Automatic control Introduction

Lecture 1

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Objectives

- After this presentation you will be able to:

- Explain the function of an automatic control system.
- Identify a block diagram representation of a physical system
- Explain the difference between an open loop and closed loop control system
- Understand the components of control systems

What is CONTROL?

- Make some object (called ***system, or plant)*** behave as we desire.
- Imagine "control" around you!
    - Room temperature control
    - Car driving
    - Voice volume control
    - Balance of bank account
    - "Control" (move) the position of the pointer
    - etc.

What is control system??

- A system Controlling the operation of another system.
- A system that can regulate itself and another system.
- A control System is a device, or set of devices to manage, command, direct or regulate the behaviour of other device(s) or system(s).

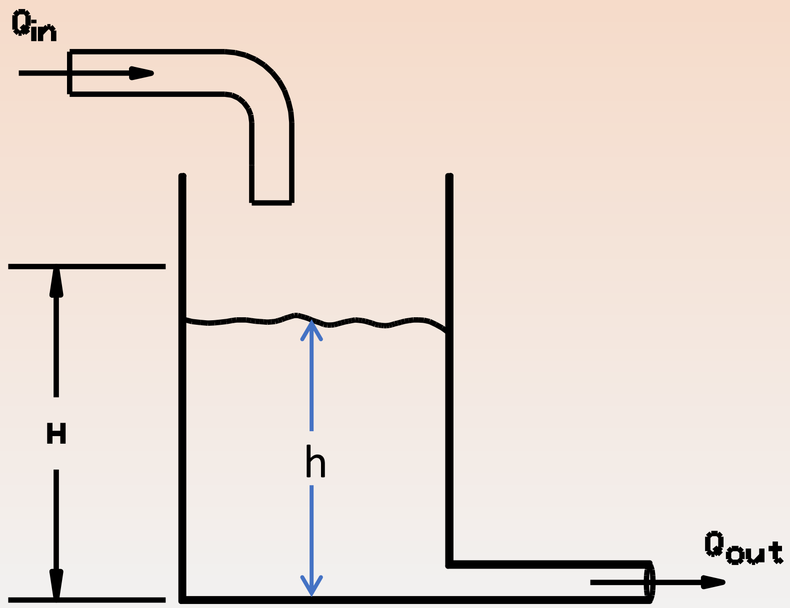
Why AUTOMATIC CONTROL?

- Not manual!
- Why do we need automatic control?
  - Convenient (room temperature, laundry machine)
  - Dangerous (hot/cold places, space, bomb removal)
  - Impossible for human (nanometer scale precision positioning, work inside the small space that human cannot enter, huge antennas control, elevator)
  - It exists in nature. (human body temperature control)

The Control Problem

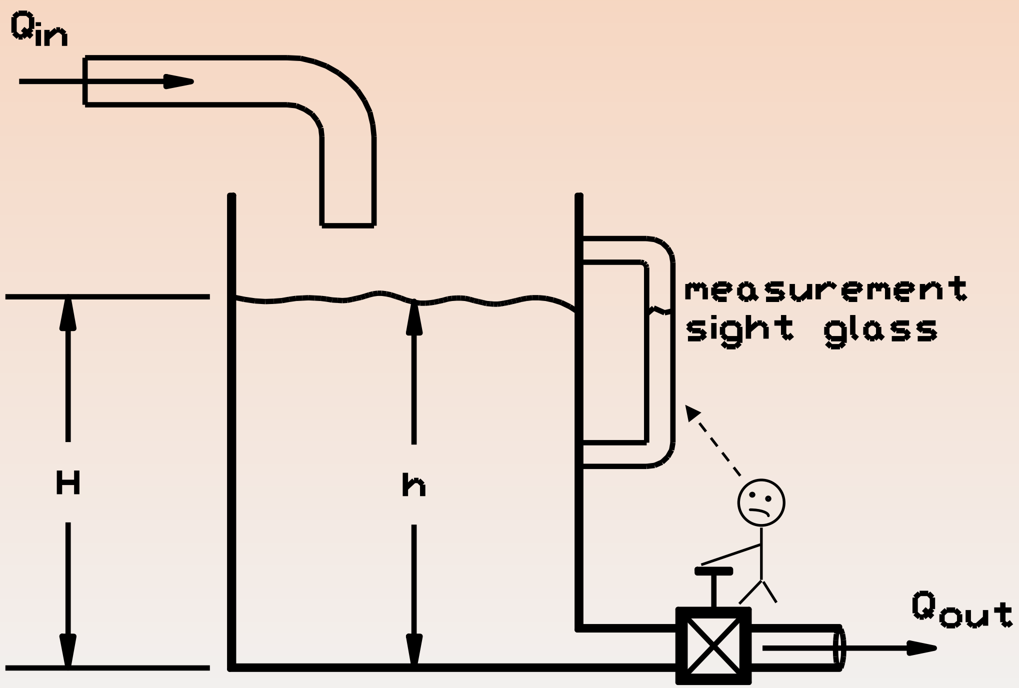
Fundamental Control Concepts

Maintain a variable of process at a desired value while rejecting the effects of outside disturbances by manipulating another system variable.



Q_{out} depends on h
If $Q_{out} = Q_{in}$, h constant
$Q_{out} > Q_{in}$, tank empties
$Q_{out} < Q_{in}$, tank overflows

