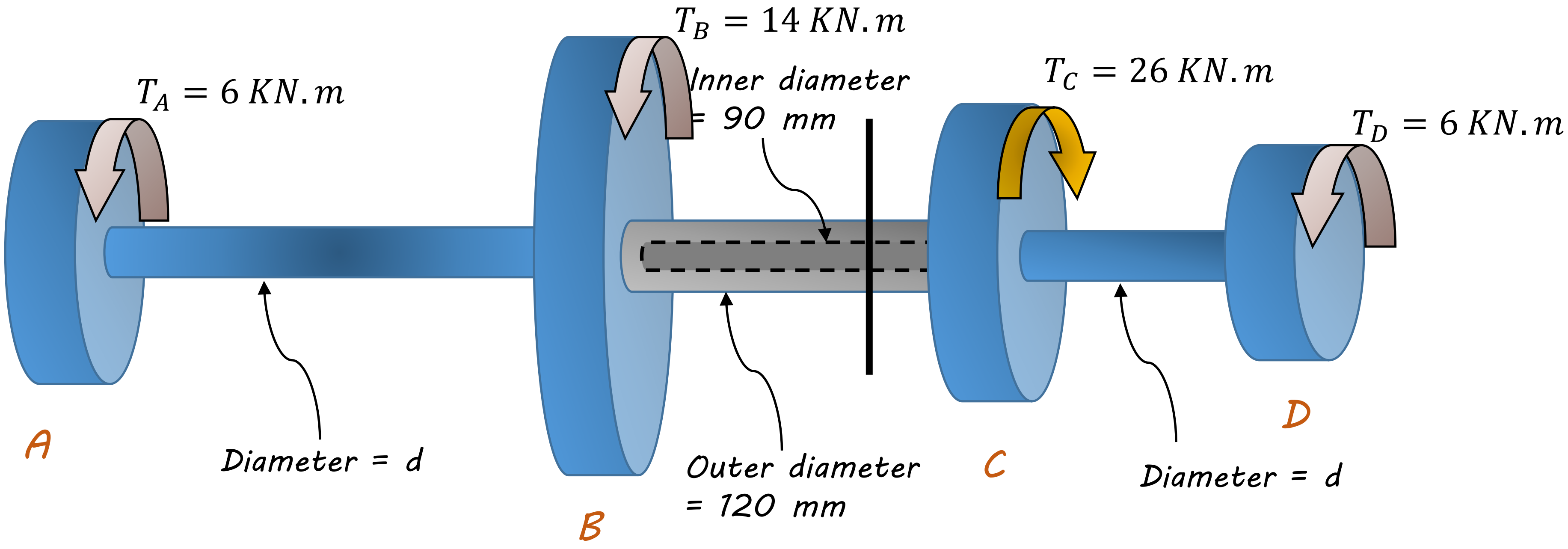
$(Torsion)^{Part2}$

Example

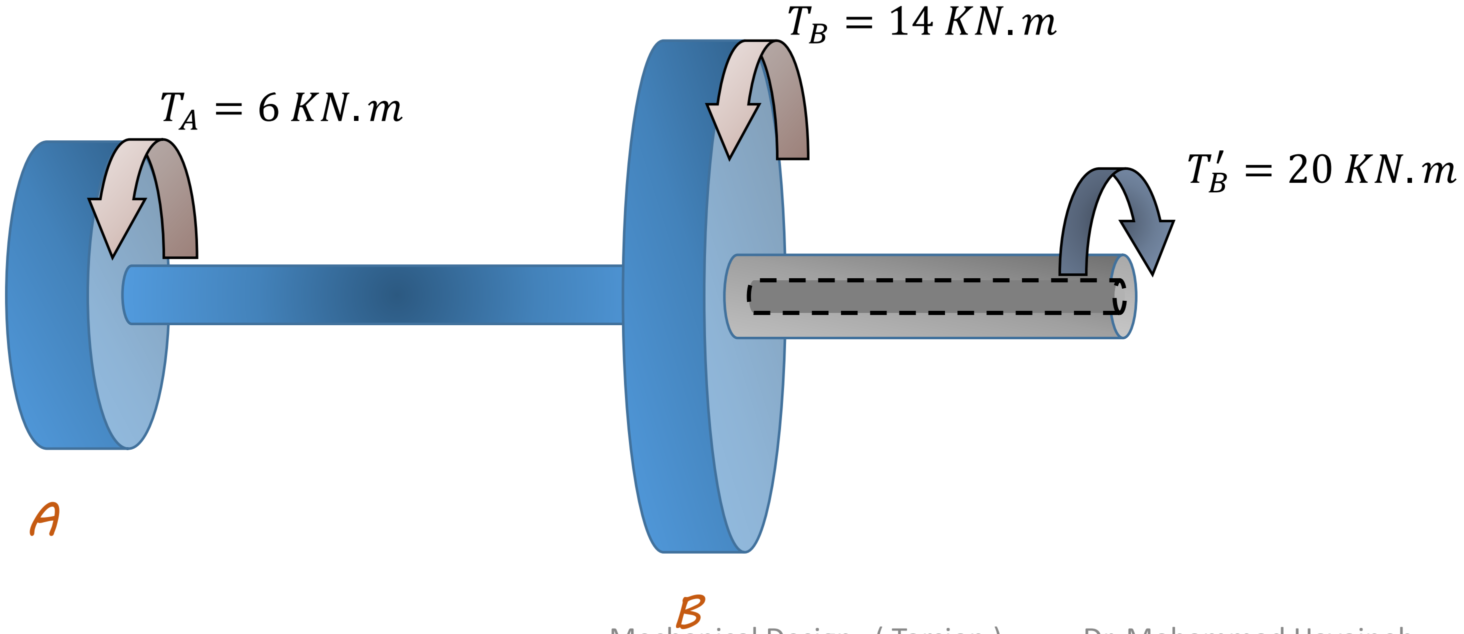




Example

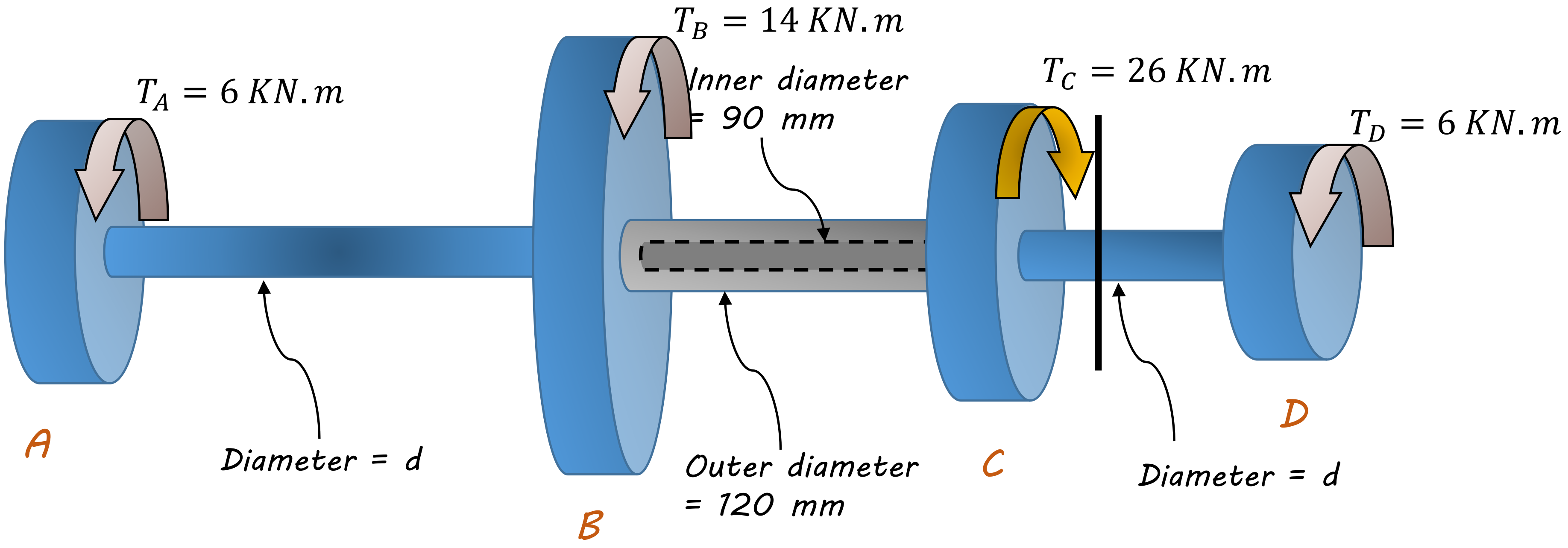


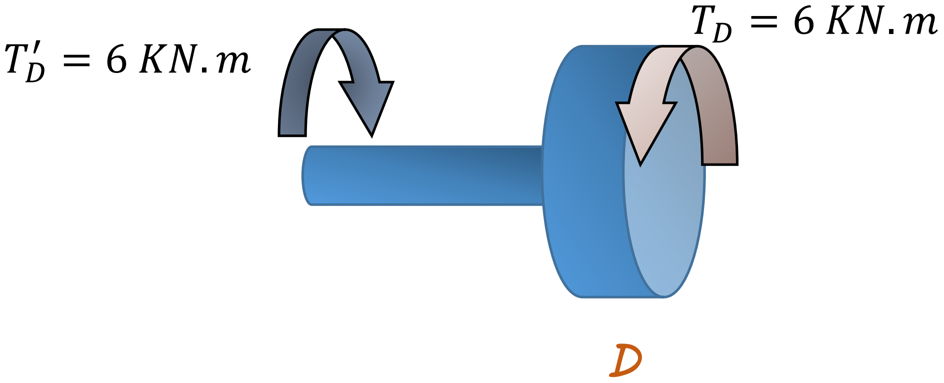
$T_B = 14\ KN.m$
Inner diameter = 90 mm
$T_C = 26\ KN.m$
$T_A = 6\ KN.m$
$T_D = 6\ KN.m$
A
Diameter = d
B
Outer diameter = 120 mm
C
D
Diameter = d



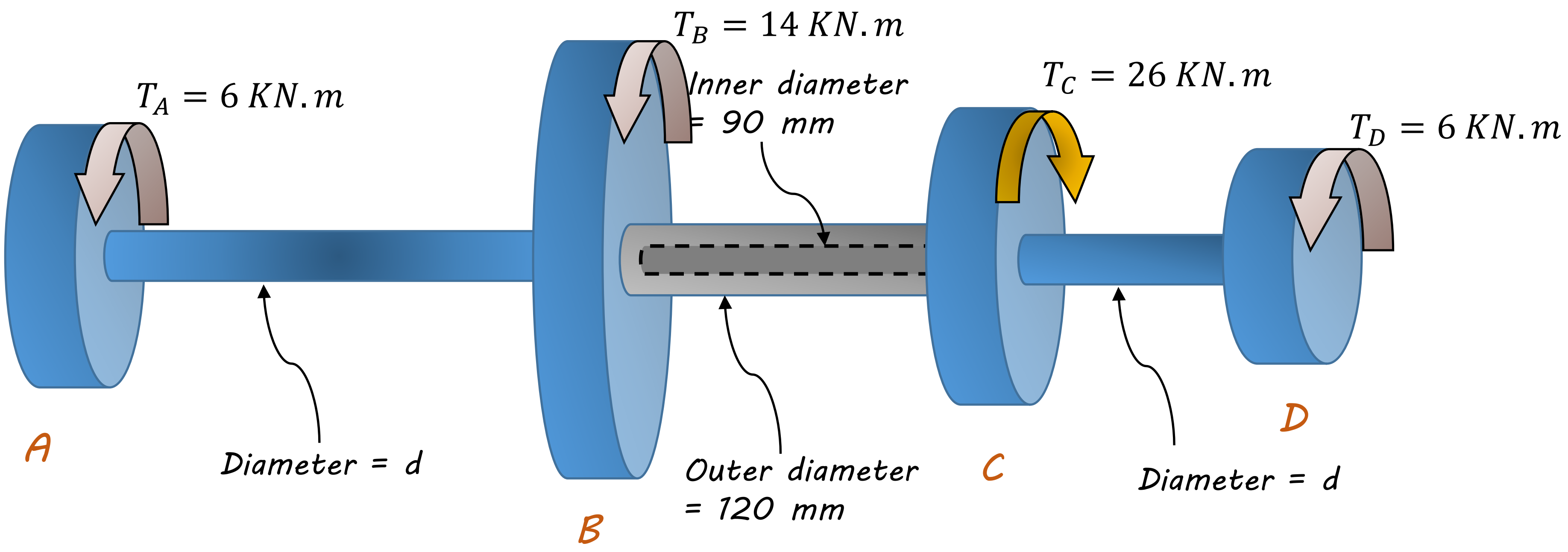
$T_B = 14\ KN.m$
$T_A = 6\ KN.m$
$T_B' = 20\ KN.m$
A
B

Example


$T_B = 14\ KN.m$
$T_A = 6\ KN.m$
Inner diameter
= 90 mm
$T_C = 26\ KN.m$
$T_D = 6\ KN.m$
A
Diameter = d
B
Outer diameter
= 120 mm
C
D
Diameter = d


$T_D = 6\ KN.m$
$T'_D = 6\ KN.m$
D

Example



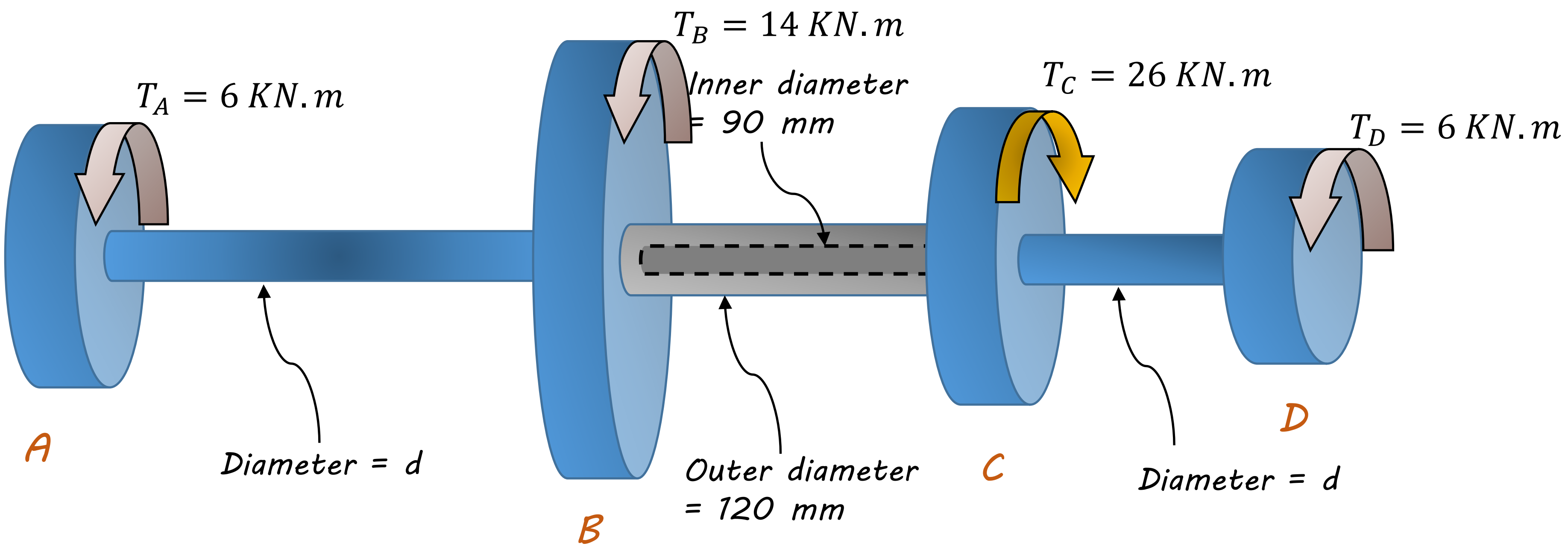
Shaft BC: For this hollow shaft we have

$$J = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}[(0.060)^4 - (0.045)^4] = 13.92 \times 10^{-6}\ \mathrm{m}^4$$

Maximum Shearing Stress On the outer surface

$$\tau_{\max} = \tau_2 = \frac{T_{BC}c_2}{J} = \frac{(20\ \mathrm{kN}\cdot\mathrm{m})(0.060\ \mathrm{m})}{13.92 \times 10^{-6}\ \mathrm{m}^4} = 86.2\ MPa$$

Example

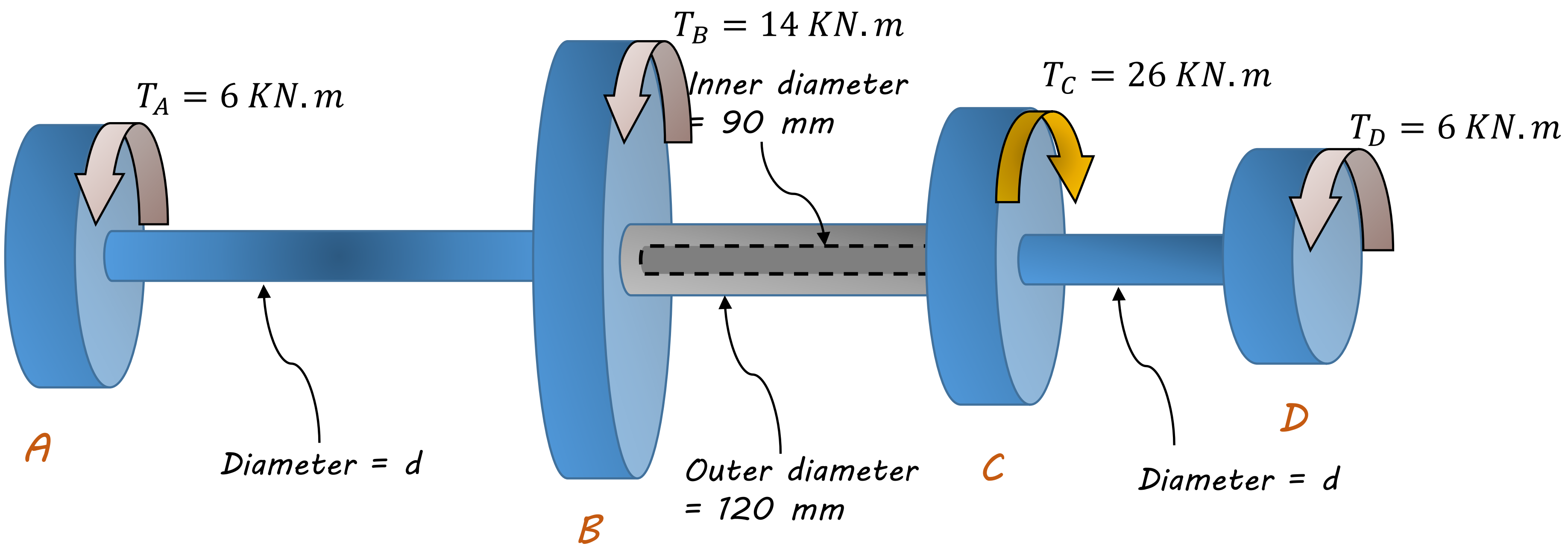


Minimum Shearing Stress On the Inner surface

$$\frac{\tau_{\min}}{\tau_{\max}} = \frac{c_1}{c_2}$$

$$\frac{\tau_{\min}}{86.2\ \text{MPa}} = \frac{45\ \text{mm}}{60\ \text{mm}} = 64.7\ MPa$$

Example



b) Diameter of shafts AB and CD

$$T = 6KN.m$$
$$\tau = 65\ MPa$$

$$\tau = \frac{Tc}{J}$$

$$65\ \text{MPa} = \frac{(6\ \text{kN} \cdot \text{m})c}{\frac{\pi}{2}c^4}$$

$$c^3 = 58.8 \times 10^{-6}\ \text{m}^3$$

$$c = 38.9 \times 10^{-3}\ \text{m}$$

$$d = 2c = 2(38.9mm) = 77.8mm$$

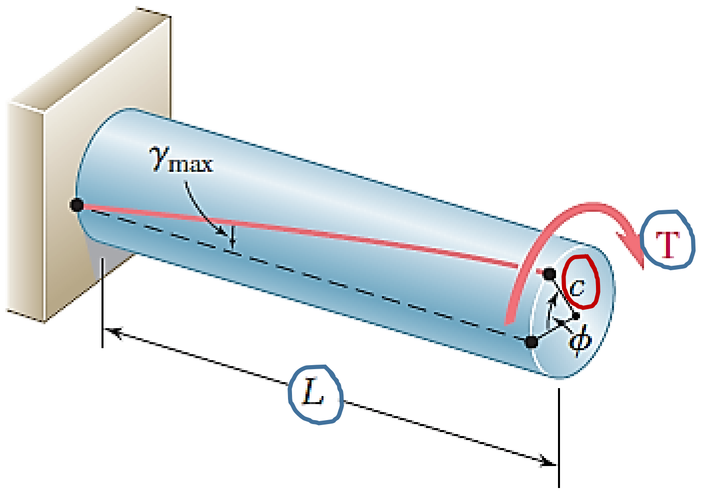
ANGLE OF TWIST IN THE ELASTIC RANGE

$$\gamma_{max} = \frac{c\phi}{L}$$

(1)

$$\gamma_{max} = \tau_{max}/G \quad \tau_{max} = \frac{Tc}{J}$$

(2)



$$\phi = \frac{TL}{JG}$$

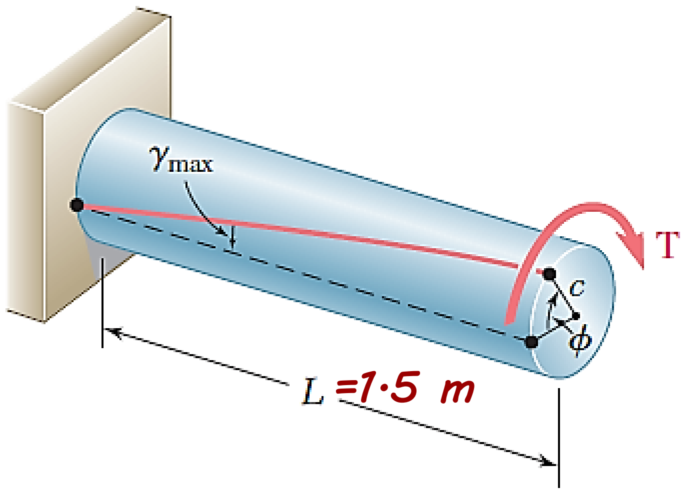
In radians

Constant

the angle of twist ϕ is proportional to the torque T applied to the shaft

Example

What torque should be applied to the end of the shaft to produce a twist of 2^o? Use the value G =77 GPa for the modulus of rigidity of steel.



