

تقدم لجنة EiCoM الاكاديمية

دفتر لمادة:

تحكم آلي

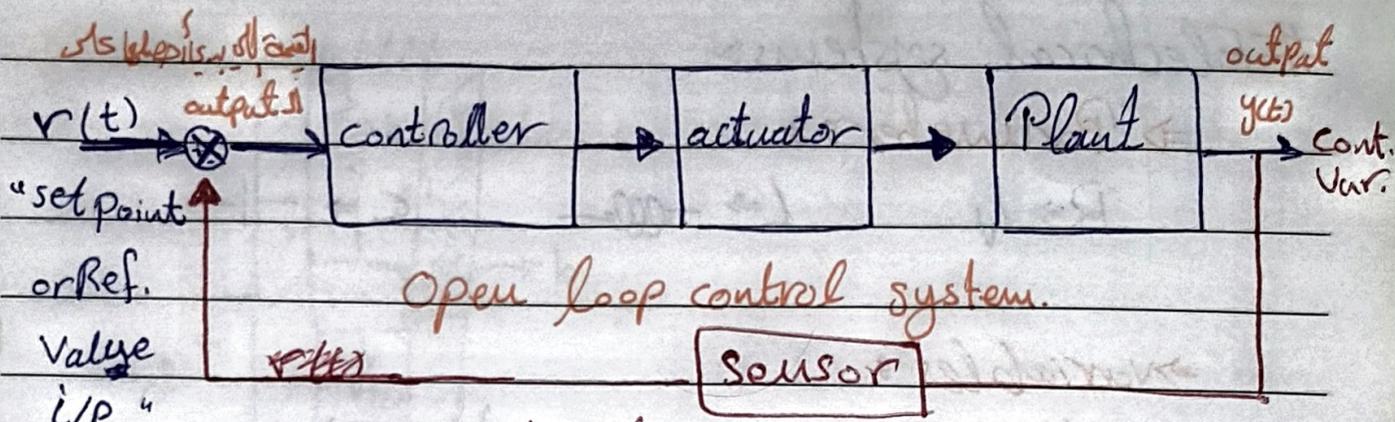
من شرح:

م. فدوى المومني

جزيل الشكر للطالبة:

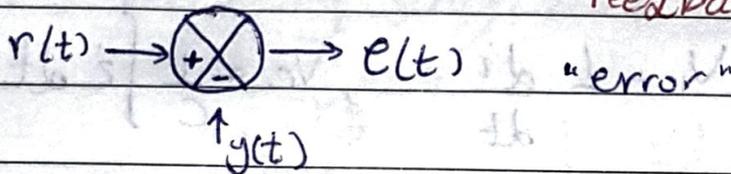
راية جهاد





Open loop control system.

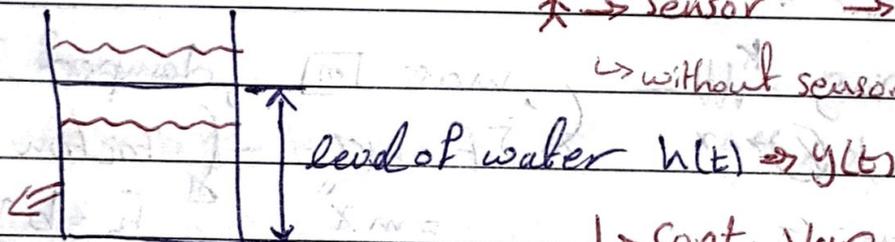
closed loop control system (with sensor) is a "Feedback control system"



$$e(t) = r(t) - y(t)$$

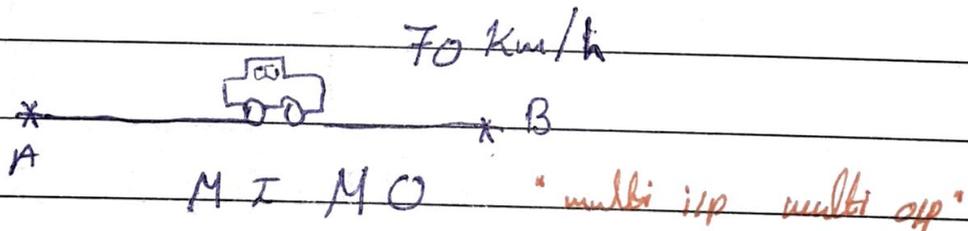
actuator → $e(t)$ → (cont.)

sensor → closed loop
 without sensor → open loop



cont. var. and ref. val.

SISO "single i/p single o/p" and act. val. $y(t)$



Electrical systems

⇒ Parameters:



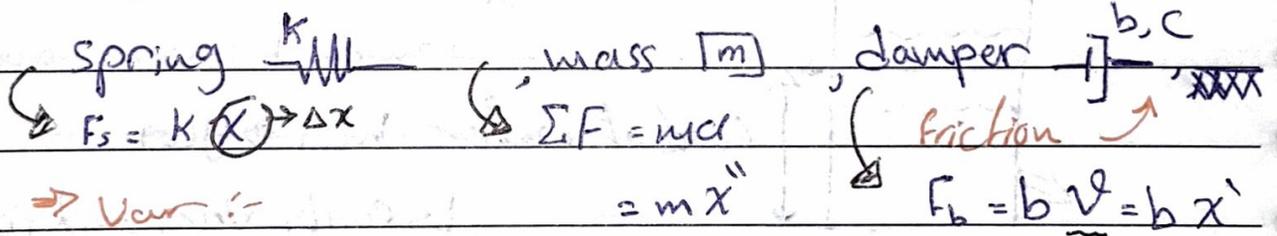
⇒ Variables:

Voltage (V), Current (I)

$$V_R = IR, \quad V_L = L \frac{di}{dt}, \quad V_C = \frac{1}{C} \int i dt$$

Mechanical systems

⇒ Par :-

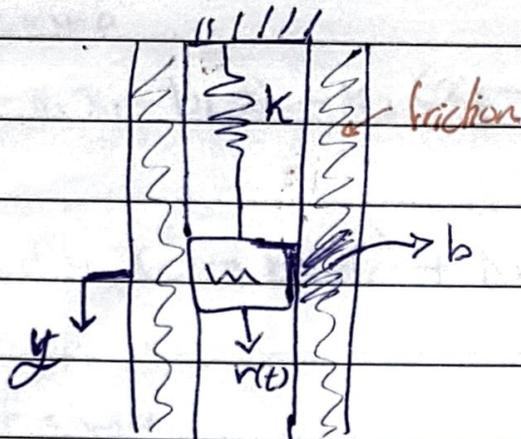


⇒ Var :-

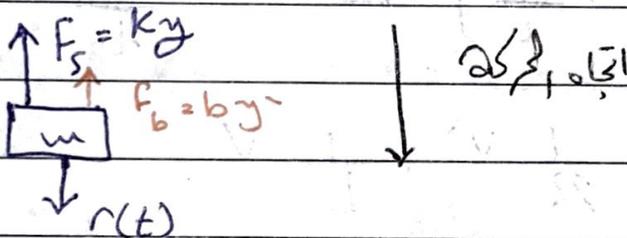
force (F), Position, Velocity, acc.

x
 x'
 x''

Exs



1) freebody diagrams

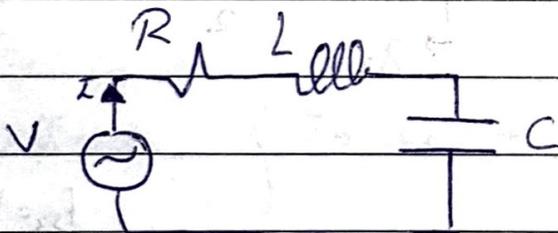


2) $\sum F = ma$

$$r(t) - by' - ky = m y''$$

$$\hookrightarrow m y'' + b y' + k y = r(t)$$

Exs



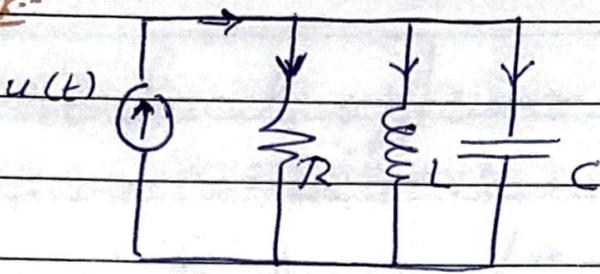
KCL: $I_R = I_L = I_C = I$

KVL: $V = V_R + V_L + V_C$

$$V = IR + L \frac{dI}{dt} + \frac{1}{C} \int I dt \quad \Rightarrow \quad \text{« Integro diff. eq. »} \quad \Rightarrow \quad L I'' + R I' + \frac{1}{C} I = V'$$

20/09/2018

Ex:-



KCL $\Rightarrow u(t) = I_R + I_L + I_C$

KVL $\Rightarrow V_L = V_R = V_C = V$

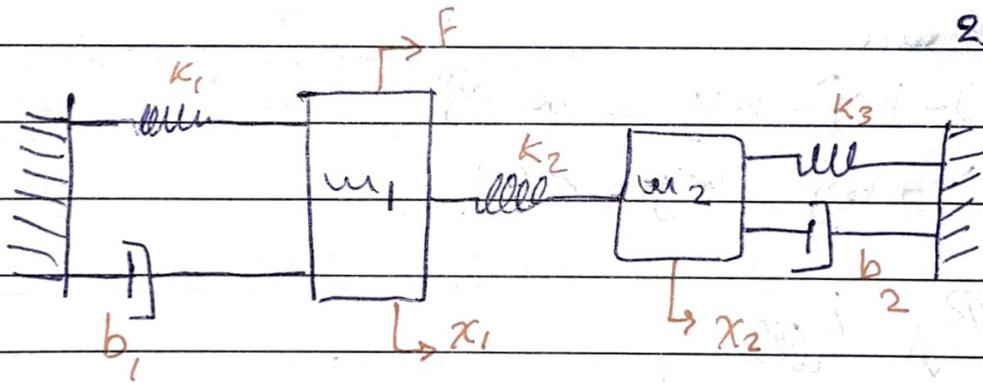
$\Rightarrow u(t) = \frac{V}{R} + \frac{1}{L} \int V dt + C \frac{dV}{dt}$

نشی

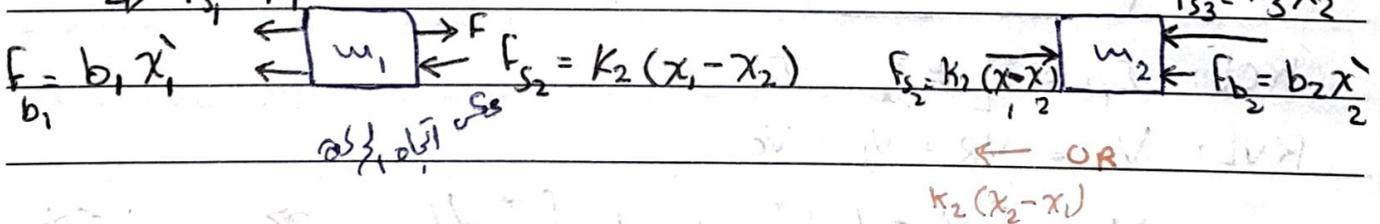
$u' = CV'' + \frac{1}{R} V' + \frac{1}{L} V$

23/09/2018

Ex:-



$\Rightarrow F_s = k_1 x_1$



$k_2 (x_2 - x_1)$

$$\Sigma F = ma$$

$$F - k_1 x_1 - b_1 \dot{x}_1 - k_2 (x_2 - x_1) = m_1 \ddot{x}_1 \quad \# \text{ for } m_1$$

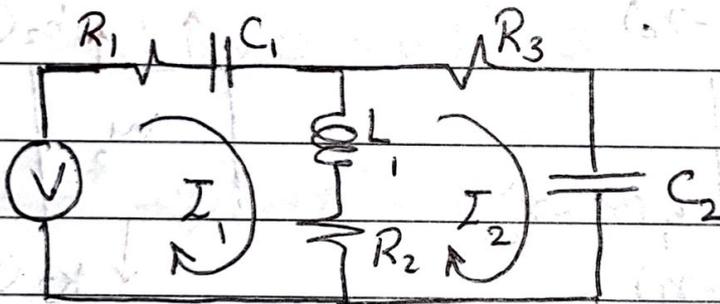
$$\rightarrow m_1 \ddot{x}_1 + b_1 \dot{x}_1 + k_1 x_1 + k_2 (x_1 - x_2) = F$$

$$\Sigma F = ma$$

$$-k_2 (x_2 - x_1) - k_3 x_2 - b_2 \dot{x}_2 = m_2 \ddot{x}_2 \quad \# \text{ for } m_2$$

$$\rightarrow m_2 \ddot{x}_2 + b_2 \dot{x}_2 + k_3 x_2 + k_2 (x_2 - x_1) = 0$$

Ex 50



mesh (1)

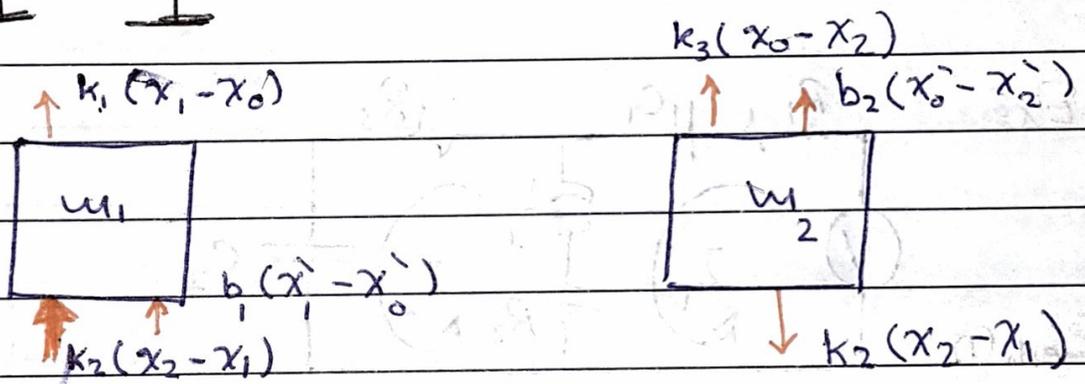
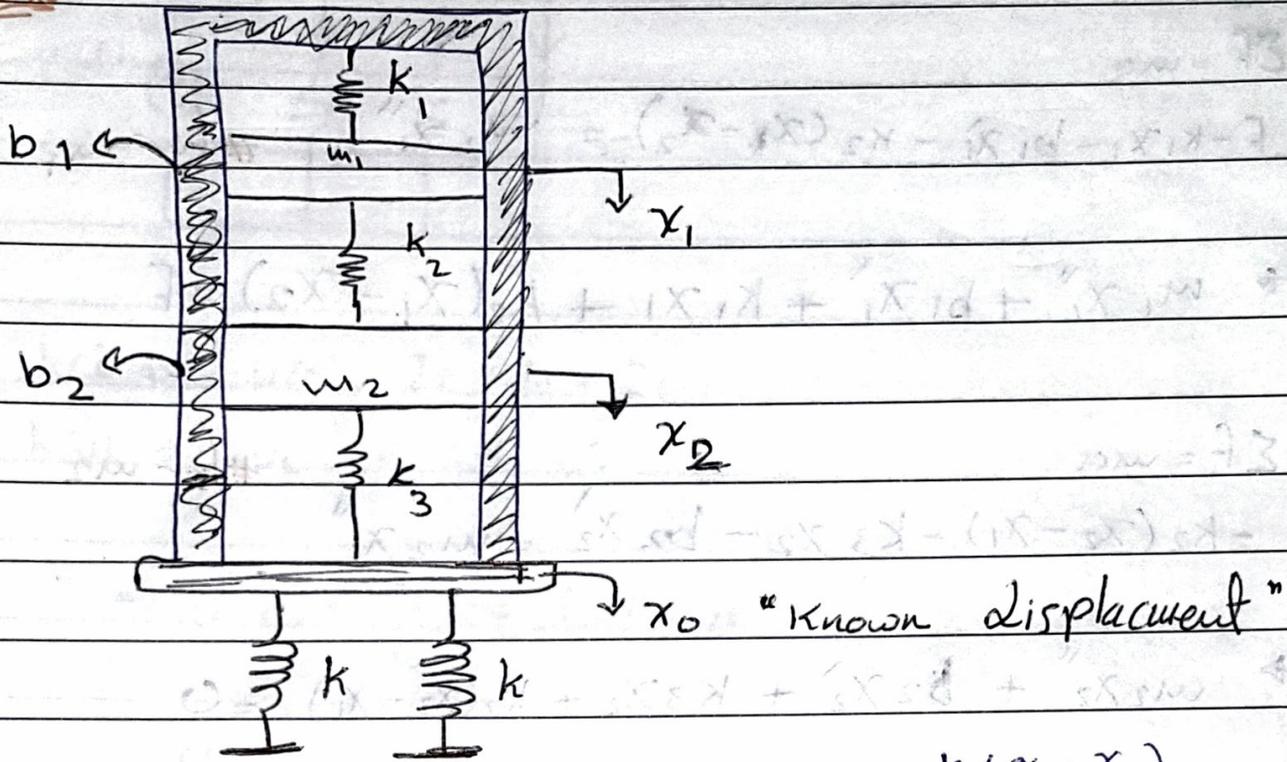
$$V = R_1 I_1 + \frac{1}{C_1} \int I_1 dt + L \frac{d(I_1 - I_2)}{dt} + R_2 (I_1 - I_2)$$

mesh (2)

$$R_3 I_2 + \frac{1}{C_2} \int I_2 dt + R_2 (I_2 - I_1) + L \frac{d(I_2 - I_1)}{dt} = 0$$

