

تقدم لجنة EiCoM الاكاديمية

دفتر لمادة:

# دوائر كهربائية (2)

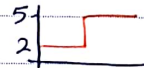
من شرح:

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جزيل الشكر للطالبة:

**نسليم كريج**

# Chapter 10 Sinusoidal analysis at steady state



For Resistance

$$v(t) = V_m \sin \omega t$$

$V_m$  : Amplitude : Peak value (V)

$$V_P - P = 2V_P = 2V_m$$

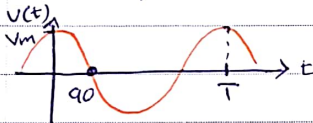
$\omega$  : Angular Frequency (rad/s)

$$\omega = 2\pi f$$

$f$  : frequency (Hz)

$$f = \frac{1}{T} \rightarrow \left( \frac{1}{s} = \text{Hz} \right)$$

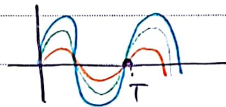
$$\omega = \frac{2\pi}{T}$$



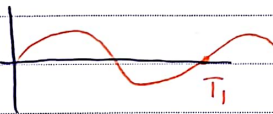
$$v(t) = V_m \cos \omega t$$

$$= V_m \cos \frac{2\pi}{T} t$$

$$i(t) = 2 \sin \omega t$$

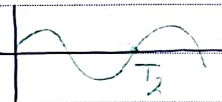


نوع الإشارة أو العاكس signal and frequency



$$T_1 > T_2 > T_3$$

$$f = \frac{1}{T}$$



$$f_3 > f_2 > f_1$$



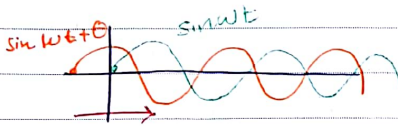
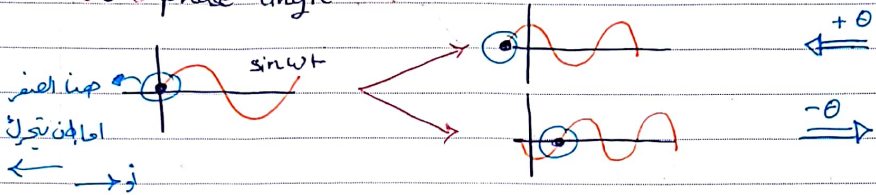
EICoM

smi)e for life



$$V(t) = V_m \sin(\omega t \pm \theta)$$

$\theta$  : phase angle



$\sin \omega t$  Lag  $\sin \omega t + \theta$  متأخر

$\sin \omega t + \theta$  lead  $\sin \omega t$  سبق

phase مختلفين في ال phase << out of phase >>

$$V_1(t) = V \sin(\omega t + \theta_1)$$

$$V_2(t) = V_2 \sin(\omega t + \theta_2)$$

IF  $\theta_1 = \theta_2$   $\triangleright$   $V_1, V_2$  in phase

IF  $\theta_1 \neq \theta_2$   $\triangleright$  phase shift  $\rightarrow$  out of phase (lead or lag)

\* To compare between two signals :

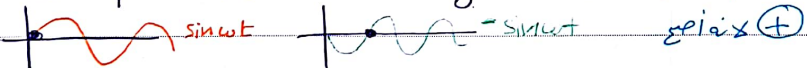
1- have the same frequency

2- Sin OR cos

$$\left\{ \begin{array}{l} \sin \omega t = \cos(\omega t - 90^\circ) \\ \cos \omega t = \sin(\omega t + 90^\circ) \end{array} \right\}$$

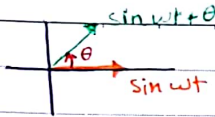


3- two positive  $\oplus$  or two negative  $\ominus$  \* لا بد أن يكونا موجبين أو سالبين



$$\left\{ \begin{array}{l} -\sin \omega t = \sin(\omega t + 180^\circ) \\ -\cos \omega t = \cos(\omega t + 180^\circ) \end{array} \right\}$$

No. \_\_\_\_\_



اتجاه الدوران على عقارب الساعة

Ex  $i_1(t) = 5 \cos(3t + 30^\circ)$

$i_2(t) = 2 \sin(3t + 20^\circ)$

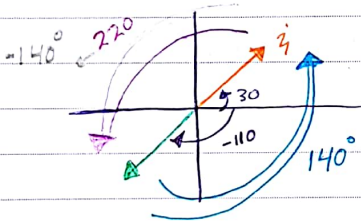
اول خطوة نتأكد من frequency وهما متساوية

$\omega = 3 \text{ rad/s} = 2\pi f$

$f = \frac{\omega}{2\pi} \text{ (Hz)}$  ,  $T = \frac{1}{f}$

$i_2(t) = 2 \cos(3t - 20^\circ - 90^\circ)$  حولنا الناتج من sin الى cos بطرح 90  
 $= 2 \cos(3t - 110^\circ)$

$i_1(t)$  and  $i_2(t)$  are out of phase



$i_1$  lead  $i_2$  by  $140^\circ$

$i_2$  lag  $i_1$  by  $140^\circ$

OR

$i_1$  lag  $i_2$  by  $220^\circ, -140^\circ$

$i_2$  lead  $i_1$  by  $220^\circ, -140^\circ$

Ex  $v_1(t) = 2 \cos(\pi t - 50^\circ)$

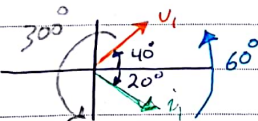
$i_1(t) = 10 \sin(\pi t - 20^\circ)$

$v_1 = 2 \sin(\pi t - 50^\circ + 90^\circ) = 2 \sin(\pi t + 40^\circ)$  حولنا sin الى cos بطرح 90

out of phase

$v_1$  lead  $i_1$  by  $60^\circ$

$i_1$  lag  $v_1$  by  $60^\circ$



$i_1$  lead  $v_1$  by  $300^\circ, -60^\circ$

$v_1$  lag  $i_1$  by  $300^\circ, -60^\circ$

Ex  $V = 3 \cos(t - 50^\circ)$

$i = 15 \cos(t - 30^\circ)$

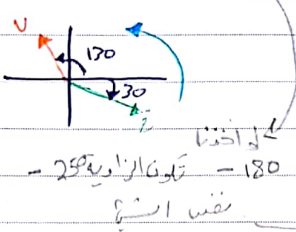
في دوائر AC

نأخذ + أو - حسب الزاوية حيث تتعامل

مع زوايا أصغر فقط ويجوز أخذ أي منها

$V = 3 \cos(t - 50^\circ \pm 180^\circ)$

$V = 3 \cos(t + 130^\circ)$  الزاوية الأصغر



$V$  lead  $i$  by  $160^\circ$

$i$  lag  $V$  by  $160^\circ$

$i$  lead  $V$  by  $200^\circ$ ,  $-160^\circ$

$V$  lag  $i$  by  $200^\circ$ ,  $-160^\circ$

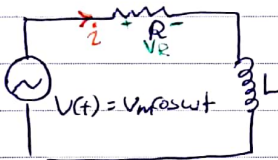
### Forced response to sin function



$i = i_p + (i_0 - i_f) e^{-t/\tau}$

$V_c = V_f + (V_0 - V_f) e^{-t/\tau}$

### For RL



$V(t) = V_R + V_L$

$V_m \cos wt = i(t)R + L \frac{di}{dt}$

first order diff equation

$i(t) = i_{Homo} + i_p$

$Ri_h + L \frac{di_h}{dt} = 0 \rightarrow$  natural response

لا يؤثر نوع مصدر التيار على شكل  $i_h$  وحسب  $i_p$

$V(t) = \cos t \rightarrow i_p = k$   
 $V(t) = e^t \rightarrow i_p = k e^t$   
 $V(t) = V_m \cos wt \rightarrow i_p = A_i \cos wt + A_i \sin wt$

$V_L = L \frac{di}{dt}$   
 $i_c = \frac{cdV}{dt}$   
 $V = iR$

كيف نعرف  $A_i$  و  $B_i$

وجدنا الوتر في المعادلة التفاضلية يتبع :-

$$i(t) = \frac{R V_m}{\omega^2 R^2 + L^2 \omega^2} \cos \omega t + \frac{L \omega V_m}{L^2 \omega^2 + R^2} \sin \omega t$$

مطابقة  $\rightarrow A \cos(\omega t - \theta) = A \cos \theta \cos \omega t + A \sin \theta \sin \omega t$

مطابقة  $\rightarrow \frac{R V_m}{R^2 + L^2 \omega^2} = A \cos \theta \rightarrow (1) \text{ eq}$

$\frac{L \omega V_m}{\omega^2 L^2 + R^2} = A \sin \theta \rightarrow (2) \text{ eq}$

$$\sqrt{(1)^2 + (2)^2} = A = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}}$$

$$\frac{(2)}{(1)} = \tan \theta = \frac{\omega L}{R}$$

$$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t - \tan^{-1} \frac{\omega L}{R})$$

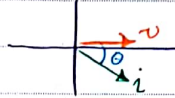
⇒ For RL

$$v(t) = V_m \cos \omega t$$

$$i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos(\omega t - \tan^{-1} \frac{\omega L}{R})$$

Frequency  $\omega$  و  
amplitude  $V_m$

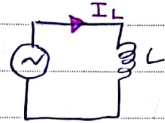
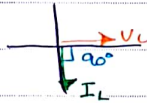
$\omega$  : excitative frequency



\*  $i_L$  lag  $v$  by  $\tan^{-1} \frac{\omega L}{R}$  for RL

- for  $R=0$

$$i(t) = \cos(\omega t - 90^\circ)$$



for pure Inductive circuit  $i_L$  lag  $V_L$  by  $90^\circ$

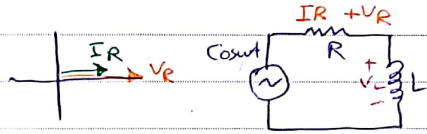
و  $V$  لags  $I$  بزاوية  $90^\circ$  ، لأن الزاوية لـ  $90^\circ$

- for  $L=0$

for pure Resistance

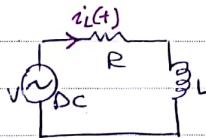
$$i(t) = \frac{V_m}{R} \cos \omega t$$

$i_R$  ,  $V_R$  in phase



- for  $\omega=0$

DC

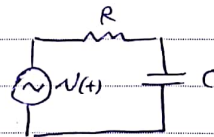


$$i_L(t) = \frac{V}{R}$$

⇒ For RC

$$v(t) = V_m \cos \omega t$$

$$i_c(t) = \frac{V_m \omega}{\sqrt{1 + \omega^2 R^2 C^2}} \cos(\omega t + \tan^{-1} \frac{1}{\omega R C})$$



$i$  lead  $v$  by  $\tan^{-1} \frac{1}{\omega R C}$





$$V(t) = V_p + V_c$$

$$\left[ V_m \cos \omega t = iR + \frac{1}{\omega C} \int i dt \right] \rightarrow \frac{d}{dt} \text{ مع } i$$

→ for RC

i lead V by  $\tan^{-1} \frac{1}{\omega RC}$

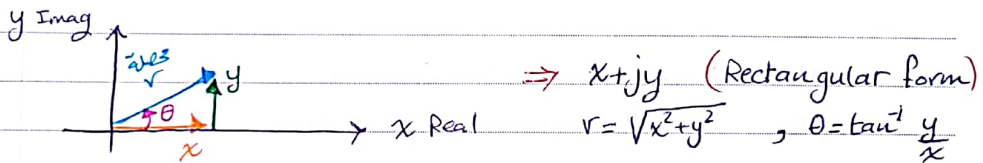
for pure capacitive

$$i(t) = \cos(\omega t + 90)$$

$i_c$  lead  $V_c$  by 90 → for pure

\* إذا كان هناك R, C, L فمجموع equivalent فئلاً لو كان ملائمة L أكثر ← RL  
أو إذا كانت C أكثر ← RC حسب الدارة

\* Complex plane



⇒ (polar form)  $r e^{j\theta}$

$\rightarrow x = r \cos \theta$  Real  
 $\rightarrow y = r \sin \theta$  Imaginary

$$V(t) = V_m \cos(\omega t + \theta)$$

$$V(t) = \text{Real} \left\{ \underbrace{V_m \cos(\omega t + \theta)}_{x \text{ Real}} + j \underbrace{V_m \sin(\omega t + \theta)}_{y} \right\} \rightarrow \text{Rectang } x + jy$$

$$= \text{Real} \left\{ V_m e^{j(\omega t + \theta)} \right\} \rightarrow \text{polar } r e^{j\theta}$$

نتجته من الدارة  $\sqrt{x^2 + y^2}$   $\downarrow$   $\tan^{-1} \frac{y}{x} = \theta$

$\tan^{-1} \frac{V_m \sin(\omega t + \theta)}{V_m \cos(\omega t + \theta)} = \theta$

$$v(t) = \text{Real} \{ V_m e^{j\omega t} e^{j\theta} \}$$

$$\vec{V} = V_m e^{j\theta} = \underline{V_m \angle \theta} \rightarrow \text{«phasor form»}$$

$$= V_m \cos \theta + j V_m \sin \theta$$

بشرط أن يكون frequency نفسه  
منا حولنا من time د في phasor د وللتأكد لآخذ  $V_m$  وار phase

Ex -  $v(t) = 2 \cos \omega t \rightarrow V = 2 \angle 0$

-  $v(t) = 2 \cos(\omega t + 20) \rightarrow V = 2 \angle 20$   $2 \angle 20 = 2 \cos 20 + j 2 \sin 20$

-  $v(t) = 3 \cos(\omega t - 50) \rightarrow V = 3 \angle -50$

-  $v(t) = 4 \sin \omega t \rightarrow v(t) = 4 \cos(\omega t - 90) \rightarrow V = 4 \angle -90$

اما نجعل المصدر  $\sin$  ولكن في النهاية نضيد او من البداية نحوله  $\cos$

-  $v(t) = -2 \sin(\pi t - 70) \rightarrow v(t) = 2 \cos(\pi t - 70 - 90 \pm 180)$

$v(t) = 2 \cos(\pi t + 20) \rightarrow V = 2 \angle 20$

Ex  $v(t) = 3 \cos 20t - 5 \sin(20t + 110)$

$v(t) = 3 \cos 20t + 5 \cos(20t + 110 - 90 \pm 180)$

$= 3 \cos 20t + 5 \cos(20t + 200)$

$V = 3 \angle 0 + 5 \angle 200 \rightarrow$  منا لا نعرفنا هو ال frequency لذلك

قبل أن نجمع نتأكد من الخطوة التي قبل ان ال frequency نفسه

$= (3 \cos 0 + j 3 \sin 0) + (5 \cos 200 + j 5 \sin 200)$

$= -1.7 - j 1.7$

$= \underline{2.4} \angle \underline{-135}$

$v(t) = \underline{2.4} \cos(20t - \underline{135})$

$\rightarrow \sqrt{(1.7)^2 + (1.7)^2}$  نتجت من  $\angle$  من  $\tan^{-1} \left( \frac{1.7}{1.7} \right)$  و 135 نتجت من  $\angle$



Ex If  $\omega = 2000 \text{ rad/s}$ , Find Instantaneous Value of Current at  $t = 1 \text{ ms}$  If:

[A-]  $I = j10 \text{ A} \rightarrow 0 + j10$   
 $= 10 \angle 90^\circ$

$x + jy \leftarrow$  صفا لا يوجد سوى  $x$  صفا

$$\left. \begin{array}{l} j + j5 = 5 \angle 90^\circ \\ -j5 = 5 \angle -90^\circ \end{array} \right\} j = 90^\circ$$

$$i(t) = 10 \cos(2000t + 90^\circ)$$

$$i(1 \text{ ms}) = 10 \cos(2 + 90^\circ)$$

$$2\pi \frac{180}{\pi}$$

دول rad الى Deg

$$i(1 \text{ ms}) = -9 \text{ A}$$

[B-]  $I = 20 + j10$

$$I = \sqrt{20^2 + 10^2} \angle \tan^{-1}\left(\frac{10}{20}\right) \equiv V_m \angle \theta$$

$$I = 22.36 \angle 26.57^\circ$$

$$i(t) = 22.36 \cos(2000t + 26.57^\circ)$$

# جميع ان تحول في النهاية الى

$$i(1 \text{ ms}) = -17.4 \text{ A}$$

Time domain

(10.5) << phasor relations for R, L, C >>

$$v(t) = i(t) R$$

$\theta \leftarrow \phi \leftarrow V$  للثابتة

$$v(t) = V_m \cos(\omega t + \theta) \quad \text{Re} \{ V_m e^{j(\omega t + \theta)} \}$$

$\phi \leftarrow \theta \leftarrow I$  للتيار

$$i(t) = I_m \cos(\omega t + \phi) \quad \text{Re} \{ I_m e^{j(\omega t + \phi)} \}$$

$$V_m e^{j(\omega t + \theta)} = R I_m e^{j(\omega t + \phi)}$$

$$V_m e^{j\theta} = R I_m e^{j\phi}$$

$$V = RI$$

$$R = \frac{V}{I}$$

$V_p$  in phase  $I_p$

\* phasors for R, L, C

$$v(t) = V_m \cos(\omega t + \theta) \\ = \operatorname{Re} \{ V_m e^{j(\omega t + \theta)} \}$$

$$i(t) = I_m \cos(\omega t + \phi) \\ = \operatorname{Re} \{ I_m e^{j(\omega t + \phi)} \}$$

\* For R  $v(t) = R i(t)$  ,  $V = IR$

V, I in phase ,  $\frac{V}{I} \rightarrow$  Impedance (Z)

$$Z_R = \frac{V}{I} = R \rightarrow Z = \frac{\text{Phasor Voltage}}{\text{Phasor Current}} , Z = (R)$$

\* For L  $v_L = L \frac{di}{dt}$   
 $V_m e^{j\omega t + \theta} = L \frac{d}{dt} I_m e^{j\omega t + \phi}$

$$V_m e^{j\omega t + \theta} = L I_m e^{j\phi} j\omega e^{j\omega t}$$

$$V = j\omega L I \rightarrow Z_L = \frac{V}{I} = j\omega L \text{ (Inductive Impedance)}$$

$$\omega = 0 \rightarrow Z_L = 0 \text{ s.c. (short circuit)}$$

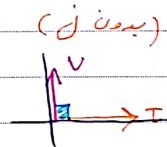
$$\omega = \infty \rightarrow Z_L = \infty \text{ o.c. (open circuit)}$$

Inductive Reactance  $\rightarrow X_L = \omega L$

$$V_L = Z_L I , V = \omega L I \angle 90^\circ$$

$I_L$  Lag  $V_L$  by  $90^\circ$

$$j 10 = 10 \angle 90^\circ$$



\* For C

$$i_C = C \frac{dv}{dt} , I_m e^{j\omega t + \phi} = C \frac{d}{dt} V_m e^{j\omega t + \theta}$$

$$I_m e^{j\omega t + \phi} = C V_m e^{j\theta} j\omega e^{j\omega t}$$

$$I = j\omega C V \rightarrow V = \frac{1}{j\omega C} I = -\frac{j}{\omega C} I$$

$$Z_C = \frac{V}{I} = \frac{1}{j\omega C} = -\frac{j}{\omega C}$$

$$\omega = 0 \xrightarrow{DC} Z_C = \infty \quad \text{o.c}$$

$$\omega = \infty \rightarrow Z_C = 0 \quad \text{s.c}$$

$$X_C = \frac{1}{\omega C} \quad \text{Reactance}$$

\* إذا كان زخمير Imp  $\ominus$  و React  $\oplus$

أو زخمير Imp  $\oplus$  و React  $\ominus$

$$Z = j2$$

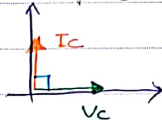
$$Z = j2$$

$$Z = 2$$

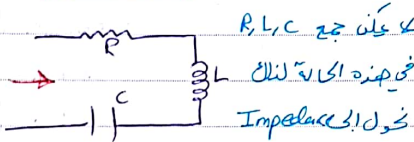
$$Z = 2 \pm j2$$

$$V = \frac{1}{\omega C} I \angle -90^\circ$$

$I_C$  lead  $V_C$  by  $90^\circ$

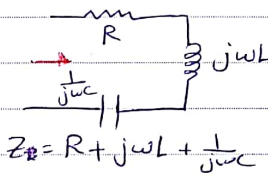


$$V = Z I$$



لا يمكن جمع R, L, C

فهيئة اى صيغة ذلك  $\Rightarrow$  Impedance



$$Z = R + j\omega L + \frac{1}{j\omega C}$$

إذا كان  $\omega L > \frac{1}{\omega C}$  يكون الدارة حثية  
إذا كان  $\omega L < \frac{1}{\omega C}$  يكون الدارة سعة  
إذا كان  $\omega L = \frac{1}{\omega C}$  يكون الدارة رنينية

$$Z = R + j(\omega L - \frac{1}{\omega C})$$

$$\text{If } R + jX \rightarrow \text{Inductive}$$

$$\text{If } R - jX \rightarrow \text{Capacitive}$$

$$\text{If } R \rightarrow \text{pure Resistance of Resonant}$$

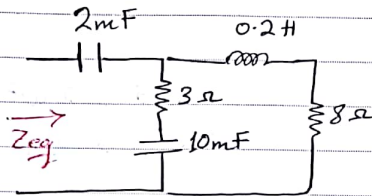
$$\text{If } +jX \rightarrow \text{pure Inductive}$$

$$\text{If } -jX \rightarrow \text{pure capacitive}$$

$$Y = \frac{1}{Z} = \frac{I}{V} \quad \text{Admittance (S)}$$

$$G = \frac{1}{R} \quad (\text{S})$$

Ex

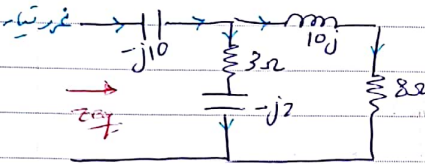


$$\omega = 50 \text{ rad/s}$$

$$Z_C = \frac{1}{j\omega C} = -j10 \Omega$$

$$Z_C = \frac{1}{j\omega C} = -j2 \Omega$$

$$Z_L = j\omega L = j10 \Omega$$



$$Z_{eq} = (8 + j10) // (3 - j2) - j10$$

$$= 3.2 - j11.07 \Omega$$

$$= A \angle \phi$$

Capacitive  $\phi$  (leading)

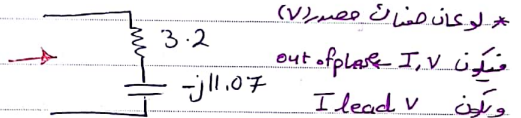
Capacitive Impedance

$$R_{eq} = 3.2$$

$$X_{eq} = 11.07$$

$$A \cos \phi = 3.2$$

$$A \sin \phi = 11.07$$

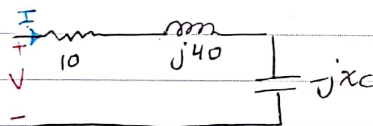


\* لو كان ضايف مع (V) فيكون I, V out of phase

فيكون I lead V

Capacitive  $Z_{eq}$   $\phi$  (leading)

Ex



$$V = V_m \angle -25^\circ$$

$$I = I_m \angle -70^\circ$$

Find  $x_c$ ?

$$Z_{eq} = 10 + j40 - jx_c = \frac{V}{I}$$

$$\frac{V_m \angle -25^\circ}{I_m \angle -70^\circ} = 10 + j[40 - x_c]$$

\* لو كان ضايف مع (V) فيكون I, V out of phase

$$\frac{V_m \angle -25^\circ}{I_m \angle -70^\circ} = 10 + j[40 - x_c]$$

$$\frac{V_m}{I_m} \cos(45^\circ) = 10$$

$$\frac{V_m}{I_m} \sin(45^\circ) = 40 - x_c$$

$$\sqrt{10^2 + (40 - x_c)^2} = \frac{V_m}{I_m} \angle 45^\circ, \tan^{-1} \frac{40 - x_c}{10} = 45^\circ$$

فيكون I lead V

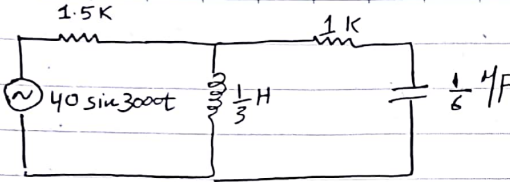
$$\frac{40 - x_c}{10} = 1 \rightarrow x_c = 30 \Omega$$

\* لو كان ضايف مع (V) فيكون I, V out of phase

$$x_c = \frac{1}{\omega C} = 30 \dots$$

smi) e for life

Ex



- Find  $i(t)$  ??

$\omega = 3000 \text{ rad/s}$

\* المصدر (sin) اما أن نبقي وبعد

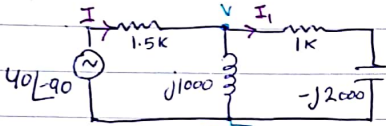
كل نتبه انه لم خوله أرفوه بك (cos)

من البداية.

$$Z_L = j\omega L = j1000 \Omega$$

$$Z_C = \frac{1}{j\omega C} = -j2000 \Omega$$

$$V(t) = 40 \cos(3000t - 90) \quad \text{phasor } \rightarrow V = 40 \angle -90$$



$$Z_{eq} = (1k - j2000) \parallel j1000 + 1.5k = 2.5 \angle 36.87^\circ \text{ K}\Omega$$

Inductive ← عااها موجبة

$127 - 90$

$$I = \frac{V}{Z_{eq}} = \frac{40 \angle -90}{2.5 \angle 36.87^\circ} = 16 \angle -127^\circ \text{ A}$$

$I = 16 \angle -127^\circ \text{ A}$  ,  $I \text{ Lag } V \text{ by } 36.87^\circ$

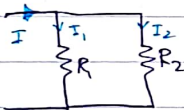
$$i(t) = 16 \cos(3000t - 127^\circ) \text{ A}$$

time domain أي

لا تطلب في السؤال  $i(t)$

$$I_1 = \frac{j1000}{-j1000 + 1k - j2000} = I$$

Current division



$$I_1 = \frac{R_2}{R_1 + R_2} \cdot I$$

$$I_2 = \frac{R_1}{R_1 + R_2} \cdot I$$

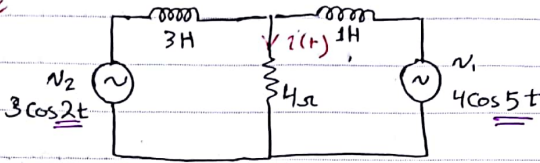
Nodal  $\frac{V}{j1000} + \frac{V}{1k - j2000} + \frac{V - 40 \angle -90}{1.5k} = 0 \rightarrow$  (Polar) خولهم أي

$$\frac{V}{j1000 \angle 90} \rightarrow 0.004 \angle -90^\circ \text{ V}$$

← نخرج الأرقام الأعلى



Ex

- Final  $i(t)$ ?

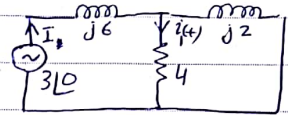
\* إذا كان هناك أكثر من مصدر وكانت  $\omega$  مختلفة لهم فيجب أن نستخدم (Super position)

■ Kill  $v_1 \rightarrow v_1 = 0 \rightarrow S.C$ ,  $I = 0 \rightarrow O.C$

$$\omega_2 = 2 \text{ rad/s}$$

$$Z_L = j\omega L$$

$$I = \frac{3 \angle 0}{j2 // 4 + j6} \rightarrow I_1 = \frac{j2}{j2 + 4} \cdot I =$$

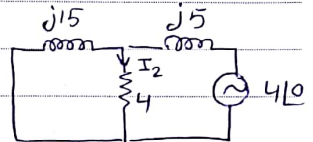


$$I_1 = 0.176 \angle -20.55$$

■ Kill  $v_2 \rightarrow v_2 = 0 \rightarrow S.C$

$$\omega_1 = 5 \text{ rad/s}$$

$$I_2 = \frac{j15}{j15 + 4} \cdot \frac{4 \angle 0}{j5 // 4 + j5} = 0.55 \angle -43.16 \text{ A}$$



$$* I = I_1 + I_2$$

$$= 0.176 \angle -20.55 + 0.55 \angle -43.16$$

$$* i(t) = i_1(t) + i_2(t)$$

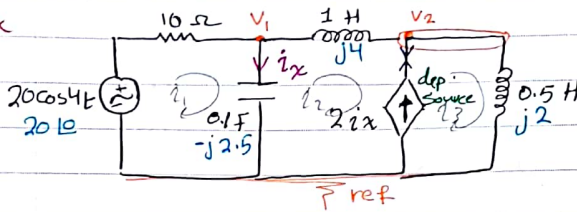
$$= 0.176 \cos(2t - 20.55) + 0.55 \cos(5t - 43.16)$$

\* جمع التيارات خطأ لأنهم

مختلفين بال  $\omega$

لذلك تبقى هكذا بأبسط صورة

Ex

- Final  $i_x(t)$  ??

$$Z_L = j\omega L$$

$$Z_C = \frac{1}{j\omega C} \text{ (المكافئة لـ)}$$

Nodal, Mesh

$$\frac{V_1 - 20\angle 0}{10} + \frac{V_1}{-j2.5} + \frac{V_1 - V_2}{j4} = 0 \rightarrow \textcircled{1}$$

نضرب المعادلة بـ (100)  $\rightarrow (1 + j1.5)V_1 + j2.5 V_2 = 20$

\* البنية البنية التيار طالما ان التردد هو نفسه

$$\frac{V_2 - V_1}{j4} + \frac{V_2}{j2} + (-2i_x) = 0 \rightarrow \textcircled{2} \quad i_x = \frac{V_1}{-j2.5}$$

لسهولة المعادلة الثانية علينا اكل بالفرص ولكن الامتثل (Matrix)

$$11 V_1 + 15 V_2 = 0$$

$$\begin{bmatrix} 1+j1.5 & j2.5 \\ 11 & 15 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 20 \\ 0 \end{bmatrix}$$

$$\Delta V_1 = \frac{\begin{bmatrix} 20 & j2.5 \\ 0 & 15 \end{bmatrix}}{\Delta} \quad \Delta V_2 = \frac{\begin{bmatrix} 1+j1.5 & 20 \\ 11 & 0 \end{bmatrix}}{\Delta}$$

$$\Delta = (1+j1.5)(15) - j2.5 \times 11 = 15 - j5$$

$$- V_1 = 19 \angle 108.43^\circ \text{ V}$$

$$- V_2 = 14 \angle 198.3^\circ \text{ V}$$

$$i_x = \frac{V_1}{-j2.5} = 7.6 \angle 108.4^\circ \text{ A}$$

$$i_x(t) = 7.6 \cos(4t + 108.4^\circ) \text{ A}$$



No.

