ElCoM
ELECTRICAL COMPUTER MECHATRONICS

تقدم لجنة ElCoM الاكاديمية

دفتر لمادة:

تفاضل و تكامل (١)

من شرح:

م.ميسم أبو دلو

جزيل الشكر للطالبة:

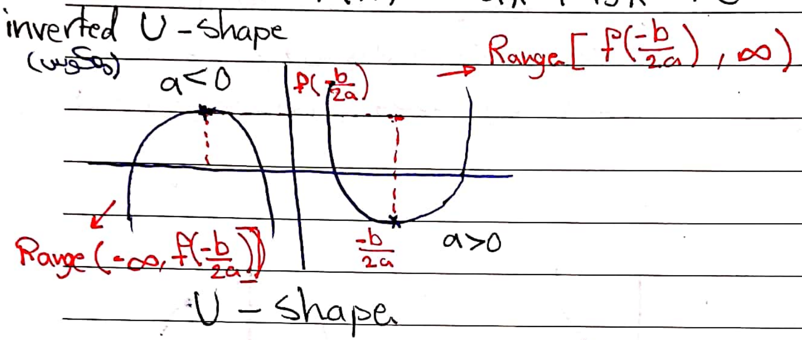
هبة كتانة

مكتب المحتوى IT

10-11 من 143

⊗ quadratic function :-

$$f(x) = ax^2 + bx + c \quad , a \neq 0$$



$$\left(\frac{-b}{2a} , f\left(\frac{-b}{2a}\right)\right)$$

vertix

Dom :- R

* كل كثيرات الحدود |Dom| فقط لإلها R (ح)

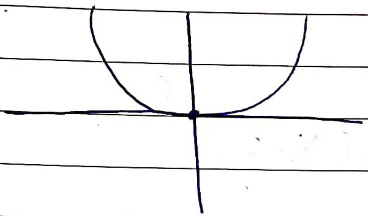
* اسم رسمته
parbola : قطع مكافئ

Five Apple

⑧ Find dom & range

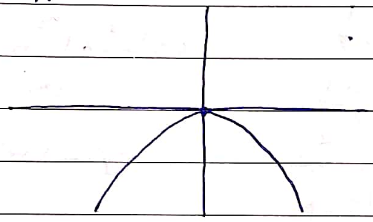
Rang:
من الصفر وإطلع
من رأس القطع

1) $f(x) = x^2$



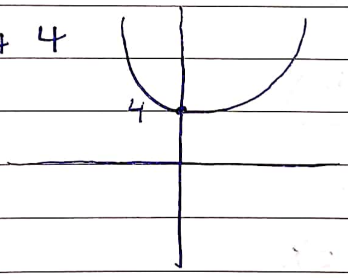
$x^2 : R \rightarrow [0, \infty)$

2) $f(x) = -x^2$



$-x^2 : R \rightarrow (-\infty, 0]$

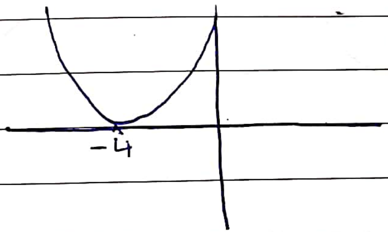
3) $f(x) = x^2 + 4$



$f(x) : R \rightarrow [4, \infty)$

4) $f(x) = (x+4)^2$

$x + 4 = 0$
$x = -4$



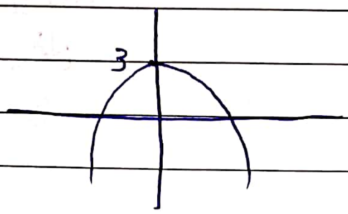
~~نعوض عن قيمة x~~
~~بـ f(x) كده~~
~~المدى و راس القطع~~
~~[0~~

$f(x) : R \rightarrow [0, \infty)$

إشارة x^2 (سالبة) إشارة 3 (موجبة)

إزاحة لأعلى بمقدار 3 =

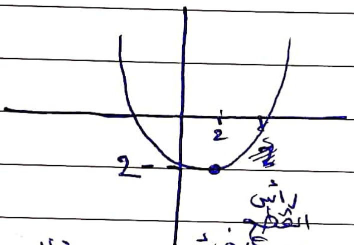
5) $f(x) = 3 - x^2$



Dom: R

rang: $(-\infty ; 3]$

6) $f(x) = -2 + (x-2)^2$



Dom: R

rang: $[-2 , \infty)$

لأنه مفتوح لأعلى من الاحداثي الصادي إلى ∞

(الإحداثي الصادي لرأس القطع بطلع منه المدى)

إحداثي ص هو الأهم عشان أطلع المدى

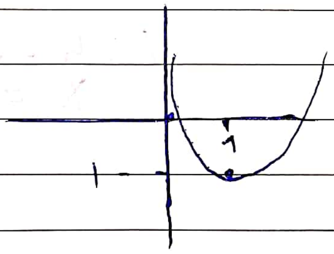
* عشان أطلع التقاطع مع السينات لأنه اساويها بالصفر

$(\text{س} + \text{ص})^{٢} = \text{س}^{٢} + ٢\text{س}\text{ص} + \text{ص}^{٢}$

راس القطع $\left(\frac{-b}{2a} , f\left(\frac{-b}{2a}\right) \right)$

1 -2

7) $f(x) = x^2 - 2x$



$\left(\frac{-b}{2a} , f\left(\frac{-b}{2a}\right) \right)$

$\left(\left(\frac{+2}{2} \right) , f\,1- \right)$

$(+1 , -1)$

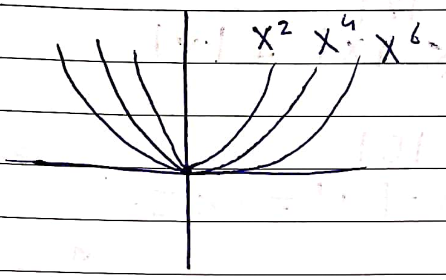
x y

Dom: R

range: $[-1 , \infty)$

parbola : قطع مكافئ

$f(x) = x^n$



$R \rightarrow [0,\infty)$

⑧ Abs value fn.

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases}$$

Dom : R
rang : $[0,\infty)$

/1\ $\sqrt{x^2} = |x|$

/2\ $(\sqrt{x})^2 = x$

/3\ $|x| = a \rightarrow x = \pm a$

* /4\ $|x| < a \rightarrow -a < x < a$

$x \in (-a, a)$

بالنسبة لل Dom
بتعلم

* /5\ $|x| \geq a$

$x \geq a$ or $x \leq -a$

$[a, \infty) \cup (-\infty, -a]$

6) $|xy| = |x|\,|y|$

7) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$

* 8) $|x+y| \leq |x| + |y|$

$|2+3| = |5|$

$|2+(-3| = |-1| = 1 < 5$

*لما يكونوا اشارتين مختلفين بكون أصغر

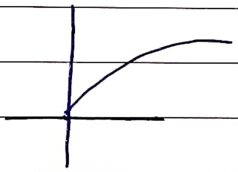
* لما يكون زي بعض بكون مساواة ←

* Root fn

1) $f(x) = \sqrt{x}$

$x \geq 0$, Dom $[0, \infty)$

Range $[0, \infty)$



2) $f(x) = \sqrt{3x - 1}$

$3x - 1 \geq 0$

$3x \geq 1$

$x \geq \frac{1}{3}$

Dom: $[\frac{1}{3}, \infty)$ / Range: $[0, \infty)$

$\boxed{3}$ $f(x) = \sqrt{16 - x^2}$

(ما داخل الجذر ≥ 0) $16 - x^2 \geq 0$

$-x^2 \geq -16$ / $x^2 \leq 16$

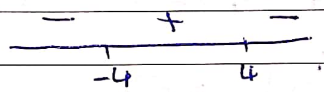
لأنه عندي تربيع تحت الجذر ← $\sqrt{x^2} \leq \sqrt{16}$

$|x| \leq 4$

$-4 \leq x \leq 4$ Dom: $[-4, 4]$

أو

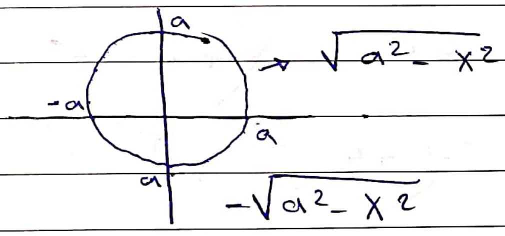
$(4-x)(4+x) \geq 0$



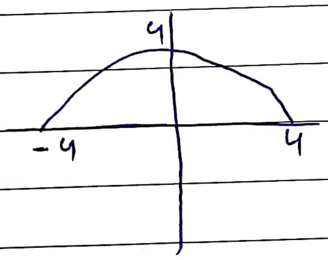
$\therefore [-4, 4]$

أو

$x^2 + y^2 = a^2 \rightarrow y^2 = a^2 - x^2$



$f(x) = \sqrt{16 - x^2}$



Dom: $[-4, 4]$

range: $[0, 4]$

* note that :-

① Dom $\left\{ f \overset{\pm}{\underset{*}{-}} g(x) \right\}$

Dom f(x) ∩ Dom g(x)

② Dom $\left\{ \frac{f(x)}{g(x)} \right\}$

= Dom f(x) ∩ Dom g(x) − { g(x) = 0 }

* تعريف دائرة :-

$f(x) = \sqrt{a-x^2}$

- Dom $[-\sqrt{a}, \sqrt{a}]$

$a \in R$

- Range $[0, \sqrt{a}]$

semi cercle

$f(x) = -\sqrt{a-x^2}$

Range [−3,0] / Dom [−3,3]

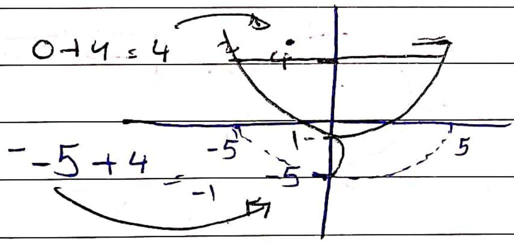
/1\ Dom $f(x) = 4 - \sqrt{25 - x^2}$ is :

−5 و 5 هو وين تتقاطع

مع ال R ؟

$R \cap [-5,5] = [-5,5]$

/2\ range $f(x) = 4 - \sqrt{25 - x^2}$



∴ [−1,4]

(إزاحة للأعلى)

H.w

Dom $f(x) = \sqrt{x^2 - 25}$

$x^2 - 25 \geq 0$

$\cancel{+25} \quad +25$

$\sqrt{x^2} \geq \sqrt{25}$ / $|x| \geq 5$

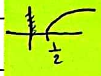
$x \geq 5 , x \leq -5$

$\therefore$ Dom: $[5,\infty) \cup (-\infty,-5]$ #

Range: $[0,\infty)$

$f(x) = \sqrt{2x-1}$

Range $[0,\infty)$



$f(x) = -\sqrt{2x-1}$

Range $(-\infty,0]$

H.w
page.3

$\boxed{1}$ $f(x) = \frac{x-1}{x^2-1}$

$\therefore$ Dom: $R - \{\pm 1\}$

$x^2 \cancel{-1} = 0$

$\cancel{+1} \quad +1$

, $\sqrt{x^2} = \sqrt{+1}$

$|x| = 1$

$\boxed{x = \pm 1}$

$\boxed{2}$ $f(x) = \frac{1}{(x-1)(x+5)}$

$f(x) = \frac{1}{x^2+5x-x-5}$ / $f(x) = \frac{1}{x^2+4x-5}$

$\therefore$ Dom: $R - \{-5, +1\}$

$x^2+4x-5 = 0$

$(x+5)(x-1) = 0$

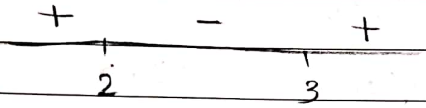
$\downarrow \qquad \downarrow$

$x = -5 \qquad x = +1$

$\boxed{3}$ $f(x) = \sqrt{x^2 - 5x + 6}$

$x^2 - 5x + 6 \geq 0$

$(x - 3)(x - 2) \geq 0$



Dom: $(-\infty, 2] \cup [3, \infty)$

$\boxed{4}$ $f(x) = \sqrt{|x-1| - 10}$

$|x-1| \cancel{-10} \geq 0$

$\cancel{+10}$ $+10$

$|x-1| \geq 10$

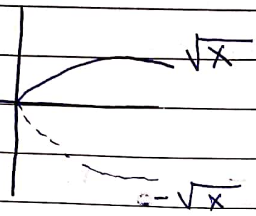
$x \cancel{-1} \geq 10$, $x - 1 \leq -10$

$\cancel{+1}$ $+1$ $+1$ $+1$

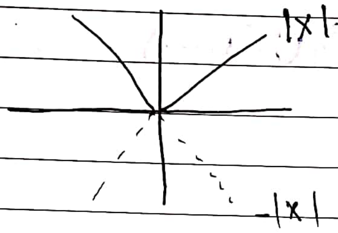
$x \geq 11$, $x \leq -9$

Dom: $[11, \infty) \cup (-\infty, -9]$

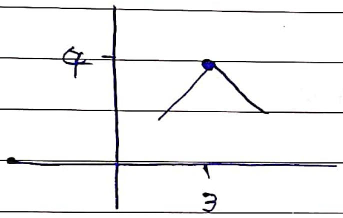
Dom: $|f(x)|$
$= \text{Dom } f(x)$



Dom: $[0, \infty)$
range: $(-\infty, 0]$



* Range of $y = 4 - |x-3|$



$\therefore (-\infty, 4]$

* Dom $\sqrt[3]{x-3}$ R

* Dom $\sqrt[3]{\frac{1}{x-1}}$ $R - \{1\}$

* Dom $\sqrt[3]{f(x)}$ = Dom $f(x)$

* Find dom :-

1) $y = \frac{2x}{|x-1|-4}$

R

$R - \{ |x-1| - 4 = 0$
$|x-1| = 4$

$|x-1| = 4 \rightarrow x-1 = 4$ or $x-1 = -4$
$x = 5$ or $x = -3$

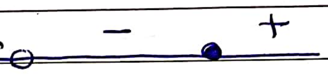
$R - \{-3, 5\}$

Five Apple

* Dom $\sqrt{\frac{x-1}{x+1}}$

$x=1$ صفر البسط ← $\frac{x-1}{x+1} \geq 0$
$x=-1$ صفرالمقام ←

حلقة مفتوحة ← +
لأنه صفر مقام



R

$(-\infty,-1) \cup [1,\infty)$

* Dom $f(x)$ $\frac{\cancel{(x-3)^2}}{x-3}$

$R-\{3\}$

* Dom $f(x) = \sqrt{x^2-5x+6}$

$x^2-5x-6 \geq 0$

$(x-2)(x-3) \geq 0$

Dom $(-\infty,2] \cup [3,\infty)$

* Dom $f(x) = \sqrt[4]{|x+2|-5}$

$|x+2| \cancel{-5} \geq 0$
$\cancel{+5} \quad +5$

$|x+2| \geq 5$

$x + \cancel{2} \geq 5$ or $x+\cancel{2} \leq -5$
$-\cancel{2} \quad -2 \qquad -\cancel{2} \quad -2$

$[3,\infty) \cup (-\infty,-7]$ $\quad$ $x \geq 3$ or $x \leq -7$

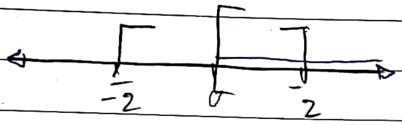
* Dom: $f(x) = \sqrt{x-2} + \frac{1}{\sqrt[3]{x+1}}$

الجذر التكعيبي
يتحمل يساوي
قيمة سالبة وتعني
الأسس الفردية

$= [2,\infty) \cap R - \{-1\} = [2,\infty)$

* Dom $f(x) = \frac{\sqrt{x}}{4+\sqrt{4-x^2}}$ $\rightarrow [0,\infty)$

$R \cap [-2,2]$



$\therefore [0,2]$ لايوجد أصفار مقام

---

* $f(x) = 3x+1$
$g(x) = \sqrt{x-1}$

fod ⓐ $f\circ g(x)$

$f\circ g(x) = f(g(x))$

$= 3\sqrt{x-1} + 1$

ⓑ $g\circ f(x) = g(3x+1)$

$= \sqrt{3x+1-1}$

$= \sqrt{3x}$

Five Apple

Scanned by CamScanner

أول
نوع
الأسئلة

Let $fog(x) = 6x^2 - 10x + 5$

$f(x) = 2x + 1$

Find $g(0)$?

