

تقدم لجنة EiCoM الأكاديمية

دفتر الفاينل لمادة:  
**فيزياء عامة (2)**

من شرح:

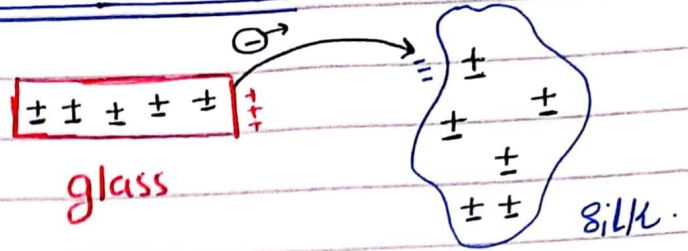
**د. غسان النعواشي**

جزيل الشكر للطالبة:

**تولين أسامة**

## ch-23 - Electric fields.

- Glass rod rubbed with silk.



مواد مختلفة مادة إيجابي فيها منها تكون طاقة  
إيجابي عاليه ومنها تكون طاقة إيجابي قليلة  
بعضها على السطح لا إلكترونات وبعضها على الخزانة  
الإلكترونات.

نحوه (1)

⇒ Electrons are transferred from glass to silk.

⇒ glass is positively charged.

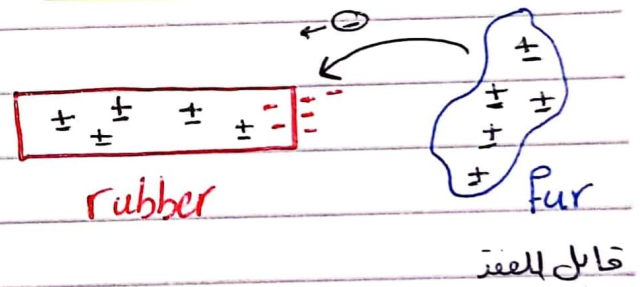
تنتقل من glass لأنه قابل للعد  
إيجابي silk لأنو قابل للعد.

silk is negatively charged.

عملية ذلك عبارة عن انتقال  
والإلكترونات هو طاقة

- Rubber rod rubbed with fur.

⇒ Electrons are transferred from  
fur to rubber



⇒ fur is positively charged.

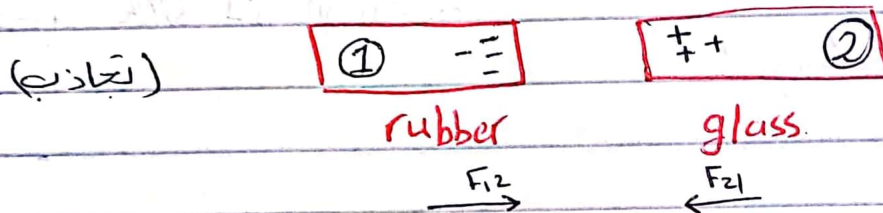
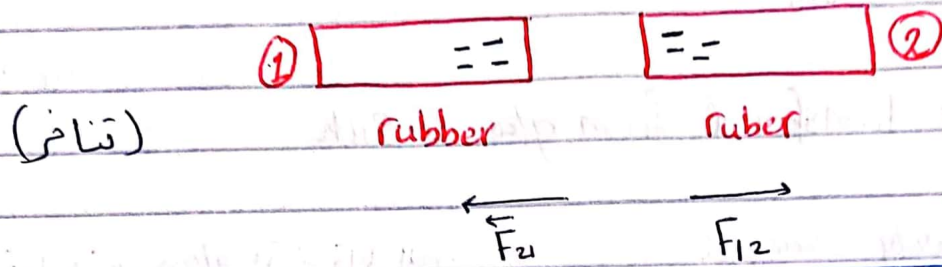
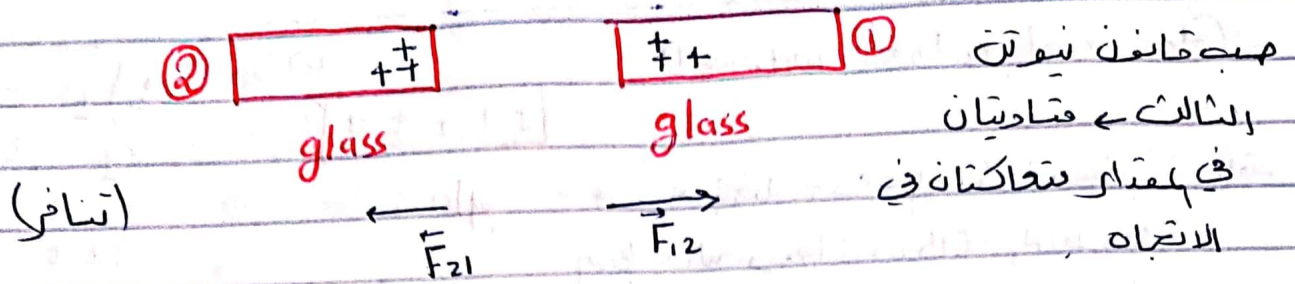
⇒ Rubber is negatively charged.

Conclusion: There are two kinds of charges.

1) positive (+) : Like protons.

2) negative (-) : Like electron.





سواء تجاذب أو تنافر (قوتان في المقدار متعاكستان في الاتجاه)  
في التجربة (١) (٢) (٣)

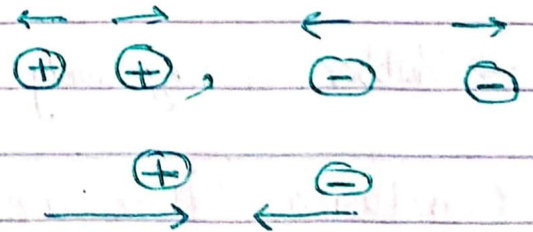
$$\vec{F}_{12} = -\vec{F}_{21}$$

Newton's 3rd Law

### Conclusion

\* Like charges repel

\* unlike charges attract



\* Conservation of charges (حفظ الشحنة)

\* Electric charge is always conserved on an isolated system

Ex  $\Rightarrow$  charge is not created in the process of rubbing two objects together but transferred from one object to another

\* Quantization of charge

Electric charge is said to be quantized (مكممة) i.e. electric charge exists as discrete packets.

• روين قيم منفصلة (لا يمكن ان نقل جزء من الالكترونات)

$q = \pm ne$	$e \times N = q$
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\*  $e = 1.6 \times 10^{-19} \text{ C}$

C: Coulomb

Electron : $q_e = -e = -1.6 \times 10^{-19} \text{ C}$
proton : $q_p = +e = 1.6 \times 10^{-19} \text{ C}$

\* Classification of materials

• materials are classified in terms of the ability of electrons to move through them.

1) Conductors (موصلات) : Some of the electrons are free (unbound)  $\rightarrow$  (غير مقيدة)

e.g. Aluminum, Silver, Copper (نحاس)



2) Insulators (العزلان) : All the electrons are bound  
لا يوجد نقل للإلكترونات لأجزاءها (مقيدة)

Example  $\Rightarrow$  glass, rubber, wood, plastic.

3) Semiconductors (أشباه موصلات) : Electric properties are  
Some when between those of conductors and insulators.

Ex  $\Rightarrow$  Silicon, germanium

\* Coulomb's law :

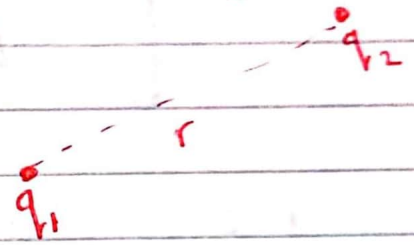
افتراض

Assumption : we have point charges

(charge of zero size) (حجمها صفر)

نعتبر الشحنة كنقطة وليس لها حجم، وبذلك لا تتداخل مع بعضها

(شحنتين)



$$F_e = k_e \frac{|q_1| |q_2|}{r^2}$$

$$\frac{1}{4\pi\epsilon_0} = k_e$$

$F$  = electric force.

$$k = \text{Coulomb's constant} = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2 \approx 9 \times 10^9$$

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2 = \text{permittivity of free space.}$$

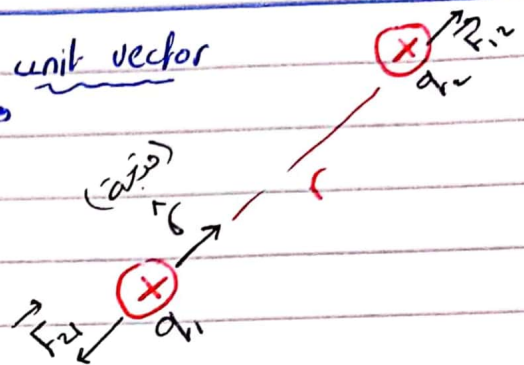


$F_e$  is attractive if charges are of opposite sign.

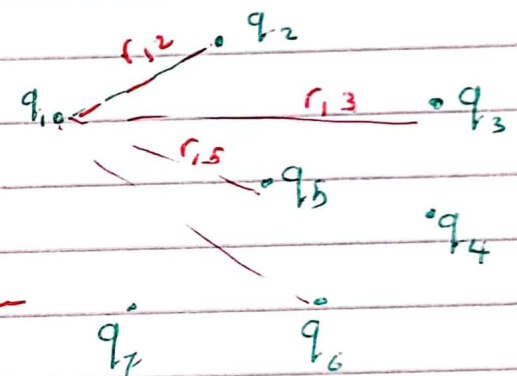
$F_e$  is negative if charges are of the same sign

$$\vec{F}_{12} = -\vec{F}_{21} = \frac{k |q_1| |q_2|}{r^2} \hat{r} \rightarrow \text{unit vector}$$

انجذاب دافع بینم سوا کان تباخر  
او تجاذب



Superposition principle :  
مبدأ التراكب



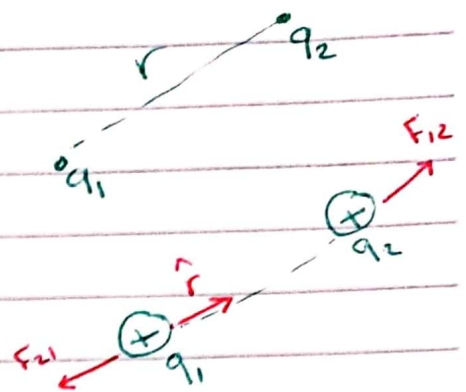
The resultant force on  $q_1$  is the vector sum of all the forces exerted on it by other charges.

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41} + \dots$$

Coulomb's Law  $\Rightarrow$

$$F_e = k_e = \frac{|q_1| |q_2|}{r^2}$$

$$\vec{F}_{12} = -\vec{F}_{21} = k \frac{|q_1| |q_2|}{r^2} \hat{r}$$



\* Superposition principle.

$$\vec{F}_1 = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41}$$

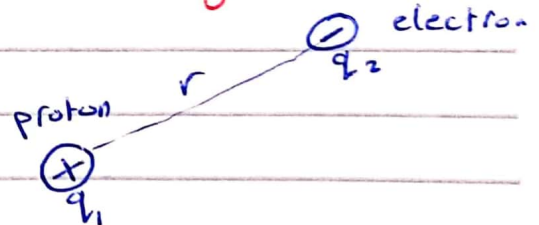
Ex  $\Rightarrow$  23.1

The electron and proton of a hydrogen atom are separated (on the average) by a distance of approximately  $5.3 \times 10^{-11} \text{ m}$ . Find the magnitudes of the electric force and the gravitational force between the two particles.

$$q_1 = +e = 1.6 \times 10^{-19} \text{ C}$$

$$q_2 = -e = -1.6 \times 10^{-19} \text{ C}$$

$$r = 5.3 \times 10^{-11} \text{ m}$$



$$m_1 = 1.67 \times 10^{-27} \text{ kg}$$

$$m_2 = 9.11 \times 10^{-31} \text{ kg}$$

$$F_e = k \frac{|q_1| |q_2|}{r^2} = (9 \times 10^9) \frac{(1.6 \times 10^{-19})(1.6 \times 10^{-19})}{(5.3 \times 10^{-11})^2}$$

$$= 8.2 \times 10^{-8} \text{ N} \leftarrow \text{قوة التجاذب الكهربائية}$$



قانون الجذب الكهلي

$$F_G = G \frac{m_1 m_2}{r^2} = \frac{(6.67 \times 10^{-11}) (1.67 \times 10^{-27}) (9.11 \times 10^{-31})}{(5.3 \times 10^{-11})^2} = 3.6 \times 10^{-47} \text{ N}$$

قوة الجذب الكهلي مفرقة جداً

$$\frac{F_e}{F_G} \approx 2.3 \times 10^{39} \Rightarrow F_G \text{ is negligible (مهملة)}$$

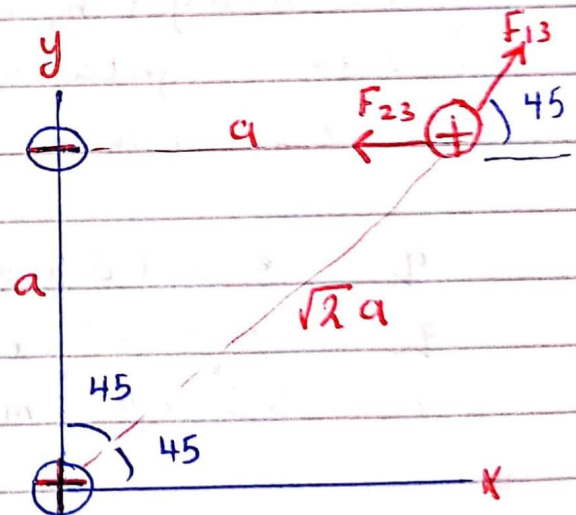
**Ex. 23.2**  $\rightarrow$  Consider three point charges located at the corners of a right triangle as shown in figure where  $q_1 = q_3 = 5.00 \mu\text{C}$ ,  $q_2 = -2.00 \mu\text{C}$  and  $a = 0.100 \text{ m}$  find the resultant force exerted on  $q_3$

$$\begin{aligned} q_1 &= q_3 = 5 \mu\text{C} \\ q_2 &= -2 \mu\text{C} \\ a &= 0.1 \text{ m} \end{aligned}$$

$$F_{13} = k \frac{|q_1| |q_3|}{(\sqrt{2} a)^2}$$

$$= (9 \times 10^9) \frac{(5 \times 10^{-6})(5 \times 10^{-6})}{(2)(0.1)^2}$$

$$F_{13} = 11.2 \text{ N}$$



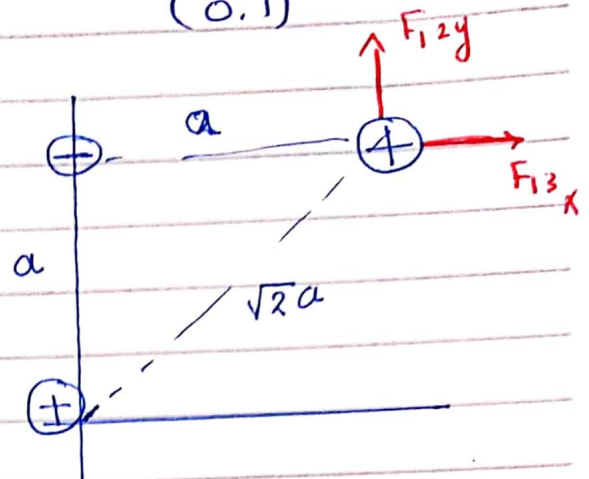


$$F_{23} = k \frac{|q_2| |q_3|}{a^2} = (9 \times 10^9) \frac{(2 \times 10^{-6})(5 \times 10^{-6})}{(0.1)^2}$$

$$F_{23} = 9 \text{ N.}$$

$$F_{13x} = F_{13} \cos 45 = (11)(0.71) = 7.9 \text{ N}$$

$$F_{13y} = F_{13} \sin 45 = (11.2)(0.71) = 7.9 \text{ N}$$



$$F_{23x} = F_{23} \cos 180 = (9)(-1) = -9 \text{ N}$$

$$F_{23y} = F_{23} \sin 180 = 0$$

$$F_{3x} = F_{13x} + F_{23x} = -1.1 \text{ N}$$

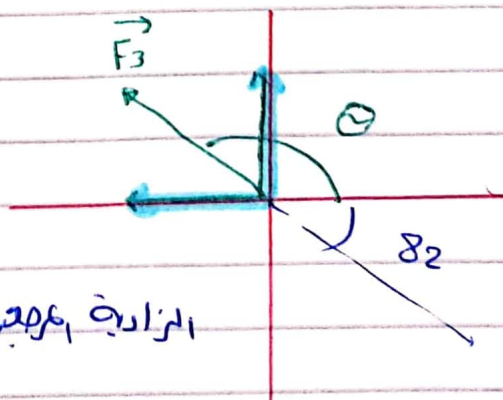
$$F_{3y} = F_{13y} + F_{23y} = 7.9 \text{ N}$$

$$\vec{F}_3 = (-1.1 \hat{i} + 7.9 \hat{j}) \text{ N}$$

$$F_3 = \sqrt{(-1.1)^2 + (7.9)^2} \approx 8 \text{ N}$$

$$\Theta = \tan^{-1} \left( \frac{F_{3y}}{F_{3x}} \right) = \tan^{-1} \left( \frac{7.9}{-1.1} \right)$$

$$= -82^\circ + 180^\circ = 98^\circ$$



الزاوية المقاسة ← الزاوية المقاسة بتطلع يا الرابع  
أو الرابع

ex. 23.3  $\Rightarrow$  Three point charges lie along the x axis as shown in Figure. The positive charge  $q_1 = 15.0 \mu\text{C}$  is at  $x = 2.00 \text{ m}$ , the positive charge  $q_2 = 6.00 \mu\text{C}$  is at the origin, and the net force acting on  $q_3$  is zero. What is the x coordinate of  $q_3$ ?

$$q_1 = 15 \mu\text{C}$$

$$q_2 = 6 \mu\text{C}$$

$$\Sigma F_{3x} = 0$$

$$F_{13} - F_{23} = 0$$

$$F_{13} = F_{23}$$

$$\cancel{k} \frac{|q_1| |q_3|}{(2-x)^2} = \cancel{k} \frac{|q_2| |q_3|}{x^2}$$

$$\frac{|q_1|}{(2-x)^2} = \frac{|q_2|}{x^2}$$

$$\textcircled{5} \frac{15 \times 10^{-6}}{(2-x)^2} = \textcircled{2} \frac{6 \times 10^{-6}}{x^2}$$

$$\Rightarrow \frac{5}{(2-x)^2} = \frac{2}{x^2} \Rightarrow \frac{5}{4-4x+x^2} = \frac{2}{x^2}$$

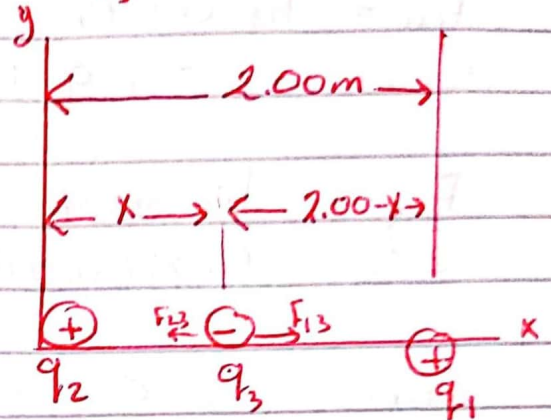
$$5x^2 = 8 - 8x + 2x^2$$

$$3x^2 + 8x + 8 = 0$$

$$x = \frac{-8 \pm \sqrt{8^2 - (4)(3)(-8)}}{6}$$

$$x = 0.775 \text{ m} \checkmark$$

$$x = -3.44 \text{ m} \times$$





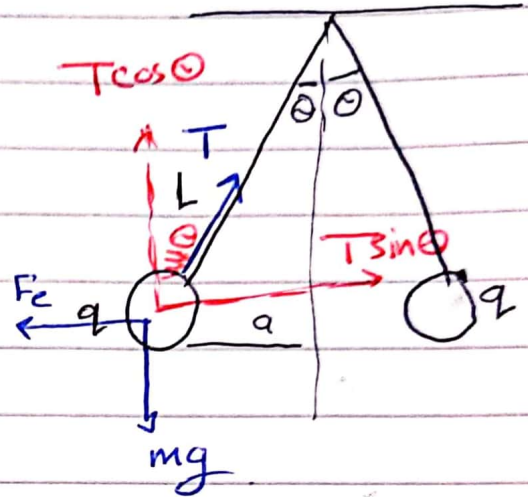
Ex 23.4

Two identical small charged spheres, each having a mass of  $3.00 \times 10^{-2} \text{ kg}$ , hang in equilibrium as shown in figure. The length  $L$  of each string is  $0.150 \text{ m}$  and the angle  $\theta$  is  $5.00^\circ$ . Find the magnitude of the charge on each sphere.

$$m = 3 \times 10^{-2} \text{ kg}, L = 0.15 \text{ m}$$

$$\theta = 5^\circ$$

$$F_e = k \frac{|q|^2}{(2a)^2} = k \frac{|q|^2}{4a^2}$$



$$T \sin \theta = F_e$$

$$T \cos \theta = mg$$

Divide

$$\sin \theta = \frac{a}{L}$$

$$a = L \sin \theta$$

$$\frac{\sin \theta}{\cos \theta} = \frac{F_e}{mg}$$

$$\tan \theta = \frac{F_e}{mg}$$

$$|q| = \sqrt{\frac{4L^2 mg \sin^2 \theta \tan \theta}{k}}$$

$$F_e = mg \tan \theta$$

$$|q| = 4.4 \times 10^{-8} \text{ C}$$

$$\frac{k |q|^2}{4a^2} = mg \tan \theta$$

$$|q| = \sqrt{\frac{4a^2 mg \tan \theta}{k}}$$



## \* The Electric field \*

$$\vec{F} = k \frac{q_1 q_2}{r^2} \hat{r}$$



- \* The electric force is a field force  
(Can act through space with no physical contact)

ایہ شے تین شے کے درمیان  
دفعہ یا جاذبہ قوت کے ذریعے ہوتی ہے

- \* Electric field exists in a region of space around charged objects.

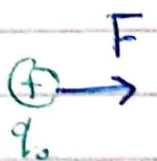
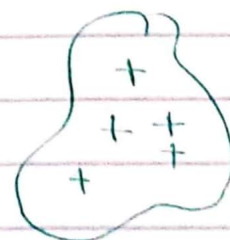
- \* When a charge enters the electric field, an electric force acts on it.

- \* The electric field is defined as the electric force acting on a positive per unit charge.

- \* The electric field ( $\vec{E}$ ) at a point is defined as the electric force ( $\vec{F}$ ) acting on a positive test charge ( $q_0$ ) placed at that point divided by the test charge.

$\vec{E} = \frac{\vec{F}}{q_0}$	$\frac{N}{C} = \frac{N}{C}$	definition of $\vec{E}$
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ایہ شے ہر جگہ ہوتی ہے  
جہاں کچھ شے یا اجسام ہوں



$$[\vec{F} = q \vec{E}]$$

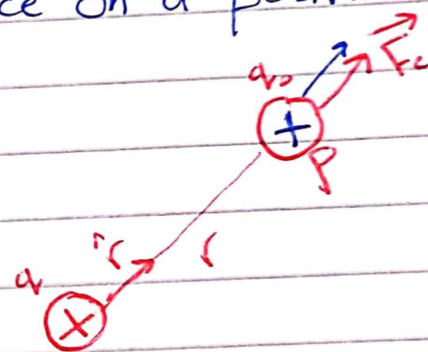
↳ The electric force  $\vec{F}$  on a charge  $q$  placed on an electric field.

SI unit of  $\vec{E}$  is  $N/C$

$$[E] = \frac{[F]}{|q|} = \frac{N}{C}$$

Comments

1) Direction of  $\vec{E}$  is that of the force on a positive charge.

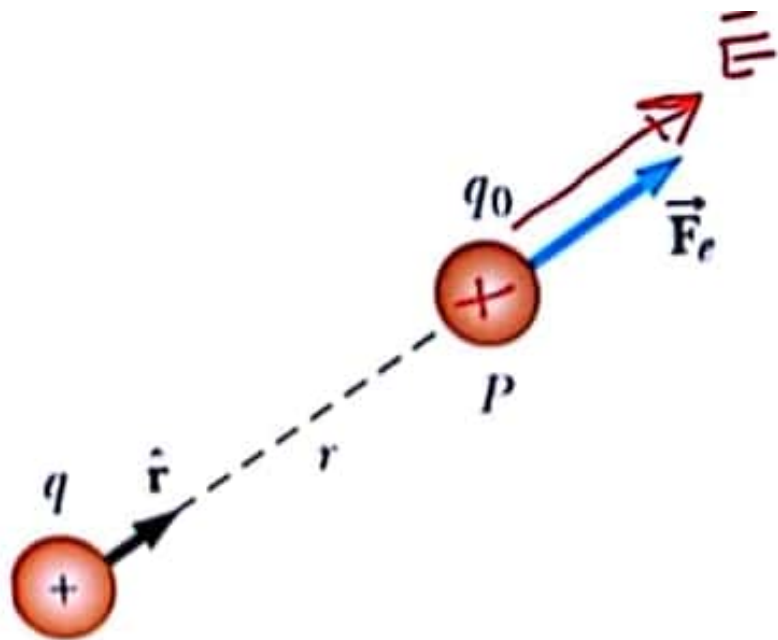


2) SI unit of  $E$  is  $N/C$

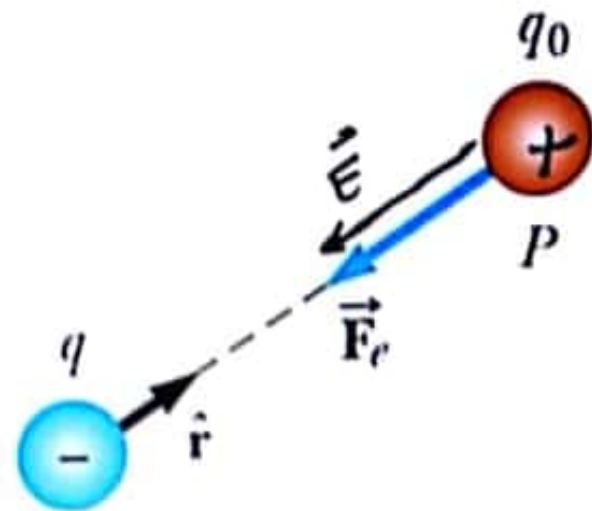
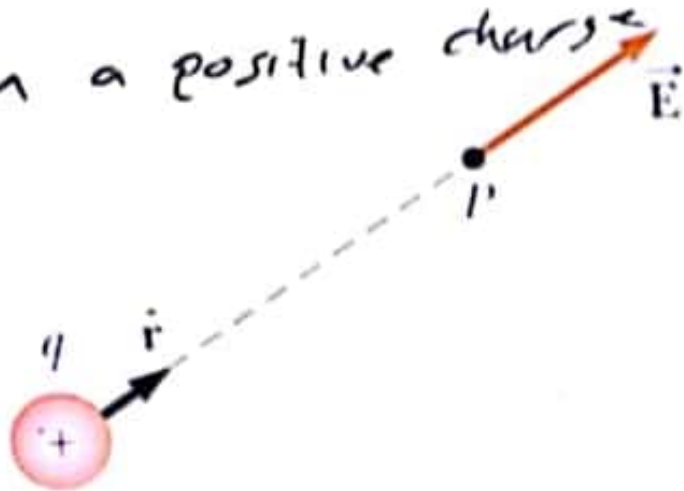
3)  $\vec{F} = q\vec{E}$  is valid for point charges.

4)  $\vec{E}$  is directed away from a positive charge.

5)  $E$  is directed toward a negative.



$\vec{E}$  is directed away from a positive charge.

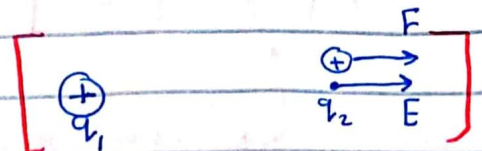


$\vec{E}$  is directed toward a negative charge.

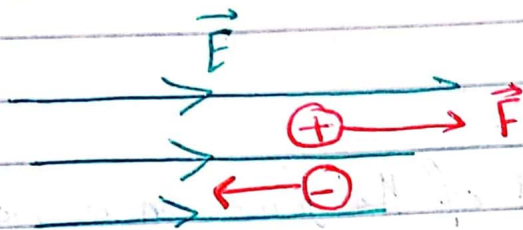
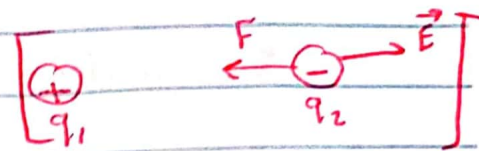




6) The force on a positive charge is in the same direction of  $\vec{E}$



7) The force on a negative charge is opposite to  $\vec{E}$



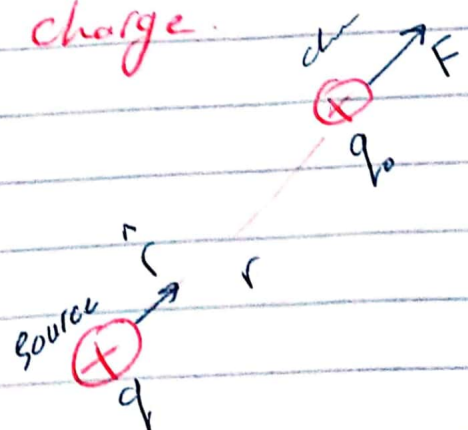
The force between the source and the test charge is

$$F = k \frac{q q_0}{r^2}$$

The electric field is then  $\vec{E} = \frac{\vec{F}}{q_0} = \frac{k q}{r^2} \hat{r}$

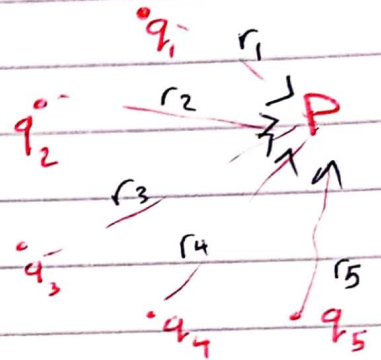
$$\vec{E} = \frac{k q}{r^2} \hat{r}$$

$\vec{E}$  due to  $r$  from a point charge.



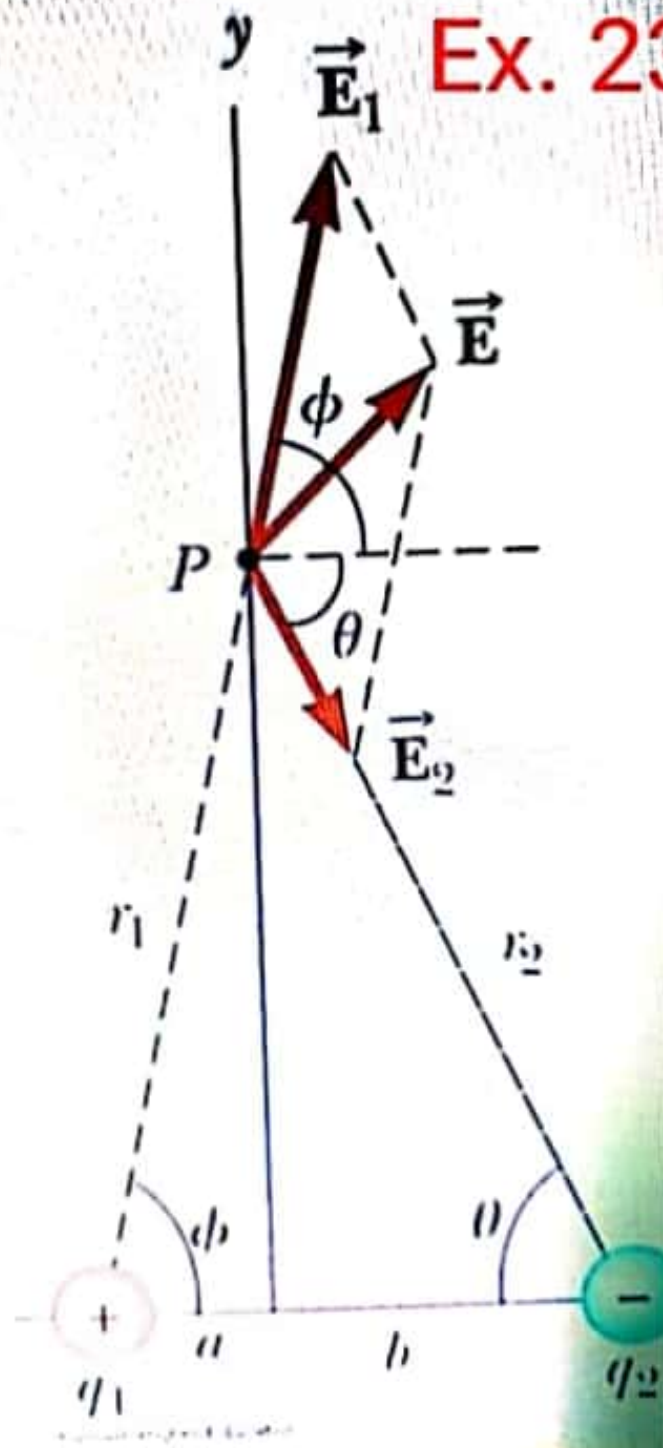
The total electric field at a point  $P$  due to a group of charges is the vector sum of the electric fields of all the charges.

$$\begin{aligned} \vec{E}_P &= \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \dots \\ &= \frac{k q_1}{r_1^2} \hat{r}_1 + k \frac{q_2}{r_2^2} \hat{r}_2 + \dots \\ &= \sum k \frac{q_c}{r_c^2} \hat{r}_c \end{aligned}$$





# Ex. 23.6



EX. 23.6 p. 702

Charges  $q_1$  and  $q_2$  are located on the  $x$  axis, at distances  $a$  and  $b$ , respectively, from the origin, as shown in the figure.