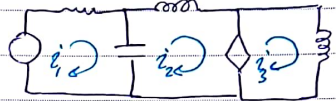
✗ يمكن حل السؤال السابق بطريقة أخرى (Mesh) :-

■ Mesh ①:  $20\angle 0 = 10 I_1 + (I_1 - I_2)(-j2.5) \rightarrow ①$

■ Super Mesh:  $-j2.5(I_2 - I_1) + j4 I_2 + j2 I_3 \rightarrow ②$

$I_3 - I_2 = 2 I_x \rightarrow ③$

$I_x = I_1 - I_2$

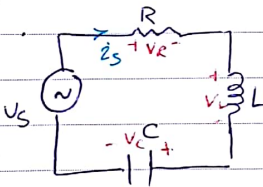
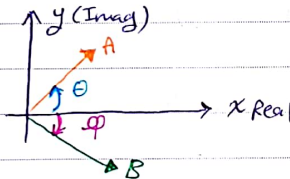


sec (10.10) << phasor diagram >>

sketch in a complex plane for all voltages and currents

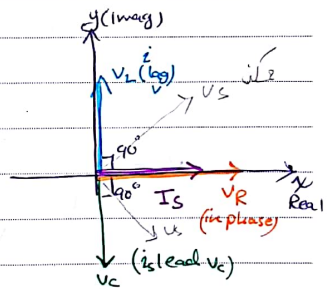
A  $\angle \theta$

B  $\angle \phi$



■ Choose  $(I_s)$  as a reference:

$I_s = 1 \angle 0$



Ex  $R = 10 \Omega$

$Z_L = j50$

$Z_C = -j50$

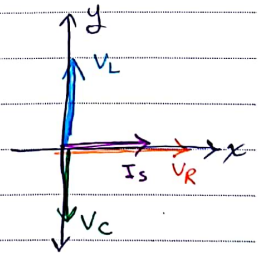
■ Choose  $I_s = 1 \angle 0$

-  $V_R = I_s R = 10 \angle 0 \text{ V}$

-  $V_L = Z_L I_s = j50 \angle 0 = j50 = 50 \angle 90^\circ \text{ V}$

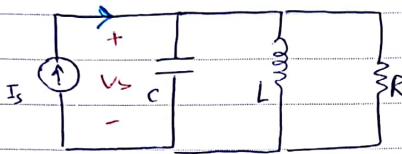
-  $V_C = Z_C I_s = -j50 \angle 0 = -j50 = 50 \angle -90^\circ \text{ V}$

$V_s = V_R + V_C + V_L = 10 \angle 0$

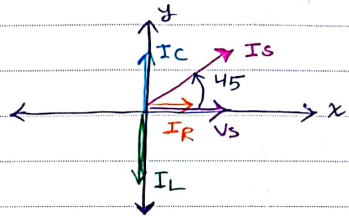


7<sup>th</sup> exp8<sup>th</sup> exp23, 24, 25, 26, 31, 32, 53, 56  
57, 74, 34, 39, 81

No.

18, 19, 24, 25, 32, 34, 43  
59, 73, 79Choose  $V_s$  as reference  $\phi$ 

$$V_s = 1 \angle 0$$



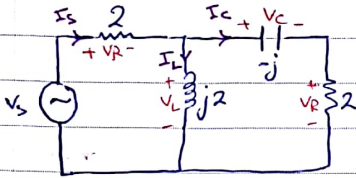
Ex  $Z_C = j0.3 \, \Omega$   
 $Z_L = -j0.1 \, \Omega$   
 $Z_R = 0.2 \, \Omega$

$$I_C = \frac{V_s}{Z_C} = Y_C V_s = (j0.3)(1 \angle 0) = j0.3 = 0.3 \angle 90$$

$$I_L = \frac{V_s}{Z_L} = Y_L V_s = (-j0.1)(1 \angle 0) = -j0.1 = 0.1 \angle -90$$

$$I_R = \frac{V_s}{Z_R} = Y_R V_s = 0.2 \angle 0$$

$$I_s = I_R + I_C + I_L = 0.283 \angle 45 \, A$$



Choose  $I_c$  as a reference

$$I_c = 1 \angle 0$$

$$I_c = 1 \angle 0$$

$$V_c = I_c Z_c = -j = 1 \angle -90$$

$$V_R = R I_c = 2 \angle 0$$

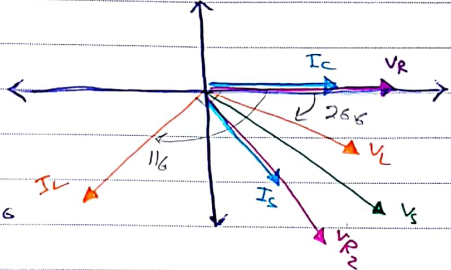
$$V_L = V_R + V_c = 2j = 2.2 \angle -26.6$$

$$I_L = \frac{V_L}{Z_L} = 1.12 \angle -116.6$$

$$I_s = I_c + I_L = 1.12 \angle -63.4 \text{ A}$$

$$V_{R2} = 2 I_s = 2.24 \angle -63.4$$

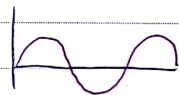
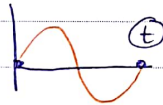
$$V_s = V_{R2} + V_L = 4.24 \angle -45 \text{ V}$$



## Chapter 11 << AC power Analysis >>

$$p(t) = v(t) i(t)$$

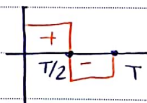
$$p = VI$$



## Average value and RMS values

\* Average value  $y(t)$

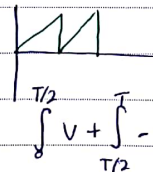
$$Y_{avg} = \frac{1}{T} \int_0^T y(t) dt$$



• for half symmetry function

$$Y_{avg} = \text{Zero}$$

(sin, cos.)



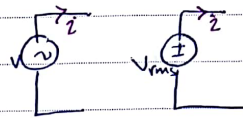
$$Y_{avg} = \frac{1}{2\pi} \int_0^{2\pi} \cos \omega t dt = 0$$

\* RMS: Root mean square value effective value

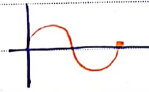
$$Y_{eff} = Y_{rms}$$

The value of the DC current that delivers the same power as an AC current

$$Y_{rms} = Y_{eff} = \sqrt{\frac{1}{T} \int_0^T y^2(t) dt}$$



$$v(t) = V_m \cos(\omega t + \theta)$$



$$V_{avg} = 0$$

$$V_{rms} = \sqrt{\frac{1}{2\pi} \int_0^{2\pi} V_m^2 \cos^2(\omega t + \theta) d\omega t} \quad \text{where } \int_0^{2\pi} \cos^2 \omega t d\omega t = \pi$$

$$V_{rms}^2 = \frac{V_m^2}{2\pi} \int_0^{2\pi} \frac{1}{2} (1 + \cos(2\omega t + 2\theta)) d\omega t$$

{  $\int_0^{2\pi} \cos^2 A = \frac{1}{2} + \frac{1}{2} \cos 2A$  }

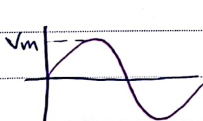
$$V_{rms}^2 = \frac{V_m^2}{2} + 0$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

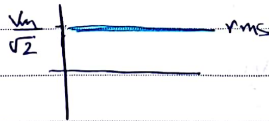
$$I_{rms} = \frac{I_m}{\sqrt{2}}$$

$$V_{avg} = 0 \quad \begin{matrix} \cos \\ \sin \end{matrix}$$

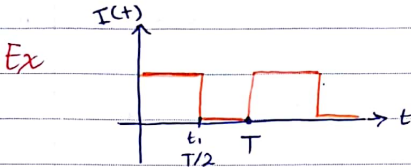
V will be zero



AC



DC



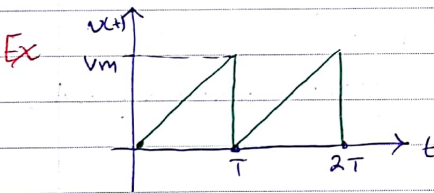
$$I_{avg} = \frac{1}{T} \int_0^T i(t) dt = \frac{1}{T} \left[ \int_0^{t_1} 0 dt + \int_{t_1}^T I_m dt \right]$$

$$I_{avg} = \frac{t_1}{T} I_m$$

$$I_{rms} = I_{eff} = \sqrt{\frac{1}{T} \int_0^T i^2(t) dt}$$

$$I_{rms}^2 = \frac{1}{T} \left[ \int_0^{t_1} 0^2 dt + \int_{t_1}^T I_m^2 dt \right] \quad \sim \quad I_{rms}^2 = \frac{t_1}{T} I_m^2$$

$$I_{rms} = \sqrt{\frac{t_1}{T}} I_m$$



نقطه ۱ و ۲ را در نظر بگیرید \*

$$y - y_0 = \text{slope} (x - x_0)$$

$$\text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$v - 0 = \frac{V_m - 0}{T - 0} (t - 0)$$

$$v(t) = \frac{V_m}{T} t$$

نقطه ۱ و ۲ را در نظر بگیرید

$$V_{avg} = \frac{1}{T} \int_0^T v(t) dt$$

$$V_{avg} = \frac{1}{T} \int_0^T \frac{V_m}{T} t dt$$

$$V_{avg} = \frac{V_m}{2}$$

$$V_{rms}^2 = \frac{1}{T} \int_0^T \frac{V_m^2}{T^2} t^2 dt$$

$$V_{rms}^2 = \frac{V_m^2}{3}$$

$$V_{rms} = \frac{V_m}{\sqrt{3}}$$

$$V(t) = \cos \omega t + \sin \omega t + A$$

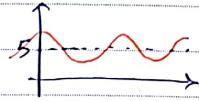
الطريقة خالص

$$V = 5 \rightarrow DC, V = V_{avg} = V_{rms}$$

Ex  $5 + 4 \cos \omega t$

$$V_{avg} = 5$$

$$V_{rms}^2 = \frac{1}{2\pi} \int_0^{2\pi} (5 + 4 \cos \omega t)^2 dt$$



$$V_{rms}^2 = \frac{1}{2\pi} \int_0^{2\pi} (25 + 40 \cos \omega t + 16 \cos^2 \omega t) dt$$

$$V_{rms}^2 = \frac{1}{2\pi} \int_0^{2\pi} 25 + 40 \cancel{\cos \omega t} + 16 \left[ \frac{1}{2} + \frac{1}{2} \cancel{\cos 2\omega t} \right] dt$$

$$V_{rms}^2 = \frac{1}{2\pi} \int_0^{2\pi} 33 dt = 33 \Rightarrow V_{rms} = \sqrt{33} \text{ V}$$

\* If  $y(t) = y_1(t) + y_2(t) + y_3(t)$  to find RMS  $y(t)$  :-

$$(1) Y_{rms} = \sqrt{\frac{1}{T} \int_0^T y^2(t) dt}$$

(2) if  $y_1, y_2, y_3$  have the same Frequency

(3) if  $y_1, y_2, y_3$  have the different Frequency :-

$$Y_{rms} = \sqrt{Y_{rms1}^2 + Y_{rms2}^2 + Y_{rms3}^2}$$

Ex  $V(t) = \overset{0}{5} + 4 \overset{\omega}{\cos \omega t}$

different frequency

$$V_{rms} = \sqrt{5^2 + \left(\frac{4}{\sqrt{2}}\right)^2} = \sqrt{33} \neq$$



Ex  $v(t) = 3 \cos \pi t + 4 \sin^2 \pi t$

$$= 3 \cos \pi t + \frac{4}{2} (1 - \cos 2\pi t)$$

$$v(t) = \underbrace{3 \cos \pi t}_{\pi} + \underbrace{2}_0 - \underbrace{2 \cos 2\pi t}_{2\pi} \quad \text{different frequency}$$

$$V_{avg} = 2 \text{ V}$$

$$V_{rms} = \sqrt{\left(\frac{3}{\sqrt{2}}\right)^2 + 2^2 + \left(\frac{2}{\sqrt{3}}\right)^2}$$

Ex  $v(t) = 10 + 3 \cos 10t + 4 \cos(10t - 120^\circ)$

$$V_{avg} = 10 \text{ V}$$

$$v(t) = 10 + 3 \angle 0^\circ + 4 \angle -120^\circ \quad \leftarrow \begin{array}{l} \text{2 phasors plus (w) is} \\ \text{2.30 phasor plus (w) is} \end{array} = 10 + 3.6 \angle -73.8^\circ$$

$$v(t) = 10 + 3.6 \cos(10t - 73.8^\circ)$$

$$V_{rms} = \sqrt{10^2 + \left(\frac{3.6}{\sqrt{2}}\right)^2} = 10.3$$

Ex  $v(t) = \cos 5t + 3 \sin(5t + 10^\circ)$

$$= 1 \angle 0^\circ + 3 \cos(5t + 10^\circ - 90^\circ)$$

$$= 1 \angle 0^\circ + 3 \cos(5t - 80^\circ) = 1 \angle 0^\circ + 3 \angle -80^\circ$$

$$v = 3.36 \angle -63.13^\circ$$

$$V_{rms} = \frac{3.36}{\sqrt{2}}$$



"power calculation"

$P = VI \rightarrow p(t) = v(t) i(t) \rightarrow \text{Instantaneous power } (P_t)$   
(w)

$p(t) = V_m \cos(\omega t + \theta) I_m \cos(\omega t + \phi)$

$\cos A \cos B = \frac{1}{2} [\cos(A-B) + \cos(A+B)]$  ← متطابقة

■  $P_t = p(t) = \underbrace{\frac{V_m I_m}{2} \cos(\theta - \phi)}_{\text{constant}} + \underbrace{\frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)}_{\text{Double } (\omega)}$

$P_{avg} = \frac{1}{T} \int_0^T P_t dt$

■  $P_{avg} = \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \cos(\theta - \phi) d\omega t + \frac{1}{2\pi} \int_0^{2\pi} \frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi) d\omega t$

■  $P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi)$  (Average power, Active power, Real power)

$P_t = P_{avg} + \frac{V_m I_m}{2} \cos(2\omega t + \theta + \phi)$

$V_{rms} = \frac{V_m}{\sqrt{2}}, I_{rms} = \frac{I_m}{\sqrt{2}}$

■  $P_{avg} = V_{rms} I_{rms} \cos(\theta - \phi)$  (w)

Ex:  $P = 2 + 2 \cos 2\omega$   
 $P = 2 \text{ w}$

■  $P_{app} = S = |S| = \frac{V_m I_m}{2} = V_{rms} I_{rms}$   
(Apperant power)  $V_A \rightarrow$  وحدتها فولت أمبير للتيارين  
ال (P) والس بقة.

■ Power Factor  $\rightarrow PF = \cos(\theta - \phi)$  Impedance ال (P) ال  
 $Z = \frac{V \angle \theta}{I \angle \phi} = \frac{V}{I} \angle \theta - \phi$  وتحدد نوعه في الدارة

$$0 \leq \text{PF} = \cos(\theta - \phi) \leq 1 \quad (\text{Unity PF})$$

$\cos(\pm 90) = 0$   
 $-\cos(90) \rightarrow \text{pure Inductive}$   
 $-\cos(-90) \rightarrow \text{pure capacitive}$

$\cos(0) = 1$   
 $\theta = \phi \rightarrow \text{In phase}$   
 $-\text{pure Resistance}$

Ex  $\cos(\pm 36) = 0.8$   
 $(\theta - \phi) \rightarrow \oplus$  Inductive  $\rightarrow$  lagging PF  
 $(\theta - \phi) \rightarrow \ominus$  capacitive  $\rightarrow$  leading PF

نظير الى  $(\theta - \phi)$  قبل اخذ الـ cos

\* For R  $\rightarrow \theta = \phi$

$$\cos(\theta - \phi) = 1 = \text{PF}$$

$P_{\text{avg}} = \frac{V_m I_m}{2} = P_{\text{app}} = \frac{V_m^2}{2R} = \frac{I_m^2 R}{2} \rightarrow \text{peak Value}$   
 $= V_{\text{rms}} I_{\text{rms}} = P_{\text{app}} = \frac{V_{\text{rms}}^2}{R} = I_{\text{rms}}^2 R \rightarrow \text{RMS}$

\* For pure C, pure L  $\rightarrow (\theta - \phi) = \pm 90$

$\text{PF} = 0$ ,  $P_{\text{avg}} = 0 \text{ W}$

$$P_{\text{app}} = V_{\text{rms}} I_{\text{rms}} = \frac{V_m I_m}{2}$$

\* (L, C)  $\rightarrow$  مخزنوا الطاقة

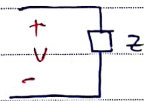
\* (R)  $\rightarrow$  تبدل عن تحويل الطاقة وإحياءاً تبدل عن اضاعتها عن شكل حرارة.

Ex  $V(t) = 4 \cos \pi t$ ,  $Z = 2 \angle 60 = 1 + j1.73 \Omega$

- Find  $P_t$ ,  $P_{\text{avg}}$  ??

$V = 4 \angle 0$   
 $I = \frac{V}{Z} = 2 \angle -60 \text{ A}$

$Z = 2 \angle 60$   
 $i(t) = 2 \cos(\frac{\pi}{6} t - 60)$



$P_t = v(t) i(t) = 4 \cos \frac{\pi}{6} t \cdot 2 \cos(\frac{\pi}{6} t - 60)$   
 Rad Deg

$= 8 \cos \frac{\pi}{6} t \cos(\frac{\pi}{6} t - 60)$   
 مطابق  $\rightarrow 4 \cos(60) + 4 \cos(2\frac{\pi}{6} t - 60)$

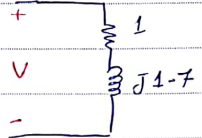
$P_{\text{avg}} = 2 \text{ W}$

طريقة اخرى  
(Pavg) حساب  $P_{avg} = \frac{VI}{2} \cos(\theta - \phi)$

(Imped)  $\theta$  زاوية  $= \frac{4 \times 2}{2} \cos(0 - -60) = 2W$

طريقة ثانية  
(Pavg) حساب  $P_{avg} L = 0W$

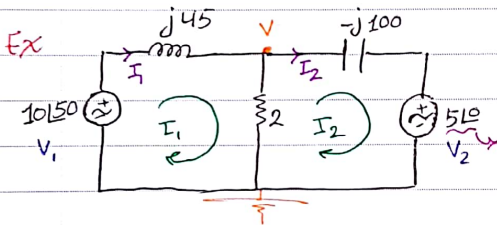
$P_{avg} = \frac{V_m I_m}{2} \cos(\theta - \phi) = \frac{VI}{2} = \frac{I^2 R}{2} = \frac{V^2}{2R}$



$P = \frac{I^2 R}{2} = \frac{4 \times 1}{2} = 2W$

generating power = dissipated power

وكان  $P_{avg}$  فقط في المقاومة (R) و (L) ليس في.



- Calculate average power delivered by each elements??

صالحه حولة في phasor لا يمكن في Time ان قول اريد قبل اى

Node  $\triangleright \frac{V - 10/50}{j45} + \frac{V}{2} + \frac{V - 5/10}{-j100} = 0$

$I_1 = \frac{10/50 - V}{j45} = 0.22 \angle -37.8$

$I_2 = \frac{V - 5/10}{-j100} = 0.0466 \angle 87.8$

$-P_L = 0W$

$-P_C = 0W$

$P_{V1} = \frac{V I_1}{2} \cos(\theta - \phi) = \frac{(10)(0.22)}{2} \cos(50 - (-37.8))$

$P_{V1} = 42 \text{ mw (generating) or } P_{V1} = -42 \text{ mw (dissipating)}$

$P_{V2} = \frac{(5)(0.0466)}{2} \cos(0.87.8) = 4.4 \text{ mw (dissipating, absorbed)}$

$$I_R = I_1 - I_2 = A \angle 0$$

$$P_R = \frac{|I_1 - I_2|^2 R}{2} = \frac{A^2 R}{2} = 37.6 \text{ mW dissipated}$$

$$\sum P_{\text{generating}} = \sum P_{\text{dissipating}}$$


$$\sum p = 0$$

\*  $\boxed{PF=1}$   $\rightarrow$  pure resistance

$$Z_{eq} = R_{eq} + jX_{eq}, \quad X_{eq}=0$$

(Resonant frequency)  $\rightarrow \omega = \omega_0$   
 $\rightarrow f = f_0$

■ for series RLC:-

$$Z_{eq} = R + j\omega L + \frac{1}{j\omega C}$$


$$Z_{eq} = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

$$\omega = \omega_0$$

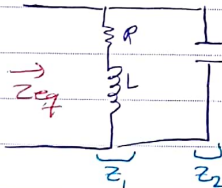
Resonant frequency  $\boxed{PF=1}$ ,  $X=0$

$$\omega_0 L - \frac{1}{\omega_0 C} = 0 \rightarrow \omega_0 = \frac{1}{\sqrt{LC}} \rightarrow Z_{eq} = R$$

■ for parallel RLC:-

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

$$Z_{eq} = (R + j\omega L) \parallel \frac{1}{j\omega C}$$



$$PF = 0.8 = \cos(\theta - \phi)$$

$$= \cos 1 \quad \omega C = \omega L$$

$$\frac{R}{R^2 + \omega^2 L^2}$$

$$Y_{eq} = \frac{1}{Z_{eq}} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

PF=

$$Y_{eq} = \left( \frac{1}{R + j\omega L} \right) + \frac{1}{1/j\omega C} \rightarrow PF=1$$

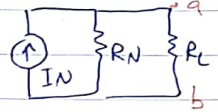
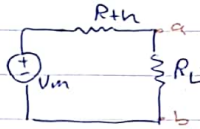
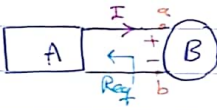
$$\frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

$$Y_{eq} = \frac{R}{R^2 + \omega^2 L^2} + j \left[ \omega C - \frac{\omega L}{R^2 + \omega^2 L^2} \right] \quad X=0$$

$$\omega_0 C - \frac{\omega_0 L}{R^2 + \omega_0^2 L^2} = 0 \rightarrow \omega_0 = \frac{\sqrt{L - R^2 C}}{L^2 C}$$

at  $\omega = \omega_0$   $PF=1 \rightarrow X=0 \rightarrow Z_{eq} = \frac{R^2 + \omega^2 L^2}{R}$

## &lt;&lt; Thevenin and Norton &gt;&gt;



$$V_{th} = V_{(o.c)}$$

$$V_{th} = I_N R_N$$

$$R_L = R_{th}$$

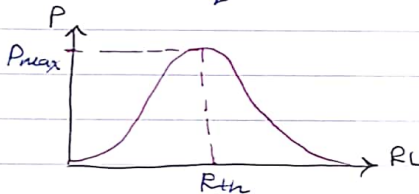
$$I_N = I_{(s.c)}$$

$$I_N = \frac{V_{th}}{R_{th}}$$

► To get max power

$$R_{eq} = R_{th} = R_N$$

$$P_{max} = \frac{V_{th}^2}{4 R_{th}}$$

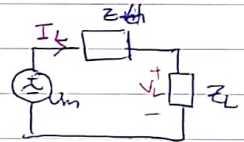


∴ Load resistance should be equal to the source resistance (Rth = RL)

$$Z_{eq} = Z_{th} = Z_N$$

$$V_{th} = I_N Z_N$$

$$I_N = \frac{V_{th}}{Z_{th}} \rightarrow Z_{th} \text{ to get max}$$



$$P_L = |V_L| |I_L| \cos(\theta - \phi)$$

$$I_L = \frac{V_{th}}{Z_L + Z_{th}}, \quad V_L = I_L Z_L$$

$$\frac{dP_L}{dZ_L} \rightarrow \boxed{Z_L = Z_{th}^*}$$

$$Z_{th} = R_{th} + jX_{th}$$

$$Z_L = Z_{th}^* = R_{th} - jX_{th}$$

\* If  $Z_L = Z_{th}^*$  :-

$$Z_{eq} = 2R_{th} \rightarrow (\text{Resistance}) \quad P.F. = 1, \quad \omega = \omega_0$$

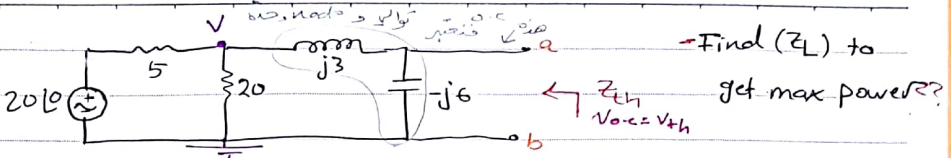
$$I_L = \frac{V_{th}}{2R_{th}}$$

$$P_{Lmax} = P_L \Big|_{Z_L = Z_{th}^*} = \frac{|V_{th}|_{max}^2}{8 R_{th}} \rightarrow P_{Lmax} = \frac{|V_{th}|_{max}^2}{4 R_{th}}$$

$$P_{Lmax} = \frac{|I_L|^2 R_{th}}{2} = \frac{|I_L|_{rms}^2 R_{th}}{2}$$



Ex



To Find ( $Z_{th}$ ) kill the source  $V_s = 0$  s.c

$$Z_{th} = (5 // 20) + j3 // -j6$$

$$Z_{th} = 5.76 - j1.68 \Omega$$

$$Z_L = Z_{th}^* = 5.76 + j1.68 \Omega$$

\* أو حسب  $Z_{eq}$  وحسب (I) الحالية في  $Z_L$   
( $V_{th}$ ) وحسب division

Nodal  $\rightarrow \frac{V-20}{5} + \frac{V}{20} + \frac{V}{j3-j6} = 0 \rightarrow I = \frac{V}{j3-j6}$

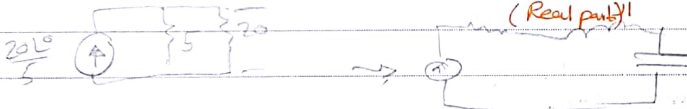
$$V_{th} = I(-j6) \rightarrow V_{th} = 19.2 \angle -53.13^\circ V$$

Amplitude (مقدار الجهد)

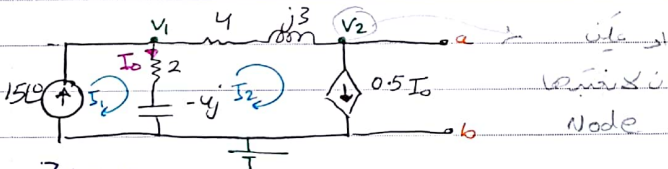
$$P_{max} = \frac{|V_{th}|^2}{8 R_{th}} = \frac{(19.2)^2}{8 \times 5.76} = 8 W$$

(Real part)

complex part, i



Ex



$$Z_{th} = \frac{V_{th}}{I_N}$$

~~At node 1~~  $\frac{V_1}{2-j4} - 15 \angle 0 + 0.5(I_1) = 0$

$$\frac{V_1}{2-j4}$$

$$V_{th} = 55 \angle -90^\circ = -j55$$

$$(4+j3)(0.5 I_1) + V_{th} - V_1 = 0$$



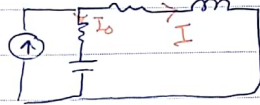
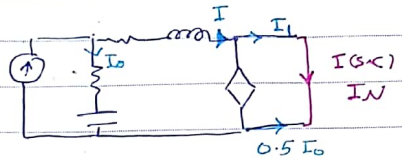
$$I_N = I_{s.c}$$

$$I_N = I - 0.5 I_0$$

$$I = \frac{2-j4}{2-j4+4+j3} \cdot 15 \angle 0^\circ$$

$$I_0 = 15 - I$$

$$I_N = 13.56 \angle -80.5^\circ$$



$$Z_{Th} = \frac{V_E}{I_N} = \frac{4.055 \angle -9.467^\circ}{13.56 \angle -80.5^\circ} = \textcircled{4} - j0.667 \Omega$$

$$Z_L = Z_{Th}^* = \textcircled{4} + j0.667 \Omega$$

$$P_{L \max} = \frac{(55)^2}{8(\textcircled{4}) R_{Th}}$$

Sec (11.5) (Complex power)

$$P_{avg} = VI \cos(\theta - \phi)$$

$$S = \underbrace{VI \cos(\theta - \phi)}_{\text{avg (Real)}} + j \underbrace{VI \sin(\theta - \phi)}_{Q \text{ (Imag.)}} \quad (\text{complex power}) S$$

$$\blacksquare P_{avg} = \text{Re}\{S\}$$

$$\blacksquare Q = \text{Im}\{S\}$$

\* Complex power (S)

$$S = P_{avg} + jQ$$

$\rightarrow Q \rightarrow$  Reactive power  
 $\rightarrow P_{avg} \rightarrow$  active power

$$S = VI \cos(\theta - \phi) + jVI \sin(\theta - \phi)$$

$$S = VI e^{j(\theta - \phi)} = VI \angle \theta - \phi$$

$$P_{app} = S = |S|$$

$$\left. \begin{array}{l} \blacksquare S \rightarrow \text{VA} \\ \blacksquare S = |S| = P_{app} \rightarrow \text{VA} \\ \blacksquare P_{avg} \rightarrow W \\ \blacksquare Q \rightarrow \text{VAR} \end{array} \right\}$$

