تقدم لجنة ElCoM الاكاديمية

ELECTRICAL COMPUTER MECHATRONICS
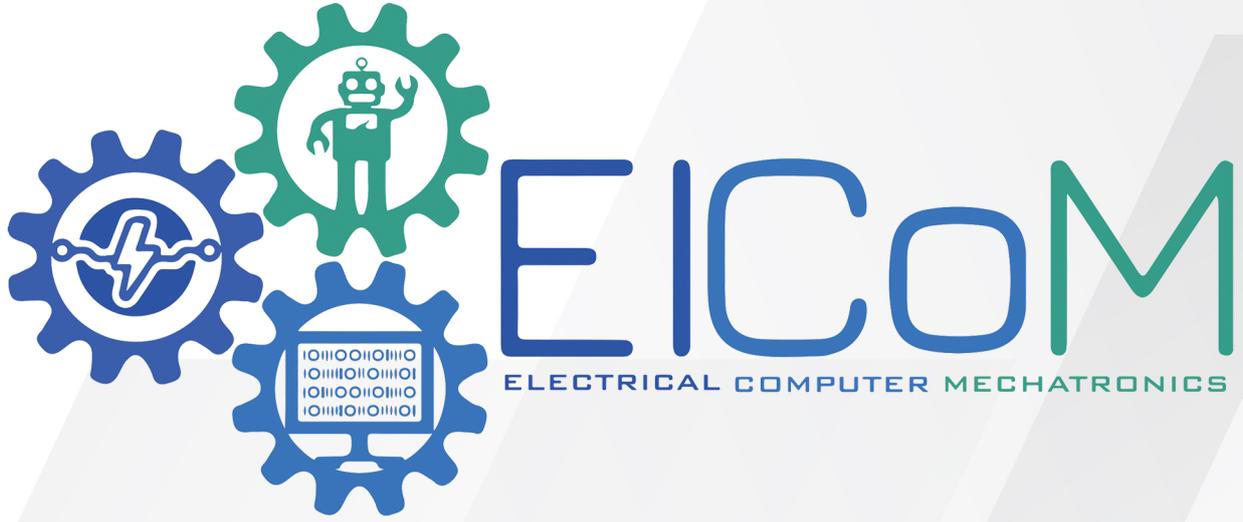
تلخيص لمادة:

رياضيات تطبيقية

جزيل الشكر للطالبة:

رزان قرمش

* Applied / first !

$$\underset{\text{complex number}}{\underline{Z}} = \underset{\text{Real part}}{\underline{x}} + \underset{\text{imaginary part}}{\underline{jy}} \quad yi$$

$$j = \frac{1}{j}$$
$$(j)^2 = -1$$
$$(j)^3 = -\sqrt{-1}$$

$$Z = (0, 1)$$

$$Z_1 = x + yi \quad , \quad Z_2 = A + Bi$$
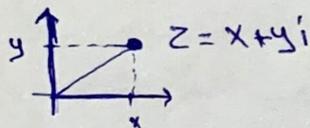
$$Z_1 + Z_2 = (x+A) + (y+B)i$$

$$Z_1 - Z_2 = (x-A) + (y-B)i$$

$$Z_1 Z_2 = (x+yi)(A+Bi)$$

$$= (xA - yB) + (xB + Ay)$$

$$\frac{Z_1}{Z_2} = \frac{(x+yi) \times (A-Bi)}{(A+Bi) \times (A-Bi)} \rightarrow \text{بضرب} \quad \text{نضرب قيمة} \quad \text{المقام} \quad \text{بمرافقه}$$

* اواخذ المرافقة الحالية نضعها

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* complex plane $\quad z = x + yi$

$$
\begin{array}{c}
y \\ \uparrow \\ \end{array} \quad \bullet \quad z = x + yi
$$

عند الجمع بطرح بطرح بطرح

مقدار الاعداد

* conjugate - complex conjugate Number

$$z = x \oplus yi$$

$$\bar{z} = x \ominus yi$$

$$z \cdot \bar{z} = x^2 + y^2 \quad / \quad z + \bar{z} = 2x$$

$$z - \bar{z} = 2yi$$

$$x = \frac{1}{2}(z + \bar{z})$$

$$y = \frac{1}{2i}(z - \bar{z})$$
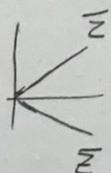
if → z real $x = z$
$$z = \bar{z}$$

if → z imaginary
$$z = yi$$
$$z = -\bar{z}$$

2

$$\overline{(Z_1 + Z_2)} = \bar{Z}_1 + \bar{Z}_2 \quad / \quad \overline{(Z_1 - Z_2)} = \bar{Z}_1 - \bar{Z}_2$$

$$\cdot \overline{(Z_1 Z_2)} = \bar{Z}_1 \bar{Z}_2 \quad / \cdot \overline{(Z_1 / Z_2)} = \bar{Z}_1 / \bar{Z}_2$$

