EICoM
ELECTRICAL COMPUTER MECHATRONICS

تحكم ألي

م. فدوى المومني

للطالب المبدع
حمزة اسماعيل

إرادة - ثقة - تغيير

Subject Date No.

✱ Automatic control :—

* Introduction about automatic control :—

* What is control :—

1- make some object (system) behave as we desire. يتصرف كما أريد

Imagine control around you :

room temperature control , Car driving , Voice Volume control

Balance of bank account , etc.

* What is control system :—

1- A system Controlling the operation of another system

2- A system that can regulate itself and another system.

3- A control system is a device , or set of devices to manage the behaviour of other devices or systems.

* Why Automatic control :—

- Not manual . يدوي ليس

Why do we need automatic control?

1. Convenient (room temperature , laundry machine).
2. Dangerous (hot/cold places , space , bomb removal (إزالة القنابل))
3. Impossible for human (nanometer scale precision positioning).
4. It exists in nature (human body temperature control).

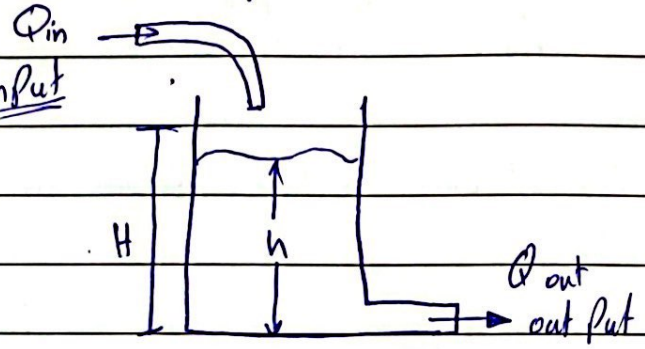
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Subject Date No.

✱ Automatic Control :-

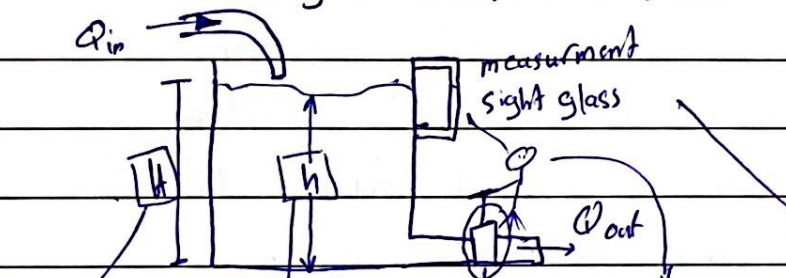
* Introduction about automatic control :-

* The control problem :-



Q out depends on h
If Q out = Q in , h constant.
Q out > Q in , tank empties
Q out < Q in , tank overflows.

* Basic subsystems of control :-



Feedback control subsystem
1. measurment.
2 control decision
3- system modification

Reference setPoint Input	(Process) maintain tank level	final control element Valve	control Decision (controller)	measurement (sensor)

(desired value) ↓ (actuator)
(controlled variable)

N O T E B O O K

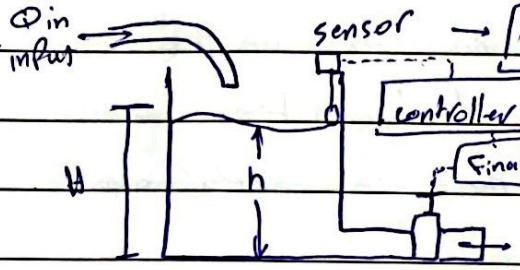
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Subject Date No.

✱ Automatic control :-

* Introduction about automatic control :-

* Automatic control systems :-

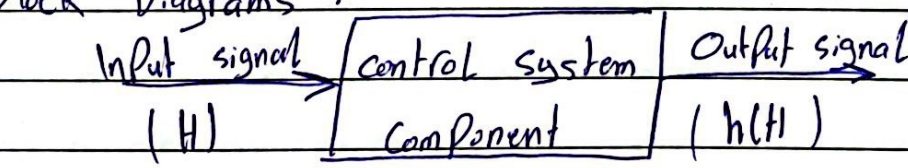


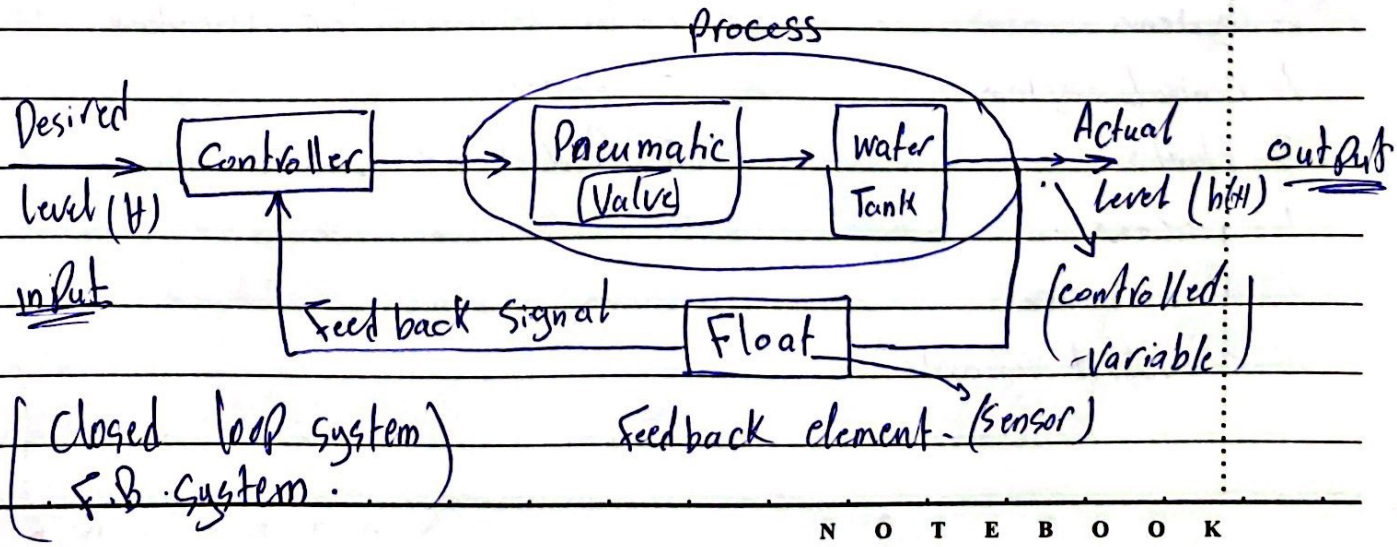
Use sensors and analog or digital electronics to monitor and adjust system.

* Elements of Automatic control :-

- Process - single or multiple variables.
- measurment - [sensors] (operator eyes)
- Error Detection - compare (H) to (h).
- [Controller] - generate corrections. (operator brain)
- Final control Element - modify process [actuater] → Valve
- [plant] - water tank system.

* Block Diagrams :-





(closed loop system)
F.B system.

Feedback element - (sensor)

NOTEBOOK

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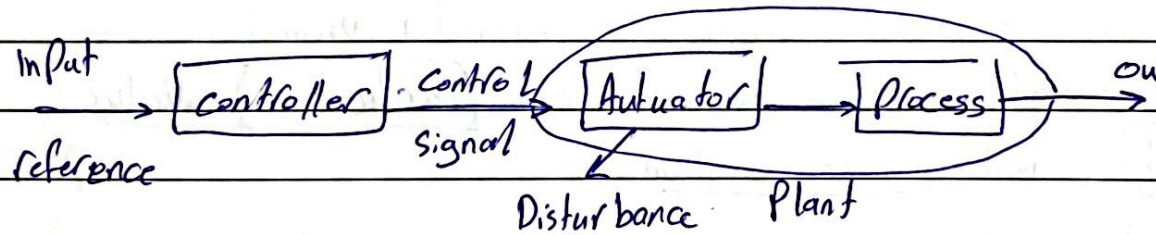
* Automatic control :-

* Introduction about automatic control :-

* Open - loop control systems :-

- Those systems in which the output has no effect on the control action are called open - loop control systems.
- The output is neither measured nor fed back for comparison with the input.
- In the presence of disturbances, an open - loop control system will not perform the desired task.

* Block Digram for open - loop control systems :-



* Definitions :-

1- Systems.
2- Control System
3- Plants
4- Process.
5- Disturbances
6- Controlled Variable.
7- The manipulated Variable.
8- control
9. feed back control

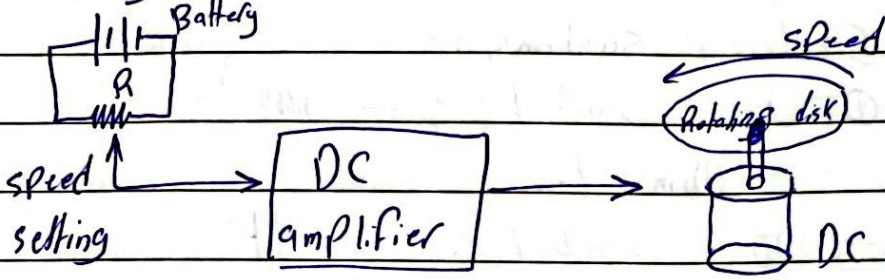
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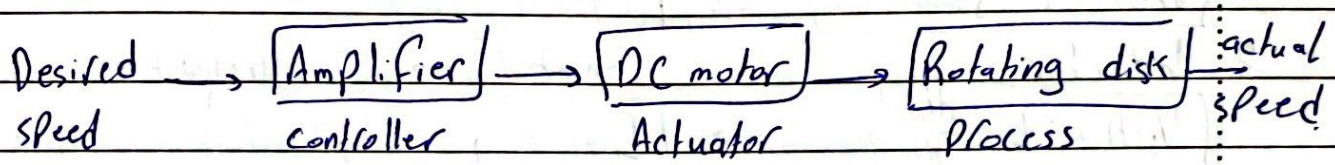
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* Automatic control :-

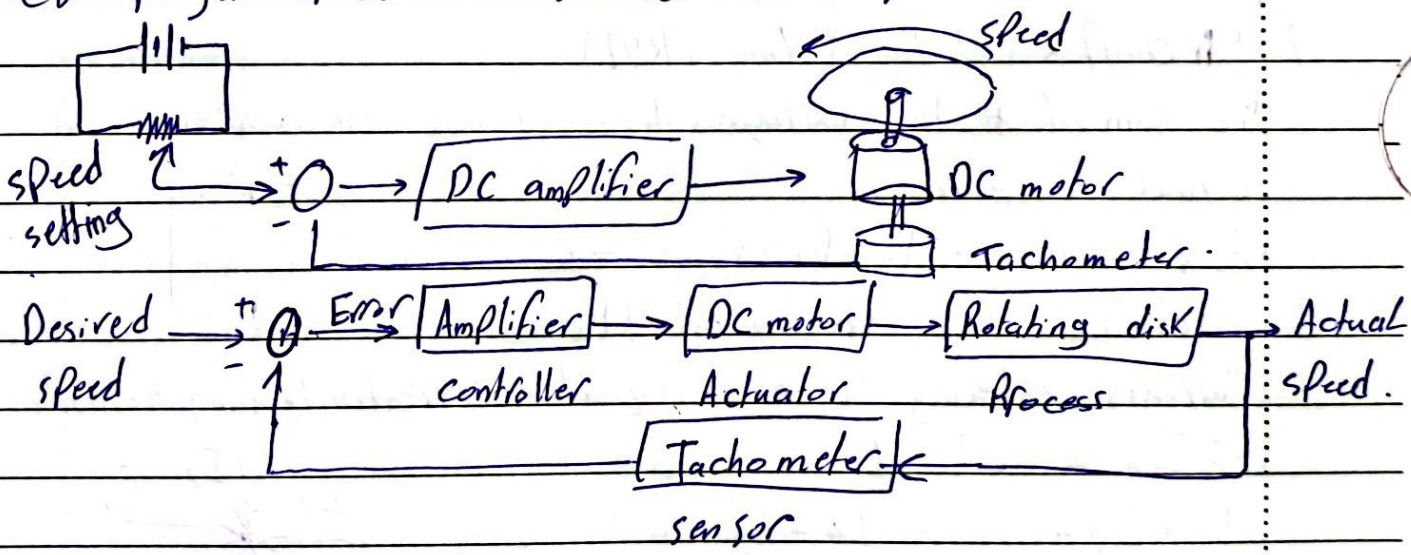
* Introduction about automatic control :-

* CD player speed control : Open-loop :-

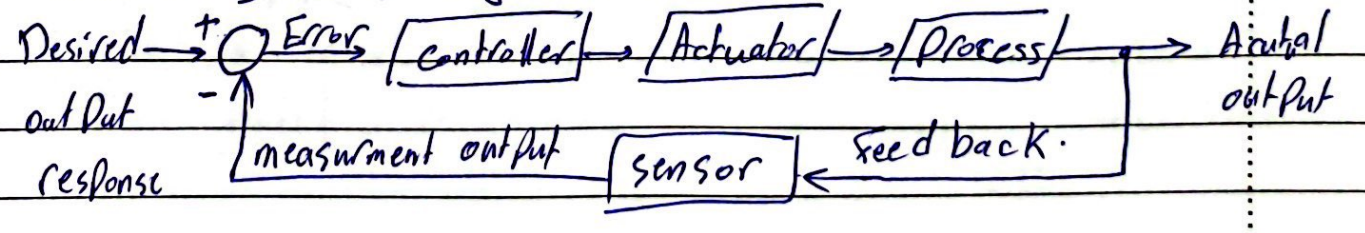




* CD player speed control : closed-loop.



* Control systems objectives :-



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✱ Automatic control :-

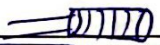
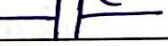
* Ch 2 :- Mathematical modling :-

* Modeling of electrical and mechanical systems :-

* Basic Elements of Electrical Systems :-

2- Resistor :-  symbol → 

$$V_R(t) = I_R(t) \cdot R$$ → Ohm's law

2- Capacitor :-  symbol → 

$$V_c(t) = \frac{1}{C} \int I_c(t)\, dt \Rightarrow I_c(t) = C \frac{dV_c(t)}{dt}$$

3- Inductor :- symbol → 

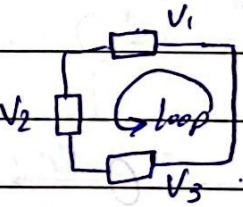
$$V_L(t) = L \frac{di_L(t)}{dt} \Rightarrow I_L(t) = \frac{1}{L} \int V_L(t)\, dt$$

* Important laws :-

2- Kirchoff's Voltage law (KVL) :-

The sum of all the voltage drops around the loop is equal to zero.

$$\sum V = 0 \rightarrow V_1 + V_2 + V_3 \dots = 0$$

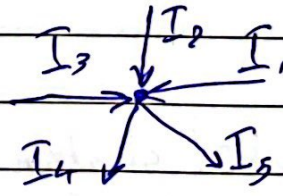


2- Kirchoff's current law (KCL) :-

Currents entering the node equals currents leaving the node.

$$\sum I_{in} = \sum I_{out}$$

$$I_1 + I_2 + I_3 = I_4 + I_5$$



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Subject Date No.

Automatical control :-

* Ch 2 :- Mathmatical modeling :-

* Modeling of electrical and mechanical systems :-

* Basic Elements of Translational Mechanical systems :-

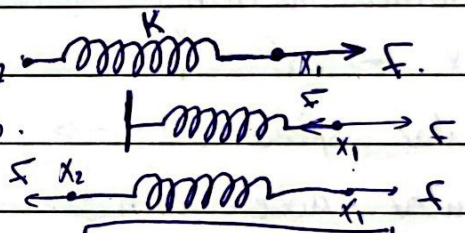
1- Translational spring (نابض) —WWW—

2- Translational Mass (كتلة) —[M]—

3- Translational Damper —|—

* Translational spring :-

If (F) is the applied force. Then x_1 is the deformation if $x_2 = 0$. or $(x_1 - x_2)$ is the deformation



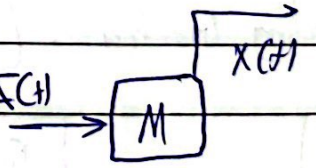
The equation of motion is given as $\boxed{F_s = K(x_1 - x_2)}$

Where (K) is stiffness of spring expressed in N/m.

* Translational Mass :-

- Translational mass is an inertia element.
- A mechanical system without mass d.n.e.
- If a force (F) is applied to a mass and it is displaced to (X) meters then the relation force and displacements is given by Newton's second law :

$\boxed{\sum \vec{F} = M\vec{a}}$ $a = x''$.



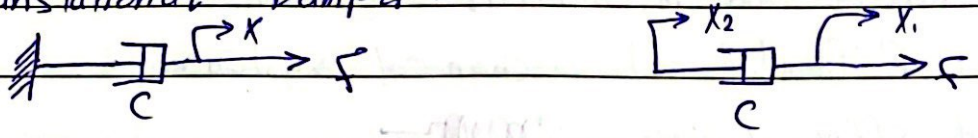
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Automatic control :-

* Ch 2 :- Mathematical modeling :-

* Translational Damper :-



$F = C x' \Rightarrow F = Cv$ $\quad F = C(x_1' - x_2') = C \Delta x'$

Where C is damping coefficient. (N/ms^{-1})

* Rotational motion :-

$\theta, x \rightarrow \theta = \frac{x}{r}$ $\quad \theta \rightarrow$ الإزاحة الزاوية ، $x \rightarrow$ الإزاحة الخطية

angular speed (w) $\rightarrow w = \frac{v}{r}$. $\quad r :-$ نصف القطر

angular acceleration $\rightarrow \alpha = \frac{a_t}{r}$

* Rotational systems :-

Analogous to linear mechanical systems

Torsional spring (resilient shaft) $(T = K\theta)$

Torsional viscous damping $\left(T = B\frac{d\theta}{dt} = Bw\right)$

Rotating Inertia . $\left(T = J\frac{d^2\theta}{dt^2} = J\frac{dw}{dt} = J\alpha\right)$

Translation	Rotational
F	T
x	θ
$V \rightarrow x'$	w
$a \rightarrow x''$	α

	Translation	Rotational
spring	$\sum F = Kx$	$T = K\theta$.
mass	$\sum F = ma = mx''$	$\sum T = m\alpha$
Damper	$F = Cx' = Cv$	$T = Cw$

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Subject Date No.

* Automatic control :-

* Ch 2 : Mathematical modeling :-

* Example :-

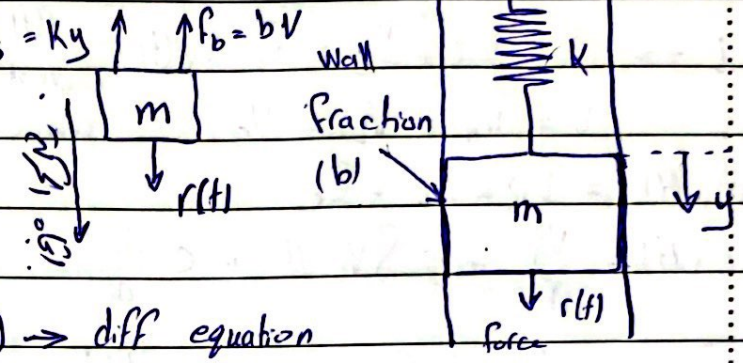
① free body diagram

② $\sum \vec{F} = m\vec{a}$.

$r(t) - f_s - f_b = ma$.

$r(t) - Ky - by' = my''$:

$my'' + by' + Ky = r(t)$ → diff equation of the system.



* Example :- spring - mass system

fig 1 : free body diagram

$\sum \vec{F} = m\vec{a}$. at equ. constant

$mg - K\delta = m(0)$

$mg = K\delta$.

fig 2 free body diagram.

$\sum \vec{F} = m\vec{a}$.

$F + mg - Kx - K\delta = ma$

but $mg = K\delta$.

$F - Kx = mx''$:

$mx'' + Kx = F$ → diff equation of the system.

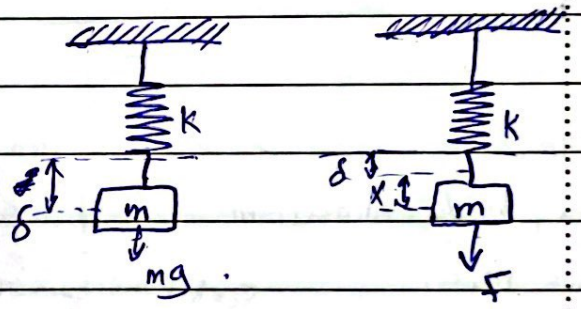
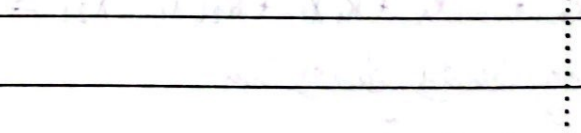


fig 1 fig 2



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Subject Date No.

✱✱ Automatic control :-

✱ Ch 2 :- Mathematical modeling :-

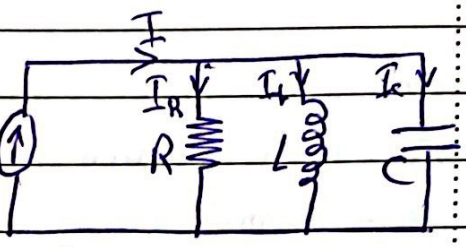
✱ Example :- ✓

KCL → $I = I_R + I_L + I_C = r(t)$.

KVL → $V = V_R = V_L = V_C$.

$\Rightarrow r(t) = I_R + I_L + I_C$.

$r(t) = \frac{V_R}{R} + \frac{1}{L}\int V_L\,dt + C\frac{dV_C}{dt}$.



(parallel combination)

$V = V_1 = V_2 = V_3$

$\frac{d}{dt}\left(r(t) = \frac{V}{R} + \frac{1}{L}\int V\,dt + C\frac{dV}{dt}\right)$ → Integro diff. equation

$r'(t) = \frac{V'}{R} + \frac{1}{L}V + CV''$ → diff equation of the system.

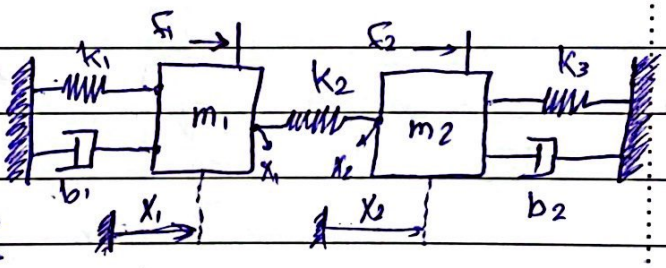
✱ Example :-

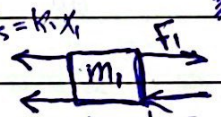
free body diagram

$\sum \vec{F} = m_1 \vec{a}_1$

$F_1 - K_1x_1 - b_1x_1' - K_2(x_1 - x_2) = m_1x_1''$

$m_1x_1'' + b_1x_1' + K_1x_1 + K_2(x_1 - x_2) = F_1$ ..... ①.



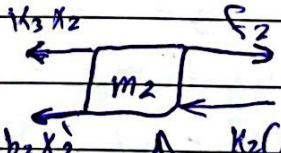


free body diagram

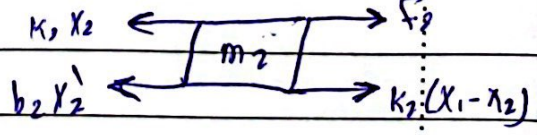
$\sum \vec{F} = m_2 \vec{a}_2$.

$F_2 - K_3x_2 - K_2(x_2 - x_1) - b_2x_2' = m_2x_2''$.

$m_2x_2'' + b_2x_2' + K_3x_2 + K_2(x_2 - x_1) = F_2$ ..... ②.



or



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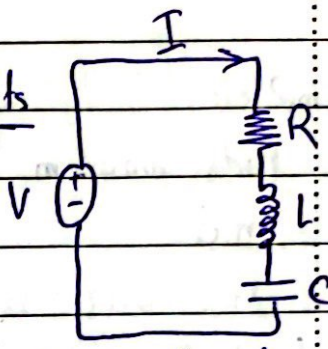
Subject Date No.

✱ Automatic control :-

* Ch 2 : Mathematical modeling :-

* Example :- ✓

RLC ccts



(series combination)

$I = I_R = I_C = I_L$

KCL → $I = I_R = I_L = I_C$.

KVL → $V = V_R + V_L + V_C$.

$V = V_R + V_L + V_C$.

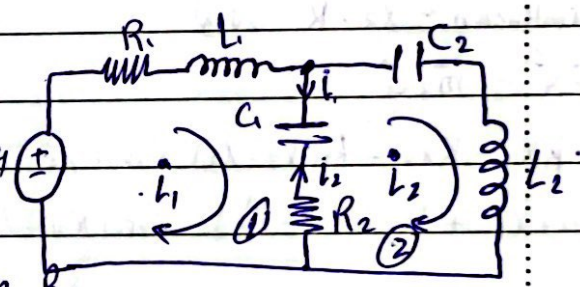
$V = IR + L\frac{dI_L}{dt} + \frac{1}{C}\int I_C\,dt$.

but $I = I_R = I_L = I_C$.

$V = IR + L\frac{dI}{dt} + \frac{1}{C}\int I\,dt$.

$V' = I'R + LI'' + \frac{1}{C}I$ → diff equation of the system.

* Example : ✓



KVL → loop ①

$V(t) = V_{R_1} + V_{L_1} + V_C + V_{R_2}$.

$V(t) = I_{R_1}R + L_1\frac{di_1}{dt} + \frac{1}{C_1}\int(I_1 - I_2)\,dt + I_{R_2}R_2$.

$(V(t) = Ri_1 + L_1\frac{di_1}{dt} + \frac{1}{C_1}\int(I_1 - I_2)\,dt + R_2(I_1 - I_2)) \cdot \frac{d}{dt}$.

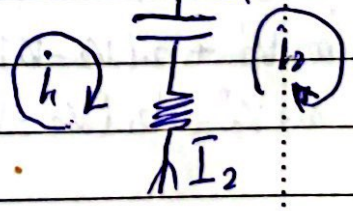
$V' = I_1'R + L_1I'' + \frac{1}{C_1}(I_1 - I_2) + R_2(I_1' - I_2')$ — diff eqn.

KVL → loop ②.

$V_{C_2} + V_{L_2} + V_{R_2} + V_{C_1} = 0$.

$(\frac{1}{C_2}\int I_2\,dt + L_2\frac{dI_2}{dt} + R_2(I_2 - I_1) + \frac{1}{C_1}\int(I_2 - I_1)\,dt = 0) \cdot \frac{d}{dt}$.

$\frac{1}{C_2}I_2 + L_2I_2'' + R_2(I_2' - I_1') + \frac{1}{C_1}(I_2 - I_1) = 0$.



NOTEBOOK

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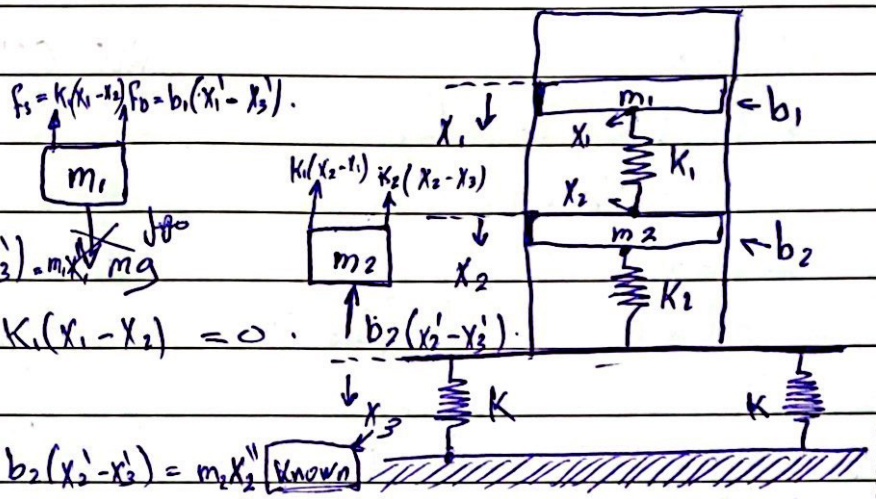
Subject Date No.

✱ Automatic control :-

✱ Ch2: Mathematical modeling :-

✱ Example :-

free body diagram



$\Sigma \vec{F} = m_1 \vec{a}$.

$-K_1(x_1 - x_2) - b_1(x_1' - x_3') - m_1 g = m_1 x_1''$

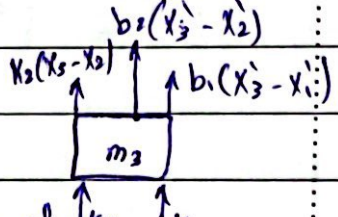
$m_1 x_1'' + b_1(x_1' - x_3') + K_1(x_1 - x_2) = 0$.

$\Sigma \vec{F} = m_2 \vec{a}$.

$-K_1(x_2 - x_1) - K_2(x_2 - x_3) - b_2(x_2' - x_3') = m_2 x_2''$ [known]

$m_2 x_2'' + K_1(x_2 - x_1) + K_2(x_2 - x_3) + b_2(x_2' - x_3') = 0$.

unknown $= x_3 \cdot K$ ✓

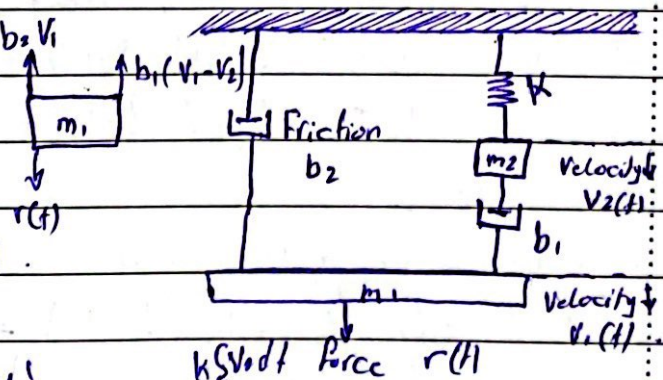


$\Sigma \vec{F} = \vec{m_3 a}$

$-Kx_3 - Kx_3 - K_2(x_3 - x_2) - b_2(x_3' - x_2') - b_1(x_3' - x_1') = m_3 x_3''$

$m_3 x_3'' + b_2(x_3' - x_2') + b_1(x_3' - x_1') + K_2(x_3 - x_2) + Kx_3 + Kx_3 = 0$.

✱ Example :-



$\Sigma \vec{F} = m_1 \vec{a}$

$r(t) - b_2 V_1 - b_1(V_1 - V_2) = m_1 V_1'$

$m_1 V_1' + b_2 V_1 + b_1(V_1 - V_2) = r(t)$.

$\Sigma \vec{F} = m_2 \vec{a}$

$-b_1(V_2 - V_1) - K\int V_2 dt = m_2 V_2'$.

$m_2 V_2' + b_1(V_2 - V_1) + K\int V_2 dt = 0$) $\frac{d}{dt}$

$m_2 V_2'' + b_1(V_2' - V_1') + KV_2 = 0$.

$V = x'$

$x = \int V dt$

NOTEBOOK

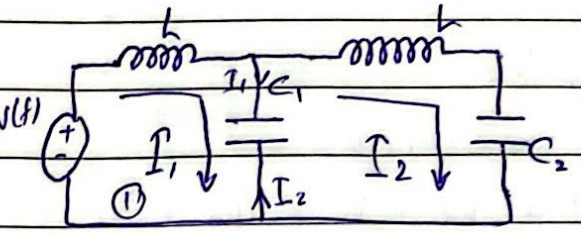
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Subject Date No.