\* اختتام المحاضرة

No.

$$S = VI e^{j(\theta - \phi)} = V e^{j\theta} \cdot I e^{-j\phi}$$

جول (rms) الجهد \*  
(2) تيار (rms)

$$S = V \cdot I^*$$

(2) تيار (rms)

$$S = V I \cos \theta - \phi$$

$$S = P_{avg} + jQ = VI \cos \theta - \phi$$

$$VI = \sqrt{P_{avg}^2 + Q^2}$$

$$\tan^{-1} \frac{Q}{P_{avg}} = \theta - \phi$$

\* Complex power (S)

$$S = P_{avg} + jQ = VI \angle \theta - \phi = VI^*$$

$$Q = VI \sin(\theta - \phi) \rightarrow \text{Reactive power}$$

► For pure R :

$$\theta = \phi \rightarrow \theta - \phi = 0 \rightarrow \sin(0) = 0$$

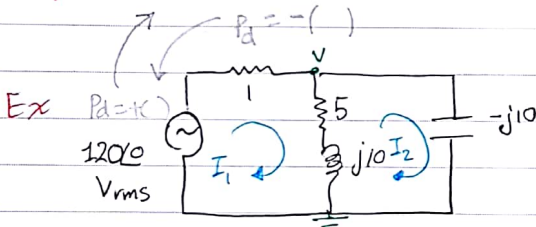
$$Q_R = 0 \text{ VAR}$$

$$S_R = P_{avg} + jQ = P_{avg} = VI = P_{app}$$

► For pure CL :

$$\theta - \phi = \pm 90 \rightarrow \cos \pm 90 = 0 \rightarrow \sin(\theta - \phi) = \pm 1$$

$$\left. \begin{aligned} \square Q_C &= -VI \\ \square Q_L &= VI \end{aligned} \right\} S_{CL} = 0 \pm jVI$$



- Find total complex power ??

$$\text{Nodal} \rightarrow \frac{V - 120}{1} + \frac{V}{5 + j10} + \frac{V}{-j10} = 0$$

$$\begin{aligned} \square I_1 &= 120 - V = 5.16 \angle 25.46^\circ \text{ A} \\ \square I_2 &= \frac{V}{-j10} = 11.5 \angle 88.9^\circ \text{ A} \end{aligned}$$

$$\square V = 115 \angle -1.1^\circ \text{ V}$$

تغير زاوية التيار  $I^*$

$$\square S_{120\Omega} = VI_1^* = (120)(5.16 \angle 25.46^\circ) = (559 - j266) \text{ VA}$$

$$\text{I} \quad P_d = (+)$$

$$\text{I} \quad P_d = (-)$$

generating  
ولـ زوئـ ديسـ ولسـ (جـ) 120

$$\square S_1 = VI^* = \frac{V^2}{R} = I^2 R = P_{avg} = P_{app}$$

$$S_1 = |I_1|^2 R = (5.16)^2 (1) = 26.6 + j0 \text{ VA}$$

$$S_c = V_c I_c^* = V_c I_2^* = (115 \angle -1.1) (11.5 \angle -88.9) - 90^\circ \text{ موجه } (C) \text{ في } \angle$$

$$S_c = 0 - j 1331 \text{ VA}$$

$$I = I_1 - I_2 = 10.3 \angle -64.5 \text{ Arms}$$

$$S_{5+j10} = V I^* = (115 \angle -1.1) (10.3 \angle 64.5) = 532 + j 1065 \text{ VA}$$

$$S_{\text{Total}} = S_1 + S_2 + S_3$$

$$= (P_{avg1} + P_{avg2} + P_{avg3}) + j(Q_1 + Q_2 + Q_3)$$

$$= P_{avg \text{ total}} + j Q_{\text{total}}$$

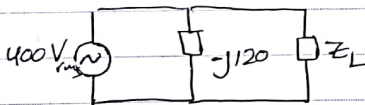
$$S_t = S_1 + S_2 + S_3$$

$$V I \angle \theta - \phi = V_1 I_1 \angle \theta - \phi + V_2 I_2 \angle \theta - \phi + V_3 I_3 \angle \theta - \phi$$

$$A \angle \theta = B \angle \theta + C \angle \theta + F \angle \theta$$

الـ (Papp) لا يمكن أن نخرج  
فصل  $S_t$  هذا، إذاً الجمع في كل  $\angle$  في Rect فـ مجموع

Q51  
من الكتاب



$$S_{\text{Source}} = 1.6 + j 0.5 \text{ kVA}$$

$$A \angle \theta - \phi$$

1) Find complex power delivered to  $Z_L$  ??

$$I_c = \frac{400}{-j120} = \frac{40}{12} j$$

$$S_c = V I_c^* = 1.33 \angle -90 \text{ kVA}$$

$$S_L = S_{\text{Source}} - S_c$$

$$= 1.6 + j 1.8 \text{ kVA} = 2.4 \angle 48.8 \text{ kVA}$$

\* ليست (pure Inductive) لانه الزاوية ليست  $90^\circ$ ، وهذا يعني أنه يوجد مكونات أخرى بعض النظر عنها مقاومات، ملفات ...

2) P.F of  $Z_L$

$P.F = \cos(48.8) = 0.658$   $\xrightarrow{\text{lag}}$  Inductive (+) الزاوية (+)  
\* بعد حساب ال P.F دائماً نكتب  $\ominus$  /  $\text{lead}$  أو  $\oplus$  /  $\text{lag}$

3) P.F of the source

$$P.F. = \cos(\theta - \phi) = \cos(17.35) = 0.954 \text{ Lag}$$

$$\Theta - \Phi = \tan^{-1} \frac{0.5}{1.6}$$

Find  $Z_L$  (4 خري اضافي)

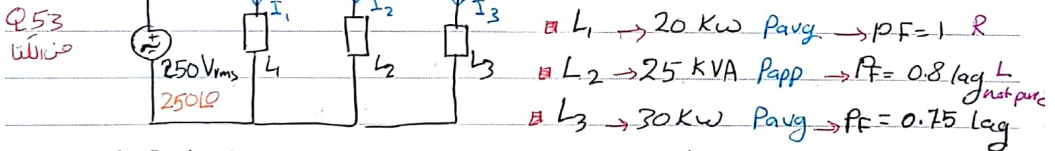
$$S_L = V I_L^*$$

$$2.41488 = 400 I_L^* \rightarrow I_L = 4 - j4.58$$

$$Z_L = \frac{400}{I_L} \rightarrow$$

$$S_t = V I_t^*$$

Ex



1) Find the total Average power supplied by the source??

$$P_{avg_s} = P_{avg_1} + P_{avg_2} + P_{avg_3}$$

$$= (20\text{ kW}) + (25\text{ kW} \times 0.8) + 3\text{ kW} \rightarrow P_{\text{avg}} = 70\text{ kW}$$

$P_{avg} = VI \cos(\theta - \phi)$  avg. or true P.F.  $\rightarrow \cos \theta$

$$P_{avg} = P_{app} - P_F$$

2)

$$P_{avg1} = |V| |I_1| \text{ P.F} \rightarrow 20 \text{ K} = (250) (|I_1|) (1)$$

$$|I_1| = 80 \text{ A}$$

\* لأن لاخراج زاوية (I) نستخدم P.F

$$\rightarrow \text{P.F} = \cos(\theta - \phi) = 1 \rightarrow \overset{0}{\theta} - \phi = 0 \rightarrow \phi = 0$$

(زاوية (V))

$$I_1 = 80 \angle 0$$

$$P_{app} = |V| |I_2| \rightarrow 25 \text{ K} = (850) (|I_2|)$$

$$|I_2| = 100 \text{ A}$$

$$\text{P.F} = \cos(\theta - \phi) = 0.8$$

$$(\theta - \phi) = \overset{\text{lag}}{+} 36.87 \rightarrow \text{lead} \rightarrow \phi = -36.87$$

$$I_2 = 100 \angle -36.87$$

$$P_{avg3} = |V| |I_3| \text{ P.F}$$

$$|I_3| = 160 \text{ A}$$

$$\text{P.F} = \cos(\theta - \phi) = 0.75 \rightarrow \theta - \phi = +41.4 \rightarrow \phi = -41.4$$

$$I_3 = 160 \angle -41.4$$

$$I = I_1 + I_2 + I_3$$

$$I = 325.4 \angle -30.635 \text{ A}$$

$$P_{app} = |S| = (400) (325.4) = 81.3 \text{ KVA}$$

$$\text{P.F} = \cos(\theta' - \phi) = \cos(0 - -30.635) = 0.86 \text{ Lag}$$

\* لا يمكن جمع ال P.F لأنها تحتوي زوايا بعين من Papp خطأ ان يجمع

$$S_t = S \angle \theta - \phi = S_1 \angle \theta - \phi + S_2 \angle \theta - \phi$$

\* قد يطلب  $\phi$  للعرض فنقوم باخراج  $V|I|$  لكه عنصر ثم نضرب به  $\sin(\theta - \phi)$ 

$$Q_t = VI \sin(\theta - \phi)$$

\* وإذا طلب (Q) ←

$$= (400) (325.4) \sin(0 + 30.6)$$

\* أو قد يطلب (S)

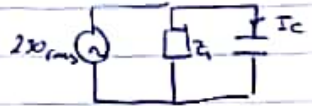


Ex load  $Z_L$  operating at 50 kW, 0.8 lagging P.F.  
a parallel capacitor is added to increase p.f to 0.95  
Find (C):

$$Z_L = \frac{V}{I}$$

$$Z_{eq} = Z_L \parallel \frac{1}{j\omega C} = R_{eq} + jI_{mg}$$

$$PF = \cos(\theta - \phi) = \cos\left(\tan^{-1} \frac{I_{mg}}{R_{eq}}\right)$$



$$S = VI^* = VI \angle 0 - \phi$$

$$P_{app} = VI = \frac{P_{av}}{PF}$$

$$S_t = S_i + S_C$$

$$\frac{50k}{0.95} \angle \cos^{-1} 0.95 = \frac{50k}{0.8} \angle \cos^{-1} 0.8 + S_C$$

$$50 + j16.4k = 50 + j27.5k + S_C$$

$$S_C = -j21.07k \text{ kVA}$$

$$S_C = VI_C^*$$

$$-j21.07k = (230) I_C^* \rightarrow \frac{21.07k \angle -90}{230} = I_C^*$$

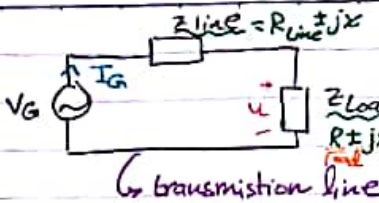
$$I_C = 91.6 \angle 90 \text{ A}$$

$$Z_C = \frac{V}{I_C} = \frac{230}{91.6 \angle 90}$$

$$Z_C = -j2.5 \Omega = \frac{1}{j\omega C} = \frac{-j}{\omega C}$$

$$\omega 2.5 = \frac{1}{\omega C} \quad \omega = 2\pi f, \quad f = 60$$

$$C = 1056 \text{ HF}$$



$$P_{in} = P_{source} = |V_G| |I_G| \cos \phi$$

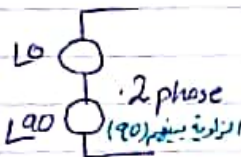
$$P_o = P_{load} = |I_G|^2 R_{load}$$

$$P_{loss} = P_{line} = |I_G|^2 R_{line}$$

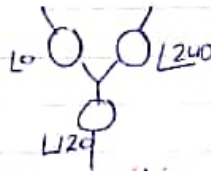
$$(P_{input} = P_{output} + P_{loss})$$

$$\text{efficiency} = \eta = \frac{P_o}{P_{in}} \times 100\%$$

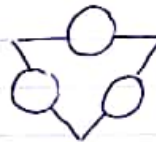
## chapter (12) (poly phase)



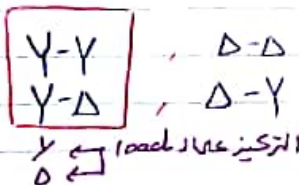
2-phase



3-phase (Y)



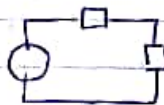
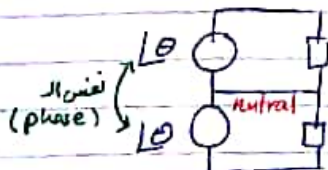
3phase (Δ)



((Balanced system))

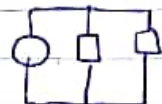
## Single phase - 3wires system

يعني انهم نفس ال phase



لما صا واحد load

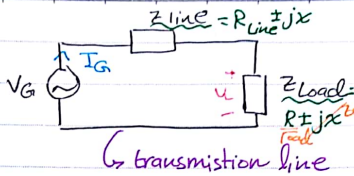
لم يعد يعمل تقف الدارة كلها



لما صا فتح تيار

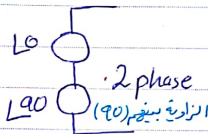
كبير حتى نوصل

للموتيرة المطلوبة

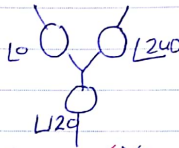


$$\begin{aligned}
 P_{in} &= P_{source} = |V_G| |I_G| \cdot P.F. \\
 P_o &= P_{load} = |I_G|^2 R_{load} \\
 P_{loss} &= P_{line} = |I_G|^2 R_{line} \\
 (P_{input} &= P_{output} + P_{loss}) \\
 \text{efficiency} &= \eta = \frac{P_o}{P_{in}} \times 100\%
 \end{aligned}$$

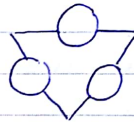
## chapter (12) (poly phase)



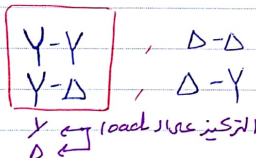
2 - phase



3 - phase (Y)

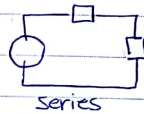
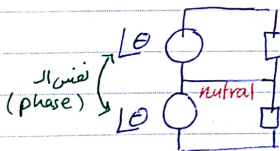


3 phase (Δ)



((Balanced system))

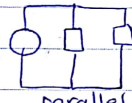
Single phase - 3 wires system > يعني إلهم نفس ال phase



series

لم صا إذا load

لم بعد جعل توقف البارة كلها

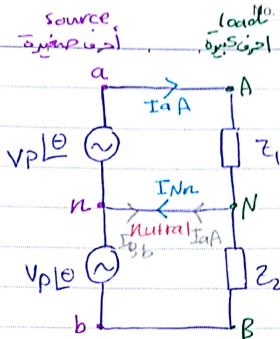


parallel

لم صا فحتاج تبار

كيس حتى نوصل

للشوتية العلوية



- $I_{aA}$
- $I_{nN}$
- $V_{an} = V_{p\angle 0}$
- $V_{bn} = -V_{nb} = -V_{p\angle 0}$
- $V_{NB} = V_{nb} = V_{p\angle 0}$   
( $V_{AN} = V_{an}$ )

$$I_{aA} = \frac{V_{an}}{Z_1} = \frac{V_{p\angle 0}}{Z_1}$$

- $V_{AN} = V_{an} = V_{p\angle 0}$
- $V_{NB} = V_{nb} = V_{p\angle 0}$

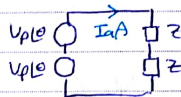
$$I_{nN} = I_{Bb} - I_{aA}$$

$$I_{P \text{ } Z_1} = Z_2$$

$$I_{3b} = I_{aA}$$

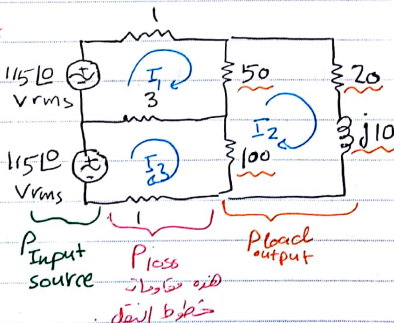
$$I_{nN} = 0 \quad (\text{Balanced system})$$

$$I_{Bb} = \frac{V_{nb}}{Z_2} = \frac{V_{p\angle 0}}{Z_2}$$



$$I_{aA} = \frac{Z}{Z+Z} V_{p\angle 0} = \frac{V_{p\angle 0}}{2}$$

Ex



$$115 = I_1 + 50(I_1 - I_2) + 3(I_1 - I_3)$$

$$115 = 3(I_3 - I_1) + 100(I_3 - I_2) + I_3$$

$$50(I_2 - I_1) + (20 + j10)I_2 + 100(I_2 - I_3) = 0$$

$$I_1 = 11.24 \angle -19.8^\circ \text{ A}$$

$$I_2 = 9.4 \angle -24.47^\circ \text{ A}$$

$$I_3 = 10.37 \angle -2.48^\circ \text{ A}$$

$$I_1 - I_2 = 1.84 \angle 10^\circ \text{ A}$$

عند تربع  $(I_1 - I_2)^2$  نأخذ  $(A)^2$  فقط بين الساتج بين طرفي

$$P_{load} = P_o$$

$$\blacksquare P_{50} = |I_1 - I_2|^2 50 = 206 \text{ W}$$

$$\blacksquare P_{100} = |I_3 - I_2|^2 100 = 117 \text{ W}$$

$$\blacksquare P_{20} = (I_2)^2 20 = (9.4)^2 (20) = 17.63 \text{ W}$$

$$P_o = P_{load} = 2086 \text{ W}$$

مجموعه

$$\blacksquare P_{loss} \rightarrow P_1 = |I_1|^2 (1) = (11.24)^2 (1) = 126 \text{ W}$$

$$\rightarrow P_2 = |I_1 - I_3|^2 (3) = 3 \text{ W}$$

$$\rightarrow P_3 = |I_3|^2 (1) = 108 \text{ W}$$

$$P_{loss} = 126 + 108 + 3$$

$$= 237 \text{ W}$$

مجموعه

$$\blacksquare P_{in} = P_{S1} + P_{S2}$$

$$- P_{S1} = |V_1| |I_1| \cos(\theta - \phi) = (115)(11.24) \cos(0 + 19.8^\circ) = 1216 \text{ W}$$

$$- P_{S2} = |V_1| |I_3| \cos(\theta - \phi) = 1107 \text{ W}$$

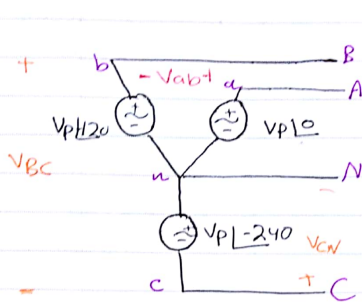
$$\Rightarrow P_{in} = P_o + P_{loss} = \underline{2323 \text{ W}}$$

$$\eta = \frac{P_o}{P_{in}} \times 100\% = \frac{2086}{2323} \times 100\% = \underline{89.8\%}$$

الرجاء تحويل الكفاءة إلى  
40 - 50



### (12-3) 3Φ - Y connection



phase voltage

- Voltage between line and neutral  
 $V_{an}$ ,  $V_{bn}$ ,  $V_{cn}$

⊕ sequence (abc)

⊖ sequence (cba)

$$V_{an} = V_p \angle 0^\circ$$

$$V_{an} = V_p \angle 0^\circ$$

$$V_{bn} = V_p \angle -120^\circ$$

$$V_{bn} = V_p \angle 120^\circ$$

$$V_{cn} = V_p \angle -240^\circ$$

$$V_{cn} = V_p \angle 240^\circ$$

$$|V_{an}| = |V_{bn}| = |V_{cn}| = V_p$$

$$V_{an} + V_{bn} + V_{cn} = 0$$

Line Voltage :-

- Voltage between two line

$V_{ab}$ ,  $V_{bc}$ ,  $V_{cn}$  ⊕

$$V_{ab} = V_{an} - V_{bn}$$

$$V_{ab} = V_p \angle 0^\circ - V_p \angle -120^\circ$$

$$V_{ab} = \sqrt{3} V_p \angle 30^\circ$$

$$V_{bc} = V_{bn} - V_{cn}$$

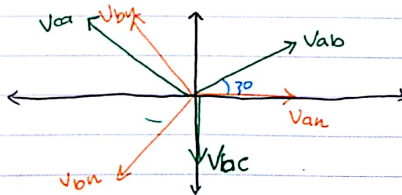
$$V_{bc} = \sqrt{3} V_p \angle -90^\circ$$

$$V_{ca} = V_{cn} - V_{an}$$

$$V_{ca} = \sqrt{3} V_p \angle -210^\circ$$

$$|V_{ab}| = |V_{bc}| = |V_{cn}| = \sqrt{3} V_p = V_L \rightarrow V_L = \sqrt{3} V_p$$

➤ line voltage lead phase voltage by 30°



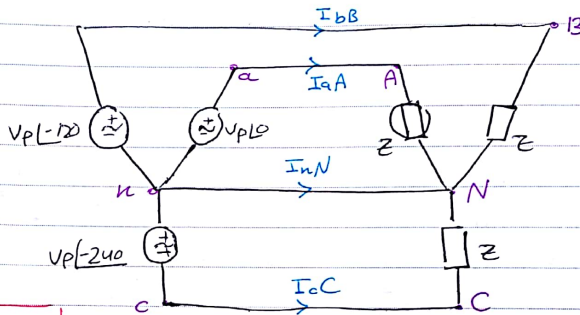


No.

⊖ sequence

$$\begin{aligned} V_{ab} &= \sqrt{3} V_P \angle -30^\circ + 120^\circ \\ V_{bc} &= \sqrt{3} V_P \angle +90^\circ \\ V_{ca} &= \sqrt{3} V_P \angle 210^\circ \end{aligned}$$

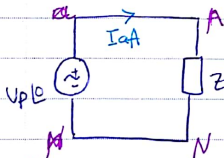
line Voltage lags Phase Voltage by  $30^\circ$  for ⊖ sequence



$$V_L = V_P$$

$$(I_L = I_P)$$

per phase



$$I_{aA} = \frac{V_{an}}{Z} = \frac{V_p/120}{Z}$$

$$I_{bB} = \frac{V_{bn}}{Z} = \frac{V_p/120}{Z}$$

$$I_{cC} = \frac{V_{cn}}{Z} = \frac{V_p/240}{Z}$$

$$|I_{aA}| = |I_{bB}| = |I_{cC}| = I_L = I_P = \frac{V_P}{|Z|}$$

$$V_{an} = V_{AN}$$

(R)  $V_{an} \neq V_{AN}$   $\rightarrow$   $V_{an} \neq V_{AN}$   $\rightarrow$   $V_{an} \neq V_{AN}$

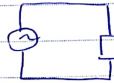
$$\rightarrow I_{nN} = I_{aA} + I_{bB} + I_{cC} = 0 \quad (\text{For balanced})$$

$$P_{loss-N} = 0 \rightarrow I_N = 0$$

Ex Find phase, line voltages and currents, Total power dissipated by the load for  $3\phi$  (Y-Y) connection.  
 IF  $V_{an} = 200 \angle 0^\circ$  Vrms,  $Z_L = 100 \angle 60^\circ \Omega$   $\oplus$  sequence.

phase  
 $V_{an} = 200 \angle 0^\circ$   
 $V_{bn} = 200 \angle -120^\circ$   
 $V_{cn} = 200 \angle -240^\circ$

phase Voltage



$$V_L = \sqrt{3} V_p = \sqrt{3} 200$$

$$V_{ab} = \sqrt{3} \times 200 \angle 30^\circ$$

$$V_{ab} = \sqrt{3} 200 \angle 30^\circ \rightarrow -120^\circ$$

$$V_{bc} = \sqrt{3} 200 \angle -90^\circ \rightarrow -120^\circ$$

$$V_{ca} = \sqrt{3} 200 \angle -210^\circ \rightarrow -120^\circ$$

Line Voltage

$$I_L = I_p$$

$$I_{aA} = \frac{V_{an}}{Z} = \frac{200 \angle 0^\circ}{100 \angle 60^\circ}$$

$$I_{aA} = 2 \angle -60^\circ$$

$$I_{bB} = 2 \angle -180^\circ \rightarrow -120^\circ$$

$$I_{cC} = 2 \angle -300^\circ \rightarrow -120^\circ$$

$$P_{\text{phase}} = |V_p| |I_p| \cos(\theta - \phi) = (200)(2) \cos(60^\circ) = 200 \text{ W}$$

$$P_{\text{total}} = 3 \times 200 = 600$$

$$\Rightarrow P_{\text{phase}} = V_p I_p \cos(\theta - \phi)$$

$$\Rightarrow P_{\text{total}} = 3 \times V_p I_p \cos(\theta - \phi) = 3 \times P_{\text{phase}}$$

(OR)

$$\Rightarrow P_{\text{phase}} = \frac{V_L}{\sqrt{3}} I_L \cos(\theta - \phi)$$

$$\Rightarrow P_{\text{total}} = 3 P_{\text{phase}} = \sqrt{3} V_L I_L \cos(\theta - \phi)$$

Ex Balanced  $3\phi$  (Y-Y)  $V_L = 300\text{ V}$  supplying total load with  $1200\text{ W}$  at  $0.8$  Leading P.F. Find  $I_L$ , per phase Impedance.  $\rightarrow (Z)$

(1)  $V_L = \sqrt{3} V_P$

$$V_P = \frac{V_L}{\sqrt{3}} = \frac{300}{\sqrt{3}}$$

$$P_{\text{phase}} = \frac{P_{\text{total}}}{3} = V_P I_P \text{ p.f.}$$

$$\frac{1200}{3} = \frac{300}{\sqrt{3}} I_P (0.8) \rightarrow I_P = I_L = 2.9\text{ A}$$

(2)  $Z = \frac{V_P}{I_P} \angle \theta - \phi \rightarrow Z = \frac{300/\sqrt{3}}{2.9} \angle -\cos^{-1}(\text{P.F.})$

$\left\{ \begin{array}{l} \rightarrow \text{lag} \\ \oplus \cos^{-1}(\text{P.F.}) \\ \rightarrow \text{lead} \end{array} \right\}$

$$Z = 60 \angle -36.87^\circ \Omega$$

part (3)  $I_a A, I_b B, I_c C, V_{ab}, V_{bc}, V_{ca}$  sequence.

$$\{V_L = 300\}$$

$$\square V_{ab} = 300 \angle 0$$

$$\square V_{bc} = 300 \angle -120$$

$$\square V_{ca} = 300 \angle -240$$

$$\{V_P = \frac{300}{\sqrt{3}}\}$$

$$\square V_{an} = \frac{300}{\sqrt{3}} \angle -30$$

$$\square V_{bn} = \frac{300}{\sqrt{3}} \angle -150$$

$$\square V_{cn} = \frac{300}{\sqrt{3}} \angle -270$$

$\left. \begin{array}{l} \text{line Voltage} \\ \text{phase Voltage} \end{array} \right\}$

$$\theta_P + 30^\circ = \theta_L$$

$$I_L = I_P$$

$$I_a A = 2.9 \angle 6.87^\circ$$

$$I_b B = 2.9 \angle -113.13^\circ$$

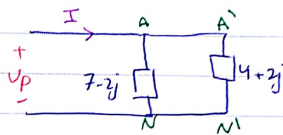
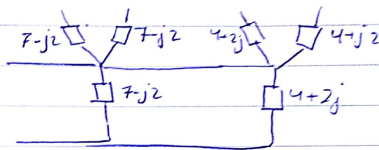
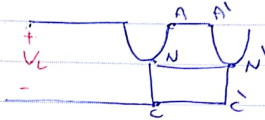
$$\cos(\theta - \phi) = 0.8$$

$$\angle \theta - \phi = -36.87^\circ \rightarrow \text{leading}$$

$$\text{Phase } V_{an} = -30^\circ \quad \phi = 6.87^\circ$$

No. 22.10.2018

Ex ( $V_L = 500$  V), 2 balanced parallel Y connected load to 3 $\phi$  system, one is capacitive ( $7-j2 \Omega$ ) the other is Inductive ( $4+j2 \Omega$ ) Find  $V_P$ ,  $I_L$ , total power by the source, PF



$$V_P = \frac{V_L}{\sqrt{3}} = \frac{500}{\sqrt{3}} \text{ V}$$

$$I_L = I_P$$

$$Z_{eq} = (7-j2) \parallel (4+j2) = 3 \angle 10.65^\circ \Omega$$

$$V_L = 500 \angle 0^\circ \rightarrow V_P = \frac{500}{\sqrt{3}} \angle -30^\circ$$

$$I_L = I_P = \frac{V_P}{Z_{eq}} = \frac{500/\sqrt{3}}{3} = 97.5$$

$$I_{aA} = \frac{V_P \angle -30^\circ}{Z} = \frac{500/\sqrt{3} \angle -30^\circ}{3 \angle 10.65^\circ} = 97.5 \angle -40.65^\circ$$

$$P_T = 3 \text{ phase} = (3)(V_P)(I_P) \cos(\theta) = (3) \left( \frac{500}{\sqrt{3}} \right) (97.5) \cos(10.65^\circ) = 83 \text{ kW}$$

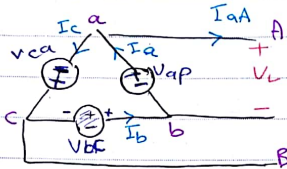
$$PF = \cos(10.65^\circ) = 0.983 \text{ lagging}$$

smile for life



No.