$\therefore g(0) = 2$

$fog(x) = 6x^2 - 10x + 5$

$f(g(x)) = 6x^2 - 10x + 5$

$2g(x) + 1 \xrightarrow{} = 6x^2 - 10x + 5$

$2(g(x) = 6x^2 - 10x + 4$

$g(x) = 3x^2 - 5x + 2$

* if & $f(\overset{g}{\underline{3x + 5}}) = \overset{fog}{\underline{6x + 11}}$

Find $f(x)$?

استبدال $y = 3x + 5$ $\rightarrow f(y) = 6\left(\frac{4-5}{3}\right) + 11$

$x = \frac{y-5}{3}$

$= 2y - 10 + 11$

$f(y) = 2y + 1$

$f(x) = 2x + 1$

Five Apple

بيبع لا يمكن عني مرضين زي بعض

*** note that:**

$$\text{Dom } f\circ g(x) = \begin{cases} x \in g(x) \\ g(x) \in \text{Dom} f \end{cases} \cap \rightarrow (\cap)$$

$f(g(x))$

* لما يكون جوا الاقتران
في Dom اختلاف هون
بكون تركيب اقترانين.

*** let** $f(x) = \sqrt{1-x}$

$g(x) = 3x$

find dom $f\circ g(x)$

① Dom الاقتران الداخلي

② Dom $f\circ g$ (كله)

③ تقاطعهم مع بعض

① Dom $g = \mathbb{R}$

② $f\circ g = \sqrt{1-3x}$

③ $\mathbb{R} \cap (-\infty, \frac{1}{3}] = (-\infty, \frac{1}{3}]$

*** Dom** $f = \sqrt{4-\sqrt{x}}$

$h = \sqrt{x}$

$g = \sqrt{4-x}$

$4-\sqrt{x} \geq 0$

$4 \geq \sqrt{x}$

$16 \geq x$

$(-\infty, 16] \cap [0, \infty) = [0, 16]$

Five Apple

if Dom $f(x) = [1, 4]$

then dom $f(\overset{g}{3x+4})$ is:-

① Dom $(g(x) = R$

$3x + 4 \ni [1,4] \rightarrow 1 \leqslant 3x + 4 \leqslant 4$

$ -4 \qquad -4 \qquad -4$

$-3 \leqslant 3x \leqslant 0$

$-1 \leqslant x \leqslant 0$

$[-1, 0] \cap R = [-1, 0]$

* Even fn

$f(-x) = f(x)$ like: x^2, $|x|$, $\cos x$

$f(-x) = f(x)$

* odd fn

$f(-x) = -f(x)$ like: x^3, $\sin x$

* determine whethere the following fn's even, odd neither even nor odd

1) $f(x) = \frac{7 - x^6}{|x|}$

$f(-x) = \frac{7-(-x)^6}{|-x|} = \frac{7 - x^6}{|x|} = f(x)$ ∴ even

اذا ما بساوي f بجيب السالب
عامل مشترك

2) $f(x) = \frac{\sin x + x^5}{\cos x}$, $f(-x) = \frac{\sin(-x) + (-x)^5}{\cos(-x)}$

$= \frac{-\sin x - x^5}{\cos x} = -\left(\frac{\sin x + x^5}{\cos x}\right) = -f(x)$

بطلع السالب يا اما من البسط او المقام

odd

على اقتران زوجي

/3| $f(x) = x + 7$ يكون متماثل حول محور الصادات

$f(-x) = -x+7 \neq f$ origin: نقطة الأصل

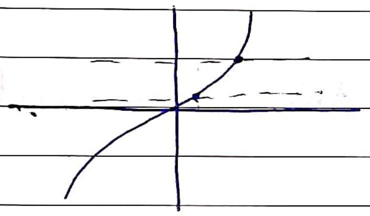
$= -(x-7) \neq -f$

عن طريق

* 1-1 fn "one to one fn" (اختبار الخط الأفقي)

$f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$



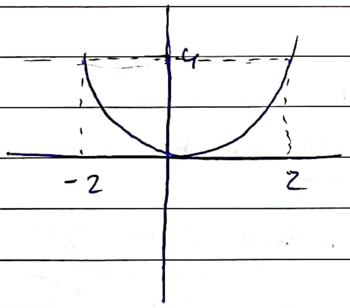
$f(x_1) = f(x)_2$

$x_1^3 = x_2^3$

$\sqrt[3]{x_1^3} = \sqrt[3]{x_2^3}$

$x_1 = x_2$

* عشان يكون للاقتران اقتران عكسي لازم يكون 1-1



not 1-1

* كل اقتران ومعكوسه لازم يكونوا متماثلين حول $y = x$

$f(2) = f(-2)$

$4 = 4$

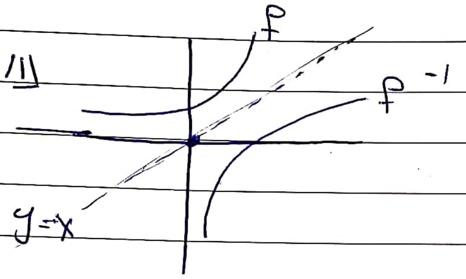
* note that

$f(x)$ 1-1 $\longleftrightarrow$ $f(x)$ has an inverse "invertable"

* Dom f^{-1} = range f

the inverse of $f(x) = f(x)^{-1} \neq \frac{1}{f(x)}$

/1/



/2/ $f: A \longrightarrow B$ Dom f^{-1} = Range f

$f^{-1}: B \longrightarrow A$ rang f^{-1} = Dom f

/3/ $f \circ f^{-1}(x) = x$

$f^{-1} \circ f(x) = x$

* Find the range of :

① Find $f^{-1}(x)$

$y = \frac{x-2}{x+1} \longrightarrow$ بدي أخلي X موضوع قانون

(بطلع X بدلالة y)

$f(x) = \frac{x-2}{x+1}$

$y(x+1) = x-2$

$yx + y = x-2$

$yx - x = -2-y$

$x(y-1) = -2-y \quad \therefore \ x = \frac{-2-y}{y-1}$

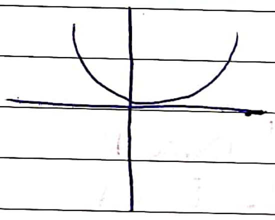
$$f^{-1}(x) = \frac{-2-x}{x-1}$$

② Range f = Dom f^{-1}

$= \mathbb{R} - \{1\}$

***** $f(x) = x^2$

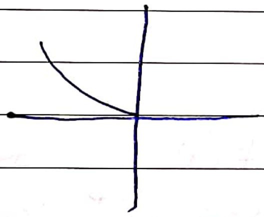
* كيف أعرف إذا الاقتران 1-1 من دون رسم ؟



not 1-1

has no invers

***** $f(x) = x^2$ on $[-3, 0]$



1-1 → inverse

$y = x^2$

$\sqrt{y} = \sqrt{x^2}$

$\sqrt{y} = |x|$

$\sqrt{y} = -x$

$x = -\sqrt{y}$ $\therefore$ $f^{-1}(x) = -\sqrt{x}$

لما بدي اتخلص من
f^{-1} بضربه f
والعكس صحيح

* let $f(x) = x^3 + 4x + 1$

if $f^{-1}(c^3) = c$ then $c =$

$$f^{-1}(c^3) = c$$

$$f(f^{-1}(c^3)) = f(c)$$

$$c^3 = c^3 + 4c + 1$$

$$0 = 4c + 1$$

$$4c = -1$$

$$c = \frac{-1}{4}$$

* if $f(x) = \dfrac{2x^3}{x^4 + 1}$, find x such that

$1 = f^{-1}(x)$

$$f(1) = f(f^{-1}(x))$$

$$f(1) = x$$

$$\frac{2}{1+1} = x$$

$$\boxed{x = 1}$$

31/10

3-4

بين العدد الي بدي أعوضه بدِه يعطيني ٦.

* $f(x) = x^3 + 4x + 1$

Find $f^{-1}(6)$

f^{-1} (العدد)

الحل بالتجريب بالأول

$f(?) = 6$

$f(1) = 6$

$f^{-1}(6) = 1$

$f^{-1}(6) = y$

$f(f^{-1}(6) = f(y)$

$6 = f(y)$

$6 = y^3 + 4y + 1$

$y^3 + 4y - 5 = 0$

عالقسمة التركيبية.

$y - 1$

* $f(x) = x - 5x^2$, $x \geq 1$ Find $f^{-1}(x)$

1) $y = x - 5x^2$

$y = -5(x^2 - \frac{x}{5})$

(إكمال مربع)

$\frac{1}{100}$ $\frac{1}{100}$

بضيف وبطرح ($\frac{1}{2}$ معامل س)^2

$y = -5(x^2 - \frac{x}{5} + (\frac{1}{2*5})^2 - (\frac{1}{2*5})^2$

$y = -5(x - \frac{1}{10})^2 + (\frac{5}{100})$

$(y - \frac{5}{100}) = -5(x - \frac{1}{10})^2$

$-\frac{1}{5}(y - \frac{5}{100}) = (x - \frac{1}{10})^2$

$\frac{1}{100} - \frac{y}{5} = (x - \frac{1}{10})^2$

$\sqrt{(\frac{1}{100} - \frac{y}{5})} = \sqrt{(x - \frac{1}{10})^2}$

* تحليل العبارة التربيعية

(عددين حاصل ضربهم $\frac{1}{100}$ وحاصل جمعهم $-\frac{1}{5}$)

$\sqrt{\frac{1}{100} - \frac{y}{5}} = \left|x - \frac{1}{10}\right|$

$\sqrt{\frac{1}{100} - \frac{y}{5}} = x - \frac{1}{10}$ because $x \geq 1$

$x = \sqrt{\frac{1}{100} - \frac{y}{5}} + \frac{1}{10}$

$f^{-1}(x) = \sqrt{\frac{1}{100} - \frac{x}{5}} + \frac{1}{10}$ //

⚠ if the range of $f(x)$ is $[-1, 7]$ then the dom of $y = 2 - f^{-1}(3-x)$

*موضوع غ Dom أغير غ شكل الإقتران

is $R \cap \text{Dom } f^{-1}(5-x)$

$\cup \equiv$ or

$\cap \equiv$ &

$\text{Dom } f^{-1}(3-x)$

R & ~~$3 - x \geq$~~ $-1 \leq 3 - x \leq 7$

$-4 \leq -x \leq 4$

$4 \geq x \geq -4$ $[-4, 4]$

Five Apple

دراسة / 18, 19, 20

$\sin x \rightarrow \csc x = \frac{1}{\sin x}$

$\cos x \rightarrow \sec x \quad \frac{1}{\cos x}$

$\tan x \rightarrow \cot x = \frac{1}{\tan x}$

$>$ not (1-1)

ما إلهم inverse

(قا ، قتا ، ظتا)

$\pi-\theta$	θ
$\pi+\theta$	$2\pi-\theta$

* always :

Rnge

$-1 \leq \sin x \leq 1$
$\cos x$

$0 \leq \frac{|\sin x|}{|\cos x|} \leq 1$

$-\frac{\pi}{2} \leq \tan^{-1} \leq \frac{\pi}{2}$

$0 \leq \frac{\sin^2 x}{\cos^2 x} \leq 1$

(مهم) H.w 16

[1] dom of $f(x) = \cos\left(\frac{1}{x-7}\right)$

$R - \{x-7=0\}$

$R - \{7\}$

[2] Range of $f(x) = 4 + 2\cos x + |\cos x|$

$-1 \leq \cos x \leq 1$ ~~cos x ∈~~

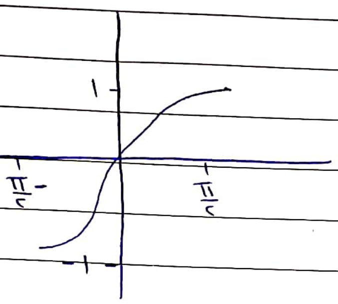
Range أكبر قيمة $4 + 2(1) + |1| = 4+3 = 7$

Rnge أصغر قيمة $4 + 2(-1) + |-1| = 4-2+1 = 3$

Range $[3,7]$

مثلثية

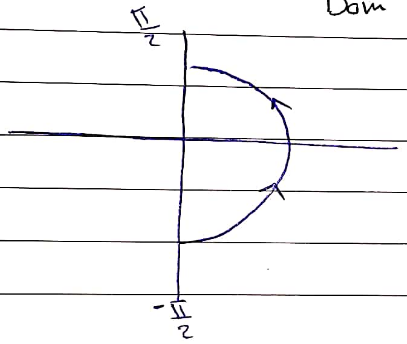
* Inverse Trig fn.



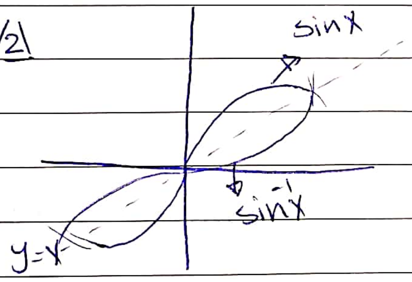
$\sin x : [\frac{-\pi}{2}, \frac{\pi}{2}] \rightarrow [-1,1]$

بجواب دائماً بالربع الأول والرابع

/1/ $\sin^{-1}x : \underbrace{[-1,1]}_{\text{Dom}} \rightarrow \underbrace{[\frac{-\pi}{2}, \frac{\pi}{2}]}_{\text{Range}}$



/2/



/3/ $\sin(\sin^{-1}x) = x \quad , \quad x \in [-1,1]$

$\sin^{-1}(\sin x) = x \, , \quad x \in [\frac{-\pi}{2}, \frac{\pi}{2}]$

$\underline{4}$ $\sin(-x) = -\sin x$ "odd fn"

$\sin^{-1}(-x) = -\sin^{-1}x$ "odd fn"

ترتيب لأنه من x

⚠ Find dom $\sin^{-1}(\underline{3x+1})$

$\mathbb{R} \cap$

$-1 \leq 3x+1 \leq 1$

$-2 \leq 3x \leq 0$

$\frac{-2}{3} \leq x \leq 0$

$x \in [\frac{-2}{3}, 0]$

⚠ $\sin^{-1}(1) = \frac{\pi}{2}$

⚠ $\sin^{-1}(-1) = -\sin^{-1} 1 = -\frac{\pi}{2}$

⚠ $\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{4}$

⚠ $\sin^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$

⚠ $\sin^{-1}(\sin(\frac{\pi}{6})) = \frac{\pi}{6} \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

⚠ $\sin^{-1}(\sin(\frac{2\pi}{3})) = +\frac{\pi}{3}$

(اطلع الزاوية المرجعية لأكم)

① ربع ثاني

② زاوية مرجعية $\frac{2\pi}{3}$

③ sin موجب

ثاني

أول سؤال بسأله

عينا مكانه

$١٢٠ = \frac{\cancel{١٨٠}^{٦٠}}{\cancel{\pi}} \times \frac{\cancel{\pi}٢}{\cancel{٣}_{١}}$

$\frac{\cancel{١٨٠}^{٤٥}}{\cancel{\pi}} \times \frac{\cancel{\pi}٥}{\cancel{٤}}$ ٢٢٥

$\begin{array}{r} ٢٢٥ \\ -١٨٠ \\ \hline ٤٥ \end{array}$

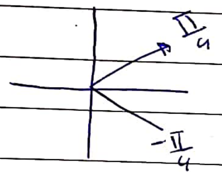
$\ast\ \sin^{-1}\left(\sin \frac{5\pi}{4}\right) = -\frac{\pi}{4}$

* ثالث

\+ زاوية مرجعية $\frac{\pi}{4}$

\+ sin سالب

* اذا كانت بالربع الأول .
بوخدها موجبة
والرابع بوخدها سالبة .