** Automatic control :-

* Ch 2 :- Mathematical modeling :-

* Example :- ✓



KVL → loop ①.

$V(t) = V_L + V_{C_1}$

$V(t) = L_1 \frac{dI_1(t)}{dt} + \frac{1}{C_1}\int (I_1 - I_2)\,dt.$

$V'(t) = L_1 I_1'' + \frac{1}{C}(I_1 + I_2)$ --- ①.

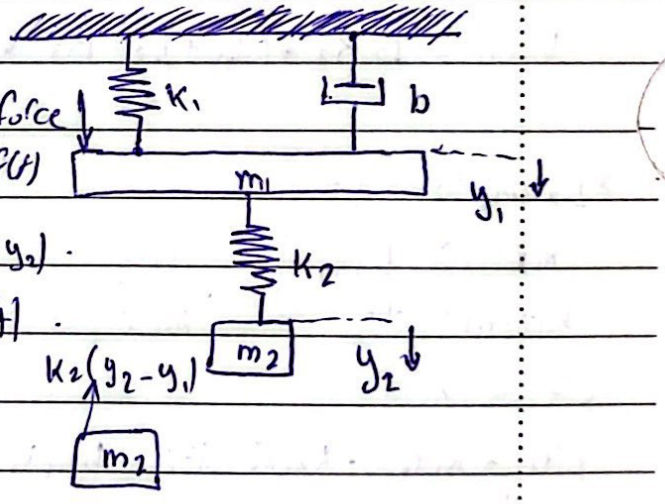
KVL → loop ②.

$V_{L_2} + V_{C_2} + V_{C_1} = 0.$

$L_2 \frac{dI_2(t)}{dt} + \frac{1}{C_2}\int I_2\,dt + \frac{1}{C_1}\int (I_2 - I_1)\,dt = 0$

$L_2 I_2'' + \frac{1}{C_2} I_2 + \frac{1}{C_1}(I_2 - I_1) = 0$ -- ②.

* Example :- P2.2



$\Sigma \vec{F} = m_1 \vec{a}$

$f(t) - K_1 y_1 - b y_1' - K_2(y_1 - y_2) = m_1 y_1''$

$\Rightarrow m_1 y_1'' + b y_1' + K_2(y_1 - y_2) + K_1 y_1 = f(t).$

$\Sigma F = m_2 \vec{a}.$

$-K_2(y_2 - y_1) = m_2 y_2''.$

$m_2 y_2'' + K_2(y_2 - y_1) = 0.$

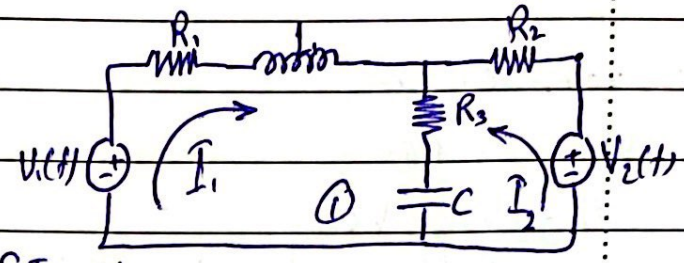
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Subject Date No.

✱ Automatic control :-

* Ch 2 :- Mathematic modeling :-

* Example :-



KVL → loop ①

$V_1(t) = V_{R_1} + V_{L_1} + V_{R_3} + V_C$.

$V_1(t) = I_{R_1} R_1 + L \frac{di_1(t)}{dt} + I_{R_3} R_3 + \frac{1}{C} \int I_C \, dt$.

$V_1(t) = I_1 R_1 + L_1 \frac{dI_1}{dt} + (I_1 + I_2) R_3 + \frac{1}{C} \int (I_1 + I_2) \, dt$.

$V_1'(t) = I_1' R_1 + L_1 I_1 + (I_1' + I_2') R_3 + \frac{1}{C} (I_1 + I_2)$ ... ①

KVL → loop ②

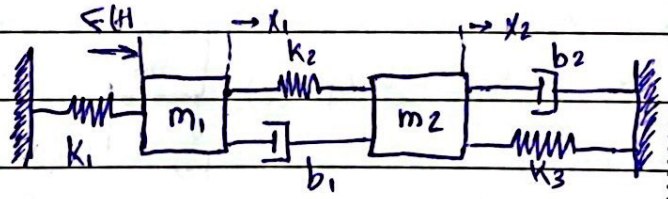
$V_2(t) = V_{R_2} + V_{R_3} + V_C$.

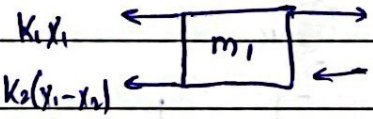
$V_2(t) = I_{R_2} R_2 + I_{R_3} R_3 + \frac{1}{C} \int I_C \, dt$.

$V_2(t) = I_2 R_2 + (I_1 + I_2) R_3 + \frac{1}{C} \int (I_1 + I_2) \, dt$.

$V_2'(t) = I_2' R_2 + (I_1' + I_2') R_3 + \frac{1}{C} (I_1 + I_2)$ ... ②.

* Example :- P 2.6





$\sum \vec{F} = m_1 \vec{a}$.

$f(t) - K_1 x_1 - K_2 (x_1 - x_2) - b_1 (x_1' - x_2') = m_1 x_1''$.

$m_1 x_1'' + b_1 (x_1' - x_2') + K_1 x_1 + K_2 (x_1 - x_2) = f(t)$. ... ①

$\sum \vec{F} = m \vec{a}$.

$-K_2 (x_2 - x_1) - K_3 x_2 - b_1 (x_2' - x_1') - b_2 x_2' = m_2 x_2''$.



$m_2 x_2'' + b_1 (x_2' - x_1') + b_2 x_2' + K_3 x_2 + K_2 (x_2 - x_1) = 0$. ... ②

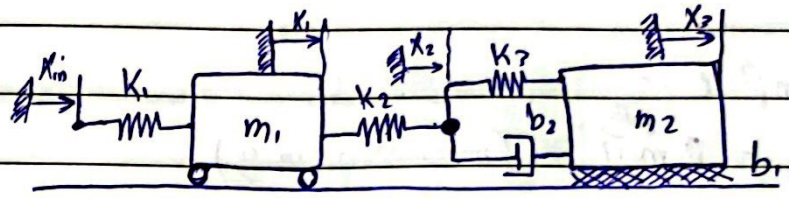
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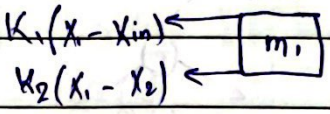
Subject Date No.

✱ Automatic control :-

✱ Ch 2 : Mathematical modeling :-

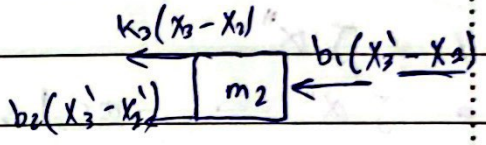
✱ Example :-





$\Sigma \vec{F} = m_1 \vec{a}_1$

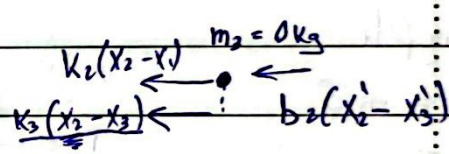
$-K_1(x_1 - x_{in}) - K_2(x_1 - x_2) = m_1 x_1''$



$\Sigma \vec{F} = m_2 \vec{a}_2$

$-K_3(x_3 - x_2) - b_2(x_3' - x_2') - b_1(x_3' - x_2') = m_2 x_3''$

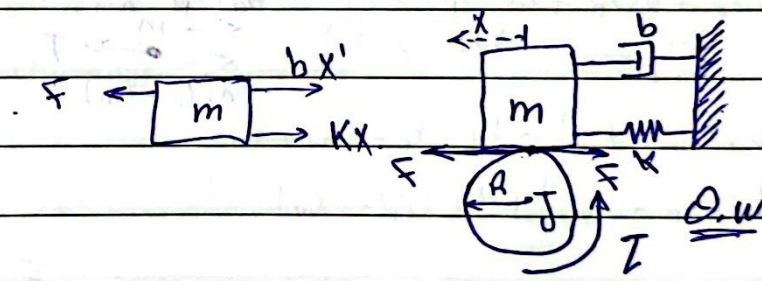
$\Sigma \vec{F} = \cancel{m_3 \vec{a}_3}^{\,0}$



$-K_2(x_2 - x_1) - K_3(x_2 - x_3) - b_2(x_2' - x_3') = 0$

$b_2(x_2' - x_3') + K_3(x_2 - x_3) + K_2(x_2 - x_1) = 0$

✱ Example :-



$\Sigma F = m\vec{a}$

$F - bx' - Kx = mx''$

$mx'' + bx' + Kx = F$

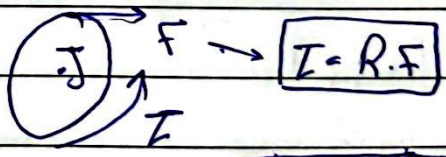
$\Sigma T = J\alpha$

$T - F \cdot R = J\theta''$

$T = J\theta'' + FR$

$T = J\theta'' + (mx'' + bx' + Kx)R$

$T = J\frac{x''}{R} + (mx'' + bx' + Kx)R$



$T = F \cdot R$

$x = R\theta$

$x' = R\theta'$

$x'' = R\theta''$

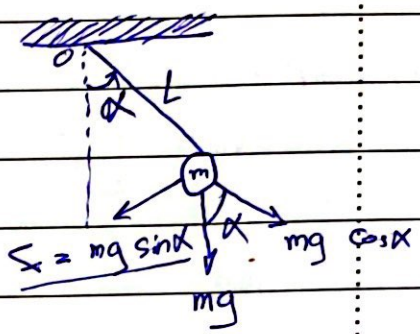
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Subject Date No.

** Automatic control :-

* Ch 2 :- Mathematical modeling :-

* Example :-



$\Sigma F_y = \cancel{m\vec{a}}^{=0}$ (no motion in y).

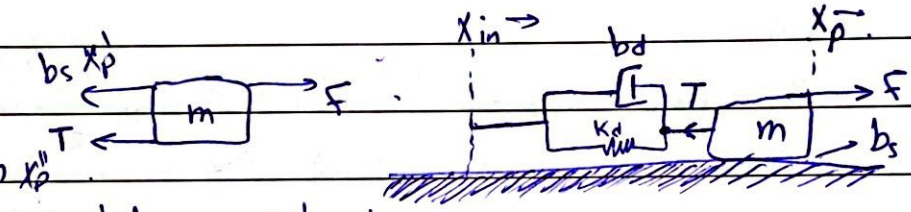
$T - mg\cos\alpha = 0 \rightarrow T = mg\cos\alpha$

$\Sigma T_x = J\alpha''$

$-F_x L = J\alpha'' \Rightarrow -mgL\sin\alpha = J\alpha''$.

non linear system.

* Example :-



$\Sigma F = m\vec{a}$.

$F - T - b_s \dot{x}_p = m\ddot{x}_p$

$F = m\ddot{x}_p + b_s\dot{x}_p + b_d(\dot{x}_p - \dot{x}_{in}) + K_d(x_p - x_{in})$

$b_d(\dot{x}_p - \dot{x}_{in})$ ← m → T

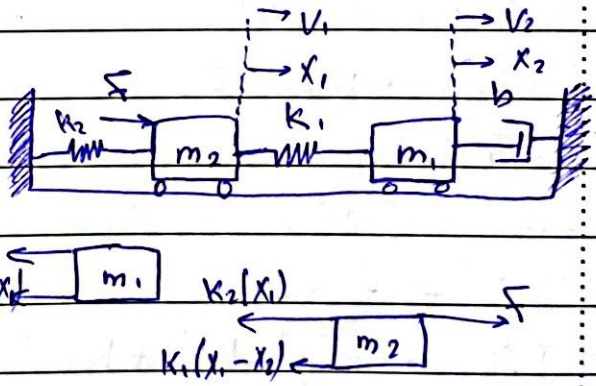
$K_d(x_p - x_{in})$

$\Sigma F = \cancel{m\vec{a}}^{=0}$.

$T - b_d(\dot{x}_p - \dot{x}_{in}) - K_d(x_p - x_{in}) = 0$.

$T = b_d(\dot{x}_p - \dot{x}_{in}) + K_d(x_p - x_{in})$

* Example :- P2.3



$\Sigma F = m_1\vec{a}_1$

$-b\dot{x}_2' - K_1(x_2 - x_1) = m_1\ddot{x}_2$.

$b\dot{x}_2'$ ← $K_1(x_2 - x_1)$ m_1 $K_2(x_1)$ → F

$K_1(x_1 - x_2)$ ← m_2

$\Sigma F = m_2\vec{a}_2$

$F - K_2x_1 - K_1(x_1 - x_2) = m_2\ddot{x}_1$.

$F = m_2\ddot{x}_1 + K_2x_1 + K_1(x_1 - x_2)$.

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Subject Date No.

** Automatic control :-

* Ch 2 :- Mathematical modeling

* The Laplace Transform of linear system :-

Taylor series Expansion :-

$$y = g(x) = g(x_0) + \frac{dg}{dx}\Big|_{x=x_0} \frac{(x-x_0)^1}{1!} + \frac{d^2g}{dx^2}\Big|_{x=x_0} \frac{(x-x_0)^2}{2!} + \cdots\cdots$$

Definition :-

$$F(s) = \mathcal{L}(f(t)) = \int_0^\infty e^{-st} \cdot f(t)\, dt. \quad \text{for } f(t),\ t>0.$$

$$f(t) = \mathcal{L}^{-1}(F(s))$$

* laplace Trance form of time Differentiation :-

$$\mathcal{L}\frac{df(t)}{dt} = sF(s) - f(0).$$

$$\mathcal{L}\frac{d^2f(t)}{dt^2} = s^2F(s) - sf(0) - f'(0)$$

$$\mathcal{L}\frac{d^3f(t)}{dt^3} = s^3F(s) - s^2f(0) - sf'(0) - f''(0).$$

general case :-

$$\mathcal{L}\frac{d^nf(t)}{dt^n} = s^nF(s) - s^{n-1}f(0) - s^{n-2}f'(0) - \cdots - f^{(n-1)}(0).$$

* Trasform non linear system to linear system by Taylor series :-

$$y = g(x_0) + \frac{dg}{dx}\Big|_{x=0} \frac{(x-x_0)^1}{1!} + \frac{d^2g}{dx^2}\Big|_{x=0} \frac{(x-x_0)^2}{2!} + \cdots\cdots$$

NOTEBOOK

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Subject Date No.

* Automatic control :-

* Ch 2:- Mathematical modeling :-

* laplace Transform

f(t)	$\mathcal{L}f(t)$	f(t)	$\mathcal{L}f(t)$
A	A/s	$\cos at$	$\frac{s}{s^2+a^2}$
At	A/s^2		
At^2	$A2!/s^3$	$\sin at$	$\frac{a}{s^2+a^2}$
At^n	$An!/s^{n+1}$		
e^{at}	$\frac{1}{s-a}$	$e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}$
te^{at}	$\frac{1}{(s-a)^2}$	$e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}$

* laplace transform :-

differential operator $\Rightarrow s = d/dt$.

integral operator $\Rightarrow 1/s = \int_0^\infty dt$.

s-operator is a complex number has areal and Imag Parts

$s = a + jb$ $\quad a \rightarrow$ real Part $\quad b \rightarrow$ Imaginary Part.

- Initial Value Theorem $\longrightarrow$ (بلش بالبداية)

$$f(0) = \lim_{s\to\infty} sF(s) = \lim_{t\to 0} f(t)$$

- Final Value Theorem (اخير بالنهاية)

$$f(\infty) = \lim_{s\to 0} sF(s) = \lim_{t\to\infty} f(t)$$

NOTEBOOK

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✱ Automatic control :-

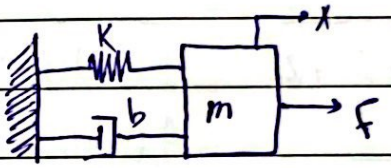
✱ Ch 2 :- Mathematical modeling :-

✱ differential equation ⟶ algebric equation

$\frac{d}{dx}, \frac{d^2}{dx^2}, \frac{d^3}{dx^3}$ ⟵ laplace transform.

(laplace) التحويل (algebric) ← (diff) يحول من

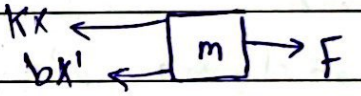
✱ Example :



$\sum \vec{F} = m\vec{a}$

$F - Kx - bx' = mx''$

$mx'' + bx' + Kx = F$ ⟶ diff equ.



$m(s^2X(s) - sX(0) - X'(0)) + b(sX(s) - X(0)) + KX(s) = F(s)$.

If initial condition = 0.

$X(0) = 0 \quad X'(0) = 0 \quad X''(0) = 0 \ldots$

$ms^2X(s) + bsX(s) + KX(s) = F(s)$.

$X(s)[ms^2 + bs + K] = F(s)$.

If $\frac{b}{m} = 3 \quad \frac{K}{m} = 2 \quad \frac{F(t)}{m} = 1$.

$$X(s) = \frac{\frac{F(s)}{m}}{s^2 + \frac{b}{m}s + \frac{K}{m}} = \frac{1}{s^2 + 3s + 2} \longrightarrow X(t) \cdot ??!!$$

$X(t) = \mathcal{L}^{-1} X(s)$

Partial fraction :-

قبل ① real different. ② complex

③ real multiple root.

N O T E B O O K

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Subject Date No.

** Automatic control :-

* Ch 2: Mathematical modeling :-

* Example :-

$X(s) = \frac{1}{s^2+3s+2}$ → Case 2: real different.

$\frac{1}{s^2+3s+2} = \frac{1}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}$

$\frac{1}{s^2+3s+2} = \frac{A(s+1)+B(s+2)}{s^2+3s+2} \Rightarrow 1 = A(s+1)+B(s+2)$

$\Rightarrow s=-1 \rightarrow \boxed{B=1}$

$s=-2 \rightarrow \boxed{A=-1}$

$X(s) = \frac{-1}{s+2} + \frac{1}{s+1} \Rightarrow x(t) = \mathcal{L}^{-1} X(s)$.

$x(t) = -1\, e^{-2t} + e^{-t}$.

* Case 2: real multiple root.

$y(s) = \frac{2}{(s+4)(s+2)^2} = \frac{A}{s+4} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$

$\frac{2}{(s+4)(s+2)^2} = \frac{A(s+2)^2 + B(s+4)(s+2) + C(s+4)}{(s+4)(s+2)^2}$.

$2 = A(s+2)^2 + B(s+4)(s+2) + C(s+4)$.

$\Rightarrow s=-2 \rightarrow C=1$.

$\Rightarrow s=-4 \rightarrow A=\frac{1}{2}$

$\Rightarrow s=0 \rightarrow 2 = 4A + 8B + 4C \rightarrow B = -\frac{1}{2}$.

NOTEBOOK

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Subject Date No.

* Automatic control :-

* Ch 2 :- Mathematical modeling :-

* Case 2 :- e.v

$$y(s) = \frac{1/2}{s+4} + \frac{-1/2}{s+2} + \frac{1}{(s+2)^2} \Rightarrow y(t) = \mathcal{L}^{-1} y(s)$$

$$y(t) = \frac{1}{2} e^{-4t} - \frac{1}{2} e^{-2t} + t e^{-2t}$$

* Case 3 :- complex number

$$y(s) = \frac{3}{s(s^2+2s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+5}$$

$$\frac{3}{s(s^2+2s+5)} = \frac{A(s^2+2s+5) + (Bs+C)s}{s(s^2+2s+5)}$$

$$3 = A(s^2+2s+5) + (Bs+C)s$$

$$3 = As^2 + 2sA + 5A + Bs^2 + Cs$$

$$3 = s^2(A+B) + s(2A+C) + 5A$$

$A+B=0$. $2A+C=0$ $5A=3 \rightarrow \boxed{A = \frac{3}{5}}$

$\boxed{B = \frac{-3}{5}}$ $\boxed{C = \frac{-6}{5}}$

$$y(s) = \frac{3/5}{s} + \frac{\frac{-3}{5}s + \frac{-6}{5}}{s^2+2s+5} = \frac{3/5}{s} + \frac{-3}{5}\frac{s+2}{s^2+2s+5}$$

$$= \frac{3/5}{s} + \frac{-3}{5}\frac{s+1+1}{s^2+2s+1+4} = \frac{3/5}{s} + \frac{-3}{5}\left[\frac{s+1}{(s+1)^2+4} + \frac{1}{(s+1)^2+4}\right]$$

$$y(t) = \mathcal{L}^{-1} y(s) = \frac{3}{5} - \frac{3}{5} e^{-t}\cos 2t - \frac{3}{10} e^{-t}\sin 2t .$$

NOTEBOOK

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Subject Date No.

* Automatic control :

* Ch 2 :- Mathematical modeling :-

* Transfer function of linear systems :-

The Transfer function of a linear system :- the ratio of the laplace transform of the output to the laplace transform of the input, with all initial condition assumed to be Zero

$T.F = \frac{\text{out Put}}{\text{in Put}}$, T.F → represents system dynamics in s-domain.

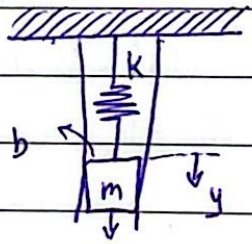
$T.F = \frac{\text{out Put}}{\text{in Put}} = \frac{P(s)}{q(s)}$ where $P(s), q(s)$ are polynomials

The root of $P(s)$ are called the zeros of the system. (O)

where the root of $q(s)$ are called the poles of the system. (X)

* Example :-

$\Sigma F = m\bar{a}$



In Put → R(s)

out Put → y(s).

$r(t) - ky - by' = my''$.

$my'' + by' + ky = r(t)$ → laplace transform.

$m(s^2 y(s)) + b(s\,y(s)) + Ky(s) = r(s)$.

$T.F = \frac{\text{out Put}}{\text{in Put}} = \frac{y(s)}{R(s)} = \frac{1}{ms^2 + bs + K}$.

$y(s)[ms^2 + bs + K] = r(s)$.

in Put → R(s) , output → Velocity $(y') \rightarrow V$

$mV' + bV + K\int V = r(t)$

$ms\,V(s) + b\,V(s) + K\frac{V(s)}{s} = r(s)$

$\frac{V(s)}{R(s)} = \left[\frac{1}{ms + b + \frac{K}{s}}\right]$

NOTEBOOK

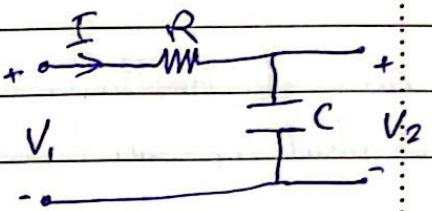
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Subject Date No.

** Automatic control

* Ch 2 :- Mathematical modeling :-

* Example :- Find the transfer function



if input → V_1 . output → V_2

$V_1 = V_R + V_C$

but $V_2 = V_C = \frac{1}{C}\int I\,dt \rightarrow I = C\frac{dV}{dt}$ $\boxed{I(s) = Cs\,V_2(s)}$

$V_1 = IR + \frac{1}{C}\int I\,dt.$

$V_1(s) = Cs\,V_2(s)\,R + V_2(s).$

$V_1(s) = V_2(s)[Cs R + 1] \Rightarrow \frac{V_2(s)}{V_1(s)} = \frac{1}{RCs+1} = T.F.$

if input → V_1 output → I

$V_1 = IR + \frac{1}{C}\int I\,dt.$ → laplace transforme.

$V_1(s) = I(s)R + \frac{1}{Cs}I(s)$

$V_1(s) = I(s)\left[R + \frac{1}{Cs}\right] \Rightarrow \frac{I(s)}{V_1(s)} = \frac{1}{R+\frac{1}{Cs}} = \frac{Cs}{RCs+1}$

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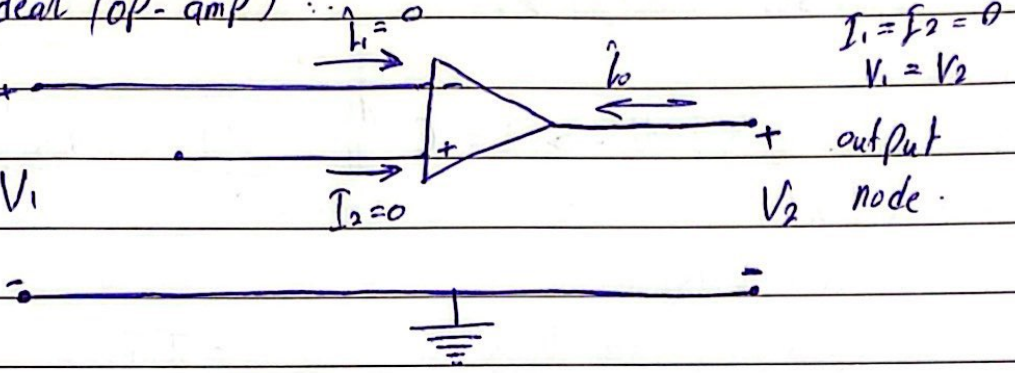
Subject Date No.

* Automatic control :-

* Ch2 : Mathematical modeling :-

* Operational Amplifier Transfer function (OP-Amp) :-

The ideal (op-amp) :



* Inverting amplifier : ⟵ العاكس (neg) يعكس الإشارة V_{in}

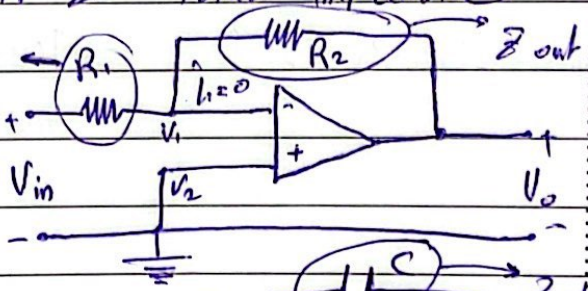
Transfer function just for Inverting amplifier

$T.F = -\frac{Z_{out}}{Z_{in}}$. $Z \rightarrow$ Impedance.

in general $Z \rightarrow$ total Impedance.

$Z_{out} = R_2$, $Z_{in} = R_1$

$T.F = -\frac{Z_{out}}{Z_{in}} = -\frac{R_2}{R_1}$

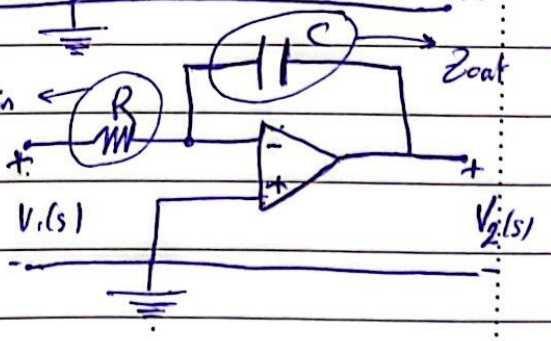


* Integrating circuit (OP-Amp)

$Z_{out} = \frac{1}{Cs}$. $Z_{in} = R$.

$T.F = \frac{V_2(s)}{V_1(s)} = \frac{-Z_{out}}{Z_{in}} = -\frac{\frac{1}{Cs}}{R}$

$= \frac{-1}{RCs}$.



NOTEBOOK

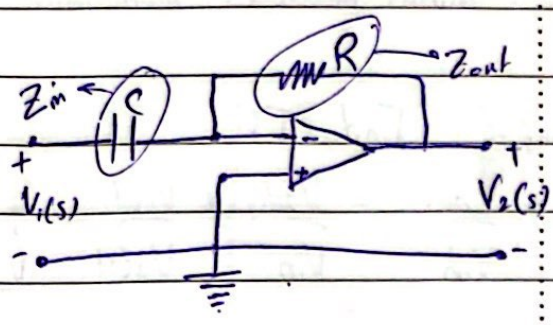
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Subject Date No.

✱ Automatic control

* Ch 2 :- Mathmatical modeling :-

* Differentiating Circuit (Op-Amp) :-

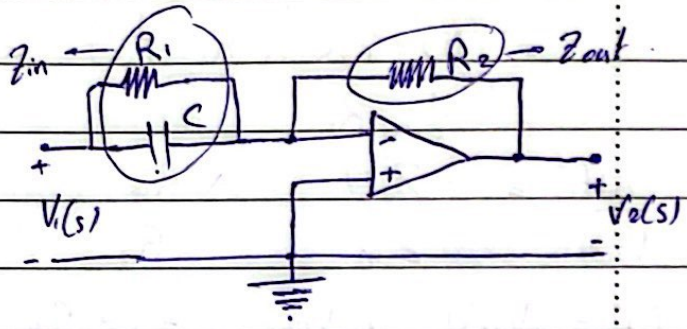


$Z_{out} = R \qquad Z_{in} = \frac{1}{Cs}$

$T.F = \frac{V_2(s)}{V_1(s)} = \frac{-Z_{out}}{Z_{in}} = \frac{-R}{\frac{1}{Cs}}$

$T.F = -RCs$

* Example :-

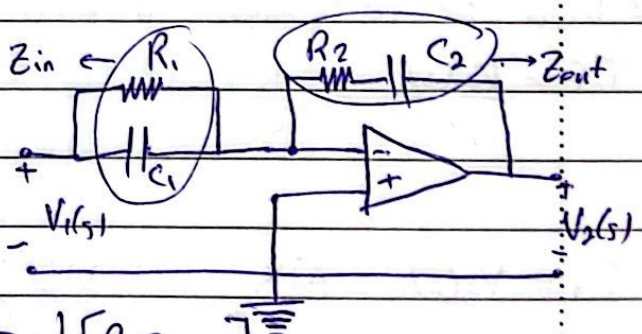


$Z_{out} = R_2 \qquad Z_{in} = R_1 // C$

$Z_{in} = \frac{\frac{1}{Cs} R_1}{R_1 + \frac{1}{Cs}} = \frac{R_1}{1 + R_1 Cs}$

$\frac{V_2(s)}{V_1(s)} = \frac{-Z_{out}}{Z_{in}} = \frac{-R_2[1 + R_1 Cs]}{R_1}$

* Example :-



$Z_{out} = R_2 + \frac{1}{C_2 s}$

$Z_{in} = \frac{\frac{1}{C_1 s} R_1}{R_1 + \frac{1}{C_1 s}} = \frac{R_1}{R_1 C_1 s + 1}$

$T.F = \frac{V_2(s)}{V_1(s)} = \frac{-Z_{out}}{Z_{in}} = \frac{-\left(R_2 + \frac{1}{C_2 s}\right)\left[R_1 C_1 s + 1\right]}{R_1}$

$= \frac{-(R_2 C_2 s + 1)(R_1 C_1 s + 1)}{R_1 C_2 s}$

NOTEBOOK

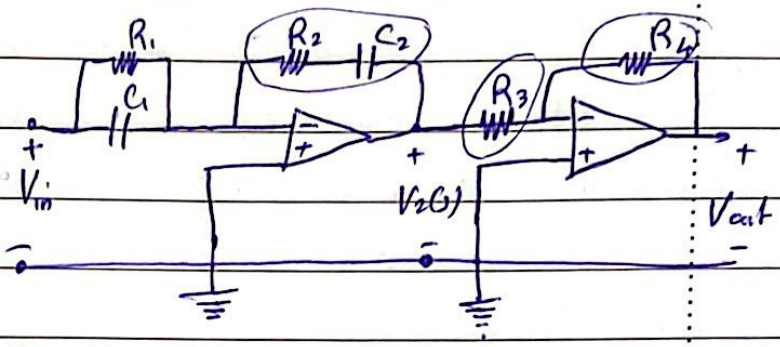
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Subject Date No.