* polar form

$$x = r\cos\theta \quad , \quad y = r\sin\theta$$

$$r = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$z = x + yi = r[\cos\theta + \sin(\theta)i] = r e^{\theta i}$$

* polar form conjugate

$$z = r e^{\theta i} \quad , \quad \bar{z} = r e^{-\theta i} \quad | \quad |z| = |\bar{z}|$$

$$\boxed{\arg \bar{z} = -\arg z}$$

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polar form

* ضرب وقسمة الاعداد عقدية

* Multiplication / Divisions

$$Z_1 = r_1[\cos\theta_1 + j\sin\theta_1]$$
$$Z_2 = r_2[\cos\theta_2 + j\sin\theta_2]$$

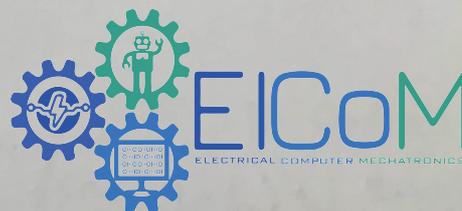$$Z_1 \ast Z_2 = r_1 r_2 [\cos(\theta_1 + \theta_2) + \sin(\theta_1 + \theta_2)]$$

$$Z_1 / Z_2 = (r_1 / r_2)[\cos(\theta_1 - \theta_2) + \sin(\theta_1 - \theta_2)]$$

مهم جدا للحل

$$\cos(A \mp B) = \cos(A)\cos(B) \pm \sin(A)\sin(B)$$
$$\sin(A \mp B) = \sin(A)\cos(B) \mp \sin(B)\cos(A)$$

$$\sqrt[n]{x + jy} = \sqrt[n]{r}\left[\cos\left(\frac{\theta + 2\pi K}{n}\right) + j\sin\left(\frac{\theta + 2\pi K}{n}\right)\right]$$

$$r = \sqrt{x^2 + y^2}$$

$$\theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$K = 0, 1, 2 \cdots n-1$$

نسبة

$$K = 0 \rightarrow \text{first root}$$

هو بحدد

$$K = n-1 \quad \text{J} \quad K = 0 \quad \text{بين قوس نحصل}$$
او بتغير بين قيم

Exponential function (e^z)

$z = x + yi$

$e^z = e^{x+yi} = e^x * e^{yi}$

$e^{iy} = (\cos y + j \sin y)$

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properties of e^z

- $(e^z)' = e^z$ / $e^{z_1} e^{z_2} = e^{z_1+z_2}$

$e^{2\pi i} = 1$ / $e^{(\pi/2)i} = i$ / $e^{i\pi} = -1$ / $e^{(-\frac{\pi}{2})i} = -i$

$|e^{iy}| = 1$ / $|e^z| = e^x$ and $\arg e^z = y + 2\pi n$

$e^{z+2n\pi i} = e^z$

$e^{R-Bi} = e^R (\cos B - j \sin B)$

$|e^{R-Bi}| = e^R$ / $\text{Ang } e^{R-Bi} = -B$

$\cos -\theta = \cos \theta \quad \leftarrow$ مهم *

$\sin -\theta = [-] \sin \theta$

$\cos x = \frac{1}{2}(e^{xi} + e^{-xi})$

$\sin x = \frac{1}{2i}(e^{xi} - e^{-xi})$

$|\cos z|^2 = \cos^2 x + \sin^2 y$

$\tan z = \frac{\sin z}{\cos z}$

$\cot z = \frac{\cos z}{\sin z} = \frac{1}{\tan z}$

$\sec z = \frac{1}{\cos z}$

$\csc(z) = \frac{1}{\sin z}$

$e^{in\theta} = \cos(n\theta) + i \sin(\theta n)$

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* Hyperbolic function

$\sinh(zi) = \frac{1}{2}(e^{zi} - e^{-zi})$

$\cosh(zi) = \frac{1}{2}(e^{zi} + e^{-zi})$

$\sinh(z) = \frac{1}{2}(e^z - e^{-z})$

$\cosh(z) = \frac{1}{2}(e^z + e^{-z})$

$\begin{array}{c} \cos(zi) \\ = \\ \cosh z \\ \sin(zi) \\ = \\ i\sinh(z) \end{array}$

$\tanh z = \frac{\sinh z}{\cosh z}$ / $\coth(z) = \frac{\cosh z}{\sinh z}$

$\text{sech}(z) = \frac{1}{\cosh(z)}$ / $\text{csch} z = \frac{1}{\sinh z}$

$\cos(z) = \cos(x)\cosh(y) \ominus i \sin(x) \sinh(y)$

$\sin(z) = \sin(x)\cosh y + i \cos(x) \sinh y$

$|\cos(z)|^2 = \cos^2(x) + \sinh^2(y)$

$|\sin(z)|^2 = \sin^2(x) + \sinh^2(y)$

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* Natural logarithm function

