* Applied / first

$\underset{\text{complex number}}{Z} = \underset{\text{Real part}}{x} + \underset{\text{imaginary part}}{jy}$ yi

$j = \frac{-1}{j}$

$(j)^2 = -1$

$(j)^3 = -\sqrt{-1}$

$Z = (0, 1)$

$Z_1 = x + yi$, $Z_2 = A + Bi$

$Z_1 + Z_2 = (x+A) + (y+B)i$

$Z_1 - Z_2 = (x-A) + (y-B)i$

$Z_1 Z_2 = (x+yi)(A+Bi)$

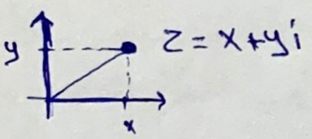
$= (xA - yB) + (xB + Ay)$

$\frac{Z_1}{Z_2} = \frac{(x+yi)}{(A+Bi)} \times \frac{(A-Bi)}{(A-Bi)} \rightarrow$ بدنضرب بمرافق المقام وبنكمل

* أو استخدم الحاسبة وخلصنا

---

* complex plane $z = x + yi$

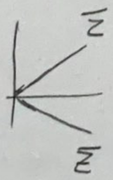


عند الجمع بنتعامل معاملة المتجهات

* conjugate - Complex conjugate Number

$z = x + yi$

$\bar{z} = x - yi$



$z \cdot \bar{z} = x^2 + y^2$ / $z + \bar{z} = 2x$

$z - \bar{z} = 2yi$

$x = \frac{1}{2}(z + \bar{z})$

$y = \frac{1}{2i}(z - \bar{z})$

if → z real $x = z$, $z = \bar{z}$

if → z imaginary $z = yi$, $z = -\bar{z}$

- $\overline{(z_1 + z_2)} = \bar{z}_1 + \bar{z}_2$ / $\overline{(z_1 - z_2)} = \bar{z}_1 - \bar{z}_2$
- $\overline{(z_1 z_2)} = \bar{z}_1 \bar{z}_2$ / $\overline{(z_1 / z_2)} = \bar{z}_1 / \bar{z}_2$

---

* polar form

$x = r\cos\theta$, $y = r\sin\theta$

$r = \sqrt{x^2 + y^2}$ $\theta = \tan^{-1}\left(\frac{y}{x}\right)$

$z = x + yi = r[\cos\theta + \sin(\theta) i] = re^{\theta i}$

* polar form conjugate

$z = re^{\theta i}$, $\bar{z} = re^{-\theta i}$ | $|z| = |\bar{z}|$

$\boxed{\arg \bar{z} = -\arg z}$

---

polar form

* بالجمع والطرح عادي

* Multiplication / Divisions

$z_1 = r_1[\cos\theta_1 + j\sin\theta_1]$

$z_2 = r_2[\cos\theta_2 + j\sin\theta_2]$

$z_1 * z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + \sin(\theta_1 + \theta_2)]$

$z_1 / z_2 = (r_1 / r_2)[\cos(\theta_1 - \theta_2) + \sin(\theta_1 - \theta_2)]$

ممكن نحتاجهم

$\cos(A \mp B) = \cos(A)\cos(B) \pm \sin(A)\sin(B)$

$\sin(A \mp B) = \sin(A)\cos(B) \mp \sin(B)\cos(A)$

$\sqrt[n]{x + jy} = \sqrt[n]{r}\left[\cos\left(\frac{\theta + 2\pi k}{n}\right) \pm j\sin\left(\frac{\theta + 2\pi k}{n}\right)\right]$

$r = \sqrt{x^2 + y^2}$

$\theta = \tan^{-1}\left(\frac{y}{x}\right)$ ← عكسي

$k = 0, 1, 2 \ldots\ldots n-1$

$k = 0 \rightarrow$ first root

هو بحدد لك

او بتختار تعوض من $k = 0$ ل $k = n-1$

[2]

Exponential function (e^z)

$z = x + yi$

$e^z = e^{x+yi} = e^x * e^{yi}$

$e^{iy} = (\cos y + j \sin y)$

---

properties of e^z

$(e^z)' = e^z$ / $e^{z_1} e^{z_2} = e^{z_1 + z_2}$

$e^{2\pi i} = 1$ / $e^{(\pi/2)i} = i$ / $e^{i\pi} = -1$ / $e^{(-\frac{\pi}{2})i} = -i$