23/09/2018

Exo



$$\Sigma F = ma$$

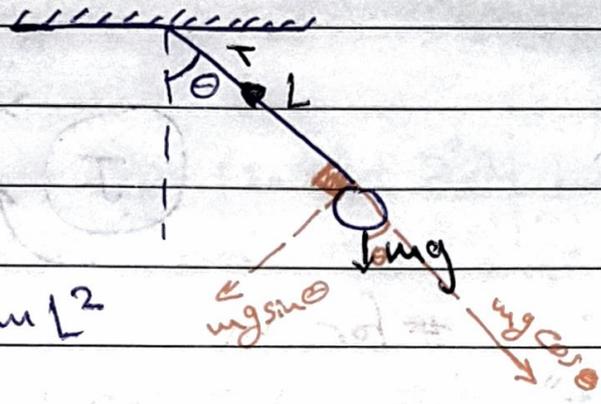
$$-b_1(x_1 - x_0') - k_1(x_1 - x_0) - k_2(x_2 - x_1) = m_1 x_1'' \quad \# m_1$$

$$\hookrightarrow m_1 x_1'' + b_1(x_1 - x_0') + k_1(x_1 - x_0) + k_2(x_2 - x_1) = 0$$

$$-k_2(x_2 - x_1) + k_3(x_0 - x_2) + b_2(x_0 - x_2') + m_2 x_2'' = 0$$

m2

Ex 2



$J = mL^2$

بالغوا بعين " $mg \cos \theta = T$ "

$\Rightarrow F = mg \sin \theta$

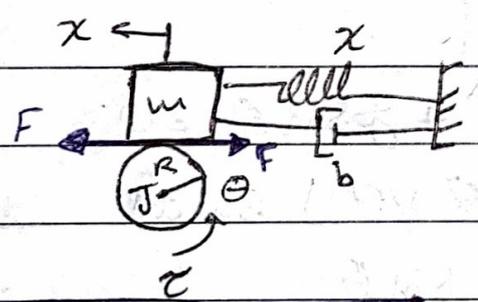
$\Sigma F = ma \rightarrow \Sigma \tau = J\alpha$

$-mgL \sin \theta = J\alpha = J\theta''$

$\hookrightarrow -mgL \sin \theta = mL^2 \theta''$

$\hookrightarrow L\theta'' + g \sin \theta = 0$

Ex 3



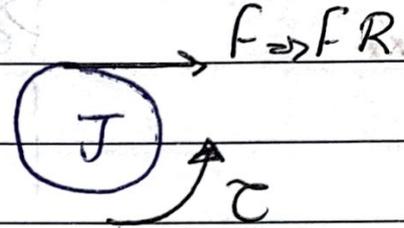
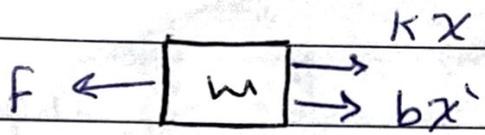
$\Sigma F = ma \rightarrow \Sigma \tau = J\alpha$

$\Rightarrow x = R\theta$

$\Rightarrow \tau = F'R$

linear	circular
x	θ
v, x'	ω, θ'
a, x''	α, θ''
F	τ
m	J

25/09/2018



$$\Sigma F = ma$$

for m

$$F - bx' - kx = mx''$$

$$F = mx'' + bx' + kx$$

$$\Sigma \tau = J\alpha$$

for J

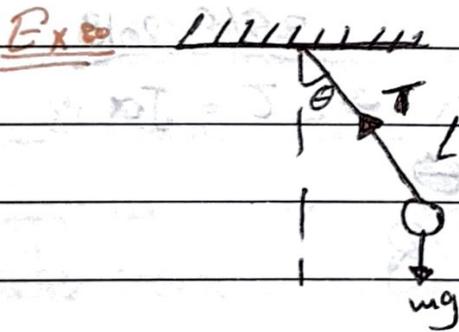
$$\tau = F \cdot R = J\theta''$$

$$\tau = J\theta'' + FR$$

$$\hookrightarrow \tau = J\theta'' + [mx'' + bx' + kx]R$$

where $x = R\theta \rightarrow \theta = x/R$

$$\tau = \frac{J}{R} x'' + R[mx'' + bx' + kx]$$



$$L\theta'' + g \sin \theta = 0$$

* If θ very small

$$\theta \approx \sin \theta$$

$$\hookrightarrow L\theta'' + g\theta = 0$$

25/09/2018

Taylor Series expansion of

$$y = g(x_0) + \frac{\partial g}{\partial x} \Big|_{x=x_0} (x-x_0) + \frac{\partial^2 g}{\partial x^2} \Big|_{x=x_0} \frac{1}{2!} (x-x_0)^2 + \dots + \frac{\partial^n g}{\partial x^n} \Big|_{x=x_0} \frac{1}{n!} (x-x_0)^n$$

↳ $\left[\sin \theta_0 + \cos \theta_0 (\theta - \theta_0) - \frac{1}{2} \sin \theta_0 (\theta - \theta_0)^2 \right] \Big|_{\theta_0 = \theta}$

$0 + \theta - 0 = \theta \Rightarrow 3$ terms accurate.

Laplace Transform

$$F(s) = \mathcal{L}[f(t)] = \int_0^{\infty} e^{-st} f(t) dt \quad t > 0$$

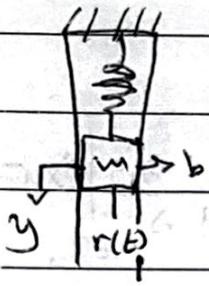
$$f(t) = \mathcal{L}^{-1}[F(s)]$$

Table 2-3

$f(t)$	$F(s)$	$f(t)$	$F(s)$
step fun $\Rightarrow f(t) = 1$	$\frac{A}{s}$	$f^{(k)}(t) = \frac{d^k f}{dt^k}$	$s^k F(s) - s^{k-1} f(0^-)$
Ae^{-at}	$\frac{A}{s+a}$	$\int_{-\infty}^t f(t) dt$	$\frac{F(s)}{s}$
$\sin \omega t$	$\frac{\omega}{s^2 + \omega^2}$		
$\cos \omega t$	$\frac{s}{s^2 + \omega^2}$		
t^n	$\frac{n!}{s^{n+1}}$		
impulse fun. δ	1		

27/09/2018

$$m y'' + b y' + k y = P(t)$$



$$\Rightarrow m [s^2 y(s) - s y(0) - y'(0)] + b [s y(s) - y(0)]$$

$$+ k y(s) = P(s)$$

⊗ if all initial conditions = 0, $y(0), y'(0) = 0$

$$m s^2 y(s) + b s y(s) + k y(s) = P(s)$$

$$\hookrightarrow y(s) [m s^2 + b s + k] = P(s)$$

$$\Rightarrow y(s) = \frac{1}{m s^2 + b s + k} \cdot P(s)$$

Ex $y(s) = \frac{s+3}{s^2+3s+2} \Rightarrow$ find $y(t)$

$$\left[\frac{s+3}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} \right] \quad * (s+1)(s+2)$$

$$s+3 = A(s+2) + B(s+1)$$

$$\Rightarrow s = -1 \Rightarrow A = 2$$

$$\Rightarrow s = -2 \Rightarrow B = -1$$

21/09/18
29/09/2018

$$Y(s) = \frac{2}{s+1} + \frac{-1}{s+2}$$

$$\rightarrow Y(t) = 2e^{-t} - e^{-2t}$$

Ex 3 $Y(s) = \frac{2}{(s+1)(s+2)^2}$ find $Y(t)$

$$= \frac{A}{s+1} + \frac{B}{(s+2)^2} + \frac{C}{s+2} \quad * (s+1)(s+2)^2$$

$$\Rightarrow 2 = A(s+2)^2 + B(s+1) + C(s+1)(s+2)$$

$$s = -1 \Rightarrow A = 2$$

$$s = -2 \Rightarrow B = -2$$

$$s = 0 \Rightarrow C = -2$$

$$Y(s) = \frac{2}{s+1} + \frac{-2}{(s+2)^2} + \frac{-2}{s+2}$$

$$\rightarrow Y(t) = 2e^{-t} - 2te^{-2t} - 2e^{-2t}$$

27/09/2018

*Complex

$$Ex 80 \quad Y(s) = \frac{3}{s(s^2+2s+5)}$$

$$s=0, -1 \pm j2$$

$$= \frac{A}{s} + \frac{Bs+C}{s^2+2s+5} \quad] * s(s^2+2s+5)$$

$$\Rightarrow 3 = A(s^2+2s+5) + s(Bs+C)$$

$$\hookrightarrow 3 = As^2 + 2As + 5A + Bs^2 + Cs$$

$$5A = 3 \Rightarrow *A = \frac{3}{5}, \quad *B = -\frac{3}{5}$$

$$2A + C = 0 \Rightarrow 2\left(\frac{3}{5}\right) + C = 0 \Rightarrow *C = -\frac{6}{5}$$

$$\Rightarrow Y(s) = \frac{3}{5s} + \frac{\left(-\frac{3}{5}\right)s + \left(-\frac{6}{5}\right)}{s^2+2s+5}$$

$$\frac{-3}{5} \left(\frac{s+1+1}{(s+1)^2+4} \right)$$

$$\frac{s+1}{(s+1)^2+4} + \frac{1}{(s+1)^2+4} * \frac{1}{2}$$

$$\frac{3}{5s} + \frac{-3}{5} \left[\frac{s+1}{(s+1)^2+4} + \frac{1}{2} \frac{1}{(s+1)^2+4} \right]$$

$$Y(t) = \frac{3}{5} - \frac{3}{5} e^{-t} \cos(2t) - \frac{3}{10} e^{-t} \sin(2t)$$

30/09/2018

Transfer function (linear systems)

The ratio of Laplace transform of the O/P variable to the Laplace transform of the I/P variable with all initial conditions = 0

$$G(s) = T(s) = \frac{Y(s)}{R(s)} = \frac{\text{Output}}{\text{Input}}$$

$$m y'' + b y' + k y = r(t)$$

O/P $\Rightarrow y$

I/P $\Rightarrow r$

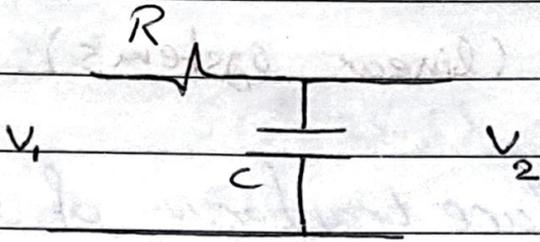
$$m s^2 y(s) + b s y(s) + k y(s) = R(s)$$

$$y(s) [m s^2 + b s + k] = R(s)$$

$$\frac{Y(s)}{R(s)} = \frac{1}{m s^2 + b s + k}$$

02/10/2018

Ex 80



$$T.F = \frac{O/P}{I/P}$$

$$\Rightarrow O/P = T.F \times I/P$$

$$y(s) = G(s) \cdot R(s)$$

$$\hookrightarrow y(t) \quad \# \text{note}$$

I/P V_1 , O/P I
= Find:

$$T_o f = T(s), \quad I(s) \\ V_1(s)$$

$$V_2(s) = ?! \Rightarrow V_1 = V_R + V_C$$

$$V_1(s) = IR + \frac{1}{C} \int I dt$$

$$\Rightarrow V_2 = V_C = \frac{1}{C} \int I dt$$

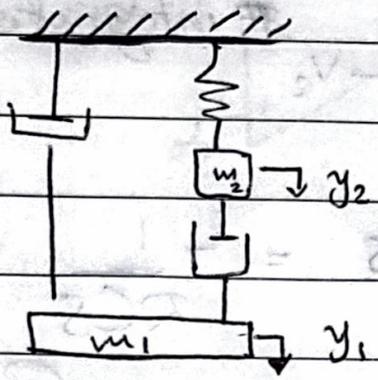
$$V_1(s) = I(s)R + \frac{1}{Cs} I(s)$$

$$V_1(s) = I(s) \left[R + \frac{1}{Cs} \right]$$

$$\Rightarrow \frac{I(s)}{V_1(s)} = \frac{Cs}{RCs + 1}$$

Ex: ۱۰۰۰ ی رکتاب

H.W



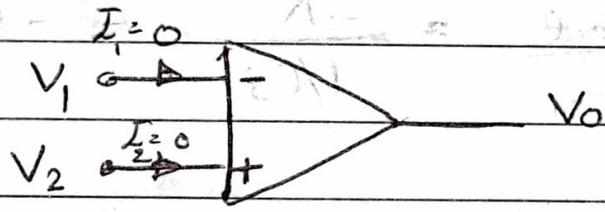
O/P $\Rightarrow y_1, y_2$

I/P $\Rightarrow v(t)$

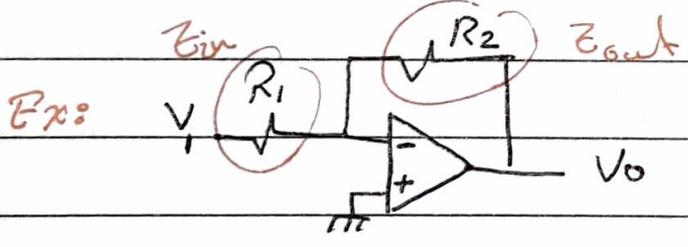
$y_2(s), y_1(s)$

$R(s) \quad R(s)$

OP - Amp transfer function so
 ↳ operational amplifier controller



for Ideal op-amp $I_1 = I_2 = 0$



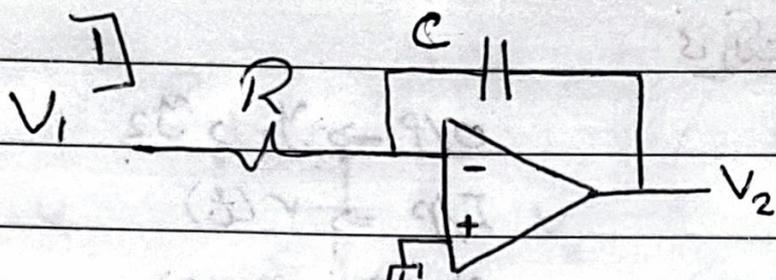
$Z_C = \frac{1}{Cs}$

$\frac{V_o(s)}{V_i(s)} = -\frac{Z_{out}}{Z_{in}} = -\frac{R_2}{R}$

$Z_L = LS$

Proportional (P)

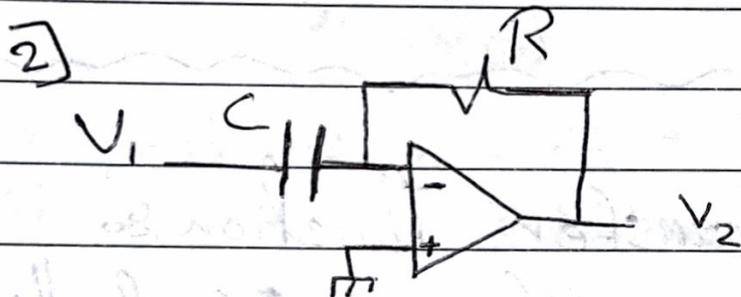
Ex 8



integrator

$$\frac{V_2(s)}{V_1(s)} = \frac{-Z_{out}}{Z_{in}} = \frac{-1/c s}{R} = -\frac{1}{RCS}$$

(I)



(D)

$$\frac{V_2(s)}{V_1(s)} = \frac{-Z_{out}}{Z_{in}} = \frac{-R}{1/c s} = -RCS$$

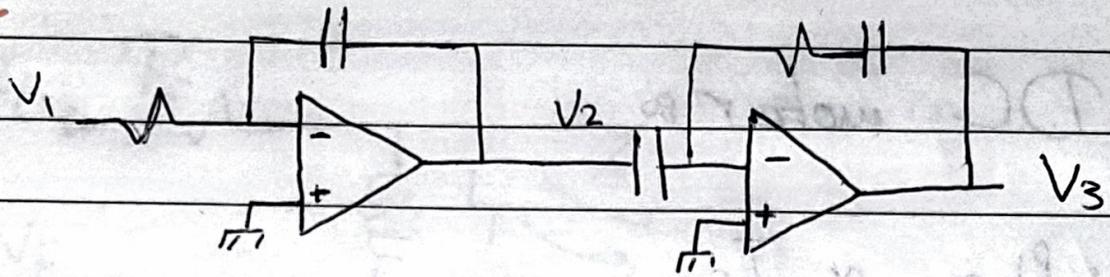
$$1] \int f(t) = \frac{f(s)}{s}$$

$$2] f(t) = s \cdot f(s) \text{ diff}$$

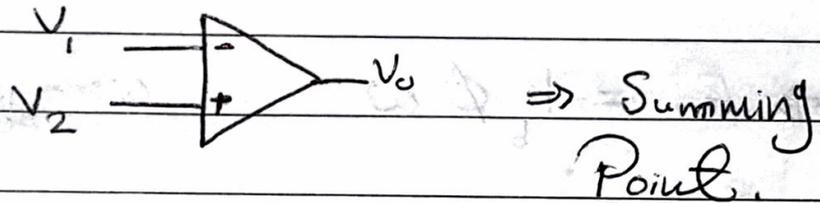
PID

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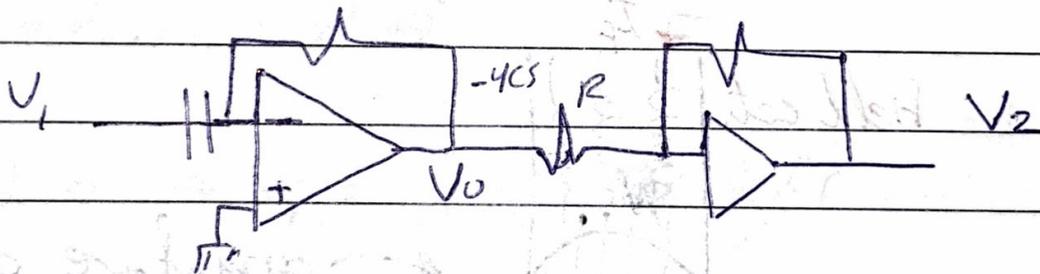
Exo