$$T = \frac{JG}{L}\phi$$

$$\phi = 2^\circ\left(\frac{2\pi \text{ rad}}{360^\circ}\right) = 34.9 \times 10^{-3} \text{ rad}$$

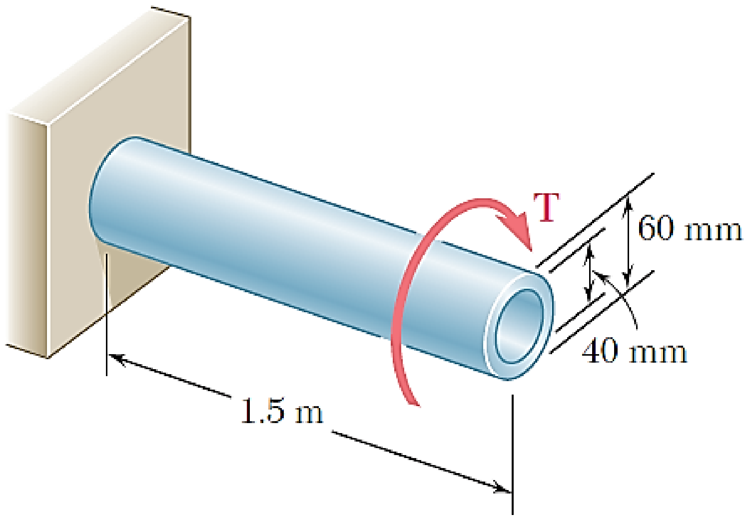
$$J = 1.021 \times 10^{-6} \text{ m}^4$$

$$T = \frac{JG}{L}\phi = \frac{(1.021 \times 10^{-6} \text{ m}^4)(77 \times 10^9 \text{ Pa})}{1.5 \text{ m}}(34.9 \times 10^{-3} \text{ rad})$$

$$T = 1.829 \times 10^3 \text{ N} \cdot \text{m} = 1.829 \text{ kN} \cdot \text{m}$$

Example

What angle of twist will create a shearing stress of 70 MPa on the inner surface of the hollow steel shaft?
Use the value $G = 77$ Gpa.



$$\gamma_{\min} = \frac{\tau_{\min}}{G} = \frac{70 \times 10^6 \text{ Pa}}{77 \times 10^9 \text{ Pa}} = 909 \times 10^{-6}$$

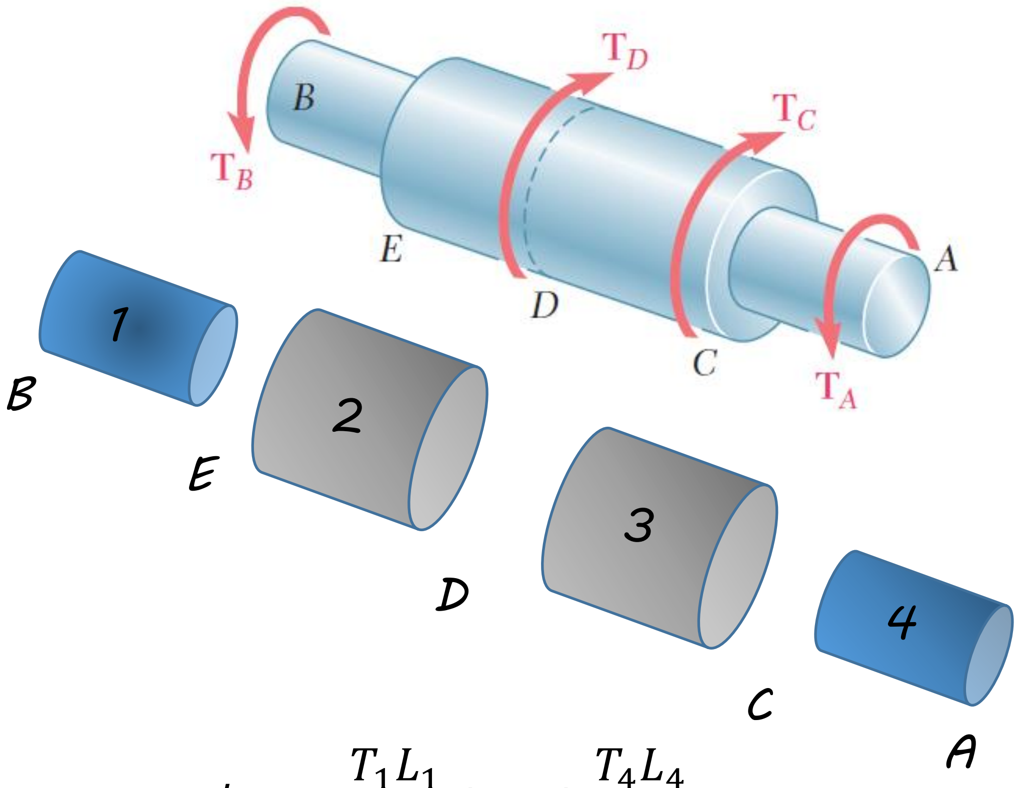
$$\phi = \frac{L\gamma_{\min}}{c_1} = \frac{1500 \text{ mm}}{20 \text{ mm}}(909 \times 10^{-6}) = 68.2 \times 10^{-3} \text{ rad}$$

To find ϕ in degree

$$\phi = (68.2 \times 10^{-3} \text{ rad})\left(\frac{360°}{2\pi \text{ rad}}\right) = 3.91°$$

Angle of Twist for complex shaft

$$\phi = \sum_i \frac{T_i L_i}{J_i G_i}$$

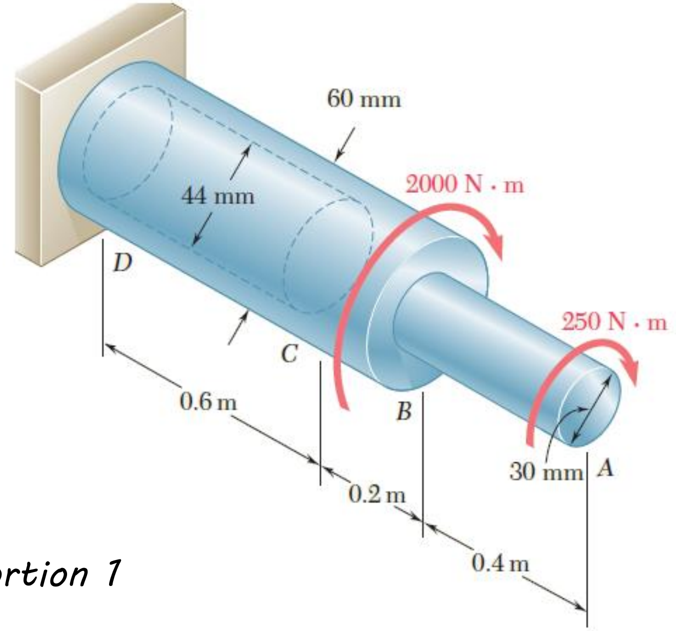


The total angle of twist of the shaft, i.e., the angle through which end A rotates with respect to end B, is obtained by adding algebraically the angles of twist of each component part.

$$\phi_{AB} = \frac{T_1 L_1}{J_1 G_1} + \cdots + \frac{T_4 L_4}{J_4 G_4}$$

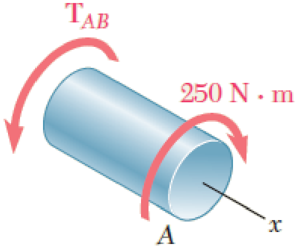
Example

The horizontal shaft AD is attached to a fixed base at D and is subjected to the torques shown. A 44-mm-diameter hole has been drilled into portion CD of the shaft. Knowing that the entire shaft is made of steel for which G = 77 GPa, determine the angle of twist at end A.



$$\phi = \sum_i \frac{T_i L_i}{J_i G_i}$$

$\Sigma M = 0$: $(250 \text{ N} \cdot \text{m}) - T_{AB} = 0$ $\quad T_{AB} = 250 \text{ N} \cdot \text{m}$ Portion 1



section BC

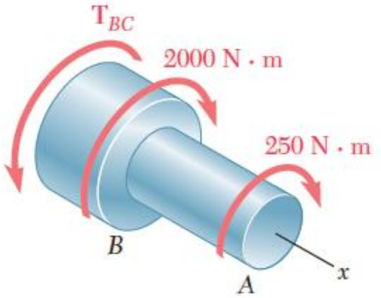
$\Sigma M = 0$: $(250 \text{ N} \cdot \text{m}) + (2000 \text{ N} \cdot \text{m}) - T_{BC} = 0$ $\quad T_{BC} = 2250 \text{ N} \cdot \text{m}$ Portion 2

$T_{CD} = T_{BC} = 2250 \text{ N} \cdot \text{m}$ Portion 3

$$J_{AB} = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.015 \text{ m})^4 = 0.0795 \times 10^{-6} \text{ m}^4$$ Portion 1

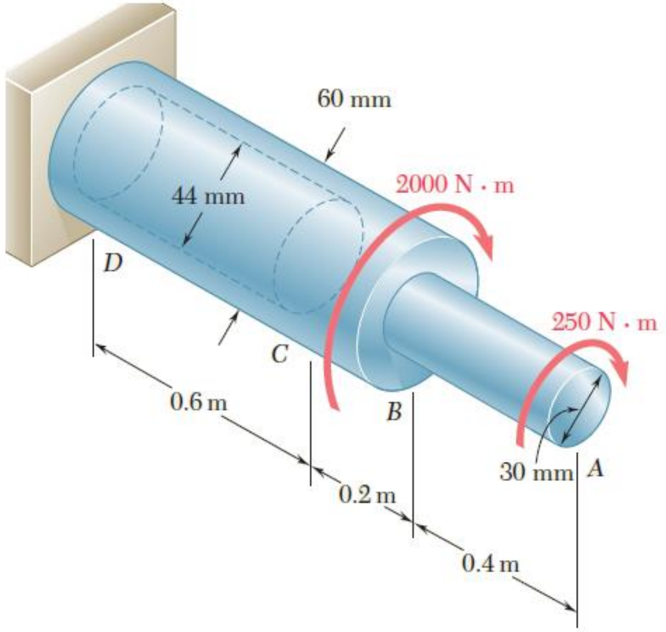
$$J_{BC} = \frac{\pi}{2} c^4 = \frac{\pi}{2}(0.030 \text{ m})^4 = 1.272 \times 10^{-6} \text{ m}^4$$ Portion 2

$$J_{CD} = \frac{\pi}{2}(c_2^4 - c_1^4) = \frac{\pi}{2}[(0.030 \text{ m})^4 - (0.022 \text{ m})^4] = 0.904 \times 10^{-6} \text{ m}^4$$ Portion 3



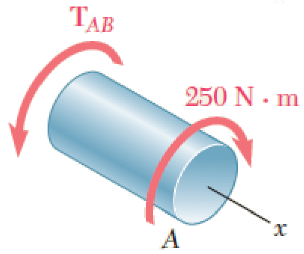
Example

The horizontal shaft AD is attached to a fixed base at D and is subjected to the torques shown. A 44-mm-diameter hole has been drilled into portion CD of the shaft. Knowing that the entire shaft is made of steel for which G = 77 GPa, determine the angle of twist at end A.



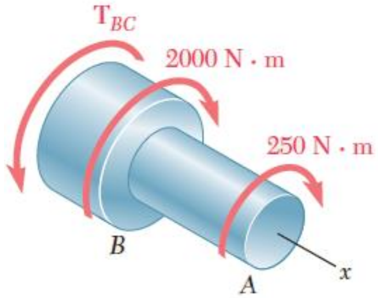
$$\phi = \sum_i \frac{T_i L_i}{J_i G_i}$$

$$\phi_A = \frac{T_1 L_1}{J_1 G_1} + \frac{T_2 L_2}{J_2 G_2} + \frac{T_3 L_3}{J_3 G_3} = \frac{1}{G}\left(\frac{T_1 L_1}{J_1} + \frac{T_2 L_2}{J_2} + \frac{T_3 L_3}{J_3}\right)$$



$$\phi_A = \frac{1}{77\ \text{GPa}}\left[\frac{(250\ \text{N}\cdot\text{m})(0.4\ \text{m})}{0.0795 \times 10^{-6}\ \text{m}^4} + \frac{(2250)(0.2)}{1.272 \times 10^{-6}} + \frac{(2250)(0.6)}{0.904 \times 10^{-6}}\right]$$

$$= 0.01634 + 0.00459 + 0.01939 = 0.0403\ \text{rad}$$



In degrees $\boxed{\phi_A = 2.31^o}$

Mechanical Deign (Torsion) Problem Solution

Mechatronics Engineering
Hashemite University
Dr. Mohammad Hayajneh

Problem and solution

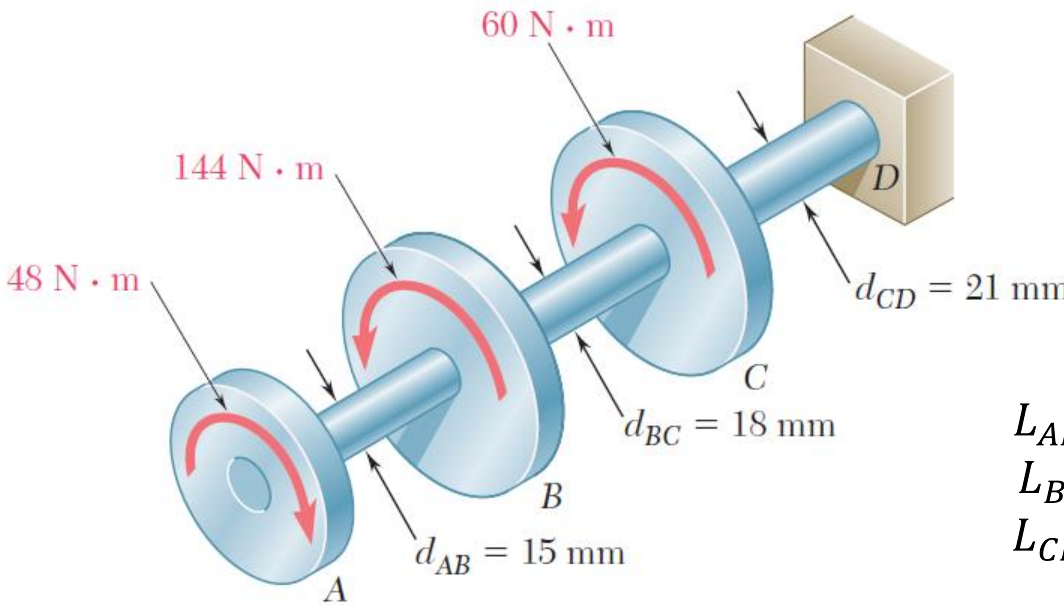
Knowing that each of the shafts AB, BC, and CD consists of a solid circular rod made of brass (G=39GPa), determine

(a) the shaft in which the maximum shearing stress occurs.

(b) the magnitude of that stress.

(c) The angle of twist between A and B.

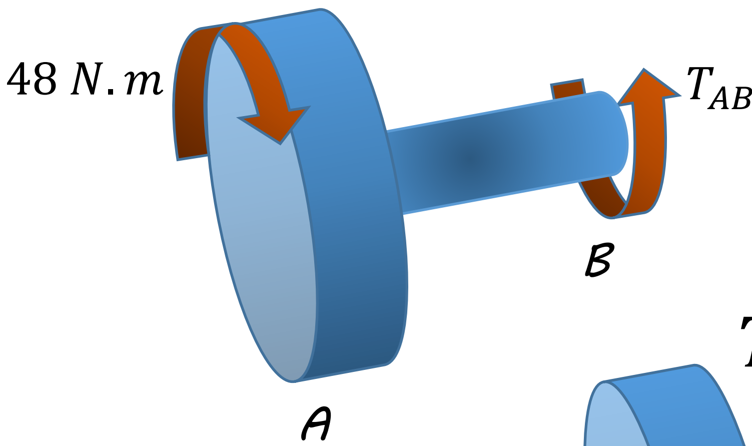
(d) The angle of twist between A and D.



$$L_{AB} = 300\ mm$$
$$L_{BC} = 400mm$$
$$L_{CD} = 350\ mm$$

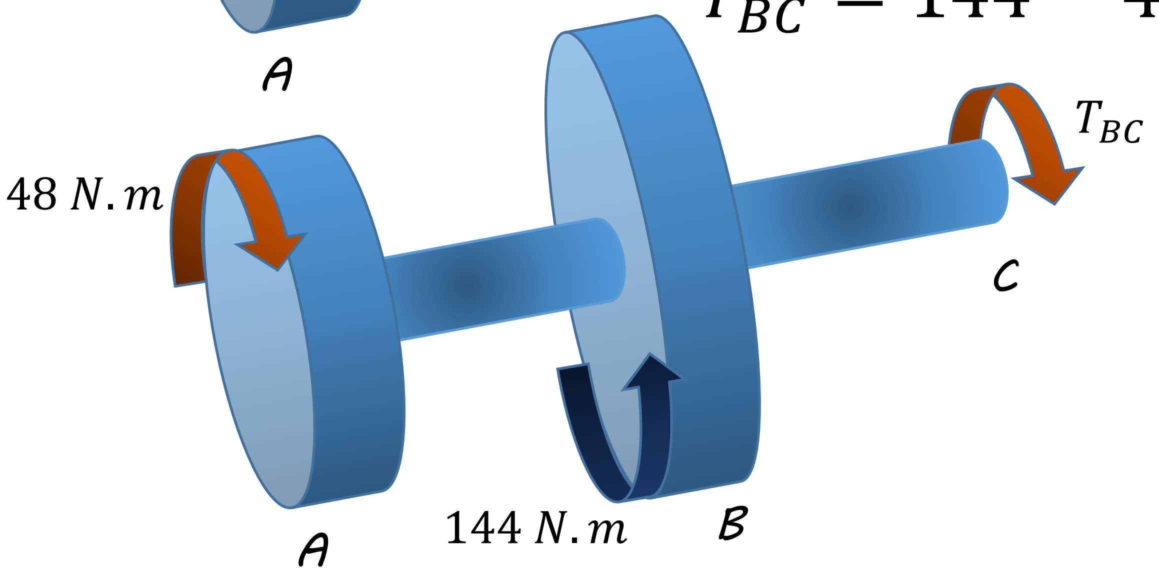
Problem and solution

Calculating torque on each shaft



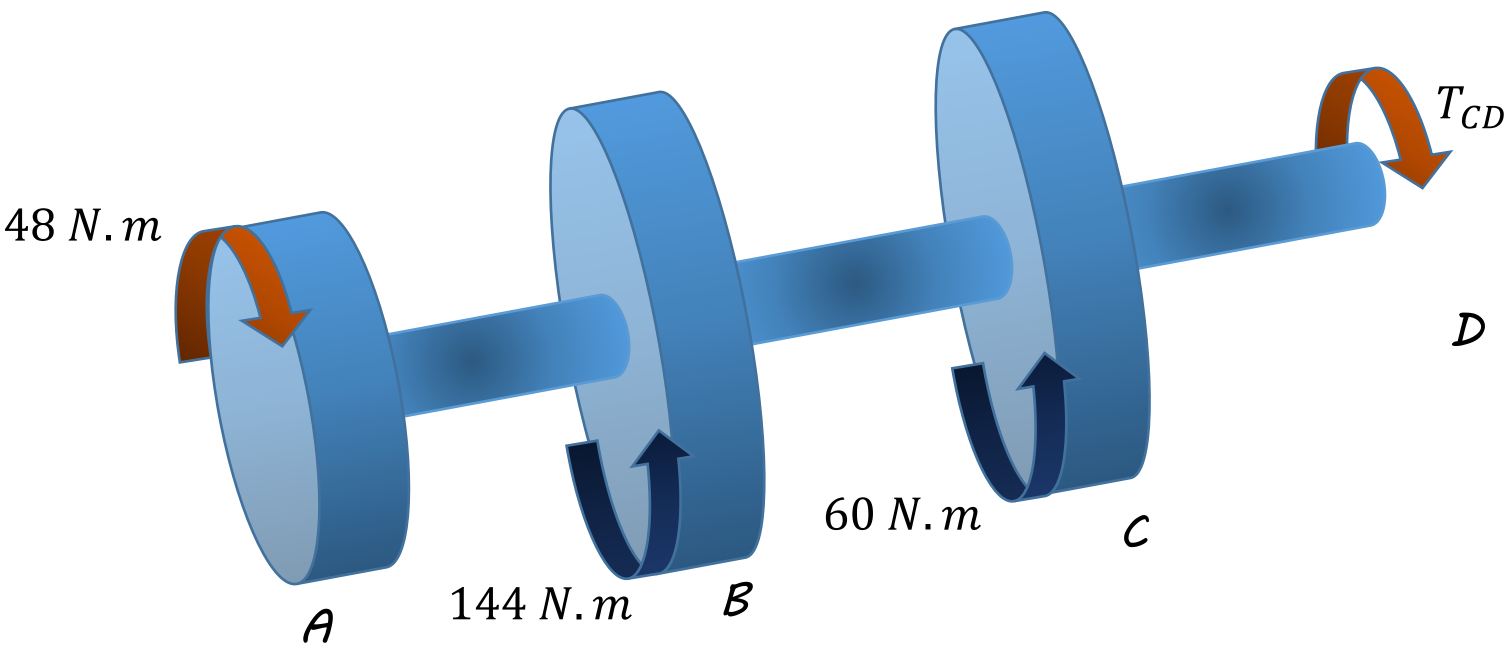
$$T_{AB} = 48\ N.m$$

$$T_{BC} = 144 - 48 = 96\ N.m$$



Problem and solution

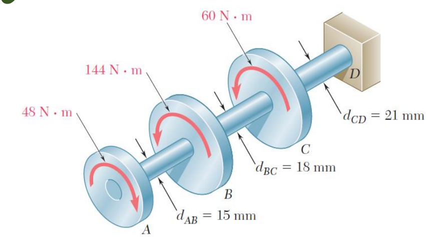
Calculating torque on each shaft



$$T_{CD} = 144 + 60 - 48 = 156\ N.m$$

Problem and solution

Calculating Max shearing stress on each shaft



$$\tau_{max} = \frac{Tc}{J}$$

Shaft AB

$$T_{AB} = 48\ N.m \qquad c_{AB} = \frac{d_{AB}}{2} = \frac{15}{2} = 7.5\ mm \qquad J_{AB} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (7.5)^4 \times 10^{-12}$$

$$\tau_{max} = \frac{T_{AB} c_{AB}}{J_{AB}} = \frac{48 \times 7.5 \times 10^{-3}}{\frac{\pi}{2} (7.5)^4 \times 10^{-12}} = \boxed{72.4\ MPa}$$

