ElCoM
ELECTRICAL COMPUTER MECHATRONICS

ديناميكا واهتزازات

د.أيات الجراح

للطالب المبدع

حمزة اسماعيل

إرادة - ثقة - تغيير

Subject Date No.

* Dynamics and Vibrations :-

* Ch 2 :- Kinematics of Particles :-

* Definitions :-

1. Dynamics :- deals with motion and It's effect on body.
   → Kinematics or → Kinetics.
2. static :- deals with bodies at rest and equilibruim.

[A] Kinematics :
→ displacement.
→ Velocity.
→ acceleration
⇒ with out force.

[B] Kinetics : force and mass effect on motion
→ Vibration.

3. Particle : Point or body in Zero dimensions.
4. Rigiod body :- distance between Points before and after motion the same.

* The motion :-

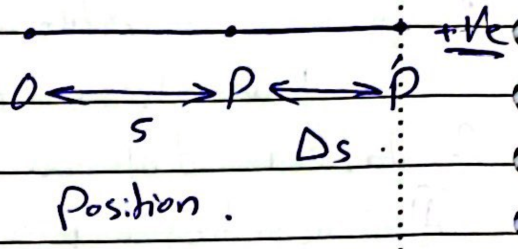
1. Rectilinear → straight line motion.
2. Curvilinear → motion along a curve Path in a single Plane.
3. Rotational. → motion along a circule Path.

* Dynamics and Vibrations :-

* Ch2 :- Kinematics of Particles :-

* Rectilinear motion :-

1- distance = +Ve scalar quantity that represents the length (total length) of Path over which the Particle travels (s).



2 displacement :- Vector quantity that represents the change in Particles Position (Δs).
+ Ve → to the right.
- Ve → to the left.

3. Speed :- +Ve scalar quantity that represents

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{\text{total distance } (s)}{\text{total time } (t)} = \boxed{m/s}$$

4- Velocity :- Vector quantity that represents

$$\text{Velocity} = \frac{\text{displacement}}{\text{time}} = \boxed{m/s}$$

Velocity → +Ve or -Ve.

Subject Date No.

Dynamics and Vibrations :-

Ch2 :- Kinematics of particles :-

Rectilinear motion

1- At non-constant acceleration and at constant :-

Velocity $\Rightarrow$ $\boxed{V = \frac{ds}{dt}}$. السرعة = مشتقة المسافة

acceleration $\Rightarrow$ $\boxed{a = \frac{dV}{dt}}$ التسارع = مشتقة السرعة

acceleration $\Rightarrow$ $\boxed{a = \frac{d^2s}{dt^2}}$. التسارع = المشتقة الثانية للمسافة

$$\left(a = \frac{dV}{dt}\right)\frac{ds}{ds} \Rightarrow a\,ds = \frac{ds}{dt}\,dV$$

$$\boxed{a\,ds = V\,dV} \quad \text{--- } (4).$$

2. At constant acceleration such as gravity ($g = a_c$)

- $V_f = V_o + a_c t$ --- (5).
- $V_f^2 = V_o^2 + 2a_c \Delta s$ --- (6).
- $\Delta s = V_o t + \frac{1}{2}a_c t^2$. --- (7).

$\Rightarrow$ Average Velocity $V_{avg} = \frac{\Delta s}{\Delta t}$

Instantaneous Velocity $V = \frac{ds}{dt}$.

Average speed $V_{speed} = \frac{\text{total Path}}{\text{time}}$

$\Rightarrow$ Average acceleration

$a_{avg} = \frac{\Delta V}{\Delta t}$.

$\Rightarrow$ Instantaneous accel

$a = \frac{dV}{dt} = \frac{d^2s}{dt^2}$

N O T E B O O K

Subject Date No.

* Dynamics and Vibrations :-
* Ch 2 :- Kinematics of Particles :-

* Sample Problem 2/1 :-
The position coordinate of a particle which is confined to move along a straight line is given by $s = 2t^3 - 24t + 6$ where (s) in (m) and (t) in (sec). Determine.

1. the time required for the particles to reach a velocity of 72 m/s from its initial condition at $t = 0$.

$s = 2t^3 - 24t + 6$.

$V = 6t^2 - 24 \Rightarrow 6t^2 - 24 = 72$.

$6t^2 = 96 \Rightarrow t^2 = 16 \Rightarrow t = 4$ sec.

2. the acceleration of the particle when $V = 30$ m/s.

$V = 6t^2 - 24 \Rightarrow 6t^2 - 24 = 30$.

$6t^2 = 54 \Rightarrow t^2 = 9 \Rightarrow \boxed{t = 3 \text{ sec}}$

$a|_{t = 3\,sec} = 12t \Rightarrow a = 12(3) = 36$ m/s^2.

3. the net displacement of the particle during the interval from $t = 1$ s to $t = 4$ s.

$S(4) - S(1) = (2(4)^3 - 24(4) + 6) - (2(1)^3 - 24(1) + 6)$.

$\Delta s = 54$ m.

N O T E B O O K

Subject Date No.

* Dynamics and Vibrations :-
* Ch2:- Kinematic of Particles :-

* Problems 2/8 :-

A particle moves along a straight line with a velocity in millimeters per second given by $V = 400 - 16t^2$, where (t) in (sec), Calculate the net displacement (Δs) and total distance (d) traveled during the first 6 sec of motion :-

$$V = \frac{ds}{dt} \Rightarrow \int_0^6 V\,dt = \int_0^{\Delta s} ds$$

$$\Delta s = 400t - \frac{16}{3}t^3 \Big]_0^6$$

$$\Delta s = 2400 - 1152 = 1248 \text{ mm.}$$

total distance

$$V = 400 - 16t^2 = \text{Zero.}$$

$$16t^2 = 400 \Rightarrow t^2 = 25 \Rightarrow \boxed{t = 5 \text{ sec}}.$$

$$\text{total distance} = |s_5 - s_0| + |s_6 - s_5|$$

$$= 1333.3 + 85.3$$

$$\text{total distance} = 1418.6 \text{ mm.}$$

* **Notes** :-

1 feet → 30.48 cm → 0.3048 m.

1 inch → 2.54 cm → 0.0254 m

1 mil → 1.609 Km → 1609 m.

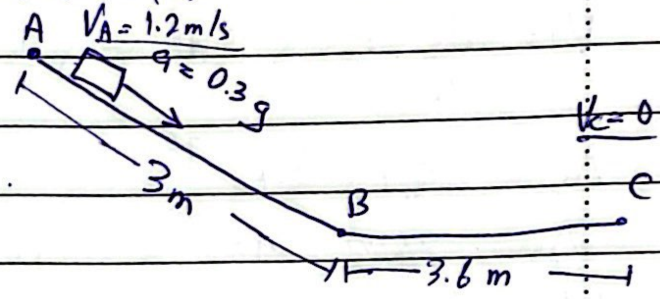
N O T E B O O K

* Dynamics and Vibrations :-

* Ch2 : Kinematics of Particles :-

* Problems 2/35 :-

Packages enter the 3m chute at (A) with a speed of 1.2 m/s and have a 0.3g acceleration from (A) to (B). If the Packages come to rest at (C), Calculate the constant acceleration (a) of the Packages from (B) to (C). Also find the time required for the Packages to go from (A) to (C)



A → B

$$V_B^2 = V_A^2 + 2a\,\Delta s$$

$$V_B^2 = (1.2)^2 + 2(0.3 * 9.81)(3)$$

$$V_B = 4.37 \text{ m/s}$$

B → C

$$V_C^2 = V_B^2 + 2a\,\Delta s$$

$$0 = (4.37)^2 + 2a(3.6) \Rightarrow a_{B\to C} = -2.65 \text{ m/s}^2$$

A → B

$$V_B = V_A + at$$

$$4.37 = 1.2 + (0.3)(9.81)t$$

$$t = 1.077 \text{ sec}$$

B → C

$$V_C = V_B + at$$

$$0 = 4.37 + (-2.65)t$$

$$t = 1.649 \text{ sec}$$

* 4 feet/sec → 1.2 m/s

4 (0.3048) = 1.2 m/s

* 10 feet → 3 m

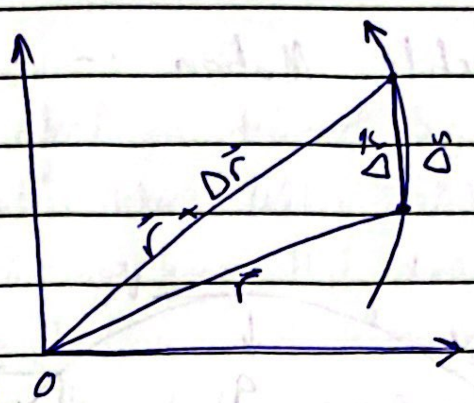
10 (0.3048) → 3.048

* 12 feet → 3.6

12 (0.3048) → 3.6576

Problem 2/35

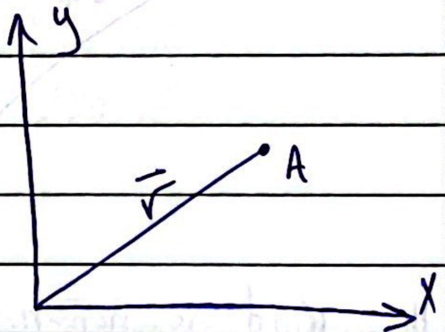
Dynamics and Vibrations :-
* Ch 2: Kinematics of Particles :-



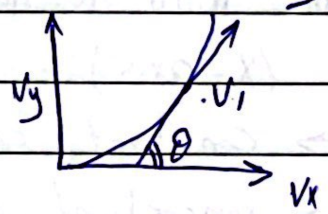
* Plane Curvilinear motion :-
It's motion along a curve path in a single plane.

* There are three coordinate system :-
1. Rectangular coordinate (X-y).
2. Normal-Tangential (N-t) coordinate.
3. Polar coordinate (r-θ) X.

* Rectangular Coordinates (X-y)



displacement $(\vec{r}) = x\hat{i} + y\hat{j}$.
Velocity $(\dot{\vec{r}}) = \vec{V} = \dot{x}\hat{i} + \dot{y}\hat{j}$.
acceleration $(\ddot{\vec{r}}) = \vec{a} = \ddot{x}\hat{i} + \ddot{y}\hat{j}$.



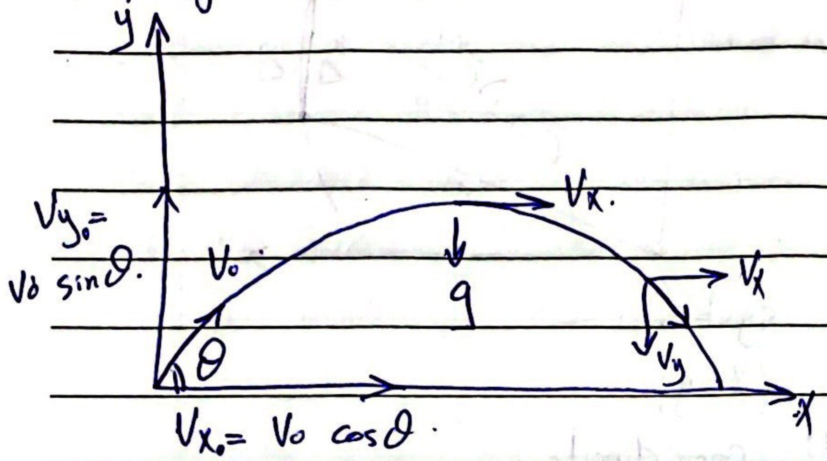
$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{\dot{x}^2 + \dot{y}^2} \quad m/s$$
$$\tan\theta = \frac{V_y}{V_x} \Rightarrow \theta = \tan^{-1}\left(\frac{V_y}{V_x}\right).$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{\ddot{x}^2 + \ddot{y}^2} \quad m/s^2.$$
$$\tan\theta = \frac{a_y}{a_x} \Rightarrow \theta = \tan^{-1}\left(\frac{a_y}{a_x}\right).$$

Dynamics and Vibrations :-

Ch2 : Kinematics ~~and~~ of Particles :-

Projectile Motion :-



For $+y \uparrow$ and $\downarrow -g$.

$V_x = \text{const} \Rightarrow a_x = 0$.

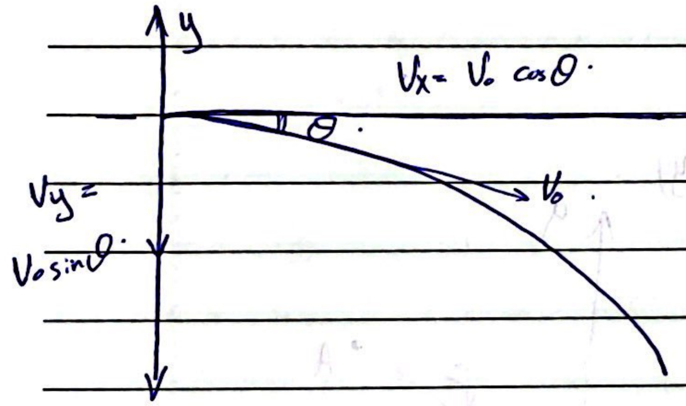
If wind is negligible.

$x = x_0 + V_{x_0} t$.

$a_y = -g$.

$V_y = V_{y_0} - gt$.

$y = y_0 + V_{y_0} t - \frac{1}{2} g t^2$.



For $+y \downarrow$ and $\downarrow g$.

$V_x = \text{cont} \Rightarrow a_x = 0$.

If wind is negligible.

$x = x_0 + V_{x_0} t$.

$a_y = g \Rightarrow V_y = V_{y_0} + gt$.

$y = y_0 + V_{y_0} t + \frac{1}{2} g t^2$.

If the wind is non-negligible there is an acceleration an (x-axis).

- Constant acceleration $\Rightarrow$ نحل على معادلات الحركة
- noncostant acceleration $\Rightarrow$ نحل على اشتقاقات .

Subject Date No.

* Dynamics and Vibrations :-

* Ch 2 :- Kinematics of particles :-

* Sample problem 2/5 :-

The curvilinear motion of a particle is defined by: $V_x = 50 - 16t$ and $y = 100 - 4t^2$ where (V_x) in (m/s) (y) in (m) and (t) in (sec). It is also known that $x = 0$ when $t = 0$. plot the path of the particle and determine it's velocity and acceleration when the position $y = 0$ reached.

$y = 100 - 4t^2 \Rightarrow 100 - 4t^2 = 0$.

$100 = 4t^2 \Rightarrow t^2 = 25 \Rightarrow \boxed{t = 5}$ sec

$V_x = 50 - 16t = 50 - 16(5) = -30$ m/s.

$V_y = \dot{y} = -8t = -8(5) = -40$ m/s.

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(-30)^2 + (-40)^2} = \boxed{50}$ m/s.

$\theta = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{-40}{-30}\right) = 53.1^\circ + 180$.

$= 230.1^\circ$.

$a_x = \dot{V}_x = -16$ m/s^2

$a_y = \ddot{y} = \dot{V}_y = -8$ m/s^2.

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-16)^2 + (-8)^2} = \boxed{17.89}$ m/s^2

$\theta = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{-8}{-16}\right) = 26.5 + 180$.

206.5°.

N O T E B O O K

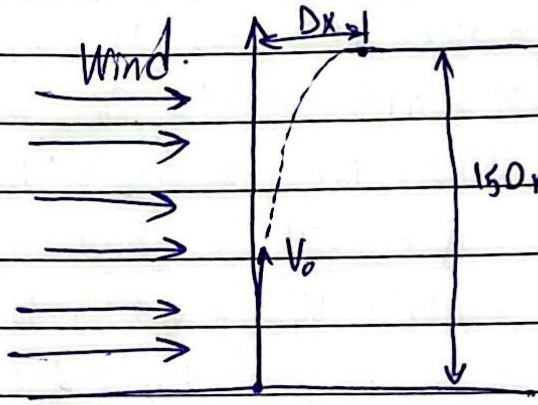
Subject Date No.

* Dynamics and Vibrations :-

* Ch 2 : Kinematics of Particles :-

* Problems 2/87 :-

A fire works shell is launched vertically from Point (A) with speed sufficient to reach a maximum altitude of (150) m. A steady horizontal wind causes a constant horizontal acceleration of (0.15 m/s²) but does not affect the vertical motion. Determine the deviation (Δx) at the top of the trajectory caused by the wind.



Vertically $\Rightarrow V_o = V_y$, $V_{x_o} = 0$.

constant accel $\Rightarrow a_x = 0.15 \; m/s^2$

$\Delta x = ??$ at max hight $\Rightarrow V_y = 0$.

$V_y^2 = V_{y_o}^2 - 2gh$.

$0 = V_{y_o}^2 - 2(9.81)(150)$

$V_{y_o} = \sqrt{2(9.81)(150)} = 54.2 \; m/s$.

$V_y = V_{y_o} - gt$.

$0 = 54.2 - 9.81\,t \Rightarrow \boxed{t = 5.53}$ sec

$x = x_o + \frac{1}{2} a_x t^2$.

$= 0 + \frac{1}{2}(0.15)(5.53)^2$

$\Rightarrow \boxed{\Delta x = 2.29}$ m.

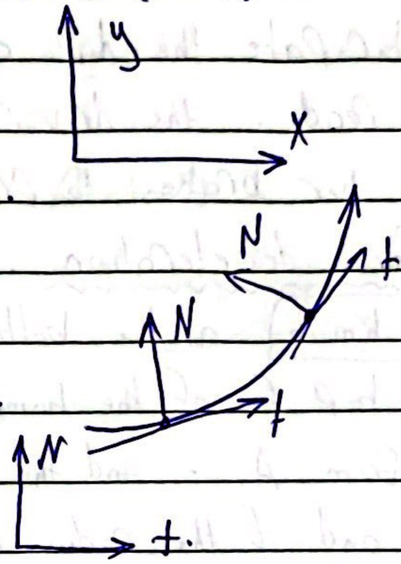
N O T E B O O K

Dynamics and Vibrations :-

Ch2 :- Kinematic of Particles :-

Normal and Tangential Coordinates (N-t) :-

(X-y) coordinates are fixed but (N-t) coordinates move along the path with the particle.



n-axis → towards the centers of Curvature of the path.

t-axis → tangent to the path

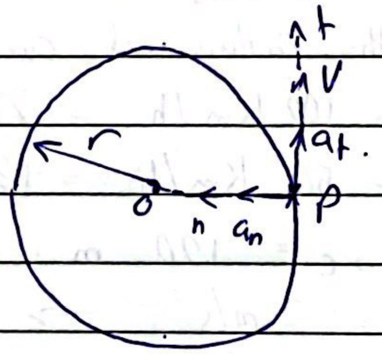
n-t axes are perpendicular

Circular motion :-

$V = r\dot{\theta} = rW$

$a_t = \dot{V} = r\ddot{\theta} = r\alpha$

$a_n = \frac{V^2}{\rho} = \frac{V^2}{r} = r\dot{\theta}^2 = rW^2$



Where : $a_t \rightarrow$ tangent acceleration.

$a_n \rightarrow$ normal acceleration.

$\dot{\theta} \rightarrow$ anglar speed.

$\ddot{\theta} \rightarrow$ angular acceleration.

$(a)_{Total} = \sqrt{(a_t)^2 + (a_n)^2}$

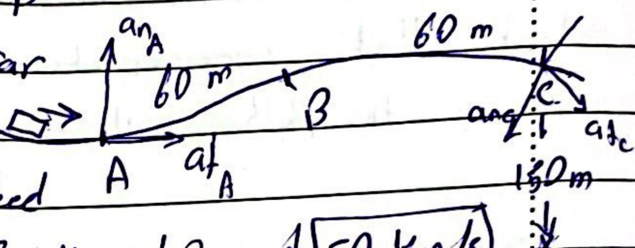
Subject Date No.

* Dynamics and Vibrations :-

* Ch 2 :- Kinematics of Particles :

* Sample problem 2/7 :-

to anticipate the dip and hump in the road . the driver of a car applies her brakes to produce a uniform deceleration . Her speed is $\boxed{100\ km/h}$ at the bottom A of the dip and $\boxed{50\ km/h}$ at the top C of the hump. , Which is $\boxed{120\ m}$ along the road from A . and the total acceleration of $\boxed{3\ m/s^2}$ at A and if the radius of curvature of the hump at C is $\boxed{150\ m}$. Calculate



A) the radius of curvature ρ at A :-

$V_A = 100\ Km/h = 27.7\ m/s$

$V_C = 50\ Km/h = 13.8\ m/s$

$S_{A \to C} = 120\ m$.

$a_A = 3\ m/s^2 \longrightarrow$ total.

$\rho_A = ??$.

$(a_n)_A = \frac{V_A^2}{\rho_A}$.

$V_C^2 = V_A^2 + 2 a_t \Delta s$.

$(13.89)^2 = (27.8)^2 + 2\, a_t\, (120)$

$(a_A)^2_{Total} = (a_n)_A^2 + (a_t)_A^2$.

$a_t = -2.41\ m/s^2$.

uniform deceleration

$(a_n)_A^2 = 3.19$

$(a_t)_A = (a_t)_B = (a_t)_C = a_t$.

$a_{n_A} = 1.785\ m/s^2$.

Dynamics and Vibrations :-

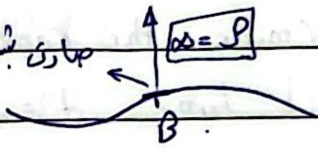
* Ch 2 :- Kinematics of particles :-

* Sample Problems 2/7 :-

$$(a_n)_A = \frac{V_A^2}{\rho} \Rightarrow \rho = \frac{V_A^2}{(a_n)_A} = \frac{(27.78)^2}{1.785} = \underline{432\ m}$$

B) the acceleration at inflection point B :-

B = inflection point . طاري شبه مستقيم



$\rho_B = \infty$.

$$(a_n)_B = \frac{V_B^2}{\rho_B} = \frac{V_B^2}{\infty} = \text{Zero}.$$

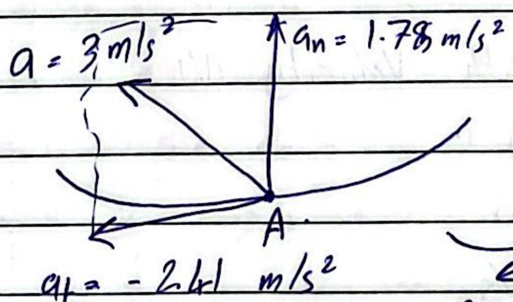
$a_B = a_t = -2.41\ m/s^2$.

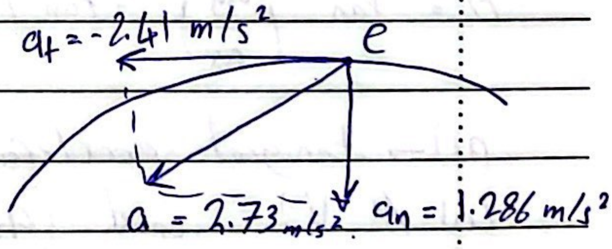
C) the total acceleration at point C.

$$(a_C)_{Total} = \sqrt{(a_n)_C^2 + (a_t)_C^2}$$

$$(a_n)_C = \frac{V_C^2}{\rho_C} = \frac{(13.89)^2}{150} = 1.286\ m/s^2.$$

$$(a_C)_{Total} = \sqrt{(1.286)^2 + (-2.41)^2} = \underline{2.73\ m/s^2}.$$



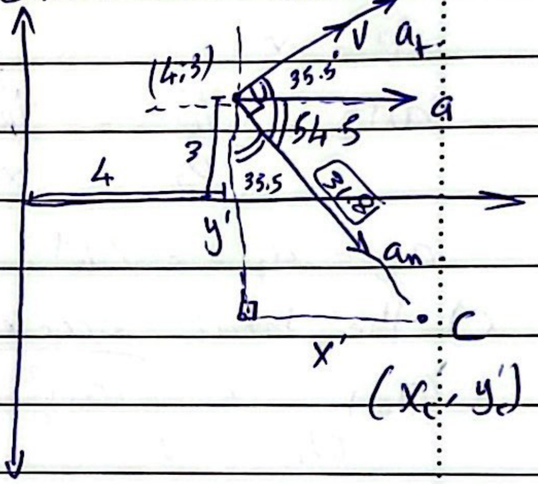


* Dynamics and Vibrations :-

* Ch 2 : Kinematics of Particles :-

* Problems 2/130 :

A Particle which moves with curvilinear motion has coordinates in meters which vary with time (t) in sec according to $x = 2t^2 + 3t - 1$ and $y = 5t - 2$. Determine the coordinates of the center of curvature C at time $t = 1$ s.



$x(1) = 2 + 3 - 1 = 4$

$y(1) = 5 - 2 = 3$ x , y (4, 3)

$\dot{x} = V_x = 4t + 3 \Rightarrow V_x = 7$.

$\dot{y} = V_y = 5 \Rightarrow V_y = 5$.

$V = \sqrt{V_x^2 + V_y^2} \Rightarrow V = 8.6 \ m/s^2$.

$\theta_v = \tan^{-1}\left(\frac{V_y}{V_x}\right) = \tan^{-1}\left(\frac{5}{7}\right) = 35.5^\circ$.

$\ddot{x} = a_x = 4$ $\ddot{y} = a_y = 0$.

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{4^2 + 0^2} = 4 \ m/s^2$.

$\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{0}{4}\right) = 0$.

$(a_t) \rightarrow$ tangent acceleration is with Velocity (V).

$(a_t \parallel V)$ and $(a_t \perp a_n)$.

$a_n = a \sin 35.5 = a \cos 54.5$

$= 4 \sin 35.5 = \underline{2.32} \ m/s$

Subject Date No.

* Dynamics and Vibrations :-
* Ch 2: Kinematics of Particles :-

* Problems 2/130 :- . تابع

$a_n = \frac{V^2}{\rho} \Rightarrow \rho = \frac{V^2}{a_n} = \frac{8.6^2}{2.32} = 31.8\,m$.

Coordinate of Center (X_c, y_c) :-

$X' = \rho \sin(35.5) = 31.8 \sin 35.5 = 18.46$

$y' = \rho \cos(35.5) = 31.8 \cos 35.5 = 25.88$.

$X_c = X' + 4 = 18.4 + 4 = 22.46$ m.

$y_c = -y' + 3 = -25.8 + 3 = -22.9$ m.

$(X_c, y_c) \Rightarrow (22.4, -22.9)$.

* Problem 2/130 :- $\boxed{t=2}$ حل مرة أخرى بفرض أن الزمن

$X(2) = 2(2)^2 + 3(2) - 1 = 13$

$y(2) = 5(2) - 2 = 8$

$\dot{X} = V_x = 4t + 3 \Rightarrow V_x = 11\,m/s$ $\rightarrow V = \sqrt{V_x^2 + V_y^2} = \sqrt{11^2 + 5^2}$

$\dot{y} = V_y = 5 \Rightarrow V_y = 5\,m/s$ $V = 12.08$ m/s.

$\theta_v = \tan^{-1}\left(\frac{V_y}{V_x}\right) = 24.4^\circ$.

$\ddot{X} = a_x = 4$ m/s $\ddot{y} = a_y = 0$

$a = \sqrt{a_x^2 + a_y^2} \Rightarrow a = 4\,m/s$ $\theta_a = \tan^{-1}\left(\frac{a_y}{a_x}\right) = 0$.

$a_n = a \sin 24.4 = 1.65$.

