ElCoM
ELECTRICAL COMPUTER MECHATRONICS

تقدم لجنة ElCoM الاكاديمية

دفتر الفاينل لمادة:

# فيزياء عامة (2)

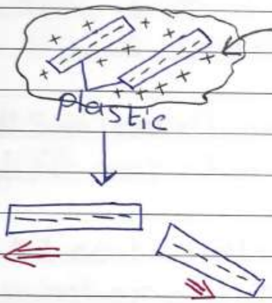
من شرح:

## د. سفيان النمرات

جزيل الشكر للطالبة:

## تسنيم بركات

* Plastic rods and fur are particularly good for demonstrating electrostatics, (interactions between electric charges that are at rest).



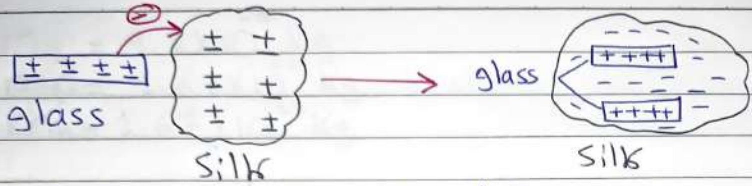

* plain plastic rods neither attract nor repel each other

* but after being rubbed with fur, the rods repel each other

* After we charge both plastic rods by rubbing them with piece of fur, we find that the rods repel each other

* There are exactly two Kinds of electric Charge: negative charge and positive charge
(plastic is negatively charged)
(fur is positively charged)

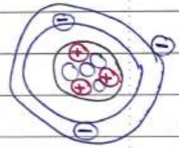
* Electrons are transfered from fur to plastic rod




- Electrons are transfered from glass to silk, glass is positively charged and silk is negatively charged

- Similar charges repel each other while different charges attracts each other

- A neutral atom has the same number of protons as electrons.



- A positive ion is an atom with one or more electrons removed.

- A negative ion is an atom with an excess of electrons.

e or $q_e = -1.6 \times 10^{-19}$ C

P or $q_p = +1.6 \times 10^{-19}$ C

n or $q_n = 0$C

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$m_e = 9.1 \times 10^{-31}$ Kg
$m_p = 1.67 \times 10^{-27}$ Kg
$m_n = 1.67 \times 10^{-27}$ Kg

* The magnitude of charge of the electron or proton is a natural unit of charge. All observable charge is quantized in this unit.

* The universal Principle of charge conservation states that the algabric sum of all the electric charges in any closed system is constant. (charges are not created or destroyed, rather it tranferred from one object to another).

* Quantization → the charge of any object $q$ is always an integer of electron charge.

$q = \pm Ne$ N: is an integer
e: is the charge of electron in unit of coulomb (C)

* Coulomb is a large quantity, smaller units are used

NOTEBOOK

Milli coulomb = mC = $10^{-3}$ C
Micro coulomb = $\mu$C = $10^{-6}$ C
Nano Coulomb = nC = $10^{-9}$ C
Pico coulomb = PC = $10^{-12}$ C

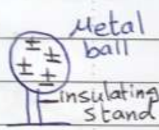
* Materials are classefied in terms of the ability of electrons to move through into:

1) Conductors: Some of electrons are free (unbound) ex. Aluminium, Copper

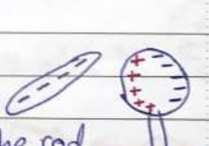
2) Insulators: All the electrons are bound (no free electrons) ex. wood, plastic

3) Semiconductors: Electric properties are some where between those of conductors and insulators. ex. Silicon, germanium

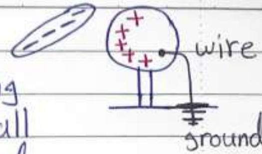
## * Charging by induction



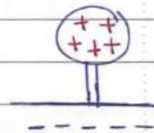

* when you bring a negatively charged rod into an uncharged metal ball without touching it, the free electrons in the metal ball are repelled by the excess electrons on the rod and they shift toward the right, away from the rod



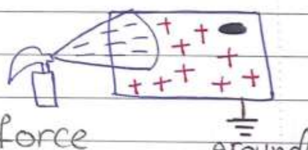
While the plastic rod is nearby you touch one end of a conducting wire to the right surface of the ball and the other end to the ground



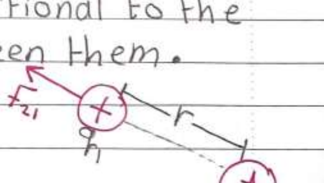

Now disconnect the wire, and remove the rod. A net positive charge is left on the ball.



Electrostatic painting induced positive charge on the metal object attracts the negatively charged paint droplets

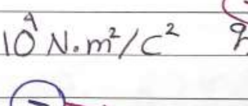



Coulomb's Law: The magnitude of the electric force between two points charge is directly proportional to the product of their charges and inversely proportional to the square of the distance between them.



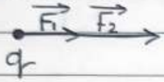

$$F = \frac{K\,|q_1 q_2|}{r^2}$$

K is Coulomb constant $= 9 \times 10^9\ N.m^2/C^2$



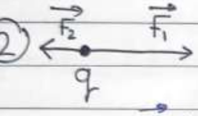
Force is a vector quantity

(1)



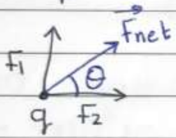

$\vec{F}_{net} = \vec{F_1} + \vec{F_2}$

(2)




$\vec{F}_{net} = \vec{F_1} + \vec{F_2} = |\vec{F_1}| - |\vec{F_2}|$

(3)




$\vec{F}_{net} = \vec{F_1} + \vec{F_2} = \sqrt{|F_2|^2 + |F_1|^2}$

$\theta = \tan^{-1}\left(\frac{F_1}{F_2}\right)$

$K = 9 \times 10^9 \ N.m^2/C^2$

$K = \frac{1}{4\pi \epsilon_0}$

The constant $\epsilon_0$ is known as permittivity of free space and has the value of

$\epsilon_0 = 8.85 \times 10^{-12} \frac{C^2}{N.m^2}$

Example 23.1:

$r = 5.3 \times 10^{-11} m$, $q_p = +1.6 \times 10^{-19} C$, $q_e = -1.6 \times 10^{-19} C$

Find: Fe and Fg:

$F_e = \frac{K|q_1 q_2|}{r^2} = \frac{9 \times 10^9 \times 1.6 \times 10^{-19} \times 1.6 \times 10^{-19}}{(5.3 \times 10^{-11})^2} = 8.2 \times 10^{-8} N$

$F_g = \frac{G m_1 m_2}{r^2}$ , $G = 6.67\times10^{-11}\ N.m^2/kg^2$

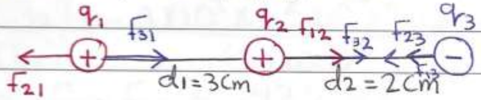
$m_e = 9.1\times10^{-31}\ kg$

$m_p = 1.67\times10^{-27}\ kg$

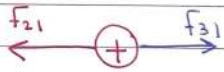
$$F_g = \frac{(6.67\times10^{-11})(9.1\times10^{-31})(1.67\times10^{-27})}{(5.3\times10^{-11})^2} = 3.6\times10^{-47}\ N$$

Ex:12: $q_1 = 6\times10^{-6} C$, $q_2 = 1.5\times10^{-6} C$, $q_3 = -2\times10^{-6} C$

$d_1 = 3\times10^{-2} m$, $d_2 = 2\times10^{-2} m$



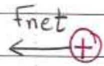

Net force on $q_1$:



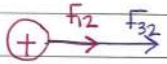

$$F_{21} = \frac{k q_1 q_2}{r_{12}^2} = \frac{9\times10^9 \times 6\times10^{-6} \times 1.5\times10^{-6}}{(3\times10^{-2})^2} = 90 N$$

$$F_{31} = \frac{k q_1 q_3}{(r_{13})^2} = \frac{9\times10^9 \times 6\times10^{-6} \times 2\times10^{-6}}{(5\times10^{-2})^2} = 43.2 N$$

$F_{net} = F_{31} - F_{21} = 43.2 - 90 = -46.8 N$




Net force on $q_2$:




$$F_{12} = \frac{k q_1 q_2}{(r_{12})^2} = \frac{9\times10^9 \times 6\times10^{-6} \times 1.5\times10^{-6}}{(3\times10^{-2})^2} = 90 N$$

$$f_{32} = \frac{K q_2 q_3}{(r_{23})^2} = \frac{9\times10^9 \times 1.5\times10^{-6} \times 2\times10^{-6}}{(2\times10^{-2})^2} = 67.5\,N$$

$F_{net} = f_{12} + f_{32} = 90 + 67.5 = 157.5\,N$ $\quad \oplus \xrightarrow{F_{net}}$

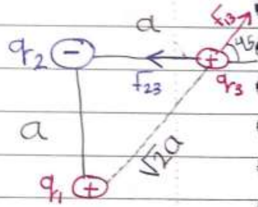
Net force on $q_3$: $\quad \xleftarrow{f_{13}} \xleftarrow{f_{23}} \ominus$

$$f_{13} = \frac{K q_1 q_3}{(r_{13})^2} = \frac{9\times10^9 \times 6\times10^{-6} \times 2\times10^{-6}}{(3\times10^{-2} + 2\times10^{-2})^2} = 43.2\,N$$

$$f_{23} = \frac{K q_2 q_3}{(r_{23})^2} = \frac{9\times10^9 \times 1.5\times10^{-6} \times 2\times10^{-6}}{(2\times10^{-2})^2} = 67.5\,N$$

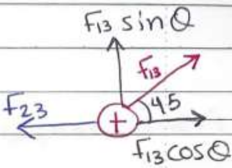
$F_{net} = -f_{13} - f_{23} = -43.2 - 67.5 = -110.7$ $\quad \xleftarrow{F_{net}} \ominus$

Example 23.2: $q_1 = q_3 = 5\times10^{-6}\,C$
$q_2 = -2\times10^{-6}\,C$, $a = 0.1\,m$




$$f_{23} = \frac{K q_2 q_3}{r^2} = \frac{9\times10^9 \times 2\times10^{-6} \times 5\times10^{-6}}{(0.1)^2}$$
$$= 9\,N$$

$$F_{13} = \frac{K q_1 q_3}{r^2} = \frac{9\times10^9 \times (5\times10^{-6})^2}{(\sqrt{2}\times0.1)^2} = 11.2\,N$$

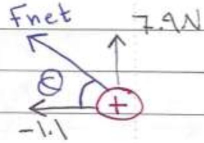



## Resultant force on $q_3$

$F_{x13} = F_{13}\cos(45) = 7.92\,N$
$F_{y,13} = F_{13}\sin(45) = 7.92\,N$

$F_{x\,net} = F_{x13} - F_{23} = 7.92 - 9 = -1.08\,N \approx -1.1\,N$

$F_{y\,net} = F_{y13} = 7.92\,N \approx 7.9\,N$

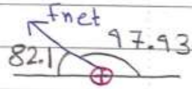



$\vec{F} = -1.1\,\hat{i} + 7.9\,\hat{j}$

$F_{net} = \sqrt{(1.1)^2 + (7.9)^2} = 8\,N$

$\theta = \tan^{-1}\left(\frac{7.9}{-1.1}\right) + 180^\circ = -82.1 + 180 = 97.93^\circ$

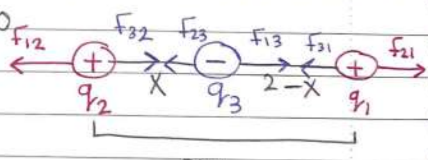
عندما يكون في مقام الـ $\tan^{-1}$ اشارة سالب نضيف على قيمة الزاوية الناتجة $180^\circ$ لتصبح قيمة $\theta$ النهائية في الزاوية الناتجة بين محور x الموجب ومتجه Fnet




Example 23.3: $q_1 = 15\times10^{-6}\,C$, $x = 2\,m$, $q_2 = 6\times10^{-6}\,C$

$F_{net}$ on $q_3$ = zero, $F_{13} - F_{23} = 0$

so $|F_{13}| = |F_{23}|$




$$\frac{\cancel{K} q_1 \cancel{q_3}}{(2-x)^2} = \frac{\cancel{K} q_2 \cancel{q_3}}{x^2}$$

$$\frac{15\times10^{-6}}{4-4x+x^2} = \frac{6\times10^{-6}}{x^2}$$

$$15\times10^{-6}x^2 = 6\times10^{-6}x^2 - 2.4\times10^{-5}x + 2.4\times10^{-5}$$

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$$\underset{a}{9\times10^{-6}}x^2+\underset{b}{2.4\times10^{-5}}x\underset{c}{-2.4\times10^{-5}}=0$$

1) نستخدم قانون

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$

$\checkmark$ $x=0.775\ m$ or $x=-3.44\ m$ ✗

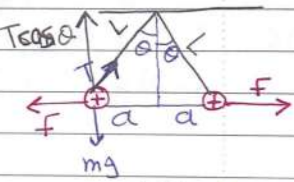
حل غير مقبول بسبب السالب

أو

2) نستخدم الآلة الحاسبة نضغط mode ثم 5 ثم نختار 3 ونعوض القيم

Example 23.4: $m=3\times10^{-2}\ kg$, $L=0.15\ m$

$\theta=5^\circ$




at equilibrium $\Sigma f=0$

$\sin\theta=\frac{a}{L}\Rightarrow\sin(5)=\frac{a}{0.15}$

$a=0.15\sin5=0.01\ m$

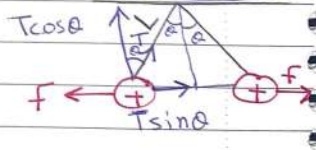
$r=2a=2\times0.01$

$r=0.02\ m$

$\Sigma F_x=0$

$T\sin\theta-f=0$

$T\sin\theta=f=\frac{Kqq}{r^2}$




$\Sigma f_y=0$

$T\cos\theta=mg$

$$\frac{\cancel{T}\sin\theta=\frac{Kq^2}{r^2}}{\cancel{T}\cos\theta=mg}\Rightarrow\frac{\tan\theta}{1}=\frac{Kq^2}{mgr^2}$$

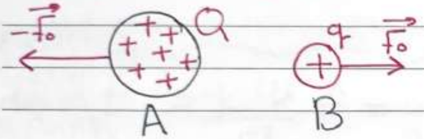
$$\Rightarrow\frac{\tan(\theta)\cdot mgr^2}{K}=\frac{\cancel{K}q^2}{\cancel{K}}$$

$$\sqrt{q^2}=\sqrt{\frac{\tan(5)\times3\times10^{-2}\times10\times(0.02)^2}{9\times10^9}}$$

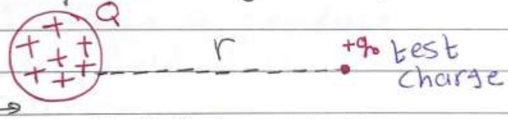
N O T E B O O K

$q = \pm 3.4 \times 10^{-8} C \Rightarrow |q| = 3.4 \times 10^{-8} C$

## Electric field:



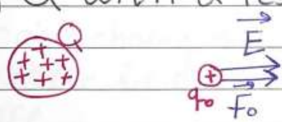

- To introduce the concept of electric field, first consider the mutual repulsion of two positively charged bodies



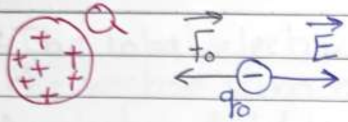

$$\vec{E} = \frac{\vec{F}}{q_0} = \frac{\frac{KQ\cancel{q_0}}{r^2}}{\cancel{q_0}} = \frac{KQ}{r^2}\hat{r}$$

$$[E] = \frac{[F]}{[q_0]} = N/C$$

- We can measure the electric field produced by Q with a test charge.




The force on a positive test charge $q_0$ points in the direction of the electric field.




The force on a negative test charge $q_0$ points opposite to the electric field.

electric field due to a point Charge

$$\vec{E} = \frac{kq}{r^2}\hat{r} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2}\hat{r}$$

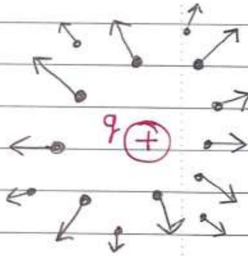
value of point charge

Unit vector

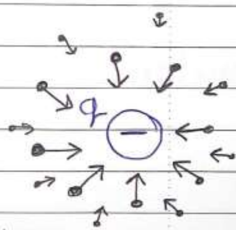
Distance from Point charge to where field is measured

electric constant

- A point charge $q$ produces an electric field at all points in space.
- The field strength decreases with increasing distance.
- The field produced by a positive point charge points away from the charge

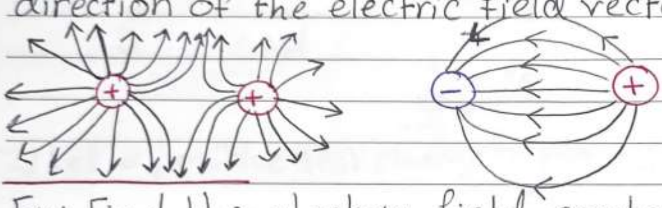



$E = 0$ only when $r \to \infty$

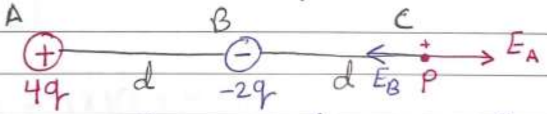



- A point charge $q$ produces an electric field at all points in space
- The field produced by a negative charge points toward the charge

- The total electric field at a point a is the vector sum of the fields
- An electric field line is an imaginary line or curve whos tangent at any point is the direction of the electric field vector



Ex: Find the electric field created by the charges A and B at point C as shown if $q = 6\mu C$ and $d = 2\,Cm$?

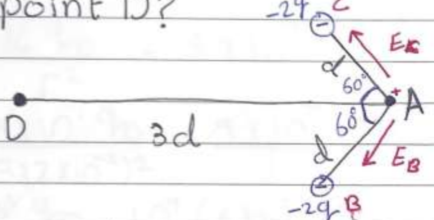



$$E_A = \frac{K q_A}{r^2} = \frac{9\times10^9 \times 4 \times 6\times10^{-6}}{(4\times10^{-2})^2} = 13.5\times10^7\ N/C$$

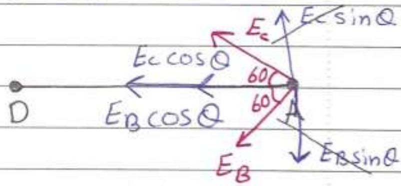
$$E_B = \frac{K q_B}{r^2} = \frac{9\times10^9 \times 2 \times 6\times10^{-6}}{(2\times10^{-2})^2} = 27\times10^7\ N/C$$

$$E_{net} = E_A - E_B = 13.5\times10^7 - 27\times10^7 = -13.5\times10^7\ N/C$$

Ex: If the electric field at point A is zero $q = 2\,nC$ and $d = 2\,cm$, find the charge at point D?




1) Let a positive test charge is placed at point A




$q_B = -2q = -4\,nC$

$q_C = -2q = -4\,nC$

$|E_c| = |E_B|$

$$E_c = \frac{k\,q_c}{r^2} = \frac{9\times10^{9}\times 4\times10^{-9}}{(2\times10^{-2})^2} = 9\times10^{4}\ N/C$$

$$E_B = \frac{k\,q_B}{r^2} = \frac{9\times10^{9}\times 4\times10^{-9}}{(2\times10^{-2})^2} = 9\times10^{+4}\ N/C$$

$$E_x = -\left(E_c \cos(60) + E_B \cos(60)\right)$$

$$E_x = -2E_c \cos(60)$$

$$\Sigma E_{net} = 0$$

$$E_D = 2E_c \cos(60) = 2\times 9\times10^{+4}\times\cos(60)$$
$$= 9\times10^{4}\ N/C$$

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$-2E_c\cos(60) \longleftarrow \bullet \longrightarrow E_D = 2E_c\cos(60)$

$E_{net} = 0$

$E_D = 9\times10^4\,N/C$

$$E_D = \frac{k\,q_D}{r^2} = 9\times10^4$$

$$\frac{\cancel{9}\times10^9\,q_D}{(3\times2\times10^{-2})^2} = \frac{\cancel{9}\times10^4}{1}$$

$$\frac{\cancel{10^9}\,q_D}{\cancel{10^9}} = \frac{10^4\,(6\times10^{-2})^2}{10^9} \Rightarrow q_D = 36\times10^{-9} = +36\,nC$$

* The electric field exists in a region of space around charged objects

* The electric force is a field force (can act through space with no physical contact)

* when a charge enters the electric field an electric force acts on it.

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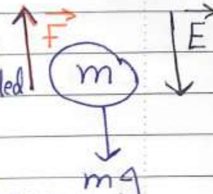
Ex 23.5: $m = 3\times10^{-12}$ Kg, $\vec{E} = 6\times10^{3}$ N/C downward

$\Sigma F = 0$ because it was written that the droplet remains suspended at rest in the air.




* To remain constant → another force must be in opposite to weight.

$$F - mg = 0 \rightarrow mg = F = -qE$$

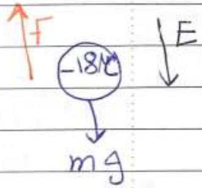
* بما أن اتجاه القوة معاكس لاتجاه المجال الكهربائي، إذا شحنة قطرة الماء سالبة. وفي القانون عوضنا بإشارة سالب $(-qE)$ لأن اتجاه المجال الكهربائي للأسفل.

$$mg = -qE \Longrightarrow q = -\frac{mg}{E} = -\frac{3\times10^{-12}\times10}{6\times10^{3}}$$

$$\therefore q = -5\times10^{-15}\ C$$

Ex 24: $m = 3.8g = 3.8 \times 10^{-3} Kg$, $q = -18\mu C = -18 \times 10^{-6} C$

* بما أن الشحنة مستقرة في الهواء ويوجد قوة جاذبية تؤثر عليها إلى الأسفل إذاً يوجد قوة F تؤثر عليها إلى الأعلى بعكس اتجاه المجال الكهربائي لأن الشحنة سالبة الشحنة




$F - mg = 0 \Rightarrow F = mg = qE \Rightarrow E = \frac{mg}{q}$

$E = \frac{3.8 \times 10^{-3} \times 10}{18 \times 10^{-6}} = 2.1 \times 10^{3} N/C$

* electric field points downward.

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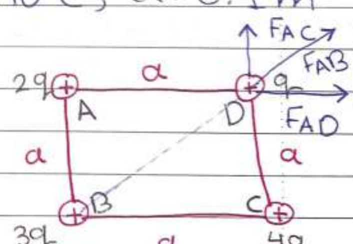
Ex 25: * Let $q = 2\mu C = 2 \times 10^{-6} C$, $a = 0.1 m$

(a) Find the electric field at charge q?

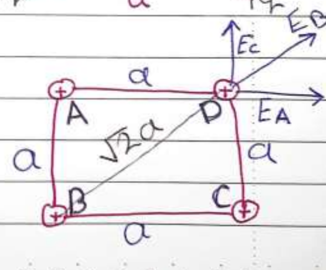


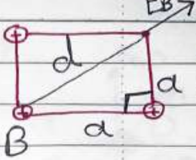

1) $E_A = \frac{K q_A}{a^2} = \frac{9 \times 10^9 \times 4 \times 10^{-6}}{(0.1)^2}$

$E_A = 3.6 \times 10^6 N/C$

2) $E_C = \frac{K q_C}{a^2} = \frac{9 \times 10^9 \times 8 \times 10^{-6}}{(0.1)^2}$

$= 7.2 \times 10^6 N/C$

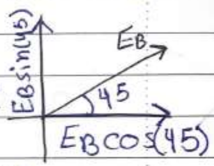





3) $E_B = \frac{K q_B}{d^2}$

$d = \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2}\, a$

$E_B = \frac{K q_B}{(\sqrt{2}\, a)^2} = \frac{9 \times 10^9 \times 6 \times 10^{-6}}{(\sqrt{2} \times 0.1)^2} = 2.7 \times 10^6 \, N/C$




$E_{Bx} = 2.7 \times 10^6 \cos(45) = 1.91 \times 10^6 \, N/C$

$E_{By} = 2.7 \times 10^6 \sin(45) = 1.91 \times 10^6 \, N/C$

$E_x = E_{Bx} + E_A = 1.91 \times 10^6 + 3.6 \times 10^6 = 5.51 \times 10^6 \, N/C$

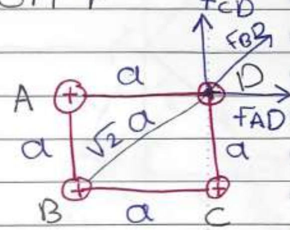
$E_y = E_{By} + E_C = 1.91 \times 10^6 + 7.2 \times 10^6 = 9.11 \times 10^6 \, N/C$

$E_{net} = \sqrt{(5.51 \times 10^6)^2 + (9.11 \times 10^6)^2} = 10.6 \times 10^6 \, N/C$

direction: $\theta = \tan^{-1}\left(\frac{E_y}{E_x}\right) = \tan^{-1}\left(\frac{9.11 \cancel{\times 10^6}}{5.51 \cancel{\times 10^6}}\right)$

$\theta = 58.8^\circ$

(b) the total electric force on q




1) $F_{AD} = K\dfrac{q_A q_D}{a^2}$

$= \dfrac{9\times10^{9}\times2\times10^{-6}\times4\times10^{-6}}{(0.1)^2}$

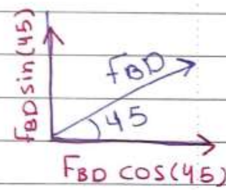
$= 7.2\ \text{N}$

2) $F_{CD} = \dfrac{K q_C q_D}{a^2} = \dfrac{9\times10^{9}\times8\times10^{-6}\times2\times10^{-6}}{(0.1)^2}$

$= 14.4\ \text{N}$

3) $F_{BD} = \dfrac{K q_B q_D}{(\sqrt{2}a)^2} = \dfrac{9\times10^{9}\times6\times10^{-6}\times2\times10^{-6}}{(\sqrt{2}\times0.1)^2}$

$= 5.4\ \text{N}$




$F_{BDx} = 5.4\cos(45) = 3.81\ \text{N}$

$F_{BDy} = 5.4\sin(45) = 3.81\ \text{N}$

$F_x = F_{BDx} + F_{AD} = 3.81 + 7.2 = 11\ \text{N}$

$F_y = F_{BDy} + F_{CD} = 3.81 + 14.4 = 18.2\ \text{N}$

$F_{net} = \sqrt{(11)^2 + (18.2)^2} = 21.3\ \text{N}$

$\theta = \tan^{-1}\left(\dfrac{18.2}{11}\right) = 58.8^\circ$

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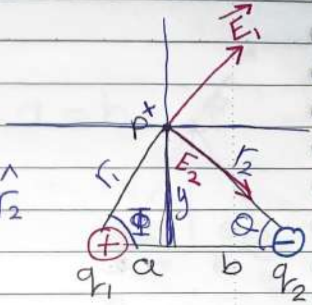
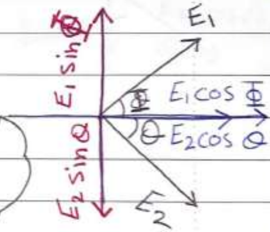



Ex 23.6:

a) $E_1 = \frac{K q_1}{(r_1)^2} \hat{r}_1$ | $E_2 = \frac{K q_2}{(r_2)^2} \hat{r}_2$

$(r_1)^2 = a^2 + y^2$ | $(r_2)^2 = b^2 + y^2$

$E_1 = \frac{K q_1}{a^2+y^2} \hat{r}_1$ | $E_2 = \frac{K q_2}{b^2+y^2} \hat{r}_2$




$E_1 = E_1 \cos\Phi \,\hat{i} + E_1 \sin\Phi \,\hat{j}$

$$E_1 = \frac{Kq_1}{a^2+y^2} \cos\Phi \,\hat{i} + \frac{Kq_1}{a^2+y^2} \sin\Phi \,\hat{j}$$

$E_2 = E_2 \cos\theta \,\hat{i} + E_2 \sin\theta \,\hat{j}$

$$E_2 = \frac{Kq_2}{b^2+y^2} \cos\theta \,\hat{i} + \frac{Kq_2}{b^2+y^2} \sin\theta \,\hat{j}$$

$$E_{net} = \vec{E}_1 + \vec{E}_2$$
$$= \left(\frac{Kq_1}{a^2+y^2}\cos\Phi + \frac{Kq_2}{b^2+y^2}\cos\theta\right)\hat{i} +$$
$$\left(\frac{Kq_1}{a^2+y^2}\sin\Phi + \frac{Kq_2}{b^2+y^2}\sin\theta\right)\hat{j}$$

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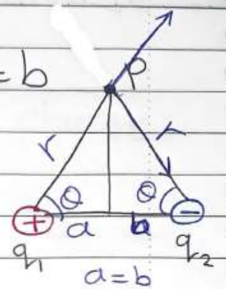
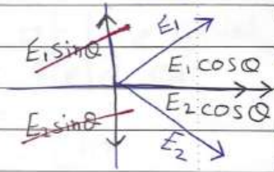
b) when $|q_1| = |q_2|$ and $a = b$

so $\theta = \Phi$



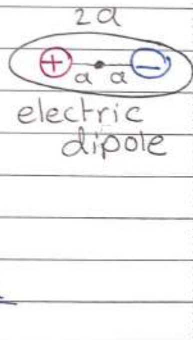

$$E_{net} = \frac{2Kq}{a^2+y^2}\cos\theta\,\hat{i} + 0\,\hat{j}$$




$$\cos\theta = \frac{a}{r} = \frac{a}{(a^2+y^2)^{\frac{1}{2}}}$$

$$E_{net} = \frac{2Kq}{(a^2+y^2)} \cdot \frac{a}{(a^2+y^2)^{\frac{1}{2}}}$$

$$= \frac{2Kqa}{(a^2+y^2)^{\frac{3}{2}}}$$




c) Limit $y \gg a$

$$E_{net} = \frac{2Kqa}{(a^2+y^2)^{\frac{3}{2}}}, \quad y \gg a$$

$$E_{net} = \frac{2Kqa}{(y^2)^{\frac{3}{2}}} = \frac{2Kqa}{y^3}$$

D) Limit $y \ll a$

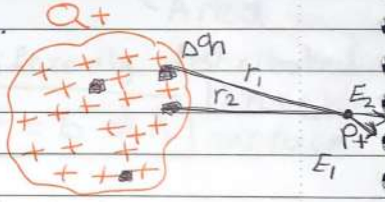
$$E_{net} = \frac{2Kqa}{(a^2)^{\frac{3}{2}}} = \frac{2Kqa}{a^3} = \frac{2Kq}{a^2}$$

# Electric field of continuous charge distribution:

* The electric field due to the charge element $\Delta q_i$ is $\Delta E_i$

$$\Delta \vec{E_i} = \frac{K \Delta q_1}{r_i^2} \hat{r}_1$$

* The total electric field $\vec{E}$ at the point $P$ is the vector of electric field of all charge element.




$$E_1 = \frac{K \Delta q_1}{r_1^2} \hat{r}_1$$

$$E_2 = \frac{K \Delta q_2}{r_2^2} \hat{r}_2$$

$$\Delta E_{net} = K \sum_{i=1}^{\infty} \frac{\Delta q_i}{r_i^2} \hat{r}_i$$

$$\lim_{\Delta q \to 0} \Delta E_{net} = K \lim_{\Delta q \to 0} \frac{\Delta q_i}{r_i^2} \hat{r}_i$$

* $E_{net} = K \int \frac{dq}{r^2} \hat{r}$


$$\frac{Q}{\text{volume}}, \quad \frac{Q}{V} = \rho \qquad [\rho] = \frac{C}{m^3}$$

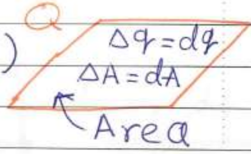
→ volume charge distribution

1) volume charge distribution
(charge per unit volume) ($\rho$)

$$\rho = \frac{\Delta q}{\Delta V} = \frac{dq}{dv} \Rightarrow \boxed{dq = \rho\, dv}$$

2) Surface charge distribution

(charge per unit area) ($\sigma$)



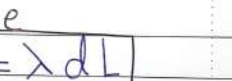

$\frac{Q}{A} = \sigma$ , $[\sigma] = \frac{C}{m^2}$

* If the charge is uniformaly distributed through surface

$$\sigma = \frac{dq}{dA} = \frac{Q}{A} \Rightarrow \boxed{dq = \sigma\, dA}$$

3) Linear charge distribution
(charge per unit Length) ($\lambda$)

$$\lambda = \frac{\Delta q}{\Delta L} = \frac{dq}{dL}$$




$[\lambda] = C/m$

* If the total charge Q is uniformaly distributed along Line

$$\lambda = \frac{dq}{dL} = \frac{Q}{L} \Rightarrow \boxed{dq = \lambda\, dL}$$

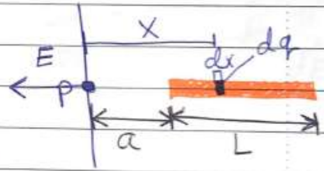
$$\vec{E} = K \int \frac{dq}{r^2} \hat{r}$$

$dq = \lambda \, dl \rightarrow$ Linear charge

$dq = \sigma \, dA \rightarrow$ Surface charge

$dq = \rho \, dv \rightarrow$ volume Charge

Ex 23.7:




$$\lambda = \frac{Q}{L}$$

$$\lambda = \frac{dq}{dx} \Rightarrow dq = \lambda dx$$

$$dE = \frac{K dq}{x^2} = \frac{K \lambda dx}{x^2}$$

$$\int dE = \int \frac{K \lambda dx}{x^2}$$

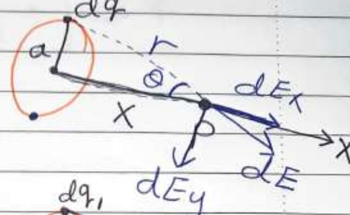
$$E = K\lambda \int_a^{L+a} \frac{1}{x^2} dx = K\lambda \left[ \frac{-1}{x} \right]_a^{L+a}$$

$$E = K\lambda \left[ \frac{-1}{L+a} + \left( \frac{1}{a} \right) \right]$$

$$E = K\lambda \left[ \frac{1}{a} - \frac{1}{L+a} \right]$$

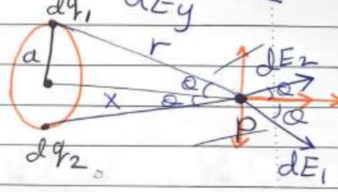
$$= K \left( \frac{Q}{L} \right) \left( \frac{1}{a} - \frac{1}{L+a} \right)$$

Ex 23.8:




$dE_{x1} = dE_1 \cos\theta$

$dE_{x2} = dE_2 \cos\theta$




$dE_{net} = dE_x = dE\cos\theta$

$dE_y = 0$
because $dE_{y2} - dE_{y1} = 0$

$dE_{net} = dE_x = \frac{K\,dq}{r^2}\cos\theta$

$r^2 = a^2 + x^2 \Rightarrow r = (a^2+x^2)^{\frac{1}{2}}$

$dE_x = \frac{K\,dq}{a^2+x^2}\cos\theta$

$\cos\theta = \frac{x}{r} = \frac{x}{(a^2+x^2)^{\frac{1}{2}}}$

$dE_x = \frac{K\,dq}{(a^2+x^2)} \cdot \frac{x}{(a^2+x^2)^{\frac{1}{2}}}$

$\int dE_x = \int \frac{K\,x\,dq}{(a^2+x^2)^{\frac{3}{2}}}$

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$$E_{net} = E_x = \frac{K x}{(a^2+x^2)^{\frac{3}{2}}} \cdot \int dq$$

$$E_{net} = E_x = \frac{K x Q}{(a^2+x^2)^{\frac{3}{2}}}$$

$$E_{net} = E_x = \frac{K Q x}{(a^2+x^2)^{\frac{3}{2}}}$$

$x \gg a$ then $E_x = \frac{K Q x}{(x^{\cancel{2}})^{\frac{3}{\cancel{2}}}} = \frac{K Q x}{x^3}$

$$\boxed{E_x = \frac{K Q}{x^2}}$$

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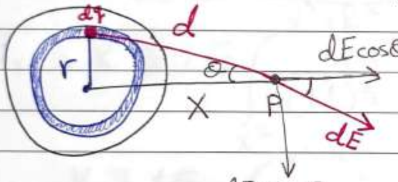
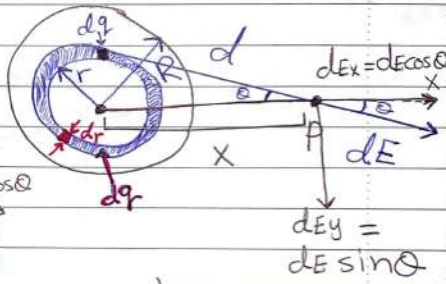
Ex 23.9: The electric field of a uniformly Charged disk:

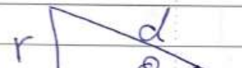
$dE_x = dE\cos\theta$

$dE_x = \frac{k\,dq}{d^2}\cos\theta$




$\cos\theta = \frac{x}{d}$




$dE_x = \frac{k\,dq}{d^2}\left(\frac{x}{d}\right) = \frac{k\,dq\,x}{d^3}$

$d = (x^2+r^2)^{\frac{1}{2}}$

$dE_x = \frac{k\,dq\,x}{(x^2+r^2)^{\frac{3}{2}}}$ , $dq = \sigma\,dA$

$dq = \sigma(2\pi r\,dr)$

عوضنا محل A بقانون
مساحة الحلقة

$\int dE_x = \int \frac{k(2\pi r\,dr)\sigma x}{(x^2+r^2)^{\frac{3}{2}}}$

$R \rightarrow$ أكبر نصف قطر

$$E_x = \underbrace{K\pi x\sigma}_{\text{ثوابت}} \int_{0}^{R} \frac{2r\,dr}{(x^2+r^2)^{\frac{3}{2}}}$$

$0 \leftarrow$ أقل نصف قطر

$$U = \underset{\text{ثابت}}{x^2} + \underset{\text{متغير}}{r^2}$$

$du = 2r\,dr$ (الثابت مشتقته صفر)

when $r=0$, then $U = x^2$

when $r=R$, then $U = x^2+R^2$

عوضنا قيم r في المعادلة $U=x^2+r^2$ والقيمة الناتجة نعوضها مكان حدود التكامل

$$E_x = K\pi x\sigma \int_{x^2}^{x^2+R^2} \frac{du}{(U)^{\frac{3}{2}}} \neq \int_{\cancel{x^2}}^{\cancel{x^2+R^2}}$$

$$E_x = K\pi x\sigma \int_{x^2}^{x^2+R^2} U^{-\frac{3}{2}}\,du = K\pi\sigma x\left[-2U^{-\frac{1}{2}}\right]_{x^2}^{x^2+R^2}$$

$$E_x = K\pi\sigma x\left(-2(x^2+R^2)^{-\frac{1}{2}} + 2(x^2)^{-\frac{1}{2}}\right)$$

$$E_x = K\pi\sigma x\left(\frac{-2}{\sqrt{x^2+R^2}} + \frac{2}{x}\right)$$

$$E_x = +2K\pi\sigma x\left(\frac{-1}{\sqrt{x^2+R^2}} + \frac{1}{x}\right)$$

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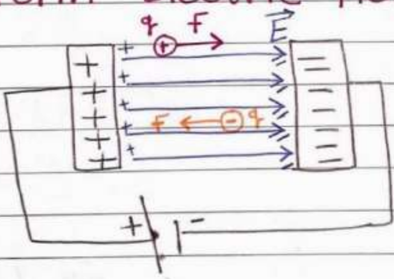
$$E_x = 2K\pi\sigma\left(\frac{-x}{\sqrt{x^2+R^2}} + 1\right)$$

$$\boxed{E_x = 2K\pi\sigma\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)}$$

Limit $x \gg R$

$$E_x = 2K\pi\sigma\left(1 - \frac{x}{\sqrt{x^2}}\right) = 0$$

## * Uniform Electric Field:




$$\Sigma F = qE = ma$$

$$a = \frac{qE}{m}$$

$$v_f = v_i + at$$

$$v_f^2 = v_i^2 + 2a\Delta x$$

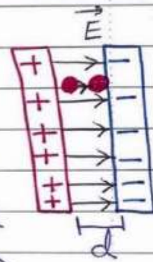
$$\Delta x = v_i t + \frac{1}{2}at^2$$

**Example 23.10** **An Accelerating Positive Charge: Two Models** **AM**

A uniform electric field $\vec{\mathbf{E}}$ is directed along the $x$ axis between parallel plates of charge separated by a distance $d$ as shown in Figure 23.23. A positive point charge $q$ of mass $m$ is released from rest at a point Ⓐ next to the positive plate and accelerates to a point Ⓑ next to the negative plate.

