

تقدّم لجنة ElCoM الأكاديمية

دفتر الفاينل لماقة:
فيزياء عامة (2)

من شرح:
د. عسان النعواشبي

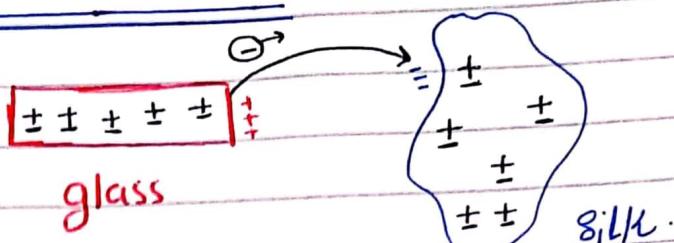
جزيل الشكر للطالبة:
تولين أسامة



Ch - 23 - Electric Fields

- Glass rod rubbed with silk.

مواد بتحتاج طاقة لربط فيها منها تكون طاقة
لربط كالب و فيها تكون طاقة لربط قليلة
بعضها يقل لاتtraction دفعهم على الحادة
الاتractionات.



نحو (1)

⇒ Electrons are transferred from glass to silk.

⇒ glass is positively charged.

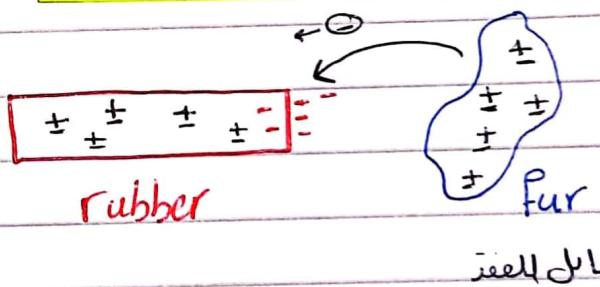
تنتقل در glass ذئب قابل العقد
إيجي در silk لذئب قابل لذئب

silk is negatively charged.

عذبة لذئب كباره ذئب ايجي
و لذئب كاله هو طاقة

- Rubber rod rubbed with fur.

⇒ Electrons are transferred from fur to rubber



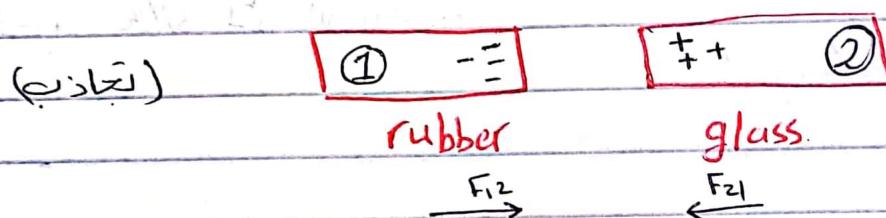
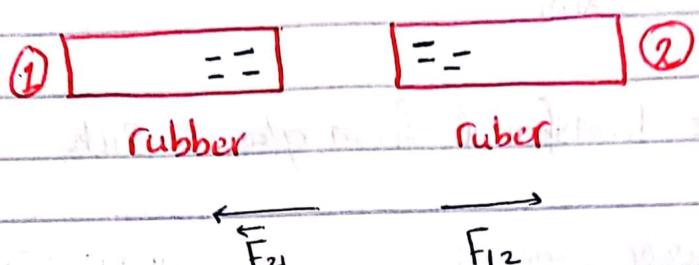
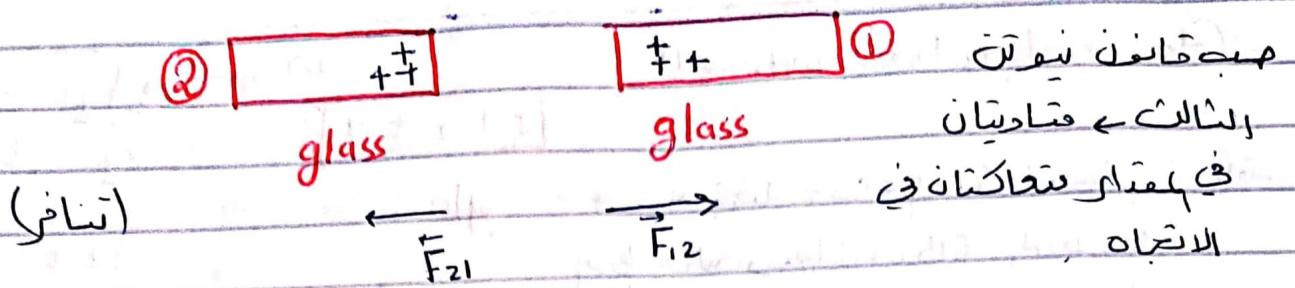
⇒ fur is positively charged.

⇒ Rubber is negatively charged.

Conclusion : There are two kinds of charges.

1) positive (+) : Like protons.

2) negative (-) : Like electron.



السماء تجاذب أو تنافر (مصادِر في المقدار)
معاكِشَةِ فِي الاتِّجاهِ

$$\vec{F}_{12} = -\vec{F}_{21}$$

Newton's 3rd Law

Conclusion

* Like charges repel



* unlike charges attract



* Conservation of charges (conservation)

* Electric charge is always conserved on an isolated system

Ex \Rightarrow charge is not created in the process of rubbing two objects together but transferred from one object to another

* Quantization of charge +

Electric charge is said to quantized. \Leftrightarrow (مقدار) ie Electric charge exist as discrete packets.

مقدار قيمه مركب (لا يمكن ان ينفصل عن مجموعه اجزائه)

$$q = \pm ne$$

$$e \times n = n$$

* $e = 1.6 \times 10^{-19}$ C

| C: Coulomb

Electron: $q_e = -e = -1.6 \times 10^{-19}$ C.

proton: $q_p = +e = 1.6 \times 10^{-19}$ C.

* Classification of materials. +

materials are classified in terms of the ability of electrons to move through insp.

1) Conductors (conductors) + Some of the electrons are free (unbound) \rightarrow (مقيمة)

e.g. Aluminum, Silver, Copper (الفضي)

2) Insulators (العزلاء) : All the electrons are bound
لَا يوجّه نظر الأكسنةات لها (عقيدة)
عقيدة

Example \Rightarrow glass, rubber, wood, plastic.

3) Semiconductors (الإيجاب) : Electric properties are some when between those of conductors and insulators.

Ex \Rightarrow Silicon, germanium

(الإيجاب)

* Coulomb's law :

افتراض

Assumption : we have point charges

(مُنْجَلِّب) (charge of zero size)

$$F_e = k_e \frac{|q_1||q_2|}{r^2}$$

$$\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$$

F = electric force.

$$k_e = \text{Coulomb's constant} = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2 = \text{permittivity of free space.}$$

F_e is attractive if charges are of ~~the~~ opposite sign.

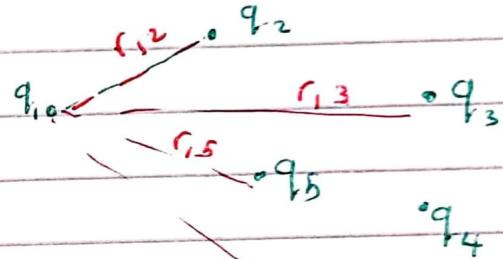
F_e is negative if charges are of the same sign

$$\vec{F}_{12} = -\vec{F}_{21} = \frac{k_e |q_1|q_2|}{r^2} \hat{r} \rightarrow \text{unit vector}$$

جذب معاً معاً معاً معاً

Superposition principle:

مقدار

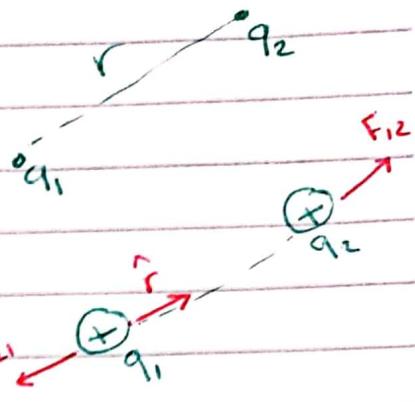


The resultant force on q_1 is the vector sum of all the forces exerted on it by other charges.

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41} + \dots$$

Coulomb's Law \Rightarrow

$$F_e = k_e = \frac{|q_1||q_2|}{r^2}$$



$$\vec{F}_{12} = -\vec{F}_{21} = k \frac{|q_1||q_2|}{r^2} \hat{r}$$

* Superposition principle.

$$\vec{F}_i = \vec{F}_{2i} + \vec{F}_{3i} + \vec{F}_{4i}$$

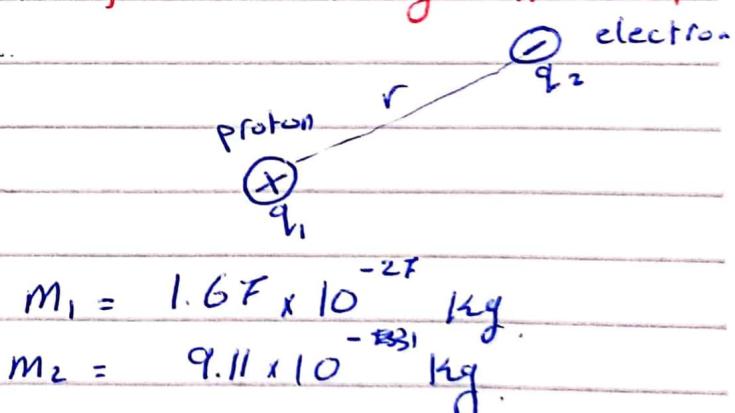
Ex \Rightarrow 23.1

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately 5.3×10^{-11} m. Find the magnitudes of the electric force and the gravitational force between the two particles.

$$q_1 = +e = 1.6 \times 10^{-19} \text{ C}$$

$$q_2 = -e = -1.6 \times 10^{-19} \text{ C}$$

$$r = 5.3 \times 10^{-11} \text{ m}$$



$$m_1 = 1.67 \times 10^{-27} \text{ kg}$$

$$m_2 = 9.11 \times 10^{-31} \text{ kg}$$

$$F_e = k \frac{|q_1||q_2|}{r^2} = (9 \times 10^9) \frac{(1.6 \times 10^{-19})(1.6 \times 10^{-19})}{(5.3 \times 10^{-11})^2}$$

\therefore by $\vec{F}_1 = -\vec{F}_2$ \therefore $\vec{F}_1 = 8.2 \times 10^{-8} \text{ N}$

dist, size, nisba

$$F_G = G \frac{m_1 m_2}{r^2}$$

$$= (6.67 \times 10^{-11}) \frac{(1.67 \times 10^{-27})(9.11 \times 10^{-31})}{(5.3 \times 10^{-11})^2}$$
$$= 3.6 \times 10^{-39} \text{ N}$$

مقدار الجاذبية قوية جداً

$$\frac{F_G}{F_e} \approx 2.3 \times 10^{39} \Rightarrow F_G \text{ is negligible (also)}$$

Ex. 23.2] Consider three point charges located at the corners of a right triangle as shown in figure where $q_1 = q_3 = 5.00 \mu\text{C}$, $q_2 = -2.00 \mu\text{C}$ and $a = 0.100 \text{ m}$ find the resultant force exerted on q_3

$$q_1 = q_3 = 5 \mu\text{C}$$

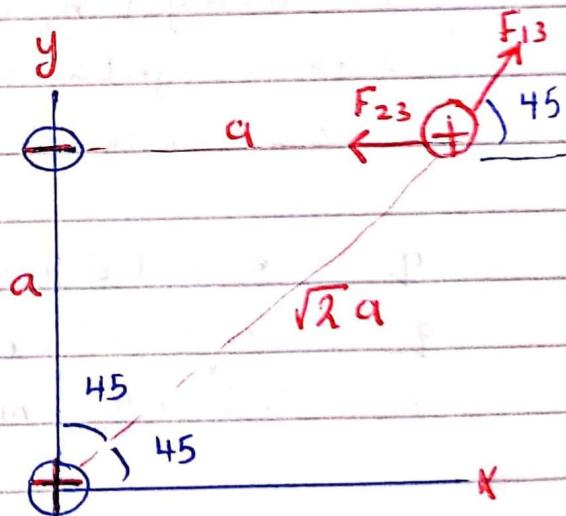
$$q_2 = -2 \mu\text{C}$$

$$a = 0.1 \text{ m}$$

$$F_{13} = k \frac{|q_1||q_3|}{(\sqrt{2}a)^2}$$

$$= (9 \times 10^9) \frac{(5 \times 10^{-6})(5 \times 10^{-6})}{(2)(0.1)^2}$$

$$F_{13} = 11.2 \text{ N}$$



$$F_{23} = k \frac{|q_2| |q_3|}{a^2} = (9 \times 10^9) \frac{(2 \times 10^{-6})(5 \times 10^{-6})}{(0.1)^2}$$

$$F_{23} = 9 \text{ N.}$$

$$F_{13x} = F_{13} \cos 45^\circ = (11)(0.71) = 7.9 \text{ N}$$

$$F_{13y} = F_{13} \sin 45^\circ = (11.2)(0.71) = 7.9 \text{ N}$$

$$F_{23x} = F_{23} \cos 180^\circ = (9)(-1) = -9 \text{ N}$$

$$F_{23y} = F_{23} \sin 180^\circ = 0$$

$$F_{3x} = F_{13x} + F_{23x} = -1.1 \text{ N}$$

$$F_{3y} = F_{13y} + F_{23y} = 7.9 \text{ N}$$

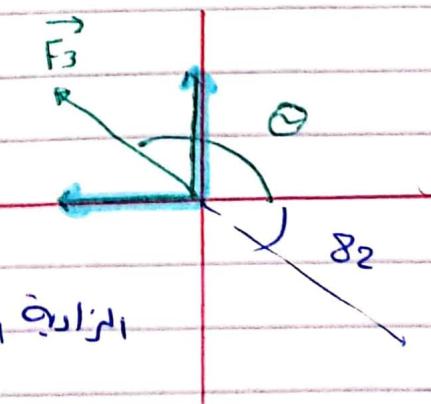
$$\vec{F}_3 = (-1.1 \hat{i} + 7.9 \hat{j}) \text{ N}$$

$$F_3 = \sqrt{(-1.1)^2 + (7.9)^2} \approx 8 \text{ N.}$$

$$\theta = \tan^{-1} \left(\frac{F_{3y}}{F_{3x}} \right) = \tan^{-1} \left(\frac{7.9}{-1.1} \right)$$

$$= -82^\circ + 180^\circ \\ = 98^\circ$$

الرابع الراحي \leftarrow الراحي، الراحي يتطلع إلى الخارج



ex. 23.3 \Rightarrow Three point charges lie along the X axis as shown in Figure. The positive charge $q_1 = 15.0 \mu C$ is at $X = 2.00 \text{ m}$, the positive charge $q_2 = 6.00 \mu C$ is at the origin, and the net force acting on q_3 is zero. What is the X coordinate of q_3 ?

$$q_1 = 15 \mu C$$

$$q_2 = 6 \mu C$$

$$\sum F_{3x} = 0$$

$$F_{13} - F_{23} = 0$$

$$F_{13} = F_{23}$$

$$\cancel{K} \frac{|q_1| |q_3|}{(2-x)^2} = \cancel{K} \frac{|q_2| |q_3|}{x^2}$$

$$\frac{|q_1|}{(2-x)^2} = \frac{|q_2|}{x^2}$$

$$\cancel{(5)} \frac{15 \times 10^{-6}}{(2-x)^2} = \cancel{(2)} \frac{6 \times 10^{-6}}{x^2}$$

$$\Rightarrow \frac{5}{(2-x)^2} = \frac{2}{x^2} \Rightarrow \cancel{4} \frac{5}{4-4x+x^2} = \frac{2}{x^2}$$

$$5x^2 = 8 - 8x + 2x^2$$

$$3x^2 + 8x + 8 = 0$$

$$x = \frac{-8 \pm \sqrt{8^2 - (4)(3)(-8)}}{6}$$

$$x = 0.775 \text{ m} \checkmark$$

$$x = -3.44 \text{ m}$$

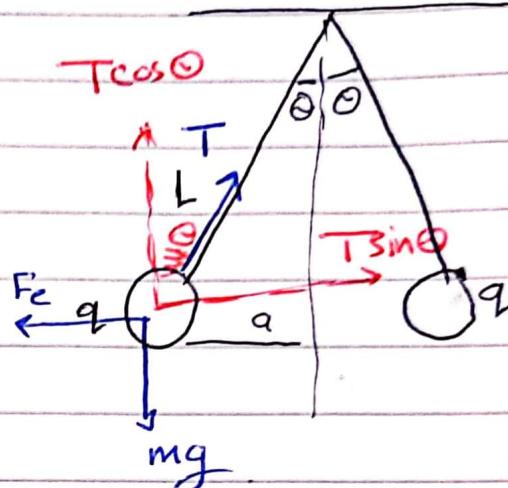
Q8(b)

Ex 23.4 \Rightarrow Two identical small charged spheres, each having a mass of 3.00×10^{-2} kg, hang in equilibrium as shown in Figure. The length L of each string is 0.150 m and the angle Θ is 5.00. Find the magnitude of the charge on each sphere.

$$m = 3 \times 10^{-2} \text{ kg}, L = 0.15 \text{ m}$$

$$\Theta = 5^\circ$$

$$F_e = k \frac{|q|^2}{(2a)^2} = k \frac{|q|^2}{4a^2}$$



$$T \sin \Theta = F_e$$

$$T \cos \Theta = mg$$

~~Divide~~

$$\sin \Theta = \frac{a}{L}$$

$$a = L \sin \Theta$$

$$\frac{\sin \Theta}{\cos \Theta} = \frac{F_e}{mg}$$

$$\tan \Theta = \frac{F_e}{mg}$$

$$|q| = \sqrt{\frac{4L^2 mg \sin^2 \Theta \tan \Theta}{k}}$$

$$F_e = mg \tan \Theta$$

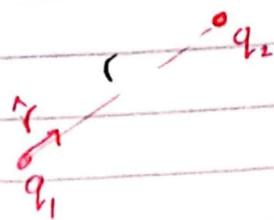
$$|q| = 4.4 \times 10^{-8} \text{ C.}$$

$$\frac{k |q|^2}{4a^2} = mg \tan \Theta$$

$$|q| = \sqrt{\frac{4a^2 mg \tan \Theta}{k}}$$

* The Electric field *

$$\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r}$$



- * The electric force is a field force.
(Can act through space with no physical contact)

ال之力能作用于空间中的任何物体，而不需要物理接触。

- * Electric field exists in a region of space around charged objects.

- * When a charge enters the electric field, an electric force acts on it.

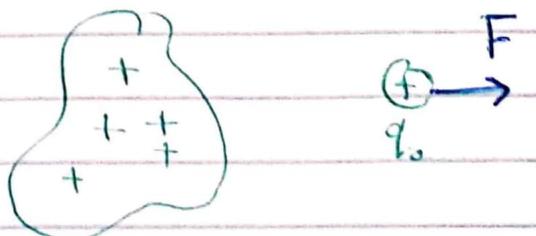
- * The electric field is defined as the electric force acting on a positive per unit charge.

- * The electric field (\vec{E}) at a point is defined as the electric force (\vec{F}) acting on a positive test charge (q_0) placed at that point divided by the test charge.

$$\vec{E} = \frac{\vec{F}}{q_0}$$

definition of \vec{E}

电场强度的定义是将一个正电荷放在该点时所受的电场力与该电荷的比值。



$$[\vec{F} = q\vec{E}]$$

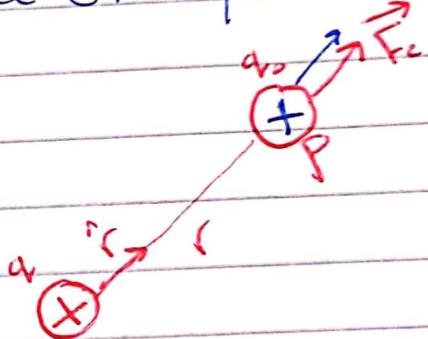
↳ The electric force \vec{F} on a charge q placed on an electric field

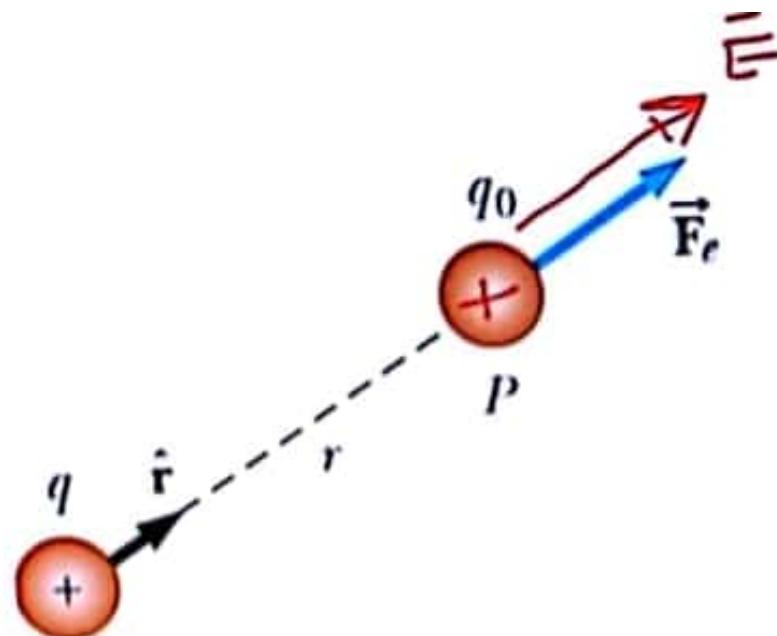
SI unit of \vec{E} is N/C

$$[E] = \frac{[F]}{|q|} = \frac{N}{C}$$

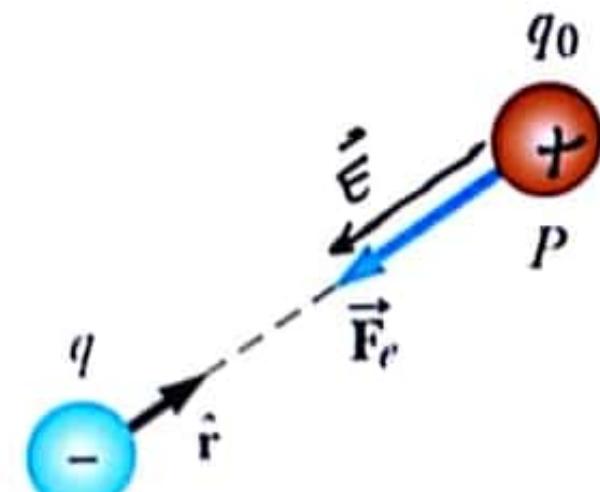
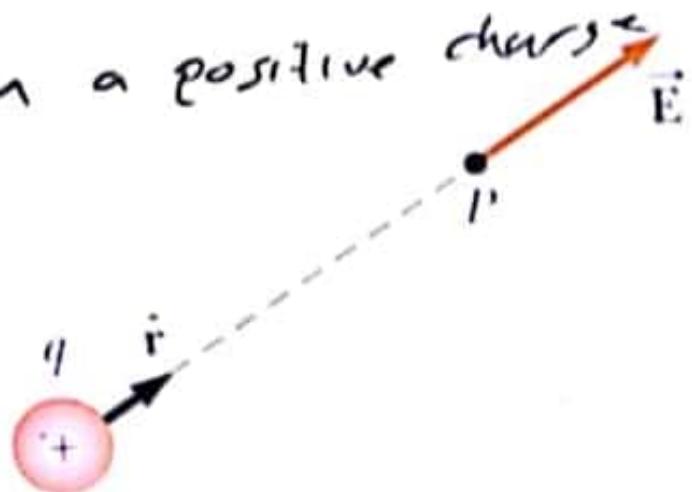
Comments:

- 1) Direction of \vec{E} is that of the force on a positive charge.
- 2) SI unit of E is N/C
- 3) $\vec{F} = q\vec{E}$ is valid for point charges.
- 4) \vec{E} is directed away from a positive charge.
- 5) E is directed toward a negative.





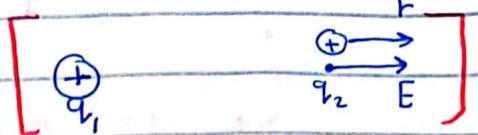
\vec{E} is directed away from a positive charge



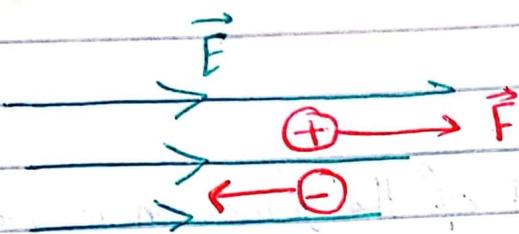
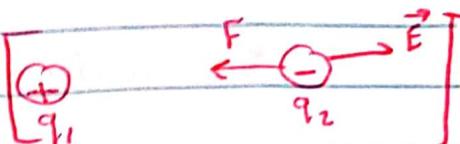
\vec{E} is directed toward a negative charge



6) The force on a positive charge is in the same direction of \vec{E}



7) The force on a negative charge is opposite to \vec{E}



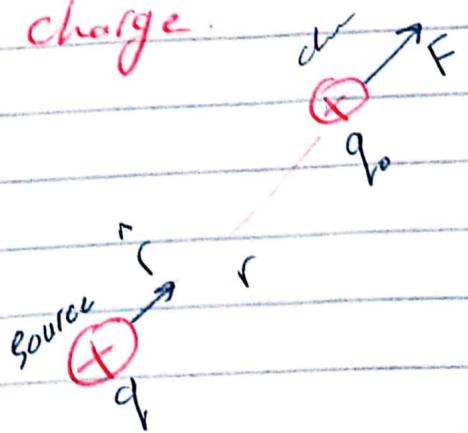
The force between the source and the test charge is

$$F = k \frac{q q_0}{r^2} \hat{r}$$

The electric field is then $\vec{E} = \vec{F} = \frac{kq}{q_0 r^2} \hat{r}$

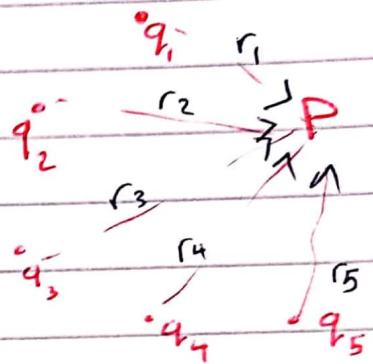
$$\boxed{\vec{E} = \frac{kq}{r^2} \hat{r}}$$

\vec{E} due to r from a point charge

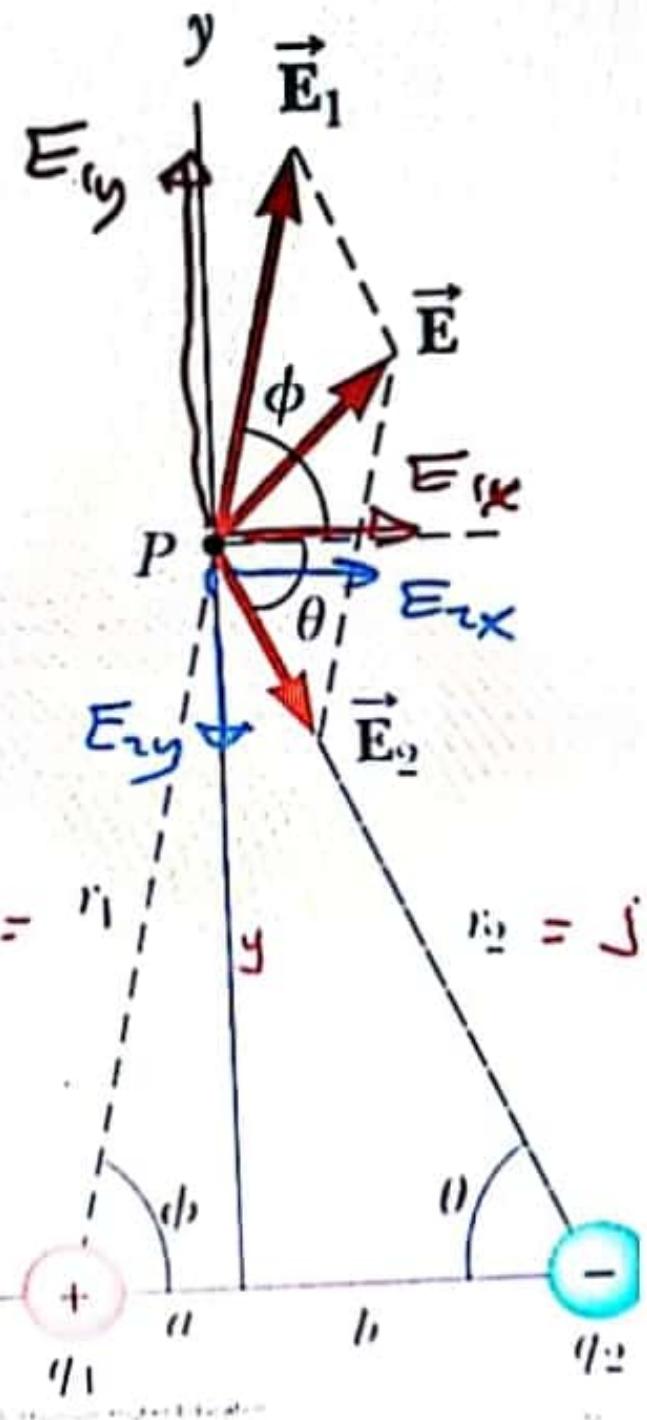
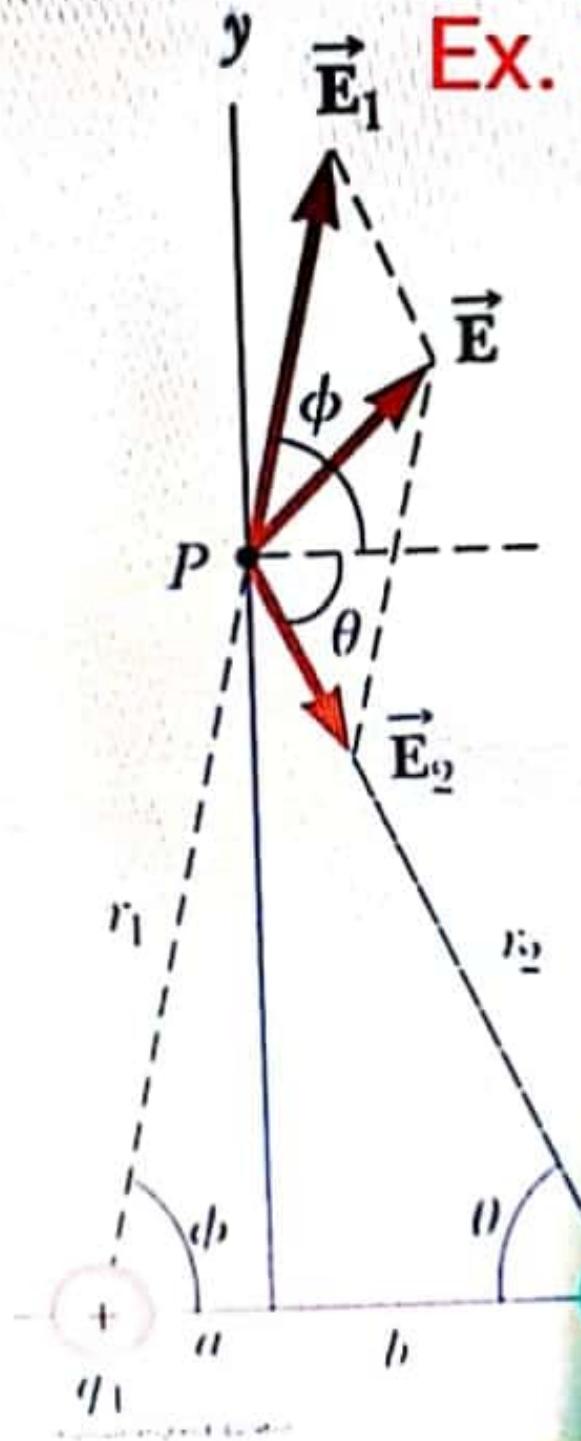


The total electric field at a point \vec{P} due to a group of charges is the vector sum of the electric fields of all the charges.

$$\begin{aligned}
 \vec{E}_P &= \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots \\
 &= \frac{kq_1}{r_1^2} \hat{r}_1 + \frac{kq_2}{r_2^2} \hat{r}_2 + \dots \\
 &= \sum k \frac{q_c}{r_c^2} \hat{r}_c
 \end{aligned}$$



Ex. 23.6



EX. 23.6 p. 702

Charges q_1 and q_2 are located on the X axis, at distances a and b , respectively, from the origin, as shown in the figure.