$a_n = \frac{V^2}{\rho} \Rightarrow \rho = \frac{V^2}{a_n} = \frac{12.08^2}{1.65} = 88.4$ m

$X_c = X' + 13 = \boxed{49.5}$ $y_c = -y' + 8 = \boxed{-72.5}$

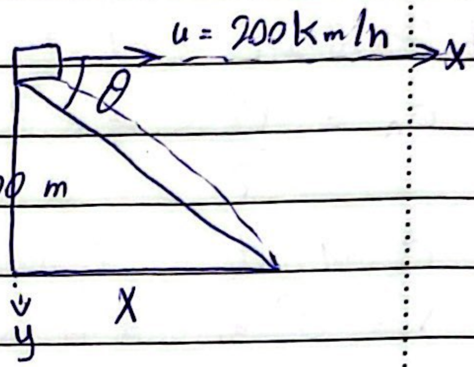
NOTEBOOK

Subject Date No.

* Dynamics and Vibrations :-

* Ch2 :- Kinematics of Particles :-

* Problems :-



* Problems 2/75

$V = 200\ Km/h = 55.5\ m/s$.

$y = 100\ m \qquad g = 9.81 = a_y$

$y = \frac{1}{2} g t^2 \qquad$ — for y-motion.

$X = Vt \qquad a_x = 0 \qquad$ — for x-motion.

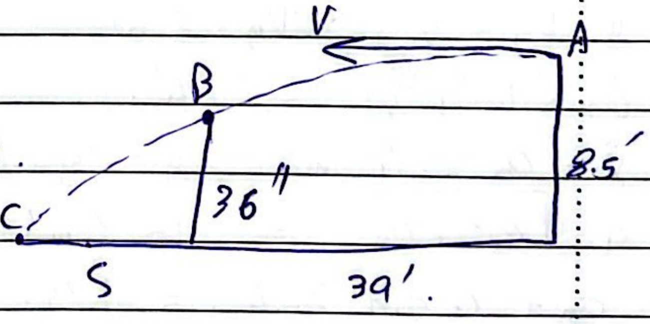
$y = \frac{1}{2} g t^2 \qquad t = \frac{X}{V} \Rightarrow y = \frac{g X^2}{2 V^2} \Rightarrow X = V\sqrt{\frac{2y}{g}}$

$X = \frac{200}{3.6}\sqrt{\frac{2(100)}{9.81}} = 251\ m$

or $\quad y = \frac{1}{2} g t^2 \Rightarrow t^2 = 20.3 \Rightarrow t = 4.5$.

$\qquad X = Vt \Rightarrow X = (55.5)(4.5) = 250\ m$

$\theta = \tan^{-1}\left(\frac{y}{X}\right) = \tan^{-1}\left(\frac{100}{251}\right) = 21.7$.



* Problems 2/77 :-

$\theta = 0$ horizantally $\Rightarrow V_{y_0} = 0$.

$a_x = 0 \rightarrow V_x = const.$

$X = V_{x_0} t \Rightarrow 39 = V_x t_B$.

$a_y = g \qquad y = \cancel{V_{y_0} t} + \frac{1}{2} g t^2$. at B.

$8.5 - 3.5 = \frac{1}{2}(32.2)\, t_B^2 \Rightarrow t_B = 0.557$ sec.

NOTEBOOK

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* Dynamics and Vibrations :-

*Ch2 : Kinematics of Particles :-

* Problem 2/77 :- عا

$V_x = \frac{X}{t_B} = \frac{39}{0.557} = 70$ ft / sec.

47.7 mi / h.

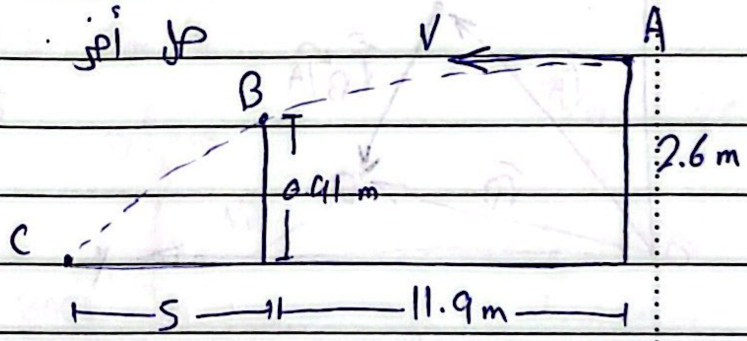
$AC \Rightarrow y = \frac{1}{2} g t_c^2$.

$8.5 = \frac{1}{2}(32.2) t_c^2 \Rightarrow t_c = 0.727$ sec.

$X = V_c t_c$.

$s + 39 = 70 \, (0.727) \Rightarrow s = 11.85$ ft.

* Problem 2/77 : حل أخر .



$\theta = 0 \Rightarrow V_{y_0} = 0$

$a_x = 0 \rightarrow V_x = \text{Const}$

$X = V_{x_0} t + \cancel{\frac{1}{2} a_x t^2}$

$11.9 = V_{x_0} t_B$ ~ ①

$y = \cancel{V_{y_0} t} + \frac{1}{2} g t^2$

$2.6 - 0.91 = \frac{1}{2}(9.81) t_B^2 \Rightarrow t_B = 0.586$ sec

$11.9 = V_{x_0} t_B \Rightarrow \boxed{V_{x_0} = 20.3 \text{ m/s}}$

from (A) to (C) :

$y = \cancel{V_{y_0} t} + \frac{1}{2} g t_c^2$

$2.6 = \frac{1}{2}(9.81) t_c^2 \Rightarrow t_c = 0.728$ sec

$X = V_c t_c + \cancel{\frac{1}{2} a_x t_c^2}$

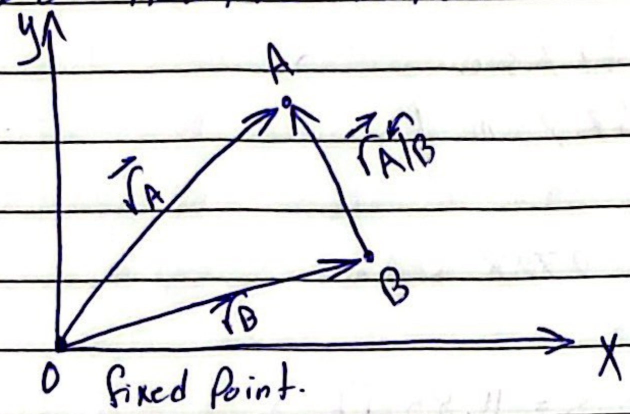
$s + 11.9 = (20.3)(0.728)$

$\boxed{s = 2.87 \text{ m}}$

Subject Date No.

* Dynamics and Vibrations :-
* Ch2:- Kinematics of Particles :-

* 2.8 : Relative motion :-



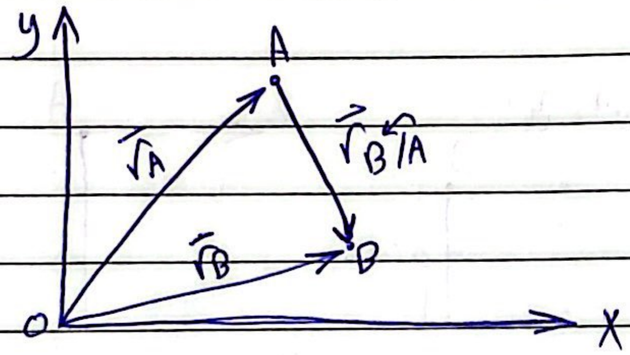
$\vec{r}_A = \vec{r}_B + \vec{r}_{A/B}$

$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$

$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$

$\vec{r}_A, \vec{r}_B \rightarrow$ absolute motion

$\vec{r}_{A/B} \rightarrow$ relative motion.



$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A}$

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

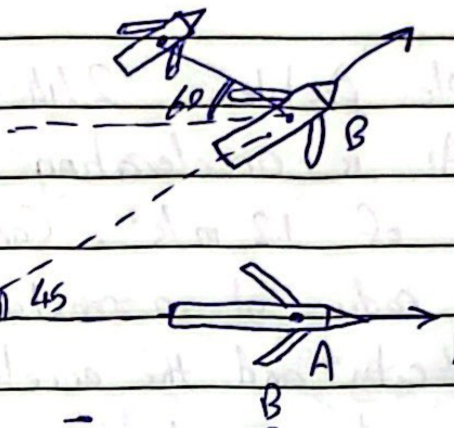
$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

* Sample Problems 2/13 :-

Passengers in the jet transport (A) flying east at a speed of 800 Km/h observe a second jet Plane (B) that Passes under the transport in horizontal flight. Although the nose of (B) is Pointed in the 45° northeast direction, Plane (B) appers to the Passengers in (A) to be moved away from the transport at the 60° angle as shown. Determine the true Velocity of B

* Dynamics and Vibrations :-

* Ch2 :- Kinematics of Particles :-



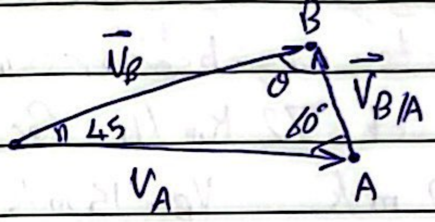
* Sample Problems 2/13 :-

$V_A = 800$ Km/h

$V_B = ??$

$45 + 60 + \theta = 180 \Rightarrow \boxed{\theta = 75}$

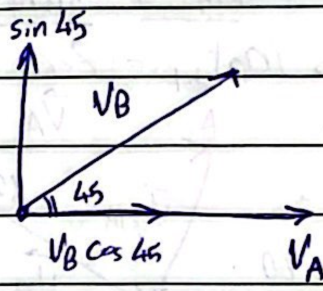
$$\frac{V_B}{\sin 60} = \frac{V_A}{\sin 75} = \frac{V_{B/A}}{\sin 45}$$

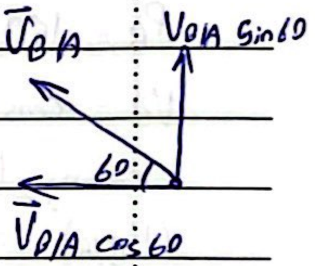


$$V_B = \frac{(\sin 60)}{(\sin 75)}(800) = 717 \text{ Km/h}$$

$$\vec{V}_{B/A} = \frac{(\sin 45)}{(\sin 75)}(800) = 585.6 \text{ Km/h}$$

another solution :-





$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

$(V_B \cos 45)\,i + (V_B \sin 45)\,j = 800\,i + -(\vec{V}_{B/A} \cos 60)\,i + (V_{B/A} \sin 60)\,j$

$\Rightarrow V_B \cos 45 = 800 - \vec{V}_{B/A} \cos 60$

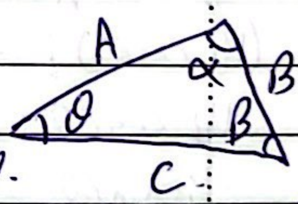
$\Rightarrow V_B \sin 45 = V_{B/A} \sin 60$

$\vec{V}_{B/A} = 585.6$ Km/h

$\vec{V}_B = 717$ Km/h

Sine law :-

$$\frac{A}{\sin\beta} = \frac{C}{\sin\alpha} = \frac{B}{\sin\theta}$$

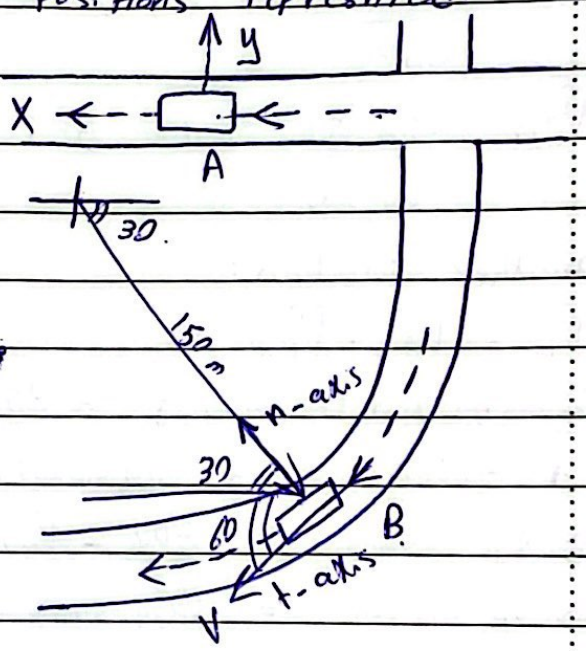


* Dynamics and Vibrations :—

* Ch 2 :- Kinematics of Particles :—

* Sample Problems 2/14 :-

Car (A) is accelerating in the direction of its motion at the rate of 1.2 m/s^2. Car (B) is rounding a curve of 150 m radius at a constant speed of 54 km/h. Determine the velocity and the acceleration which car (B) appears to have to an observer in car (A) if car (A) has reached a speed of 72 km/h for the positions represented.



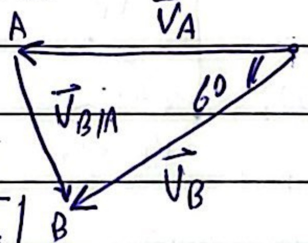
$V_A = 20$ m/s $\quad V_B = 15$ m/s

$a_A = 1.2\ m/s^2 \quad a_B = 1.5\ m/s^2$

$\rho_B = 150$ m. $\quad \underline{\vec{V}_{B/A}\ ??},\ \underline{\vec{a}_{B/A}\ ??}$

$V_B = \text{const} \Rightarrow (a_B)_{tot} = \text{Zero}$.

cosine law.



$(V_{B/A})^2 = V_A^2 + V_B^2 - 2V_A V_B \cos 60$.

$\Rightarrow \boxed{\vec{V}_{B/A} = 18.03\ m/s}$

anther solution :-

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

$(V_B \cos 60)\ \hat{i} - (V_B \sin 60)\ \hat{j} = V_A\ \hat{i} + -(V_{B/A})\ \hat{i} - (V_{B/A})\ \hat{j}$

$V_B \cos 60 = 20 - \vec{V}_{B/A} \quad \Rightarrow V_{B/A} = 12.5\ \hat{i}$

$-V_B \sin 60 = -V_{B/A} \quad \Rightarrow V_{B/A} = 1.29\ \hat{j}$.

$\vec{V}_{B/A} = 18$ m/s

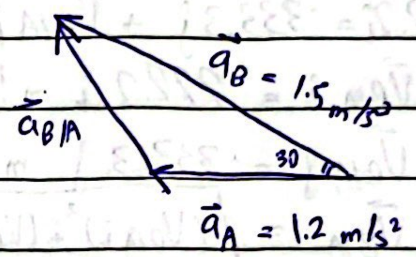
Dynamics and Vibrations :-

Ch2: Kinematics of Particles :-

* Sample Problems 2/14 :-

$V_B = \text{const} \Rightarrow (a_B)_{t} = \text{Zero}.$

$(a_B)_n = \frac{V_B^2}{\rho} = \frac{15^2}{150} = 1.5 \ m/s^2.$



$(\vec{a}_{B/A})^2 = (a_B)^2 + (a_A)^2 - 2(a_B)(a_A)\cos 30.$

$\Rightarrow (\vec{a}_{B/A})^2 = (1.5)^2 + (1.2)^2 - 2(1.5)(1.2)\cos 30.$

$\vec{a}_{B/A} = 0.756 \ m/s^2.$

another solution :-

$\cancel{(a_B \cos 30)\hat{i}}$ $\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$-(a_B \cos 30)\hat{i} + (a_B \sin 30)\hat{j} = -\vec{a}_A \hat{i} - \vec{a}_{B/A}\hat{i} + \vec{a}_{B/A}\hat{j}.$

$\Rightarrow -a_B \cos 30 = -a_A - a_{B/A} \Rightarrow \vec{a}_{B/A}\hat{i} = 0.099$

$\Rightarrow a_B \sin 30 = a_{B/A} \Rightarrow \vec{a}_{B/A}\hat{j} = 0.75.$

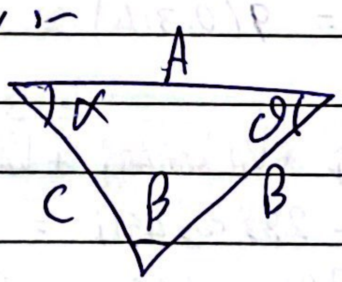
$|\vec{a}_{B/A}| = \sqrt{(\vec{a}_{B/A})_{\hat{i}}^2 + (a_{B/A})_{\hat{j}}^2} = \sqrt{0.099^2 + 0.75^2}$

$\vec{a}_{B/A} = 0.756 \ m/s^2.$

* Sine and Cosine law :-

Sine law :-

$\frac{A}{\sin \beta} = \frac{B}{\sin \alpha} = \frac{C}{\sin \theta}.$

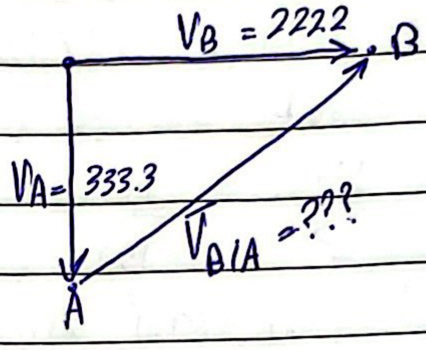


cosine law :-

$C^2 = A^2 + B^2 - 2AB\cos\theta.$

* Dynamics and Vibrations :-

* Ch 2 : Kinematics of Particles :-



* Problem 2/185 :-

$V_A = 333.3\ m/s$, $V_B = 222.2\ m/s$.

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$.

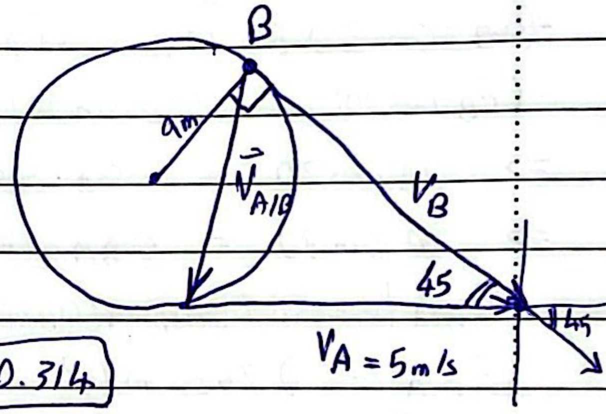
$222.2\hat{i} = -333.3\hat{j} + \vec{V}_{B/A}\hat{i} + \vec{V}_{B/A}\hat{j}$.

$\Rightarrow \vec{V}_{B/A}\hat{i} = 222.2\,\hat{i}\ m/s$

$\Rightarrow \vec{V}_{B/A}\hat{j} = 333.3\,\hat{j}\ m/s$.

$\therefore \vec{V}_{B/A} = \sqrt{(V_{B/A}\hat{i})^2 + (V_{B/A}\hat{j})^2} = 400.6$.

$\theta = \tan^{-1}\left(\dfrac{V_{B/A}\hat{j}}{V_{B/A}\hat{i}}\right) = 56.3^\circ$.

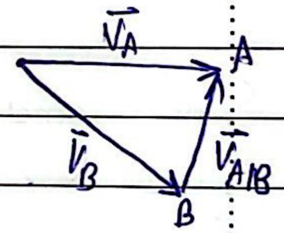


* Problem 2/188 :-

$V_A = 5\ m/s$ $\vec{a}_A = 3\ m/s^2$.

The angle rate = 3 rev/min

$\Omega = \dfrac{3\ rev}{min} = \dfrac{3 * 2\pi}{60} = \dfrac{\pi}{10} = \boxed{0.314}$



$\vec{V}_B = r\theta' = rW = 9(0.314) = 2.83\ m/s$

$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$

$5\hat{i} = \vec{V}_B \cos 45\,\hat{i} - V_B \sin 45\,\hat{j} + \vec{V}_{A/B}\,\hat{i} + \vec{V}_{A/B}\,\hat{j}$.

$\Rightarrow \vec{V}_{A/B}\,\hat{i} = (5 - 2.83\cos 45)\hat{i} = 3\hat{i}$.

$\vec{V}_{A/B}\,\hat{j} = +2.83 \sin 45\,\hat{j} = 2\hat{j}$.

$|\vec{V}_{A/B}| = \sqrt{3^2 + 2^2} = 3.6$.

* Dynamics and Vibrations :-

* Ch2 : Kinematics of Particles :-

* Problem 2/188 :-

$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$.

$\vec{a}_A = 3 m/s^2$.

$\vec{a}_B = (\vec{a}_n)_B = \frac{V_B^2}{r} = \frac{(2.83)^2}{9} = 0.89 \, m/s^2$.

$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$.

$3\hat{i} = a_B \cos 45 \, \hat{i} - a_B \sin 45 \, \hat{j} + \vec{a}_{A/B} \hat{i} + \vec{a}_{A/B} \hat{j}$.

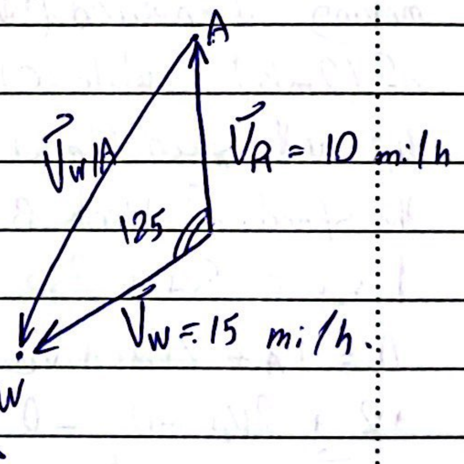
$\Rightarrow \vec{a}_{A/B} \hat{i} = (3 - 0.89 \cos 45) \hat{i} = 2.37 \hat{i} \; m/s^2$

$\Rightarrow \vec{a}_{A/B} \hat{j} = 0.89 \sin 45 \, \hat{j} = 0.63 \, \hat{j} \; m/s^2$.

$|\vec{a}_{A/B}| = \sqrt{2.37^2 + 0.63^2} = 2.45 \; m/s^2$.

$\alpha = \tan^{-1}\left(\frac{0.63}{2.37}\right) = 14.88^\circ$.

* Problem 2/186 :-



$\vec{V}_A = 10 \; mi/h \qquad V_W = 15 \; mi/h$.

$(\vec{V}_{W/A})^2 = V_A^2 + V_W^2 - 2 V_A V_W \cos 125$.

$= (10)^2 + (15)^2 - 2(10)(15) \cos 125$.

$(\vec{V}_{W/A})^2 = 497.07 \; mi/h \Rightarrow \boxed{\vec{V}_{W/A} = 22.29}$

Or $\vec{V}_W = \vec{V}_A + \vec{V}_{W/A}$.

$- V_W \cos(35) \hat{i} - V_W \sin(35) \hat{j} = \overleftarrow{10 \hat{i}} + \vec{V}_{W/A}$.

$\vec{V}_{W/A} = -8.6 \hat{j} - 22.2 \hat{i}$.

$\vec{V}_{W/A} = \sqrt{-8.6^2 + -22.2^2} \approx 23 \; mi/h$.

* Dynamics and Vibrations :—

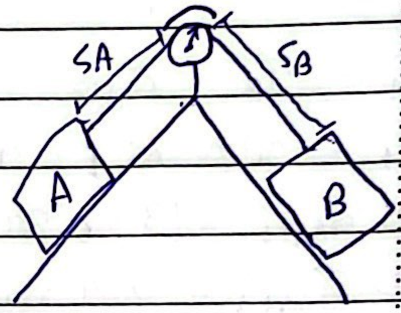
* Ch2 :· Kinematics of Particles :—

* Constrained Motion of Connected Particles :—

Dependant motion of two Particles

motion of one Particle depends on corresponding motion of another Particle interconnected by inextensible cords.

L_{Total} = total length of rope



$L_{Total} = S_A + S_B + \pi r \rightarrow$ const.

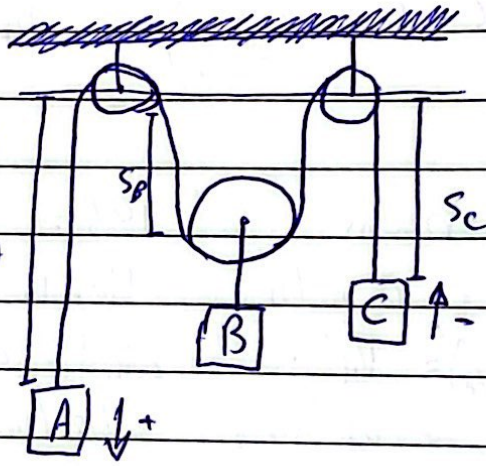
$\boxed{0 = V_A + V_B}$ ---- ①

$\vec{V}_A = -\vec{V}_B$.

$\boxed{0 = a_A + a_B}$ ---- ②

$\vec{a}_A = -\vec{a}_B$

* Example :- if Block (A) is moving downward (+↓) with speed of (2 m/s) while (C) is moving up with speed (1 m/s) · determine the speed of block B :



$L_{tot} = S_A + 2S_B + S_C +$ Const

$0 = V_A + 2V_B + V_C$

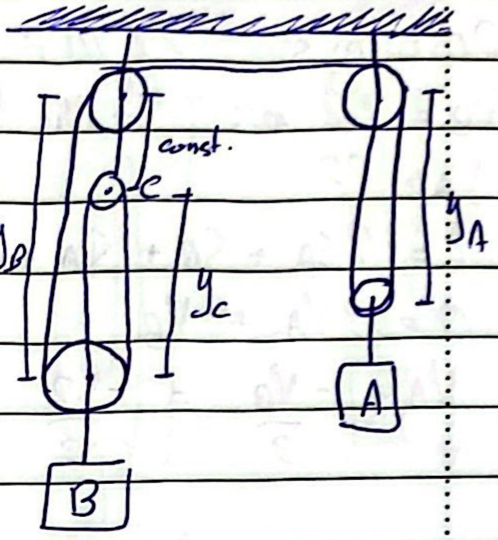
$+2 + 2V_B (-1) = 0 \Rightarrow \vec{V}_B = -0.5$ m/s

$\therefore \vec{V}_B = 0.5$ ↑ up.

Subject Date No.

* Dynamics and Vibrations :-

* Ch2 :- Kinematic of Particles :-



*** Sample Problem 2/15 :-**

In the pulley configuration shown: Cylinber (A) has a downward velocity of (0.3 m/s). determine the Velocity of (B). V_B ??.

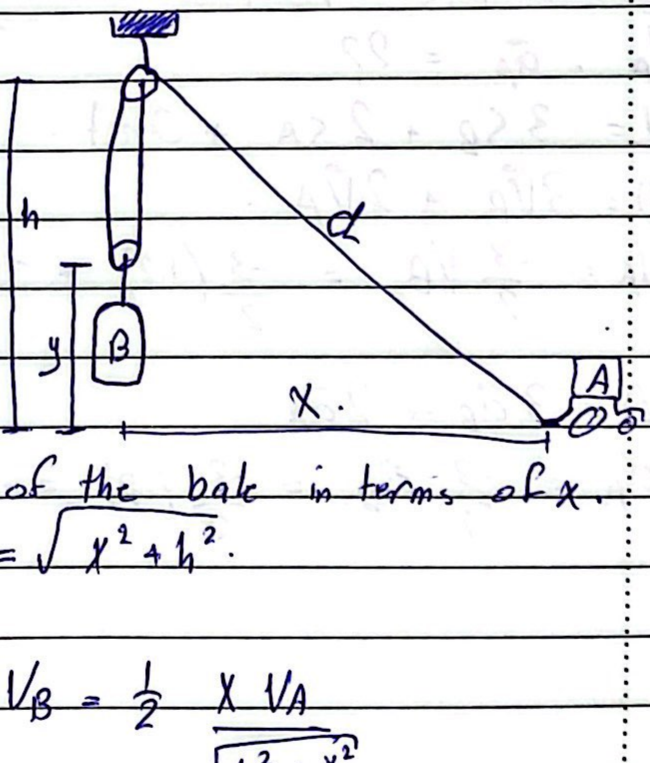
$L_{tot} = 2y_A + \underline{y_B} + 2\underline{y_c}$

$L_{tot} = 2y_A + 3y_B$.