$|e^{iy}| = 1$ / $|e^z| = e^x$ and $\arg e^z = y \mp 2\pi n$

$e^{z+2n\pi i} = e^z$

$e^{R - Bi} = e^R (\cos(B) - j \sin(B))$

$|e^{R-Bi}| = e^R$, $\operatorname{Ang} e^{R \ominus Bi} = \ominus B$

$\cos -\theta = \cos\theta$ ← تنبيه *

$\sin -\theta = \boxed{-}\sin\theta$

$\cos x = \frac{1}{2}(e^{xi} + e^{-xi})$

$\sin x = \frac{1}{2i}(e^{xi} - e^{-xi})$

$|\cos z|^2 = \cos^2 x + \sin^2 y$

$\tan z = \frac{\sin z}{\cos z}$

$\cot z = \frac{\cos z}{\sin z} = \frac{1}{\tan z}$

$\sec z = \frac{1}{\cos z}$

$\csc(z) = \frac{1}{\sin(z)}$

$e^{in\theta} = \cos(n\theta) + i \sin(\theta n)$

* hyperbolic function

$\sinh(zi) = \frac{1}{2}(e^{zi} - e^{-zi})$

$\cosh(zi) = \frac{1}{2}(e^{zi} + e^{-zi})$

$\sinh(z) = \frac{1}{2}(e^{z} - e^{-z})$

$\cosh(z) = \frac{1}{2}(e^{z} + e^{-z})$

$\cos(zi) = \cosh z$

$\sin(zi) = i\sinh(z)$

$\tanh z = \frac{\sinh z}{\cosh z}$ / $\coth(z) = \frac{\cosh z}{\sinh z}$

$\operatorname{sech}(z) = \frac{1}{\cosh(z)}$ / $\operatorname{csch} z = \frac{1}{\sinh z}$

$\cos(z) = \cos(x)\cosh(y) \ominus i \sin(x)\sinh(y)$

$\sin(z) = \sin(x)\cosh y + i \cos(x)\sinh(y)$

$|\cos(z)|^2 = \cos^2(x) + \sinh^2(y)$

$|\sin(z)|^2 = \sin^2(x) + \sinh^2(y)$

---

* Natural Logarithm function

$\operatorname{Ln}(z) = \ln(r) + j \operatorname{Ang} z$ دورة واحدة

$\ln(z) = \ln(r) + j \arg z$ عدة دورات

$\operatorname{Ln} z = \ln|z| + j \operatorname{Ang} z$

$\ln(z) = \operatorname{Ln} z \mp jn2\pi$

$\arg(e^z) = y \mp 2n\pi$

$\ln e^z = z \pm in2\pi$

$\ln(z_1 z_2) = \ln(z_1) + \ln(z_2)$

$\ln(z_1/z_2) = \ln(z_1) - \ln(z_2)$

---

* general power

$z^c = e^{c\ln z} = e^{c(\ln r + \theta i)}$

بالتوفيق ♡

$e^{z_1 + z_2} = e^{x_1} e^{x_2} [\cos(y_1 + y_2) + j \sin(y_1 + y_2)]$

Second / Applied

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} , \quad B = \begin{bmatrix} 4 & 0 \\ 3 & -1 \end{bmatrix}$$

row, column

size = 2×2
rows = 2
columns = 2

$A = B$

[1] have same size

[2] $a_{11} = 4$, $a_{12} = 0$
$a_{21} = 3$, $a_{22} = -1$

row vector
صف واحد
column vector
عامود واحد

*Addition of Matrics

$$A + B = \begin{bmatrix} (a_{11}+4) & (a_{12}+0) \\ (a_{21}+3) & (a_{22}-1) \end{bmatrix}$$

الجمع/بجمع العناصر المتناظرة

$$A - B = \begin{bmatrix} (a_{11}-4) & (a_{12}-0) \\ (a_{21}-3) & (a_{22}+1) \end{bmatrix}$$