Shaft BC

$$T_{BC} = 96\ N.m \qquad c_{BC} = \frac{18}{2} = 9\ mm \qquad J_{BC} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (9)^4 \times 10^{-12}$$

$$\tau_{max} = \frac{T_{BC} c_{BC}}{J_{BC}} = \frac{96 \times 9 \times 10^{-3}}{\frac{\pi}{2} (9)^4 \times 10^{-12}} = \boxed{83.8\ MPa}$$

Shaft CD

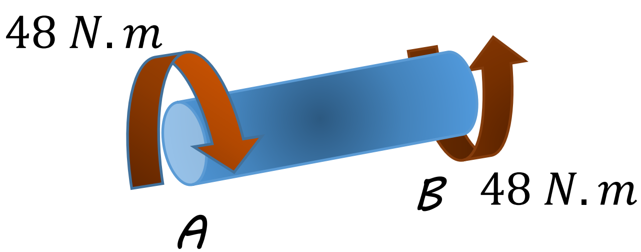
$$T_{CD} = 96\ N.m \qquad c_{CD} = \frac{21}{2} = 10.5\ mm \qquad J_{CD} = \frac{\pi}{2} c^4 = \frac{\pi}{2} (10.5)^4 \times 10^{-12}$$

$$\tau_{max} = \frac{T_{BC} c_{BC}}{J_{BC}} = \frac{156 \times 10.5 \times 10^{-3}}{\frac{\pi}{2} (10.5)^4 \times 10^{-12}} = \boxed{85.8\ MPa}$$

maximum shearing stress occurs on Shaft CD

Problem and solution

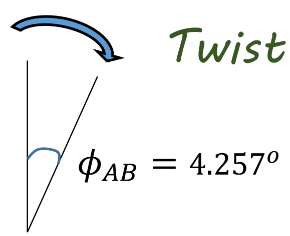
Calculating angle of twist on shaft AB



$$\phi_{AB} = \frac{T_{AB}L_{AB}}{GJ_{AB}} = \frac{48 \times 0.3}{39 \times 10^9 \times \frac{\pi}{2}(7.5)^4 \times 10^{-12}} = 0.0743\, rad$$

$$= 4.257^o$$

Twist to the right



Problem and solution

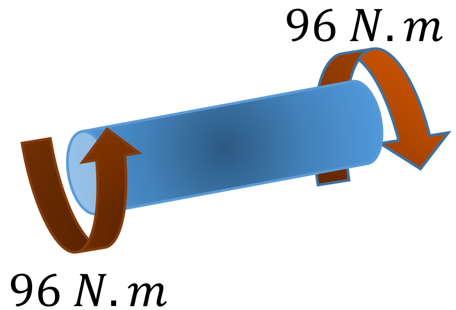
Calculating angle of twist between A and D

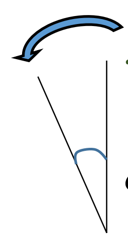
1) Angle of twist between A and B? Done

2) Angle of twist between B and C?

$$\phi_{BC} = \frac{T_{BC}L_{BC}}{GJ_{BC}} = \frac{96 \times 0.4}{39 \times 10^9 \times \frac{\pi}{2}(9)^4 \times 10^{-12}} = 0.0955\, rad$$

$$= 5.474^o$$





Twist to the left

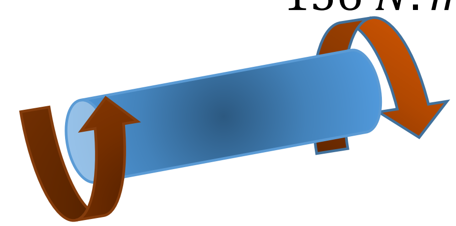
$\phi_{BC} = -5.474^o$

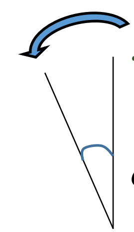
Problem and solution

3) Angle of twist between C and D?

$$\phi_{CD} = \frac{T_{CD}L_{CD}}{GJ_{CD}} = \frac{156 \times 0.35}{39 \times 10^9 \times \frac{\pi}{2}(10.5)^4 \times 10^{-12}} = 0.0733\, rad$$

$$= 4.201^o$$





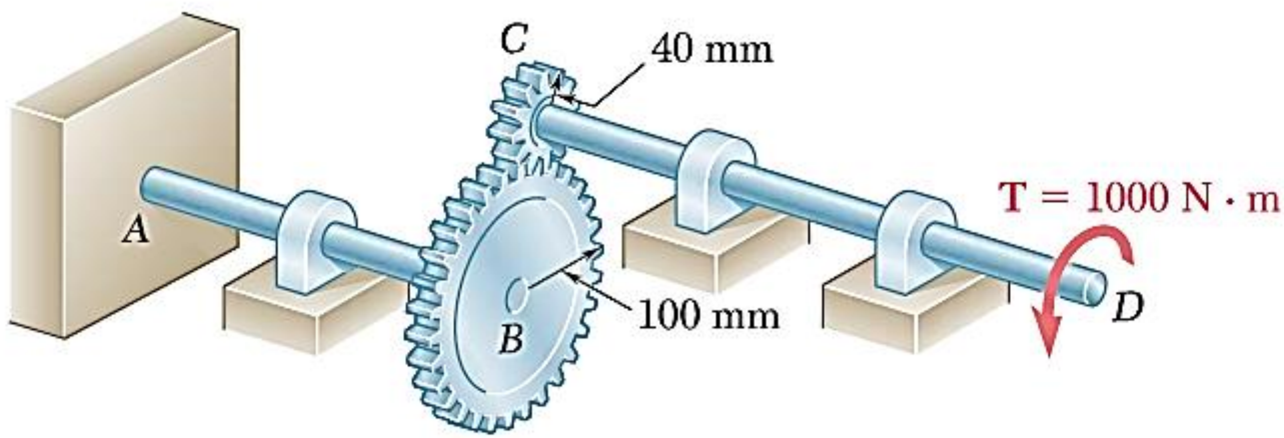
The overall twist angle between A and D is

We consider the angle which twist to the right is +ve and the angle which twist to the left is -ve

$$\phi_{AD} = 4.257^o - 5.474^o - 4.201^o = -5.418^o$$

Shafts with gears

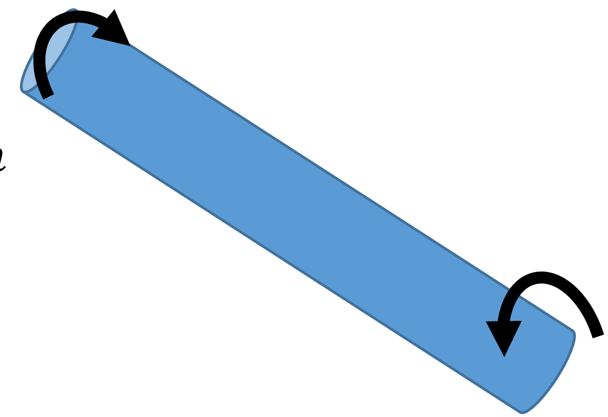
3.21 A torque of magnitude $T = 1000$ N · m is applied at D as shown. Knowing that the diameter of shaft AB is 56 mm and that the diameter of shaft CD is 42 mm, determine the maximum shearing stress in (*a*) shaft AB, (*b*) shaft CD.



Shafts with gears

$$\tau_{max} = \frac{Tc}{J}$$

$$T_{AB} = \frac{r_B}{r_C} T_{CD} = \frac{100}{40}(1000) = 2500N.m$$



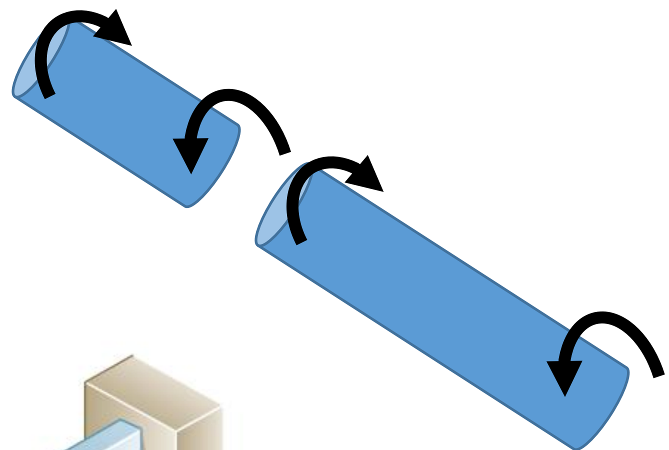
Max. Shearing stress in AB

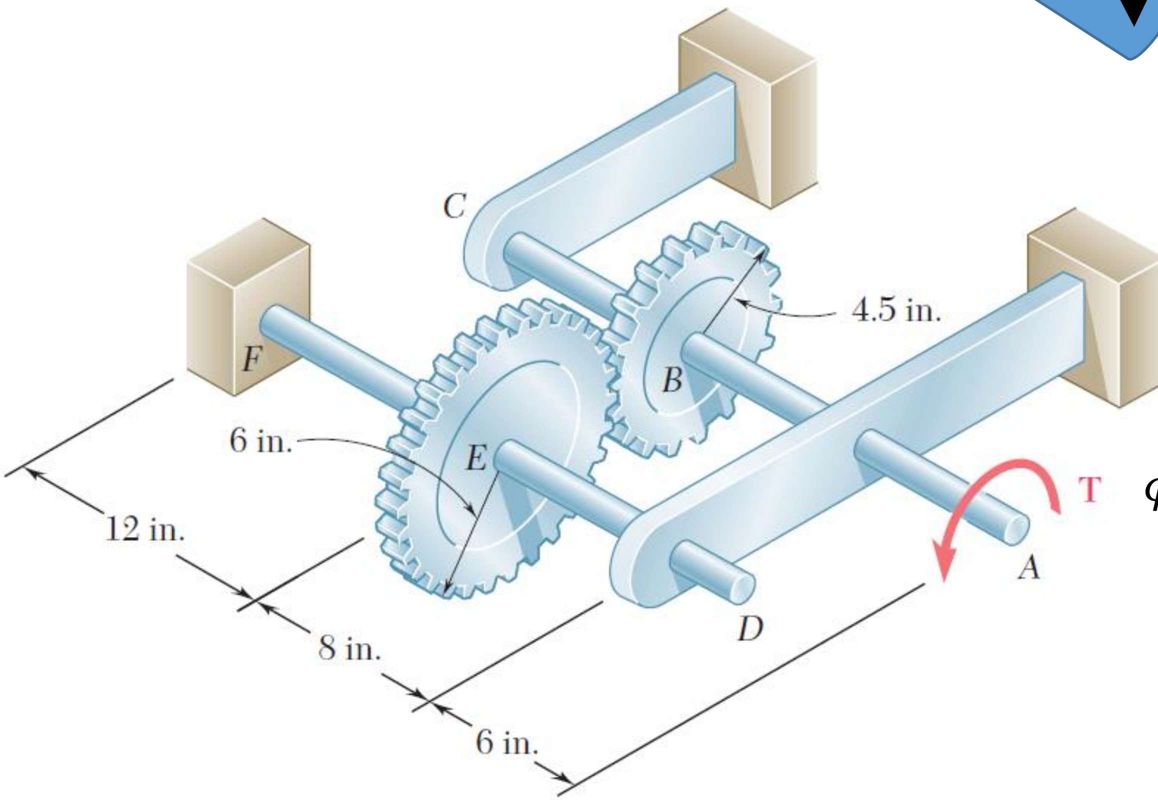
$$\tau_{max} = \frac{2500\ (0.028)}{\frac{\pi}{2}(0.028)^4} = 72.5\ MPa$$

Max. Shearing stress in CD

$$\tau_{max} = \frac{1000\ (0.021)}{\frac{\pi}{2}(0.021)^4} = 68.7\ MPa$$

Shafts with gears (twist angle)



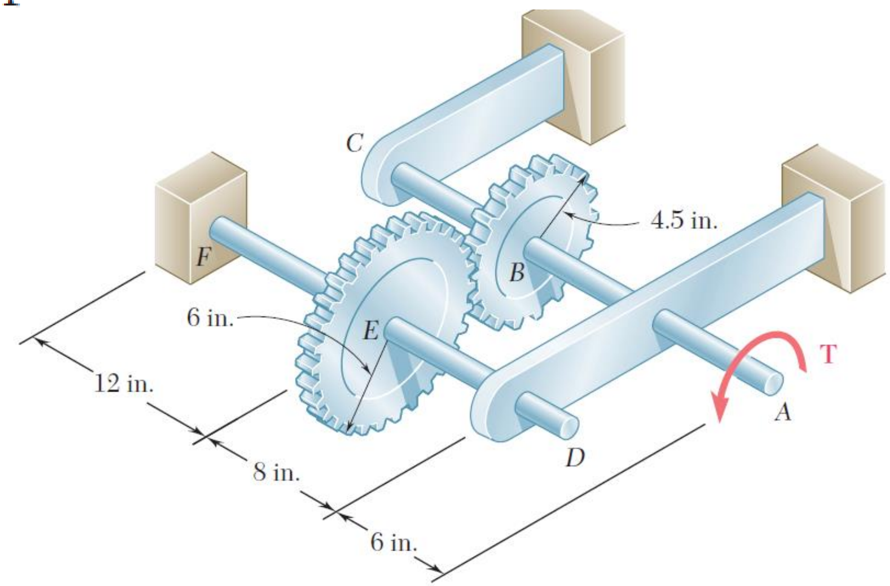


$$\phi_{AC} = \phi_{AB} + \phi_{BC} = \frac{TL_{AC}}{GJ}$$

$$\phi_{EF} = \frac{T_E L_{EF}}{GJ}$$

Shafts with gears (twist angle)

Two shafts, each of $\frac{7}{8}$-in. diameter, are connected by the gears shown. Knowing that $G = 11.2 \times 10^6$ psi and that the shaft at F is fixed, determine the angle through which end A rotates when a 1.2 kip · in. torque is applied at A.



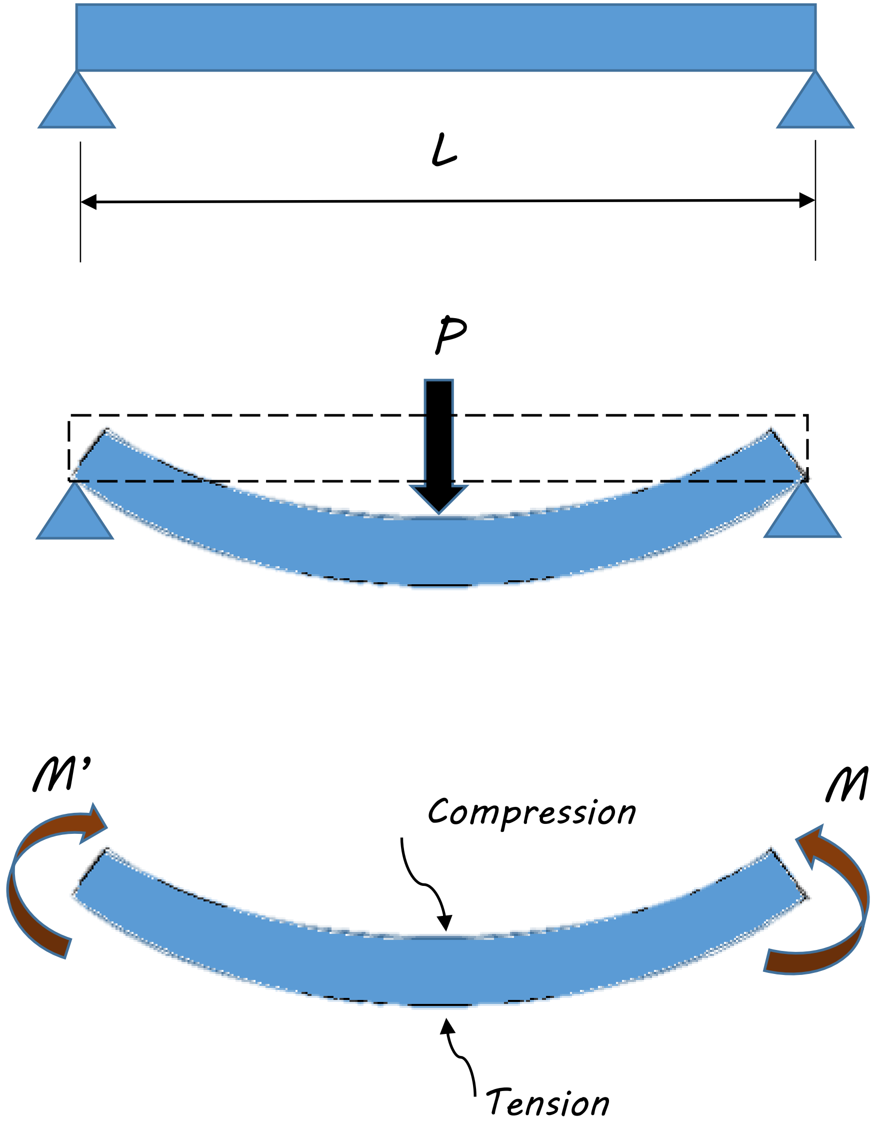
Pure Bending

Dr. Mohammad Hayajneh

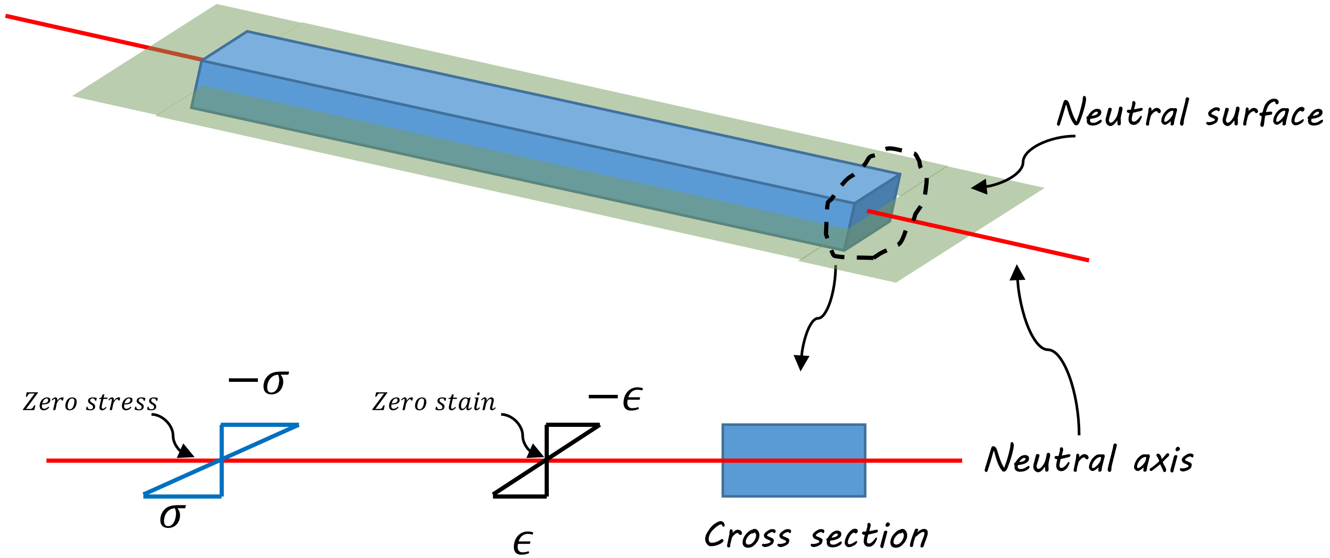
Mechanical Design

Mechatronics Engineering, The Hashemite University

Bending definition



prismatic members subjected to equal and opposite couples M and M' acting in the same longitudinal plane. Such members are said to be in ***pure bending***.



There must exist a surface parallel to the upper and lower faces of the member, where σ and ϵ are zero. This surface is called the ***neutral surface***.