$\text{Ln}(z) = \ln(r) + j \text{Ang} z \quad$ القيمة الرئيسية

$\ln(z) = \ln(r) + j \text{ang} z \quad$ عدد القيم

$\text{Ln } z = \ln|z| + j \text{Ang} z$

$\ln(z) = \text{Ln} z \mp jn2\pi$

$\text{ang}(e^z) = y \mp 2n\pi$

$\ln e^z = z \pm in2\pi$

$\ln(z_1 z_2) = \ln(z_1) + \ln(z_2)$

$\ln(z_1/z_2) = \ln(z_1) - \ln(z_2)$

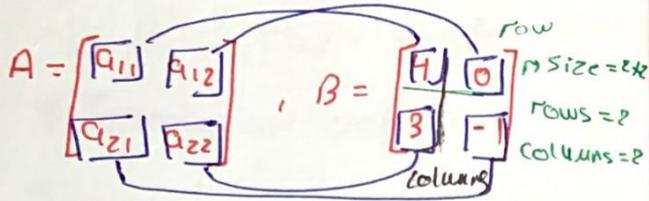## * general power $z^c = e^{c \ln z} = e^{c(\ln r + \theta i)}$

$c = z^c = e^{c \ln z} = e^{c(\ln r + \theta i)}$ is single valued

النهايه ♡

$e^{z_1 + z_2} = e^{x_1} e^{x_2} [\cos(y_1 + y_2) + j \sin(y_1 + y_2)]$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ \end{bmatrix}$$ الضرب الأول

الضرب الثاني
$$\begin{bmatrix} a_{21} & a_{22} & a_{23} \end{bmatrix}$$

$$\times \begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix} \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix} \begin{bmatrix} a_{13} \\ a_{23} \\ a_{33} \end{bmatrix}$$

عمود (1) عمود (2) عمود (3)

$$[2 \times 3] \qquad [3 \times 3]$$

* second / Applied

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad , \quad B = \begin{bmatrix} 4 & 0 \\ 3 & -1 \end{bmatrix}$$

row
n size = 2×2
rows = 2
columns = 2
columns

$$A = B$$

1 have same size

2 $a_{11} = 4$, $a_{12} = 0$

$a_{21} = 3$, $a_{22} = -1$

row vector
حجز واحد
column vector
عامود واحد

الضرب الأول	الضرب الأول	الضرب الأول
الصف الأول	الصف الأول	الصف الأول
عامود (1)	عامود (2)	عامود (3)
الضرب الثاني	الضرب الثاني	الضرب الثاني
الصف الثاني	الصف الثاني	الصف الثاني
عامود (1)	عامود (2)	عامود (3)

*Addition of Matrics

$$A + B = \begin{bmatrix} (a_{11}+4) & (a_{12}+0) \\ (a_{21}+3) & (a_{22}-1) \end{bmatrix}$$

بجمع /جمع الاضافة المتشابهة

* Matrix Multiplication Rules

$$(kA)B = k(AB) = A(kB)$$

$$A(BC) = (AB)C$$

$$(A+B)C = AC + BC$$

$$C(A+B) = CA + CB$$

انتبه للترتيب

$$A - B = \begin{bmatrix} (a_{11}-4) & (a_{12}-0) \\ (a_{21}-3) & (a_{22}+1) \end{bmatrix}$$
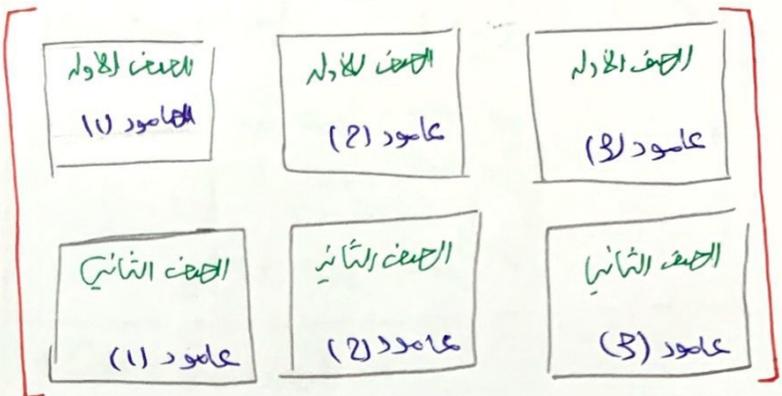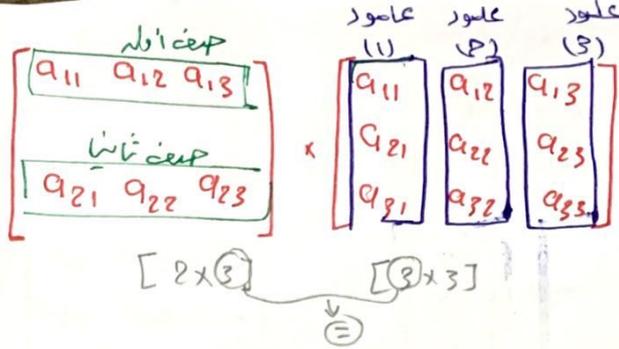
لازم يكون نفس الـ size لنفس

