تقدم لجنة ElCoM الاكاديمية

ELECTRICAL COMPUTER MECHATRONICS

دفتر لمادة:

تحكم آلي

جزيل الشكر للطالب:

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Chapter 2 : Mathematical Model of Systems :

is discribing the System by Set of differential equations that relate between the input and the Output.

→ physical Laws :

1) Mechanical System : Newton's Second Law :

a. Translational motion :

$$\sum F = ma \longrightarrow a : \text{linear acceleration.}$$

b. Rotational motion :

$$\sum T = J\ddot{\theta} \longrightarrow \ddot{\theta} : \text{angular acceleration.}$$

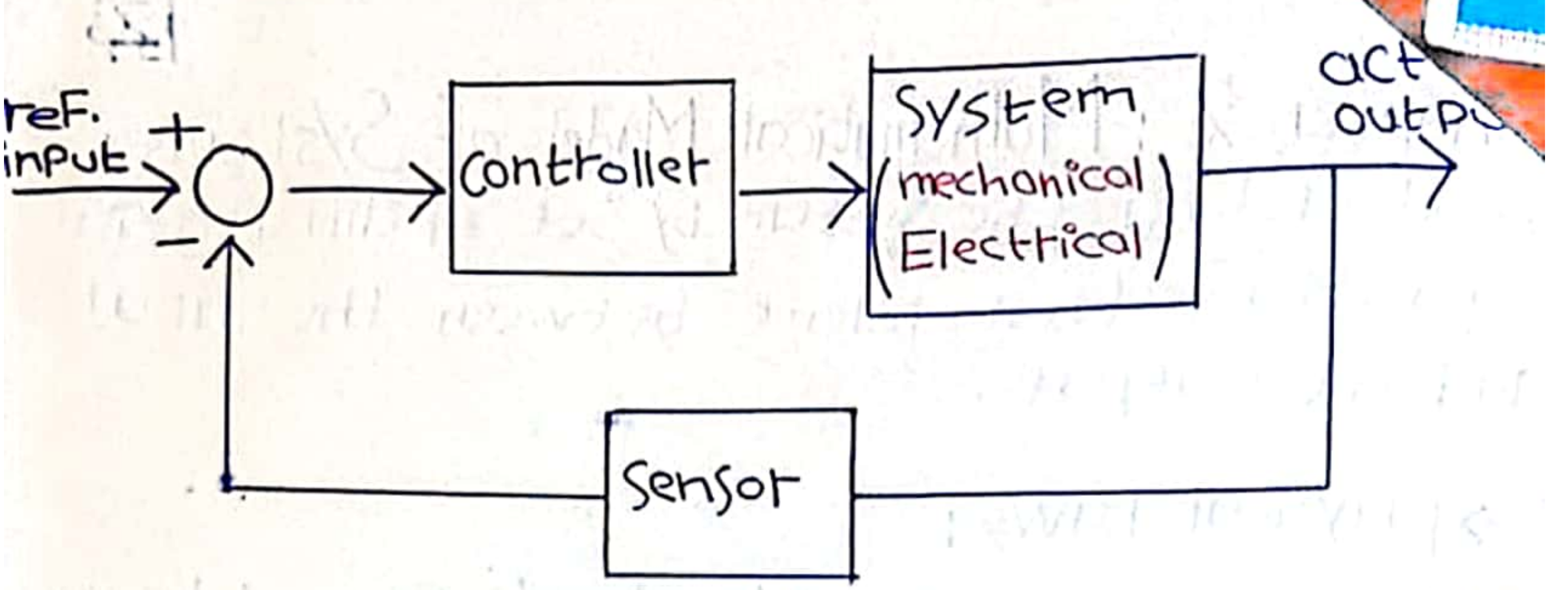
2) Electrical System : Kirchhoff Law :

a. KVL : Kirchhoff's Voltage Law.

$$\sum V_{loop} = 0$$

b. KCL : Kirchhoff's Current Law.

$$\sum I_{node} = 0$$



Basic elements of mechanical System:

1. mass . الكتلة

$$\Sigma F = m\ddot{x}$$

2. damper ; (يمتص الطاقة والFriction) $F_b = b\dot{x}$

F_b or F_d — b: damper const.

x : displacement.

$\dot{x}$: Velocity.

$\ddot{x}$: acceleration.

direction : against speed direction.

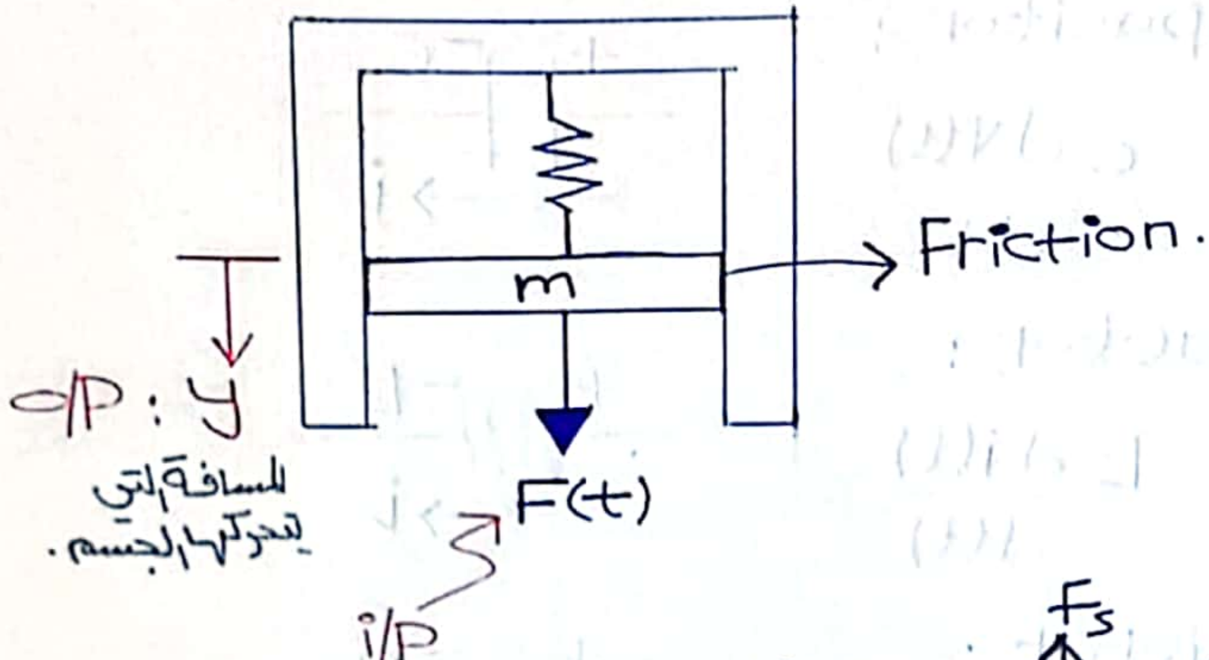
3. Spring ; $F_s = k\,x$

k: Stiffness — x: مقدار الاستطالة

(كل Spring له k الخاصة به)

direction : against deformation direction.

...mple : mathematical model System discribing the system. :- المطلوب



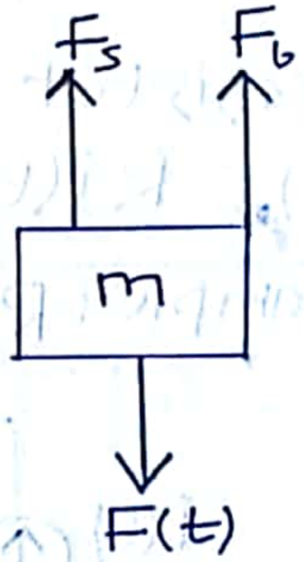
→ Translational motion.

$$\Sigma F = ma = m\ddot{y}$$

$$F - F_b - F_s = m\ddot{y}$$

اتجاه حركة الجسم

عكس اتجاه حركة الجسم



الاتجاه الموجب هو اتجاه حركة الجسم.

$$F - b\dot{y} - ky = m\ddot{y}$$

$$F = m\ddot{y} + b\dot{y} + ky$$

second order diff. eqn. #

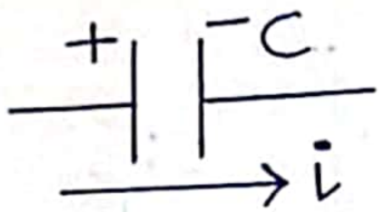
تصف العلاقة بين الـ i/p والـ o/p.

$\underbrace{\text{i/p}}_{F}$ $\underbrace{\text{o/p}}_{y}$

EICoM ELECTRICAL COMPUTER MECHATRONICS

Basic elements of electrical Syst

1. capacitor :

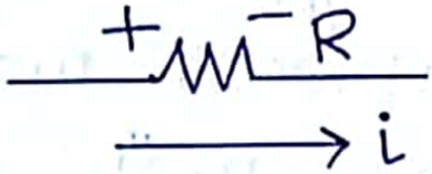


$$i(t) = C\frac{dV(t)}{dt}$$

2. inductor :

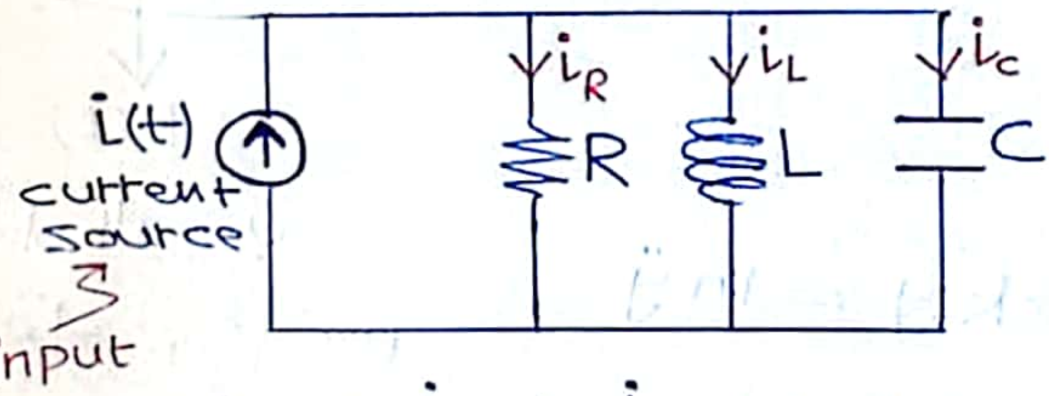
$$V(t) = L\frac{d\,i(t)}{d(t)}$$

3. resistor :



$$V(t) = R\,i(t)$$

Example : RLC -parallel connection :



$$i(t) = i_R + i_L + i_c$$

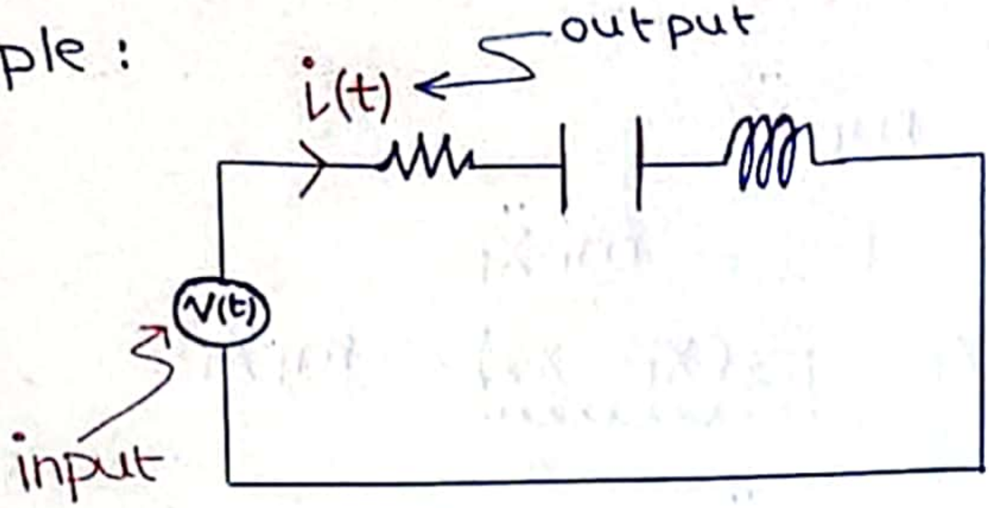
$$= \frac{V(t)}{R} + \frac{1}{L}\int V(t)\,dt + C\frac{dV(t)}{dt}$$

نشتق الطرفين :

$$\dot{I}(t) = \frac{\dot{v}(t)}{R} + \frac{1}{L}v(t) + C\,\ddot{v}(t)$$

→ Second order diff. equ.

Example :

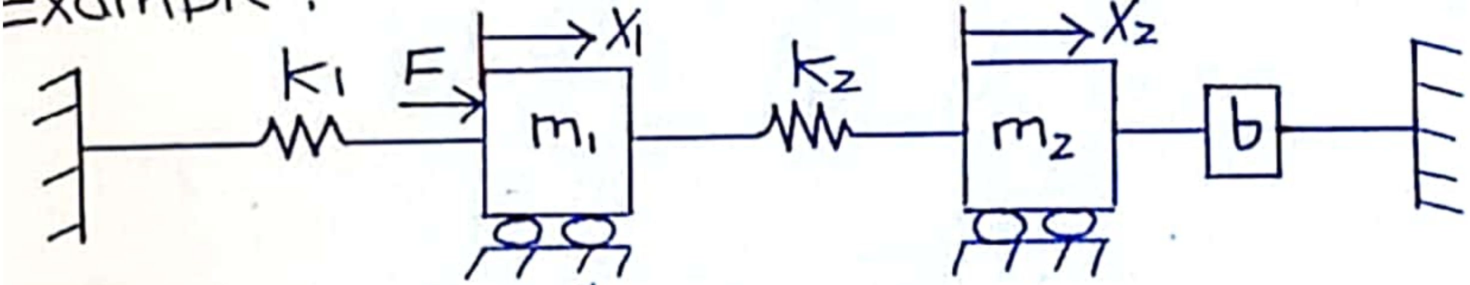


$$V(t) = V_R + V_c + V_L$$

$$= IR + \frac{1}{C}\int i(t)\,dt + L\,\frac{di(t)}{dt}$$

$$\dot{V}(t) = \dot{I}R + \frac{1}{C}\, i(t) + L\,\ddot{i}(t)$$

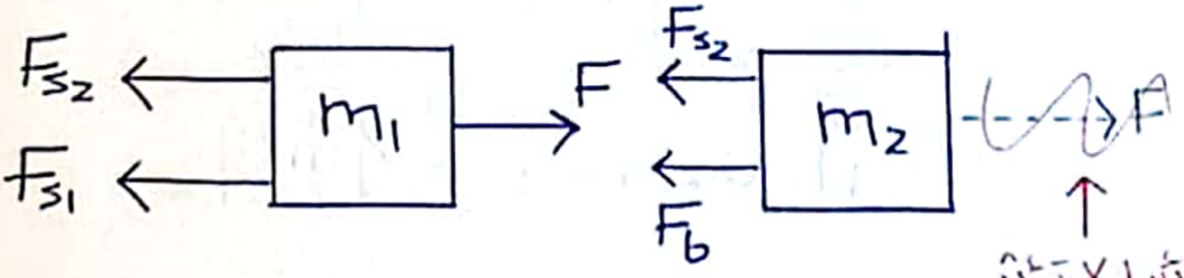
Example :



F : input.

X_1, X_2 : Out put (2 diff. equ's).

↳ X_1, X_2 are independent.



لا نرسمها لأنها لا تؤثر بشكل direct على الجسم وإنما m_2 تتحرك بتأثير الـ F على m_1.

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1) $\Sigma F = m_1 \ddot{x}_1$

$F - F_{s1} - F_{s2} = m_1 \ddot{x}_1$

$F - k_1 x_1 - k_2(x_1 - x_2) = m_1 \ddot{x}_1$

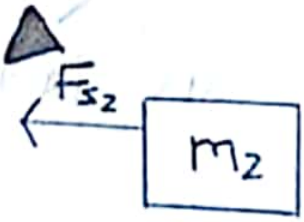
2) $\Sigma F = m_2 \ddot{x}_2$

$-F_{s2} - F_b = m_2 \ddot{x}_2$

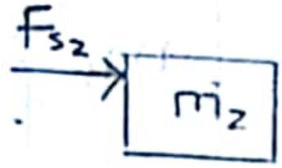
$-k_2(x_2 - x_1) - b\dot{x}_2 = m_2 \ddot{x}_2$

نفس المقدار وعكس
الإتجاه .

$-k_2(x_1 - x_2)$:



$+k_2(x_2 - x_1)$:



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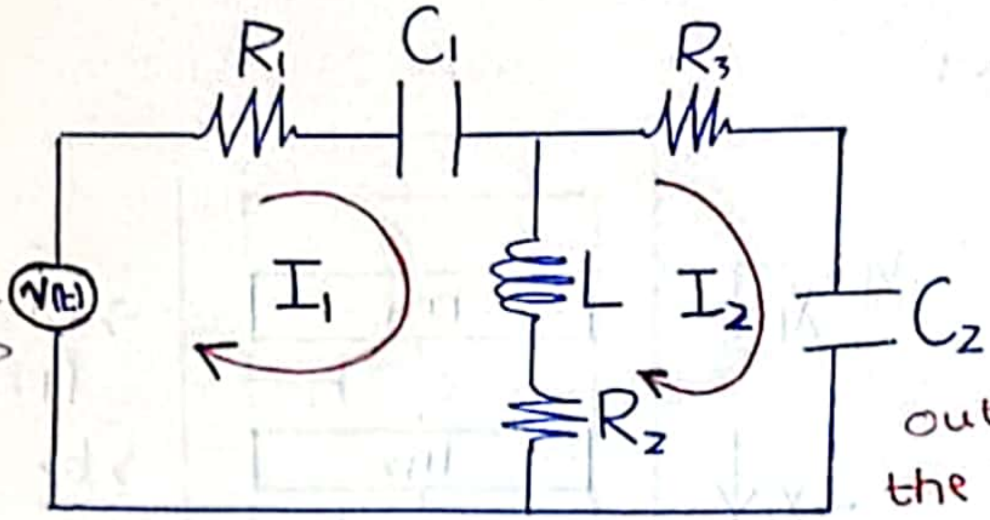
Example :

input



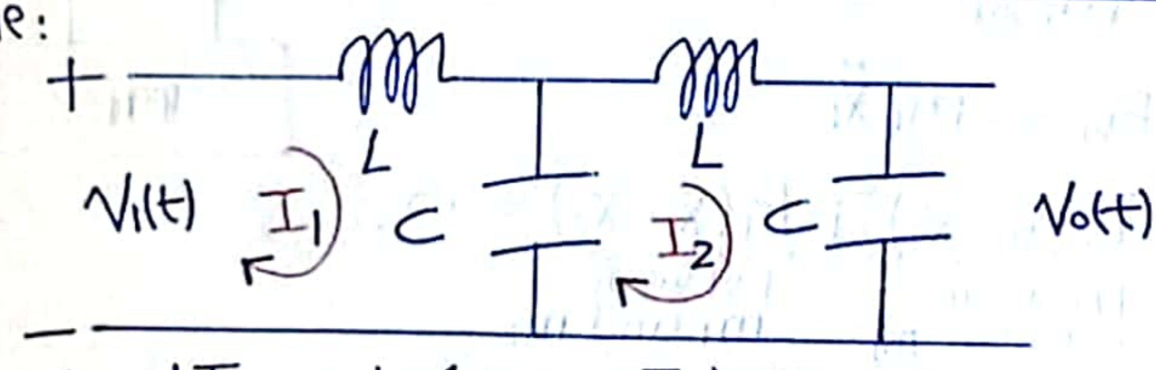
out put : the voltage on C_2.

$$V(t) = I_1R_1 + \frac{1}{C_1}\int I_1\,dt + L\frac{d(I_1-I_2)}{dt} + R_2(I_1-I_2).$$

$$0 = R_3I_2 + \frac{1}{C_2}\int I_2\,dt + R_2(I_2-I_1) + L\frac{d(I_2-I_1)}{dt}$$

$$V_o = \frac{1}{C_2}\int I_2\,dt .$$

Example:



$$V_1(t) = L\frac{dI_1}{dt} + \frac{1}{C}\int (I_1-I_2)\,dt$$

$$0 = L\frac{dI_2}{dt} + \frac{1}{C}\int I_2\,dt + \frac{1}{C}\int (I_2-I_1)\,dt .$$

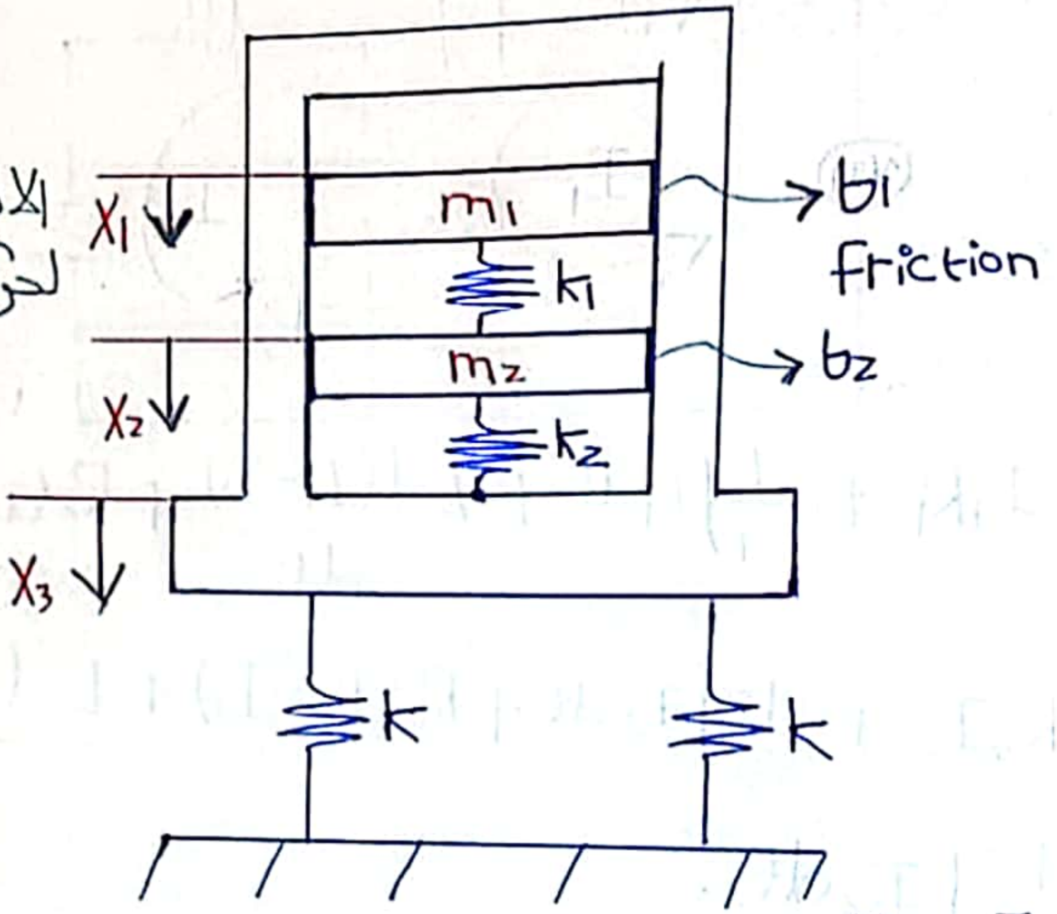
$$V_o = \frac{1}{C}\int I_2(t)\,.dt.$$

→ mech. or Elec. sys. → 2 masses or 2 loops → Coupling

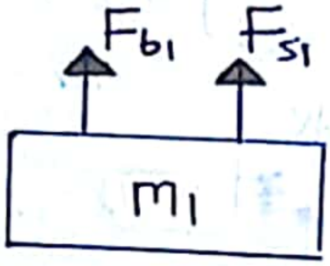
بينهم spring أو damper مشترك .

Example:

الاتجاه الموجب لحركة الجسم.



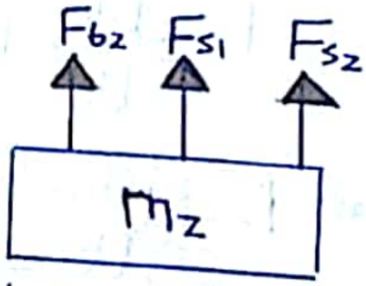
Solution:



$$\Sigma F = m_1 \ddot{X}_1$$

$$-F_{s_1} - F_{b_1} = m_1 \ddot{X}_1$$

$$m_1\ddot{X}_1 + \underbrace{b_1(X_1 - X_3)}_{\text{Friction between } m_1 \text{ and the Frame.}} + \underbrace{k_1(X_1 - X_2)}_{\text{between } m_1 \text{ and } m_2.} = 0$$



$$\Sigma F = m_2 \ddot{X}_2$$

$$-F_{b_2} - F_{s_2} - F_{s_1} = m_2 \ddot{X}_2$$

$$m_2\ddot{X}_2 + b_2(\dot{X}_2 - \dot{X}_3) + k_2(X_2 - X_1) + k_2(X_2 - X_3) = 0$$

Cases: without Friction $\rightarrow b_1 ; b_2 = 0$

without Friction + Frame is static $\rightarrow b_1, b_2, X_3 = 0$

Linear System :- linear system Should satisfy two conditions :-

1) Scaling :

$$y(\alpha X_1 + \beta X_2) = y(\alpha X_1) + y(\beta X_2).$$

2) Super position :

$$y(x_1 + x_2) = y(x_1) + y(x_2).$$

$y = x$

$\alpha = 1 \quad , x_1 = 1$

$\beta = 2 \quad , x_2 = 4$

$\rightarrow y(1+8) \overset{?}{=} y(1) + y(8)$

$9 = 9$ ✓

$\rightarrow y(1+4) \overset{?}{=} y(1) + y(4)$

$y(5) \overset{?}{=} 1 + 4$

$5 = 5$ ✓

∴ linear system.

$y(x) = e^{-x}$: non linear sys.

↓

نحولها الى linear

عن طريق Taylor series.

