ElCoM

ELECTRICAL COMPUTER MECHATRONICS

تقدم لجنة ElCoM الاكاديمية

دفتر لمادة:

تفاضل و تكامل (١)

من شرح:

م.رانيا شقبوعة

جزيل الشكر للطالبة:

بتول محمد

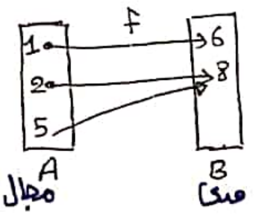
٥٠

تفاضل وتكامل
(١) دوسيه

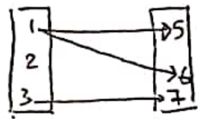
HASHEMITE UNIVERSITY

CALCULUS I / CHAPTER ONE (functions and models)

Definition: $f: A \to B$ (f is a function from A to B). f is a relation that assigns every element of $a \in A$ to a unique element $b \in B$ denoted by $f(x) = y$. A is called domain and B is called Range.
x-axis
y-axis



f is a function
Domain = $\{1,2,5\}$, Range = $\{6,8\}$
$f(1) = 6$, $f(2) = 8$, $f(5) = 8$.

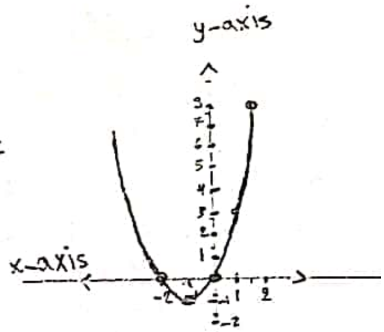


Not function
relation

Example: Graph the function $f(x) = x^2 + 2x$

x	0	1	-1	2	-2
$f(x)$	0	3	-1	8	0

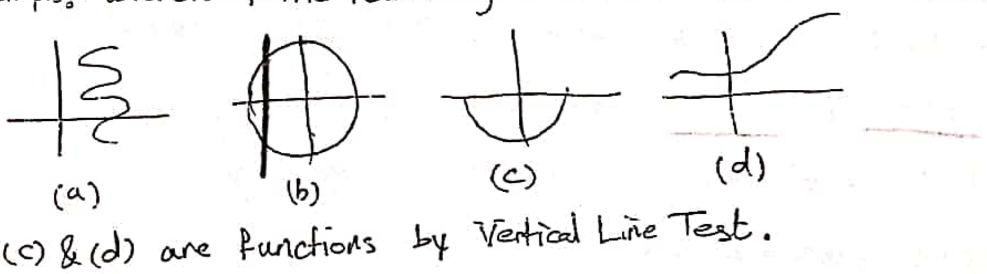
(x, y)
$(0,0), (1,3)$
$(-1,-1), (2,8)$
$(-2,0)$.



H.W(1) Graph the functions
(a) $f(x) = 2x + 1$ (b) $g(x) = x^3 + x^2 + 1$ (c) $h(x) = \sqrt{x-1}$

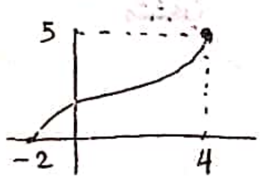
Bonus
Vertical Line Test: If the vertical line intersects the graph at exactly One point then y is a function of x. $y = f(x)$

Example: which of the following is a function?



(c) & (d) are functions by Vertical Line Test.

Example: Find domain and Range.



Domain $[-2,4]$
Range $[0,5]$

Domain : D_f : The set of all possible input (x-value)
Range : D_f : The set of all possible output (y-value)

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First Semester 2018/2

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Types of Functions:

(1) polynomials: $f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_1 x + a_0$, $n = 1, 2, 3, \dots$ (Integers), $a_n \in \mathbb{R}$ (Real numbers)

(2) Rational functions = $\frac{\text{Polynomial}}{\text{Polynomial}}$

(3) Absolute value function: $|f(x)|$

(4) root functions: $f(x) = \sqrt[n]{g(x)}$ $n: 2, 3, 4, \dots$

(5) Exponential functions

(6) Logarithmic functions

(7) Trigonometric functions & Inverse Trigonometric functions.

Examples: Classify each functions

(1) $f(x) = 5x^6 + 8x^2 + 10x + 1$ (polynomial)

(2) $h(x) = x^2 + 2x + 7$ (polynomial [quadratic function])

(3) $g(x) = x^3 + 5x^2 + 7$ (poly, cubic function)

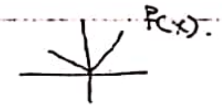
(4) $h(x) = \frac{2x+1}{5x^3+7}$ Rational function.

(5) $g(x) = \sqrt{2x+1}$, root function

(6) $h(x) = x^{1/2} + x^{-2} + 5x^2 + 3$ Not poly.

Absolute Value function:

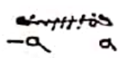
$$f(x) = |x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$



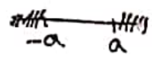
properties ~~value~~ of Absolute value:

(1) $|-a| = |a|$ (2) $|ab| = |a||b|$. (3) $\left|\frac{a}{b}\right| = \frac{|a|}{|b|}$ (4) $|a+b| \leq |a| + |b|$

(5) $|x| = a \Rightarrow x = \pm a$ (6) $|x| \leq a \Rightarrow -a \leq x \leq a$



(7) $|x| \geq a \Rightarrow x \geq a$ or $x \leq -a$



Example: $|2x - 1| < 7$

$-7 < 2x - 1 < 7 \underset{+1}{\Rightarrow} -6 < 2x < 8 \underset{\div 2}{\Rightarrow} -3 < x < 4$ $(-3, 4)$

Example: Express the following as picewise function

$f(x) = |8 - 2x|$

$8 - 2x = 0 \Rightarrow x = 4$

$\underset{8-2x}{++} \quad 4 \quad \underset{-(8-2x)}{---}$

$$f(x) = \begin{cases} 8 - 2x, & x \leq 4 \\ -(8 - 2x), & x > 4 \end{cases}$$

*Domain Rules:-

Example: Find the domain for the following ..

(1) $f(x)= x^{10}+7x^{9}+\frac{1}{2}x^{8}-\sqrt{2}x^{7}+5x^{6}+10x+1$ .. $D_f=\mathbb{R}$

$\boxed{\text{Rule}}$ Domain(polynomial) $=\mathbb{R}$ [illegible]

(2) $f(x)=\sqrt{x-7}$

$x-7\geq 0 \Rightarrow x\geq 7$ $D_f=[7,\infty)$

$\boxed{\text{Rule}}$ $f(x)=\sqrt[n]{g(x)}$ n: even number, domain (f); $\{x: g(x)\geq 0\}$

(3) $f(x)=\frac{10}{x-5}$, $x-5=0 \Rightarrow x=5$, Domain $=\mathbb{R}-\{5\}$

$\boxed{\text{Rule}}$: denominator is never Zero.

[illegible]

(4) $f(x)=\sqrt[3]{\frac{1}{x+7}}$, $x+7=0 \Rightarrow x=-7 \Rightarrow$ Domain $=\mathbb{R}-\{7\}$.

$\boxed{\text{Rule}}$ $f(x)=\sqrt[n]{g(x)}$, n: odd number

Domain(f) = Domain(g).

(5) $f(x)=\frac{x^2}{x}$

$x=0 \Rightarrow D_f=\mathbb{R}-\{0\}$

$\boxed{\text{Rule}}$: Don't simplify.

(6) $f(x)=|x^3+7x^2+5|$

Domain $=\mathbb{R}$

$\boxed{\text{Rule}}$ $g(x)=|f(x)|$, $f(x)$ polynomial

$D_g=\mathbb{R}$.

H.W: Find the domain

(1) $f(x)=\frac{x-1}{x^2-1}$

(2) $f(x)=\frac{1}{(x-1)(x+5)}$

(3) $f(x)=\sqrt{x^2-5x+6}$

(4) $f(x)=\sqrt{|x-1|-10}$

New functions From old

Given functions f & g we defined

1. $(f \pm g)(x) = f(x) \pm g(x)$
2. $(fg)(x) = f(x)\, g(x)$
3. $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, $g(x) \neq 0$.

Domain $(f \pm g,\ fg) = \underbrace{D_f \cap D_g}_{\text{الاشتراك}}$ & Domain $\left(\frac{f}{g}\right) = D_f \cap D_g - \{x : g(x) = 0\}$

Example: $f(x) = 1 - \sqrt{x-2}$, $g(x) = x - 4$

Find (1) $(f+g)(7)$, (2) $(f-g)(x)$ (3) $\left(\frac{f}{g}\right)(x)$ (4) $7f(x)$

(5) find domain $(fg)(x)$ (6) find domain $\left(\frac{g}{f}\right)(x)$.

Sol:- (1) $(f+g)(7) = f(7) + g(7)$
$= 1 - \sqrt{7-2} + 3 = 4 - \sqrt{5}$

(2) $(f-g)(x) = f(x) - g(x)$
$= (1 - \sqrt{x-2}) - (x-4) = -\sqrt{x-2} - x + 5$

(3) $\left(\frac{f}{g}\right)(x) = \dfrac{1 - \sqrt{x-2}}{x-4}$

(4) $7f(x) = 7 - 7\sqrt{x-2}$

(5) domain (f) $x - 2 \geqslant 0 \Rightarrow x \geqslant 2$ $[2, \infty)$
domain $(g) = \mathbb{R}$.
Domain $(fg) = [2, \infty) \cap \mathbb{R} = [2, \infty)$

(6) Domain $\left(\frac{g}{f}\right) = D_g \cap D_f - \{x : f(x) = 0\}$
$[2, \infty) - \{x : 1 - \sqrt{x-2} = 0\}$
$[2, \infty) - \{3\} = [2, 3) \cup (3, \infty)$.

$1 - \sqrt{x-2} = 0$
$1 = \sqrt{x-2}$ ^2
$1 = x - 2$
$\boxed{x = 3}$

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H.W: Find Domain $\dfrac{\sqrt{x^2-4}}{\sqrt{x-4}}$

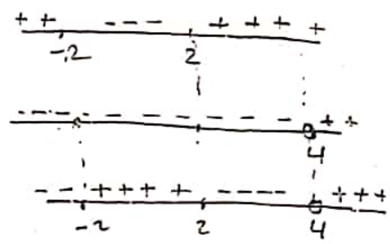
Example: Find the domain

(1) $f(x)=\sqrt{\dfrac{x^2-4}{x-4}}$

$\dfrac{x^2-4}{x-4} \geq 0 \Rightarrow$ $x^2-4=0$, $x=\pm 2$

$x-4=0$, $x=4$

$\dfrac{x^2-4}{x-4}$



Domain(f). $[-2,2] \cup (4,\infty)$.

(2) $f(x)=\sqrt{|x-1|-4}+\dfrac{\sqrt{2x-1}}{3-|x|}$

* $\sqrt{|x-1|-4} \Rightarrow |x-1|-4 \geq 0 \Rightarrow |x-1| \geq 4 \Rightarrow x-1 \geq 4$ or $x-1 \leq -4$
$x \geq 5$ or $x \leq -3$
$(-\infty,-3] \cup [5,\infty)$

* $\sqrt{2x-1} \Rightarrow 2x-1 \geq 0 \Rightarrow x \geq \frac{1}{2}$ $\quad [1/2,\infty)$

* $3-|x| \Rightarrow 3-|x|=0 \Rightarrow x=\pm 3 \Rightarrow \mathbb{R}-\{3,-3\}$

Domain (f) $= \big((-\infty,-3] \cup [5,\infty)\big) \cap \big(\mathbb{R}-\{3,-3\}\big) \cap [\frac{1}{2},\infty) = [5,\infty)$

H.W (2): Find the domain
① $f(x)=\dfrac{x^2-3}{2x^3-4x}$ ② $g(x)=\sqrt{5-x}+\dfrac{1}{\sqrt{x-1}}+\dfrac{1}{x-3}$.

* $\boxed{\text{Range}}$:

Example: Find the Range.

(1) $f(x)=\sqrt{2x-1} \Rightarrow$ Range $[0,\infty)$

(2) $f(x)=-\sqrt{2x-1} \Rightarrow$ Range $(-\infty,0]$

(3) $f(x)=\sqrt{2x-1}+5 \Rightarrow$ Range $[5,\infty)$

(4) $f(x)=x^3+2x^2+10 \Rightarrow$ Range: $\mathbb{R}$.

(5) $f(x)=\sqrt{2x-1}-3 \rightarrow$ Range $[-3,\infty)$

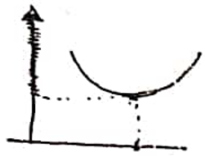
(6) $f(x)=-2-\sqrt{2x-1} \rightarrow$ Range $(-\infty,-2]$

• Range ← القيمة المطلقة
و الجذر نفس المعادلة

Example: Find the Range $f(x) = 2x^2 + 6x - 1$.

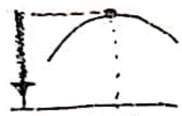
Sol: to find the Range of quadratic function $f(x) = ax^2 + bx + c$.

$a > 0$



$\frac{-b}{2a}$

$a < 0$



$\frac{-b}{2a}$

Range $[f(\frac{-b}{2a}), \infty)$ Range $(-\infty, f(\frac{-b}{2a})]$

$f(x) = 2x^2 + 6x + 1$ $a = 2 > 0$

$\frac{-b}{2a} = \frac{-6}{4} = -\frac{3}{2}$, Range $[f(-\frac{3}{2}), \infty) = [-\frac{11}{2}, \infty)$.

Example: Find the Range $f(x) = \sqrt{4 - x^2}$

Sol: to find range of $f(x) = \sqrt{a - x^2}$ (Semi Circle)

Range $[0, \sqrt{a}]$

Range $(\sqrt{4 - x^2}) = [0, 2]$

center $(0, 0)$

radius $\sqrt{a}$

Range $[0, \sqrt{a}]$

Domain $[-\sqrt{a}, \sqrt{a}]$

H.W (3): Find the Range.

(1) $f(x) = |7x - 1| - 4$

(2) $f(x) = -|5x + 8| + 9$

(3) $f(x) = -x^2 + 10x + 2$

(4) $f(x) = \sqrt{9 - x^2} + 3$ $[0, 3] \xrightarrow{+3} [3, 6]$

(5) $f(x) = -\sqrt{4 - x^2}$ $[-2, 0]$

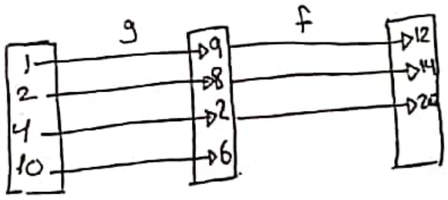
(6) $f(x) = x^2 + 7$.

and → $\cap$

Composition of functions :

Given Two functions f and g, the composite $(f\circ g)$ defined by $(f\circ g)(x) = f(g(x))$; $f\circ g$: f circle g, g is applied first then f is applied second.

Example :



* $f(g(1)) = f(9) = 12$
* $f(g(2)) = f(8) = 14$
* $f(g(4)) = f(2) = 20$
* $f(g(10)) = f(6)$ undefined.
* $\text{Domain}(g) = \{1, 2, 4, 10\}$, $\text{Domain}(f) = \{9, 8, 2\}$
* $\text{Domain}(f\circ g) = \{1, 2, 4\}$.

$\text{Domain } f\circ g = \{x \in D_g \text{ and } g(x) \in D_f\}$.