Basic Subsystems of Control



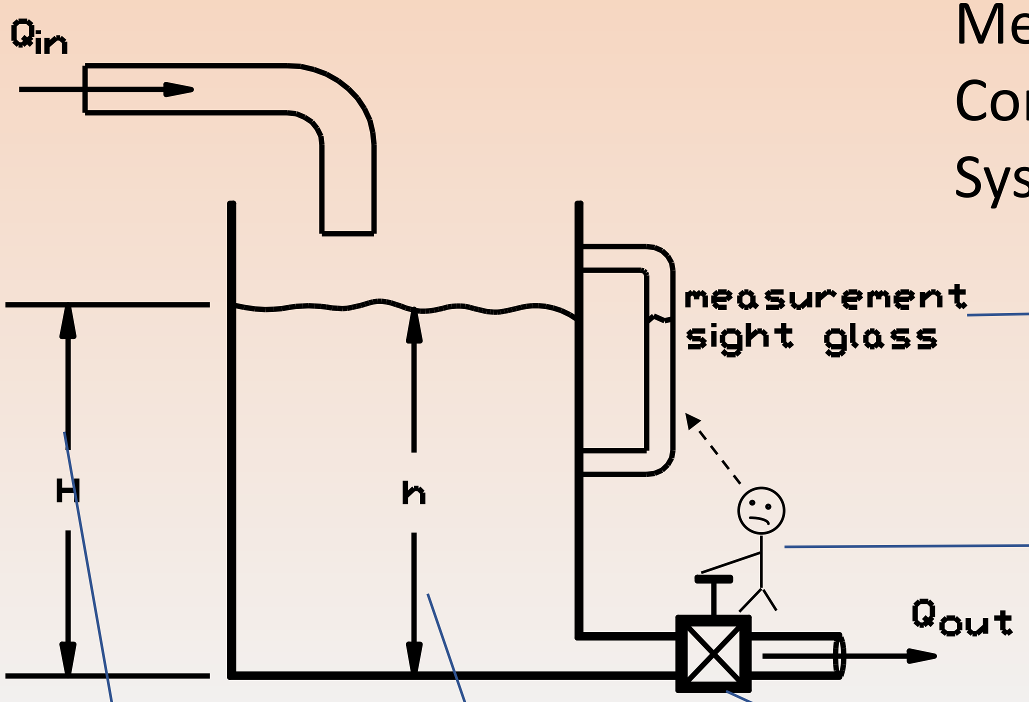
Basic Subsystems of Control

Feedback Control Subsystems

Measurement

Control decision

System modification



Measurement- sight glass

Control Decision
Human adjusts Q_{out}
to maintain h =H

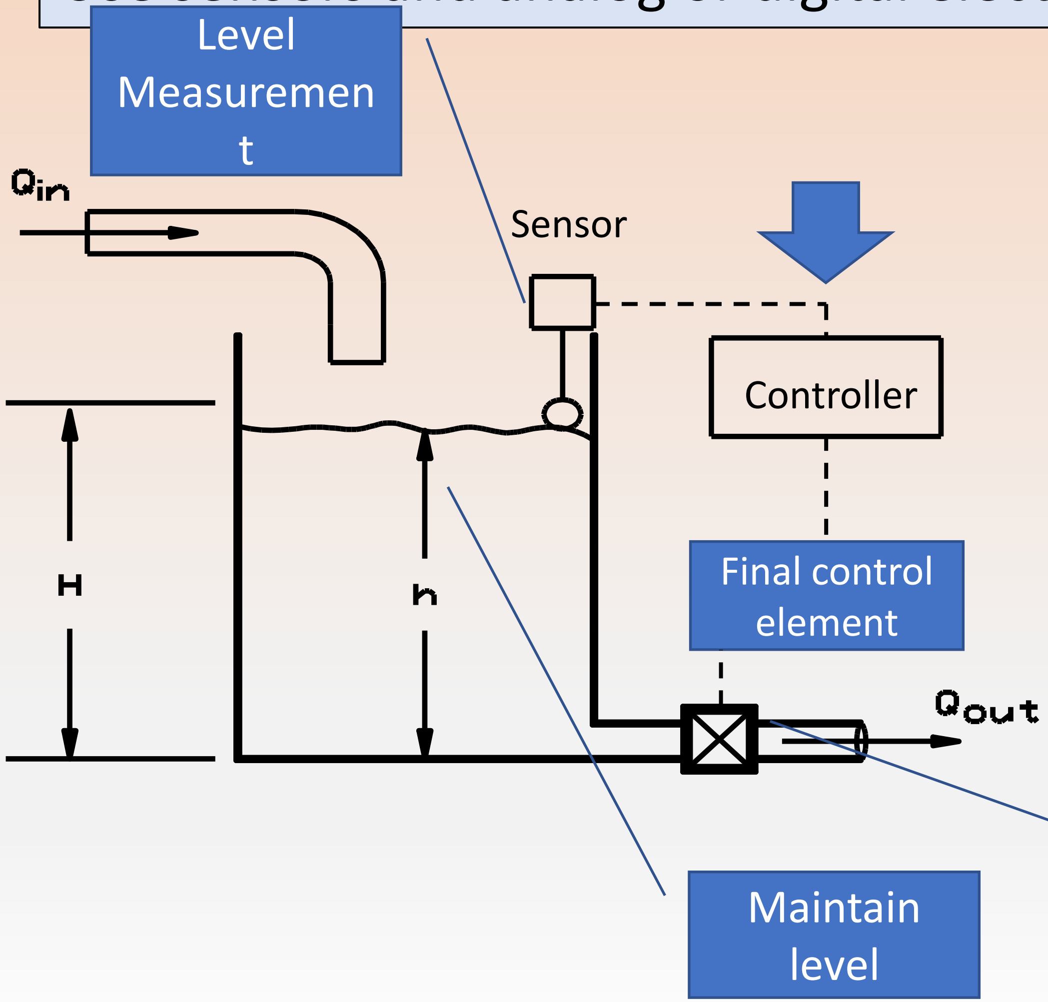
Reference (setpoint) Input
h = control variable

Process- Maintain tank level

Final Control Element
Valve

Automatic Control Systems

Use sensors and analog or digital electronics to monitor and adjust system



<u>Elements of Automatic Control</u>

Process – single or multiple variables

Measurement – sensors

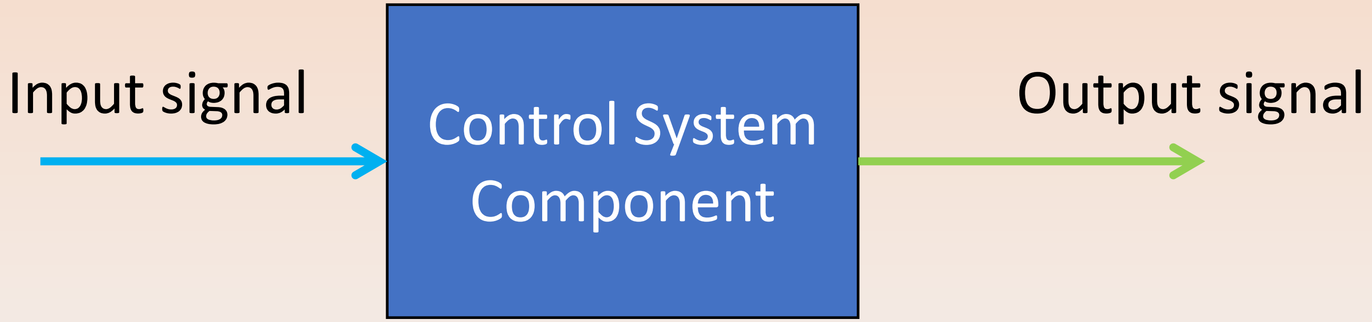
Error Detection – compare H to h

Controller – generate corrections

Final Control Element – modify process

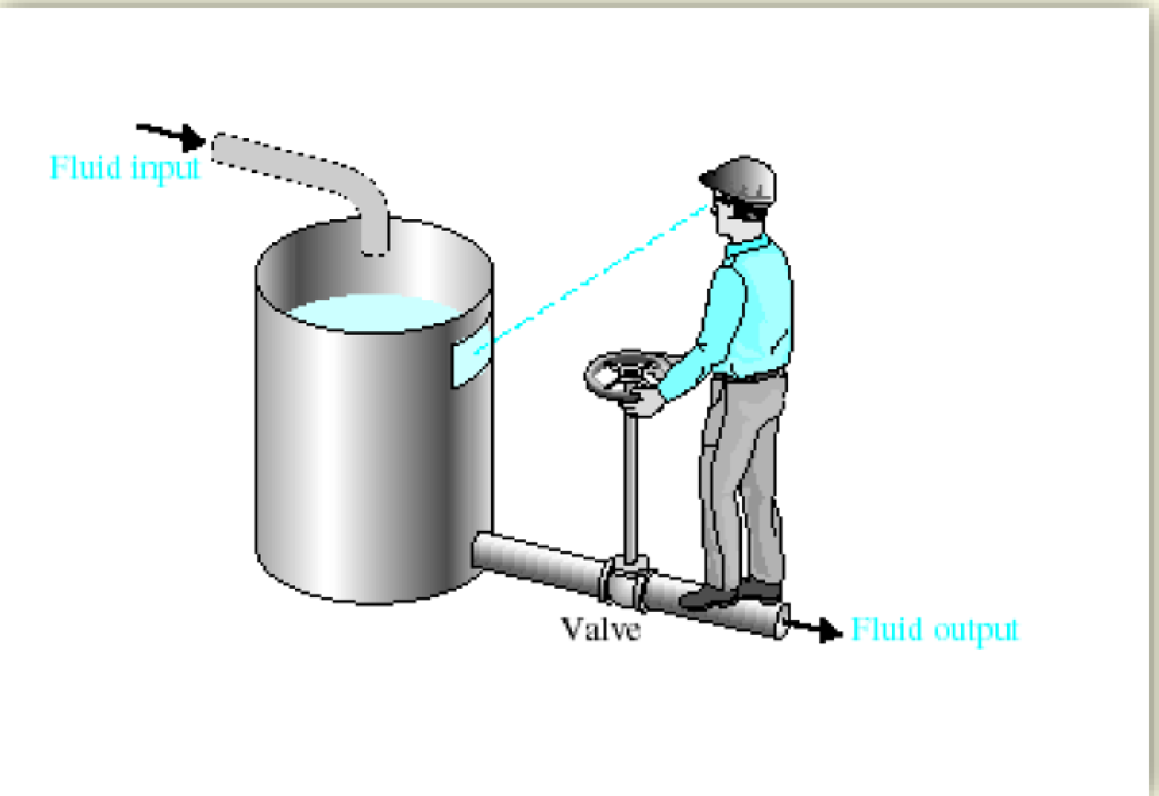
Block Diagrams

Automatic control systems use mathematical descriptions of subsystems to reduce complex components to inputs and outputs



