

تفاضل وتكامل 1

د. آلاء قدومي

للطالبة المبدعة

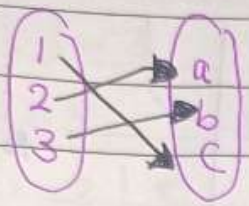
حنان صوان

إرادة - ثقة - تغيير

Subject Calculus (1) Lecture (1) Date No.

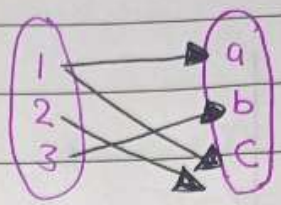
* الاقتران: هو علاقة تربط كل عنصر بالمجال بعنصر واحد فقط بالمدى

1)



function

2)



not function

Domain = مجال = input = x

Range = مدى = output = y

Example:- $f(x) = \sqrt{x+2}$

Find:

1) $f(2)$

$f(2) = \sqrt{2+2} = \sqrt{4} = 2$

2) $f(-4)$

$f(-4) = \sqrt{-4+2} = \sqrt{-2}$ undefined

3) $f(m+3)$

$f(m+3) = \sqrt{m+3+2} = \sqrt{m+5}$

4) Domain f

$f(x) = \sqrt{x+2}$

$x+2 \geq 0$

$x \geq -2$

$[-2, \infty)$

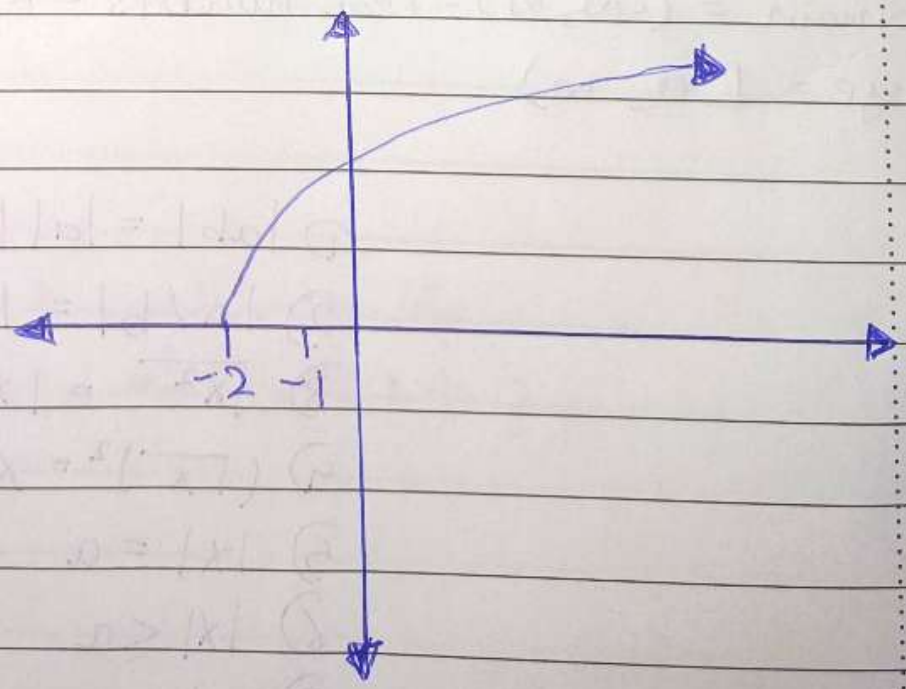
5) Range f

$[0, \infty)$

ملاحظة:-

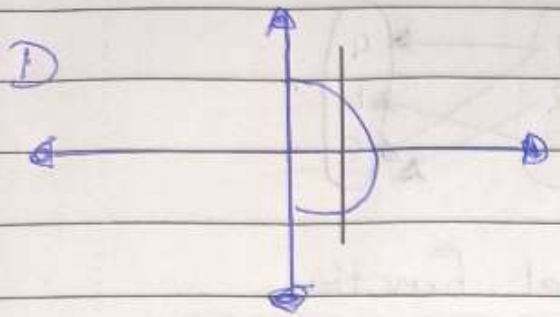
* لإيجاد المجال من الرسم نسقط رسعة الاقتران على محور السينات

* لإيجاد المدى من الرسم نسقط رسعة الاقتران على محور العادات



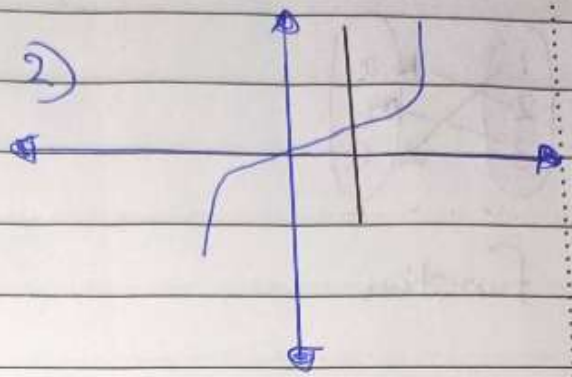
* The vertical line test : اختبار الخط العامودي

1)



not function

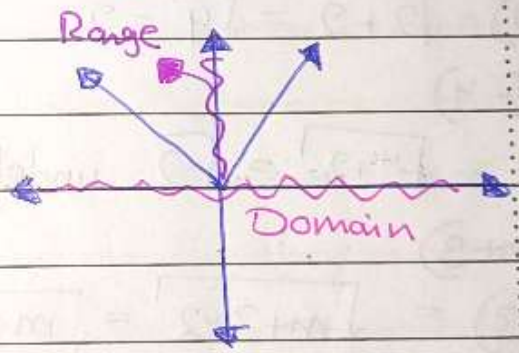
2)



function

* The absdute value function : اقران القيمة المطلقة

$$f(x) = |x| = \begin{cases} -x, & x \le 0 \\ x, & x > 0 \end{cases}$$



$f(0) = |0| = 0$

$f(2) = |2| = 2$

$f(-2) = |-2| = 2$

Domain $= (-\infty, \infty) =$ real numbers $= R$ الأعداد الحقيقية

Range $= [0, \infty)$

خصائص المطلق :-

1) $|ab| = |a||b|$
2) $|a/b| = |a| / |b|$
3) $\sqrt{x^2} = |x|$
4) $(\sqrt{x})^2 = x$
5) $|x| = a$ $\quad x = -a$ or $x = a$
6) $|x| \le a$ $\quad -a \le x \le a$
7) $|x| \ge a$ $\quad x \ge a$ or $x \le -a$

Subject Calculus (1) Lecture (2) Date No.

Examples: find the value of :-

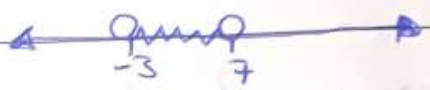
1) $|x| = 3$

$x = 3 \qquad x = -3$

2) $|x-2| < 5$

$-5 < x-2 < 5$

$-3 < x < 7 \qquad x \in (-3, 7)$



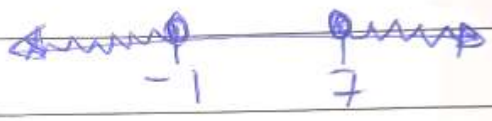
3) $|x-3| \geq 4$

$x-3 \geq 4 \qquad x-3 \leq -4$

$x \geq 7 \qquad x \leq -1$

$x \in (-\infty, -1] \cup [7, \infty)$

اتحاد.

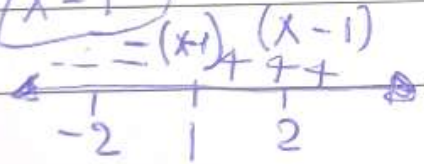


Example: Rewrite.

1) $f(x) = |x-1|$

$x - 1 = 0$

$x = 1$



$$f(x) = \begin{cases} x-1 & , x > 1 \\ -(x-1) & , x \leq 1 \end{cases}$$

يمكن وضع المساواة في واحدة من القوسين

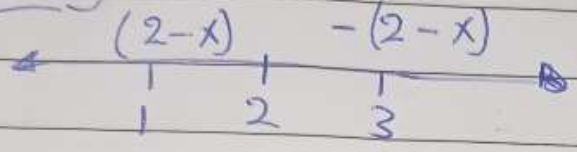
Subject Calculus (1) Lecture (2). Date No.

homework: Rewrite

2) $f(x) = |2-x|$.

$2-x=0$

$\boxed{x=2}$



$$f(x) = \begin{cases} 2-x & , x \leq 2 \\ -(2-x) & , x > 2 \end{cases}$$

Lecture (3)

* Polynomials : كثيرات الحدود

$f(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + a_2 + a_1 + a_0 \quad a_n \neq 0$

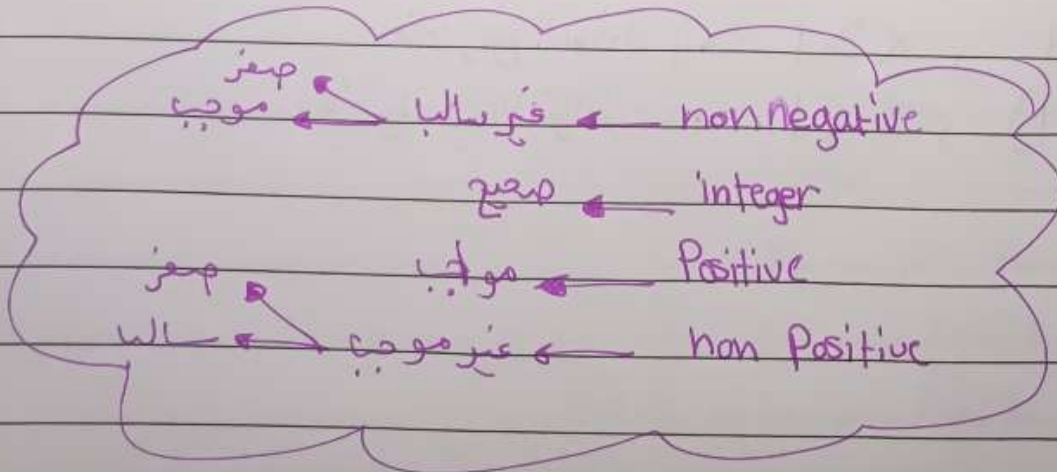
$a_n, a_{n-1}, a_{n-2}, a_{n-3} \dots a_2, a_1, a_0 \in \mathbb{R}$

* The numbers $a_n, a_{n-1}, a_2, a_1, a_0$ are constant Called Coefficients of the Polynomial.

* The degree of the Polynomial is n

(n) is nonnegative integer. ← عدد صحيح غير سالب

* The domain of any Polynomial is $\mathbb{R} = (-\infty, \infty)$



Example:

1) $f(x) = 3x^5 + 7x - 4$ Polynomial with degree 5 / constant term = -4

2) $f(x) = \sqrt{x} + 9x^4 + 1$ Not Polynomial

3) $f(x) = 9x^{-5} - 4$ Not Polynomial

4) $f(x) = 9$ Polynomial

5) $f(x) = |x| + 5x + 7$ $\begin{cases} -x + 5x + 7, & x \leq 0 \\ x + 5x + 7, & x > 0 \end{cases}$

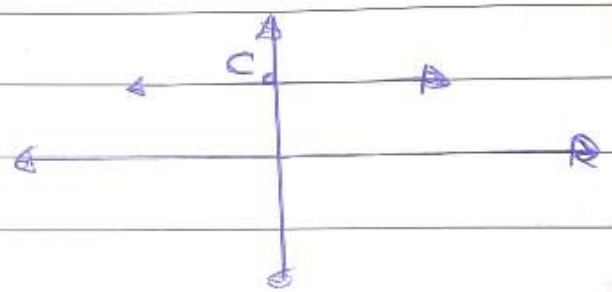
$f(x) = \begin{cases} 4x + 7, & x \leq 0 \\ 6x + 7, & x > 0 \end{cases}$ Not Polynomial

* Constant functions:-

$f(x) = C$

Domain: $(-\infty, \infty)$

Range: C



Example: $f(x) = 5$

$f(x) = 0$

$f(x) = -1$

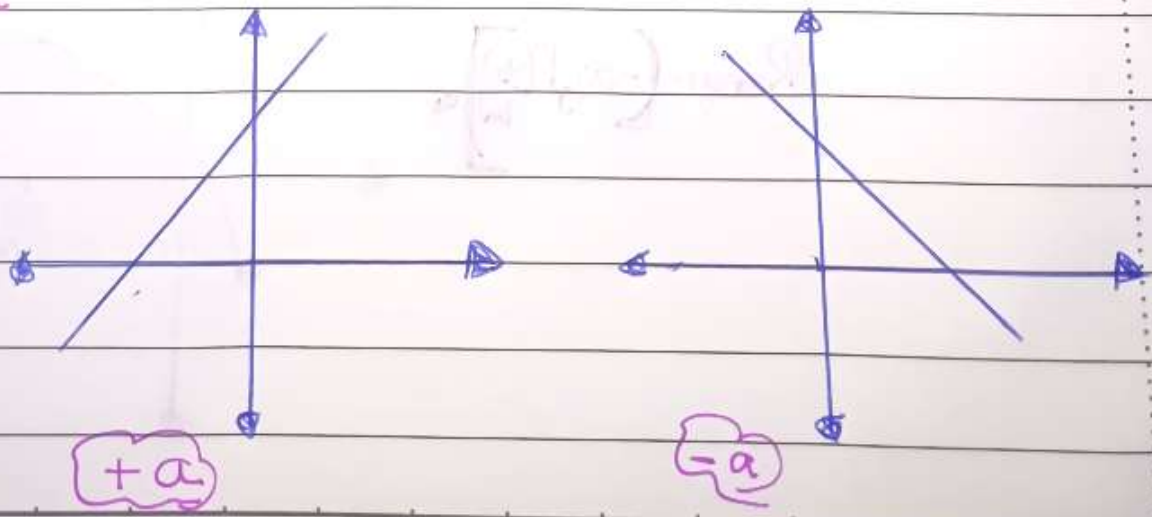
Lecture (4)

* liner function

$f(x) = ax + b$

Domaine R

Range R



Subject Calculus (1) Lecture (4) Date No.

Example :-

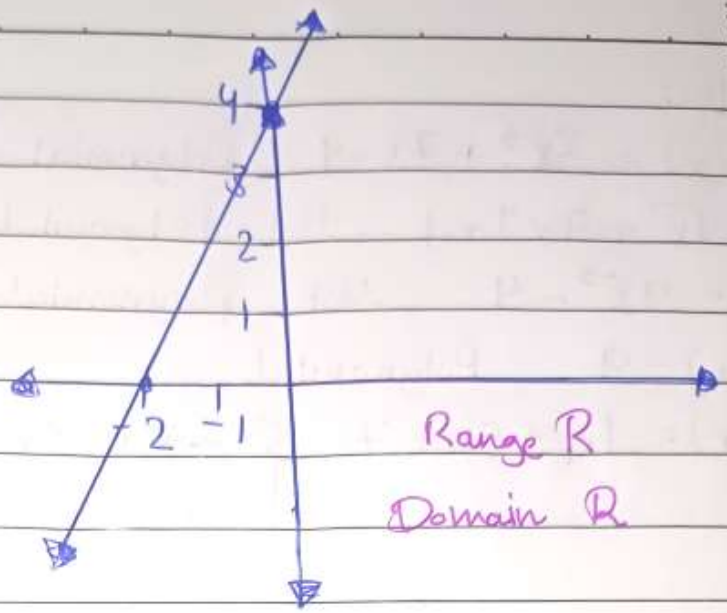
$f(x) = 2x + 4$

X-intercept $(y = 0)$

$2x + 4 = 0$

$2x = -4$

$\boxed{x = -2}$



y-intercept $(x = 0)$

$f(0) = 0 + 4$

$f(0) = 4$

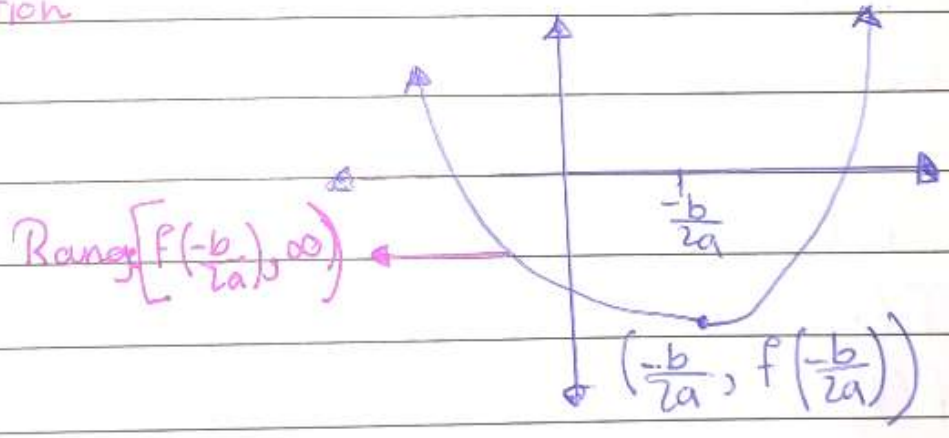
$\boxed{y = 4}$

* Quadratic function

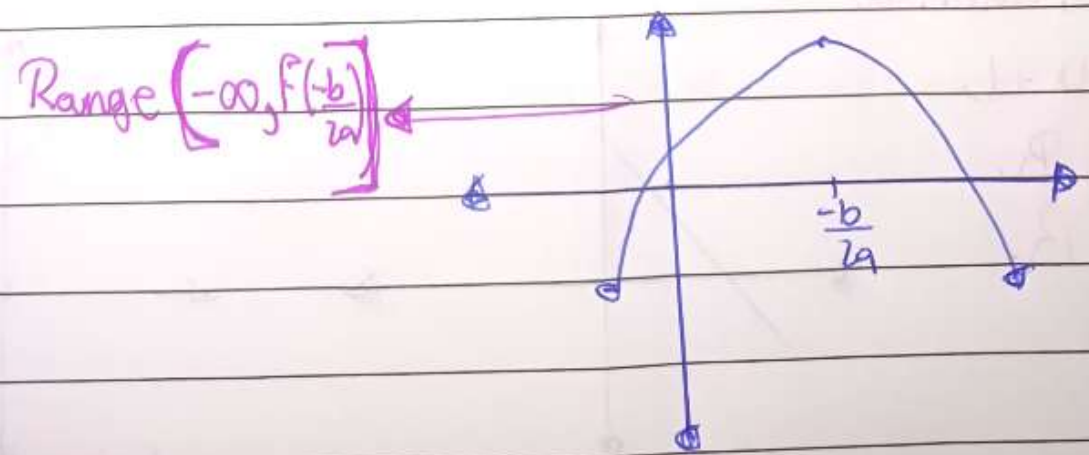
$f(x) = ax^2 + bx + c$

Domain R

if $a > 0$



if $a < 0$



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Example :-

① $f(x) = x^2 - 6x + 7$

$a = 1 \quad b = -6 \quad c = 7$

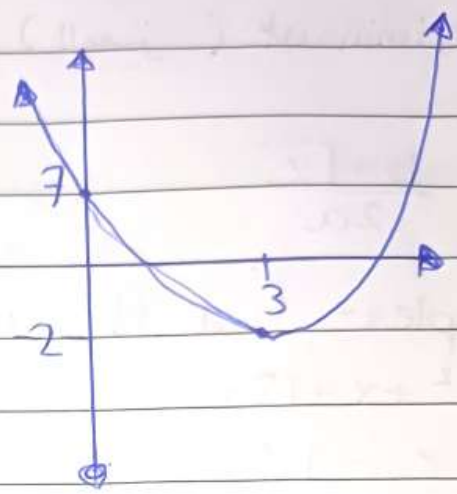
$a > 0$ مفتوح للأعلى

Domain R

Range $[f(\frac{-b}{2a}), \infty)$

$[f(\frac{--6}{2(1)}), \infty)$

$[-2, \infty)$



② $f(x) = 3 + 2x - x^2$

$a = -1 \quad b = 2 \quad c = 3$

$a < 0$ للأسفل مفتوح

Domain R

Range $(-\infty, f(\frac{-b}{2a})]$

$(-\infty, f(\frac{-2}{2(-1)})]$

$(-\infty, f(1)]$

$(-\infty, 4]$

Subject Calculus (1) Lecture (5) Date No.

$ax^2+bx+c=0 \quad a \neq 0$

Discriminant (المميز) $= \Delta = b^2-4ac$

$\Delta \geq 0$ لها حلول — حلين $x \geq 0$ / حل واحد $x=0$

$\Delta \leq 0$ ليس لها حل

$$x = \frac{-b \pm \sqrt{\Delta}}{2a}$$

Examples:- find the value of x:-

1) $x^2+x+12=0$

$\Delta = b^2-4ac$

$= 1^2-4(1)(12)$

$= 1-48$

$= -47$ ليس لها حل.

2) $x^2-3x-4=0$

$\Delta = b^2-4ac$

$= (-3)^2-4(1)(-4)$

$= 9+16$

$= 25$ لها حلين

$$x = \frac{3 \pm \sqrt{25}}{2}$$

$$x = \frac{3+5}{2} \qquad x = \frac{3-5}{2}$$

$x=4$ $\qquad$ $x=-1$

طريقة التحليل

$(x-4)(x+1)=0$

$x=4$ $\qquad$ $x=-1$

Subject Calculus (1) Lecture(5) Date No.

* Power function : $f(x) = x^n$, n Positive integer

*ملاحظة :- هناك صور لرسمات الاقترانات تمت إضافتها في الصورة القادمة.

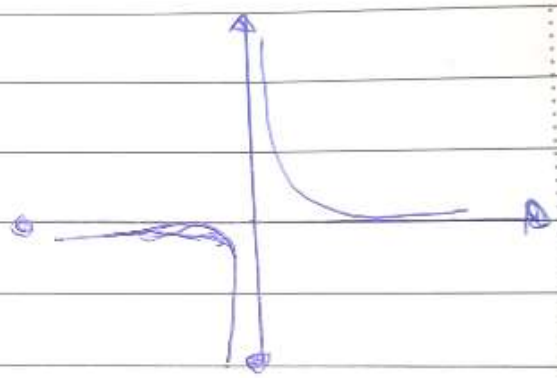
*ملاحظة :- الاقترانات المرفوعة لأس زوجي كلما زادت القوة الرسمة من الأعلى تقترب من المحور لا و تتوسع / والاقتران الفردي أيضا يتوسع بزيادة القوة.

* Rational function :-

$$f(x) = \frac{Q(x)}{P(x)} = \frac{\text{كثيرحدود}}{\text{كثيرحدود}}$$

Example: ① $f(x) = \frac{x^2+5x}{x^7-6x+1}$ Rational function

② $f(x) = \frac{1}{x}$ rational function.

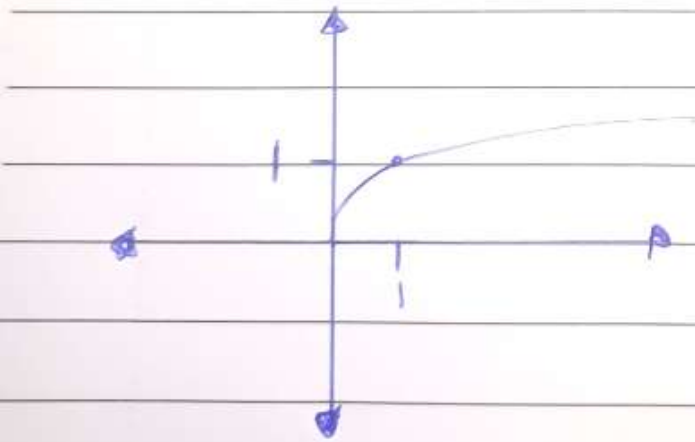


Domain $R \setminus \{0\}$

Range $R \setminus \{0\}$

* Root function:

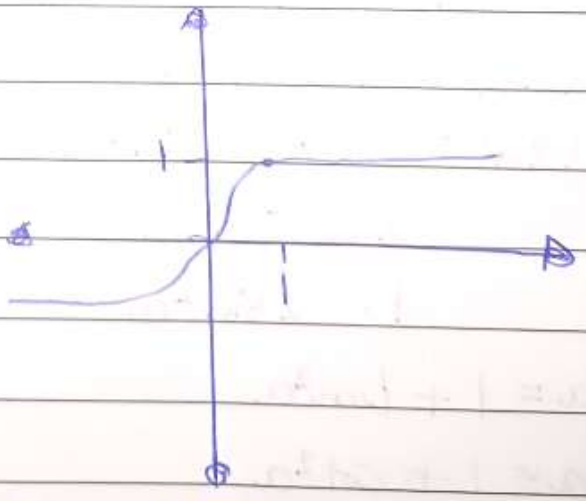
$f(x) = x^{\frac{1}{n}} = \sqrt[n]{x}$, n position integer.



$f(x) = \sqrt{x}$

Domain $[0, \infty)$

Range $[0, \infty]$

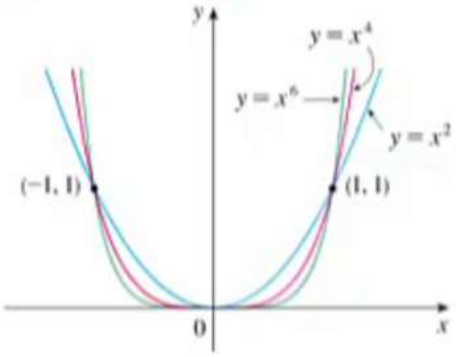


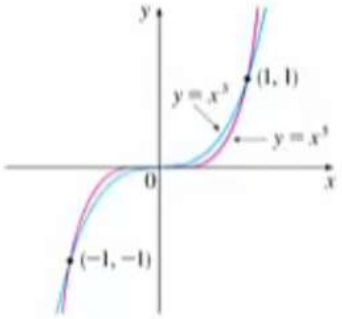
$f(x) = \sqrt[3]{x}$

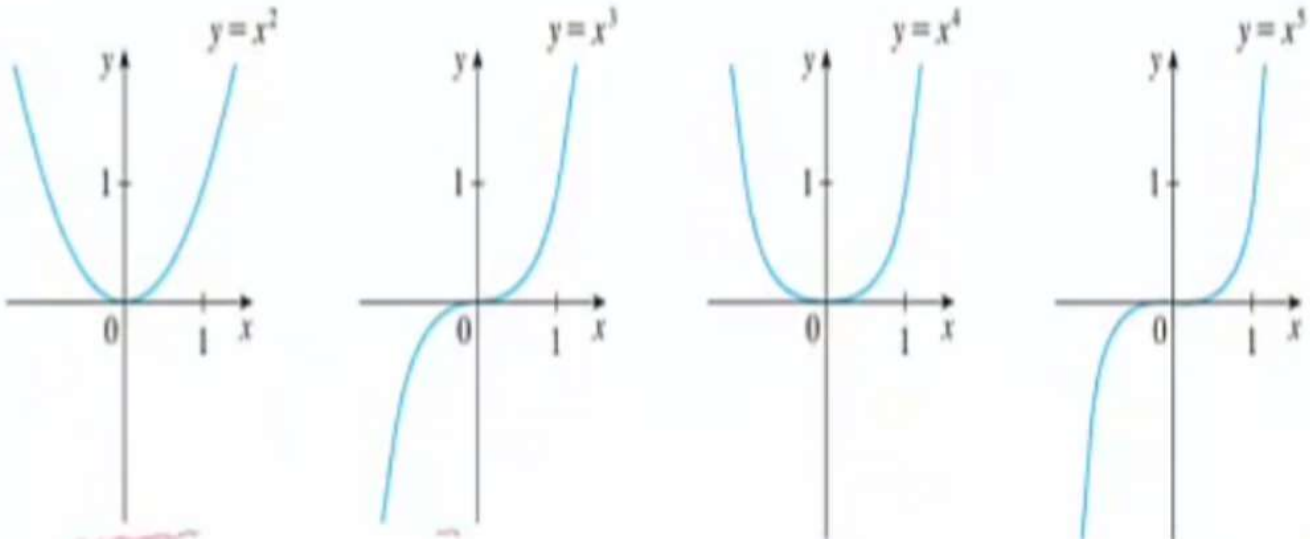
Domain = R

Range = R

NOTEBOOK


y
y = x^4
y = x^6
y = x^2
(−1, 1)
(1, 1)
0
x


y
y = x^3
(1, 1)
y = x^5
0
x
(−1, −1)


y = x²
y = x³
y = x⁴
y = x⁵
y
x
0
1

Subject Calculus (1) Lecture (6) Date No.