Ex: $f(x) = x^2 - 1$, $g(x) = \sqrt{3-x}$

Find (1) $(f\circ g)(-1)$ (2) $(g\circ f)(x)$ (3) $(f\circ g)(x)$

(4) Domain $f\circ g$ (5) Domain $(g\circ f)$

Sol: (1) $(f\circ g)(-1) = f(g(-1)) = f(\sqrt{3+1}) = f(2) = 3$

(2) $(g\circ f)(x) = g(f(x)) = g(x^2-1) = \sqrt{3-(x^2-1)} = \sqrt{4-x^2}$

(3) $(f\circ g)(x) = f(g(x)) = f(\sqrt{3-x}) = (\sqrt{3-x})^2 - 1 = 3 - x - 1 = 2 - x$

(4) $\text{domain}(f) = \mathbb{R}$, $\text{domain}(g) = (-\infty, 3]$

$\text{Domain}(f\circ g) = \{x \in D_g \ \& \ g(x) \in D_f\}$

$= \{x \in (-\infty, 3] \ \& \ \underbrace{\sqrt{3-x} \in \mathbb{R}}_{\text{True.}}\}$

$\text{Domain}(f\circ g) = (-\infty, 3]$

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(5) Domain $g\circ f = \{x \in D_f \ \& \ f(x) \in D_g\}$

$= \{x \in \mathbb{R} \ \& \ x^2-1 \in (-\infty, 3]\}$

Now, $x^2-1 \in (-\infty, 3]$

$x^2-1 \leq 3 \Rightarrow x^2-4 \leq 0$

$x = \pm 2$ $+ \quad - - \quad +$; $-2 \quad 2$

$x \in [-2, 2]$

$\Rightarrow \{x \in \mathbb{R} \cap x \in [-2,2]\} = [-2,2]$

Example: $f(x) = \frac{1+x}{1-x}$, $g(x) = \frac{x}{1-x}$, find domain $f\circ g$, $g\circ f$.

Sol: $D_f = \mathbb{R} - \{1\}$, $D_g = \mathbb{R} - \{1\}$

$D_{f\circ g} = \{x \in D_g \ \& \ g(x) \in D_f\}$

$= \{x \in \mathbb{R}-\{1\} \ \& \ \frac{x}{1-x} \in \mathbb{R}-\{1\}\}$

Now, $\frac{x}{1-x} \in \mathbb{R}-\{1\} \Rightarrow \frac{x}{1-x} \neq 1$

$\frac{x}{1-x} = 1 \Rightarrow x = 1-x \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

$\Rightarrow x \in \mathbb{R} - \{\frac{1}{2}\}$

$\Rightarrow = \{x \in \mathbb{R}-\{1\} \ \& \ x \in \mathbb{R}-\{\frac{1}{2}\}\} = \mathbb{R} - \{1, \frac{1}{2}\}$.

H.W (4) Domain $g\circ f$

Example: Find domain $f(x) = \sqrt{2-\sqrt{x}}$

$\sqrt{x} \Rightarrow x \geq 0$ and $2-\sqrt{x} \geq 0$

$2-\sqrt{x} = 0 \Rightarrow x = 4$ $+ + \quad - -$; 4

$x \in (-\infty, 4]$

$[0,\infty) \cap (-\infty, 4] = [0,4]$.

0   4

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Example: If $(f\circ g)(x) = x^2+6x+6$ and $g(x)=x+1$, find $f(x)$.

Sol: $(f\circ g)(x) = x^2+6x+6$

$f(x+1) = x^2+6x+6$, $y = x+1 \Rightarrow y-1 = x$

$f(y) = (y-1)^2 + 6(y-1) + 6 \rightarrow f(y) = y^2+4y+1 \rightarrow f(x) = x^2+4x+$

Example: If $(f\circ g)(x) = 3x^2+3x+2$, $f(x) = 3x+5$ find $g(x)$?

Sol: $f(g(x)) = 3g(x)+5 = 3x^2+3x+2$

$3g(x) = 3x^2+3x+2-5$

$g(x) = \frac{1}{3}(3x^2+3x-3) = x^2+x-1$.

H.W (5): (1) $f(x) = x^2+5x-7$, $g(x) = \frac{1}{x-1}$ find

(a) $(f\circ g)(x)$ (2) $(g\circ f)(x)$

(2) $f(2x-3) = x^2+5$, find $f(10)$?

→

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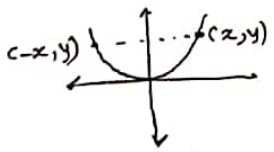
Tuesday

Even and Odd Functions

Definition: (1) A function f is said to be an even function if $f(-x) = f(x)$

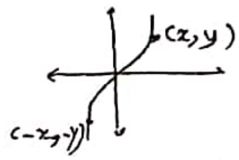
Example: $|x|, x^2, x^4, x^6, \dots$



symmetry about y-axis.

(2) A function f is said to be an odd function if $f(-x) = -f(x)$

Example: $x, x^3, x^5, \dots$



symmetry about origin

Rule:

÷ *	Even	odd
Even	Even	odd
odd	odd	Even

Example: Determine whether each of the following functions is even, odd or neither even nor odd.

(1) $f(x) = x^5 + x$

Sol: $f(-x) = (-x)^5 + (-x)$
$= -x^5 - x = -(x^5 + x) = -f(x)$
odd function.

(2) $f(x) = 1 - x^4$

Sol: $f(-x) = 1 - (-x)^4 = 1 - x^4$ Even function

(3) $h(x) = 2x - x^2$

$h(-x) = -2x - x^2 = -(2x + x^2) \neq f(x)$ Not even
and $\neq -f(x)$ Not odd $\Big\}$ neither

(4) $f(x) = \dfrac{x^5 + x}{1 - x^4}$

$x^5 + x$ odd & $1 - x^4$ even

then $f(x) = \dfrac{odd}{even}$ $\Rightarrow$ odd function.

• $f^{-1} \neq (f(x))^{-1}$

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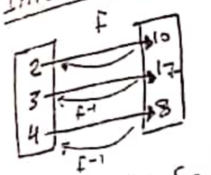
Inverse functions



$f(2)=10$, $f^{-1}(10)=2$
$f(3)=17$, $f^{-1}(17)=3$
$f(4)=8$, $f^{-1}(8)=4$

Domain $(f)=\{2,3,4\}$, Range $f\{10,17,8\}$
Domain $(f^{-1})=\{10,17,8\}$, Range $f^{-1}\{2,3,4\}$

نظرية
Theorem: Domain (f^{-1}) = Range (f)
Domain (f) = Range (f^{-1})

Examples: Find $f^{-1}(x)$.

$\cancel{x}=$

(1) $f(x)=5x^3+7$

Sol: $y=5x^3+7 \Rightarrow y-7=5x^3 \Rightarrow \frac{y-7}{5}=x^3$ $\wedge \frac{1}{3}$

$f^{-1}(x)=\left(\frac{x-7}{5}\right)^{1/3} \Leftarrow \left(\frac{y-7}{5}\right)^{1/3}=x$

(2) $f(x)=\sqrt[5]{2x-1}$

$y=(2x-1)^{1/5}$ $\wedge 5$

$y^5=2x-1 \Rightarrow y^5+1=2x \Rightarrow \frac{y^5+1}{2}=x$

$f^{-1}(x)=\frac{x^5+1}{2}$

Example: $f(x)=\frac{3x+5}{2x-1}$

Find (1) $f^{-1}(x)$, (2) domain (f) (3) Range (f)

Sol:

$y=\frac{3x+5}{2x-1}$

$2xy-y=3x+5$

$2xy-3x=y+5$

$x(2y-3)=y+5$

$x=\frac{y+5}{2y-3} \Rightarrow f^{-1}(x)=\frac{x+5}{2x-3}$

(2) domain (f) :-

$2x-1=0 \Rightarrow x=\frac{1}{2}$ $D_f=\mathbb{R}-\{\frac{1}{2}\}$

(3) Range (f) , to find Range (f) find domain f^{-1}

$f^{-1}(x)=\frac{x+5}{2x-3} \Rightarrow 2x-3=0 \Rightarrow x=\frac{3}{2}$

Range f = Domain $f^{-1}=\mathbb{R}-\{\frac{3}{2}\}$

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Thurs day 10. Oct.

One-to-One function :

A function f is called a one-to-one function if it never take On the same value twice, that is,

$$f(x_1) \neq f(x_2) \text{ whenever } x_1 \neq x_2 .$$

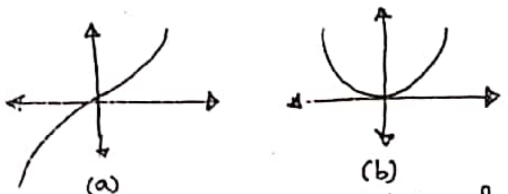
Example:

(1) $|x|, x^2, x^4, \dots$ Not 1-1 $\qquad f(G) = f(1)$

(2) $x^3, x^5, x^7, \dots$ is 1-1 $\qquad 1 = G$

افقي Horizontal line Test: A function is 1-1 iff no horizontal line intersect its graph more than once.

Ex:



(a) 1-1 function $\qquad$ (b) Not 1-1 function

Thm: A function f has an inverse iff is 1-1.

Restricting Domain for invertibility.

Example: find $f^{-1}(x)$.

اكمال المربع (1) $f(x) = 3x^2+6x-6$, $x \geq -1$.

Sol:

$$y = 3x^2+6x-6$$

$$\frac{y}{3} = x^2+2x-2 \qquad +\left(\frac{b}{2}\right)^2 = \left(\frac{2}{2}\right)^2 = 1$$

$$\frac{y}{3}+1 = x^2+2x+1-2$$

$$\frac{y}{3}+1 = (x+1)^2-2 \Rightarrow \frac{y}{3}+3 = (x+1)^2 \quad \wedge \tfrac{1}{2}$$

$$\sqrt{\frac{y}{3}+3} = x+1$$

$$\sqrt{\frac{y}{3}+3} - 1 = x$$

$$f^{-1}(x) = \sqrt{\frac{x}{3}+3} - 1 .$$

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H.W (6): Find $f^{-1}(x)$.

(1) $f(x) = (x+2)^4 , \ x \geq 0$

(2) $f(x) = -\sqrt{3-2x} , \ x \leq 0$

(3) $f(x) = 2x^2 - 16x + 5 , \ x \leq 4$.

Theorem: (1) $(f \circ f^{-1})(x) = x , \ \forall x \in D_{f^{-1}}$

(2) $(f^{-1} \circ f)(x) = x , \ \forall x \in D_f$.

Ex: Determine whether f & g are inverse functions

$f(x) = x^3 + 3x^2 + 3x + 1 , \ g(x) = x^{1/3} - 1$

Sol: $f(x) = x^3 + 3x^2 + 3x + 1 = (x+1)^3$

$\rightarrow (f \circ g)(x) = f(x^{1/3} - 1) = (x^{1/3} - 1 + 1)^3 = x$ ✓

$\rightarrow (g \circ f)(x) = g((x+1)^3) = ((x+1)^3)^{1/3} - 1 = x$ ✓

$\Rightarrow$ f and g are inverse functions.

Example 2:

(1) If $f(x) = x^3 + 5x - 2$, find $f^{-1}(4)$

Sol: $x^3 + 5x - 2 = 4$

$x^3 + 5x - 6 = 0 \Rightarrow$ Try numbers $(\pm 1, \pm 2, \pm 3, \pm 6)$

$\Rightarrow x = 1$

$f^{-1}(4) = 1$.

H.W (7): (1) $f(x) = \dfrac{x^3}{x+1}$, find x if $f^{-1}(x) = 2$?

(2) $f^{-1}(x) = 2x^3 + 5x + 3$, find x such that $f(x) = 1$.

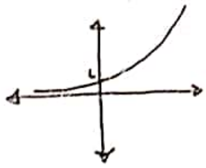
(3) $f(x) = \dfrac{(x+1)^3}{x^3} , x \neq 0$. find $f^{-1}(x)$ and range (f).

(3) $f(x) = 2x^3 + 5x + 3$, find $f^{-1}(0)$?

Exponential Functions

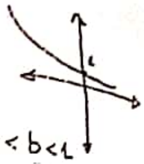
$f(x) = b^x$, $b > 0$ and $b \neq 1$

* $b^x : \mathbb{R} \longrightarrow (0, \infty)$



$b > 1$

Ex: 2^x, 3^x, ...
e^x



$0 < b < 1$

Ex: $(\frac{1}{2})^x$, $(\frac{1}{3})^x$,
e^{-x}

* The Natural Exponential Function

$f(x) = e^x$, $e = 2.7182$ ...

Example: ■ $f(x) = 2^{3x-1}$

Find ① $f(0) = 2^{-1}$ ② $f(2) = 2^5$ ③ $f(\frac{1}{3}) = 2^0 = 1$

■ $g(x) = e^{3x}$

Find ① $g(\frac{1}{2}) = e^{3/2}$ ② $g(0) = e^0 = 1$.

Laws of exponents:

If a & b are positive numbers and x & y any real numbers, then

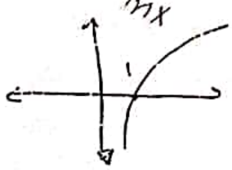
① $a^x . a^y = a^{x+y}$ ② $(a^x)^y = a^{xy}$ ③ $\frac{a^x}{a^y} = a^{x-y}$ ④ $a^{-x} = \frac{1}{a^x}$

⑤ $a^0 = 1$ ⑥ $a^{x/y} = \sqrt[y]{a^x}$ ⑦ $(a.b)^x = a^x b^y$

Logarithmic Function

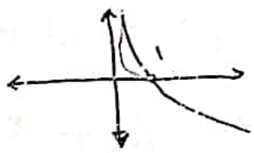
$f(x) = \log_b x$, $b > 0$ and $b \neq 1$

* $\log_b x : (0, \infty) \longrightarrow \mathbb{R}$.



$b > 1$

* The Natural Log Function



$0 < b < 1$

$f(x) = \log_e x = \ln x$ $\log_e x = \ln x$

* Note: $\log_{10} x = \log x$

Example

① $\log_2 8 = 3$ $(2^3 = 8)$ ③ $\log \frac{1}{1000} = -3$ $(10^{-3} = \frac{1}{1000})$

② $\log_9 \boxed{3} = \frac{1}{2}$ $(9^{\frac{1}{2}} = 3)$ ④ $\log_7 1 = 0$ $(7^0 = 1)$

⑤ $\log_{12} 12 = 1$ $(12^1 = 12)$.

مشابهات log هي نفسها مشابهات ln

Theorem: If $b>0$ and $b\neq 1$, Then b^x and $\log_b x$ are inverse functions

Example: Find $f^{-1}(x)$

(1) $f(x) = 5^x \Rightarrow f^{-1}(x) = \log_5 x$

(2) $f(x) = \log_2 x \Rightarrow f^{-1}(x) = 2^x$

Algebric properties of Logarithms.

If $b>0$ and $b\neq 1$ and $a, c>0$ and $r\in\mathbb{R}$, then.

(1) $\log_b(ac) = \log_b a + \log_b c$

(2) $\log_b\left(\frac{a}{c}\right) = \log_b a - \log_b c$

(3) $\log_b a^r = r\log_b a$

(4) $\log_b \frac{1}{c} = -\log_b c$

(5) $\log_b a = \frac{\log_c a}{\log_c b} = \frac{\ln a}{\ln b}$, $c\neq 1$.

(6) $\log_b b^x = x$ (10) $\ln e^x = x$

(7) $b^{\log_b x} = x$ (11) $e^{\ln x} = x$

(8) $\log_b b = 1$ (12) $\ln e = 1 \rightarrow \log_e e = 1$

(9) $\log_b 1 = 0$ (13) $\ln 1 = 0 \rightarrow \log_e 1 = 0$

• مرفوع لل log
او مرفوع لل اي اس $= x$

• ln مع e يشطبوا بعض

Ex: Simplify. بسط

(1) $\log \frac{xy^5}{\sqrt{z}}$

$= \log xy^5 - \log\sqrt{z}$

$= \log x + \log y^5 - \log z^{1/2}$

$= \log x + 5\log y - \frac{1}{2}\log z$.

Example: Find the exact value of each expression.

(1) $\log_2 6 \div \log_2 15 + \log_2 20$

$= \log_2(\frac{6}{15}) + \log_2 20 = \log_2(\frac{6}{15} \times 20) = \log_2 8 = 3$

(2) $e^{-2\ln 5}$

$= e^{\ln 5^{-2}} = 5^{-2} = \frac{1}{25}$

Example: Find the domain

(1) $f(x) = 2^{\sqrt{x^2-9}}$

$x^2 - 9 \geq 0 \Rightarrow$ $+ \quad - \quad +$; $-3 \quad 3$

$D_f = (-\infty, -3] \cup [3, \infty)$.