$$\frac{V_3}{V_1} = \frac{V_3}{V_2} * \frac{V_2}{V_1}$$



Exo $\frac{V_2}{V_1} = 4CS$



07/10/2018

DC motor

* ϕ : Flux $\propto I_f$

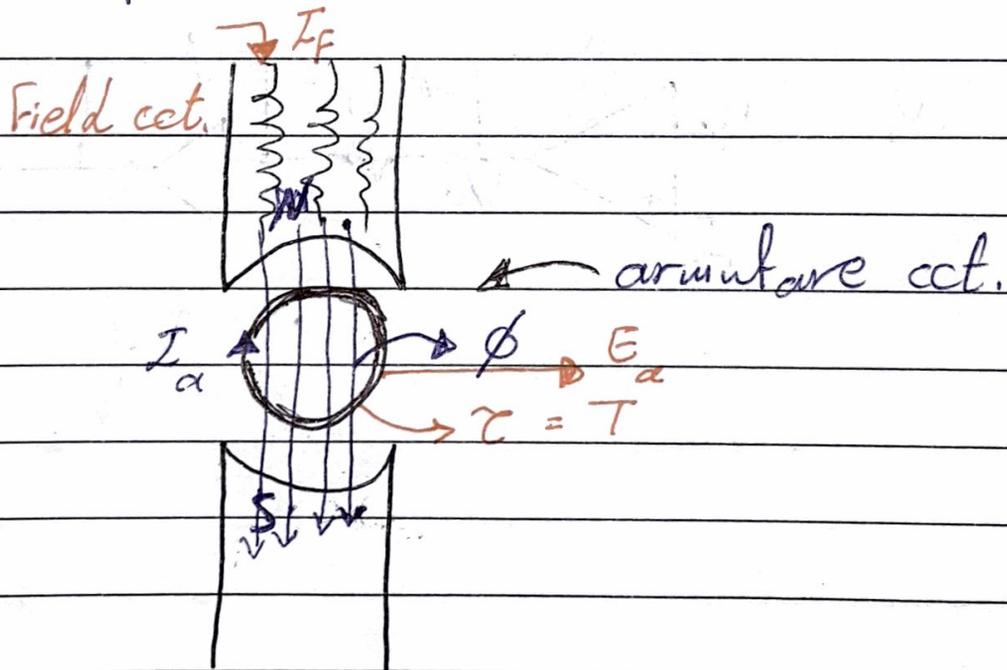
$$\Rightarrow \phi = k_f I_f$$

* E = emf $\propto \phi \omega$

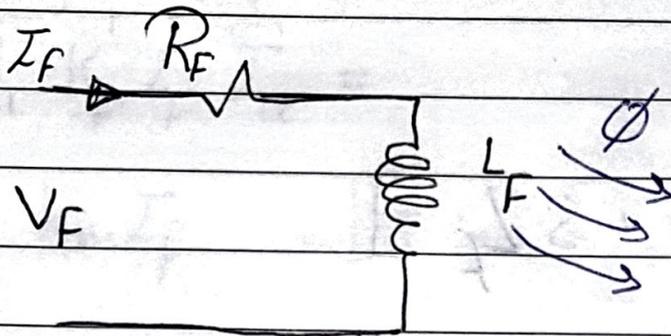
$$\Rightarrow E_a = k_b \phi \omega \quad \omega \text{ angular speed}$$

* $T \propto I_a \phi$

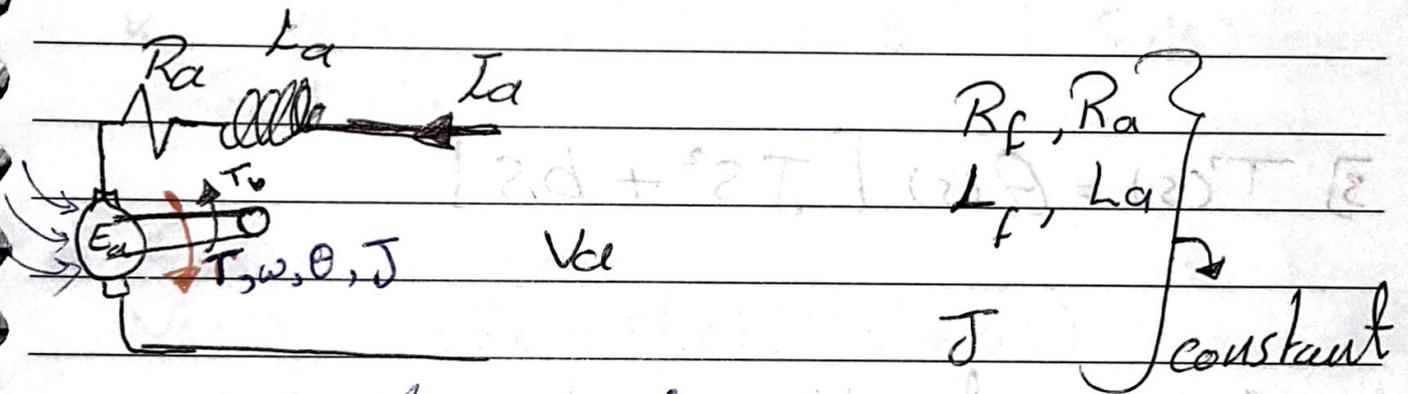
$$\Rightarrow T = k_t \phi I_a$$



07/10/2018



Field cct. \Rightarrow



armature cct.

$$T_B = B \omega = B \dot{\theta}$$

$$\# \text{ field cct} \Rightarrow V_f = I_f R_f + L_f \frac{dI_f}{dt} \quad \text{--- (1)}$$

$$\# \text{ armature cct} \Rightarrow V_a = I_a R_a + L_a \frac{dI_a}{dt} + E_a \quad \text{--- (2)}$$

$$\Sigma T = J \theta''$$

$$\hookrightarrow T - T_b = J \theta'' \Rightarrow T - b \theta' = J \theta''$$

$$T = J \theta'' + b \theta' \quad \text{--- (3)}$$

07/10/2018

To Laplace \Rightarrow

$$1] V_f(s) = I_f [R_f + s L_f]$$

$$2] V_a(s) = I_a [R_a + s L_a] + E_a(s)$$

$$3] T(s) = \Theta(s) [J s^2 + b s]$$

If the input is $V_f(s)$ and o/p is $\Theta(s)$

V_f : Field Voltage, Θ : Position

$$\frac{\cancel{V_f(s)}}{\Theta(s)} \cdot \frac{O/P}{I/P} = \frac{\Theta(s)}{V_f(s)}$$

$$T(s) = K_t \Theta(s) I_a(s)$$

$T_{elec.}$

$$= \Theta(s) [J s^2 + b s]$$

$T_{mech.}$

07/10/2018

constant K_m

$$K_f I_f I_a =$$

$$K_m I_f = [J s^2 + b s] \theta(s)$$

from eq. 1 so

$$\frac{K_m V_f(s)}{R_f + s L_f} = [J s^2 + b s] \theta(s)$$

$$\Rightarrow \frac{\theta(s)}{V_f(s)} = \frac{K_m}{(R_f + s L_f)(J s^2 + b s)}$$

09/10/2018

If the input V_a , output θ is
 "armature voltage controlled DC motors"

$$T_{elec} = T_{mech}$$

const. = K_m

$$K_m \phi I_a(s) = \theta(s) [J s^2 + b s]$$

$$K_m I_a(s) = \theta [J s^2 + b s]$$

09/10/2018

* where $I_a = \frac{V_a - E_a}{R_a + S L_a}$

* $E_a = K_b \phi \omega \Rightarrow K_b \dot{\theta} \Rightarrow K_b S \theta(s)$

$\rightarrow K_m \left[\frac{V_a(s) - K_b S \theta(s)}{R_a + S L_a} \right] - \theta(s) [J S^2 + b S]$

$\theta(s) = \frac{K_m}{V_a(s) S [(R_a + L_a S)(J S + b) + K_b K_m]}$

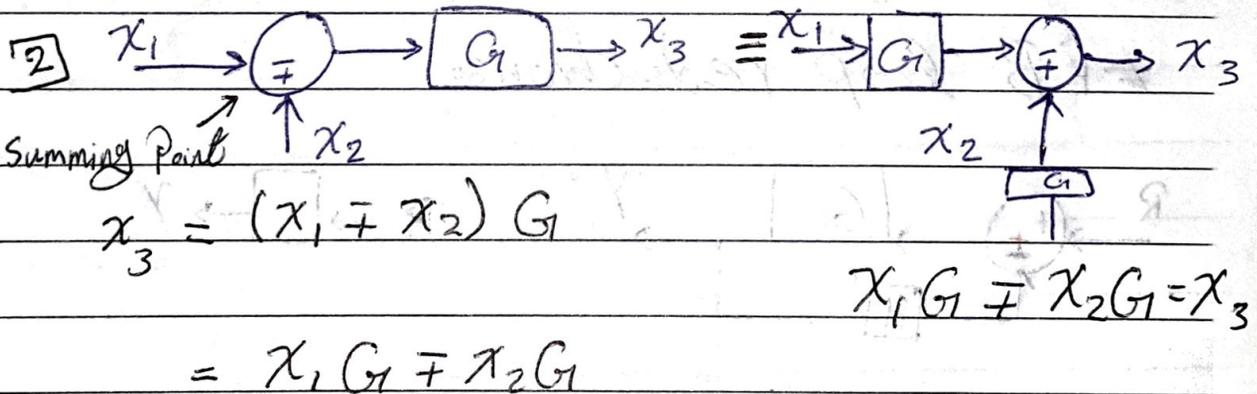
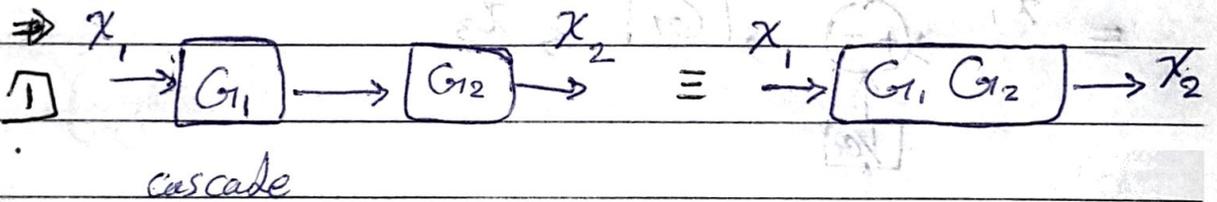
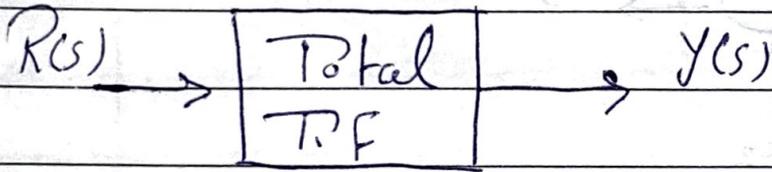
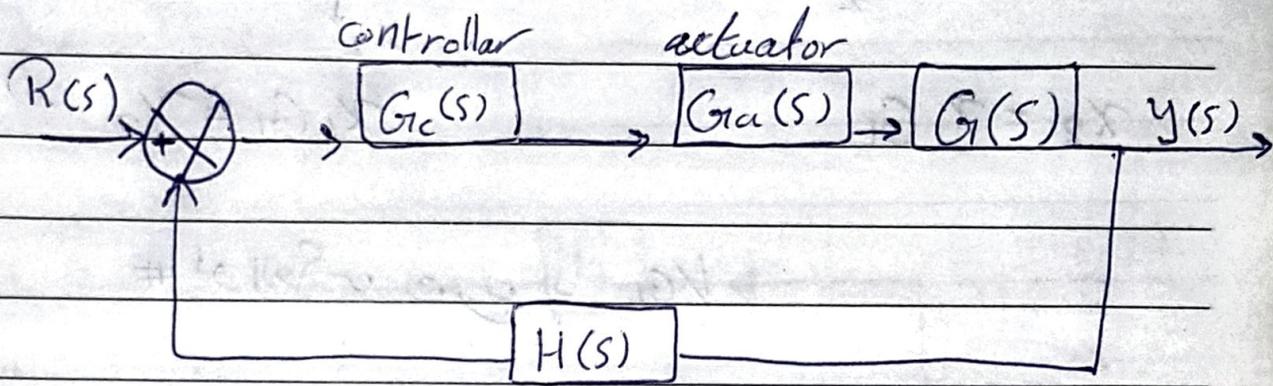
notes 2 inputs \Rightarrow one out put \Rightarrow 2 T.F \Rightarrow sum

Ex: $V_a \xrightarrow{(1)} \theta \Rightarrow 1 + 2$

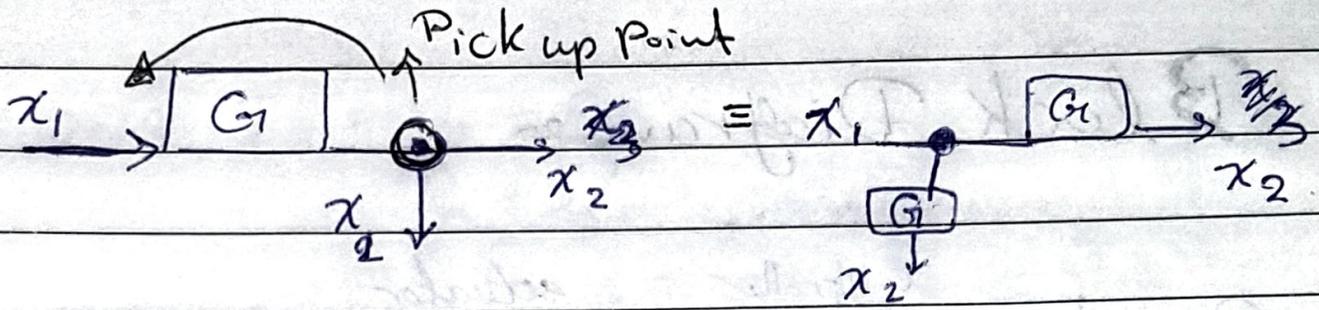
$V_f \xrightarrow{(2)}$

09/10/2018

Block Diagram



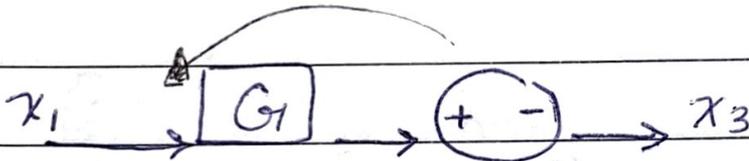
09/10/2018



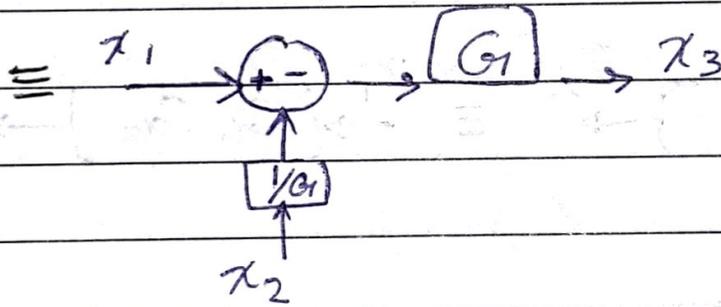
$$x_2 = x_1 G_1$$

$$x_1 G_1 = x_2$$

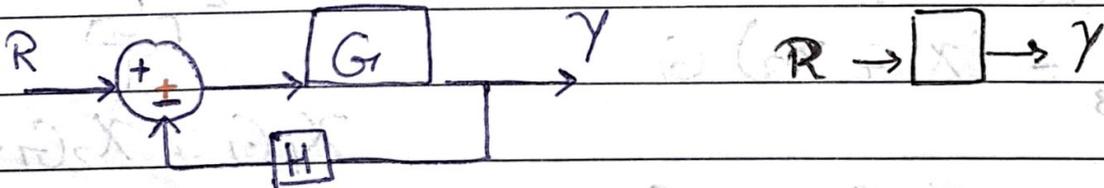
لوالعكس بصرى د $1/G_1$



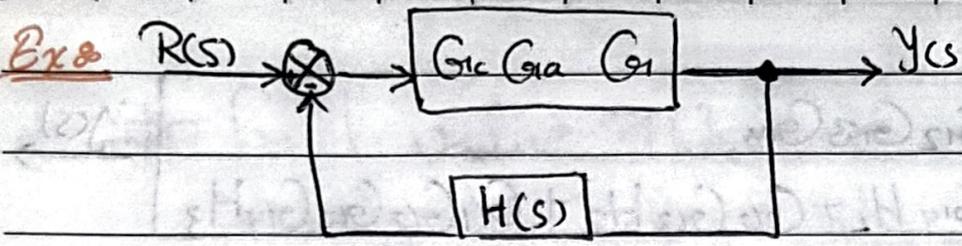
$$x_1 G_1 = x_2 = x_3$$



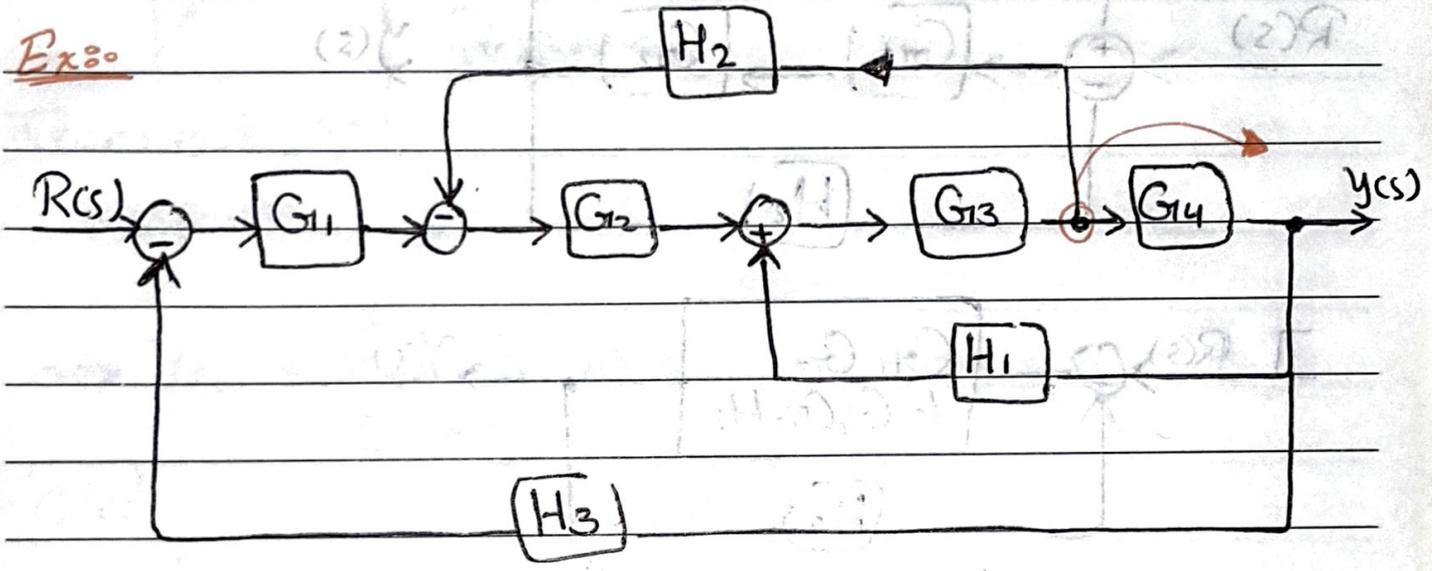
FB loop reduction:



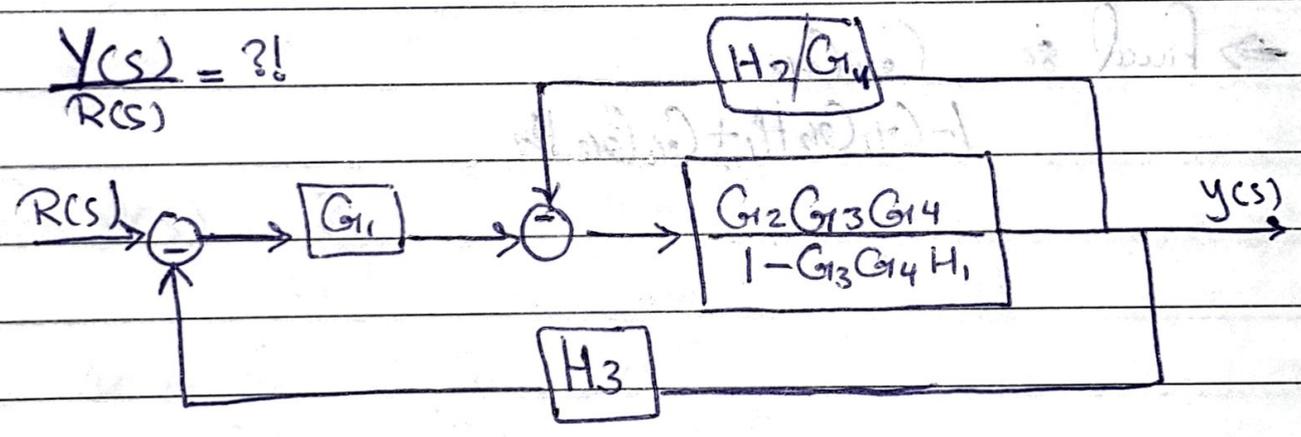
$$\frac{Y(s)}{R(s)} = \frac{G_1}{1 + G_1 H}$$



$$\frac{Y(s)}{R(s)} = \frac{G_c G_a G_1}{1 + G_c G_a G_1 H}$$



$\frac{Y(s)}{R(s)} = ?!$



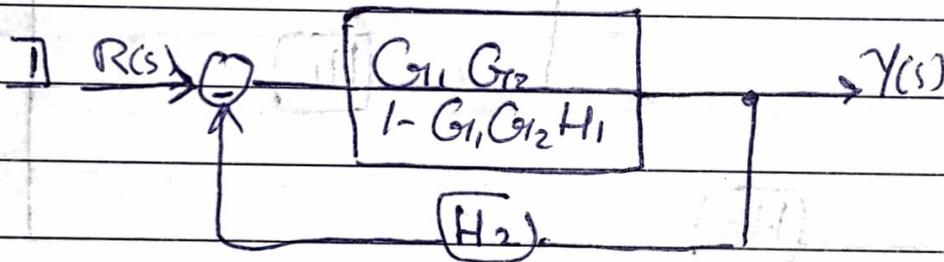
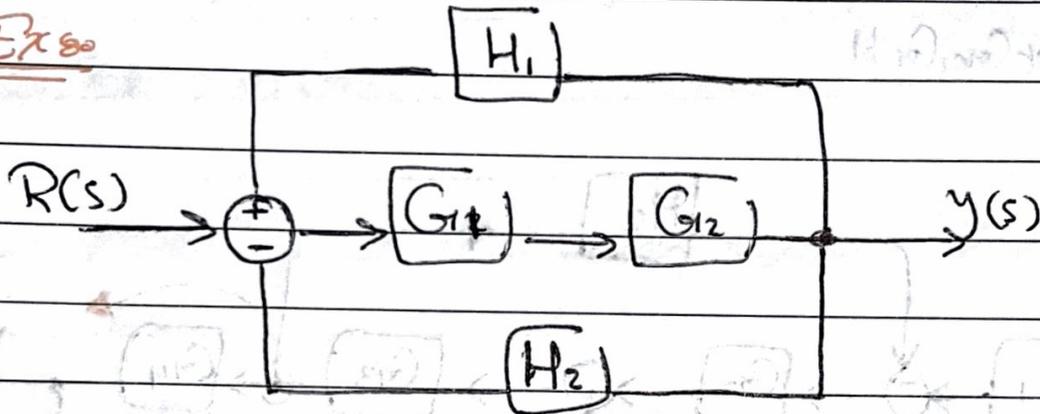
$$\frac{G_{12} G_{13} G_{14}}{1 - G_{13} G_{14} H_1}$$

$$1 + \frac{G_{12} G_{13} G_{14}}{1 - G_{13} G_{14} H_1} \cdot \frac{H_2}{G_{14}} = \frac{1 - G_{13} G_{14} H_1 + G_{12} G_{13} H_2}{1 - G_{13} G_{14} H_1}$$

11/10/2018

$$\Rightarrow R(s) \left| \begin{array}{c} G_1 G_2 G_3 G_4 \\ 1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3 \end{array} \right| y(s)$$

Ex 80

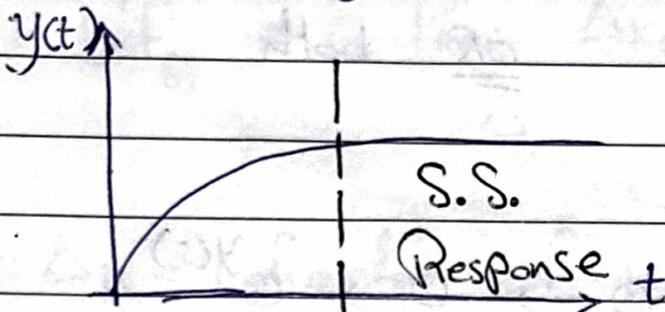


⇒ final 80 $\frac{G_1 G_2}{1 - G_1 G_2 H_1 + G_1 G_2 H_2}$

11/10/2018

Final Value Theorem

* steady state value *



Transient ←

Response →

$$\Rightarrow Y_{ss} = \lim_{s \rightarrow 0} s Y(s)$$

OR

$$\Rightarrow Y_{ss} = \lim_{t \rightarrow \infty} y(t)$$

$$\text{Ex: } Y(s) = \frac{3}{s+4}$$

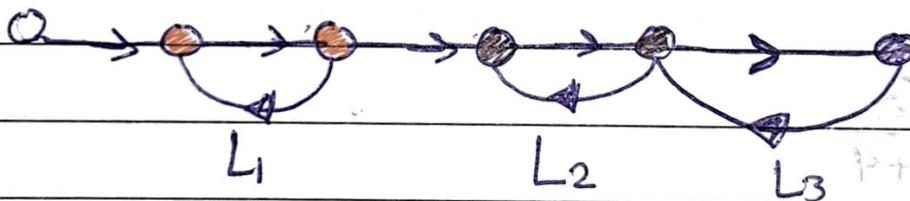
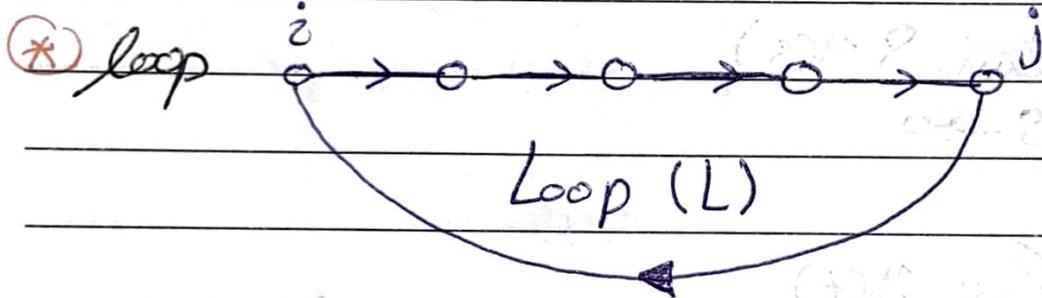
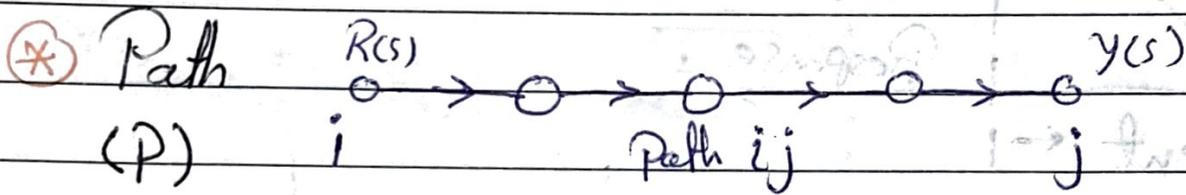
$$Y_{ss} = \lim_{s \rightarrow 0} s \frac{3}{s+4} = 0$$

$$\text{OR } y(t) = 3e^{-4t}$$

$$Y_{ss} = \lim_{t \rightarrow \infty} y(t) = \lim_{t \rightarrow \infty} 3e^{-4t} = 0$$

Signal flow Graph

G_i → branch
 \circ node → I/P OR O/P
OR both



non-touching loops = $L_1 \& L_2$, $L_1 \& L_3$
 2#

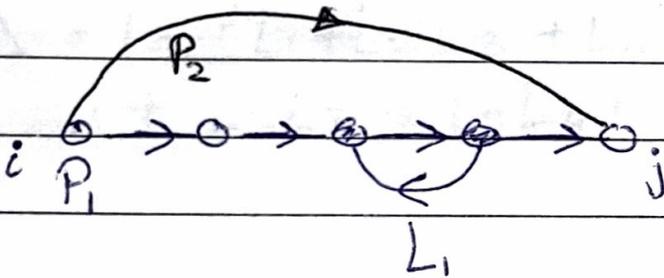
Touching loop = $L_2 \& L_3$
 1#

14/10/2018

Mason's Rule 80

$$T.F. = \sum_{\Delta} P_{k i-j} \Delta_{k i-j}$$

(*) $\Delta_k = \text{Cofactor of Path}(k)$
 $= 1 - \text{Loops that don't touch path } k.$



* $\Delta_1 = 1 - 0 = 1$

* $\Delta_2 = 1 - L_1$

(*) Δ 80 Determinant of the Graph

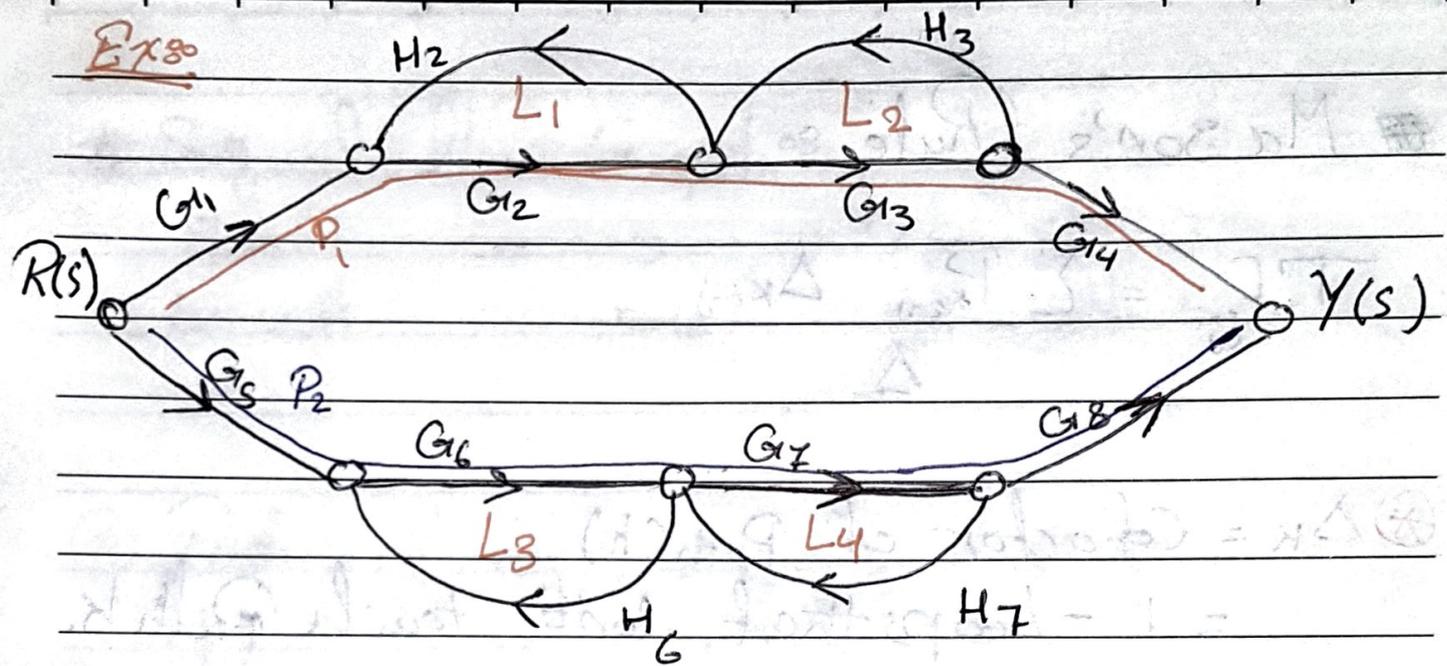
$$\Delta = 1 - (\text{Sum of all loops Gain})$$

$$+ [\text{Sum of the product of all (2 n.t.L)}]$$

$$- [\text{Sum of the product of all (3 n.t.L)}]$$

14/10/2018

Ex 80



Find T.F $\frac{Y(s)}{R(s)}$

$$P_1 = G_1 G_2 G_3 G_4$$

$$P_2 = G_5 G_6 G_7 G_8$$

$$L_1 = G_2 H_2$$

$$L_2 = G_3 H_3$$

$$L_3 = G_6 H_6$$

$$L_4 = G_7 H_7$$

⊕ 2. No. T. Ls 80

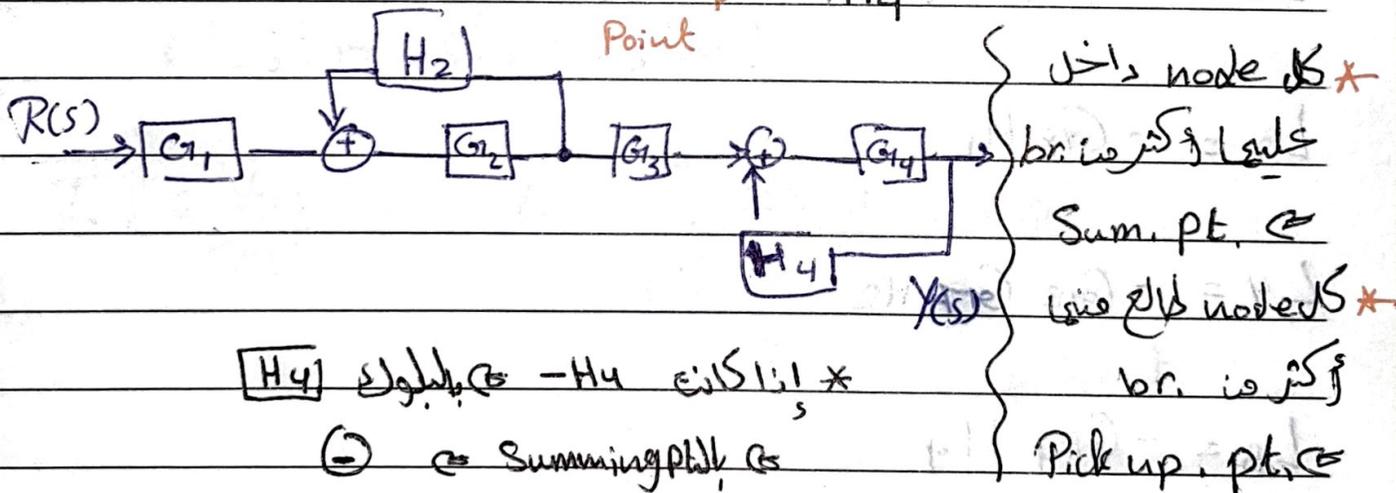
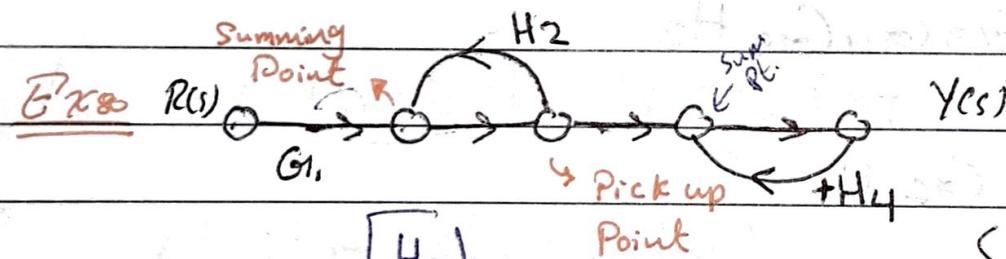
$$L_1, L_3 \quad / \quad L_2, L_3 \quad / \quad L_2, L_3 \quad / \quad L_2, L_4$$

$$* \Delta_1 = 1 - (L_3 + L_4) = 1 - L_3 - L_4$$

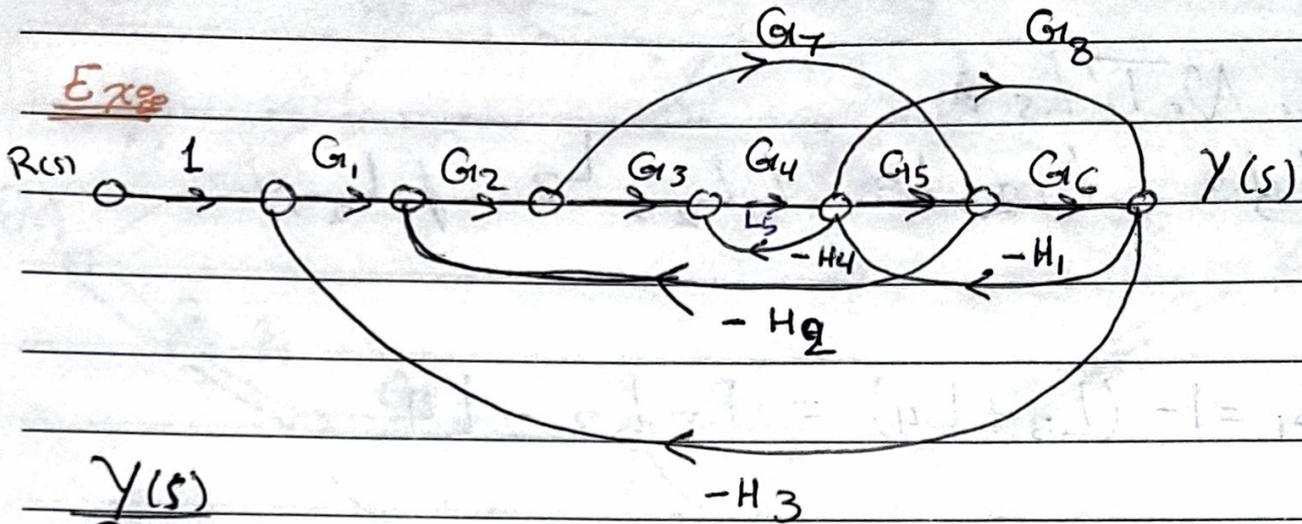
$$* \Delta_2 = 1 - L_1 - L_2$$

$$* \Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1 L_3 + L_1 L_4 + L_2 L_3 + L_2 L_4)$$

$$\Rightarrow \frac{\sum P_k \Delta_k}{\Delta} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$