**(A)** Find the speed of the particle at Ⓑ by modeling it as a particle under constant acceleration.

**(B)** Find the speed of the particle at Ⓑ by modeling it as a nonisolated system in terms of energy.




a) $v_A = 0$ , $v_B = ?$ , $\Delta x = d = ?$

$$v_B^2 = \cancel{v_A^2}^{\,0} + 2a\underbrace{\Delta x}_{d}$$

$$v_B^2 = 2ad$$

$$v_B^2 = \frac{2qEd}{m}$$

$$v_B = \sqrt{\frac{2qEd}{m}}$$

$$\Sigma F = ma$$

$$qE = ma$$

$$a = \frac{qE}{m}$$

b) $W = \Delta K$

$$\Sigma F \Delta x = \frac{1}{2} m v_B^2 - \cancel{\frac{1}{2} m v_A^2}^{\,0}$$

$$\Sigma F \Delta x = \frac{1}{2} m v_B^2$$

$$v_B^2 = \frac{2\overbrace{\Sigma F}^{qE}\,\overbrace{\Delta x}^{d}}{m} = \frac{2qEd}{m}$$

$$\Rightarrow v_B = \sqrt{\frac{2qEd}{m}}$$

## Example 23.11 An Accelerated Electron AM

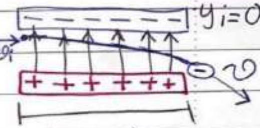
No.

An electron enters the region of a uniform electric field as shown in Figure 23.24, with $v_i = 3.00 \times 10^6$ m/s and $E = 200$ N/C. The horizontal length of the plates is $\ell = 0.100$ m.

**(A)** Find the acceleration of the electron while it is in the electric field.

**(B)** Assuming the electron enters the field at time $t = 0$, find the time at which it leaves the field.

**(C)** Assuming the vertical position of the electron as it enters the field is $y_i = 0$, what is its vertical position when it leaves the field?




$v_i = 3\times10^6\ m/s$, $E = 200\ N/C$

$L = 0.1\ m$, $a = ?$, $\Delta t = ?$ $y_f = ?$

$q = -1.6\times10^{-19}\ C$, $m_e = 9.11\times10^{-31}\ kg$

$L = 0.1m$

1) $\Sigma F = ma$

$$-qE = ma \Rightarrow a_y = \frac{-qE}{m_e} = \frac{-1.6\times10^{-19}\times200}{9.11\times10^{-31}}$$

$$\boxed{a_y = -3.5\times10^{13}\ m/s^2}$$

2) $$v = \frac{\Delta x}{\Delta t} = \frac{d}{t} \Rightarrow t = \frac{d}{v} = \frac{0.1}{3\times10^6}$$

$$\boxed{t = 0.3\times10^{-7}\ s}$$

3) $$\Delta y = y_f - \cancel{y_i}^{\,0} = \cancel{v_{iy}t}^{\,0} + \frac{1}{2}a_y t^2$$

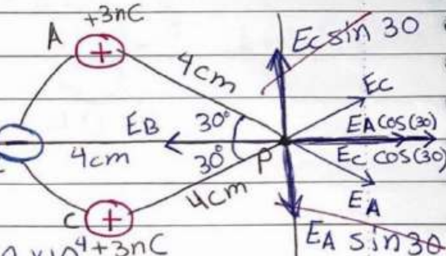
$$y_f = \frac{1}{2}a_y t^2 = \frac{1}{2}(-3.5\times10^{13})(0.3\times10^{-7})^2$$

$$\boxed{y_f = -0.019m}$$

**31.** Three point charges are located on a circular arc as shown in Figure P23.31. (a) What is the total electric field at *P*, the center of the arc? (b) Find the electric force that would be exerted on a −5.00-nC point charge placed at *P*.

$E_A = \frac{K q_A}{r^2} = \frac{9\times10^9 \times 3\times10^{-9}}{(4\times10^{-2})^2}$

$E_A = 1.69\times10^4 \, N/C$




$E_C = \frac{K q_C}{r^2} = 1.69\times10^4 \, N/C$

$E_B = \frac{K q_B}{r^2} = \frac{9\times10^9 \times 2\times10^{-9}}{(4\times10^{-2})^2} = 1.13\times10^4 \, N/C$

$E_{net} = E_C \cos(30) + E_A \cos(30) - E_B$

$E_{net} = 1.69\times10^4 \cos(30) + 1.69\times10^4 \cos(30) - 1.13\times10^4$

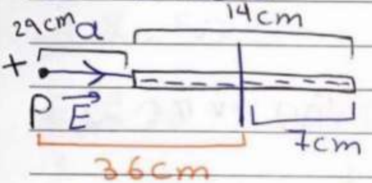
$E_{net} = 1.79\times10^4 \, N/C$

b) $F = qE = -5\times10^{-9}(1.79\times10^4) = -8.95\times10^{-5}$

**37.** A rod 14.0 cm long is uniformly charged and has a total **W** charge of $-22.0\ \mu C$. Determine (a) the magnitude and (b) the direction of the electric field along the axis of the rod at a point 36.0 cm from its center.

$L = 14\times10^{-2}\,m$, $\vec{E} = ?$, $Q = -22\times10^{-6}\,C$

$$\boxed{E = \frac{KQ}{l}\left(\frac{1}{a} - \frac{1}{L+a}\right) \qquad E = \frac{KQ}{a(L+a)}}$$



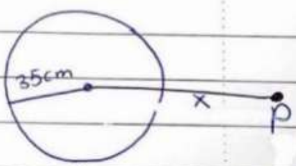

$a = 36 - 7 = 29\,cm$

$$E = \frac{KQ}{a(L+a)} = \frac{9\times10^{9}\times22\times10^{-6}}{(29\times10^{-2})(43\times10^{-2})} = 1.59\times10^{6}\,N/C$$

**38.** A uniformly charged disk of radius 35.0 cm carries charge with a density of $7.90 \times 10^{-3}\ C/m^2$. Calculate the electric field on the axis of the disk at (a) 5.00 cm, (b) 10.0 cm, (c) 50.0 cm, and (d) 200 cm from the center of the disk.

$\sigma = 7.9\times10^{-3}\,C/m^2$, $r = 35\,cm$

$$E_x = 2\pi K\sigma\left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$




a) $x = 5\,cm$: $E_x = 2\pi\times9\times10^{9}\times7.9\times10^{-3}\left(1 - \frac{5\times10^{-2}}{\sqrt{(5\times10^{-2})^2+(35\times10^{-2})^2}}\right)$

$E_x = 383\times10^{6}\,N/C$

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b) $x = 10\,cm$

$$E_x = 2\pi \times 9\times10^{9} \times 7.9\times10^{-3}\left(1 - \frac{10\times10^{-2}}{\sqrt{(10\times10^{-2})^2+(35\times10^{-2})^2}}\right)$$

$E_x = 324\times10^{6}\,N/C$

c) $x = 50\,cm$

$$E_x = 2\pi \times 9\times10^{9} \times 7.9\times10^{-3}\left(1 - \frac{50\times10^{-2}}{\sqrt{(50\times10^{-2})^2+(35\times10^{-2})^2}}\right)$$

$= 80.7\times10^{6}\,N/C$

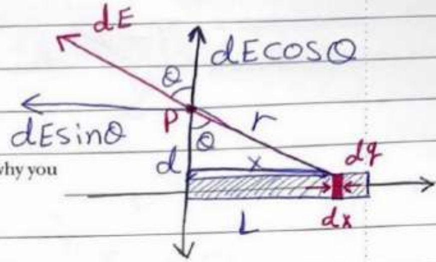
d) $x = 200\,cm$

$$E_x = 2\pi \times 9\times10^{9} \times 7.9\times10^{-3}\left(1 - \frac{200\times10^{-2}}{\sqrt{(200\times10^{-2})^2+(35\times10^{-2})^2}}\right)$$

$E_x = 6.68\times10^{6}\,N/C$

N O T E B O O K

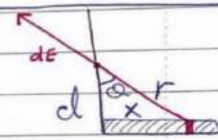
**42.** A uniformly charged rod of length $L$ and total charge $Q$ lies along the $x$ axis as shown in Figure P23.42. (a) Find the components of the electric field at the point $P$ on the $y$ axis a distance $d$ from the origin. (b) What are the approximate values of the field components when $d >> L$? Explain why you would expect these results.




$$dE_y = dE\cos\theta$$

$$dE_x = -dE\sin\theta = -\frac{K\,dq}{r^2}\sin\theta$$

$$dE_x = -\frac{K\,dq}{r^2}\left(\frac{x}{r}\right) = -\frac{K\,dq\,x}{r^3}$$




$$\sin\theta = \frac{x}{r}$$

$$r = \sqrt{x^2+d^2}$$

$$dE_x = \frac{-K\,dq\,x}{(x^2+d^2)^{\frac{3}{2}}} \quad \boxed{dq = \lambda\,dx}$$

$$\int dE_x = \int \frac{-K\,\lambda\,x\,dx}{(x^2+d^2)^{\frac{3}{2}}}$$

$$E_x = -K\lambda \int_0^L \frac{x\,dx}{(x^2+d^2)^{\frac{3}{2}}}$$

$$u = x^2+d^2$$
$$du = 2x\,dx$$

when $x = 0 \rightarrow u = d^2$
when $x = L \rightarrow u = L^2+d^2$ } حدود التكامل

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$$E_x = \frac{-K\lambda}{2}\int_{d^2}^{d^2+L^2} \frac{1}{u^{\frac{3}{2}}}\,du = \frac{\cancel{-}K\lambda}{\cancel{2}}\left[\frac{\cancel{-2}}{\sqrt{u}}\right]_{d^2}^{d^2+L^2}$$

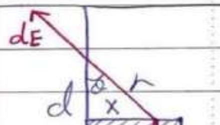
$$E_x = K\lambda\left[\frac{1}{\sqrt{d^2+L^2}} - \frac{1}{d}\right]$$

Lim $d \gg L$ :

$$E_x = K\lambda\left(\frac{1}{d} - \frac{1}{d}\right) \Longrightarrow E_x = 0$$

$$E_y = dE\cos\theta = \frac{K\,dq}{r^2}\cos\theta$$

$$dE_y = \frac{K\,dq\,(d)}{r^3} \qquad \boxed{dq = \lambda\,dx}$$




$$\cos\theta = \frac{d}{r}$$

$$r = \sqrt{x^2+d^2}$$

$$dE_y = \frac{K\,d\,(\lambda\,dx)}{(x^2+d^2)^{\frac{3}{2}}}$$

$$\int dE_y = \int \frac{K\,d\,\lambda\,dx}{(x^2+d^2)^{\frac{3}{2}}} = K\lambda d\int \frac{dx}{(x^2+d^2)^{\frac{3}{2}}}$$

$$E_y = K\lambda d\int_0^L \frac{1}{(x^2+d^2)^{\frac{3}{2}}}\,dx$$

$$E_y = K\lambda d \int_0^L \frac{1}{(x^2+d^2)^{\frac{3}{2}}}\, dx$$

Let $x = d\tan\theta$ | $\theta = \tan^{-1}\left(\frac{x}{d}\right)$

$dx = d\sec^2\theta\, d\theta$

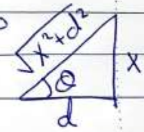

$$E_y = K\lambda d \int_0^L \frac{1}{(d^2\tan^2\theta + d^2)^{\frac{3}{2}}} \cdot d\sec^2\theta\, d\theta$$

$$E_y = K\lambda d \int_0^L \frac{d\sec^2\theta}{(d^2)^{\frac{3}{2}}(\tan^2\theta+1)}\, d\theta$$

$$E_y = K\lambda d \int_0^L \frac{d\sec^2\theta}{d^3(\sec^2\theta)^{\frac{3}{2}}}\, d\theta \quad , \quad \tan^2\theta + 1 = \sec^2\theta$$

$$E_y = K\lambda d \int_0^L \frac{\sec^2\theta}{d^2\sec^3\theta}\, d\theta = \frac{K\lambda d}{d^2}\int_0^L \frac{1}{\sec\theta}\, d\theta$$

$$E_y = \frac{K\lambda}{d}\int_0^L \cos\theta\, d\theta = \frac{K\lambda}{d}\sin\theta \Big]_0^L$$




$$E_y = \frac{K\lambda}{d}\left(\frac{x}{\sqrt{x^2+d^2}}\right)\Big]_0^L$$

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$$E_y = \frac{K\lambda}{d}\left(\frac{L}{\sqrt{L^2+d^2}}\right) - \frac{K\lambda}{d}(0)$$

$$E_y = \frac{K\lambda L}{d\sqrt{L^2+d^2}} \quad , \quad \boxed{\lambda = \frac{Q}{L}}$$

$$E_y = \frac{KQL}{Ld\sqrt{L^2+d^2}} \Rightarrow E_y = \frac{KQ}{d\sqrt{L^2+d^2}}$$

$\lim d >> L$

$$E_y = \frac{KQ}{d\sqrt{d^2}} = \frac{KQ}{d^2}$$

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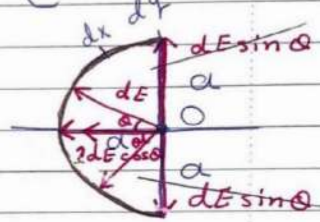
**45.** A uniformly charged insulating rod of length 14.0 cm is bent into the shape of a semicircle as shown in Figure P23.45. The rod has a total charge of $-7.50\ \mu C$. Find (a) the magnitude and (b) the direction of the electric field at $O$, the center of the semicircle.

No.

$L = 14\ cm$, $Q = -7.5 \times 10^{-6}\ C$



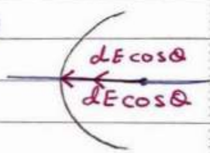

$L = a\theta = \pi a$

$2\pi r$ يساوي $(2L)$ المحيط

$(r)$ نصف القطر وهو يساوي $a$ في السؤال

$\cancel{2}L = \cancel{2}\pi r$

$L = \pi a$ : لأنها نصف حلقة




$a = \dfrac{L}{\pi} = \dfrac{14\ cm}{3.14}$

$\boxed{a = 4.5\ cm}$

$dE_x = -dE\cos\theta$ السالب لأن الاتجاه إلى اليسار

$dE_x = -\dfrac{K\,dq}{a^2}\cos\theta$ $\boxed{dq = \lambda\,dx}$

$dE_x = -\dfrac{K\,dx\,\lambda}{a^2}\cos\theta$

x هنا في L

$x = L = a\theta$

$dx = a\,d\theta$

$dE_x = -\dfrac{K\lambda(\cancel{a}\,d\theta)\cos\theta}{a^{\cancel{2}}}$

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$$\int dE_x = \int -\frac{K\lambda \cos\theta \, d\theta}{a}$$

$$E_x = -\frac{2K\lambda}{a} \int_0^{\frac{\pi}{2}} \cos\theta \, d\theta$$

نضرب بـ 2 لحساب القيمة للمجال على نصف الحلقة

$$= -\frac{2K\lambda}{a} \sin\theta \Big]_0^{\frac{\pi}{2}}$$

$$= -\frac{2K\lambda}{a} (\sin(90) - \sin(0))$$

$$= -\frac{2K\lambda}{a} \qquad \boxed{\lambda = \frac{Q}{L}}$$

$$= -\frac{2KQ}{La} = \frac{-2 \times 9 \times 10^{9} \times 7.5 \times 10^{-6}}{(14 \times 10^{-2})(4.5 \times 10^{-2})}$$

$$E = -2.14 \times 10^{7} \, N/C$$

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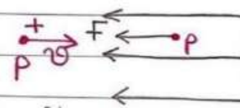
**52.** A proton is projected in the positive $x$ direction into a region of a uniform electric field $\vec{\mathbf{E}} = (-6.00 \times 10^5)\hat{\mathbf{i}}$ N/C at $t = 0$. The proton travels 7.00 cm as it comes to rest. Determine (a) the acceleration of the proton, (b) its initial speed, and (c) the time interval over which the proton comes to rest.

W

$\vec{E} = -6 \times 10^5 \,\hat{i}\, N/C, \; q_p = +1.6 \times 10^{-19} C$

$m_p = 1.67 \times 10^{-27} Kg, \; \Delta x = 7 \times 10^{-2} m, \; v_f = 0$

a) $\vec{F} = m\vec{a} = q\vec{E}$




$\vec{a} = \frac{q\vec{E}}{m} = \frac{1.6 \times 10^{-19} \times (-6 \times 10^5)}{1.67 \times 10^{-27}}$

$\vec{a} = -6 \times 10^{13} m/s^2$

b) $v_f^2 = v_i^2 + 2a\Delta x$

$0 = v_i^2 + 2(-6 \times 10^{13})(7 \times 10^{-2})$

$0 = v_i^2 - 8.4 \times 10^{12}$

$\sqrt{v_i^2} = \sqrt{8.4 \times 10^{12}}$

$v_i = 2.9 \times 10^6 m/s \;\hat{i}$

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c) $v_f = v_i + at$

$0 = 2.9\times10^{6} - 6\times10^{13}\, t$

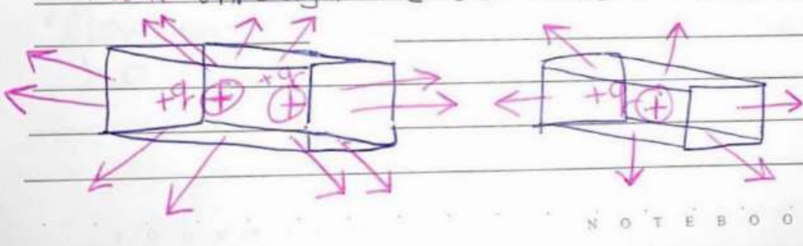
$\frac{\cancel{6\times10^{13}}\, t}{\cancel{6\times10^{13}}} = \frac{2.9\times10^{6}}{6\times10^{13}}$

$\boxed{t = 4.83\times10^{-8}\, s}$

# Chapter 24: Gauss's Law

- Gauss's Law is a relationship between the field at all the points on the surface and the total charge enclosed within the surface

- In both boxes below, there is a positive charge within the box, which produces an outward pointing electric flux through the surface of the box

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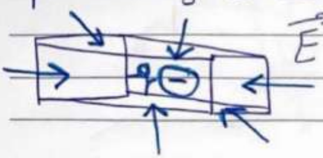
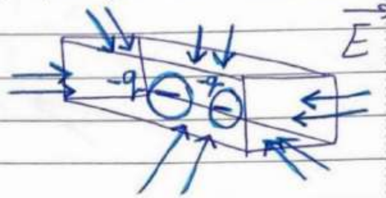
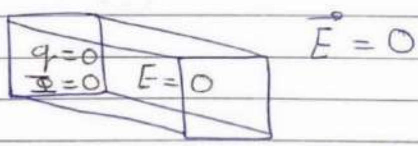
* When there is a negative charge inside the box, there is an inward pointing electric flux on the surface



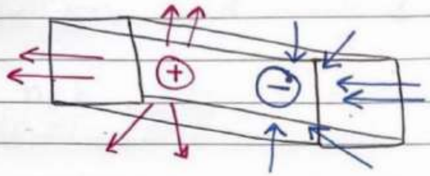

## * Zero net charge inside a box cases:

1) If the box is empty and the electric field is zero everywhere, then there is no electric flux into or out of the box.




2) There is an electric field, but it "flows into" the box on half of its surface and "flows out of" the box on the other half

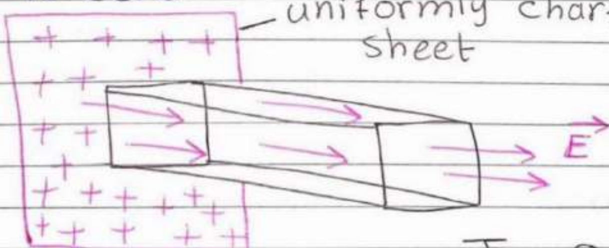
* Hence there is no net electric flux int or out of the box



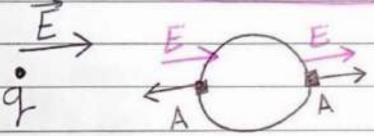
$\Phi_{net} = 0$

3) on one end of the box, the flux points into the box; on the opposite end, the flux points out of the box; and on the sides, the field is parallel to the surface and so the flux is zero.

* The net electric flux through the box is zero




$\Phi = 0$




$\Phi = 0$

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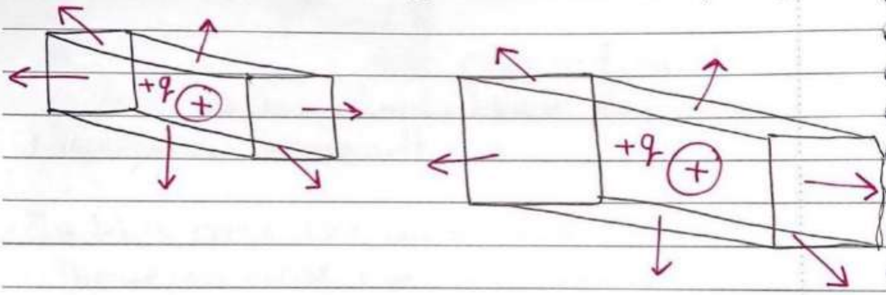
- The net electric flux is directly proportional to the net amount of charge enclosed within the surface

- The net electric flux is independent of the size of the closed surface




## Calculating electric flux

- consider a flat area prependicular to a uniform electric field

- Increasing the area means that more electric field lines pass through the area, increasing flux

- A stronger field means more closely spaced lines, and therefore more flux

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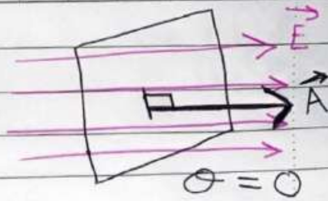
$\Phi = \vec{E} \cdot \vec{A}$

$\Phi = |E||A| \cos\theta$

$\theta$ ↓ the angle between $\vec{E}$ and $\vec{A}$



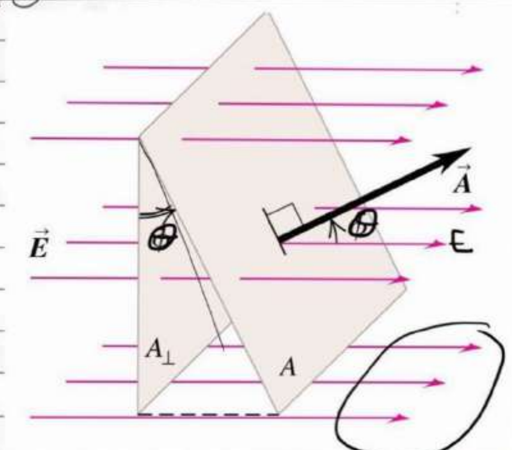

$\theta = 0$

$\Phi_{max} = |E||A|$

- If the area is not prependicular to a uniform electric field, then fewer field Lines pass through it.

- In this case the area that counts is the silhouette area that we see when looking in the direction of the field

$\Phi = |E||A| \cos\theta$



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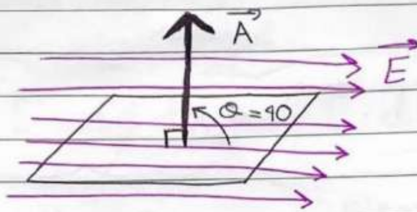
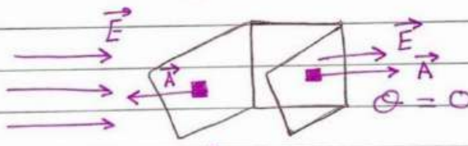
• If the area is edge-on to the field, then the area is prependicular to the field and the flux is zero




$$\Phi = |E||A| \cos(90) = 0$$

• If the angle between $\vec{E}$ and $\vec{A}$ is $180°$ then we will get $\Phi$ minimum




$$\Phi = |E||A| \cos(180)$$

$$\Phi_{min} = -|E||A|$$

$$\Phi = \int E.dA = \int E\, dA \cos\theta$$

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# Flux of a nonuniform electric field:

In general, the flux through a surface must be computed using a surface intergal over the area.

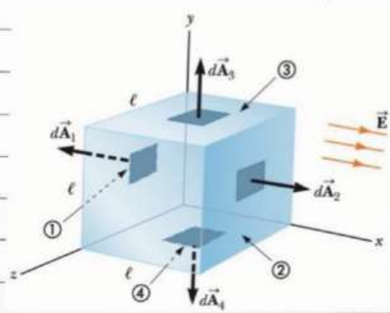
$$\underbrace{\Phi}_{\text{electric flux through a surface}} = \int \overbrace{E}^{\text{magnitude of electric field}} \underbrace{\cos\theta}_{\text{angle between } \vec{E} \text{ and normal to surface}} \underbrace{dA}_{\text{element of surface area}} = \int \vec{E}\cdot \underbrace{d\vec{A}}_{\text{vector element of surface area}}$$

$$[\Phi] = [E][A] = \frac{N\cdot m^2}{C}$$

**Example 24.1** **Flux Through a Cube**

Consider a uniform electric field $\vec{\mathbf{E}}$ oriented in the $x$ direction in empty space. A cube of edge length $\ell$ is placed in the field, oriented as shown in Figure 24.5. Find the net electric flux through the surface of the cube.

$$\Phi_1 = \int E\,dA\cos(180)$$
$$= -E\int dA = -EA$$
$$= -E\underbrace{L^2}_{\text{مساحة المربع}}$$

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$\Phi_2 = \int E\, dA \cos\theta = E \int dA = EL^2$

$\Phi_3 = \Phi_4 = \Phi_5 = \Phi_6 = 0$

لأن الزاوية بين E و A تكون $90^\circ$

$\Phi_{net} = \sum \Phi = -EL^2 + EL^2 + 0 + 0 + 0 + 0$

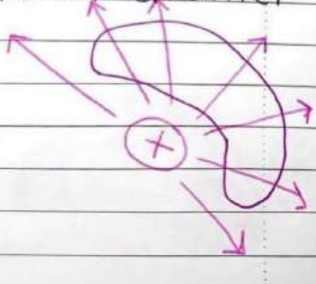
$\Phi_{net} = 0$

* The flux is independent of the surface and depends only on the charge inside.

* For a closed surface enclosing no charge

$\Phi_{encl} = \oint \vec{E}\, d\vec{A} = 0$

* If an electric field line from the external charge enters the surface at one point, it must leave at another



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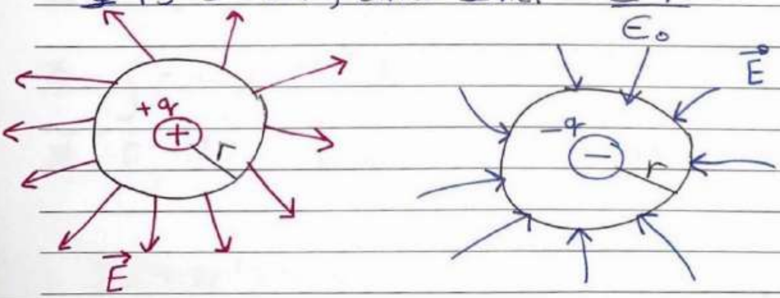
* Let $Q_{encl}$ be the total charge enclosed by a surface

* Gauss's Law states the total electric flux through a closed surface is equal to the total (net) electric charge inside the surface, divided by $\varepsilon_0$:

$$\Phi_E = \oint \vec{E}\, d\vec{A} = \frac{Q_{encl}}{\varepsilon_0}$$

total charge enclosed by surface

$\Phi$ is salar, and $\Phi_{net} = \frac{\sum q}{\epsilon_0}$



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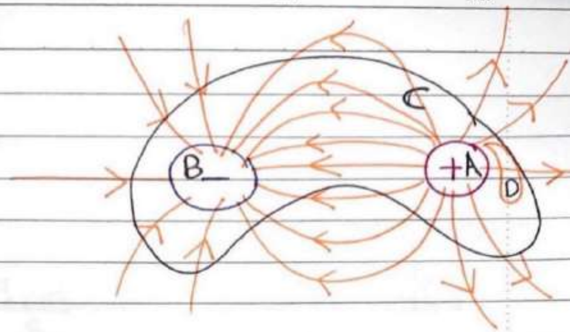
$$\Phi_{EA} = \frac{+q}{\varepsilon_0}$$

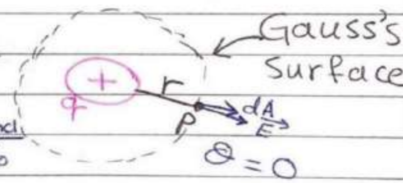
$$\Phi_{EB} = \frac{-q}{\varepsilon_0}$$

$$\Phi_{EC} = 0$$

$$\Phi_{ED} = 0$$




Ex: Find electric field of point P located at r from point charge q?




$$\Phi = \oint E \, dA = \frac{q_{encl}}{\varepsilon_0}$$

$$\Phi = \int E \cos\theta \, dA = \frac{q_{encl}}{\varepsilon_0}$$

$$\Phi = E \int dA = \frac{q_{encl}}{\varepsilon_0} = E\underbrace{(4\pi r^2)}_{\text{مساحة سطح الكرة}} = \frac{q}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$E = \frac{q}{4\pi\varepsilon_0 r^2} = \frac{Kq}{r^2}$$

$$\boxed{E = \frac{F}{q_0} \qquad E = \frac{Kq}{r^2} \qquad K = \frac{1}{4\pi\varepsilon_0}}$$

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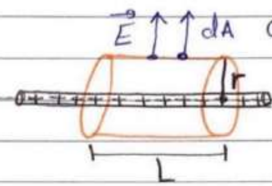
Find the electric field a distance $r$ from a line of positive charge of infinite length and constant charge per unit length $\lambda$ (Fig. 24.12a).




$\theta = 0$

$$\Phi = \oint E\,dA = \frac{q_{encl}}{\varepsilon_0}$$

$$\oint E\cos\theta\,dA = \frac{q_{encl}}{\varepsilon_0}$$

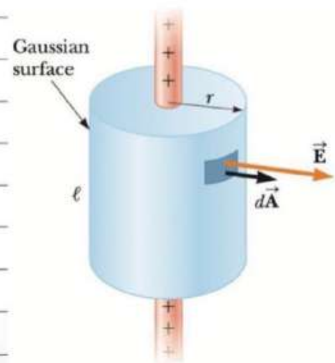
$\boxed{q = \lambda L}$

$$E(\underline{2\pi r L}) = \frac{q_{encl}}{\varepsilon_0} = \frac{\lambda L}{\varepsilon_0}$$

المساحة الجانبية للأسطوانة

$$E(2\pi r \cancel{L}) = \frac{\lambda \cancel{L}}{\varepsilon_0}$$

$$E = \frac{\lambda}{2\pi\varepsilon_0 r}\,\frac{\times 2}{\times 2} = \frac{2\lambda}{4\pi\varepsilon_0 r} = \frac{2K\lambda}{r}$$

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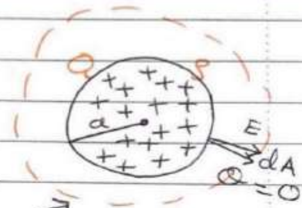
## Example 24.3 A Spherically Symmetric Char

An insulating solid sphere of radius $a$ has a uniform volume charge density $\rho$ and carries a total positive charge $Q$ (Fig. 24.10).

**(A)** Calculate the magnitude of the electric field at a point outside the sphere.

**(B)** Find the magnitude of the electric field at a point inside the sphere.

No.




a) $\oint E\,dA = \dfrac{q_{encl}}{\varepsilon_0}$

$E(\underline{4\pi r^2}) = \dfrac{q_{encl}}{\varepsilon_0}$

مساحة سطح الكرة

$r > a$

$E(4\pi r^2) = \dfrac{Q}{\varepsilon_0} \Longrightarrow \boxed{E = \dfrac{1}{4\pi\varepsilon_0}\dfrac{Q}{r^2} = \dfrac{kQ}{r^2}}$

$\rho = \dfrac{Q}{V} \Longrightarrow Q = \rho V = \rho\left(\dfrac{4}{3}\pi a^3\right)$

$r > a$

$$\boxed{E = \frac{1}{\cancel{4\pi}\varepsilon_0 r^2}\,\rho\left(\frac{\cancel{4}}{3}\cancel{\pi} a^3\right) = \frac{\rho a^3}{3\varepsilon_0 r^2}}$$

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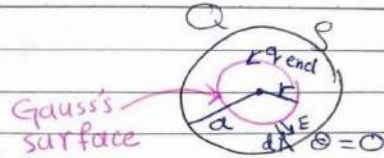
b) $r < a$




$$\oint E\,dA\cos\theta = \frac{q_{encl}}{\varepsilon_0}$$

$$\boxed{\begin{aligned} q_{encl} &= \rho V \\ &= \rho\left(\tfrac{4}{3}\pi r^3\right) \end{aligned}}$$

$$E(4\pi r^2) = \frac{q_{encl}}{\varepsilon_0}$$

$$E(\cancel{4\pi} r^2) = \frac{\rho(\cancel{\tfrac{4}{3}\pi} r^3)}{\varepsilon_0}$$

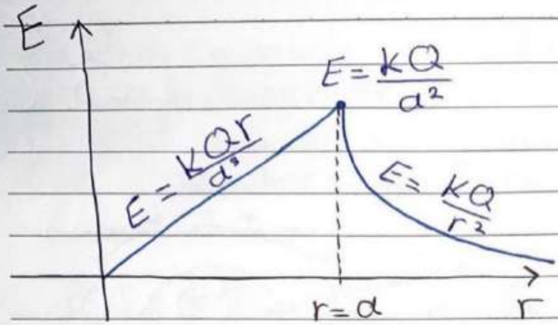
$$\boxed{E = \frac{\rho r}{3\varepsilon_0}} \qquad \rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi a^3}$$

$$E = \frac{Qr}{\cancel{3}\varepsilon_0(\tfrac{4}{\cancel{3}}\pi a^3)} = \frac{Qr}{4\pi\varepsilon_0 a^3}$$

$$E = \frac{kQr}{a^3}$$

at surface $r = a$

$$E = \frac{kQr}{a^3} = \frac{kQa}{a^3} = \frac{kQ}{a^2}$$




**Example 24.5** **A Plane of Charge**

Find the electric field due to an infinite plane of positive charge with uniform surface charge density $\sigma$.

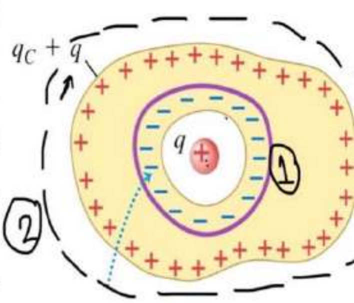
. non-conductivity $\oint E\,dA = \frac{q_{encl}}{\varepsilon_0}$

$2E\cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0} \Longrightarrow \boxed{E = \frac{\sigma}{2\varepsilon_0}}$

- Suppose we construct a Gaussian surface inside a conductor.