$\Delta \rightarrow$  Source



$$V_L = V_p$$

$\oplus$  sequence  $\Delta$

phase current

$$I_a = I_p \angle 0^\circ$$

$$I_b = I_p \angle -120^\circ$$

$$I_c = I_p \angle -240^\circ$$

line current

$$I_{aA} = \sqrt{3} I_p \angle -30^\circ$$

$$I_{bB} = \sqrt{3} I_p \angle -150^\circ$$

$$I_{cC} = \sqrt{3} I_p \angle -270^\circ$$

$$I_L = \sqrt{3} I_p$$

$\Rightarrow$  line currents lag phase current by  $30^\circ$

$\ominus$  sequence

$$I_a = I_p \angle 0^\circ$$

$$I_b = I_p \angle 120^\circ$$

$$I_c = I_p \angle 240^\circ$$

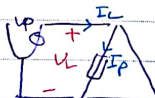
$$I_{aA} = \sqrt{3} I_p \angle 30^\circ$$

$$I_{bB} = \sqrt{3} I_p \angle 150^\circ$$

$$I_{cC} = \sqrt{3} I_p \angle 270^\circ$$



$(V_p) \angle 0^\circ$

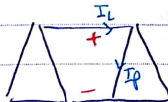
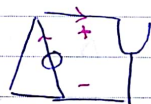


$2 \angle 150^\circ$  (line)

$(V_{an})$

$$V_L, I_p = \frac{V_L}{\sqrt{3}}$$

$$I_L = \sqrt{3} I_p$$



smile for life



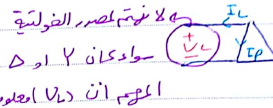
No.

In 3 $\phi$  system Line Voltage of (300V) supplied total power (1200W) to  $\Delta$  load at 0.8 lagging P.F

Find  $I_L$ ,  $Z$ .

$$V_L = V_P = 300 \text{ V}$$

$$P_{\text{phase}} = \frac{P_T}{3} = V_P I_P \text{ P.F}$$



$$\frac{1200}{3} = (300) I_P (0.8)$$

$$I_P = 1.67 \text{ A}$$

$$I_L = \sqrt{3} I_P = 2.9 \text{ A}$$

$$Z = \frac{V_P}{I_P} \frac{1}{\cos^{-1} \text{P.F.}} \xrightarrow{\text{lagging}} Z = \frac{300}{1.67} \frac{1}{\cos^{-1} \text{P.F}}$$

$$Z = 180 \angle 36.87^\circ \Omega$$



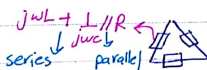
$$I_{AB} = 1.67 \angle \text{P.F.} \rightarrow \text{P.F.} = \cos(\theta - \phi) = 0.8$$

$$\theta - \phi = \cos^{-1} 0.8 \rightarrow \phi = -36.87^\circ$$

$$I_{AA} = 2.9 \angle -36.87^\circ$$

Ex  $\Delta$  connected load, consist of 200 mH in series with parallel 5  $\mu$ F and 200  $\Omega$ ,  $V_P = 200 \text{ V}$ ,  $\omega = 400 \text{ rad/s}$

Find  $I_P$ ,  $I_L$ , total power



$$Z_{eq} = j\omega L + \frac{1}{\frac{1}{j\omega C} \parallel R}$$

$$Z_{eq} = 172.75 \angle 3.66^\circ \Omega$$

$$V_P = V_L = 200 \text{ V}$$

$$I_P = \frac{V_P}{|Z|} = \frac{200}{172.75} = 1.158 \text{ A}$$

$$I_{AB} = 1.158 \angle 3.66^\circ$$

$$I_L = \sqrt{3} I_P \rightarrow I_L = 2 \text{ A}$$

$$I_{AA} = 2 \angle -33.66^\circ$$

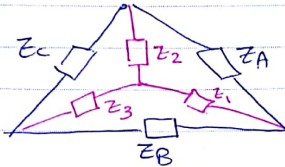
$$P_T = 3 P_{\text{phase}} = 3 (200) (1.158) \cos(3.66^\circ)$$

$$P_T = 693 \text{ W}$$

sm) e for file



No.



\* الأفضل ان تحول من (Y → Δ) لأن Y أسهل التحل عليها

Y → Δ

$$\blacksquare Z_A = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_3}$$

$$\blacksquare Z_B = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_2}$$

$$\blacksquare Z_C = \frac{Z_1 Z_2 + Z_1 Z_3 + Z_2 Z_3}{Z_1}$$

$$Z_D = 3 Z_Y$$

Δ → Y

$$\blacksquare Z_1 = \frac{Z_A Z_B}{Z_A + Z_B + Z_C}$$

$$\blacksquare Z_2 = \frac{Z_A Z_C}{Z_A + Z_B + Z_C}$$

$$\blacksquare Z_3 = \frac{Z_B Z_C}{Z_A + Z_B + Z_C}$$

$$Z_Y = \frac{Z_\Delta}{3}$$

Q(16)

7<sup>th</sup> back

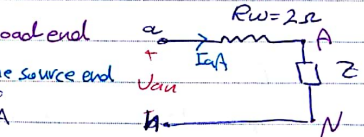
If  $V_{an} = 230 \angle 0^\circ$  balanced Y-Y  $R_w = 2 \Omega$  sequence total power supplied by the source  $100 + j30 \text{ kVA}$  find  $I_{aA}$ ,  $V_{AN}$ ,  $Z$

(1)  $I_{aA} = \frac{V_{an}}{Z + R_w}$  Voltage at the load end  $V_{an} \rightarrow$  Voltage at the source end

Perphase =  $\frac{S_{total}}{3} = V_{an} I_{aA}$

$\frac{100 + j30 \text{ k}}{3} = (230 \angle 0^\circ) I_{aA}$

$I_{aA} = 15.13 \angle -16.7^\circ \text{ A}$



KVL → (2)  $V_{AN} = V_{an} - I_{aA} R_w \rightarrow V_{AN} = 227.1 \angle 0.29^\circ \text{ V}$

(3)  $Z = \frac{V_{AN}}{I_{aA}} = 150 \angle 17^\circ \Omega = 143.3 + j44.3$

(4)  $\eta = \frac{\text{out power}}{\text{input power}} = \frac{P_{load}}{P_{in}} \times 100\%$   $P_{load} = V_{AN} I_{aA} \cos(\theta) = (227.1)(15.13) \cos(17^\circ)$

$\eta = 98.6\%$

OR  $P_{in} = (15.13)^2 (143.3)$

$P_{loss} = I_{aA}^2 R_w \approx 3$

$= 32.87 \text{ k per phase (smi) e for the}$

$P_{in} = 100 \text{ kVA}$

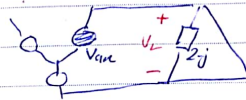
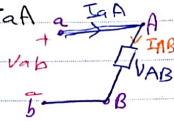
No. 24. 10. 2018

Q.25  
7th Back

Balanced 3wires, 3 $\phi$  Y- $\Delta$   $R_w=0 \rightarrow R_{wire}$

$V_{an}=200 \angle 60^\circ$ , power per phase = 2-j KVA  $\oplus$  Sequence

Find  $V_{bc}$ ,  $Z$ ,  $I_{aA}$



phase  $\Rightarrow V_{an}=200 \angle 60^\circ$

line  $\Rightarrow V_{ab}=200\sqrt{3} \angle 90^\circ$   
 $V_{bc}=200\sqrt{3} \angle -30^\circ$

$I_{aA} = \frac{V_{ab}}{Z}$  لا نستخرج هذا القانون لأن غير موجود

$S = V_{AB} I_{AB}^* \rightarrow 2000 - j1000 = (200\sqrt{3} \angle 90^\circ) I_{AB}^*$

$I_{AB} = 6.47 \angle 116.56^\circ \rightarrow$  phase current

$I_{aA} = \sqrt{3} \cdot 6.47 \angle 116.56^\circ - 30^\circ$  لأننا نريد Line current لذلك نخرج -30

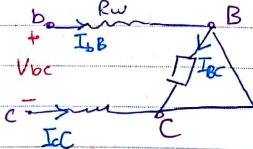
$I_{aA} = 11.2 \angle 86.57^\circ$  A  $\rightarrow$  line current

$Z = \frac{V_{AB}}{I_{AB}} = \frac{200\sqrt{3} \angle 90^\circ}{6.47 \angle 116.56^\circ} = 53.67 \angle -26.57^\circ$

Q.26  $\Delta$  load Requires = 15 KVA at 0.8 lagging P.F  $\oplus$  Sequence

$V_{BC}=180 \angle 30^\circ$ ,  $R_w=0.75 \Omega$  لا نأخذ هذا لأننا نريد Source  $\Delta$  Source لا نأخذها لأننا نريد Source  $\Delta$

Find  $V_{bc}$ , total complex power generating by the source



من نص السؤال  
 سبب وجود  
 (source)  
 لأن لم تكن موجودة  
 $V_{BC}=V_{bc}$   
 لأنها  $\Delta$

7<sup>th</sup>  $\Rightarrow$  11/17/21/23/26/28/30/31

8<sup>th</sup>  $\Rightarrow$  3/4/5/12/15/19/27/30/31

Q 12.13

No.

S - small  
= P<sub>app</sub>

$$S_{load \text{ per phase}} = \frac{15 \text{ K}}{3} = 5 \text{ K}$$

S - Capital  
= complex power

$$S_{load} = P_{app} \left( \cos \phi \right) \begin{matrix} \text{lag} \\ \text{lead} \end{matrix}$$

$$= 5 \sqrt{3} \cdot 87 = V_{BE} I_{BC}$$

phase (I<sub>AB</sub>, I<sub>CA</sub>)

$$I_{BC} = 27.78 \angle -6.87^\circ \rightarrow \text{phase current}$$

$$I_{BB} = 27.78 \sqrt{3} \angle -6.87^\circ - 30^\circ \text{ line current}$$

$$I_{BB} = 48 \angle -36.87^\circ - 120^\circ$$

$$I_{CC} = 48 \angle -56.87^\circ$$

$$V_{BC} = I_{BB} R_w + V_{BC} - I_{CC} R_w$$

$$V_{BC} = 233 \angle 120.74^\circ \text{ V}$$

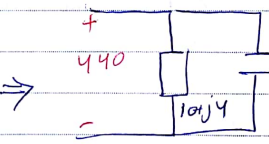
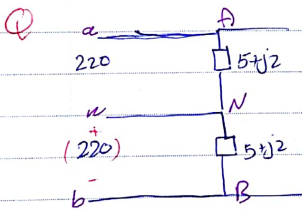
Complex

$$S_{input} = S_{load} + S_{line}$$

$$= 15 \text{ K} \cos \phi + 3 I_L R_w$$

$$= 15 \text{ K} \cos \phi + 3 (48)^2 (0.75) = 15 \text{ K} \cos \phi + 5208$$

$$S_{in} = 17.2 + j 9 \text{ KVA}$$



\* Balanced single phase

V<sub>AN</sub> = 220V, 60 Hz

N is balanced circuit (load) and source

$$Z_{eq} = R + jX_w$$

X=0 at ω<sub>0</sub>

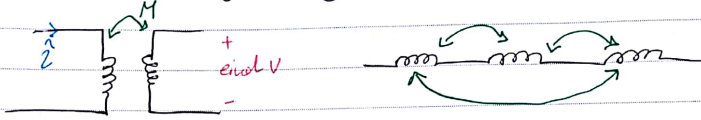
Z<sub>eq</sub> = R is (unity P.F.)

$$Y_{in} = \frac{1}{10+j4} + j\omega C \rightarrow \frac{10-j4}{10-j4} \cdot \frac{1}{10+j4} + j\omega C$$

$$\frac{10-j4}{116} + j\omega C \rightarrow \frac{10}{116} + j(\omega C - \frac{4}{116})$$

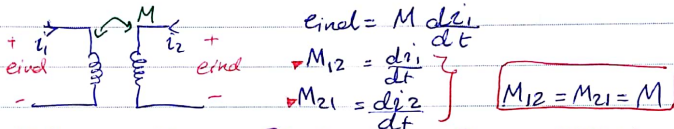
$$\frac{2\pi f}{2\pi \cdot 60} \omega C - \frac{4}{116} = 0 \rightarrow C = 91.5 \text{ MF}$$

### CH 13 << Magnetically capled circuit >>



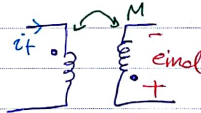
#### ■ M: Mutual Inductance

To describe the voltage in a coil due to the current Pass through another coil (current-carrying wire) Produce flux around



>> Dot convention → وضعنا نقطة لتحدد اتجاه التيار والقولبة

■ If the current enters the dotted-end → eind ⊕ at dotted-end

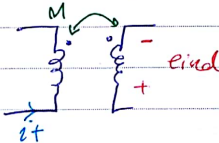


هنا التيار داخل في ال dotted

فنتج موجب عند ال dotted

■ If the current enters undotted-end

eind ⊖ at undotted-end



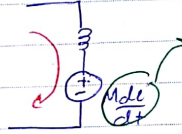
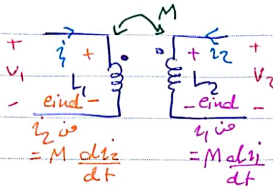
هنا التيار داخل في undott

فنتج الموجب عند ال undott



No.

Ex



\* أصبحت الـ  $e_{1.ind}$   
 مكانها مصدر موجود  
 على التوالي مع الملف

$$V_1(t) = L_1 \frac{di_1}{dt} + M \frac{di_2}{dt}$$

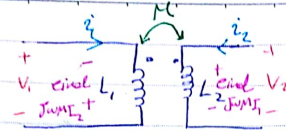
مؤلفة من تأثير (تأثير  $i_1$ )  
 على (الـ  $e_{1.ind}$ )  
 \* تأثير (الـ  $i_2$ )  
 مؤلفة من تأثير (تأثير  $i_2$ )  
 على (الـ  $e_{2.ind}$ )

$$V_2(t) = L_2 \frac{di_2}{dt} + M \frac{di_1}{dt}$$

$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

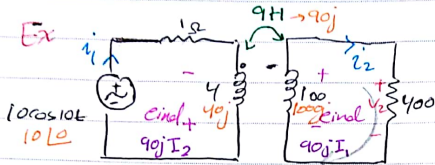
$$V_2 = j\omega L_2 I_2 + j\omega M I_1$$

No. 31. 10. 2018



$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

$$V_2 = j\omega L_2 I_2 - j\omega M I_1$$

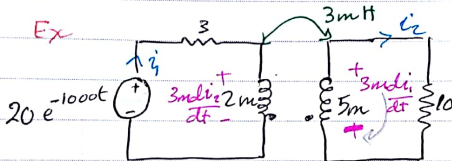


Find  $V_2$ ??  
 $\omega = 10$

$$400 i_2 + j1000 I_2 - j90 I_1 = 0 \quad \text{---} \rightarrow (1)$$

$$I_1 + 40j I_1 - 90j I_2 = 10 \angle 0 \quad \text{---} \rightarrow (2)$$

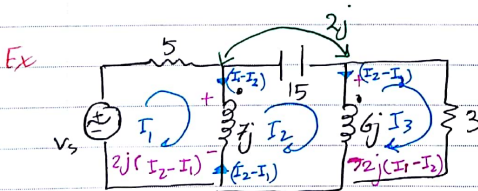
$$V_2 = 400 I_2$$



فيه المعادلات كقولنا ب diff-equ  
ولا يمكن ان نحول الى phasor  
مذالك لان ال source ليس (Sinusoidal)

$$(1) \leftarrow 20 e^{-1000t} = 3 i_1 + 2m \frac{di_1}{dt} - 3m \frac{di_2}{dt} \quad (L)$$

$$(2) \leftarrow 10 i_2 + 5m \frac{di_2}{dt} - 3m \frac{di_1}{dt} = 0$$



المعادلات في (Time domain)

$$V_s = 5i_1 + 3 \frac{d(i_1 - i_2)}{dt} + 2 \frac{d(i_1 - i_2)}{dt}$$

$$\frac{1}{2} \int i_2 + 6 \frac{d(i_2 - i_3)}{dt} + 7 \frac{d(i_2 - i_3)}{dt}$$

$$\text{المعادلات} \left\{ \begin{aligned} V_s &= 5I_1 + j7(I_1 - I_2) + j2(I_2 - I_3) \quad \text{---} \rightarrow (1) \\ -jI_2 + j6(I_2 - I_3) + j7(I_2 - I_1) + j2(I_1 - I_2) - j2(I_2 - I_3) &= 0 \quad \text{---} \rightarrow (2) \\ 3I_3 + j6(I_3 - I_2) - j2(I_1 - I_2) &= 0 \quad \text{---} \rightarrow (3) \end{aligned} \right.$$

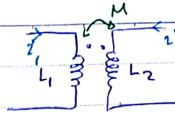
phasor في  
Domain



No.

$$W_C = \frac{1}{2} C V^2$$

$$W_L = \frac{1}{2} L I^2$$



(W)

هي الطاقة المخزنة في L

$$W_L(t) = \frac{1}{2} L_1 i_1^2(t) + \frac{1}{2} L_2 i_2^2(t) + M i_1(t) i_2(t)$$

⊕ إذا كانا متساويين سواء داخلين أو طالعين تكون

⊖ إذا كانا مختلفين غير واحد داخل والآخر طالع أو العكس

⊕  $i_1, i_2 \rightarrow$  enter or leave dot  
⊖  $i_1$  enter dot and  $i_2$  leaves dot  
OR  $i_1$  leaves dot and  $i_2$  enter dot

$$( ) ( ) \geq 0$$

$$M = k \sqrt{L_1 L_2}$$

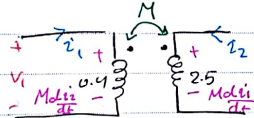
$k$ : constant coupling coefficient

$$0 \leq k \leq 1$$

No coupling

Tightly coupled.

Ex



$$K = 0.6$$

(متردد)

$$i_1 = 4i_2 = 20 \cos(500t - 30^\circ) \text{ mA}$$

$$i_2(t) = 5 \cos(500t - 20^\circ) \text{ mA}$$

$$i_2(0) = 4.7 \text{ mA}$$

الانتباه: لعمد الزوايا  
(rad - Deg) لأنهم مختلفين

$$M = k \sqrt{L_1 L_2} \rightarrow M = 0.6 \text{ H}$$

$$v_1 = 0.4 \frac{di_1}{dt} + 0.6 \frac{di_2}{dt} \rightarrow v_1(0) = 1.88 \text{ V}$$

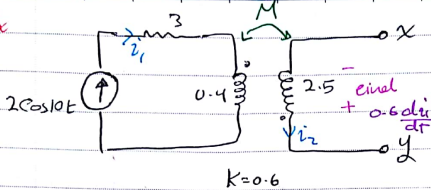
$$W_L(t) = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 + M di_1 i_2$$

$$= \left(\frac{1}{2}\right) (0.4) (20 \cos(500t - 30^\circ))^2$$

$$W_L(0) = 151 \text{ J}$$

No. \_\_\_\_\_

Ex



$$M = K \sqrt{L_1 L_2} \\ = 0.6 \text{ H}$$

- (1)  $w_L(0)$ , If  $x, y$  open  $\boxed{i_2 = 0}$   $\Rightarrow$  (open)  $\Rightarrow$   $w_L(0) = \frac{1}{2} L_1 i_1^2$   $\Rightarrow$   $w_L(0) = 0.8 \text{ J}$

- (2)  $w_L(0)$ , If  $x, y$  shorted.

$$e_{ind} = 0.6 \frac{di_1}{dt} = -12 \sin 10t$$

$$2.5 \frac{di_2}{dt} - 0.6 \frac{di_1}{dt} = 0$$

$$2.5 \frac{di_2}{dt} = -12 \sin 10t \quad \frac{dw}{dt} \quad i_2 = -\frac{12}{2.5} \int \sin 10t$$

$$\boxed{i_2(t) = 0.48 \cos 10t}$$

$$w_L(t) = \frac{1}{2} L_1 i_1^2 + \frac{1}{2} L_2 i_2^2 - M d_1 L_2$$

$$w_L(0) = 0.512 \text{ J}$$

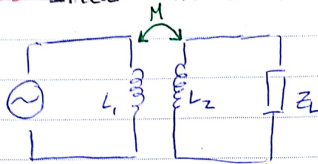
قلت ان  $x$  و  $y$  التيارات بنفس الاتجاه



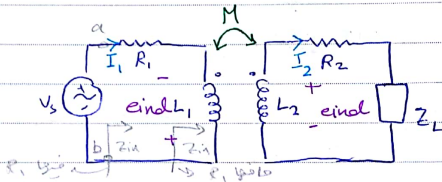
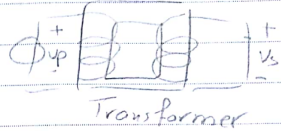
$$\left\{ \begin{array}{l} j 25 I_2 - j 6 I_1 = 0 \\ e_{ind} = j 6 I_1 \end{array} \right\} \rightarrow \text{in phasor}$$

No. 5.11.2018

### B3 Linear Transformer



primary side      secondary side  
Transformer



$$Z_{in} = R_1 + j\omega L_1$$

(M) also added

... of ...

(Z) also added

(X)

$$V_s = I_1 R_1 + j\omega L_1 I_1 - j\omega M I_2 \quad \text{--- (1)}$$

$$R_2 I_2 + Z_L I_2 + j\omega L_2 I_2 - j\omega M I_1 = 0 \quad \text{--- (2)}$$

$$(R_2 + j\omega L_2 + Z_L) I_2 = j\omega M I_1$$

$$V_s = (R_1 + j\omega L_1) I_1 - j\omega M \frac{j\omega M I_1}{R_2 + j\omega L_2 + Z_L}$$

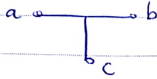
$$\frac{V_s}{I_1} = (R_1 + j\omega L_1) + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L} \quad \text{Reflected Impedance from Secondary to primary}$$

$$\square M=0 \rightarrow Z_{in} = R_1 + j\omega L_1$$

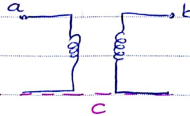
$$Z_{in} = Z_{11} + \frac{\omega^2 M^2}{Z_{22}}$$

$$Z_{in} = \left( \frac{j\omega L_1 + \frac{\omega^2 M^2}{j\omega L_2 + R_2 + Z_L}} \right) // Z_L$$

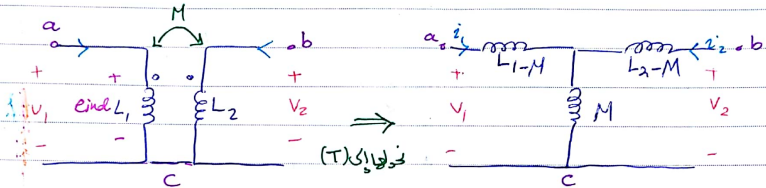
» T-model «



» TC-model «



[1]

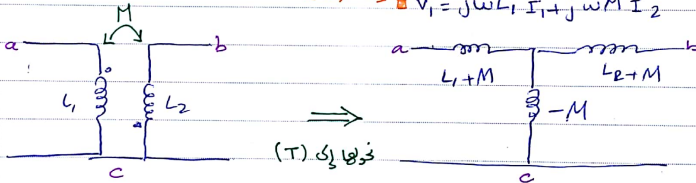


$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

$$V_1 = j\omega(L_1 - M) I_1 + j\omega M (I_1 + I_2)$$

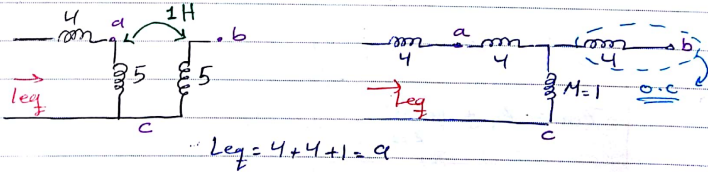
$$V_1 = j\omega L_1 I_1 + j\omega M I_2$$

[2]

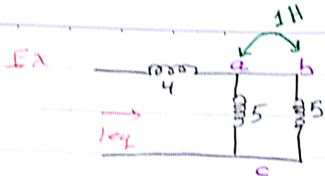


في الكمية [1] إذا كانا (dot) في نفس الاتجاه  $\oplus M$  ونضع  $(L-M)$   
 في الكمية [2] إذا كانا (dot) مختلفين باتجاه  $\ominus M$  ونضع  $(L+M)$

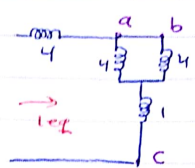
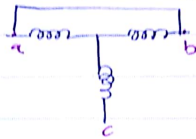
Ex



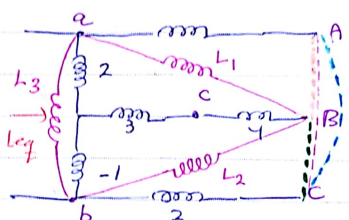
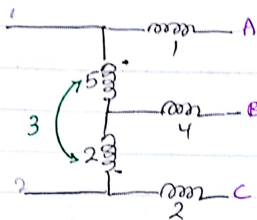
No.



$$L_{eq} = 4 // 4 + 1 + 4 = 7H$$

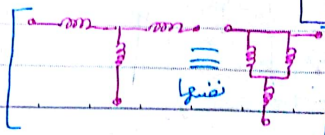
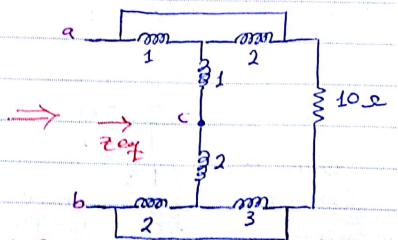
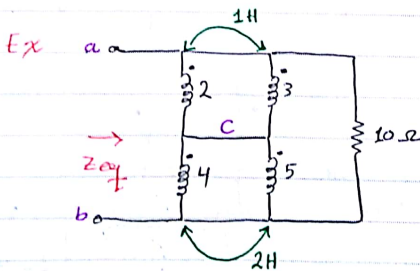


Ex  
Q35  
7th bank



$$L_{eq} = 2 - 1 = 1H$$

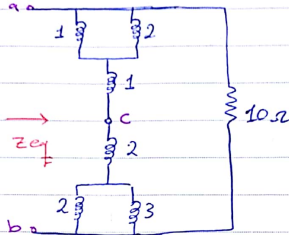
- AB shorted  $\rightarrow L_{eq} = (1 + 4 + 3) // 2 = 1 = 0.6H$
- BC shorted  $\rightarrow L_{eq} = (3 + 4 + 2) // -1 + 2 = 0.875H$
- AC shorted  $\rightarrow L_{eq} = (1 + 2) // (3 - 1) = 0.75H$
- ABC shorted  $\rightarrow L_{eq} = (L_1 // 1) + (L_2 // 2) // L_3$  بعد آن خود اکی (Δ)



smi)e for life



No.