$0 = 2V_A + 3V_B$.

$V_A = 0.3$ m/s downward

$V_B = -0.2$ m/s upward.



*** Sample Problem 2/16 :-**

The tracter (A) is used to hoist the bale (B) with the pully arrangement shown. If (A) has a forward velocity (V_A). determine the expression for the upward velocity (V_B) of the bale in terms of x.

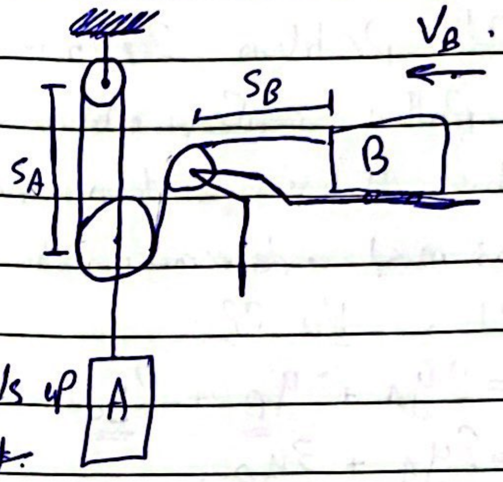
$L_{tot} = 2(h-y) + d \Rightarrow d = \sqrt{x^2+h^2}$.

$L = 2(h-y) + \sqrt{x^2+h^2}$.

$0 = -2\dot{y} + \frac{x\dot{x}}{\sqrt{x^2+h^2}}$, $\dot{y} \rightarrow V_B$, $\dot{x} \rightarrow V_A$ $\Rightarrow V_B = \frac{1}{2}\frac{x V_A}{\sqrt{h^2+x^2}}$

Dynamics and Vibrations :-
Ch 2 - Kinematics of Particles :-

Problems 2/207 :-



$V_B = 1.2$ m/s to the leftward

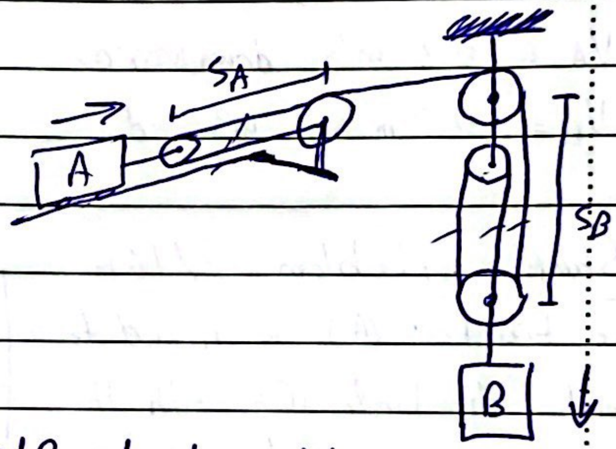
$V_A = ??$

$L = 2S_A + S_B + S_A + Const.$

$0 = 3V_A + V_B$

$V_A = \frac{-V_B}{3} = \frac{-1.2}{3} = -0.4$ m/s up ~~to right.~~

Problems 2/208 :-



$\vec{V}_B = 1.2$ m/s down.

$\vec{a}_B = 2$ m/s² up.

$V_A, \vec{a}_A = ??$

$L = 3S_B + 2S_A + Const.$

$0 = 3\vec{V}_B + 2\vec{V}_A$

$V_A = \frac{-3}{2} V_B = \frac{-3}{2}(1.2) = -1.8$ m/s to right

$0 = 3\vec{a}_B + 2\vec{a}_A$

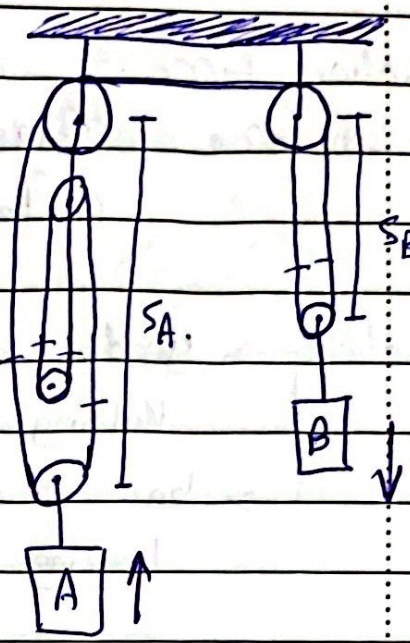
$\vec{a}_A = \frac{-3}{2}\vec{a}_B = \frac{-3}{\cancel{2}}(\cancel{2}) = -3$ m/s². down.

Subject Date No.

* Dynamics and Vibrations :-

* Ch 2 : Kinematics of Particles :-

* Problems 2/209 :-



$V_B = \frac{1}{2}t^2 + \frac{1}{6}t^3$ feet/sec downward

$a_A |_{t=2sec}$??.

$L = 4S_A + 2S_B + Const.$

$0 = 4V_A + 2V_B$

$0 = 4\vec{a}_A + 2\vec{a}_B$.

$V_B = \frac{1}{2}t^2 + \frac{1}{6}t^3$.

$\vec{a}_B = t + \frac{1}{2}t^2$

$\vec{a}_B(2) = 2 + \frac{1}{2}(2)^2 = 4$ feet/sec^2.

$0 = 4\vec{a}_A + 2\vec{a}_B$.

$\vec{a}_A = \frac{-2\vec{a}_B}{4} = \frac{-1}{2}(4) = -2$ feet/sec^2.

N O T E B O O K

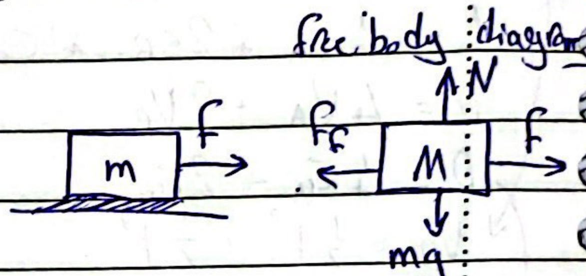
* Dynamics and Vibrations :-

* Ch 3 :- Kinetics of Particles :-

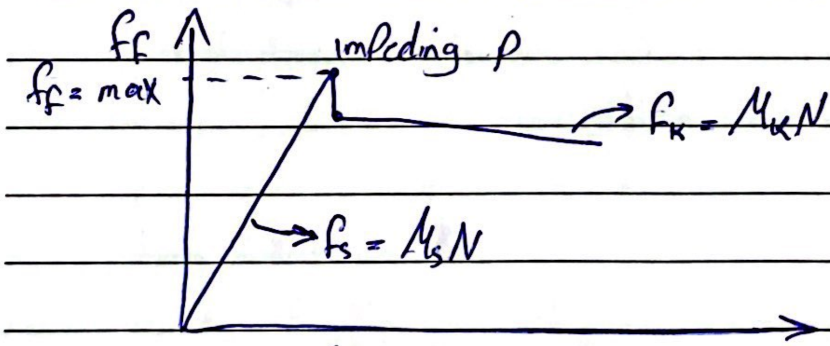
* friction force :- قوة الاحتكاك

friction force :- Tangential forces generated between contacting surface.

* friction → good → e.g. brakes, walking, belt-drives.
  → bad → e.g. gears, bearing



* friction force :-
- at rest : good contact between two surfaces tips are interconnected.
- at motion : motion is near tops.



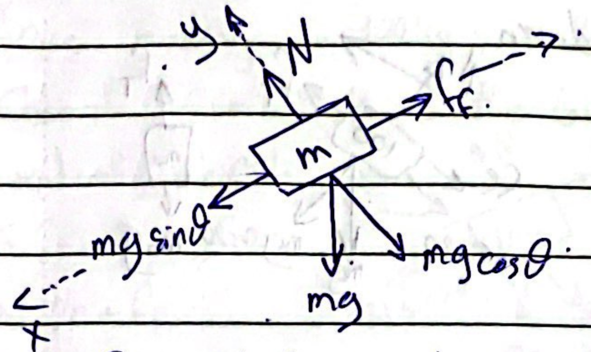
$\boxed{\mu_s > \mu_k}$

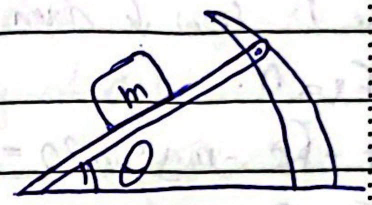
μ_s : coefficient of static friction.
μ_k : coefficient of kinetic friction.

\# Dynamics and Vibrations :-
* Ch 3 :- Kinetics of Particles :-

* Sample Problems 6/1 :

Find θ_{max} before block of mass (m) begins to slip.





before block (m) begins to slip.

at rest $\Rightarrow \Sigma F = 0$.

$\Sigma F_x = 0 \Rightarrow mg\sin\theta = f_f$. --- ①

$\Sigma F_y = 0 \Rightarrow N = mg\cos\theta$. --- ②

$F_{max} = \mu_s N = \mu_s mg \cos\theta$.

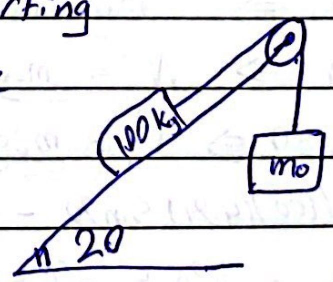
$mg\sin\theta = f_f \Rightarrow \cancel{mg}\sin\theta = \mu_s \cancel{mg}\cos\theta$.

$\sin\theta = \mu_s \cos\theta \Rightarrow \theta = \tan^{-1}(\mu_s)$.

* Sample problem 6/2 :-

Determine the range of the Value which m_o may have So that the 100 Kg will neither starting moving up nor slip down the Plane

$\mu_s = 0.3$.

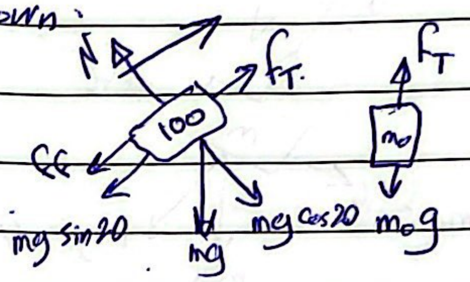


Dynamics and Vibrations :-
Ch 3 :- Kinetics of Particles :-

Sample Problems 6/2 :- ٥٤٦

maximum (m_o) when the motion impending up the plane.

- Case 1 :- (m_o) ↓ down so f_f = down.



$\Sigma F_x = 0$.

$f_T - f_f - mg\sin 20 = 0$.

$\Sigma F_y = 0$.

$N = mg\cos 20 = (100)(9.81)\cos 20$

$f_f = f_T - mg\sin 20$.

$\Sigma F_y |_{m_o} = 0$

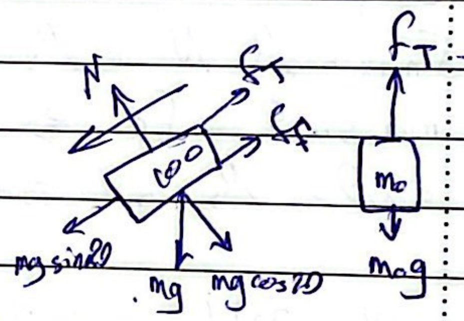
$f_T = m_o g$

$f_f = m_o g - mg\sin 20$.

$\mu_s N = m_o g - (100)(9.81)\sin 20$.

$(0.3)(921.8) = m_o g - 981\sin 20$.

$m_o = 62.39\ Kg$



- Case 2 :- minimum value of (m_o) occures when motion is impending down the plane.

$\Sigma F_x = 0 \Rightarrow mg\sin 20 = f_T + f_f$.

$\Sigma F_y = 0 \Rightarrow N = mg\cos 20 = 921.8$.

$\Sigma F_y |_{m_o} = 0 \Rightarrow f_T = m_o g$.

$f_f = (100)(9.81)\sin 20 - m_o g \Rightarrow \mu_s N = 335.5 - m_o g$.

$m_o = \frac{335.5 - (0.3)(921.8)}{9.81} \Rightarrow m_o = 6.01\ Kg$

Subject Date No.

* Dynamics and Vibrations :-

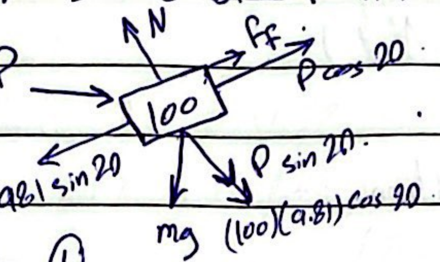
* Ch3 :- Kinetics of Particles :-

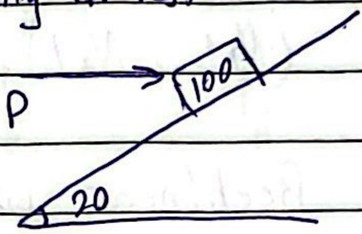
* Sample Problems 6/3 :-

Determine the magnitude and direction of the friction force acting on 100 kg , $\mu_s = 0.2$, $\mu_k = 0.17$.

Case 1 : $P = 500\ N \Rightarrow$ the block initially at rest

initially at rest





$\Sigma F_x = 0.$

$P\cos 20 + f_f - mg \sin 20 = 0$ ... ①

$\Sigma F_y = 0.$

$N = P\sin 20 + mg \cos 20 = 1092.8$

$F_{max} = \mu_s N = (0.2)(500 \sin 20 + 981 \cos 20) = \boxed{218.5}$

$f_f = mg \sin 20 - P\cos 20$

$f_f = 981 \sin 20 - 500 \cos 20 \Rightarrow \boxed{f_f = -134.3} \rightarrow$ negative sign

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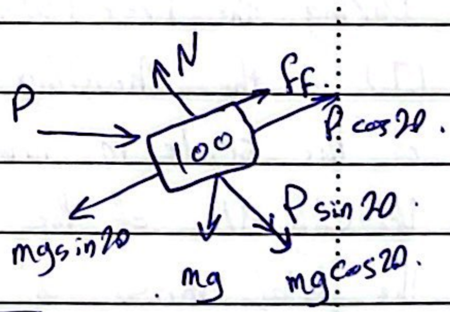
$\boxed{f_{max} > f_f}$

Case 2 , $P = 100\ N$



$\Sigma F_x = 0 \Rightarrow P\cos 20 + f_f - mg \sin 20 = 0.$

$\Sigma F_y = 0 \Rightarrow N = mg \cos 20 + P \sin 20.$

$f_{max} = \mu_s N = (0.2)(956) = \boxed{191.2\ N}$

$f_f = mg \sin 20 - P\cos 20 \Rightarrow \boxed{f_f = 242\ N}$

$f_f > f_{max}.$

N O T E B O O K

Subject Date No.

Dynamics and Vibrations :-

Ch3 :- Kinetics of Particles :-

Kinetics of Particles :-

relation ship between force acting on a body and the change in it's motion.

section (3.1 + 3.2 + 3.3) → $\Sigma \vec{F} = m\vec{a}$.

(N1L, N2L, N3L).

3.4 : Rectilinear motion :-

$\Sigma F_x = m a_x$.

$\Sigma F_y = m a_y$.

$\Sigma F_z = m a_z$

$\vec{a} = a_x \hat{i} + a_y \hat{j} + a_z \hat{k}$.

Sample Problem 3/1 :-

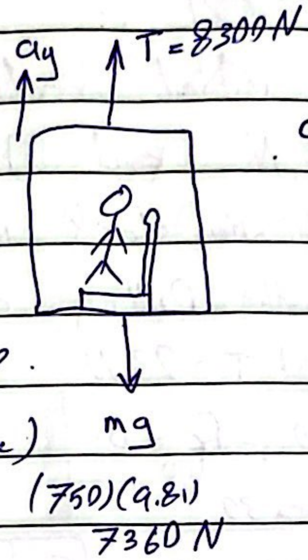
A 75 Kg man stands on a spring scale in an elevator. During the first (3) second of motion from (rest). the tension (T) in the hoisting cable is 8300 N. find the reading (R) of the scale in newtons during this interval and the upward Velocity (V) of the elevator at the end of the 3 seconds. The total mass of the elevator, man, and ~~the~~ scale is 750 Kg.

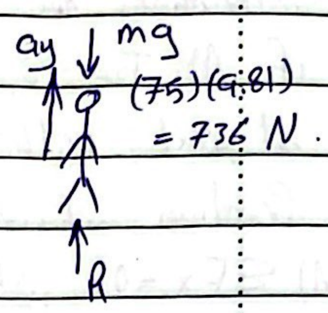
N O T E B O O K

* Dynamics and Vibrations:-

* Ch3 :- Kinetics of Particles :-

* Sample Problems 3/1 :-





Mass total = 750 Kg.

M_{man} = 75 kg.

during 3 second ⇒ $t = 3$ sec.

from rest ⇒ $V_0 = 0$.

$T = 8300$ N ⇒ find R and V??

(1) R = Reaction force (Normal force)

a_y elevator = a_y man.

$* \sum F_y = m\vec{a}_y$.

$T - m_{tot}\, g = M_{tot}\, a_y$.

$8300 - 7360 = 750\, a_y$.

$\Rightarrow a_y = \boxed{1.257 \ m/s^2}$.

$* \sum F_y = m\vec{a}_y$

$R - m_{man}\, g = m_{man}\, \vec{a}_y$

$R - 736 = 75(1.257)$

$\Rightarrow \boxed{R = 830 \ N}$

(2) V = ??

$V_f = V_o + at$.

$V = 0 + (1.257)(3) \Rightarrow V = 3.77$ m/s.

Or $a = \frac{dv}{dt} \Rightarrow \int_0^V dv = \int_0^3 a\, dt$.

$V = (1.257)(3) = 3.77$ m/s.

Subject Date No.

Dynamics and Vibrations :-

Ch3 :- Kinetics of Particles :-

Problems 3/21 :-

Determine the initial acceleration of the 15 Kg block if A) T = 23 N and B) T = 26 N. The system is initially at rest with no slack in the cable. and the mass and friction of the Pulleys are negligible.

A) $\Sigma F_x = 0$. at $T = 23\,N$.

$2T + T\cos 30 - f_f = 0$.

$f_f = 2T + T\cos 30$

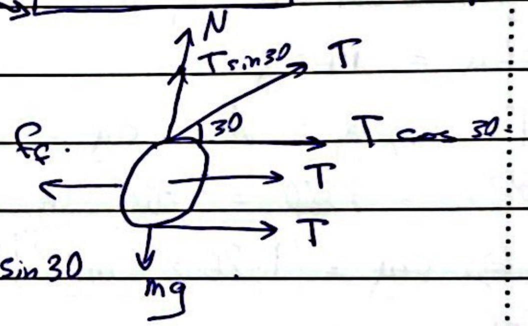
$f_f = 2(23) + 23\cos 30 = \boxed{f_f = 65.9}$

$\Sigma F_y = 0$.

$N + T\sin 30 - mg = 0$.

$N = mg - T\sin 30 = (15)(9.81) - (23)\sin 30$

$\boxed{N = 135.6\ Net}$



$f_{max} = \mu_s N = (0.5)(135.6) = 67.8\,N$.

$f_f < f_{max} \Rightarrow \boxed{a = 0}$

B) $\Sigma F_x = 0$ at $T = 26\,N$

$2T + T\cos 30 - f_f = 0. \Rightarrow \boxed{f_f = 74.5\,N}$

$\Sigma F_y = 0 \Rightarrow N + T\sin 30 - mg = 0 \Rightarrow \boxed{N = 134.2\,N}$

$f_{max} = \mu_s N = (0.5)(134.2) = 67.1$

$f_f > f_{max} \Rightarrow$ we have an acceleration.

$\therefore \Sigma F_x = m\vec{a} \Rightarrow 2T + T\cos 30 - f_k = m\vec{a}$.

$\Rightarrow 52 + 26\cos 30 - \mu_k N = 15\,a \Rightarrow \boxed{a = 1.39\ m/s^2 \rightarrow}$

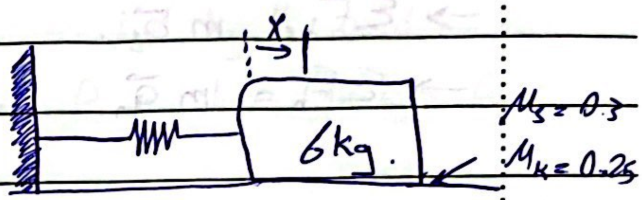
NOTEBOOK

Dynamics and Vibrations :-

* Ch3:- Kinetics of Particles :-

* Problem 3/35:-

The nonlinear spring has a tensile force deflection relationship given by $f_s = 150x + 400x^2$ where (x) in meters and (f_s) in newtons. Determine the acceleration of the (6kg) block if it ~~has~~ is released from rest at a) $x = 50\ mm$ and b) $x = 100\ mm$



a) at $x = 50\ mm$

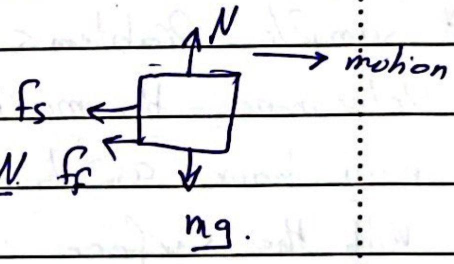
$\Rightarrow f_s = 8.5\ N$

$\Sigma F_x = 0.$

$-f_s - f_f = 0 \Rightarrow \boxed{f_f = -8.5\ N}$

$\Sigma f_y = 0.$

$N - mg = 0 \Rightarrow N = (6)(9.81) = \underline{58.9N}$



$f_{max} = \mu_s N = (0.3)(58.9) = 17.66.$

$f_{max} > f_f \Rightarrow \boxed{a = 0}$

b) at $x = 100\ mm.$

$\Rightarrow f_s = 19\ N.$

$\Sigma f_y = 0$

$N - mg = 0 \Rightarrow N = mg = 58.9\ N.$

$\Sigma F_x = 0 \Rightarrow -f_s - f_f = 0 \Rightarrow \boxed{f_f = -19\ N}$

$f_{max} = \mu_s N = (0.3)(58.9) = 17.66.$ $\boxed{f_f > f_{max}}$

$\Sigma F_x = ma \Rightarrow -f_s - f_k = m\vec{a}$

$-19 - \mu_k N = (6)a \Rightarrow \boxed{\vec{a} = -6.11\ m/s^2}$

Dynamics and Vibrations :-

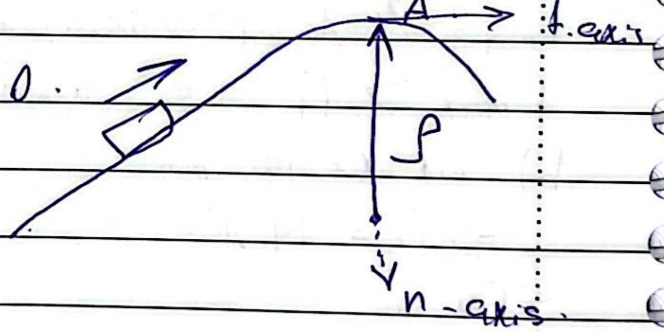
Ch3 :- Kinetics of Particles :-

3/5 Curvilinear motion :-

1. Rectangular coordinates
   - $\rightarrow \sum F_x = m\vec{a}_x$
   - $\rightarrow \sum F_y = m\vec{a}_y$.
2. N-T coordinates :-
   - $\rightarrow \sum F_t = m\vec{a}_t$.
   - $\rightarrow \sum F_n = m\vec{a}_n$.

Sample Problems 3/6 :-

Determine the maximum speed ($\mathring{V}$) which the sliding block may have as it passes point (A) without losing contact with the surface.



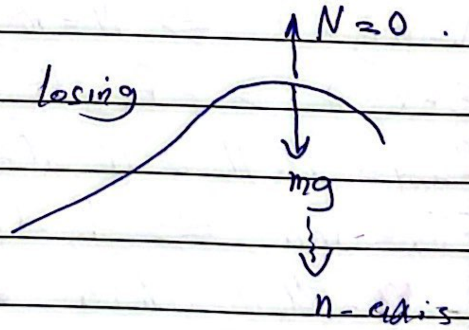
no contact $\Rightarrow$ Normal force = 0.

$\sum F_n = m a_n$.

$\cancel{m}g = \cancel{m}\dfrac{V^2}{\rho} \Rightarrow V = \sqrt{\rho g}$

$V_{max} < \sqrt{\rho g}$.

↳ to passes point (A) without losing contat with the surface.



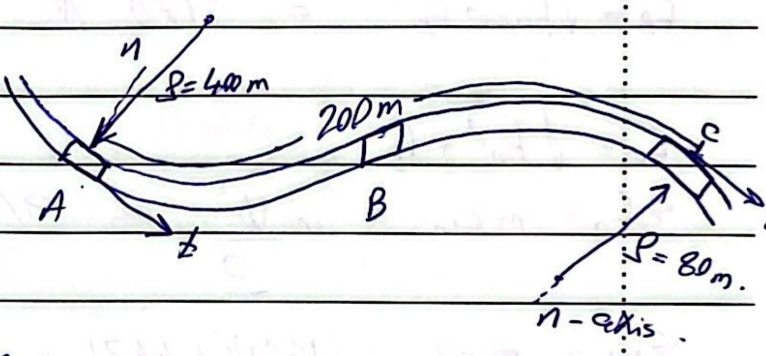
Dynamics and Vibrations:-
* Ch 3: Kinetics of Particles :-

* Sample Problems 3/8 :-

A 1500 Kg car enters a section of $\boxed{\text{curved road}}$ the horizontal plane and slows down at a uniform rate from a speed of 100 Km/h at A to a speed of 50 Km/h as it passes C. The raduis of curvature (ρ) of the road at (A) is 400 m and at C is 80 m. Determine the total horizonatal force exerted by the road on the tires at positions A, B and C. Point B is the inflection point where the curvature changes direction.

$m_{car} = 1500\ Kg$.

slows down at a uniform rate



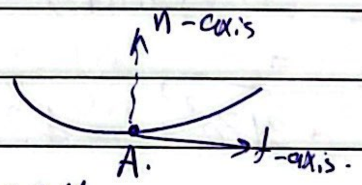
$\Rightarrow a =$ constant.

$V_A = 100\ Km/h = 27.7\ m/s$

$V_C = 50\ Km/h = 13.8\ m/s$.

$\rho_A = 400\ m \qquad \rho_C = 80\ m$

Find F_A, F_B, F_C :-



$F_A = \sqrt{F_n^2 + F_t^2}$

$\Sigma F_n = m\,\bar{a}_n = m\dfrac{V^2}{\rho} = \underline{2890\ N}$

$\Sigma F_t = m\,\vec{a}_t \quad \Rightarrow \quad \Sigma F_t = (1500)(1.447) = \underline{2170\ N}$

$V_C^2 = V_A^2 + 2a_t\,\Delta s$.

$13.8^2 = 27.7^2 + 2a_t(200) \quad \Rightarrow a_t = 1.447\ m/s^2$.