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* Taylor Series Expansion :-

$$y = \underset{\text{term}}{\underline{y(x_0)}} + \underline{\left.\frac{dy}{dx_1}\right|_{x=x_0}(x-x_0)} + \underline{\frac{1}{2!}\frac{d^2y}{dx_2^2}(x-x_0)^2}$$

$$+ \cdots \left.\frac{d^n y}{n!\,dx^n}\right|_{x=x_0}(x-x_0)^n$$

$$= \sum_{n=0}^{n} \frac{(x-x_0)^n \cdot F^n(x_0)}{n!}$$

Example: $y(x) = e^{-x}$, $x_0 = 0$,

using Taylor Series (for 3 terms) :

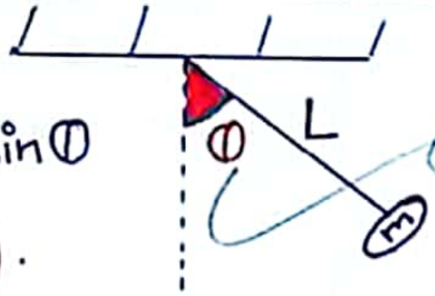
$$y = 1 + (-1)(x-0) + \frac{1}{2}(1)(x-0)^2$$

$$\boxed{y = 1 - x + \frac{x^2}{2}}$$

$$\frac{dy}{dx} = -e^{-x}\Big|_{x=0} = -1 \checkmark \#$$

$$\frac{d^2y}{dx^2} = e^{-x}\Big|_{x=0} = 1 \checkmark \#$$

Example :



لو ذكر بالسؤال : θ is small value

$\hookrightarrow = 0$

$T(\theta) = mgL\sin\theta$

(for 2 terms).

$\therefore T(\theta) = mgL\theta$

$$\boxed{\begin{array}{l} \sin 0 = 0 \\ \sin\theta = \theta \end{array}}$$

$T(0) = 0$

$$\frac{dT}{d\theta}\Big|_{\theta=\theta_0} = mgL\cos\theta = mgL$$

$$y = 0 + mgL\theta = mgL\theta$$

#.

Laplace Transform :-

a mathematical tool for solving linear invarient differential equation.

$$F(s) = \mathcal{L}\{f(t)\} = \int_0^\infty e^{-st} \cdot f(t) \cdot dt \;;$$

for $f(t), t > 0$

$$f(t) = \mathcal{L}^{-1}\{F(s)\}$$

→ laplace transform of time differential :

$$\mathcal{L}\left\{\frac{dF(t)}{dt}\right\} = sF(s) - F(0).$$

$$\mathcal{L}\left\{\frac{d^2f(t)}{dt^2}\right\} = s^2F(s) - sF(0) - F(0).$$

→ Table 2.3 :

	f(t)	$\mathcal{L}(f)$		f(t)	$\mathcal{L}(f)$
1	Unit-impulse $\delta(t)$	1	7	$\cos \omega t$	$\frac{s}{s^2+\omega^2}$
2	Unit-step 1	$1/s$	8	$\sin \omega t$	$\frac{\omega}{s^2+\omega^2}$
3	Unit-ramp t	$1/s^2$	9	cosh at	$\frac{s}{s^2-a^2}$
4	t^2	$2!/s^3$	10	$t e^{at}$	$\frac{1}{(s-a)^2}$
5	t^n (n is +ve integer)	$\frac{n!}{s^{n+1}}$	11	$e^{at}\cos \omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
6	e^{at}	$\frac{1}{s-a}$	12	$e^{at}\sin \omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$

For most engineering purposes the inverse Laplace transformation can be accomplished simply by referring to Laplace transform tables

Example :

$$m\ddot{y} + b\dot{y} + ky = f(t) \; ; \; y(0) = 0 , \; \dot{y}(0) = 0 ,$$

$$\frac{F}{m} = 1 \; , \; \frac{k}{m} = 2 \; , \; \frac{b}{m} = 3 \; .$$

→ Solution: $\frac{m\ddot{y}}{m} + \frac{b\dot{y}}{m} + \frac{ky}{m} = \frac{F(t)}{m}$

$$\ddot{y} + 3\dot{y} + 2y = 1$$

$$S^2 Y(S) + 3S\,Y(S) + 2Y(S) = \frac{1}{S}$$

$$Y(S)\,(S^2 + 3S + 2) = \frac{1}{S}$$

$$\boxed{Y(S) = \frac{1}{S(S^2+3S+2)}}$$

$$y(t) = L^{-1}\{y(S)\} = L^{-1}\left\{\frac{1}{S(S^2+3S+2)}\right\}$$

$$\rightarrow Y(S) = \frac{1}{S(S+2)(S+1)} = \frac{A}{S} + \frac{B}{S+1} + \frac{C}{S+2}$$

Real Roots

$$1 = A(S+1)(S+2) + B(S)(S+2) + C(S)(S+1)$$

$\rightarrow S = 0 \rightarrow 1 = 2A \rightarrow A = \frac{1}{2}$

$\rightarrow S = -1 \rightarrow 1 = -B \rightarrow B = -1$

$\rightarrow S = -2 \rightarrow 1 = 2C \rightarrow C = \frac{1}{2}$

$$\rightarrow Y(s) = \frac{1/2}{s} - \frac{1}{s+1} + \frac{1/2}{s+2}$$

$$\rightarrow y(t) = \frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t} \quad \#.$$

Example : $Y(s) = \dfrac{2}{(s+1)(s+2)^2}$

Repeated

$$Y(s) = \frac{2}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$$A(s+2)^2 + B(s+1)(s+2) + C(s+1) = 2$$

$\rightarrow s=-1 \rightarrow A=2.$

$\rightarrow s=-2 \rightarrow C=-2$

$\rightarrow s=0 \rightarrow 2 = 4A+2B+C$

$$2 = 4(2) + 2B - 2$$

$$B = -2.$$

$$Y(s) = \frac{2}{s+1} - \frac{2}{s+2} - \frac{2}{(s+2)^2}$$

$$y(t) = 2e^{-t} - 2e^{-2t} - 2te^{-2t} \quad \#.$$

$$L^{-1}\left\{\frac{A}{s+a}\right\} = Ae^{-at}$$

$$L^{-1}\left\{\frac{A}{(s+a)^n}\right\} = At^{n-1}e^{-at}$$

Example: $Y(S) = \dfrac{3}{S(S^2+2S+5)}$

$$\frac{3}{\underbrace{S(S^2+2S+5)}} = \frac{A}{S} + \frac{BS+C}{S^2+2S+5}$$

$S = \dfrac{-2 \mp \sqrt{4-4(5)}}{1}$

$\rightarrow A(S^2+2S+5) + (BS+C)(S) = 3$

$AS^2 + 2AS + 5A + BS^2 + CS = 3$ ← نجمع الحدود المتشابهة ، ثم نقارن المعاملات بين الطرفين .

$A+B=0$

$2A+C=0$

$5A=3 \rightarrow A = \frac{3}{5}$.

$\rightarrow B = -\frac{3}{5}$.

$\rightarrow C = \frac{-6}{5}$.

$\boxed{S^2+2S+5 = S^2+2S+4+1 = (S+1)^2+4}$

$$Y(S) = \frac{3/5}{S} - \frac{3/5\,S}{S^2+2S+5} - \frac{6/5}{S^2+2S+5}$$

$$= \frac{3/5}{S} - \frac{3/5\,S}{(S+1)^2+4} - \frac{6/5}{(S+1)^2+4}$$

$$= \frac{3/5}{S} - \frac{3}{5}\frac{S+1}{(S+1)^2+4} - \underbrace{\frac{6}{5}\frac{1}{(S+1)^2+4} - \frac{-\frac{3}{5}}{(S+1)^2+4}}$$

$$= \frac{3/5}{S} - \frac{3}{5}\frac{S+1}{(S+1)^2+4} - \frac{3}{2\times 5}\frac{1\times 2}{(S+1)^2+4}$$

$$y(t) = \frac{3}{5} - \frac{3}{5}(e^{-t}\cos 2t) - \frac{3}{10}(e^{-t}\sin 2t)$$

3

$$L\{e^{-t}\cos wt\} = \frac{S+a}{(S+a)^2+w^2}.$$

$$L\{e^{-at}\sin wt\} = \frac{w}{(S+a)^2+w^2}.$$

$$L\{\cos wt\} = \frac{S}{S^2+w^2}.$$

$$L\{\sin wt\} = \frac{w}{S^2+w^2}.$$

Transfer Function :-

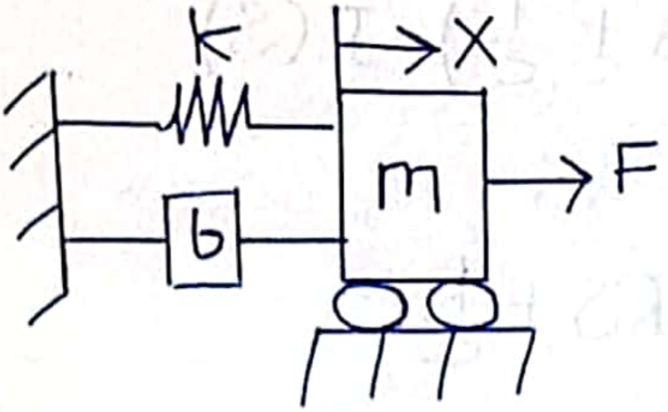
of linear System :

The Ratio of Laplace transform of the output variable to the Laplace transform of the input variable with the intial conditions assume to be Zero.

$Y(S) \rightarrow$ output
$R(S) \rightarrow$ input

$$\rightarrow T(S) = \frac{Y(S)}{R(S)}.$$

ample:



ransfer fun. : $\frac{\text{output}}{\text{input}} : \frac{X(S)}{F(S)}$

$$\rightarrow m\ddot{X} + b\dot{X} + kX = F(t)$$

$$mS^2X(S) + bSX(S) + kX(S) = F(S)$$

$$X(S)\left[mS^2 + bS + k\right] = F(S)$$

$$\frac{X(S)}{F(S)} = \frac{1}{mS^2 + bS + k} \quad \#$$

Example:

$$\frac{V_o(S)}{V_I(S)} = ?$$

$$V_1(t) = RI(t) + \frac{1}{c}\int I(t).dt$$

$$\dot{V}_1(t) = R\dot{I}(t) + \frac{1}{c}I(t)$$

$$SV_1(S) = RSI(S) + \frac{1}{c}I(S)$$

$$SV_1(S) = \left(RS + \frac{1}{C}\right) I(S)$$

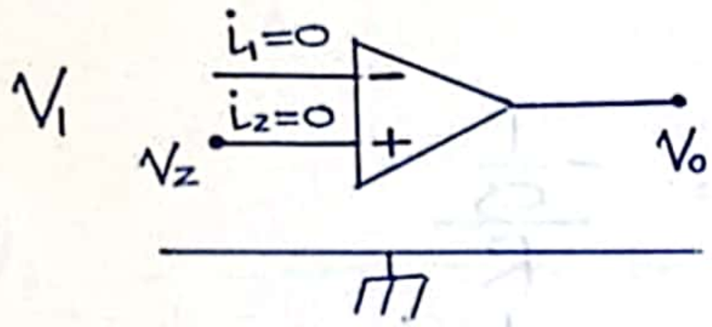
$$\frac{I(S)}{V_1(S)} = \frac{S}{RS + \frac{1}{C}}$$

$$\rightarrow I(t) = C\frac{dV_o}{dt} \rightarrow \boxed{I(S) = CSV_o(S)}$$

$$\frac{CSV_o(S)}{V_1(S)} = \frac{S}{RS + \frac{1}{C}}$$

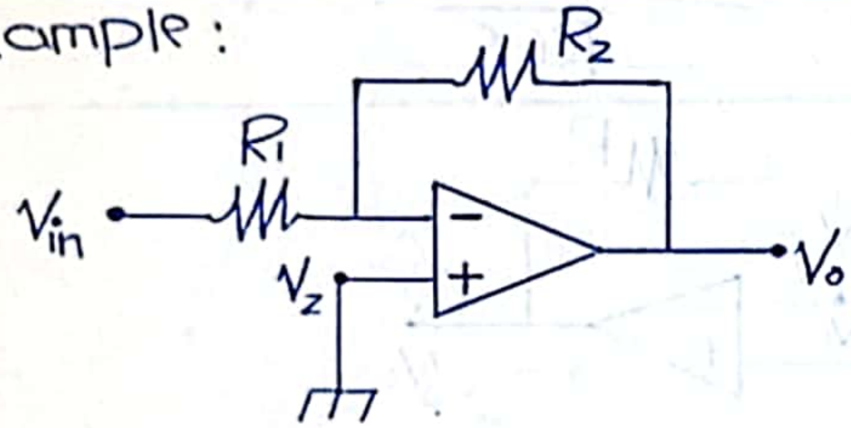
$$\frac{V_o(S)}{V_1(S)} = \frac{1}{CRS + 1} \quad \#.$$

The Transfer function of an Op.Amp. Circuit :-



$$\left.\begin{aligned} i_1 &= 0 \\ i_2 &= 0 \\ V_1 &= V_2 \end{aligned}\right\} \text{Ideal}$$

Example :



$$\frac{V_o(S)}{V_{in}(S)} = T \cdot F$$

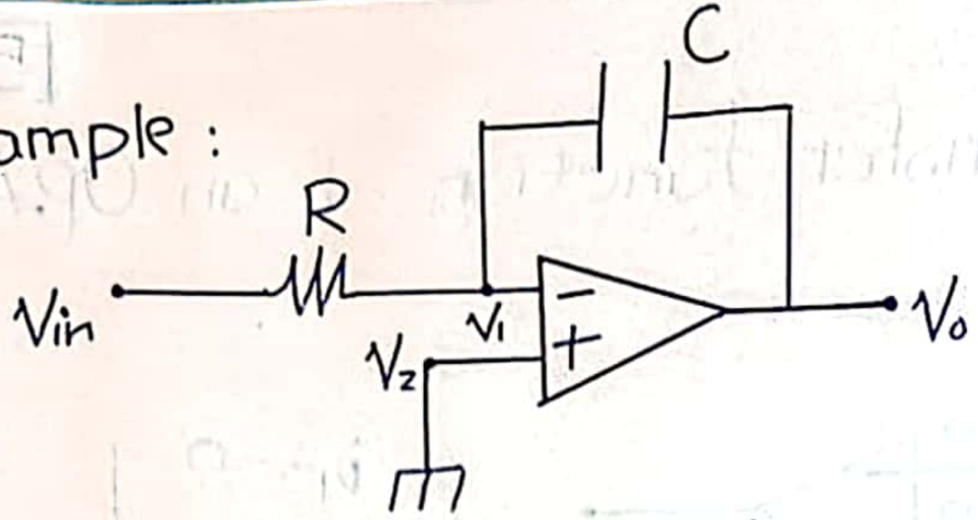
$$= \frac{-Z_{R_2}}{Z_{R_1}}$$

$$= \frac{-R_2}{R_1} \quad \# .$$

$$\boxed{\begin{aligned} Z_R &= R \\ Z_c &= \frac{1}{CS} \\ Z_L &= LS \end{aligned}}$$

Table 2.5 ص٥

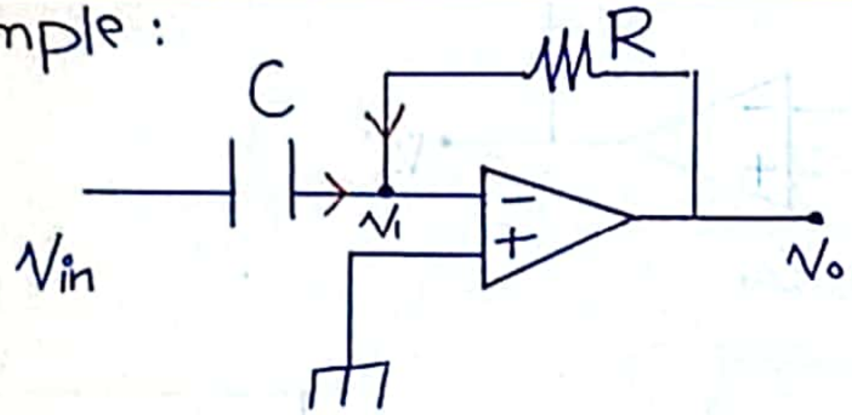
Example :



$$\frac{V_o(S)}{V_{in}(S)} = -\frac{Z_c}{Z_R} = \frac{-\frac{1}{CS}}{R}$$

$$= -\frac{1}{RCS} \,. \quad \#$$

Example :



$$\frac{V_o(S)}{V_{in}(S)} = -\frac{Z_R}{Z_c} = \frac{-R}{\frac{1}{CS}} = -RCS \,.$$

$\#$.

OR :

$$C\frac{d(\cancelto{0}{V_1} - V_{in})}{dt} + \frac{\cancelto{0}{V_1} - V_o}{R} = 0 \quad ; V_1 = 0 \ldots$$

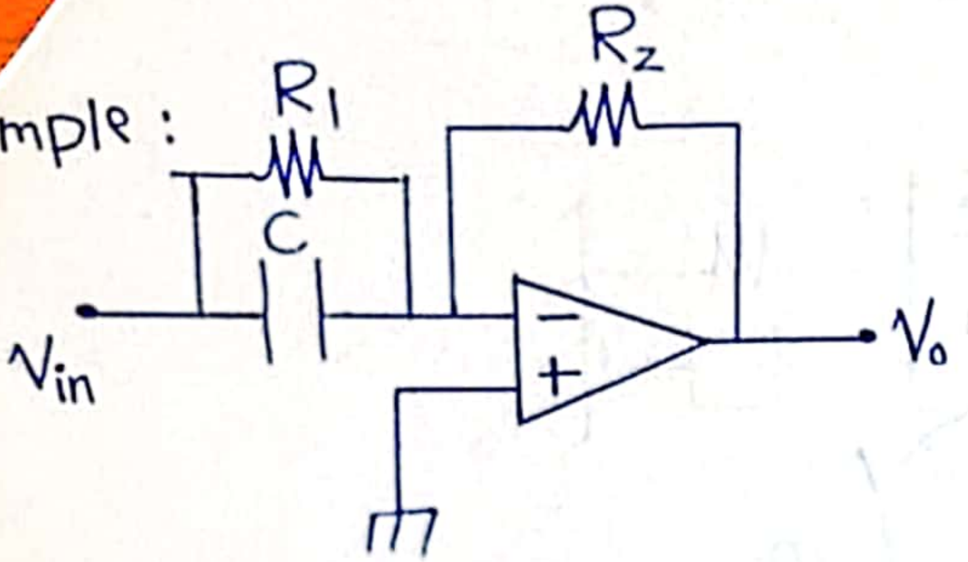
Virtual ground .

$$-C\frac{dV_{in}}{dt} = \frac{V_o}{R}$$

$$-CS\,V_{in}(S) = \frac{V_o(S)}{R} \rightarrow \frac{V_o(S)}{V_{in}(S)} = -RCS$$

$\#$.

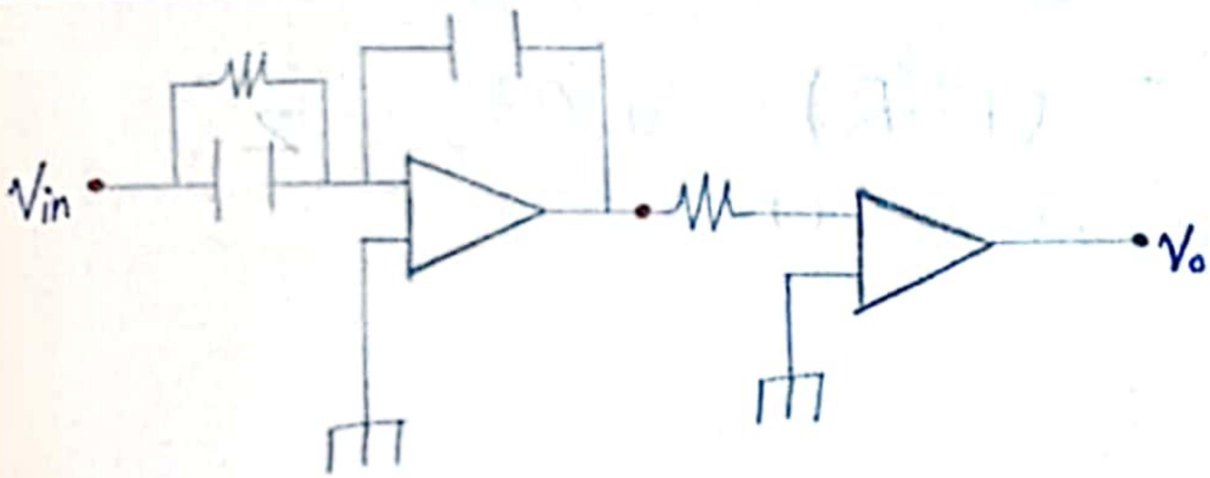
xample :



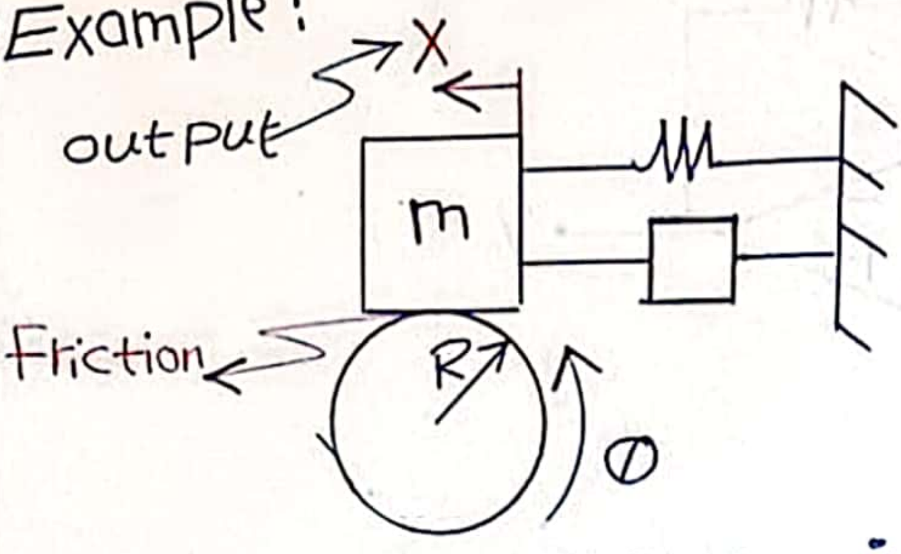
$$\frac{V_o(S)}{V_{in}(S)} = \frac{-Z_{R_2}}{Z_{R_1 \parallel C}}$$

$$\left.\begin{aligned} Z_{R_1} &= R_1 \\ Z_C &= \frac{1}{CS} \end{aligned}\right\} \quad \begin{aligned} \frac{1}{Z_{R_1C}} &= \frac{1}{Z_{R_1}} + \frac{1}{Z_C} \\ \frac{1}{Z_{R_1C}} &= \frac{1}{R_1} + \frac{CS}{R_1} R_1 \end{aligned}$$

$$\therefore \frac{V_o(S)}{V_{in}(S)} = \frac{-R_2}{\frac{R_1}{1+R_1CS}} = -R_2\left[\frac{1}{R_1} + \frac{R_1CS}{R_1}\right].$$



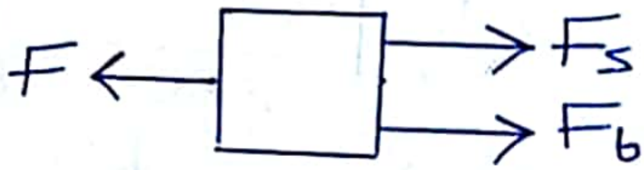
Example:



T : Torque : input

$$\frac{X(S)}{T(S)} = ?$$

$$\sum F = m\ddot{x}$$
$$\sum T = J\ddot{\theta}$$



$$\rightarrow F - F_s - F_b = m\ddot{x}$$

$$\boxed{F - b\dot{x} - kx = m\ddot{x}} \quad \text{----- } 1$$

$$\rightarrow \boxed{T - \underbrace{(F\cdot R)}_{\cdot\ \text{friction}\ (T)} = J\ddot{\theta}} \quad \text{---- } 2$$

$$\rightarrow F = m\ddot{x} + b\dot{x} + kx$$

عوّض

$$2) \rightarrow T_{in} - (m\ddot{x} + b\dot{x} + kx)R = J\ddot{\theta}$$

$$\boxed{\begin{aligned} x &= R\theta \\ \dot{x} &= R\dot{\theta} \\ \ddot{x} &= R\ddot{\theta} \end{aligned}} \rightarrow \ddot{\theta} = \frac{\ddot{x}}{R}.$$

$$\rightarrow T_{in} - (m\ddot{x} + b\dot{x} + kx)R = \frac{J\ddot{x}}{R}$$

$$T_{in} = \left(Rm + \frac{J}{R}\right)\ddot{x} + bR\dot{x} + kRx$$

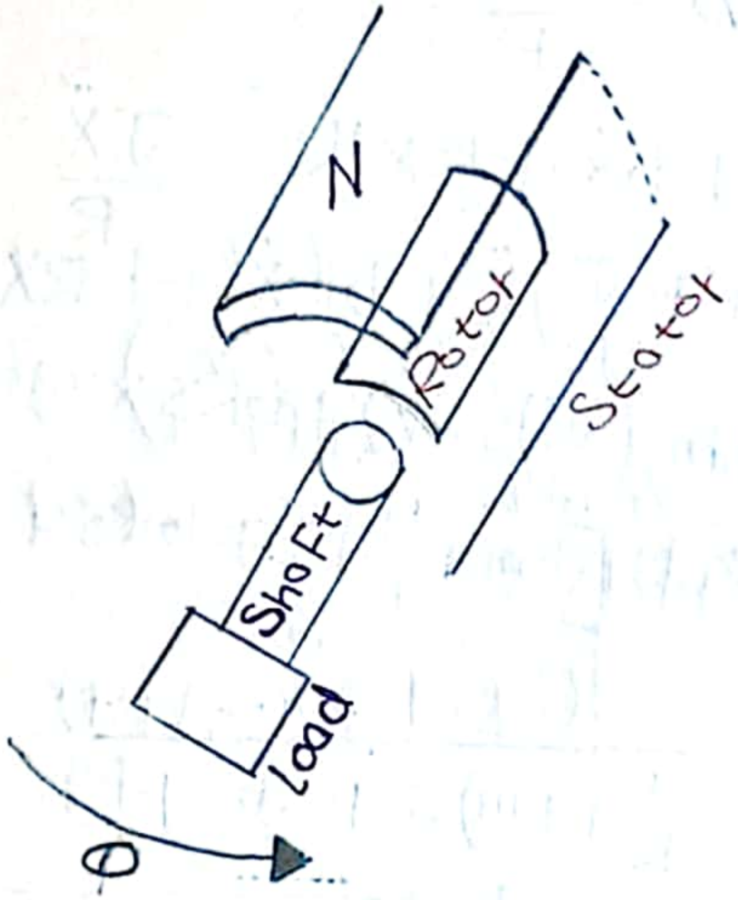
$$T(s) = \left(Rm + \frac{J}{R}\right)s^2 X(s) + bRsX(s) + kRX(s)$$

$$= X(s)\left[\left(Rm + \frac{J}{R}\right)s^2 + bRs + kR\right]$$

$$\therefore \frac{X(s)}{T(s)} = \frac{1}{\left(\frac{J}{R} + Rm\right)s^2 + Rbs + Rk} \quad \#.$$

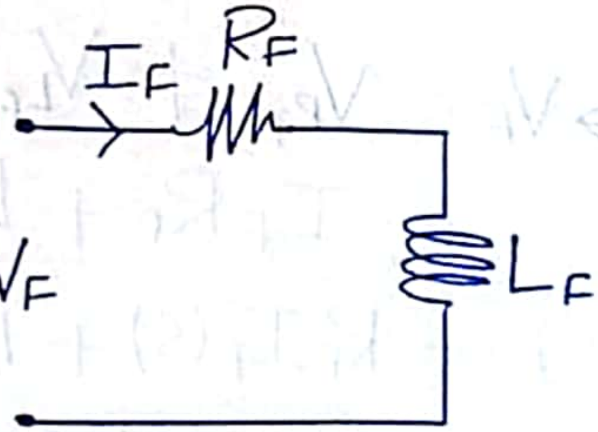
DC-motor :-

is used to move a load, it's called an actuator,
it converts direct current (DC) electrical energy into Rotational energy



$$T \cdot F = \frac{\theta(s)}{V_i(s)} \;?$$

"Rotor"
(armature).



