Physics 2 — Chapter 23 — د. جميل حجا

* properties of electric charges

1) two kind of charges → positive (glass rubbed with silk)
زجاج — حرير
→ negative (rubber rubbed with fur)
مطاط — فرو

2) like charges repel one another and unlike charges attrack one another
تتنافر — تجذب

3) electric charge is conserved: the electrified state due to charge transfer from object to another

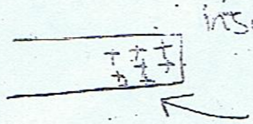
4) uncharged matter (nutral matter): contains # of negative charges (electrons) equal to # of positive charges (protons)
المادة المتعادلة

5) electric charge is quantized: electric charge exists as discreate packets $q = Ne$; N: integer
e: the charge of the smallest charge $\equiv$ electron charge $\equiv$ proton charge

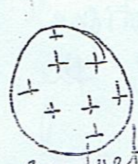
6) the ability for conducting charges (قابلية التوصيل)

material
→ conductors (q moves freely)
→ insulators (q cannot move freely)
→ semi conductors (Si, Ge, C, ...)

insulator



only this place is charged

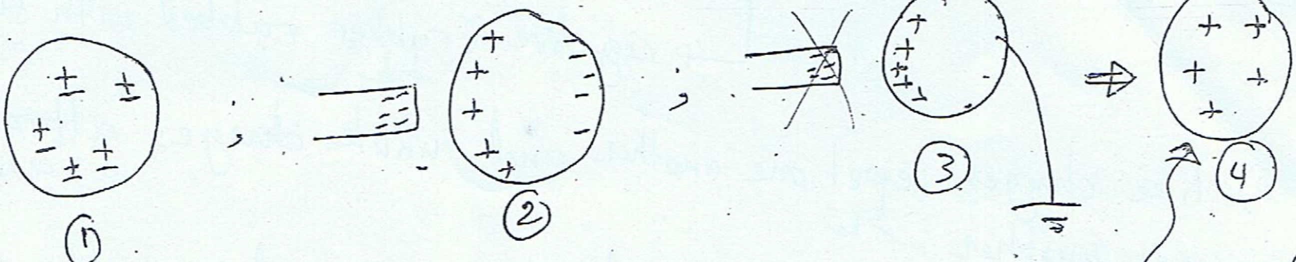


conductor

the charge distribute its self over the entire surface
الشحنة توزع نفسها على السطح الخارجي

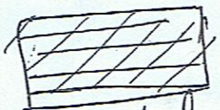
charging processes → conduction توصيل
→ induction حث

<u>example</u> :- how we can charge <u>a metalic</u> object by induction



uniformly distributed over the surface (repulsion force)

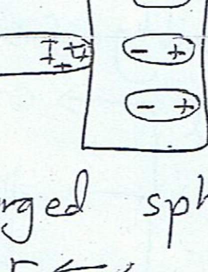
[illegible] : what is the kind of force between q and the metalic plate



q metal

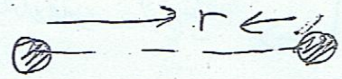
a) repulsion $\boxed{\text{b) attraction}}$ c) repulsion if q positive
d) attraction if q negative e) No force.

* Insulators charged with conduction which require contact with the object



<u>Coulomb Law</u>:

* the electric force between small charged sphere (point charge) proportional to the product of the charges q_1 and q_2 and $\frac{1}{r^2}$



q_2 q_1

$F \alpha \frac{1}{r^2}$; $F \alpha q_1 q_2 / r^2$

$$F_e = k_e \frac{q_1 q_2}{r^2} \; ; \; k_e : \text{Coulomb constant} = \frac{1}{4\pi \epsilon_0}$$

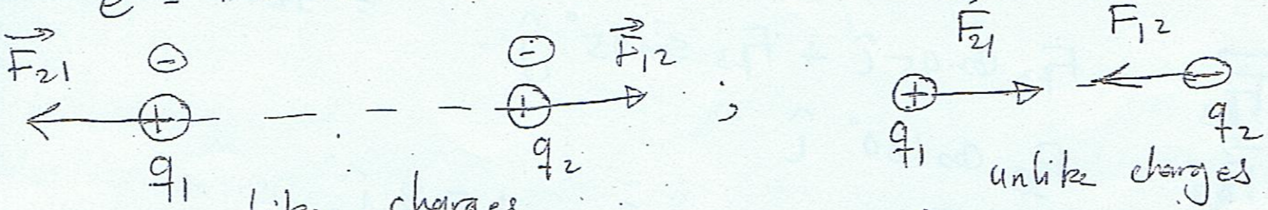
ϵ_0 : permitivity of free space

$$\epsilon_0 = 8.8542 \times 10^{-12} \; C^2/N^2 m^2$$

$$\therefore k_e = 8.987 \times 10^{9} \; Nm^2/C^2$$

* the smallest charge known is the electron charge

$$e = 1.6 \times 10^{-19} \; C \quad ; \quad q = Ne$$



$$\vec{F}_{12} = -\vec{F}_{21}$$

$\vec{F}_{12}$: force exerted from q_1 to q_2

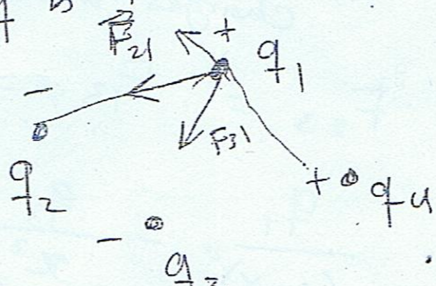
$\vec{F}_{21}$: ~ ~ ~ q_2 to q_1

* if $q_1 * q_2$ = positive (force is repulsion)

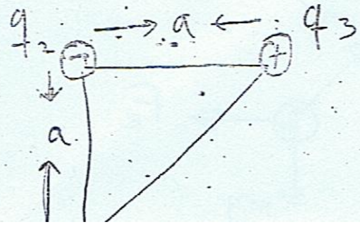
if $q_1 * q_2$ = negative (force is attractive)

* the total force exerted on q is the vector sum

$$\vec{F}_{on\, q_1} = \vec{F}_{21} + \vec{F}_{31} + \vec{F}_{41}$$



<u>example</u>: consider three point charges located at the corners of a right triangle (مثلث قائم) as shown

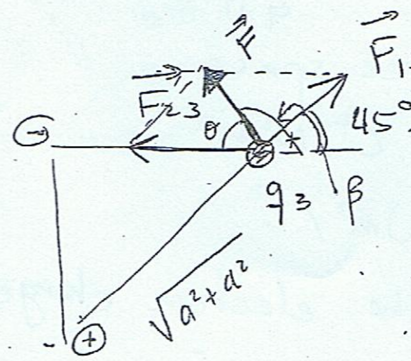


$q_1 = q_3 = 5 \mu C$

$q_2 = -2 \mu C$

$a = 10 \text{ cm} = 0.1 \text{ m}$

find the resultant force exerted on q_3



$$F_{23} = \frac{k_e q_2 q_3}{r_{23}^2} = \frac{9\times10^{9} * 5\times10^{-6} * 2\times10^{-6}}{(0.1)^2}$$

$$= 9N$$

$$F_{13} = \frac{k_e q_1 q_3}{r_{13}^2} = \frac{9\times10^{9} * 5\times10^{-6} * 5\times10^{-6}}{(\sqrt{2} * 0.1)^2}$$

$$= 11N$$

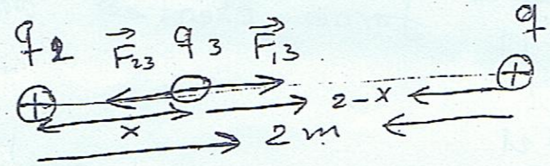
$$\vec{F}_{13} = F_{13} \cos 45\,\hat{i} + F_{13} \sin 45°\,\hat{j}$$

$$\vec{F}_{23} = F_{23} \cos 180°\,\hat{i}$$

$$\vec{F} = \vec{F}_{13} + \vec{F}_{23} = -1.22\,\hat{i} + 7.78\,\hat{j}$$

$$\theta = \tan^{-1}\left(\frac{F_y}{F_x}\right) = \frac{7.78}{1.22} = 81° \; ; \; \beta = 99° = (180° - 81°)$$

<u>example</u>: where we can place a negative charge between two positive charges where the resultant force on q_3



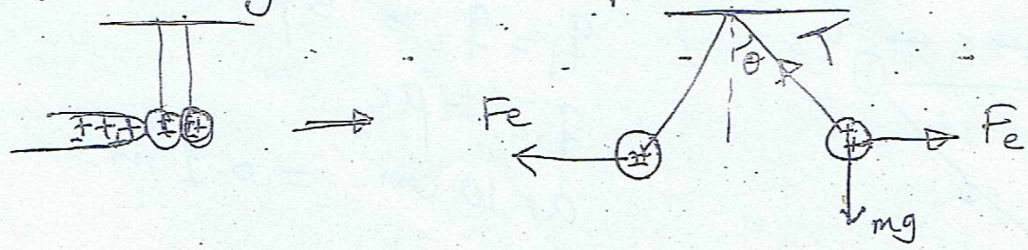
$$F_{23} = F_{13} \Rightarrow \frac{\cancel{k_e} q_2 \cancel{q_3}}{r_{23}^2} = \frac{\cancel{k_e} q_1 \cancel{q_3}}{r_{13}^2}$$

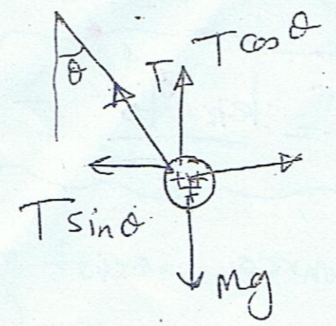
$$\frac{q_1}{(2-x)^2} = \frac{q_2}{x^2} \Rightarrow \text{solve the equation}$$

$$Ax^2 + Bx + C = 0$$

$$x = \frac{-B \pm \sqrt{B^2 - 4AC}}{2A}$$

<u>example</u>: two charged identical spheres





$$\sum \vec{F} = m\vec{a} = 0$$

$$\sum F_x = 0 \Rightarrow F_e = T\sin\theta \cdots (1)$$

$$\sum F_y = 0 \Rightarrow T\cos\theta = mg \cdots (2)$$

divide (1)/(2) $\Rightarrow$

$$\boxed{\tan\theta = F_e / mg}$$

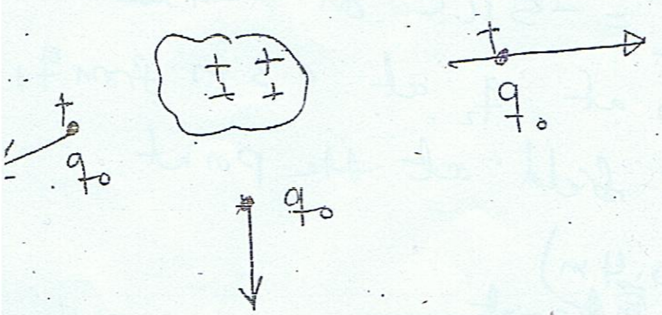
find $q \Rightarrow F_e = k_e \frac{q_1 q_2}{r^2} = k_e q^2 / r^2$

$r = 2l\sin\theta$

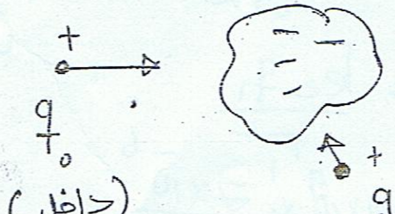
* The Electric Feilds

field forces : such as gravitional force, magnetic force and electric force.

electric field : the region around the charge in which you can feel the electric force



: the direction of electric field outward (خارج) the region of the charge



the direction of electric field inward (داخل) the region of the charge.

* the electric field ($\vec{E}$) at a point in space : is the electric force $\vec{F}_e$ acting on a positive test charge q_0 placed at the point divided by the magnitude of the test charge.

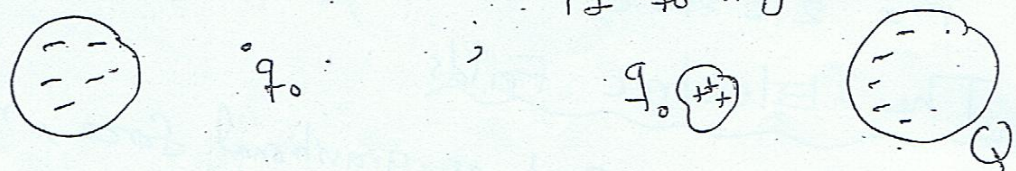
$$\vec{E} = \frac{F_e}{q_0} \quad \text{in units of } (N/C)$$

$$= k_e \frac{q\,\cancel{q_0}}{r^2\,\cancel{q_0}}\,\hat{r} \Rightarrow \boxed{\vec{E} = \frac{k_e\, q_0}{r^2}\,\hat{r}}$$

Note: ✓ $\vec{E}$ is a property of the source exist regardless (بغض النظر)
q_0 is located at the point or no.

✓ q_0: test charge should be very small so it will not disturb the charge distribution

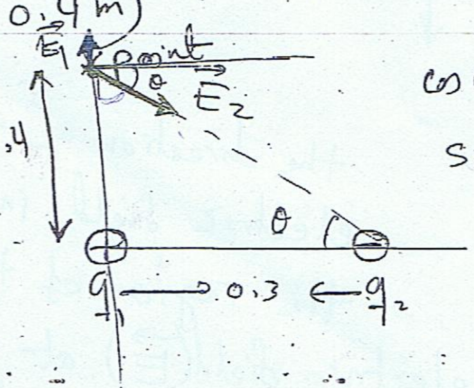
if q_0 large:



✓ if we have many charges we use the superposition principle (vector sum) جمع متجهات

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots$$

example: $q_1 = 7\,\mu C$; $q_2 = -5\,\mu C$ on x-axes
q_1 at the origin at q_2 at 0.3 m from q_1
find the electric field at the point
(0, 0.4 m)



$$\cos\theta = \frac{0.4}{0.5} = \frac{4}{5}$$

$$\sin\theta = \frac{0.3}{0.5} = \frac{3}{5}$$

$$E_1 = \frac{k_e q_1}{r_1^2} = \frac{9\times10^9 \times 7\times10^{-6}}{(0.4)^2}$$

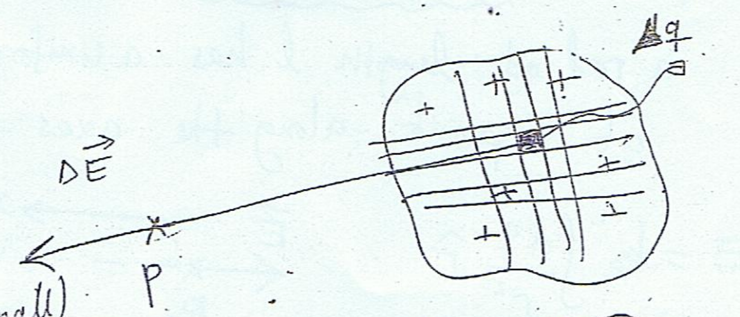
$$E_2 = k_e \frac{q_2}{r_2^2} = \frac{9\times10^9 \times 5\times10^{-6}}{\left(\sqrt{(0.4)^2+(0.3)^2}\right)^2}$$

$$\vec{E}_2 = E_2 \cos\theta\,\hat{i} - E_2 \sin\theta\,\hat{j} \quad , \quad \vec{E} = E_1\,\hat{j} \ldots$$

continue

Electric field due to a continuous charge distribution ⑦

$$\Delta \vec{E} = k_e \frac{\Delta q}{r^2}\,\hat{r}$$



$$E_{tot} = \sum k_e \frac{\Delta q_i}{r_i^2}\,\hat{r}_i$$

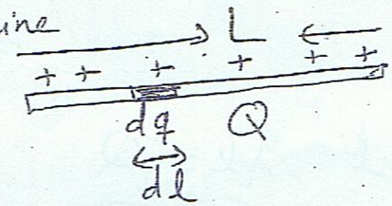
for $\Delta q_i \longrightarrow dq_i$ (very small)

$$\vec{E} = k_e \lim_{\Delta q_i \to 0} \sum \frac{\Delta q_i}{r_i^2}\,\hat{r}_i = \boxed{k_e \int \frac{dq}{r^2}\,\hat{r} = \vec{E}}$$

* Assume the charge distributed uniformly (موزعة بانتظام) on the charged object.

1) the charge distributed on a line

linear charge distribution

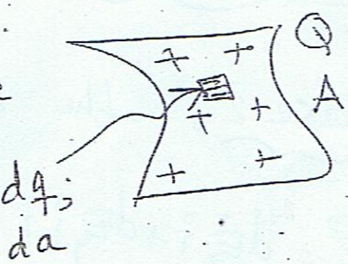


$$\frac{Q}{L} = \frac{dq}{dl} = \lambda \ (C/m)$$

λ: linear charge density $\Rightarrow dq = \lambda\, dl$

2) the charge distributed on a surface

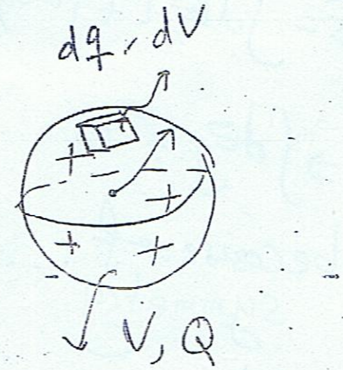
surface charge distribution



$$\frac{Q}{Area} = \frac{dq}{da} = \sigma$$

σ: surface charge density (C/m^2)

3) the charge distributed on a volume

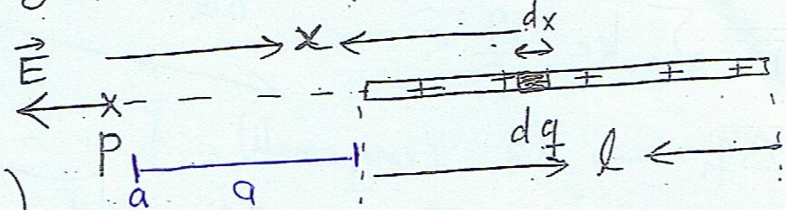


$$\frac{Q}{V} = \frac{dq}{dV} = \rho \ (C/m^3)$$

ρ: volume charge density.

Case 1: linear charge density.

a rod of length l has a uniform charge distribution find E at a point along the axes of the charge



$$E = k_e \int \frac{dq}{r^2}\,\hat{r}$$

$$= k_e \int \frac{dq}{x^2}\,(-\hat{i})$$

$$\lambda = \frac{Q}{l} = \frac{dq}{dl} \Rightarrow dq = \lambda\, dl \quad ; \quad \boxed{dl = dx}\ ?$$

$$E = k_e \int_a^{a+l} \frac{\lambda\, dx}{x^2} = k_e \lambda \left.\frac{x^{-1}}{-1}\right|_a^{a+l} = k_e \lambda \left\{\frac{1}{a} - \frac{1}{a+l}\right\}$$

$$= k_e \lambda \left\{\frac{\cancel{a}+l-\cancel{a}}{a(a+l)}\right\} = \frac{k_e \lambda l}{a(a+l)}$$

but $\lambda l = Q$

$$\boxed{E = \frac{k_e Q}{a(a+l)}} \qquad \text{if } a \gg l \Rightarrow E = \frac{k_e Q}{a^2} \ \text{(point charge)}$$

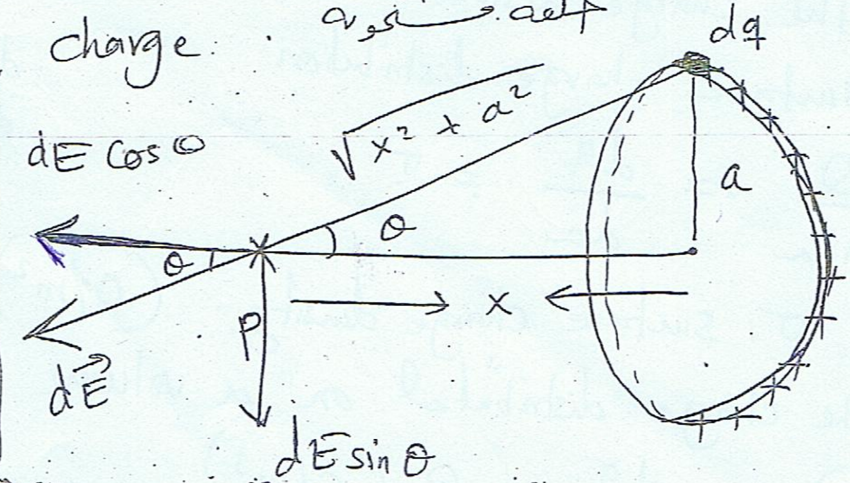
Case 2: the electric field due to a uniform ring of charge. حلقة مشحونة

$$d\vec{E} = dE_x\,\hat{i} + dE_y\,\hat{j}$$

$$\vec{E} = \int dE_x\,\hat{i} + \int dE_y\,\hat{j}$$

$$\Rightarrow \int dE_y = 0$$

because of symmetry بسبب التماثل



(نصف الحلقة العلوي يعطي مركبة لـ dE_y لأسفل والنصف العلوي يعطي مركبة للأعلى تلغي بعضها)

* the components of E_y will cancel each others (half positive and half negative).

$$E = \int_0^Q dE \cos\theta = \int_0^Q k_e \frac{dq}{r^2} \cos\theta = k_e \int_0^Q \frac{dQ}{(x^2+a^2)} \cdot \frac{x}{(x^2+a^2)^{1/2}}$$

$$E = \frac{k_e x}{(x^2+a^2)^{3/2}} \int_0^Q dq = \frac{k_e x Q}{(x^2+a^2)^{3/2}}$$

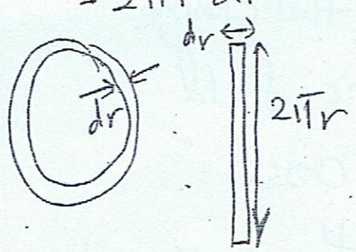
for $x \gg a \Rightarrow E = \frac{k_e \cancel{x} Q}{x^{\cancel{3}}} = \frac{k_e Q}{x^2}$ (like point charge)

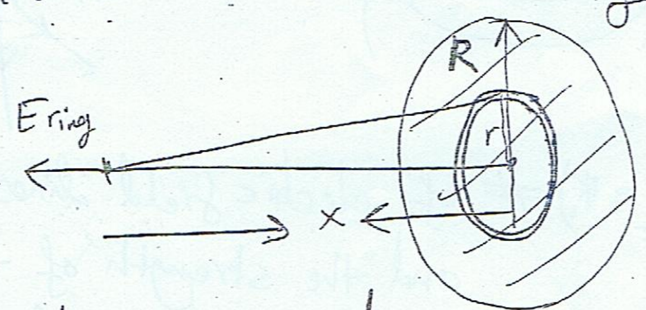
Case 3: Electric field of a uniformly charged disk

surface charge distribution σ

area of the ring

$= 2\pi r dr$





$$dq = \sigma da = \sigma * 2\pi r dr$$

$$E_{ring} = \frac{k_e x Q}{(x^2+a^2)^{3/2}} \; ; \; a = r \Rightarrow Q = dq$$

$$E_{ring} = k_e \frac{x\, dq}{(x^2+r^2)^{3/2}} \equiv dE \text{ for the disk}$$

$$E_{disk} = \int dE_{ring} = \int_0^R \frac{k_e x}{(x^2+r^2)^{3/2}} * \sigma * 2\pi r dr$$

$$= 2\pi \sigma k_e x \int_0^R \frac{r\, dr}{(x^2+r^2)^{3/2}}$$

let $u = x^2 + r^2$

$2r dr = du \Rightarrow E_{disk} = \pi \sigma k_e x \int \frac{du}{u^{3/2}}$

$$E_{disk} = \pi \sigma k_e x \frac{u^{-1/2}}{-1/2} = -2\pi\sigma k_e x \left. \frac{1}{\sqrt{x^2+r^2}} \right|_0^R$$

$$= 2\pi k_e \sigma \left\{ \frac{x}{|x|} - \frac{x}{\sqrt{R^2+x^2}} \right\}$$

* for $R \gg x \Rightarrow E_{disk} = \frac{\sigma}{2\epsilon_0} \equiv E$ for infinite sheet.

* Electric field lines

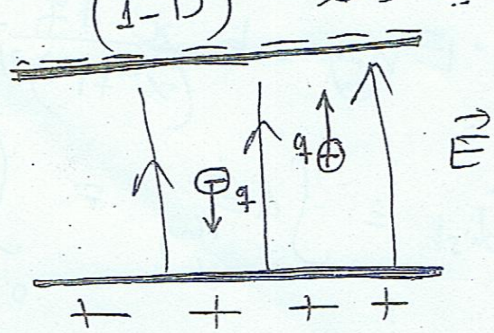
Note :*) # of electric field lines $\propto$ to the charge and the strength of the electric field

* No two field lines can cross

لا يمكن لخطوط المجال الكهربائي ان تتقاطع

Motion of charged particle in a uniform electric field

(1-D) الحركة ببعد واحد



uniform electric field

a charge q of mass m placed in a uniform electric field

$$\vec{F}_e = q\vec{E} = m\vec{a}$$

$$\rightarrow \vec{a} = \frac{q\vec{E}}{m} \text{ : the charge acceleration}$$

(تسارع ثابت)

we can apply the equations of motion with constant acceleration

$$v_f^2 = v_i^2 + 2a\Delta x \quad \text{---} \; (1)$$

$$v_f = v_i + at \quad \text{----} \; (2)$$

$$x_f = x_i + v_{ix}t + \frac{1}{2}at^2 \quad \text{--} \; (3)$$

<u>or</u> we may apply the conservation of energy.

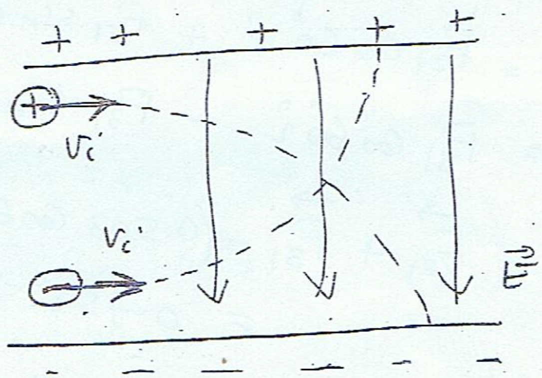
$$W = \Delta K = K_f - K_i = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2$$

$$W = F\Delta x = \boxed{qE\Delta x = \frac{1}{2}mv_f^2 - \frac{1}{2}mv_i^2}$$

* the motion may be in two dimensions

$$x_f = x_i + v_{ix}t + \cancel{\frac{1}{2}a_x t^2}^{\,0}$$

$$y_f = y_i + v_{iy}t + \frac{1}{2}a_y t^2$$



for simplicity $x_i, y_i = 0$ unless it is defined

if the charge moved from rest $v_{xi} = 0$

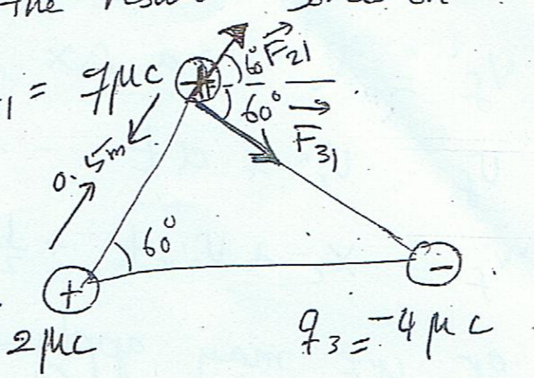
$$\vec{a} = \frac{q\vec{E}}{m} = -\frac{qE}{m}\hat{j} \quad ; a_x = 0 \quad ; a_y = -\frac{qE}{m}$$

$$x = v_i t + \cancel{x_i}^{\,0} + \cancel{\frac{1}{2}a_x t^2}^{\,0} \Longrightarrow x = v_i t \quad \text{--} \; (1)$$

$$y = \cancel{y_i}^{\,0} + \cancel{v_{iy}t}^{\,0} + \frac{1}{2}a_y t^2 = -\frac{1}{2}\frac{qE}{m}t^2 \quad \text{--} \; (2)$$

problems :-

1) three point charges are located at the corners of an equilateral triangle as shown in the figure. Calculate the resultant force on the $7\,\mu C$ charge.



$$F_{21} = k_e \frac{q_1 q_2}{r_{12}^2}$$

$$= \frac{9\times10^{9} * 7\times10^{-6} * 2\times10^{-6}}{(0.5)^2}$$

$$= 0.503\ N$$

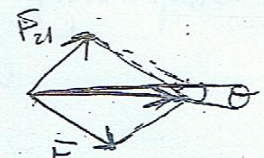
$$F_{31} = k_e \frac{q_1 q_3}{r_{13}^2} = \frac{9\times10^{9} * 7\times10^{-6} * 4\times10^{-6}}{(0.5)^2} = 1.01 N$$

$$\vec{F}_{21} = F_{21}\cos 60^\circ\,\hat{i} + F_{21}\sin 60\,\hat{j}$$

$$\vec{F}_{31} = F_{31}\cos 60^\circ\,\hat{i} - F_{31}\sin 60\,\hat{j}$$

$$\vec{F} = \vec{F}_{21} + \vec{F}_{31} = (0.503\cos 60^\circ + 1.01\cos 60^\circ)\hat{i} + (0.503\sin 60^\circ - 1.01\sin 60^\circ)\hat{j}$$

$$= 0.755\,\hat{i} - 0.436\,\hat{j}\,N$$



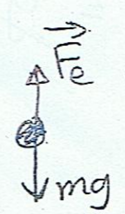
$$|F| = \sqrt{(0.755)^2 + (-0.436)^2} = 0.872\ N$$

$$\theta = \tan^{-1}\frac{F_y}{F_x} = \tan^{-1}\left(\frac{0.436}{0.755}\right) \approx 30^\circ$$

β = the angle with positive x-axes $= 360^\circ - 30^\circ = 330^\circ$

problem 2 what are the magnitude and direction of the electric field that will balance (تعادل) the weight (وزن) of (a) an electron

(b) proton



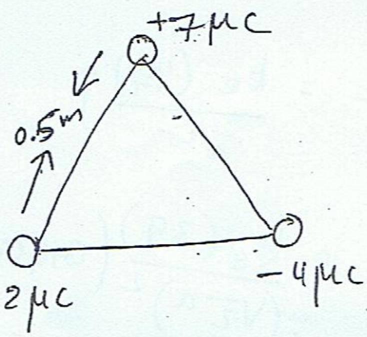
$$\vec{F_e} = \vec{mg}$$

(a) electron $\quad qE = mg \Rightarrow \vec{E} = \frac{mg}{q}\hat{j}$

$$\vec{E} = \frac{9.11\times10^{-31} * 9.8}{-1.6\times10^{-19}}\hat{j} = -5.58\times10^{-11}\ N/C\ (\hat{j})$$

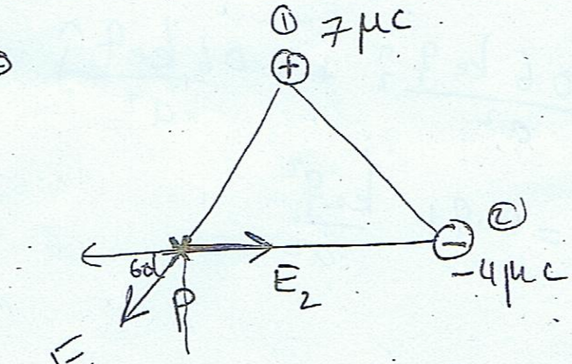
(b) proton $\Rightarrow \vec{E} = \frac{1.67\times10^{-27} * 9.8}{+1.6\times10^{-19}}\hat{j} = +1.02\times10^{-7}\ N/C(\hat{j})$

problem 3: Three charges are at the corners of an equiateral triangle (مثلث متساوي الاضلاع) as shown in the figure



(a) Calculate the electric field at the point of the position of 2μC charge

⇒



$$E_1 = \frac{k_e q_1}{r_1^2} = \frac{9\times10^9 * 7\times10^{-6}}{(0.5)^2} = 2.52\times10^5\ N/C$$

$$E_2 = \frac{k_e q_2}{r_2^2} = \frac{9\times10^9 * 4\times10^{-6}}{(0.5)^2} = 1.44\times10^5\ N/C$$

$\vec{E}_1 = -E_1\cos60^\circ\,\hat{i} + E_1\sin60^\circ\,\hat{j}$

$\vec{E}_2 = E_2\,\hat{i}$

$\vec{E} = \vec{E}_1 + \vec{E}_2 = (18\times10^3\,\hat{i} - 218\times10^3\,\hat{j})\ N/C$

$|E| = \sqrt{(18)^2 + (218)^2} * 10^3\ N/C$

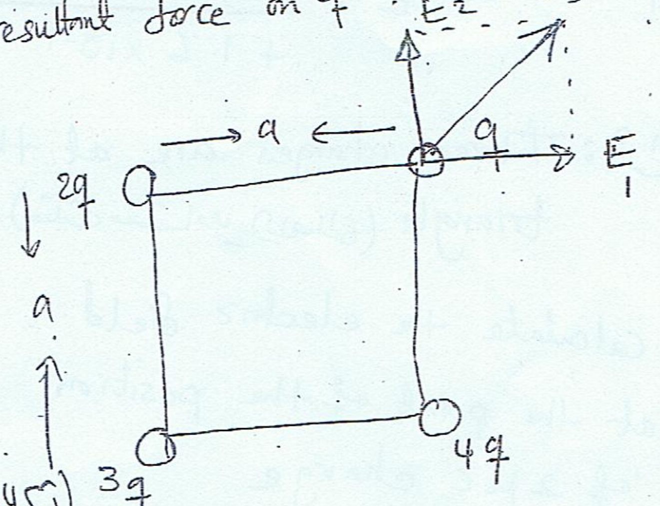
(b) determine the force on 2μC charge

$\vec{F} = q\vec{E} = 2\times10^{-6}(18\times10^3\,\hat{i} - 218\times10^3\,\hat{j}) = (36\hat{i} - 436\hat{j})\underbrace{\times10^{-3}\,N}_{mN}$

problem 4 : four point charges are at the corners of a squre of side a as shown in the figure

(a) determine the magnetude and direction of the electric field at the location of charge q

(b) what is the resultant force on q



$$\vec{E}_1 = \frac{k_e(2q)}{a^2}\hat{i}$$

$$\vec{E}_2 = \frac{k_e(4q)}{a^2}\hat{j}$$

$$\vec{E}_3 = \frac{k_e(3q)}{(\sqrt{2}a)^2}(\cos 45\hat{i} + \sin 45\hat{j})$$

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 = 3.06\frac{k_e q}{a^2}\hat{i} + 5.06\frac{k_e q}{a^2}\hat{j}$$

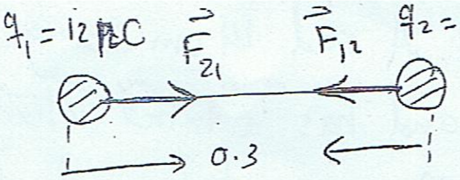
$$|E| = \sqrt{(3.06)^2 + (5.06)^2} * \frac{k_e q}{a^2} = 5.91\frac{k_e q^2}{a^2}$$

$$\theta = \tan^{-1}\left(\frac{5.06}{3.06}\right) = 58.8^{\circ}$$

problem 5 : Two identical conducting small spheres (كرات) are placed with their centers 0.3 m apart. One is given a charge of 12 nC and the other a charge of −18 nC

(a) find the electric force exerted by one sphere on the other

(b) if the ~~sphers~~ sphers are connected by a conducting wire find the electric force each exerts on the other after they have come to equilibrium.



(a) $\vec{F}_{12} = -\vec{F}_{21}$

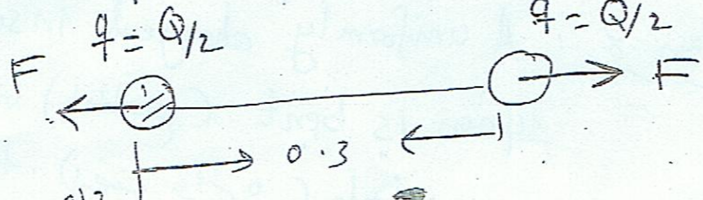
$$|F_{12}| = |F_{21}| = k_e \frac{q_1 q_2}{r_{12}^2}$$

$$= \frac{9\times10^{9} * 12\times10^{-9} * 18\times10^{-9}}{(0.3)^2} = 2.16\times10^{-5}\ \text{N}$$ (tward each others) متقاربات من بعض

(b) if we connect the spheres by a wire

the total charge $Q = q_1 + q_2 = (12 - 18)$ nC $= -6$ nC

will be divided on the two spheres because they are identical

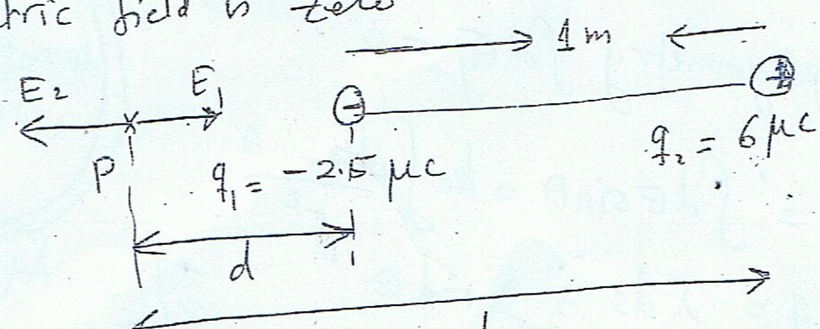


$$q = \frac{Q}{2} = -3\text{ nC}$$

$$F = \frac{k_e q^2}{r^2} = \frac{9\times10^{9} * (3\times10^{-9})^2}{(0.3)^2} = 8.98\times10^{-7}\ \text{N}$$

away from each others متباعدات عن بعض

problem 6: determine the point other than infinity at which the electric field is zero



$$E_1 = E_2$$

$$\cancel{k_e} \frac{q_1}{r_1^2} = k_e \frac{q_2}{r_2^2}$$

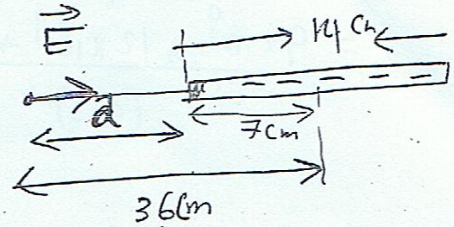
$$\frac{2.5\times\cancel{10^{-6}}}{d^2} = \frac{6\times\cancel{10^{-6}}}{(1+d)^2} \Rightarrow \boxed{(1+d)^2 = 2.4\,d^2}$$

$$\Rightarrow 1+d = \pm\sqrt{2.4}\,d = \pm 1.55d$$

$$\Rightarrow 1+d = 1.55d \Rightarrow d = 1.82\text{ m}$$

$$1+d = -1.55d \Rightarrow \boxed{d = -0.392\ \cancel{m}}$$ between the charges so it is wrong

problem 7: A rod 14 cm long is uniformly charged and has a total charge of $-22\,\mu C$. Determine the magnetude and direction of the electric field along the axes of the rod at a point 36 cm from the center



$$E = \frac{k_e Q}{a(a+l)} = \frac{9\times10^{9} \times 22\times10^{-6}}{0.29\,(0.14+0.29)}$$

$$= 1.59\times10^{6}\ N/C$$

$$a = 36\ cm - 7\ cm = 29\ cm$$

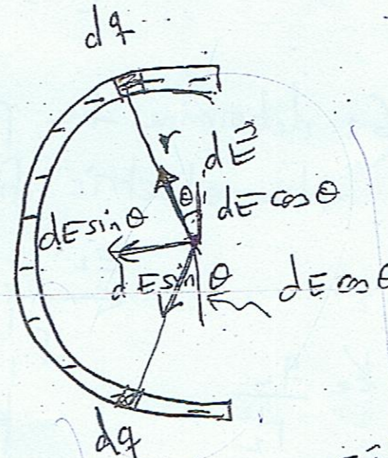
$\vec{E}$ toward the rod .إتجاه السالب أخترى

problem 8, A uniformly charged insulating rod of length 14 cm is bent (إنبعج) in to the shape (شكل) of a semi~~circle~~ circle (نصف دائرة). the rod has a total charge of $-7.5\,\mu C$. find the magnetude and direction of the electric field at the center of the semicircle.

$$\vec{E} = \int d\vec{E} = \int dE_x\,\hat{i} + \int dE_y\,\hat{j}$$

by symmetry $\int dE_y = 0$

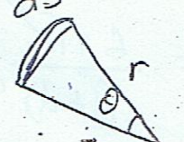
$$\vec{E} = \int dE\sin\theta = k_e \int \frac{dq\,\sin\theta}{r^2}$$



$$dq = \lambda\,ds = \lambda r\,d\theta$$

$$E = \frac{k_e}{2r}$$

ds ← طول القوس



$$\vec{E} = k_e \frac{\lambda r}{r^2}\int_0^{\pi} \sin\theta\,d\theta$$

$$= \frac{k_e\lambda}{r}\cos\theta\Big|_0^{\pi} = \frac{2k_e\lambda}{r} = \frac{2k_e\,q\ast\pi}{L\ast L}$$

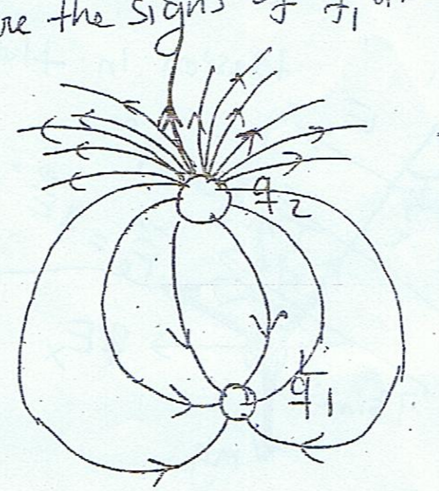
$$\lambda = \frac{q}{L},\quad L = \frac{2\pi r}{2}$$

$$\boxed{E = \frac{2k_e \pi q}{L^2}} = \frac{2 \times 9 \times 10^9 \times 3.14 \times 7.5 \times 10^{-6}}{(0.14)^2}$$

$$= 2.16 \times 10^7 \ N/C$$

$$\vec{E} = -21.6 \underbrace{M}_{Mega = 10^6} N/C \ (\hat{i})$$

problem 9: figure shows the electric field lines for two point charges seperated by a small distance. determine the ratio q_1/q_2; what are the signs of q_1 and q_2.