Automatic control :-

*Ch2 : Mathematical modeling :-

*Example : Find T.F.



$$T.F = \frac{V_{out}}{V_{in}} = \frac{V_2(s)}{V_{in}} * \frac{V_{out}}{V_2(s)}$$

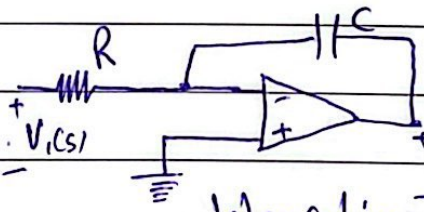
$$\frac{V_2(s)}{V_{in}(s)} = -\frac{Z_{out}}{Z_{in}} = -\frac{\left(R_2 + \frac{1}{C_2 s}\right)\left(R_1 + \frac{1}{C_1 s}\right)}{\frac{1}{C_1 s} R_1} = -\frac{(R_2 C_2 s + 1)(R_1 C_1 s + 1)}{R_1 C_2 s} \quad *$$

$$\frac{V_{out}}{V_2(s)} = -\frac{Z_{out}}{Z_{in}} = -\frac{R_4}{R_3} \quad *$$

$$T.F = \frac{V_2(s)}{V_{in}(s)} * \frac{V_{out}}{V_2(s)} = -\frac{R_4}{R_3}\left[-\frac{(R_2 C_2 s + 1)(R_1 C_1 s + 1)}{R_1 C_2 s}\right]$$

*Draw the (op-Amp) if :

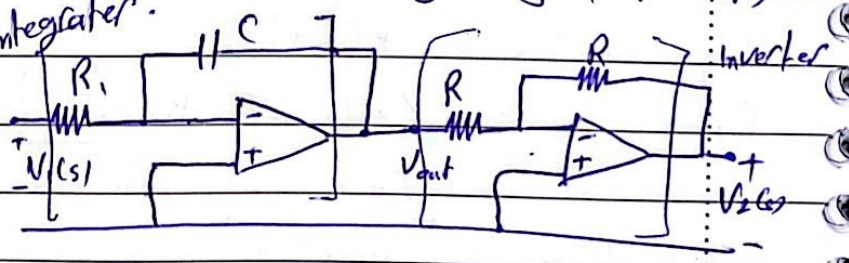
$$T.F = \frac{V_2(s)}{V_1(s)} = \frac{-K}{s}$$



Integrating (op-amp)

$$T.F = \frac{V_2(s)}{V_1(s)} = \frac{+K}{s}$$

Integrater.



$$= \frac{V_2(s)}{V_{out}} * \frac{V_{out}}{V_1(s)}$$

$$= -1 * \frac{-1}{R_1 C s} = \frac{+1}{R_1 C s} = \frac{K}{s}$$

$$\frac{V_2}{V_{out}} = -\frac{R}{R} = -1$$

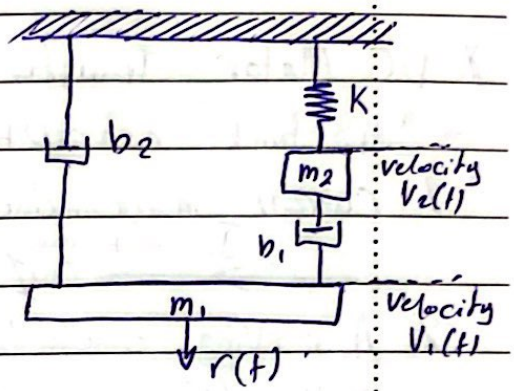
NOTEBOOK

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Subject Date No.

✱ Automatic control :-

* Ch 2 : Mathematical modeling :-



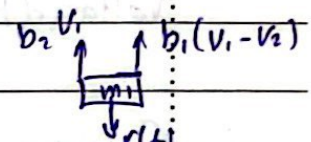
* Example : Find the total Transfer function

in Put → R(t) , out Put → $V_2(t)$, $V_1(t)$

2 out Put ⟶ 2 T.F.

$$T.F = \frac{\text{out Put}}{\text{in Put}} = \frac{V_2(s)}{R(s)}$$

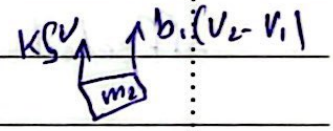
Free body diagram m_1



$$r(t) - b_2 V_1(t) - b_1(V_1(t) - V_2(t)) = m_1 V_1'(t) .$$

$$m_1 V_1'(t) + b_2 V_1(t) + b_1(V_1(t) - V_2(t)) = r(t) \quad \cdots ① .$$

Free body diagram m_2



$$-b_1(V_2(t) - V_1(t)) - K\int V_2(t) = m_2 V_2'(t) .$$

$$m_2 V_2'(t) + b_1(V_2(t) - V_1(t)) + K\int V_2(t) = 0 \quad \cdots ②$$

laplace transforme form the two masses :-

$$m_1 s\, V_1(s) + b_2 V_1(s) + b_1(V_1(s) - V_2(s)) = R(s) .$$

$$m_2 s\, V_2(s) + b_1(V_2(s) - V_1(s)) + \frac{K}{s} V_2(s) = 0 .$$

$$(m_1 s + b_2 + b_1) V_1(s) + (-b_1) V_2(s) = R(s)$$

$$(m_2 s + b_1 + \frac{K}{s}) V_2(s) + (b_1) V_1(s) = 0 .$$

$$\begin{bmatrix} (m_1 s + b_2 + b_1) & (-b_1) \\ (b_1) & (m_2 s + b_1 + \frac{K}{s}) \end{bmatrix} \begin{bmatrix} V_1(s) \\ V_2(s) \end{bmatrix} = \begin{bmatrix} R(s) \\ 0 \end{bmatrix} .$$

$$V_1(s) = \frac{R(s)\left[m_2 s + b_1 + \frac{K}{s}\right]}{(m_1 s + b_2 + b_1)(m_2 s + b_1 + \frac{K}{s}) - b_1^2} .$$

$$V_2(s) = \frac{R(s)\, b_1}{(m_1 s + b_2 + b_1)(m_2 s + b_1 + \frac{K}{s}) - b_1^2}$$

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Subject Date No.

✱ Automatic control :-

* Ch 2 :- Mathematical modeling :-

* DC Motor Transfer function :-

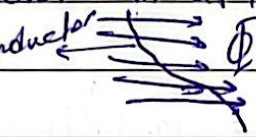
- Important concepts :- (DC motor → Actuator)

1- Current in a conductor will produce electromagnetic flux (ϕ).



$\phi \propto I$ (التدفق يتناسب طرديًا مع التيار)

2- A moving conductor in a flux will produce induced Voltage (E_{ind})



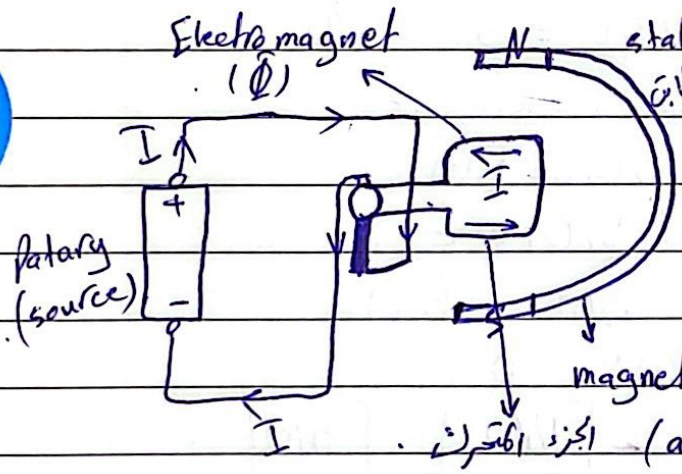
$E \propto w \phi$

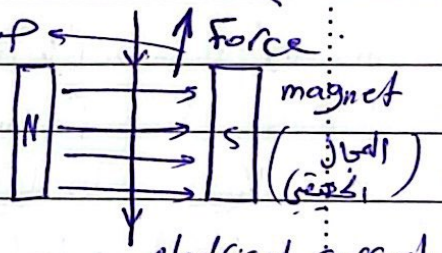
speed

3- Current holding conductor exists in a flux will be affected by Torque (T). $T \propto \phi I$.

↳ عزم







. concept (3) .

. الجزء الدوار (armature) : (rotor) .

DC Motor have two Part → Electrical Part.
→ Mechanical Part.

DC Motor → Electro mechanical system.

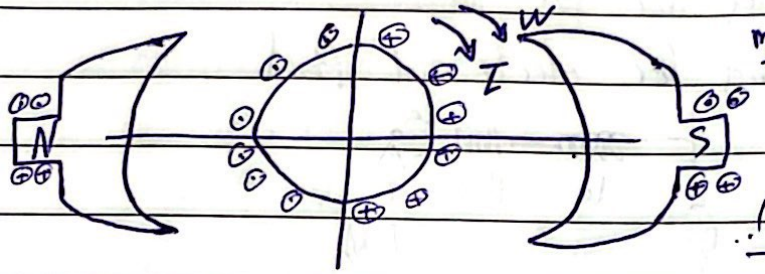
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Subject Date No.

** Automatic control :-

* Ch 2 : Mathematical modeling :-

*



magnet V_f $I_f \rightarrow$ $L_f + R_f$

field.

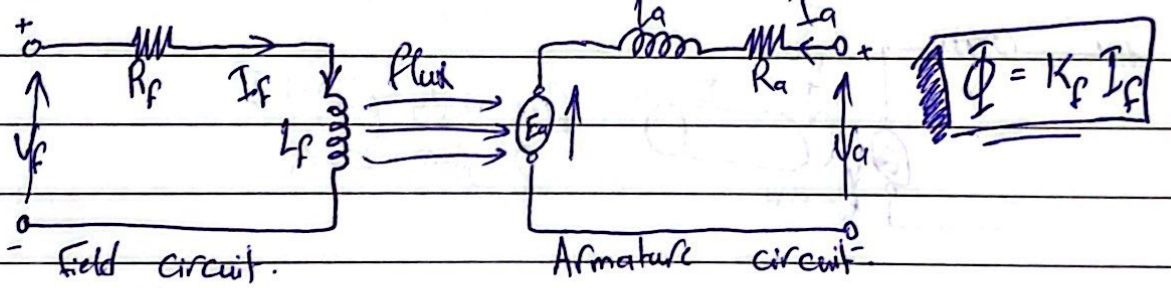
armature $\rightarrow$ $L_a + R_a$.

V_a I_a.

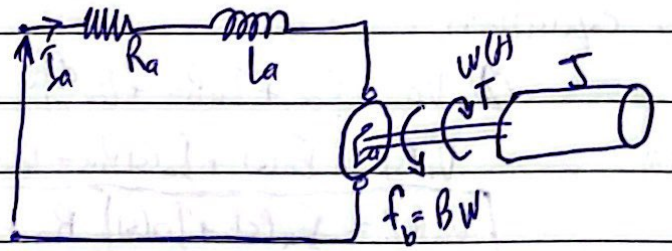
DC motor (electrical energy $\xrightarrow{\text{into}}$ mechanical energy).

* Electromechanical system (DC motor) :

1- Electrical Part.



2- Mechanical Part.



$$T = K \Phi I_a$$

$$E_a = K_b \Phi W.$$

NOTEBOOK

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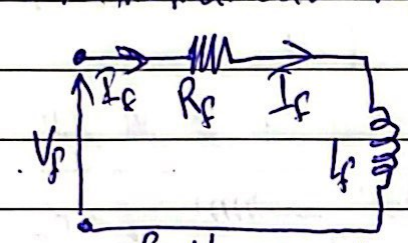
Subject Date No.

* Automatic control :-

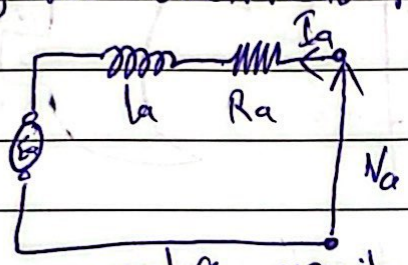
* Ch 2 :- Mathematical modeling :-

* Mathematical modeling for DC Motors :-

* Mathematical modeling for electrical part.



field circuit



armature circuit.

KVL at loop ①

$V_f = V_R + V_L$.

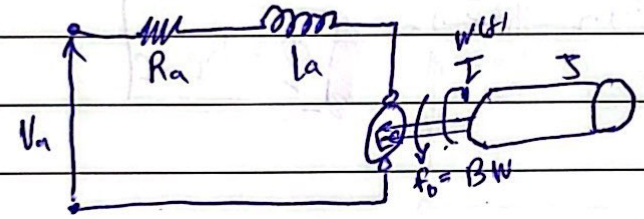
$$\boxed{V_f = I_f R_f + L_f \frac{dI_f}{dt}} \quad -①$$

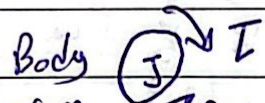
KVL at loop ②

$V_a = E_a + V_R + V_L$

$$\boxed{V_a = E_a + I_a R_a + L_a \frac{dI_a}{dt}} \quad -②$$

* Mathematical modeling for mechanical part.





$\Sigma I = J W'$

$I - BW = J W' \rightarrow \boxed{I - B\theta' = J\theta''}$ -③

$\boxed{W = \theta'}$

$\boxed{\alpha = W' = \theta''}$

Laplace transform for the equation.

① $V_f = I_f R_f + L_f \frac{dI_f}{dt}$.

$\rightarrow V_f(s) = I_f(s) R_f + L_f s I_f(s)$

$\boxed{V_f(s) = I_f(s)[R_f + L_f s]}$.

② $V_a = E_a + I_a R_a + L_a \frac{dI_a}{dt}$.

$V_a(s) = E_a(s) + I_a(s) R_a + L_a s I_a$

$\boxed{V_a(s) = E_a(s) + I_a(s)[R_a + L_a s]}$.

③ $I - B\theta' = J\theta''$.

$I(s) - Bs\theta(s) = Js^2\theta(s) \rightarrow \boxed{I(s) = \theta(s)[Js^2 + Bs]}$.

NOTEBOOK

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Subject Date No.

** Automatic control :-

* Ch2 : Mathematical modeling :-

* Transfer function of DC motor :-

- What is output and input :-

output → V_f , I_f , V_a , I_a .

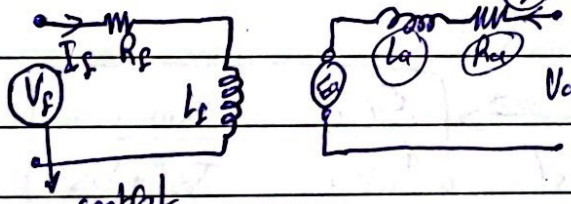
input → θ , w

$$T.F = \frac{output}{input} \overset{①}{=} \frac{\theta(s)}{V_f(s)} \overset{②}{=} \frac{\theta(s)}{V_a(s)}$$

* Field control motor → Input = V_f .

current field control motor → Input = I_f .

$$T.F = \frac{output}{input} = \frac{\theta(s)}{V_f(s)}$$



note :- $K_1 = K K_f \overset{\text{constant}}{I_a(s)}$

$$\Rightarrow K_1 I_f(s) = (Js^2 + bs)\theta(s)$$

$$K_1 \frac{V_f(s)}{R_f + L_f s} = (Js^2 + bs)\theta(s)$$

$$\boxed{\frac{\theta(s)}{V_f(s)} = \frac{K_1}{(R_f + L_f s)(Js^2 + bs)}} \quad \#$$

If w → input .

$$\boxed{\frac{W(s)}{V_f(s)} = \frac{K_1}{(R_f + L_f s)(Js + b)}} \quad \#$$

V_a & I_a = constant

* Telectrical = Tmechanical .

$$K \phi(s) I_a(s) = (Js^2 + bs)\theta(s)$$

$$\phi(s) = K_f I_f(s) .$$

$$K K_f I_f(s) I_a(s) = (Js^2 + bs)\theta(s)$$

$$K_1 I_f(s) = (Js^2 + bs)\theta(s)$$

but $V_f(s) = I_f(s)[R_f + L_f s]$.

$$I_f(s) = \frac{V_f(s)}{R_f + L_f s}$$

$$\boxed{\frac{\theta(s)}{I_f(s)} = \frac{K_1}{Js^2 + bs}}$$

$$\boxed{\frac{W(s)}{I_f(s)} = \frac{K_1}{Js + b}}$$

NOTEBOOK

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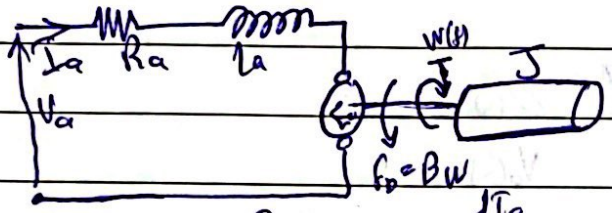
* Automatic control :-

* Ch 2 :- Mathematical modeling :-

* Transfer function of DC motor :-

* Armature control motor ⟶ input = V_a. $V_f . I_f = const$

Armature current control motor ⟶ input = I_a.



$V_a = E_a + I_a R_a + L_a \frac{dI_a}{dt}$.

$V_a(s) = E_a(s) + I_a(s)[R_a + L_a s]$.

$T = K \Phi I_a$

$\Phi = K_f I_f$.

$E_a = K_b \Phi \omega$.

* Electrical = T mechanical.

$K \Phi(s) I_a(s) = (Js^2 + bs)\,\theta(s)$

$K K_f I_f(s) I_a(s) = (Js^2 + bs)\,\theta(s)$

$K_m I_a(s) = (Js^2 + bs)\,\theta(s)$ $\Rightarrow$ $\frac{\theta(s)}{I_a(s)} = \frac{K_m}{Js^2 + bs}$

$\Rightarrow V_a(s) = E_a(s) + I_a(s)[R_a + L_a s]$.

$\frac{\omega(s)}{I_a(s)} = \frac{K_m}{Js + b}$

$I_a(s) = \frac{V_a(s) - E_a(s)}{R_a + L_a s}$

$K_m \left(\frac{V_a(s) - E_a(s)}{R_a + L_a s} \right) = (Js^2 + bs)\,\theta(s)$

$\Rightarrow E_a(s) = K_b \Phi \omega = K_b \Phi \theta' = K_{b_1} \theta'$

$E_a(s) = K_{b_1} s\,\theta(s)$.

$K_m \left(\frac{V_a(s) - K_{b_1} s\theta(s)}{R_a + L_a s} \right) = (Js^2 + bs)\,\theta(s)$

$$\frac{\theta(s)}{V_a(s)} = \left[\frac{K_m}{K_m K_{b_1} s + (R_a + L_a s)(Js^2 + bs)} \right]$$

NOTEBOOK

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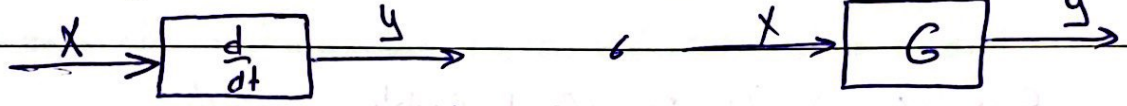
** Automatic control :-

* Ch 2 : Mathematical modeling :-

* Bloge Diagram fundamentals and reduction techniques :-

Bloge Diagram :- is a shorthand, graphical representation of a physical system.

* The arrows represent the direction of information or signal flow.

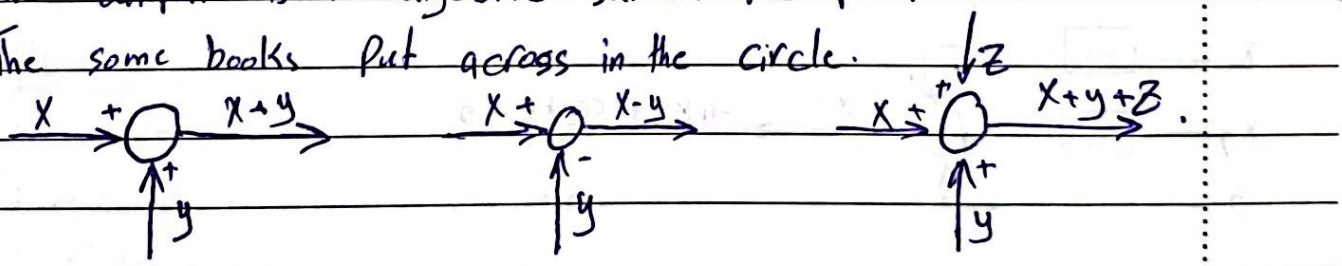


$y = X \cdot \frac{dx}{dt} = \frac{dy}{dt}$ $\qquad$ $y = GX$. #

* The operations of addition and subtraction have a special representation.

The output is the algebric sum of the point.

The some books put across in the circle.



* Components of a Bloge Diagram (elements) :-

1. signals.
2. system block.
3. Summing junction
4. Pick off / Take off point.

NOTEBOOK

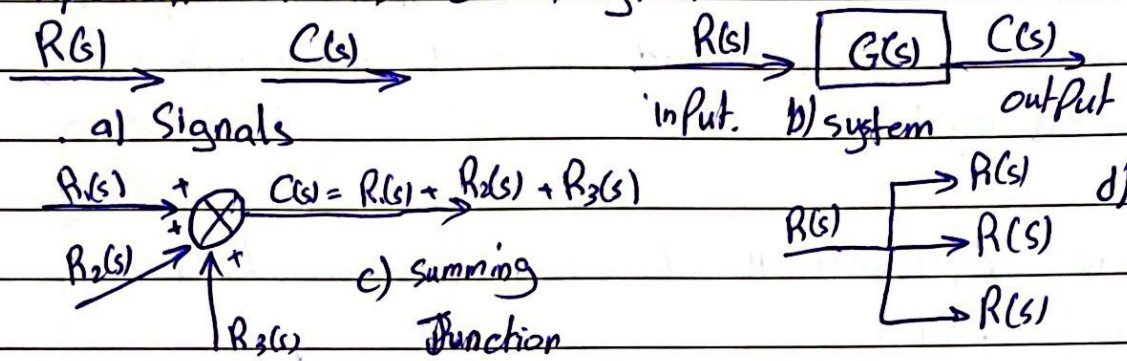
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Subject Date No.

✱ Automatic control :-

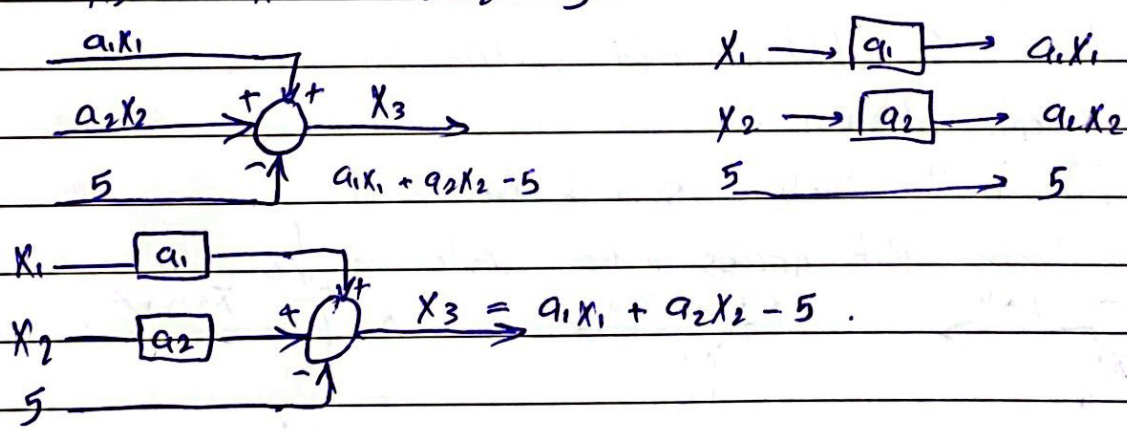
* Ch2 :- Mathe matical modeling :-

* Components of a Block Diagram :-



* Example :- Consider X_1, X_2, X_3 are variables :- and a_1, a_2 are general coefficients or mathematical oper.

$$X_3 = a_1X_1 + a_2X_2 - 5.$$



* Topologies and Cascade :-

NOTEBOOK

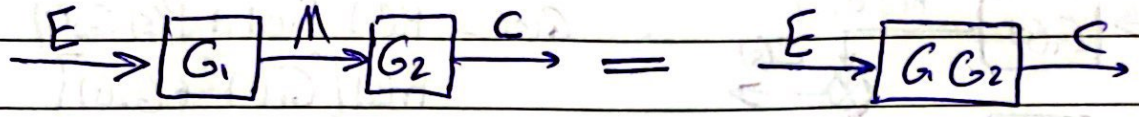
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Subject Date No.

** Automatic control :-

* Ch 2 :- Mathematical modeling :-

* Example



$M = EG_1 \quad C = MG_2$.

$\therefore C = EG_1G_2$.

$C = EG_1G_2$.

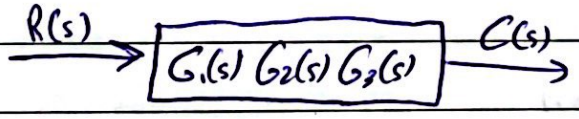
* Example :- Find the cascaded subsystem and the equivalent transfer function.



$X_1(s) = R(s)\,G_1(s) \qquad X_2(s) = R(s)\,G_1(s)\,G_2(s)$.

$X_2(s) = X_1(s)\,G_2(s)$, $C(s) = X_2(s)\,G_3(s)$

$\Rightarrow C(s) = R(s)\,G_1(s)\,G_2(s)\,G_3(s)$ #.



→ Cascaded subsystem.

2- equivalent transfer function

$G_e(s) = G_1(s)\,G_2(s)\,G_3(s)$.

* Parallel form :-

Parallel subsystems have a common input and an output formed by the algebric sum of the output from all of the subsystems.

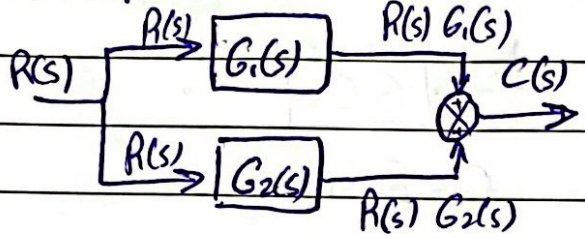
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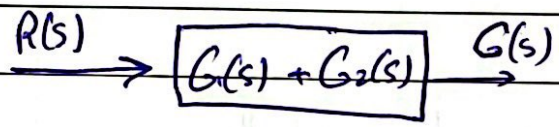
* Automatic control :-

* Ch 2 : Mathematical modeling :-

* Example :-

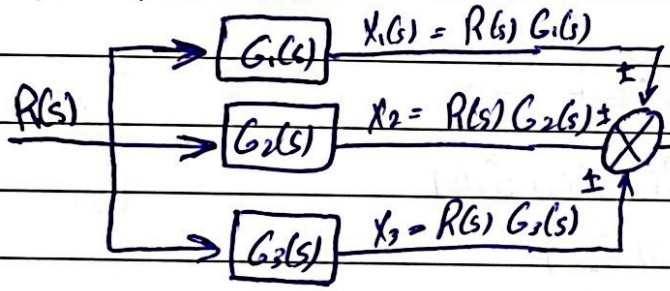


$$C(s) = R(s) G_1(s) + R(s) G_2(s)$$
$$R(s)[G_1(s) + G_2(s)]$$



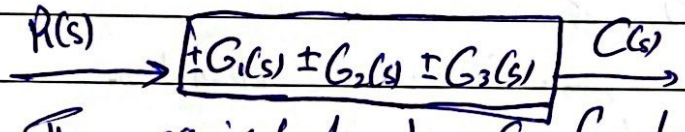
$$C(s) = R(s)[G_1(s) + G_2(s)]$$

* Parallel form :-



$$C(s) = R(s) G_1(s) \pm R(s) G_2(s) \pm R(s) G_3(s)$$
$$= R(s)[G_1(s) \pm G_2(s) \pm G_3(s)]$$

Parallel form.



The equivalent transfer function

$$G_e(s) = \pm G_1(s) \pm G_2(s) \pm G_3(s).$$

* Reduction techniques :-

1. Combining blocks in cascade.



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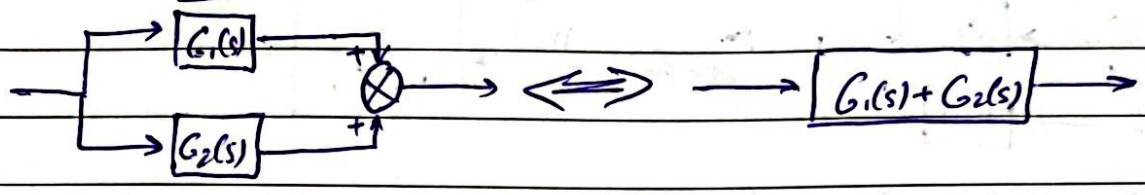
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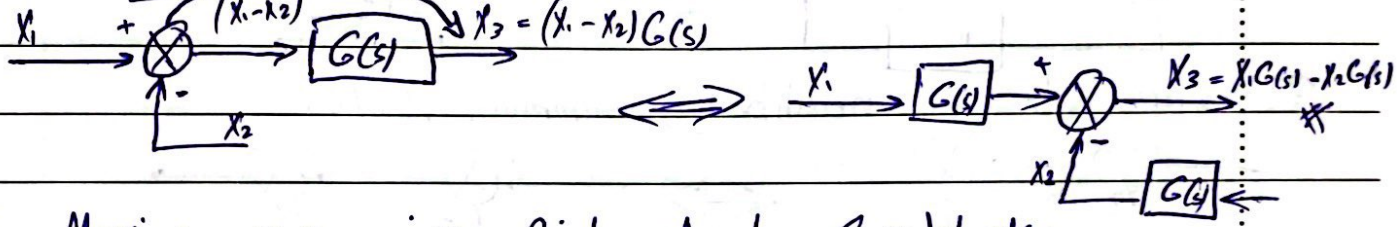
* Ch2 :- Mathematical modeling :-

* Reduction techniques :-

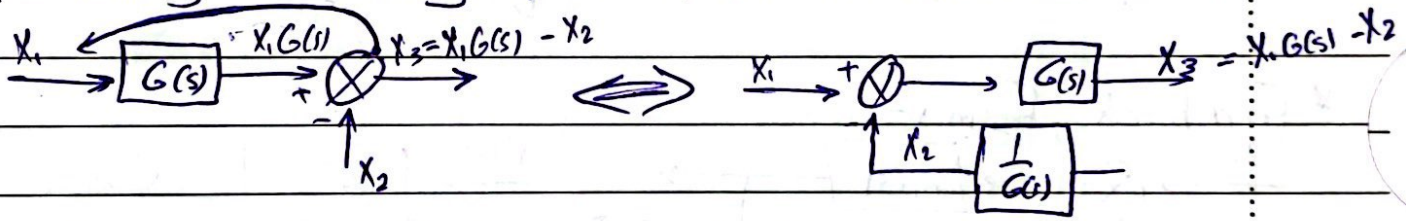
2- Combining blocks in parallel



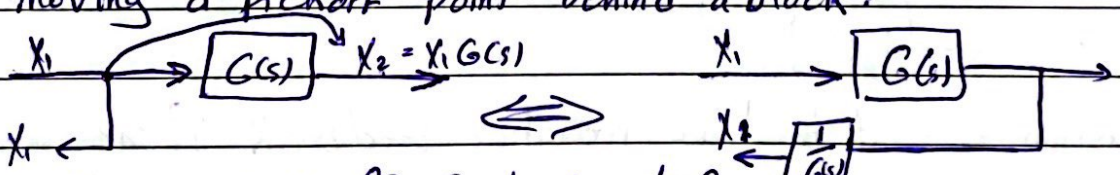
3- Moving a summing point behind a block :-



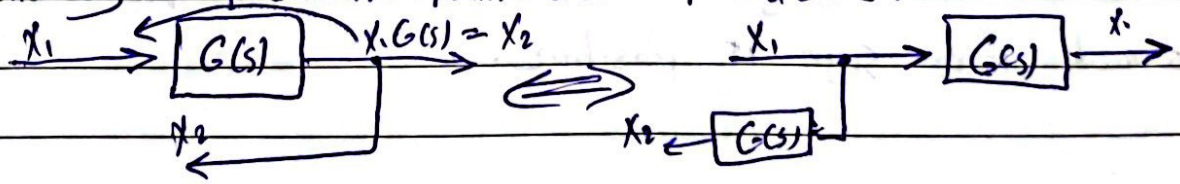
4- Moving a summing point ahead of a block :-



5- Moving a pickoff point behind a block :-



6- Moving a pick off point ahead of a block :-



NOTEBOOK

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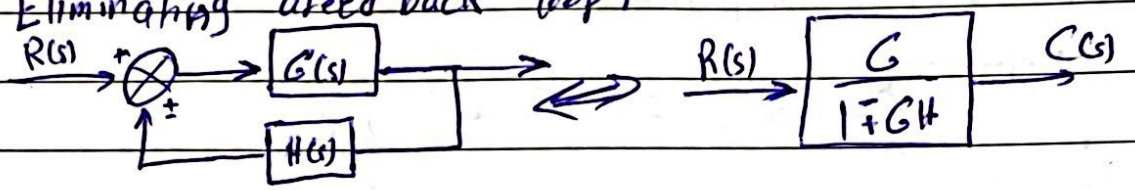
Subject Date No.