"Stator"
(Field).

1- Field controlled DC-motor :-

$$\underset{\substack{\uparrow \\ \text{airgap flux}}}{T_m = k_1 \phi} \overset{\substack{\text{field curr.} \\ \downarrow}}{I_F} \underset{\substack{\uparrow \\ \text{armature current}}}{I_a}$$

$$T_m = J\ddot{\theta} + b\dot{\theta}$$

$$\boxed{\begin{aligned} \Sigma T &= J\ddot{\theta} \\ T_m - b\dot{\theta} &= J\ddot{\theta} \end{aligned}}$$

$$T_m(S) = JS^2\theta(S) + bS\theta(S)$$

$$\boxed{T_m(S) = \theta(S)\left[JS^2 + bS\right]}$$

$k_1 \phi I_a = k_m$: motor constant.

ثبتّه لأني
بشتغل field.

$$\therefore T_m = k_m I_F$$

$$\boxed{T_m(S) = k_m I_F(S).}$$

$$\rightarrow V_F = V_{R_F} + V_{L_F}$$

$$= I_F R_F + L_F \frac{dI_F}{dt}$$

$$V_F(S) = R_F I_F(S) + L_F S I_F(S)$$

$$= I_F(S)\left[R_F + L_F S\right]$$

$$\boxed{I_F(S) = \frac{V_F(S)}{R_F + L_F S}}$$

$$\because \; T_m = (JS^2 + bS)\,\theta(S).$$

$$T_m = K_m \left(\frac{V_F(S)}{R + L_F S}\right).$$

$$\rightarrow (JS^2 + bS)\,\theta(S) = \frac{K_m V_F(S)}{R + L_F S}$$

$$\boxed{\frac{\theta(S)}{V_F(S)} = \frac{K_m}{(R + L_F S)(JS^2 + bS)}} \quad \#.$$

$$\rightarrow \boxed{\frac{\theta(S)}{I_F(S)} = \frac{K_m}{JS^2 + bS}}$$

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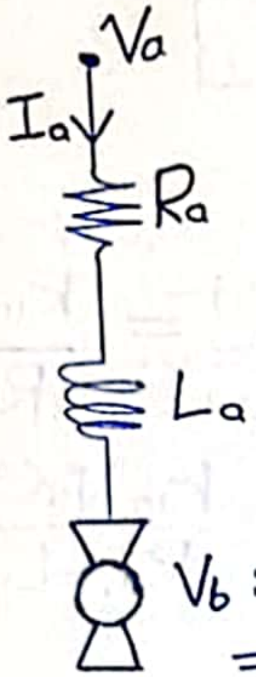
2. Armature – Controlled DC-motor :-



→ Electromagnetic Force.

V_b : induced Voltage

$= k_b\dot{\Phi} = k_b\omega$; Φ, ω : Angular Velocity.

$$\frac{\Phi(S)}{V_a(S)} = ?$$

$$T_m = J\ddot{\Phi} + b\dot{\Phi} \longrightarrow T_m(S) = (JS^2 + bS)\Phi(S).$$

$$T_m = k_m I_m \longrightarrow T_m(S) = k_m I_a(S).$$

→ back to circuit :-

$$V_a = I_a R_a + L_a\frac{dI_a}{dt} + V_b \quad \rightarrow k_b\dot{\Phi}$$

$$V_a(S) = I_a(S)R_a + L_aS\,I_a(S) + k_bS\Phi(S)$$

$$= I_a(R_a + L_aS) + k_bS\Phi(S)$$

$$\boxed{I_a = \frac{V_a(S) - k_bS\Phi(S)}{R_a + L_aS}}$$

$$\therefore T_m(S) = K_m\left[\frac{V_a(S) - K_b S \Phi(S)}{R_a + L_a S}\right]$$

$$T_m(S) = (JS^2 + bS)\Phi(S)$$

$$\rightarrow (JS^2 + bS)\Phi(S) + \frac{K_b K_m S \Phi(S)}{R_a + L_a S} = \frac{K_m V_a(S)}{R_a + L_a S}$$

$$\Phi(S)\left[(JS^2 + bS) + \frac{K_b K_m S}{R_a + L_a S}\right] = \frac{K_m V_a(S)}{R_a + L_a S}.$$

$$\boxed{\frac{\Phi(S)}{V_a(S)} = \frac{K_m}{S(R_a + L_a S)(JS + b) + K_b K_m}} \#$$

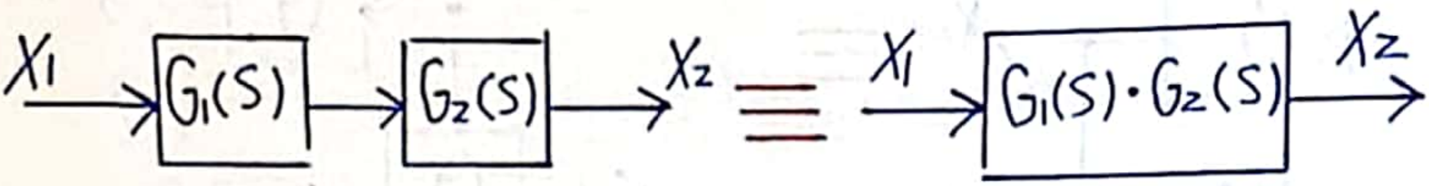
Block diagram model :-

Block diagram consists of undirctional operational block that represent the transfer function of the variable of interest.

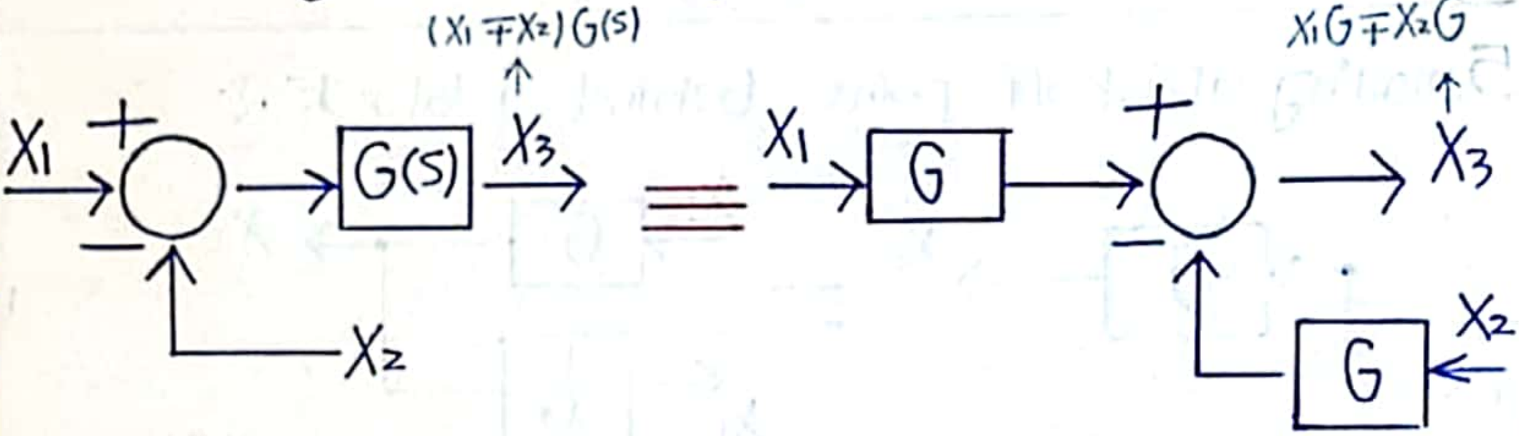
Table 2.6

→ Block diagram transformation :-

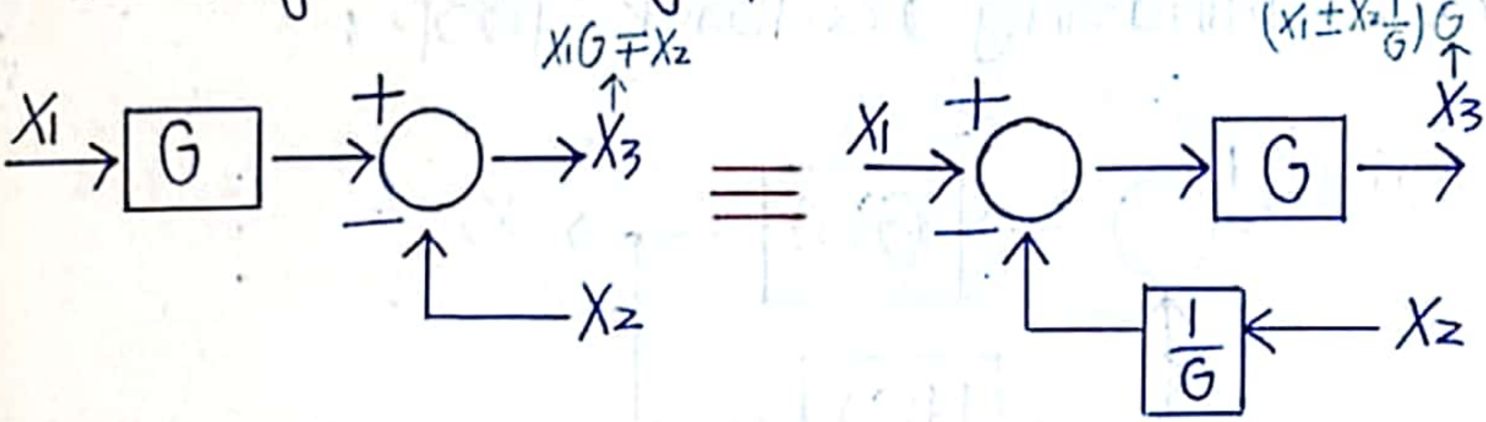
1. Cascade :

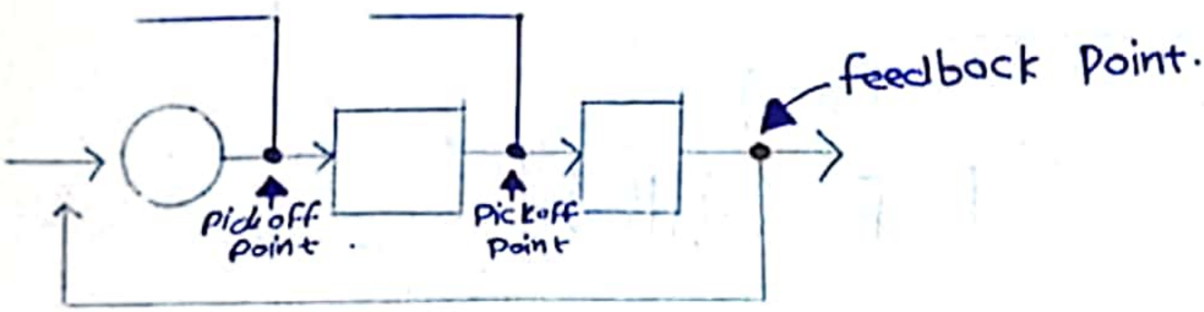


2. moving a Summing point behind a block:



3. moving a Summing point ahead of a block:





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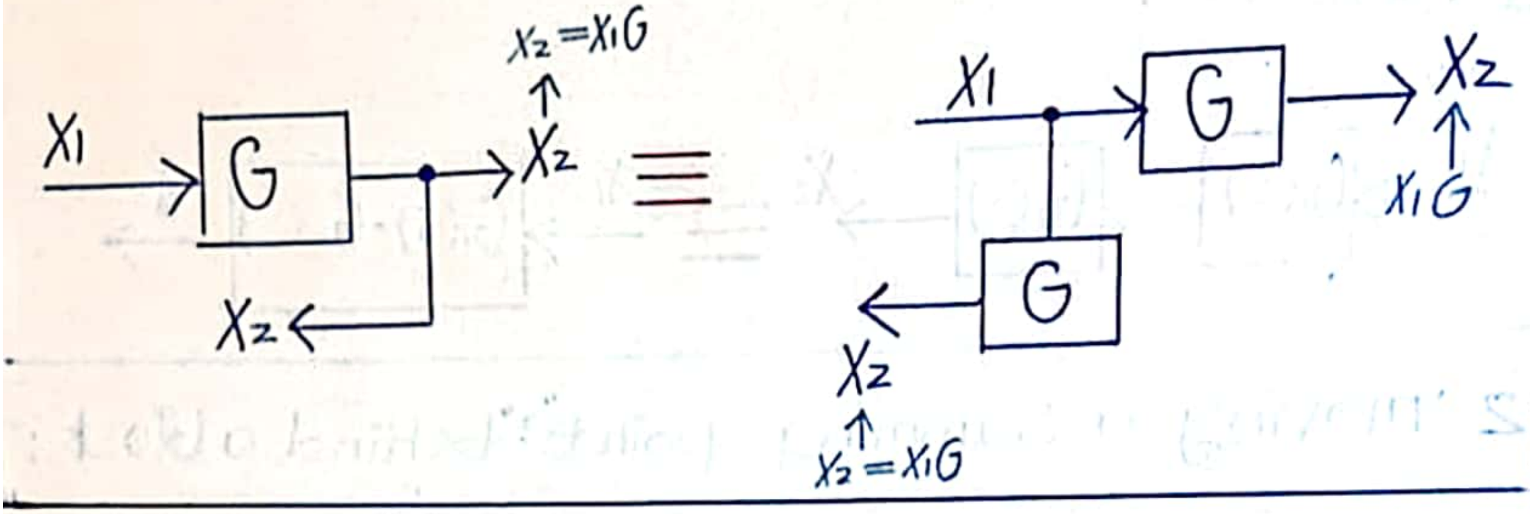
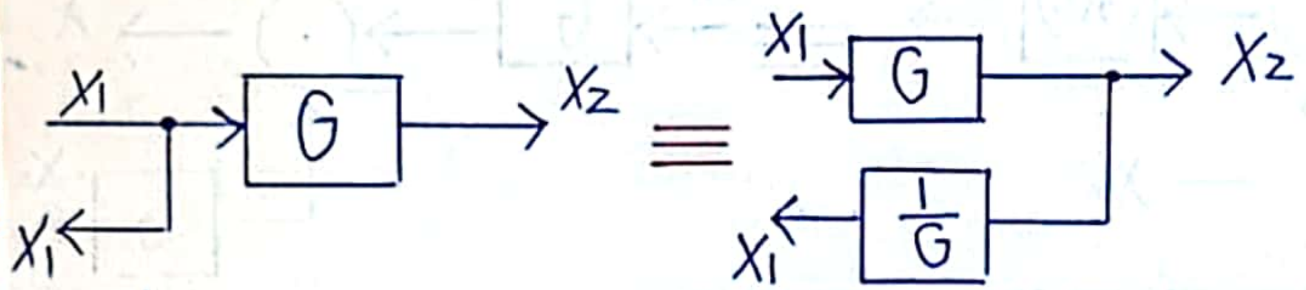
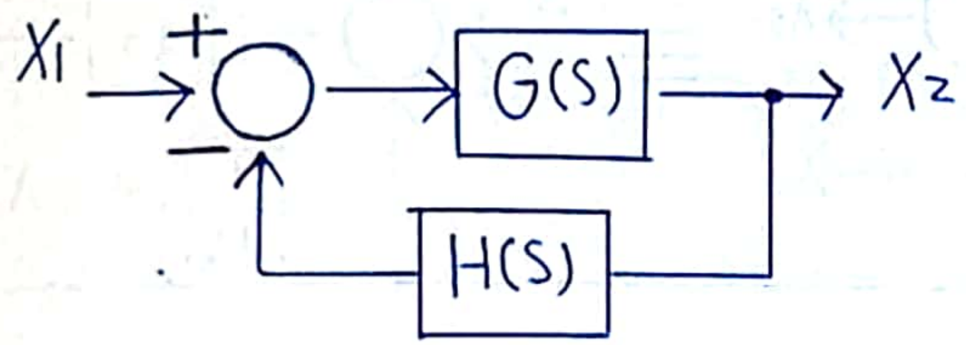
4. moving a pickoff point ahead of block :



5. moving a pickoff point behind a block :



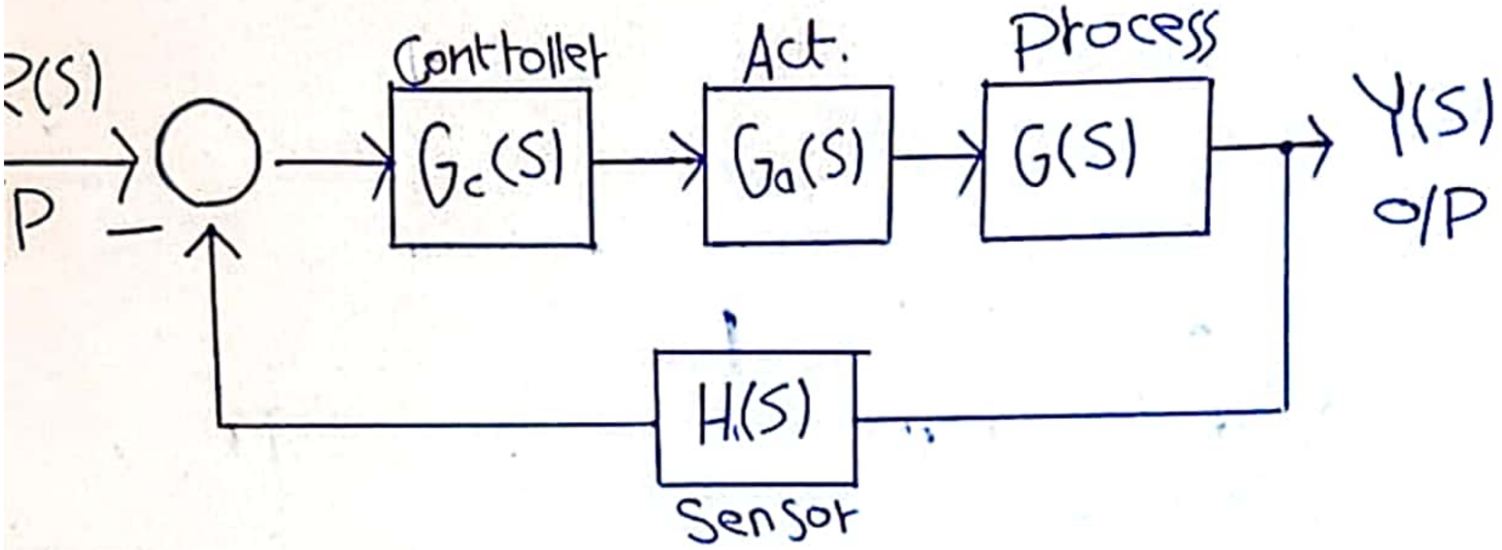
6. eleminating feedback loop :



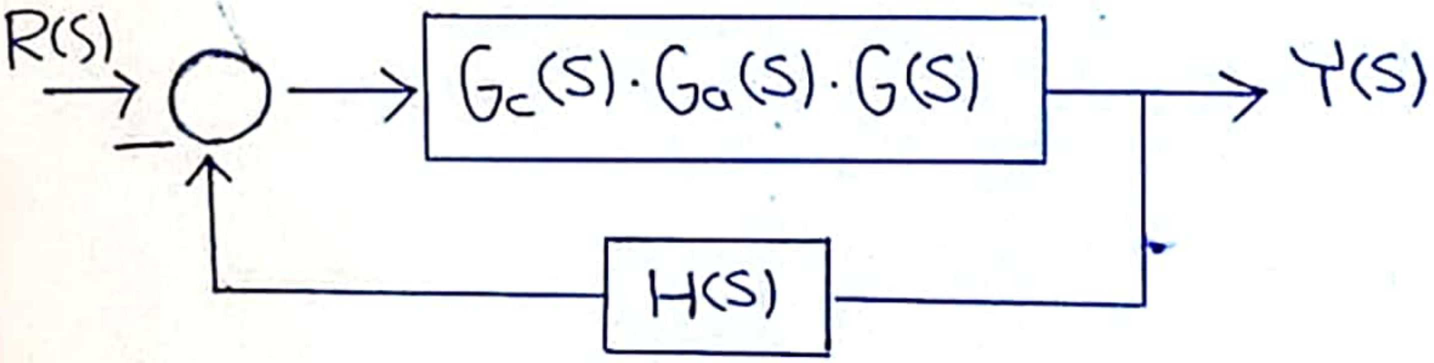
$$\boxed{\frac{X_2}{X_1} = \frac{G(S)}{1 \mp G(S)H(S)}}$$

EICoM
ELECTRICAL COMPUTER MECHATRONICS

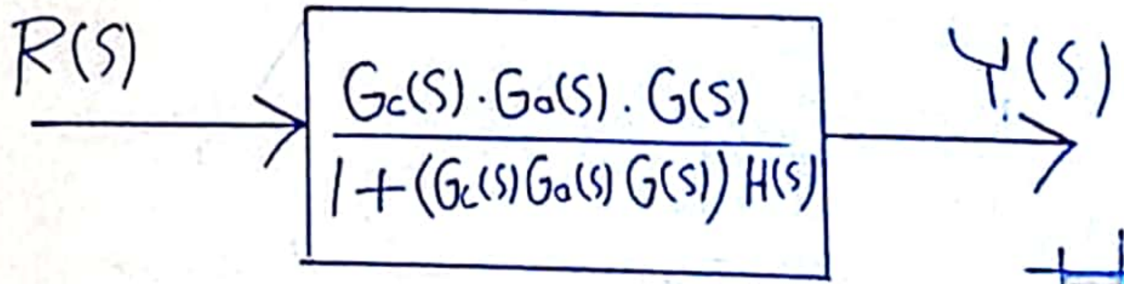
Example :



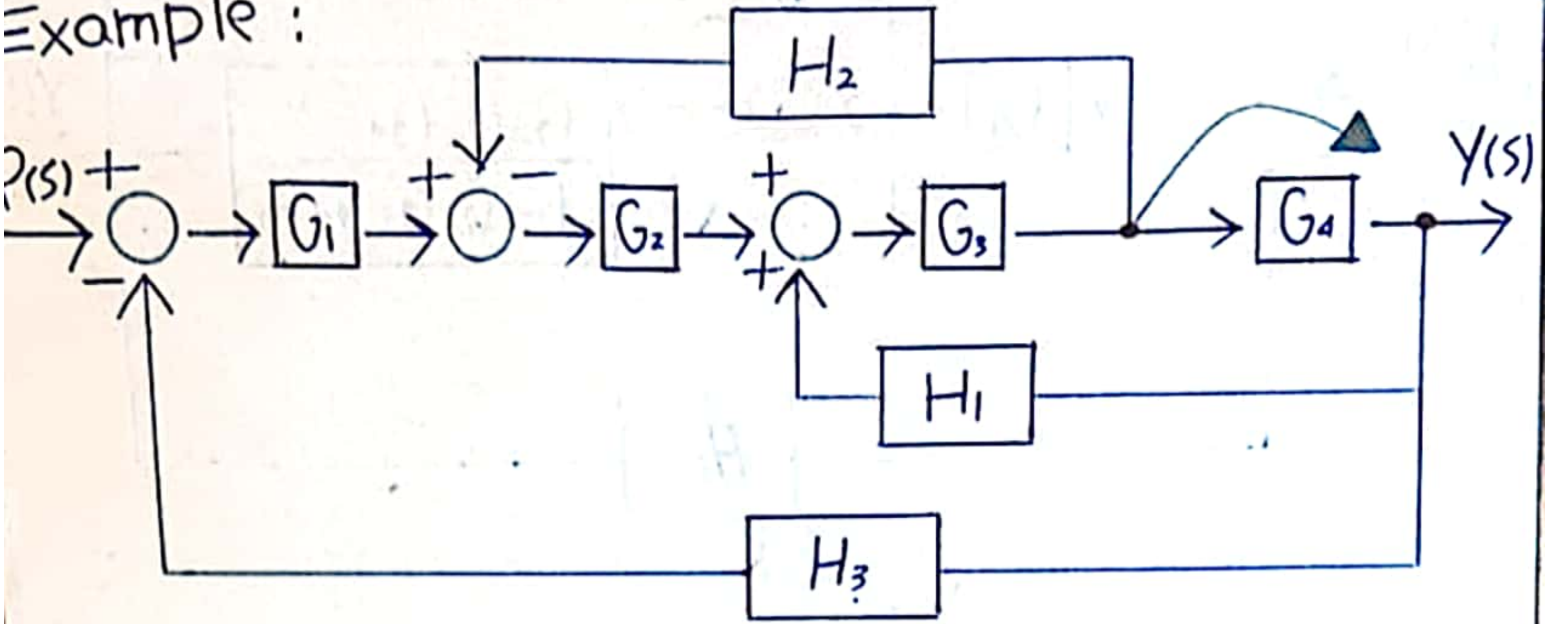
$$\frac{Y(S)}{R(S)} = ?$$



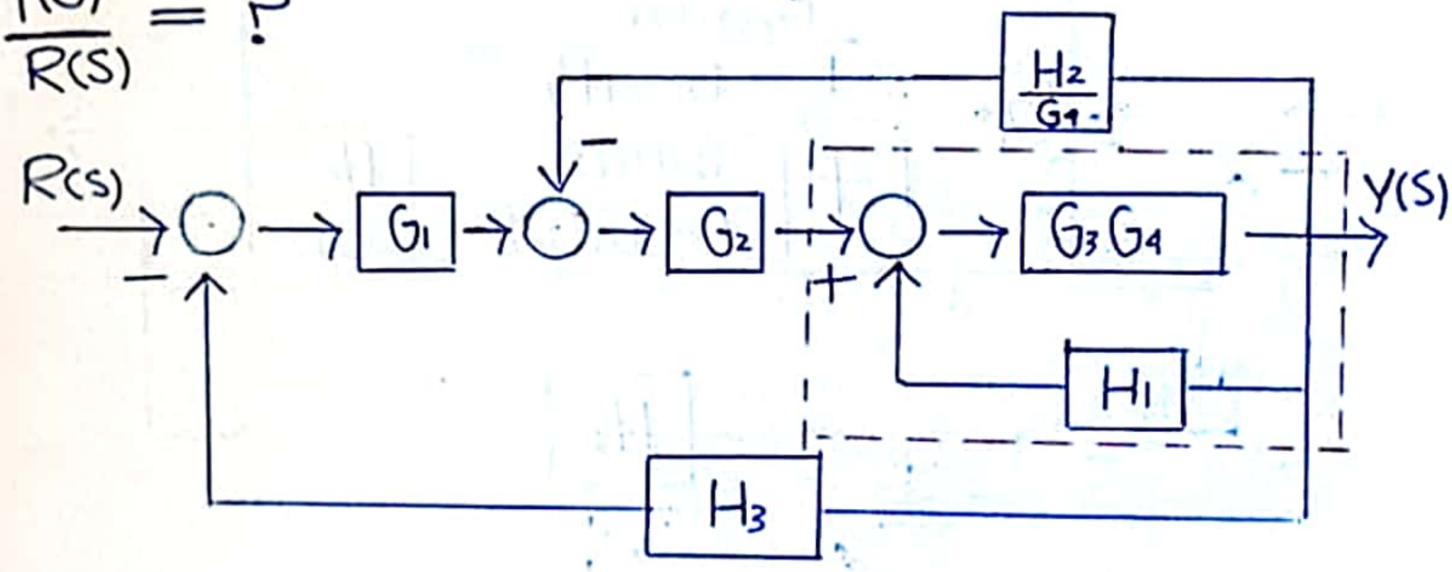
$$\frac{Y(S)}{R(S)} = \frac{G_c(S) \cdot G_a(S) \cdot G(S)}{1 + (G_c(S) \cdot G_a(S) \cdot G(S))\, H(S)}$$