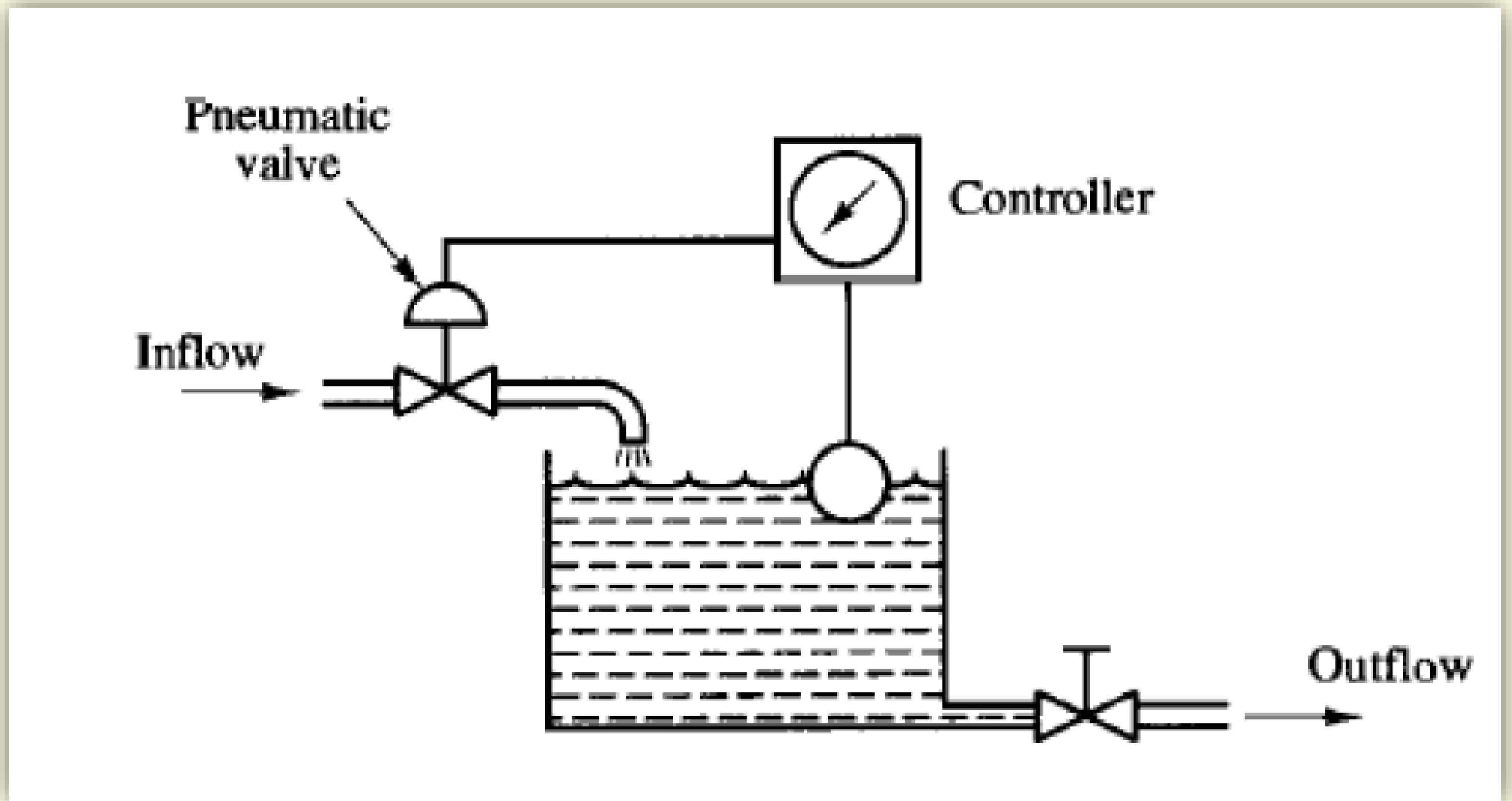
Signals flow between components in system based on arrow direction

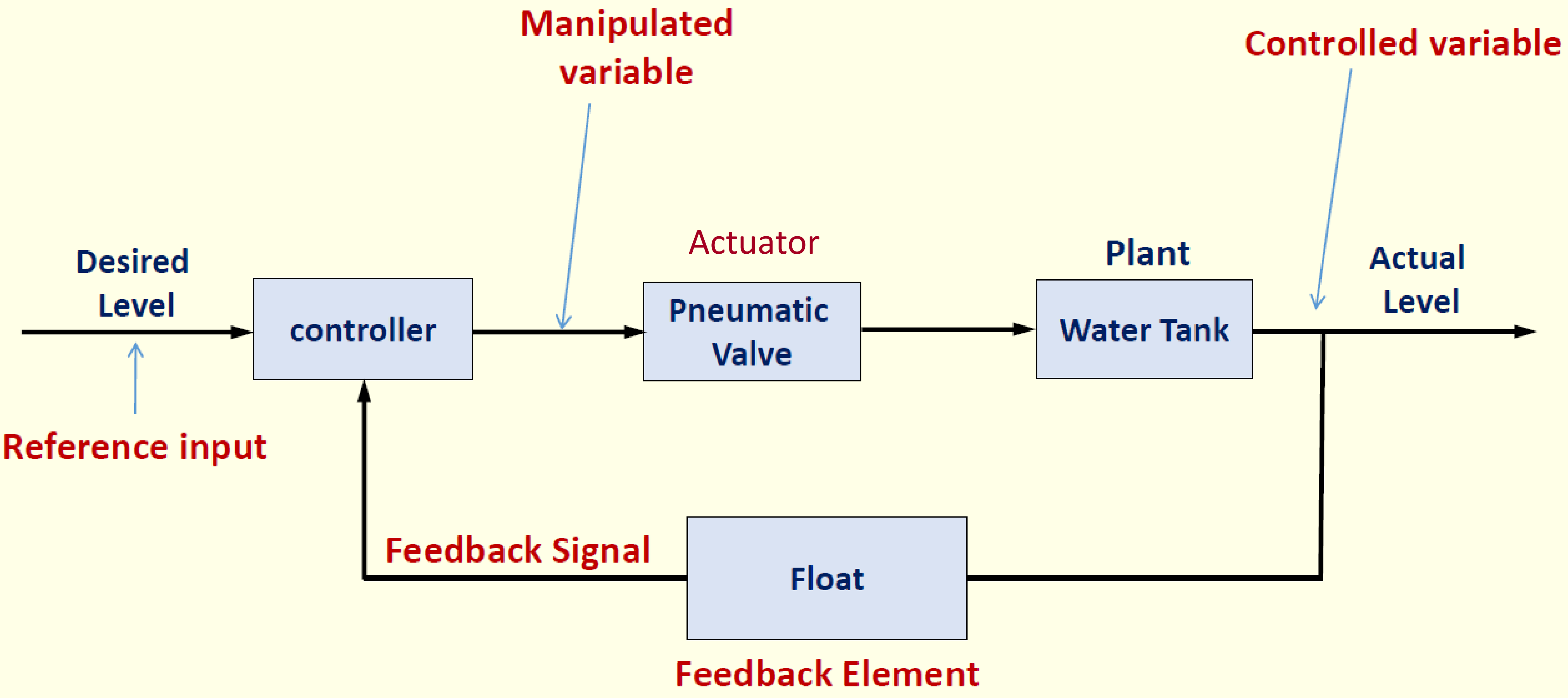
Manual Liquid-level control system



A manual Control Systems for regulating the level of fluid in a tank by adjusting the output valve. The operator views the level of fluid through a port in the side of the tank.