لازم يكونوا نفس size

* scalar multiplication

$$-A = \begin{bmatrix} -a_{11} & -a_{12} \\ -a_{21} & -a_{22} \end{bmatrix}$$

$A+B = B+A$	$C(A+B) = CA+CB$
$(A+B)+C = A+(B+C)$	$(C+K)A = CA+KA$
$A+0 = A$ matrix	$C(KA) = (CK)A$
$A+(-A) = 0$	$1A = A$

$A \times B = C$ ≠ $A\times B \neq B \times A$

$[M \times n] \times [n \times P] = [M \times P]$

لازم يكونوا متساويان

$BA = 0$
$A = 0$
or $B = 0$

$$\begin{array}{c} \text{صف أولى} \\ \text{صف ثاني} \end{array} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix} \times \begin{array}{ccc} \text{عامود (1)} & \text{عامود (2)} & \text{عامود (3)} \\ \end{array} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

[2×3] [3×3]
=

الصف الأول العامود (1)	الصف الأول عامود (2)	الصف الأول عامود (3)
الصف الثاني عامود (1)	الصف الثاني عامود (2)	الصف الثاني عامود (3)

* Matrix multiplication Rules

$(KA)B = K(AB) = A(KB)$

$A(BC) = (AB)C$

$(A+B)C = AC + BC$

$C(A+B) = CA + CB$

انتبه للترتيب

*Transposition:-

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \end{bmatrix} \rightarrow A^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ \vdots & \vdots \\ a_{1n} & a_{2n} \end{bmatrix}$$

* قلبنا كل صف عامود

* Rules of Transposition.

$(A^T)^T = A$

$(A+B)^T = A^T + B^T$

$(CA)^T = C A^T$ — C scalar

$(AB)^T = \cancel{A^T B^T}$ $B^T A^T$

انتبه للترتيب حولة
لازم تقلب

$A^T = A \rightarrow$ symmetric matrix

$A^T = -A \rightarrow$ skew-symmetric matrix

* Triangular matrix

* upper triangular matrix
(المصفوفة المثلث الفوقي)

*مثال G

$$\begin{bmatrix} 1 & 4 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 6 \end{bmatrix} \qquad \begin{bmatrix} 1 & 3 \\ 0 & 8 \end{bmatrix}$$

* Lower triangular matrix
(المصفوفة المثلث التحتي)

*مثال

$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -1 & 0 \\ 7 & 6 & 8 \end{bmatrix}$$

1. Diagonal matrix (D) $\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

2. Scalar matrix (S) $\begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix}$

*عكسا زي عن (Diagonal)

3. Unit or identity matrix (U) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

* عشان احل معادلتين من متغيرين عندي 3 خيارات

1 اذا تقاطعوا بنقطة ← حل واحد

2 اذا انطبقوا على بعض ← عدد لا نهائي

3 اذا توازوا ← صفر من الحلول

لـ مستحيل يكون (homogeneous)

$$a_{11}x_1 + a_{12}x_2 \; --- + a_{1n}x_{1n} = b_1$$
$$a_{21}x_1 + a_{22}x_2 + \; --- \; a_{2n}x_{2n} = b_2$$
$$a_{31}x_1 + a_{32}x_3 + \; --- \; a_{3n}x_{3n} = b_3$$

$$\begin{bmatrix} a_{11} & a_{12} - a_{1n} \\ a_{21} & a_{22} - a_{2n} \\ a_{31} & a_{32} -- a_{3n} \end{bmatrix}, \; x = \begin{bmatrix} x_{1n} \\ x_{2n} \\ \vdots \\ x_n \end{bmatrix} \text{ and } b \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} a_{11} & a_{12} & --- a_{1n} & b_1 \\ a_{21} & a_{22} & -- a_{2n} & b_2 \\ a_{31} & a_{32} & -- a_{3n} & b_3 \end{array}\right]$$

* Gaues Elimination

* الفكرة انو بدي اطلع شي تحول من معادلة matrix جديدة بالحالة تغير بتشكل الـ matrix لتوصل لشكل معين تطلع منو احد المتغيرات

مثال

$$2x_1 + 5x_2 = 2$$
$$-4x_1 + 3x_2 = -30$$

$$\left[\begin{array}{cc|c} 2 & 5 & 2 \\ -4 & 3 & -30 \end{array}\right] \rightarrow row(2) + 2\,row(1)$$

$$\left[\begin{array}{cc|c} 2 & 5 & 2 \\ 0 & 13 & -26 \end{array}\right]$$

$$13x_2 = -26$$

$$\boxed{x_2 = -2}$$

مسموح اضرب / اجمع / اقسم / اطرح / اقلب بين الـ (rows) الصفوف

* هنا ممنوع تقلب الأعمدة

* والعمليات الحسابية بس للـ (rows) ؟

* Rank of Matrix(A):