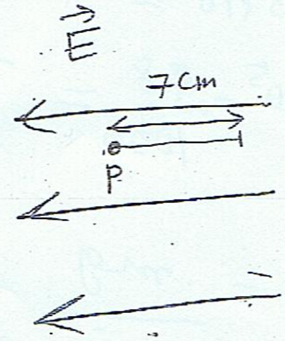


$$\frac{\# E_1}{\# E_2} = \frac{q_1}{q_2} = \frac{-6}{18} = \frac{-1}{3}$$

$$q_1 = \frac{-1}{3} q_2$$

* q_2 is positive, q_1 is negative

problem 10: A proton is projected in the positive x-direction in to a region of a uniform electric $\vec{E} = -6 \times 10^5 \hat{i}$ N/C at $t=0$. The proton travels 7 cm before comming to rest. determine the acceleration of the proton, its initial speed, the time at which the proton comes to rest.



$$\vec{a} = \frac{q\vec{E}}{m} = \frac{1.602 \times 10^{-19} \times -6 \times 10^5}{1.67 \times 10^{-27}}$$

$$= -5.76 \times 10^{13} \ m/s \ \hat{i}$$

$$\vec{a} = -5.76 \times 10^{13} \ m/s^2 \ \hat{i}$$

$$* \ v_f^2 = v_i^2 + 2a\Delta X = 0 = v_i^2 - 2 \times 5.76 \times 10^{13} \times 0.07$$

$$v_i = 2.84 \times 10^6 \ \hat{i} \ (m/s)$$

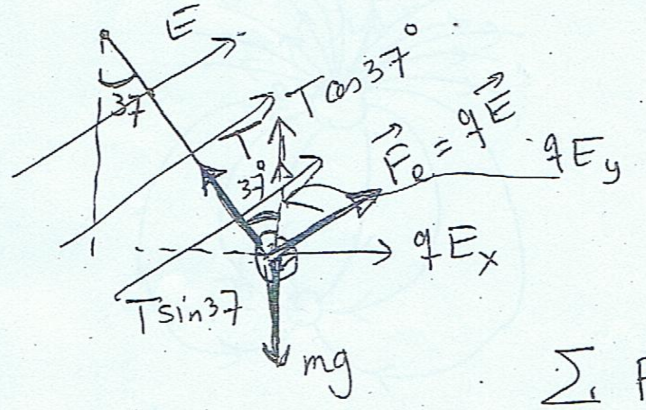
$* \; v_f = v_i + at$

$0 = 2.84 \times 10^{6} - 5.76 \times 10^{13} * t \Rightarrow t = 4.93 \times 10^{-8}$ s

$= 49.3 \times 10^{-9}$ s

$= 49.3$ ns

problem 11: A charged cork ball of mass 1g is suspended on a light string (خيط خفيف) in the presence of a uniform electric field as shown in the figure $\vec{E} = (3\hat{i} + 5\hat{j}) \times 10^{5}$ N/C; the ball in equilibrium at $\theta = 37°$. find the charge on the ball and the tension in the string.



$$\vec{F} = F_x \hat{i} + F_y \hat{j}$$

$$\left.\begin{array}{l} \sum F_x = 0 \\ \sum F_y = 0 \end{array}\right\} \text{in equilibrium.}$$

$$\sum F_x = qE_x - T \sin 37° = 0 \quad \text{--(1)}$$

$$\sum F_y = T\cos 37 + qE_y - mg = 0 \quad \text{--(2)}$$

Sub. the numbers :-

$E_x = 3 \times 10^{5}$ N/C ; $E_y = 5 \times 10^{5}$ N/C , $mg = \frac{1}{100} * 9.8$

$$\left.\begin{array}{l} q * 3 \times 10^{5} = T \sin 37 \quad \text{(1)} \\ q * 5 \times 10^{5} - \frac{9.8}{1000} = T \cos 37 \quad \text{--(2)} \end{array}\right\} \text{divide (1)/(2)} \Rightarrow \text{to find } q.$$

$$q = \frac{mg}{E_x \cot\theta + E_y} = \frac{1 \times 10^{-3} * 9.8}{3 \times 10^{5} \cot 37° + 5 \times 10^{5}} = 1.09 \times 10^{-8} C.$$

$= 10.9$ nC.

from (1) or (2) find $T = \frac{qE_x}{\sin 37} = 5.44 \times 10^{-3}$ N $= 5.44$ mN old exams: 88

1) each of two small non-conducting spheres is charged positively the combined charge is being $40\mu C$. if each sphere is repelled from the other by a force having a magnetude of 2 N when the two spheres are 50 cm apart, determine the charge on the sphere having the smaller charge.

$\boxed{a)\ 1.4\mu C}$ b) $1.1\mu c$ c) $2\mu c$ d) $3.3\mu c$ e) $17\mu c$

2) A point charge (Q) is placed at the origion, a second charge (2Q) is placed on the x-axes at $x=-3m$. if $Q=50\mu C$ what is the magnetude of the electrostatic force on a third point (−Q) placed on the y-axes at $y=4m$?

a) 2.5 N $\boxed{b)\ 3N}$ c) 3.7 N d) 4.4 N e) 1.8 N

3) A particle (charge $=+40\mu C$) is located on the x-axes at the point $x=-20$ cm and a second particle (charge $=-50\mu C$) is placed on the x-axes at $x=+30$ cm. what is the magnetude of the total electrostatic force on a third charge $-4\mu C$ placed at the origion ($x=0$)?

a) 41 N b) 16 N $\boxed{c)\ 56N}$ d) 35 N e) 72 N

$$F_e=\frac{k_e q_1 q_2}{r^2}$$

$$2=\frac{9\times10^9}{0.25}q_1q_2$$

$$q_1q_2=5.5\times10^{-13}$$

$$q_2=\frac{5.5\times10^{-3}}{q}$$

$$q_2+q_1=40\times10^{-6}$$

$$5.5\times10^{-3}$$

$$5.5\times10^{-13}+q_1^2=40\times10^{-6}q_1$$

$$q_1=\frac{-(40\times10^{-6})\pm\sqrt{16\times10^{-10}-22\times10^{-13}}}{2}$$

ADDITIONAL PROBLEMS

CHAPTER 23

1) Each of two small non-conducting spheres is charged positively, the combined charge being 40 μC. If each sphere is repelled from the other by a force having a magnitude of 2.0 N when the two spheres are 50 cm apart, determine the charge on the sphere having the smaller charge.

a. 1.4 μC b. 1.1 μC c. 2.0 μC d. 3.3 μC e. 17 μC

Combined charge (المجموع) $= \boxed{q_1 + q_2 = 40\,\mu C}$ ...(1); $F = 2N$ (rep) ∴ same kind of charges.

$F = k_e \frac{q_1 q_2}{r^2} = 2 = \frac{9\times10^9 * q_1 q_2}{(0.5)^2} \Rightarrow \boxed{q_1 q_2 = 5.56\times10^{-11}}$ ...(2)

Solve equs (1) & (2) $\Rightarrow q_1 = 40\times10^{-6} - q_2$ put in (2) $\Rightarrow$

$(40\times10^{-6} - q_2)\, q_2 = 5.56\times10^{-11} \Rightarrow q_2^2 - 40\times10^{-6}\, q_2 + 5.56\times10^{-11} = 0$

$q_2 = \frac{+40\times10^{-6} \mp \sqrt{(40\times10^{-6})^2 - 4*5.5\times10^{-11}}}{2} = \frac{4\times10^{-5} \pm 3.7\times10^{-5}}{2}$

$q_2 = 38.6\times10^{-6}$ or 1.4×10^{-6} $\Rightarrow$ the smallest charge equal $\boxed{1.4\times10^{-6}\,C}$ الجواب $\underline{a}$

2) A particle (m = 50 g, q = 5.0 μC) is released from rest when it is 50 cm from a second particle (Q = −20 μC). Determine the magnitude of the initial acceleration of the 50-g particle.

a. 54 m/s² b. 90 m/s² c. 72 m/s² d. 65 m/s² e. 36 m/s²

$m = 50g = \frac{50}{1000}\,kg = 0.05\,kg$

$F = ma = k_e \frac{q_1 q_2}{r^2}$

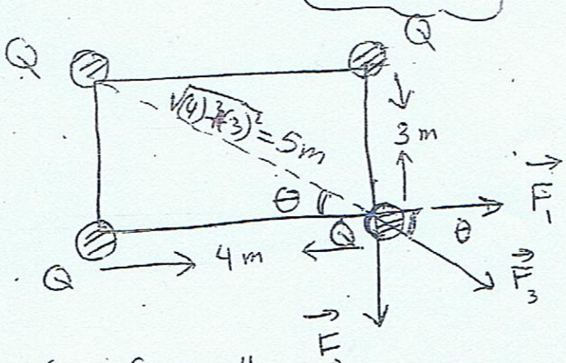
$0.05 * a = \frac{9\times10^9 * 5\times10^{-6} * 20\times10^{-6}}{(0.5)^2} \Rightarrow \boxed{a = 72\,N}$

الجواب $\underline{C}$

مثال هام جداً

3) 2. Identical point charges Q are placed at each of the four corners of a 3.0 m x 4.0 m rectangle. If Q = 40 μC, what is the magnitude of the electrostatic force on any one of the charges?

a. 3.0 N b. 2.4 N c. 1.8 N d. 3.7 N e. 2.0 N



$\vec{F}_1 = \frac{k_e Q^2}{r_1^2}(\hat{i}) = \frac{9\times10^9 * (40\times10^{-6})^2}{(4)^2} = 900\times10^{-3}\,\hat{i} = 0.9\,N(\hat{i})$

$\vec{F}_2 = \frac{9\times10^9 * (40\times10^{-6})^2}{(3)^2} = 1.6\,N\,(-\hat{j})$

$\vec{F}_3 = \frac{9\times10^9 * (40\times10^{-6})^2}{(5)^2}(\cos\theta\,\hat{i} - \sin\theta\,\hat{j})$

$= 0.576\left(\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}\right) = 0.46\,\hat{i} - 0.35\,\hat{j}$

$\cos\theta = \frac{4}{5}$

$\sin\theta = 3/5$

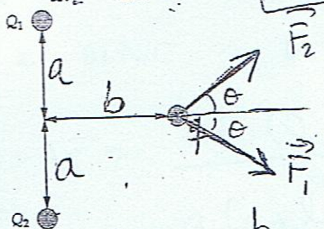
$\vec{F}_{net} = (0.9 + 0.46)\hat{i} - (1.6 + 0.35)\hat{j} = 1.36\,\hat{i} - 1.95\,\hat{j}$

$|F_{net}| = \sqrt{(1.36)^2 + (1.95)^2} = 2.38\,N \approx \boxed{2.4\,N}$

الجواب b

(4) If a = 3.0 mm, b = 4.0 mm, $Q1$ = 60 nC, $Q2$ = 80 nC, and q = 32 nC in the figure, what is the magnitude of the total electric force on q?

a. 1.6 N $\boxed{\text{b. } 1.3 \text{ N}}$ c. 1.9 N d. 2.2 N e. 0.04 N



$\cos\theta = \frac{b}{\sqrt{a^2+b^2}} = \frac{4}{5}$

$\sin\theta = \frac{a}{\sqrt{a^2+b^2}} = \frac{3}{5}$

$F_x = F_1\cos\theta + F_2\cos\theta$

$= \frac{9\times10^9 * 60\times10^{-9} * 32\times10^{-9}}{(5\times10^{-3})^2} * \frac{4}{5} + \frac{9\times10^9 * 80\times10^{-9} * 32\times10^{-9}}{(5)^2} * \frac{4}{5}$

$F_x = 0.55 + 0.737 = 1.28 N\,\hat{i}$

$F_y = F_2\sin\theta - F_1\sin\theta = \frac{9\times10^9 * q}{(5\times10^{-3})^2}\{Q_2 - Q_1\} * \frac{3}{5}$

$F_y = 0.138\,\hat{j}$

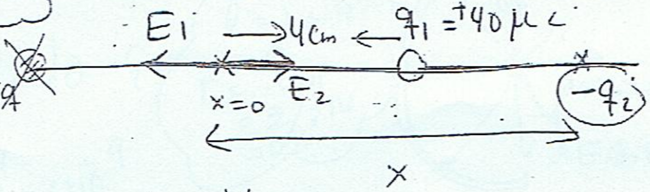
$\vec{F} = \sqrt{F_x^2 + F_y^2} = 1.29\ N \approx 1.3\ N$

الجواب b

(5) A 40-μC charge (q_1) is positioned on the x axis at x = 4.0 cm. To produce a net electric field of zero at the origin where should a −60-μC charge (q_2) be placed?

a. −5.3 cm b. 5.7 cm c. 4.9 cm d. −6.0 cm e. +6.0 cm

توضع الشحنة في q_2 ما على محور x الموجب
لأنها إذا وضعت على السالب لا يمكن أنه تكون المجال
صفر عند المركز



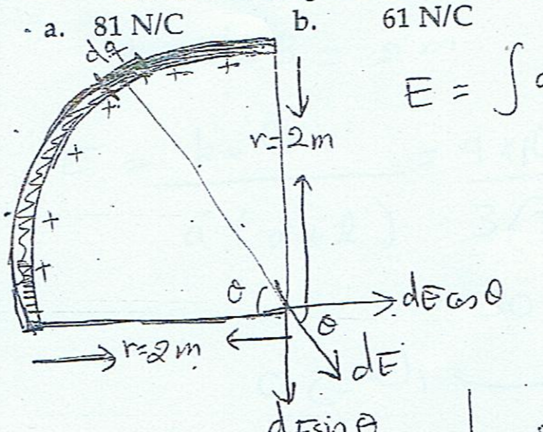
$E_1 = E_2$

$\frac{\cancel{k_e} q_1}{(4)^2} = \frac{\cancel{k_e} q_2}{x^2} \Rightarrow \frac{40}{16} = \frac{60}{x^2} \Rightarrow x^2 = \frac{60 * 16}{40} \Rightarrow$

$\boxed{x = 4.9}$ cm from the center

(6) A charge of 25 nC is uniformly distributed along a circular arc (radius = 2.0 m) that is subtended by a 90-degree angle. What is the magnitude of the electric field at the center of the circle along which the arc lies?

a. 81 N/C b. 61 N/C c. 71 N/C $\boxed{\text{d. } 51 \text{ N/C}}$ e. 25 N/C



$E = \int dE = \int dE\cos\theta\,\hat{i} + \int dE\sin\theta\,\hat{j}$

$= \frac{k_e\lambda}{r^2}\int ds\cos\theta\,\hat{i} + \frac{k_e\lambda}{r^2}\int ds\sin\theta\, d\theta$

$= \frac{k_e\lambda\cdot \cancel{r}}{r^{\cancel{2}}}\int_0^{\pi/2}\cos\theta\, d\theta(\hat{i}) + \frac{k_e\lambda \cancel{r}}{r^{\cancel{2}}}\int_0^{\pi/2}\sin\theta\, d\theta(\hat{j})$

$= \frac{k_e\lambda}{r}\left\{\sin\theta\Big|_0^{\pi/2}(\hat{i}) - \cos\theta\Big|_0^{\pi/2}\hat{j}\right\} = \frac{k_e\lambda}{r}\{\hat{i}+\hat{j}\}$

$dq = \lambda ds = \lambda r d\theta$

$\lambda = \frac{Q}{l} = \frac{Q}{2\pi r/4}$ ← ربع دائرة

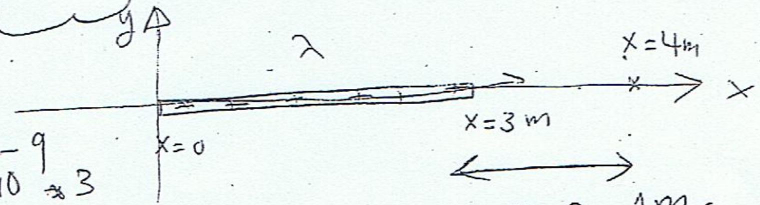
$|E| = \frac{9\times10^9 * 25\times10^{-9}}{\frac{2\pi * \cancel{2}}{4} * \cancel{2}} * \left\{\sqrt{(1)^2+(1)^2}\right\} = 50.7\ N/C$

$\approx 51\ N/C$

d. الجواب

7. A charge (uniform linear density = 9.0 nC/m) is distributed along the x axis from $x = 0$ to $x = 3.0$ m. Determine the magnitude of the electric field at a point on the x axis with $x = 4.0$ m.

a. 81 N/C b. 74 N/C c. 61 N/C d. 88 N/C e. 20 N/C



$$E = \frac{k_e Q}{a(a+l)} = \frac{k_e \lambda l}{a(a+l)} = \frac{9\times10^{9} * 9 \times10^{-9} * 3}{1(1+3)} = 6.07\ N/m \simeq 6.1\ N/m$$

$a = 4 - 3 = 1m$.

$\lambda = 9.0 nC/m$

$l = 3m$

$a = 1m$

C الجواب

8. A uniformly charged rod (length = 2.0 m, charge per unit length = 3.0 nC/m) is bent to form a semicircle. What is the magnitude of the electric field at the center of the circle?

a. 64 N/C b. 133 N/C c. 48 N/C d. 85 N/C e. 34 N/C

$$E = \int dE = \int dE \sin\theta\ \hat{i} + \cancel{\int dE\cos\theta\ \hat{j}}$$

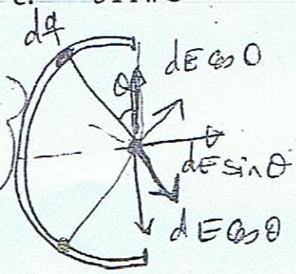
$$= \frac{k_e}{r^2}\int \lambda ds = \frac{k_e \lambda r}{r^2}\int_0^{\pi} \sin\theta\, d\theta\ \hat{i}$$

$$= \frac{2 k_e \lambda}{r} = \frac{2\times9\times10^{9} * 3\times10^{-9}}{0.637}$$

$= 85\ N/C$ d الجواب

$l = \frac{1}{2} * 2\pi r$

$r = l/\pi = 2/3.14 = 0.6$



$ds = r d\theta$

طول القوس

$dq = \lambda ds$

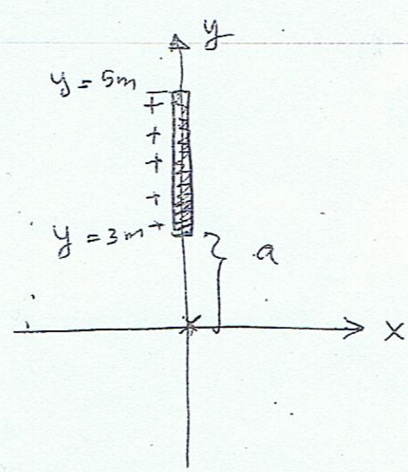
9. A charge of 50 nC is uniformly distributed along the y axis from $y = 3.0$ m to $y = 5.0$ m. What is the magnitude of the electric field at the origin?

a. 18 N/C b. 50 N/C c. 30 N/C d. 15 N/C e. 90 N/C

$a = 3 - 0 = 3$ m ; $Q = \lambda l$

$l = 5 - 3 = 2$ m

$$E = \frac{k_e Q}{a(a+l)} = \frac{9\times10^{9} * 50\times10^{-9}}{3(3+2)} = 30\ N/C$$



c الجواب الصحيح

10. A particle (mass = 4.0 g, charge = 80 mC) moves in a region of space where the electric field is uniform and is given by $E_x = -2.5$ N/C, $E_y = E_z = 0$. If the velocity of the particle at $t = 0$ is given by $v_x = 80$ m/s, $v_y = v_z = 0$, what is the speed of the particle at $t = 2.0$ s?

a. 40 m/s b. 20 m/s c. 60 m/s d. 80 m/s e. 180 m/s

$m = 4 \times 10^{-3}$ kg

$q = 80 \times 10^{-3}$ C

$\vec{E} = E_x = -2.5$ N/C

$\vec{v_i} = 80$ m/s

$v_f = ??$

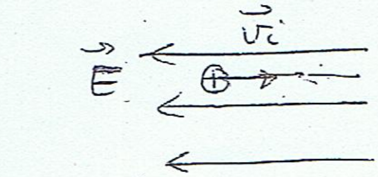
$t = 2$ s

$v_f = v_i + at$

$= v_i - \frac{qE}{m} t$

$= 80 - \frac{80 \times 10^{-3} \times 2.5 \times 2}{4 \times 10^{-3}}$

$= 80 - 100 = -20$ m/s



السرعه والمجال الكهربائي
(متعاكسات في الاتجاه)

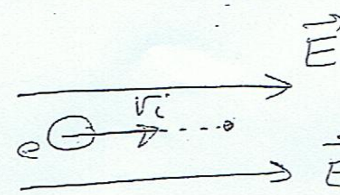
the speed $= |v_f| = |-20| = 20$ m/s

الجواب b

11. An electron enters a region of uniform electric field ($E = 50$ N/C) with an initial velocity of 40 km/s directed the same as the electric field. What is the speed of the electron 1.5 ns after entering this region?

a. 53 km/s b. 27 km/s c. 18 km/s d. 62 km/s e. 42 km/s

$v_f = v_i + at$; $a = -\frac{qE}{m} = -\frac{1.6 \times 10^{-19} \times 50}{9.11 \times 10^{-31}}$



$v_f = 40 \times 10^{3} - \frac{1.6 \times 10^{-19} \times 50}{9.11 \times 10^{-31}} \times 1.5 \times 10^{-9}$

$v_f = (40 - 13.2) \times 10^{3} = 26.8$ km/s $= \underline{27 \text{ km/s}}$

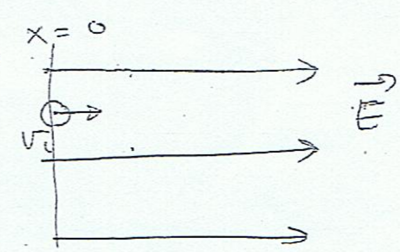
الجواب (b)

12. A particle ($m = 20$ mg, $q = -5.0\,\mu$C) moves in a uniform electric field of 60 N/C in the positive x direction. At $t = 0$, the particle is moving 25 m/s in the positive x direction and is passing through the origin. How far is the particle from the origin at $t = 2.0$ s?

a. 80 m b. 20 m c. 58 m d. 10 m e. 30 m

$x_f = x_i + v_i t + \frac{1}{2} a t^2$

$a = -\frac{qE}{m} = \frac{-5 \times 10^{-6} \times 60}{20 \times 10^{-6}} = 15.0$ m/s^2



$x_f = 0 + 25 \times 2 - \frac{1}{2} \times 15 \times (2)^2$

$= 50 - 30 = 20$ m

Note $20 \text{ mg} = 20 \times 10^{-3} \text{ g} = 20 \times 10^{-3} \times 10^{-3}$ kg

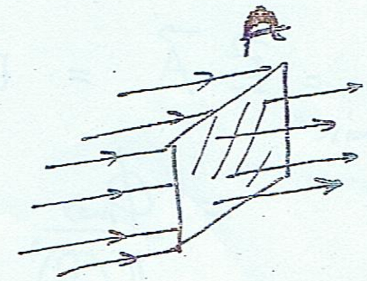
$= 20 \times 10^{-6}$ kg

chapter 24

Guass's Law قانون جاوس

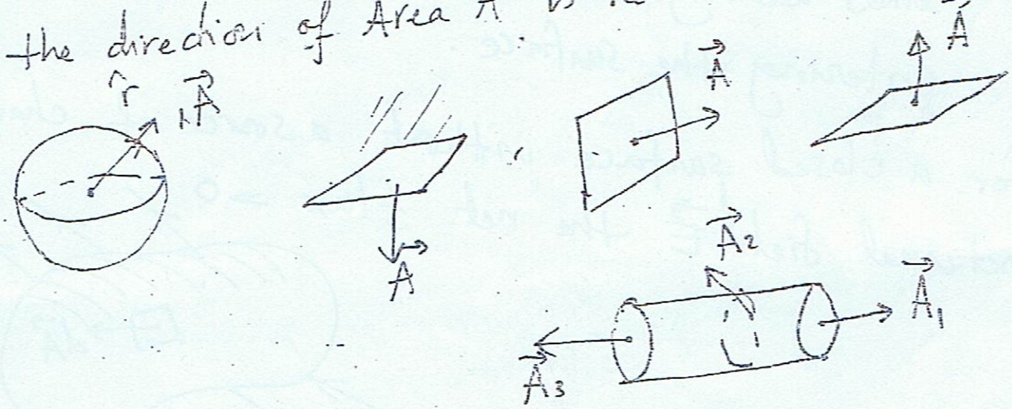
* Electric flux ϕ_E through area is

$$\phi_E = \vec{E} \cdot \vec{A} \quad \left(\frac{N}{C} \cdot m^2\right)$$



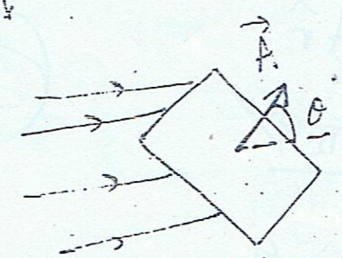
electric flux $\propto$ to the number of electric field lines penetrating (تخترق) the surface.

Note: the direction of Area $\vec{A}$ is normal to it

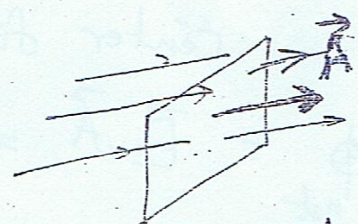


* flux $\phi_E = \vec{E} \cdot \vec{A} = EA\cos\theta$

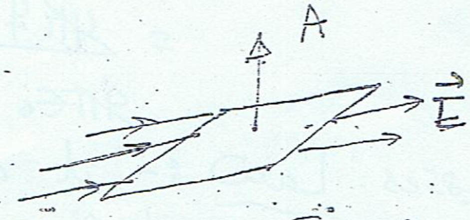
$\Phi_{max} = EA \quad (\theta = 0$ between $\vec{A}$ and $\vec{E})$



$\Phi_E = EA\cos\theta$



$\Phi_E = EA$



$\Phi_E = EA\cos\frac{\pi}{2} = 0$

In general

$$\Phi_E = \int_{surface} E \cdot dA$$

<u>example</u>: a 40 cm diameter (قطر) loop rotated until the maximum flux found to be $\phi_{max} = 5.2 \times 10^5 \, Nm^2/c$

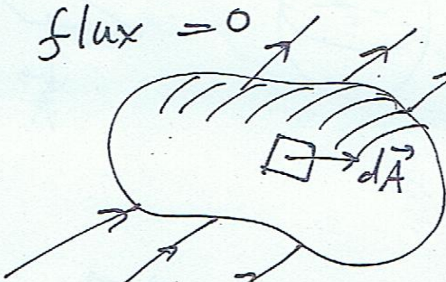
what is $\vec{E}$

$$\phi_{E_{max}} = \vec{E} \cdot \vec{A} = EA \Rightarrow E = \frac{\phi_{max}}{A} = \frac{\phi_{max}}{\pi r^2}$$

$$= \frac{\phi_{max}}{\pi \left(\frac{D}{2}\right)^2} = \frac{5.2 \times 10^5}{3.14 \left(\frac{0.4}{2}\right)^2} = 4.14 \times 10^6 \, N/c$$

<u>Definition</u>: the net flux: the number of electric field lines leaving the surface — the number of lines entering the surface.

Note: for a closed surface without a source of charge in external field $\vec{E}$ the net flux $= 0$



<u>example</u>: a sphere of radius R has a charge q in its center find the net flux through the surface.

$$\phi_{net} = \vec{E} \cdot \vec{A} = \frac{k_e q}{R^2} \hat{r} \cdot A \hat{r}$$

$$= \frac{k_e \, q \times 4\pi \cancel{R^2}}{\cancel{R^2}}$$

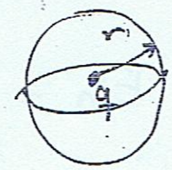
$$= \frac{4\pi q}{4\pi \epsilon_0} = \frac{q}{\epsilon_0}$$

<u>Guasses Law</u> :- it describe the relationship between the net flux through a closed surface (Guassian surface سطح جاوس) and the charge enclosed by that surface.

$$\phi = \oint E \cdot da = \oint E_n \, da = E_n \oint da$$

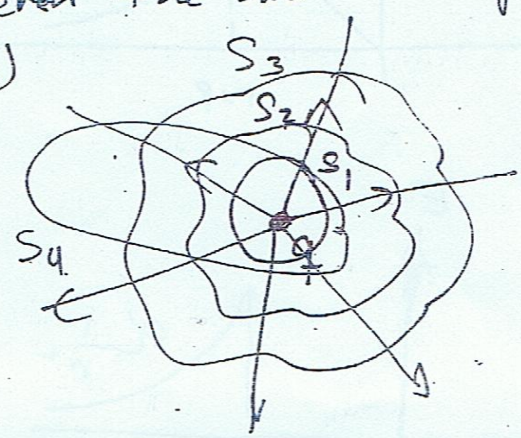
integral over a closed surface.

$$\boxed{\phi_E = \frac{q}{\epsilon_0}}$$



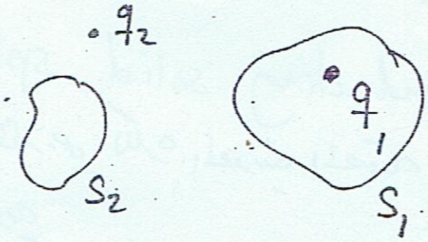
or $$\boxed{\oint E_n \, da = \frac{q_{in}}{\epsilon_0}}$$ Guass's Law

Note: the net flux is the same even the surface shape is different (closed surface)

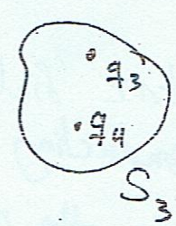


- # of electric field lines through the surfaces S_1, S_2, S_3, S_4 is the same for all surfaces:

$$\phi_E = q/\epsilon_0$$



<u>example</u>: find the net flux through S_1, S_2, S_3.



$$\phi_{s_1} = \frac{q_1}{\epsilon_0} \; ; \; \phi_{s_2} = 0$$

$$\phi_{s_3} = \frac{q_3 + q_4}{\epsilon_0}$$

Note that q_{in}: the net charge inside the surface.

if q_{in} is a distribution of charge then

$q_{in} = \int \lambda \, dl$ <u>or</u> $q_{in} = \underbrace{\int \sigma \, da}_{\text{surface}}$ <u>or</u> $\underbrace{\int \rho \, dV}_{\text{volume}}$

Notes * ϕ_E is independent (لا تعتمد) on the guassian shape

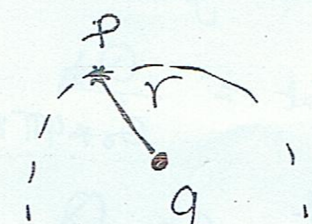
* if the charges move or rearranged inside the guassian surface ϕ_E doesnot change.

Applications of Guass's Law

* Guass's Law can be solved to find $\vec{E}$ due to a system of continuous distribution of charge of highly symmetric situations (الحالات عالية التماثل)

$\vec{E} \parallel \vec{A}$ only.

example ① : find the electric field due to a point charge



نفترض وجود سطح جاوسي يمر بالنقطة p
(سطح كروي)

$$\phi_E = EA = q_{in}/\epsilon_0 = q/\epsilon_0$$

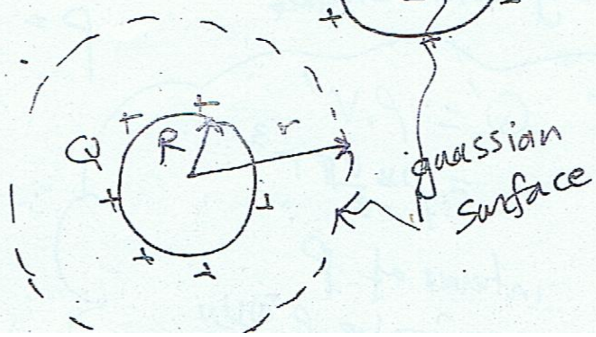
$$E = \frac{q}{\epsilon_0} \cdot \frac{1}{A} = \frac{q}{\epsilon_0 * 4\pi r^2} = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \leftarrow \text{Colomb.}$$

example ② : the electric field due to a spherical shell (thin رقيقة) inside and outside the shell which has a charge Q

1) inside $\Rightarrow \oint_{\text{Guassian}} E\,da = \frac{q_{in}}{\epsilon_0} = 0$

$\therefore E_{inside} = 0$

b) outside $\Rightarrow$



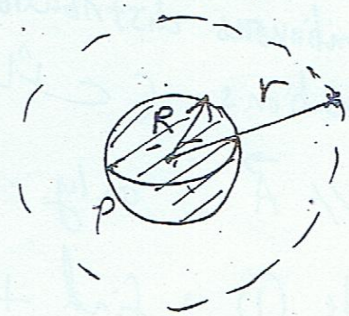
$$\oint_{\text{guassian}} E_{out} \cdot da = \frac{q_{in}}{\epsilon_0} = \frac{Q}{\epsilon_0}$$

$$E_{out} * 4\pi r^2 = Q/\epsilon_0 \Rightarrow E_{out} = \frac{Q}{4\pi\epsilon_0}\frac{1}{r^2} = \frac{k_e Q}{r^2}$$

as a point charge.

example 3: an insulating solid sphere of radius R has a uniform charge density ρ and total charge Q calculate E

(a) out side the sphere (b) inside the sphere.



$$\oint_{\text{guassian}} E.da = \frac{Q_{in}}{\epsilon_0}$$

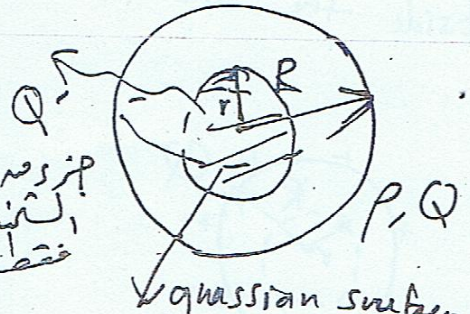
$$E_{out} * A_{\text{guassian}} = \frac{q_{in}}{\epsilon_0}$$

$$E_{out} = \frac{Q_{in}}{\epsilon_0 * 4\pi r^2} = k_e\frac{Q}{r^2}$$

but $\rho = \frac{Q}{V} = \frac{Q}{\frac{4}{3}\pi R^3} \Rightarrow Q = \frac{4}{3}\pi R^3 \rho$

$$E_{out} = \frac{1}{\cancel{4\pi}\epsilon_0} * \frac{\cancel{4}}{3}\cancel{\pi} R^3 \rho / r^2 = \frac{\rho R^3}{3\epsilon_0}\frac{1}{r^2}$$

(b) inside the solid sphere



$$E_{in} * A_{\text{guassian}} = \frac{q_{in}}{\epsilon_0}$$

$$E_{in} * 4\pi r^2 = \frac{Q'}{\epsilon_0}$$

find Q'.

$$\rho = \frac{Q}{V} = \frac{Q'}{V'}$$

$$\frac{Q}{\frac{4}{3}\pi R^3} = \frac{Q'}{\frac{4}{3}\pi r^3}$$

$$\boxed{Q' = \frac{r^3}{R^3} Q}$$

or $Q' = \rho V'$

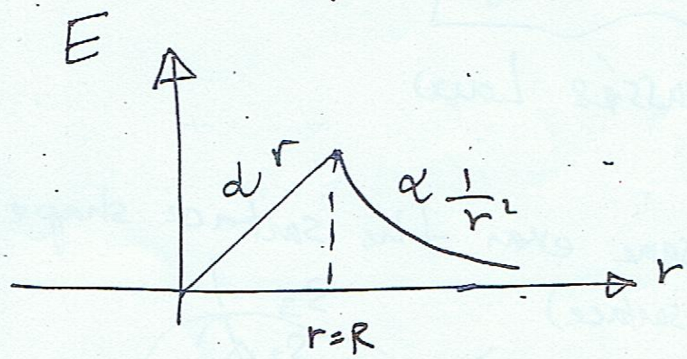
$= \rho * \frac{4}{3}\pi r^3$

interms of ρ

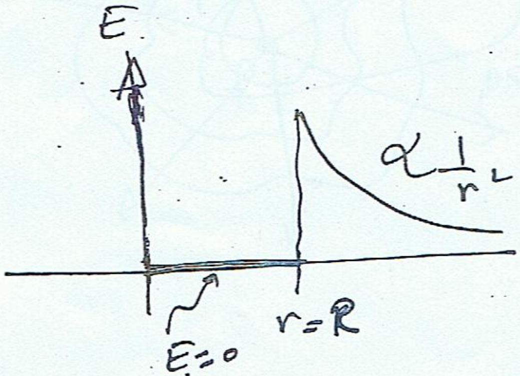
$E_{in} * 4\pi r^2 = \dfrac{Q'}{\epsilon_0} = \dfrac{r^3}{R^3 \epsilon_0} Q$

$E_{in} = k_e \dfrac{Q}{R^3} r$ or interms of ρ $E_{in} = k_e \cdot \frac{4}{3}\pi \rho r$

$= \underbrace{\frac{4}{3}\pi k_e \rho}_{\text{Constant}} r$



⇐ solid insulating sphere.



⇐ shell.

Note : conducting solid sphere equivalent to shell
كقشرة
لأن الشحنة تتواجد على السطح الخارجي للكرة المعدنية المصمتة
فتعاملها معاملة القشرة الكروية

example:- the electric field $\vec{E}$ every where on a surface of a thin conducting shell of $r = 0.75$ cm is 980 N/c
(a) what is the net charge within the sphere surface
(b) inside the sphere.

shell → charge on the surface

(a) $E = \dfrac{Q}{4\pi\epsilon_0 r^2} = k_e \dfrac{Q}{r^2}$

$890 = 9\times10^9 * Q / (0.75)^2 \Rightarrow Q = \dfrac{(0.75)^2 * 980}{9\times10^9} = \ldots$

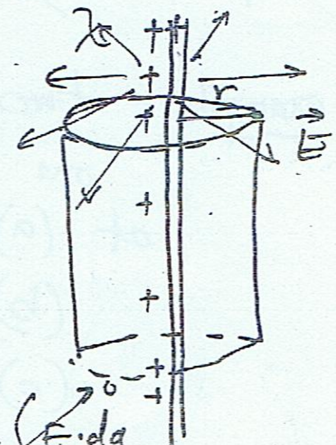
(b) inside the sphere $Q = 0$. (shell)

Cylindrically symmetric charge distribution

توزيع شحني متماثل اسطوانياً.

Case 1: infinite line of charge — سلك طويل جداً مشحون

Find $\vec{E}$ at a point at a distance r from the line of charge of constant linear charge density.



نأخذ سطح جاوسي وهمي محيط بالسلك ويمر بالنقطة
مغلق على شكل اسطوانة

$$\Phi_E = \oint \vec{E}\cdot d\vec{a} = \underbrace{\int E\,da}_{\text{Curved surface}} + \cancel{\underbrace{\int E\,da}_{\text{upper surface}}} + \cancel{\underbrace{\int E\,da}_{\text{lower surface}}}$$

* خطوط المجال عمودية على اتجاه مساحة الغطاء العلوي والغطاء السفلي إذا التدفق من خلالها = صفر.

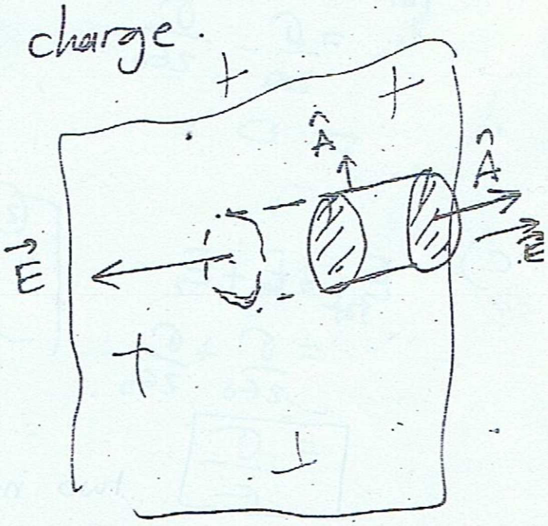
$$\Phi_E = \frac{q_{in}}{\epsilon_0} = E * A_{\text{Curved surface}} = E * 2\pi r l$$

$$E * 2\pi r \cancel{l} = \frac{q_{in}}{\epsilon_0} = \frac{\lambda \cancel{l}}{\epsilon_0} \Rightarrow E = \frac{\lambda}{2\pi\epsilon_0 r} = \frac{2 k_e \lambda}{r}$$

Note $E \propto \frac{1}{r}$ but for spherical symmetric $\propto \frac{1}{r^2}$

Case 2: Non-Conducting plane of charge

Find E due to a non-conducting infinite plane of +ve charge with uniform charge density σ



نأخذ اسطوانة كسطح جاوسي وهمي داخله
في الصفيحة المشحونة

$$\oint \vec{E}\cdot d\vec{a} = \frac{q_{in}}{\epsilon_0} = \frac{\sigma A}{\epsilon_0}$$

$$\oint E \cdot da = \cancel{\int_{\text{curved surface}} \vec{E}} + \int_{\text{upper}} \vec{E} \cdot d\vec{a} + \int_{\text{lower}} E \cdot da = EA + EA$$

$$\phi = 2E\cancel{A} = \frac{\sigma \cancel{A}}{\epsilon_0} \Rightarrow \boxed{E = \frac{\sigma}{2\epsilon_0}}$$

non-conducting

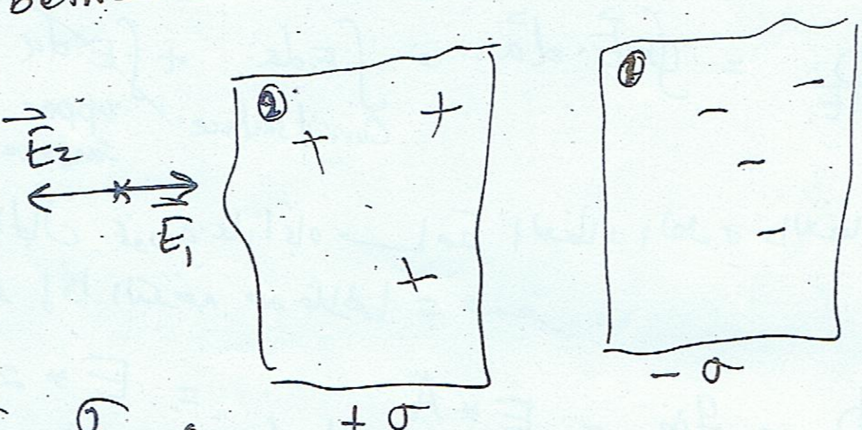
في منطقة قريبة من الصفيحة المستوية

<u>example</u> : two infinite non-conducting parallel sheets one with $+\sigma$ and the other $-\sigma$ find $\vec{E}$ at (a) to the left of the two sheets

(b) to the right - "

(c) between them.