* scalar multiplication

$$-A = \begin{bmatrix} -a_{11} & -a_{12} \\ -a_{21} & -a_{22} \end{bmatrix}$$

* Transposition :-

$$A = \begin{bmatrix} a_{11} & a_{12} & -a_{1n} \\ a_{21} & a_{22} & -- a_{2n} \end{bmatrix} \rightarrow A^T = \begin{bmatrix} a_{11} & a_{21} \\ a_{12} & a_{22} \\ a_{1n} & a_{2n} \end{bmatrix}$$

$$A + B = B + A \qquad \qquad C(A+B) = CA + CB$$

$$(A+B)+C = A+(B+C) \qquad (C+B)A = CA + BA$$

$$A + 0 = A \quad \text{matrix}$$

$$A + (-A) = 0 \qquad \qquad C(kA) = (Ck)A$$

$$1A = A$$

مقلوب صف إلى عامود

* Rules of Transposition.

$$(A^T)^T = A$$

$$(A+B)^T = A^T + B^T$$

$$A \times B = C \quad + \quad A \times B \neq B \times A$$

$$[m \times n] \times [n \times p] = [m \times p]$$

$$(CA)^T = C \, A^T$$

scalar

انتبه للترتيب محلولة

لازم تطلب

$$BA = 0$$
$$A = 0$$
or $$B = 0$$

يكونوا متساويان

$$(AB)^T = B^T A^T$$

$A^T = A \rightarrow$ symmetric matrix

$\bar{A}^T = -A \rightarrow$ skew-symmetric matrix

* Triangular matrix

* upper triangular matrix

(متساوية المثلث العلوي)

* مثال G

$$\begin{bmatrix} 1 & 4 & 2 \\ 0 & 3 & 2 \\ 0 & 0 & 6 \end{bmatrix} \quad \begin{bmatrix} 1 & 3 \\ 0 & 8 \end{bmatrix}$$

* Lower triangular matrix

(وحصني تحت المثلث السفلي)

* مثال G

$$\begin{bmatrix} 2 & 0 & 0 \\ 8 & -1 & 0 \\ 7 & 6 & 8 \end{bmatrix}$$

1. Diagonal matrix (D)
$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

2. Scalar matrix (S)
$$\begin{bmatrix} c & 0 & 0 \\ 0 & c & 0 \\ 0 & 0 & c \end{bmatrix}$$

* جميع تكون نفسها (Diagonal)

3. Unit or identity matrix (U)
$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

* عندنا اشكال معادلتين هادلتين نستفاد متغيرين
عندها ٣ خيارات

① اذا تقاطعوا بنقطة ← حل واحد

② اذا انطبقوا على بعض ← عدد لا نهائي

③ اذا توازوا ← صفر من الحلول
(homogeneous) مسألة تجانس

$$a_{11}x_1 + a_{12}x_2 \quad --- + a_{1n}x_{1n} = b_1 \quad \boxed{5}$$
$$a_{21}x_1 + a_{22}x_2 + \quad --- a_{2n}x_{2n} = b_2$$
$$a_{31}x_1 + a_{32}x_3 + \quad --- a_{3n}x_{3n} = b_3$$

$$\begin{bmatrix} a_{11} & a_{12} & - & a_{1n} \\ a_{2\dagger} & a_{22} & - & a_{2n} \\ a_{31} & a_{32} & -- & a_{3n} \end{bmatrix}, \quad x = \begin{bmatrix} x_{1n} \\ x_{2n} \\ : \\ x_n \end{bmatrix} \quad \text{and } b \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$$

$$\begin{bmatrix} a_{11} & a_{12} & ---a_{1m} & b_1 \\ a_{21} & a_{22} & -- a_{2n} & b_2 \\ a_{31} & a_{32} & -- a_{3n} & b_3 \end{bmatrix}$$

* Gaues Elimination

* الفكرة انو بدك لعملشي تحولهن معادلة matrix

جدهم بتحادل تغير بتشكل لل matrix

لتوصل لشكل معين تطلع منو اخر متغيرات

مثلاً

$$2x_1 + 5x_2 = 2$$
$$-4x_1 + 3x_2 = -30$$

$$\begin{bmatrix} 2 & 5 & | & 2 \\ -4 & 3 & | & -30 \end{bmatrix} \rightarrow \text{row}(2) + 2\,\text{row}(1)$$

$$\begin{bmatrix} 2 & 5 & | & 2 \\ 0 & 13 & | & -20 \end{bmatrix}$$

$$13x_2 = -26$$
$$\boxed{x_2 = -2}$$

مسموح تخرب / تجمع / تقسم / تضرب /
تكتب بين در (rows) الصفوف

* نشان فقط تحط على اخر الأخيرة

وفي العلاقات الاساسية بين الصفوف (rows)
؟

* Rank of matrix (A):

هو عدد الصفوف التي لا تساوي صفراً

بعد اجراء (Gaus elimination)