* Dynamics and Vibrations :-

* Ch3 :- Kinetics of Particles :-

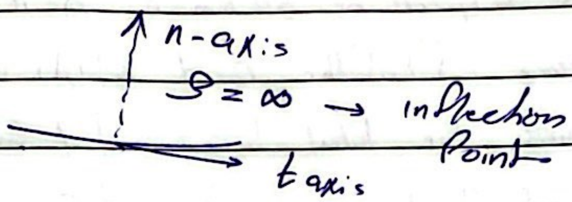
* Sample Problems 3/8 :- تابع

$F_A = \sqrt{F_n^2 + F_t^2}$

$= \sqrt{2890^2 + 2170^2} = 3614 N.$

$* F_B = \sqrt{F_n^2 + F_t^2}$

$\Sigma F_n = m a_n = m \frac{V^2}{\rho_\infty} = \text{Zero}$



$\Sigma F_t = m \bar{a}_t = (1500)(1.447) = \underline{2170 N}$

$F_B = \sqrt{F_n^2 + F_t^2} = 2170 N.$

$* F_C = \sqrt{F_n^2 + F_t^2}$

$\Sigma F_n = m a_n = m \frac{V_C^2}{\rho} = 3620 N.$

$\Sigma F_t = m a_t = (1500)(1.447) = 2170 N$

$F_C = \sqrt{F_n^2 + F_t^2} =$

$= \sqrt{3620^2 + 2170^2} = 4220 N$

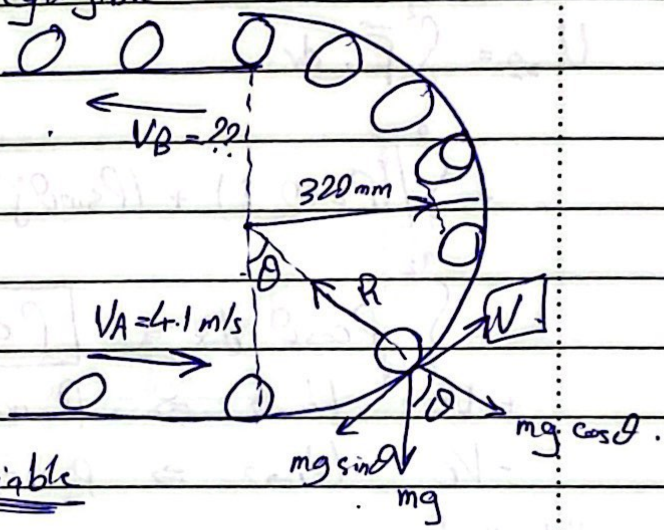
Dynamics and Vibrations :-

Ch3 - Kinetics of particles :-

* Problems 3/77 :-

Small steel balls each with a mass of 65 g enter the semicircular trough in the vertical plane with a horizontal velocity of (4.1) m/s at (A). find the force R exerted by the trough on each ball in terms of θ and the velocity (V_B) of the balls at B. friction is negligible.

$m_{ball} = 65\,g = 0.65\,Kg.$



$V_A = 4.1\,m/s$

$\Sigma F_t = m\vec{a}_t$

$-mg\sin\theta = m a_t$

$\boxed{a_t = -g\sin\theta}$

$\Sigma F_n = m\vec{a}_n$

$R - mg\cos\theta = m\frac{V^2}{\rho} \rightarrow$ <u>Variable</u>

$V\,dV = a_t\,ds.$ but $s = \rho\theta \Rightarrow ds = \rho\,d\theta.$

$\int_{4.1}^{V} V\,dV = \int_0^{\theta} a_t\,\rho\,d\theta.$ $V = \rho\theta'$

$\rightarrow (-g\sin\theta)\rho.$

$\frac{V^2}{2} - 8.4 = +\rho g\cos\theta.$

$V^2 = 10.53 + 6.28\cos\theta.$

When $\theta = 180 \Rightarrow V_B = 2.06\,m/s.$

$R = mg\cos\theta + \frac{m}{\rho}(10.53 + 6.28\cos\theta) \Rightarrow R = 21.38 + 19.1\cos\theta.$

※ Dynamics and Vibrations :-

* Ch3 :- Kinetics of Particles :-

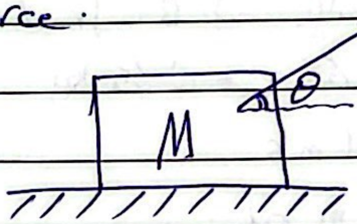
* 3/6 : Work and Kinetic energy :-

$$U_{1\to2} = \int_1^2 \vec{F} \cdot dr \quad \text{in Joule} = N.m.$$

$$= \int_1^2 f_x\,dx + \int_1^2 f_y\,dy + \int_1^2 f_z\,dz.$$

* Work :-

1. work associated with a constant force.



$$U_{1\to2} = \int_1^2 \vec{F} \cdot dr$$

$$= \int_1^2 \left[(P\cos\theta\, \hat{i}) + (P\sin\theta\, \hat{j}) \right] ds\, \hat{i}$$

$$= \int_{x_1}^{x_2} P\cos\theta\, dx = \boxed{P\cos\theta\,(x_2 - x_1)}$$

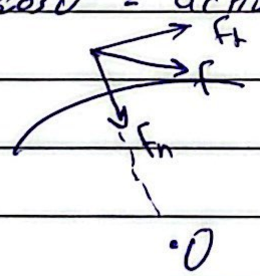
$+Ve$ $U_{1\to2}$ $\Rightarrow$ P with (X) direction

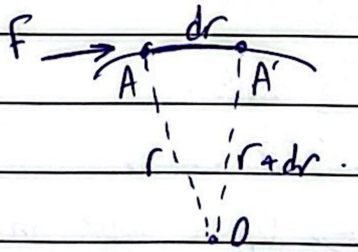
$-Ve$ $U_{1\to2}$ $\Rightarrow$ P opposite (X) direction.

Note that

$P\sin\theta$ = reactive force does not work.

$P\cos\theta$ = active force does work.





- work positive if F_t in the direction of the displacement.
- work negative if F_t in the opposite of the displacement.

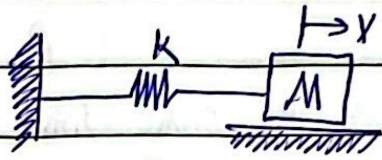
F_n : does no work (reactive force)

F_t : do work. (active force).

Dynamics and Vibrations :-
Ch3 :- Kinetics of Particles :-

Work :-

2- Work associated with a spring force.

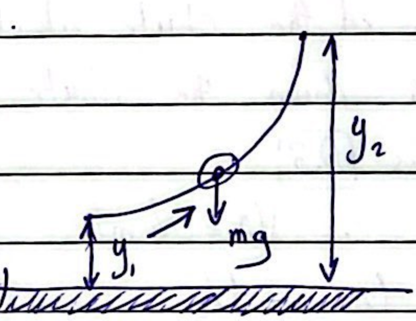


$$U_{1\to2} = \int_{x_1}^{x_2} -Kx\,dx$$

$$= -\frac{K}{2}x^2\Big]_{x_1}^{x_2} = \frac{1}{2}K(x_1^2 - x_2^2).$$

$f_s = Kx$

3- Work associated with weight.



$$U_{1\to2} = \int_{y_1}^{y_2} -mg\,dy$$

$$= mg(y_1 - y_2)$$

if $y_2 > y_1$ -ve U (body rised)

$y_2 < y_1$ +ve U (body falls)

Kinetic energy :- (T)

$T = \frac{1}{2}m(V_2^2 - V_1^2)$

work - energ equation :

$T_1 + U_{1\to2} = T_2$.

$U_{1\to2}$ → constant force
→ spring force
→ weight.

Power = $F \cdot V = \frac{U_{1\to2}}{\text{time}}$.

Power : time rate at which it can do work or deliver energy.

Subject Date No.

Dynamics and Vibrations :-

Ch3 - Kinetics of Particles :-

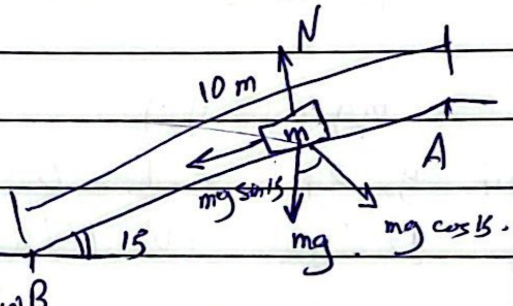
Kinetic energy (T) :-

efficiency $\varepsilon = \frac{P_{out}}{P_{input}} = \frac{\text{Work done by a machine}}{\text{work done on a machine}}$

during the same time intervals

Sample Problems 3/11 :-

Calculate the Velocity (V) of the 50 kg crate when it reaches the bottom of the chute at (B) if it is given an initial Velocity of 4 m/s down the chute at (A) The coefficient of kinetic friction is $\boxed{0.3}$



V_B ?? , $V_A = 4$ m/s

$U_{A\to B} = U_W - U_F$

$= (mg\sin 15)(10) - \mu_k N (10)$

$= (mg\sin 15)(10) - (0.3)(mg\cos 15)(10)$

$= (50)(9.81)(10)\sin 15 - (0.3)(50)(9.81)(\cos 15)(10)$

$= 1269.5 - 1421.3 = \boxed{-151.9\,J}$

$T_A + U_{A\to B} = T_B$

$\frac{1}{2}mV_A^2 + (-151.9) = \frac{1}{2}mV_B^2$

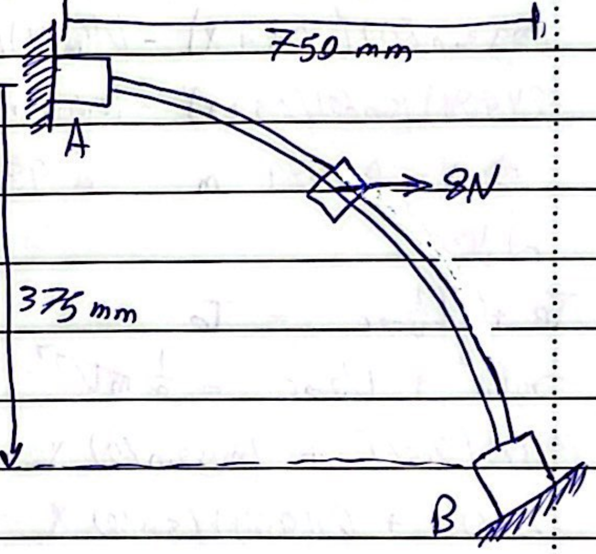
$V_B = \sqrt{\frac{2}{m}\left(\frac{1}{2}mV_A^2 - 151.9\right)} \Rightarrow V_B = 3.15$ m/s

N O T E B O O K

* Dynamics and Vibrations :-
* Ch 3:- Kinetics of Particles :-

* Problem 3/100 :-

The 0.8 kg collar slides with negligible friction on the fixed rod in the vertical plane. If the collar starts from rest at (A) under the action of the constant 8 N horizontal force. Calculate it's velocity (V) as it hits the stop at B.



$f_f = 0$, $m = 0.8$ kg

$V_A = 0$ $\quad V_B = ??$

$T_A + U_{A \to B} = T_B$

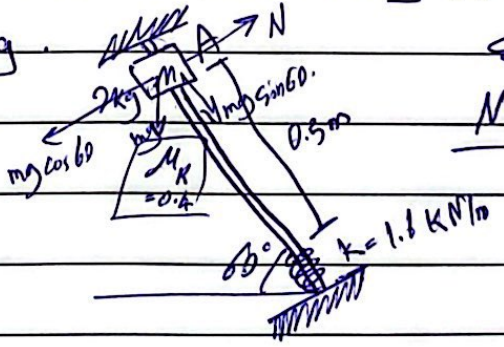
$\frac{1}{2} m V_A^2 + 8(0.75) + mg(0.375) = \frac{1}{2} m V_B^2$

$V_B = \sqrt{\frac{2}{m}\left(8(0.75) + mg(0.375)\right)}$

$\Rightarrow V_B = 4.73$ m/s.

* Problem 3/109 :-

The 2 kg collar is released from rest at (A) and slides down the inclined fixed rod in the vertical plane. The coefficient of kinetic friction is 0.4. Calculate a) the velocity (V) of the collar as it strikes the spring.



$\Sigma F_y = 0$

$N = mg \cos 60$

$T_A + U_{A \to B} = T_B$

$\frac{1}{2} m V_A^2 + (mg \sin 60)(0.5) - (\mu_k N)(0.5) = \frac{1}{2} m V_B^2$

$V_B = 2.56$ m/s

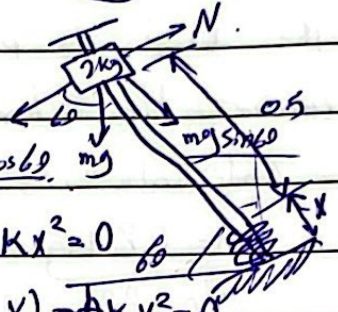
Subject　　　　Date　　　　No.

Dynamics and Vibrations :-

* Ch 3 :- Kinetics of Particles :-

* Problems 3/109 :- تابع

b) the maximum deflection (X) of the spring.



$T_A + U_{A\to C} = T_C$

$\cancel{\frac{1}{2} m V_A^2} + U_{A\to C} = \cancel{\frac{1}{2} m V_C^2}$.

$U_{A\to C} = 0$.

$(mg\sin 60)(0.5+X) - (\mu_k N)(0.5+X) - \frac{1}{2}KX^2 = 0$

$(2)(9.81)(\sin 60)(0.5+X) - (0.4)(mg\cos 60)(0.5+X) - \frac{1}{2}KX^2 = 0$.

$\Rightarrow X = 0.0989$ m $= 98.9$ mm.

. حل آخر

$T_B + U_{B\to C} = T_C$.

$\frac{1}{2} m V_B^2 + U_{B\to C} = \cancel{\frac{1}{2} m V_C^2}$..

$(\frac{1}{2})(2)(2.56)^2 + (mg\sin 60)X - (\mu_k N)X - \frac{1}{2}Kx^2 = 0$

$(2.56)^2 + (2)(9.81)(\sin 60)X - (0.4)(mg\cos 60)X - \frac{1}{2}X^2K = 0$.

$6.55 + 16.99X - 3.92X - 0.8\times 10^3 X^2 = 0$.

$\Rightarrow 6.55 + 13.07X - 800X^2 = 0$.

$X = 0.0989$ m $= 98.9$ mm.

* Dynamics and Vibrations :-
* Ch 3:- Kinetics of Particles.

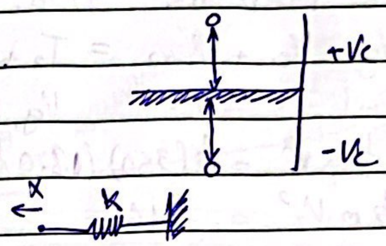
* 3.7:- Potential energy:-

1. Gravitational Potential energy.

$V_g = mgh$

2. Elastic Potential energy:-

$V_e = \frac{1}{2} k x^2$ always +Ve.



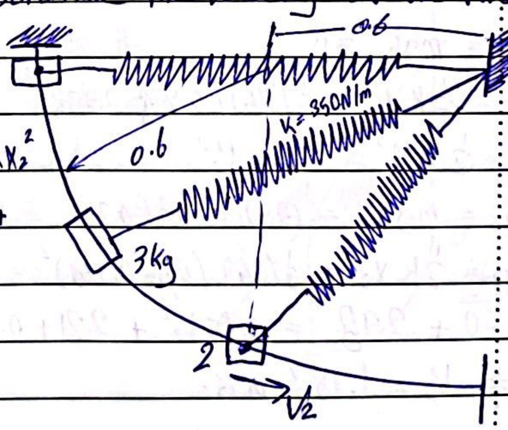
* Work - energy equation

$T_1 + V_{g_1} + V_{e_1} + U'_{1\to2} = T_2 + V_{g_2} + V_{e_2}$

$U'_{1\to2} \Rightarrow$ Work done by constant force.

* Sample problem 3/16:-

The 3 kg slider is released from rest at position 1 and slides with negligible friction in a vertical plane along the circular rod. The attached spring has a stiffness of 350 N/m and has an unstretched length of 0.6 m. Determine the velocity of the slider as it passes position 2.



$T_1 + V_{g_1} + V_{e_1} + U'_{1\to2} = T_2 + V_{g_2} + V_{e_2}$

$0 - mgh + \frac{1}{2} k x_1^2 = \frac{1}{2} m V_2^2 + -mgh + \frac{1}{2} k x_2^2$

$\frac{1}{2}(350)(1.2-0.6)^2 = \frac{1}{2}(3) V_2^2 - (3)(9.81)(0.6) + \frac{1}{2}(350)(0.6\sqrt{2}-0.6)^2$

$\Rightarrow V_2 = 6.82$ m/s.

* Dynamics and Vibrations :-

* Ch3 :- Kinetics of Particles :-

* Sample Problems 3/16.

$T_1 + V_{g_1} + V_{e_1} + U'_{1\to2} = T_2 + V_{g_2} + V_{e_2}$.

$T_1 = \frac{1}{2} m V_1^2 = 0$. $\quad V_{g_1} = mgh = mg(0) = 0$.

$V_{e_1} = \frac{1}{2} K x_1^2 = \frac{1}{2}(350)(1.2 - 0.6)^2 = 63$

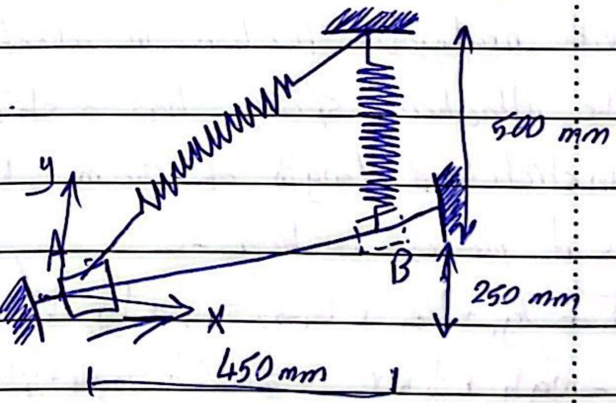
$T_2 = \frac{1}{2} m V_2^2 = \frac{3}{2} V_2^2$.

$V_{g_2} = {}^-mgh = (3)(9.81)(0.6) = {}^-17.65$

$V_{e_2} = \frac{1}{2} K x_2^2 = \frac{1}{2}(350)(0.6\sqrt{2} - 0.6)^2 = 10.8$

* Problem 3/139 :-

The 0.9 kg collar is relased from rest at A and slides freely up the inclined rod, striking the stop at B with avelocity (V). The spring of stiffness K = 24 N/m has an unstretched lengh of 375 mm Calculate V :-



$\cancel{T_1} + \cancel{V_{g_1}} + V_{e_1} + \cancel{U'_{1\to2}} = T_2 + V_{g_2} + V_{e_2}$.

$T_1 = \frac{1}{2} m V_1^2 = 0 \qquad U'_{1\to2} = 0$.

$V_{g_1} = mgh = 0$

$V_{e_1} = \frac{1}{2} K x_1^2 = \frac{1}{2}(24)(0.874 - 0.375)^2$

$T_2 = \frac{1}{2} m V_2^2 = 0.45\ V_2^2$.

$V_{g_2} = mgh = (0.9)(9.81)(0.25) = 2.2$. $\quad$ A is refrence ل V_{g1}

$V_{e_2} = \frac{1}{2} K x_2^2 = \frac{1}{2}(24)(0.5 - 0.375)^2 = 0.187$

$0 + 0 + 2.98 = 0.45 V_2^2 + 2.21 + 0.187$

$\Rightarrow V_2 = 1.156$ m/s.

NOTEBOOK

* Dynamics and Vibrations:-

* Ch 3:- Kinetics of Particles:-

* Problem 3/139:-

$T_1 + V_{g_1} + V_{e_1} + U'_{1\to 2} = T_2 + V_{g_2} + V_{e_2}$.

$T_1 = \frac{1}{2} m V_1^2 = 0 \qquad U'_{1\to 2} = 0$.

$V_{g_1} = mgh = -(0.9)(9.81)(0.25)$

$V_{e_1} = \frac{1}{2} K x_1^2 = \frac{1}{2}(24)(0.874 - 0.375)^2$

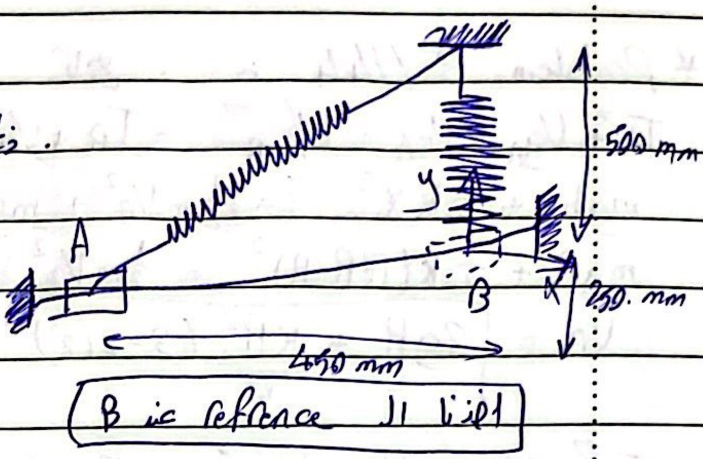
$V_{g_2} = mgh = mg(0) = 0$.

$V_{e_2} = \frac{1}{2} K x_2^2 = \frac{1}{2}(24)(0.5 - 0.375)^2$.

$\cancel{T_1} + V_{g_1} + V_{e_1} + \cancel{U'_{1\to 2}} = T_2 + \cancel{V_{g_2}} + V_{e_2}$.

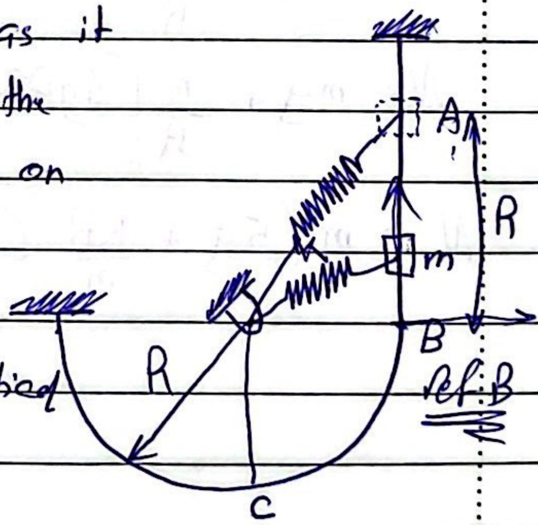
$-2.2 + 2.98 = \frac{1}{2}(0.9) V_2^2 + 0.187$

$\Rightarrow V_2 = 1.156 \text{ m/s}$



* Problem 3/144:-

The spring of constant (K) is unstretched when the slider of mass (m) passes position B. If the slider is released from rest in position A, determine its speed as it passes points B and C. What is the normal force exerted by the guide on the slider at position C? Neglect friction between the mass and the circular guide which lies in a vertical plane.



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Subject   Date   No.

* Dynamics and Vibrations :-

* Ch 3 :- Kinetics of Particles :-

* Problem 3/144 :-

$$\cancel{T_A} + V_{g_A} + V_{e_A} + \cancel{U'_{A \to B}} = T_B + \cancel{V_{g_B}} + V_{e_B}$$ ref at B

$$mgh + \frac{1}{2}kx_1^2 = \frac{1}{2}mV_B^2 + \cancel{mg(0)} + \frac{1}{2}kx_2^2$$

$$mgR + \frac{1}{2}k(\sqrt{2}R-R)^2 = \frac{1}{2}mV_B^2 + \cancel{\frac{1}{2}k(R-R)^2}$$

$$V_B = \sqrt{2gR + \frac{kR^2}{m}(3-2\sqrt{2})}$$

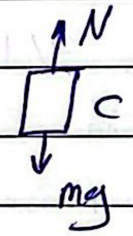
$$\cancel{T_A} + V_{g_A} + V_{e_A} + \cancel{U'_{A \to C}} = T_C + \cancel{V_{g_C}} + V_{e_C}$$ ref at C

$$mgh + \frac{1}{2}kx_1^2 = \frac{1}{2}mV_C^2 + \cancel{mg(0)} + \frac{1}{2}kx_2^2$$

$$mg(2R) + \frac{1}{2}k(\sqrt{2}R-R)^2 = \frac{1}{2}mV_C^2 + \cancel{\frac{1}{2}k(R-R)^2}$$

$$V_C = \sqrt{4gR + \frac{kR^2}{m}(3-2\sqrt{2})}$$

$$\Sigma F_n = ma_n \Rightarrow N - mg = \frac{mV_C^2}{R}$$



$$\Rightarrow N = mg + \frac{mV_C^2}{R}$$

$$N = mg + \frac{m}{\cancel{R}}\left[4g\cancel{R} + \frac{kR^{\cancel{2}}}{m}(3-2\sqrt{2})\right]$$

$$N = m\left(5g + \frac{kR}{m}(3-2\sqrt{2})\right)$$ #

* Dynamics and Vibrations :-

* Ch 5 :- Plane Kinematics of Rigid Bodies :-

* 5.1 :- Introduction :-

We use the same relationship governing the displacement Velocity and acceleration of Particles but we must account for rotational motion of the body.

* Plane motion :-

1- Translational : every line in the body remains Parallel to its origional Position at all times.

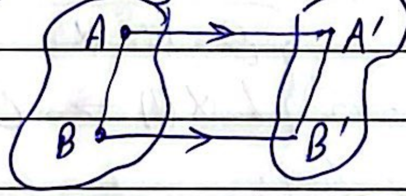
a- Recetilinear. b- Curvilinear.

2. Rotation about fixed axis.

3- General Plane motion : (combination between 1 and 2)

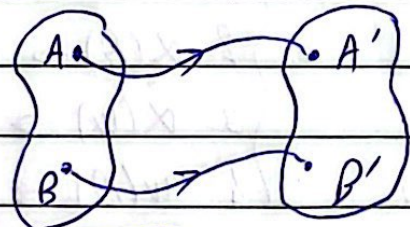
* Type of Rigid Body Plane motion:-

1. Recetilinear translation



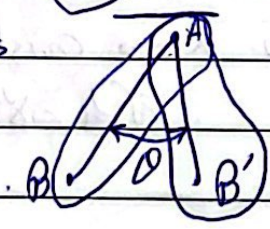
Example :- Rocket test sled.

2. Curvilinear translation



Example :- Parallel - link swinging Plate.

3- Fixed axis rotation



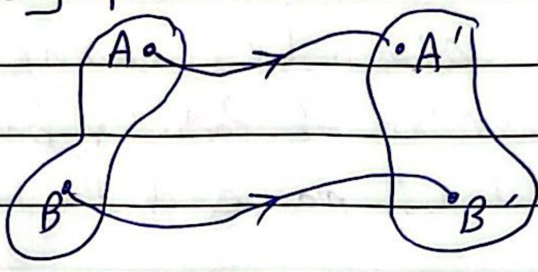
Example compound Pendulum

Dynamics and Vibrations:-

Ch5: Plane Kinematics of Rigid Bodies:-

Types of Rigid-Body Plane motion:-

4- General Plane motion

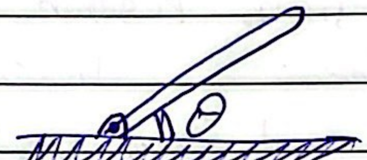


Example: Connecting rod in a reciprocating engine.

5/2: Rotation motion:-

$$w = \frac{d\theta}{dt} = \dot{\theta} \quad \text{--- (1)}$$

$$\alpha = \frac{d^2\theta}{dt^2} = \frac{dw}{dt} = \ddot{\theta} = \dot{w} \quad \text{--- (2)}$$



$$w\,dw = \alpha\,d\theta \quad \text{--- (3)} \qquad \dot{\theta}\,d\dot{\theta} = \ddot{\theta}\,d\theta$$

if $\alpha = \alpha_c$ → constant angular acceleration.

$$w = w_0 + \alpha_c t \quad \text{--- (4)}$$

$$w^2 = w_0^2 + 2\alpha_c(\theta - \theta_0) \quad \text{--- (5)}$$

$$\theta = \theta_0 + w_0 t + \tfrac{1}{2}\alpha_c t^2 \quad \text{--- (6)}$$

w_0, θ_0 → at $t = 0$.