$\ast\ \sin\left(\sin^{-1}\left(\frac{1}{2}\right)\right) = \frac{1}{2}$

~~غلط اسئلة~~
غلط أسئلة عن اشي اكبر من ١+ او أقل من ١-

$\ast\ \cos(\sin^{-1}x) =$

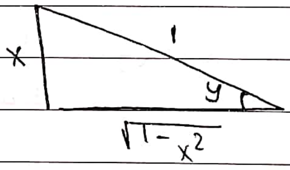
لما يكونوا مختلفين (مثلاً sin, cos)
بفرض الـ ١- العكسي y

$\sin^{-1}x = y$

$\sin(\sin^{-1}x) = \sin y$

$x = \sin y$

$\left(\frac{\text{المقابل}}{\text{الوتر}}\right) \frac{x}{1} = \sin y$

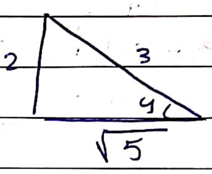


* يرجع للمطلوب

$\cos(y) = \frac{\text{مجاور}}{\text{الوتر}} = \frac{\sqrt{1-x^2}}{1}$ #

⚠ $\sec\left(\sin^{-1}\left(\frac{2}{3}\right)\right) =$

$$\sin^{-1}\left(\frac{2}{3}\right) = y$$



$$\sin\left(\sin^{-1}\left(\frac{2}{3}\right)\right) = \sin y$$

$$\frac{2}{3} = \sin y$$

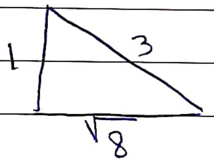
$$\sec(y) = \frac{3}{\sqrt{5}}$$

* $\tan\left(\sin^{-1}\left(\frac{-1}{3}\right)\right) =$

odd $\tan\left(-\sin^{-1}\left(\frac{1}{3}\right)\right) =$

$$-\tan\left(\sin^{-1}\left(\frac{1}{3}\right)\right) =$$

$\sin^{-1}\frac{1}{3} = y$ $\qquad$ $\frac{1}{3} = \sin y$



$$-\tan(y) = \frac{-1}{\sqrt{8}}$$

⚠ H.w $\cos^{-1}\left(\frac{-\sqrt{3}}{2}\right) = \pi - \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$$= \pi - \frac{\pi}{6}$$

$$= \frac{5\pi}{6}$$

⚠ $\tan^{-1}\left(\frac{-1}{\sqrt{3}}\right) = -\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{-\pi}{6}$

Find :-

$\circledast$ $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right) \quad , \pi - \frac{\pi}{4} = \frac{3\pi}{4}$

$\circledast$ $\cos^{-1}\left(\cos\left(\frac{12\pi}{7}\right)\right) = \quad 2\pi - \theta = \frac{12\pi}{7} \qquad \theta = \cancel{12} \quad 2\pi - \frac{12\pi}{7} = \frac{2\pi}{7}$

رابع

+

أول

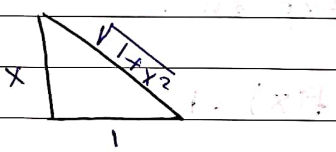
$\circledast$ $\tan^{-1}\left(\tan \frac{7\pi}{5}\right) =$

له ثالث / موجب

$\pi + \theta = \frac{7\pi}{5} \quad , \quad \theta = \frac{7\pi}{5} - \pi \qquad = \frac{2\pi}{5}$

$\circledast$ $\sin\left(2 \tan^{-1} x\right) =$

$\tan^{-1} x = y \rightarrow x = \tan y$



$\sin(2y) = 2 \sin y \cos y$

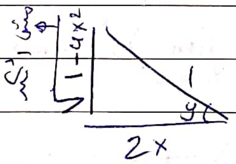
$= 2 \cdot \frac{x}{\sqrt{1+x^2}} * \frac{1}{\sqrt{1+x^2}} = \frac{2x}{1+x^2}$

$\circledast$ $\sin\left(2 + \underbrace{\cos^{-1}(2x)}_{y}\right) =$

$\sin(2+y) = \sin 2 \cos y + \cos 2 \sin y$

$\cos^{-1}(2x) = y$

$2x = \cos y$



$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}$

* Range of $y = \pi + 2\,|\tan^{-1}x|$

$0 \leq |\tan^{-1}x| < \frac{\pi}{2}$ *

$0 \leq |\tan^{-1}x| < \frac{\pi}{2}$

مش ضرورية

$0 < 2|\tan^{-1}x| < \pi$

$\pi < \pi + 2|\tan^{-1}x| < 2\pi$

* الاقتران يكون متناقص لما تزيد قيمه س تقل قيمه ص .

Exp fn .

$f(x) = b^x \quad , 0<b \quad , b\neq 1$ ال Dom تعتمد ع الاقتران

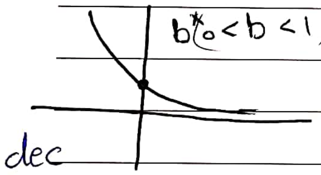
/11 $\quad 0<b<1$

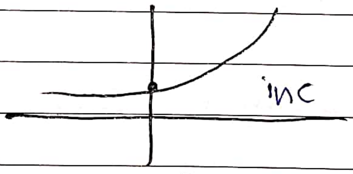
$f(x) = \frac{1}{2}^x$

2^{-x}

x	-3	-2	-1	0	1	2	3	~~4~~
f(x)	8	4	2	1	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	

$b > 1$





Dom : R

range: $(0,\infty)$

$b>1 \quad \lim_{x\to\infty} b^x = \infty$

$x \to -\infty = 0$

$0<b<1$

$\lim_{x\to\infty} b^x = 0$

$x \to -\infty = \infty$

دائماً

$\frac{\text{عدد}}{\infty}$ = صفر

$\therefore \text{Dom}\{b^{f(x)}\} \equiv \text{Dom} f(x)$

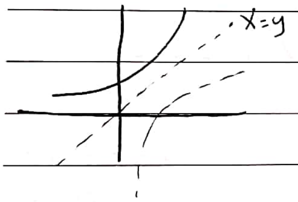
$* \ \text{dom}\{3^{x-1}\} = R$

$\text{dom}\left\{\frac{1}{3^{x-1}}\right\} = R-\{1\}$

$* \ e = \varnothing =$ العدد النيبري $= 2.718$

$e^x \qquad / \ e^{-x} = \frac{1}{e^x}$

* log function:-



/1\ $b^x : R \rightarrow (0,\infty)$

$\log_b X : (0,\infty) \rightarrow R$

/2\ $f(f^{-1}(x)) = X$

$\log_b b^x = X$

$* \ \log_{10} x = \log x$

$* \ \log_e x = \ln x$

$b^{\log_b x} = X$

* find dom:-

① $f(x) = \frac{\sqrt{x}}{e^{3x-1}}$ Dom$\sqrt{x} \cap$ Dom$\{3x-1\}$

$[0,\infty) \cap R - \{\frac{1}{3}\}$

$[0,\infty) - \{\frac{1}{3}\}$

② dom $f(x) = \log(7x-1)$

$7x-1>0$

$7x>1$ ~~6X~~ $x > \frac{1}{7}$ $(\frac{1}{7},\infty)$

(حل المعادلة)

✱ Solve for x :-

+ (اللوغاريتم = 0 لما يكون داخل اللوغاريتم يساوي ١)

① $(x^2-9)(e^{-x}-2)\log_{(x-2)}(x-6) = 0$

+ أول خطوة :

① أطلع Dom

②

لل Dom $R \cap R \cap$

عالهامش

Dom $\log_{(x-2)}(x-6) = \frac{\log(x-6)}{\log(x-2)}$

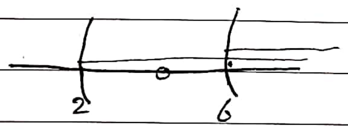
، $x-6>0 \rightarrow x>6 \rightarrow (6,\infty)$

$x-2>0 \rightarrow x>2 \rightarrow (2,\infty)$

$\log(x-2)=0$

$x-2=1$

$x=3$



$\therefore (6,\infty)$

$\therefore$ Dom $R \cap R\ (6,\infty) = (6,\infty)$

/2\ $X^2 - 9 = 0 \rightarrow X = \pm 3$ $\notin$

/3\ $e^{-x} - 2 = 0 \rightarrow$

$e^{-x} = 2 \rightarrow \ln e^{-x} = \ln^2$

$-x = \ln 2$ $-\ln \notin$

/4\

* ② $\frac{1}{3 - e^{2x}} = 4$

$1 = 12 - 4e^{2x}$

$4e^{2x} \rightarrow = 11$

$e^{2x} = \frac{11}{4}$

$\ln e^{2x} = \ln 11/4$

$2x = \ln 11/4 = \ln \sqrt{11}/2$

$x = \frac{1}{2} \ln 11/4 = \ln \sqrt{\frac{11}{4}}$

③ $3e^{-2x} = 5$

$e^{-2x} = \frac{5}{3} \rightarrow \ln e$

$\log_2 3x + \log_4 9x^2 = 4$ $\frac{\log_2 x^2}{\log_2 4}$

dom

④ $x^2 \ln x - 16 \ln x = 0$ $(0,\infty)$

$\ln x [x^2 - 16] = 0$

$\swarrow$

$\ln x = 0 \rightarrow x = 1 \in$

$x^2 - 16 = 0 \rightarrow x = \pm 4$

$\therefore x \in \{1, 4\}$

⑤ $\log x^{3/2} - \log \sqrt{x} = 5$ $(0,\infty)$

$\log \left(\frac{x^{3/2}}{x^{1/2}} \right) = 5$

$\log_{10} x = 5 \rightarrow x = 10^5$

* Find dom

[1] $f(x) = \frac{3 \rightarrow \mathbb{R}}{\ln(x-2) \rightarrow (2,\infty)}$

$\ln(x-2) = 0$

$x - 2 = 1 \quad \rightarrow x = 3$ $(2,\infty) - \{3\}$

② $f(x) = \ln|x|$ $R - \{0\}$ → دائماً عندي $R - \{$ صفر اعطاه $\}$

③ $f(x) = \ln|x-4|$, $R - \{4\}$

④ $f(x) = \ln(x^2-4)$

$x^2-4>0$

$x^2>4$, $|x|>2$

$\therefore (-\infty,-2)\cup(2,\infty)$

⑤ $f(x) = \ln(x^2+4)$ $: R$

*H.w

① dom : $f(x) = \dfrac{1-e^{x^2}}{1-e^{1-x^2}} + \cos^{-1}(2x-1)$

② Range $f(x) = \dfrac{e^x}{1+2e^x}$

التمرين 1/202
الامتحان

* range $f(x) = 2^x + 5$

$y = 2^x + 5$

$y - 5 = 2^x$

$\log_e(y-5) = \log 2^x$

$\therefore f(x)^{-1} = \log_2(x-5)$ ، Range f = Dom $f^{-1} = (5,\infty)$

* if $y = e^{2x+10}$

then $y^{-1}(x)$:

$\ln y - \ln e^{2x+10}$

$\ln y = 2x + 10$

$\frac{\ln y - 10}{2} = x$ $\qquad \therefore y^{-1} = \frac{(\ln x) - 10}{2}$

تكملة
H.w

0 ≤ ما داخله

③ dom : $f(x) = \sqrt{\ln(x^2-5)}$

الي برا ∩ الي برا ∩ اجعل Dom

④ dom : $f(x) = \ln(\ln(\ln x))$

⑤ dom : $f(x) = \ln\left(\frac{x+2}{x-1}\right)$

معادلة تربيعية بعد التعويض

⑥ Solve : $e^{-2x} - 3e^{-x} = -2$

⑦ Solve : $\ln\left(\frac{1}{x}\right) + \ln(2x^3) = \ln 3$

بستخدم الخواص

رقم المتسلسل/ اصيا/ اسم الدكتورة.

The limits of Functions

$$\lim_{x \to a} \frac{f}{g} = \frac{\text{عدد}}{\text{عدد}} , \frac{\text{صفر}}{\text{عدد}} , \frac{\text{صفر}}{\text{صفر}} , \frac{\text{عدد}}{\text{صفر}}$$

$$* \lim_{x \to 1} \frac{3-x}{x+2} = \frac{2}{3}$$

$$* \lim_{x \to 3} \frac{3-x}{x+2} = \frac{0}{5} = 0$$

$$* \lim_{x \to 1} \frac{2-\sqrt{x+3}}{x^2+2x-3} = \frac{0}{0}$$

$$= \lim_{x \to 1} \frac{2-\sqrt{x+3}}{x^2+2x-3} * \frac{2+\sqrt{x+3}}{2+\sqrt{x+3}}$$

$$= \lim_{x \to 1} \frac{4-(x+3)}{x^2+2x} * \frac{1}{4}$$

$$= \frac{1}{4} \lim_{x \to 1} \frac{1-x}{(x-1)(x+3)} = \frac{1}{4} * \frac{-1}{4} = \frac{-1}{16}$$

$$* \lim_{x \to 3} \frac{|x-3|}{x-3} = \frac{0}{0}$$

$$= \lim \begin{cases} \frac{x-3}{x-3} = 1 , & x \geq 3 \\ \frac{-(x-3)}{x-3} = -1 , & x < 3 \end{cases}$$

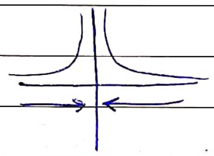
H.w

$$\lim_{x \to 2} \frac{\sqrt{6-x} -2}{\sqrt{3-x} -1} = \frac{1}{2}$$

* $(x^2 - a^2) = (x-a)(x+a)$
* $ax^2 + bx + c = 0$ ...
* $(x^3 \mp y^3) = (x \mp y)(x^2 \pm xy + y^2)$

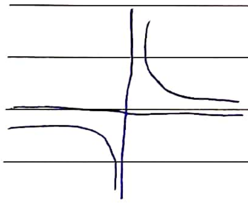
* [illegible] $\frac{\text{عدد}}{\text{صفر}}$

$$\lim_{x \to 0} \frac{1}{x^2} = \infty$$



$x \to 0^+ = +\infty$

$x \to 0^- = +\infty$



$\lim_{x \to 0^+} \frac{1}{x} = \infty \Rightarrow x \to 0^- = -\infty$

$\therefore \lim_{x \to 0} \frac{1}{x} = d.n.e$

* $\lim_{x \to 2} \frac{x+3}{x-2} = \frac{5}{0}$

الخط العمودي المتقارب بعد ما اختصر

لما سائلين عن الخط العمودي كنق علشي

افقي

$+ \quad - \quad +$

$-3 \quad 2$

$\lim_{x \to 2^+} = +\infty$

$x \to 2^- = -\infty \qquad = d.n.e$

$* \lim_{x \to 4} \frac{1}{(x-4)^2}$

$+ \quad +$ over 4

$= \infty$

$* \lim_{x \to 4} \frac{-1}{(x-4)^2}$

$= -\infty$

$* \lim_{x \to 4} \frac{2-x}{x^2-2x-8} \qquad = \frac{-2}{0}$

$= \lim_{x \to 4} \frac{2-x}{(x-4)(x+2)} \qquad$ d.n.e

$+ \quad -$

$-2 \quad 2 \quad 4$

* أصفار المقام بعد الإختصار

*vertical asy V.A

the line $x = a$ is called vertical asy if $\lim_{x \to a^{+}, a^{-}} f(x) = \infty, -\infty$ d.n.e

* find the virtical a.sy of :-

1) $f(x) = \frac{10}{x^2-4} \qquad x = \pm 2$

الخط العمودي هو أصفار

المقام بعد الإختصار.