① اعمل gaus elimination

② الصفوف التي لا تساوي صفر $[0\ 0\ 0\ 0\ 2]$
عد

يكون مقدار (A) أي ,, Rank

* يُقال للصفوف والأعمدة بأنها مترابطة
(dependent) يعني

* ولو ما تساوي غير نفسها ($A = 0$)
يعني مستقلة (independent)

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* Determinants (def)

$$D = \det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = (a_{11}a_{22}) \ominus (a_{12}a_{21})$$

$$D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix} = (a_{11}(a_{22}a_{33}) \ominus (a_{23}a_{32}))$$

$$\ominus (a_{12}(a_{21}a_{33} - a_{23}a_{31})) + (a_{13}(a_{21}a_{32} - a_{22}a_{31}))$$

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* Minors and cofactors

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

* The minor matrix

$$M = \begin{bmatrix} M_{11} & M_{12} & M_{13} \\ M_{21} & M_{22} & M_{23} \\ M_{31} & M_{32} & M_{33} \end{bmatrix}$$

$$M_{11} = \begin{vmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{vmatrix}$$

$$M_{12} = \begin{vmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{vmatrix}$$

وهذا يجري على كل عنصر

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* The cofactor matrix

$$C = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & a_{22} & a_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}$$

$$C = \begin{bmatrix} + & - & + \\ - & + & - \\ + & - & + \end{bmatrix}$$

$$C_{jk} = (-1)^{j+k} (M_{jk})$$

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* Diagonal matrix

$$\begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{32} \end{bmatrix}$$ بتعتبر gaus elimination

$$\text{Diagonal} = a_{11} * a_{22} * a_{33}$$

$$a_{11}x_1 + a_{12}x_2 = b_1 \quad , \quad a_{21}x_1 + a_{22}x_2 = b_2$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{D} \quad x_2 \quad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}}{D}$$

نحذف عمود
x_1

$$D = \det A = \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

x_1 نحذف عمود

نحذف عمود
x_2

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 = b_1$$
$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 = b_2$$
$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 = b_3$$

$$x_1 = \frac{\begin{vmatrix} b_1 & a_{12} & a_{13} \\ b_2 & a_{22} & a_{23} \\ b_3 & a_{32} & a_{33} \end{vmatrix}}{D} \quad x_2 = \frac{\begin{vmatrix} a_{11} & b_1 & a_{13} \\ a_{21} & b_2 & a_{23} \\ a_{31} & b_3 & a_{33} \end{vmatrix}}{D}$$

$$x_3 = \frac{\begin{vmatrix} a_{11} & a_{21} & b_1 \\ a_{21} & a_{22} & b_2 \\ a_{31} & a_{23} & b_3 \end{vmatrix}}{D} \quad D = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

* ~~Matrix~~ **inverse** of matrix (A^{-1})

* كل مصفوفة تكون لها inverse إذا ,و

كان rankها = الحالة إذا / (inverse)ها rank = n
det ≠ 0
(non-singular /invertible) بنسميها

$\cancel{=}$ / إذا ما كان الها (inverse) أو (matrix)ها (inverse) لنسميها
(singular)

* مثال على إيجاد العكس

$$A = \begin{bmatrix} -1 & 1 & 2 \\ 3 & -1 & 1 \\ -1 & 3 & 4 \end{bmatrix}$$

$$\left[\begin{array}{ccc|ccc} -1 & 1 & 2 & 1 & 0 & 0 \\ 3 & -1 & 1 & 0 & 1 & 0 \\ -1 & 3 & 4 & 0 & 0 & 1 \end{array}\right]$$

نطبق (gaus elimination) عشان نخليها على فكرة هذا الجزء

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & \square & \square & \square \\ 0 & 1 & 0 & \square & \square & \square \\ 0 & 0 & 1 & \square & \square & \square \end{array}\right]$$

(inverse) هذا الجزء بكون الـ لهذا

$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ is $A^{-1} = \dfrac{1}{det} \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$

$A^{-1} = \dfrac{1}{det A} [C_{jk}]^{T \#}$ * طريقة أسهل لجعل (inverse)

$(AC)^{-1} = C^{-1} A^{-1}$

$(AC \text{ -- } PG)^{-1} = G^{-1} P^{-1} \cdots C^{-1} A^{-1}$

* انتبه للعلاقات

$AB = 0 \rightsquigarrow A = 0$ or $B = 0$

$AB \neq BA$

---

let A, B, C be $n \times n$

↳ rank$(A) = n$ and $AB = AC \rightsquigarrow A = B$

↳ rank$(A) = n$ and $AB = 0 \begin{Bmatrix} \end{Bmatrix} B = 0$

↳ $AB = 0$ but $A \neq 0$ and $B \neq 0$

↦ rank$(A) < n$ and rank$(B) < 0$

if A is singular $\rightsquigarrow AB$ and BA
singular

$det(AB) = det(BA) = det A \cdot det B$

$adj(A) = (cof A)^T$

Orthogonal matrix

$$\boxed{A^T = A^{-1}}$$

* Matrix Eigenvalue problems

$$[A]\, x = [\lambda x] \rightarrow \text{unknow vector}$$
(احنا بنجيبها)

square matrix   unknown scalar
مصفوفة مربعة   (احنا بنجيبها)
غالباً تعبر بالسمبول Δ