θ, w, α . α حسب جي

1. $\alpha(t) \rightarrow \theta, w, \alpha$ ($\xrightarrow{\text{اشتق}}$, $\xleftarrow[\text{كامل}]{}$)

2. $\alpha(\theta) \rightarrow \alpha\,d\theta = w\,dw$.

3. $\alpha(w) \rightarrow \alpha = dw/dt$.

4. $w(\theta) \rightarrow w = d\theta/dt$.

5. α → constant . uniform

. [illegible]

Rotation about fixed axis:-

$$V = rw$$

$$a_t = \alpha r = \dot{V} = r\dot{w}$$

$$a_n = rw^2 = \frac{V^2}{r} = Vw.$$

Subject Date No.

* Dynamics and Vibrations :-

* Ch5 :- Plane Kinematics of Rigid Bodies :-

* Using Vector notation :-

$V = W \times r$

use Right - hand Rule.

$\vec{W}$: normal to the Plane of rotation

$\vec{a}_n = \vec{W} \times \vec{V} = \vec{W} \times (\vec{W} \times \vec{r})$

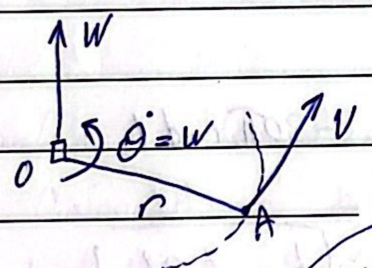
$\vec{a}_t = \vec{\alpha} \times \vec{r}$.

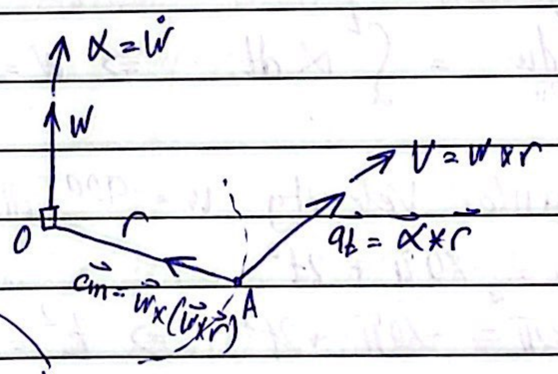
$\vec{W}$: is increasing $\rightarrow$ $\vec{\alpha}$ is in the direction of $\vec{W}$.

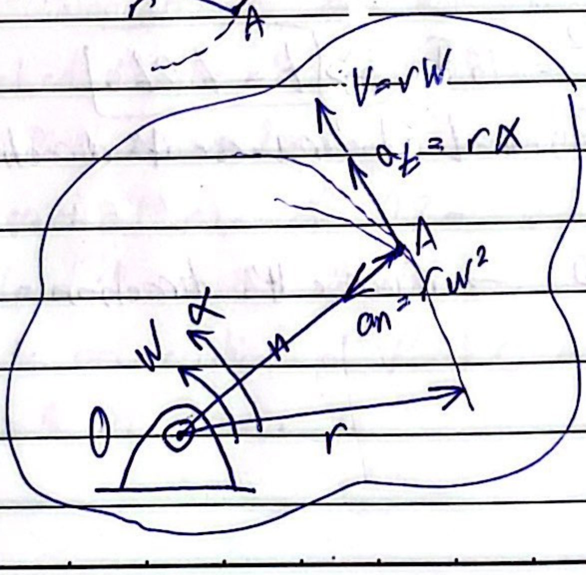
$\vec{W}$: is decreasing $\rightarrow$ $\vec{\alpha}$ is in the opposite direction of $\vec{W}$.

$\vec{W}$: is increasing $\rightarrow$ $\vec{a}_t$ is in the same direction of $\vec{V}$.

$\vec{W}$: is decreasing $\rightarrow$ $\vec{a}_t$ is in the opposite direction of $\vec{V}$







Subject Date No.

* Dynamics and Vibrations:-

* Ch5 : Plane Kinematics of Rigid Bodies :-

→ clockwise ⊖
→ Counterclockwise ⊕

* Sample Problems 5/1 :-

A fly wheel rotating freely at 1800 rev/min clockwise is subjected to a variable counterclockwise torque which is first applied at time $t = 0$. the torque produce a counterclockwise angular acceleration $\alpha = 4t$ rad/s^2. Determine

a) the time required for the fly wheel to reduce it's clockwise angular speed to 900 rev/min.

$$\alpha = \frac{dw}{dt} \Rightarrow \int dw = \int \alpha \, dt$$

angular velocity $w = \frac{-1800\,(2\pi)}{60} = -60\pi$ rad/s.

$$\int_{-60\pi}^{w} dw = \int_0^t \alpha \, dt \Rightarrow w = -60\pi + 2t^2.$$

angular velocity $w = -900\,(2\pi)/60 = -30\pi$ rad/s.

$$w = -60\pi + 2t^2.$$

$$-30\pi = -60\pi + 2t^2 \Rightarrow t^2 = 15\pi \Rightarrow \boxed{t = 6.86\,s}$$

b) the time required for the fly wheel to reverse it's direction of rotation.

$w = -60\pi + 2t^2$. $w = 0$ → reverse it's direction.

$$2t^2 = 60\pi$$

$$t^2 = 30\pi \Rightarrow \boxed{t = 9.71\,s}$$

* Dynamics and Vibrations :-
* Ch5 - Plane Kinematics of Rigid Bodies :-

* Sample Problem 5/1 :-
C) the total number of revelutions, clockwise plus Counterclockwise, turned by the flywheel during the first 14 seconds of torque application.

$$w = \frac{d\theta}{dt} \Rightarrow \int_0^{\theta} d\theta = \int_0^{N_1} w\,dt \qquad \boxed{N_1 = 9.71\,s}$$

$$\theta_1 = \int_0^{9.71} (-60\pi + 2t^2)\,dt \Rightarrow \theta_1 = -1220 \text{ rad.}$$

$N_1 = 1220 / 2\pi = 194.2$ revolutions, clockwise

$$\int_0^{\theta_2} d\theta = \int_{9.71}^{14} (-60\pi + 2t^2)\,dt \Rightarrow \theta_2 = 410 \text{ rad.}$$

$N_2 = 410 / 2\pi = 65.3$ revolutions counterclockwise.

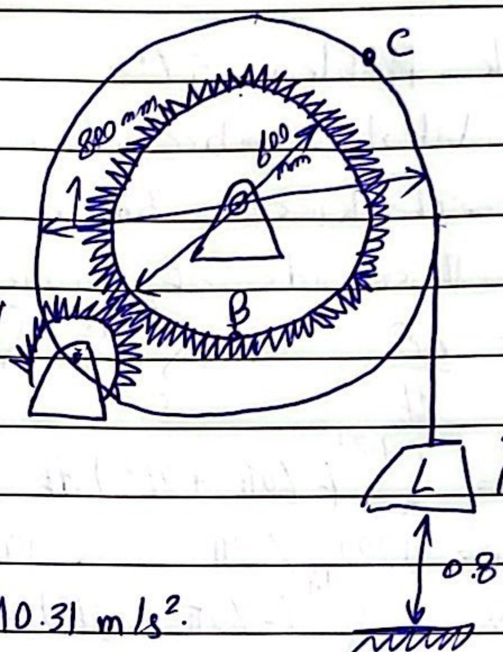
$N = N_1 + N_2 = 194.2 + 65.3 = 259$ rev.

* Sample Problem 5/2 :-
The Pinion A of the hoist motor drives gear (B) which is attached to the hoisting drum. The load (L) is lifted from its rest position and acquires an upward velocity of 2m/s in a vertical rise of 0.8m with constant acceleration. As the load passes this position, compute

a) the acceleration of Point C on the cable in constant with the drum.

* Dynamics and Vibrations :-

* Ch5 :- Plane Kinematics of Rigid Bodies :-



* Sample Problem 5/2 :-

Constant acceleration

$V_o = 0 \qquad V_f = 2\ m/s \qquad \Delta s = 0.8m$

$V_f^2 = V_o^2 + 2a\Delta s$

$4 = 0 + 2(a)(0.8) \Rightarrow \boxed{a = 2.5\ m/s^2}$

$a = a_t = 2.5\ m/s^2$

$a_n = \dfrac{V^2}{r} = \dfrac{2^2}{0.400} = 10\ m/s^2$

$a_c = \sqrt{a_t^2 + a_n^2} = \sqrt{2.5^2 + 10^2} = 10.31\ m/s^2$

b) the angular Velocity and angular acceleration of the Pinion A :-

$V = rW \Rightarrow W_B = V/r = 2/0.4 = 5\ rad/s$

$a_t = r\alpha \Rightarrow \alpha_B = a_t/r = 2.5/0.4 = 6.25\ rad/s^2$

$V_1 = r_A W_A = r_B W_B \Rightarrow$ gears rule

$W_A = \dfrac{r_B}{r_A} W_B = \dfrac{0.3}{0.1}(5) = 15\ rad/s$ CW

$a_i = r_A \alpha_A = r_B \alpha_B \Rightarrow$ gears rule

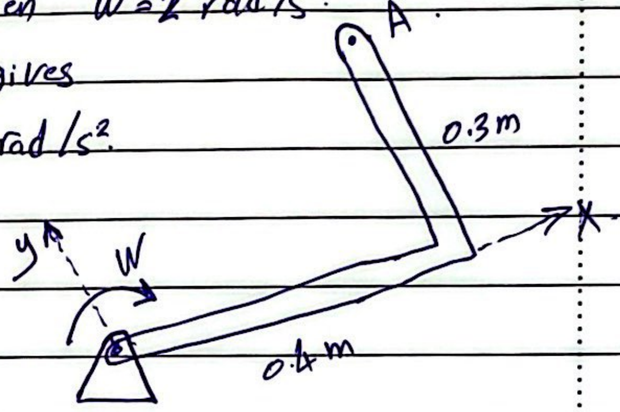
$\alpha_A = \dfrac{r_B}{r_A}\alpha_B = \dfrac{0.3}{0.1}(6.25) = 18.75\ rad/s^2$ CW

Subject Date No.

Dynamics and Vibrations :-
Ch5 :- Plane Kinematics of Rigid Bodies :-

Sample Problem 5/3 :-

The right angle bar rotates clockwise with an angular velocity which is decreasing at the rate of $4\ rad/s^2$. Write the vector expressions for the velocity and acceleration of point (A) when $w = 2\ rad/s$.



Using the right-hand rule gives

$W = -2K\ rad/s$ $\alpha = +4K\ rad/s^2$

$V = W \times r$

$V = -2\hat{k} * (0.4\hat{i} + 0.3\hat{j})$

$V = (0.6\hat{i} - 0.8\hat{j})\ m/s$

$a_n = W \times (W \times r)$

$a_n = -2\hat{k} * (0.6\hat{i} - 0.8\hat{j})$

$a_n = (-1.6\hat{i} - 1.2\hat{j})\ m/s^2$

$a_t = \alpha \times r$ $a = \vec{a_n} + \vec{a_t}$

$a_t = 4k * (0.4\hat{i} + 0.3\hat{j})$ $\vec{a} = (-2.8\hat{i} + 0.4\hat{j})\ m/s^2$

$a_t = (1.6\hat{j} - 1.2\hat{i})\ m/s^2$

The magnitudes of V and a are

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{0.6^2 + 0.8^2} = 1\ m/s$

$a = \sqrt{a_x^2 + a_y^2} = \sqrt{2.8^2 + 0.4^2} = 2.83\ m/s^2$

N O T E B O O K

* Dynamics and Vibrations :-

* Ch5 :- Plane Kinematics of Rigid Bodies :-

* Problems 5/17 :-

The belt driven Pulley and attached disk are rotating with increasing angular Velocity. At a certain instant the speed V of the belt is 1.5 m/s. and the total accele of Point A is 75 m/s². for this instant determine

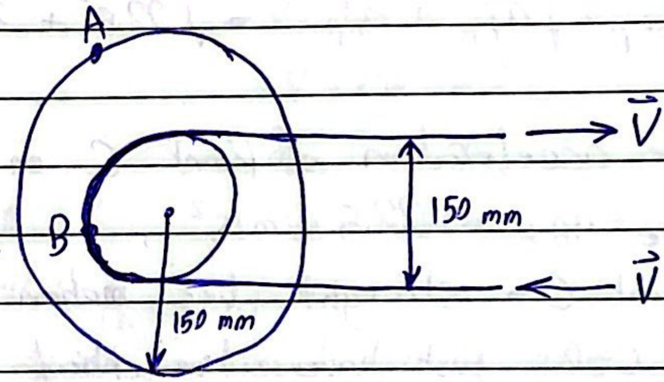
a) the angular acceleration α of the Pulley and disk.

$V_B = 1.5 \ m/s$

$\vec{a}_{tot\,A} = 75 \ m/s^2.$

$V_B = W_B * r_B$

$W_B = \frac{V_B}{r_B} = \frac{1.5}{0.075} = 20 \ rad/s$



$a_{tot\,A} = \sqrt{a_{tA}^2 + a_{nA}^2}$

$a_{tA} = \alpha r_A = 0.15 \alpha.$

$a_{nA} = \frac{V^2}{r} = W^2 r = (20)^2 (0.15) = 60$

$a_{tot\,A} = \sqrt{a_{tA}^2 + a_{nA}^2}$

$a_{tA} = \sqrt{a_{tot\,A}^2 - a_{nA}^2} \Rightarrow 0.15 \alpha = \sqrt{75^2 - 60^2}.$

$\alpha = 300 \ rad/s^2.$

Subject Date No.

Dynamics and Vibrations :-

Ch5 :- Plane Kiniematics of Rigid Bodies :-

Problem 5/17 :-

b) the total acceleration of Point B ~~and~~.

$W_B = 20$ rad/s. $\alpha_B = 300$ rad/s^2.

$a_{n_B} = \frac{V^2}{r} = W_B^2 r_B = (20)^2 (0.075) = 30$ m/s^2.

$a_{t_B} = \alpha r = (300)(0.075) = 22.5$ m/s^2.

$a_{tot_B} = \sqrt{a_{t_B}^2 + a_{n_B}^2} = \sqrt{22.5^2 + 30^2} = 37.5$ m/s^2.

c) the acceleration of Point C on the belt.

$a_c = a_{B_t} = 22.5$ m/s^2.

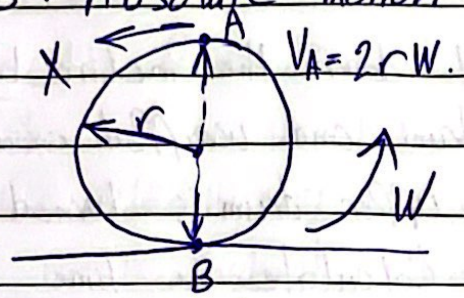
Point C → straight line motion normal acceleration at C = 0 just have a tangential acceleration.

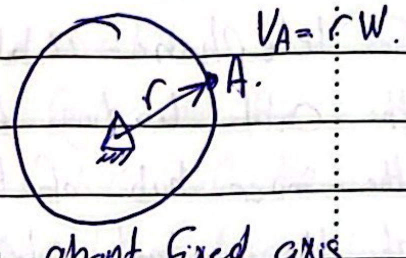
Subject Date No.

Dynamics and Vibrations :-

* Ch5: Plane Kinematics of Rigid Bodies :-

* 5/3 : Absolute motion :





rolling without slipping. ⇒ rotation about fixed axis

⇒ A: without S ⇒ $V_B = 0$. ⇒ at center.

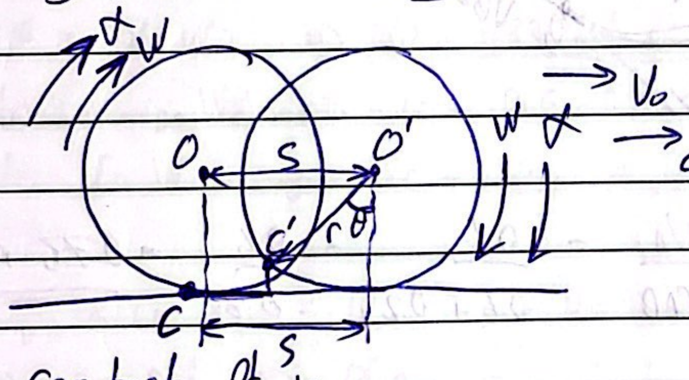
⇒ at center $V_{center} = 0$.

$V = Wr \neq$ zero. $V = rW = 0$.

⇒ at Point B. ⇒ $V_B = 0$. ⇒ at Point A

⇒ at Point A ⇒ $V_A = 2rW$. (rotation) $V_A = r_A W$.

rolling without slipping ⇒ motion about moving axis.



$S = r\theta$.

$V_o = rW$

$a_o = r\alpha$.

C – Contact pt.

new coordinates C' ⟶ $X = S - r\sin\theta$; $y = r - r\cos\theta$.

$\dot{X} = V_o(1 - \cos\theta)$; $\dot{y} = V_o \sin\theta$.

$\ddot{X} = a_o(1-\cos\theta) + rW^2 \sin\theta$; $\ddot{y} = a_o \sin\theta + rW^2\cos\theta$.

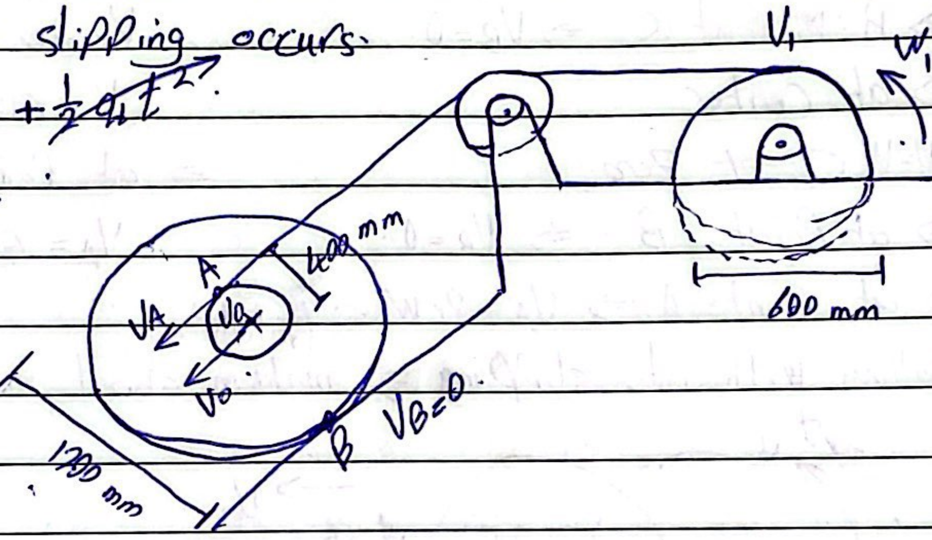
Subject   Date   No.

Dynamics and Vibration

*Ch5:- Plane Kiniemcatics of Rigid Bodies :-

* Problems 5/31 :-

The telephone-cable reel is rolled down the incline by the cable leading from the upper drum and wrapped around the inner hub of the reel. If the upper drum is turned at the constant rate $W_1 = 2$ rad/s. Calculate the time required for the center of the reel to move 30m along the incline. No slipping occurs.



$\Delta s = V_0 t + \frac{1}{2} a_t t^2$??

$V_0 = \frac{S}{t} = \frac{30}{t}$

$t = \frac{30}{V_0} \rightarrow$??

$V_1 = V_A$

$V_A = r_1 W_1$

$V_A = (0.3)(2) = 0.6$ m/s

$V_A = r_{AB} W_2 \Rightarrow W_2 = \frac{V_A}{r_{AB}} = \frac{0.6}{0.6 + 0.2} = \frac{0.6}{0.8} = 0.75$ rad/s

$V_0 = r_{OB} W_2$

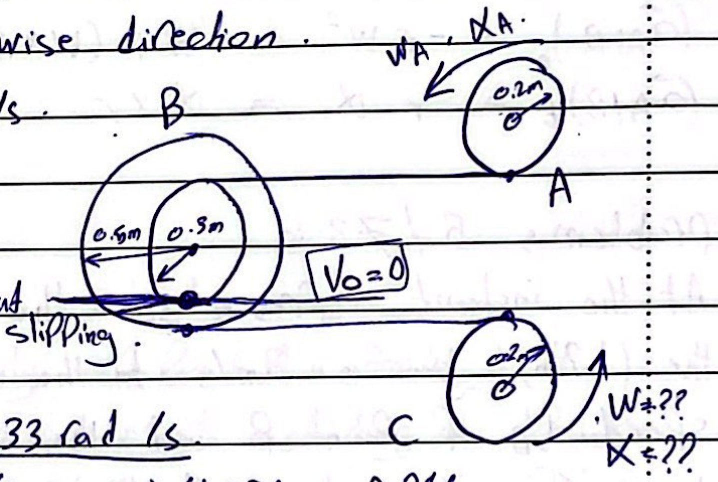
$= (0.6)(0.75) = 0.45$ m/s

$t = \frac{30}{V_0} = \frac{30}{0.45} = 66.7$ sec

Subject Date No.

* Dynamic and Vibrations
* Ch5 :- Plane Kiniematics of Rigid Bodies :-

* Problems 5/47 :-
The Cable from drum A turns the double Wheel B, which rolls on its hubs without slipping. Determine the anglar Velocity (W) and angular acceleration (α) of drum (C) for the instant when the angular Velocity and angular acceleration of A are 4 rad/s and 3 rad/s² respectively both in the counterclockwise direction.



$W_A = 4$ rad/s $\quad \alpha_A = 3$ rad/s.

$V_A = W_A\, r_A \Rightarrow$ [crossed out]

$V_A = (4)(0.2) = \underline{0.8}$

$V_A = V_B$ ⟶ rolling without slipping.

$V_A = (0.3 + 0.3)\, W_B$.

$0.8 = 0.6\, W_B \Rightarrow \underline{W_B = 1.33 \text{ rad/s}}$

$V_c = r_{BO} W_B \Rightarrow V_c = (0.5 - 0.3)(1.33) = 0.266$

$V_c = r_c W_c \Rightarrow W_c = \dfrac{V_c}{r_c} = \dfrac{0.266}{(0.2)} = \underline{1.33 \text{ rad/s}}$.

$a_{tA} = r_A\, \alpha_A = (0.2)(3) = 0.6$ rad/s²

$a_{tB} = 0.6 \Rightarrow \alpha_B = \dfrac{0.6}{0.6} = 1$ rad/s².

$a_{tc} = r\, \alpha_B \Rightarrow a_{tc} = (0.5 - 0.3)(1) = 0.2$ rad/s².

$a_{tc} = r_c\, \alpha_c \Rightarrow \alpha_c = \dfrac{a_{tc}}{r_c} = \dfrac{0.2}{0.2} = 1$ rad/s².

* Dynamics and Vibrations :-

* Ch 5 :- Plane Kinematics of Rigid Bodies :-

* 5/4 and 5/6 → Relative motion

Relative Velocity + Relative acceleration.

$\vec{V}_A = \vec{V}_B + \vec{V}_{A/B}$

$\vec{V}_{A/B} = r\,W = \vec{W} \times \vec{r}$

$\vec{a}_A = \vec{a}_B + \vec{a}_{A/B}$.

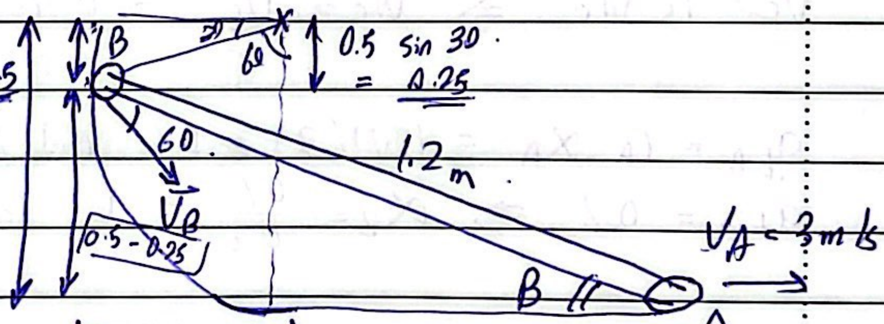
$\vec{a}_{A/B} = (\vec{a}_{A/B})_n + (\vec{a}_{A/B})_t$.

$(\vec{a}_{A/B})_n = -r W^2 = W \times (W \times r)$.

$(\vec{a}_{A/B})_t = r\,\alpha = \alpha \times r$.

* problems 5/73 :-

At the instant represented, the velocity of point A of the (1.2m) bar is 3m/s to the right. Determine the speed V_B of point B and the angular velocity W of the bar. The diameter of the small end wheels may be neglected.



$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$

$V_B \cos 60\,\hat{i} - V_B \sin 60\,\hat{j}$

$= 3\hat{i} + \vec{V}_{A/B}$.

$\vec{V}_{B/A} = \vec{W} \times \vec{r}_{B/A}$

$= W\hat{k} \times (-1.2 \cos\beta\,\hat{i} + 1.2 \sin\beta\,\hat{j})$

$\sin\beta = \frac{0.5 - 0.25}{1.2} \Rightarrow \beta = 12.02^\circ$.

* Dynamics and Vibrations :-

* Ch5 :- Plane of Kinematics of Rigid Bodies :-

* Problem 5/73 :- تابع

$\vec{V}_{B/A} = \vec{W} \times \vec{r}_{B/A} = W\hat{k} \times (-1.2 \cos 12.02 \hat{i} + 1.2 \sin 12.02 \hat{j})$

$= -1.174 W\hat{j} - 0.25 W\hat{i}$.

$\vec{V}_B = \vec{V}_A + \vec{V}_{B/A}$.

$V_B \sin 30 \hat{i} - V_B \cos 30 \hat{j} = 3\hat{i} - 1.174 W\hat{j} - 0.25 W\hat{i}$.

$V_B \sin 30 = 3 - 0.25 W$. --- ①

$-V_B \cos 30 = -1.174 W$ --- ②

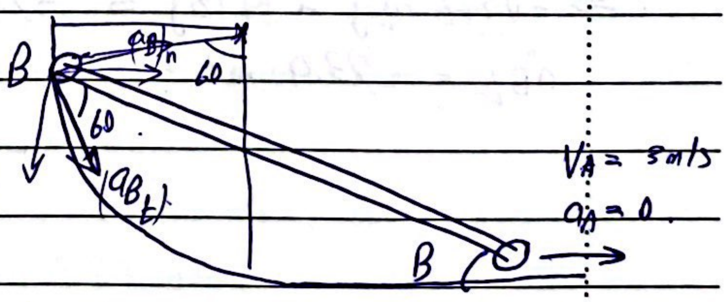
$\Rightarrow V_B = 4.38$ m/s .

$W = 3.23$ rad/s .

* Problems 5/140 :-

The bar AB. if the Velocity of Point A is 3m/s to the right and is constant for an interval including the Position shown. determine the tangential acceleration of Point B along its Path and the angular acceleration of the bar

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$;



$\vec{a}_B = (\vec{a}_B)_t + (a_B)_n$

$\vec{a}_B = (a_{t_B} \cos 60 \hat{i} - (a_{t_B}) \sin 60 \hat{j})$

$+ (\vec{a}_{n_B} \cos 30 \hat{i} + a_{n_B} \sin 30 \hat{j})$

$\vec{a}_{B/A} = (\vec{a}_{B/A})_n + (\vec{a}_{B/A})_t$

$(\vec{a}_{B/A})_t = \alpha \hat{k} \times \vec{r}_{B/A} = \alpha\hat{k} \times (-1.2 \cos \beta \hat{i} + 1.2 \sin \beta \hat{j})$.