(A) Find the components of the net electric field at the point P , which is at position.

$$A) E_1 = \frac{k |q_1|}{r_1^2} = k \frac{|q_1|}{a^2 + y^2}$$

$$E_2 = \frac{k |q_2|}{r_2^2} = \frac{k |q_2|}{b^2 + y^2}$$

$$E_{1x} = E_1 \cos \theta = \frac{k |q_1| \cos \theta}{a^2 + y^2}$$

$$E_{1y} = E_1 \sin \theta = \frac{k |q_1| \sin \theta}{a^2 + y^2}$$

$$E_{2x} = E_2 \cos \theta = \frac{k |q_2| \cos \theta}{b^2 + y^2}$$

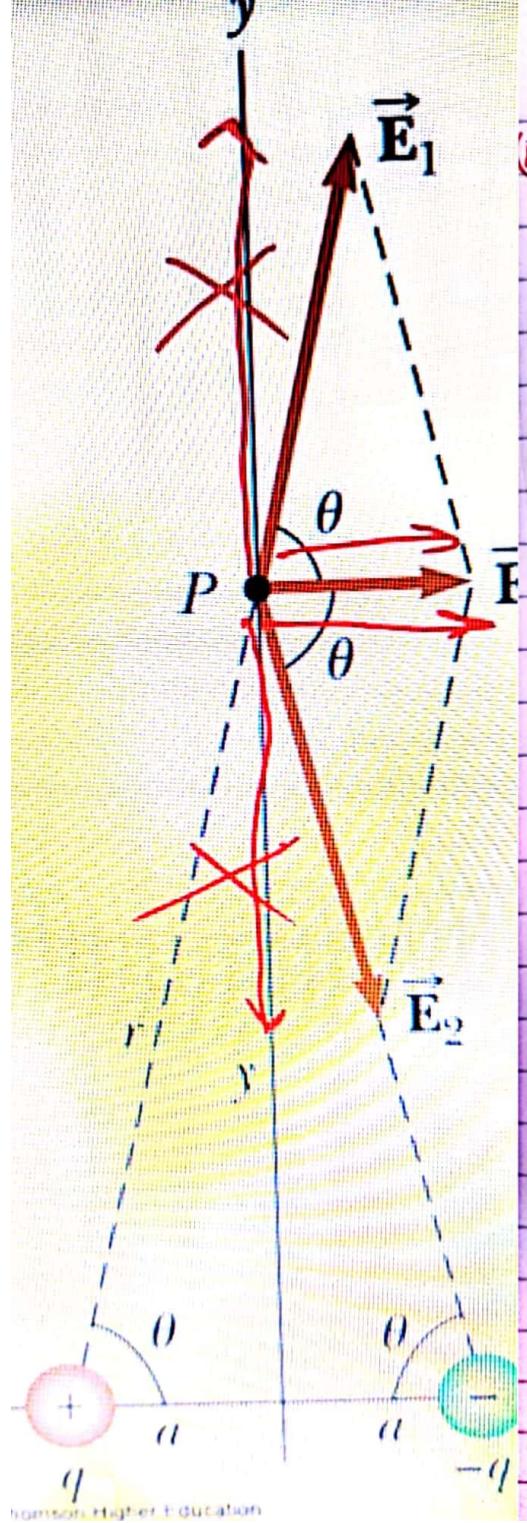
$$E_{2y} = -E_2 \sin \theta = \frac{-k |q_2| \sin \theta}{b^2 + y^2}$$

$$E_x = E_{1x} + E_{2x} = k \left[\frac{|q_1| \cos \theta}{a^2 + y^2} + \frac{|q_2| \cos \theta}{b^2 + y^2} \right]$$

$$E_y = E_{1y} + E_{2y} = k \left[\frac{|q_1| \sin \theta}{a^2 + y^2} - \frac{|q_2| \sin \theta}{b^2 + y^2} \right]$$

$$\vec{E} = E_x \hat{i} + E_y \hat{j}$$

$$E = \sqrt{}$$



(B) Evaluate the electric field of point (p) in the special case that $|q_1| = |q_2|$ and $a = b$.

$$|q_1| = |q_2| = q$$

$$a = b \Rightarrow \Theta = \phi$$

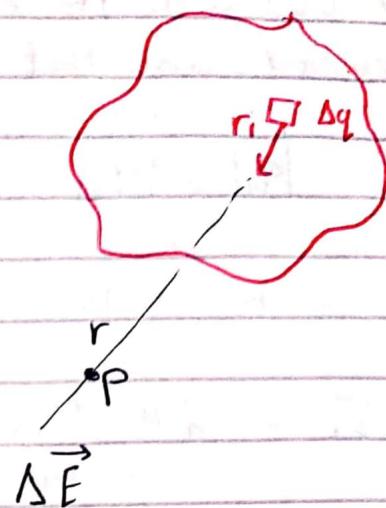
$$E_x = \frac{2kq \cos \Theta}{a^2 + y^2}$$

$$E_y = 0$$

Electric field of a continuous charge distribution

The electric field due to the charge element Δq_i is ΔE_i

$$\Delta E_i = k \frac{\Delta q_i}{r_i^2} \hat{r}_i$$



The total electric field \vec{E} at the point P is the vector of electric field of all charge elements.

$$\begin{aligned} \vec{E} &= \vec{\Delta E}_1 + \vec{\Delta E}_2 + \dots \\ &= k \frac{\Delta q_1}{r_1^2} \hat{r}_1 + k \frac{\Delta q_2}{r_2^2} \hat{r}_2 + \dots \\ &= k \sum_n \frac{\Delta q_n}{r_n^2} \hat{r}_n \end{aligned}$$

If we make the elements very small ($\Delta q \rightarrow 0$)

then

$$\vec{E} = k \lim_{\Delta q_n \rightarrow 0} \sum_n \frac{\Delta q_n}{r_n^2} \hat{r}_n = k \int \frac{dq}{r^2} \hat{r}$$

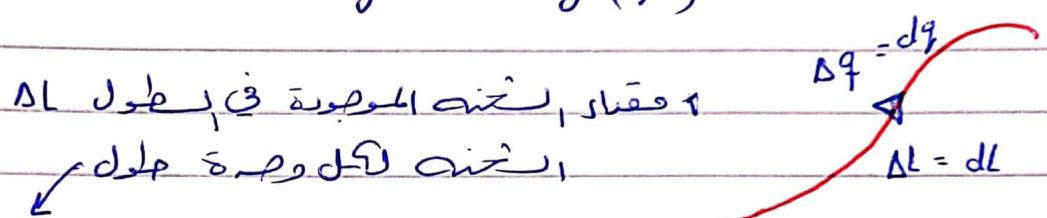
~~log~~

The electric field due to a continuous charge distributions.

$$\boxed{\vec{E} = k \int \frac{dq}{r^2} \hat{r}} = \vec{E} \text{ due to a continuous charge}$$

There are three kinds of charge distributions.

1) Linear charge distribution (أبعاد ازوجية)
we define the linear charge density (λ)



(charge per unit length)

$$\boxed{\lambda = \frac{\Delta q}{\Delta L} = \frac{dq}{dx}} ; [\lambda] \equiv C/m$$

If the total charge Q is uniformly distributed along line.
then

$$\boxed{\lambda = \frac{dq}{dx} = \frac{Q}{L} = \text{Const}}$$

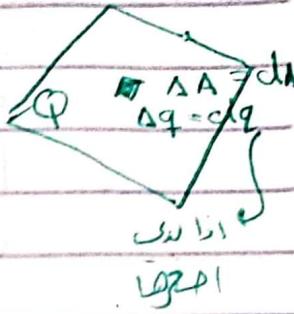
$$\boxed{dq = \lambda dx}$$

2) Surface charge distribution

We define the surface charge density (σ)

(charge per unit area)

$$\sigma = \frac{\Delta q}{\Delta A} = \frac{dq}{dA} ; [\sigma] = C/m^2$$



If the charge is uniformly distributed through the surface, then

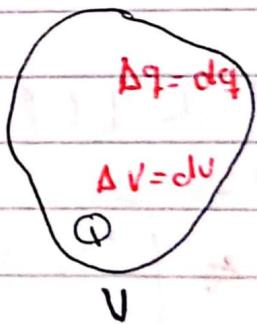
$$\sigma = \frac{dq}{dA} = \frac{Q}{A} = \text{Const}$$

$$dq = \sigma dA$$

3) Volume charge distribution

We define the volume charge density (ρ)

(charge per unit volume)



$$\rho = \frac{\Delta q}{\Delta V} = \frac{dq}{dV} ; [\rho] = C/m^3$$

If the charge is uniformly distributed (per unit size) through the volume then

$$\rho = \frac{\Delta q}{\Delta V} = \frac{Q}{V} = \text{Const}$$

$$dq = \rho dV$$

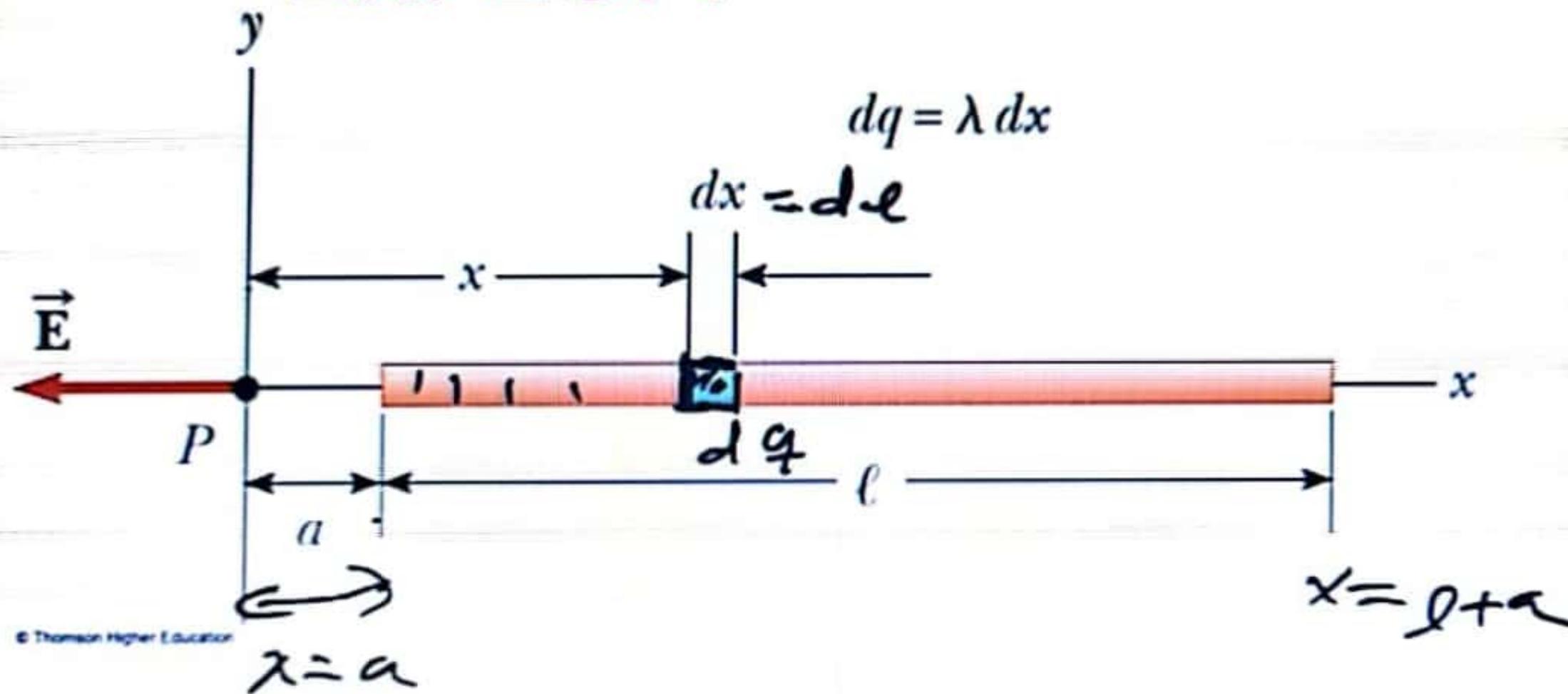
$$E = k \int \frac{dq}{r^2} \hat{r}$$

$dq = \lambda dl$ = Linear charge. (one point)

$dq = \sigma dA$ = Surface charge.

$dq = \rho \int dV$ = Volume charge.
(ρ = ~~charge~~ density)

Ex. 23.7



Ex 23.7

A rod of length L has a uniform positive charge per unit length λ and a total charge Q . Calculate the electric field at a point P that is located along the long axis of the rod and a distance a from one end

$$\lambda = \frac{Q}{L} = \text{const}$$

$$E = k \int \frac{dq}{r^2}$$

$$dq = \lambda dl = \lambda dx = \left(\frac{Q}{L} \right) dx$$

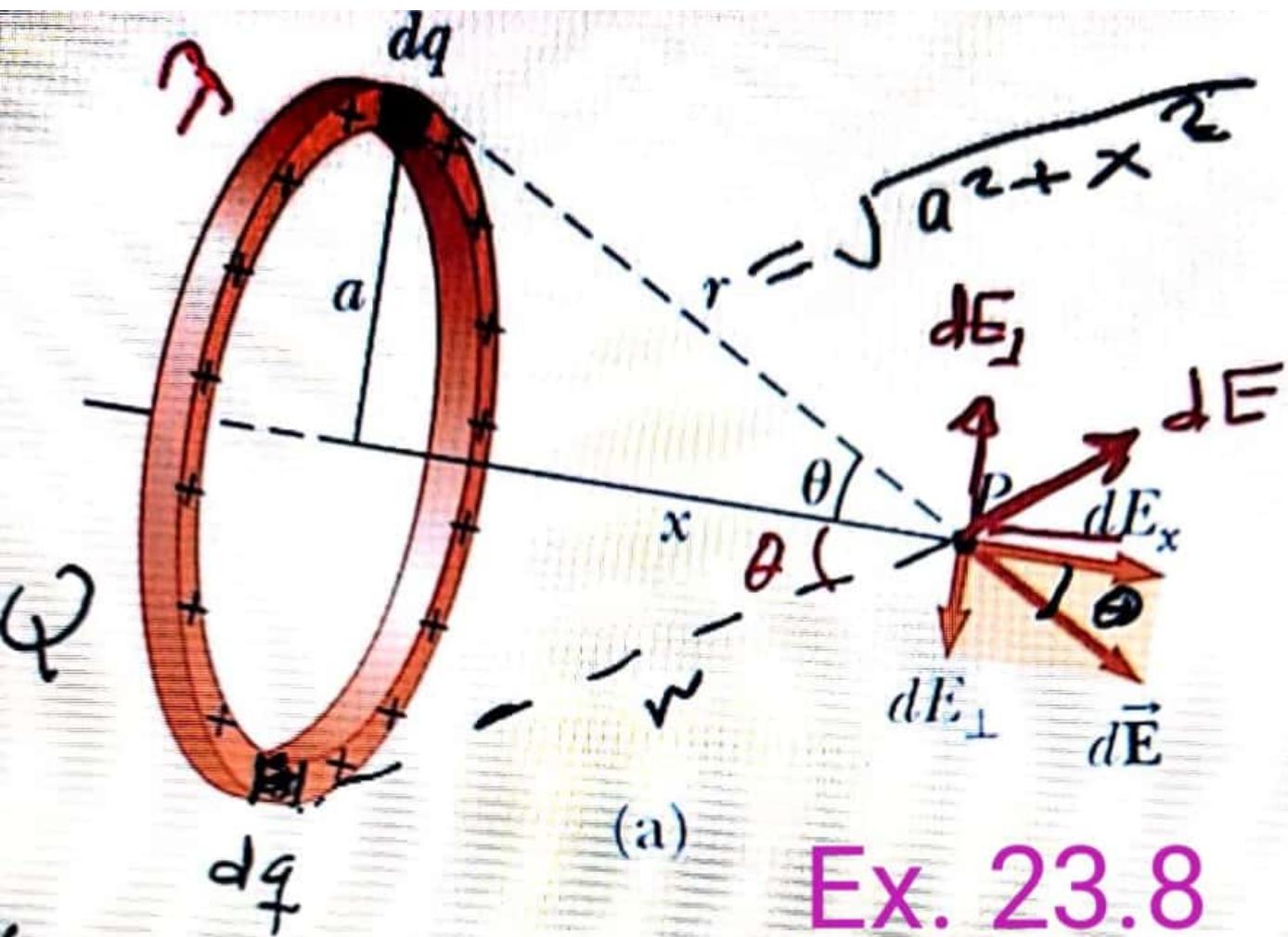
$$r = x$$

$$E = k \int_a^{L+a} \frac{dq}{r^2} = k \int_a^{L+a} \frac{\frac{Q}{L} dx}{x^2}$$

dq is the charge of dx, r is the distance of dx, dx is the length of dx

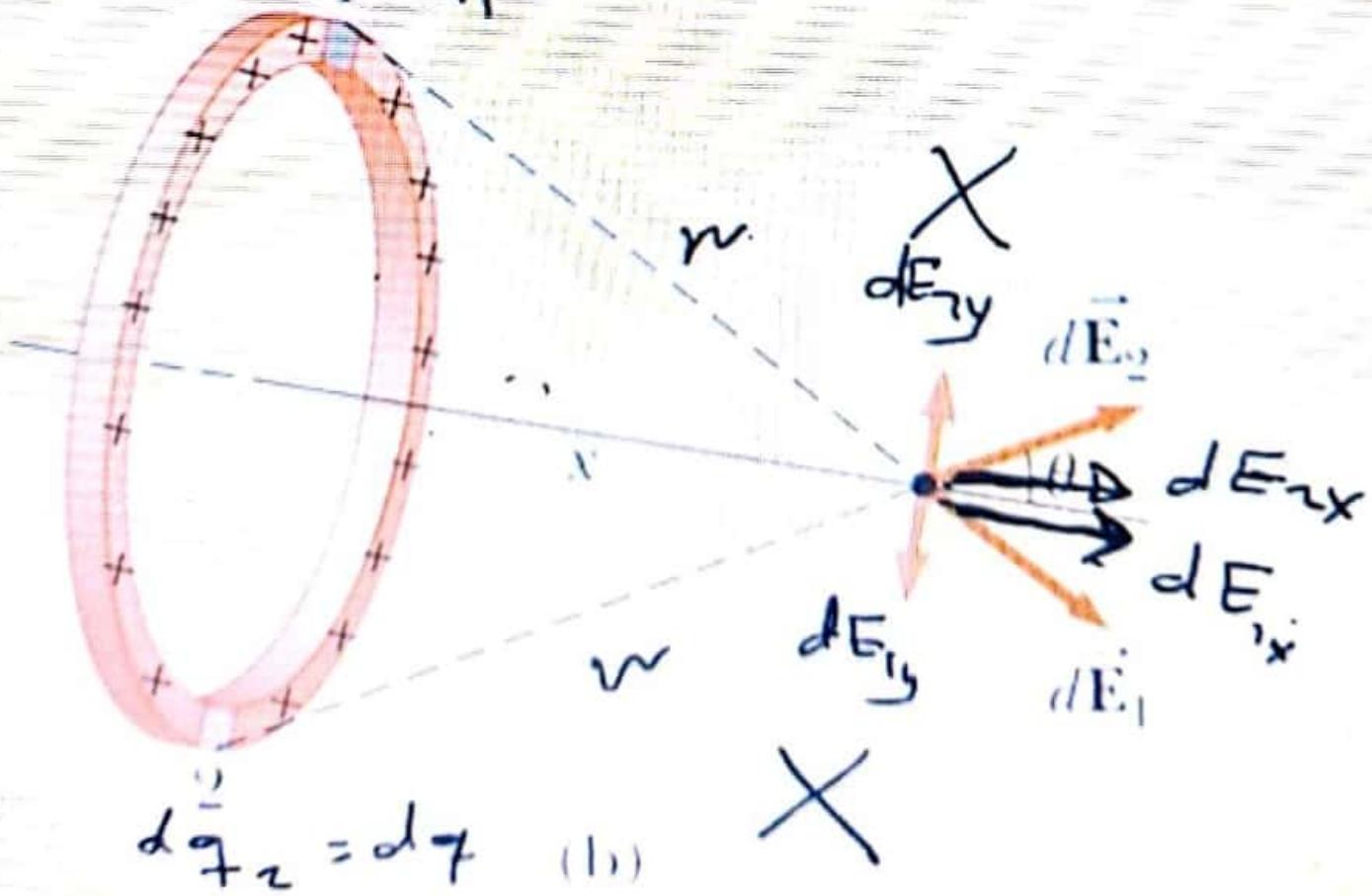
$$= \frac{kQ}{L} \int_a^{a+L} \frac{dx}{x^2} = \frac{kQ}{L} \left[\frac{-1}{x} \right]_a^{a+L}$$

$$E = \frac{kQ}{L} \left[\frac{1}{a} - \frac{1}{a+L} \right]$$



Ex. 23.8

$$d\vec{q}_1 = d\vec{q}$$



$$d\vec{q}_2 = d\vec{q}$$

Ex 23.8

A ring of radius a carries a uniformly distributed positive total charge Q . Calculate the electric field due to the ring at a point P lying a distance x from its center along the central axis perpendicular to the plane of the ring.

$\lambda = \text{uniform}$

$$\lambda = \frac{Q}{2\pi a}$$

$$dE_1 = dE_2$$

vertical components, dE_{\perp} , cancel. (each other)

$$\frac{dE}{\text{normal}} = 0$$

$$dE_{1x} = dE_{2x} = dE \cos \theta$$

$$dE = k \frac{dq}{r^2} = k \frac{dq}{a^2 + x^2}$$

$$dE_x = dE \cos \theta = k \frac{dq}{a^2 + x^2} = \frac{k \cancel{dq} x}{\sqrt{a^2 + x^2}}$$

$$\int dE_x = \int \frac{k x \, dq}{(a^2 + x^2)^{\frac{3}{2}}}$$

$$E_x = k \int \frac{x \cdot dq}{(a^2 + x^2)^{\frac{3}{2}}} \rightarrow \frac{k x}{(a^2 + x^2)^{\frac{1}{2}}} \int \frac{dq}{\cancel{x}} \quad (\text{cancel the } a^2) \\ = \frac{k Q}{(a^2 + x^2)^{\frac{1}{2}}} \quad \text{كتبه توابع} *$$

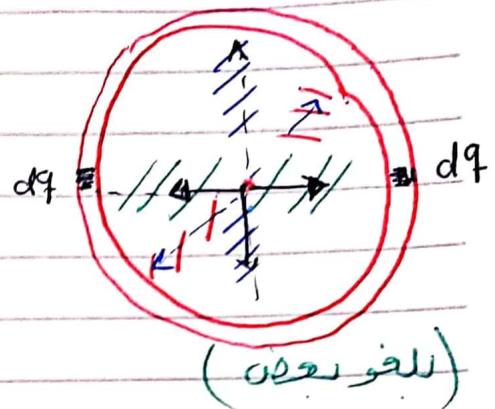
$$E_x = \frac{k Q x}{(a^2 + x^2)^{\frac{3}{2}}}$$

$\rightarrow \vec{E}$ a distance x from the center of a ring of radius a along the perpendicular axis.

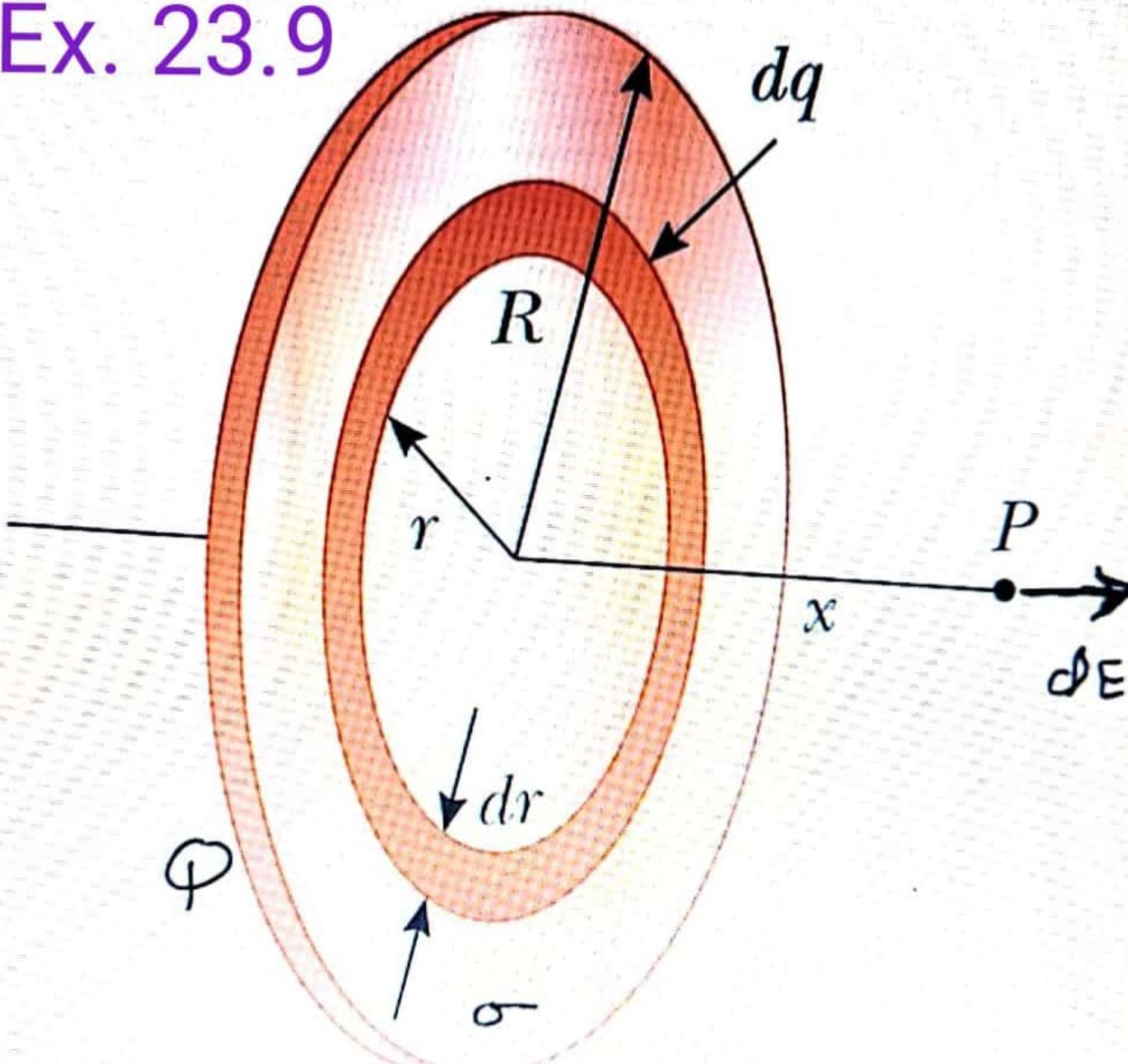
• The electric field at the center of the ring. ①
is zero

• At the center of the ring. ②
($x = 0$)

$$E = \frac{kq(x)}{(a^2+x^2)^{\frac{3}{2}}} \xrightarrow{x=0} \text{zero}$$



Ex. 23.9



[EX. 23.9]

A disk of radius R has a uniform surface charge density σ . Calculate the electric field at a point (P) that lies along the central perpendicular axis of the disk a distance X from the center of the disk.

σ is uniform
 \Rightarrow Go back to EX. 23.8 \leftarrow

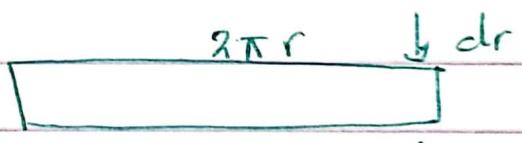
$$E = \frac{kQX}{(a^2 + X^2)^{\frac{3}{2}}}$$

with replacing

$$Q \equiv dq \text{ and } a \equiv r \rightarrow [E_x \equiv dE]$$

The electric field dE due to the charge element dq distribution along a ring of radius r is

$$\int dE = \int \frac{k dq X}{(X^2 + r^2)^{\frac{3}{2}}} \quad (\text{anti-clockwise})$$



$$dq = \sigma dA = \sigma (2\pi r dr)$$

$$dA = (2\pi r) \cdot dr$$

$$\int dE = \int_A \frac{k X dq}{(X^2 + r^2)^{\frac{3}{2}}}$$

$$E_x = \int_0^R \frac{k X \sigma (2\pi r dr)}{(X^2 + r^2)^{\frac{3}{2}}}$$

$$= \pi k \sigma X \int \frac{2r dr}{(X^2 + r^2)^{\frac{3}{2}}}$$

(integration by substitution) \therefore easily

$$E_x = \pi \kappa \sigma x \left[\left(\frac{x^2 + r^2}{-\frac{1}{2}} \right)^{-\frac{1}{2}} \right]_0^R$$

$$E_x = 2\pi \kappa \sigma x \left[\frac{1}{x} - \frac{1}{\sqrt{x^2 + R^2}} \right]$$

$$E_x = 2\pi \kappa \sigma \left[1 - \frac{x}{\sqrt{x^2 + R^2}} \right]$$

↓
The electric field due to a disk

$$E_x = \pi k \sigma x \left[\frac{(x^2 + r^2)^{-\frac{1}{2}}}{-\frac{1}{2}} \right]_0^R$$

$$E_x = 2\pi k \sigma x \left[\frac{1}{x} - \frac{1}{\sqrt{x^2 + R^2}} \right]$$

$$E_x = 2\pi k \sigma \left[1 - \frac{x}{\sqrt{x^2 + R^2}} \right]$$

The electric field due to uniformly charged disk of radius (R) a distance x from the center along the perpendicular axis.

* If x is very small ($x \rightarrow 0$)

$$E_x = 2\pi k \sigma$$

$$= \cancel{2\pi} \frac{1}{2} \cancel{4\pi} \sigma \epsilon_0$$

$$E_x = \frac{\sigma}{2\epsilon_0}$$

→ very close to the surface of the disk

* Electric field lines *

الخطوط الكهربائية

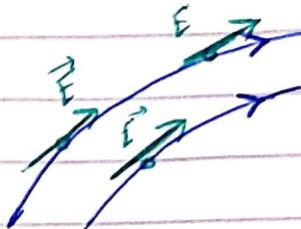
⇒ Graphical representation of the electric field. (رسومات)

* properties of the electric field lines :

خصائص الخطوط

1 The electric field \vec{E} is tangent to the field lines.

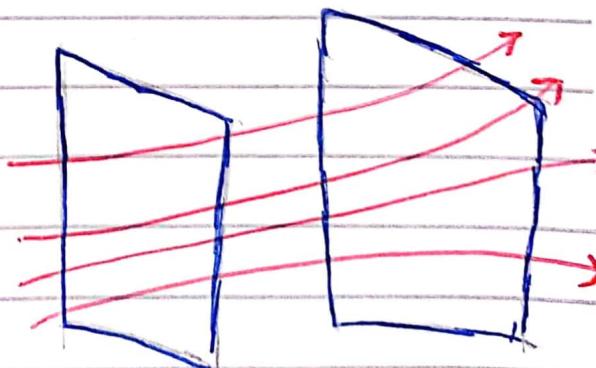
(الخطوط الكهربائية ملائمة لاتجاه \vec{E})



2 number of field lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of E

Density of lines > density of lines (كثافة الخطوط)
at (A) at (B)

$$E_A > E_B$$

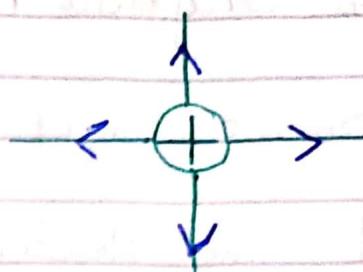


الاتجاهات ترتبط ببعضها البعض (الاتجاهات)
الاتجاهات تحدد الكثافة، لأن الكثافة تختلف من موضع إلى آخر
كما تزداد الكثافة، لخطوط في سطح ما
ذلك يدل على

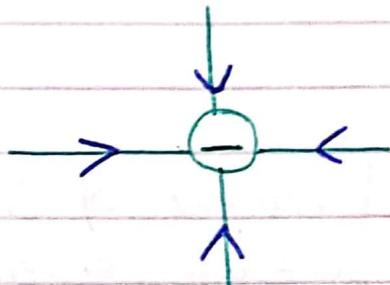
الاتجاهات (الاتجاهات) امرأة مواتية (الاتجاهات)
↑ امرأة العارضة على عاصفة (الاتجاهات)

3 Field lines are directed away from a positive charge.

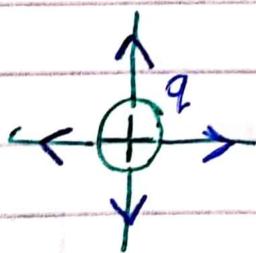
(+) *and is same direction*



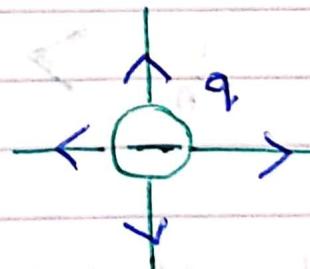
4 Field lines are directed toward a negative charge.



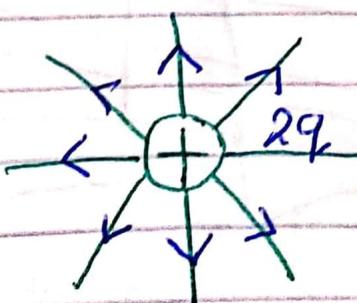
5 Number of field lines leaving a positive charge or approaching a negative charge is proportional to the magnitude of charge.