* خطوات حل الأسئلة اللي على
  الموضوع ع

1 $D(\lambda) = \det (A - \lambda I)$

$$\begin{bmatrix} [\square - \lambda] & \square & \square \\ \square & [\square - \lambda] & \square \\ \square & \square & [\square - \lambda] \end{bmatrix}$$ هون بنقص من (العناصر اللي على القطر)

بجديت بنطلع الـ det عادي بتطلع مع معادلة
منها بنطلع قيم الـ λ هي دي بتكون الـ (eigenvalue)

2 $A - \lambda I$

هون بجيب الـ A بعد وضع قيم الـ (λ) اللي
[خلعت معي] واحنا بنعوض من عناصر القطر

بعدها ونطلع قيم X₁, X₂ ---- Gauss
elimination (بجد علاقات بينع)

بالآخر بعرض خليت لوحدة من (X₁, Xₚ ---)
ونطلع الفكتور X

eigenvector   هي بتكون طلعت الـ
(AX = λX إذا بدك تتأكد)

$A - \lambda I \rightarrow$ the charactristc
                                    matrix

$D(\lambda) = \det(A - \lambda I) \rightarrow$
the charactristic determinta
                        of A

$D(\lambda) = 0 \rightarrow$ the charactristc
equation of A

A^T and A has the
same eigenvalue

Symmetric matrix
$$A^T = A$$
بتقلب الصفوف أعمدة والعكس

Skew symmetric matrix
$$A^T = -A$$

orthoganal matrix
$$A^T = A^{-1}$$

$$R = \frac{1}{2}(A + A^T),$$

$$S = \frac{1}{2}(A - A^T)$$

$$R + S = A$$
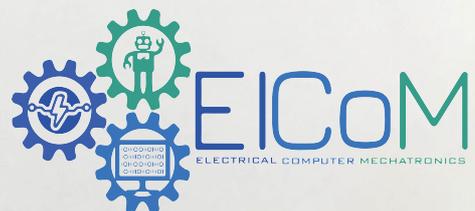
* Similar matrix ($\hat{A}$)

$$\hat{A} = \bar{P}^{1} A \boxed{P} \rightarrow \text{non-singular}$$

$\hat{A}$ is similar to A
→ $\hat{A}$ has the same eigenvalues
and eigenvector as $\underline{\underline{A}}$

* بإضافة بنعطيني matrix n P
matrix n A ,

$\hat{A} = \bar{P} A P$ بنجيب عكس P^{-1} بطلع
الاول بنوجد
بنجيب عكس (P)

_ _ _ _ _ _ _ _ _

* Diagonalization of matrix

$$D = X^{-1} A X$$

الحل :-

بنطلع eigenvector يعني قيم X_1 , X_2 نحطها
خطوات الحل بالورقة الي جبل

* اجمع ال matrix وحدة
* فظ X^{-1} ئي
* صعبا : $D = X^{-1} A X$

بكون مصفوفة A matrix n
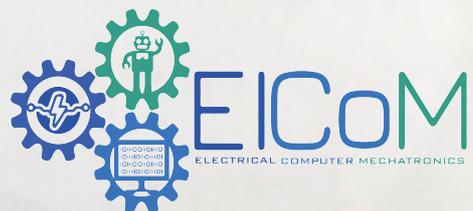
* Vector Calculus :-

Scalar ⟶ determined by magnitude

Vector ⟶ Magnitude and direction.

Tail → initial point

tip → terminal point

$\vec{a} = \vec{b}$

↳ $\vec{a}$ and $\vec{b}$ Have same direction
when and same magnitude

*A vector of length (1) is called a unit vector

- - - - - - - - - -

initial point → $P: (x_1, y_1, z_1)$

terminal point → $q: (x_2, y_2, z_2)$

$a = [\underset{a_1}{(x_2-x_1)}, \underset{a_2}{(y_2-y_1)}, \underset{a_3}{(z_2-z_1)}]$

| terminal − initial |

$|a| = \sqrt{(a_1)^2 + (a_2)^2 + (a_3)^2}$

* A vector with the origin (0,0,0) as the initial point is called position vector