Bending stress

$$F_T = F_c = \sigma A = \sigma \frac{1}{2}\frac{d}{2} b = \frac{\sigma\, bd}{4}$$

After we have the forces, we can calculate the applied Moment M

$$M = \frac{\sigma\, bd}{4}\frac{d}{3} + \frac{\sigma\, bd}{4}\frac{d}{3} = \frac{\sigma\, bd}{4}\frac{2d}{3} = \frac{\sigma\, bd^2}{6}$$

Section modulus (S)

Then we have the stress on the member is:

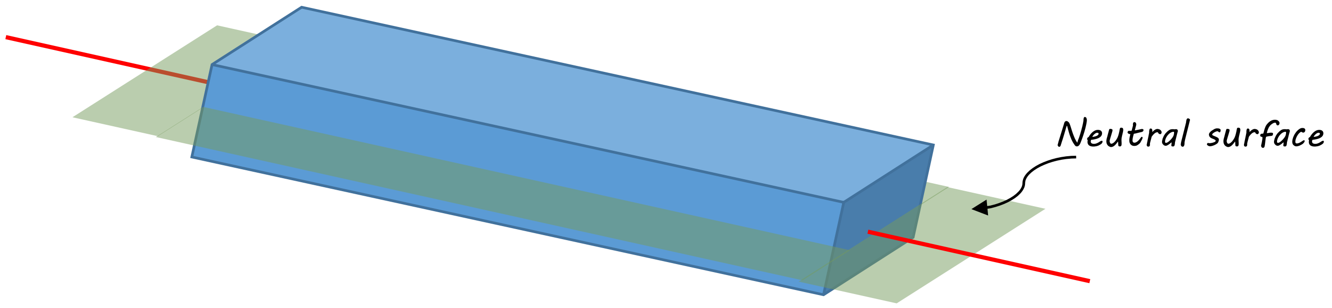
$$\boxed{\sigma = \frac{M}{S}}$$

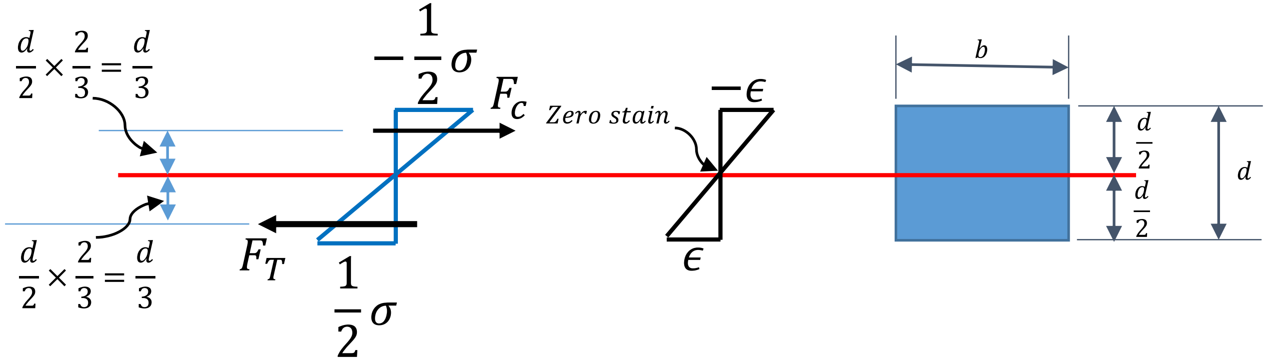
This is the maximum stress

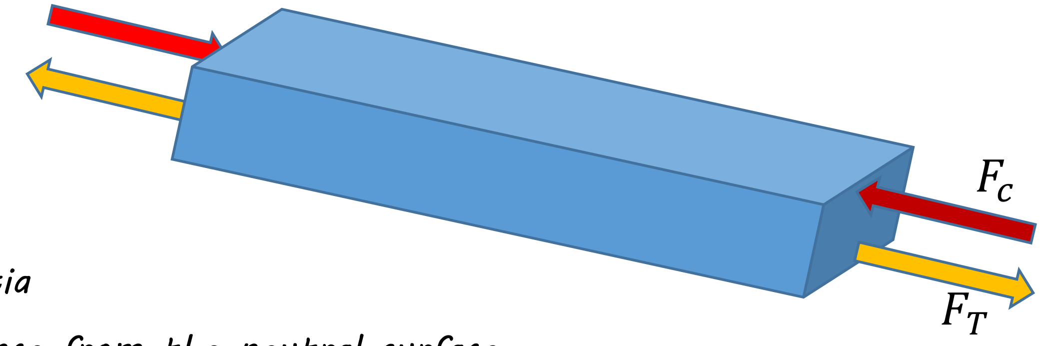
$$S = \frac{I}{c}$$

I Moment of inertia

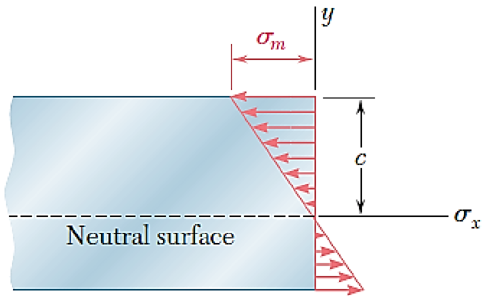
c the largest distance from the neutral surface







Bending stress



$$\sigma_m = \frac{M}{S}$$

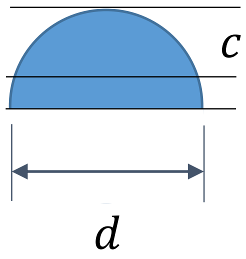
$$\sigma_x = y\frac{\sigma_m}{c}$$

$$S = \frac{I}{c}$$

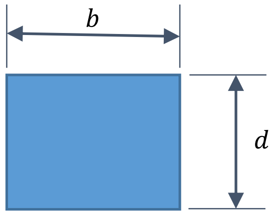
I — Moment of inertia

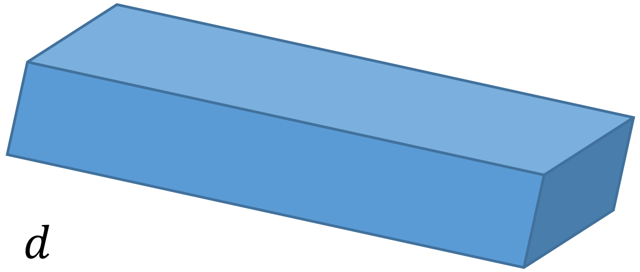
c — the largest distance from the neutral surface

Moment of inertia or second moment refers to rotational inertia

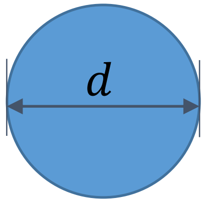


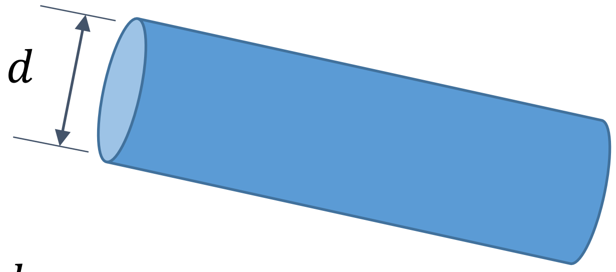
$$I = \frac{\pi}{8}\left(\frac{d}{2}\right)^4 \qquad c = \frac{d}{2} - \frac{2d}{3\pi}$$



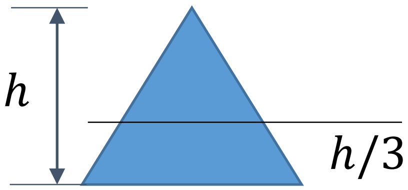


$$I = \frac{bd^3}{12} \qquad c = \frac{d}{2}$$



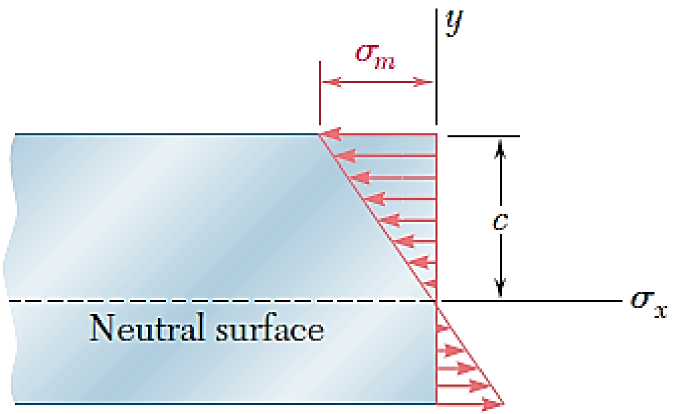


$$I = \frac{\pi}{4}\left(\frac{d}{2}\right)^4 \qquad c = \frac{d}{2}$$



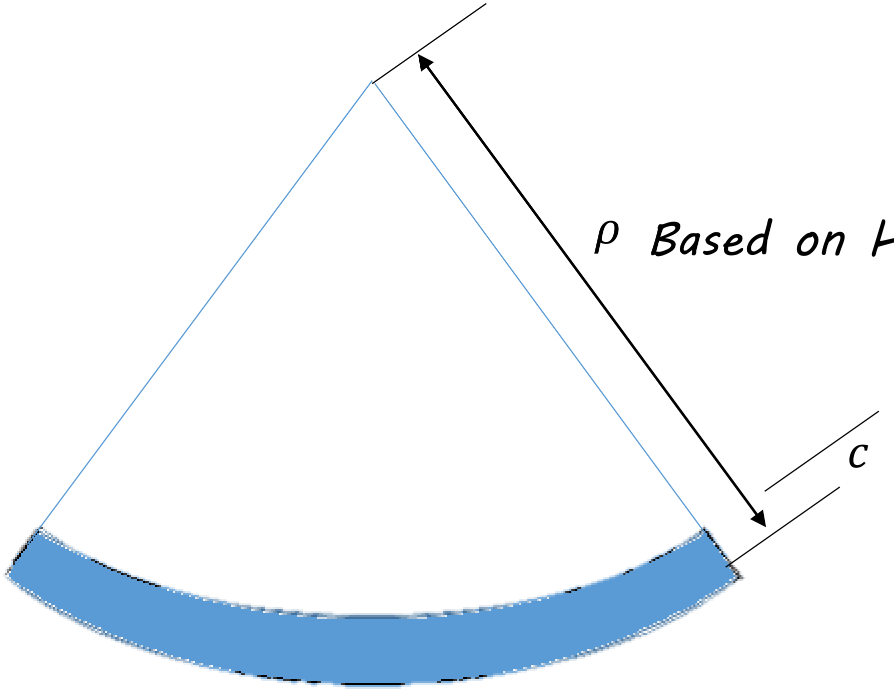
$$I = \frac{bh^3}{36} \qquad c = \frac{2h}{3}$$

Stresses and deformations in the Elastic range



$$\sigma_x = y\frac{\sigma_m}{c}$$

In the elastic range, the strain varies linearly with the distance from the neutral surface



Based on Hook's law

$$\epsilon_m = \frac{c}{\rho}$$

$$\epsilon_x = \frac{y}{\rho}$$

Based on Hook's law

$$\sigma_x = E\epsilon_x \qquad \sigma_m = E\epsilon_m$$

Then

$$\epsilon_x = \frac{y}{c}\epsilon_m$$

Example

*An aluminum rod with a semicircular cross section of radius r = 12 mm is bent into the shape of a circular arc of mean radius r = 2.5 m. **Knowing that the flat face of the rod is turned toward the center of curvature** of the arc, determine the maximum tensile and compressive stress in the rod. Use E = 70 GPa.*

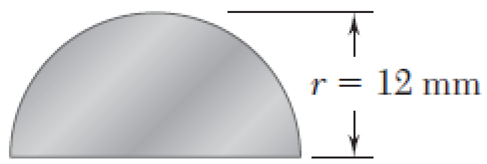
$$\bar{y} = \frac{4r}{3\pi} = \frac{4(12\text{ mm})}{3\pi} = 5.093\text{ mm}$$

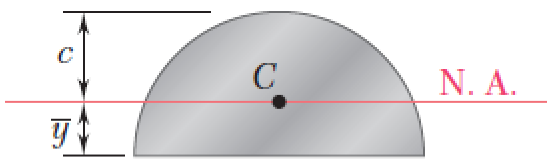
$$c = r - \bar{y} = 12\text{ mm} - 5.093\text{ mm} = 6.907\text{ mm}$$

$$\epsilon_m = \frac{c}{\rho} = \frac{6.907 \times 10^{-3}\text{ m}}{2.5\text{ m}} = 2.763 \times 10^{-3}$$

$$\sigma_m = E\epsilon_m = (70 \times 10^9\text{ Pa})(2.763 \times 10^{-3}) = 193.4\text{ MPa}$$

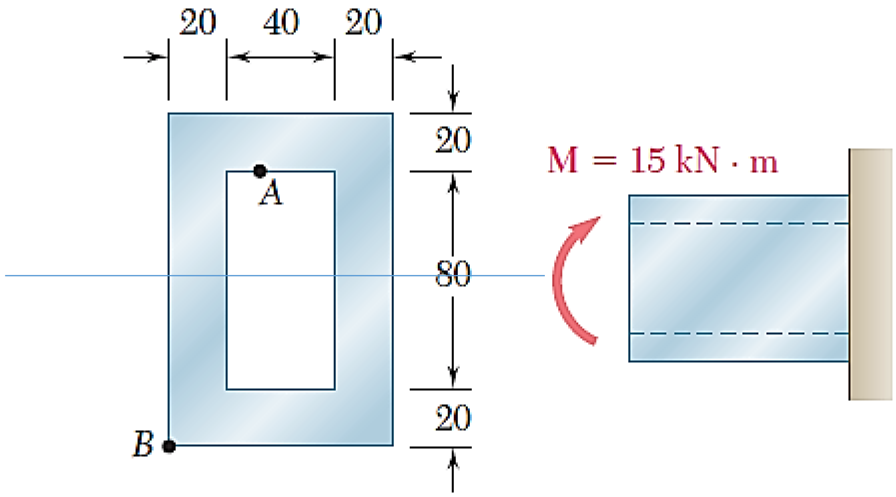
$$\sigma_{\text{comp}} = -\frac{\bar{y}}{c}\sigma_m = -\frac{5.093\text{ mm}}{6.907\text{ mm}}(193.4\text{ MPa})$$
$$= -142.6\text{ MPa}$$





Problem: Knowing that the couple shown acts in a vertical plane, determine the stress at (a) point A, (b) point B.

$$I = \frac{bd^3}{12}$$



Dimensions in mm

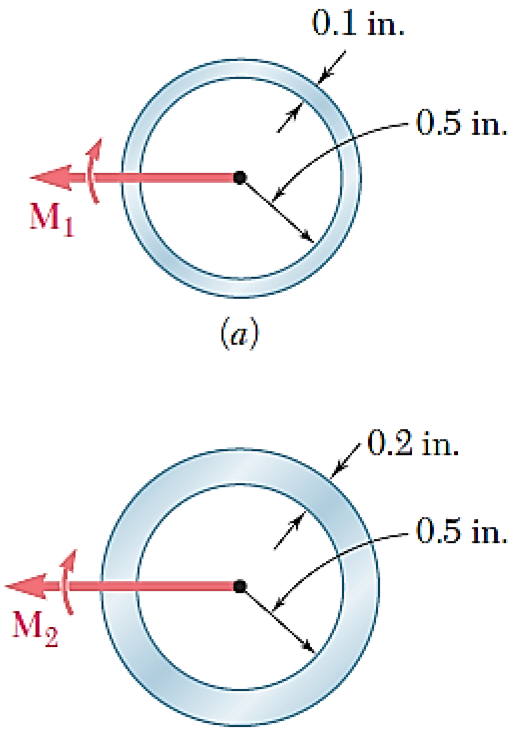
$$I_{all} = I_l - I_s = \frac{80(120)^3}{12} - \frac{40(80)^3}{12} = 9.8133 \times 10^6 mm^4$$

$$c = 60\ mm$$

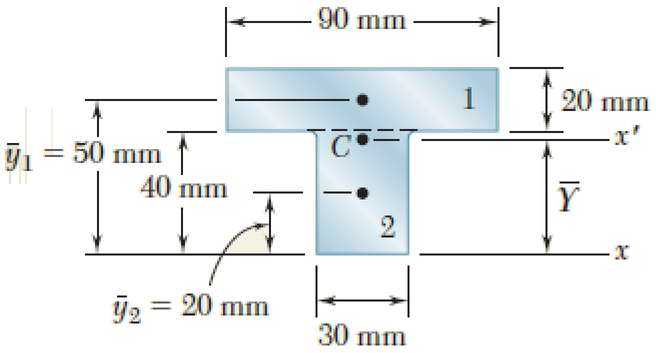
$$\sigma_m = \sigma_B = \frac{M}{\frac{I}{c}} = \frac{15000}{\frac{9.8133 \times 10^6 mm^4}{60\ mm}} = 91.7MPa$$

$$\sigma_x = \sigma_A = -40\frac{91.7MPa}{60} = -61.1\ MPa$$

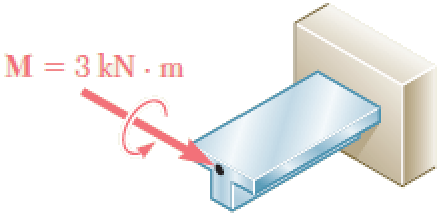
Problem: Knowing that the couple shown acts in a vertical plane, determine the stress at (a) point A, (b) point B.

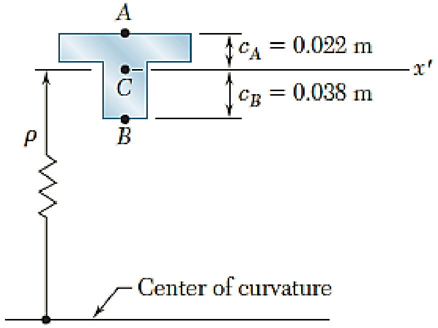


Problem: A cast-iron machine part is acted upon by the 3 kN ? m couple shown. Knowing that E = 165 GPa and neglecting the effect of fillets, determine (a) the maximum tensile and compressive stresses in the casting, (b) the radius of curvature of the casting.



$$I = 868 \times 10^{-9}\ \mathrm{m}^4$$





a. Maximum Tensile Stress. Since the applied couple bends the casting downward, the center of curvature is located below the cross section. The maximum tensile stress occurs at point A, which is farthest from the center of curvature.

$$\sigma_A = \frac{Mc_A}{I} = \frac{(3\ \mathrm{kN \cdot m})(0.022\ \mathrm{m})}{868 \times 10^{-9}\ \mathrm{m}^4} \qquad \sigma_A = +76.0\ \mathrm{MPa}$$

Maximum Compressive Stress. This occurs at point B; we have

$$\sigma_B = -\frac{Mc_B}{I} = -\frac{(3\ \mathrm{kN \cdot m})(0.038\ \mathrm{m})}{868 \times 10^{-9}\ \mathrm{m}^4} \qquad \sigma_B = -131.3\ \mathrm{MPa}$$

b. Radius of Curvature. From Eq. (4.21), we have

$$\frac{1}{\rho} = \frac{M}{EI} = \frac{3\ \mathrm{kN \cdot m}}{(165\ \mathrm{GPa})(868 \times 10^{-9}\ \mathrm{m}^4)}$$
$$= 20.95 \times 10^{-3}\ \mathrm{m}^{-1} \qquad \rho = 47.7\ \mathrm{m}$$

Screws, Fasteners, and the Design of Nonpermanent Joints

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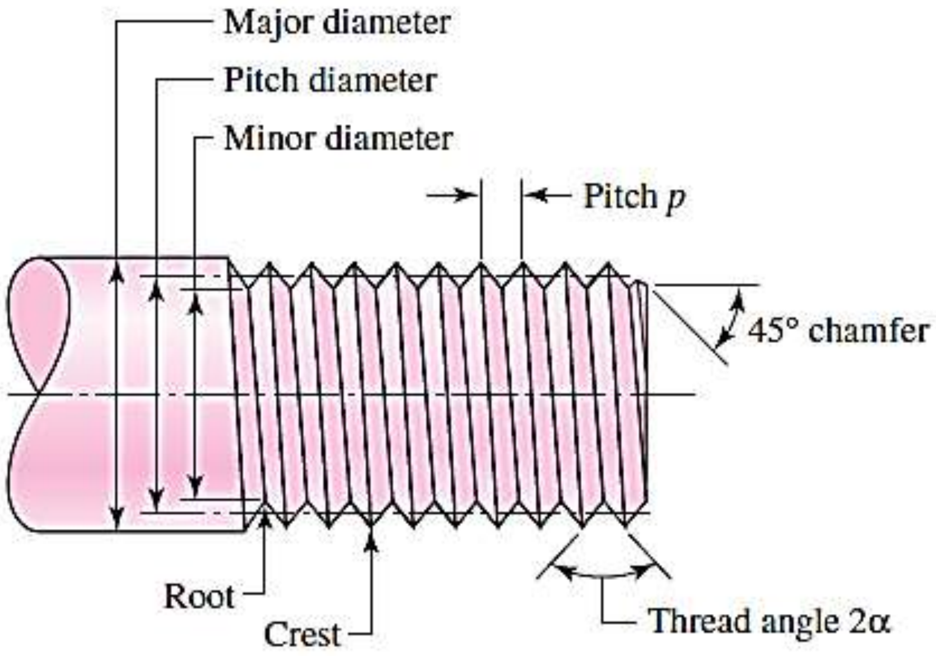
Thread Standards and Definitions

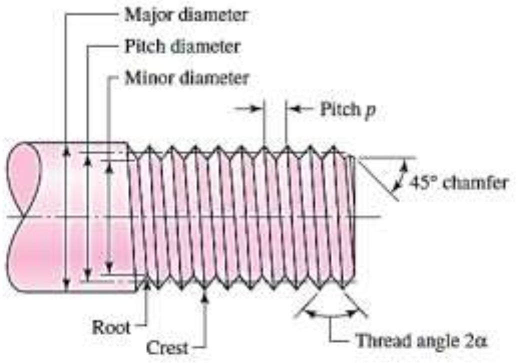
The terminology of screw threads explained as follows:

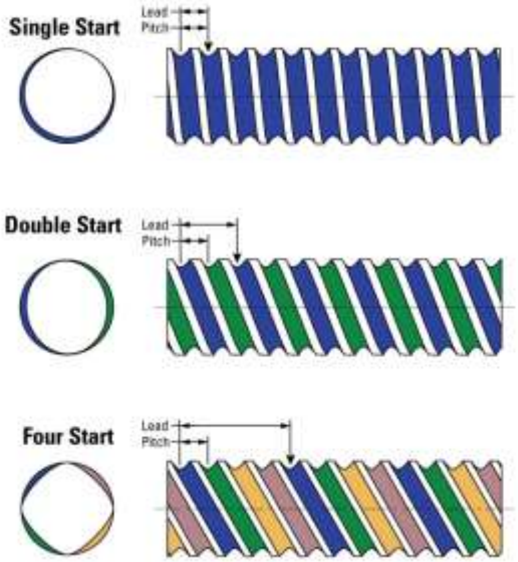
- The **pitch** is the distance between adjacent thread forms measured parallel to the thread axis.
- **The pitch in metric is the reciprocal of the number of thread forms per inch N .**
- The **major diameter d** is the largest diameter of a screw thread.
- The **minor (or root) diameter** $\mathbf{d_r}$ is the smallest diameter of a screw thread.
- The **pitch diameter** $\mathbf{d_p}$ is a theoretical diameter between the major and minor diameters.