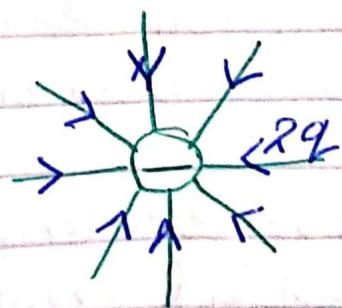
decreasing this direction



increasing this direction

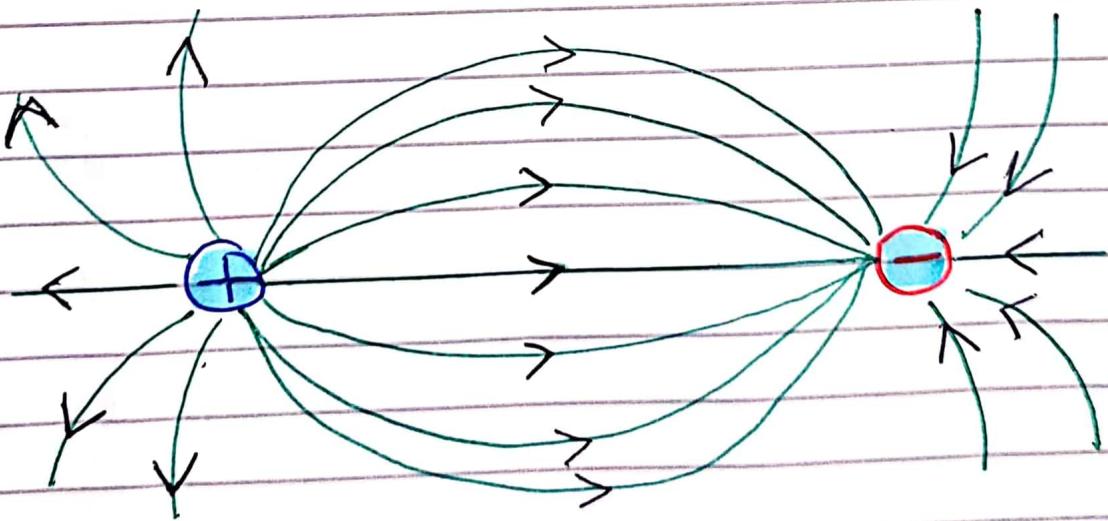
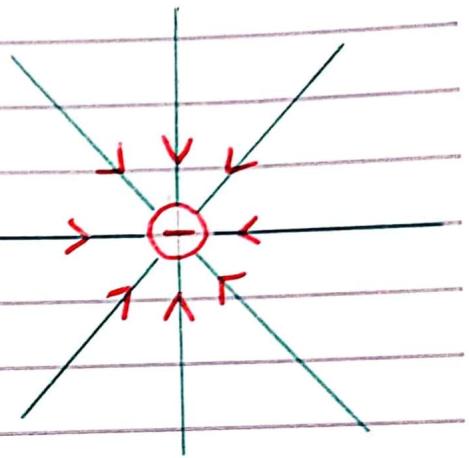
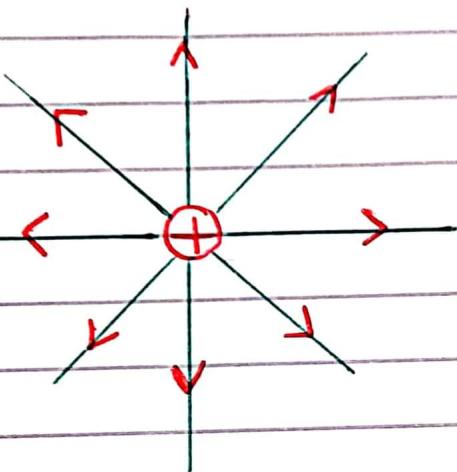
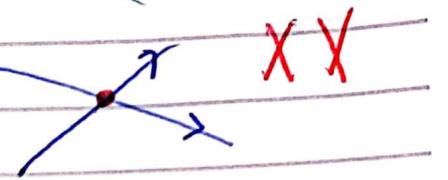


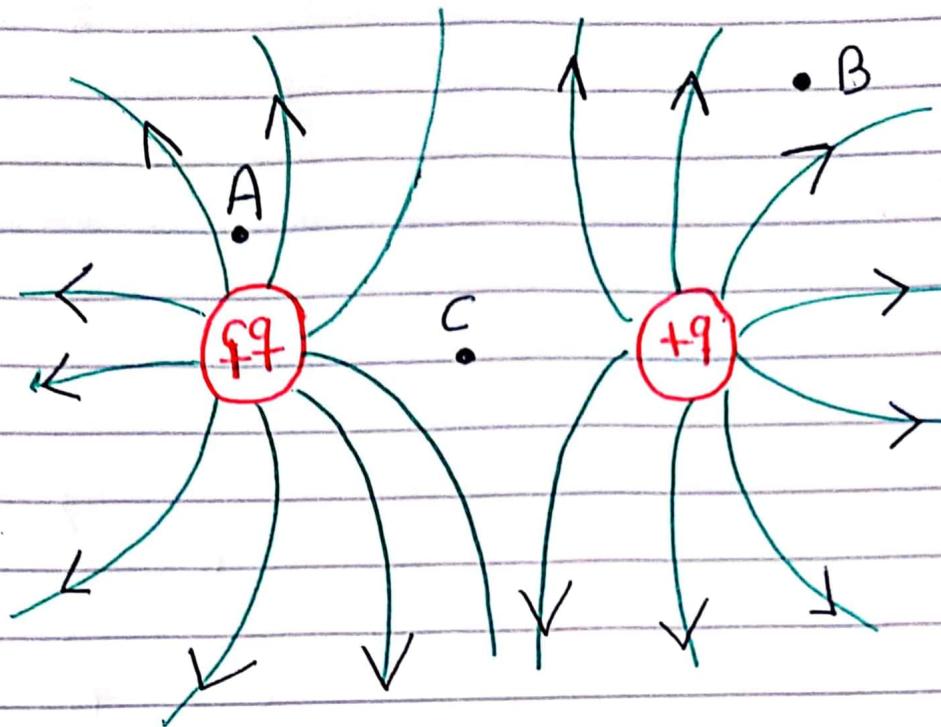
(proportional)



6 No two field lines can cross. (لا يمكن لخطين معاً أن يتقاطعاً)

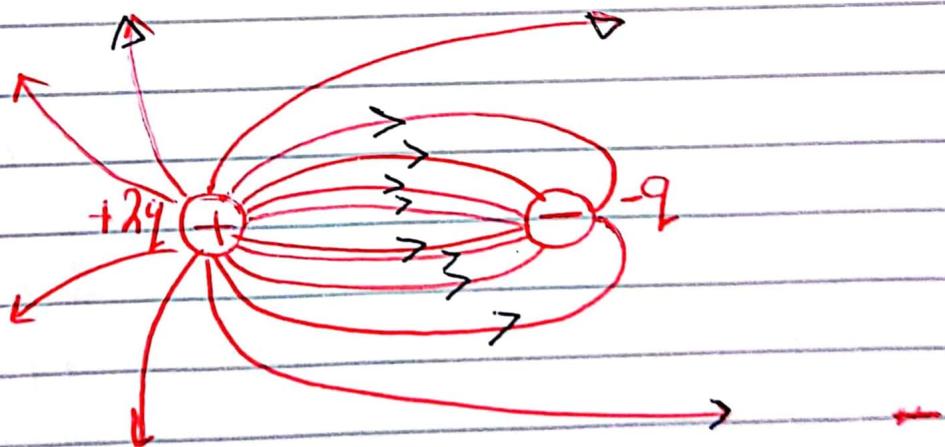
طبعاً كل خط من اتجاهه متعارض





$$E_A > E_B \quad (B \text{ is closer to } A \text{ than } C)$$

$$E_C = 0 \text{ (zero) } \text{none line. (no lines)} \quad (C \text{ is a neutral point})$$



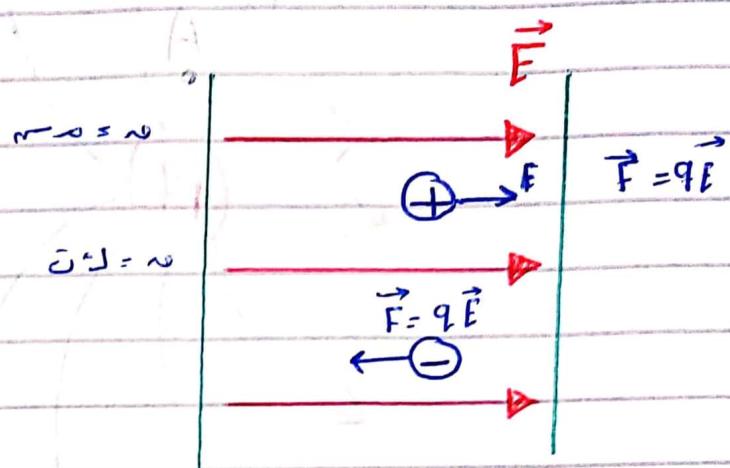
$$\frac{\text{Number of lines leaving } (+2q)}{\text{Number of lines approaching } (-q)} = \frac{16}{8} = (2)$$

* motion of charged particle in a uniform electric field \rightarrow مُتحركة داخل مجال كهربائي

$$\vec{F} = q \vec{E}$$

$$\vec{F} = m \vec{a}$$

$$q \vec{E} = m \vec{a}$$



(acceleration)

$$a = \frac{qE}{m}$$

التسارع الذي يكتسبه جسم

• هنا كان عجلة منظم، ثابتة (اعتبار الاتجاه)

• إذا اتساع ثابتة

• تغير لبراسة في هنا لجعل على مُتحركة مسمى تثبيت داخل مجال منظم (ثابتة)

If E is uniform then $\vec{a} = \text{constant}$

عجلة ثابتة

ويمكن أن اتساع ثابتة نسخ مقطبة اتساع في مُتحركة ثابتة

Comments

1) If $\vec{E} = \text{constant}$ (\vec{E} is uniform) then

$$\vec{a} = \frac{q\vec{E}}{m} = \text{constant}$$

⇒ Kinematic equations can be used.

1) $v_{x_f} = v_{x_i} + a_{x,t}$

2) $x_f - x_i = v_{x,i}t + \frac{1}{2}a_{x,t}^2$

3) $v_{x_f}^2 = v_{x_i}^2 + 2a_{x,t}(x_f - x_i)$

]

same for y.

2) If q is positive $\Rightarrow \vec{a}$ is in the direction of \vec{E}

3) If q is negative $\Rightarrow \vec{a}$ is opposite to \vec{E}

الله يحييكم بحاجة الى مراجعة سريعة (الى انتهاء الامتحان) \Rightarrow (الى انتهاء الامتحان)

P. 11

جواب

Ex - 23.10

⇒ A uniform electric field \vec{E} is directed along the x axis between parallel plates of charge separated by a distance d as shown in figure. A positive point charge q of mass m is released from rest at a point (A) next to the positive plate and accelerates to a point (B) next to the negative plate.

$$\vec{E} = \text{Constant} \quad (\text{uniform})$$

$$E_x = E, \quad E_y = 0$$

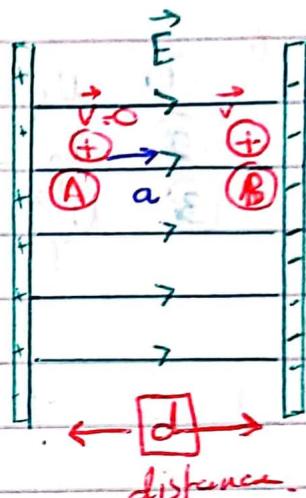
$$q, m.$$

$$U_i = U_A = 0$$

$$U_B = ??$$

$$x_i = x_A = 0$$

$$x_F = x_B = d$$



(A) Find the speed of the particle at (B) by modeling it as a particle under constant acceleration.

$$\vec{a} = \frac{q \vec{E}}{m}$$

$$a_y = \frac{q E_y}{m} = 0$$

$$a_x = \frac{q E_x}{m} = \frac{q E}{m}$$

$$U_{x_F}^2 = U_{x_i}^2 + 2a_x (x_F - x_i)$$

$$U_B^2 = U_A^2 + 2 \frac{q E}{m} (d - 0)$$

$$U_B = \sqrt{\frac{2 q E d}{m}}$$

(B) find the speed of the particle at B by modeling it as a nonisolated system in terms of energy.

أولاً، نحلل نظاماً مفتوحاً

Non-isolated system.

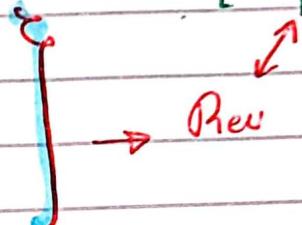
(أولاً، نحلل نظاماً مفتوحاً)

System = charge.

[Chapter 7
Physics(1)]

$$W_{\text{ext}} = K_B - K_A$$

$$F dx \cos\theta = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2$$



Prev

$$F d \underbrace{\cos\theta}_{(1)} = \frac{1}{2} m v_B^2$$

$$q Ed = \frac{1}{2} m v_B^2$$

$$v_B = \sqrt{\frac{2q Ed}{m}}$$

Ex. 23.11 p. 712.

⇒ An electron enters the region of a uniform electric field as shown, with $v_i = 3.00 \times 10^6 \text{ m/s}$ and $E = 200 \text{ N/C}$. The horizontal length of the plates is $L = 0.100 \text{ m}$.

A) Find the acceleration of the electron while it is in the electric field.

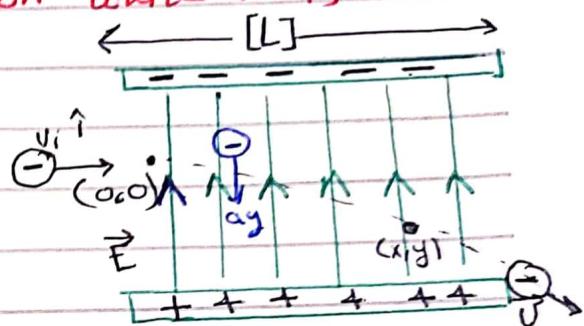
$$q = -1.6 \times 10^{-19} \text{ C}$$

$$m_{\text{mass}} = 9.11 \times 10^{-31} \text{ kg}$$

$$v_i = 3 \times 10^6 \text{ m/s}$$

$$E = (200 \hat{j})$$

$$E_x = 0 \quad E_y = 200 \quad , \quad L = 0.1 \text{ m} \quad (\text{arz})$$



$$F = q \vec{E} = ma$$

$$\vec{a} = \frac{q \vec{E}}{m}$$

$$a_x = \frac{q E_x}{m} = 0$$

$$a_y = \frac{q E_y}{m} = - \frac{(1.6 \times 10^{-19})(200)}{9.11 \times 10^{-31}} \Rightarrow a_y = -3.15 \times 10^{13} \text{ m/s}^2$$

$$\vec{a} = (-3.15 \times 10^{13} \hat{j}) \text{ m/s}^2$$

(B) Assuming the electron enters the field at time $t=0$, find the time at which it leaves the field.

الرقة لدى محتاجه هو يخرج عن منفعته - بخلاف) ١٩٨
المعنى (يقطع افتراضنا (المطلب ٧)

$$U_x = U_i (\cos \theta)^{(1)} = U_i = 3 \times 10^6 \text{ m/s}$$

$$Uy_i = v_i \sin \theta = 0.$$

$$X_f - X_i = V_x t + \frac{1}{2} a_x t^2$$

$$L = U_x E$$

$$t = \frac{L}{U_x} = \frac{0.1 \text{ m}}{3 \times 10^6 \text{ m/s}} = 3.33 \times 10^{-8} \text{ s.}$$

c) Assuming the vertical position of the electron as it enters the field is $y_i = 0$, what is its vertical position when it leaves the field? 30.5 \text{ cm} \text{ in } 1.5 \text{ s}

$$y_F - y_i = \cancel{Ug_i t} + \frac{1}{2} \cancel{a y t^2}$$

$$y_F = \frac{1}{2} a y t^2 = \frac{1}{2} (-3.51 \times 10^{13} \text{ m/s}^2) (3.33 \times 10^{-8} \text{ s})^2$$

$$y_p = -1.59 \times 10^{-2} \text{ m} = -1.59 \text{ cm}$$

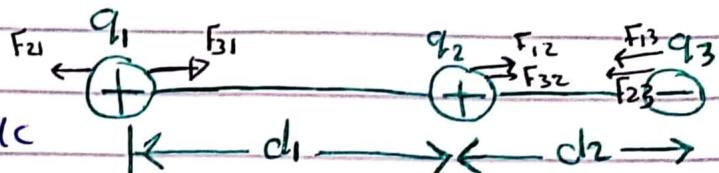
prob 12

Three point charges lie along a straight line as shown in figure, where $q_1 = 6.00 \mu C$, $q_2 = 1.50 \mu C$, and $q_3 = -2.00 \mu C$. The separation distance are $d_1 = 3.00 \text{ cm}$ and $d_2 = 2.00 \text{ cm}$. Calculate the magnitude and direction of the net electric force on (a) q_1 (b) q_2 and (c) q_3

$$q_1 = 6 \mu C$$

$$q_2 = 1.5 \mu C$$

$$d_1 = 3 \text{ cm}, \quad d_2 = 2 \text{ cm}$$



$$F_{12} = F_{21} = \frac{k |q_1| |q_2|}{d_1^2} = (9 \times 10^9) \frac{(6 \times 10^{-6})(1.5 \times 10^{-6})}{(0.03)^2} = 89.5 \text{ N}$$

$$F_{13} = F_{31} = \frac{k |q_1| |q_3|}{(d_1 + d_2)^2} = 43.2 \text{ N}$$

(الخطوة الثانية)

$$F_{23} = F_{32} = \frac{k |q_2| |q_3|}{d_2^2} = (9 \times 10^9) \frac{(1.5 \times 10^{-6})(2 \times 10^{-6})}{(2 \times 10^{-2})^2} = 67.4 \text{ N}$$

A) $\vec{F}_1 = (F_{31} - F_{21})\hat{i} = (-46.3\hat{i}) \text{ N}$

B) $\vec{F}_2 = (F_{12} + F_{32})\hat{i} = (157\hat{i}) \text{ N}$

C) $\vec{F}_3 = -(F_{13} + F_{23})\hat{i} = (-111\hat{i}) \text{ N}$

[prob. 15] #

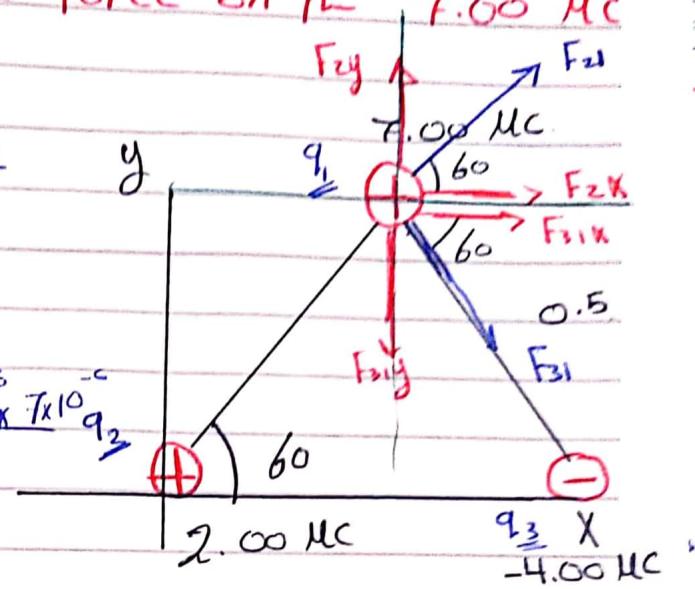
Three charged particles are located at the corners of an equilateral triangle as shown in Figure. Calculate the total electric force on the 7.00 nC charge.

$$q_1 = 7 \text{ nC}, q_2 = 2 \text{ nC}, q_3 = -4 \text{ nC}$$

$$d = 0.5 \text{ m}$$

$$F_{21} = k \frac{|q_2||q_1|}{d^2} = \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 7 \times 10^{-9}}{(0.5)^2}$$

$$= 0.504 \text{ N}$$



$$F_{31} = k \frac{|q_3||q_1|}{d^2} = \frac{9 \times 10^9 \times 4 \times 10^{-6} \times 7 \times 10^{-9}}{(0.5)^2}$$

$$= 1.008 \text{ N}$$

$$\Rightarrow F_{21x} = F_{21} \cos 60 = 0.252 \text{ N}$$

$$F_{21y} = F_{21} \sin 60 = 0.436 \text{ N}$$

$$\Rightarrow F_{31x} = F_{31} \cos 60 = 0.504 \text{ N}$$

$$F_{31y} = -F_{31} \sin 60 = -0.872 \text{ N}$$

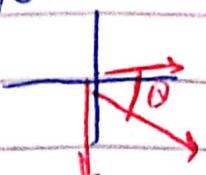
$$F_{1x} = F_{21x} + F_{31x} = 0.776 \text{ N}$$

$$F_{1y} = F_{21y} + F_{31y} = -0.436 \text{ N}$$

$$\vec{F}_1 = F_{1x} \hat{i} + F_{1y} \hat{j} = \vec{F}_1 = (0.776 \hat{i} - 0.436 \hat{j}) \text{ N}$$

$$F_1 = \sqrt{0.776^2 + (-0.436)^2} = 0.89 \text{ N}$$

$$\theta = \tan^{-1} \left(\frac{-0.436}{0.776} \right) = -30^\circ$$



Prob 23.16

P. 717

Two small metallic spheres, each of mass $m = 0.200\text{g}$ are suspended as pendulums by light strings of length L as shown in Figure. The spheres are given the same electric charge of 7.2nC , and may come to equilibrium when each string is at an angle of $\Theta = 5.00^\circ$ with the vertical. How long are the strings?

$$m = 0.2\text{ kg}, \Theta = 5^\circ$$

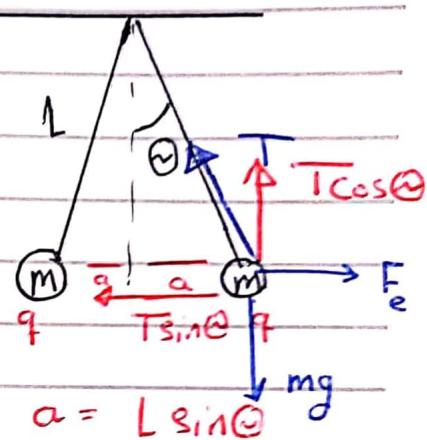
$$q = 7.2\text{nC}$$

$$a = L \sin \Theta$$

$$F_e = T \sin \Theta$$

$$mg = T \cos \Theta$$

$$\frac{F_e}{mg} = \tan \Theta \rightarrow F_e = mg \tan \Theta$$



$$k \frac{|q|^2}{(2a)^2} = mg \tan \Theta$$

$$\frac{k |q|^2}{4L^2 \sin^2 \Theta} = mg \tan \Theta$$

$$L = \sqrt{\frac{k |q|^2}{4mg \tan \Theta \sin^2 \Theta}} = \sqrt{\frac{9 \times 10^9 \times (7.2 \times 10^{-9})^2}{4(0.2)(10) \tan(5) \sin^2(5)}}$$

$$= 0.299\text{ m}$$
$$= 29.9\text{ cm.}$$

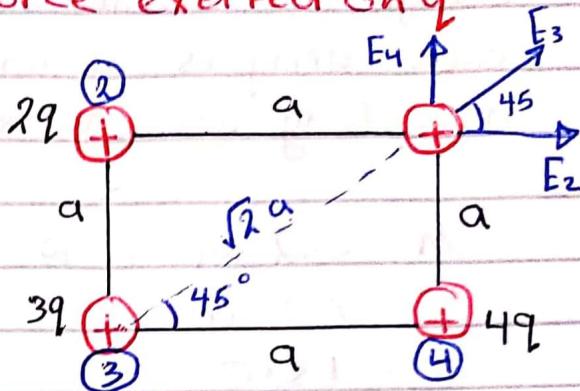
Prob. 25

P. 718

four charged particles are at the corners of a square of side a as shown in figure. Determine

(a) the electric field at the location of charge q and (b) the total electric force exerted on q

$$A) E_2 = \frac{k(2q)}{a^2}$$



$$E_3 = \frac{k(3q)}{2a^2}$$

$$E_4 = k \frac{4q}{a^2}$$

$$E_{2x} = E_2 \cos 0 = \frac{k(2q)}{a^2}$$

$$E_{2y} = E_2 \sin 0 = 0$$

$$E_{3x} = E_3 \cos 45 = \frac{k(3q)}{2a^2} \cos 45$$

$$E_{3y} = E_3 \sin 45 = \frac{k(3q)}{2a^2} \sin 45$$

$$E_{4x} = E_4 \cos 90 = 0$$

$$E_{4y} = E_4 \sin 90 = \frac{k(4q)}{a^2}$$

$$E_x = E_{2x} + E_{3x} + E_{4x} = \frac{kq}{a^2} \left(2 + \frac{3}{2} \cos 45 + 0 \right)$$

$$E_x = 3.06 \frac{kq}{a^2}$$

$$E_y = E_{2y} + E_{3y} + E_{4y} = \frac{kq}{a^2} \left(0 + \frac{3}{2} \sin 45 + 4 \right)$$

$$E_y = 5.06 \frac{kq}{a^2}$$

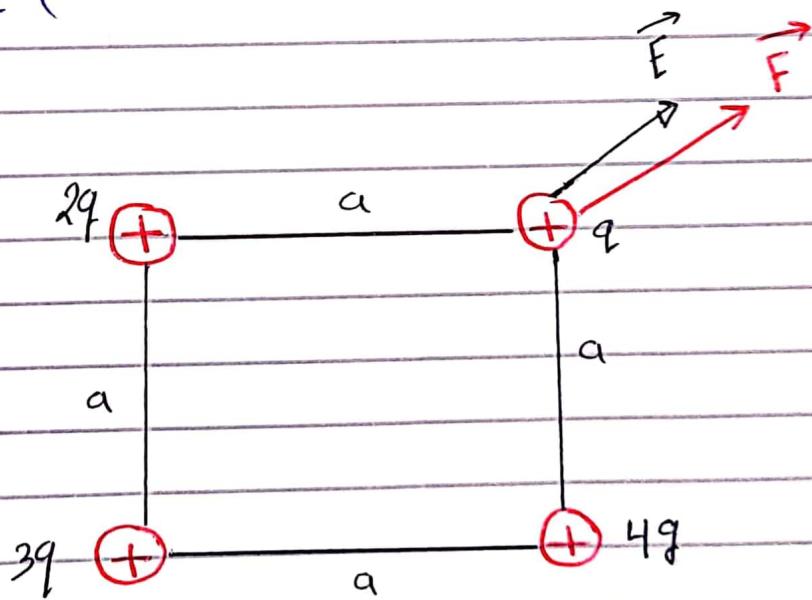
$$\vec{E} = E_x \hat{i} + E_y \hat{j}$$

$$= \frac{kq}{a^2} (3.06 \hat{i} + 5.06 \hat{j})$$

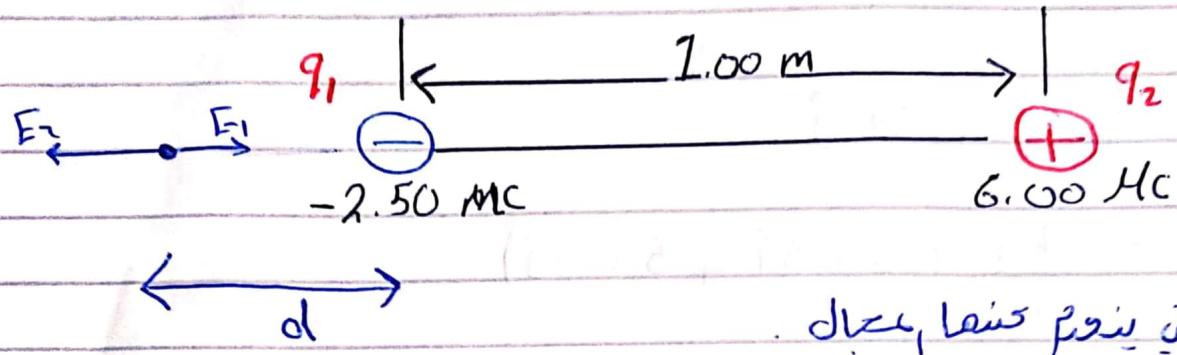
B)

$$\vec{F} = q \vec{E}$$

$$= \frac{kq^2}{a^2} (3.06 \hat{i} + 5.06 \hat{j})$$



In figure, determine the point (other than infinity) at which the electric field is zero



النقطة التي ينبع منها الحقل الكهربائي

$$\sum E_x = 0$$

$$E_1 - E_2 = 0$$

$$E_1 = E_2$$

نقطة التوازن
النقطة التي ينبع منها الحقل الكهربائي

$$k \frac{|q_1|}{d^2} = k \frac{|q_2|}{(1+d)^2}$$

$$\frac{2.5 \times 10^{-6}}{d^2} = \frac{6 \times 10^{-6}}{d^2 + 2d + 1}$$

$$6d^2 = 2.5d^2 + 5d + 2.5$$

$$3.5d^2 - 5d - 2.5 = 0$$

$$d = \frac{+5 \pm \sqrt{25 - 5(3.5)(-2.5)}}{2(3.5)}$$

$$d = 1.82 \text{ m } \checkmark \text{ } \#$$

$$= -0.392 \text{ m } \times$$

٤٤ في السؤال السابعة هن نحرف اين ينتمي المجال.

الحالات الشحنتان متاثرتان في الاتسارة

قناديلان في المقادير - تقع نقطه التبادل بينهما في منتصف المدى

$\text{---}^{\oplus} \text{---}^{\ominus}$ (هي نقطه التبادل)

مختلفةان بالمقادير - تقع نقطه التبادل بينها اقرب للشحنة الپوزيسيون

$(\text{---}^{\ominus} \text{---}^{\oplus})$ $\text{---}^{\ominus} \text{---}^{\oplus}$

الشحنتان مختلفةان في الاتسارة.

قناديلان بالمقادير \Leftarrow لا يوجد نقطه تبادل

مختلفةان بالمقادير - ~~النقطه التبادل هي اقرب~~ تقع نقطه التبادل ~~هي اقرب~~ (اقرب للشحنة الپوزيسيون)

$\text{---}^{\ominus} \text{---}^{\oplus}$ $\text{---}^{\oplus} \text{---}^{\ominus}$
نقطه التبادل
الماخرا

Prob. 33

A small, 2.00-g plastic ball is suspended by a 20.0-cm-long string in a uniform electric field as shown. If the ball is in equilibrium when the string makes a 15.0° angle with the vertical, what is the net charge on the ball?

q is positive, $m = 2g = 0.002 \text{ kg}$

$$E = 1 \times 10^3 \text{ N/C}$$

$$\theta = 15^\circ$$

$$qE = T \sin \theta$$

$$mg = T \cos \theta \quad \text{Divide}$$

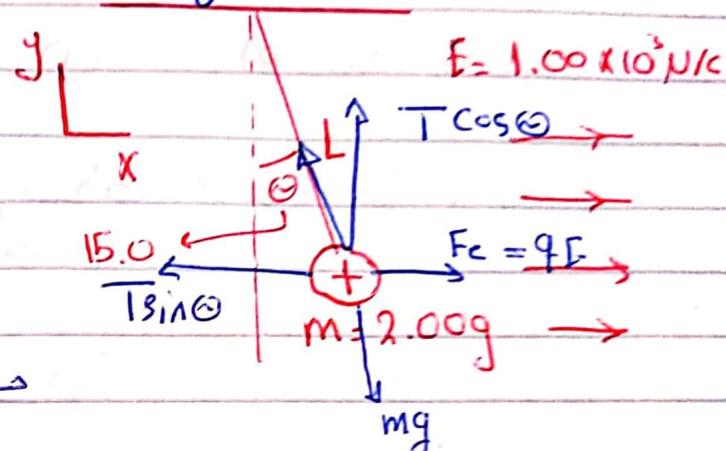
$$\frac{qE}{mg} = \tan \theta$$

$$q = \frac{mg \tan \theta}{E}$$

$$= \frac{(0.002) \times (10) (\tan 15)}{1 \times 10^3} =$$

$$5.25 \times 10^{-6} \text{ C}$$

$$= 5.25 \mu\text{C}$$

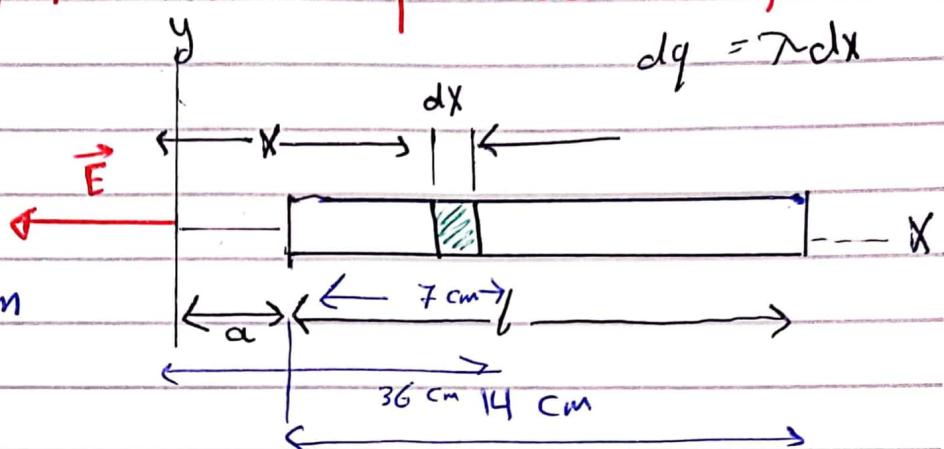


prob 37

A rod 14.0 cm long is uniformly charged and has a total charge of $-22.0 \mu C$. Determine (a) the magnitude and (b) the direction of the electric field along the axis of the rod at a point 36.0 cm from its center.

$$L = 14 \text{ cm} = 0.14 \text{ m}$$

$$Q = -22 \mu C$$

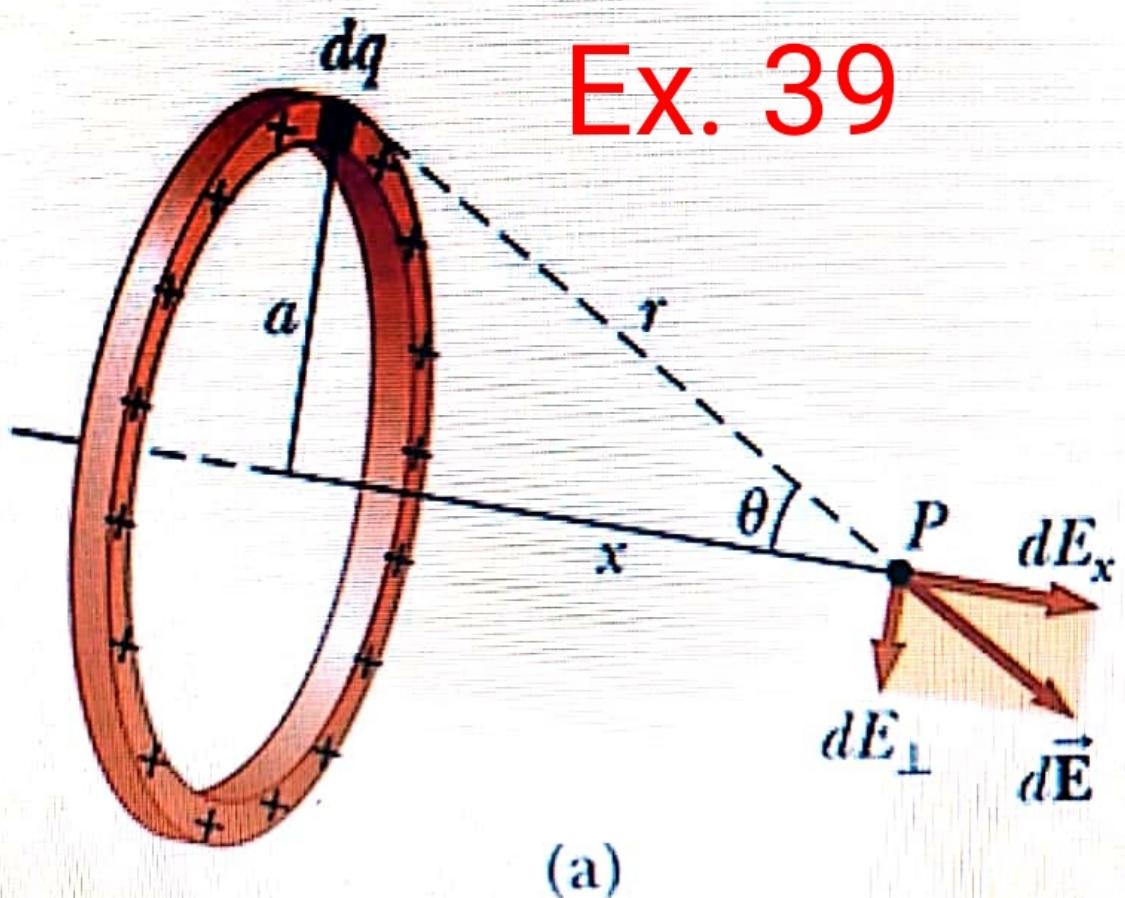


$$a = 36 \text{ cm} - 7 \text{ cm} = 29 \text{ cm}$$

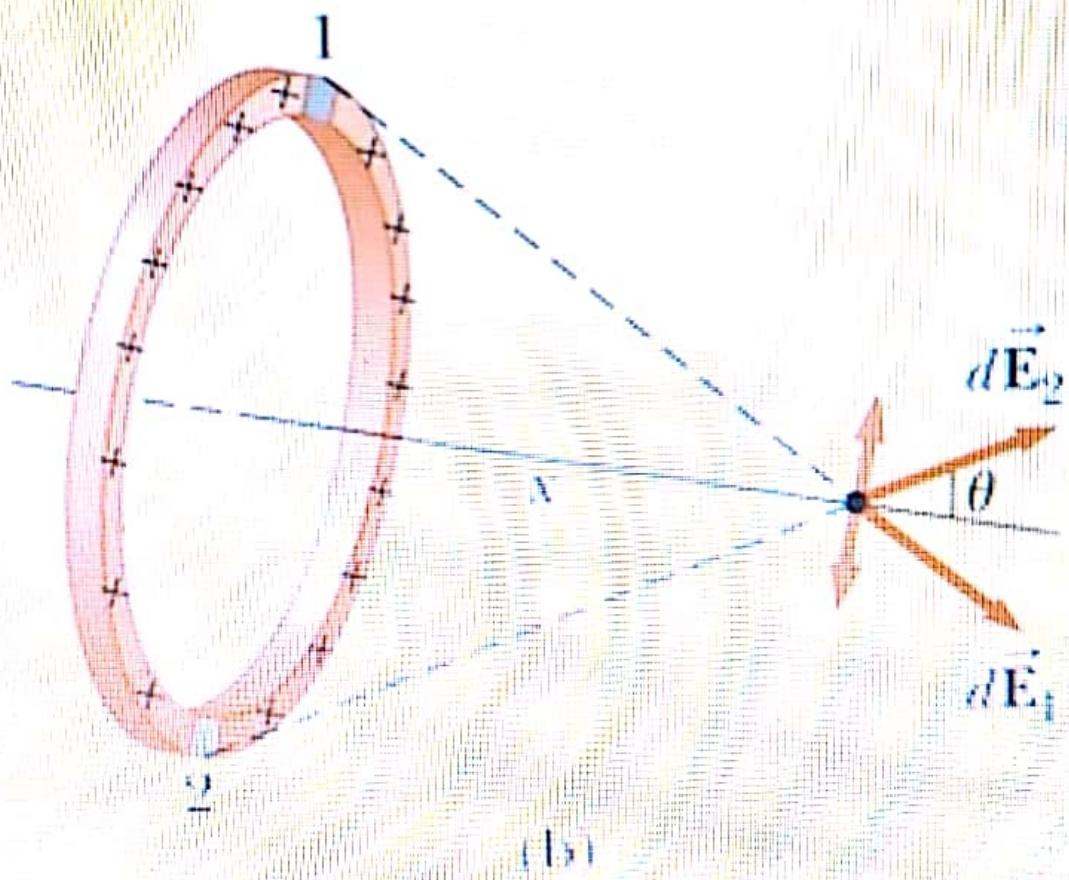
$$E = \frac{kQ}{a(L+a)}$$

$$= \frac{(9 \times 10^9)(22 \times 10^{-6})}{(0.29)(0.41 + 0.29)} = 1.59 \times 10^6 \text{ N/C}$$

Ex. 39



(a)



(b)

© Thomson Right Stuff Inc.