(A) Find the components of the net electric field at the point  $P$ , which is at position.

$$A) E_1 = \frac{k |q_1|}{r_1^2} = k \frac{|q_1|}{a^2 + y^2}$$

$$E_2 = \frac{k |q_2|}{r_2^2} = \frac{k |q_2|}{b^2 + y^2}$$

$$E_{1x} = E_1 \cos \theta = \frac{k |q_1| \cos \theta}{a^2 + y^2}$$

$$E_{1y} = E_1 \sin \theta = \frac{k |q_1| \sin \theta}{a^2 + y^2}$$

$$E_{2x} = E_2 \cos \theta = \frac{k |q_2| \cos \theta}{b^2 + y^2}$$

$$E_{2y} = -E_2 \sin \theta = \frac{-k |q_2| \sin \theta}{b^2 + y^2}$$

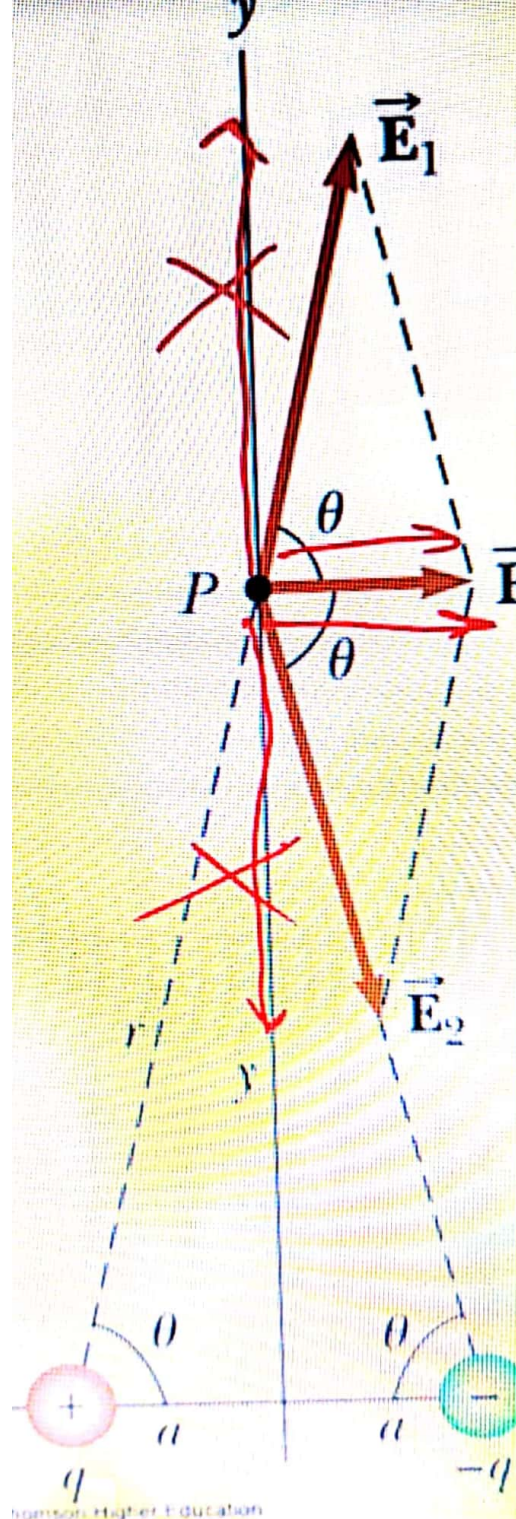
$$E_x = E_{1x} + E_{2x} = k \left[ \frac{|q_1| \cos \theta}{a^2 + y^2} + \frac{|q_2| \cos \theta}{b^2 + y^2} \right]$$

$$E_y = E_{1y} + E_{2y} = k \left[ \frac{|q_1| \sin \theta}{a^2 + y^2} - \frac{|q_2| \sin \theta}{b^2 + y^2} \right]$$

$$\vec{E} = E_x \hat{i} + E_y \hat{j}$$

$$E = \sqrt{E_x^2 + E_y^2}$$





(B) Evaluate the electric field of point (p) in the special case that  $|q_1| = |q_2|$  and  $a = b$ .

$$|q_1| = |q_2| = q$$

$$a = b \Rightarrow \alpha = \phi$$

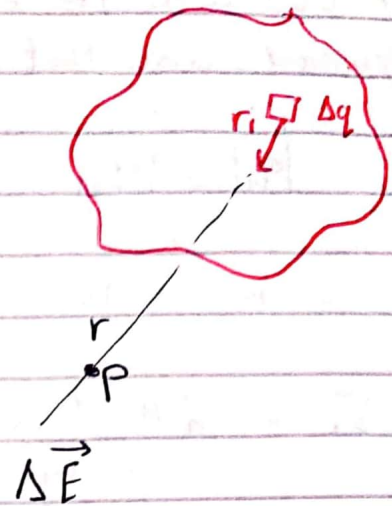
$$E_x = \frac{2kq \cos \theta}{a^2 + y^2}$$

$$E_y = 0$$

## \* Electric field of a continuous charge distribution

The electric field due to the charge element  $\Delta q_i$  is  $\Delta \vec{E}_i$

$$\Delta \vec{E}_i = k \frac{\Delta q_i}{r_i^2} \hat{r}_i$$



The total electric field  $\vec{E}$  at the point  $P$  is the vector of electric field of all charge elements.

$$\vec{E} = \Delta \vec{E}_1 + \Delta \vec{E}_2 + \dots$$

$$= k \frac{\Delta q_1}{r_1^2} \hat{r}_1 + k \frac{\Delta q_2}{r_2^2} \hat{r}_2 + \dots$$

$$= k \sum_n \frac{\Delta q_n}{r_n^2} \hat{r}_n$$

If we make the elements very small ( $\Delta q \rightarrow 0$ )

then

$$\vec{E} = k \lim_{\Delta q_n \rightarrow 0} \sum_n \frac{\Delta q_n}{r_n^2} \hat{r}_n = k \int \frac{dq}{r^2} \hat{r}$$

~~60/11~~



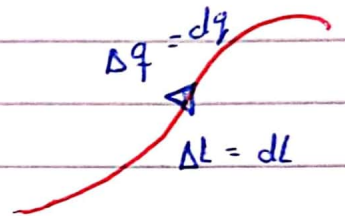
The electric field due to a continuous charge distributions.

$$\boxed{\vec{E} = k \int \frac{dq}{r^2} \hat{r}} = \vec{E} \text{ due to a continuous charge.}$$

There are three kinds of charge distributions.

1) Linear charge distribution. (نقطة، خط، سطح)  
we define the linear charge density ( $\lambda$ )

نسبة الشحنة الموجبة في طول  $\Delta L$   
نسبة الشحنة السالبة في طول  $\Delta L$



(charge per unit length).

$$\boxed{\lambda = \frac{\Delta q}{\Delta L} = \frac{dq}{dL}}; [\lambda] \equiv C/m.$$

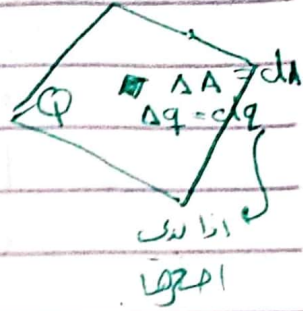
If the total charge  $Q$  is uniformly distributed along line.

$$\lambda = \frac{dq}{dL} = \frac{Q}{L} = \text{Const}$$

$$\boxed{dq = \lambda dL.}$$

## 2) Surface charge distribution

we define the surface charge density ( $\sigma$ )  
(charge per unit area)



$$\sigma = \frac{\Delta q}{\Delta A} = \frac{dq}{dA} ; [\sigma] = C/m^2$$

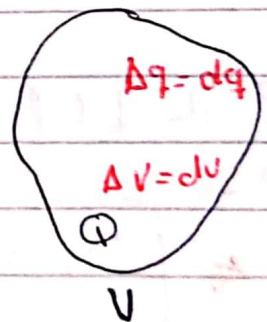
If the charge is uniformly distributed through the surface, then

$$\sigma = \frac{dq}{dA} = \frac{Q}{A} = \text{Const}$$

$$dq = \sigma dA$$

## 3) Volume charge distribution

we define the volume charge density ( $\rho$ )  
(charge per unit volume)



$$\rho = \frac{\Delta q}{\Delta V} = \frac{dq}{dV} ; [\rho] = C/m^3$$

If the charge is uniformly distributed (پخش در حجم) through the volume then

$$\rho = \frac{\Delta q}{\Delta V} = \frac{Q}{V} = \text{Const}$$

$$dq = \rho dV$$



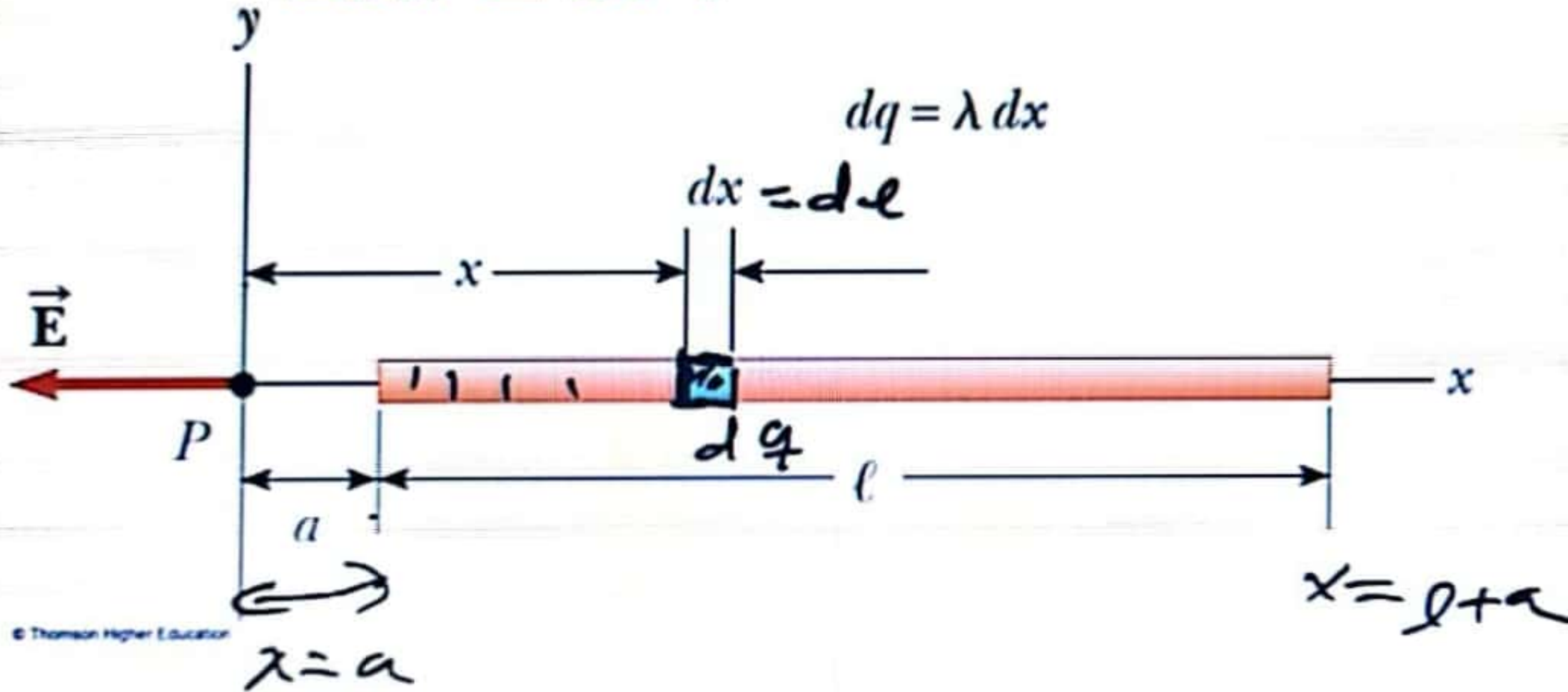
$$E = k \int \frac{dq}{r^2} \hat{r}$$

$dq = \lambda dL \equiv$  Linear charge. (charge)

$dq = \sigma dA \equiv$  Surface charge.

$dq = \rho dV \equiv$  Volume charge.  
(pp) ~~charge~~  $\leftarrow$

# Ex. 23.7





Ex ⇒ 23.7

A rod of length  $L$  has a uniform positive charge per unit length  $\lambda$  and a total charge  $Q$ . Calculate the electric field at a point  $P$  that is located along the long axis of the rod and a distance  $a$  from one end

$$\lambda = \frac{Q}{L} = \text{Const}$$

$$E = k \int \frac{dq}{r^2}$$

$$dq = \lambda dl = \lambda dx = \left( \frac{Q}{L} \right) dx$$

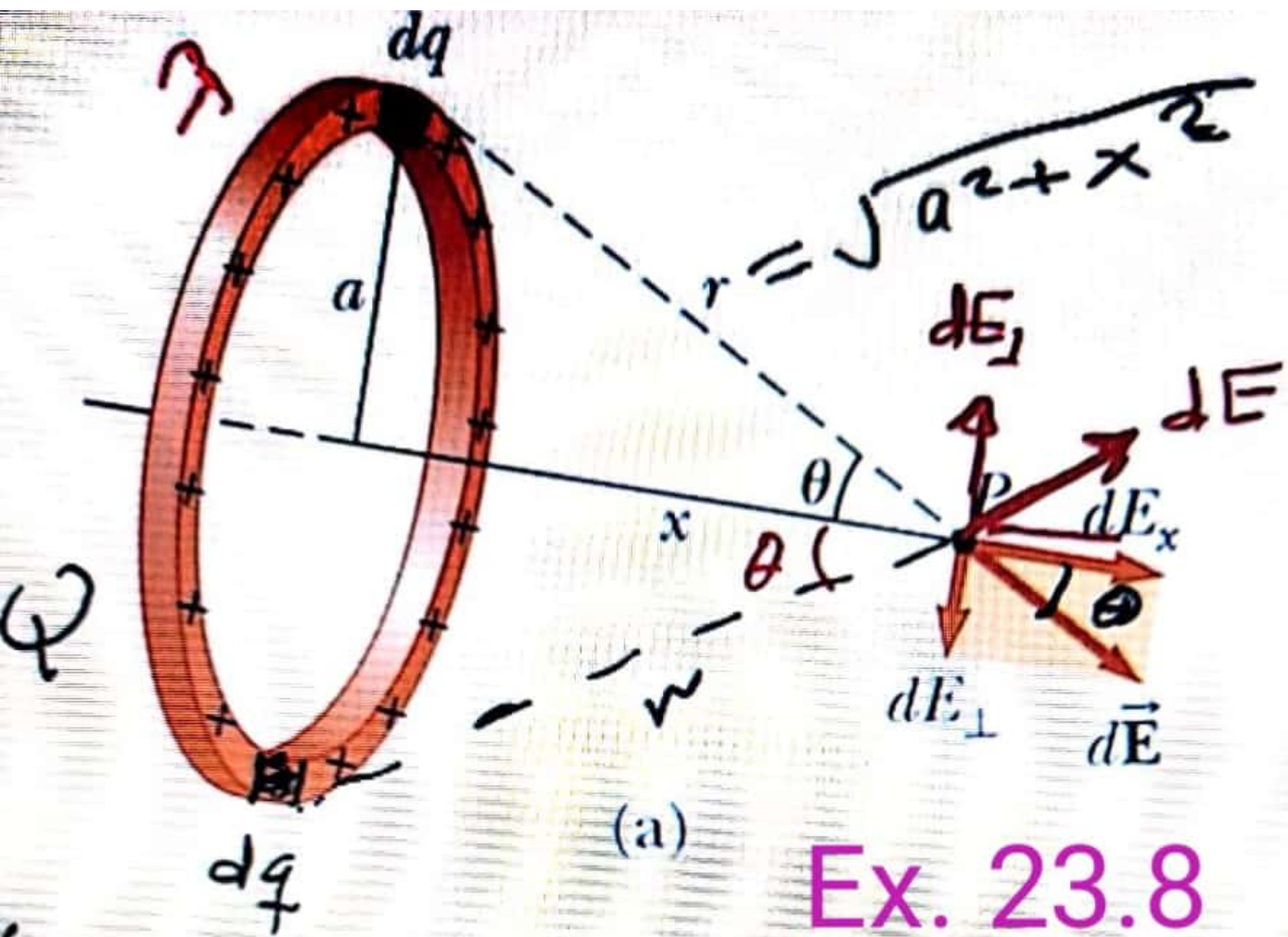
$$r = x$$

$$E = k \int \frac{dq}{r^2} = k \int_a^{L+a} \frac{\frac{Q}{L} dx}{x^2}$$

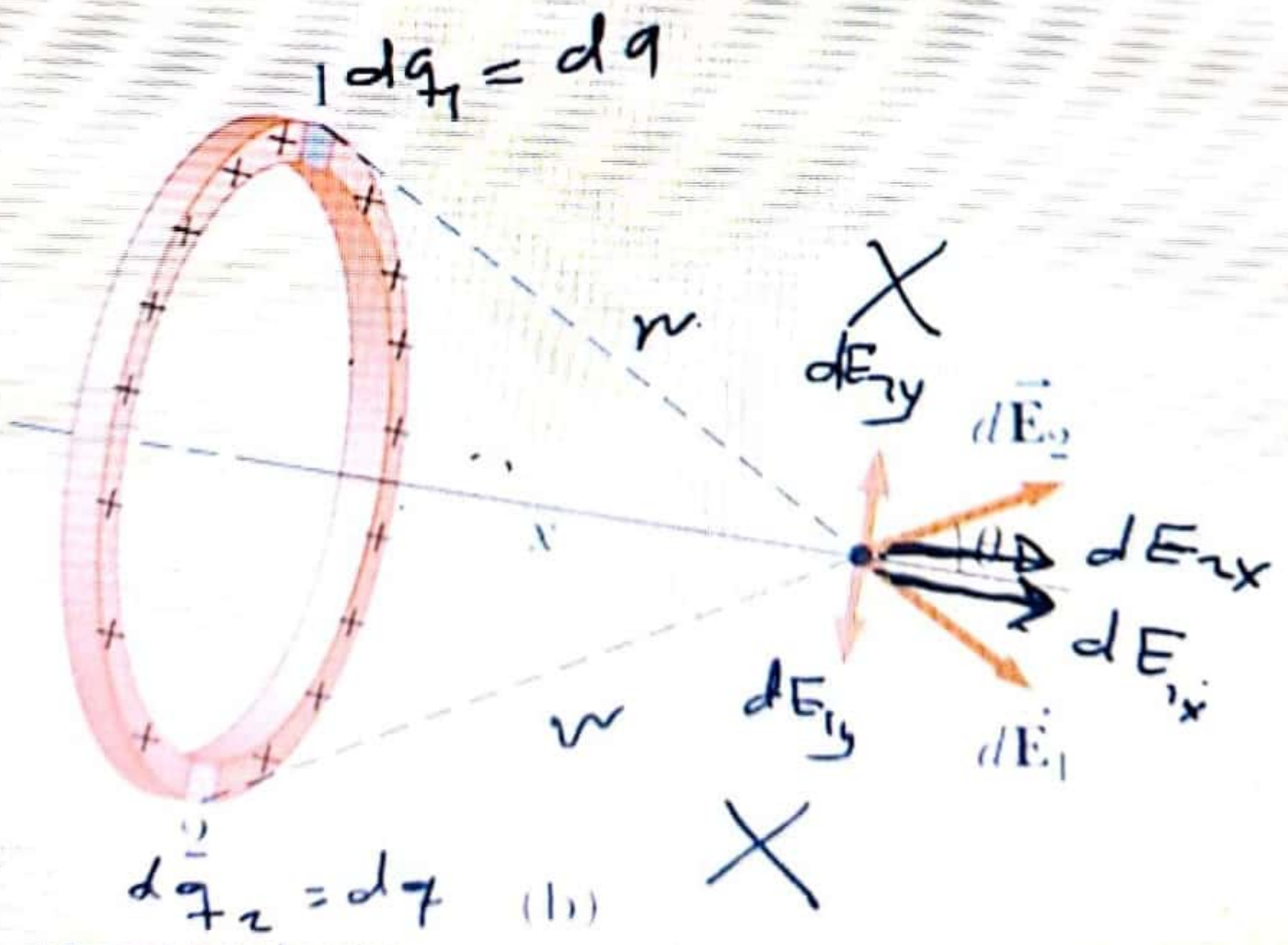
الـ  $r$  في البعد  $x$  من  
رأس الشحنة إلى نقطة  
الحقل

$$= \frac{kQ}{L} \int_a^{L+a} \frac{dx}{x^2} = \frac{kQ}{L} \left[ \frac{-1}{x} \right]_a^{L+a}$$

$$E = \frac{kQ}{L} \left[ \frac{1}{a} - \frac{1}{L+a} \right]$$



Ex. 23.8





EX  $\Rightarrow$  23.8

A ring of radius  $a$  carries a uniformly distributed positive total charge  $Q$ . Calculate the electric field due to the ring at a point ~~lying~~ (P) lying a distance  $x$  from its center along the central axis perpendicular to the plane of the ring.

$\lambda = \text{uniform}$

$$\lambda = \frac{Q}{2\pi a}$$

$$dE_1 = dE_2$$

vertical components,  $dE_{\perp}$ , cancel (each other)

$$dE_{\text{net } \perp} = 0$$

$$dE_{1x} = dE_{2x} = dE \cos \theta$$

$$dE = k \frac{dq}{r^2} = k \frac{dq}{a^2 + x^2}$$

$$dE_x = dE \cos \theta = k \frac{dq}{a^2 + x^2} = \frac{\cancel{k} x}{\sqrt{a^2 + x^2}}$$

$$\int dE_x = \int \frac{kx dq}{(a^2 + x^2)^{\frac{3}{2}}}$$

$$E_x = k \int \frac{x \cdot dq}{(a^2 + x^2)^{\frac{3}{2}}} \rightarrow \frac{kx}{(a^2 + x^2)^{\frac{3}{2}}} \int dq \quad \begin{matrix} \text{(لأنه نفس المتغيرات)} \\ \text{كلمة ثابتة} \end{matrix}$$

$= Q$

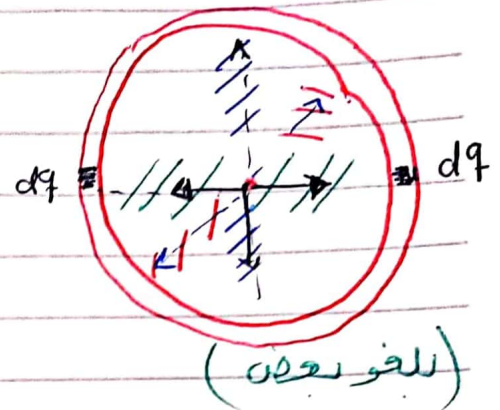
$$E_x = \frac{kQx}{(a^2 + x^2)^{\frac{3}{2}}}$$

$\vec{E}$  a distance  $x$  from the center of a ring of radius  $a$  along the perpendicular axis.

Q1 The electric field at the center of the ring is zero. (1)

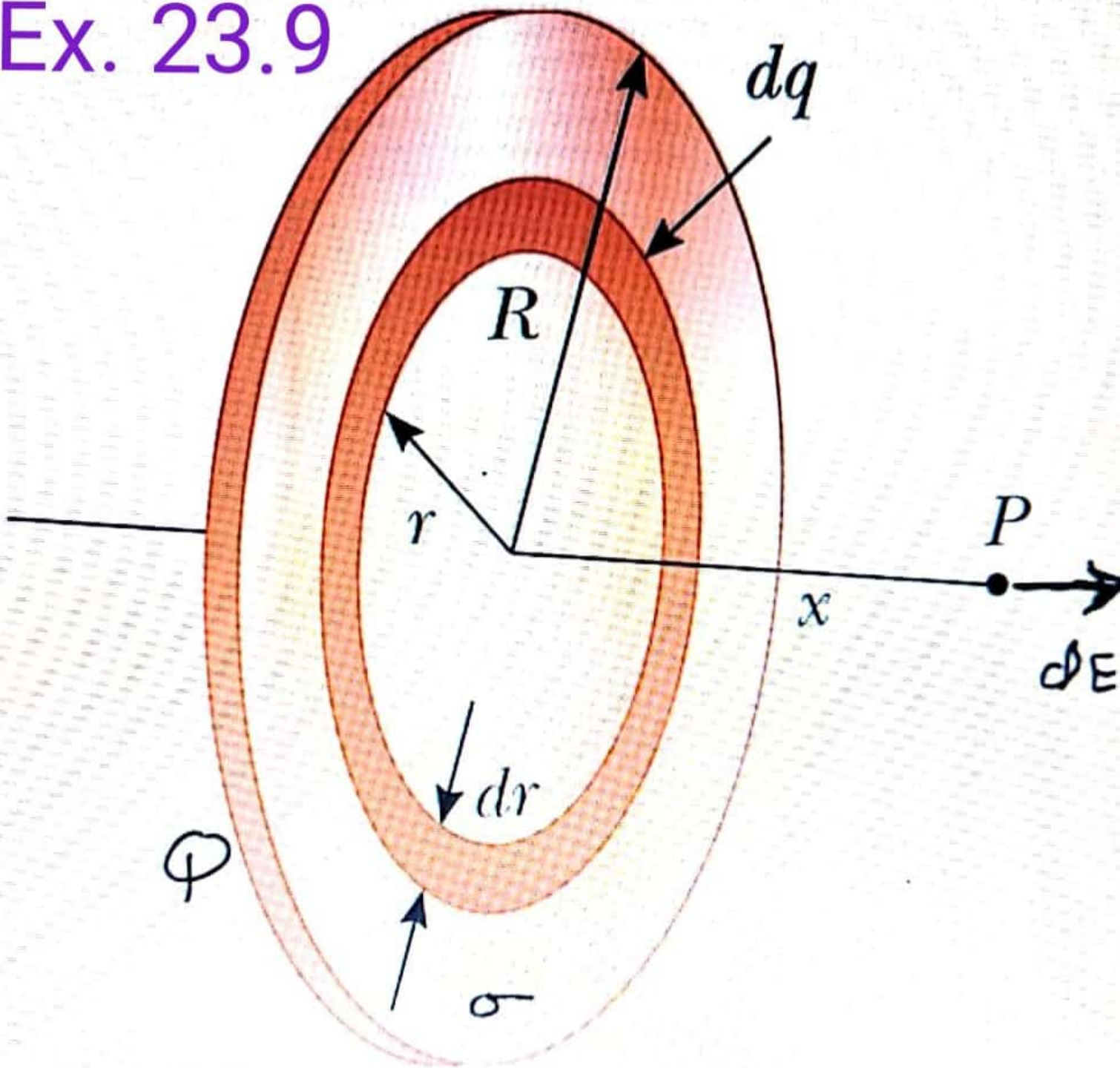
Q2 At the center of the ring (x=0) (2)

$$E = \frac{kQx}{(a^2 + x^2)^{\frac{3}{2}}} = 0 \quad \text{zero}$$





# Ex. 23.9



[Ex. 23.9]

A disk of radius  $R$  has a uniform surface charge density  $\sigma$ . Calculate the electric field at a point (P) that lies along the central perpendicular axis of the disk a distance  $x$  from the center of the disk.

$\sigma$  is uniform

⇒ Go back to Ex. 23.8

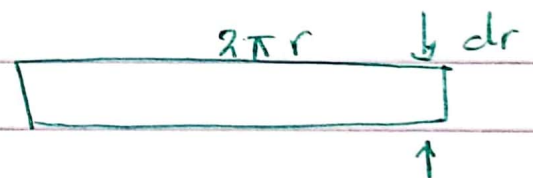
$$E = \frac{kQx}{(a^2 + x^2)^{\frac{3}{2}}}$$

with replacing

$Q \equiv dq$  and  $a \equiv r$  ,  $[E_x \equiv dE]$

The electric field  $dE$  due to the charge element  $dq$  distribution along a ring of radius  $r$  is

$$\int dE = \int \frac{k dq x}{(x^2 + r^2)^{\frac{3}{2}}} \quad (\text{دائرة صغيرة})$$



$$dq = \sigma dA = \sigma (2\pi r dr)$$

$$dA = (2\pi r) \cdot dr \quad (\text{مساحة})$$

$$\begin{aligned} \int dE &= \int \frac{k x dq}{(x^2 + r^2)^{\frac{3}{2}}} \\ E_x &= \int \frac{k x \sigma (2\pi r dr)}{(x^2 + r^2)^{\frac{3}{2}}} \\ &= \pi k \sigma x \int \frac{2r dr}{(x^2 + r^2)^{\frac{3}{2}}} \end{aligned}$$

(integration by substitution) بالحدس



$$E_x = \pi k \sigma x \left[ \frac{(x^2 + r^2)^{-\frac{1}{2}}}{-\frac{1}{2}} \right]_0^R$$

$$E_x = 2\pi k \sigma x \left[ \frac{1}{x} - \frac{1}{\sqrt{x^2 + R^2}} \right]$$

$$\left[ E_x = 2\pi k \sigma \left[ 1 - \frac{x}{\sqrt{x^2 + R^2}} \right] \right]$$



The electric field due to a disk

$$E_x = \pi k \sigma x \left[ \frac{(x^2 + r^2)^{-\frac{1}{2}}}{-\frac{1}{2}} \right]_0^R$$

$$E_x = 2\pi k \sigma x \left[ \frac{1}{x} - \frac{1}{\sqrt{x^2 + R^2}} \right]$$

$$E_x = 2\pi k \sigma \left[ 1 - \frac{x}{\sqrt{x^2 + R^2}} \right]$$



The electric field due to uniformly charged disk of radius ( $R$ ) a distance  $x$  from the center along the perpendicular axis.

\* IF  $x$  is very small ( $x \rightarrow 0$ )

$$E_x = 2\pi k \sigma$$

$$= \cancel{2\pi} \frac{1}{\cancel{2} 4\pi \epsilon_0} \sigma$$

$$E_x = \frac{\sigma}{2\epsilon_0}$$

→ very close the surface of the disk



## \* Electric field lines \*

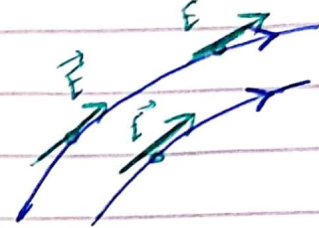
خطوط المجال الكهربائي

⇒ Graphical representation of the electric field. (تمثيل رسومي)

\* properties of the electric field lines :

[1] The electric field  $\vec{E}$  is tangent to the field lines.

(اتجاه المجال الكهربائي هو مماس)



[2] number of field lines per unit area through a surface perpendicular to the lines is proportional to the magnitude of  $E$  متناسب

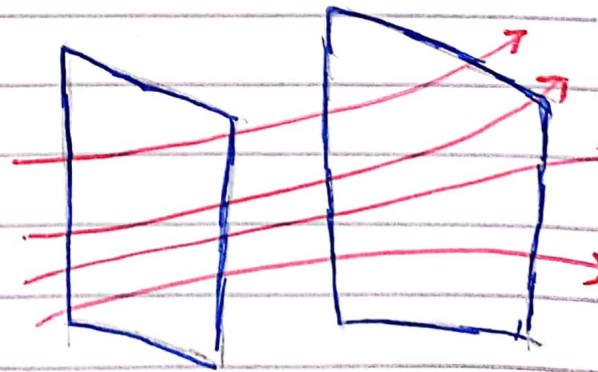
Density of Lines at (A) > density of Lines at (B) كثافة الخطوط

$$E_A > E_B$$

\* كثافة (سعة) المجال ← تربط مع الكثافة

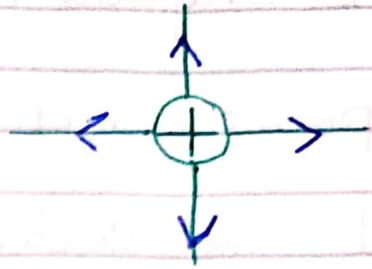
و الكثافة عدد الخطوط التي تخترق المساحة عمودياً  
كلما زادت كثافة الخطوط في منطقة ما  
زاد المجال

المجال (مركبة عمودية) المركبة موازية للمساحة  
← المركبة العمودية على المساحة (المسطح)

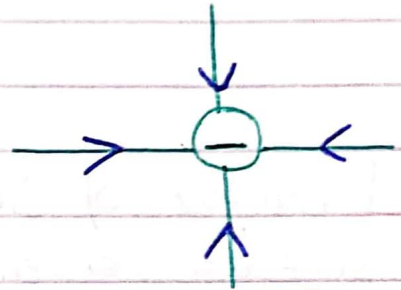


**3** Field lines are directed away from a positive charge.

(+)  $\vec{E}$  is same direction

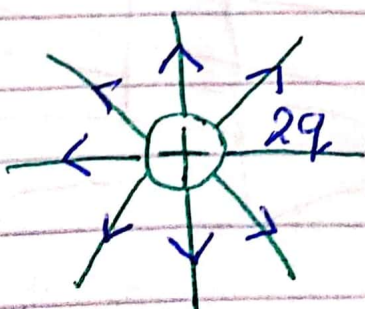
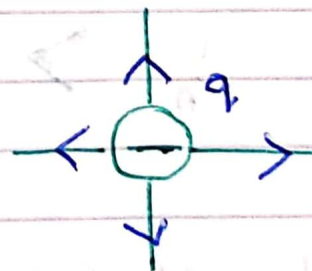
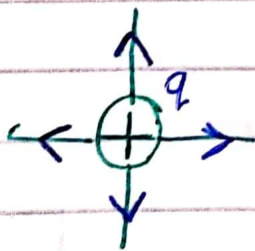


**4** Field lines are directed toward a negative charge.

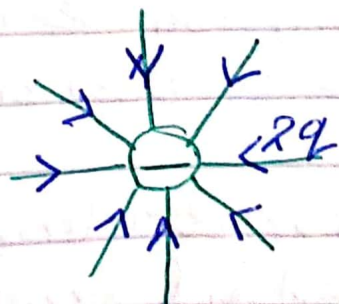


**5** Number of field lines leaving a positive charge or approaching a negative charge is proportional to the magnitude of charge.

مقدار شدة المجال  $\propto$  مقدار الشحنة



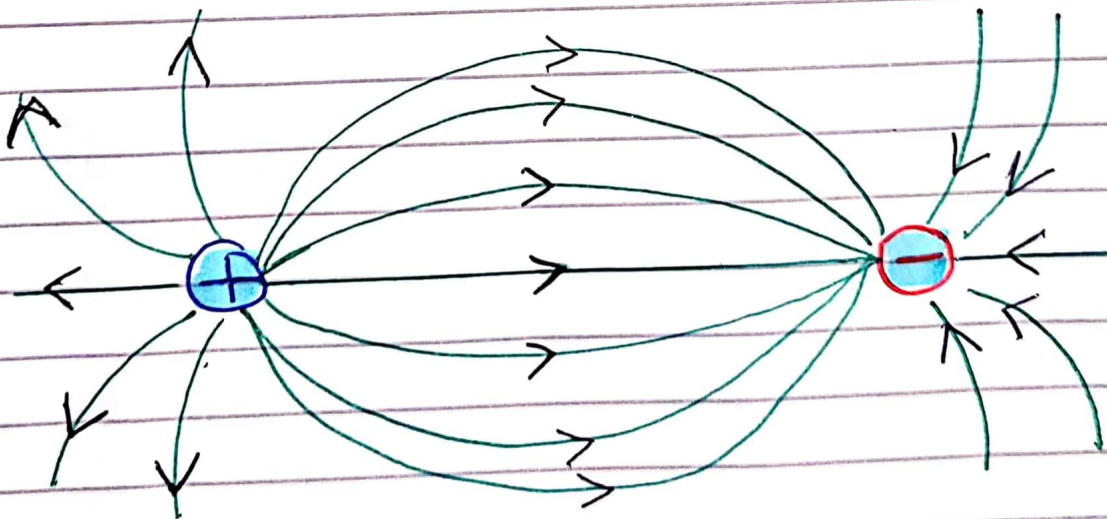
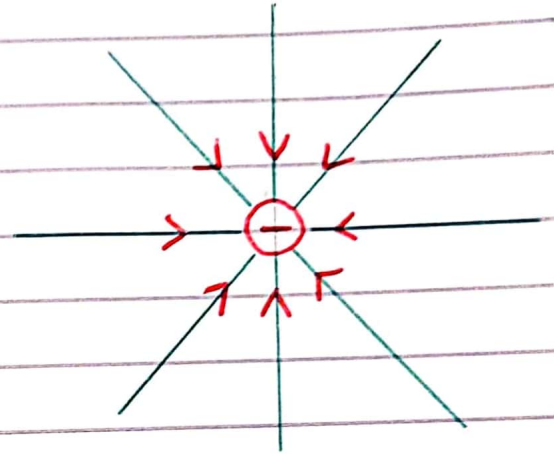
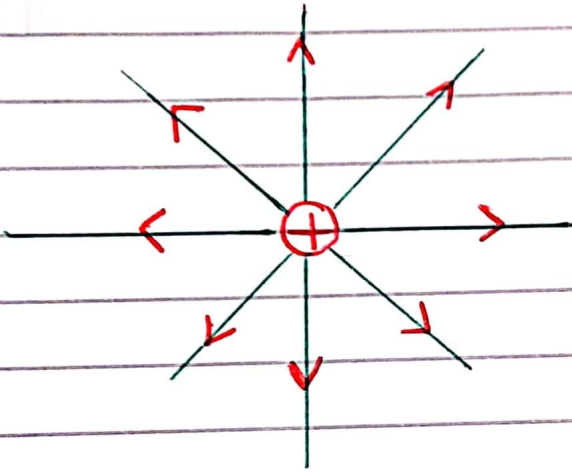
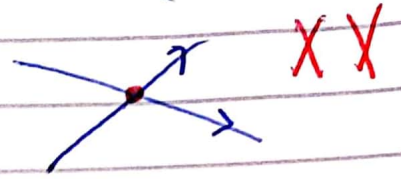
(ضعف، ضعف)

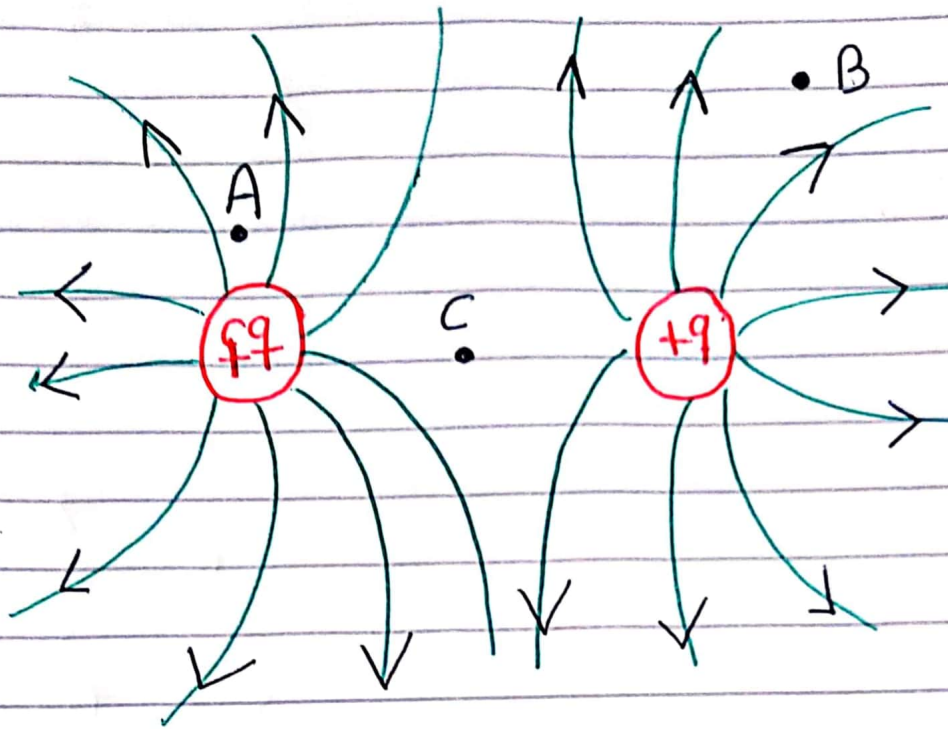




**6** No two field lines can cross. (لا يمكن أن يتقاطعا)

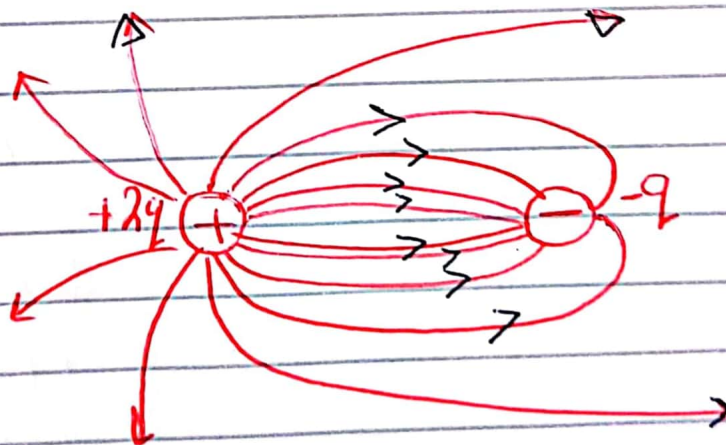
جميع الأسهم في اتجاه للمجال وهذا مستحيل





$E_A > E_B$  (لأن كثافة خطوط عند A أكبر من عند B)

$E_C = 0$  (zero) none line. (لا يوجد مجال)



$$\frac{\text{Number of lines leaving } (2q)}{\text{Number of lines approaching } (-q)} = \frac{16}{8} = (2)$$



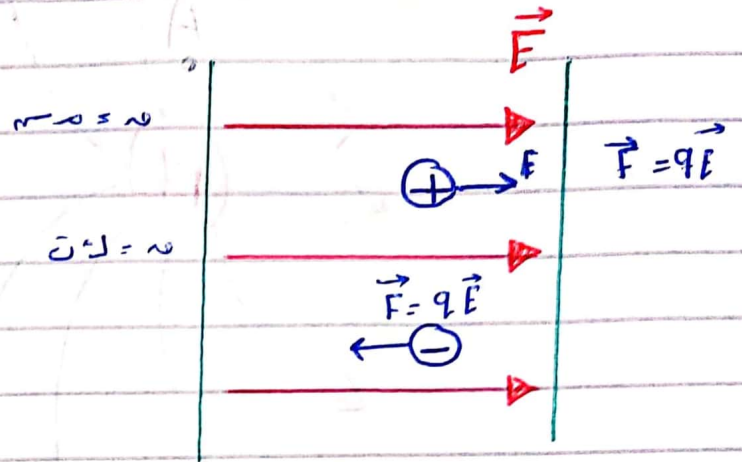
## \* motion of charged particle in a uniform electric field

حركة جسيم مشحون داخل مجال كهربائي

$$\vec{F} = q\vec{E}$$

$$\vec{F} = m\vec{a}$$

$$q\vec{E} = m\vec{a}$$



(acceleration)

$$a = \frac{qE}{m}$$

التسارع الذي (كتسبه الجسيم)

إذا كان المجال منتظم، ثابت المقدار والاتجاه

له إذا التسارع ثابت

تعتبر الدراسة في هذا الفصل على حركة جسيم مشحون داخل مجال منتظم (ثابت)

If  $\vec{E}$  is uniform then  $\vec{a} = \text{constant}$

مجال ثابت

وبما أن التسارع ثابت نستخدم قوانين الحركة في حركة ثابتة

## [Comments]

1) If  $\vec{E} = \text{Constant}$  ( $\vec{E}$  is uniform) then

$$\vec{a} = \frac{qE}{m} = \text{Constant}$$

$\Rightarrow$  Kinematic equations can be used.