2) $f(x) = \frac{x-3}{x^2-9} = f(x) = \frac{1}{x+3} \qquad x = -3$

3) $f(x) = \frac{x-1}{|x|-1} = \begin{cases} \frac{x-1}{x-1} = 1, & x \geq 0 \\ \frac{x-1}{-x-1}, & x < 0 \end{cases}$

$|x|+1$ لو كانت

V.a ما في ارسم

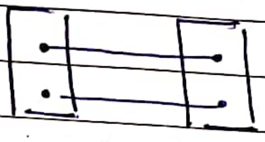
$-x-1=0$

$x = -1$

V. A

The domain of polynomials Fn = R
كثيرات الحدود

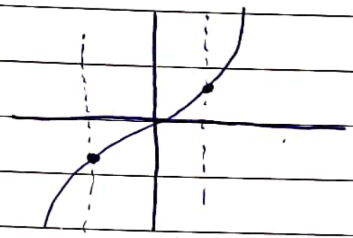
✱ Functions :- (الاقترانات)



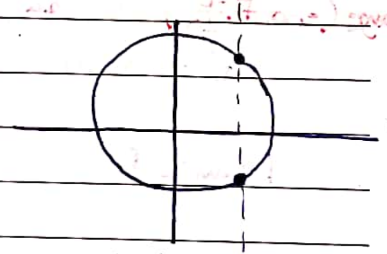
★ إذا عل صورة بالمجال ارتبطت بأكثر من صورة بالمدى بتكون علاقة
إذا ارتبطت بصورة وحدة بيكون اقتران

المجال
Domain
(Dom)
له الإختصار

المدى
Range



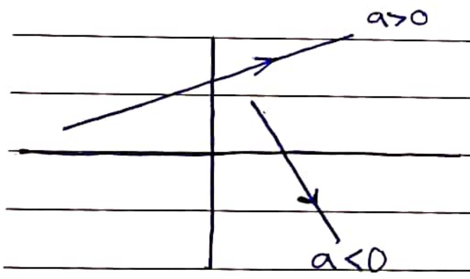
Functions
اقتران



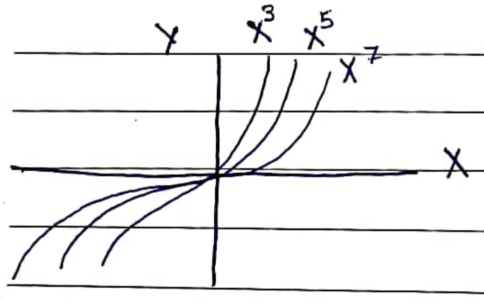
Relation
علاقة

⊗ Liner Function

$$f(x) = ax + b$$



(Real num)
Dom :- R (ح)
Range :- R (ح)



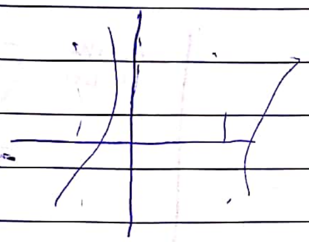
(فردي / odd)
X^3 : R _ R

ملاحظة هاي

Log ، ln +

الاقترانات يكون ما بداخله يساوي صفر يكون في عندي

V.A



$* \tan x = \frac{\sin x}{\cos x}$

$x = \frac{\pi}{2} + n\pi$ V. asy (∞)

$* f(x) = \log_b x$

$x = 0$ V. asy

$* f(x) = \ln x$

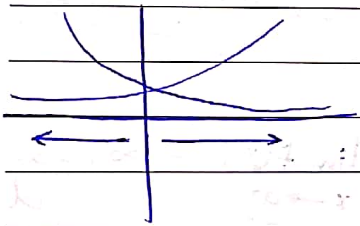
$x = 0$ V. asy

*Limits at ∞ :-

$\triangle \lim_{x \to \infty} \frac{\text{عدد} \pm}{x} = 0 \qquad \left(\frac{\text{عدد}}{\infty}\right)$

$x \to -\infty$

$\infty = \left(\frac{\infty}{\text{عدد}}\right)$



$a > 1 : a^{\infty} = \infty$

$: a^{-\infty} = 0$

$\triangle \lim_{x \to \infty} 3^x = \infty$

$\lim_{x \to \infty} \left(\frac{1}{3}\right)^x = 0$

$0 < a < 1 : a^{\infty} = 0$

$a^{-\infty} = \infty$

$\lim_{x \to \infty} 3^{-x} = 0 \quad ; \quad 3^{-\infty} = \frac{1}{3^{\infty}}$

$= \frac{1}{\infty} = 0$

$\triangle \lim_{x \to -\infty} 3x^5 = -\infty$

$\triangle \lim_{x \to \infty} -3x^5 = -\infty$

$\triangle \lim_{x \to -\infty} -x^2 = -\infty$

Five Apple

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(عند كثير حدود باخذ ال limit للأعلى
قوة مع الإشارة)

في حالة كثير حدود
كثير حدود

*Hint
* درجة البسط > المقام
$X \to \infty$ أو $X \to -\infty$
دائماً جوابي = صفر

$\lim_{X \to -\infty} 4X^2 - X^5 + 3X^8$

$= \lim_{X \to -\infty} 3X^8 = +\infty$

*Hint
* درجة البسط = المقام
معامل أعلى أس في البسط
معامل أعلى أس بالمقام

$\lim_{X \to -\infty} 3X^2 - 5X^3 = \lim_{X \to -\infty} -5X^3 = \infty$

*Hint
* درجة البسط > درجة المقام
الجواب ∞ أو $-\infty$

$\lim_{X \to \infty} \frac{4X^2 - 1}{2X - X^5}$

$= \lim_{X \to \infty} \frac{4X^2}{-X^5} = \lim_{X \to \infty} \frac{-4}{X^3} = 0$

$\lim_{X \to \infty} \frac{3X - 2X^2}{4X^2 + 5} = \lim_{X \to \infty} \frac{-2X^2}{4X^2} = \frac{-1}{2}$

$\lim_{X \to \infty} \frac{7X^7 - 2X + 1}{3X^2 - X^5} = \lim_{X \to \infty} \frac{7X^7}{-X^5} = \lim_{X \to \infty} -7X^2 = -\infty$

$\lim_{X \to \infty} \frac{4X^3 - 2X}{3X^2 + 1} = \lim_{X \to -\infty} \frac{4X^3}{3X^2} = \lim_{X \to -\infty} \frac{4}{3} X = \infty$

X بين الصفر والواحد مرفوع لـ ∞ = 0

* $e^{\infty} = \infty$, $e^{-\infty} = 0$, $1^{\infty} = \infty$, $\frac{1}{٢}^{\infty} = 0$, $\frac{1}{٢}^{-\infty} = \infty$
* $\tan^{-1} 0 = 0$, $\tan^{-1} 1 = \frac{\pi}{4}$, $\tan^{-1} \infty = \frac{\pi}{2}$, $\tan^{-1} -\infty = -\frac{\pi}{2}$
* $\ln \infty = \infty$, $\ln 0^+ = -\infty$

* $\lim_{X \to \infty} \frac{1}{X} = 0$
* $\lim_{X \to \infty} e^X = \infty$
* $\lim_{X \to -\infty} \frac{1}{X} = \infty$
* $\lim_{X \to -\infty} e^X = 0$
* $\lim_{X \to 0^+} \ln X = -\infty$
* $\lim_{X \to \infty} 2^{-X} = 0$
* $\lim_{X \to \infty} \ln X = \infty$
* $\lim_{X \to \infty} e^{-X} = 0$

$\lim_{X \to \infty} 2^X = \infty$

* $\lim_{X \to \infty} \tan^{-1}(X) = \frac{\pi}{2}$

$\lim_{X \to -\infty} 2^X = 0$

* $\lim_{X \to -\infty} \tan^{-1}(X) = \frac{-\pi}{2}$

$$\boxed{*}\ \lim_{x\to\infty} \frac{\sqrt{16x^2-5x}}{x+3}$$

* بوخذ أعلى درجة بالبسط والمقام ←

$$= \lim_{x\to\infty} \frac{\sqrt{16x^2}}{x}$$

لأنها كانت تقترب من $+\infty$ بوخذ $+x$ ←

$$= \lim_{x\to\infty} \frac{4|x|}{x} = \lim \frac{4x}{x} = 4$$

$$\boxed{*}\ \lim_{x\to-\infty} \frac{x+3}{\sqrt{16x^2-5x}} = \lim_{x\to-\infty} \frac{x}{\sqrt{16x^2}} = \lim_{x\to-\infty} \frac{x}{4|x|}$$

← لأنها كانت تقترب من $-\infty$ بوخذ $-x$

(عدد * $\frac{\infty}{\infty}$) = ∞ (حسب الإشارة)

وهالأسئلة تعويض مباشر

$$\boxed{*}\ \lim_{x\to\infty} \frac{e^{-x}-3e^{2x}}{e^{-2x}+5e^{x}} = \lim_{x\to\infty} \frac{-3e^{2x}}{5e^{x}} = \lim_{x\to\infty} \frac{-3}{5}e^{x} = -\infty$$

$$\boxed{*}\ \lim_{x\to-\infty} \frac{5e^{-2x}-\cancel{3e^{x}}^{\,0}}{\cancel{e^{2x}}^{\,0}-4e^{-2x}} = \lim_{x\to-\infty} \frac{5e^{-2x}}{-4e^{-2x}} = \frac{-5}{9}$$

* The horizontal asymptote for $f(x)$ is:-

$$y = \lim_{x\to\infty} f(x)$$
$$x \to -\infty$$

* find the horizantal asy :-

1) $f(x) = \frac{\sqrt{x^4-1}}{x+5}$

① $y = \lim_{x\to\infty} \frac{\sqrt{x^4-1}}{x+5} = \lim_{x\to\infty} \frac{\sqrt{x^4}}{x} = \lim_{x\to\infty} \frac{x^2}{x} = \infty$

② $y = \lim\limits_{x \to -\infty} \dfrac{\sqrt{x^4 - 1}}{x + 5} = \lim\limits_{x \to -\infty} \dfrac{x^2}{x} = -\infty$

∴ theres no H.A

*Hint

$\dfrac{\text{عدد}}{\infty} = 0$

$\dfrac{\infty}{\text{عدد}} = \infty$

/2 $f(x) = \dfrac{x - 7}{3x + 1}$

① $y = \lim\limits_{x \to \infty} \dfrac{x - 7}{3x + 1} = \left|\dfrac{1}{3}\right|$, ② $y = x \to -\infty = \left|\dfrac{1}{3}\right|$

∴ H.A , $y = 1/3$

/3 $f(x) = 2 + \tan^{-1}(2x)$

*لو كانت $\frac{1}{\infty}$ بتساوي 0

① $y = \lim\limits_{x \to \infty} 2 + \tan^{-1}(2x) = 2 + \dfrac{\pi}{2}$

② $y = \lim\limits_{x \to -\infty} + \tan^{-1}(2x) = 2 - \dfrac{\pi}{2}$

H.w

/4 $f(x) = \dfrac{1}{3} \tan^{-1}(e^x)$

H.w

/5

$f(x) = \sqrt{x^2 - 3x} - x$

*note that :-

/1\ $\lim_{x\to 0} \frac{\sin x}{x} = 1$

/2\ $\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \frac{a}{b}$

/3\ $\lim_{x\to 0} \frac{\tan x}{x} = 1$

/4\ $\lim_{x\to 0} \frac{\tan ax}{\tan bx} = \frac{a}{b}$

(*) Continuous Fn . (cont fn)

$f(x)$ cont at x_0 :-

/1\ $f(x_0)$ defined

/2\ $\lim_{x\to x_0} f(x)$ exist

/3\ $\lim_{x\to x_0} f(x) = f(x_0)$

(*) note that

$\underline{\underline{f}}$ cont , $\underline{\underline{g}}$ cont

/1\ $f \pm g(x) \to$ cont

/2\ $\frac{f(x)}{g(x)} \longrightarrow$ cont (but $g(x) \neq 0$)

/3\ $f \circ g(x) \longrightarrow$ cont

|4\ $f(x)$ cont every where (R) $\Longrightarrow$ $f \circ g$ cont every where (R)
$g(x)$ 〃 〃 〃 (R)

|5\ $f(x)$ cont on it's dom & 1-1 then $f^{-1}(x)$ cont on range f.

⊛ $f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 4, & x = 3 \end{cases}$, is f(x) cont ?

① $f(3) = 4$

② $\lim_{x \to 3} \frac{x^2-9}{x-3} = \lim_{x \to 3} x+3 = 6$

③ $4 \neq 6 \Longrightarrow$ discont at ~~R-{3}~~ $x = 3$

⊛ $f(x) = \begin{cases} x^2-1, & x \geq 0 \\ 3-\left(\frac{x+8}{2}\right), & x < 0 \end{cases}$, is f(x) cont ?

① $f(0) = -1$

② $\lim_{x \to 0^+} f(x) = -1$, $\lim_{x \to 0^-} f(x) = -1$

~~Son~~ So the fun cont at $x = 0$

⊛ $$f(x) = \begin{cases} a(\tan^{-1}x + 2) , & x < 0 \\ 2e^{bx} + 1 , & 0 \leq x \leq 3 \\ \ln(x-2) + x^2 , & x > 3 \end{cases}$$

*Find a & b ∍ f(x) cont every where ?

/1\ f(x) cont on $x = 0$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$$

$$2e^0 + 1 = a(\tan^{-1}0 + 2)$$

$$\frac{3}{2} = \frac{2}{2} a$$

$$a = 3/2$$

/2\

f(x) cont on $x = 3$

$$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^-} f(x)$$

$$\ln(3-2) + 9 = 2e^{2b} + 1$$

$$\ln^{4} = \ln^{e^{3b}} \Rightarrow \ln\frac{4}{3} = \frac{3}{\cancel{3}} b \quad \#$$

⊛ $f(x) = \frac{x+3}{x^2+ax+1}$ cont every where find the values of a?

صافي أصفار للمقام

$x^2+ax+1 \neq 0$

$b^2-4ac<0$

$a^2-4(1)(1)<0$

$a^2-4<0$

$a^2<4 \Longrightarrow |a|<2 \Longrightarrow -2<a<2$

$\therefore a \ni (-2,2)$

⊛ discuss the continuaily

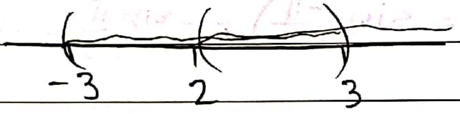
:Hint

* زي حكاي بوجد Dom

1) $f(x) = 10x^2-4x+1$ (cont every where)

2) $f(x) = \frac{10x^2-1}{x^2+1}$ (cont every where)

3) $\frac{\ln(2x-4)}{\sqrt{9-x^2}}$ $\longrightarrow 2x-4>0$, $x>2$; $\longrightarrow [-3,3]-\{\pm 3\}$



$\therefore (2,3)$ cont

4) $f(x) = \frac{x+3}{|x|+2}$ (cont every where)

5) $f(x) = \frac{x+3}{|x|-2}$ $(R-\{\pm 2\})$

6) $f(x) = |\cos x|$ (cont on R)

7) $f(x) = \sin\left(\frac{2\pi}{x-\pi}\right)$ (cont on $R-\{\pi\}$)

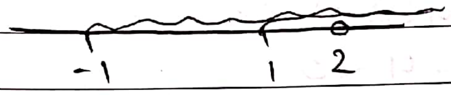
8) $F(x) = \text{Log}_{x-1}^{\overset{x+1}{\downarrow}}$

$$\frac{\text{Log}(x+1) \rightarrow x>-1}{\text{Log}(x-1) \rightarrow x>1}$$

طالما

$x-1=1$

$x=2$



$\therefore\ (1,\infty)-\{2\}$

(*) The points of discont of :-

$F(x) = \dfrac{x^2+2x-3}{(x-2)(x^2-1)}$ is $x \in \{-1, 1, 2\}$

نظرية

(*) $\lim\limits_{x\to a} f(g(x)) = f(\lim\limits_{x\to a} g(x))$

نوجد ←

القيمة بعدين بعوضها بال f(x)

eq) $\lim\limits_{x\to 0^+} e^{-2/x} = e^{\lim\limits_{x\to 0^+} -3/x} = e^{-\infty} = 0$

eq) $\lim\limits_{x\to\infty} \sin\left(\dfrac{2-\pi x}{3x+1}\right) = \sin \lim\limits_{x\to\infty}\left(\dfrac{2-\pi x}{3x+1}\right) = \sin\left(\dfrac{-\pi}{3}\right) = -\sin\dfrac{\pi}{3} = \dfrac{-\sqrt{3}}{2}$

eq) $\lim\limits_{x\to\infty} \ln\left(\dfrac{x^6+1}{-1+x^6}\right) = \ln\left(\lim\limits_{x\to\infty} \dfrac{x^6+1}{-1+x^6}\right) = \ln 1 = 0$

eq) $\lim\limits_{x\to\infty} \sin(\tan^{-1} x) = \sin(\frac{\pi}{2}) = 1$

eq) $\lim\limits_{x\to\infty} \dfrac{x^3+1}{(1-2x)^2} = \dfrac{-1}{8}$

eq) if $\lim\limits_{x\to 0} \dfrac{\sin(2ax)}{\sin(8x)} = \dfrac{14}{8}$

find a ?

$\dfrac{2a}{8} = \dfrac{14}{8}$, $2a = 14$, $a = 7$

Five Apple

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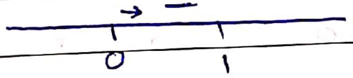
eq) $\lim\limits_{x\to 0} x^2 \csc x = \lim\limits_{x\to 0} \frac{x^2}{\sin x} = \lim\limits_{x\to 0} \frac{x}{\sin x} = 1.0 = 0$

eq) $\lim\limits_{x\to\infty} \tan^{-1}(\ln x) = \tan^{-1}(\ln\infty) = \tan^{-1}(\infty) = \frac{\pi}{2}$

eq) $\lim\limits_{x\to 0^+} \frac{1}{x} - \frac{1}{x^2}$

$\infty - \infty \Rightarrow \lim\limits_{x\to 0^+} \frac{x-1}{x^2} = \frac{-1}{0} = -\infty$



eq) if $\lim\limits_{x\to 0} f(x) = 4$

then $\lim\limits_{x\to 0} \frac{\tan(8x)}{x\, f(4x)}$ $= 4x = y$

$\therefore \lim\limits_{y\to 0} \frac{4 \tan(8\cdot\frac{y}{4})}{y\cdot f(y)} = \lim\limits_{y\to 0} \frac{4\cdot\tan 2y}{f(y)\cdot y} = \frac{4\cdot 2}{4} = 2$

~~eq) lim~~ H.w

⊛ $f(x) = \begin{cases} \frac{2\sin x}{x}, & x \overset{\ominus}{<} 0 \\ a, & x = 0 \\ b\cos x, & x \overset{\oplus}{>} 0 \end{cases}$, find a, b ?