* Trigonometric functions :-

[1] $f(x) = \sin x$ المقابل / الوتر

Domain : R

Range : $[-1, 1]$

[2] $f(x) = \cos x$ المجاور / الوتر

Domain : R

Range : $[-1, 1]$

[3] $f(x) = \tan x = \frac{\sin x}{\cos x}$ المقابل / المجاور

Domain : $R \setminus \{\pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2} \dots\}$

Range : R

[4] $\cot x = \frac{1}{\tan x} = \frac{\cos x}{\sin x}$ المجاور / المقابل

[5] $f(x) = \frac{1}{\cos x} = \sec x =$ الوتر / المجاور

[6] $f(x) = \csc x = \frac{1}{\sin x}$ الوتر / المقابل

متطابقات هامة :-

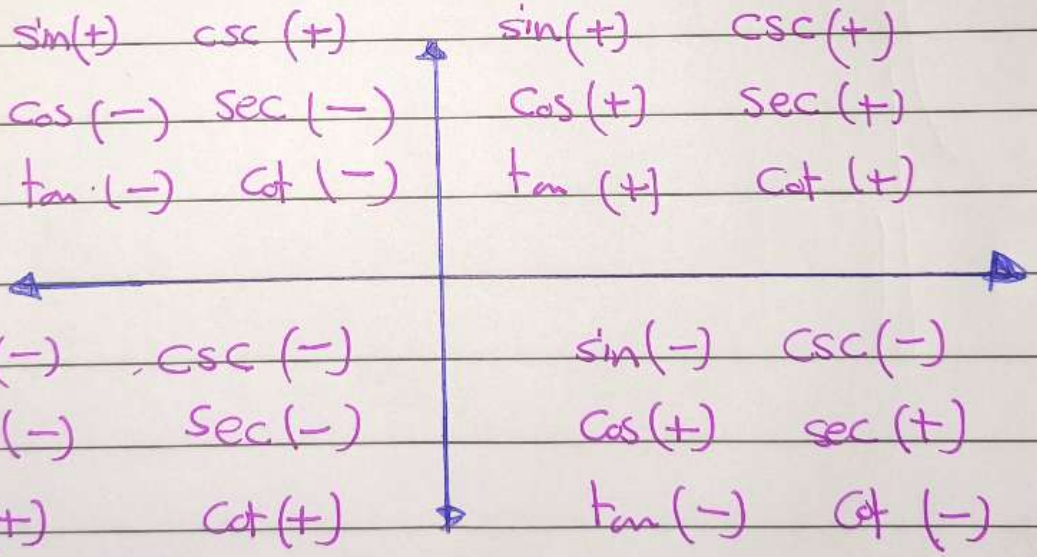
* $\sin^2 a + \cos^2 a = 1$
* $\sin(2a) = 2 \sin a \cos a$
* $\cos(2a) = \cos^2 a - \sin^2 a$
  $2\cos^2 a - 1$
  $1 - 2\sin^2 a$
* $\sec^2 a = 1 + \tan^2 a$
* $\csc^2 a = 1 + \cot^2 a$

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* $\sin^2 a = \frac{1}{2}(1 - \cos(2a))$
* $\cos^2 a = \frac{1}{2}(1 + \cos(2a))$
* $\sin a \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$
* $\sin a \sin b = -\frac{1}{2}[\cos(a+b) - \cos(a-b)]$
* $\cos a \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$
* $\sin(a+2\pi) = \sin a$
* $\cos(a+2\pi) = \sin b$

* Note :-

1) $\cos x$, $\sec x$ even functions زوجية
2) $\sin x$, $\tan x$, $\cot x$, $\csc x$, odd functions. فردية



Example :-

1) $\cos\left(\frac{3\pi}{4}\right)$ ← الربع الثاني سالب

$-\cos\left(\frac{3\pi}{4}\right)$

$= -\cos\left(\pi - \frac{3\pi}{4}\right)$

$= -\cos\frac{\pi}{4}$

$= \frac{-1}{\sqrt{2}}$

2) $\sin\left(\frac{11\pi}{6}\right)$ ← الربع الرابع سالب

$\frac{11\pi}{6} \times \frac{180}{\pi} = 330^\circ$

$-\sin(360 - 330)$ أو $-\sin(2\pi - \frac{11\pi}{6})$

$-\sin(30)$ $-\sin(\frac{\pi}{6})$

$= -\frac{1}{2}$ $= \frac{1}{2}$

3) $\tan\left(\frac{4\pi}{3}\right)$ ← الربع الثالث موجب

$\frac{4\pi}{3} \times \frac{180}{\pi} = 240^\circ$

$\tan\left(\frac{4\pi}{3} - \pi\right)$

$\tan\left(\frac{\pi}{3}\right)$

$= \sqrt{3}$

Homework :-

1) $\sin\theta = \frac{1}{2}$, $\cos\theta = \frac{\sqrt{3}}{2}$ Find $\sin(2\theta)$.

$\sin 2\theta = 2\sin\theta\cos\theta$

$= 2 \times \frac{1}{2} \times \frac{\sqrt{3}}{2}$

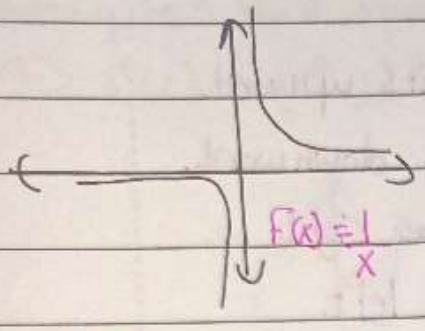
$= \frac{\sqrt{3}}{2}$

2) Find $\sin(3x)\cos(5x)$

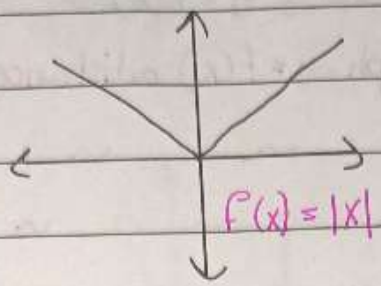
$= \frac{1}{2}[\sin(3x+5x) + \sin(3x-5x)]$

$\frac{1}{2}[\sin 8x + \sin(-2x)]$

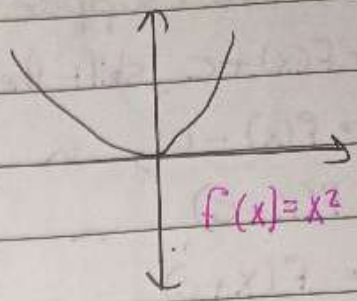
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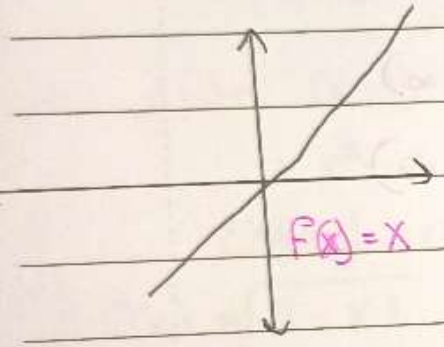
Domain = R - {0}
Range = R - {0}



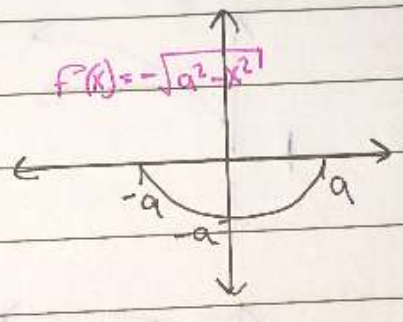
Domain R
Range [0, ∞)



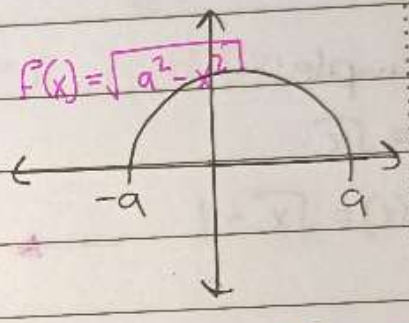
Domain = R
Range = [0, ∞)



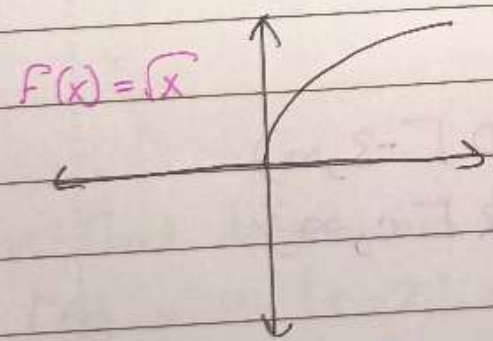
Domain R
Range R



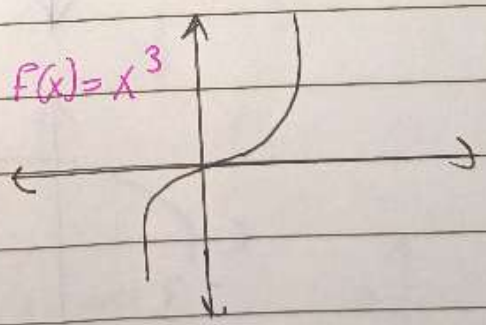
Domain [-a, a]
Range [-a, ∞]



Domain [-a, a]
Range [0, a]



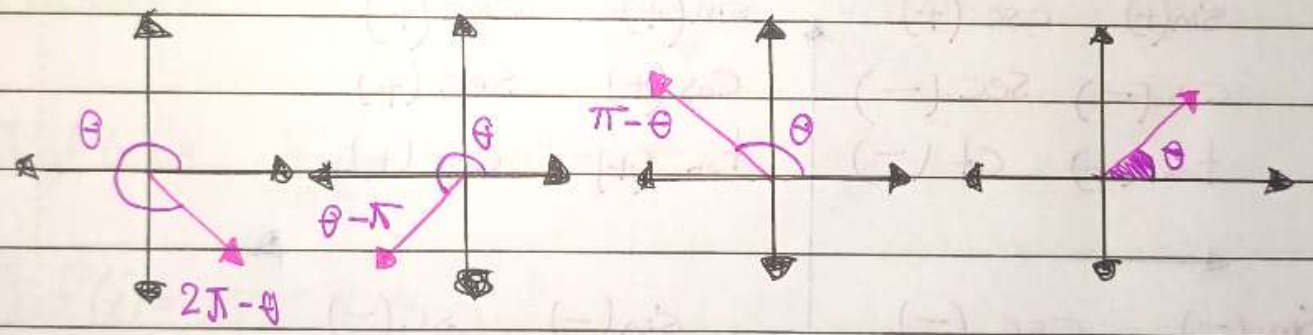
Domain [0, ∞)
Range [0, ∞)



Domain R
Range R

Subject Calculus (1) Lecture (7) Date No.

X	sinx	cosx	tanx	cotx	secx	cscx
$0, 2\pi$	0	1	0	غير معرف	1	غير معرف
$\frac{\pi}{2}$	1	0	غير معرف	0	غير معرف	1
π	0	-1	0	غير معرف	-1	غير معرف
$\frac{3\pi}{2}$	-1	0	غير معرف	0	غير معرف	-1
$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	1	$\sqrt{2}$	$\sqrt{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$



* زاوية المرجع هي زاوية حادة محصورة بين محور السينات وضلع انتهاء الزاوية.

* درجات ← راديان ← نضرب $\frac{\pi}{180}$

* راديان ← درجات ← نضرب $\frac{180}{\pi}$

Example :-

* $120^\circ \rightarrow 120 \times \frac{\pi}{180} = \frac{2\pi}{3}$ rad

* $\frac{3\pi}{4} \rightarrow \frac{3\pi}{4} \times \frac{180}{\pi} = 135^\circ$

bject Calculus (1) Lecture (7) Date No.

③ If $\sin\theta = \frac{1}{2}$ $\quad \frac{\pi}{2} \leq \theta \leq \pi$

Find:

$\cos\theta = -\frac{\sqrt{3}}{2}$ $\quad \sec\theta = \frac{-2}{\sqrt{3}}$ $\quad \cot\theta = -\sqrt{3}$

$\csc\theta = 2$ $\quad \tan\theta = \frac{-1}{\sqrt{3}}$

④ Find:-

1) $\sin\left(\frac{5\pi}{6}\right) = \frac{1}{2}$

2) $\sin\left(\frac{3\pi}{4}\right) = \frac{-1}{\sqrt{2}}$

3) $\cot\left(\frac{4\pi}{3}\right) = \frac{1}{\sqrt{3}}$

4) $\csc\left(\frac{5\pi}{4}\right) = -\sqrt{2}$

5) $\cos\left(\frac{11\pi}{6}\right) = \frac{1}{2}$

6) $\sin^2\left(\frac{\pi}{12}\right) = \frac{1}{2}\left(1 - \cos\frac{2\pi}{12}\right)$

$= \frac{1}{2}\left(1 - \cos\frac{\pi}{6}\right)$

$= \frac{1}{2}\left(1 - \frac{\sqrt{3}}{2}\right)$

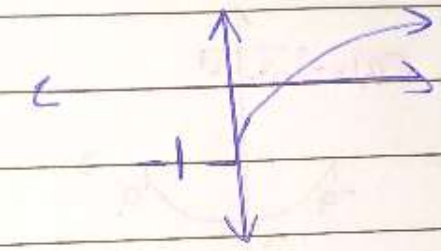
* Definition : suppose $C > 0$, $y = f(x)$

1) $y = f(x) + C$, shift the graph $y = f(x)$ a distance C units upward.
2) $y = f(x) - C$, " " " " " " downward.
3) $y = f(x - C)$ " " " " to the right
4) $y = f(x + C)$ " " " " to the left
5) $y = -f(x)$ " reflect " " " about the x-axis
6) $y = f(-x)$ " reflect " " " " y-axis

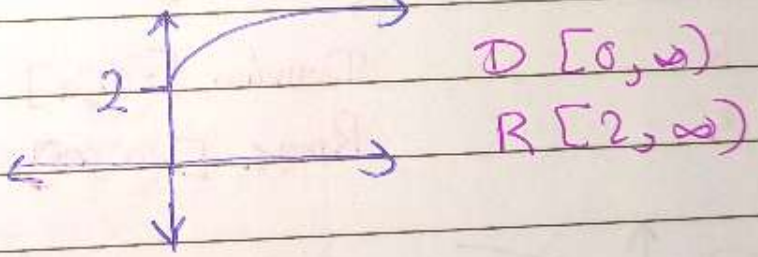
Example:-

$y = \sqrt{x}$

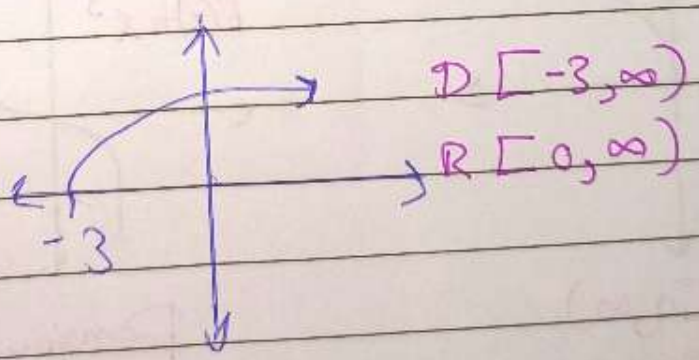
* $f(x) = \sqrt{x} - 1$ →



* $f(x) = \sqrt{x} + 2$ →

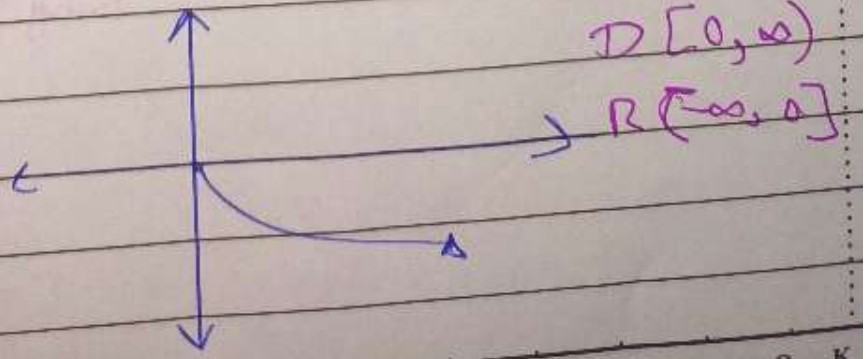


* $f(x) = \sqrt{x+3}$



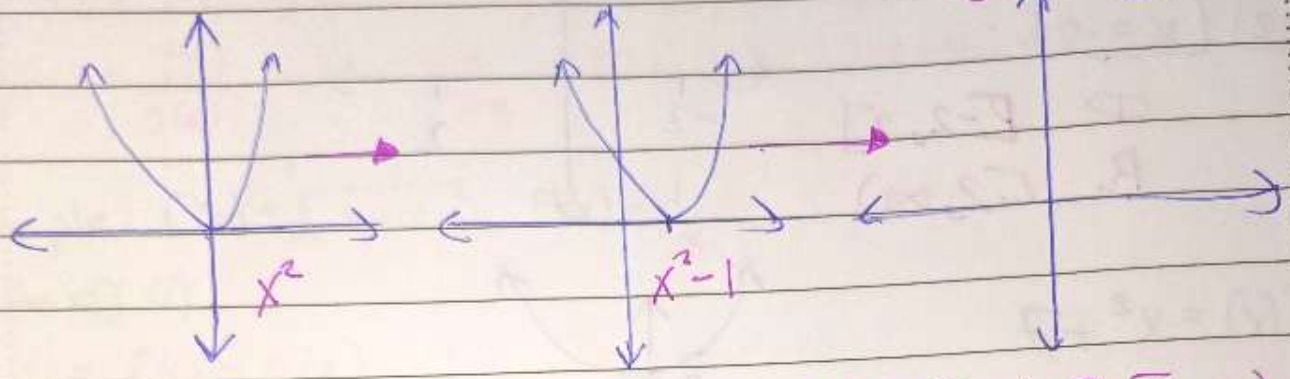
~~* $f(x) = \sqrt{x}$~~

* $f(x) = -\sqrt{x}$



Example : find domain and Range

① $f(x) = (x^2-1)^2 + 3$



$D = R \;/\; R[3, \infty)$

* بدل $(x-1)^2+3$

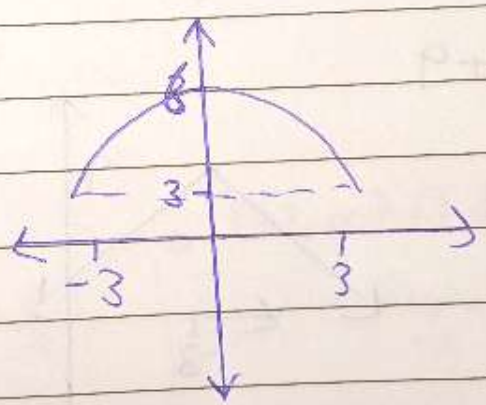
$= x^2 - x + 4$

بدل الطريقة!

② $f(x) = \sqrt{9 - x^2} + 3$

$D\ [-3, 3]$

$R\ [0, 3]$



H.W : find domain and Range

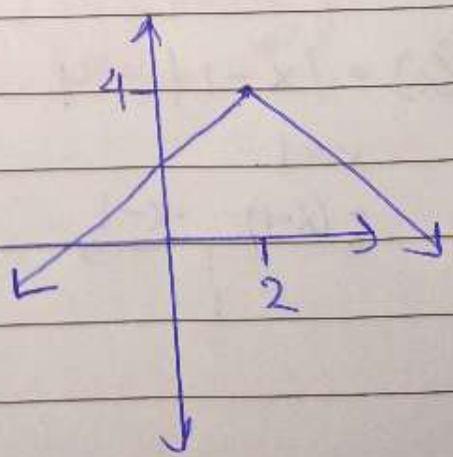
① $f(x) = 4 - |x-2| = -|x-2| + 4$

$x - 2 = 0$

$x = 2$

$2-x \quad | \quad x-2$

2



$$f(x) = \begin{cases} x-2, & x \geq 2 \\ 2-x, & x < 2 \end{cases}$$

~~$D\ [4, \infty)$~~

~~$R\ [4, -$~~

$D\ \{R\}$

$R\ (-\infty, 4]$

2) $f(x) = -\sqrt{4-x^2}$

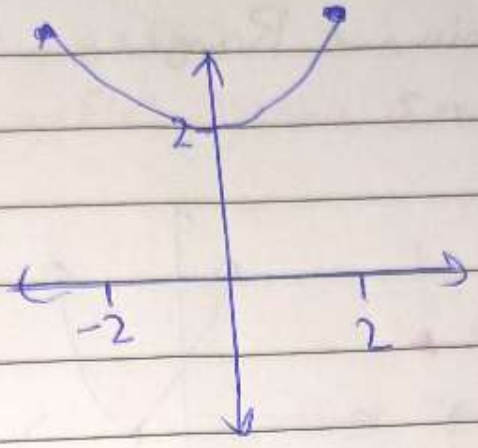
$4-x^2=0$

$x^2-4=0$

$x=2$ $x=-2$

D $[-2,2]$

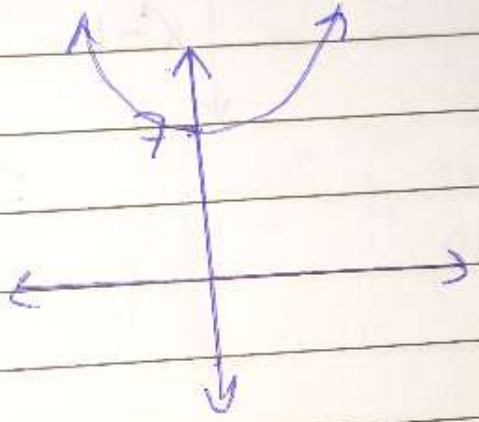
R $[-2,0]$



3) $f(x) = x^2+7$

D: R

R $[7,\infty)$

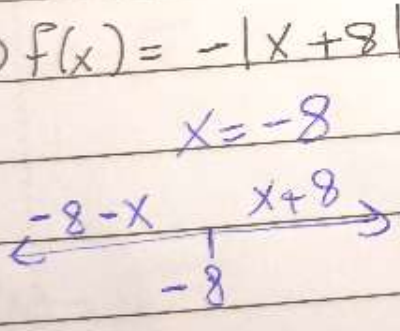


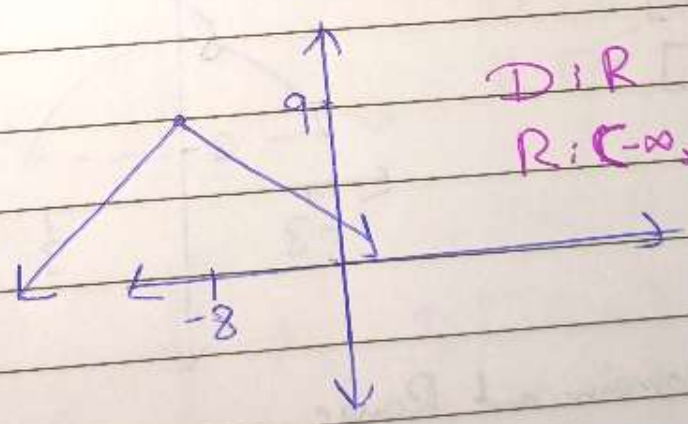
4) $f(x) = -|x+8|+9$

$x=-8$

$-8-x$ $x+8$

-8





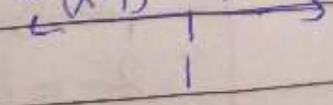
D: R

R: $[-\infty,9]$

5) $f(x) = |x-1|-4$

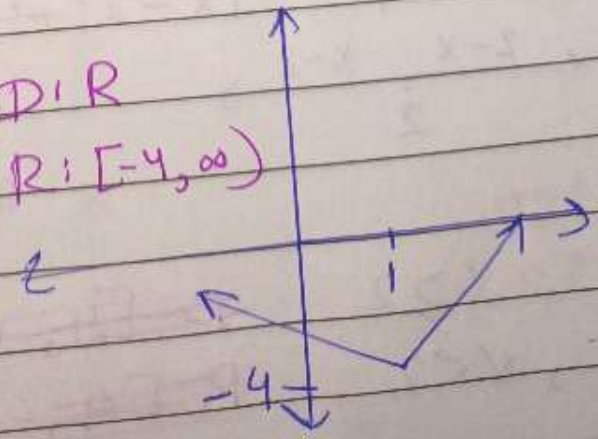
$x=1$

$-(x-1)$ $x-1$



D: R

R: $[-4,\infty)$



* Definition: If f and g functions with domains D_f, D_g.

1) $D(f \pm g)(x) = f(x) \pm g(x)$ with $D_{f+g} = D_f \cap D_g$

2) $(f.g)(x) = f(x).g(x)$ with $D_{f.g} = D_f \cap D_g$

3) $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$ with $\frac{D_f}{D_g} = D_f \cap D_g \setminus \{x : g(x) = 0\}$

* Example: $f(x) = \sqrt{x+2}$ $g(x) = \frac{1}{x}$

$D(f - g)(x)$

$= f(x) - g(x)$

$\sqrt{x+2} - \frac{1}{x}$

2) $(f - g)(4)$

$= \sqrt{4+2} - \frac{1}{4}$

$= \sqrt{6} - \frac{1}{4}$

3) $D_{fg} =$

$f(x) = \sqrt{x+2}$ $g(x) = \frac{1}{x}$

$x \geq -2$

$D_f = [-2, \infty)$ $D_g = R - \{0\}$

$[-2, \infty) \cap R \setminus \{0\}$



$[-2, 0) \cup (0, \infty)$

$[-2, 0) \setminus \{0\}$

* Example: Find domain

$D\ f(x) = \frac{x}{x^2 - 9}$ $\longrightarrow D_1 = R$ $\longrightarrow D_2 = R$

$-3/3 =$ [illegible]

$D_f = D_1 \cap D_2 \setminus \{$[illegible]$\}$

$= R \cap R \setminus \{\pm 3\} = R \setminus \{\pm 3\}$

2) $f(x) = \frac{1}{\sqrt{1-x}}$ $\rightarrow D_1 = R$

$\rightarrow D_2 = 1 - x \geq 0$

$(-\infty, 1]$ $1 \geq x$

$x = 1$ اصفار المقام

$D_f = D_1 \cap D_2$ | اصفار المقام

$= R \cap (-\infty, 1]$ | $\{1\}$

$= (-\infty, 1)$

Lecture (10)

Example1 - find domain

D $f(x) = \frac{x^2 - 1}{x - 1}$ $\rightarrow R$ $\rightarrow R$

$x = 1$ اصفار المقام

$D_f = R - \{0\}$

$= (-\infty, 1) \cup (1, \infty)$

3) $f(x) = \sqrt{\frac{x}{x+1}}$ $\rightarrow R$ $\rightarrow R$

~~$x \neq 1$~~ f

$\frac{x}{x+1} \geq 0$

$x = 0$ $x = -1$

$x = 0$ ← − − − | + + + → (0)

2) $f(x) = \sqrt{|x-5| - 2}$

$|x-5| - 2 \geq 0$

$|x-5| \geq 2$

$x - 5 \geq 2$ $x - 5 \leq -2$

$x \geq 7$ $x \leq 3$

$D_f = (-\infty, 3) \cup (7, \infty)$

$x + 1 \neq 0$

$x = -1$ ← − − | + + + → (−1)

$\frac{x}{x+1}$ + + + | − − − | + + + (−1, 0)

$D_f = (-\infty, -1) \cup [0, \infty)$

4) $f(x) = \frac{\sqrt{x}}{\sqrt{x+1}}$ → (H.W)

مختلف عن السابق بطريقة المقام ليس جذر واحد

* $\sqrt{x}$ → $x \geq 0$ اصفار المقام $x = -1$

* $\sqrt{x+1}$ → $x+1 \geq 0$

$x \geq -1$

$\sqrt{x}$ ← 0

$\sqrt{x+1}$ ← -1

← -1 0

$D_f = (-1, \infty)$

5) $f(x) = \sqrt[3]{\frac{1}{x-1}}$ → R

$x-1$ → R

اصفار المقام $x = 1$

$D_f = R - \{1\}$

Lecture (11)

Example: Find domain

D $f(x) = \frac{2x+1}{3-|x|}$ → R

→ R

اصفار المقام -3/3

$3 - |x| = 0$

$|x| = 3$

$x = 3/-3$

$D_f = R - \{3, -3\}$

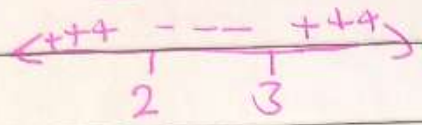
Subject Calculus (1) Lecture (11) Date No.

② $f(x) = \sqrt{x^2 - 5x + 6}$

$x^2 - 5x + 6 \geq 0$

$(x-2)(x-3) \geq 0$

$x = 2$ $x = 3$

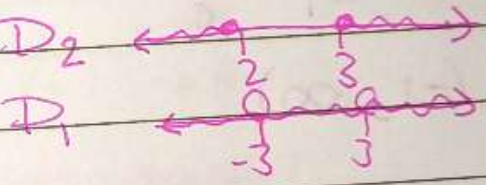


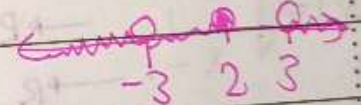
$D_f = (-\infty, 2] \cup [3, \infty)$

③ $f(x) = \frac{\sqrt{x^2 - 5x + 6} + 2x + 1}{3 - |x|}$

$D_2 \cap D_1 = (-\infty, 2] \cup [3, \infty) \cap R - \{3, -3\}$

$= (-\infty, -3) \cup (-3, 2] \cup (3, \infty)$





Lecture (12).

Example:- find domain & Range:-

$Df(x) = \sqrt{x-1} + 5 \rightarrow R$

$x - 1 \geq 0$

$x \geq 1$

$D_f = R \cap [1, \infty)$

$D_f = [1, \infty)$

$R_f = [5, \infty)$

② $f(x) = \frac{3}{5+\cos x}$: $\mathbb{R} \to \mathbb{R}$

اصغر القيم : لا يوجد

$D_f = \mathbb{R}$

R_f = $-1 < \cos x \leq 1$

$4 \leq \cos x + 5 \leq 6$

$\frac{1}{4} \geq \frac{1}{\cos x + 5} \geq \frac{1}{6}$

$\frac{3}{4} \geq \frac{3}{5+\cos x} \geq \frac{3}{6}$

$R_f = \frac{1}{2} \leq f(x) \leq \frac{3}{4}$

③ $f(x) = -3\sin^2 x$

$D_f = \mathbb{R}$

$-1 \leq \sin x \leq 1$

$0 \leq \sin^2 x \leq 1$

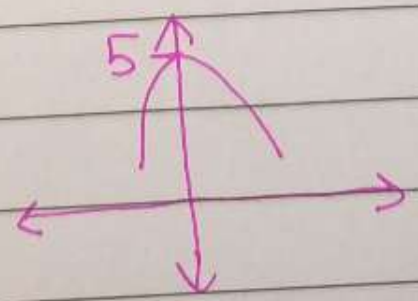
$0 \geq \sin^2 x \geq -3$

$R_f = [-3, 0]$

④ $f(x) = -x^2 + 5$

$D_f = \mathbb{R}$

$R_f = (-\infty, 5]$

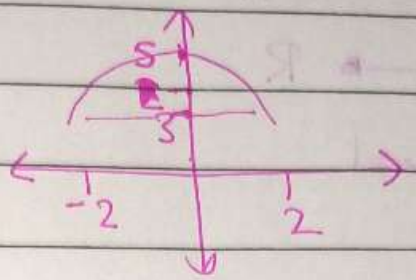


Subject Calculus (1) Lecture (12) Date No.