$\Delta$

①	$U_{xf} = U_{xi} + a_x t$	} same for y.
②	$x_f - x_i = U_{xi} t + \frac{1}{2} a_x t^2$	
③	$U_{xf}^2 = U_{xi}^2 + 2 a_x (x_f - x_i)$	

2) If  $q$  is positive  $\Rightarrow \vec{a}$  is in the direction of  $\vec{E}$

3) If  $q$  is negative  $\Rightarrow \vec{a}$  is opposite to  $\vec{E}$

دالة الشحنة الموجبة تتأثر بقوة مع المجال إذا كانت مع المجال  
(لأن القوة بنفس اتجاه المجال)



## Ex. 23.10

⇒ A uniform electric field  $\vec{E}$  is directed along the x axis between parallel plates of charge separated by a distance  $d$  as shown in figure. A positive point charge  $q$  of mass  $m$  is released from rest at a point (A) next to the positive plate and accelerates to a point (B) next to the negative plate.

$$\vec{E} = \text{Constant (uniform)}$$

$$E_x = E, \quad E_y = 0$$

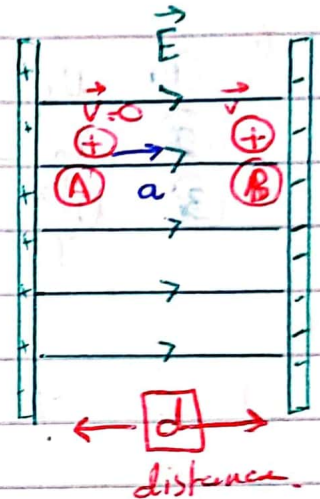
$$q, m$$

$$v_i = v_A = 0$$

$$v_B = ??$$

$$x_i = x_A = 0$$

$$x_f = x_B = d$$



(A) Find the speed of the particle at (B) by modeling it as a particle under constant acceleration.

$$\vec{a} = \frac{q\vec{E}}{m}$$

$$a_y = \frac{qE_y}{m} = 0$$

$$a_x = \frac{qE_x}{m} = \frac{qE}{m}$$

$$v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$v_B^2 = \cancel{v_A^2} + 2 \frac{qE}{m} (d - 0)$$

$$v_B = \sqrt{\frac{2qEd}{m}}$$

(B) find the speed of the particle at B by modeling it as a nonisolated system in terms of energy.

نستخدم نظرية الشغل والطاقة الحركية

Non-isolated system.  $\xrightarrow{\text{الشغل}}$

(ما يتأثر عليه قوة خارجية)

System  $\equiv$  charge.

[chapter 7  
physics(1)]

$$W_{\text{ext}} = K_B - K_A$$

$$F \Delta x \cos \theta = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2$$

$\rightarrow$  Rev

$$F = d \cos \theta \xrightarrow{\text{zero}} = \frac{1}{2} m v_B^2$$

(1)

$$q E d = \frac{1}{2} m v_B^2$$

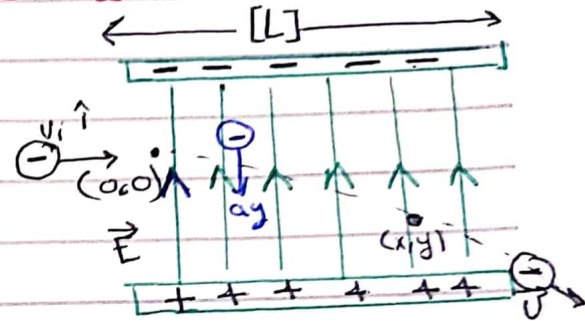
$$v_B = \sqrt{\frac{2 q E d}{m}}$$



Ex. 23.11 p. 712.

⇒ An electron enters the region of a uniform electric field as shown, with  $v_i = 3.00 \times 10^6 \text{ m/s}$  and  $E = 200 \text{ N/C}$ . The horizontal length of the plates is  $L = 0.100 \text{ m}$ .

A) Find the acceleration of the electron while it is in the electric field.



$$q = -1.6 \times 10^{-19} \text{ C}$$

$$m = 9.11 \times 10^{-31} \text{ kg}$$

$$v_i = 3 \times 10^6 \text{ m/s}$$

$$E = (200 \hat{j})$$

$$E_x = 0 \quad E_y = 200 \quad , \quad L = 0.1 \text{ m} \quad (\text{طول الصفحة})$$

الناتج في (x) ناتج صفر

$$F = qE = ma$$

$$\vec{a} = \frac{qE}{m}$$

$$a_x = \frac{qE_x}{m} = 0$$

$$a_y = \frac{qE_y}{m} = - \frac{(1.6 \times 10^{-19})(200)}{9.11 \times 10^{-31}} \Rightarrow a_y = -3.51 \times 10^{13} \text{ m/s}^2$$

$$\vec{a} = (-3.51 \times 10^{13} \hat{j}) \text{ m/s}^2$$

(B) Assuming the electron enters the field at time  $t=0$ , find the time at which it leaves the field.

الزمن الذي يحتاجه إلكترون من لحظة دخوله المجال  
المعقد (يقطع أفقياً المسافة  $L$ )

$$U_x = U_i (\cos 0) = U_i = 3 \times 10^6 \text{ m/s} \quad (1)$$

$$U_{y_i} = U_i \sin 0 = 0.$$

$$\boxed{x_i = 0} \quad \boxed{x_f = L}$$

$$x_f - \cancel{x_i} = U_x t + \cancel{\frac{1}{2} a_x t^2} \quad \text{zero}$$

$$L = U_x t$$

$$t = \frac{L}{U_x} = \frac{0.1 \text{ m}}{3 \times 10^6 \text{ m/s}} = 3.33 \times 10^{-8} \text{ s}.$$

(C) Assuming the vertical position of the electron as it enters the field is  $y_i = 0$ , what is its vertical position when it leaves the field?

كم الزنر من عتري يكون  
 $\Delta y = ?$

$$y_i = 0 \quad y_f = ?$$

$$y_f - \cancel{y_i} = \cancel{U_{y_i} t} + \frac{1}{2} a_y t^2 \quad \text{zero zero}$$

$$y_f = \frac{1}{2} a_y t^2 = \frac{1}{2} (-3.51 \times 10^{13} \text{ m/s}^2) (3.33 \times 10^{-8} \text{ s})^2$$

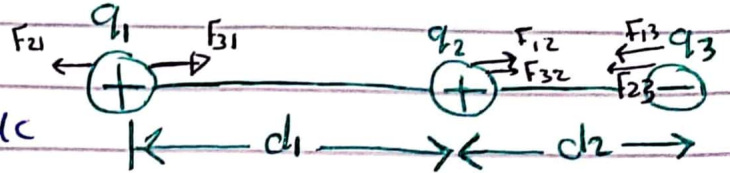
$$y_f = -1.59 \times 10^{-2} \text{ m} = -1.59 \text{ cm}$$



## Prob 12 #

Three point charges lie along a straight line as shown in figure, where  $q_1 = 6.00 \mu\text{C}$ ,  $q_2 = 1.50 \mu\text{C}$ , and  $q_3 = -2.00 \mu\text{C}$ . The separation distances are  $d_1 = 3.00 \text{ cm}$  and  $d_2 = 2.00 \text{ cm}$ . Calculate the magnitude and direction of the net electric force on (a)  $q_1$ , (b)  $q_2$  and (c)  $q_3$ .

$$\begin{aligned} q_1 &= 6 \mu\text{C} \\ q_2 &= 1.5 \mu\text{C} \quad q_3 = -2 \mu\text{C} \\ d_1 &= 3 \text{ cm}, \quad d_2 = 2 \text{ cm} \end{aligned}$$



$$F_{12} = F_{21} = \frac{k |q_1| |q_2|}{d_1^2} = \frac{(9 \times 10^9) (6 \times 10^{-6}) (1.5 \times 10^{-6})}{(0.03)^2} = \boxed{89.5 \text{ N}}$$

$$F_{13} = F_{31} = \frac{k |q_1| |q_3|}{(d_1 + d_2)^2} = \boxed{43.2 \text{ N}}$$

(لأنه نفس الشحنة سالبة)

$$F_{23} = F_{32} = \frac{k |q_2| |q_3|}{d_2^2} = \frac{(9 \times 10^9) (1.5 \times 10^{-6}) (2 \times 10^{-6})}{(2 \times 10^{-2})^2} = \boxed{67.5 \text{ N}}$$

لأنها سالبة  
السالبة

$$\text{A) } \vec{F}_1 = (F_{31} - F_{21}) \hat{i} = (-46.3 \hat{i}) \text{ N}$$

$$\text{B) } \vec{F}_2 = (F_{12} + F_{32}) \hat{i} = (157 \hat{i}) \text{ N}$$

$$\text{C) } \vec{F}_3 = -(F_{13} + F_{23}) \hat{i} = (-111 \hat{i}) \text{ N}$$

# [Prob. 15] #

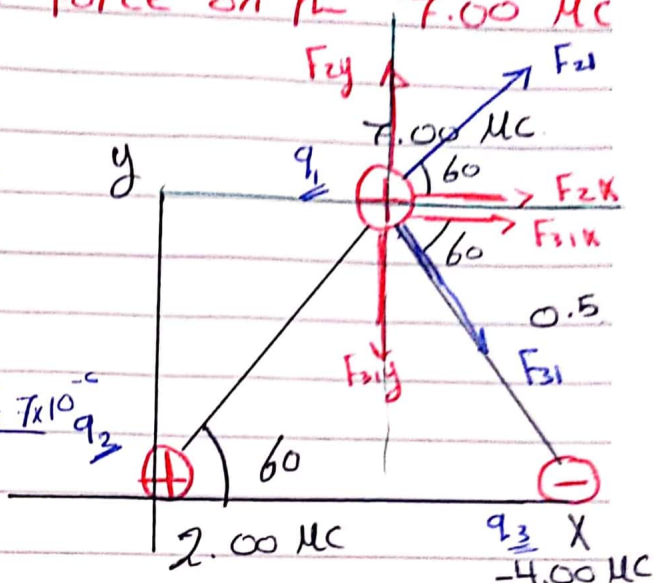
Three charged particles are located at the corners of an equilateral triangle as shown in figure. Calculate the total electric force on the  $7.00 \mu\text{C}$  charge.

$$q_1 = 7 \mu\text{C}, \quad q_3 = -4 \mu\text{C}$$

$$q_2 = 2 \mu\text{C}, \quad d = 0.5 \text{ m}$$

$$F_{21} = \frac{k |q_2| |q_1|}{d^2} = \frac{9 \times 10^9 \times 2 \times 10^{-6} \times 7 \times 10^{-6}}{(0.5)^2}$$

$$= 0.504 \text{ N}$$



$$F_{31} = \frac{k |q_3| |q_1|}{d^2} = \frac{9 \times 10^9 \times 4 \times 10^{-6} \times 7 \times 10^{-6}}{(0.5)^2} = 1.008 \text{ N}$$

$$\Rightarrow F_{21x} = F_{21} \cos 60 = 0.252 \text{ N}$$

$$F_{21y} = F_{21} \sin 60 = 0.436 \text{ N}$$

$$\Rightarrow F_{31x} = F_{31} \cos 60 = 0.504 \text{ N}$$

$$F_{31y} = -F_{31} \sin 60 = -0.872 \text{ N}$$

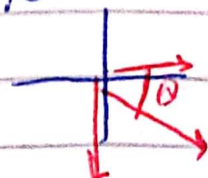
$$F_{1x} = F_{21x} + F_{31x} = 0.756 \text{ N}$$

$$F_{1y} = F_{21y} + F_{31y} = -0.436 \text{ N}$$

$$\vec{F}_1 = F_{1x} \hat{i} + F_{1y} \hat{j} = \vec{F}_1 = (0.756 \hat{i} - 0.436 \hat{j}) \text{ N}$$

$$F_1 = \sqrt{0.756^2 + (-0.436)^2} = 0.89 \text{ N}$$

$$\theta = \tan^{-1} \left( \frac{-0.436}{0.756} \right) = -30^\circ$$





Prob 23.16

p. 717

Two small metallic spheres, each of mass  $m = 0.200 \text{ g}$  are suspended as pendulums by light strings of length  $L$  as shown in figure. The spheres are given the same electric charge of  $7.2 \text{ nC}$ , and they come to equilibrium when each string is at an angle of  $\theta = 5.00^\circ$  with the vertical. How long are the strings?

$$m = 0.2 \text{ kg}, \quad \theta = 5^\circ$$

$$q = 7.2 \text{ nC}$$

$$a = L \sin \theta$$

$$F_e = T \sin \theta$$

$$mg = T \cos \theta$$

$$\frac{F_e}{mg} = \tan \theta \rightarrow F_e = mg \tan \theta$$

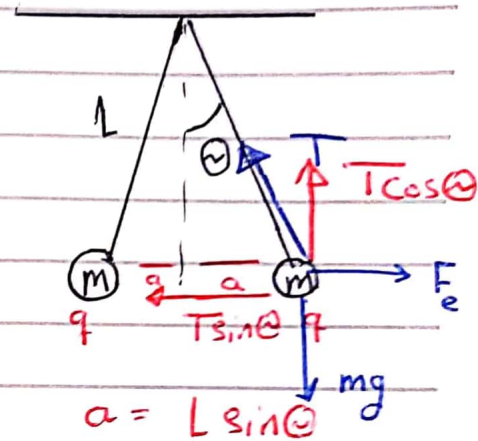
$$k \frac{|q|^2}{(2a)^2} = mg \tan \theta$$

$$\frac{k |q|^2}{4L^2 \sin^2 \theta} = mg \tan \theta$$

$$L = \sqrt{\frac{k |q|^2}{4 mg \tan \theta \sin^2 \theta}} = \sqrt{\frac{9 \times 10^9 \times (7.2 \times 10^{-9})^2}{4 (0.2)(10) \tan(5) \sin^2(5)}}$$

$$= 0.299 \text{ m}$$

$$= 29.9 \text{ cm}$$



prob. 25

p. 718

four charged particles are at the corners of a square of side  $a$  as shown in figure. Determine (a) the electric field at the location of charge  $q$  and (b) the total electric force exerted on  $q$ .

A)  $E_2 = \frac{k(2q)}{a^2}$

$$E_3 = \frac{k(3q)}{2a^2}$$

$$E_4 = k \frac{4q}{a^2}$$

$$E_{2x} = E_2 \cos 0 = \frac{k(2q)}{a^2}$$

$$E_{2y} = E_2 \sin 0 = 0$$

$$E_{3x} = E_3 \cos 45 = \frac{k(3q)}{2a^2} \cos 45$$

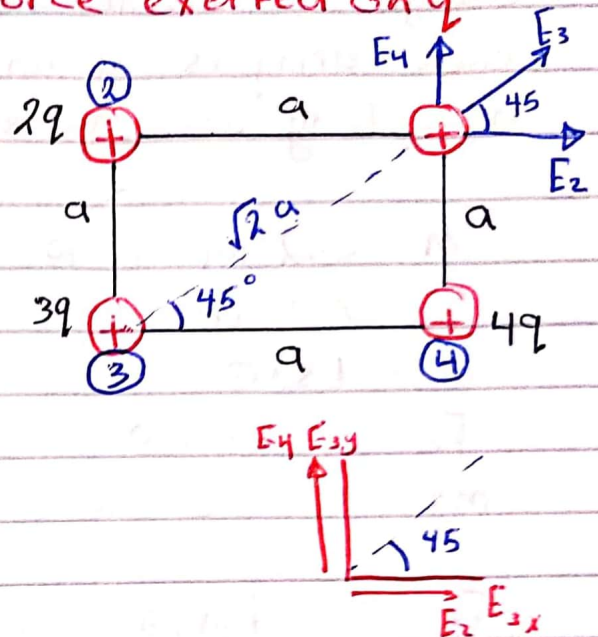
$$E_{3y} = E_3 \sin 45 = \frac{k(3q)}{2a^2} \sin 45$$

$$E_{4x} = E_4 \cos 90 = 0$$

$$E_{4y} = E_4 \sin 90 = \frac{k(4q)}{a^2}$$

also  $E_x = E_{2x} + E_{3x} + E_{4x} = \frac{kq}{a^2} \left( 2 + \frac{3}{2} \cos 45 + 0 \right)$

$$E_x = 3.06 \frac{kq}{a^2}$$





$$E_y = E_{2y} + E_{3y} + E_{4y} = \frac{kq}{a^2} \left( 0 + \frac{3}{2} \sin 45 + 4 \right)$$

$$E_y = 5.06 \frac{kq}{a^2}$$

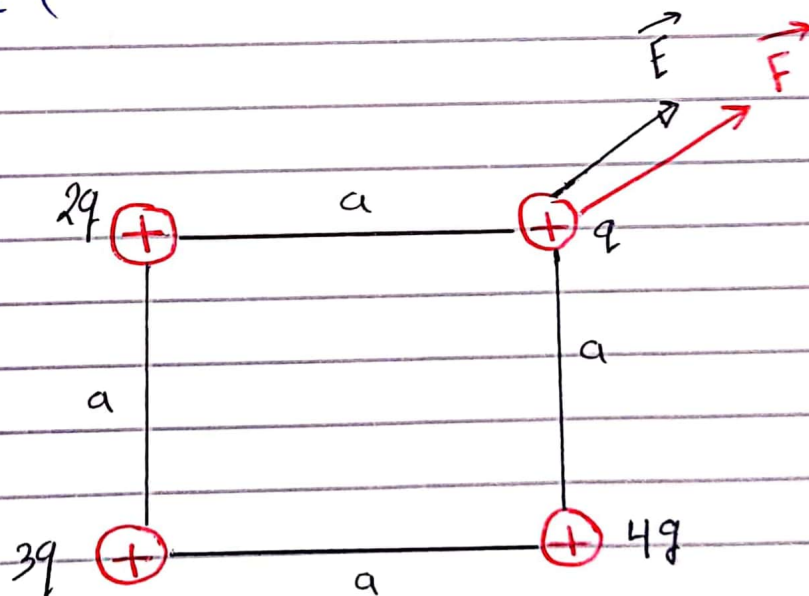
$$\vec{E} = E_x \hat{i} + E_y \hat{j}$$

$$= \frac{kq}{a^2} (3.06 \hat{i} + 5.06 \hat{j})$$

B)

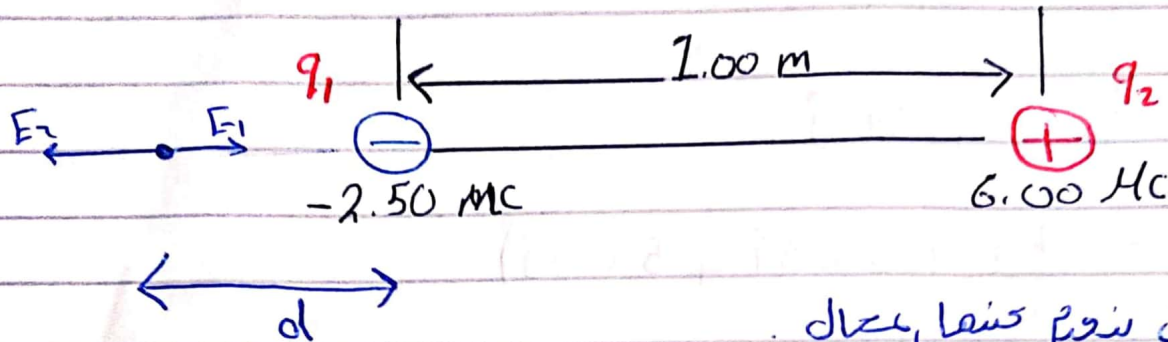
$$\vec{F} = q \vec{E}$$

$$= \frac{kq^2}{a^2} (3.06 \hat{i} + 5.06 \hat{j})$$



Prob 29 . P. 718

In figure, determine the point (other than infinity) at which the electric field is zero



النقطة التي يكون فيها المجال

$$\sum E_x = 0$$

$$E_1 - E_2 = 0$$

$$E_1 = E_2$$

$$k \frac{|q_1|}{d^2} = k \frac{|q_2|}{(1+d)^2}$$

$$\frac{2.5 \times 10^{-6}}{d^2} = \frac{6 \times 10^{-6}}{d^2 + 2d + 1}$$

$$6d^2 = 2.5d^2 + 5d + 2.5$$

$$3.5d^2 - 5d - 2.5 = 0$$

$$d = \frac{+5 \pm \sqrt{25 - 5(3.5)(-2.5)}}{2(3.5)}$$

$$d = 1.82 \text{ m} \quad \checkmark$$

$$= -0.392 \text{ m} \quad \times$$

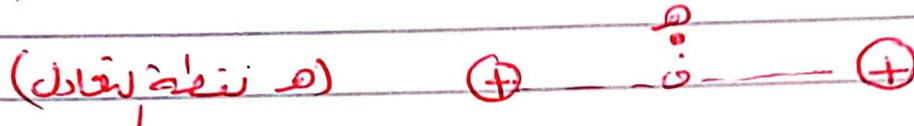
نقطة المجال في  
الصفر التالية



4 في السؤال السابقة من تعرف اين ينعم المجال .

الحالات [1] الشحنتان متماثلتان في الإشارة .

[2] متماثلتان في المقدار - تقع نقطة التعادل بينهما في منتصف المسافة



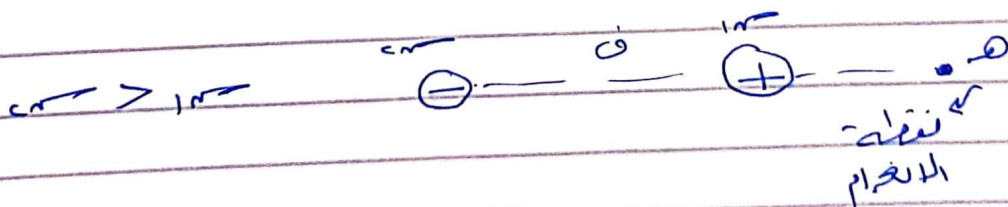
[3] مختلفتان بالمقدار - تقع نقطة التعادل بينهما اقرب للشحنة الاكبر



[4] الشحنتان مختلفتان في الإشارة .

[5] متماثلتان بالمقدار ← لا يوجد نقطة تعادل

[6] مختلفتان بالمقدار - تقع نقطة التعادل خارجهما (اقرب للشحنة الاكبر)



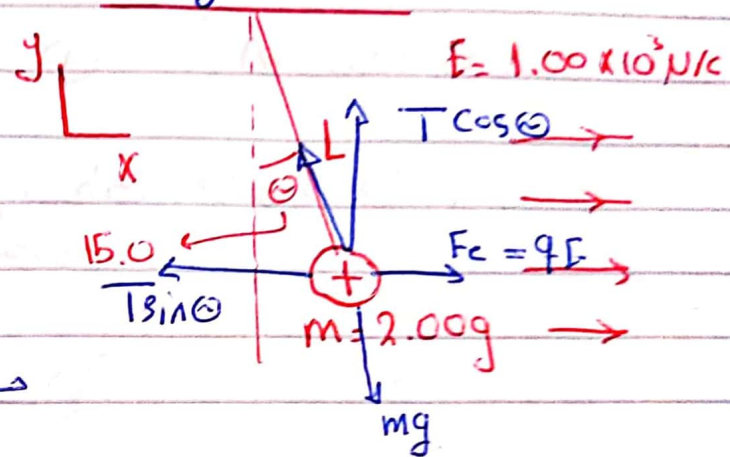
### Prob. 33

A small, 2.00-g plastic ball is suspended by a 20.0-cm-long string in a uniform electric field as shown. If the ball is in equilibrium when the string makes a 15.0° angle with the vertical, what is the net charge on the ball?

$q$  is positive. ,  $m = 2g = 0.002 \text{ kg}$

$$E = 1 \times 10^3 \text{ N/C}$$

$$\theta = 15^\circ$$



$$qE = T \sin \theta$$

$$mg = T \cos \theta \quad \text{Divide}$$

$$\frac{qE}{mg} = \tan \theta$$

$$q = \frac{mg \tan \theta}{E}$$

$$= \frac{(0.002) \times (10) (\tan 15)}{1 \times 10^3} =$$

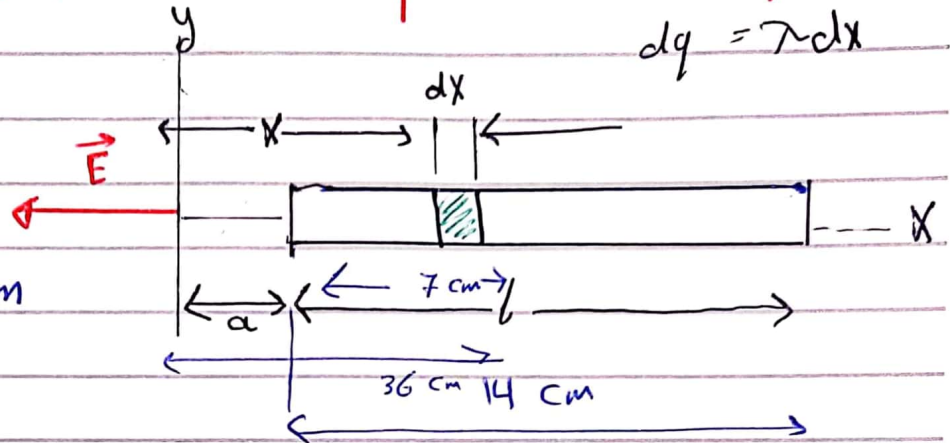
$$5.25 \times 10^{-6} \text{ C}$$

$$= 5.25 \text{ } \mu\text{C}$$



prob 37

A rod 14.0 cm long is uniformly charged and has a total charge of  $-22.0 \mu\text{C}$ . Determine (a) the magnitude and (b) the direction of the electric field along the axis of the rod at a point 36.0 cm from its center.



$$L = 14 \text{ cm} = 0.14 \text{ m}$$

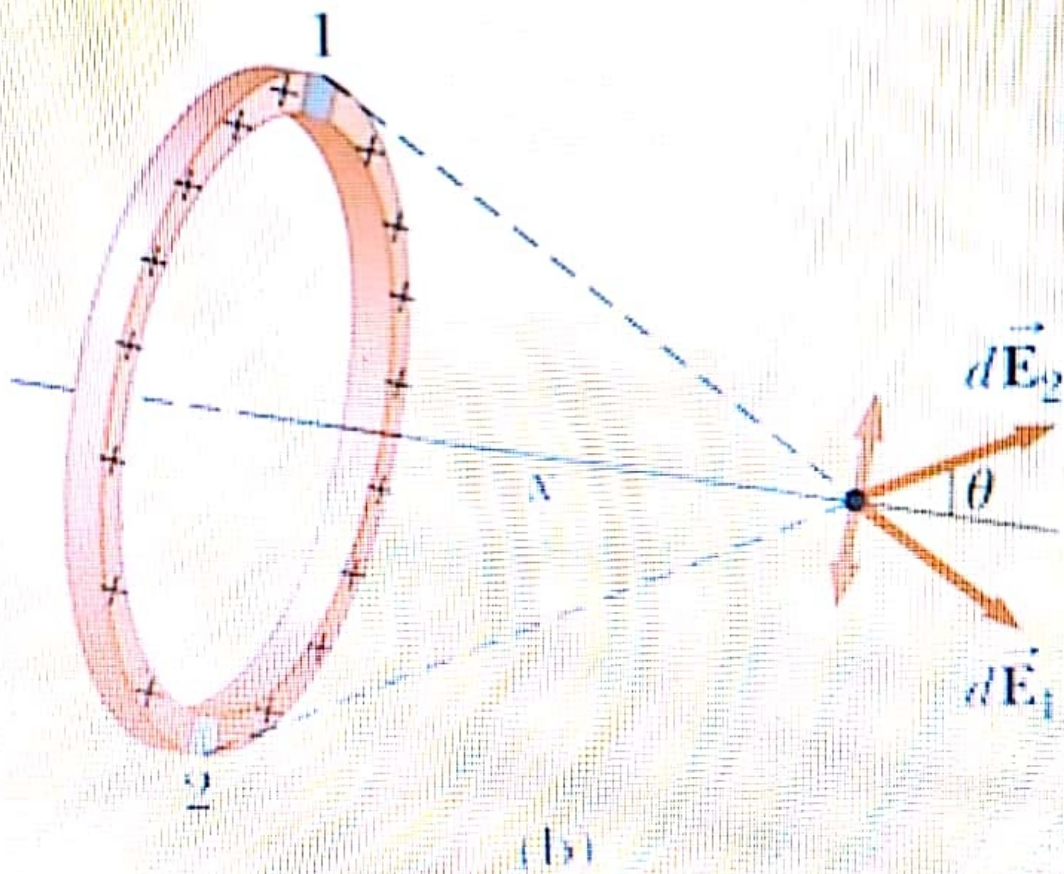
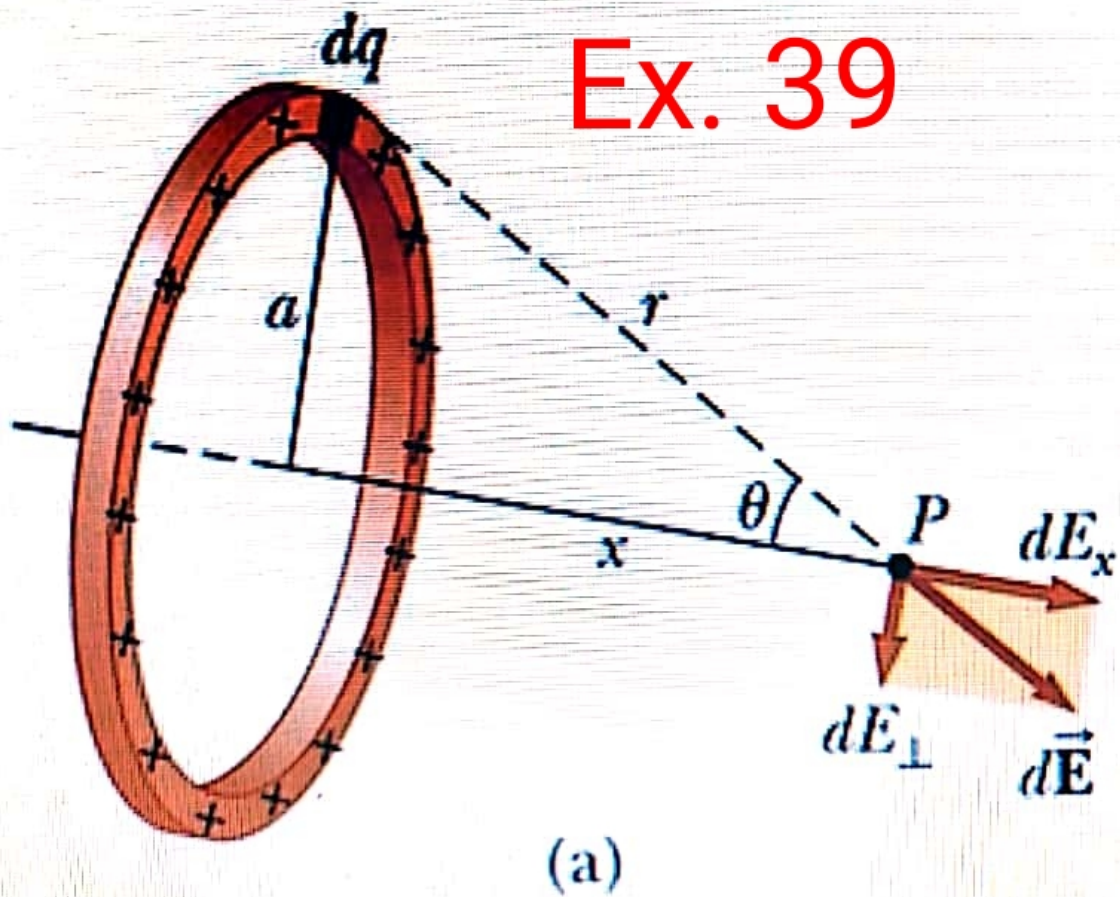
$$Q = -22 \mu\text{C}$$

$$a = 36 \text{ cm} - 7 \text{ cm} = 29 \text{ cm}$$

$$E = \frac{kQ}{a(L+a)}$$

$$= \frac{(9 \times 10^9)(22 \times 10^{-6})}{(0.29)(0.41 + 0.29)} = 1.59 \times 10^6 \text{ N/C}$$

# Ex. 39



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Prob 39 -

A uniformly charged ring of radius 10.0 cm has a total charge of 75.0  $\mu\text{C}$ . Find the electric field on the axis of the ring at (a) 1.00 cm (b) 5.00 cm (c) 30.0 cm and (d) 100 cm from the center of the ring

$$a = 10 \text{ cm} = 0.1 \text{ m}$$

$$q = 75 \mu\text{C} = 75 \times 10^{-6} \text{ C}$$

$$E_y = \frac{k q x}{(x^2 + a^2)^{\frac{3}{2}}}$$

$$\text{A) } x = 1 \text{ cm} = 0.01 \text{ m}$$

$$E_x = \frac{(9 \times 10^9)(75 \times 10^{-6})(0.01)}{(0.01^2 + 0.1^2)^{\frac{3}{2}}}$$

$$E_x = 6.64 \times 10^6 \text{ N/C}$$

$$\Rightarrow \vec{E} = (6.64 \times 10^6 \hat{i}) \text{ N/C}$$

P. 45 . P. 720

A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle as shown.