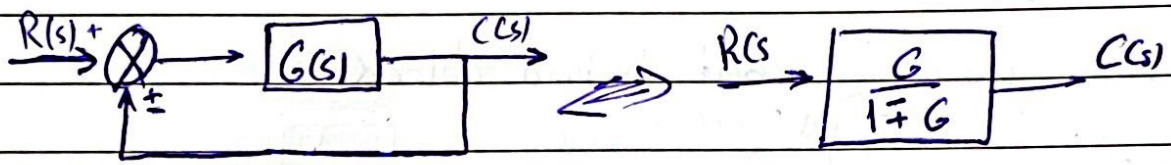
* Automatic control :-

* Ch2 :- Mathematical modeling :-

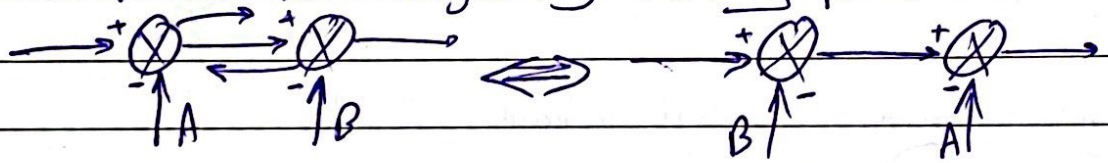
* Reduction techniques :-

7- Eliminating a feed back loop :-

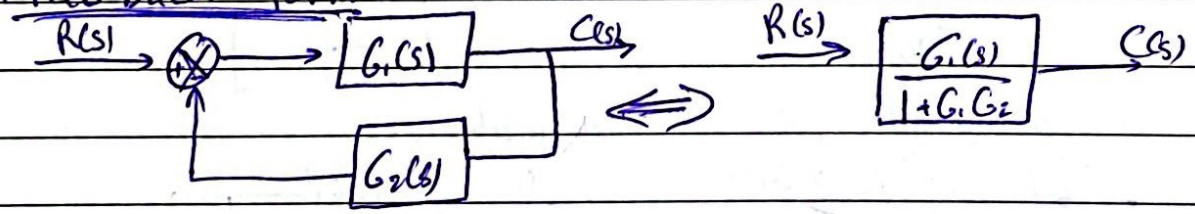




8- Swap with two neighboring summing points :-



* Feed back form :-



The system is said to have negative feed back if the sign at the summing junction is negative and positive feedback if the sign is positive.

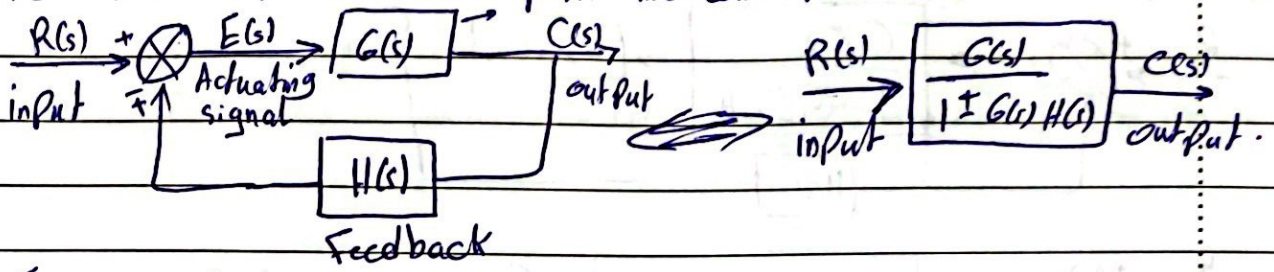
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Subject Date No.

* Automatic control :-

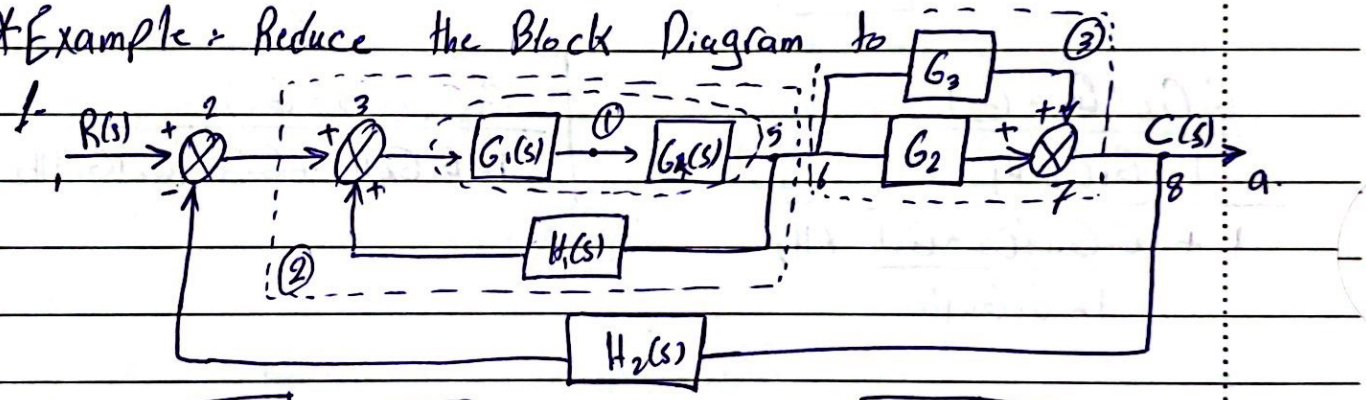
* Ch2 : Mathematical modeling :-

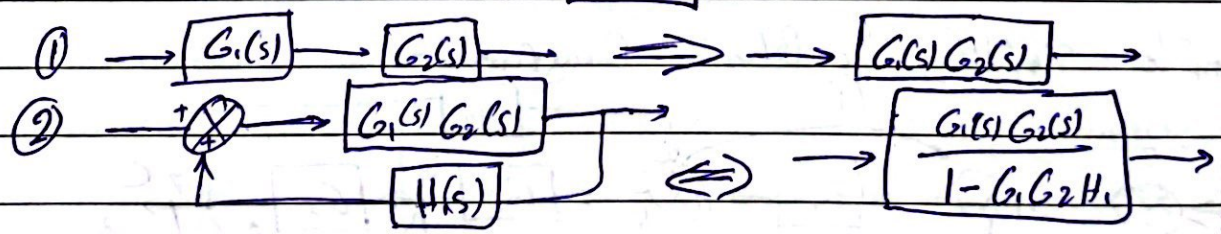
* Feed back form :- plant and controller

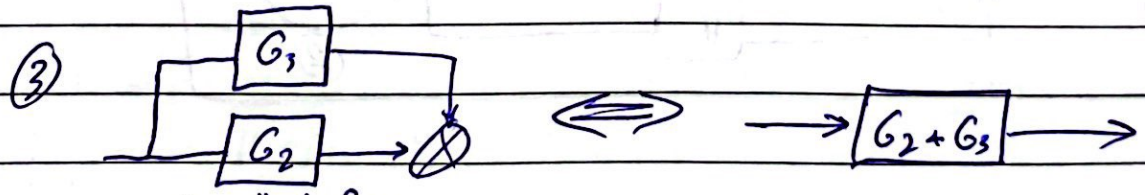


The equivalent or closed-loop transfer function is $G_c(s) = \frac{G(s)}{1 \mp G(s)\,H(s)}$

* Example : Reduce the Block Diagram to 1.







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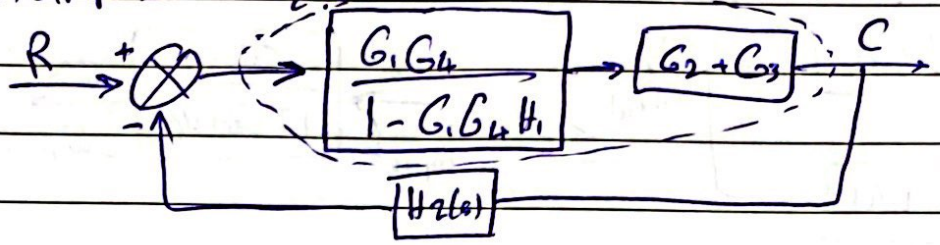
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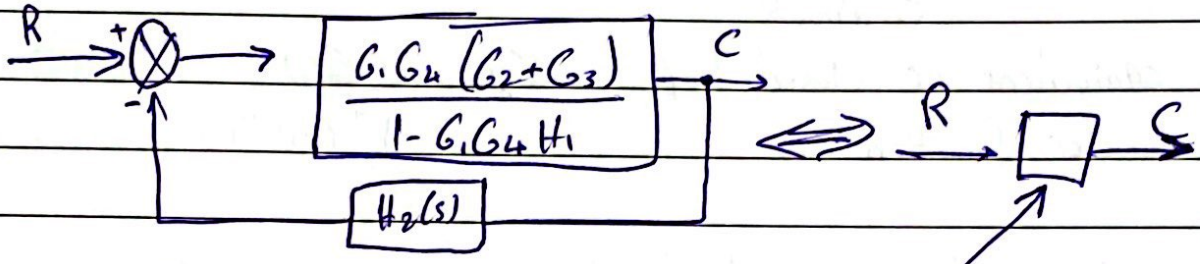
Subject Date No.

* Automatic control :-

* Ch2 :- Mathematical modeling :-

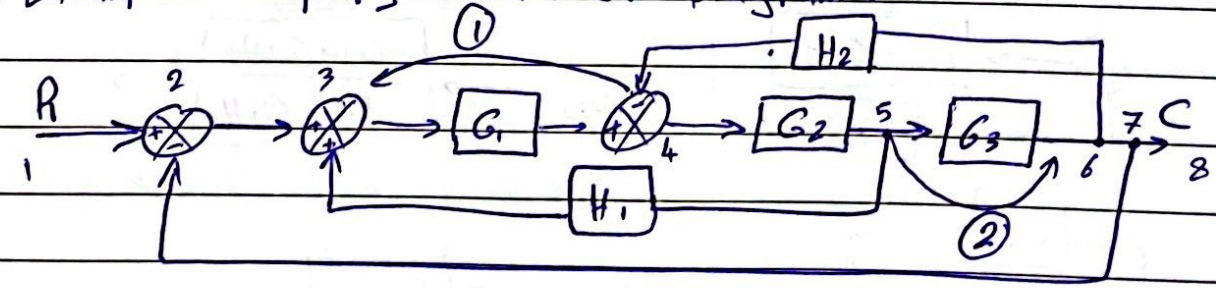
* Example :-





$$\frac{\frac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1}}{1+\frac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1}*H_2} = \frac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1+G_1G_4(G_2+G_3)H_2}$$

* Example :- Simplify the Block Diagram :-



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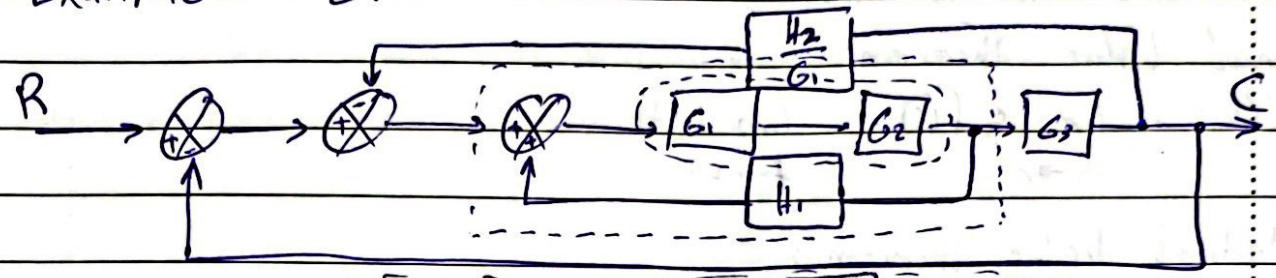
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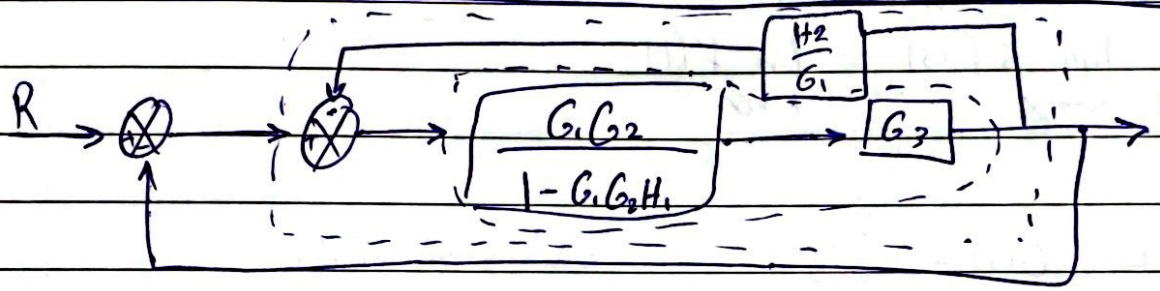
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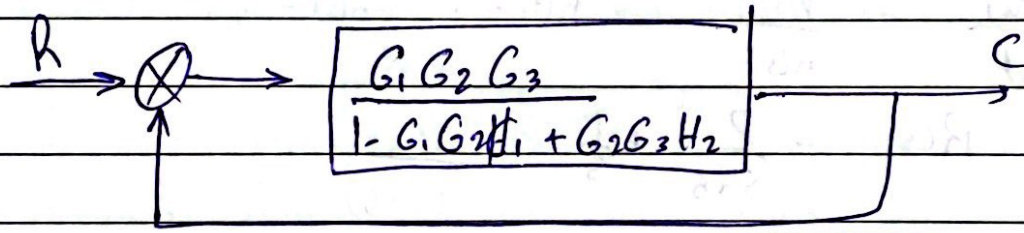
Automatic control :-

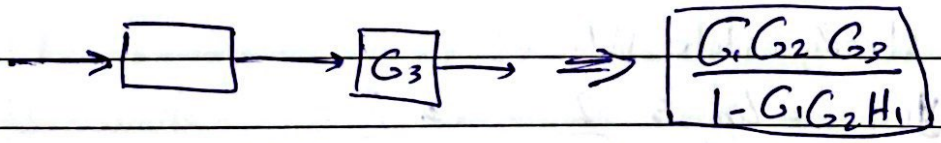
Ch 2 :- Mathematical modeling :-

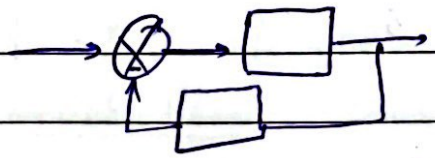
Example :-





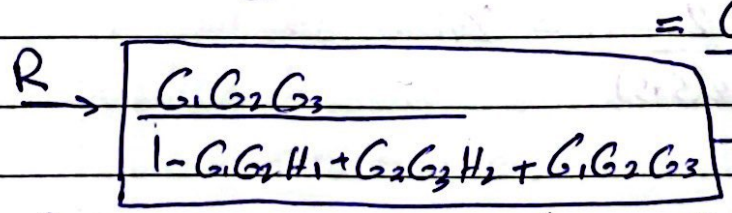






$$\Leftrightarrow \left(\frac{G_1G_2G_3}{1-G_1G_2H_1}\right) \div \left(1 - \frac{G_1G_2G_3}{1+G_1G_2H_1} \times \frac{H_2}{G_1}\right)$$

$$= \frac{G_1G_2G_3}{1-G_1G_2H_1+G_2G_3H_2}$$



$$\left(\frac{G_1G_2G_3}{1-G_1G_2H_1+G_2G_3H_2}\right) \div \left(1 - \frac{G_1G_2G_3}{1-G_1G_2H_1+G_2G_3H_2} \times 1\right)$$

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* Automatic control :-

* Ch2 :- Mathematical modeling :-

* Final and initial Valuse theorems :-

- Final Value Theorem :-

$$f(\infty) = \lim_{s \to 0} s f(s) = \lim_{t \to \infty} f(t)$$

- Initial Value Theorem :-

$$f(0) = \lim_{s \to \infty} s f(s) = \lim_{t \to 0} f(t)$$

* Example :- $G(s) = \frac{2}{s+5}$. unit step input.

$$G(s) = \frac{\text{out Put}}{\text{in Put}} = \frac{Y(s)}{R(s)} \qquad R(t) = 1 \rightarrow R(s) = \frac{1}{s}.$$

$$Y(s) = G(s)\, R(s) = \frac{2}{s+5} * \frac{1}{s} = \frac{2}{s(s+5)}.$$

Final Value : steady-state Value

$$y_{s.s} = \lim_{s \to 0} s Y(s) = \lim_{s \to 0} \cancel{s} \frac{2}{\cancel{s}(s+5)} = \frac{2}{5}.$$

Inital Value.

$$y_{In} = \lim_{s \to \infty} s Y(s) = \lim_{s \to \infty} \cancel{s} \frac{2}{\cancel{s}(s+5)} = \text{Zero}.$$

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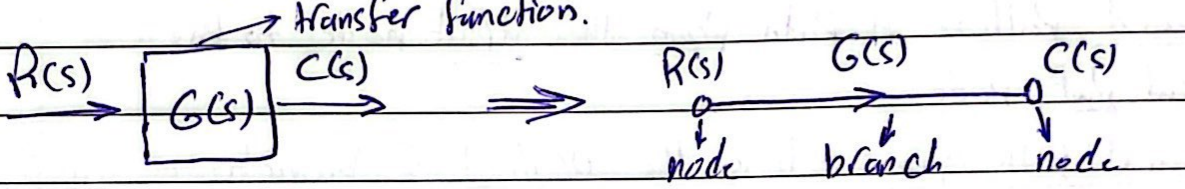
** Automatic control :-

* Ch 2 :- Mathematical modeling :-

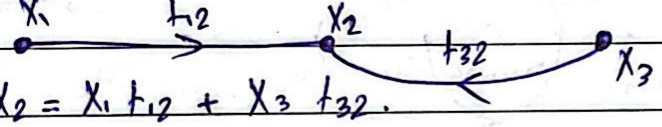
* Signal Flow Graph :-

Signal flow Graph :- is a diagram which represents aset of simultaneous equations.

* block diagram ⟹ signal flow graph



* Node :- It is a point representing a variable.



$X_2 = X_1 t_{12} + X_3 t_{32}$.

We have three node.

- Branch :- A line joining two nodes.



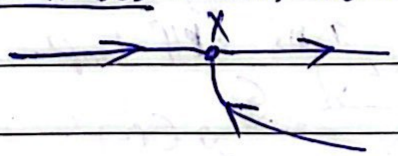
- Input node : Node which has only outgoing branches.

X_1 , X_3 Input node. . ﻫﻮ اللي طالع ← input

- Output node : only incoming branches.

X_2 out put node. . ﻫﻮ اللي داخل ← output

- Mixed nodes : Has both incoming and outgoing branches.



X is a mixed nodes.

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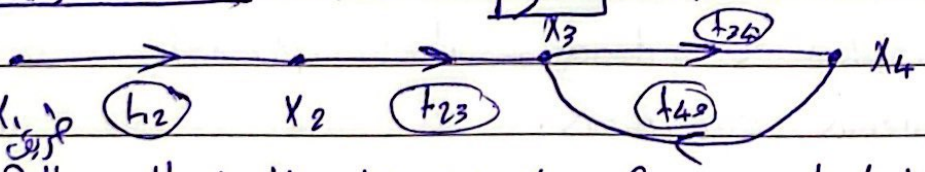
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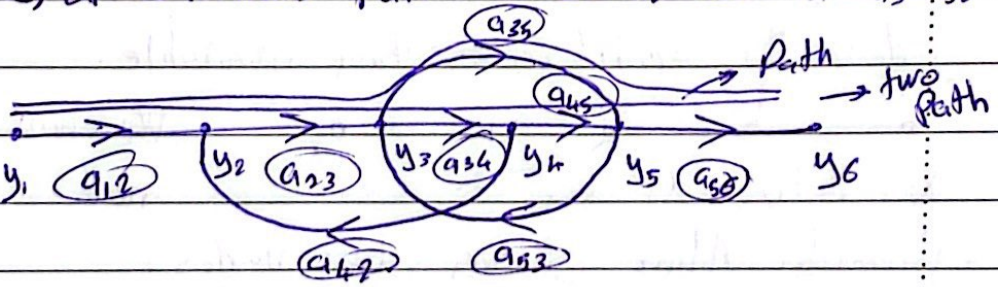
✱ Automatic control :-

* Ch 2 :- Mathematical modeling :-

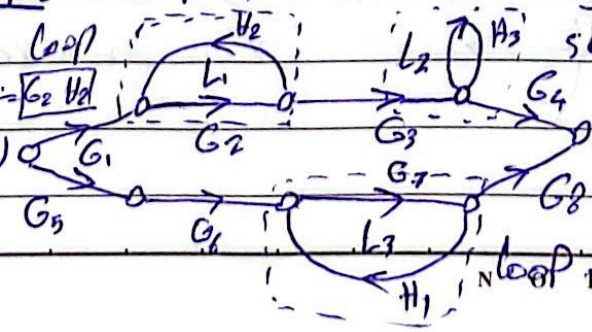
* Transmittance :- It is the gain between two nodes.



- Path :- It is the tranversal of connected branches in the direction of branch arrows. (input → output)
- forward path :- A path from the input node to the out put node.
- forward path gain :- It is the product of branch transmittances of a forward path. $G = a_{12} \cdot a_{23} \cdot a_{34} \cdot a_{45} \cdot a_{56}$



- loop :- Path that originates and terminates at the same node and along which no other node is traversed more than once.
- slef loop :- Path that originates and terminates at the same node.
- loop gain :- It is the Product of branch transmittance of a loop.
- Non Touching loops :- loops that don't have any common node or branch.



L_1, L_2 non T.L
L_1, L_3 non T.L
L_2, L_3 non T.L

NOTEBOOK

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※ Automatic control :-

* Ch 2 :- Mathematical modeling :-

* Mason's Rule :-

The Transfer function of a system represented by a signal flow graph is :-

$$G(s) = \frac{\text{out put}}{\text{in put}} = \frac{C(s)}{R(s)} = \frac{\sum P_K \Delta_K}{\Delta}$$

(P_K → Path, Δ_K → cofactor, Δ → determinant.)

$$\Rightarrow G(s) = \frac{P_1 \Delta_{1K} + P_2 \Delta_{2K} + P_3 \Delta_{3K} \cdots}{\Delta}$$

K :- number of forward path.

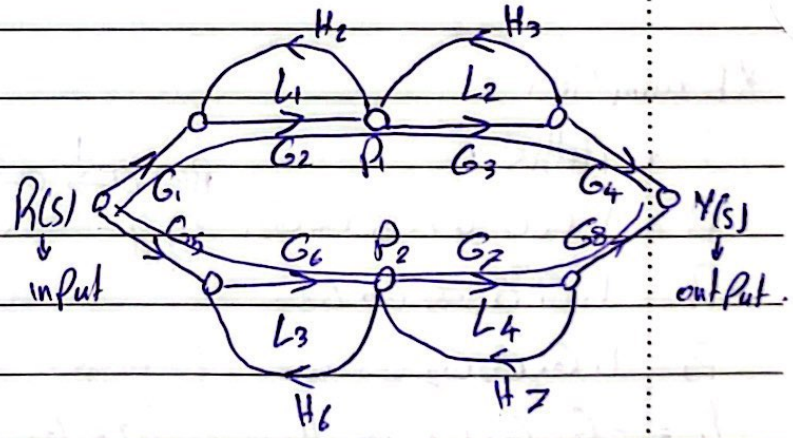
P_K :- The Kth forward path gain.

Δ_K = 1 - (loop gain which does not touch the forward path).

Δ = 1 - (Σ loop gain) + (Σ non touching loop gains taken Two at a time) - (Σ non toching loog gains taken three at a time).

* Example :-

$$G(s) = \frac{\text{out put}}{\text{in put}} = \frac{Y(s)}{R(s)}$$



$P_1 = G_1 \cdot G_2 \cdot G_3 \cdot G_4$

$P_2 = G_5 \cdot G_6 \cdot G_7 \cdot G_8$

$L_1 = G_2 H_2$. $L_2 = G_3 H_3$

$L_3 = G_6 H_6$. $L_4 = G_7 H_7$.

Two non touching loops → $L_1 L_3$. $L_1 L_4$.
$L_2 L_3$. $L_2 L_4$.

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Subject Date No.

* Automatic control :-

* Ch 2 :- Mathematical modeling :-

* Example :-

Cofactors :-

$\Delta_1 = 1 - (L_3 + L_4)$.

$\Delta_2 = 1 - (L_1 + L_2)$

Determinant (Δ)

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4)$.

$T.F = G(s) = \frac{\sum_k P_k \Delta_k}{\Delta} = \frac{(P_1\Delta_1 + P_2\Delta_2)}{\Delta}$

$= \frac{G_1 \cdot G_2 \cdot G_3 \cdot G_4 \cdot (1 - (L_3 + L_4)) + G_5 \cdot G_6 \cdot G_7 G_8 \cdot (1 - (L_1 + L_2))}{1 - (L_1 + L_2 + L_3 + L_4) + (L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4)}$

$= \frac{G_1 \cdot G_2 \cdot G_3 G_4 \cdot (1 - L_3 - L_4) + G_5 G_6 G_7 G_8 \cdot (1 - L_1 - L_2)}{1 - L_1 - L_2 - L_3 - L_4 + L_1L_3 + L_1L_4 + L_2L_3 + L_2L_4}$.

* Example :-

3 → Paths.

$P_1 = 1 \cdot G_1 G_2 G_3 G_4 G_5 G_6$.

$P_2 = 1 \cdot G_1 G_2 G_7 G_6$.

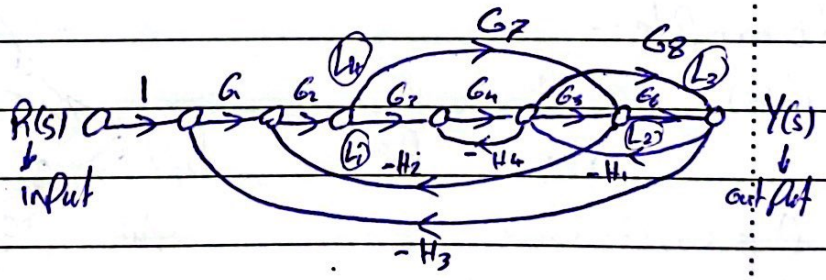
$P_3 = 1 \cdot G_1 G_2 G_3 G_4 G_8$.

$L_1 = -G_2 G_3 G_4 G_5 H_2$, $L_2 = -G_5 G_6 H_1$

$L_3 = -G_8 H_1$, $L_4 = -G_2 G_7 H_2$. $L_5 = -G_4 H_4$

$L_6 = -G_1 G_2 G_3 G_4 G_5 G_8 H_3$, $L_7 = -G_1 G_2 G_7 G_6 H_3$.

$L_8 = -G_1 G_2 G_3 G_4 G_8 H_3$.



NOTEBOOK

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Subject Date No.

* Automatic control :-

* Ch 2 :- Mathematical modeling :-

* Example : مثال.

Two non Touching loop → L_5L_4 , L_5L_7 , L_3L_4.

cofactor

$\Delta_1 = 1$.

$\Delta_2 = 1 - L_5$ $\Delta_3 = 1$

Determinant (Δ)

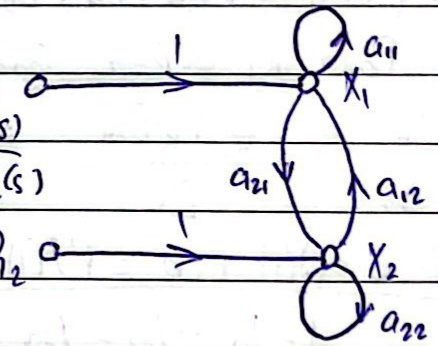
$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5 + L_6 + L_7 + L_8) + (L_5L_7 + L_5L_4 + L_3L_4)$.

$$T.F = \frac{Y(s)}{R(s)} = \frac{\sum_k P_k \Delta_k}{\Delta} = \frac{P_1\Delta_{1k} + P_2\Delta_{2k} + P_3\Delta_{3k}}{\Delta}.$$

$$= \frac{(G_1G_2G_3G_4G_5G_6)(1) + (G_1G_2G_7G_6)(1-L_5) + (G_1G_2G_3G_4G_8)(1)}{1 - L_1 - L_2 - L_3 - L_4 - L_5 - L_6 - L_7 - L_8 + L_5L_7 + L_5L_4 + L_3L_4}.$$

* Example :- 4 Transfer Function

① $\frac{X_1(s)}{R_1(s)}$, ② $\frac{X_2(s)}{R_1(s)}$, ③ $\frac{X_1(s)}{R_2(s)}$, ④ $\frac{X_2(s)}{R_2(s)}$



input (R_1) → output (X_1) → one Path : R_2

$P = 1$, $L_1 = a_{11}$ $\Delta_K = 1 - a_{22}$.

$\Delta = 1 - (a_{11} + a_{12}a_{21} + a_{22}) + (a_{11}a_{22})$.

$$G(s) = \frac{X_1}{R_1} = \frac{\sum_k P_k \Delta_k}{\Delta} = \frac{(1)(1 - a_{22})}{1 - a_{11} - a_{12}a_{21} - a_{22} + a_{11}a_{22}}.$$

NOTEBOOK

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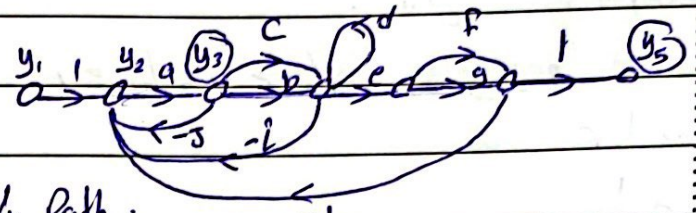
Subject Date No.

* Automatic control :-

* Ch 2 :- Mathematical modeling :-

* Example :- Find :

$T.F = \frac{y_5}{y_3}$



input (y_3) → out put (y_5) → 4 Path.