Automatic Liquid-level control system




Manipulated variable
Controlled variable
Actuator
Desired Level
controller
Pneumatic Valve
Plant
Water Tank
Actual Level
Reference input
Feedback Signal
Float
Feedback Element

Example: laundry machine

- A laundry machine washes clothes, by setting a program.
- A laundry machine does not measure how clean the clothes become.
- Control without measuring devices (sensors) are called ***open-loop control***.

Open-loop control systems

- Open-loop control systems. Those systems in which the output has no effect on the control action are called open-loop control systems.
- In other words, in an open-loop control system the output is neither measured nor fed back for comparison with the input.
- In the presence of disturbances, an open-loop control system will not perform the desired task.
- Open-loop control can be used, in practice, only if the relationship between the input and output is known and if there are neither internal nor external disturbances.

Open-loop control systems

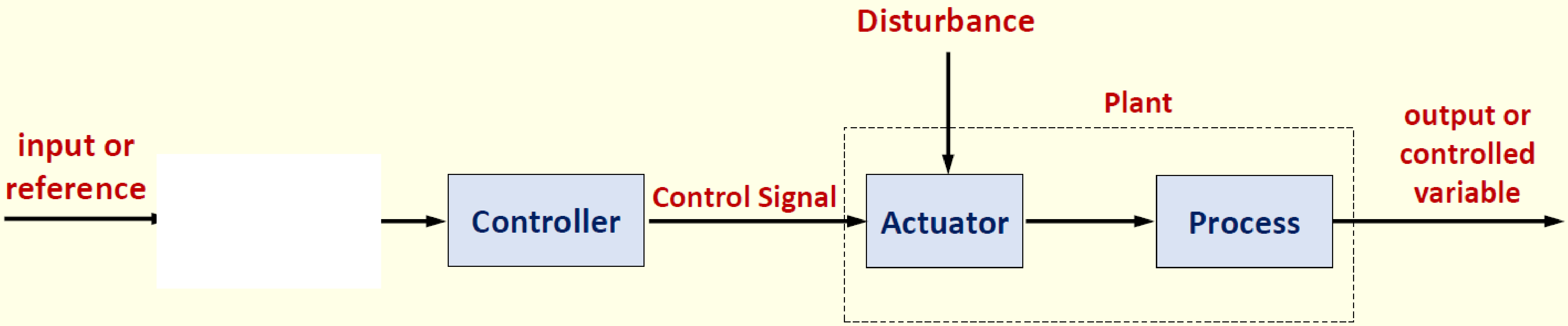


Fig. 1.2 An open-loop system

Open-loop control systems

Advantages:

- Simple construction, ease of maintenance, and less expensive.
- There is no stability concern.
- Convenient when output is hard to measure or measuring the output precisely is economically not feasible. (For example, in the washer system, it would be quite expensive to provide a device to measure the quality of the washer's output, cleanliness of the clothes).

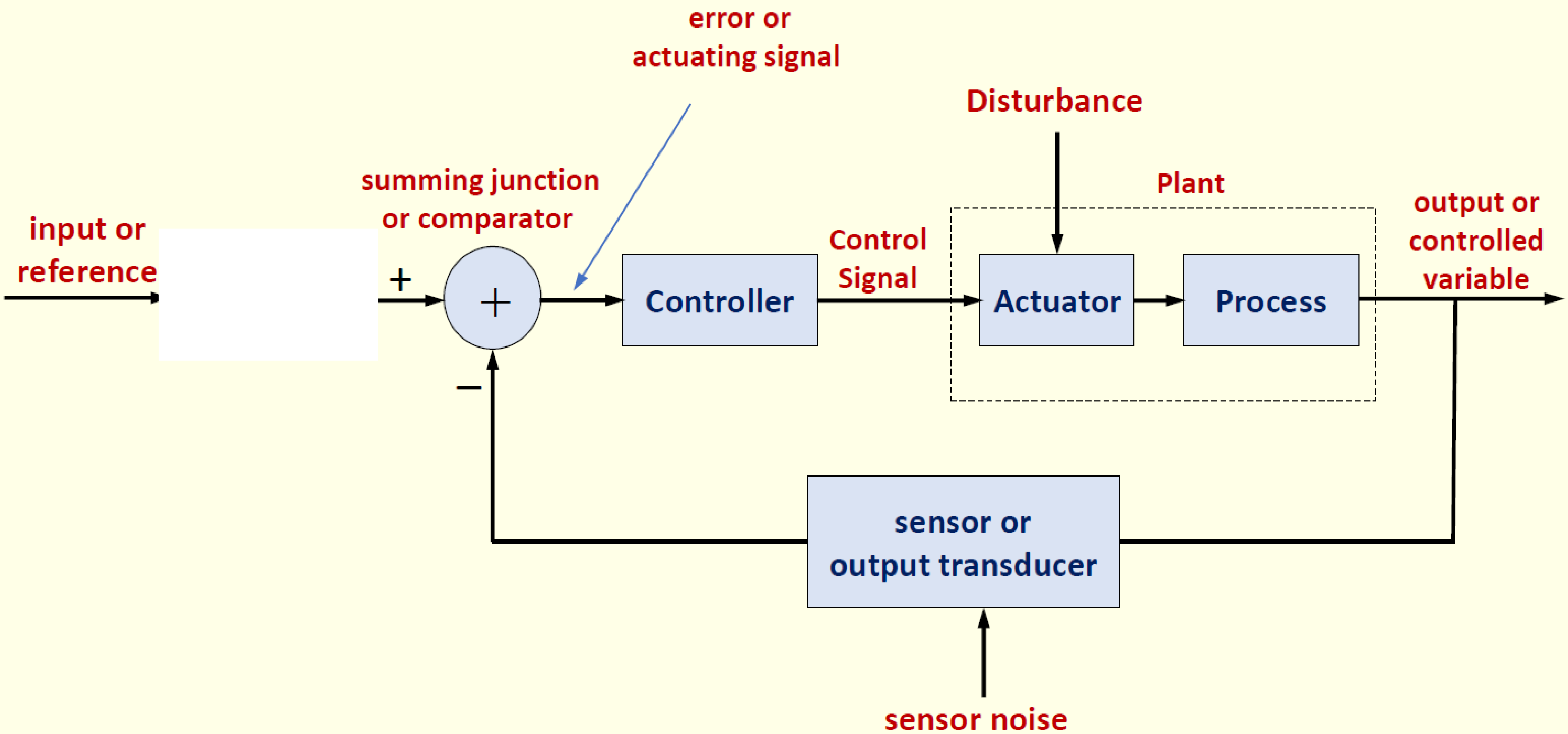
Disadvantages:

- Disturbances and changes in calibration cause errors, and the output may be different from what is desired.
- Recalibration is necessary from time to time.

Closed-loop control systems

- Closed-loop control systems. Feedback control systems are often referred to as closed-loop control systems.
- In practice, the terms feedback control and closed-loop control are used interchangeably.
- In a closed-loop control system the actuating error signal, which is the difference between the input signal and the feedback signal , is fed to the controller so as to reduce the error and bring the output of the system to a desired value.

Closed-loop (feedback) control



Closed-loop control systems

Advantages:

- High accuracy
- Not sensitive to disturbance
- Controllable transient response
- Controllable steady state error

Disadvantages:

- More Complex, and More Expensive.
- Possibility of instability.
- Need for output measurement.
- Recalibration is necessary from time to time.

Definitions.