Thread Standards and Definitions

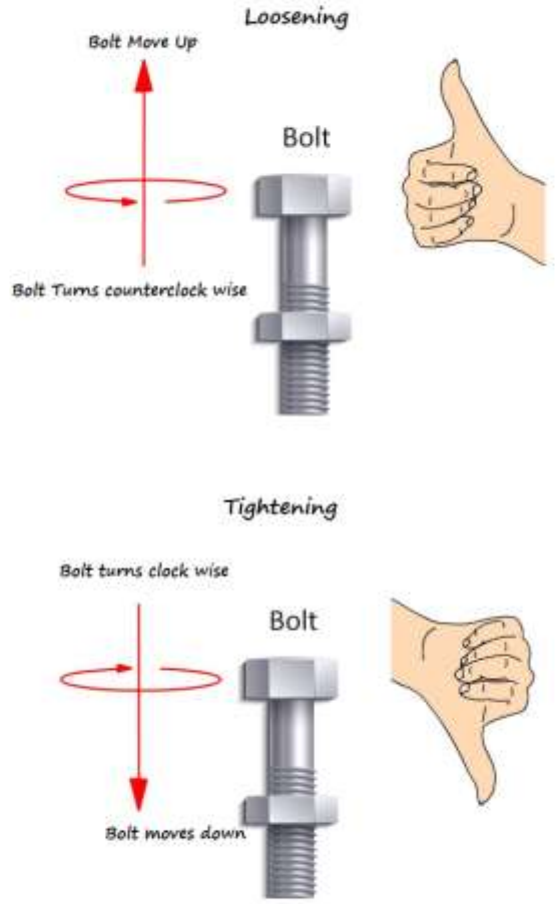
- *The* ***lead (l)****, not shown, is the distance the nut moves parallel to the screw axis when the nut is given one turn.*
  - *For example: For a single thread, the lead is the same as the pitch value. (for single thread screw $l = p$)*
- *A multiple-threaded product is one having* ***two or more threads cut beside each other*** *(imagine two or more strings wound side by side around a pencil).*
- ***Standardized products such as screws, bolts, and nuts all have single threads.***
- *a double-threaded screw has a lead equal to twice the pitch, a triple-threaded screw has a lead equal to 3 times the pitch, and so on.*





Thread Standards and Definitions

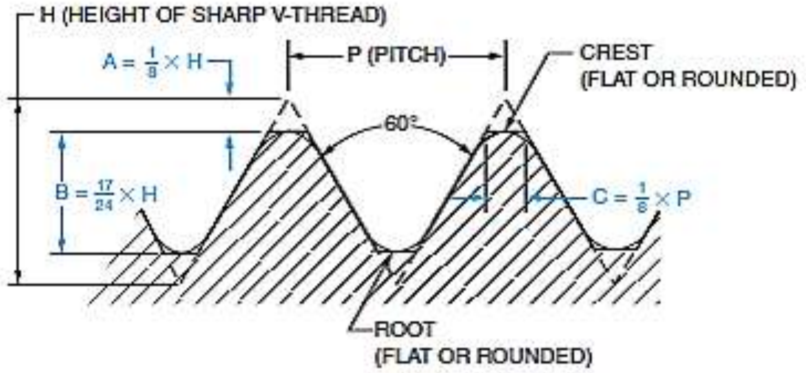
- *All threads are made according to the right-hand rule unless otherwise noted.*
- *That is, if the bolt is turned clockwise, the bolt advances toward the nut.*

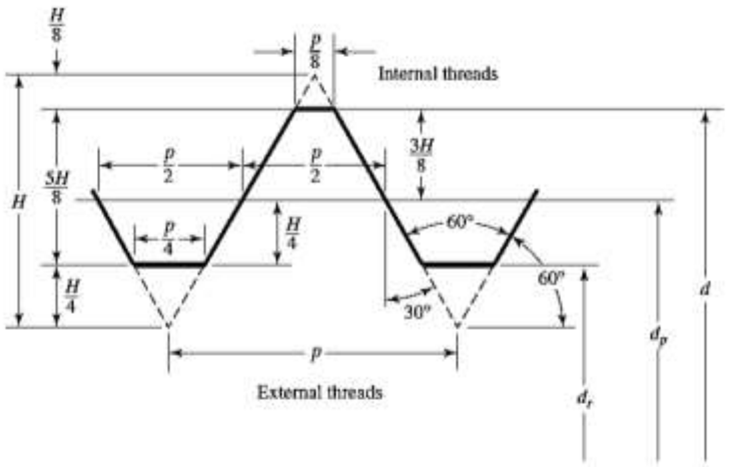


Thread standard

- The *American National (Unified)* thread standard has been approved in this country and in Great Britain for use on all standard threaded products.

- The thread angle is 60^0 and the crests of the thread may be either **flat** or **rounded**.
- Thread geometry of the metric M and MJ profiles.
- The M profile replaces the inch class and is the basic ISO 68 profile with 60◦ symmetric threads
- The MJ profile has a **rounded fillet at the root** of the external thread and a larger minor diameter of both the internal and external threads. This profile is especially useful where high fatigue strength is required.





Metric screw Threads (millimeters)

- Tables 8-1 will be useful in specifying and designing threaded parts. It is used in metric sizes.
- The thread size is specified by giving the pitch p for metric sizes.
- This table provide important information on each screw size such as pitch, Tensile stress area, and minor diameter area.
  - For example: For 14 mm diameter (Major) bolt, you can read the following:
    - Pitch =2 mm
    - Tensile stress Area (A_T)=115 mm^2
    - Minor diameter Area = 104 mm^2
  - Major diameter Area = $\frac{\pi}{4} \times 14^2 = 153.94\, mm^2$

Table 8-1

Diameters and Areas of Coarse-Pitch and Fine-Pitch Metric Threads.*

Nominal Major Diameter d mm	Coarse-Pitch Series: Pitch p mm	Tensile-Stress Area A_t mm²	Minor-Diameter Area A_r mm²	Fine-Pitch Series: Pitch p mm	Tensile-Stress Area A_t mm²	Minor-Diameter Area A_r mm²
1.6	0.35	1.27	1.07			
2	0.40	2.07	1.79			
2.5	0.45	3.39	2.98			
3	0.5	5.03	4.47			
3.5	0.6	6.78	6.00			
4	0.7	8.78	7.75			
5	0.8	14.2	12.7			
6	1	20.1	17.9			
8	1.25	36.6	32.8	1	39.2	36.0
10	1.5	58.0	52.3	1.25	61.2	56.3
12	1.75	84.3	76.3	1.25	92.1	86.0
14	2	115	104	1.5	125	116
16	2	157	144	1.5	167	157
20	2.5	245	225	1.5	272	259
24	3	353	324	2	384	365
30	3.5	561	519	2	621	596
36	4	817	759	2	915	884
42	4.5	1120	1050	2	1260	1230
48	5	1470	1380	2	1670	1630
56	5.5	2030	1910	2	2300	2250
64	6	2680	2520	2	3030	2980
72	6	3460	3280	2	3860	3800
80	6	4340	4140	1.5	4850	4800
90	6	5590	5360	2	6100	6020
100	6	6990	6740	2	7560	7470
110				2	9180	9080

Metric screw Threads (millimeters)

- Tables 8-1 Provide information for two series
  - Coarse Pitch Series
  - Fine Pitch Series
- For the same bolt or screw diameter, The pitch value in coarse series is larger then in fine series

- Metric threads are specified by writing the diameter and pitch in millimeters, in that order.
  - For example: M12 × 1.75 is a thread having a nominal major diameter of 12 mm and a pitch of 1.75 mm.
  - The letter M means metric standard.

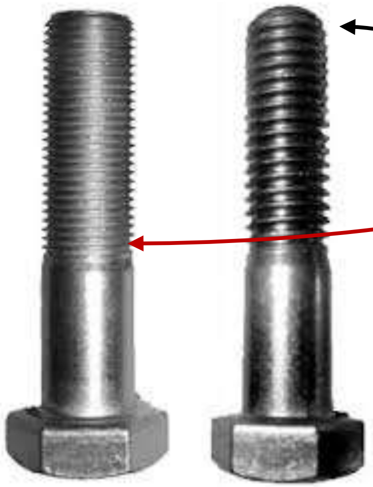


Table 8-1
Diameters and Areas of Coarse-Pitch and Fine-Pitch Metric Threads.*

Nominal Major Diameter d mm	Coarse-Pitch Series: Pitch p mm	Coarse-Pitch Series: Tensile-Stress Area A_t mm²	Coarse-Pitch Series: Minor-Diameter Area A_r mm²	Fine-Pitch Series: Pitch p mm	Fine-Pitch Series: Tensile-Stress Area A_t mm²	Fine-Pitch Series: Minor-Diameter Area A_r mm²
1.6	0.35	1.27	1.07			
2	0.40	2.07	1.79			
2.5	0.45	3.39	2.98			
3	0.5	5.03	4.47			
3.5	0.6	6.78	6.00			
4	0.7	8.78	7.75			
5	0.8	14.2	12.7			
6	1	20.1	17.9			
8	1.25	36.6	32.8	1	39.2	36.0
10	1.5	58.0	52.3	1.25	61.2	56.3
12	1.75	84.3	76.3	1.25	92.1	86.0
14	2	115	104	1.5	125	116
16	2	157	144	1.5	167	157
20	2.5	245	225	1.5	272	259
24	3	353	324	2	384	365
30	3.5	561	519	2	621	596
36	4	817	759	2	915	884
42	4.5	1120	1050	2	1260	1230
48	5	1470	1380	2	1670	1630
56	5.5	2030	1910	2	2300	2250
64	6	2680	2520	2	3030	2980
72	6	3460	3280	2	3860	3800
80	6	4340	4140	1.5	4850	4800
90	6	5590	5360	2	6100	6020
100	6	6990	6740	2	7560	7470
110				2	9180	9080

Unified Screw Threads (Inches)

- *Tables 8-2 will be useful in specifying and designing threaded parts.* ***It is used in Unified sizes.***
- *The thread size is specified by giving the number of threads per inch N .*
- *Two major Unified thread series are in common use: UN and UNR. The difference between these is simply that a root radius must be used in the UNR series.*
- *Unified threads are specified by stating the* ***nominal major diameter, the number of threads per inch, and the thread series.***
  - ***Example:*** $\frac{5}{8}$ ***in-18 UNF or*** *0.625 in-18 UNF*
  - ***F means fine series***

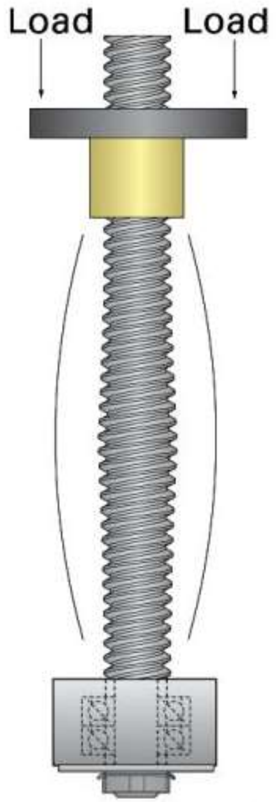
Table 8-2

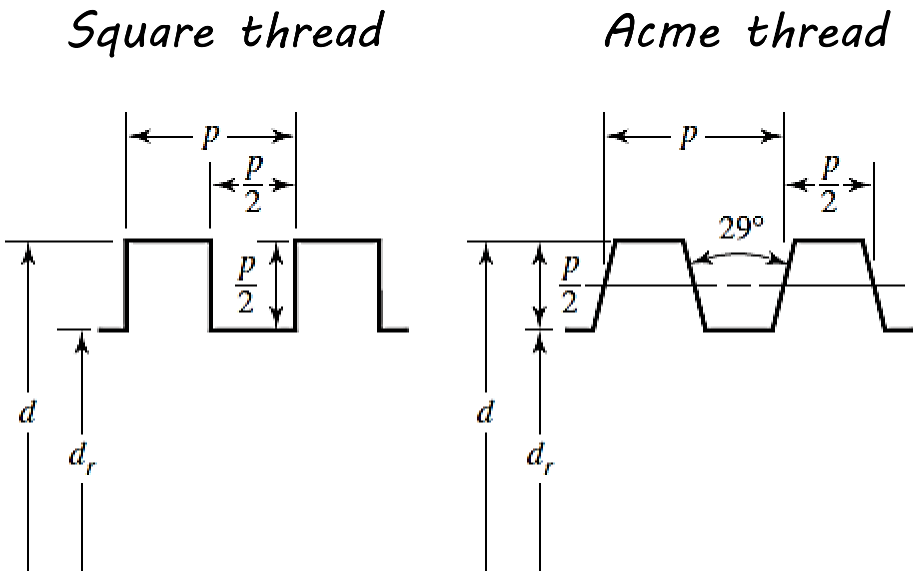
Diameters and Area of Unified Screw Threads UNC and UNF*

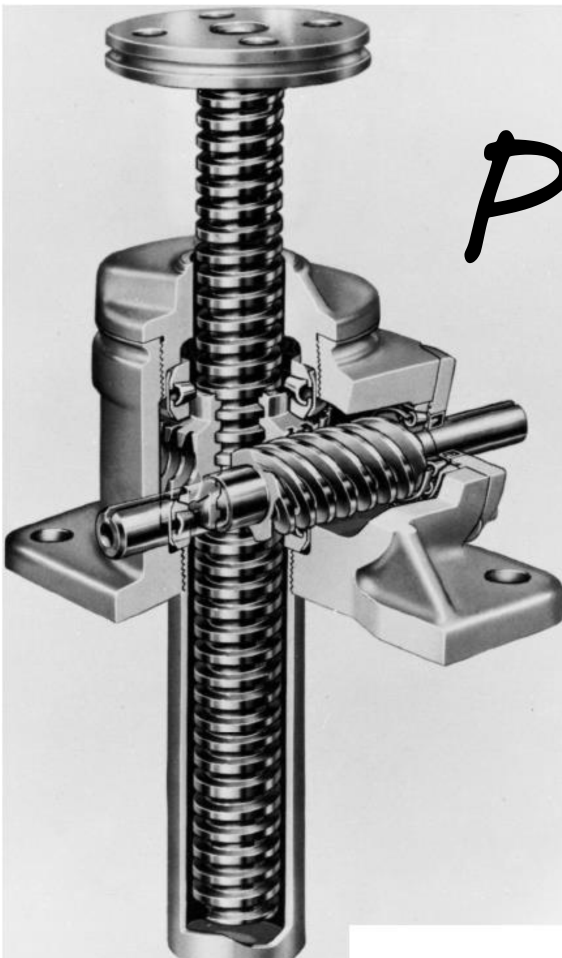
Size Designation	Nominal Major Diameter in	Coarse Series—UNC Threads per Inch N	Coarse Series—UNC Tensile-Stress Area A_t, in²	Coarse Series—UNC Minor-Diameter Area A_r, in²	Fine Series—UNF Threads per Inch N	Fine Series—UNF Tensile-Stress Area A_t, in²	Fine Series—UNF Minor-Diameter Area A_r, in²
0	0.0600				80	0.001 80	0.001 51
1	0.0730	64	0.002 63	0.002 18	72	0.002 78	0.002 37
2	0.0860	56	0.003 70	0.003 10	64	0.003 94	0.003 39
3	0.0990	48	0.004 87	0.004 06	56	0.005 23	0.004 51
4	0.1120	40	0.006 04	0.004 96	48	0.006 61	0.005 66
5	0.1250	40	0.007 96	0.006 72	44	0.008 80	0.007 16
6	0.1380	32	0.009 09	0.007 45	40	0.010 15	0.008 74
8	0.1640	32	0.014 0	0.011 96	36	0.014 74	0.012 85
10	0.1900	24	0.017 5	0.014 50	32	0.020 0	0.017 5
12	0.2160	24	0.024 2	0.020 6	28	0.025 8	0.022 6
$\frac{1}{4}$	0.2500	20	0.031 8	0.026 9	28	0.036 4	0.032 6
$\frac{5}{16}$	0.3125	18	0.052 4	0.045 4	24	0.058 0	0.052 4
$\frac{3}{8}$	0.3750	16	0.077 5	0.067 8	24	0.087 8	0.080 9
$\frac{7}{16}$	0.4375	14	0.106 3	0.093 3	20	0.118 7	0.109 0
$\frac{1}{2}$	0.5000	13	0.141 9	0.125 7	20	0.159 9	0.148 6
$\frac{9}{16}$	0.5625	12	0.182	0.162	18	0.203	0.189
$\frac{5}{8}$	0.6250	11	0.226	0.202	18	0.256	0.240
$\frac{3}{4}$	0.7500	10	0.334	0.302	16	0.373	0.351
$\frac{7}{8}$	0.8750	9	0.462	0.419	14	0.509	0.480
1	1.0000	8	0.606	0.551	12	0.663	0.625
$1\frac{1}{4}$	1.2500	7	0.969	0.890	12	1.073	1.024
$1\frac{1}{2}$	1.5000	6	1.405	1.294	12	1.581	1.521

Square and Acme Thread

- Square and Acme threads are used on screws when power is to be transmitted.
- This type is called power screw.







Power screws

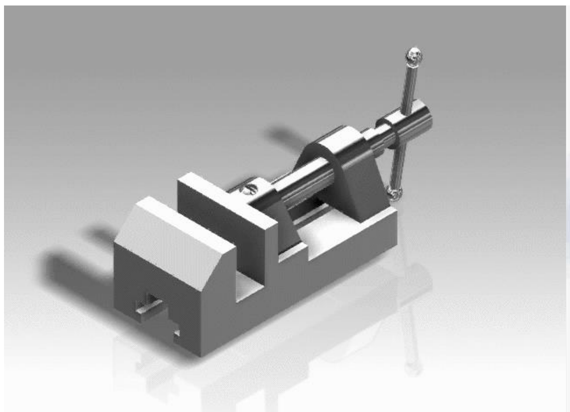
Mechanical design

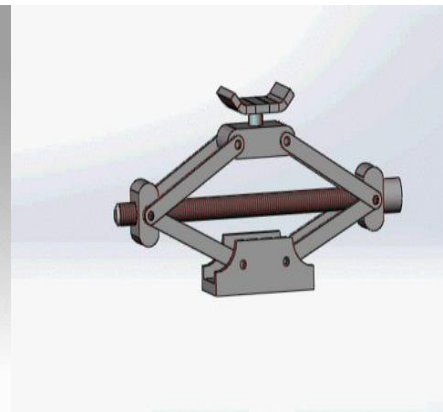
Dr. Mohammad Hayajneh

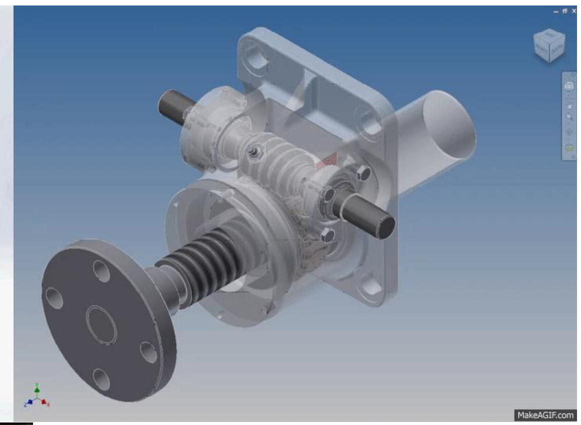
Mechantronics engineering

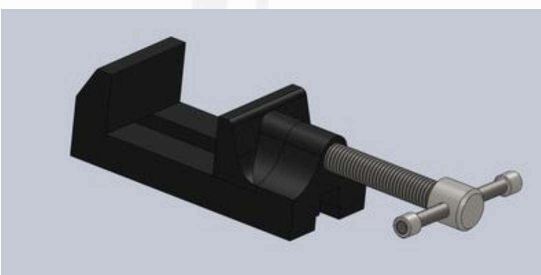
Hashemite University

Power screw applications



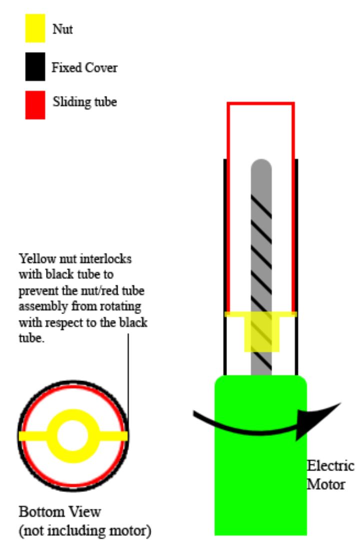






Bench vise

car jack



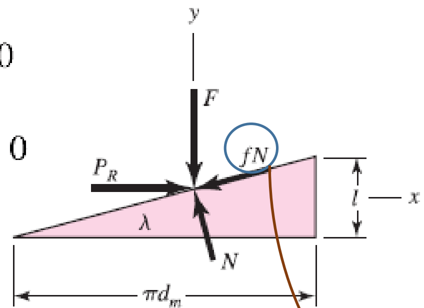
Raising and lowering forces

1) The force to raise the load (P_R)

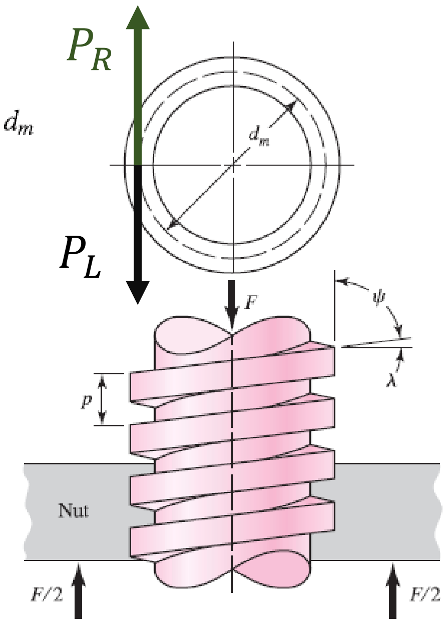
$$\sum F_x = P_R - N\sin\lambda - fN\cos\lambda = 0$$

$$\sum F_y = -F - fN\sin\lambda + N\cos\lambda = 0$$

$$P_R = \frac{F(\sin\lambda + f\cos\lambda)}{\cos\lambda - f\sin\lambda}$$



mean diameter d_m
pitch p
lead angle λ
helix angle ψ

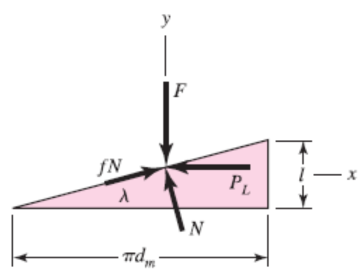


2) The force to lower the load (P_L)

$$\sum F_x = -P_L - N\sin\lambda + fN\cos\lambda = 0$$

$$\sum F_y = -F + fN\sin\lambda + N\cos\lambda = 0$$

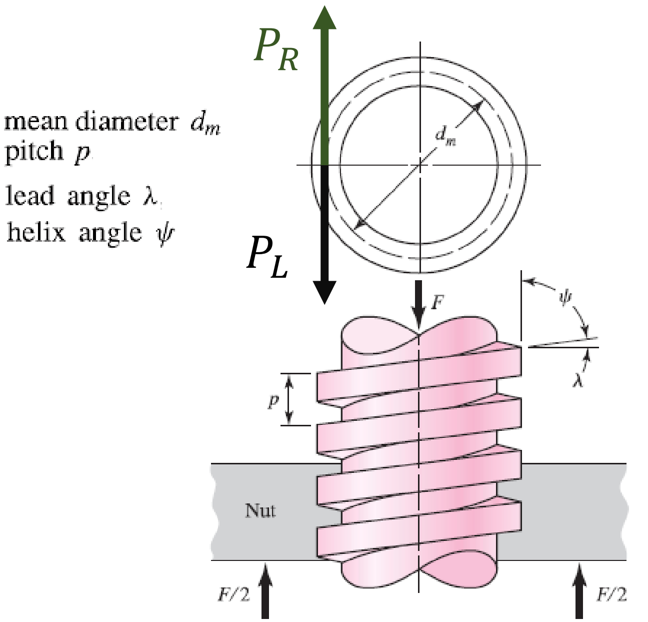
$$P_L = \frac{F(f\cos\lambda - \sin\lambda)}{\cos\lambda + f\sin\lambda}$$



The friction force is the product of the coefficient of friction f with the normal force N

Raising and lowering forces

By dividing the numerator and the denominator of these equations by $\cos\lambda$ *and using the relation* $\tan\lambda = \frac{l}{\pi d_m}$*, We then have, respectively,*



$$P_R = \frac{F[(l/\pi d_m) + f]}{1 - (fl/\pi d_m)}$$

$$P_L = \frac{F[f - (l/\pi d_m)]}{1 + (fl/\pi d_m)}$$

Raising and lowering torques

the torque is the product of the force P and the mean radius $\frac{d_m}{2}$, for raising the load we can write

$$T_R = \frac{d_m}{2} \times P_R = \frac{F d_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - f l}\right)$$

$$T_L = \frac{d_m}{2} \times P_L = \frac{F d_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + f l}\right)$$

When a positive torque is obtained from T_L equation, the screw is said to be self-locking.