$\vec{a} = [a_1, a_2, a_3]$

↳ $\vec{a} = a_1 \hat{i} + a_2 \hat{j} + a_3 \hat{k}$

$a = [a_1, a_2, a_3]$

$b = [b_1, b_2, b_3]$

$a+b = [(a_1+b_1), (a_2+b_2), (a_3+b_3)]$

$a-b = [(a_1-b_1), (a_2-b_2), (a_3-b_3)]$

$\underset{\text{scalar}}{c}a = [ca_1, ca_2, ca_3]$

* properties

* $\vec{a}+\vec{b} = \vec{b}+\vec{a}$

* $(\vec{a}+\vec{b})+\vec{k} = (\vec{k}+\vec{a})+\vec{b}$

* $\vec{a} + (-\vec{a}) = 0$

* $\vec{a} + 0 = \vec{a}$

c → scalar

* $c(\vec{a}+\vec{b}) = c\vec{a}+c\vec{b}$

* $(c+k)\vec{a} = c\vec{a}+k\vec{a}$

* $c(k\vec{a}) = (ck)\vec{a}$

* $1\vec{a} = \vec{a}$

---

* Dot product ↘ (inner product)
The result is always a scalar

$\vec{a}.\vec{b} = |a||b| \cos y$

$a.b > 0 → y ←$ حادة

b و a متجهين الزاوية

$a.b = 0 → y ←$ قائمة

$a.b < 0 → y ←$ منفرجة

أو

$a = [a_1, a_2, a_3]$, $b = [b_1, b_2, b_3]$

$a.b = [a_1b_1 + a_2b_2 + a_3b_3]$

$\cos y = \dfrac{a.b}{|a||b|}$ ↝ $y = \cos^{-1}\left(\dfrac{a.b}{|a||b|}\right)$

Scanned with CamScanner

* Properties of (inner /Dot) product.

- $(q_1 \vec{a} + q_2 \vec{b}) \cdot \vec{c} = q_1 (\vec{a} \cdot \vec{c}) + q_2 (\vec{b} \cdot \vec{c})$

- $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

- $\vec{a} \cdot \vec{a} \geq 0$

  $\rightarrow \vec{a} \cdot \vec{a} = 0 \rightarrow \vec{a} = 0$

* since $|\cos y| \leq 1$

$|a \cdot b| \leq |a| \, |b|$

$|a + b| \leq |a| + |b|$

$|a+b|^2 + |a-b|^2 = 2(|a|^2 + |b|^2)$

$\vec{\imath} \cdot \vec{\imath} = \vec{\jmath} \cdot \vec{\jmath} = \vec{k} \cdot \vec{k} = \boxed{1}$ فريق متجهين متشابهين

$\vec{\imath} \cdot \vec{\jmath} = \vec{\jmath} \cdot \vec{\imath} = \vec{\jmath} \cdot \vec{k} = \vec{k} \cdot \vec{\jmath} = \vec{k} \cdot \vec{\imath} = \vec{\imath} \cdot \vec{k} = \boxed{0}$

---

* Vector /Cross product

$a \times b = |a| \, |b| \sin \theta$

$a \times b = 0 \rightarrow \begin{cases} a = 0 \\ b = 0 \\ \theta = 180, 0 \end{cases}$

الزاوية بين a و b

* عمليا احد المتجهات بيستخدم قاعدة اليد اليمنى
يمط اليد اليمنى باتجاه المتجه ($\vec{a}$) و تدور

حتى تطابق المتجه ($\vec{b}$) النتيجه properties المتجهات
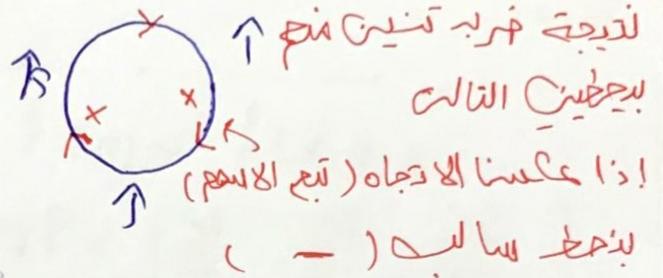النتائج عمودي على $\vec{a}$ و $\vec{b}$

$a = [a_1, a_2, a_3]$

$b = [b_1, b_2, b_3]$

$$a \times b = \begin{vmatrix} \vec{\imath} & \vec{\jmath} & \vec{k} \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix}$$

$$= \begin{vmatrix} a_2 & a_3 \\ b_2 & b_3 \end{vmatrix} \vec{\imath} \ominus \begin{vmatrix} a_1 & a_3 \\ b_1 & b_3 \end{vmatrix} \vec{\jmath}$$
$$+ \begin{vmatrix} a_1 & a_2 \\ b_1 & b_2 \end{vmatrix} \vec{k}$$

$\vec{\imath} \times \vec{\imath} = \vec{\jmath} \times \vec{\jmath} = \vec{k} \times \vec{k} = \boxed{0}$

لنتيجة ضرب متجهين مع نفسه بتطلع الناتج صفر

إذا عكسنا الاتجاه (تتبع الاسهم)
بتطلع بالسالب ($-$)

---

* Properties of Cross product

$(L \vec{a}) \times \vec{b} = L(\vec{a} \times \vec{b})$

$(\vec{a} + \vec{b}) \times \vec{c} = \vec{a} \times \vec{c} + \vec{b} \times \vec{c}$

$(\vec{b} \times \vec{a}) = -(\vec{a} \times \vec{b})$

$\vec{a} \times (\vec{b} \times \vec{c}) \neq (\vec{a} \times \vec{b}) \times \vec{c}$