Prob 39 -

A uniformly charged ring of radius 10.0 cm has a total charge of 75.0 μC . Find the electric field on the axis of the ring at (a) 1.00 cm (b) 5.0 cm (c) 30.0 cm and (d) 100 cm from the center of the ring.

$$a = 10 \text{ cm} = 0.1 \text{ m}$$

$$q = 75 \mu\text{C} = 75 \times 10^{-6} \text{ C}$$

$$E_y = \frac{kqX}{(X^2 + a^2)^{\frac{3}{2}}}$$

$$\text{A) } X = 1 \text{ cm} = 0.01 \text{ m} \quad E_x = \frac{(q \times 10^9)(75 \times 10^{-6})(0.01)}{(0.01^2 + 0.1^2)^{\frac{3}{2}}}$$

$$E_x = 6.64 \times 10^6 \text{ N/C}$$
$$\Rightarrow \vec{E} = (6.64 \times 10^6 \hat{i}) \text{ N/C}$$

P. 45 . P. 720

A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle as shown.

The rod has a total charge of $-7.50 \mu C$. Find (a) the magnitude and (b) the direction of the electric field at \odot , the center of the semicircle.

$$L = 14 \text{ cm} = 0.14 \text{ m}$$

$$q = -7.5 \mu C \equiv \text{uniform}$$

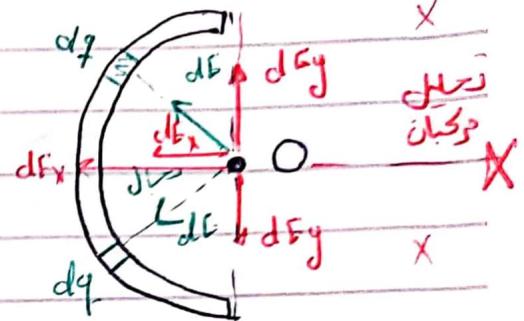
$$\Rightarrow E_y \text{ cancel.} \quad dF = k \frac{dq}{r^2}$$

$$dE_x = dE \sin \theta$$

$$dE_x = \frac{k dq \sin \theta}{r^2} \quad \text{outward}$$

$$E_x = \frac{k}{r^2} \int dq \sin \theta$$

$$\begin{aligned} dL &= r d\theta & \lambda &= \frac{q}{L} \\ dq &= \lambda dL & L &= \pi r \\ &= \lambda r d\theta & r &= \frac{L}{\pi} \\ E_x &= \frac{k}{r^2} \int dq \sin \theta \end{aligned}$$



$$E_x = \frac{k}{r^2} \int_0^{\pi} dq \sin \theta$$

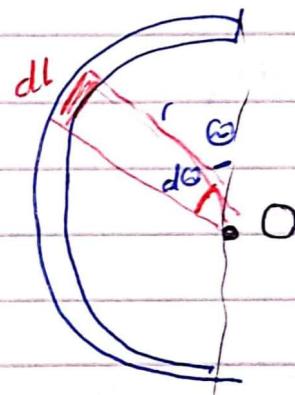
$$= \frac{k}{r^2} \int_0^{\pi} \lambda r d\theta \sin \theta$$

$$= \frac{k \lambda r}{r^2} \int \sin \theta d\theta$$

$$= \frac{k \lambda}{r} \left[-\cos \theta \right]_0^{\pi}$$

$$E_x = \frac{2k\lambda}{r}$$

$$= \frac{2k \left(\frac{q}{L} \right)}{\frac{L}{\pi}}$$



$$E_x = \frac{2\pi k q}{L^2}$$

$$= \frac{(2\pi)(9 \times 10^9)(7.5 \times 10^{-6})}{(0.14)^2}$$

$$E_x = 2.16 \times 10^7 \text{ N/C}$$

اجزاء
الحصص $\leftarrow \vec{E} = (-2.16 \times 10^7 \hat{i}) \text{ N/C}$

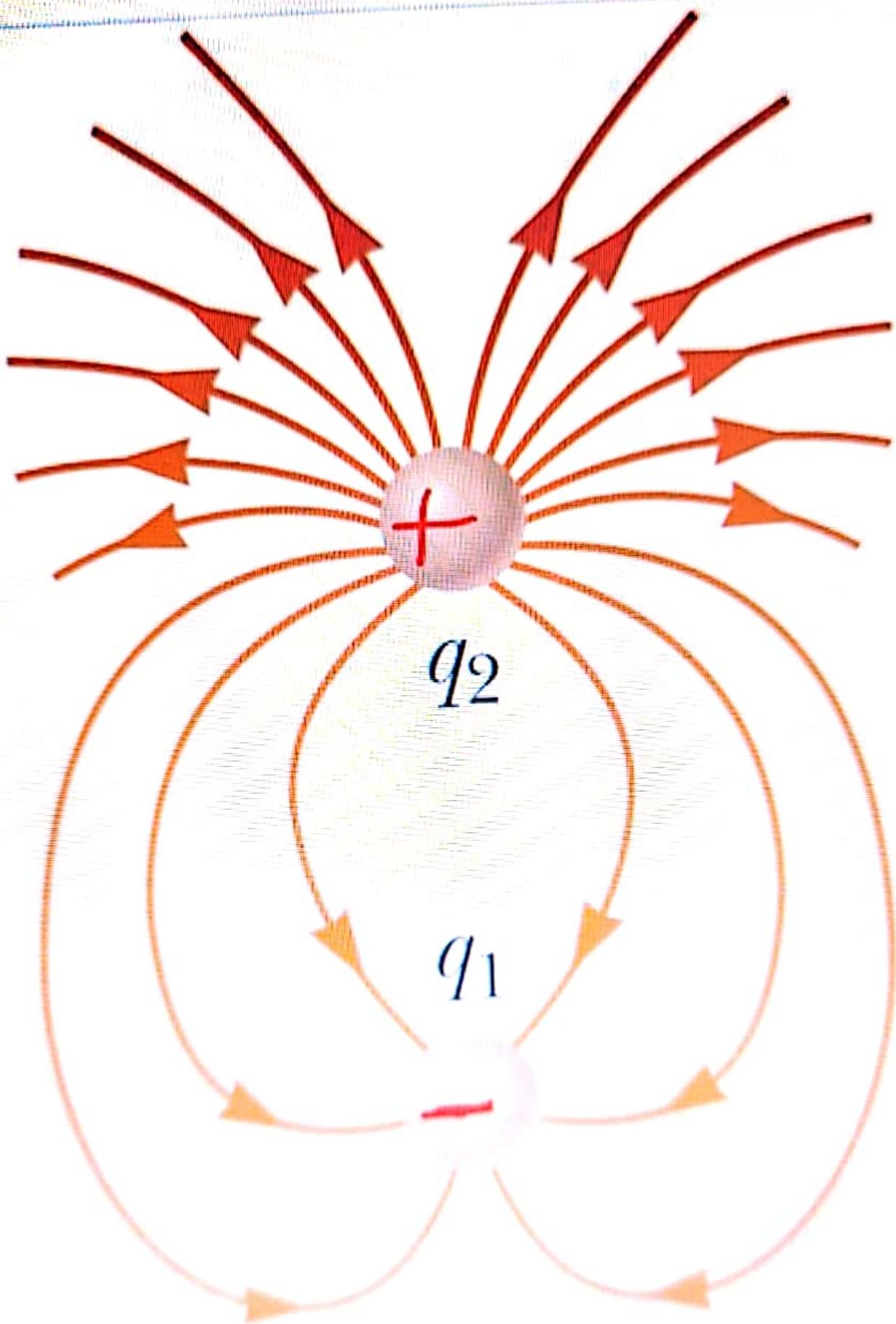


Figure shows the electric field lines for two charged particles separated by a small distance

(a) determine the ratio q_1/q_2 (b) what are the signs of q_1 and q_2

$$\left| \frac{q_1}{q_2} \right| = \frac{6}{18} = \frac{1}{3}$$

$$\frac{q_1}{q_2} = -\frac{1}{3}$$

$q_2 \rightarrow (+)$

$q_1 \rightarrow (-)$

Prob. 52

A proton is projected in the positive x direction into a region of a uniform electric field

$\vec{E} = (-6.00 \times 10^5) \hat{i}$ N/C at $t=0$. The proton travels 7.00 cm as it comes to rest. Determine (a) the acceleration of the proton, (b) its initial speed, and (c) the time interval over which the proton comes to rest

$$q = 1.6 \times 10^{-19} \text{ C}$$

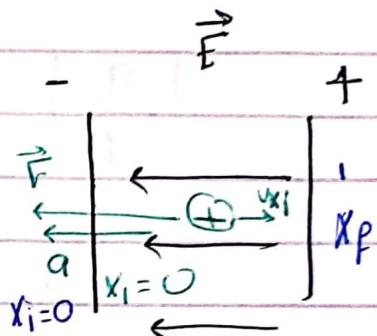
$$m = 1.67 \times 10^{-27} \text{ kg}$$

$$E = (-6 \times 10^5 \hat{i}) \text{ N/C}$$

$$x_f = 7 \text{ m}$$

$$u_{x_f} = 0$$

$$u_{x_i} = 0$$



$$E_x = -6 \times 10^5 \text{ N/C}, E_y = 0$$

$$(A) q\vec{E} = m\vec{a} \Rightarrow \vec{a} = \frac{q\vec{E}}{m}$$

$$a_x = \frac{qE_x}{m} = -5.76 \times 10^{13} \text{ m/s}^2$$

$$a_y = \frac{qE_y}{m} = 0$$

$$(B) U_{x_f}^2 = U_{x_i}^2 + 2a_x(x_f - x_i)$$

$$0 = U_{x_i}^2 + 2(-5.76 \times 10^{13})(0.07 - 0)$$

$$U_{x_i} = 2.84 \times 10^6 \text{ m/s.}$$

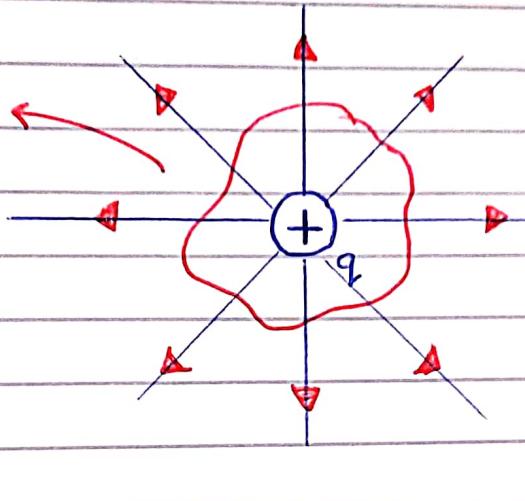
$$(C) U_{x_f} = U_{x_i} + a_x t$$

$$0 = 2.84 \times 10^6 + (-5.76 \times 10^{13}) t$$

$$t = 4.93 \times 10^{-8} \text{ s}$$

- chapter 24 -

Gauss's Law



في المكمل \Rightarrow سواء كانت موجبة او سالبة (كره) لم يطرأ على المواجهة مللاجأة في المواجهة تتساءل
جزئياً في عمق المواجهة

لذوق في الع

* Electrical flux \rightarrow is a measure of the number of field lines penetrating some surface.

وَمِنْكُمْ الْمُجْرِمُونَ إِنَّمَا تَنْهَىٰ رَسُولُنَا

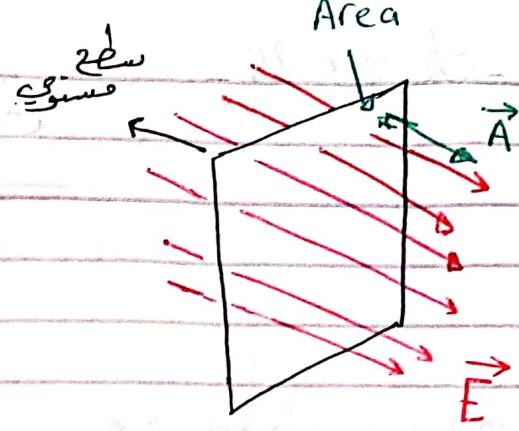
• إذا كانت النسبة الكهربائية تتناسب مع حجم الخلط ونوع الماء فـ
• إذا كانت النسبة تتناسب مع الخلط \rightarrow أي سخنة ١٩
لهم في داخل كل

- * If the surface is closed and encloses some net charge, then the net number of lines that go through the surface (electric flux) is proportional to the net charge within the surface

أولاً كان المفعول مختلفاً عن المفعول الذي تصرّف هذا المفعول
لما ينبع عنه، لذا فهو في المقام

* Area is a vector, ~~not~~ ^{2D} ~~2D~~ normal to the Surface

جهاز فلکسیبل \vec{E} the area (\vec{A}) \perp (أول (1))



* The electric flux (Φ) of a uniform electric field (\vec{E}) normal to the surface ($\vec{E} \parallel \vec{A}$) parallel (موازي)

If $\vec{E} \perp$ ^{normal} surface ($\vec{E} \parallel \vec{A}$)

then

$$\boxed{\Phi = EA}$$

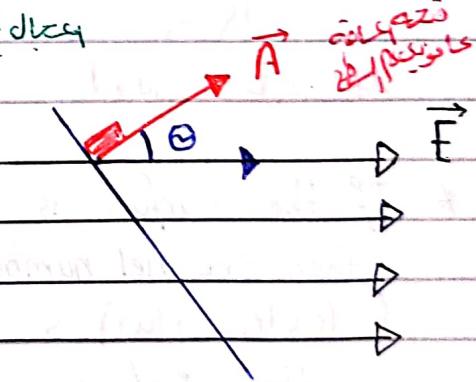
* If \vec{E} is not parallel to \vec{A} (\vec{E} is not normal to the surface)

أول \vec{E} \perp \vec{A} \Rightarrow $\Phi = EA$

$$\boxed{\Phi = EA \cos \theta}$$

$$= \vec{E} \cdot \vec{A}$$

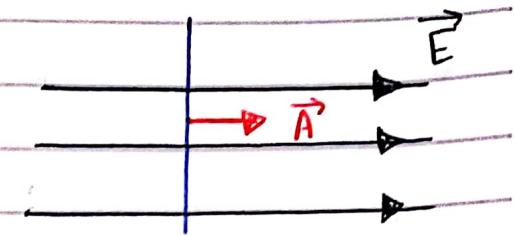
أول \vec{E} \perp \vec{A} \Rightarrow $\Phi = 0$



If $\Theta = 0$

$$\Rightarrow \Phi = EA \cos 0$$

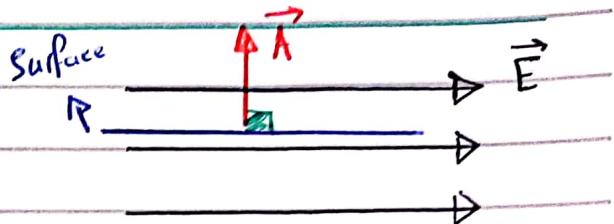
$$\Phi = EA$$



If $\Theta = \frac{\pi}{2}$

$$\Rightarrow \Phi = EA \cos \frac{\pi}{2}$$

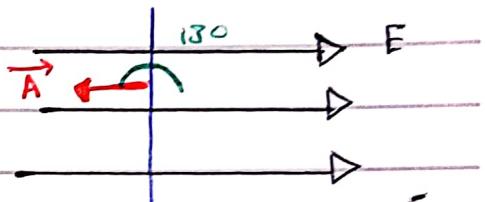
$$\Phi = 0$$



If $\Theta = \pi$

$$\Rightarrow \Phi = EA \cos \pi$$

$$\Phi = -EA$$



<next page>

In the figure \Rightarrow

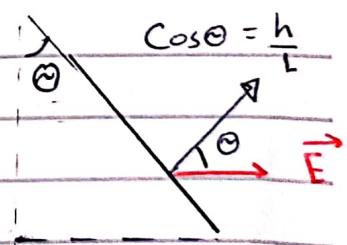
$$|\Phi_A| = |\Phi_{A\perp}|$$

$$|\Phi_{A\perp}| = EA_{\perp} = Ewh$$

$$A_{\perp} = wh, A = WL$$

$$= WL \cos \Theta$$

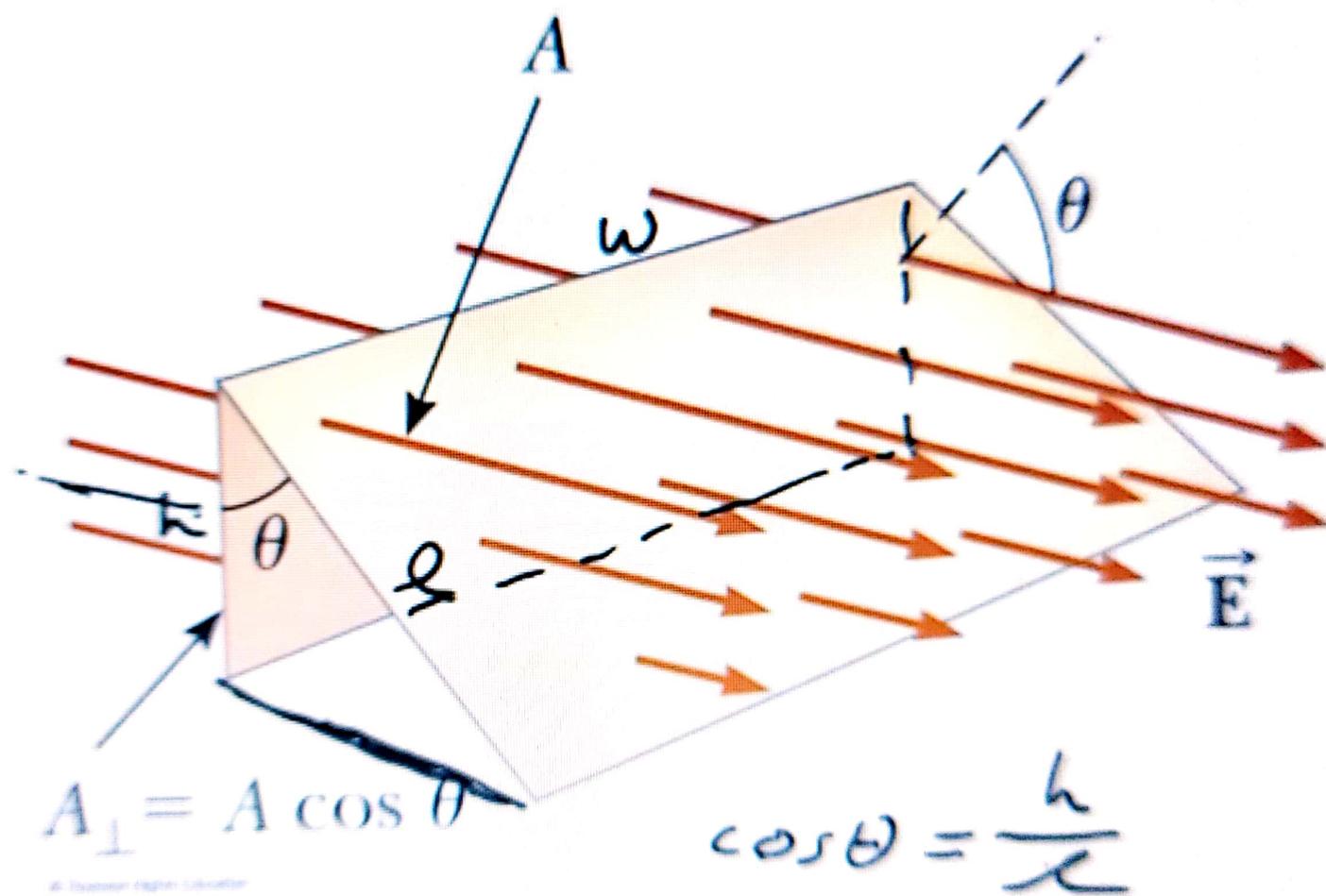
$$A_{\perp} = A \cos \Theta$$



$$|\Phi_A| = |\Phi_{A\perp}| = EA_{\perp} = EA \cos \Theta$$

$$|\Phi_A| = EA \cos \Theta = \vec{E} \cdot \vec{A}$$

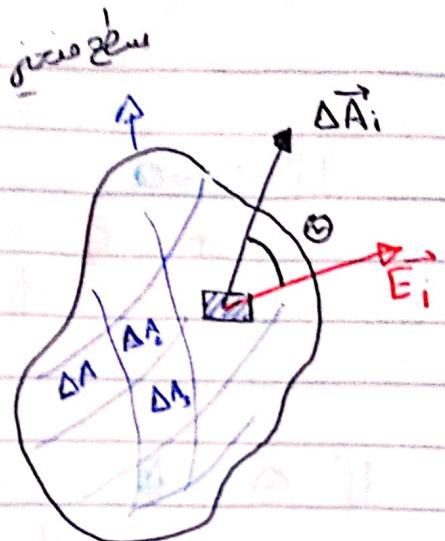
Normal



$$\cos \theta = \frac{h}{\ell}$$

for the most case:

\vec{E} is not uniform and
 \vec{A} is curved (not plane)



The electric flux, $\Delta\Phi$, of the electric field through the area element ΔA_i is

$$\Delta\Phi_i = \vec{E}_i \cdot \vec{\Delta A}_i$$

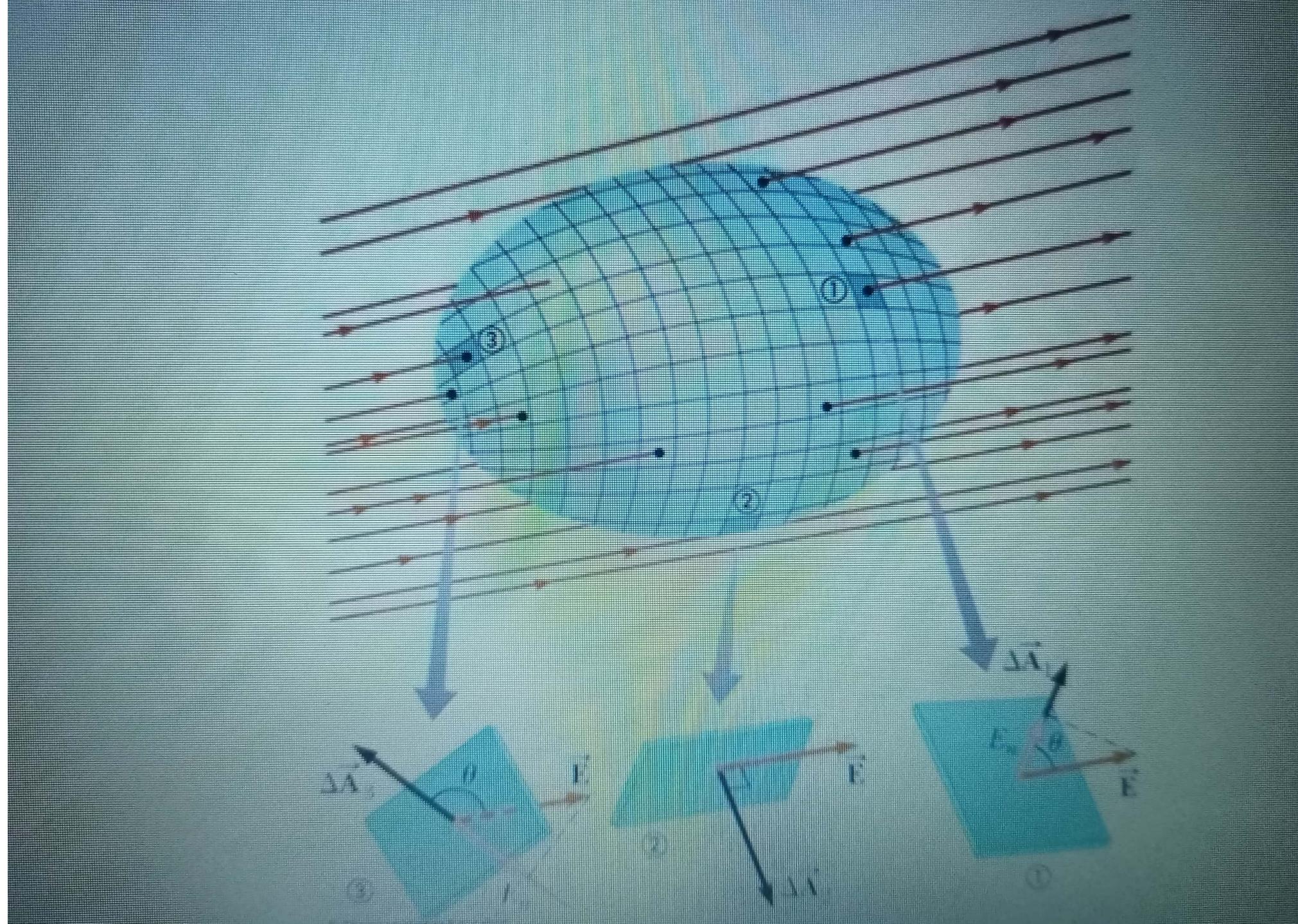
Summing the contributions of all area elements

(إذن بعده)

$$\Phi = \sum_i \Delta\Phi_i = \sum_i \vec{E}_i \cdot \vec{\Delta A}_i$$

$$\Phi = \lim_{\Delta A_i \rightarrow 0} \sum_i \vec{E}_i \cdot \vec{\Delta A}_i = \int \vec{E} \cdot d\vec{A}$$

$$\boxed{\Phi = \int \vec{E} \cdot d\vec{A}} \text{ Most general case.}$$



مقدار مساحة المثلث

If the surface closed

$$\Phi_c = \oint \vec{E} \cdot d\vec{A}$$

\oint

الدائرة تدور
أو التكامل
مغلق.

$$\Delta \Phi_1 = \vec{E} \cdot \Delta \vec{A}_1 = E \Delta A_1 \cos \theta$$

$\theta > 0$

$$0 \leq \theta \leq \frac{\pi}{2}$$

(+) $\cos \theta > 0$

1 always

$$\Delta \Phi_2 = \vec{E} \cdot \Delta \vec{A}_2 = E (\Delta A_2) \cos \frac{\pi}{2}$$

$= 0$

$$\theta_2 = \frac{\pi}{2}$$

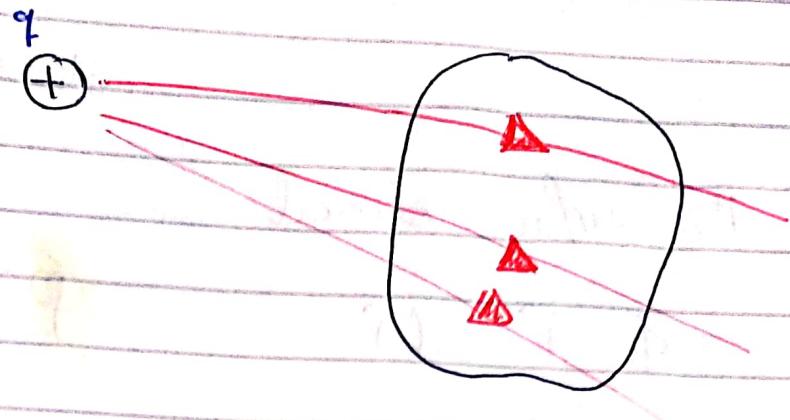
2 always

$$\frac{\pi}{2} \leq \theta \leq \pi$$

$$\Delta \Phi_3 = E \cdot \Delta \vec{A}_3 = E (\Delta A_3) \cos \theta,$$

$\theta < 0$

3 always

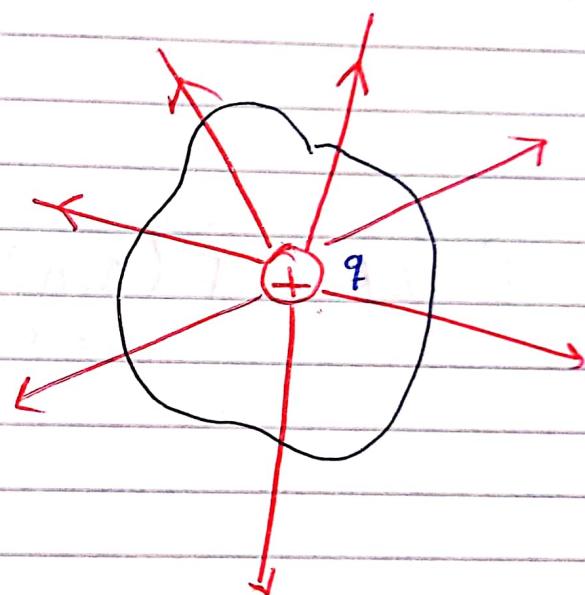


$$\Phi = 0 \rightarrow$$

لأن الحبة
ليست بالداخل

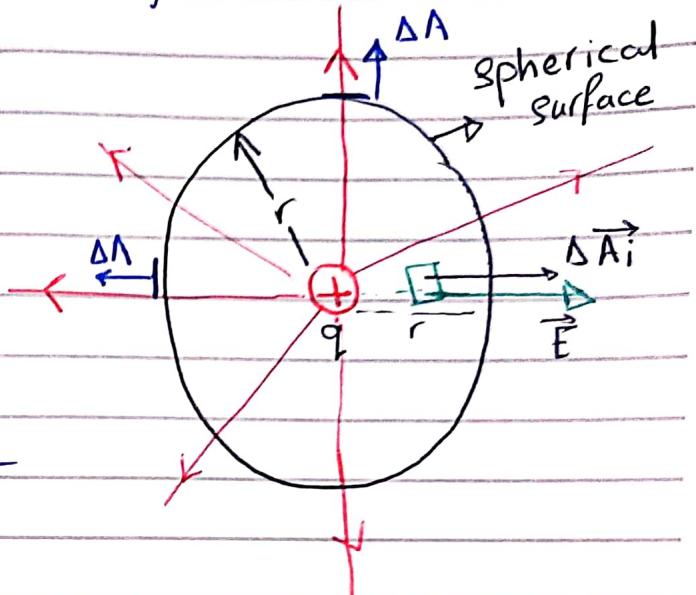
$$\Phi \neq 0$$

ويجب في الخ



* Gauss's Law relates the net electric flux through a closed surface and the net charge enclosed by the surface.

$$\text{areas } (A) \rightarrow (F) \perp E \text{ (i.e.)} \\ \rightarrow \int \vec{F} \cdot d\vec{A}$$



* The electric field at the surface is

$$\vec{E} = \frac{kq}{r^2} \hat{r}$$

$\epsilon_0 = 0$ between \vec{E} and $d\vec{A}$

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$= \oint E dA = \oint \frac{kq}{r^2} dA$$

$$= \frac{kq}{r^2} \oint dA = \frac{kq}{r^2} A = \frac{kq}{r^2} (4\pi r^2)$$

$$4\pi kq = \frac{4\pi}{4\pi kE} q = \frac{q}{E}$$

$$\Phi = \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon}$$

in \rightarrow inside the
Surface
($2e$, $10e$)

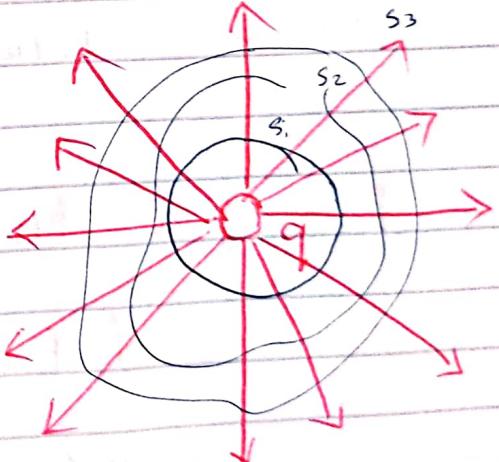
Gauss's Law

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C/N.m}^2$$

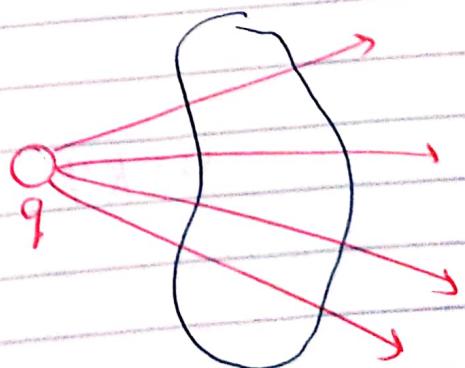
Gauss's Law \Rightarrow The net electric flux through a closed surface is equal to the net charge inside the surface divided by ϵ_0 .