- **Systems** - A system is a combination of components that act together and perform a certain objective.
- **Control System** - An interconnection of components forming a system configuration that will provide a desired response.
- **Plants** - A plant may be a piece of equipment, perhaps just a set of machine parts functioning together, the purpose of which is to perform a particular operation.
- **Process** - The device, plant, or system under control. The input and output relationship represents the cause-and-effect relationship of the process.

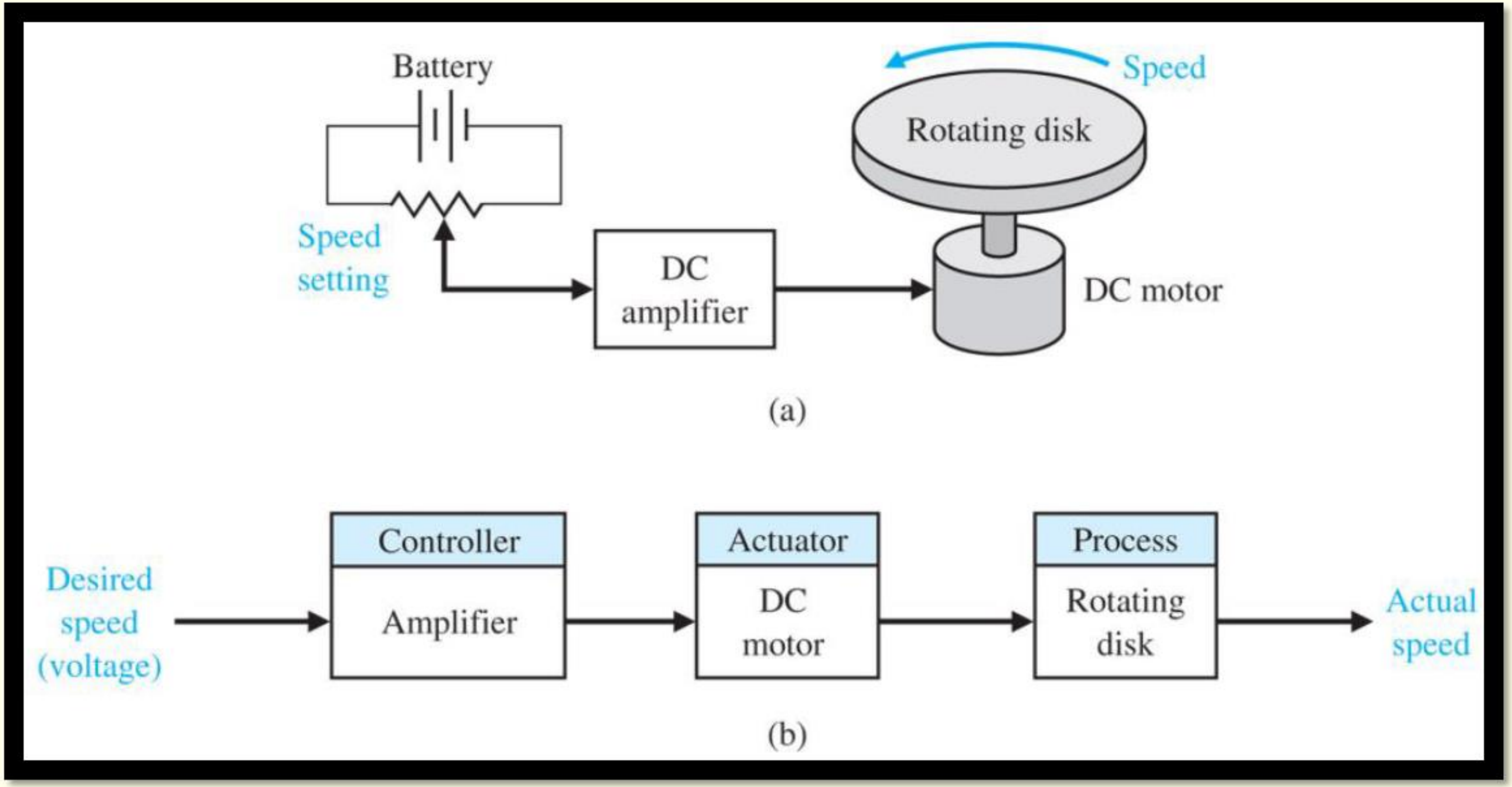
Definitions.

- **Disturbances** - A disturbance is a signal that tends to adversely affect the value of the output of a system. If a disturbance is generated within the system, it is called internal, while an external disturbance is generated outside the system and is an input.
- **Controlled Variable** - is the quantity or condition that is measured and controlled. the controlled variable is the output of the system.
- **The Manipulated Variable** - is the quantity or condition that is varied by the controller so as to affect the value of the controlled variable. Normally,
- **Control** - means measuring the value of the controlled variable of the system and applying the manipulated variable to the system to correct or limit deviation of the measured value from a desired value.

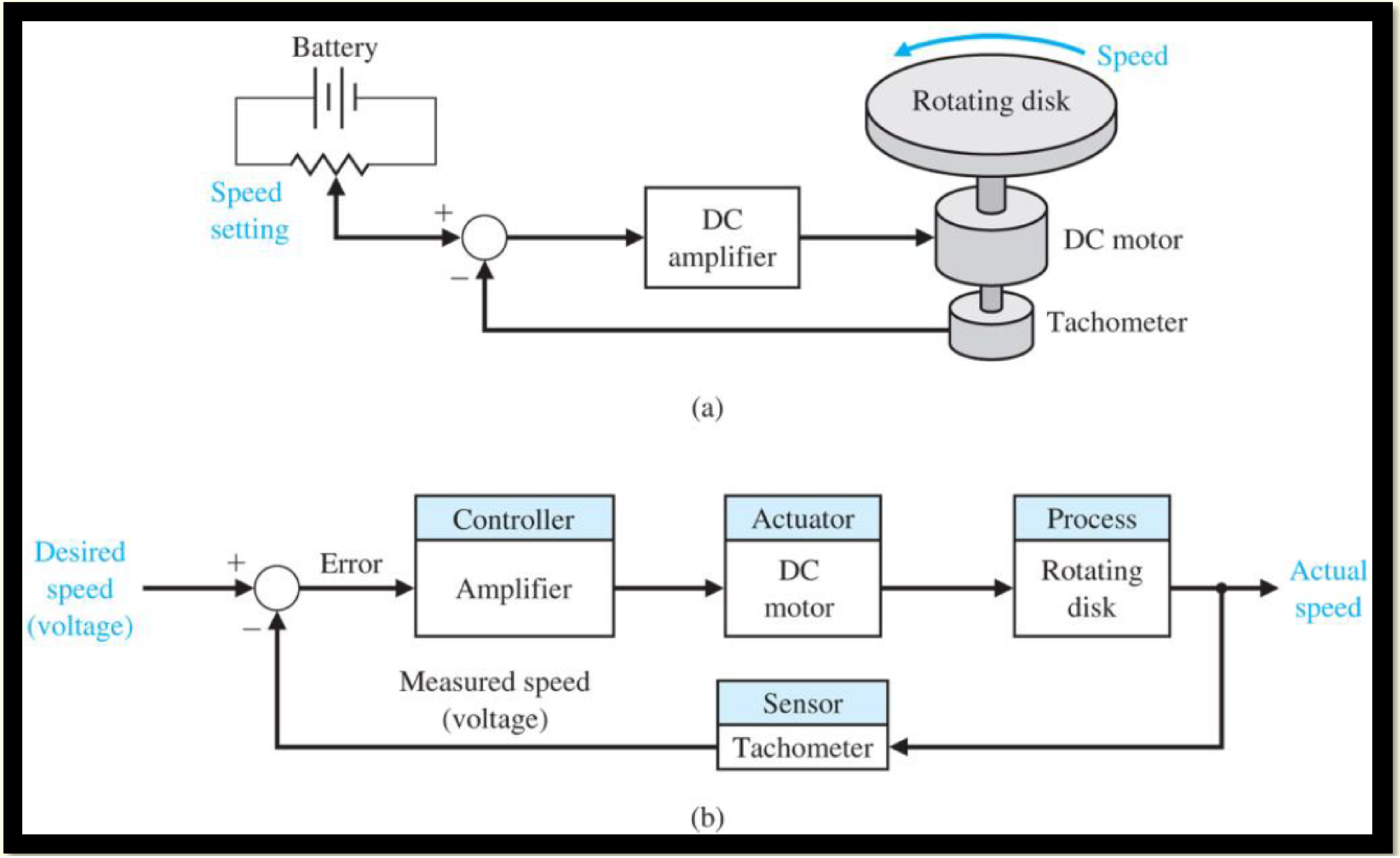
Definitions.