هو عدد الصفوف التي ما تصفرت

بعد ما عملنا (Gaus elimination)

1 اعمل gaus elimination

2 عدد الصفوف التي ما صارت (0 0 0 0)

بكون هو (A) أو الـ Rank

* لو تصفر عندي لو صف واحد بس

بنسميها (dependent)

* لو ما تصفر ولا صف (A = 0)

بنسميه (independent)

---

* Determinants (det)

$$D = \det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = (a_{11}a_{22}) - (a_{12}a_{21})$$

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (a_{11}(a_{22}a_{33}) - (a_{23}a_{32}))$$

$$-(a_{12}(a_{21}a_{33} - a_{23}a_{31})) + \boxed{(a_{13}(a_{21}a_{32} - a_{22}a_{31}))}$$

---

* Minors and cofactors

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

* The minor matrix

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

وهيك بطلع كلهم

* The cofactor Matrix

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix}$$

$$C = \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$C_{jk} = (-1)^{j+k}(M_{jk})$$

---

* Diagonal matrix

$$\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix}$$ بتعمل (gaus elimination)

$$\text{Diagonal} = a_{11} * a_{22} * a_{33}$$

$$a_{11}x_1 + a_{12}x_2 = b_1, \quad a_{21}x_1 + a_{22}x_2 = b_2$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{D} \qquad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{D}$$

معاملات x_2 — معاملات x_1

$$D = \det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

معاملات x_1 — معاملات x_2

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{D} \qquad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{D}$$

$$x_3 = \frac{\begin{vmatrix} a_{11} & a_{21} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{83} & b_3 \end{vmatrix}}{D} \qquad D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

* ~~Matrix~~ Inverse of matrix (A^{-1})

* لازم تكون المصفوفات مربعة

اذا كان الها (inverse) / $rank = n$

$\det \neq 0$

بنسميها (non-singular / invertible)

اذا ما كان لل (matrix) ما الها (inverse) / ~~rank~~

بنسميها (singular)

* مثال على الحل

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 3 & -1 & 1 & 0 & 1 & 0 \\ -1 & 3 & 4 & 0 & 0 & 1 \end{array}\right]$$

بنعمل (gauss elimination) عشان نخلي هاد الجزء،

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \square & \square & \square \\ 0 & 1 & 0 & \square & \square & \square \\ 0 & 0 & 1 & \square & \square & \square \end{array}\right]$$

هاد الجزء بكون ال (inverse)

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is $A^{-1} = \frac{1}{\det} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$

$$A^{-1} = \frac{1}{\det A} [C_{jk}]^{T}$$ *

* طريقة أسهل لايجاد (inverse)

$(AC)^{-1} = C^{-1} A^{-1}$

$(AC \cdots PQ)^{-1} = Q^{-1} P^{-1} \cdots C^{-1} A^{-1}$

* انتبه للقلب هون

$AB = 0 \rightsquigarrow A = 0$ or $B = 0$

$AB \neq BA$

---

let A, B, C be $n \times n$

↳ $rank(A) = n$ and $AB = AC \rightsquigarrow A = B$

↳ $rank(A) = n$ and $AB = 0 \rightsquigarrow B = 0$

↳ $AB = 0$ but $A \neq 0$ and $B \neq 0$

$\mapsto rank(A) < n$ and $rank(B) < n$

if A is singular $\rightsquigarrow$ AB and BA singular

$\det(AB) = \det(BA) = \det A \cdot \det B$

$adj(A) = (\operatorname{cof} A)^{T}$

Orthogonal matrix

$$\boxed{A^{T} = A^{-1}}$$

* Matrix Eigenvalue problems

$\boxed{A}\, x = \boxed{\lambda}\boxed{x}$ → unknown vector (احنا بنطلعه)

A → square matrix (مصفوفة مربعة) غالباً تعطى بالسؤال

λ → unknown scalar (احنا بنطلعها)