- Because $\vec{E} = 0$ everywhere on this surface, Gauss's Law requires that the net charge inside the surface is zero

## Charges on conductor:




★ The total charge on the conductor must remain zero. So a charge $+q$ must appear on its outer surface.

For $E$ to be zero at all points on the Gaussian surface, the surface of the cavity must have a total charge $-q$

① $E \cdot dA = \frac{q_{encl}}{\varepsilon_0}$

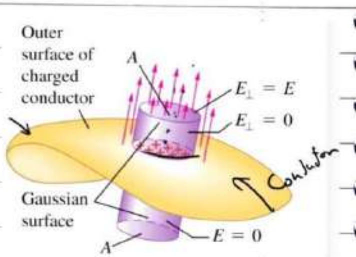
$q_{encl} = q + (-q) = 0$

$E = 0$

② $E \cdot dA = \frac{q_c + q}{\varepsilon_0}$

## Field at the surface of a conductor:

★ Gauss's Law can be used to show that the direction of the electric field at the surface of any conductor is always prependicular to the surface

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The magnitude of the electric field just outside a charged conductor is proportional to the surface charge density $\sigma$.

Electric field at surface of a conductor, $\rightarrow$ $\boxed{E_{\perp} = \frac{\sigma}{\varepsilon_0}}$ $\leftarrow$ surface charge density; $\varepsilon_0$ $\leftarrow$ Electric constant

$\vec{E}$ is prependicular to surface

conducting

A

$\int E \cdot dA = \frac{q_{en}}{\varepsilon_0}$

$E \cancel{A} = \frac{\sigma \cancel{A}}{\varepsilon_0}$

$E = \frac{\sigma}{\varepsilon_0}$

insulating

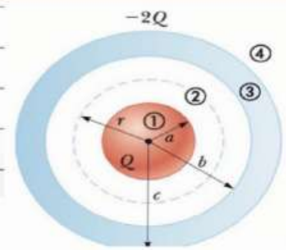
A

$E(2\cancel{A}) = \frac{\sigma \cancel{A}}{\varepsilon_0}$

$E = \frac{\sigma}{2\varepsilon_0}$

## Example 24.7 A Sphere Inside a Spherical Shell

A solid insulating sphere of radius *a* carries a net positive charge *Q* uniformly distributed throughout its volume. A conducting spherical shell of inner radius *b* and outer radius *c* is concentric with the solid sphere and carries a net charge $-2Q$. Using Gauss's law, find the electric field in the regions labeled ①, ②, ③, and ④ in Figure 24.19 and the charge distribution on the shell when the entire system is in electrostatic equilibrium.



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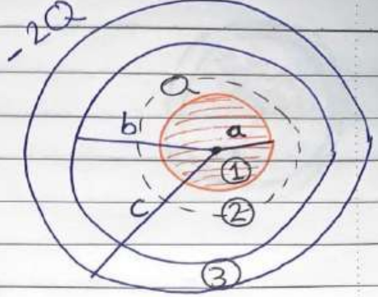
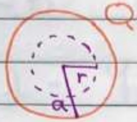



① $r < a$




$$\int E \cdot dA = \frac{q_{encl}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{q_{en}}{\varepsilon_0} \quad , \quad q_{enc} = \rho V \quad ④$$

مساحة سطح جاوس الكروي

$$q_{enc} = \rho\left(\frac{4}{3}\pi r^3\right)$$

الحجم لسطح جاوس

$$E(\cancel{4\pi r^2}) = \frac{\rho\left(\cancel{\frac{4}{3}\pi}\cancel{r^3}^{\,r}\right)}{\varepsilon_0}$$

$$E = \frac{\rho r}{3\varepsilon_0} \quad , \quad \rho = \frac{Q}{V}$$

الحجم للكرة sphere

$$E = \frac{r}{\cancel{3}\varepsilon_0} \cdot \left(\frac{Q}{\cancel{\frac{4}{3}}\pi a^3}\right) \quad , \quad \rho = \frac{Q}{\frac{4}{3}\pi a^3}$$

$$\boxed{E = \frac{Qr}{4\pi\varepsilon_0 a^3} = \frac{kQr}{a^3}}$$

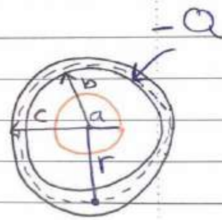
② $a < r < b$


$$E(4\pi r^2) = \frac{q_{enc}}{\varepsilon_0}$$

$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$\boxed{E = \frac{Q}{4\pi\varepsilon_0 r^2} = \frac{KQ}{r^2}}$$

③ $E(4\pi r^2) = \dfrac{q_{enc}}{\varepsilon_0}$

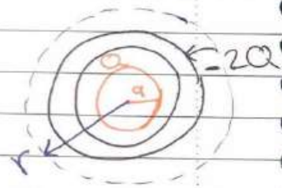



$$q_{enc} = Q_{sphere} + Q_{shell}$$

$$q_{enc} = Q - Q = 0$$

$$E = 0$$

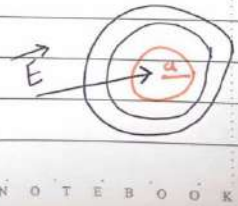
④ $r > c$



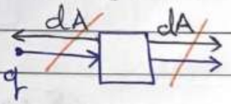

$$E(4\pi r^2) = \frac{q_{enc}}{\varepsilon_0}$$

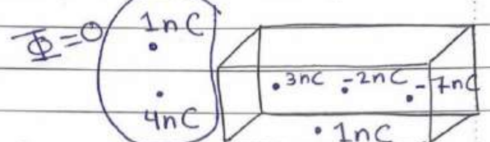
$$q_{enc} = Q + (-2Q) = -Q$$

$$\boxed{E = \frac{-Q}{4\pi\varepsilon_0 r^2} = \frac{-KQ}{r^2}}$$



Ex:5: charges of 3nC, -2nC, -7nC, and 1nC are contained inside a rectangular box with Length 1m, width 2m, and hight 2.5m. Outside the box are charges of 1nC and 4nC. What is the electric flux through the surface of the box?




$$\Phi_{net} = \frac{\Sigma q}{\varepsilon_0} = \frac{(3-2-7+1)\times 10^{-9}}{8.8\times 10^{-12}}$$

$$= \frac{-5\times 10^{-9}}{8.8\times 10^{-12}} = -5.68\times 10^{2}\ N.m^2/C$$

Ex: 7: Two solid spheres, both of radius 5cm, Carry identical total charges of 2 μC. spere A is a good conductor. sphere B is an insulator, and its charge is distributed uniformaly throughout Its volume.
(i) How do the magnitudes of the electric fields they separately create at a radial distance of 6cm compare?

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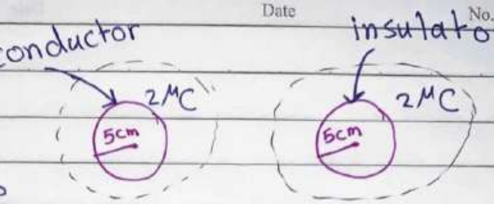



$$E(4\pi r^2) = \frac{Q}{\varepsilon_0}$$

$$E = \frac{KQ}{r^2} \qquad E_A = \frac{KQ}{r^2} \qquad E_B = \frac{KQ}{r^2}$$

$$E_A = E_B > 0$$

**3.** A 40.0-cm-diameter circular loop is rotated in a uniform electric field until the position of maximum electric flux is found. The flux in this position is measured to be $5.20 \times 10^5$ N $\cdot$ m$^2$/C. What is the magnitude of the electric field?

$r = 20 \times 10^{-2}$ m , $\Phi_{net} = 5.2 \times 10^5$ N.m$^2$/C

$$\Phi = \frac{q_{enc}}{\varepsilon_0} = \int E \cdot dA$$


$$5.2 \times 10^5 = EA$$

$$5.2 \times 10^5 = E(\pi r^2)$$

$\Phi_{max} \rightarrow \theta = 0$

$$E = \frac{5.2 \times 10^5}{\pi (20 \times 10^{-2})^2} = 4.14 \times 10^6 \text{ N/C}$$

**6.** A nonuniform electric field is given by the expression

$$\vec{\mathbf{E}} = ay\,\hat{\mathbf{i}} + bz\,\hat{\mathbf{j}} + cx\,\hat{\mathbf{k}}$$

where $a$, $b$, and $c$ are constants. Determine the electric flux through a rectangular surface in the $xy$ plane, extending from $x = 0$ to $x = w$ and from $y = 0$ to $y = h$.

$$\vec{E} = ay\,\hat{i} + bz\,\hat{j} + cx\,\hat{k}$$

$$d\vec{A} = dx\,dy\,\hat{k}$$

$$d\Phi = E \cdot dA = (ay\,\hat{i} + bz\,\hat{j} + cx\,\hat{k}) \cdot (dx\,dy\,\hat{k})$$

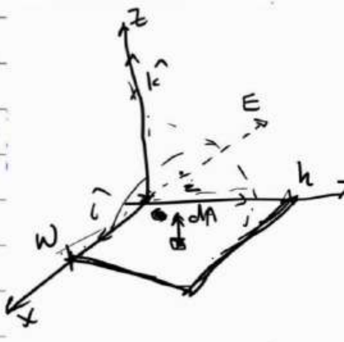
$$d\Phi = cx\,dx\,dy$$

$$\int d\Phi = \iint cx\,dx\,dy$$

$$\Phi = c\int_0^w x\,dx \cdot \int_0^h 1\,dy$$

$$\Phi = c\,y\Big]_0^h \cdot \frac{x^2}{2}\Big]_0^w$$

$$\Phi = \frac{c\,hw^2}{2}$$



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Let $\vec{E} = ay\hat{i} + bz\hat{j} + cxy^2\hat{k}$

$$d\Phi = dE \cdot dA = (ay\hat{i} + bz\hat{j} + cxy^2\hat{k}) \cdot (dx\,dy\,\hat{k})$$

$$\int d\Phi = \iint cxy^2\,dx\,dy$$

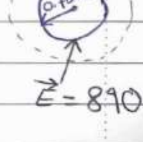
$$\Phi = c\int_0^w x\,dx \int_0^h y^2\,dy$$

$$\Phi = c\frac{x^2}{2}\Big]_0^w \frac{y^3}{3}\Big]_0^h \Rightarrow \Phi = \frac{cw^2h^3}{(2)(3)}$$

$$\Phi = \frac{cw^2h^3}{6}$$

Ex10: The electric field everywhere on the surface of a thin, spherical shell of radius 0.75m is of magnitude 890 N/C and points radially toward the center of the sphere. (a) What is the net charge within the sphere's surface?
(b) What is the distribution of the charge inside the spherical shell?

$r = 0.75\,m$, $E = 890\,N/C$




b) inside the spherical shell

$Q = 0$, $E = 0$

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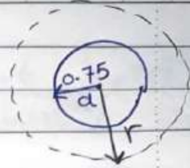
a) Outside sphere




$$E(4\pi r^2) = \frac{q_{enc}}{\varepsilon_0}$$

$$E(4\pi r^2) = -\frac{Q}{\varepsilon_0}$$

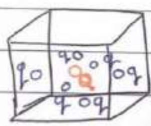
الـ Q سالبة لأن المجال الكهربائي
في اتجاه المركز للكرة

$$E = \frac{-KQ}{r^2} = \frac{-KQ}{a^2}$$

$$Q = \frac{-Ea^2}{K} = \frac{-890 \times (0.75)^2}{9 \times 10^9}$$

$$\boxed{Q = -5.56 \times 10^{-8} C}$$

Ex 19: A particle with charge $Q = 5\mu C$ is located at the center of a cube of edge $L = 0.1 m$. In addition, six other identical charged particles having $q = -1\mu C$ are positioned symmetrically around $Q$ as shown in figure. Determine the electric flux through one face of cube.




$$\Phi_{net} = \int E \cdot dA = \frac{q_{enc}}{\varepsilon_0}$$

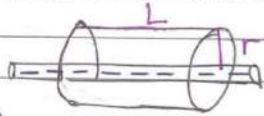
$$\Phi_{net} = \frac{\Sigma q}{\varepsilon_0} = \frac{Q + 6q}{\varepsilon_0}$$

$$\Phi_{1face} = \frac{1}{6}\left(\frac{Q+6q}{\varepsilon_0}\right) = \frac{1}{6}\left(\frac{5-6}{8.8\times10^{-12}}\right)\times10^{-6}$$

$$= \frac{-1\times10^{-6}}{6\times8.8\times10^{-12}} = 1.89\times10^{4}\ N.m^2/C$$

Ex 24: The charge per unit Length on a long straight filament is $-90\ \mu C/m$. Find the electric field (a) 10 cm (b) 20 cm, (c) 100 cm from the filament, where distances are measured prependicular to the length of the filament?

$$\oint E.dA = \frac{q_{en}}{\varepsilon_0}$$




$\lambda = -90\ \mu C/m$

$$E(2\pi rL) = \frac{\lambda L}{\varepsilon_0}$$

$$E = \frac{\lambda \cancel{L}}{2\pi r \cancel{L} \varepsilon_0}\ \frac{\times 2}{\times 2} = \frac{2K\lambda}{r}$$

(a) $r = 10\times10^{-2}\ m$

$$E = \frac{2\times9\times10^{9}\times(-90\times10^{-6})}{10\times10^{-2}} = -1.62\times10^{7}\ N/C$$

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(b) $r = 20 \times 10^{-2}\, m$

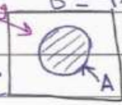
$$E = \frac{2 \times 9 \times 10^{9} \times (-9 \times 10^{-6})}{20 \times 10^{-2}} = -8.1 \times 10^{6}\, N/C$$

(c) $r = 100 \times 10^{-2}\, m$

$$E = \frac{2 \times 9 \times 10^{9} (-9 \times 10^{-6})}{100 \times 10^{-2}} = -1.62 \times 10^{6}\, N/C$$

Ex 27: A large, flat, horizontal sheet of charge has a charge per unit area of $9\,\mu C/m^2$. Find the electric field just above the middle of the sheet.

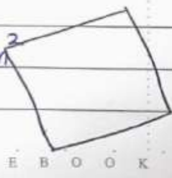
non conducting $\sigma = 9\,\mu C/m^2$




$$E = \frac{\sigma}{2\varepsilon_0} = \frac{9 \times 10^{-6}}{2 \times 8.8 \times 10^{-12}} = 5.11 \times 10^{5}\, N/C$$

Ex 30: A non conducting wall carries charge with a uniform density of $8.6\,\mu C/cm^2$. (a) what is the electric field 7cm in front of the wall if 7cm is small compared with the dimensions of the wall? (b) Does your result change as the distance from the wall varies?

$$\sigma = \frac{8.6\,\mu C}{cm^2} \times \frac{10^{4}\, cm^2}{m^2} = 8.6 \times 10^{-6} \times 10^{4}\, C/m^2$$



$$= 8.6 \times 10^{-2}\, C/m^2$$



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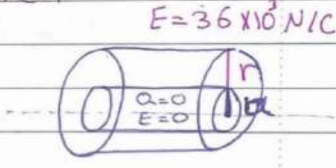
$$E = \frac{\sigma}{2\varepsilon_0} = \frac{8.6\times10^{-2}}{2(8.8\times10^{-12})} = 4.89\times10^{9}\ N/C$$

E is independent of r

**34.** A cylindrical shell of radius 7.00 cm and length 2.40 m **W** has its charge uniformly distributed on its curved surface. The magnitude of the electric field at a point 19.0 cm radially outward from its axis (measured from the midpoint of the shell) is 36.0 kN/C. Find (a) the net charge on the shell and (b) the electric field at a point 4.00 cm from the axis, measured radially outward from the midpoint of the shell.

b) $r = 4cm$ inside the shell

$q = 0 \Rightarrow E = 0$




a) $\oint E \cdot dA = \frac{q_{enc}}{\epsilon_0}$

$E(2\pi rL) = \frac{Q}{\epsilon_0}$

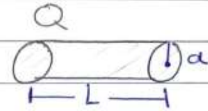
$E = \frac{Q}{2\pi\epsilon_0 rL} \Rightarrow Q = 2\pi rL\epsilon_0$

$Q = 2\pi(0.19)(2.4)(36\times10^{3})(8.8\times10^{-12})$

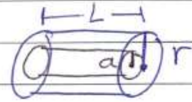
$Q = 9.077\times10^{-7}\ C$

## insulating Cylinder:

① $r<a$ ② $r>a$




2) $r>a$




$$\int E \cdot dA = \frac{q_{enc}}{\epsilon_0}$$

$$E(2\pi rL) = \frac{q_{enc}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

$$E = \frac{Q}{2\pi rL\epsilon_0}, \quad \rho = \frac{Q}{V} = \frac{Q}{\pi a^2 L}$$

$$Q = (\pi a^2 L)\rho$$

$$E = \frac{\cancel{\pi} a^2 \cancel{L} \rho}{2\cancel{\pi} r \cancel{L} \epsilon_0} = \frac{\rho a^2}{2\epsilon_0 r}$$

$r<a$


$$\int E \cdot dA = \frac{q_{enc}}{\epsilon_0}, \quad q_{enc} = \rho V \leftarrow$$ لسطح جاوسي

$$E(2\pi rL) = \frac{q_{enc}}{\epsilon_0}, \quad q_{enc} = \rho(L\pi r^2)$$

$$E(2\cancel{\pi}\cancel{r}\cancel{L}) = \frac{\rho(\cancel{\pi} r^{\cancel{2}} \cancel{L})}{\epsilon_0}$$

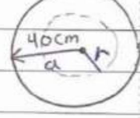
$$\boxed{E = \frac{\rho r}{2\epsilon_0}}$$

$$\rho = \frac{Q}{V}$$

$$\rho = \frac{Q}{(\pi a^2)L}$$

$$E = \frac{r}{2\epsilon_0}\left(\frac{Q}{\pi a^2 L}\right) = \frac{Qr}{2\pi\epsilon_0 L a^2}$$

Ex 35: A solid sphere of radius 40cm has a total positive charge of 26 $\mu C$ uniformly distributed throughout its volume. Calculate the magnitude of the electric field (a) 0cm, (b) 10cm, (c) 40cm, and (d) 60cm, from the center of the sphere.




$r<a$: $\quad E(4\pi r^2) = \dfrac{q_{enc}}{\epsilon_0}$

$$E(\cancel{4}\cancel{\pi}\cancel{r^2}) = \frac{\rho(\frac{\cancel{4}}{3}\cancel{\pi} r^{\cancel{3}})}{\epsilon_0}$$

$q_{enc} = \rho V$

$q_{enc} = \rho\left(\frac{4}{3}\pi r^3\right)$

$$\boxed{E = \frac{\rho r}{3\epsilon_0}}$$

(a) $r = 0\,cm \Rightarrow E = 0$

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$$Q = \rho V \longrightarrow \rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi a^3}$$

$$E = \frac{r}{3\epsilon_0}\left(\frac{Q}{\frac{4\pi}{3}a^3}\right) \longrightarrow \boxed{E = \frac{KQr}{a^3}}$$

(b) at $r = 10cm$

$$E = \frac{9\times10^{9}\times26\times10^{-6}\times(10\times10^{-2})}{(40\times10^{-2})^3}$$

$$E = 3.66\times10^{5}\,N/C$$

(c) at $r = 40\,cm \Rightarrow r = a$

$$E = \frac{KQ}{a^2} = \frac{9\times10^{9}\times26\times10^{-6}}{(40\times10^{-2})^2} = 1.46\times10^{6}\,N/C$$

(d) at $r = 60\,cm$

$$E = \frac{KQ}{r^2} = \frac{9\times10^{9}\times26\times10^{-6}}{(60\times10^{-2})^2} = 6.5\times10^{5}\,N/C$$

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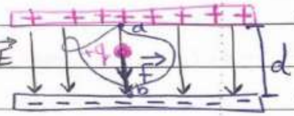
# Chapter 25: Electric potential energy in a uniform field:

- A pair of charged parallel metal plates sets up a uniform, downward electric field.
- As the charge moves from point a to point b, the work done by the field is independent of the path the particle takes.

$q_0 \rightarrow \vec{F} = q_0 \vec{E}$




$W_{a \to b} = \vec{F} \cdot \Delta \vec{X} = |F||\Delta X| \cos\theta$

$= |F| d$

$\boxed{W_{a \to b} = q E d}$

done by the field

- The work done by the electric force is the same for any path from a to b

$W_{a \to b} = -\Delta U$

If the charge is +

$\Delta U \rightarrow$ negativ

electric potential energy

$U_b < U_a$

$\Delta U$ is negative

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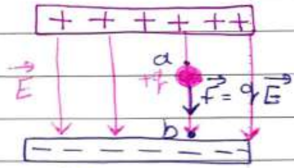
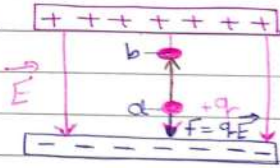
- If the positive charge moves in the direction of the field, the field does positive work on the charge, and the potential energy decreases.

$\Delta U$ is negative




$U_b < U_a$

- If the positive charge moves opposite the direction of the field, the field does negative work on the charge, and the potential energy increases




$$W_{a\to b} = \vec{F}\cdot\Delta\vec{x} = |F||\Delta x|\underbrace{\cos\theta}_{-1}$$

$$= -|F||\Delta x|$$

$$= -q_0 . E d$$

- work done on the charge

$\Delta U$ is positive

$U_b > U_a$

$$\boxed{W = -\Delta U}$$

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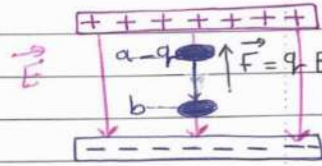
A negative charge moving in a uniform field.

* If the negative charge moves in the direction of the field, the field does negative work on the charge, and the potential energy increases.

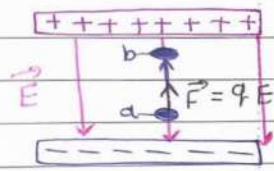
$$W_{a\to b} = F \cdot \Delta x$$
$$= |F||\Delta x| \cos\theta$$
$$= -qEd$$




$\Delta U$ is positive

* If the negative charge moves opposite the direction of the field, the field does positive work on the charge, and the potential energy decreases.

$$W_{a\to b} = qEd$$




$\Delta U$ is negative

$$U_b < U_a$$

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* Electric potential is defined as:

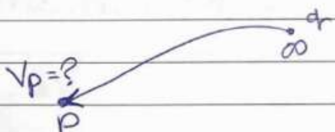
$$V = \frac{U}{q}$$

* The potential difference between two points A and B in an electric field is defined as the change in electric potential energy of the system when a charge q is moved between two points

$$\Delta V = \frac{\Delta U}{q} \quad J/C$$

$$[V] \rightarrow J/C = \text{volts } (V)$$

$$\Delta V = V_b - V_a = \frac{\Delta U}{q} = \frac{U_b - U_a}{q}$$




$$\Delta V = V_p - \cancel{V_\infty}^{\,0} = \frac{U_p - \cancel{U_\infty}^{\,0}}{q}$$

$$\boxed{V_p = \frac{U_p}{q}}$$



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* Moving with the direction of the electric field means moving in the direction of decreasing V

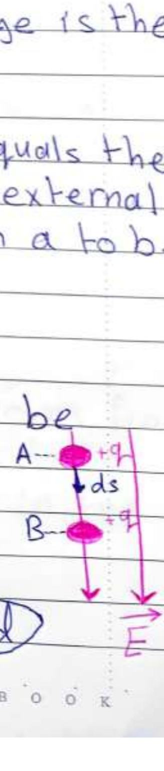
* To move a unit charge slowly against the electric force, we must apply an external force per unit charge equal and opposite to the electric force per unit charge

* The electric force per unit charge is the electric field.

* The potential difference $V_b - V_a$ equals the work done per unit charge by this external force to move a unit charge from a to b.

$$\Delta V = V_b - V_a = -\int_A^B \vec{E} \cdot d\vec{s}$$

* The unit of electric field can be expressed as $N/C = V/m$




$$\Delta V = -\int_A^B E\, ds\,(\cos 0) = -\int_A^B E\, ds$$

$$\Delta V = -Ed \quad , \quad \Delta U = q\Delta V = -qEd$$

$V_B < V_A$

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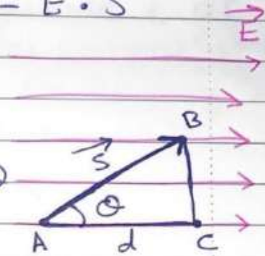
$$\Delta V = -\int_A^B \vec{E}\, d\vec{s} = -\vec{E}\int_A^B d\vec{s} = -\vec{E}\cdot\vec{S}$$

$$\Delta U = q\Delta V = -q\vec{E}\cdot\vec{S}$$
$$= -q|\vec{E}||\vec{s}|\cos\theta$$



$$\Delta V_{AB} = \Delta V_{AC} + \Delta V_{CB}$$
$$= -\int_A^C E\,ds - \int_C^B E\,ds$$
$$= -\int_A^C |E|\,ds\cos 0 - \overset{0}{\cancel{\int_C^B |E|\,ds\cos(90)}}$$
$$= -\int_A^C E\,ds = -Ed$$

## The electron volt:

When a particle with charge $q$ moves from a point where the potential is $V_b$ to a point where it is $V_a$, the change in the potential energy $U$ is

$$U_b - U_a = q(V_b - V_a)$$

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If charge q equals the magnitude e of the electron charge, and the potential difference is 1V, the change in energy is defined as one electron volt (ev)

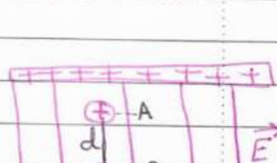
$$1\,eV = 1.602 \times 10^{-19}\,J$$

energy

Ex 25.2: A proton is released from rest at point A in a uniform electric field that has a magnitude of $8 \times 10^4$ (V/m). The proton undergoes a displacement of magnitude $d = 0.5\,m$ to point B in the direction of $\vec{E}$. Find the speed of the proton after completing the displacement

$q_p = +1.6 \times 10^{-19}\,C$

$m_p = 1.67 \times 10^{-27}\,Kg$




$\Delta K + \Delta U = 0$

$K_f - \cancel{K_i}^{0} + U_f - U_i = 0$

$\frac{1}{2} m v_B^2 + q\Delta V = 0$



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$$\frac{1}{2} m v_B^2 = -q\Delta V \quad , \quad \Delta V = -\int_A^B E \cdot ds$$

$$\frac{1}{2} m v_B^2 = \cancel{-}q(\cancel{-}Ed) \quad , \quad \Delta V = -Ed$$

$$v_B^2 = \frac{2qEd}{m}$$

$$v_B = \sqrt{\frac{2qEd}{m}}$$

$\Delta U = q\Delta V$

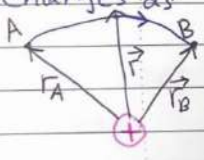
$J = CV$

$$v_B = \sqrt{\frac{2(1.6\times10^{-19})(8\times10^{4})(0.5)}{1.67\times10^{-27}}} = 2.8\times10^{6}\ m/s$$

Electric Potential and potential energy Due to point charges:

- The work done by the electric field of one point charge on another doesn't depend on the path taken.

- The electric potential energy only depends on the distance between the charges




$$\Delta V = V_B - V_A = -\int_A^B \vec{E} \cdot d\vec{s}$$

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$$\vec{E}\cdot d\vec{s} = \frac{Kq}{r^2}\,\hat{r}\cdot d\vec{s}$$

$$\hat{r}\cdot d\vec{s} = d\vec{r}$$

$$V_B - V_A = -Kq\int_{r_A}^{r_B}\frac{dr}{r^2}$$

$$V_B - V_A = \frac{Kq}{r}\Big]_{r_A}^{r_B} = Kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

$$\boxed{V = K\sum_{n=1}\frac{q_i}{r_i}} \qquad V = \frac{Kq_1}{r_1} + \frac{Kq_2}{r_2} + \text{------}$$

* The electric potential energy of two point charges only depends on the distance between the charges

$$\boxed{U = \frac{1}{4\pi\epsilon_0}\left(\frac{q\,q_0}{r}\right)}$$

* This equation is valid no matter what the signs of the charges are

* Potential energy is defined to be zero when the charges are infinitely far apart



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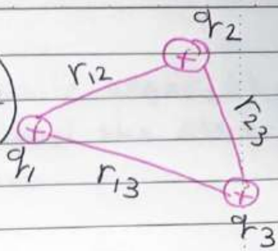
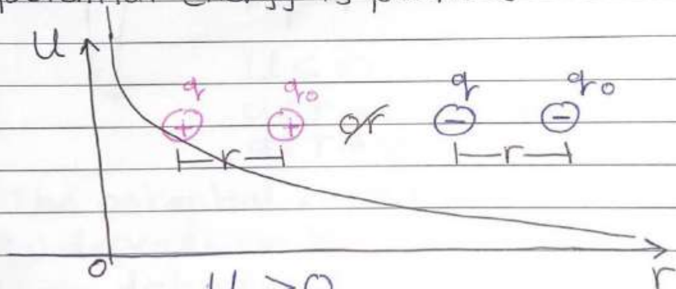
$$U = K\left(\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$




$$V_p = K \sum \frac{q_i}{r_i}$$

$$U_{\infty \to p} = K q_0 \sum \frac{q_i}{r_i}$$

* If two charges have the same sign the interaction is repulsive, and the electric potential energy is positive




$U > 0$

as $r \to 0$, $U \to +\infty$

as $r \to \infty$, $U \to 0$

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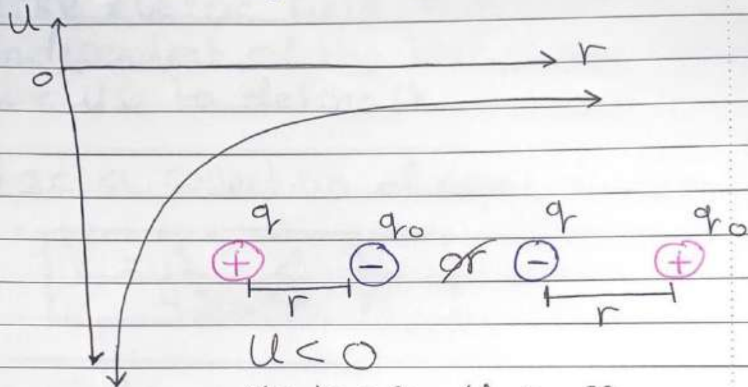
- If two charges have opposite signs, the interaction is attractive and the electric potential energy is negative




$U < 0$

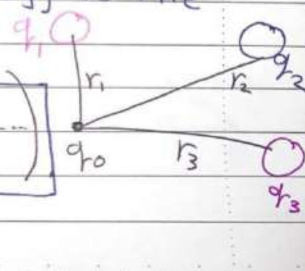
as $r \to 0$, $U \to -\infty$

as $r \to \infty$, $U \to 0$

- The potential energy associated with $q_0$ depends on the other charges and their distances from $q_0$

- The electric potential energy is the algabric sum

$$\boxed{U = \frac{q_0}{4\pi\varepsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right)}$$



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The potential due a single point charge

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\frac{q}{r}}$$

Like electric field, Potential is independent of the test charge that we use to define it

For a collection of point charges

$$\boxed{V = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}}$$

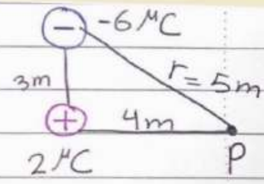
**Example 25.3** **The Electric Potential Due to Two Point Charges**

As shown in Figure 25.10a, a charge $q_1 = 2.00\ \mu C$ is located at the origin and a charge $q_2 = -6.00\ \mu C$ is located at (0, 3.00) m.

**(A)** Find the total electric potential due to these charges at the point *P*, whose coordinates are (4.00, 0) m.

**(B)** Find the change in potential energy of the system of two charges plus a third charge $q_3 = 3.00\ \mu C$ as the latter charge moves from infinity to point *P* (Fig. 25.10b).

$q_1 = 2\times10^{-6}\,C$, $q_2 = -6\times10^{-6}\,C$




a) $V_P = K\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$

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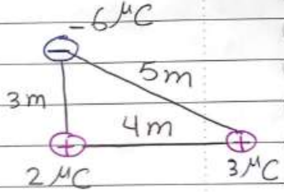
$$V_p = 9\times10^{9}\left(\frac{2\times10^{-6}}{4} + \frac{-6\times10^{-6}}{5}\right)$$

$$V_p = -6.29\times10^{3}\,V$$



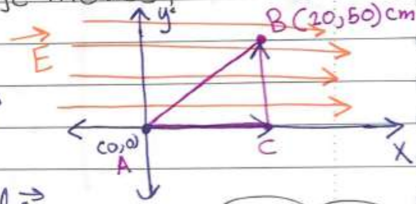

b)

$$\Delta U = U_f - \cancel{U_i}^{\,0} = q_3 V_p - 0$$

$$\Delta U = (3\times10^{-6})(-6.29\times10^{3})$$

$$\Delta U = -1.89\times10^{-2}\,J \qquad U_p < U_\infty$$

Ex 2: A uniform electric field of magnitude 250 V/m is directed in the positive x direction. A +12 $\mu C$ charge moves from the origin to the point (x,y) = (20cm, 50cm)
(a) What is the change in the potential energy of the charge-field system? (b) Through what potential difference does the charge moves?




$$|\vec{E}| = 250 \text{ V/m}$$

$$\Delta V = V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s}$$

$$\Delta V = -\int_A^C \vec{E}\cdot ds - \int_C^B \vec{E}\cdot d\vec{s}$$

$$\Delta V = -\int_A^C E.ds\cos(0) - \cancelto{0}{\int_C^B E.ds\cos(90)}$$

$$\Delta V = -E\int_0^{0.2} ds = -Ed\Big]_0^{0.2}$$

$$\Delta V = -250(0.2) = -50V$$

$$\Delta U = q\Delta V = -50(12\times10^{-6}) = -6\times10^{-4} J$$

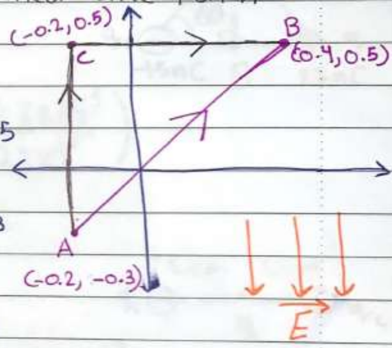
$$q\Delta V = \Delta U$$

$$V_p = \frac{Kq}{r}$$

$$V_p = K\sum_{i=1} \frac{q_i}{r_i}$$

Ex5: A uniform electric field of magnitude 325 V/m is directed in the negative y direction. The cordinates of point (A) are (-0.2, -0.3) m and those of point (B) are (0.4, 0.5) m. Calculate the electric potential difference $V_B - V_A$ Using dashed-Line Path

$$\Delta V = V_B - V_A = -\int_A^C E\cdot ds - \cancelto{0}{\int_C^B E\cdot ds}$$




$$\Delta V = 325 \int_{-0.3}^{0.5} ds = 325\, s \Big]_{-0.3}^{0.5}$$

$$\Delta V = 325\,(0.5 - (-0.3))$$

$$\Delta V = 325\,(0.8) = 260\ V$$

or

$$\vec{E} = 325\,(-\hat{j}) = -325\,\hat{j}$$

$$d\vec{s} = dx\,\hat{i} + dy\,\hat{j}$$

$$\vec{E}\cdot d\vec{s} = (-325\,\hat{j})\cdot(dx\,\hat{i} + dy\,\hat{j})$$

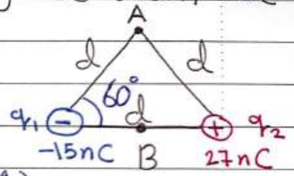
$$= -325\, dy$$

$$\Delta V = -\int_{-0.3}^{0.5} -325\, dy = 325\, y \Big]_{-0.3}^{0.5}$$

$$\Delta V = 325\,(0.5 + 0.3) = 325\,(0.8)$$

$$\Delta V = 260\ V$$

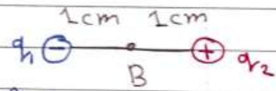
Ex 14: The two charges are seperated by $d = 2\,cm$. Find the electric potential at (a) point A and (b) point B, which is half way between the charges.




$$V_A = K\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_A = 9\times10^{9}\left(\frac{-15\times10^{-9}}{2\times10^{-2}} + \frac{27\times10^{-9}}{2\times10^{-2}}\right)$$

$$V_A = 54\times10^{2}\,V$$




$$V_B = K\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$V_B = 9\times10^{9}\left(\frac{-15\times10^{-9}}{1\times10^{-2}} + \frac{27\times10^{-9}}{1\times10^{-2}}\right)$$

$$V_B = 108\times10^{+2}\,V$$

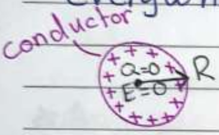
$$V \propto \frac{1}{r}$$

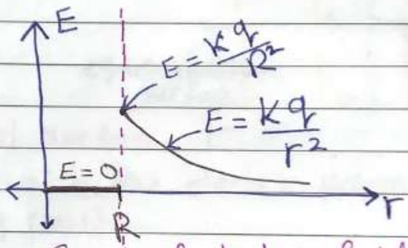
$$V_B = 2\,V_A = 108\times10^{+2}\,V$$

# Electric potential and field of a charged conductor:

- A solid conducting sphere of radius R has a total charge q.

- The electric field inside the sphere is zero everywhere.