Power screw efficiency

Ideal system works at zero friction (f= 0), then we will have:

$$T_R\big|_{f=0} = \frac{F d_m}{2}\left(\frac{l + \cancel{\pi f d_m}}{\pi d_m - \cancel{f l}}\right) = \frac{F \cancel{d_m}}{2}\left(\frac{l}{\pi \cancel{d_m}}\right)$$

$$T_0 = \frac{F l}{2\pi}$$

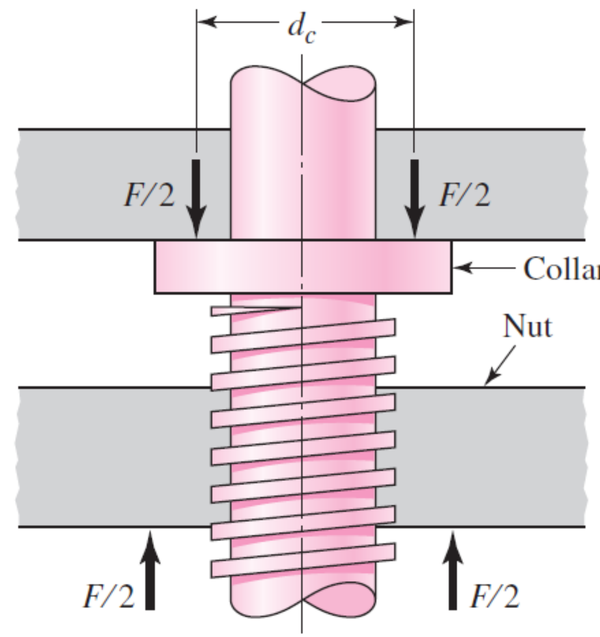
The efficiency is therefore,

$$e = \frac{T_0}{T_R} = \frac{F l}{2\pi T_R}$$

Adding collar torque

When the screw is loaded axially, a thrust or collar bearing must be employed between the rotating and stationary members in order to carry the axial component.
A typical thrust collar in which the load is assumed to be concentrated at the mean collar diameter d_c. If f_c is the coefficient of collar friction, the torque required is

$$T_c = \frac{F f_c d_c}{2}$$



When collar is exist, T_c should be added to the raising and lowering torques, such that

$$T_R = \frac{F d_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - f l}\right) + T_c$$

$$T_L = \frac{F d_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + f l}\right) + T_c$$

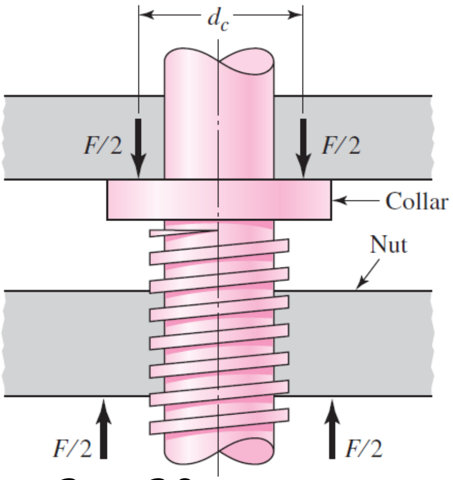
Example

A square-thread power screw has a major diameter of 32 mm and a pitch of 4 **mm** with <u>double threads</u>. The given data include $f = f_c$= 0.08, d_c = 40 mm, and F = 6.4 kN per screw.

(a) Find the thread depth, thread width, pitch diameter, minor diameter, and lead.

(b) Find the torque required to raise and lower the load.

(c) Find the efficiency during lifting the load.

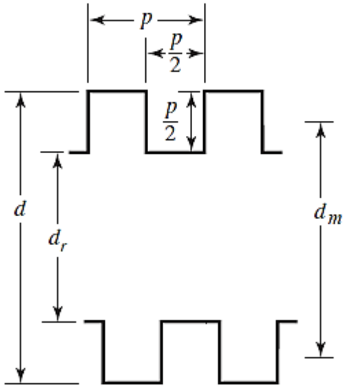


Solution:

(a) For square thread

$$depth = \frac{p}{2} = \frac{4}{2} = 2\,mm$$

$$width = \frac{p}{2} = \frac{4}{2} = 2\,mm$$



$$pitch\ diameter\ d_m = d - 0.5\frac{p}{2} - 0.5\frac{p}{2} = 32 - 2 = 30\,mm$$

$$minor\ diameter\ d_r = d - \frac{p}{2} - \frac{p}{2} = 32 - 4 = 28mm$$

$$l = 2p = 8mm$$

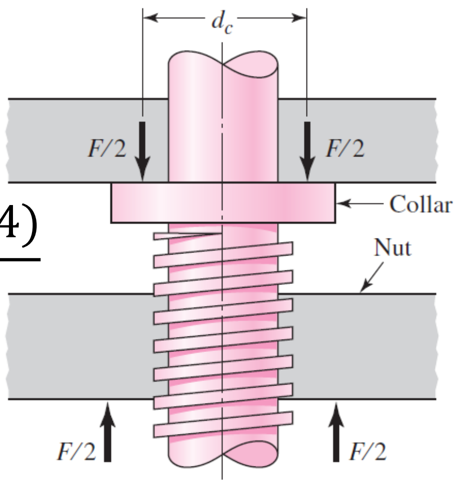
Example

(b) Find the torque required to raise and lower the load.

Solution:

(b) T_R and T_L

$$T_R = \frac{F d_m}{2}\left(\frac{l + \pi f d_m}{\pi d_m - f l}\right) + T_c \qquad T_c = \frac{F f_c d_c}{2}$$



$$= \frac{6.4 \times 10^3 (0.03)}{2}\left(\frac{0.008 + \pi(0.08)(0.03)}{\pi(0.03) - 0.08(0.008)}\right) + \frac{6.4 \times 10^3 (0.08)(0.04)}{2}$$

$$= 15.94 + 10.24 = 26.18\, N.m$$

$$T_L = \frac{F d_m}{2}\left(\frac{\pi f d_m - l}{\pi d_m + f l}\right) + T_c$$

$$= \frac{6.4 \times 10^3 (0.03)}{2}\left(\frac{\pi(0.08)(0.03) - 0.008}{\pi(0.03) + 0.08(0.008)}\right) + \frac{6.4 \times 10^3 (0.08)(0.04)}{2}$$

$$= -0.466 + 10.24 = 9.77 N.m$$

Example

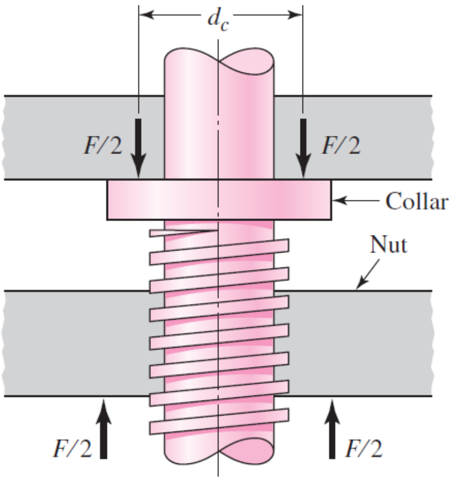
(c) Find the efficiency during lifting the load.

Solution:

(c) efficiency

$$e = \frac{T_0}{T_R} = \frac{F\,l}{2\pi T_R}$$

$$= \frac{6.4 \times 10^3 (0.008)}{2\pi (26.18)} = 0.31 = 31\%$$



Threaded Fasteners and stiffness

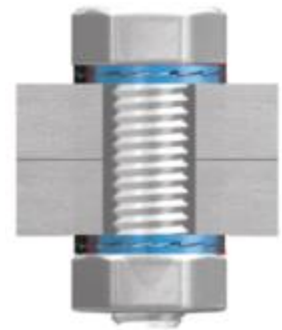
Mechanical design

Dr. Mohammad Hayajneh

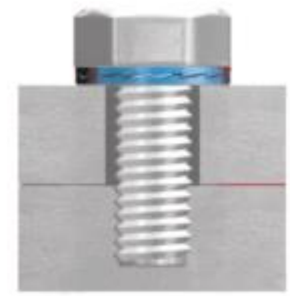
Mechantronics engineering

Hashemite University

Applications

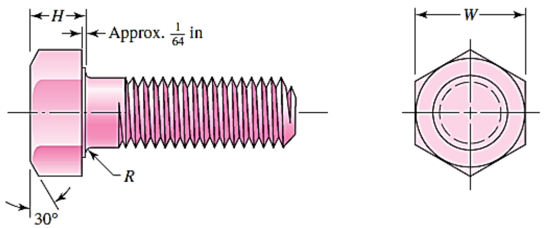


Through hole (bolt)

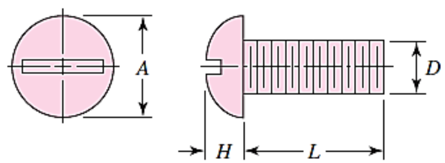


Blind hole

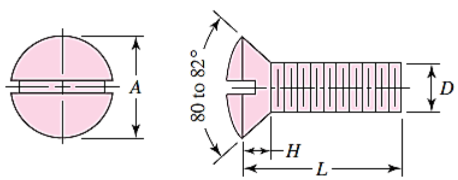
Types of screws



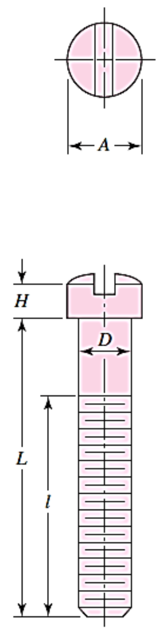
Hexagon-head bolt



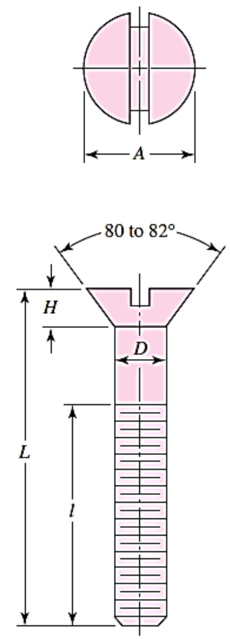
Round head



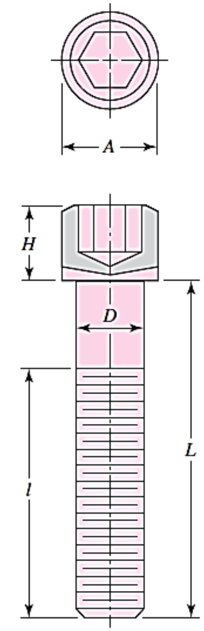
Oval head



fillister head



flat head



hexagonal socket head

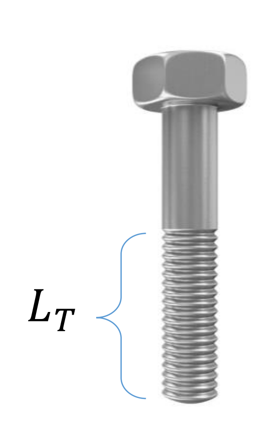
Standard bolt length

The thread length of inch-series bolts, where d is the nominal diameter

$$L_T = \begin{cases} 2d + \frac{1}{4} \text{ in} & L \leq 6 \text{ in} \\ 2d + \frac{1}{2} \text{ in} & L > 6 \text{ in} \end{cases}$$

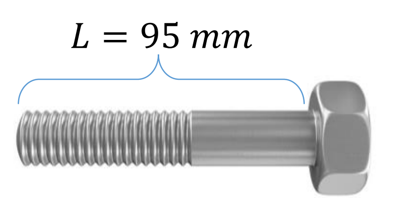
And for metric bolts, where the dimensions in millimeters

$$L_T = \begin{cases} 2d + 6 & L \leq 125 \\ 2d + 12 & 125 < L \leq 200 \\ 2d + 25 & L > 200 \end{cases}$$



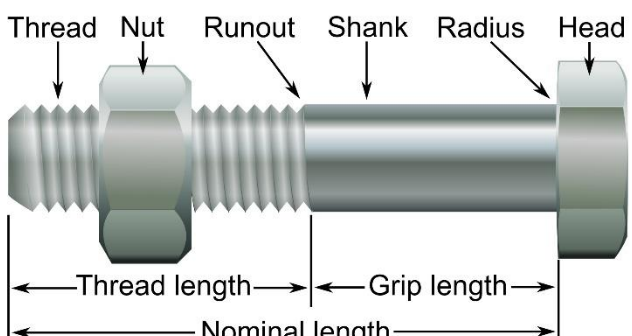
Example: for a 95mm-length bolt with d= 15 mm, calculate the threaded length.

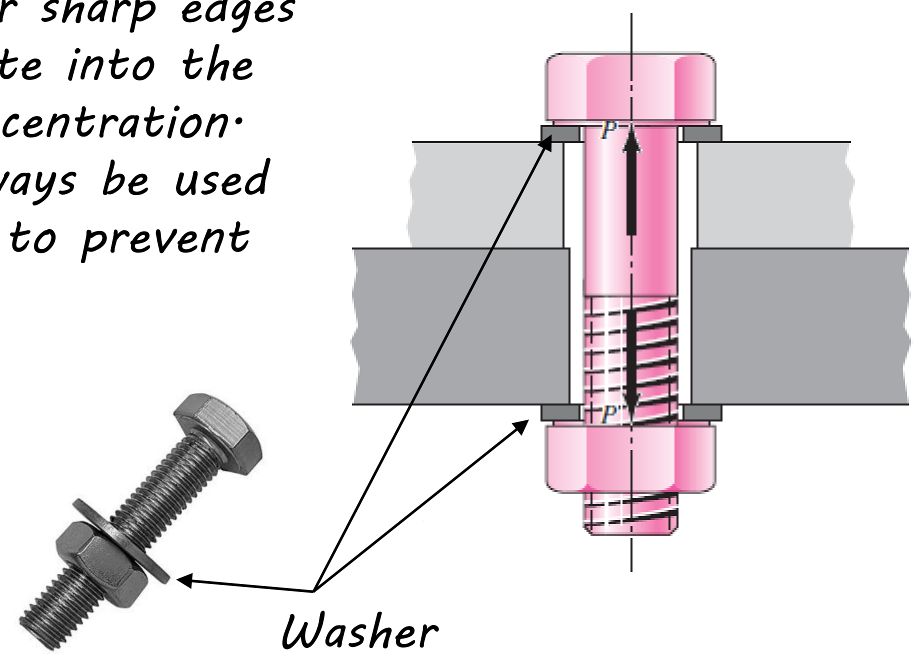
L=95mm ≤ 125 → L_T=2d+6=2(15)+6=36 mm



Joints and Fastener

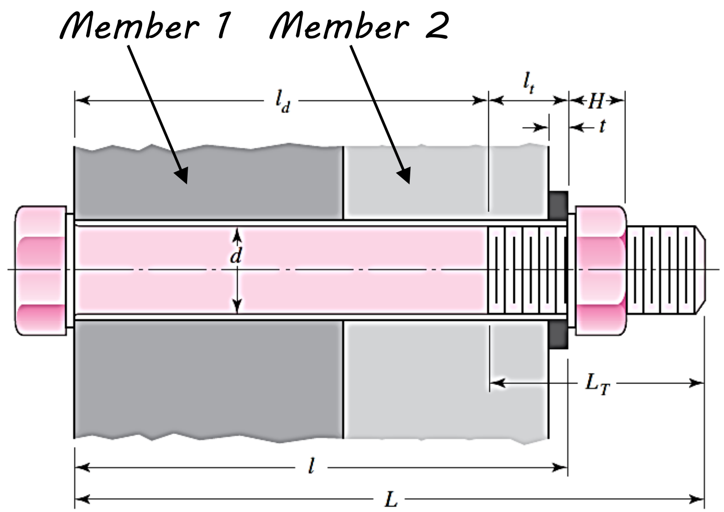
- *The purpose of the bolt is to clamp two, or more, parts together*
- *Twisting the nut stretches the bolt to produce the clamping force.*
- *This clamping force is called the pretension or bolt preload.*
- *Bolt holes may have burrs or sharp edges after drilling. These could bite into the fillet and increase stress concentration. Therefore, washers must always be used under the bolt head or nut to prevent this.*





Joints and Fastener Terminology

$$L > l + H$$



L: Bolt nominal length
L_T: Threaded length of the bolt
d: bolt nominal diameter
t: Washer thickness
H: Nut height

l: Thickness of all material squeezed between face of bolt and face of nut

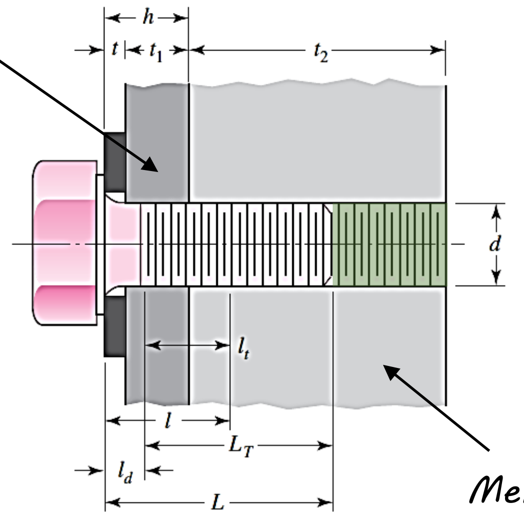
l_d: Unthreaded portion of the bolt in grip (inside the squeezed material)

$$l_d = L - L_T$$

l_t: Threaded portion of the bolt in grip (inside the squeezed material)

$$l_t = l - l_d$$

Joints and Fastener Terminology



$$L > h + 1.5d$$

L: Bolt nominal length
L_T: Threaded length of the bolt
d: bolt nominal diameter
t: Washer thickness

$$l = \begin{cases} h + t_2/2, & t_2 < d \\ h + d/2, & t_2 \geq d \end{cases}$$

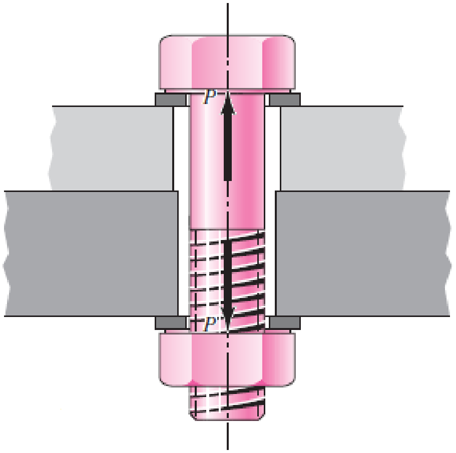
l_d: Unthreaded portion of the bolt in the grip

$$l_d = L - L_T$$

$$l_t = l - l_d$$

Bolt Stiffness

The stiffness of the portion of a bolt or screw within the clamped zone will generally consist of two parts, that of the unthreaded shank portion and that of the threaded portion. Thus the stiffness constant of the bolt is equivalent to the stiffnesses of two springs in series.