$(\vec{a}_{B/A})_n = -W^2 \vec{r}_{B/A} = -W^2(-1.2 \cos\beta \hat{i} + 1.2 \sin\beta \hat{j})$.

* Dynamics and Vibrations :—

* Ch5 : Plane Kiniematics of Rigid Bodies :—

* Problem 5/140 : حل .

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$.

$\Rightarrow \vec{a}_B = (\vec{a}_B)_t + (\vec{a}_B)_n$

$\vec{a}_B = (a_t \cos 60 \hat{i} - a_t \sin 60 \hat{j}) + (a_n \cos 30 \hat{i} + a_n \sin 30 \hat{j})$

$a_t \rightarrow$ ~~[illegible]~~ $\quad a_{n_B} = V_B^2 / r = 38.36$.

$\vec{a}_B = \frac{1}{2} a_t \hat{i} - 0.866 a_t \hat{j} + 33.2 \hat{i} + 19.18 \hat{j}$.

* $\vec{a}_{B/A} = (\vec{a}_{B/A})_n + (a_{B/A})_t$

$\Rightarrow (\vec{a}_{B/A})_n = -W^2 r_{B/A} = -W^2(-1.2 \cos 12.02 \hat{i} + 1.2 \sin 12.02 \hat{j})$

$(\vec{a}_{B/A})_n = 12.24 \hat{i} - 2.6 \hat{j}$.

$\Rightarrow (\vec{a}_{B/A})_t = \alpha \hat{k} \times \vec{r}_{B/A} = \alpha \hat{k} \times (-1.2 \cos 12.02 \hat{i} + 1.2 \sin 12.02 \hat{j})$

$(\vec{a}_{B/A})_t = 1.17 \alpha \hat{j} - 0.24 \alpha \hat{i}$.

$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A}$

$\underline{\frac{1}{2} a_t \hat{i}} - 0.866 a_t \hat{j} + \underline{33.2 \hat{i}} + 19.18 \hat{j} = 0 + \underline{12.24 \hat{i}} - 2.6 \hat{j}$

$+ 1.17 \alpha \hat{j} - \underline{0.24 \alpha \hat{i}}$.

$\Rightarrow \frac{1}{2} a_t \hat{i} + 33.2 \hat{i} = 12.24 \hat{i} - 0.24 \alpha \hat{i}$ — (1) .

$\Rightarrow -0.866 a_t \hat{j} + 19.18 \hat{j} = -2.6 \hat{j} + 1.17 \alpha \hat{j}$ — (2) .

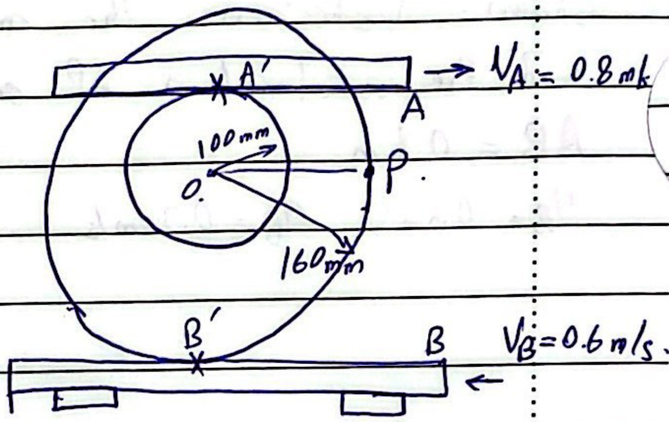
$\vec{a}_{B_t} = -23.9 \ m/s^2 \quad , \quad \alpha = -36.2 \ rad/s$.

Subject Date No.

* Dynamics and Vibrations :-
* Ch5 : Plane Kinematics of Rigid Bodies :-

* Problem 5/75 :-

Each of the sliding bars (A) and (B) engages its respective rim of the two riveted wheels without slipping. Determine the magnitude of the velocity of point P for the position shown :



$V_{A'} \neq 0$, $V_{B'} \neq 0$, $V_o \neq 0$.

$\vec{V}_P = \vec{V}_o + \vec{V}_{P/O}$.

$\vec{V}_P = \vec{V}_o + \vec{W} \times \vec{r}_{P/O}$.

$\vec{V}_P = V_o + \vec{W} \times (0.16\,\hat{i})$

$W = \dfrac{V_A + V_B}{r_A + r_B} = \dfrac{0.8 + 0.6}{0.1 + 0.16}$

$W = \dfrac{1.4}{0.26} = 5.38$ CW $\rightarrow \boxed{W = -5.38\hat{k}}$

$\vec{V}_P = \vec{V}_o + (-5.38\hat{k})(0.16\hat{i}) \Rightarrow \vec{V}_P = \vec{V}_o - 0.862\hat{j}$.

$\vec{V}_o = \vec{V}_A + \vec{V}_{O/A} \Rightarrow \vec{V}_o = \vec{V}_A + \vec{W} \times \vec{r}_{O/A}$

$\vec{V}_o = 0.8\hat{i} - 5.38\hat{k} \times 0.1(-\hat{j})$

$= 0.8\hat{i} - 0.38\hat{i}$. $= 0.262\hat{i}$. $\Rightarrow \boxed{\vec{V}_o = 0.262\hat{i}}$

$\rightarrow \vec{V}_P = \vec{V}_o - 0.862\hat{j}$.

$\therefore \vec{V}_P = 0.262\hat{i} - 0.862\hat{j}$.

$|\vec{V}_P| = \sqrt{0.262^2 + 0.862^2} = 0.90$ m/s .

$\theta_P = \tan^{-1}\left(\dfrac{V_{P_y}}{V_{P_x}}\right) = \tan^{-1}\left(\dfrac{-0.862}{0.262}\right) = -73.09°$.

N O T E B O O K

Subject Date No.

Dynamics and Vibrations :-

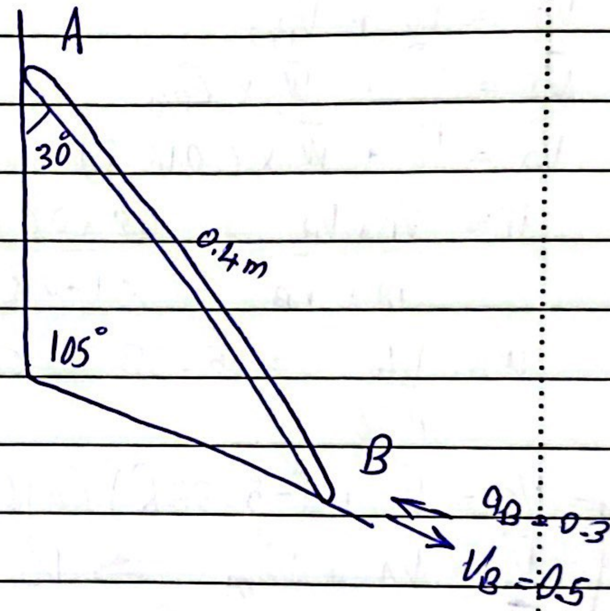
* Ch5 :- Plane Kiniematics of Rigid Bodies :-

* Problems 5/127 :-

The ends of the 0.4 m bar remain in contact with their respective support surfaces. End B has a velocity of 0.5 m/s and an acceleration of 0.3 m/s^2 in the direction shown. Determine the angular acceleration of the bar and the acceleration of end A.

$AB = 0.4 m$

$V_B = 0.5 m/s$ $a_B = 0.3 m/s$



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Subject Date No.

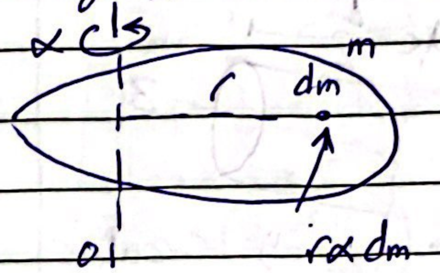
* Dynamics and Vibrations :-

* Ch6 :- plane Kinetics of Rigid Bodies :-

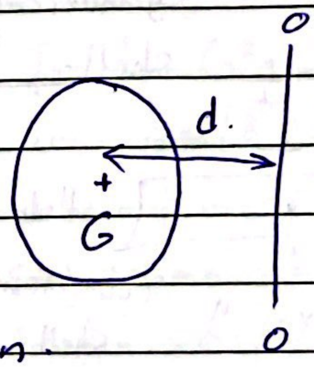
* Appendix B :- Mass moments of Inertia about an axis :-

Mass moment of Inertia (I) of the body about the axis O-O :-

$$\boxed{I = \int r^2 \, dm}$$



* Theorem : Parallel axis theorem



$$I_{oo} = I_G + m d^2 .$$

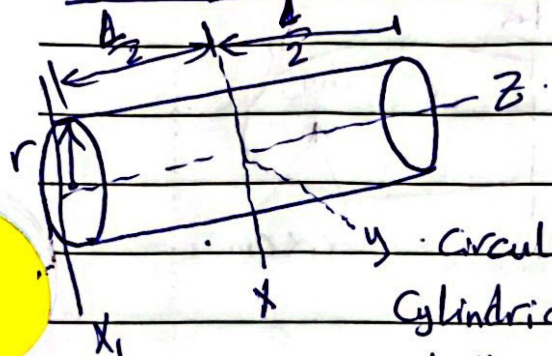
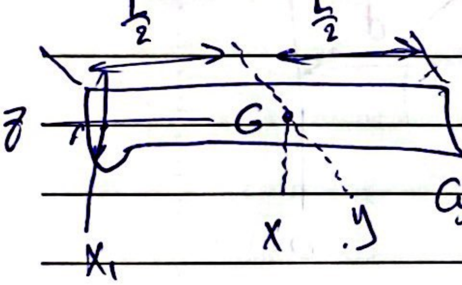
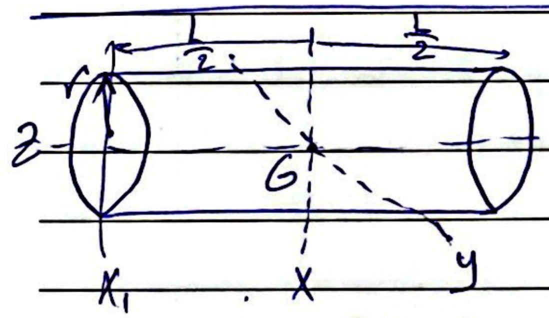
$$I = I_{oo} - I_G .$$

$$I = m d^2 .$$

$d = \sqrt{\frac{I}{m}}$, d : radius of Gyration.

* Dynamics and Vibration
* Ch6:- Plane Kinetics of Rigid body:-

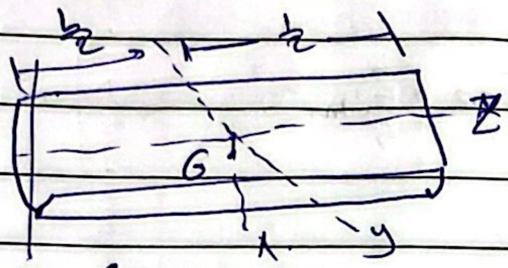
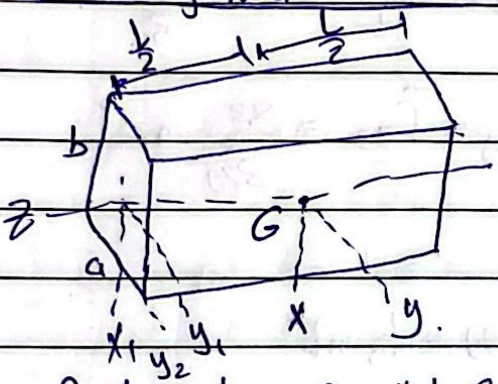
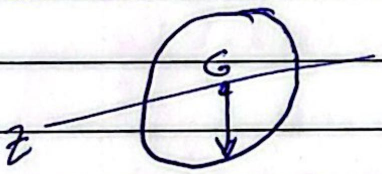
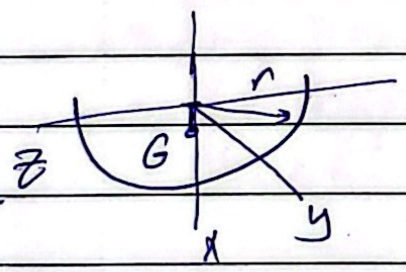
* Properties of homogeneous solids:-

Body	mass center	mass moment of Inertia
 Circular Cylindrical shell.	—	$I_{xx} = \frac{1}{2}mr^2 + \frac{1}{12}mL^2$ $I_{x_1x_1} = \frac{1}{2}mr^2 + \frac{1}{3}mL^2$ $I_{zz} = mr^2$.
 Half Cylindrical shell.	$\bar{X} = \frac{2r}{\pi}$	$I_{xx} = I_{yy} = \frac{1}{2}mr^2 + \frac{1}{12}mL^2$ $I_{x_1x_1} = I_{y_1y_1} = \frac{1}{2}mr^2 + \frac{1}{3}mL^2$ $I_{zz} = mr^2$. $\bar{I}_{zz} = \left(1 - \frac{4}{\pi^2}\right) mr^2$
 Circular Cylinder	—	$I_{xx} = \frac{1}{4}mr^2 + \frac{1}{12}mL^2$ $I_{x_1x_1} = \frac{1}{4}mr^2 + \frac{1}{3}mL^2$ $I_{zz} = \frac{1}{2}mr^2$.

* Dynamics and Vibrations :-

* Ch6 :- Plane kinetics of Rigid body :-

* Properties of homogeneous solids.

Body	mass Center	mass moment of inertia
 Semicylinder	$\bar{X} = \frac{4r}{3\pi}$	$I_{xx} = \frac{1}{4}mr^2 + \frac{1}{12}ml^2$ $I_{x_1x_1} = \frac{1}{4}mr^2 + \frac{1}{3}ml^2$. $I_{zz} = \frac{1}{2}mr^2$ $\bar{I}_{zz} = \left(\frac{1}{2} - \frac{16}{9\pi^2}\right)mr^2$
 Rectangular Parallelepiped	—	$I_{xx} = \frac{1}{12}m(a^2+l^2)$. $I_{yy} = \frac{1}{12}m(b^2+l^2)$ $I_{zz} = \frac{1}{12}m(a^2+b^2)$ $I_{y_1y_1} = \frac{1}{12}mb^2 + \frac{1}{3}ml^2$ $I_{y_2y_2} = \frac{1}{3}m(b^2+l^2)$.
 spherical shell	—	$I_{zz} = \frac{2}{3}mr^2$.
 hemispherical shell	$\bar{X} = \frac{r}{2}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{3}mr^2$ $\bar{I}_{yy} = \bar{I}_{zz} = \frac{5}{12}mr^2$.

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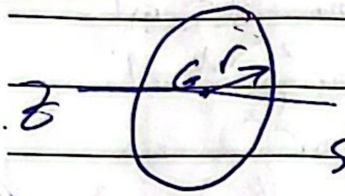
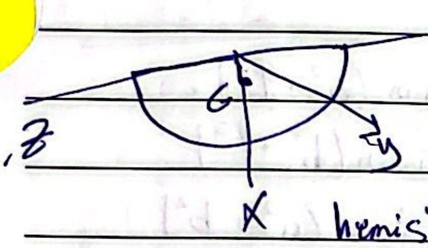
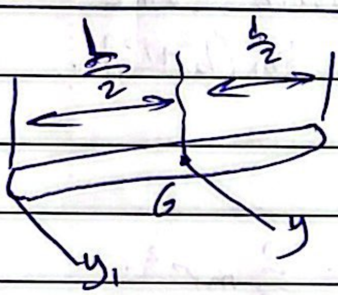
* Dynamics and Vibrations :-

* Ch6 :- Plane Kinetics of Rigid body :-

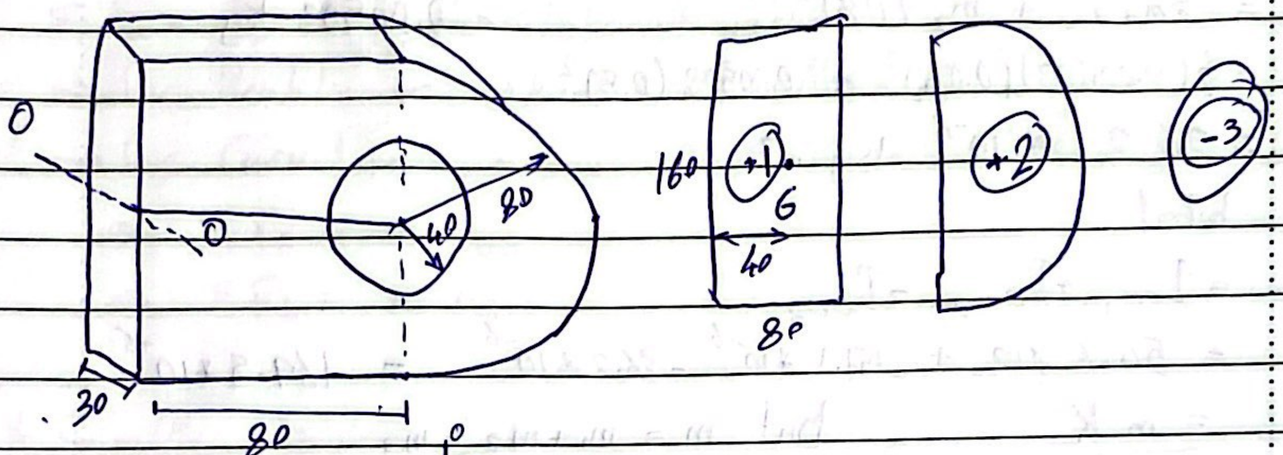
* Properties of homogeneous solids :-

Body	mass center	mass moment of inertia
 sphere	—	$I_{zz} = \frac{2}{5} m r^2$
 hemisphere	$\bar{x} = \frac{3r}{8}$	$I_{xx} = I_{yy} = I_{zz} = \frac{2}{5} m r^2$ $\bar{I}_{yy} = \bar{I}_{zz} = \frac{83}{320} m r^2$
 Uniform slender Rod	—	$I_{yy} = \frac{1}{12} m L^2$ $I_{y_1 y_1} = \frac{1}{3} m L^2$

* Dynamics and Vibrations :-

* Ch6 :- Plane Kinetics of Rigid body :-

* Example B/54 :- The machine element is made of steel and is designed to rotate about axis O-O. Calculate it's radius of gyration (K) about this axis :-



for 1 :-

$$I_{oo_1} = I_{G_1} + m_1 d_1^2 \qquad \boxed{I_G = I_{y_1 y_1}}$$

$b = 160$. $m_1 = a * b * L$.

$L = 80$ $\boxed{m_1 = 0.0128\ kg}$

$a = 30$.

$$I_{oo_1} = I_{y_1 y_1}$$

$$= \frac{1}{12} m_1 b^2 + \frac{1}{3} m_1 L^2$$

$$= \frac{1}{12}(0.0128)(0.16)^2 + \frac{1}{3}(0.0128)(0.08)^2 = \underline{54.6 * 10^{-6}\ Kg \cdot m^2}.$$

for 2 :-

$$I_{oo_2} = I_{G_2} + m_2 (d_2 + 80)^2 \qquad \boxed{I = I_G + m_2 d_2^2}$$

d_2 : Centroid.

$$= \frac{1}{2} m_2 r^2 - m_2 (0.034)^2 + m_2 (0.114)^2.$$

$d_2 = \frac{4r}{3\pi} = 0.034\ m.$

$$= \frac{1}{2}(0.01005)(0.8)^2 - 0.01005(0.034)^2 + 0.01005(0.114)^2.$$

$$= 151.1 * 10^{-6}\ Kg \cdot m^2.$$

$m_2 = \frac{1}{2}\pi (0.8)^2$

$= 0.01005.$

Subject Date No.

Dynamics and Vibrations :-
Ch 6 :- Plane Kinetics of Rigid Body :-

*Example B/54 :- 2)

for 3 :-

$I_{oo_3} = I_{G_3} + m_3 d_3^2$ $\qquad m = \pi (0.04)^2$

$= \frac{1}{2} m_3 r^2 + m_3 (0.8)^2$ $\qquad = 0.00503$ Kg.

$= \frac{1}{2}(0.00503)(0.04)^2 + 0.00503 (0.8)^2$.

$= 36.2 * 10^{-6}$ Kg.m².

for total :

$I_{oo} = I_{oo_1} + I_{oo_2} - I_{oo_3}$.

$= 54.6 * 10^{-6} + 151.1 * 10^{-6} - 36.2 * 10^{-6} = 169.5 * 10^{-6}$.

$I_{oo} = m K_o^2$ $\qquad$ but $m = m_1 + m_2 - m_3$

$K = \sqrt{\dfrac{169.5 * 10^{-6}}{0.01783}} = 97.5$ mm $\qquad m = 0.01783$ Kg.

N O T E B O O K

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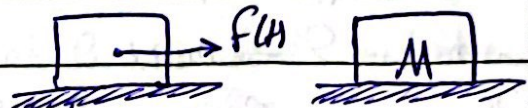
Subject Date No.

* Dynamics and Vibrations :-
* Ch6 - Plane Kinetics of Rigid Body :-

* Plane Kinetics of Rigid Body :- $\boxed{6.1,\ 6.2,\ 6.3}$

$\Sigma F_x = m\, a_{G,x}$

$\Sigma F_y = m\, a_{G,y}$



$\Sigma M = I_G\, \alpha$ for Rotational motion

→ for Rectilinear motion ⇒ $\Sigma M_G = 0$.

→ for Curvilinear motion ⇒ $\Sigma M_G = 0$ $\boxed{\alpha = 0,\ \omega = 0}$

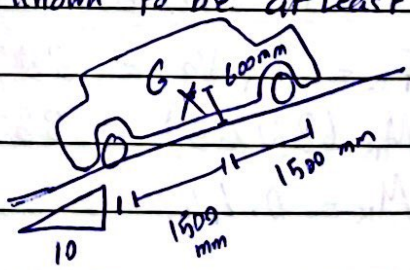
⇒ $\Sigma F_n = m\, a_{G,n}$

⇒ $\Sigma F_t = m\, a_{G,t}$

* Sample Problem 6/1 :-

The 1500 kg Pickup truck reaches a speed of 50 Km/h from rest in a distance of 60 m up the 10-Percent incline with constant acceleration. Calculate the normal force under each pair of wheels and the friction force under the rear driving wheels. The effective coefficient of friction between the tires and the road is known to be at least 0.8

$\theta = \tan^{-1}\left(\frac{1}{10}\right) = 5.71°$



$V_f = 50\ \text{Km/h} = 13.8\ \text{m/s}$

$V_i = 0$ $\qquad \Delta s = 60\ \text{m}$

$V_f^2 = V_i^2 + 2a_x \Delta s$

$(13.8)^2 = 0 + 2a(60)$

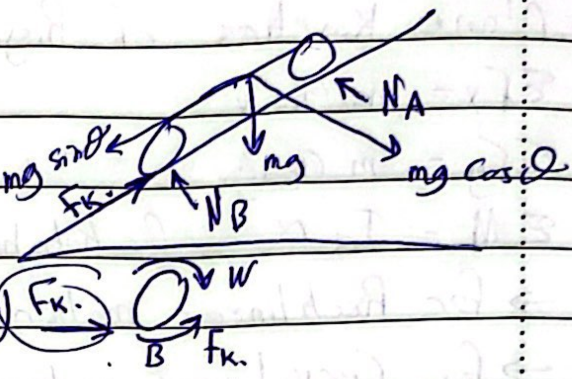
$\boxed{a_x = 1.607\ \text{m/s}^2}$

Subject Date No.

Dynamics and Vibrations :-

*Ch 6 :- Plane Kinetics of Rigid Body :-

* Sample problem 6/1 :-



$\Sigma F_x = m a_x$.

$F_x - mg\sin\theta = m a_x$.

$\underline{\mu_k N_B} - mg\sin\theta = m(1.607)$

$\underline{\mu_k N_B} - (1500)(9.81)\sin(5.71) = (1500)(1.607)$

$F_k = \mu_k N_B = 3875$ ---- ①

$\Sigma F_y = 0$.

$N_A + N_B - mg\cos\theta = 0$.

$N_A + N_B = 14641.9$ ---- ②

$\Sigma M_G = 0$.

$N_A(1.5) - N_B(1.5) + F_k(0.6) = 0$. ---- ③.

$1.5 N_A - 1.5 N_B + 3875(0.6) = 0$.

$N_A = N_B - 2583.3$. ---- ④

$N_A + N_B = 14641.9$.

$N_B - 2583.3 + N_B = 14641.9$.

$\boxed{N_B = 8612.6 \text{ N}}$ $N_A = 6029.3$ N.

$F_k = \mu_k N_B = 3875$.

$\mu_k(8612.6) = 3875$

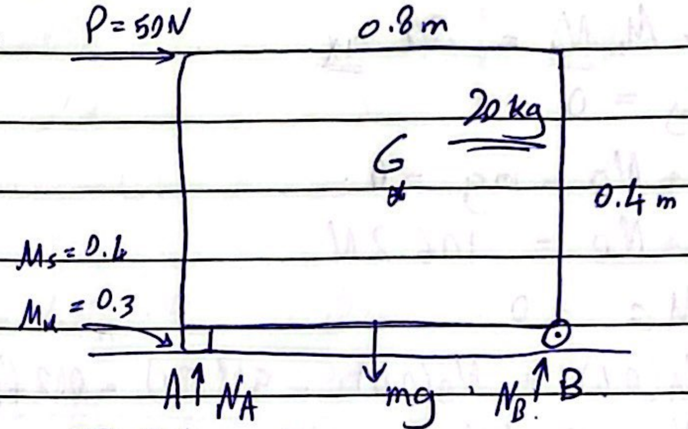
$\mu_k = 0.44$.

Subject Date No.

Dynamics and Vibrations :-

Ch 6 :- Plane Kinetics of Rigid Body :-

* Problem 6/19 :- Determine the acceleration of the initially stationary 20 kg body when the 50 N force P is applied as shown. The small wheels at B are ideal and the feet at A are small.



$\Sigma F_x = 0$.

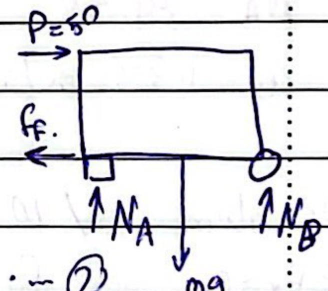
$50 - f_f = 0$.

$f_f = 50\ N$.

$F_{f\,max} = M_k N_A$:

~~$F_{f\,max} = M_k (mg)$~~

$= 0.3\cancel{(20)(9.81)} = \cancel{58.86}$.



$\Sigma F_y = 0$.

$N_A + N_B - mg = 0$. ... ①

$\Sigma M_G = 0$.

$N_B(0.4) - N_A(0.4) - f_f(0.2) - 50(0.2) = 0$. ... ②

$0.4 N_B - 0.4 N_A - (50)(0.2) = 0$.

$\underline{N_A = 73.1\ N}$ $\underline{N_B = 123\ N}$

$F_{f\,max} = M_k N_A = (0.3)(73.1) = 29.93$.