(2) $f(x) = \frac{1}{e^x - e^{2x}}$

$e^x - e^{2x} = 0 \Rightarrow e^x(1 - e^x) = 0 \Rightarrow 1 = e^x \Rightarrow x = 0$

$D_f = \mathbb{R} - \{0\}$.

(3) $f(x) = \ln(x-9)$

$\ln >$
$\log >$

$x - 9 > 0 \Rightarrow x > 9 \quad D_f = (9, \infty)$.

H.W (8) (1) Find domain

(a) $f(x) = \ln(\frac{4x-2}{2+x}) \rightarrow$ شرط البسط
شرط المقام

(b) $f(x) = \log_2(5-x)$

(c) $f(x) = \sqrt{1-2^x}$

(d) $f(x) = \ln(\ln x)$

(2) find domain and Range, $f(x) = \frac{e^x - 1}{e^x + 3}$

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اوجد قيمة
Example: Solve the following equations

(1) $\log x^2 + \log x = 30$

$2\log x + \log x = 30 \Rightarrow 3\log x = 30 \Rightarrow \log x = 10 \Rightarrow x = 10^{10}$

(2) $2^{x-5} = 3$

$\log_2 2^{x-5} = \log_2 3 \Rightarrow x-5 = \log_2 3 \Rightarrow x = \log_2 3 + 5 \Rightarrow x = \frac{\ln 3}{\ln 2} + 5.$

(3) $\ln(x+1) = 5$

$x+1 = e^5 \Rightarrow x = e^5 - 1$

(4) $e^{2x} - e^x = 6$ $\qquad y = e^x \rightarrow y^2 = e^{2x}$ gl

$e^{2x} - e^x - 6 = 0 \Rightarrow$

$(e^x+2)(e^x-3) = 0 \Rightarrow e^x = -2$ X and $e^x = 3 \Rightarrow \boxed{x = \ln 3}$

(5) $(x^2-1)(x-5)x^3 \log_x 2x = 0$

$x^2 - 1 = 0 \Rightarrow x = \pm 1$
$x - 5 = 0 \Rightarrow x = 5$
$x^3 = 0 \Rightarrow x = 0$
$\log_x 2x = 0 \Rightarrow 2x = 1 \Rightarrow x = \frac{1}{2}$

$\Rightarrow x = \{5, \frac{1}{2}\}$

H.W: Solve the following equations
(9) حل عادي

(1) $e^x - 2xe^x = 0$

(2) $\ln x + \ln(x-1) = 1$

(3) $\log_x (x^2 - 2) = 1$

(4) $\log x^{3/2} - \log\sqrt{x} = 5$

(5) $xe^{-x} - 3e^{-x} = -2$ $\boxed{-x = y}$

(6) $\frac{e^x - e^{-x}}{2} = 1$ $\qquad$ (8) $\log_2 3x + \log_4 9x^2 = 4$

(7) $\log_{x^x}(x^2-6) = 1$ X $\qquad$ ﻋﺎﺩﻱ (9) $9(3)^x = 4^{-x}$ $\quad \frac{1}{4^x}$

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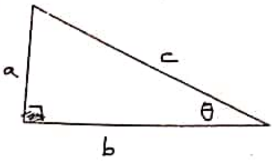
Trigonometric functions



(1) $\sin\theta = \frac{a}{c}$

(2) $\cos\theta = \frac{b}{c}$

(3) $\tan\theta = \frac{a}{b} = \frac{\sin\theta}{\cos\theta}$

(4) $\csc\theta = \frac{1}{\sin\theta} = \frac{c}{a}$

(5) $\sec\theta = \frac{1}{\cos\theta} = \frac{c}{b}$

(6) $\cot\theta = \frac{1}{\tan\theta} = \frac{b}{a} = \frac{\cos\theta}{\sin\theta}$

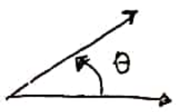
Pythagorean Thm: $c^2 = a^2 + b^2$

degrees	0	30°	45°	60°	90°	180°	270°	360°
radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1

Function	Domain	Range	Even/odd	1-1 / Not 1-1
$\sin\theta$	$\mathbb{R}$	$[-1,1]$	odd $\sin(-\theta) = -\sin\theta$	Not 1-1
$\cos\theta$	$\mathbb{R}$	$[-1,1]$	even $\cos(-\theta) = \cos\theta$	Not 1-1
$\tan\theta$	$\mathbb{R} - \{\frac{\pi}{2} + n\pi$ $n = 0, \pm1, \pm2, \dots\}$	$\mathbb{R}$	odd $\tan(-\theta) = -\tan\theta$	Not 1-1

Identities:

- $\sin^2 x + \cos^2 x = 1$, $1 + \tan^2 x = \sec^2 x$, $1 + \cot^2 x = \csc^2 x$.
- $\sin 2x = 2\sin x \cos x$.
- $\cos 2x = \cos^2 x - \sin^2 x$, $\cos 2x = 2\cos^2 x - 1$, $\cos 2x = 1 - 2\sin^2 x$
- $\sin^2 x = \frac{1}{2}(1 - \cos 2x)$, $\cos^2 x = \frac{1}{2}(1 + \cos 2x)$
- $\sin(x+y) = \sin x \cos y + \cos x \sin y$
- $\cos(x+y) = \cos x \cos y - \sin x \sin y$



Counterclockwise
(positive angle)

clockwise
(negative angle)

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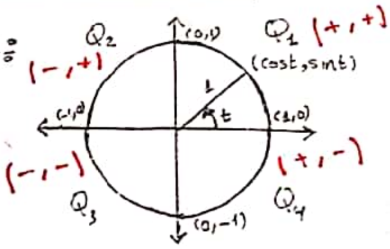
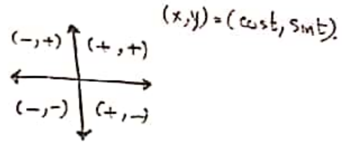
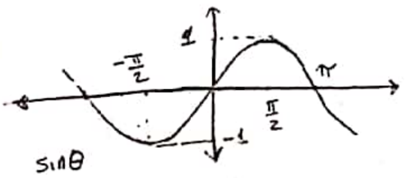
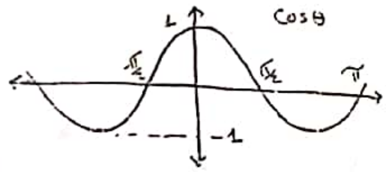
Unit Circle :

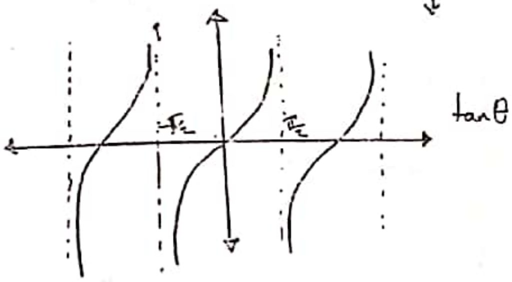




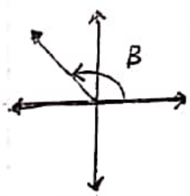
Q : Quadrant.

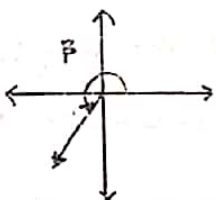


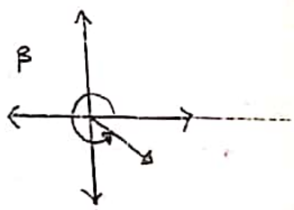




not 1-1







$Q_2 \to Q_1 : \theta = \pi - \beta$ $\quad$ $Q_3 \to Q_1 : \theta = \beta - \pi$ $\quad$ $Q_4 \to Q_1 : \theta = 2\pi - \beta$

Ex: Find $\sin\beta$ and $\cos\beta$ for..

(1) $\beta = \frac{5\pi}{6}$

$\theta = \pi - \frac{5\pi}{6} = \frac{\pi}{6}$

$\cos\frac{5\pi}{6} = -\cos\frac{\pi}{6} = -\frac{\sqrt{3}}{2}$

$\sin\frac{5\pi}{6} = \sin\frac{\pi}{6} = \frac{1}{2}$

(2) $\beta = \frac{4\pi}{3}$

$\theta = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$

$\cos\frac{4\pi}{3} = -\cos\frac{\pi}{3} = -\frac{1}{2}$

$\sin\frac{4\pi}{3} = -\sin\frac{\pi}{3} = -\frac{\sqrt{3}}{2}$

(3) $\beta = \frac{7\pi}{4}$.

$\theta = 2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$

$\cos\frac{7\pi}{4} = \cos\frac{\pi}{4} = \frac{1}{\sqrt{2}}$

$\sin\frac{7\pi}{4} = -\sin\frac{\pi}{4} = \frac{-1}{\sqrt{2}}$

Example: (1) Find the domain $f(x) = \sin(\sqrt{x-5})$?

$x - 5 \geq 0 \Rightarrow x \geq 5 \quad D_f : [\cancel{-5}\,5, \infty)$

(2) Find the Range $\quad D = R$

(a) $f(x) = \dfrac{3}{5 + \cos\theta}$

$-1 \leq \cos\theta \leq 1 \quad \boxed{+5}$

$4 \leq 5 + \cos\theta \leq 6$

$\dfrac{1}{4} \geq \dfrac{1}{5+\cos\theta} \geq \dfrac{1}{6} \xrightarrow{*3} \dfrac{3}{4} \geq \dfrac{3}{5+\cos\theta} \geq \dfrac{3}{6}$ Range $[\frac{1}{2}, \frac{3}{4}]$.

(b) $f(x) = 2\sin^2 x + 3$

$-1 \leq \sin x \leq 1 \quad$ ^2

$0 \leq \sin^2 x \leq 1 \quad *2$

$0 \leq 2\sin^2 x \leq 2 \quad +3$

$3 \leq 3 + 2\sin^2 x \leq 5$ Range $[3, 5]$.

H.W (10): (1) Find the domain $f(x) = \cos\left(\dfrac{1}{x-7}\right)$.

(2) Find the Range of $f(x) = 4 + 2\cos \cancel{x} + |\cos x|$ أكبر قيمة وأقل قيمة

$[-1, 1]$

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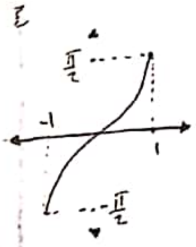
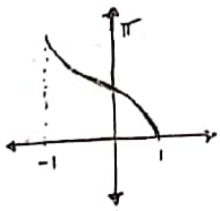
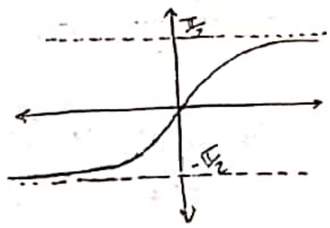
Inverse Trigonometric Functions

function	Domain	Range	Even/odd neither	$(f\circ f^{-1})(x)=x$, $(f^{-1}\circ f)(x)=x$
$\sin^{-1}x$	$[-1,1]$	$[-\frac{\pi}{2},\frac{\pi}{2}]$	odd	$\sin(\sin^{-1}x)=x$, $\forall x\in[-1,1]$ $\sin^{-1}(\sin x)=x$, $\forall x\in[-\frac{\pi}{2},\frac{\pi}{2}]$
$\cos^{-1}x$	$[-1,1]$	$[0,\pi]$	neither	$\cos(\cos^{-1}x)=x$, $\forall x\in[-1,1]$ $\cos^{-1}(\cos x)=x$, $\forall x\in[0,\pi]$
$\tan^{-1}x$	$\mathbb{R}$	$(-\frac{\pi}{2},\frac{\pi}{2})$	odd	$\tan(\tan^{-1}x)=x$, $\forall x\in\mathbb{R}$ $\tan^{-1}(\tan x)=x$, $\forall x\in(-\frac{\pi}{2},\frac{\pi}{2})$







Example:

(1) $\sin^{-1}(\frac{1}{2})=\frac{\pi}{6}$, (2) $\sin^{-1}(-1)=-\sin^{-1}(1)=-\frac{\pi}{2}$ (3) $\tan^{-1}(\frac{1}{\sqrt{3}})=\frac{\pi}{6}$

(4) $\cos^{-1}(\frac{1}{2})=\frac{\pi}{3}$, (5) $\cos^{-1}(-\frac{1}{2})=\pi-\cos^{-1}(\frac{1}{2})$
$=\pi-\frac{\pi}{3}=\frac{2\pi}{3}$

Note: $\cos^{-1}(-x)=\pi-\cos^{-1}(x)$

Example: Find domain $f(x)=\sin^{-1}(2x+1)$

$-1\le 2x+1\le 1$ $\boxed{-1}$

$-2\le 2x\le 0$ $\boxed{\div 2}$

$-1\le x\le 0$ $D_f=[-1,0]$

Example: Find the exact value.

(1) $\sin(\sin^{-1}\frac{1}{2})=\frac{1}{2}$ $\frac{1}{2}\in[-1,1]$

(2) $\sin^{-1}(\sin\frac{\pi}{4})=\frac{\pi}{4}$ $\frac{\pi}{4}\in[-\frac{\pi}{2},\frac{\pi}{2}]$

sin, $\sin^{-1}$ odd

(3) $\sin^{-1}(\sin\frac{2\pi}{3})$

$\frac{2\pi}{3}\in Q_2 \Rightarrow \theta=\pi-\frac{2\pi}{3}=\frac{\pi}{3}$

$\sin^{-1}(\sin\frac{\pi}{3})=\frac{\pi}{3}$

الربع الثاني يرجع اول
الثالث = رابع

$$(4)\ \sin^{-1}\left(\sin \frac{4\pi}{3}\right)$$

sin إشارة [illegible] سالب

$$\frac{4\pi}{3} \in Q_3 \Rightarrow \theta = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$$

$$\sin^{-1}\left(-\sin\frac{\pi}{3}\right) = -\frac{\pi}{3}$$

- $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

$Q_2 \longrightarrow Q_1$ $= \frac{\pi}{3}$

$\tan^{-1}\left(-\tan\frac{\pi}{3}\right) = -\frac{\pi}{3}$ odd

$$(5)\ \cos^{-1}\left(\cos\frac{2\pi}{3}\right) = \frac{2\pi}{3}$$

$$(6)\ \cos^{-1}\left(\cos\frac{7\pi}{4}\right)$$

$$\frac{7\pi}{4} \in Q_4 \Rightarrow \theta = 2\pi - \frac{7\pi}{4} = \frac{\pi}{4}$$

$$\cos^{-1}\left(\cos\frac{\pi}{4}\right) = \frac{\pi}{4}$$

$$(7)\ \cos^{-1}\left(\cos\frac{4\pi}{3}\right)$$

$$\frac{4\pi}{3} \in Q_3 \Rightarrow \theta = \frac{4\pi}{3} - \pi = \frac{\pi}{3}$$

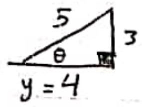
$$\cos^{-1}\left(-\cos\frac{\pi}{3}\right) = \pi - \cos^{-1}\left(\cos\frac{\pi}{3}\right) = \frac{2\pi}{3}$$

Example: Find the value of the following

(1) $\sec\left[\sin^{-1}\frac{3}{5}\right] = \theta$

sin

$$\theta = \sin^{-1}\frac{3}{5} \Rightarrow \sin\theta = \frac{3}{5}$$



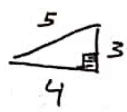
$y^2 = 5^2 - 3^2$

$y = 4$.

$$\sec\theta = \frac{5}{4}$$

(2) $\sin\left[2\sin^{-1}\frac{3}{5}\right]$

$$\theta = \sin^{-1}\frac{3}{5} \Rightarrow \sin\theta = \frac{3}{5}$$



$$\sin(2\theta) = 2\sin\theta\cos\theta$$
$$= 2\left(\frac{3}{5}\right)\left(\frac{4}{5}\right) = \frac{24}{25}$$

H.W

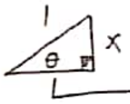
(1) (1) $\cos\left(\tan^{-1}\frac{1}{4}\right)$

(2) $\sec\left(\sin^{-1}\left(-\frac{3}{4}\right)\right)$.