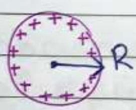


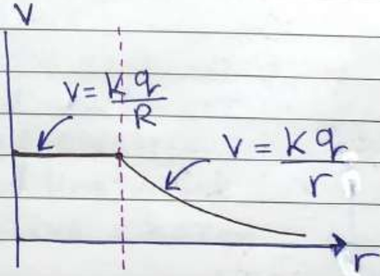




$$V = \frac{KQ}{R}$$

Graph of electric field

- The potential is the same at every point inside the sphere and is equal to its value at the surface.




Graph of potential

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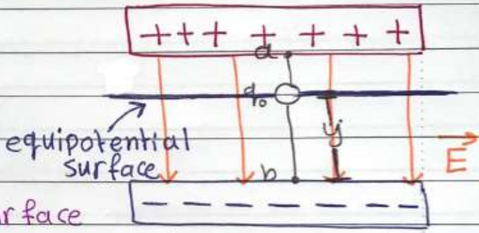
# Oppositely charged parallel plates:

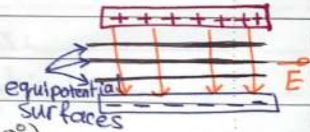
* The potential at any hight y between the two large oppositely charged parallel plates is $V = Ey$

$$\Delta V = -\int_a^b E \cdot ds$$

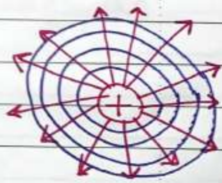



* An equipotential surface is a surface on which the electric potential is the same at every point.

* Field lines and equipotential surfaces are always mutually prependicular ($\theta = 90°$)




* Shown are cross sections of equipotential surface (blue lines) and electric field lines (red lines) for a single positive charge



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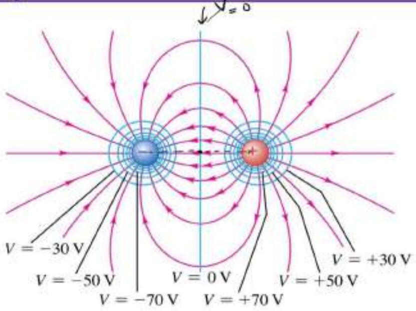
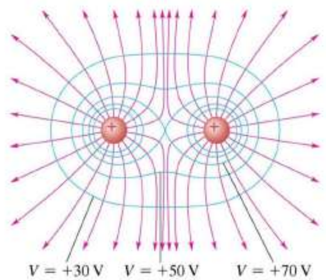
## Equipotential surfaces and field lines for a dipole




## Field and potential of two equal positive charges

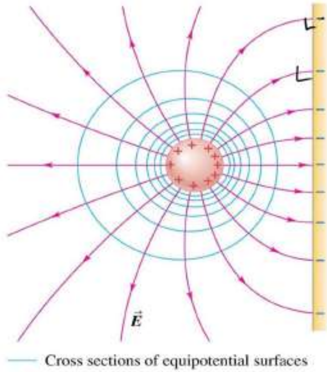



Equipotentials and conductors:

When all charges are at rest:

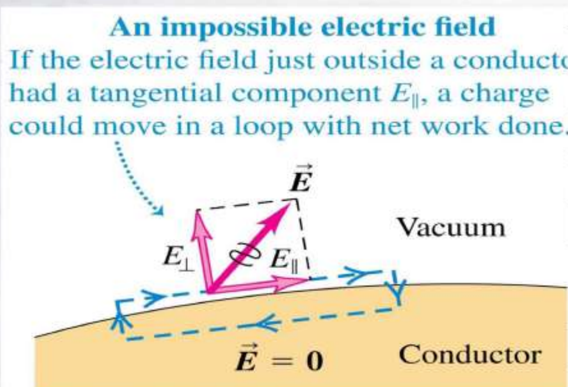
- The surface of a conductor is always an equipotential surface

- The electric field just outside a conductor is always prependicular to the surface.



# Equipotentials and conductors:

* If the electric field had a tangential component at the surface of a conductor, a net amount of work would be done on a test charge by moving it around a loop, which is impossible because the electric force is conservative.




$$\Delta V = -\int \vec{E} \cdot d\vec{s}$$

$$E = -\frac{dV}{d}$$

$$E_x = -\frac{\partial V}{\partial x}\,\hat{i}\,, \quad E_y = -\frac{\partial V}{\partial y}\,\hat{j}, \quad E_z = -\frac{\partial V}{\partial z}\,\hat{k}$$



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$$\vec{E} = -\left[ \frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k} \right]$$

gradient

$$\vec{E} = -\vec{\nabla} V$$

Ex: $V(x,y) = x^2 y^3$

$$\frac{\partial V}{\partial x} = 2xy^3, \quad \frac{\partial V}{\partial y} = x^2(3y^2) = 3x^2y^2$$

$$\frac{\partial V}{\partial z} = 0$$

## Potential gradient:

- The components of the electric field can be found by taking partial derivatives of the electric potential.

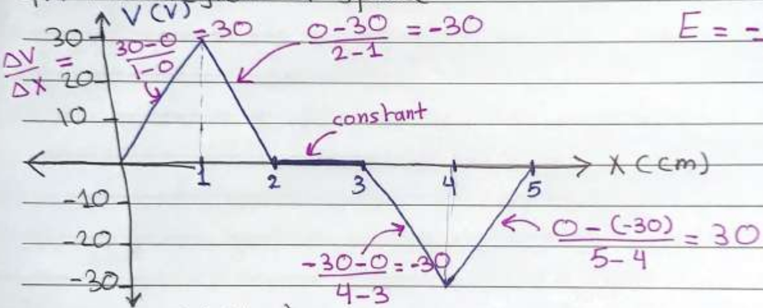
$$E_x = -\frac{\partial V}{\partial x}, \quad E_y = -\frac{\partial V}{\partial y}, \quad E_z = -\frac{\partial V}{\partial z}$$

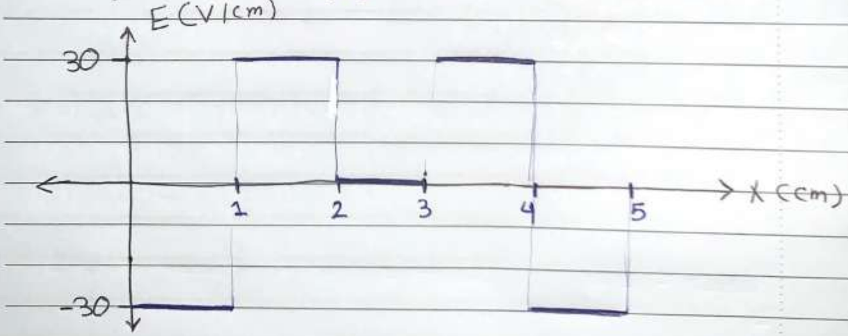
- The electric field is the negative gradient of the potential: $\vec{E} = -\vec{\nabla} V$

$$\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}$$

Ex 38: An electric field in a region of space is parallel to the x axis. The electric potential varies with position as shown, Graph the x components of the electric field versus position in this region of space

$$E = -\frac{dV}{dx}$$



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Ex 39: Over a certain region of space, the electric potential is $V = 5x - 3x^2y + 2yz^2$
(a) Find the expressions for the x, y and z components of the electric field over this region. (b) What is the magnitude of the field at the point P that has cordinates (1, 0, -2) m?

$$V(x, y, z) = 5x - 3x^2y + 2yz^2$$

$$E_x = -\frac{dv}{dx} = -[5 - 3y(2x) + 0] = 6xy - 5$$

$$E_y = -\frac{dv}{dy} = -[0 - 3x^2 + 2z^2] = 3x^2 - 2z^2$$

$$E_z = -\frac{dv}{dz} = -[0 - 0 + 2y(2z)] = -4yz$$

$$\vec{E} = (6xy - 5)\hat{i} + (3x^2 - 2z^2)\hat{j} + (-4yz)\hat{k}$$

$$P(\underset{x}{\underline{1}}, \underset{y}{\underline{0}}, \underset{z}{\underline{-2}})$$

$$\vec{E} = -5\hat{i} - 5\hat{j}$$

$$|\vec{E}| = \sqrt{(-5)^2 + (-5)^2} = 7.07 \text{ V/m}$$

$$\theta = \tan^{-1}\left(\frac{-5}{-5}\right) + 180^\circ = 45^\circ + 180^\circ = 225^\circ$$

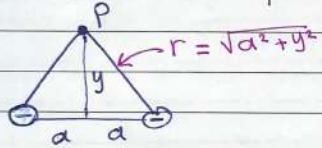
# The electric potential due a dipole

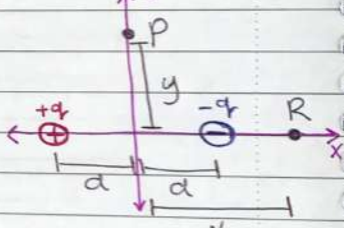
Ex 25.4: An electric dipole consists of two charges of equal magnitude and opposite sign seperated by a distance $2a$. The dipole is along the $x$ axis and is centered at the origin

(a) Calculate the electric potential at point $P$ on the $y$ axis.

(b) Calculate the electric potential at point $R$ on the positive $x$ axis

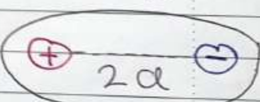
(c) Calculate $V$ and $E_x$ at a point on the $x$ axis far from the dipole.




a) $V_P = K\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$

$$V_P = K\left(\frac{q}{\sqrt{a^2+y^2}} - \frac{q}{\sqrt{a^2+y^2}}\right) = 0$$




electric dipole

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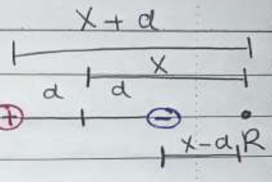


b) $V_R = K\left(\frac{q}{x+a} - \frac{q}{x-a}\right)$

$V_R = Kq\left(\frac{1}{x+a} - \frac{1}{x-a}\right)$

$V_R = Kq\left(\frac{x-a-x-a}{(x+a)(x-a)}\right)$

$V_R = Kq\left(\frac{-2a}{(x+a)(x-a)}\right) = Kq\left(\frac{-2a}{x^2-a^2}\right)$

c) $V_R = Kq\left(\frac{-2a}{x^2}\right) \longrightarrow$ zero  when $x \gg a$

$E = \frac{-dv}{dx} = -Kq(-2a)\left(-\frac{1}{x}\right)$

$E = -\frac{Kq(2a)}{x}$

when $x \gg a$

$E = -\frac{Kq(2a)}{x} \longrightarrow$ zero

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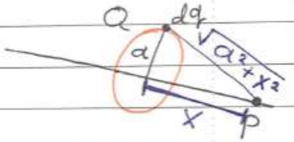
# Electric potential due to a uniformly charged ring

Ex 25.5: (A) Find an expression for the electric potential at point P located on the perpendicular central axis of a uniformly charged ring of radius a and total charge Q

(B) Find an expression for the magnitude of the electric field at point P.




$$\int dV_p = \int \frac{K\,dq}{r} = \int \frac{K\,dq}{\sqrt{a^2+x^2}}$$

$$V_p = \frac{K}{\sqrt{a^2+x^2}} \int_0^Q dq = \frac{KQ}{\sqrt{a^2+x^2}}$$

$$V_p = \frac{KQ}{(a^2+x^2)^{\frac{1}{2}}}$$

$$E_x = -\left[\frac{0 - KQ(\cancel{\tfrac{1}{2}})(a^2+x^2)^{-\frac{1}{2}}(\cancel{2}x)}{a^2+x^2}\right]$$

$$E_x = \frac{KQx}{(a^2+x^2)^{\frac{1}{2}}(a^2+x^2)} = \frac{KQx}{(a^2+x^2)^{\frac{3}{2}}}$$

$$\text{Lim } x \gg a \Rightarrow E_x = \frac{KQx}{x^3} = \frac{KQ}{x^2}$$

$$E_y = -\frac{dV}{dy} = 0, \quad E_z = 0$$

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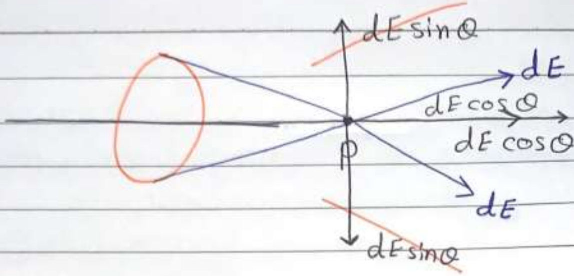
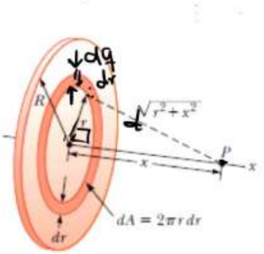
$E_y = 0$




Ex 25.6 : Electric potential due to a uniformly Charged disk :

A uniformly charged disk has a radius R and surface charge density σ. (A) Find the electric potential at a point P along the prependicular central axis of the disk. (B) Find the x components of the electric field at a point P along the prependicular central axis of the disk.




$$dV_p = \frac{K\,dq}{d} = \frac{K\,dq}{\sqrt{r^2+x^2}}$$

$$dq = \sigma\,dA = \sigma\,(2\pi r\,dr)$$

$$\int dV_p = \int \frac{k\,(2\pi r\,dr)\,\sigma}{\sqrt{r^2+x^2}}$$

$$V_p = K\sigma\pi \int \frac{2r\,dr}{\sqrt{r^2+x^2}}$$

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$$V_p = \pi \sigma K \int_0^R \frac{2r\,dr}{(r^2+x^2)^{\frac{1}{2}}}$$

$$u = r^2 + x^2$$
$$du = 2r\,dr$$
$$r = 0 \rightarrow u = x^2$$
$$r = R \rightarrow u = R^2 + x^2$$

$$V_p = \pi \sigma K \int_{x^2}^{x^2+R^2} \frac{du}{u^{\frac{1}{2}}}$$

$$V_p = 2\pi \sigma K \sqrt{u} \Big]_{x^2}^{x^2+R^2}$$

$$V_p = 2\pi \sigma K \left(\sqrt{x^2+R^2} - x\right)$$

$$E_x = -\frac{dv}{dx}$$

$$E = -2\pi \sigma K \left(\frac{1}{\cancel{2}} (x^2+R^2)^{-\frac{1}{2}} (\cancel{2}x) - 1\right)$$

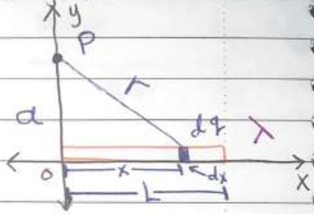
$$E = -2\pi \sigma K \left(\frac{x}{\sqrt{x^2+R^2}} - 1\right)$$

$$E = 2\pi \sigma K \left(1 - \frac{x}{\sqrt{x^2+R^2}}\right)$$

$$\frac{Q}{A} = \frac{Q}{\pi R^2}$$

# Electric potential due to a finite line of charge

Ex 25.7: A rod of length L located along the x axis has a total charge Q and a uniform linear charge density λ. Find the electric potential at point P located on the y axis distance a from the origin.




$$dV_p = \frac{K\,dq}{r} \quad , \quad r = \sqrt{a^2+x^2}$$

$$dV_p = \frac{K\,dq}{\sqrt{a^2+x^2}} \quad , \quad dq = \lambda\,dx$$

$$\int dV_p = \int \frac{K\lambda\,dx}{\sqrt{a^2+x^2}} \Rightarrow V_p = K\lambda \int_0^L \frac{dx}{\sqrt{a^2+x^2}}$$

$$\Rightarrow \int \frac{dx}{\sqrt{a^2+x^2}} = \ln\left(x+\sqrt{x^2+a^2}\right)$$

Let $x = a\tan\theta$

$dx = a\sec^2\theta$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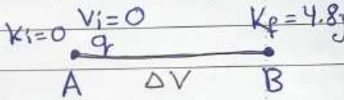
$$V_p = K\lambda \ln\left(x+\sqrt{x^2+a^2}\right)\Big]_0^L$$

$$V_p = K\lambda \cdot \ln\left(L+\sqrt{L^2+a^2} - a\right)$$

Example: A charged particle ($q = -8\,mC$), which moves in a region where the only force acting on the particle is an electric force, is released from rest at a point A. at point B the Kinetic energy of the Particle is equal to 48J. what is the electric potential difference?




$\Delta U = q\Delta V$ , $\Delta V = V_B - V_A$

$$V_B - V_A = \frac{\Delta U}{q}$$

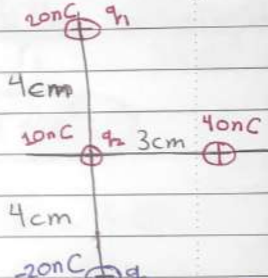
$$\Delta K + \Delta U = 0$$

$$K_f - \cancel{K_i}^{\,0} + \Delta U = 0 \Rightarrow \Delta U = -K_f$$

$$\boxed{\Delta U = -4.8}$$

$$\Delta V = \frac{-4.8}{-8\times10^{-3}} = 600\,V$$

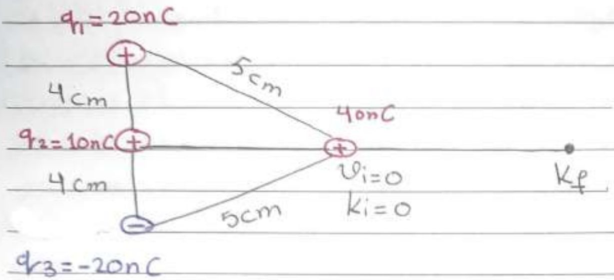
Ex 17: Two particles, with charges of 20nC and -20nC, are placed at the points with cordinates (0, 4cm) and (0, -4cm). A particle with charge 10nC is located at the origin. (a) find the electric potential energy of the configuration of the three fixed charges. (b) A fourth particle with a mass of $2\times10^{-13}$ Kg and a charge of 40nC is released from rest at the point (3cm, 0). Find its speed after it has moved freely to a very large distance away




a) $U = K\left(\frac{q_1 q_2}{r_{12}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_3}{r_{13}}\right)$

$U = 9\times10^{9}\left(\frac{20\times10^{-9}\times10\times10^{-9}}{0.04} - \frac{10\times10^{-9}\times20\times10^{-9}}{0.04} - \frac{20\times10^{-9}\times20\times10^{-9}}{0.08}\right)$

$U = -4.5\times10^{-5}$ J




$$\Delta K + \Delta U = 0$$

$$K_f - \cancel{K_i}^{0} = -\Delta U$$

$$K_f = \frac{1}{2} m v_f^2 = -\Delta U$$

$$\frac{1}{2} m v_f^2 = -\Delta U$$

$$\Delta U = U_f - U_i = \cancel{U_\infty}^{0} - U_i$$

$$-\Delta U = U_i$$

$$\frac{1}{2} m v_f^2 = q \Delta V$$

$$\Delta V = \cancel{\frac{K q_1}{r_1}} + \frac{K q_2}{r_2} + \cancel{\frac{K q_3}{r_3}}$$

$$\Delta V = \frac{K q_2}{r_2} = \frac{9 \times 10^{9} \times 10 \times 10^{-9}}{3 \times 10^{-2}}$$

$$\Delta V = 3 \times 10^{3} V$$

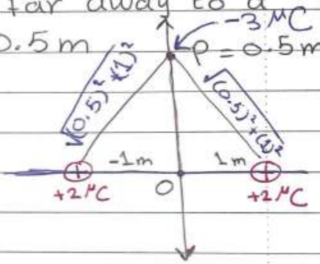
$$\frac{1}{2} m v_f^2 = q \Delta V$$

$$\cancel{\frac{1}{2}} (\cancel{2} \times 10^{-13}) v_f^2 = 40 \times 10^{-9} (3 \times 10^{3})$$

$$v_f^2 = \frac{40 \times 10^{-9} \times 3 \times 10^{3}}{10^{-13}}$$

$$v_f = 3.46 \times 10^{4} m/s$$

Ex 25: Two particles each with charge $+2\mu C$ are Located on the x axis. Once is at $x = 1m$, and the other is at $x = -1m$. (a) Determine the electric potential on the y axis at $y = 0.5m$ (b) Calculate the change in electric potential energy of $-3\mu C$ is brought from infinitely far away to a position on the y axis at $y = 0.5m$




$$V_p = \frac{Kq_1}{r_1} + \frac{Kq_2}{r_2}$$

$$V_p = \frac{2Kq}{r} = \frac{2\times10^9 \times 2\times10^{-6}}{\sqrt{(0.5)^2+(1)^2}}$$

$$V_p = 32.2\times10^3 V$$

$$\Delta U = q\Delta V = -3\times10^{-6}(32.2\times10^3)$$

$$\Delta U = -0.1 J$$

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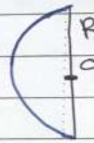
Ex 44: A uniformly charged insulating rod of length 14cm is bent into the shape of a semicircle. The rod has a total charge of $-7.5\mu C$. Find the electric potential at O, the center of the semicircle.

$$\int dV_o = \int \frac{K\,dq}{R}$$




$$V_o = \frac{K}{R}\int_0^Q dq = \frac{KQ}{R}$$

$L = R\theta$

$L = R\pi$

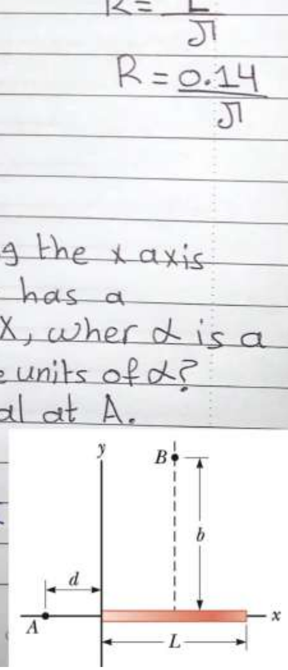
$R = \frac{L}{\pi}$

$R = \frac{0.14}{\pi}$

$$V_o = \frac{9\times10^9 \times (-7.5\times10^{-6})}{\frac{0.14}{\pi}}$$

$$V_o = -1.51\times10^6\,V$$

Ex 45: A rod of length L Lies along the x axis with its left end at the origin. it has a nonuniform charge density $\lambda = \alpha x$, wher $\alpha$ is a positive constant (a) what are the units of $\alpha$? (b) Calculate the electric potential at A.

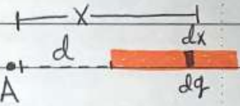



a) $[\lambda] = [\alpha][x]$, $[\alpha] = \frac{[Q]}{[L]^2}$

$\alpha = C/m^2$

b) $dV_A = \frac{K\,dq}{x}$

$dq = \lambda\,dx$
$\lambda = \alpha x$
$dq = \alpha x\,dx$




$\int dV_A = \int \frac{K\alpha \cancel{x}\,dx}{\cancel{x}}$

$V_A = K\alpha \int_d^{d+L} dx = K\alpha x\Big]_d^{d+L}$

$V_A = K\alpha(d+L-d) \Rightarrow \boxed{V_A = K\alpha L}$

$\Rightarrow$ If $\lambda$ is constant then $\lambda \neq \alpha x$

$dV_P = \frac{K\,dq}{x}$ $\quad dq = \lambda\,dx$

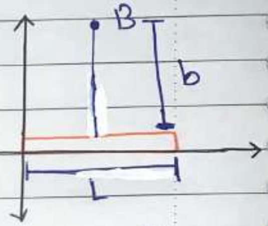
$dV_P = \frac{K\lambda\,dx}{x} \Rightarrow V_P = K\lambda \int_d^{d+L} \frac{dx}{x}$

$V_P = K\lambda \ln x\Big]_d^{d+L}$

$V_P = K\lambda\left(\ln(d+L) - \ln(d)\right)$

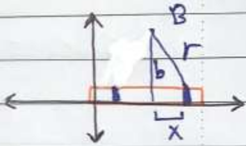
$\boxed{V_P = K\lambda \ln\left(\frac{d+L}{d}\right)}$

Ex 46: For the arrangement described in problem 45, calculate the electric potential at point B, which lies on the prependicular bisector of the rod a distance b above the x axis.




$$dV_B = \frac{K\,dq}{r} = \frac{K\,dq}{\sqrt{b^2+x^2}}$$

$$dV_B = \frac{K\lambda\,dx}{\sqrt{b^2+x^2}} \qquad \boxed{dq = \lambda\,dx}$$




$$\int dV_B = \int \frac{K\alpha\,x\,dx}{\sqrt{b^2+x^2}} \qquad \boxed{\lambda = \alpha x}$$

$$V_B = 2K\lambda \int_0^{L/2} \frac{x\,dx}{\sqrt{b^2+x^2}} = K\alpha \int_0^{L/2} \frac{2x\,dx}{\sqrt{b^2+x^2}}$$

$$V_B = K\alpha \int_{b^2}^{b^2+L^2/4} \frac{1}{\sqrt{u}}\,du$$

Let $u = b^2+x^2$
$du = 2x\,dx$
$x = 0 \rightarrow u = b^2$
$x = \frac{L}{2} \rightarrow u = b^2 + \frac{L^2}{4}$

$$V_B = 2K\alpha\sqrt{u}\,\Big]_{b^2}^{b^2+L^2/4}$$

$$V_B = 2K\alpha\left(\sqrt{b^2+\frac{L^2}{4}} - b\right)$$

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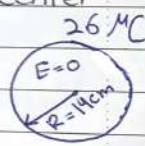
$\Rightarrow$ If $\lambda$ is constant $\lambda \neq \alpha x$

$$V_B = 2K\lambda \int_0^{L/2} \frac{dx}{\sqrt{b^2+x^2}}$$

$$V_B = 2K\lambda \ln\left(x+\sqrt{b^2+x^2}\right)\Big]_0^{L/2}$$

$$V_B = 2K\lambda \ln\left(\frac{L}{2}+\sqrt{b^2+\frac{L^2}{4}}-b\right)$$

Ex 50: A spherical conductor has a radius of 14 cm and a charge of 26 $\mu C$. Calculate the electric field and the electric potential at (a) $r = 10$ cm (b) $r = 20$ cm, and (c) $r = 14$ cm from the center




(a) $r = 10$ cm (inside conductor):

$$E = 0, \quad V = \frac{KQ}{R} = \frac{9\times10^{9}\times26\times10^{-6}}{0.14}$$

$$V = 1.67\times10^{6} \text{ volt}$$

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(b) $r = 20\,cm$:

$$E = \frac{KQ}{r^2} = \frac{9\times10^{9}\times26\times10^{-6}}{(0.2)^2} = 5.85\times10^{6}\ N/C$$

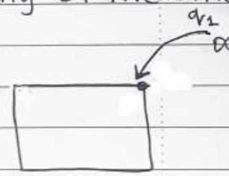
$$V = \frac{KQ}{r} = \frac{9\times10^{9}\times26\times10^{-6}}{0.2} = 1.17\times10^{6}\ Volt$$

(c) $r = 14\,cm$:

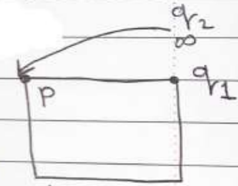
$$E = \frac{KQ}{r^2} = \frac{9\times10^{9}\times26\times10^{-6}}{(0.14)^2} = 1.19\times10^{6}\ N/C$$

$$V = \frac{KQ}{r} = \frac{9\times10^{9}\times26\times10^{-6}}{0.14} = 1.67\times10^{6}\ volt$$

Example: Four identical point charges ($+4\mu C$) are placed at the corners of a square which has 20 cm sides. How much work is required to assemble this charge arrangement starting with each of the charges a very large distance from any of the other charges?




① $q_1 \rightarrow W = 0 \ , \ \Delta U = 0$

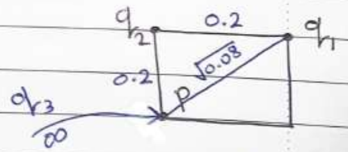



② $q_2$:

$$V_p = \frac{kq}{r} = \frac{9\times10^9 \times 4\times10^{-6}}{0.2} = 18\times10^4 \text{ volt}$$

$$\Delta U = W = q\Delta V = 4\times10^{-6} \times 18\times10^4 = 0.72 \text{ J}$$

③ $q_3$:

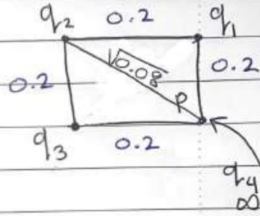



$$V_p = \frac{kq_1}{r_1} + \frac{kq_2}{r_2}$$

$$V_p = 9\times10^9 \left( \frac{4\times10^{-6}}{0.2} + \frac{4\times10^{-6}}{\sqrt{0.08}} \right) = 307.3\times10^3 \text{ volt}$$

$$\Delta U = W = q_3 \Delta V = 4\times10^{-6} \times 307.3\times10^{3}$$

$$\Delta U = 1.23\ J$$




(4) $q_4$:

$$V_p = K\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3}\right)$$

$$V_p = 9\times10^{9}\left(\frac{4\times10^{-6}}{0.2} + \frac{4\times10^{-6}}{\sqrt{0.08}} + \frac{4\times10^{-6}}{0.2}\right)$$

$$V_p = 4.87\times10^{5}\ \text{volt}$$

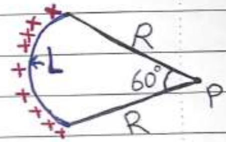
$$W = \Delta U = q\Delta V = 4\times10^{-6}\times 4.87\times10^{5}$$

$$W = 1.95\ J$$

$$W_{total} = 0 + 0.72 + 1.23 + 1.95$$

$$W_{total} = 3.9\ J$$

Example: Charge of uniform density ($\lambda$ = 3.5 nC/m) is distributed along the circular arc. Determine the electric potential (relative to zero at infinity) at point p.




$$dV_p = \frac{K\,dq}{R}$$

$$\int dV_p = \int \frac{K\,dq}{R} = \frac{K}{R}\int_0^Q dq = \frac{KQ}{R}$$

$$V_p = \frac{KQ\pi}{3L}$$

$$\boxed{\lambda = \frac{Q}{L}}$$

$L = R\theta$

$\theta = 60^\circ = \frac{\pi}{3}$

$L = R\left(\frac{\pi}{3}\right)$

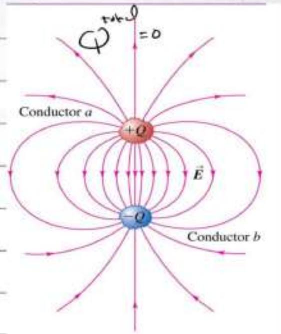
$R = \frac{3L}{\pi}$

$$V_p = \frac{\pi K \lambda}{3} = \frac{\pi(9\times10^{9})(3.5\times10^{-9})}{3}$$

$$\boxed{V_p = 33\ \text{volt}}$$

# Chapter 26: Capacitance and Dielectrics:

* Any two conductors seperated by an insulator (or a vacuum) form a Capacitor.




* when the capacitor is charged, it means the two conductors have charges with equal magnitude and opposite Sign, and the net charge on the Capacitor as a whole is zero.

* One common way to charge a capacitor is to connect the two conductors to opposite terminals of a battery, this gives a potential difference $V_{ab}$ between the conductors that is equal to the voltage of the battery.

* If we change the magnitude of charge on each Conductor, the potential difference between conductors changes; however, the ration of charge to potential difference doesn't change

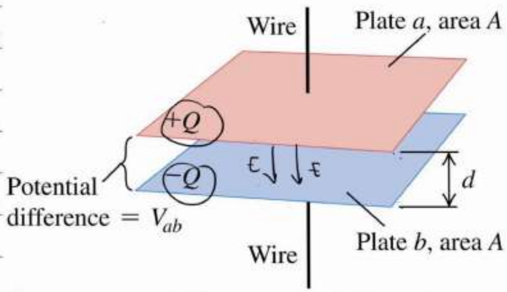
Subject Parallel-plate Capacitor Date No.

Capacitance $\rightarrow C = \dfrac{Q}{V_{ab}}$ — $Q$: magnitude of charge on each conductor; $V_{ab}$: potential difference between conductors

❀ A Parallel-plate Capacitor consists of two parallel conducting plates seperated by a distance that is small compared to their dimensions.

$$E = \frac{\sigma}{\epsilon_0} = \frac{Q}{\epsilon_0 A}$$

$$\Delta V = Ed = \frac{Qd}{\epsilon_0 A}$$




❀ The field between the plates of a parallel-plate capacitor is essentially uniform, and the charges on the plates are uniformly distributed over their opposing surface

❀ When the region between the plates is empty, the capacitance is:

Capacitance of a parallel plate capacitor in Vaccum $\rightarrow$

$$C = \frac{Q}{V_{ab}} = \epsilon_0 \frac{A}{d}$$

$\epsilon_0$: electric constant; $A$: area of each plate; $d$: Distance between plates

 NOTEBOOK

- The capacitance depends on only the geometry of the capacitor

- The quantities A and d are constants for a given capacitor, and $\epsilon_0$ is a universal constant.

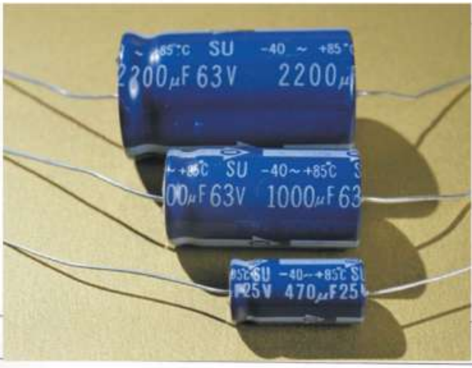
## Units of Capacitance:

- The SI unit of capacitance is the farad, F

$$1F = 1C/V = 1C^2/N \cdot m = 1C^2/J$$

- One farad is a very large capacitance

$$1\mu\mu F = 1\times10^{-12} F = PF$$





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# Geometry of Capacitors:



## ① Cylindrical capacitors:

A solid cylindrical conductor of radius a and charge Q is Coaxial with a cylindrical shell of negligible thickness, radius b, and charge $-Q$. Find the Capacitance of this cylindrical capacitor if its length is L?


$E=0$

$$V_b - V_a = -\int_a^b \vec{E}\cdot d\vec{s}\,\cos(0)$$

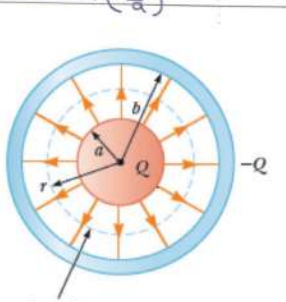
$$V_b - V_a = -\int_a^b \vec{E_r}\,d\vec{r} = -2K\lambda\int_a^b \frac{dr}{r} = \cancel{-2K\lambda \ln}$$

$$V_b - V_a = -2K\lambda \ln\left(\frac{b}{a}\right)$$

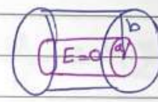
$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{(2K\cancel{Q}/L)\ln(b/a)} = \frac{L}{2K\ln\left(\frac{b}{a}\right)}$$

$$\Rightarrow \frac{C}{L} = \frac{1}{2K\ln\left(\frac{b}{a}\right)}$$

capacitance per unit length



$$E(2\pi r L) = \frac{Q_{en}}{\epsilon_0}$$




$$E(2\pi r \cancel{L}) = \frac{\lambda \cancel{L}}{\epsilon_0}$$

$$E = \frac{2\lambda}{4\pi\epsilon_0 r} = \frac{2K\lambda}{r}$$

## ② Spherical capacitors:

A spherical capacitor consists of a spherical conducting shell of radius b and charge $-Q$ concentric with a smaller conducting sphere of radius a and charge Q. Find the capacitance of this device.

$$\Delta V = V_b - V_a = -\int_a^b \vec{E} \cdot d\vec{s} \cos(0)$$

$$V_b - V_a = -\int_a^b E_r dr = -KQ\int_a^b \frac{dr}{r^2} = KQ\left(\frac{1}{r}\right)\Big]_a^b$$

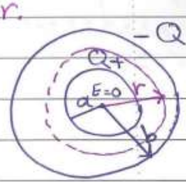
$$C = \frac{Q}{|\Delta V|} = \frac{Q}{|V_b - V_a|} = \frac{ab}{K(b-a)}$$

$$V_b - V_a = KQ\left(\frac{1}{b} - \frac{1}{a}\right) = KQ\frac{(a-b)}{ab}$$

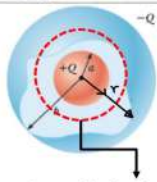
$$C = \lim_{b \to \infty} \frac{ab}{K(b-a)} = \frac{ab}{K(b)} = \frac{a}{K} = 4\pi\epsilon_0 a$$

The capacitance of an isolated spherical conductor.

$$E(4\pi r^2) = \frac{Q_{enc}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$




$$\boxed{E = \frac{KQ}{r^2}}$$



Gauss's Surface

## Capacitors in series:

- Capacitors are in Series if they are connected one after the other, as illustrated

- Capacitors in series:

1) have the same Charge Q
2) Their potential difference add:

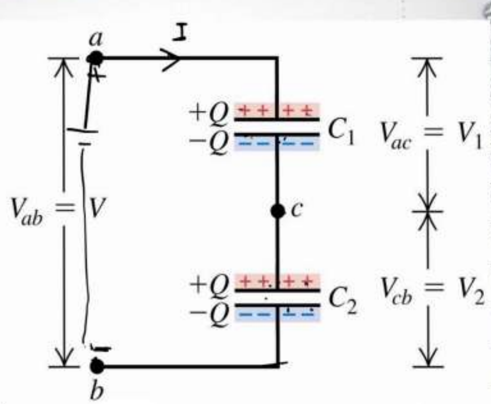
$$V_{ac} + V_{cb} = V_{ab}$$

Subject: Capacitors in series  Date  No.