The rod has a total charge of  $-7.50 \mu\text{C}$ . Find (a) the magnitude and (b) the direction of the electric field at  $\odot$ , the center of the semicircle.

المجال في مركز النصف

$$L = 14 \text{ cm} = 0.14 \text{ m}$$

$$q = -7.5 \mu\text{C} \equiv \text{uniform}$$

$$\Rightarrow E_y \text{ cancel.}, \quad dE = k \frac{dq}{r^2}$$

$$dE_x = dE \sin \theta$$

$$dE_x = \frac{k dq \sin \theta}{r^2} \quad \text{تكال دافيس}$$

$$E_x = \frac{k}{r^2} \int dq \sin \theta$$

$$\begin{array}{l|l} dL = r d\theta & \lambda = \frac{q}{L} \\ dq = \lambda dL & \\ = \lambda r d\theta & L = \pi r \\ & r = \frac{L}{\pi} \end{array}$$

$$E_x = \frac{k}{r^2} \int dq \sin \theta$$



$$E_x = \frac{k}{r^2} \int dq \sin \theta$$

$$= \frac{k}{r^2} \int_0^\pi \lambda r d\theta \sin \theta$$

$$= \frac{k\lambda r}{r^2} \int \sin \theta d\theta$$

$$= \frac{k\lambda}{r} \left( -\cos \theta \right) \Big|_0^\pi$$

$$E_x = \frac{2k\lambda}{r}$$

$$= \frac{2k \left( \frac{q}{L} \right)}{\frac{L}{\pi}}$$

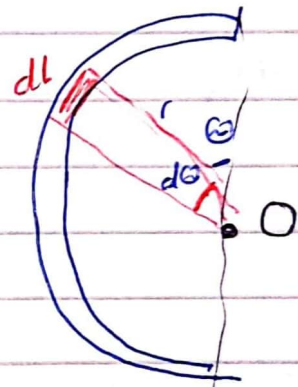
$$E_x = \frac{2\pi k q}{L^2}$$

$$= \frac{(2\pi)(9 \times 10^9)(7.5 \times 10^{-6})}{(0.14)^2}$$

$$E_x = 2.16 \times 10^7 \text{ N/C}$$

لأنه لا يوجد  
للمركز

$$\vec{E} = (-2.16 \times 10^7 \hat{i}) \text{ N/C}$$



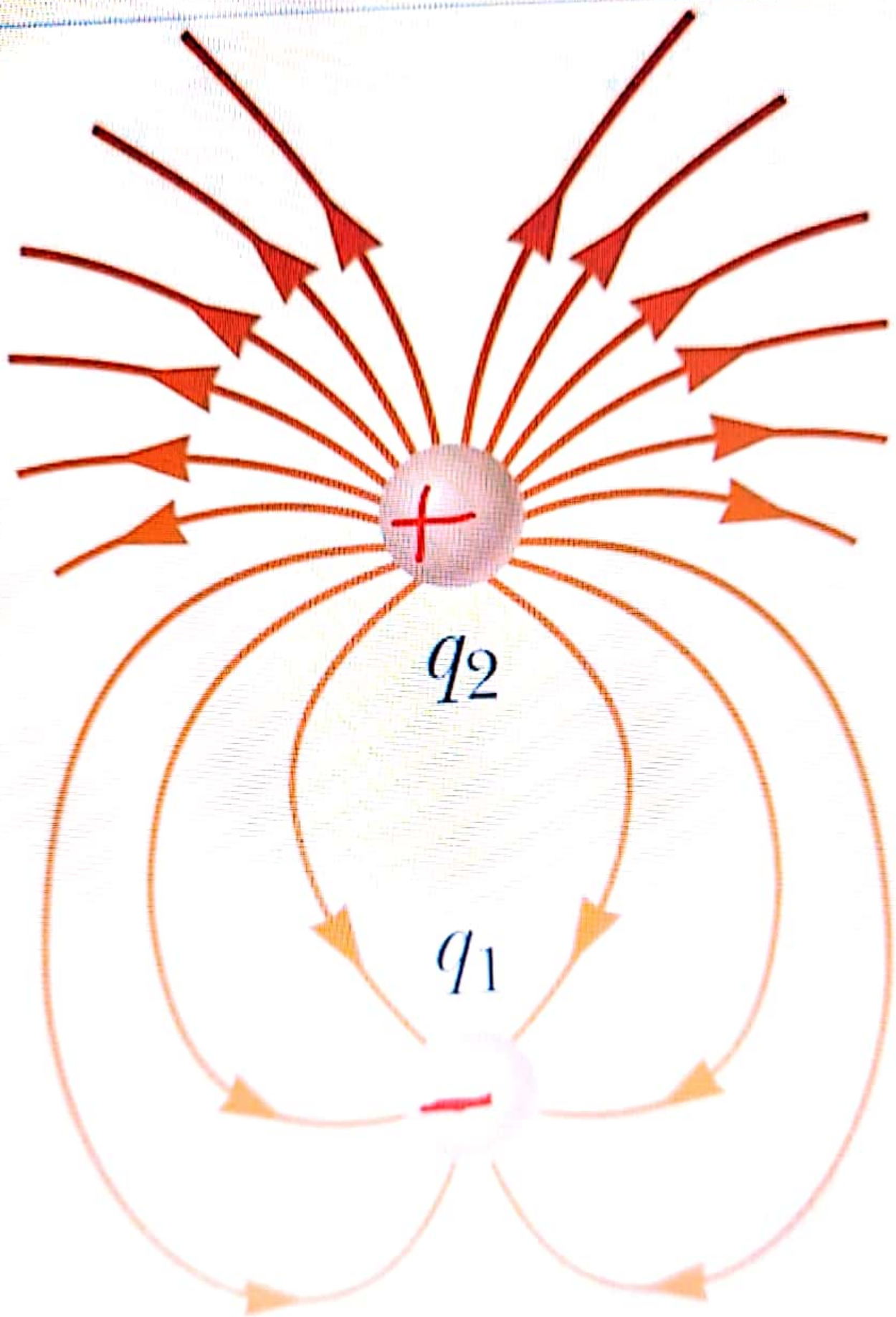




Figure shows the electric field lines for two charged particles separated by a small distance  
(a) determine the ratio  $q_1/q_2$  (b) what are the signs of  $q_1$  and  $q_2$

$$\left| \frac{q_1}{q_2} \right| = \frac{6}{18} = \frac{1}{3}$$

$$\frac{q_1}{q_2} = -\frac{1}{3}$$

$$q_2 \longrightarrow (+)$$

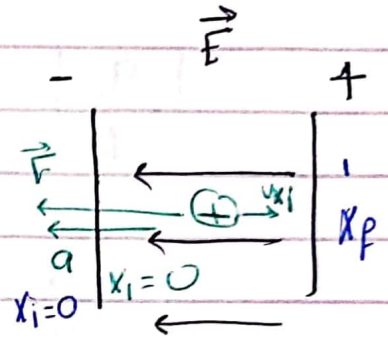
$$q_1 \longrightarrow (-)$$

### Prob. 52

A proton is projected in the positive  $x$  direction into a region of a uniform electric field

$\vec{E} = (-6.00 \times 10^5) \hat{i} \text{ N/C}$  at  $t=0$ . The proton travels 7.00 cm as it comes to rest. Determine (a) the acceleration of the proton, (b) its initial speed, and (c) the time interval over which the proton comes to rest.

$$\begin{aligned} q &= 1.6 \times 10^{-19} \text{ C} \\ m &= 1.67 \times 10^{-27} \text{ kg} \\ E &= (-6 \times 10^5 \hat{i}) \text{ N/C} \end{aligned} \quad \left| \quad \begin{aligned} x_f &= 7 \text{ m} \\ v_{xf} &= 0 \end{aligned} \right.$$



$$E_x = -6 \times 10^5 \text{ N/C}, \quad E_y = 0$$

$$(A) \quad q\vec{E} = m\vec{a} \Rightarrow \vec{a} = \frac{q\vec{E}}{m}$$

$$a_x = \frac{qE_x}{m} = -5.76 \times 10^{13} \text{ m/s}^2$$

$$a_y = \frac{qE_y}{m} = 0$$

$$(B) \quad v_{xf}^2 = v_{xi}^2 + 2a_x(x_f - x_i)$$

$$0 = v_{xi}^2 + 2(-5.76 \times 10^{13})(0.07 - 0)$$

$$v_{xi} = 2.84 \times 10^6 \text{ m/s}$$

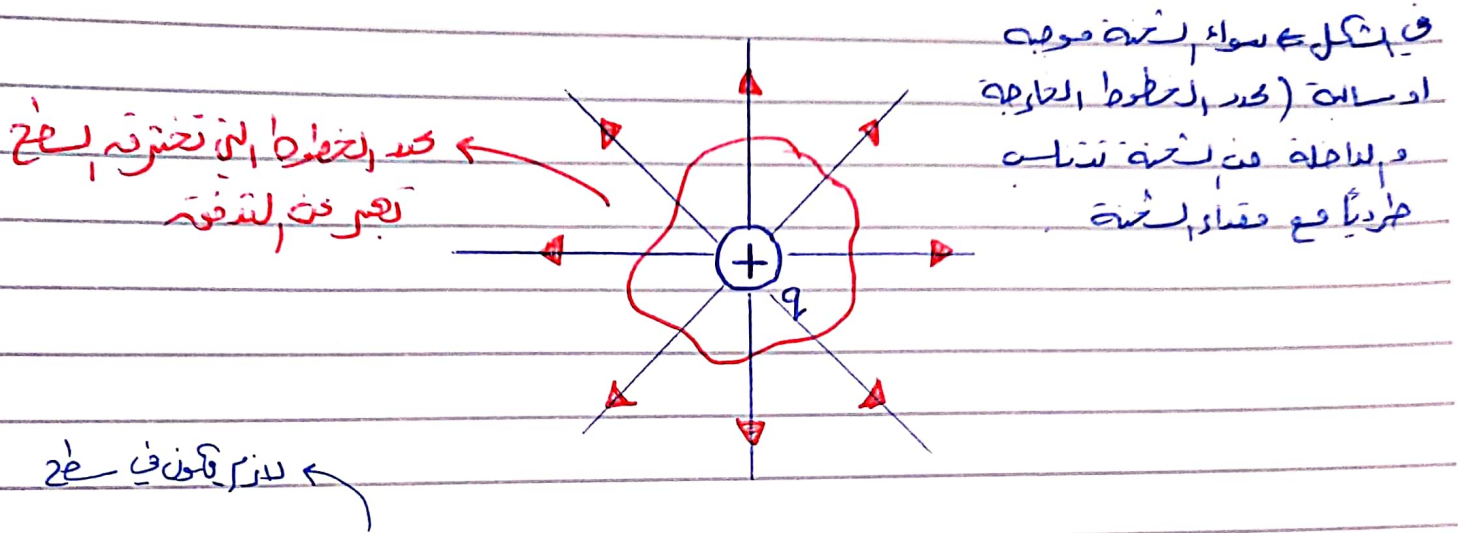
$$(C) \quad v_{xf} = v_{xi} + a_x t$$

$$\begin{aligned} 0 &= 2.84 \times 10^6 + (-5.76 \times 10^{13}) t \\ t &= 4.93 \times 10^{-8} \text{ s} \end{aligned}$$



# - chapter 24 -

## Gauss's Law



\* **Electrical flux** is a measure of the number of field lines penetrating some surface.  
تخترق

عدد الخطوط التي تخترق سطح معين

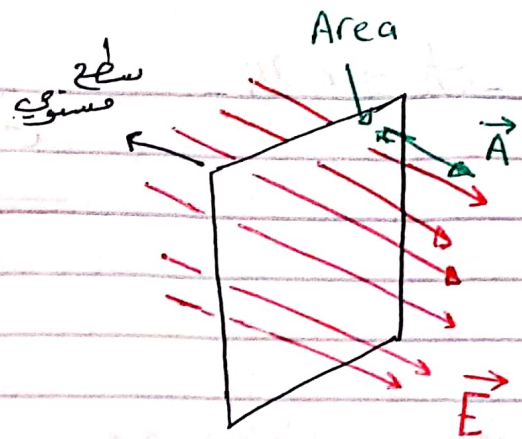
\* بما أن الشحنة الكهربائية تتناسب مع عدد الخطوط التي تخترق السطح، فإن الشحنة تتناسب مع عدد الخطوط التي تخترق السطح.  
لذلك فإن عدد الخطوط التي تخترق السطح

\* If the surface is closed and encloses some net charge, then the net number of lines that go through the surface (**electric flux**) is proportional to the net charge within the surface

\* إذا كان السطح مغلقاً ويحيط بشحنة معينة، فإن عدد الخطوط التي تخترق هذا السطح يتناسب مع الشحنة التي في الداخل.

\* Area is a vector ,  
normal to the surface

الـ (A) متجه ولاء انتظامها عودى كل السطح  
(الـ (A))



\* The electric flux ( $\Phi$ ) of a  
uniform electric field ( $E$ )  
normal to the surface

( $\vec{E} \parallel \vec{A}$ ) parallel (موازي)

If  $E \perp$  surface ( $\vec{E} \parallel \vec{A}$ )  
then

$$\Phi = EA$$

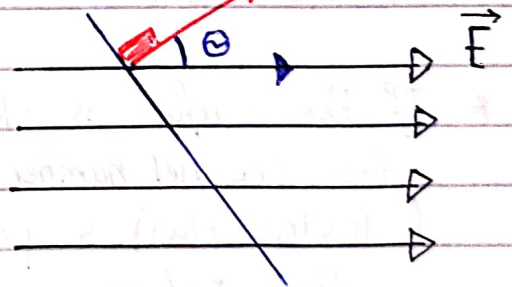
\* If  $\vec{E}$  is not parallel to  $\vec{A}$  ( $\vec{E}$  is not normal to the  
surface)

بجاء غير عادي في السطح

الزاوية  $\theta$  بين  $E$  و  $A$  متجه

$$\Phi = EA \cos \theta$$

$$= \vec{E} \cdot \vec{A}$$



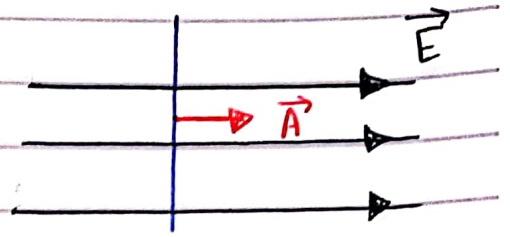
Dot products  
منه المنتجات



If  $\theta = 0$

$$\Rightarrow \Phi = EA \cos 0$$

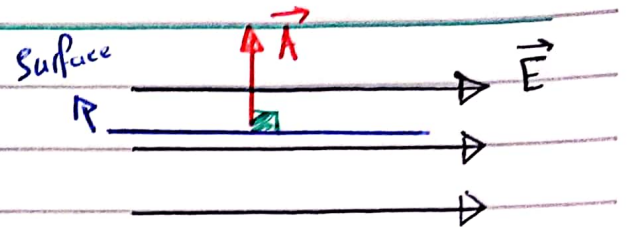
$$\Phi = EA$$



If  $\theta = \frac{\pi}{2}$

$$\Rightarrow \Phi = EA \cos \frac{\pi}{2}$$

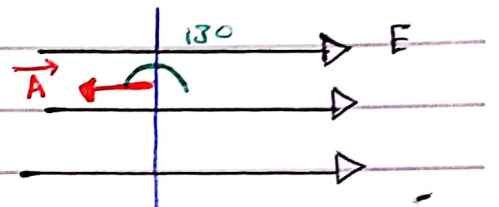
$$\Phi = 0$$



If  $\theta = \pi$

$$\Rightarrow \Phi = EA \cos \pi$$

$$\Phi = -EA$$



<next page>

In the figure  $\Rightarrow$

$$|\Phi_A| = |\Phi_{A_\perp}|$$

$$|\Phi_{A_\perp}| = EA_\perp = Ewh$$

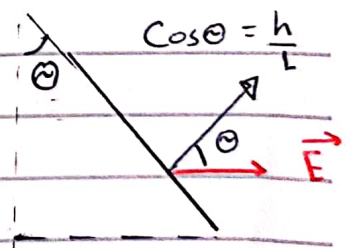
$$A_\perp = wh, \quad A = WL$$

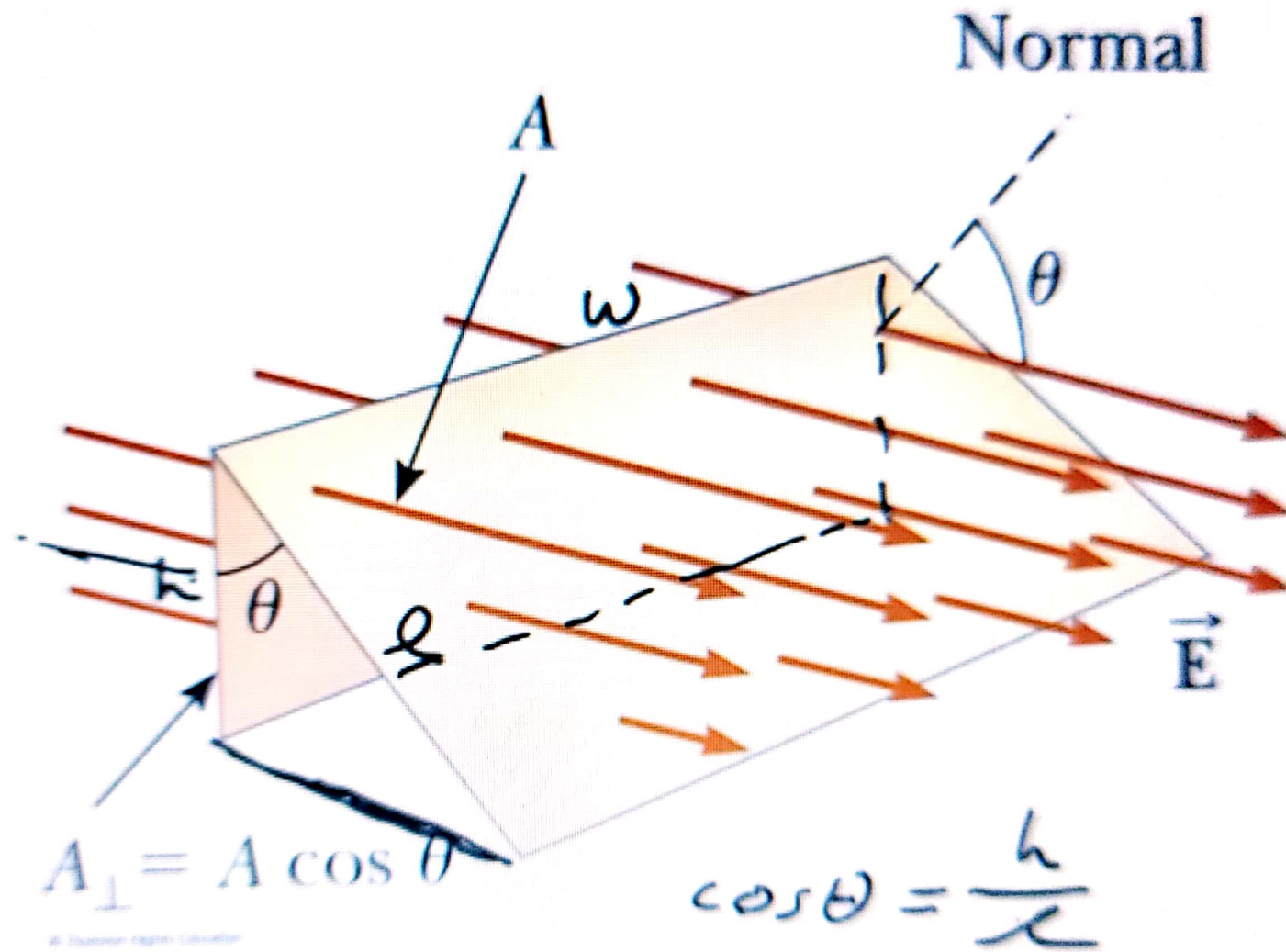
$$= WL \cos \theta$$

$$A_\perp = A \cos \theta$$

$$|\Phi_A| = |\Phi_{A_\perp}| = EA_\perp = EA \cos \theta$$

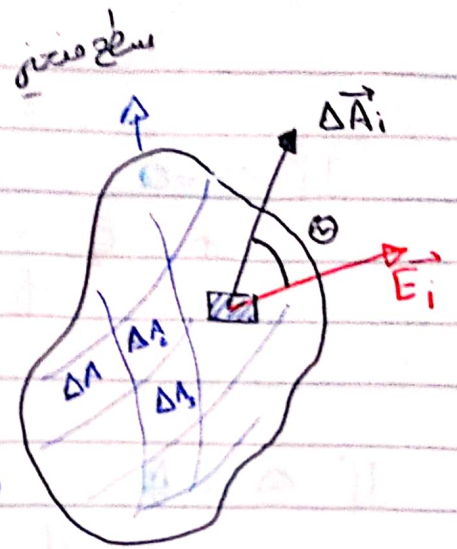
$$\Phi_A = EA \cos \theta = \vec{E} \cdot \vec{A}$$







For the most case:  
 $\vec{E}$  is not uniform and  
 $\vec{A}$  is Curved (not plane)



The electric flux,  $\Delta \Phi$ , of the electric field through the area element  $\Delta A_i$  is

التي تسببها  
 المجال الكهربائي  
 الصغيرة

$$\Delta \Phi_i = \vec{E}_i \cdot \Delta \vec{A}_i$$

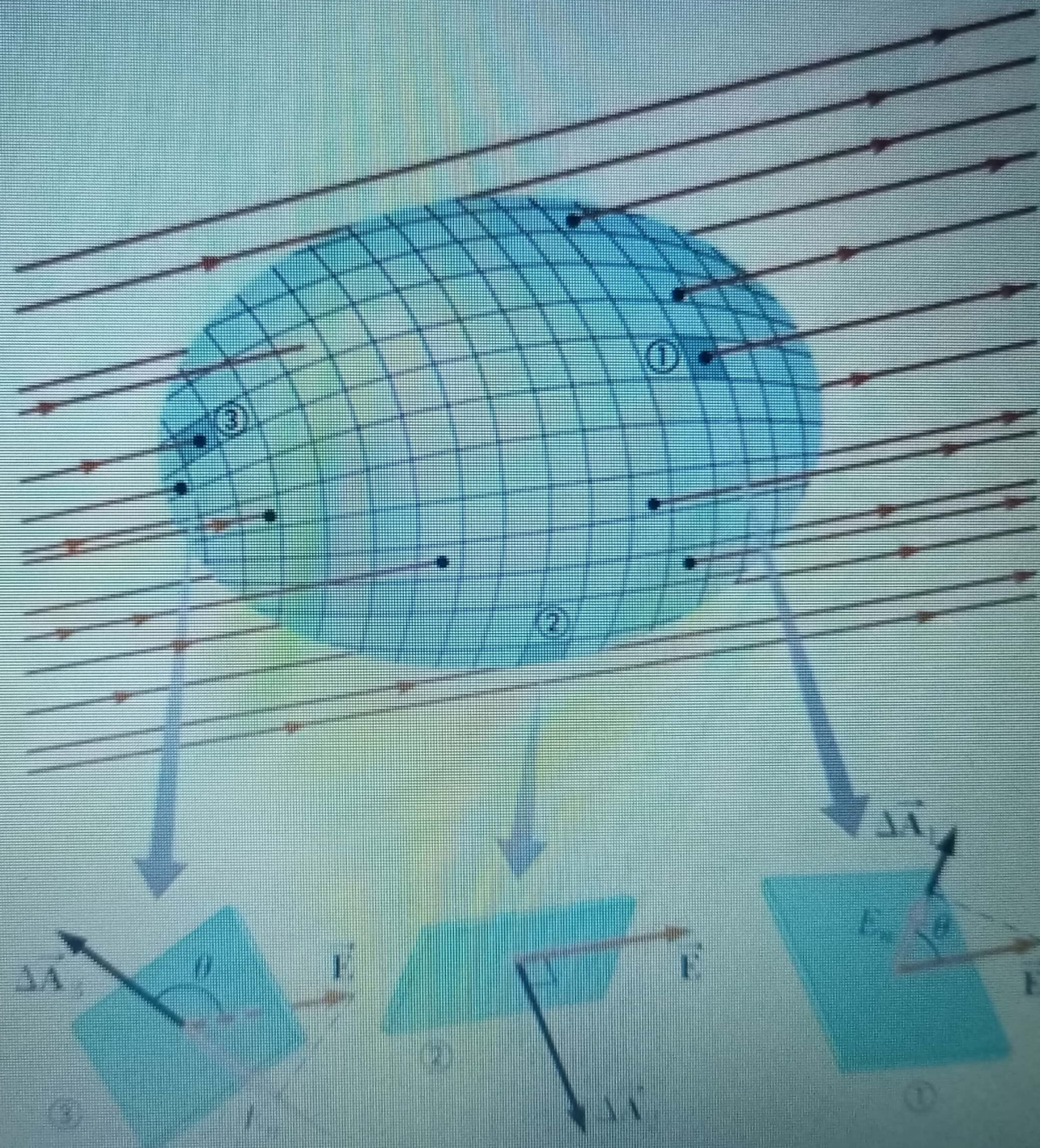
Summing the contributions of all area elements  
 (نجمع كل مساهمات)

$$\Phi = \sum_i \Delta \Phi_i = \sum_i \vec{E}_i \cdot \Delta \vec{A}_i$$

$$\Phi = \lim_{\Delta A_i \rightarrow 0} \sum_i \vec{E}_i \cdot \Delta \vec{A}_i = \int_s \vec{E} \cdot d\vec{A}$$

$$\left[ \Phi = \int \vec{E} \cdot d\vec{A} \right] \text{ most general case.}$$







في هذه البرصة السطح مغلق

If the surface closed

$$\Phi_c = \oint \vec{E} \cdot d\vec{A}$$

$\oint$

الدائرة تعني  
ان التكامل على سطح  
مغلق

$$\Delta\Phi_1 = \vec{E} \cdot \Delta A_1 = E \Delta A_1 \underbrace{\cos\theta}_{>0}$$

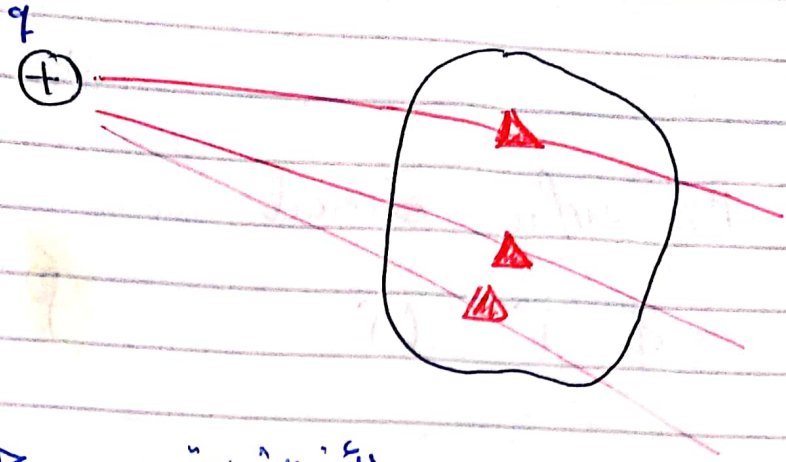
رسمه ①  
 $0 \leq \theta_1 \leq \frac{\pi}{2}$   
له تعني الـ  $\cos$  (+)

$$\Delta\Phi_2 = \vec{E} \cdot \Delta A_2 = E (\Delta A_2) \cos \frac{\pi}{2} = 0$$

رسمه ②  
 $\theta_2 = \frac{\pi}{2}$

$$\Delta\Phi_3 = E \cdot \Delta A_3 = E (\Delta A_3) \underbrace{\cos\theta_3}_{<0}$$

رسمه ③  
 $\frac{\pi}{2} \leq \theta_3 \leq \pi$



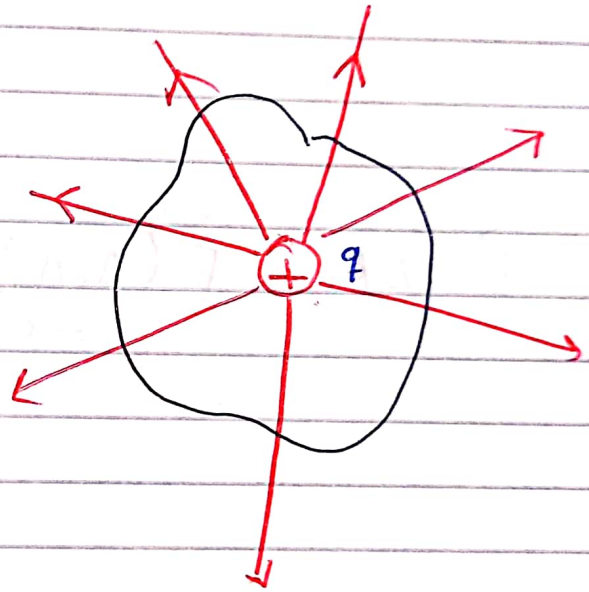
$$\Phi = 0 \rightarrow$$

الصافي

لأن الشحنة  
ليست بالداخل

$$\Phi \neq 0 \rightarrow$$

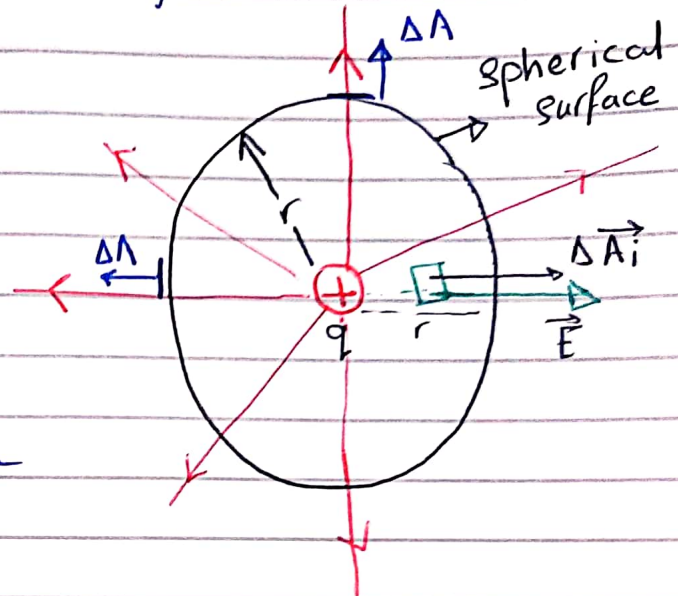
في الشحنة





\* Gauss's Law relates the net electric flux through a closed surface and the net charged enclosed by the surface.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q}{\epsilon_0}$$



\* The electric field at the surface is

$$\vec{E} = \frac{kq}{r^2} \hat{r}$$

∴ = 0 between  $\vec{E}$  and  $\Delta \vec{A}_i$

$$\Phi = \oint \vec{E} \cdot d\vec{A}$$

$$= \oint E dA = \oint \frac{kq}{r^2} dA$$

$$= \frac{kq}{r^2} \oint dA = \frac{kq}{r^2} A = \frac{kq}{r^2} (4\pi r^2)$$

← المجال

$$4\pi kq = \frac{1}{\epsilon_0} q = \frac{q}{\epsilon_0}$$

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

in  $\rightarrow$  inside the surface  
(20, 10, 15)

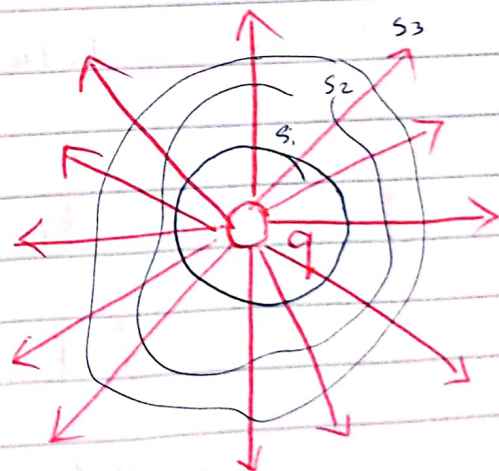
Gauss's Law

$$\epsilon_0 = 8.85 \times 10^{-12} \text{ C/V.m}^2$$

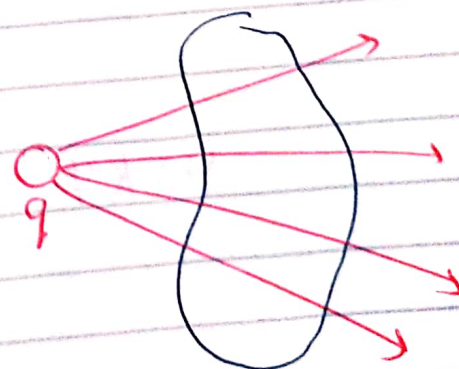
Gauss's Law  $\Rightarrow$  The net electric flux through a closed surface is equal to the net charge inside the surface divided by  $\epsilon_0$ .

### Comments

$\Rightarrow$  The net electric flux through each closed surface surrounding the same charge is the same regardless the shape and size.

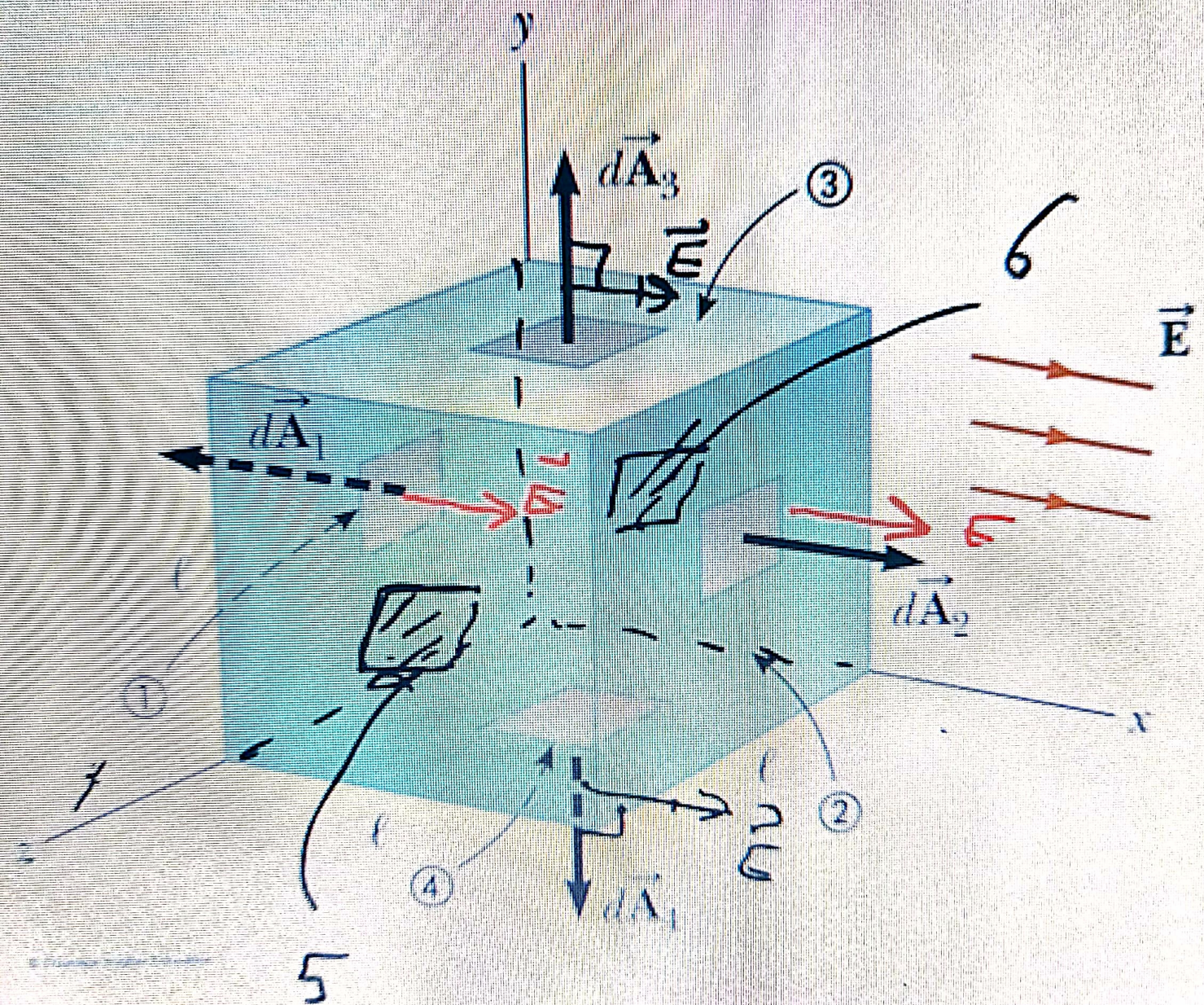


$\Rightarrow$  The net electric flux through any closed surface is zero if there is no charge inside.