- **Feedback Control** - Feedback control refers to an operation that, in the presence of disturbances, tends to reduce the difference between the output of a system and some reference input and that does so on the basis of this difference.

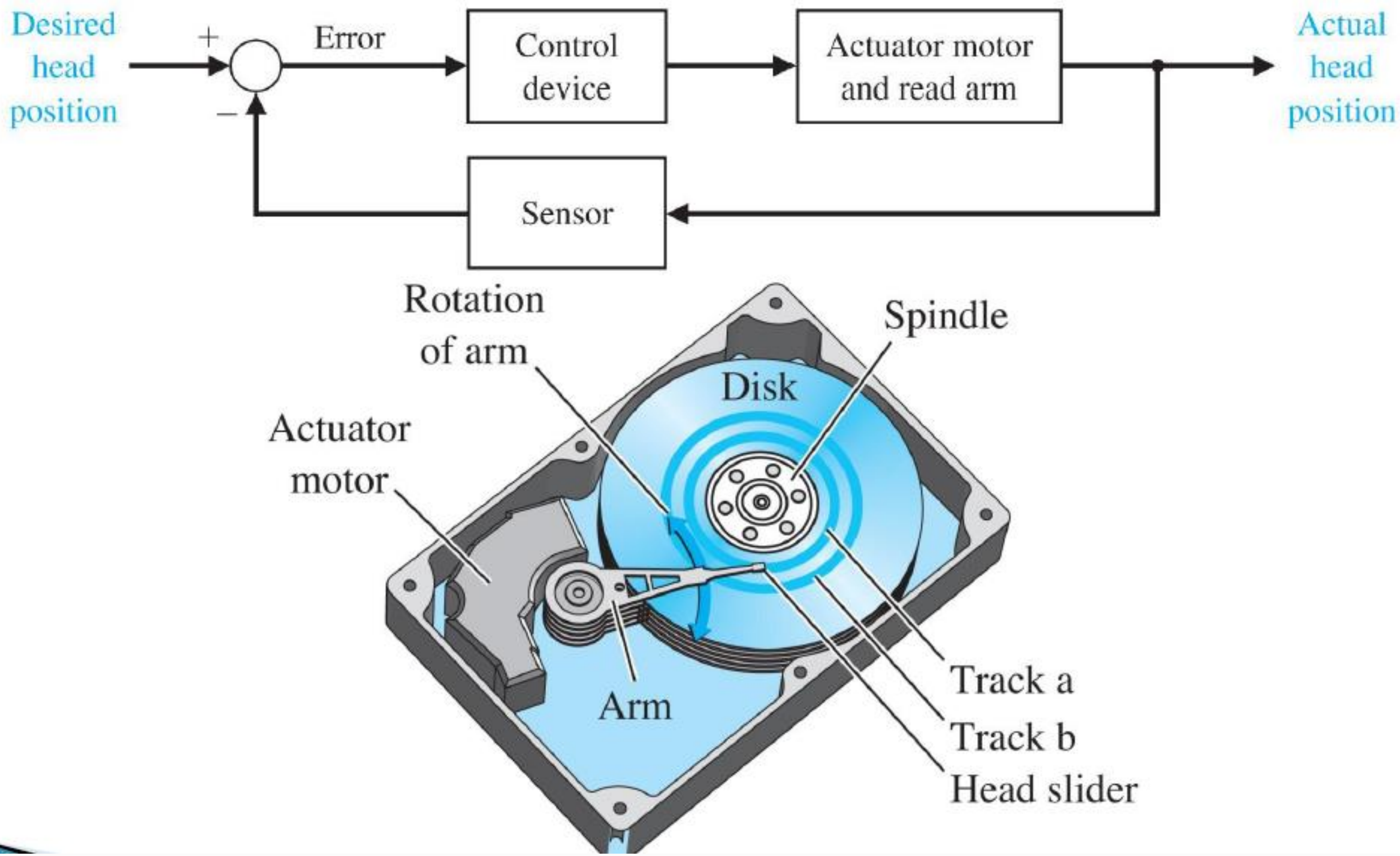
CD player speed control: Open-Loop



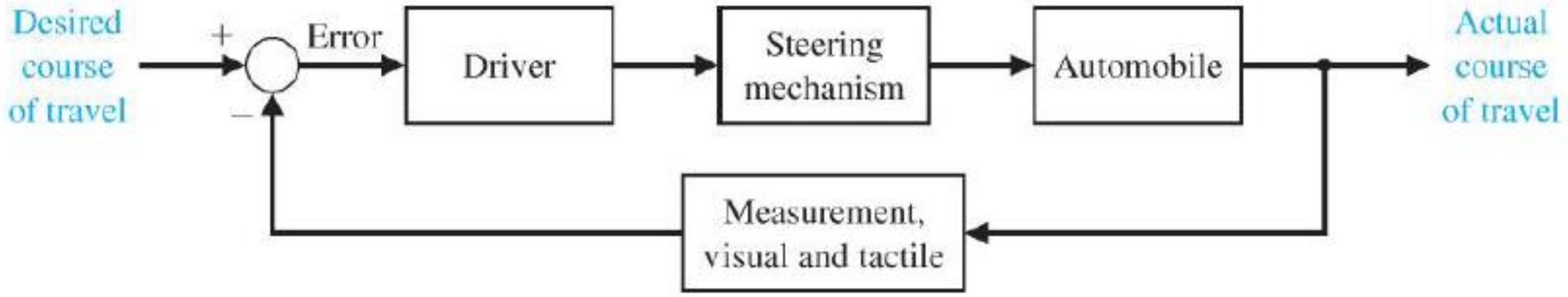
CD player speed control: Closed-Loop



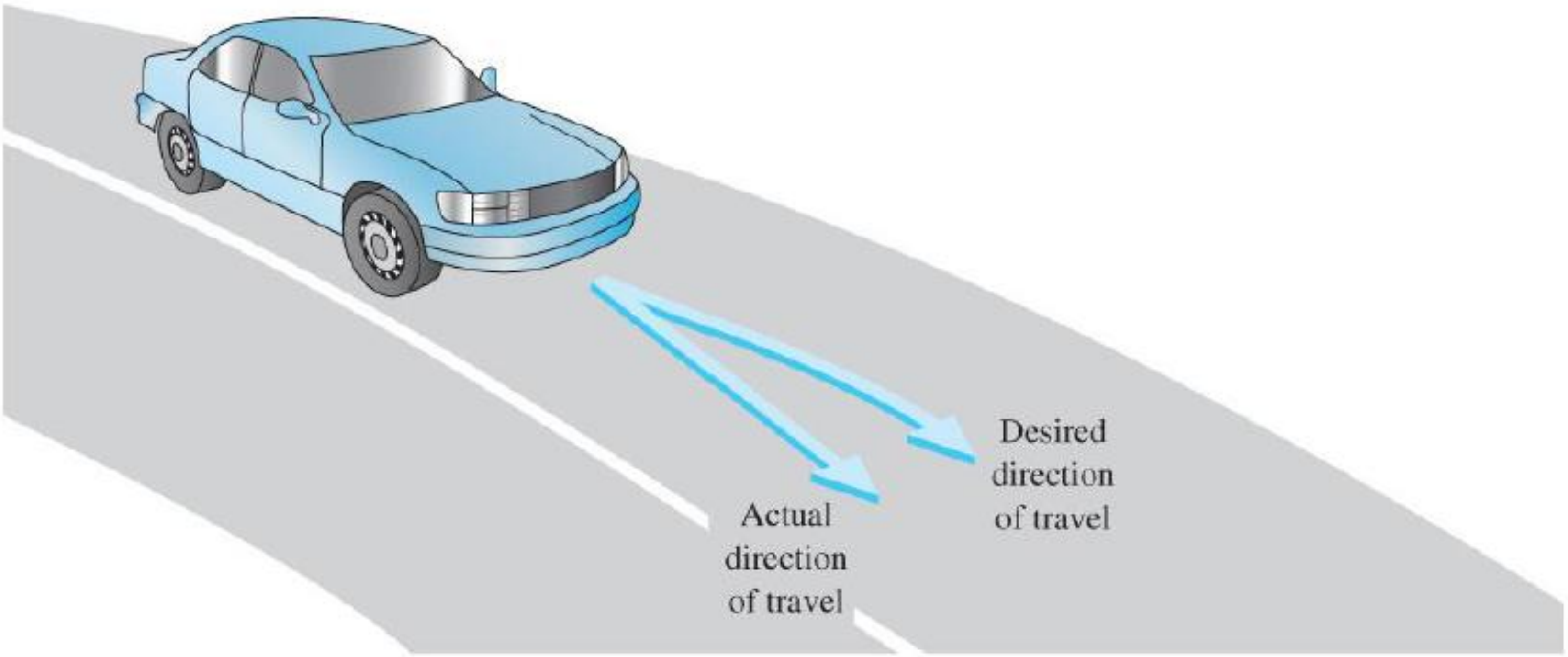
Example: Disk Drive



Example: Feedback in everyday life

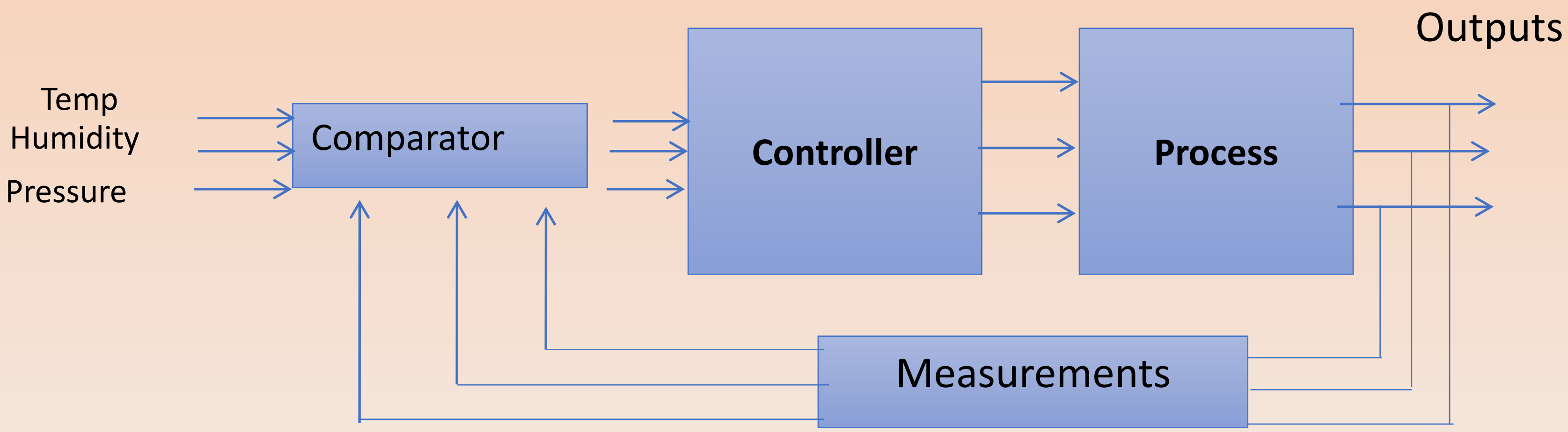


(a)

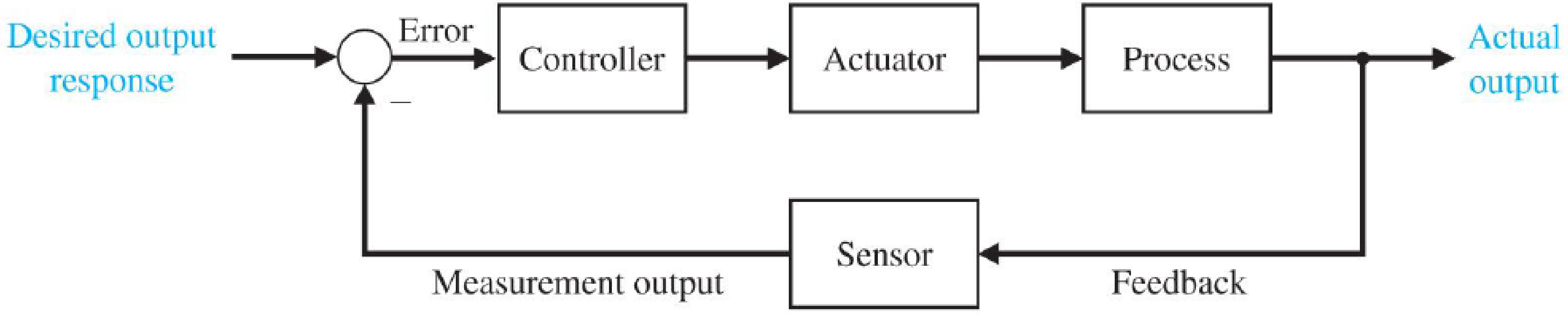


Types of Control System

Multivariable Control System



Control Systems Objectives



- Stabilizing closed loop system
- Accuracy
- Achieving proper transient and steady-state response
- Reduction of sensitivity to process parameters
- Disturbance rejection
- Performance and robustness

The control system design process

1. Establishment of goals, variables to be controlled, and specifications.
2. System definition and modeling.
3. Control system design, simulation, and analysis.
4. If the performance meets the specifications, then finalize the design.
5. Otherwise iterate.

THANK YOU

Automatic Control
CH2-Part 1
Mathematical Modling

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Before starting

- Mathematical models of physical systems are key elements in the design and analysis of control systems.
- We will consider electrical and mechanical systems
- Obtain the input-output relationship for components and subsystems of the system in the form of transfer functions using Laplace transforms.
- Forming Different graphical representations of the system model (Block diagram and signal flow).