Comments

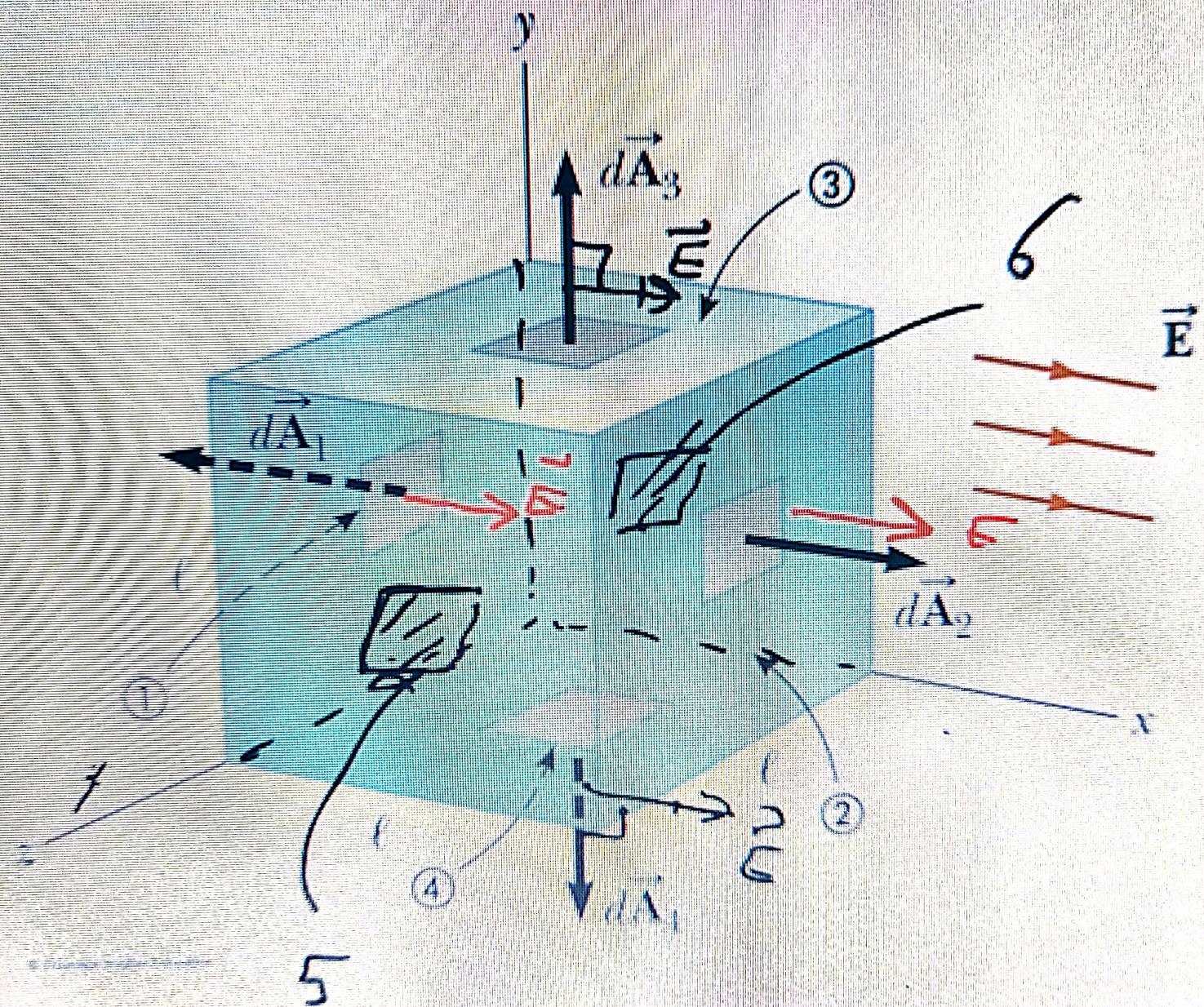
\Rightarrow The net electric flux through each closed surface surrounding the same charge is the same regardless the shape and size.



\Rightarrow The net electric flux through any closed surface is zero if there is no charge inside.



Ex. 24.1



⊗ Gauss's Law

$$\left[\Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \right]$$

(zero) \rightarrow \leftarrow zero q_{in} \rightarrow zero flux \rightarrow $\Phi_e = 0$

Ex. 24.1]

Consider a uniform electric field \vec{E} oriented in the X direction in empty space. A cube of edge length L is placed in the field, oriented as shown in figure. Find the net electric flux through the surface of the cube.

$$\Phi_1 = EA_1 \cos 180^\circ = -EL^2$$

$$\Phi_2 = EA_2 \cos 0^\circ = EL^2$$

for A_3, A_4, A_5 , and A_6

we have $\Theta = \frac{\pi}{2}$ between the area vector and \vec{E}

$$\Rightarrow \Phi_3 = \Phi_4 = \Phi_5 = \Phi_6 = 0$$

Total flux

$$\Phi_{\text{total}} = \Phi_1 + \Phi_2 + \Phi_3 + \Phi_4 + \Phi_5 + \Phi_6 = 0$$

EX) Electric field due to a distance r from a point charge q

$$\vec{E} \cdot d\vec{A} = E dA, \quad \textcircled{O} = 0$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

$$\oint E dA = \frac{q_{in}}{\epsilon}$$

$$E \underbrace{\oint dA}_{=A} = \frac{q}{\epsilon}$$

$$EA = \frac{q}{\epsilon}$$

$$E(4\pi r^2) = \frac{q}{\epsilon} \rightarrow E = \frac{1}{4\pi\epsilon} \frac{q}{r^2}$$

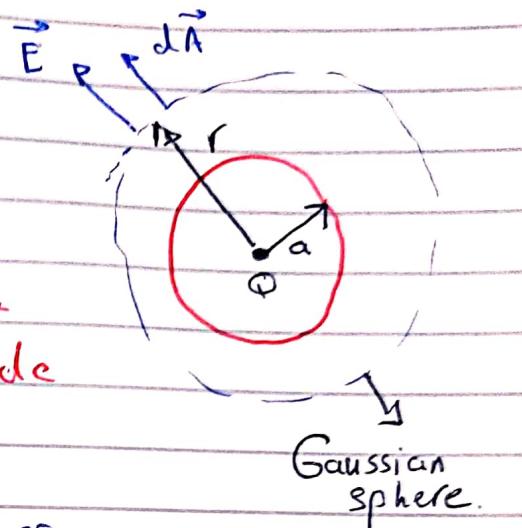
$$E = \frac{kq}{r^2}$$

EX. 24.3 \Rightarrow An insulating solid sphere of radius a has a uniform volume charge density ρ and carries a total positive charge Q

$$Q: \text{total charge (uniform)}$$

$$\rho = \frac{Q}{\frac{4}{3}\pi a^3}$$

A) Calculate the magnitude of the electric field at a point outside the sphere. ($r > a$)



$$\oint_{\text{out}} \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$EA = \frac{Q}{\epsilon_0}$$

$$\oint_{\text{out}} E dA = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E (4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$\frac{\oint_{\text{out}} dA}{E} = \frac{Q}{\epsilon_0}$$

$$E_{\text{out}} (4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E_{\text{out}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}$$

plenty of space \leftarrow

$$E_{\text{out}} = k \frac{Q}{r^2}$$

\vec{E} out side the sphere.

B) find the magnitude of the electric field at a point inside the sphere. ($r < a$)

$$\oint \vec{E}_{in} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

$$\oint E_{in} dA = \frac{q_{in}}{\epsilon}$$

$$E_{in} \oint dA = \frac{q_{in}}{\epsilon}$$

$$E_{in} (4\pi r^2) = \frac{q_{in}}{\epsilon}$$

$$E_{in} = \frac{1}{4\pi\epsilon} \frac{q_{in}}{r^2} = k \frac{q_{in}}{r^2} = k \frac{q_{in}}{r^2}$$



Gaussian Sphere

$$\Rightarrow \rho = \frac{Q}{\frac{4}{3}\pi a^3} = \frac{q_{in}}{\frac{4}{3}\pi r^3} \quad \boxed{E_{in} = k \frac{Qr}{a^3}}$$

$$q_{in} = \frac{Qr^3}{a^3}$$

\vec{E} inside the sphere.

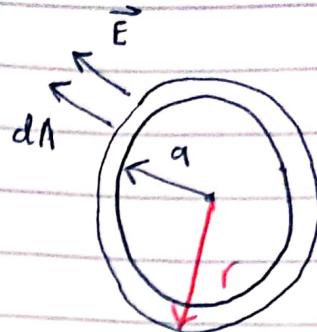
Example Find the electric field inside and outside a thin spherical shell of radius a and uniform charge Q .

A) Outside ($r > a$)

$$\oint \vec{E}_{\text{out}} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$E_{\text{out}} (4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$\boxed{E_{\text{out}} = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2}}$$



B) Inside ($r < a$)

$$\oint \vec{E}_{\text{in}} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0} = \frac{0}{\epsilon_0}$$

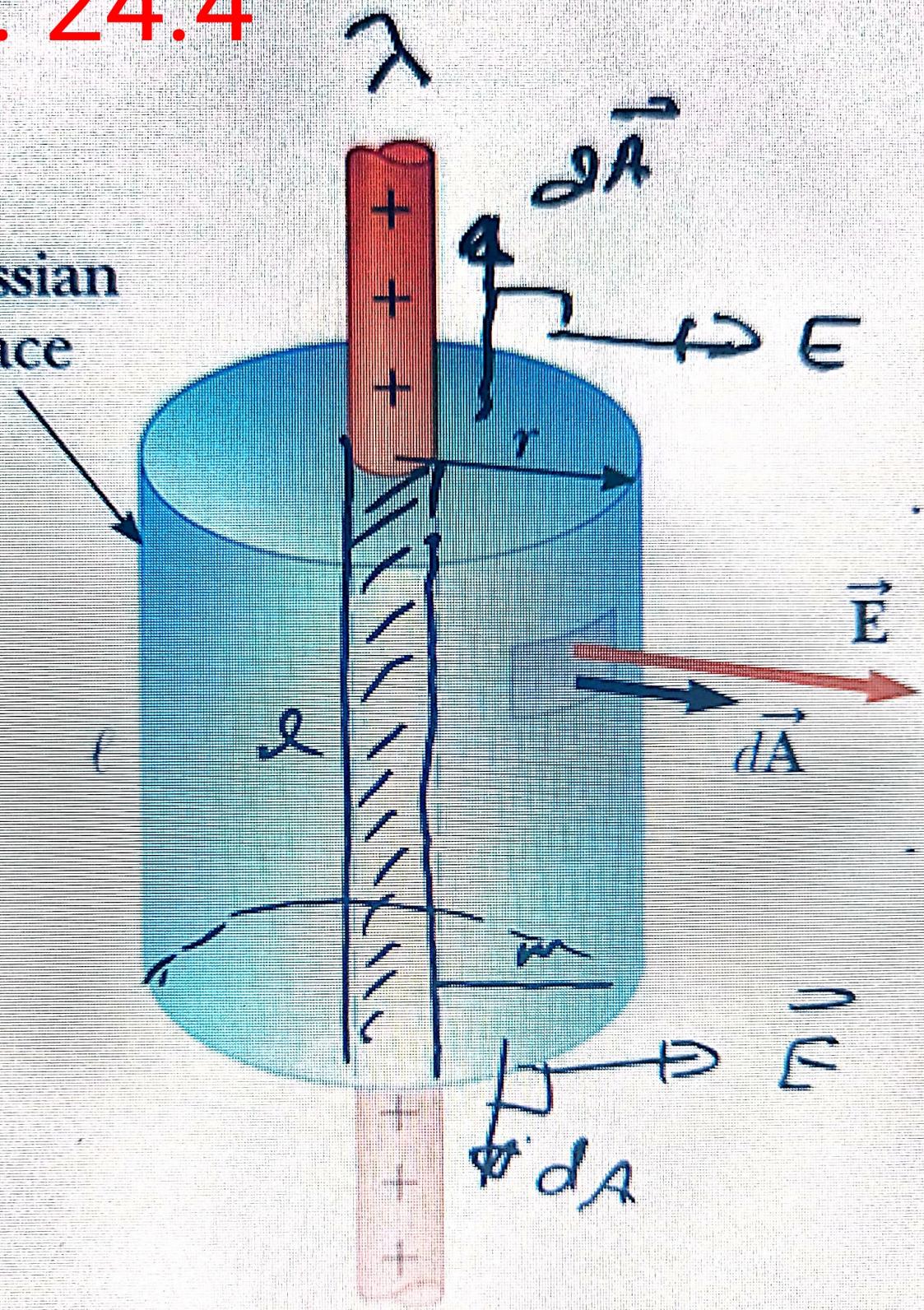
$$\oint \vec{E}_{\text{in}} \cdot d\vec{A} = 0$$

$$\boxed{E_{\text{in}} = 0}$$

لما
 يكون
 في
 الماء
 تكون
 الماء
 في
 الماء
 الماء
 الماء

Ex. 24.4

Gaussian surface



(.1)

Ex. 24.4)

Find the electric field a distance r from a line of positive charge of infinite length and constant charge per unit length λ

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon}$$

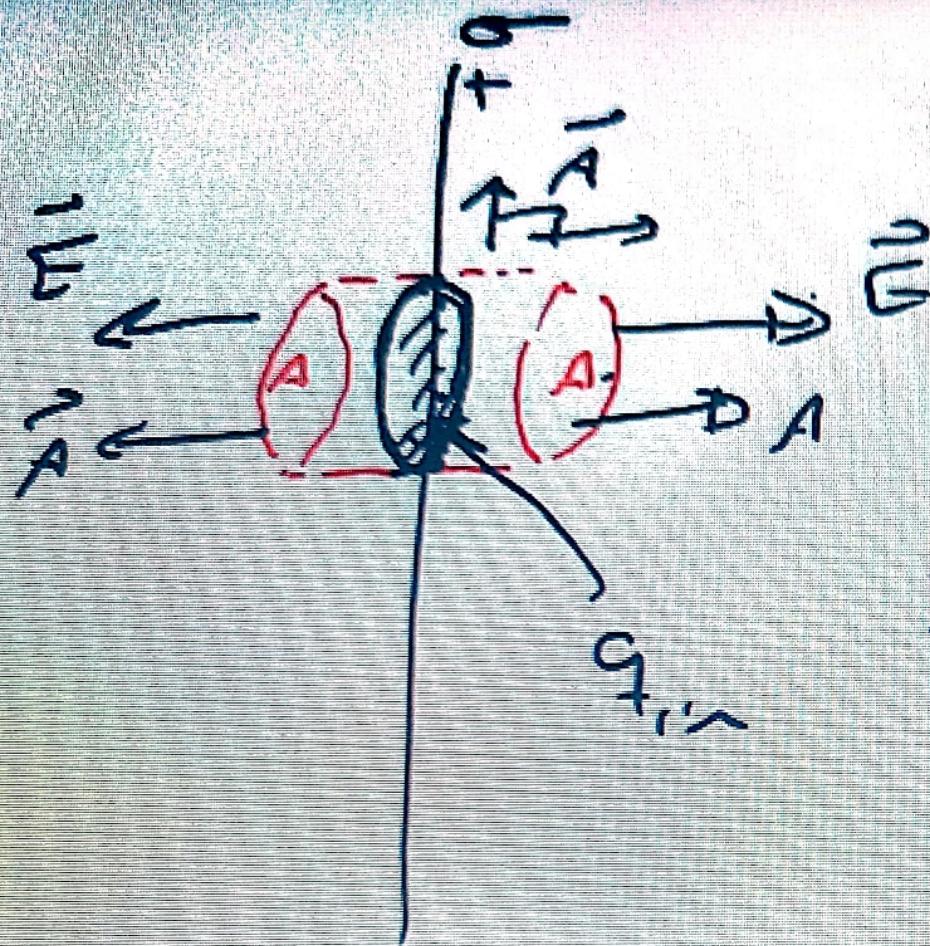
$$E(2\pi r l) = \frac{\lambda l}{\epsilon}$$

$$= E \left(\frac{1}{2\pi\epsilon} \frac{\lambda}{r} \right) \times \frac{2}{2}$$

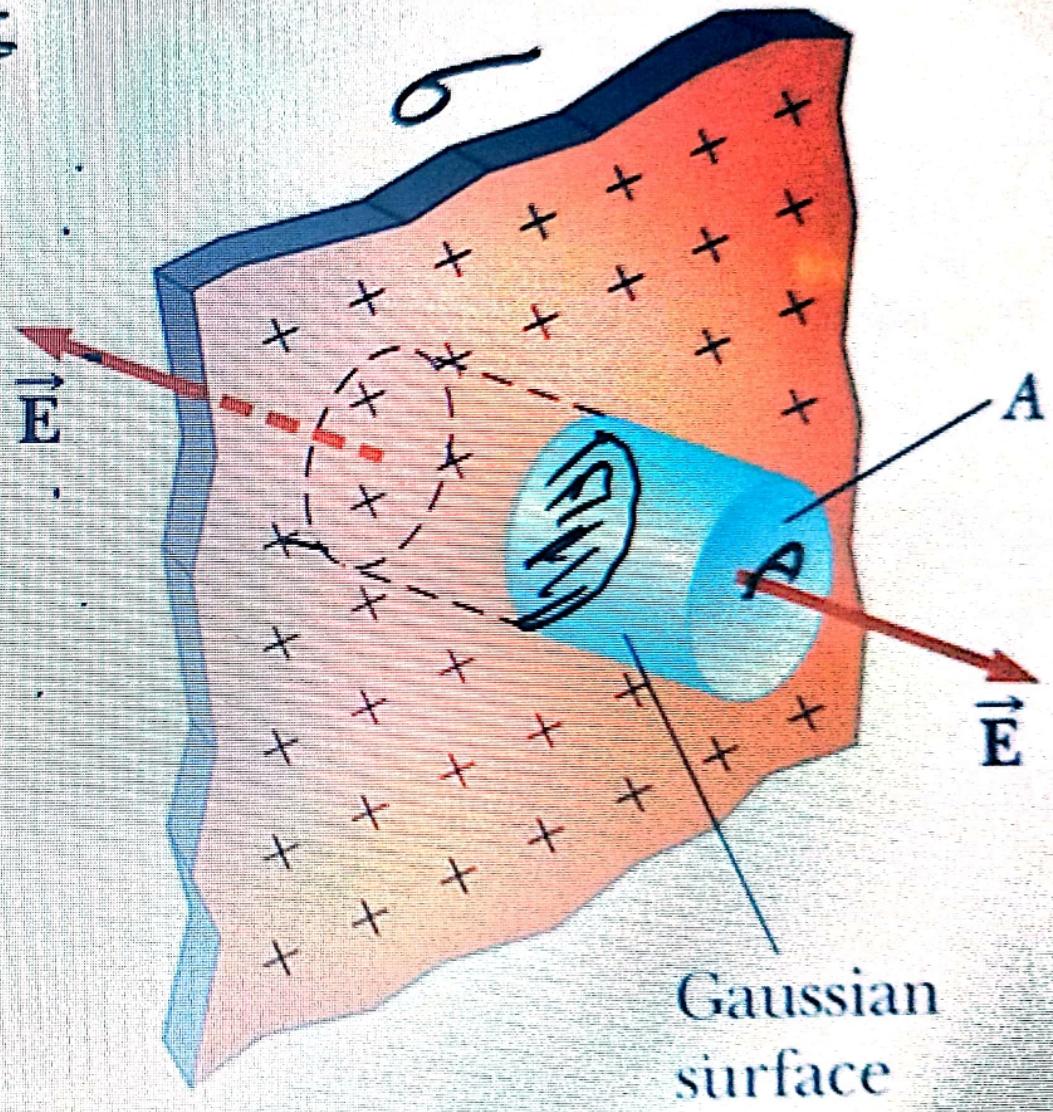
$$= \frac{1}{4\pi\epsilon} \frac{\lambda}{r}$$

$$\boxed{\vec{E} = \frac{2\lambda \hat{r}}{r}}$$

\vec{E} due to a line charge λ of infinite length



Ex. 24.5



EX 24.5) Find the electric field due to an infinite plane of positive charge with uniform surface charge density σ .

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

$$EA + EA = \frac{\sigma A}{\epsilon}$$

$$= 2EA = \frac{\sigma A}{\epsilon}$$

$$E = \frac{\sigma}{2\epsilon}$$

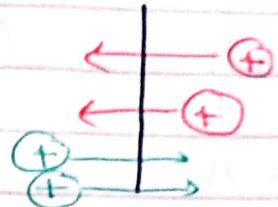
\vec{E} of non conducting sheet of uniform charge density σ

الكتل المعدنية في التوازن الكهربائي Conductors in electrostatic equilibrium

الكتل المعدنية في التوازن الكهربائي \equiv لا توجد حركة لجزيئات الشحنة \equiv no net motion of charge

الكتل المعدنية في التوازن الكهربائي \equiv لا توجد حركة لجزيئات الشحنة \equiv no net motion of charge

Zero = $\vec{E}_{\text{in}} = 0$



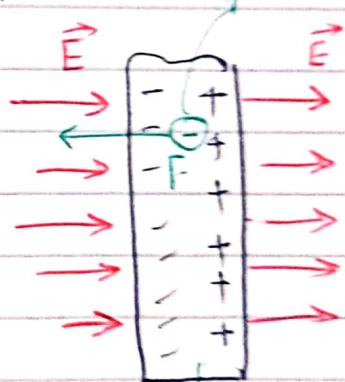
Properties of Conductors in electrostatic equilibrium.

□ $\vec{E}_{\text{in}} = 0$ inside the conductor

$$\vec{E}_{\text{in}} = \vec{E} - \vec{E}'$$

الكتل المعدنية في التوازن الكهربائي
تحتاج إلى توزيع الشحنة
الداخلية.

$$F = q_a$$



الكتل المعدنية في التوازن الكهربائي
تحتاج إلى توزيع الشحنة
الداخلية.

The process (redistribution) of charge continue until

$$F = q_a E_{\text{in}} = 0$$

الكتل المعدنية في التوازن الكهربائي

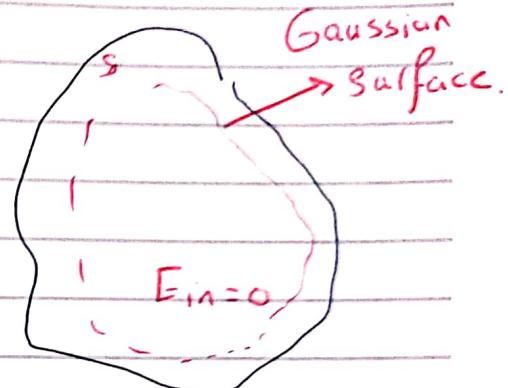
2 If an isolated conductor is charged then the charge resides on its surface.

لوكان بعزم مفتح دمحرك
الشحن لا يترك في الداخل

$$\oint \vec{E}_{in} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

$$0 = \frac{q_{in}}{\epsilon}$$

$$q_{in} = 0$$



⇒ The charge resides on the surface.

3 Just outside the conductor

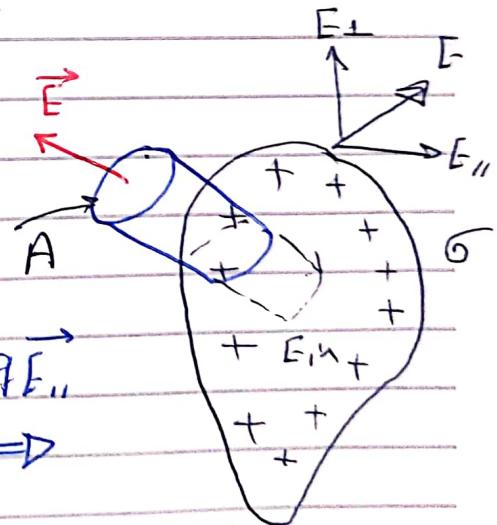
a) $\vec{E} \perp \text{Surface}$

b) $E_{\text{surface}} = \frac{q}{\epsilon}$

Let \vec{E} is not normal

to the surface ⇒

E_{\parallel} affects before of $\vec{F}_{\parallel} = q\vec{E}_{\parallel}$
parallel to the surface ⇒



There is a net motion of charge parallel to the surface ⇒

The conductor is not in electrostatic equilibrium

⇒ We must have $\vec{E}_{\parallel} = 0$

$E_{\parallel} \parallel$ is 0

For a conductor to be in electrostatic equilibrium we must have:

$\vec{E} \perp \text{surface}$

$$\vec{E}_{in} = 0$$

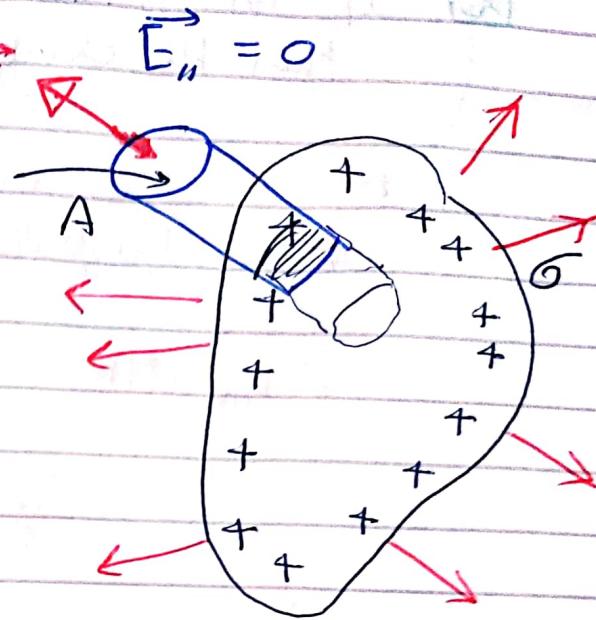
(B)

Apply Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

$$E_A - \frac{E_{in} A}{\text{zero}} = \frac{G A}{\epsilon}$$

$$E_A = \frac{G A}{\epsilon}$$

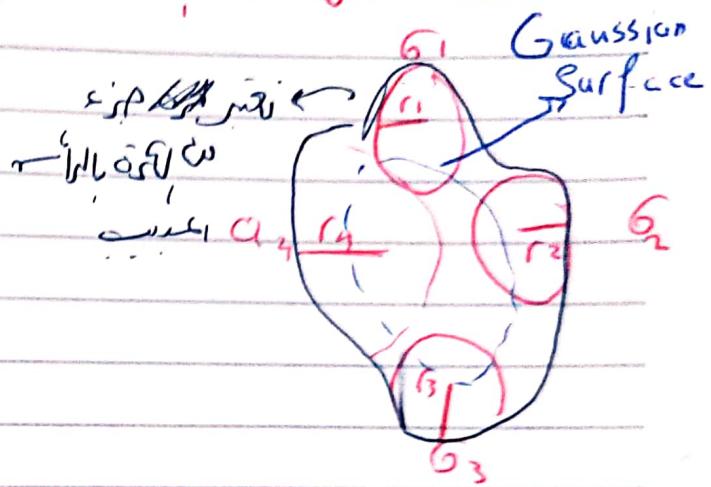


$$E = \frac{G}{\epsilon}$$

2210

④ On an irregularly shaped conductor, σ and E are large where the radius of curvature is small

$Q_{LS} \leftarrow \sigma \rightarrow E \rightarrow$

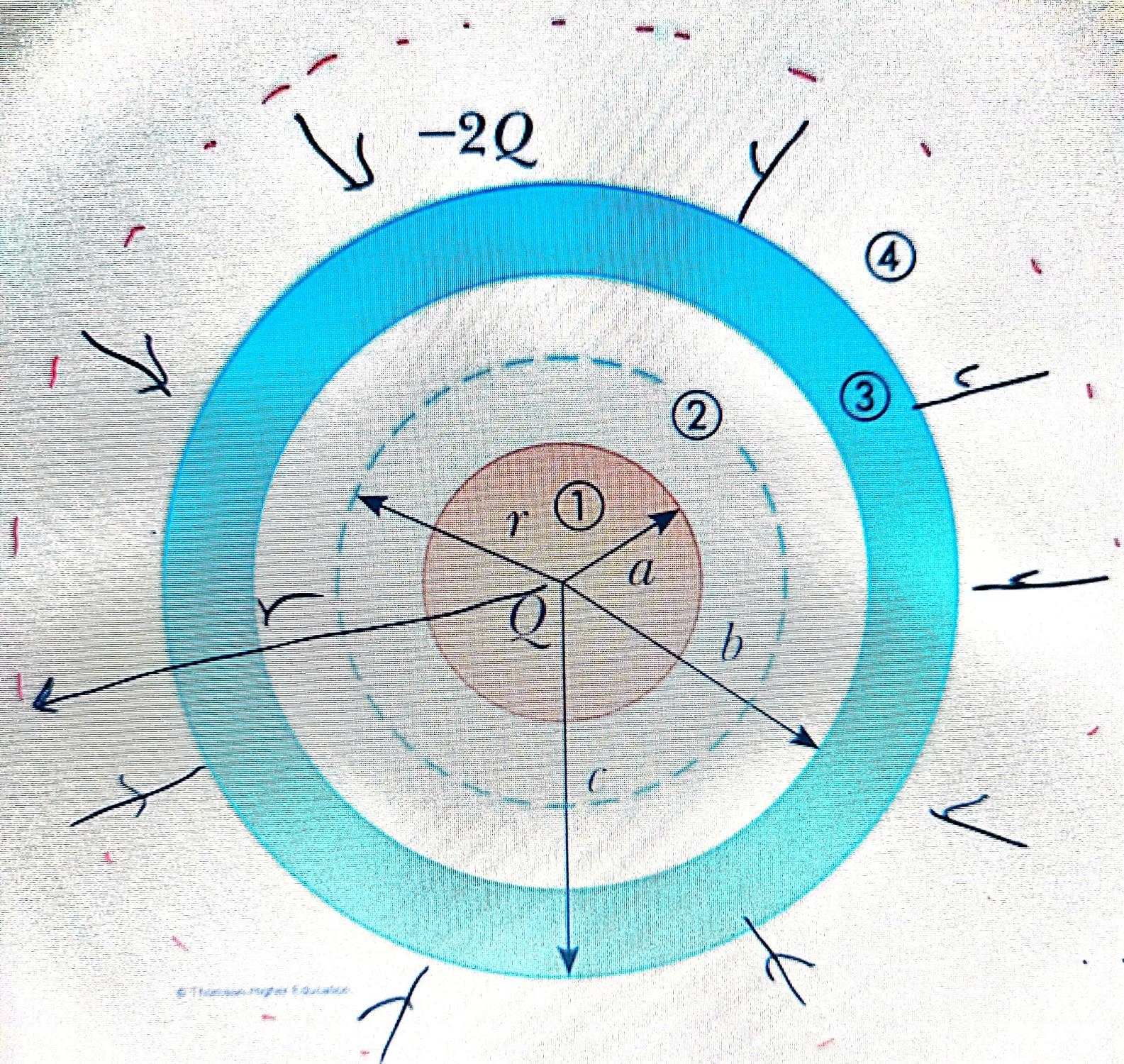


$$r_1 < r_2 < r_3 < r_4$$

$$\sigma_1 > \sigma_2 > \sigma_3 > \sigma_4$$

$$E = \frac{\sigma}{\epsilon_0}$$

$$E_1 > E_2 > E_3 > E_4$$



Ex... 24.7

Ex. 24.7 A solid insulating sphere of radius a carries a net positive charge Q uniformly distributed throughout its volume. A conducting spherical shell of inner radius b and outer radius c is concentric with the solid sphere and carries a net charge $-2Q$. Using Gauss's law find the electric field in the regions labeled ①, ②, ③ and ④ in figure and the charge distribution on the shell when the entire system is in electrostatic equilibrium.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

region ① ($r < a$)

$$\oint \vec{E}_1 \cdot d\vec{A} = \frac{q_{in}}{\epsilon}, \rho = \text{constant}$$

$$E_1 (4\pi r^2) = \frac{q_{in}}{\epsilon}$$

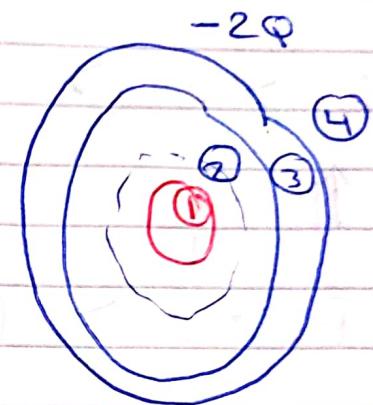
$$E_1 = \frac{1}{4\pi\epsilon} \frac{q_{in}}{r^2} = k \frac{q_{in}}{r^2}$$

$$\rho = \frac{Q}{\frac{4}{3}\pi a^3} = \frac{q_{in}}{\frac{4}{3}\pi r^3}$$

$$q_{in} = \frac{Qr^3}{a^3}$$

$$E_{in} = k \frac{Qr^3/a^3}{r^2} = \frac{kQr}{a^3}$$

$$\Rightarrow \boxed{E_1 = \frac{kQr}{a^3}}$$



Region ② $a < r < b$

$$\oint \vec{E}_2 \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

$$E_2 (4\pi r^2) = \frac{Q}{\epsilon}$$

$$\boxed{E_2 = \frac{1}{4\pi\epsilon} \frac{Q}{r^2} = k \frac{Q}{r^2}}$$

Region ③ $b < r < c$

$$\boxed{E_3 = 0} \rightarrow \text{inside the conductor}$$

Region ④

$$\oint \vec{E}_4 \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

$$E_4 (4\pi r^2) = -\frac{2Q + Q}{\epsilon}$$

$$E_4 (4\pi r^2) = -\frac{Q}{\epsilon}$$

$$E_4 = -\frac{1}{4\pi\epsilon} \frac{Q}{r^2} = -\frac{kQ}{r^2}$$

$$q' + q'' = -2\varphi$$

$$\oint \vec{E}_3 \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0} ; E_3 = 0$$

$$0 = \frac{q_{\text{in}}}{\epsilon_0}$$

$$q_{\text{in}} = 0$$

$$q'' + Q = 0$$

$$\boxed{q'' = -Q}$$

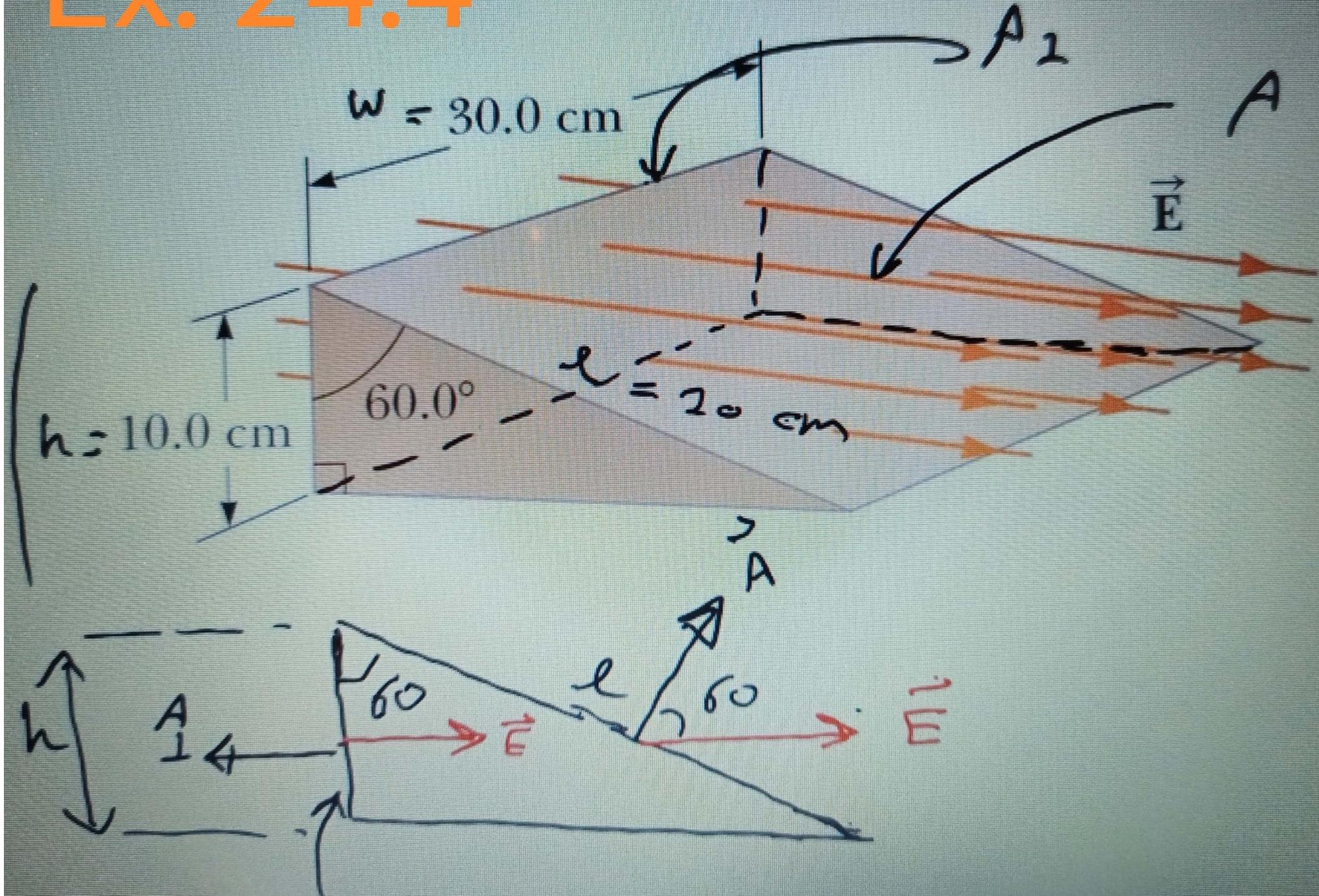
$$q' + q'' = -2\varphi \Rightarrow$$

$$q' = -2\varphi - q'' = -Q$$

$$\Rightarrow \boxed{q' = -Q}$$

Die \vec{E} -Feldlinien verlaufen parallel zu \vec{E}

Ex. 24.4

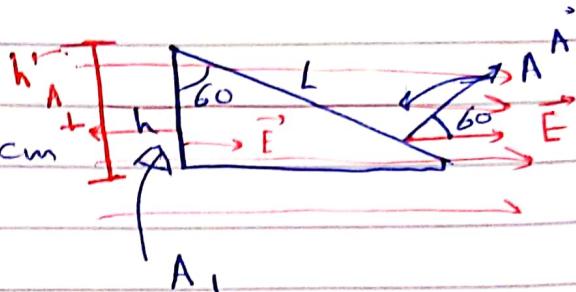


prob. 24.4 \Rightarrow Consider a closed triangle box resting within a horizontal electric field of magnitude $E = 7.80 \times 10^4 \text{ N/C}$ as shown. Calculate the electric flux through (a) the vertical rectangular surface, (b) the slanted surface, and (c) the entire surface of the Box

$$E = 7.8 \times 10^4 \text{ N/C}$$

$$h = 10 \text{ cm} \quad , \quad w = 30 \text{ cm}$$

$$L = \frac{h}{\cos 60} = \frac{10 \text{ cm}}{0.5} = 20 \text{ cm}$$



$$\begin{aligned}
 \text{a) } \Phi_{A_L} &= \vec{E} \cdot A_L \cos 180 \\
 &= (7.8 \times 10^4) (0.3 \times 0.1) (-1) \\
 &= -2.34 \times 10^3 \text{ N.m}^2/\text{C} \\
 &= -2.34 \text{ kN.m}^2/\text{C}
 \end{aligned}$$

التدفق من لمح بار
 هو سالب نسبة
 إلى ملحوظ، مما يعني لاثن
 مفهوم معاوين مثل ملحوظ
 العادي رخصته لمح

$$\begin{aligned}
 \text{b) } \Phi_A &= \vec{E} \cdot A \cos \overbrace{\theta}^{60} \\
 &= (7.8 \times 10^4) (0.3 \times 0.2) \left(\frac{1}{2}\right) \\
 &= 2.34 \times 10^3 \text{ N.m}^2/\text{C}
 \end{aligned}$$

$$\Phi_{\text{Total}} = \Phi_{A_L} + \Phi_A \rightarrow = 0$$

لأن لا يوم
 رحمة في
 السافر

Prob - 24-10

The electric field everywhere on the surface of a thin, spherical shell of radius 0.750 m is of magnitude 890 N/C and points radially toward the center of the sphere.