(a)

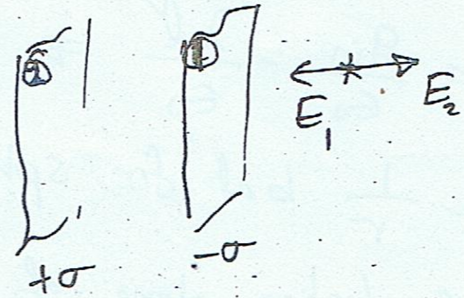


$$|E_1| = \frac{\sigma}{2\epsilon_0}$$

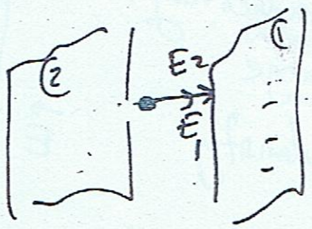
$$|E_2| = \frac{\sigma}{2\epsilon_0}$$

$$E_{net} = \vec{E}_1 - \vec{E}_2 = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$

(b) to the right



$$E_{tot} = E_2 - E_1 = \frac{\sigma}{2\epsilon_0} - \frac{\sigma}{2\epsilon_0} = 0$$



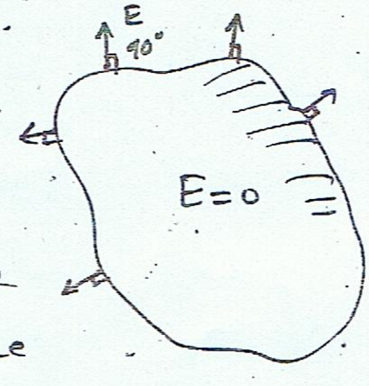
c) $$E_{tot} = E_1 + E_2 = \frac{\sigma}{2\epsilon_0} + \frac{\sigma}{2\epsilon_0} = \boxed{\frac{\sigma}{\epsilon_0}}$$

two non-conducting

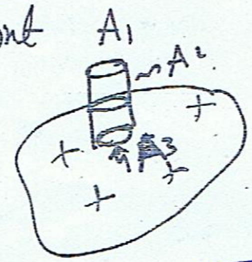
Conductors in electrostatic equilibrium

* the conductors have charger are freely to move. when their is no net motion of charges, the conductor in electrostatic equlibrium (اتزان كهروستاتيكي)

properties of Conductors



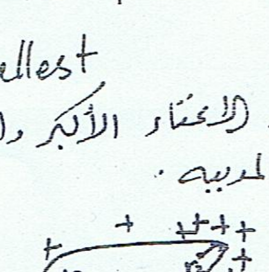
1) $E = 0$ everywhere inside the conductor
لان الشحنة تستقر على السطح المجال في الداخل صفر
2) the charges resides (تستقر) on the surface
3) E just outside a charged conductor ⊥ to the surface and equal to $\frac{\sigma}{\epsilon_0}$; σ the surface charge density at that point



proof:

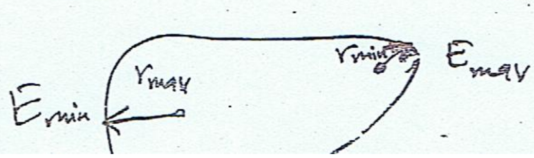
$$\phi_E = \frac{q_{in}}{\epsilon_0}$$
$$= \vec{E}\cdot\vec{A}_1 + \cancel{\vec{E}\cdot\vec{A}_2} + \cancel{\vec{E}\cdot\vec{A}_3}$$
عمودي 0, E=0
$$=$$
$$= E\cancel{A} = \frac{Q_{in}}{\epsilon_0} = \frac{\sigma \cancel{A}}{\epsilon_0} \Rightarrow \boxed{E_{outside} = \sigma/\epsilon_0}$$ conductor

4) in irrgular shaped (الاشكال الغير منتظمة) conductors: the surface charge density at the locations where the radius of curviture gratest is smallest
نصف قطر الانحناء
الشحنات تكون أقل عند نصف قطر الانحناء الأكبر والعكس صحيح.
أي الشحنات تتوزع على الرؤوس المدببة.



example: in an irregular shaped conductor maximum $E = 56\ kN/C$ and minimum $E = 28\ kN/C$ find $\sigma_{r_{max}}$; $\sigma_{r_{min}}$

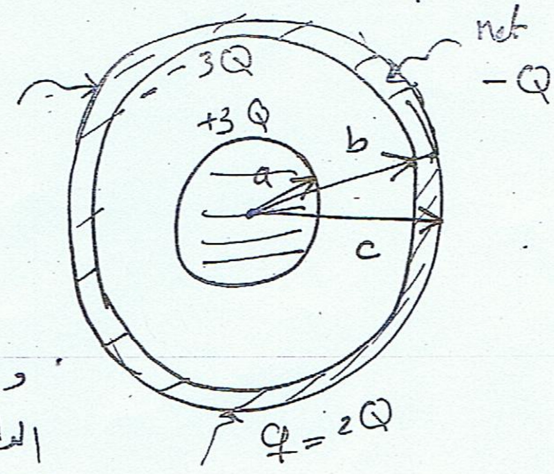
$$E_{max} = \frac{\sigma_{max}}{\epsilon_0} = \frac{\sigma_{at\ r_{min}}}{\epsilon_0}$$



Conducting spherical shell of inner radius b and outer radius c and net charge $-Q$. find E at

a) $r>c$
b) $b<r<c$
d) $a<r<b$
e) $r<a$

القشرة الكروية المعدنية السميكه شحنتها $-Q$ وسيتم إعادة توزيع الشحنات على الكره الداخليه والخارجيه بناءً على الشحنه التأثيريه على الصفحه الداخليه بسبب الكره المشحونه في الداخل.



* the charge on the outer shell will be

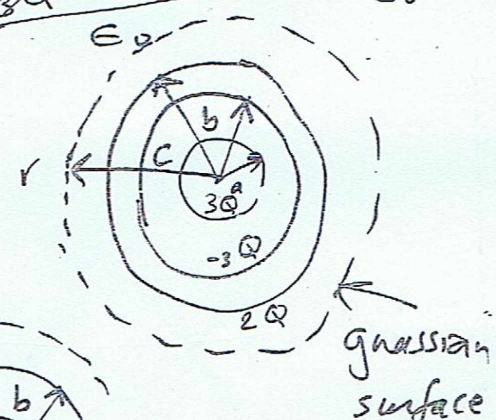
$-3Q+q=-Q \Rightarrow \boxed{q=2Q}$

a) $r>c \Rightarrow E_1 * A_{gaussian} = \frac{q_{in}}{\epsilon_0} = \frac{\cancel{3Q}-\cancel{3Q}+2Q}{\epsilon_0} = \frac{2Q}{\epsilon_0}$

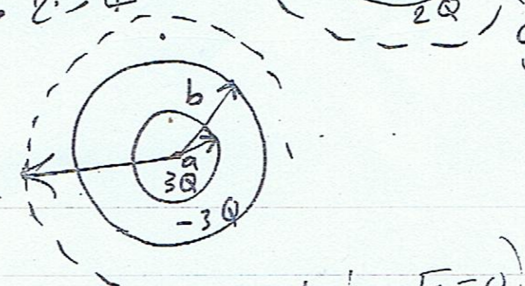
$E_1 = \frac{2Q}{\epsilon_0 * 4\pi r^2} = \frac{2k_e Q}{r^2}$

radially out ward

خارج قطر 2.



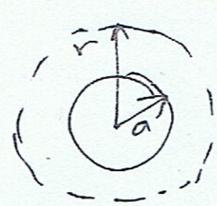
(b) $b\le r\le c$



$E_2 * A_{gaussian} = \frac{q_{in}}{\epsilon_0}$

$= \frac{-3Q+3Q}{\epsilon_0} = 0$ (inside conductor $E=0$)

$E_2 = 0$



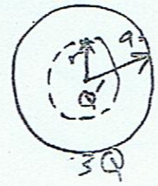
(c) $a\le r\le b$

$E_3 * 4\pi r^2 = \frac{q_{in}}{\epsilon_0} = \frac{3Q}{\epsilon_0}$

$E_3 = \frac{3k_e Q}{r^2}$ radially out ward.

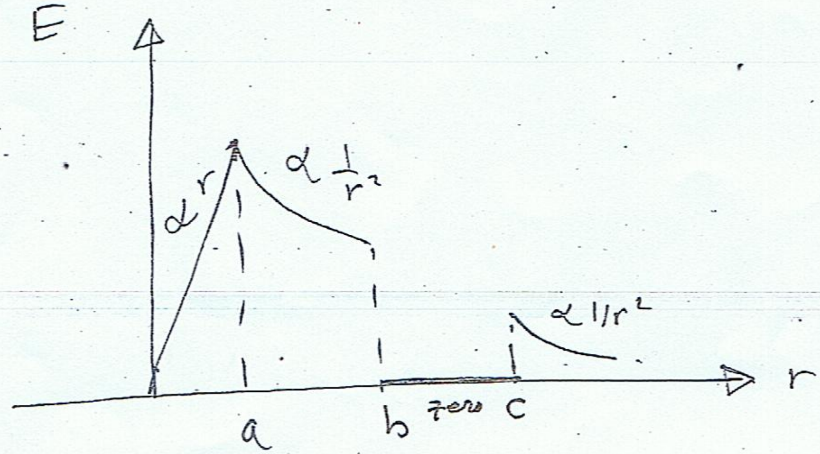
(d) $r < a$

$$E_4 * A_{guassian} = \frac{q_{in}}{\epsilon_0} = \frac{Q'}{\epsilon_0}$$



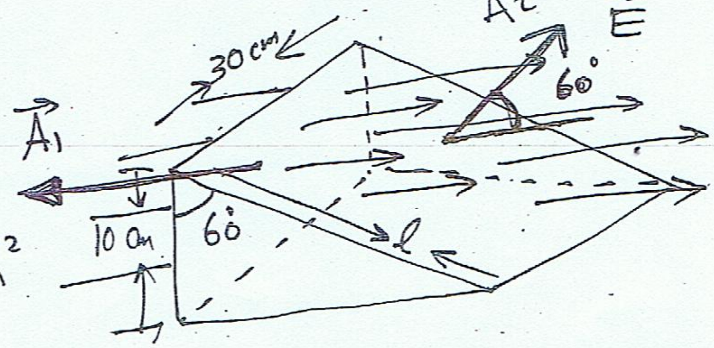
$$\rho = \frac{3Q}{\frac{4}{3}\pi a^3} = \frac{Q'}{\frac{4}{3}\pi r^3} \Rightarrow Q' = \frac{3r^3}{a^3} Q$$

$$E_4 * 4\pi \cancel{r^2} = \frac{3 r^{\cancel{3}} Q}{a^3} \Rightarrow E_3 = \frac{3Q\, k_e}{a^3} r$$



problems

① Consider a closed triangular box resting within a horizontal electric field of magnitude $E = 7.8 \times 10^4$ N/C as shown. Calculate the electric flux through (a) the vertical rectangular surface (b) the slanted (مائل) surface (c) the entire surface of the box.



(a) vertical area

$A_1 = 10\text{ cm} * 30\text{ cm}$
$= 300\text{ cm}^2$
$= 0.1 * 0.3 = 0.03\text{ m}^2$

$$\Phi_{A_1} = \vec{E}\cdot\vec{A}_1 = EA_1\cos 180°$$
$$= 7.8\times10^4 * 0.03\cos 180 = -2.34\times10^3\ \text{N.m}^2/\text{C}$$

$\cos 60 = \frac{10}{l}$

$l = \frac{10}{\cos 60}$

(b) Slanted surface

$$\Phi_{A_2} = EA_2\cos\theta = 7.8\times10^4 * A_2 * \cos 60°$$

$A_2 = l * 30\text{ cm} = l * 0.3$
$= \frac{10 * 0.3}{\cos 60°}$

$$\Phi_{A_2} = 7.8\times10^4 * \frac{10 * 0.3}{\cancel{\cos 60°}} * \cancel{\cos 60°} = +2.34\times10^3\ \text{Nm}^2/\text{C}$$

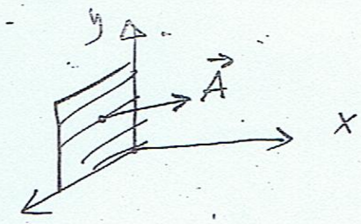
c) Φ on the sides $= EA_{sides} * \cos\frac{\pi}{2} = 0$

$\Phi_{on\ the\ bottom} = 0$ also.

$$\Phi_{total} = \Phi_{A_1} + \Phi_{A_2} + \cancelto{0}{\Phi_{sides\ and\ bottom}}$$
$$= -2.34\times10^3 + 2.34\times10^3 + 0 = 0$$

closed surface with No source charge inside it then the $\Phi_{total} = 0$

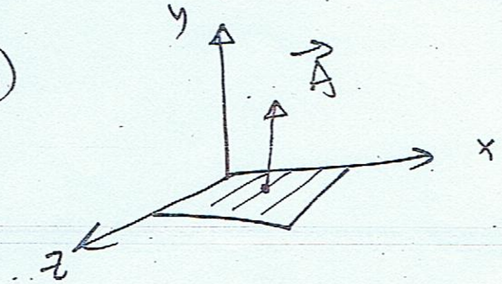
(2) a uniform electric field $a\hat{i} + b\hat{j}$ intersects a surface of area A. what is the flux through this area if the surface lies. (a) in the yz plane (b) in the xz plane c) in the xy plane



(a) $\vec{E} = a\hat{i} + b\hat{j}$

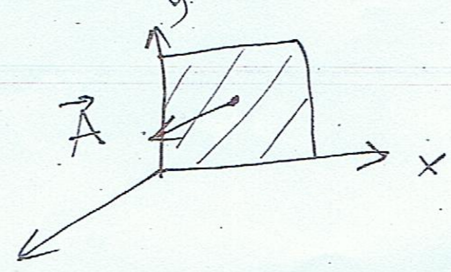
$$\phi_E = \vec{E} \cdot \vec{A} = (a\hat{i} + b\hat{j}) \cdot A\hat{i}$$
$$= aA$$

(b)



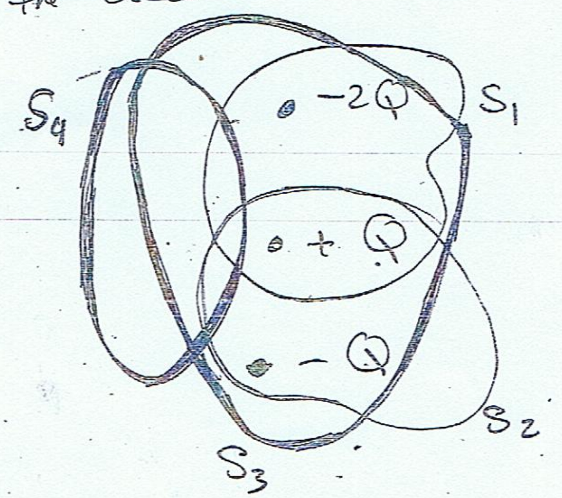
$\vec{A} = A\hat{j}$

$$\phi_E = (a\hat{i} + b\hat{j}) \cdot A\hat{j} = bA$$



c) $$\phi_E = \vec{E} \cdot \vec{A} = (a\hat{i} + b\hat{j}) \cdot A\hat{k}$$
$$= 0$$

problem 3 : four closed surfaces S_1 through S_4 together with the charges $-2Q$, Q and $-Q$ are sketched in the figure. ~~the ello~~ find the electric flux through each surface.



$$\phi_E = \frac{q_{in}}{\epsilon_0}$$

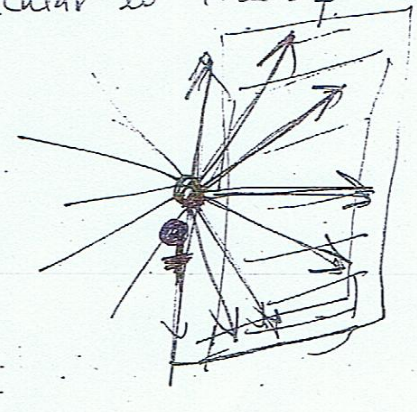
$$\phi_{S_1} = \frac{-2Q + Q}{\epsilon_0} = \frac{-Q}{\epsilon_0}$$

$$\phi_{S_2} = \frac{+Q - Q}{\epsilon_0} = 0$$

$$\phi_{S_3} = \frac{-2Q + Q - Q}{\epsilon_0} = \frac{-2Q}{\epsilon_0}$$

$$\phi_{S_4} = 0$$

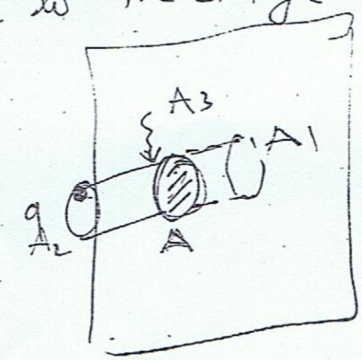
problem 4 : A point charge q is located a distance d from an infinit plane . determine the electric flux through the plane due to the point charge . what if the point charge located a very small distance from the center of a very large squre on the line perpendicular to the squre and going through its center .



(a) one half of the total flux created by the charge q goes throug the plane

$$\phi_{E\,plane} = \frac{1}{2}\phi_{E\,tot} = \frac{1}{2}\left(\frac{q}{\epsilon_0}\right) = \frac{q}{2\epsilon_0}$$

(b) the squre looks like an infinite plane to the charge (very close)



flux throug guassian surface

$$\phi_{net} = \phi_{A_1} + \phi_{A_2} + \cancel{\phi_{A_3}} \to 0 =$$

$$A_1 = A_2$$

$$\frac{q_{in}}{\epsilon_0} = 2\phi \Rightarrow \phi = \frac{q}{2\epsilon_0} \text{ (infinite plane)}$$

(c) the plane and the squre look the same to the charge.

problem 5 : a particle with charge of $12\,\mu C$ is placed at the center of a spherical shell of radius 22 cm. what is the total electric flux through

(a) the surface of the shell

b) any hemispherical ~~shell~~ (نصف كروي) surface to the shell

$$\text{(a)}\ \phi_{shell} = \frac{q_{in}}{\epsilon_0} = \frac{12\times10^{-6}}{8.85\times10^{-12}} = 1.36\times10^{6}\ \frac{N\,m^2}{C} = 1.36\ M\ Nm^2/C$$

(b) $\Phi_{E(\text{half shell})} = \frac{1}{2}\Phi_{E\,shell} = \frac{1}{2}\left(1.36\times10^{\dots}\ \frac{N\dots}{C}\right)$

$= 6.78\times10^{5}\ Nm^2/C = 678\ k.Nm^2/C$

* the result independent of the radius. the same number of field lines will pass through each surface, no matter how the radius changes.

problem 6: Determine the magnetude of the electric field at the surface of a leadd 208 nucleus (نواة الرصاص) which contain 82 protons and 126 neutrons. Assume the lead nucleus has a volume 208 times that of one proton, and consider a proton to be a sphere of radius 1.2×10^{-15} m

$$q_{nucleus} = n_p * q_p = 82 * 1.6\times10^{-19}\ C$$

$$E = \frac{k_e q_n}{r_n^2} = \frac{9\times10^{9} * 82 * 1.6\times10^{-19}}{r_n^{2}}$$

(assume the volumes are spheres)

$$V_{nucleus} = 208\, V_{proton}$$

$$\frac{4}{3}\pi r_n^3 = 208 * \frac{4}{3}\pi r_p^2 \Rightarrow r_n = \left(208 * 1.2\times10^{-15}\right)^{1/3}$$

$$E = 2.33\times10^{+21}\ N/C \text{ away from the nucleus.}$$

problem 7: A solid sphere of radius 40 cm has total positive charge of 26 μC uniformly distributed throughout its volume. Calculate the magnetude of the electric field at:

(a) 0 cm (b) 10 cm (c) 40 cm d) 60 cm from the center of the sphere.

a) $E = \frac{k_e Q r}{a^3} = 0$ (inside)

b) $E = k_e \frac{Q r}{a^3} = \frac{9\times10^{9} * 26\times10^{-6} * 0.1}{(0.4)^3} = 365\ kN/C$ radially outward

c) $E = \frac{k_e Q}{r^2} = \frac{9\times10^{9} * 26\times10^{-6}}{(0.4)^2} = 1.46\ M\,N/C$ radially outward

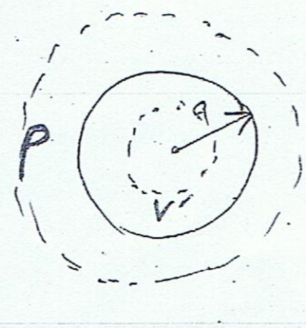
(d) ...

problem 8 : An insulating solid sphere of radius a has a uniform volume charge density and carries a total positive charge Q. A spherical guassian surface of radius r which shares a common center with the insulating sphere.

(a) find an expression for the electric flux passing through the surface of the guassian sphere as a function of $r < a$

b) $r > a$ (c) plot the flux versus r

في الداخل

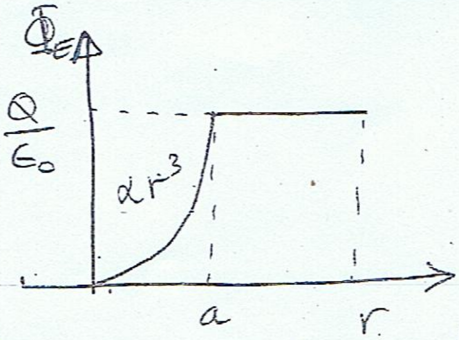


(a) $\rho = \frac{Q}{V} = \frac{Q}{\frac{4\pi a^3}{3}} = \frac{3Q}{4\pi a^3}$

$\phi_E = \frac{q_{in}}{\epsilon_0} = \frac{\rho V'}{\epsilon_0} = \frac{3Q}{4\pi a^3 \epsilon_0} * \frac{4}{3}\pi r^3$

$= \frac{Q r^3}{\epsilon_0 a^3}$

(b) out side the sphere $\Phi_E = \frac{Q}{\epsilon_0}$ = constant



problem 9 : A solid Copper (conductor) sphere (كرة نحاس) of radius 15 cm carries a charge of 40 nC. find the electric field 15cm from the center of the sphere

a) 12 cm b) 17 cm c) 75 cm

d) what if we change the sphere with hollow (مجوف كرة مجوفة)

1) * all charge sits on the surface $\Rightarrow E_{inside} = 0$ (at $r = 12cm$)

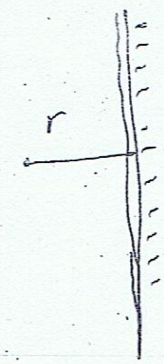
2) $E = \frac{k_e Q}{r^2} = \frac{9\times10^9 * 40\times10^{-9}}{(0.17)^2} = 1.24 \times 10^4$ N/C outward

d) the solid conducting sphere $\equiv$ hollow sphere or shell

لا شيء تتغير على الاجابة

problem 10: the electric charge per unit length on a long, straight filament is $-90\ \mu C/m$ find the electric field

a) 10 cm   b) 20 cm   c) 100 cm from the filament



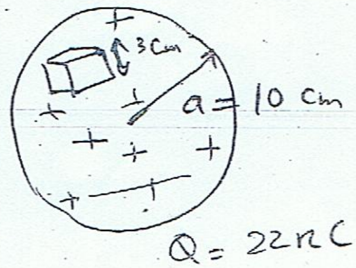
a) $E = \frac{2k_e\lambda}{r} = \frac{2*9\times10^{9} * 90\times10^{-6}}{0.1} = 16.2\times10^{6}\ N/C$

b) $E = \frac{2k_e\lambda}{r} = \frac{2*9\times10^{9} * 90\times10^{-6}}{0.2} = 8.09\times10^{6}\ N/C$

c) $E = \frac{2k_e\lambda}{r} = \frac{2(9\times10^{9}) * 90\times10^{-6}}{1} = 1.62\times10^{6}\ N/C$

old exam problem

a solid sphere with charge Q and radius a find the flux on a cube of side 3cm inside the sphere



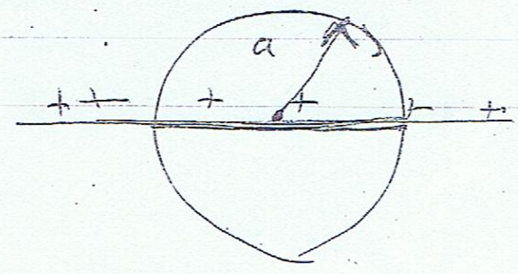
$$\Phi = \frac{q_{in}}{\epsilon_0} = \frac{\rho * V_{cube}}{\epsilon_0} = \rho * \frac{L^3}{\epsilon_0}$$

$$= \rho * \frac{(3\times10^{-2})^3}{8.85\times10^{-12}} = \ldots$$

$$\rho = Q / \frac{4}{3}\pi a^3 = 22\times10^{-9} / \frac{4}{3}*\pi * (10\times10^{-2})^3 = \ldots$$

* old exam problem

a sphere of radius $a = 10$ cm covers a part of line of charge of $\lambda = 50\ nC/m$



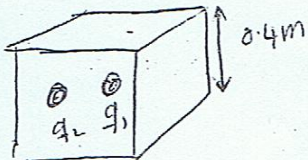
find Φ (flux) on the sphere

$$\Phi = \frac{q_{in}}{\epsilon_0} = \frac{\lambda l}{\epsilon_0} = \frac{\lambda * 2a}{\epsilon_0} = \frac{50\times10^{-9} * 2*(10\times10^{-2})}{8.85\times10^{-12}} = \ldots$$

CHAPTER 24

1. Two charges of 15 pC and −40 pC are inside a cube with sides that are of 0.40 m length. Determine the net electric flux through the surface of the cube.

a. +2.8 N m²/C b −1.1 N m²/C c. +1.1 N m²/C d. −2.8 N m²/C e. −0.47 N m²/C



$$\phi = \frac{q_{in}}{\epsilon_0} = \frac{-40\times10^{-12} + 15\times10^{-12}}{8.85\times10^{-12}} = -2.82\ \mathrm{N\,m^2/C}$$

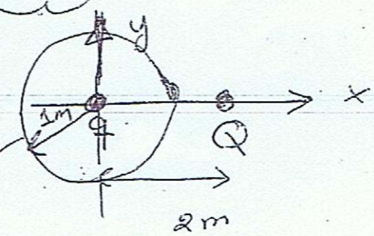
d. الحل

2. Charges q and Q are placed on the x axis at x = 0 and x = 2.0 m, respectively. If q = −40 pC and Q = +30 pC, determine the net flux through a spherical surface (radius = 1.0 m) centered on the origin.

a. −9.6 N m²/C b. −6.8 N m²/C c. −8.5 N m²/C d. −4.5 N m²/C e. −1.1 N m²/C

$$\phi = \frac{q_{in}}{\epsilon_0} = \frac{q}{\epsilon_0} = \frac{-40\times10^{-12}}{8.85\times10^{-12}} = -4.5\ \mathrm{Nm^2/C}$$

Note: Q out side the sphere



Surface of radius $r = 1\,m$, مركز حول أ، ر

3. A point charge +Q is located on the x axis at x = a, and a second point charge −Q is located on the x axis at x = −a. A Gaussian surface with radius r = 2a is centered at the origin. The flux through this Gaussian surface is

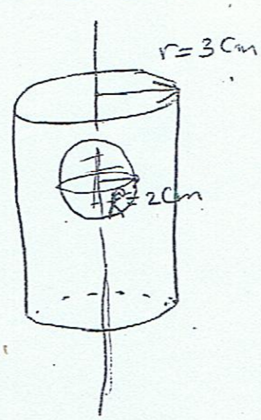
نصف كرة

a. zero because the negative flux over one hemisphere is equal to the positive flux over the other.
b. greater than zero.
c. zero because at every point on the surface the electric field has no component perpendicular to the surface.
d. zero because the electric field is zero at every point on the surface.
e. none of the above

4. A long cylinder (radius = 3.0 cm) is filled with a nonconducting material which carries a uniform charge density of 1.3 µC/m³. Determine the electric flux through a spherical surface (radius = 2.0 cm) which has a point on the axis of the cylinder as its center.

a. 5.7 N m²/C b. 4.9 Nm²/C c. 6.4 Nm²/C d. 7.2 N m²/C e. 15 Nm²/C

ϕ through the sphere which is inside the solid cylinder $= \frac{q_{in}}{\epsilon_0} = \frac{\rho * V_{charge}}{\epsilon_0}$



$$\phi = \frac{\rho * \frac{4}{3}\pi R^3}{\epsilon_0} = \frac{1.3\times10^{-6} * 4 * 3.14 * (2\times10^{-2})^3}{3 * 8.85\times10^{-12}}$$

$= 4.9\ \mathrm{Nm^2/C}$

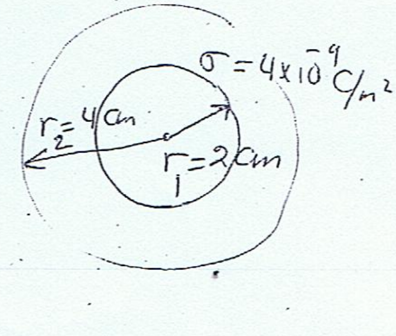
5. Charge of uniform surface density (4.0 nC/m²) is distributed on a spherical surface (radius = 2.0 cm). What is the total electric flux through a concentric spherical surface with a radius of 4.0 cm?

a. 2.8 Nm²/C b. 1.7 Nm²/C c. 2.3 Nm²/C d. 4.0 Nm²/C e. 9.1 Nm²/C

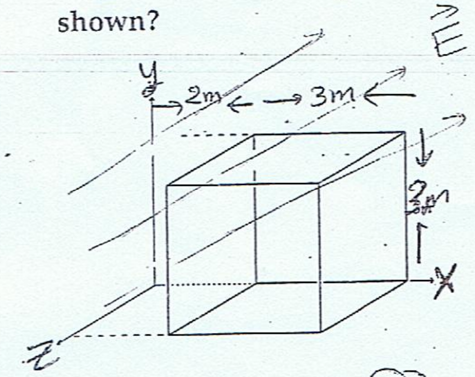
$$\phi_{\text{large sphere}} = q_{in}/\epsilon_0 = \frac{\sigma A}{\epsilon_0} = \frac{\sigma \times 4\pi r_1^2}{\epsilon_0}$$

$$= \frac{4\times10^{-9} * 4 * 3.14 * (2\times10^{-2})^2}{8.85\times10^{-12}}$$

$$= 2.27 \simeq 2.8\ \mathrm{N\,m^2/C}$$



6. The electric field in the region of space shown is given by E = (8i + 2yj) N/C where y is in m. What is the magnitude of the electric flux (in Nm²/C) through the top face of the cube shown?



$$\vec{A}_{\text{top face}} = 3 * 3\ m^2 = 9\ m^2\ \hat{j}$$

$$\phi = \vec{E}\cdot\vec{A} = (8\hat{i} + 2y\hat{j})\cdot 9\hat{j}$$

$$= 2 * 9 * y = 18y$$

$$y\big|_{\text{top surface}} = 3m \Rightarrow \phi = 18 * 3 = 54\ \mathrm{Nm^2/C}$$

a. 90 b. 6.0 c. 54 d. 12 e. 126

7. Charge of uniform surface density (0.20 nC/m²) is distributed over the entire xy plane. Determine the magnitude of the electric field (in N/C) at any point having z = 2.0 m.

a. 17 b. 11 c. 23 d. 28 e. 40

infinite sheet $\Rightarrow E = \frac{\sigma}{2\epsilon_0} = \frac{0.2\times10^{-9}}{2*8.85\times10^{-12}} = 11.2 \simeq 11\ N/C$

8. Two infinite parallel surfaces carry uniform charge densities of 0.20 nC/m² and −0.60 nC/m². What is the magnitude of the electric field at a point between the two surfaces?

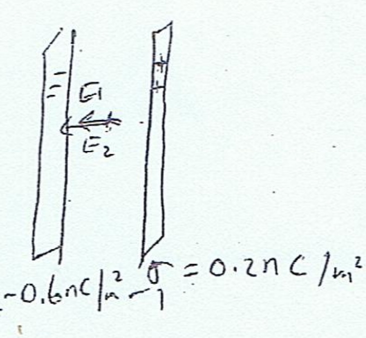
a. 34 N/C b. 23 N/C c. 45 N/C d. 17 N/C e. 90 N/C

$$\vec{E} = \vec{E}_1 + \vec{E}_2$$

$$= \left(\frac{\sigma_1}{2\epsilon_0} + \frac{\sigma_2}{2\epsilon_0}\right)(-\hat{i}) = \frac{1}{2\epsilon_0}\{\sigma_1 + \sigma_2\}(-\hat{i})$$

$$= \frac{0.2\times10^{-9}}{2*8.85\times10^{-12}} + \frac{0.6\times10^{-9}}{2*8.5\times10^{-12}} = 45.2\ N/C$$

$$\simeq 45\ N/C$$



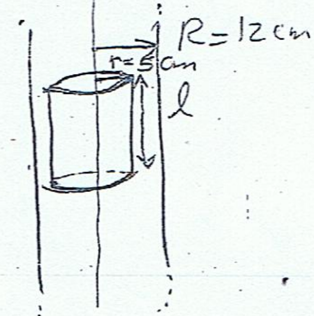
9. A long non-conducting cylinder (radius = 12 cm) has a charge of uniform density (5.0 nC/m³) distributed throughout its column. Determine the magnitude of the electric field 5.0 cm from the axis of the cylinder.

a. 25 N/C b. 20 N/C c. 14 N/C d. 31 N/C e. 34 N/C

$$E A_{gaussian} = q_{in}/\epsilon_0 = \frac{\rho * V_{charge}}{\epsilon_0}$$

$$E * 2\pi r l = \frac{\rho * \pi r^2 l}{\epsilon_0}$$

$$E = \frac{\rho r}{2\epsilon_0} = \frac{5\times10^{-9} * 5\times10^{-2}}{2 * 8.85\times10^{-12}} = 14.1 \cong 14\ N/C$$



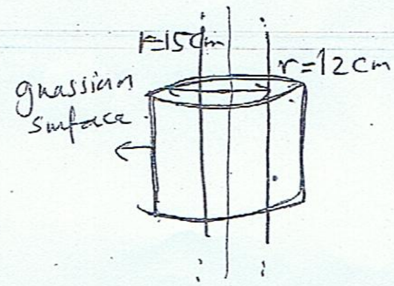
10. A long nonconducting cylinder (radius = 12 cm) has a charge of uniform density (5.0 nC/m³) distributed throughout its volume. Determine the magnitude of the electric field 15 cm from the axis of the cylinder.

a. 20 N/C b. 27 N/C c. 16 N/C d. 12 N/C e. 54 N/C

$$E A_{gaussian} = q_{in}/\epsilon_0 = \rho * V_{charge}/\epsilon_0$$

$$E * 2\pi r l = \frac{\rho * \pi R^2 l}{\epsilon_0} \Rightarrow E = \frac{\rho R^2}{2\epsilon_0} * \frac{1}{r}$$

$$E = \frac{5\times10^{-9} * (0.12)^2}{2 * 8.85\times10^{-12}} * \frac{1}{0.15} = 27\ N/C$$



11. Charge of uniform density (80 nC/m³) is distributed throughout a hollow cylindrical region formed by two coaxial cylindrical surfaces of radii, 1.0 mm and 3.0 mm. Determine the magnitude of the electric field at a point which is 2.0 mm from the symmetry axis.

a. 7.9 N/C b. 9.0 N/C c. 5.9 N/C d. 6.8 N/C e. 18 N/C

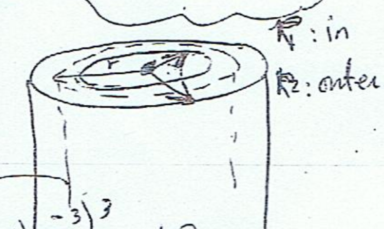
المطلوب الحال في نقطة تقع بين الاسطوانتين متحدتي المركز :

$$E * A_{gaussian} = \frac{q_{in}}{\epsilon_0}$$

$$E * 2\pi r l = \frac{\rho V_{charge}}{\epsilon_0} = \frac{\rho(V_2 - V_1)}{\epsilon_0}$$

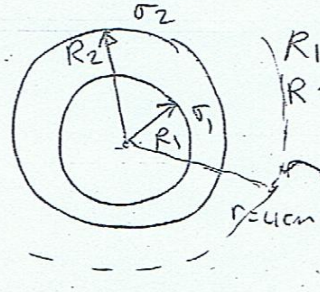
$$E * 2\pi r l = \frac{\rho \pi l (R_2^2 - R_1^2)}{\epsilon_0} \Rightarrow E = \frac{\rho (R_2^2 - R_1^2)}{2\epsilon_0 r} = \frac{80\times10^{-9}((3-1)\times10^{-3})^3}{2*8.85 * 2\times10^{-3}} = 18\ N/C$$

الجواب e



12. Charge of uniform density (40 pC/m²) is distributed on a spherical surface (radius = 1.0 cm), and a second concentric spherical surface (radius = 3.0 cm) carries a uniform charge density of 60 pC/m². What is the magnitude of the electric field at a point 4.0 cm from the center of the two surfaces?

a. 3.8 N/C b. 4.1 N/C c. 3.5 N/C d. 3.2 N/C e. 0.28 N/C



$R_1 = 1\ cm$; $\sigma_1 = 40\times10^{-12}\ C/m^2$
$R_2 = 3\ cm$; $\sigma_2 = 60\times10^{-12}\ C/m^2$

$$E * A_{gaussian} = \frac{q_{in}}{\epsilon_0} = (Q_1 + Q_2)/\epsilon_0$$

$$E * 4\pi r^2 = 4\pi(\sigma_2 R_2^2 - \sigma_1 R_1^2)/\epsilon_0$$

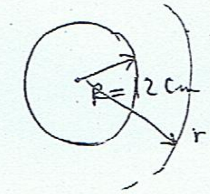
$Q_1 = \sigma_1 A_1 = \sigma_1 * 4\pi R_1^2$
$Q_2 = \sigma_2 A_2 = \sigma_2 * 4\pi R_2^2$

$$E = \frac{(\sigma_2 R_2^2 - \sigma_1 R_1^2)}{\epsilon_0 r^2} = \frac{(60*(3)^2 - 40*1)\times10^{-12}\times10^{-4}}{8.85\times10^{-12}(4\times10^{-2})^2}$$

(4)

13. A solid nonconducting sphere (radius = 12 cm) has a charge of uniform density (30 nC/m3) distributed throughout its volume. Determine the magnitude of the electric field 15 cm from the center of the sphere.

a. 22 N/C b. 49 N/C c. 31 N/C d. 87 N/C e. 26 N/C



$E * A_{gaussian} = q_{in}/\epsilon_0 = \rho * V_{charge}/\epsilon_0$

$E * 4\pi r^2 = \rho * \frac{4\pi R^3}{3\,\epsilon_0} \Rightarrow E = \frac{\rho R^3}{3\epsilon_0 r^2}$

$E = \frac{30\times10^{-9} * (0.12)^3}{3*8.85\times10^{-12} * (0.15)^2} = 86.8\ N/C \approx 87 N/C$

14. The electric field just outside the surface of a hollow conducting sphere of radius 20 cm has a magnitude of 500 N/C and is directed outward. An unknown charge Q is introduced into the center of the sphere and it is noted that the electric field is still directed outward but has decreased to 100 N/C. What is the magnitude of the charge Q?

a. 1.5 nC b. 1.8 nC c. 1.3 nC d. 1.1 nC e. 2.7 Nc

E_1

$E_{1out} = 500\ N/m \Rightarrow E_1 = k_e q/r^2 \Rightarrow q = \frac{r^2 E_1}{k_e} = \frac{(0.2)^2 * 500}{9\times10^9}$

$+q = 2.22\times10^{-9}\,C$ الشحنة الاولى قبل الاضافة

$E_2(out) = 100\ N/C$

Q_0 E_2 Q_1

$E_2 = 100 = \frac{k_e(q+Q)}{r^2} \Rightarrow q+Q = 100*(0.2)^2/9\times10^9 = 4.44\times10^{-10}\,C$

$Q = 4.44\times10^{-10} - 2.2\times10^{-9} = -1.78\times10^{-9}\,C \Rightarrow$ magnitude $\simeq 1.8\ nC$

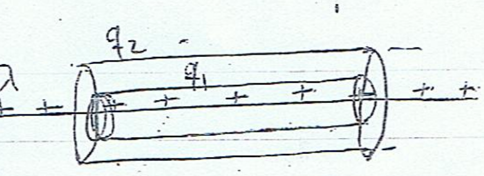
15. The axis of a long hollow metallic cylinder (inner radius = 1.0 cm, outer radius = 2.0 cm) coincides with a long wire. The wire has a linear charge density of +8.0 nC/m, and the cylinder has a net charge per unit length of +4.0 nC/m. Determine the surface charge density on the outer surface of the cylinder. اسطوانة سمكية مجموعها سالب شحنة

a. 95 nC/m² b. 64 nC/m² c. 48 nC/m² d. 38 nC/m² e. 32 nC/m²

$\lambda_1 = +8\times10^{-9}\,C$ للسلك

$-\lambda_1$ السلك سيكون شحنة تأثيره على الاسطوانة الداخلية

تساوي شحنة السلك بالاشارة السالبة وبالتالي مجموع الشحنتين



$12\,nC/m = q_2 \Leftarrow 4\,nC/m = q_2 + q_1/l$

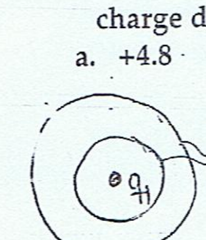
كثافة شحنة السطحية على الاسطوانة الخارجية

$\sigma_{outer} = \frac{q_2}{A_2} = \frac{12\times10^{-9} * l}{2\pi r_2 l} = 95.5\,nC \simeq 95\,nC/m^2$

16. A point charge of 6.0 nC is placed at the center of a hollow spherical conductor (inner radius = 1.0 cm, outer radius = 2.0 cm) which has a net charge of –4.0 nC. Determine the resulting charge density (µC/m²) in on the inner surface of the conducting sphere.

a. +4.8 b. –4.8 c. –9.5 d. +9.5 e. –8.0

سوف تكون شحنة تأثيره مقدارها $-q_1$ على الكرة الداخلية وبالتالي



$Q_{net} = -4nC$

تكون الشحنة على الكرة الخارجية q_2 وبالتالي

$Q_{net} = -q_1 + q_2$ (الكرة الخارجية)

$q_2 = Q_{net} + q_1 = -4nC + 6nC = +2nC$

$q = 6nC$

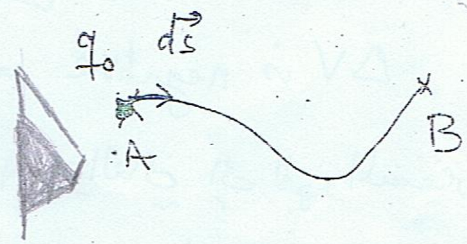
وبالتالي $\sigma_1 = \frac{q_1}{4\pi r_1^2} = \frac{-6\times10^{-9}}{4\pi * (0.01)^2} = -4.8\,\mu C/m^2$

phy 2 — chapter 25: Electric Potential — د. صفية حسن

* the electrostatic phenomin (ظواهر) can be described by defining a scalar quantity called electric potential V

* for a test charge q_0 placed in $\vec{E}$ the force acting on on $q_0 = q_0\vec{E}$.