$V_{ab} = V_1 + V_2$

$\frac{Q_{tot}}{C_{eq}} = \frac{Q_1}{C_1} + \frac{Q_2}{C_2}$

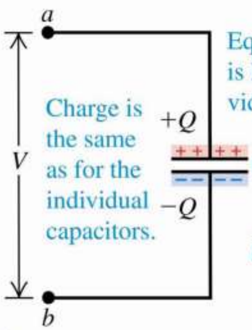
$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$




✲ Equivalent capacitance is less than the individual Capacitances:

$C_{eq} = \frac{Q_{tot}}{V_{ab}}$

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$




Equivalent capacitance is less than the individual capacitances:

$C_{eq} = \frac{Q_{total}}{V_{ab}}$

$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$

✲ When several capacitors are connected in Series, the magnitude of charge is the same on all plates of all the capacitors



NOTEBOOK

- The potential differences of the individual capacitors add to give the total potential difference across the series combination:

$$V_{total} = V_1 + V_2 + V_3 + \cdots$$

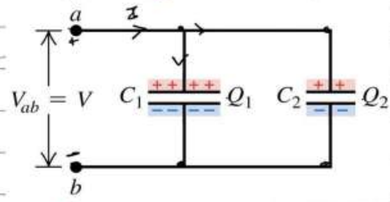
## Capacitors in Parallel:

- Capacitors are connected in parallel between a and b if the potential difference $V_{ab}$ is the same for all capacitors

Capacitors in parallel:

1) have the same potential $V$
2) The charge on each capacitor depends on its Capacitance : $Q_1 = C_1 V$ , $Q_2 = C_2 V$

$$Q_{total} = Q_1 + Q_2 + Q_3 + \cdots$$

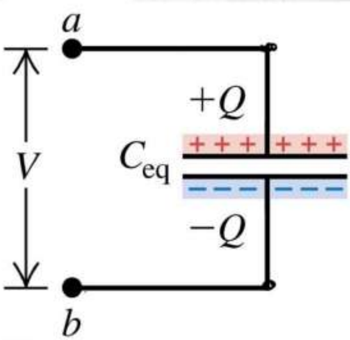



$$V_{ab} = V_1 = V_2 = V_3 = \cdots$$

$$C_{eq}\cancel{V_{eq}} = C_1\cancel{V_1} + C_2\cancel{V_2} + C_3\cancel{V_3} + \cdots$$

$$C_{eq} = C_1 + C_2 + C_3 + \cdots$$

Subject Capacitors in parallel Date No.




- Charge is the sum of the individual charges:

$Q_{total} = Q_1 + Q_2$

- Equivalent capacitance:

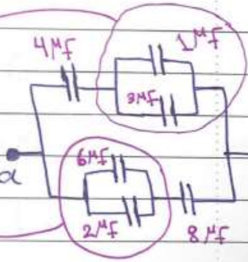
$C_{eq} = C_1 + C_2$

- when several capacitors are connected in parallel, the potential differences are the same for all the capacitors.

Ex 26.3: find the equivalent capacitance between a and b for the combination of capacitors shown. All capacitances are in microfarad

$C_{eq} = C_1 + C_2$
$C_{eq} = 1\mu F + 3\mu F = 4\mu F$

$C_{eq} = C_1 + C_2$
$C_{eq} = 6\mu F + 2\mu F = 8\mu F$






N O T E B O O K

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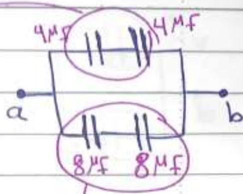
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$



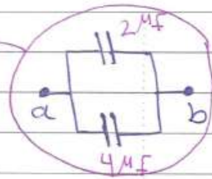

$$\frac{1}{C_{eq}} = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}$$

$$C_{eq} = 2\,\mu f$$

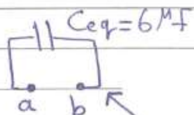
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

$$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{8} = \frac{2}{8} \Rightarrow C_{eq} = 4\,\mu f$$

$$C_{eq} = C_1 + C_2 = 2\mu f + 4\mu f$$




$$C_{eq} = 6\,\mu f \Longrightarrow$$




Let $V_{ab} = 12\,V$

$$Q_{total} = CV = 6\mu f \times 12 = 72\,\mu C$$

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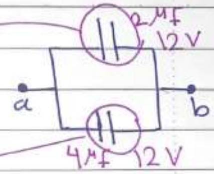
$Q = CV = 2\mu F \times 12$

$Q = 24\mu C$

$Q = CV = 4\mu F \times 12$

$Q = 48\mu C$

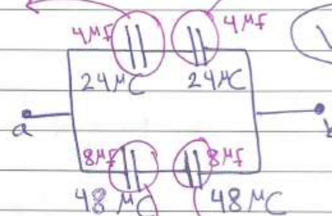



$V = \frac{Q}{C} = \frac{24}{4}$

$V = 6V$

$V = \frac{Q}{C} = \frac{24}{4}$

$V = 6V$




$V = \frac{Q}{C} = \frac{48}{8}$

$V = 6V$

$V = \frac{Q}{C} = \frac{48}{8}$

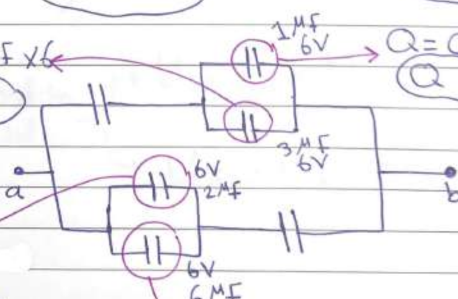
$V = 6V$

$Q = CV = 3\mu F \times 6$

$Q = 18\mu C$

$Q = CV = 1\mu F \times 6$

$Q = 6\mu C$




$Q = CV$

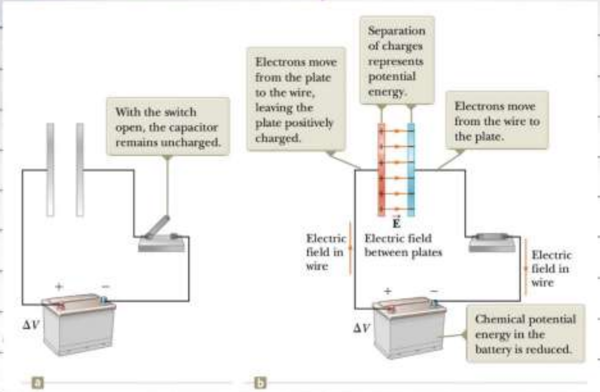
$Q = 2\mu F \times 6$

$Q = 12\mu C$

$Q = CV = 6\mu F \times 6$

$Q = 36\mu C$

Subject Energy stored in Capacitor Date No.




$$du = dw = \Delta V\, dq = \frac{q}{C}\, dq \quad , \quad \boxed{V = \frac{q}{C}}$$

$$U = W = \int_0^Q \frac{q}{C}\, dq = \frac{1}{C}\int_0^Q q\, dq = \frac{Q^2}{2C}$$

$$U_E = \frac{Q^2}{2C} = \frac{1}{2} Q \Delta V = \frac{1}{2} C \Delta V^2$$

Energy density stored in electric field

$$U_E = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(E d)^2 = \frac{1}{2}(\epsilon_0 A d) E^2$$

$$U_E = \frac{1}{2} \epsilon_0 E^2$$

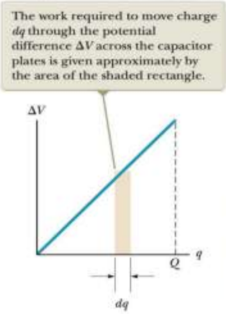



**Figure 26.11** A plot of potential difference versus charge for a capacitor is a straight line having slope $1/C$.

Subject Date No.

$$W = U = \frac{1}{2} C \Delta V^2 = \frac{1}{2}\left(\frac{\epsilon_0 A}{d}\right)(Ed)^2$$

$$U = \frac{1}{2}\frac{\epsilon_0 A E^2 d^{\cancel{2}}}{\cancel{d}} = \frac{1}{2}\epsilon_0 \underbrace{(Ad)}_{\text{Volume}} E^2$$

$$\frac{U}{V} = \frac{\frac{1}{2}\epsilon_0 \cancel{V} E^2}{\cancel{V}}$$

$$u = \frac{U}{V} = \frac{1}{2}\epsilon_0 E^2$$

energy density
stored in electric field $(J/m^3)$

Ex 26.4: Rewiring two charged Capacitors:

Two capacitors $C_1$ and $C_2$ (where $C_1 > C_2$) are charged to the same initial potential difference $\Delta V_i$. The charged capacitors are removed from the battery, and their plates are connected with opposite polarity. The switches $S_1$ and $S_2$ are then closed
(A) Find the potential difference $\Delta V_f$ between a and b after the switches are closed
(B) Find the total energy stored in the capacitors before and after the switches are closed and determine the ratio of the final energy to the initial energy

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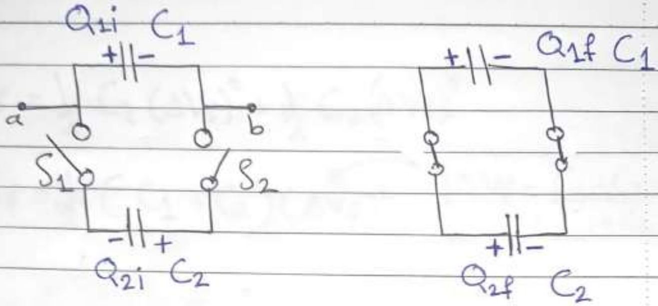


① $Q_i = Q_{1i} + Q_{2i} = C_1 \Delta V_i - C_2 \Delta V_i$

$$\boxed{Q_i = (C_1 - C_2)\Delta V_i}$$

② $Q_f = Q_{1f} + Q_{2f} = C_1 \Delta V_f + C_2 \Delta V_f$

$$\boxed{Q_f = (C_1 + C_2)\Delta V_f}$$

$Q_f = Q_i \Longrightarrow (C_1 + C_2)\Delta V_f = (C - C_2)\Delta V_i$

③ $$\Delta V_f = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)\Delta V_i$$

④ $U_i = \frac{1}{2} C_1 (\Delta V_i)^2 + \frac{1}{2} C_2 (\Delta V_i)^2$

$U_i = \frac{1}{2}(C_1 + C_2)(\Delta V_i)^2$

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$$U_f = \frac{1}{2} C_1 (\Delta V_f)^2 + \frac{1}{2} C_2 (\Delta V_f)^2$$

$$U_f = \frac{1}{2} (C_1 + C_2)(\Delta V_f)^2 \qquad \boxed{\Delta V_f = \frac{C_1 - C_2}{C_1 + C_2} \Delta V_i}$$

$$U_f = \frac{1}{2} (C_1 + C_2)\left(\left(\frac{C_1 - C_2}{C_1 + C_2}\right) \Delta V_i\right)^2$$

$$U_f = \frac{1}{2} \frac{(C_1 - C_2)^2 (\Delta V_i)^2}{(C_1 + C_2)}$$

$$\frac{U_f}{U_i} = \frac{\cancel{\frac{1}{2}} (C_1 - C_2)^2 \cancel{(\Delta V_i)^2} / (C_1 + C_2)}{\cancel{\frac{1}{2}} (C_1 + C_2) \cancel{(\Delta V_i)^2}}$$

☆ $$\frac{U_f}{U_i} = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2$$

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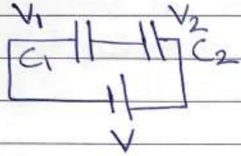
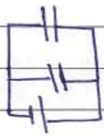
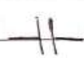
$$C = \frac{\epsilon_0 A}{d}, \quad C = \frac{Q}{V}$$




$$\left[\begin{aligned} V &= V_1 + V_2 \\ Q &= Q_1 = Q_2 \\ \frac{1}{C_{eq}} &= \frac{1}{C_1} + \frac{1}{C_2} \end{aligned}\right]$$



$$\left[\begin{aligned} V &= V_1 = V_2 \\ Q &= Q_1 + Q_2 \\ C_{eq} &= C_1 + C_2 \end{aligned}\right]$$



$$C = \frac{Q}{V}, \quad U = \frac{1}{2} C V^2$$

$$\text{energy density} = \frac{U}{V} = \frac{1}{2} \epsilon_0 E^2$$

Ex 4: An air-filled spherical capacitor is constructed with inner and outer-shell radii of 7cm and 14cm, respectively
(a) Calculate the capacitance of the device
(b) What potential difference between the spheres results in a $4\mu C$ charge on the capacitor?

$$C = \frac{ab}{K(b-a)}$$


$$C = \frac{7\times10^{-2}\times14\times10^{-2}}{9\times10^{9}(14\times10^{-2}-7\times10^{-2})} = 1.56\times10^{-11}\ F$$

$$V = \frac{Q}{C} = \frac{4\times10^{-6}}{1.56\times10^{-11}} = 256.4\times10^{3}\ \text{Volt}$$

Ex 7: When a potential difference of 150 V is applied to the plates of a parallel-plate capacitor the plates carry a surface charge density of $30nC/cm^2$. What is the spacing between the plates?

$$\sigma = \frac{30nC}{cm^2} = \frac{30nC}{cm^2}\times\frac{10^{4}cm^2}{1m^2}\times\frac{10^{-9}C}{1nC}$$

$$\sigma = 30\times10^{-5}\ C/m^2$$



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Subject Date No.

$$C = \frac{\epsilon_0 A}{d} = \frac{Q}{V}$$

$$\frac{\epsilon_0 V}{d} = \frac{Q}{A} \quad , \quad \sigma = \frac{Q}{A}$$

$$\sigma = \frac{\epsilon_0 V}{d} \rightarrow d = \frac{\epsilon_0 V}{\sigma} = \frac{8.85 \times 10^{-12} \times 150}{30 \times 10^{-5}}$$

$$\boxed{d = 4.425 \times 10^{-6}\, m}$$

Ex 4: An air filled capacitor consists of two parallel plates, each with an area of $7.6\, cm^2$, seperated by a distance of 1.8 mm. A 20 V potential difference is applied to these plates Calculate (a) the electric field between the plates (b) the surface charge density, (c) the capacitance and (d) the charge on each plate

a) $E = \frac{V}{d} = \frac{20}{1.8 \times 10^{-3}} = 1.11 \times 10^{4}\, V/m$

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b) $C = \frac{\epsilon_0 A}{d} = \frac{Q}{V} \Rightarrow \frac{\epsilon_0 V}{d} = \frac{Q}{A} = \sigma$

$\sigma = \frac{\epsilon_0 V}{d} = \frac{8.85 \times 10^{-12} \times 20}{1.8 \times 10^{-3}} = 9.83 \times 10^{-8}\ C/m^2$

c) $C = \frac{\epsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 7.6 \times 10^{-4}}{1.8 \times 10^{-3}}$

$C = 3.74 \times 10^{-12}\ F$

d) $Q = \sigma A = \quad 10^{-8} \times 7.6 \times 10^{-4}$

$Q = 74.7 \times 10^{-12}\ C$

Ex13: Two capacitors, $C_1 = 5\mu F$ and $C_2 = 12\ \mu F$ are connected in parallel, and the resulting combination is connected to a 9V battery. Find
(a) the equivalent capacitance of the combination
(b) the potential difference across each capacitor
(c) the charge stored on each capacitor.

$C_1 = 5\mu F$

$C_2 = 12\mu F$

9V

a) $C_{eq} = C_1 + C_2 = 5 \times 10^{-6} + 12 \times 10^{-6}$
$C_{eq} = 17 \times 10^{-6}\ F$

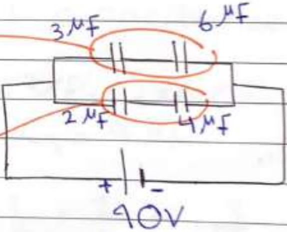
b) $V_{battery} = V_1 = V_2 = 9V$

c) $Q_1 = C_1 V_1 = 5 \times 10^{-6} \times 9 = 45 \mu C$

$Q_2 = C_2 V_2 = 12 \times 10^{-6} \times 9 = 108 \mu C$

$Q_{total} = Q_1 + Q_2 = 45 \mu C + 108 \mu C = 153 \mu C$

* Find equivalent capacitance

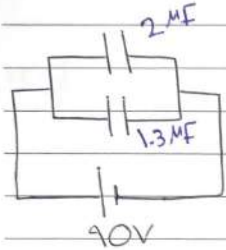

$$\frac{1}{C_{eq}} = \frac{1}{3} + \frac{1}{6} = \frac{3}{6}$$

$C_{eq} = 2 \mu F$

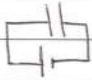



$$\frac{1}{C_{eq}} = \frac{1}{2} + \frac{1}{4} = \frac{3}{4}$$

$$C_{eq} = \frac{4}{3} \mu F = 1.33 \mu F$$



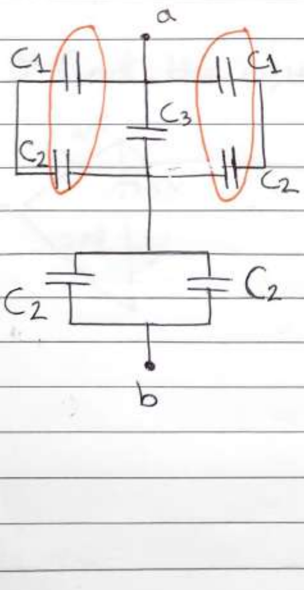

$C_{eq} = C_1 + C_2 = 3.3\,\mu F$



$Q_{total} = C_{eq} V = 3.3 \times 10^{-6} \times 90$

$Q_{total} = 2.97 \times 10^{-4}\ C$

★ Find the equivalent capacitance:

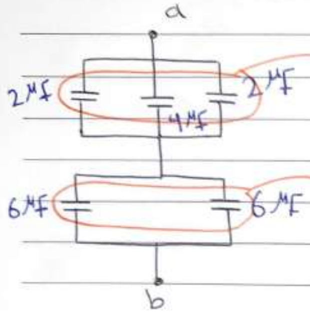



$C_1 = 3\mu F,\ C_2 = 6\,\mu F$

$C_3 = 4\,\mu F$

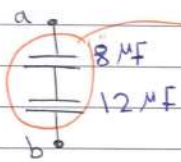
$\frac{1}{C_{eq}} = \frac{1}{3} + \frac{1}{6} = \frac{3}{6}$

$C_{eq} = 2\,\mu F$




$$C_{eq} = 2+2+4 = 8\,\mu F$$

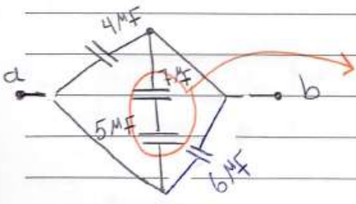
$$C_{eq} = 6+6 = 12\,\mu F$$




$$\frac{1}{C_{eq}} = \frac{1}{8} + \frac{1}{12} = \frac{5}{24}$$

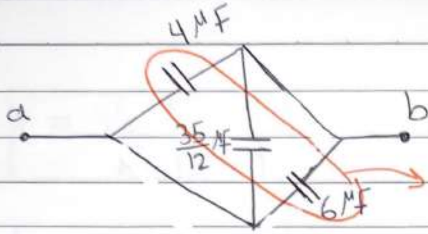
$$C_{eq} = \frac{24}{5}\,\mu F$$

☆ Find the equivalent capacitance




$$\frac{1}{C_{eq}} = \frac{1}{7} + \frac{1}{5} = \frac{12}{35}$$

$$C_{eq} = \frac{35}{12}\,\mu F$$

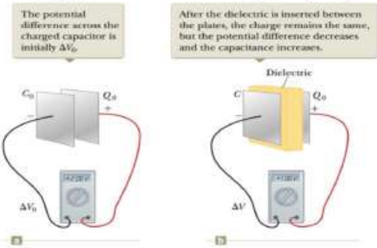



$$C_{eq} = 6 + 4 + \frac{35}{12}$$

$$C_{eq} = 12.9\,\mu F$$

## Capacitors with Dielectrics:

A dielectric is a nonconducting material such as rubber, glass, or waxed paper.




$$\boxed{\Delta V = \frac{\Delta V_0}{K}}$$

$$C = \frac{Q_0}{\Delta V} = \frac{Q_0}{\Delta V_0 / K} = \frac{K Q_0}{\Delta V_0}$$

$$\boxed{C = K C_0}$$

- Charge does not change
- K is called the dielectric constant
- The capacitance increases by K if the dielectric fills the capacitor $C = \frac{K \epsilon_0 A}{d}$

Subject Capacitors with Dielectrics Date No.

$$\boxed{V = \frac{V_0}{K}}, \quad \boxed{C = KC_0} = \frac{K\epsilon_0 A}{d}$$

$$K\epsilon_0 = \epsilon \rightarrow \boxed{C = \frac{\epsilon A}{d}}$$

$$U = \frac{1}{2}CV^2 = \frac{1}{2}(KC_0)\left(\frac{V_0}{K}\right)^2$$

$$U = \frac{1}{2}\cancel{K}C_0\frac{V_0^2}{K^{\cancel{2}}} \Longrightarrow U = \frac{1}{2}\frac{C_0V_0^2}{K}$$

$$\boxed{U = \frac{U_0}{K}}$$

☆ A parallel-plate capacitor is charged with a battery to a charge $Q_0$. The battery is then removed, and a slab of material that has a dielectric constant $K$ is inserted between the plates. Identify the system as the capacitor and the dielectric. Find the energy stored in the system before and after the dielectric is inserted.

$$U = \frac{Q_0^2}{2C_0}, \quad U = \frac{Q_0^2}{2C} = \frac{Q_0^2}{2KC_0} = \frac{U_0}{K}$$

before after

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Example: Determine the capacitance and maximum potential difference that can be applied to a teflon-filled parallel plate capacitor of plate $E_{max} = 6 \times 10^{7}$ V/m, with area $1.75\ cm^2$ and a plate seperation of 0.04 mm, $K = 2.1$

$$C = KC_0 = \frac{K\epsilon_0 A}{d} = \frac{2.1 \times 8.85 \times 10^{-12} \times 1.75 \times 10^{-4}}{0.04 \times 10^{-3}}$$

$$C = 81.3 \times 10^{-12} F = 81.3\ pF$$

$$V_{max} = E_{max} d = (6 \times 10^{7})(0.04 \times 10^{-3})$$

$$V_{max} = 2.4 \times 10^{3} V = 2.4\ KV$$

Ex 47: Parallel plate capacitor in air has a plate seperation of 1.5 cm and area of $25\ cm^2$ the plates are charged to 250 V and then disconnected from the source. The capacitor is then immersed in water assume water is insulator, determine: (a) charge on the plates before and after immersion. (b) Capacitance and potential after immersion. (c) Change in energy of the capacitor

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a) The charge before and after is the same:

$$Q = CV = \frac{\epsilon_0 A V}{d} = \frac{8.85\times10^{-12}\times25\times10^{-4}\times250}{1.5\times10^{-2}}$$

$$Q = 364\times10^{-12}\,C = 364\,pC$$

b) $$C_{after} = K_{water}\frac{\epsilon_0 A}{d} = \frac{80\times8.85\times10^{-12}\times25\times10^{-4}}{1.5\times10^{-2}}$$

$$C_{after} = 1.18\times10^{-10}\,F$$

$$V_{after} = \frac{Q}{C_{after}} = \frac{V_i}{K} = \frac{250}{80} = 3.1\,V$$

c) $$U_{before} = \frac{1}{2}C_i(V_i)^2 = \frac{1}{2}\frac{\epsilon_0 A}{d}(V_i)^2$$

$$U_{before} = \frac{1}{2}\times\frac{8.85\times10^{-12}\times25\times10^{-4}\times(250)^2}{1.5\times10^{-2}}$$

$$U_i = 4.61\times10^{-8}\,J$$

$$U_f = \frac{1}{2}C_f(V_f)^2 = \frac{1}{2}KC_i\left(\frac{V_i}{K}\right)^2 = \frac{1}{2}\frac{C_iV_i^2}{K}$$

$$U_f = \frac{1}{2}\frac{\epsilon_0 A}{d}\frac{V_i^2}{K} = \frac{1}{2}\frac{(8.85\times10^{-12}\times25\times10^{-4})(250)^2}{(1.5\times10^{-2})(80)}$$

$$U_f = 5.76\times10^{-10}$$

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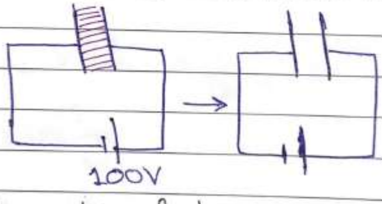
$$\Delta U = U_f - U_i = 5.76 \times 10^{-10} - 4.61 \times 10^{-8}$$

$$\Delta U = -4.55 \times 10^{-8} \, J = -45.5 \, nJ$$

Ex 49: A 2nF parallel-plate capacitor is charged to an initial potential difference $\Delta V_i = 100\,V$ and is then isolated. The dielectric material between the plates is mica, with a dielectric constant of 5. (a) How much work is required to withdraw the mica sheet? (b) What is the potential difference across the capacitor after the mica is withdrawn?




$C = K C_0$

$V = \frac{V_0}{K}$

$U = \frac{U_0}{K}$

* $C = 2nF \leftarrow$ with dielectric
* $V = 100V \leftarrow$ with dielectric

$K = 5$

$$W = \Delta U = U_f - U_i$$

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$U_i = \frac{1}{2} C_i V_i^2 = \frac{1}{2}(2\times10^{-9})(100)^2$

$U_i = 1\times10^{-5}\,J$

$U_f = \frac{1}{2} C_f V_f^2$

$C = K C_o \rightarrow C_o = \frac{C}{K}$

$V = \frac{V_o}{K} \longrightarrow V_o = VK$

$U_f = \frac{1}{2}\left(\frac{C}{K}\right)(VK)^2$

$U_f = \frac{1}{2} C V^2 K = \frac{1}{2}(2\times10^{-9})(100)^2(5)$

$U_f = 5\times10^{-5}\,J$

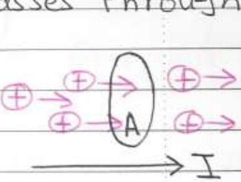
$W = \Delta U = U_f - U_i = 5\times10^{-5} - 1\times10^{-5} = 4\times10^{-5}\,J$

b) $V = \frac{V_o}{K} \Rightarrow V_o = VK = 100\times5 = 500$ volt

# Chapter 27: Current, Resistance and Electromotive

* The **current** is defined as the rate at which charge flows through this surface. If $\Delta Q$ is the amount of charge that passes through this surface in a time interval $\Delta t$, the **average current** $I_{avg}$ is equal to the charge that passes through A per unit time

$$I_{avg} = \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$$




* The SI unit of current is the **ampere (A)**

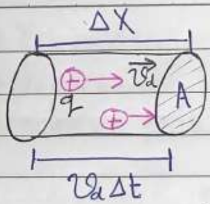
$$1A = 1C/s$$

Ex: The charge that passes through a conductor is given as a function of time as: $Q(t) = 2t - 1$ where Q is the charge in Coloum and t is in second, find the current at $t = 2s$?

$$I = \frac{dQ}{dt} = 4t = 4(2) \Rightarrow \boxed{I = 8A}$$

# Current:






$$\Delta Q = nq$$

$$\Delta Q = (\underbrace{n}_{\text{number charge carrier per volume}} \underbrace{A \Delta X}_{\text{Volume}})\, q$$

$$\Delta Q = (n A v_d \Delta t)\, q$$

$$I_{avg} = \frac{\Delta Q}{\Delta t} = n q v_d A$$

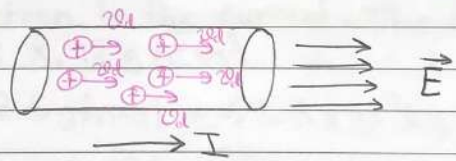
cross-sectional area

Drift speed

current through an area

concentration of moving charged particle

charge per particle

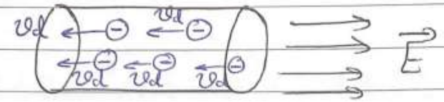



- A current can be produced by positive or negative charge flow

- Conventional current is treated as a flow of positive charges

Subject: Direction of current flow   Date   No.

* In metalic conductor, the moving charges are electrons, but the current still points in the direction positive charges would flow.




$\xrightarrow{I}$ convential current

Ex 27.1: The 12-guage copper wire in typical residential building has a cross-sectional area of $3.31\times10^{-6}\,m^2$. It carries a constant current of 10 A. What is the drift speed of the electrons in the wire? Assume each copper atom contributes one free electron to the current. The density of copper is $8.92\,g/cm^3$.

$M_m\ Cu = 63.5\,g/mol = 63.5\times10^{-3}\,Kg/mol$

$I = 10A,\ N_A = 6.022\times10^{23}$ particle/mole

$q_e = 1.6\times10^{-19},\ A = 3.31\times10^{-6}\,m^2$

$\rho = 8.92\,g/cm^3 = 8920\,Kg/m^3$

$$V = \frac{M}{\rho} = \frac{63.5\times10^{-3}}{8920} = 7.12\times10^{-6}\,m^3$$

N O T E B O O K

$$n = \frac{N_A}{V} = \frac{6.022 \times 10^{+23}}{7.12 \times 10^{-6}} = 8.46 \times 10^{28}$$

$$I = n q v_d A$$

$$10 = 8.46 \times 10^{28} \times 1.6 \times 10^{-19} \times v_d \times (3.31 \times 10^{-6})$$

$$v_d = \frac{10}{8.46 \times 10^{28} \times 1.6 \times 10^{-19} \times 3.31 \times 10^{-6}}$$

$$\boxed{v_d = 2.23 \times 10^{-4}\ m/s}$$

* We can define a vector current density that includes the direction of the drift velocity:

$$\vec{J} = n q \vec{v}_d$$

Drift velocity

vector current density

charge per particle

concentration of moving charged particles

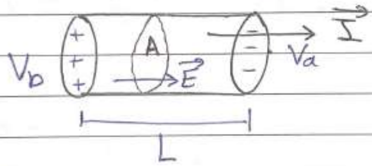
Subject: Current density    Date    No.

- The vector current density is always in the same direction as the electric field, no matter what the signs of the charge carriers are.

$$\boxed{J = \sigma E}$$

$\sigma$ is called the conductivity
$E$ is the electric field

**Ohm's Law:** For many materials (including most metals), the ratio of the current density to the electric field is a constant $\sigma$ that is independent of the electric field producing the current.




$$\Delta V = EL \rightarrow E = \frac{\Delta V}{L}$$

$$J = \sigma E \longrightarrow J = \sigma \frac{\Delta V}{L}$$

$$\Delta V = \frac{L}{\sigma} J = \left(\frac{L}{\sigma A}\right) I = RI$$

$$J = I/A$$

Subject: Ohm's Law  Date  No.

* The quantity $R = L/\sigma A$ is called the resistance of the conductor. we define the resistance as the ratio of the potential difference across a conductor to the current in the conductor

$$\boxed{R = \frac{\Delta V}{I}} \qquad 1\,\Omega = 1\,V/A$$

* The inverse of conductivity is resistivity

$$\boxed{\rho = \frac{1}{\sigma}} \qquad \Omega.m$$

$$R = \frac{\rho L}{A}$$

Ex: 27.2: The radius of 22-guage Nichrome wire is 0.32mm (a) Calculate the resistance per unit length of this wire. (B) If a potential difference of 10V is maintained across a 1m length of the Nichrome wire, what is the current in the wire

$\rho = 1 \times 10^{-6}\,\Omega.m$, $r = 0.32 \times 10^{-3}\,m$

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(a) $A = \pi r^2 = \pi (0.32 \times 10^{-3})^2 = 3.22 \times 10^{-7}\ m^2$

$\frac{R}{L} = \frac{\rho}{A} = \frac{1 \times 10^{-6}}{3.22 \times 10^{-7}} = 3.1\ \Omega$

(b) $I = \frac{\Delta V}{R} = \frac{\Delta V}{(R/L)L} = \frac{10}{3.1 \times 1} = 3.2A$

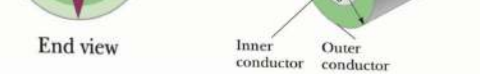
**Example 27.3** **The Radial Resistance of a Coaxial Cable**

Coaxial cables are used extensively for cable television and other electronic applications. A coaxial cable consists of two concentric cylindrical conductors. The region between the conductors is completely filled with polyethylene plastic as shown in Figure 27.8a. Current leakage through the plastic, in the *radial* direction, is unwanted. (The cable is designed to conduct current along its length, but that is *not* the current being considered here.) The radius of the inner conductor is $a = 0.500$ cm, the radius of the outer conductor is $b = 1.75$ cm, and the length is $L = 15.0$ cm. The resistivity of the plastic is $1.0 \times 10^{13}\ \Omega \cdot$ m. Calculate the resistance of the plastic between the two conductors.

$dR = \frac{\rho}{A} dr$

$\int dR = \int \frac{\rho}{2\pi r L} dr$

$R = \frac{\rho}{2\pi L} \int_a^b \frac{1}{r} dr$



Subject Resistivity Date No.

$$R = \frac{\rho}{2\pi L} \ln(r) \Big]_a^b$$

$$R = \frac{\rho}{2\pi L}(\ln b - \ln a) = \frac{\rho}{2\pi L}\ln\left(\frac{b}{a}\right)$$

$$R = \frac{10^{13}}{2\pi(15\times10^{-2})}\ln\left(\frac{1.75}{0.5}\right)$$

$$R = 1.33\times10^{13}\,\Omega$$

★ The resistivity of a material is the ratio of the electric field in the material to the current density it causes:

Resistivity of a material → $\boxed{\rho = \frac{E}{J}}$

$E$ ← magnitude of electric field in material

$J$ ← magnitude of current density caused by electric field

★ The conductivity is the reciprocal of the resistivity

$$I = \frac{\Delta Q}{\Delta t} = n q v_d A$$

$$J = \frac{I}{A} = n q v_d$$

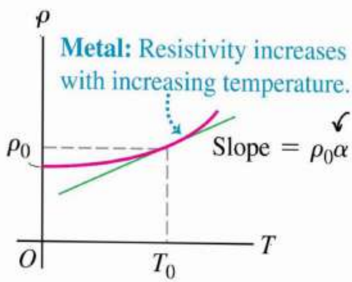
$$\frac{J}{E} = \sigma = \frac{1}{\rho}, \quad R = \frac{V}{I} = \frac{\rho L}{A} = \frac{L}{\sigma A}$$

## Resistivities at room temperature (20°C)

| | Substance | $\rho\ (\Omega \cdot m)$ |
|---|---|---|
| Conductors | Copper | $1.72 \times 10^{-8}$ |
| | Gold | $2.44 \times 10^{-8}$ |
| | Lead | $22 \times 10^{-8}$ |
| Semiconductor: | Pure carbon (graphite) | $3.5 \times 10^{-5}$ |
| Insulators | Glass | $10^{10} - 10^{14}$ |
| | Teflon | $>10^{13}$ |
| | Wood | $10^{8} - 10^{11}$ |

## Resistivity and temperature:

* The resistivity of a metallic conductor nearly always increases with increasing temperature



Over a small temperature range, the resistivity of a metal can be represented approximately

Temperature dependence of resistivity

$$\rho(T) = \rho_0[1+\alpha(T-T_0)]$$

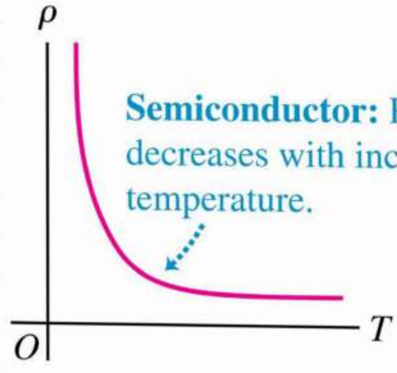
resistivity at temperature T

resistivity at reference temperature $T_0$

Temperature coefficient of resistivity

$$\boxed{\alpha = \frac{1}{\rho_0}\left(\frac{\Delta\rho}{\Delta T}\right)}$$ slope

measuring the resistivity of a small Semiconductor crystal is a sensitive measure of temperature; this is the principle of a type of thermometer called a thermistor.

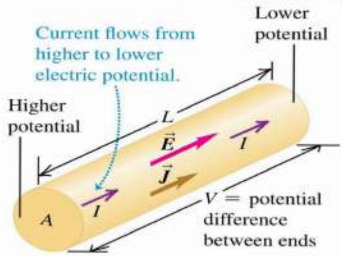


Subject Resistance and ohm's law Date No.

$$R = R_0 [1 + \alpha (T - T_0)]$$

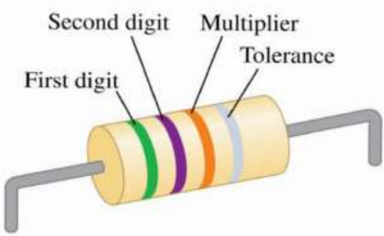
* The resltance of a conductor is $R = \rho L / A$

* The potential across a conductor is given by Ohm's Law: $V = IR$




* Resistors are color-coded for easy identification:

| Color | Value as Digit | Value as Multiplier |
|---|---|---|
| Black | 0 | 1 |
| Brown | 1 | 10 |
| Red | 2 | $10^2$ |
| Orange | 3 | $10^3$ |
| Yellow | 4 | $10^4$ |
| Green | 5 | $10^5$ |
| Blue | 6 | $10^6$ |
| Violet | 7 | $10^7$ |
| Gray | 8 | $10^8$ |
| White | 9 | $10^9$ |



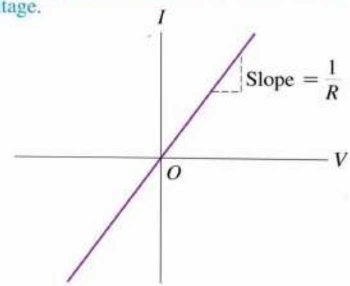

$5.7 \times 10^3\ \Omega \pm 10\%$

$5.7\ k\Omega \pm 10\%$

## ✲Ohmic resistors:

✲For a resistor that obeys Ohm's Law, a graph of current as a function of potential difference (voltage) is a straight line

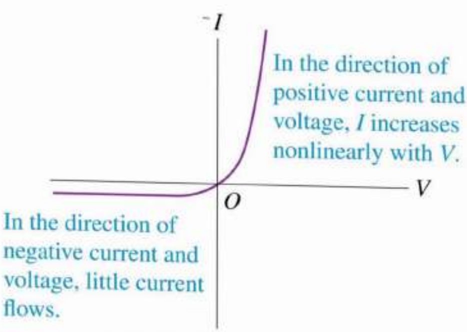
**Ohmic resistor** (e.g., typical metal wire): At a given temperature, current is proportional to voltage.