# Ex. 24.1





## \*\*\* Gauss's Law

$$\left[ \Phi_e = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0} \right]$$

لو كان ما في سطح بين zero  $q_{in}$  ← والتفئة (zero)

### EX. 24.1

Consider a uniform electric field  $\vec{E}$  oriented in the  $x$  direction in empty space. A cube of edge length  $L$  is placed in the field, oriented as shown in figure. Find the net electric flux through the surface of the cube.

$$\Phi_1 = EA_1 \cos 180 = -EL^2$$

$$\Phi_2 = EA_2 \cos 0 = EL^2$$

for  $A_3, A_4, A_5$ , and  $A_6$ .

we have  $\theta = \frac{\pi}{2}$  between the area vector and  $\vec{E}$

$$\Rightarrow \Phi_3 = \Phi_4 = \Phi_5 = \Phi_6 = 0$$

The total flux

$$\Phi_{total} = \Phi_1 + \Phi_2 + \Phi_3 + \dots + \Phi_6 = 0$$



EX) Electric field due a distance  $r$  from a point charge  $q$

$$\vec{E} \cdot d\vec{A} = E dA, \quad \odot = 0$$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

✓

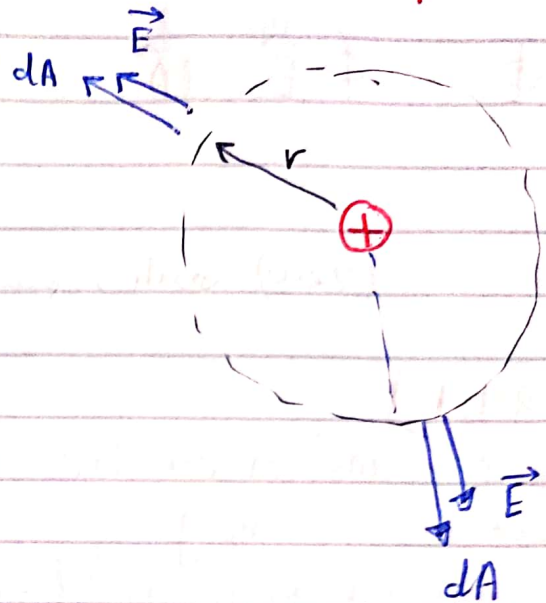
$$\oint E dA = \frac{q_{in}}{\epsilon}$$

$$E \underbrace{\oint dA}_{=A} = \frac{q}{\epsilon}$$

$$EA = \frac{q}{\epsilon}$$

$$E(4\pi r^2) = \frac{q}{\epsilon} \quad \leadsto \quad E = \frac{1}{4\pi\epsilon} \frac{q}{r^2}$$

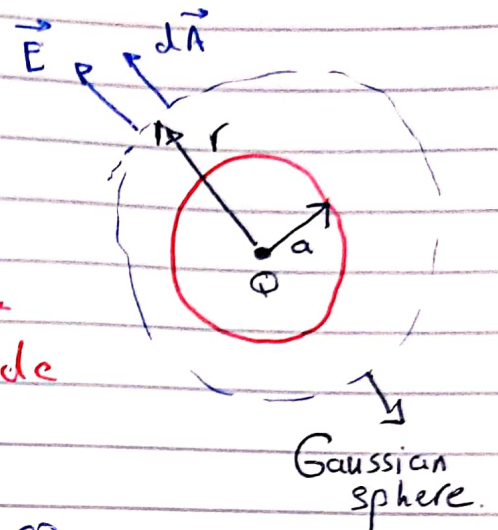
$$E = \frac{kq}{r^2}$$



EX. 24.3  $\Rightarrow$  An insulating solid sphere of radius  $a$  has a uniform volume charge density  $\rho$  and carries a total positive charge  $Q$  distributed uniformly.

$Q$ : total charge (uniform)  
 $\rho = \frac{Q}{\frac{4}{3}\pi a^3}$

A) Calculate the magnitude of the electric field at a point outside the sphere. ( $r > a$ )



$$\oint_{\text{out}} \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon}$$

$$EA = \frac{Q}{\epsilon}$$

$$\oint_{\text{out}} E dA = \frac{q_{\text{in}}}{\epsilon}$$

$$E (4\pi r^2) = \frac{Q}{\epsilon}$$

$$E \oint_{\text{out}} dA = \frac{Q}{\epsilon} \quad \nearrow$$

$$E_{\text{out}} (4\pi r^2) = \frac{Q}{\epsilon}$$

$$E_{\text{out}} = \frac{1}{4\pi\epsilon} \frac{Q}{r^2}$$

←  $E_{\text{out}} = k \frac{Q}{r^2}$

$\vec{E}$  out side the sphere.



**B** find the magnitude of the electric field at a point inside the sphere. ( $r < a$ )

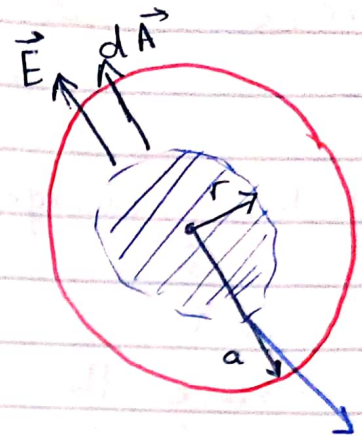
$$\oint \vec{E}_{in} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

$$\oint E_{in} dA = \frac{q_{in}}{\epsilon}$$

$$E_{in} \oint dA = \frac{q_{in}}{\epsilon}$$

$$E_{in} (4\pi r^2) = \frac{q_{in}}{\epsilon}$$

$$E_{in} = \frac{1}{4\pi\epsilon} \frac{q_{in}}{r^2} = k \frac{q_{in}}{r^2} = k \frac{q_{in}}{r^2}$$



Gaussian sphere.

$$\Rightarrow \rho = \frac{Q}{\frac{4}{3}\pi a^3} = \frac{q_{in}}{\frac{4}{3}\pi r^3}$$

$$q_{in} = \frac{Q r^3}{a^3}$$

$$E_{in} = k \frac{Q r}{a^3}$$

$\vec{E}$  inside the sphere.

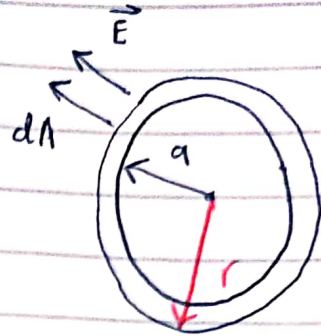
**Example** Find the electric field inside and outside a thin spherical shell of radius  $a$  and uniform charge  $Q$ .

A) outside ( $r > a$ )

$$\oint \vec{E}_{out} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

$$E_{out} (4\pi r^2) = \frac{Q}{\epsilon}$$

$$\boxed{E_{out} = \frac{1}{4\pi\epsilon} \frac{Q}{r^2}}$$



لما كنت خارج  
تكون ساحة في القانون  
لأنه لا يتم ان يعطى  
للداخل

B) Inside ( $r < a$ )

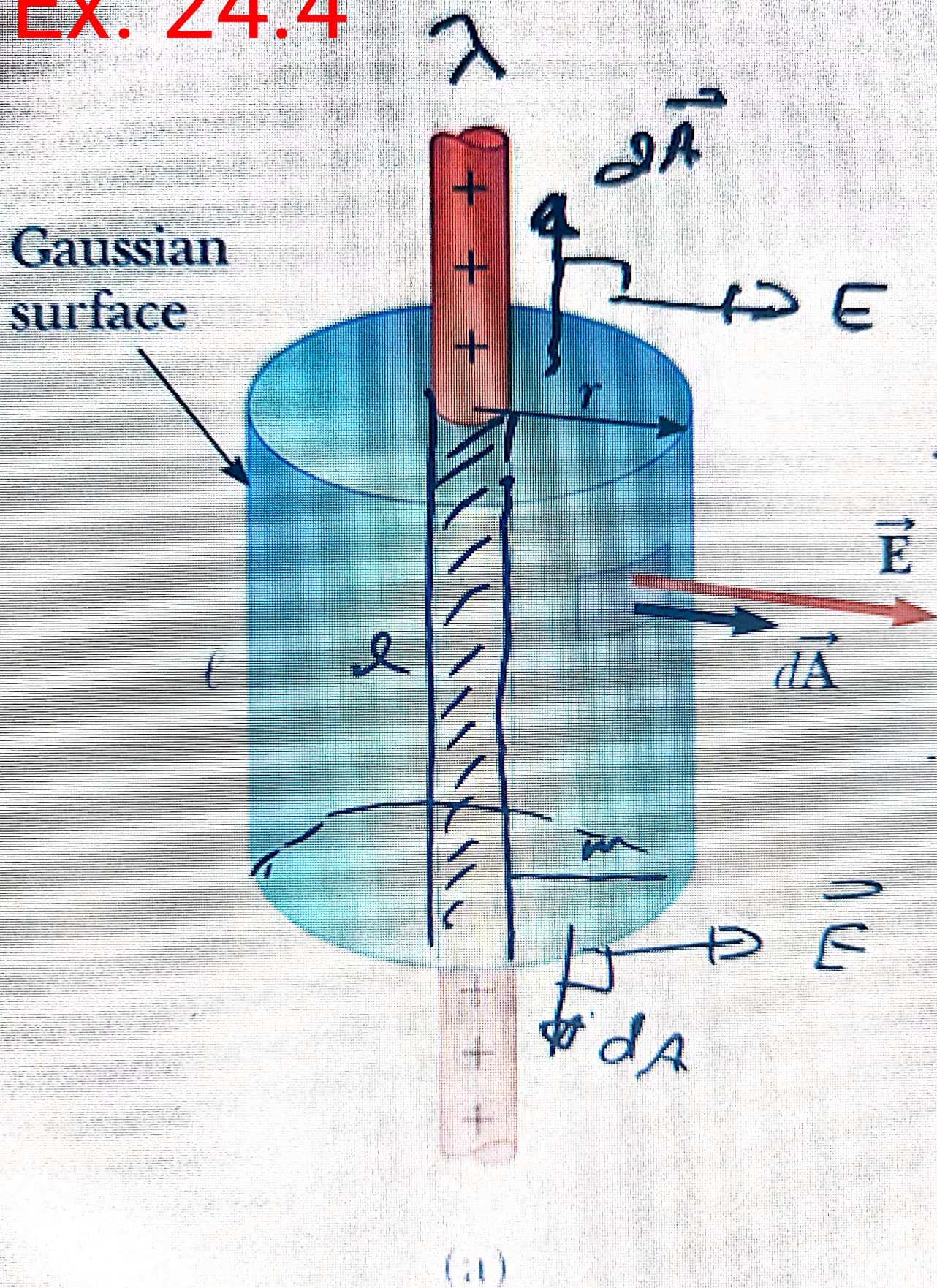
$$\oint \vec{E}_{in} \cdot d\vec{A} = \frac{q_{in}}{\epsilon} = \frac{0}{\epsilon}$$

$$\oint \vec{E}_{in} \cdot d\vec{A} = 0$$

$$\boxed{E_{in} = 0}$$



# Ex. 24.4



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EX. 24.4)

Find the electric field a distance  $r$  from a line of positive charge of infinite length and constant charge per unit length  $\lambda$

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon}$$

$$E(2\pi r L) = \frac{\lambda L}{\epsilon}$$

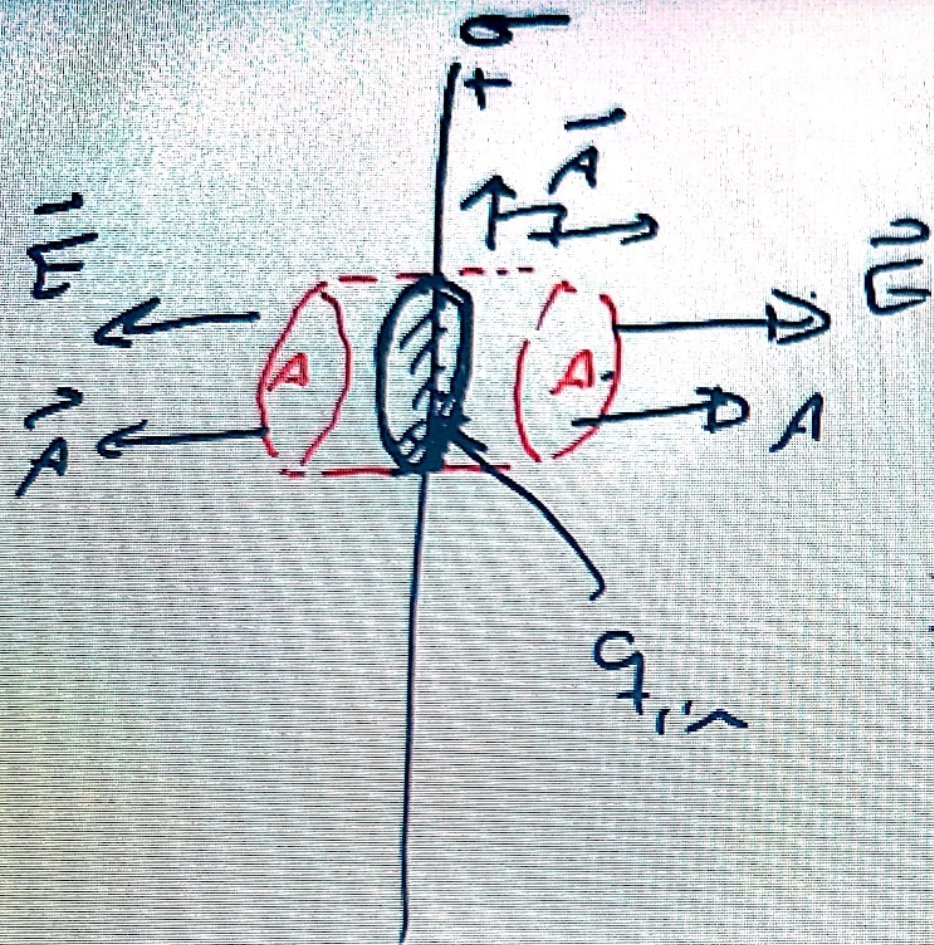
$$= E \left( \frac{1}{2\pi\epsilon} \frac{\lambda}{r} \right) \neq \frac{2}{2}$$

$$= \frac{1}{4\pi\epsilon} \frac{2\lambda}{r}$$

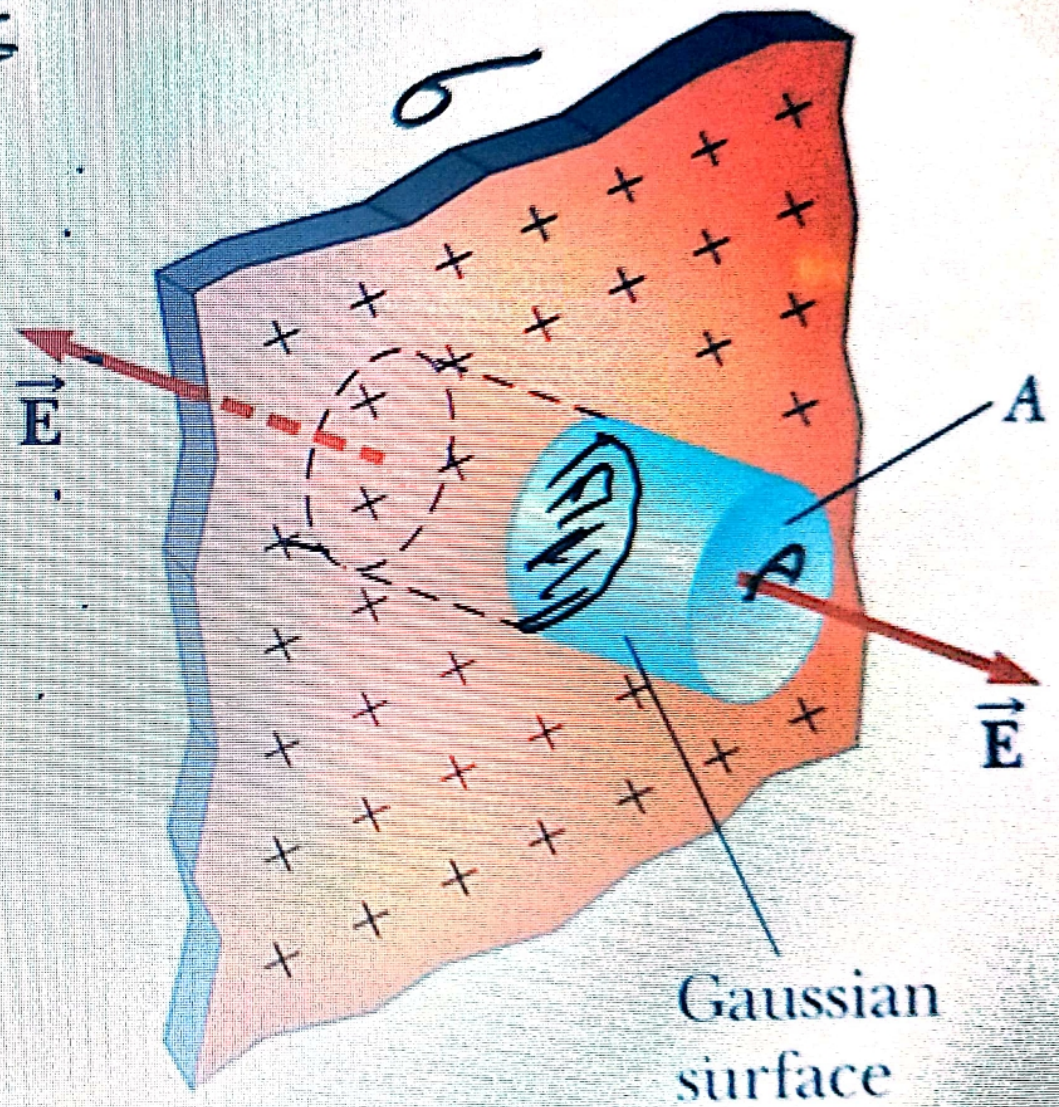
$$\boxed{\vec{E} = \frac{2k\lambda}{r}}$$

$\rightarrow \vec{E}$  due to a line charge  $\lambda$  of infinite length





Ex. 24.5





EX 24.5) Find the electric field due to an infinite plane of positive charge with uniform surface charge density  $\sigma$ .

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon_0}$$

$$EA + EA = \frac{\sigma A}{\epsilon_0}$$

$$= 2EA = \frac{\sigma A}{\epsilon_0}$$

$$E = \frac{\sigma}{2\epsilon_0}$$

$\vec{E}$  of non conducting sheet of uniform charge density  $\sigma$

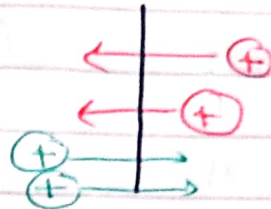


الموصلات في حالة التوازن الكهروستاتيكي Conductors in electrostatic equilibrium

Electrostatic equilibrium  $\equiv$  no net motion of charge

الخضري راحة اليدين = العضلات التي راحة اليدين

Zero = ohne, kein, Lein



## properties of Conductors in electrostatic equilibrium.

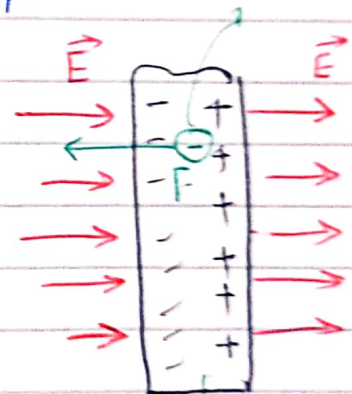
۶. دائماً یکبار در هر ثانیه داخل می شود  $Zero =$

$E_{in} = 0$  inside the conductor

$$\vec{E}_{in} = \vec{E} - \vec{E}'$$

$$\leftarrow F = 9 E_{in}$$

التواضع  
الحق  
بالداخل



معيناً فعالاً في العمل  
الذي في العمل

The process (redistribution) of charge continues until

$$\vec{F} = q \vec{E}_{in} = 0$$

صنایع تعاونی معمال اسلامی مع معیاد

2 If an isolated conductor is charged then the charge resides on its surface.

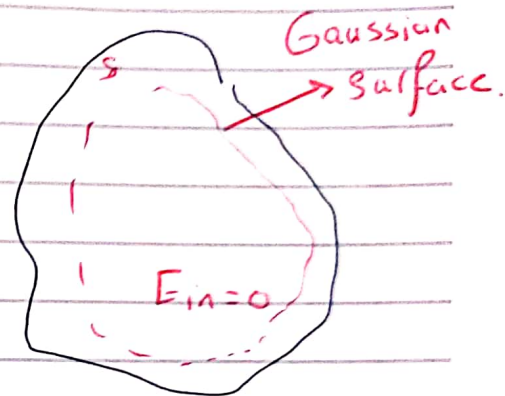
لو كان الموصل مشحون ومغلق  
الشحن لا يتراكم في الداخل تتراكم كل على السطح

$$\oint \vec{E}_{in} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

$$0 = \frac{q_{in}}{\epsilon}$$

$$q_{in} = 0$$

⇒ The charge resides on the surface.



3 Just outside the conductor

a)  $\vec{E} \perp$  surface

b)  $E_{surface} = \frac{\sigma}{\epsilon}$

Let  $\vec{E}$  is not normal

to the surface ⇒

$E_{||}$  affects before of  $\vec{E}_{||} = q\vec{E}_{||}$

parallel  
موازي  
السطح

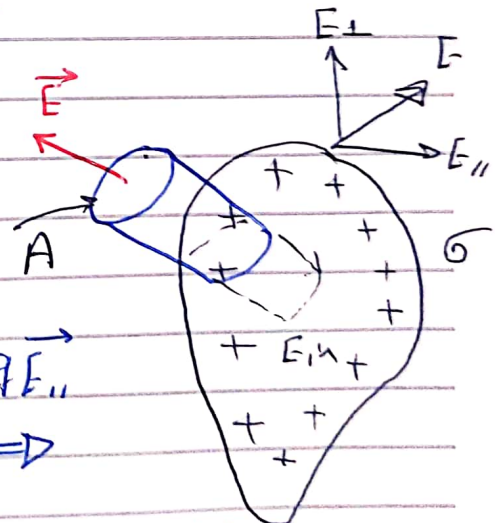
parallel to the surface ⇒

There is a net motion of charge parallel to the surface ⇒

The ~~conductor is not~~ Conductor is not in electrostatic equilibrium

⇒ We must have  $\vec{E}_{||} = 0$

بحيث ان يكون  $E_{||}$  zero





For a conductor to be in electrostatic equilibrium we must have

$\Rightarrow \vec{E} \perp \text{surface}$

$$\vec{E}_{\parallel} = 0$$

(B)

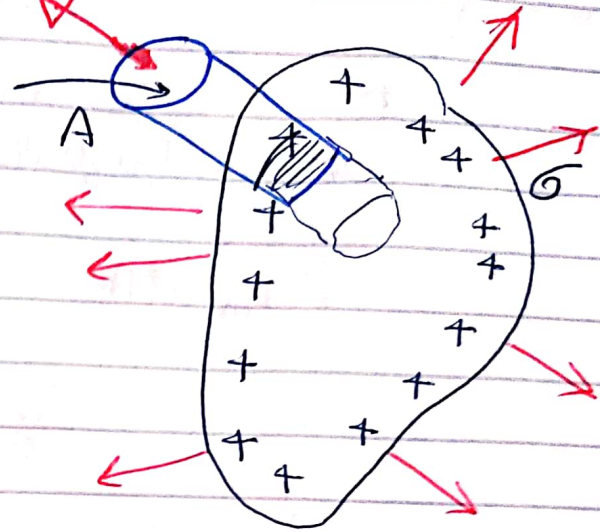
Apply Gauss's Law

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon}$$

$$E_A - \cancel{E_{\text{in}} A} = \frac{QA}{\epsilon}$$

zero

$$E_A = \frac{QA}{\epsilon}$$

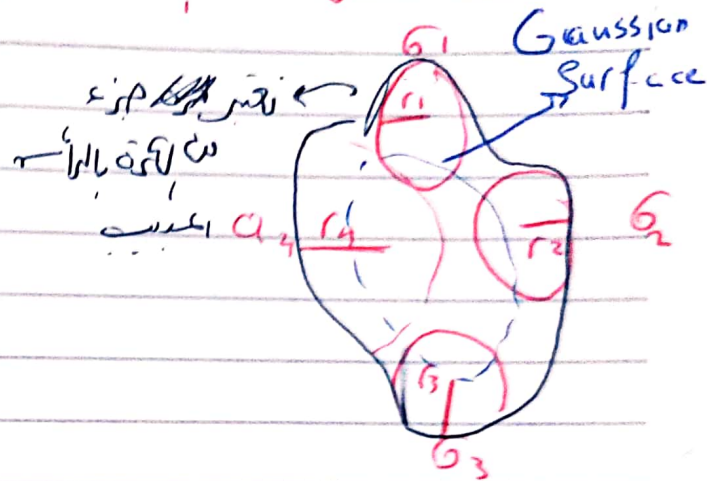


$$E = \frac{\sigma}{\epsilon}$$

فيلد  
فيلد

④ on an irregularly shaped conductor,  $\sigma$  and  $E_{\text{surface}}$  are large where the radius of curvature is small

كلما كان نصف القطر صغيراً، كلما كان  $\sigma$  و  $E$  أكبراً



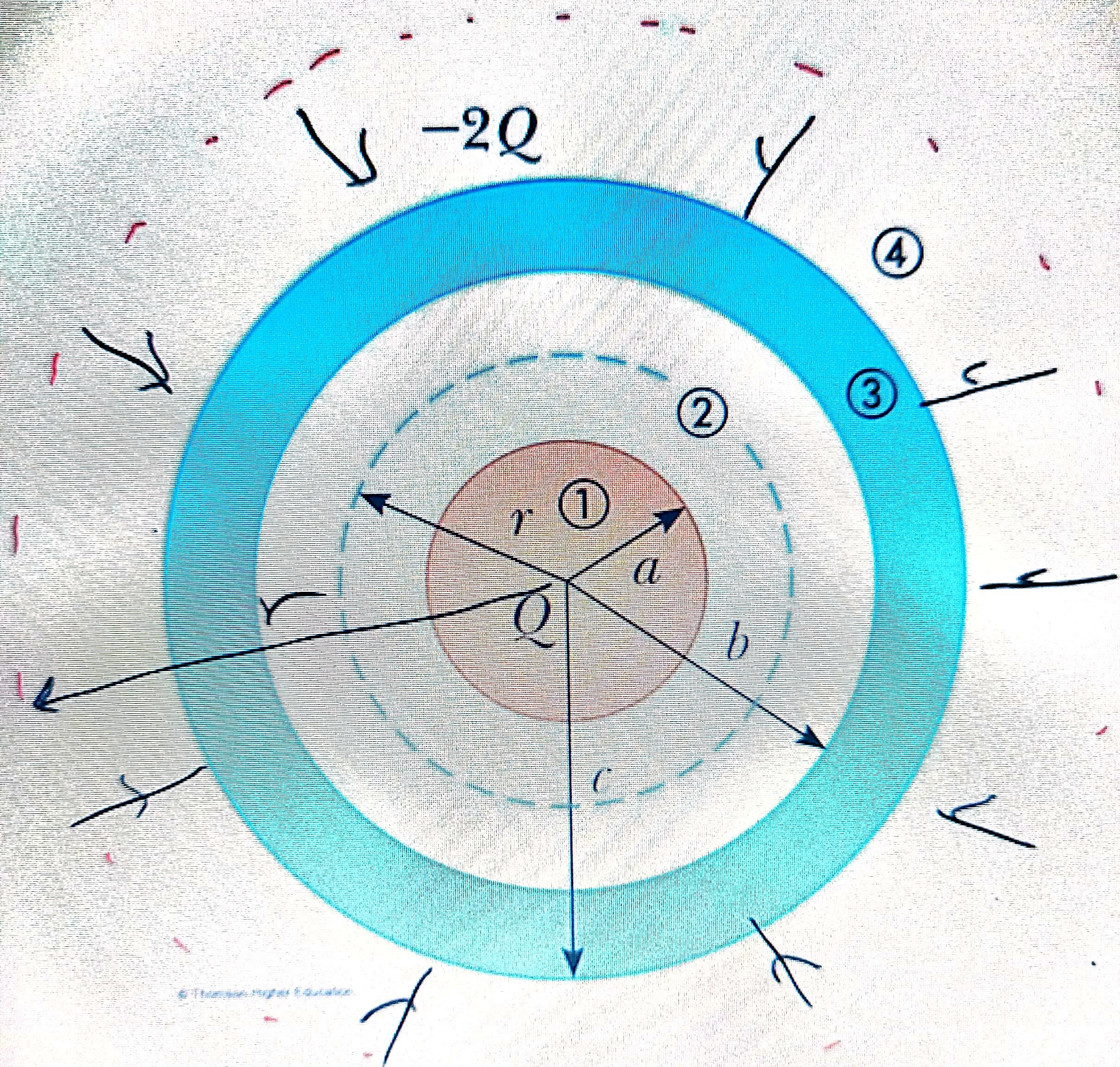
$$r_1 < r_2 < r_3 < r_4$$

$$\sigma_1 > \sigma_2 > \sigma_3 > \sigma_4$$

$$\uparrow E = \frac{\uparrow \sigma}{\downarrow \epsilon}$$

$$E_1 > E_2 > E_3 > E_4$$





Ex... 24.7



**Ex. 24.7** A solid insulating sphere of radius  $a$  carries a net positive charge  $Q$  uniformly distributed throughout its volume. A conducting spherical shell of inner radius  $b$  and outer radius  $c$  is concentric with the solid sphere and carries a net charge  $-2Q$ . Using Gauss's law find the electric field in the regions labeled ①, ②, ③ and ④ in figure and the charge distribution on the shell when the entire system is in electrostatic equilibrium.

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

region ① ( $r < a$ )

$$\oint \vec{E}_1 \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}, \rho = \text{constant (uniform)}$$

$$E_1 (4\pi r^2) = \frac{q_{in}}{\epsilon_0}$$

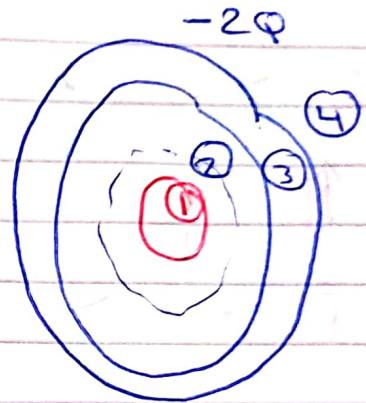
$$E_1 = \frac{1}{4\pi\epsilon_0} \frac{q_{in}}{r^2} = k \frac{q_{in}}{r^2}$$

$$\rho = \frac{Q}{\frac{4}{3}\pi a^3} = \frac{q_{in}}{\frac{4}{3}\pi r^3}$$

$$q_{in} = \frac{Q r^3}{a^3}$$

$$E_{in} = k \frac{Q r^3 / a^3}{r^2} = \frac{k Q r}{a^3}$$

$$\Rightarrow \boxed{E_1 = \frac{k Q r}{a^3}}$$





Region ②  $a < r < b$

$$\oint \vec{E}_2 \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$E_2 (4\pi r^2) = \frac{Q}{\epsilon_0}$$

$$E_2 = \frac{1}{4\pi\epsilon_0} \frac{Q}{r^2} = k \frac{Q}{r^2}$$

Region ③  $b < r < c$

$$E_3 = 0 \rightarrow \text{inside the conductor}$$

Region ④

$$\oint \vec{E}_4 \cdot d\vec{A} = \frac{q_{in}}{\epsilon_0}$$

$$E_4 (4\pi r^2) = \frac{-2Q + Q}{\epsilon_0}$$

$$E_4 (4\pi r^2) = \frac{-Q}{\epsilon_0}$$

$$E_4 = \frac{-1}{4\pi\epsilon_0} \frac{Q}{r^2} = -\frac{kQ}{r^2}$$

$$q' + q'' = -2Q$$

$$\oint \vec{E}_3 \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon} \quad ; \quad E_3 = 0$$

$$Q = \frac{q_{in}}{E}$$

$$q'' + q = 0$$

$$q'' = -q$$

$$q' + q'' = -2\phi \Rightarrow$$

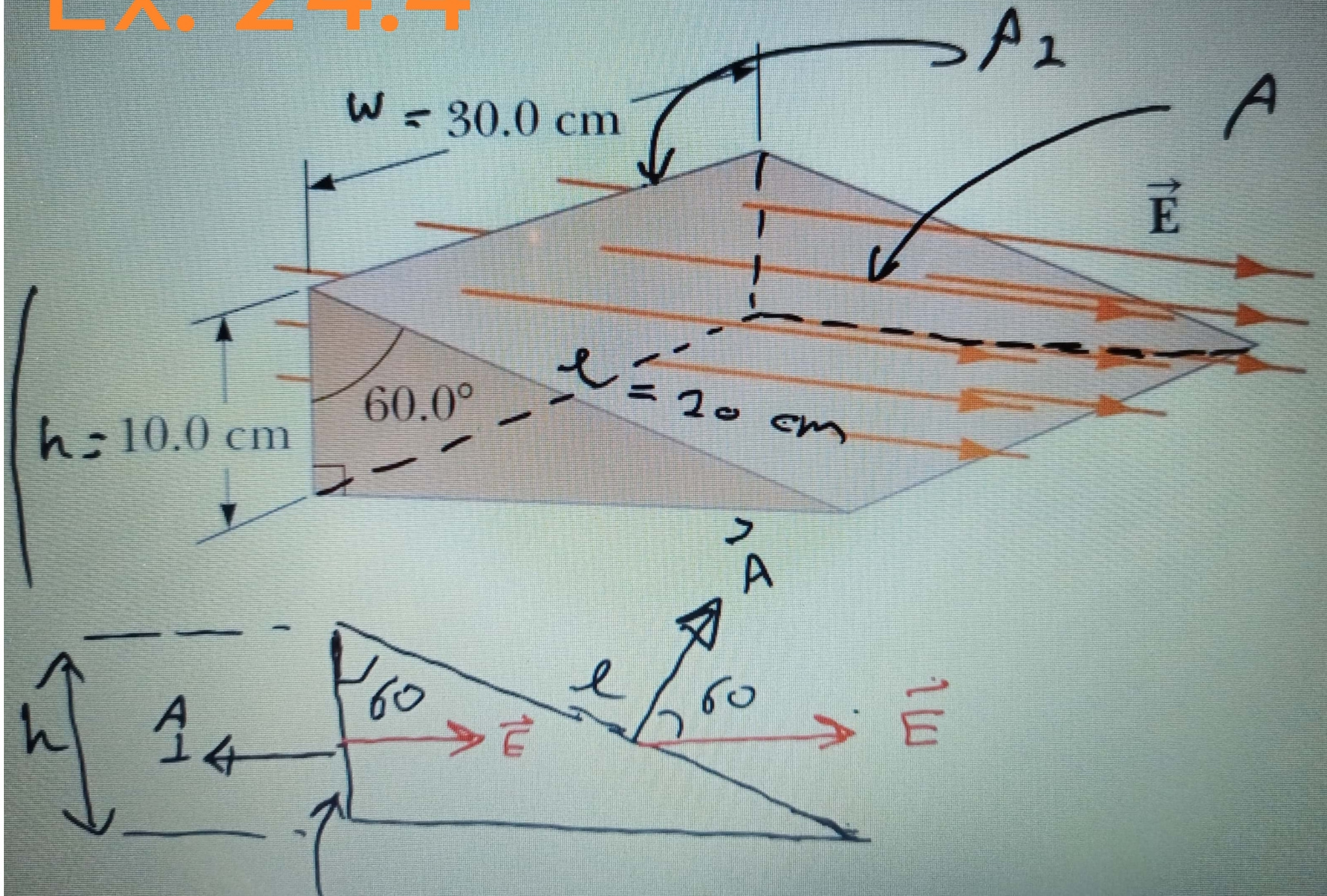
$$q' = -2\phi - q'' = -\phi$$

⇒  $q' = -Q$

وہ داء کا علاج ایسا ہے۔ ناقص بحث ہے فی المکرر



# Ex. 24.4



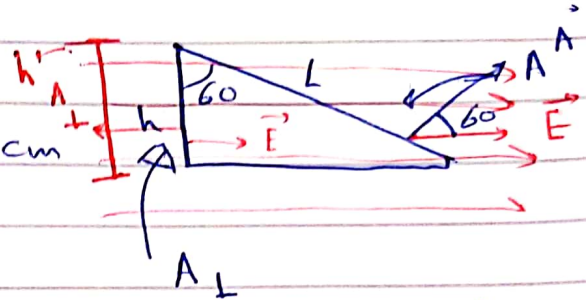


Prob. 24.4  $\Rightarrow$  Consider a closed triangle box resting within a horizontal electric field of magnitude  $E = 7.80 \times 10^4 \text{ N/C}$  as shown. Calculate the electric flux through (a) the vertical rectangular surface, (b) the slanted surface, and (c) the entire surface of the box.

$$E = 7.8 \times 10^4 \text{ N/C}$$

$$h = 10 \text{ cm}, \quad w = 30 \text{ cm}$$

$$L = \frac{h}{\cos 60} = \frac{10 \text{ cm}}{0.5} = 20 \text{ cm}$$



$$\begin{aligned} \text{a) } \Phi_{A_{\perp}} &= \vec{E} A_{\perp} \cos 180 \\ &= (7.8 \times 10^4) (0.3 \times 0.1) (-1) \\ &= -2.34 \times 10^3 \text{ N} \cdot \text{m}^2 / \text{C} \\ &= -2.34 \text{ kN} \cdot \text{m}^2 / \text{C} \end{aligned}$$

الدفقة من سطح  $A_{\perp}$  هو سالب الدفقة من سطح  $A_{\perp}$  لأن  $\vec{E}$  و  $\vec{A}_{\perp}$  في اتجاهين متعاكسين.

$$\begin{aligned} \text{b) } \Phi_A &= EA \cos \theta (60) \\ &= (7.8 \times 10^4) (0.3 \times 0.2) \left(\frac{1}{2}\right) \\ &= 2.34 \times 10^3 \text{ N} \cdot \text{m}^2 / \text{C} \end{aligned}$$

$$\Phi_{\text{total}} = \Phi_{A_{\perp}} + \Phi_A \rightarrow 0$$

لأنه لا يوجد شحنة في الداخل.