* Work done by the field by the external agent المؤثر الخارجي
 = − Work done by the field



$dW_{(\text{by the } (E) \text{ on the charge})} = \vec{F}\cdot d\vec{s}$
$= q_0\vec{E}\cdot d\vec{s}$

$W = \int dW = q_0\int \vec{E}\cdot d\vec{s}$ (by the field)

$\therefore W_{\text{by the external agent}} = -q_0\int \vec{E}\cdot d\vec{s}$

* the potential energy ΔU is defined through the work of conservative force (chapter 7)

$W_{\text{conservative}} = -\Delta U = -(U_f - U_i) = U_i - U_f$

$\Rightarrow \boxed{\Delta U = -W = -q_0\int_A^B \vec{E}\cdot d\vec{s}}$

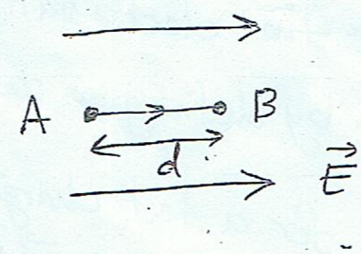
q_0E is conservative force $\Rightarrow$ the integral is independent of path (لاتعتمد التكامل على ا‍ـــن) تعتمد على نقطة البداية ونقطة النهاية فقط

* Define: the electric potential is the potential energy per unit charge

$V = U/q_0 \Rightarrow \frac{\Delta U}{q_0} = \boxed{\Delta V = -\int_A^B \vec{E}\cdot d\vec{s}}$

* <u>the potential difference in a uniform electric field</u>

a charged moved from A to $\vec{B}$



$$\Delta V = V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s}$$

$$= -\int_A^B E\,ds\cos\theta = -E\int_A^B ds$$

$$\boxed{\Delta V = -Ed}$$

ΔV is negative $\Rightarrow$ means $V_B < V_A$ (means the charge r from high potential lower potential)

الـــــشحنات تتحرك من الجهد العالي إلى الجهد المنخفض.

مهم * the electric field lines always points in the direction of decreasing potential.

* positive charge loses potential energy when it moves in the direction of the electric field (means: E does not any work to move positive charges in its direction)

* $V = U/q_0 \Rightarrow \Delta V = \dfrac{\Delta U}{q_0} \Rightarrow \boxed{\Delta U = q_0 \Delta V}$

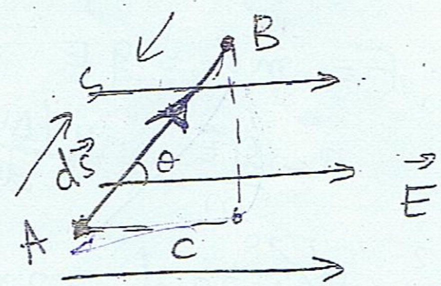
ΔV is a scalar quantity in units of $(J/C) \equiv$ Volt.

* the potential difference ΔV between two points A, B is the work per unit charge that an external agent must perform to move the test charge from A to B

* electric potential at any arbitrary (إفتراضية) point: الجهد المطلق لأي نقطة.
is the work required per unit charge to bring +ve charge from ∞ to that point P. $(V_\infty = 0)$

$V_P = -\int_{\infty}^{P} E \cdot ds$

* potential difference in a uniform electric field :



$$\Delta V_{(A \to B)} = -\int_A^B \vec{E} \cdot \vec{ds}$$

$$= -\left(\vec{E} \cdot \vec{S}\right)_{A \to B}$$

$$= -ES\cos\theta$$

$\cos\theta = \dfrac{\overline{AC}}{S}$

$\overline{AC} = S\cos\theta$

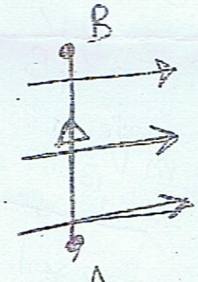
$$\Delta V_{A \to B} = \Delta V_{(A \to C)} + \Delta V_{C \to B}$$

$$= \cancel{V_C} - V_A + V_B - \cancel{V_C} = V_B - V_A$$

$$\Delta V_{A \to C} = -\vec{E} \cdot \vec{AC} = -ES\cos\theta$$

$$\Delta V_{C \to B} = -\vec{E} \cdot \vec{CB} = -E\,\vec{CB}\cos\frac{\pi}{2} = 0$$

* Note : $\Delta V_{A \to B} = V_B - V_A$
$= 0$



الانتقال العمودي على خطوط المجال : فرق الجهد على السطح = صفر

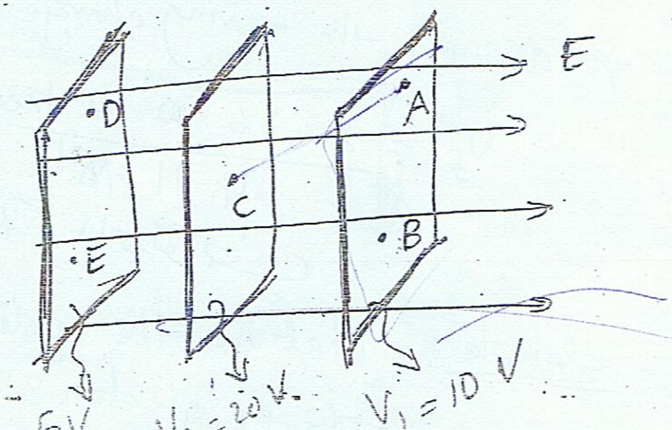
all points in the plane $\perp$ to a uniform $\vec{E}$ are at the same electric potential called equipotential surface سطح تساوي جهد

ex a) find $V_{AB} = V_B - V_A$
$= 0$

b) $V_{AC} = V_C - V_A = 20 - 10$
$= 10\,V$

c) $V_{BD} = V_D - V_B = 5 - 10$
$= -5\,V$

d) $V_{ED} = V_E - V_D = 0$

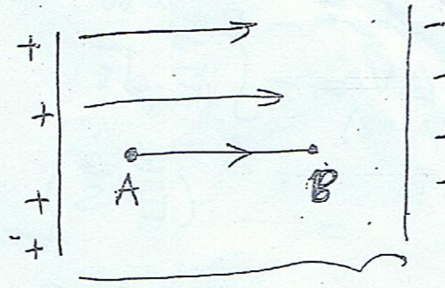


example: Calculate the speed of a proton that is accelerated from the rest through a potential difference $= 120\,V$

④

$$F = ma = qE$$

$$a = \frac{qE}{m} = \frac{q\,|\Delta V|}{d * m_p}$$



$\Delta V = 120\,V$

$$V_f^2 = \cancel{V_i^2}^{\,0} + 2ad = 2 * \frac{q\,\Delta V}{\cancel{d}\, m_p}\cancel{d}$$

$$= \frac{2q\Delta V}{m_p} = \frac{2 * 1.6\times10^{-19} * 120}{1.67\times10^{-27}}$$

$$= \dots$$

or: by conservation of energy.

$$E_{tot})_A = E_{tot})_B$$

$$qV_A + \cancel{\tfrac{1}{2}mv_A^2}^{\,0} = qV_B + \tfrac{1}{2}mv_B^2$$

$$q(V_A - V_B) = \tfrac{1}{2}mv_B^2 \quad ; V_A > V_B$$

$$v_B = \sqrt{\frac{2q\,\Delta V}{m}} = \sqrt{\frac{2 * 1.6\times10^{-19} * 120}{1.67 * 10^{-27}}} = \dots$$

* assume the moving charge is **electron**

$$v_f = \sqrt{\frac{2 * 1.6\times10^{-19} * 120}{9.11\times10^{-31}}} = \dots$$

انتبه: اتجاه حركة الالكترون عكس حركة البروتون

example: what is the potential difference is needed to stop an electron with initial speed $4.2\times10^5\,m/s$

$$(v_f = 0)\ ;\ V_f^2 = V_i^2 + 2ad = 0 = V_i^2 + \frac{2q\Delta V}{m_e}$$

$$\Delta V = -V_i^2\,\frac{m_e}{2q} = \frac{(4.2\times10^5)^2 * 9.11\times10^{-31}}{-2 * 1.6\times10^{-19}} = 0.501\ \text{Volt}$$

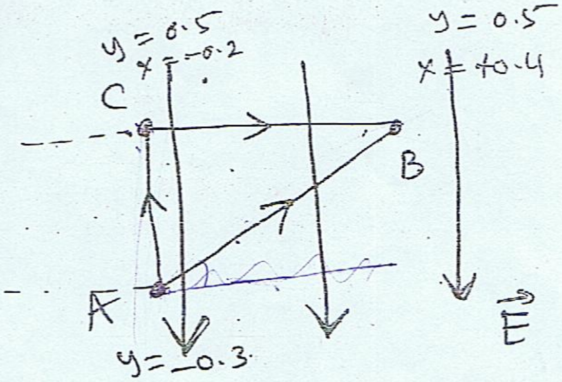
example: Calculate $V_B - V_A$ in the figure.

$E = 325\ V/m$

A: $(-0.2, -0.3)$

B: $(0.4, 0.5)$

$\Rightarrow$ C: $(-0.2, 0.5)$



1) $V_B - V_A = V_{A\to B} = V_{A\to C} + V_{C\to B}$

$= (V_C - V_A) + (V_B - V_C)$

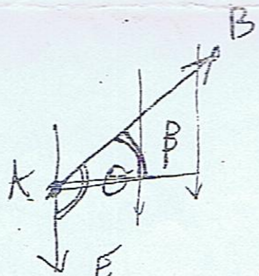
$= -E\,\overline{AC}\cos 180^\circ - \cancel{E\,\overline{CB}\cos\frac{\pi}{2}}^{\,0}$

$= +E\,\overline{AC}$

$\overline{AC} = \sqrt{(y_2 - y_1)^2 - (x_2 - x_1)^2} = \sqrt{(0.5 - -0.3)^2 - \cancel{(-0.2 - -0.02)^2}^{\,0}}$

$= 0.5 + 0.3 = 1$ (أو مباشرة على الـ y)

$V_B - V_A = 325 * 0.8 = 260$ Volt.



* by long path

$V_B - V_A = -\vec{E}\cdot\vec{S} = -ES\cos\theta$

$\beta = \tan^{-1}\left(\frac{0.8}{0.6}\right) = 53^\circ$

$S = \sqrt{(0.5 - -0.3)^2 + (0.4 - -0.2)^2}$

$= \sqrt{(0.8)^2 + (0.6)^2} = 1$

$\theta = \beta + \pi/2$

$\theta = 53^\circ + 90^\circ$

$= 143^\circ$

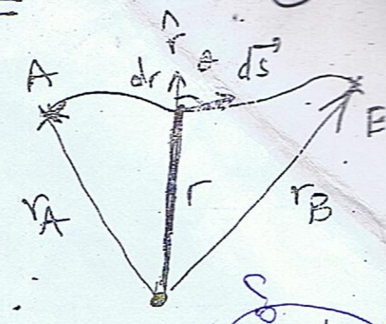
$V_B - V_A = -325 * 1 * \cos 143^\circ$

$= 260$ Volt. same result.

* Electric potential due to a point charge

$$V_B - V_A = -\int_A^B \vec{E}\cdot d\vec{s}$$

move a charge from $A \to B$



δ $dr = ds\cos\theta$

$$\vec{E} = k_e \frac{q}{r^2}\hat{r}$$

$$V_B - V_A = -\int_A^B \frac{k_e q}{r^2}\hat{r}\cdot d\vec{s} = -\int_A^B \frac{k_e q}{r^2}\, ds\cos\theta$$

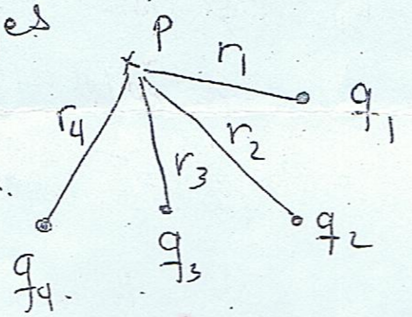
$$= -k_e q \int \frac{dr}{r^2} = kq\left(\frac{1}{r_B} - \frac{1}{r_A}\right)$$

if $r_A \to \infty$; $\frac{1}{r_A} \to 0$

$$\Rightarrow V_B = \frac{k_e q}{r_B}$$

$\Rightarrow$ in general the potential of an point charge $\boxed{V = \frac{k_e q}{r}}$

* if we have several charges



$$V_p = k_e \sum_i \frac{q_i}{r_i}$$

$$= k_e\frac{q_1}{r_1} + k_e\frac{q_2}{r_2} + k_e\frac{q_3}{r_3} + k_e\frac{q_4}{r_4} + \dots$$

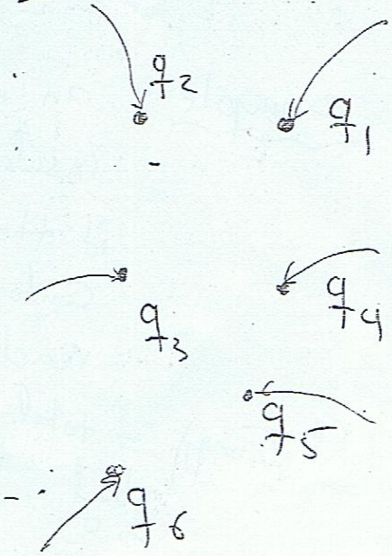
* if we bring a charge (Q) from ∞ the point p then the potential energy $= QV_p = k_e Q \sum_i \frac{q_i}{r_i}$

$$= k_e\frac{Qq_1}{r_1} + k_e\frac{Qq_2}{r_2} + k_e\frac{Qq_3}{r_3} + \dots$$

$\therefore$ the potential energy stored in configuration two point charges q_1, q_2 is $\boxed{U = k_e\frac{q_1 q_2}{r_{12}}}$

* the total potential energy stored in a system of point charges is equivalant (شغل) to bring all the charges from ∞ to their points. (7)

$$U_{tot} = k_e \frac{q_1 q_2}{r_{12}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_1 q_4}{r_{14}} + \cdots$$
$$+ k_e \frac{q_2 q_3}{r_{23}} + k_e \frac{q_2 q_4}{r_{24}} + \cdots$$
$$+ k_e \frac{q_3 q_4}{r_{34}} + k_e \frac{q_3 q_5}{r_{35}} + \cdots$$
$$+ \frac{q_4 q_5}{r_{45}} + \cdots$$



<u>example</u>: find the potential at a distance of 1cm from a proton and 2cm

r=1cm) $V_1 = k_e \frac{q}{r_1} = \frac{9\times10^{9} * 1.6\times10^{-19}}{1\times10^{-2}} = 1.44\times10^{-7}$ Volt.

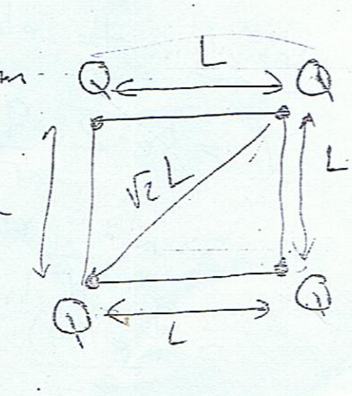
r=2cm) $V_2 = \frac{9\times10^{9} * 1.6\times10^{-19}}{2\times10^{-2}} = 7.2\times10^{-8}$

$\Delta V_{12} = V_2 - V_1 = 7.2\times10^{-8} - 1.44\times10^{-7} = -7.2\times10^{-8}$ Volt.

* for electron $V_1 = -1.44\times10^{-7}$ V
$V_2 = -7.2\times10^{-8}$ V

$\Delta V = V_2 - V_1 = +7.2\times10^{-8}$ V

* find the total energy stored in the system

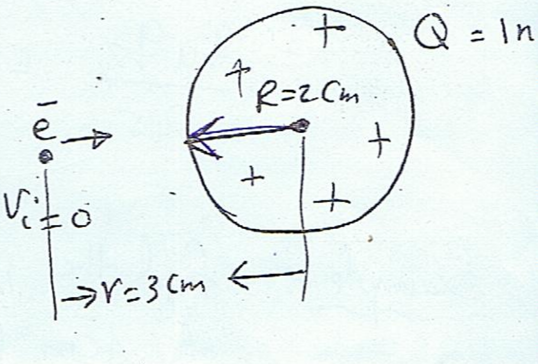


$$U_{tot} = k_e \frac{q_1 q_2}{r_{12}} + k_e \frac{q_1 q_3}{r_{13}} + k_e \frac{q_1 q_4}{r_{14}}$$
$$+ k_e \frac{q_2 q_3}{r_{23}} + k_e \frac{q_2 q_4}{r_{24}} + k_e \frac{q_3 q_4}{r_{34}}$$

$$U_{tot} = k_e \left\{ \frac{Q^2}{L} + \frac{Q^2}{\sqrt{2}L} + \frac{Q^2}{L} + \frac{Q^2}{L} + \frac{Q^2}{\sqrt{2}L} + \frac{Q^2}{L} \right\}$$

$$= \frac{k_e Q^2}{L} \left\{ 4 + \frac{2}{\sqrt{2}} \right\} = 5.41 \, \frac{k_e Q^2}{L}$$

example: an electron moves from rest toward a charged insulating sphere with $Q = 1\,nC$ and $R = 2\,cm$ if the electron started at $r = 3\,cm$ from the center of the sphere. find its speed when it reaches the surface of the sphere.



total Energy)$_{initial}$ = total Energy)$_{final}$

$$U_i + \cancel{\tfrac{1}{2} m v_i^2}^{\,0} = U_f + \frac{1}{2} m v_f^2$$

$$U_i = q V_i = q \frac{k_e Q}{r_i} = \frac{k_e q Q}{r_i}$$

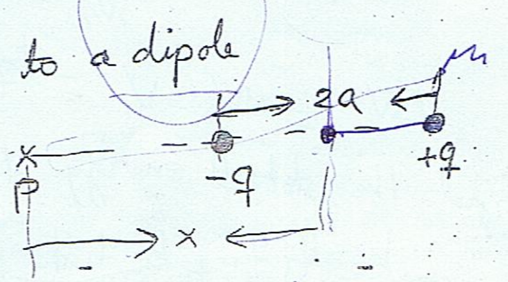
$$= \frac{9\times10^9 * -1.6\times10^{-19} * 1\times10^{-9}}{3\times10^{-2}}$$

$$U_f = q V_f = q \frac{k_e Q}{r_f} = \frac{9\times10^9 * -1.6\times10^{-19} * 1*10^{-9}}{2\times10^{-2}}$$

$$\frac{1}{2} m v_f^2 = U_i - U_f = \cdots$$

$$v_f = 7.26\times10^6 \; m/s$$

example: the electric potential due to a dipole



$$V = k_e \sum \frac{q_i}{r_i}$$

$$= k_e \frac{q_1}{r_1} + k_e \frac{q_2}{r_2}$$

$$= \frac{k_e(-q)}{x-a} + \frac{k q}{x+a} = k_e \frac{(-x-a+x-a)}{(x-a)(x+a)} = -2ak_e \frac{q}{x^2-a^2}$$

Obtaining the value of the electric field from the electric potential:

$$dV = -\vec{E}.d\vec{s}$$ (9)

$$\vec{E} = E_x\hat{i} + E_y\hat{j} + E_z\hat{k} \; ; \; d\vec{s} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$dV = -(E_x\,dx + E_y\,dy + E_z\,dz)$$

$$\therefore E_x = -\frac{dV}{dx} \Rightarrow \text{or } -\frac{\partial V}{\partial x}$$

$$E_y = -\frac{dV}{dy} \Rightarrow \text{or } -\frac{\partial V}{\partial y}$$

$$E_z = -\frac{dV}{dz} \Rightarrow \text{or } -\partial V/\partial z$$

$3x^2+2y+z$

$6yx$

example: $V(x,y,z) = 3x^2y + y^2 + yz$ find the electric field $\vec{E}$ at the point $(1,1,2)$

$$E_x = -\frac{\partial V}{\partial x} = -\frac{\partial}{\partial x}(3x^2y + y^2 + yz) = -6xy$$

$$E_x(1,1,2) = -6*1*1 = -6 \text{ V/m}$$

$$E_y = -\frac{\partial V}{\partial y} = -\frac{\partial}{\partial y}(3x^2y + y^2 + yz) = -3x^2 - 2y - z$$

$$E_y(1,1,2) = -3*1^2 - 2(1) - 2 = -3 - 4 = -7 \text{ Volt}$$

$$E_z = -\frac{\partial V}{\partial z} = -\frac{\partial}{\partial z}(3x^2y + y^2 + yz) = -y \Rightarrow$$

$$E_z(1,1,2) = -1 \text{ Volt}$$

$$\vec{E} = -6\hat{i} - 7\hat{j} - \hat{k}$$

$$|E| = \sqrt{(6)^2 + (7)^2 + (1)^2} = ---$$

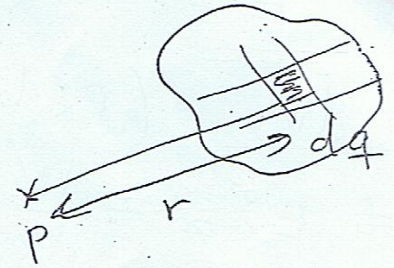
* problem: over a certain region in space $V = 5x - 3x^2y + 2yz^2$ find expression x, y, z find $\vec{E}$ at $(1, 0, -2)$

$\boxed{\text{same way}}$

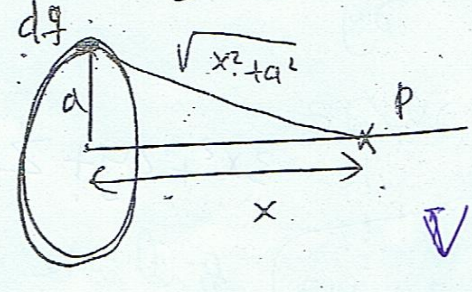
Electric potential due to a continuous charge distribution

(10)

$$dV = k_e \frac{dq}{r} \rightarrow V = k_e \int \frac{dq}{r}$$



example: find the electric potential due to a point on the axes of a ring



$$V = k_e \int \frac{dq}{r} = k_e \int_0^q \frac{dq}{\sqrt{x^2+a^2}}$$

$$V \quad \boxed{= \frac{k_e q}{\sqrt{x^2+a^2}}}$$

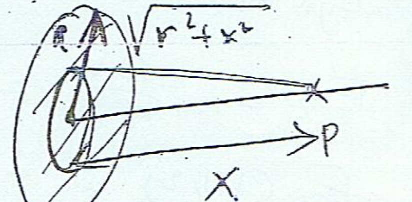
at the center of the ring $x=0 \Rightarrow V = \frac{k_e q}{a}$ (center)

$$E_x = -\frac{dV}{dx} = -k_e q \frac{d}{dx}\{x^2+a^2\}^{-1/2}$$

$$E \quad \boxed{= \frac{k_e q x}{(x^2+a^2)^{3/2}}} \; ; \; E_x(\text{the center } x=0) = 0$$

example: find the electric potential due to a uniformly charged disk ; $dq = \sigma da = \sigma \, 2\pi r dr$

$$V = k_e \int \frac{\sigma da}{\sqrt{r^2+x^2}} = 2\pi k_e \sigma \int_0^R \frac{r dr}{\sqrt{r^2+x^2}}$$



let $u = r^2 + x^2 \Rightarrow du = 2r\,dr$

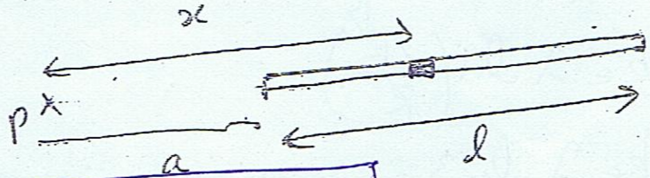
$$V = \frac{2\pi k_e \sigma}{2} \int du \,(u^{-1/2}) = \frac{\pi k_e \sigma u^{1/2}}{1/2} \quad V$$

$$= 2\pi k_e \sigma \sqrt{r^2+x^2}\,\Big|_0^R \; \boxed{= 2\pi k_e \sigma \{\sqrt{a^2+x^2} - x\}}$$

$$E_x = -\frac{dV}{dx} = -2\pi k_e \sigma \frac{d}{dx}\left(\sqrt{a^2+x^2} - x\right) \; E \; \boxed{= 2\pi k\sigma \left\{1 - \frac{x}{\sqrt{a^2+x^2}}\right\}}$$

example : (1) electric potential due to a line of charge on its axes

(11)

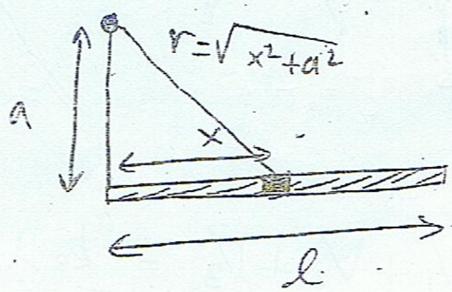


$$V_p = k_e \int \frac{dq}{r}$$

$$= k_e \int \frac{\lambda\, dx}{x}$$

$$= k_e \lambda \ln x \Big|_a^{a+l} = \boxed{k_e \lambda \ln\left(\frac{a+l}{a}\right)}$$

(2) on the y-axes :



$$V = k_e \int \frac{dq}{r}$$

$$= k_e \int \frac{\lambda\, dx}{\sqrt{x^2+a^2}}$$

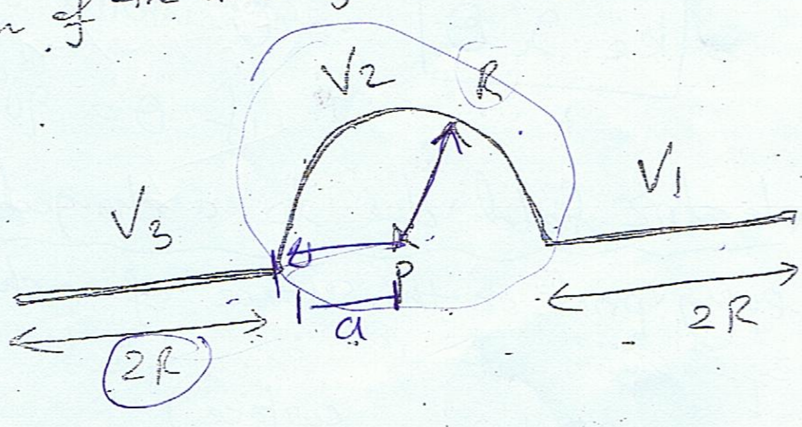
$$= \frac{k_e Q}{l} \int_0^l \frac{dx}{(x^2+a^2)^{1/2}}$$

$$= \frac{k_e Q}{l} \ln\left(x + \sqrt{x^2+a^2}\right)\Big|_0^l$$

$$= \boxed{k_e \frac{Q}{l} \ln\left(\frac{l+\sqrt{l^2+a^2}}{a}\right)}.$$

example : find the electric potential at the point p in the center of the wire of the following shape

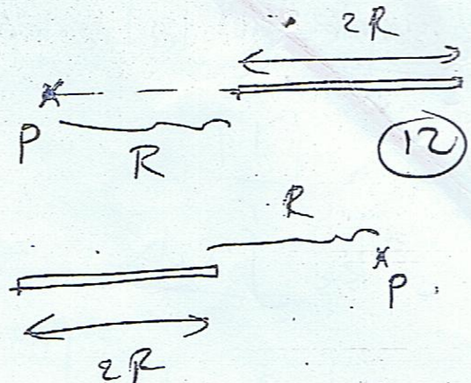
the charge is λ.



* we divide the shape to three potentials

$$V = V_1 + V_2 + V_3$$

$$V_1 = V_3 = k_e \int_R^{3R} \frac{\lambda\, dx}{x}$$



$$= k_e \lambda \ln\left(\frac{3\cancel{R}}{\cancel{R}}\right)$$

$$= k_e \lambda \ln 3$$

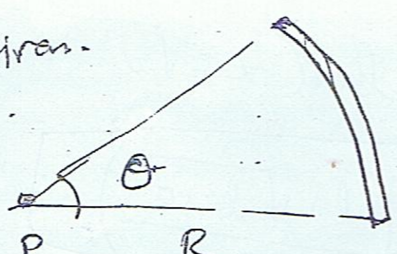
$$V_2 = k_e \int \frac{\lambda\, dl}{R} = \frac{\lambda k_e}{R} \int_0^{\pi} R\, d\theta$$

$$= \frac{\lambda k_e}{\cancel{R}} \times \cancel{R}\, (\pi - 0) = \pi \lambda k_e$$

$$V = V_1 + V_2 + V_3 = k_e \lambda \ln 3 + \pi \lambda k_e + k_e \lambda \ln 3$$

$$\boxed{= k_e \lambda\, (2 \ln 3 + 3.14)} \text{ Volts}$$

<u>Note</u>: if you given.



$$V_P = \frac{k_e \lambda}{R} \int_0^{\theta} R\, d\theta$$

$$= \boxed{k_e \lambda \theta}$$; θ interms of π ;

if $\theta = 90^\circ \Rightarrow \theta = \frac{\pi}{2} = \frac{3.14}{2}$;

* electric field due to a charged conductor:

every point on the surface of a charged conductor in equilibrium at the same electric potential (equipotential surface)

$\vec{E} \perp d\vec{s}$

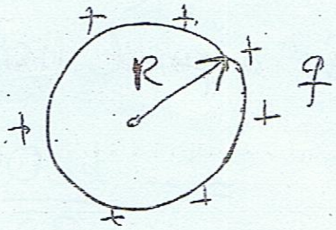
$\vec{E} \cdot d\vec{s} = 0$

$$V_B - V_A = \int \vec{E} \cdot d\vec{s} = 0 \Rightarrow \boxed{V_B = V_A}$$

[illegible]

* the electric potential every where inside the conductor = potential ~~at~~ on the surface = constant.

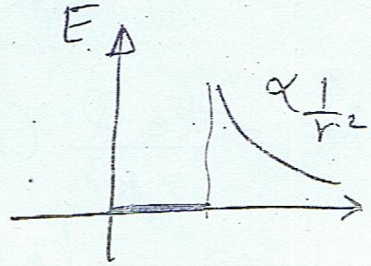
ex : spherical conducting shell

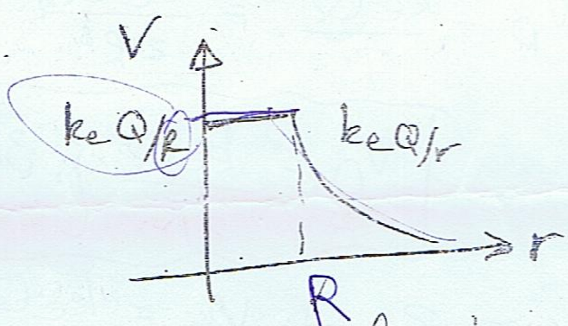


$$V = \frac{k_e q}{r}$$

on the surface $r = R$

$$V = \text{constant} = \frac{k_e q}{R}$$

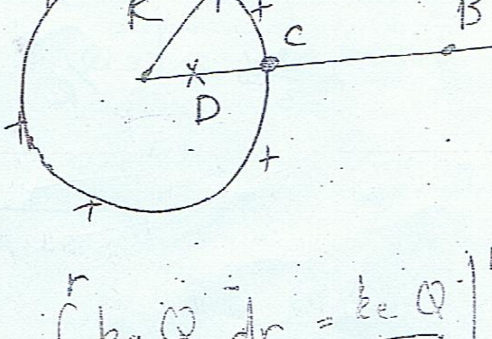




* Electric potential due to a uniform charged sphere

an insulating charged sphere, ρ uniform with total charge

Q find the potential at a point outside the sphere and inside the sphere.



$(V_\infty = 0)$ reference potential

$$E = \begin{cases} k_e \dfrac{Q}{r^2} & r > R \text{ (outside)} \\ k_e \dfrac{Q}{R^3} r & r < R \text{ inside} \end{cases}$$

1) outside

$$V_B = -\int_{\infty}^{r} E_r \, dr = -\int_{\infty}^{r} k_e \frac{Q}{r^2} dr = \frac{k_e Q}{r}\Big|_{\infty}^{r} = \boxed{k_e \frac{Q}{r}}$$

on the surface of the sphere at c $r = R$

$$V_c = \frac{k_e Q}{R}$$

b) for a point inside the sphere $V_D - V_c = -\int_R^r E_r\,dr$

$$= -\frac{k_e Q}{R^3}\int_R^r r\,dr = -\frac{k_e Q}{2R^3}\left(r^2 - R^2\right)$$

$$= \frac{k_e Q}{2R^3}\left(R^2 - r^2\right)$$

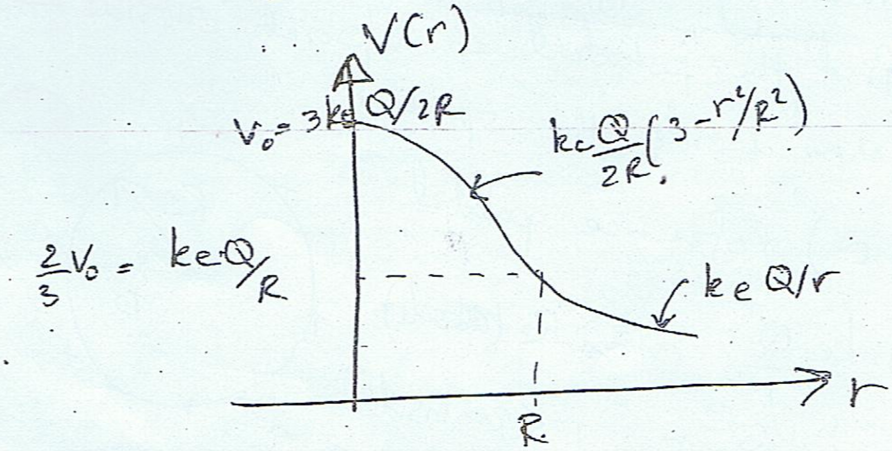
$$V_D - V_c = \frac{k_e Q}{2R^3}\left(R^2 - r^2\right)$$

$$V_D - \frac{k_e Q}{R} = \frac{k_e Q R^2}{2R^3} - \frac{k_e Q r^2}{2R^3} \Rightarrow V_D = \frac{3k_e Q}{2R} - \frac{k_e Q r^2}{2R^3}$$

$$\boxed{V_D = \frac{k_e Q}{2R}\left\{3 - r^2/R^2\right\}}$$ in ~~out~~side.

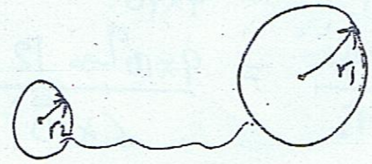
at $r = R \Rightarrow V_c = \frac{k_e Q}{2R}(3 - 1) = \frac{k_e Q}{R}$

at the Center of the sphere $V_0(r=0) = \frac{3k_e Q}{2R}$



* <u>example</u>: two spheres of radii r_1, r_2 and charges q_1, q_2 are connected by a light conducting wire find E_1, E_2 after connection. (15)

* the charge will flow from one sphere to another untill the voltage equal



$V_1 = V_2$

$\cancel{k_e}\frac{q_1}{r_1} = \cancel{k_e}\frac{q_2}{r_2}$

$$\boxed{\frac{q_1}{q_2} = \frac{r_1}{r_2}}$$

Note: the total charge before connection $Q = Q_1 + Q_2$

than after connection $Q = q_1 + q_2$

find q_1, q_2

<u>example</u>: a charge $Q = 20\mu C$ placed on the combination of the two conductors. $r_1 = 4\,cm$, $r_2 = 6\,cm$

(find V_1, V_2) find E_1, E_2 near the surface of each sphere after connected by a conducting wire.

$Q = q_1 + q_2 = 20\times10^{-6}$ --- (1)

$\frac{q_1}{q_2} = \frac{r_1}{r_2} = \frac{\cancel{4}^{2}}{\cancel{6}_{3}} \Rightarrow q_1 = \frac{2}{3} q_2$ -- (2)

eqn (2) in (1) $\Rightarrow \frac{2}{3}q_2 + q_2 = 20\times10^{-6} = \frac{5}{3} q_2$

$q_2 = \frac{3 * \cancel{20}^{4} * 10^{-6}}{\cancel{5}_{1}} = 1.2\times10^{-5}\,C$

$= 12\mu C$

$q_1 = \frac{2}{3} q_2 = \frac{2}{3} * 12 = 8\mu C$

(16)

$$E_1 = k_e \frac{q_1}{r_1^2} = \frac{9\times10^9 * 8\times10^{-6}}{(4\times10^{-2})^2} = 4.5\times10^7 \ N/C$$

$$E_2 = k_e \frac{q_2}{r_2^2} = \frac{9\times10^9 * 12\times10^{-6}}{(6\times10^{-2})^2} = 3\times10^7 \ N/C$$

$\Delta V = \frac{\Delta U}{q}$

$= \frac{-\Delta K}{q}$

$-\Delta V \cdot q = \Delta K$

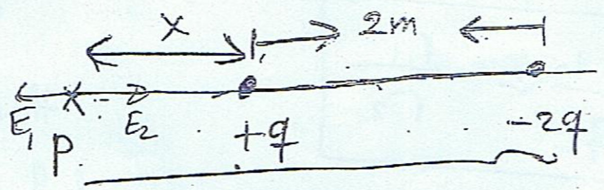
$K_B - K_A =$

5) $$V_1 = \frac{k_e q_1}{r_1} = \frac{9\times10^9 * 8\times10^{-6}}{4\times10^{-2}} = 18\times10^5 \ V$$

$$V_2 = k_e \frac{-q_2}{r_2} = \frac{9\times10^9 * 12\times10^{-6}}{6\times10^{-2}} = 18\times10^5 \ V$$

problem: a charge $+q$ at the origion, a charge $-2q$ at $x = 2m$ on the x-axes. for what finite values of x is

a) $E = 0$ b) $V = 0$



$$\frac{\cancel{k_e q}}{x^2} = \frac{\cancel{k_e}(2\cancel{q})}{(x+2)^2} \Rightarrow 2x^2 = x^2 + 4 + 4x$$

$$2x^2 - x^2 - 4x - 4 = 0$$

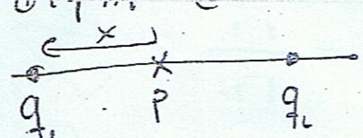
$$x^2 - 4x - 4 = 0$$

$$x = \frac{+4 \pm \sqrt{16 + 4*4}}{2*1} = \frac{4 \pm \sqrt{32}}{2} =$$

$$= \frac{4 \pm 5.8}{2} = 4.83 \ m$$

or $0.9m$ (not accepted)

b) $$V_p = \frac{\cancel{k_e} q_1}{x} + \frac{\cancel{k_e} q_2}{(2-x)}$$



$$\frac{k_e \cancel{q}}{x} - \frac{2\cancel{q} k_e}{2-x} = 0$$

$$2x = (2-x) \Rightarrow x = 0.667 \ m$$

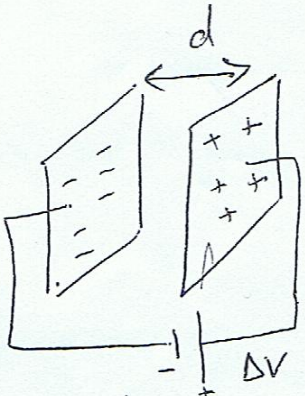
$$0 \leq x \leq 2.$$

and for $x < 0 \Rightarrow x = -2m$.

Chapter 26: Capacitance and Dielectrics

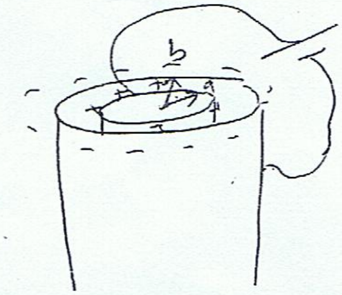
تعريف:

Capacitors (المكثفات): are devices that store electric charge consists of two conductors seperated by an insulator; the capacitance (السعة) depends on the capacitor geometry (الشكل الهندسي) and the dielectric (العازل) material between the conductors



parallel-plates capacitors

Spherical Capacitors



Cylindical Capacitors.

* the capacity of the capacitor $\boxed{C = Q/\Delta V}$ (C/Volt ≡ Farad)

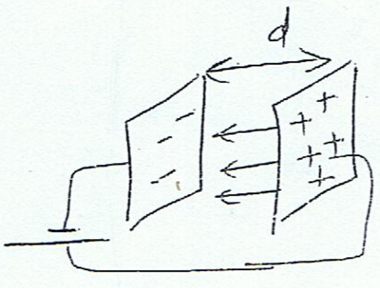
$$Q = C\,\Delta V$$

C. the ratio of the charge (magnitude) on either conductor to the potential difference between them (always positive)

1 Farad is large ⇒ $C \sim 10^{-6}\,F$ or 10^{-12} or 10^{-9}

* if we have a capacitor of 4pF capacity means we can store 4pC of charge for each 1 Volt of potential difference between it's plates. if the battery is 9 Volt then we can store $4 * 9 = 36\,pC$ in the capacitor.

Calculating the capacitance



① parallel plates capacitors

$|\Delta V| = |E d|$; $\sigma = Q/A$; $E = \sigma/\epsilon_0$

$$\Delta V = \frac{\sigma}{\epsilon_0} d = \frac{Q d}{\epsilon_0 A}$$

$$\Rightarrow C = \frac{Q}{\Delta V} = \frac{\cancel{Q}\,\epsilon_0 A}{\cancel{Q}\,d} = \boxed{\frac{\epsilon_0 A}{d} = C}$$

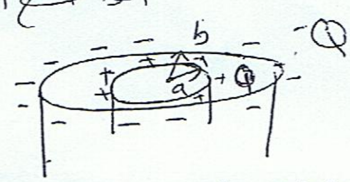
$C \propto A$; $C \propto \frac{1}{d}$ (if we increase A C increases
" " decrease d C increase)

زيادة A أو زيادة d تزيد من السعه.