Ex 20



$$\frac{Y(s)}{R(s)}$$

$$P_1 = 1 G_1 G_2 G_3 G_4 G_5 G_6$$

$$P_2 = 1 G_1 G_2 G_7 G_8$$

$$P_3 = 1 G_1 G_2 G_3 G_4 G_8$$

$$L_1 = -G_2 G_3 G_4 G_5 H_2$$

$$L_2 = -G_5 G_6 H_1$$

$$L_3 = -G_8 H_1$$

$$L_4 = -G_2 G_7 H_2$$

$$L_5 = -G_4 H_4$$

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$$L_6 = -G_1 G_2 G_3 G_4 G_5 G_6 H_3$$

$$L_7 = -G_1 G_2 G_7 G_6 H_3 \quad \text{st. 13, Loop 11#}$$

$$L_8 = -G_1 G_2 G_3 G_4 G_8 H_3$$

2 n.T.L.

$$L_5 L_4$$

$$L_5 L_7$$

$$L_3 L_4$$

$$\Delta_1 = 1$$

$$\Delta_2 = 1 - L_5$$

$$\Delta_3 = 1$$

$$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8)$$

$$+ (L_5 L_4 + L_5 L_7 + L_3 L_4)$$

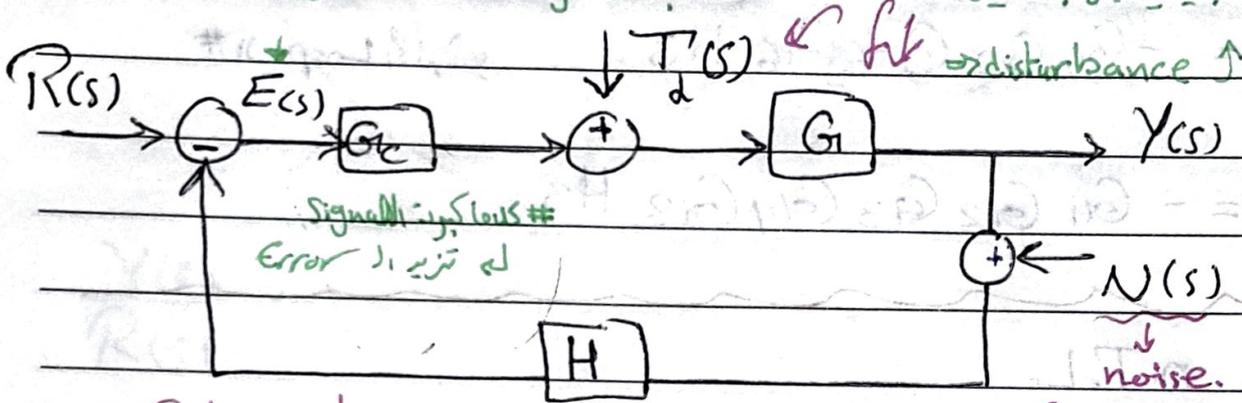
$$T.F = \frac{Y(s)}{R(s)} = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3}{\Delta}$$

→ First Exam

CH4 @ FB Sys. Charac.

Error signal \rightarrow Summing Pt. \downarrow \rightarrow $E(s)$

مؤثر خارجي في نقطة الجمع \rightarrow disturbance \uparrow



Signal الكبر: $\#$ Error له تزييد \uparrow

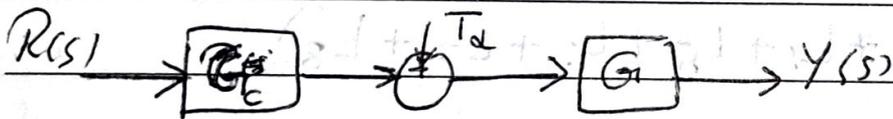
noise \downarrow

Closed sys.

$\hookrightarrow f \uparrow$

Error \downarrow \Rightarrow accuracy \uparrow , O.L, Co.L \downarrow
 \hookrightarrow steady state \downarrow \Rightarrow E. of sys. \downarrow \Rightarrow sys. \downarrow

Open Sys. ∞



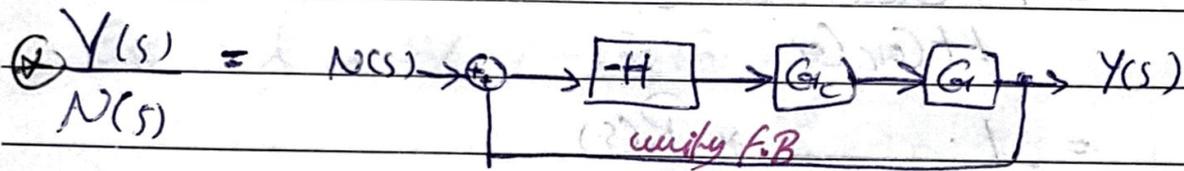
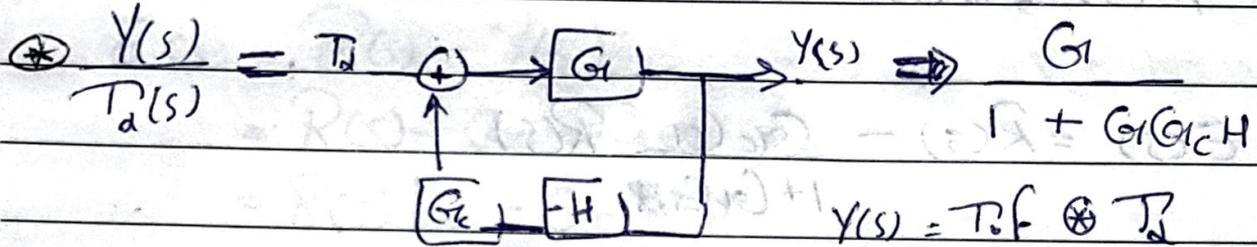
The disadvantages of C.L systems \rightarrow cost \uparrow $\&$ complexity \uparrow

* for closed loop syst. ∞

$$\textcircled{*} \frac{Y(s)}{R(s)} = \frac{G_c G_1}{1 + G_c G_1 H}$$

eg. Sum $e=0=T_d, N$ 11 jw 11
PE.

$$\Rightarrow Y(s) = T.F. * R(s)$$



$$= \frac{-G_c G_1 H}{1 + G_c G_1 H} \Rightarrow Y(s) = T.F. * N(s)$$

Total $y = \sum Y(s)$

for Open loop syst. ∞

$$\textcircled{*} \frac{Y(s)}{R(s)} = G_c G_1 \Rightarrow Y(s) = G_c G_1 R(s)$$

$$\frac{Y(s)}{T_d(s)} = G_1 \Rightarrow Y(s) = G_1 T_d(s)$$

$$\text{Total } Y = G_c G_1 R(s) + G_1 T_d(s)$$

$$E(s) = R(s) - Y(s)$$

for closed loop

Error due to $R(s)$ is "if $H(s) = 1$

$$N(s), T_d(s) = 0$$

$$\rightarrow E(s) = R(s) - \frac{G_c G_1}{1 + G_c G_1} R(s)$$

$$= \frac{1 + G_c G_1 - G_c G_1}{1 + G_c G_1} * R(s)$$

$$= \frac{1}{1 + G_c G_1} * R(s)$$

$$e_{ss} = \lim_{s \rightarrow 0} s \frac{1}{1 + G_c G_1} * R(s) \quad R(s) = \frac{1}{s}$$

→ steady state error → unit step i/p

$$\lim_{s \rightarrow 0} s \frac{1}{1 + G_c G_1} * \frac{1}{s}$$

$$e_{ss} = \frac{1}{1 + G_c G_1(0)}$$

* عند التحويل إلى $s=0$ وكانت قيمة $(-ve)$ e_{ss} هو $(-)$ الإشارة، أما إذا كانت
بيضاء ال (0) و (∞)

23/10/2018

Error for Open loop sys :

$$\text{Error due to } R(s) = \frac{1}{s}$$

$$\begin{aligned} E(s) &= R(s) - Y(s) \\ &= R(s) - G_c G_1 R(s) \\ &= R(s) [1 - G_c G_1] \end{aligned}$$

$$e_{ss} = \lim_{s \rightarrow 0} s (1 - G_c G_1) \frac{1}{s}$$

$$e_{ss, o.l.} = 1 - G_c(0) G_1(0)$$

$$\textcircled{50} \quad e_{ss, c.l.} < e_{ss, o.l.}$$

$$\therefore \alpha_{c.l.} > \alpha_{o.l.}$$

$$E(s) = R(s) - Y(s) \Rightarrow \text{due to } T_d(s), N(s), R(s) = 0, H(s) = 1$$

$$E(s) = 0 - \frac{G_1 T_d(s)}{1 + G_c G_1} = - \frac{G_1 T_d(s)}{1 + G_c G_1} \rightarrow \text{const step i/p}$$

$$\Rightarrow e_{ss, c.l.} = \frac{-G_1(0)}{1 + G_c(0) G_1(0)}$$

* for O.L.

Due to $T_d(s)$

$$E(s) = R(s) - Y(s)$$

$$= 0 - G_1 T_d(s) = -G_1 T_d(s)$$

$$e_{ss \text{ O.L.}} = \lim_{s \rightarrow 0} s (-G_1) \cdot \frac{1}{s} = -G_1(0) = -103$$

(50) $e_{ss \text{ O.L.}} < e_{ss \text{ O.L.}}$ due to T_d

disturbance.

Rejection \rightarrow d.R.O.L

C.O.L noise

* Noise Rejection $\&$ $H(s) = 1$

error due to $N(s)$ $(T_d, R_{ss}) = 0$

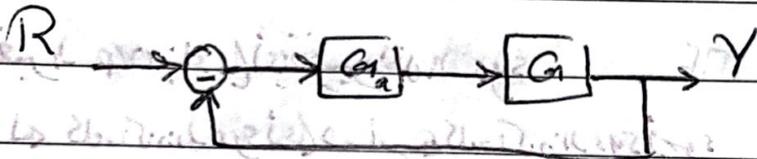
$$\frac{Y(s)}{N(s)} = \frac{-G_c G_1}{1 + G_c G_1}$$

$$E(s) = R(s) - Y(s) \quad N(s) = \frac{1}{s}$$

$$e_{ss} = \lim_{s \rightarrow 0} \left(0 - \frac{-G_c G_1}{1 + G_c G_1} \right) = \lim_{s \rightarrow 0} s \frac{G_c G_1}{1 + G_c G_1} \cdot \frac{1}{s} = \frac{G_c(0) G_1(0)}{1 + G_c(0) G_1(0)}$$

Ex 80 $G(s) = \frac{10}{s(0.001s + 1)}$, $G_d = K$

for unit step function input $\frac{1}{s}$, find e_{ss}



$$\Rightarrow E(s) = R(s) - Y(s) = R(s) \left[\frac{1 - G_d G}{1 + G_d G} \right] = R(s) \frac{1}{1 + G_d G}$$

$$\frac{Y(s)}{R(s)} = \frac{G_d G}{1 + G_d G} \Rightarrow Y(s) = \frac{G_d G}{1 + G_d G} R(s)$$

$$\Rightarrow e_{ss} = \lim_{s \rightarrow 0} s \cdot \frac{1}{1 + G_d G} \cdot \frac{1}{s} = 0 \quad \left[\begin{array}{l} \text{no error -} \\ \text{Perfect sys.} \end{array} \right]$$

* If $r(t) = 10t = \frac{10}{s^2}$, $e_{ss} = 0.1 \text{ mm}$, find K ?

$$\lim_{s \rightarrow 0} s \cdot \frac{1}{1 + G_d G} \cdot \frac{10}{s^2} = 0.1 * 10^{-3}$$

$$\Rightarrow \lim_{s \rightarrow 0} \frac{10}{s + K} \cdot \frac{10}{s(0.001s + 1)} = 0.1 * 10^{-3} = \frac{1}{K}$$

Sensitivity

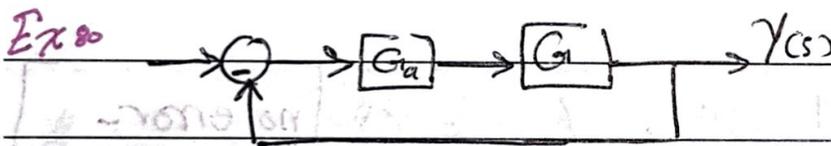
→ The Ratio of the change in the sys. T.o.f. to the change of a process T.o.f. for Small Change.

$$S_G^T = \frac{\Delta T}{\Delta G}$$

* تغییر در O/P بنیادی (تغییر داخلی در sys) که کلمات کنده در sys زود
 * تغییر در T.o.f.

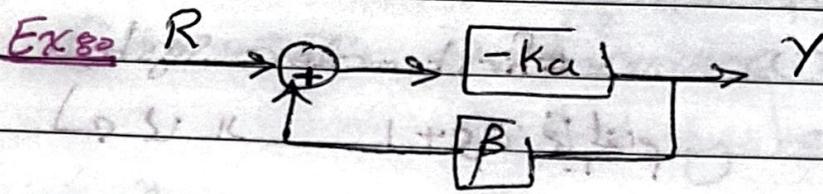
$$S_G^T = \frac{\frac{\partial T}{\partial G}}{\frac{\partial G}{G}} = \frac{\frac{\partial T}{T} \cdot G}{\frac{\partial G}{G}}$$

تغییر در T.o.f. / تغییر در Process T.o.f. (استفاده بالنسبه)



$$T = \frac{Y(s)}{R(s)} = \frac{G_c G}{1 + G_c G}$$

$$S_G^T = \frac{G_c}{(1 + G_c G)^2} \cdot \frac{G}{1 + G_c G} = 1$$



$$-K_a = T$$

$$1 + K_a \beta$$

$$S_{K_a}^T = \frac{\partial T}{\partial K_a} \cdot \frac{K_a}{T}$$

Closed Loop

$$= \frac{-1}{(1 + K_a \beta)^2} \cdot \frac{K_a}{\frac{-K_a}{1 + K_a \beta}} = \frac{1}{1 + K_a \beta}$$

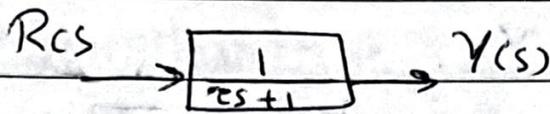


$$T = -K_a$$

$$S_{K_a}^T = -1 \cdot \frac{K_a}{-K_a} = 1$$

∞ S.O.L. sys. = 1 ⇒ O.L. sys. sensitivity

1 sensitivity

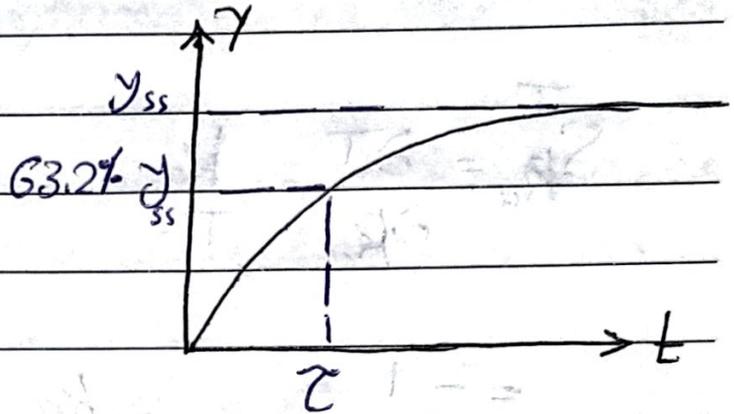


first order system

ماتر في $\tau s + 1$

τ : Time constant

open loop



* for closed loop *

$$\frac{1}{\tau s + 1} = \frac{1/2}{\frac{\tau}{2}s + \frac{2}{2}}$$

⇒ Closed loop

⇒ so the advantages of CLT through the role of signal error

1] Decreased sensitivity of system to

① ageing ② model uncertainty

③ variations in parameters of the process

④ environmental change sensor ⇒ "الطوبى، ا، ا، ا"