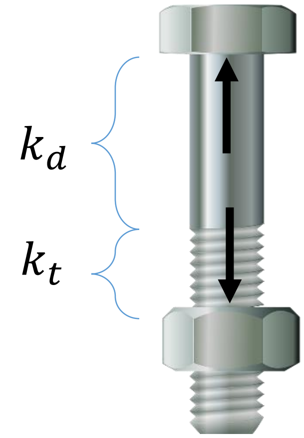
$$\frac{1}{k_b} = \frac{1}{k_d} + \frac{1}{k_t} = \frac{k_d + k_t}{k_d k_t}$$

We know that, the spring force is

$$F = k\delta_l$$

We know also that, the elongation on a member under axial load is

$$\delta_l = \frac{Fl}{AE} \Rightarrow \delta_l = \frac{k\delta_l l}{AE} \Rightarrow k = \frac{AE}{l}$$



Bolt Stiffness

Then we have:

$$k_t = \frac{A_t E}{l_t} \qquad k_d = \frac{A_d E}{l_d}$$

$$k_b = \frac{k_d k_t}{k_d + k_t}$$

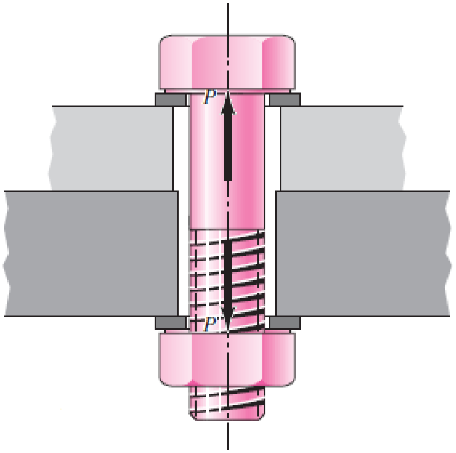
$$\boxed{k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d}}$$

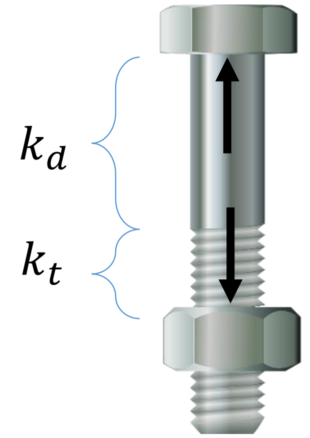
A_t = tensile-stress area (Tables 8–1, 8–2)

l_t = length of threaded portion of grip

A_d = major-diameter area of fastener

l_d = length of unthreaded portion in grip

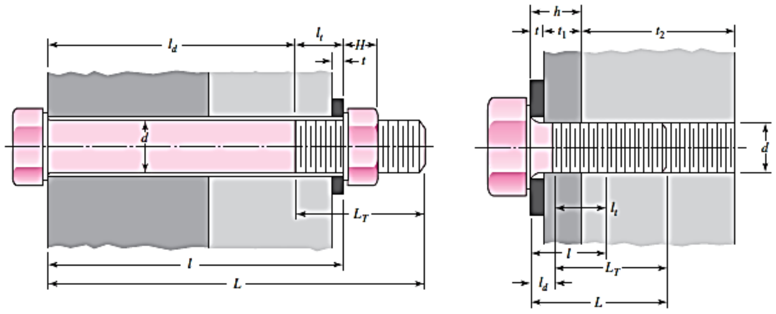




Procedure for Finding Fastener Stiffness

Table 8–7

Suggested Procedure for Finding Fastener Stiffness



(a) (b)

Given fastener diameter d and pitch p in mm or number of threads per inch

Washer thickness: t from Table A–32 or A–33

Nut thickness [Fig. (a) only]: H from Table A–31

Grip length:

For Fig. (a): l = thickness of all material squeezed between face of bolt and face of nut

For Fig. (b): $l = \begin{cases} h + t_2/2, & t_2 < d \\ h + d/2, & t_2 \geq d \end{cases}$

Fastener length (round up using Table A–17*):

For Fig. (a): $L > l + H$

For Fig. (b): $L > h + 1.5d$

Threaded length L_T: Inch series:

$$L_T = \begin{cases} 2d + \frac{1}{4}\text{ in}, & L \leq 6\text{ in} \\ 2d + \frac{1}{2}\text{ in}, & L > 6\text{ in} \end{cases}$$

Metric series:

$$L_T = \begin{cases} 2d + 6\text{ mm}, & L \leq 125\text{ mm}, d \leq 48\text{ mm} \\ 2d + 12\text{ mm}, & 125 < L \leq 200\text{ mm} \\ 2d + 25\text{ mm}, & L > 200\text{ mm} \end{cases}$$

Length of unthreaded portion in grip: $l_d = L - L_T$

Length of threaded portion in grip: $l_t = l - l_d$

Area of unthreaded portion: $A_d = \pi d^2/4$

Area of threaded portion: A_t from Table 8–1 or 8–2

Fastener stiffness: $k_b = \dfrac{A_d A_t E}{A_d l_t + A_t l_d}$

Tables in finding fasteners stiffness

Table A-31

Dimensions of Hexagonal Nuts

Nominal Size, in	Width W	Height H: Regular Hexagonal	Height H: Thick or Slotted	Height H: JAM
$\frac{1}{4}$	$\frac{7}{16}$	$\frac{7}{32}$	$\frac{9}{32}$	$\frac{5}{32}$
$\frac{5}{16}$	$\frac{1}{2}$	$\frac{17}{64}$	$\frac{21}{64}$	$\frac{3}{16}$
$\frac{3}{8}$	$\frac{9}{16}$	$\frac{21}{64}$	$\frac{13}{32}$	$\frac{7}{32}$
$\frac{7}{16}$	$\frac{11}{16}$	$\frac{3}{8}$	$\frac{29}{64}$	$\frac{1}{4}$
$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{16}$	$\frac{9}{16}$	$\frac{5}{16}$
$\frac{9}{16}$	$\frac{7}{8}$	$\frac{31}{64}$	$\frac{39}{64}$	$\frac{5}{16}$
$\frac{5}{8}$	$\frac{15}{16}$	$\frac{35}{64}$	$\frac{23}{32}$	$\frac{3}{8}$
$\frac{3}{4}$	$1\frac{1}{8}$	$\frac{41}{64}$	$\frac{13}{16}$	$\frac{27}{64}$
$\frac{7}{8}$	$1\frac{5}{16}$	$\frac{3}{4}$	$\frac{29}{32}$	$\frac{31}{64}$
1	$1\frac{1}{2}$	$\frac{55}{64}$	1	$\frac{35}{64}$
$1\frac{1}{8}$	$1\frac{11}{16}$	$\frac{31}{32}$	$1\frac{5}{32}$	$\frac{39}{64}$
$1\frac{1}{4}$	$1\frac{7}{8}$	$1\frac{1}{16}$	$1\frac{1}{4}$	$\frac{23}{32}$
$1\frac{3}{8}$	$2\frac{1}{16}$	$1\frac{11}{64}$	$1\frac{3}{8}$	$\frac{25}{32}$
$1\frac{1}{2}$	$2\frac{1}{4}$	$1\frac{9}{32}$	$1\frac{1}{2}$	$\frac{27}{32}$
Nominal Size, mm				
M5	8	4.7	5.1	2.7
M6	10	5.2	5.7	3.2
M8	13	6.8	7.5	4.0
M10	16	8.4	9.3	5.0
M12	18	10.8	12.0	6.0
M14	21	12.8	14.1	7.0
M16	24	14.8	16.4	8.0
M20	30	18.0	20.3	10.0
M24	36	21.5	23.9	12.0
M30	46	25.6	28.6	15.0
M36	55	31.0	34.7	18.0

Table A-32

Basic Dimensions of American Standard Plain Washers (All Dimensions in Inches)

Fastener Size	Washer Size	Diameter ID	Diameter OD	Thickness
#6	0.138	0.156	0.375	0.049
#8	0.164	0.188	0.438	0.049
#10	0.190	0.219	0.500	0.049
#12	0.216	0.250	0.562	0.065
$\frac{1}{4}$ N	0.250	0.281	0.625	0.065
$\frac{1}{4}$ W	0.250	0.312	0.734	0.065
$\frac{5}{16}$ N	0.312	0.344	0.688	0.065
$\frac{5}{16}$ W	0.312	0.375	0.875	0.083
$\frac{3}{8}$ N	0.375	0.406	0.812	0.065
$\frac{3}{8}$ W	0.375	0.438	1.000	0.083
$\frac{7}{16}$ N	0.438	0.469	0.922	0.065

Table A-33

Dimensions of Metric Plain Washers (All Dimensions in Millimeters)

Washer Size*	Minimum ID	Maximum OD	Maximum Thickness	Washer Size*	Minimum ID	Maximum OD	Maximum Thickness
1.6 N	1.95	4.00	0.70	10 N	10.85	20.00	2.30
1.6 R	1.95	5.00	0.70	10 R	10.85	28.00	2.80
1.6 W	1.95	6.00	0.90	10 W	10.85	39.00	3.50
2 N	2.50	5.00	0.90	12 N	13.30	25.40	2.80
2 R	2.50	6.00	0.90	12 R	13.30	34.00	3.50
2 W	2.50	8.00	0.90	12 W	13.30	44.00	3.50
2.5 N	3.00	6.00	0.90	14 N	15.25	28.00	2.80
2.5 R	3.00	8.00	0.90	14 R	15.25	39.00	3.50
2.5 W	3.00	10.00	1.20	14 W	15.25	50.00	4.00

Tables in finding fasteners stiffness

Table A–17

Preferred Sizes and Renard (R-Series) Numbers (When a choice can be made, use one of these sizes; however, not all parts or items are available in all the sizes shown in the table.)

Fraction of Inches

$\frac{1}{64}, \frac{1}{32}, \frac{1}{16}, \frac{3}{32}, \frac{1}{8}, \frac{5}{32}, \frac{3}{16}, \frac{1}{4}, \frac{5}{16}, \frac{3}{8}, \frac{7}{16}, \frac{1}{2}, \frac{9}{16}, \frac{5}{8}, \frac{11}{16}, \frac{3}{4}, \frac{7}{8}, 1, 1\frac{1}{4}, 1\frac{1}{2}, 1\frac{3}{4}, 2, 2\frac{1}{4}, 2\frac{1}{2}, 2\frac{3}{4}, 3, 3\frac{1}{4}, 3\frac{1}{2}, 3\frac{3}{4}, 4, 4\frac{1}{4}, 4\frac{1}{2}, 4\frac{3}{4}, 5, 5\frac{1}{4}, 5\frac{1}{2}, 5\frac{3}{4}, 6, 6\frac{1}{2}, 7, 7\frac{1}{2}, 8, 8\frac{1}{2}, 9, 9\frac{1}{2}, 10, 10\frac{1}{2}, 11, 11\frac{1}{2}, 12, 12\frac{1}{2}, 13, 13\frac{1}{2}, 14, 14\frac{1}{2}, 15, 15\frac{1}{2}, 16, 16\frac{1}{2}, 17, 17\frac{1}{2}, 18, 18\frac{1}{2}, 19, 19\frac{1}{2}, 20$

Decimal Inches

0.010, 0.012, 0.016, 0.020, 0.025, 0.032, 0.040, 0.05, 0.06, 0.08, 0.10, 0.12, 0.16, 0.20, 0.24, 0.30, 0.40, 0.50, 0.60, 0.80, 1.00, 1.20, 1.40, 1.60, 1.80, 2.0, 2.4, 2.6, 2.8, 3.0, 3.2, 3.4, 3.6, 3.8, 4.0, 4.2, 4.4, 4.6, 4.8, 5.0, 5.2, 5.4, 5.6, 5.8, 6.0, 7.0, 7.5, 8.5, 9.0, 9.5, 10.0, 10.5, 11.0, 11.5, 12.0, 12.5, 13.0, 13.5, 14.0, 14.5, 15.0, 15.5, 16.0, 16.5, 17.0, 17.5, 18.0, 18.5, 19.0, 19.5, 20

Millimeters

0.05, 0.06, 0.08, 0.10, 0.12, 0.16, 0.20, 0.25, 0.30, 0.40, 0.50, 0.60, 0.70, 0.80, 0.90, 1.0, 1.1, 1.2, 1.4, 1.5, 1.6, 1.8, 2.0, 2.2, 2.5, 2.8, 3.0, 3.5, 4.0, 4.5, 5.0, 5.5, 6.0, 6.5, 7.0, 8.0, 9.0, 10, 11, 12, 14, 16, 18, 20, 22, 25, 28, 30, 32, 35, 40, 45, 50, 60, 80, 100, 120, 140, 160, 180, 200, 250, 300

Renard Numbers*

1st choice, R5: 1, 1.6, 2.5, 4, 6.3, 10

2d choice, R10: 1.25, 2, 3.15, 5, 8

3d choice, R20: 1.12, 1.4, 1.8, 2.24, 2.8, 3.55, 4.5, 5.6, 7.1, 9

4th choice, R40: 1.06, 1.18, 1.32, 1.5, 1.7, 1.9, 2.12, 2.36, 2.65, 3, 3.35, 3.75, 4.25, 4.75, 5.3, 6, 6.7, 7.5, 8.5, 9.5

*May be multiplied or divided by powers of 10.

Tables in finding fasteners stiffness

Table 8–1

Diameters and Areas of Coarse-Pitch and Fine-Pitch Metric Threads.*

Nominal Major Diameter d mm	Coarse-Pitch Series: Pitch p mm	Coarse-Pitch Series: Tensile-Stress Area A_t mm²	Coarse-Pitch Series: Minor-Diameter Area A_r mm²	Fine-Pitch Series: Pitch p mm	Fine-Pitch Series: Tensile-Stress Area A_t mm²	Fine-Pitch Series: Minor-Diameter Area A_r mm²
1.6	0.35	1.27	1.07			
2	0.40	2.07	1.79			
2.5	0.45	3.39	2.98			
3	0.5	5.03	4.47			
3.5	0.6	6.78	6.00			
4	0.7	8.78	7.75			
5	0.8	14.2	12.7			
6	1	20.1	17.9			
8	1.25	36.6	32.8	1	39.2	36.0
10	1.5	58.0	52.3	1.25	61.2	56.3
12	1.75	84.3	76.3	1.25	92.1	86.0
14	2	115	104	1.5	125	116
16	2	157	144	1.5	167	157

Table 8–2

Diameters and Area of Unified Screw Threads UNC and UNF*

Size Designation	Nominal Major Diameter in	Coarse Series—UNC: Threads per Inch N	Coarse Series—UNC: Tensile-Stress Area A_t in²	Coarse Series—UNC: Minor-Diameter Area A_r in²	Fine Series—UNF: Threads per Inch N	Fine Series—UNF: Tensile-Stress Area A_t in²	Fine Series—UNF: Minor-Diameter Area A_r in²
0	0.0600				80	0.001 80	0.001 51
1	0.0730	64	0.002 63	0.002 18	72	0.002 78	0.002 37
2	0.0860	56	0.003 70	0.003 10	64	0.003 94	0.003 39
3	0.0990	48	0.004 87	0.004 06	56	0.005 23	0.004 51
4	0.1120	40	0.006 04	0.004 96	48	0.006 61	0.005 66
5	0.1250	40	0.007 96	0.006 72	44	0.008 80	0.007 16
6	0.1380	32	0.009 09	0.007 45	40	0.010 15	0.008 74
8	0.1640	32	0.014 0	0.011 96	36	0.014 74	0.012 85
10	0.1900	24	0.017 5	0.014 50	32	0.020 0	0.017 5

Example 1:

An M14 x 2 hex-head bolt with a nut is used to clamp together two 15-mm steel plates.

(a) Determine a suitable length for the bolt, rounded up to the nearest 5 mm.

(b) Determine the bolt stiffness.

Solution:

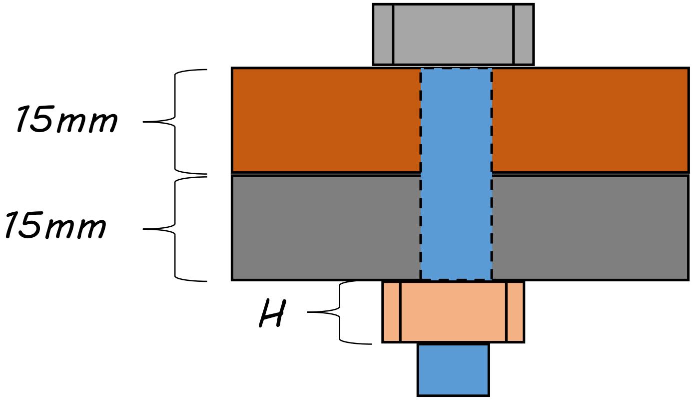
(a) The bolt length $L \geq 15+15+H$

From table A.31, for regular hexagonal nut $H=12.8$ mm

$L \geq 42.8$ mm

When we round the length L up to the nearest 5 mm

$L=45$ mm



Example 1:

(b) Determine the bolt stiffness.

$$L_T = \begin{cases} 2d + 6 & L \le 125 \\ 2d + 12 & 125 < L \le 200 \\ 2d + 25 & L > 200 \end{cases}$$

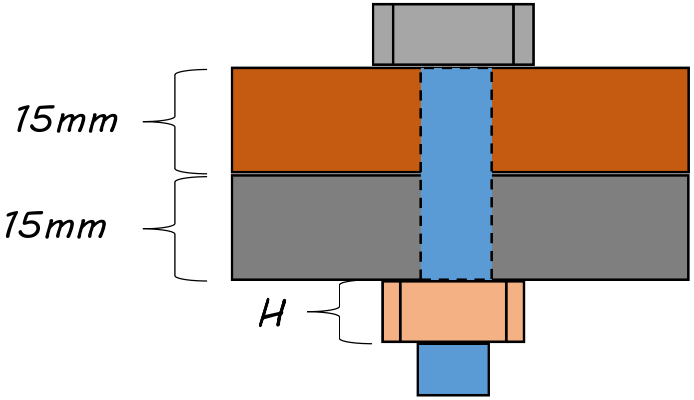
Solution:

1) Grip length $l = 15 + 15 = 30mm$

2) Threaded length $L_T = 2 \times 14 + 6 = 34mm$

3) Unthreaded portion length in grip
$l_d = L - L_T = 45 - 34 = 11\ mm$

4) Threaded portion length in grip
$l_t = l - l_d = 30 - 11 = 19\ mm$



5) Area of unthreaded portion
$A_d = \frac{\pi}{4} d^2 = \frac{\pi}{4} (14)^2 = 153.94\ mm^2$

5) Area of threaded portion
$A_t = 115\ mm^2$ (from table 8.1)

6) $k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{153.94\times10^{-6}\times115\times10^{-6}\times207\times10^{9}}{153.94\times10^{-6}\times19\times10^{-3}+115\times10^{-6}\times11\times10^{-3}} = 874.6 \times 10^6\ N/m$

Example 2:

Repeat previous example with the addition of one 14R metric plain washer under the nut.

Solution:

(a) The bolt length $L \geq 15+15+H+t$

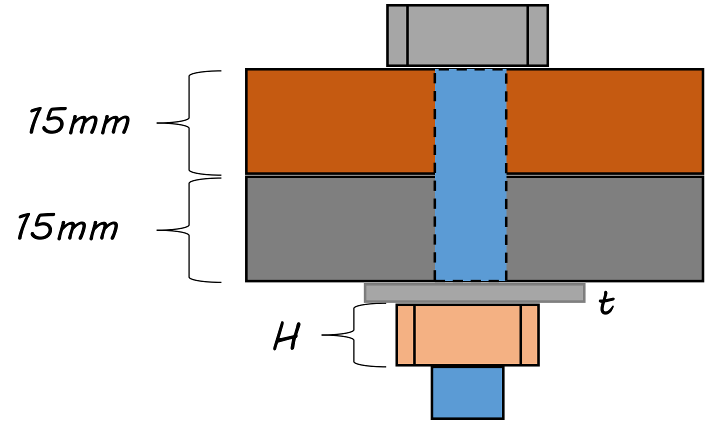
From table A.31, for regular hexagonal nut $H=12.8$ mm

From table A.33, for regular hexagonal nut $t=3.5$ mm

$L \geq (15+15+12.8+3.5)=46.3$ mm

When we round the length L up to the nearest 5 mm

$L=50$ mm



Example

$$L_T = \begin{cases} 2d+6 & L \le 125 \\ 2d+12 & 125 < L \le 200 \\ 2d+25 & L > 200 \end{cases}$$

(b) Determine the bolt stiffness.