إذا كانت
متصلة

$b = a = 2$

(1) $\lim\limits_{x\to 0^+} \frac{2\sin x}{x} = \lim\limits_{x\to 0} b\cos x$

$\boxed{2 = b}$

(2) $\lim\limits_{x\to 0^+} \frac{2\sin x}{x} = f(0)$ $\therefore a = b = 2$ #

$\boxed{2 = a}$

eq) $\lim\limits_{x \to 3} \sqrt{x-2} = 1$

eq) $\lim\limits_{x \to 1} \sqrt{x-2}$ = d.n.e

eq) $\lim\limits_{x \to 2} \sqrt{x-2}$ = d.n.e $\quad \underset{2}{\overset{- \quad\quad +}{\overline{\quad\quad|\quad\quad}}}$

$x \to 2^+ = 0$

$x \to 2^-$ = d.n.e

eq) $\lim\limits_{x \to 1} \sqrt{x^2-2x+1} = \lim\limits_{x \to 1} \sqrt{(x-1)^2} = 0 \quad \underset{1}{\overset{+ \quad\quad +}{\overline{\quad\quad|\quad\quad}}}$

⊛ Sequeezing Thrm

$\lim\limits_{x \to a} f(x)$

$$h(x) < f(x) < g(x)$$

$$\lim_{x \to a} h(x) = l \leftarrow h(x), \quad f(x) \to l, \quad g(x) \to \lim_{x \to a} g(x) = l$$

eq) $\lim\limits_{x \to 0} \frac{\sin x}{x} = 1$

eq) $\lim\limits_{x \to \infty} \frac{\sin x}{x}$, $-1 < \sin x < 1$

$$\underset{0}{\underset{\downarrow}{\lim_{x \to \infty} \frac{-1}{x}}} < \underset{0}{\underset{\downarrow}{\frac{\sin x}{x}}} < \underset{0}{\underset{\downarrow}{\lim_{x \to \infty} \frac{1}{x}}}$$

eq) $\lim\limits_{x \to 0^+} x^4 \cos\left(\frac{2}{x}\right) = 0$

$-1 < \cos\left(\frac{2}{x}\right) < 1$

$$\underset{0}{\underset{\downarrow}{\lim_{x \to 0^+} -x^4}} < \underset{0}{\underset{\downarrow}{x^4 \cos\left(\frac{2}{x}\right)}} < \underset{0}{\underset{\downarrow}{\lim_{x \to 0^+} x^4}}$$

/ eq) $\lim\limits_{x \to \infty} e^{-3x} \sin^2 x = 0$

$0 < \sin^2 x < 1$

$0 < e^{-3x} \sin^2 x < e^{-3x}$

$\lim\limits_{x \to \infty} \swarrow 0 \qquad \lim\limits_{x \to \infty} \swarrow 0$

eq) $\lim\limits_{x \to \infty} \dfrac{2x + \sin x}{\tan x + 10x} = \lim\limits_{x \to 0} \dfrac{\dfrac{2x}{x} + \dfrac{\sin x}{x}}{\dfrac{\tan x}{x} + \dfrac{10x}{x}} = \dfrac{2+1}{1+10} = \dfrac{3}{11}$

eq) $\lim\limits_{x \to \infty} \dfrac{2x + \sin 2x}{x + \cos x}$

$\lim\limits_{x \to \infty} \dfrac{\dfrac{2x}{x} + \cancel{\dfrac{\sin 2x}{x}} \to 0}{\dfrac{x}{x} + \cancel{\dfrac{\cos x}{x}} \to 0} = \dfrac{2+0}{1+0} = 2$

$\dfrac{-1}{x} < \dfrac{\sin x}{x} < \dfrac{1}{x} \qquad \swarrow 0 \quad \swarrow 0 \quad \swarrow 0$

$\dfrac{-1}{x} < \dfrac{\cos x}{x} < \dfrac{1}{x} \qquad \swarrow 0 \quad \swarrow 0 \quad \swarrow 0$

eq) $f(x) = \sqrt{9 - x^2}$ show that $f(x)$ cont on $[-3, 3]$

1) $\lim\limits_{x \to -3^+} f(x) = f(-3) = 0$ ∴ cont at $x = -3^+$

2) $\lim\limits_{x \to 3^-} f(x) = f(3) = 0$ ∴ cont at $x = 3^-$

3) $a \in (-3, 3)$, $\lim\limits_{x \to a} = f(a) = \sqrt{9 - a^2}$

* الدالة يكون غير قابل للاشتقاق
عند رؤوس الدالة $|x|$

⊛ Differentiation :-

$$f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$$

$f'(x)$, $\frac{dy}{dx}$, $\frac{d}{dx} f(x)$, y' , slope

1\ $f(x)$ diffble at $x = 0 \implies f(x)$ cont at $x = 0$
← قابل للاشتقاق

2\ $f(x)$ discont at $x = a \implies f(x)$ not diffble at $x = a$

eq) $f(x) = |x|$, cont at $x = 0$ but not diffble at $x = 0$.

$$|x| = \begin{cases} x, & x \geq 0 \\ -x, & x < 0 \end{cases} \quad , \quad f'(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \\ d.n.e, & x = 0 \end{cases}$$

eq) $f(x) = \begin{cases} 5x - 1, & x > 1 \\ x^2 + 9, & x \leq 1 \end{cases}$

is $f'(1)$ exist ?

find $f'(2)$?

find $f'(0)$?

① $f(1) = 10$

② $\lim_{x \to 1^+} f(x) = 4$

$x \to 1^- \quad = 10$

$\therefore$ discont at $x = 1 \implies f'(1)$ d.n.e

Five Apple

Scanned by CamScanner

$f'(x) = \begin{cases} 5 , & x > 1 \\ 2x , & x < 1 \\ d.n.e , & x = 1 \end{cases}$, $f'(2) = 5$, $f'(0) = 0$

eq) $f(x) = \begin{cases} x^2 \cos\left(\frac{2}{x}\right) , & x \neq 0 \\ 5 , & x = 0 \end{cases}$, is $f'(x)$ exist ?

1| $f(0) = 5$

2| $\lim_{x \to 0} x^2 \cos\left(\frac{2}{x}\right) = 0 \longrightarrow \underset{\underset{0}{\downarrow}}{-x^2} < \underset{\underset{0}{\downarrow}}{x^2 \cos\left(\frac{2}{x}\right)} < \underset{\underset{0}{\downarrow}}{x^2}$

sequeezing

$\therefore$ discont $\Rightarrow$ $f'(0)$ d.n.e

⚠ if $f(2x - 3) = y$

such that $f'(1) = 2$, find $\left.\frac{dy}{dx}\right|_{x=2}$

$\frac{dy}{dx} = f'(2x - 3) * 2 \Big|_{x=2}$

$= f'(1) * 2$

$= 2 * 2 = \boxed{4}$

⚠ if $f(x^3 - 1) = \frac{3x^2}{x^2 + 1}$ find $f'(7)$?

$f'(x^3 - 1) * 3x^2 = \frac{(x^2 + 1)(6x) - (3x^2)(2x)}{(x^2 + 1)^2}$ ⊛

$f'(7) = ?$

$x^3 - 1 = 7$

$x^3 = 8 \Rightarrow x = 2$

sub $x=2$ in (*)

$f'(7) * 12 = \frac{(5)(12)-(12)(4)}{25}$

$f'(7) = \frac{1}{25}$

△* $\frac{d}{dx}\left[f(x^2)\right] = 4x^2$ then $f'(x) =$

$f'(x^2) \frac{* \cancel{2x}}{\cancel{2x}} = \frac{4x^2}{2x}$

$x^2 = y$

$x = \sqrt{y}$

$f'(x^2) = 2x \longrightarrow$

$f'(y) = 2\sqrt{y} \Rightarrow \therefore f'(x) = 2\sqrt{x}$

△* $\frac{d}{dx}\left[f(2x+4)\right] = 4x+2$

find $\left.\frac{d}{dx}\left[f(x)\right]\right|_{x=4}$?

$f'(2x+4) * 2 = 4x+2$

$2x+4 = 4$

$x = 0 \Rightarrow f'(4) * 2 = 0+2$

$f'(4) = 1$

H.w

① $f'(9) = 5$, $g(2) = 9$

$g'(2) = -3$

$(f\circ g)'(2) =$

H.w

② $f'(x) = \frac{x}{x^2+1}$, $g(x) = \sqrt{3x-1}$. find $(f\circ g)'(x)$

⚠ $\frac{d}{dx}\left[\tan^3\sqrt{1+e^x}\right] =$

$3\tan^2\sqrt{1+e^x} * \sec^2\sqrt{1+e^x} * \frac{e^x}{2\sqrt{1+e^x}}$

⚠ $\frac{d}{dx}\left[\sqrt[5]{\csc^2(3x)-\ln(3x+1)}\right]$

$\frac{1}{5}\left(\csc^2(3x)-\ln(3x+1)\right)^{-4/5} * \left(2\csc(3x) * -\csc(3x)\cot(3x)*3\right.$

$- \left(\frac{3}{3x+1}\right)$

⚠ $f(x) = -\sin^2(x) + (\sin 2)x$, $f'(\frac{\pi}{4})$?

① $f'(x) = -2\sin x\cos x + \sin 2$

$= -\sin(2x) + \sin 2$

② $f'(\frac{\pi}{4}) = -1 + \sin 2$

$\triangle$ * the slope of $f(x) = 3\tan x - 2\csc x$ at $x = \frac{\pi}{3}$ is:-

$f'(x) = 3\sec^2 x + 2\csc x \cancel{*}\cot x$

$f'(\frac{\pi}{3}) = \frac{3}{\cos^2(\frac{\pi}{3})} + 2 * \frac{1}{\sin(\frac{\pi}{3})} * \frac{\cos(\frac{\pi}{3})}{\sin(\frac{\pi}{3})} = \boxed{16}$

*Hint

* دائماً لما تجيني سؤال فيه Log أحوله $\frac{\ln}{\ln}$ ~~بعض الأشكال~~ لما تكون الأساس اقتران

$\triangle$ * $f(x) = \text{Log}_{(x+3)}(x+1)$, $f'(0) = ?$

$= \frac{\ln(x+1)}{\ln(x+3)}$

$f'(x) = \frac{\ln(x+3) * \frac{-1}{x+1} - \ln(x+1) * \frac{1}{x+3}}{(\ln(x+3))^2}$

$f'(0) = \frac{\ln(3) * 1 - 0}{(\ln 3)^2} = 1/\ln(3)$. #

$\triangle$ * $\frac{d}{dx}\left[\ln\left(\frac{\sqrt{x+1}\,\cdot x^5}{(7+x)^2}\right)^3\right] =$

* لما يكون ln جواه أشياء كثير بستخدم خصائص ال ln بعدين بشتقه (بنزل الأسس)

/1\ $y = 3\left[\ln(x+1)^{\frac{1}{2}} + \cancel{5}\ln x^5 - \ln(7+x)^3\right]$

$= 3\left[\frac{1}{2}\ln(x+1) + 5\ln x - 3\ln(7+x)\right]$

/2\ $y' = 3\left[\frac{1}{2} \cdot \frac{1}{x+1} + 5 \cdot \frac{1}{x} - 3 \cdot \frac{1}{7+x}\right]$

*اقتران قوة اقتران
① بفرض
② بناخذ ln

* $\frac{d}{dx}\left[\sqrt[3]{e^{6x} \cdot x^3}\right] = \frac{d}{dx}\left[(e^{6x} \cdot x^3)^{\frac{1}{3}}\right]$

$= \frac{d}{dx}\left[e^{2x} \cdot x\right] = e^{2x} \cdot 1 + 2e^{2x} \cdot x$

* $\frac{d}{dx}\left[\pi^{3x+1}\right] = \pi^{3x+1} \cdot 3 \cdot \ln \pi$

* $\frac{d}{dx}\left[x^{\sin x}\right]$

① $y = x^{\sin x}$

② $\ln y = \sin x \ln x$

③ $\frac{y'}{y} = \sin x \cdot \frac{1}{x} + \ln x \cdot \cos x$

يوجد اشتقاق ضمني

$y' = y * \left[\frac{\sin x}{x} + \ln x \cdot \cos^{x}\right]$

$= x^{\sin x}\left[\ 〃 \quad 〃 \quad 〃\ \right]$

* if $f(x) = \cos(\log_2 x)$ then $f'(x)$

$= \cos\left(\frac{\ln x}{\ln 2}\right)$

$f'(x) = \sin(\log_2 x) * \frac{1}{\ln 2} * \frac{1}{x}$

* the slope of the tangent line to the curve $2x^2y + y^2 + \cos(\frac{\pi}{2}x) = 3$ at $(1,1)$ is?

$2x^2 \cdot y' + y * 4x + 2y \cdot y' - \frac{\pi}{2}\sin(\frac{\pi}{2}x) = 0$

$2y' + 4 + 2y' - \frac{\pi}{2} = 0$

$4y' = \frac{\pi}{2} - 4$

$y' = \frac{\pi}{8} - 1$ #

x	$f(x)$	$f'(x)$
1	3	0
2	1	4
3	5	6

x	$g(x)$	$g'(x)$
1	2	7
2	3	8
3	1	0

/1\ $h(x) = f(g(x))$ find $h'(1)$

$h'(x) = f'(g(x)) * g'(x)$

$h'(1) = f'(2) * g'(1) = 4*7 = 28$

/2\ $h(x) = \sqrt{3 + f(x)}$

$h'(2) = \dfrac{f'(2)}{2\sqrt{3 + f(2)}} = \dfrac{4}{2*2} = 1$

/3\ $h(x) = g(x) \cdot \cos\left(\frac{\pi}{4} x\right)$

$h'(2) = g(2) * -\frac{\pi}{4} \sin\left(\frac{\pi}{4} * 2\right) + \cos\left(\frac{\pi}{4} * 2\right) * g'(2)$

$= 3 * -\frac{\pi}{4} * 1 + 0$

/4\ $h(x) = \dfrac{4^x}{f(x)}$, $h'(1)$

$h'(x) = \dfrac{f(x) * 4^x \cdot \ln 4 - 4^x \cdot f'(x)}{(f(x))^2}$

$h'(1) = \dfrac{3*4*\ln 4 - 4*0}{9} = \dfrac{4 \ln 4}{3}$

/5\ $h(x) = x^{g(x)}$, $h'(3)$?