Example: Find the value

(1) $\cos(\sin^{-1} x)$

$\theta = \sin^{-1} x \Rightarrow \sin\theta = x$



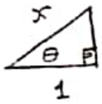
$\rightarrow \sqrt{1-x^2}$ by pythagorean Thm

$\cos(\theta) = \dfrac{\sqrt{1-x^2}}{1}$

(2) $\tan(\sin^{-1} x) = \dfrac{x}{\sqrt{1-x^2}}$

(3) $\sin(\sec^{-1} x)$

$\theta = \sec^{-1} x \Rightarrow \sec\theta = x$



$\rightarrow \sqrt{x^2-1}$ by pythagorean Thm

$\sin(\theta) = \dfrac{\sqrt{x^2-1}}{x}$

H.W: (12)

(1) $\cos(\tan^{-1} x)$

(2) $\sin(\cos^{-1} x)$

(3) $\cos(\sin^{-1} x + \sin^{-1} y)$.

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Sunday
Nov, 3, 2019

* Continuity الاتصال

A function f is continuous at $x=c$ if

① $f(c)$ is defined موجودة

② $\lim_{x \to c} f(x)$ exists

③ $\lim_{x \to c} f(x) = f(c)$

Remarks:

1. if f not continuous at $x=c$ we said that discontinuous at $x=c$

2. If f cts on $\mathbb{R}$ we say f cts everywhere.

Thm: The following types of functions are cts at every number in their domains ploy, rational, root, trigonometric, Inverse trigonometric, exponentia logarithmic.

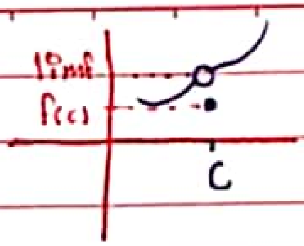
Thm: If f and g are cts at $x=c$

a) $f \pm g$ is cts at $x=c$

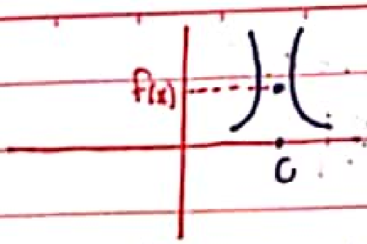
b) fg is cts at $x=c$

c) $\frac{f}{g}$ is cts at $x=c$ $g(c) \neq 0$

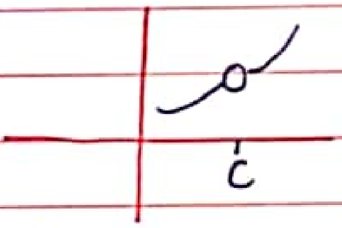
Sunday
Nov, 3, 2019



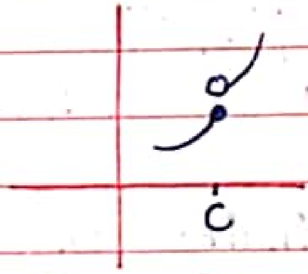
discts at x=c
③ x



discts at x=c.
② x



discts at $x=c$
① x



discts at $x=c$

- cts every where $\longrightarrow$ ploy, sin, cos, $\tan^{-1}$, b^x, e^x

$\rightarrow$ Where f(x) continuous :-

① $f(x) = \dfrac{\ln x + \tan^{-1} x}{x^2 - 1}$

$\ln x \longrightarrow (0,\infty) \longrightarrow (0,\infty)$ البسط

$\tan^{-1} x \longrightarrow R$

$x^2 - 1 \longrightarrow R - \{\pm 1\}$

$D_f = (0,\infty) \cap R - \{\pm 1\}$

f(x) is cts at $(0,1) \cup (1,\infty)$

② $f(x) = \sin^{-1}(1+2x)$

$-1 \le 1+2x \le 1$
$\quad -1 \quad\; -1 \quad\; -1$

$-2 \le 2x \le 0$

$-1 \le x \le 0$

$D_f = [-1, 0]$

$f(x)$ cts at $[-1, 0]$

* Continuity on an Interval.

f cts at $[a, b]$ if

① f cts at (a, b)

② $\lim_{x \to a^+} f(x) = f(a)$ (f cts from right at $x = a$)

③ $\lim_{x \to b^-} f(x) = f(b)$ (f cts from left at $x = a$)

Ex: $f(x) = \begin{cases} x^2 - 1 & , 0 \le x < 1 \\ 5 & , x = 1 \end{cases}$

Is f cts at $[0, 1]$? $\longrightarrow$ No, cts $[0, 1)$

① Is f cts at $(0, 1)$? yes, $x^2 - 1$ poly

② $\lim_{x \to 0^+} f(x) \stackrel{?}{=} f(0)$

$\lim_{x \to 0^+} x^2 - 1 = -1 = -1$ ✓

③ $\lim_{x \to 1^-} f(x) \stackrel{?}{=} f(1)$

$\lim_{x \to 1^-} x^2 - 1 = f(1)$

$0 \ne 5$

dis cts at $x = 1$ L.D.

Tuesday
5. Nov. 2019

Thm: If f and g are cts every where Then $f \circ g$ and $g \circ f$ cts everywhere.

Ex: 2^{x^2+1} , $|\sin x|$, $\tan^{-1}(\cos x)$

Thm: If g cts at $x=c$ and $f(x)$ cts at $g(c)$,
Then $\underline{\underline{f \circ g}}$ cts at $x=c$. ← الخارجي يقيم بجوار $g(C)$

Ex: $f(x) = \frac{1}{x - \frac{1}{2}}$, $g(x) = \frac{1}{2x}$

① where $f(x)$ cts? $\mathbb{R} - \{\frac{1}{2}\}$

② where $g(x)$ cts? $\mathbb{R} - \{0\}$

③ Is $f \circ g$ cts at $x=0$? $\longrightarrow$ No $\qquad f(g(x))$
Is g cts at $x=0$
$f \circ g$ dis cts at $x=0$

④ Is $f \circ g$ cts at $x=\frac{1}{2}$? $\qquad f(g(x))$
Is g cts at $x=\frac{1}{2}$? yes $\qquad f(g(\frac{1}{2}))$
Is $g(x)$ cts at $g(\frac{1}{2}) = 1$? yes $\qquad f(1)$

$f \circ g$ cts at $x = \frac{1}{2}$

Tuesday
5, Nov, 2019.

⑤ Is $f \circ g$ cts at $x = 1$?
Is g cts at $x = 1$? yes
Is $f(x)$ cts at $g(1) = \frac{1}{2}$? No

$f \circ g$ dis cts $x = 1$

Thm: If $\lim_{x \to c} g(x) = l$ and $f(x)$ cts at l, then

$$\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right) = f(l)$$

H.W:

1) $\lim_{x \to 2} \tan^{-1}\left(\frac{x^2 - 4}{3x^2 - 6x}\right)$

2) $\lim_{x \to 1} |x^2 - 4|$

3) find K such that $f(x)$ cts everywhere

$$f(x) = \begin{cases} 9 - x^2, & x \geq 0 \\ x + 3K & x < 0 \end{cases}$$

Ex: $\lim_{x\to 1} \sin^{-1}\left(\frac{1-\sqrt{x}}{1-x}\right)$ $\quad D \to [-1, 1]$

$= \sin^{-1}\left(\lim_{x\to 1} \frac{1-\sqrt{x}}{1-x}\right)$

$= \sin^{-1}\left(\lim_{x\to 1} \frac{\cancel{1-\sqrt{x}}}{\cancel{(1-\sqrt{x})}\,(1+\sqrt{x})}\right)$

$= \sin^{-1}\left(\lim_{x\to 1} \frac{1}{1+\sqrt{x}}\right) = \sin^{-1}\frac{1}{2} = \frac{\pi}{6}$

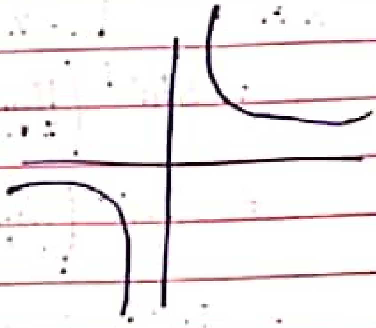
Ex: $\lim_{x\to 1} \cos\left(\frac{x^2-1}{x-1}\right)$ $\quad D \to R$

$= \cos\left(\lim_{x\to 1} \frac{\cancel{(x-1)}\,(x+1)}{\cancel{(x-1)}}\right)$

$= \cos(2)$

* limits : at Infinity :-

$x \to \infty$ $\qquad$ $x \to -\infty$



$$\lim_{x \to +\infty} \frac{1}{x} = \frac{1}{\infty} = \text{Zero}$$

$$\lim_{x \to -\infty} \frac{1}{x} = \frac{1}{-\infty} = \text{Zero}$$

Rules :

$$\lim_{x \to \infty} K = K$$

عدد على ∞ الجواب = Zero

Ex: ① $\lim_{x \to -\infty} \pi e^2 = \pi e^2$

بطلع نفس الجواب لانه مافي X حتى نعوض

② $\lim_{x \to +\infty} x^n = +\infty$ $\qquad$ $n = 1, 2, 3 \ldots$

③ $\lim_{x \to +\infty} x^9 = +\infty$

④ $\lim_{x \to +\infty} x^4 = +\infty$

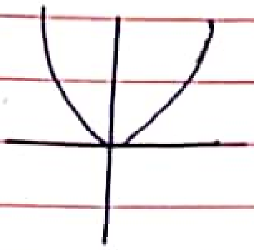
⑤ $\lim_{x \to -\infty} x^n = \begin{cases} +\infty & n : \text{even} \\ -\infty & n : \text{odd} \end{cases}$

Ex: ① $\lim_{x \to -\infty} x^9 = -\infty$

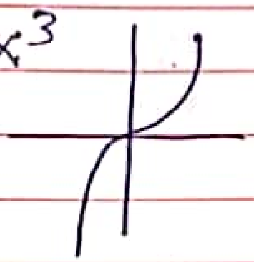
② $\lim_{x \to -\infty} x^4 = +\infty$

لان $X \to -\infty$ الها جوابين اذا كان الاس فردي يكون $\leftarrow -\infty$ واذا كان زوجي يكون $\leftarrow +\infty$

x^2



x^3



$$\lim_{x \to \infty} K x^n = K \lim_{x \to \infty} x^n$$

Ex: ① $\lim_{x \to -\infty} 10x^4 = +\infty$

② $\lim_{x \to -\infty} -7x^6 = -\infty$

③ $\lim_{x \to +\infty} 8x^8 = +\infty$

$\lim_{x \to \infty} (\text{poly})$

$$\lim (a_n x^n + a_{n-1} x^{n-1} + \dots + a x + a) = \lim_{x \to \infty} a_n x^n$$

Ex:

① $\lim_{x \to +\infty} (10x^7 - 8x^2 + x - \frac{1}{2}) = \lim_{x \to \infty} 10x^7 = +\infty$

② $\lim_{x \to -\infty} (1 - x - x^2 - x^3) = \lim_{x \to -\infty} -x^3 = +\infty$

Sunday Nov. 10

* limits of rational functions:

① $\lim_{x\to\infty} \frac{3x+5}{6x-3} = \lim_{x\to-\infty} \frac{3x}{6x} = \lim \frac{1}{2} = \frac{1}{2}$

② $\lim_{x\to\infty} \frac{5x^3-2x^2+1}{1-3x} = \lim_{x\to+\infty} \frac{5x^3}{-3x} = \lim_{x\to+\infty} \frac{-5}{3}x^2 = -\infty$

③ $\lim_{x\to\infty} \frac{4x^2-x}{2x^3-5} = \lim_{x\to-\infty} \frac{4x^2}{2x^3} = \lim_{x\to-\infty} \frac{2}{x} = 0$

Ex:

① $\lim_{x\to-\infty} \frac{3x+5}{6x-8} = \lim \frac{x\left(3+\frac{5}{x}\right)}{x\left(6-\frac{8}{x}\right)} = \frac{3+0}{6-0} = \frac{3}{6} = \frac{1}{2}$

② $\lim_{x\to\infty} \sqrt{\frac{2x-1}{5x+7}} = \sqrt{\lim_{x\to\infty} \frac{2x-1}{5x+7}} = \sqrt{\lim_{x\to\infty} \frac{2x}{5x}}$

③ $\lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2+2}}{3x-6}$

$= \lim\limits_{x \to -\infty} \dfrac{\sqrt{x^2\left(1+\frac{2}{x^2}\right)}}{x\left(3-\frac{6}{x}\right)}$

$= \lim\limits_{x \to -\infty} \dfrac{|x|\sqrt{1+\frac{2}{x^2}}}{x\left(3-\frac{6}{x}\right)}$

$= \lim \dfrac{-\cancel{x}\sqrt{1+\frac{2}{x^2}}}{\cancel{x}\left(3-\frac{6}{x}\right)}$ $= \lim \dfrac{-\sqrt{1+\frac{2}{x^2}}}{3-\frac{6}{x}}$

$= \dfrac{-\sqrt{1+0}}{3-0} = \dfrac{-1}{3}$

H.W:-

① $\lim\limits_{x \to \infty} \dfrac{\sqrt{x^2+2}}{3x-6} = \dfrac{1}{3}$

② $\lim\limits_{x \to +\infty} \sqrt{x^6+5x^3} - x^3 = \dfrac{5}{2}$

* Horizontal asymptotes:

$y = L$ is H. asy if $\lim\limits_{x \to +\infty} f(x) = L$ or $\lim\limits_{x \to -\infty} f(x) = L$

Sunday. Nov. 10

Find the H. asys

① $f(x) = \dfrac{x^2 - 10x}{3x^2 + 5}$

$\rightarrow \lim_{x \to \infty} \dfrac{x^2 - 10x}{3x^2 + 5} = \dfrac{1}{3}$

$\rightarrow \lim_{x \to -\infty} \dfrac{x^2 - 10x}{3x^2 + 5} = \dfrac{1}{3} \longrightarrow y = \dfrac{1}{3}$ is H. asy

② $f(x) = \dfrac{x - 2}{|x| - 2}$

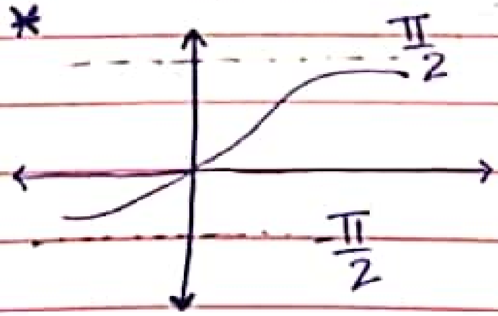
$\lim_{x \to \infty} \dfrac{x - 2}{x - 2} = 1$

$\lim_{x \to -\infty} \dfrac{x - 2}{2 - x} = -1$

$y = 1, -1$ is H. asy

$\boxed{*}$ ① No H. asy $\longrightarrow$ poly
② One H. asy $\longrightarrow$ e^x
③ 2 H. asy $\longrightarrow$ $\tan^{-1} x$

*

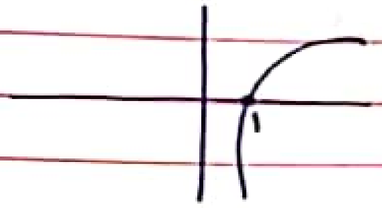


$\tan^{-1} x$

$\lim_{x \to \infty} \tan^{-1} x = \dfrac{\pi}{2}$

$\lim_{x \to -\infty} \tan^{-1} x = -\dfrac{\pi}{2}$

$y = \dfrac{\pi}{2}, -\dfrac{\pi}{2}$ are H. asy



$\ln x$

$\log_b x \quad b>1$

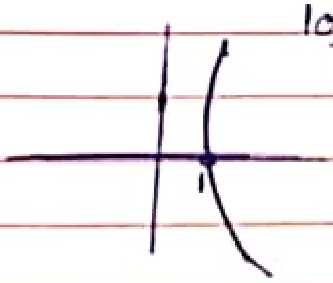
ln x Cute log

$\lim_{x\to\infty} \ln x = +\infty$

$\lim_{x\to+\infty} \log_5 x = +\infty$

$\lim_{x\to{}^+0} \ln x = 0 \; {}^{-\infty}$

$\lim_{x\to{}^+0} \log x = -\infty$



$\log_b x$

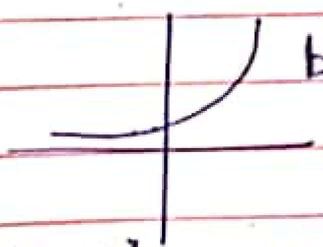
$0<b<1$

$\lim_{x\to+\infty} \log_b x = -\infty$

$\lim_{x\to+\infty} \log_{\frac{1}{2}} x = -\infty$

$\lim_{x\to 0^+} \log_b x = +\infty$

$\lim_{x\to+0} \log_{0.7} x = +\infty$



$b^x \quad b>1$

e^x

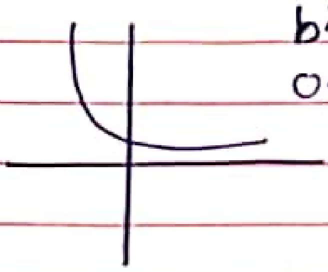
$\lim_{x\to+\infty} b^x = +\infty$

Ex: ① $\lim_{x\to+\infty} e^x = +\infty$

$\lim_{x\to-\infty} b^x = 0$

② $\lim_{x\to-\infty} 2^x = 0$

$y=0$ is H.asy



b^x

$0<b<1$

$\lim_{x\to+\infty} b^x = 0$.