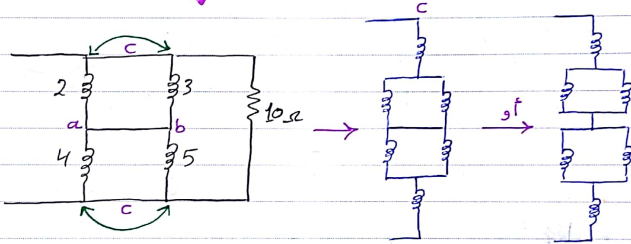


$$L_{eq} = 1/2 + 1 + 2 + 2/3 = 4.8 H$$

$$Z_{eq} = j\omega L_{eq} // 10$$

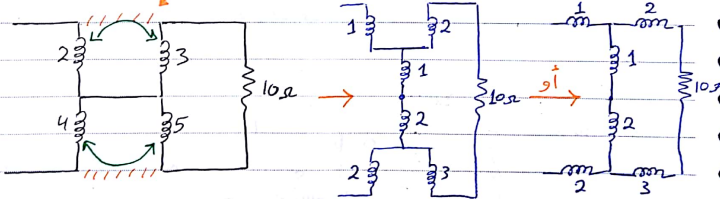
\* لو اعتبرنا (c) فوق و (b) تحت نفس الرسمية ولكن بالعكس

Ex

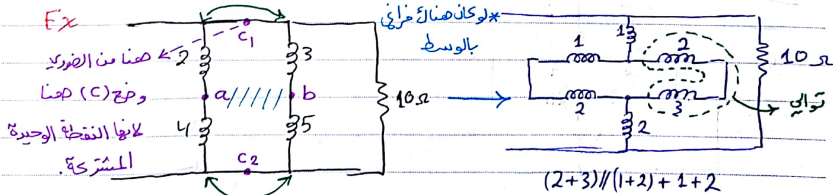


\* لو كان لا يوجد خطوط من الاعلى والاسفل فصل بينهم

Ex



Ex

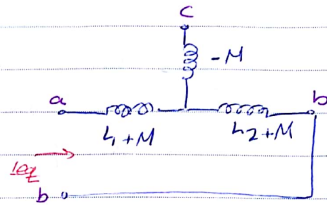
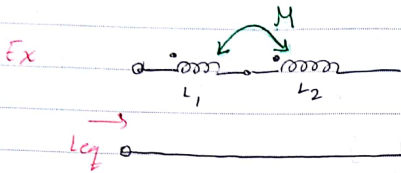


$$(2+3)/(1+2) + 1 + 2$$

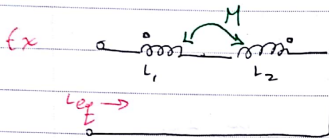
smi)e for 80



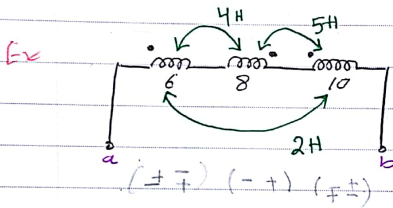
No. 7.11.2018



$$L_{eq} = L_1 + M + L_2 + M = L_1 + L_2 + 2M$$

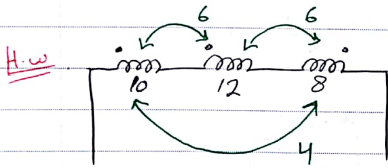


$$L_{eq} = L_1 - M + L_2 - M = L_1 + L_2 - 2M$$



$$L_{eq} = 6 - 4 + 2 + 8 - 4 - 5 + 10 - 5 + 2 = 10 \text{ H}$$

or use  $V_L = L \frac{di}{dt}$



$$= 22 \text{ H}$$

## Chapter (14) (Complex Frequency)

$$v(t) = V_m \cos \omega t + \theta$$

$$v(t) = \text{Real} \{ V_m e^{j(\omega t + \theta)} \}$$

$$= \text{Real} \{ V_m e^{j\omega t} e^{j\theta} \}$$

$$A + jB$$

$$s = \sigma + j\omega$$

■  $s$  : complex Frequency ( $s^{-1}$ )

■  $\sigma$  : neper Frequency (neper/s)

■  $\omega$  : angular Frequency (rad/s)  $\rightarrow \omega = 2\pi f$

■  $f$  : Frequency (Hz) or ( $\frac{1}{s}$ )

$$\Rightarrow F(t) = K e^{st} \quad s = \sigma + j\omega$$

[1]  $s = \sigma$  :

$$F(t) = K \cos \omega t \quad (\text{DC source})$$

[2]  $s = \sigma$  :

$$P(t) = K e^{\sigma t} \quad \begin{matrix} \rightarrow +\sigma \text{ (increasing)} \\ \text{exponential} \quad \rightarrow -\sigma \text{ (decreasing)} \end{matrix}$$

[3]  $s = j\omega$  :

$$P(t) = K e^{j\omega t} = K [\cos \omega t + j \sin \omega t] \rightarrow \sin$$



[4]  $s = \sigma + j\omega$  :

$$P(t) = K e^{(\sigma + j\omega)t} = K e^{\sigma t} [\cos \omega t + j \sin \omega t]$$

Damped sin

No.

$$\begin{aligned} \Rightarrow V(t) &= V_m \cos(\omega t + \theta) \\ &= \frac{V_m}{2} [e^{j(\omega t + \theta)} + e^{-j(\omega t + \theta)}] \\ &= [e^{j\omega t} e^{j\theta} + e^{-j\omega t} e^{-j\theta}] \end{aligned}$$

$$* S = \pm j\omega \quad \begin{array}{l} \rightarrow s_1 = +j\omega \\ \rightarrow s_2 = -j\omega \end{array}$$

$$- V(t) = 2 \sin 5t \quad \rightarrow S = \pm 5j$$

$$Ex \quad \cos(10t + 50) \quad \rightarrow S = \pm j10$$

$$S(t) = \left( \frac{3e^{-100t}}{-100} + \frac{e^{-200t}}{-200} \right) \sin 2000t \quad \pm j2000$$

$$\left( \frac{e^{j2000t} - e^{-j2000t}}{2j} \right)$$

$\underline{S} \rightarrow$

$$\begin{aligned} &(-100 + j2000), \quad (-100 - j2000) \\ &(-200 + j2000), \quad (-200 - j2000) \end{aligned}$$

$\times e^{j2} \rightarrow \text{phase}$   
 $\times e^{j\omega t} \rightarrow \text{frequency } \omega$   
 $\times e^{2t} \rightarrow \delta$   
 $\times e^2 \rightarrow \text{constant}$

$$Ex \quad (2 \cdot e^{10t}) \cos(4t + 30) \quad \pm j4$$

$$2 \cos(4t + 30) = e^{j10t} \cos 4t + 30$$

$$(+j4), (-j4), (-10 + j4), (-10 - j4)$$

Ex constant a time domain signals

$$(1) S = 0, 10^5, -10^5 \quad s^{-1}$$

$$V(t) = A + B e^{10t} + C e^{-10t}$$

$$(2) S = -5^5, j\omega 8, -5-j8 \quad s^{-1}$$

$$[s = (\delta + j\omega)t]$$

$$V(t) = A e^{-5t} + B \cos(8t + \theta) + C e^{-5t} \cos(8t)$$

$$B \sin(8t)$$

smile for life

Ex  $V = 12 \angle 35^\circ$  V , Final  $v(t)$  ??

(1)  $s = 0 \rightarrow \delta = 0, \omega = 0$

$$v(t) = A e^{st} \cos(\omega t + \theta)$$

$$v(t) = 12 e^{0t} \cos(0t + 35)$$

$$v(t) = 12 \cos(35) = 9.8 \text{ V}$$

(2)  $s = -20 \text{ s}^{-1}$

$$v(t) = 12 e^{-20t} \cos(0t + 35)$$

$$v(t) = 9.8 e^{-20t}$$

(3)  $s = j5$

$$v(t) = 12 \cos(5t + 35)$$

(4)  $s = 20 + j5$

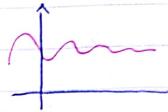
$$v(t) = 12 e^{-20t} \cos(5t + 35)$$

### (14.3) Laplace

Convert from time to frequency (s) domain

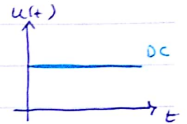
$$F(t) \longleftrightarrow F(s)$$

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$



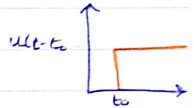
$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t < 0 \end{cases}$$



$$A u(t) \quad \int_0^{\infty} A e^{-st} dt \quad \frac{A}{s}$$

$$A u(t-t_0) \quad \int_{t_0}^{\infty} A e^{-st} dt \quad \frac{-A e^{-t_0 s}}{s}$$



$$A e^{-at} u(t) \quad \int_0^{\infty} A e^{-at} e^{-st} dt \quad \frac{A}{s+a}$$

$$A e^{at} u(t) \quad \frac{A}{s-a}$$

$$A t u(t) \quad \int_0^{\infty} A t e^{-st} dt \quad \frac{A}{s^2}$$

$$A t^n u(t) \quad \frac{A n!}{s^{n+1}}$$

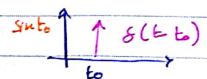
$$A t e^{-at} u(t) \quad \frac{A}{(s+a)^2}$$

$$A \delta(t) \quad A$$

$$A \delta(t-t_0) \quad A e^{-t_0 s}$$

$$\begin{aligned} A \sin \omega t &\rightarrow \frac{A \omega}{s^2 + \omega^2} \\ A \cos \omega t &\rightarrow \frac{A s}{s^2 + \omega^2} \end{aligned}$$

cos, sin transform  
applied to  
Laplace transform





No.

Ex  $V(t) = 5 \delta(t) - 2 u(t)$

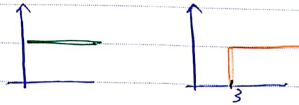
$V(s) = 5 - \frac{2}{s}$

Ex  $V(t) = 4 \delta(t-2) - 2 t u(t)$

$V(s) = 4 e^{-2s} - \frac{2}{s^2}$

Ex  $u(t) u(t-3) = u(t-3)$

$\frac{e^{-3s}}{s}$



Ex  $N(s) = 10$

$N(t) = 10 \delta(t)$

Ex  $V(s) = \frac{10}{s} \rightarrow 10 u(t)$

Ex  $V(s) = \frac{10}{s^2} \rightarrow 10 t u(t)$

$\left( \frac{N}{D} \right)$

- $N < D$  partial fraction
- $N \geq D$  long division

Ex  $\Rightarrow V(s) = \frac{10}{s(s+10)} = \frac{A}{s} + \frac{B}{s+10}$

$A s + 10 A + B s = 10 \quad \blacksquare \quad 10 A = 10 \rightarrow \boxed{A=1}$

$\blacksquare \quad A + B = 0 \rightarrow \boxed{B=-1}$

$V(s) = \frac{1}{s} + \frac{-1}{s+10} \rightarrow N(t) = u(t) - e^{-10t} u(t)$

Ex  $V(s) = \frac{10s}{s+10} \rightarrow \text{long division}$

$V(s) = 10 - \frac{100}{s+10}$

$$\begin{array}{r} 10 \\ s+10 \overline{) 10s} \\ \underline{-10s+100} \\ -100 \end{array}$$

$V(t) = 10 \delta(t) - 100 e^{-10t} u(t)$

smile for life



No.

Ex  $V(s) = \frac{3s^2 - 4}{s^2} \rightarrow = \frac{3 - 4}{s^2}$

$V(t) = 3\delta(t) - 4t u(t)$

Ex  $V(s) = \frac{11s + 30}{s^2 + 3s} \rightarrow = \frac{11s + 30}{s(s+3)}$

$V(s) = \frac{A}{s} + \frac{B}{(s+3)}$

$A=10$

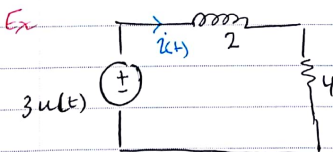
$B=1$

$V(t) = A u(t) + B e^{-2t} u(t)$

$As + 3A + Bs = 11s + 30$

$\left. \begin{aligned} \mathcal{L}\left(\frac{dv}{dt}\right) &\Leftrightarrow s V(s) - v(0^-) \\ \mathcal{L}\left(\frac{d^2v}{dt^2}\right) &\Leftrightarrow s^2 V(s) - s v(0^-) - v'(0^-) \end{aligned} \right\}$

20+30(t) = 0, 7=0, initial conditions



Find  $i(t)$ ,  $i(0^-) = 5A$

$\mathcal{L}(3u(t) - 2 \frac{di}{dt} + 4i)$

$I_L = I_f + (I_0 - I_f) e^{-t/\tau}$   
 $I_f(t \rightarrow \infty) \rightarrow (L: s.c) \rightarrow I_f = \frac{3}{4}$

$\tau = \frac{L}{R} = \frac{2}{4}$

$\frac{3}{s} = 2 [s I(s) - I(0)] + 4 I(s)$

$\frac{3}{s} = 2 s I(s) - 10 + 4 I(s)$

$\frac{\frac{3}{s} + 10}{2s + 4} = I(s) = \frac{3 + 10s}{2s(s+2)}$

No. \_\_\_\_\_

$$\frac{A}{2s} + \frac{B}{s+2}$$

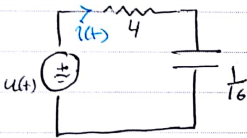
$$I(s) = \frac{3/4}{s} + \frac{4.25}{s+2}$$

$$i(t) = \frac{3}{4} u(t) + 4.25 e^{-2t} u(t)$$

$$i(t) = \left( \frac{3}{4} + 4.25 e^{-2t} \right) u(t)$$

$$\left[ \begin{aligned} \mathcal{L} \left[ \int_{-\infty}^t f(t) dt \right] &= \mathcal{L} \left[ \int_{-\infty}^0 f(t) dt \right] + \mathcal{L} \left[ \int_0^t f(t) dt \right] \\ &= \frac{F(0)}{s} + \frac{F(s)}{s} \end{aligned} \right]$$

Ex



$$i(t) \rightarrow v_c(0^+) = 9V$$

$$\mathcal{L} \left( u(t) = 4I + \frac{1}{s} \int_{-\infty}^t i(t) dt \right)$$

$$\frac{1}{s} = 4I(s) + \mathcal{L} \left( \underbrace{16 \int_{-\infty}^t i(t) dt}_{v_c(0)} + \int_0^t i(t) dt \right)$$

$$\frac{1}{s} = 4I(s) + \frac{v_c(0)}{s} + \frac{16I(s)}{s}$$

$$\frac{\frac{1}{s} - \frac{9}{s}}{4 + \frac{16}{s}} = I(s) \rightarrow I(s) = \frac{-\frac{8}{s}}{4 + \frac{16}{s}} = \frac{-8}{4s + 16} = \frac{-2}{s + 4}$$

$$i(t) = -2e^{-4t} u(t)$$

No. \_\_\_\_\_

▣ Initial value  $P(0)$

▣ Final value  $P(\infty)$

$$\Rightarrow \text{Initial value } P(0) = \lim_{t \rightarrow 0} P(t) \stackrel{\text{OR}}{=} \lim_{s \rightarrow \infty} s F(s)$$

$$\Rightarrow \text{Final value } P(\infty) = \lim_{t \rightarrow \infty} P(t) \stackrel{\text{OR}}{=} \lim_{s \rightarrow 0} s F(s)$$

Ex  $V(s) = \frac{5s^2 + 10}{2s(s^2 + 3s + 5)}$

$$f(0) = \lim_{s \rightarrow \infty} s F(s) = \lim_{s \rightarrow \infty} \frac{5s^2 + 10}{2(s^2 + 3s + 5)}$$

$$\lim_{s \rightarrow \infty} \frac{5 + \frac{10}{s^2}}{2(1 + \frac{3}{s} + \frac{5}{s^2})} \rightarrow P(0) = \frac{5}{2}$$

## Chapter (14) (Complex Frequency)

$$v(t) = V_m \cos \omega t + \theta$$

$$v(t) = \text{Real} \{ V_m e^{j(\omega t + \theta)} \}$$

$$= \text{Real} \{ V_m e^{j\omega t} e^{j\theta} \}$$

$$A + jB$$

$$s = \sigma + j\omega$$

■  $\sigma$  : complex frequency ( $s^{-1}$ )

■  $\sigma$  : neper frequency (neper/s)

■  $\omega$  : angular frequency (rad/s)  $\rightarrow \omega = 2\pi f$

■  $f$  : frequency (Hz) or ( $\frac{1}{s}$ )

$$\gg f(t) = K e^{st} \quad s = \sigma + j\omega$$

[1]  $s = 0$ :

$$f(t) = K \cos \omega t \quad (\text{DC source})$$

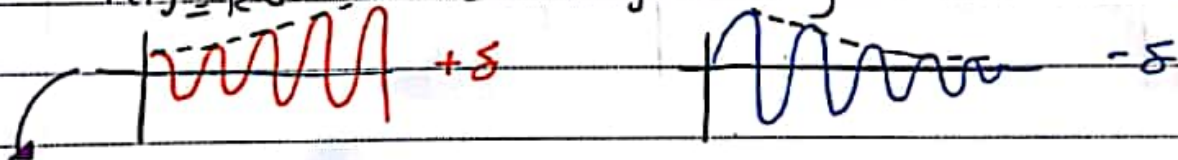
[2]  $s = \sigma$ :

$$f(t) = K e^{\sigma t}$$

exponential  $\rightarrow +\sigma$  (increasing)  
 $\rightarrow -\sigma$  (decreasing)

[3]  $s = j\omega$ :

$$f(t) = K e^{j\omega t} = K [\cos \omega t + j \sin \omega t] \rightarrow \sin$$



[4]  $s = \sigma + j\omega$ :

$$f(t) = K e^{(\sigma + j\omega)t} = K e^{\sigma t} [\cos \omega t + j \sin \omega t]$$

Damped Sin



$$\begin{aligned} \Rightarrow V(t) &= V_m \cos(\omega t + \theta) \\ &= \frac{V_m}{2} \left[ e^{j(\omega t + \theta)} + e^{-j(\omega t + \theta)} \right] \\ &= \left[ e^{j\omega t} e^{j\theta} + e^{-j\omega t} e^{-j\theta} \right] \end{aligned}$$

$$* S = \pm j\omega \rightarrow \begin{matrix} s_1 = +j\omega \\ s_2 = -j\omega \end{matrix}$$

$$v(t) = 2 \sin 5t \quad s = \pm 5j$$

$$f_x \cos(10t + 50) \rightarrow s = \pm j10$$

$$S(t) = \left( \frac{e^{-100t}}{-100} + \frac{e^{-200t}}{-200} \right) \sin 2000t + \frac{j2000}{2j} (e^{j2000t} - e^{-j2000t})$$

5 ↓

$$\begin{aligned} &(-100 + j2000), \quad (-100 - j2000) \\ &(-200 + j2000), \quad (-2000 - j2000) \end{aligned}$$

$j^2 \rightarrow \text{phase}$   
 $j\omega t \rightarrow \text{Frequency}$   
 $\omega^2 \rightarrow \delta$   
 $e^2 \rightarrow \text{constant}$

$$\text{Ex } (2 \cdot e^{+j\omega t}) \cos(4t + 30^\circ) + j^4$$

$$2 \cos(4t + 30^\circ) - e^{-j/6t} \cos 4t + 30^\circ$$

$$(+j4), (-j4), (-10+j4), (-10-j4)$$

Ex construct a time domain signals

(1)  $s = 0$  /  $10^5$  /  $-10^5$   $s^{-1}$

$$v(t) = A + B e^{10t} + C e^{-10t}$$

(2)  $s = -5 \pm j\omega 8, -5 - j\omega 8 \quad s^{-1} \quad [s = (\sigma + j\omega) t]$

$$v(t) = A e^{-st} + \underbrace{B \cos(8t + \theta)}_{B \sin(8t)} + C e^{-st} \cos(8t)$$



## / 14-3) Laplace

Convert from time to frequency (s) domain

$$f(t) \longleftrightarrow F(s)$$

$$F(s) = \int_{-\infty}^{\infty} f(t) e^{-st} dt$$



method  
partial

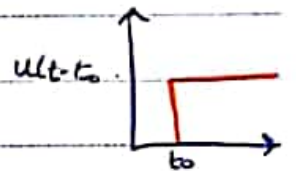
$$F(s) = \int_0^{\infty} f(t) e^{-st} dt$$

$$u(t) = \begin{cases} 1, & t > 0 \\ 0, & t \leq 0 \end{cases}$$



$$A u(t) \Rightarrow \int_0^{\infty} A e^{-st} dt \Rightarrow \frac{A}{s}$$

$$A u(t-t_0) \Rightarrow \int_{t_0}^{\infty} A e^{-st} dt \Rightarrow \frac{-A e^{-t_0 s}}{s}$$



$$A e^{-at} u(t) \Rightarrow \int_0^{\infty} A e^{-at} e^{-st} dt \Rightarrow \frac{A}{s+a}$$

$$A e^{at} u(t) \Rightarrow \frac{A}{s-a}$$

$$A t u(t) \Rightarrow \int_0^{\infty} A t e^{-st} dt \Rightarrow \frac{A}{s^2}$$

$$\begin{cases} A \sin \omega t \rightarrow \frac{A \omega}{s^2 + \omega^2} \\ A \cos \omega t \rightarrow \frac{A s}{s^2 + \omega^2} \end{cases}$$

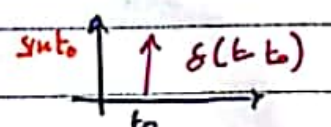
$$A t^n u(t) \Rightarrow \frac{A n!}{s^{n+1}}$$

cos, sin functions  
في L تحويل  
L وظيفه

$$A t e^{-at} u(t) \Rightarrow \frac{A}{(s+a)^2}$$

$$A \delta(t) \Rightarrow A$$

$$A \delta(t-t_0) \Rightarrow A e^{-t_0 s}$$



Ex  $V(t) = 5\delta(t) - 2u(t)$

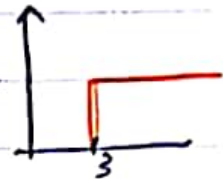
$$V(s) = 5 - \frac{2}{s}$$

Ex  $V(t) = 4\delta(t-2) - 2tu(t)$

$$V(s) = 4e^{-2s} - \frac{2}{s^2}$$

Ex  $u(t)u(t-3) = u(t-3)$

$$\frac{e^{-3s}}{s}$$



Ex  $N(s) = 10$

$$N(t) = 10\delta(t)$$

Ex  $V(s) = \frac{10}{s} \rightarrow 10u(t)$

Ex  $V(s) = \frac{10}{s^2} \rightarrow 10tu(t)$

$$\left( \frac{N}{D} \right)$$

$\blacksquare N < D$  partial fraction  
 $\blacksquare N \geq D$  long division

Ex  $\gg V(s) = \frac{10}{s(s+10)} = \frac{A}{s} + \frac{B}{s+10}$

$$As + 10A + Bs = 10 \quad \blacksquare 10A = 10 \rightarrow \boxed{A=1}$$

$$\blacksquare A + B = 0 \rightarrow \boxed{B=-1}$$

$$V(s) = \frac{1}{s} + \frac{-1}{s+10} \rightarrow N(t) = u(t) - e^{-10t}u(t)$$

Ex  $V(s) = \frac{10s}{s+10} \rightarrow$  long division

$$V(s) = 10 - \frac{100}{s+10}$$

$$\begin{array}{r}
 10 \\
 s+10 \overline{) 10s} \\
 \underline{-10s+100} \\
 -100
 \end{array}$$

$$V(t) = 10\delta(t) - 100e^{-10t}u(t)$$

Ex  $V = 12 \angle 35^\circ \text{ V}$  , Final  $v(t)$  ??

(1)  $s = 0 \rightarrow \delta = 0, \omega = 0$

$$v(t) = A e^{\delta t} \cos(\omega t + \theta)$$

$$v(t) = 12 e^{0t} \cos(0t + 35)$$

$$v(t) = 12 \cos(35) = 9.8 \text{ V}$$

(2)  $s = -20 \text{ s}^{-1}$

$$v(t) = 12 e^{-20t} \cos(0t + 35)$$

$$v(t) = 9.8 e^{-20t}$$

(3)  $s = j5$

$$v(t) = 12 \cos(5t + 35)$$

(4)  $s = 20 + j5$

$$v(t) = 12 e^{-20t} \cos(5t + 35)$$



$$\frac{A}{s+2} + \frac{B}{s+2}$$

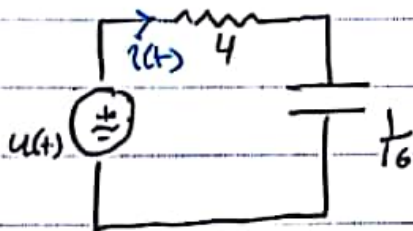
$$I(s) = \frac{3/4}{s} + \frac{4.25}{s+2}$$

$$i(t) = \frac{3}{4} u(t) + 4.25 e^{-2t} u(t)$$

$$i(t) = \left( \frac{3}{4} + 4.25 e^{-2t} \right) u(t)$$

$$\left[ \begin{aligned} \mathcal{L} \left[ \int_{-\infty}^t f(t) dt \right] &= \mathcal{L} \left[ \underbrace{\int_{-\infty}^0 f(t) dt}_{F(0)} \right] + \mathcal{L} \left[ \underbrace{\int_0^t f(t) dt}_t \right] \\ &= \frac{F(0)}{s} + \frac{F(s)}{s} \end{aligned} \right]$$

Ex



$$i(t) \rightarrow v_C(0^+) = 9V$$

$$\mathcal{L}(u(t)) = 4I + \frac{1}{s} \int_{-\infty}^t i(t) dt$$

$$\frac{1}{s} = 4I(s) + \mathcal{L} \left( \underbrace{16 \int_{-\infty}^t i(t) dt}_{v_C(0)} + \underbrace{t \int_0^t i(t) dt} \right)$$

$$\frac{1}{s} = 4I(s) + \frac{v(0)}{s} + \frac{16I(s)}{s}$$

$$\frac{\frac{1}{s} - \frac{9}{s}}{4 + \frac{16}{s}} = I(s) \rightarrow I(s) = \frac{-\frac{8}{s}}{4 + \frac{16}{s}} = \frac{-8}{4s + 16} = -\frac{2}{s + 4}$$

$$i(t) = -2e^{-4t} u(t)$$

Ex  $V(s) = \frac{3s^2 - 4}{s^2} \rightarrow = 3 - \frac{4}{s^2}$

$$V(t) = 3\delta(t) - 4t u(t)$$

Ex  $V(s) = \frac{11s + 30}{s^2 + 3s} \rightarrow = \frac{11s + 30}{s(s+3)}$

$$V(s) = \frac{A}{s} + \frac{B}{(s+3)}$$

$$A=10$$

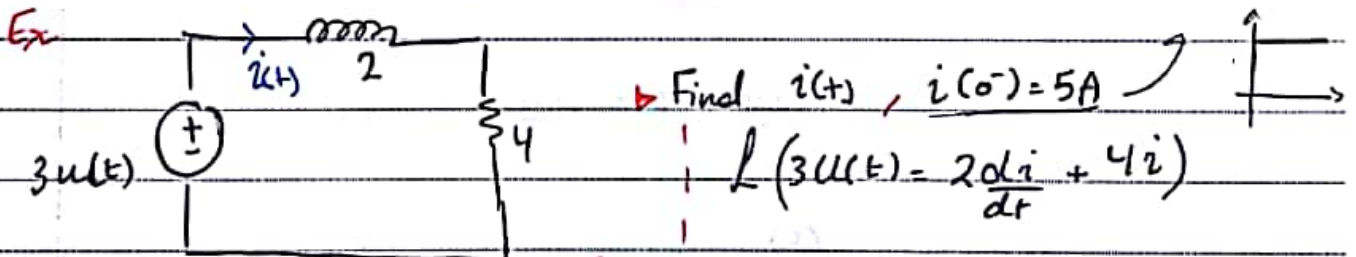
$$B=1$$

$$V(t) = A u(t) + B e^{-2t} u(t)$$

$$As + 3A + Bs = 11s + 30$$

$$\left. \begin{aligned} \mathcal{L}\left(\frac{dv}{dt}\right) &\Leftrightarrow s V(s) - v(0^-) \\ \mathcal{L}\left(\frac{d^2v}{dt^2}\right) &\Leftrightarrow s^2 V(s) - s v'(0^-) - v(0^-) \end{aligned} \right\}$$

لو لم نذكرها السؤال كمنسب Partial = نأخذ المعاد مخلق بغير  $t=0$  ولأن يجب ان تكون  $20+3u(t)$



$$I_L = I_f + (I_0 - I_f) e^{-t/\tau}$$

$$I_f (t=\infty) \rightarrow (L.s.c) \rightarrow I_f = \frac{3}{4}$$

$$\tau = \frac{L}{R} = \frac{2}{4}$$

$$\frac{3}{s} = 2 [s I(s) - I(0)] + 4 I(s)$$

$$\frac{3}{s} = 2 s I(s) - 10 + 4 I(s)$$

$$\frac{\frac{3}{s} + 10}{2s + 4} = I(s) = \frac{3 + 10s}{2s(s+2)}$$



No.