Note: d (the seperation) must be small enough to neglect (ليهمل) the edges effects (أثر الحواف)

حتى يكون المجال الكهربائي منتظم عند الحواف

② The cylindrical capacitor المواسع الاسطواني



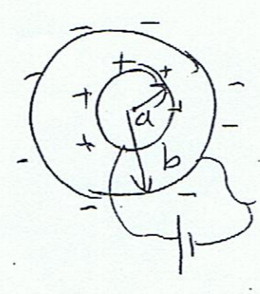
$E_{in} = \frac{2k_e \lambda}{r}$ (between the cylinders) $(a<r<b)$

$$V_b - V_a = |\Delta V| = -\int_a^b \vec{E} \cdot d\vec{r} = \left| 2k_e\lambda \int_a^b \frac{1}{r} dr \right| = 2k_e\lambda \ln(b/a)$$

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q}}{2k_e \frac{\cancel{Q}}{l} \ln b/a} = \frac{\cancel{4}^{2}\pi l \epsilon_0}{\cancel{2}\ln b/a} = \frac{2\pi\epsilon_0 l}{\ln(b/a)}$$

$$\boxed{C = \frac{2\pi\epsilon_0 l}{\ln(b/a)}} \; ; \; \boxed{\frac{C}{l} = \frac{2\pi\epsilon_0}{\ln b/a}}$$

③ The spherical capacitors المواسع الكروي



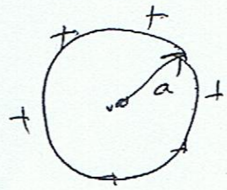
$E = k_e Q/r^2$

$$\Delta V = -\int_a^b E \cdot dr = -k_e Q \int_a^b \frac{1}{r^2} dr$$

$$|\Delta V| = k_e Q \left(\frac{1}{a} - \frac{1}{b} \right)$$

$$C = \frac{Q}{\Delta V} = \frac{\cancel{Q} * 4\pi\epsilon_0}{\cancel{Q}\left(\frac{1}{a}-\frac{1}{b}\right)} = \boxed{\frac{4\pi\epsilon_0}{1/a - 1/b}}$$

spical, case : if $b \to \infty$ (one shell only)
حالة خاصة



$$C = \frac{4\pi\epsilon_0}{1/a - 1/\infty} = 4\pi\epsilon_0 a$$

* energy stored in the capacitor

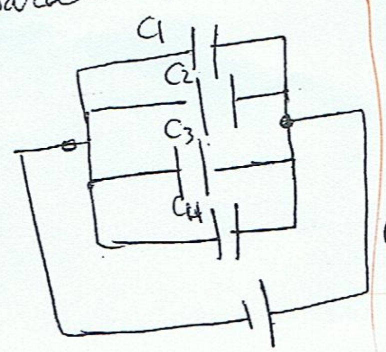
$$U = \frac{1}{2} C (\Delta V)^2 \quad ; \quad \boxed{C = Q/\Delta V}$$

$$= \frac{1}{2}\frac{Q}{\cancel{\Delta V}}(\Delta V)^{\cancel{2}} \quad \boxed{= \frac{1}{2} Q \Delta V}$$

$$= \frac{1}{2} C \left(\frac{Q}{C}\right)^2 = \boxed{\frac{1}{2}\frac{Q^2}{C}}$$

* Combination of capacitors :-

1) parall Combination: التوصيل على التوازي



$$\Delta V_1 = \Delta V_2 = \Delta V_3 = \cdots$$

$$Q = Q_1 + Q_2 + Q_3 + \cdots$$

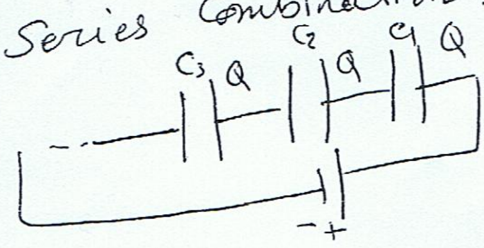
$$C_{eq}\cancel{\Delta V} = C_1\cancel{\Delta V_1} + C_2\cancel{\Delta V_2} + C_3\cancel{\Delta V_3} + \cdots$$

$$C_{eq} = C_1 + C_2 + C_3 + \cdots = \sum_{i=1}^{N} C_i$$

C_{eq} : equivilant capacitance
السعة المكافئة.

* Note that C_{eq} (parallel) > any capacitance.

2) Series Combination: التوصيل على التوالي



$$\Delta V = \Delta V_1 + \Delta V_2 + \Delta V_3 + \cdots$$

$$\frac{\cancel{Q}}{C_{eq}} = \frac{\cancel{Q}}{C_1} + \frac{\cancel{Q}}{C_2} + \frac{\cancel{Q}}{C_3} + \cdots$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots = \sum \frac{1}{C_i}$$

Note: $C_{eq} <$ any C_i

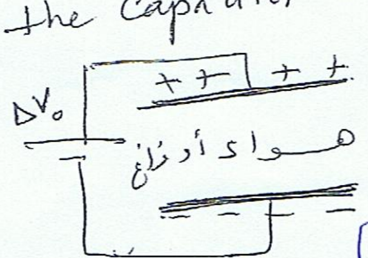
Capacitors with dielectrics المواسعات والعازلات.

a dielectrics : non - conducting material (rubber, glass, ...)

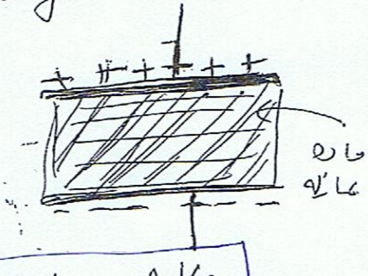
if we insert it between the plates of the capacitors

$\boxed{\text{it increases the capacity}}$.

* the capacitor without dielectric has a charge Q_0



$$\Delta V_0 = Q_0 / C_0$$



$$\boxed{\begin{array}{l} \text{(فرق الجهد ينخفض بوضع العازل)} \quad \Delta V = \dfrac{\Delta V_0}{K} \\ \text{(الشحنة تبقى ثابتة)} \quad Q = Q_0 \end{array}}$$

K : called dielectric constant (ثابت العزل)

$K > 1$

$$C_{after} = \frac{Q}{\Delta V} = \frac{Q_0 K}{\Delta V_0} = K \frac{Q_0}{\Delta V_0} = K C_0$$

$$\boxed{C = K C_0}$$

السعة تزداد بوضع العازل داخل المواسع.

* the dielectric material increases the capacitance untill it reaches the maximum approching voltage (أقصى جهد)

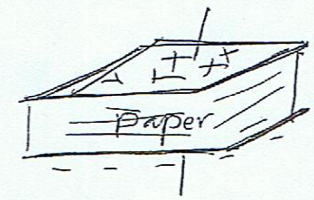
* if we increase ΔV more than $V_{max.}$ then the dielectric material will discharge the capacitor.

* the dielectric strength: the maximum electric field can pass through the capacitor (E_{max})

* for parallel plate capacitors $\Delta V_{max} = E_{max} * d$

* κ and E_{max} are given in table 26.1 in the book for some dielectrics.

* example : a parallel plates capacitor has a plate dimentions 2cm x 3cm sperated by 1mm thickness of paper



find (a) $C = \kappa C_0$

$$= \kappa \frac{\epsilon_0 A}{d}$$

$\kappa_{paper} = 3.7$

$$= \frac{3.7 * 8.85\times10^{-12} * 2\times10^{-2} * 3\times10^{-2}}{1\times10^{-3}} = 20\times10^{-12}\,F$$

$$= 20\,pF$$

(b) maximum charge can be placed on the capacitor

(dilectric strength of paper $= 16\times10^{6}\,V/m$)

$$\Delta V_{max} = E_{max}\, d = 16\times10^{6} * 1\times10^{-3} = 16\times10^{3}\,V$$

$$Q_{max} = C\,\Delta V_{max} = 20\times10^{-12} * 16\times10^{3} = 0.32\,\mu C$$

(c) maximum energy can be stored in the capacitor

$$U_{max} = \frac{1}{2} C (\Delta V)^2_{max} = \frac{1}{2} * 20\times10^{-12} * (16\times10^{3})^2$$

$$= 2.56\times10^{-3}\,J$$

$$= 2.56\,mJ$$

* the energy stored in the capacitor after inserting a dielectric material.

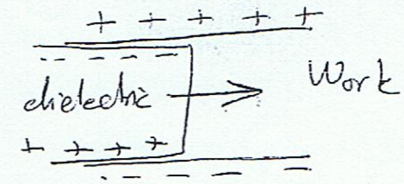
* $U_0 = \frac{Q_0^2}{2C_0}$ (before) ; $U(after) = \frac{Q^2}{2C} = \frac{Q_0^2}{2C}$

$$\boxed{U(after) = \frac{Q_0^2}{2\kappa C_0} = \frac{1}{\kappa} U_0}$$

(الطاقة تقل المخزونة)

∴ the final energy is less than the initial energy.

خزنت الطاقة ذهبت على شكل شغل يجعل المادة العازلة تدخل داخل ~~المواسع~~ المواسع نفسها

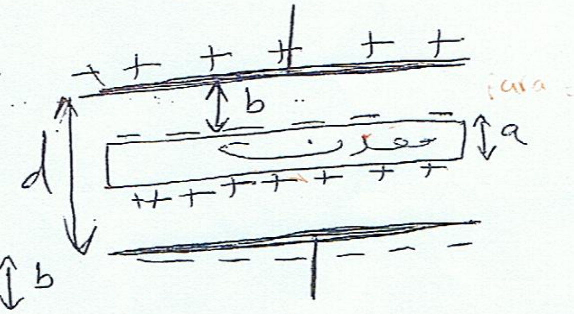


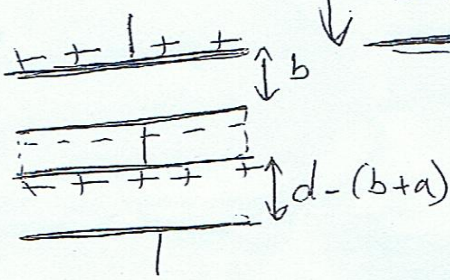
induced charge made a driving force

example : find the capacitance of a parallel plate capacitor of metalic slab inside it

قطعة معدنية داخل مواسع ذي الصفيحتين المتوازيتين دون أن تلمس أي من الصفيحتين .

سوف تشحن القطعة الداخلية بالتأثير فيصبح المواسع كأنه اثنين موصولين على التوالي





$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

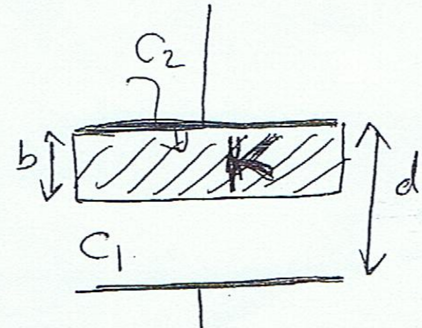
$$= \frac{1}{\frac{\epsilon_0 A}{b}} + \frac{1}{\frac{\epsilon_0 A}{d-b-a}}$$

$$= \frac{b}{\epsilon_0 A} + \frac{d-b-a}{\epsilon_0 A} = \frac{d-\cancel{b}-a+\cancel{b}}{\epsilon_0 A} = \frac{d-a}{\epsilon_0 A}$$

$$\boxed{C_{eq} = \frac{\epsilon_0 A}{d-a}}$$

example : partially filled capacitor

كأنه مواسعين على التوالي



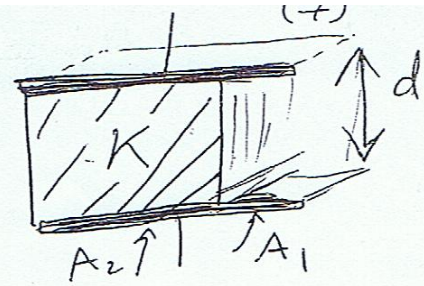
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{\frac{\epsilon_0 A}{d-b}} + \frac{1}{\frac{K\epsilon_0 A}{b}}$$

$$= \frac{d-b}{\epsilon_0 A} + \frac{b}{K\epsilon_0 A}$$

السمك مختلف في المسافات فقط

example

المادة العازلة موضوعة بشكل عرضي



Same voltage $\rightarrow$ different charge

$\equiv$ parallel combination

(حصلنا على التوازي)

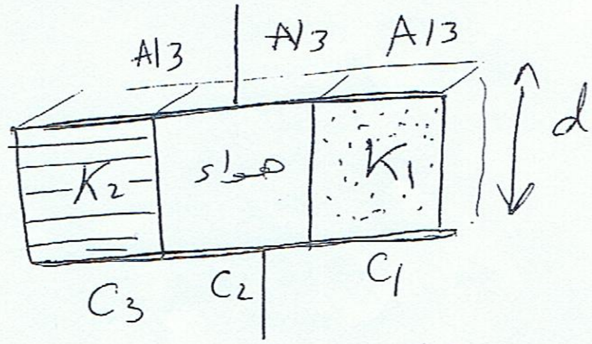
المساحات مختلفة
والسمك نفسه

$$C_{eq} = C_1 + C_2 = \frac{\epsilon_0 A_1}{d} + \frac{\epsilon_0 A_2}{d}$$

$$= \frac{\epsilon_0}{d}(A_1 + A_2)$$

سؤال امتحان سابق

find C_{eq}.



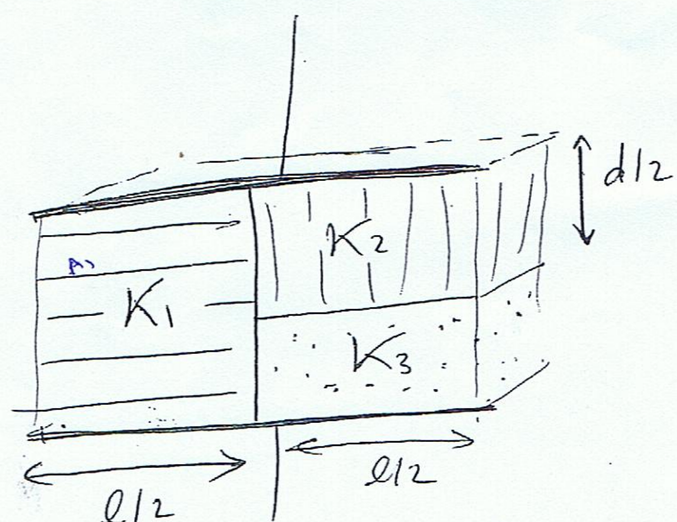
$$C_{eq} = C_1 + C_2 + C_3$$

$$= K_1 \frac{\epsilon_0 A_1}{d} + \frac{\epsilon_0 A_2}{d} + K_2 \frac{\epsilon_0 A_3}{d}$$

$A_1 = A_2 = A_3 = A/3$

$$= \frac{\epsilon_0 A}{3d}\left\{ K_1 + 1 + K_2 \right\}.$$

example



$$C_1 = K_1 \epsilon_0 \frac{A_1}{d_1} = \frac{K_1 \epsilon_0 A}{2d}$$

$$C_2 = K_2 \frac{\epsilon_0 A_2}{d_2} = \frac{\epsilon_0 K_2 A}{\cancel{2} \cdot d/\cancel{2}}$$

$$C_3 = K_3 \frac{\epsilon_0 A_3}{d_3} = \frac{\epsilon_0 K_3 A}{d}$$

$A_1 = \frac{1}{2}A$

$A_2 = \frac{1}{2}A$

$A_3 = \frac{1}{2}A$

$d_1 = d$

$d_2 = d/2$

$d_3 = d/2$

C_2 and C_3 in series (على التوالي)

$$C_{eq(2,3)} \Rightarrow \frac{1}{C_{eq(2,3)}} = \frac{1}{C_2} + \frac{1}{C_3}$$

$$\frac{1}{C_{eq(2,3)}} = \frac{d}{\epsilon_0 K_2 A} + \frac{d}{\epsilon_0 K_3 A} = \frac{d}{\epsilon_0 A}\left(\frac{1}{K_2} + \frac{1}{K_3}\right)$$

$$= \frac{d}{\epsilon_0 A}\left(\frac{K_2 + K_3}{K_2 \cancel{*} K_3}\right)$$

$$C_{eq(2,3)} = \frac{\epsilon_0 A}{d}\left(\frac{K_2 K_3}{K_2 + K_3}\right)$$

Now $C_{eq}(2,3)$ and C_1 in parall $\Rightarrow C_{eq} = C_{eq}(2,3) + C_1$

$$= \frac{\epsilon_0 A}{d}\left(\frac{K_2 K_3}{K_2 + K_3}\right) + K_1 \frac{\epsilon_0 A}{2d}$$

$$C_{eq} = \frac{\epsilon_0 A}{d}\left\{\frac{K_2 K_3}{K_2 + K_3} + \frac{K_1}{2}\right\}$$

if $A = 1 cm^2$; $K_1 = 4.9$; $K_3 = 2.1$

$d = 2 mm$; $K_2 = 5.6$

$\Rightarrow C_{eq} = 1.76 \times 10^{-12} F = 1.76 pF$.

problems :

1) when a potential difference of 150 V is applied to the plates of a parallel plates capacitor, the plates carry a surface charge density of $30 nC/cm^2$ what is the spacing between plates ?

$$C = \frac{\epsilon_0 A}{d} = \frac{Q}{\Delta V} \Rightarrow \frac{Q}{A} = \frac{\Delta V \epsilon_0}{d} = \sigma$$

$$\Rightarrow d = \frac{\Delta V \epsilon_0}{\sigma} = \frac{8.85 \times 10^{-12} * 150}{30 \times 10^{-9} * 1 \times 10^{4}} = 4.42 \times 10^{-6} m$$

$$\boxed{\text{Note : } 30 nC/cm^2 = \frac{30 \times 10^{-9}}{(1 \times 10^{-2})^2} = 30 \times 10^{-9} * 1 \times 10^{4}}$$

$d = 4.42 \mu m$

(2) two conductors having net charges of $10\mu C$ and $-10\mu C$ have a potential difference of $10V$ between them. determine (a) the capacitance (b) the potential difference between the two conductors if the charges on each are increased to $+100\mu C$ and $-100\mu C$?

(a) $C = \frac{Q}{\Delta V} = \frac{10 \times 10^{-6}}{10} = 1 \times 10^{-6}\ F = 1\mu F$

(b) $\Delta V = \frac{Q}{C} = \frac{100 \times 10^{-6}}{1 \times 10^{-6}} = 100\ V$

(3) an air filled capacitor consists of two parallel plates, each with an area of $7.6\ cm^2$, seperated by a distance of $1.8\ mm$. A $20\ V$ potential difference is applied to these plates calculate

(a) the electric field between plates

$\Delta V = E d \Rightarrow E = \frac{\Delta V}{d} = \frac{20}{1.8 \times 10^{-3}} = 11.1 \times 10^{3}\ V/m$
$= 11.1\ kV/m$

(b) the surface charge density

$E = \frac{\sigma}{\epsilon_0} \Rightarrow \sigma = \epsilon_0 E = 1.11 \times 10^{4} * 8.85 \times 10^{-12} = 98.3 \times 10^{-9}\ C/m^2$
$= 98.3\ nC/m^3$

(d) the charge on each plate?

$\Delta V = \frac{Q}{C} \Rightarrow Q = C\,\Delta V$
$= 3.74 \times 10^{-12} * 20$
$= 74.7 \times 10^{-12}\ C$
$Q = 74.7\ pC$

(4) A isolated charged conducting sphere of radius 12 cm creates an electric field of 4.9×10^4 N/c at a distance 21 cm from its center

(a) what is its surface charge density

$$E = \frac{ke\, q}{r^2} \Rightarrow q = \frac{E * r^2}{ke} = \frac{4.9 \times 10^4 * 0.21}{9 \times 10^9} = 0.24\ \mu C$$

$$\sigma = \frac{q}{A} = \frac{q}{4\pi R^2} = \frac{0.24 \times 10^{-6}}{4\pi\,(0.12)^2} = 1.33\ \mu C/m^2$$

(b) what is its capacitance ?

$$C = 4\pi \epsilon_0 R = 4\pi * 8.85 \times 10^{-12} * 0.12 = 13.3\,pF$$

(5) two capacitors $C_1 = 5\,\mu F$ and $C_2 = 12\,\mu F$ are connected in parallel. the resulting combination is connected to 9V battrey

(a) what is the equivilant capacitance

توازي parallel $\Rightarrow$ $C_{eq} = C_1 + C_2 = 5 + 12 = 17\ \mu F$

(b) the potential difference across each capacitor.

parallel $\Rightarrow$ $\Delta V_1 = \Delta V_2 = \Delta V$ (battrey)

$= 9$ Volts.

(c) the charged stored in each capacitor

$$Q_1 = C_1 \Delta V = 5 * 9 = 45\ \mu C$$

$$Q_2 = C_2 \Delta V = 12 * 9 = 108\ \mu C$$

(e) the two capacitors are now connected in series find: equivalant capacatance $\Rightarrow$ $\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{5} + \frac{1}{12}$

$$C_{eq} = \frac{5 * 12}{5 + 12} = 3.53\ \mu F$$

* the charge on the equivalent capacitor.

$$Q_{eq} = C_{eq}\,\Delta V = 3.53\,\mu F * 9V = 31.8\,\mu C$$

* the charge on each capacitor

$$Q_1 = Q_2 = Q_{eq} = 31.8\,\mu C$$

* the potential difference across each capacitor

$$\Delta V_1 = \frac{Q_1}{C_1} = \frac{31.8\,\mu C}{5\,\mu F} = 6.35\,V$$

$$\Delta V_2 = \frac{Q_2}{C_2} = \frac{31.8\,\mu C}{12\,\mu F} = 2.65\,V$$

(6) two capacitors when connected in parallel give an equivalent capacitance of 9 pF and give an equivalent capacitance of 2 pF when connected in series. what is the capacitance of each capacitor?

$C_p = C_1 + C_2$ (parallel) ; $\frac{1}{C_s} = \frac{1}{C_1} + \frac{1}{C_2}$ (series)

$9 = C_1 + C_2$ ... (1)

$C_2 = 9 - C_1$

sub in the other eqn.

$\frac{1}{2} = \frac{1}{C_1} + \frac{1}{C_2}$ ... (2)

$= \frac{C_2 + C_1}{C_1 C_2} = \frac{1}{2}$

$C_1 C_2 = 2(C_2 + C_1)$

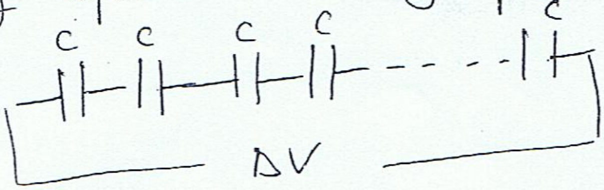
$C_1(9 - C_1) = 2(9 - C_1) + 2C_1$

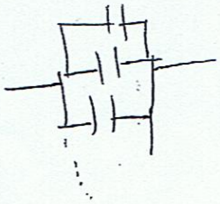
$9C_1 - C_1^2 = 18 - \cancel{2C_1} + \cancel{2C_1} \Rightarrow C_1^2 - 9C_1 + 18 = 0$

$$C_1 = \frac{9 \pm \sqrt{(9)^2 - 4*1*18}}{2} = 6\,pF \text{ or } 3\,pF$$

(7) a group of 2 identical capacitors is connected first in series and then in parallel. the combined capacitance in parallel is 100 times larger than for series connection

how many capacitor in the group.?





C_s

$\boxed{C_p = 100\ C_s}$ $\quad \frac{1}{C_s} = \underbrace{\frac{1}{C_1} + \frac{1}{C_2} + \dots + \frac{1}{C}}_{n\text{-times}}$

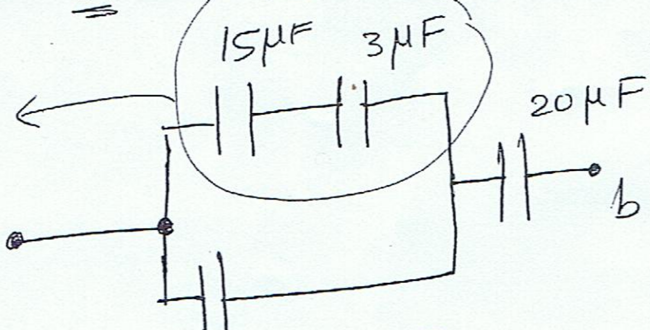
n: عدد المواسعات

$$C_p = \underbrace{C_1 + C_2 + \dots + C_n}_{n\text{-times}} = nC$$

$$n\cancel{C} = 100\left(\frac{1}{\underbrace{\frac{1}{C} + \frac{1}{C} + \dots + \frac{1}{C}}_{n\text{-times}}}\right) = \frac{100}{n/C} = \frac{100}{n}\cancel{C}$$

$$n = \frac{100}{n} \Rightarrow n^2 = 100 \Rightarrow \boxed{n = 10}$$ عشرة مواسعات.

(8) Four capacitors are connected as shown find the equivalent between a and b



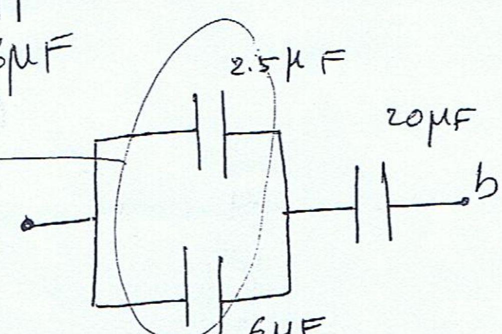
Series على التوالي

$$\frac{1}{C_s} = \frac{1}{15} + \frac{1}{3}$$

$C_s = 2.5\,\mu F$

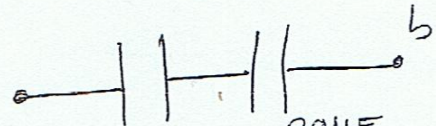
parallel

$C_p = 2.5 + 6 = 8.5\,\mu F$



$$\frac{1}{C_{eq}} = \frac{1}{8.5\,\mu F} + \frac{1}{20\,\mu F}$$

$C_{eq} = 5.96\,\mu F$



Calculate the charge on each capacitor if $\Delta V_{ab} = 15V$

$$Q_{eq} = C_{eq}\,\Delta V_{ab} = 5.96\times10^{-6} * 15 = 89.5\,\mu C$$

this charge is the same on $8.5\,\mu F$ and on $20\,\mu F$

* the parallel combination $C_p = 8.5\,\mu F$ on same voltage

$$\Delta V_p = \Delta V_{on\ 2.5\mu F} = \Delta V_{on\ 6\mu F}$$

$$\Delta V_p = \frac{Q}{C_p} = \frac{89.5\,\mu C}{8.5\,\mu F} = 10.53 \text{ Volt}$$

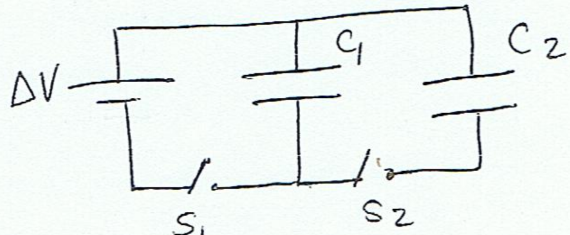
$$\Delta V_{20\mu F} = \frac{Q}{20\mu} = \frac{89.5}{20} = 4.47 \text{ Volts}$$

Note $\Delta V_p + \Delta V_{20\mu C} = 10.53 + 4.47 = \underline{19 \text{ Volts}}$. as given

$$Q\,(6\mu F) = C\,\Delta V_p$$
$$= 6\times10^{-6} * 10.53 = 63.2\,\mu C$$

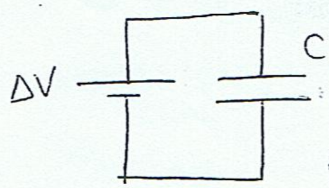
$$Q_{series} = Q_{3\mu F} = Q_{15\mu F} = 89.5\,\mu C - 63.2\,\mu C$$
$$= 26.3\,\mu C$$

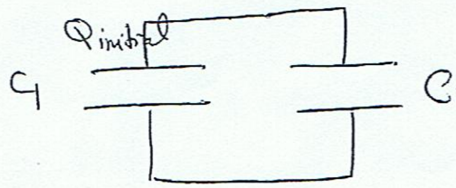
(9) Consider the circuit shown in the figure ; $C_1 = 6\,\mu F$, $C_2 = 3\,\mu F$, $\Delta V = 20\,V$. Capacitor C_1 first charged by the closing of switch S_1, S_1 then oppened, and the charged capacitor is connected to the ancharged capacitor by closing S_2. Calculate the initial charge acquired by C_1 and the final charge on each capacitor.



$Q_{initial} = C_1 \Delta V$

$= 6 * 10^{-6} * 20 = 120 \mu C$





; the capacitor C_1 will charge the capacitor C_2

after connection $Q_{tot} = 120 \mu C = Q_1 + Q_2$

the final voltage on both capacitor will be equal (charging will stop when the potential will be equal on both C_1, C_2

$$\Delta V_1 = \Delta V_2$$

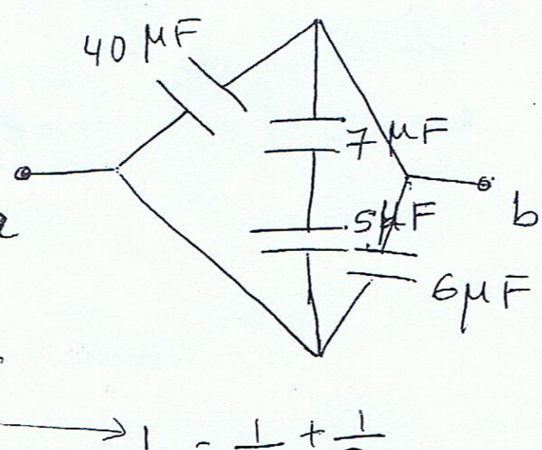
$$\frac{Q_1}{C_1} = \frac{Q_2}{C_2} \Rightarrow Q = \frac{C_1}{C_2} Q_2 = \frac{6}{3} Q_2 = 2 Q_2$$

$\boxed{Q_1 = 2 Q_2}$; $\boxed{120 \mu C = Q_1 + Q_2}$

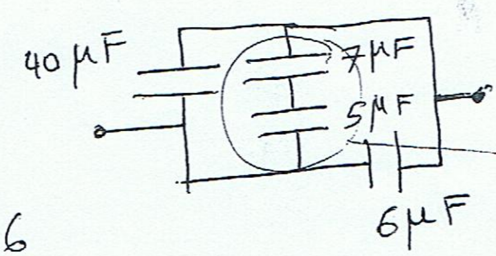
$$120 = 2Q_2 + Q_2 = 3Q_2 \Rightarrow Q_2 = \frac{120}{3} = 40 \mu C$$

$$Q_1 = 2 * 40 = 80 \mu C.$$

problem 10 : find the equivalent capacitance between points a, b in the combination of capacitors shown in the figure.



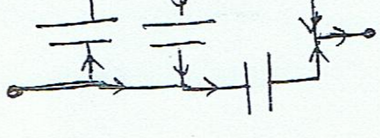
هذا الرسم يمكن أن نرسمه على الشكل التالي



$$\frac{1}{C_s} = \frac{1}{7} + \frac{1}{5}$$

$$C_s = 2.92 \mu F$$

$C_p = 2.92 + 4 + 6$
$= 12.9 \mu F$



(11) A parallel plate capacitor is charged and then disconnected from a battery. by what fraction does the stored energy change (increase or decrease) when the plate seperation is doubled (تتضاعف) (15)

$$U = \frac{1}{2}\frac{Q^2}{C} \quad ; \quad C = \frac{\epsilon_0 A}{d} \quad ; \quad U_i = \frac{1}{2}\frac{Q^2 d_i}{\epsilon_0 A}$$

d_i: initial seperation

$$U_f = \frac{1}{2}\frac{Q^2}{\epsilon_0 A} d_f$$

; d_f: بعد مضاعفة المسافه الفاصله بين الصفيحتين final seperation

$$d_f = 2 d_i .$$

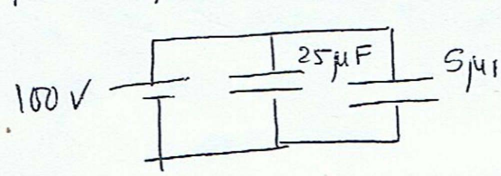
* الشحنه تبقى لان المواسع فُصل عن البطاريه.

$$U_f = \frac{1}{2}\frac{Q^2 (2 d_i)}{\epsilon_0 A} = 2\left(\frac{1}{2}\frac{Q^2 d_i}{\epsilon_0 A}\right) = 2 U_i$$

* سوف تتضاعف الطاقه

* stored energy doubles

(12) two capacitors $C_1 = 25\,\mu F$ and $C_2 = 5\,\mu F$ are connected in parallel and charged with 100 V power supply



(a) Calculate the energy stored in the two capacitors.

$$C_{eq} = C_1 + C_2 = 25 + 5 = 30\,\mu F$$

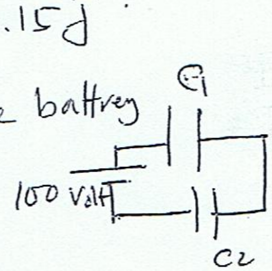
$$U = \frac{1}{2} C_{eq} (\Delta V)^2 = \frac{1}{2} * 30 \times 10^{-6} * (100)^2 = 0.15\,J$$

$$\text{or} \quad U = \frac{1}{2} C_1 (\Delta V_1)^2 + \frac{1}{2} C_2 (\Delta V_2)^2 = \frac{1}{2} C_1 (\Delta V)^2 + \frac{1}{2} C_2 (\Delta V)^2$$

$\Delta V_1 = \Delta V_2$ (parallel)

$$U = \frac{1}{2} * 25 * (100)^2 + \frac{1}{2} * 5 \times (100)^2 = 0.15\,J.$$

(b) what if connected in series with same battrey

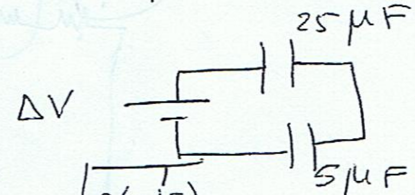


$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{25} + \frac{1 * 5}{5 * 5}$$

$$C_{eq} = 4.17\,\mu F$$

$$U = \frac{1}{2} C_{eq} (\Delta V)^2 = \frac{1}{2} * 4.17\mu * (100)^2 = 2.08 \times 10^{-2} \text{ J}$$

(c) what if C_1, C_2 ~~are~~ connected in series then store the same energy in parallel find the required potential difference



$C_s = 4.17\ \mu F$

$$U = \frac{1}{2} C (\Delta V)^2 \Rightarrow \Delta V = \sqrt{\frac{2U}{C}} = \sqrt{\frac{2(0.15)}{4.17 \times 10^{-6}}}$$

$$= 268 \text{ V}$$

(13) A parallel-plate capacitor in air has a plate seperation of 1.5 cm and a plate area of 25 cm². the plates are charged to the potential difference of 250V and disconnected from the source. the capacitor then immersed in distilled water Determine (a) the charge on the plates before and after immersion (الغطس) (b) C_{before} and after

$$C = \frac{\epsilon_0 A}{d} = \frac{Q}{(\Delta V)_i} \Rightarrow Q = \frac{\epsilon_0 A (\Delta V)_i}{d}$$

Q is the same before and after immersion

$$Q = \frac{8.85 \times 10^{-12} * 25 \times (10^{-2})^2 * 250}{1.5 \times 10^{-2}} = 369 \text{ pC}$$

* after immersion $\quad C_f = \frac{\kappa \epsilon_0 A}{d} = \frac{Q}{(\Delta V)_f}$

κ (distilled water) = 80.

(b) $$C_f = \frac{80 * 8.85 \times 10^{-12} * 25 \times (10^{-2})^2}{1.5 \times 10^{-2}} = 118 \text{ pF}$$

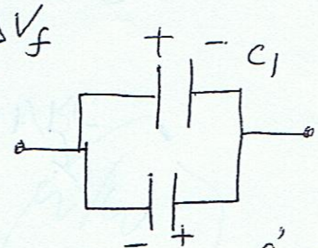
(c) $$C_f = \frac{Q}{\Delta V_f} \Rightarrow \Delta V_f = \frac{Q}{C_f} = \frac{369 \times 10^{-12}}{118 \times 10^{-12}} = 3.12 \text{ V}$$

(d) the change of energy of the capacitor: $\Delta U = U_f - U_i$

$$\Delta U = \frac{1}{2} C_i (\Delta V_i)^2 - \frac{1}{2} C_f (\Delta V_f)^2 = -45.5 \times 10^{-9} \text{ J}$$

(14) Rewiring two charged Capacitors

C_1 and C_2 $(C_1 > C_2)$ are charged to ΔV_i with opposite polarity, then they connected find ΔV_f



(a) charge is conserved find ΔV_f

$$Q = Q_{1i} + Q_{2i} = Q_{1f} + Q_{2f}$$

$$C_1 \Delta V_i - C_2 \Delta V_i = C_1 \Delta V_f + C_2 \Delta V_f$$

$$(C_1 - C_2)\, \Delta V_i = (C_1 + C_2)\, \Delta V_f$$

$$\Delta V_f = \frac{C_1 - C_2}{C_1 + C_2}\, \Delta V_i$$

كل مواسع شُحن على حده
ثم تم وصلهم مع بعض
بعكس القطبية الموجب مع السالب
والسالب مع الموجب

(b) find the total energy stored in the Capacitors before and after the rewiring (إعادة توصيل) of the capacitors.

$$U_i = \frac{1}{2} C_1 (\Delta V_i)^2 + \frac{1}{2} C_2 (\Delta V_i)^2 = \frac{1}{2}(C_1 + C_2)\, \Delta V_i^2$$

$$U_f = \frac{1}{2} C_1 (\Delta V_f)^2 + \frac{1}{2} C_2 (\Delta V_f)^2 = \frac{1}{2}(C_1 + C_2)\, \Delta V_f^2$$

$$U_f = \frac{1}{2}(C_1 + C_2) \left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2 \Delta V_i^2$$

$$U_f = \frac{1}{2} \frac{(C_1 - C_2)^2}{C_1 + C_2} (\Delta V_i)^2$$

find $\frac{U_f}{U_i} = \frac{\cancel{\frac{1}{2}} \frac{(C_1 - C_2)^2}{C_1 + C_2} \cancel{(\Delta V_i)^2}}{\cancel{\frac{1}{2}} (C_1 + C_2) \cancel{\Delta V_i^2}} = \left(\frac{C_1 - C_2}{C_1 + C_2}\right)^2$

$\frac{U_f}{U_i} < 1$ (the initial energy larger than the final energy)

Chapter 27 : Current and Resistance

د. خضير [illegible]

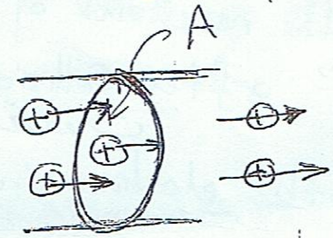
* In the last chapters we treated elect charges at rest. when we talk about motion of electric charges, we talk about electric current.

* the electric current (I) : the rate of flow of charges through ~~some~~ region in space

* average current $I_{av.} = \dfrac{\Delta Q}{\Delta t}$ ($C/sec \equiv$ Ampere)

ΔQ : the amount of charges that pass throug area A

Δt ; the time period.



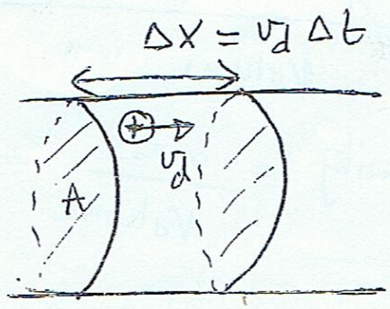
* the instantanous current

$$I = \frac{dQ}{dt} \quad \text{(Ampere)}$$

* by convention (اصطلاحا) the direction of I is the same of flow of positive charges.

* Note ; if there is no potential difference ($\Delta V = 0$) in a wire or conductor then No charge flow i,e (no current)

* Microscopic model of current



ΔQ in the cross section is the number of charge carriers (حاملات الشحنة)

* charge per carrier

* the charge carrier density = # of charges / Volume. (2)

$$n = \Delta Q / \text{Volume} \Rightarrow \Delta Q = n * \text{Volume}$$

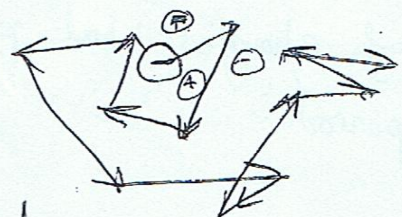
$$\Delta Q = n * A \Delta x * q \;;\; \text{but } \Delta x = v_d \Delta t$$

v_d: the drift velocity (السرعة الانسياقية) متوسط السرعة average speed.

$\therefore \Delta Q = n A v_d \Delta t\, q$ divide by $\Delta t \Rightarrow$

$$I = \frac{\Delta Q}{\Delta t} = \frac{n A v_d \cancel{\Delta t} q}{\cancel{\Delta t}} = n q v_d A$$

v_d: the resultant of $\vec{v}$ after Collisions تصادمات with electrons in atoms.



Example: a copper wire has across sectional area of مقطع عرضي $3.31 \times 10^{-6}\ m^2$. it carries a constant current of 10A what is the drift velocity of the electrons in the wire (assume each Cu atom contributes one free electron to the current. the mass density of Cu $= 8.92\ g/cm^3$

$$I_{av} = n q v_d A \Rightarrow v_d = \frac{I_{av}}{n q A}$$

$$n = \frac{\text{Avogadro's number of atoms}}{\text{Volume}} = \frac{6.02 \times 10^{23} / \text{mole}}{\text{Volume}}$$

$$\text{mass density} = \frac{\text{mass}}{\text{Volume}} \Rightarrow \text{Volume} = \frac{m}{\text{density}} = \frac{\text{molar mass}}{\text{density}}$$

$$v_d = \frac{I_{av} * \text{Volume}}{6.02\times10^{23} * q * A} = \frac{I_{av} * \text{molar mass}}{6.02\times10^{23} * q * A * \text{density}}$$

$$= \frac{10 * 0.0635\ \text{kg/mole}}{1.6\times10^{-19} * 3.31\times10^{-6}\,(m^2) * 6.02\times10^{23}\ /\text{mole} * \frac{8920\times10^{-3}}{(10^{-2})^3}}$$

$$= 2.23\times10^{-4}\ m/s$$

Note: the molar mass is constant لكل عنصر for $Cu = 63.5$ g/mole

example: the electron in the ground state of H-atom at radius of $r = 5.29\times10^{-11}$ m. find the speed of the electron and the effective current (التيار الكهربي) associating مصاحب with orbiting electron (الالكترون الدوار)

$$\frac{m_e v^2}{r} = k_e \frac{q_1 q_2}{r^2} = k_e \frac{e^2}{r^2} \Rightarrow v^2 = \frac{k_e e^2}{m_e r}$$

قوة المركزية

$$v = \sqrt{\frac{9\times10^9 * (1.6\times10^{-19})^2}{9.11\times10^{-31} * 5.29\times10^{-11}}} = 2.19\times10^6\ m/s.$$

$$I = nqvA = 1 * 1.6\times10^{-19} * 2.19\times10^6 * \pi(5.29\times10^{-11})^2 = \ldots$$

$A = \pi r^2$ (مساحة الدائرة)

example: an electric current is given by: $I(t) = 100\sin(120\pi t)$ what is the total charge carried by the current from $t=0$ to $t = \frac{1}{240}$ seconds.

$$I = \frac{dq}{dt} \Rightarrow dq = I\,dt \Rightarrow q = \int_{t_i}^{t_f} I(t)\,dt$$

$$q = \int_0^{1/240} 100\sin(120\pi t)\,dt = \frac{-100\cos t}{120\pi}\Big|_0^{1/240}$$

$$= \frac{-100}{120\pi}\left(\cos\frac{\pi}{2} - \cos 0\right) = 0.265\ A$$

example : for $q(t) = 100 \cos \pi t$ find the average current I_{av} between $t=0$ to $t = 1/2$ sec. (4)

$$I_{av} = \frac{q(t=1/2) - q(t=0)}{1/2 - 0} = \frac{100 \cos \pi/2 - 100 \cos 0}{1/2}$$

$$= \frac{0 - 100}{1/2} = -200 \, A$$

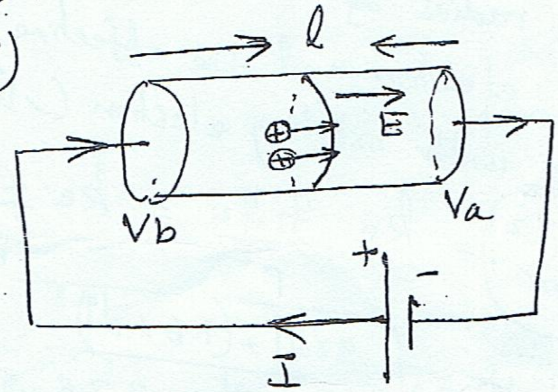
* find the instantaneous at $t = 1/2$.

$$I = \frac{dq}{dt} = \frac{d}{dt}(100 \cos \pi t) = -100\pi \sin \pi t \Big|_{t=1/2}$$

$$= -100 \pi \sin \pi/2 = -100\pi$$

Resistance and Ohm's Law

Define the current density (J) (كثافة التيار)



$$\boxed{J = \frac{I}{A}} \; (\text{Amper}/m^2)$$

$$= \frac{nqv_d \cancel{A}}{\cancel{A}} = \boxed{nqv_d}$$

J parallel to the current

Note : the electric field $E = 0$ inside the conductor when the charges are not moving (electrostatic) only.