5) $f(x)=\sqrt{4-x^2}+3$

$4-x^2 \geq 0$

$x=2$ $x=-2$



$D_f = [-2,2]$

$R_f = [3,5]$

Example: If Range of $[1,2]$ find Range the Range of

$g(x) = \frac{1}{1+(f(x))^2} \rightarrow R$

$1 \leq f(x) \leq 2$

$1 \leq (f(x))^2 \leq 4$

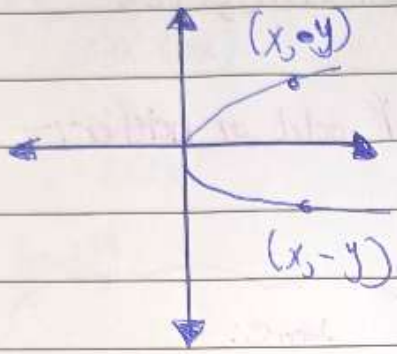
$2 \leq (f(x))^2 \leq 5$

$\frac{1}{2} \geq \frac{1}{(f(x))^2} \geq \frac{1}{5}$

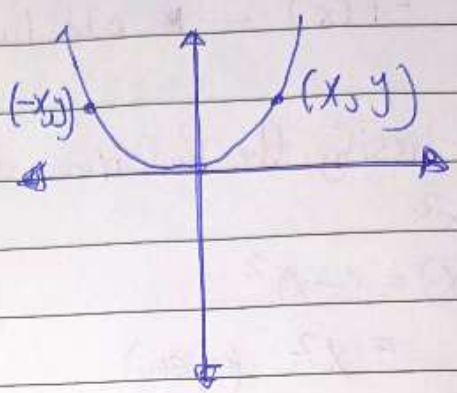
Range of $[\frac{1}{5}, \frac{1}{2}]$

*Symmetry (تناظر)

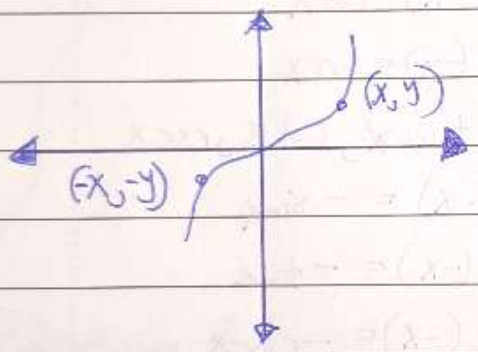
1) about x-axis ($y \to -y$)



2) about y-axis ($x \to -x$)



3) About the origin ($x \to -x$), ($y \to -y$)



Examples:-

1) Is $x^3 = y^4 + 3$ symmetric about x-axis:-

$x^3 = y^4 + 3$

$x^3 = (-y)^4 + 3$

$x^3 = y^4 + 3$ ✓

2) Is $x^4 + 5y^2 = 9$ symmetric about the origin

$(-x)^4 + 5(-y)^2 = 9$

$x^4 + 5y^2 = 9$ ✓

3) Is $y^3 = x^7 - 7$ symmetric about origin

$(-y)^3 = (-x)^7 - 7$

$-y^3 = -x^7 - 7$ ✗

* Even (دوج) and odd (فرد) function:-

1) $f(-x) = f(x)$ → even function متماثل حول المحاور

2) $f(-x) = -f(x)$ → odd function متماثل حول نقطة الأصل

Example: classify the function as even or odd or neither:-

1) $f(x) = x^2$

$f(-x) = (-x)^2$

$= x^2$ (even)

2) $f(x) = \frac{x^5 - x}{1+x^2}$

$f(-x) = \frac{(-x)^5 - (-x)}{1+(-x)^2}$

$= \frac{-x^5 + x}{1+x^2}$

$= \frac{-(x^5 - x)}{1+x^2}$ (odd)

Note:

1) $\cos x, \sec x$ even functions

$\cos(-x) = \cos x$

$\sec(-x) = \sec x$

2) $\sin x, \tan x, \cot x, \csc x$

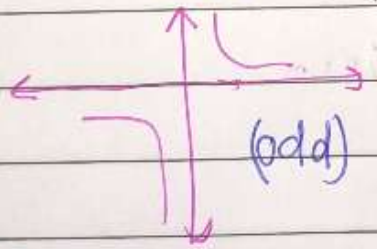
$\sin(-x) = -\sin x$

$\tan(-x) = -\tan x$

$\csc(-x) = -\csc x$

$\cot(-x) = -\cot x$

3)



4) $F(x) = x^5 + x$

$f(-x) = (-x)^5 + (-x)$

$= -x^5 - x$

$= -(x^5 + x)$

(odd)

5) $F(x) = \frac{\tan x}{x^3 - x}$

$f(-x) = \frac{\tan(x)}{(-x)^3 - (-x)}$

$= \frac{-\tan x}{-x^3 + x}$

$= \frac{-\tan x}{-(x^3 - x)}$ (even)

$= \frac{\tan x}{x^3 - x}$

E B O O K

6) $f(x) = \frac{x^4+1}{\csc x}$

$f(-x) = \frac{(-x)^4+1}{\csc(-x)}$

$= \frac{x^4+1}{-\csc x}$

(odd)

7) $f(x) = 2x - x^2$

$f(-x) = 2(-x) - (-x)^2$

$= -2x - x^2$

$= -(2x + x^2)$

(Neither)

* ملاحظة:- (استنتاج).

- كسر مشتمل اقترن $\frac{\text{even}}{\text{even}}$ ← even

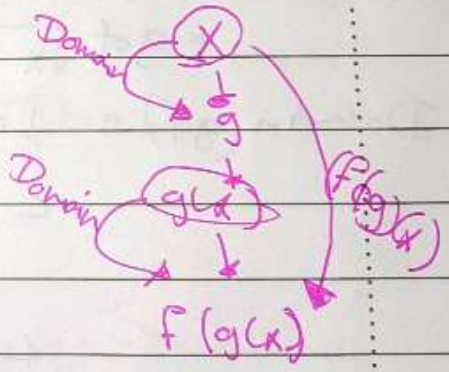
- كسر مشتمل اقترن $\frac{\text{even}}{\text{odd}}$ ← odd

- كسر مشتمل اقترن $\frac{\text{odd}}{\text{odd}}$ ← even

Lecture (15).

* Composition of functions (تركيب الاقترانات)

- The composition of f with g $(f \circ g)(x) = f(g(x))$
- The composition of g with f $(g \circ f)(x) = g(f(x))$



1* Domain $f \circ g = \{x \in D_{g(x)} \text{ and } g(x) \in D_{f(x)}\}$

$= \{x$ المجال D_f and $g(x) \in$ المجال $D_g\}$

2* Domain $f \circ g =$ المجال $D_f \cap$ المجال D_g الناتج عن التركيب بعد التبسيط

* Example: $f(x) = \sqrt{x}$, $g(x) = \sqrt{2-x}$ function and Domain

$D(f \circ g)(x) = f(g(x))$

$= f(\sqrt{2-x}) = \sqrt{\sqrt{2-x}}$

$= \sqrt[4]{2-x}$

الباقي في الصفحة التالية

Subject Date No.

$Domain(fog) = \{ x \in D_{\text{الداخلي}}(g(x)) \text{ and } g(x) \in D_{\text{الخارجي}}(f(x)) \}$

$= \{ x \in (-\infty, 2] \cap \sqrt{2-x} \in [0, \infty) \}$

$\{ x \in (-\infty, 2] \text{ and } \sqrt{2-x} \geq 0 \}$

$= \{ x \in (-\infty, 2] \text{ and } 2 \geq x \}$

$= x \in (-\infty, 2] \text{ and } (-\infty, 2]$

$= x \in (-\infty, 2] \cap (-\infty, 2] = (-\infty, 2]$

طريقة أخرى -

$Domain(fog) = D_{\text{الداخلي}} \cap D_{\text{التركيب بعد التعويض}}$

$= (-\infty, 2] \cap [-\infty, 2]$

$= [-\infty, 2]$

تابع السؤال

* $f(x) = \sqrt{x}$

$D_f = [0, \infty)$

* $g(x) = \sqrt{2-x}$

$2 \geq 0$

$D_g = (-\infty, 2]$

D ← التركيب

$\sqrt[4]{2-x}$

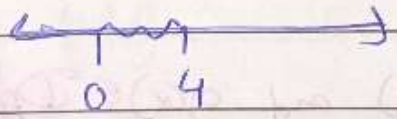
$(-\infty, 2]$

2) $(gof)(x) = g(f(x))$

$= g(\sqrt{x}) = \sqrt{2-\sqrt{x}}$

$Domain(gof) = [0, \infty) \cap (-\infty, 4]$





$Domain(gof) = [0, 4]$

* D بعد التركيب

$\sqrt{2-\sqrt{x}} \geq 0$

$2 \geq \sqrt{x}$

$4 \geq x$

$(-\infty, 4]$

إذا طلب $(gof)(9)$ نتأكد انها من المجال

$(gof)(9) = g(f(9)) = g(3) = \sqrt{2-3} = \text{undefined}$

معناها بعض الأسئلة يطلع رقم لذلك يجب إيجاد المجال أول

3) $(ff)(x) = f(f(x))$

$= f(\sqrt{x}) = \sqrt{\sqrt{x}}$

$= \sqrt[4]{x}$

$D_{ff} = [0,\infty) \cap [0,\infty)$

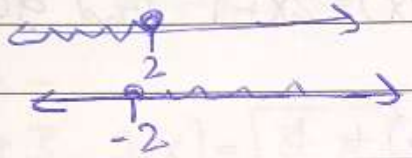
$= [0,\infty)$

4) $(gog) = g(g(x))$

$= g(\sqrt{2-x})$

$= \sqrt{2-\sqrt{2-x}}$

$D_{gog} = (-\infty, 2] \cap [-2,\infty)$



$D_{gog} = [-2,2]$

* D_{gog}

$\sqrt{2-\sqrt{2-x}} \geq 0$

$2-\sqrt{2-x} \geq 0$

$2 \geq \sqrt{2-x}$

$4 \geq 2-x$

$x \geq -2$

$[-2,\infty)$

Lecture (16)

Example:- $f(x) = \frac{3}{x}$, $g(x) = \frac{5x}{x-2}$ $(gof)(x)$ and Domain

$(gof)(x) = g(f(x)) = g(\frac{3}{x}) = \frac{5(\frac{3}{x})}{\frac{3}{x}-2}$

$= \frac{\frac{15}{x}}{\frac{3-2x}{x}}$

$= \frac{15}{3-2x}$

Domain $= R-\{0\} \cap R-\{\frac{3}{2}\}$

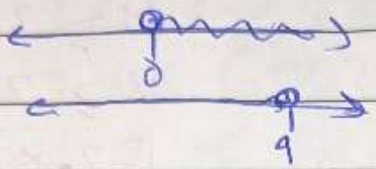
$= R-\{0, \frac{3}{2}\}$

Example: find domain $f(x)=\sqrt{3-\sqrt{x}}$

$g(x)=\sqrt{x}$ $h(x)=\sqrt{3-x}$

$D_{h(g(x))} = D_{\sqrt{x}} \cap D_{\sqrt{3-\sqrt{x}}}$ * $D_{\sqrt{3-\sqrt{x}}}$

$= [0,\infty) \cap (-\infty, 9]$ $\sqrt{3-\sqrt{x}} \geq 0$



$3-\sqrt{x} \geq 0$

$3 \geq \sqrt{x}$

$9 \geq x$

$D_f = [0,9]$

Lecture (17)

Example: $(f \circ g)(x) = 6x^2-10x+5$, $f(x)=2x+1$ find $g(0)$?

$(f \circ g)(x) = f(g(x))$

$6x^2-10x+5 = 2g(x)+1$

$6x^2-10x+4 = 2g(x)$

$3x^2-5x+2 = g(x)$

$\boxed{g(0)=2}$

Example: $(f \circ g)(x) = x^2+5$, $g(x)=2x+3$ find $f(x)$?

$(f \circ g)(x) = f(g(x)) = x^2+5$

$f(2x+3) = x^2+5$

~~$(2x+3)^2+5$~~

Let $y = 2x+3$ $f(y) = \left(\frac{y-3}{2}\right)^2+5$

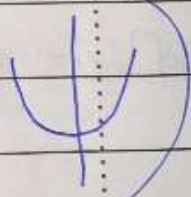
$y-3 = 2x$

$x = \frac{y-3}{2}$ $f(y) = \frac{(y-3)^2}{4}+5$

$f(x) = \frac{(x-3)^2}{4}+5$

$f(x)=x^2$

$f(7)=7^2$



Subject Calculus(1) Lecture(17) Date No.

Example: If domain $f=[1,3]$ and $g(x)=-2x+1$ find domain fog?

$D_{fog} = \{x \in D_{g(x)} \text{ and } g(x) \in D_{f(x)}\}$

$= \{x \in R \text{ and } -2x+1 \in [1,3]\}$

$= \{x \in R \text{ and } 1 \leq -2x+1 \leq 3\}$

$= \{x \in R \text{ and } 0 \leq -2x \leq 2\}$

$= R \cap [-1,0]$

$= [-1,0]$

Homework:-

1) $f(x)=x^2+3$, $g(x)=\sqrt{3-x}$ Find $(fog)(x)$, D_{fog}, $f.g(2)$, $f(g(4))$ $(gog)(x)$, D_{gog}, gof, D_{gof}

$(fog)(x) = f(g(x))$

$= f(\sqrt{3-x}) = (\sqrt{3-x})^2 + 3$

$= 3-x+3$

$f(g(x)) = 6-x$

$D_{fog} = (-\infty, 3] \cap R$

$= (-\infty, 3]$

$f(g(2)) = g(2) = \sqrt{3-2}$

$= 1$

$f(1) = 1+3$

$= 4$

$f(g(4)) =$ undefined

D_g: $\sqrt{3-x} \geq 0$

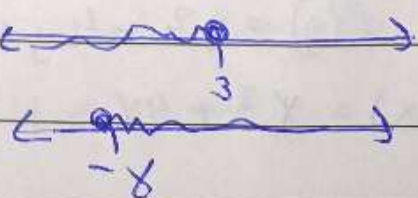
$3-x \geq 0$

$3 \geq x$

$(gog)(x) = g(g(x)) = \sqrt{3-\sqrt{3-x}}$

$D_{gog} = 3 \geq x \cap -6 \leq x$

$= (-\infty, 3] \cap [-6, \infty)$



$D_{gog} = [-6, 3]$

* $g(f(x)) = g(x^2+3)$

$= \sqrt{3-(x^2+3)}$

$= \sqrt{3-x^2-3}$

$= \sqrt{-x^2}$ undefinted

Dgog

$\sqrt{3-\sqrt{3-x}} \geq 0$

$3-\sqrt{3-x} \geq 0$

$3 \geq \sqrt{3-x}$

$9 \geq 3-x$

$x \geq -6$

2) $(fog)(x) = 3x^2+3x+2$, $f(x) = 3x+5$ find $g(x)$?

$f(g(x)) = (fog)(x)$

$3(g(x))+5 = 3x^2+3x+2$

$3(g(x)) = 3x^2+3x-3$

$g(x) = x^2+x-1$

3) $(fog)(x) = x^2+6x+6$, $g(x) = x+1$ find $f(x)$?

$f(g(x)) = (fog)(x)$

$f(x+1) = x^2+6x+6$

Let $y = x+1$

$x = y-1$

$f(y) = (y-1)^2+6(y-1)+6$

$= y^2-2y+1+6y-6+6$

$f(y) = y^2+4y+1$

$f(x) = x^2+4x+1$

4) $f(x) = \frac{x}{x+1}$, $g(x) = x^3$, $h(x) = x+2$

- $(f \circ g \circ h) = f(g(h(x)))$

$g(h(x)) = (x+2)^3$

$f(g(h(x)) = \frac{(x+2)^3}{(x+2)^3+1}$

- $(h \circ g)(x) = h(g(x)) = h(x^3)$

$= x^3 + 2$

- $(f \circ g)(x) = f(g(x))$

$= \frac{x^3}{x^3+1}$

- $(f \circ f)(x)$

$= f(f(x))$

$= \frac{\frac{x}{x+1}}{\frac{x}{x+1} + 1}$

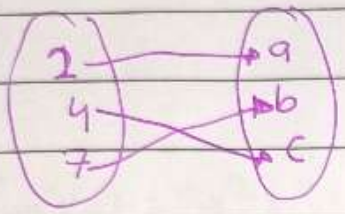
Subject Calculus (1) Lecture (18) Date No.

One to One function

f is called one to one function if $f(x_1) \neq f(x_2)$ whenever $x_1 \neq x_2$

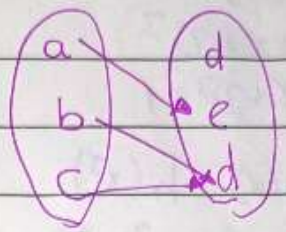
Example :-

1)



one to one function

2)



$b = c$

but $f(b) = f(c) = f$

Not one to one function

Remark : Test for one to one function : If $f(a) = f(b) \Rightarrow a = b$ then f is one to one function

Example :- Is $f(x) = x^2$ one to one function?

$-1 \neq 1$ but $f(1) = f(1) = 1$

Not one to one function

Example : Is $f(x) = 3x - 2$ one to one function?

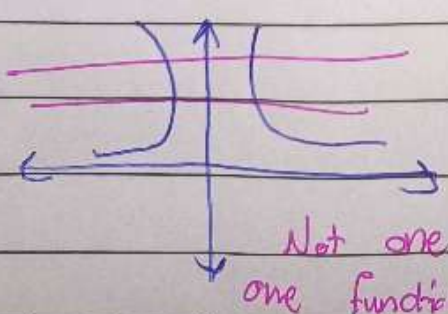
$f(a) = f(b)$

$3a - 2 = 3b - 2$

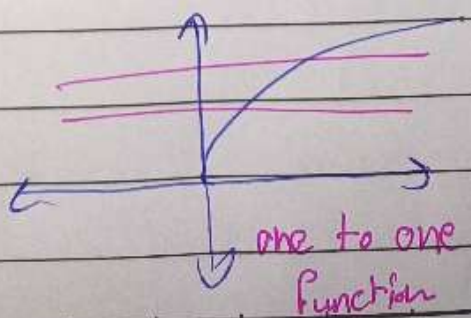
$a = b$

one to one function

* Horizontal line test اختبار الخط الأفقي



Not one to one function



one to one function

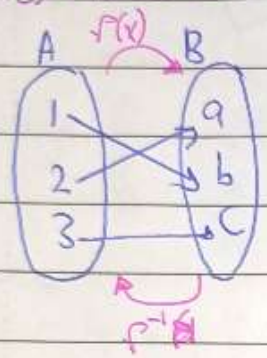
Subject Calculus (1) Lecture (9) Date No.

Inverse function الاقتران العكسي

Theorem: f has inverse iff f one to one function
(if and only if)

Definition: Let f is one to one function, then its inverse is defined by:

$f^{-1}(y) = x \iff y = f(x)$



$f(1) = b$ $\quad f^{-1}(a) = 2$

$f(2) = a$ $\quad f^{-1}(b) = 1$

$f(3) = c$ $\quad f^{-1}(c) = 3$

$f(1) = b \longleftrightarrow f^{-1}(b) = 1$

$f(2) = a \longleftrightarrow f^{-1}(a) = 2$

$f(3) = c \longleftrightarrow f^{-1}(c) = 3$

$D_f \longrightarrow R_f$ $\quad f(2) = a$

$R_f^{-1} \longrightarrow D_f$ $\quad f^{-1}(f(2)) = f^{-1}(a)$

$2 = f^{-1}(a)$

$f^{-1}(c) = 3$

$f(f^{-1}(c)) = f(3)$

$c = f(3)$

*Remark:-

1) Domain f^{-1} = Range f (B)

2) Rang f^{-1} = Domain f (A)

3) $(f \circ f^{-1})(x) = x = f(f^{-1}(x)) = x \quad \forall x \in B$ ← مجال الاقتران الرأسي

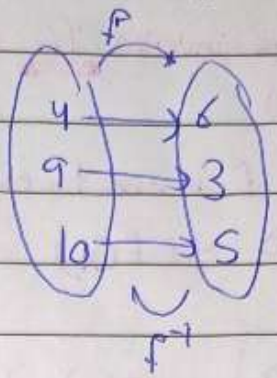
4) $(f^{-1} \circ f)(x) = x = f^{-1}(f(x)) = x \quad \forall x \in A$ ← مجال الاقتران الداخلي

5) f and f^{-1} Symmetric about $y = x$

$\forall$ ← for all الكل

$\exists$ ← there exists يوجد

Example: Given that f has an inverse $f(4)=6$, $f(9)=3$
$f^{-1}(5)=10$ find



1) $f^{-1}(6)=4$

~~$f(f^{-1}(6))=f^{-1}(4)$~~

$f(4)=6$

$f^{-1}(f(4))=f^{-1}(6)$

$4=f^{-1}(6)$

2) $f(10)=5$

3) $f^{-1}(3)=9$

Lecture (20)

Example: Find $f^{-1}(x)$ for:

1) $f(x)=\sqrt{3x-2}$

$y=\sqrt{3x-2}$

$y^2=3x-2$

$y^2+2=3x$

$x=\frac{y^2+2}{3}$

$f^{-1}(x)=\frac{x^2+2}{3}$

2) $f(x)=\frac{x+1}{x-1}$

$y(x-1)=x+1$

$yx-y=x+1$

$yx-x=1+y$

$x(y-1)=y+1$

$x=\frac{y+1}{y-1}$

$f^{-1}(x)=\frac{x+1}{x-1}$

* Find domain f^{-1}

$f^{-1}(x)=\frac{x+1}{x-1}\rightarrow R$

$x-1\rightarrow R$

$D_{f^{-1}}=R-\{1\}$

$D_{f^{-1}}=$ Range f $R-\{1\}$

* Find Range f^{-1}

D_f $R-\{1\}$

Subject Calculus (1) Lecture (20) Date No.

Example: Find domain & Range for $f(x) = \frac{x+2}{2x-1}$?

$f(x) = \frac{x+2}{2x-1} \to \mathbb{R}$

$D_f = \mathbb{R} \cap \mathbb{R} - \{\frac{1}{2}\}$

$= \mathbb{R} - \{\frac{1}{2}\}$

Range f = Domain f^{-1}

$y = \frac{x+2}{2x-1}$

$y(2x-1) = x+2$

$2xy - y = x + 2$

$2xy - x = 2 + y$

$x = \frac{y+2}{2y-1}$

$f^{-1}(x) = \frac{x+2}{2x-1} \to \mathbb{R}$

$D_{f^{-1}} = \text{Range } f$

$= \mathbb{R} \cap \mathbb{R} - \{\frac{1}{2}\}$

Range $f = \mathbb{R} - \{\frac{1}{2}\}$

Subject Calculus (1) Lecture (21) Date No.

Example: $f(x)=6x-x^2$, $x\geq 3$ Find $f^{-1}(x)$?

$y=6x-x^2$

$-y=-6x+x^2$

$-y=x^2-6x$

$-y=x^2-6x+9-9$

$-y=(x-3)(x-3)+9$

$-y+9=(x-3)^2$

$\sqrt{9-y}=|x-3|$

$x=\sqrt{9-y}+3$

$f^{-1}(x)=\sqrt{9-x}+3$

$(\frac{1}{2}\times -6)^2$

9

$f^{-1}(x)$ فقط للاقتران one to one

* اكمال المربع لجعل x موضوع القانون:

1) يجب أن يكون معامل x^2 موجب 1

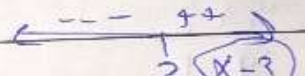
2) نضيف ونطرح المقدار $(\frac{1}{2}\times$ معامل $x)^2$

3) نحلل

* إعادة تعريف المطلقة:

$x-3=0$

$x=3$



اكملوا في المنزل

Example: Determine whether f and g are inverse functions:-

$f(x)=x^3+3x^2+3x+1=(x+1)^3$

$g(x)=x^{\frac{1}{3}}-1$

$f(g(x))=f(x^{\frac{1}{3}}-1)$

$=(x^{\frac{1}{3}}-1+1)^3$

$=x$

$g(f(x))=g((x+1)^3)$

$=((x+1)^3)^{\frac{1}{3}}-1$

$=x+1-1$

$=x+1-1$

$=x$

f, g are inverse

$f=g^{-1}$ $g=f^{-1}$

NOTEBOOK

Subject Calculus (1) Lecture (21) Date No.

متطابقات هامة:-

$(a+b)^3 = (a+b)(a^2+2ab+b^2)$

$(a-b)^3 = (a-b)(a^2-2ab+b^2)$

$(a+b)^2 = a^2+2ab+b^2$

$(a-b)^2 = a^2-2ab+b^2$

$a^2+b^2 =$ لا يحلل

$a^2-b^2 = (a-b)(a+b)$

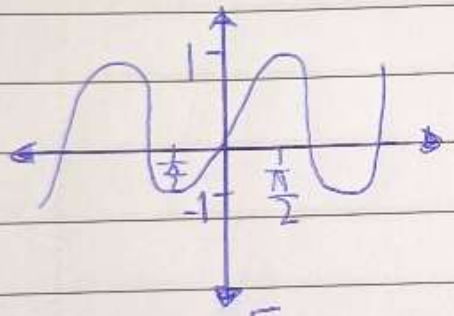
$a^3+b^3 = (a+b)(a^2-ab+b^2)$

$a^3-b^3 = (a-b)(a^2+ab+b^2)$

Lecture (22)

* Inverse Trigometric function (الاقترانات العكسية المثلثية)

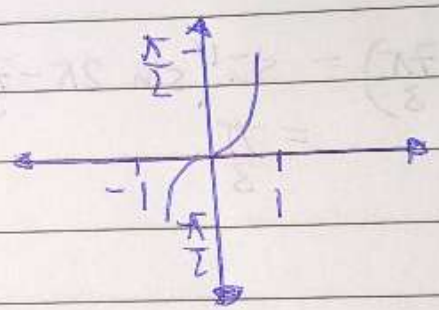
1



$f(x) = \sin x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$D_{\sin x} = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$R_{\sin x} = [-1, 1]$



$f(x) = \sin^{-1} x \quad , x \in (-1, 1)$

$= \arcsin x$

$D_{\sin^{-1} x} = [-1, 1]$

$R_{\sin^{-1} x} = [-\frac{\pi}{2}, \frac{\pi}{2}]$

Remark:-

$D_{\sin^{-1} x} = R_{\sin x} = [-1, 1]$

$R_{\sin^{-1} x} = D_{\sin x} = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\sin x\,(\sin^{-1} x) = x \quad \forall x \in [-1, 1]$

$\sin^{-1}(\sin x) = x \quad \forall x \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

$\sin^{-1}(-x) = -\sin^{-1}(x)$ odd function

Subject Calculus (1) Lecture (22) Date No.

Example:-

1) $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

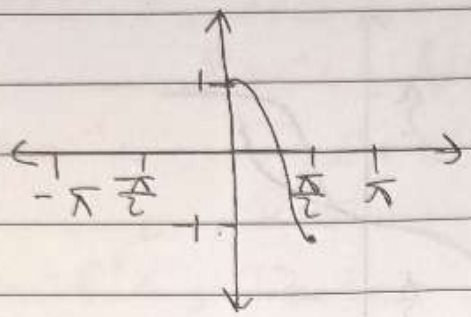
2) $\sin^{-1}\left(\frac{-1}{2}\right) = -\sin^{-1}\left(\frac{1}{2}\right)$
$= -\frac{\pi}{6}$

3) $\sin^{-1}\left(\sin\frac{\pi}{7}\right) = \frac{\pi}{7}$

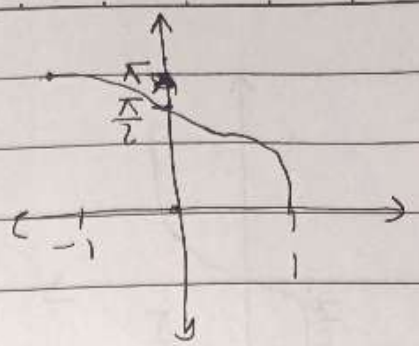
4) $\sin^{-1}\left(\sin\frac{11\pi}{6}\right) = \sin^{-1}\left(-\sin\left(2\pi - \frac{11\pi}{6}\right)\right)$
$= \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$
$= -\frac{\pi}{6}$

5) $\sin^{-1}\left(\sin\frac{7\pi}{3}\right) = \sin^{-1}\left(\sin 2\pi - \frac{7\pi}{3}\right)$
$= \frac{\pi}{3}$

Subject Calculus (1) Lecture (23) Date No.