Initial value  $P(0)$

Final value  $P(\infty)$

$$\gg \text{Initial value } P(0) = \lim_{t \rightarrow 0} P(t) \stackrel{\text{OR}}{=} \lim_{s \rightarrow \infty} s F(s)$$

$$\gg \text{Final value } P(\infty) = \lim_{t \rightarrow \infty} P(t) \stackrel{\text{OR}}{=} \lim_{s \rightarrow 0} s F(s)$$

Ex  $V(s) = \frac{5s^2 + 10}{2s(s^2 + 3s + 5)}$

$$P(0) = \lim_{s \rightarrow \infty} s F(s) = \lim_{s \rightarrow \infty} \frac{5s^2 + 10}{2(s^2 + 3s + 5)}$$

$$\lim_{s \rightarrow \infty} \frac{5 + \frac{10}{s^2}}{2(1 + \frac{3}{s} + \frac{5}{s^2})} \rightarrow P(0) = \frac{5}{2}$$

## chapter (15) complex analysis

►► For R :

$$\mathcal{L}(V(t) = R i(t))$$

$$V(s) = R I(s)$$

$$R = \frac{V(s)}{I(s)} \Rightarrow Z$$

►► For L :

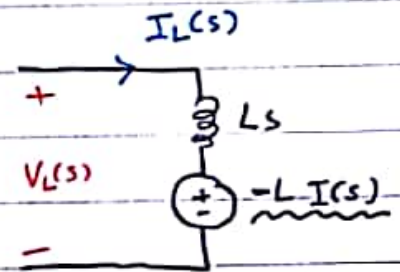
$$\mathcal{L}(V_L = L \frac{di}{dt})$$

$$V_L(s) = L s I(s) - L I(0), \quad I(0) = 0$$

$$\frac{V_L(s)}{I(s)} = L s \Rightarrow Z_L(s)$$

$$I(s) = \frac{V_L(s)}{L s} + \frac{L I(0)}{L s}$$

[Voltage Model]

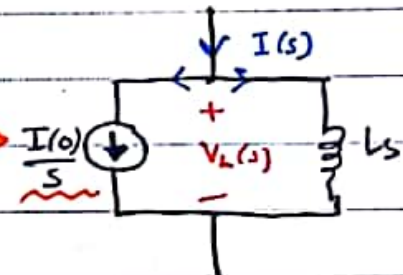


(L) و (CM) و (V.M) \*

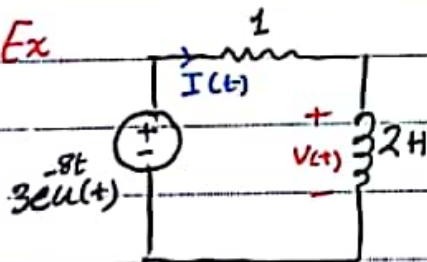
ال (L) دائما يعمل على التآكل

لذلك في الحالة يوجد (I) للقياس

[Current Model]



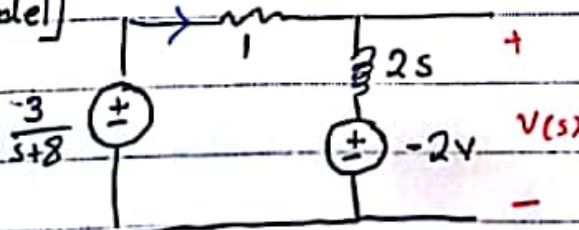
Ex



Find  $V(t)$ , if  $i(0) = 1A$  لأننا لم نجد  
 حسبها وإذا ذكر السؤال (no stored energy) فإنه لا يوجد

$$I = \frac{\left(\frac{3}{s+8}\right) - (-2)}{1 + 2s}$$

[Voltage Model]

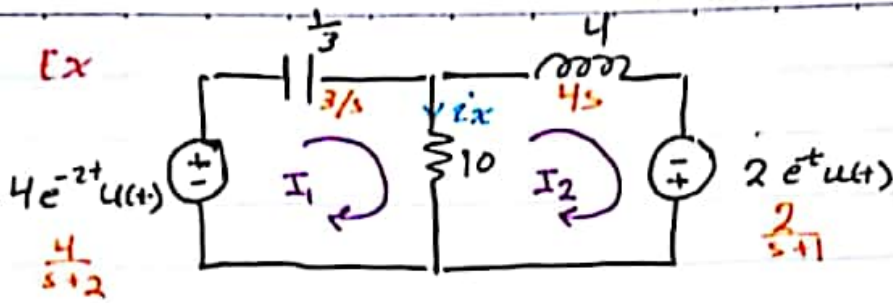


$$V(s) = 2s I + (-2)$$

$$V(s) = \frac{3 \cdot 2}{s+8} - \frac{1 \cdot 2}{s+0.5}$$

$$V(t) = \left(3 \cdot 2 e^{-8t} + (-1 \cdot 2) e^{-0.5t}\right) u(t)$$

No

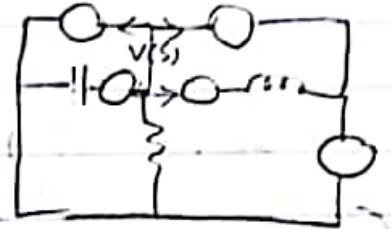


Find  $i_x$

No stored energy

$$\frac{3}{s} I_1 + 10(I_1 - I_2) = \frac{4}{s+2} \quad \text{---} \rightarrow \textcircled{1}$$

$$4s I_2 - \frac{2}{s+1} + 10(I_2 - I_1) = 0 \quad \text{---} \rightarrow \textcircled{2}$$

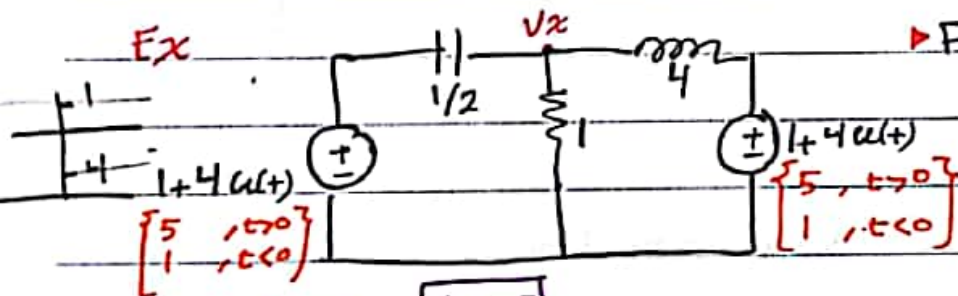


لم يذكر السؤال أن  
(No energy stored.)

$$I_1 = \frac{2s(4s^2 + 19s + 20)}{20s^4 + 66s^3 + 73s^2 + 57s + 30}$$

$$i_x(t) = 9.6e^{-2t} - 344.8e^{-t} + 841e^{-0.15t} + 197.7e^{-0.15t} \sin 0.8t$$

\* تيار (L) دائماً مع التيار الداخل في الـ Node و تيار (C) عاكسه



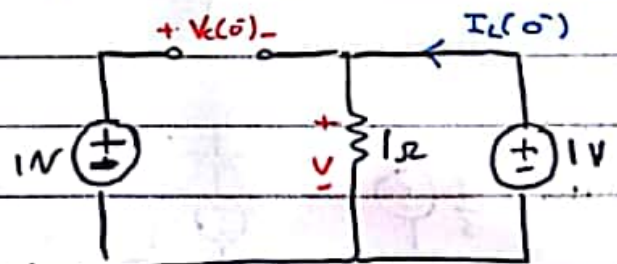
Find  $v_x(t)$

لم يذكر السؤال أنه (no energy)

$\Rightarrow t < 0$   $t = 0^-$

$$v_c(0^-) = v_c(0) = v_c(0^+)$$

$$I_L(0^-) = I_L(0) = I_L(0^+)$$



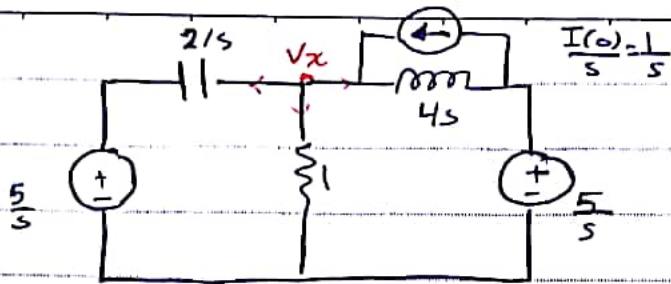
$$v_c(0^-) = 1 - 1 = 0V$$

$$-1 + v_c + v_R = 0$$

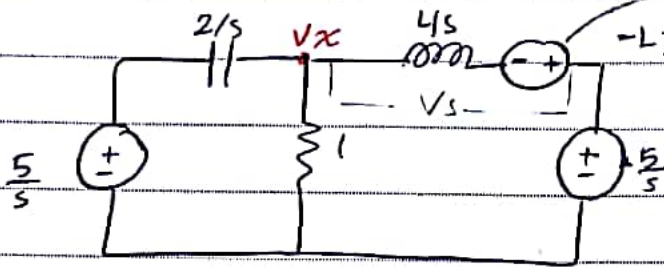
$$I_L(0^-) = \frac{1}{1} = 1A$$



No.



$$\frac{vx - \frac{5}{s}}{2/s} + \frac{vx}{1} + \frac{vx - 5/s}{4/s} - \frac{1}{s} = 0$$



$$\frac{vx - \frac{5}{s}}{2/s} + \frac{vx}{1} + \frac{vx - 5/s - (-4)}{4/s} = 0$$

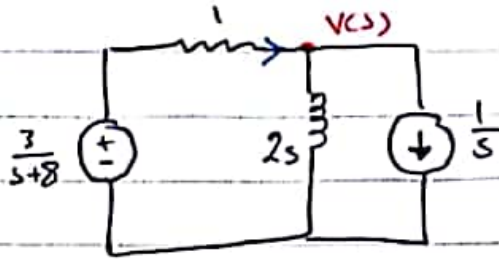
$$vx - \frac{1}{s} = 0$$

$$vx - \left( \frac{5}{s} + (-4) \right) = 0$$

$$vx - \frac{5}{s} = -4$$

طريقة أخرى

[Current Model]



$$\frac{V}{2s} + \frac{V - \frac{3}{s+8}}{1} + \frac{1}{s} = 0$$

$$V(s) = \frac{3.2}{s+8} - \frac{1.2}{s+0.5}$$

► For C:

$$\mathcal{L}(i_c = C \frac{dV}{dt})$$

$$i_c(s) = CSV(s) - CV(0^-)$$

$$\text{* If } V(0^-) = 0 \rightarrow i_c(s) = CSV(s)$$

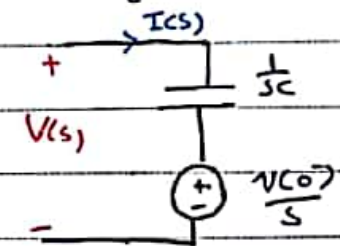
$$\frac{V(s)}{I(s)} = \frac{1}{sC}$$

(C) و (V, M), (C, M) و \*

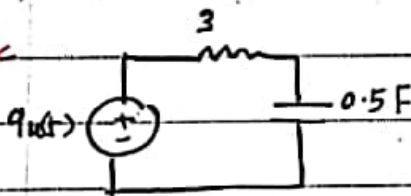
I(s) و V(s)

$$V(s) = \frac{I(s)}{Cs} + \frac{V(0^-)}{s}$$

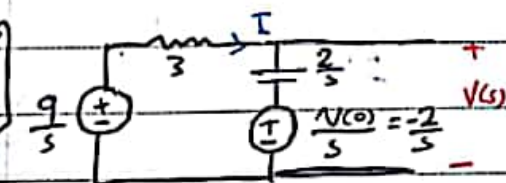
[Voltage Model]



Ex

► Find  $V(t)$  if  $V_C(0) = -2V$ 

[Voltage Model]

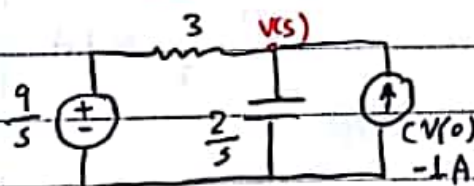


$$V(s) = \frac{2}{s} I + \left(-\frac{2}{s}\right)$$

$$I = \frac{9/s - (-2/s)}{3 + 2/s}$$

OR

[Current Model]



$$\frac{V(s)}{2/s} + \frac{V - (9/s)}{3} - (-1) = 0$$

$$V(s) = \frac{9}{s} - \frac{11}{s+2/3}$$

$$V(t) = 9u(t) - 11e^{-2/3t}u(t)$$

نفس الجواب النهائي في الطريقتين



### (15.4) Transfer Function $H(s)$

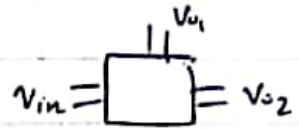
- ratio between Input and output voltage in s-domain

$$H(s) = \frac{V_{out}(s)}{V_{in}(s)}, \quad \blacksquare H(s) \therefore \text{voltage gain } A_v$$

$$\blacksquare H(s) = \frac{I_o(s)}{I_{in}(s)}$$

$$\blacksquare H(s) = \frac{V_o(s)}{I_{in}(s)}$$

$$\blacksquare H(s) = \frac{I_o(s)}{V_{in}(s)}$$



$\Rightarrow H(s)$  has zero at  $s=s_0$  if  $H(s_0) = 0$

$$H(s) = \frac{V_o(s)}{V_{in}(s)}$$

$\Rightarrow H(s)$  has poles at  $s=s_0$  if  $H(s_0) = \infty$

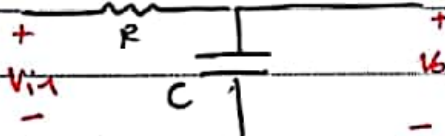
Ex.  $H(s) = \frac{7s}{s(3s^2 - 9s + 4)}$ , Find poles, zero?

$$\blacksquare H(s) = \frac{7s}{(3s-2)(s-2)}$$

$\rightarrow H(s_0) = 0 \rightarrow \text{Zero}, s_0 = \infty$   
 $\rightarrow H(s_0) = \infty \rightarrow \text{Poles}$

$$\left. \begin{array}{l} s-2=0 \rightarrow s_0=2 \\ 3s-2=0 \rightarrow s_0=\frac{2}{3} \end{array} \right\} \text{poles}$$

Ex



$$\blacksquare H(s) = \frac{V_o}{V_{in}} ??$$

$$V_o = \frac{Z_c}{Z_c + R} V_{in}$$

$H(j\omega)$

$$\frac{V_o(s)}{V_{in}(s)} = \frac{\frac{1}{sC}}{\frac{1}{sC} + R} \rightarrow \boxed{H(s) = \frac{1}{1 + sCR}} \rightarrow |H(s)| = A_v$$

$\blacksquare \text{Zero} \rightarrow H(s) = 0 \rightarrow s = \infty$

$\blacksquare \text{poles} \rightarrow H(s) = \infty \rightarrow 1 + sCR = 0 \rightarrow s = -\frac{1}{RC} \rightarrow \underline{\underline{\tau = RC}}$

$$\Rightarrow H(j\omega) = A + Bj = A \angle \theta$$

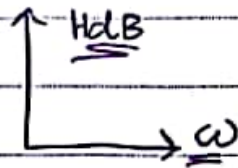
$V_{in} = \square = V_o$  frequency ( $\omega$ ) // line loss & Impedance

## (16.6) Bode plot

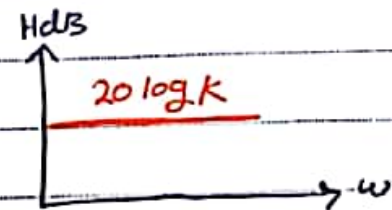
⇒ sketch of a phase and magnitude of the transfer function with " $\omega$ "



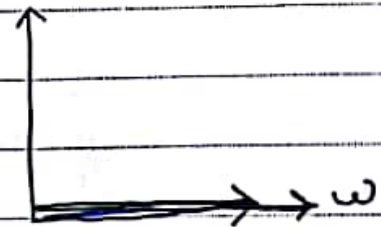
$|H(j\omega)| \rightarrow H_{dB} = 20 \log |H(j\omega)|$  عدد  $|H(j\omega)|$  يكون باعداد كبيرة جداً لذلك نستخدم  $\{ \log \}$  ونستخدم وحدة (dB) (decibel) فتصبح الرسمة بين  $H_{dB}$  و  $\omega$



(1)  $H(s) = \frac{K}{K + 0j}$   
 $H(j\omega) = K \rightarrow |H(j\omega)| = K$   
 $H_{dB} = 20 \log K$



$\phi = \tan^{-1} \frac{0}{K} = 0$



(2)  $H(s) = s$

$H(j\omega) = j\omega \quad |H(j\omega)| = \omega$

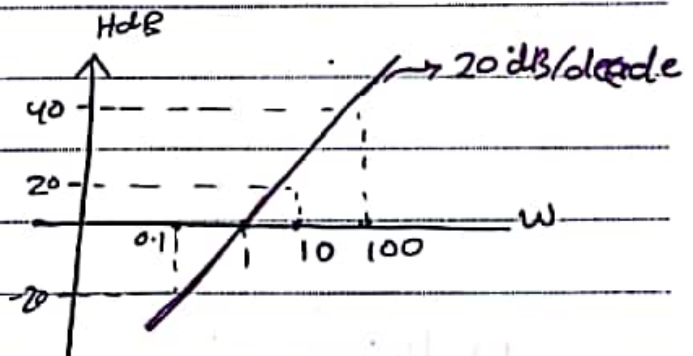
$H_{dB} = 20 \log \omega$

■  $\omega = 1 \rightarrow H_{dB} = 0$

■  $\omega = 10 \rightarrow H_{dB} = 20$

■  $\omega = 100 \rightarrow H_{dB} = 40$

■  $\omega = 0.1 \rightarrow H_{dB} = -20$



$\phi = \tan^{-1} \frac{\omega}{0} = 0 = \frac{\pi}{2} = 90^\circ$

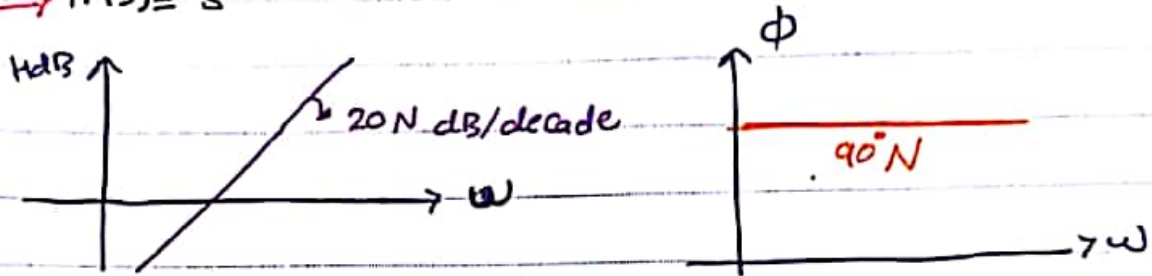


(3)  $H(s) = s^2$

$$H(j\omega) = (j\omega)^2 \rightarrow |H(j\omega)| = \omega^2$$

$$H_{dB} = 20 \log \omega^2 = 40 \log \omega$$

In general  $\rightarrow H(s) = s^N$



Ex  $H(s) = \frac{1}{s}$

$$H(j\omega) = \frac{1}{j\omega} = \frac{-j}{\omega}$$

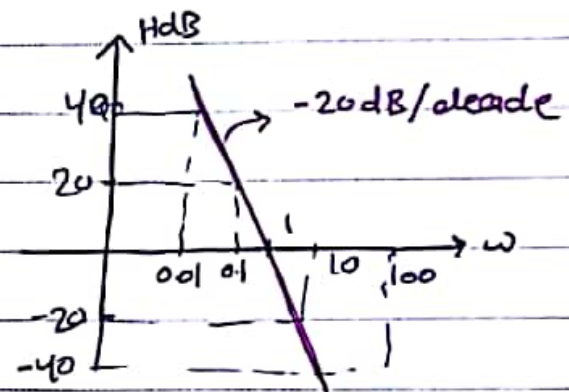
$$|H(j\omega)| = \frac{1}{\omega}$$

$$H_{dB} = 20 \log \frac{1}{\omega} = -20 \log \omega$$

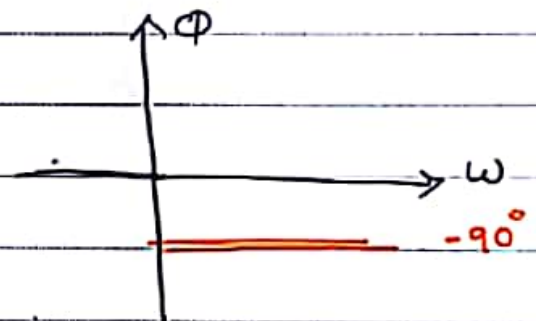
$\omega = 1 \rightarrow 0$        $\omega = 0.1 \rightarrow 20$

$\omega = 10 \rightarrow -20$        $\omega = 0.01 \rightarrow 40$

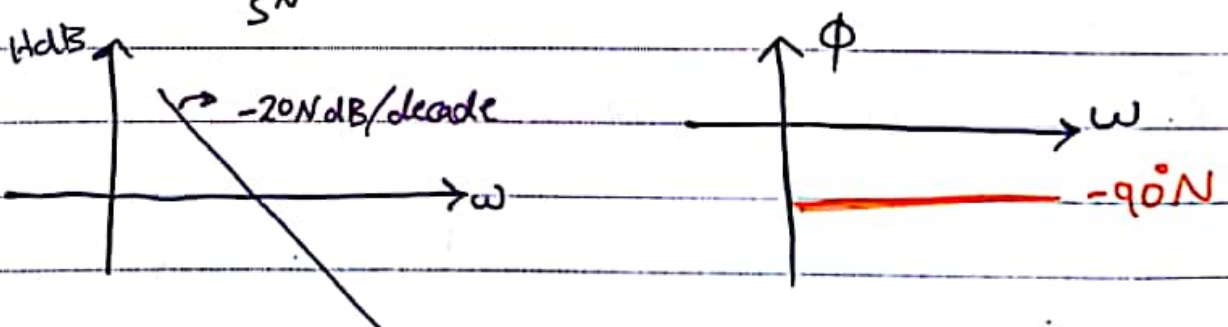
$\omega = 100 \rightarrow -40$



$$\phi = \tan^{-1} -\infty = -90^\circ$$



In general  $\rightarrow H(s) = \frac{1}{s^N}$

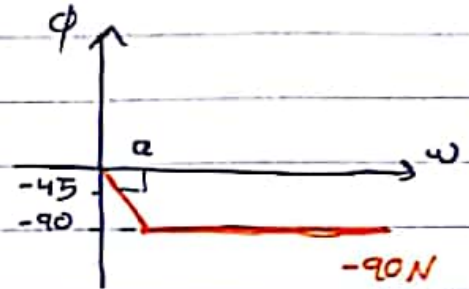
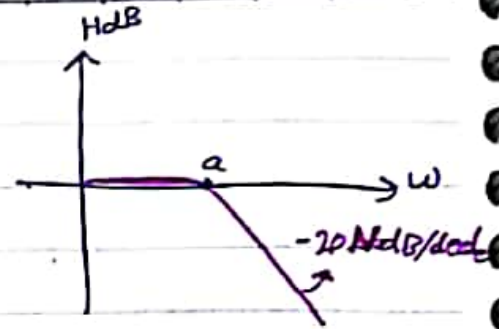




Ex  $H(s) = \frac{1}{(\frac{s}{a} + 1)^N}$

$$H(j\omega) = \frac{1}{1 + j\frac{\omega}{a}} \rightarrow 20 \log \frac{1}{\sqrt{1 + \frac{\omega^2}{a^2}}}$$

$$20 \log 1 - 20 \log \sqrt{1 + \frac{\omega^2}{a^2}}$$

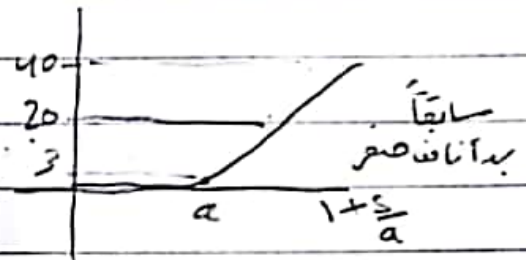


Ex  $H(s) = 50 + s$

$$H(s) = 50(1 + \frac{s}{50})$$

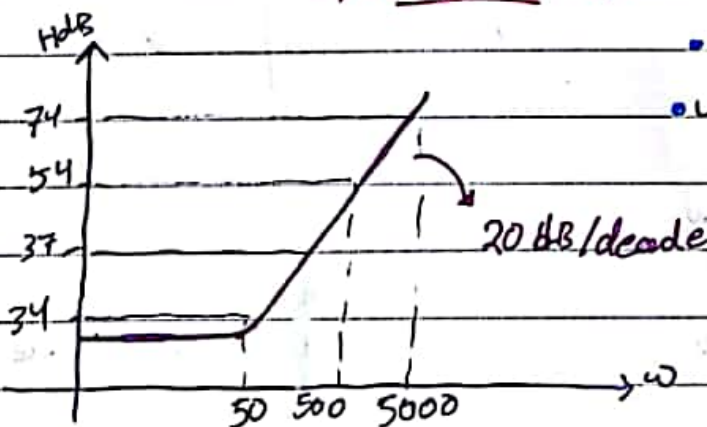
$$H(j\omega) = 50(1 + j\frac{\omega}{50})$$

$$|H(j\omega)| = 50\sqrt{1 + \frac{\omega^2}{50^2}}$$

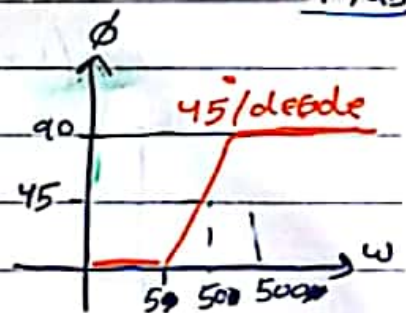


$$HdB = 20 \log 50 \sqrt{1 + \frac{\omega^2}{50^2}}$$

- $\omega = 50 \rightarrow HdB = 20 \log 50 \sqrt{2} = 37 \text{ dB}$
- $\omega \ll 50 \rightarrow HdB = 20 \log 50 = 34 \text{ dB}$
- $\omega \gg 50 \rightarrow HdB = 20 \log \omega$



- $\omega = 50 \times 10 \rightarrow HdB = 20 \log 500 = 54 \text{ dB}$
- $\omega = 50 \times 100 \rightarrow HdB = 20 \log 5000 = 74 \text{ dB}$
- 94 dB



$$\phi = \tan^{-1} \frac{\omega}{50}$$

- $\omega \ll 50 \approx 0.1 \rightarrow 0$

- $\omega = 50 \rightarrow \phi = 45$

- $\omega \gg 50 \approx 10 \rightarrow \phi = 90$

$$G(s) = 1 + \frac{s}{a}$$

$$H(j\omega) = 1 + j\frac{\omega}{a}$$

$$|H(j\omega)| = \sqrt{1 + \frac{\omega^2}{a^2}} \rightarrow a \rightarrow \text{Frequency}$$

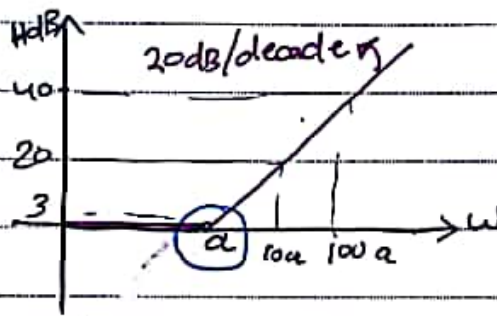
$$\therefore H_{dB} = 20 \log \sqrt{1 + \frac{\omega^2}{a^2}}$$

$$\bullet \omega = a \rightarrow H_{dB} = 20 \log \sqrt{2} = 3 \text{ dB}$$

$$\bullet \omega \ll a \rightarrow \frac{\omega}{a} \ll 1 \rightarrow H_{dB} = 20 \log 1 = 0 \text{ dB}$$

$$\bullet \omega \gg a \rightarrow \frac{\omega}{a} \gg 1 \rightarrow H_{dB} = 20 \log \frac{\omega}{a} \rightarrow \omega = 10a \rightarrow H_{dB} = 20 \log 10 = 20 \text{ dB}$$

$$\bullet \omega = 100a \rightarrow H_{dB} = 20 \log 100 = 40 \text{ dB}$$



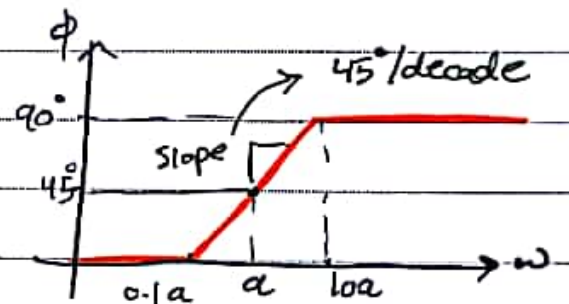
- cut off frequency
- critical frequency
- corner frequency

$$\phi = \tan^{-1} \frac{\omega}{a}$$

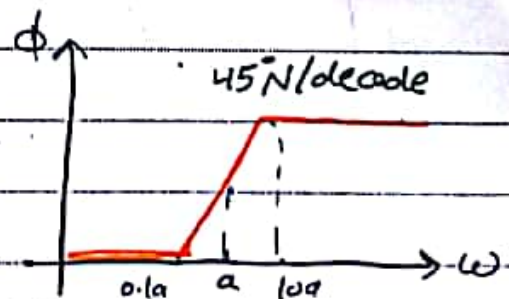
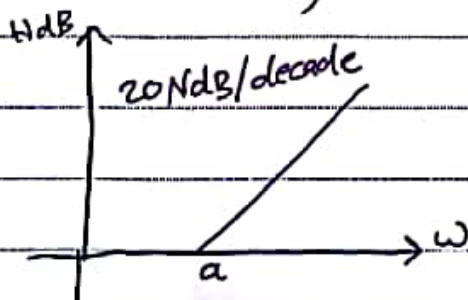
$$\omega \ll a \rightarrow \omega \leq 0.1a \rightarrow \phi = 0$$

$$\omega = a \rightarrow \phi = \tan^{-1} 1 = 45^\circ$$

$$\omega \gg a \rightarrow \omega \geq 10a \rightarrow \phi = 90^\circ$$

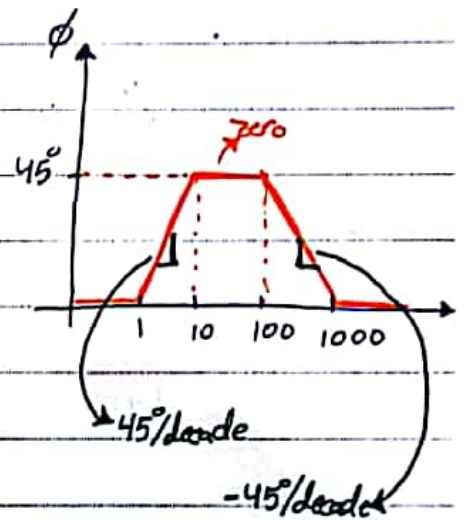
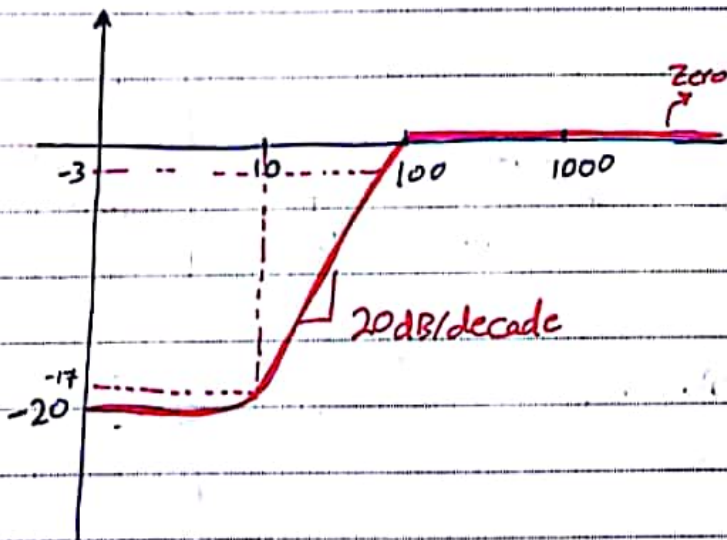
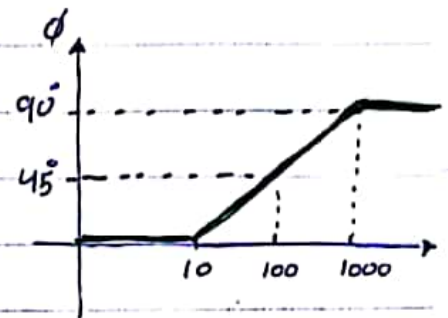
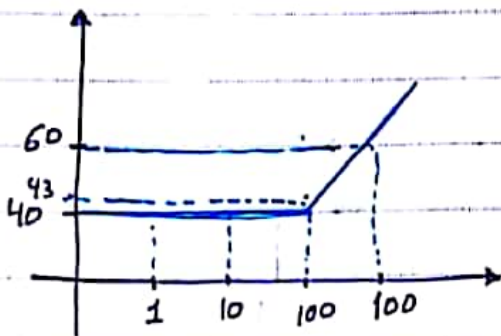
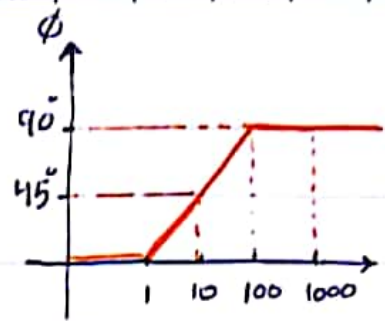
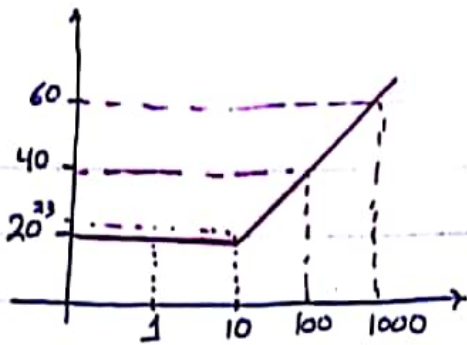


$$\hookrightarrow H(s) = \left(1 + \frac{s}{a}\right)^N$$





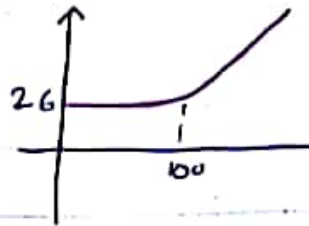
No



$$\phi = \frac{\tan^{-1}\left(\frac{\omega}{10}\right)}{\tan^{-1}\left(\frac{\omega}{100}\right)}$$

$$\phi = \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{100}\right)$$

1.6.0  $H(s) = 20 + 0.25$   
 $20(1 + \frac{s}{100})$



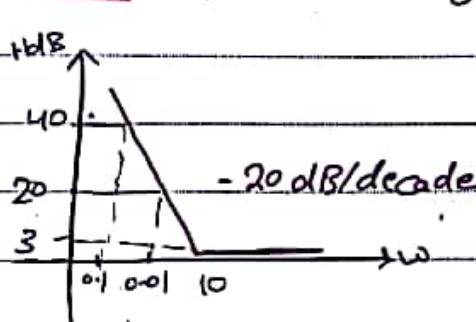
Ex  $H(s) = \frac{s+10}{s}$

$H(s) = 1 + \frac{10}{s}$

$H(j\omega) = 1 - j\frac{10}{\omega} \rightarrow |H(j\omega)| = \sqrt{1 + \frac{10^2}{\omega^2}}$