$y = x^{g(x)}$

$\ln y = g(x) \cdot \ln x$

$\dfrac{y'}{y} = \dfrac{g(x)}{x} + \ln x \cdot g'(x) \Rightarrow y' = y[\quad]$

$h'(3) = h(3) * \left[\dfrac{g(3)}{3} + \ln 3 * g'(3)\right]$

$= 3^{g(3)} * [*] = 1$

△* $f(x) = \cos^{-1}(\tan 2x)$

$$f'(x) = \frac{-2\sec^2 2x}{\sqrt{1-\tan^2(2x)}}$$

△* $f(x) = e^{2\tan^{-1}x}$, $f'(0) = ?$

$$f'(x) = e^{2\tan^{-1}x} * \frac{2}{1+x^2} \Rightarrow f'(0) = e^0 * \frac{2}{1+0} = 2$$

△* $$\frac{d}{dx}\left[3^{\cot^{-1}(5x)}\right] = 3^{\cot^{-1}(5x)} * \frac{-5}{1+25x^2} * \ln 3$$

△* $$\frac{d}{dx}\left[\sec^{-1}(\ln x) + \sin^{-1}(\cos(5x))\right]$$

$$= \frac{1/x}{|\ln x|\sqrt{(\ln x)^2-1}} + \frac{-5\sin 5x}{\sqrt{1-(\cos^2(5x))}}$$

△* $x^3 + x\tan^{-1}y = e^y$ Find $\frac{dy}{dx}$

$$3x^2 + \left[\frac{x\cdot y'}{1+y^2} + \tan^{-1}y\right] = e^y\cdot y'$$

$$\frac{x}{1+y^2}y' - e^y\cdot y' = -3x^2 - \tan^{-1}y$$

$$y'\left[\frac{x}{1+y^2} - e^y\right] = -3x^2 - \tan^{-1}y$$

$$y' = \frac{-3x^2-\tan^{-1}y}{\frac{x}{1+y^2} - e^y}$$

* note that:-

If the tangent is horizantal then the slope = 0

⚠ Let $f(x) = \ln(x - 4x^2)$, then $f(x)$ has a horizantal tangent line at $x = ?$

$$f'(x) = \frac{1-8x}{x-4x^2} = 0 \Rightarrow 1-8x = 0$$

$$x = 1/8$$

⚠ the points of the tangent line to the curve $x^2+y^2=4$ are horizantal are :-

$$2x + 2y \cdot y' = 0$$

$$y' = \frac{-x}{y} \quad \therefore y' = 0 \Rightarrow x = 0$$

$$0 + y^2 = 4 \Rightarrow y = \pm 2 \qquad (0,2),(0,-2)$$

⚠ Horizantal tangent line $f(x) = \sec x$

$$f'(x) = \sec x \tan x = \frac{\sin x}{\cos^2 x}$$

$$\sin x = 0$$

$$x = 0 + n\pi \ , \ n \in Z$$

integer no

(أعداد صحيحة)

$\triangle\!\!*\;\; f(x) = 3x^4 + 2$

$f^5(0) = ??$

$f'(x) = 12x^3 \quad , \quad f''(x) = 36x^2$

$f^{(3)}(x) = 72x \quad , \quad f^4(x) = 72x \quad , \quad f^{(5)}(x) = 0$

$f^5(0) = 0$

H.w

$f(x) = 3^x \quad , \quad f^{10}(0) = ??$

$f'(x) = 3^x \cdot \ln(3)$

$f''(x) = 3^x (\ln 3)^2 \quad ---- \quad f^{10}(0) = (\ln 3)^{10}$

⊛ the slope of tangent line at $x_0 = f'(x)$ $\quad y_0 \rightarrow$ هي صورة x_0

⊛ the slope of the normal (perpendicular) tangent line at $x_0 = \dfrac{-1}{f'(x_0)}$.

⊛ the equ of the tangent line at x_0 : ?

$$y - y_0 = m(x - x_0)$$

$\swarrow f(x_0 \quad\quad \swarrow f'(x_0) \quad\quad \swarrow$ معطى

⚠ find the equ of the tangent line for:-

$x^2y^2 - 2x = 4 - 4y$ at $(2,-2)$

1) $x_o = 2$

2) $y_o = -2$

3) $m = ?$

$x^2 \cdot 2y \cdot y' + y^2 \cdot 2x - 2 = -4y'$

$(4)(-4)y' + 4(4) - 2 = -4y'$

$-16y' + 14 = 4y'$

$y' = \frac{14}{12} = m$

4) $y+2 = \frac{14}{12}(x-2)$

⚠ find the equ of the normal tangent line for $f(x) = xe^{-x}$ at $x=0$?

$x_o = 0$

$y_o = f(0) = 0 \cdot 1 = 0$

slope $= f'(x) = -xe^{-x} + e^{-x}$

$\searrow f'(0) = 1$

slope of normal $= \frac{-1}{1}$

$y = -x$

* $f(x) = \begin{cases} x^2-1, & x \geq 1 \\ kx-k, & x<1 \end{cases}$

find the valus of k ∋ f(x) cont but not diffble ?

cont $\longrightarrow \lim_{x\to 1^+} = \lim_{x\to 1^-}$

$0=0$ ∴ cont on R

$f'(x) = \begin{cases} 2x & , x>1 \\ k & , x<1 \end{cases}$

$f'(1)^+ \neq f'(1)^-$

$2 \neq k$

$R-\{2\}$

* Maximum and Minimum value :-

*ffinl

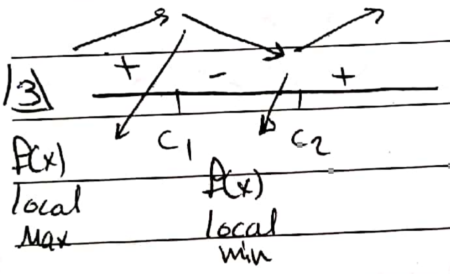
Local محلية
absluote مطلقة

/1\ Dom $\longleftrightarrow$

/2\ critical no x
point (x,y)
$\overrightarrow{f(x)}$

$f'(x) = 0$
= d.n.e ∈ Domain

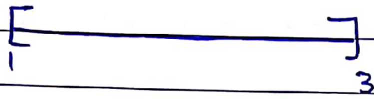
/3\



$f(x) > 0$ inc
$f(x) < 0$ dec
$f(x) = 0$ constant

* $f(x) = x^3 - 3x^2 + 1$, [1,3]

/1\



/2\ crtical

$f'(x) = 0 \longrightarrow 3x^2 - 6x = 0$

$x = 0$ ∉ Dom

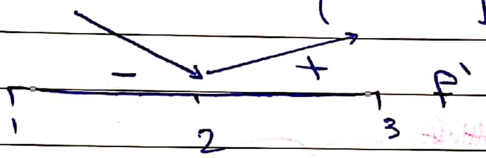
$x = 2$ ∈ Dom

$f'(x)$ d.n.e

$x = 1$, $x = 3$

crtical no = {1, 2, 3}

فترات التزايد والتناقص أكتبها دائماً مفتوحة



increezing (2,3)

dec (1,2)

*extermum valu :-

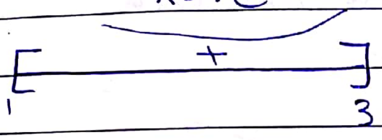
(2, f(2)) abs Mm

f(1) = −1 , f(3) = 1

(3, f(x)) abs Max

$f''(x) = 6x-6 = 0$

$x = 1 \in$



concave up (1,3)
down x
inflection pt x

*relativ. معناها محلي
local نفس

* $f(x) = x^3 - 6x^2 + 1$

/1\ dom : R

/2\ $f'(x) = 3x^2 - 12x$

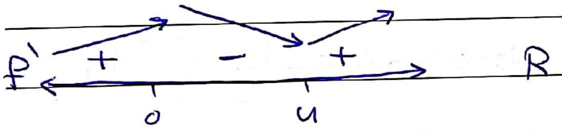
$f'(x) = 0 \Rightarrow 3x^2 - 12x = 0 \Rightarrow 3x(x-4) = 0$

$x=0 \in$ $\quad$ $x=4 \in$

$\therefore$ critical no $x \in \{0, 4\}$

*Hint

فترات التزايد والتناقص
دائمًا مفتوحة

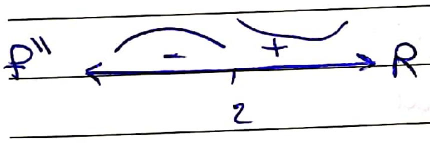


inc $(-\infty, 0), (4, \infty)$

dec $(0, 4)$

/3\ $f''(x) = 6x - 12 = 0$

$x = 2$



concave down $(-\infty, 2)$
" up $(2, \infty)$
$(2, f(2))$ inflection point

$(0, f(0))$ local max
$(4, f(4))$ local min

* if $f(x) = ax^3+3x^2$, has inflection point at $x=1$ then $a=??$

$f'(x) = 3ax^2+6x$

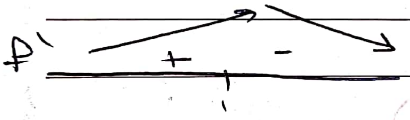
$f''(x) = 6ax+6 \Longrightarrow f''(1) = 0 \Rightarrow 6a+6=0 \therefore \boxed{a=-1}$

* $f(x) = xe^{-x}$

① Dom $= \mathbb{R}$

② $f'(x) = -xe^{-x}+e^{-x} \Rightarrow e^{-x}(-x+1) = 0$

$\neq 0$ $\boxed{x=1}$ critical no

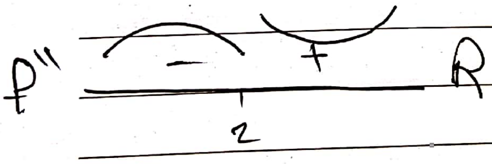


inc $(-\infty,1)$, dec $(1,\infty)$

$(1,f(1))$ abs Max & Local

③ $f''(x) = (e^{-x})(-1)+(-x+1)(-e^{-x})$

$= e^{-x}(-1+x-1) \Rightarrow e^{-x}(x-2) = 0$



* concave down $(-\infty,2)$
* concave up $(2,\infty)$

$(2,f(2))$ inflection pt.

* L'H Rule :-

$$\lim_{x\to a}\frac{f(x)}{g(x)} \quad \frac{0}{0} \quad \frac{\infty}{\infty} \quad \frac{-\infty}{-\infty} \quad \frac{\infty}{-\infty} \Rightarrow \lim_{x\to a}\frac{f'(x)}{g'(x)}$$

$$\triangle\!\!* \lim_{x\to\infty}\frac{\ln(3x)}{\ln(2x)} = \frac{\infty}{\infty} \Rightarrow L'H \quad \frac{\lim_{x\to\infty}\frac{3}{3x}}{\lim_{x\to\infty}\frac{2}{2x}}$$

(رتب ثم عوّض)

$$= \lim_{x\to\infty}\frac{x}{x} = 1$$

$$\triangle\!\!* \lim_{x\to 0}\frac{1-\cos(4x)}{2x^2} = \frac{0}{0} \Rightarrow L'H \quad \lim_{x\to 0}\frac{4\sin 4x}{4x} = 4$$

$$\triangle\!\!* \lim_{x\to 0^+}\frac{\ln(\sin x)}{\ln(\tan x)} = \frac{-\infty}{-\infty} \Rightarrow L'H \quad \lim_{x\to 0^+}\frac{\frac{\cos x}{\sin x}}{\frac{\sec^2 x}{\tan x}} = \lim_{x\to 0^+}\frac{\cos x}{\sin x} * \frac{\sin x}{\cos x} * \cos^2 = 1$$

$$\triangle\!\!* \lim_{x\to 0^+}\frac{1-\ln x}{e^{1/x}} = \frac{\infty}{\infty} \Rightarrow \lim \frac{\frac{-1}{x}}{e^{1/x}\cdot\frac{-1}{x^2}} = \lim_{x\to 0^+}\frac{x}{e^{1/x}} = \frac{0}{\infty} = 0$$

$$\triangle\!\!* \lim_{x\to 0}\frac{5x}{\tan^{-1}x} = \frac{0}{0} \Rightarrow \lim_{x\to 0}\frac{5}{\frac{1}{1+x^2}} = 5$$

$$\triangle\!\!* \lim_{x\to 0^+} \csc - \frac{1}{x} \quad \infty - \infty$$

* Hint
$(\infty - \infty)$
نوحد مقامات

$$= \lim_{x\to 0^+}\frac{x-\sin x}{x\sin x} = \frac{0}{0} \Rightarrow L'H \quad \lim_{x\to 0^+}\frac{1-\cos x}{x\cos x + \sin x} = \frac{0}{0}$$

$$\lim_{x\to 0^+}\frac{\sin x}{-x\sin x + \cos x + \cos x} = \frac{0}{2} = 0$$

$$\triangle\!\!* \lim_{x\to 0^+}\frac{1}{\ln(x+1)} - \frac{1}{x} = \infty - \infty \longrightarrow$$

نوحد مقامات

$$= \lim_{x\to 0^+}\frac{x-\ln(1+x)}{x\ln(x+1)} = \frac{0}{0} \Rightarrow L'H \quad \lim_{x\to 0^+}\frac{1-\frac{1}{1+x}}{x\cdot\frac{1}{1+x} + \ln(1+x)}$$

$$= \lim_{x\to 0^+}\frac{\frac{1+x-1}{\cancel{1+x}}}{\frac{x+(1+x)\ln(1+x)}{\cancel{1+x}}} = \lim_{x\to 0^+}\frac{x}{x+(1+x)\ln(1+x)} = \frac{0}{0}$$

$$L'H \quad \lim_{x\to 0^+}\frac{1}{1+(1+x)\cdot\frac{1}{1+x}+\ln(1+x)} = \frac{1}{2}$$

⚠* $\lim_{x\to\infty} \ln(3x+1) - \ln(5x+1) = \infty - \infty$ → ع خصائص ln

$= \lim_{x\to\infty} \ln\left(\frac{3x+1}{5x+1}\right) = \ln\left(\frac{3}{5}\right)$ #

⚠* $\lim_{x\to\infty} 2x - \ln(5+3e^{2x})$ $\infty - \infty$

$= \lim_{x\to\infty} \ln e^{2x} - \ln(5+3e^{2x}) = \lim_{x\to\infty} \ln\left(\frac{e^{2x}}{5+3e^{2x}}\right)$

$\ln \lim_{x\to\infty} \left(\frac{e^{2x}}{5+3e^{2x}}\right)$ $\frac{\infty}{\infty}$ $\Rightarrow \ln\left(\lim_{x\to\infty} \frac{2e^{2x}}{6e^{2x}}\right) = \lim \frac{1}{3}$ #

* If $\lim_{x\to 0} f(x) \cdot g(x) = \infty \cdot 0$

$= \lim_{x\to\infty} \frac{f(x)}{1/g(x)} = \frac{\infty}{\infty}$ OR $\lim_{x\to 0} \frac{g(x)}{1/f(x)} = \frac{0}{0}$

⚠* $\lim_{x\to\infty} xe^{-x} = \infty \cdot 0$

* Hint

إذا طلع بالتعويض $(\infty.0)$
تقلب واحد من الاقترانات
عشان أقدر استخدم لوبيتال

$\lim_{x\to\infty} \frac{x}{e^{x}} = \frac{\infty}{\infty} \Rightarrow$ L'H $\lim_{x\to\infty} \frac{1}{e^{x}} = 0$

⚠* $\lim_{x\to\infty} x \sin(\pi/x) = \infty \cdot 0 \Rightarrow \lim_{x\to\infty} \frac{\sin(\pi/x)}{1/x} = \frac{0}{0}$

L'H $\lim_{x\to\infty} \frac{\frac{-\pi}{x^2}\cos(\pi/x)}{\frac{-1}{x^2}} = \lim_{x\to\infty} \pi \cos\left(\frac{\pi}{x}\right) = \pi$ #

$* \text{ If } \lim f(x)^{g(x)} = 0^0 , \infty^0 , 1^\infty$

نفرض → $y = f(x)^{g(x)} = 0^0 , \infty^0 , 1^\infty$

ناخذ ln → $\ln y = g(x) \ln f(x) = \infty \cdot 0 , \frac{\infty}{\infty} , \frac{0}{\infty}$

⚠ $\lim_{x \to 0^+} (1 + \sin x)^{1/x} = 1^\infty$

① $y = (1 + \sin x)^{1/x}$

② $\ln y = \frac{1}{x} \ln(1 + \sin x)$

③ $\lim_{x \to 0^+} \ln y = \lim_{x \to 0^+} \frac{1}{x} \ln(1 + \sin x) \Rightarrow \ln y = \lim_{x \to 0^+} \frac{\ln(1 + \sin x)}{x} = \frac{0}{0}$

$L'H \quad \ln y = \lim_{x \to 0^+} \frac{\frac{\cos x}{1 + \sin x}}{1} \Rightarrow \underset{e}{\ln y} = \underset{e}{1} \quad \therefore y = e$

⚠ $\lim_{x \to 0^+} (e^{2x} - 1)^x = 0^0$

$\ln y = x \ln(e^{2x} - 1) \Rightarrow \ln y = \lim_{x \to 0^+} x \ln(e^{2x} - 1) = 0 \cdot \infty$

$\ln y = \lim_{x \to 0^+} \frac{\ln(e^{2x} - 1)}{1/x} = \frac{-\infty}{\infty}$

$L'H \quad \ln y = \lim_{x \to 0^+} \frac{\frac{2e^{2x}}{e^{2x-1}}}{\frac{-1}{x^2}} = \lim_{x \to 0^+} \frac{-2x^2 e^{2x}}{e^{2x} - 1} = \frac{0}{0}$

$L'H \quad \ln y = \lim_{x \to 0^+} \frac{-2x^2 * 2e^{2x} - e^{2x} * -4x}{2e^{2x}} \Rightarrow \underset{e}{\ln y} = \underset{e}{0} \quad \therefore y = 1$