$f(x) = \cos x \;,\; x \in [0, \pi]$



$f(x) = \cos^{-1} x \quad x \in [-1, 1]$

Remarks:-

1) $D_{\cos^{-1} x} = [-1, 1]$

2) $R_{\cos^{-1} x} = [0, \pi]$

3) $\cos(\cos^{-1} x) = x \;,\; \forall x \in [-1, 1]$

4) $\cos^{-1}(\cos x) = x \;,\; \forall x \in [0, \pi]$

5) $\cos^{-1}(-x) = \pi - \cos x$ ($\cos^{-1} x$ not even / not odd)

Examples: Find

1) $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

2) $\cos^{-1}\left(\frac{-1}{\sqrt{2}}\right) = \pi - \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$= \pi - \frac{\pi}{4}$

$= \frac{3\pi}{4}$

3) $\cos^{-1}\left(\cos \frac{11\pi}{6}\right) = \frac{11\pi}{6}$

$\cos^{-1}\left(\cos\left(2\pi - \frac{11\pi}{6}\right)\right)$

$\cos^{-1}\left(\cos \frac{\pi}{6}\right)$

$= \frac{\pi}{6}$

4) $\cos^{-1}\left(\cos \frac{7\pi}{6}\right)$

$= \cos^{-1}\left(-\cos\left(\frac{7\pi}{6} - \pi\right)\right)$

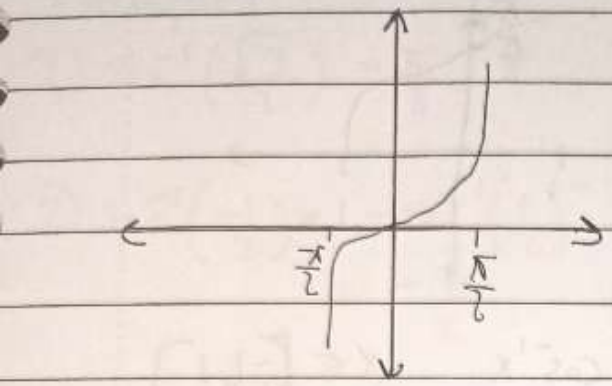
$= \cos^{-1}\left(-\cos \frac{\pi}{6}\right)$

$= \pi - \cos^{-1}\left(\cos \frac{\pi}{6}\right)$

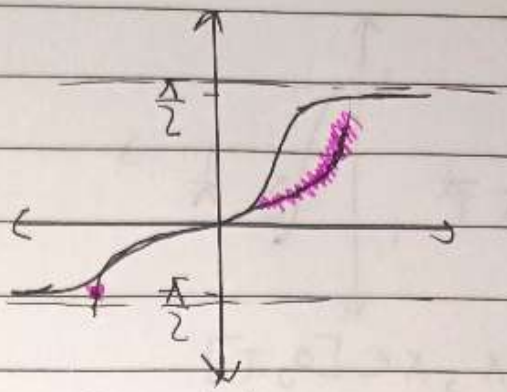
$= \pi - \frac{\pi}{6}$

$= \frac{5\pi}{6}$

Subject Lecture (24) Date No.



$f(x) = \tan x$, $x \in [-\frac{\pi}{2}, \frac{\pi}{2})$



$f(x) = \tan^{-1} x$, $x \in R$

Remark :-

1) $D_{\tan^{-1} x} = R$

2) $R_{\tan^{-1} x} = (-\frac{\pi}{2}, \frac{\pi}{2})$

3) $\tan(\tan^{-1} x) = x$, $\forall x \in R$

4) $\tan^{-1}(\tan x) = x$ $\forall x \in (-\frac{\pi}{2}, \frac{\pi}{2})$

5) $\tan^{-1}(-x) = -\tan^{-1} x$ odd function.

Example : find

1) $\tan^{-1}(1) = \frac{\pi}{4}$

2) $\tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$

3) $\tan^{-1}\left(\tan \frac{\pi}{6}\right) = \frac{\pi}{6}$

4) ~~$\tan^{-1}$~~(

Subject Calculus(1) Lecture(24) Date No.

* $f(x)=\csc^{-1}x$, $D_{\csc^{-1}x}=(-\infty,-1]\cup[1,\infty)$

$R_{\csc^{-1}x}=(0,\frac{\pi}{2}]\cup[\pi,\frac{3\pi}{2}]$

* $f(x)=\sec^{-1}x$, $D_{\sec^{-1}x}=(-\infty,-1]\cup[1,\infty)$

$R_{\sec^{-1}x}=[0,\frac{\pi}{2})\cup[\pi,\frac{3\pi}{2})$

* $f(x)=\cot^{-1}x$, $D_{\cot^{-1}x}=R$

$R_{\cot^{-1}x}=(0,\pi)$

Example: Find

1) $\sec^{-1}(2)$ → $\frac{1}{\cos\theta}=2$

$=\frac{\pi}{3}$ $\frac{1}{\frac{1}{2}}=\sec\theta$

$\cos\theta=\frac{1}{2}$

2) $\cot^{-1}(+\sqrt{3})$ → $\frac{1}{\tan\theta}=\sqrt{3}$

$=\frac{\pi}{6}$ $\tan\theta=\frac{1}{\sqrt{3}}$

$\theta=\frac{\pi}{6}$

3) $\cot^{-1}(-\sqrt{3})$

$=\frac{5\pi}{6}$ او $\frac{11\pi}{6}$

استخدم الحل الذي

$\frac{5\pi}{6}$ بدي

$\cot\theta=-1$ → $\frac{1}{\tan\theta}$

$\tan\theta=\sqrt{3}$ $\tan\theta=\frac{-1}{\sqrt{3}}$

$\theta=\frac{5\pi}{6}$,

Lecture (25)

Example: Find domain $f(x)=\sin^{-1}(3x+1)$?

$-1\le 3x+1\le 1$

$-2\le 3x\le 0$

$\frac{-2}{3}\le x\le 0$

Example: Find domain & rang $f(x)=\frac{\pi}{2}+\tan^{-1}x$

R ← $\frac{\pi}{2}$ + $\tan^{-1}x$ → R

$D_f=R\cap R=R$

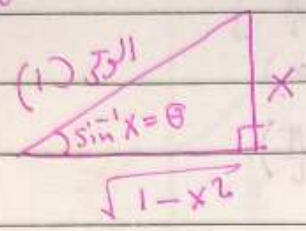
$R_f=(0,\pi)$

$-\frac{\pi}{2}<\tan^{-1}x<\frac{\pi}{2}$

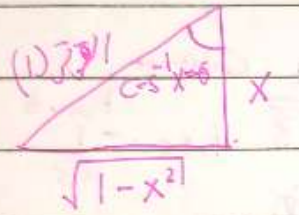
$0<\tan^{-1}x+\frac{\pi}{2}<\pi$

Subject Calculus (1) Lecture (25) Date No.

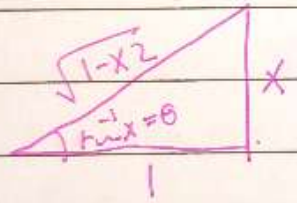
* $\sin(\sin^{-1}x) = x =$ المقابل / الوتر



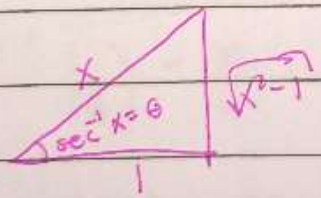
* $\cos(\cos^{-1}x) = x =$ المجاور / الوتر



* $\tan(\tan^{-1}x) = x =$ المقابل / المجاور



* $\sec(\sec^{-1}x) = x =$ ~~المقابل~~ الوتر / المجاور



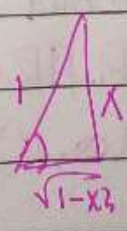
Example:-

1) $\sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$ أو لا بد من ذكر النظرية

2) $\tan(\sin^{-1}x) =$

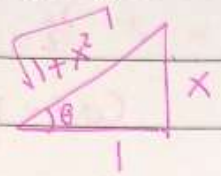
$\sin(\sin^{-1}x) = x = x$

$\tan\theta = \frac{x}{\sqrt{1-x^2}}$



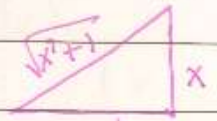
Subject Calculus(1) Lecture(25) Date No.

3) $\sec(\tan^{-1}x)$



$\sec\theta = \frac{\sqrt{x^2+1}}{1}$

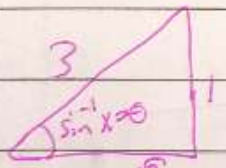
4) $\cos^2(\tan^{-1}x)$



$\left(\cos\left(\frac{1}{\sqrt{x^2+1}}\right)\right)^2$

$= \frac{1}{x^2+1}$

5) $\cot(\sin^{-1}\frac{1}{3})$



$3^2 = 1^2 + x^2$

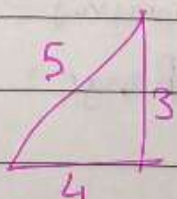
$9 = 1 + x^2$

$x = \sqrt{8}$

$\cot = \sqrt{8}$

6) $\sin(2\sin^{-1}(\frac{3}{5}))$ $= 2\sin(\sin^{-1}(\frac{3}{5}))\cos(\sin^{-1}(\frac{3}{5}))$

$= 2\left(\frac{3}{5}\right)\cos(\sin^{-1}\frac{3}{5})$



$= \left(\frac{6}{5}\right)\left(\frac{4}{5}\right)$

$= \frac{24}{}$

ملاحظة

$\cos^2 x = (\cos x)^2$

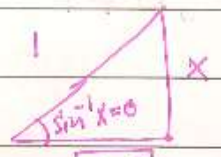
$\cos x^2 \neq \cos^2 x$

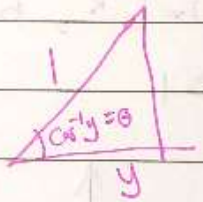
* $\sin 2a = 2\sin a \cos a$
* $\sin(a+b) = \sin a\cos b + \cos a \sin b$
* $\cos(a+b) = \cos a\cos b - \sin a \sin b$

7) $\sin(\sin^{-1}x + \cos^{-1}y)$

$\sin(\sin^{-1}x)\cos(\cos^{-1}y) + \cos(\sin^{-1}x)\sin(\cos^{-1}y)$

$xy + \cos(\sin^{-1}x)\sin\cos^{-1}y$

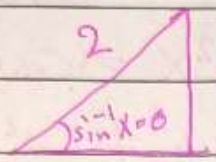




$xy + \sqrt{1-x^2}(y)$

Subject Calculus (1) Lecture (25) Date No.

8) $\tan(\sin^{-1}(\frac{1}{2}))$



$2^2 = 1^2 + x^2$

$3 = x^2$

$x = \sqrt{3}$

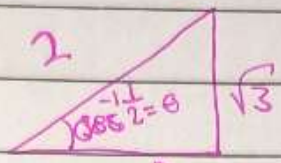
$= \frac{1}{\sqrt{3}}$

9) $\sin(\sec^{-1}(2))$

$\sec^{-1}(2) = \frac{1}{\cos\theta}$

$= \frac{1}{\frac{1}{2}}$

$\cos\theta = \frac{1}{2}$



$2^2 = 1^2 + x^2$

$x = \sqrt{3}$

$= \frac{\sqrt{3}}{2}$

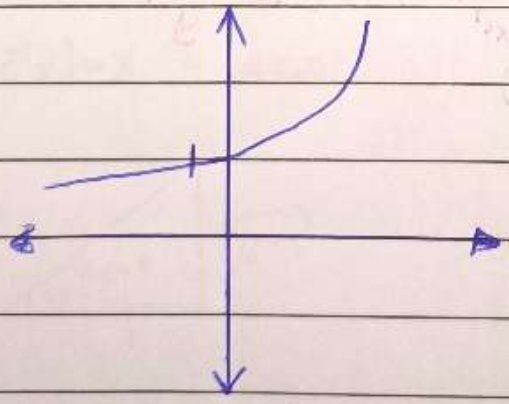
Lecture (26)

Exponential (اي) and Logarithmic (لوغاريثمي) Functions: -

* Definition: Exponential function

$f(x) = b^x$, $b > 0$, $b \neq 1$

→ If $b > 1$

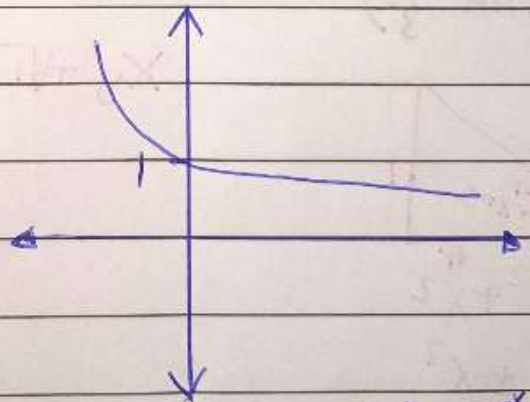


Ex: $f(x) = 3^x$

Domain $f = R$

Range $= (0, \infty)$

If $0 < b < 1$



Ex: $f(x) = (\frac{1}{2})^x = (2^{-1})^x = 2^{-x}$

Domain $f = R$

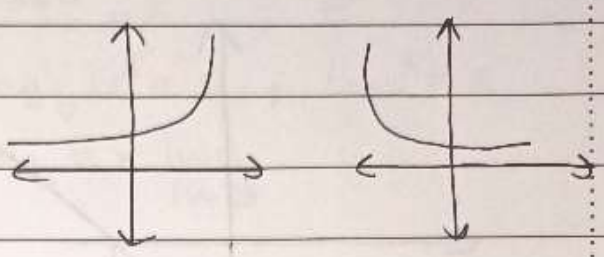
Range $= (0, \infty)$

Subject Calculus (1) Lecture (26) Date No.

* The natural exponential function الاقتران الأسي الطبيعي

$f(x) = e^x$ $e \simeq 2.718$

$f(x) = e^{-x} = \left(\frac{1}{e}\right)^x$



Example: find domain

1) $f(x) = e^{\sqrt{x}}$

$D_f = D_{\sqrt{x}} = [0, \infty)$

2) $F(x) = \frac{x \longrightarrow R}{1 + e^x \longrightarrow R}$

$D_f = R \cap R = R$

$e^x + 1 = 0$

$e^x \neq -1$ لا يوجد أصفار للمقام

Domain $(e^{g(x)}, b^{g(x)})$ = Domain $(g(x))$

Lecture (27)

* Logarithmic function الاقتران اللوغاريتمي

$2^5 = 32 \iff \log_2 32 = 5$

$\log_2 16 = 4 \iff 2^4 = 16$

$x^y = z \iff \log_x z = y$

* Example: find the value $\log_9 3 = \frac{1}{2}$

$9^x = 3$

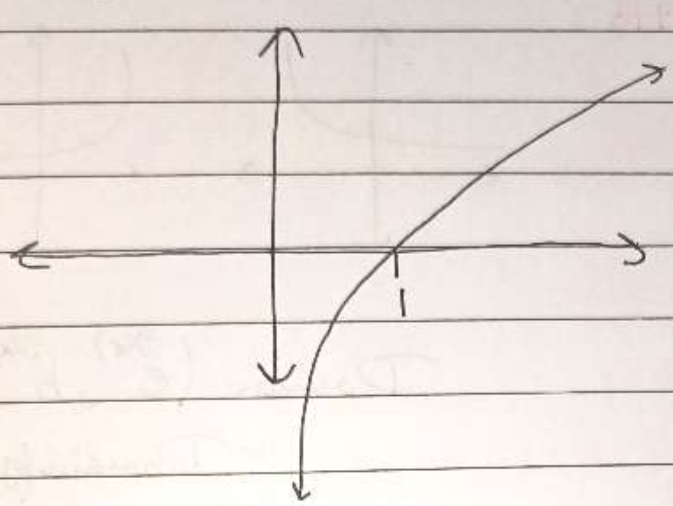
$3^{2x} = 3$

$2x = 1$

$x = \frac{1}{2}$

Subject Calculus (1) Lecture (27) Date No.

* $f(x) = \log_b x$, $b > 0$, $b \neq 1$, $x > 0$



$\log_b 1 = 0$

$\log_b b = 1$

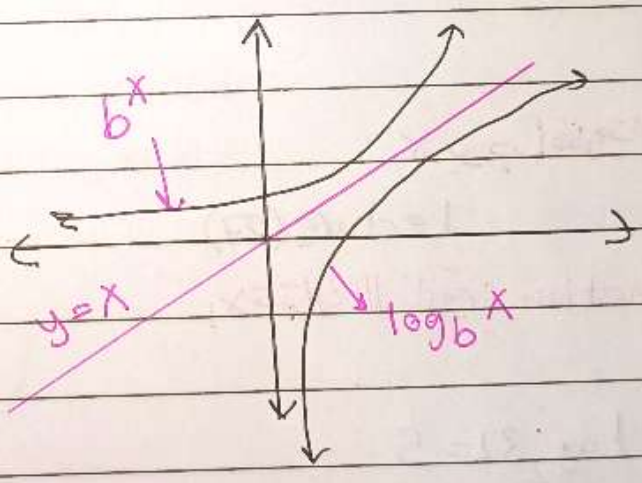
$D_f = (0, \infty)$

$R_f = R$

* Theorem: If $b > 0$, $b \neq 1$ Then $f(x) = b^x$, $f(x) = \log_b x$ are inverse.

$f^{-1}(x)$ الـ $f(x)$ الـ

$y = x$ حول متماثلين



* $\log_{10} x = \log x$ Common log (العشري)

* $\log_e x = \ln x$ natural log (الطبيعي)

* Example:-

1) $\log 10 = \log_{10} 10 = 1$

2) $\ln e = \log_e e = 1$

3) $\ln 1 = \log_e 1 = 0$

Subject | Lecture (27+28) | Date | No.

(27)

* Example: Find domain:-

1) $f(x) = \log(x^2 - 9)$

$x^2 - 9 > 0$

$(x+3)(x-3) > 0$

$\Longleftrightarrow$



$D_f = (-\infty, -3) \cup (3, \infty)$

2) $f(x) = \frac{1}{\ln(4-x)} \rightarrow R$

* $\ln(4-x)$

$4 - x > 0$

$4 > x$

$(-\infty, 4)$

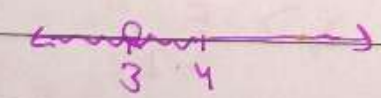
* $\ln(4-x) = 0$

$e^0 = 4 - x$

$1 = 4 - x$

$\boxed{x = 3}$

$R_f = R \cap (-\infty, 4) - \{3\}$

$\frac{1}{\ln(4-x)}$ 

$R = (-\infty, 3) \cup (3, 4)$

(28)

* Logarithm properties:-

1) $b^{\log_b x} = x$, $e^{\ln x} = x$

2) $\log_b b^x = x$, $\ln e^x = x$

3) $\log_b a = \frac{\ln a}{\ln b}$

4) $\log_b ac = \log_b a + \log_b c$

5) $\log_b \frac{a}{c} = \log_b a - \log_b c$

6) $\log_b a^r = r \log_b a$

7) $\log_b x = \log_b y \Longrightarrow x = y$

Example:-

1) $\log \sqrt{x} = 2$

$10^2 = \sqrt{x}$

$100 = \sqrt{x}$

$x = 10000$

2) $\log(x-2) + \log(2x-3) = 2\log x$

$\log(x-2)(2x-3) = \log x^2$

$(x-2)(2x-3) = x^2$

$2x^2 - 3x - 4x + 6 = x^2$

$x^2 - 7x + 6 = 0$

$(x-6)(x-1) = 0$

$\boxed{x = 6}$ $\boxed{x = 1}$

$\log(1-2)$ ✗ ~~$\boxed{1}$~~

$\boxed{6}$

NOTE

Subject Lecture(29) Date No.

Example : Find the value of x :-

1) $\log_x 4 = 2$

$\sqrt{x^2} = \sqrt{4} \rightarrow |x| = 2$

$x = \pm 2$ $\quad x = -2$ مرفوضة

$x = 2$

2) $5^x = 7$

$x \log_5 5 = \log_5 7$

$x = \log_5 7$ طريقة أولى

$\ln 5^x = \ln 7$

$x = \frac{\ln 7}{\ln 5}$

3) $(x^2-1)(x-5)(x^3)\log_x 2x = 0$

$x^2 - 1 = 0$

$x = \pm 1$ مرفوض

$x = 5$ مقبول

$x = 0$ مرفوضة

$\log_x 2x = 0$

$x^0 = 2x$

$1 = 2x$

$x = \frac{1}{2}$ مقبول

4) $e^{2x} - 6e^x - 7 = 0$ $\quad e^x = y$

$(e^x)^2 - 6e^x - 7 = 0$

$(y-7)(y+1) = 0$

$y = 7$ $\quad y = -1$

$e^x = 7$ $\quad e^x \neq -1$

$\ln e^x = \ln 7$

$x = \ln 7$

5) $1 - 2\ln x < 3$

$-2\ln x < 2$

$\ln x > -1$

$e^{\ln x} > e^{-1}$

$x > e$

$x \in (e, \infty)$

Remark

$\ln x \geq 0 \rightarrow x \geq 1$

$\ln x < 0 \rightarrow 0 < x < 1$

$(\ln 2)x > 5$

$x > \frac{5}{\ln 2}$

$(\ln \frac{1}{2})x > 5$

$x < \frac{5}{\ln \frac{1}{2}}$

NOTEBOOK

Subject Calculus (1) Lecture (29) Date No.

Example:-

$f(x) = \ln(x-1) + 5$ Find $f^{-1}(x)$?

$y = \ln(x-1) + 5$

$y - 5 = \ln(x-1)$

$e^{y-5} = x - 1$

$e^{y-5} + 1 = x$

$f^{-1}(x) = e^{x-5} + 1$

Example: Find the exact value of

1) $\log_2 8 + \log_4 2$

* $\log_2 8 \longrightarrow 2^x = 8$

$x = 3$

* $\log_4 2$

$4^x = 2$

$x = \frac{1}{2}$

$3 + \frac{1}{2}$

$\frac{6}{2} + \frac{1}{2} = \frac{7}{2}$

2) $\ln(\ln e^{e^b})$

$\ln e^{e^b}$

$e^x = \ln e^{e^b}$

$e^x = e^b \ln e$

$e^x = e^b$

$x = b$

Chapter (2)

Subject lecture(30) Date No.

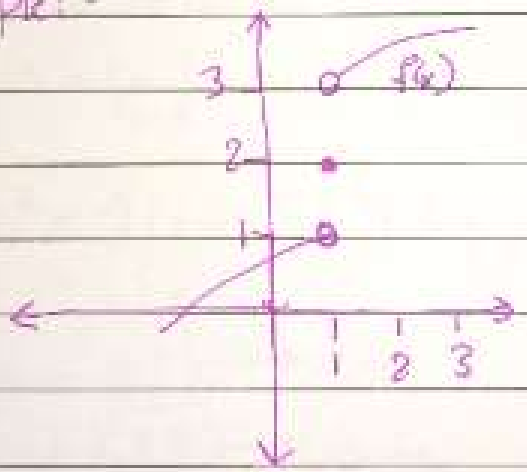
The limit of a function

Definition: Suppose $f(x)$ is defined when x is near the number a. then we write $\lim_{x \to a} f(x) = L$ and say "The limit of $f(x)$ as x goes to a, equal L"

If $\lim_{x \to a^+} f(x) = L$ and $\lim_{x \to a^-} f(x) = L$ then $\lim_{x \to a} f(x) = L$ exist

If $\lim_{x \to a^+} f(x) \neq \lim_{x \to a^-}$ then $f(x)$ does not exist $x \to a$

Example:-



find:-

1) $f(1) = 2$
2) $\lim_{x \to 1^+} f(x) = 3$
3) $\lim_{x \to 1^-} f(x) = 1$
4) $\lim_{x \to 1} f(x)$ = does not exist

* Limits laws: Suppose that c is a constant and the limits $\lim_{x \to a} f(x)$, $\lim_{x \to a} g(x)$ exist, then:-

1) $\lim_{x \to a} (f(x) \pm g(x)) = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x)$

2) $\lim_{x \to a} cf(x) = c \lim_{x \to a} f(x)$

3) $\lim_{x \to a} (f(x) \cdot g(x)) = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$

Subject lecture(30) Date No.

4) $\lim_{x\to a}\left(\frac{f(x)}{g(x)}\right) = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$ if $\lim_{x\to a} g(x) \neq 0$

5) $\lim_{x\to a} c = c$

6) $\lim_{x\to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x\to a} f(x)}$ n Positive integer (If n even we assure that $\lim_{x\to a} f(x) > 0$)

7) $\lim_{x\to a} f(x) = f(a)$ If f is a polynomial or rational functions and $a \in D_f$

8) If $f(x) \leq g(x)$ then $\lim_{x\to a} f(x) \leq \lim_{x\to a} g(x)$

Example: Find

1) $\lim_{x\to 5} 9 = 9$

2) $\lim_{x\to 1} (x^3 - 2x + 1) = 0$

lecture(31)

* $\lim_{x\to a} \frac{f}{g}$ → DNE / ∞ / DNE

→ $\lim_{x\to a} \frac{f}{g}$ = DNE

Example: $\lim_{x\to -2} \frac{2x+4}{3x-2} = \frac{(2)(-2)+4}{3(-2)-2} = 0$

Subject Lecture (3D Date No.

$\lim_{x\to a} \frac{f}{g} = \frac{f'}{g'}$ * تحليل المقدار بالتحليل

(المقدار) (المشتقة الأولى) (المشتقة)



Examples find

1) $\lim_{x\to -4} \frac{2x+8}{x^2+x-12} = \frac{0}{0}$

$\lim_{x\to -4} \frac{2(x+4)}{(x+4)(x-3)} = \frac{-2}{7}$

2) $\lim_{x\to 0} \frac{x}{1-\sqrt{x+1}} = \frac{0}{0}$ ضرب بمرافق

$\lim_{x\to 0} \frac{x}{1-\sqrt{x+1}} \cdot \frac{1+\sqrt{x+1}}{1+\sqrt{x+1}}$

$= \lim_{x\to 0} \frac{x(1+\sqrt{x+1})}{1-(x+1)}$

$= \lim_{x\to 0} \frac{x(1+\sqrt{x+1})}{-x}$

$= -2$

3) $\lim_{x\to -4} \frac{\frac{1}{4}+\frac{1}{x}}{4+x} = \frac{0}{0}$ توحيد مقامات

$= \lim_{x\to -4} \frac{x+4}{4x} \cdot \frac{1}{4+x}$

$= \lim_{x\to -4} \frac{1}{4x}$

$= \frac{-1}{16}$

NOTEBOOK

Subject Lecture (32) Date No.

Example: Find

1) $\lim_{x\to 1} \frac{\sqrt[4]{x+15}-2}{x-1} = \frac{0}{0}$ الحل

$y = \sqrt[4]{x+15}$ $x \to 1$

$y^4 = x+15$

$x = y^4 - 15$ $y \to 2$

$\lim_{x\to 1} \frac{y-2}{y^4-15-1}$

$\lim_{x\to 1} \frac{y-2}{y^4-16}$

$\lim_{x\to 1} \frac{(y-2)}{(y^2+4)(y-2)(y+2)}$

$= \lim_{x\to 1} \frac{1}{(y^2+4)(y+2)}$

$= \frac{1}{32}$

2) $\lim_{x\to 0} \frac{|x|}{x}$ (اعادة تعريف المقام)



$\lim_{x\to 0^+} \frac{x}{x} = \lim_{x\to 0^+} 1 = 1$

$\lim_{x\to 0^-} \frac{-x}{x} = \lim_{x\to 0^-} -1 = -1$

$\therefore \lim_{x\to 0} \frac{|x|}{x}$ = does not exist

3) $\lim_{x\to 2} \frac{|x-3|}{2x-6} = \frac{1}{-2}$

4) $\lim_{x\to 3^+} \frac{|x-3|}{2x-6} = \frac{0}{0}$

$= \lim_{x\to 3^+} \frac{(x-3)}{2(x-3)} = \frac{1}{2}$



$\lim_{x\to 3^-} \frac{(3-x)}{2(x-3)} = \frac{-1}{2}$

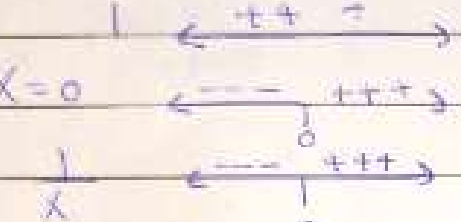
$\lim_{x\to 3} \frac{|x-3|}{2x-6}$ = does not exist

Subject Lecture (33) Date No.

- $\lim_{x \to a} \frac{f}{g} = \frac{\text{شيء}}{\text{صفر}}$ → $+\infty$ / $-\infty$ / diverge

Example: Find

1) $\lim_{x \to 0^+} \frac{1}{x} = \frac{1}{0} = +\infty$



$\lim_{x \to 0^-} \frac{1}{x} = -\infty$

$\lim_{x \to 0} \frac{1}{x} = \text{diverge}$

2) $\lim_{x \to 2} \frac{x}{x^2-4} = \frac{2}{0}$





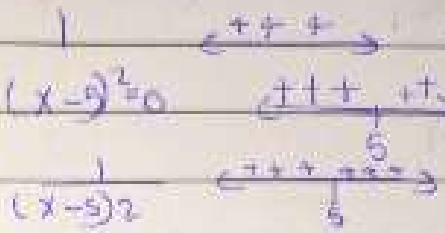


$\lim_{x \to 2^+} \frac{x}{x^2-4} = +\infty$

$\lim_{x \to 2^-} \frac{x}{x^2-4} = -\infty$

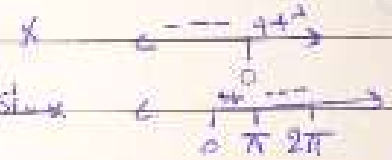
$\lim_{x \to 2} \frac{x}{x^2-4} = \text{diverge}$

3) $\lim_{x \to 5^-} \frac{1}{(x-5)^2} = \frac{1}{0}$



$\lim_{x \to 5^-} \frac{1}{(x-5)^2} = +\infty$

4) $\lim_{x \to \pi^-} x \csc x = \lim_{x \to 2\pi^-} \frac{x}{\sin x}$



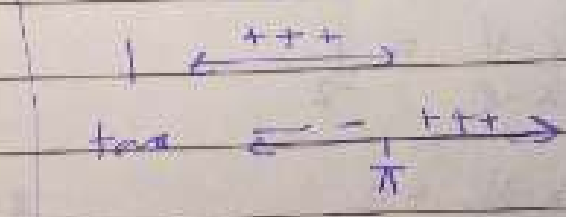
$= \infty$

لو كانت

$\lim_{x \to 0} \frac{x}{x^2-4}$

$= 0$

5) $\lim_{x \to \pi^-} \cot x = \lim_{x \to \pi^-} \frac{1}{\tan x}$



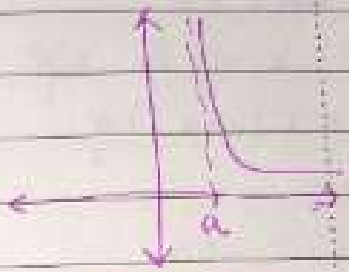
$= -\infty$

Subject Lecture (34) Date No.