$HdB = 20 \log \sqrt{1 + \frac{10^2}{\omega^2}}$

$\omega \ll 10 \rightarrow HdB = 20 \log \frac{10}{\omega}$ 
 $\omega = 10 \rightarrow HdB = 20 \log \sqrt{2} = 3dB$ 
 $\omega \gg 10 \rightarrow HdB = 20 \log 1 = 0dB$

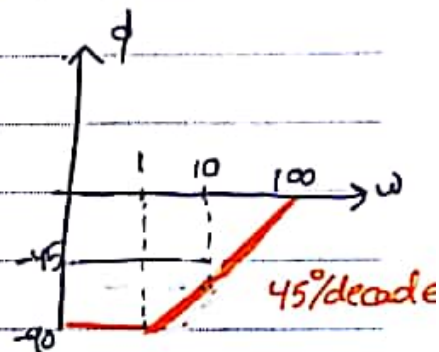


$\phi = \tan^{-1} \frac{10}{\omega}$

$\omega = 10 \rightarrow \phi = -45^\circ$

$\omega \ll 10 \rightarrow \phi = -90^\circ$

$\omega \gg 10 \rightarrow \phi = 0^\circ$



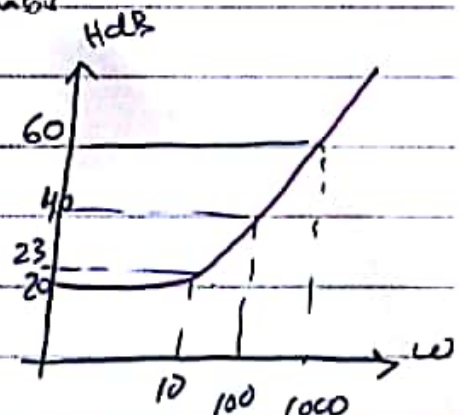
Ex  $H(s) = \frac{s+10}{s+100}$

$10 \sqrt{1 + \frac{\omega^2}{10^2}} \left| \tan^{-1} \frac{\omega}{10} \right|$   
 $100 \sqrt{1 + \frac{\omega^2}{100^2}} \left| \tan^{-1} \frac{\omega}{100} \right|$

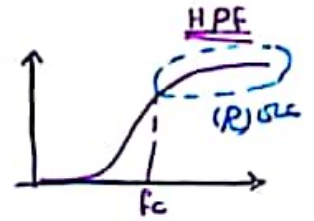
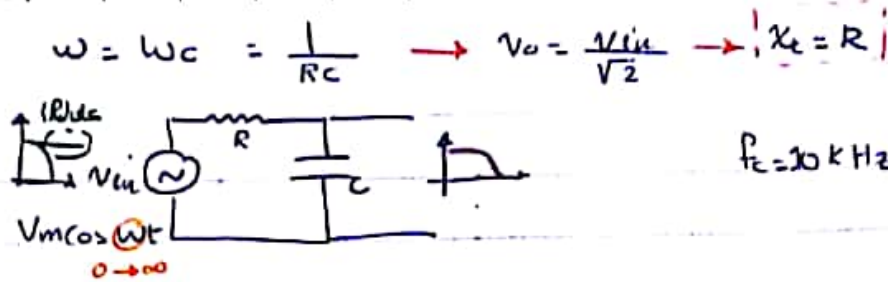
$H(j\omega) = \frac{10+j\omega}{100+j\omega} \rightarrow |H(j\omega)| = 10(1 + j\frac{\omega}{10})$  phase

$|H(j\omega)| = \frac{10 \sqrt{1 + \frac{\omega^2}{10^2}}}{100 \sqrt{1 + \frac{\omega^2}{100^2}}}$

$HdB = 20 \log 10 \sqrt{1 + \frac{\omega^2}{10^2}} - 20 \log 100 \sqrt{1 + \frac{\omega^2}{100^2}}$

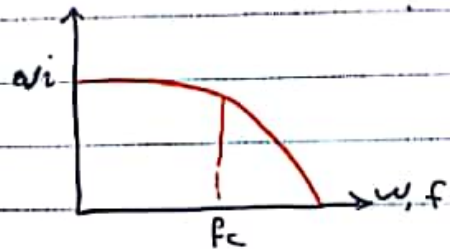
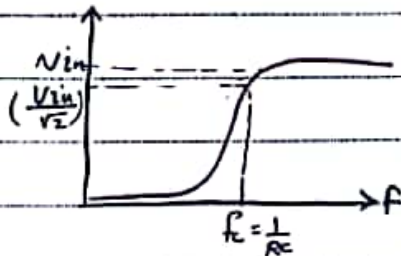


No.

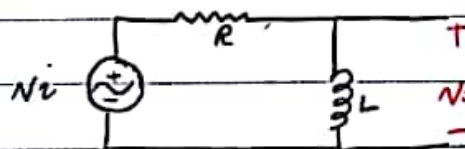


$V_{or} \rightarrow \omega = 0 \rightarrow Z_c = \frac{1}{j\omega C} = \infty \text{ (o.c.)}$   
 $V_{or} = 0, V_{oc} = V_{in}$

$\omega = \infty \rightarrow Z_c = \frac{1}{j\omega C} = 0 \text{ (s.c.)}$   
 $V_{or} = V_{in}, V_{oc} = 0$

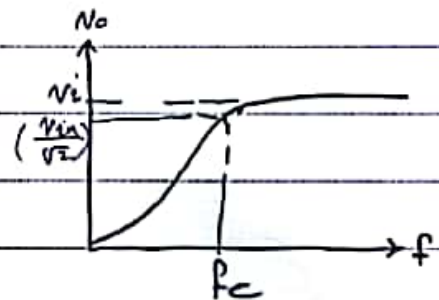


RL



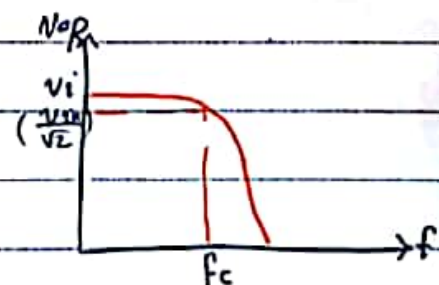
$\omega = 0 \rightarrow X_L = \omega L = 0 \text{ (s.c.)}$   
 $V_{oL} = 0, V_{oR} = V_{in}$

$\omega = \infty \rightarrow X_L = \omega L = \infty \text{ (o.c.)}$   
 $V_{oL} = V_{in}, V_{oR} = 0$



$Z = \frac{L}{R}$   
 $\omega = \frac{R}{L} = \frac{1}{Z}$

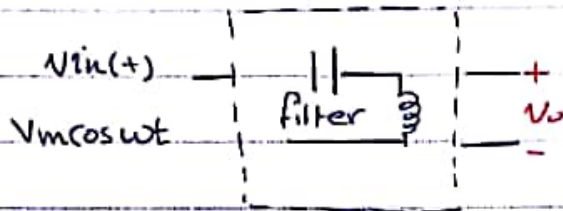
$f_c = \frac{R}{2\pi L}$



في (RC) أفضل من (RL) لأن (L) مقارنة كبيرة واهم مجال  
 كما أن حجمه كبير مقارنة مع (C)




## Sec (16.7) &lt;&lt; Filters &gt;&gt;



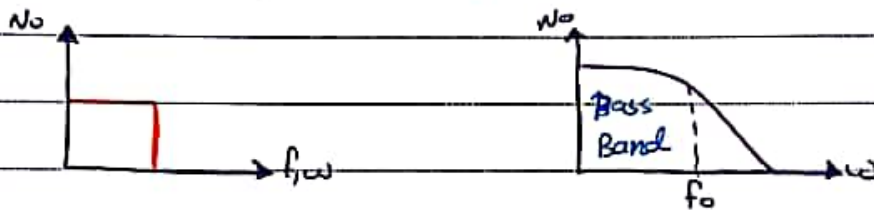
\* Filters types according to Component :-

1) passive Filters ( $R, L, C$ )

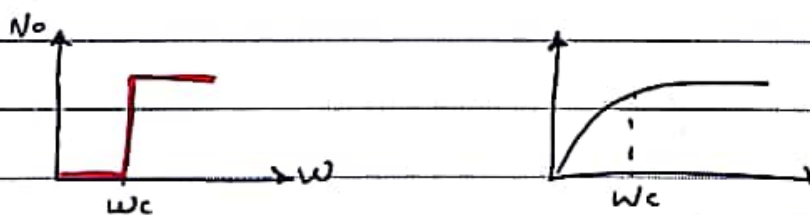
2) Active Filters ( $R, L, C$ ) + Op-Amp   $V_o$

\* Filters types according to Frequency :-

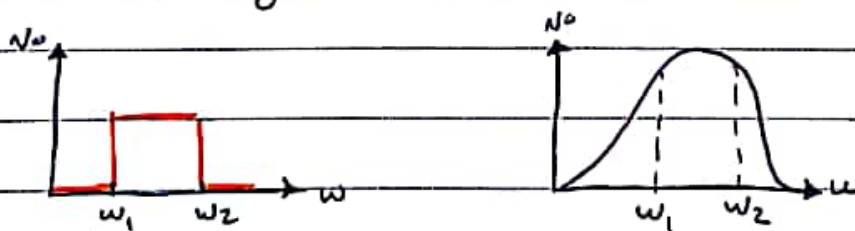
1) Low pass filter (LPF)



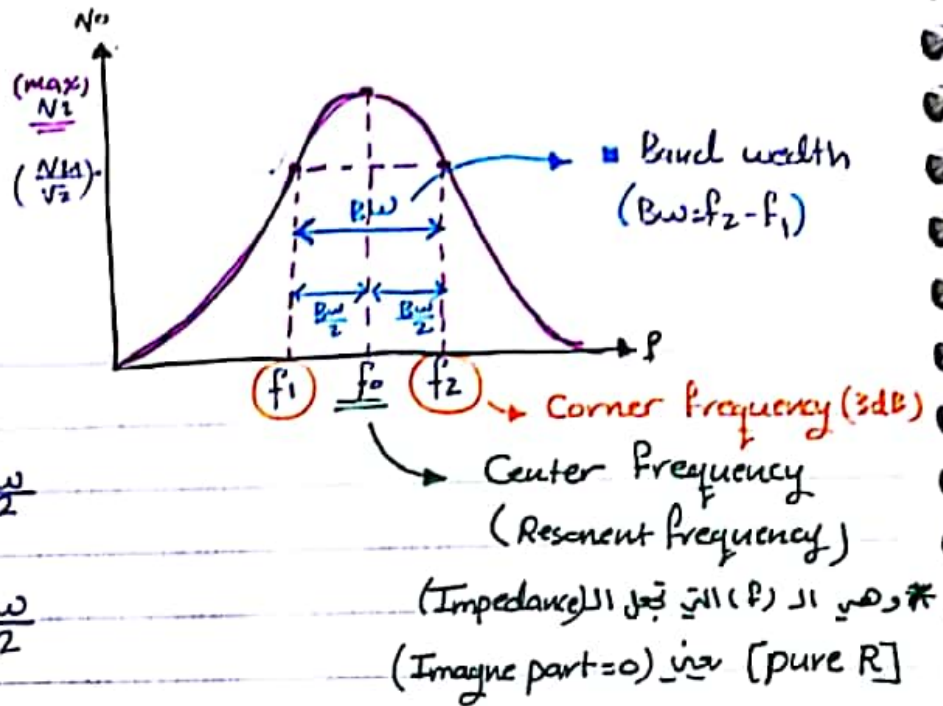
2) High pass Filter (HPF)



3) Band passing Filter (BPF)



BPF



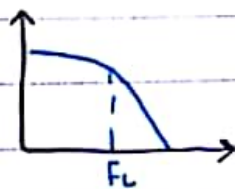
$$f_0 = f_1 + \frac{Bw}{2}$$

(OR)

$$f_0 = f_2 - \frac{Bw}{2}$$

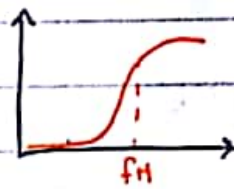
BPF

(1) LPF + HPF



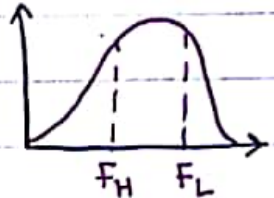
(LPF)

+



(HPF)

=



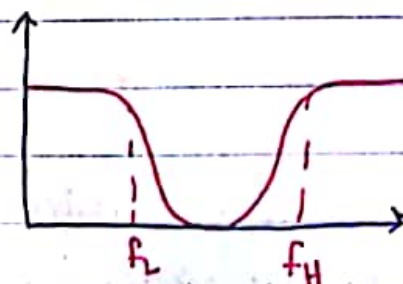
(BPF)

شروط أن تكون  $(f_L > f_H)$

لو كانت  $(f_H > f_L)$  تصبح الرسمة (BSF)

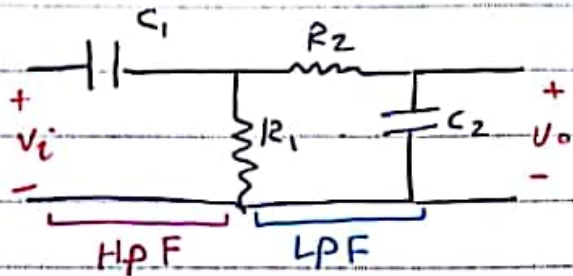
\*Resonant circuit  
 RLC series  
 RLC parallel

(BSF)



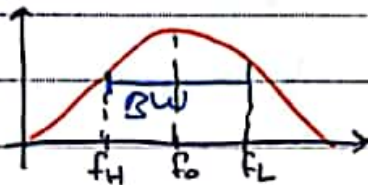


## LPF + HPF



$$F_H = \frac{1}{2\pi R_1 C_1}$$

$$F_L = \frac{1}{2\pi R_2 C_2}$$



$$f_0 = \frac{f_H + f_L}{2}$$

$$BW = f_L - f_H$$

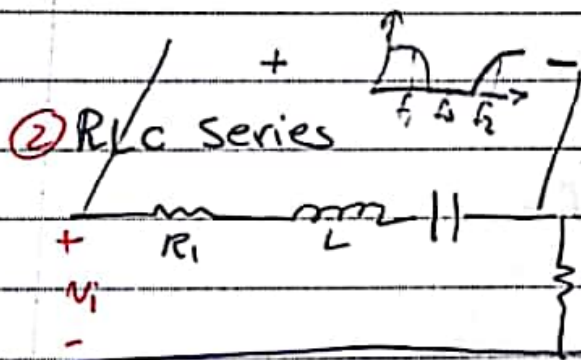
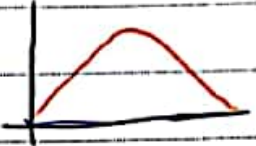
$$f_0 = f_H + \frac{BW}{2} \rightarrow f_L - \frac{BW}{2}$$

∴ LPF + HPF كیفی فرد آنه

$$\begin{aligned} \omega = 0 & \rightarrow X_{C1} = \frac{1}{\omega C_1} = \infty \\ & \rightarrow X_{C2} = 0 \\ & \rightarrow V_o = 0 \end{aligned}$$

$$\begin{aligned} \omega = \infty & \rightarrow X_{C1} = 0 = s.c \\ & \rightarrow X_{C2} = \infty = s.c \\ & \rightarrow V_o = 0 \end{aligned}$$

بدان (صفر)  
وانتهای (باصفر)



Find  $f_0$  resonant frequency

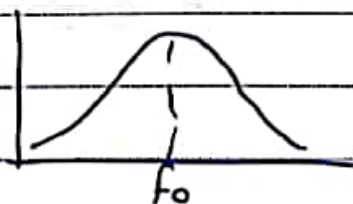
$$Z_{eq} = R_1 + j\omega L + \frac{1}{j\omega C} + R_2$$

$$Z_{eq} = R_1 + R_2 + j(\omega L - \frac{1}{\omega C})$$

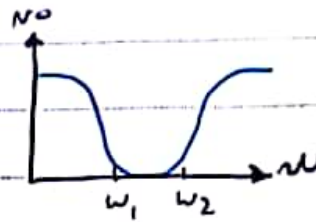
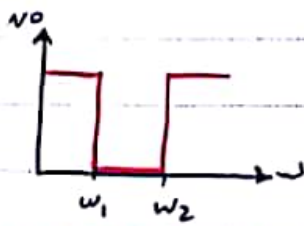
$$\begin{aligned} \omega = 0 & \rightarrow X_C = \infty (o.c) \\ & \rightarrow X_L = 0 (s.c) \\ & \rightarrow V_o = 0 \end{aligned}$$

$$\begin{aligned} \omega = \infty & \rightarrow X_C = 0 s.c \\ & \rightarrow X_L = \infty o.c \\ & \rightarrow V_o = 0 \end{aligned}$$

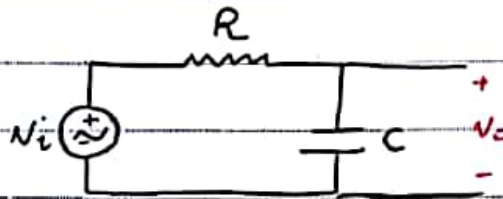
BPF



## 4) Band stop filter (BSF)



RC

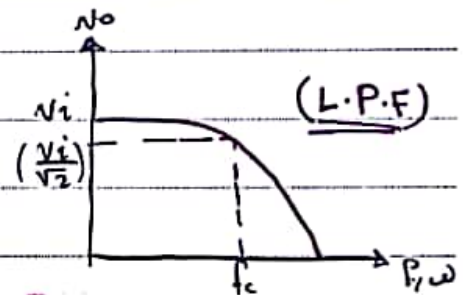


$$V_o = \frac{Z_c}{Z_c + R} \cdot V_{in} \rightarrow \frac{V_o}{V_i} = \frac{Z_c}{Z_c + R} \rightarrow \frac{V_o}{V_i} = \frac{1}{1 + j\omega RC}$$

$$\left| \frac{V_o}{V_i} \right| = \frac{1}{\sqrt{1 + \omega^2 R^2 C^2}} = \frac{1}{\sqrt{1 + \frac{\omega^2}{1/R^2 C^2}}}$$

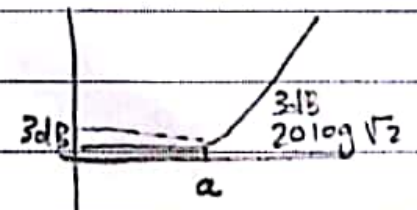
$$\square \underline{\omega = 0} \rightarrow \left| \frac{V_o}{V_{in}} \right| = 1 \rightarrow V_o = V_{in}$$

$$Z_c = \frac{1}{j\omega C} = \infty \text{ (o.c.)}$$



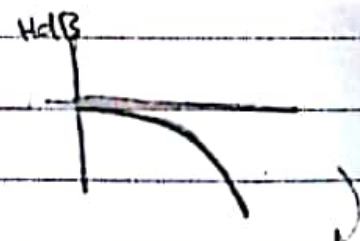
$$\square \underline{\omega = \infty} \rightarrow \left| \frac{V_o}{V_{in}} \right| = 0 \rightarrow V_o = 0$$

$$Z_c = \frac{1}{j\omega C} = 0 \text{ (s.c.)}$$



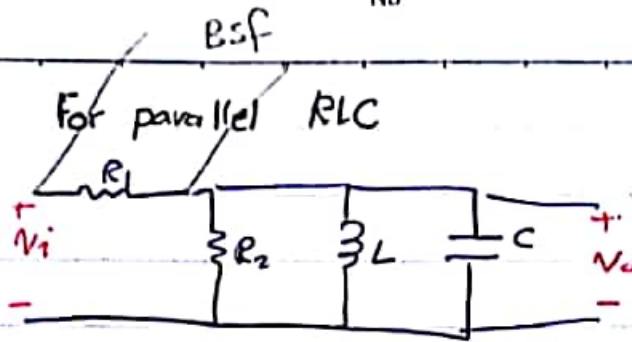
$$\omega_c = \frac{1}{RC} = 2\pi f \rightarrow f_c = \frac{1}{2\pi RC}$$

$$\left| \frac{V_o}{V_{in}} \right| = \frac{1}{\sqrt{2}} \rightarrow V_o = \frac{V_{in}}{\sqrt{2}}$$



(Bode plot) المخطط

No



$$Z_{eq} = R_1 + R_2 \parallel \frac{1}{j\omega C} \parallel j\omega L$$

■ at  $(f=f_0)$  resonant frequency  $\rightarrow Z_{eq} = \text{Real} \rightarrow \text{Imag} = 0$

■  $\omega_0 = \frac{1}{\sqrt{LC}}$

■  $f_0 = \frac{1}{2\pi\sqrt{LC}}$

**BPF**

■  $\omega=0 \rightarrow C \rightarrow \text{o.c.} \left. \begin{array}{l} \text{L} \rightarrow \text{s.c.} \end{array} \right\} V_o=0$      ■  $\omega=\infty \rightarrow C \rightarrow \text{s.c.} \left. \begin{array}{l} \text{L} \rightarrow \text{o.c.} \end{array} \right\} V_o=0$

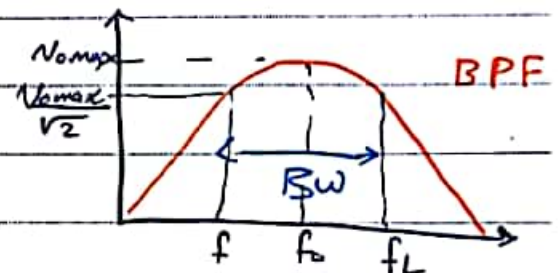
نختبر العنصر

$P_{xc}$

$$Q = \omega_0 RC$$

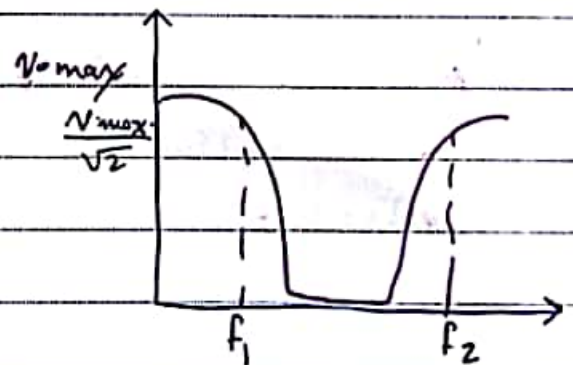
$$BW = \frac{\omega_0}{Q} = \frac{1}{RC} \text{ (rad/s)}$$

$$BW = \frac{f_0}{Q} = \frac{1}{2\pi RC} \text{ (Hz)}$$



■  $f_1 = f_0 - \frac{BW}{2}$

■  $f_2 = f_0 + \frac{BW}{2}$



3dB  $\rightarrow \sqrt{2}$



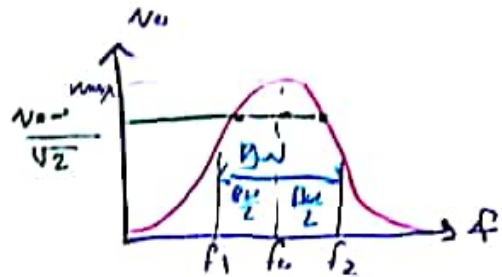
At  $f = f_0$  Resonant

$Z_{eq} = \text{pure Resistance} / \text{Imag} = 0$

$$\omega_0 L - \frac{1}{\omega_0 C} = 0 \rightarrow \boxed{\omega_0 = \frac{1}{\sqrt{LC}}}$$

$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

At  $f = f_0$   $\boxed{Z_{eq} = R_1 + R_2}$



$$\boxed{V_{o\max} = \frac{R_2}{R_1 + R_2} V_{in}}$$

►► Quality Factor (Q)

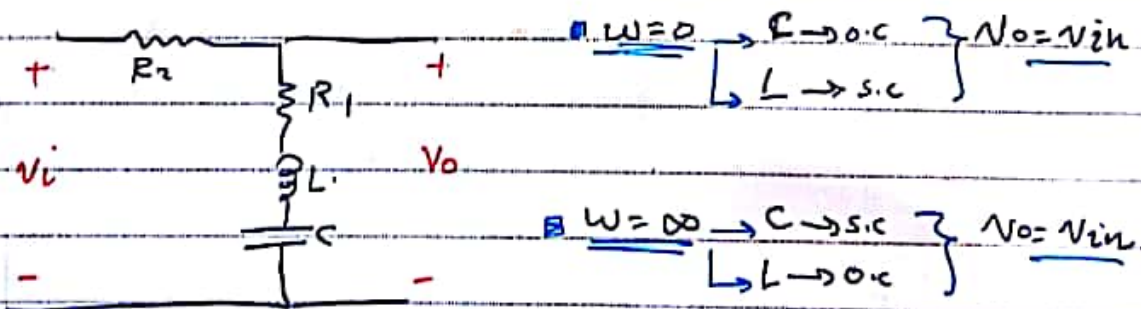
$$Q = \frac{\omega_0 L}{R_{eq}} = \frac{\omega_0 L}{R_1 + R_2}$$

$$BW = \frac{\omega_0}{Q} = \frac{\omega_0}{\frac{\omega_0 L}{R}} = \frac{R}{L} \text{ (rad/s)}$$

$$BW = \frac{f_0}{Q} = \frac{f_0}{\frac{f_0}{2\pi f_0 L}} = \frac{R}{2\pi L} \text{ (Hz)}$$

$$\blacksquare f_1 = f_0 - \frac{BW}{2}$$

$$\blacksquare f_2 = f_0 + \frac{BW}{2}$$



**BSF**

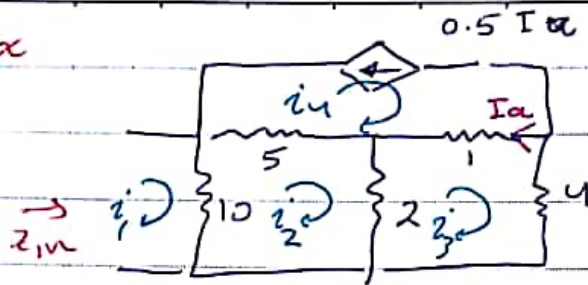
$$\blacksquare \omega_0 = \frac{1}{\sqrt{LC}}$$

$$\blacksquare Q = \frac{\omega_0 L}{R}$$

$$\blacksquare BW = \frac{\omega_0}{Q}$$

No. \_\_\_\_\_

Ex



$$\Delta Z = \begin{bmatrix} 10 & -10 & 0 & 0 \\ -10 & 17 & -2 & -5 \\ 0 & -2 & 7 & -1 \end{bmatrix}$$

$$i_4 = -0.5 I_a = -0.5 (I_4 - I_3)$$

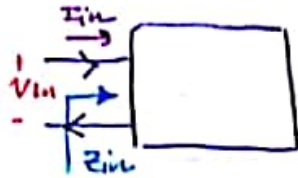
$$i_4 = -0.5 I_4 + 0.5 I_3 \rightarrow \boxed{1.5 I_4 - 0.5 I_3 = 0}$$

$$Z_{in} = 3.7\ \Omega$$



## ❖ Port network

pair of Input and output terminals.