## ✲Nonohmic resistors:

✲In devices that don't obey Ohm's Law, the relationship of voltage to current may not be a direct proportion, and it may be different for the two directions of current

**Semiconductor diode: a nonohmic resistor**



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$$\frac{dU}{dt} = \frac{d}{dt}(Q\Delta V) = \frac{dQ}{dt}\Delta V = I\Delta V$$

$$P = I\Delta V = I^2 R = \frac{(\Delta V)^2}{R}$$

Ex 27.4: An electric heater is constructed by applying a potential difference of 120 V across a Nichrome wire that has a total resistance of 8 Ω. Find the current carried by the wire and the power rating of the heater.

$$I = \frac{\Delta V}{R} = \frac{120}{8} = 15 \text{ A}$$

$$P = I^2 R = (15)^2(8) = 1.8 \times 10^3 \text{ W} = 1.8 \text{ KW}$$

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Subject Date No.

Ex 6: A copper wire has a circular cross section with a radius of 1.25mm (a) If the wire carries a current of 3.7 A, find the drift speed of the electrons in this wire. (b) All other things being equal, what happens to the drift speed in wires made of metal having a larger number of conduction electrons per atom than copper?

* density of charge carrier $n_{copper} = 8.5 \times 10^{28}$ electron/$m^3$

a) $A = \pi r^2 = \pi (1.25 \times 10^{-3})^2 = 4.91 \times 10^{-6}\ m^2$

$I = n q A v_d$

$3.7 = 8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 4.91 \times 10^{-6}\, v_d$

$v_d = \dfrac{3.7}{8.5 \times 10^{28} \times 1.6 \times 10^{-19} \times 4.91 \times 10^{-6}}$

$v_d = 5.54 \times 10^{-5}\ m/s$

b) The drift speed inversly proportional to density number

If n Increased then $v_d$ decreased

N O T E B O O K

Ex 9: The quantity of charge q (in coulombs) that has passed through a surface of area $2\,cm^2$ varies with time acording to the equation $q = 4t^3 + 5t + 6$, where t is in seconds. (a) What is the instantaneous current through the surface at $t = 1s$? (b) What is the value of the current density?

$q = 4t^3 + 5t + 6$, $A = 2\,cm^2 = 2\times10^{-4}\,m^2$

a) $I = \frac{dq}{dt} = 12t^2 + 5$

$I(1) = 12(1)^2 + 5 = 17\ A$

b) $J = \frac{I}{A} = \frac{17}{2\times10^{-4}} = 85\times10^{3}\ A/m^2 = 85\,K\,A/m^2$

Ex 16: A 0.9V potential difference is maintained across a 1.5m length of tungsten wire that has a cross-sectional area of $0.6\,mm^2$. What is the current in the wire?

$V = 0.9V$, $L = 1.5m$, $A = 0.6\times10^{-6}\,m^2$, $\rho = 5.6\times10^{-8}$

$I = \frac{\Delta V}{R} = \frac{\Delta V A}{\rho L} = \frac{0.9\times0.6\times10^{-6}}{5.6\times10^{-8}\times1.5} = 6.4\ A$

Ex 23: A curren density of $6\times10^{-13}$ A/m² exists in the atmosphere at a location where the electric field is 100 V/m. Calculate the electrical conductivity of the Earth's atmosphere in this region?

$J = 6\times10^{-13}\ A/m^2,\ E = 100\ V/m$

$J = \sigma E$

$\sigma = \frac{J}{E} = \frac{6\times10^{-13}}{100} = 6\times10^{-15}\ (\Omega.m)^{-1}$

Ex 26: A certain lightbulb has a tungsten filament with a resistance of 19 Ω when at 20°C and 140 Ω when hot. Assume the resistivity of tungsten varies linearly with temperature even over the large temperature range involved here. Find the temperature of the hot filament?

$R_o = 19\ \Omega$ at $T = 20^\circ C$ and $R = 140\ \Omega$ when

$\alpha_{tungsten} = 4.5\times10^{-3}$

Subject Date No.

$$R = R_0 [1 + \alpha (T - T_0)]$$

$$\frac{140}{19} = \frac{\cancel{19}(1 + 4.5\times10^{-3}(T-20))}{\cancel{19}}$$

$$\frac{140}{19} = 1 + 4.5\times10^{-3}(T-20)$$

$$\frac{140}{19} - 1 = 4.5\times10^{-3}\,T - 0.09$$

$$\frac{6.37 + 0.09}{4.5\times10^{-3}} = \frac{\cancel{4.5\times10^{-3}}\,T}{\cancel{4.5\times10^{-3}}}$$

$$T = 1435.2\,°C = 1.43\times10^{3}\,°C$$

**31.** M (a) A 34.5-m length of copper wire at 20.0°C has a radius of 0.25 mm. If a potential difference of 9.00 V is applied across the length of the wire, determine the current in the wire. (b) If the wire is heated to 30.0°C while the 9.00-V potential difference is maintained, what is the resulting current in the wire?

$\alpha_{copper} = 3.4\times10^{-3}$

N O T E B O O K

Subject Date No.

a) $R_o = \frac{V}{I} = \frac{\rho L}{A}$

$A = \pi r^2 = \pi (0.25 \times 10^{-3})^2 = 1.96 \times 10^{-7} m^2$

$R_o = \frac{1.7 \times 10^{-8} \times 34.5}{1.96 \times 10^{-7}} = 2.99 \approx 3 \Omega$

$I = \frac{\Delta V}{R_o} = \frac{9}{3} = 3A$ at $T = 20^\circ C$

b) at $T = 30^\circ C$

$R = R_o (1 + \alpha \Delta T)$

$R = 3 (1 + 3.4 \times 10^{-3} (30 - 20))$

$R = 3.1 \Omega$

$I = \frac{\Delta V}{R} = \frac{9}{3.1} = 2.9A$

**39.** A certain waffle iron is rated at 1.00 kW when connected to a 120-V source. (a) What current does the waffle iron carry? (b) What is its resistance?

$P = IV \rightarrow I = \frac{P}{V} = \frac{1 \times 10^3}{120} = 8.3\,A$

$P = \frac{V^2}{R} \rightarrow R = \frac{V^2}{P} = \frac{(120)^2}{1 \times 10^3} = 14.5\,\Omega$

**49.** A coil of Nichrome wire is 25.0 m long. The wire has a diameter of 0.400 mm and is at 20.0°C. If it carries a current of 0.500 A, what are (a) the magnitude of the electric field in the wire and (b) the power delivered to it? (c) **What If?** If the temperature is increased to 340°C and the potential difference across the wire remains constant, what is the power delivered?

$R = \frac{\rho L}{A} = \frac{1.5 \times 10^{-6} \times 25}{\pi (0.2 \times 10^{-3})^2} = 298\,\Omega$ at $T = 20°C$

$\Delta V = IR = 0.5 \times 298 = 149\,V$ at $T = 20°C$

a) $E = \frac{\Delta V}{L} = \frac{149}{25} = 5.9\,V/m$

b) $P = IV = 0.5 \times 149 = 74.6\,W$

c) $R = R_0(1 + \alpha \Delta T) = 298(1 + 0.4 \times 10^{-3}(340 - 20))$
$R = 336\,\Omega$

Subject Date No.

$$P = \frac{V^2}{R} = \frac{(144)^2}{336} = 66$$

## Chapter 28: Electromotive force and circuits

- The influence that makes current flow from lower to higher potential is called electromotive force (emf) and a circuit device that provides emf is called a Source of emf.

- electromotive force is a poor term because emf is not a force but an energy per unit charge quantity, like potential.

- The SI unit of emf is the same as that for potential, the volt (1V = 1 J/C)

- A typical flashlight battery has an emf of 1.5V this means that the battery does 1.5J of work on every Coulomb of charge that passes through it.

- The symbol $\mathcal{E}$ (a script capital E) for emf

Subject **Internal resistance** Date No.

* Real sources of emf actually contain some internal resistance $r$.

* The terminal voltage of the 12V battery is less than 12V when it is connected to the light bulb

$$\boxed{V_{ab} = \mathcal{E} - Ir}$$

Terminal voltage source with internal resistance

emf of source

current through Source

Internal resistance of source

## * Symbols for circuit diagrams



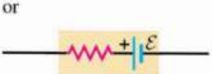
Conductor with negligible resistance

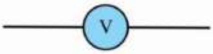

Resistor


Source of emf (longer vertical line always represents the positive terminal, usually the terminal with higher potential)

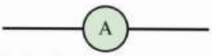

Source of emf with internal resistance $r$ ($r$ can be placed on either side)

or




Voltmeter (measures potential difference between its terminals)

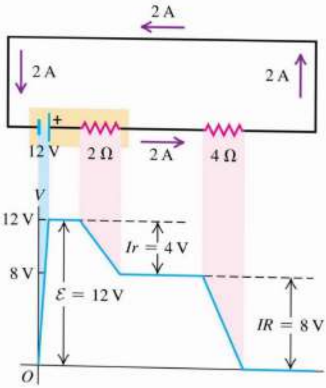



Ammeter (measures current through it)

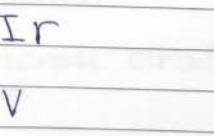
Subject **Potential changes** Date No.

★ The potential rises when the current goes through a battery, and drops when it goes through a resistor.

★ Going all the way around the loop brings the potential back to where it started.




Ex 28.1: A battery has an emf of 12 V and an internal resistance of 0.05 Ω. Its terminals are connected to load resistance of 3 Ω. (A) Find the current in the circuit and the terminal voltage of the battery. (B) Calculate the power delivered to the load resistor, the power deliverd to the internal resistance of the battery, and the power delivered by the battery. $V = IR$, $I = \frac{V}{R}$




$I = \frac{\varepsilon}{r+R} = \frac{12}{0.05+3} = 3.93\,A$

★ terminal voltage $V_{ab} = \varepsilon - Ir$

$V_{ab} = 12 - 3.93 \times 0.05 = 11.8\,V$

NOTEBOOK

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Subject Energy and power Date No.

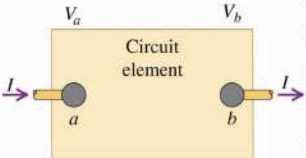
$P_R = I^2 R = (3.93)^2 (3) = 46.3\,W$

$P_r = I^2 r = (3.93)^2 (0.05) = 0.8\,W$

$P_{source} = P_R + P_r = 46.3 + 0.8 = 47.1\,W$

Energy and power in electric circuits:

* The box represents a circuits element with potential difference $V_{ab} = V_a - V_b$ between its terminals and current passing throug it in the direction from a toward b.




* If the potential at a is lower than at b, then there is a net transfer of energy out of the circuit element.

* The time rate of energy transfer is power, denoted by P:

$$P = V_{ab} I$$

Power delivered to or extracted from a circuit element

Current in circuit element

Voltage across circuit element

Subject dc versus ac Date No.

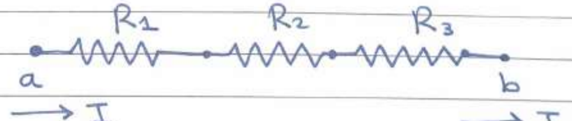
Our principal concern in this chapter is with direct-current (dc) circuits, in which the direction of the current doesn't change with time.

Flashlights and automobile wiring systems are examples of direct current circuits.

Household electrical power is supplied in the form of alternating current (ac), in which the current oscillates back and forth.

## Resistors in Series:

Resistors are in series if they are connected one after the other so the current is the same in all of them




$I_1 = I_2 = I_3$

$V_{ab} = V_1 + V_2 + V_3$

The equivalent resistance of a series combination is the sum of the individual resistances:

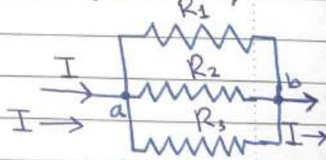
Resistors in series

$$R_{eq} = R_1 + R_2 + R_3 + \cdots\cdots$$

equivalent resistance of series combination

## Resistors in parallel:

If the resistors are in parallel, the current through each resistor need not to be the same, but the potential difference between the terminals of each resistor must be the same, and equal to $V_{ab}$.




$$V_{ab} = V_1 = V_2 = V_3$$

$$I = I_1 + I_2 + I_3$$

The reciprocal of the equivalent resistance of a parallel combination equals the sume of the reciprocals of the individual resistances

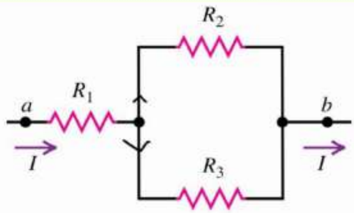
Resistors in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

equivalent resistance of parallel combination

## Series and parallel combinations: Example 1

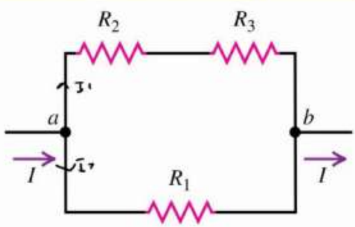
- Resistors can be connected in *combinations* of series and parallel, as shown.
- In this case, try reducing the circuit to series and parallel combinations.




- For the example shown, we first replace the parallel combination of $R_2$ and $R_3$ with its equivalent resistance; this then forms a series combination with $R_1$.

## Series and parallel combinations: Example 2

- Resistors can be connected in *combinations* of series and parallel, as shown.
- In this case, try reducing the circuit to series and parallel combinations.




- For the example shown, we first replace the series combination of $R_2$ and $R_3$ with its equivalent resistance; this then forms a parallel combination with $R_1$.

$I_1 + I_2 = 3\,A$

$I_1 + 2I_1 = 3\,A \Rightarrow \boxed{I_1 = 1\,A}$

$\boxed{I_2 = 2I_1 = 2 \times 1 = 2\,A}$

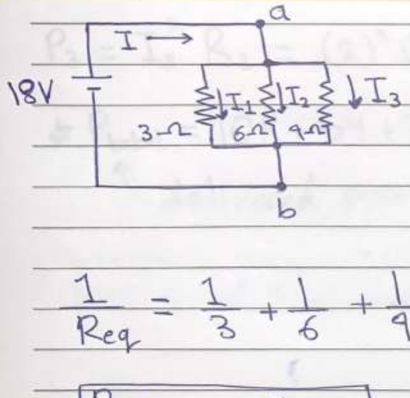
**Example 28.5** **Three Resistors in Parallel**

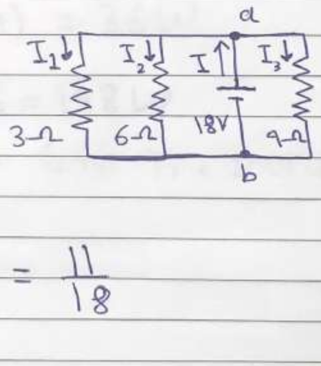
Three resistors are connected in parallel as shown in Figure 28.11a. A potential difference of 18.0 V is maintained between points *a* and *b*.

**(A)** Calculate the equivalent resistance of the circuit.

**(B)** Find the current in each resistor.

**(C)** Calculate the power delivered to each resistor and the total power delivered to the combination of resistors.




$$\frac{1}{R_{eq}} = \frac{1}{3} + \frac{1}{6} + \frac{1}{9} = \frac{11}{18}$$

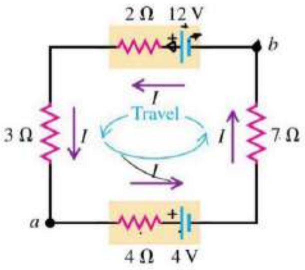
$\boxed{R_{eq} = 1.64\,\Omega}$

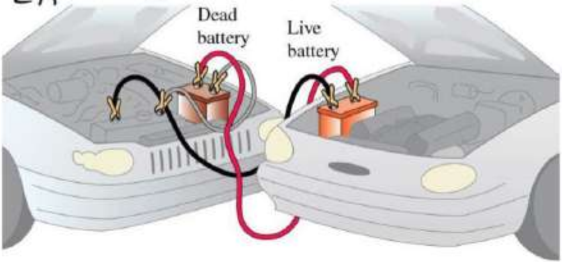
# A single-loop circuit

- The circuit shown contains two batteries, each with an emf and an internal resistance, and two resistors.

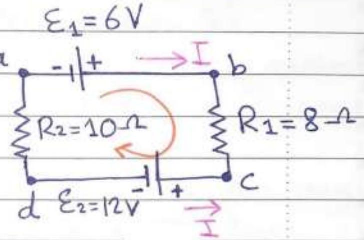
- Using Kirchhoff's rules, you can find the current in the circuit, the potential difference $V_{ab}$, and the power output of the emf of each battery.

$$I = \frac{\sum \varepsilon}{\sum R} = \frac{12-4}{18} = \frac{8}{18} = 0.5\,A$$



Ex 28.6: A single loop circuit contains two resistors and two battries (Neglect the internal resistances of the battries). Find the current in the circuit?




$$\sum \Delta V = 0$$

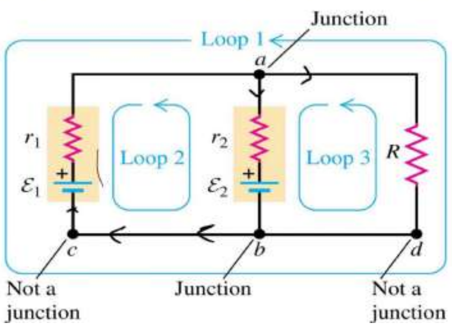
$$\mathcal{E}_1 - IR_1 - \mathcal{E}_2 - IR_2 = 0$$

$$I = \frac{\mathcal{E}_1 - \mathcal{E}_2}{R_1 + R_2} = \frac{6-12}{8+10}$$

$$\boxed{I = -0.33\,A}$$

## Kirchhoff's rules:

Many particle resistor networks can't be reduced to simple series-parallel combinations, To analyze these networks, we will use the techniques developed by Kirchhoff.



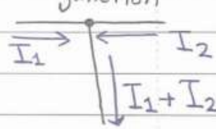
$$P = V_{ab} I = \mathcal{E} I - I^2 r$$

## Kirchhoff's junction rule:

- A junction is a point where three or more conductors meet.

$$\Sigma I = 0$$

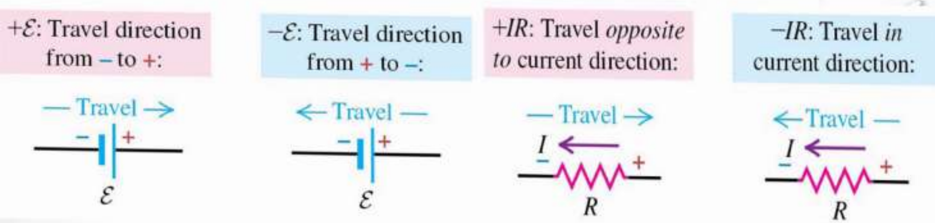
the sum of currents into any junction equals zero




- A Loop is any closed conducting path

- The loop rule is a statement that the electrostatic force is conservative

- Sign conventions for the loop rule:

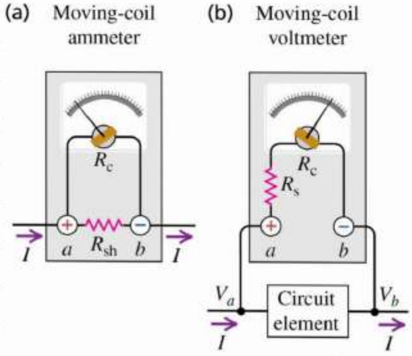


Subject Ameters and voltmeters Date No.

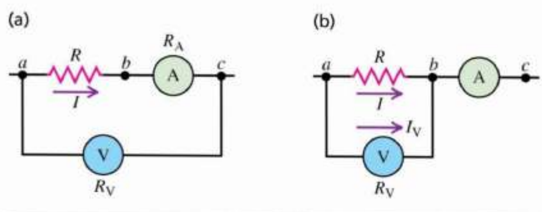
* An Ameter measures the current passing through it

* A voltmeter measures the potential difference between two points

* Both instruments contain a galvanometer.




* An ameter and a Voltmeter may be used together to measure resistance and power.




* An Ohmmeter consists of a meter, a resistor, and a source (often a flashlight battery) Connected in series.

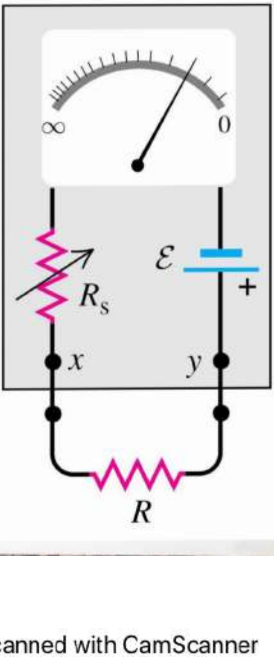
NOTEBOOK

Subject Ohmmeter and digital multimeter Date No.

* The resistor $R_s$ has a variable resistance, as is indicated by the arrow through the resistor symbol.

* To use the ohmmeter, first connect x directly to y and adjust $R_s$ until the meter reads zero, then connect x and y across the resistor R and read the scale

* A digital multimeter can measure voltage, current, or resistance over a wide range.

Subject The potentiometer Date No.

- The potentiometer is an instrument that can be used to measure the emf of a source without drawing any current from the source. It balances an unknown potential difference against an adjustable, measurable potential difference.

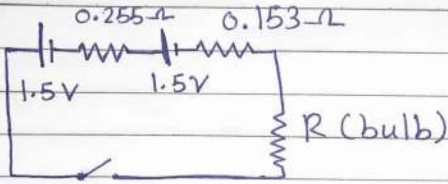
- The term potentiometer is also used for any variable resistor, usually having a circular resistance element and a sliding contact controlled by a rotating shaft and Knob.

- The circuit symbol for a potentiometer is

**2.** AMT Two 1.50-V batteries—with their positive terminals in the same direction—are inserted in series into a flashlight. One battery has an internal resistance of 0.255 Ω, and the other has an internal resistance of 0.153 Ω. When the switch is closed, the bulb carries a current of 600 mA. (a) What is the bulb's resistance? (b) What fraction of the chemical energy transformed appears as internal energy in the batteries?

N O T E B O O K

Subject Date No.




$I = 600\,mA$

$I = 0.6\,A$

$$R_{total} = \frac{V}{I} = \frac{3}{0.6} = 5\,\Omega$$

internal resistance

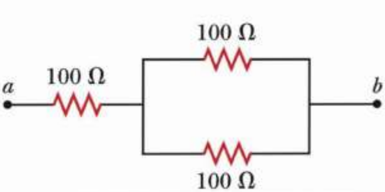
$$R_{bulb} = R_{total} - (r_1 + r_2)$$

$$R_{bulb} = 5 - (0.255 + 0.153) = 4.59\,\Omega$$

$$P_{total} = R_{total} I^2 \quad , \quad P_r = (r_1 + r_2) I^2$$

$$\frac{P_r}{P_{total}} = \frac{(0.255 + 0.153)\cancel{I^2}}{5\cancel{I^2}} = 8.2\%$$

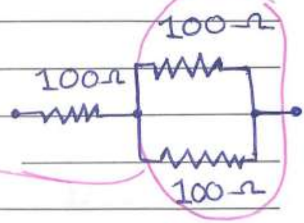
**5.** Three 100-Ω resistors are connected as shown in Figure P28.5. The maximum power that can safely be delivered to any one resistor is 25.0 W. (a) What is the maximum potential difference that can be applied to the terminals *a* and *b*? (b) For the voltage determined in part (a), what is the power delivered to each resistor? (c) What is the total power delivered to the combination of resistors?



Subject ______ Day ______ Date ______

a) $P_{max} = I_{max}^2 R \Rightarrow I_{max} = \sqrt{\frac{P_{max}}{R}}$

$I_{max} = \sqrt{\frac{25}{100}} = 0.5\,A$




$\frac{1}{R_{eq}} = \frac{1}{100} + \frac{1}{100}$

$R_{eq} = 50\,\Omega$


$R_{eq} = 100 + 50$

$R_{eq} = 150\,\Omega$

$\Delta V_{max} = R_{eq} I_{max} = 150\,(0.5)$

$\Delta V_{max} = 75\,V$

b) Parallel resistors:

$P = I \Delta V = 0.5\,(75) = 37.5\,W$

Series resistors:

$P = 25\,W$

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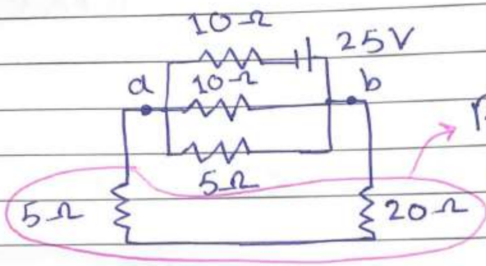
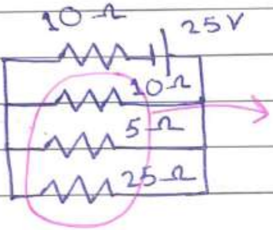
Ex 9: find the current in 20 Ω resistor and the potential difference between a and b




$R_{eq} = 5 + 20$

$R_{eq} = 25\ \Omega$

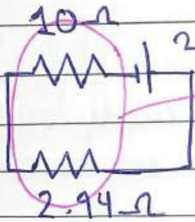



$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{5} + \frac{1}{25}$$

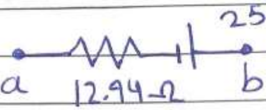
$$\frac{1}{R_{eq}} = \frac{85}{250}$$

$$R_{eq} = \frac{250}{85} = 2.94\ \Omega$$

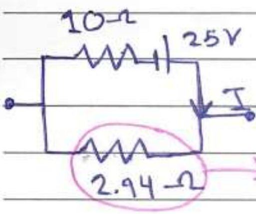
Subject ________ Day ________ Date ________




$R_{eq} = 10 + 2.94 = 12.94\,\Omega$

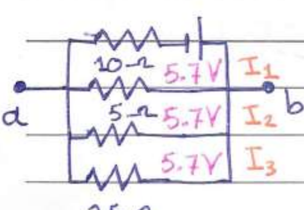



$$I = \frac{V}{R_{eq}} = \frac{25}{12.94} = 1.94\,A$$




$V = IR = 1.94 \times 2.94$

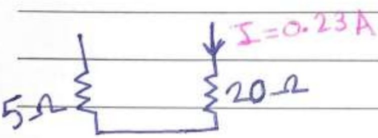
$V = 5.7\,V$




$$I_1 = \frac{V_1}{R_1} = \frac{5.7}{10} = 0.57\,A$$

$$I_2 = \frac{V}{R_2} = \frac{5.7}{5} = 1.1\,A$$

$$I_3 = \frac{V}{R_3} = \frac{5.7}{25} = 0.23\,A$$




$V_{ab} = 5.7\,V$

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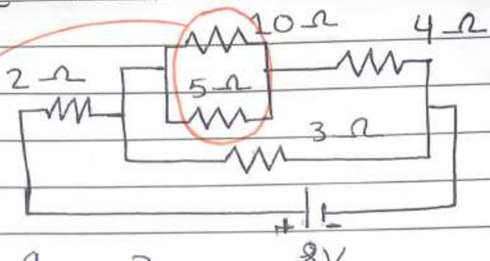
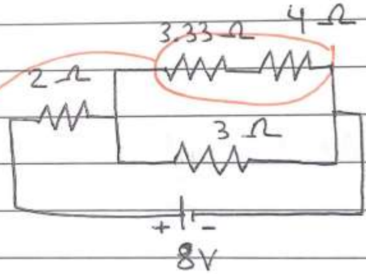
Subject ____________ Day ____________ Date ____________

Ex 21: consider the circuit below
(a) Find the voltage across the 3 Ω resistor
(b) Find the current in the 3 Ω resistor



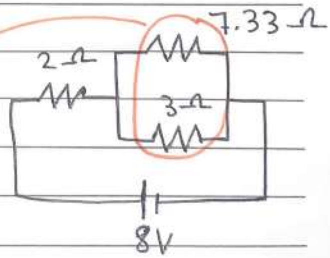

$$\frac{1}{R_{eq}} = \frac{1}{10} + \frac{1}{5} = \frac{3}{10}$$




$$R_{eq} = \frac{10}{3} = 3.33\,\Omega$$

$$R_{eq} = 4 + 3.33$$

$$R_{eq} = 7.33\,\Omega$$




$$\frac{1}{R_{eq}} = \frac{1}{7.33} + \frac{1}{3} = \frac{10.3}{21.9}$$

$$R_{eq} = 2.13\,\Omega$$

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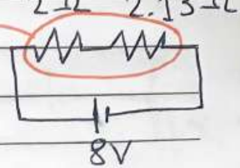
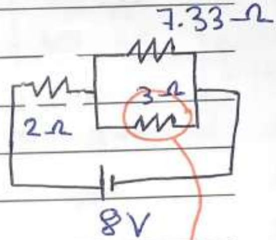



$R_{eq} = 2 + 2.13$

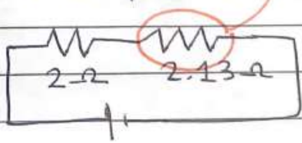
$R_{eq} = 4.13\,\Omega$

$I = \frac{V}{R} = \frac{8}{4.13} = 1.9\,A$




$V = IR = 1.9 \times 2.13$

$V = 4.1\,V$




$I = \frac{V}{R}$

$I = \frac{4.1}{3} = 1.4A$

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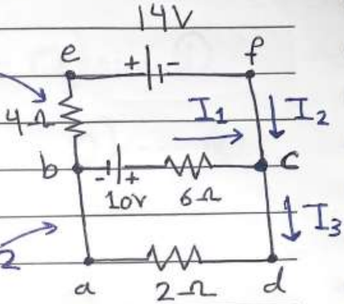
Ex 28.7: Find the currents $I_1, I_2, I_3$

$\Sigma I = 0$

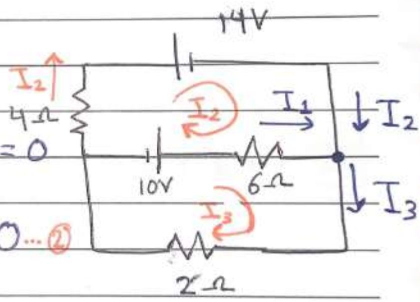
$I_3 = I_2 + I_1 \cdots (1)$




**Loop 1:**

$\Sigma V = 0$

$\underbrace{\quad}$ $\mathcal{E}$ $IR$




$-4I_2 - 14 + 6I_1 - 10 = 0$

$6I_1 - 4I_2 - 24 = 0 \cdots (2)$

**Loop 2:**

$-2I_3 + 10 - 6I_1 = 0$

$10 - 6I_1 - 2I_3 = 0 \cdots (3)$

$10 - 6I_1 - 2(I_1 + I_2) = 0$

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$10 - 6I_1 - 2I_1 - 2I_2 = 0$

$2\times(10 - 8I_1 - 2I_2 = 0)$ ---- ①

$-1\times(6I_1 - 4I_2 - 24 = 0)$ ---- ②

$$
\begin{aligned}
&20 - 16I_1 - \cancel{4I_2} = 0\\
+\ &\underline{-6I_1 + \cancel{4I_2} + 24 = 0}\\
&44 - 22I_1 = 0
\end{aligned}
$$

$$\frac{22I_1}{22} = \frac{44}{22} \Rightarrow \boxed{I_1 = 2A}$$

$10 - 8(2) - 2I_2 = 0$

$-6 - 2I_2 = 0$

$2I_2 = -6 \Rightarrow \boxed{I_2 = -3A}$

* magnitude of $|I_2| = 3A$

$I_3 = I_2 + I_1 = -3 + 2$

$\boxed{I_3 = -1A}$ * magnitude of $|I_3| = 1A$

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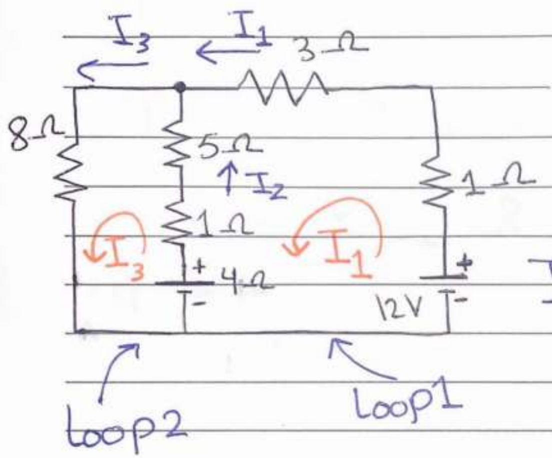
**22.** In Figure P28.22, show how to add just enough ammeters to measure every different current. Show how to add just enough voltmeters to measure the potential difference across each resistor and across each battery.




$I_1 + I_2 = I_3$ ①

Loop 1:

$12 - 1I_1 - 3I_1 + 5I_2 + 1I_2 - 4 = 0$

$8 - 4I_1 + 6I_2 = 0$ ..... ②

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## Loop 2:

$$-8I_3 + 4 - 1I_2 - 5I_2 = 0$$

$$4 - 6I_2 - 8I_3 = 0 \quad \text{.....} \quad (3)$$

$$4 - 6I_2 - 8(I_1 + I_2) = 0$$

$$4 - 6I_2 - 8I_1 - 8I_2 = 0$$

$$4 - 8I_1 - 14I_2 = 0$$

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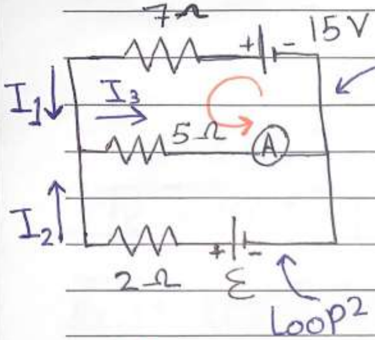
Ex 29: The ammeter reads 2A. Find $I_1$, $I_2$ and $\mathcal{E}$




$I_3 = I_1 + I_2 = 2A$

Loop 1:

$15 - 7I_1 - 5I_3 = 0$

$15 - 7I_1 - 5(2) = 0$

$5 - 7I_1 = 0 \rightarrow \boxed{I_1 = \frac{5}{7} = 0.7A}$

$I_1 + I_2 = 2$

$0.7 + I_2 = 2$

$\boxed{I_2 = 1.3A}$

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**Loop 2:**

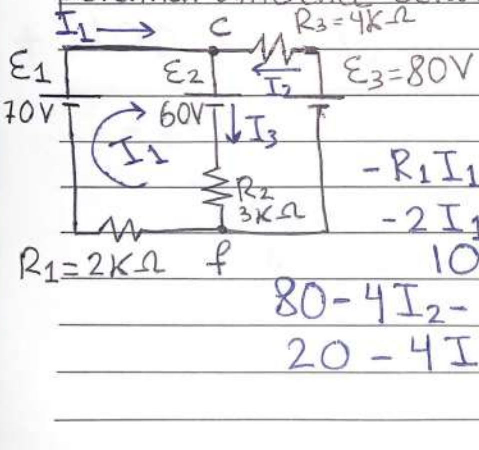
$\varepsilon - 2I_2 - 5I_3 = 0$

$\varepsilon = 2I_2 + 5I_3$

$\varepsilon = 2(1.3) + 5(2)$

$\boxed{\varepsilon = 12.6V}$

Ex 31: Using Kirchhoff's rules, find the current in each resistor and find the potential difference between points c and f.