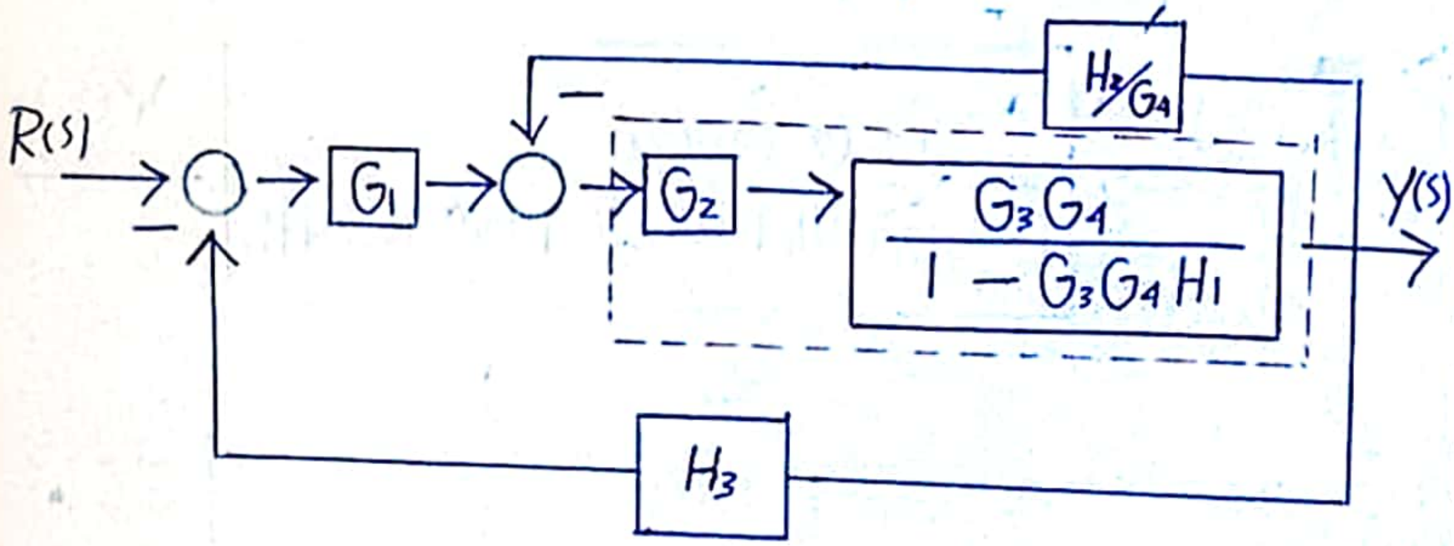


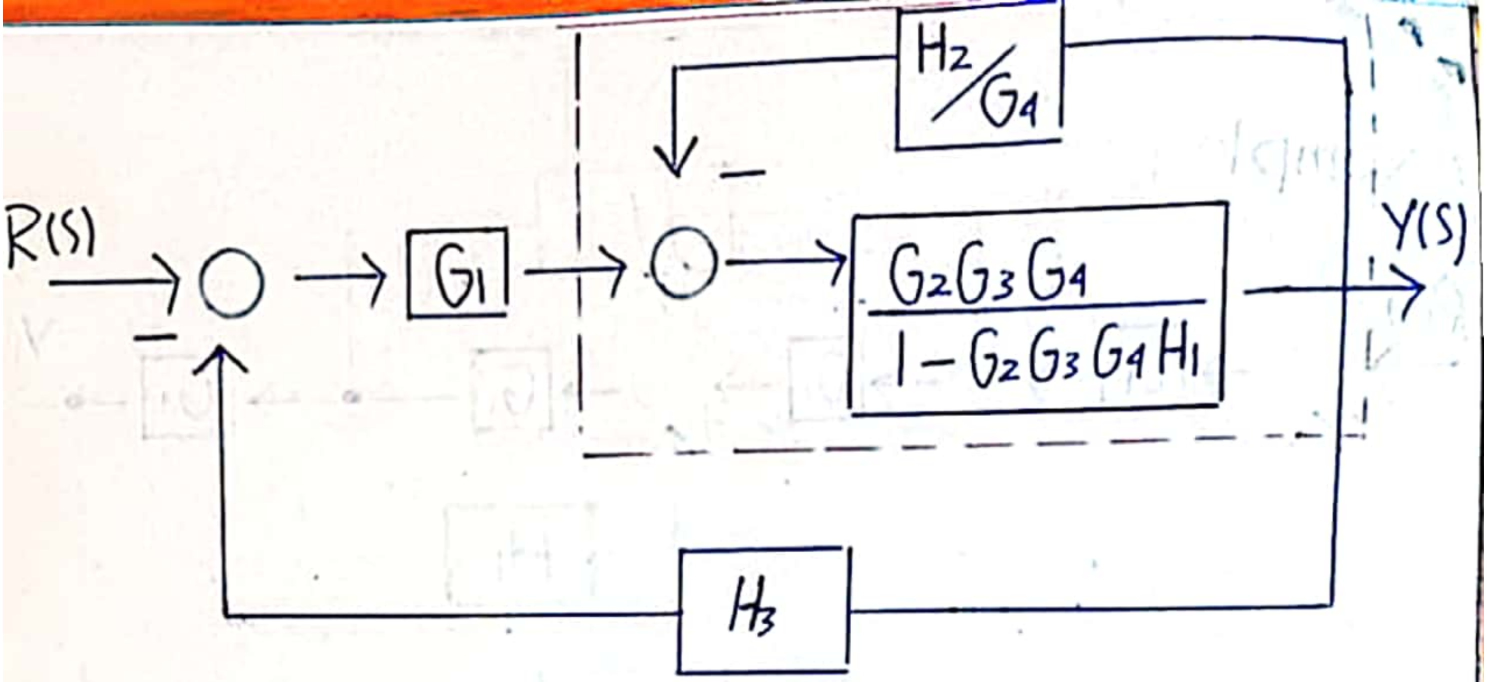
Example :

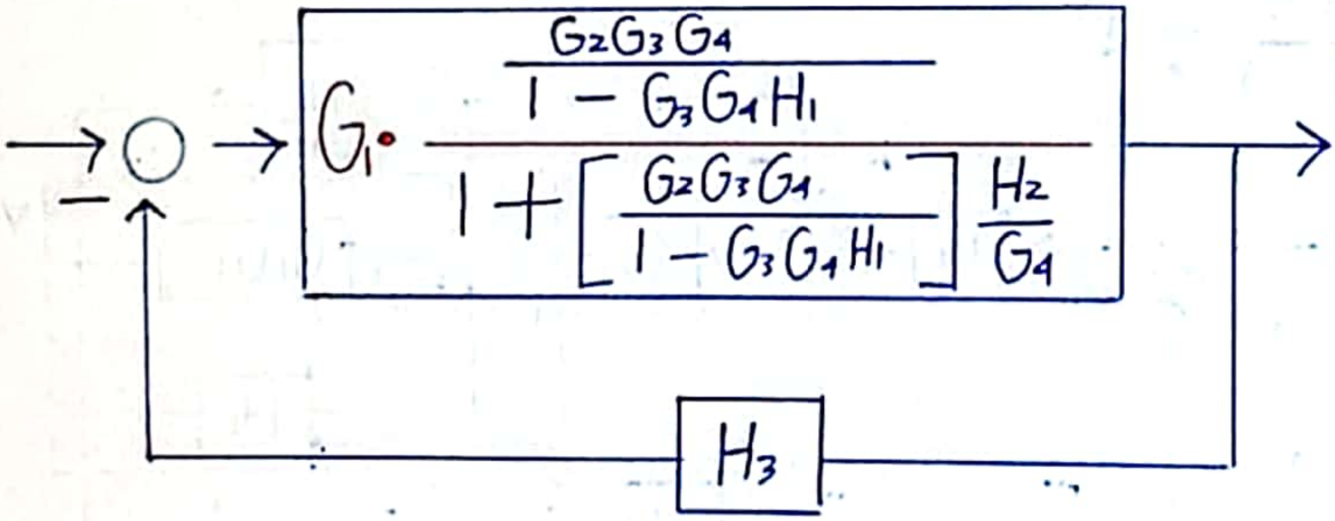


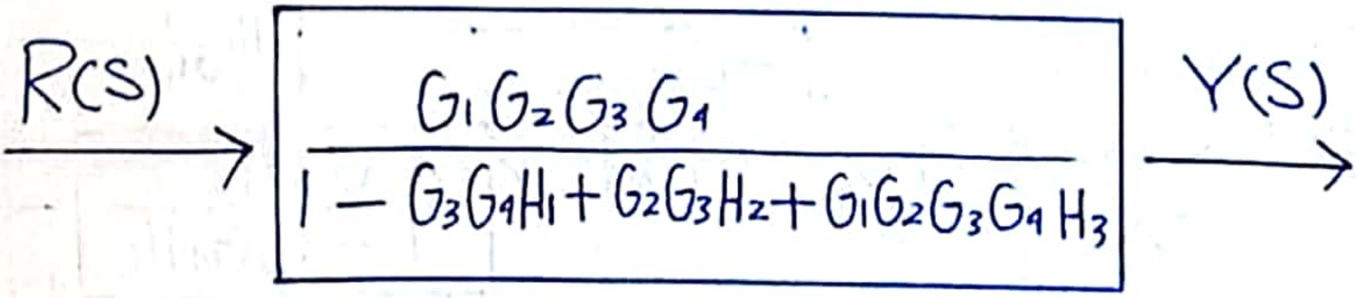
$$\frac{Y(S)}{R(S)} = ?$$






$R(S)$
G_1
H_2/G_4
$\frac{G_2G_3G_4}{1-G_2G_3G_4H_1}$
$Y(S)$
H_3


$G_1 \cdot \frac{\frac{G_2G_3G_4}{1-G_3G_4H_1}}{1+\left[\frac{G_2G_3G_4}{1-G_3G_4H_1}\right]\frac{H_2}{G_4}}$
H_3


$R(S)$
$\frac{G_1G_2G_3G_4}{1-G_3G_4H_1+G_2G_3H_2+G_1G_2G_3G_4H_3}$
$Y(S)$

#.

Signal Flow graph :-

is a diagram consisting of nodes that are connected by several directed branches and it's a graphical representation of a set of linear relations.

1 branch "transfer fun"

2 node (i/p / o/p / Junction)

3 Loop : is a closed path that originate and terminate on the same node.

L = G G G

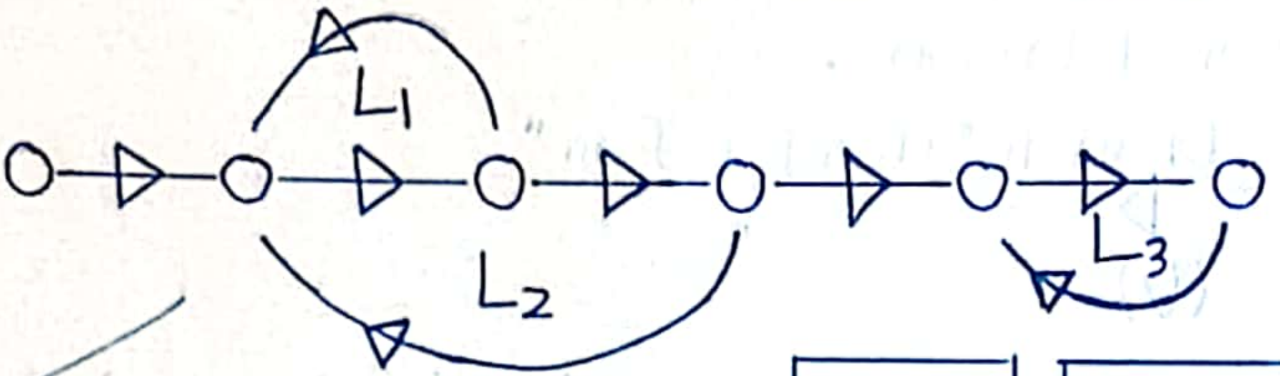
4 path :

$P_1 = G G$

$P_2 = G G$

5 Non-touching Loops:

They doesn't posses any common nodes, and branches.

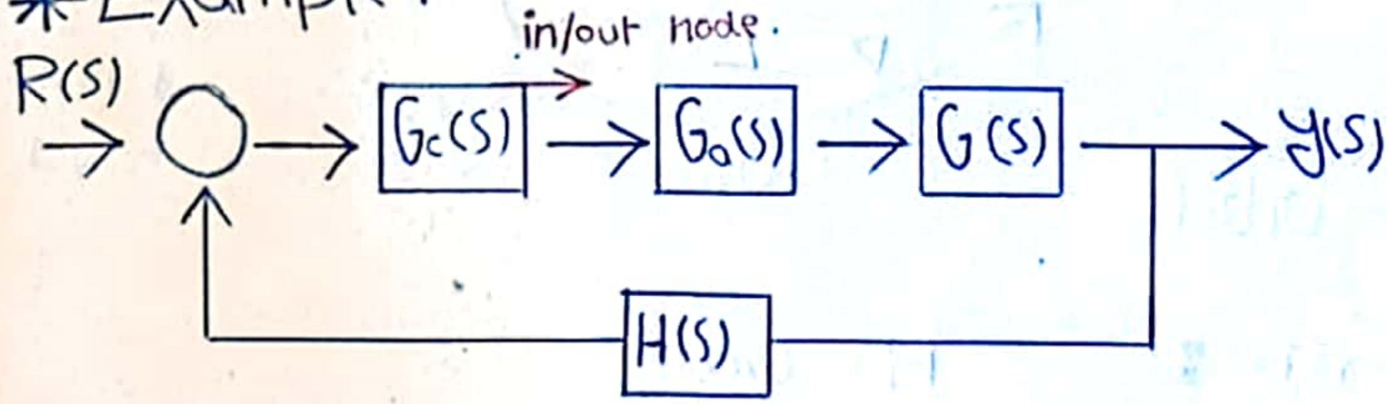


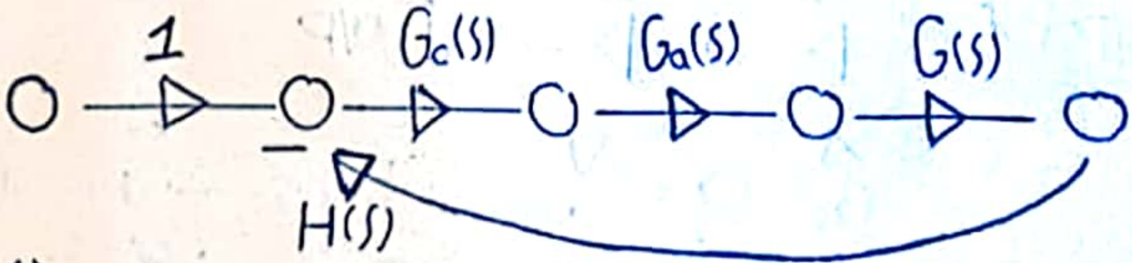
$\boxed{\text{1 path}}$ $\boxed{\text{3 Loops}}$

$\left.\begin{array}{l} L_1, L_3 \\ L_2, L_3 \end{array}\right\}$ non-touching Loops.

L_1, L_2 : touching Loop.

***Example:-**





1 Path
1 loop

$\frac{Y(S)}{R(S)}$? Using Mason's Rule #. 2

Mason's Rule :

$$\boxed{TF_{ij} = \frac{\sum P_{ijk}\,\Delta_{ijk}}{\Delta}}$$

k: # of paths.

P_{ijk} = gain of k^{th} path from variable x_i to x_j

Δ_{ijk} = Cofactor of the path P_{ijk}.

Δ = Determinant of graph.

= 1 − [Sum of all different Loop gains]

+ [Sum of the gain products of all Combinations of two non-touching Loops.]

− [Sum of the gain products of all Combinations of three non-touching Loops.]

+ (. . .

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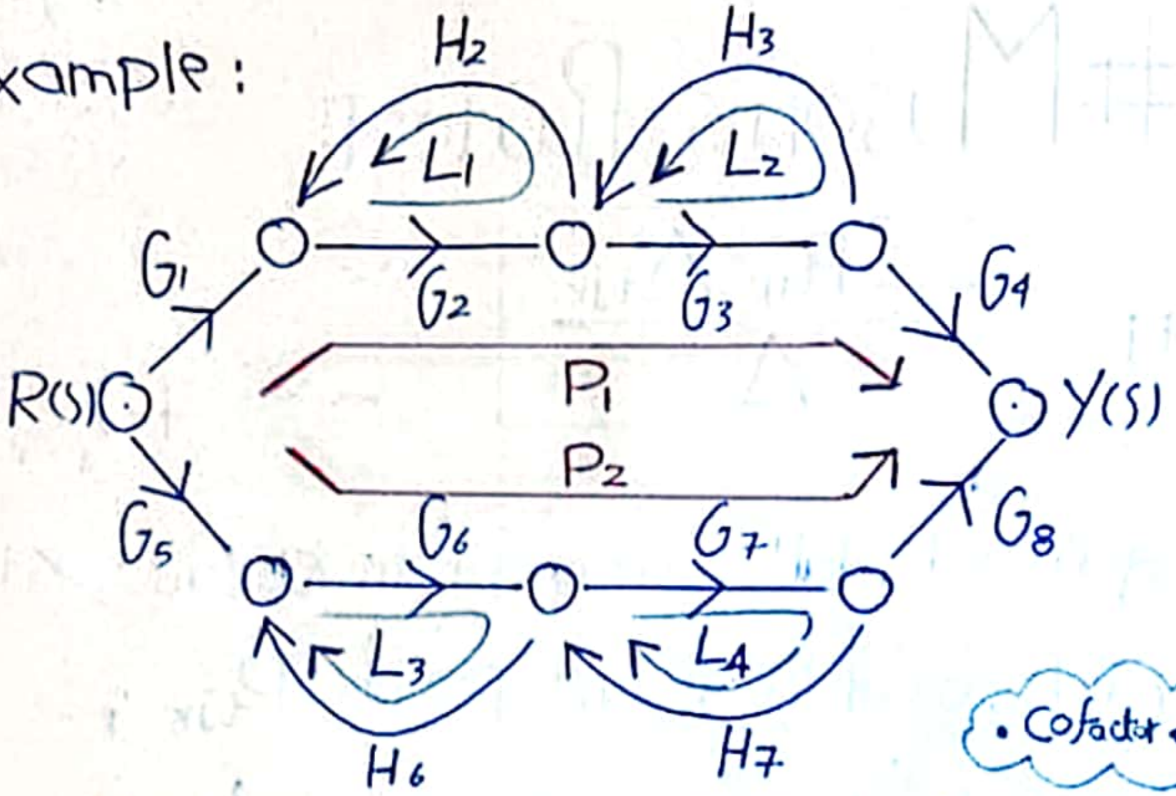
Example :



• Cofactor ← Path لـ

$P_1 = G_1G_2G_3G_4 \Longrightarrow \Delta_1 = 1 - (L_3 + L_4)$

$P_2 = G_5G_6G_7G_8 \Longrightarrow \Delta_2 = 1 - (L_1 + L_2)$

$(k = 2)$

# of Loops = 4 :

$L_1 = G_2H_2$

$L_2 = G_3H_3$

$L_3 = G_6H_6$

$L_4 = G_7H_7$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4) - (0)$$

→ two non-touching loops :-

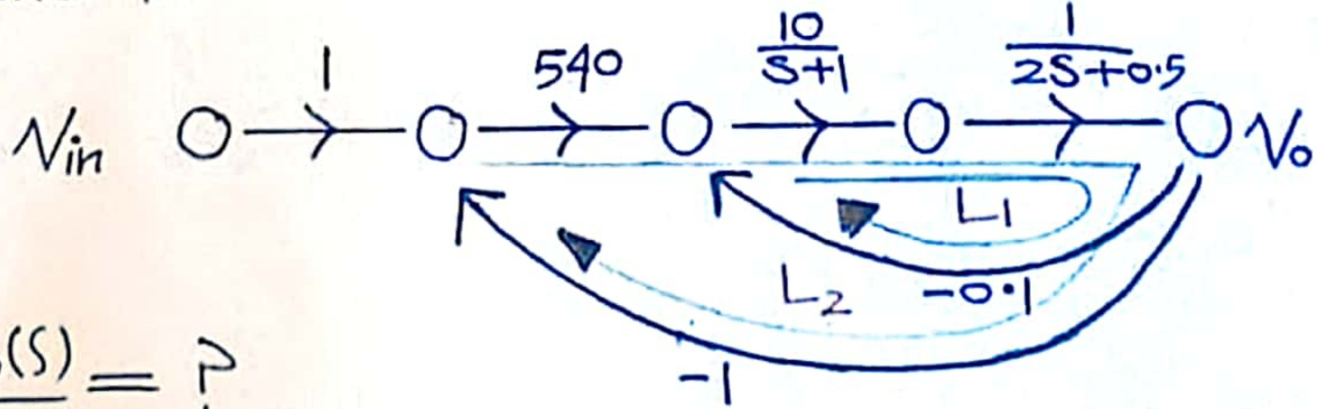
L_1L_3 ; L_1L_4 ; L_2L_3 ; L_2L_4 .

$$\frac{Y(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} \rightarrow$$

بدلالة G و H .

$$= \frac{(G_1G_2G_3G_4)(1-G_6H_6-G_7H_7)+(G_5G_6G_7G_8)(1-G_2H_2-G_3H_3)}{1-(G_2H_2+G_3H_3+G_6H_6+G_7H_7)+(G_2G_6H_2H_6+G_2G_7H_2H_7+G_3H_3G_6H_6+G_3G_7H_3H_7}$$

Example:



$$\frac{V_o(S)}{V_{in}(S)} = ?$$

$$P_1 = (1)(540)\left(\frac{10}{S+1}\right)\left(\frac{1}{2S+0.5}\right)$$

$$\Delta_1 = 1 - 0 = 1$$

$$L_1 = (540)\left(\frac{10}{S+1}\right)\left(\frac{1}{2S+0.5}\right)(-0.1)$$

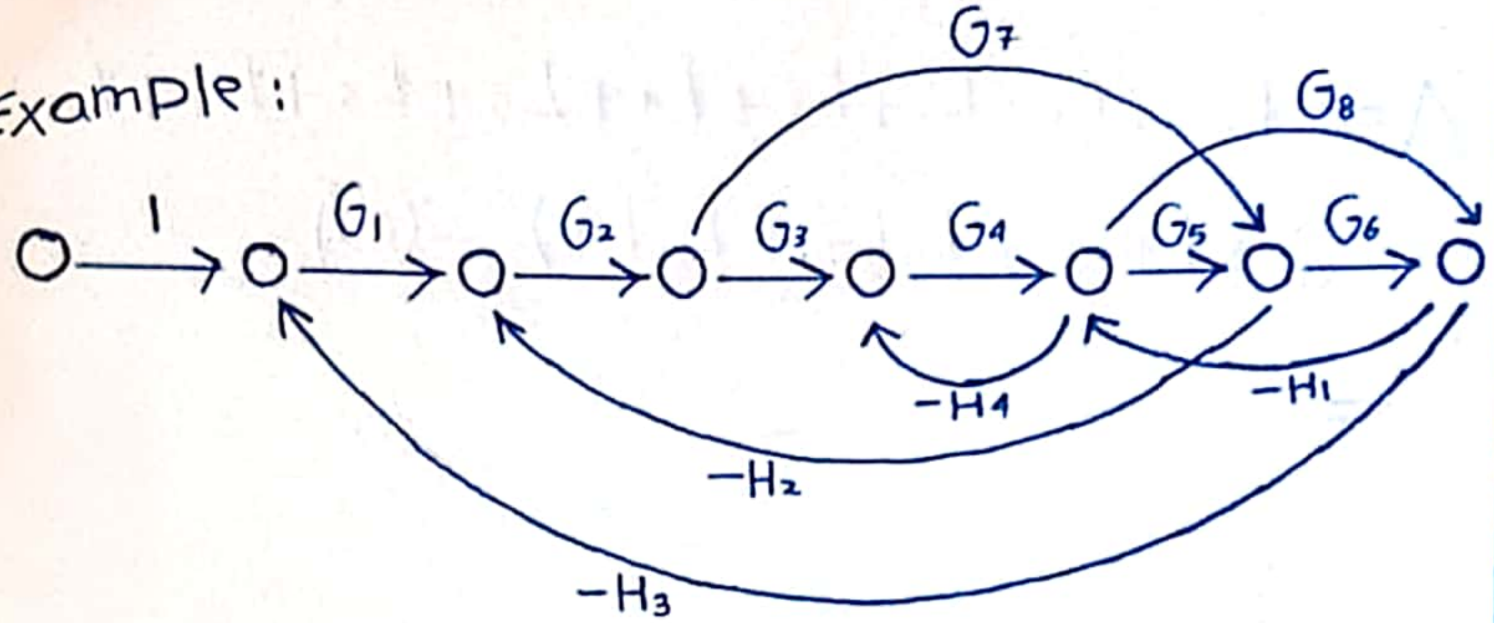
$$L_2 = (540)\left(\frac{10}{S+1}\right)\left(\frac{1}{2S+0.5}\right)(-1)$$

$$\Delta = 1 - (L_1 + L_2) + (0)$$

$$\frac{V_o(S)}{V_{in}(S)} = \frac{P_1\Delta_1}{\Delta}$$

$$= \frac{5400}{2S^2+2.5S+5400.5}$$

Example :