$f_{f\,max} < f_f \Rightarrow$ motion

N O T E B O O K

*Dynamics and Vibrations :-
*Ch6: Plane Kinetics of Rigid Body :-

*Problem 6/9 :- ٢٠V

$\Sigma F_x = m a_x$

$50 - f_k = m a_x$

$50 - \mu_k \underline{N_A} = 20 \underline{a_x}$

$\Sigma F_y = 0$

$N_A + N_B - mg = 0$

$\underline{N_A} + \underline{N_B} = 196.2\,N$

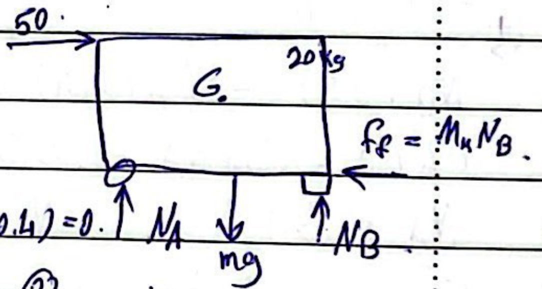
$\circlearrowleft^{+} \Sigma M_G = 0$

$-N_A(0.4) + N_B(0.4) - 50(0.2) - 0.2\underline{f_k} = 0$

$N_A = 79.6\,N \qquad N_B = 116.6\,N$

$a_x = 1.306\ m/s^2 \longrightarrow (+)$

*Problem 6/10 :-



$\Sigma F_x = 0 \Rightarrow f_f = 50$

$\Sigma F_y = 0 \Rightarrow N_A + N_B = mg = 196.2$ — (1)

$\circlearrowleft^{+} \Sigma M_G = 0 \Rightarrow N_B(0.4) - 50(0.2) - f_f(0.2) - N_A(0.4) = 0$

$0.4 N_B - 0.4 N_A = 20 \Rightarrow N_B = N_A + 50$ — (2)

$\boxed{N_A = 73.1\,N} \qquad \boxed{N_B = 123.1\,N}$

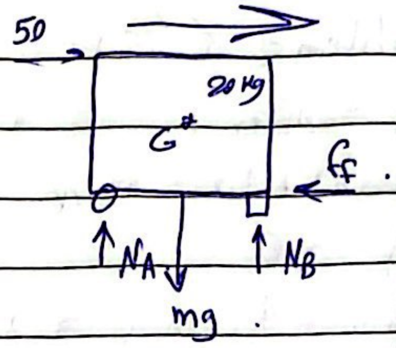
$F_{f\,max} = \mu_s N_B = (0.4)(123.1) = 49.24$

$f_{f\,max} < f_f \Rightarrow$ motion occurs.

* Dynamics and Vibrations :-

* Ch 6 :- Plane Kinetics of Rigid Body.

* Problem 6/10 :- . ૨૫



$\Sigma F_x = m a_x$.

$50 - f_f = m a_x$.

$50 - \mu_K N_B = 20 a_x$.

$\Sigma F_y = 0$

$N_A + N_B = mg = 196.2$. ---①

$\circlearrowleft + \Sigma M_G = 0$.

$0.4 N_B - 50(0.21) - 0.2 f_f - 0.4 N_A = 0$. ---②

$0.34 N_B - 0.4 N_A = 10 \Rightarrow N_A = 0.85 N_B - 25$.

$\boxed{N_B = 119.5\ N}$ $\boxed{N_A = 76.57\ N}$

$50 - 0.3(119.5) = 20 a_x$.

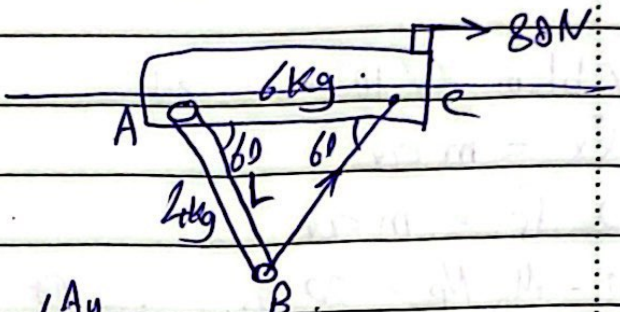
$a_x = 0.7075\ m/s^2$.

* Problems 6/13 :-

The 6 Kg frame AC and 4 kg uniform slender bar AB of length (l) slide with negligible friction along the fixed horizontal rod under the action of the 80 N force. Calculate the tension T in wire BC and the x and y components of the force exerted on the bar by the pin at A. The x-y plane is vertical.

Dynamics and Vibrations :-

Ch6 :- Plane Kinetics of Rigid Body :-



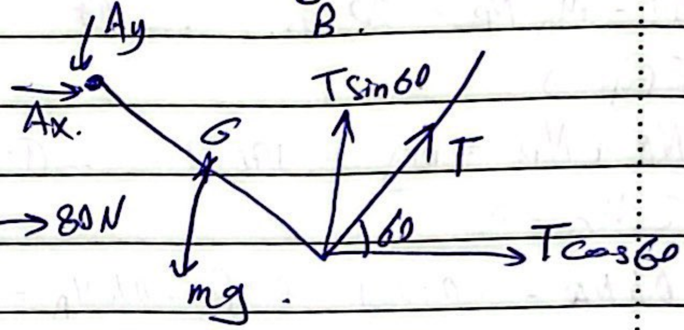
Problems 6/13 :- حلها

$\overset{+}{\rightarrow}\Sigma F_x = m a_x$

$T\cos 60 + A_x = m a_x \quad \sim (1)$

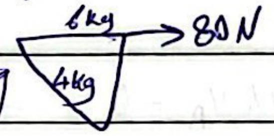
$+\!\uparrow \Sigma F_y = 0$

$T\sin 60 - A_y - mg = 0 \quad \sim (2)$



$\Sigma F_x = m a_x$ للجسم كامل

$80 = m a_x$



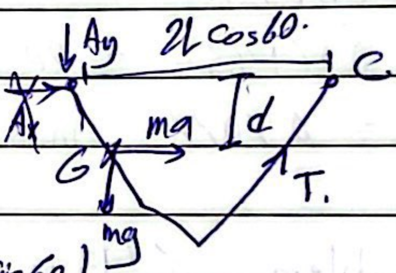
$a_x = 80/10 = \boxed{8\ m/s^2}$

from (1) : $T\cos 60 + A_x = 8m$

(2) : $T\sin 60 - A_y - mg = 0$

$T\cos 60 + A_x = 32 \quad \sim (1)$

$T\sin 60 - A_y = 39.24 \quad \sim (2)$



$\overset{+}{\circlearrowleft} \Sigma M_C = ma \cdot d$

$A_y(2L\cos 60) + mg\left(\tfrac{3}{2}L\cos 60\right) = ma\left(\tfrac{1}{2}L\sin 60\right)$

$\cancel{L}(2A_y\cos 60) + 4(9.81)\left(\tfrac{3}{2}\cos 60\right) = \cancel{L}4(8)\left(\tfrac{1}{2}\sin 60\right)$

$\Rightarrow A_y = -15.57\ N$ عكس الاتجاه المفروض

(2) $T\sin 60 - A_y = 39.24$

$T = 27.3\ N$

(1) $T\cos 60 + A_x = 32$

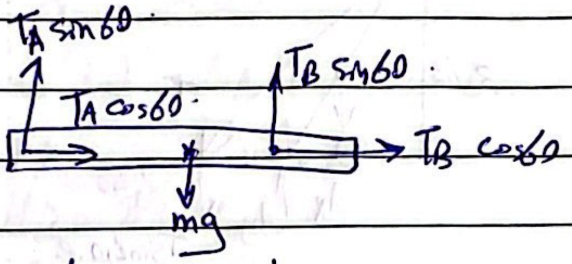
$A_x = 18.34\ N$

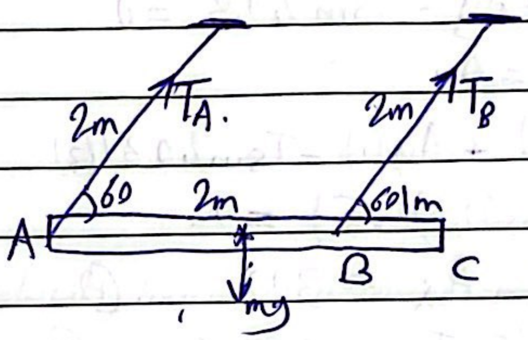
Subject Date No.

* Dynamics and Vibrations :-
* Ch6 :- Plane kinetics of Rigid Body :-

* Problems 6/19 :-

The uniform 100 kg log is supported by the two cables and used as a battering ram. if the log is released from rest in the position shown. Calculate the initial tension induced in each cable immediately after release and the corresponding angular acceleration (α) of the cables.



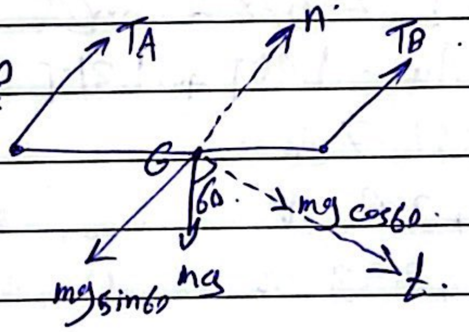


Curvelinear motion :-

$* \ \Sigma F_n = m a_n$

but $a_n = V^2/r \quad V=0 \Rightarrow \underline{\underline{a_n = 0}}$

$T_A + T_B - mg \sin 60 = 0 \quad \sim (1)$



$* \ \Sigma F_t = m a_t$

$\cancel{m} g \cos 60 = \cancel{m} \, r \alpha$

$\boxed{\alpha = 2.45 \text{ rad/s}}$

$\circlearrowleft + \ \Sigma M_G = 0$

$(T_B \sin 60)(0.5) - (T_A \sin 60)(1.5) = 0 \quad \sim (2)$

$T_B + T_A = 849.57 \quad \sim (1)$

$0.43 T_B - 1.29 T_A = 0 \quad \sim (2)$

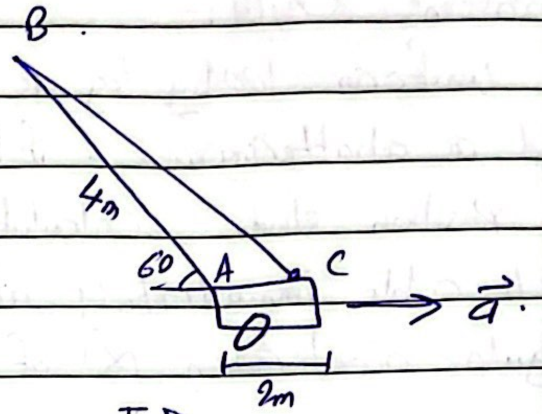
$\boxed{\begin{aligned} T_A &= 212 \text{ N} \\ T_B &= 637 \text{ N} \end{aligned}}$

Subject Date No.

* Dynamics and Vibrations :-

* Ch6 :- Plane kinetics of Rigid Body :

* problems 6/16 :-



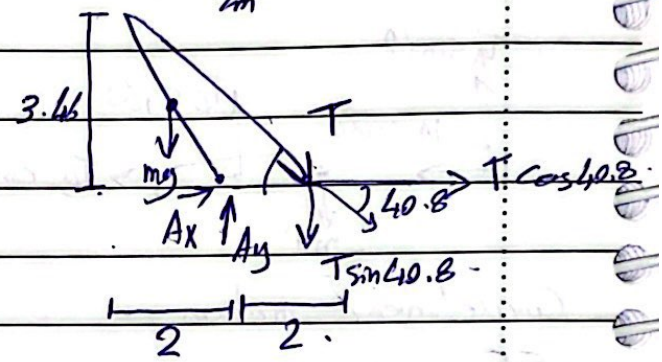
$\theta = \tan^{-1}\left(\frac{3.46}{4}\right) = 40.85$

$\Sigma F_x = m a_x$

$T\cos 40.8 + A_x = (60)(5)$ ... ①

$\Sigma F_y = 0$

$-mg + A_y - T\sin 40.8 = 0$



$\Sigma M_G = 0$

$A_x(\sqrt{3}) + A_y(1) - (T\sin 40.8)(3) + (T\cos 40.8)(\sqrt{3}) = 0$

$0.756T + A_x = 300$ .... ①

$A_y - 0.653T = 588.6$ ... ②

$1.73A_x + A_y - 1.96T + 1.31T = 0$ ... ③

$A_x = -341.7\ N$

$A_y = 1142.8\ N$

$T = 848.8\ N$

N O T E B O O K

Subject Date No.

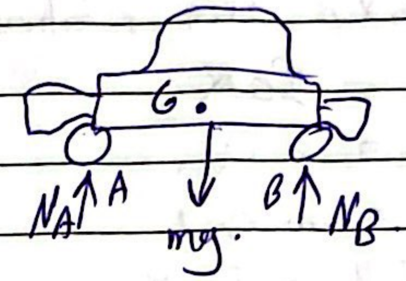
* Dynamics and Vibrations :-

* Ch6 :- Plane Kinetics of Rigid Body :-

* Problems 6/14 :-

$m = 1500\ Kg$.

A) being stationary.



$\Sigma F_y = 0$.

$N_A + N_B - mg = 0$.

$N_A + N_B = 14715$.

$\circlearrowleft \Sigma M_G = 0$.

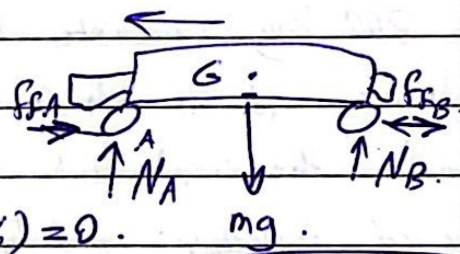
$N_B(1.1) - N_A(1.65) = 0$.

$N_A = 5886\ N$

$N_B = 8829\ N$.

b) braking from a forward Velocity V with all wheels locked.

$\Sigma F_y = 0$.



$N_A + N_B = 14715$.

$\Sigma M_G = 0$.

$N_B(1.1) - N_A(1.65) + f_{fB}(0.6) + f_{fA}(0.6) = 0$.

$1.1 N_B - 1.65 N_A + 0.54 N_B + 0.54 N_A = 0$.

breaking بكون عكس الحركة

$1.64 N_B - 1.11 N_A = 0$.

$N_A = 8775.4\ N$.

$N_B = 5939.5\ N$.

Dynamics and Vibrations :-

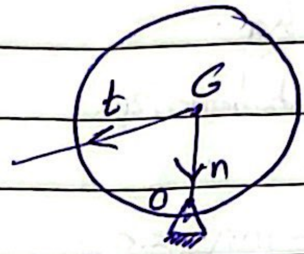
Ch6 :- Plane Kinetics of Rigid Body :-

6/4 :- Fixed axis rotation :-

$$\circlearrowleft + \sum M_G = I_G \alpha$$

$$\sum F_n = m\, a_{G,n}$$

$$\sum F_t = m\, a_{G,t}$$



also $\sum M_O = I_O \alpha$.

$$= (I_G + m d^2)\, \alpha$$

note that.

$$a_n = r w^2 = V^2/r.$$

$$a_t = r \alpha$$

Sample Problem 6/3 :-

The 300 Kg concrete block is elevated by the hoisting mechanism shown where the cables are securely wrapped around , the respective drums. The drums which are fastened together and turn as a single unit about (O) of 450 mm. If a constant tension P of 1.8KN is maintained by the power unit at A. determine the vertical acceleration of the block and the resultant force on the bearing at O.

Subject Date No.

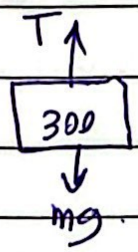
Dynamics and Vibrations :—

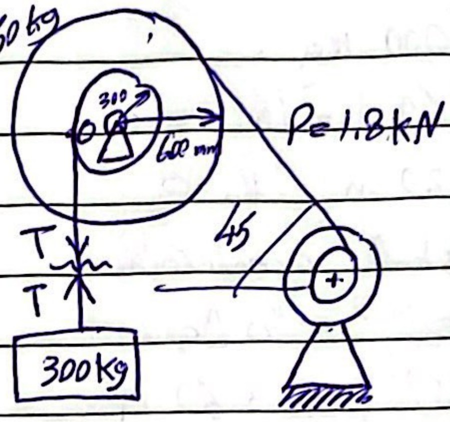
Ch6 :- Plane Kinetics of Rigid Body :-

* Sample Problem 6/3 :—

radius of gyration = 450 mm

$P = 1.8 KN$





$\uparrow \Sigma F_y = m a_y$.

$T - mg = m a_y$.

$a_y = \frac{T - (300)(9.81)}{300}$.

$\circlearrowleft \Sigma M_o = I_o \alpha$

$-P(0.6) + T(0.3) = m d^2 \alpha$.

$-1800(0.6) + T(0.3) = (150)(0.45)^2 \alpha$.

$0.3 T - 30.4 \alpha = 1080$. —— ①.

$a_y = a_t = r \alpha = \frac{T - 2943}{300}$.

$T - 90 \alpha = 2943$. —— ②.

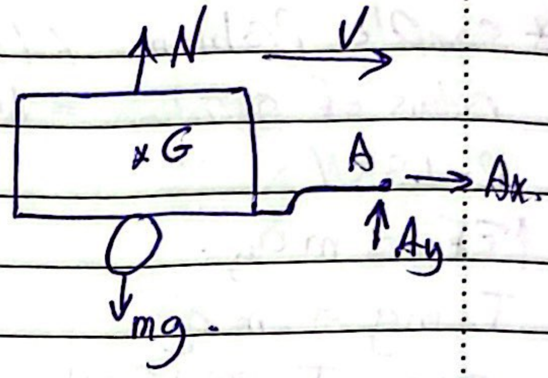
$T = 3250 N$ $\alpha = 3.44 rad/s$.

$a_y = r\alpha = (0.3)(3.44) = 1.031 \ m/s^2$.

Dynamics and Vibrations :-

Ch6 : Plane Kinetics of Rigid Body :-

Problems 6/17 :-



$m = 900\ kg$

$V = 60\ km/h$

$S = 30\ m \quad V_0 = 0$

constant acceleration

$V^2 = V_0^2 + 2a\Delta s$

$(16.66)^2 = 0 + 60\, a$

$\boxed{a = 4.63\ m/s^2}$

$\Sigma f_y = 0$

$N + A_y - mg = 0$

$N + A_y = 8829 \quad -①$

$\circlearrowleft \Sigma M_G = 0$

$A_y(1.2) + A_x(0.4) = 0$

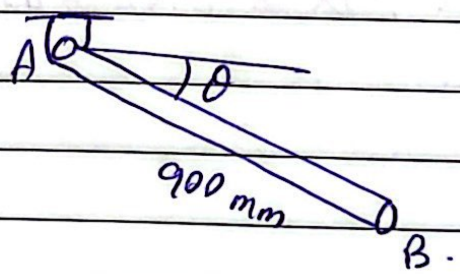
$\underline{A_y = -1389\ N}$

$\underline{N = 10218\ N}$

$\Sigma f_x = m a_x$

$A_x = (900)(4.63) = \underline{4167\ N}$

Problems 6/40 :-



$m = 8kg$

$W = \dot{\theta} = 2\ rad/s \quad \theta = 30$

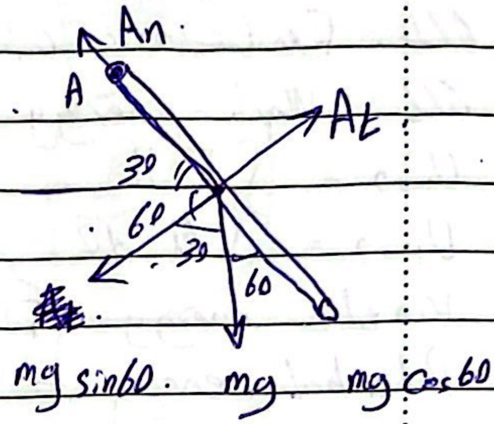
find A_n, A_t ??

Subject Date No.

* Dynamics and Vibrations :-

* Ch6 :- Plane Kinetics of Rigid Body :-

* Problems 6/40 :-



$\sum F_n = m a_n$

$A_n - mg\cos 60 = m r w^2$

$A_n = 8(0.45)(2)^2 + 8(9.81)\cos 60$

$\boxed{A_n = 53.64\ N}$

$\sum F_t = m a_t$

$-A_t + mg\sin 60 = m r \alpha$

$A_t = mg\sin 60 - m r \alpha$

$\sum M_A = I_A \alpha$

$mg(0.45\cos 30) = \frac{1}{3}\, 8\,(0.9)^2 \alpha$

$\alpha = 14.16\ rad/s^2$

$\boxed{A_t = 16.99\ N}$

Subject Date No.

* Dynamics and Vibrations :-

* Ch6 :- Plane Kinetics of Rigid Bodies :-

* ~~6/4 :- fixed axis rotation~~ :-

* 6/6 : Work - energy relations :-

$U_{1\to2} = \int F\, dr$.

$U_{1\to2} = \int M\, d\theta$.

Kinetic energy $(T) = \frac{1}{2} m V^2 + \frac{1}{2} I w^2$

Potential energy $= V_e + V_g$.

⇒ Rotational motion.

$T_1 + V_{e_1} + V_{g_1} + U'_{1\to2} = T_2 + V_{e_2} + V_{g_2}$

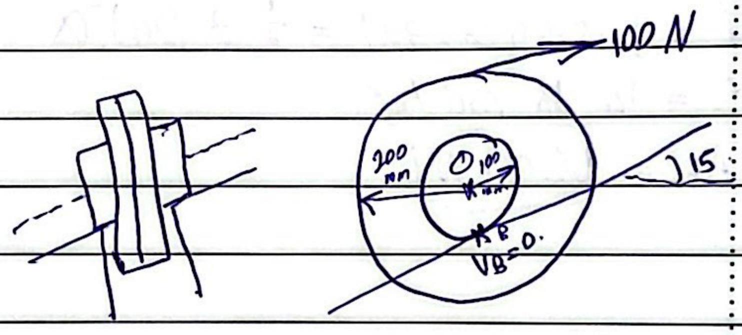
Power $= F \cdot V + M w$.

* Sample Problem 6/9 :-

Rolling without slipping.

$V_1 = 0 \Rightarrow T_1 = 0$. $m = 40$ kg

$K = 150$ mm



$\cancel{T_1} + U_{1\to2} = T_2$.

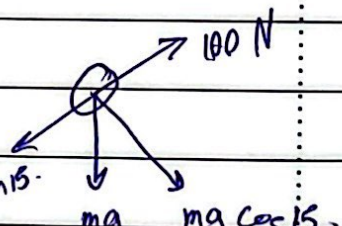
$0 + U_{1\to2} = \frac{1}{2} m V^2 + \frac{1}{2} I_o w^2$

two motion translational + Rotational

$U_{1\to2} = \frac{1}{2}(40)(0.1w)^2 + \frac{1}{2}(m k_o^2)\, w^2$.

$\left(\frac{300}{100}(100)\right)(3) - (mg \sin 15)(3) = 0.2 w^2 + 0.45 w^2$.

$595 = 0.65 w^2 \Rightarrow w = 30.2$ rad/s.



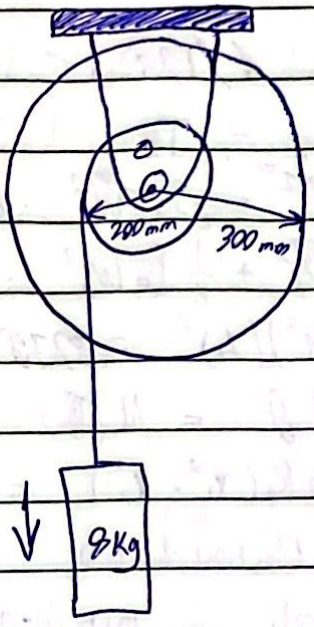
$P = F.V = F r w$

$= (100)(0.3)(30.3) = 908 W$.

Subject Date No.

* Dynamics and Vibrations :-

* Ch6 :- Plane Kinetics of Rigid Bodies :-



* Sample Problems 6/113 :-

$V_{8kg} = 0.3$ m/s.

V after 1.5 m $= ??$

m grooved $= 12$ kg

$k = 210$ mm $\quad r_i = 200$ mm

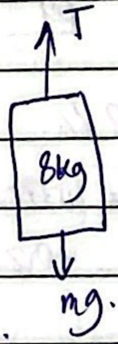
$F_{f\,moment_O} = 3$ N.m.

$T_1 \neq 0 \rightsquigarrow V_1 = 0.3$.

$\Sigma F_y = m a_y$.

$I_O = m K^2 = 12(0.21)^2 = 0.5292$

$W_1 = \frac{V_1}{r_i} = \frac{0.3}{0.2} = 1.5$ rad/s.



$M = F \cdot r_i$.

$3 = F(0.2) \Rightarrow \underline{F_f = 15\text{ N}}$

$T_1 + U_{1\to2} = T_2$.

$\frac{1}{2} m V_1^2 + \frac{1}{2} I_O W_1^2 + mg(1.5) - 15(1.5) = \frac{1}{2} m V_2^2 + \frac{1}{2} I_O W_2^2$.

$\frac{1}{2}(12)(0.3)^2 + \frac{1}{2}(0.5292)(1.5)^2 + 12(9.81)(1.5) - 15(1.5) = \frac{1}{2}(12) V_2^2 + \frac{1}{2}(0.5292)\left(\frac{V_2}{0.2}\right)^2$

$0.54 + 0.59535 + 176.58 - 22.5 = 6V_2^2 + 6.615 V_2^2$.

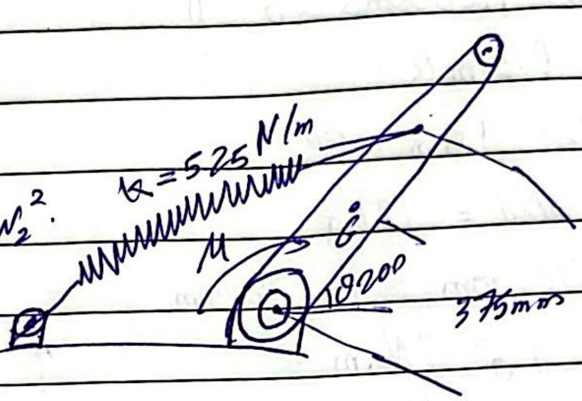
$\Rightarrow V_2 = 3.01$ m/s.

Subject Date No.

* Dynamics and Vibrations :-

* Ch6 : Plane Kinetics of Rigid Bodies :—

* problems 6/123 :-



$T_1 + U_{1\to2} = T_2$

$T_1 = \cancel{\frac{1}{2}m V_1^2} + \cancel{\frac{1}{2} I_o W_1^2} = 0$.

$T_2 = \cancel{\frac{1}{2}m V_2^2} + \frac{1}{2} I_o W_2^2 = \frac{1}{2} I_o W_2^2$.

$T_2 = \frac{1}{2}(m k^2)(4)^2 = 2.75$ Kg m/s

$U_{1\to2} = {}^{+}M\theta = M(\frac{\pi}{2})$.

$U_{1\to2} = \frac{1}{2}K(X_1^2 - X_2^2) = -12.66$.

$X_1 = X_o - \text{Unstretch} = 0$ $X_2 = (0.375 + 0.375) - \sqrt{2}(0.375) = 0.2196$

$U_{1\to2} = mgh. = mg(0 - 0.2) = -0.2mg$.

$T_1 + U_{1\to2} = T_2$.

$0 + M(\frac{\pi}{2}) - 12.66 - 0.2mg = 2.75$.

$\Rightarrow M = 2.95$ N.m.

NOTEBOOK

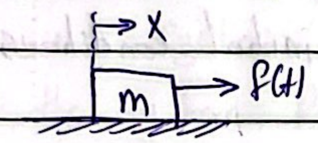
Subject Date No.

* Dynamics and Vibrations :-

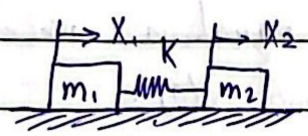
* Ch8 :- Vibration and time response :-

* It is a class of dynamics in which motion of the bodies oscillate, or respond to external effect in the presence of restoring forces.