$I_3 = I_1 + I_2 \cdots (1)$

$-R_1 I_1 + 70 - 60 - I_3 R_2 = 0$

$-2I_1 + 10 - 3I_3 = 0$

$10 - 2I_1 - 3I_3 = 0 \cdots (2)$

$80 - 4I_2 - 60 - 3I_2 = 0$

$20 - 4I_2 - 3I_3 = 0 \cdots (3)$

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$$10 - 2I_1 - 3(I_1 + I_2) = 0$$

$$10 - 2I_1 - 3I_1 - 3I_2 = 0$$

$$3\times \quad 10 - 5I_1 - 3I_2 = 0 \; \cdots\cdots \; (2)$$

$$20 - 4I_2 - 3(I_1 + I_2) = 0$$

$$20 - 4I_2 - 3I_1 - 3I_2 = 0$$

$$-5\times \quad 20 - 7I_2 - 3I_1 = 0 \; \cdots\cdots \; (1)$$

$$30 - \cancel{15I_1} - 9I_2 = 0$$

$$+ \quad -100 + 35I_2 \cancel{+15I_1} = 0$$

$$-70 + 26I_2 = 0$$

$$\frac{\cancel{26}I_2}{\cancel{26}} = \frac{70}{26}$$

$$\boxed{I_2 = 2.7A}$$

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$30-15I_1-9(2.7)=0$

$30-24-15I_1=0$

$\frac{6}{15}=\frac{\cancel{15}I_1}{\cancel{15}} \Rightarrow \boxed{I_1=0.4A}$

$I_3=I_1+I_2=0.4+2.7$

$\boxed{I_3=3.1A}$

$\Delta V=V_f-V_c=-60-3I_3$

$\Delta V=-60-3(3)=-69V$

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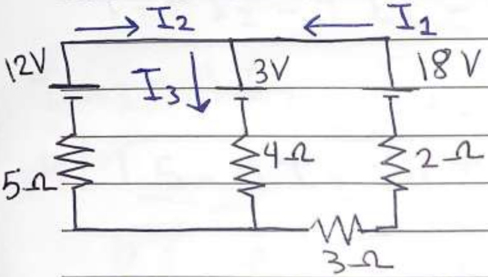
Ex 35: Find the potential difference across each resistor




$$I_3 = I_1 + I_2 \quad \text{--- (1)}$$

$$-2I_1 + 18 - 3 - 4I_3 - 3I_1 = 0$$

$$15 - 5I_1 - 4I_3 = 0 \quad \text{--- (2)}$$

$$-5I_2 + 12 - 3 - 4I_3 = 0$$

$$9 - 5I_2 - 4I_3 = 0 \quad \text{--- (3)}$$

$$\star\; 15 - 5I_1 - 4(I_1 + I_2) = 0$$

$$15 - 5I_1 - 4I_1 - 4I_2 = 0$$

$$15 - 9I_1 - 4I_2 = 0 \quad \text{--- (2)}$$

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$$9 - 5I_2 - 4(I_1 + I_2) = 0$$

$$9 - 5I_2 - 4I_1 - 4I_2 = 0$$

$$9\times \; 9 - 9I_2 - 4I_1 = 0 \quad \text{---- } (3)$$

$$-4\times \; 15 - 9I_1 - 4I_2 = 0 \quad \text{---- } (2)$$

$$81 - 81I_2 - \cancel{36I_1} = 0$$

$$+ \quad -60 + \cancel{36I_1} + 16I_2 = 0$$

$$21 - 65I_2 = 0$$

$$\frac{65I_2}{65} = \frac{21}{65} \Rightarrow \boxed{I_2 = 0.3A}$$

$$9 - 9(0.3) - 4I_1 = 0$$

$$\frac{6.3}{4} = \frac{\cancel{4}I_1}{\cancel{4}} \Rightarrow \boxed{I_1 = 1.6A}$$

$$I_3 = 0.3 + 1.6 = 1.9A$$

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$V_1 = 5I_2 = 5(0.3) = 1.5V$

$V_2 = 4I_3 = 4(1.9) = 7.6V$

$V_3 = 2I_1 = 2(1.6) = 3.2V$

$V_4 = 3I_1 = 3(1.6) = 4.8V$

$P_1 = I^2R = IV = 1.5(0.3) = 0.45W$

$P_2 = IV = 7.6(1.9) = 14.4W$

$P_3 = IV = 3.2(1.6) = 5.1W$

$P_4 = IV = 4.8 \times 1.6 = 7.7W$

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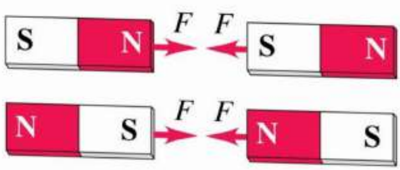
# Chapter 29: Magnetic Fields :

## Magnetic Poles:

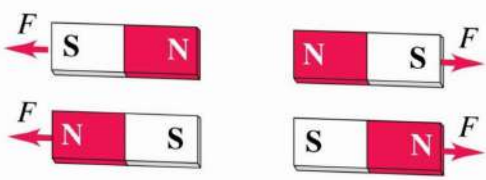
- If a bar-shaped permanent magnet is free to rotate, One end points north, this end is called north pole or n pole. The other end is a South pole or S pole.

- Opposite poles attract each other, and like poles repel each other

(a) Opposite poles attract.




(b) Like poles repel.



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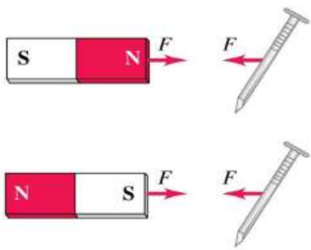
An object that contains iron but is not itself magnetized (shows no tendency to point north or south) is attracted by either pole of a permanent magnet.




The earth itself is a magnet, Its north geographich pole is close to a magnetic south pole, which is why the north pole of a compass needle points north.

The earth's magnetic axis is not quite parallel to its geographic axis (The axis of rotation), so a compass reading deviates somewhat from geographic north, this devation is called **magnetic declination or magnetic variation.**

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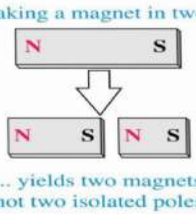
* The magnetic field is not horizontal at most points on the earth's surface; its angle up or down is called **magnetic inclination**

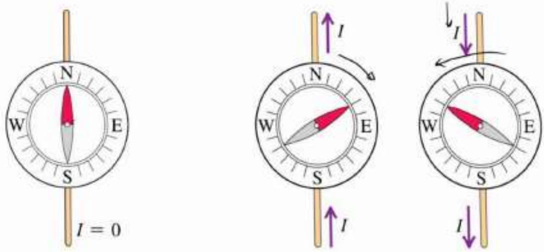
* Magnetic poles always comes in pairs.

* There is no experimental evidence for magnetic monopoles

In contrast to electric charges, magnetic poles always come in pairs and can't be isolated.




* A compass near a wire with no current Points north, However if an electric current runs through the wire, the compass needle deflects somewhat



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# The magnetic field:

- A moving charge (or current) creates a magnetic field in the surrounding space, The magnetic field exerts a force on any other moving charge (or current) that is present in the field.

- Like an electric field, a magnetic field is a vector field, that is a vector quantity associated with each point in space.

- The symbol $\vec{B}$ is for magnetic field.

- At any position the direction of $\vec{B}$ is defined as the direction in which the north pole of a compass needle tends to point.

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## Similarities and differences:

### 1) Similarities between magnetic and electric forces:

- The magnetic force is proportional to the charge $q$ of the particle.
- The magnetic force on a negative charge is directed opposite to the force on a positive charge moving in the same direction.
- The magnetic force is proportional to the magnitude of the magnetic field vector $\vec{B}$.

### 2) Differences between magnetic and electric forces:

- The magnetic force is proportional to the speed $v$ of the particle.
- If the velocity vector makes an angle $\theta$ with the magnetic field, the magnitude of the magnetic force is proportional to $\sin\theta$

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* When a charged particle moves parallel to the magnetic field field vector, the magnetic force on the charge is zero.

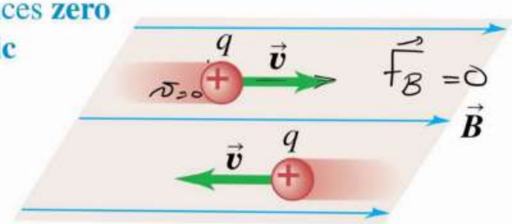
* When a charged particle moves in a direction not parallel to the magnetic field vector, the magnetic force acts in a direction prependicular to both $\vec{v}$ and $\vec{B}$; that is, the magnetic force is prependicular to the plane formed by $\vec{v}$ and $\vec{B}$.

* The magnetic force on a moving charge:

* The magnitude of the magnetic force on a moving particle is proportional to the component of the particle's velocity prependicular to the field.

* If the particle is at rest, or moving parallel to the field, it experiences zero magnetic force.

A charge moving **parallel** to a magnetic field experiences **zero magnetic force.**




* The magnetic force is best represented as a vector product.

Particle's charge

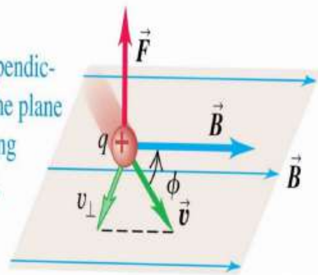
$$\vec{F} = q\,\vec{v} \times \vec{B}$$

magnetic force on a moving charged particle

Particle's velocity

magnetic field

$\vec{F}$ is perpendicular to the plane containing $\vec{v}$ and $\vec{B}$.




$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}, \quad \vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$$

$$\vec{C} = \vec{A} \times \vec{B} = |\vec{A}|\,|\vec{B}|\sin\theta$$

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

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$\vec{A} \times \vec{B} = \hat{i}(A_y B_z - A_z B_y) - \hat{j}(A_x B_z - A_z B_x)$
$+ \hat{k}(A_x B_y - A_y B_x)$

Example: Let $\vec{A} = 2\hat{i} - \hat{j} + 2\hat{k}$ and
$\vec{B} = \hat{i} + 3\hat{k}$, Find $\vec{A} \times \vec{B}$ ?

$$\vec{C} = \vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 2 \\ 1 & 0 & 3 \end{vmatrix}$$

$\vec{C} = \hat{i}(-1 \times 3 - 2 \times 0) - \hat{j}(2 \times 3 - 2 \times 1) +$
$\hat{k}(2 \times 0 - 1 \times -1)$

$\vec{C} = -3\hat{i} - 4\hat{j} + \hat{k}$

or $\vec{A} \times \vec{B} = (2\hat{i} - \hat{j} + 2\hat{k}) \times (\hat{i} + 3\hat{k})$

$\vec{C} = (2\hat{i} \times \hat{i}) + (2\hat{i} + 3\hat{k}) + (-\hat{j} \times \hat{i}) + (-\hat{j} \times 3\hat{k}) +$
$(2\hat{k} \times \hat{i}) + (2\hat{k} \times 3\hat{k})$

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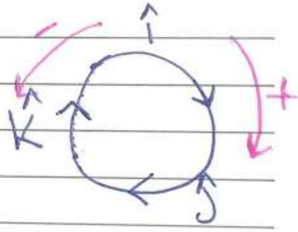
$\vec{C} = -6\hat{k} + \hat{k} - 3\hat{i} + 2\hat{j}$

$\vec{C} = -4\hat{j} + \hat{k} - 3\hat{i}$

$\vec{A} \times \vec{B} \neq \vec{B} \times \vec{A}$

$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A}$

Ex: Let $\vec{A} = 2\hat{i} + \hat{j}$ and $\vec{B} = \hat{i} + \hat{k}$




$\vec{A} \times \vec{B} = (2\hat{i} + \hat{j}) \times (\hat{i} + \hat{k})$

$\vec{A} \times \vec{B} = (2\hat{i} \times \hat{i}) + (2\hat{i} \times \hat{k}) + (\hat{j} \times \hat{i}) + (\hat{j} \times \hat{k})$

$\vec{A} \times \vec{B} = -2\hat{j} - \hat{k} + \hat{i}$

★ $\hat{i} \times \hat{i} = \hat{j} \times \hat{j} = \hat{k} \times \hat{k} = 0$

| | |
|---|---|
| $\hat{i} \times \hat{j} = +\hat{k}$ | $\hat{i} \times \hat{k} = -\hat{j}$ |
| $\hat{j} \times \hat{k} = +\hat{i}$ | $\hat{k} \times \hat{j} = -\hat{i}$ |
| $\hat{k} \times \hat{i} = +\hat{j}$ | $\hat{j} \times \hat{i} = -\hat{k}$ |

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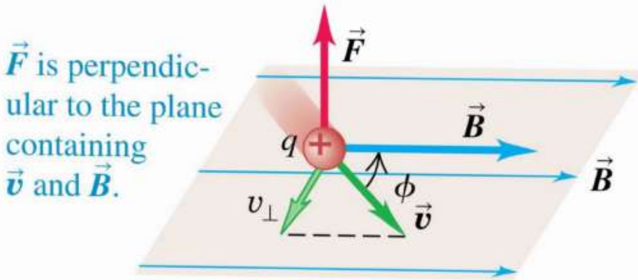
## ★ The magnetic force on a moving charge:

★ A charge moving at an angle $\phi$ to a magnetic field experiences a magnetic force with magnitude $F = |q| v_\perp B = |q| v B \sin\phi$




## ★ Right-hand rule for magnetic force:

★ The right hand rule gives the direction of the force on a positive charge

1) Place the velocity and magnetic field vectors tail to tail

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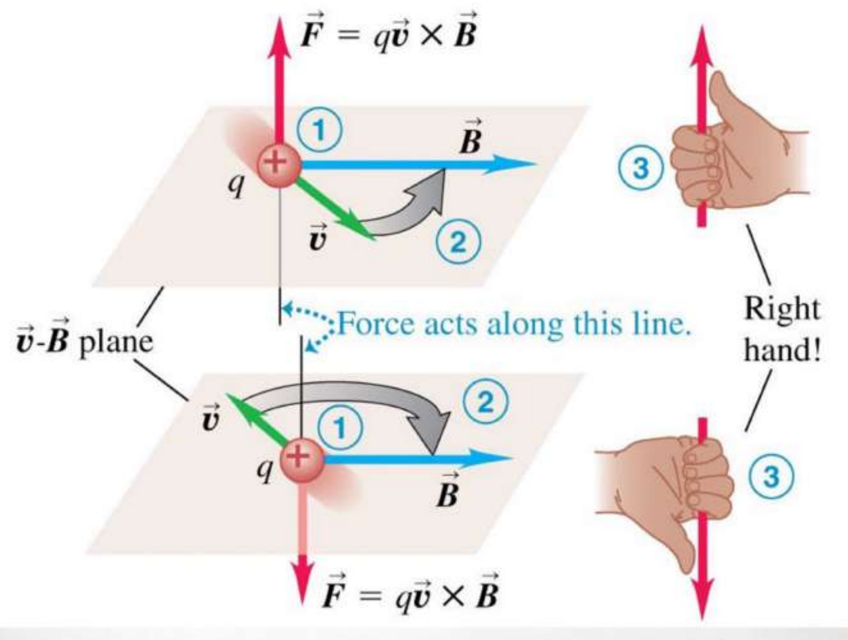
2) Imagine turning $\vec{v}$ toward $\vec{B}$ in the $\vec{v}$-$\vec{B}$ plane (through the smaller angle)

3) The force acts along a line prependicular to the $\vec{v}$-$\vec{B}$ plane. Curl the fingers of your right hand around this line in the same direction you rotated $\vec{v}$. Your thumb now points in the direction the force acts.



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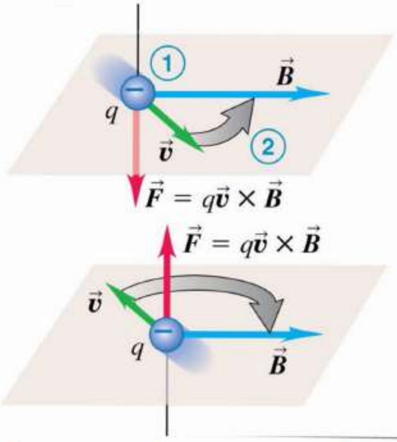
* If the charge is negative, the direction of the force is opposite to that given by the right-hand rule.




* Equal velocities but opposit signs:

* Imagine two charges of the same magnitude but opposite sign moving with the same velocity in the same magnetic field.

* The magnetic forces on the charges are equal in magnitude but opposite in direction

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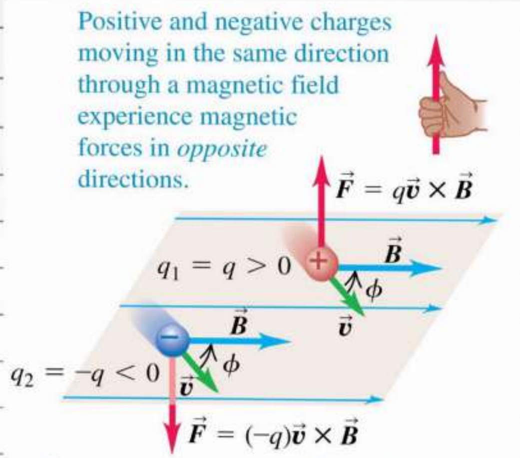


## ☆ Units of magnetic field:

☆ The SI unit of magnetic field $\vec{B}$ is called the tesla (1T)

$$1 \text{ tesla} = 1T = 1 N/A \cdot m$$

☆ Another unit of B, the gauss ($1G = 10^{-4} T$)

☆ The magnetic field of the earth is on the order of $10^{-4}$ T or 1 G

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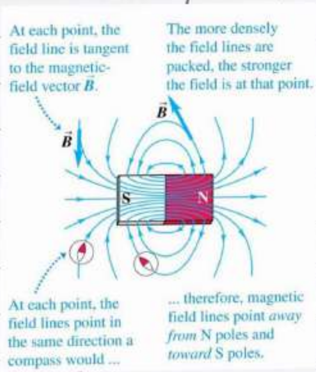
## Magnetic field lines:

- We can represent any magnetic field by magnetic field lines

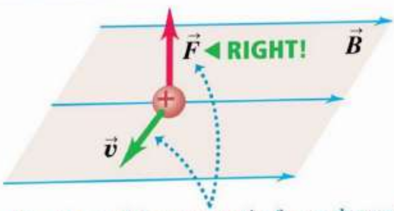
- We draw the lines so that the line through any point is tangent to the magnetic field vector at that point

- Field lines never intersect.




- magnetic field lines are not lines of magnetic force.

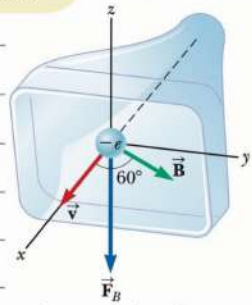
- The force on a charged particle is not along the direction of a field line




The direction of the magnetic force depends on the velocity $\vec{v}$, as expressed by the magnetic force law $\vec{F} = q\vec{v} \times \vec{B}$.

**Example 29.1** **An Electron Moving in a Magnetic**

An electron in an old-style television picture tube moves toward the front of the tube with a speed of $8.0 \times 10^6$ m/s along the $x$ axis (Fig. 29.6). Surrounding the neck of the tube are coils of wire that create a magnetic field of magnitude 0.025 T, directed at an angle of 60° to the $x$ axis and lying in the $xy$ plane. Calculate the magnetic force on the electron.




$F_B = |q| v B \sin\theta$

$F_B = 1.6 \times 10^{-19} \times 8 \times 10^{6} \times 0.025 \times \sin(60)$

$F_B = 2.8 \times 10^{-14}$ N

Ex 8: A proton moves with a velocity of $\vec{v} = (2\hat{i} - 4\hat{j} + \hat{k})$ m/s in a region in which the magnetic field is $\vec{B} = (\hat{i} + 2\hat{j} - \hat{k})$ T. What is the magnitude of the magnetic force this particle experiences?

$\vec{v} = 2\hat{i} - 4\hat{j} + \hat{k}, \vec{B} = \hat{i} + 2\hat{j} - \hat{k}$

$\vec{F} = q\vec{v} \times \vec{B}$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 1 \\ 1 & 2 & -1 \end{vmatrix}$$

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$$\vec{v} \times \vec{B} = \hat{i}(4-2) - \hat{j}(-2-1) + \hat{k}(4-(-4))$$

$$\vec{v} \times \vec{B} = 2\hat{i} + 3\hat{j} + 8\hat{k}$$

$$\vec{F} = 1.6 \times 10^{-19}(2\hat{i} + 3\hat{j} + 8\hat{k})$$

$$\vec{F} = 3.2 \times 10^{-19}\hat{i} + 4.8 \times 10^{-19}\hat{j} + 1.28 \times 10^{-18}\hat{k}$$

$$|\vec{F}| = \sqrt{(3.2 \times 10^{-19})^2 + (4.8 \times 10^{-19})^2 + (1.28 \times 10^{-18})^2}$$

$$|\vec{F}| = 1.4 \times 10^{-18}\,N$$

Ex9: A proton travels with a speed of $5.02 \times 10^6$ m/s in a direction that makes angle of $60°$ with the direction of a magnetic field of magnitude 0.18 T in the positive x direction. What are the magnitudes of (a) the magnetic force on the proton and (b) the proton's acceleration?

$v = 5.02 \times 10^5$ m/s, $\theta = 60°$, $|B| = 0.18$ T

(a) $F_B = qvB\sin\theta$

$F_B = 1.6 \times 10^{-19} \times 5.02 \times 10^6 \times 0.18 \sin(60)$

$F_B = 1.25 \times 10^{-13}$ N

(b) $\Sigma F = F_B = ma$

$1.25 \times 10^{-13} = 1.67 \times 10^{-27}\, a$

$a = 7.5 \times 10^{13}$ m/s²

Ex 11: A proton moves prependicular to a uniform magnetic field $\vec{B}$ at a speed of $1 \times 10^7$ m/s and experiences an acceleration of $2 \times 10^{13}$ m/s² in the positive x direction when its velocity is in the positive z direction. Determine the magnitude and direction of the field?

$v = 1 \times 10^7$ m/s, $a = 2 \times 10^{13}$ m/s²

$F = qvB\sin\theta = ma$

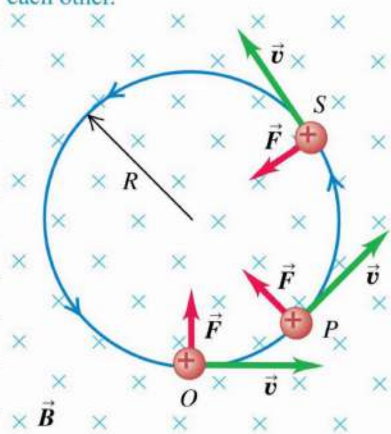
$B = \dfrac{ma}{qv\sin(90)}$

$$B = \frac{1.67\times10^{-27} \times 2\times10^{13}}{1.6\times10^{-19} \times 1\times10^{7}} = 2\times10^{-2}\ T$$

## Motion of charged particles in a magnetic field

when a charged particle moves in a magnetic field, it is acted on by the magnetic force, the force is always prependicular to the velocity, so It can't change the speed of the particle

A charge moving at right angles to a uniform $\vec{B}$ field moves in a circle at constant speed because $\vec{F}$ and $\vec{v}$ are always perpendicular to each other.




$$\Sigma F = F_B = m a_c$$

$$F_B = q\cancel{v}B = \frac{m v^{2}}{R}$$

$$qB = \frac{mv}{R}$$

$$R = \frac{mv}{qB}$$

$$\omega = \frac{v}{R} = \frac{qB}{m}$$

$$T = \frac{2\pi R}{v} = \frac{2\pi}{\omega} = \frac{2\pi m}{qB}$$ Periodic Time

Ex 29.2: A proton is moving in a circular orbit of radius 14 cm in a uniform 0.35 T magnetic field prependicular to the velocity of the proton. Find the speed of the proton?

$$F_B = q\cancel{v}B = \frac{m v^{\cancel{2}}}{R}$$

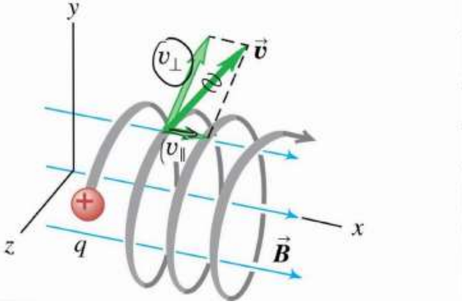
$$qB = \frac{mv}{R} \Rightarrow v = \frac{qBR}{m}$$

$$v = \frac{1.6\times10^{-19}\times0.35\times0.14}{1.67\times10^{-27}} = 4.7\times10^{6}\ \text{m/s}$$

## Helical motion:

- If the particle has velocity components parallel to and prependicular to the field, its path is a helix.
- The speed and kinetic energy of the particle remain constant

This particle's motion has components both parallel ($v_\parallel$) and perpendicular ($v_\perp$) to the magnetic field, so it moves in a helical path.



**Example 29.3** **Bending an Electron Beam** AM

In an experiment designed to measure the magnitude of a uniform magnetic field, electrons are accelerated from rest through a potential difference of 350 V and then enter a uniform magnetic field that is perpendicular to the velocity vector of the electrons. The electrons travel along a curved path because of the magnetic force exerted on them, and the radius of the path is measured to be 7.5 cm. (Such a curved beam of electrons is shown in Fig. 29.10.)

**(A)** What is the magnitude of the magnetic field?

**(B)** What is the angular speed of the electrons?

$\Delta K + \Delta U = 0$

$(\frac{1}{2} m v_f^2 - 0) + (q \Delta V) = 0$

$\frac{1}{2} m v_f^2 = -q \Delta V$

$v = \sqrt{\frac{-2q\Delta V}{m}} = \sqrt{\frac{-2 \times -1.6 \times 10^{-19} \times 350}{9.11 \times 10^{-31}}} = 1.11 \times 10^{7}$ m/s

$B = \frac{mv}{qR} = \frac{9.11 \times 10^{-31} \times 1.11 \times 10^{7}}{1.6 \times 10^{-19} \times 0.075} = 8.4 \times 10^{-4}$ T

$\omega = \frac{v}{R} = \frac{1.11 \times 10^{7}}{0.075} = 1.5 \times 10^{8}$ rad/s

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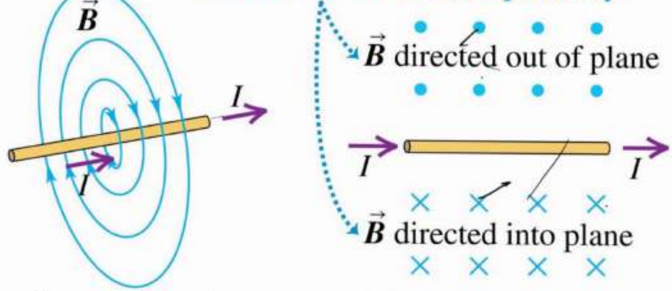
Ex 13: An electron moves in a circular path prependicular to a uniform magnetic field with a magnitude of 2mT. If the speed of the electron is $1.5 \times 10^{7}$ m/s, determine (a) the radius of the circular path and (B) the time interval required to complete one revolution?

a) $R = \frac{mv}{qB} = \frac{9.11 \times 10^{-31} \times 1.5 \times 10^{7}}{1.6 \times 10^{-19} \times 2 \times 10^{-3}} = 4.3 \times 10^{-2}\,m$

b) $T = \frac{2\pi R}{v} = \frac{2\pi \times 4.3 \times 10^{-2}}{1.5 \times 10^{7}} = 1.8 \times 10^{-8}\,s$

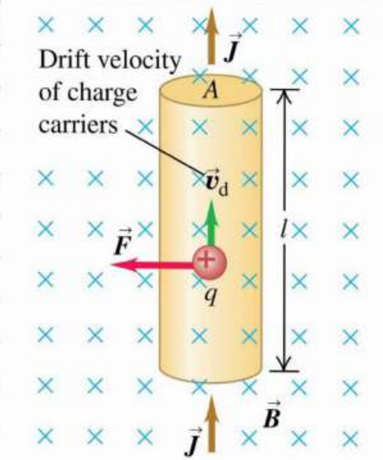
* Magnetic field of a straight current-carrying wire:

To represent a field coming out of or going into the plane of the paper, we use dots and crosses, respectively.




Perspective view Wire in plane of paper

## The magnetic force on a Current-Carring conductor :




- The figure shows a straight Segment of a conducting wire, with Length L and cross-sectional area A.
- The magnitude of the force on a single charge is $F = q v_d B$
- If the number of charges per unit volume is n, then the total force on all the charges in this segment is :

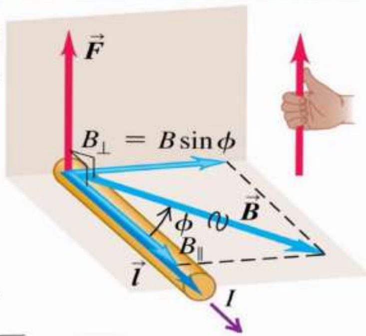
$$F = \underbrace{(nAL)}_{\text{number of charges}}(q v_d B) = \underbrace{n q v_d A}_{\text{Current (I)}} L B$$

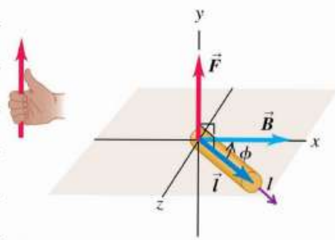
$$F = I L \times B$$

- The force is always prependicular to both the conductor and the field, with the direction determined by the same right-hand rule we used of a moving positive charge.

Force $\vec{F}$ on a straight wire carrying a positive current and oriented at an angle $\phi$ to a magnetic field $\vec{B}$:

- Magnitude is $F = IlB_{\perp} = IlB\sin\phi$.
- Direction of $\vec{F}$ is given by the right-hand rule.




The magnetic force on a segment of a straight wire can be represented as a vector product

$$\vec{F} = I\,\vec{L} \times \vec{B}$$

$\vec{F}$: Magnetic force on a straight wire segment

$I$: current

$\vec{L}$: vector length of segment (points in current direction)

$\vec{B}$: Magnetic field

Ex 39: A wire having a mass per unit length of 0.5g/cm carries a 2A current horizontally to the south. What are (a) the direction and (b) the magnitude of the minimum magnetic field needed to lift this wire vertically upward?

$$\frac{m}{L} = \frac{0.5\cancel{g}}{\cancel{cm}} \times \frac{10^{-3}\,kg}{1\cancel{g}} \times \frac{10^{2}\cancel{cm}}{m} = 0.05\,kg/m$$

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$B_{min} = F_{B\,min} = mg$

$ILB\cancel{\sin(90)}^{1} = mg$

$B = \frac{mg}{IL} = \frac{0.05 \times 10}{2} = 2.5 \times 10^{-3}$ T east direction

**33.** A conductor carrying a current $I = 15.0$ A is directed along the positive $x$ axis and perpendicular to a uniform magnetic field. A magnetic force per unit length of 0.120 N/m acts on the conductor in the negative $y$ direction. Determine (a) the magnitude and (b) the direction of the magnetic field in the region through which the current passes.

$I = 15A, \frac{F}{L} = 0.12\, N/m$

$F = I\vec{L} \times \vec{B} = ILB\sin(90)$

$\left(\frac{F}{L}\right) = IB$

$\frac{0.12}{15} = \frac{\cancel{15}B}{\cancel{15}} \Rightarrow B = 8 \times 10^{-3}$ T

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**34.** A wire 2.80 m in length carries a current of 5.00 A in a region where a uniform magnetic field has a magnitude of 0.390 T. Calculate the magnitude of the magnetic force on the wire assuming the angle between the magnetic field and the current is (a) 60.0°, (b) 90.0°, and (c) 120°.

$L = 2.8\,m,\ I = 5A,\ B = 0.39T,\ \theta = 60^\circ$

$F = I\vec{L} \times \vec{B} = ILB \sin(\theta)$

$F_1 = 5 \times 2.8 \times 0.39 \sin(60) = 4.13\,N$

$F_2 = 5 \times 2.8 \times 0.39 \sin(90) = 5.46\,N$

$F_3 = 5 \times 2.8 \times 0.39 \times \sin(120) = 4.73\,N$

Ex 35: A wire carries a steady current of 2.4 A. A straight section of the wire is 0.75m long and lies along the x axis within a uniform magnetic field, $\vec{B} = 1.6\hat{k}\,T$. If the current is in the positive x direction. What is the magnetic force on the section of the wire?

$I = 8.4A,\ L = 0.75m,\ B = 1.6\hat{k}\,T$

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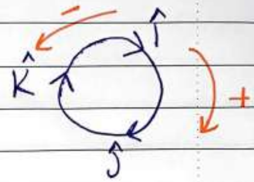
$$F = I\vec{L}\times\vec{B} = IL\hat{i}\times B\hat{k}$$




$$F = ILB(\hat{i}\times\hat{k})$$

$$F = (2.4)(0.75)(1.6)(-\hat{j})$$

$$F = -2.9\,\hat{j}\ N$$

---

$$F_B = q\vec{v}\times\vec{B} = q|v||B|\sin\theta$$

$$\theta = 90^\circ$$

$$qvB = \frac{mv^2}{R} \Longrightarrow R = \frac{mv}{qB}$$

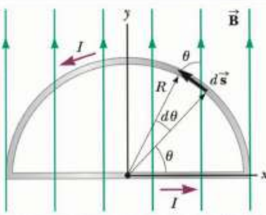
$$\omega = \frac{v}{R} = \frac{qB}{m}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi m}{qB} = \frac{2\pi R}{v}$$

$$F = I\vec{L}\times\vec{B} = ILB\sin\theta$$

**Example 29.4** **Force on a Semicircular Conductor**

A wire bent into a semicircle of radius $R$ forms a closed circuit and carries a current $I$. The wire lies in the $xy$ plane, and a uniform magnetic field is directed along the positive $y$ axis as in Figure 29.20. Find the magnitude and direction of the magnetic force acting on the straight portion of the wire and on the curved portion.




① Straight Portion:

$$F = I\int d\vec{L}\times\vec{B} = I\int dL\,B\sin(90)$$

$$= IB\int dL = IBL = IB(2R)$$

$L = 2R$

$$\boxed{F = 2IBR\,\hat{k}}$$

② Curve portion:

$$F = I\int d\vec{L}\times\vec{B} = -I\int dL\,B\sin\theta\,\hat{k}$$

$$= -IB\int dL\sin\theta\,\hat{k}$$

arc length $dL = R\,d\theta$

$$= -IBR\int_0^\pi \sin\theta\,d\theta\,\hat{k}$$

$$= -IBR\,(-\cos\theta)_0^\pi\,\hat{k}$$

$$= IBR\,(\cos\pi - \cos 0)\,\hat{k}$$

$$= IBR(-1-1) = -2IRB\,\hat{k}$$

$$\boxed{F = -2IRB\,\hat{k}}$$

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$$f_{net} = f_1 + f_2 = 2IRB\hat{k} - 2IRB\hat{k} = 0$$

**37. Review.** A rod of mass 0.720 kg and radius 6.00 cm rests on two parallel rails (Fig. P29.37) that are $d$ = 12.0 cm apart and $L$ = 45.0 cm long. The rod carries a

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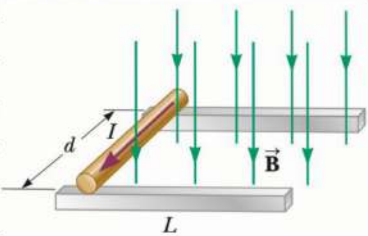
$v_i = 0,\ L = 45\times10^{-2}\,m,\ m = 0.72\,Kg,\ I = 48\,A$

$d = 12\times10^{-2}\,m,\ B = 0.24\,T,\ v_f = ?$

$$v_f^2 = \cancel{v_i^2}^{\,0} + 2a\Delta x$$

$$v_f^2 = 2aL = 0.9a$$

$$\Sigma F = IdB = ma$$

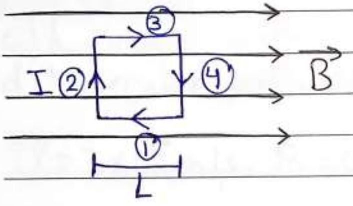



$$a = \frac{IdB}{m} = \frac{48\times12\times10^{-2}\times0.24}{0.72}$$

$$\boxed{a = 1.92\,m/s^2}$$

$$v_f = \sqrt{0.9a} = \sqrt{0.9\times1.92} = 1.31\,m/s$$

Ex: There is a current $I$ flowing in a clockwise direction in a square loop of wire that is in the plane of the paper. If the magnetic field $B$ is toward the right, and if each side of the loop has length $L$, then the net magnetic force acting on the loop is:




① $F = ILB\sin\theta$, $\theta = 180$

$F = 0$

② $F = ILB\sin\theta$, $\theta = 90$

$F = IBL(-\hat{k})$

$F = -ILB\hat{k}$

③ $F = ILB\sin\theta$, $\theta = 0$

$F = 0$

④ $F = ILB\sin\theta$, $\theta = 90$

$F = ILB(+\hat{k})$

a) $2ILB$

b) $ILB$

c) $IBL^2$

d) zero

$\therefore F_{net} = -ILB\hat{k} + ILB\hat{k} = 0$

Ex: A proton moving with a speed of $3\times10^{5}$ m/s prependicular to a uniform magnetic field of 0.2T will follow which of the paths described below? ($q_p = 1.6\times10^{-19}$ C, $m_p = 1.67\times10^{-27}$ Kg)

a) A straight line path
b) A circular path of 1.6 cm radius
c) A circular path of 3.1 cm radius
d) A circular path of 0.78 cm radius

$v = 3\times10^{5}$ m/s, $B = 0.2$T

$$R = \frac{mv}{qB} = \frac{1.67\times10^{-27}\times3\times10^{5}}{1.6\times10^{-19}\times0.2} = 0.0156\,m$$

$R = 1.56$ cm $\approx 1.6$ cm

Ex: The path of a charged particle moving parallel to a uniform magnetic field will be a:

a) straight line
b) Circle
c) ellipse
d) parabola

$\xrightarrow{} v$ $\quad$ $\xleftarrow{} v$

$\xrightarrow{} B$ $\quad$ $\xrightarrow{} B$

$\theta = 0$ $\quad$ $\theta = 180$

$F = 0$

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Ex: If a charged particle is moving in a uniform magnetic field, its path can be:

a) a straight line
b) a circle
c) a helix
d) any of the above

Ex: A proton with initial Kinetic energy E is moving in circular motion in a uniform magnetic field. when it has completed one eighth of a revolution. What is its Kinetic energy?

a) 1.4 E
b) 0.71E
c) E
d) The value is not given

$$E = \frac{1}{2} m v^2$$

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Ex: A proton which moves prependicular to a magnetic field of 1.2T in a circular path of radius 0.08 m, has what speed?
$q_p = 1.6 \times 10^{-19}$ C, $m_p = 1.67 \times 10^{-27}$ Kg

a) $3.4 \times 10^{6}$ m/s
b) $4.6 \times 10^{6}$ m/s
c) $9.6 \times 10^{6}$ m/s
d) $9.2 \times 10^{6}$ m/s

$$v = \frac{RqB}{m}$$

$$v = \frac{0.08 \times 1.6 \times 10^{-19} \times 1.2}{1.67 \times 10^{-27}}$$

$$v = 9.2 \times 10^{6} \text{ m/s}$$

## Chapter 30: Sources of the magnetic field

The immense cylinder in this photograph is a current carrying coil, or solenoid, that generates a uniform magnetic field in its interior as part of an experiment at CERN, the European Organization for Nuclear Res arch

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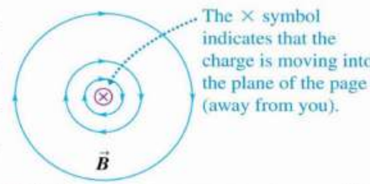
# The magnetic field of a moving charge:

(b) View from behind the charge




* A moving charge generates a magnetic field that depends on the velocity of the charge and the distance from the charge.

magnetic constant — charge — velocity

magnetic field due to a point charge with constant velocity

$$\boxed{\vec{B} = \frac{\mu_0}{4\pi}\frac{q\vec{v}\times\hat{r}}{r^2}}$$

unit vector from point charge toward where field is measured

distance from point charge to where field is measured

* $\mu_0$ is a constant called permability of free space. $\mu_0 = 4\pi \times 10^{-7}\ T.m/A$

## Magnetic field of a current element:

* The total magnetic field of several moving charges is the vector sum of each field.