Solution :

$P_1 = G_1 G_2 G_3 G_4 G_5 G_6$

$P_2 = G_1 G_2 G_7 G_6$

$P_3 = G_1 G_2 G_3 G_4 G_8$

$L_1 = -G_2 G_3 G_4 G_5 H_2$

$L_2 = -G_5 G_6 H_1$

$L_3 = -G_8 H_1$

$L_4 = -G_2 G_7 H_2$

$L_5 = -G_4 H_4$

$L_6 = -G_1 G_2 G_3 G_4 G_5 G_6 H_3$

$L_7 = -G_1 G_2 G_7 G_6 H_3$

$L_8 = -G_1 G_2 G_3 G_4 G_8 H_3$

$\Delta_1 = 1 - 0 = 1$

$\Delta_2 = 1 - L_5 = 1 + G_4 H_4$

$\Delta_3 = 1 - 0 = 1$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8) + (L_1L_5 + L_5L_7 + L_3L_4) - (0)$$

$$=$$

\# two non-touching loops :

L_1L_5 ; L_5L_7 ; L_3L_4

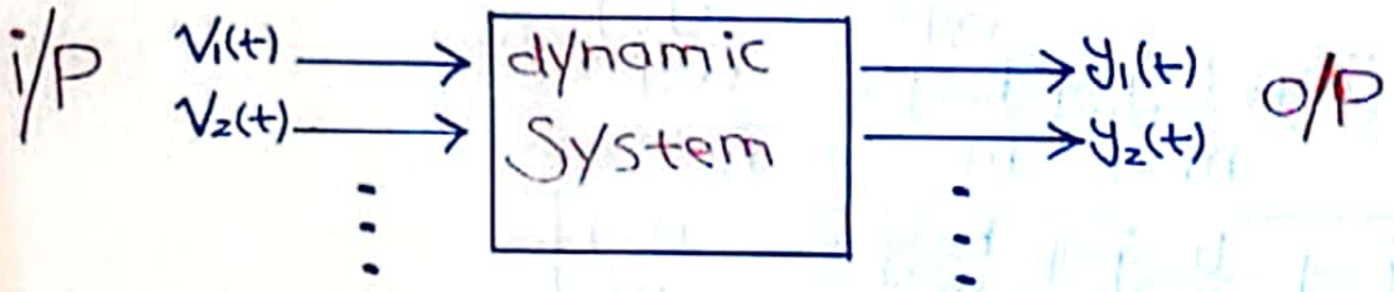
$$\rightarrow \frac{Y(s)}{R(s)} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}$$

$$= \frac{P_1 + P_2\Delta_2 + P_3}{\Delta}$$

$$=$$

Chapter 3: State Variable Model :-

is a set of variables whose values together with the i/P signals and the equations describing the dynamics will provide the future state and O/P of the system.



$$\boxed{\begin{aligned}\dot{X} &= AX + Bu(t)\\ y &= CX + Du(t)\end{aligned}}$$

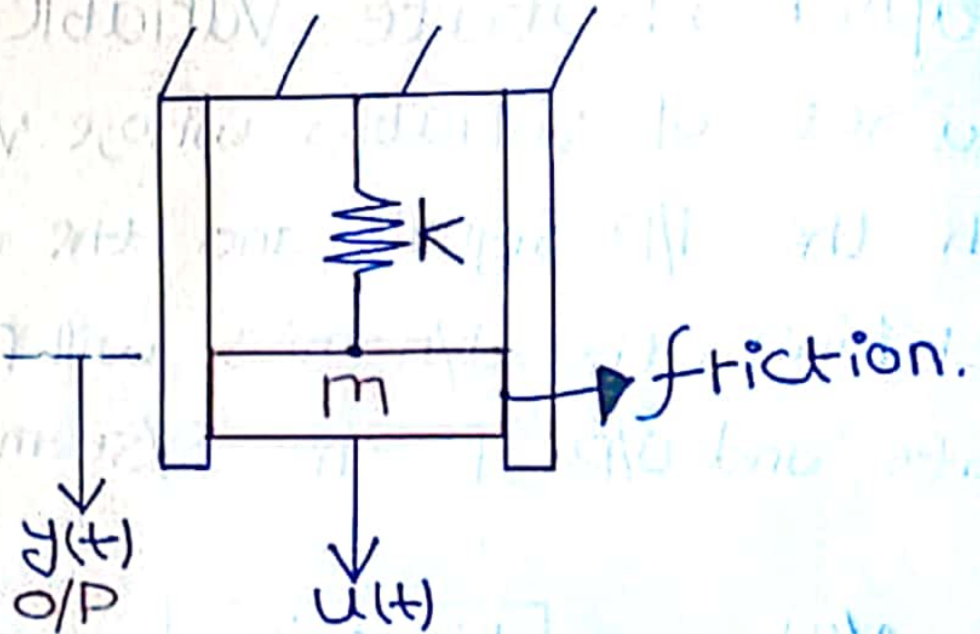
i/P

O/P

$$\dot{X} = \begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \vdots\end{bmatrix}, \quad u = \begin{bmatrix}u_1\\ u_2\\ \vdots\end{bmatrix}$$

$$X = \begin{bmatrix}x_1\\ x_2\\ \vdots\end{bmatrix} = \text{State variable}.$$

Example :



$$\frac{m}{m}\ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y = \frac{u(t)}{m}$$

$$\boxed{\ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y = \frac{u(t)}{m}}$$

1) assume two state variables :

$Z_1 = y \rightsquigarrow$ displacement.

$Z_2 = \dot{y} \rightsquigarrow$ velocity .

2) نشتق

$\dot{Z}_1 = \dot{y} = Z_2$.

$\dot{Z}_2 = \ddot{y}$

3) $\ddot{y} + \frac{b}{m}\dot{y} + \frac{k}{m}y = \frac{u(t)}{m}$

بدلالة State ال Var.

$$\dot{Z}_2 + \frac{b}{m}Z_2 + \frac{k}{m}Z_1 = \frac{u(t)}{m}$$

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1 $\rightarrow \dot{z} = AZ + Bu$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} u(t)$$

$$\rightarrow \boxed{\dot{z}_1 = z_2}$$

$$\rightarrow \boxed{\dot{z}_2 = \frac{u}{m} - \frac{b}{m} z_2 - \frac{k}{m} z_1}$$

$\therefore$

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix} ; B = \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix}.$$

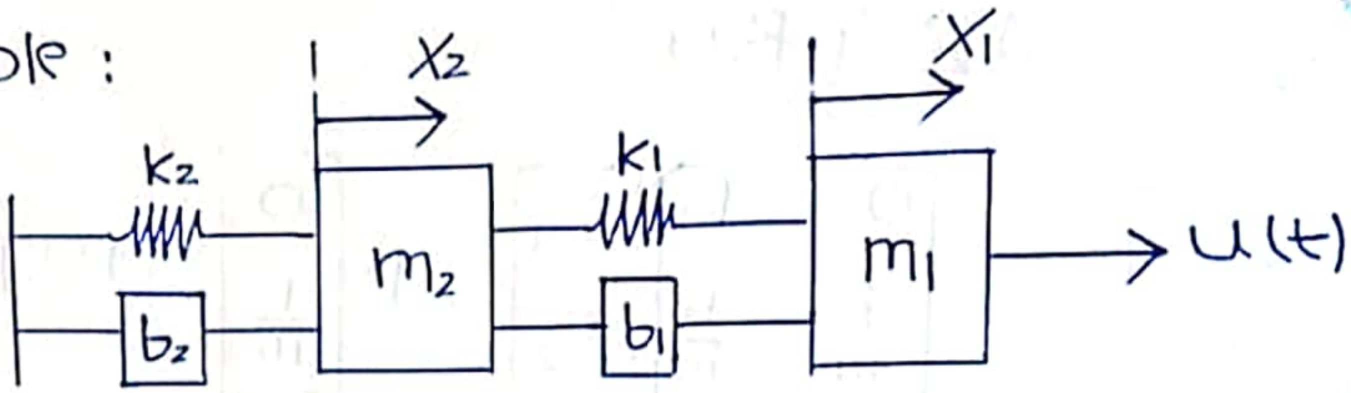
② $\rightarrow y = CZ + Du$

$$\rightarrow y(t) = z_1$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 \end{bmatrix}}_{C} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u(t)$$

$\#$.

Example :



assume $y = x_1$;

① $\rightarrow m_2 \ddot{x}_2 + b_2 \dot{x}_2 + k_2 x_2 + k_1 (x_2 - x_1) + b_1 (\dot{x}_2 - \dot{x}_1) = 0$

② $\rightarrow m_1 \ddot{x}_1 + k_1 (x_1 - x_2) + b_1 (\dot{x}_1 - \dot{x}_2) = u(t)$.

Let : $z_1 = x_1 \longrightarrow \boxed{\dot{z}_1 = \dot{x}_1 = z_2}$.

$z_2 = \dot{x}_1 \longrightarrow \dot{z}_2 = \ddot{x}_1$

$z_3 = x_2 \longrightarrow \boxed{\dot{z}_3 = \dot{x}_2 = z_4}$.

$z_4 = \dot{x}_2 \longrightarrow \dot{z}_4 = \ddot{x}_2$

① $\ddot{x}_2 + \frac{b_2}{m_2} \dot{x}_2 + \frac{k_2}{m_2} x_2 + \frac{k_1}{m_2} (x_2 - x_1) + \frac{b_1}{m_2} (\dot{x}_2 - \dot{x}_1) = 0$

$\rightarrow \boxed{\dot{z}_4 = -\frac{b_2}{m_2} z_4 - \frac{k_2}{m_2} z_3 - \frac{k_1}{m_2} (z_3 - z_1) - \frac{b_1}{m_2} (z_4 - z_2)}$

② $\ddot{x}_1 + \frac{k_1}{m_1} (x_1 - x_2) + \frac{b_1}{m_1} (\dot{x}_1 - \dot{x}_2) = \frac{u(t)}{m_1}$.

$\rightarrow \boxed{\dot{z}_2 = -\frac{k_1}{m_1} (z_1 - z_3) - \frac{b_1}{m_1} (z_2 - z_4) + \frac{u(t)}{m_1}}$

$$\underbrace{\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{bmatrix}}_{} = \underbrace{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_1}{m_1} & -\frac{b_1}{m_1} & \frac{k_1}{m_1} & \frac{b_1}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k_1}{m_1} & \frac{b_1}{m_1} & \frac{-k_2-k_1}{m_2} & \frac{-b_1-b_2}{m_2} \end{bmatrix}}_{A} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ \frac{1}{m_1} \\ 0 \\ 0 \end{bmatrix}}_{B} u(t)$$

$$\longrightarrow y = x_1 \longrightarrow y = z_1$$

$$y = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}}_{C} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u(t)$$

#.

Transfer function from State Space

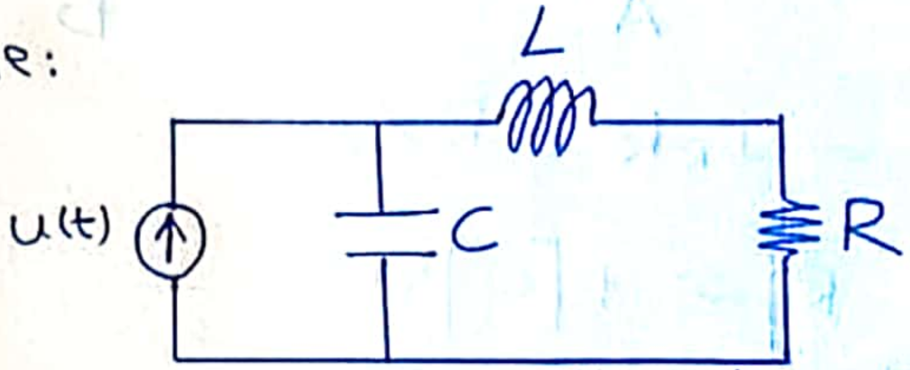
$$\frac{Y(S)}{U(S)} = C[SI - A]^{-1}B + D$$

$\hookrightarrow$ identity matrix.

$\longrightarrow$ output in time domain:

$$\mathcal{L}^{-1}\{X(S)\} = \underbrace{\phi(t)}_{\text{transition matrix}} X(0) + \int_0^t \phi(t-\tau)B\,u(\tau)\,d\tau$$

Example:



assume $z_1 = V_c$, $z_2 = i_L$, $y = V_R$ ---

$$\dot{z}_1 = \dot{V}_c$$
$$\dot{z}_2 = \dot{I}_L$$

$$V_c = \frac{1}{C}\int I_c \cdot dt \rightarrow \dot{V}_c = \frac{I_c}{C} = \dot{z}_1$$

$$\dot{z}_1 = \frac{u(t) - I_L}{C} = \frac{u(t) - z_2}{C}.$$

$$z_1 = L\frac{di_1}{dt} + I_R R \quad , I_R = I_L = I_2$$

$$= L\dot{z}_2 + R z_2$$

$$\dot{z}_2 = \frac{z_1 - z_2 R}{L}.$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & -\frac{1}{C} \\ \frac{1}{L} & -\frac{R}{L} \end{bmatrix}}_{A.} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix}}_{B.} u(t)$$

$$y = V_R = I_R R = R z_2$$

$$y = \underbrace{\begin{bmatrix} 0 & R \end{bmatrix}}_{C} z + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{D} u$$

Let. $R = 3$, $L = 1$, $c = \frac{1}{2}$:-

$$T(s) = \begin{bmatrix} 0 & 3 \end{bmatrix} [sI - A]^{-1} * \begin{bmatrix} 2 \\ 0 \end{bmatrix} + [0]$$

$$SI-A=\begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix}-\begin{bmatrix} 0 & -2 \\ 1 & -3 \end{bmatrix}=\begin{bmatrix} S & 2 \\ -1 & S+3 \end{bmatrix}$$

$$[SI-A]^{-1}=\begin{bmatrix} S+3 & -2 \\ 1 & S \end{bmatrix} * \frac{1}{\Delta}$$

$$\longrightarrow \Delta = S(S+3)+2 = S^2+3S+2$$

$$T(S)=\frac{1}{\Delta}\begin{bmatrix} 0 & 3 \end{bmatrix}\begin{bmatrix} S+3 & -2 \\ 1 & S \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix}+0$$

$$=\begin{bmatrix} \frac{3}{\Delta} & \frac{3S}{\Delta} \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$=\frac{6}{S^2+3S+2}$$ #.

→ transition matrix :-

$$\phi(S) = [SI - A]^{-1}$$

$$\phi(S) = \frac{1}{\Delta}\begin{bmatrix} S+3 & -2 \\ 1 & 5 \end{bmatrix}$$

$$\mathcal{L}^{-1}\{\phi(S)\} = \mathcal{L}^{-1}\begin{bmatrix} \frac{S+3}{S^2+3S+2} & \frac{-2}{S^2+3S+2} \\ \frac{1}{S^2+3S+2} & \frac{5}{S^2+3S+2} \end{bmatrix}$$

$$\frac{\quad}{S^2+3S+2} = \frac{A}{S+1} + \frac{B}{S+2}$$

→ partial fraction 4 times :

$$\phi(t) = Ae^{-t} + Be^{-2t}$$

$$\phi(t) = \begin{bmatrix} 2e^{-t} - e^{-2t} & -2e^{-t} + 2e^{-2t} \\ e^{-t} - e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

#.

Chapter 4 ;Feedback Control System

Characteristics :-

1. open loop Vs Closed loop System.
2. Error Signal (closed loop) and Steady-State error.
3. Sensitivity of control system to Parameter variation.
4. Disturbance Signals in a feedback Control System.

---

a. An open loop Signal operates without feedback and directly generates the output in response to an input Signal.

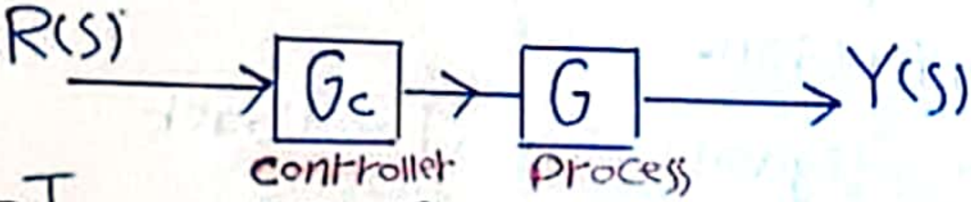
b. A Closed loop system uses a measurment of the output signal and a Comparison with the desired output to generate an error Signal that is used by the Controller to adjust the actuator.

---

1) Sensitivity :- is the ratio of the Change in the System Transfer fun. to change in process Transfer fun. for a small incremental change.

$$\boxed{S_G^T = \frac{dT}{dG} \cdot \frac{G}{T}}$$

a. open loop:



$$S_G^T = \frac{dT}{dG} \cdot \frac{G}{T}$$

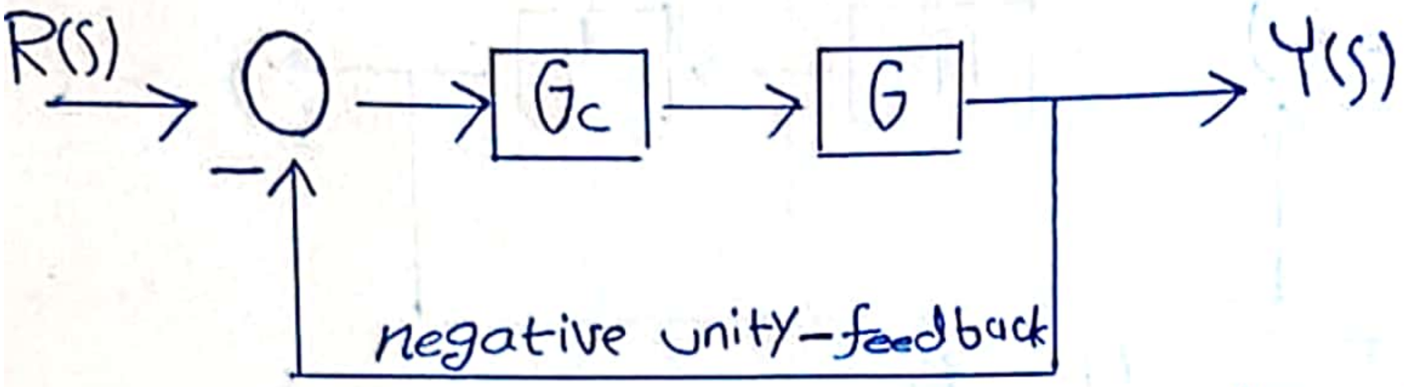
$$T = G_c G$$

$$\frac{dT}{dG} = G_c$$

$$\rightarrow S_G^T = G_c \cdot \frac{G}{G_c G} = 1$$, high Sensitivity (100%)

بتغير (بتأثر) بشكل
كبير بأي عامل
جواته.

b. Closed loop :



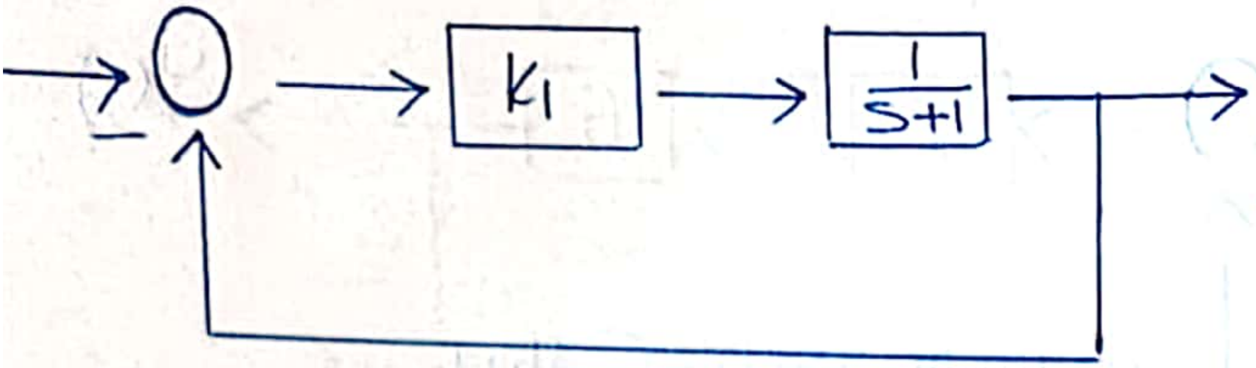
$$S_G^T = \frac{dT}{dG} \cdot \frac{G}{T}$$

$$T = \frac{GG_c}{1+GG_c}$$

$$\frac{dT}{dG} = \frac{(1+GG_c)(G_c) - (GG_c)(G_c)}{(1+GG_c)^2} = \frac{G_c}{(1+GG_c)^2}$$

$$\rightarrow S_G^T = \frac{G_c}{(G_cG+1)^2} \cdot \frac{G}{\frac{G_cG}{1+G_cG}} = \frac{1}{1+G_cG}$$; low Sensit..

Example :



$$S_{K_1}^{T} = \frac{dT}{dK_1} \cdot \frac{K_1}{T}$$

$$T = \frac{\frac{K_1}{S+1}}{\frac{K_1}{S+1} + 1} = \frac{K_1}{K_1 + S + 1}$$

$$\frac{dT}{dK_1} = \frac{(K_1+S+1)(1) - K_1(1)}{(K_1+S+1)^2} = \frac{S+1}{(S+1+K)^2}$$

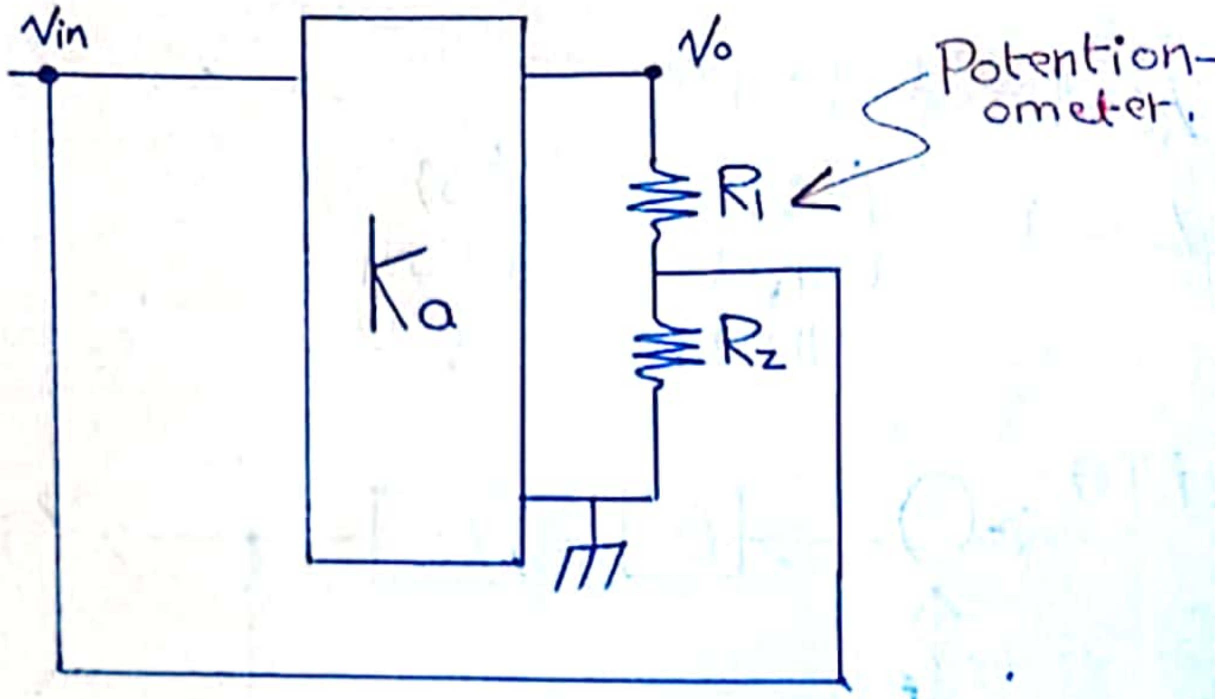
$$\rightarrow S_{K_1}^{T} = \frac{S+1}{(S+1+K)^2} \cdot \frac{K_1}{\frac{K_1}{K_1+S+1}} = \frac{S+1}{S+1+K_1}$$

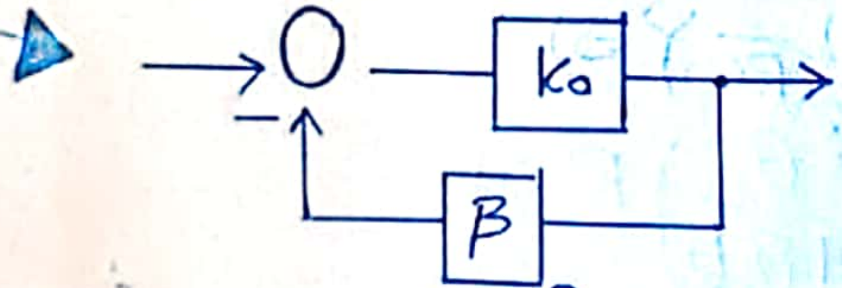
Example : Find S_{Ka}^{T} ?

$$T = \frac{V_o}{V_{in}} = K_a$$

$$S_{Ka}^{T} = \frac{dT}{dK_a} \cdot \frac{K_a}{T} = 1 \cdot 1 = 1$$

Example :





$\beta = \frac{R_2}{R_1}$

$$T = \frac{K_a}{1 + K_a \beta}$$

$$\frac{dT}{dK_a} = \frac{(1 + K_a\beta)(1) - K_a(\beta)}{(1 + K_a\beta)^2} = \frac{1}{(1 + K_a\beta)^2}$$