⚠ $\lim_{x \to \infty} x^{\frac{1}{\ln x}} = \infty^0$

$y = x^{-1/\ln x} \Rightarrow \ln y = \frac{-1}{\ln x} \cdot \ln x \Rightarrow \ln y = \lim_{x \to \infty} \frac{-1}{\ln x} \cdot \ln x$

$\underset{e}{\ln y} = \underset{e}{-1} \Rightarrow y = e^{-1}$

$\triangle\!\!*\ \lim_{x\to\infty} (\ln x)^{1/x} = \infty^0$

$y = (\ln x)^{1/x} \Rightarrow \ln y = \frac{1}{x} \ln(\ln x) = \lim_{x\to\infty} \ln y = \lim_{x\to\infty} \frac{\ln(\ln x)}{x} = \frac{\infty}{\infty}$

$L'H \quad \ln y = \lim_{x\to\infty} \frac{\frac{1/x}{\ln x}}{1} \Longrightarrow \ln y = \lim_{x\to\infty} \frac{1}{x \ln x} = 0$

$y = e^0 = 1$

قاعدة

$$\boxed{\lim_{x\to\infty} \left(1 + \frac{n}{x}\right)^x = e^n}$$

$\triangle\!\!*\ \lim_{x\to\infty} \left(1 + \frac{3}{x}\right)^x = e^3$

$\triangle\!\!*\ \lim_{x\to 0^+} (1 + x)^{1/x} \qquad \begin{matrix} 1/x = y \\ x\to 0^+ \\ y\to\infty \end{matrix}$

$\therefore \lim_{y\to\infty} \left(1 + \frac{1}{y}\right)^y = e^1$

$\triangle\!\!*\ \lim_{x\to\infty} \left(1 - \frac{2}{x}\right)^{8x} = \lim_{x\to\infty} \left(\left(1 - \frac{2}{x}\right)^x\right)^8$

$(e^{-2})^8 = e^{-16}$

$\triangle\!\!*\ \lim_{x\to\infty} \left(1 + \frac{1}{4x}\right)^{5x} = \lim_{x\to\infty} \left(\left(1 + \frac{1/4}{x}\right)^x\right)^5 = e^{5/4}$

$\triangle\!\!*\ \lim_{x\to\infty} \left(\frac{x+1}{x-1}\right)^x \Rightarrow \lim_{x\to\infty} \left(\frac{x(1 + \frac{1}{x})}{x(1 - \frac{1}{x})}\right)^x = \frac{e^1}{e^{-1}} = e^2$

H.w

$*\ \lim_{x\to 0^+} x^{\sin x} \qquad 0^0$

$y = x^{\sin x} \longrightarrow \ln y = \sin x \ln x \Longrightarrow \lim_{x\to 0^+} \ln y = \lim_{x\to 0^+} \sin x \ln x = 0\cdot\infty$

$\ln y = \lim_{x\to 0^+} \frac{\ln x}{1/\sin x} \quad \frac{\infty}{\infty}$

$L'H \quad \ln y = \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1 \cos x}{(\sin x)^2}} \Longrightarrow \ln y = \lim_{x\to 0^+}$

Maysam Abu-Dalo

6.9 Hyperbolic Functions

7.8.1 DEFINITION.

Hyperbolic sine $\sinh x = \dfrac{e^x - e^{-x}}{2}$

Hyperbolic cosine $\cosh x = \dfrac{e^x + e^{-x}}{2}$

مهم
جدا

cosh x + sinh = e

Hyperbolic tangent $\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

مقلوب
tanh

Hyperbolic cotangent $\coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

Hyperbolic secant $\operatorname{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}}$

Hyperbolic cosecant $\operatorname{csch} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}}$

7.8.2 THEOREM.

مهمين

$\cosh x + \sinh x = e^x$

$\cosh x - \sinh x = e^{-x}$

$\cosh^2 x - \sinh^2 x = 1$

$1 - \tanh^2 x = \operatorname{sech}^2 x$

$\coth^2 x - 1 = \operatorname{csch}^2 x$

$\cosh(-x) = \cosh x$

$\sinh(-x) = -\sinh x$

$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$

$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$

$\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$

$\cosh(x - y) = \cosh x \cosh y - \sinh x \sinh y$

$\sinh 2x = 2 \sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\cosh 2x = 2\sinh^2 x + 1 = 2\cosh^2 x - 1$

even

* حكم ال limit مهم

$\lim_{x\to\infty} \cosh x = \infty$

$x \to -\infty = \infty$

sinh (+)
0 < x / Dom لما

لما sinh (-)
Dom x < 0

~~origin~~ متماثل حول

بعرف إنه (-1,1)
عن طريق اختبار ال
الأفقي

6.9 Hyperbolic Functions

Maysam Abu-Dalo

even

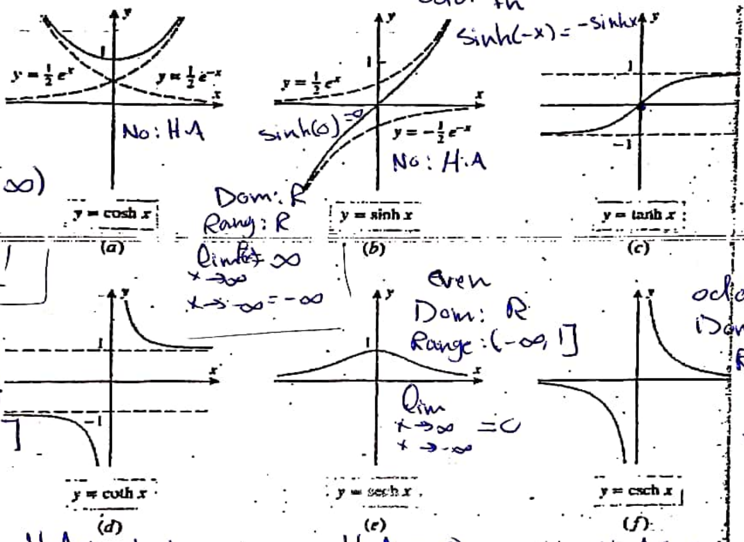
اما cosh متماثل
حول ال y-axis
يكون even

Dom : R
Range : [1,∞)

Range :-
دائماً | cosh x ≥ 1 |

odd fn
sinh(-x) = -sinh x
sinh(0) = 0
No : H.A
Dom : R
Rang : R
$\lim_{x\to\infty} = \infty$
$x \to -\infty = -\infty$

Dom : R
Range : (-1,1)
tanh(0) = 0
odd
$\lim_{x\to\infty} = 1$
$x \to -\infty = -1$
* H.A : 1, -1



coth x odd
dom : R - {0}
Rang : R - [-1,1]
$\lim_{x\to\infty} = 1$
$\lim_{x\to-\infty} = -1$
H.A : -1, 1

Even
Dom : R
Range : (-∞, 1]
$\lim_{x\to\infty} = 0$
$x \to -\infty$
H.A = 0

odd
Dom R - {0}
Range R - {0}
$\lim_{x\to\infty} = 0$
$\lim_{x\to-\infty} = 0$
H.A : 0
V.A = 0

* ممكن كرابعة
يوجدها H-A
انها تقترب من
-∞ , ∞

7.8.3 THEOREM.

عكسهم
موجب

$$\frac{d}{dx}[\sinh u] = \cosh u \frac{du}{dx} \qquad \int \cosh u\, du = \sinh u + C$$

$$\frac{d}{dx}[\cosh u] = \sinh u \frac{du}{dx} \qquad \int \sinh u\, du = \cosh u + C$$

$$\frac{d}{dx}[\tanh u] = \operatorname{sech}^2 u \frac{du}{dx} \qquad \int \operatorname{sech}^2 u\, du = \tanh u + C$$

$$\frac{d}{dx}[\coth u] = -\operatorname{csch}^2 u \frac{du}{dx} \qquad \int \operatorname{csch}^2 u\, du = -\coth u + C$$

$$\frac{d}{dx}[\operatorname{sech} u] = -\operatorname{sech} u \tanh u \frac{du}{dx} \qquad \int \operatorname{sech} u \tanh u\, du = -\operatorname{sech} u +$$

$$\frac{d}{dx}[\operatorname{csch} u] = -\operatorname{csch} u \coth u \frac{du}{dx} \qquad \int \operatorname{csch} u \coth u\, du = -\operatorname{csch} u +$$

نفس مشتقة
ال sin
و cos
بس بحوطهم
سالب

(٨)

⚠* find the value

$$\cosh(2\ln x) = \cosh(\ln x^2) = \frac{e^{\ln x^2} + e^{-\ln x^2}}{2} = x^2 + \frac{1}{x^2}/2$$

Hint

* الفرق بينهم وبين sin/cos ---

إنه حصول جايين من قطع زائد

(sinh / cosh ---)

إنما (cos / sin ---) جايين

من دائرة

⚠* Find the value of :-

$$\sinh(\ln x - 3\ln x^2) = \sinh\left(\ln \frac{x}{x^6}\right)$$

$$= \sinh(\ln x^{-5}) = \frac{e^{\ln x^{-5}} - e^{-\ln x^{-5}}}{2} = \frac{1}{x^2} - x^5/2$$

⚠* Solve for x

/1\ $\cosh x - \sinh = 6 \Rightarrow e^{-x} = 6 \Rightarrow -x = \ln 6$

$x = \ln 1/6$

/2\ $\sinh = \frac{e^x}{2} - 2 \Rightarrow \frac{e^x - e^{-x}}{2} - \frac{e^x}{2} + 2 = 0 \Rightarrow -\frac{e^{-x}}{2}\overrightarrow{+2=0}$ ---

⚠* $\frac{d}{dx}[\cosh \ln x] = \frac{\sinh \ln x}{x}$

⚠* $\frac{d}{dx}[3^{\tanh x}] = 3^{\tanh x} . \ln 3 . \operatorname{sech}^2 x$

⚠* $\frac{d}{dx}[\tanh(\tan^{-1} 3x)] = \operatorname{sech}^2(\tan^{-1}(3x)) * \frac{3}{1+9x^2}$

⚠* $\frac{d}{dx}[\sqrt{\coth x + \operatorname{sech} x}] = \frac{-\operatorname{csh}^2 - \operatorname{sech} x \tanh x}{2\sqrt{\coth x + \operatorname{sech} x}}$

⚠* $\lim_{x\to\infty} \tan^{-1}(\tanh x) \Rightarrow \tan^{-1}(1) = \frac{\pi}{4}$

أهم سؤال

H.w

/1\ $\lim_{x\to\infty}(\tanh x)$ cosh * --

/2\ $\lim_{x\to\infty} \frac{\sinh x}{e^x} = \frac{\infty}{\infty} \Rightarrow -\lim_{x\to\infty} \frac{e^x - e^{-x}}{2e^x}$

$\triangle$ * given that $\text{csh}\,x = \frac{-1}{3}$

Hint * بيهمني أعرف $\frac{\sinh x}{\cosh x}$ عشان اطلع الباقي

Find ① $\sinh x$
② $\cosh x$
③ $\tanh x$
④ $\coth x$
⑤ $\text{sech}\,x$

Sol: ① $\text{csch}\ x = \frac{-1}{3} \Rightarrow \sinh x = -3$

② $\cosh^2 - \sinh^2 = 1$

$\cosh^2 x = 1 + 9 \Rightarrow \cosh^2 x = 10 \Rightarrow \cosh x = \pm\sqrt{10}$

③ $\tanh x = \frac{-3}{\sqrt{10}}$ ④ $\coth x = \frac{-\sqrt{10}}{3}$ ⑤ $\text{sech}\,x = \frac{1}{\sqrt{10}}$

* السالبة تهمل لأنه $\cosh \geq 1$
$\cosh x = \sqrt{10}$

$\triangle$ * given that

$\cosh x = \frac{5}{4}$, $x < 0$ / Find $\sinh x$, $\tanh x$

$\cosh^2 x - \sinh^2 x = 1 \Rightarrow \sinh x = \pm \frac{3}{4}$

$\tanh = \frac{-3/4}{5/4} = -3/5$

$x > 0$
قيمة sinh موجبة

$x < 0$
قيمة sinh سالبة

⊛ Integration:-

$\triangle$ * $\int 3x - x^3 + 5 \,.\, dx = \frac{3x^2}{2} - \frac{x^4}{4} + 5x + c$

$\triangle$ * $\int 2x(3x-1)^2 \,.\, dx = \int 2x(9x^2 - 6x + 1) = \int 18x^3 - 12x^2 + 2x$

$= \frac{18x^4}{4} - \frac{12x^3}{3} + \frac{2x^2}{2} + c$

$\triangle$ * $\int \frac{5x^3+4}{\sqrt{x}}\, dx = \int x^{-1/2}(5\ x^3 + 4) = \int 5x^{5/2} + 4x^{-1/2} = \frac{5x^{7/2} * 2}{7} + \frac{4x^{1/2} * 2}{1} + c$

$\triangle$ * $\int \cos(7x+5) \,.\, dx = \frac{\sin(7x+5)}{7} + c$

$\triangle$ * $\int \sec(2x)(\sec(2x) + \tan(2x))\, dx$

$= \int \sec^2(2x) + \sec(2x)\tan(2x)\, dx = \frac{\tan 2x}{2} + \frac{\sec(2x)}{2} + c$

△ $\int \frac{\sin(4x)}{\cos^2(4x)} dx = \int \frac{\sin 4x}{\cos 4x} \cdot \frac{1}{\cos 4x} = \int \tan 4x \cdot \sec 4x = \frac{\sec 4x}{4} + c$

△ $\int \frac{\sin 2x}{\cos x} dx = \int \frac{2\sin x \cos x}{\cos x} = -2\cos x + c$

△ $\int 1 + \sin^2 x \underbrace{\csc x}_{\frac{1}{\sin x}} dx = \int 1 + \sin x = x - \cos x + c$

△ $\int \sin 3x \cot 3x \; dx = \int \sin 3x \cdot \frac{\cos 3x}{\sin 3x} = \frac{\sin 3x}{3} + c$

△ $\int (\sin x + \cos x)^2 = \int \sin^2 x + 2\sin x \cos x + \cos^2 x = \int 1 + \sin 2x = x - \frac{\cos 2x}{2} + c$

△ $\int \cos^2(4x) + \tan^2(5x) \, dx = \int (\frac{1}{2} + \frac{1}{2}\cos 8x) + \int \sec^2(5x) - 1 =$

$= \frac{1}{2} x + \frac{1}{2} \frac{\sin 8x}{8} + \frac{\tan 5x}{5} - x + c$

$* \int e^{ax+b} \cdot dx = \frac{e^{ax+b}}{a} + c$

$* \int a^{bx+c} \cdot dx = \frac{a^{bx+c}}{b * \ln a}$

△ $\int 5^{2x-1} \cdot dx = \frac{5^{2x-1}}{2 * \ln 5} + c$ / △ $\int e^{2x-1} \cdot dx = \frac{e^{2x-1}}{2} + c$

تكامل بداخله كسر

Hint

*أول اشي بسأل حالي هل البسط مشتقة للمقام؟

/1\ $\int \frac{f(x)}{g(x)}$ $\frac{\text{poly}}{\text{poly}}$ $\Rightarrow \int \frac{f'(x)}{f(x)} = \ln|f(x)| + c$

/2\ $\int \frac{f'(x)}{a^2 + (f(x))^2} = \frac{1}{a} \tan^{-1}(\frac{f(x)}{a}) + c$

△ $\int \frac{5x}{x^2+1} = \frac{5}{2} \int \frac{2x}{x^2+1} = \frac{5}{2} \ln(x^2+1) + c$

△ $\int \frac{3}{x^2+1} \cdot dx = \int \frac{3}{(x)^2 + (1)^2} = 3\int \frac{1}{x^2+1^2} = 3\tan^{-1} x + c$

△* $\int \frac{\cos(2x)}{7+\sin^2(2x)} \cdot dx = \int \frac{\cos 2x}{(\sqrt{7})^2+(\sin(2x))^2} = \frac{1}{2}\int \frac{2\cos 2x}{(\sqrt{7})^2+(\sin 2x)^2} =$

$\frac{1}{2} * \frac{1}{\sqrt{7}} \tan^{-1}\left(\frac{\sin 2x}{\sqrt{7}}\right)+c$

△* $\int \frac{e^x+3}{e^x} \cdot dx = \int 1+3e^{-x} = x-3e^{-x}+c$

△* $\int \frac{e^x}{e^{2x}+3} = \int \frac{e^x}{(e^x)^2+(\sqrt{3})^2} = \frac{1}{\sqrt{3}} * \tan^{-1}\left(\frac{e^x}{\sqrt{3}}\right)+c$