(a) what is the net charge within the spheres surface? (b) what is the distribution of the charge inside the spherical shell?

$$a = 0.75 \text{ m}$$

$$E = 890 \text{ N/C} \text{ at } r = a = 0.75 \text{ m}$$

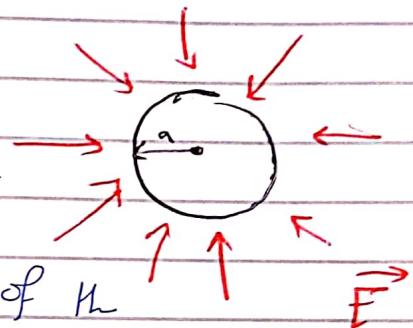
$$E = \text{point radially toward the center}$$

b) Because \vec{E} points radially toward the center

$\Rightarrow Q$ inside the spherical shell is negative

Because $|\vec{E}| = \text{constant}$ at the surface of the

spherical shell then the distribution of charge inside is uniform and concentric with the spherical shell



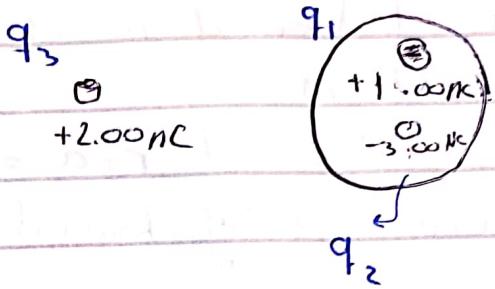
$$\text{a) } E = \frac{kQ}{a^2} \rightarrow Q = \frac{Ea^2}{k}$$

$$= \frac{(-890)(0.75)^2}{9 \times 10^9}$$

$$Q = -5.57 \times 10^{-8} \text{ C}$$
$$= -55.7 \text{ nC}$$

prob 24.8

Find the net electric flux through the spherical closed surface shown in Figure. The two charges on the right are inside the spherical surface.



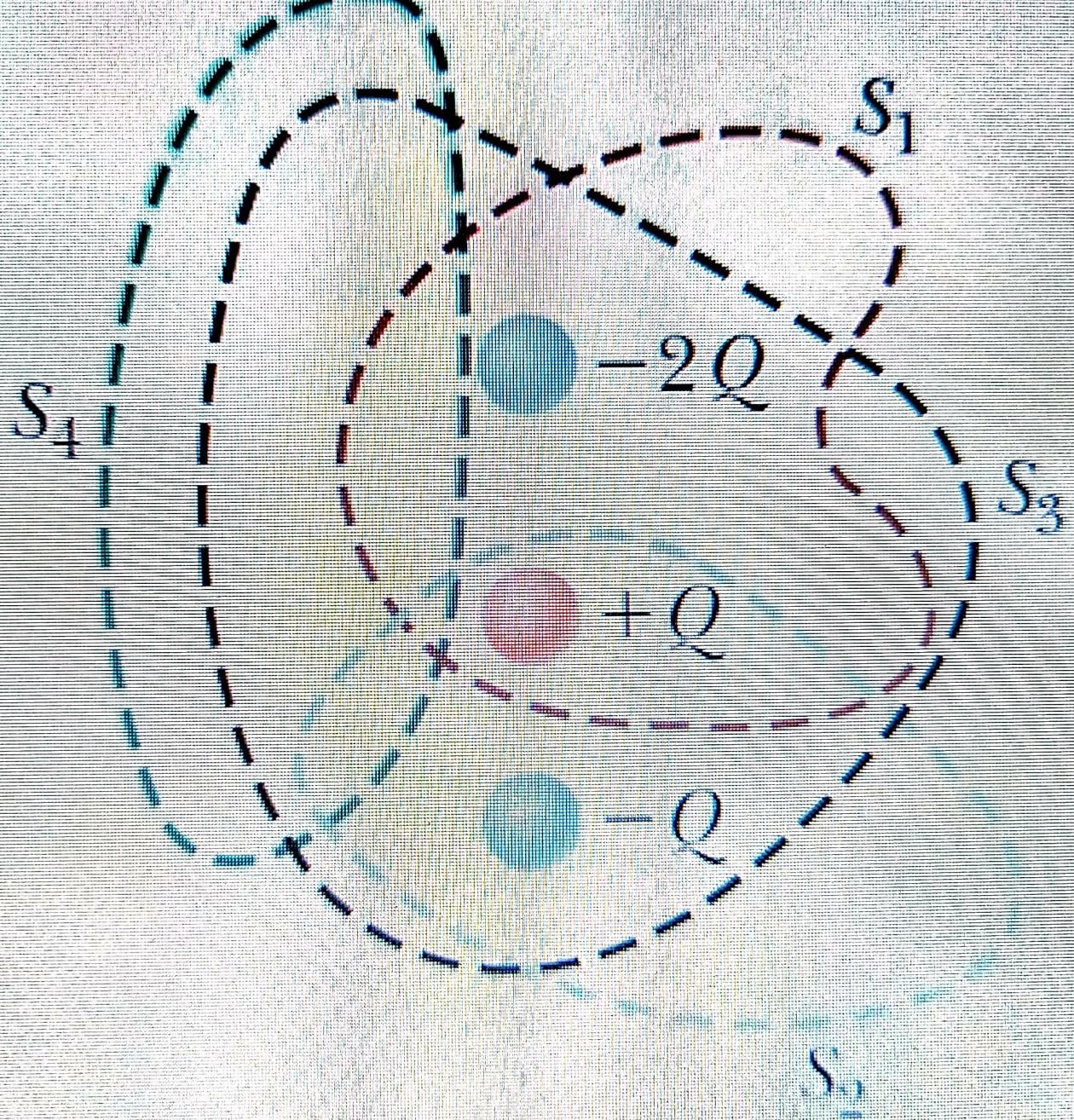
$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

$$\Phi_{tot} = \frac{q_{in}}{\epsilon} = \frac{q_1 + q_2}{\epsilon}$$

$$\Phi_{tot} = \frac{(1 \times 10^{-9}) + (-3 \times 10^{-9})}{8.85 \times 10^{-12}}$$

$$\Phi_{tot} = -226 \text{ N.m}^2/\text{C}$$

Ex. 24.11



prob 24.11

Four closed surfaces, S_1 through S_4 , together with the charges $-2Q$, Q , and $-Q$ are sketched in figure (the colored lines are the intersections of the surfaces with the page) find the electric flux through each surface.

$$\bar{\Phi} = \frac{q_{in}}{\epsilon}$$

$$S_1 = \bar{\Phi}_1 = \frac{-2Q + Q}{\epsilon} = -\frac{Q}{\epsilon}$$

$$S_2 = \bar{\Phi}_2 = \frac{+Q - Q}{\epsilon} = 0$$

$$S_3 = \bar{\Phi}_3 = \frac{-2Q + Q - Q}{\epsilon} = -\frac{2Q}{\epsilon}$$

$$S_4 = \bar{\Phi}_4 = 0 \quad \text{and} \quad \begin{matrix} \text{Only } Q \text{ is } \\ \text{inside } S_4 \end{matrix}$$

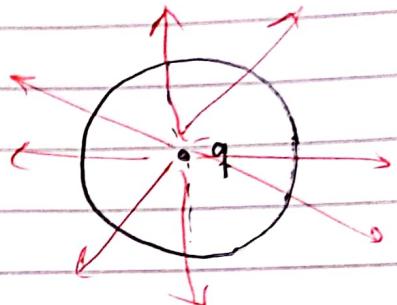
Prob \Rightarrow 24-14)

A particle with charge of $12.0 \mu C$ is placed at the center of a spherical shell of radius 22.0 cm . What is the total electric flux through (a) the surface of the shell and (B) any hemispherical surface of the shell? (C) Do the results depend on the radius? Explain \Rightarrow $\nabla \Phi_{\text{sh}}$

$$q = 12 \mu C = 12 \times 10^{-6} \text{ C}$$

$$a) \Phi_{\text{shell}} = \frac{q_{\text{in}}}{\epsilon} = \frac{q}{\epsilon} = \frac{12 \times 10^{-6}}{8.85 \times 10^{-12}}$$

$$\Phi_{\text{shell}} = 1.36 \times 10^6 \text{ N.m}^2/\text{C}$$

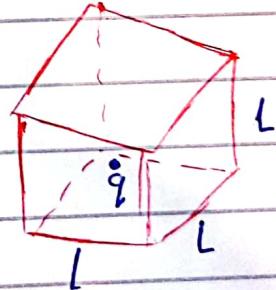


$$b) \bar{\Phi}_{\text{half shell}} = \frac{1}{2} \Phi_{\text{shell}} = 6.78 \times 10^5 \text{ N.m}^2/\text{C}$$

Prob \Rightarrow دالة لثالة
(ج) Cube لكان

q is at the center

$$\Phi_{\text{tot}} = \frac{q}{\epsilon}$$



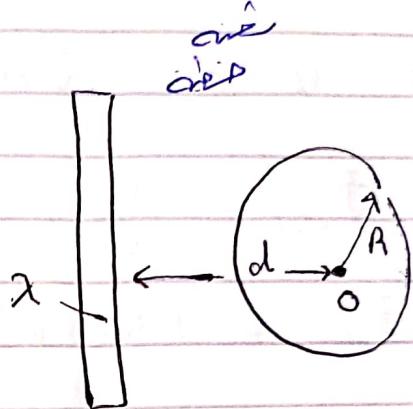
$$\Phi_{\text{one-face}} = \frac{\Phi_{\text{tot}}}{6} = \frac{q}{6\epsilon}$$

prob. 24-17

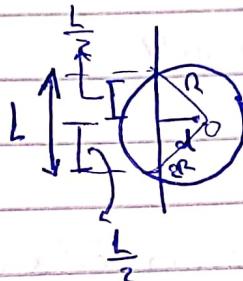
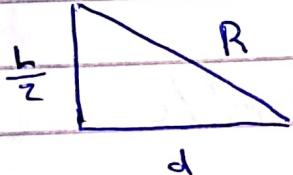
An infinitely long line charge having a uniform charge per unit length λ lies a distance d from point O as shown. Determine the total electric flux through the surface of a sphere of radius R centered at O resulting from this line charge. Consider both cases where (a) $R < d$, and (b) $R > d$.

A) $R < d \Rightarrow \Phi = \frac{q_{in}}{C} = 0$

Explain $\int \lambda dl$
at $R < d$



B) $R > d$

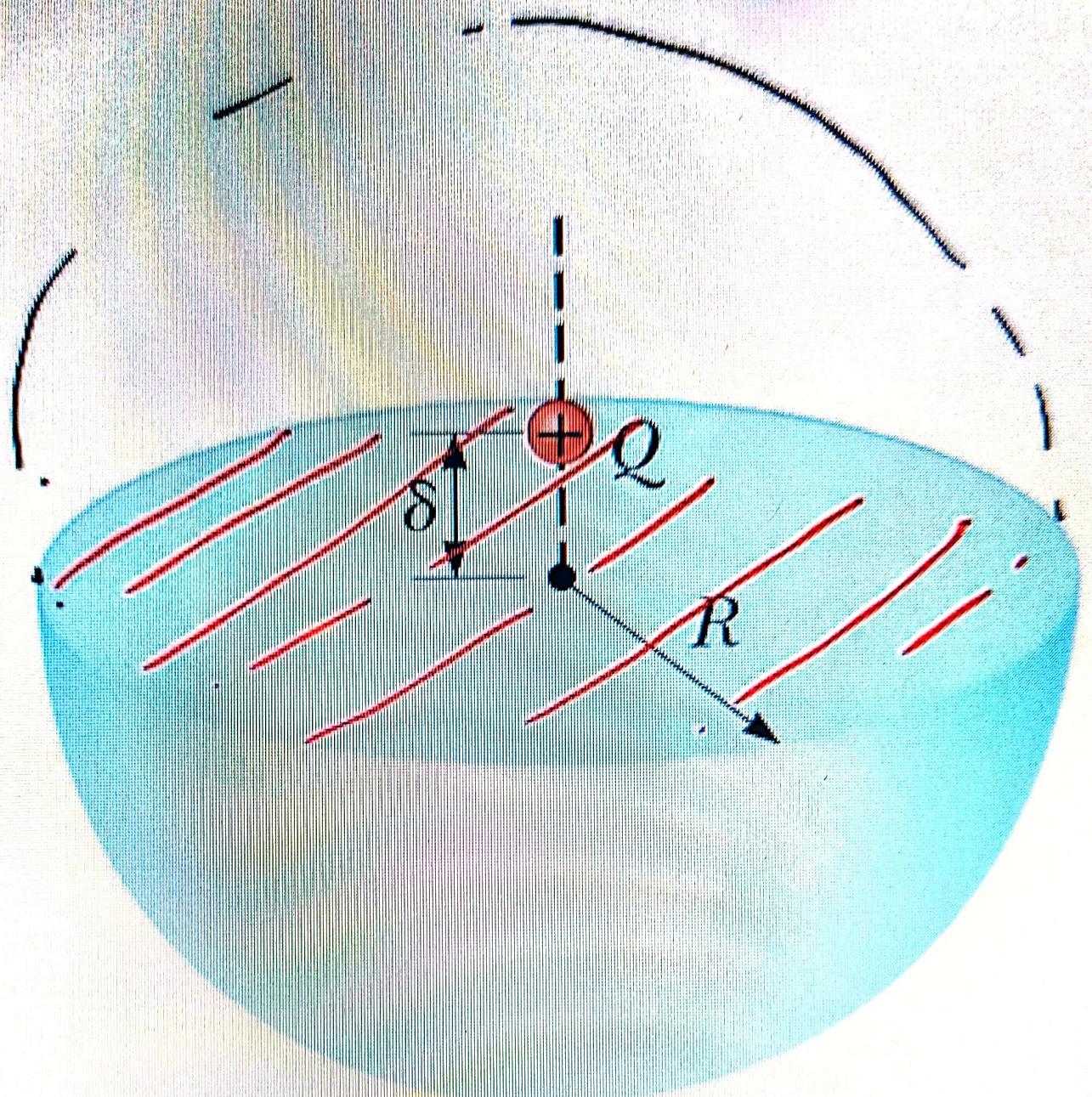


$\frac{L}{2} = \sqrt{R^2 - d^2}$

$L = 2 \sqrt{R^2 - d^2}$

$q_{in} = \lambda L = 2 \lambda \sqrt{R^2 - d^2}$

$\Phi = \frac{q_{in}}{C} = \frac{2 \lambda \sqrt{R^2 - d^2}}{C}$



Ex... 24.21

Prob. 24.21

A particle with charge Q is located a small distance δ immediately above the center of the flat face of a hemisphere of radius R as shown what is the electric flux (a) through the curved surface and (b) through the flat face as $\delta \rightarrow 0$?

$$\delta \rightarrow 0$$

A)

$$\begin{aligned}\Phi_{\text{hemisphere}} &= \frac{1}{2} \Phi_{\text{sphere}} \\ &= \frac{1}{2} \frac{Q}{\epsilon_0} = \frac{Q}{2\epsilon_0} = \frac{Q}{2\epsilon_0}\end{aligned}$$

B) $\Phi_{\text{flat}} = -\Phi_{\text{hemisphere}} = -\frac{Q}{2\epsilon_0}$

prob. 24.24 The charge per unit length on a long, straight filament is -90.0 nC/m . Find the electric field (a) 10.0 cm , (b) 20.0 cm and (c) 100 cm from the filament where distance are measured perpendicular to the length of the filament

$$\begin{aligned}\text{a) } r = 10 \text{ cm} &= 0.1 \text{ m} & \lambda = -90 \text{ nC} \\ E &= \frac{2k\lambda}{r} = 16.2 \times 10^6 \text{ N/C} & m = -90 \times 10^{-9} \text{ C/m} \\ &= -16.2 \text{ MN/C} & E = \frac{2k\lambda}{r}\end{aligned}$$

b) $r = 20 \text{ cm}$

$$E = \frac{2k\lambda}{r} = 8.09 \text{ MN/C.}$$

Prob

29) Consider a thin spherical shell of radius 14.0 cm with a total charge of 32.0 nC distributed uniformly on its surface. Find the electric field (a) 10.0 cm and (b) 20.0 cm from the center of the charge distribution.

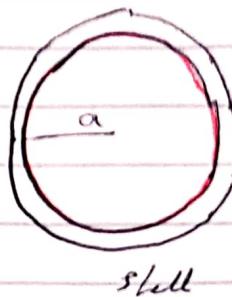
$$a = 0.14 \text{ m}$$

$$Q = 32 \text{ nC}$$

$$E_{in} = 0 \rightarrow E_{out} = \frac{kQ}{r^2}$$

$$a) r = 10 \text{ cm} < a$$

$$\Rightarrow E = E_{in} = 0$$



shell

$$b) r = 20 \text{ cm} = 0.2 \text{ m} > a$$

$$E = E_{out} = \frac{kQ}{r^2} = 7.19 \times 10^{16} \text{ N/C}$$

Prob 35] A solid sphere of radius 40.0 cm has a total positive charge of 26.0 nC uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0 cm (b) 10.0 cm (c) 40 cm, and (d) 60.0 cm from the center of the sphere.

$$a = 40 \text{ cm}, Q = 26 \text{ nC}$$

$$E_{in} = \frac{kQr}{a^3}, E_{out} = \frac{kQ}{r^2}$$

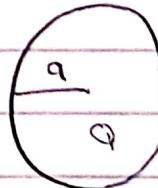
$$A) r = 0 \quad 0 < a \quad \Rightarrow E = E_{in} =$$

$$\frac{kQr}{a^3} = 0 \quad 0.1 \text{ m}$$

$$B) r = 10 \text{ cm} = 10 \text{ cm} < a \Rightarrow E = E_{in} = \frac{kQr}{a^3}$$

$$= 365 \text{ kN/C}$$

$$C) r = 40 \text{ cm} \Rightarrow E = E_{out} = \frac{kQ}{r^2} = 1.46 \text{ MN/C}$$

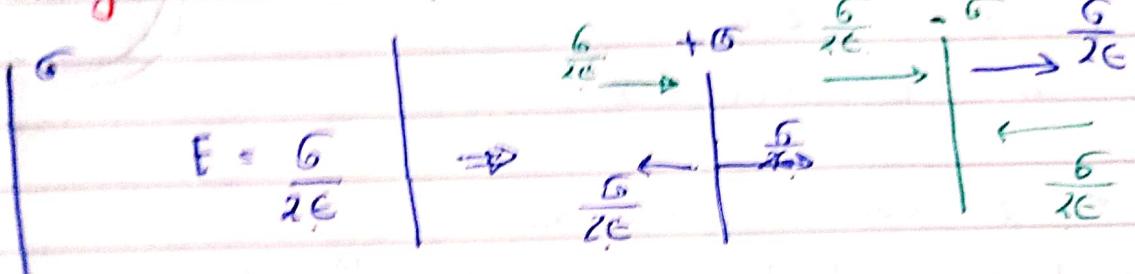


solid insulating

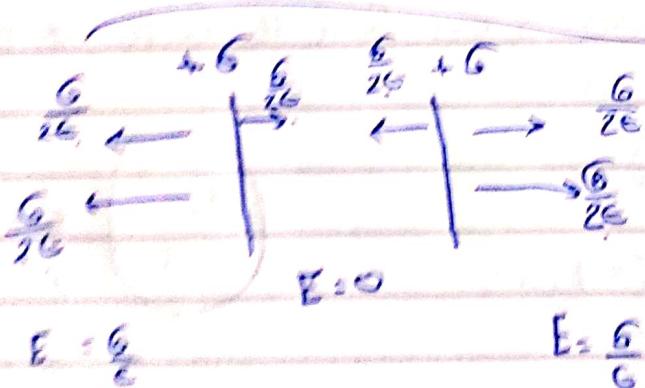
$$D) r = 60 \text{ cm} \Rightarrow E = E_{out} = \frac{kQ}{r^2} = 649 \text{ kN/C}$$

Prob. 56)

Two infinite, nonconducting sheets of charge are parallel to each other, as shown. The sheet on the left has a uniform surface of charge density σ and the one on the right has a uniform charge density $+\sigma$. Calculate the electric field at points (a) to the left of, (b) in between and (c) to the right of the two sheets. (d) What if? Find the electric fields in all three regions if both sheets have positive, uniform surface charge densities of value σ .

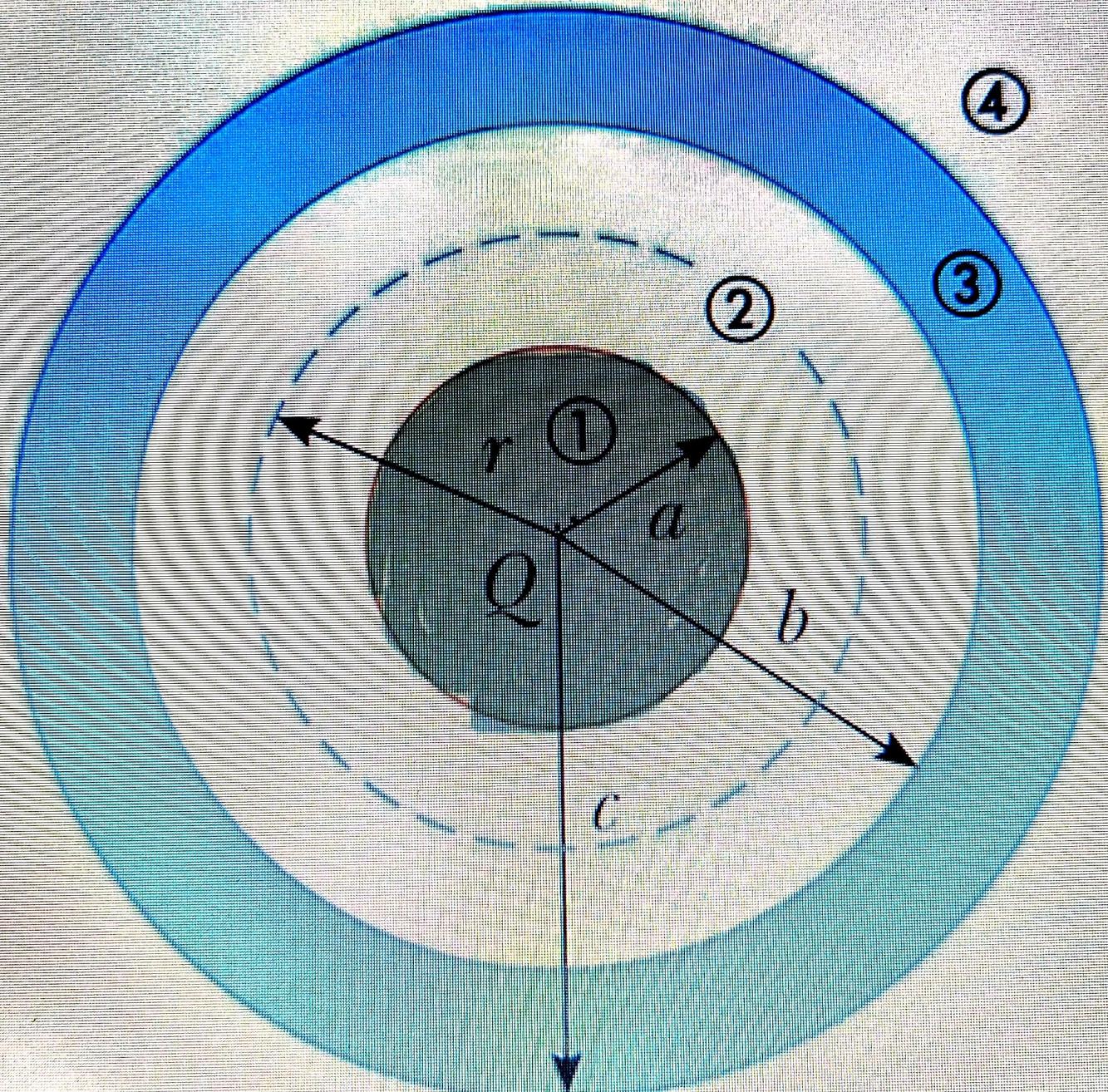


$$E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0 \quad | \quad E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \frac{\sigma}{\epsilon_0} \quad | \quad E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$



Ex. 47

q



Prob. 47 :

A solid conducting sphere of radius 2.00 cm has a charge of 8.00 μC . A conducting spherical shell of inner radius 4.00 cm and outer radius 5.00 cm is concentric with the solid sphere and has a charge of -4.00 μC .

Find the electric field at (a) $r = 1.00 \text{ cm}$ (b) $r = 3.00 \text{ cm}$

(c) $r = 4.50 \text{ cm}$ and (d) $r = 7.00 \text{ cm}$ from the center of this charge configuration

① $r < a \Rightarrow$ (inside the conductor)

$$E_1 = 0$$

② $a < r < b$

$$E_2 = \frac{KQ}{r^2}$$

③ $b < r < c$

$$E_3 = 0 \quad (\text{inside conductor})$$

$r > c$.

④ ~~$E_4 =$~~ $E_4 = \frac{K(Q+q)}{r^2}$

ch-25- Electric potential.

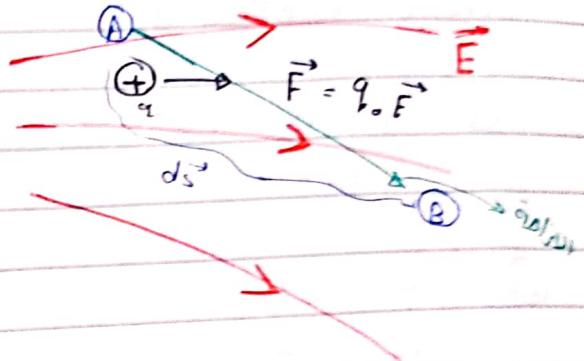
Electric force = conservative force

(work done does not depend on the path)

أيضاً، التغير في طاقة الحقل لا يعتمد على المسار

$$\vec{F} = q \cdot \vec{E}$$

نرسان بـ (B) في (A) في خط مستقيم



$$W = \int_A^B \vec{F} \cdot d\vec{s}$$

الخط
المستقيم

↓ The work done by the field on moving the charge from point A to B

$$W = \int_A^B \vec{F} \cdot d\vec{s} = q \int_A^B \vec{E} \cdot d\vec{s}$$

$$\vec{F} = q \vec{E} \text{ is conservative.} \Rightarrow W = -\Delta U = -(U_B - U_A)$$

$$= U_A - U_B$$

$$W = -\Delta U = q \int_B^A \vec{E} \cdot d\vec{s}$$

$$U_B - U_A = -q \int_A^B \vec{E} \cdot d\vec{s}$$

$$\frac{U_B - U_A}{q} = - \int_A^B \vec{E} \cdot d\vec{s}$$

Example 25.7

A rod of length L located along the x axis has a total charge Q and a uniform linear charge density λ . Find the electric potential of a point p located on the y axis a distance a from the origin.

$$\lambda = \frac{Q}{L}$$

$$dq = \lambda dl = \frac{Q}{L} dx$$

$$r = \sqrt{x^2 + a^2}$$

$$\begin{aligned} V_p &= k \int_{-L}^L \frac{dq}{r} \\ &= k \int_{0}^L \frac{\frac{Q}{L} dx}{\sqrt{x^2 + a^2}} \\ &= \frac{kQ}{L} \int_{0}^L \frac{dx}{\sqrt{x^2 + a^2}} \end{aligned}$$

$$V_p = \frac{kQ}{L} \int \frac{a \sec u \cdot du}{\sqrt{a^2 \sec^2 u}}$$

$$V_p = \frac{kQ}{L} \int \frac{a \sec^2 u \cdot du}{a \sec u}$$

$$V_p = \frac{kQ}{L} \int_{x=0}^{x=L} \sec u \cdot du$$

$$= \frac{kQ}{L} \ln [\sec u + \tan u]$$

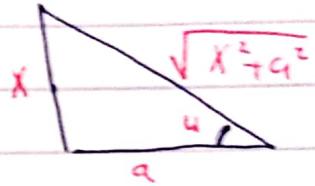
$$= \frac{kQ}{L} \ln \left[\frac{\sqrt{x^2 + a^2}}{a} + \frac{x}{a} \right] \Big|_0^L = \frac{kQ}{L} \ln \left[\frac{\sqrt{L^2 + a^2}}{a} + L \right]$$

$$V_p = \frac{kQ}{L} \ln \left[\frac{\sqrt{L^2 + a^2}}{a} + L \right]$$

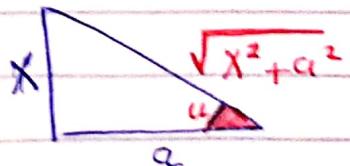
or

$$x = a \tan u \Rightarrow \frac{\tan u}{a} = \frac{x}{a}$$

$$dx = a \sec^2 u \cdot du$$



$$\begin{aligned} x^2 + a^2 &= a^2 \tan^2 u + a^2 \\ &= a^2 (\tan^2 u + 1) \\ &= a^2 \sec^2 u \end{aligned}$$



- ~~ln(1)~~ $2\pi k$

ch- 27 - Currents and resistance.

Electric Currents is the rate of flow of charge through some region of space.

If ΔQ is the charge that passes the A in time Δt then the average current is.

$$\boxed{I_{\text{avg}} = \frac{\Delta Q}{\Delta t}} \quad \text{average current}$$

If the rate at which the charge flows varies then the instantaneous current is

$$I = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

$$\boxed{I = \frac{dQ}{dt}} \quad \text{instantaneous current}$$

SI of current is Ampere (A)

$$1 \text{ A} = 1 \text{ C/s}$$

current could be due to a flow of positive or negative charge.

⇒ Conventional current :- is the current due to the flow of positive charge

⇒ Direction of current is opposite to the direction of flow of electrons.

Microscopic Model of Current

* Consider a section of conductor of uniform cross sectional area A

n :- number of charge carriers per unit volume.