Ohm's Law : In some materials (ohmic materials) the current density is proportional to the electric field inside the conductor

$$\vec{J} \propto \vec{E} \Rightarrow \boxed{\vec{J} = \sigma \vec{E}}$$; σ : constant called (الموصلية) conductivity

$$\boxed{\sigma = \frac{1}{\rho}}$$; ρ : resistivity (المقاومية)

$\rho \, (\Omega \cdot m)$

the potential difference $\Delta V = V_b - V_a = -\int_a^b E \cdot dl$

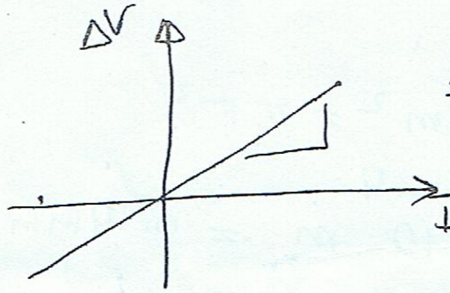
$|\Delta V| = E * l \Rightarrow E = \Delta V / l$

$J = \sigma E$

$\frac{I}{A} = \sigma \frac{\Delta V}{l} \Rightarrow \Delta V = \frac{l}{A\sigma} I = \frac{\rho l}{A} I$

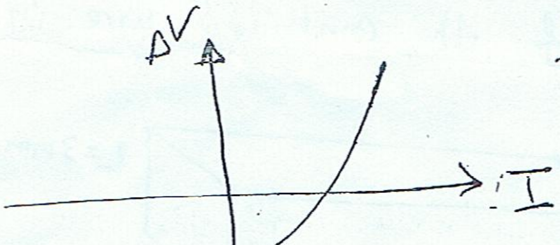
$\Delta V = \underbrace{\frac{\rho l}{A}}_{\text{called } R} I$; $\boxed{R = \frac{\rho l}{A}}$ (resistance المقاومة
تقاس بوحدة أوم Ω)

$\boxed{\Delta V = I R}$ or $\boxed{R = \frac{\Delta V}{I}}$ = Volt/Ampere $\boxed{\equiv \text{Ohm} \equiv \Omega}$



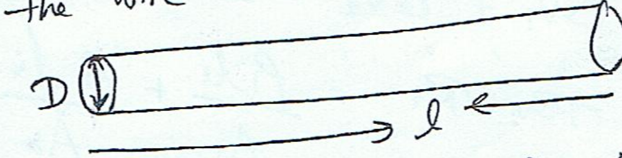
$\frac{\Delta V}{I} = R \equiv$ constant

ohmic material



$\frac{\Delta V}{I} \neq$ constant (not ohmic material)

example: Suppose you wish to fabricate (تصنيع) a uniform wire of resistance $R = 0.5\,\Omega$ from a 1 g of copper. all copper should be used what is the length and diameter of the wire



$D = 2r$

Area $(A) = \pi r^2 = \pi \left(\frac{D}{2}\right)^2$

density of copper $= 8.92 \times 10^3$ kg/m

$m = 1\,g = 1 \times 10^{-3}$ kg ; density of copper $= 8.92 \times 10^3$ kg/m

ρ for copper (Cu) $= 1.7 \times 10^{-8}\ \Omega \cdot m$

$$R = \frac{\rho l}{A} \Rightarrow \frac{l}{A} = \frac{R}{\rho} = \frac{0.5}{1.7 \times 10^{-8}} = 0.294 \times 10^{8}\ (1/m)$$

(6)

$$\boxed{\frac{l}{A} = 2.94 \times 10^{7}} \quad \text{---} \ (1)$$

(حجم الاسطوانة = مساحة المقطع * الارتفاع
$= A * l$)

but the density $= \frac{mass}{volume} = \frac{mass}{A * l}$

$$8.92 \times 10^{3} = \frac{1 \times 10^{-3}}{Al} \Rightarrow Al = \frac{1 * 10^{-3}}{8.92 \times 10^{3}} = 0.112 \times 10^{-6} = 1.12 \times 10^{-7}\ (m^3)$$

$$\boxed{Al = 1.12 \times 10^{-7}} \quad \cdots \ (2)$$

$$A = 1.12 \times 10^{-7} / l \Longrightarrow \text{sub. in } (1) \Rightarrow \frac{l}{1.12 \times 10^{-7} / l} = 2.94 \times 10^{7}$$

$$l^2 = 2.94 \times 10^{7} * 1.12 \times 10^{-7} = 3.29$$

$$l = 1.81\ m.$$

$$A = 1.12 \times 10^{-7} / 1.81 = 62 * 10^{-8}\ m^2 = \pi r^2$$

$$r = \sqrt{\frac{A}{\pi}} = \sqrt{\frac{6.2 \times 10^{-8}}{3.14}} = 1.4 \times 10^{-4}\ m = 0.14\ mm$$

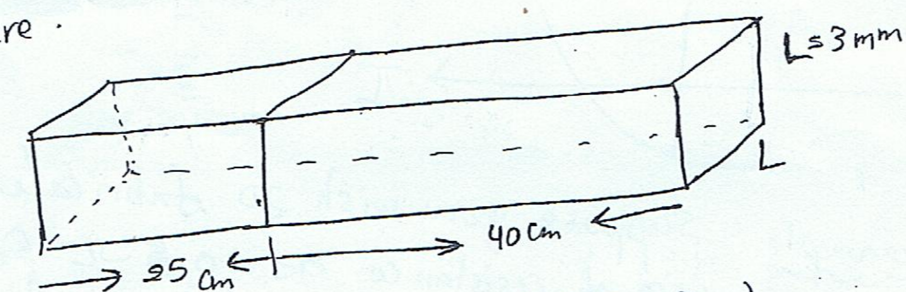
example: find the resistance of the combined wire in the figure.

$\rho_1 = 4 \times 10^{-3}\ \Omega \cdot m$

$\rho_2 = 6 \times 10^{-3}\ \Omega \cdot m$

$l_1 = 25\ cm$

$l_2 = 40\ cm$



$$\Delta V = \Delta V_1 + \Delta V_2 = I R_1 + I R_2 = I(R_1 + R_2) = I R_{tot}.$$

$$R_{tot} = R_1 + R_2 = \frac{\rho_1 l_1}{A_1} + \frac{\rho_2 l_2}{A_2} = \frac{\rho_1 l_1 + \rho_2 l_2}{L^2}$$

$$A_1 = A_2 = L^2$$

$$R_{tot} = \frac{4 \times 10^{-3} * 0.25 + 6 \times 10^{-3} * 0.4}{3 \times 10^{-3} * 3 \times 10^{-3}} = 377.77\ \Omega$$

problem: the quantity of charge q that passed through a surface of area 2 cm² varies with time according to the equation

$q = 4t^3 + 5t + 6$

(a) what is the instantaneous current through the surface at $t = 1$ sec.

$$I = \left.\frac{dq}{dt}\right|_{t=1s} = \left.(12t^2+5)\right|_{t=1} = 17A$$

(b) what is the value of the current density

$$J = \frac{I}{A} = \frac{17}{2(10^{-2})^2} = 8.5\times10^{4} A/m^2$$

problem: the figure represents a section of circular conductor of non-uniform diameter carrying a current of 5 A. the radius of cross section A_1 is 0.4 cm

(a) what is the magnitude of the current density across A_1

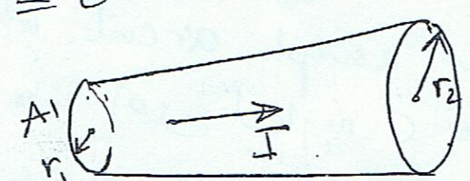
$$J = \frac{I}{A} = \frac{5}{\pi r^2} = \frac{5}{\pi(4\times10^{-3})^2} = 9.95\times10^{4} A/m^2$$

(b) if $J_2 = \frac{1}{4}J_1$ find the radius of A_2

$$J_2 = \frac{1}{4}J_1 \Rightarrow \frac{\cancel{I}}{A_2} = \frac{1}{4}\frac{\cancel{I}}{A_1} \Rightarrow 4A_1 = A_2$$

$$\Rightarrow 4 * \cancel{\pi} r_1^2 = \cancel{\pi} r_2^2 \Rightarrow r_2 = \sqrt{4 r_1^2} = 2 * r_1$$

$$r_2 = 2 * 4 \times 10^{-3} = 8\times10^{-3} m = 8 mm$$



problem: a 9-Volt potential difference is maintained across a 1.5 m length of tungsten wire that has a cross sectional area of 0.6 mm² what is the current in the wire

$$\Delta V = IR \Rightarrow I = \frac{\Delta V}{R} = \frac{9}{R_1}$$

$$R = \frac{\rho l}{A} = \frac{5.6\times10^{-7} * 1.5}{0.6*(10^{-3})^2} = 15\times10^{-1} = 1.4\ \Omega$$ (8)

$$I = \frac{9}{1.4} = 6.43\ A$$

Resistance and Temperature

the resistivity of a metal varies with temperature as in the relation $\rho = \rho_0\{1+\alpha(T-T_0)\}$ --- (I)

ρ : resistivity at some temperature

ρ_0 : ~ ~ refrence temperature ($20^\circ C$ غالبا)

α : temperature Coefficient of resistivity.

$$\alpha = \frac{1}{\rho_0}\frac{\Delta\rho}{\Delta T} = \frac{1}{\rho_0}\frac{\rho-\rho_0}{T-T_0}$$

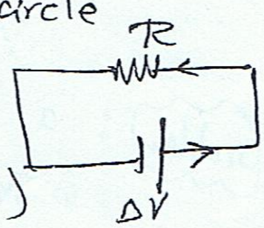
if multiply equation (I) by l and divide by A $\Rightarrow$

$$\frac{\rho l}{A} = \frac{\rho_0 l}{A}\{1+\alpha(T-T_0)\} \quad ; \quad T-T_0 = \Delta T$$

$$\boxed{R = R_0\{1+\alpha(T-T_0)\}}$$

Electrical power القدرة الكهربائية

Consider a simple circuit where energy is dilivered to the resistor (neglect اهمال the resistance of wires). the amount of energy to move a one charge around the circle is $\Delta U = q\,\Delta V$. to move a ΔQ of charge



$\Delta U = \Delta Q\,\Delta V$: divide by Δt (time period)

$$\frac{\Delta U}{\Delta t} = \frac{\Delta Q}{\Delta t}\Delta V \rightarrow \boxed{\text{Power} = I\,\Delta V}$$

Define power $= \dfrac{\Delta U}{\Delta t} = J/s \equiv$ Watt.

$\boxed{\text{power} = I * \Delta V}$. the power dissipated (تستهلك) in a resistor equal to

$$P_R = I\Delta V = I * IR = I^2 R .$$

$$\text{or} \quad P = I\,\Delta V = \frac{\Delta V}{R} * \Delta V = \frac{(\Delta V)^2}{R}$$

Note * for resistor the power is dissipated (مستنفذة) and from the battrey the power is produced (تنتج).

* the battrey is the source of electromotive force (emf) (القوة الدافعة [illegible])

the potential diffrence on the baltrey $\equiv \mathcal{E} = \Delta V \Rightarrow P_{\text{battrey}} = I\mathcal{E}$

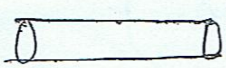
examplp: an electric heater is constructed by applaying a $\Delta V = 120\,V$ to a Nichrome wire ($R = 8\,\Omega$) find the current carried by the wire and the power rating of the heater

✓ $\Delta V = IR \Rightarrow I = \dfrac{\Delta V}{R} = \dfrac{120}{8} = 15\,A$

✓ $P = I^2 R = (15)^2 * 8 = 1.8\times10^3\ W = 1.8\,kW$

or $P = \dfrac{(\Delta V)^2}{R} = \dfrac{(120)^2}{8} = 1.8\times10^3\ W = 1.8\ kW$.

problem: a heating coil has power $p = 500\,W$ and $\Delta V = 110\,V$ of Nichrome wire. if its diameter

 $D = 0.5\,mm$ find the length of the wire

(a) at $T = 20^\circ C$

$$P = \frac{(\Delta V)^2}{R} \Rightarrow R = \frac{(\Delta V)^2}{P} = \frac{(110)^2}{500} = 24.2\,\Omega$$

$$R = \frac{\rho l}{A} = \frac{\rho l}{\pi (D/2)^2} \Rightarrow l = \frac{\pi (D/2)^2 * R}{\rho}$$

$$l = \frac{3.14 * \left(\frac{0.5 \times 10^{-3}}{2}\right)^2 * 24.2}{1.5 \times 10^{-6}} =$$

(b) if we heat up to $1200°C$ find the power needed

$$R = R_0 \{ 1 + \Delta T \} = 24.2 \{ 1 + 0.4 \times 10^{-3} (1200° - 20°) \}$$

$$R(T=1200°) = 35.6\ \Omega$$

$$P = \frac{(\Delta V)^2}{R(T=1200)} = \frac{(110)^2}{35.6} = 340\ W$$

problem: a certain light pulb has $R_0 = 19\ \Omega$ of Tangsten when cold and $140\ \Omega$ when hot find the temperature when it is hot. $(T_0 = 20°C)$

$$R(T) = R_0 \{ 1 + \alpha \Delta T \}$$

$$= 19 \{ 1 + 4.5 \times 10^{-3} \Delta T \} = 140$$

$$\Delta T = 1.42 \times 10^3\ C = T_2 - T_0 = T_2 - 20°$$

$$T_2 = \Delta T + 20° = 1.42 \times 10^3 + 20$$

$$= 1.44 \times 10^3\ °C$$

problem: Nichrome resistance wire of toster has $\Delta V = 120\ V$ at $T_0 = 20°C$ and $I_0 = 1.8\ A$ find the temprature and R when hot if $I = 1.53\ A$, find the powe of the toster.

$$(\rho_0 = 1.5 \times 10^{-6} ;\ T_0 = 20° ,\ \alpha = 0.4 \times 10^{-3})$$

$$R = R_0 \{ 1 + \alpha (\Delta T) \}$$

$\Delta V = I_o R_o \Rightarrow R_o = \Delta V / I_o$

$= \frac{120}{1.8} = 66.67\ \Omega$

$R_{hot} = \frac{\Delta V}{I_{hot}} = \frac{120}{1.53} = 78.43\ \Omega$

$P = \frac{(\Delta V)^2}{R} = I^2 R = \frac{(120)^2}{78.43} = 183.6$ watt.

$R = R_o \{ 1 + \alpha (T - 20^\circ) \}$

$78.43 = 66.67 \{ 1 + 0.4 \times 10^{-3} (T - 20^\circ) \quad ; \Delta T = 441^\circ C$

$T = 461^\circ C$

Problem: compute the cost per day of operating a lamp that draws a current of 1.7 A from 110 V line assume the energy from the power company is شركة الكهرباء

$0.06\ \$ / kWh$.

$P = I \Delta V = 1.7 * 110 = 187 W$.

energy in 24 hours $= 187 * 24 = 4490$ W hour

energy in kWh $= 4.49$ kWh
كيلوواط ساعة

$1\ kWh \xrightarrow{\text{Cost}} 0.06\ \$$

$4.490\ kWh \longrightarrow ??$ cost

$\text{Cost} = \frac{4.490 * 0.06}{1} = 0.269\ \$$

hapter 27/ Current and Resistance

A rod of 2.0-m length and a square (2.0 mm x 2.0 mm) cross section is made of a material with a resistivity of 6.0×10^{-8} Ω · m. If a potential difference of 0.50 V is placed across the ends of the rod, at what rate is heat generated in the rod?

a. 3.0 W
b. 5.3 W
c. 8.3 W
d. 1.3 W
e. 17 W

2. How much energy is dissipated as heat during a two-minute time interval by a 1.5-kΩ resistor which has a constant 20-V potential difference across its leads?

a. 58 J
b. 46 J
c. 32 J
d. 72 J
e. 16 J

A 4.0-Ω resistor has a current of 3.0 A in it for 5.0 min. How many electrons pass through the resistor during this time interval?

a. 7.5×10^{21}
b. 5.6×10^{21}
c. 6.6×10^{21}
d. 8.4×10^{21}
e. 2.1×10^{21}

4. What maximum power can be generated from an 18-V emf using any combination of a 6.0-Ω resistor and a 9.0-Ω resistor?

a. 54 W
b. 71 W
c. 90 W
d. 80 W
e. 22 W

A rod (length = 80 cm) with a rectangular cross section (1.5 mm x 2.0 mm) has a resistance of 0.20 Ω. What is the resistivity of the material used to make the rod?

a. 6.0×10^{-7} Ω · m
b. 3.8×10^{-7} Ω · m
c. 7.5×10^{-7} Ω · m
d. 3.0×10^{-7} Ω · m
e. 4.8×10^{-7} Ω · m

. Light bulb A is rated at 60 W and light bulb B is rated at 100 W. Both are designed to operate at 110 V. Which statement is correct?

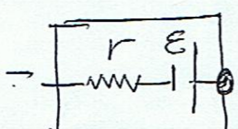
a. The 60 W bulb has a greater resistance and greater current than the 100 W bulb.
b. The 60 W bulb has a greater resistance and smaller current than the 100 W bulb.
c. The 60 W bulb has a smaller resistance and smaller current than the 100 W bulb.
d. The 60 W bulb has a smaller resistance and greater current than the 100 W bulb.
e. We need to know the resistivities of the filaments to answer this question

Chapter 28 Direct Current Circuits

* we will concentrate (نركز) on analysing (تحليل) of simple Circuits that contains batteries, resistors and capacitors (with their combinations)
* the steady state current : I constant in magnetude and direction.
* electromotive force (emf) $\equiv \mathcal{E}$ is the device who produces an electric field and cause the charges to move (charge pumb)
  القوه الدافعه الكهربائيه

emf ($\mathcal{E}$) is the work done per unit charge.

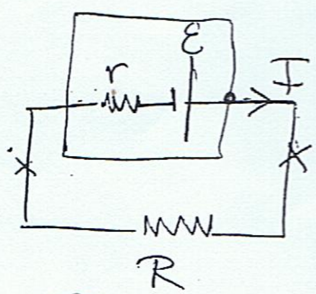
* r: internal resistor
  مقاومه داخليه



battrey

- if we neglect (نهمل) the internal resistance the terminal voltage (جهد الاقطاب) equal to emf ($\Delta V = \mathcal{E}$)
  (open Circuit voltage)

* Simple Circuit



$$\boxed{\Delta V = V_b - V_a = \mathcal{E} - Ir}$$
$$= IR$$

$$\boxed{I = \frac{\mathcal{E}}{R+r}} \quad \text{or} \quad \boxed{\mathcal{E} = IR + Ir} \quad \text{multiply by } I \Rightarrow$$

$$\boxed{\begin{aligned} \underbrace{I\mathcal{E}}_{P_{\mathcal{E}}} &= \underbrace{I^2R}_{P_R} + \underbrace{I^2 r}_{P_r} \end{aligned}}$$

* the total out put power of $\mathcal{E}$ is delivered (يعطى) to the load resistor and internal resistor
* the maximum power delivered to the load when $r = R$

2

example: a battrey has emf = 12V and internal resistance $r = 0.05\,\Omega$, connected with a load resistance $R = 3\,\Omega$ find (a) current (b) total power (c) P_R, P_r, P_ε (d) terminal voltage of the battrey.

✓ $\varepsilon = IR + Ir \Rightarrow I = \dfrac{\varepsilon}{r+R} = \dfrac{12}{3+0.05} = 3.93\,A$

✓ ΔV on the load resistor $= IR = 3.93 * 3 = 11.8\,V$

(terminal voltage) $= \varepsilon - Ir = 12 - 3.93 * 0.05 = 11.8\,V$

✓ $P_\varepsilon = I\varepsilon$ (power delivered by the battray)

$= 12 * 3.93 = 47.1\,W$

✓ $P_R = I^2 R = (3.93)^2 * 3 = 46.3\,W$

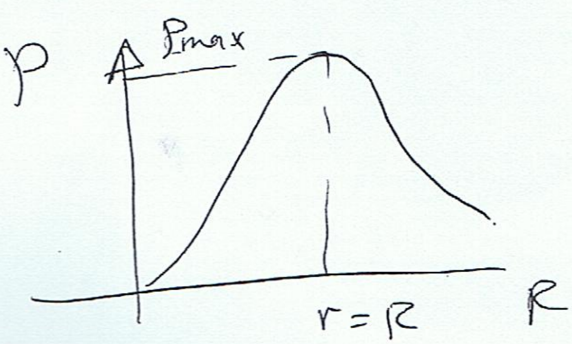
✓ $P_r = I^2 r = (3.93)^2 * 0.05 = 0.772\,W$

Note $P_\varepsilon = 47.1 = P_R + P_r$

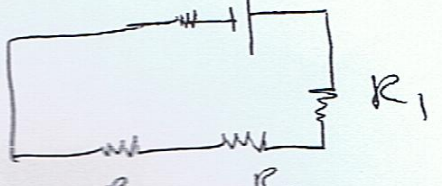
* the maximum power delivered to the load

$$P = I^2 R = \frac{\varepsilon^2}{(r+R)^2} R$$

$$\frac{dP_R}{dR} = 0 \text{ (بالتفاضل)} \Longrightarrow r = R$$



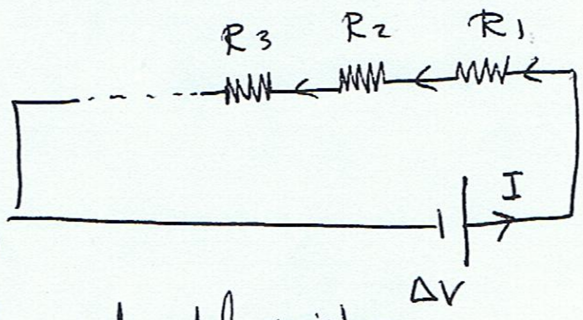
* if there is more than on load resistors



$$I = \frac{\varepsilon}{R_1 + R_2 + R_3 + r}$$

Combination of resistors

1) series combination

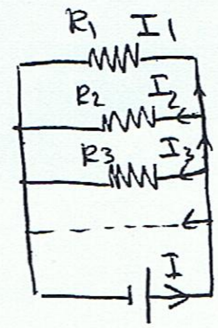


$$\Delta V = IR_1 + IR_2 + IR_3 + \cdots$$

$$IR_{eq} = I(R_1 + R_2 + R_3 + \cdots)$$

$$R_{eq} = \sum_{i=1}^{N} R_i \quad ; \quad R_{eq} > \text{any indivisulal resistor}$$

2) parallel combination



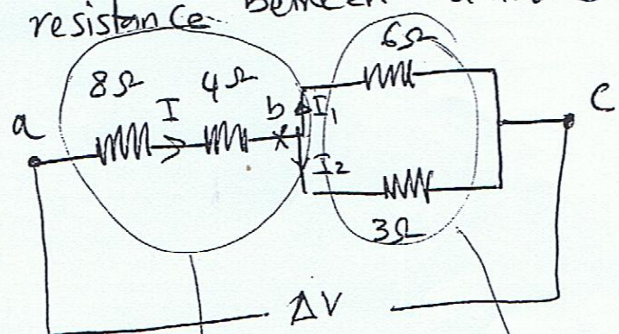
$$I = I_1 + I_2 + I_3 + \cdots$$

$$\frac{\Delta V}{R_{eq}} = \frac{\Delta V}{R_1} + \frac{\Delta V}{R_2} + \frac{\Delta V}{R_3} + \cdots$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots$$

R_{eq} < any one of them.

example: (1) find the equirilant resistance between a and c



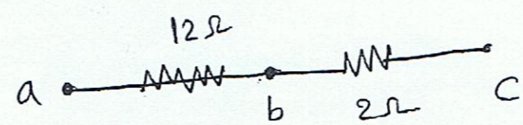
(2) If $\Delta V_{ac} = 42\ V$ find the total current and the current in 6Ω and 3Ω resistors

(3) find ΔV_{ab}, ΔV_{bc} ..

$8 + 4 = 12\ \Omega$

$\frac{1}{6} + \frac{1\times 2}{3\times 2} = \frac{3}{6}$

$R = 2\ \Omega$

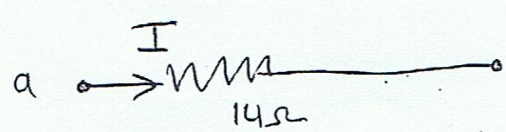


(1) $R_{eq} = 14\ \Omega$

(2) $\Delta V_{ac} = I R_{eq}$

$I = \Delta V_{ac} / R_{eq}$

$= 42 / 14 = 3\ A$



$\cdot\ I = I_1 + I_2 = 3A \cdots (\mathrm{I})$

(3) $\Delta V_{on\ 6\Omega} = \Delta V_{on\ 3\Omega}$

$I_1 * 6 = I_2 * 3 \Rightarrow I_1 = \frac{1}{2} I_2 \cdots (\mathrm{II})$

sub. (II) in (I) $\Rightarrow 3 = \frac{1}{2} I_2 + I_2 = \frac{3}{2} I_2$

$I_2 = 2A\ ;\ I_1 = 1A$

(4) $\Delta V_{ab} = I * R_{eq}(a,b) = 3 * 12 = 36V$

$\Delta V_{bc} = I_1 * 6\Omega = 1 * 6 = 6V$

$= I_2 * 3\Omega = 2 * 3 = 6V$

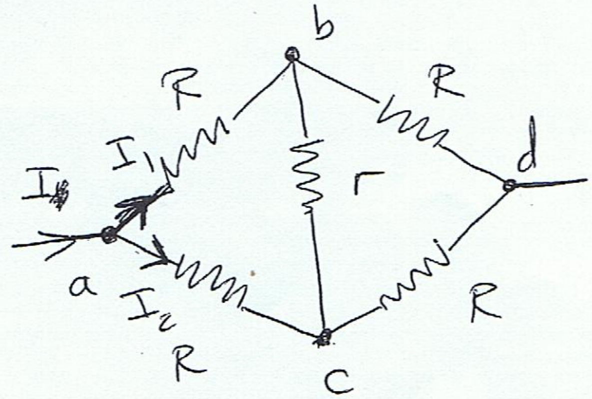
or

or $\Delta V_{ac} = \Delta V_{ab} + \Delta V_{bc} \Rightarrow \Delta V_{bc} = \Delta V_{ac} - \Delta V_{ab}$

$= 42 - 36 = 6V$

* example: 4 equal resistors (R)

المقاومات متساوية إذا سوف ينقسم التيار I إلى قسمين متساويين



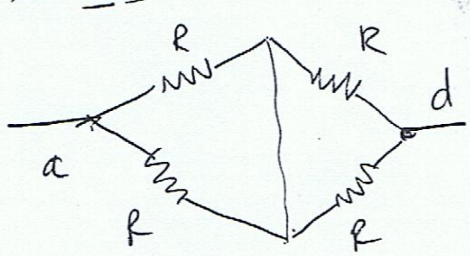
$I_1 = I_2$

$\Rightarrow \Delta V_{ab} = \Delta V_{ac}$

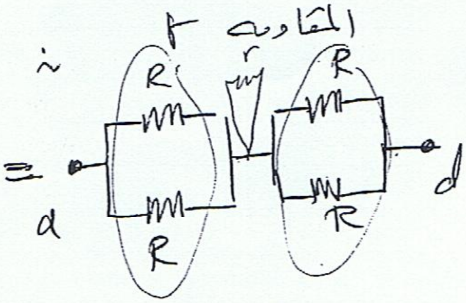
$V_b - \cancel{V_a} = V_c - \cancel{V_a} \Rightarrow V_b = V_c$

* جهد النقطتين b , c متساوي ∴ لا يمر تيار في المقاومة r

∴



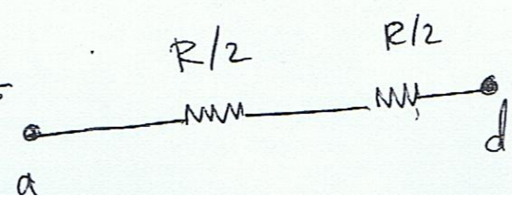
$=$



$\frac{1}{R} + \frac{1}{R} = \frac{2}{R}$

$R_{eq}(1) = \frac{1}{2} R$

$\Leftarrow$



$\Leftarrow$

$\boxed{R_{eq} = R}$

Example: a TV repair man (مصلح تلفزيونات) needs 100 Ω resistor ; he has 500 Ω, 250 Ω and 250 Can he get 100 Ω resistor ??

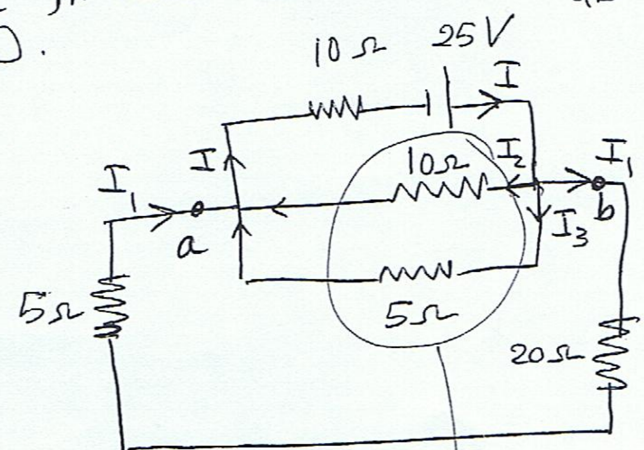
Connect in parallel.

100 < any resistor he has ⇒ Connect in parallel

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{500} + \frac{1*2}{250*2} + \frac{1*2}{250*2}$$

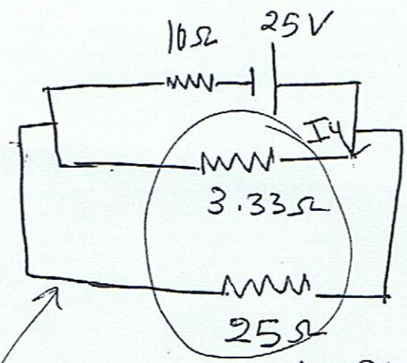
$$= \frac{5}{500} = \frac{1}{100} \Rightarrow \boxed{R_{eq} = 100\ \Omega}$$

example: find the current I in 20 Ω resistor and ΔV_{ab} in the circuit below.



$$\frac{1}{10} + \frac{1\times2}{5\times2} = \frac{3}{10}$$

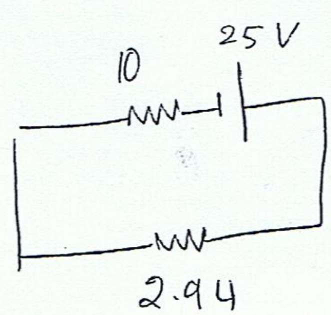
$$R_{eq(1)} = \frac{10}{3} = 3.33\ \Omega$$



in parallel.

$$\frac{1}{R_{eq(2)}} = \frac{1}{3.33} + \frac{1}{25} = \frac{3}{10} + \frac{1}{25}$$

$$R_{eq(2)} = 2.94\ \Omega$$



$$I = \frac{\mathcal{E}}{10+2.94} = \frac{25}{12.94}$$

$$\boxed{I = 1.93\ A}$$

$$\Delta V_{(3.3\Omega)} = \Delta V_{(25\ \Omega)}$$

$$I_4 * 3.3 = I_1 * 25 \quad ; \quad I_4 = I_2 + I_3$$

$$I = I_1 + I_2 + I_3 = 1.93 = I_1 + I_4$$

$1.93 = I_1 + I_4 \quad ; \quad I_4 = \frac{25}{3.3} I_1 = 7.57 I_1$

$1.93 = I_1 + 7.57 I_1$

$I_1 = \frac{1.93}{8.57} = 0.225\,A \simeq 0.23\,A$

$I_4 = 7.57 * 0.23 = 1.72 A$

$I_4 = I_2 + I_3 = 1.72$

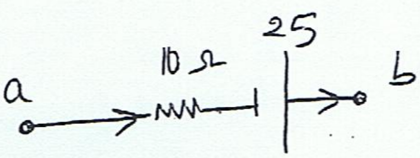
ΔV on 10Ω = ΔV on 5Ω

$I_2 * 10 = I_3 * 5 \Rightarrow I_2 = \frac{1}{2} I_3$

$I_4 = \frac{1}{2} I_3 + I_3 = \frac{3}{2} I_3 = 1.72 \Rightarrow I_3 = \frac{2 * 1.72}{3}$

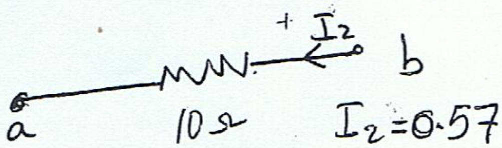
$= 1.14\ A$

$I_2 = 1.71 - 1.14 = 0.57\ A$



$\Delta V_{ab} = V_b - V_a$

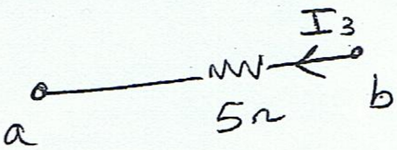
$= 25 - 10 * I = 25 - 10 * 1.93 = 5.7\ V$



or $\Delta V_{ab} = V_b - V_a = I_2 * 10$

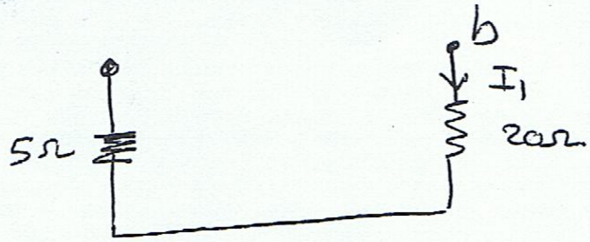
$= 0.57 * 10$

$= 5.7\,V$



$\Delta V_{ab} = I_3 * 5 = 1.14 * 5$

$= 5.7\ V$



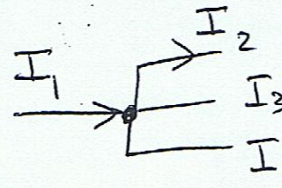
or $\Delta V_{ab} = I_1 (20+5)$

$= 0.2\cancel{26} * 25$

$\simeq 5.7$ Volts.

Kirchhoff's Rules

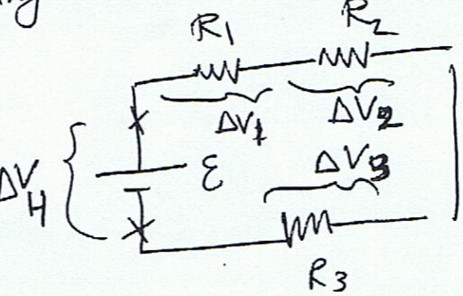
① Conservation of charge ⟹ the sum of the currents entering any junction in a circuite must equal the sum of the current leaving that junction



$$\boxed{\begin{gathered}\sum_i I_{in} = \sum_i I_{out} \\ I_1 = I_2 + I_3 + I_4\end{gathered}}$$

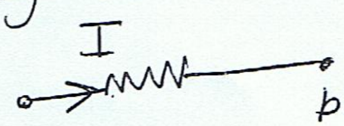
② Conservation of energy ⟹ the sum of the potential difference across all elements around any closed circuit loop must be zero.

$$\sum_{\text{loop}} \Delta V = 0$$

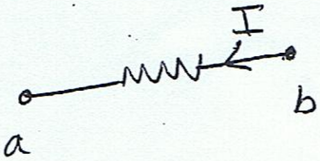


$$\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$$

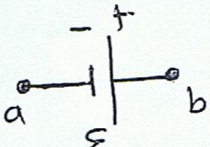
Note: current moves from high voltage to lower voltage in resistor



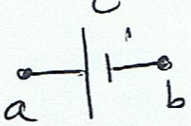
$V_b - V_a = -IR$ $\qquad V_a > V_b$



$V_b - V_a = +IR$

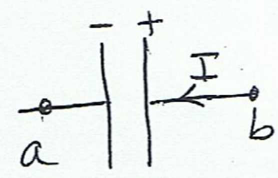


$V_b - V_a = \mathcal{E}$

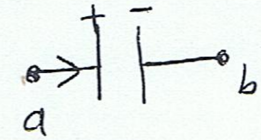


$V_b - V_a = -\mathcal{E}$

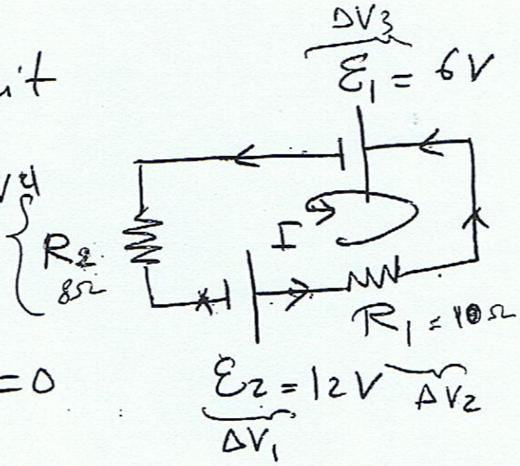
$V_b - V_a = Q/C$



$V_b - V_a = -Q/C$



example: single loop circuit



الأفضل أخذ التيار من البطارية الأكبر

$$\sum \Delta V = 0$$

$$\mathcal{E}_2 - IR_1 - \mathcal{E}_1 - IR_2 = 0$$

$$\mathcal{E}_2 - \mathcal{E}_1 - I(R_1 + R_2) = 0$$

$$I(R_1 + R_2) = \mathcal{E}_2 - \mathcal{E}_1 \Rightarrow I = \frac{\mathcal{E}_2 - \mathcal{E}_1}{R_2 + R_1} = \frac{12 - 6}{8 + 10}$$

$$I = \frac{1}{3} = 0.33\,A$$

* what is the power delivered to each resistor

$$P_1 = I^2 R_1 = 1.1\,W \qquad P_{\mathcal{E}_1} = I\mathcal{E}_1 = 2W$$

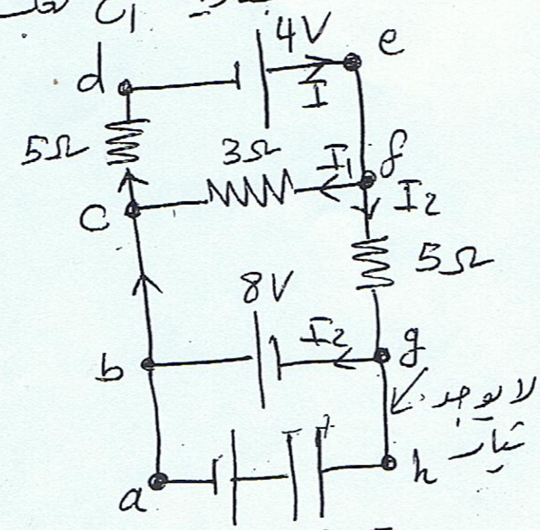
$$P_2 = I^2 R_2 = 0.87\,W \qquad P_{\mathcal{E}_2} = 4W$$

$$P_{\mathcal{E}_2} = P_1 + P_2 + P_{\mathcal{E}_1} = 0.87 + 1.1 + 2 \cong 4W$$

البطارية $\mathcal{E}_1$ تعكس اتجاه $\mathcal{E}_1$ وهذا يجعلها تستهلك طاقة

example: Multi loop circuit

find the unknown currents in the circuit in the steady state (the capacitor does not have a current passing it)



$$I = I_1 + I_2 \quad \text{--- (1)}$$

the loop cdef $\Rightarrow (V_c - V_d) + (V_d - V_e) + (V_f - V_c) = 0$

$$5I - 4 + 3I_1 = 0 \quad \text{--- (2)}$$

loop cfgb $\Rightarrow (V_c - V_f) + (V_f - V_g) + (V_g - V_b) + (V_b - V_c) = 0$ (9)

$$-3I_1 + 5I_2 - 8 + 0 = 0$$

$$-3I_1 + 5I_2 - 8 = 0 \quad \text{---} \quad (3)$$

sub (1) in (2) $\Rightarrow 5(I_1 + I_2) + 3I_1 - 4 = 0$

$$8I_1 + \cancel{5I_2} - 4 = 0$$

$$(-3I_1 + \cancel{5I_2} - 8) = 0 \quad * -1$$

$$11 I_1 + 4 = 0 \Rightarrow I_1 = -4/11 = -0.364 \text{ A}$$

اتجاه التيار عكس الاتجاه المفروض كده نستمر بالتعويض بالقيمة السالبة

$$8I_1 + 5I_2 - 4 = 0$$

$$8 * -0.364 + 5I_2 - 4 = 0 \Rightarrow I_2 = 1.382$$

$$I = I_1 + I_2 = -0.364 + 1.382 = 1.02 \text{ A}$$

b) what is the charge on the capacitor

$$\Delta V_{gb} = \Delta V_{ha}$$

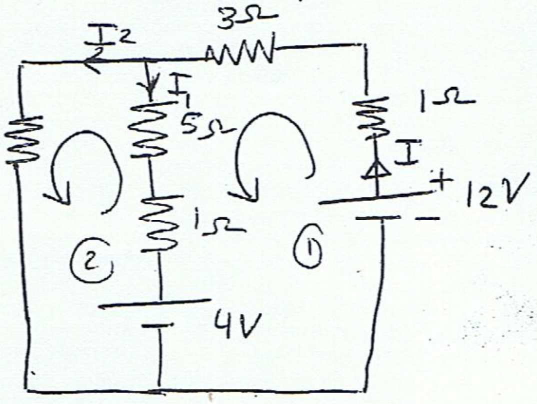
$$V_b - V_g = V_a - V_h$$

$$8 = -3 + \Delta V_c \Rightarrow \Delta V_c = 11 \text{ Volts.}$$

$$\Delta V_c = \frac{Q}{C} \Rightarrow Q = C\Delta V_c = 6 \times 10^{-6} * 11$$

$$= 66 \mu C$$

example: Determine the circuits currents



$$I = I_1 + I_2$$

loop (1) $\Rightarrow 12 - 4I - 6I_1 - 4 = 0$

$$\Rightarrow 8 - 4I_1 - 4I_2 - 6I_1 = 0$$

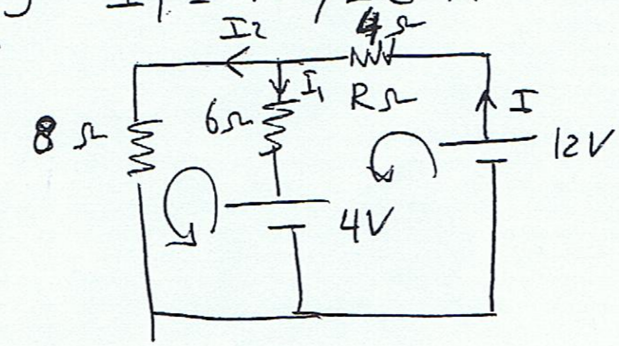
$$8 - 10I_1 - 4I_2 = 0 \quad \cdots \quad (1)$$

loop (2) $\Rightarrow 4 + 6I_1 - 8I_2 = 0 \quad \dots (2)$

$(8 - 10I_1 - 4I_2 = 0) * -2$

$$\left.\begin{array}{l} 4 + 6I_1 - \cancel{8I_2} = 0 \\ -16 + 20I_1 + \cancel{8I_2} = 0 \end{array}\right\} \quad -12 = -26I_1$$

$I_1 = +12/26\ A$



$I_1 = +0.462\ A$ (نفس الاتجاه المفروض)

$4 + 6I_1 - 8I_2 = 0$

$4 + 0.462 \times 6 - 8I_2 = 0 \Rightarrow I_2 = 0.846\ A$

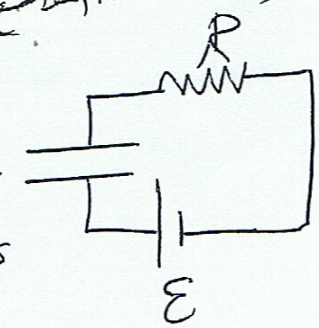
$I = I_1 + I_2 = 0.462 + 0.846 = 1.31\ A$

R- C circuit:

the circuits containing Resistance and capacitor in series called R- C circuits

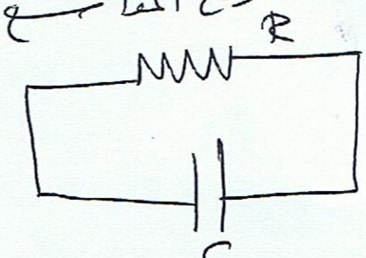
(1) capacitor charging circuit (دائرة شحن المواسع)

تحتوي بطارية ومقاومة ومواسع



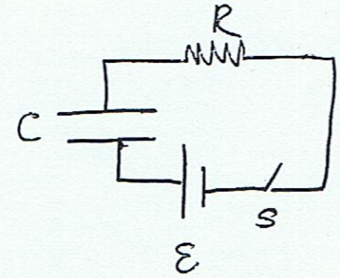
(2) discharging the capacitor circuits

دائرة تفريغ المواسع



تحتوي بطارية ومواسع مشحون فقط

* يبدأ الشحن أو التفريغ تدريجياً مع الزمن حتى تشحن المواسع تماماً أو تفرغ تماماً حسب الدائرة المستخدمة.