△* $\int \frac{1}{1+e^x} \cdot dx = \int \left(\frac{1}{e^x(1+e^{-x})}\right) = -\ln|1+e^{-x}|+c$

△* $\int \frac{5x}{3+x^4} \cdot dx = \int \frac{5x}{(\sqrt{3})^2+(x^2)^2} = \frac{5}{2}\int \frac{2x}{(\sqrt{3})^2+(x^2)^2}$

$= \frac{5}{2} * \frac{1}{\sqrt{3}} * \tan^{-1}\left(\frac{x^2}{\sqrt{3}}\right)+c$

△* $\int \frac{1}{x^3+x} \cdot dx = \int \frac{1}{x^3(1+x^{-2})} \cdot dx = \int \frac{x^{-3}}{1+x^{-2}} = -\frac{1}{2}\int \frac{-2x^{-3}}{1+x^{-2}}$

$= -\frac{1}{2} \ln|1+x^{-2}|+c$

△* $\int \frac{1}{x\ln x} \cdot dx = \int \frac{\frac{1}{x}}{\ln x} \cdot dx = \ln|\ln x|+c$

△* $\int \frac{1}{(x^2+1)\tan^{-1}x} = \int \frac{\frac{1}{x^2+1}}{\tan^{-1}x} = \ln|\tan^{-1}x|+c$

قانون (*) $\boxed{\int \frac{f'(x)}{\sqrt{a^2-(f(x))^2}} = \sin^{-1}\left(\frac{f(x)}{a}\right)+c}$

قانون (*) $\boxed{\int \frac{f'(x)}{|f(x)|\sqrt{(f(x))^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{f(x)}{a}\right)+c}$

△* $\int \frac{e^x}{\sqrt{-e^{2x}+4}} \cdot dx = \sin^{-1}\left(\frac{e^x}{2}\right)+c$

△* $\int \frac{2}{\sqrt{3-x^2}} \cdot dx = 2\sin^{-1}\left(\frac{x}{\sqrt{3}}\right)+c$

△* $\frac{1}{2}\int \frac{2\cos 2x}{\sin 2x\sqrt{\sin^2(2x)-1}} = \frac{1}{2}\sec^{-1}\left(\frac{\sin 2x}{1}\right)+c$

7.1 An Overview of Integration Methods

Maysam Abu-Dalo

CONSTANTS, POWERS, EXPONENTIALS

1. $\int du = u + C$ 2. $\int a\,du = a\int du = au + C$

3. $\int u^r\,du = \frac{u^{r+1}}{r+1} + C,\ r \neq -1$ 4. $\int \frac{du}{u} = \ln|u| + C$

5. $\int e^u\,du = e^u + C$ 6. $\int b^u\,du = \frac{b^u}{\ln b} + C,\ b > 0, b \neq 1$

TRIGONOMETRIC FUNCTIONS

7. $\int \sin u\,du = -\cos u + C$ 8. $\int \cos u\,du = \sin u + C$

9. $\int \sec^2 u\,du = \tan u + C$ 10. $\int \csc^2 u\,du = -\cot u + C$

11. $\int \sec u \tan u\,du = \sec u + C$ 12. $\int \csc u \cot u\,du = -\csc u + C$

13. $\int \tan u\,du = -\ln|\cos u| + C$ 14. $\int \cot u\,du = \ln|\sin u| + C$

HYPERBOLIC FUNCTIONS

15. $\int \sinh u\,du = \cosh u + C$ 16. $\int \cosh u\,du = \sinh u + C$

17. $\int \operatorname{sech}^2 u\,du = \tanh u + C$ 18. $\int \operatorname{csch}^2 u\,du = -\coth u + C$

19. $\int \operatorname{sech} u \tanh u\,du = -\operatorname{sech} u + C$ 20. $\int \operatorname{csch} u \coth u\,du = -\operatorname{csch} u + C$

ALGEBRAIC FUNCTIONS ($a > 0$)

21. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1}\frac{u}{a} + C \quad (|u| < a)$

22. $\int \frac{du}{a^2 + u^2} = \frac{1}{a}\tan^{-1}\frac{u}{a} + C$

23. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}\left|\frac{u}{a}\right| + C \quad (0 < a < |u|)$

X 24. $\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{u^2 + a^2}) + C$

25. $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln\left|u + \sqrt{u^2 - a^2}\right| + C \quad (0 < a < |u|)$

26. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a}\ln\left|\frac{a+u}{a-u}\right| + C$

27. $\int \frac{du}{u\sqrt{a^2 - u^2}} = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 - u^2}}{u}\right| + C \quad (0 < |u| < a)$

28. $\int \frac{du}{u\sqrt{a^2 + u^2}} = -\frac{1}{a}\ln\left|\frac{a + \sqrt{a^2 + u^2}}{u}\right| + C$

حاصل ضرب/ قسمة
اقترانات واحد منهم
جزء من الثاني أو
مشتقة الثاني
كل ع التعويض.

H.w

$$\int \frac{1}{1+\sin x} * \frac{1-\sin x}{1-\sin x} \ldots$$

⚠ $$\int_0^{\ln 2} \frac{e^{4x}+3}{e^{2x}}\, dx = \int_0^{\ln 2} e^{2x}+3e^{-2x} \cdot dx = \frac{e^{2x}}{2} + \frac{3e^{-2x}}{-2}\Big|_0^{\ln 2}$$

$$= \left(\frac{e^{2\ln 2}}{2} + \frac{3}{-2} e^{-2\ln 2}\right) - \left(\frac{1}{2} - \frac{3}{2}\right) = \frac{4}{2} + \left(\frac{3}{-2}\right)\left(\frac{1}{4}\right) - (-1)$$

⚠ $$\int \frac{1}{3-x} + \frac{1}{x+5} \cdot dx = -\ln|3-x| + \ln|x+5| = \ln\left|\frac{x+5}{3-x}\right| + c$$

⚠ $$\int 3^{3-2\log_3 x} = \int 3^3 \cdot 3^{-2\log_3 x} = \int 27 \cdot 3^{\log_3 x^{-2}} = 27\int x^{-2} = 27 \cdot \frac{x^{-1}}{-1} + c$$

التحكم ← خصائص ال ln

⚠ $$\int_0^{\ln\sqrt{3}} \frac{e^{-x}}{1+e^{-2x}} \cdot dx = -\int_0^{\ln\sqrt{3}} \frac{-e^{-x}}{1+(e^{-x})^2} = -\tan^{-1}(e^{-x})\Big|_0^{\ln\sqrt{3}}$$

$$= -\tan^{-1}(e^{-\ln\sqrt{3}}) + \tan^{-1}(e^0) = -\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) + \tan^{-1}(1) = \frac{-\pi}{6} + \frac{\pi}{4} = \frac{2}{24}\pi$$

⚠ $$\int_0^{\ln 3} \tanh \cdot dx = \int_0^{\ln 3} \frac{e^x - e^{-x}}{e^x - e^x} \cdot dx = \ln|e^x + e^{-x}|\Big|_0^{\ln 3}$$

⚠ $$\int \sinh(7-5x)\, dx = \frac{\cosh(7-5x)}{-5} + c$$

⚠ $$\int \frac{3e^x}{\cosh x + \sinh x} \cdot dx = \int \frac{3e^x}{e^x} = 3x + C$$

⚠ $$\int \frac{\cosh x^{\sqrt{5}}}{5\sinh^2 x + 2} \cdot dx = \frac{1}{\sqrt{5}} * \frac{1}{\sqrt{2}} * \tan^{-1}\left(\frac{\sqrt{5}\sinh x}{\sqrt{2}}\right) + c$$

$\frac{1}{\sqrt{5}}$

* Integration by substituation :-

- لأنني بطلع عكس
- الاقترانات المثلثية
بتطلع عكسها

$$\int f(g(x)) \cdot g'(x)$$

غير خطي

$$\int f(x) \cdot e^{g(x)}$$

غير خطي

$$\int f(x) \cdot \text{Trig}(g(x))$$

sin
cos
⋮

⚠ $\int \cosh^2 x \,.\, \sinh x \,.\, dx$

$\cosh = u$

$\sinh x \,.\, dx = du$

$I = \int (u)^7 \,.\, \sinh x \,.\, \frac{du}{\sinh x}$

$= \int u^7 du = \frac{u^8}{8} + C = \frac{\cosh^2 x + C}{8}$

⚠ $\int \coth^2 x \,.\, csh^2 x \,.\, dx$

$\coth x = u$

$- csh^2 x \; dx = du$

$\int - u^2 \,.\, du$

⋮

⚠ $\int 3^{\sin x} \,.\, \cos x \,.\, dx$

$I = \int 5^u \,.\, du$

$\sin x = u$

$\cos x \,.\, dx$

$= \frac{3^u}{\ln 3} + C \qquad = \frac{3^{\sin x}}{\ln 3} + C$

⚠ $\int x^2 e^{x^3} . dx$

$x^3 = u \Rightarrow 3x^2 . dx = du$

$= \int x^2 e^u \frac{.du}{3x^2} = \frac{1}{3} e^u + C = R...$

⚠ $\int \frac{\cos^2(\ln x)}{x} . dx$

$\ln x = u \Rightarrow \frac{dx}{x} = du$

$\int \cos^2 u \,.du = \int \frac{1}{2} + \frac{1}{2} \cos 2 \times u \,. du = ...$

⚠ $\int \frac{\sqrt{1 + \tan x}}{\cos^2 x} . dx$

$1 + \tan x = u$

$\sec^2 x \,. dx = du$

$\int \frac{\sqrt{u}}{\cos^2 x} \frac{.du}{\sec^2 x} = \frac{2u^{3/2}}{3} + c = R.$

⚠ $\int (1 + \frac{1}{x})^5 \,.\, \frac{1}{x^2} \,. dx$

$1 + \frac{1}{x} = u$

$\frac{-1}{x^2} \,. dx = du$

$\int (u)^5 . \frac{1}{\cancel{x^2}} . du \times \cancel{- x^2}$

$= \frac{-u^6}{6} + c = R..$

⚠ $\int \frac{e^{-x}}{\sqrt{1 - e^{-x}}} . dx$

$1 - e^{-x} = u$

$e^{-x} dx = du$

$\int \frac{1}{\sqrt{u}} \cdot du = \int u^{-1/2} . du$

⋮

⚠ $\int \frac{e^{-x}}{\sqrt{1-e^{-2x}}} = -\sin^{-1}(e^{-x}) + c$

$e^{-2x} \to (e^{-x})^2$

⚠ $\int \frac{dx}{x\sqrt{1-(\ln x)^2}} = \int \frac{1/x \; dx}{\sqrt{1-(\ln x)^2}} = \sin^{-1}(\ln x) + c$

⚠ $\int \frac{dx}{\sqrt{x}(x+1)} = \int \frac{1/\sqrt{x}}{1+(\sqrt{x})^2} = 2\int \frac{1/2\sqrt{x}}{1+(\sqrt{x})^2} = 2\tan^{-1}(\sqrt{x}) + c$

H.w

① $\int_0^{\pi/2} e^{\sin x} \cos x \, . dx$

② $\int_1^{\sqrt{3}} \frac{\sqrt{\tan^{-1} x}}{1+x^2}$

⚠ $\int_0^1 x\sqrt{1-x} \, . dx$

$1-x=u$
$-dx=du$
$\Big\} \Rightarrow$ $x=0 \Rightarrow u=1$
$x=1 \Rightarrow u=0$

$I = \int_1^0 (1-u)\sqrt{u} \, . -du$

$= \int_0^1 (1-u)u^{1/2} . du = \int_0^1 u^{1/2} - u^{3/2} . du = \frac{2u^{3/2}}{3} - \frac{2u^{5/2}}{5} \Big|_0^1$

$= \left(\frac{2}{3} - \frac{2}{5}\right) - (0) = \frac{4}{15}$

* note that:-

1\ $\int_a^b k f(x) \, dx = k \int_a^b f(x) \, . dx$

2\ $\int_a^b f(x) \pm g(x) . dx = \int_a^b f(x) \pm \int_a^b g(x)$

3\ $\int_a^a f(x) . dx = 0$

4\ $\int_a^b f(x) . dx = -\int_b^a f(x) . dx$

5\ $\int_a^b f(x) . dx = \int_a^{c} f(x) . dx + \int_c^b f(x) . dx$ $\longrightarrow$ لنفس الإقتران

6\ if $f(x)$ odd $\int_{-a}^{a} f(x) . dx = 0$

7\ if $f(x)$ even $\int_{-a}^{a} f(x) . dx = 2\int_0^a f(x) . dx$

⚠ $\int_{5}^{5} \frac{}{\sqrt{x+\sin\sqrt{x}}} .dx = 0$

⚠ $\int_{-3}^{3} \frac{\sin x}{\underbrace{x^4+x^2+1}_{odd}} .dx = 0$

⚠ $\int_{1}^{2} 2f(x) .dx = 4$, $\int_{5}^{1} 3f(x).dx = 12$ Find $\int_{5}^{2} 7f(x) .dx$

$= 7\int_{5}^{2} f(x) .dx = 7 * \left[\int_{5}^{1} f(x) + \int_{1}^{2} f(x)\right] = 7 * \left[\frac{12}{3} + \frac{4}{2}\right] = 42$

⚠ $\int_{-1}^{2} |2x-2| .dx$

$2x - 2 = 0$
$x = 1$

$= \int_{-1}^{1} -(2x-2).dx$

$+ \int_{1}^{2} +(2x-2).dx = ------$

$-$ $+$
-1 1 2

⚠ if $\int_{3}^{2x-1} f(y)\, dy = 5ax^2 - 4$, find a?

$3 = 2x - 1$
$2x = 4 \Rightarrow x = 2$

$\int_{3}^{3} f(y).dy = 5a*4 - 4 \Rightarrow 0 = 20a - 4 \Rightarrow a = \frac{1}{5}$
$20a = 4$

⊛ Fundemntal Thrm of Cal :-

$\frac{d}{dx}\left[\int_{f(x)}^{g(x)} h(t).dt = h(g(x)) * g'(x) - h(f(x)) * f'(x)\right.$

⚠ $\frac{d}{dx}\left[\int_{\cos(3x)}^{7^x} \sin t .dt\right] = \sin(7^x) * 7^x * \ln 7 - \sin(\cos 3x) * -\sin(3x) * 3$

⚠ if $f(x) = x \int_{16}^{x^2} \frac{t}{t^2+1} .dt$, find $f'(4)$?

$f'(x) = x * \left[\frac{x^2}{x^4+1} * 2x - 0\right] + \int_{16}^{x^2} \frac{t}{t^2+1} .dt * 1$

$f'(4) = 4 * \left[\frac{16}{(4)^4+1} * 8\right] + \underbrace{\cancel{\int_{16}^{16} \frac{t}{t^2+1} .dt}}_{zero}$ 16

Hw

$x=2$
$f(2x)=2$

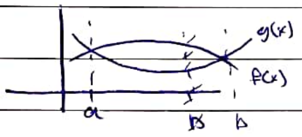
① if $\int_{3}^{2x} f(y)\,.dy = \frac{1}{x^3+3} + c$, find $f(4)$?

② if $f'(x) = x\sqrt{2x^2+4}$, find $f(x)$ when $f(0)=1$

⊛ Area between two curves

if f & g are cont's functions on the interval $[a,b]$ such that $f(x) \geq g(x)$ for all $x \in [a,b]$ then the area of the region bounded above by $f(x)$, below by $g(x)$ on the left $x=a$, on the right $x=b$ is

$$A = \int_{a}^{b} f(x) - g(x)\,.dx$$

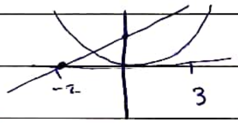


⚠ find the area of the regin that enclosed by $y=x^2$ & $y=x+6$

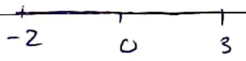
$x^2 = x+6 \Rightarrow x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x=3$ / $x=-2$



$y = x^2 \Rightarrow y\Big|_{x=1} = 1$



$y = x+1 \quad y\Big|_{x=1} = 7$

$\therefore A = \int_{-2}^{3} (x+6) - x^2\,.dx$. - - -

H.w

Area:-

① $y = x^2$, $y = \sqrt{x}$, $x = \frac{1}{4}$, $x = 1$

② $y = e^{2x}$, $y = e^x$, $x = 0$, $x = \ln 2$

③ $y = \cos x$, $y = \sin x$, $x = 0$, $x = \pi/4$

$\cos x = \sin x$

$\tan x = 1$

$x = \pi/4$