Definition: The line $x = a$ is called (a vertical asymptote) of the curve $y = f(x)$ at least one of the following statements is true:-

$\lim_{x \to a} f(x) = \infty$ $\lim_{x \to a^-} f(x) = \infty$ $\lim_{x \to a^+} f(x) = \infty$

$\lim_{x \to a} f(x) = -\infty$ $\lim_{x \to a^-} f(x) = -\infty$ $\lim_{x \to a^+} f(x) = -\infty$



$\lim_{x \to a^+} f(x) = \infty$

* Example: Find the vertical asymptote of

1) $f(x) = \frac{x-1}{x^2-1}$

$x^2 - 1 = 0$ $x = 1$ $x = -1$

at $x = 1$

$\lim_{x \to 1} \frac{x-1}{(x-1)} = \lim_{x \to 1} = \frac{1}{2}$

at $x = -1$

$\lim_{x \to -1} = \frac{-2}{0}$

$+ + \; 0 \; - -$ (over $-\infty$, -1, ∞)

is at $x = -1$ vertical asymptote

2) $f(x) = \frac{x-2}{|x|-2}$ $|x|$ $- - \; + +$ (over $-x$, 0, x)

$|x| - 2 = 0$

$|x| = 2$

$x = 2 / -2$

at $x = 2$

$\lim_{x \to 2} \frac{x-2}{|x|-2} = \lim_{x \to 2} = 1$

at $x = -2$

$\lim_{x \to -2} \frac{x-2}{|x|-2} = \frac{-4}{0}$

$x - 2 < 0$ $- - \; + +$

$\lim_{x \to -2} \frac{x-2}{-x-2}$ $-x-2$ $+ + \; 0 \; - -$ (over -2)

$\lim_{x \to -2^+} \frac{x-2}{|x|-2} = +\infty$ $- - \; + + \; - -$ (over -2, 2)

$\lim_{x \to -2} \frac{x-2}{|x|-2} = -\infty$

Subject lecture (34) Date No.

3) $f(x) = \tan x = \frac{\sin x}{\cos x}$

$\cos x = 0$

$x = \pm \frac{\pi}{2}, \pm \frac{3\pi}{2}, \pm \frac{5\pi}{2}$

$x = \pm (2n+1)\frac{\pi}{2}$

4) $f(x) = \ln x$

$\lim_{x \to 0^+} \ln x = -\infty$ vertical asymptote

5) $f(x) = \ln(x^2 - 9)$

$x = 3 \quad x = -3$

vertical asymptote

6) $f(x) = \cot x$

$f(x) = \frac{1}{\tan x}$

$\tan x = 0$

$x = 0, \pm\pi, \pm 2\pi$

Lecture (35)

Example: $f(x) = \begin{cases} \sqrt{x-4}, & x > 4 \\ 8 - 2x, & x \leq 4 \end{cases}$ find



1) $\lim_{x \to 4} f(x) = 8$

2) $\lim_{x \to 4} f(x) = 0$

$\lim_{x \to 4^+} \sqrt{x-4} = \sqrt{4-4} = 0$

$\lim_{x \to 4^-} 8 - 2x = 0$

Subject Lecture (35) Date No.

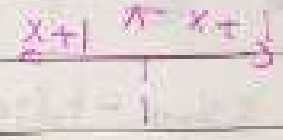
Example: $f(x)=\begin{cases} 5 & , x\le 2 \\ x+a & , x>2 \end{cases}$ find a if $\lim_{x\to 2} f(x)$ exist?



$\lim_{x\to 2^+}(x+a)=\lim_{x\to 2^-}(5)$ $\quad 2+a=5$

$\boxed{a=3}$

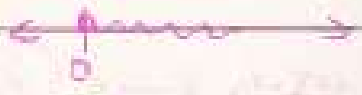
Example: $g(x)=\begin{cases} x+1, & x\neq 1 \\ \pi, & x=1 \end{cases}$ find $\lim_{x\to 1} g(x)=?$



$\lim_{x\to 1^-}(x+1)=\lim_{x\to 1^+}(x+1)=1+1=2$

Example: find

1) $\lim_{x\to 0^-}\sqrt{x}$ = d.n.e



2) $\lim_{x\to 1}\sqrt{x}=\sqrt{1}=$

3) $\lim_{x\to 7^+}\sqrt{x-7}=0$



4) $\lim_{x\to 2}\sqrt{x-7}$ = d.n.e



Theorem: $\lim_{x\to 0}\frac{\sin(ax)}{bx}=\lim_{x\to 0}\frac{ax}{\sin(bx)}=\frac{a}{b}$

$\lim_{x\to 0}\frac{\tan(ax)}{bx}=\lim_{x\to 0}\frac{ax}{\tan(bx)}=\frac{a}{b}$

Subject Lecture (35) Date No.

Example: Find

1) $\lim_{x \to 0} \frac{x}{\cos x} = \frac{0}{1} = 0$

2) $\lim_{x \to 0} \frac{\sin x}{x} = 1$

3) $\lim_{x \to 0} \frac{\tan 4x}{5x} = \frac{4}{5}$

4) $\lim_{x \to 0} \frac{x \sin x - \tan(7x)}{\sin(7x) - 5x} = \lim_{x \to 0} \frac{\frac{x \sin x}{x} - \frac{\tan 7x}{x}}{\frac{\sin(7x)}{x} - \frac{5x}{x}}$

$= \frac{-2}{-2} = 1$

5) $\lim_{x \to 5} \frac{\sin(x-5)}{x-5}$ Let $y = x-5$, $x \to 5$, $y \to 0$

$\lim_{y \to 0} \frac{\sin y}{y} = 1$

6) $\lim_{x \to 5} \frac{\sin(x^2-25)}{x-5} * \frac{(x+5)}{(x+5)} = \lim_{x \to 5} (x+5) \cdot \lim_{x \to 5} \frac{\sin(x^2-25)}{(x-5)(x+5)}$

$= 10 * 1$

$= 10$

$\lim (x^2-25)$

(x^2-5)

$y = x^2 - 25$

$x \to 5$

$y \to y$

$\lim (y)$

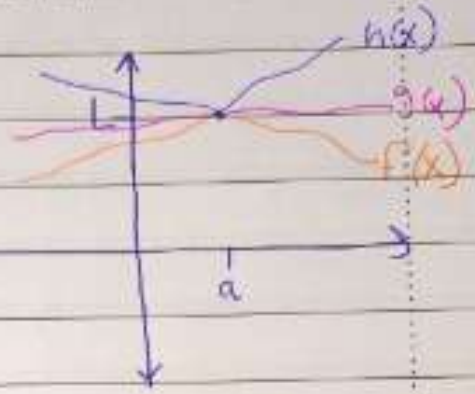
y

$= 1$

NOTEBOOK

Subject Lecture (36) Date No.

The squeeze theorem: If $f(x) \le g(x) \le h(x)$ when x is near a and $\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$ then $\lim_{x \to a} g(x) = L$



Example: If $3x \le f(x) \le x^3 + 2 \quad \forall x \in (-1, 1)$

Find $\lim_{x \to 1} f(x)$?

$\lim_{x \to 1} 3x = 3$, $\lim_{x \to 1} x^3 + 2 = 3$

by squeeze theorem $\Rightarrow \lim_{x \to 1} f(x) = 3$

Example: Find $\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$?

$-1 \le \sin\left(\frac{1}{x}\right) \le 1$

$-x^2 \le \sin\left(\frac{1}{x}\right) x^2 \le x^2$

$\lim_{x \to 0} -x^2 = 0$, $\lim_{x \to 0} x^2 = 0$

by squeeze theorem

$\lim_{x \to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$

Subject Calculus(1) Lecture (37) Date No.

Continuity الاستمرارية

Definition: Continuity at number

f is continuous at $x=a$ if

1) $f(a)$ defined

2) $\lim_{x \to a} f(x)$ exist

3) $\lim_{x \to a} f(x) = f(a)$

Example: Is $f(x)$ continuous at $x=3$

$$f(x) = \begin{cases} \frac{x^2-9}{x-3}, & x \neq 3 \\ 6, & x = 3 \end{cases}$$

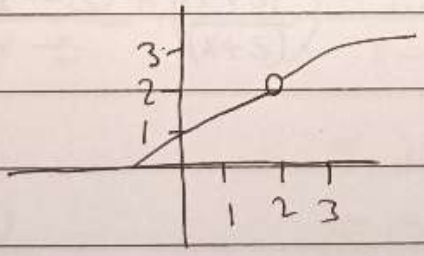
1) $f(3) = 6$

2) $\lim_{x \to 3} \frac{x^2-9}{(x-3)} = \frac{(x+3)(x-3)}{(x-3)} = (x+3) = 3+3 = 6$

3) $f(3) = \lim_{x \to 3} f = 6$

f con at $x=3$

2)



1) $f(3)$ undefined (discon)

Theorem: The following types of function are continuous at every number in there domains polynomials, rational function, root function, trigonometric functions, inverse function, trigonometric, exponential, logarithmic functions.

Subject Lecture (37) Date No.

Example: Where f is continuous :-

$f(x) = \frac{\ln x + \tan^{-1}(x)}{x^2 - 9}$ $\longrightarrow (0,\infty)$ $\longrightarrow \mathbb{R}$

$D_f = (0,\infty) \cap \mathbb{R} - \{3,-3\}$

f con $(0,\infty) - \{3\}$

Example: find k such that f continuous everywhere:-

$$f(x) = \begin{cases} kx^2 & , x \leq 2 \\ 2x+k & , x > 2 \end{cases}$$

kx^2 $\quad$ $2x+k$

$\longleftarrow \quad | \quad \longrightarrow$

$\quad\quad 2$

f continuous at $x = 2$

$\Leftrightarrow \lim_{x \to 2} f$ exist $\Rightarrow \lim_{x \to 2^+} 2x+k = \lim_{x \to 2^-} kx^2$

$2(2) + k = 4k$

$k = \frac{4}{3}$

2) $f(x) = \begin{cases} \frac{\sin(kx)}{5x} & , x < 0 \\ 2 & , x \geq 0 \end{cases}$

$\lim_{x \to 0^+} 2 = \lim_{x \to 0^-} \frac{\sin kx}{5x}$

$2 = \frac{k}{5}$

$\boxed{k = 10}$

Example: where f is not continuous

$D\ f(x) = \frac{1}{\sqrt{x-2}}$ $\longleftarrow \mathbb{R}$

$\longleftarrow [2,\infty)$

$D_f = \mathbb{R} \cap [2,\infty) - \{2\} = (2,\infty)$

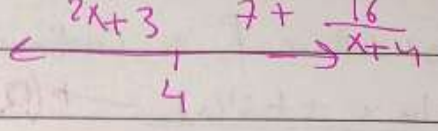
f con $(2,\infty)$

f not $(-\infty, -2]$

NOTEBOOK

Subject Lecture (37) Date No.

2) $f(x)=\begin{cases} 2x+3 & , x\le 4 \\ 7+\frac{16}{x+4} & , x>4 \end{cases}$



$(-\infty,4) \rightarrow 2x+3$ poly con

$(4,\infty)$ $7+\frac{16}{x+4}$ con $R-\{-4\}$ $(4,\infty)$

at $x=4$

$\rightarrow 2(4)+3=11$

$\lim_{x\to 4^+}\left(7+\frac{16}{x+4}\right)=7+\frac{16}{8}=9$

$\lim_{x\to 4} f \neq f(4)$

f not con at $x=4$

Lecture (38)

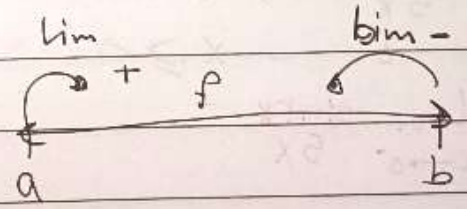
Continuity at an interval :- الاتصال على فترة

f is continuous on $[a,b]$ if

1) f continuous on (a,b)

2) $\lim_{x\to a^+} f(x)=f(a)$

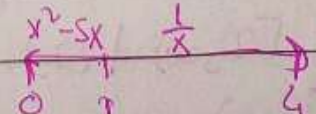
3) $\lim_{x\to b^-} f(x)=f(b)$



1) القراءة المستمرة
2) عند الأطراف
3) نقاط التعب

Example: $f(x)=\begin{cases} x^2-5x & , 0\le x\le 1 \\ \frac{1}{x} & , 1\le x\le 4 \end{cases}$

Is f continuous at $[0,4]$



القراءة المستمرة

$(0,1) \rightarrow f(x)=x^2-5x$ Poly con R $(0,1)$

$(1,4) \rightarrow f(x)=\frac{1}{x}$ con on $R-\{0\}$, $0\in(1,4)$, f on $(1,4)$

نقاط التي تكون عندها الدالة غير متصلة

at $x=1$

1) $f(1)=\frac{1}{1}=1$

2) $\lim_{x\to 1^+} \frac{1}{x}=1$

3) $\lim_{x\to 1^-} x^2-5x=-4$

$\lim_{x\to 1} f$ dne.

f not con at $x=1$

at $x=0$

1) $f(0)=0$

2) $\lim_{x\to 0} (x^2-5x)=0$

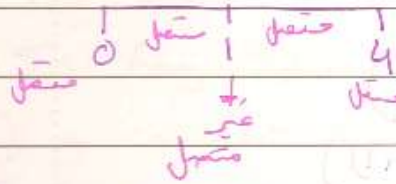
f con at $x=0$

at $x=4$

1) $f(4)=\frac{1}{4}$

2) $\lim_{x\to 4^-} = \frac{-1}{x}=\frac{1}{4}$

3) $\lim_{x\to 4^-} f = f(4)$



$\therefore$ f con $[0,4]\setminus\{1\}$

* Theorem: If f and g are continuous at a and C is a constant, then the following 1) $f+g$ 2) $f-g$ 3) $f.g$ 4) $\frac{f}{g}$ if $g(a)\neq 0$ function are also continuous at $x=a$

Example:

$f(x)=5x^2$

$g(x)=x-1$

Is $\frac{f}{g}$ continuous at $x=2$?

$f(x)=5x^2$ Poly con on $\mathbb{R}$

$g(x)=x-1$ " "

by theorem $\frac{f}{g}$ con at $x=2$ $g(2)\neq 0 \neq \emptyset$

Subject Lecture (39) Date No.

Theorem: If ① $\lim_{x \to c} g(x) = L$

② f continuous at $x = L$ then

③ $\lim_{x \to c} f(g(x)) = f\left(\lim_{x \to c} g(x)\right) = f(L)$

Example:

① $\lim_{x \to 1} \sin^{-1}\left(\frac{1-\sqrt{x}}{1-x}\right)$

$f = \sin^{-1} x$

$g = \frac{1-\sqrt{x}}{1-x}$

$\sin^{-1}\left(\lim_{x \to 1} \frac{1-\sqrt{x}}{1-x}\right)$

$\sin^{-1}\left(\lim_{x \to 1} \frac{1-\sqrt{x}}{1-x} \cdot \frac{(1+\sqrt{x})}{(1+\sqrt{x})}\right)$

$\sin^{-1}\left(\lim_{x \to 1} \frac{1}{1+\sqrt{x}}\right)$

$= \sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{6}$

② $\lim_{x \to 1} e^{\frac{x^2-1}{x-1}}$

$f \quad e^x$

$g \quad \frac{x^2-1}{x-1}$

$e^{\left(\lim_{x \to 1} \frac{x^2-1}{x-1}\right)}$

$e^{\left(\lim_{x \to 1} (x+1)\right)}$

e^2

③ $\lim_{x \to 1} \cos\left(\frac{x^2-1}{x-1}\right)$

$f \quad \cos x$

$g \quad \frac{x^2-1}{x-1}$

$\cos\left(\lim_{x \to 1} \frac{x^2-1}{x-1}\right)$

$\cos\left(\lim_{x \to 1} x+1\right)$

$\cos(2)$

NOTEBOOK

Subject lecture (39) Date No.

Theorem: If 1) g continuous at $X=c$ (fog) Con at c

2) f Continuous at $X=g(c)$

then (fog) Continuous at $X=c$

Example: $f(x)=\frac{1}{x-\frac{1}{2}}$ → Con on $R-\{\frac{1}{2}\}$

$g(x)=\frac{1}{2x}$ → Con on $R-\{0\}$

D Is fog Continuous at $X=0$

[1] g Cont at $X=0$? No

g not Con at $X=0$

fog not Con

2) Is Fog Continuous at $X=1$

[1] g Con at $X=1$? Yes

[2] f Con at $X=g(1)$? No

fog Not Con

3) Is fog Continuous at $X=2$

[1] g Con at $X=2$? Yes

[2] f Con at $X=g(2)=\frac{1}{4}$? Yes

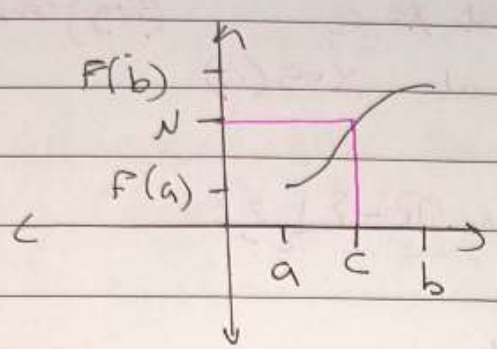
fog Con at $X=2$

Lecture (40)

The intermediate value theorem:

Suppose that f is Continuous on the closed interval $[a,b]$ and Let N any number between $f(a)$, where $f(a)\neq f(b)$ Then there exists a number (C in (a,b) such that $f(C)=N$

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Example: If $f(x) = x^3 + 5x - 1$ show that $\exists \in \in (1,2)$ such that $f(c) = 7$?

1) f con $(1,2)$ poly ✓

2) $f(1) \neq f(2)$

$5 \neq 13$ ✓

3) $5 < 7 < 13$

Then $\exists \in c \in (1,2)$ such that $f(c) = 7$

Example: Show that there is a root of the equation $4x^3 - 6x^2 + 3x = 2$ between 1 & 2 ?

$4x^3 - 6x^2 + 3x - 2 = 0$

$f(x)$, $[1,2]$

1) f con on $[1,2]$ ✓

2) $f(1) = -1$

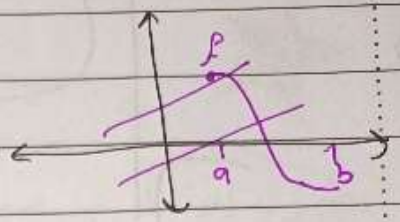
$f(2) = 12$

$-1 < \overset{N}{0} < 12$

that $\exists \in \in (1,2)$ such that $f(c)$ ∃ root between 1 & 2

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Example: Show that the equation
$x^3 - x - 1 = 0$ has at least one root?



$f(x) = x^3 - x - 1$

$f(-1) = -1 < 0$

$f(0) = -1$

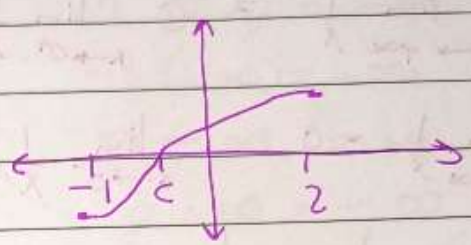
$f(1) = -1$

$f(2) = 5 > 0$

$[-1, 2]$

$f(-1) < 0 < f(2)$

$-1 < 0 < 5$



1) f con $[-1, 2]$

2) $f(-1) < 0 < f(2)$

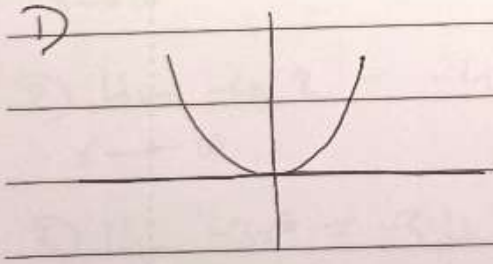
$-1 < 0 < 5$

$\exists\, c \in (-1, 2)$ such that $f(c) = 0$

Lecture (41)

Limits at infinity النهايات في اللانهاية

1)

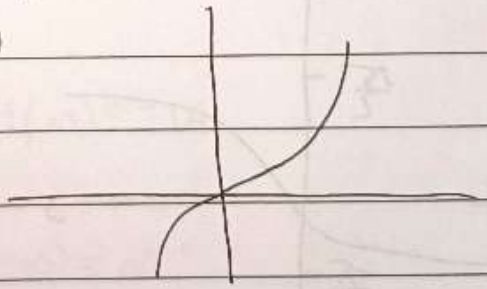


$\lim_{x \to +\infty} x^2 = +\infty$

$\lim_{x \to -\infty} x^2 = +\infty$

x^2 / x زوجي

2)



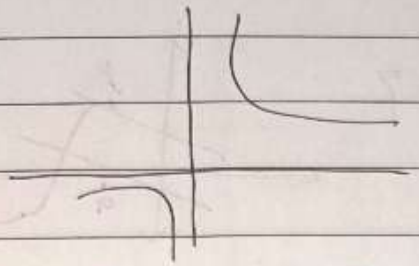
$\lim_{x \to +\infty} x^3 = +\infty$

$\lim_{x \to -\infty} x^3 = -\infty$

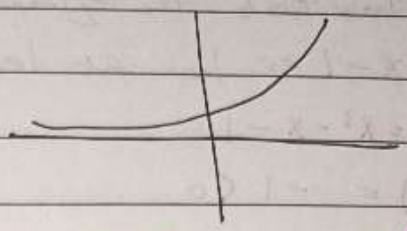
x^3 / x فردي

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3)



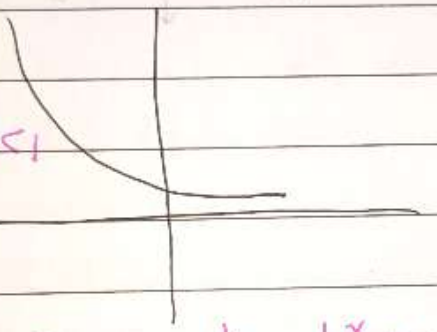
4)



$\lim_{x \to +\infty} \frac{1}{x} = 0$ $\lim_{x \to 0^+} \frac{1}{x} = +\infty$ $\lim_{x \to +\infty} b^x = +\infty$ $\lim_{y \to +\infty} e^x = +\infty$

$\lim_{x \to -\infty} \frac{1}{x} = 0$ $\lim_{x \to 0^-} \frac{1}{x} = -\infty$ $\lim_{x \to -\infty} b^x = 0$ $\lim_{x \to -\infty} e^x = 0$

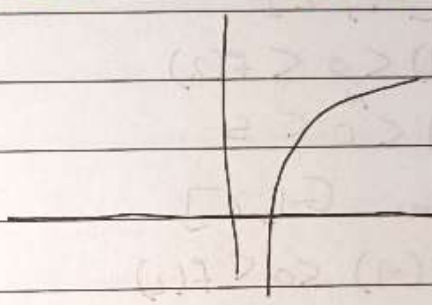
5)



b^x $0 < b < 1$

$e^{-x} = \frac{1}{e^x}$

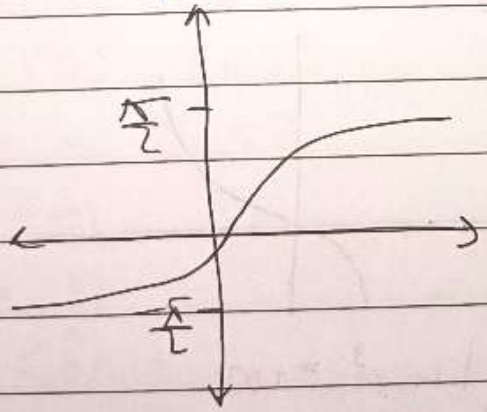
6)



$\lim_{x \to +\infty} e^{-x} = 0$ $\lim_{x \to +\infty} b^x = 0$ $\lim_{x \to +\infty} \ln x = +\infty$

$\lim_{x \to -\infty} e^{-x} = +\infty$ $\lim_{x \to -\infty} b^x = +\infty$ $\lim_{x \to 0^+} \ln x = -\infty$

7)



$\lim_{x \to +\infty} \ln^{-1}(x) = \frac{\pi}{2}$

$\lim_{x \to -\infty} \ln^{-1} x = -\frac{\pi}{2}$

Subject Lecture(41) Date No.

Theorem:-

1) $\lim_{x\to\pm\infty} (f(x))^n = \left(\lim_{x\to\pm\infty} f(x)\right)^n$ n Positive integer

2) $\lim_{x\to\pm\infty} k f(x) = k \lim_{x\to\pm\infty} f(x)$

3) $\lim_{x\to\pm\infty} k = k$

4) $\lim_{x\to\infty} x^n = \infty$, $n = 1, 2, 3, 4 \ldots$

5) $\lim_{x\to\pm\infty} x^n \begin{cases} -\infty, & n = 1/3/5 \ldots \\ +\infty, & n = 2/4/\ldots \end{cases}$

6) $\lim_{x\to\pm\infty} (a_0 + a_1 x + a_2 x^2 + \ldots + a_{n-1} x^{n-1} + a_n x^n) = \lim_{x\to\pm\infty} a_n x^n$

7) If $r > 0$ is a rational number then $\lim_{x\to\pm\infty} \frac{1}{x^r} = 0$

Example:-

1) $\lim_{x\to\infty} x^8 = \infty$

2) $\lim_{x\to-\infty} -4x^2 = -4 \lim_{x\to-\infty} x^2 = -4(\infty) = -\infty$

3) $\lim_{x\to-\infty} -3x^5 = -3 \lim_{x\to-\infty} x^5 = -3(-\infty) = \infty$

4) $\lim_{x\to-\infty} (4x^2 - 5x^6 + 1) = \lim_{x\to-\infty} -5x^6 = -5 \lim_{x\to-\infty} x^6 = -5(\infty) = -\infty$

5) $\lim_{x\to-\infty} 2^x = 0$

عزيزي الزائر ، $\pm\infty$:

$-2(\pm\infty) = -\infty$

$-3(-\infty) = +\infty$

$\infty + \infty = +\infty$

$\infty \div \infty = -\infty$

Subject Lecture (41) Date No.

6) $\lim_{x\to+\infty} 3^{-x} = \lim_{x\to+\infty} \left(\frac{1}{3}\right)^x = 0$

7) $\lim_{x\to\infty} \left(\frac{\Delta}{e}\right)^x = +\infty$

8) $\lim_{x\to-\infty} \frac{5}{x^4} = 5 \lim_{x\to-\infty} \frac{1}{x^4} = 5(0) = 0$

9) $\lim_{x\to\infty} e^{-2/x} = e^{\lim_{x\to\infty} e^{\frac{-2}{x}}}$

$= e^{-2\lim_{x\to\infty} \frac{1}{x}}$

$= e^{-2(0)}$

$= e$

$= e^0$

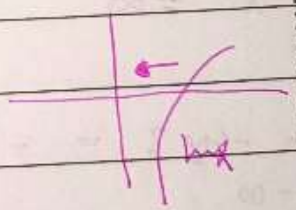
$= 1$

Lecture (42)

Example

1) $\lim_{x\to\infty} \tan^{-1}(e^x) = \tan^{-1}\left(\lim_{x\to\infty} e^x\right) = \frac{\pi}{2}$

2) $\lim_{x\to 0^+} \tan^{-1}(\ln x) = \tan^{-1}\left(\lim_{x\to 0^+} \ln x\right) = \frac{-\pi}{2}$ ($\ln x \to -\infty$)



3) $\lim_{x\to\infty} \frac{5+x^2-3x}{5+10x} = \lim_{x\to\infty} \frac{x^2}{10x} = \frac{1}{10} \lim_{x\to\infty} x = \infty$

4) $\lim_{x\to-\infty} \frac{5-x^2}{3+2x^3+10x} = \lim_{x\to-\infty} \frac{-x^2}{2x^3} = \frac{-1}{2} \lim_{x\to-\infty} \frac{1}{x} = \frac{-1}{2}(0) = 0$

5) $\lim_{x\to\infty} \ln\left(\frac{x+1}{x}\right) = \ln\left(\lim_{x\to+\infty} \frac{x+1}{x}\right) = \ln\left(\lim_{x\to+\infty} x\right) = \ln 1 = 0$

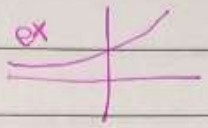
6) $\lim_{x\to\infty} \frac{3+4x^5-2x^3}{4+x^5} = \lim_{x\to\infty} \frac{4x^5}{x^5} = \lim_{x\to\infty} 4 = 4$

NOTEBOOK

Subject Leeture(42) Date No.