1. when switch S closed at $t=0$ charge begins to flow, Capacitor begin to charge until the capacitor is fully charged, ΔV across the capacitor increases to maximum. when the capacitor is fully charged (steady state) the current in the circuit $=0$

closed loop $\Rightarrow \sum \Delta V = 0 = \boxed{\mathcal{E} - \frac{q}{C} - IR = 0}$

Note: at $t=0$; $q=0 \Rightarrow \mathcal{E} = I_{max} R \Rightarrow \boxed{I_{max} = \mathcal{E}/R}$

after fully charged $I=0 \Rightarrow \mathcal{E} - \frac{q_{max}}{C} = 0 \Rightarrow$

$$\boxed{q_{max} = \mathcal{E} C}$$

* during charging (خلال الشحن اي زمن)

$$\mathcal{E} - \frac{q}{C} - IR = 0 \quad ; I = dq/dt$$

$$\frac{\mathcal{E}}{R} - \frac{q}{CR} = I = \frac{dq}{dt} \qquad \frac{\mathcal{E}C - q}{CR}$$

$$\frac{dq}{\mathcal{E}C - q} = dt/RC \Rightarrow \text{integrate the equation}$$

نكامل الطرفين $\Leftarrow$

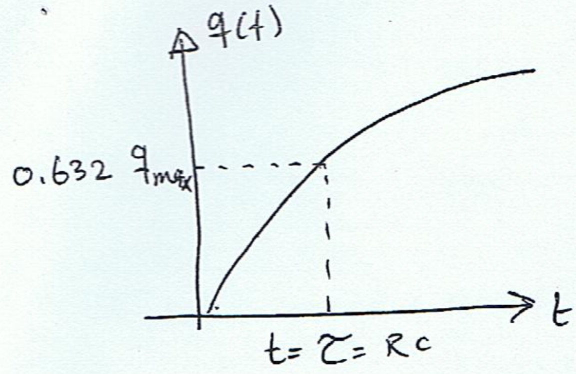
$$\int_0^q \frac{dq}{\mathcal{E}C - q} = \int_0^t dt/RC = -\int_0^q \frac{dq}{q - \mathcal{E}C} = -\ln(q - \mathcal{E}C)\Big|_0^q = -\{\ln(q - \mathcal{E}C) - \ln \mathcal{E}C\}$$

$$-\ln\left(\frac{q - \mathcal{E}C}{\mathcal{E}C}\right)\Big| = t/RC \Rightarrow \ln\left(\frac{q - \mathcal{E}C}{\mathcal{E}C}\right) = -t/RC$$

نأخذ exponential للطرفين

$$e^{\ln\left(\frac{q - \mathcal{E}C}{\mathcal{E}C}\right)} = e^{-t/RC} \Rightarrow \frac{q - \mathcal{E}C}{\mathcal{E}C} = e^{-t/RC}$$

$$\boxed{q(t) = \mathcal{E}C\{1 - e^{-t/RC}\}} = \boxed{q_{max}\{1 - e^{-t/RC}\}}$$



$$q(t) = q_{max}\left\{1 - e^{-t/RC}\right\}$$

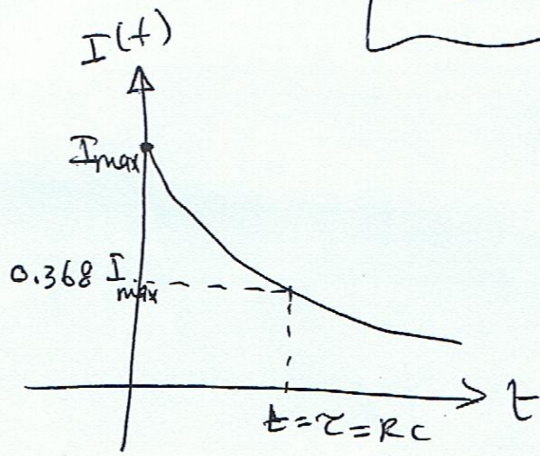
$\boxed{RC \equiv \tau}$: time constant of the circuit

ثابت الزمن للـعوي.

* the charging current $I(t) = \dfrac{dq(t)}{dt} = \dfrac{d}{dt}\left\{q_{max}\left\{1 - e^{-t/RC}\right\}\right\}$

$$= q_{max}\frac{1}{RC}e^{-t/RC} = \frac{\mathcal{E}\cancel{C}}{R\cancel{C}}e^{-t/RC}$$

$$\boxed{I(t) = I_{max}\, e^{-t/RC}}$$



$0.368\, I_{max}$ at $\tau \equiv$ the time it takes the current to decrease to $\frac{1}{e} I_{max} = 0.368\, I_{max}$.

* the charge increases from zero to $\mathcal{E}C\left(1 - \frac{1}{e}\right) = 0.632\, \mathcal{E}C = 0.632\, q_{max}$

* $[\tau] = [RC] = [R][C] = \left[\frac{\Delta V}{I}\right]\cdot\left[\frac{Q}{\Delta V}\right]$

$= [Q]/[I] = [t] =$ time.

بعد زمن τ .

* $\Delta V = \dfrac{Q}{C} \Rightarrow \Delta V(t) = \dfrac{q_{max}}{C}\left\{1 - e^{-t/RC}\right\}$

خلال الشحن $\Delta V(t) = \Delta V_{max}\left\{1 - e^{-t/RC}\right\}$

* the energy stored in the capacitor after fully charged

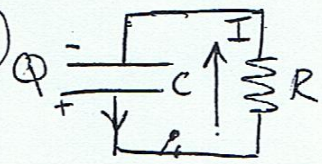
$U = \frac{1}{2} Q\mathcal{E} = \frac{1}{2} C\mathcal{E}^2 = \frac{1}{2}$ energy out put of the battrey

Discharging of the capacitor:

if we have a charged capacitor with initial charge Q

when the switch open

لا يوجد تيار في الدائرة



$\Delta V_c = Q/C$

$\Delta V_R = 0 \quad (I = 0)$

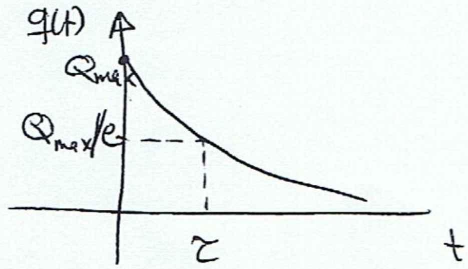
if we close S, the capacitor discharge through the resistor

$$-\frac{q}{C} - IR = 0 \Rightarrow \frac{-q}{RC} = \frac{dq}{dt}$$

$$\Rightarrow \frac{dq}{q} = -\frac{dt}{RC} \xrightarrow{\int} \int_{Q}^{q(t)} \frac{dq}{q} = -t/RC$$

$$\ln q \Big|_{Q}^{q(t)} = -t/RC \Rightarrow \frac{q(t)}{Q} = e^{-t/RC}$$

$$\boxed{q(t) = Q\, e^{-t/RC}}$$



$$I(t) = \frac{dq(t)}{dt} = \frac{-Q}{RC} e^{-t/RC}$$

$$\boxed{I(t) = -I_o\, e^{-t/RC}}$$

$I_o = \frac{Q}{RC}$ is the initial current

negative sign means discharging.

Note: the direction of charging current is the opposite of direction of discharging current.

example: uncharged capacitor and resistor are connected in series with a battrey $\mathcal{E} = 12\,V$; $C = 5\,\mu F$

$R = 8 \times 10^5\ \Omega$ find:

(a) $Q_{max} = \mathcal{E} C = 5 \times 10^{-6} \times 12 = 60\,\mu C$

(b) $I_{max} = \mathcal{E}/R = 12/8 \times 10^5 = 15\,\mu A$

(c) time constant $\tau = RC = 8\times10^{5} * 5\times10^{-6}$

$= 4 \text{ sec.}$

(d) the charge and the current as a function of time

$q(t) = q_{max}\{1 - e^{-t/RC}\}$

$= 60\mu C\{1 - e^{-t/4}\}$

$I(t) = I_0 e^{-t/RC} = 15 e^{-t/4}$

(e) after the time constant ellapsed (ثانية) find I, q

$\tau = t = 4 \text{ sec} \Rightarrow q(t=4), I(t=4)$

$q(t=4) = 60\{1 - \frac{1}{e}\} = 60 * 0.63 = 37.8 \ \mu C$

$I(t=4) = \frac{I_0}{e} = 15\times10^{-6}/e = 5.55 \mu A$

<u>example</u> : Discharging Capacitor

(a) after how many time constants is the charge on the capacitor $\frac{1}{4}$ of initial value.

$q(t) = Q_{max} e^{-t/RC}$; $q(t) = \frac{1}{4} Q_{max}$

$\frac{1}{4} \cancel{Q_{max}} = \cancel{Q_{max}} e^{-t/RC} \Rightarrow \frac{1}{4} = e^{-t/RC}$

$\ln(1/4) = \ln(e^{-t/RC}) = -t/RC = -\ln 4$

$RC \ln 4 = \boxed{t = 1.39 RC}$

* find when it reaches $\frac{1}{4}$ maximum voltage on the capacitor.

$V(t) = \frac{Q(t)}{C} = \frac{Q_{max}}{C} e^{-t/RC}$

$\frac{1}{4} \cancel{V_{max}} = \cancel{V_{max}} e^{-t/RC} \Rightarrow t = 1.39 RC$

$V(1.39RC) = \frac{Q_{max}}{C} e^{-1.39}$

(b) the energy stored in the capacitor decreases with time as the capacitor discharges. after how many time constants is the stored energy drop ~~from~~ $\frac{1}{4}$ of initial value.

$$U = \frac{Q^2}{2C} = \frac{q^2(t)}{2C} = \frac{\left(Q_{max}\, e^{-t/RC}\right)^2}{2C} = \frac{Q_{max}^2}{2C}\, e^{-2t/RC}$$

$$= U_0\, e^{-2t/RC}$$

$$\frac{1}{4}\cancel{U_0} = \cancel{U_0}\, e^{-2t/RC} \Rightarrow \ln 4 = 2t/RC$$

$$t = \frac{\ln 4}{2} RC = 0.693\, \tau$$

(c) after how many time constant the current reaches $\frac{1}{2}$ initial value.

$$I(t) = \left(-\frac{Q}{RC}\right) e^{-t/RC} = +I_0\, e^{-t/RC}$$

$$\frac{1}{2}\cancel{I_0} = \cancel{I_0}\, e^{-t/RC} \Rightarrow \frac{1}{2} = \frac{1}{e^{t/RC}}$$

$$\Rightarrow t = \ln 2\, RC = \ln 2\, \tau$$

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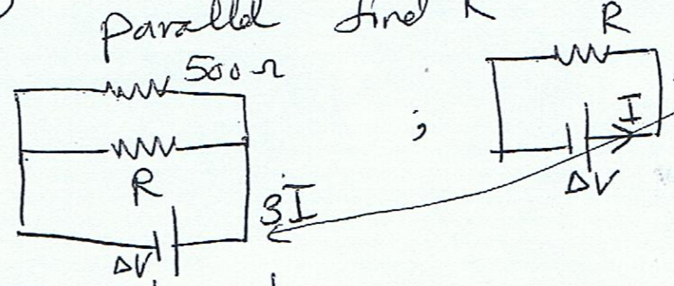
problems

(1): the current is trippled by adding 500 Ω to R in parallel find R

التيار يتضاعف ثلاث مرات بإضافة



$$\Delta V = IR = 3I R_{eq}$$

$$\cancel{I}\cancel{R} = 3\cancel{I} * \frac{500\cancel{R}}{500+R}$$

$$500 + R = 1500$$

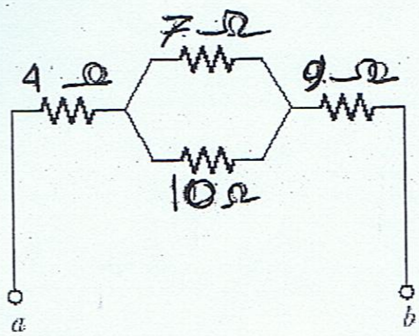
$$R = 1500 - 500$$

$$R = 1000\ \Omega$$

$$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{500}$$

$$R_{eq} = \frac{500R}{500+R}$$

2 (a) Find the equivalent resistance between points a and b in Figure . (b) A potential difference of 34.0 V is applied between points a and b. Calculate the current in each resistor.



(a) $R_p = \dfrac{1}{(1/7.00\ \Omega)+(1/10.0\ \Omega)} = 4.12\ \Omega$

$R_s = R_1 + R_2 + R_3 = 4.00 + 4.12 + 9.00 = \boxed{17.1\ \Omega}$

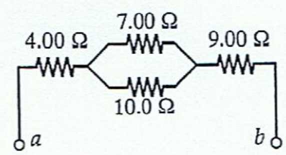


FIG. P28.6

(b) $\Delta V = IR$

$34.0\ \text{V} = I(17.1\ \Omega)$

$I = \boxed{1.99\ \text{A}}$ for $4.00\ \Omega$, $9.00\ \Omega$ resistors.

Applying $\Delta V = IR$, $(1.99\ \text{A})(4.12\ \Omega) = 8.18\ \text{V}$

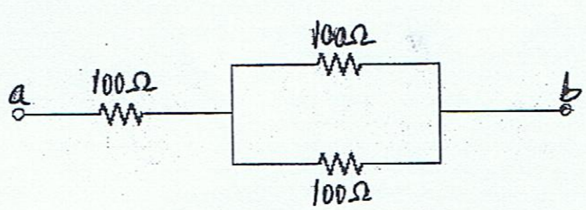
$8.18\ \text{V} = I(7.00\ \Omega)$

so $I = \boxed{1.17\ \text{A}}$ for $7.00\ \Omega$ resistor

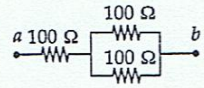
$8.18\ \text{V} = I(10.0\ \Omega)$

so $I = \boxed{0.818\ \text{A}}$ for $10.0\ \Omega$ resistor.

3 Three 100-Ω resistors are connected as shown in Figure below The maximum power that can safely be delivered to any one resistor is 25.0 W. (a) What is the maximum voltage that can be applied to the terminals a and b? For the voltage determined in part (a), what is the power delivered to each resistor? What is the total power delivered?



(a) Since all the current in the circuit must pass through the series $100\ \Omega$ resistor, $P = I^2R$



$$P_{max} = RI_{max}^2$$

so $$I_{max} = \sqrt{\frac{P}{R}} = \sqrt{\frac{25.0\text{ W}}{100\ \Omega}} = 0.500\text{ A}$$

$$R_{eq} = 100\ \Omega + \left(\frac{1}{100} + \frac{1}{100}\right)^{-1}\ \Omega = 150\ \Omega$$

$$\Delta V_{max} = R_{eq} I_{max} = \boxed{75.0\text{ V}}$$

(b) $P = I\Delta V = (0.500\text{ A})(75.0\text{ V}) = \boxed{37.5\text{ W}}$ total power

$$P_1 = \boxed{25.0\text{ W}}$$

$$P_2 = P_3 = RI^2(100\ \Omega)(0.250\text{ A})^2 = \boxed{6.25\text{ W}}$$

4 Two resistors connected in series have an equivalent resistance of 690 **Ω**. When they are connected in parallel, their equivalent resistance is 150 **Ω**. Find the resistance of each resistor.

Denoting the two resistors as x and y,

$$x + y = 690, \text{ and } \frac{1}{150} = \frac{1}{x} + \frac{1}{y}$$

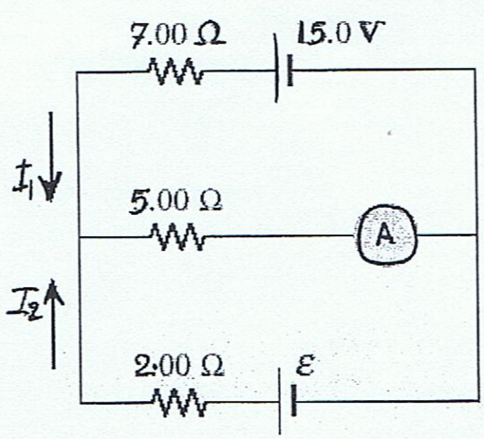
$$\frac{1}{150} = \frac{1}{x} + \frac{1}{690 - x} = \frac{(690 - x) + x}{x(690 - x)}$$

$$x^2 - 690x + 103\,500 = 0$$

$$x = \frac{690 \pm \sqrt{(690)^2 - 414\,000}}{2}$$

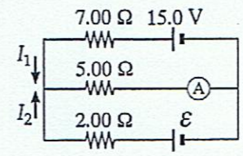
$$x = \boxed{470\ \Omega} \qquad y = \boxed{220\ \Omega}$$

5 The ammeter shown in Figure reads 2.00 A. Find I_1, I_2, and $\mathcal{E}$.



$$+15.0-(7.00)I_1-(2.00)(5.00)=0$$

$5.00 = 7.00I_1$ so $\boxed{I_1 = 0.714\text{ A}}$

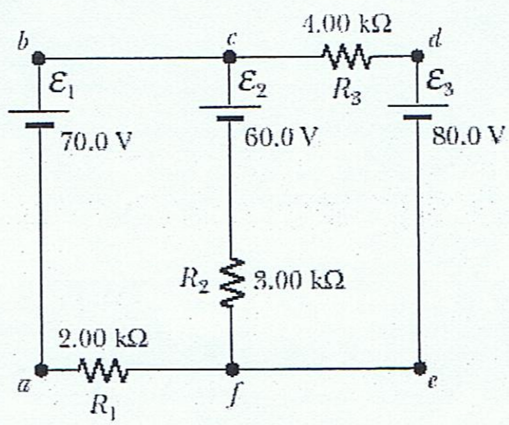


$I_3 = I_1 + I_2 = 2.00\text{ A}$

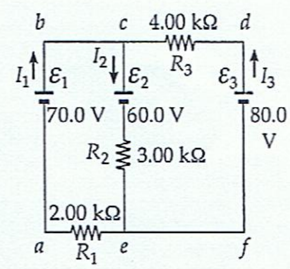
$0.714 + I_2 = 2.00$ so $\boxed{I_2 = 1.29\text{ A}}$

$+\varepsilon - 2.00(1.29) - 5.00(2.00) = 0$ $\boxed{\varepsilon = 12.6\text{ V}}$

6 Using Kirchhoff's rules, (a) find the current in each resistor in Figure - - - .. (b) Find the potential difference between points *c* and *f*. Which point is at the higher potential?



We name the currents I_1, I_2, and I_3 as shown.



[1] $70.0 - 60.0 - I_2(3.00\text{ k}\Omega) - I_1(2.00\text{ k}\Omega) = 0$

[2] $80.0 - I_3(4.00\text{ k}\Omega) - 60.0 - I_2(3.00\text{ k}\Omega) = 0$

[3] $I_2 = I_1 + I_3$

(a) Substituting for I_2 and solving the resulting simultaneous equations yields

$I_1 = \boxed{0.385\text{ mA}}$ (through R_1)

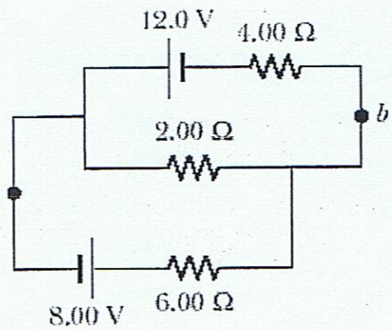
$I_3 = \boxed{2.69\text{ mA}}$ (through R_3)

$I_2 = \boxed{3.08\text{ mA}}$ (through R_2)

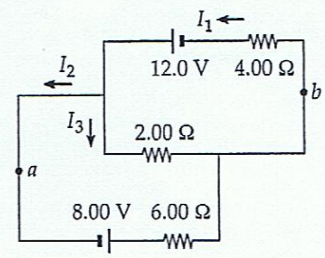
(b) $\Delta V_{cf} = -60.0\text{ V} - (3.08\text{ mA})(3.00\text{ k}\Omega) = \boxed{-69.2\text{ V}}$

$\boxed{\text{Point } c \text{ is at higher potential.}}$

7 For the circuit shown in Figure - . calculate (a) the current in the 2.00-Ω resistor and (b) the potential difference between points *a* and *b*.



We name the currents I_1, I_2, and I_3 as shown.



(a) $I_1 = I_2 + I_3$

Counterclockwise around the top loop,

$12.0\text{ V} - (2.00\ \Omega)I_3 - (4.00\ \Omega)I_1 = 0$.

Traversing the bottom loop,

$8.00\text{ V} - (6.00\ \Omega)I_2 + (2.00\ \Omega)I_3 = 0$

$I_1 = 3.00 - \frac{1}{2}I_3$, $I_2 = \frac{4}{3} + \frac{1}{3}I_3$, and $\boxed{I_3 = 909\text{ mA}}$.

(b) $V_a - (0.909\text{ A})(2.00\ \Omega) = V_b$

$V_b - V_a = \boxed{-1.82\text{ V}}$

8 A 2.00-nF capacitor with an initial charge of 5.10 μC is discharged through a 1.30-kΩ resistor. (a) Calculate the current in the resistor 9.00 μs after the resistor is connected across the terminals of the capacitor. (b) What charge remains on the capacitor after 8.00 μs? (c) What is the maximum current in the resistor?

(a) $I(t) = -I_0 e^{-t/RC}$

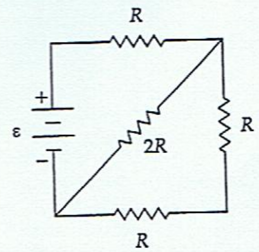
$$I_0 = \frac{Q}{RC} = \frac{5.10 \times 10^{-6}\text{ C}}{(1\,300\ \Omega)(2.00 \times 10^{-9}\text{ F})} = 1.96\text{ A}$$

$$I(t) = -(1.96\text{ A})\exp\left[\frac{-9.00 \times 10^{-6}\text{ s}}{(1\,300\ \Omega)(2.00 \times 10^{-9}\text{ F})}\right] = \boxed{-61.6\text{ mA}}$$

(b) $$q(t) = Qe^{-t/RC} = (5.10\ \mu\text{C})\exp\left[\frac{-8.00 \times 10^{-6}\text{ s}}{(1\,300\ \Omega)(2.00 \times 10^{-9}\text{ F})}\right] = \boxed{0.235\ \mu\text{C}}$$

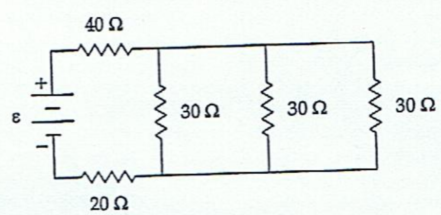
Chapter 28/ Direct Current Circuits

1. If $\varepsilon = 12$ V and $R = 3.0\ \Omega$, at what rate is thermal energy being generated in the $2R$-resistor shown?



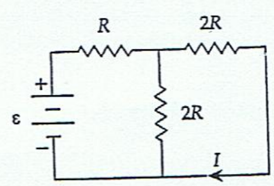
a. 12 W b. 24 W c. 6.0 W d. 3.0 W e. 1.5 W

2. If $\varepsilon = 20$ V, at what rate is thermal energy being generated in the 20-Ω resistor shown in the circuit?



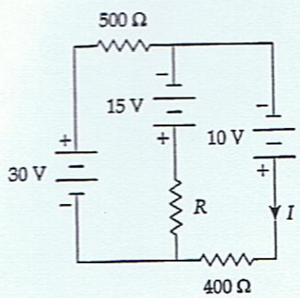
a. 6.5 W b. 1.6 W c. 15 W d. 26 W e. 5.7 W

3. In the figure shown, if $I = 0.50$ A and $R = 12\ \Omega$, determine ε.



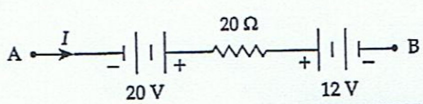
a. 12 V b. 24 V c. 30 V d. 15 V e. 6.0 V

4. In the figure, if $I = 30$ mA, determine the magnitude and sense (direction) of the current in the 500-Ω resistor.



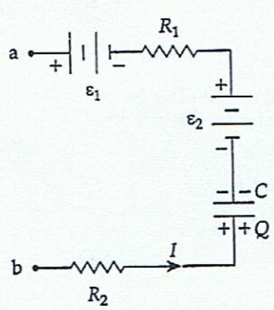
a. 56 mA left to right b. 56 mA right to left c. 48 mA left to right
d. 48 mA right to left e. 26 mA left to right

5. In the figure, if $I = 1.5$ A in the circuit segment shown, what is the potential difference
$V_B - V_A$?



a. +22 V b. −22 V c. −38 V d. +38 V e. +2.0 V

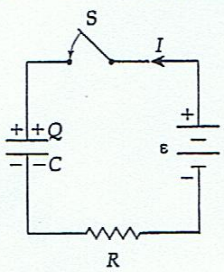
6. In the figure, if $\varepsilon_1 = 4.0$ V, $\varepsilon_2 = 12.0$ V, $R_1 = 4\ \Omega$, $R_2 = 12\ \Omega$, $C = 3\ \mu$F, $Q = 18\ \mu$C, and
$I = 2.5$ A, what is the potential difference $V_a - V_b$?



a. −30 V b. 30 V c. 5.0 V d. −5.0 V e. −1.0 V

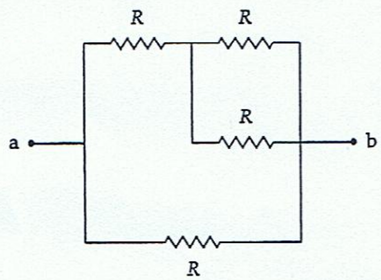
7. In an RC circuit, how many time-constants must elapse if an initially uncharged capacitor is to reach 80% of its final potential difference?
a. 2.2 b. 1.9 c. 1.6 d. 3.0 e. 5.0

8. In the figure, at $t = 0$ the switch S is closed with the capacitor uncharged. If C = 50 μF,
ε = 20 V, and R = 4.0 kΩ, what is the charge on the capacitor when I = 2.0 mA?



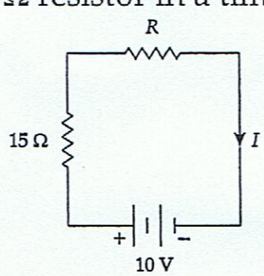
a. 360 μC b. 480 μC c. 240 μC d. 600 μC e. 400 μC

9.In the figure, if R = 30 Ω, what is the equivalent resistance between points a and b?



a. 27 Ω b. 21 Ω c. 24 Ω d. 18 Ω e. 7.5 Ω

10. A 10-V battery is connected to a 15-Ω resistor and an unknown resistor R, as shown. The current in the circuit is 0.40 A. How much heat is produced in the 15-Ω resistor in a time of 2.0 min?



a. 0.40 kJ b. 0.19 kJ c. 0.29 kJ d. 0.72 kJ e. 0.80 kJ

فيزياء ١٠ صفر ١٨/٤

chapter 29 : Magnetic fields مجالات مغناطيسية ①

* magnetic poles (الاقطاب المغناطيسية) like point charges but they are always in pairs (مزدوجة)
  الاقطاب المغناطيسية لا تأتي منفصلة عن بعضها

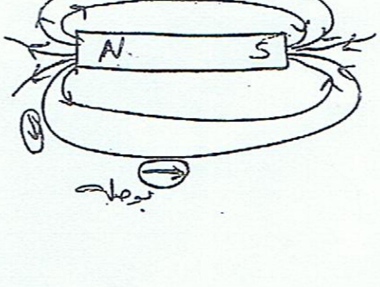
* there is a strong relationship between magnetizem and electricity (the current has a magnetic field around it and crossing the magnetic field lines produce electric current)

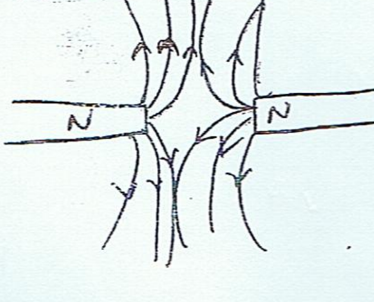
* one can magnetize an magnetized piece of iron by strocking it with a magnet (مثل عليه الدلك الكهربائي)

the magnetic field

the magnetic field surrounds any magnetic substance
المجال المغناطيسي هو ما يحيط بالمغناطيس

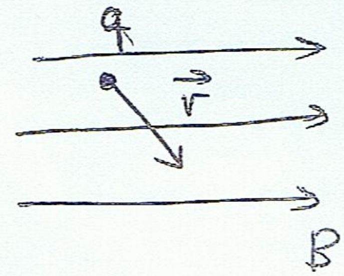
$\vec{B}$: represents the magnetic field (شدة المجال المغناطيسي)
the direction of the magnetic field at any location is the direction in which the compass needle (ابرة البوصلة) points at
(إتجاه المجال المغناطيسي عند أية نقطة هو الاتجاه الذي تؤشر عليه ابرة البوصلة)





a moving charge through a magnetic field

the magnetic force exerted on the charged particle is $\propto q, v, B$

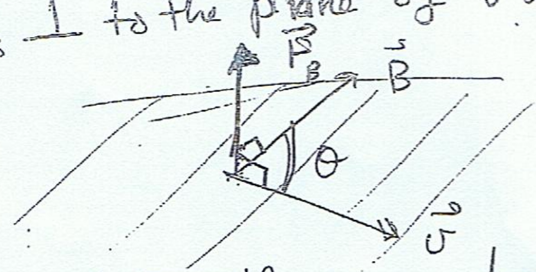


θ بين B و $\hat{v}$

$$\boxed{|\vec{F}_B| = q v B \sin\theta}$$

θ: the angle between $(\vec{B})$ and $(\vec{v})$

$\vec{F}_B = q\, \vec{v} \times \vec{B}$; $\vec{F}_B \perp$ to the plane of v and B



$\vec{F}_B$ on a positive charge $= -\vec{F}_B$ on the negative charge if both are moving in the same direction

differences between electric field and magnetic fields

\# $\vec{F}_e$ acts in the direction of $\vec{E}$
F_B acts $\perp$ to $\vec{B}$

\# F_e on charges (moving or not), but $\vec{F}_B$ on moving charges only.

\# F_e does work in displacing a charged particle but
F_B __do NO work__ on a charged displaced.

$$W = \int \vec{F}_B \cdot d\vec{s} \;;\; \vec{F}_B \perp d\vec{s}$$
$$= 0$$

* when a charged particle moves with $\vec{v}$ through the magnetic field it can change the direction of $\vec{v}$ but cannot change the speed or the kinetic energy of the particle.

$$B = \left|\frac{F_B}{qv}\right| = \frac{N}{C.\,m/s} = \frac{N}{A.m} = \boxed{\text{Tesla}}$$

$\boxed{1T = 10^{4}\ \text{Guass}}$;. Guass وحدة اخرى للمجال المغناطيسي.

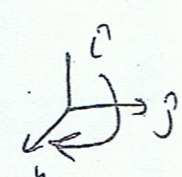
Note: strong supermagnets $\sim 30T$
strong lab magnet $\sim 2T$
earth magnetic field $0.5 \times 10^{4}\,T$

<u>example</u>: an electron moves with speed of $8 \times 10^{6}\,m/s$ along the x-axes, a magnetic field $B = 0.025T$ directed 60° to the x-axes find F_B

$\vec{v} = 8\times10^{6}\,\hat{i}$
$\vec{B} = 0.025\cos60^\circ\,\hat{i} + 0.025\sin60^\circ\,\hat{j}$

$$\vec{F}_B = q\,\vec{v}\times\vec{B} = -1.6\times10^{-19} * 8\times10^{6} * 0.025\left(\hat{i}\times(\cos60\,\hat{i} + \sin60\,\hat{j})\right)$$
$$= -1.6\times10^{-19} * 8\times10^{6} * 0.025\,(\sin60\ \hat{k})$$
$$= 2.8\times10^{-14}\,(-\hat{k})$$ داخل في الصفحة

Note $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = 0$
$\hat{i}\times\hat{j} = \hat{k}$,
$\hat{j}\times\hat{k} = \hat{i}$, $\hat{k}\times\hat{i} = \hat{j}$,



($\hat{j}\times\hat{i} = -\hat{k}$ ...) والعكس باتجاه سالب

example: an electron is moving along the positive x-axes $\perp$ to B experiences a magnetic deflection in (يعاني من انحراف مغناطيسي) the negative y-direction. what is the direction of $\vec{B}$

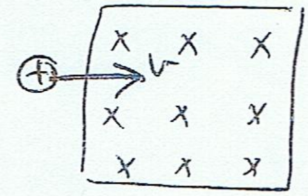
$$\vec{F}_B = q\,\vec{v}\times\vec{B} = -qvB\,(\hat{i}\times \underset{B\text{ أي}}{\underline{\quad}}) = -\hat{j}$$

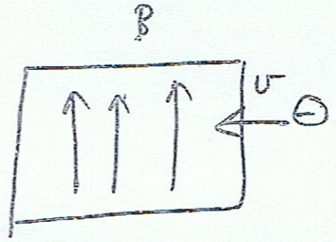
$\hat{i}\times$ B أي $= \hat{j}$ ... $\hat{i}\times\hat{k}$ ...

problem: Determine the initial direction of the deflection of charged particles as they enter the magnetic fields as shown.

$(\hat{i}, \hat{j}, \hat{k})$

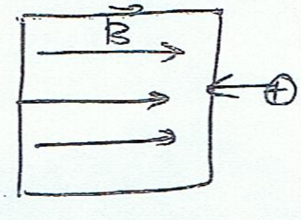
$$\vec{F}_B = q\vec{v}\times\vec{B} = {}^{+}qvB(\hat{i}\times -\hat{k})$$
$$= qvB(-\times -\hat{j})$$
$$= qvB(\underline{\hat{j}}) \text{ up}$$

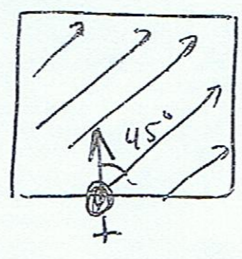




$$F_B = -qvB(-\hat{i}\times\hat{j}) = qvB(\hat{i}\times\hat{j})$$
$$= qvB(\hat{k})$$
out of page.

$$F_B = {}^{+}qvB(-\hat{i}\times +\hat{i}) = 0$$
No deflection.





$$F_B = {}^{+}qvB(\hat{j}\times(\cos 45\,\hat{i} + \sin 45\,\hat{j}))$$
$$= qvB\cos 45^\circ\,(\hat{j}\times\hat{i}) = qvB\cos 45^\circ(-\hat{k})$$
into the page

داخل الصفحة

problem: a proton moves with a velocity of $\vec{v} = (2\hat{i} - 4\hat{j} + \hat{k})\,m/s$ in a region in which the magnetic field is $\vec{B} = (\hat{i} + 2\hat{j} - 3\hat{k})\,T$ what is the magnitude of the magnetic force this charge experiences.

$$\vec{F}_B = q\vec{v}\times\vec{B}$$; first find $\vec{v}\times\vec{B}$

$\vec{v} = 2\hat{i} - 4\hat{j} + \hat{k}$; $\vec{B} = \hat{i} + 2\hat{j} - 3\hat{k}$

$$\vec{v} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -4 & 1 \\ 1 & 2 & -3 \end{vmatrix} = \hat{i}(-4*-3 - 1*2) - \hat{j}(2*-3 - 1) + \hat{k}(2*2 - -4*1)$$

$$= 10\hat{i} + 7\hat{j} + 8\hat{k}$$

$$|v \times B| = \sqrt{(10)^2 + (7)^2 + (8)^2} = 14.6 \; T.m/s$$

$$\vec{F}_B = q\,\vec{v} \times \vec{B} = 1.6 \times 10^{-19}(10\hat{i} + 7\hat{j} + 8\hat{k})$$

$$|\vec{F}_B| = 2.34 \times 10^{-18} \; N.$$

problem : a proton travels with speed of 3×10^6 m/s at an angle $37°$ with the direction of a magnetic field of $0.3T$ in the y-direction.

$|\vec{F}_B|$

(a) magnitude of magnetic force

$$F_B = qvB\sin\theta = 1.6 \times 10^{-19} * 3 * 10^6 * 0.3 * \sin 37°$$

$$= 8.67 \times 10^{-14} \; N.$$

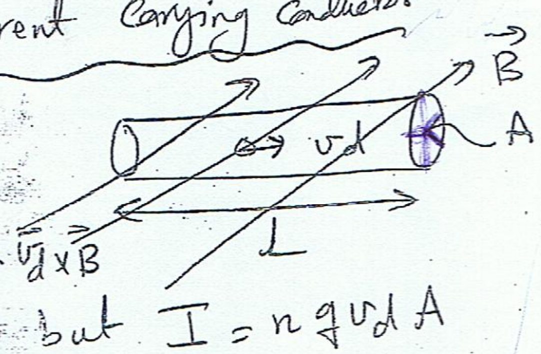
(b) the acceleration

$$F_B = ma \Rightarrow a = F/m = \frac{8.67 * 10^{-14}}{1.67 \times 10^{-27}}$$

$$= 5.19 \times 10^{13} \; m/s^2$$

Magnetic force on a current carrying conductor

$\vec{F}_B = q\vec{v}_d \times \vec{B}$ for any charge



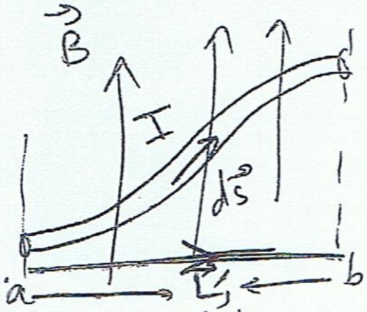
$\vec{F}_{B(Tot)} = $ # of charge carriers $* \, q * \vec{v}_d \times \vec{B}$

$= n * L * A * q\,\vec{v}_d \times \vec{B}$ but $I = n q v_d A$

$\boxed{\vec{F}_B = I\,\vec{L} \times \vec{B}}$ on all the wire.