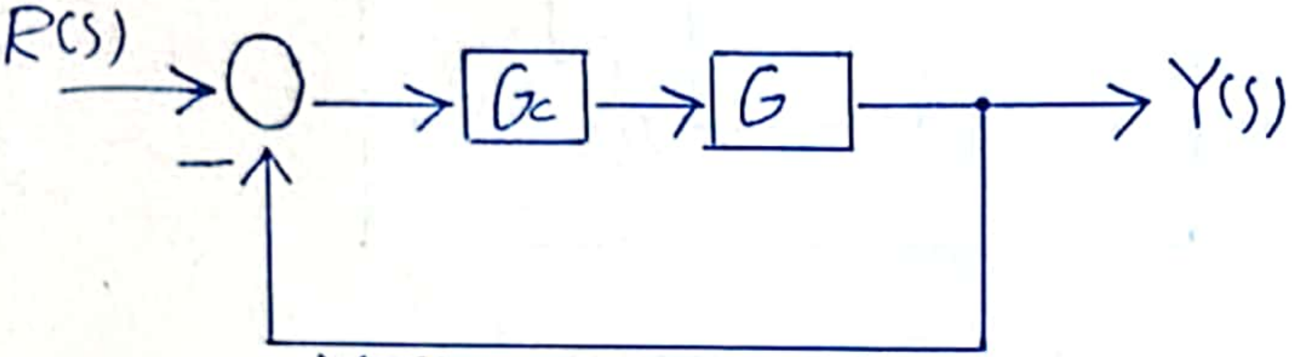
$$\rightarrow S_{K_a}^{T} = \frac{dT}{dK_a} \cdot \frac{K_a}{T} = \frac{1}{(1 + K_a\beta)^2} \cdot \frac{K_a}{\frac{K_a}{1 + K_a\beta}}$$

$$= \frac{1}{1 + K_a\beta}$$

2) Error Signal :

* Closed loop:

$$E(S) = \underset{\text{desired O/P}}{R(S)} - \underset{\text{actual O/P}}{Y(S)}$$



$$T(S) = Y(S)/R(S)$$

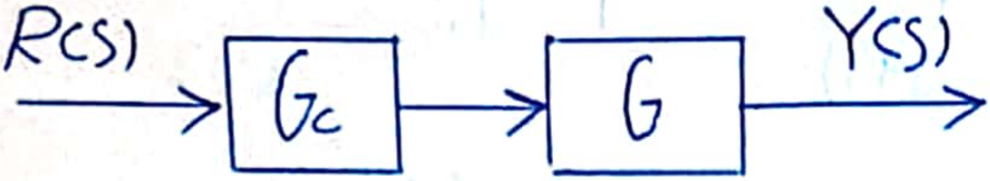
$$Y(s) = \left(\frac{G_c G}{1+G_c G}\right) R(S)$$

$$\rightarrow E(S) = R(S) - \left(\frac{G_c G}{1+G_c G}\right) R(S)$$

$$= R(S)\left(1 - \frac{G_c G}{1+G_c G}\right)$$

$$= \frac{R(S)}{1+G_c G} \quad \#$$

* open loop :



$$E(S) = R(S) - Y(S)$$
$$= R(S) - (G_c G) R_S$$
$$= R(S)(1 - G_c G).$$

---

* Final Value Theorem :

$$\lim_{S \to 0} S E(S) = \lim_{T \to \infty} E(t)$$

Steady State Error / Steady State value

Example:

$$E(S) = \frac{1}{S+1} \rightarrow E(t) = e^{-t}$$

$$\rightarrow \lim_{S \to 0} \frac{S}{S+1} = 0$$

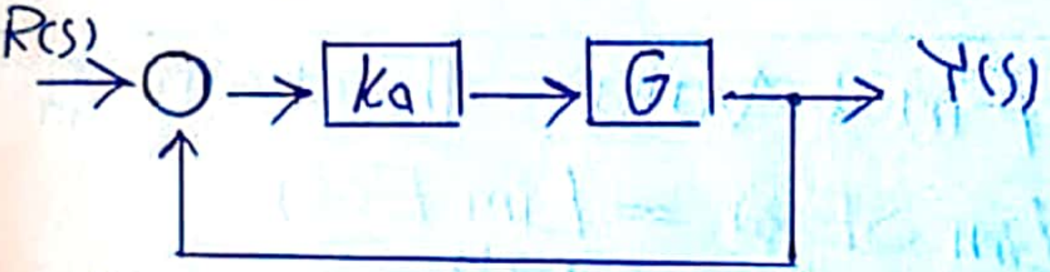
$$\rightarrow \lim_{T \to \infty} e^{-t} = 0$$

$$\frac{1}{S(S+1)}$$

$$\lim_{S \to 0} S \cdot \frac{1}{S(S+1)} = 1 \ \#$$

Example: $G(S) = \dfrac{10}{S(0.001S+1)}$

for a unit step input, Find Steady State Error ?



$$E = R(S) - Y(S)$$

$$= R(S)\left(1 - \frac{KaG}{1+KaG}\right)$$

$\rightarrow R(S) = \dfrac{1}{S}$ ...(unit step input) #

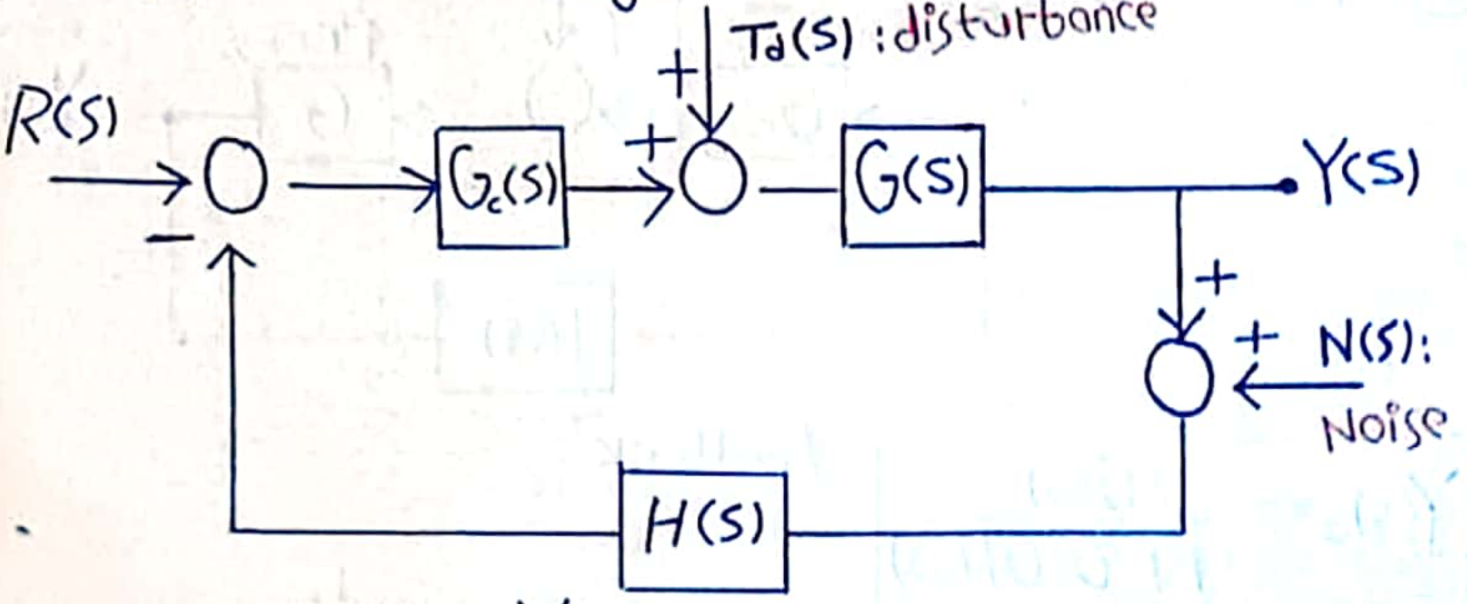
$$E = \frac{R(S)}{1+KaG(S)}$$

$$\rightarrow \lim_{S \to 0} S \cdot \frac{R(S)}{1+KaG(S)} = \lim_{S \to 0} S \frac{1/S}{1+Ka\left[\frac{10}{S(0.001S+1)}\right]}$$

$$= 0$$

#

3) Disturbance rejection :-



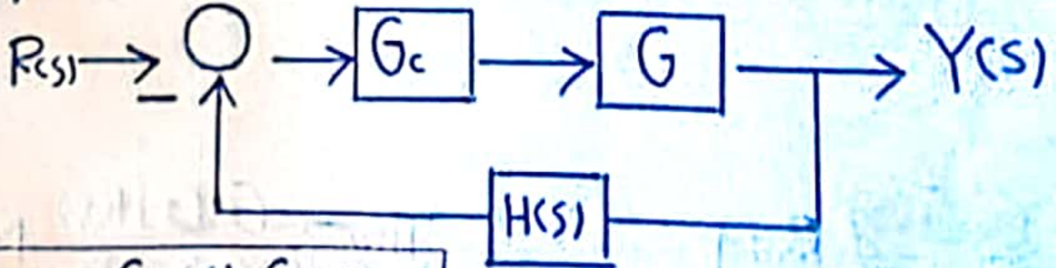
$E(S) = R(S) - Y(S)$

$\rightarrow Y(S) = \underline{\qquad} R(S) + \underline{\qquad} Td(S) + \underline{\qquad} N(S)$

T.F input
3 i/P

Using Superposition :

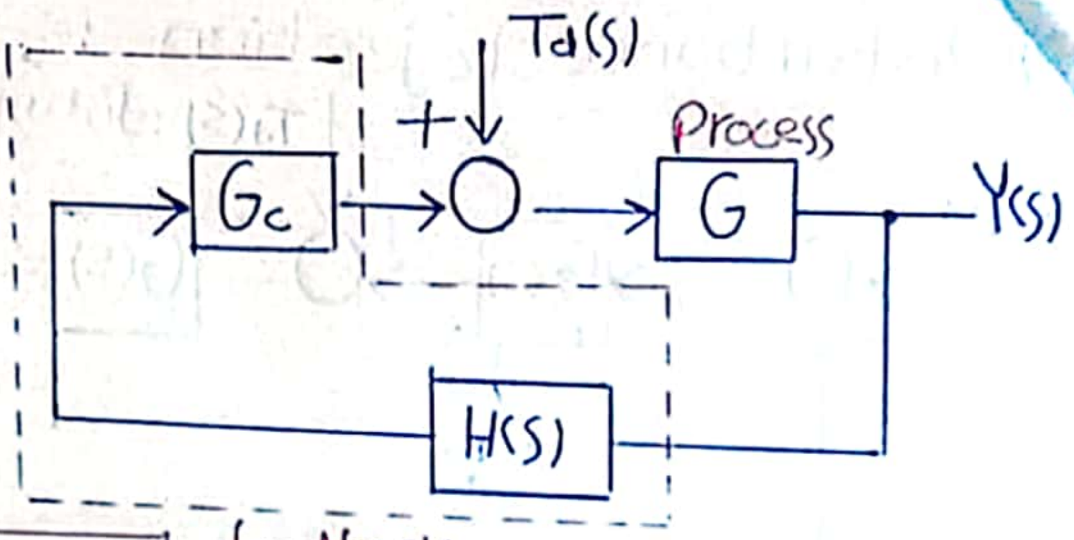
① R(S) ✓



$$\boxed{Y(S)_1 = \frac{G_c(S)\,G(S)}{1 + G_c G H(S)}}$$

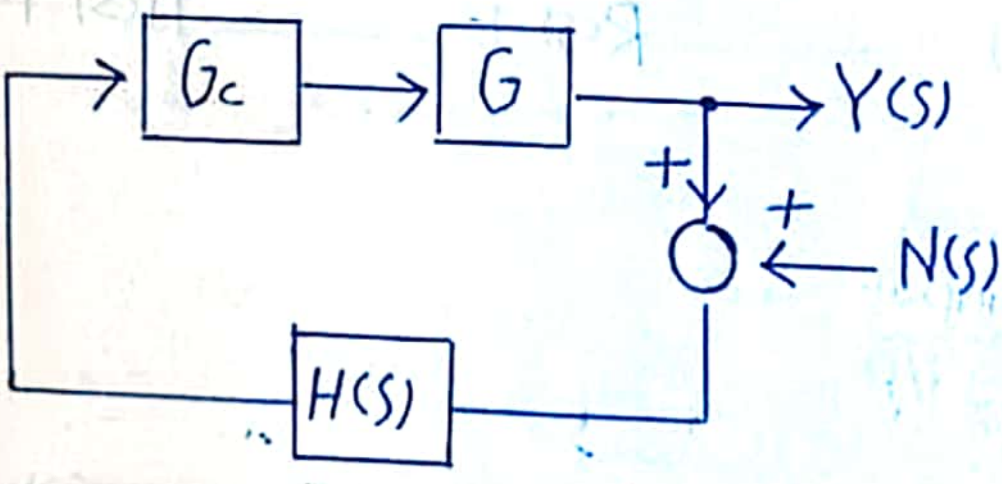
Scanned with CamScanner

② Td(S) ✓



$$\boxed{Y(S)_2 = \frac{G(S)}{1 + G_c G H(S)}}$$

③ N(S) ✓



$$\boxed{Y(S)_3 = \frac{G_c G H(S)}{1 + G_c G H(S)}}$$

∴

$$Y(s) = \underbrace{\frac{G_c G(S)}{1+G_c G H(S)}}R(s) + \underbrace{\frac{G(S)}{1+G_c G H(S)}}T_d(s) - \underbrace{\frac{G_c G H(S)}{1+G_c G H(S)}}N(S).$$

Assume $H(S) = 1$;

$$\rightarrow Y(s) = \underbrace{\frac{G_c G}{1+G_c G}}\, R(s) + \underbrace{\frac{G(s)}{1+G_c G}}\, T_d(s) - \underbrace{\frac{G_c G}{1+G_c G}}\, N(s)$$

$$\rightarrow E(s)\Big|_{R(s)} = R(s) - \frac{G_c G}{1+G_c G} R(s)$$

$$= R(s)\left[1 - \frac{G_c G}{1+G_c G}\right]$$

$$= \frac{R(s)}{1+G_c G} .$$

$$\rightarrow E(s)\Big|_{T_d(s)} = \underbrace{R(s)}_{\text{zero}} - \frac{G(s)}{1+G_c G} T_d(s)$$

$$= \frac{-G(s)}{1+G_c G} T_d(s) .$$

$$\rightarrow E(s)\Big|_{N(s)} = \underbrace{R(s)}_{\text{zero}} + \frac{G_c G}{1+G_c G} N(s)$$

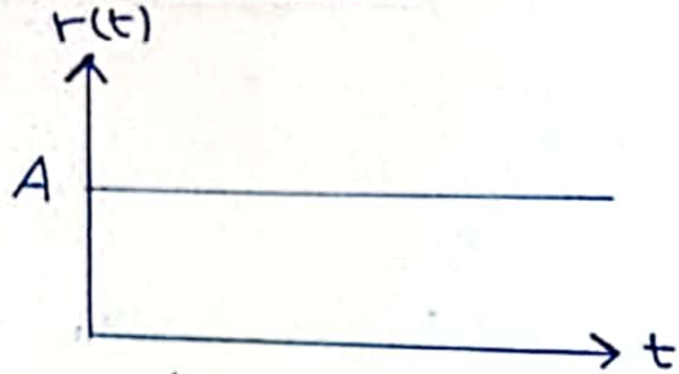
$$= \frac{G_c G}{1+G_c G} N(s) .$$

$$E(s) = \underbrace{\frac{1}{1+G_cG(s)}R(s)}_{\text{Error due to desired output.}} - \underbrace{\frac{G(s)}{1+G_cG(s)}T_d(s)}_{\text{Error due to disturbance}} + \underbrace{\frac{G_cG}{1+G_cG}N(s)}_{\text{Error due to noise}}$$

Chapter 5 : The Performance of Feedback Control System :-

* Test input Signals For the time response of Control System:

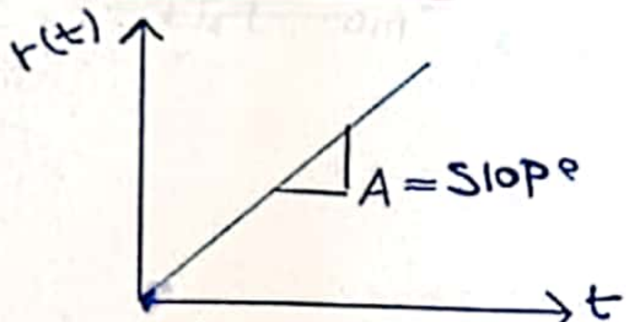
1. Step input :-



$$r(t) = \begin{cases} A & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$

$$R(s) = \frac{A}{s}.$$

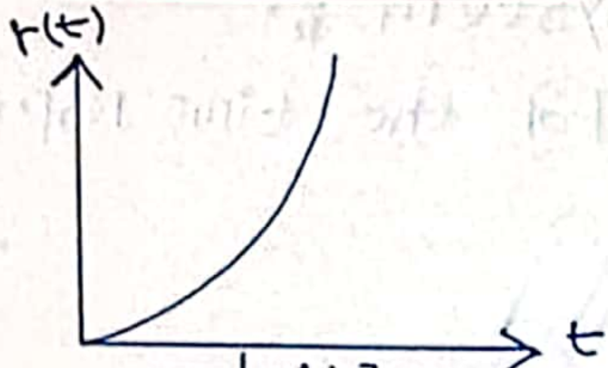
2. ramp input :-



$$r(t) = \begin{cases} At & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$

$$R(s) = \frac{A}{s^2}.$$

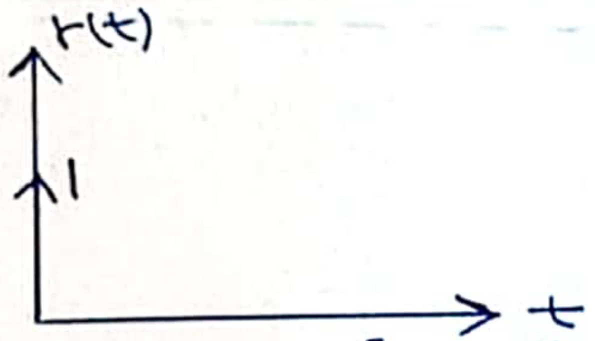
3. Parabolic input :



$$r(t) = \begin{cases} At^2 , & t \geq 0 \\ 0 , & t < 0 \end{cases}$$

$$R(s) = \frac{2A}{S^3}$$

4. unit impulse input :



$$r(t) = \begin{cases} \frac{1}{\varepsilon} , & -\frac{\varepsilon}{2} < \varepsilon < \frac{\varepsilon}{2} \\ 0 , & o.w \end{cases}$$; ε is small value.

$$R(s) = 1 .$$

1. Zero Order System : $\xrightarrow{V_1} \boxed{k} \xrightarrow{V_2}$

2. First order System : $\dfrac{Y(s)}{R(s)} = \dfrac{1}{\tau s+1}$

τ : time constant , where $\boxed{y(\tau) = 0.63\, y_{ss}}$

Steady State.

T_s : Settling time $= 4\tau$

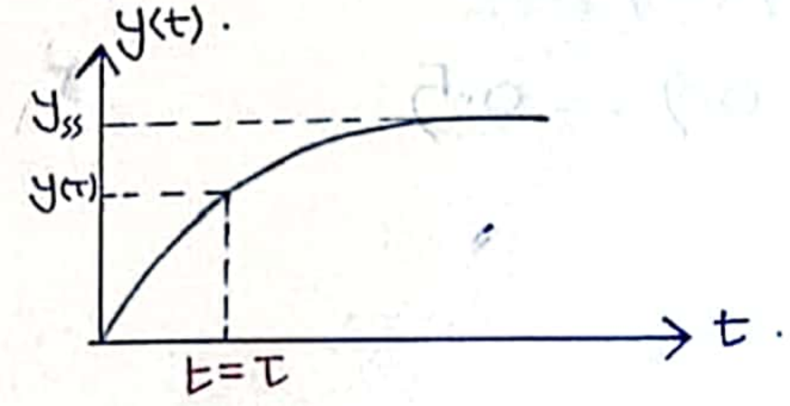
3. Second order System : $\dfrac{Y(s)}{R(s)} = \dfrac{b}{s^2+as+b}$.

4. Higher-order System.

← مقام الـ T.F هو الـ C/C وبيحدد الـ order

---

* First-Order System :-



$\dfrac{1}{S}$

$$\frac{Y(s)}{R(s)} = \frac{K}{\tau s+1}$$; for a unit Step-input

$$y(t) = K(1-e^{-\frac{t}{\tau}}) .$$

Example : $G(s) = \frac{9}{s+10}$;

for a unit Step input, Find k, τ, $\frac{Y(s)}{R(s)}$, $y(t)$?

1^{st} order . $\rightarrow G(s) = \frac{9}{10(\frac{s}{10}+1)} \rightarrow \boxed{\tau = 0.1}$ $\boxed{k = \frac{9}{10}}$

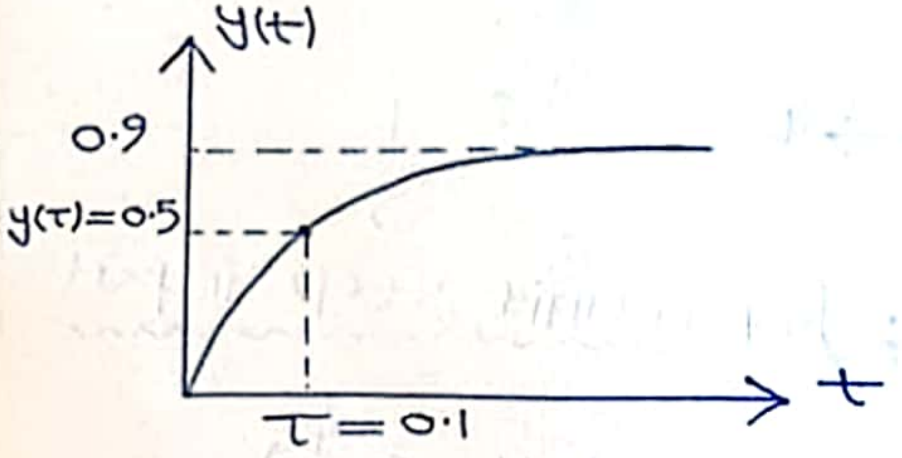
$y(\tau) = 0.63\, y_{ss}$

$y_{ss} = \lim_{s \to 0} s\, Y(s)$

$Y(s) = \left(\frac{9}{s+10}\right)\left(\frac{1}{s}\right)$

$\rightarrow y_{ss} = \lim_{s \to 0} s \left(\frac{9}{s+10}\right)\left(\frac{1}{s}\right) = \lim_{s \to 0} \frac{9}{s+10} = \frac{9}{10}$

$\rightarrow y(\tau) = 0.63 * 0.9 = 0.5$ #

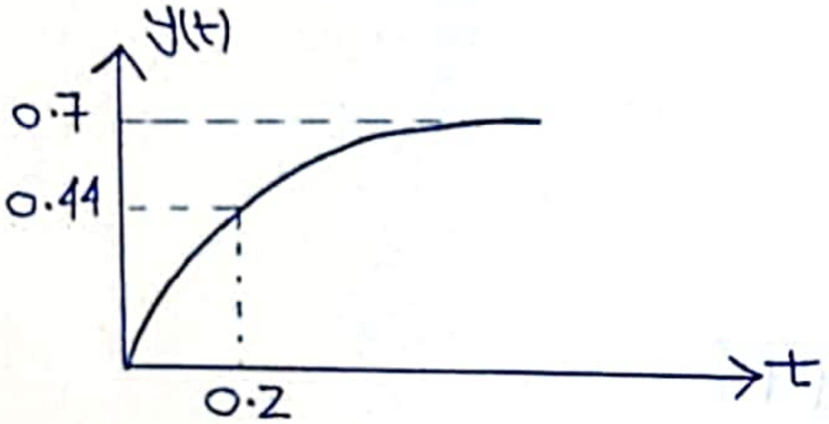


$T_s = 4\tau = 4 * 0.1 = 0.4$

↳ . Steady State الزمن اللازم للوصول لل :

$$y(t) = k(1-e^{-\frac{t}{\tau}})$$
$$= 0.9(1-e^{-\frac{t}{0.1}})$$
$$= 0.9(1-e^{-10t}) \cdot \quad \#.$$

Example:



for a unit step input,

Find $\frac{Y(S)}{R(S)}$, k, τ, T_s, $y(t)$?

Solution: $\rightarrow$ first order System:

$$\frac{Y(s)}{R(s)} = \frac{k}{\tau s+1}$$

$y_{ss} = 0.7$

$y(\tau) = (0.63)(0.7) = 0.44$

$\tau = 0.2$

$\rightarrow \boxed{K = \dfrac{y_{ss}}{R_{ss}}}$

y_{ss} : Steady-State O/P.

R_{ss} : Steady-State i/P.

$K = \dfrac{0.7}{1} = 0.7$.

$R_{ss} = \lim_{s \to 0} S \frac{1}{S} = 1$

$\therefore$

$$\frac{Y(S)}{R(S)} = \frac{0.7}{0.2S+1}$$

$T_s = 4\tau = 4 * 0.2 = 0.8S$

$y(t) = K(1 - e^{-\frac{t}{\tau}})$

$= 0.7(1 - e^{-\frac{t}{0.2}})$.

\#.

*Second - Order System :-

General Form: $\boxed{\frac{Y(S)}{R(S)} = \frac{k\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}}$, $\boxed{K = \frac{y_{ss}}{R_{ss}}}$

1. Undamped System :

$\longrightarrow \zeta = 0$

$S_{1,2} = \mp \omega_n j$

for unit step i/p : $y(t) = 1 - \cos\omega_n t$.