$P_1 = b \cdot e \cdot g \cdot 1$, $P_2 = b \cdot e \cdot f \cdot 1$

$P_3 = C \cdot e \cdot g \cdot 1$, $P_4 = C \cdot e \cdot f \cdot 1$.

$L_1 = -a \cdot j$, $L_2 = -ab\hat{i}$, $L_3 = -a \cdot c\hat{i}$.

$L_4 = -a \cdot b \cdot e \cdot g \cdot h$, $L_5 = -a \cdot b \cdot e \cdot g \cdot h$,

cofactor.

$\Delta_1 = 1$ $\Delta_2 = 1$ $\Delta_3 = 1$ $\Delta_4 = 1$

Determinant (Δ)

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5) + (\text{2nd})$.

Two non Toughing loop →

$$T.F = \frac{y_5}{y_3} = \frac{\sum P_K \Delta_K}{\Delta} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3 + P_4\Delta_4}{\Delta}.$$

$$= \frac{(b \cdot e \cdot g)(1) + (b \cdot e \cdot f)(1) + (C \cdot e \cdot g)(1) + (C \cdot e \cdot f)(1)}{1 - L_1 - L_2 - L_3 - L_4 - L_5}.$$

#

* Sugested Problem :-

P 2.13 , P 2.26 , P 2.33 , P 2.2 , P 2.3 . P 2.12

P 2.35 , P 2.37 . P 2.69 , P 2.50 , AP 21 , DP 2.1

DP 2.3 , E 2.9 . E 2.13 . E 2.15 . E 2.20 . E 2.22 , E 2.27.

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Subject Date No.

✳ Automatic control :-

* Ch 4 :- Feed back control system characteristics :-

* Open loop Vs. Closed loop :-

- An open-loop system operates without feed back and directly generates the output in response to an input signal.
- A closed-loop system uses a measurement of the output signal and a comparison with the desired output to generate an error signal that is used by the controller to adjust the actuator. (C.L exp than O.L)

* Open loop Vs. Closed loop :-

1. steady state error (Accuracy).
2. Sensitivity of the system. (حساسية النظام) . $S_{o.p} > S_{c.l}$
3. Disturbance rejection. (رفض الازعاج) . (رفض التشويش) ← $S_{o.p} = 1$
4. Noise rejection of sensor.
5. Transient response. (استجابة عابرة) . (سرعة الاستجابة) .

* Closed loop system :-

- Advantages of the closed loop system. متغيران .
1. Decreased sensitivity of the system to variations in the parameters of the Process. (تقليل)
2. Improved rejection of disturbances
3. Improved measurment noise attenuation.

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Subject Date No.

* Automatic control :-

* Ch 4 :- Feed back control system characteristics :-

* Closed loop system

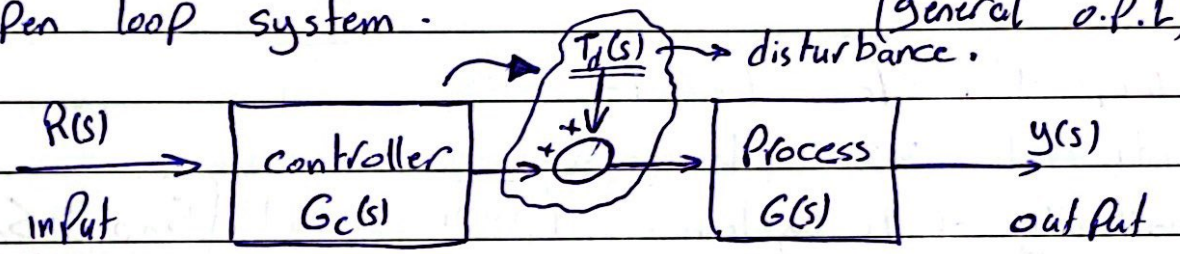
4- Improved reduction of the steady-state error of the system.

5- Easy control and adjustment of the transient response of the system.

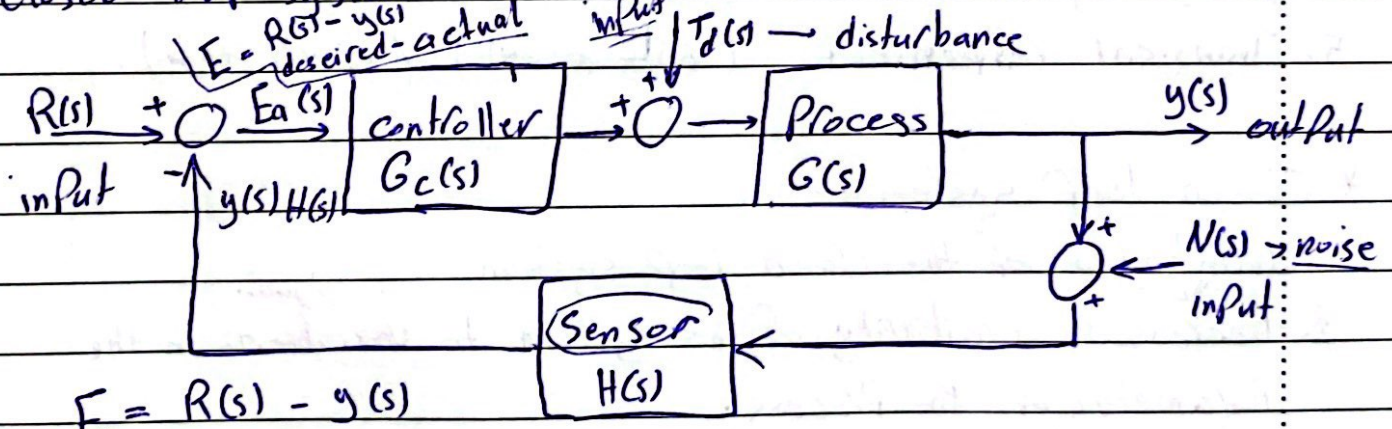
steady state error value.

$y_{s.s} = \lim_{s \to 0} s\, y(s)$ # $E_{ss} = \lim_{s \to 0} s\, E(s)$.

* Open loop system. (general o.p.l)



* Closed loop system



$E = R(s) - y(s)$

= desired - actual.

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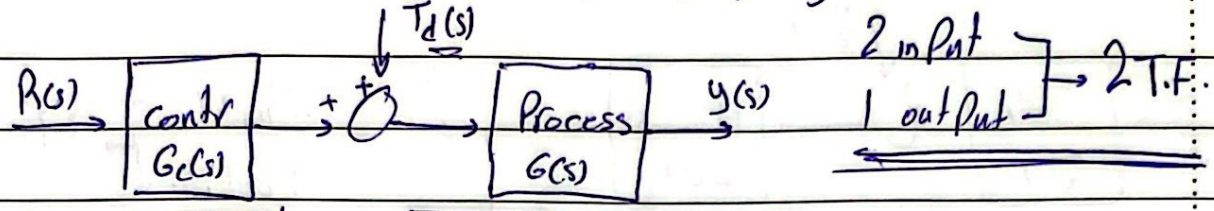
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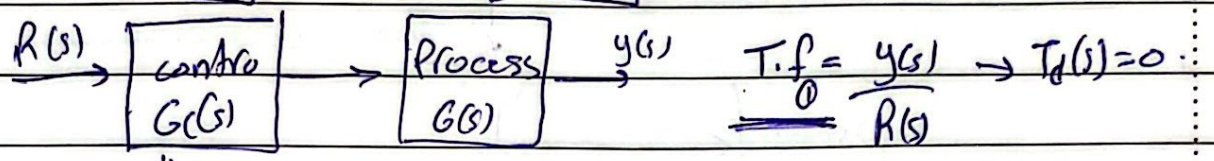
Subject Date No.

* Automatic control :-

* Ch 4 :- Feed back control system characteristics :-

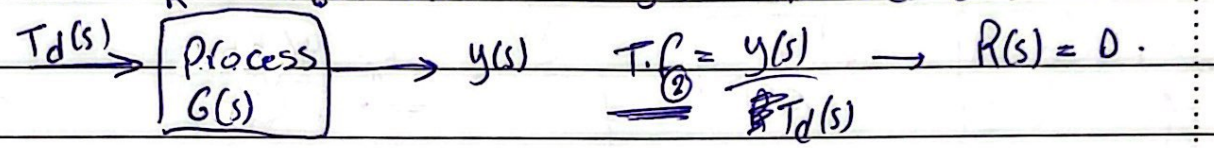
* Transfer function of the open loop system.





$T.f = \frac{y(s)}{R(s)} \rightarrow T_d(s) = 0$ ①

$T.f = \frac{y}{A} = G_c(s)\, G(s) \rightarrow y(s) = R(s)\, G_c(s)\, G(s)$.



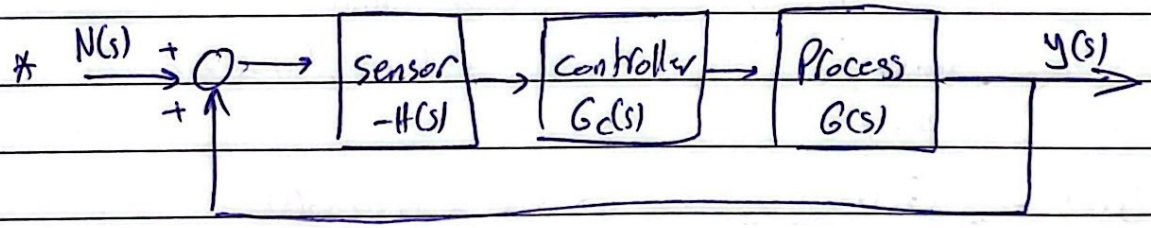
$T.f = \frac{y(s)}{T_d(s)} \rightarrow R(s) = 0$ ②

$T.f = \frac{y(s)}{T.d(s)} = G(s) \rightarrow y(s) = T_d(s)\, G(s)$.

total resfonce $y(s)$

$y(s) = G_c(s)\, G(s)\, R(s) + G(s)\, T_d(s)$.

$y(t) = \mathcal{L}^{-1}(G_c(s)\, G(s)\, R(s) + G(s)\, T_d(s))$.



$T.f = \frac{y(s)}{N(s)} = \frac{-H(s)\, G_c(s)\, G(s)}{1 + H(s)\, G_c(s)\, G(s)}$

$T.f = \frac{y(s)}{N(s)} \rightarrow T_d(s),\ R(s) = 0$ ②

$y(s) = \frac{-H(s)\, G_c(s)\, G(s)\, N(s)}{1 + H(s)\, G_c(s)\, G(s)}$.

NOTEBOOK

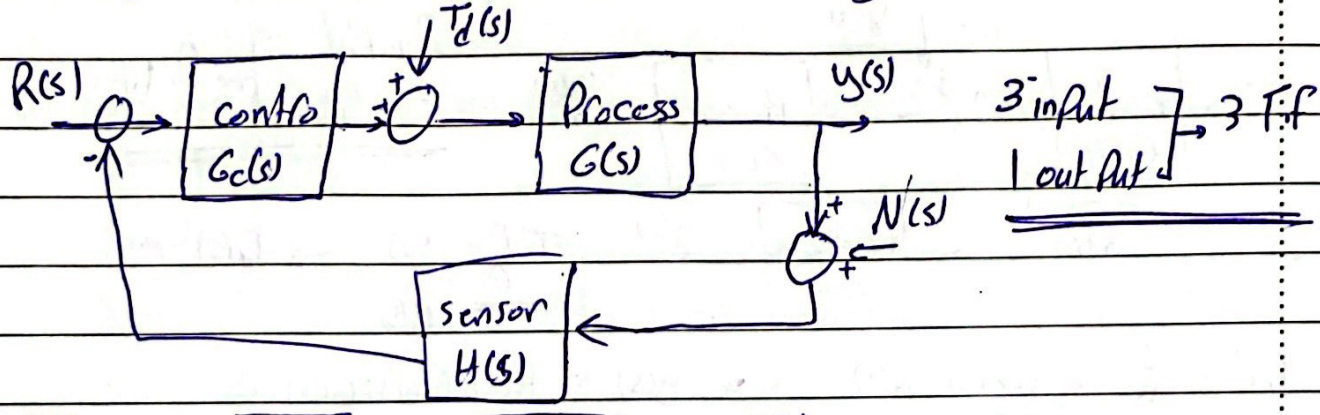
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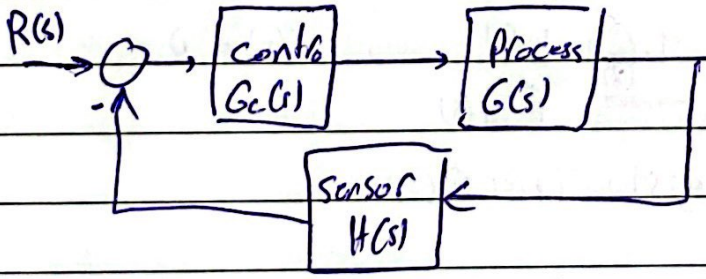
* Automatic control

* Ch4 : Feedback control system characteristics :-

* Transfer function of the closed loop system :-



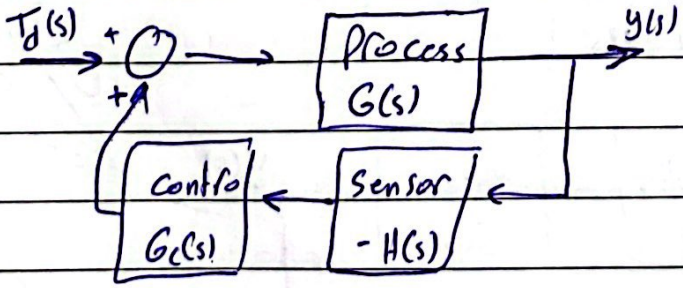
3 input
1 output
→ 3 T.f.



T.f ① = $\frac{y(s)}{R(s)}$

→ N(s), Td(s) = Zero

$$T.f = \frac{y(s)}{R(s)} = \frac{G(s)\,G_c(s)}{1 + G(s)\,G_c(s)\,H(s)} \Rightarrow y(s) = \frac{R(s)\,G(s)\,G_c(s)}{1 + G(s)\,G_c(s)\,H(s)}.$$



T.f ② = $\frac{y(s)}{Td(s)}$

→ R(s) . N(s) = Zero.

$$T.f = \frac{y(s)}{T_d(s)} = \frac{G(s)}{1 + H(s)\,G_c(s)\,G(s)} \Rightarrow y(s) = \frac{T_d(s)\,G(s)}{1 + H(s)\,G_c(s)\,G(s)}.$$

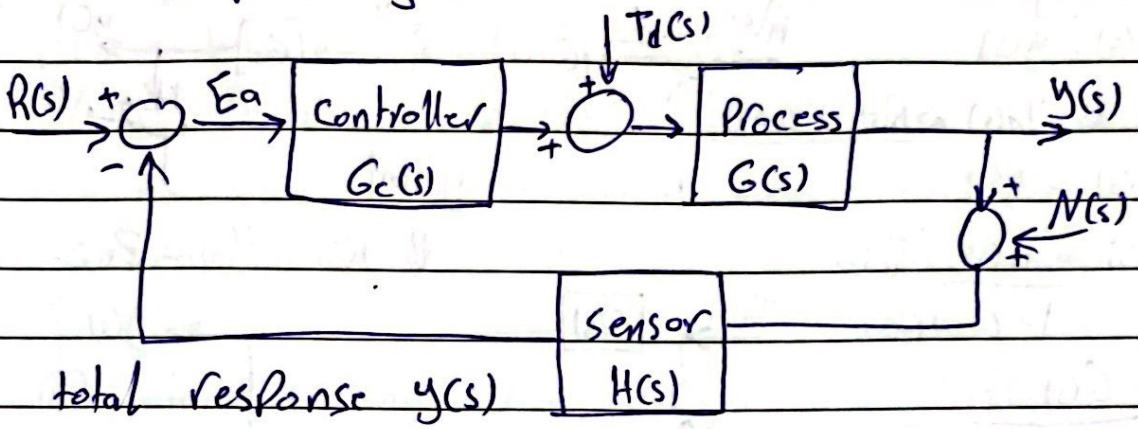
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* Automatic control :-

* Ch 4 :- Feed back control system charakterstics :-

* Total Response y(s) :-



total response y(s)

$$y(s) = \frac{G_c(s)\,G(s)\,R(s)}{1 + G_c(s)\,G(s)\,H(s)} + \frac{T_d(s)\,G(s)}{1 + H(s)\,G_c(s)\,G(s)} - \frac{H(s)\,G_c(s)\,G(s)\,N(s)}{1 + H(s)\,G_c(s)\,G(s)}$$

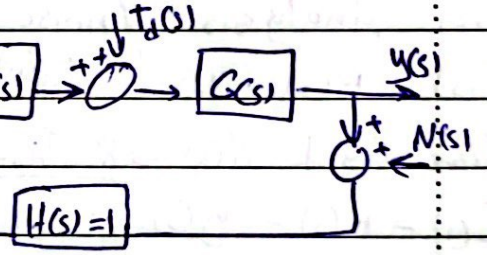
$1 + G_c(s)\,G(s)\,H(s)$ → characteristics equation.

* Error Analysis :- (Accuracy R(s))

$E(s) = R(s) - y(s)$.

Error = desired - Actual

Error due to R(s) $\Rightarrow N(s) \cdot T_d(s) = 0$



$E(s) = R(s) - y(s)$.

$$E(s) = R(s) - \frac{G_c(s)\,G(s)}{1 + G_c(s)\,G(s)}\,R(s)$$



$$E(s) = R(s)\left[\frac{1}{1 + G_c(s)\,G(s)}\right], \quad E_{s.s} = \lim_{s \to 0} s\,E(s).$$

$$\Rightarrow E_{s.s} = \lim_{s \to 0} s\,R(s)\left[\frac{1}{1 + G_c(s)\,G(s)}\right]$$

↓

~~ette~~ steady state error.

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Subject Date No.

* Automatic control :-

* Ch 4 :- feedback control system characteristics :-

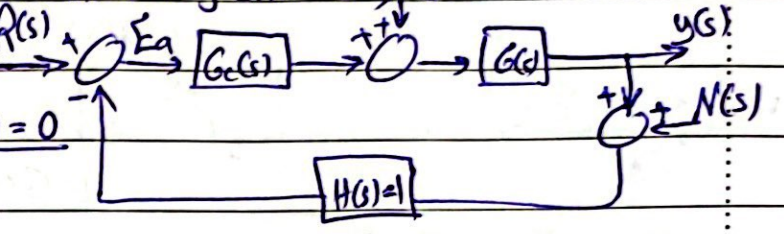
* Error Analysis :- (Disturbance rejection $T_d(s)$), $T_d(s)$

$E(s) = R(s) - y(s)$.

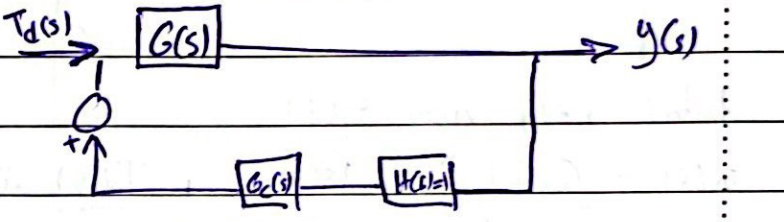
Error due to $T_d(s) \Rightarrow R(s), N(s) = 0$

$E(s) = \cancel{R(s)}^{0} - y(s)$

$= \cancel{R(s)} - \dfrac{G(s)\, T_d(s)}{1 + G_c(s) G(s)}$.



$\Downarrow R(s), N(s) = Zero$



$$E(s) = \frac{-G(s)\, T_d(s)}{1 + G_c(s) G(s)}$$

$$E_{s.s} = \lim_{s \to 0} s\, E(s) = \lim_{s \to 0} s * \frac{-G(s)\, T_d(s)}{1 + G_c(s) G(s)}$$ → DC gain.

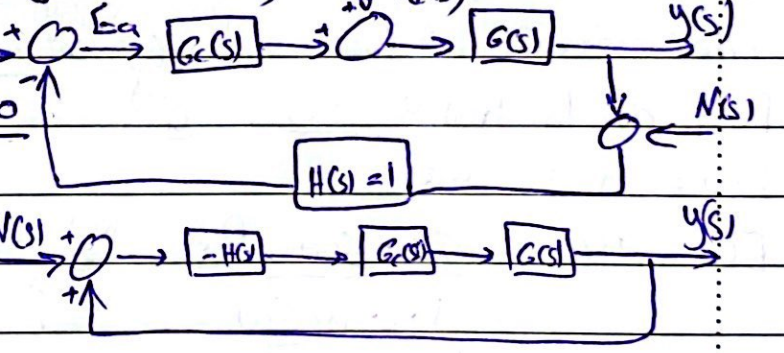
* Error Analysis :- (noise rejection $N(s)$)

$E(s) = R(s) - y(s)$

Error due to $N(s) \Rightarrow R(s), T_d(s) = 0$

$E(s) = R(s) - y(s)$

$E(s) = 0 - \dfrac{-G_c(s) G(s) N(s)}{1 + G_c(s) G(s)}$



$$E_{s.s} = \lim_{s \to 0} s\, E(s) = \lim_{s \to 0} s * \frac{G_c(s) G(s) N(s)}{1 + G_c(s) G(s)}$$

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Subject Date No.

✱ Automatic control :-

✱ Ch 4 : Feedback control system characteristics :-

✱ Total Error (E(s)) :-

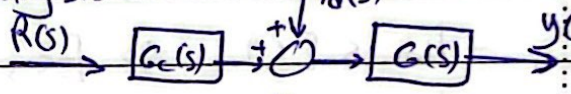
$$E(s) = E(s) + E(s) + E(s)$$

Accuracy disturbance noise

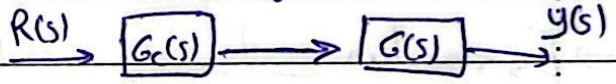
$$E(s) = \frac{1}{1+G_cG_1} R(s) - \frac{G(s)}{1+G_cG_1} T_d(s) + \frac{G_c(s)\,G(s)\,N(s)}{1+G_cG_1}$$

✱ Open loop system Error Analysis :-

$E(s) = R(s) - y(s)$.



Error due to $R(s) \Rightarrow T_d(s) = 0$

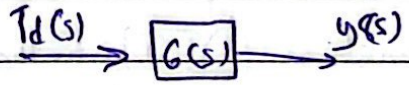


$E(s) = R(s) - y(s)$

$E(s) = R(s) - G_cG\,R(s)$

$E(s) = R(s)[1 - G_cG]$

$$E_{s.s} = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} s\,R(s)[1 - G_cG]$$



$\Rightarrow E(s) = R(s) - y(s)$

Error due to $T_d(s) \Rightarrow R(s) = 0$

$E(s) = R(s) - y(s)$

$= 0 - G(s)\,T_d(s)$

$$E_{s.s} = \lim_{s\to 0} s\,E(s) = \lim_{s\to 0} s \times -G(s)\,T_d(s).$$

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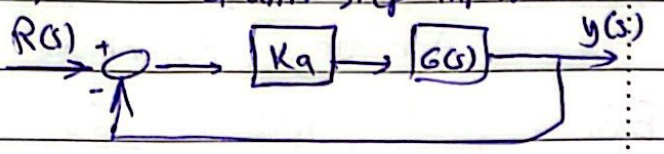
Subject Date No.

✱ Automatic control :-

* Ch 4 : Feed back control system characteristics :-

* Example :- If $G(s) = \frac{10}{s(0.001s+1)}$ find $E_{s.s}$ if $R(s)$ is a unit step input.

$$T.F = \frac{y(s)}{R(s)} = \frac{K_a G(s)}{1 + K_a G(s)}$$



$$E(s) = R(s) - y(s)$$
$$= R(s) - \frac{K_a G(s)}{1 + K_a G(s)} R(s).$$

$$E_{s.s} = \lim_{s \to 0} s\,E(s) \Rightarrow E_{s.s} = \lim_{s \to 0} s \cdot \frac{1}{s}\left[1 - \frac{K_a G(s)}{1 + K_a G(s)}\right]$$

$$\Rightarrow E_{s.s} = \lim_{s \to 0} \frac{1}{1 + K_a G(s)} = \lim_{s \to 0} \frac{s(0.001s+1)}{s(0.001s+1) + K_a 10} = 0$$

* System Sensitivity :-

System Sensitivity is the ratio of the change in the system transfer function ($\Delta T(s)$) to the change of a Process transfer function (or Parameter) for a small incremental change (ΔG)

$$S = \frac{\Delta T(s) / T(s)}{\Delta G(s) / G(s)} \Rightarrow \boxed{S_G^T = \frac{\partial T}{\partial G} \frac{G}{T}}$$

$$\boxed{0 < S \leq 1}$$

$$\boxed{\begin{aligned} &E_{ss\ o.l} > E_{ss\ c.l} \\ &E_{ss\ o.l}\big|_{T_d} > E_{ss\ c.l}\big|_{T_d} \\ &S_{o.l} > S_{c.l} \rightarrow S_{o.l} = 1 \end{aligned}}$$

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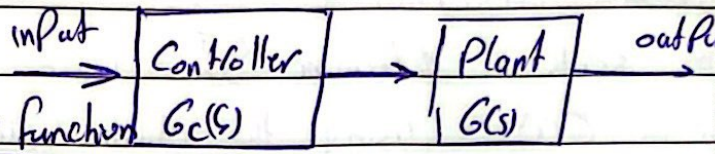
Subject Date No.

* Automatic control :-

* Ch 4 :- Feedback control system characteristics :-

* Open loop Sensitivity

$$S_G^T = \frac{\partial T}{\partial G} \cdot \frac{G}{T}$$



$T \rightarrow$ transfer function

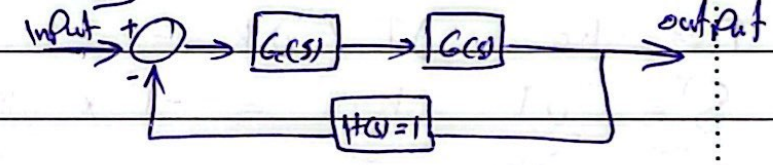
$$\Rightarrow T(s) = G_c(s)\, G(s)$$ open loop system.

$$\frac{\partial T}{\partial G} = G_c(s)$$

$$S_G^T = \frac{G_c(s) \cdot G(s)}{G_c(s)\, G(s)} = \boxed{1}$$

* Closed loop system sensitivity

$$S_G^T = \frac{\partial T}{\partial G} \cdot \frac{G}{T}$$



$$T(s) = \frac{G_c(s)\, G(s)}{1 + G_c(s)\, G(s)}$$

$$\frac{\partial T}{\partial G} = \frac{(1 + G_c(s) G(s))\, G_c(s) - (G_c(s) G(s))\, G_c(s)}{(1 + G_c(s)\, G(s))^2} = \boxed{\frac{G_c(s)}{(1 + G_c(s)\, G(s))^2}}$$

$$S_G^T = \frac{\partial T}{\partial G} \cdot \frac{G}{T} = \frac{G_c(s)}{(1 + G_c(s) G(s))^2} \cdot \frac{G(s)(1 + G_c(s)\, G(s))}{G_c(s) G(s)}$$

$$\Rightarrow S_G^T = \frac{1}{1 + G_c(s)\, G(s)}.$$

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Subject Date No.

* Automatic control :-

* Ch 4 :- Feedback control system characteristics :-

* System Sensitivity :-

If we seek to determine S_α^T where α is a parameter with in $G(s)$. using the chain rule gives

$$S_\alpha^T = S_G^T * S_\alpha^G$$

* Example :- Feed back amplifier :-

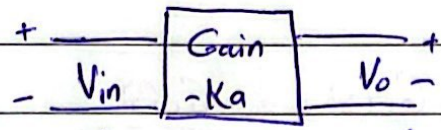
An amplifier used in many applications has a gain ($-K_a$)

gain = Transfer function.

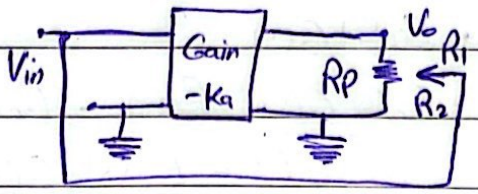
$$T.f = \frac{\text{output}}{\text{input}} \Rightarrow \frac{V_{out}}{V_{in}} = -K_a \rightarrow V_o = -K_a V_{in}$$

$T(s) = -K_a \quad \partial T / \partial K_a = -1.$

$$S_{K_a}^T = \frac{\partial T}{\partial K_a} \cdot \frac{K_a}{T} = -1 * \frac{K_a}{-K_a} = \boxed{1}$$



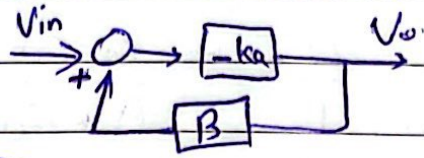
$\Rightarrow$





$T = -K_a.$

$$\frac{\partial T}{\partial K_a} = \frac{-1}{(1+K_a\beta)^2}$$



$T = -K_a$

$$S_{K_a}^T = \frac{\partial T}{\partial K_a} \cdot \frac{K_a}{T} = \frac{+K_a}{(1+K_a\beta)^2} \cdot \frac{(1+K_a\beta)}{+K_a} = \frac{1}{1+K_a\beta} \#$$

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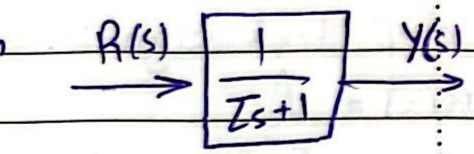
Subject Date No.

✱ Automatic control :-

* Ch 4 :- Feed back control system characteristics :-

* Control of the Transient Response :-

If $T(s) = \frac{1}{Ts+1}$ ⇒ First order system ($\rightarrow s^1 \rightarrow$ order $= 1$)



T = time constant. → 0.632 y_{ss}.

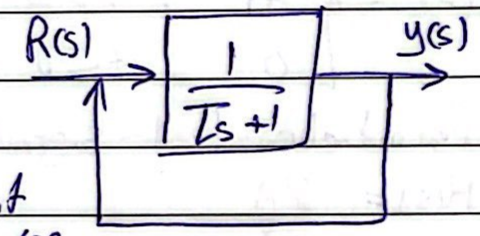
* in open loop system.

$T(s) = \frac{1}{Ts+1}$ → $T = 0.632\ y_{s.s}$.

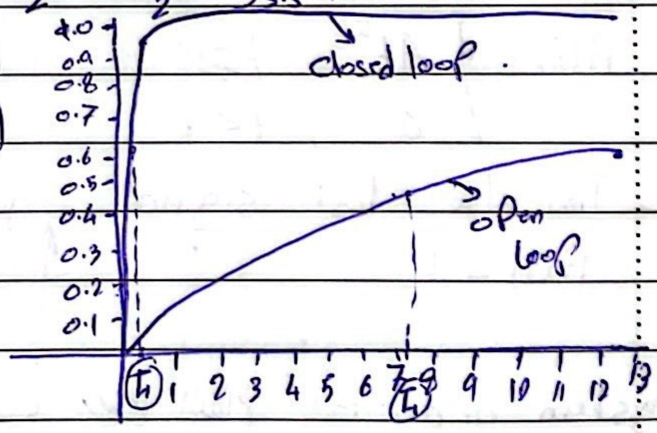
* in close loop system.

$T(s) = \frac{\frac{1}{Ts+1}}{1+\frac{1}{Ts+1}} = \frac{1}{Ts+2}$

$T(s) = \frac{1/2}{\frac{Ts}{2}+1}$ → Time constant $= \frac{T}{2} = \frac{0.632}{2}\ y_{s.s}$.



closed loop better that open loop → (C.L faster O.P)



* Suggested Problems :-

E 4.3 . E 4.4 . E 4.8 . E 4.10 . E 4.11

P 4.2 . P 4.3 . P 4.12 . P 4.17

AP 4.1 . AP 4.6 . Ap 4.7.