7) $\lim_{x\to\infty} \frac{1-e^x}{1+2e^x} = \frac{\infty}{\infty}$

$y = e^x$, $x \to \infty$, $y \to \infty$



$\lim_{y\to\infty} \frac{1-y}{1+2y} = \lim_{y\to\infty} \frac{-y}{2y} = \frac{-1}{2}$

ملاحظة :- $\frac{\text{كثير حدود}}{\text{كثير حدود}}$

1) درجة البسط < المقام ← صفر

2) درجة البسط > المقام ← $\pm\infty$

3) درجة البسط = المقام ← عدد

8) $\lim_{x\to-\infty} \frac{e^{2x}-e^{-x}}{e^{3x}-e^{-2x}}$

$y = e^x$, $x \to -\infty$, $y \to 0$

$\lim_{y\to0} \frac{y^2-y^{-1}}{y^3-y^{-2}}$

$= \lim_{y\to\infty} \frac{y^4-y}{y^5-1} = \frac{0-0}{0-1} = 0$

9) $\lim_{x\to\infty} \frac{e^{-x}-3e^x}{e^{-2x}+5e^x}$

$y = e^x$, $x \to \infty$, $y \to \infty$

$\lim_{y\to\infty} \frac{y^{-1}-3y}{y^{-2}+5y}$

$\lim_{y\to\infty} \frac{\frac{1}{y}-3y}{\frac{1}{y^2}+5y}$

$\lim_{y\to\infty} \frac{\frac{1-3y^2}{y}}{\frac{1+5y^3}{y^2}}$

$\lim_{y\to\infty} \frac{(1-3y^2)y}{1+5y^3}$

$\lim_{y\to\infty} \frac{y-3y^3}{1+5y^3}$

$= \lim_{y\to\infty} \frac{-3y^3}{5y^3}$

$= \lim_{y\to\infty} \frac{-3}{5}$

$= \frac{-3}{5}$

Subject Lecture (43) Date No.

Example

1) $\lim_{x \to -\infty} \frac{\sqrt{5x^2+2}}{x+3}$

$= \lim_{x \to -\infty} \frac{\sqrt{x^2(5-\frac{2}{x^2})}}{x(1+\frac{3}{x})}$

$= \lim_{x \to -\infty} \frac{\sqrt{x^2}\sqrt{(5-\frac{2}{x^2})}}{x(1+\frac{3}{x})}$

$= \lim_{x \to -\infty} \frac{|x|\sqrt{5-\frac{2}{x^2}}}{x(1+\frac{3}{x})}$

$= \lim_{x \to -\infty} \frac{-x\sqrt{5-\frac{2}{x^2}}}{x(1+\frac{3}{x})}$

$= \lim_{x \to -\infty} \frac{-\sqrt{5-\frac{2}{x^2}}}{(1+\frac{3}{x})}$

$= \frac{-\sqrt{5-0}}{1+0} = -\sqrt{5}$

$\sqrt{x^2} = |x| = \begin{cases} -x, & x<0 \\ x, & x \geq 0 \end{cases}$

$\sqrt{x^4} = (\sqrt{x^2})^2 = |x|^2 = x^2$

$\sqrt{x^6} = \sqrt{x^2}^3 = |x|^3 = \begin{cases} -x^3, & x<0 \\ x^3, & x \geq 0 \end{cases}$

$|x|$: $-x$ | x

$\lim_{x \to -\infty} |x| = -x$, $\lim_{x \to \infty} |x| = x$

2) $\lim_{x \to -\infty} \frac{\sqrt{4x^4+x}}{x^2-8}$

$= \lim_{x \to -\infty} \frac{\sqrt{x^4}\sqrt{(4+\frac{1}{x^2})}}{x^2(1-\frac{8}{x^2})}$

$= \lim_{x \to -\infty} \frac{x^2}{x^2} \frac{\sqrt{4+\frac{1}{x^2}}}{(1-\frac{8}{x^2})}$

$= \lim_{x \to -\infty} \frac{\sqrt{4+\frac{1}{x^2}}}{(1-\frac{8}{x^2})}$

$= \frac{\sqrt{4}}{1} = 2$

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Subject Lecture (43) Date No.

3) $\lim_{x\to+\infty} \sqrt{x^2-3x} - x = \frac{(\sqrt{x^2-3x}+x)}{(\sqrt{x^2-3x}+x)}$

$= \lim_{x\to+\infty} \frac{-3x}{\sqrt{x^2}\sqrt{1-\frac{3}{x}}+x}$ $= \lim_{x\to+\infty} \frac{-3x}{x\sqrt{1-\frac{3}{x}}+1}$ $= \lim_{x\to\infty} \frac{-3}{\sqrt{1-\frac{3}{x}}+1}$

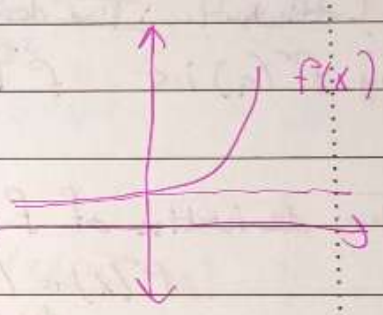
$x \rightsquigarrow |x|$

$= \frac{-3}{2}$

Definition

The line $y=L$ is called horizontal asymptote of the curve $y=f(x)$ if either

$\lim_{x\to\infty} f(x)=L$ or $\lim_{x\to-\infty} f(x)=L$



Example: find horizontal asymptote for

$Df(x) = \frac{\sqrt{x^2+7}-4x}{x+3}$

$\lim_{x\to+\infty} \frac{\sqrt{x^2+7}-4x}{x+3}$

$= \lim_{x\to+\infty} \frac{\sqrt{x^2}\sqrt{1+\frac{7}{x^2}}-4x}{x(1+\frac{3}{x})}$

$= \lim_{x\to+\infty} \frac{x\sqrt{1+\frac{7}{x^2}}-4x}{x(1+\frac{3}{x})}$

$y=-3$ hor asy

$\lim_{x\to-\infty} \frac{\sqrt{x^2+7}-4x}{x+3}$

$\lim_{x\to-\infty} \frac{-x\left(-\sqrt{1+\frac{7}{x^2}}\right)-4x}{x(1+\frac{3}{x})}$

$= -5$

at $y=-5$ hor asy

Subject Lecture (43) Date No.

2) $f(x) = e^x$

$\lim_{x \to \infty} e^x = \infty$

$\lim_{x \to -\infty} e^x = 0$

$y = 0$ hor asy

3) $f(x) = \tan^{-1} x$

$\lim_{x \to \infty} \tan^{-1} x = \frac{\pi}{2}$

$\lim_{x \to -\infty} \tan^{-1} x = -\frac{\pi}{2}$

$y = \frac{\pi}{2}$ $y = -\frac{\pi}{2}$ hor asy

Lecture (44)

Derivatives: مشتقات

Definition: The derivative of a function f at a number a denoted by $f'(a)$ is $f'(a) = \lim_{x \to a} \frac{f(x) - f(a)}{x - a}$

$f'(a) \to \left.\frac{dy}{dx}\right|_{x=a}$

The derivative of f with respect to x

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{z \to x} \frac{f(z) - f(x)}{z - x}$$

Example: $f(x) = x^2$ find $f'(x)$ & $f'(2)$?

$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

$= \lim_{h \to 0} \frac{(x+h)^2 - x^2}{h}$

$= \lim_{h \to 0} \frac{x^2 + 2xh + h^2 - x^2}{h}$

$= 2x$

$2(2) = 4$

$f'(2) = \lim_{h \to 0} \frac{(2+h) - 4}{h}$

$= \lim_{h \to 0} \frac{2 + h - 4}{h}$

Subject Date No.

[1] $f(x)=C \rightarrow f'(x)=zero$, C Constant

Example 1) $f(x)=3$ $f(x)=e^7$

$f'(x)=zero$ $f'(x)=zero$

[2] $f(x)=x$

$f'(x)=1$

[3] $f(x)=x^n$

$f'(x)=n x^{n-1}$

Example 1) $f(x)=\sqrt{x}$

$f'(x)=\frac{1}{2} x^{-\frac{1}{2}}$

$=\frac{1}{2\sqrt{x}}$

2) $f(x)=x^7$

$=7x^6$

[4] $\frac{d}{dx}(c f(x))=c f'(x)$

$\frac{d}{dx}\left(\frac{f(x)}{c}\right)=\frac{1}{c}\frac{d}{dx}f(x)$

Example:- 1) $f(x)=3x^4$

$f'(x)=12x^3$

2) $f(x)=\frac{x^6}{5}$ $\frac{6}{5}x^5$

[5] $(f\pm g)'=f'\pm g'$

$f(x)=3x^7-2x^4+x$

$f'(x)=21x^6-8x+1$

[6] $(f\cdot g)'(x)=f'\cdot g+f\cdot g'$

$y=(x^2+1)(x^3-2x)$

$y'=(x^2+1)(3x^2-2)+(x^3-2x)(2x)$

$x=1$ $=zero$

[7] $\left(\frac{f}{g}\right)'=\frac{f'\cdot g-g'\cdot f}{g^2}$

1) $y=\frac{3x+4}{x^2+1}$

$y'(x)=\frac{(x^2+1)(3)-(3x+4)(2x)}{(x^2+1)^2}$

2) $f(2)=-3$, $f'(2)=4$, $g(2)=1$, $g'(2)=5$

a) $\left(3\frac{f}{g}\right)'(2)=3\left(\frac{g\cdot f'-f\cdot g'}{(g)^2}\right)$

$=3\frac{((4)(1)-(5)(-3))}{1}$

b) $\left(\frac{3f(2)}{g(2)}\right)'$

$=\left(3\left(\frac{-3}{1}\right)\right)'=(-9)'=0$

Subject Lecture (45) Date No.

[8] $\left(\frac{c}{f}\right)' = \frac{-c \cdot f'}{f^2}$

Example:-

$Df(x) = \frac{2}{3x+1}$

$f'(x) = \frac{-2(3)}{(3x+1)^2}$

$= \frac{-6}{(3x+1)^2}$

[7] $\frac{d}{dx}\left(x^2\left(\frac{d}{dx}\frac{1}{x}\right)\right)$

$\frac{d}{dx}(x^2)\left(\frac{-1}{x^2}\right)$

$= \text{zero}$

[9]

f	f'
sinx	Cosx
Cosx	-Sinx
tanx	Sec^2x
cotx	-CscxCotx
Secx	secxtanx
Cscx	-CscxCotx

Example:

$f(x) = \tan x + 3x^2\cos x - \frac{5x}{\sec x}$

$f'(x) = \sec^2 x + (3x^2)(-\sin x) + (\cos x)(6x) - \frac{(\sec x)(5) - (5x)(\sec x \tan x)}{(\sec^2 x)}$

$= \sec^2 x + 6x\cos x - 3x^2\sin x - \frac{5}{\sec x} + \frac{5x\tan x}{\sec x}$

[10] Chain rule

If $y = f(z)$

$z = g(x)$

then $\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$

Example:

① If $y = 3z^3 + 2$

$z = x^2 + 4$

find $\frac{dy}{dx}$

$\frac{dy}{dx} = \frac{dy}{dz} \cdot \frac{dz}{dx}$

$\frac{dy}{dz} = 9z^2$

$\frac{dz}{d} = 2x$

$\frac{dy}{dx} = 9z^2 \cdot (2x)$

$= 9(x^2+4)^2(2x)$

② If $x = \cos u$, $\frac{du}{dm} = 3$ find $\frac{dx}{dm}$ when $= \frac{3\pi}{2}$

$\frac{dx}{dm} = \frac{dx}{du} \cdot \frac{du}{dm}$

$= -\sin(u) \cdot 3$

$= -3\sin\left(\frac{3\pi}{2}\right)$

$= 3$

NOTEBOOK

Lecture (46)

[11] $(f \circ g)'(x) = f'(g(x))\, g'(x)$

Example:

1) $f(x) = x^2$

$g(x) = 3x + 1$

Find $(f \circ g)'(x)$

$f'(x) = 2x$

$g'(x) = 3$

$f'(g(x))\, g'(x)$

$f'(3x+1)(3)$

$2(3x+1)(3)$

$= 6(3x+1)$

$(g \circ f)'(x)$

$g'(x^2)(2x)$

$= 6x$

2) $f'(9) = 5, g(2) = 9,$

$g'(2) = -3$ / find

$(f \circ g)'(2) = f'(g(2))\, g'(2)$

$= f'(9)(-3)$

$= -15$

3) If $f(2x) = 3x^2 + 8x$ $f'(2)$?

$g = 2x$

$\frac{d}{dx}\left(f(2x) = 3x^2 + 8x\right)$ $\quad 2x = 2$

$f'(2x)\,(2) = 6x + 8$ $\quad [x = 1]$

$f'(2x) = 3x + 4$

$f'(2) = 7$

[4] If $\frac{d}{dx}(f(3x)) = 6x$ find $\frac{d}{dx}(f(x))$

$\frac{d}{dx}(f(3x))$

$f'(3x)(3) = 6x$

$f'(3x) = 2x$

$f'(y) = 2\left(\frac{y}{3}\right)$ $\quad y = 3x$

$f'(x) = \frac{2x}{3}$ $\quad x = \frac{y}{3}$

[12] $\frac{d}{dx}(f(x))^n = n(f(x))^{n-1} f'(x)$

Example:

1) $f(x) = (x^3 + 5)^{27}$ find $f'(x)$ /

$f'(x) = (27)(x^3+5)^{26}(3x^2)$

2) $y = f^5(3x + 4)$ find $\frac{dy}{dx}$

~~$\frac{dy}{dx} = 5 f(3x+4)^4 (3)$~~

$\frac{dy}{dx} = 5(f(3x+4))^4 f'(3x+4)$

$= 15(f(3x+4))^4 f'(3x+4)$

3) $y = \csc^3 x$ Fin y'

$y = \csc^3 x$

$\frac{dy}{dx} = 3\csc^2(x)(-\csc x \cot x)$

PRENK

Lecture (46)

[13]

$f(x)$	$f'(x)$
$\sin(f(x))$	$\cos(f(x))\, f'(x)$
$\cos(f(x))$	$-\sin(f(x))\, f'(x)$
$\tan(f(x))$	$\sec^2(f(x))\, f'(x)$
$\cot(f(x))$	$-(\csc^2 f(x))\, f'(x)$
$\sec(f(x))$	$\sec(f(x))\tan(f(x))\, f'(x)$
$\csc(f(x))$	$-\csc(f(x))\cot(f(x))\, f'(x)$

Example: find $\frac{dy}{dx}$

D $f(x) = \cot(x^2)$

$f'(x) = -\csc^2(x^2)(2x)$

2) $y = \cos^2(3x^5)$

$\frac{dy}{dx} = 2\cos(3x^5)(-\sin(3x^5))(15x^2)$

$= -30\sin(3x^5)\cos(3x^5)$

$= -15x^4 \sin(6x^5)$

3) $y = \sin(\cos(\tan x))$

$\frac{dy}{dx} = \cos(\cos(\tan x))(\sec^2 x)(-\sin(\tan x))$

$= -\sec^2 x \cos(\cos(\tan x))(\sin(\tan x))$

[14] $\frac{d}{dx}\sqrt{f(x)} = \frac{f'(x)}{2\sqrt{f(x)}}$

Example:

D $y = \sqrt{\tan x}$

$y' = \frac{\sec^2 x}{2\sqrt{\tan x}}$

Lecture (47)

15 Implicit Differentiation مشتقات الضمني

$y = x^2 + x + 5$ explicit صريح

$yx + y^2 + 1 = x$ implicit ضمني

$\frac{d}{dx}(x) = 1 \cdot \frac{dx}{dx} = 1$

$\frac{d}{dx}(y) = 1 \cdot \frac{dy}{dx}$ أو y'

$\frac{d}{dx}(y^2) = 2y \cdot \frac{dy}{dx}$

$\frac{d}{dx}(x^2 + y^2) = 2x\frac{dx}{dx} + 2y\frac{dy}{dx}$

$\frac{d}{dx}(xy) = 1 \cdot \frac{dx}{dx} \cdot y + x \cdot 1 \cdot \frac{dy}{dx}$

$\frac{d}{dx}\left(\frac{x}{y}\right) = \frac{y - xy'}{y^2}$

$\frac{d}{dx}\left(\frac{y}{x}\right) = \frac{\frac{dy}{dx} \cdot x - y}{x^2}$

Example:-

1) $x^2 + xy = 1$ find $\frac{dy}{dx}$

$2x + (x)(y') + (y) = 0$

$2x + xy' + y = 0$

$2x + y = -xy'$

$y' = \frac{-(2x+y)}{x}$

2) $x^2 + xy = 1$ find $\frac{d^2y}{dx^2}$

$2x + (x)(y') + (y) = 0$

$2x + y = -xy'$

$y' = \frac{-2x-y}{x}$

$y'' = \frac{(x)(-2-y') - (-2x-y)(1)}{x^2}$

$= \frac{(2x - xy') - (-2x-y)}{x^2}$

$= \frac{-2x - x\left(\frac{-2x-y}{x}\right) - (-2x-y)}{x^2}$

$= \frac{-2x + 2x + y + 2x + y}{x^2}$

$= \frac{2x + 2y}{x^2}$

Lecture (47)

3) If $\sin(xy^2) + 3xy = 1 - y$ $\frac{dy}{dx}$ when $x = 0$

$x = 0 \longrightarrow \boxed{y = 1}$

$\cos(xy^2)\left((x)(2yy') + (y^2)\right) + (3x)(y') + (y)(3) = -y'$

$\cos(0)\ (0) + (1) + (0) + (3) = -y'$

$0 + 1 + 0 + 3 = -y'$

$\boxed{y' = -4}$

4] If $x^2 + y^2 = 9$ show that $\frac{d^2y}{dx^2} = \frac{-9}{y^3}$?

$2x + 2yy' = 0$

$2yy' = -2x$

$y' = \frac{-x}{y}$

$y'' = \frac{(y)(-1) - (-x)(y')}{(y)^2}$

$y'' = \frac{-y + xy'}{y^2}$

$= \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$

$= \frac{-y - \frac{x^2}{y}}{y^2}$

$= \frac{-\left(\frac{y^2 + x^2}{y}\right)}{y^2}$

$= \frac{-(y^2 + x^2)}{y^3}$

$= \frac{-9}{y^3}$

PRENI

Lecture (48)

[16] $f(x) = \log_a g(x) \Rightarrow f'(x) = \frac{g'(x)}{g(x) \cdot \ln a}$

$f(x) = \ln(g(x)) \Rightarrow f'(x) = \frac{g'(x)}{g(x)}$

Example:

1) $y = \log_{10}(x^2+1)$

$y' = \frac{2x}{(x^2+1)\ln 10}$

2) $y = \log_7(\cot x)$

$y' = \frac{-\csc^2 x}{(\cot x)\ln 7}$

3) $y = (\ln x)^3$

$y' = 3(\ln x)^2 \left(\frac{1}{x}\right)$

4) $\cos(\ln x)$

$\frac{dy}{dx} = -\sin(\ln x)\left(\frac{1}{x}\right)$

5) $y = \ln\left(\frac{x^2 \sin x}{x+1}\right)$

$y' = \ln(x^2) + \ln(\sin x) - \ln(x+1)$

$= \frac{2x}{x^2} + \frac{\cos x}{\sin x} - \frac{1}{x+1}$

6) $\log_x 2$

$y = \frac{\ln 2}{\ln x}$

$y' = \frac{-\ln 2 \left(\frac{1}{x}\right)}{(\ln x)^2}$

$= \frac{-\ln 2}{x(\ln x)^2}$

7) $y = (x^2+1)^{\sin x}$

$\ln y = \sin x \ln(x^2+1)$

$\frac{y'}{y} = (\sin x)\left(\frac{2x}{x^2+1}\right) + \ln(x^2+1)(\cos x)$

$y' = \left[\left(\frac{2x}{x^2+1}\right)\sin x + \cos x \ln(x^2+1)\right]\left[(x^2+1)^{\sin x}\right]$

ملاحظة:

1) (عدد) اقتران نشتق حسب القاعدة القديمة

2) (اقتران)$^{\text{عدد}}$ ← (اقتران)$^{(\text{عدد}-1)}$ * مشتقة الاقتران

3) (اقتران)$^{\text{اقتران}}$ ← ناخذ ln للاقترانين

Lecture (48)

[17] $f(x) = a^{g(x)} \longrightarrow f'(x) = g'(x) \cdot a^{g(x)} \cdot \ln a$

$f(x) = e^{g(x)} \longrightarrow f'(x) = g'(x) \cdot e^{ax} \cdot \ln e$

Examples:-

1) $y = e^{3x}$

$y' = 3e^{3x}$

2) $y = 2^{\sin x}$

$y' = \cos 2^{\sin x} \ln 2$

3) $y = \ln(\cos(e^x))$

$\frac{-\sin(e^x) \cdot e^x}{\cos(e^x)}$

4) $y = 2^{3x^2}$

$y' = 2x\, 3^{x^2} \ln 3 \left(2^{3x^2}\right) \ln 2$

$g(x) = 3^{x^2}$

$= 2x\, 3^{x^2} \ln 3$

Lecture (49)

[19]

f	f'	f	f'
$\sin^{-1}(g(x))$	$\frac{g'(x)}{\sqrt{1-(g(x))^2}}$	$\cos^{-1}(g(x))$	$\frac{-g'}{\sqrt{1-g^2}}$
$\tan^{-1}(g(x))$	$\frac{g'(x)}{1+g^2}$	$\cot^{-1}(g(x)) =$	$\frac{-g}{1+g^2}$
$\sec^{-1}(g(x))$	$\frac{g'(x)}{g\sqrt{g^2-1}}$	$\csc^{-1}(g(x))$	$\frac{-g}{g\sqrt{g^2-1}}$

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Lecture (48)

Examples:-

1) $y = \sin^{-1}(5x)$

$$y' = \frac{5}{\sqrt{1-(5x)^2}}$$

2) $y = \tan^{-1}(3x^2)$

$$y' = \frac{6x}{1+(3x^2)^2}$$

3) $y = \log(\cos^{-1}x)$

$$y' = \frac{\frac{-1}{\sqrt{1-x^2}}}{(\cos^{-1}x)\ln 10}$$

$$= \frac{-1}{\ln 10 \cdot \cos^{-1}x \cdot \sqrt{1-x^2}}$$

4) $y = 2^{\cot^{-1}x}$

$$y' = \frac{-1}{(1+x^2)} \cdot 2^{\cot^{-1}x} \cdot \ln 2$$

* Higher derivatives

$f', f'', f''', f^{(4)}, f^{(5)} \ldots$

1) Ex

$y = 5x^7 + 2x^5 + 3x$ find $f^{(4)}(x)$

$f'(x) = 35x^6 + 10x^4 + 3$

$f^{(4)}(x) = (5)(7)(6)(5)(4)x^3 + (2)(5)(4)(3)(2)x$

2) $f(x) = 2^x$ find $f^{(100)}$

$f'(x) = 2^x \ln 2$

$f''(x) = (\ln 2)^2 2^x$

$f'''(x) = (\ln 2)^3 2^x$

$f^{(100)}(x) = (\ln 2)^{100} 2^x$

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Lecture (50)

Differentiability : قابلية الاشتقاق

Theorem: The function f is differentiable at $x=a$ if $f'(a)$ exist.

1) If f is differentiable at $x=a$ then f is continuous at $x=a$

2) If f is not continuous at $x=a$ then f is not dif at $x=a$

Example:-

1) $f(x)=\begin{cases} x^2+1, & x\le 1 \\ 2x, & x>1 \end{cases}$ is dif at $x=1$?

(1) $f(1)=2$ (2a) $\lim_{x\to 1^+} 2x = 2$ $\lim_{x\to 1^-}(x^2+1)=2$ (3) $f(1)=\lim_{x\to 1} f = 2$

∴ f con at $x=1$

$$f'(x)=\begin{cases} 2x, & x\le 1 \\ 2, & x>1 \end{cases}$$

$f'_+(1)=2$

$f'_-(1)=2$

$f'_+(1)=f'_-(1) \longrightarrow$ f dif at $x=1$

2) $f(x)=\begin{cases} x^2+1, & x\le 1 \\ x^3+1, & x>1 \end{cases}$ is dif at $x=1$?

(1) $f(1)=2$ / $\lim_{x\to 1^+}(x^3+1)=\lim_{x\to 1^-}(x^2+1)=2$ / $f(1)=\lim_{x\to 1} f(x)=2$

f is con

$$f'(x)=\begin{cases} 2x, & x<1 \\ 3x^2, & x>1 \end{cases}$$

$f'_+(1)=3$

$f'_-(1)=2$

f is not dif at $x=1$

Lecture (50)

3) $f(x) = \begin{cases} x^2+1, & x > 1 \\ 2x+5, & x \le 1 \end{cases}$ is f dif at $x=1$?

$f(1) = 7$, $\lim_{x \to 1^+} (x^2+1) = 2$ / $\lim_{x \to 1^-} (2x+5) = 7$ / $f(1) \neq \lim_{x \to 1} f(x)$

f is Not con / f is Not dif

4) $f(x) = \begin{cases} 3x^2, & x \le 1 \\ ax+b, & x > 1 \end{cases}$ is dif at $x=1$? find a, b

$f(1) = 3$ / $\lim_{x \to 1^+} (ax+b) = \lim_{x \to 1^-} (3x^2)$

$a+b = 3$

$f'(x) = \begin{cases} 6x, & x \le 1 \\ a, & x > 1 \end{cases}$

$f'(1) = f'(1)$

$6 = a$

$a+b = 3$

$6+b = 3$

$b = -3$

Ex:

D $f'(4) = 3$, $f'(4) = 9$ find

$$\lim_{h \to 0} \frac{f(4+h) - 3}{5h}$$

$$\frac{1}{5} \times 9$$

$$= \frac{9}{5}$$

Lecture (50) $f'(4)$

2) If $\lim_{x \to 4} \frac{f(x) - f(4)}{x - 4} = 2$ then $f(x)$ may equal to?

If $f'(4) = 2$, $f(x) = ?$

a) $8\sqrt{x}$

b) $-8\sqrt{x}$

c) $2\cos(x-4)$

d) $x^2 - 4$

~~Lecture (51)~~

~~* Slope of normal (perpendecular)~~

Lecture (51)

* Equation of tangent line at $x = a$

- Point of tangency $(a, f(a))$
- slope of normal (perpendecular)
- equation of normal

$$y - f(a) = \frac{-1}{f'(a)}(x - a)$$

horizital line slope = 0
vertical line slope = und
Up

Remarks:-

1) If $L_1 // L_2 \rightarrow m_1 = m_2$ (Parallel //)

2) If $L_1 \perp L_2 \rightarrow m_1 \cdot m_2 = -1$

3) The slope of horizental line = Zero

The slope of horizental tangent line = $f'(a)$ = Zero

4) $f(a) = g(a)$

$f'(a) = g'(a)$

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Lecture (51)

Example:-

1) find the equation of tangent line for $y = x \tan^{-1}(x)$ at $x = 1$?