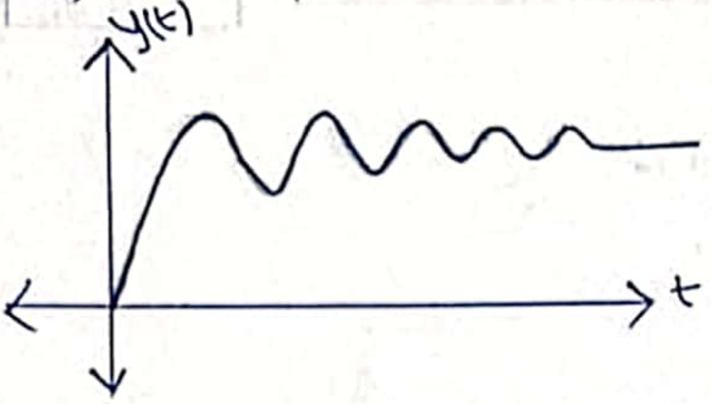
$\longrightarrow S^2+2\zeta\omega_n S+\omega_n^2 = 0$

$$S = \frac{-2\zeta\omega_n \mp \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2}$$

$$= \frac{\mp\sqrt{-4\omega_n^2}}{2} = \mp j\omega_n$$ #

2. Underdamped System :

$\rightarrow \quad 0 < \zeta < 1$



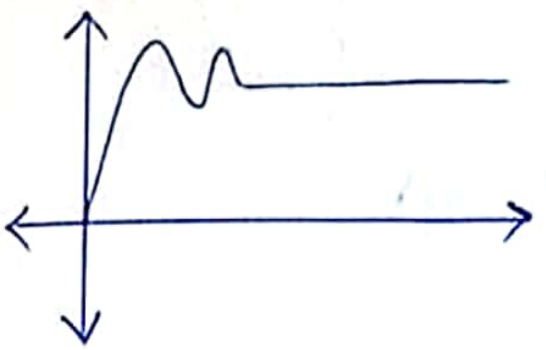
$$S_{1,2} = -\zeta \omega_n \mp j \omega_n \sqrt{1-\zeta^2} .$$

$$y(t) = 1 - \frac{e^{-\zeta \omega_n t}}{\sqrt{1-\zeta^2}} \sin(\omega_n \sqrt{1-\zeta^2}\, t + \cos^{-1} \zeta) .$$

---

3. Critically damped System :

$\longrightarrow \quad \zeta = 1$

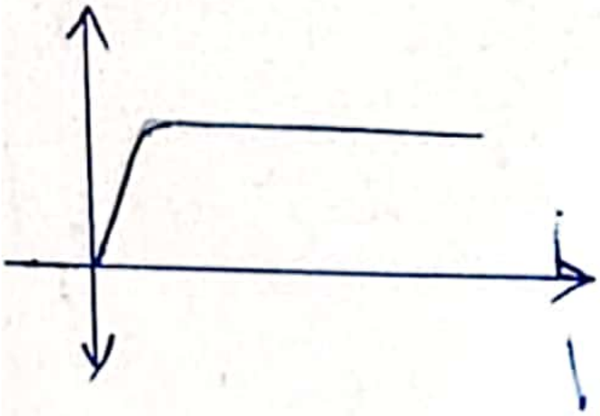


$$S = -\omega_n$$

$$y(t) = 1 - e^{-\omega_n t} (1 + \omega_n t)$$

4. Overdamped System :

$\rightarrow \zeta > 1$



$$S = -\zeta\omega_n \mp \omega_n\sqrt{\zeta^2-1}$$

$$y(t) = 1 + \frac{\omega_n}{2\sqrt{\zeta^2-1}}\left[\frac{e^{-S_1 t}}{S_1} - \frac{e^{-S_2 t}}{S_2}\right].$$

ζ : damping Ratio.

ω_n : natural freq.

Example :

Find $y(t)$ for a unit-Step input :

1 $G(S) = \dfrac{9}{S^2+2S+9}$?

$2\zeta w_n = 2 \rightarrow 2\zeta(3) = 2$

$\zeta = \frac{1}{3}$, $0<\zeta<1$

(Underdamped)

$w_n^2 = 9$

$\hookrightarrow w_n = 3$.

$S_{1,2} = -\zeta w_n \mp j w_n\sqrt{1-\zeta^2}$

$= -1 \mp j2\sqrt{2}$.

$y(t) = 1 - 1.06\, e^{-t} \cos(\sqrt{8}\,t - 19.47)$

$= 1 - \dfrac{e^{-\zeta w_n t}}{\sqrt{1-\zeta^2}} \sin(w_n\sqrt{1-\zeta^2}\,t + \cos^{-1}\zeta)$.

w_d : damping freq. $= w_n\sqrt{1-\zeta^2}$

$\rightarrow w_d = 3\sqrt{1-(\frac{1}{3})^2} = \sqrt{8} = 2\sqrt{2}$.

$\cos^{-1}(\frac{1}{3}) = 70.5$

$\rightarrow 90 - 70.5 = 19.5$

$\boxed{2}$ $G(s) = \dfrac{9}{s^2+9}$? [undamped Sys.]

$\zeta = 0 \quad , \omega_n = 3$

$S_{1,2} = \mp j\omega_n = \mp j3$

$y(t) = 1 - \cos \omega_n t$

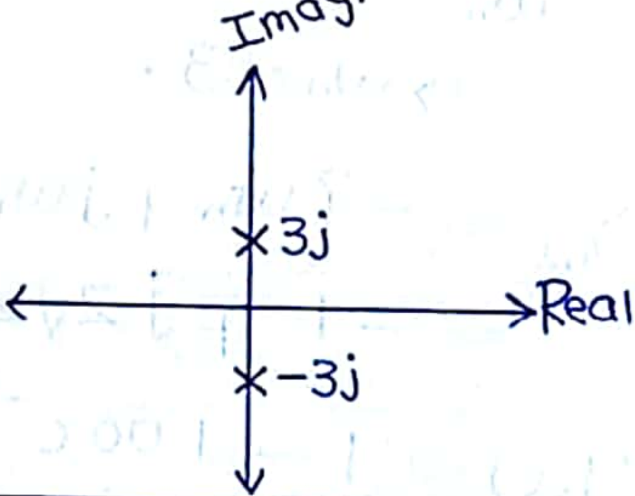
$\quad = 1 - \cos 3t$ #.

$k = 1$

→ S-plane

× : Poles. → أصفار المقام .

o : Zeros. → أصفار البسط .



$\boxed{3}$ $G(s) = \dfrac{9}{s^2+6s+9}$?

→ $2\zeta\omega_n = 6 \rightarrow \zeta = 1$: Criticaly damped.

$\omega_n = 3$

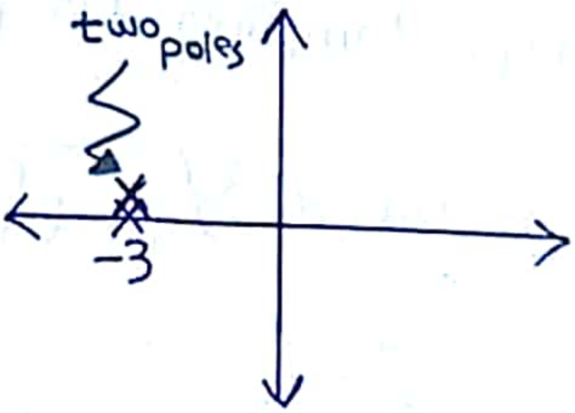
$S_{1,2} = -3$

$y(t) = 1 - e^{-\omega_n t}(1+\omega_n t)$

$\quad = 1 - e^{-3t}(1+3t)$

$\quad = 1 - e^{-3t} - 3te^{-3t}$.

#



Scanned with CamScanner

$\cos^{-1}(0)$ $\zeta=1$ ← Critically أكيد ← Real ← إذا كان ال poles

$\cos^{-1}(90)$ $\zeta=0$ ← undamped أكيد ← Imag. ← إذا كان ال poles

$\cos^{-1}(\ )$ بآمنت ← Over/under ← Real + Imag. ← إذا كان ال poles

---

$\boxed{4}$ $G(S) = \dfrac{9}{S^2+9S+9}$?

$\omega_n = 3$

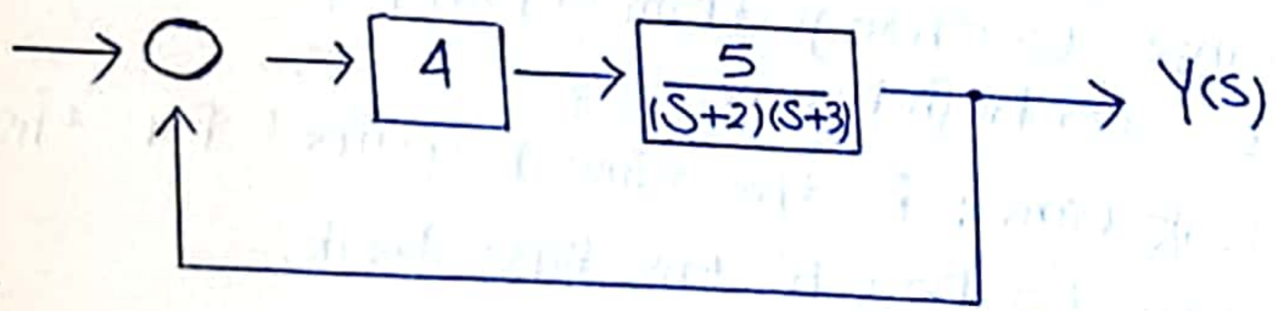
$\zeta = 1.5$: over damped.

$S_{1,2} = -(1.5)(3) \mp (3)\sqrt{(1.5)^2-1}$

$= -4.5 \mp 3.3 = -1.2, -7.8$.

$y(t) = 1 + \dfrac{3}{2\sqrt{0.5}}\left[\dfrac{-e^{+1.2t}}{1.2} + \dfrac{e^{+7.8t}}{7.8}\right]$.

Example:



Response : for Second order [over dam./....].

Order : Zero/first/... .

System Type

Find K, ω_n, ζ ?

$$\frac{k\omega_n^2}{S^2+2\zeta\omega_n S+\omega_n^2}$$

$$\rightarrow \quad \frac{\frac{20}{(S+2)(S+3)}}{1+\frac{20}{(S+2)(S+3)}} = \frac{20}{(S+2)(S+3)+20} = \frac{20}{S^2+5S+26}$$

$$\boxed{\omega_n = \sqrt{26}}$$

$$2\zeta\sqrt{26} = 5 \rightarrow \boxed{\zeta = 0.5}$$: under damped.

$$k\omega_n^2 = 20 \rightarrow \boxed{k = \frac{20}{26}}$$.

* home work :- find $y(t)$?

---

[1] Rise time : is the time required for the response to change from a lower prescribed value to higher one.

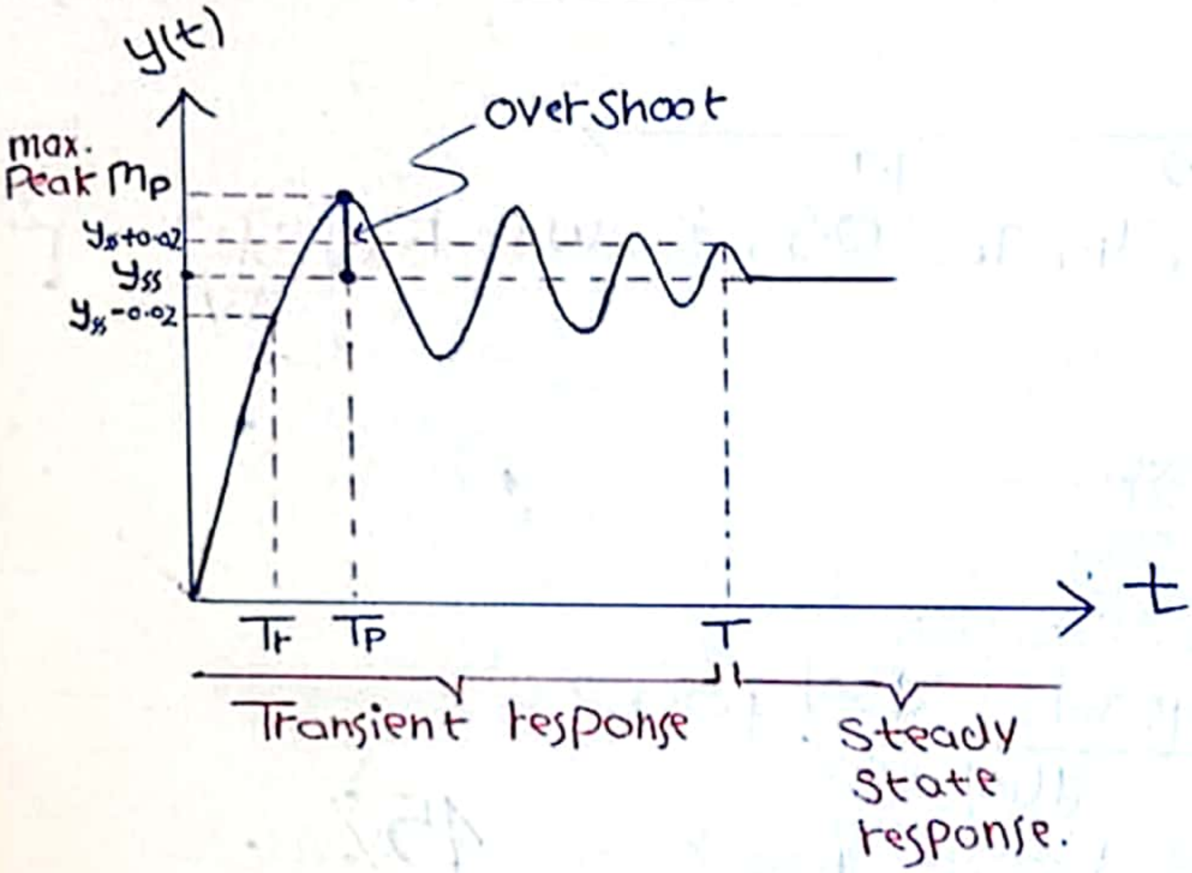
[2] Peak time : is the time required for the response to reach the first peak.

$$T_p = \frac{\pi}{\omega_n\sqrt{1-\zeta^2}}$$

[3] Settling time : is the time required for the amplitude of the sinusoid to decay to 2% or 5% of steady state value.

$$T_s = \frac{4}{\zeta \omega_n} \qquad 2\% \text{ criterian.}$$

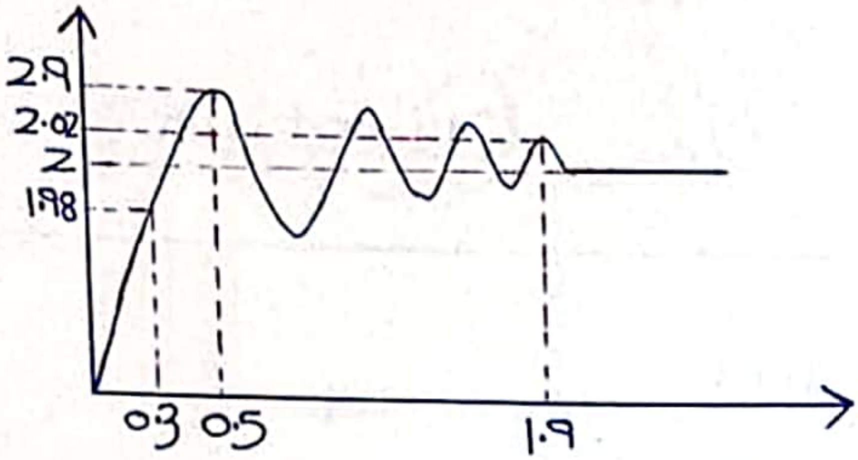
$$T_s = \frac{3}{\zeta \omega_n} \qquad 5\% \text{ criterian.}$$



$$O.S\% = \frac{y(t_p) - y(\infty)}{y(\infty)} 100\% \quad ; \; y(\infty) \to y_{ss}$$

$$= 100 e^{-\pi\zeta/\sqrt{1-\zeta^2}}.$$

Example :



find T_S, T_P, T_R, O.S, ζ, ω_n, k, Response Type ?

Solution:

$T_S = 1.9$ sec.

$T_P = 0.5$ sec.

$T_R = 0.3$ sec.

$$O.S = \frac{y(T_P) - y(\infty)}{y(\infty)} 100\%$$

$$= \frac{2.9 - 2}{2} 100\% = 45\% .$$

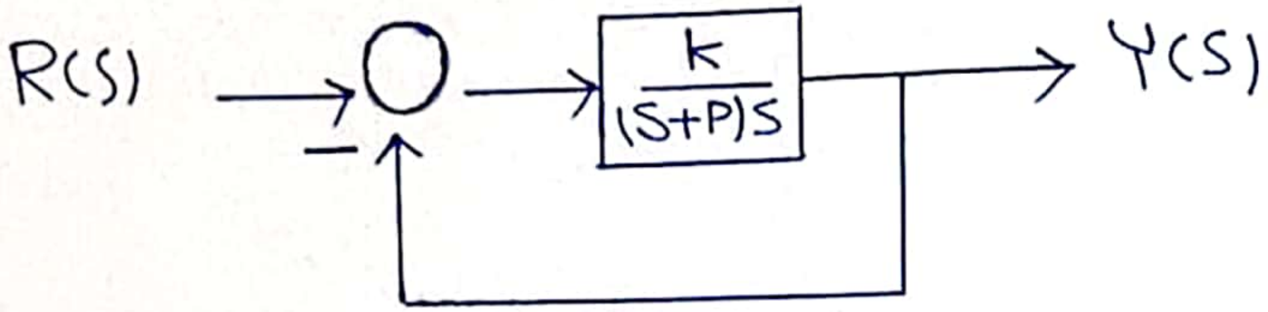
$$\rightarrow 0.45 = e^{-\pi \zeta \sqrt{1-\zeta^2}}$$

$\zeta = 0.246 \longrightarrow$ underdamped.

$$T_S = \frac{4}{\zeta \omega_n} = 1.9 \rightarrow \omega_n = 16.23 \text{ rad/s.}$$

$$k = \frac{2}{1} = 2 .$$

Example:



Select k and P where :

$T_s < 4$ Sec ; $\zeta = 0.707$?

↳ using 2% criteria .

Sol. $$\frac{Y(S)}{R(S)} = \frac{\frac{k}{(S+P)S}}{1+\frac{k}{(S+P)S}} = \frac{k}{S(S+P)+k}$$

$$= \frac{k}{S^2+\underbrace{P}_{2\zeta\omega_n}S+\underbrace{k}_{\omega_n^2}}$$

$$T_s = \frac{4}{3\omega_n} < 4$$

$$\frac{4}{0.707\,\omega_n} < 4$$

$$\omega_n < 1.4 \text{ rad/s.}$$

$$k = \omega_n^2 = 1.96 .$$

$$P = 2\zeta\omega_n = 2 * 0.707 * 1.4$$

$$= 1.97 \text{ rad/s.}$$

"Chapter (5) :- (تكملة)" Slides 26 - 33

System type :-
type zero ; type one ; type two.

$N \rightarrow$ System Type تحدد .
$S^1 \rightarrow$ type 1 .
$S^2 \rightarrow$ type 2 .
⋮

$\rightarrow$ Step input $\frac{A}{S}$. ✓

$k_p \rightarrow$ position Error constant.

$\rightarrow$ ramp $\frac{A}{S^2}$. ✓ ; $k_v \rightarrow$ Velocity error Constant

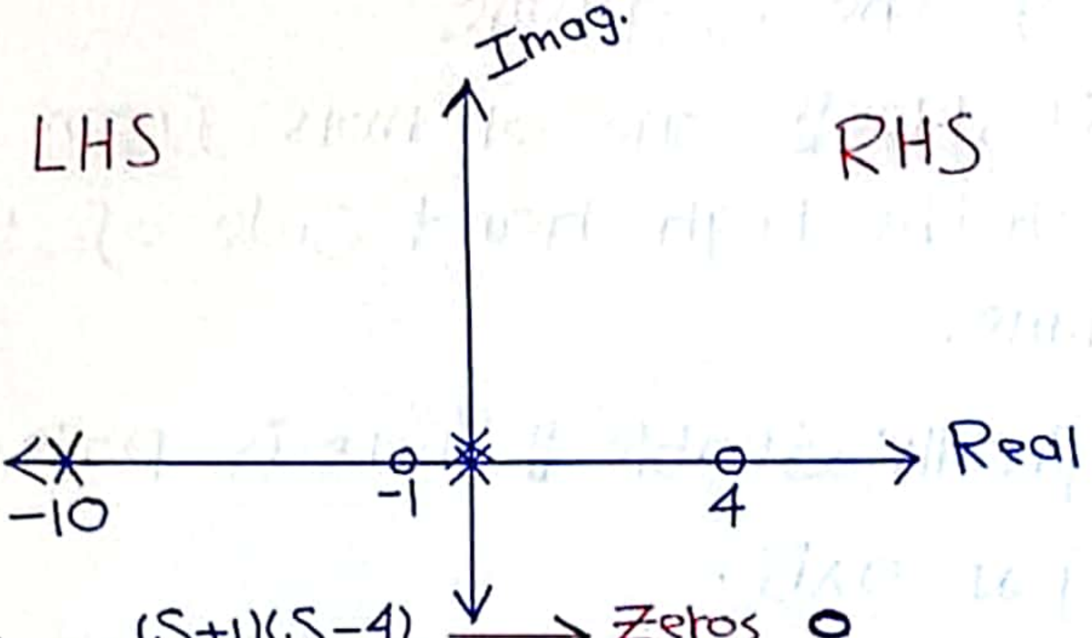
$\rightarrow$ acceleration $\frac{A}{S^3}$. ✓
$k_a \rightarrow$ Acceleration error constant.

order أقل من ال system $\rightarrow 0$.

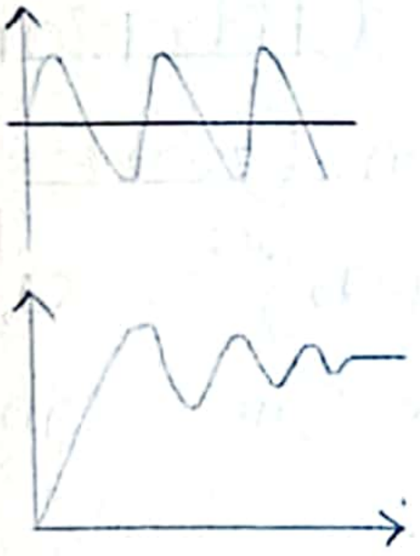
order أعلى من ال system $\rightarrow \infty$.

order = type
تطلع قيمة

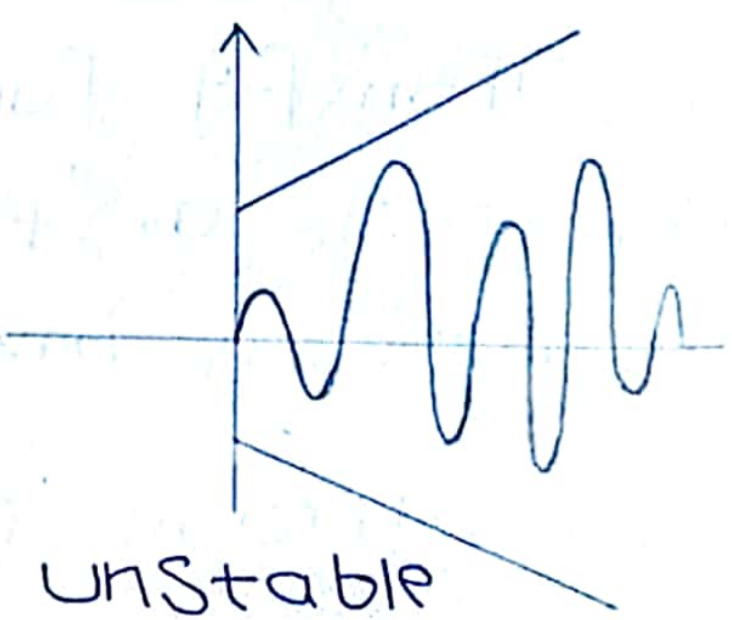
Chapter 6 : The Stability of Linear feedback System



$$T(s) = \frac{(S+1)(S-4)}{S^2(S+10)} \quad \begin{matrix} \longrightarrow \text{Zeros } o \\ \longrightarrow \text{Poles } X \end{matrix}$$



Stable



unstable

note: 1 pole at RHS $\rightarrow \therefore$ System is unstable

1. Stable :- all poles on the left hand Side of the S-plane.

2. unstable :- one or more from the poles on the righ hand Side of the S-plane.

3. Marginally Stable :- There is poles on $\pm j\omega$ axis.

Routh-Herwitz Criterian :-

for Transfer function $G(S) = \frac{q(S)}{P(S)}$

Where : $P(S) = a_0 S^N + a_1 S^{N-1} + a_2 S^{N-2} + \dots a_N$

① $a_0, a_1, a_2, \dots a_N$ have the Same Sign.

← جميعها موجودة ولها نفس الاشارة.

② No coefficient is Zero.

$P(S) = S^2 + S + 1$ ✓

$P(S) = S^3 + S^2 + 1$ X

$P(S) = S^2 - S - 1$ X

الصف الأول زوجي ← زوجي :N

الصف الأول فردي ← فردي :N

$$
\begin{array}{c|cccc}
S^{N} & a_0 & a_2 & a_4 & a_6 \\
S^{N-1} & a_1 & a_3 & a_5 & a_7 \\
S^{N-2} & b_1 & b_2 & b_3 & \\
S^{N-3} & c_1 & c_2 & c_3 & \\
\vdots & & & &
\end{array}
$$

$$b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$$

$$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$$

$$b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1}$$

$$c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

$$c_3 = \frac{b_1 a_7 - a_1 b_4}{b_1}$$

Example :

$$1 + KGH = P(S) = S^3 + 2S^2 + 4S + \underline{K}$$

S^3	1	4	0
S^2	2	K	0
S^1	$\frac{8-K}{2}$	0	0
S^0	$\underline{K}$ ← دائماً	0	0

$$b_1 = \frac{(2*4) - (K*1)}{2} = \frac{8-K}{2}$$

$$b_2 = \frac{(2*0) - (1*0)}{2} = 0$$

$$C_1 = \frac{\left(\left(\frac{8-K}{2}\right)*K\right) - (2*0)}{\left(\frac{8-K}{2}\right)} = K > 0$$

$$\frac{8-K}{2} > 0 \rightarrow K < 8$$

$$\therefore \boxed{0 < K < 8}$$

* أوّل عامود كلّه +ve ← Stable.
* أوّل عامود كلّه -ve ← Stable.
* اذا تغيرت الاشارة على الأقل مرة ← unstable.

Example :-

$$P(S) = S^5 + 2S^4 + 2S^3 + 4S^2 + 11S + 10 \;?$$

S^5	1	2	11	0
S^4	2	4	10	0
S^3	$\cancel{0}\,\varepsilon$	6	0	
S^2	$\frac{-12}{\varepsilon}$	10	0	
S^1	6	0	0	
S^0	10	0	0	

→ الحالة الثانية :
$b_1 = 0$: نستبدله ب
Small value $= \varepsilon$

0 ← ما بأثر

$$C_1 = \frac{\cancel{4\varepsilon}^{\,0} - 12}{\varepsilon} = \frac{-12}{\varepsilon}.$$

إذا عوضته صفر بأثر .

$$C_2 = \frac{10\varepsilon - 2*0}{\varepsilon} = 10.$$

$$D_1 = \frac{\frac{-12}{\varepsilon}\cdot 6 - 10\varepsilon}{\frac{-12}{\varepsilon}} = 6.$$

System is unstable.

→ 2 poles are in right hand Side. → عددهم بكم مرة تغيرت الاشارة.

→ 3 poles are in left hand Side.

Example: $S^4+S^3+S^2+S+k$;

Find k that results in marginally Stable!