2] Improved rejection of disturbance تهدئة ال noise

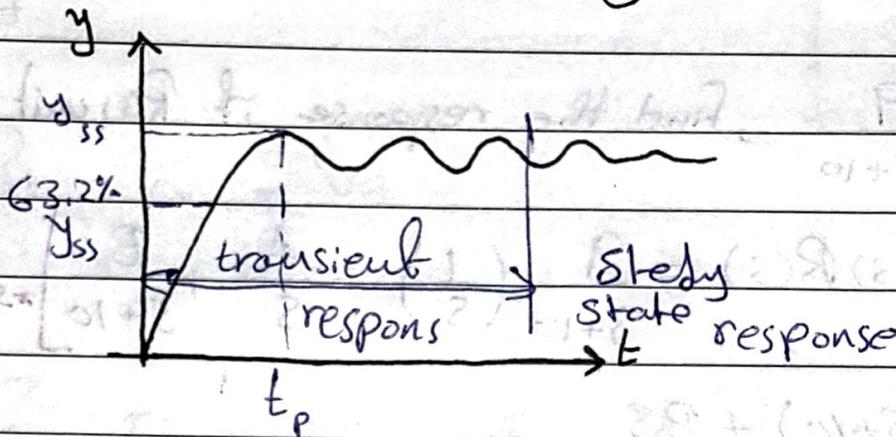
3] measurement → noise attenuation 5th sys

4] reduction of steady state error 1st order

5] easy adjustment transient response

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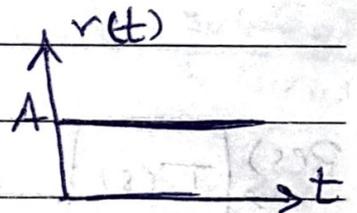
CH 5 ^{crisis of all responses} The performance of feedback control systems 80



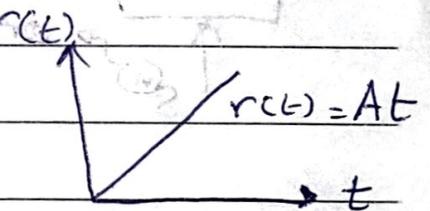
$$\frac{Y(s)}{R(s)} \Rightarrow \text{laplace}^{-1} \Rightarrow y(t)$$

Test signals (Inputs), $R(s)$

① unit step input $\Rightarrow R(s) = \frac{1}{s}$
 $\hookrightarrow A = 1$



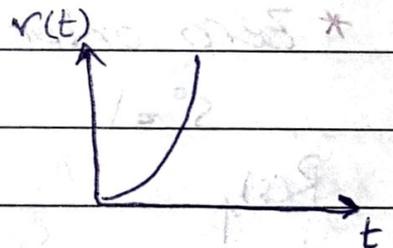
② Ramp input $\Rightarrow R(s) = \frac{A}{s^2}$



③ parabolic input

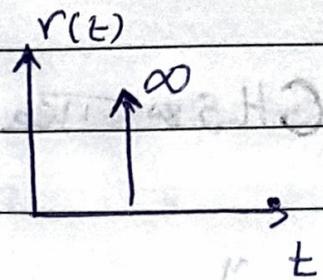
$$r(t) = At^2$$

$$R(s) = \frac{2A}{s^3}$$



(4) unit impulse input

$S(s) = 1$



Exo $G(s) = \frac{9}{s+10}$, find the response if $R(s)$ unit step

$Y(s) = G(s)R(s) = \frac{9}{s+10} \left(\frac{1}{s} \right) = \frac{A}{s} + \frac{B}{s+10}$ * $S(s+10)$

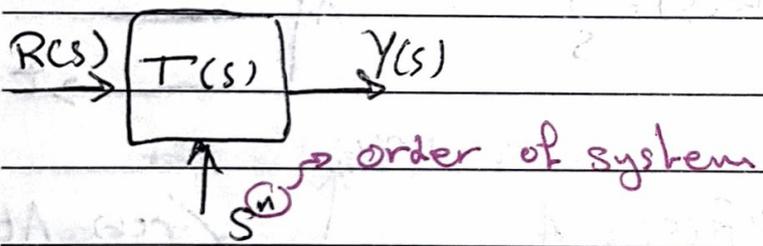
$9 = A(s+10) + BS$

$\Rightarrow A = 0.9, B = -0.9$

when $s=0$

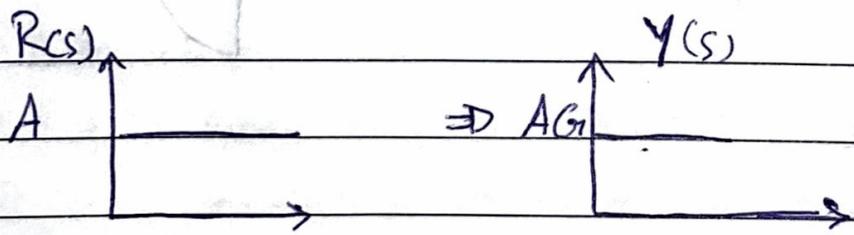
when $s=-10$

$\Rightarrow y(t) = 0.9 - 0.9e^{-10t} = 0.9(1 - e^{-10t})$



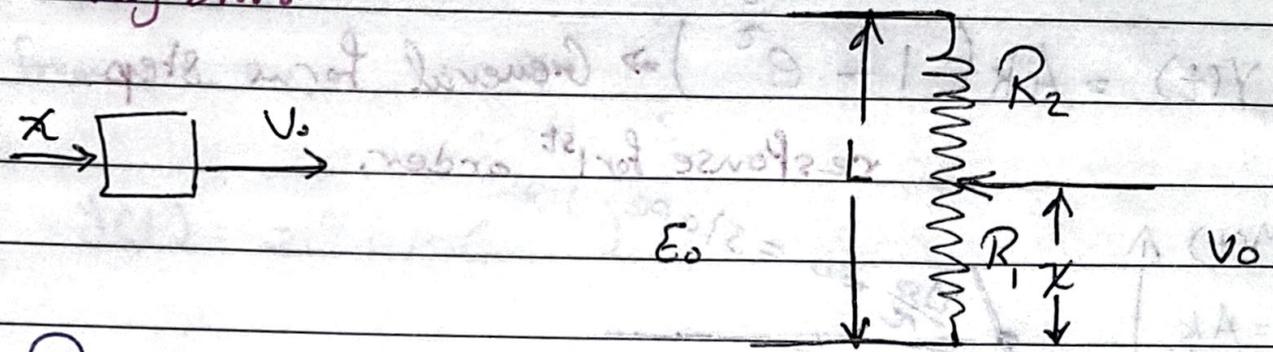
* zero order system :

$s^0 = 1, T(s) = \frac{1}{s}$



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* Voltage Div. :-



$$\frac{R_1}{R_1 + R_2} E_o = V_o$$

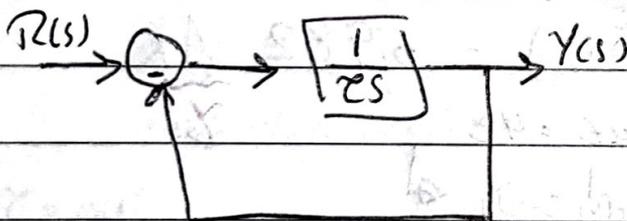
$$\frac{X}{L} E_o = V_o \Rightarrow \frac{V_o(s)}{X(s)} = \frac{E_o}{L}$$

* First order system :-

$$\frac{Y(s)}{R(s)} = \frac{1/\tau s}{1 + 1/\tau s} = \frac{K}{\tau s + 1}$$

K → Gain constant = $\frac{y_{ss}(of p_{ss})}{R_{ss}(of p_{ss})}$
 τ → time constant.

General form of 1st order system T.F



$Y(t) = ?$

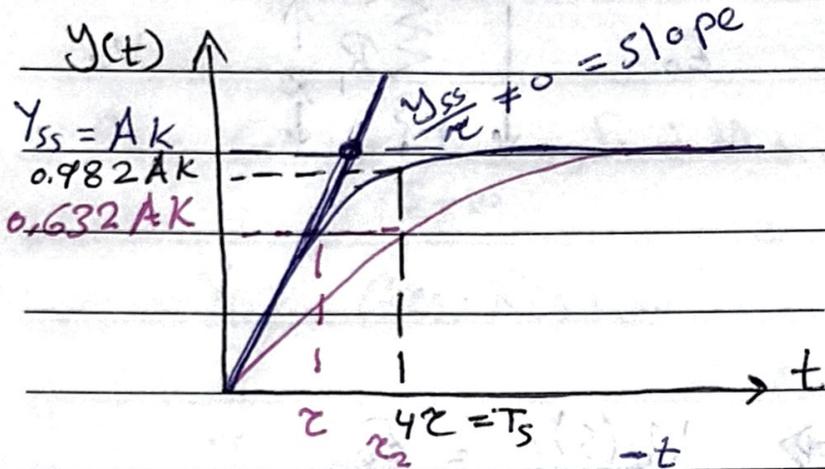
⇒ Step input response :-

$$Y(s) = T(s) R(s) = \frac{K}{\tau s + 1} \cdot \frac{A}{s} = \frac{B_1}{s} + \frac{B_2}{\tau s + 1}$$

\uparrow AK \downarrow $-AK$

$$Y(s) = AK \left(\frac{1}{s} - \frac{1}{\tau s + 1} \right)$$

$$Y(t) = AK(1 - e^{-\frac{t}{\tau}}) \rightarrow \text{General form step response for 1st order.}$$



Subst 2nd Sys Ji

$$Y_{ss} = \lim_{t \rightarrow \infty} AK(1 - e^{-\frac{t}{\tau}}) = AK$$

$$Y(t) \Big|_{t=\tau} = AK(1 - e^{-\frac{\tau}{\tau}}) = 0.632 \underbrace{AK}_{Y_{ss}}$$

$$Y(t) \Big|_{t=4\tau} = AK(1 - e^{-\frac{4\tau}{\tau}}) = 0.982 \underbrace{AK}_{Y_{ss}}$$

\neq settling time = 4τ

از وقت اولی بایزم لیوید 98% یو Y_{ss} 98% $Y_{ss} \pm 2\%$

$$Y' = \frac{AK}{\tau} e^{-\frac{t}{\tau}} \Big|_0 \Rightarrow \frac{AK}{\tau} = \frac{Y_{ss}}{\tau}$$

30/10/2018

from pre. Ex: $\frac{9}{s+10} \cdot \frac{1}{s}$

$y(t) = 0.9(1 - e^{-10t})$

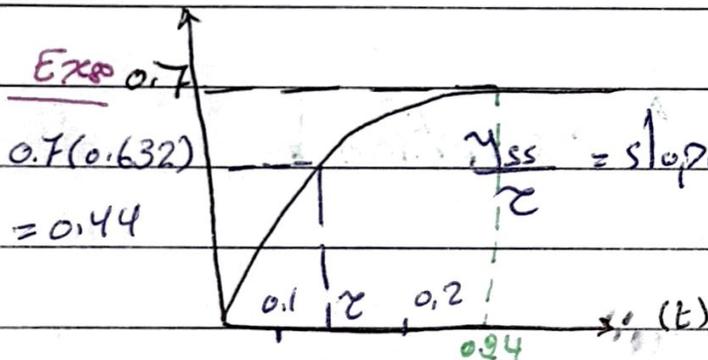
$e_{ss} \Rightarrow r(t) - y(t) = e(t)$

$E(s) = R(s) - Y(s)$

$\lim_{t \rightarrow \infty} [1 - 0.9(1 - e^{-10t})] = 1 - 0.9 = 0.1$

error 10% \rightarrow cl

04/11/2018



$T(s) = ?$

$R(s) = \frac{1}{s} \rightarrow A$

unit step response

$\frac{k}{\tau s + 1}$

$\tau = 0.15$

$Y_{ss} = \frac{1}{0.15s + 1} \cdot k = k = 0.7$

$T_s = 4\tau$

04/11/2018

Second order system

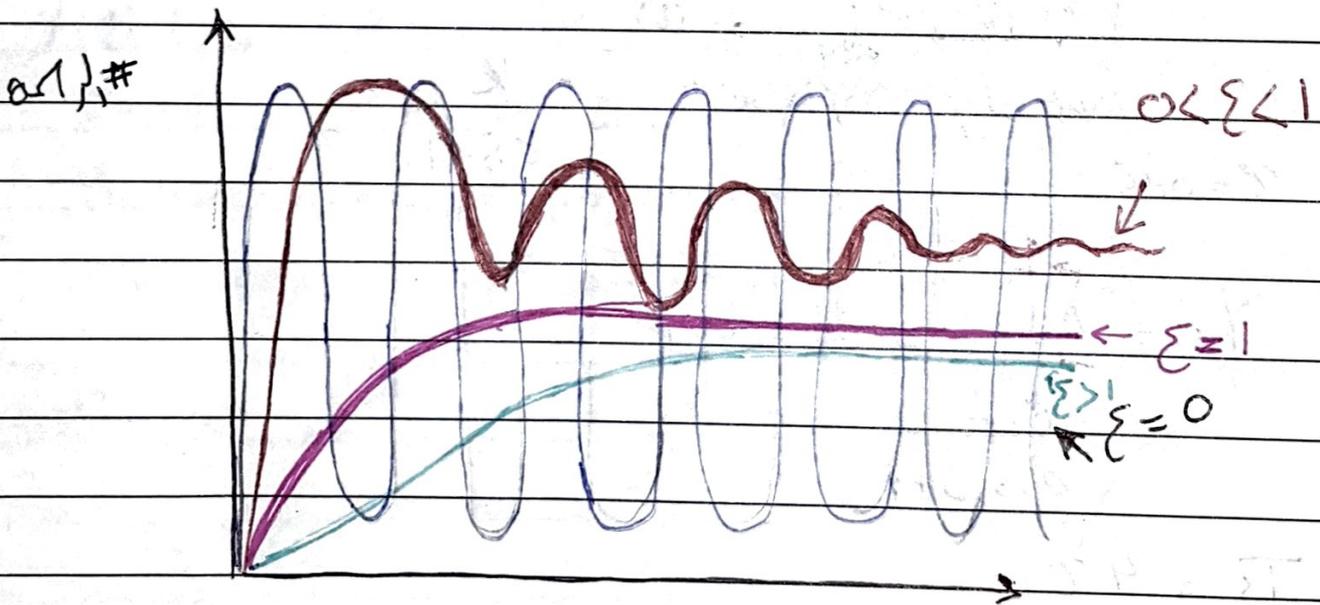
$$T(s) = \frac{K\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$$

ξ = damping ratio $0, 1, 0 < \xi < 1, \xi > 1$

ω_n = natural freq.

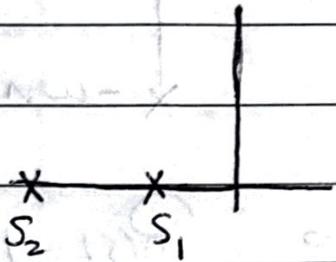
$$Y(s) = T(s)R(s)$$

Damper ↑ Oscillation ↓



1) $\xi > 1$ over damped system $R(s) = \frac{1}{s}$

roots, poles $s_{1,2} = -\xi \omega_n \pm \omega_n \sqrt{\xi^2 - 1}$ Real roots, #



$$y(t) = 1 + \frac{\omega_n}{2\sqrt{\xi^2 - 1}} \left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2} \right)$$

$\xi = 0$ to 1

2) $0 < \xi < 1$ under damped resp $0 < \xi < 1$

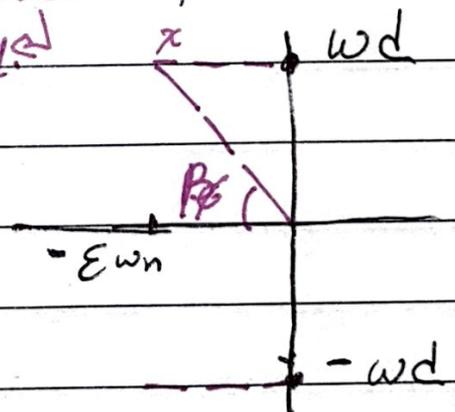
resp. poles: # $R(s) = \frac{1}{s}$

$$s_{1,2} = -\xi \omega_n \pm j \omega_n \sqrt{1 - \xi^2}$$

$$y(t) = 1 - \frac{e^{-\xi \omega_n t}}{\sqrt{1 - \xi^2}} \sin(\omega_d t + \beta)$$

damped freq. ω_d

$$\beta = \tan^{-1} \frac{\omega_d}{\xi \omega_n}$$

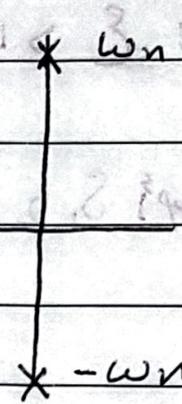


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[3] $\zeta = 0$ undamped resp

$$s_{1,2} = \pm j\omega_n$$

$$y(t) = 1 - \cos(\omega_n t)$$

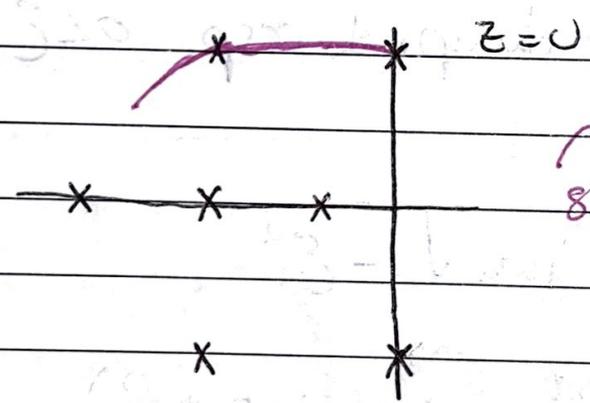
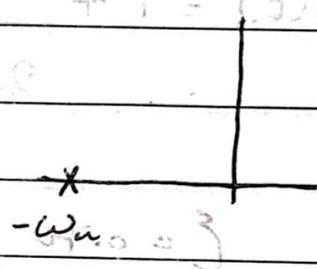