---

* Scalar Triple product

$(abc) = a \cdot (b \times c) = a \cdot V$

$= a_1 V_1 + a_2 V_2 + a_3 V_3$

* $a \cdot (b \times c) = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}$

$= (a \times b) \cdot c$

1. Volume of a box

V = (Height) (Area of the base)

$= |g| \cos\beta \qquad |b \times a|$

$Volume = |a| \; |b \times c| \; |\cos\beta|$

$\hookrightarrow (abc) = |a \cdot |b \times c| |$

علي إذا علم $\hookleftarrow$

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* Vector ~~function~~ function
y (Whose Values are Vectors)

$U(t) = [V_1(t), N_2(t), U_3(t)]$

$U'(t) = [U_1'(t), U_2'(t), U_3'(t)]$

لتشتق k بدق بالنسبة لـ (t) إحداثي

* Rules

• $(CU)' = C V'$ • $(u+v)' = u' + v'$

scalar

$(u \cdot v)' = u' \cdot v + u \cdot v'$

$(u \times v)' = u' \times v + u \times v'$

$(u \, v \, w)' = (u' \, v \, w) + (u \, v' \, w) +$

$\qquad\qquad (u \, v \, w')$

$f = [x, y, z]$

~~unit~~

unit vector f $\dfrac{[x, y, z]}{|f|}$

$U = [V_1, V_2, V_3] = U_1 \uparrow + V_2 \uparrow + V_3 \uparrow$

* Partial derivatives

$\dfrac{\partial U}{\partial t_u} = \dfrac{\partial V_1}{\partial t_u}\uparrow + \dfrac{\partial U_2}{\partial t_u}\uparrow + \dfrac{\partial V_3}{\partial t_u}\uparrow$

* Second partial derivatives

$\dfrac{\partial^2 U}{\partial t_i \partial t_u} = \dfrac{\partial^2 U_1}{\partial t_i t_u}\uparrow + \dfrac{\partial^2 V_2}{\partial t_i t_u}\uparrow + \dfrac{\partial^2 U_3}{\partial t_i t_u}\uparrow$

لو أدل شيء بنشتق بالنسبة لـ t_u بعدين

t_L بالنسبة

- - - - - - - -

$f(P) = f(x, y, z)$

$\nabla f = \left[\dfrac{\partial f}{\partial x}, \dfrac{\partial f}{\partial y}, \dfrac{\partial f}{\partial z} \right]$

بالنسبة لـ(z) بنشتق بالنسبة لـ(x) بنشتق بالنسبة لـ(y) بنشتق بالنسبة لـ(x) والباقي ثوابت   والباقي ثوابت

$\nabla f \rightarrow grad \, f$

$D_a f = \dfrac{a \cdot \nabla f(a)}{|a|}$

$div \; U = \nabla \cdot U = \dfrac{\partial U_1}{\partial x} + \dfrac{\partial U_2}{\partial y} + \dfrac{\partial U_3}{\partial z}$

$div(grad \, f) = \nabla^2 f$

$= \dfrac{\partial^2 f}{\partial x^2} + \dfrac{\partial^2 f}{\partial y^2} + \dfrac{\partial^2 f}{\partial z^2}$

let

$U(x,y,x) = U = [U_1, U_2, U_3]$

$= U_1 \hat{\imath} + U_2 \hat{\jmath} + U_3 \hat{k}$

curl $U = \Delta \times U = $

$$\begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ U_1 & U_2 & U_3 \end{vmatrix}$$

$= \left(\frac{\partial U_3}{\partial y} \ominus \frac{\partial U_2}{\partial z} \right) \hat{\imath} \ominus \left(\frac{\partial U_3}{\partial x} \ominus \frac{\partial U_1}{\partial z} \right) \hat{\jmath}$

$\oplus \left(\frac{\partial U_2}{\partial z} \ominus \frac{\partial U_3}{\partial y} \right) \hat{k}$

- - - - - - - - -

$\nabla(fg) = f\nabla g + g\nabla f$

$\nabla(f/g) = (1/g^2)(g\nabla f - f\nabla g)$

$div(fU) = f \, div \, U + U \cdot \nabla f$

$div(f\nabla g) = f\nabla^2 g + \nabla f \cdot \nabla g$

$\nabla^2 f = div(\nabla f)$

$curl(\nabla f) = 0$

$div(curl(U) = 0$

$\nabla^2(fg) = g\nabla^2 f + 2\nabla f \cdot \nabla g + f\nabla^2 g$

$curl(fU) = \nabla f \times U + f \, curl \, U$

$div(u \times V) = V \cdot curl \, u - u \cdot curl \, V$

Scalar field $\xrightarrow{\nabla f}$ Vector field

Scalar field $\xrightarrow{\nabla^2 f}$ scalar field

Vector field $\xrightarrow{div}$ scalar field

Vector field $\xrightarrow{curl}$ vector field

- - - - - -

$D_{\alpha}f = \frac{\partial f}{\partial s} = \lim_{s \to 0} \frac{f(Q) - f(P)}{s}$

$S = Q - P$

دعاء ۱♥ مونصر