DP 4.3.

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* Automatic control :-

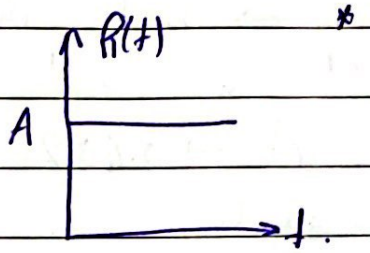
* Ch 5 :- The performance of FB control system :-

* Test signal :-

1- step test signal.

$R(s) = \frac{A}{s}$.

$$R(t) = \begin{cases} A & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$



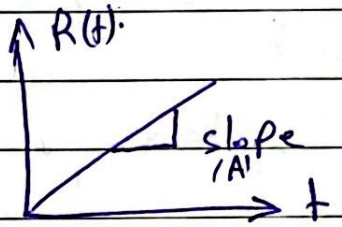
* unit step

$R(s) = \frac{1}{s}$.

2- Ramp test signal :-

$R(s) = \frac{A}{s^2}$

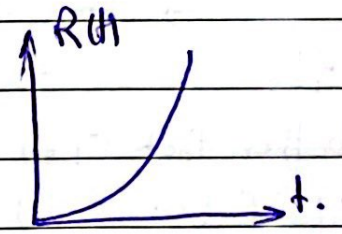
$$R(t) = \begin{cases} At & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$



3- Parabolic test signal

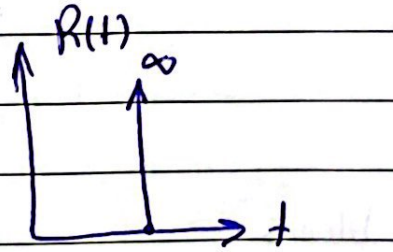
$R(s) = \frac{2A}{s^3}$

$$R(t) = \begin{cases} At^2 & , t \geq 0 \\ 0 & , t < 0 \end{cases}$$



4- Impulse test signal

$R(s) = 1$



* system order :- اعلى أس

$\frac{y(s)}{R(s)} = \frac{A}{as^2 + bs + c}$ → characteristic equ

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Subject Date No.

✱ Automatic control :-

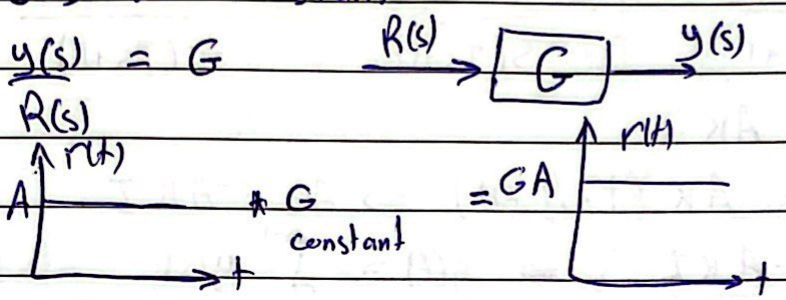
* Ch5 :- The Performance of FB control system :-

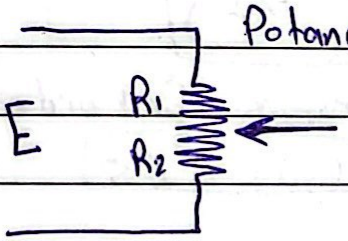
* System order :-

1. Zero order system ($s^0 = 1$) :-

$G s^0 \Rightarrow$ constant.

$\frac{y(s)}{R(s)} = G$





$V_{out} = \frac{R_2}{R_1 + R_2} E$ ⇝ mathematical equation.

$\frac{V_{out}(s)}{E(s)} = \frac{R_2}{R_1 + R_2}$ ⇝ zero order system

2. First order system :-

general transfer function :-

$\frac{y(s)}{R(s)} = \frac{K}{Ts+1}$ ⇝ general T.F.

T : time constant ⇒ 0.632 y_{ss}

$y(s) = \frac{K}{Ts+1} R(s) \Rightarrow y(t) = \mathcal{L}^{-1} y(s)$.

if $R(s)$ is step input $R(s) = \frac{A}{s}$. ⇝ standard T.S.

$y(s) = \frac{K}{Ts+1} * \frac{A}{s} = \frac{AK}{s(Ts+1)} = \frac{A_1}{s} + \frac{A_2}{Ts+1}$

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Subject Date No.

* Automatic control :-

* Ch5 :- The performance of F.B control system :-

* System order :-

2- First order system :-

$$y(s) = \frac{A_1}{s} + \frac{A_2}{Ts+1} \Rightarrow \frac{AK}{s(Ts+1)} = \frac{A_1(Ts+1) + A_2 s}{s(Ts+1)}.$$

$\Rightarrow s=0 \rightarrow A_1 = AK.$

$\Rightarrow s=1 \rightarrow A_2 = AK(K-(T+1)) \Rightarrow A_2 = -AKT$

$$y(s) = \frac{AK}{s} + \frac{-AKT}{Ts+1}. \Rightarrow y(t) = \int^{-1} y(s)$$

$$y(t) = AK - AK e^{-t/T} = \boxed{AK(1-e^{-t/T})}.$$

general response first order.

input $\boxed{R(s) = A/s}$

* First order system :-

general T.F 1st order $\frac{y(s)}{R(s)} = \frac{K}{Ts+1}$

general response

if response step response $R(s) = A/s$.

$$y(t) = AK(1-e^{-t/T}).$$

$$y_{s.s} = \lim_{t \to \infty} AK(1-e^{-t/T}) = AK \rightarrow \boxed{y_{s.s} = AK}$$

$$y(T) = AK(1-e^{-1}) = 0.632\,AK = \boxed{0.632\,y_{s.s}}.$$

$$y(4T) = AK(1-e^{-4}) = 0.98\,AK = 0.98\,y_{s.s}$$

T_s : settling time زمن الاستقرار $\boxed{T_s = 4T}$ ~ 2% error.

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Subject Date No.

* Automatic control :-

* Ch 5 :- The performance of F.B control system :-

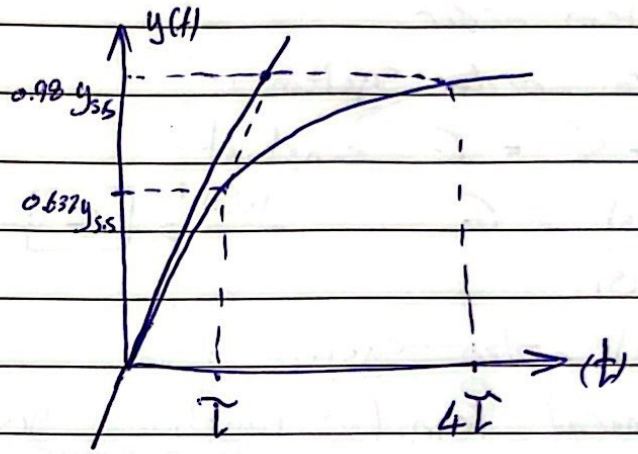
* First order system :-

$E_{ss} = R_{ss} - 0.98\, y_{ss} \Rightarrow 2\%$

$T_s = 4T \rightarrow$ **2% error.**

$T_s = 3T \rightarrow 5\%$ error

$T_s = 5T \rightarrow 1\%$ error



$\text{slope}|_{\text{Zero}} = \frac{y_{ss} - 0}{T - 0} = \frac{y_{ss}}{T}$ slop $\neq 0$.

If $R(s) = 1 \rightarrow y_{ss} = 1$

$\text{slope}|_{\text{Zero}} = \frac{1}{T} \neq 0$.

* Example :-

$\frac{y(s)}{R(s)} = \frac{2}{s+7}$ and $R(s)$ unit step $R(s) = \frac{1}{s}$.

Find T ??!!.

general form $\frac{2/7}{\frac{s}{7} + \frac{7}{7}} = \frac{2/7}{\frac{T}{7}s + 1} \rightarrow$ **$T = \frac{1}{7}$**

Find y(t) ???.

$y(t) = AK(1 - e^{-t/T}) \Rightarrow A = 1 \quad K = 2/7 \quad T = \frac{1}{7}$.

$\Rightarrow y(t) = \frac{2}{7}(1 - e^{-7t})$

$y(T) = 0.632 * \frac{2}{7}$.

$y_{s.s} = \lim_{s \to 0} s \cdot \frac{2}{s+7} \cdot \frac{1}{s} = \frac{2}{7} = AK$. $T_s = 4T = \frac{4}{7}$.

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Subject Date No.

✱ Automatic control :-

* Ch 5 :- The Performance of F.B control system :-

* System order

1. Zero order system :-

$G s^0 = G$ constant.

$\frac{y(s)}{R(s)} = G$ $\quad R(s) \rightarrow [G] \rightarrow y(s)$

2. First order system :-

general Transfer function $\frac{y(s)}{R(s)} = \frac{K}{\tau s + 1}$

τ :- time constant. $\Rightarrow \boxed{\tau = 0.632\, y_{ss}}$

if $R(s)$ is a step input $R(s) = A/s$.

$y(t) = AK(1 - e^{-t/\tau})$ ⇝ general response 1st order if $R(s) = A/s$

$y_{s.s} = \lim_{t \to \infty} y(t) = AK \rightarrow \boxed{y_{s.s} = AK}$

$y(\tau) = 0.632\, y_{s.s} = 0.632\, AK.$

$y(4\tau) = 0.98\, AK = 0.98\, y_{s.s}.$

$T_s = 4\tau \rightarrow$ if 2% error.

* first order system for different input signals :-

Graphical.	input	out put.
Ramp.	$r(t) = t.$	$R(t) = t - \tau + \tau e^{-t/\tau}$
	$r(t) = At.$	$R(t) = At - \tau + \tau e^{-t/\tau}$
step	$r(t) = 1$	$R(t) = 1 - e^{-t/\tau}$
	$r(t) = A$	$R(t) = A - e^{-t/\tau}$
Impulse	$r(t) = \begin{cases} 1 & \text{for } t = A \\ 0 & t \neq A. \end{cases}$	$R(t) = \frac{1}{\tau} e^{-t/\tau}$

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Subject Date No.

* Automatic control :-

* Ch5 :- The Performance of F.B control system :-

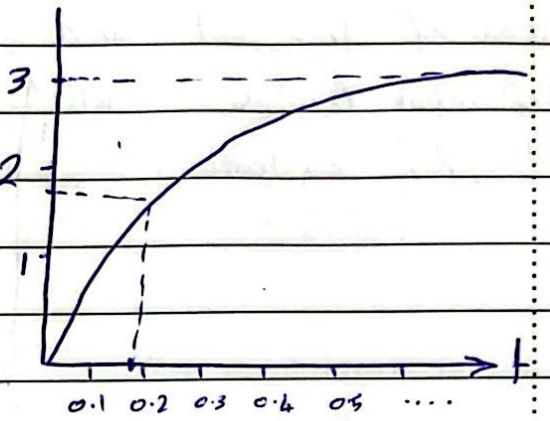
* Example :-

Find T ??

$y(T) = 0.632\ y_{ss}$ $\quad y_{ss} = 3.$

$y(T) = 0.632 * 3 = 1.896 = y(T)$

$T = 0.18$ sec.



Unit step input response.

$T.F = \dfrac{K}{Ts+1}$ $\quad$ 1st order.

$y_{ss} = AK$, $y_{ss} = 3$ $\quad A = 1.$

$3 = 1 * K \quad K = 3.$

$T.F = \dfrac{3}{0.18\,s + 1}$

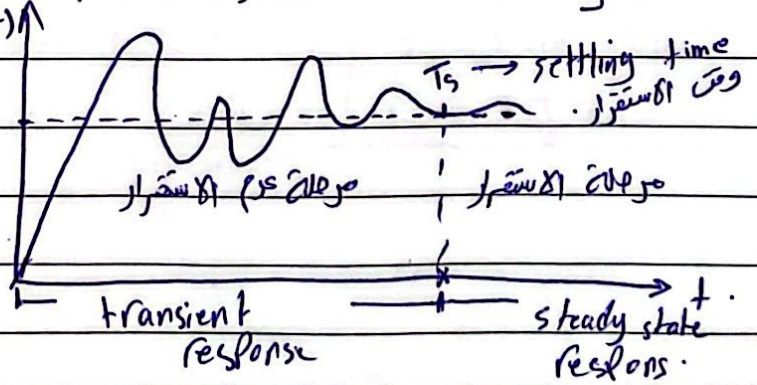
Find $y(t)$?!

$y(t) = AK(1 - e^{-t/T})$

$= 3(1 - e^{-t/0.18}).$

$T_s = 4T = 4 * 0.18 = 0.72$ sec.

* Review of transient and steady state Response :-



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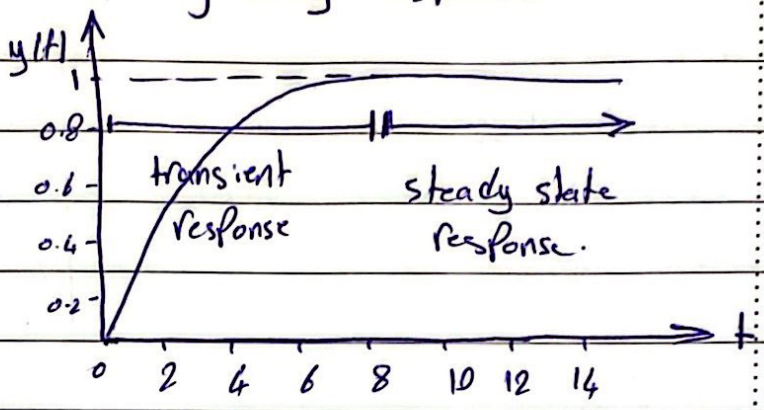
* Automatic control :-

* Ch5 : The performance of F.B control system :-

* Review of transient and steady state response :-

step input response.

1st order system.



* Review of First order response (unit step response) :-

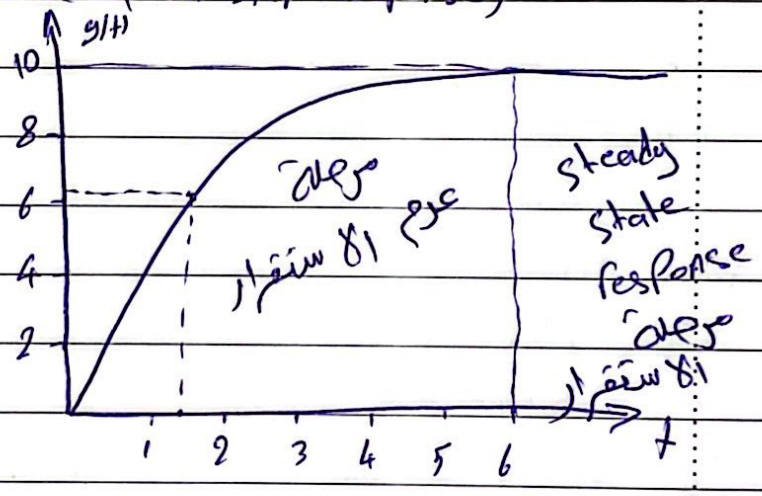
$y(T) = 0.632\ y_{s.s}$.

$\Rightarrow 0.632 * 10 = 6.32$.

$\Rightarrow T = 1.5$ sec.

$y_{ss} = AK \Rightarrow K = 10$.

$T.F = \frac{K}{Ts+1} = \frac{10}{1.5s+1}$



$y(t) = AK(1 - e^{-t/T})$

$y(t) = 10(1 - e^{-\frac{2}{3}t})$.

$T_s = 4T = 4 * .15 = 6$ sec.

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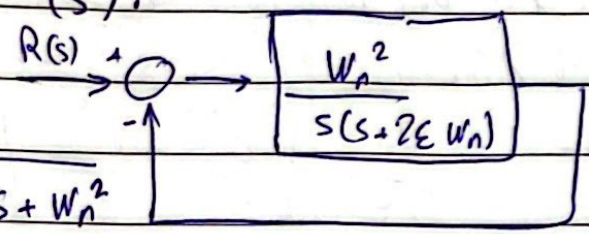
✱ Automatic control :-

* Ch5 :- The Performance of F.B control system :-

* system order :-

3- second order system (s^2) :-

Transfer function G(s)



$$G(s) = \frac{C(s)}{R(s)} = \frac{W_n^2}{s^2 + 2\xi W_n s + W_n^2}$$

General T.S 2st order system.

W_n :- natural frequancy (التردد الطبيعي)

ξ :- damping ratio (معدل التضاؤل)

$$G(s) = \frac{C}{s^2 + ds + e} = K \frac{W_n^2}{s^2 + 2\xi W_n s + W_n^2}$$

$W_n = \sqrt{e}$, $\xi = d / 2\sqrt{e}$, $K = C/e \rightarrow C = Ke$

Characteristic equation.

$$s^2 + 2\xi W_n s + W_n^2 = 0.$$

$$s_1, s_2 = \frac{-2\xi W_n \pm \sqrt{(2\xi W_n)^2 - 4W_n^2}}{2} = -\xi W_n \pm W_n \sqrt{\xi^2 - 1}$$

The values of ξ :-

Name	Value of ξ	Roots of s	Characteristics
Over damped	$\xi > 1$	$s = -\xi W_n \pm W_n \sqrt{\xi^2 - 1}$	Two real roots.
Critically Damp.	$\xi = 1$	$s = -W_n$.	single neg root.
Under damped	$0 < \xi < 1$	$s = -\xi W_n \pm j W_n \sqrt{1 - \xi^2}$	complex conjugate.
Un damped.	$\xi = 0$.	$s = \pm j W_n$.	Pure imaginary

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Subject Date No.

※ Automatic control :-

* Ch 5 :- The performance of F.B control system :-

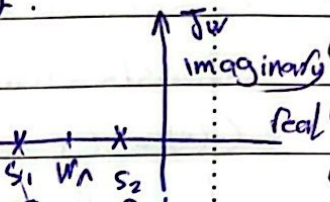
* The Values of (ξ) :-

1. Case 1 :- Over damped $\xi > 1$ Unit step input.

Real different Poles $s_1 , s_2 = -\xi w_n \pm w_n \sqrt{\xi^2 - 1}$

$$y(t) = 1 + \frac{w_n}{2\sqrt{\xi - 1}} \left[\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2} \right]$$

response

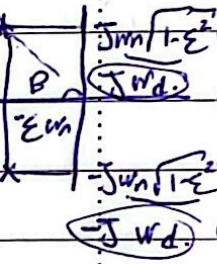


2. Case 2 :- Under damped $0 < \xi < 1$ Unit step input.

Complex Poles, $s_1 , s_2 = -\xi w_n \pm J w_n \sqrt{1 - \xi^2}$.

$$y(t) = 1 - \frac{e^{-\xi w_n t}}{\sqrt{1 - \xi^2}} \sin(w_d t + B)$$

2nd response



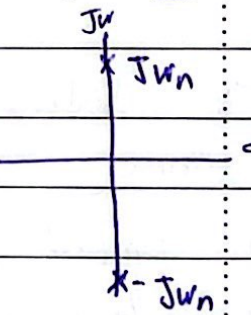
Where $B = \tan^{-1}\left(\frac{w_d}{\xi w_n}\right) = \tan^{-1}\frac{w_n\sqrt{1-\xi^2}}{\xi w_n} = \cos^{-1}\xi$

$w_d = w_n\sqrt{1-\xi^2}$ → is the damped natural frequancy

3. Case 3 :- Undamped (Zero damped) $\xi = 0$.

Imaginary Poles $s_1 , s_2 = \pm J w_n$.

$y(t) = 1 - \cos(w_n t)$.

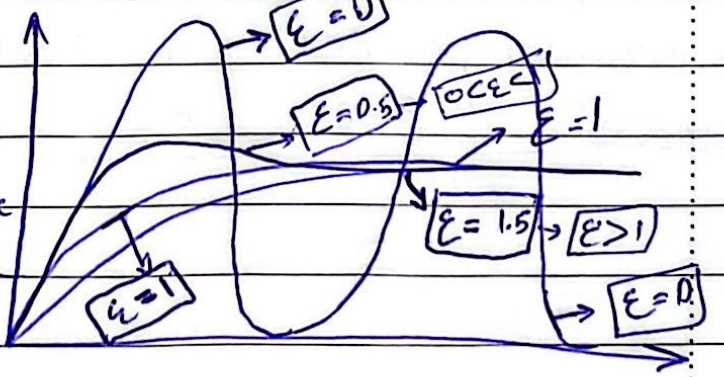


4. Case 4 :- Critically damped $\xi = 1$

Real equal double Pole $s_1 = s_2 = -w_n$.

$y(t) = 1 - e^{-w_n t}(1 + w_n t)$.

2nd order, step input different damping ratio (ξ).



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✱ Automatic control :-

* Ch5 :- The performance of F.B control system :-

* Poles locations with different damping ratio (ξ) :-

$\xi = 0 \rightarrow s_1 . s_2 = \pm j w_n$ complex . . $\xi = 0$

↳ Pure Imaginary .

$\xi = 1 \rightarrow s_1 . s_2 = -w_n .$

↳ Pure real

$0 < \xi < 1 \rightarrow s_1 . s_2 = -\xi w_n \pm j w_d$

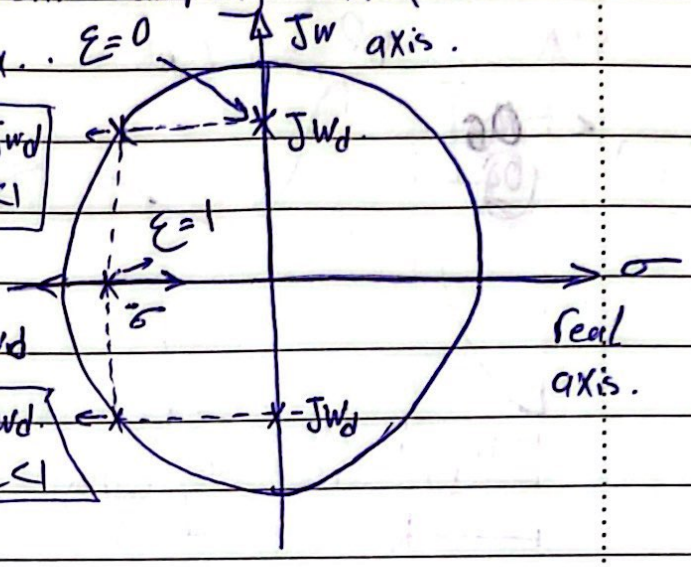
↳ complex poles .

$\xi > 1 \rightarrow s_1 . s_2 = -\sigma \pm w_d$

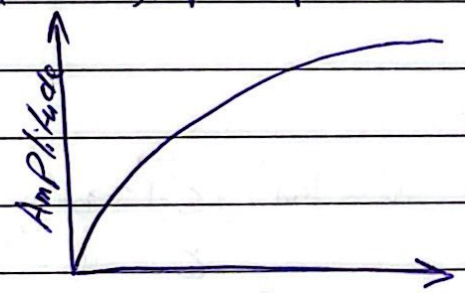
↳ real different poles .

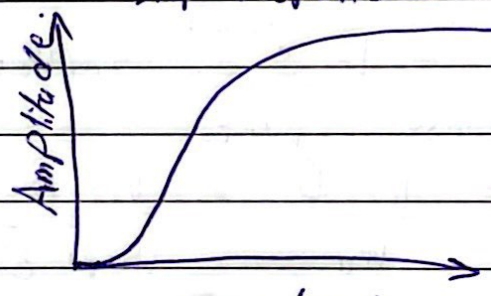
$\sigma \pm j w_d$ $0 < \xi < 1$

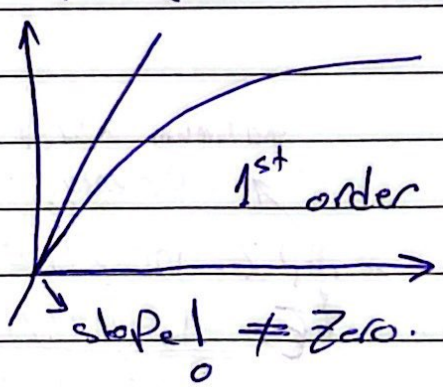
$\sigma - j w_d$ $0 < \xi < 1$

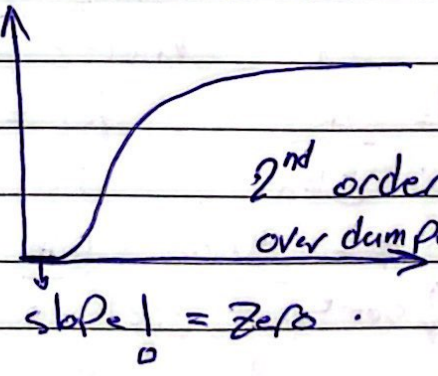


* step response .









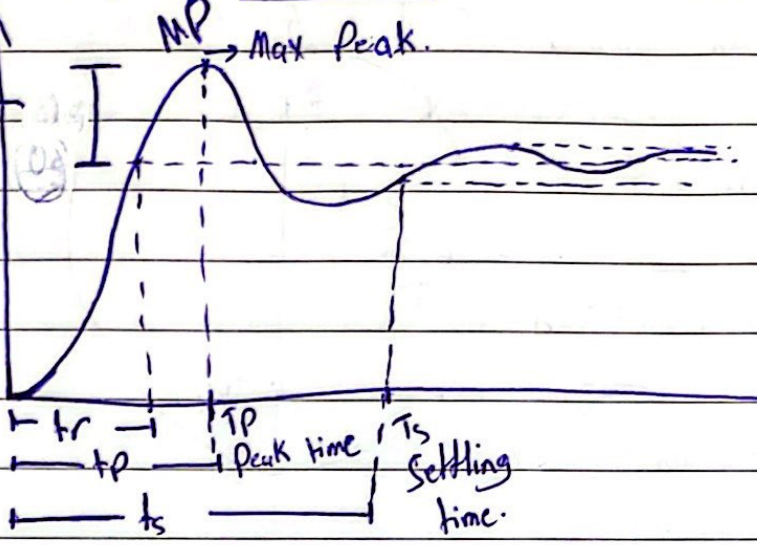
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Subject Date No.

✱ Automatic control :-

* Ch5 :- The Performance of F.B control system :-

* Under damped $\boxed{0<\xi<1}$ second order :-



response of underdamped input ⇒ A/s. $0<\xi<1$

$tr \rightarrow$ rise time , $tp \rightarrow$ Peak time.

$ts \rightarrow$ settling time.

* $y(t) = 1 - \frac{e^{-\xi w_n t}}{\sqrt{1-\xi^2}} \sin(w_d t + B)$ ↓ response

* $tr = \frac{\pi - B}{w_d} = \frac{\pi - B}{w_n\sqrt{1-\xi^2}}$ Where $B = \tan^{-1}\frac{\sqrt{\xi^2-1}}{\xi} \rightarrow$ (will $(1-\xi^2)$)

* $tp = \frac{\pi}{w_d} = \frac{\pi}{w_n\sqrt{1-\xi^2}}$

* $ts| = \frac{4}{\xi w_n}$, $ts| = \frac{3}{\xi w_n}$
  2% 5%

* $Mp\% = \frac{y(tp) - y(\infty)}{y(\infty)} * 100\%$, $Mp\% = 100\, e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$ ← maximum overshoot percentage.

  $y(\infty) \rightarrow$ steady state. $\xi = \frac{-\ln(Mp\%/100)}{\sqrt{\pi^2 + \ln^2(\%Mp/100)}}$

  over shoot ↕ same

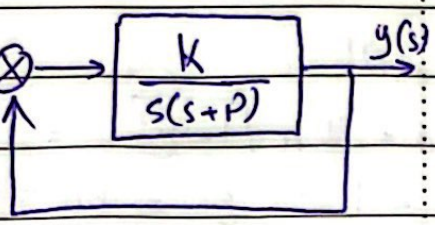
* $OS\% = \frac{y_{max} - y_{final}}{y_{final}} * 100\%$ ← overshoot percentages

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✱ Automatic control :-

* Ch5 :- The performance of F.B control system :-



* Example :- If 0.5% = 4.3%
$T_s = 4$ sec . 2% cn.

$$T.F = \frac{y(s)}{R(s)} = \frac{K}{s(s+p)+K} = \frac{K}{s^2+ps+K}$$

$$\Rightarrow \frac{K}{s^2+ps+K} = \frac{w_n^2}{s^2+2\xi w_n s + w_n^2} \qquad K = w_n^2 \qquad p = 2\xi w_n$$

$$\xi = \frac{-\ln(\%Mp/100)}{\sqrt{\pi^2+\ln^2(\%Mp/100)}} \Rightarrow \xi = \frac{-\ln(0.043)}{\sqrt{\pi^2+\ln^2(0.043)}} \Rightarrow \boxed{\xi = \frac{1}{\sqrt{2}}}$$

$$T_s = \frac{4}{\xi w_n} \Rightarrow 4 = \frac{4}{\frac{1}{\sqrt{2}} w_n} \Rightarrow w_n = \sqrt{2}.$$

$$K = w_n^2 \rightarrow K = (\sqrt{2})^2 = 2$$

$$p = 2\xi w_n = 2\frac{1}{\sqrt{2}}\sqrt{2} = 2.$$

$$T.F = \frac{y(s)}{R(s)} = \frac{2}{s^2+2s+2}$$

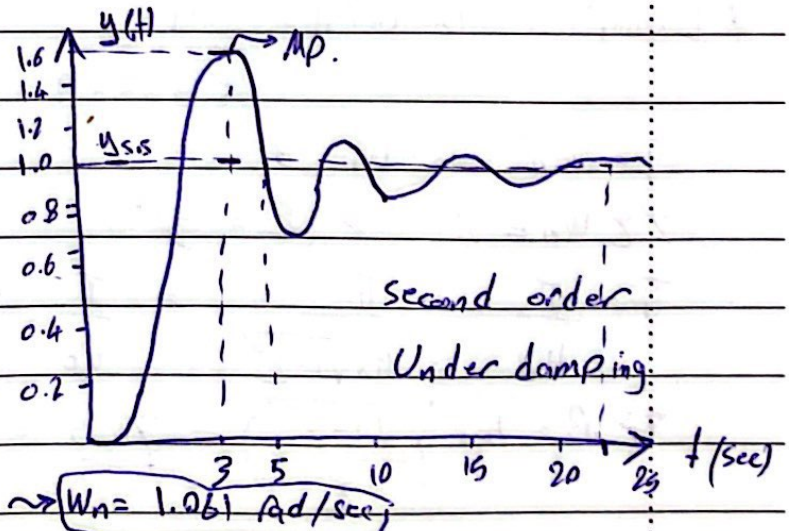
* Example :- input unit step.

0.5% = $\frac{Mp - y_{ss}}{y_{ss}} * 100\%$

$= \frac{1.6-1}{1} * 100\% = 60\%$

$$\xi = \frac{-\ln(\%Mp/100)}{\sqrt{\pi^2+\ln^2(\%Mp/100)}} = 0.16$$

$T_p = 3$ sec $\rightsquigarrow T_p = \frac{\pi}{w_n\sqrt{1-\xi^2}}$ $\rightsquigarrow \boxed{w_n = 1.061 \text{ rad/sec}}$



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Subject Date No.

Automatic control :-

Ch5 :- The performance of F.B control system :-

* Example :- sol:

$W_d = W_n\sqrt{1-\xi^2} = 1.0473$ rad/sec.