* خطوات حل الأسئلة الي على هالموضوع

1. $D(\lambda) = \det(A - \lambda I)$

$$\begin{bmatrix} (\square-\lambda) & \square & \square \\ \square & (\square-\lambda) & \square \\ \square & \square & (\square-\lambda) \end{bmatrix}$$

هون بنقص من (العناصر الي على القطر) λ

بعدين بنطلع ال det عادي بتطلع مع معادلة منها بنطلع قيم λ هاي هي ال eigenvalue

2. $A - \lambda I$

هون بجيب الكاركترستك A بعوض ال (λ) الي طلعت معي [واحنا منقصين من عناصر القطر]

ونعمل Gauss elimination ونطلع قيم x_1, x_2 .... (بنعمل علاقة بينهم)

بالآخر بنفرض قيمة لوحدة من ($x_1, x_2, ...$)

ونطلع الفكتور x

هيك بكون طلعت ال eigenvector

اذا بدك تتأكد ($Ax = \lambda x$)

* Definitions:

$A - \lambda I$ → the charactristic matrix

$D(\lambda) = \det(A - \lambda I)$ → the charactristic determinant of A

$D(\lambda) = 0$ → the charactristic equation of A

A^T and A has the same eigenvalue

symmetric matrix

$A^T = A$

بتقلب الصفوف أعمدة والعكس

skew symmetric matrix

$A^T = -A$

orthogonal matrix

$A^T = A^{-1}$

$R = \frac{1}{2}(A + A^T)$,

$S = \frac{1}{2}(A - A^T)$

$R + S = A$

* Similar Matrix ($\hat{A}$)

$$\hat{A} = P^{-1} A \boxed{P} \rightarrow \text{non-singular}$$

$\hat{A}$ is similar to A

→ $\hat{A}$ has the same eigenvalues and eigenvector as $\underline{\underline{A}}$

* بالعادة بيعطيني matrix ال P

و ال matrix A

بطلع P^{-1} بعدين بعوض ب $\hat{A} = \underbrace{P^{-1}}AP$

هولا اول

بعدين بجيب (P)

- - - - - - -

* Diagonalization of matrix

$$D = X^{-1} A X$$

الحل :-

بنطلع eigenvector يعني قيم x_1، x_2 نفس خطوات الحل بالورقة الي قبل

* اجمعهم ب ال matrix وحدة

* طلع X^{-1} هي

* عوض ب $D = X^{-1} A X$

بكون معطيك A ال Matrix

* Vector Calculus :-

Scalar ↝ determined by magnitude

Vector ↝ Magnitude and direction

Tail → initial point

tip → terminal point

$\vec{a} = \vec{b}$

↳ when $\vec{a}$ and $\vec{b}$ have same direction and same magnitude

* A vector of length (1) is called a unit vector

---

initial point → $P: (x_1, y_1, z_1)$

terminal point → $Q: (x_2, y_2, z_2)$

$$a = [\underbrace{(x_2 - x_1)}_{a_1}, \underbrace{(y_2 - y_1)}_{a_2}, \underbrace{(z_2 - z_1)}_{a_3}]$$

$\boxed{\text{terminal} - \text{initial}}$

$$|a| = \sqrt{(a_1)^2 + (a_2)^2 + (a_3)^2}$$

* A vector with the origin $(0,0,0)$ as the initial point is called position vector

$\vec{a} = [a_1, a_2, a_3]$

↳ $\vec{a} = a_1\hat{i} + a_2\hat{j} + a_3\hat{k}$

$a = [a_1, a_2, a_3]$

$b = [b_1, b_2, b_3]$

$a + b = [(a_1 + b_1), (a_2 + b_2), (a_3 + b_3)]$

$a - b = [(a_1 - b_1), (a_2 - b_2), (a_3 - b_3)]$

$\underset{\text{scalar}}{c}\,a = [ca_1, ca_2, ca_3]$

* properties

* $\vec{a} + \vec{b} = \vec{b} + \vec{a}$
* $(\vec{a} + \vec{b}) + \vec{k} = (\vec{k} + \vec{a}) + \vec{b}$
* $\vec{a} + (-\vec{a}) = 0$
* $\vec{a} + 0 = \vec{a}$

