ElCoM
ELECTRICAL COMPUTER MECHATRONICS

تلخيص

ديناميكا واهتزازات

للطالبة المبدعة
تولين صبرة

إرادة - ثقة - تغيير

Dynamics and Vibration

- **Chapter (2):**

-When the acceleration is not constant:

$$v = \frac{ds}{dt} \ldots\ldots (1)$$

$$a = \frac{dv}{dt} \ldots\ldots (2)$$

$$a = \frac{d^2s}{dt^2} \ldots\ldots (3)$$

$$a.ds = v.dv \ldots\ldots (4)$$

-At constant acceleration:

$$v = v_o + a_c t \ldots\ldots (5)$$

$$v^2 = v_o^2 + 2a_c(s - s_o) \ldots\ldots (6)$$

$$s = s_o + v_o t + \frac{1}{2}a_c t^2 \ldots\ldots (7)$$

-Rectangular coordinates (x-y):

$$\vec{r} = xi + yj$$

$$\dot{\vec{r}} = \dot{x}i + \dot{y}j$$

$$\ddot{\vec{r}} = \ddot{x}i + \ddot{y}j$$

-Projectile motion:

*If wind is negligible:

$For\ \uparrow +y, \downarrow -g$:

$$a_x = 0$$

$$v_x = v_{x,o}$$

$x = x_o + v_{x,o}t$

$a_y = -g$

$v_y = v_{y,o} - gt$

$v_y^2 = v_{y,o}^2 - 2g(y - y_o)$

$y = y_o + v_{y,o} - \frac{1}{2}gt^2$

-Circular motion:

$v = r\dot{\theta} = r\omega$

$a_t = \dot{v} = r\ddot{\theta} = r\alpha$

$a_n = \frac{v^2}{r} = r\dot{\theta}^2 = r\omega^2$

$\dot{\theta} = \omega \rightarrow angular\ speed$

$\ddot{\theta} = \alpha \rightarrow angular\ acceleration$

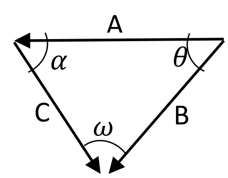
-Relative motion:

$\overrightarrow{r_A} = \overrightarrow{r_B} + \overrightarrow{r_{A,B}}$

$\overrightarrow{v_A} = \overrightarrow{v_B} + \overrightarrow{v_{A,B}}$

$\overrightarrow{a_A} = \overrightarrow{a_B} + \overrightarrow{a_{A,B}}$

*Sine law:



$\frac{A}{sin\omega} = \frac{B}{sin\alpha} = \frac{c}{sin\theta}$

*Cosine law:

$C^2 = A^2 + B^2 - 2ABcos\theta$

#Note: To change from (rpm) to (m/s) multiply by ($\frac{2\pi}{60}$)

-Constrained motion of connected particles:

$L_{total} = total\ length\ of\ rope$

$L_{total} = S_A + S_B + L_{pully}$ (In general)

$L_{total} = constant,\ \ L_{pully} = constant$

-Friction:

$F_f = ma_x$

$F_s = \mu_s N$ Static friction

$F_k = \mu_k N$ Kinetic friction

$\sum F_x = \sum F_y = 0$ (At rest)

$\sum F_x = ma_x, \sum F_y = ma_y$ (At motion)

- **Chapter (3):**

-Rectilinear motion:

$\sum F_x = ma_x$

$\sum F_y = ma_y$

$\sum F_z = ma_z$

$a = a_x i + a_y j + a_z k$

-Work and Kinetic Energy

a) Work associated with a constant force

$U_{1\to2} = Pcos\alpha(x_2 - x_1), \begin{cases} +ve, P\ with\ x \\ -ve, P\ opposite\ x \end{cases}$

b) Work associated with a spring force

$U_{1\to2} = \frac{1}{2}k(x_1^2 - x_2^2)$, opposite of body direction

$c)\ Work\ associated\ with\ weight$

$$U_{1\to2} = mg(y_1 - y_2), \begin{cases} y_2 > y_1, -ve, body\ rises \\ y_2 < y_1, +ve, body\ falls \end{cases}$$

$$Kinetic\ energy(T) = \frac{1}{2}m(v_2^2 - v_1^2)$$

$$Work - Energy\ equation\ \to\ T_1 + U_{1\to2} = T_2$$

$$Power = F.v = \frac{U_{1\to2}}{t}$$

$$Efficiency = \frac{P_{out}}{P_{in}}$$

Potential Energy

1) Gravitational potential energy

$$V_g = mgh$$

2) Elastic potential energy

$V_e = \frac{1}{2}kx^2$, always +ve

Work-Energy equation

$$T_1 + V_{g1} + V_{e1} + U'_{1\to2} = T_2 + V_{g2} + V_{e2}$$

-Curvilinear motion:

Rectangular coordinates

$$\sum F_x = ma_x$$

$$\sum F_y = ma_y$$

n-t coordinates

$$\sum F_t = ma_t \to a_t = r\alpha$$

$$\sum F_n = ma_n \to a_n = \frac{v^2}{r}$$

*Normal Force (N)=0 when there is no contact between the body and the surface

If (a_t) is not constant we can extract this equation

$s = r\theta$

$ds = rd\theta$

Also $vdv = a_t ds \rightarrow vdv = a_t r d\theta$

- **Chapter (5):**

-Rotation

$$\omega = \frac{d\theta}{dt} = \dot{\theta}$$

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \ddot{\theta}$$

$\omega d\omega = \alpha d\theta$

If $\alpha = \alpha_{const}$ →constant angular acceleration:

$\omega = w_o + \alpha_c t$

$\omega^2 = \omega_o^2 + 2\alpha_c(\theta - \theta_o)$

$$\theta = \theta_o + \omega_o t + \frac{1}{2}\alpha_c t^2$$

-Rotation about fixed axis:

$v = r\omega$

$a_t = r\alpha$

$$a_n = r\omega^2 = \frac{v^2}{r} = v\omega$$

*using vector notation (by using right hand rule):

$\vec{v} = \vec{\omega} \times \vec{r}$

$\vec{\omega}$ *normal to the plane of rotation*

$\overrightarrow{a_n} = \vec{\omega} \times (\vec{\omega} \times \vec{r}) = \vec{\omega} \times \vec{v} = -r\omega^2$

$\overrightarrow{a_t} = \vec{\alpha} \times \vec{r}$

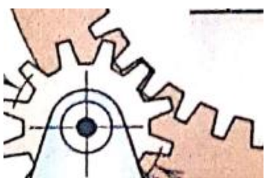
$\vec{\omega}$: *is increasing* $\rightarrow \vec{\alpha}$ *is in the direction of* $\vec{\omega}$

$\vec{\omega}$: *is decreasing* $\rightarrow \vec{\alpha}$ *is in the opposite direction of* $\vec{\omega}$

$\vec{\omega}$: *is increasing* $\rightarrow a_t$ *is in the direction of* $\vec{v}$

$\vec{\omega}$: *is decreasing* $\rightarrow a_t$ *is in the opposite direction of* $\vec{v}$

-Gear ratio:



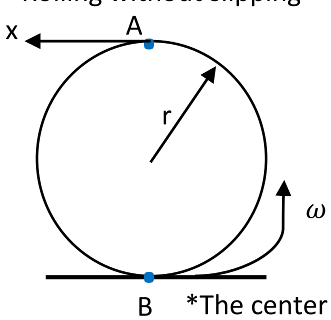
$v_1 = v_2, So: r_1\omega_1 = r_2\omega_2 \rightarrow \omega_1 = \frac{r_2}{r_1}\omega_2$

$\frac{r_2}{r_1} \rightarrow Gear\ ratio$

*Use the same method to find the angular acceleration

-Absolute motion:

Rolling without slipping



*The center is not fixed

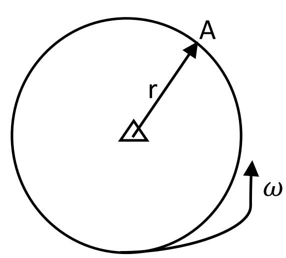
At center

$v = r\omega \neq zero$

At point B, At Point A

v=0 $v = 2r\omega$, 2r=diameter

Rotation about fixed axes



At center

$v = r\omega = 0$

At point A

$v = r\omega$

-Relative motion:

Relative velocity and relative acceleration

$\overrightarrow{v_A} = \overrightarrow{v_B} + \overrightarrow{v_{A,B}}$

$v_{A,B} = r\omega = \vec{\omega} \times \vec{r}$

$\overrightarrow{a_A} = \overrightarrow{a_B} + \overrightarrow{a_{A,B}}$

$\overrightarrow{a_{A,B}} = (\overrightarrow{a_{A,B}})_n + (\overrightarrow{a_{A,B}})_t$

$(\overrightarrow{a_{A,B}})_t = \vec{\alpha} \times \vec{r}$

$(\overrightarrow{a_{A,B}})_n = \vec{\omega} \times (\vec{\omega} \times \vec{r}) = \vec{\omega} \times \vec{v} = -r\omega^2$

- **Chapter (6):**

$I = mk^2$

$\sum F_x = ma_{G,x}$

$\sum F_y = ma_{G,y}$-

$+\curvearrowleft \sum M = I_G\alpha \ \rightarrow for\ rotational\ motion$

$+\curvearrowleft \sum M = 0 \ \rightarrow for\ rectilinear\ motion$

*For curvilinear motion

$\sum F_n = ma_{G,n}$

$\sum F_t = ma_{G,t}$

$+\curvearrowleft \sum M = 0 \rightarrow where\ \alpha = o, \omega = 0$

-Fixed axis rotation

$+\curvearrowleft \sum M_G = I_G\alpha$

$\sum F_n = ma_{G,n}$

$\sum F_t = ma_{G,t}$

$+\curvearrowleft \sum M_o = I_o\alpha = (I_G + md^2)\alpha$

$a_n = r\omega^2$

$a_t = r\alpha$

-Work-Energy relation

$$U_{1\to2} = \int f\, dr$$

$$U_{1\to2} = \int M\, d\theta$$

$$U'_{1\to2} = F(x - x_o) + M(\theta - \theta_o)$$

$$Kinetic\ energy = T = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$Potential\ energy = V_e + V_g$$

$$T_1 + V_{g1} + V_{e1} + U'_{1\to2} = T_2 + V_{g2} + V_{e2}$$

$$Power = P = Fv + m\omega$$