$\lim_{x\to-\infty} b^x = +\infty$

* Tuesday, Nov. 12

① $\lim_{x\to 0^+} \tan^{-1}(\ln x) = \tan^{-1}\left(\lim_{x\to 0^+} \ln x\right) = \tan^{-1}(-\infty) = -\frac{\pi}{2}$

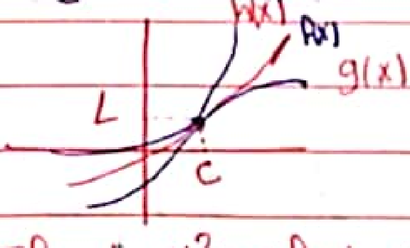
② $\lim_{x\to+\infty} \tan^{-1} e^x = \tan^{-1}\left(\lim_{x\to+\infty} e^x\right) = \tan^{-1}(+\infty) = \frac{\pi}{2}$

③ $\lim_{x\to+\infty} e^{\frac{3x-7}{6x+4}} = e^{\lim_{x\to+\infty}\frac{3x-7}{6x+4}} = e^{\frac{3}{6}} = e^{\frac{1}{2}} = \sqrt{e}$

P.80

* The squeez Theorem:

let f, g, h are functions, such that $g(x) \le f(x) \le h(x)$ and $\lim_{x\to c} g(x) = \lim_{x\to c} h(x) = L$, then $\lim_{x\to c} f(x) = L$



Ex: If $1-x^2 \le f(x) \le \cos x$ find $\lim_{x\to 0} f(x)$?

$\downarrow \lim_{x\to 0}$ $\quad\quad$ $\downarrow \lim_{x\to 0}$

$1 \quad\quad 1$

$\lim_{x\to 0} f(x) = 1$ by S.T

Tuesday

12, Nov, 2019

$\lim\limits_{\substack{x \to +\infty \\ x \to -\infty}} \sin x$] d.n.e $\qquad$ $\lim\limits_{\substack{x \to +\infty \\ x \to -\infty}} \cos x$] d.n.e

Ex: $\lim\limits_{x \to 0} x^2 \sin \frac{1}{x}$ $\longrightarrow$ $-1 \leq \sin \frac{1}{x} \leq 1$ $\quad x^2$

0 d.n.e لايوجد

$$-x^2 \leq x^2 \sin \frac{1}{x} \leq x^2$$

$$\lim_{x \to 0} \downarrow \qquad\qquad \lim_{x \to 0} \downarrow$$

$$0 \qquad\qquad\qquad 0$$

$$\therefore \lim_{x \to 0} x^2 \sin \frac{1}{x} = 0 \quad \text{by S.T}$$

H.W:

① $\lim\limits_{x \to 0} x \sin \frac{1}{x}$ = Zero

② $\lim\limits_{x \to +\infty} e^{-2x} \cos x$

③ $\lim\limits_{x \to +\infty} \dfrac{3x + \cos 5x}{x - \sin 3x}$

④ If $-x^2 + 4 \leq \dfrac{f(x)}{1 + x^2} \leq x^2 + 4$ find $\lim\limits_{x \to 0} f(x) = 4$

* Differentiation:

The derivative (مشتقة) of a function $f(x)$ at a number a denoted by $f'(a) = \left.\dfrac{dy}{dx}\right|_{x=a} = \left.\dfrac{d}{dx}\right|_{x=a}$ is

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a} = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h}$$ if this Limit exists

Ex: $f(x) = x^2$

① find $f'(8)$?

$$f'(8) = \lim_{x \to 8} \frac{f(x) - f(8)}{x - 8} = \lim_{x \to 8} \frac{x^2 - 64}{x - 8}$$

$$= \lim_{x \to 8} \frac{\cancel{(x-8)}(x+8)}{\cancel{x-8}} = 16$$

Ex: $f(x) = 3x^2$ find $f'(x)$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{3(x+h)^2 - 3x^2}{h}$$

$$= \lim \frac{\cancel{3x^2} + 6xh + 3h^2 - \cancel{3x^2}}{h}$$

$$= \lim_{h \to 0} \frac{h(6x + 3h)}{h} = 6x$$

Thursday Nov 14:-

* Differentiation Rules:-

find $y' = \frac{dy}{dx} = f'(x)$

① $y = \sqrt{x} + \sqrt[3]{x^2} - 10x^3 + \frac{4}{x} + \pi - 15$

$y = x^{\frac{1}{2}} + x^{\frac{2}{3}} - 10x^3 + 4x^{-1} + \pi - 15$

$y' = \frac{1}{2} x^{-\frac{1}{2}} + \frac{2}{3} x^{-\frac{1}{3}} - 30x^2 - 4x^{-2}$

② $(f \cdot g)' = ① ②' + ② ①'$

$y = (x^{-\frac{1}{3}} + x^2) \cdot (10x - 1)$

$y' = (x^{-\frac{1}{3}} + x^2)(10) + (10x - 1)(-\frac{1}{3} x^{-\frac{4}{3}} + 2x)$

Thursday
Nov, 14

③ $y = \frac{x^3 + 7x^{-2}}{5x+1}$

$y' = \frac{(5x+1)(3x^2 - 14x^{-3}) - (x^3 + 7x^{-2})(5)}{(5x+1)^2}$

④ $\left(\frac{c}{f}\right)' = \frac{-c f'}{(f)^2}$

$y = \frac{-2}{(x^3+3)^2}$

$y' = \frac{2(3x^2)}{(x^3+2)^2}$

Chain Rule:

$\frac{d}{dx}(f(x))^n = n(f(x))^{n-1} f'(x)$

Ex: ① $y = (3x^4 + x^2 - 1)^{-10}$

$y' = -10(3x^4 + x^2 - 1)^{-11}(12x^3 + 2x)$

② $y = \sqrt[5]{(10x^2 + x + 7)^2} = (10x^2 + x + 7)^{\frac{2}{5}}$

$y' = \frac{2}{5}(10x^2 + x + 7)^{-\frac{3}{5}}(20x+1)$

** $\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$

find $f'(2)$, $f(x) = \sqrt{x^2 + x - 1}$

$f'(x) = \frac{2x+1}{2\sqrt{x^2+x-1}} = \frac{4+1}{2\sqrt{4+2-1}} = \frac{\sqrt{5}}{2}$

Thursday
Nov, 14

$$\frac{d}{dx} f(g(x)) = f'(g(x))\, g'(x)$$

Ex: ① $f(1) = 2$, $f'(1) = 3$, $g(4) = 1$, $g'(4) = 2$

find $(f\circ g)'(4)$?

$(f\circ g)'(x) = f'(g(x))\, g'(x)$

$= f'(g(4))\, g'(4)$

$= f'(1)\, g'(4)$

$= 3 \times -2 = -6$

② $f(x) = (x^2+1)^2$, $g(x) = \frac{2x}{x-1}$

find: ⓐ $(f\circ g)'(2)$ {H.W}

ⓑ $(g\circ f)'(x) = g'(f(x))\, f'(x)$

$= g'((x^2+1)^3)\,(6x(x^2+1)^2)$

$= \frac{-2}{((x^2+1)^2-1)^2}\; 6x(x^2+1)^2$

$f'(x) = 3(x^2+1)^2(2x)$

$= 6x(x^2+1)$

$g'(x) = \frac{(x-1)(2) - 2x(1)}{(x-1)^2}$

$g'(x) = \frac{\cancel{2x} - 2 - \cancel{2x}}{(x-1)^2}$

Find:

① If $f(2x+1) = x^3+5x-10$ find $f(3)$

$f'(2x+1)*(2) = 3x^2+5$

$f'(2x+1) = \frac{3x^2+5}{2}$

$f'(3) = \frac{3(1)^2+5}{2} = 4$

$2x+1=3$

$2x=2$

$\boxed{x=1}$

مشتقة

(2) $\frac{d}{dx}\left(f(x^3)\right) = \underbrace{12x^{17}}_{\text{جواب الاشتقاق}}$ find $f'(x)$

$f'(x^3)(3x^2) = 12x^{17}$

$f'(x^3) = \frac{12x^{17}}{3x^2}$ $\qquad y = x^3 \longrightarrow x = y^{\frac{1}{3}}$

$f'(x^3) = 4x^{15}$

$f'(y) = 4(y^{\frac{1}{3}})^{15}$

$f'(y) = 4y^5$

$f'(x) = 4x^5$

find y':

(1) $y = \sqrt{\tan x + \cos x^3}$

$y' = \frac{\sec^2 x + (-\sin x^3)(3x^2)}{2\sqrt{\tan x + \cos x^3}}$

(2) $y = \sin^2 x + \sin x^2$

$= (\sin x)^2 + \sin x^2$

$y' = 2(\sin x)(\cos x) + \cancel{\sin}\ \cos x^2 (2x)$

Thursday
Nov, 14

H.W: ⓐ $f(2)=3$, $g(2)=4$, $f'(2)=-2$, $g'(2)=7$
Find $h'(2)$, $h(x)=\frac{f(x)}{g(x)}$

ⓑ find y':

① $y=(x^3+x+1)^{10}$
$y' = 10(x^3+x+1)^9(3x^2+1)$

② $y=\sqrt{3x+x^3}$
$y' = \frac{1}{2}(3x+x^3)^{-\frac{1}{2}}(3+3x^2)$

③ $y=\frac{x^2+1}{(3x-7)^{\frac{1}{3}}}$

④ $y=\sqrt[3]{(8x+10)^2}$

⑤ $y=\cos x^3+\sin^4 x$
$y'=(-\sin x^3)(3x^2)+4(\sin x)(\cos x)$

⑥ $y=5\cos x+x^2\sin x$
$y'=-5\sin x+x^2\cos x+(\sin x)(2x)$

⑦ $y=(1+\tan x)^2 \rightarrow y'=2(1+\tan x)(\sec^2 x)$

⑧ $y=\sec((\cos x)+1)^2$

Sunday

17/11/201

Find y' : "Log"

① $y = \ln(x^3 + \cos x)$

$$y' = \frac{3x^2 - \sin x}{x^3 + \cos x}$$

② $y = \log_3 (1 - \tan x)$

$$y' = \frac{-\sec^2 x}{(1 - \tan x)\ln 3}$$

③ $y = \ln(\ln x)$ find $\left.\frac{dy}{dx}\right|_{x=e^3}$

$$y' = \frac{\frac{1}{x}}{\ln x} = \frac{1}{x \ln x}$$

$$\left.\frac{dy}{dx}\right|_{x=e^3} = \frac{1}{e^3 \ln e^3} = \frac{1}{3e^3}$$

④ $y = \ln(\ln(\ln x))$

$$y' = \frac{\frac{\frac{1}{x}}{\ln x}}{\ln(\ln x)} = \frac{1}{x \ln x\,(\ln(\ln x))}$$

⑤ $y = \ln\left(\sqrt{1 + \sin^2 x}\right)$ H.W

⑥ $y = \sqrt{1 + \sqrt{1 + \sqrt{x}}}$ H.W

⑦ $y = \log_5 x$

$$y' = \frac{1}{x \ln 5}$$

⑧ $y = \log_x 5 \quad = y = \frac{\ln 5}{\ln x}$

$$y' = \frac{(-\ln 5)\left(\frac{1}{x}\right)}{(\ln x)^2}$$

⑨ $y = \log_7 5 = 0$

⑩ $y = \log_{(x^2-1)} (x^3+5x)$

$$y = \frac{\ln (x^3+5x)}{\ln (x^2-1)}$$

find y' "Exponential"

① $y = 3^{x^2}$

$y' = 3^{x^2} \quad (2x) \quad \ln 3$

② $y = e^{-x} \longrightarrow y' = (e^{-x})\,(-1)$

③ $y = 9^{\tan x} \longrightarrow y' = (9^{\tan x})\,(\sec^2 x)\,(\ln 9)$

④ $y = e^{x\cos x} \longrightarrow y' = e^{x\cos x}\,(-x\sin x + \cos x)$

Find $\left.\frac{dy}{dx}\right|_{x=\pi}$

$= e^{\pi(-1)}\,(\underbrace{-\pi \cancel{\sin \pi}}_{zero} + \underbrace{\cancel{\cos \pi}}_{-1})$

$= -e^{-\pi}$

H.W: ① $f(x) = e^x \, g(x)$, $g'(0) = 5$, find $f'(0)$

$g(0) = 2$

$f'(x) = e^x \, g'(x) + g(x)\, e^x$

$f'(0) = e^0 \, g'(0) + g(0)\, e^0 = 5+2 = 7$

Sunday

②/

② Find y' :

1) $y = \pi^{x\tan x^2}$

$y' = \pi^{x\tan x^2}\,(x\sec^2 x^2)(2x + (\tan x^2))\ln\pi$

2) $y = e^{\sqrt{1-5x^3}}$ → $y' = e^{\sqrt{1-5x^3}}\,\dfrac{-10x}{2\sqrt{1-5x}}$

3) $y = \sqrt{\ln x}$

$y' = \dfrac{\frac{1}{x}}{2\sqrt{\ln x}} = \dfrac{1}{2x\sqrt{\ln x}}$

4) $y = \log_7 x^3 \rightarrow y' = \dfrac{3x^2}{x^3 \ln 7}$

5) $y = \dfrac{\log x}{1 + \log x}$

6) $y = \ln(1 - xe^{-x})$

$y' = \dfrac{-x(-e^{-x}) + e^{-x}(-1)}{1 - xe^{x}}$

7) $y = \cos(\ln x)$

$y' = -\sin(\ln x)(\ln x)$

Ex: Find y' :

$y = \ln(\cos(e^x))$

$y' = \dfrac{[-\sin(e^x)]\,e^x}{\cos(e^x)}$

$y' = -e^x \tan e^x$

Find $\left.\dfrac{dy}{dx}\right|_{x=0} = -\tan 1$

Sunday

Find y'

① $y = \sin^{-1}(1+x^2)$

$$y' = \frac{2x}{\sqrt{1-(1+x^2)^2}}$$

② $y = \tan^{-1}(\ln x)$

$$y' = \frac{\frac{1}{x}}{1+(\ln x)^2}$$

③ $y = \sec^{-1}(3^x)$

$$y' = \frac{3^x \ln 3}{|3^x|\sqrt{(3^x)^2 - 1}}$$

$$y' = \frac{3^x \ln 3}{3^x\sqrt{3^{2x}-1}} = \frac{\ln 3}{\sqrt{3^{2x}-1}}$$

④ $y = \tan^{-1} x$

$$y' = \frac{1}{1+x^2}$$

⑤ $y = \sin^{-1} x$

$$y' = \frac{1}{\sqrt{1-x^2}}$$

⑥ $y = (\sin x)^{-1}$

$y' = -(\sin x)^{-2}(\cos x)$ or $y = (\sin x)^{-1} = \frac{1}{\sin x}$

$= \csc x$

$y' = -\csc x \cot x$

find y'