$\boxed{c \rightarrow \text{scalar}}$

* $c(\vec{a} + \vec{b}) = c\vec{a} + c\vec{b}$
* $(c + \vec{k})\vec{a} = c\vec{a} + \vec{k}\vec{a}$
* $c(\vec{k}\vec{a}) = (c\vec{k})\vec{a}$
* $1\vec{a} = \vec{a}$

---

* Dot product ↝ (inner product) The result is always a scalar

$\vec{a} \cdot \vec{b} = |a||b| \cos y$

y: الزاوية بين a و b

$a \cdot b > 0 \rightarrow y \leftarrow$ حادة

$a \cdot b = 0 \rightarrow y \leftarrow$ قائمة

$a \cdot b < 0 \rightarrow y \leftarrow$ منفرجة

$a = [a_1, a_2, a_3]$, $b = [b_1, b_2, b_3]$

$a \cdot b = (a_1 b_1 + a_2 b_2 + a_3 b_3)$

$\cos y = \dfrac{a \cdot b}{|a||b|}$ ↝ $y = \cos^{-1}\left(\dfrac{a \cdot b}{|a||b|}\right)$

* Properties of (inner/Dot) product.

- $(q_1\vec{a} + q_2\vec{b})\cdot c = q_1(\vec{a}\cdot\vec{c}) + q_2(\vec{b}\cdot\vec{c})$
- $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$
- $\vec{a}\cdot\vec{a} \geq 0$
  - ↳ $\vec{a}\cdot\vec{a} = 0 \to \vec{a} = 0$

* Since $|\cos\gamma| \leq 1$

$|a\cdot b| \leq |a||b|$

$|a+b| \leq |a|+|b|$

$|a+b|^2 + |a-b|^2 = 2(|a|^2 + |b|^2)$

$\hat{i}\cdot\hat{i} = \hat{j}\cdot\hat{j} = \hat{k}\cdot\hat{k} = \boxed{1}$

لانهم متعامدين

$\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{i} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{j} = \hat{k}\cdot\hat{i} = \hat{i}\cdot\hat{k} = \boxed{0}$

---

* Vector / cross product

$a\times b = |a||b|\sin\theta$

الزاوية بين b و a

$a\times b = 0 \to$
- $a = 0$
- $b = 0$
- $\theta = 180, 0$

* عشان احدد الاتجاه بستخدم قاعدة اليد اليمنى
بحط ايدي (اليمين) على اتجاه $(\vec{a})$ و بحرك
اصابعي باتجاه $(\vec{b})$ ← الابهام بيحدد اتجاه
الناتج عاموديا على $\vec{a}$ و $\vec{b}$

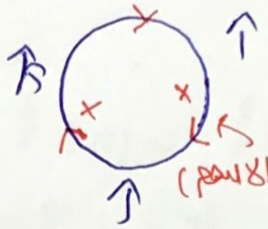
$a = [a_1, a_2, a_3]$

$b = [b_1, b_2, b_3]$

$$a\times b = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix}\hat{i} - \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix}\hat{j} + \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix}\hat{k}$$

$\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \boxed{0}$



لنتيجة ضرب تنين منهم بيعطيني التالت
إذا عكسنا الاتجاه (تبع الاسهم) بنحط سالب ($-$)

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* Properties of Cross product

$(L\vec{a})\times\vec{b} = L(\vec{a}\times\vec{b})$

$(\vec{a}+\vec{b})\times\vec{c} = \vec{a}\times\vec{c} + \vec{b}\times\vec{c}$

$(\vec{b}\times\vec{a}) = -(\vec{a}\times\vec{b})$

$\vec{a}\times(\vec{b}\times\vec{c}) \neq (\vec{a}\times\vec{b})\times\vec{c}$

---

* Scalar Triple product

$(abc) = a\cdot(b\times c) = a\cdot V$

$= a_1V_1 + a_2V_2 + a_3V_3$

* $a\cdot(b\times c) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$= (a\times b)\cdot c$