Prob - 24-10

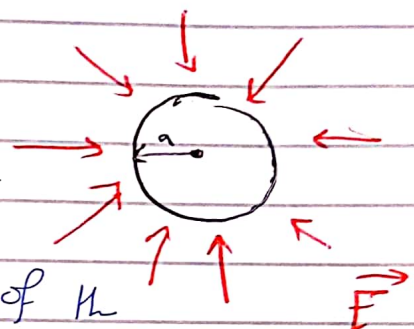
The electric field everywhere on the surface of a thin, spherical shell of radius 0.750 m is of magnitude 890 N/C and points radially toward the center of the sphere.

(a) What is the net charge within the sphere's surface? (b) What is the distribution of the charge inside the spherical shell?

$$a = 0.75 \text{ m}$$

$$E = 890 \text{ N/C at } r = a = 0.75 \text{ m}$$

$E$  = point radially toward the center



b) Because  $\vec{E}$  points radially toward the center

$\Rightarrow Q$  inside the spherical shell is negative

Because  $|\vec{E}| = \text{constant}$  at the surface of the spherical shell then the distribution of charge inside is uniform and concentric with the spherical shell

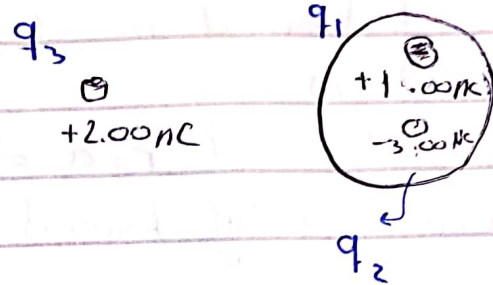
$$a) \quad E = \frac{kQ}{a^2} \rightarrow Q = \frac{Ea^2}{k}$$

$$= \frac{(-890)(0.75)^2}{9 \times 10^9}$$

$$Q = -5.57 \times 10^{-8} \text{ C}$$
$$= -55.7 \text{ nC}$$

prob 24.8

Find the net electric flux through the spherical closed surface shown in figure. The two charges on the right are inside the spherical surface.



$$\Phi = \oint \vec{E} \cdot d\vec{A} = \frac{q_{in}}{\epsilon}$$

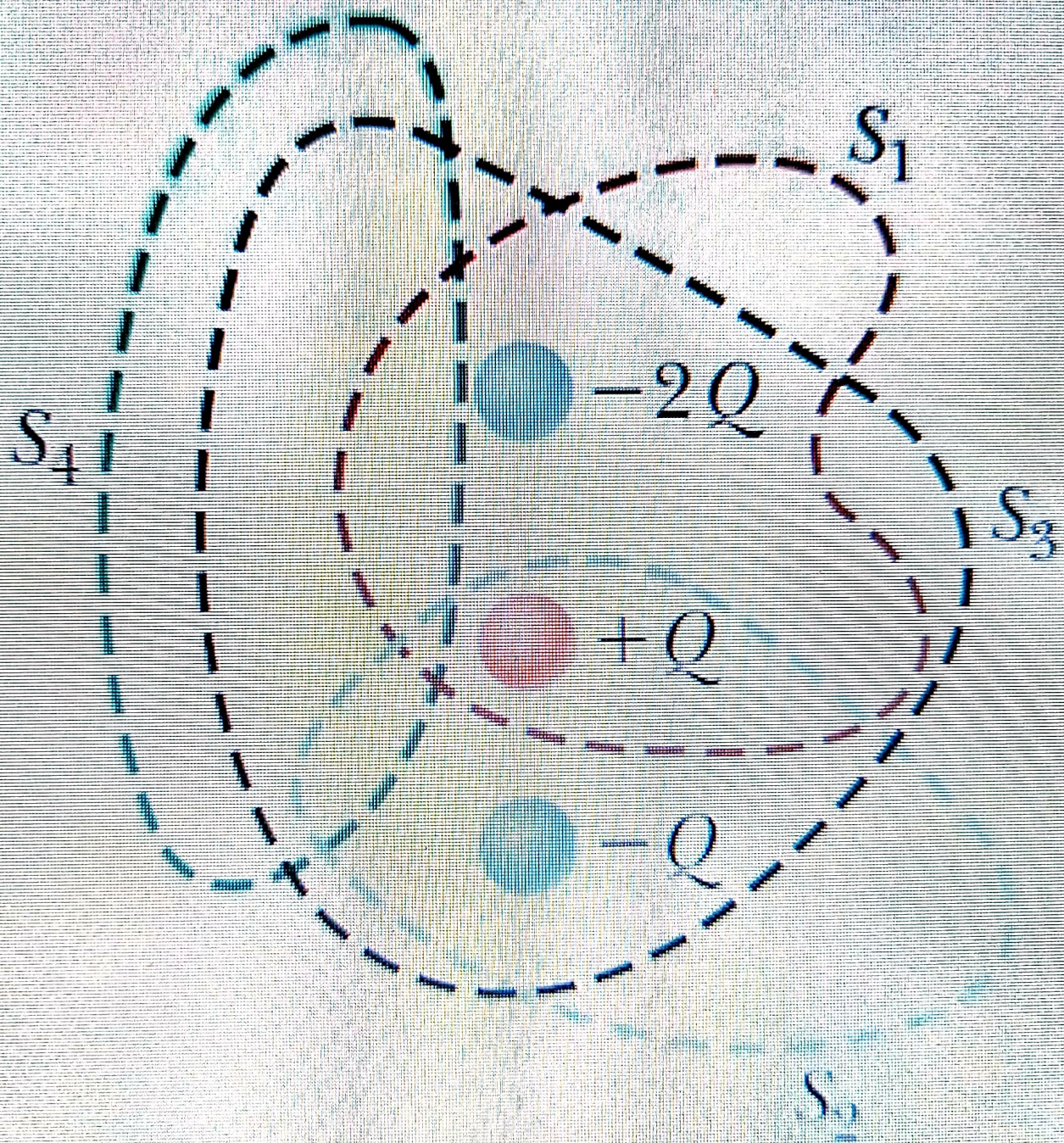
$$\Phi_{tot} = \frac{q_{in}}{\epsilon} = \frac{q_1 + q_2}{\epsilon}$$

$$\Phi_{total} = \frac{(1 \times 10^{-9}) + (-3 \times 10^{-9})}{8.85 \times 10^{-12}}$$

$$\Phi_{total} = -226 \text{ N.m}^2/\text{C}$$



## Ex. 24.11





### Prob 24.11

Four closed surfaces,  $S_1$  through  $S_4$ , together with the charges  $-2Q$ ,  $Q$ , and  $-Q$  are sketched in figure (the colored lines are the intersections of the surfaces with the page). Find the electric flux through each surface.

$$\Phi = \frac{q_{in}}{\epsilon}$$

$$S_1 = \Phi_1 = \frac{-2Q + Q}{\epsilon} = -\frac{Q}{\epsilon}$$

$$S_2 = \Phi_2 = \frac{+Q - Q}{\epsilon} = 0$$

$$S_3 = \Phi_3 = \frac{-2Q + Q - Q}{\epsilon} = -\frac{2Q}{\epsilon}$$

$$S_4 = \Phi_4 = 0 \quad \rightsquigarrow$$

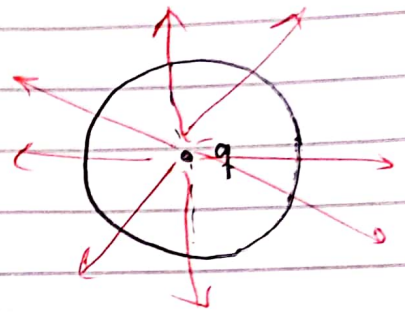
لا يتم تخزين  
في ذاكرة



Prob → 24-14

A particle with charge of  $12.0 \mu\text{C}$  is placed at the center of a spherical shell of radius  $22.0 \text{ cm}$ . What is the total electric flux through (a) the surface of the shell and (b) any hemispherical surface of the shell? (c) Do the results depend on the radius? explain →  $\text{نعم}$

$$q = 12 \mu\text{C} = 12 \times 10^{-6} \text{ C}$$



$$a) \Phi_{\text{shell}} = \frac{q_{\text{in}}}{\epsilon} = \frac{q}{\epsilon} = \frac{12 \times 10^{-6}}{8.85 \times 10^{-12}}$$

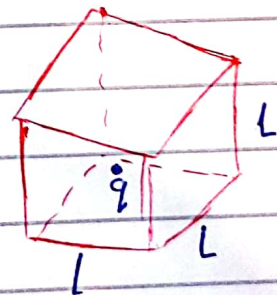
$$\Phi_{\text{shell}} = 1.36 \times 10^6 \text{ N.m}^2/\text{C}$$

$$b) \Phi_{\text{half shell}} = \frac{1}{2} \Phi_{\text{shell}} = 6.78 \times 10^5 \text{ N.m}^2/\text{C}$$

Prob → 25-14  
(25) Cube لوكان، مثال

$q$  is at the center

$$\Phi_{\text{tot}} = \frac{q}{\epsilon}$$



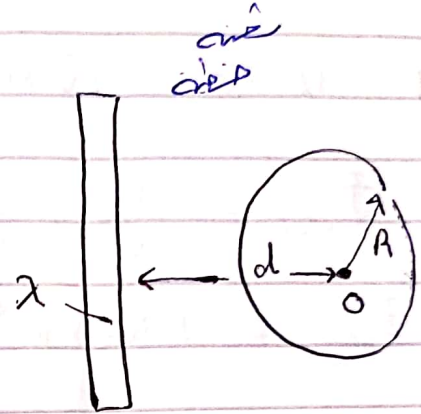
$$\Phi_{\text{one-face}} = \frac{\Phi_{\text{tot}}}{6} = \frac{q}{6\epsilon}$$

Prob. 24-17

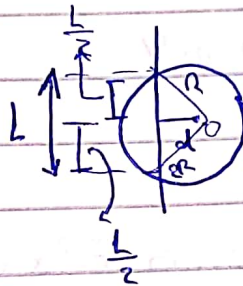
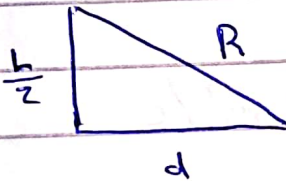
An infinitely long line charge having a uniform charge per unit length  $\lambda$  lies a distance  $d$  from point  $O$  as shown. Determine the total electric flux through the surface of a sphere of radius  $R$  centered at  $O$  resulting from this line charge. Consider both cases where (a)  $R < d$  and (b)  $R > d$ .

A)  $R < d \Rightarrow \Phi = \frac{q_{in}}{\epsilon} = 0$

لا يوجد شحنة داخل الكروي



B)  $R > d$



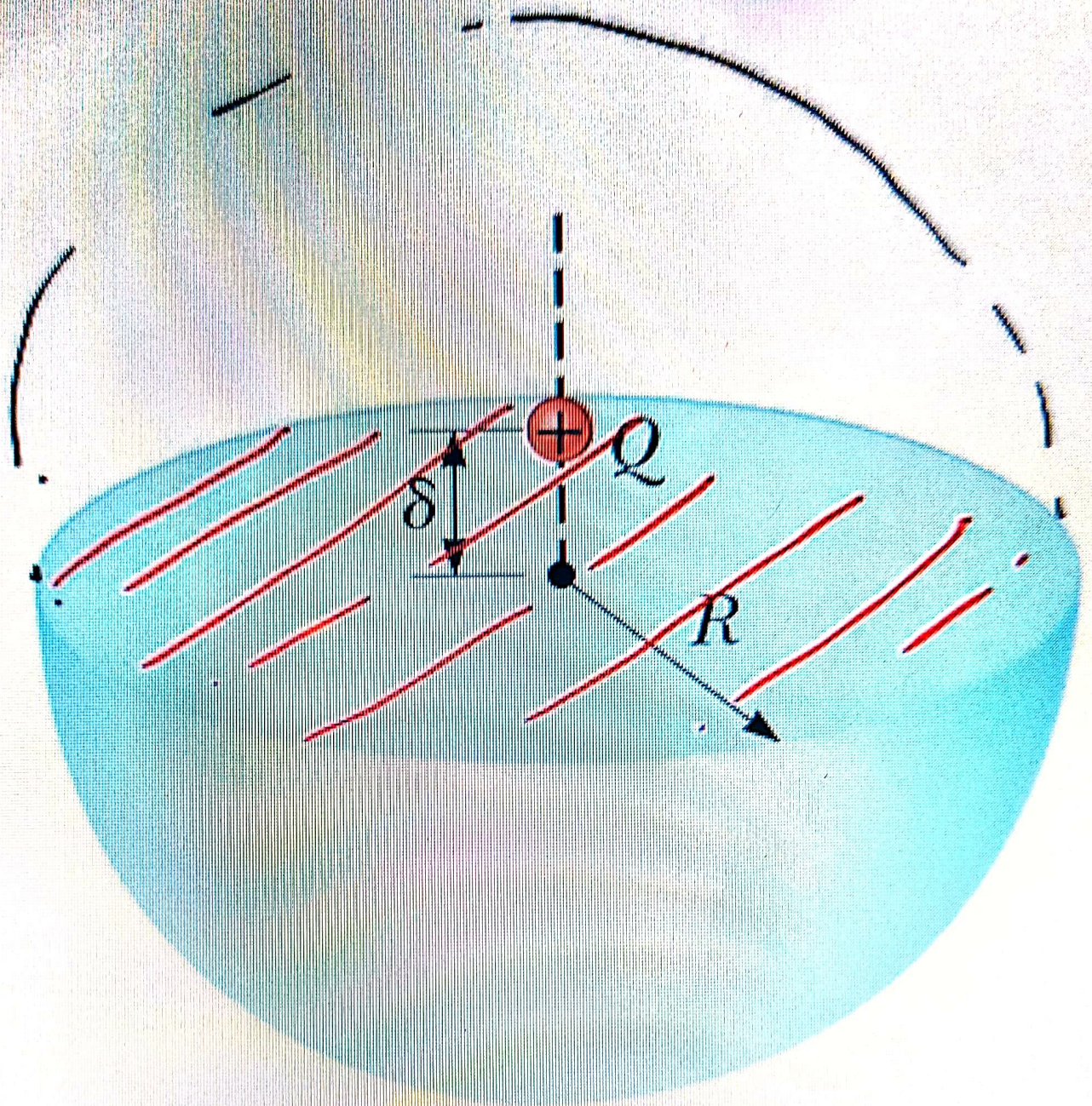
$\frac{L}{2} = \sqrt{R^2 - d^2}$

$L = 2 \sqrt{R^2 - d^2}$

$q_{in} = \lambda L = 2 \lambda \sqrt{R^2 - d^2}$

$\Phi = \frac{q_{in}}{\epsilon} = \frac{2 \lambda \sqrt{R^2 - d^2}}{\epsilon}$





Ex... 24.21



prob. 24.21

A particle with charge  $Q$  is located a small distance  $\delta$  immediately above the center of the flat face of a hemisphere of radius  $R$  as shown what is the electric flux (a) through the curved surface and (b) through the flat face as  $\delta \rightarrow 0$ ?

$$\delta \rightarrow 0$$

A)

$$\Phi_{\text{hemisphere}} = \frac{1}{2} \Phi_{\text{sphere}}$$

$$= \frac{1}{2} \frac{Q}{\epsilon} = \frac{Q}{2\epsilon} = \frac{Q}{2\epsilon}$$

$$B) \Phi_{\text{flat}} = -\Phi_{\text{hemisphere}} = -\frac{Q}{2\epsilon}$$

prob. 24.24 The charge per unit length on a long, straight filament is  $-90.0 \mu\text{C}/\text{m}$ . Find the electric field (a)  $10.0 \text{ cm}$ , (b)  $20.0 \text{ cm}$  and (c)  $100 \text{ cm}$  from the filament where distance are measured perpendicular to the length of the filament

$$a) r = 10 \text{ cm} = 0.1 \text{ m}$$

$$E = \frac{2k\lambda}{r} = 16.2 \times 10^6 \text{ N/C} = -16.2 \text{ MN/C}$$

$$\lambda = -90 \mu\text{C}$$

$$m = -90 \times 10^{-6} \text{ C/m}$$

$$E = \frac{2k\lambda}{r}$$

$$b) r = 20 \text{ cm}$$

$$E = \frac{2k\lambda}{r} = 8.09 \text{ MN/C}$$



Prob

29) Consider a thin, spherical shell of radius 14.0 cm with a total charge of 32.0  $\mu\text{C}$  distributed uniformly on its surface. Find the electric field (a) 10.0 cm and (b) 20.0 cm from the center of the charge distribution.

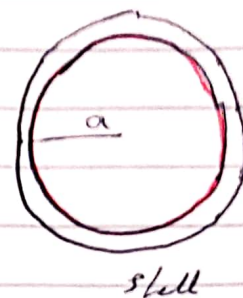
$$a = 0.14 \text{ m}$$

$$Q = 32 \mu\text{C}$$

$$E_{\text{in}} = 0, \quad E_{\text{out}} = \frac{kQ}{r^2}$$

$$a) \quad r = 10 \text{ cm} < a$$

$$\Rightarrow E = E_{\text{in}} = 0$$



$$b) \quad r = 20 \text{ cm} = 0.2 \text{ m} > a$$

$$E = E_{\text{out}} = \frac{kQ}{r^2} = 7.19 \times 10^{16} \text{ N/C}$$

Prob 35) A solid sphere of radius 40.0 cm has a total positive charge of 26.0  $\mu\text{C}$  uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0 cm (b) 10.0 cm (c) 40 cm, and (d) 60.0 cm from the center of the sphere.

$$a = 40 \text{ cm}, \quad Q = 26 \mu\text{C}$$

$$E_{\text{in}} = \frac{kQr}{a^3}, \quad E_{\text{out}} = \frac{kQ}{r^2}$$

$$A) \quad r = 0 \quad 0 < a \quad \Rightarrow E = E_{\text{in}} =$$

$$\frac{kQr}{a^3} = 0 \quad 0.1 \text{ m}$$

$$B) \quad r = 10 \text{ cm} = 10 \text{ cm} < a \Rightarrow E = E_{\text{in}} = \frac{kQr}{a^3}$$

$$= 365 \text{ kN/C}$$

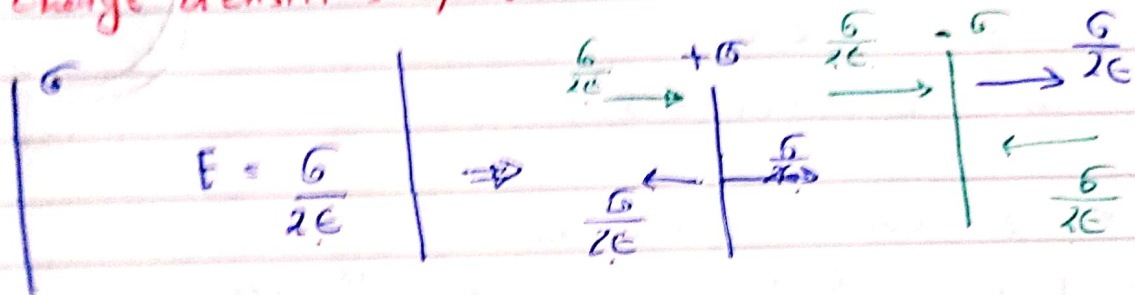
$$c) \quad r = 40 \text{ cm} \Rightarrow E = E_{\text{out}} = \frac{kQ}{r^2} = 1.46 \text{ MN/C}$$

$$d) \quad r = 60 \text{ cm} \Rightarrow E = E_{\text{out}} = \frac{kQ}{r^2} = 649 \text{ kN/C}$$

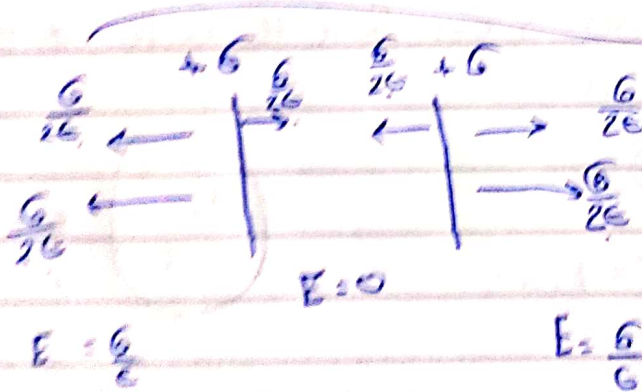


prob. 56)

Two infinite, nonconducting sheets of charge are parallel to each other as shown. The sheet on the left has a uniform surface charge density  $\sigma$  and the one on the right has a uniform charge density  $-\sigma$ . Calculate the electric field at points (a) to the left of, (b) in between and (c) to the right of the two sheets. (d) What if? Find the electric fields in all three regions if both sheets have positive, uniform surface charge densities of value  $\sigma$ .



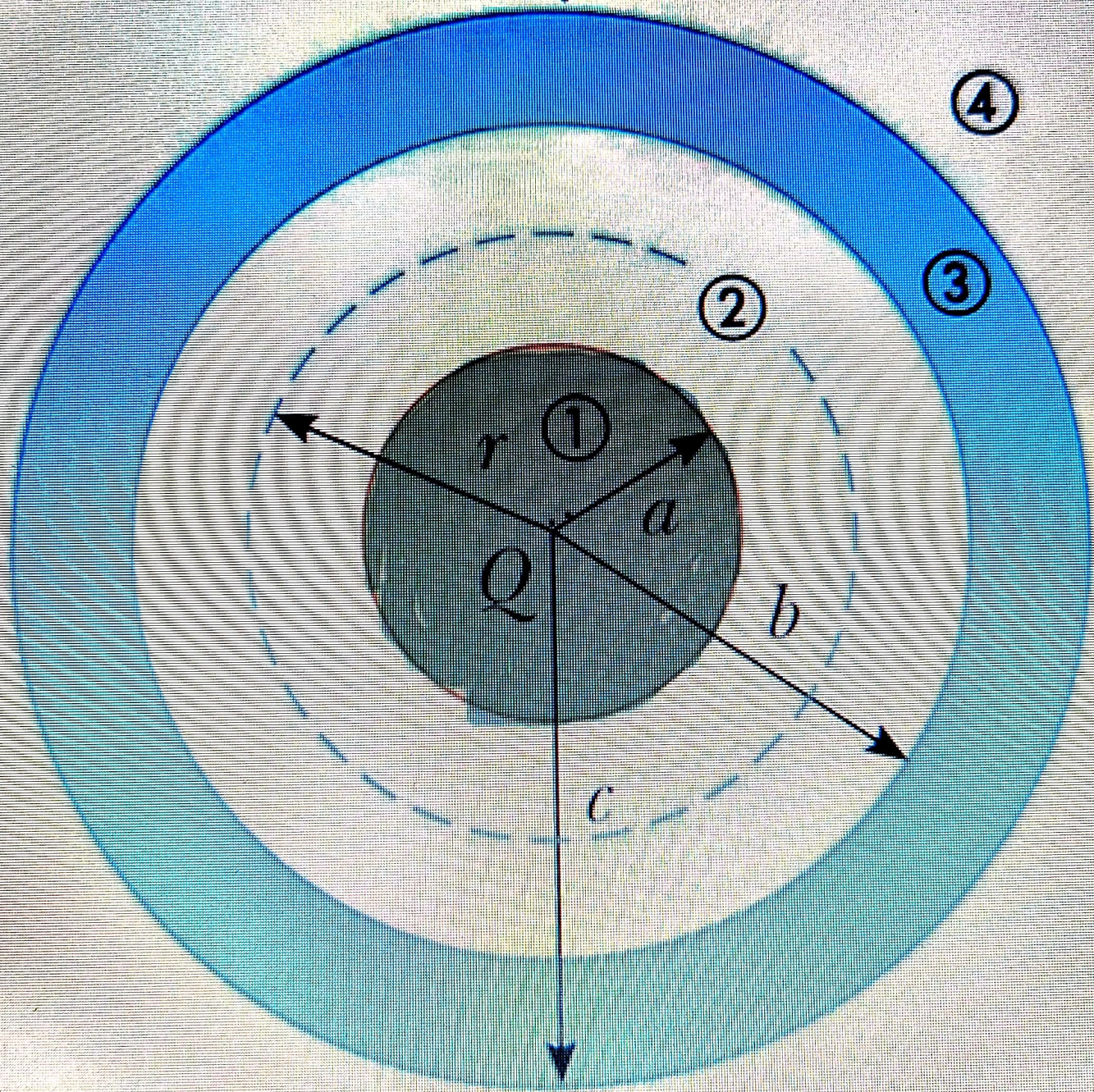
$$\begin{array}{|c|} \hline E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} \\ \hline = 0 \\ \hline \end{array}
 \quad
 \begin{array}{|c|} \hline E = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} \\ \hline = \frac{\sigma}{\epsilon_0} \\ \hline \end{array}
 \quad
 \begin{array}{|c|} \hline E = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} \\ \hline = 0 \\ \hline \end{array}$$





Ex. 47

9



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Prob. 47

A solid conducting sphere of radius 2.00 cm has a charge of 8.00  $\mu\text{C}$ . A conducting spherical shell of inner radius 4.00 cm and outer radius 5.00 cm is concentric with the solid sphere and has a charge of -4.00  $\mu\text{C}$ . Find the electric field at (a)  $r = 1.00$  cm (b)  $r = 3.00$  cm (c)  $r = 4.50$  cm and (d)  $r = 7.00$  cm from the center of this charge configuration

①  $r < a \Rightarrow$  (inside the conductor)  
 $E_1 = 0$

②  $a < r < b$   
 $E_2 = \frac{kQ}{r^2}$

③  $b < r < c$   
 $E_3 = 0$  (inside conductor)

$r > c$   
④  ~~$E_4$~~   $E_4 = \frac{k(Q + Q)}{r^2}$



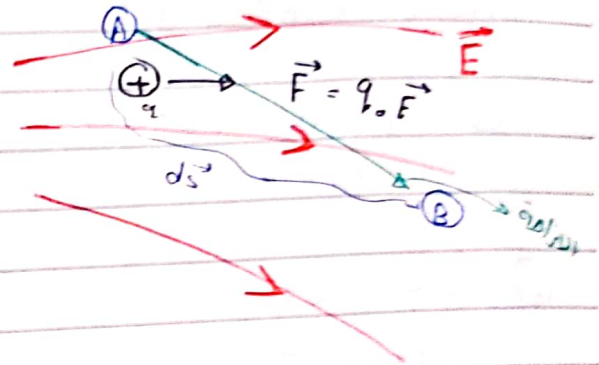
## ch-25- Electric potential

Electric force  $\equiv$  conservative force قوة محافظة  
 (work done does not depend on the path)

العمل لا يعتمد على مسار نقطة في المنطقة الحرة، بل على نقطتي البداية والنهاية

$$\vec{F} = q \cdot \vec{E}$$

نريد ان نحس الشغل من A الى B



$$W = \int_A^B \vec{F} \cdot d\vec{s}$$

كثير  
الارادة

The work done By the field on moving the charge from point A to B

$$W = \int_A^B \vec{F} \cdot d\vec{s} = q \int_A^B \vec{E} \cdot d\vec{s}$$

$$\vec{F} = q\vec{E} \equiv \text{Conservative} \Rightarrow W = -\Delta U \\ = -(U_B - U_A) \\ = U_A - U_B$$

$$W = -\Delta U = q \int_B^A \vec{E} \cdot d\vec{s}$$

$$U_B - U_A = -q \int_B^A \vec{E} \cdot d\vec{s} \\ \frac{U_B}{q} - \frac{U_A}{q} = - \int_B^A \vec{E} \cdot d\vec{s}$$

### Example 25 F

A rod of length  $L$  located along the  $x$  axis has a total charge  $Q$  and a uniform linear charge density  $\lambda$ . find the electric potential at a point  $P$  located on the  $y$  axis a distance  $a$  from the origin.

$$\lambda = \frac{Q}{L}$$

$$dq = \lambda dl = \frac{Q}{L} dx$$

$$r = \sqrt{x^2 + a^2}$$

$$V_p = k \int \frac{dq}{r}$$
$$= k \int_0^L \frac{\frac{Q}{L} dx}{\sqrt{x^2 + a^2}}$$

$$= \frac{kQ}{L} \int_0^L \frac{dx}{\sqrt{x^2 + a^2}}$$

$$V_p = \frac{kQ}{L} \int \frac{a \sec^2 u \cdot du}{\sqrt{a^2 \sec^2 u}}$$

$$V_p = \frac{kQ}{L} \int \frac{\cancel{a} \sec^2 u \cdot du}{\cancel{a} \sec u}$$

$$V_p = \frac{kQ}{L} \int_{x=0}^{x=L} \sec u \cdot du$$

$$= \frac{kQ}{L} \ln [\sec u + \tan u]$$

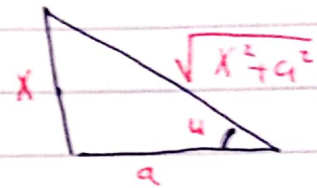
$$= \frac{kQ}{L} \ln \left[ \frac{\sqrt{x^2 + a^2}}{a} + \frac{x}{a} \right] \Big|_0^L = \frac{kQ}{L} \ln \left[ \frac{\sqrt{L^2 + a^2}}{a} + \frac{L}{a} \right]$$

$$V_p = \frac{kQ}{L} \ln \left[ \frac{\sqrt{L^2 + a^2}}{a} + \frac{L}{a} \right]$$

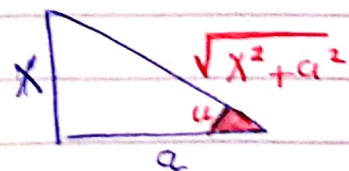
~~$-\ln(1)$~~  zero

we  $\tan u = \frac{x}{a}$

$$dx = a \sec^2 u \cdot du$$



$$x^2 + a^2 = a^2 \tan^2 u + a^2$$
$$= a^2 (\tan^2 u + 1)$$
$$= a^2 \sec^2 u$$





## ch- 27 - Currents and resistance.

Electric Currents is the rate of flow of charge through some region of space.

If  $\Delta Q$  is the charge that passes the A in time  $\Delta t$  then the average current is.

$$I_{\text{avg}} = \frac{\Delta Q}{\Delta t} \quad \text{average current}$$

If the rate at which the charge flows varies then the instantaneous current is

$$I = \lim_{\Delta t \rightarrow 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$

$$I = \frac{dQ}{dt} \quad \text{instantaneous current}$$

SI of current is Ampere (A)

$$1 \text{ A} = 1 \text{ C/s}$$

Current could be due to a flow of positive or negative charge.

⇒ Conventional current :- is the current due to the flow of positive charge

⇒ Direction of current is opposite to the direction of flow of electrons.

### Microscopic Model of Current

\* Consider a section of conductor of uniform cross sectional area  $A$

$n$  :- number of charge carriers per unit volume

\*  $v_d$  :- drift speed (average speed of charges)

$\Delta x = v_d \Delta t \equiv$  distance the charge moves in time  $\Delta t$



$$\text{Volume} = (\Delta x) A = v_d A \Delta t$$

$$\text{Total number of charges in volume} = (n) \text{ volume} \\ = n v_d A \Delta t$$

If  $q$  is the charge per charge carrier

$$\Delta Q = (q) (n v_d A \Delta t) = n q v_d A \Delta t$$

$$I = \frac{\Delta Q}{\Delta t} = \frac{n q v_d A \cancel{\Delta t}}{\cancel{\Delta t}}$$

$$[I = n q v_d A]$$

Ex. 26.2) A spherical Capacitor consists of a spherical conducting shell of radius  $b$  and charge  $-Q$  concentric with a smaller conducting sphere of radius  $a$  and charge  $Q$ . Find the capacitance of this device.

The electric field between the plates  
( $a < r < b$ )

$$\oint \vec{E} \cdot d\vec{A} = \frac{q_{\text{in}}}{\epsilon}$$

$$E (4\pi r^2) = \frac{Q}{\epsilon}$$

$$E = \frac{1}{4\pi\epsilon} \frac{Q}{r^2} = \frac{kQ}{r^2}$$

$$\Delta V = V_+ - V_- = - \int_{(-)}^{(+)} \vec{E} \cdot d\vec{s} \\ = - \int_b^a \frac{kQ}{r^2} dr$$

$$= -kQ \int_b^a \frac{dr}{r^2} \\ = -kQ \int_b^a \frac{dr}{r^2}$$

$$= -kQ \left( \frac{-1}{r} \right) \Big|_b^a$$

$$\Delta V = kQ \left( \frac{1}{a} - \frac{1}{b} \right)$$



Ex. 27.1

Copper ( $n = 8.48 \times 10^{28}$  electron /  $m^3$ )

$$q = e = 1.6 \times 10^{-19} \text{ C}$$

$$I = 10 \text{ A}$$

$$A = 3 \times 10^{-6} \text{ m}^2$$

$$I = nqV_d A$$

$$V_d = \frac{I}{nqA} = \frac{10 \text{ A}}{(8.48 \times 10^{28})(1.6 \times 10^{-19})(3 \times 10^{-6})}$$

$$V_d = 2.46 \times 10^{-4} \text{ m/s}$$

## Summary

$$I_{\text{avg}} = \frac{\Delta Q}{\Delta t}$$

$$I = \frac{dQ}{dt}$$

$$J = \frac{I}{A}$$

$$J = \sigma E = \text{Ohm's law}$$

$$\Delta V = RI = \text{Ohm's law}$$

$$R = \frac{\rho l}{A} ; \rho = \frac{1}{\sigma} , P =$$



## Electrical power

The energy gained when a charge  $\Delta Q$  from point  
to point b through a potential difference  
 $\Delta V$

$$\Delta U = (\Delta Q) \Delta V$$

The rate at which the charge  $\Delta Q$  loses a  
potential energy in going through the resistor

ex - 2F. 2

$$w = 0.32 \text{ mm} = 3.2 \times 10^{-4} \text{ m}$$

$$\rho = 1 \times 10^{-6} \Omega \cdot \text{m}$$

$$A) R = \frac{\rho L}{A}$$

$$\frac{R}{L} = \frac{\rho}{A} = \frac{\rho}{\pi r^2} = 3.1 \Omega / \text{m}$$

$$B) L = 1 \text{ m}, \Delta V = 10 \text{ V}$$

$$R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2} = 3.1 \Omega$$

$$I = \frac{\Delta V}{R}$$

$$= \frac{10 \text{ V}}{3.1 \Omega} = 3.2 \text{ A}$$



Ex. 27.4

$$\Delta V = 120 \text{ V}, \quad R = 8 \, \Omega$$

$$I = \frac{\Delta V}{R} = \frac{120 \text{ V}}{8 \, \Omega} = 15 \text{ A}$$

$$P = I \Delta V = 1.8 \text{ kW}$$

or

$$P = I^2 R = 1.8 \times 10^3 \text{ W} = 1.8 \text{ kW}$$

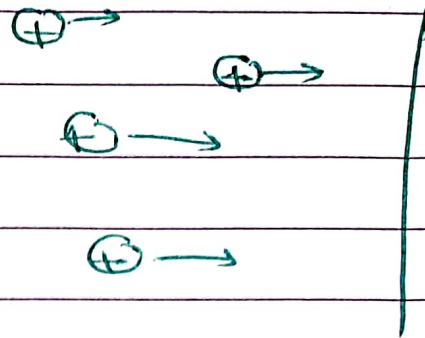
or

$$P = \frac{(\Delta V)^2}{R} = 1.8 \text{ kW}$$

prob - 27.5

$$q = e = 1.6 \times 10^{-19} \text{ C}$$

$$I = 125 \text{ } \mu\text{A} = 125 \times 10^{-6} \text{ A}$$



$$\Delta t = 23 \text{ s}$$

$$I = \frac{\Delta Q}{\Delta t}$$

$$\Delta Q = I(\Delta t)$$

$$N = \frac{\Delta Q}{q} = \frac{I \Delta t}{e} = \frac{(125 \times 10^{-6} \text{ A})(23 \text{ s})}{(1.6 \times 10^{-19} \text{ C})}$$

$$N = 1.8 \times 10^{16} \text{ proton}$$



#ch-28-

## Direct current circuits.

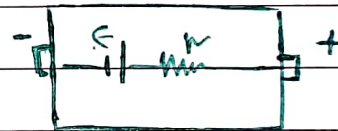
النوع  
التيار

- \* Direct current (DC)  $\equiv$  has a constant direction
- \* steady current  $\equiv$  has constant direction and constant magnitude
- \* Alternating current  $\equiv$  (AC)  $\equiv$  Alternate their direction

\* The electromotive force (emf or  $\mathcal{E}$ ) is the maximum possible voltage (potential difference) that the battery can provide between its terminals (الفرق الجهد)

$\mathcal{E}$  is electromotive force (emf)

$r$  is internal resistance



مقاومة داخلية

emf or  $\mathcal{E}$  is the potential difference between the terminals when the battery is in an open circuit

## Resistance and Ohm's Law.

The current density  $\vec{J}$  is defined as the current per unit area.

$$\vec{J} = \frac{I}{A}$$



If  $I = nqV_d A \Rightarrow \vec{J} = \frac{I}{A} = nqV_d$

SI unit of  $\vec{J}$   $A/m^2$ .