① $y = \ln\left(\dfrac{x^2 \sin x}{\sqrt{1+x}}\right)$

$y = \ln x^2 \sin x - \ln\sqrt{1+x}$

$= \ln x^2 + \ln \sin x - (1+x)^{\frac{1}{2}}$

$= 2\ln x + \ln \sin x - \frac{1}{2}\ln(1+x)$

$= \dfrac{2}{x} + \dfrac{\cos x}{\sin x} - \dfrac{1}{2}\left(\dfrac{1}{1+x}\right)$

② $y = \log_3 (\sin^{-1}(e^x))(\cos x^2)$

$y = \log_3 (\sin^{-1}(e^x)) + \log_3 \cos x^2$

$y' = \dfrac{\frac{e^x}{\sqrt{1-e^{2x}}}}{(\sin^{-1} e^x)\ln 3} + \dfrac{(-\sin x^2)(2x)}{(\cos x^2)\ln 3}$

H.W: find y'

① $y = \sec^{-1}(e^x)$

② $y = \sin^{-1}(\cos x^2)$

③ $y = \tan^{-1}\sqrt{x}$

④ $y = \ln|\sin x + x|$

⑤ $y = \log_5\left(\dfrac{x^2 \sin e^x}{\sqrt{1+5x}}\right)$

Tuesday
Nov. 19

* Implicit differentiation :-

الاشتقاق الضمني

$y \rightarrow y'$

① $x^3 + y^4 = 10$

$3x^2 + 4y^3 y' = 0$

$4y^3 y' = -3x^2$

$y' = \dfrac{-3x^2}{4y^3}$

② $e^y \sin x = \tan(x-y) + e^{y^2}$

$e^y * \cos x + \sin x * e^y y' = \sec^2(x-y)(1-y') + 2yy' e^{y^2}$

$e^y \cos x + (e^y \sin x) y' = \sec^2(x-y) - y' \sec^2(x-y) + 2yy' e^{y^2}$

$e^y \cos x - \sec^2(x-y) = -y' \sec^2(x-y) + 2yy' e^{y^2} - (e^y \sin x) y'$

$e^y \cos x - \sec^2(x-y) = y'\left(-\sec^2(x-y) + 2y e^{y^2} - e^y \sin x\right)$

$y' = \dfrac{e^y \cos x - \sec^2(x-y)}{-\sec^2(x-y) + 2y e^{y^2} - e^y \sin x}$

③ Ex: $f(1) = 1$, $f'(1) = 3$

$f(y^2 x) = 2y f(x) - 1$

find $\left.\dfrac{dy}{dx}\right|_{(x,y)=(1,1)}$

$f'(y^2 x)\,(y^2 + x\,2yy') = 2y f'(x) + f(x)\,2y'$

$f'(1)\,(1 + 2y') = 2 f'(1) + 2 f(1) y'$

$3\,(1 + 2y') = 6 + 2y'$

$3 + 6y' = 6 + 2y'$

$y' = \dfrac{3}{4}$

Tuesday

Ex: find y'

① $y = x^2 \longrightarrow y' = 2x$ $\qquad (f(x))^n$

② $y = 2^x \longrightarrow y' = 2^x \ln 2$ $\qquad b^{f(x)}$

③ $y = x^x$: $\qquad (f(x))^{g(x)}$

$\ln y = \ln x^x$

$\ln y = x \ln x$

$\frac{y'}{y} = x\left(\frac{1}{x}\right) + \ln x$

$\frac{y'}{y} = 1 + \ln x$

$y' = y(1+\ln x)$

$y' = x^x(1+\ln x)$

④ $x^y = y^x$

$\ln x^y = \ln y^x$

$y \ln x = x \ln y$

$y\left(\frac{1}{x}\right) + \ln x \, y' = x \frac{y'}{y} + \ln y$

$\ln x \, y' - x \frac{y'}{y} = \ln y - \frac{y}{x}$

$y'\left(\ln x - \frac{x}{y}\right) = \ln y - \frac{y}{x} \longrightarrow y' = \frac{\ln y - \frac{y}{x}}{\ln x - \frac{x}{y}}$

Tuesday
Nov. 19, 2019

Higher Derivatives:

$f'\ ,\ f''\ ,\ f^{(3)}\ ,\ f^{(4)}\ ,\ \ldots\ldots\ ,\ f^{(n)}$

$\frac{d}{dx}\ ,\ \frac{d^2}{dx^2}\ ,\ \frac{d^3}{dx^3}\ ,\ \ldots\ldots\ ,\ \frac{d^n}{dx^n}$

$\frac{d}{dy}\ ,\ \frac{d^2}{dy^2}\ ,\ \frac{d^3}{dx^3}\ ,\ \ldots\ldots\ ,\ \frac{d^n}{dy^n}$

$f^{(4)}(x) \neq f^4(x)$

$f^{(4)}(x) = \frac{d^4y}{dx^4}$

$\frac{d^4y}{dx^4} \neq \left(\frac{dy}{dx}\right)^4$

Ex: $f(4) = 3\ ,\ f'(4) = 3\ ,\ f''(4) = 2$

find $\left(\frac{f'}{f}\right)'(4)$:

عوض

$$\left(\frac{f'}{f}\right)' = \frac{f(x)\, f''(x) - f'(x)\, f'(x)}{(f(x))^2} = -\frac{1}{3}$$

H.W:

① $f(x) = x^5$ find

$f^{(4)}(x)\ ,\ f^{(5)}(x)\ ,\ f^{(6)}(x)\ ,\ f^{(7)}(x)$

$\downarrow$ $120x$, $\downarrow$ 120 , $\downarrow$ zero , $\downarrow$ zero

② $f(x) = 3^x$

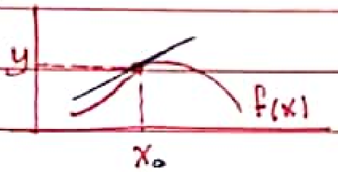
find $f^{(10)}(x)$?

③ $y = \cos^3 x + 5\tan x$ find y''

Sunday

Nov. 24, 2018

* Equation of tangent line:



$m = \text{Slope} = f'(x_0)$ الميل

→ equ of tangent line at (x_0, y_0) is $y - y_0 = m(x - x_0)$

→ equ of normal Line (perpendicular to the tangent line) عمودي على المماس ⊥
at (x_0, y_0)

$$y - y_0 = \frac{-1}{m}(x - x_0)$$

Ex: find the equ of tangent line, normal line at $x = \pi$ / $y = \sin x$

$y = \sin \pi = 0$ $(\pi, 0)$

$y' = \cos x \longrightarrow \cos \pi = -1 = m$

$y - 0 = -1(x - \pi) \longrightarrow y = \pi - x$ T.L

N.L → $y - 0 = \frac{-1}{-1}(x - \pi)$

$y = x - \pi$

Ex: find the values of x, such that the tangent line is horizontal

إذا المماس أفقي

$\boxed{\text{slope} = 0}$

$y = 2x^3 - 6x$ ← Domain

$y' = 6x^2 - 6$

$6x^2 = 6$

$\boxed{x = \pm 1}$

at $x = 1$, $x = -1$ H.T.L

Thm: If $f(x)$ is differentiable at $x=c$ then $f(x)$ is continuous at $x=c$

Remark: If f discts $c \rightarrow$ not diff at c

Ex: Is $f(x)$ diff at $x=1$

$$f(x) = \begin{cases} x^2+2 & , x \geq 1 \\ x+2 & , x < 1 \end{cases}$$

$\lim_{x \to 1^+} x^2+2 = 3$

$\lim_{x \to 1^-} = 3$

$f(1) = 3$

$\rightarrow f(x)$ Cts at $x=1$

$$f'(x) = \begin{cases} 2x & , x > 1 \\ 1 & , x < 1 \end{cases} \rightarrow \begin{matrix} f'_+(1) = f'_-(1) \\ 2 \neq 1 \end{matrix}$$

not diff at $x=1$

Ex: $$f(x) = \begin{cases} x^2-1 & , x \leq 1 \\ Kx-K & , x > 1 \end{cases}$$

find the Values of K, If f

1. Continuous
2. differentiable
3. Continuous but not diff

① $\lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x) = 0 \longrightarrow K \in R$ لما يطلع أيكوان zero

$\downarrow$

$\in R$

② $$f'(x) = \begin{cases} 2x & , x \leq 1 \\ K & , x > 1 \end{cases}$$ $f'_+(1) = f'_-(1)$

$\boxed{K = 2}$ $f(x)$ diff at $x=1$

③ $R - \{2\}$

Tuesday
26.Nov.2019

$f(x) = ***$ finds

متزايد   متناقص

① The Intervals of Incrasing and decreasing.
② The cirtical valuse
③ The extrema point (local, Absolut.) · (مطلق)
④ The max value of $f(x)$ / the min value of $f(x)$
⑤ The Intervals of concavity
⑥ The Inflection point

* كل الفترات مفتوحة ماعدا اذا هما جابولنا مغلقة

Ex: $f(x) = 3x^2 - 6x + 5$, $D = R$

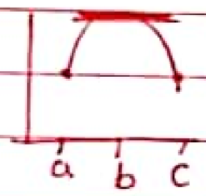
$f'(x) = 6x - 6$

$6x - 6 = 0$

$\boxed{x = 1}$

− | +

In $[1, \infty)$
dec $(-\infty, 1)$

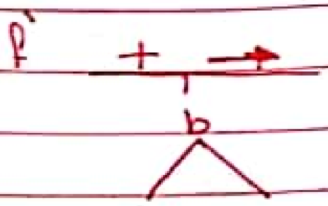


Domain $= [a, b]$
Inc (a, b) $f' > 0$
dec (b, c) $f' < 0$

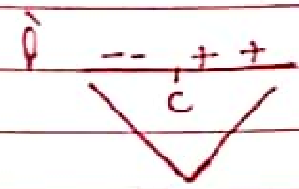
Critical
→ $f' \neq 0$ → H.T → b
→ f' d.n.e

* اطراف الفترات المغلقة
* اصفار مقام المشتقة
* احيانًا نقاط التشعب

اطراف الفترة بتكون absolute



local max



local Absolute

at $x = d$ Absolute max

Critical $\{a, b, c, d\}$

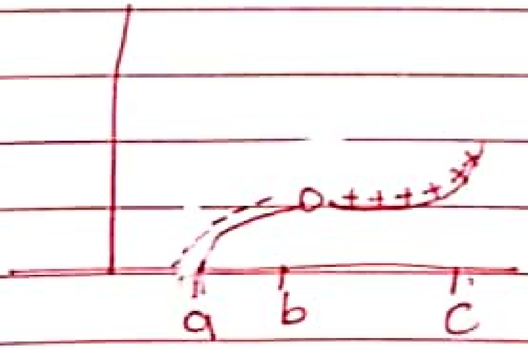
extreme $\{b, c, d\}$

min value of $f(x)$

$f(1) = 3(1)^2 - b(1) + 5 = 2$

$\frac{-b}{2a}$

Range $[2, \infty)$



$(a, b) \longrightarrow$ Concave down تقعر للاسفل

$(b, c) \longrightarrow$ concave up تقعر للاعلى

$f''(x) = 6 \quad f'' \; \underline{+\;+}$ Concave up $(-\infty, \infty)$

* $f(x) = x e^{-x}$ Domain $= \mathbb{R}$

$f'(x) = -x e^{-x} + e^{-x}$

$-x e^{-x} - e^{-x} = 0$

$e^{-x}(-x-1) = 0$

$\boxed{x = 1}$ critical value

$+\;+ \quad \underset{1}{|} \quad -\;-\;-$

Inc $(-\infty, 1)$

dec $(1, \infty)$

at $x=1$ Absolute local max

max value of $f(x) = e^{-1} = \frac{1}{e}$

Range $(-\infty, \frac{1}{e})$

$f''(x) = e^{-x}(-1) + (-x+1)e^{-x}(-1)$

$f''(x) = -2e^{-x} + xe^{x}$

تعويضها دائماً موجب

$-2e^{-x} + xe^{x} = 0$

$-e(-2+x)$

$\cancel{-2+x=0}$ $\quad -2-x=0$

* $\quad x=2$

$- \quad \underset{2}{|} \quad +$

$\cap \quad \cup$

Con.. up $(2, \infty)$

Con down $(-\infty, 2)$

at $x=2$ inflec point

Differentiation Rules

- $\frac{d}{dx}(c)=0 \quad , c \in \mathbb{R}$
- $\frac{d}{dx}(x^n)=n\,x^{n-1}$
- $\frac{d}{dx}(cf)=cf'$
- $\frac{d}{dx}(f \pm g)=f' \pm g'$
- $\frac{d}{dx}(f.g)=fg'+gf'$
- $\frac{d}{dx}\left(\frac{f}{g}\right)=\frac{gf'-fg'}{g^2}$
- $\frac{d}{dx}\left(\frac{c}{g}\right)=-\frac{c.g'}{g^2}$

Chain Rule

قاعدة السلسلة

- $\frac{d}{dx}(f(g(x)))=f'(g(x))g'(x)$
- $\frac{d}{dx}(f(x))^n=n(f(x))^{n-1}f'(x)$
- $\frac{d}{dx}\sqrt{f(x)}=\frac{f'(x)}{2\sqrt{f(x)}}$
- $\frac{dx}{dy}=\frac{dx}{du}\cdot\frac{du}{dy}$

Trigonometric functions

$\frac{d}{dx}(\sin(f(x)))=\cos(f(x))f'(x)$

$\frac{d}{dx}(\cos(f(x)))=-\sin(f(x))f'(x)$

$\frac{d}{dx}(\tan(f(x)))=\sec^2(f(x))f'(x)$

$\frac{d}{dx}(\cot(f(x)))=-\csc^2(f(x))f'(x)$

$\frac{d}{dx}(\sec(f(x)))=\sec(f(x))\tan(f(x))\cdot f'(x)$

$\frac{d}{dx}(\csc(f(x)))=-\csc(f(x))\cot(f(x))\cdot f'(x)$

Logarithmic functions

- $\frac{d}{dx}\left(\ln(f(x))\right) = \frac{f'(x)}{f(x)}, \quad f(x) > 0$
- $\frac{d}{dx}(\ln x) = \frac{1}{x}, \quad x > 0$
- $\frac{d}{dx}(\ln|x|) = \frac{1}{x}$
- مهم $\frac{d}{dx}\left(\log_b(f(x))\right) = \frac{f'(x)}{f(x)\ln b}, \quad f(x) > 0$
- $\frac{d}{dx}(\log_b x) = \frac{1}{x \ln b}, \quad x > 0$

Exponential functions

- $\frac{d}{dx}\left(e^{f(x)}\right) = f'(x)\, e^{f(x)}$
- $\frac{d}{dx}(e^x) = e^x$
- للأس $\frac{d}{dx}\left(b^{f(x)}\right) = b^{f(x)} \cdot \ln b \cdot f'(x)$
- $\frac{d}{dx}(b^x) = b^x \ln b$

Inverse Trigonometric Functions

- $\frac{d}{dx}\left(\sin^{-1}(f(x))\right) = \frac{f'(x)}{\sqrt{1-(f(x))^2}}$
- $\frac{d}{dx}\left(\cos^{-1}(f(x))\right) = \frac{-f'(x)}{\sqrt{1-(f(x))^2}}$
- $\frac{d}{dx}\left(\tan^{-1}(f(x))\right) = \frac{f'(x)}{1+(f(x))^2}$
- $\frac{d}{dx}\left(\cot^{-1}(f(x))\right) = \frac{-f'(x)}{1+(f(x))^2}$
- $\frac{d}{dx}\left(\sec^{-1}(f(x))\right) = \frac{f'(x)}{|f(x)|\sqrt{(f(x))^2-1}}$
- $\frac{d}{dx}\left(\csc^{-1}(f(x))\right) = \frac{-f'(x)}{|f(x)|\sqrt{(f(x))^2-1}}$

Ch4 Applications of Differentiation

Def c is critical number of a function f. if $c \in D_f$, such that $f'(c) = 0$ or $f'(c)$ does not exists.

Increasing and decreasing Test

- If $f'(x) > 0$ for all $x \in (a,b)$, then f increasing on (a,b).
- If $f'(x) < 0$ for all $x \in (a,b)$, then f decreasing on (a,b).