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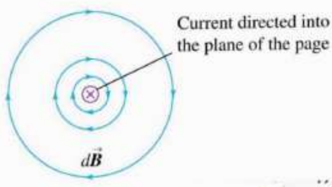
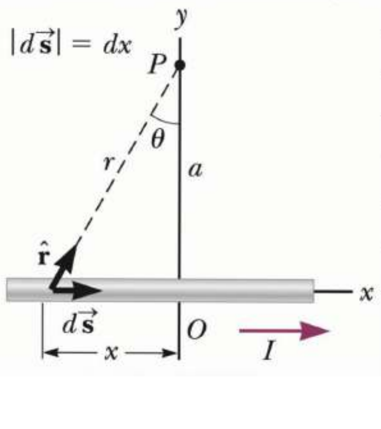
(b) View along the axis of the current element




The magnetic field caused by a short segment of a current-carring conductor is found using the Law of Biot and savart.

$$\underbrace{d\vec{B}}_{\text{magnetic field due to an infinitesimal current element}} = \frac{\overbrace{\mu_0}^{\text{magnetic constant}}\ \overbrace{I}^{\text{current}}\ \overbrace{d\vec{L}}^{\text{vector length of element (points in current direction)}} \times \underbrace{\hat{r}}_{\text{unit vector from element toward where field is measured}}}{4\pi \underbrace{r^2}_{\text{Distance from element to where field is measured}}}$$

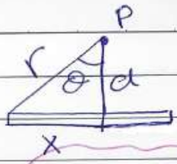
Ex 30.1: Consider a thin, slight wire of finite length carring a constant current I and placed along the x axis Determine the magnitude and direction of the magnetic field at point P due to this current




$$d\vec{B} = \frac{\mu_0 I \, d\vec{s} \times \hat{r}}{4\pi r^2}$$

$$d\vec{s} \times \hat{r} = |ds|\,|\hat{r}|\sin(\Phi)$$
$$= ds\cos(\theta)$$

$$d\vec{B} = \frac{\mu_0 I \, ds \cos(\theta)}{4\pi r^2}$$




$$d\vec{B} = \frac{\mu_0 I \, dx \cos\theta}{4\pi r^2} \hat{k}$$

$$\cos\theta = \frac{a}{r} \rightarrow r = \frac{a}{\cos\theta}, \quad r^2 = \frac{a^2}{\cos^2\theta}$$

$$\tan\theta = \frac{-x}{a} \rightarrow x = -a\tan\theta$$

$$dx = -a\sec^2\theta \, d\theta$$

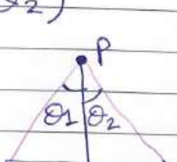
$$dx = \frac{-a \, d\theta}{\cos^2\theta}$$

$$d\vec{B} = \frac{\mu_0 I}{4\pi} \left(\frac{-a \, d\theta}{\cos^2\theta}\right)\left(\frac{\cos^2\theta}{a^2}\right) \cdot \cos\theta$$

$$d\vec{B} = \frac{-\mu_0 I}{4\pi a} \cos\theta \, d\theta$$

$$B = -\frac{\mu_0 I}{4\pi a} \int_{\theta_1}^{\theta_2} \cos\theta \, d\theta$$

$$B = \frac{\mu_0 I}{4\pi a} (\sin\theta_1 - \sin\theta_2)$$



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$\Rightarrow$ In the limit of infinite wire

$\theta_1 \to \frac{\pi}{2}$ , $\theta_2 \to -\frac{\pi}{2}$

$$B = \frac{\mu_0 I}{4\pi a}\left(\sin\left(\frac{\pi}{2}\right) - \sin\left(-\frac{\pi}{2}\right)\right)$$

$$B = \frac{\mu_0 I}{4\pi a}(1+1) = \frac{2\mu_0 I}{4\pi a} = \frac{\mu_0 I}{2\pi a}$$

## Example 30.2 Magnetic Field Due to a Curved Wire Segment

Calculate the magnetic field at point $O$ for the current-carrying wire segment shown in Figure 30.4. The wire consists of two straight portions and a circular arc of radius $a$, which subtends an angle $\theta$.

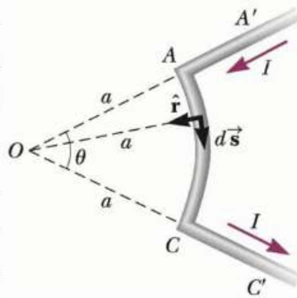
$$d\vec{B} = \frac{\mu_0 I \, d\vec{s} \times \hat{r}}{4\pi r^2}$$

$$d\vec{s} \times \hat{r} = |ds|\,|\hat{r}| \sin(90) = ds$$

$$d\vec{B} = \frac{\mu_0 I \, ds}{4\pi a^2}$$

$$B = \frac{\mu_0 I}{4\pi a^2}\int ds = \frac{\mu_0 I}{4\pi a^2} s$$

$$s = \text{arc length} = a\theta$$

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$$B = \frac{\mu_0 I}{4\pi a^2}(a\theta) = \frac{\mu_0 I \theta}{4\pi a}$$

⇒ If this is a quarter of a circle

$$\theta = \frac{1}{4}(2\pi) = \frac{\pi}{2}$$

$$B = \frac{\mu_0 I}{4\pi a}\left(\frac{\pi}{2}\right) = \frac{\mu_0 I}{8a}$$

⇒ If semi circle

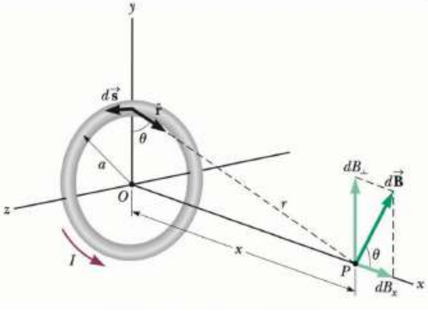
$$\theta = \frac{1}{2}(2\pi) = \pi$$

$$B = \frac{\mu_0 I}{4\pi a}(\pi) = \frac{\mu_0 I}{4a}$$

Ex 30.3: Consider a circular wire loop of radius a located in the yz plane and carring a steady current I. Calculate the magnetic field at an axial point P a distance x from the center of the loop?

$$d\vec{B} = \frac{\mu_0 I \, d\vec{s} \times \hat{r}}{4\pi r^2}$$

$$d\vec{s} \times \hat{r} = |ds||\hat{r}| \sin\left(\frac{\pi}{2}\right)$$
$$= ds \cos\theta$$

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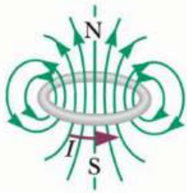
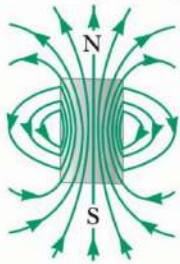
$$r = \sqrt{a^2+x^2}, \quad r^2 = a^2 + x^2$$

$$d\vec{B}_x = \frac{\mu_0 I \, ds}{4\pi(a^2+x^2)} \cos\theta$$

$$B_x = \frac{\mu_0 I}{4\pi} \int \frac{ds \cos\theta}{a^2+x^2}$$

$$\cos\theta = \frac{a}{(a^2+x^2)^{\frac{1}{2}}}$$




$$B_x = \frac{\mu_0 I}{4\pi} \oint \frac{a \, ds}{(a^2+x^2)^{\frac{3}{2}}}$$

$$B_x = \frac{\mu_0 I a}{4\pi (a^2+x^2)^{\frac{3}{2}}} \oint ds$$

$$B_x = \frac{\mu_0 I a}{4\pi (a^2+x^2)^{\frac{3}{2}}} (S) = \frac{\mu_0 I a}{4\pi (a^2+x^2)^{\frac{3}{2}}} (2\pi a)$$

$$\boxed{B_x = \frac{\mu_0 I a^2}{2 (a^2+x^2)^{\frac{3}{2}}}}$$

$$dB_y = dB \sin\theta = \frac{\mu_0 I \sin\theta \, ds}{4\pi (x^2+a^2)}$$

$$B_x = \frac{\mu_0 I a^2}{2(a^2+x^2)^{\frac{3}{2}}}$$

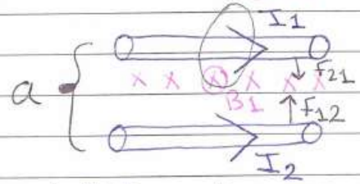
- What is the magnetic field at point P such that $x >> a$

$$B_x = \frac{\mu_0 I a^2}{2(x^2)^{\frac{3}{2}}} = \frac{\mu_0 I a^2}{2x^3}$$

- what is the magnetic field at the center of the loop $(x = 0)$

$$B_x = \frac{\mu_0 I a^2}{2(a^2)^{\frac{3}{2}}} = \frac{\mu_0 I a^2}{2a^3} = \frac{\mu_0 I}{2a}$$

The magnetic force between two parallel conductors:




$$f_{12} = I_2 L_2 \times B_1$$

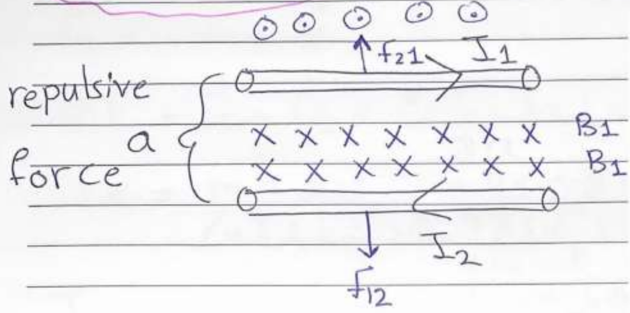
$$\theta = 90$$

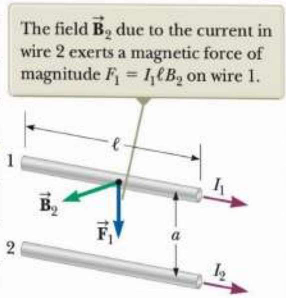
$$f_{12} = I_2 L_2 B_1$$

$$B_1 = \frac{\mu_0 I_1}{2\pi a}$$

$$f_{12} = \frac{I_2 L \mu_0 I_1}{2\pi a} = \frac{\mu_0 I_1 I_2 L}{2\pi a}$$

$$\boxed{\frac{f_{12}}{L} = \frac{\mu_0 I_1 I_2}{2\pi a}} \qquad \frac{|f_{21}|}{L} = \frac{|f_{12}|}{L} = \frac{\mu_0 I_1 I_2}{2\pi a}$$

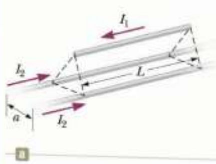


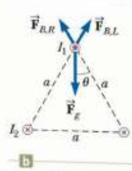




$$|f_{12}| = |f_{21}| = \frac{\mu_0 I_1 I_2}{2\pi a}$$

**Example 30.4** **Suspending a Wire** AM

Two infinitely long, parallel wires are lying on the ground a distance $a = 1.00$ cm apart as shown in Figure 30.8a. A third wire, of length $L = 10.0$ m and mass 400 g, carries a current of $I_1 = 100$ A and is levitated above the first two wires, at a horizontal position midway between them. The infinitely long wires carry equal currents $I_2$ in the same direction, but in the direction opposite that in the levitated wire. What current must the infinitely long wires carry so that the three wires form an equilateral triangle?




$a = 10 \times 10^{-2}$ m, $L = 10$ m, $m = 0.4$ Kg, $I_1 = 100$ A

$$\vec{F}_B = 2\left(\frac{\mu_0 I_1 I_2 L}{2\pi a}\right)\cos\theta\,\hat{k}$$

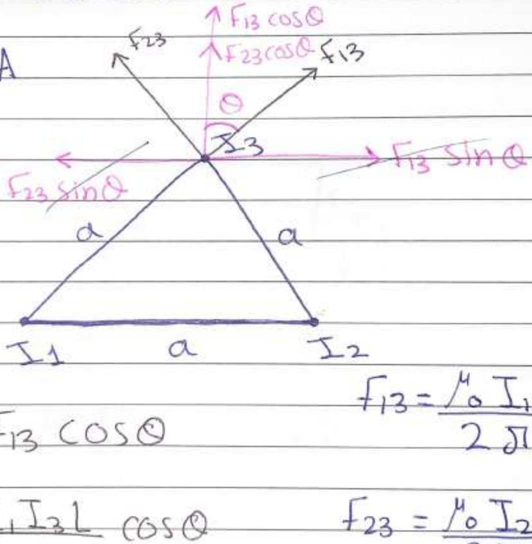
$F_B = \frac{\mu_0 I_1 I_2}{\pi a} L\cos\theta \, \hat{k}$

$F_g = -mg\,\hat{k}$

$\sum \vec{F} = \vec{F}_B + \vec{F}_g = \frac{\mu_0 I_1 I_2}{\pi a} L\cos\theta - mg = 0$

$I_2 = \frac{mg\pi a}{\mu_0 I_1 L\cos\theta} = \frac{0.4\times10\pi\times10\times10^{-2}}{4\pi\times10^{-7}\times100\times10\cos(30)}$

$I_2 = 113A$




$F_{13} = \frac{\mu_0 I_1 I_3 L}{2\pi a}$

$F_{net} = 2F_{13}\cos\theta$

$F_{net} = \frac{\mu_0 I_1 I_3 L}{\pi a}\cos\theta$

$F_{23} = \frac{\mu_0 I_2 I_3 L}{2\pi a}$

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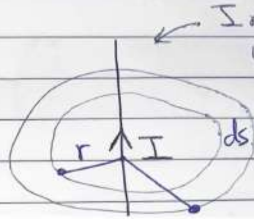
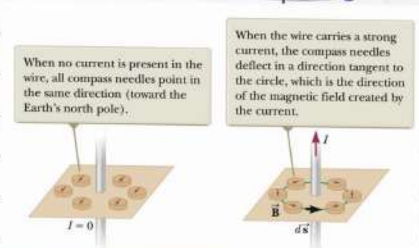



$$\oint B \cdot ds = \mu_0 I$$

$$B(2\pi r) = \mu_0 I$$

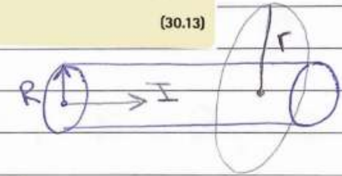
$$B = \frac{\mu_0 I}{2\pi r}$$




The line integral of $\vec{\mathbf{B}} \cdot d\vec{\mathbf{s}}$ around any closed path equals $\mu_0 I$, where $I$ is the total steady current passing through any surface bounded by the closed path:

$$\oint \vec{\mathbf{B}} \cdot d\vec{\mathbf{s}} = \mu_0 I \qquad (30.13)$$

☆ Find B:




$$\oint B \cdot ds = \mu_0 I$$

$$B(2\pi r) = \frac{\mu_0 I}{2\pi r}, \quad r > R$$

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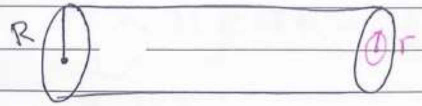
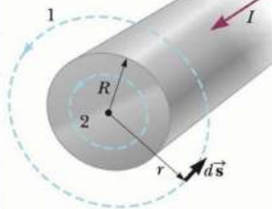
A long, straight wire of radius $R$ carries a steady current $I$ that is uniformly distributed through the cross section of the wire (Fig. 30.13). Calculate the magnetic field a distance $r$ from the center of the wire in the regions $r \geq R$ and $r < R$.




when $r < R$:

$$\oint B \cdot ds = \mu_0 I'$$

$$B(2\pi r) = \mu_0 I'$$

$$B = \frac{\mu_0 I'}{2\pi r}$$

$$I \neq I'$$

$$\frac{I'}{\pi r^2} = \frac{I}{\pi R^2}$$

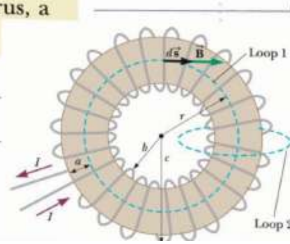
$$\frac{I'}{I} = \frac{r^2}{R^2} I$$

$$B = \frac{\mu_0}{2\pi r}\left(\frac{r^2 I}{R^2}\right)$$

$$\boxed{B = \frac{\mu_0 I}{2\pi R^2} r} \quad r < R, \; I' < I$$

## Example 30.6 The Magnetic Field Created by a Toroid

A device called a *toroid* (Fig. 30.15) is often used to create an almost uniform magnetic field in some enclosed area. The device consists of a conducting wire wrapped around a ring (a *torus*) made of a nonconducting material. For a toroid having $N$ closely spaced turns of wire, calculate the magnetic field in the region occupied by the torus, a distance $r$ from the center.

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$$\oint B \cdot ds = B \oint ds = B(2\pi r) = \mu_0 N I$$

$$\boxed{B = \frac{\mu_0 N I}{2\pi r}}$$

## The magnetic field of solenoid

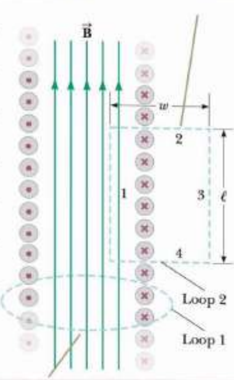
$$\oint \vec{B} \cdot d\vec{s} = B \int ds = BL \leftarrow \text{length}$$

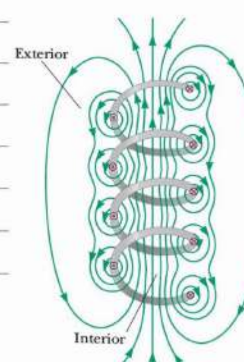
$$\oint \vec{B} \cdot d\vec{s} = BL = \mu_0 N I$$

number of turns

$$\boxed{B = \frac{\mu_0 N I}{L} = \mu_0 n I}$$

$n = \frac{N}{L} \rightarrow$ number of turns per unit length



# Gauss's Law in Magnetism:

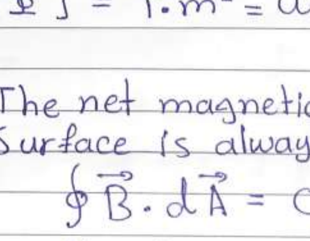
$$\Phi_B = \int \vec{B} \cdot d\vec{A} = BA\cos\theta$$

$$[\Phi] = T \cdot m^2 = \text{weber} = Wb$$

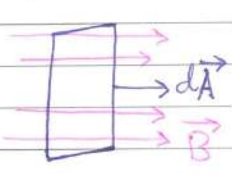
- The net magnetic flux through any closed Surface is always zero

$$\oint \vec{B} \cdot d\vec{A} = 0$$

- The flux through the plane is zero when the magnetic field is parallel to the plane surface




- The flux through the plane is a maximum when the magnetic field is prependicular to the plane



## Example 30.7 Magnetic Flux Through a Rectangular Loop

A rectangular loop of width $a$ and length $b$ is located near a long wire carrying a current $I$ (Fig. 30.21). The distance between the wire and the closest side of the loop is $c$. The wire is parallel to the long side of the loop. Find the total magnetic flux through the loop due to the current in the wire.

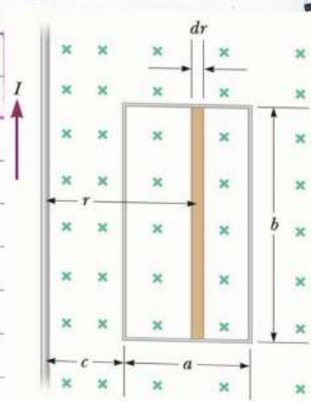
$$\Phi_B = \int \vec{B} \cdot d\vec{A} = \int \frac{\mu_0 I}{2\pi r} dA$$

$$\Phi_B = \int \frac{\mu_0 I}{2\pi r} b \, dr$$

$$\Phi_B = \frac{\mu_0 I b}{2\pi} \int_c^{a+c} \frac{dr}{r}$$

$$\Phi_B = \frac{\mu_0 I b}{2\pi} \ln r \Big]_a^{a+c} = \frac{\mu_0 I b}{2\pi} \ln \left(\frac{a+c}{c}\right)$$

$$\boxed{\Phi_B = \frac{\mu_0 I b}{2\pi} \ln \left(1 + \frac{a}{c}\right)}$$



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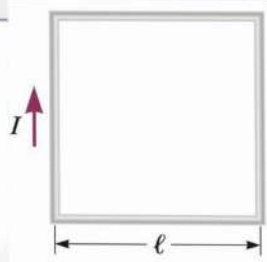
**5.** (a) A conducting loop in the shape of a square of edge length $\ell = 0.400$ m carries a current $I = 10.0$ A as shown in Figure P30.5. Calculate the magnitude and direction of the magnetic field at the center of the square. (b) **What If?** If this conductor is reshaped to form a circular loop and carries the same current, what is the value of the magnetic field at the center?




a) $B = \frac{\mu_0 I}{4\pi a}(\sin\theta_1 - \sin\theta_2)$

where $\theta_1 = 45^\circ$, $\theta_2 = -45^\circ$, $a = \frac{L}{2}$

$B_{total} = \frac{4\mu_0 I(2)}{4\pi L}(\sin\theta_1 - \sin\theta_2)$

$B_{total} = \frac{2\times 4\pi\times 10^{-7}\times 10}{\pi\times 0.4}(\sin(45) - \sin(-45))$

$B_{total} = 2.83\times 10^{-5}$ T into the page

b) For a single turn with $4L = 2\pi R$

$B = \frac{\mu_0 I}{2R}$, $R = \frac{4L}{2\pi}$

$B = \frac{\mu_0 \pi I}{4L} = \frac{4\pi\times 10^{-7}\times 10\times\pi}{4(0.4)}$

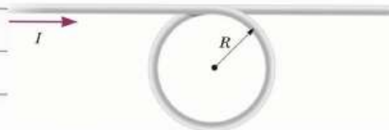
$B = 2.47\times 10^{-5}$ T into the page

N O T E B O O K

**7.** A conductor consists of a circular loop of radius $R$ = 15.0 cm and two long, straight sections as shown in Figure P30.7. The wire lies in the plane of the paper and carries a current $I$ = 1.00 A. Find the magnetic field at the center of the loop.

$B_{total}$ = B straight wire + B ring

$$B = \frac{\mu_0 I}{2\pi R} + \frac{\mu_0 I}{2R}$$



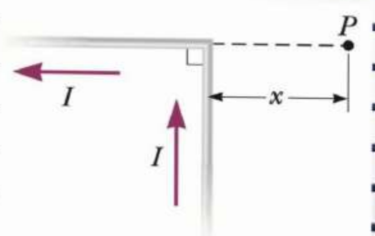

$$B = \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 0.15} + \frac{4\pi \times 10^{-7} \times 1}{2 \times 0.15}$$

$B = 5.52 \times 10^{-6} = 5.52\ \mu T$ into the page

**10.** An infinitely long wire carrying a current $I$ is bent at a right angle as shown in Figure P30.10. Determine the magnetic field at point $P$, located a distance $x$ from the corner of the wire.

The vertical section of wire:

$$B = \frac{\mu_0 I}{2(2\pi x)}$$



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The horizontal section of wire $d\vec{s} \times \hat{r} = 0$

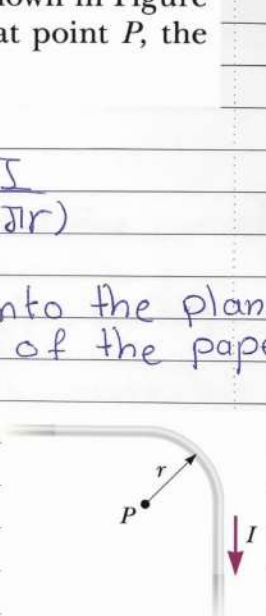
$$B = \frac{\mu_0 I}{4\pi x} \text{ into the paper}$$

**11.** A long, straight wire carries a current *I*. A right-angle bend is made in the middle of the wire. The bend forms an arc of a circle of radius *r* as shown in Figure P30.11. Determine the magnetic field at point *P*, the center of the arc.

$$\vec{B} = \frac{\mu_0 I}{2(2\pi r)} + \frac{\mu_0 I}{4(2r)} + \frac{\mu_0 I}{2(2\pi r)}$$

$$\vec{B} = \frac{\mu_0 I}{2r}\left(\frac{1}{\pi} + \frac{1}{4}\right) \text{ into the plane of the paper}$$

$$\vec{B} = 0.28415 \frac{\mu_0 I}{r}$$

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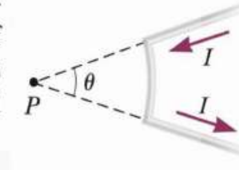
**13.** A current path shaped as shown in Figure P30.13 produces a magnetic field at $P$, the center of the arc. If the arc subtends an angle of $\theta = 30.0°$ and the radius of the arc is 0.600 m, what are the magnitude and




$$B = \int d\vec{B} = \int \frac{\mu_0 I |d\vec{s} \times \hat{r}|}{4\pi r^2} = \frac{\mu_0 I}{4\pi r^2} \int 1 |ds|$$

$$B = \frac{\mu_0 I}{4\pi r^2} s$$

* where s is the arc length of the curved wire:

$$s = r\theta = 0.6(30°)\left(\frac{\pi}{180°}\right) = 0.314$$

* what is the direction of the magnetic field at the center of the arc if $I = 3A$?

$$B = \frac{4\pi \times 10^{-7} \times 3 \times 0.314}{4\pi (0.6)^2} = 2.62 \times 10^{-7}\ T$$

into the page

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Ex: 19

(a) At the point half way between the two wires:

$$B_{net} = -B_1 - B_2 = -\left(\frac{\mu_0 I_1}{2\pi r_1} + \frac{\mu_0 I_2}{2\pi r_2}\right)$$

$$B_{net} = -\left(\frac{\mu_0}{2\pi r}\right)(I_1 + I_2)$$

$$B_{net} = -\left(\frac{4\pi \times 10^{-7}}{2\pi \times 5 \times 10^{-2}}\right)(10) = 40 \times 10^{-6}\ T$$

into the page

(b) At point $P_1$:

$$B_{net} = B_1 - B_2 = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right)$$

$$B_{net} = \frac{4\pi \times 10^{-7}}{2\pi}\left(\frac{5}{0.1} - \frac{5}{0.2}\right) = 5 \times 10^{-6}\ T$$

out of the page

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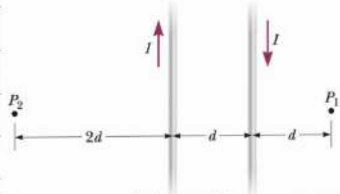
**19.** The two wires shown in Figure P30.19 are separated by $d = 10.0$ cm and carry currents of $I = 5.00$ A in opposite directions. Find the magnitude and direction of the net magnetic field (a) at a point midway between the wires; (b) at point $P_1$, 10.0 cm to the right of the wire on the right; and (c) at point $P_2$, $2d = 20.0$ cm to the left of the wire on the left.




(a) At the point half way between the two wires:

$$B_{net} = -B_1 - B_2 = -\left(\frac{\mu_0 I_1}{2\pi r_1} + \frac{\mu_0 I_2}{2\pi r_2}\right)$$

$$B_{net} = -\left(\frac{\mu_0}{2\pi r}\right)(I_1 + I_2)$$

$$B_{net} = -\left(\frac{4\pi \times 10^{-7}}{2\pi \times 5 \times 10^{-2}}\right)(10) = 40 \times 10^{-6}\ T$$

into the page

(b) At point $P_1$:

$$B_{net} = B_1 - B_2 = \frac{\mu_0}{2\pi}\left(\frac{I_1}{r_1} - \frac{I_2}{r_2}\right)$$

$$B_{net} = \frac{4\pi \times 10^{-7}}{2\pi}\left(\frac{5}{0.1} - \frac{5}{0.2}\right) = 5 \times 10^{-6}\ T$$

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(c) At point $P_2$:

$$B_{net} = -B_1 + B_2 = \frac{\mu_0}{2\pi}\left(-\frac{I_1}{r_1} + \frac{I_2}{r_2}\right)$$

$$B_{net} = \frac{4\pi \times 10^{-7}}{2\pi}\left(\frac{-5}{0.3} + \frac{5}{0.2}\right) = 1.67 \times 10^{-6} T$$

out of the page

**21.** Two long, parallel conductors, separated by 10.0 cm, W carry currents in the same direction. The first wire carries a current $I_1 = 5.00$ A, and the second carries $I_2 = 8.00$ A. (a) What is the magnitude of the magnetic field created by $I_1$ at the location of $I_2$? (b) What is the force per unit length exerted by $I_1$ on $I_2$? (c) What is the magnitude of the magnetic field created by $I_2$ at the location of $I_1$? (d) What is the force per length exerted by $I_2$ on $I_1$?

(a) $\vec{B} = \frac{\mu_0 I}{2\pi r}\hat{k} = \frac{4\pi \times 10^{-7} \times 5}{2\pi(0.1)}\hat{k}$

$\vec{B} = 1 \times 10^{-5}$ T out of the page

(b) $\vec{F_B} = I_2 \vec{L} \times \vec{B} = 8(1\hat{i} \times (10^{-5})\hat{k})$

$\vec{F_B} = 8 \times 10^{-5}$ N toward the first wire

N O T E B O O K

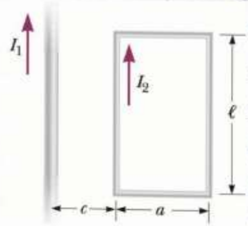
(c) $\vec{B} = \frac{\mu_0 I}{2\pi r}(-\hat{k}) = \frac{4\pi \times 10^{-7} \times 8}{2\pi \times 0.1}(-\hat{k})$

$\vec{B} = 1.6 \times 10^{-5}$ T into the page

(d) $\vec{F_B} = I_1 \vec{l} \times \vec{B} = 5[(1)\hat{i} \times (1.6 \times 10^{-5})(-\hat{k})]$

$\vec{F_B} = 8 \times 10^{-5}$ N $\hat{j}$ towards the second wire

**25.** **M** In Figure P30.25, the current in the long, straight wire is $I_1 = 5.00$ A and the wire lies in the plane of the rectangular loop, which carries a current $I_2 = 10.0$ A. The dimensions in the figure are $c = 0.100$ m, $a = 0.150$ m, and $\ell = 0.450$ m. Find the magnitude and direction of the net force exerted on the loop by the magnetic field created by the wire.

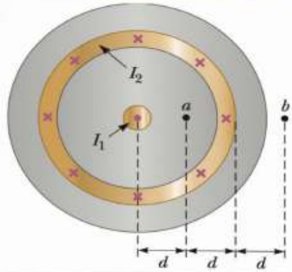



$\vec{F} = \vec{F_1} + \vec{F_2} = \frac{\mu_0 I_1 I_2 l}{2\pi}\left(\frac{1}{c+a} - \frac{1}{c}\right)\hat{i}$

$\vec{F} = \frac{4\pi \times 10^{-7} \times 5 \times 10 \times 0.45}{2\pi}\left(\frac{1}{0.25} - \frac{1}{0.1}\right)$

$\vec{F} = -27 \times 10^{-6}\,\hat{i}$ N

**31.** W Figure P30.31 is a cross-sectional view of a coaxial cable. The center conductor is surrounded by a rubber layer, an outer conductor, and another rubber layer. In a particular application, the current in the inner conductor is $I_1 = 1.00$ A out of the page and the current in the outer conductor is $I_2 = 3.00$ A into the page. Assuming the distance $d = 1.00$ mm, determine the magnitude and direction of the magnetic field at (a) point $a$ and (b) point $b$.




(a) from Ampere's law, the magnetic field at point a is given by $B_a = \frac{\mu_0 I_a}{2\pi r_a}$, where $I_a$ is the net current through the area of the circle radius $r_a$, In this case $I_a = 1$ A Out of the page

$\vec{B}_a = \frac{4\pi \times 10^{-7} \times 1}{2\pi \times 10^{-3}} = 200 \times 10^{-6}$ T toward top of page

(b) Similarly at point b: $B_b = \frac{\mu_0 I_b}{2\pi r_b}$

where $I_b$ is the net current through the area of the circle having radius $r_b$ Taking out of the page as positive, $I_b = 1 - 3 = -2$ A

$I_b = 2$ A into the page

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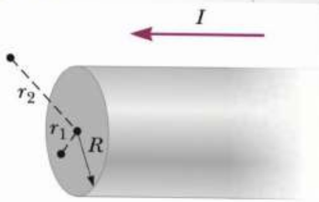
$\vec{B}_b = \frac{4\pi \times 10^{-7} \times 2}{2\pi(3\times 10^{-3})} = 133 \times 10^{-6}\,T$ toward bottom of page

**38.** A long, cylindrical conductor of radius $R$ carries a current $I$ as shown in Figure P30.38. The current density $J$, however, is not uniform over the cross section of the conductor but rather is a function of the radius according to $J = br$, where $b$ is a constant. Find an expression for the magnetic field magnitude $B$ (a) at a distance $r_1 < R$ and (b) at a distance $r_2 > R$, measured from the center of the conductor.

$\oint \vec{B}\cdot d\vec{s} = \mu_o I \quad , \quad I = \int J\, dA$

$\oint \vec{B}\, d\vec{s} = \mu_o \int J\, dA$




(a) $r_1 < R$

$2\pi r_1 B = \mu_o \int_0^{r_1} br(2\pi r\, dr)$

$2\pi r_1 B = \mu_o (2\pi b)\left[\frac{r_1^3}{3} - 0\right]$

$\Rightarrow B = \frac{1}{3}\mu_o b r_1^2$ (inside)

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(b) $r_2 > R$

$$2\pi r_2 B = \mu_0 \int_0^R br(2\pi r dr)$$

$$2\pi r_2 B = \mu_0 (2\pi b)\left[\frac{R^3}{3} - 0\right]$$

$$\vec{B} = \frac{\mu_0 b R^3}{3 r_2} \text{ (outside)}$$

→ If $J$ is constant:

$$\int B\,ds = J \int dA$$

$$B(2\pi r_1) = \mu_0 J A$$

$$B(2\pi r_1) = \mu_0 J (\pi r^2)$$

$$\boxed{B = \frac{J r_1 \mu_0}{2}}$$

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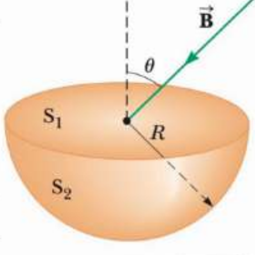
**41.** A long solenoid that has 1 000 turns uniformly distributed over a length of 0.400 m produces a magnetic field of magnitude $1.00 \times 10^{-4}$ T at its center. What current is required in the windings for that to occur?

The magnetic field at the center of a Solenoid is $B = \mu_0 \frac{N}{L} I$

$n = \frac{N}{L} = \frac{1000}{0.4} = 2500$

$I = \frac{B}{\mu_0 n} = \frac{10^{-4}}{4\pi \times 10^{-7} \times 2500} = 31.8 \times 10^{-3}$ A

**46.** Consider the hemispherical closed surface in Figure P30.46. The hemisphere is in a uniform magnetic field that makes an angle $\theta$ with the vertical. Calculate the magnetic flux through (a) the flat surface $S_1$ and (b) the hemispherical surface $S_2$.




$\Phi = \vec{B} \cdot \vec{A} = BA\cos\theta = -B(\pi R^2)\cos\theta$

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(a) The magnetic flux through the flat Surface $S_1$ is:

$$(\Phi_B)_{flat} = \vec{B} \cdot \vec{A} = B\pi R^2 \cos(180-\theta)$$

$$(\Phi_B)_{flat} = -B\pi R^2 \cos\theta$$

(b) The net flux out of the closed surface is zero

$$(\Phi_B)_{flat} + (\Phi_B)_{curved} = 0$$

$$(\Phi)_{curved} = -(\Phi)_{flat}$$

$$(\Phi)_{curved} = B\pi R^2 \cos(\theta)$$

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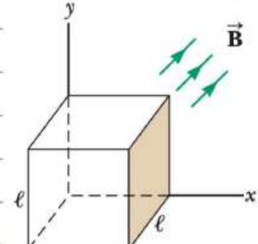
**47.** A cube of edge length $\ell = 2.50$ cm is positioned as shown in Figure P30.47. A uniform magnetic field given by $\vec{\mathbf{B}} = (5\hat{\mathbf{i}} + 4\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ T exists throughout the region. (a) Calculate the magnetic flux through the shaded face. (b) What is the total flux through the six faces?




(a) $\Phi = \vec{B} \cdot \vec{A}$

$\Phi = (5\hat{i} + 4\hat{j} + 3\hat{k})\,(l^2\,\hat{i})$

$\Phi = 5L^2 = 5(2.5 \times 10^{-2})^2 = 3.12 \times 10^{-3}$ Wb

(b) For a closed surface, $\oint B \cdot d\vec{A} = 0$

$(\Phi_B)_{total} = \oint B \cdot dA = 0$