* v_d = drift speed (average speed of charges)

$\Delta x = v_d \Delta t$ = distance the charge moves in time Δt

$$\text{Volume} = (\Delta x) A = V_d A \Delta t$$

$$\text{Total number of charges in volume} = (n) \text{ volume} \\ = n V_d A \Delta t$$

If q is the charge per charge carrier

$$\Delta Q = (q) (n V_d A \Delta t) = n q V_d A \Delta t$$

$$I = \frac{\Delta Q}{\Delta t} = \frac{n q V_d A \Delta t}{\Delta t}$$

$$I = n q V_d A$$

Ex. 26.2) A spherical capacitor consists of a spherical conducting shell of radius b and charge $-Q$ concentric with a smaller conducting sphere of radius a and charge Q . Find the capacitance of this device.

the electric field between the plates
($a < r < b$)

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon}$$

$$E(4\pi r^2) = \frac{Q}{\epsilon}$$

$$E = \frac{1}{4\pi\epsilon} \frac{Q}{r^2} = \frac{kQ}{r^2}$$

$$\Delta V = V_+ - V_- = - \int_{(-)}^{(+)} \vec{E} \cdot d\vec{s}$$

$$= - \int_b^a \frac{kQ}{r^2} dr$$

$$= -kQ \int_b^a \frac{dr}{r^2}$$

$$= -kQ \int_b^a \frac{dr}{r^2}$$

$$= -kQ \left(\frac{-1}{r} \right) \Big|_b^a$$

$$\Delta V = kQ \left(\frac{1}{a} - \frac{1}{b} \right)$$

Ex. 27.1

Copper ($n = 8.48 \times 10^{28}$ electron/m³)

$$q = e = 1.6 \times 10^{-19} \text{ C}$$

$$I = 10 \text{ A}$$

$$A = 3 \times 10^{-6} \text{ m}^2$$

$$I = n q V_d \text{ A}$$

$$V_d = \frac{I}{n q A} = \frac{10 \text{ A}}{(8.48 \times 10^{28})(1.6 \times 10^{-19})(3 \times 10^{-6})}$$

$$V_d = 2.46 \times 10^{-4} \text{ m/s.}$$

Summary

$$I_{\text{avg}} = \frac{\Delta Q}{\Delta t}$$

$$I = \frac{dQ}{dt}$$

$$J = \frac{I}{A}$$

$$J = \sigma E = \text{Ohm's law}$$

$$\Delta V = RI \equiv \text{Ohm's law}$$

$$R = \frac{PL}{A} ; \rho = \frac{L}{\sigma} \rightarrow P \propto$$

Electrical power

The energy gained when a charge ΔQ from point a to point b through a potential difference ΔV

$$\Delta U = (\Delta Q) \Delta V$$

The rate at which the charge ΔQ loses a potential energy in going through the resistor

ex - 2F. 2

$$w = 0.32 \text{ mm} = 3.2 \times 10^{-4} \text{ m}$$

$$f = 1 \times 10^{-6} \text{ N} \cdot \text{m}$$

A) $R = \frac{\rho L}{A}$

$$\frac{R}{L} = \frac{\rho}{A} = \frac{\rho}{\pi r^2} = 3.1 \text{ } \Omega / \text{m}$$

B) $L = 1 \text{ m} , \Delta V = 10 \text{ V}$

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = 3.1 \text{ } \Omega$$

$$I = \frac{\Delta V}{R}$$

$$= \frac{10 \text{ V}}{3.1 \text{ } \Omega} = 3.2 \text{ A}$$

Ex. 27.4

$$\Delta V = 120 \text{ V}, R = 8 \Omega$$

$$I = \frac{\Delta V}{R} = \frac{120 \text{ V}}{8 \Omega} = 15 \text{ A}$$

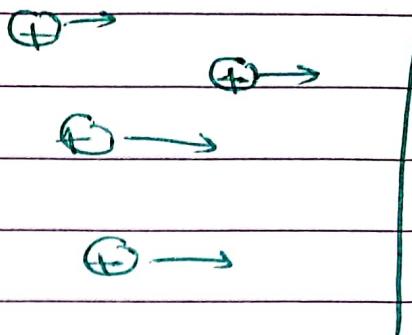
$$P = I \Delta V = 1.8 \text{ kW}$$

or $P = I^2 R = 1.8 \times 10^3 \text{ W} = 1.8 \text{ kW}$

or $P = \frac{(\Delta V)^2}{R} = 1.8 \text{ kW}$

prob - 27.5

$$q = e = 1.6 \times 10^{-19} \text{ C}$$



$$I = 125 \text{ mA} = 125 \times 10^{-3} \text{ A}$$

$$\Delta t = 23 \text{ s}$$

$$I = \frac{\Delta Q}{\Delta t}$$

$$\Delta Q = I(\Delta t)$$

$$N = \frac{\Delta Q}{q} = \frac{I \Delta t}{e} = \frac{(125 \times 10^{-3} \text{ A})(23 \text{ s})}{(1.6 \times 10^{-19}) \text{ C}}$$

$$N = 1.8 \times 10^{16} \text{ proton}$$

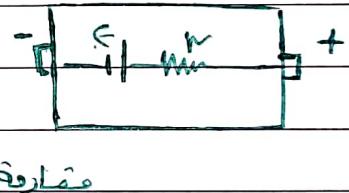
Direct current circuits.

النوع
المصدر

- * Direct current (DC) \equiv has a constant direction
 - * steady current \equiv has constant direction and constant magnitude
 - * Alternating current \equiv (AC) \equiv Alternate their direction
- * The electromotive force (emf or E) \equiv is the maximum possible voltage (potential difference) that (ممكن) the battery can provide between its terminals

E \equiv electromotive Force . (emf)

r : internal resistance



emf or E is the potential difference between the terminals when the battery is in an open circuit

Resistance and Ohm's Law.

The current density J is defined as the current per unit area.

$$J = \frac{I}{A}$$

$$(I \rightarrow \oplus \rightarrow A)$$

$$\text{If } I = nqV_dA \Rightarrow J = \frac{I}{A} = nqV_d$$

SI unit of J A/m^2

J is a vector that has a direction of the flow of positive charge (conventional current)

Ohm's law : The current density in a conductor is proportional to the electric field

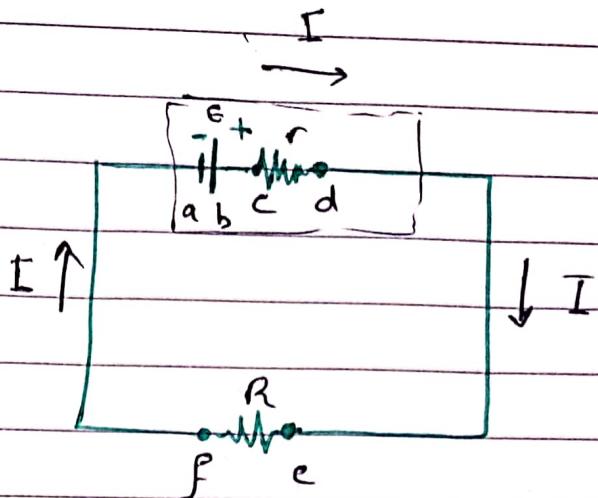
$$\vec{J} \propto \vec{E}$$

$$[\vec{J} = \sigma \vec{E}] \text{ Ohm's law}$$

σ = electrical conductivity of the material
(depends on the material)

comments

- The current flows from the higher to the lower potential
- The positive terminal is at higher potential and the negative terminal is at lower potential
- We consider the wires to have no resistance.



ϵ = emf (electromotive force)

r = internal resistance

R : load resistance

$$V_d - V_a = \text{term}$$

→ The terminal voltage is

$$\Delta V = V_d - V_a = \epsilon - Ir$$

$$\Delta V = V_d - V_a = \epsilon - Ir$$

$$\Delta V = V_d - V_a = IR$$

$$\underbrace{\epsilon - Ir}_{I = \frac{\epsilon}{r+R}} = IR \Rightarrow \epsilon = I(r+R)$$

[Power]

* The total power output of the battery is

$$P = I \epsilon$$

~~dis~~

* The power disipated in the internal resistance (P_r) and in the load resistance P_R are

$$P_r = I^2 r$$

$$P_R = I^2 R$$

$$P = P_r + P_R$$

$$I\epsilon = I^2 R + I^2 r$$

Ex. 28.1

$$\mathcal{E} = 12 \text{ V}, r = 0.05 \text{ } \Omega$$

$$R = 3 \text{ } \Omega$$

A) $I = \frac{\mathcal{E}}{r+R} = \frac{12}{0.05+3} = 3.93 \text{ A}$

$$\Delta V = \mathcal{E} - Ir = 12 - (3.93)(0.05)$$

$$\Delta V = 11.8 \text{ V}$$

or

$$\Delta V = IR = (3.93 \text{ A})(3 \text{ } \Omega) = 11.8 \text{ V}$$

B) $P_r = I^2 r = (3.93 \text{ A})^2 (0.05 \text{ } \Omega) = 0.772 \text{ W}$

$$P_R = I^2 R = (3.93 \text{ A})^2 (3) = 46.3 \text{ W}$$

$$P = I\mathcal{E} = (3.93 \text{ A})(12 \text{ V}) = 47.1 \text{ W}$$

$$P = P_r + P_R = 47.1 \text{ W}$$

Consider a section of conductor at uniform cross section A

$$\Delta V = E L, \quad \sigma = \frac{I}{A}$$

$$J = \sigma E$$

$$\frac{I}{A} = \sigma \frac{\Delta V}{L}$$

$$\Delta V = \left(\frac{L}{\sigma A} \right) I$$

$$\boxed{\Delta V = R I} \quad \text{ohm's law}$$

$$\boxed{R = \frac{L}{\sigma A} = \frac{\rho L}{A}}, \quad R : \text{resistance.}$$

ρ : resistivity

$$\boxed{\rho = \frac{1}{\sigma}}$$

ρ and σ depends on the material R depends on the material and the geometry (size and shape)

SI unit of R is ohm (Ω)

$$1 \Omega = 1 V/A$$

$$I = \left(\frac{1}{R} \right) \Delta V \equiv \text{slope} = \frac{1}{R}$$

I is linear with ΔV for ohmic material.

Ex. 28.2

Find the load resistance R for which the maximum power is delivered to the load resistance in figure.

$$I = \frac{E}{r+R}$$

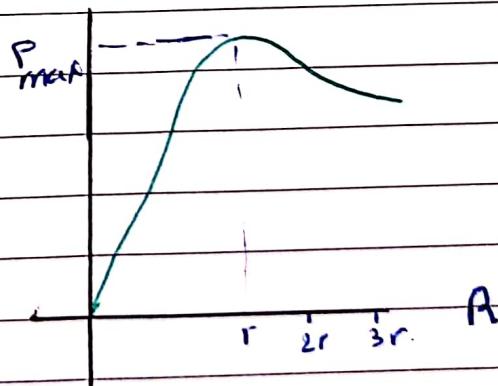
$$P_R = I^2 R$$
$$P_R = \frac{E^2 R}{(r+R)^2}$$

$$\frac{dP_R}{dR} = E^2 \left[\frac{(r+R)^2 - 2R(r+R)}{(r+R)^4} \right] = 0$$

$$\Rightarrow (r+R)^2 - 2R(r+R) = 0$$

$$r - R = 0$$
$$R = r$$
 maximum.

$$P_{R\max} = \frac{E^2 r}{(r+r)^2} = E^2 / 4r$$



Resistors in series and parallel

D Resistors in series.

$$I_1 = I_2 = I \text{ (current is the same)}$$

$$\Delta V = \Delta V_1 + \Delta V_2$$
$$IR_{eq} = IR_1 + IR_2$$

$$R_{eq} = R_1 + R_2$$

For more than two resistors connected in series.

$$R_{eq} = R_1 + R_2 + R_3$$

R_{eq} is greater than the greatest resistor in the group.

2) Resistors in parallel

$$\Delta V = \Delta V_1 = \Delta V_2$$

The current at the junction is

$$I = I_1 + I_2$$

$$\frac{\Delta V}{R_{eq}} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2}$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\text{or } R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

For more than two resistors connected in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

resistors in parallel

R_{eq} in parallel is smaller than the smallest resistor in the group

Ex. 28.4

Four resistors are connected as shown

A) find the equivalent resistance between points a and c

A) R_1, R_2 , parallel

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2} = 2 \Omega$$

R_3, R_4 in series

~~$$R_{34} = R_3 + R_4 = 12 \Omega$$~~

R_{12}, R_{34} in series

$$R_{eq} = R_{12} + R_{34} = 14 \Omega$$

B) what is the current in each resistor if a potential difference of 42 V is maintained between a and c?

$$\Delta V = 42 V$$

$$I = \frac{\Delta V}{R_{eq}} = \frac{42 V}{14 \Omega} = 3 A$$

The current passing R_3 and R_4 is the same current
($I = 3 A$)

$$I_3 = I_4 = 3 A$$

$$\Delta V_{bc} = IR_{12} = (3 A)(2 \Omega) = 6 V$$

$$I_1 = \frac{\Delta V_{bc}}{R_1} = \frac{6 V}{6 \Omega} = 1 A$$

$$I_2 = \frac{\Delta V_{bc}}{R_2} = \frac{6 V}{3 \Omega} = 2 A$$

$$I = I_1 + I_2 \Rightarrow 3 A = 1 A + 2 A$$

Ex. 28. 5) Three resistors are connected in parallel as shown. A potential difference of 18.0 V is maintained between points a and b.

A) Calculate the equivalent resistance of the circuit.

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{6} + \frac{1}{9} = \frac{11}{18}$$

$$R_{eq} = \frac{18}{11} \Omega = 1.64 \Omega$$

B) Find the current in each resistor.

$$I_1 = \frac{\Delta V}{R_1} = \frac{18V}{3\Omega} = 6A$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18V}{6\Omega} = 3A$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18V}{9\Omega} = 2A$$

$$I = I_1 + I_2 + I_3 = 11A$$

$$I = \frac{\Delta V}{R_{eq}} = \frac{18V}{(8/11)} = 11A$$

c) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.

$$P_1 = I_1^2 R_1 = 108W \quad P_2 = I_2^2 R_2 = 54W$$

$$P_3 = I_3^2 R_3 = 36W$$

$$P = P_1 + P_2 + P_3 = 198W$$

$$\text{or } P = I \Delta V = (11A)(18V) = 198W$$

(Kirchhoff's Rules)

- # In some circuits, resistors can not be reduced to a ~~single~~ single equivalent resistance -
- Therefore the Kirchhoff's rules can be applied

1) Junction Rule.

It is a consequence of conservation of charge

- The sum of the current at the junction must equal zero.

$$\sum I = 0$$

Junction

Convention of current entering the junction is positive
(+) current leaving the junction is negative.

Comments

a) If the resistor is traversed in direction of current

$$\Delta V = V_b - V_a = -IR$$

b) If the resistor is traversed opposite to the current I

$$\Delta V = V_b - V_a = IR$$

c) If the emf is traversed from the negative to the positive terminal

$$\Delta V = V_b - V_a = +\epsilon$$

d) If the emf is traversed from (+) to (-)

$$\Delta V = V_b - V_a = -\epsilon$$

RC circuit

RC - circuit contains a series combination of a resistor and capacitor

I) charging a capacitor

$$\sum \Delta V = 0$$

$$\mathcal{E} - \frac{q}{C} - IR = 0 \quad \text{--- (1)}$$

Two extreme cases &

1) At $t = 0$, $q = 0 \Rightarrow$

$$\mathcal{E} - IR = 0$$

$$I_i = \frac{\mathcal{E}}{R} \quad \text{Initial (maximum) current}$$

In the intermediate case

$$\mathcal{E} - \frac{q}{C} - IR = 0 \quad ; \quad I = \frac{dq}{dt}$$

$$\mathcal{E} - \frac{q}{C} - R \frac{dq}{dt} = 0$$

$$\frac{dq}{dt} = - \frac{q - C\mathcal{E}}{RC}$$

$$\int \frac{dq}{q - C\varepsilon} = -\frac{1}{RC} \int dt$$

$$\ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = -\frac{t}{RC}$$

$$\frac{q - C\varepsilon}{-C\varepsilon} = e^{-t/RC}$$

$$q(t) = C\varepsilon (1 - e^{-t/RC}) = Q_{\max} (1 - e^{-t/\tau})$$

$$\boxed{\tau = RC} = \text{time constant of RC-circuit}$$

The current is

$$I = \frac{dq}{dt}$$

$$I = \frac{\varepsilon}{R} e^{-t/RC} = I_i e^{-t/\tau}$$

The voltage on the capacitor

$$\frac{q}{C} = \frac{C\varepsilon}{C} (1 - e^{-t/RC})$$

$$V_C = \varepsilon (1 - e^{-t/RC}) = \varepsilon (1 - e^{-t/\tau})$$

Prob 28.1 A ~~leaking~~ battery has an emf of 15.0 V

The terminal voltage of the battery is 11.6 V when it is delivering 20.0 W of power to an external load resistor R (a) what is the value of R ? (b) what is the internal resistance of the battery?

$$\mathcal{E} = 15 \text{ V}$$

$$\Delta V = V_d - V_a = 11.6 \text{ V} \text{ when } P_B = 20 \text{ W}$$

$$\text{A) } P_R = \frac{(\Delta V)^2}{R}$$

$$R = \frac{(\Delta V)^2}{P_R} = \frac{(11.6 \text{ V})^2}{20 \text{ W}} = 6.73 \Omega$$

$$I = \frac{\Delta V}{R} = \frac{11.6 \text{ V}}{6.73 \Omega} = 1.72 \text{ A}$$

$$\Delta V = \mathcal{E} - Ir$$

$$r = \frac{\mathcal{E} - \Delta V}{I} = \frac{15 \text{ V} - 11.6 \text{ V}}{1.72 \Omega} = 1.97 \Omega$$

B)

$$R_{12} = R_1 + R_2 = 9 \Omega \text{ (Series)}$$

$$R_{34} = R_3 + R_4 = 6 \Omega \text{ (Series)}$$

$$R_{eq} = \frac{R_{12} R_{34}}{R_{12} + R_{34}} = 3.6 \Omega$$

$$\frac{q}{C} = \frac{Q_i}{C} e^{-t/RC}$$

$$V_c = \Delta V_c e^{-t/RC_{eq}}$$

$$V_c = \frac{1}{10} \Delta V_c$$

$$\frac{1}{10} \Delta V_i = \Delta V_i e^{-t/RC}$$

$$= -\ln 10 = \frac{-t}{RC}$$

$$t = (R_{eq} C) \ln 10$$

$$= (3.6 \Omega) (1 \times 10^{-6} F) \ln (10)$$

$$t = 8.29 \mu s$$

Prob. 28.43 The circuit in Figure has been connected for a long time (a) What is the potential difference across the capacitor? (b) If the battery is disconnected from the circuit, over what time interval does the capacitor discharge to one-tenth its initial voltage?

For long time \Rightarrow the current in the branch of the capacitor is zero because it is max charged

$$\Rightarrow R_1, R_3 \text{ in series} \Rightarrow R_{13} = R_1 + R_3 = 5 \Omega$$

$$R_2, R_4 \text{ in series} \Rightarrow R_{24} = R_2 + R_4 = 10 \Omega$$

$$I_2 = \frac{\epsilon}{R_{13}} = \frac{10 \text{ V}}{5 \Omega} = 2 \text{ A}$$

$$I_3 = \frac{\epsilon}{R_{24}} = \frac{10 \text{ V}}{10 \Omega} = 1 \text{ A}$$

$$V_a - V_b = I_3 R_2 - I_2 R_1 \\ = (1)(8) - (2)(1) =$$

$$\boxed{V_a - V_b = \Delta V_1 = 6 \text{ V}}$$

\equiv Potential across the capacitor

$$\Delta V_1 = 6 \text{ V}$$

Prob. 28.9] Consider the circuit shown in Fig. Find
 (a) The current in the $20\text{-}\Omega$ resistor and (b) The
 potential difference between points a and b.

R_4, R_5 series

$$R_{45} = R_4 + R_5 = 20\text{ }\Omega + 5\text{ }\Omega = 25\text{ }\Omega$$

R_2, R_3, R_{45} : parallel

$$\frac{1}{R_{2345}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_{45}} = \frac{1}{10\text{ }\Omega} + \frac{1}{5\text{ }\Omega} + \frac{1}{25\text{ }\Omega} = \frac{17}{50\text{ }\Omega}$$

$$R_{2345} = \frac{50}{17}\text{ }\Omega = 2.94\text{ }\Omega$$

R_1, R_{2345} : series.

$$R_{\text{eq}} = R_1 + R_{2345} = 12.94\text{ }\Omega$$

$$I = \frac{\epsilon}{R_{\text{eq}}} = \frac{20\text{ V}}{12.94} = 1.93\text{ A}$$

$$\Delta U_{ab} = I R_{2345} = (1.93\text{ A})(2.94\text{ }\Omega)$$

$$\Delta U_{ab} = 5.68\text{ V}$$

$$I_4 = \frac{\Delta U_{ab}}{R_{45}} = \frac{5.68\text{ V}}{25\text{ }\Omega} = 0.227\text{ A} = 227\text{ mA}$$

$$I_2 = \frac{\Delta U_{ab}}{R_2} = \frac{5.68\text{ V}}{10\text{ }\Omega} = 0.568\text{ A}$$

$$I_3 = \frac{\Delta U_{ab}}{R_3} = \frac{5.68\text{ V}}{5} = 1.136\text{ A}$$

Prob - 28.14)

- (a) when the switch S in the circuit of figure is closed, will the equivalent resistance between points a and b increase or decrease? State your reasoning.
- (b) Assume the equivalent resistance drops by 50.0%, when the switch is closed. Determine the value of R .

A) If S is closed

R_1, R_2 : parallel

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2} = 9 \Omega$$

R_3, R_4 : parallel

$$R_{34} = \frac{R_3 R_4}{R_3 + R_4} = 9 \Omega$$

R_1, R_{12}, R_{34} : series

$$\begin{aligned} R_{eq} &= R + R_{12} + R_{34} \\ &= R + 9 \Omega + 9 \Omega \\ &= R + 18 \Omega \end{aligned}$$

If S is open R_1, R_3 : series $R_{13} = R_1 + R_3 = 100 \Omega$

R_2, R_4 : series.

$$R_{24} = R_2 + R_4 = 100 \Omega$$

R_{13}, R_{24} : parallel

$$R_{1234} = \frac{R_{13} R_{24}}{R_{13} + R_{24}} = 50 \Omega$$

R, R_{1234} : series

$$R_{eqc} = R + R_{1234} = R + 50 \Omega$$

If S is closed the R_{eq} decreases

$$R_{eq} (S \text{ closed}) = 0.5 (R_{eq} (S \text{ open}))$$

$$R + 18 \Omega = 0.5 (R + 50 \Omega)$$

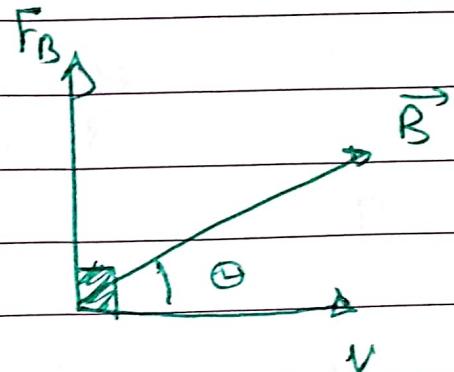
$$R + 18 = 0.5 R + 25$$

$$R = 14 \Omega$$

Q1 The magnetic force on a moving charge q is

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

$$F_B = q v B \sin \theta$$



Ex. 29.1]

$$q = -e = -1.6 \times 10^{-19} \text{ C}$$

$$v = 8 \times 10^6 \text{ m/s}$$

$$\beta = 0.025 \text{ T}$$

$$\theta = 60^\circ$$

$$F_B = |q| v B \sin \theta$$

$$= (1.6 \times 10^{-19}) (8 \times 10^6) (0.025) \sin 60$$

$$= 2.8 \times 10^{-4} \text{ N}$$

Motion of a charged particle in a uniform magnetic field

$$\vec{B} = \text{Const (uniform)}$$
$$\vec{U} \perp \vec{B}$$

It is clear that

1. The path followed by the particle is circular.

2. The magnetic force (\vec{F}_3) is toward the center of the circle.

3. \vec{F}_3 causes centripetal acceleration ($a_c = \frac{v^2}{r}$)

4. \vec{F}_3 changes the direction of \vec{v} only

$\Rightarrow \vec{F}_3$ does not change the magnitude of \vec{v}

$\Rightarrow F_3$ does not change the kinetic energy.

\Rightarrow magnetic force (\vec{F}_3) does no work.

$$\vec{v} \perp \vec{B} \Rightarrow \theta = \frac{\pi}{2}$$

$$F_B = qvB \sin\left(\frac{\pi}{2}\right) = qvB$$

$$F_B = qvB = ma_c$$

$$qvB = \frac{mv^2}{r}$$

$$r = \frac{mv}{qB} \quad \text{The radius of the path.}$$

Angular Speed (ω) :

$$s = \theta r$$

$$\frac{ds}{dt} = \frac{d\theta}{dt} r$$

$$v = \omega r \rightarrow \omega : \text{angular speed}$$

$$\boxed{\omega = \frac{v}{r}}$$

$$\omega = \frac{v}{r} = \frac{v}{m/qB} = \frac{qB}{m}$$

$$\boxed{\omega = \frac{v}{r} = \frac{qB}{m}} \quad \text{Angular Speed.}$$

The period (T) : Time for one revolution

$$v = \frac{2\pi r}{T} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$

period of
the motion.

The frequency (f) : Number of revolutions in one second

$$f = \frac{1}{T} = \frac{v}{2\pi r} = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}$$

ch-29. magnetic fields.

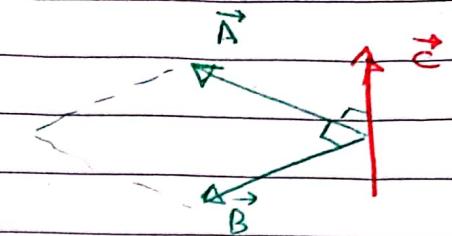
Cross (vector) product

$$\vec{C} = \vec{A} \times \vec{B}$$

$$|\vec{C}| = |\vec{A}| |\vec{B}| \sin \theta$$

$$C = AB \sin \theta$$

Direction of \vec{C} is determined by the right hand rule
 $\vec{C} \perp \vec{A}$, and $\vec{C} \perp \vec{B}$



$$\cdot \vec{B} \times \vec{A} = -\vec{A} \times \vec{B} \quad (\text{not commutative})$$

$$\cdot \vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

$$\cdot \frac{d}{dt} (\vec{A} \times \vec{B}) = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d\vec{B}}{dt}$$

If $\theta = 0$ ($\vec{A} \perp \vec{B}$) or π ($\vec{A} \perp \vec{B}$) then

$$\vec{A} \times \vec{B} = 0 \quad (\theta = 0, \pi)$$

$$\Rightarrow \vec{A} \times \vec{A} = 0$$

$$\text{If } \theta = \frac{\pi}{2} \quad (\vec{A} \perp \vec{B}) \Rightarrow |\vec{A} \times \vec{B}| = AB$$

unit Vectors.

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{i} \times \hat{j} = \hat{k}$$

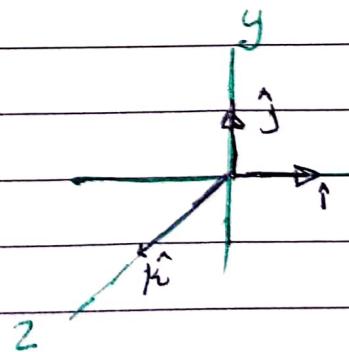
$$\hat{k} \times \hat{i} = -\hat{j}$$

$$|\hat{i} \times \hat{j}| = (1)(1) \sin \frac{\pi}{2} = 1$$

$$\hat{i} \times \hat{k} = -\hat{j}$$

$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$



In terms of unit vectors

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - A_z B_y) \hat{i} - (A_x B_z - A_z B_x) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$

ch- 29- ~~magnitude~~ magnitude field

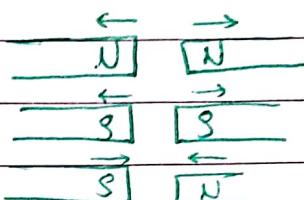
* Every magnet, regardless of its shape, has two poles

- 1) North pole (N)
- 2) South pole(s)

N S

* Magnetic poles exert forces on one another

- Like poles repel each other
- Unlike poles attract each other



* If a bar magnet is suspended as it can move freely, it will rotate such that the north pole points toward the north geographic pole.

* Magnetic poles are always found in pairs (N S)

* The region of space surrounding a moving charge (current) contains a magnetic field.

* Region of space surrounding magnetic materials (made up permanent magnets) contains a magnetic field

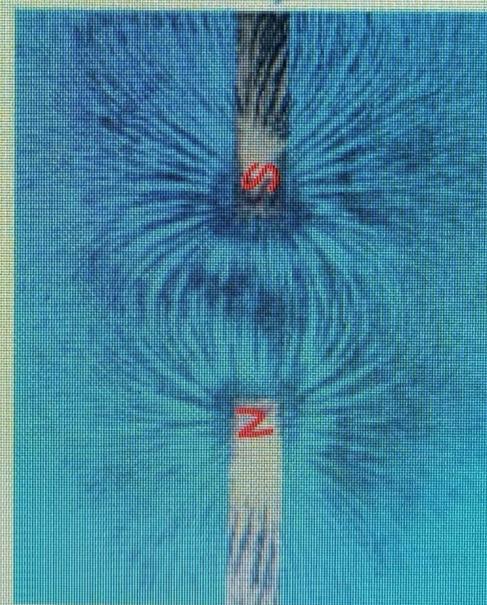
\vec{B} = magnetic field.

Magnetic field lines

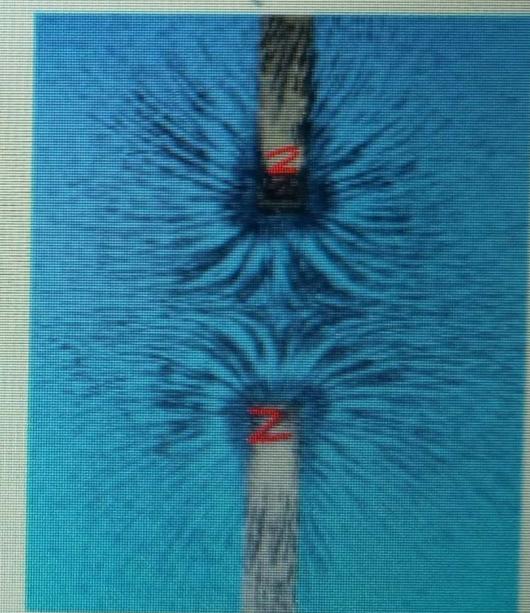
Magnetic field pattern surrounding a bar magnet



Magnetic field pattern between *opposite* poles (N-S) of two bar magnets



Magnetic field pattern between *like* poles (N-N) of two bar magnets



Properties of field lines.

1. The lines outside the magnet point from the north pole (N) to the south pole (S)
2. The magnetic field (\vec{B}) is tangent to the field lines.
3. Number of field lines per unit (intensity of lines) is proportional to the magnitude of the field, $|\vec{B}|$.
4. The field lines form closed loops.

SI unit of \vec{B} is Tesla (T)

$$1 \text{ T} = 1 \text{ wb/m}^2 = 1 \text{ N/A.s}$$

The cgs unit of \vec{B} is Gause (G)

$$1 \text{ G} = 10^{-4} \text{ T}$$

Magnetic force on a moving charge.

$\vec{F}_3 \equiv$ magnet force.

It was found

- $F_B \propto q$
- $F_B \propto v$, ($F_B = 0$ if $v = 0$)
- $F_B \propto B$
- $F_B \propto \sin \theta$ (If $\theta = 0$ or π then $F_B = 0$)
 $\vec{F}_B \perp \vec{v}$, $\vec{F}_B \perp \vec{B}$

The properties are summarised in

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

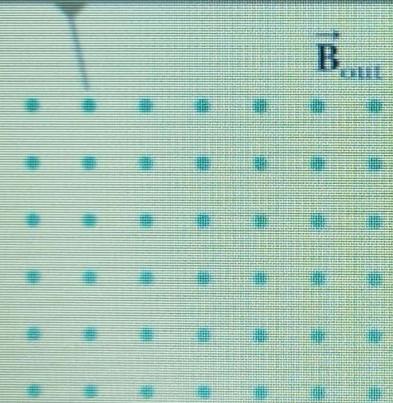
$$F_B = q v B \sin \theta$$

- The magnetic force on a negative charge is opposite to the magnetic force on a positive charge.

$$\vec{F}_3 = q \vec{v} \times \vec{B}$$

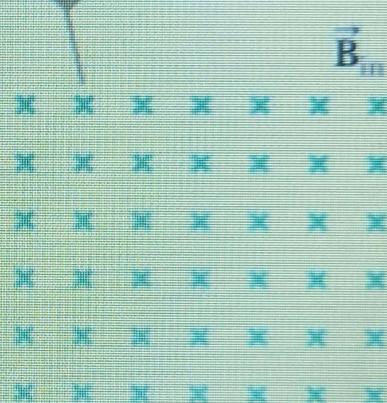
Convention

Magnetic field lines coming out of the paper are indicated by dots, representing the tips of arrows coming outward.



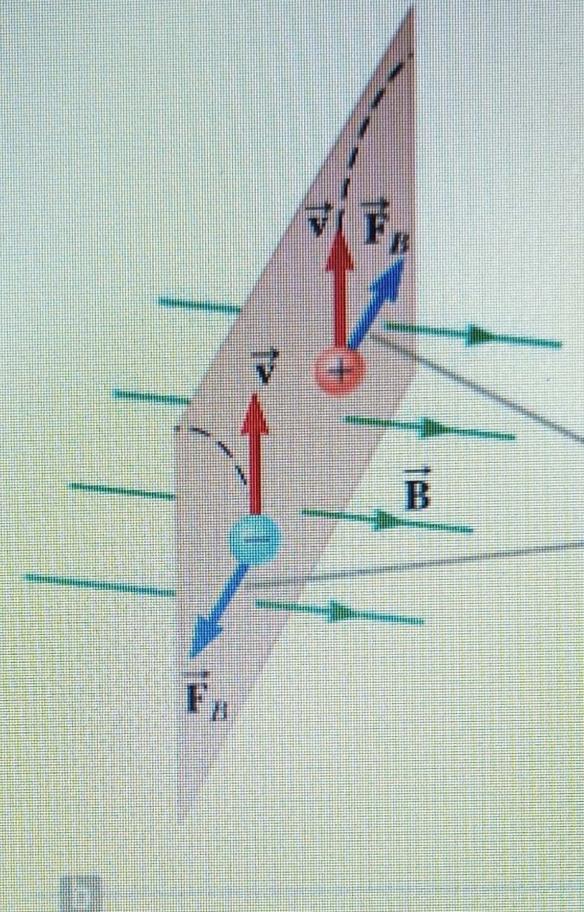
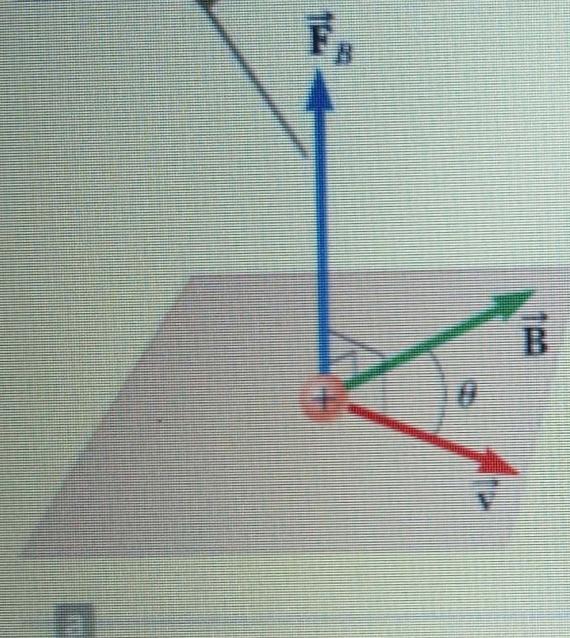
\vec{B}_{out} (out of the page)

Magnetic field lines going into the paper are indicated by crosses, representing the feathers of arrows going inward.