Thm If $f(c)$ is extreme value. then c must be a critical number for $f(x)$. but the converse is not true.

$f(c)$ extreme value

- maximum
  - local: $f(c) > f(x)$ $\forall x$ near to c
  - Absolute: $f(c) > f(x)$ $\forall x \in D_f$
- minimum
  - local: $f(c) < f(x)$ $\forall x$ near to c
  - Absolute: $f(c) < f(x)$ $\forall x \in D_f$

First Derevative Test

Suppose that $f(x)$ is continuous at a critical number $c \in [a,b]$

- If $f'(x) > 0$ for all $x \in (a,c)$ and $f'(x) < 0$ for all $x \in (c,b)$ then f has local max at $x = c$

- If $f'(x) < 0$ for all $x \in (a,c)$ and $f'(x) > 0$ for all $x \in (c,b)$ then f has local min at $x = c$

Concavity Test

- If $f''(x) > 0$ for all $x \in (a,b)$. then f concave up on (a,b)
- If $f''(x) < 0$ for all $x \in (a,b)$, then f concave down on (a,b)

Def at $x = c$ is inflection point ($f''(c) = 0$ or $f''(c)$ d.n.e) if

1. f continuous at $x = c$
2. f changes the direction of its concavity at $x = c$

Rana Shaqboua

[redacted]

H.W:

ⓐ find all:

① $f(x) = \dfrac{x^2}{x^2-1}$ ② $f(x) = x^2 - x - \ln x$

③ $f(x) = \sqrt{a - x^2}$

ⓑ find the critical values $f(x) = \begin{cases} -x^2 + x + 1, & -1 < x < 0 \\ x + x^2, & 0 \leq x < 1 \end{cases}$

ⓒ find the min value of $f(x) = |x^2 - 3|$ at $x \in [2, 4]$

H.W ① Is $f(x) = \begin{cases} x^2 + 1, & x \leq 1 \\ 2x, & x > 1 \end{cases}$ differentiable at $x = 1$?

② $f(x) = \begin{cases} x^2, & x \leq 2 \\ ax + b, & x > 2 \end{cases}$ find a, b if $f(x)$ diff ?

③ Is $f(x) = |x - 5|$ diff at $x = 5$?

Thursday
Nov. 28, 2019

* Ex: find the critical values $f(x) = 4x^{3/5} - x^{8/5}$

$f(x) = 4x^{3/5} - x^{8/5}$ $D \to R$

$f'(x) = \frac{12}{5} x^{-\frac{2}{5}} - \frac{8}{5} x^{\frac{3}{5}}$

$= \frac{12}{5x^{\frac{2}{5}}} - \frac{8x^{\frac{3}{5}} * x^{\frac{2}{5}}}{5 * x^{\frac{2}{5}}}$

$f'(x) = \frac{12 - 8x}{5x^{\frac{2}{5}}}$

$f'(x) = 0$

$12 - 8x = 0$

$x = \frac{12}{8}$

f' d.n.e

$5x^{\frac{2}{5}} = 0$

$x = 0$

critical Values

Ex: $f(x) = ax^2 - bx$ has an $\boxed{\text{extreme value}}$ at $(3, -9)$ find the values of a, b?

critical values

$f'(x) = 2ax - b$

$6a - b = 0$ ... (2)

$f(3) = -9$

$9a - 3b = -9$ ... (1)

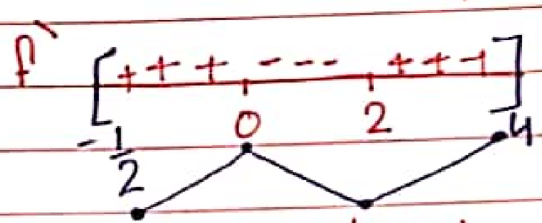
Ex: $f(x) = x^3 - 3x^2 + 1 \quad [-\frac{1}{2}, 4]$

$f'(x) = 3x^2 - 6x$

$3x(x-2) = 0$

$x = 0 \ , \ x = 2$

Critical values $= \{0, 2, -\frac{1}{2}, 4\}$

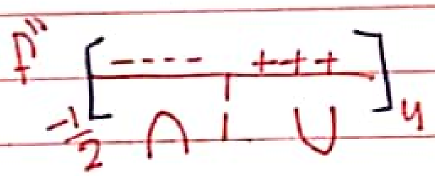


Inc $(-\frac{1}{2}, 0) \cup (2, 4)$

dec $(0, 2)$

$f''(x) = 6x - 6 = 0$

$\boxed{x = 1}$



Concave up $(1, 4)$

Concave down $(-\frac{1}{2}, 1)$

	x	f(x)	
Max	0	$f(0) = 1$	Local max
	4	$f(4) = 17$	Absolute max
min	$-\frac{1}{2}$	$f(-\frac{1}{2}) = \frac{1}{8}$	xxx
	2	$f(2) = -3$	Local min Absolute

Thursday

5, Dec, 2019

*Hyperbolic function:

① $(\sinh)(x) = \dfrac{e^x - e^{-x}}{2}$

② $\cosh(x) = \dfrac{e^x + e^{-x}}{2}$

imp.

③ $\tanh x = \dfrac{\sinh x}{\cosh x}$

④ $\operatorname{sech} x = \dfrac{1}{\cosh x}$

⑤ $\operatorname{csch} x = \dfrac{1}{\sinh x}$

⑥ $\coth x = \dfrac{1}{\tanh x}$

Ex: H.W

① $\sinh(\ln 3)$

② $\cosh(0)$

③ $\tanh(\ln 5)$

④ $\operatorname{sech}(0)$

⑤ if $\sin x = \frac{1}{3}$ find $\cosh x$, $\operatorname{sech} x$, $\tanh x$, $\coth x$, $\operatorname{csch} x$.

* Cosh x

Cosh x
Domain : R
Range : $[1, \infty)$
even

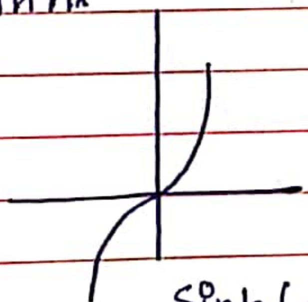
$\cosh(-x) = \cosh x$

- $\lim_{x \to +\infty} \cosh x = +\infty$

- $\lim_{x \to -\infty} \cosh x = +\infty$

$[-1, 0]$ -ne $[0, 1]$

* sinh x



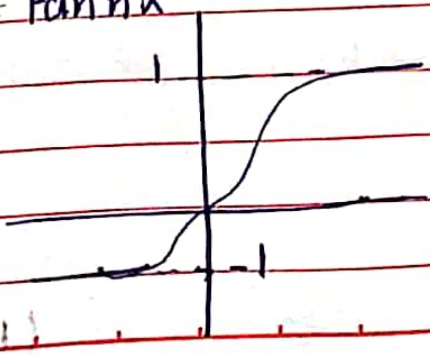
Domain : R
Range : R
odd

$\sinh(-x) = -\sinh(x)$

- $\lim_{x \to +\infty} \sinh x = +\infty$

- $\lim_{x \to -\infty} \sinh x = -\infty$

* tanh x



$R \to (-1, 1)$ odd

$\tanh(-x) = -\tanh x$

horizontal asy $(y = \pm 1)$

- $\lim_{x \to +\infty} \tanh x = 1$

- $\lim_{x \to -\infty} \tanh y = -1$

Idearites :-

① $\cosh^2 x - \sinh^2 x = 1$

② $1 - \tanh^2 x = \operatorname{Sec}^2 h x$

③ $\cosh x + \sinh x = e^x$

④ $\cosh - \sinh x = e^{-x}$

$$\frac{d}{dx}(\sinh x) = \cosh x$$

$$\frac{d}{dx}(\cosh x) = \sinh x$$

$$\frac{d}{dx}(\tanh x) = \operatorname{Sech}^2 x$$

$$\frac{d}{dx}(\operatorname{csch} x) = -\operatorname{Scs} h x * \coth x$$

$$\frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$\frac{d}{dx}(\coth x) = \operatorname{Csch}^2 x$$

Ex: find X

① $\cosh x + \sinh x = 4$

$$e^x = 4$$

$$x = \ln 4$$

or $\frac{e^x + e^{-x}}{2} + \frac{e^x - e^{-x}}{2} = 4$

$$\frac{2e^x}{2} = 4$$

$$e^x = 4 \longrightarrow x = \ln 4$$

② $\sinh x = \frac{e^x}{2} - 3$

$$\frac{e^x - e^{-x}}{2} = \frac{e^x}{2} - 3$$

$$\cancel{\frac{e^x}{2}} - \frac{e^{-x}}{2} - \cancel{\frac{e^x}{2}} = -3$$

$$\frac{-e^{-x}}{2} = -3 \longrightarrow e^{-x} = -6$$

$$-x = \ln 6$$

$$x = -\ln 6$$

③ $\cosh x - \sinh x = 8$

$$e^{-x} = 8$$

$$-x = \ln 8$$

$$x = -\ln 8$$

Ex 8 find y'

① $y = e^{\sinh(x)^3}$

$$y' = e^{\sinh(x)^3} (\cosh x)(3x^2)$$

② $y = \cosh(\ln x)$

$$y' = \sinh(\ln x)\frac{1}{x} \rightarrow \frac{e^{\ln x} - e^{-\ln x}}{2} * \frac{1}{x} \rightarrow \cancel{\frac{x^2 - 1}{2x^2}}$$

③ $y = \ln(\tan h)$

$$y' = \frac{\operatorname{sech}^2 x}{\tanh x}$$

(4) $y = \tan^{-1}(\sinh^2 x)$

$y' = \dfrac{\cosh x}{1 + (\sinh x)}$

$y' = \dfrac{\cosh x}{\cosh x^2} = \operatorname{sech} x$

- $\cosh^2 x - \sinh^2 = 1$
- $\cosh^2 x = 1 + \sinh^2 x$

(5) $y = x^2 \sin^{-1}(\sinh x)$

$y' = x^2 \dfrac{\cosh x}{\sqrt{1 + \sinh^2 x}} + \sin^{-1}(\sinh x)\ 2x$

(6) $y = 7^{\tanh x^4}$

$y' = 7^{\tanh x^4}\ \operatorname{sech}^2 x^4\ (4x^3)\ \ln 7$

(7) $y = x^{\sinh x}$

$y' = x^{\sinh x}\ \cosh x$

(8) $y = (1 + \coth x)^4$

$y' = 4(1 + \coth x)^3 - \operatorname{csch}^2 x$

Sunday

8.12.2019m

* Indeterminate forms and L'Hôpital Rules:-

if f,g diff and $\lim_{x\to a} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$, then

$\lim_{x\to a} \frac{f(x)}{g(x)}$

$= \lim_{x\to a} \frac{f'(x)}{g'(x)}$

① $\lim_{x\to \frac{\pi}{2}} \frac{1-\sin x}{\cos x} = L'H = \lim_{x\to\frac{\pi}{2}} \frac{-\cos x}{-\sin x} = 0$

② $\lim_{x\to+\infty} \frac{e^x}{x^2} = \lim_{x\to+\infty} \frac{e^x}{2x} \xrightarrow{L'H} \lim_{x\to+\infty} \frac{e^x}{2} = +\infty$ #

Ex: $0 \cdot \infty$ • $f \cdot g = \frac{f}{1/g}$

① $\lim_{x\to 0^+} x\ln x = 0 \cdot \infty$

$\lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}}$ $\frac{-\infty}{+\infty}$ $\xrightarrow{L'H}$ $\lim \frac{\frac{1}{x}}{-\frac{1}{x^2}} = \lim_{x\to 0^+}(-x) = 0$

② $\lim_{x\to\frac{\pi}{4}} (1-\tan x)\sec 2x$

$\lim_{x\to\frac{\pi}{4}} \frac{1-\tan x}{\cos 2x}$ $\frac{0}{0}$

$\lim_{x\to\frac{\pi}{4}} \frac{-\sec^2 x}{-2\sin 2x} = \frac{\sec^2(\pi/4)}{2\sin(\pi/2)} = \frac{(\sqrt{2})^2}{2} = 1$

Sunday
8, 12, 2019

Ex: $\infty - \infty$

① $\lim\limits_{x \to 0^+} \frac{1}{x} - \frac{1}{\sin x}$

$\lim\limits_{x \to 0^+} \frac{\sin x - x}{x \sin x} \xrightarrow{L'H} \lim\limits_{x \to 0^+} \frac{\cos x - 1}{x\cos x + \sin x}$

$\xrightarrow{L'H} \lim\limits_{x \to 0} \frac{-\sin x}{-x\sin x + \cos x + \cos x} = \frac{0}{2} = 0$

② $\lim\limits_{x \to +\infty} x - \ln(x^2+1)$ $\boxed{*}$ $x = \ln e^x$

$\lim\limits_{x \to +\infty} \ln e^x - \ln(x^2+1)$

$\lim\limits_{x \to +\infty} \frac{\ln e^x}{\ln(x^2+1)} = \ln\left(\lim\limits_{x \to +\infty} \frac{e^x}{x^2+1}\right)$

$= \lim \frac{e^x}{x^2+1} \xrightarrow{L'H} \lim\limits_{x \to +\infty} \frac{e^x}{2x} \xrightarrow{L'H}$

$= \lim \frac{e^x}{2} = +\infty$

$\ln(+\infty) = +\infty$

Ex: $\lim\limits_{x \to +\infty} \frac{e^{3x} - e^{-3x}}{e^{3x} + e^{-3x}} = \frac{\infty}{\infty}$

$\lim\limits_{x \to +\infty} \frac{e^{3x}(1 - e^{-6x})}{e^{3x}(1 + e^{-6x})}$

or $\lim\limits_{x \to +\infty} \tanh(3x) = \tanh\left(\lim\limits_{x \to +\infty} 3x\right) = \tanh(+\infty) = 1$

Tuesday
Dec. 10, 2014

* 0^0, ∞^0, 1^∞

① $\lim_{x \to +\infty} \sqrt[x]{x} = \lim_{x \to +\infty} x^{\frac{1}{x}}$

$y = x^{\frac{1}{x}}$

$\ln y = \frac{\ln x}{x}$

$\lim_{x \to +\infty} \ln y = \lim_{x \to +\infty} \frac{\ln x}{x}$

$\ln y = \lim_{x \to +\infty} \frac{\ln x}{x} \xrightarrow{L.H} \lim_{x \to +\infty} \frac{\frac{1}{x}}{1} = 0$

$\ln y = 0 \longrightarrow e^{\ln y} = e^0$

$\boxed{y = 1}$

② $\lim_{x \to 0^+} (1+x)^{\frac{1}{x}} \longrightarrow 1^{+\infty}$

$y = (1+x)^{\frac{1}{x}}$

$\ln y = \ln (1+x)^{\frac{1}{x}}$

$\ln y = \frac{\ln(1+x)}{x}$

$\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} \frac{\ln(1+x)}{x} = \frac{0}{0} \xrightarrow{L.H}$

$\ln y = \lim_{x \to 0^+} \frac{\frac{1}{1+x}}{1} = 1$

$\ln y = 1$

$y = e^1$

$\infty - \infty \rightarrow$ الوا لجثرده امراج

- $\lim_{x\to 0} (1+ax)^{\frac{b}{x}} = e^{ab}$

- $\lim_{x\to \infty} \left(1+\frac{a}{x}\right)^{bx} = e^{ab}$

Ex:

① $\lim_{x\to 0} (1-2x)^{\frac{5}{x}} = e^{-10}$

② $\lim_{x\to +\infty} \left(1+\frac{3}{x}\right)^{-2x} = e^{-6}$

③ $\lim_{x\to +\infty} \left(\frac{x}{x-7}\right)^{-2x}$

$\lim_{x\to +\infty} \left(\frac{x-7}{x}\right)^{2x}$

$\lim_{x\to +\infty} \left(1-\frac{7}{x}\right)^{2x} = e^{-14}$

④ $\lim_{x\to +\infty} \left(\frac{x-3}{x-4}\right)^{x}$

$\lim_{x\to +\infty} \left(\frac{x\left(1-\frac{3}{x}\right)}{x\left(1-\frac{4}{x}\right)}\right)^{x}$