[4] $\zeta = 1$ Critically damped resp

$$P(s) = \frac{1}{s}$$

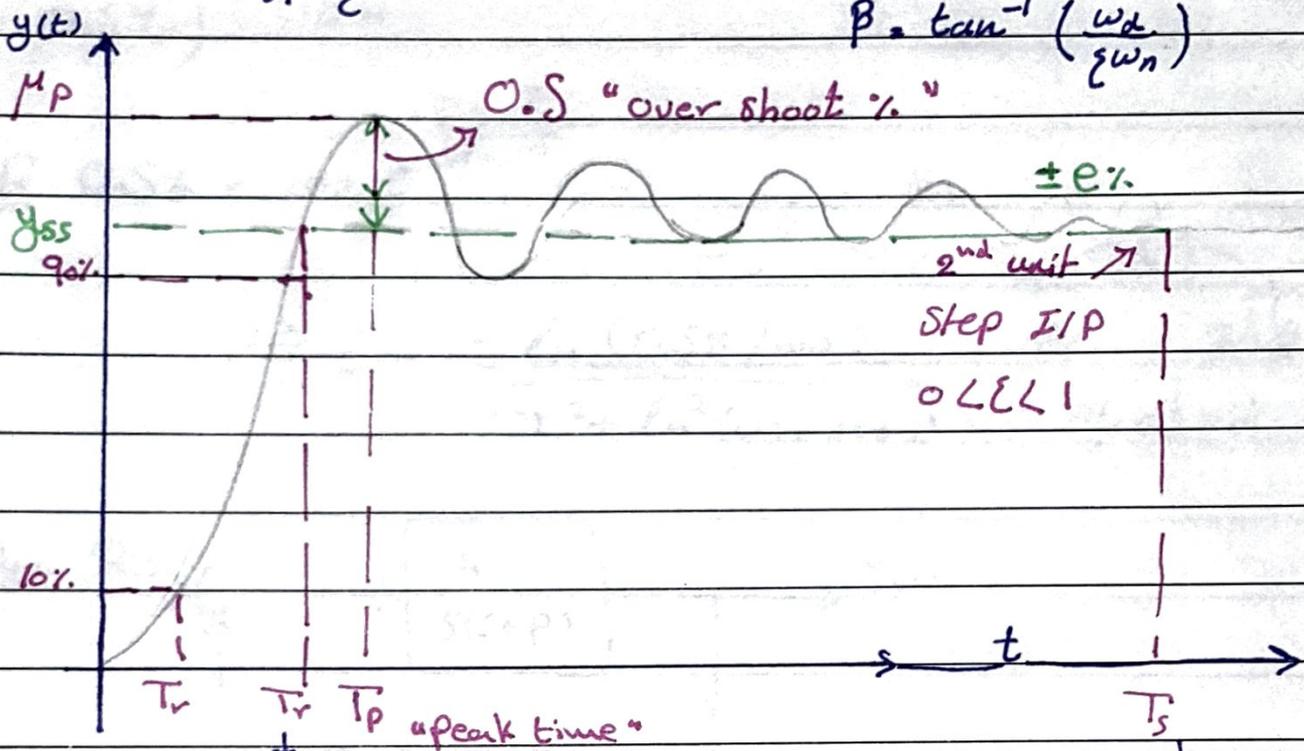
$$s_{1,2} = -\omega_n$$

$$y(t) = 1 - e^{-\omega_n t} (1 + \omega_n t)$$



plant resp. is resp. to 1/s ds
stable as sys. 1/s ds

Exo $y = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \beta)$, $\omega_d = \omega_n \sqrt{1-\xi^2}$
 $\beta = \tan^{-1} \left(\frac{\omega_d}{\xi\omega_n} \right)$



الوقت اللازم للوصول إلى y_{ss}

الوقت الذي يستقر عنده النظام

OR
 وقت الوصول إلى 90%

في الارتفاع T_r = الوقت عند الانتقال من 10% إلى 90% y_{ss}

* $T_r = \text{Rise time} = \frac{\pi - \beta}{\omega_d}$

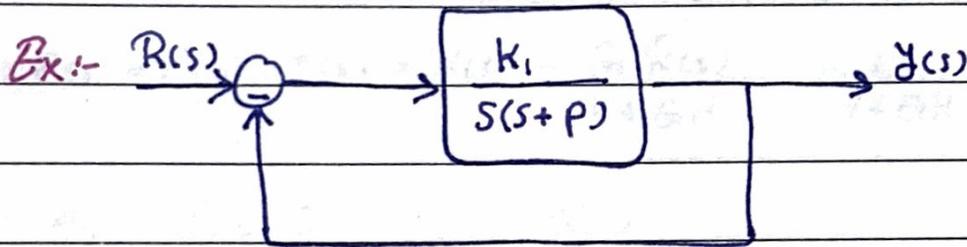
* $T_p = \text{Peak time} = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\xi^2}}$

* $T_s = \text{settling time} \rightarrow \frac{4}{\xi\omega_n} \pm 2\%$
 $\rightarrow \frac{3}{\xi\omega_n} \pm 5\%$

$$* \text{Mp\%} = \frac{y_{\max} - y_{\text{ss}}}{y_{\text{ss}}} \times 100\% \quad \left[\begin{array}{l} y_{\text{ss}} = y(\infty) \\ \text{O.S.T.} \end{array} \right] \quad [1]$$

$$\text{OR O.S.T.} = 100 e^{\frac{-\epsilon \pi}{\sqrt{1-\epsilon^2}}} \quad [2]$$

$$\rightarrow \epsilon = \frac{-\ln(\text{O.S.T.}/100)}{\sqrt{\pi^2 + \ln^2(\text{O.S.T.}/100)}} \Rightarrow \text{مثال}$$



select K, P $T_s = 4 \text{ sec}$ $\epsilon = 0.707$

$$\text{Sol:- } T(s) = \frac{K_1 / (s+p) s}{\frac{s(s+p)}{s(s+p)} + \frac{K_1}{s(s+p)}} = \frac{K_1}{s(s+p) + K_1} = \frac{K_1}{s^2 + ps + K_1}$$

$$\frac{K \omega_n^2}{s^2 + 2\zeta \omega_n s + \omega_n^2} \Rightarrow \begin{array}{l} P = 2\zeta \omega_n \\ K_1 = \omega_n^2 \end{array}$$

$$T_s = \frac{4}{\zeta \omega_n} \Rightarrow \omega_n = \frac{4}{T_s \zeta} = \frac{4}{4 \times 0.707} = \frac{1}{0.707}$$

settling time $\epsilon \times 2$

$$P = 2, \quad K_1 = \left(\frac{1}{0.707} \right)^2$$

Ex: $S^3 + S^2 + S \Rightarrow S^1 (S^2 + S + 1) \Rightarrow N=1$

$S^3 + S^2 + S + 1 \Rightarrow N=0$

$$\frac{1}{1+GH} = \frac{1}{1 + K \prod (s+z_i)} \cdot \frac{S^N \prod (s+p_j)}{S^N \prod (s+p_j) + K \prod (s+z_i)}$$

$\lim_{s \rightarrow 0} s \cdot \frac{S^N \prod (s+p_j)}{S^N \prod (s+p_j) + K \prod (s+z_i)} \cdot R(s)$

- 1) $R(s) = \frac{A}{s}$ "step"
- 2) $R(s) = \frac{A}{s^2}$ "ramp"
- 3) $R(s) = \frac{A}{s^3}$ "acc"

Type zero $\Rightarrow N=0$

$\frac{\prod p_j}{\prod p_j + K \prod z_i} \cdot A = A$
 \downarrow
 $\frac{K \prod z_i}{\prod p_j}$

$\prod p_j \in$ * K_p = Position error constant

	A/s	A/s^2	A/s^3	---	---
0	$\frac{A}{1+K_p}$	∞	∞	--	∞
1	0	$\frac{A}{K_v}$	∞	---	∞
2	0	0	$\frac{A}{K_a}$		
	i	i			
	0	0			

Type one : $N=1$ s^1

$$\text{for } R(s) = \frac{A}{s^2} \Rightarrow \frac{\sum \pi P_j}{K \sum \pi Z_i} \cdot A = \frac{A}{K_v}$$

$$K_v = \frac{K \sum \pi Z_i}{\sum \pi P_j} = \text{velocity error const.}$$

Type two :- $N=2$

$$\text{for } R(s) = \frac{A}{s^2} \Rightarrow \frac{\sum \pi P_j}{K \sum \pi Z_i} \cdot A = \frac{A}{K_a}$$

$$K_a : \text{acc error constant} = \frac{K \sum \pi Z_i}{\sum \pi P_j}$$

error in the sys is of s^2 type

Routh Hurwitz Criteria

$f(s) = A(s) = \text{Charac. eqn.}$ # s^n ...

بال first row $a_0, a_1, a_2, a_3, a_4, a_5$... # a_0 a_1 a_2 a_3 a_4 a_5 ...

s^n	a_0	a_2	a_4	---
s^{n-1}	a_1 ①	a_3 ②	a_5	---
s^{n-2}	b_1	b_2	b_3	}
s^{n-3}	c_1	c_2	c_3	
\vdots				
s^0	a_n	0	0	---

إذا كان جميع الأجزاء الحقيقية سالبة \rightarrow stable

where $b_1 = \frac{a_1 a_2 - a_0 a_3}{a_1}$

a_1 column

$b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}$

$c_1 = \frac{b_1 a_3 - a_4 b_2}{b_1}$

وهكذا

مجرد فاصل، تغير بالإشارة \rightarrow unstable

right side \rightarrow تغير بالإشارة \rightarrow right side

Exo $q(s) = s^3 + 2s^2 + 4s + 4$

Sol \Rightarrow

s^3	1	4	} الأعداد الفردية
s^2	2	4	
s^1	2	0	} الأعداد الزوجية
s^0	4	0	

أند، المطلق

* لها 3 أصفاء على الـ left side كونها كل قيم $co. +ve$ و 1^{st} $co. +ve$ ما تبقي zero
 stable system

$$b_1 = \frac{2(4) - 1(4)}{2} = 2$$

$$b_2 = \frac{2(0) - 1(0)}{2} = 0$$

لو كان $\Rightarrow q(s) = s^3 + 2s^2 + 4s + k \Rightarrow$ for stable sys.

s^3	1	4
s^2	2	k
s^1	$\frac{8-k}{2}$	0
s^0	k	0

$$b_1 = \frac{2(4) - 1(k)}{2} = \frac{8-k}{2}$$

so $\Rightarrow \frac{8-k}{2} > 0$

Ex

$$k > 0$$

} $k > +ve$ و s^0 $+$
 } stable و 3 أصفاء على الـ left side
 } zero و 1^{st} $co. +ve$ و 1^{st} $co. +ve$
 } polarity و 1^{st}

$$k < 8$$

$$0 < k < 8$$

Ex:- $f(s) = s^3 + 2s^2 + 4s + 8$

كالتالي، الثاني

s^3	1	4
s^2	2	8
s^1	4 → 4	0
s^0	8	0

معنا هاني 2 poles على الـ s axis
marginally stable

لهذا الـ row الـ s^1 row all صفرا

كل row ومنه يعني عن 2 poles على الـ s axis
لا عرف إذا في الـ صفرا الـ right zo

بموجب الـ row الـ قبل الـ row الـ s^1

↳ $2s^2 + 8$ aux. eq.
 له بعض هون تركب الـ s

اشتقاق الـ aux. eq بالنسبة لـ s

↳ $\frac{d}{ds} = 4s + 0$

3] نقارن كونه كالتالي +ve هون
 ↳ فاذا صفرا على الـ right side

لو طلع فيه الـ s الـ unstable

15/11/2018

Ex: $f(s) = s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10$

s^5	1	2	11	$b_2 = 2(11) - 1(10) = 6$
s^4	2	4	10	$\frac{12}{2} = 6$
s^3	$+ve \rightarrow E$	6	0	$0 = \text{element on right side}$
s^2	$-ve \rightarrow E$	$c_1 = -12$	10	رقم صغیر کثیر یقیناً
s^1	$+ve \rightarrow E$	$d_1 = 6$	0	من ال zero وک مساوی zero +ve
s^0	(10)	0	0	$c_1 = 4E - 12 = \frac{-12}{E}$

$c_2 = \frac{10E - 0}{E} = 10$, $d_1 = \frac{-12}{E} \times 6 - 10/E = 6$

2 zeros at right side

↳ unstable

↳ 3 zeros at left side

↳ no poles at $j\omega$

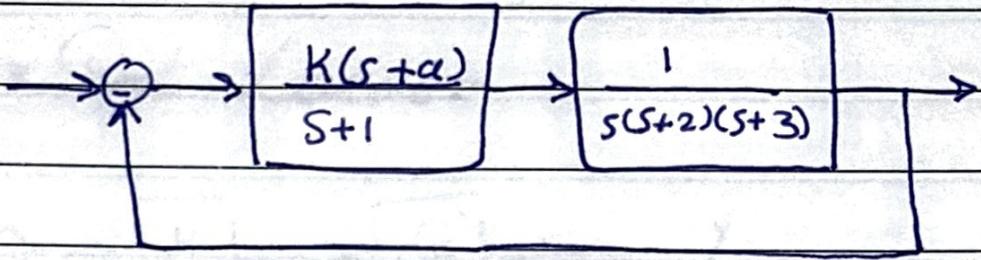
Ex: $f(s) = s^4 + s^3 + s^2 + s + K$

s^4	1	1	K	$\frac{-K}{E} > 0$
s^3	1	1	0	
s^2	$0 \rightarrow E$	K	0	$K > 0$
s^1	$\frac{E-K}{E} = \frac{-K}{E}$	0	0	$\frac{d_1}{E} > 0$
s^0	K			

there is no value of K that's make the sys. stable

15/11/20/8

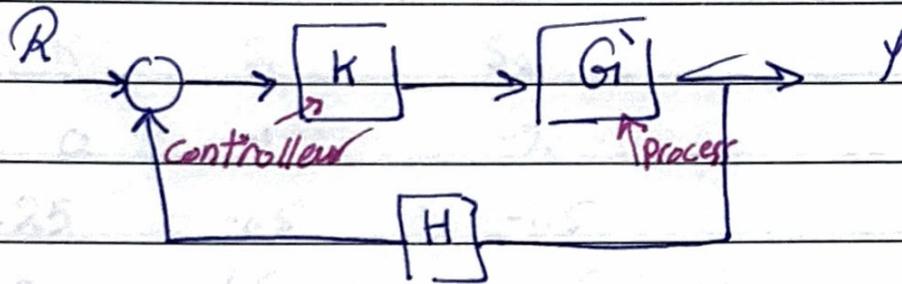
Ex 7.8



find the value of K, a that's make the system stable.

H.W

CH 7:8 Root Locus



$$\frac{Y(s)}{R(s)} = \frac{K G_i}{1 + G_i K H} \rightarrow G = \frac{G_i}{1 + G_i H}$$

↳ loop gain

$$G_i H = \frac{k \frac{a_0 s^n + a_1 s^{n-1} + \dots + a_n}{b_0 s^n + b_1 s^{n-1} + \dots + b_n}}{1 + \frac{k \frac{a_0 s^n + a_1 s^{n-1} + \dots + a_n}{b_0 s^n + b_1 s^{n-1} + \dots + b_n}}{1 + G_i H}}$$

$$\Delta(s) = 1 + G_i H = 1 + \frac{k \frac{a_0 s^n + a_1 s^{n-1} + \dots + a_n}{b_0 s^n + b_1 s^{n-1} + \dots + b_n}}{1 + G_i H}$$

↑ poles

Ex 80 loop gain = $\frac{k}{(s+1)(s+2)} = G_i H$

$$\Delta(s) \Rightarrow 1 + \frac{k}{(s+1)(s+2)} = \frac{(s+1)(s+2) + k}{(s+1)(s+2)}$$

Charac. equ.

$$\text{Charac equation} = s^2 + 3s + 2 + k = \Delta(s)$$

From Pre. Ex:-

$$1 + GH = 0$$

$$1 + \frac{K}{(s+1)(s+2)} = 0$$

$$\Delta(s) = s^2 + 3s + 2 + K$$

if $s = -1.5 \pm j1.3 \Rightarrow$ نعوذنا لوجود قيمه K التي تحقق المعادله

لو كان α غير تحقق α المعادله

لازم يكون للعزيم شرطين

1] mag condition.

$$|GH| = 1$$

2] angle condition

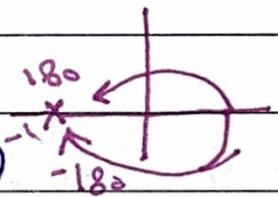
$$1 + GH = 0 \Rightarrow GH = -1$$

$$|GH| = 1$$

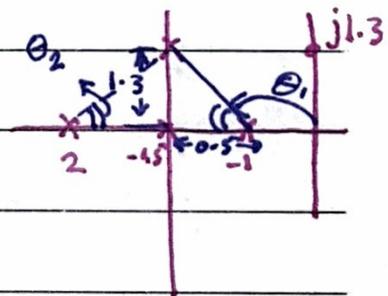
$$\angle GH = -1$$

$$\angle GH = \angle \text{Zeros} - \angle \text{Poles} = \mp 180(2Z+1)$$

for loop Gain \leftarrow



Sol] $\frac{K}{(s+1)(s+2)}$, $s = -1.5 + j1.3$



1] angle condition: $\angle \text{Zeros} - \angle \text{Poles}$

$$\theta_1 = \tan^{-1} \frac{1.3}{0.5} = 70^\circ$$

$$\Rightarrow 180 - 70 = 110$$

$$\theta_2 = \tan^{-1}\left(\frac{1.3}{0.5}\right) = 70^\circ$$

$\angle \text{Zeros} = 0 \Rightarrow \angle \text{Zeros} - \angle \text{Poles}$

$$0 - 110 - 70 = -180 \quad \therefore \text{تحقق الشرط}$$

$$\mp 180 (2Z+1) \leftarrow$$

2] mag. condition :-

$$|GH| = 1 \Rightarrow \text{تعريف بقوى الس}$$

$$\left| \frac{K}{(1.5 + j1.3 + 1)(-1.5 + j1.3 + 2)} \right| = 1$$

$$\rightarrow \ll K = 2 \gg$$

A] * root locus / من حيث بيده أو بيده

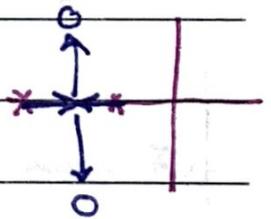
$$K \frac{(s+Z_1)(s+Z_2) \dots (s+Z_m)}{(s+P_1)(s+P_2) \dots (s+P_n)} = 1$$

$$\Rightarrow \underset{\substack{\downarrow \\ \text{تسمى} \\ \text{منتهي عند } \infty}}{K} (s+Z_1)(s+Z_2) \dots (s+Z_m) = (s+P_1)(s+P_2) \dots (s+P_n)$$