$\beta = \cos^{-1}(\xi) = 80.79^\circ = 1.41$ rad.

or $\beta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) = \tan^{-1}\left(\frac{\sqrt{1-0.16^2}}{0.16}\right) = 80.79^\circ$.

$$y(t) = 1 - \frac{e^{-\xi W_n t}}{\sqrt{1-\xi^2}} \sin(W_d t + \beta)$$

$$y(t) = 1 - \frac{e^{-(0.16)(1.061)t}}{\sqrt{1-0.16^2}} \sin(1.0473t + 80.79)$$

$$T_s = \frac{4}{\xi W_n} = \frac{4}{(0.16)(1.061)} = 23.578 \text{ sec.}$$

$$T_r = \frac{\pi - \beta}{W_d} = \frac{3.14 - 1.41}{1.0473} = 1.65 \text{ sec.}$$

* Example :- If $G(s) = \frac{9}{s^2+2s+9}$ unit step input.
Find $y(t), T_p, T_r, T_s, y_{ss}$.

$9 = W_n^2 \rightarrow W_n = 3$

$2\xi W_n = 2 \rightarrow \xi = \frac{1}{3}$.

$$T_p = \frac{\pi}{W_d} = \frac{\pi}{W_n\sqrt{1-\xi^2}} = \frac{\pi}{\sqrt{8}} = 1.11 \text{ sec}$$

$\beta = \tan^{-1}\left(\frac{\sqrt{1-\xi^2}}{\xi}\right) = 70.5^\circ = 1.23$ (degree, radian)

NOTEBOOK

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Subject Date No.

* Automatic control :-

* Ch 5 :- The performance of F.B control system :-

* Example :-

$$T_r = \frac{\pi - \beta}{W_d} = \frac{3.14 - 1.23}{\sqrt{8}} = 0.6753 \text{ sec.}$$

$$T_s = \frac{4}{\xi W_n} = \frac{4}{(\frac{1}{3})(3)} = 4 \text{ sec.}, \quad T_p = \frac{\pi}{W_d} = \frac{3.14}{\sqrt{8}} = 1.11 \text{ sec.}$$

$$y(t) = 1 - \frac{e^{-\xi W_n t}}{\sqrt{1-\xi^2}} \sin(W_d t + \beta)$$

$$y(t) = 1 - \frac{e^{-(\frac{1}{3})(3)t}}{\sqrt{1-(\frac{1}{3})^2}} \sin(\sqrt{8}t + 70.5)$$

$$y_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s} \left(\frac{9}{s^2 + 2s + 9} \right) = 1$$

* sugested problems :-

E5.2 . E5.4 . E5.6 . P5.1 . P5.3 . P5.16

AP5.2 . AP5.4 . DP5.3 . DP5.6 .

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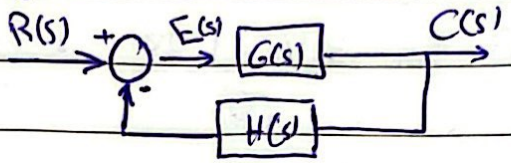
Subject Date No.

Automatic control :-

* Ch 5 :- The Performance of F.B control system :-

* Steady state Error Analysis :-

$$T.F = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$



$$E(s) = R(s) - C(s)H(s)$$

$$= R(s) - \frac{G(s)H(s)}{1+G(s)H(s)} R(s) = R(s)\left[1 - \frac{G(s)H(s)}{1+G(s)H(s)}\right]$$

$$E(s) = \frac{1}{1+G(s)H(s)} R(s)$$

$G(s)H(s)$ → loop gain.

The loop gain $G(s)H(s)$ can be expressed in the general form.

$$G(s)H(s) = \frac{K(s+z_1)(s+z_2)(s+z_3)\cdots(s+z_m)}{s^N(s+p_1)(s+p_2)(s+p_3)\cdots(s+p_n)}$$

$z \rightarrow$ Zeros. $p \rightarrow$ Poles.

$$= \frac{K\prod_{i=1}^{i=m}(s+z_i)}{s^N\prod_{i=1}^{i=n}(s+p_i)}.$$

The error in this case would be given by

$$E(s) = \frac{1}{1+G(s)H(s)} R(s) = \frac{s^N\prod_{i=1}^{i=n}(s+p_i)}{s^N\prod_{i=1}^{i=n}(s+p_i) + K\prod_{i=1}^{i=m}(s+z_i)} R(s)$$

$$E_{ss} = \lim_{s\to 0} s \cdot E(s) = \lim_{s\to 0} s\left[\ \nearrow\ \right]$$

$s^N \rightarrow N$ (system ال type هو كده)

$N = 0 \rightarrow$ The system is a type Zero.

$N = 1 \rightarrow$ The system is a type one.

$N = 2 \rightarrow$ The system is a type Two.

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* Automatic control :-

* Ch5 : The performance of F.B control system :-

* Type Zero system ($s^0 = 1$) :-

- The steady state error for a step input (A/s) :-

$$E_{s.s} = \lim_{s \to 0} s \cdot \frac{A}{s} \left[\frac{\prod_{i=1}^{n} (s + P_i)}{\prod (s + P_i) + K \prod (s + Z_i)} \right] = \frac{\prod P_i}{\prod P_i + K \prod Z_i} A.$$

$$= \frac{A}{1 + K_p} \quad \text{for } K_p = \frac{K \prod Z_i}{\prod P_i}$$

Position error constant

$K_p = \lim_{s \to 0} G(s) H(s)$.

- The steady state error for a ramp input (A/s^2) :-

$$E_{s.s} = \lim_{s \to 0} s \cdot \frac{A}{s^2} \left[\frac{\prod (s + P_i)}{\prod (s + P_i) + K \prod (s + Z_i)} \right] = \infty.$$

- The steady state error for a acceleration (A/s^3) :-

$$E_{s.s} = \lim_{s \to 0} s \cdot \frac{A}{s^3} \left[\frac{\prod (s + P_i)}{\prod (s + P_i) + K \prod (s + Z_i)} \right] = \infty$$

* Type One system (s^1) :-

- The steady state error for a step input (A/s) :-

$$E_{s.s} = \lim_{s \to 0} s \cdot \frac{A}{s} \left[\frac{s \prod (s + P_i)}{s \prod (s + P_i) + K \prod (s + Z_i)} \right] = 0.$$

- The steady state error for a ramp input (A/s^2) :-

$$E_{s.s} = \lim_{s \to 0} s \frac{A}{s^2} \left[\frac{s \prod (s + P_i)}{s \prod (s + P_i) + K \prod (s + Z_i)} \right] = \frac{\prod P_i}{K \prod Z_i} A$$

$$= \frac{A}{K_v} \quad \text{for } K_v = \frac{K \prod Z_i}{\prod P_i}$$

Velocity error constant.

$K_v = \lim_{s \to 0} s\, G(s) H(s)$

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Subject Date No.

✱ Automatic control :-

✱ Ch5 :- The Performance of F.B control system :-

✱ Type one system :-

- The steady state error for an acceleration (A/s^3) :-

$$E_{s.s} = \lim_{s\to 0} s \cdot \frac{A}{s^3}\left[\frac{s\,\Pi(s+P_i)}{s\,\Pi(s+P_i)+K\,\Pi(s+Z_i)}\right] = \infty$$

✱ Type Two system :-

- The steady state error for a step input (A/s).

$$E_{s.s} = \lim_{s\to 0} s \cdot \frac{A}{s}\left[\frac{s^2\,\Pi(s+P_i)}{s^2\,\Pi(s+P_i)+K\,\Pi(s+Z_i)}\right] = \underline{0}.$$

- The steady state error for a ramp input (A/s^2) :-

$$E_{s.s} = \lim_{s\to 0} s \cdot \frac{A}{s^2}\left[\frac{s^2\,\Pi(s+P_i)}{s^2\,\Pi(s+P_i)+K\,\Pi(s+Z_i)}\right] = \underline{0}$$

- The steady state error for an acceleration (A/s^3) :-

$$E_{s.s} = \lim_{s\to 0} s \cdot \frac{A}{s^3}\left[\frac{s^2\,\Pi(s+P_i)}{s^2\,\Pi(s+P_i)+K\,\Pi(s+Z_i)}\right] = \frac{\Pi P_i}{K\,\Pi Z_i}\,A.$$

$$= \frac{A}{K_a} \quad \text{for } K_a = K\,\frac{\Pi Z_i}{\Pi P_i}$$

[Accel error constant
$K_a = \lim_{s\to 0} s^2\,G(s)\,H(s)$]

$$K_p = K_v = K_a = K\,\frac{\Pi Z_i}{\Pi P_i}$$

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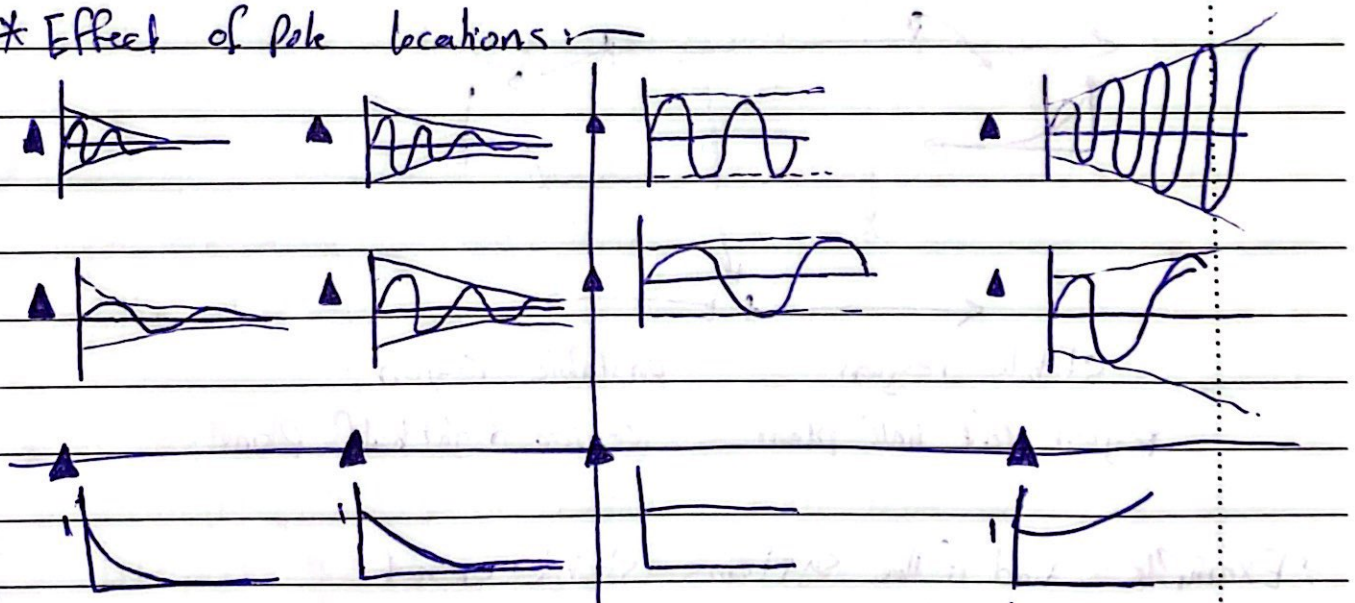
✱ Automatic control

* Ch5 :- The performance of FB control system :-

* Types of systems :-

Type	$R(s) = A/s$	$R(s) = A/s^2$	$R(s) = A/s^3$
0	$\frac{A}{1+K_p}$	∞	∞
1	0	$\frac{A}{K_v}$	∞
2	0	0	$\frac{A}{K_a}$

* Effect of pole locations :-



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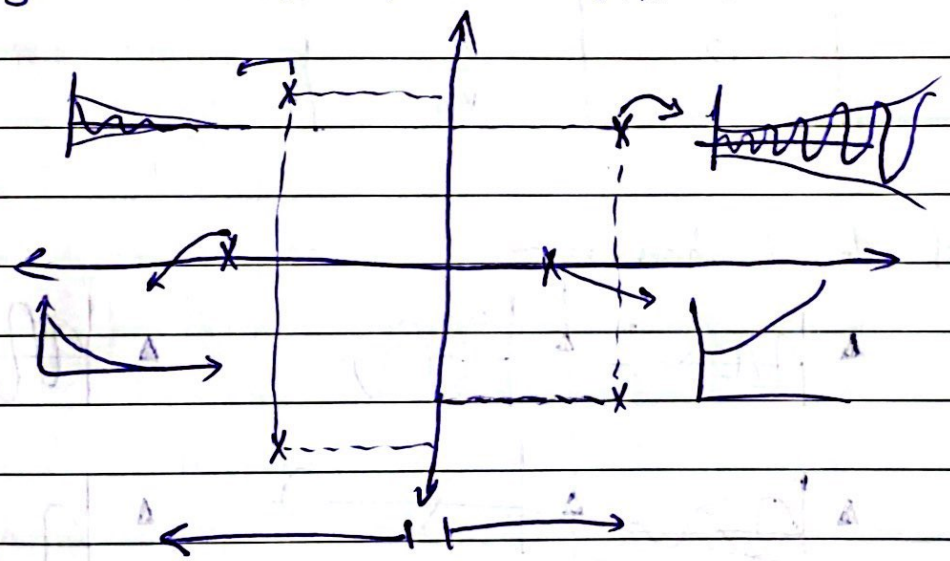
* Automatic control :-

* Ch5 :- The performance of F.B control system :-

* Stability :-

A BIBO (bounded-input bounded output) stable system is a system for which the outputs will remain bounded for all time . for any finite initial condition and inputs .

∴ The system is BIBO stable if and only if all the poles of the system have real parts less than Zero.



stable region region left half plane

unstable region. region Right half plane.

* Example :- Find if the systems stable or not.

1. $G(s) = \frac{s+3}{(s+5)(s+2)}$ Poles ⇒ $s = -5$. $s = -2$. (L.H.P) stable system.

2. $G(s) = \frac{s+3}{(s+5)(s-2)}$ Poles ⇒ $s = -5$ $s = 2 \rightarrow$ (R.H.P) unstable system.

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Subject Date No.

Automatic control :-

* Ch 6 :- Routh - Herwitz stability Criterion :-

* Routh - Hurwiz stability Criterion :-

It is method for determining continuous system stability.

* The Routh - Hurwiz criterion states that "the number of root of the characteristic equation with positive real part is equal to the number of changes in sign of the first column of the Routh array.

* This method yields stability information without the need to solve for the closed - loop system poles.

* Using this method we can tell how many close loop system poles are in the left half - plane, in the right half - plane, and on the JW - axies

* The method requires two steps :-

1. Generate a data table called a Routh table.
2. interpret the Routh table to tell how many closed loop system poles are in the LHP, the RHP, and on the JW - axis.

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Subject Date No.

* Automatic control :-

* Ch6 :- Routh - Hurwitz stability criterion :-

* Example :- General abasic Routh Table.

$$R(s) \rightarrow \boxed{\frac{N(s)}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}} \rightarrow C(s)$$

Row

Column

s^4	a_4	a_2	a_0	← العوامل الزوجية
s^3	a_3	a_1	0	← العوامل الفردية
s^2	$b_1 = -\frac{\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3}$	$b_2 = -\frac{\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3}$	$b_3 = -\frac{\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$	
s	$c_1 = -\frac{\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1}$	$c_2 = -\frac{\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$c_3 = -\frac{\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	
s^0	$d_1 = -\frac{\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = a_0$	$d_2 = -\frac{\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	$d_3 = -\frac{\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	

* Four special Cases or Configurations in the first Column Array of the Routh's table.

1. Case 1 :- No element in the first column is Zero.
2. Case 2 :- A Zero in the first column but some other elemint of the row are non Zero.
3. Case 3 :- Entire Row is Zero.

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✱ Automatic control :-

*Ch 6 :- Routh-Hurwitz stability criterion :-

*Example :- Find the stability of the continues system having the characteristic equation of $s^3 + 6s^2 + 12s + 8 = 0$.

s^3	1	12	0
s^2	6	8	0
s	64/6	0	0
s^0	8	0	0

$$b_1 = -\frac{\begin{vmatrix} 1 & 12 \\ 6 & 8 \end{vmatrix}}{6} = \frac{-8 + 6(12)}{6} = \frac{64}{6}$$

There are no sign changes in the first column of the Routh table, It means that all the root of the characteristic equation have negative real parts and this system is stable.

*Example:- $s^3 + s^2 + 2s + 24 = 0$.

s^3	1	2	0
s^2	1	24	0
s	-22	0	0
s^0	24	0	0

$$b_1 = -\frac{\begin{vmatrix} 1 & 2 \\ 1 & 24 \end{vmatrix}}{1} = \frac{-24 + 2}{1} = -22$$

There are two changes in sign appear in the first column we find that two roots of the characteristic equation lie in the right hand side of the s-Plane, and this system unstable.

2 change in sign → 2 Poles in RHS.

characteristic eq(s^3) → 3 sols → 2 RHS, 1 LHS.

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Subject Date No.

* Automatic control :—

* Ch6 :- Routh-Hurwiz Stability Criterion :-

* Example :- Determine the rang of values of a system parameter (K) for which the system is stable. $s^3 + 3s^2 + 3s + 1 + K = 0$.

s^3	1	3	0
s^2	3	$1+K$	0
s	$\frac{8-K}{3}$	0	0
s^0	$1+K$	0	0

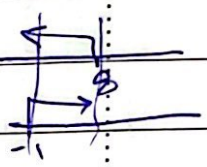
system stable

first column ⇒ all values Positive.

$\frac{8-K}{3} > 0 \Rightarrow 8-K > 0$

$\boxed{K < 8}$

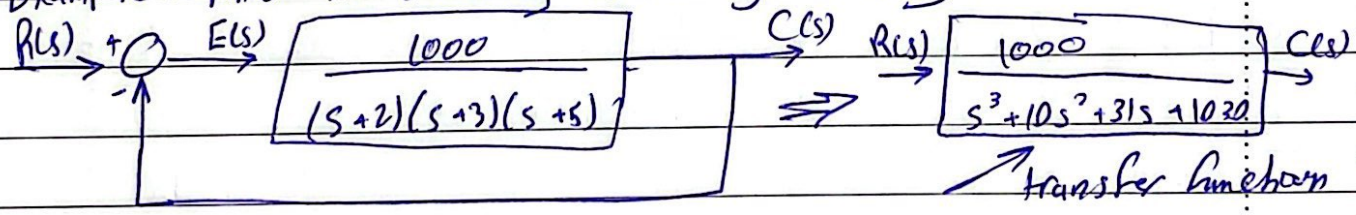
$1+K > 0 \Rightarrow \boxed{K > -1}$



$K < 8$, $K > -1$

$\boxed{-1 < K < 8}$

* Example :- Find the stability of the system using Routh criterion



characteristic equation ⇒ $s^3 + 10s^2 + 31s + 1030 = 0$

s^3	1	31	0
s^2	10	1030	0
s	-72	0	
s^0	103		

Two changes in sign ⇒ system is unstable.

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Subject Date No.

Automatic control :-

Ch 6 : Routh-Hurwiz stability criterion :-

* Example :- Find the stability of the system using Routh criterion

$2s^4 + s^3 + 3s^2 + 5s + 10 = 0$.

s^4	2	3	10
s^3	-1	5	0
s^2	-7	10	0
s	6.43	0	0
s^0	10	0	0

2 changes in sign → 2 Poles in RHS.
characteristic equ (s^4) → 2 Poles in LHS.
system is unstable.

Two Change in sign ⇒ the system is unstable.

Case II : A zero only in the first column (stability via Epsilon Method) :

- If the first element of a row is zero, division by zero would be required to form the next row.
- to avoid this Phenomenon an epsilon (ϵ) (a small positive number) is assigned to replace the zero in the first column
- The Value (ϵ) is then allowed to approach zero from either the positive or the negative side, after which the signs of the entries in the first column can be determined.

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Subject Date No.

✱ Automatic control :-

* Ch 6:- Routh-Hurwitz stability criterion :-

* Example :- Case II

Determine the stability of the system having a characteristic equ

$q(s) = s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10$

s^5	1	2	11	0
s^4	2	4	10	0
s^3	0ε	6	0	
s^2	c_1	10	0	
s	$d_1 6$	0	0	
s^0	10	0	0	

$$c_1 = \frac{4\varepsilon - 4(3)}{\varepsilon} = \frac{-12}{\varepsilon}$$

$$d_1 = \frac{6c_1 - 10\varepsilon}{c_1} \rightarrow 6.$$

Two changes in sign → system is unstable.

* Example :- Determine the rang of parameter (K) for which the system is stable

$q(s) = s^4 + s^3 + s^2 + s + K.$

s^4	1	1	K
s^3	1	1	0
s^2	0ε	K	0
s	c_1	0	0
s^0	K	0	0

$$c_1 = \frac{\varepsilon - K}{\varepsilon} \rightarrow \frac{-K}{\varepsilon}$$

* for any value of K greater than Zero the system unstable.

Also because the last term in the first column is equal to (K) a negative value of (K) will result in an unstable system. Consequently, the system is unstable for all value of gain (K).

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ubject Date No.

* Automatic control :-

* Ch 6 :- Routh-Hurwiz stability criterion :-

* Example :- Determine the stability of the closed-loop transfer function : $T(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3}$ → chars eqn.

s^5	1	3	5	0
s^4	2	6	3	0
s^3	0ε	$\frac{7}{2}$	0	
s^2	c_1	3	0	
s	d_1	0	0	
s^0	3	0	0	

$$c_1 = \frac{6\varepsilon - 7}{\varepsilon} = \frac{-7}{\varepsilon}$$

$$d_1 = \frac{c_1 \frac{7}{2} - 3\varepsilon}{c_1} = \frac{42\varepsilon - 49 - 6\varepsilon^2}{12\varepsilon - 14}$$

$$d_1 = \frac{7}{2}$$

Two changes in sign → The system is unstable.

* Case (III) : Entire Row is Zero :-

- Some times while making routh table, we find that an entire row consists of zeros.
- This happen because there is an even polynomial that is a factor of the original polynomial.
- This case must be handled differently from the case of a zero in only the first column of a row.

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Subject Date No.

* Automatic control

* Ch 6 :- Routh- Herwiz stability criterion :-

* Example :- Determine the number of right half plane poles in the closed loop transfer function

$$T(s) = \frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56}$$ → Characteristic equ.

s^5	1	6	8	0
s^4	7	42	56	0
s^3	~~0~~ 4	~~0~~ 12	~~0~~	~~0~~
s^2	3	8	0	
s	$\frac{1}{3}$	0	0	
s^0	56	0	0	

Jolge (s+1) Chara ||

* find the auxiliary polynomial

$P(s) = 7s^4 + 42s^2 + 56$.

$P(s) = s^4 + 6s^2 + 8$

* Next differentiate the polynomial

$$\frac{dP(s)}{ds} = 4s^3 + 12s + 0$$

All the entires in the first column are positive.

there are no right-half plane poles, the system → ~~stable~~ marginare stable.

2 poles in Jw axis. 3 poles are in (LHP).

* Example :- Determine the stability of the system

$q(s) = s^3 + 2s^2 + 4s + K$.

s^3	1	4	0
s^2	2	K	0
s	$\frac{8-K}{2}$	0	
s^0	K	0	

* for stable system the Value of (K) must be $0 < K < 8$.

Also. when K=8 rows of ~~Zeros~~.

2 Poles in Jw axis.

NOTEBOOK

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Subject Date No.

✱ Automatic control :-

* Ch6 :- Routh-Herwiz stability criterions :-

* Example :- for the transfer function tell How many Poles are in right half-Plane, in the left half Plane, and on the Jw axis. $T(s) = \frac{20}{s^8 + s^7 + 12s^6 + 22s^5 + 39s^4 + 59s^3 + 48s^2 + 38s + 20}$

s^8	1	12	39	48	20
s^7	1	22	59	38	0
s^6	-10 ①	-20 ②	+10 ①	20 ②	0
s^5	20 ①	60 ③	40 ②	0	0
s^4	1	3	2	0	0
s^3	0 4	0 6	0 0.	0	0] → Row of zero
s^2	3/2 ③	2 ④	0		
s	2/3	0	0		
s^0	4	0	0.		

entires there are 2 Poles in (Jw)

$P(s) = s^4 + 3s^2 + 2$

$P'(s) = 4s^3 + 6s + 0$

2 Poles → Jw axis.

Two change of sing → 2Poles in (RHP)

s^8 → 4 Poles in LHP.

* Suggested Problems :-

E 6.19 . E 6.24 . P 6.1 . P 6.3 . P 6.4 .

AP 6.3 . AP 6.2 . DP 6.3 . DP 6.7

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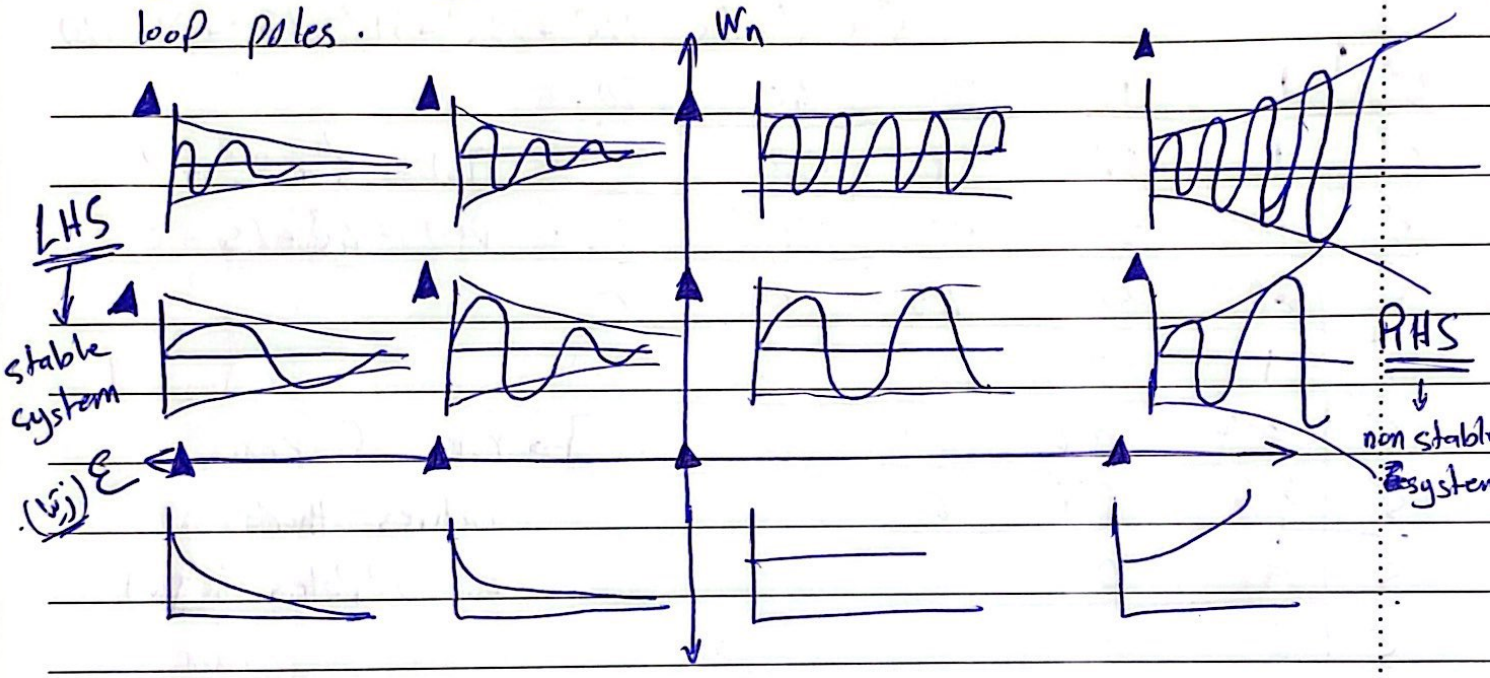
Subject Date No.

* Automatic control :-

* Ch7 :- Root - locus Method :-

* Introduction :-

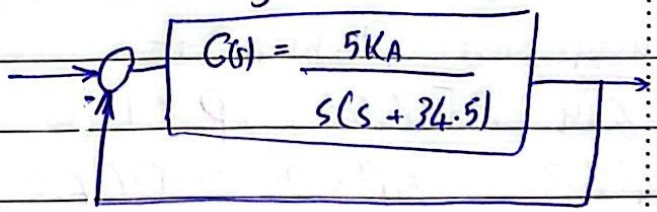
It is well known that the transient response of a feedback system is closely related to the locations of the closed loop poles.



- from the design viewpoint for some systems simple gain adjustment may move the close-loop poles to desired locations

for example: consider a unity feedback system with open loop transfer function.

$$G(s) = \frac{5KA}{s^2 + 34.5s + 5KA}$$

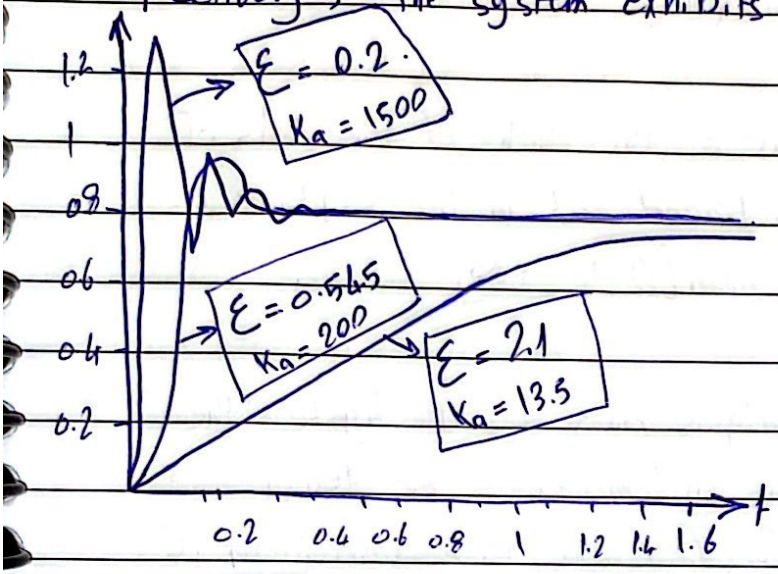


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* Automatic Control :-

* Ch7 : Root-locus Methods :-

* for same example :- for 2nd order unit step response.
Take open loop gain with the values $K_A = 13.5$, 200, 1500 respectively, the system exhibits different responses :-



$\varepsilon = 2.1 \Rightarrow Mp = 0$.

$\varepsilon = 0.545 \Rightarrow Mp = 13\%$.

$\varepsilon = 0.2 \Rightarrow Mp = 52.7\%$.

$K_a = 1500$.

$W_n^2 = 5 * 1500 = 7500 \Rightarrow W_n = 50\sqrt{3}$.

$2\varepsilon W_n = 34.5 \Rightarrow \varepsilon = \frac{34.5}{173.2} = 0.2$

$Mp\% = 100\, e^{\frac{-\varepsilon\pi}{\sqrt{1-\varepsilon^2}}} \Rightarrow Mp\% = 52.7\%$.

$K_a = 200$.

$W_n^2 = 5 * 200 = 1000 \Rightarrow W_n = 10\sqrt{10}$.

$2\varepsilon W_n = 34.5 \Rightarrow \varepsilon = \frac{34.5}{20\sqrt{10}} = 0.545$

$Mp\% = 100\, e^{\frac{-\varepsilon\pi}{\sqrt{1-\varepsilon^2}}} \Rightarrow Mp\% = 13\%$.

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Subject Date No.

Automatic control :-

* Ch7 :- Root-locus Method :-

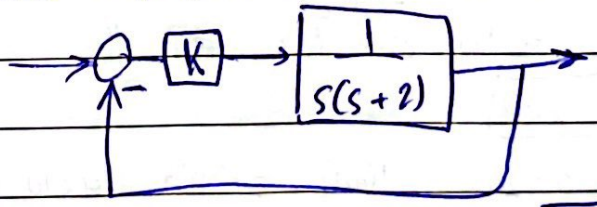
* Therefore it is important to determine how the roots of the characteristic equation moved around the s-plane as we change the open loop gain (Ka)

* Definition :- The Root locus is the path of the roots of the characteristic equation traced out in the s-plane as a system parameter varies from [0 to +∞]

* Remark :- The Root locus analysis aims to investigate the closed-loop stability and the system controller design through the openloop transfer function with variation of a certain system parameter commonly the open loop gain.