Solution:

1) Grip length $l = 15 + 15 + 3.5 = 33.5\ mm$

2) Threaded length $L_T = 2 \times 14 + 6 = 34mm$

3) Unthreaded portion length in grip

$l_d = L - L_T = 50 - 34 = 16\ mm$

4) Threaded portion length in grip

$l_t = l - l_d = 33.5 - 16 = 17.5\ mm$

5) Area of unthreaded portion

$A_d = \frac{\pi}{4} d^2 = \frac{\pi}{4} (14)^2 = 153.94\ mm^2$

5) Area of threaded portion

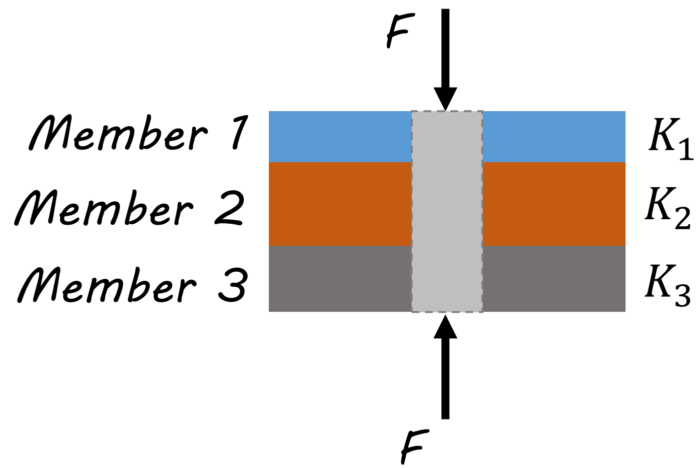
$A_t = 115\ mm^2$ (from table 8.1)

6) $k_b = \frac{A_d A_t E}{A_d l_t + A_t l_d} = \frac{153.94\times10^{-6}\times115\times10^{-6}\times207\times10^{9}}{153.94\times10^{-6}\times17.5\times10^{-3}+115\times10^{-6}\times16\times10^{-3}} = ??\ N/m$

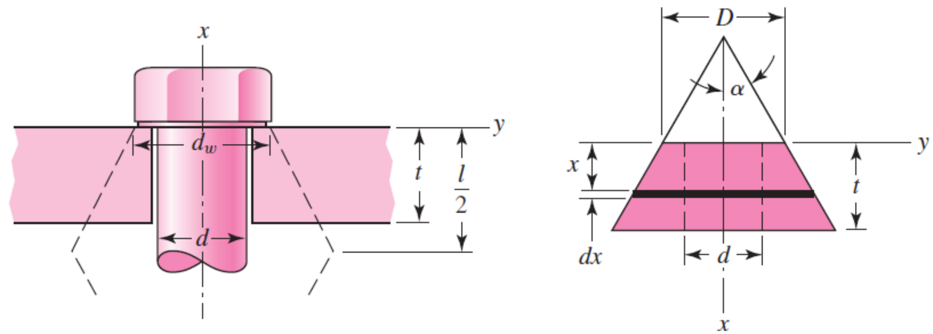
Joints—Member Stiffness

- Here, we wish to study the stiffnesses of the members in the clamped zone.
- There may be more than two members included in the grip of the fastener. All together these act like compressive springs in series, and hence the total spring rate of the members is

$$\frac{1}{k_m} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3} + \cdots + \frac{1}{k_i}$$



Joints—Member Stiffness



$$k = \frac{P}{\delta} = \frac{\pi E d \tan\alpha}{\ln \dfrac{(2t\tan\alpha + D - d)(D + d)}{(2t\tan\alpha + D + d)(D - d)}}$$

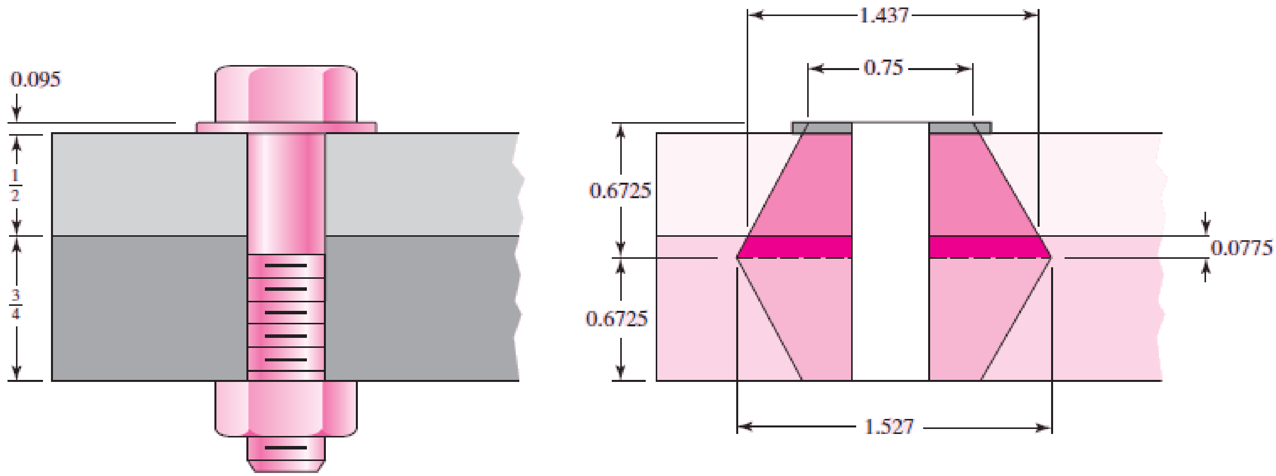
In this book we shall use $\alpha = 30^o$ *then:*

$$k = \frac{0.5774\pi E d}{\ln \dfrac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}}$$

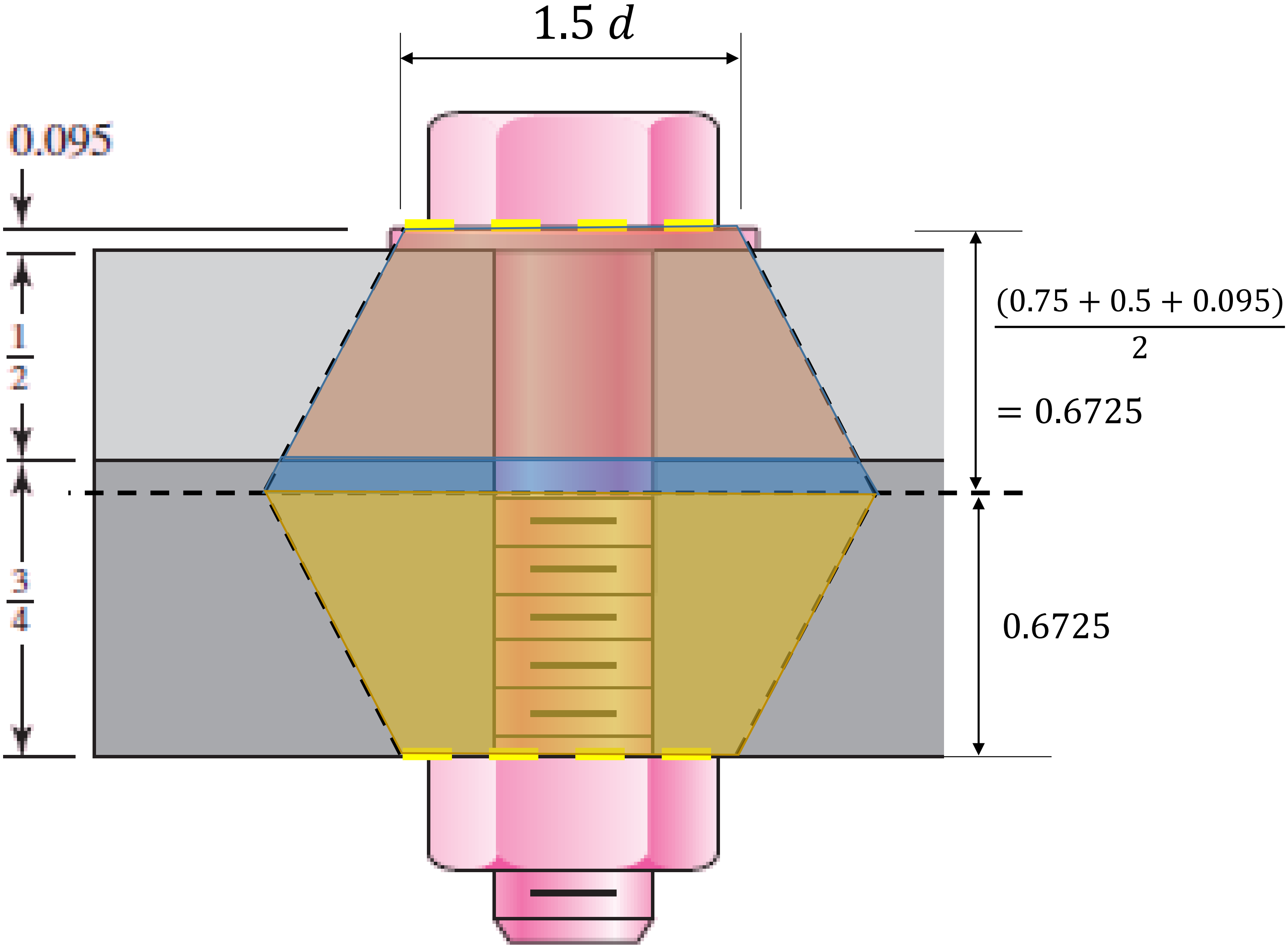
must be solved separately for each frustum in the joint

Example

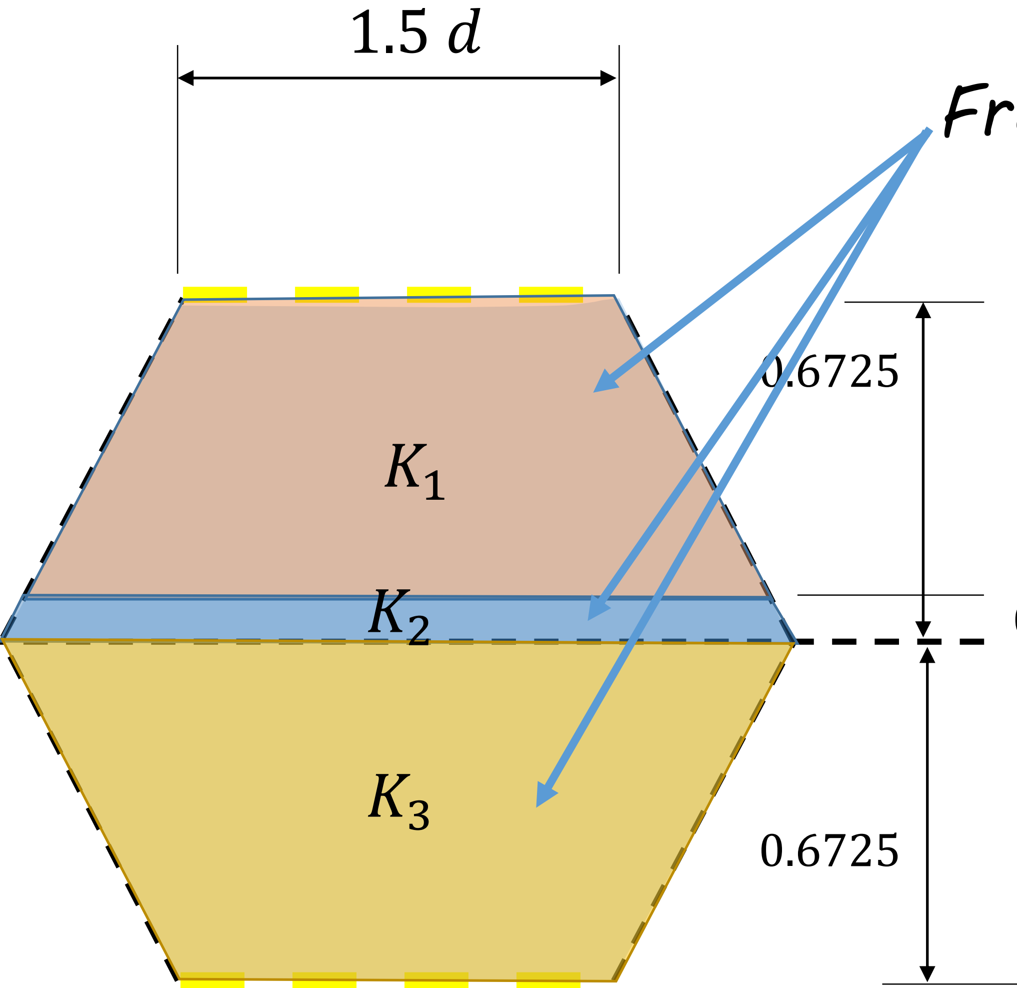
As shown in figure, two plates are clamped by washer-faced ½ in-20 UNF x 1 ½ In SAE grade 5 bolts each with a standard ½ N steel plain washer. Determine the member spring rate k_m if the top plate is steel and the bottom plate is gray cast iron.



Material Used	Poisson Ratio	Elastic GPa	Modulus Mpsi	A	B
Steel	0.291	207	30.0	0.787 15	0.628 73
Aluminum	0.334	71	10.3	0.796 70	0.638 16
Copper	0.326	119	17.3	0.795 68	0.635 53
Gray cast iron	0.211	100	14.5	0.778 71	0.616 16
General expression				0.789 52	0.629 14



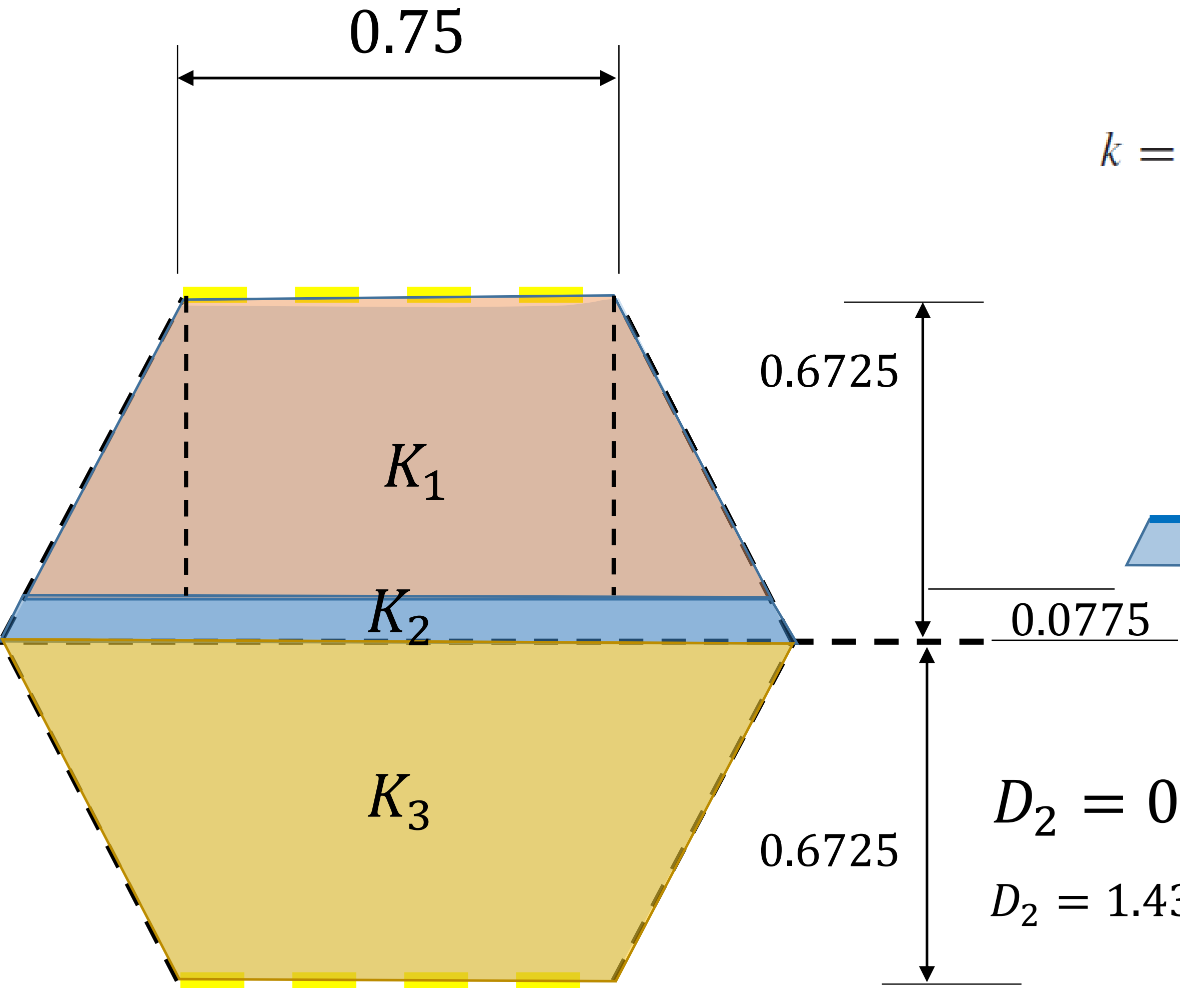
1.5 d
0.095
1/2
3/4
(0.75 + 0.5 + 0.095) / 2
= 0.6725
0.6725



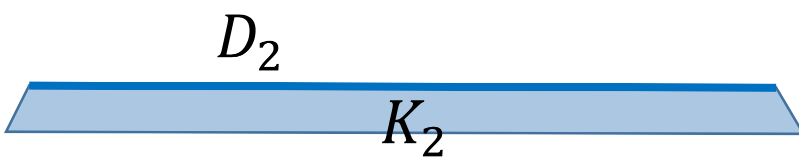
$0.6725 - 0.5 - 0.095 = 0.0775$

$$k = \frac{0.5774\pi E d}{\ln \dfrac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}}$$

$$K_1 = \frac{0.5774\pi(E_{steel})(0.5)}{\ln\left[\dfrac{(1.155(0.595) + 0.75 - 0.5)(0.75 + 0.5)}{(1.155(0.595) + 0.75 + 0.5)(0.75 - 0.5)}\right]} = 30.8 \times 10^6 lbf\ /in$$



$$k = \frac{0.5774\pi E d}{\ln \dfrac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}}$$

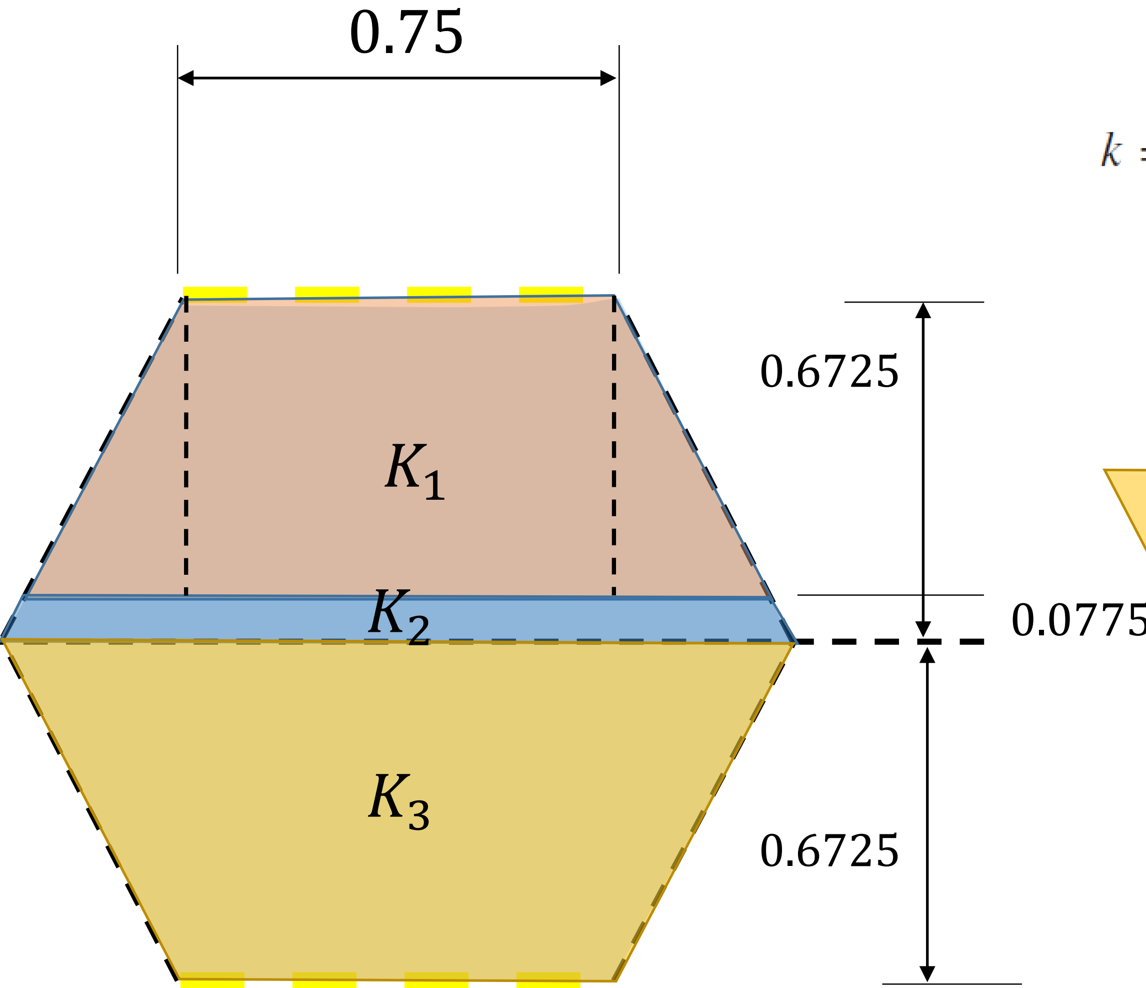


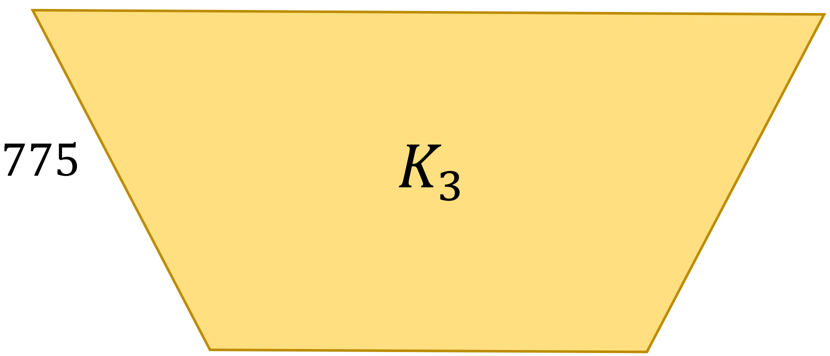
$$D_2 = 0.75 + 2(0.595\, tan\, 30^o)$$

$D_2 = 1.437$ in

$$K_2 = \frac{0.5774\pi(E_{cast\ iron})(0.5)}{\ln\left[\dfrac{(1.155(0.0775) + 1.437 - 0.5)(1.437 + 0.5)}{(1.155(0.0775) + 1.437 + 0.5)(1.437 - 0.5)}\right]} = 285.5 \times 10^6 lbf\ /in$$

$$k = \frac{0.5774\pi E d}{\ln \dfrac{(1.155t + D - d)(D + d)}{(1.155t + D + d)(D - d)}}$$





$D_3 = 0.75$

$$K_3 = \frac{0.5774\pi(E_{cast\ iron})(0.5)}{\ln\left[\dfrac{(1.155(0.6725) + 0.75 - 0.5)(0.75 + 0.5)}{(1.155(0.0.6725) + 0.75 + 0.5)(0.75 - 0.5)}\right]} = 14.15 \times 10^6 lbf\ /in$$

$$\frac{1}{k_m} = \frac{1}{30.80(10^6)} + \frac{1}{285.5(10^6)} + \frac{1}{14.15(10^6)}$$

$$k_m = 9.378\ (10^6)\ \text{lbf/in}$$