S^4	1	1	k
S^3	1	1	0
S^2	$\cancel{0}\ \varepsilon$	k	0
S^1	$\frac{0-k}{\varepsilon}$	0	0
S^0	k	0	0

$\frac{-k}{\varepsilon} > 0 \rightarrow -k > \varepsilon$

$k < 0$ → System is unstable for all value of k.

Example:

$P(S) = S^5 + S^4 + 2S^3 + 2S^2 + S + 1$.

S^5	1	2	1
S^4	1	2	1
S^3	$\cancel{0}\ 4$	$\cancel{0}\ 4$	0
S^2	1	1	0
S^1	$\cancel{0}\ 2$	0	0
S^0	1	0	0

$S^4 + 2S^2 + 1$

نشتق

$4S^3 + 4S$

الحالة الثالثة

$S^2 + 1$

$2S$

System is stable .

Example :

$$P(s) = s^5 + s^4 + 4s^3 + 24s^2 + 3s + 63 \quad ?$$

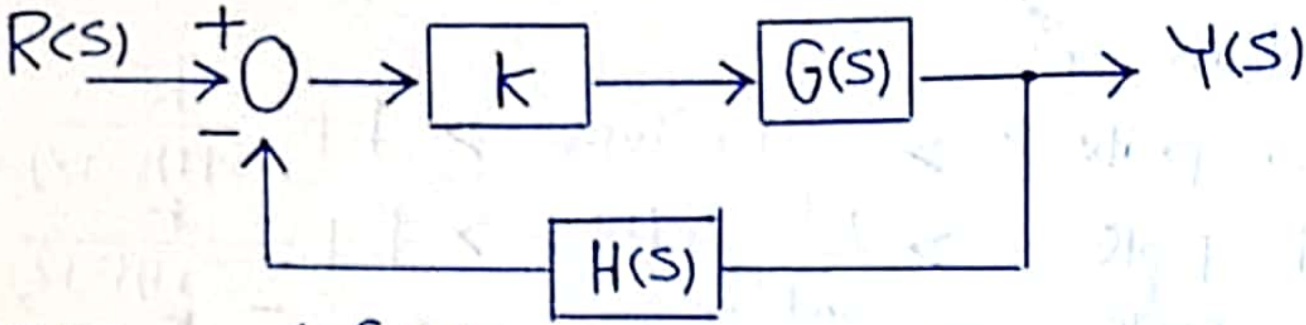
s^5	1	4	3
s^4	1	24	63
s^3	-20	-60	0
s^2	21	63	0
s^1	$\cancel{0}$ 42	0	0
s^0	63	0	0

$\rightarrow 21s^2 + 63$

$42s$

System is unstable.

Chapter 7 : The Root Locus Method :-



$$T.F = \frac{kG(S)}{1+kGH(S)}$$

$$1 + kGH = 0$$

Example :

$$1 + \frac{k}{(S+1)(S+2)}$$

$\rightarrow (S+1)(S+2) + k = 0$

$k = 0 \rightarrow S_{1,2} = -1, -2$

$k = 0.25 \rightarrow S_{1,2} = -1.5$

$k = 0.5 \rightarrow S_{1,2} = -1.5 \pm j0.5$

$k = 1 \rightarrow S_{1,2} = -1.5 \mp 0.866$

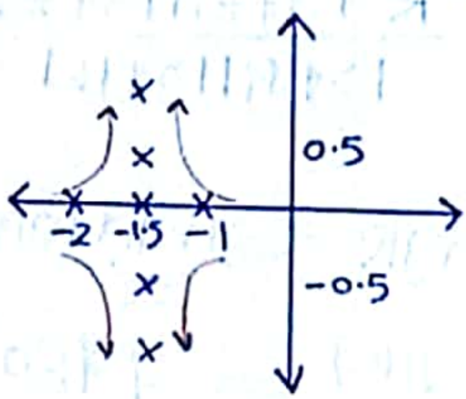


Table 1

System Type :

How many poles at zero-zero point in S-plane ?

→ 0 pole → Zero Type → $1+\frac{k}{(S+1)(S+2)}$

→ 1 pole → 1^{st} Type → $1+\frac{k}{S(S+1)(S+2)}$

→ 2 pole → 2^{nd} Type → $1+\frac{k}{S^2(S+1)(S+2)}$

---

* The Conditions that must prevail for any point on the root locus :-

1) Magnitude condition :

gain → $$\frac{k\,|S+Z_1|\,|S+Z_2|\cdots|S+Z_m|}{|S+P_1|\,|S+P_2|\cdots|S+P_n|}=1$$

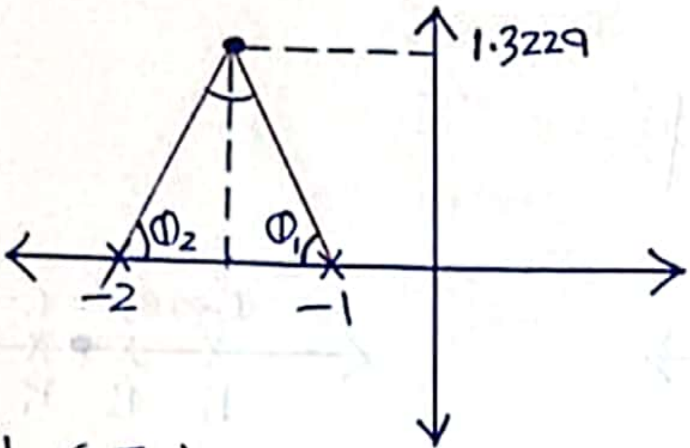
2) Angle condition :

$$\angle G(S)H(S) = \mp 180^\circ(2k+1)\ . \text{ where } k=0,1,2,\ldots$$

$$\angle(S+Z_1)(S+Z_2)\cdots\cdots\angle S+Z_m - \angle S+P_1 - \angle S+P_2 - \cdots$$

$$\cdots\ \angle S+P_n = \mp 180^\circ(2k+1)$$

Example: $S = -1.5 + 1.3229j$; $k = 2$



$\rightarrow \dfrac{k(1)}{|S+P_1||S+P_2|} = \dfrac{k}{|S+1||S+2|}$

$= \dfrac{2}{|-1.5+1.3229j+1||-1.5+1.3229j+2|}$

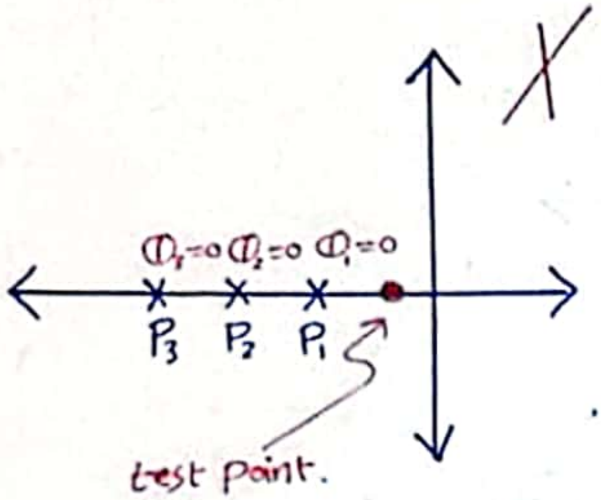
$= 1$ ✓ conition 1 #.

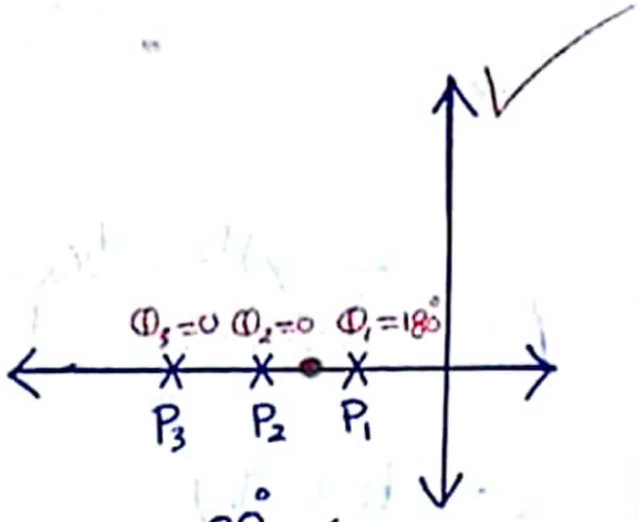
$\rightarrow 0 - \angle S+1 - \angle S+2 \overset{?}{=} \pm 180(2k+1)$

$\Phi_1 = \tan^{-1}\left(\dfrac{1.3229}{-0.5}\right) = 110.7°$

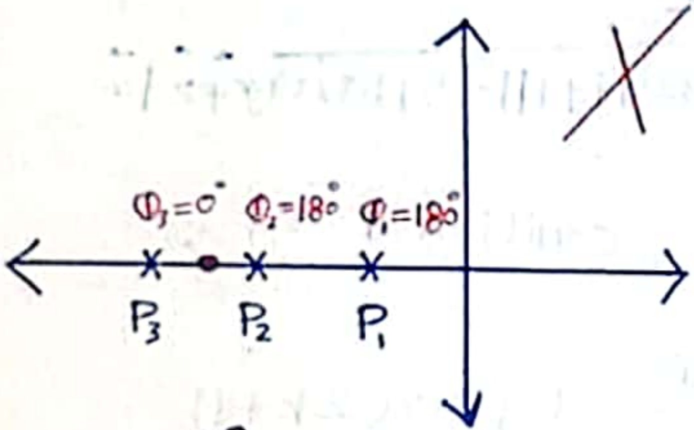
$\Phi_2 = \tan^{-1}\left(\dfrac{1.3229}{0.5}\right) = 69.29°$ ✓ condition 2 #

Example : $\left[\angle Z - \angle P = \mp 180^\circ\right]$

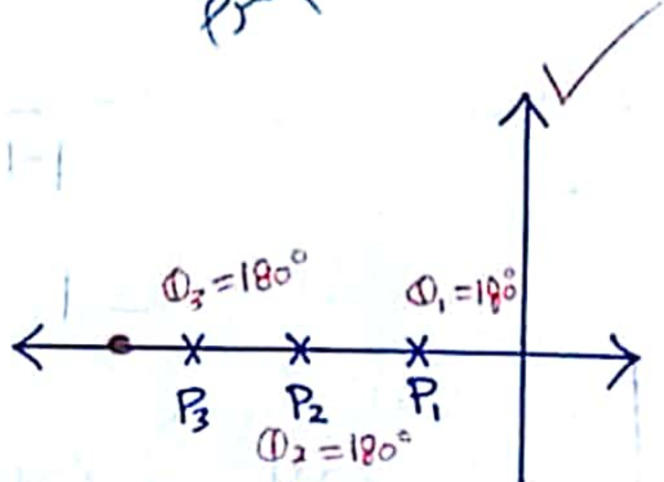




180°
تحقق الشرط



360
لا تحقق الشرط



450°
تحقق الشرط

Summary of General Rules For Constructing Root Loci

. Write the characteristic equation in the form:

$$1 + K\frac{(s+z_1)(s+z_2)\ldots.(s+z_m)}{(s+p_1)(s+p_2)\ldots.(s+p_n)}$$

K is the parameter of interest

$-z_1$, $-z_2$, ..., $-z_m$ are the zeros of the open-loop and $-p_1$, $-p_2$, ..., $-p_n$ are the poles of the open-loop.

2. Locate the poles and zeros of the open-loop on the s-plane: The root-locus branches start from the open-loop poles and terminate at the open-loop zeros.

3. Determine the asymptotes of the root loci: If the number of open-loop poles (n) is greater than the number of open-loop zeros (m), then the root loci has n-m asymptotes.

   a. Angles of the asymptotes:

$$\alpha_k = \frac{\pm 180(2k+1)}{n-m} \qquad k = 0, 1, 2, \ldots, n-m-1$$

   b. Intersection of the asymptotes with the real axis

$$\sigma_a = \frac{(sum\ of\ poles) - (sum\ of\ zeros)}{n-m}$$

4. Find the breakaway and break-in points (if any).

5. Find the points of intersection of the root loci with the imaginary axis (if any).

6. Determine the angle of departure (angle of arrival) if the open-loop has complex poles (complex zeros).

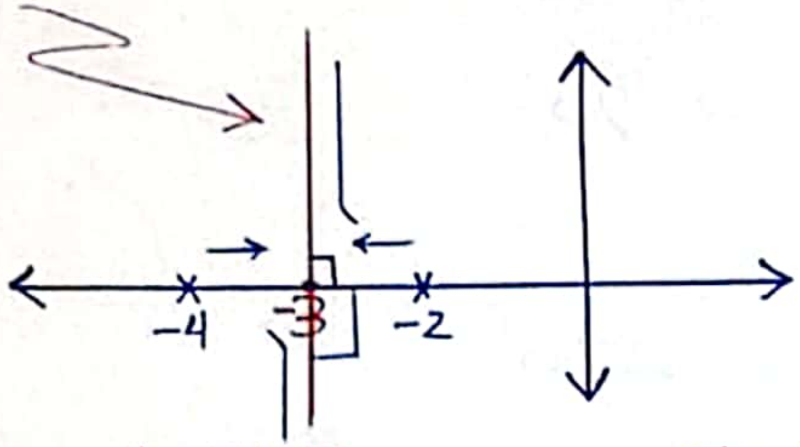
7. Find the gain K that corresponds to a particular (desired) closed-loop pole s.

$$K = \frac{product\ of\ lengths\ between\ s\ and\ the\ open\text{-}loop\ poles}{product\ of\ lengths\ between\ s\ and\ the\ open\text{-}loop\ zeros}$$

Example: $1 + \frac{k}{(S+2)(S+4)}$ ... zero Type system.

Draw a root locus ?

(asymp. axis)



σ_A : [نقطة تقاطع ال asym. مع ال real axis.] ; $\sigma_A = \frac{\Sigma \text{poles} - \Sigma \text{zeros}}{n - m}$

(n ↑ poles, m ↑ zeros)

$$\alpha_A = \frac{\mp 180(2k+1)}{n-m}$$

$\rightarrow k = n - m - 1 \rightarrow k = 2 - 0 - 1 = 1$

# of asymptotes $= n - m = 2 - 0 = 2$

$$\sigma_A = \frac{(-2-4)-0}{2-0} = -3$$

$$\alpha_A = \frac{\pm 180^\circ}{2} = \pm 90^\circ$$

→ zero تبدأ من ال

$(2k+1)$

→ $k = n - m - 1 = 1$ وتنتهي عند ال

$k = 0$:

$$\alpha_0 = \frac{180(1)}{2-0} = 90^\circ$$

$k = 1$:

$$\alpha_1 = \frac{180(3)}{2-0} = 270^\circ$$

To Find breakaway points:

$$\frac{dk}{ds} = 0 \longrightarrow (S+2)(S+4) + k = 0$$

$$k = -(S+2)(S+4)$$

$$\frac{dk}{ds} = -2S - 6 = 0$$

$$\boxed{S = -3}$$

في هذه الحالة ال poles تطلع على ال .asym نفسه.

* الأصل أن هناك breakpoints تطلع من عندها ال poles بمحاذاة ال .asym

→ To find intersection with imaginary axis

↳ From Routh - Herwitz .

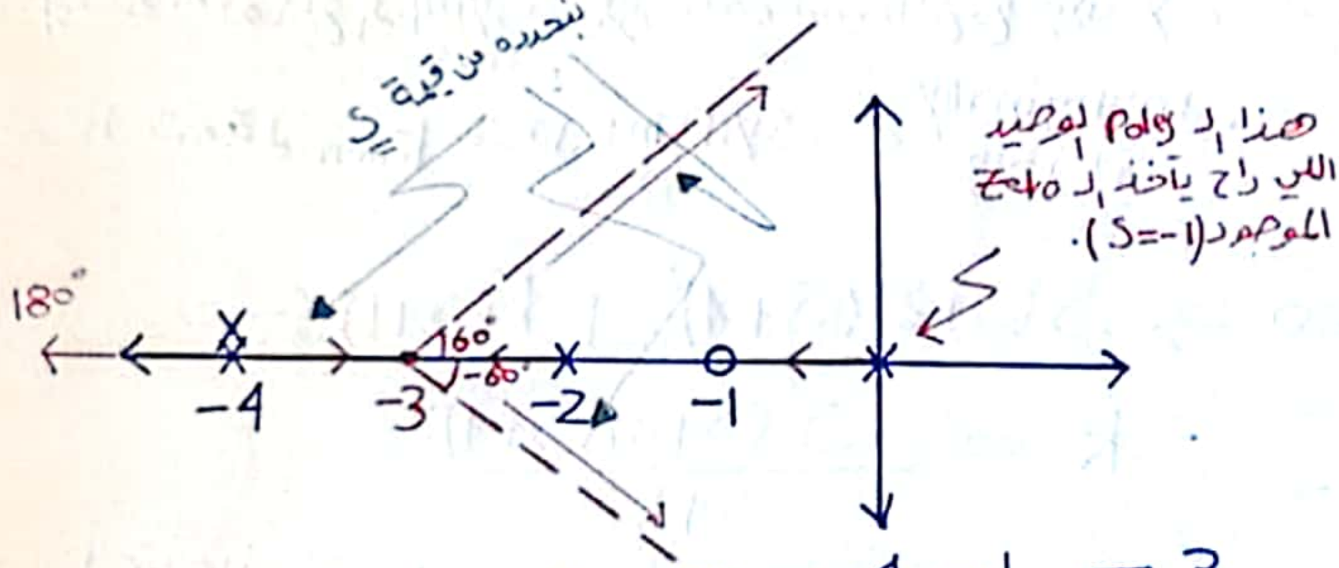
→ Angle of departure :-

لو كان هناك Poles ع .Imag ←

↳ بدو يمشي مع الـ asym. axis

. angle of departure : بزاوية تسمى

Example: $1+\frac{k(S+1)}{S(S+2)(S+4)^2}$ ... 4^{th} order, 1^{st} Type System



$$\# \text{ of asym.} = n - m = 4 - 1 = 3$$

$$\sigma_A = \frac{(0-2-4-4)-(-1)}{4-1} = -3$$

$$k = n - m - 1 = 4 - 1 - 1 = 2$$

$k = 0$:

$$\alpha_0 = \frac{180(1)}{3} = 60^\circ$$

$k = 1$:

$$\alpha_1 = \frac{180(3)}{3} = 180^\circ$$

$k = 2$

$$\alpha_2 = \frac{180(5)}{3} = 300^\circ$$

* نقطة تقاطع الـ asymp. axis بالزاوية 60 ستتقاطع مع y-axis ، وبالتالي

هناك قيمة لـ k_{max} تجعل الـ system ← marginally stable .

$$\frac{dk}{ds} = 0 \rightarrow S(S+2)(S+4)^2 + k(S+1) = 0$$

$$k = \frac{-S(S+2)(S+4)^2}{S+1}$$

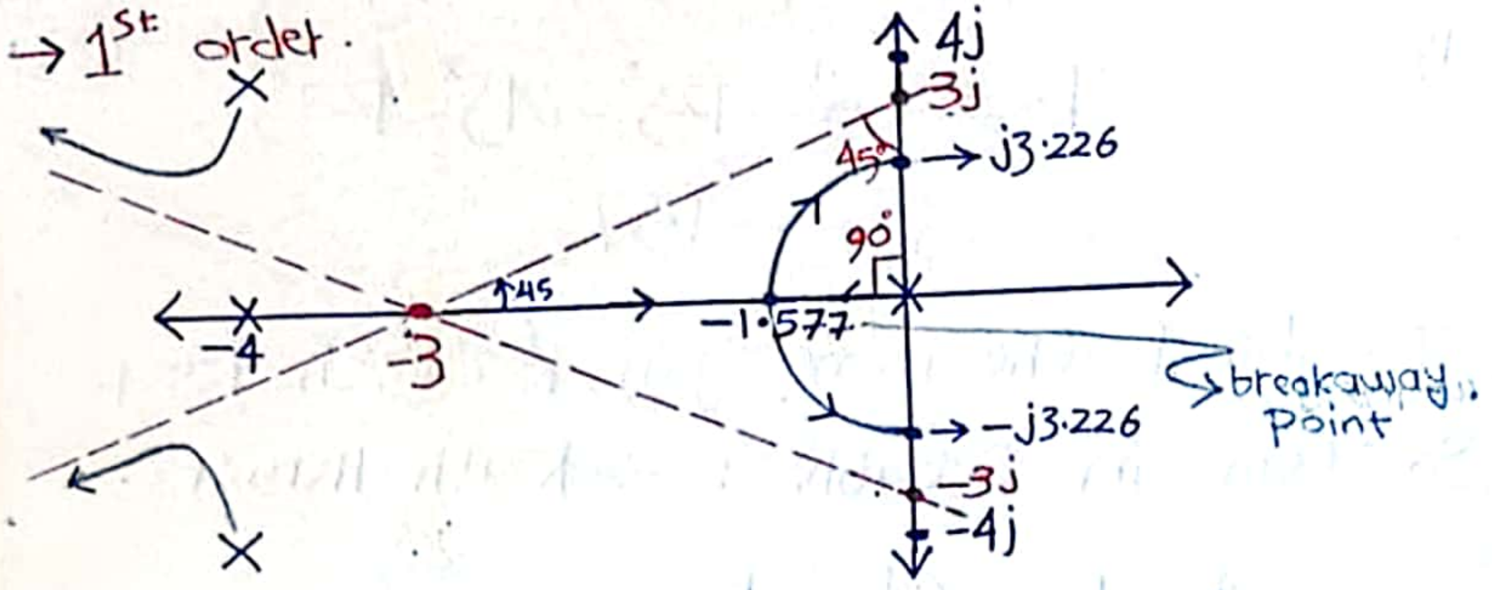
$S = \underline{\underline{-3}}$ → نقطة التقاطع ، عندها يلتقي الـ poles ويطلعوا بمحاذاة الـ asymp. axis .

Check !!

Example: $1 + \frac{K}{S^4 + 12S^3 + 64S^2 + 128S} = 0$

→ 4^{th} type.

→ 1^{st} order.



$\# \text{ of asym.} = 4 - 0 = 4$

$$\sigma_A = \frac{(0 - 4 - 4 - 4j + 4 + 4j) - 0}{4} = -3$$

$k = n - m - 1 = 4 - 0 - 1 = 3$

$$\alpha_0 = \frac{180(1)}{4} = 45^\circ.$$

$$\alpha_1 = \frac{180(3)}{4} = 135^\circ.$$

$$\alpha_2 = \frac{180(5)}{4} = 225^\circ.$$

$$\alpha_3 = \frac{180(7)}{4} = 315^\circ.$$

# of angle of departure = 2

$\frac{dk}{ds} = 0 \rightarrow S^4 + 12S^3 + 64S^2 + 128S + k = 0$

$k = -S^4 - 12S^3 - 64S^2 - 128S$

$\rightarrow S = -1.57$.

To find the max. gain k which keep System in Stable : $\rightarrow$ Routh-Herwitz.

S^4	1	64	k
S^3	12	128	0
S^2	53.3	k	0
S^1	C_1	0	0
S^0	$k > 0$	0	0

$\rightarrow 53.3S^2 + 568.89 = 0$

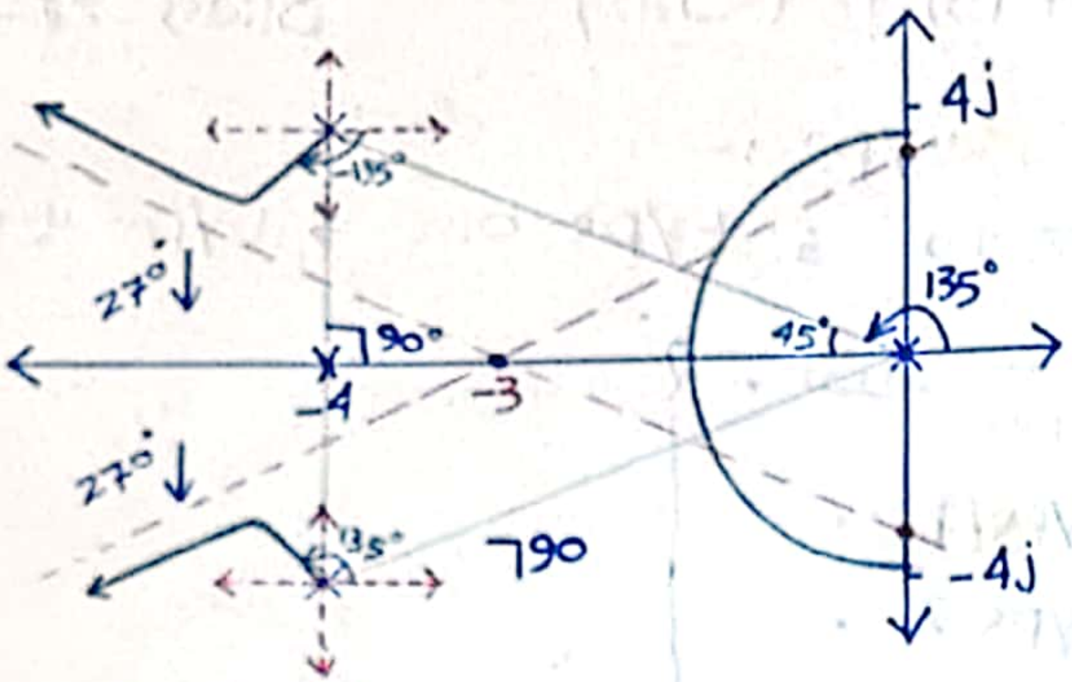
$\boxed{S = \pm j3.229}$

نقطة التقاطع مع ال y-axis .

$$C_1 = \frac{53.3(128) - 12k}{53.3} > 0$$

$k = 568.89$

Scanned with CamScanner



Angle of departure :

Φ_1 :

$\Phi_1 + 90 + 90 + 135 = 180°$

$\Phi_1 = 180 - 90 - 90 - 135 = -135° \rightarrow$ الربع الثالث.

Φ_2 :

$\Phi_2 + 270° + 270° + 225° = 180°$

$\Phi_2 = -585° + 360°$

$\rightarrow \Phi_2 = -225° \equiv 135° \rightarrow$ الربع الثاني .

"Chapter (5) :- (تكملة)" Slides 26-33

System type :-
type zero ; type one ; type two.

N → System Type تحدد .
S^1 → type 1 .
S^2 → type 2 .
⋮

→ Step input $\frac{A}{S}$. ✓

k_p → position Error constant.

→ ramp $\frac{A}{S^2}$. ✓ ; k_v → Velocity error Constant

→ acceleration $\frac{A}{S^3}$. ✓
k_a → Acceleration error constant.

order أقل من ال system → 0 .

order أعلى من ال system → ∞ .

order = type
نطلع قيمة