* Example :- Determine the closed loop root loci when K varies from [0 to +∞]

T.F = $\frac{K}{s^2+2s+K}$.



characteristic equation $s^2+2s+K=0 \Rightarrow \underline{s_{1,2} = -1 \pm \sqrt{1-K}}$

$K=0 \rightarrow s_1=0 \;,\; s_2=-2$ $\quad K \rightarrow \infty \cdot s_{1,2} = -1 \pm \infty j$.

$K=1 \rightarrow s_1 = s_2 = -1$.

$K=2 \rightarrow s_{1,2} = -1 \pm j1$

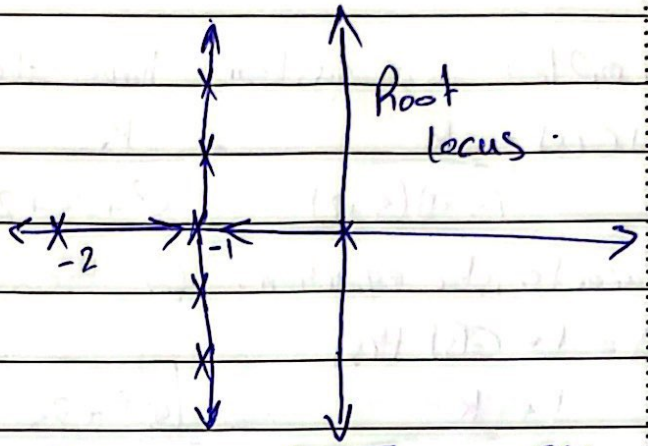
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Subject Date No.

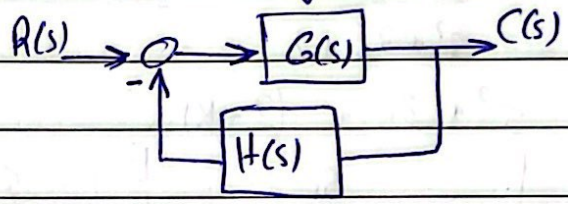
* Automatic control :-

* Ch 7 : Root-locus Method :-

* هي
عبارة عن رسم Root-locus لل
ال Poles اللي بتغير قيم
الـ closed loop system
. Ka



* General system



Total transfer function

$$T.F = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}.$$

let us write GH (loop gain) as following :-

$$G(s)H(s) = \frac{K(s+Z_1)(s+Z_2)\cdots(s+Z_m)}{(s+P_1)(s+P_2)\cdots(s+P_n)}$$

So the characteristic equation is equal to.

$$1 + G(s)H(s) = 0.$$

$$1 + \frac{K(s+Z_1)(s+Z_2)\cdots(s+Z_m)}{(s+P_1)(s+P_2)\cdots(s+P_n)} = 0. \quad Z \rightarrow \text{Zeros}. \quad P \rightarrow \text{Poles}.$$

$$\frac{K(s+Z_1)(s+Z_2)\cdots(s+Z_m)}{(s+P_1)(s+P_2)\cdots(s+P_n)} = -1$$

Angle condition :- $\angle G(s)H(s) = 180(2K+1)$. $K = 0, \pm 1, \pm 2$

Magnitude condition :- $|G(s)H(s)| = 1$

N O T E B O O K

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Subject Date No.

* Automatic control:-

* Ch 7 : Root - locus Method..

* Example:- an system have the following loop gain.

$$H(s)G(s) = \frac{K}{(s+1)(s+2)} = \frac{K}{s^2+3s+2}$$

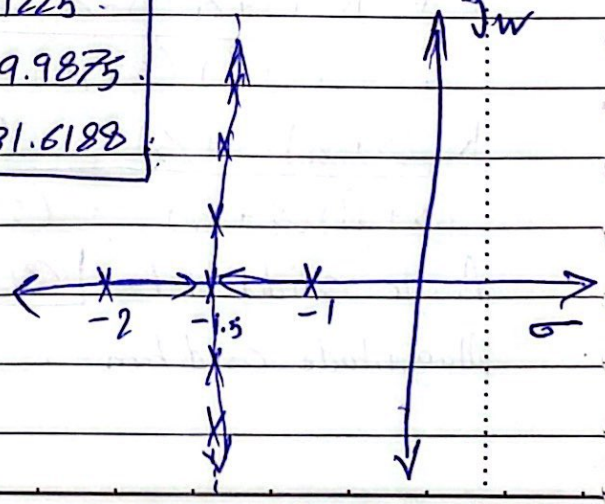
characteristic equation. for the system.

$$\Delta = 1 + G(s)H(s)$$

$$1 + \frac{K}{s^2+3s+2} = \frac{\boxed{s^2+3s+(2+k)}}{s^2+3s+2} \rightarrow \text{characteristic equation for the system.}$$

$$\Delta = s^2 + 3s + (2+K).$$

K	S_1	S_2
0	-2	-1
0.25	-1.5	-1.5
0.5	-1.5 - J0.5	-1.5 + J0.5
1	-1.5 - J0.866	-1.5 + J0.866
2	-1.5 - J1.3229	-1.5 + J1.3229
4	-1.5 - J1.9365	-1.5 + J1.9365
10	-1.5 - J3.1225	-1.5 + J3.1225
100	-1.5 - J9.9875	-1.5 + J9.9875
1000	-1.5 - J31.6188	-1.5 + J31.6188



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Subject Date No.

* Automatic control :-

* Ch 7 :- Root-locus Methode :-

* Magnitude and Angle condition :-

- Magnitude condition :-

$$|G(s)H(s)| = 1.$$

$$K \frac{|(s+z_1)|\,|(s+z_2)| \cdots |(s+z_m)|}{|(s+p_1)|\,|(s+p_2)| \cdots |(s+p_n)|} = 1$$

من خلال ال magnitude cond بنعرف اذا كان النقطة واقعة على ال (Root-locus) عن طريق حساب K معينة.

$$\frac{|(s+p_1)|\,|(s+p_2)| \cdots |(s+p_n)|}{|(s+z_1)|\,|(s+z_2)| \cdots |(s+z_m)|}\Bigg|_{s=a+jb} = K$$

for a given Point on Root locii.

- Angle condition :-

$$\angle G(s)H(s) = -1$$

$$= \pm 180(2k+1) \quad \text{for } k = 0, 1, 2.$$

This equation can be written as :

$$\angle G(s)H(s) = \angle(s+z_1) + \angle(s+z_2) + \angle(s+z_3) \cdots + \angle(s+z_m) - \angle(s+p_1) - \angle(s+p_2) - \angle(s+p_3) \cdots - \angle(s+p_n),$$

$$= \pm 180(2k+1)$$

for a given Pain on Root locii.

$s = a + jb$.

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Subject Date No.

* Automatic control :-

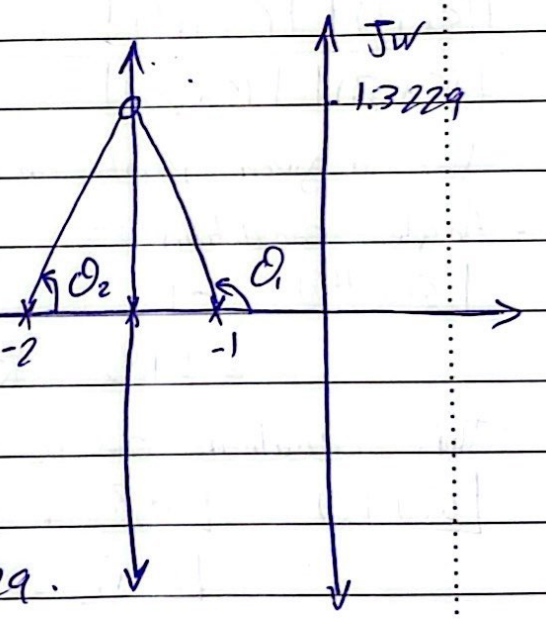
* Ch 7 :- Root - locus Method :—

* Example :- If the loop gain be $G(s)H(s) = \frac{K}{(s+1)(s+2)}$

$$G(s)H(s) = \frac{K}{s^2+3s+2}$$

$$1 + G(s)H(s) = 1 + \frac{K}{s^2+3s+2} = \boxed{\frac{s^2+3s+(2+K)}{s^2+3s+2}} \rightarrow \text{Char eqn.}$$

for a system analyzed at the point given by $-1.5 \pm j\,1.3229$ the angles of the vectors from that point to the two poles are calculated as follows.



$$\theta_1 = 180 - \tan^{-1}\left(\frac{1.3229}{+0.5}\right)$$

$$= 180 - 69.29 = 110.71^\circ$$

$$\theta_2 = \tan^{-1}\left(\frac{1.3229}{+0.5}\right) = 69.29^\circ$$

$$|G(s)H(s)| = |\text{Zeros}| - |\text{Poles}|$$

$$= 0 - |s+1| - |s+2|$$

$$= -\theta_1 - \theta_2 = -110.71 - 69.29$$

$$= -180$$

$$|G(s)H(s)| = 1$$

$$\left|\frac{K|(s+Z_1)(s+Z_2)|}{|s+P_1)(s+P_2)|}\right| = 1 \Rightarrow K = |(s+1)(s+2)|_{s=-1.5+j1.3229}$$

$$\boxed{K = 2}$$

NOTEBOOK

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* Automatic control.
* Ch7:- Root locus Method:-

* Before starting the steps of sketching Root locus, We have to know
1. The start and End Point of a Root locus.
2. Nember of segments (branches) of root locus.
3. location of root locus segments on real axis.

* The characteristic equation:

$$1 + \frac{K(s+z_1)(s+z_2)(s+z_3)\cdots(s+z_m)}{(s+p_1)(s+p_2)(s+p_3)\cdots(s+p_n)} = 0$$

$$(s+p_1)(s+p_2)\cdots(s+p_n) = K(s+z_1)(s+z_2)\cdots(s+z_m)$$

When $K=0$ The equation becomes.

$$(s+p_1)(s+p_2)\cdots(s+p_n) = \text{Zero}$$

The root locii starts at the **Poles** of the system

When $K=\infty$ The equation becomes.

$$\frac{(s+p_1)(s+p_2)\cdots(s+p_n)}{K} = (s+z_1)(s+z_2)\cdots(s+z_m)$$

$$(s+z_1)(s+z_2)\cdots(s+z_m) = \text{Zero}$$

The root locii ends at the **Zeros** of the system.

Root locus starts at the system Poles (when $K=0$) and ends at the system Zeros (when $K=\infty$).

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Subject Date No.

* Automatic control :-

* Ch 7 :- Root - locus Method :-

* Example :- Second order system :-

$$G(s)\,H(s) = \frac{K(s+2)}{s(s+4)}$$

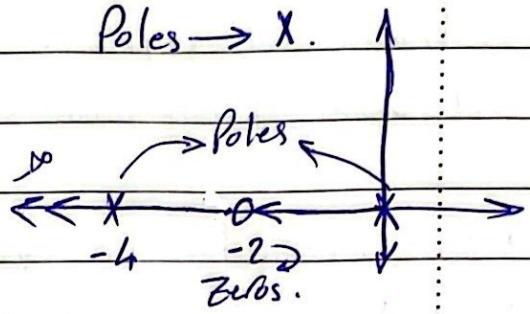
$\boxed{\frac{\text{Zeros}}{\text{Poles}}}$

Zeros → O.

Poles → X.

$s+2=0 \Rightarrow s=-2$

$s(s+4)=0 \Rightarrow s=0 \;,\; s=-4$



* Example :- for a unity FB system

The open loop transfer function is

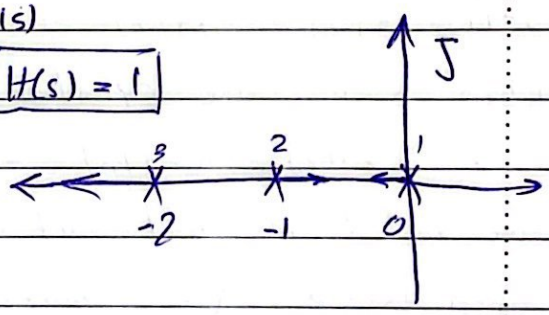
$$G(s) = \frac{K}{s(s+1)(s+2)} = H(s)\,G(s)$$

but $\boxed{H(s)=1}$

no zeros.

$s(s+1)(s+2)=0$

$s=0 \quad s=-1 \quad s=-2.$



* Root loci Construction Rules :-

If the number of $\boxed{\text{Poles}}$ in the loop gain equation is lager than the number of $\boxed{\text{Zeros}}$ this means that a number of root locii segments equal to the number of Poles and Zeros will be ending at infinity.

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Subject Date No.

Automatic control :-

Ch7 :- Root locus Method :-

* Root locus Construction Rules :-

Note that for root loci the following fact are true :-

1- The number of root locus branches is equal to the order of the characteristic equation → number of Poles

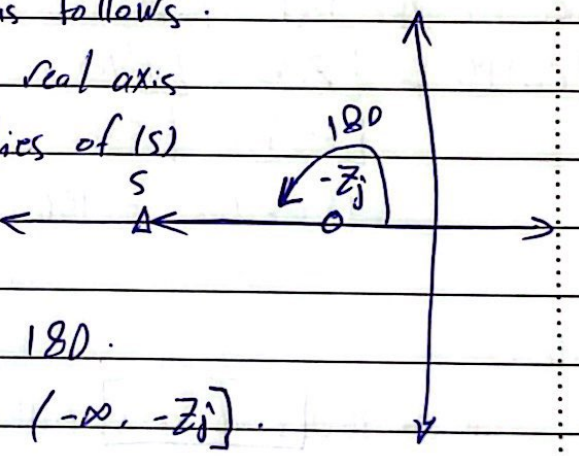
2- The loci are symmetrical about the real axis.

* Root locii segement on the real axis can be found by applying the angle criteria as follows.

let (s) be a test point on the real axis

since Zero ($-z_j$) of G(s) H(s) lies of (s)

it follow that



$$|G(s)H(s) = |Zeros - |Poles$$

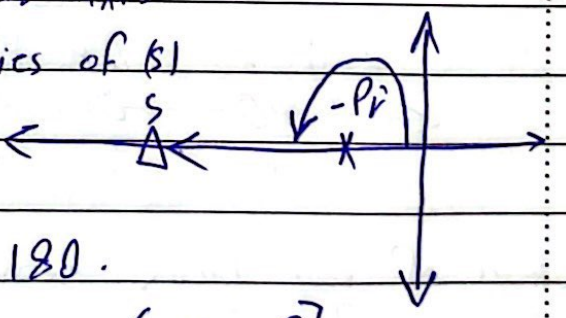
$$|(s+z_j) - 0 = 180.$$

Therefore, root locus exist on $(-\infty, -z_j]$.

* let (s) be a test point on the real axis

since Poles ($-p_i$) of G(s) H(s) lies of (s)

it follow that :-



$$|G(s)H(s| = |Zeros - |Poles$$

$$= 0 - |(s+p_i) = -180.$$

Therefor. root locus exists on $(-\infty, -p_i]$.

NOTEBOOK

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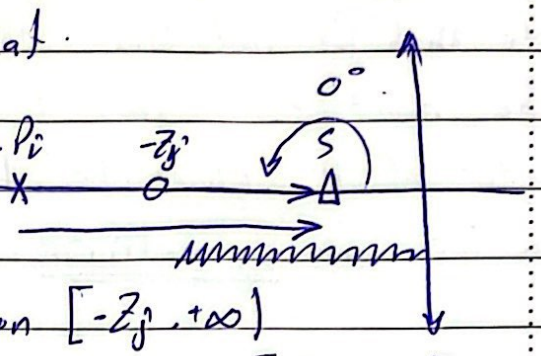
Subject Date No.

Automatic control :-

* Ch 7 :- Root-locus Method :-

* let (s) be a test point (the pole $-p_i$ and zeros $-z_i$) lies to the left of (s). it follows that.

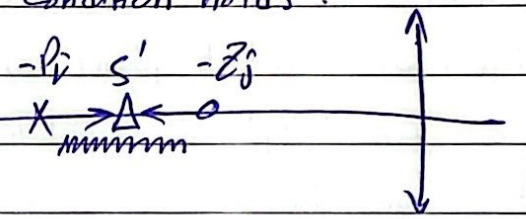
$$\angle G(s)H(s) = \angle \text{Zeros} - \angle \text{Poles}$$
$$= \angle s+z_j - \angle s+p_i$$
$$= 0 - 0 = 0^\circ.$$



Therefore no root locus exists on $[-z_j, +\infty)$

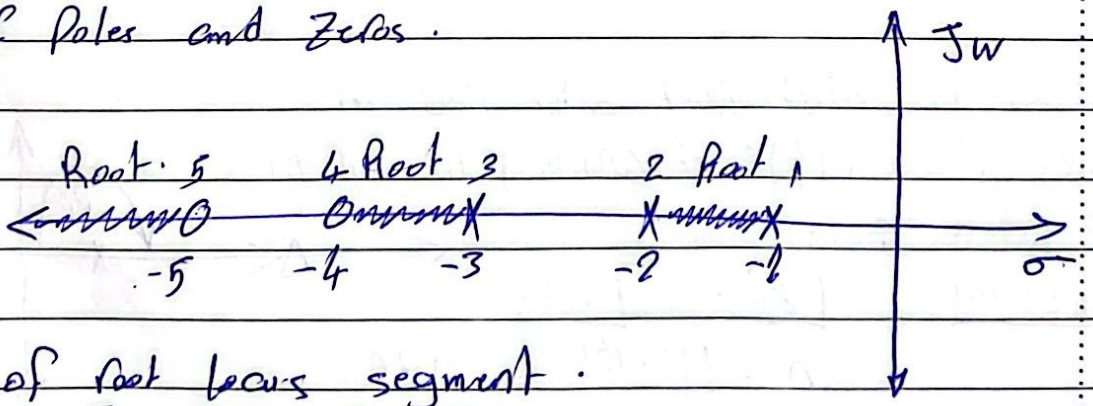
However. by the rule. root locus exists on $[-p_i, -z_j]$ since for the test point s' angle condition holds.

$$\angle G(s)H(s) = \angle \text{Zeros} - \angle \text{Poles}$$
$$= \angle s+z_j - \angle s+p_i$$
$$= 180 - 0 = 180^\circ$$



* Rule :-

Root locii on the real axis will occur to the left of odd number of poles and zeros.



location of root locus segment.

$(-\infty, -5] \cdot [-4, -3] \cdot [-2, -1]$.

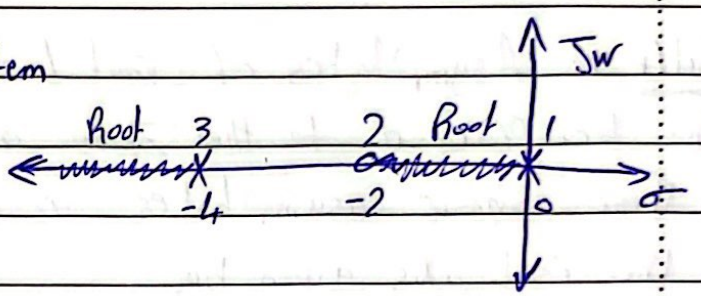
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* Automatic control :-

* Ch 7 :- Root - locus Method :-

* Example:- Second order system

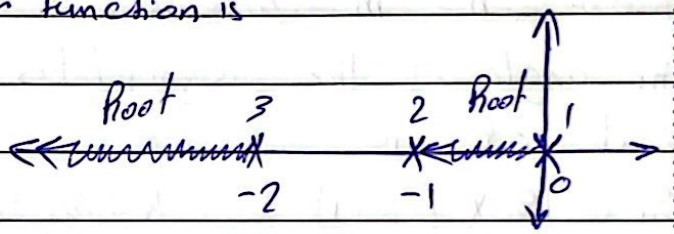
$$G(s)\,H(s) = \frac{K(s+2)}{s(s+4)}$$



* Example:- for a unity FB system

the open loop transfer function is

$$G(s) = \frac{K}{s(s+1)(s+2)}$$



* Steps of constructing Root locus of a system:-

1- Write the characteristic equation of the system in the following standard form

$$\Delta = 1 + K\,\frac{(s+Z_1)(s+Z_2)(s+Z_3)\cdots(s+Z_m)}{(s+P_1)(s+P_2)(s+P_3)\cdots(s+P_n)} = 0.$$

where (K) might be a controller gain or system gain and is the parameter of interest.

2- locate all poles $P_1, P_2 \cdots P_n$ and zeros $Z_1, Z_2 \cdots Z_m$ in s-plane.

3- Determine the root locus segments on the real axis.

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Subject Date No.

* Automatic control :-

* Ch 7 :- Root-locus Method :-

* Rule 1 :- Asymptotes of root loci.

The loci proceed to the zeros at infinity along asymptotes.

⇒ These linear asymptotes are centered at a point on the real axis given by.

$$\sigma_a = \frac{\sum(-P_i) - \sum(-Z_j)}{\text{Poles} \leftarrow n - m \rightarrow \text{Zeros}} = \frac{\sum \text{Poles of GH} - \sum \text{Zero of GH}}{n-m}.$$

⇒ The angle of the asymptotes with respect to the real axis is

$$\theta_a = 180^\circ \frac{(2K+1)}{n-m} \qquad K = (0, \pm 1, \dots, \pm n-m-1)$$

$$\alpha_a = \pm \frac{180(2K+1)}{n-m}$$

* Example :- An open loop transfer function of a unity feed back system is

$$G(s) = \frac{K(s+2)}{s(s+1)(s+3)} \Rightarrow 1 + G(s) = 1 + \frac{K(s+2)}{s(s+1)(s+3)}$$

① Sketch the root locus plot

Zeros → $s = -2$ $\boxed{m=1}$

Poles → $s = 0, s = -1, s = -3$ $\boxed{n=3}$

root loci exist on $[-1, 0]$ and $[-3, -2]$

$$s^3 + 3s^2 + s^2 + 3s + s + 2.$$

$$s^3 + 4s^2 + 4s + 2$$

NOTEBOOK

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Subject Date No.

* Automatic control :-

*Ch7 :- Root locus Method :-

*Example :- .

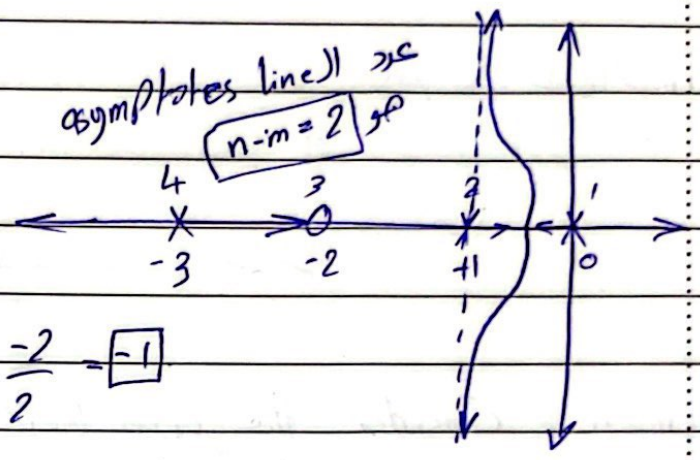
for the asymptotes :-

$$\sigma_a = \frac{\Sigma \text{ Poles} - \Sigma \text{ Zeros}}{n-m}$$

$$= \frac{(-1-3+0) - (-2)}{3-1} = \frac{-2}{2} = \boxed{-1}$$

asymptotes line // x-axis

$n-m=2$



$$\theta_a = 180 * \frac{2K+1}{3-1} = \begin{cases} 90 & K=0 \\ 270 & K = n-m-1 = 3-1-1 = \boxed{1} \end{cases}$$

*Example :- $G(s) = \frac{K}{s(s+1)(s+2)} \Rightarrow 1 + \frac{K}{s(s+1)(s+2)} = 0$

sketch the root locus plot

Zeros → no zeros $\boxed{m=0}$

Poles → $s=0$, $s=-1$, $s=-2$ $\boxed{n=3}$

root loci exist on $[-1, 0]$, and $[-\infty, -2]$

- for the asymptotes

$$\sigma_a = \frac{(-1-2+0) - (0)}{3-0} = \boxed{-1}$$

$\boxed{3} = n-m$ = asymptote line // x-axis

$$\theta_a = 180 * \frac{2K+1}{3-0} = \begin{cases} 60 & K=0 \\ 180 & K=1 \\ 300 & K = n-m-1 = 3-0-1 = \boxed{2} \end{cases}$$

NOTEBOOK

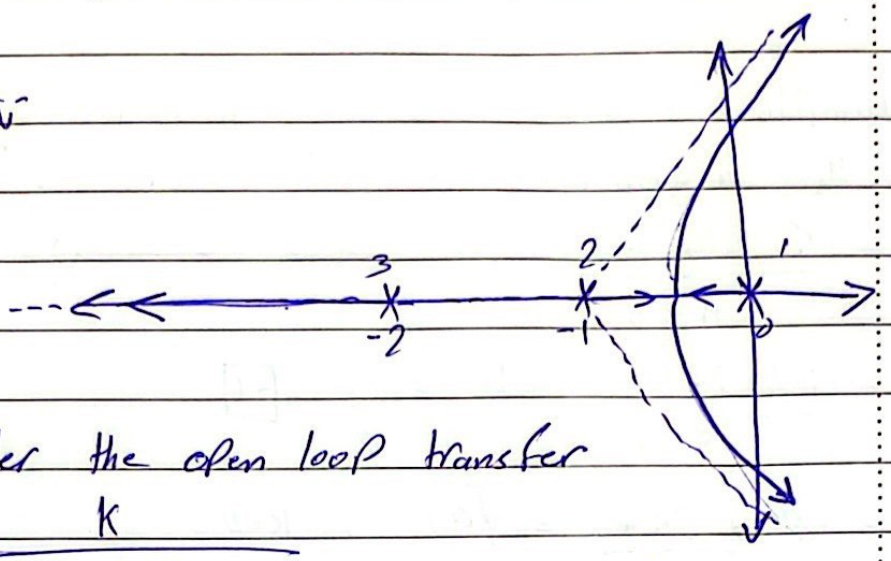
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Subject Date No.

* Automatic control

* Ch 7 :- Root locus Method :-

* Example :- ∴



* Example :- Consider the open loop transfer function $G(s) = \frac{K}{s(s+4)(s^2+4s+20)} \Rightarrow 1 + \frac{K}{s(s+4)(s^2+4s+20)} = 0$

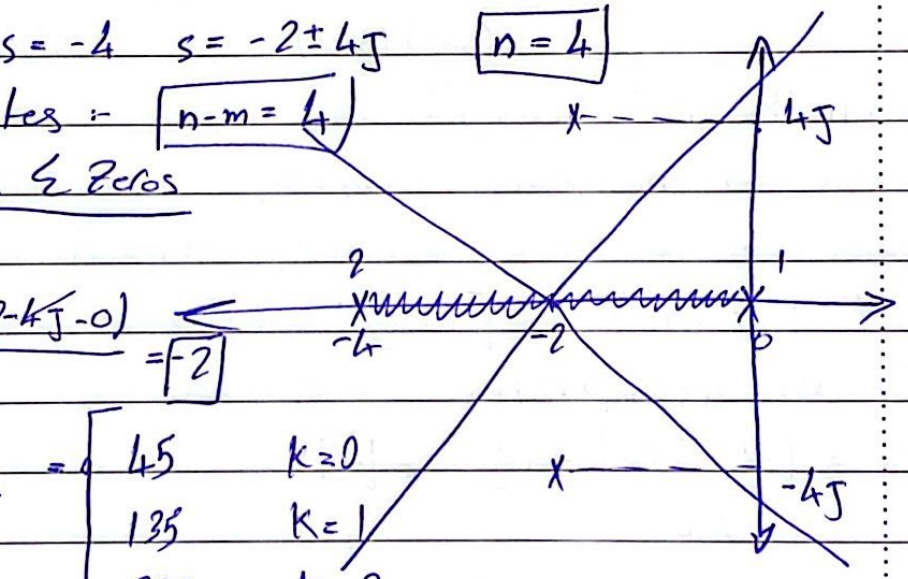
- sketch the root locus plot

Zeros → no zeros $m = 0$.

Poles → $s = 0$, $s = -4$, $s = -2 \pm 4j$ $\boxed{n = 4}$

for the asymptotes :- $\boxed{n - m = 4}$

$$\sigma_a = \frac{\Sigma \text{ Poles} - \Sigma \text{ Zeros}}{n - m} = \frac{(-4 - 2 + 4j - 2 - 4j - 0)}{4} = \boxed{-2}$$



$$\theta_a = 180 * \frac{2K+1}{4} = \begin{cases} 45 & K = 0 \\ 135 & K = 1 \\ 225 & K = 2 \\ 315 & K = n - m - 1 = 4 - 0 - 1 = \boxed{3} \end{cases}$$

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bject Date No.

* Automatic control :-

* Ch 7 :- Root - locus Method :-

* Rule 5 :- Break away (break in) Point on the root loci

Rearrange the characteristic equation then find the break away (break in) Points that should result from

$$\boxed{\frac{dK}{ds} = 0}$$

* Example :- Consider the open loop transfer function

$$G(s) = \frac{K}{s(s+1)(s+2)} \Rightarrow 1 + \frac{K}{s(s+1)(s+2)} = 0.$$

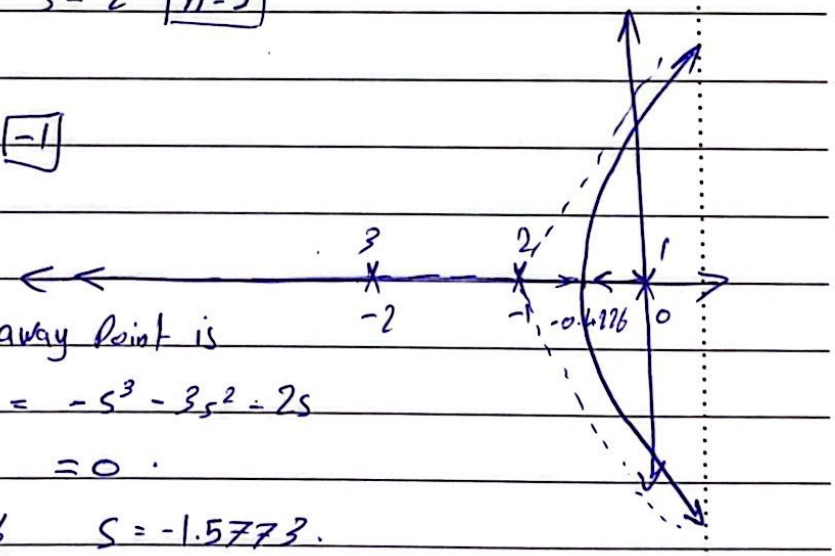
Zeros → ~~s~~ no Zeros $\boxed{m=0}$.

Poles → $s=0$, $s=-1$ $s=-2$ $\boxed{n=3}$

for an asymptotes

$$\sigma_a = \frac{(0-1-2)-(0)}{3-0} = \boxed{-1}$$

$\theta_a = \{60, 180, 30\}$



By rule 4 :- Break away Point is

$$K = -[s(s+1)(s+2)] = -s^3 - 3s^2 - 2s$$

$$\frac{dK}{ds} = -3s^2 - 6s - 2 = 0.$$

$s = -0.4226$ $s = -1.5773$.

✓

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