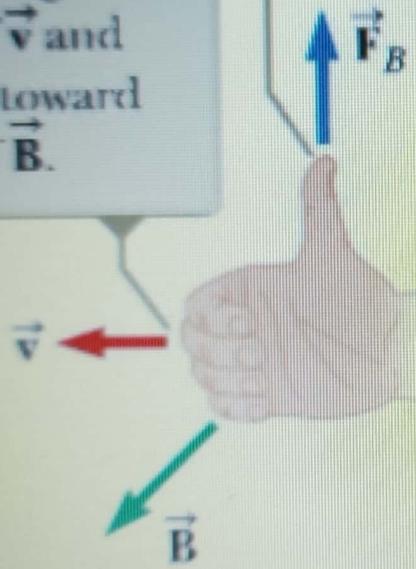
(\vec{B}_{in} into the page)

The magnetic force is perpendicular to both \vec{v} and \vec{B} .



The magnetic forces on oppositely charged particles moving at the same velocity in a magnetic field are in opposite directions.

(1) Point your fingers in the direction of \vec{v} and then curl them toward the direction of \vec{B} .



(2) Your upright thumb shows the direction of the magnetic force on a positive particle.

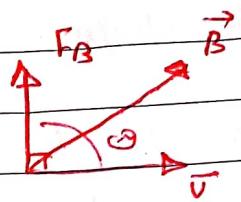


(2) The magnetic force on a positive particle is in the direction you would push with your palm.

The magnetic force on a moving charge q is

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

$$F_B = q v B \sin \theta \quad \textcircled{1}$$



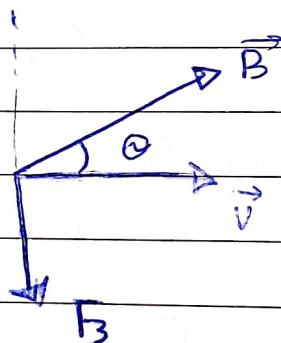
Ex. 29.1 \Rightarrow An electron in an old-style television picture tube moves toward the front of the tube with a speed of $8.0 \times 10^6 \text{ m/s}$ along the x axis. Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude 0.025 T , directed at an angle of 60° to the x -axis and lying in the xy plane. Calculate the magnetic force on the electron.

$$q = -e = -1.6 \times 10^{-19} \text{ C}$$

$$v = 8 \times 10^6 \text{ m/s}$$

$$B = 0.025 \text{ T}$$

$$\theta = 60^\circ$$



$$F_B = |q| v B \sin \theta$$

$$= (1.6 \times 10^{-19}) (8 \times 10^6) (0.025) \sin 60^\circ$$

$$= 2.8 \times 10^{-4} \text{ N}$$

Motion of a charged particle in a uniform magnetic field

$$\vec{B} = \text{const (uniform)}$$

$$\vec{v} \perp \vec{B}$$

It is clear that :

1. The path followed by the particle is circular
2. The magnetic force (\vec{F}_B) is toward the center of the circle
3. \vec{F}_B causes centripetal acceleration ($a_c = \frac{v^2}{r}$)
4. \vec{F}_B changes the direction of \vec{v} only.

$\Rightarrow \vec{F}_B$ does not change the magnitude of \vec{v}

$\Rightarrow \vec{F}_B$ does not change the kinetic energy

\Rightarrow Magnetic force (\vec{F}_B) does no work.

$$\vec{v} \perp \vec{B} \Rightarrow \theta = \frac{\pi}{2}$$

$$F_B = qvB \sin\left(\frac{\pi}{2}\right) = qvB$$

$$F_B = qvB = mac$$

$$qvB = m \frac{v^2}{r}$$

$$\boxed{r = \frac{mv}{qB}} \quad \text{The radius of the path}$$

Angular Speed (ω)

$$s = \omega r$$

$$\frac{ds}{dt} = \frac{d\theta}{dt} r$$

$$v = wr \quad ; \quad \omega : \text{angular speed}$$

$$\boxed{\omega = \frac{v}{r}}$$

$$\omega = \frac{v}{r} = \frac{v}{\cancel{mv/qB}} = \frac{qB}{m}$$

$$\boxed{\omega = \frac{v}{r} = \frac{qB}{m}} \quad \text{angular speed.}$$

* The period (T): Time for one revolution

$$v = \frac{2\pi r}{T}$$

$$\boxed{T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}}$$

period of 1Hz
motion.

The Frequency (F) :- number of revolutions in one second.

$$\boxed{F = \frac{1}{T} = \frac{v}{2\pi r} = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}}$$

Frequency
($1\text{Hz} \equiv \text{s}^{-1}$)

Ex 29.2: A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35 T magnetic field perpendicular to the velocity of the proton. Find the speed of the proton.

$$\text{Proton} \Rightarrow q = 1.6 \times 10^{-19} \text{ C}, m = 1.67 \times 10^{-27} \text{ kg}$$

$$r = 14 \text{ cm} = 0.14 \text{ m}$$

$$B = 0.35 \text{ T} \text{ (uniform)}$$

$$\vec{v} \perp \vec{B} \quad (\theta = \pi/2)$$

Speed : $F_B = ma_c$

$$\cancel{qvB = m \frac{v^2}{r}}$$

$$v = \frac{qBr}{m} = 4.7 \times 10^6 \text{ m/s}$$

Find the angular speed

$$\omega = \frac{v}{r} = 3.36 \times 10^7 \text{ rad/s. or}$$

$$\omega = \frac{qB}{m} = 3.36 \times 10^7 \text{ rad/s}$$

Find the period of the motion

$$T = \frac{2\pi r}{v} = 1.9 \times 10^{-7} \text{ s. or}$$

$$T = \frac{2\pi}{\omega} = 1.9 \times 10^{-7} \text{ s. or}$$

$$T = \frac{2\pi m}{qB} = 1.9 \times 10^{-7} \text{ s.}$$

Ex. 29.3

$$\text{electron} \therefore q = -1.6 \times 10^{-19} \text{ C}$$
$$m = 9.11 \times 10^{-31} \text{ kg.}$$

$$v_i = 0, \quad v_f = v$$

$$\Delta V = 350 \text{ V}$$

$$r = 7.5 \text{ cm} = 0.075 \text{ m.}$$

A) $\Delta K + \Delta U = 0$

$$\left(\frac{1}{2}mv^2 - \frac{1}{2}mu_i^2 \right) + q\Delta U = 0$$

$$v = \sqrt{\frac{-2q\Delta V}{m}} = 1.11 \times 10^7 \text{ m/s.}$$

$$F_B = m a_c$$

$$qvB = \frac{mv^2}{r}$$

$$B = \frac{mv}{qr} = 8.4 \times 10^{-4} \text{ T}$$

B) $\omega = \frac{v}{r} = 1.5 \times 10^5 \text{ rad/s.}$

$$\text{or } \omega = \frac{qB}{m} = 1.5 \times 10^5 \text{ rad/s}$$

Calculate PL period (T)

$$T = \frac{2\pi}{\omega} = 4.18 \times 10^{-5} \text{ s.}$$

or

$$T = \frac{2\pi r}{v} = 4.18 \times 10^{-5} \text{ s.}$$

or

$$T = \frac{2\pi m}{qB} = 4.18 \times 10^{-5} \text{ s}$$

Magnetic force on a current carrying conductor

- A magnetic force is ~~is~~ exerted on a ~~is~~ current carrying conductor when placed in a magnetic field.
- Direction of the magnetic force is determined by ~~the~~ right-hand-rule.
- The magnetic force on a single charge moving in the wire with the drift speed, v_d , in a magnetic \vec{B} is

$$\vec{F}_{\text{single-charge}} = q \vec{v}_d \times \vec{B}$$

The total force is

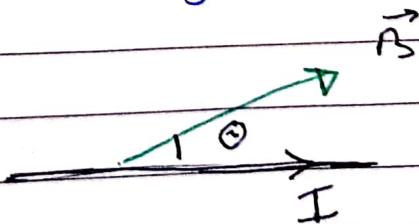
$$\vec{F}_B = (\vec{F}_{\text{single-charge}}) (\text{number of charges})$$
$$= (q \vec{v}_d \times \vec{B}) (n A L)$$

$$\vec{F}_B = \left[\underbrace{(n q \vec{v}_d A) L}_{I} \right] \times \vec{B}$$

$$\vec{F}_B = (I L) \times \vec{B}$$

$\vec{F}_B = I L \times \vec{B}$ The force on the straight wire.

$$|F_B| = I L B \sin \theta$$

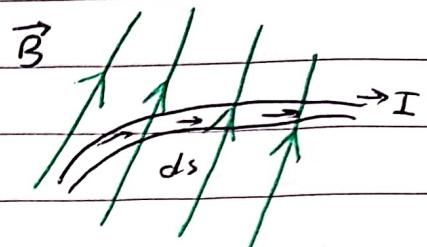


for an irregular shaped wire we consider a ~~small~~ segment \vec{ds}

$$d\vec{F}_B = I \vec{ds} \times \vec{B} \equiv \text{Force on segment } \vec{ds}$$

The total force is the sum over all segment.

$$\vec{F}_B = I \int \vec{ds} \times \vec{B}$$



Ex. 29.4 The Force on the straight motion

$$\vec{F}_1 = I \vec{L} \times \vec{B}$$

$$L = 2R, \Theta = \frac{\pi}{2}$$

\vec{F}_B is out the page ($+\hat{k}$)

$$\begin{aligned} F_1 &= ILB \sin \Theta \hat{k} \\ &= I(2R)B \sin \frac{\pi}{2} \hat{k} \end{aligned}$$

$$F_1 = 2IRB \hat{k}$$

The force on the current portion

$$\begin{aligned} \vec{F}_2 &= I \int \vec{ds} \times \vec{B} + \vec{ds} \times \vec{B} = (ds) B \sin \Theta (-\hat{k}) \\ &= -I \int B (ds) \sin \Theta \hat{k} & &= -B (ds) \sin \Theta \hat{k} \\ &= -I \int_0^\pi B (R d\Theta) \sin \Theta \hat{k} & ds = R d\Theta \\ &= -IRB \int_0^\pi \sin \Theta d\Theta \hat{k} & \underbrace{\pi - 0}_{=2} \end{aligned}$$

$$F_2 = -2IRB \hat{k}$$

The force on the current portion.

we have

$$\vec{F}_1 = 2IRB \hat{k}$$

$$\vec{F}_2 = -2IRB \hat{i}$$

Two Conclusions

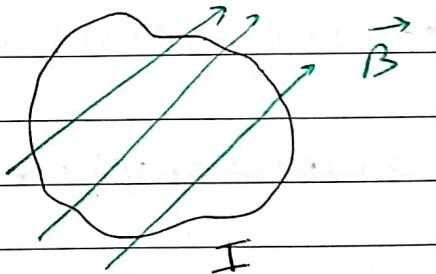
$$\vec{F}_1 = -\vec{F}_2 \Rightarrow \vec{F}_1 + \vec{F}_2 = 0.$$

Conclusion

The force on the curved portion is equal in magnitude and opposite direction to the force on the straight portion.

The magnetic force on a closed current loop in a uniform magnetic field is zero.

$$\sum \vec{F} = 0$$

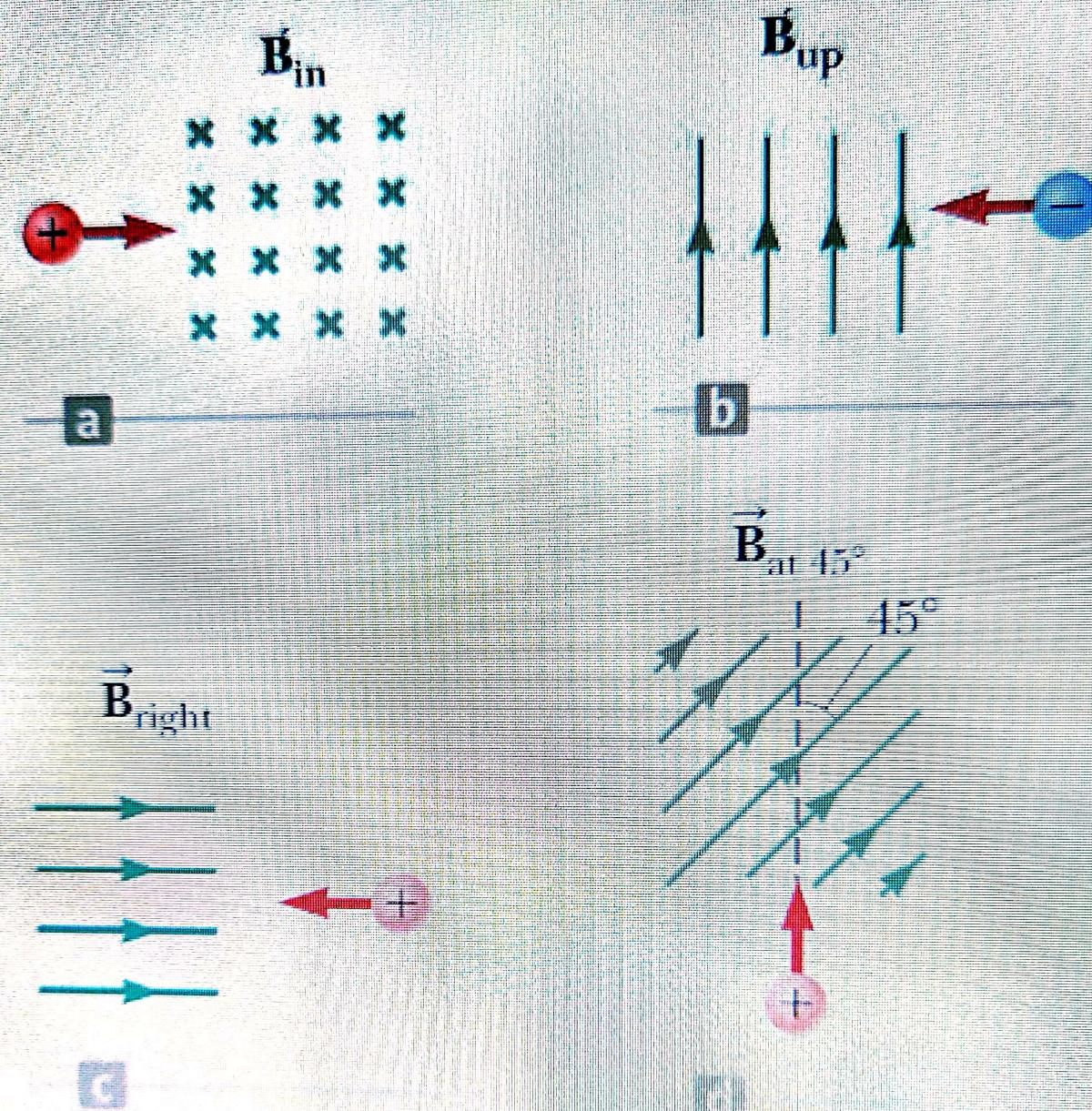


$$\vec{F}_1 + \vec{F}_2 = 0$$

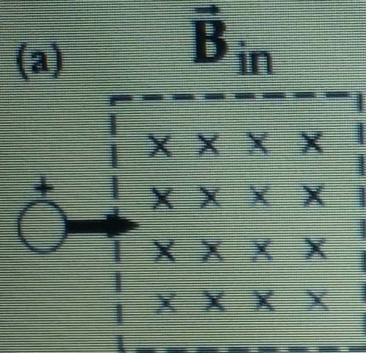
$$\vec{F}_1 = -\vec{F}_2 = -IL \times \vec{B}$$

In general the force on the curved portion in a uniform magnetic field is equal to that on the straight portion.

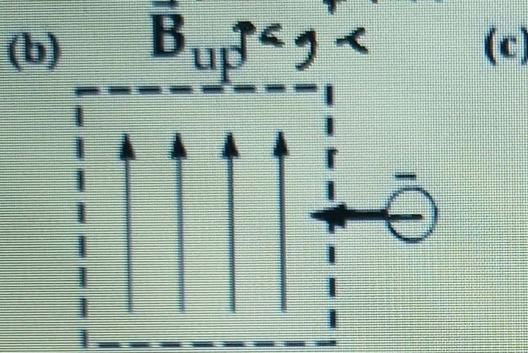
Prob. 29.2



deflection is zero

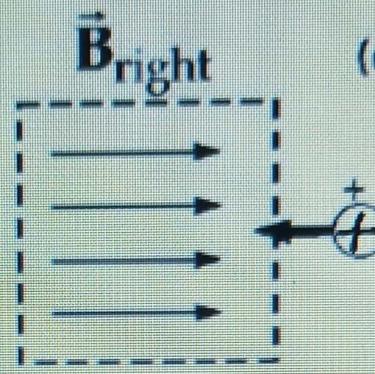


deflection is out of the page



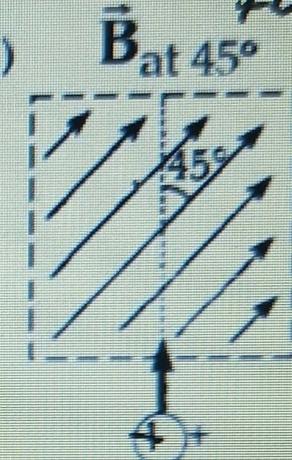
(c)

no deflection



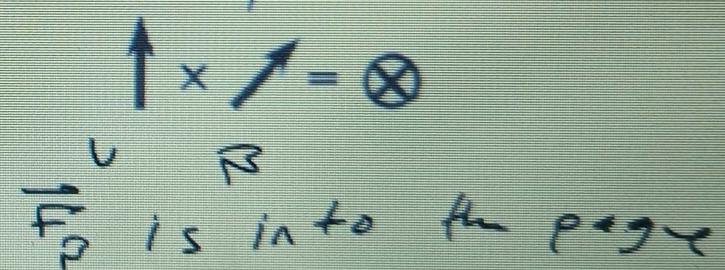
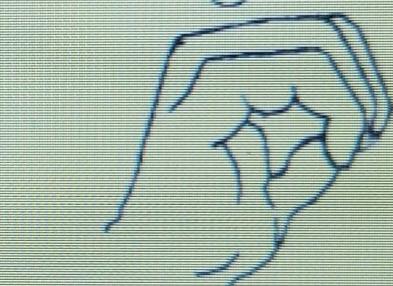
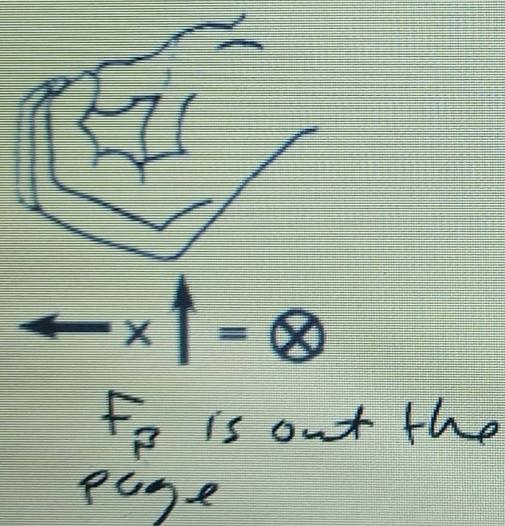
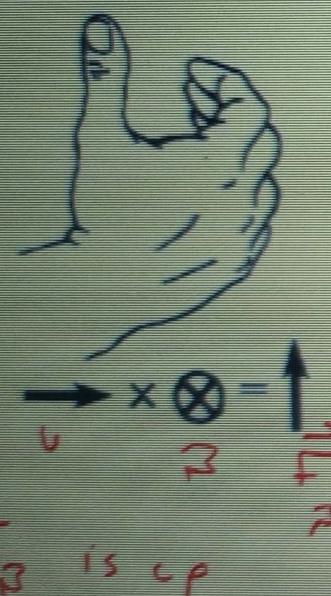
(d)

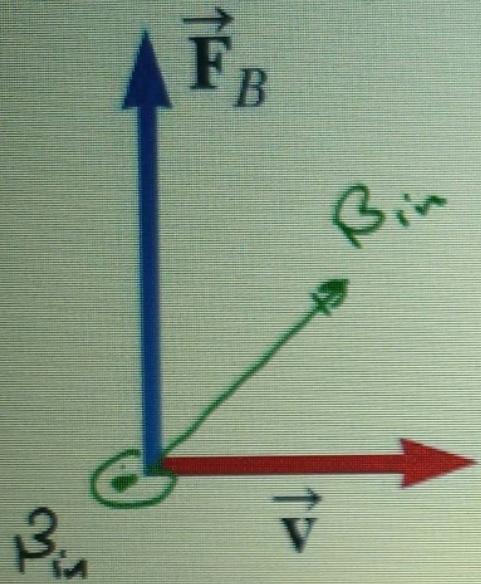
deflection is into the page



$$\theta = \pi$$

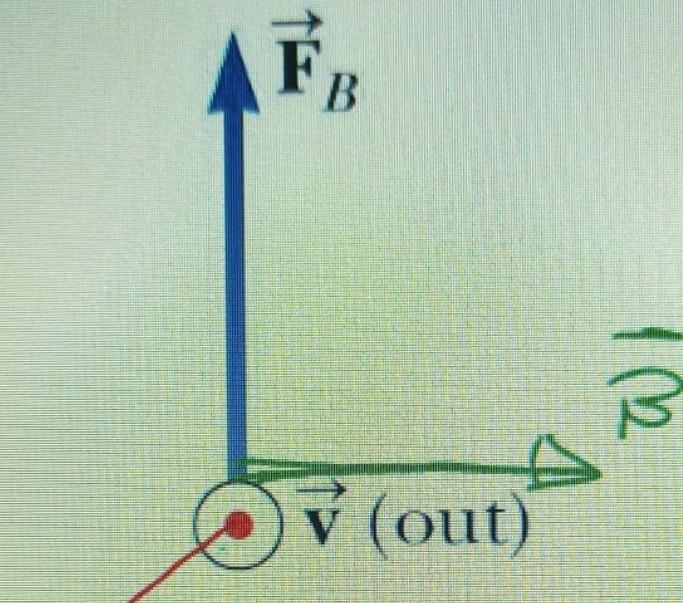
$$F_B = 0$$





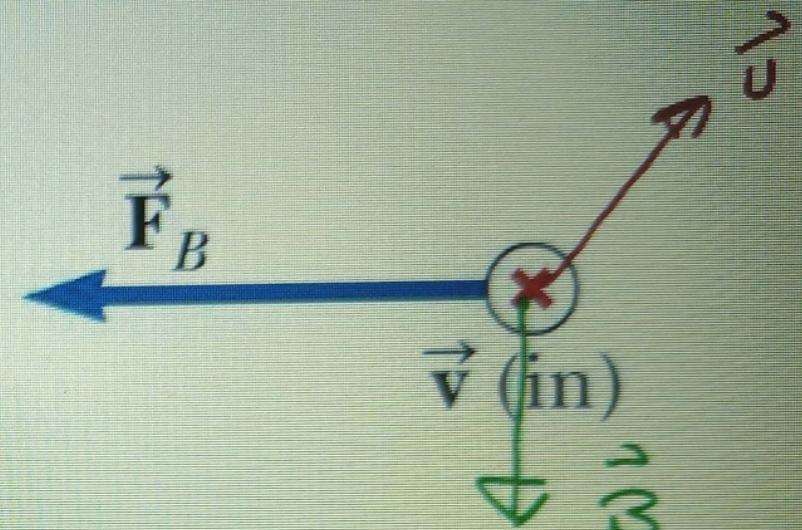
a

\vec{B} is into the page



b

\vec{v}_{out} \vec{B} is to the right



c

\vec{B} is to the bottom

Prob. 29.3

Prob. 29.8 \Rightarrow A proton moves with a velocity of $\vec{v} = (2\hat{i} - 4\hat{j} + \hat{k})$ m/s in a region in which the magnitude of the magnetic field is $\vec{B} = (\hat{i} + 2\hat{j} - \hat{k})$ T. What is the magnitude of the magnetic force this particle experiences?

proton: $q = 1.6 \times 10^{-19}$ C, $m = 1.67 \times 10^{-27}$ kg
 $\vec{v} = (2\hat{i} - 4\hat{j} + \hat{k})$ m/s.

$$\vec{B} = (\hat{i} + 2\hat{j} - \hat{k})$$
 T

$$\vec{F}_B = q\vec{v} \times \vec{B} = (1.6 \times 10^{-19})$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

$$\vec{F}_B = (1.6 \times 10^{-19}) [(4-2)\hat{i} - (-2-1)\hat{j} + (4-4)\hat{k}]$$

$$\vec{F}_B = (3.2\hat{i} + 4.8\hat{j} + 12.8\hat{k}) \times 10^{-19}$$
 N

$$|\vec{F}_B| = \sqrt{3.2^2 + 4.8^2 + 12.8^2} \times 10^{-19}$$
 N

$$= 13.2 \times 10^{-19}$$
 N

prob - 29.13] An electron moves in a circular path perpendicular to a uniform magnetic field with a magnitude of 2.00 mT . If the speed of the electron is $1.50 \times 10^7 \text{ m/s}$ determine (a) the radius of the circular path (b) the time interval required to complete one revolution.

$$q = -e = -1.6 \times 10^{-19}, m = 9.11 \times 10^{-31} \text{ kg}$$

$$B = 2 \text{ mT} = 2 \times 10^{-3} \text{ T}$$

$$v = 1.5 \times 10^7$$

$$\vec{J} \perp \vec{B}$$

$$A) qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB} = 0.0427 \text{ m} = 4.27 \text{ cm}$$

$$B) T = \frac{2\pi r}{v} = 1.79 \times 10^{-8} \text{ s}$$

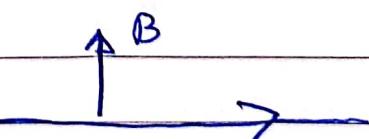
$$T = \frac{2\pi m}{qB} = 1.79 \times 10^{-8} \text{ s}$$

Prob. 29.32.

$$I = 3 \text{ A}, B = 0.23 \text{ T}, \theta = \frac{\pi}{2}$$

$$A) L = 14 \text{ cm} = 0.14 \text{ m}$$

$$F_B = ILB \sin \theta \\ = 0.118 \text{ N}$$



Chapter 30 - Sources of magnetic field.

There are two sources of magnetic field :

1) Electric currents (moving charges)

2) Magnetic materials.

Biot and Savart conducted experiments to measure the magnetic fields and magnetic forces due to a current I .

The results are :-

The magnetic $d\vec{B}$ at point P due to a current I in the length segment $d\vec{s}$ is.

$$1) d\vec{B} \perp d\vec{s}$$

$$2) d\vec{B} \perp \hat{r} ; (\hat{r} \text{ is unit vector that points from } d\vec{s} \text{ to the point } P)$$

$$3) dB \propto \frac{1}{r^2} ; (r : \text{ is the distance from } d\vec{s} \text{ to } P)$$

$$4) dB \propto I$$

$$5) dB \propto |d\vec{s}| = ds$$

$$6) dB \propto \sin \theta, (\theta : \text{ the angle between } d\vec{s} \text{ and } \hat{r})$$

The results can be summarized in the Biot-Savart law.

$$\left[d\vec{B} = \frac{\mu_0}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2} \right]$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T.m/A}$$

μ_0 = magnetic permeability of free space.

The total magnetic field can be found by summing up all contributions from all current elements (integral)

$$\left[\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2} \right] \text{Biot, Savart law}$$

Ex. 30.2.

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

The magnetic fields due to the sections A 'A and cc' is zero because.

$$d\vec{s} \times \hat{r} = 0$$

The sections A A' and cc' do not contribute to the magnetic field at O

The field at O is due to the section Ac only

$$\Theta = \frac{\pi}{2}$$

$$|d\vec{s} \times \hat{r}| = |d\vec{s}| \underbrace{|r|}_{=1} \underbrace{\sin\left(\frac{\pi}{2}\right)}_{=1} = ds$$

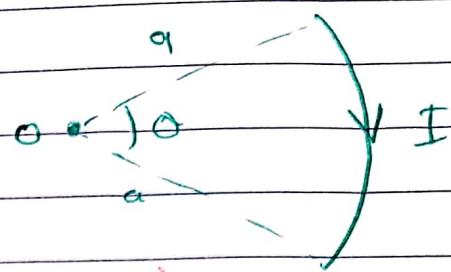
$$B = \frac{\mu_0 I}{4\pi} \int \frac{|d\vec{s} \times \hat{r}|}{r^2} = \frac{\mu_0 I}{4\pi} \int \frac{ds}{a^2} = \frac{\mu_0 I}{4\pi a^2} \int \underbrace{ds}_{=S} = \Theta$$

$$B = \frac{\mu_0 I}{4\pi a^2} (d\Theta)$$

$$\boxed{B = \frac{\mu_0 I}{4\pi a^2} \Theta}$$

$$B_0 = \frac{\mu_0 I}{4\pi a} \Theta$$

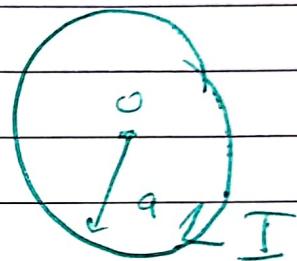
Special case of



If $\Theta = 2\pi$ (closed circular loop)

$$B = \frac{\mu_0 I}{2a}$$

Magnetic field at the center of a closed circular current loop of radius a



$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$

Special Case 1

If $x=0$

$$B = \frac{\mu_0 I a^2}{2(0+a^2)^{3/2}} = \frac{\mu_0 I}{2a}$$

$$B = \frac{\mu_0 I}{2a}$$

Magnetic field at the center of a closed circular loop of current

Ex - 30.6.

Take loop 2

$$\oint \vec{B}_{\text{out}} \cdot d\vec{s} = \mu_0 I_{\text{in}} = 0$$

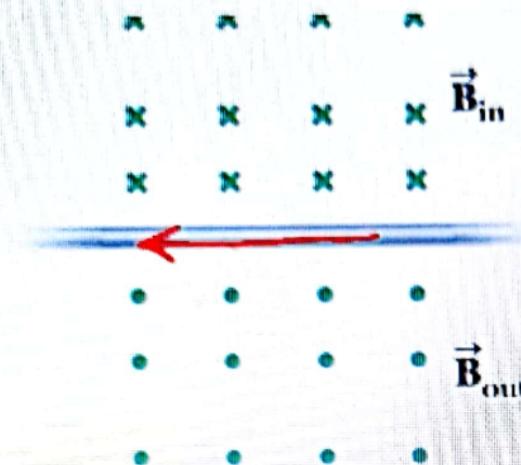
$$B_{\text{out}} = 0, r < b, r > c$$

Take loop 1 ($b < r < c$)

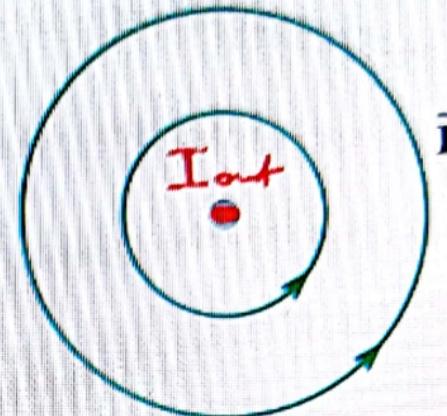
$$\oint \vec{B}_{\text{in}} \cdot d\vec{s} = \mu_0 I_{\text{in}}$$

$$B_{\text{in}} (2\pi r) = \mu_0 NI$$

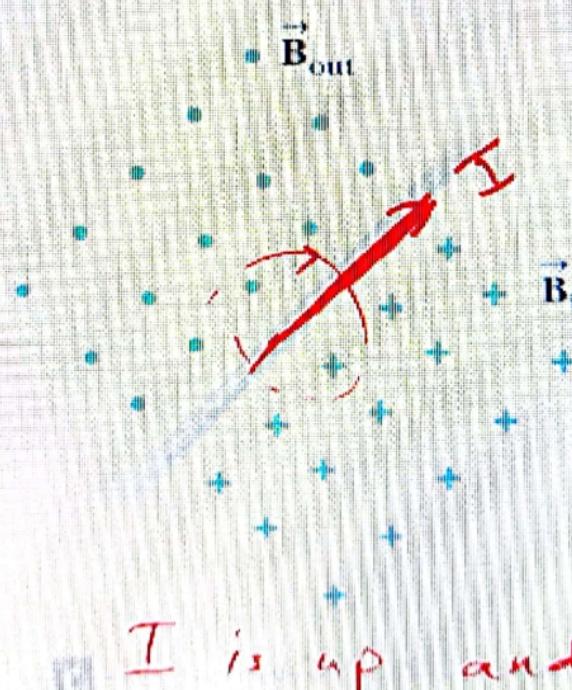
$$B_{\text{in}} = \frac{\mu_0 NI}{2\pi r}, b < r < c$$



a I to the left



b I is out of the page



c I is up and right

Prob. 2

prob. 19

$$I = 5 \text{ A} , d = 10 \text{ cm} = 0.1 \text{ m}$$

a) At the point midway between the two wires

$$B_1 = \frac{\mu_0 I}{2\pi \left(\frac{d}{2}\right)} \quad (\text{into the page})$$

$$B_2 = \frac{\mu_0 I}{2\pi \left(\frac{d}{2}\right)} \quad (\text{into the page})$$

$$B = B_1 + B_2 = \frac{\mu_0 I}{2\pi \frac{d}{2}} + \frac{\mu_0 I}{2\pi \frac{d}{2}} = \frac{2\mu_0 I}{\pi d} = 40 \times 10^{-6} \text{ T}$$

(into the page)

$$= (-40 \times 10^{-6} \hat{R}) \text{ T}$$

$$B_1 = \frac{\mu_0 I}{2\pi d} \quad (\text{out of the page})$$

$$B_2 = \frac{\mu_0 I}{2\pi (2d)} \quad (\text{into the page})$$

$$B_{P_1} = (B_1 - B_2) \hat{k} = \frac{\mu_0 I}{2\pi d} \left(1 - \frac{1}{2}\right) \hat{k}$$

$$B_{P_1} = (5 \times 10^{-6} \hat{R}) \text{ T}$$

$$= 5 \times 10^{-6} \text{ T} \quad (\text{out of the page})$$

C) At point P₂

$$B_1 = \frac{\mu_0 I}{2\pi(3d)} \text{ (into the page)}$$

$$B_2 = \frac{\mu_0 I}{2\pi(2d)} \text{ (out of the page)}$$

$$\vec{B}_{P_2} = \vec{B}_2 - \vec{B}_1$$

$$= \frac{\mu_0 I}{2\pi d} \left(\frac{1}{2} - \frac{1}{3} \right) \hat{k}$$

$$= (1.67 \times 10^{-6} \text{ T}) \hat{k}$$
$$= 1.67 \times 10^{-6} \text{ T} \text{ (out of the page)}$$

prob. 31

$$I_1 = 1A, I_2 = 3A$$

$$d = 1mm = 0.001m$$

A) B at a (take loop 1)

$$\oint \vec{B}_0 \cdot d\vec{s} = \mu_0 I_{in}$$

$$B_a (2\pi d) = \mu_0 I_1$$

$$B_a = \frac{\mu_0 I_1}{2\pi d} = 200 \times 10^{-6} T \text{ (counter clockwise)}$$

B) B at b (take loop 2)

$$\oint \vec{B}_b \cdot d\vec{s} = \mu_0 I_{in}$$

$$B_b (2\pi (3d)) = \mu_0 (I_1 - I_2)$$

$$B_b = \frac{\mu_0 (I_1 - I_2)}{6\pi d}$$

$$B = -133 \times 10^{-6} T$$

(-) = clockwise.

77