$y = (1)\tan^{-1}(1)$

$* \left(1, \frac{\pi}{4}\right)$ equation of normal $\frac{-1}{f'(a)}$

$y' = \tan^{-1}x + \frac{x}{1+x^2}$

$= \tan^{-1}(1) + \frac{1}{1+1}$

$* = \frac{\pi}{4} + \frac{1}{2}$

$* \; y - \frac{\pi}{4} = \frac{\pi+2}{4}(x-1)$

2) find the slope of the tangent line and slope of normal

$2x^2 + 3e^{x+1} + 3xy + y^2 = 9$ at $(-6, 1)$

$4x + 3e^{x+1} + (3x)(y') + (y)(3) + 2yy' = 0$

$-4 + 3 + -3y' + 3 + 2y' = 0$

$2 - y' = 0$

$\boxed{y' = 2}$ ← slope of tangent

$\boxed{\frac{-1}{2}}$ slope of normal

3) find all values of x at which the tangent line of the curve $y = x^5 e^x$ horizontal

$y' = (x^5)(e^x) + (e^x)(5x^4)$

$0 = e^x(x^5 + 5x^4)$

$e^x \neq 0$

$x^5 + 5x^4 = 0$

$x^4(x+5) = 0$

$\boxed{x = 0}$ $\boxed{x = -5}$

PRENKO

Lecture (52)

Hyperbolic functions:-

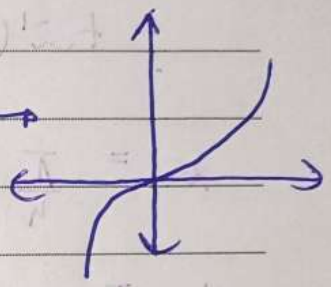
(1) Hyperbolic Sine $\sinh(x) = \frac{e^x - e^{-x}}{2}$

$D = \mathbb{R}$

$R = \mathbb{R}$

$\lim_{x \to +\infty} \sinh x = +\infty$

$\lim_{x \to -\infty} \sinh x = -\infty$



$\sinh(0) = 0$

odd fun

$\sinh x \geq 0 \quad \forall \; x \geq 0$

$\sinh x \leq 0 \quad \forall \; x \leq 0$

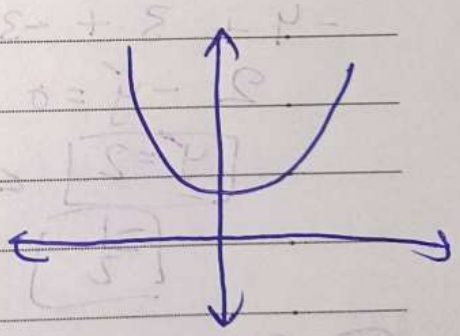
(2) Hyperbolic cosine $\cosh x = \frac{e^x + e^{-x}}{2}$

$D = \mathbb{R}$

$R = [1, \infty)$

$\lim_{x \to +\infty} \cosh x = +\infty$

$\lim_{x \to -\infty} \cosh x = -\infty$



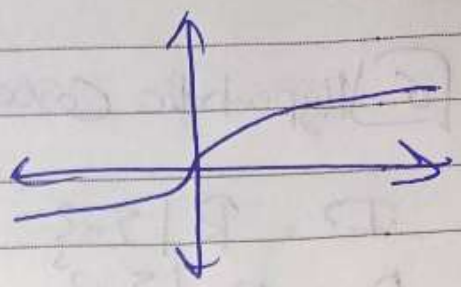
$\cosh(0) = 1$

$\cosh x \geq 1 \quad \forall x$

even function

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Lecture (32)



(3) Hyperbolic tangent

$$\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$$

$D = R$

$R = (-1, 1)$

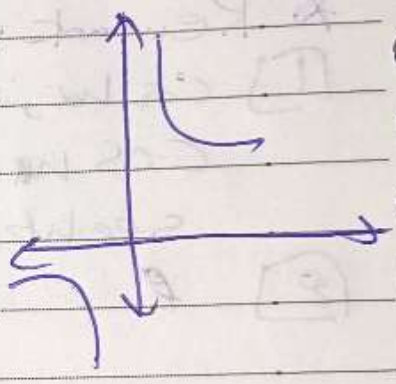
$\lim_{x \to +\infty} \tanh x = 1$

$\lim_{x \to -\infty} \tanh x = -1$

$\tanh(0) = 0$

$x > 0 \longrightarrow 0 < \tanh \leq 1$

$x < 0 \longrightarrow -1 < \tanh x < 0$



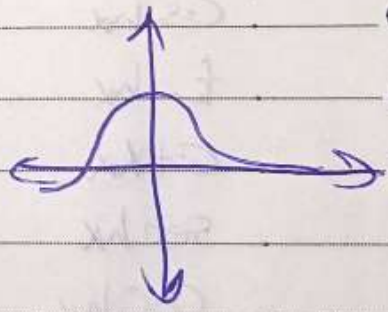
(4) Hyperbolic cotangent $\coth x = \frac{e^x + e^{-x}}{e^x - e^{-x}}$

$D = R \setminus \{0\}$

$R = (-\infty, -1) \cup (1, \infty)$

$\lim_{x \to +\infty} \coth x = 1$

$\lim_{x \to -\infty} \coth x = -1$



(5) Hyperbolic secant $\operatorname{sech} x = \frac{1}{e^x + e^{-x}}$

$D = R$

$R = (0, 1)$

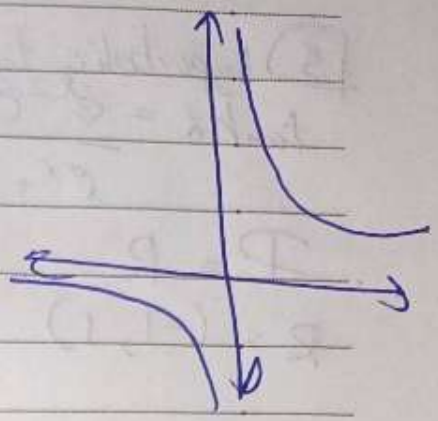
$\lim_{x \to +\infty} \operatorname{sech} x = 0$

$0 < \operatorname{sech} x \leq 1$

$\lim_{x \to -\infty} \operatorname{sech} x = 0$

Lecture (52)

(6) Hyperbolic Cosecant $\operatorname{csch} x = \frac{2}{e^x - e^{-x}}$



$D = R \setminus \{0\}$

$R = R \setminus \{0\}$

$\lim_{x \to +\infty} \operatorname{csch} x = 0$

$\lim_{x \to -\infty} \operatorname{csch} x = 0$

* Remark:

(1) $\cosh x$, $\operatorname{sech} x$, even function

$\cosh(-x) = \cosh x$

$\operatorname{sech}(-x) = \operatorname{sech} x$

(2) $\sinh$, $\tanh$, $\coth$, csch odd (ลี่)

f	f'
$\sinh x$	$\cosh x$
$\cosh x$	$\sinh x$
$\tanh x$	$\operatorname{sech}^2 x$
$\coth x$	$-\operatorname{csch}^2 x$
$\operatorname{sech} x$	$-\operatorname{sech} x \tanh x$
$\operatorname{csch} x$	$-\operatorname{csch} x \coth x$

Lecture (52)

* Theorems:-

1) $\cosh x + \sinh x = e^x$
2) $\cosh x - \sinh x = e^{-x}$
3) $\cosh^2 x - \sinh^2 x = 1$
4) $1 - \tanh^2 x = \operatorname{sech}^2 x$
5) $\coth^2 x - 1 = \operatorname{csch}^2 x$
6) $\sinh(x+y) = \sinh x \cosh y + \cosh x \sinh y$
7) $\cosh(x+y) = \cosh x \cosh y + \sinh x \sinh y$
8) $\sinh(x-y) = \sinh x \cosh y - \cosh x \sinh y$
9) $\cosh(x-y) = \cosh x \cosh y - \sinh x \sinh y$
10) $\cosh(2x) = \cosh^2 x + \sinh^2 x$
11) $\cosh(2x) = 2\sinh^2 x + 1 = 2\cosh^2 x - 1$
12) $\sinh^2 x = \dfrac{\cosh(2x) - 1}{2}$
13) $\cosh^2 x = \dfrac{\cosh(2x) + 1}{2}$

Lecture (53)

Example

1) Find $\sinh(2\ln 2)$

$= \sinh(\ln 2^2)$

$= \sinh(\ln 4)$

$= \dfrac{e^{\ln 4} - e^{-\ln 4}}{2}$

$= \dfrac{15}{8}$

2) $\cosh(\ln x)$

$= \dfrac{e^{\ln x} + e^{-\ln x}}{2}$

$= \dfrac{x^2 + 1}{2x}$

Lecture (53)

3) If $\sinh x + \cosh x = 5$ find

$e^x = 5$

$x = \ln 5$

4) Given that $\operatorname{csch} x = -\frac{1}{3}$ find $\sinh x$, $\cosh x$, $\coth x$, $\tanh x$, $\operatorname{sech} x$

$\sinh x = -3$

$\cosh^2 x - \sinh^2 x = 1$

$\cosh^2 x = 10$

$\cosh x = \pm\sqrt{10}$

$\operatorname{sech} x = \frac{1}{\sqrt{10}}$

$\coth x = -\frac{\sqrt{10}}{3}$

$\tanh x = \frac{-3}{\sqrt{10}}$

Examples:- find $\frac{dy}{dx}$ for

1) $y = \sinh(4x-8)$

$\frac{dy}{dx} = \cosh(4x-8)(4)$

2) $y = \cosh^3(2x)$

$\frac{dy}{dx} = 3\cosh^2(2x)\sinh(2x)(2)$

3) $y = \ln(\tanh(2x))$

$\frac{dy}{dx} = \frac{2\cdot\operatorname{sech}^2(2x)}{\tanh(2x)}$

4) $y = \operatorname{sech}(e^{2x})$

$\frac{dy}{dx} = -\operatorname{sech}(e^{2x})\tanh(e^{2x})\,2e^{2x}$

5) $y = \sinh(\cos 3x)$

$= \cosh(\cos 3x)(-\sin 3x)(3)$

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Lecture (53)

6) $y = (\cosh x)^{\sinh x}$

$\ln y = \sinh x \ln \cosh x$

$\frac{y'}{y} = \cosh x \cdot \ln(\cosh x) + (\sinh x)\frac{\sinh x}{\cosh x}$

7) $y = \frac{(\sinh x)(\ln x)(\tanh^{-1} x)}{\cos 2x}$

$\ln y = \ln(\sinh x) + \ln(\ln x) + \ln(\tanh^{-1} x) - \ln(\cos 2x)$

$\frac{y'}{y} = \frac{\cosh x}{\sinh x} + \frac{1}{x \ln x} + \frac{1}{(1+x^2)(\tanh^{-1} x)} + \frac{2\sin(2x)}{\cos(2x)}$

{Examples}

1) $f(x) = \sqrt{x^x}$

$y = \sqrt{x^x}$

$y = x^{\frac{x}{2}}$

$\ln y = \frac{x}{2} \ln x$

$\frac{y'}{y} = \left(\frac{x}{2}\right)\left(\frac{1}{x}\right) + (\ln x)\left(\frac{1}{2}\right)$

2) $y = \sin^2(\cos(\sqrt{\sinh(\ln x)}))$

$= 2\sin(\cos(\sqrt{\sinh(\ln x)}))\cos(\cos(\sqrt{\sinh(\ln x)}))(-\sin(\sqrt{\sinh(\ln x)}))\frac{\cosh(\ln x)}{2\sqrt{\sinh(\ln x)}}$

3) Find a such that $y = a\sin(3t) \rightarrow y'' + 2y = 4\sin(3t)$

$y' = 3a\cos(3t)$

$y'' = -9a\sin(3t)$

$-9a\sin(3t) + 2a\sin(3t) = 4\sin(3t)$

$-7a = 4$

$a = \frac{-4}{7}$

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Examples

3) If $g(x) = xf(x)$, $f(3) = 4$, $f'(3) = -2$ find the equation of tangent line of $g(x)$ at $x = 3$

$(3, 12)$

$g'(x) = f(x) + xf'(x)$

Slope $= 4 + (3)(-2)$

$= -2$

$y - 12 = -2(x - 3)$

4) If $f(x) = \frac{1}{2-x}$ find $f^{(n)}(x)$?

$f'(x) = \frac{-1(-1)}{(2-x)^2}$

$f'(x) = \frac{1}{(2-x)^2}$

$f''(x) = \frac{2}{(2-x)^3}$

$f'''(x) = \frac{6}{(2-x)^4}$

$f^{(4)} = \frac{24}{(2-x)^5}$

$f^{(n)} = \frac{n!}{(2-x)^{n+1}}$

5) Show that $\frac{d}{dx}\sqrt[4]{\frac{1+\tanh x}{1-\tanh x}} = \frac{1}{2}e^{\frac{x}{2}}$

$y = \left(\frac{1 + \frac{\sinh x}{\cosh x}}{1 - \frac{\sinh x}{\cosh x}}\right)^{\frac{1}{4}}$

$y = \left(\frac{\cosh x + \sinh x}{\cosh x - \sinh x}\right)^{\frac{1}{4}}$

$y = \left(\frac{e^x}{e^{-x}}\right)^{\frac{1}{4}}$

$y = \left(e^{2x}\right)^{\frac{1}{4}}$

$y = e^{\frac{x}{2}}$

$y' = \frac{1}{2}e^{\frac{x}{2}}$

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Lecture (54)

* Applications of Differentiation

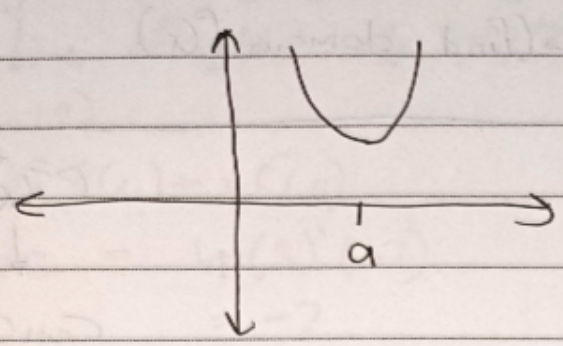
$f(x)$ → (find domain $f(x)$)

- $f'(x)$
  - a critical number $\in D_f$
    - $f'(a) = 0$ (has horizontal tangent line)
    - $f'(a)$ d.n.e
  - $f'(a)$ d.n.e
  - $f'(a)$ d.n.e
  - $f'(a) = 0$
  - * f Increasing :-
    If $x_2 > x_1 \rightarrow f(x_2) > f(x_1)$
    * + + + + $f'(x) > 0$
  - * f Decreasing :-
    If $x_2 > x_1 \rightarrow f(x_2) < f(x_1)$
    - - - - $f'(x) < 0$
  - * f Constant
    $f(x_2) = f(x_1) \forall x$ f
    - * a local max if $f(a) \geq f(x)$ when x is near a
    - * a local min if $f(a) \leq f(x)$ when x is near a
    - * a absolute max if $f(a) \geq f(x) \forall x \in D_f$
    - * a absolute min if $f(a) \leq f(x) \forall x \in D_f$
- $f''(x)$
  - Concavity
    * f Concave upward
    + + + + f'' =
    (the graph of f lies above all of its tangets on an interval)
    * f Concave downward
    (the graph of f lies below ∩)
  - * a inflection Points
    1) $a \in D_f$
    2) f cont at $x = a$
    3) the curve changes from concave upward to concave downward

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Example:-

1)



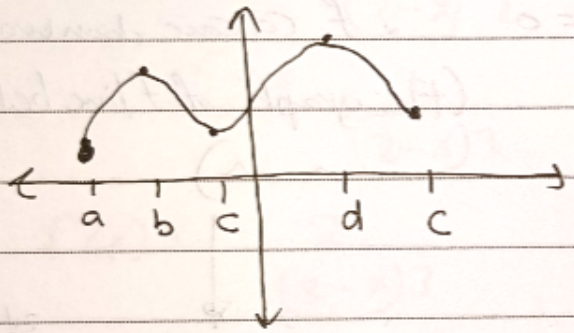
$D_f = R$

at $x = a$ local min $= f(a)$

$x = a$ absolute min $= f(a)$

No absolute max

2)



$D_f [a, b]$

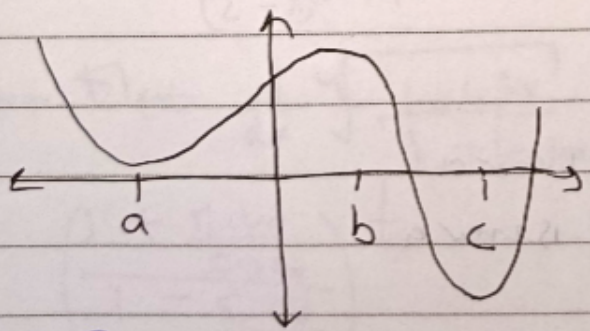
at $x = c$ local min $= f(c)$

at $x = b$ local max $= f(b)$

at $x = d$ absolute & local max $= f(d)$

at $x = a$ absolute min $= f(a)$

3)



$D_f = R$

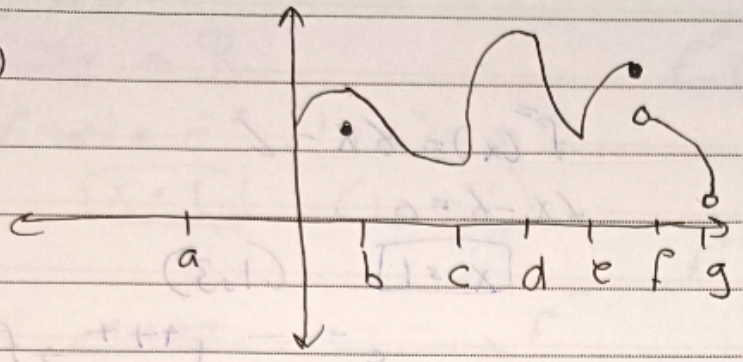
$x = a$ local min $= f(a)$

$x = c$ no

$x = b$ local max $f(b)$

Lecture(54)

4)



$D_f = [a, g]$

ab $x = c$ local min $f(c)$

$x = e$ " " $f(e)$

$x = b$ local min $= f(b)$

ab ← at $x = d$ local max $f(d)$
max

$x = f$ " " $f(f)$

* The extreme value theorem:-

If f continuous on a closed interval. then f attains an absolute maximum value $f(c)$ & an absolute minimum value $f(a)$ at some number c & d in $[a, b]$

* Fermats Theorem:-

If f has local maximum or minimum at c & $f'(c)$ exists then $f'(c) = 0$

* Theorem:-

If f has local maximum or minimum at c then c is a critical number of f.

Lecture (55)

Example

1) $f(x) = x^3 - 3x^2 + 5$

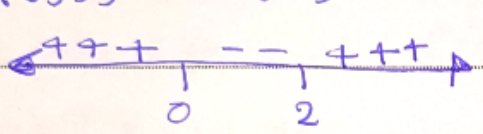
$D_f = \mathbb{R}$

$f'(x) = 3x^2 - 6x$

$3x(x-2) = 0$

$\boxed{x=0}$ $\boxed{x=2}$ Critical Number

$(0,5)$ $(2,0)$



f Increasing $(-\infty, 0) \cup (2, \infty)$

f Decreasing $(0, 2)$

at $x = 0$ local max $f(0) = 5$

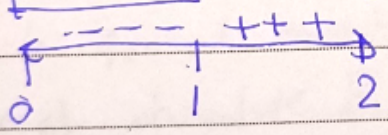
at $x = 2$ " min $f(2) = 0$

2) $f(x) = x^4 - 4x$, $x \in [0, 2]$

$f'(x) = 4x^3 - 4$

$\boxed{x=1}$ $(1, -3)$

$(0,0)$



$(2,1)$

f Decreasing $(0, 1)$

f Increasing $(1, 2)$

$x = 1$ local min $f(1) = -3$

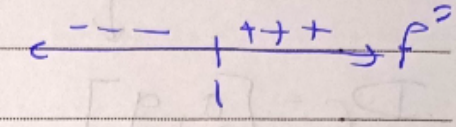
$x = $ absolute min " "

$x = 2$ absolute max $f(2) = 1$

$f''(x) = 6x - 6$

$6x - 6 = 0$

$\boxed{x=1}$ $(1,3)$



f Concave upward $(1, \infty)$

f " downward $(-\infty, 1)$

1) $D_f = \mathbb{R}$

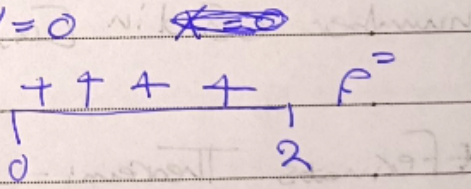
2) f Con at $x = 1$

2) f'' Changes about $x = 1$

$(1,3)$ → Inflection Point

$f''(x) = 12x^2$

$x = 0$ ~~$x = 0$~~



f Concave upward $(0, 2)$

No Inflection Point

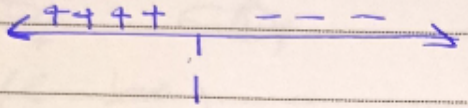
Lecture (55)

3) xe^{-x}

$D_f = \mathbb{R}$

$f'(x) = e^{-x}(1-x) = 0$

$\boxed{x=1}$ $(1, e^{-1})$



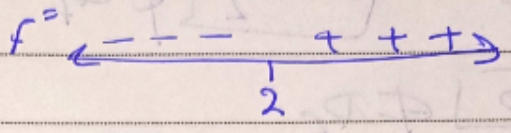
f Increasing $(-\infty, 1)$

f decreasing $(1, \infty)$

at $x=1$ local & absolute max $= e^{-1}$

$f''(x) = -2e^{-x} + xe^{-x} = 0$

$\boxed{x=2}$ $(2, 2e^{-2})$



f Concave down $(-\infty, 2)$

f Concave up $(2, \infty)$

Inflection Point

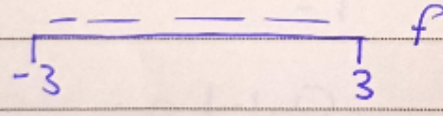
Lecture (56)

Example: $f(x) = \sqrt{9-x^2}$ find the interval of Concavity

$f'(x) = \dfrac{-x}{\sqrt{9-x^2}}$ $D_f = [-3, 3]$

$f''(x) = \dfrac{-9}{\sqrt{(9-x^2)^3}}$

$x = 3 / -3$



Concave downward $(-3, 3)$

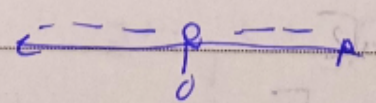
Example:-

$f(x) = \dfrac{1}{x^3}$ find the intervals of Increasing & decreasing

$D_f = \mathbb{R} / \{0\}$

$f'(x) = \dfrac{-3}{x^4}$

$\boxed{x=0}$



f Decreasing $(-\infty, 0) \cup (0, \infty)$

$\mathbb{R} / \{0\}$

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Lecture (56)

Example: $f(x) = x^2 - x - \ln x$ find the critical numbers of function

$D_f = (0, \infty)$

$f'(x) = \frac{2x^2 - x - 1}{x}$

$x = 1$ $\quad$ $x = -\frac{1}{2} \notin D_f$

$x = 0 \notin D_f$

$(1, 0)$

Example: $f(x) = (x+1)^5 (x-2)^5$ find critical numbers:-

$D_f = R$

$f'(x) = 5(x+1)^4 (x-2)^4 + (x+1)^5 (4(x-2)^3 = 0$

$x = -1$

$x = 2$

$x = \frac{2}{3}$

→ Critical numbers

Lecture (57)

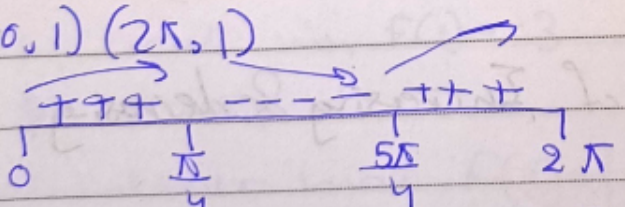
Example $f(x) = \cos x + \sin x \quad x \in (0, 2\pi)$

$f'(x) = -\sin x + \cos x = 0$

$\sin x = \cos x$

$x = \frac{\pi}{4}, \frac{5\pi}{4} \quad (\frac{\pi}{4}, \sqrt{2}) \quad (\frac{5\pi}{4}, -\sqrt{2})$

End Points $(0, 1) \quad (2\pi, 1)$



at $x = \frac{\pi}{4}$ local max $f(\frac{\pi}{4}) = \sqrt{2}$

$x = \frac{5\pi}{4}$ " min $f(\frac{5\pi}{4}) = -\sqrt{2}$

at $x = \frac{\pi}{4}$ absolute max $f(\frac{\pi}{4}) = \sqrt{2}$

at $x = \frac{5\pi}{4}$ " min

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Lecture (57)

Example: $f(x)=\frac{\ln x}{x}$ find the Intervals of Increasing & decreasing

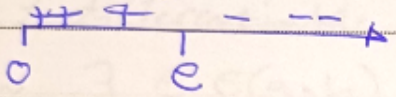
$D_f (0,\infty)$

$f'(x)=\frac{1-\ln x}{x^2}$

$x=0$ $x=e$

$\notin D_f$ $\in D_f$

at $(e, \frac{\ln e}{e})$



f Increasing $(0,e)$

f Decreasing (e,∞)

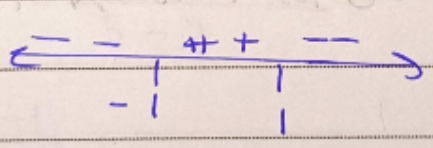
Example: $f(x)=\frac{x^2}{x^2+3}$ find the Interval of Concavity

$D_f = \mathbb{R}$

$f'(x)=\frac{6x}{(x^2+3)^2}$

$f''(x)=\frac{18-18x^2}{(x^2+3)^3}$

$x=\pm 1$



f Concave up $(-1,1)$

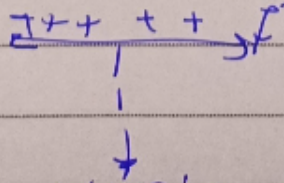
f Concave down $(1,\infty)\cup(-\infty,-1)$

at $(-1,f(-1))$, $(1,f(1))$ Inflection Point

Example: $f(x)=(x-1)^3$ find Interval of Increasing & Decreasing

$f'(x)=3(x-1)^2$

$x=1$



f Increasing on $\mathbb{R}$

~~Asymptote~~ Critical Point

Lecture (57)

Example: $f(x) = |2x-8|$ find Critical Number

$D_f = \mathbb{R}$

$$f(x) = \begin{cases} 2x-8, & x>4 \\ 8-2x, & x<4 \end{cases}$$

$$f'(x) = \begin{cases} 2, & x>4 \\ -2, & x<4 \\ \text{dine}, & x=4 \end{cases}$$

$f'_+(4) = 2$

$f'_-(4) = -2$

$f'(4) = \text{dine}$

at $x=4$ critical numbers $(4, 0)$

Example: If at $x=1$ we have extrema values for $f(x) = ax^3 - 2x + 3$

$f(x) = ax^3 - 2x + 3$ $\quad f'(1) = 0$

$f'(x) = 3ax^2 - 2$

$0 = 3a - 2$

$a = \frac{2}{3}$

Example: If $(1, 3)$ extrema Points for $f(x) = x^2 + ax + b$?

$f(x) = x^2 + ax + b$

$f'(x) = 2x + a$

$0 = 2 + a$

$\boxed{a = -2}$

$f(1) = 3$

$3 = 1 + -2 + b$

$3 = -1 + b$

$\boxed{b = 4}$

Lecture (59)

* Rolles Theorem: Let y = f(x) that satisfies

[1] f continuous on [a, b]

[2] f differentiable on (a, b)

[3] f(a) = f(b)

Then ∃ c ∈ (a, b) such that f'(c) = 0

* The mean value Theorem: Let y = f(x) that satisfies:-

[1] f continuous on [a, b]

[2] f differentiable on (a, b)

Then ∃ c ∈ (a, b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

Example: find the value(s) of c that satisfy the inclusion of M.V.T?

1) $f(x) = \frac{1}{x-1}$ on [2, 5]

[1] f con on [2, 5]

[2] f dif on [2, 5]

[3] $f'(c) = \frac{f(5) - f(2)}{5 - 2}$

$\frac{-1}{(c-1)^2} = \frac{-1}{4}$

$4 = (c-1)^2$

$c - 1 = 2 \qquad c - 1 = -2$

$\boxed{c = 3} \qquad \boxed{c = -1}$

Lecture (59)

2) $f(x) = \sqrt{25 - x^2}$ on $[-3, 5]$

[1] f on $[-3, 5]$

[2] $f(x) = \frac{x}{\sqrt{25 - x^2}}$ dif $(-3, 5)$

[3] $f'(c) = \frac{f(5) - f(-3)}{5 - -3}$

$\frac{-c}{\sqrt{25 - c^2}} = \frac{-4}{8}$

$2c = \sqrt{25 - c^2}$

$4c^2 = 25 - c^2$

$5c^2 = 25$

$c = \pm\sqrt{5}$

$\sqrt{5}$ مقبول $-\sqrt{5}$ مرفوض

Lecture (60)

Indeterminate forms and L'Hopital's Rule:-

[1] $\frac{0}{0}, \frac{+\infty}{+\infty}, \frac{+\infty}{-\infty}, \frac{-\infty}{+\infty}, \frac{-\infty}{-\infty}$ ...