(6)

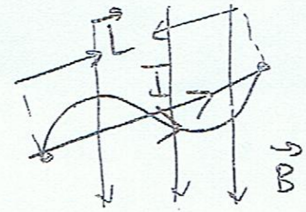
for arbitrary shaped wire of a uniform cross section :



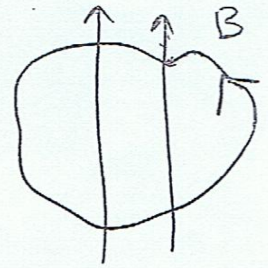
$$\vec{F}_B = I \int_a^b d\vec{s} \times \vec{B} .$$

$$= I \vec{L}' \times \vec{B} \; , \quad \vec{L}' :$$ اقصر مسافة بين النقطتين

example



$$\vec{F}_B = I \vec{L} \times \vec{B}$$



ring of current $\quad \vec{F}_B = 0$

تيار يسري في حلقة مغلقة
داخل مجال مغناطيسي تكون
القوه المغناطيسيه عليها صفراً .

problem : a wire 2.8 m in length carries a current
of 5 A in a region where a uniform magnetic
field has a magnitude of 0.39T. Calculate the magnitude
of the magnetic force on the wire assuming the
angle between the magnetic field and the current
is (a) 60° (b) 90° (c) 120°

$$\vec{F}_B = I \vec{L} \times \vec{B} \; ; \; |F_B| = I L B \sin\theta$$

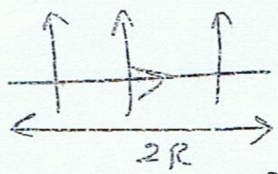
(a) $F_B = 5 * 2.8 * 0.39 \sin 60 = 4.73\ N$

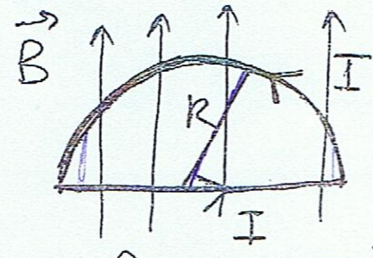
(b) $F_B = 5 * 2.8 * 0.39 \sin 90 = 5.46\ N$

(c) $F_B = 5 * 2.8 * 0.39 \sin 120^\circ = 4.73\ N$

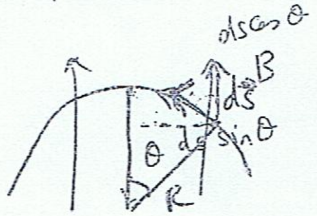
example : find the force on a hemicircle of radius ⑦
R through a magnetic field in y-direction نصف دائرة

نقسمها الى جزئية





$\vec{F}_{B_1} = I\vec{L} \times \vec{B} = ILB(\hat{i} \times \hat{j}) = ILB\,\hat{k} = 2RIB\,\hat{k}$
(out of page)



$\vec{F}_{B_2} = I\int d\vec{s} \times \vec{B} = I\int ds\,B\sin\theta(-\hat{i} \times \hat{j})$

but $ds = R\,d\theta$

$$\vec{F}_{B_2} = IBR\int_0^{\pi} \sin\theta\, d\theta\,(-\hat{k})$$

$$= IBR\cos\theta\Big|_0^{\pi}(\hat{k}) = IBR(-1-1)(\hat{k})$$

$$= \boxed{-2IBR}(\hat{k})$$

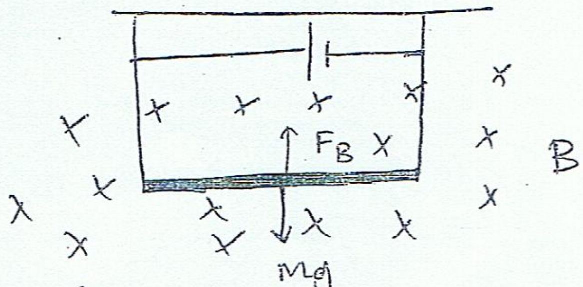
Note that $\vec{F}_{B_1} + \vec{F}_{B_2} = \vec{F}_B = 2RIB\hat{k} - 2RIB\hat{k}$
$= 0$

F_B(on closed loop) = Zero

problem : A conductor suspended by two flexible wires as shown in the figure. the wire has mass per unit length of 0.04 kg/m what current must exist in the conductor in order for the tension in the supporting wires to be zero when the magnetic field is 3.6 T into the page?

$\vec{B} = 3.6T(-\hat{k})$

$\frac{m}{L} = 0.04 \ kg/m$



$F_B = mg = 0.04 \ell g$

$0.04 \cancel{\ell} g = I \cancel{\ell} B \Rightarrow I = \frac{0.04 g}{B} = \frac{0.04 * 9.8}{3.6} = 0.109 A.$

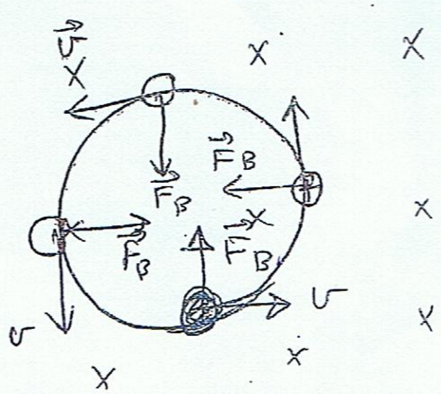
$F_B = I \vec{L} \times \vec{B}$

$\vec{F}_B(\hat{j}) = (\otimes \times -\hat{k}) = (ILB)\hat{i} \times -\hat{k} = -(-\hat{j}) = \hat{j}$

$\vec{I} = I \hat{i}$ الاتجاه هو x-axis الموجب

Motion of a charged particle in a uniform magnetic field

magnetic force $\vec{F}_B \perp \vec{v}$, work done by the magnetic force $= 0$



Consider a positively charged particle moving in a uniform $\vec{B}$ with initial velocity $\vec{v} \perp \vec{B}$

$\vec{B} = B(-\hat{k})$

$\vec{v} = v\hat{i}$

$F_B = q\vec{v} \times \vec{B} = |F_B|(\hat{j})$ ينحرف باتجاه y

وتستمر بالانحراف حتى تسير مسار دائري.

$|F_B| = qvB = \frac{mv^2}{r} \Rightarrow r = \frac{mv^{\cancel{2}}}{q\cancel{v}B} = \frac{mv}{qB}$

r: radius of circulation ; angular speed $\omega = \frac{v}{r}$

نصف قطر الحركة الدائرية

$\omega = \frac{qB}{m}$ (angular speed)

problem: an electron moves in a circular path $\perp$ to a constant magnetic field of magnitude 1mT. the angular momentum of the electron about the center of the circle is 4×10^{-25} J.S. determine the radius of the circular path and the speed of the electron (10)

angular momentum $\boxed{L = mvR} = 4\times10^{-25}$ J.S
الزخم الزاوي

$\vec{B} = 1mT = 1\times10^{-3}$ T

(a) $qvB = \frac{mv^{2}}{R} \Rightarrow (qRB = mv) * R$

$qR^2B = mvR = L = 4\times10^{-25}$

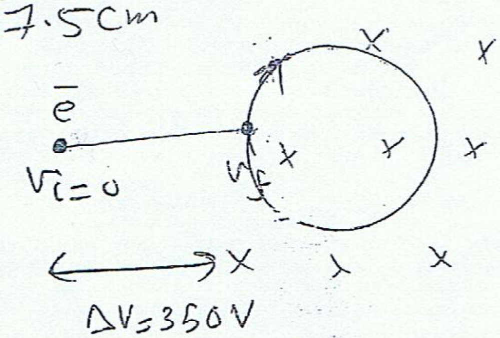
$R = \sqrt{\frac{L}{qB}} = \sqrt{\frac{4\times10^{-25}}{1.6\times10^{-19} * 1\times10^{-3}}} = 0.05$ m $= 5$ cm.

(b) $mv = qRB \Rightarrow v = \frac{qBR}{m} = \frac{1.6\times10^{-19} * 1\times10^{-3} * 5\times10^{-2}}{9.11\times10^{-31}}$

$v = \frac{qBR}{m}$, $\omega = \frac{qB}{m}$, $T = \frac{2\pi m}{qB}$

$= 8.78\times10^{6}$ m/s.

example: an electron beam accelerated through 350V then enter a magnetic field $\perp \vec{v}$. the radius of the path = 7.5cm



لمعرفة السرعة النهائية نطبق حفظ
الطاقة أو معادلات الحركة

$|e|\Delta V = \frac{1}{2} m_e v^2$

$\Rightarrow v = \sqrt{\frac{2|e|\Delta V}{m_e}}$

or $v_f^2 = \cancel{v_i^2}^{\,0} + 2a\Delta X$; $a = \frac{F_e}{m_e} = \frac{qE}{m_e} = \frac{q\Delta V}{\Delta X m_e}$

$= 2 * \frac{q\Delta V}{m_e} \Rightarrow v_f = \sqrt{\frac{2 * q\Delta V}{m}}$

(11)

$$v_f = v = \sqrt{\frac{2 * 1.6 \times 10^{-19} * 350}{9.11 \times 10^{-31}}} = 1.11 \times 10^{7}\ m/s.$$

(a) $\cancel{q}\cancel{v} B = \frac{m v^{\cancel{2}}}{r} \Rightarrow B = \frac{m v}{r q} = \frac{9.11 \times 10^{-31} * 1.11 \times 10^{7}}{7.5 \times 10^{-2} * 1.6 \times 10^{-19}} = 8.4 \times 10^{-4}\ T$

(b) $\omega = \frac{v}{r} = \frac{1.11 \times 10^{7}}{7.5 \times 10^{-2}} = 1.5 \times 10^{8}\ rad/s$

(c) $T = \frac{2\pi}{\omega} = \frac{2\pi}{1.5 \times 10^{8}} = 43 \times 10^{-9}\ s = 43\ n\,sec$

Chapter 29/ suggested problems

1.An electron has a velocity of 6.0 x 10^6 m/s in the positive x direction at a point where the magnetic field has the components, B_x = 3.0 T, B_y = 1.5 T and B_z = 2.0 T. What is the magnitude of the acceleration of the electron at this point?

a. 2.1 x 10^{18} m/s^2 b. 1.6 x 10^{18} m/s^2 c. 2.6 x 10^{18} m/s^2 4.12×10^{18}
d. 3.2 x 10^{18} m/s^2 e. 3.7 x 10^{18} m/s^2

2.An electron moving in the positive x direction experiences a magnetic force in the positive z direction. If B_x = 0, what is the direction of the magnetic field?

a. negative y direction b. positive y direction c. negative z direction
d. positive z direction e. negative x direction

3.A 2.0 C charge moves with a velocity of (2.0i + 4.0j + 6.0k) m/s and experiences a magnetic force of (4.0i – 20j + 12k) N. The x component of the magnetic field is equal to zero. Determine the y component of the magnetic field.

a. –3.0 T b. +3.0 T c. +5.0 T d. –5.0 T e. +6.0 T

4.A particle (mass 6.0 mg) moves with a speed of 4.0 km/s and a direction that makes an angle of 37_ above the positive x axis in the xy plane. A magnetic field of (5.0i) mT produced an acceleration of (8.0k) m/s^2. What is the charge of the particle?

a. –4.8 μC b. 4.0 μC c. –4.0 μC d. 4.8 μC e. –5.0 μC

5. A positively charged particle has a velocity in the negative z direction at point P. The magnetic force on the particle at this point is in the negative y direction. Which one of the following statements about the magnetic field at point P can be determined from this data?

a. B_x is positive. b. B_z is positive. c. B_y is negative d. B_y is positive.
e. B_x is negative.

6.A charged particle (mass = M, charge = Q > 0) moves in a region of space where the magnetic field has a constant magnitude of B and a downward direction. What is the magnetic force on the particle at an instant when it is moving horizontally toward the north with a speed V?

a. QVB toward the east b. Zero c. QVB toward the west d. QVB upward
e. QVB toward the south

7. A straight wire carries a current of 40 A in a uniform magnetic field (magnitude = 80 mT). If the force per unit length on this wire is 2.0 N/m, determine the angle between the wire and the magnetic field.

a. either 39° or 141° b. either 25_ or 155° c. either 70° or 110°
d. either 42° or 138° e. either 65° or 115°

8. A segment of wire carries a current of 25 A along the x axis from $x = -2.0$ m to $x = 0$ and then along the z axis from $z = 0$ to $z = 3.0$m. In this region of space, the magnetic field is equal to 40 mT in the positive z direction. What is the magnitude of the force on this segment of wire? B

a. 1.0 N b. 5.0 N c. 2.0 N d. 3.6 N e. 3.0 N

9. A straight wire of length L carries a current I in the positive z direction in a region where the magnetic field is uniform and specified by $B_x = 3B$, $B_y = -2B$, and $B_z = B$, where B is a constant. What is the magnitude of the magnetic force on the wire?

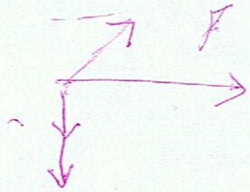
a. 3.2 ILB b. 5.0 ILB c. 4.2 ILB d. 3.6 ILB e. 1.0 ILB

10. A 500-eV electron and a 300-eV electron are trapped in a magnetic field. What is the ratio of the radii of their orbits?

a. 2.8 b. 1.7 c. 1.3 d. 4.0 e. 1.0

11. A charged particle ($m = 5.0$ g, $q = -70\ \mu$C) moves horizontally at a constant speed of 30 km/s in a region where the free fall gravitational acceleration is 9.8 m/s^2 downward, the electric field is 700 N/C upward, and the magnetic field is perpendicular to the velocity of the particle. What is the magnitude of the magnetic field in this region?

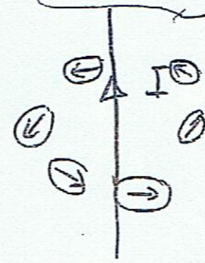
a. 47 mT b. zero c. 23 mT d. 35 mT e. 12 mT



F $mg = qvB$

Chapter 30 :- Sources of the magnetic fields ①

(تجربة أورستد)

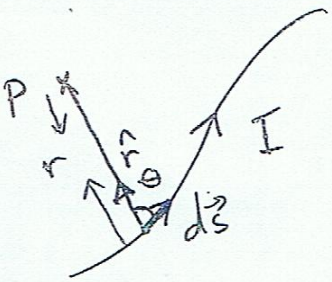


the mathmatical expression that give the magnetic field at any point in space interms of the electric current that produces the field is

$$B = \frac{\mu_0 I}{4\pi} \int \frac{d\vec{s} \times \vec{r}}{r^2}$$

الشيار الكهربائي ينتج مجال مغناطيسي حوله .

نستخدم ابهام اليد اليمنى ليكون اتجاه التيار واتجاه حركة الاصابع هي اتجاه المجال المغناطيسي .



μ_0 : the permeability of free space

النفاذيه المغناطيسيه.

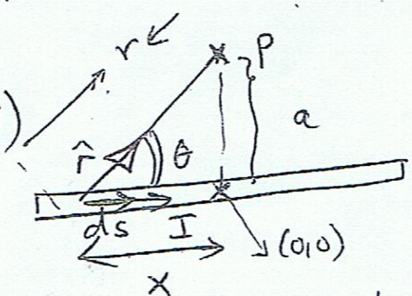
$$\mu_0 = 4\pi \times 10^{-7} \; T.m/A$$

<u>example</u> the magnetic field surrounding a thin streight conductor

$$d\vec{s} \times \vec{r} = dx\,\hat{i} \times (|\hat{r}|\cos\theta\,\hat{i} + |\hat{r}|\sin\theta\,\hat{j})$$

$$= dx\,\hat{i} \times \sin\theta\,\hat{j}$$

$$= dx \sin\theta\,(\hat{k})$$



; Note $|\hat{r}| = 1$ (unit vector)

$$B = \frac{\mu_0 I}{4\pi} \int \frac{dx \sin\theta}{r^2} \;\hat{k}$$

; $\sin\theta = \frac{a}{r} \;\; r = \frac{a}{\sin\theta}$

$\tan\theta = a/{-x}$

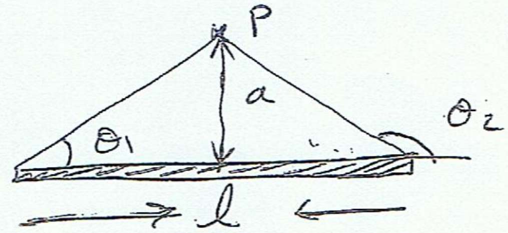
$x = \frac{a}{\tan\theta} = -a\cot\theta$

$dx = a\csc^2\theta\, d\theta$

$$= \frac{\mu_0 I}{4\pi} \int \frac{a\csc^2\theta\, d\theta \sin\theta}{a^2\csc^2\theta}$$

$$B = \frac{\mu_0 I}{4\pi} (\cos\theta_1 - \cos\theta_2)$$

$$\boxed{\vec{B} = \frac{\mu_0 I}{4\pi a}(\cos\theta_1 - \cos\theta_2)}$$



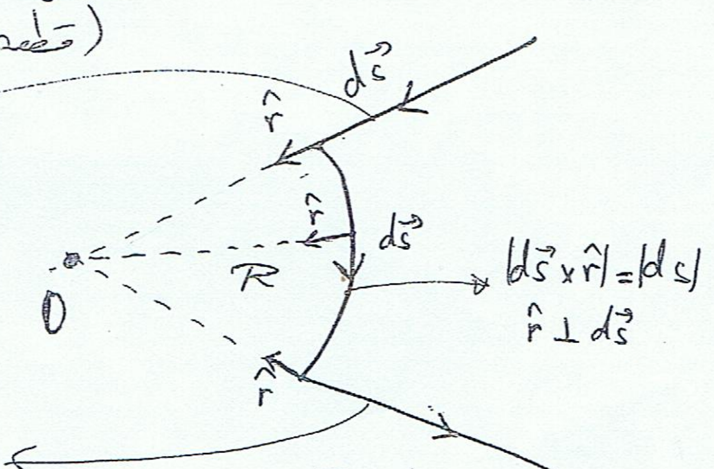
* if the line is too long

$\theta_1 \sim = 0$; $\theta_2 \simeq 180^\circ$

$$\vec{B} = \frac{\mu_0 I}{4\pi a}(\cos 0 - \cos\pi) = \boxed{\frac{\mu_0 I}{2\pi a} = \vec{B}}$$ for long wire

<u>example</u>: the magnetic field due to a curved wire segment (قطعة)

$d\vec{s} \times \hat{r} = 0$



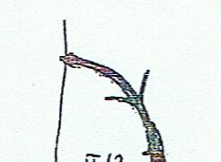
$d\vec{s} \times \hat{r} = 0$

$$B = \int \frac{\mu_0 I}{4\pi}\frac{ds}{R^2} = \frac{\mu_0 I}{4\pi R^2} * S = \frac{\mu_0 I}{4\pi R^{\cancel{2}}} R\theta$$

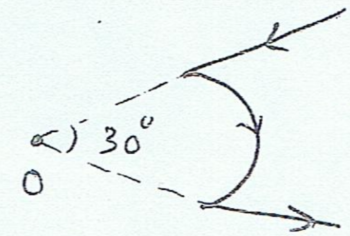
$$\boxed{B = \frac{\mu_0 I}{4\pi R}\theta}$$ θ interms of π

المجال المغناطيسي عند النقطة O

$$B_0 = \frac{\mu_0 I}{4\cancel{\pi} R} * \frac{\cancel{\pi}}{2} = \frac{\mu_0 I}{8R}$$



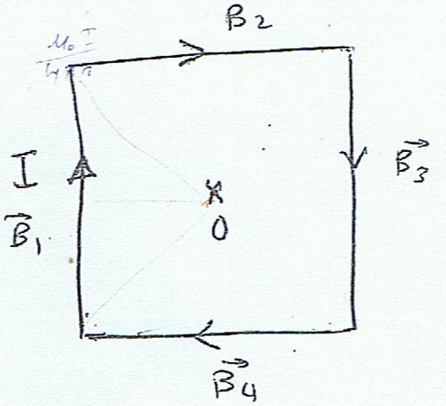
problem : find the magnetic field at point P in the figure if $\theta = 30^\circ$; $R = 0.6\,m$; $I = 3A$ (3)

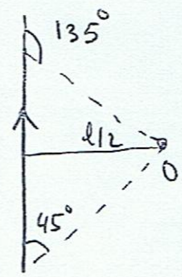


$$B = \frac{\mu_0 I}{4\pi R}\,\theta = \frac{\mu_0 * 3}{\cancel{4\pi} * 0.6} * \frac{\cancel{\pi}}{6}$$

$$= \frac{4\pi \times 10^{-7} * 3}{0.6 * 6} = \frac{4 * 3.14 \times 10^{-7}}{0.6 * 6}\,T = 2.61 \times 10^{-7}\,T$$

problem find the magnetic field at a center of a squre of edge $l = 0.4\,m$ and electric current $= 10A$ as in the figure.



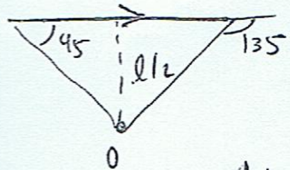


$$B_1 = \frac{\mu_0 I}{4\pi a}(\cos 45^\circ - \cos 135^\circ)$$

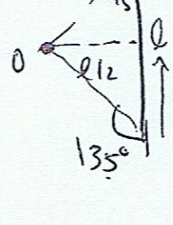
$$= \frac{\mu_0 I}{2\,4\pi \frac{l}{2}}(0.7 + 0.7)$$

$$\vec{B}_1 = \frac{1.4\,\mu_0 I}{2\pi l}\,(-\hat{k})$$

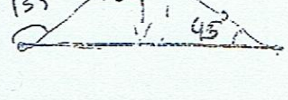
$$|B_1| = |B_2| = |B_3| = |B_4| = 28.3\,\mu T$$



$$\vec{B}_2 = \frac{1.4\,\mu_0 I}{2\pi l}\,(-\hat{k})$$



$$\vec{B}_3 = \frac{1.4\,\mu_0 I}{2\pi l}\,(-\hat{k})$$



$$\vec{B}_4 = \frac{1.4\,\mu_0 I}{2\pi l}\,(-\hat{k})$$

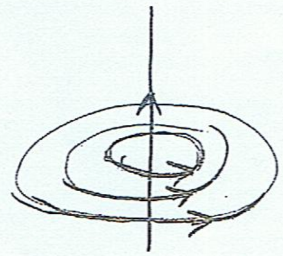
$$\vec{B}_{tot} = 4 * \frac{1.4\,\mu_0 I}{2\pi l}\,(-\hat{k}) = 24.7\,\mu T$$

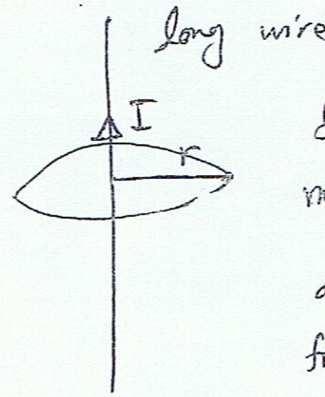
Amper's Law

$$\oint B \cdot dl = B \oint dl = \mu_0 I$$

$$= B * 2\pi r = \mu_0 I$$

$$\boxed{B = \frac{\mu_0 I}{2\pi r}}$$





find the magnetic field at point (r) from the wire.

* ملاحظة: إذا كان السلك سميك نحسب إيجاد المجال المغناطيسي في الداخل وفي الخارج كما في المثال الآتي.

example a long straight wire of radius R carries a steady state current I_0 uniformly distributed through the cross section of wire. Calculate $\vec{B}$ at a distance r from the center of the wire

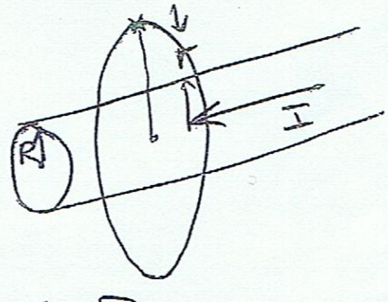
(a) $r \geq R$ (b) $r < R$

Amperes Law

$$B * l = \mu_0 I$$

l: line around the wire

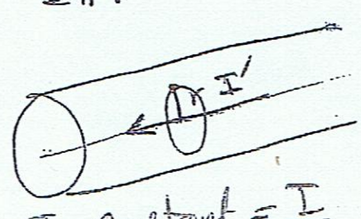
I: current inside the line.



$$B * 2\pi r = \mu_0 I_0 \Rightarrow B = \frac{\mu_0 I_0}{2\pi r}$$

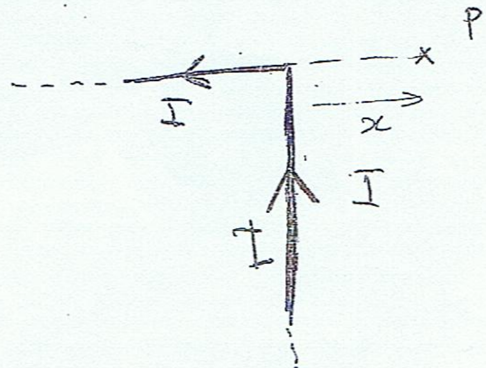
$$B * l = \mu_0 I' \; ; \; \frac{I_0}{A_0} = \frac{I'}{A'}$$

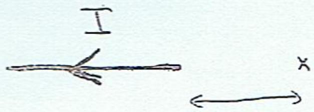
$$B = \frac{\mu_0 I}{2\pi r} \; ; \; \frac{I_0}{\pi R^2} = \frac{I'}{\pi r^2}$$



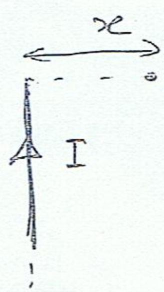
$J = \text{constant} = \frac{I}{A}$

problem: find the magnetic field at the point P in the figure. (5)





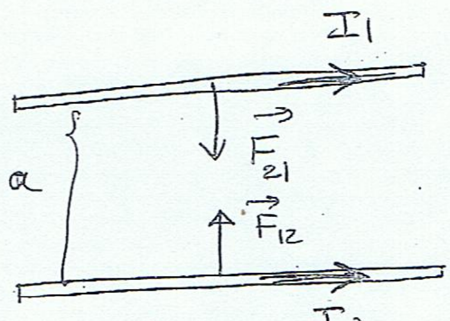
the magnetic field from this part = 0



هذا السلك نصف لانهائي

$$B = \frac{1}{2}\left(\frac{\mu_0 I}{2\pi x}\right) \text{ into the paper}$$

* the magnetic force between two parallel conductors



$B_1 = \frac{\mu_0 I_1}{2\pi a}$

$B_2 = \frac{\mu_0 I_2}{2\pi a}$

$\vec{F}_{21} = I_1 \ell \times \vec{B}_2 = I_1 \ell B_2 (-\hat{j})$

$B_2 = \frac{\mu_0 I_2}{2\pi a}$. at the location of I_1

$\vec{F}_{21} = \frac{\mu_0 I_1 I_2}{2\pi a} \ell \; (-\hat{j})$

$\frac{\vec{F}_{21}}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a} (-\hat{j})$

$$\boxed{\frac{|\vec{F}_{21}|}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}}$$

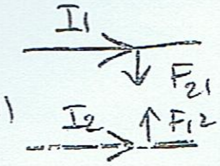
* $\vec{F}_{12} = I_2 \ell \times \vec{B}_1 = I_2 \ell B_1 (\hat{j})$

$\vec{B}_1$ (at the location I_2) $= \frac{\mu_0 I_1}{2\pi a} (\hat{j})$

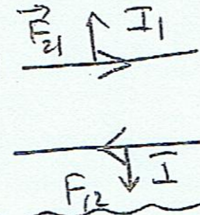
في موقع

$\frac{\vec{F}_{12}}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a} (\hat{j})$ or $\boxed{\frac{|\vec{F}_{12}|}{\ell} = \frac{|\vec{F}_{21}|}{\ell} = \frac{\mu_0 I_1 I_2}{2\pi a}}$

Note: السيارات بنفس الاتجاه القوة تجاذب



في الحالتين: $\vec{F}_{12} = -\vec{F}_{21}$ فعل ورد فعل



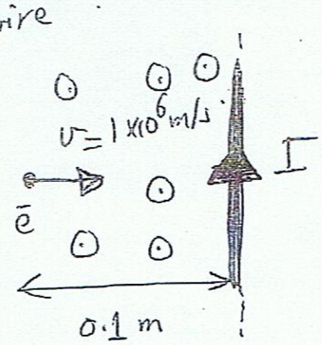
التيارات متعاكسة في الاتجاه (تنافر)

سؤال امتحان سابق: a charge q is moving toward a wire carrying current I. find the magnetic force on the charge when it was at $r = 10$ cm from the wire.

$q = 1.6\times10^{-19}$ شحنة الالكترون

$I = 2$ A

$v = 1\times10^{6}$ m/s



$$B = \frac{\mu_0 I}{2\pi r}(\hat{k})$$

$$\vec{F}_B = q\vec{v}\times\vec{B}$$

$$= -evB(\hat{i}\times\hat{k}) = evB(\hat{j}) = \frac{ev\mu_0 I}{2\pi r}(\hat{j})$$

$$\vec{F}_B = \frac{1.6\times10^{-19} * 1\times10^{6} * \cancel{4\pi}^{2}\times10^{-7} * 2}{\cancel{2\pi} * 0.1}(\hat{j})$$

$= - - -$

problem: two long parallel conductors seperated (مفصولة) by 10cm. the current in the same direction

(a) what is the magnetude of the magnetic field created by I_1 acting on I_2 location (موقع) where $I_1 = 5$ A, $I_2 = 8$ A

$$B_1 = \frac{\mu_0 I_1}{2\pi a} = \frac{4\pi\times10^{-7} * 5}{2\pi * 0.1} = 1\times10^{-5}\ T$$

(b) the force per unit length exerted on I_2 by I_1

$$|\vec{F}_{21}| = |I_2\vec{l}\times\vec{B}_1| = I_2 l B_1 \Rightarrow \frac{F_{21}}{l} = I_2 * B_1$$

$$\frac{F_{21}}{l} = 8\times10^{-5}\ N/m$$

(c) what is the magnetude of the magnetic field created by I_2 acting on I_1 ~~location~~ (موضع) ⑦

$$B_2 = \frac{\mu_0 I_2}{2\pi a} = \frac{4\pi \times 10^{-7} * 8}{2\pi * 0.1} = 1.6 \times 10^{-5}\ T$$

(d) the force per unit length exerted on I_1 by wire ②

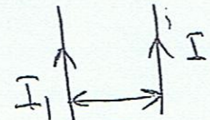
$$|\vec{F}_{12}| = I_1 |\vec{l} \times \vec{B}_2| = I_1\, l\, B_2$$

$$\frac{F_{12}}{l} = I_1 * B_2 = 5 * 1.6 \times 10^{-5} = 8 \times 10^{-5}\ N/m$$

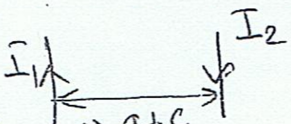
problem :- In the figure the current in the long straight wire is $I_1 = 3\ A$ and the wire lies in the plane of the rectangular loop, which carries the current $I_2 = 10\ A$. the dimentions $c = 0.1\ m$, $a = 0.15\ m$ and $l = 0.45\ m$. find the magnetude and direction of the net force exerted on the loop by the magnetic field created by the wire.

$$\vec{F}_{tot} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4$$

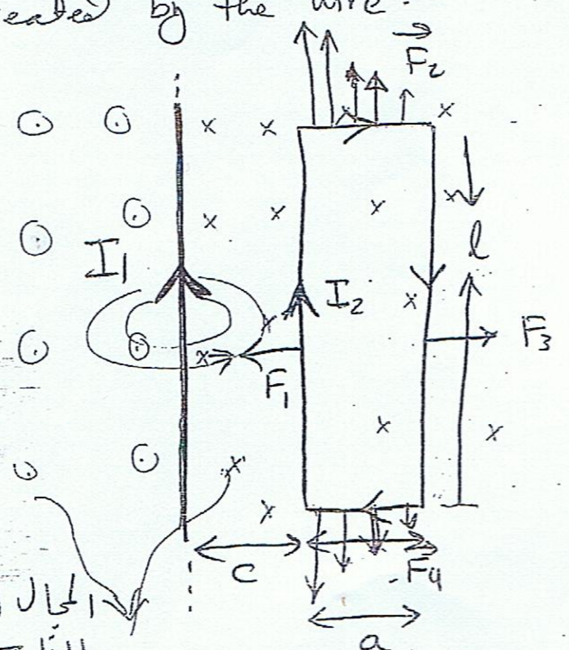
$\vec{F}_1$



$$\frac{\vec{F}_1}{l} = \frac{\mu_0 I_1 I_2}{2\pi c}(-\hat{i})$$



$$\frac{\vec{F}_3}{l} = \frac{\mu_0 I_1 I_2}{2\pi(a+c)}(+\hat{i})$$



$\frac{\vec{F}_2}{l} = -\frac{\vec{F}_4}{l}$: متساويات مقداراً ومتعاكسات اتجاهاً

لأن اتجاه التيار متعاكس في الجزء ٢

والجزء ٤ لكنهما يقعان على نفس الأبعاد

من التيار I_1 ولذلك محصلة $\vec{F}_2+\vec{F}_4$ تساوي صفر (بالتماثل)

$$\vec{F}_{tot} = \vec{F}_1 + \vec{F}_3 + 0 = \frac{\mu_0 I_1 I_2 l}{2\pi(a+c)}(\hat{i}) - \frac{\mu_0 I_1 I_2 l}{2\pi c}(\hat{i})$$

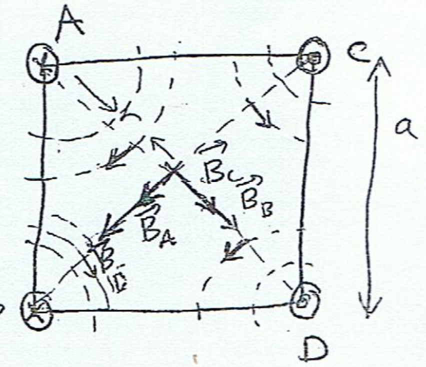
$$= \frac{\mu_0 I_1 I_2 l}{2\pi}\left\{\frac{1}{a+c} - \frac{1}{c}\right\}\hat{i}$$

$$= \frac{\mu_0 I_1 I_2 l}{2\pi}\left\{\frac{\cancel{c}-a-\cancel{c}}{c(c+a)}\right\}(\hat{i})$$

$$= \frac{\cancel{4\pi}\times 10^{-7} * 5 * 10 * 0.45}{2\cancel{\pi}} * \frac{-0.15}{0.1 * 0.25}\hat{i}$$

$$= (-2.7\times 10^{-5}\,\hat{i})\,N \quad \text{(toward the left)}$$

★ problem: four long parallel conductors carry equal currents of $I = 5A$ as in the figure below. the current direction is into the page at point A and B and out of the page at C and D calculate the magnitude and direction of the magnetic field at a point p located at the center of the squre of edge (ضلع) length 0.2 m



$$(2r)^2 = a^2 + a^2 \Rightarrow r = \frac{a}{\sqrt{2}}$$

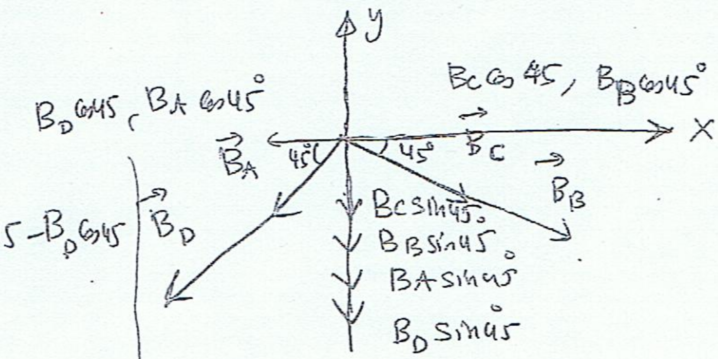
$$r = \frac{0.2}{\sqrt{2}} = 0.14\,m.$$

$$|\vec{B}_A| = |\vec{B}_B| = |\vec{B}_C| = |\vec{B}_D| = \frac{\mu_0 I}{2\pi r}$$

$$|\vec B_A| = |\vec B_B| = |\vec B_C| = |\vec B_D| = \frac{4\pi\times10^{-7} * 5}{2\pi * 0.14} = 7.07\times10^{-6}\,T$$ ⑨

$\vec B_A$ و $\vec B_D$ نفس الاتجاه ، $\vec B_C$ ، $\vec B_B$ نفس الاتجاه

على محور X سوف
تكون المحصلة صفر .



$$\sum B_x = B_C\cos45 + B_B\cos45 - B_A\cos45 - B_D\cos45$$

$$B_C = B_B = B_A = B_D$$

$$\sum B_x = 0$$

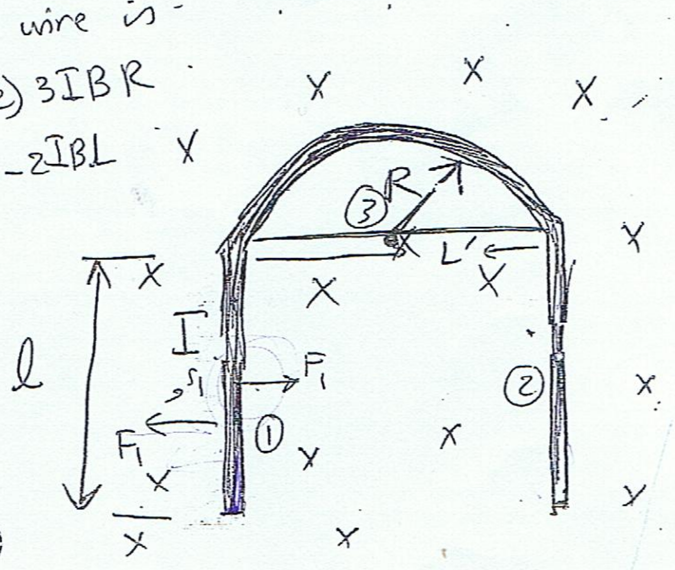
$$\sum B_y = 4 * B_A(-\hat j) = 4 * 7.07\times10^{-6} = 20\times10^{-6}\,T$$
$$= 20\,\mu T\,(-\hat j)$$

problem :- سؤال سنوات سابقة

A rigid (صلب) wire (the shape as in the figure below) consists of a semicircle (نصف دائرة) and two straight portions (جزء). the wire lies in a plane ⊥ to a uniform magnetic field. the magnetic force on the wire is

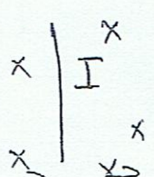
a) IBR ⓑ 2IBR c) 3IBR

d) 2IBL+2IBR e) 2IBR − 2IBL



$$\vec F_{tot} = \vec F_1 + \vec F_2 + \vec F_3$$

$\vec F_1$

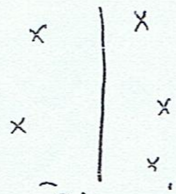


$$\vec F_1 = I\vec L \times \vec B = ILB(\pm\hat i)$$

حسب اتجاه التيار

$\vec{F}_2 = I\vec{L} \times \vec{B}$

$= ILB(\mp\hat{i})$



$\vec{F}_1 = -\vec{F}_2$ (لان التيار متعاكس في الاتجاه سواء كان التيار صاعداً في السلك الأول أو السلك الثاني)

$\vec{F}_3 = I\vec{L}' \times \vec{B} = IL'B(\pm\hat{j})$

$\vec{F}_{tot} = \underbrace{\vec{F}_1 + \vec{F}_2}_{\text{المحصلة = صفر}} + \vec{F}_3 = \vec{F}_3$

$|\vec{F}_{tot}| = |F_3| = IL'B$, but $L' = 2R$

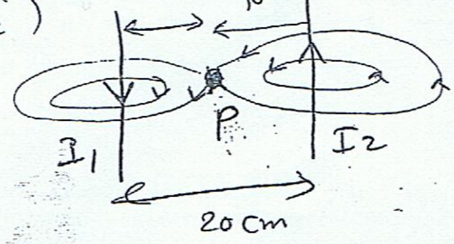
$|\vec{F}_{tot}| = |F_3| = 2IRB$ ✓ (b) الجواب الصحيح

سؤال امتحان سابق : two long parallel conductors carry currents in opposite direction seperated $I_1 = I_2 = 2\,A$ by 20 cm. the magnetude of the magnetic field at the midpoint (نقطة المنتصف) between the two conductors is

$\frac{4\,2\times10^{-7}}{0.1}$

a) 0 $\boxed{\text{(b) } 8\,\mu T}$ (c) $4\,\mu T$ (d) $16\,\mu T$ (e) $2\,\mu T$

$B_1 = \frac{\mu_0 I_1}{2\pi r_1}(\hat{k})$; $B_2 = \frac{\mu_0 I_2}{2\pi r_2}(\hat{k})$



but $I_1 = I_2 = 2\,A$

$r_1 = r_2 = 0.1\,m = r$

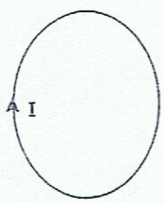
$\vec{B}_{tot} = \vec{B}_1 + \vec{B}_2 = \frac{2 * \mu_0 I}{2\pi r}(\hat{k}) = \frac{\cancel{2} * 4\cancel{\pi} \times 10^{-7} * 2}{\cancel{2\pi} * 0.1}$

$= 8\times10^{-6}\,T = 8\,\mu T$ (b) الجواب الصحيح فرع

Dr. Safaa

Chapter30/ suggested problems

1. The direction of the magnetic field at a point directly below a horizontal wire carrying a current to the east and the direction of the magnetic field at the center of the loop shown are



a. north and into the paper.   b. north and out of the paper.   c. south and into the paper
d. south and out of the paper   e. none of the above

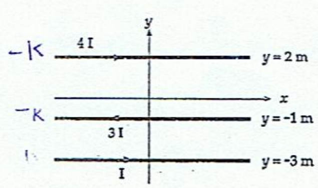
2. A wire carries a current of 20 A along the x-axis from $x = -3.0$ cm to $x = +3.0$ cm. Determine the magnitude of the resulting magnetic field at the point $y = 4.0$ cm on the y axis.

a. 96 μT   b. 72 μT   c. 84 μT   d. 60 μT   e. 100 μT

3. A 2.0-cm length of wire centered on the origin carries a 20-A current directed in the positive y direction. Determine the magnetic field at the point $x = 5.0$ m on the x-axis.

a. 1.6 nT in the negative z direction   b. 1.6 nT in the positive z direction
c. 2.4 nT in the negative z direction   d. 2.4 nT in the negative z direction
e. None of the above

4. Three long wires parallel to the x axis carry currents as shown. If $I = 20$ A, what is the magnitude of the magnetic field at the origin?



a. 37 μT   b. 28 μT   c. 19 μT   d. 47 μT   e. 58 μT

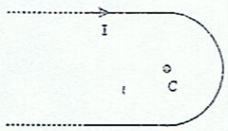
5. Two long parallel wires carry unequal currents in the same direction. The ratio of the currents is 3 to 1. The magnitude of the magnetic field at a point in the plane of the wires and 10 cm from each wire is 4.0 μT. What is the larger of the two currents?

a. 5.3 A   b. 3.0A   c. 4.5 A   d. 3.8 A   e. 0.5 A

6. A long straight wire carries a current of 40 A in a region where there is a uniform external magnetic field which has a 30-μT magnitude and is parallel to the current. What is the magnitude of the resultant magnetic field at a point that is 20 cm from the wire?

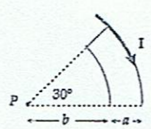
a. 70 μT b. 40 μT c. 10 μT d. 50 μT e. 36 μT

7. A long straight wire is bent as shown to form two parallel straight wires and a semicircle (radius = 2.0 m). A current of 40 A is directed as shown. What is the magnitude of the magnetic field at point C, the center of the circle along which the semicircle lies.



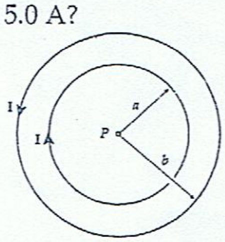
a. 10 μT b. 6.3 μT c. 4.0 μT d. 2.3 μT e. 14 μT

8. In the figure shown, if a = 2.0 cm, b = 5.0 cm, and I = 20 A, what is the magnitude of the magnetic field at the point P?



a. 4.5 μT b. 7.5 μT c. 9.0 μT d. 6.0 μT e. 3.6 μT

9. What is the magnitude of the magnetic field at point P in the figure if a = 2.0 cm, b = 4.5 cm, and I = 5.0 A?



a. 87 μT, into the paper b. 87 μT, out of the paper
c. 0.23 mT, out of the paper d. 0.23 mT, into the paper
e. 23 μT, into the paper

10. The figure shows a cross section of three parallel wires each carrying a current of 5.0 A out of the paper. If the distance R = 6.0 mm, what is the magnitude of the magnetic force on a 2.0-m length of any one of the wires?