(One port)



(Two port)

$$R_{in} = \frac{V_i}{I_{in}}$$

► Input for one port network using determinant method:

$$a_{11} I_1 + a_{12} I_2 + \dots + a_{1N} I_N = V_1 \rightarrow \text{Mesh ①}$$

$$a_{21} I_1 + a_{22} I_2 + \dots + a_{2N} I_N = V_2 \rightarrow \text{Mesh ②}$$

$$a_{N1} I_1 + a_{N2} I_2 + \dots + a_{NN} I_N = V_N \rightarrow \text{Mesh ④}$$

$$\begin{bmatrix} a_{11} & a_{12} & \dots & a_{1N} \\ a_{21} & a_{22} & \dots & a_{2N} \\ \vdots & \vdots & \ddots & \vdots \\ a_{N1} & a_{N2} & \dots & a_{NN} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

$$I_1 = \frac{\begin{vmatrix} V_2 & a_{12} & \dots & a_{1N} \\ V_N & a_{N2} & \dots & a_{NN} \end{vmatrix}}{\Delta a}$$

► To find  $R_{in}$  → Kill sources

$$\begin{cases} V_s = 0 \rightarrow (\text{S.C}) \\ I_s = 0 \rightarrow (\text{O.C}) \end{cases} \quad \begin{cases} V_2 = V_3 = \dots = V_N = 0 \end{cases}$$

$$I_1 = \frac{\begin{vmatrix} V_1 & a_{12} & \dots & a_{1N} \\ 0 & a_{22} & \dots & a_{2N} \\ 0 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \end{vmatrix}}{\Delta a}$$

$$\frac{\begin{vmatrix} a_{22} & \dots & a_{2N} \\ \vdots & \ddots & \vdots \\ a_{N2} & \dots & a_{NN} \end{vmatrix}}{\Delta a} V_1$$

No.

$$I_1 = \frac{\Delta_{11}}{\Delta a} V_1$$

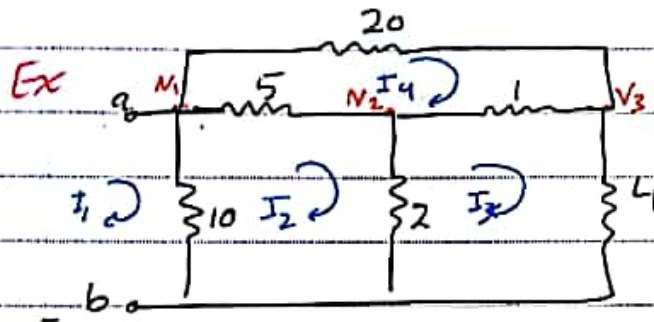
■  $\Delta_{11} \rightarrow$  minor m

$$\boxed{\frac{V_1}{I_1} = \frac{\Delta a}{\Delta_{11}}} = R_{in}$$

$$\blacksquare G_{in} = \frac{\Delta_{11}}{\Delta a} \rightarrow = \frac{\Delta Y}{\Delta \Phi}$$

$$\blacksquare Z_{in} = \frac{\Delta a}{\Delta_{11}}$$

$$\blacksquare Y_{in} = \frac{\Delta Y}{\Delta_{11}}$$



► Find  $R_{in} ??$

$$V_1 = 10(i_1 - i_2) = 10i_1 - 10i_2 \quad \text{Mesh}$$

$$10(i_2 - i_1) + 5(i_2 - i_3) + 2(i_2 - i_3) = 0$$

$$\text{Nodal} \rightarrow \frac{V_1}{10} + \frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{20}$$

$$R_{in} = \frac{\Delta R}{\Delta_{11}}$$

$$\Delta R = \begin{bmatrix} 10 & -10 & 0 & 0 \\ -10 & 17 & -2 & -5 \\ 0 & -2 & 7 & -1 \\ 0 & -5 & -1 & 26 \end{bmatrix}$$

$$= 9680$$

$$\blacksquare R_{in} = \frac{9680}{2778} = \boxed{3.48 \Omega}$$

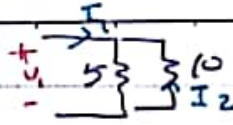
$$\Delta_{11} = \begin{bmatrix} 17 & -2 & -5 \\ -2 & 7 & -1 \\ -5 & -1 & 26 \end{bmatrix}$$

$$= 2778$$

$$\blacksquare G_{in} = \frac{1}{R_{in}} = \boxed{0.347 \Omega}$$

No.

$$Y_{21} = \frac{I_2}{V_1} \big|_{V_2=0}$$



$$Y_{21} = -V_1/10$$

$$Y_{21} = -0.1 \text{ S}$$

$$\Delta Y = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} = \begin{bmatrix} 0.3 & -0.1 \\ -0.1 & 0.5 \end{bmatrix}$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} 0.3 & -0.1 \\ -0.1 & 0.5 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$$I_1 = 0.3 V_1 - 0.1 V_2$$

$$I_2 = -0.1 V_1 + 0.5 V_2$$

■ Z - parameter

■ Impedance-parameter

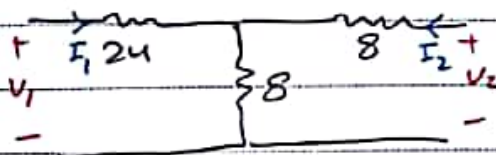
■ open circuit-parameter

$$Z_{11} = \frac{V_1}{I_1} \big|_{I_2=0}$$

$$Z_{12} = \frac{V_1}{I_2} \big|_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1} \big|_{I_2=0}$$

$$Z_{22} = \frac{V_2}{I_2} \big|_{I_1=0}$$

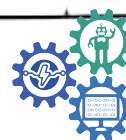


$$Z_{11} = \frac{V_1}{I_1} \big|_{I_2=0} = 24 + 8 = 32 \Omega$$

(open circuit)

■  $Y_{11} = \frac{1}{(8 \parallel 8 + 24)} \Rightarrow$  (parameter)  $Z$  ←  $Z$  تسمى عن  $Y$  وليسا على نفس في ال (parameter)

(short circuit)

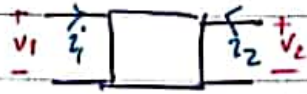


EICoM  
ELECTRICAL COMPUTER MECHANISMS

smile for life

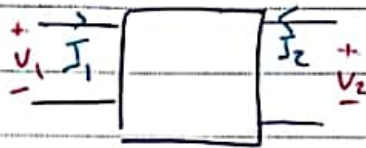


No.



$$Y = \frac{I}{V}$$

- Y - parameter
- Admittance - parameter
- Short circuit

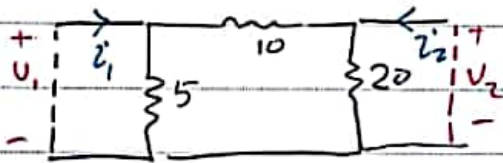


$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$

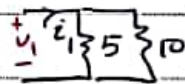
$$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0}$$

$$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}$$

$$Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}$$

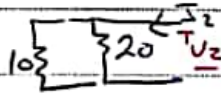


$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$$



$$Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0} = \frac{1}{5 \parallel 10} = 0.3 \text{ S}$$

$$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0}$$



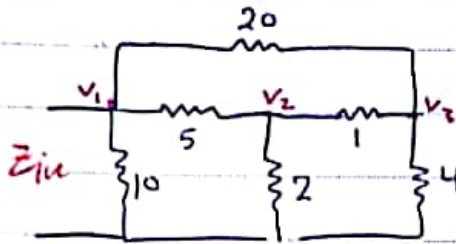
$$Y_{22} = \frac{I_2}{V_2} \Big|_{V_1=0} = \frac{1}{10 \parallel 20} = 0.15 \text{ S}$$

$$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0}$$



$$Y_{12} = \frac{I_1}{V_2} \Big|_{V_1=0} = -\frac{V_2/10}{V_2} = -0.15$$

Ex



$$\frac{V_1}{10} + \frac{V_1 - V_2}{5} + \frac{V_1 - V_3}{20}$$

$$\left(\frac{1}{10} + \frac{1}{5} + \frac{1}{20}\right) - \frac{V_2}{5} - \frac{V_3}{20} = 0$$

$$\Delta Y = \begin{bmatrix} \frac{1}{10} + \frac{1}{5} + \frac{1}{20} & (-\frac{1}{5}) & (-\frac{1}{20}) \\ (-\frac{1}{5}) & (\frac{1}{2} + \frac{1}{5} + 1) & -1 \\ -\frac{1}{20} & -1 & (\frac{1}{4} + 1 + \frac{1}{20}) \end{bmatrix}$$

$$\begin{bmatrix} 0.3 & -0.2 & -0.05 \\ -0.2 & 1.7 & -1 \\ -0.05 & -1 & 1.3 \end{bmatrix} = 0.347 \text{ Siemens}$$

$$\Delta_{11} = \begin{bmatrix} 1.7 & -1 \\ -1 & 1.3 \end{bmatrix} = 1.21$$

$$Y = \frac{\Delta Y}{\Delta_{11}} = 0.287 \text{ S}$$

$$Z = \frac{1}{Y} = 3.48$$

هذه الطريقة عندما يكون كل ال (source) (independent)



$$\square Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0} = 8 + 8 = 16 \Omega$$

$$\square Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0} = \frac{I_2 \cdot 8}{I_2} = 8 \Omega \quad V_{20} \text{ من } I_{20} = 11$$

$$\square Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0} = \frac{8 I_1}{I_1} = 8 \Omega$$

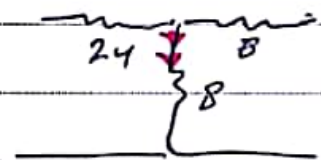
$$\Delta Z = \begin{bmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{bmatrix} = \begin{bmatrix} 32 & 8 \\ 8 & 16 \end{bmatrix}$$

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 32 & 8 \\ 8 & 16 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

$$V_1 = 32 I_1 + 8 I_2$$

$$V_2 = 8 I_1 + 16 I_2$$

الاشارة موجبة لان التيارات ايضا الاشارة



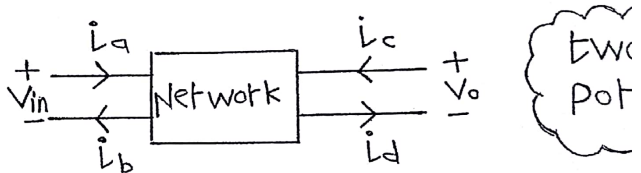
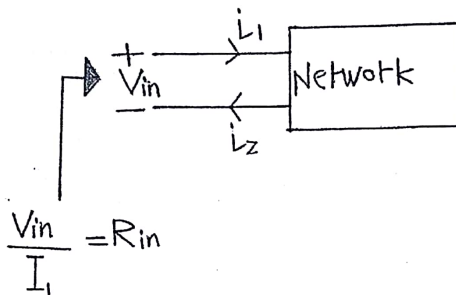
$$Y_{22} = \frac{1}{(24/8) + 8} \rightarrow \frac{1}{16}$$

$$Y_{22} \neq \frac{1}{Z_{22}} \neq 16$$

# Chapter (17) : Two - Port Network.

## 17.1 : ONE Port Networks :

Port  $\rightarrow$  pair of terminals.



### \* One port network :

$$R_{in} = \frac{V_{in}}{I_{in}}$$

$$V_1 = a_{11}X + a_{12}Y + a_{13}Z$$

$$V_2 = a_{21}X + a_{22}Y + a_{23}Z$$

$$V_3 = a_{31}X + a_{32}Y + a_{33}Z$$

(1)

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix}$$

$$K = \frac{\Delta a}{\begin{vmatrix} V_1 & a_{12} & a_{13} \\ V_2 & a_{22} & a_{23} \\ V_3 & a_{32} & a_{33} \end{vmatrix}}$$

$$= \frac{\Delta a}{\begin{vmatrix} a_{11} & V_1 & a_{13} \\ a_{21} & V_2 & a_{23} \\ a_{31} & V_3 & a_{33} \end{vmatrix}}$$

$$= \frac{\Delta a}{\begin{vmatrix} a_{11} & a_{12} & V_1 \\ a_{21} & a_{22} & V_2 \\ a_{31} & a_{32} & V_3 \end{vmatrix}}$$

Mesh Equation :-

$$Z_{11} I_1 + Z_{12} I_2 + \dots + Z_{1N} I_N = V_1$$

$$Z_{21} I_1 + Z_{22} I_2 + \dots + Z_{2N} I_N = V_2$$

$$Z_{N1} I_1 + Z_{N2} I_2 + \dots + Z_{NN} I_N = V_N$$

Kill Sources (without dependent source).

$$V_2 = V_3 = \dots = V_N = 0$$

$$\underbrace{\begin{bmatrix} Z_{11} & Z_{12} & \dots & Z_{1N} \\ Z_{21} & Z_{22} & \dots & Z_{2N} \\ Z_{31} & Z_{32} & \dots & Z_{3N} \\ \vdots & \vdots & \ddots & \vdots \\ Z_{N1} & Z_{N2} & \dots & Z_{NN} \end{bmatrix}}_{\Delta Z} \begin{bmatrix} I_1 \\ I_2 \\ \vdots \\ I_N \end{bmatrix} = \begin{bmatrix} V_1 \\ V_2 \\ \vdots \\ V_N \end{bmatrix}$$

(3)



$$= \frac{\begin{bmatrix} V_1 & Z_{12} & \dots & Z_{1N} \\ V_2 & Z_{22} & & \\ \vdots & \vdots & & \\ V_N & Z_{NZ} & \dots & Z_{NN} \end{bmatrix}}{\Delta Z}$$

$$= \frac{\begin{bmatrix} Z_{12} & Z_{13} & \dots & Z_{1N} \\ \vdots & & & \\ Z_{NZ} & \dots & \dots & Z_{NN} \end{bmatrix}}{\Delta Z}$$

$\Rightarrow$  minor  $\Delta_{11}$

$$I_1 = \frac{V_1 \Delta_{11}}{\Delta Z}$$

$$\frac{V_1}{I_1} = \frac{\Delta Z}{\Delta_{11}} = Z_{in}$$

Mesh .

$$Y_{in} = \frac{1}{Z_{in}}$$

$$Y_{in} = \frac{\Delta Y}{\Delta_{11}}$$

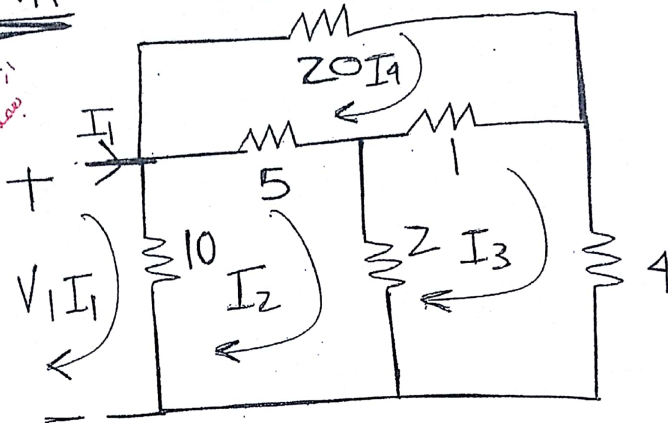
for Nodal analysis:

$$\frac{\Delta Y}{\Delta_{11}} = Y_{in}$$

EXA-

(Source) في 1:1  
kill  
سوال 1:1

s.c. ③  
o.c. ①  
سوال 1:1



find Input Resistance for one port using determinant theorem  
\* By mesh:-

Sol.  $Z_{in} = \frac{\Delta Z}{\Delta I_1}$

$\Delta Z = \begin{vmatrix} I_1 & I_2 & I_3 & I_4 \\ 10 & -10 & 0 & 0 \\ -10 & 17 & -2 & -5 \\ 0 & -2 & 7 & -1 \\ 0 & -5 & -1 & 20 \end{vmatrix} = 9680 \Omega$

$Z_{ij} \Rightarrow i=j \Rightarrow \text{positive}$   
 $i \neq j \Rightarrow \text{negative}$

(15)

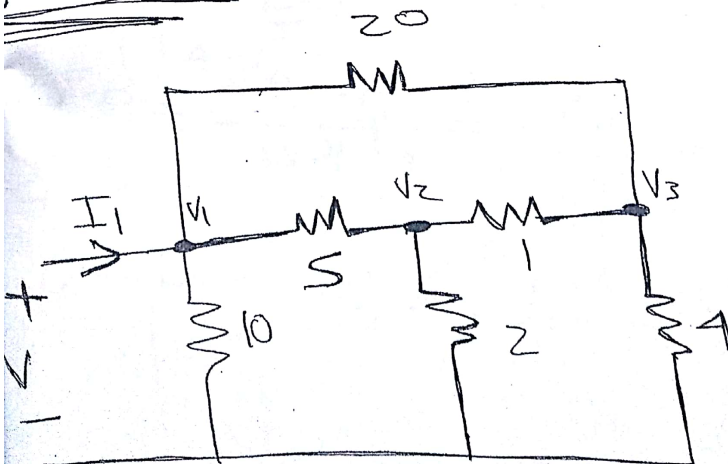
$$= \begin{bmatrix} 17 & -2 & -5 \\ -2 & 7 & -1 \\ -5 & -1 & 26 \end{bmatrix}$$

$$= 2778$$

$$= 3.48 \, \Omega.$$

$$= 0.287 \, S.$$

Nodal:



$$\Delta Y = \begin{bmatrix} V_1 & V_2 & V_3 \\ \frac{1}{10} + \frac{1}{5} + \frac{1}{20} & -\frac{1}{5} & -\frac{1}{20} \\ -\frac{1}{5} & \frac{1}{1} + \frac{1}{5} + \frac{1}{2} & -1 \\ -\frac{1}{20} & -1 & \frac{1}{20} + \frac{1}{4} + 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.35 & -0.2 & -0.05 \\ -0.2 & 1.7 & -1 \\ -0.05 & -1 & 1.3 \end{bmatrix}$$

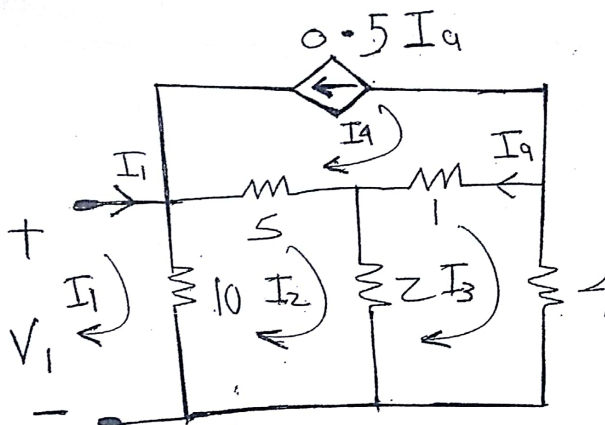
$$= 0.347$$

$$\Delta_{11} = \begin{bmatrix} 1.7 & -1 \\ -1 & 1.3 \end{bmatrix} = 1.2$$

$$Y_{in} = \frac{\Delta Y}{\Delta_{11}} = \frac{0.347}{1.2} = 0.289 \text{ S}$$

$$Z_{in} = \frac{1}{Y} = 3.48 \text{ } \Omega \cdot \# \quad (7)$$





نودال ؟

$$I_1 = -0.5 I_4$$

$$I_2 = (I_4 - I_3)$$

$$0.5 I_3 + 1.5 I_4 = 0$$

$$= \begin{bmatrix} 10 & -10 & 0 & 0 \\ -10 & 17 & -2 & -5 \\ 0 & -2 & 7 & -1 \\ 0 & 0 & 0.5 & 1.5 \end{bmatrix} = 590 \Omega$$

(8)

$$\Delta_{11} = \begin{bmatrix} 17 & -2 & -5 \\ -2 & 7 & -1 \\ 0 & -0.5 & 1.5 \end{bmatrix} = 15^\circ$$

$$Z_{in} = 3.71 \Omega.$$


---

## 17.2) Two port network:



Y-Parameters / Short circuit parameters  
/ Admittance.

$$I_1 = Y_{11} V_1 + Y_{12} V_2$$

$$I_2 = Y_{21} V_1 + Y_{22} V_2$$

$$\begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} \\ Y_{21} & Y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

(10)

$$Y_{11} = \left. \frac{I_1}{V_1} \right|_{V_2=0}$$

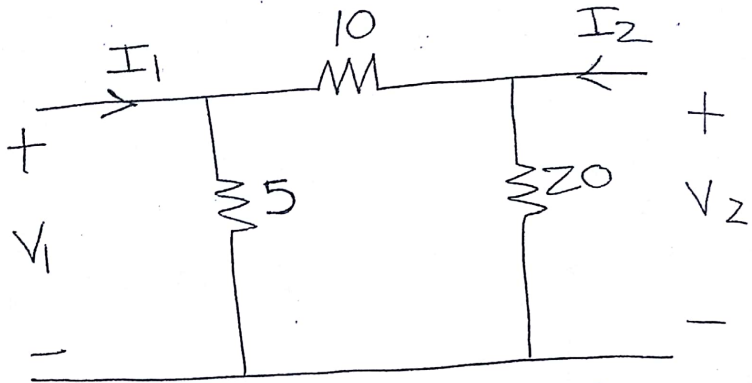
$$Y_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0}$$

$$Y_{21} = \left. \frac{I_2}{V_1} \right|_{V_2=0}$$

$$Y_{22} = \left. \frac{I_2}{V_2} \right|_{V_1=0}$$

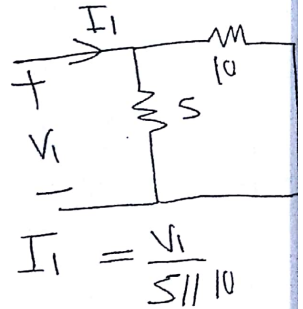


EXA:



find Y-parameters  
Short circuit parameters

Sol.  $Y_{11} = \frac{I_1}{V_1} \Big|_{V_2=0}$



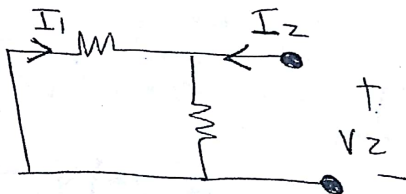
$$I_1 = \frac{V_1}{5 \parallel 10}$$

$$\frac{I_1}{V_1} = Y_{11} = \frac{1}{5 \parallel 10} = 0.3 \text{ S.}$$

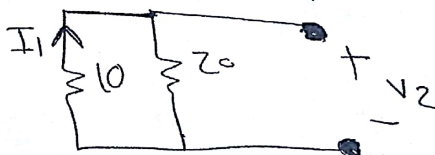
$$z = \left. \frac{I_2}{V_2} \right|_{V_1=0} = \frac{1}{z_0 // 10}$$

$$z = (z_0 // 10) I_2 \rightarrow \frac{I_2}{V_2} = \frac{1}{z_0 // 10} = Y_{22} = 0.15 \text{ S}$$

$$z_{12} = \left. \frac{I_1}{V_2} \right|_{V_1=0}$$

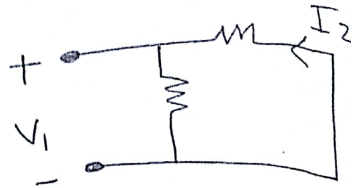


$$= \frac{-V_2}{10}$$



$$z_{12} = \frac{-1}{10} = Y_{12} = -0.1$$

$$Y_{21} = \frac{I_2}{V_1} \Big|_{V_2=0}$$



$$I_2 = \frac{-V_1}{10}$$

$$\frac{I_2}{V_1} = -\frac{1}{10} = -0.1 = Y_{21} \quad \#$$

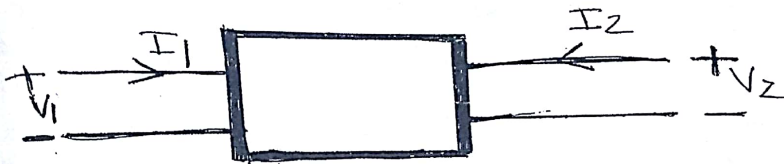
$$\mathbf{Y} = \begin{bmatrix} 0.3 & -0.1 \\ -0.1 & 0.15 \end{bmatrix}$$

$$I_1 = 0.3 V_1 - 0.1 V_2$$

$$I_2 = -0.1 V_1 + 0.15 V_2$$

(1)

4) Impedence,  
 $Z$ , open circuit  
 parameters



$$V_1 = Z_{11} I_1 + Z_{12} I_2$$

$$V_2 = Z_{21} I_1 + Z_{22} I_2$$

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}$$

$$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}$$

$$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}$$

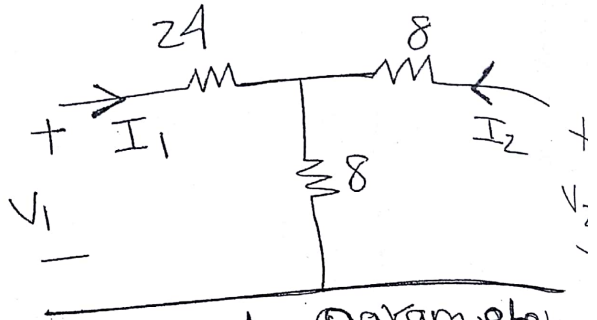
$$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0}$$

4  
 سیم  
 ها  
 بسته  
 است

(15)



EXA:



find open ckt parameter  
(Z parameter) ? :-

Sol.

$$Z_{11} = \frac{V_1}{I_1} \Big|_{I_2=0}$$

$$= 24 + 8 = \underline{\underline{32}}$$

or;

$$V_1 = I_1 (24 + 8)$$

$$\frac{V_1}{I_1} = Z_{11} = \underline{\underline{32 \Omega}}$$

$$Z_{22} = \frac{V_2}{I_2} \Big|_{I_1=0} = 8 + 8 = 16 \Omega$$

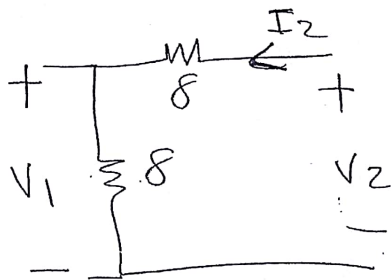
$$\frac{V_2}{I_2} = Z_{22} = 16 \Omega \quad (11)$$

$$V_2 = I_2 (8+8) = 16 \Omega$$

$$Z_{12} = \frac{V_1}{I_2} \Big|_{I_1=0}$$

$$V_1 = 8 I_2$$

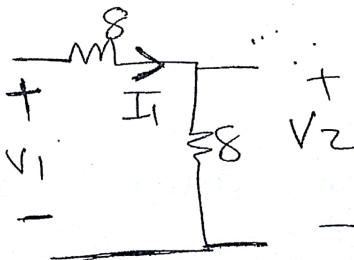
$$\frac{V_1}{I_2} = Z_{12} = 8 \Omega$$



$$Z_{21} = \frac{V_2}{I_1} \Big|_{I_2=0}$$

$$V_2 = 8 I_1$$

$$\frac{V_2}{I_1} = Z_{21} = 8 \Omega$$



$$Z = \begin{bmatrix} 32 & 8 \\ 8 & 16 \end{bmatrix}$$

$$V_1 = 32 I_1 + 8 I_2$$

$$V_2 = 8 I_1 + 16 I_2$$

$$Z_{\text{parameter}} \neq \frac{1}{Y_{\text{parameter}}}$$

\* Short cct.  $\rightarrow Y_{\text{parameter}}$ .

\* Open cct.  $\rightarrow Z_{\text{parameter}}$ .

z port ✓

filter ✓

3-phase ✓

S ✓

(1)