$\vec{J}$  is a vector that has a direction of the flow of positive charge (conventional current).

Ohm's law: The current density in a conductor is proportional to the electric field.

$$\vec{J} \propto \vec{E}$$

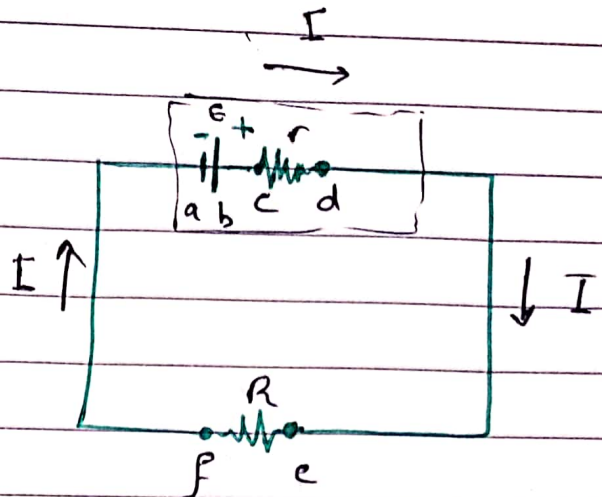
$$\vec{J} = \sigma \vec{E} \quad \text{Ohm's law}$$

$\sigma$  = electrical conductivity of the material (depends on the material).



## # comments

- \* The current flows from the higher to the lower potential
- \* The positive terminal is at higher potential and the negative terminal is at lower potential
- \* We consider the wires to have no resistance.



$\epsilon \equiv \text{emf (electromotive force)}$

$r \equiv \text{internal resistance.}$

$R : \text{load resistance}$

$V_d - V_a \equiv \text{term}$

→ The terminal voltage is

$$\Delta V = V_d - V_a = \epsilon - Ir$$

$$\Delta V = V_d - V_a = \epsilon - Ir$$

$$\Delta V = V_d - V_a = IR$$

$$\epsilon - Ir = IR \Rightarrow \epsilon = I(r + R)$$

$$\boxed{I = \frac{\epsilon}{r + R}}$$



## [power]

\* The total power output of the battery is

$$P = I \mathcal{E}$$

or

\* The power dissipated in the internal resistance ( $P_r$ ) and in the load resistance  $P_R$  are

$$P_r = I^2 r$$

$$P_R = I^2 R$$

$$P = P_r + P_R$$

$$I \mathcal{E} = I^2 R + I^2 r$$

Ex. 28.1

$$\mathcal{E} = 12 \text{ V}, \quad r = 0.05 \, \Omega$$

$$R = 3 \, \Omega$$

$$A) \quad I = \frac{\mathcal{E}}{r+R} = \frac{12}{0.05+3} = 3.93 \text{ A}$$

$$\Delta V = \mathcal{E} - Ir = 12 - (3.93)(0.05)$$

$$\Delta V = 11.8 \text{ V}$$

or

$$\Delta V = IR = (3.93 \text{ A})(3 \, \Omega) = 11.8 \text{ V}$$

$$B) \quad P_r = I^2 r = (3.93 \text{ A})^2 (0.05 \, \Omega) = 0.772 \text{ W}$$

$$P_R = I^2 R = (3.93 \text{ A})^2 (3) = 46.3 \text{ W}$$

$$P = I\mathcal{E} = (3.93 \text{ A})(12 \text{ V}) = 47.1 \text{ W}$$

$$P = P_r + P_R = 47.1 \text{ W}$$



Consider a section of conductor at uniform cross section  $A$

$$\Delta V = E L, \quad J = \frac{I}{A}$$

$$J = \sigma E$$

$$\frac{I}{A} = \sigma \frac{\Delta V}{L}$$

$$\Delta V = \left( \frac{L}{\sigma A} \right) I$$

$$\boxed{\Delta V = R I} \quad \text{ohm's law}$$

$$\boxed{R = \frac{L}{\sigma A} = \frac{\rho L}{A}}, \quad R : \text{resistance.}$$

$\rho$  - resistivity

$$\boxed{\rho = \frac{1}{\sigma}}$$

$\rho$  and  $\sigma$  depends on the material  $R$  depends on the material and the geometry (size and shape)

SI unit of  $R$  is Ohm ( $\Omega$ )

$$1 \Omega = 1 \text{ V/A}$$

$$I = \left( \frac{1}{R} \right) \Delta V \equiv \text{slope} = \frac{1}{R}$$

$I$  is linear with  $\Delta V$  for ohmic material.

### Ex. 28.2

Find the load resistance  $R$  for which the maximum power is delivered to the load resistance in figure.

$$I = \frac{\mathcal{E}}{r+R}$$

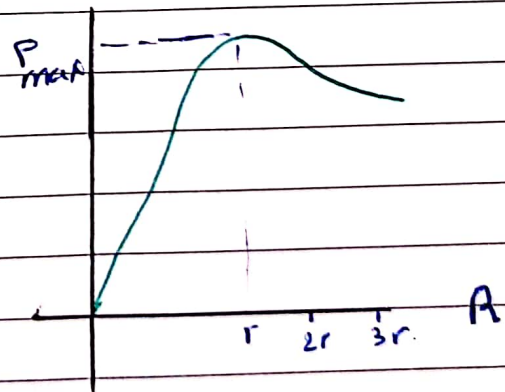
$$P_R = I^2 R$$
$$P_R = \frac{\mathcal{E}^2 R}{(r+R)^2}$$

$$\frac{dP_R}{dR} = \mathcal{E}^2 \left[ \frac{(r+R)^2 - 2R(r+R)}{(r+R)^4} \right] = 0$$

$$\Rightarrow (r+R)^2 - 2R(r+R) = 0$$

$$r - R = 0$$
$$\boxed{R=r} \text{ maximum.}$$

$$P_{R\max} = \frac{\mathcal{E}^2 r}{(r+r)^2} = \mathcal{E}^2 / 4r$$





## Resistors in series and parallel

### 1) Resistors in series.

$$I_1 = I_2 = I \text{ (Current is the same)}$$

$$\Delta V = \Delta V_1 + \Delta V_2$$
$$IR_{eq} = IR_1 + IR_2$$

$$R_{eq} = R_1 + R_2$$

For more than two resistors connected in series.

$$R_{eq} = R_1 + R_2 + R_3$$

$R_{eq}$  is greater than the greatest resistor in the group.

## 2) Resistors in parallel

$$\Delta V = \Delta V_1 = \Delta V_2$$

The current at the junction is

$$I = I_1 + I_2$$

$$\frac{\cancel{\Delta V}}{R_{eq}} = \frac{\cancel{\Delta V}}{R_1} + \frac{\cancel{\Delta V}}{R_2}$$

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}}$$

$$\text{or } \boxed{R_{eq} = \frac{R_1 R_2}{R_1 + R_2}}$$

For more than two resistors connected in parallel

$$\boxed{\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots}$$

resistors in  
parallel

$R_{eq}$  in parallel is smaller than the smallest resistor in the group



### Ex. 28.4

Four resistors are connected as shown

A) find the equivalent resistance between points a and c.

A)  $R_1, R_2$ , parallel

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2} = 2 \Omega$$

$R_3, R_4$  + series

$$R_{34} = R_3 + R_4 = 12 \Omega$$

$R_{12}, R_{34}$  + series

$$R_{eq} = R_{12} + R_{34} = 14 \Omega$$

B) what is the current in each resistor if a potential difference of 42 V is maintained between a and c?

$$\Delta V = 42 \text{ V}$$

$$I = \frac{\Delta V}{R_{eq}} = \frac{42 \text{ V}}{14} = 3 \text{ A}$$

The current passing  $R_3$  and  $R_4$  is the same current  
( $I = 3 \text{ A}$ )

$$I_3 = I_4 = 3 \text{ A}$$

$$\Delta V_{bc} = I R_{12} = (3 \text{ A})(2 \Omega) = 6 \text{ V}$$

$$I_1 = \frac{\Delta V_{bc}}{R_1} = \frac{6 \text{ V}}{6 \Omega} = 1 \text{ A}$$

$$I_2 = \frac{\Delta V_{bc}}{R_2} = \frac{6 \text{ V}}{3 \Omega} = 2 \text{ A}$$

$$I = I_1 + I_2 \Rightarrow 3 \text{ A} = 1 \text{ A} + 2 \text{ A}$$

Ex. 28.5) Three resistors are connected in parallel as shown  
A potential difference of 18.0 V is maintained between points a and b

A) Calculate the equivalent resistance of the circuit

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{6} + \frac{1}{9} = \frac{11}{18}$$

$$R_{eq} = \frac{18}{11} \Omega = 1.64 \Omega$$

B) Find the current in each resistor

$$I_1 = \frac{\Delta V}{R_1} = \frac{18V}{3\Omega} = 6A$$

$$I_2 = \frac{\Delta V}{R_2} = \frac{18V}{6\Omega} = 3A$$

$$I_3 = \frac{\Delta V}{R_3} = \frac{18V}{9\Omega} = 2A$$

$$I = I_1 + I_2 + I_3 = 11A$$

$$I = \frac{\Delta V}{R_{eq}} = \frac{18V}{(18/11)} = 11A$$

c) Calculate the power delivered to each resistor and the total power delivered to the combination of resistors

$$P_1 = I_1^2 R_1 = 108W \quad P_2 = I_2^2 R_2 = 54W$$

$$P_3 = I_3^2 R_3 = 36W$$

$$P = P_1 + P_2 + P_3 = 198W$$

$$\text{or } P = I\Delta V = (11A)(18V) = 198W$$



## (Kirchhoff's Rules)

\* In some circuits, resistors can not be reduced to a ~~single~~ single equivalent resistance.

\* Therefore the Kirchhoff's rules can be applied

### 1) Junction Rule.

It is a consequence of conservation of charge

- The sum of the current at the junction must equal zero.

$$\sum_{\text{Junction}} I = 0$$

Convention & current entering the junction is positive  
(+) current leaving the junction is negative.

### Comments

- a) If the resistor is traversed in direction of current

$$\Delta V = V_b - V_a = -IR$$

- b) If the resistor is traversed opposite to the current  $I$

$$\Delta V = V_b - V_a = IR$$

- c) If the emf is traversed from the negative to the positive terminal

$$\Delta V = V_b - V_a = +\mathcal{E}$$

- d) If the emf is traversed from (+) to (-)

$$\Delta V = V_b - V_a = -\mathcal{E}$$



## RC - circuit

RC - circuit contains a series combination of a resistor and capacitor

I) charging a capacitor

$$\sum_{\text{order}} \Delta V = 0$$

$$\mathcal{E} - \frac{q}{C} - IR = 0 \quad \text{--- (1)}$$

Two extreme cases :

1) At  $t = 0$ ,  $q = 0 \Rightarrow$

$$\mathcal{E} - IR = 0$$

$$\boxed{I_i = \frac{\mathcal{E}}{R}} \equiv \text{Initial (maximum) current}$$

In the intermediate case

$$\mathcal{E} - \frac{q}{C} - IR = 0 \quad ; \quad I = \frac{dq}{dt}$$

$$\mathcal{E} - \frac{q}{C} - R \frac{dq}{dt} = 0$$

$$\frac{dq}{dt} = - \frac{q - C\mathcal{E}}{RC}$$

$$\int_0^q \frac{dq}{q - C\varepsilon} = -\frac{1}{RC} \int_0^t dt$$

$$\ln\left(\frac{q - C\varepsilon}{-C\varepsilon}\right) = \frac{-t}{RC}$$

$$\frac{q - C\varepsilon}{-C\varepsilon} = e^{-t/RC}$$

$$q(t) = C\varepsilon(1 - e^{-t/RC}) = Q_{\max}(1 - e^{-t/\tau})$$

$$\boxed{\tau = RC} \equiv \text{time constant of RC-circuit}$$

The current is

$$I = \frac{dq}{dt}$$

$$\boxed{I = \frac{\varepsilon}{R} e^{-t/RC} = I_i e^{-t/\tau}}$$

The voltage on the capacitor

$$\frac{q}{C} = \frac{C\varepsilon}{C}(1 - e^{-t/RC})$$

$$\boxed{V_c = \varepsilon(1 - e^{-t/RC}) = \varepsilon(1 - e^{-t/\tau})}$$



Prob 28.1 A battery has an emf of 15.0 V

The terminal voltage of the battery is 11.6 V when it is delivering 20.0 W of power to an external load resistor R (a) what is the value of R? (b) what is the internal resistance of the battery?

$$\mathcal{E} = 15 \text{ V}$$

$$\Delta V = V_d - V_a = 11.6 \text{ V when } P_R = 20 \text{ W}$$

$$A) \quad P_R = \frac{(\Delta V)^2}{R}$$

$$R = \frac{(\Delta V)^2}{P_R} = \frac{(11.6 \text{ V})^2}{20 \text{ W}} = 6.73 \, \Omega$$

$$I = \frac{\Delta V}{R} = \frac{11.6 \text{ V}}{6.73 \, \Omega} = 1.72 \text{ A}$$

$$\Delta V = \mathcal{E} - Ir$$

$$r = \frac{\mathcal{E} - \Delta V}{I} = \frac{15 \text{ V} - 11.6 \text{ V}}{1.72 \text{ A}} = 1.97 \, \Omega$$

B)

$$R_{12} = R_1 + R_2 = 9 \Omega \text{ (series)}$$

$$R_{34} = R_3 + R_4 = 6 \Omega \text{ (series)}$$

$$R_{eq} = \frac{R_{12} R_{34}}{R_{12} + R_{34}} = 3.6 \Omega$$

$$\frac{q}{C} = \frac{Q_i}{C} e^{-t/RC}$$

$$V_c = \Delta V_c e^{-t/RC_{eq}}$$

$$V_c = \frac{1}{10} \Delta V_c$$

$$\frac{1}{10} \cancel{\Delta V_i} = \cancel{\Delta V_i} e^{-t/RC}$$

$$= -\ln 10 = \frac{-t}{RC}$$

$$t = (RC_{eq}) \ln 10$$

$$= (3.6 \Omega) (1 \times 10^{-6} \text{ F}) \ln(10)$$

$$t = 8.29 \mu\text{s}$$



Prob. 28.43 The circuit in Figure has been connected for a long time (a) What is the potential difference across the capacitor? (b) If the battery is disconnected from the circuit, over what time interval does the capacitor discharge to one-tenth its initial voltage?

For long time  $\Rightarrow$  the current in the branch of the capacitor is zero because it is max charged

$$\Rightarrow R_1, R_3 : \text{series} \Rightarrow R_{13} = R_1 + R_3 = 5 \Omega$$

$$R_2, R_4 : \text{series} \Rightarrow R_{24} = R_2 + R_4 = 10 \Omega$$

$$I_2 = \frac{\mathcal{E}}{R_{13}} = \frac{10 \text{ V}}{5 \Omega} = 2 \text{ A}$$

$$I_3 = \frac{\mathcal{E}}{R_{24}} = \frac{10 \text{ V}}{10 \Omega} = 1 \text{ A}$$

$$V_a - V_b = I_3 R_2 - I_2 R_1 \\ = (1)(8) - (2)(1) =$$

$$\boxed{V_a - V_b = \Delta V_c = 6 \text{ V}}$$

$\equiv$  potential across the capacitor

$$\Delta V_c = 6 \text{ V}$$

Prob. 28.9 Consider the circuit shown in Fig. find  
(a) the current in the  $20\text{-}\Omega$  resistor and (b) the  
potential difference between points a and b.

$R_4, R_5$  series

$$R_{45} = R_4 + R_5 = 20\text{ }\Omega + 5\text{ }\Omega = 25\text{ }\Omega$$

$R_2, R_3, R_{45}$  parallel

$$\frac{1}{R_{2345}} = \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_{45}} = \frac{1}{10\text{ }\Omega} + \frac{1}{5\text{ }\Omega} + \frac{1}{25\text{ }\Omega} = \frac{17}{50\text{ }\Omega}$$

$$R_{2345} = \frac{50}{17}\text{ }\Omega = 2.94\text{ }\Omega$$

$R_1, R_{2345}$  series.

$$R_{eq} = R_1 + R_{2345} = 12.94\text{ }\Omega$$

$$I = \frac{\mathcal{E}}{R_{eq}} = \frac{20\text{ V}}{12.94} = 1.93\text{ A}$$

$$\Delta V_{ab} = I R_{2345} = (1.93\text{ A})(2.94\text{ }\Omega)$$

$$\Delta V_{ab} = 5.68\text{ V}$$

$$I_4 = \frac{\Delta V_{ab}}{R_{45}} = \frac{5.68\text{ V}}{25\text{ }\Omega} = 0.227\text{ A} = 227\text{ mA}$$

$$I_2 = \frac{\Delta V_{ab}}{R_2} = \frac{5.68\text{ V}}{10\text{ }\Omega} = 0.568\text{ A}$$

$$I_3 = \frac{\Delta V_{ab}}{R_3} = \frac{5.68\text{ V}}{5} = 1.136\text{ A}$$



### Prob - 28.14]

- a) when the switch  $S$  in the circuit of figure is closed, will the equivalent resistance between points  $a$  and  $b$  increase or decrease? state your reasoning.
- b) Assume the equivalent resistance drops by 50.0% when the switch is closed. Determine the value of  $R$ .

A) If  $S$  is closed

$R_1, R_2$  : parallel

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2} = 9 \Omega$$

$R_3, R_4$  : parallel

$$R_{34} = \frac{R_3 R_4}{R_3 + R_4} = 9 \Omega$$

$R_1, R_{12}, R_{34}$  : Series

$$\begin{aligned} R_{eq} &= R + R_{12} + R_{34} \\ &= R + 9 \Omega + 9 \Omega \\ &= R + 18 \Omega \end{aligned}$$

If  $S$  is open  $R_1, R_3$  : Series  $R_{13} = R_1 + R_3 = 100 \Omega$

$R_2, R_4$  : Series

$$R_{24} = R_2 + R_4 = 100 \Omega$$

$R_{13}, R_{24}$  : parallel

$$R_{1234} = \frac{R_{13} R_{24}}{R_{13} + R_{24}} = 50 \Omega$$

$R, R_{1234}$  : Series

$$R_{eqc} = R + R_{1234} = R + 50 \Omega$$

If  $S$  is closed the  $R_{eq}$  decreases

$$R_{eq}(S \text{ closed}) = 0.5 (R_{eq}(S \text{ open}))$$

$$R + 18 \Omega = 0.5 (R + 50 \Omega)$$

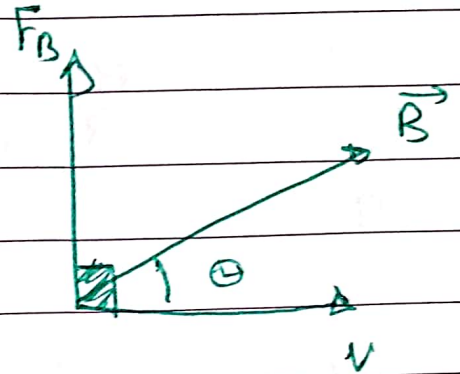
$$R + 18 = 0.5 R + 25$$

$$R = 14 \Omega$$

~~Q~~ The magnetic force on a moving charge  $q$  is

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

$$F_B = q v B \sin \theta$$





Ex. 29.1

$$q = -e = -1.6 \times 10^{-19} \text{ C}$$

$$v = 8 \times 10^6 \text{ m/s}$$

$$B = 0.025 \text{ T}$$

$$\theta = 60^\circ$$

$$F_B = |q|vB \sin \theta$$

$$= (1.6 \times 10^{-19}) (8 \times 10^6) (0.025) \sin 60^\circ$$

$$= 2.8 \times 10^{-4} \text{ N}$$

## Motion of a charged particle in a uniform magnetic field

$$\vec{B} = \text{Const (uniform)}$$

$$\vec{v} \perp \vec{B}$$

It is clear that

1. The path followed by the particle is circular.
2. The magnetic force ( $\vec{F}_3$ ) is toward the center of the circle.
3.  $\vec{F}_3$  causes centripetal acceleration ( $a_c = \frac{v^2}{r}$ )
4.  $\vec{F}_3$  changes the direction of  $\vec{v}$  only  
 $\Rightarrow \vec{F}_3$  does not change the magnitude of  $\vec{v}$   
 $\Rightarrow F_3$  does not change the kinetic energy.  
 $\Rightarrow$  magnetic force ( $\vec{F}_3$ ) does no work.



$$\vec{v} \perp \vec{B} \Rightarrow \theta = \frac{\pi}{2}$$

$$F_B = qvB \sin\left(\frac{\pi}{2}\right) = qvB$$

$$F_B = qvB = ma_c$$

$$qvB = m \frac{v^2}{r}$$

$r = \frac{mv}{qB}$	The radius of the path.
---------------------	-------------------------

Angular Speed ( $\omega$ ) :

$$s = \odot r$$

$$\frac{ds}{dt} = \frac{d\odot}{dt} r$$

$$\boxed{v = \omega r} \quad \omega : \text{angular speed}$$

$$\boxed{\omega = \frac{v}{r}}$$

$$\omega = \frac{v}{r} = \frac{v}{\cancel{mv} qB} = \frac{qB}{m}$$

$$\boxed{\omega = \frac{v}{r} = \frac{qB}{m}} \quad \text{angular speed.}$$

The period ( $T$ ) : Time for one revolution

$$\boxed{v = \frac{2\pi r}{T} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}} \quad \text{period of the motion.}$$

The Frequency ( $f$ ) : Number of revolutions in one second

$$\boxed{f = \frac{1}{T} = \frac{v}{2\pi r} = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}}$$



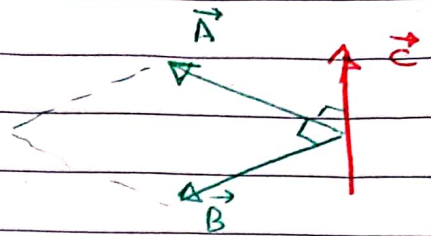
## Ch-29. magnetic fields.

Cross (vector) product

$$\vec{C} = \vec{A} \times \vec{B}$$

$$|\vec{C}| = |\vec{A}| |\vec{B}| \sin \Theta$$

$$C = AB \sin \Theta$$



Direction of  $\vec{C}$  is determined by the right hand rule  
 $\vec{C} \perp \vec{A}$ , and  $\vec{C} \perp \vec{B}$

$$\vec{B} \times \vec{A} = -\vec{A} \times \vec{B} \quad (\text{not commutative})$$

$$\vec{A} \times (\vec{B} + \vec{C}) = \vec{A} \times \vec{B} + \vec{A} \times \vec{C}$$

$$\frac{d}{dt} (\vec{A} \times \vec{B}) = \frac{d\vec{A}}{dt} \times \vec{B} + \vec{A} \times \frac{d\vec{B}}{dt}$$

If  $\Theta = 0$  (  $\vec{A}$  and  $\vec{B}$  parallel ) or  $\pi$  (  $\vec{A}$  and  $\vec{B}$  antiparallel ) then

$$\vec{A} \times \vec{B} = 0 \quad (\Theta = 0, \pi)$$

$$\Rightarrow \vec{A} \times \vec{A} = 0$$

$$\text{If } \Theta = \frac{\pi}{2} \left( \vec{A} \perp \vec{B} \right) \Rightarrow |\vec{A} \times \vec{B}| = AB$$

unit vectors.

$$|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$$

$$\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$$

$$|\hat{i} \times \hat{j}| = (1)(1) \sin \frac{\pi}{2} = 1$$

$$\hat{i} \times \hat{j} = \hat{k}$$

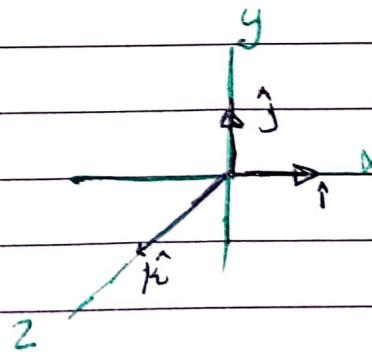
$$\hat{j} \times \hat{k} = \hat{i}$$

$$\hat{k} \times \hat{i} = \hat{j}$$

$$\hat{j} \times \hat{i} = -\hat{k}$$

$$\hat{k} \times \hat{j} = -\hat{i}$$

$$\hat{i} \times \hat{k} = -\hat{j}$$



In terms of unit vectors

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

$$\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$= (A_y B_z - A_z B_y) \hat{i} - (A_x B_z - A_z B_x) \hat{j} + (A_x B_y - A_y B_x) \hat{k}$$



## ch-29- ~~mag~~ magnitude field

\* Every magnet, regardless of its shape, has two poles

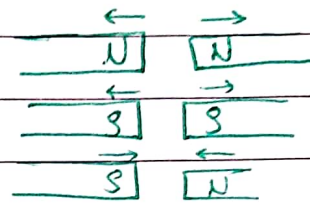
1) North pole (N)

2) South pole (S)



\* Magnetic poles exert forces on one another

- Like poles repel each other
- Unlike poles attract each other.



\* If a bar magnet is ~~sub~~ suspended as it can move freely, it will rotate such that the north pole points toward the north geographic pole.

\* Magnetic poles are always found in pairs (N S)

\* The region of space surrounding a moving charge (current) contains a magnetic field.

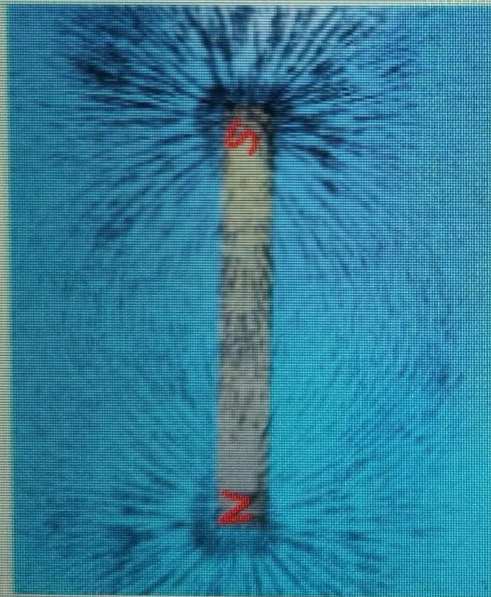
\* Region of space surrounding magnetic materials (make up permanent magnetic) contains ~~mag~~ magnetic field

$\vec{B} \equiv$  magnetic field.



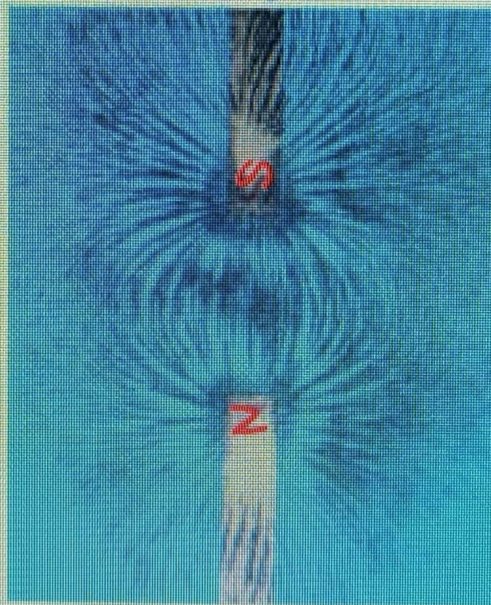
# magnetic field lines

Magnetic field pattern surrounding a bar magnet



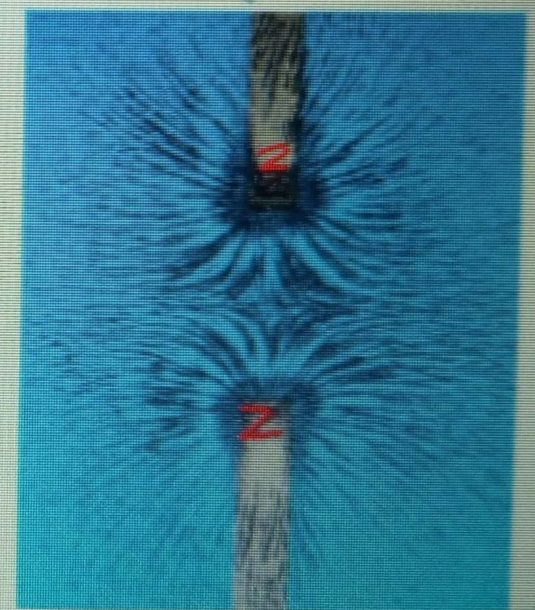
a

Magnetic field pattern between *opposite* poles (N-S) of two bar magnets



b

Magnetic field pattern between *like* poles (N-N) of two bar magnets



c



## Properties of field lines.

1. The lines outside the magnet point from the north pole (N) to the south pole (S).
2. The magnetic field ( $\vec{B}$ ) is tangent to the field lines.
3. Number of field lines per unit (intensity of lines) is proportional to the magnitude of the field,  $|\vec{B}|$ .
4. The field lines form closed loops.

SI unit of  $\vec{B}$  is Tesla (T)

$$1 \text{ T} = 1 \text{ Wb/m}^2 = 1 \text{ N/A}\cdot\text{m}$$

The cgs unit of  $\vec{B}$  is Gauss (G)

$$1 \text{ G} = 10^{-4} \text{ T}$$

## magnetic force on a moving charge.

$\vec{F}_B \equiv$  magnet force.

It was found

- $F_B \propto q$
- $F_B \propto v$ , ( $F_B = 0$  if  $v = 0$ )
- $F_B \propto B$
- $F_B \propto \sin \theta$  (If  $\theta = 0$  or  $\pi$  then  $F_B = 0$ )  
 $\vec{F}_B \perp \vec{v}$ ,  $\vec{F}_B \perp \vec{B}$

The properties are summeried in

$$\vec{F}_B = q \vec{v} \times \vec{B}$$

$$F_B = q v B \sin \theta$$

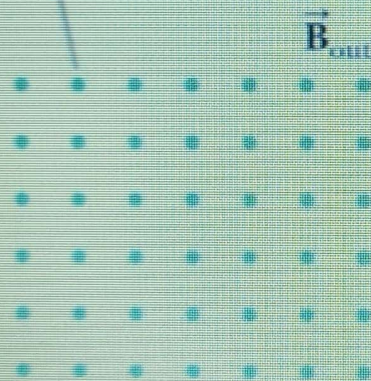
\* The magnetic force on a negative charge is opposite to the magnetic force on a positive charge.

$$\vec{F}_B = q \vec{v} \times \vec{B}$$



# Convention

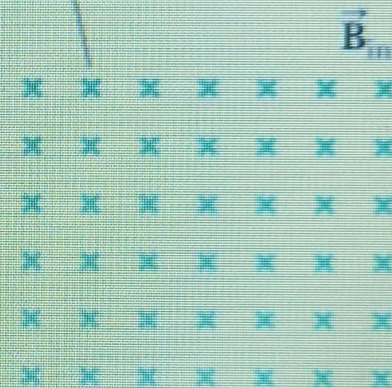
Magnetic field lines coming out of the paper are indicated by dots, representing the tips of arrows coming outward.



a

$B_{out}$  (out of the page)

Magnetic field lines going into the paper are indicated by crosses, representing the feathers of arrows going inward.

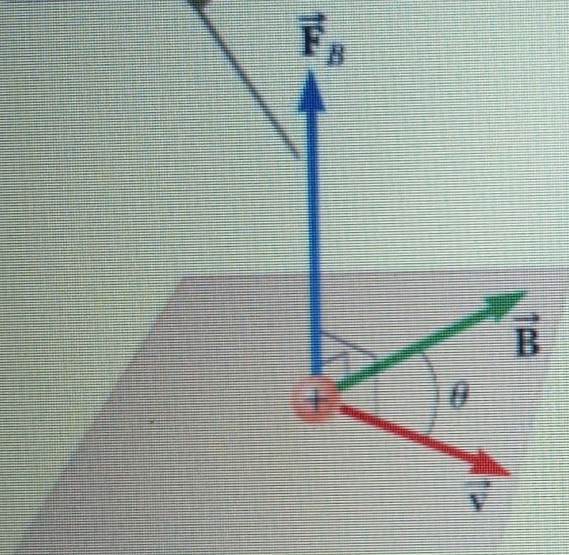


b

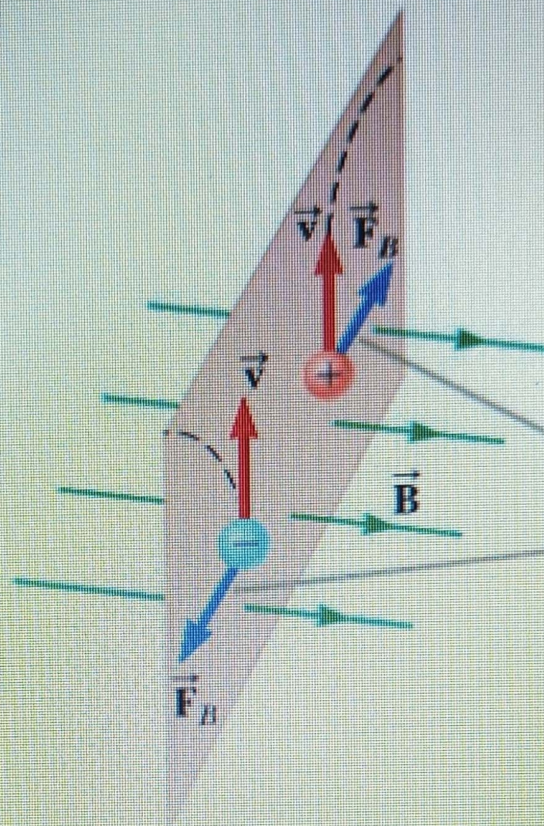
$B_{in}$  (into the page)



The magnetic force is perpendicular to both  $\vec{v}$  and  $\vec{B}$ .



a



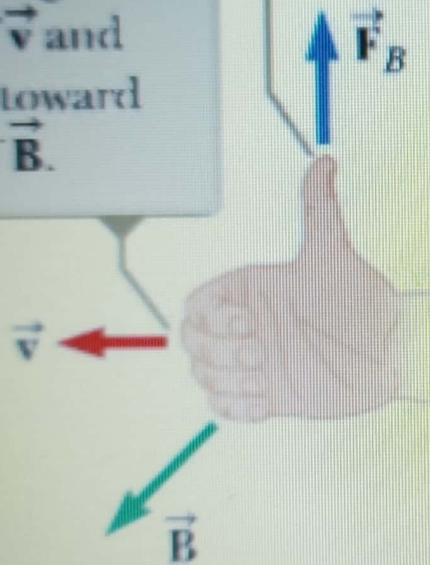
The magnetic forces on oppositely charged particles moving at the same velocity in a magnetic field are in opposite directions.

b

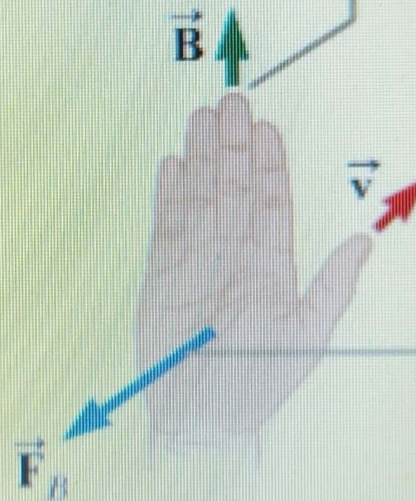


(2) Your upright thumb shows the direction of the magnetic force on a positive particle.