\downarrow عدد Zeros
 \downarrow عدد Poles

$\rightarrow K=0 \Rightarrow \therefore \text{sol. of this eq.} \Rightarrow \text{poles}$

1] R.L. starts at poles "X"



$\rightarrow K = \infty \Rightarrow \therefore \text{sol. of the eq.} \Rightarrow \text{zeros}$

2] R.L. ends at zeros "O"

B) کم خط R.L. بی آر ام
R.L. segment

Poles عدد = R.L. seg. عدد

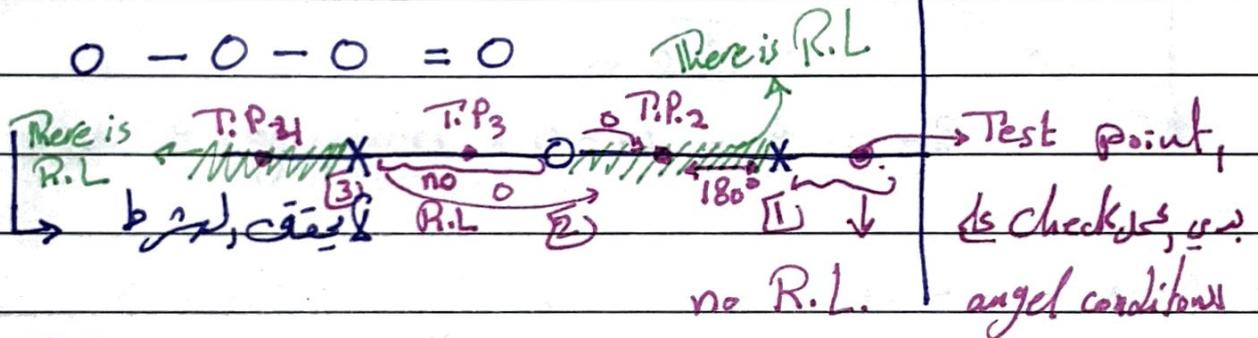
لیس مو نسیه # zeros !?

له لاینه سکا ما یسون فیه zeros

$\square \# \text{ of RL segments} = \# \text{ of poles}$

T.P. (1) $\angle \text{Zeros} - \angle \text{Poles}$

$0 - 0 - 0 = 0$



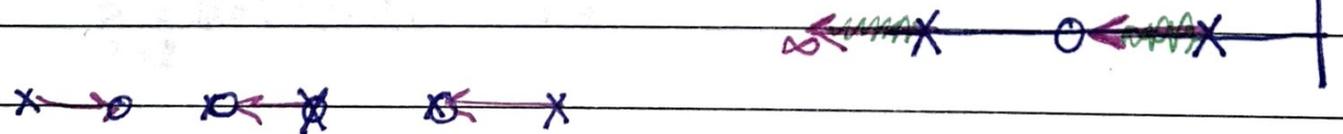
T.P. (2) $\angle \text{Zeros} - \angle \text{Poles} = 0 - 180 - 0 = -180$ تقف لبرط

T.P. (3) $\angle \text{Zeros} - \angle \text{Poles} = 180 - 180 - 0 = 0$ لا یقف لبرط

T.P. (4) $\angle \text{Zeros} - \angle \text{Poles} = 180 - 180 - 180 = -180$ تقف لبرط

* ال R.L يقع على الجانب الأيسر من ال Zero ال Pole الفردي

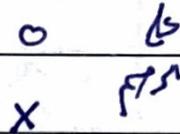
RL at real axis is on the left side of ODD Poles or Zeros



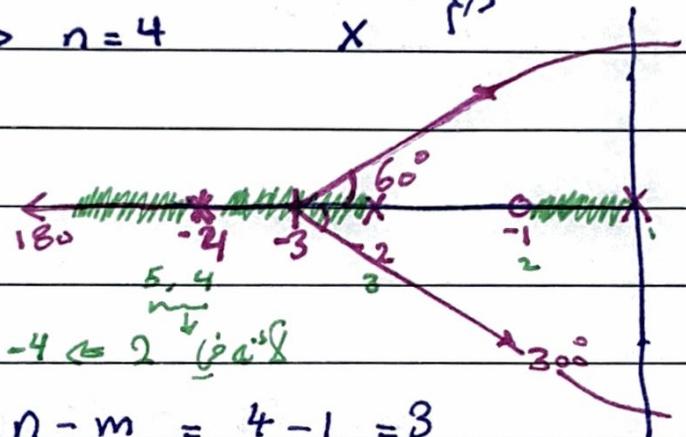
Exr $\Delta(s) = 1 + K \frac{(s+1)}{s(s+2)(s+4)^2} = 0$

Zeros = -1 $\rightarrow m=1$

Poles = 0, -2, -4, -4 $\rightarrow n=4$



R.L. \rightarrow \sum odd # of poles



of asymptotes = $n - m = 4 - 1 = 3$

$\sigma = \frac{\sum \text{Poles} - \sum \text{Zeros}}{n - m} = \frac{0 + (-2) + (-4) + (-4) - (-1)}{4 - 1} = \frac{-9}{3} = -3$

$\sigma = -3 \rightarrow$ asymptote (خط مقارب) في النصف الأيسر

$z = 0, 1, 2, \dots, n - m - 1 \rightarrow 0, 1, 2$ (2)

$\alpha = \pm \frac{180(2z + 1)}{n - m}$

في النصف الأيسر، خط مقارب \rightarrow R.L. line \neq asym. line

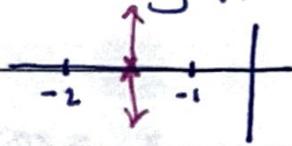
$\alpha_{z=0} = \frac{180(2(0) + 1)}{3} = 60^\circ$

$\alpha_{z=1} = \frac{180(2(1) + 1)}{3} = 180^\circ$

$\alpha_{z=2} = \frac{180(2(2) + 1)}{3} = 300^\circ$

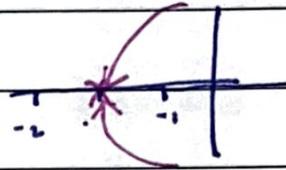
واقعة على المحور الحقيقي

Break away point → النقطة التي تنفر فيها من المحور الحقيقي



وتنصب خارج المحور الحقيقي

Break in → النقطة التي يجتمعوا عندها القطب



$$\frac{dk}{ds} = 0$$

$$k = \frac{-s(s+2)(s+4)^2}{s+1}$$

$$\frac{dk}{ds} = \frac{-(s+4)(3s^3 + 12s^2 + 14s + 8)}{(s+1)^2} = 0$$

$$s = -4, -2.599, -0.7 \mp j0.73$$

RL * لا يتم تكون منقطه على RL → complex Break in away

→ نأخذ الـ -2.599 لأنها باربعة

الفترة ح و زقره لا a sym.

متى يتقاطع مع الـ axis ω ?

له قوسه k التي يتغير في row كالمعاد

By Routh Hurwitz له

Ex "book" $\Delta(s) = 1 + \frac{k}{s^4 + 12s^3 + 64s^2 + 128s} = 0$

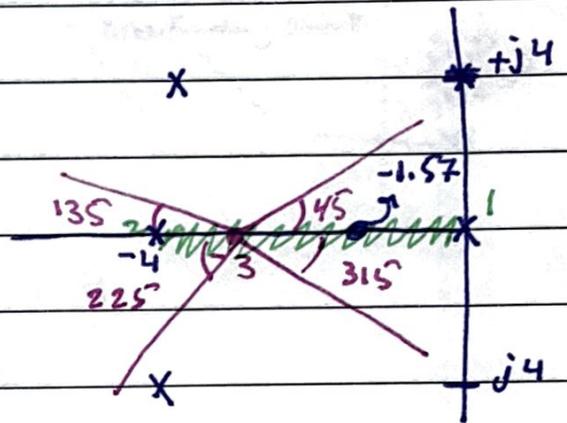
* Zeros = x $m = 0$

* Poles = $0, -4, -4 \pm j4$ $n = 4$

* # of asym = $n - m = 4$

$$\sigma = \frac{0 + (-4) + (-4) + j4 + (-4) - j4 - 0}{4}$$

* $\sigma = \frac{-12}{4} = -3$



$$\alpha = \frac{\mp 180(2z + 1)}{n - m}$$

$$z = 0 \rightarrow n - m - 1 = 4 - 0 - 1 = 3$$

$$z = 0, 1, 2, 3$$

* $\alpha = 45^\circ, 135^\circ, 225^\circ, 315^\circ$

* $k = -s^4 - 12s^3 - 64s^2 - 128s$

$$\frac{dk}{ds} = -4s^3 - 36s^2 - 128s - 128 = 0$$

$$s = -1.57, -3.7 \pm j2.5$$

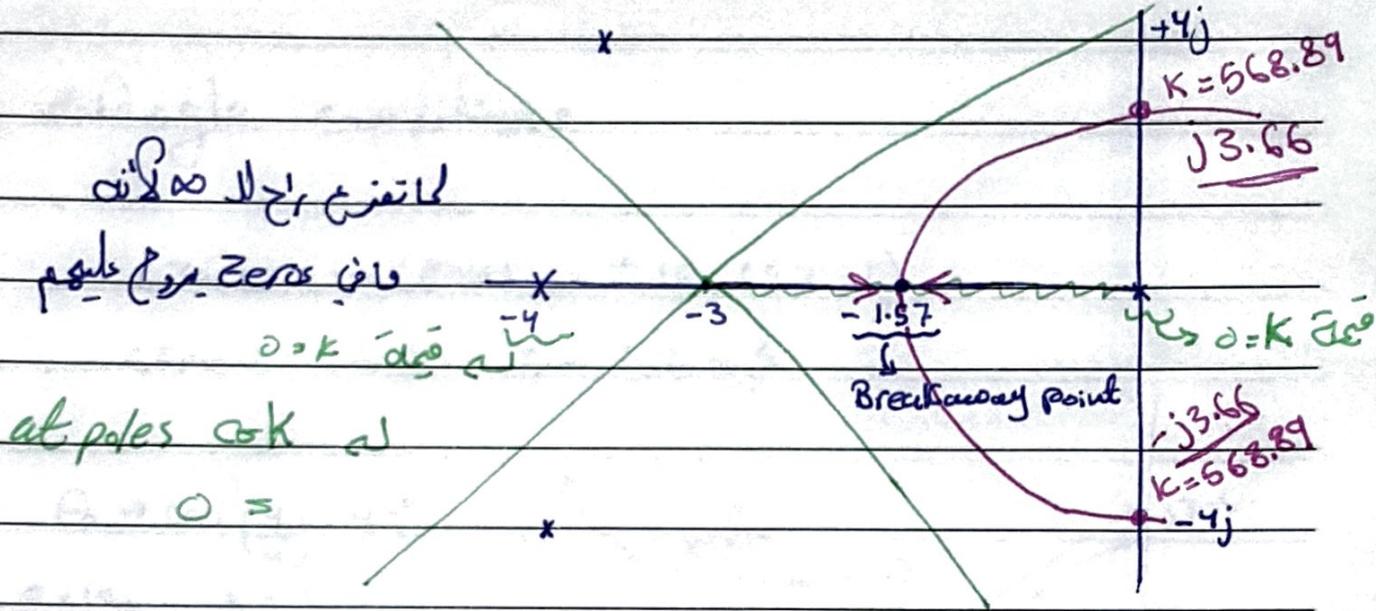
complex not break away

\therefore Break away point = -1.57 .



Happy National Day

2/12/2018



$$\Delta s = s^4 + 12s^3 + 64s^2 + 128s + k$$

s^4	1	64	k	
s^3	12	128	0	$C_1 = 53.33(128) - 12(k)$
s^2	53.33	k	0	53.33
s^1	C_1	0	0	
s^0	k			

كبري يطوع (تصغير) الى اليمين

$$C_1 = 0 \Rightarrow 53.33(128) - 12k = 0$$

$$\hookrightarrow k = 568.89 \rightarrow \text{نقطة تقاطع المحاور}$$

R.S. له رجع غير مستقر $k = 600$ فرقا $k = 600$ غير مستقر لا

نقطة تقاطع المحاور $k = 568.89$ s^2 ويطوع الاضمار

$$\hookrightarrow 53.33s^2 + 568.89 = 0$$

$$s = \pm j3.66$$

↓
 "Pure imag. تطوع"



Angle condition:

$$\angle \text{Zeros} - \angle \text{Poles} = \pm 180(2Z+1)$$

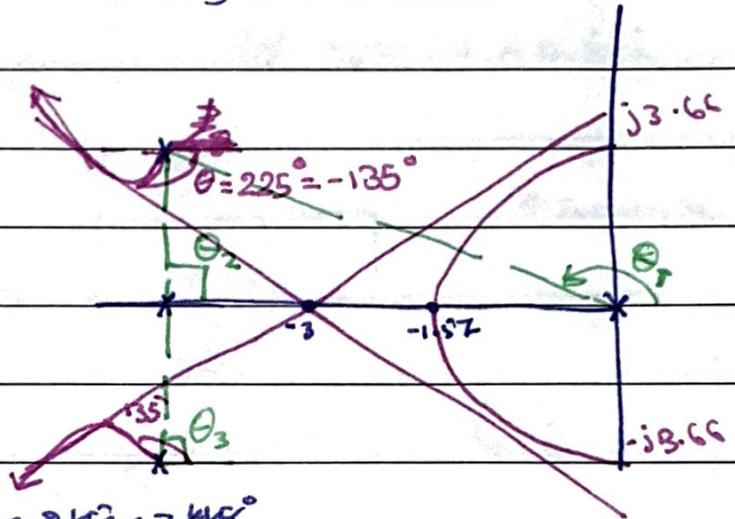
$$\angle \text{Zeros} = 0 \quad \text{Zeros list is } 0$$

$$\theta_1 \Rightarrow \tan^{-1}\left(\frac{4}{4}\right) = 45^\circ$$

$$\rightarrow \theta_1 = 180 - 45 = 135^\circ$$

$$\theta_2 = 90^\circ$$

$$\theta_3 = 90^\circ$$



$$\Rightarrow 0 - 135 - 90 - 90 = -315 \Rightarrow 45^\circ$$

$$0 - 135 - 90 - 90 - \theta = 180(2Z+1)$$

$$\theta = -135^\circ = 225^\circ$$

$$1 + \frac{k(\text{Zeros})}{\text{Poles}} = \text{Charac. Eqn.} \quad \text{poles} = 0, -4, -4 \pm j4$$

$$\left[\begin{array}{c} 1 + k \\ \hline \end{array} \right]$$

$$S(S+4)(S+4+j4)$$

unstable, marg. or sys

0 = Pole عند 0

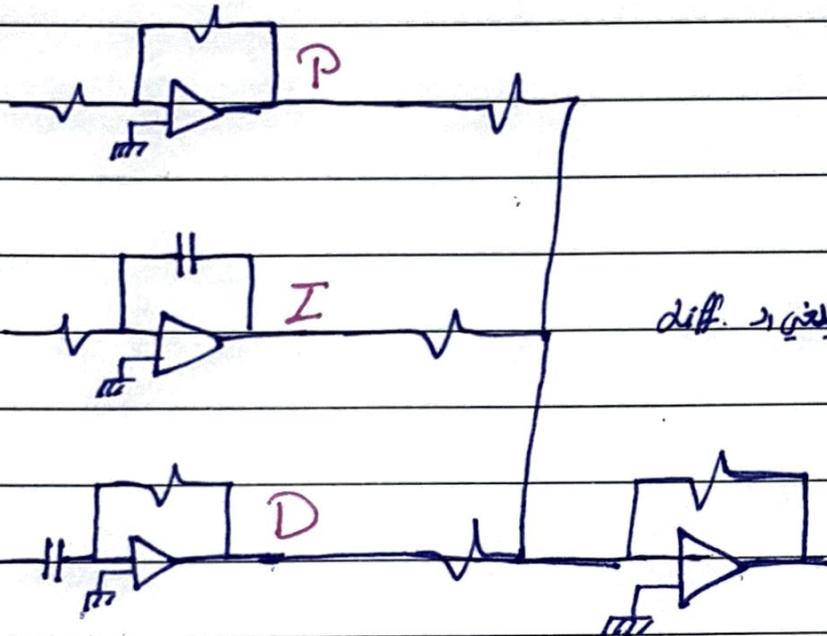
قيمة k عند نقاط التقاطع مع الحز

عدد الأصفار على الحز = 2 = R.S

عدد الأصفار على الحز = 2 = R.S

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PID Controller



ID controller is a diff. & int. of error

Summing

$$PID = K_P + K_I \frac{1}{s} + K_D s$$

one pole

$$\Rightarrow K_D s^2 + K_P s + K_I$$

complex roots & zero



* K_I : eliminate of s.s.e. $\Rightarrow e_{ss} = 0$

e_{ss} is disturbance type 1, 2, 3

* K_D : Dec. T_s

* K_P : Dec. $e_{ss} \Rightarrow$ dist. zero \Rightarrow zero of type 1, 2

stable dist. \Rightarrow PID

	T_r	O.S.	T_s	e_{ss}
K_P	dec.	dec.	S.C.	dec.
K_I	dec.	dec.	inc.	eliminate
K_D	Small Change.	Inc. (disadv.)	dec. (adv.)	S.C.