$\lim_{x\to +\infty} \frac{\left(1-\frac{3}{x}\right)^{x}}{\left(1-\frac{4}{x}\right)^{x}} = \frac{e^{-3}}{e^{-4}} = e$

⑤ $\lim_{x \to +\infty} 3^x - 2^x$ $\qquad \infty - \infty$

$b^x \quad 0<b<1$

$\lim_{x \to +\infty} 3^x \left(1 - \underbrace{\left(\frac{2}{3}\right)^x}_{\text{zero}}\right) = +\infty$

H.W:

① $\lim_{x \to 0^+} (\tan 2x)^x$

② $\lim_{x \to +\infty} (\tanh x)^x$

③ $\lim_{x \to 0^+} x^{\sqrt{x}}$

④ $\lim_{x \to} (e^x + x)^{\frac{1}{x}}$

⑤ $\lim_{x \to 0} (1-2x)^{\frac{1}{x}}$

[1,0]

Thursday
12, Dec, 2019

* Integrals:

① $\int K\,dx = Kx + C$ $K \in \mathbb{R}$

② $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$, $n \neq -1$

③ $\int (f(x) \pm g(x))\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

④ $\int K.\,f(x)\,dx = K\int f(x)\,dx$

⑤ $\int x^{-1}\,dx = \ln|x| + C$

Exs

① $\int 5\,dx = 5x + C$

② $\int x^7\,dx = \frac{x^8}{8}\,dx$

③ $\int x^{-2}\,dx = \frac{x^{-1}}{-1} + C$

④ $\int (3x^{-3} - \frac{1}{2}x + \frac{10}{x} + 5x^{-\frac{1}{2}} - 2x^{\frac{1}{4}} + \pi)\,dx$

$= \frac{3x^{-3}}{-2} - \frac{1}{2}\frac{x^2}{2} + 10\ln|x| + \frac{5x^{\frac{1}{2}}}{1/2} - \frac{2x^{5/4}}{5/4} + \pi x + C$

$= \frac{-3}{2x^2} - \frac{x^2}{4} + 10\ln|x| + 10\sqrt{x} - \frac{8x^{5/4}}{5} + \pi x + C$

⑤ $\int_e^{e^4} 3x^{-1}\,dx = 3\ln|x|\Big]_e^{e^4} = 3\ln e^4 - 3\ln e$

$12 - 9 = 3$

Thursday
12, Dec, 2019

(6) $\int_1^4 \frac{x^3 - 2\sqrt{x}}{x}\, dx = \int_1^4 x^2 - 2x^{-\frac{1}{2}}\, dx$

$= \left.\frac{x^3}{3} - 4x^{\frac{1}{2}}\right]_1^4$

$= \left(\frac{4^3}{3} - 4\sqrt{4}\right) - \left(\frac{1}{3} - 4\sqrt{1}\right)$

- $\int (ax+b)^n\, dx = \frac{(ax+b)^{n+1}}{a(n+1)} + C, \; n \neq -1$

Ex: $\int (1-3x)^{\frac{12}{5}}\, dx = \frac{(1-3x)^{\frac{17}{5}}}{(-3)\left(\frac{17}{5}\right)} + C$

$\rightarrow \int (1+x^2)^2\, dx$

$= \int (1+2x^2+x^4)\, dx = x + \frac{2x^3}{3} + \frac{x^5}{5} + C$

- $\int \frac{g'(x)}{g(x)}\, dx = \ln|g(x)| + C$

Ex:

$\rightarrow \int \frac{1}{x+5}\, dx = \ln|x+5| + C$

$\rightarrow \int \frac{2x+2}{x^2+2x+1}\, dx = \ln|x^2+2x+1| + C$

$\rightarrow \int \frac{x^2}{x^3+5}\, dx = \frac{1}{3}\int \frac{3x^2}{x^3+5} = \frac{1}{3}\ln|x^3+5| + C$

$= \ln\sqrt[3]{|x^3+5|} + C$

• مهمين الـ٣ دول

- $\int e^{ax+b}\,dx = \dfrac{e^{ax+b}}{a} + C$
- $\int b^{ax+k}\,dx = \dfrac{b^{ax+k}}{(a)\ln b} + C$

Ex 8

① $\int e^{-2x+7}\,dx = \dfrac{e^{-2x+7}}{-2} + C$

② $\int 7^{5x+1}\,dx = \dfrac{7^{5x+1}}{(5)\ln 7} + C$

③ $\int_0^1 3^{2x+1}\,dx = \left.\dfrac{3^{2x+1}}{2\ln 3}\right]_0^1$

$= \dfrac{1}{2\ln 3}\left[3^3 - 3\right] = \dfrac{12}{\ln 3}$

- $\int \cos(ax+b)\,dx = \dfrac{\sin(ax+b)}{a} + C$
- $\int \sin(ax+b)\,dx = \dfrac{-\cos(ax+b)}{a} + C$
- $\cancel{\int \tan(ax+b)\,dx}$ $\int \csc^2(ax+b)\,dx = \dfrac{-\cot(ax+b)}{a} + C$
- $\int \sec^2(ax+b)\,dx = \dfrac{\tan(ax+b)}{a} + C$
- $\int \sec(ax+b)\tan(ax+b)\,dx = \dfrac{\sec(ax+b)}{a} + C$
- $\int \csc(ax+b)\cot(ax+b)\,dx = \dfrac{-\csc(ax+b)}{a} + C$

Ex:

① $\int \cos 2x - \sec^2(1-7x)\, dx$

$= \frac{\sin 2x}{2} - \frac{\tan(1-7x)}{-7} + C$

② $\int \frac{\sin x}{\cos^2 x}\, dx = \int \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \int \tan \cdot \sec x\, dx$

$= \sec x + C$

③ $\int \cos^2 x\, dx$

$= \int \frac{1}{2}(1+\cos 2x)\, dx = \frac{1}{2}\left(x + \frac{\sin 2x}{2}\right) + C$

④ $\int \tan x\, dx = \int \frac{\sin x}{\cos x}\, dx = -\int \frac{-\sin x}{\cos x}\, dx$

$= -\ln|\cos x| + C$

$= \ln|\cos x|^{-1} + C$

$= \ln|\sec x| + C$

Sunday
15, Dec, 2019

Ex8

1 $\int \sinh(2x-1) - \operatorname{Sech}^2(5x+4)\, dx$

$\frac{\cosh(2x-1)}{2} - \frac{\tanh(5x+4)}{5} + C$

2 $\int \tanh\, dx = \int \frac{\sinh}{\cosh} = \ln|\cosh| + C$

3 $\int \frac{1}{4+x^2}\, dx = \frac{1}{2} \tan^{-1}\left(\frac{x}{a}\right) + C$

$f^2(x) = \cancel{x}^2$

$f(x) = x$

$a^2 = 4$

$a = 2$

4 $\int \frac{1}{\sqrt{9-x^2}}\, dx = \sin^{-1}\left(\frac{x}{3}\right) + C$

$\int \frac{f'(x)}{\sqrt{a^2-(f(x))^2}} = \sin^{-1} \frac{f(x)}{a} + C$

$a^2 = 9$

$f(x) = x$

$\int \frac{f'(x)}{a^2+(f(x))^2} = \frac{1}{a} \tan^{-1} \frac{f(x)}{a} + C$

6 $\int \frac{1}{1+x^2}\, dx = \tan^{-1}(x) + C$

7 $\int \frac{1}{\sqrt{1-x^2}}\, dX = \sin^{-1}(x) + C$

~~(10)~~ (15)

Sunday
15, Dec, 2019

8) $\int \frac{x}{5+x^4}\,dx = \frac{1}{2}\int \frac{2x}{5+x^4} = \frac{1}{2}\left(\tan^{-1}\frac{x^2}{\sqrt{5}}\right) + C$

$a^2 = 5$

$a = \sqrt{5}$

$f^2(x) = x^4$

$f(x) = x^2$

$f'(x) = 2x$

9) $\int \frac{e^x}{\sqrt{10-e^{2x}}}\,dx - \sin^{-1}\left(\frac{e^x}{\sqrt{10}}\right) + C$

$a^2 = 10$ $\qquad f(x) = e^x$

$a = \sqrt{10}$ $\qquad f'(x) = e^x$

Ex:

10) $\int_0^7 f(x)\,dx = 14$ and $\int_5^3 f(x)\,dx = 9$ find:-

① $\int_0^3 f(x)\,dx = \frac{14}{7} = 2$

② $\int_0^5 f(x)\,dx = \int_0^3 f(x)\,dx + \int_3^5 f(x)$

$= 2 - 9 = -7$

③ $\int_3^0 4f(x) = 4(-2) = -8$

④ $\int_5^3 f(x)\,dx + x\,dx = \int_5^3 f(x)\,dx + \int_5^3 x\,dx$

$9 + \left.\frac{x^2}{2}\right]_5^3$

$9 + \left(\frac{3}{2} - \frac{5}{2}\right)$

(16)

Tuesday
17, Dec, 2019

Ex: $\int_0^2 |x-1|\,dx$

$1-x \quad x-1$; $0 \quad 1 \quad 2$

$\int_0^1 (1-x) + \int_1^2 (x-1)$

* Integration by Substitution. التكامل بالتعويض

① $\int x\sqrt{3+x^2}\,dx = \int x\, y^{\frac{1}{2}} \frac{dy}{2x}$

$y = 3 + x^2$ $\qquad = \frac{1}{2}\int y^{\frac{1}{2}}\,dy$

$\frac{dy}{2x} = \frac{2x\,dx}{2x}$ $\qquad = 3\left(y^{\frac{3}{2}}\right) + C$

$dx = \frac{dy}{2x}$ $\qquad = 3\left(3+x^2\right)^{\frac{3}{2}} + C$

② $\int_0^{\frac{\pi}{2}} e^{\sin x} \cos\,dx = \int_0^1 e^y \cancel{\cos x} \frac{dy}{\cancel{\cos x}}$

$y = \sin x$ $\qquad = \int_0^1 e^y\,dy = e^y \Big]_0^1$

$dy = \cos x\,dx$

$\frac{dy}{\cos x} = dx$ $\qquad \longrightarrow y = \sin 0 = 0$

$y = \sin\frac{\pi}{2} = 1$

Tuesday
17, Dec, 2019

③ $\int_{1}^{\sqrt{3}} \frac{\sqrt{\tan^{-1}x}}{1+x^2} = \int_{\pi/4}^{\pi/2} \frac{y^{1/2}}{\cancel{1+x^2}} \cancel{(1+x^2)}\, dy$

$y = \tan^{-1} x$

$dy = \frac{1}{1+x^2} dx$ $\qquad = \int_{\pi/4}^{\pi/2} y^{\frac{1}{2}}\, dy$

$dx = (1+x^2)\, dy$

$\to y = 1 \to \tan^{-1} 1 = \frac{\pi}{4}$

$\to y = \sqrt{3} \to \tan^{-1}\sqrt{3} = \frac{\pi}{2}$

* The Fundamental Theorem of Calculus:

$\frac{d}{dx}\int_{g(x)}^{h(x)} f(t)\, dt = f(h(x))\, h'(x) - f(g(x))\, g(x)$

1. $\frac{d}{dx}\int_{x^2}^{5x^2} \sin^2 t\, dt = \sin(5x^2)^2 (15x^2)] - \sin(x^2)^2\, 2x$

2. $\frac{d}{dx}\int_{e^x}^{\sin x} \frac{1}{t}\, dt = \frac{1}{\sin x}(\cos x) - \frac{1}{e^x}(e^x)$

3. $\frac{d}{dx}\int_{8}^{\sin^{-1}x} \tan^{-1} t\, dt = \tan^{-1}(\sin^{-1} x)\left[\frac{1}{\sqrt{1-x^2}} - 0\right.$

4. $\frac{d}{dx}\int_{\tan^{-1}x}^{3} e^t \sin t\, dt = 0 - e^{\tan^{-1}x}(\sin(\tan^{-1}x)) \frac{1}{1+x^2}$

5. $\int_{3}^{10} \frac{e^t}{1+\sin t}\, dt = 0$

Thursday

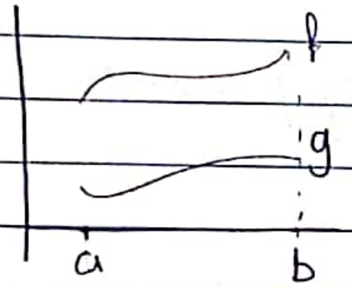
$f(x) = x^2 \int_2^{2x} \frac{\sin t}{t}$ find $f'(1)$?

$f'(x) = x^2 \frac{d}{dx} \int_2^{2x} \frac{\sin t}{t}\, dt + \int_2^{2x} \frac{\sin t}{t}\, dt\, (2x)$

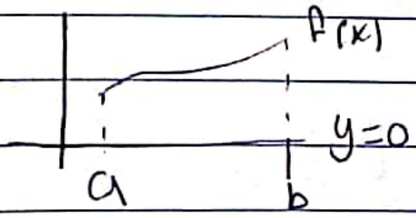
$x^2 \left(\frac{\sin(2x)}{2x} \right) 2 - 0 + 2x \int_2^{2x} \frac{\sin t}{t}\, dt$

$f'(x) = \sin 2 + 2 \int_2^{2} \frac{\sin t}{t} = \sin 2$

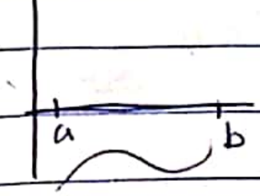
H.W $\lim_{x \to 0} \frac{1}{x} \int_0^{x} (1 - \tan 2t)^{\frac{1}{t}}\, dt$?

① 

area $= \int_a^b f(x) - g(x)$

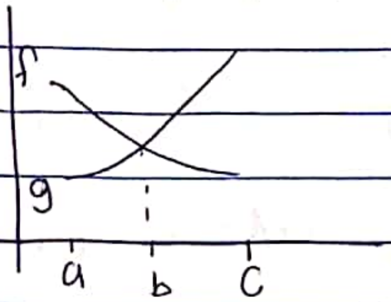
② 

$\int_a^b (f(x) - 0)$

③ 

$\int_a^b (0 - f(x))$

(5)



$$\int_a^b f-g + \int_b^c g-f$$

Ex: (1) $\underset{f(x)}{y = x^2}$ and $\underset{g(x)}{x+6}$

$x^2 = x+6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3$

$x = -2$

$-2 \quad 0 \quad 2$

نأخذ أي رقم ونعوض في fg

f g

0 6

$$A = \int_{-2}^{3} x+6 - \int$$

(2) $\underset{f(x)}{x^3 - 4x}$ and $\underset{y=0}{x\text{-axis}}$ $\quad x=0 \quad x=2$

$x^3 - 4x = 0$

$x(x^2-4) = 0$

$x=0 \quad x = \pm 2$

بدنا نشوف مين الاعلى نختار رقم ونعوض

0 1 2

$f(x)$ $g(x)$

-3 0

$$A = \int_0^2 0 - (x^2 - 4x)\, dx$$

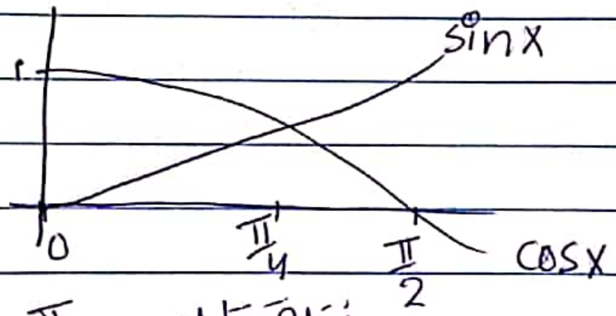
$$= \int_0^2 4x - x^2 \, dx$$

③ $y = \cos x$ $y = \sin x$ $[0, \frac{\pi}{2}]$

$\cos x = \sin x$

$x = \frac{\pi}{4}$

$$A = \int_0^{\pi/4} (\cos - \sin x)\, dx + \int_{\pi/4}^{\pi/2} (\sin x - \cos x)\, dx$$



$\frac{\pi}{4} \rightarrow$ نقطة تقاطع