(1) Point your fingers in the direction of  $\vec{v}$  and then curl them toward the direction of  $\vec{B}$ .



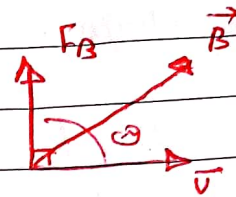
(1) Point your fingers in the direction of  $\vec{B}$ , with  $\vec{v}$  coming out of your thumb.



(2) The magnetic force on a positive particle is in the direction you would push with your palm.

The magnetic force on a moving charge  $q$  is

$$\vec{F}_B = q \vec{v} \times \vec{B}$$
$$F_B = q v B \sin \theta$$



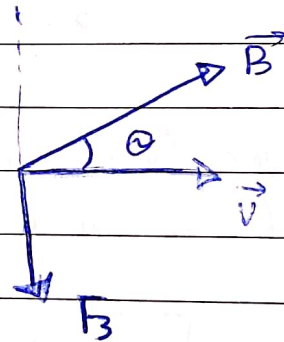
**EX. 29.1**  $\Rightarrow$  An electron in an old-style television picture tube moves toward the front of the tube with a speed of  $8.0 \times 10^6$  m/s along the x axis. Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude 0.025 T, directed at an angle of  $60^\circ$  to the x-axis and lying in the xy plane. Calculate the magnetic force on the electron.

$$q = -e = -1.6 \times 10^{-19} \text{ C}$$

$$v = 8 \times 10^6 \text{ m/s}$$

$$B = 0.025 \text{ T}$$

$$\theta = 60^\circ$$



$$F_B = |q| v B \sin \theta$$
$$= (1.6 \times 10^{-19}) (8 \times 10^6) (0.025) \sin 60^\circ$$
$$= 2.8 \times 10^{-4} \text{ N}$$



## Motion of a charged particle in a uniform magnetic field

$$\vec{B} = \text{const (uniform)}$$

$$\vec{v} \perp \vec{B}$$

It is clear that:

1. The path followed by the particle is circular
2. The magnetic force ( $\vec{F}_B$ ) is toward the center of the circle
3.  $\vec{F}_B$  causes centripetal acceleration ( $a_c = \frac{v^2}{r}$ )
4.  $\vec{F}_B$  changes the direction of  $\vec{v}$  only.  
 $\Rightarrow \vec{F}_B$  does not change the magnitude of  $\vec{v}$   
 $\Rightarrow \vec{F}_B$  does not change the kinetic energy  
 $\Rightarrow$  magnetic force ( $\vec{F}_B$ ) does no work.

$$\vec{v} \perp \vec{B} \Rightarrow \theta = \frac{\pi}{2}$$

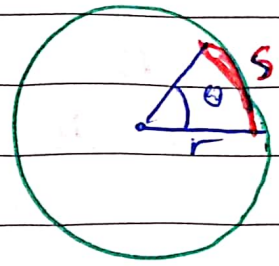
$$F_B = qvB \sin\left(\frac{\pi}{2}\right) = qvB$$

$$F_B = qvB = ma_c$$

$$qvB = \frac{mv^2}{r}$$

$$\boxed{r = \frac{mv}{qB}} \quad \text{The radius of the path.}$$

## Angular speed ( $\omega$ )



$$s = \omega r$$

$$\frac{ds}{dt} = \frac{d\theta}{dt} r$$

$$\boxed{v = \omega r} \quad ; \quad \omega : \text{angular speed}$$

$$\boxed{\omega = \frac{v}{r}}$$

$$\omega = \frac{v}{r} = \frac{\cancel{v}}{mv/qB} = \frac{qB}{m}$$

$$\boxed{\omega = \frac{v}{r} = \frac{qB}{m}} \quad \text{angular speed.}$$

→ The period ( $T$ ) : Time for one revolution

$$v = \frac{2\pi r}{T}$$

$$\boxed{T = \frac{2\pi r}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}}$$

period of the motion.

The Frequency ( $F$ ) :- number of revolutions in one second.

$$\boxed{F = \frac{1}{T} = \frac{v}{2\pi r} = \frac{\omega}{2\pi} = \frac{qB}{2\pi m}}$$

Frequency  
( $\text{Hz} \equiv \text{s}^{-1}$ )



**Ex] 29.2]** A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35 T magnetic field perpendicular to the velocity of the proton. Find the speed of the proton.

Proton :  $q = 1.6 \times 10^{-19} \text{ C}$ ,  $m = 1.67 \times 10^{-27} \text{ Kg}$ .

$r = 14 \text{ cm} = 0.14 \text{ m}$

$B = 0.35 \text{ T}$  (uniform)

$\vec{v} \perp \vec{B}$  ( $\theta = \pi/2$ )

Speed :  $F_B = m a_c$   
 ~~$q v B = m \frac{v^2}{r}$~~

$$v = \frac{q B r}{m} = 4.7 \times 10^6 \text{ m/s}$$

Find the angular speed.

$$\omega = \frac{v}{r} = 3.36 \times 10^7 \text{ rad/s. or}$$

$$\omega = \frac{q B}{m} = 3.36 \times 10^7 \text{ rad/s}$$

Find the period of the motion.

$$T = \frac{2\pi r}{v} = 1.9 \times 10^{-7} \text{ s. or}$$

$$T = \frac{2\pi}{\omega} = 1.9 \times 10^{-7} \text{ s. or}$$

$$T = \frac{2\pi m}{q B} = 1.9 \times 10^{-7} \text{ s.}$$

### Ex. 29.3

$$\text{electron : } q = -1.6 \times 10^{-19} \text{ C.}$$

$$m = 9.11 \times 10^{-31} \text{ kg.}$$

$$v_i = 0, \quad v_f = v$$

$$\Delta V = 350 \text{ V}$$

$$r = 7.5 \text{ cm} = 0.075 \text{ m.}$$

$$A) \quad \Delta K + \Delta U = 0$$

$$\left( \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2 \right) + q\Delta V = 0.$$

$$v = \sqrt{\frac{-2q\Delta V}{m}} = 1.1 \times 10^7 \text{ m/s.}$$

$$F_B = mac$$

$$qvB = \frac{mv^2}{r}$$

$$B = \frac{mv}{qr} = 8.4 \times 10^{-4} \text{ T}$$

$$B) \quad \omega = \frac{v}{r} = 1.5 \times 10^5 \text{ rad/s.}$$

$$\text{or } \omega = \frac{qB}{m} = 1.5 \times 10^5 \text{ rad/s}$$

Calculate ML period (T)

$$T = \frac{2\pi}{\omega} = 4.18 \times 10^{-5} \text{ s.}$$

or

$$T = \frac{2\pi r}{v} = 4.18 \times 10^{-5} \text{ s.}$$

or

$$T = \frac{2\pi m}{qB} = 4.18 \times 10^{-5} \text{ s}$$



## Magnetic force on a current carrying conductor

- A magnetic force is ~~ex~~ exerted on a ~~current~~ carrying conductor when placed in a magnetic field.
- Direction of the magnetic force is determined by the right-hand-rule.
- The magnetic force on a single charge moving in the wire with the drift speed,  $v_d$ , in a magnetic  $\vec{B}$  is

$$\vec{F}_{\text{single-charge}} = q \vec{v}_d \times \vec{B}$$

The total force is

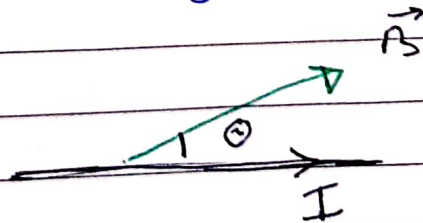
$$\vec{F}_B = (\vec{F}_{\text{single-charge}}) (\text{number of charges})$$
$$= (q \vec{v}_d \times \vec{B}) (nAL)$$

$$\vec{F}_B = \left[ \underbrace{(nq \vec{v}_d A)}_I \right] \times \vec{B}$$

$$\vec{F}_B = (I \vec{L}) \times \vec{B}$$

$\vec{F}_B = I \vec{L} \times \vec{B}$  The force on the straight wire.

$$|\vec{F}_B| = ILB \sin \theta$$

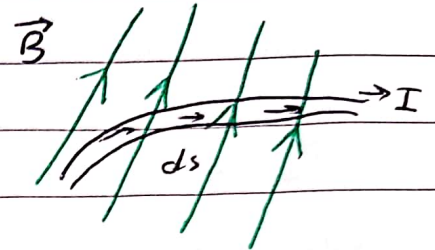


For an irregular shaped wire we consider a ~~segment~~ segment  $d\vec{s}$

$$d\vec{F}_B = I d\vec{s} \times \vec{B} \equiv \text{Force on segment } d\vec{s}$$

The total force is the sum over all segment.

$$\vec{F}_B = I \int d\vec{s} \times \vec{B}$$



**EX. 29.4** The force on the straight motion

$$\vec{F} = I \vec{L} \times \vec{B}$$

$$L = 2R, \quad \theta = \frac{\pi}{2}$$

$\vec{F}_B$  is out the page  $(+\hat{k})$

$$F_1 = ILB \sin \theta \hat{k}$$

$$= I(2R)B \sin \frac{\pi}{2} \hat{k}$$

$$\boxed{F_1 = 2IRB \hat{k}}$$

The force on the current portion

$$\begin{aligned} \vec{F}_2 &= I \int d\vec{s} \times \vec{B} \quad + \quad d\vec{s} \times \vec{B} = (ds) B \sin \theta (-\hat{k}) \\ &= -I \int B(ds) \sin \theta \hat{k} \quad = -B(ds) \sin \theta \hat{k} \\ &= -I \int_0^{\pi} B(R d\theta) \sin \theta \hat{k} \quad ds = R d\theta \end{aligned}$$

$$= -IRB \int_0^{\pi} \sin \theta d\theta \hat{k}$$

$$\boxed{F_2 = -2IRB \hat{k}} \quad \text{The force on the curved portion.}$$



we have

$$\vec{F}_1 = 2IRB \hat{k}$$

$$\vec{F}_2 = -2IRB \hat{k}$$

Two Conclusions

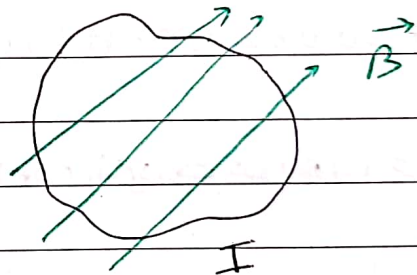
$$\vec{F}_1 = -\vec{F}_2 \Rightarrow \vec{F}_1 + \vec{F}_2 = 0.$$

10 [Conclusion]

The force on the curved portion is equal in magnitude and opposite direction to the force on the straight portion.

11 The magnetic force on a closed current loop in a uniform magnetic field is zero.

$$\boxed{\Sigma \vec{F} = 0}$$



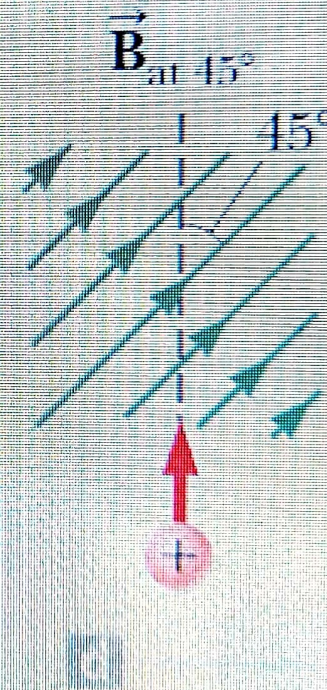
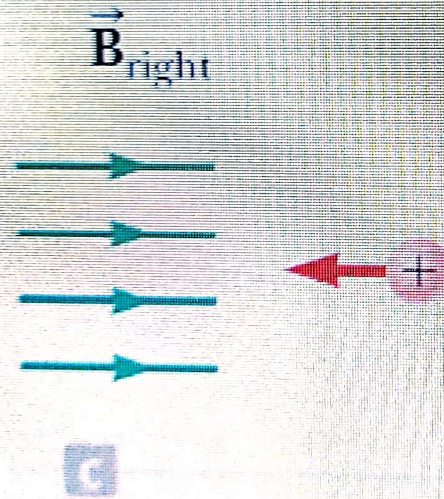
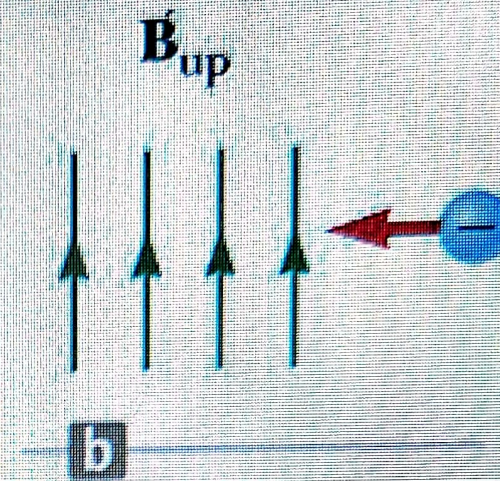
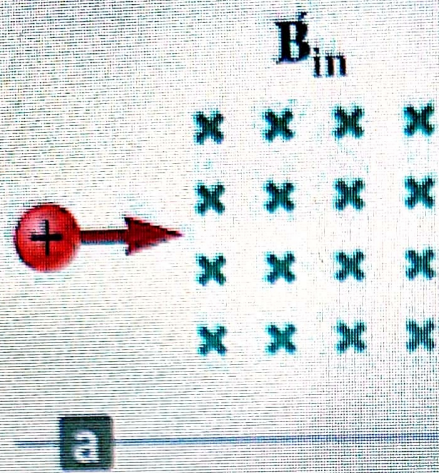
$$\vec{F}_1 + \vec{F}_2 = 0$$

$$F_1 = -F_2 = -I\vec{L} \times \vec{B}$$

In general the force on the curved portion in a uniform magnetic field is equal to that on the straight portion.

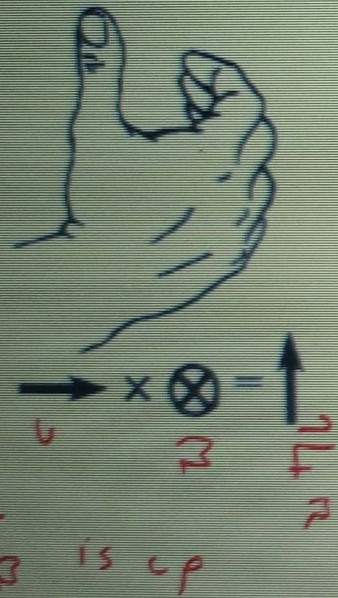
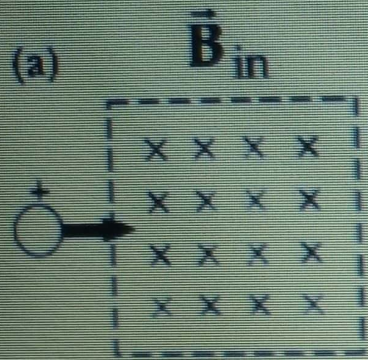


# Prob. 29.2

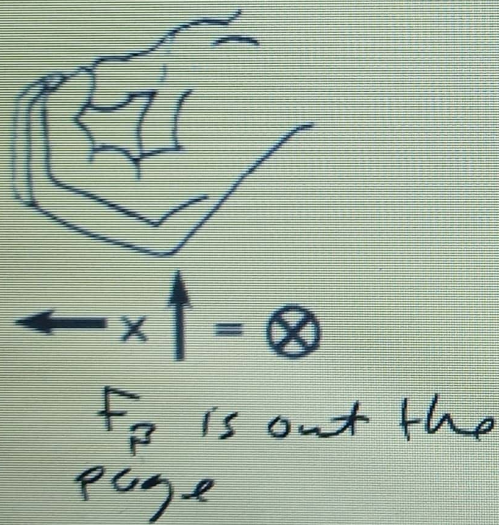
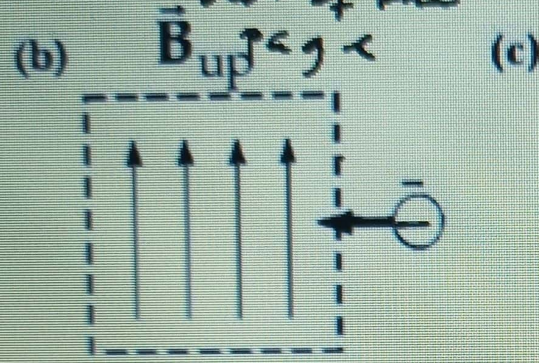




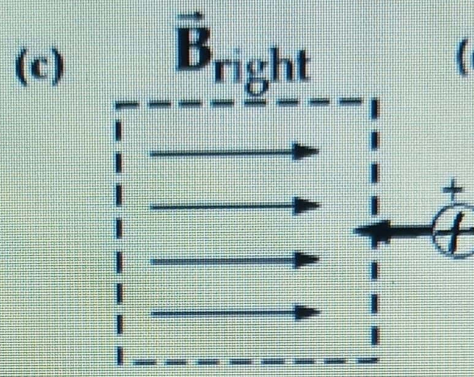
deflection is up



deflection is out of the page



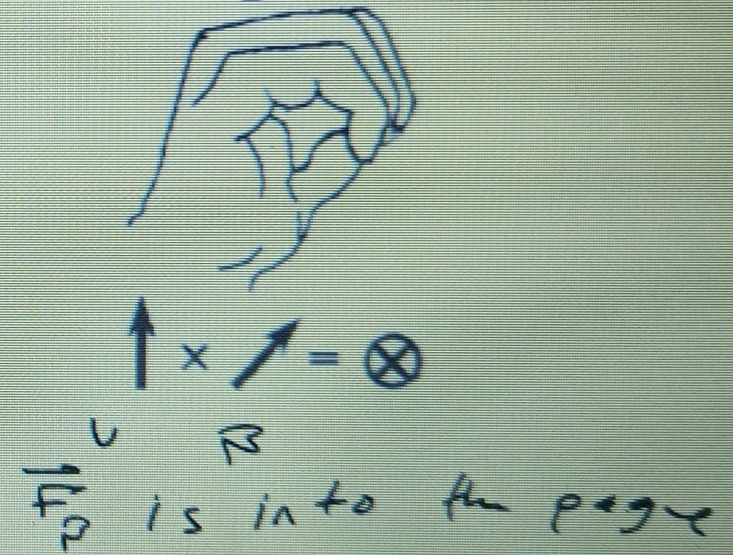
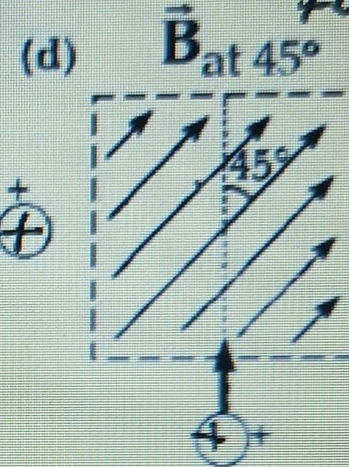
No deflection



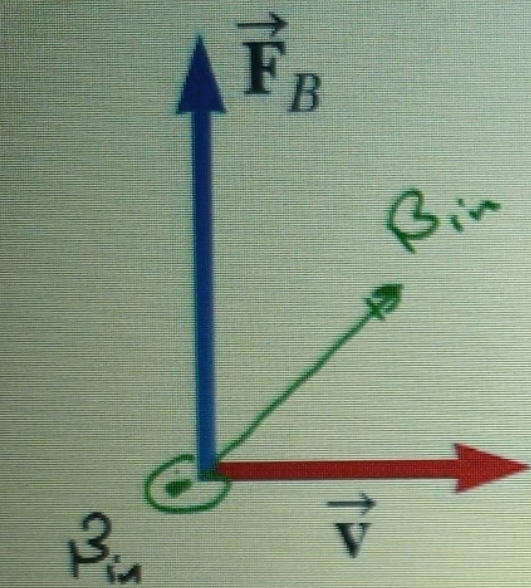
$$\theta = \pi$$

$$F_B = 0$$

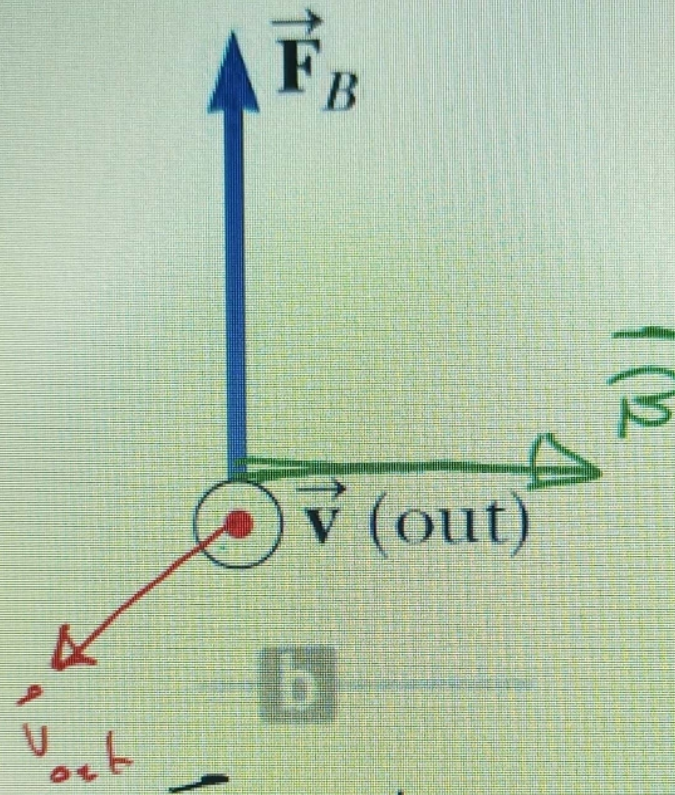
deflection is into the page



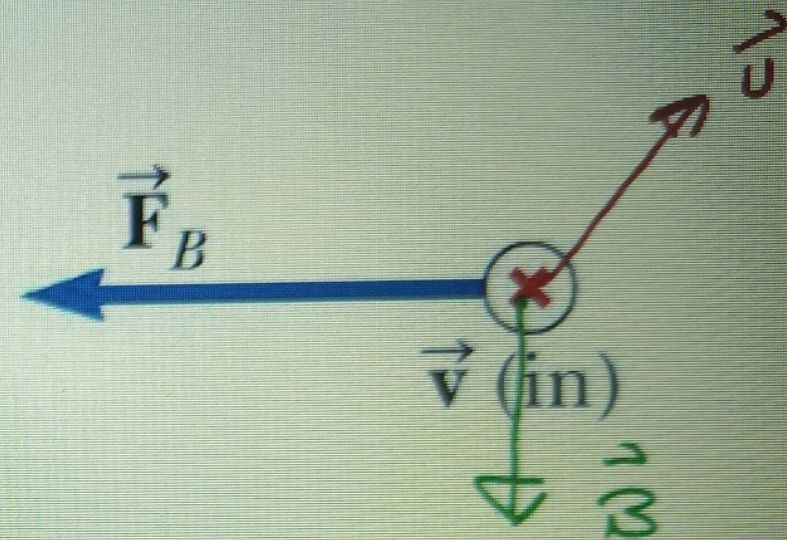




a  
 $\vec{B}$  is into the page  $\leftarrow$



b  
 $\vec{B}$  is to the right



c  
 $\vec{B}$  is to the bottom

# Prob. 29.3



Prob. 29.8  $\Rightarrow$  A proton moves with a velocity of  $\vec{v} = (2\hat{i} - 4\hat{j} + \hat{k})$  m/s in a region in which the magnetic field is  $\vec{B} = (\hat{i} + 2\hat{j} - \hat{k})$  T. What is the magnitude of the magnetic force this particle experiences?

proton:  $q = 1.6 \times 10^{-19}$ ,  $m = 1.67 \times 10^{-27}$  kg

$$\vec{v} = (2\hat{i} - 4\hat{j} + \hat{k}) \text{ m/s.}$$

$$\vec{B} = (\hat{i} + 2\hat{j} - \hat{k}) \text{ T}$$

$$\vec{F}_B = q\vec{v} \times \vec{B} = (1.6 \times 10^{-19})$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

$$\vec{F}_B = (1.6 \times 10^{-19}) [(4-2)\hat{i} - (-2-1)\hat{j} + (4--4)\hat{k}]$$

$$\vec{F}_B = (3.2\hat{i} + 4.8\hat{j} + 12.8\hat{k}) \times 10^{-19} \text{ N}$$

$$|\vec{F}_B| = \sqrt{3.2^2 + 4.8^2 + 12.8^2} \times 10^{-19} \text{ N}$$
$$= 13.2 \times 10^{-19} \text{ N}$$

**prob. 29.13** An electron moves in a circular path perpendicular to a uniform magnetic field with a magnitude of  $2.00 \text{ mT}$ . If the speed of the electron is  $1.50 \times 10^7 \text{ m/s}$  determine (a) the radius of the circular path (b) the time interval required to complete one revolution.

$$q = -e = -1.6 \times 10^{-19}, \quad m = 9.11 \times 10^{-31} \text{ kg}$$

$$B = 2 \text{ mT} = 2 \times 10^{-3} \text{ T}$$

$$v = 1.5 \times 10^7$$

$$\vec{v} \perp \vec{B}$$

$$\text{A) } qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv}{qB} = 0.0427 \text{ m} = 4.27 \text{ cm}$$

$$\text{B) } T = \frac{2\pi r}{v} = 1.79 \times 10^{-8} \text{ s}$$

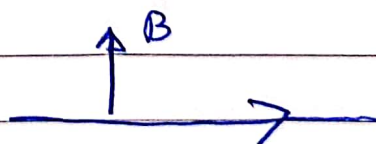
$$T = \frac{2\pi m}{qB} = 1.79 \times 10^{-8} \text{ s}$$

**prob. 29.32**

$$I = 3 \text{ A}, \quad B = 0.28 \text{ T}, \quad \theta = \frac{\pi}{2}$$

$$\text{A) } L = 14 \text{ cm} = 0.14 \text{ m}$$

$$F_B = ILB \sin \theta \\ = 0.118 \text{ N}$$





## Chapter 30 - Sources of magnetic field.

There are two sources of magnetic field :

1) Electric currents (moving charges)

2) magnetic materials.

\* Biot and Savart conducted experiments to measure the magnetic fields and magnetic forces due to a current  $I$ .

The results are :-

The magnetic  $d\vec{B}$  at point  $P$  due to a current  $I$  in the length segment  $d\vec{s}$  is.

1)  $d\vec{B} \perp d\vec{s}$

2)  $d\vec{B} \perp \hat{r}$  ; ( $\hat{r}$  is unit vector that points from  $d\vec{s}$  to the point  $P$ )

3)  $dB \propto \frac{1}{r^2}$  ; ( $r$  is the distance from  $d\vec{s}$  to  $P$ )

4)  $dB \propto I$

5)  $dB \propto |d\vec{s}| = ds$

6)  $dB \propto \sin \theta$  , ( $\theta$  : the angle between  $d\vec{s}$  and  $\hat{r}$ )

The results can be summarized in the Biot-Savart law.

$$\left[ d\vec{B} = \frac{\mu_0 I}{4\pi} \frac{d\vec{s} \times \hat{r}}{r^2} \right]$$

$$\mu_0 = 4\pi \times 10^{-7} \text{ T.m/A}$$

$\mu_0$  = magnetic permeability of free space.

The total magnetic field can be found by summing up the contributions from all current elements (integral)

$$\left[ \vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2} \right] \text{ Biot, Savart law}$$

EX. 30.2

$$\vec{B} = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \hat{r}}{r^2}$$

The magnetic fields due to the sections AA' and CC' is zero because.

$$d\vec{s} \times \hat{r} = 0$$

The sections AA' and CC' do not contribute to the magnetic field at O

The field at O is due to the section AC only

$$\Theta = \frac{\pi}{2}$$

$$|d\vec{s} \times \hat{r}| = |d\vec{s}| \underbrace{|\hat{r}|}_{=1} \underbrace{\sin\left(\frac{\pi}{2}\right)}_{=1} = ds$$

$$B = \frac{\mu_0 I}{4\pi} \int \frac{|d\vec{s} \times \hat{r}|}{r^2} \quad r=a \quad = \frac{\mu_0 I}{4\pi} \int \frac{ds}{a^2} = \frac{\mu_0 I}{4\pi a^2} \int ds \quad \text{--- } s = \Theta$$

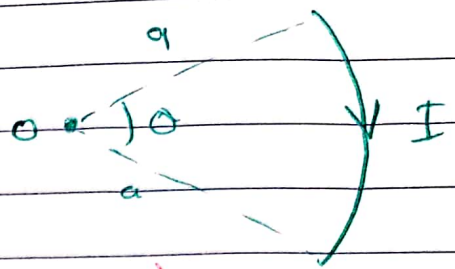
$$B = \frac{\mu_0 I}{4\pi a} (\Theta)$$

$$B = \frac{\mu_0 I}{4\pi a} \Theta$$



$$B_0 = \frac{\mu_0 I \Theta}{4\pi a}$$

Special case :-

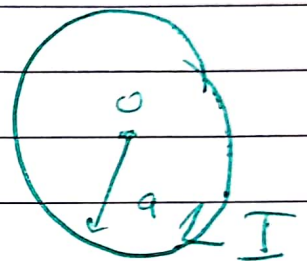


If  $\Theta = 2\pi$  (closed circular loop)

$$B = \frac{\mu_0 I}{2a}$$

Magnetic field at the center of a closed circular current loop of radius  $a$

$$B_x = \frac{\mu_0 I a^2}{2(x^2 + a^2)^{3/2}}$$



Special Case. 1

If  $x=0$

$$B = \frac{\mu_0 I a^2}{2(0 + a^2)^{3/2}} = \frac{\mu_0 I}{2a}$$

$$B = \frac{\mu_0 I}{2a}$$

Magnetic field at the center of a closed circular loop of current

### EX - 30.6

Take loop 2

$$\oint \vec{B}_{out} \cdot d\vec{s} = \mu_0 I_{in} = 0$$

$$\boxed{B_{out} = 0}, \quad r < b, \quad r > c$$

Take loop 1 ( $b < r < c$ )

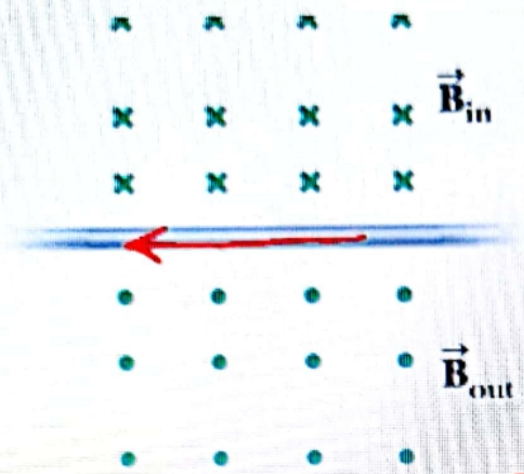
$$\oint \vec{B}_{in} \cdot d\vec{s} = \mu_0 I_{in}$$

$$B_{in}(2\pi r) = \mu_0 NI$$

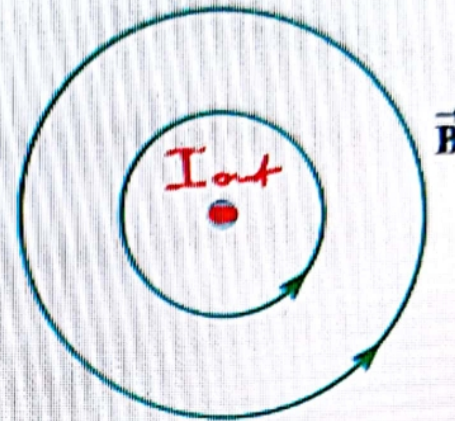
$$\boxed{B_{in} = \frac{\mu_0 NI}{2\pi r}}, \quad b < r < c$$



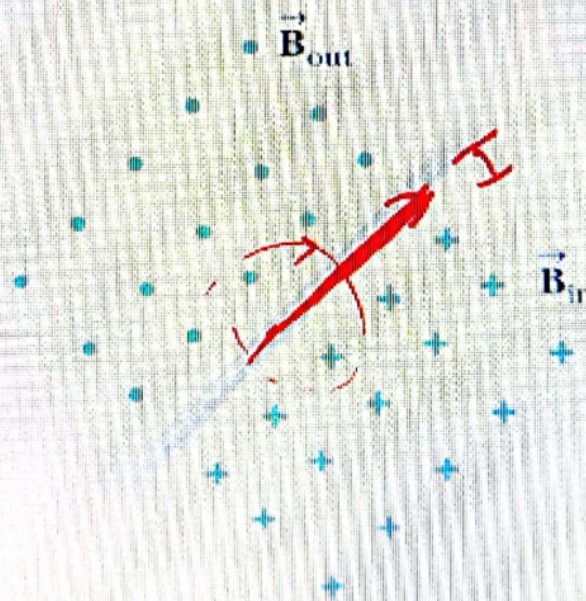
# Prob. 2



a  $I$  to the left



b  $I$  is out of the page



c  $I$  is up and right

### prob. 19

$$I = 5 \text{ A} \quad , \quad d = 10 \text{ cm} = 0.1 \text{ m}$$

a) At the point midway between the two wires

$$B_1 = \frac{\mu_0 I}{2\pi(\frac{d}{2})} \quad (\text{into the page})$$

$$B_2 = \frac{\mu_0 I}{2\pi(\frac{d}{2})} \quad (\text{into the page})$$

$$B = B_1 + B_2 = \frac{\mu_0 I}{2\pi \frac{d}{2}} + \frac{\mu_0 I}{2\pi \frac{d}{2}} = \frac{2\mu_0 I}{\pi d} = 40 \times 10^{-6} \text{ T} \quad (\text{into the page})$$

b) at point  $P_1$   $= (-40 \times 10^{-6} \hat{k}) \text{ T}$

$$B_1 = \frac{\mu_0 I}{2\pi d} \quad (\text{out of the page})$$

$$B_2 = \frac{\mu_0 I}{2\pi(2d)} \quad (\text{into the page})$$

$$B_{P_1} = (B_1 - B_2) \hat{k} = \frac{\mu_0 I}{2\pi d} \left(1 - \frac{1}{2}\right) \hat{k}$$

$$B_{P_1} = (5 \times 10^{-6} \hat{k}) \text{ T}$$

$$= 5 \times 10^{-6} \text{ T} \quad (\text{out of the page})$$



c) At point  $P_2$

$$B_1 = \frac{\mu_0 I}{2\pi(3d)} \quad (\text{into the page})$$

$$B_2 = \frac{\mu_0 I}{2\pi(2d)} \quad (\text{out of the page})$$

$$\vec{B}_{P_2} = \vec{B}_2 - \vec{B}_1$$

$$= \frac{\mu_0 I}{2\pi d} \left( \frac{1}{2} - \frac{1}{3} \right) \hat{k}$$

$$= (1.67 \times 10^{-6} \hat{k}) \text{ T}$$
$$= 1.67 \times 10^{-6} \text{ T} \quad (\text{out of the page})$$

prob. 31

$$I_1 = 1 \text{ A}, I_2 = 3 \text{ A}$$

$$d = 1 \text{ mm} = 0.001 \text{ m}$$

A) B at a (take loop 1)

$$\oint \vec{B}_a \cdot d\vec{s} = \mu_0 I_{in}$$

$$B_a (2\pi d) = \mu_0 I_1$$

$$B_a = \frac{\mu_0 I_1}{2\pi d} = 200 \times 10^{-6} \text{ T (counter clock wise)}$$

B) B at b (take loop 2)

$$\oint \vec{B}_b \cdot d\vec{s} = \mu_0 I_{in}$$

$$B_b (2\pi (3d)) = \mu_0 (I_1 - I_2)$$

$$B_b = \frac{\mu_0 (I_1 - I_2)}{6\pi d}$$

$$B = -133 \times 10^{-6} \text{ T}$$

(-)  $\equiv$  clock wise.

#