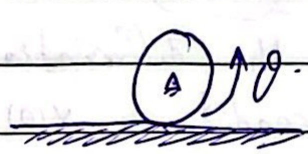
* Degree of freedom (DOF) :-
(number of impedendent Variables)



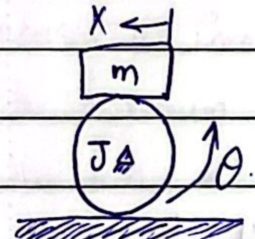
one DOF



two DOF.



one DOF.



two Variables $\underline{X}$ and $\underline{\theta}$.
but $X = r\theta$.
two dependent Variables.
One DOF.

* 8/2 :- Free Vibration of Particles :-
↓
without external force.

without damping → undamped response (Zero damped)

with damping → damped response:
- under damped $0 < \xi < 1$
- critically $\xi = 1$
- over damped $\xi > 1$

Dynamics and Vibrations :-

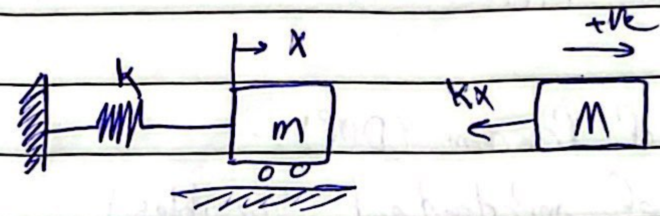
Ch 8 :- Vibration and time response :-

Free Vibration of Particles (without external force) :-

1. Undamped - free - Vibrations :-

damped بدون ← force بدون

$\Sigma F_x = m a_x$.



$-Kx = m\ddot{x}$.

$$mx'' + Kx = 0$$

to solve the differential equation we need initial conditions

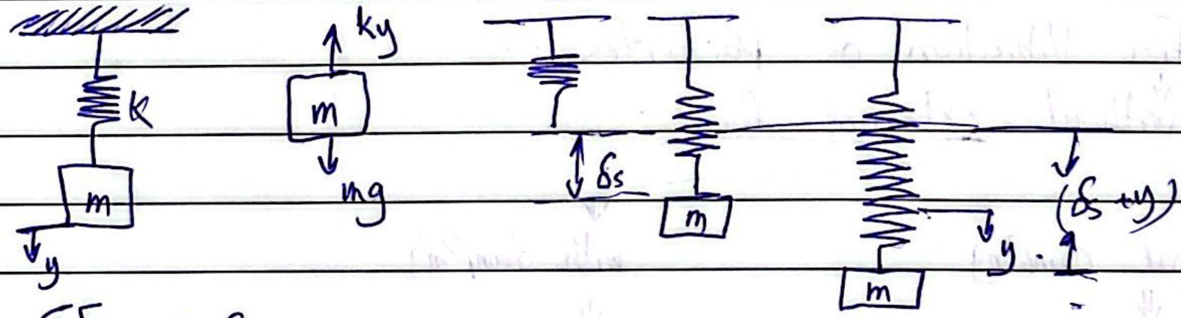
initial conditions $x(0)$ and $\dot{x}(0)$.

$\frac{m\ddot{x}}{m} + \frac{Kx}{m} = 0 \Rightarrow \ddot{x} + \frac{K}{m}x = 0$.

$\ddot{x} + W_n^2 x = 0$. $W_n = \sqrt{\frac{K}{m}}$ rad/s

natural frequency

$x(t) = \underline{\underline{C}} \sin(W_n t + \underline{\underline{\varphi}}) \Rightarrow$ Undamped free Vibrations.



$\Sigma F_y = m a_y$.

$mg - K(\delta_s + y) = m\ddot{y}$

$\cancel{mg} - \cancel{K\delta_s} - Ky = m\ddot{y}$

$\boxed{m\ddot{y} + Ky = 0} \Rightarrow \ddot{y} + \frac{K}{m}y = 0$.

* Dynamics and Vibrations :-

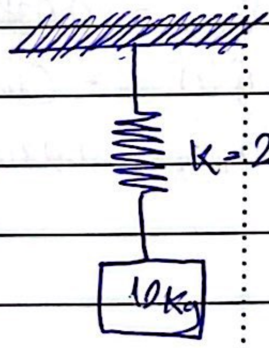
* Ch8 :- Vibration and time response :-

* Sample Problem 8/1 :-

A 10 kg body is suspended from a spring of constant $k = 2.5\,kN/m$. At time $t = 0$ it has a downward velocity of $0.5\,m/s$ as it passes through the position of static equilibrium. Determine :-

1- the static spring deflection (δ_s)
2- the natural frequency of the system in both rad/s (W_n) and cycles/s (f_n).
3- the system period (T).
4- the displacement (x) as a function of time, when (y) is measured from the position of static equilibrium.
5- the maximum velocity V_{max} attained by the mass.
6- the maximum acceleration a_{max} attained by the mass.

The Solution :- $\boxed{m\ddot{y} + ky = 0}$



1- $mg = k\delta_s$.

$$\delta_s = \frac{mg}{k} = \frac{(10)(9.81)}{2.5k} = 0.03924\ m.$$

2- $$W_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{2.5k}{10}} = 15.8\ rad/s.$$

$$f_n = \frac{W_n}{2\pi} = \frac{15.8}{2\pi} = 2.52\ cycles/s.$$

3- $$T = \frac{1}{f_n} = \frac{1}{2.52} = 0.397\ s.$$

Subject Date No.

* Dynamics and Vibrations :-
* Ch 8 : Vibration and time response :-

* Sample Problem 8/1 :

4- $\dot{y}(0) = 0.5$ $y(0) = 0$.

$y(t) = C \sin(W_n t + \psi)$

$y(0) = C \sin(\psi) = 0$.

$\Rightarrow \sin\psi = 0 \Rightarrow \boxed{\psi = 0}$

$\dot{y}(t) = C W_n \cos W_n t = 0.5$.

$\dot{y}(0) = C W_n \underbrace{\cos 0}_{1} = 0.5$.

$C = \frac{0.5}{15.8} = 0.0316$ $\boxed{C = 0.0316}$

$y(t) = 0.0316 \sin(15.8 t)$

5- $\dot{y}(t) = C W_n \cos(W_n t)$

$\dot{y}(t) = 0.0316 W_n \cos(15.8 t) \rightarrow$ equal to 1.

$V_{max} = 0.0316 W_n = 0.0316 (15.8) = 0.5$ m/s.

6- $\ddot{y}(t) = - C W_n^2 \sin(W_n t) \rightarrow$ equal to (-1)

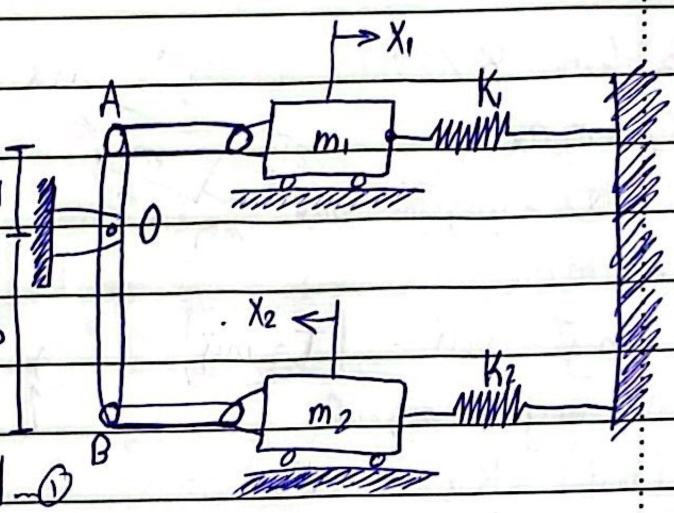
$a_{max} = 0.0316 (15.8)^2 = 7.91$ m/s^2.

Subject Date No.

* Dynamics and Vibrations :-

* Ch 8 :- Vibration and time response :-

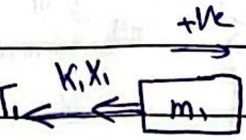
* Problem 8/24 :-



$K_1 = K_2 = K$

$m_1 = m_2 = m$

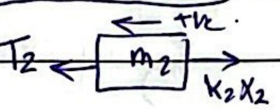
$x_2 = \frac{b}{a} x_1$



$\Sigma F_x = m_1 a_{x_1}$

$-T_1 - K_1 x_1 = m_1 \ddot{x}_1 \Rightarrow \boxed{m\ddot{x}_1 + Kx_1 + T_1 = 0}$ — (1)

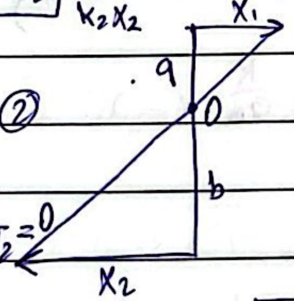
$\Sigma F_x = m_2 a_{x_2}$



two Variable x_1, x_2

$T_2 - K_2 x_2 = m\ddot{x}_2$

$\boxed{m\ddot{x}_2 + K_2 x_2 - T_2 = 0}$ — (2)



$\frac{x_1}{x_2} = \frac{a}{b}$

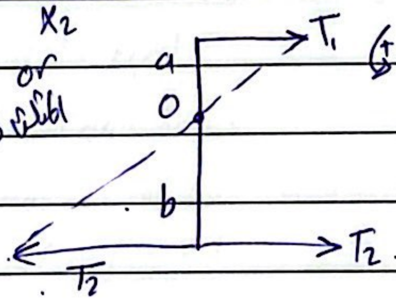
$\boxed{x_2 = \frac{b}{a} x_1}$ dependent

$\boxed{1 \text{ DOF}}$

from (1) + (2)

$m\ddot{x}_1 + Kx_1 + T_1 + m\ddot{x}_2 + Kx_2 - T_2 = 0$

$T_2 = \frac{a}{b} T_1 \qquad x_2 = \frac{b}{a} x_1$



$\circlearrowleft \Sigma M_O = 0$

$-T_1(a) + bT_2 = 0$

$\boxed{T_2 = \frac{a}{b} T_1}$

$m\ddot{x}_1 + Kx_1 + T_1 + m\left(\frac{b}{a}\ddot{x}_1\right)$

$+ K\left(\frac{b}{a}x_1\right) - \frac{a}{b}T_1 = 0$

$\left[m + \frac{b}{a}m\right]\ddot{x}_1 + \left[K + \frac{b}{a}K\right]x_1$

$+ T_1\left[1 - \frac{a}{b}\right] = 0$

$T_1 = -Kx_1 - m\ddot{x}_1$

$\left[m + \frac{b}{a}m\right]\ddot{x}_1 + \left[K + \frac{b}{a}K\right]x_1 + \left(-Kx_1 - m\ddot{x}_1\right)\left[1 - \frac{a}{b}\right] = 0$

$\Rightarrow \left[m_1 + \frac{b^2}{a^2}m_2\right]\ddot{x}_1 + \left[K_1 + \frac{b^2}{a^2}K_2\right]x_1 = 0$

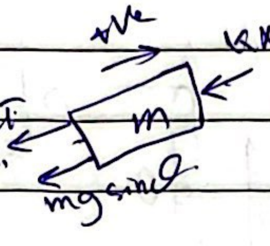
$$W_n = \sqrt{\frac{K_1 + \frac{b^2}{a^2}K_2}{m_1 + \frac{b^2}{a^2}m_2}}$$

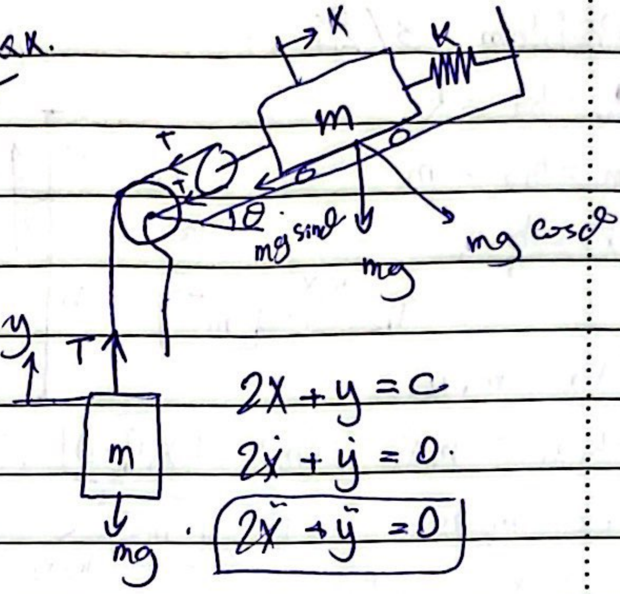
N O T E B O O K

Dynamics and Vibrations :-

* Ch8 :- Vibration and time response :-

* Problem 8/23.





$\Sigma F_x = m a_x$.

$-2T - Kx - \cancel{mg\sin\theta} = m\ddot{x}$

$\Sigma F_y = m a_y$

$T - \cancel{mg} = m\ddot{y}$. $\boxed{T = m\ddot{y}}$

but $mg + mg\sin\theta$

$-2(m\ddot{y}) - Kx = m\ddot{x}$.

$-2(m(+2\ddot{x})) - Kx = m\ddot{x}$.

$-4m\ddot{x} - m\ddot{x} - Kx = 0$.

$+5m\ddot{x} + Kx = 0$. $\Rightarrow$ $\ddot{x} + \frac{K}{5m}x = 0$.

$W_n = \sqrt{\frac{K}{5m}}$

$2x + y = c$

$2\dot{x} + \dot{y} = 0$.

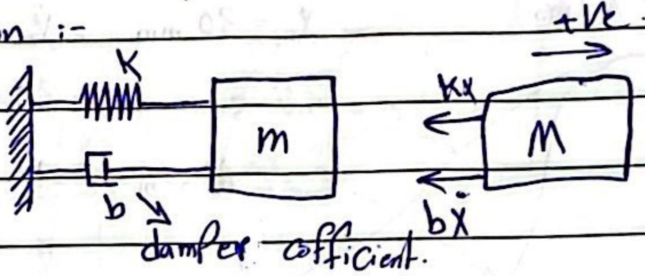
$\boxed{2\ddot{x} + \ddot{y} = 0}$

Dynamics and Vibrations :-

* Ch8 :- Vibration and time response :-

* Free Vibration of Particles (without external force) :-

2. damped - free - Vibration :-



$\Sigma F_x = m a_x$.

$-Kx - b\dot{x} = m\ddot{x}$.

$m\ddot{x} + b\dot{x} + Kx = 0$.

$$\boxed{\ddot{x} + \frac{b}{m}\dot{x} + \frac{K}{m}x = 0}$$

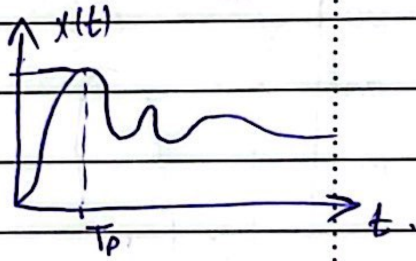
$2\varepsilon W_n = \frac{b}{m}$, $W_n^2 = \frac{K}{m}$

$$\boxed{\varepsilon = \frac{b}{2m W_n}}$$

$$\boxed{\ddot{x} + 2\varepsilon W_n \dot{x} + W_n^2 x = 0}$$

damping ratio natural frequency.

* Case 1 :- Underdamped $\boxed{0 < \varepsilon < 1}$

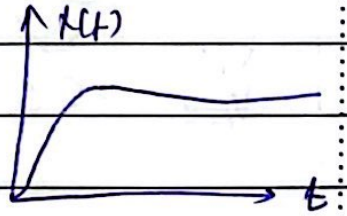


$X(t) = D\, e^{-\varepsilon W_n t} \sin(W_d t + \theta)$

$D, \theta \rightarrow$ from initial conditions.

$W_d = W_n\sqrt{1-\varepsilon^2}$ W_d : damped frequency.

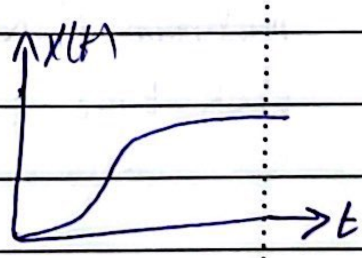
* Case 2 :- Critically damped $\boxed{\varepsilon = 1}$



$X(t) = (A_1 + A_2 t)\, e^{-W_n t}$.

$A_1, A_2 \rightarrow$ from initial conditions

* Case 3 :- Overdamped $\boxed{\varepsilon > 1}$



$X(t) = A_1 e^{((-\varepsilon + \sqrt{\varepsilon^2-1})W_n t)} + A_2 e^{((-\varepsilon - \sqrt{\varepsilon^2-1})W_n t)}$

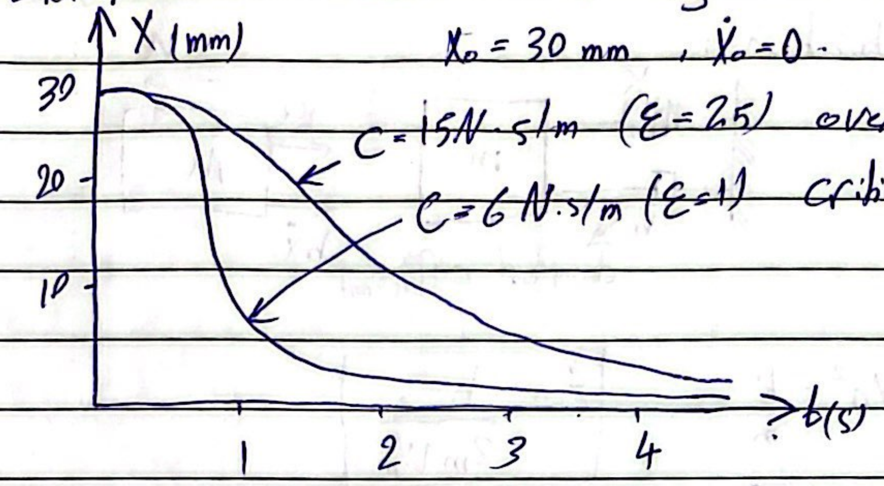
$A_1, A_2 \rightarrow$ from initial conditions

*Dynamics and Vibrations :-

*Ch8 :- Vibration and time Response :—

*Example :- Conditions $m = 1 Kg$, $K = 9 N/m$

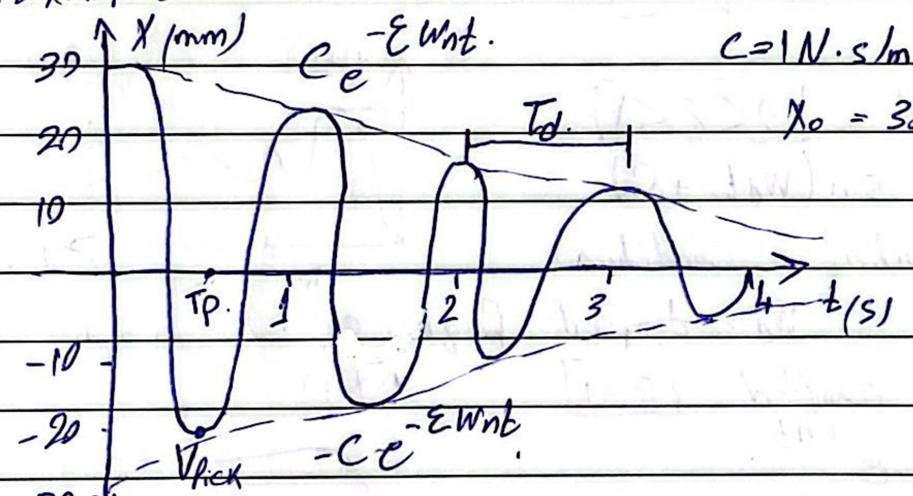
$X_o = 30 mm$, $\dot{X}_o = 0$.



*Example :- Conditions :- $m = 1 Kg$, $K = 36 N/m$

$C = 1 N.s/m$ ($\xi = 0.0833$)

$X_o = 30 mm$ $\dot{X}_o = 0$.



maximum pick ≅ −22.

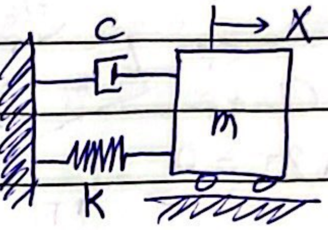
pick time ≅ 0.5 s.

* Dynamics and Vibrations :-

* Ch8 :- Vibration and time response :-

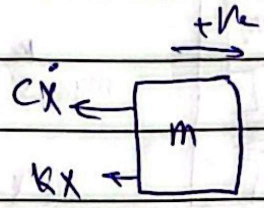
* Sample problem 8/12 :-

$m = 8\,kg$



$C = 20\ N.s/m \qquad k = 32\ N/m$

$\Sigma F_x = m a_x$



$-C\dot{x} - kx = m\ddot{x}$

$m\ddot{x} + C\dot{x} + kx = 0.$

$\ddot{x} + \frac{C}{m}\dot{x} + \frac{k}{m}x = 0.$

$W_n^2 = \frac{k}{m} \Rightarrow W_n = \sqrt{\frac{32}{8}} = 2\ rad/s$

$2\xi W_n = \frac{C}{m} \Rightarrow \xi = \frac{C}{2m W_n} = \frac{20}{2(8)(2)} = 0.625 < 1$ Underdamped.

Underdamped free Vibration system response

$x(t) = C\, e^{-\xi W_n t} \sin(W_d t + \theta)$

conditions : $\dot{x}_o = 0. \qquad x(0) = 0.2$

$x(0) = C \sin\theta = 0.2. \qquad C \sin\theta = 0.2$ --- ①.

$\dot{x}(t) = C W_d\, e^{-\xi W_n t} \cos(W_d t + \theta) + -C\xi W_n\, e^{-\xi W_n t} \sin(W_d t + \theta) = 0$

$C W_d \cos\theta - C\xi W_n \sin\theta = 0.$

$\cancel{C}\, W_n\sqrt{1-\xi^2} \cos\theta = \cancel{C}\xi W_n \sin\theta.$

$\tan\theta = \frac{\sqrt{1-\xi^2}}{\xi} \Rightarrow \theta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) \Rightarrow \theta = 0.896\ rad.$

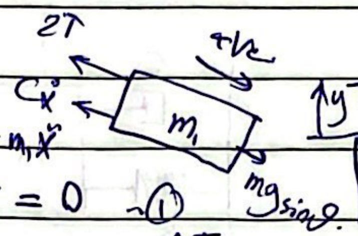
$C = 0.256\ m.$

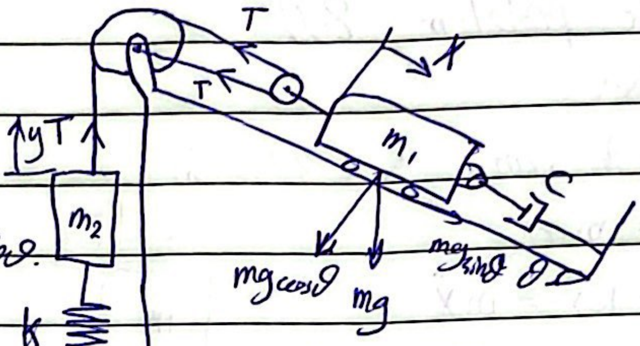
$C \sin\left(\frac{0.896 * 180}{\pi}\right) = 0.2 \qquad x(t) = 0.256\, e^{-1.25t} \sin(1.561t + 0.896)$

~~Dynamics and Vibrations :-~~

~~Ch 8 :- Vibrations and time response :-~~

~~Problem~~

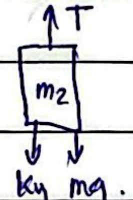




$\sum F_x = m a_x$

$\cancel{mg\sin\theta} - 2T - C\dot{x} = m_1\ddot{x}$

$m_1\ddot{x} + C\dot{x} + 2T = 0 \quad -(1)$



$\sum F_y = m_2 a_y$

$T - Ky - \cancel{m_2 g} = m_2\ddot{y}$

$\boxed{T = m_2\ddot{y} + Ky}$

$2x + y = C$

$2\dot{x} + \dot{y} = 0$

$2\ddot{x} + \ddot{y} = 0$

$m_1\ddot{x} + C\dot{x} + 2(m_2\ddot{y} + Ky) = 0$

$m_1\ddot{x} + C\dot{x} + 2m_2(+2\ddot{x}) + 2K(+2x) = 0$

$m_1\ddot{x} + C\dot{x} + +4m_2\ddot{x} + 4Kx = 0$

$[m_1 + 4m_2]\ddot{x} + C\dot{x} + 4Kx = 0$

$\ddot{x} + \dfrac{C}{m_1 + 4m_2}\dot{x} + \dfrac{+4K}{m_1 + 4m_2}x = 0$

$W_n = \sqrt{\dfrac{-4K}{m_1 - 4m_2}} \qquad \xi = \dfrac{C}{2W_n[m_1 - 4m_2]}$

$m_1\ddot{x} + C\dot{x} + 2m_2\ddot{y} + 2Ky = 0$

$m_1\left(\frac{\pm 1}{2}\ddot{y}\right) + C\left(\frac{\pm 1}{2}\dot{y}\right) + 2m_2\ddot{y} + 2Ky = 0$

$\left[2m_2 + \frac{1}{2}m_1\right]\ddot{y} + \frac{C}{2}\dot{y} + 2Ky = 0$

$\ddot{y} + \dfrac{C}{2\left[2m_2 + \frac{1}{2}m_1\right]}\dot{y} + \dfrac{2K}{2m_2 - \frac{1}{2}m_1}y = 0$

$W_n = \sqrt{\dfrac{2K}{2m_2 + \frac{1}{2}m_1}} = \sqrt{\dfrac{4K}{4m_2 + m_1}}$

$2\xi W_n = \dfrac{C}{4m_2 - m_1}$

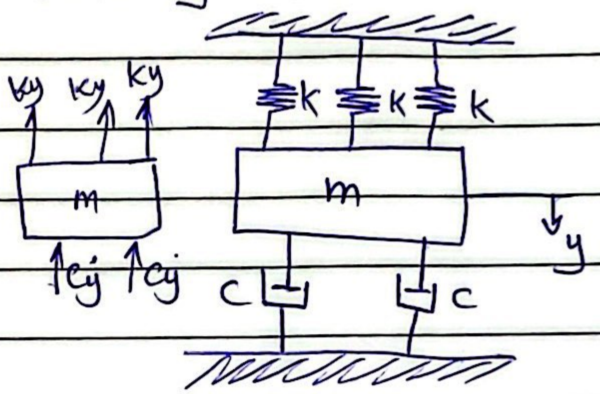
$\xi = \dfrac{C}{2W_n[4m_2 + m_1]}$

Subject Date No.

* Dynamics and Vibrations:-

* Ch8:- Vibrations and time responses:-

* Example: Determine the values of the damping coefficent (C) for which the system is critically damped if $K = 50K\ N/m$ and $m = 10Kg$.



$\Sigma F_y = m a_y$

$-Ky - Ky - Ky - C\dot{y} - C\dot{y} = m\ddot{y}$

$m\ddot{y} + 2C\dot{y} + 3Ky = 0$

$\ddot{y} + \frac{2C}{m}\dot{y} + \frac{3K}{m}y = 0$

Critically damped $\Rightarrow \varepsilon = 1$.

$W_n = \sqrt{\frac{3K}{m}} = \sqrt{\frac{3(100k)}{10}} = 173.2$

$\cancel{2}\varepsilon W_n = \frac{\cancel{2}C}{m} \Rightarrow C = \varepsilon W_n m = 1(173.2)(10)$

$= 1732\ N.s/m$.

N O T E B O O K