Examples:-

1) $\lim_{x \to +\infty} \frac{e^{3x}}{x^2} = \frac{+\infty}{+\infty}$ نشتق البسط والمقام

$\lim_{x \to +\infty} e^{3x} = +\infty$

$\lim_{x \to +\infty} x^2 = +\infty$

$\lim_{x \to +\infty} \frac{3e^{3x}}{2x} = \frac{+\infty}{+\infty}$

$\lim_{x \to +\infty} \frac{9e^{3x}}{2} = \frac{+\infty}{2} = +\infty$

lecture (60)

2) $\lim_{x \to 0^+} \frac{\ln(\sin x)}{\ln(\tan x)}$

$= \frac{-\infty}{-\infty}$

$\lim_{x \to 0^+} \cos^2 x = 1$

$+\infty + \infty = +\infty$

$-\infty - \infty = -\infty$

2 $0 \cdot \pm\infty$, $+\infty - \infty$, $-\infty + \infty$ نغير بالرتابة المحافظة على القيمة

Example:-

1) $\lim_{x \to 0^+} x \ln x$ $\quad 0 \cdot -\infty$

$= \lim_{x \to 0^+} \frac{\ln x}{\frac{1}{x}}$

$= \frac{-\infty}{+\infty}$

$= \lim_{x \to +\infty} \frac{\frac{1}{x}}{\frac{-1}{x^2}}$

$= 0$

$\frac{a}{b} = a \cdot \frac{1}{b}$

$a \cdot b = \frac{a}{\frac{1}{b}}$

$a \cdot b = \frac{b}{\frac{1}{a}}$

$x = e^{\ln x}$

$x = \ln e^x$

$\frac{a}{b} + \frac{c}{d} = \frac{ad + bc}{bd}$

$\sec x = \frac{1}{\cos x}$

2) $\lim_{x \to \frac{\pi}{2}^-} (\sec x - \tan x)$

$\lim_{x \to \frac{\pi}{2}^-} \frac{1}{\cos x} = \frac{\text{كبير}}{\text{صفر}}$

$\cos x = 0$ $\quad$ ++ + $\quad$ 1

$\lim_{x \to \frac{\pi}{2}^-} \tan x = +\infty$

$= +\infty - +\infty$

$\frac{\pi}{2}$ $\quad$ +++ --- $\quad$ $\frac{\pi}{2}$

$\lim_{x \to \frac{\pi}{2}^-} \left(\frac{1}{\cos x} - \frac{\sin x}{\cos x} \right)$

$= \frac{0}{0}$

lecture (80)

$$\lim_{x \to \frac{\pi}{2}} \frac{-\cos x}{-\sin x} = \frac{0}{1} = 0$$

3) $\lim_{x \to \frac{\pi}{2}^-} \sec(3x)\cos(5x)$

$$\lim_{x \to \frac{\pi}{2}^-} \sec(3x) = \lim_{x \to \frac{\pi}{2}^-} \frac{1}{\cos(3x)} = \begin{cases} +\infty \\ -\infty \end{cases}$$

$$\lim_{x \to \frac{\pi}{2}^-} \cos(5x) = 0$$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{1}{\cos(3x)} \cdot \cos(5x)$$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{\cos(5x)}{\cos(3x)} = \frac{0}{0}$$

$$\lim_{x \to \frac{\pi}{2}^-} \frac{-5\sin(5x)}{-3\sin(3x)}$$

$$= \frac{-5}{3}$$

4) $\lim_{x \to +\infty} (x - \ln(x^2+1))$ $(+\infty - +\infty)$

$$\lim_{x \to +\infty} (\ln e^x - \ln(x^2+1))$$

$$\lim_{x \to +\infty} \ln\left(\frac{e^x}{x^2+1}\right)$$

$$= \frac{+\infty}{+\infty}$$

$$\ln\left(\lim_{x \to +\infty} \left(\frac{e^x}{2x}\right)\right)$$

$+\infty$

5) $\lim_{x \to \infty} (5^x - 2^x) = +\infty - +\infty$

$$\lim_{x \to +\infty} 5^x \left(1 - \frac{2^x}{5^x}\right)$$

$$+\infty(1-0)$$

$+\infty$

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Lecture (61)

3 $1^{\pm\infty}$, ∞^{0} , 0^{0}

Example :-

1 $\lim_{x\to 0^+} (1+3x)^{\frac{-5}{x}}$

$\lim_{x\to 0^+} (1+3x) = 1$

$\lim_{x\to 0^+} \frac{-5}{x} = -\infty$

$= 1^{-\infty}$

$y = (1+3x)^{\frac{-5}{x}}$

$\ln y = \frac{-5}{x} \ln(1+3x)$

$\lim_{x\to 0^+} \frac{-5}{x} \cdot \ln(1+3x)$

$= -\infty * 0$

$= \lim_{x\to 0^+} \frac{-5\ln(1+3x)}{x}$

$= \frac{0}{0}$

$= \lim_{x\to 0^+} \frac{-5\left(\frac{3}{1+3x}\right)}{1}$

$= \frac{-15}{1}$

$= -15$

$= e^{-15}$

1 ما داخل $y = \lim$

2 نأخذ ln

3 نجد الـ lim للمقدار الناتج

4 الجواب = الجواب في خطوة 3

2 $\lim_{x\to 0^+} x^{x}$

$= 0^{0}$

$y = x^{x}$

$\ln y = x \ln x$

$\lim_{x\to 0^+} x \ln x$

$= 0 \cdot -\infty$

$= \lim_{x\to 0^+} \frac{\ln x}{\frac{1}{x}}$

$= \frac{-\infty}{+\infty}$

$= \lim_{x\to 0^+} \frac{\frac{1}{x}}{\frac{-1}{x^2}}$

$= \lim_{x\to 0^+} -x = 0$

$\lim_{x\to 0^+} x^{x} = e^{0} = 1$

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Lecture (61)

* **Remark:-**

1. $\lim_{x \to \pm\infty} \left(1 + \frac{a}{x}\right)^{bx} = e^{ab}$

2. $\lim_{x \to 0} (1 + ax)^{\frac{b}{x}} = e^{ab}$

Example:-

1. $\lim_{x \to +\infty} \left(1 + \frac{5}{x}\right)^{x} = e^{5(1)} = e^{5}$

2. $\lim_{x \to 0} (1 - x)^{\frac{3}{x}} = e^{(-1)(3)} = \frac{1}{e^3}$

3. $\lim_{x \to \infty} \left(\frac{x+4}{x}\right)^{5x} = \lim_{x \to \infty} \left(\frac{x}{x} + \frac{4}{x}\right)^{5x}$

$= \lim_{x \to \infty} \left(1 + \frac{4}{x}\right)^{5x}$

$= e^{20}$

4. $\lim_{x \to \infty} \left(\frac{x+2}{x+3}\right)^{x} = \lim_{x \to \infty} \left(\frac{\frac{x+2}{x}}{\frac{x+3}{x}}\right)^{x}$

$= \lim_{x \to 0} \left(\frac{1 + \frac{2}{x}}{1 + \frac{3}{x}}\right)^{x}$

$= \frac{e^2}{e^3} = \frac{1}{e}$

Lecture (62)

Indefinite Integrals: التكامل الغير محدود

$\int f(x)\,dx = F(x) + C$, C constant

Examples: - Find general antiderivative of:-

1) $h(x) = \sec^2 x$

$\int \sec^2 x\,dx$

$= \tan x + C$

2) $g(x) = \cos x + \frac{1}{1+x^2}$

$\int \cos x + \frac{1}{1+x^2}$

$\sin x + \tan^{-1} x + C$

* Definite Integrals:-

$\int_a^b f(x)\,dx = F(x)\Big]_a^b = F(b) - F(a)$

1) $\int k\,dx = kx + C$, k, C Constant

Example: $\int 5x\,dx = 5x + C$

2) $\int x^r\,dx = \frac{x^{r+1}}{r+1} + C$, $r \neq -1$

Example:-

$\int_1^2 \frac{1}{\sqrt{x}}\,dx$

$\int_1^2 x^{-\frac{1}{2}}\,dx$

$\frac{x^{\frac{1}{2}}}{\frac{1}{2}}$

$2\sqrt{x}\,\Big|_1^2$

$2(\sqrt{2} - \sqrt{1})$

$2\sqrt{2} - 2$

Lecture (62)

2) $\int \frac{5x^3 - x}{x} \cdot dx$

$\int 5x^2 - 1 \cdot dx$

$\frac{5x^3}{3} - x + C$

3) $\int \sqrt{x}\,(1+x) \cdot dx$

$\int (\sqrt{x} + x^{\frac{3}{2}}) \cdot dx$

$\int (x^{\frac{1}{2}} + x^{\frac{3}{2}}) \cdot dx$

$= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} + \frac{x^{\frac{5}{2}}}{\frac{5}{2}} + C$

$= \frac{2}{3}\sqrt{x^3} + \frac{2}{5}\sqrt{x^5} + C$

[3] $\int (ax+b)^r \cdot dx = \frac{(ax+b)^{r+1}}{a(r+1)} + C, \quad r \neq -1$

Example:

1) $\int (5x+3)^3 \cdot dx$

$\frac{(5x+3)^4}{20} + C$

2) $\int_0^1 \frac{1}{\sqrt{7-3x}} \cdot dx$

$\int_0^1 (7-3x)^{-\frac{1}{2}} \cdot dx$

$\frac{(7-3x)^{\frac{1}{2}}}{-3 \times \frac{1}{2}} \Big|_0^1$

$\frac{2}{-3}\sqrt{(7-3x)} \Big|_0^1$

$\frac{-2}{3}(\sqrt{4} - \sqrt{7})$

PRENKO

Lecture (83)

4) $\int \cos(ax+b)\,dx = \frac{1}{a}\sin(ax+b)+C$

$\int \sin(ax+b)\,dx = -\frac{1}{a}\cos(ax+b)+C$

$\int \sec^2(ax+b)\,dx = \frac{1}{a}\tan(ax+b)+C$

$\int \csc^2(ax+b)\,dx = -\frac{1}{a}\cot(ax+b)+C$

$\int \sec(ax+b)\tan(ax+b)\,dx = \frac{1}{a}\sec(ax+b)+C$

$\int \csc(ax+b)\cot(ax+b)\,dx = -\frac{1}{a}\csc(ax+b)+C$

Examples:-

1) $\int \sin(5x+7)\,dx$

$-\frac{1}{5}\cos(5x+7)+C$

$\cos^2 x = \frac{1+\cos(2x)}{2}$

$\sin^2 x = \frac{1-\cos(2x)}{2}$

$\sin 2x = 2\sin x\cos x$

2) $\int 2\sin x\cos x\,dx$

$\int \sin 2x\,dx$

$-\frac{1}{2}\cos(2x)+C$

3) $\int \frac{\sin 2x}{\sin x}\,dx$

$\int 2\cos x\,dx$

$2\sin x + C$

5) $\int c f(x)\,dx = c\int f(x)\,dx$

$\int (f \pm g)\,dx = \int f(x)\,dx \pm \int g(x)\,dx$

Example:

1) $\int \sec^2 x - 5\cos(9x)\,dx$

$\tan x + \frac{5}{9}\sin(9x)+C$

2) $\int \sin^2(3x)\,dx$

$\int \frac{1-\cos 6x}{2}\,dx$

$\frac{1}{2}x - \frac{1}{12}\sin 6x + C$

PRENKO

lecture (63)

3] $\int \frac{\sec x + \cos x}{2\cos x} \, dx$

$\int 2\sec^2 x + \frac{1}{2} \, dx$

$2\tan x + \frac{1}{2}x + C$

4] $\int \frac{2}{1-\cos x} \, dx$

$\int \frac{2}{1-\cos x} \cdot \frac{(1+\cos x)}{(1+\cos x)} \, dx$

$\int \frac{2+2\cos x}{1-\cos^2 x} \, dx$

$\int \frac{2+2\cos x}{\sin^2 x} \, dx$

$\int 2\csc^2 x + 2\cot x \csc x \, dx$

$-2\cot x + 2\csc x + C$

5) $\int \frac{\cos x}{\sin^2 x} \, dx$

$\int \cot x \csc x \, dx$

$-\csc x + C$

6] $\int_a^a f(x) \, dx = 0$

$\int_{-a}^{a}$ odd function $= 0$

$\int_a^b C \, dx = C(b-a)$

$\int_{-a}^{a}$ even function $= 2\int_0^a$ even function

$\int_a^b f(x) \, dx = -\int_b^a f(x) \, dx$

$\int_a^b f(x) \, dx = \int_a^c f(x) + \int_c^b f(x) \, dx$

PRENKO

Lecture (64)

Example:

① If $\int_1^2 3f(x)\,dx = 6$, $\int_3^2 5 - f(x)\,dx = 4$

find $\int_1^3 f(x)\,dx$?

$$\int_1^2 3f(x)\,dx = 6$$

$$\int_1^2 f(x)\,dx = 2$$

$$\int_3^2 5\,dx - \int_3^2 f(x)\,dx = 4$$

$$5(2-3) - \int_3^2 f(x)\,dx = 4$$

$$-5 - \int_3^2 f(x)\,dx = 4$$

$$\int_3^2 f(x)\,dx = -9$$

$$\int_1^2 f(x) + \int_2^3 f(x) = \int_1^3 f(x)$$

$$2 + 9 = 11$$

② Find $\int_{-3}^{3} \frac{\sin 2x}{x^4+x^2+1}\,dx$

$$\frac{\sin(-2x)}{(-x)^4+(-x)^2+1} = \frac{-\sin(2x)}{x^4+x^2+1} = -f(x)$$ odd fn $\rightarrow 0$

② If $\int_{-1}^{a} x\,dx = 0$ find a

$$\frac{x^2}{2}\Big|_{-1}^{a} = 0 \qquad \frac{a^2}{2} = \frac{1}{2}$$

$$\frac{a^2}{2} - \frac{1}{2} = 0$$

$a = \pm 1$

Lecture (64)

4) If $f(x) = \begin{cases} 2x, & 0 \le x \le 2 \\ x^2, & 2 \le x \le 3 \end{cases}$ Find $\int_0^3 f(x)\,dx$

$$\int_0^3 f(x)\,dx = \int_0^2 2x + \int_2^3 x^2\,dx$$

$$= x^2 \Big|_0^2 + \frac{x^3}{3}\Big|_2^3$$

Lecture (65)

7) $\int e^{ax+b}\,dx = \frac{e^{ax+d}}{a} + C$

$\int b^{ax+d}\,dx = \frac{b^{ax+d}}{a \ln b} + C$

Examples:-

1) $\int_0^{\ln 2} e^x\,dx$

$e^x \Big|_0^{\ln 2}$

$e^{\ln 2} - e^0$

$2 - 1$

$= 1$

2) $\int 7^{4-7x}\,dx$

$\frac{7^{4-3x}}{-3 \ln 7} + C$

3) $\int e^{6x+2}\,dx$

$\frac{e^{6x+2}}{6} + C$

4) $\int 2^x \cdot e^x\,dx$

$\int (2e)^x\,dx$

$= \frac{(2e)^x}{\ln 2e} + C$

$= \frac{(2e)^x}{\ln 2 + 1} + C$

Lecture (65)

8. $\int \frac{1}{x} \cdot dx = \ln|x| + C$

$\int \frac{f'(x)}{f(x)} \cdot dx = \ln f(x) + C$

Examples:-

1) $\int \frac{4}{x} \cdot dx = 4\ln x + C$

2) $\int_{-1}^{2} \frac{5}{4+x} \cdot dx = 5\ln|4+x| \Big]_{-1}^{2}$

$= 5\ln\frac{6}{3} = 5\ln 2 = \ln 2^5 = \ln 36$

3) $\int \frac{x}{1+x^2} \cdot dx$

$= \frac{1}{2}\ln|1+x^2| + C$

4) $\int \tan x \cdot dx$

$= \int \frac{\sin x}{\cos x} \cdot dx$

$-\ln|\cos x| + C \longrightarrow \ln|\sec x| + C$

5) $\int \frac{x^2}{7+x^3} \cdot dx$

$\frac{1}{3}\ln|7+x^3| + C$

6) $\int \frac{e^x + e^{-x}}{e^x - e^{-x}} \cdot dx$

$\ln|e^x - e^{-x}| + C$

Lecture (65)

7) $\int (\ln e^x + \ln e^{-x}) \, dx$

$= \int (x + -x) \, dx$

$\int 0 \, dx$

$0 + C$

$= C$

8) $\int \frac{\cos 2x}{1 + \sin 2x} \, dx$

$\frac{1}{2} \ln|1 + \sin 2x| + C$

Lecture (66)

[9] $\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C$

$\int \frac{g'(x)}{1 + (g(x))^2} \, dx = \tan^{-1}(g(x)) + C$

Example :-

1) $\int \frac{1}{1 + x^2} \, dx = \tan^{-1}(x) + C$

2) $\int \frac{1}{16 + x^2} \, dx = \frac{1}{4} \tan^{-1}\left(\frac{x}{4}\right) + C$

3) $\int \frac{1}{5 + x2} \, dx = \frac{1}{\sqrt{5}} \tan^{-1}\left(\frac{x}{\sqrt{5}}\right) + C$

4) $\int \frac{3}{1 + 9x^2} \, dx = \tan^{-1}(3x) + C$

5) $\int \frac{1}{1 + 16x^2} \, dx = \frac{1}{4} \int \frac{4}{1 + (4x)^2} \, dx$

$= \frac{1}{4} \tan^{-1}(4x) + C$

6) $\int \frac{t}{1 + t^4} \, dt = \int \frac{t}{1 + (t^2)^2} \, dt = \frac{1}{2} \int \frac{2t}{1 + (t^2)^2} \, dt$

$= \frac{1}{2} \tan^{-1}(t^2) + C$

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Lecture (66)

7) $\int \frac{x^2}{1+x^6}\,dx$

$= \frac{1}{3}\int \frac{3x^2}{1+(x^3)^2}\,dx$

$= \frac{1}{3}\tan^{-1}(x^3) + C$

8) $\int \frac{e^{2x}}{1+e^{4x}} = \frac{1}{2}\int \frac{2e^{2x}}{1+(e^{2x})^2}$

$= \frac{1}{2}\tan^{-1}(e^{2x}) + C$

9) $\int \frac{e^x}{1+e^{2x}}\,dx$

$\int \frac{e^x}{1+(e^x)^2}\,dx$

$\tan^{-1}(e^x) + C$

10) $\int \frac{t^2-1}{t^4-1}\,dt$

$\int \frac{1}{(t^2+1)}\,dt$

$\tan^{-1}(t) + C$

Lecture (67)

10) $\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{a}\right) + C$

$\int \frac{g'(x)}{\sqrt{1-g(x)^2}}\,dx = \sin^{-1}(g(x)) + C$

Examples -

1) $\int_{\ln(2/\sqrt{3})} \frac{1}{\sqrt{9-x^2}}\,dx = \int \frac{1}{\sqrt{3^2-x^2}}\,dx = \sin^{-1}\left(\frac{x}{3}\right) + C$

2) $\int_{\ln 2} \frac{e^{-x}}{\sqrt{1-e^{-2x}}} = -\sin^{-1}(e^{-x})\Big]_{\ln 2}^{\ln 2/\sqrt{3}}$

$-\sin^{-1}\left(e^{-\frac{\ln 2}{\sqrt{3}}}\right) - -\sin^{-1}\left(e^{-\ln 2}\right)$

$-\sin^{-1}\left(e^{\ln\frac{1}{2}}\right) + \sin^{-1}\left(e^{\ln\frac{1}{2}}\right)$

$= -\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$

$\frac{-\pi}{3} + \frac{\pi}{6}$

$= -\frac{\pi}{6}$

Lecture (67)

3) $\int \frac{\sec^2\theta}{\sqrt{1-\tan^2\theta}} \, d\theta$

$= \sin^{-1}(\tan\theta) + C$

4) $\int \frac{dx}{x\sqrt{1+(\ln x)^2}}$

$= \int \frac{\frac{1}{x}}{\sqrt{1-(\ln x)^2}} \, dx$

$= \sin^{-1}(\ln x) + C$

[1] $\int \frac{1}{x\sqrt{x^2-a^2}} \, dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + C$

$\int \frac{g'(x)}{g(x)\sqrt{(g(x))^2-1}} \, dx = \sec^{-1}(g(x)) + C$

Example:-

1) $\int \frac{1}{x\sqrt{x^2-4}} \, dx = \frac{1}{2}\sec^{-1}\left(\frac{x}{2}\right) + C$

2) $\int \frac{1}{x\sqrt{x^2-1}} \, dx = \sec^{-1}(x) + C$

Lecture (68)

[1] Integration by substitution : تكامل بالتعويض

* Examples:-

1) $\int \frac{\sqrt{\tan^{-1}x}}{1+x^2} \, dx$

$y = \tan^{-1}x$

$\frac{dy}{dx} = \frac{1}{1+x^2} \rightarrow dx = (1+x^2)\,dy$

$\int \frac{\sqrt{y}}{1+x^2} \cdot (1+x^2)\, dy$

$\int \sqrt{y} \, dy$

$\frac{2}{3}y^{\frac{3}{2}} + C$

$= \frac{2}{3}\sqrt{(\tan^{-1}x)^3} + C$

PRENKO

lecture (68)

2) $\int_0^{\frac{\pi}{2}} \sin^2 x \cos x \cdot dx$

$y = \sin x \qquad y = \sin 0 \qquad y = \sin \frac{\pi}{2}$

$\frac{dy}{dx} = \cos x \qquad y = 0 \qquad y = 1$

$\int_0^1 y^2 \cos x \cdot \frac{dy}{\cos x}$

$\int_0^1 y^2 \cdot dy$

$\frac{y^3}{3} \Big|_0^1$

$\frac{1}{3} - 0$

$= \frac{1}{3}$

3) $\int_0^1 x\sqrt{1-x} \cdot dx$

$y = \sqrt{1-x}$

$y^2 = 1 - x$

$2y \frac{dy}{dx} = -1$

$dx = -2y\,dy$

$\int_1^0 x \cdot y \cdot \frac{dy}{-1} (-2y)$

$\int_1^0 -2xy^2 \cdot dy$

$2 \int_0^1 y^2 (1-y^2) \cdot dy$

$2 \int_0^1 y^2 - y^4 \cdot dy$

~~$2 \left[(0) - (0-0) \right] y^3$~~

~~$= 0$~~

$2 \left(\frac{y^3}{3} - \frac{y^5}{5} \right) \Big|_0^1$

$2 \left[\frac{1}{3} - \frac{1}{5} \right)$

$2 \left(\frac{5-3}{15} \right) = \frac{4}{15}$

PRENKO

Lecture (69)

1) $\int (1+\sqrt{x})^{20} \cdot dx$

$y = 1+\sqrt{x}$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x}}$

$2\sqrt{x}\, dy = dx$

$\int y^{20} \cdot dy\, 2\sqrt{x}$

$\int 2y^{20}(y-1) \cdot dy$

$\int 2y^{21} - 2y^{20} \cdot dy$

$\frac{2y^{22}}{22} - \frac{2y^{21}}{21} + C$

$\frac{y^{22}}{11} - \frac{2}{21} y^{21} + C$

$\frac{(\sqrt{x}+1)^{22}}{11} - \frac{2}{21}(1+\sqrt{x})^{21} + C$

2) $\int \frac{\sin x}{16 + 9\cos^2 x} \cdot dx$

$y = \cos x$

$\frac{dy}{dx} = -\sin x$

$dx = \frac{-dy}{\sin x}$

$\int \frac{\sin x}{16+9y^2} \cdot \frac{-dy}{\sin x}$

$\int \frac{-1}{16+9y^2} \cdot dy$

$-\int \frac{1}{9\left(\frac{16}{9}+y^2\right)} \cdot dy$

$\frac{-1}{9} \int \frac{1}{\left(\frac{16}{9}+y^2\right)} \cdot dy$

$\frac{-1}{9}\left(\frac{1}{\frac{4}{3}}\right) \tan^{-1}\left(\frac{y}{\frac{4}{3}}\right) + C$

$\frac{-3}{36} \tan^{-1}\left(\frac{3y}{4}\right) + C$

$\frac{-1}{12} \tan^{-1}\left(\frac{3}{4}y\right) + C$

3) If $\int_1^5 f(x) \cdot dx = 10$ find $\int_0^2 x f(x^2+1) \cdot dx$

$y = x^2 + 1$

$dy = 2x \cdot dx$

$dx = \frac{dy}{2x}$

$\int_1^5 x\, f(y) \cdot \frac{dy}{2x}$

$\frac{1}{2}\int_1^5 f(y) \cdot dy$

$= 5$

PRENKO

Lecture (70)

[13] $\int \cosh(ax+b)\,dx = \frac{1}{a}\sinh(ax+b) + C$

$\int \sinh(ax+b)\,dx = \frac{1}{a}\cosh(ax+b) + C$

$\int \operatorname{sech}^2(ax+b)\,dx = \frac{1}{a}\tanh(ax+b) + C$

$\int \operatorname{csch}^2(ax+b)\,dx = -\frac{1}{a}\coth(ax+b) + C$

$\int \operatorname{sech}(ax+b)\tanh(ax+b)\,dx = -\frac{1}{a}\operatorname{sech}(ax+b) + C$

$\int \operatorname{csch}(ax+b)\coth(ax+b)\,dx = -\frac{1}{a}\operatorname{csch}(ax+b) + C$

Example :-

1) $\int \sinh(5x+4)\,dx = \frac{1}{5}\cosh(5x+4) + C$

2) $\int \operatorname{sech}(9x)\tanh(9x)\,dx = -\frac{1}{9}\operatorname{sech}(9x) + C$

3) $\int \operatorname{sech}^2(3-9x)\,dx = -\frac{1}{9}\tanh(3-9x) + C$

4) $\int (\sinh x + \cosh x)\,dx = \cosh x + \sinh x + C$

$e^x + C$

5) $\int \frac{\operatorname{sech}(5x)\tanh(5x)}{4+\operatorname{sech}(5x)}\,dx$

$y = \operatorname{sech}(5x)$

$dy = -5\sec(5x)\tan(5x)\,dx$

$dx = \frac{-dy}{5\sec(5x)\tan(5x)}$

$= \int \frac{\operatorname{sech}(5x)\tanh(5x)}{4+y} \cdot \frac{dy}{-5\sec(5x)\tan(5x)}$

$\frac{-1}{5}\int \frac{1}{4+y}\,dy$

$\frac{-1}{5}\ln|y+4| + C$

[14] $\frac{dy}{dx} \int_a^b f(x).dx = 0$

$\frac{d}{dx} \int_a^x f(x).dx = f(x)$

Examples:-

1) find $\frac{d}{dx} \int_0^4 \sqrt{5+\cos x} = 0$

2) If $y = \int_0^x \sqrt{5+\cos x}$ find $\frac{dy}{dx} = \sqrt{5+\cos x}$

3) If $\int f(x).dx = x^4 + x + 4$ find $f(1)$, $f'(1)$

$\frac{d}{dx} \int f(x)\,dx = \frac{d}{dx}(x^4 + x + 4)$

$f(x) = 4x^3 + 1$

$f(1) = 5$

$f'(x) = 12x^2$

$= 12$

4) If $y = e^{2x} + \ln(x^2+1) + \int_0^4 \frac{5x^2}{\cos x}.dx$ find $\frac{dy}{dx}$

$\frac{dy}{dx} = e^{2x} + \frac{2x}{x^2+1} + 0$

[15] $\int f'(x)\,dx = f(x) + c$

$\int f'(x).dx = f(x) + c$

Examples:-

1) If $f'(x) = 5x^4 + 2x + 3$, $f(2) = 3$, find $f(x)$?

$\int f'(x) = \int 5x^4 + 2x + 3$

$c + f(x) = x^5 + x^2 + 3x$ $\quad f(x) = x^5 + 2x^2 + 3x - 39$

$f(2) + c = 39$

$c = 39$

$C = 39$

PRENKO

Lecture (70)

2) $\int f'(x)\,dx = x^2+3x+3$, $f(1)=?$, find $f(4)$?

$F(x) + C = x^2+3x+3$

$f(1) + C = 1+3+3$

$2 + C = 7$

$\boxed{C = 5}$

$f(x) = x^2+3x+3-5$

$= x^2+3x-2$

$F(4) = 4^2+3(4)-2$

Lecture (71)

16 The fundemental Theorem of Calculus :-

$$\frac{d}{dx}\int_{f(a)}^{g(x)} h(t)\,dt = h(g(x))\cdot g'(x) - h(f(a))\,f'(x)$$

Examples:-

1) $\frac{d}{dx}\int_{1}^{x} \sin t\,dt = \sin(x)\cdot(1) - \sin(1)(0) = \sin x$

2) $\frac{d}{dx}\int_{2x}^{x^2} (t^2-1)\,dt = ((x^2)^2-1)(2x) + ((2x)^2-1)(2)$

$= 2x(x^4-1) - 2(4x^2-1)$

3) $\frac{d}{dx}\int_{0}^{2x} \sqrt{1+t^4}\,dt = \sqrt{1+(2x)^4}\,(2) - 0$

4) If $g(x)=x^2\cdot\int_{3}^{3x} \frac{t^5}{t^2+5}\,dt$, find $g'(2)$?

$g'(x) = 2x\cdot\int_{3}^{3x} \frac{t^5}{t^2+5}\,dt + x^2\cdot\frac{d}{dx}\int_{3}^{3x} \frac{t^5}{t^2+5}\,dt$

$= \quad\quad + x^2\left(\frac{(3x)^5}{(3x)^2+5}(3) - \frac{6^5}{6^2+5}(0)\right)$

$g'(2) = 12\left(\frac{6^5}{36+5}\right)$

PRENKO

Lecture (7b)

5) If $\int_3^{2x} f(t)\,dt = \frac{5}{x^2+3}$ find $f(x)$?

$$\frac{d}{dx}\int_3^{2x} f(t)\,dt = \frac{-5(2x)}{(x^2+3)^2}$$

$$f(2x)(2) - f(3)(0) = \frac{-10x}{(x^2+3)^2}$$

$$f(2x) = \frac{-5x}{(x^2+3)^2}$$

let $y = 2x$ $\quad f(y) = \frac{-5y}{2}$

$x = \frac{y}{2}$ $\quad \overline{\left(\left(\frac{y}{2}\right)^2+3\right)^2}$

$$f(x) = \frac{\frac{-5x}{2}}{\left(\left(\frac{x}{2}\right)^2+3\right)^2}$$

6) find $\lim_{x\to 1} \frac{\int_2^{2x}(t+3)\,dt}{x-1} = \frac{0}{0}$

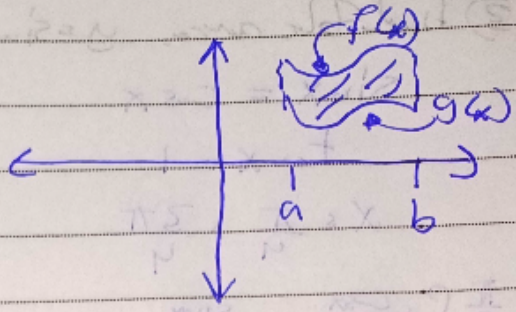
$$\lim_{x\to 1} \frac{\frac{d}{dx}\int_2^{2x}(t+3)\,dt}{\frac{d}{dx}(x-1)} = \lim_{x\to 1} \frac{(2x+3)(2)-(2+3)(0)}{1}$$

$$= \lim_{x\to 1} 4x+6 = \boxed{10}$$

Lecture (72)

Area between tow curves :-

Theorem: $A = \int_a^b (f(x) - g(x)).dx$



ملاحظة: لازم يكون نقاط التقاطع

$g(x) = f(x)$

Example :-

Find the area $y = x^2$, $y = x + 6$?

$x^2 = x + 6$

$x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$\boxed{x = 3}$ $\boxed{x = -2}$

$$\int_{-2}^{3} |x^2 - x - 6|.dx$$

$$\frac{x^3}{3} - \frac{x^2}{2} - 6x \Big|_{-2}^{3}$$

2) Find the area $y = e^x$ $y = e^{2x}$, $x = 0$, $x = \ln 2$

$$\int_0^{\ln 2} |e^{2x} - e^x|.dx$$

$$\frac{e^{2x}}{2} - e^x \Big|_0^{\ln 2}$$

$$\left(\frac{e^{2\ln 2}}{2} - e^{\ln 2}\right) - \left(\frac{1}{2} - 1\right)$$

$$\frac{e^{\ln 2^2}}{2} - e^{\ln 2} - \left(-\frac{1}{2}\right)$$

$$\frac{4}{2} - \frac{4}{2} + \frac{1}{2}$$

$$= \frac{1}{2}$$

PRENKO

Lecture (72)

3) Find the area $y = \sin x$, $y = \cos x$, $x = 0$, $x = \frac{\pi}{2}$

$\sin x = \cos x$

$\tan x = 1$

$x = \frac{\pi}{4}, \frac{3\pi}{4}$

$$\int_0^{\frac{\pi}{2}} |\underbrace{\cos x}_{\cos x} - \underbrace{\sin x}_{\sin x}| \, dx$$

$$= \sin x + \cos x \Big|_0^{\frac{\pi}{2}}$$