1. Volume of a box

$V = (\text{Height})(\text{Area of the base})$

$= |a|\cos\beta \quad |b\times a|$

$\text{Volume} = |a|\,|b\times c|\,|\cos\beta|$

↳ $(abc) = |a\cdot|b\times c||$

↳ يعني اذا طلع

---

* Vector ~~function~~ function

↳ (Whose Vales are Vectors)

$$U(t) = [U_1(t), U_2(t), U_3(t)]$$

$$U'(t) = [U_1'(t), U_2'(t), U_3'(t)]$$

↳ نشتق كل حدة بدلالة (t) لحالها

*Rules

- $(cU)' = cU'$ — c: scalar
- $(u+v)' = u'+v'$

$$(u\cdot v)' = u'\cdot v + u\cdot v'$$

$$(u\times v)' = u'\times v + u\times v'$$

$$(u\,v\,w)' = (u'\,v\,w) + (u\,v'\,w) + (u\,v\,w')$$

$f = [x, y, z]$

unit vector f $= \dfrac{[x, y, z]}{|f|}$

$$U = [V_1, V_2, V_3] = V_1 i + V_2 j + V_3 k$$

* Partial derivatives

$$\frac{\partial U}{\partial t_m} = \frac{\partial V_1}{\partial t_m} i + \frac{\partial V_2}{\partial t_m} j + \frac{\partial V_3}{\partial t_m} k$$

* Second partial derivatives

$$\frac{\partial^2 U}{\partial t_l \partial t_m} = \frac{\partial^2 V_1}{\partial t_l t_m} i + \frac{\partial^2 V_2}{\partial t_l t_m} j + \frac{\partial^2 V_3}{\partial t_l t_m} k$$

↳ اول شي بنشتق بالنسبة لـ t_m بعدين بالنسبة لـ t_L

$f(p) = f(x, y, z)$

$$\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right]$$

بنشتق بالنسبة لـ(x) والباقي ثوابت

بنشتق بالنسبة لـ(y) والباقي ثوابت

بالنسبة لـ(z)

$\nabla f \to \text{grad } f$

$$D_a f = \frac{a\cdot\nabla f(p)}{|a|}$$

$$\text{div } U = \nabla\cdot U = \frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z}$$

$$\text{div}(\text{grad } f) = \nabla^2 f$$

$$= \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Scanned with CamScanner

let

$V(x,y,x) = V = [V_1, V_2, V_3]$

$= V_1 \hat{i} + V_2 \hat{j} + V_3 \hat{k}$

curl $V = \Delta \times V =$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ V_1 & V_2 & V_3 \end{vmatrix}$$

$$= \left(\frac{\partial V_3}{\partial y} - \frac{\partial V_2}{\partial z}\right)\hat{i} - \left(\frac{\partial V_3}{\partial x} - \frac{\partial V_1}{\partial z}\right)\hat{j}$$

$$+ \left(\frac{\partial V_2}{\partial z} - \frac{\partial V_3}{\partial y}\right)\hat{k}$$

$\nabla(fg) = f\nabla g + g\nabla f$

$\nabla(f/g) = (1/g^2)(g\nabla f - f\nabla g)$

$\operatorname{div}(fV) = f \operatorname{div} V + V \cdot \nabla f$

$\operatorname{div}(f\nabla g) = f\nabla^2 g + \nabla f \cdot \nabla g$

$\nabla^2 f = \operatorname{div}(\nabla f)$

$\operatorname{curl}(\nabla f) = 0$

$\operatorname{div}(\operatorname{curl}(V)) = 0$

$\nabla^2(fg) = g\nabla^2 f + 2\nabla f \cdot \nabla g + f\nabla^2 g$

$\operatorname{curl}(fV) = \nabla f \times V + f \operatorname{curl} V$

$\operatorname{div}(U \times V) = V \cdot \operatorname{curl} U - U \cdot \operatorname{curl} V$

Scalar field $\xrightarrow{\nabla f}$ vector field

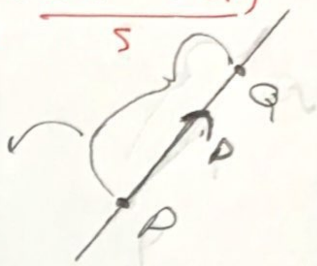
Scalar field $\xrightarrow{\nabla^2 f}$ scalar field

Vector field $\xrightarrow{\text{div}}$ scalar field

Vector field $\xrightarrow{\text{curl } V}$ vector field

$$D_a f = \frac{\partial f}{\partial s} = \lim_{s \to 0} \frac{f(Q) - f(P)}{s}$$

$S = Q - P$



دبس

موفقين ❤!