Modeling of electrical and mechanical systems

Basic Elements of Electrical Systems

- The time domain expression relating voltage and current for the resistor is given by Ohm's law

$$v_R(t) = i_R(t)R$$

Basic Elements of Electrical Systems

- The time domain expression relating voltage and current for the Capacitor is given as:

$$v_c(t) = \frac{1}{C}\int i_c(t)dt$$

Basic Elements of Electrical Systems

- The time domain expression relating voltage and current for the inductor is given as:

$$v_L(t) = L\frac{di_L(t)}{dt}$$

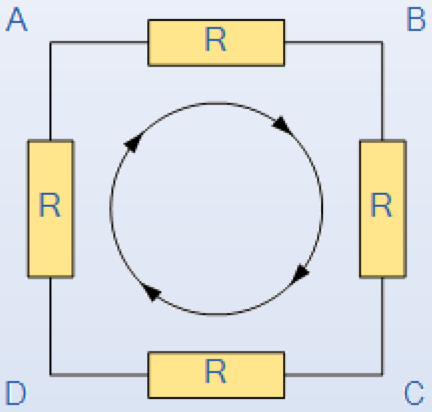
V-I and I-V relations

Component	Symbol	V-I Relation	I-V Relation
Resistor		$v_R(t) = i_R(t)R$	$i_R(t) = \frac{v_R(t)}{R}$
Capacitor		$v_c(t) = \frac{1}{C}\int i_c(t)dt$	$i_c(t) = C\frac{dv_c(t)}{dt}$
Inductor		$v_L(t) = L\frac{di_L(t)}{dt}$	$i_L(t) = \frac{1}{L}\int v_L(t)dt$

Important laws:

KVL

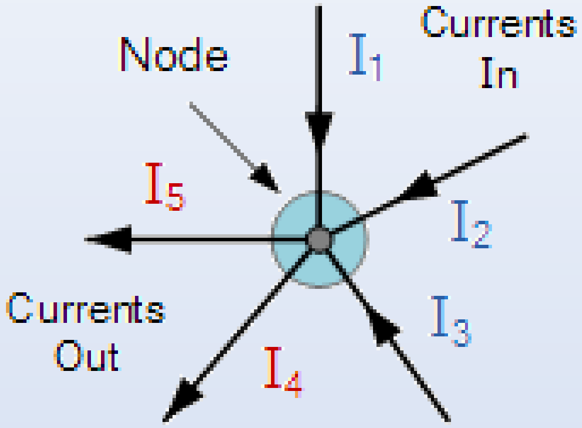
The sum of all the Voltage
Drops around the loop
is equal to Zero



$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$

KCL

Currents Entering the Node
Equals
Currents Leaving the Node



$I_1 + I_2 + I_3 + (-I_4 + -I_5) = 0$

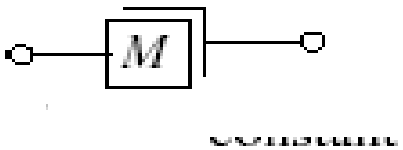
Basic Elements of Translational Mechanical Systems

Translational Spring

i)

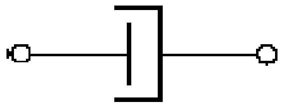
Translational Mass

ii)



Translational Damper

iii)

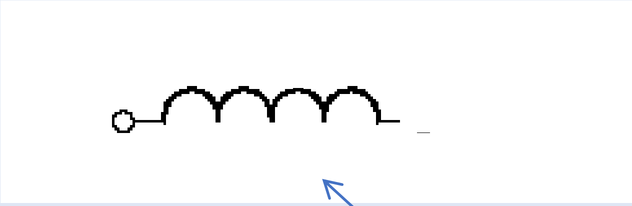


Translational Spring

- A translational spring is a mechanical element that can be deformed by an external force such that the deformation is directly proportional to the force applied to it.

Translational Spring

i)

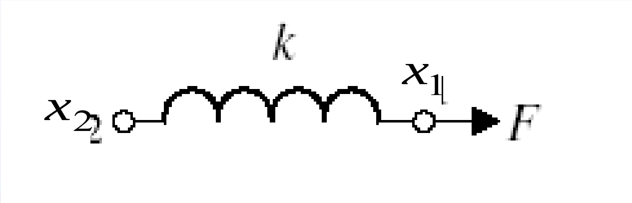


Circuit Symbols

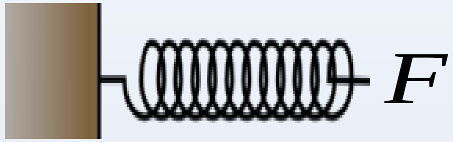
Translational Spring

Translational Spring

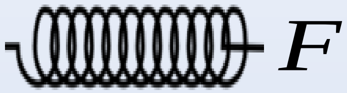
- If *F* is the applied force



- Then x_1 is the deformation if $x_2 = 0$



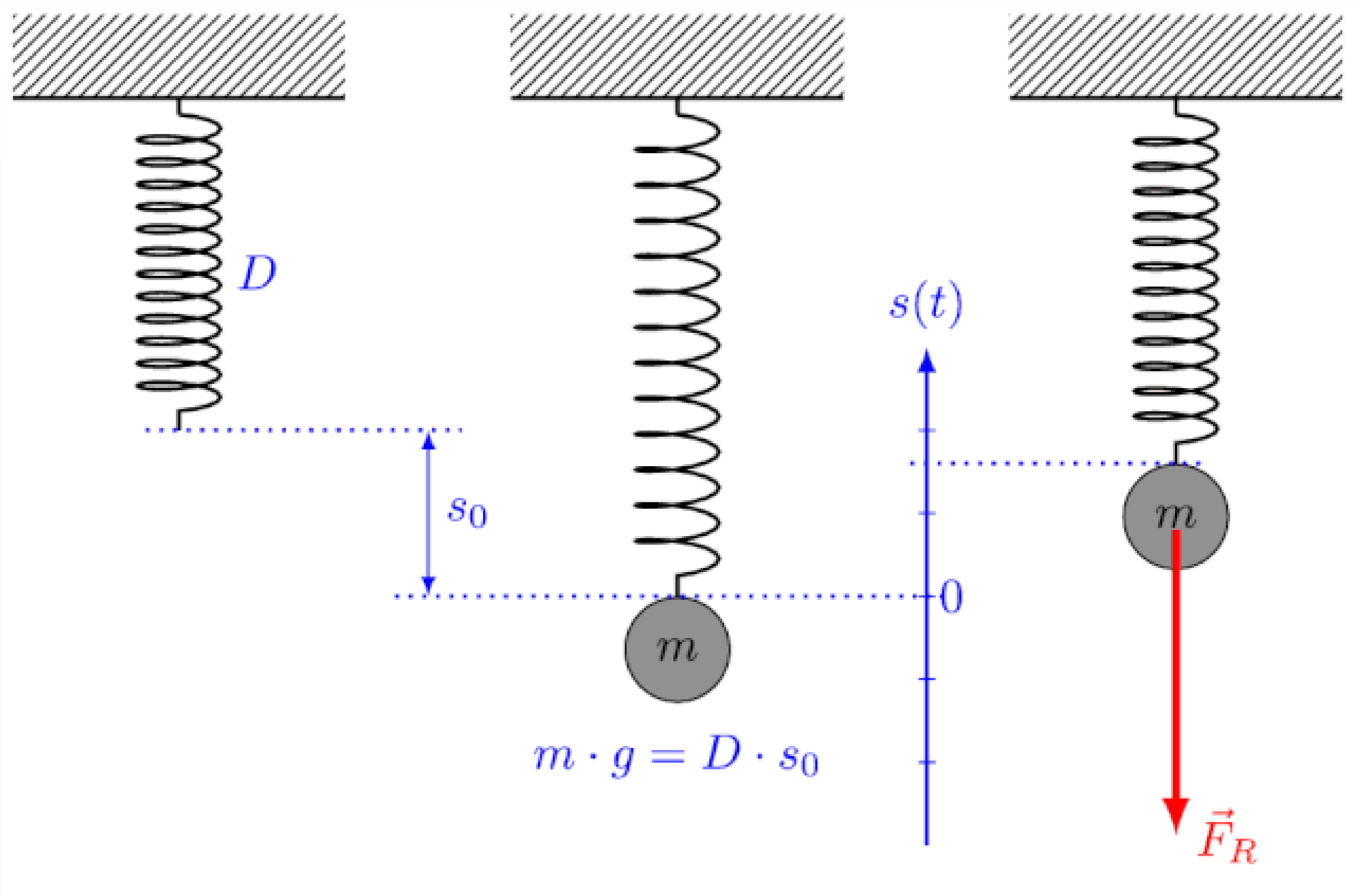
- Or $(x_1 - x_2)$ is the deformation.



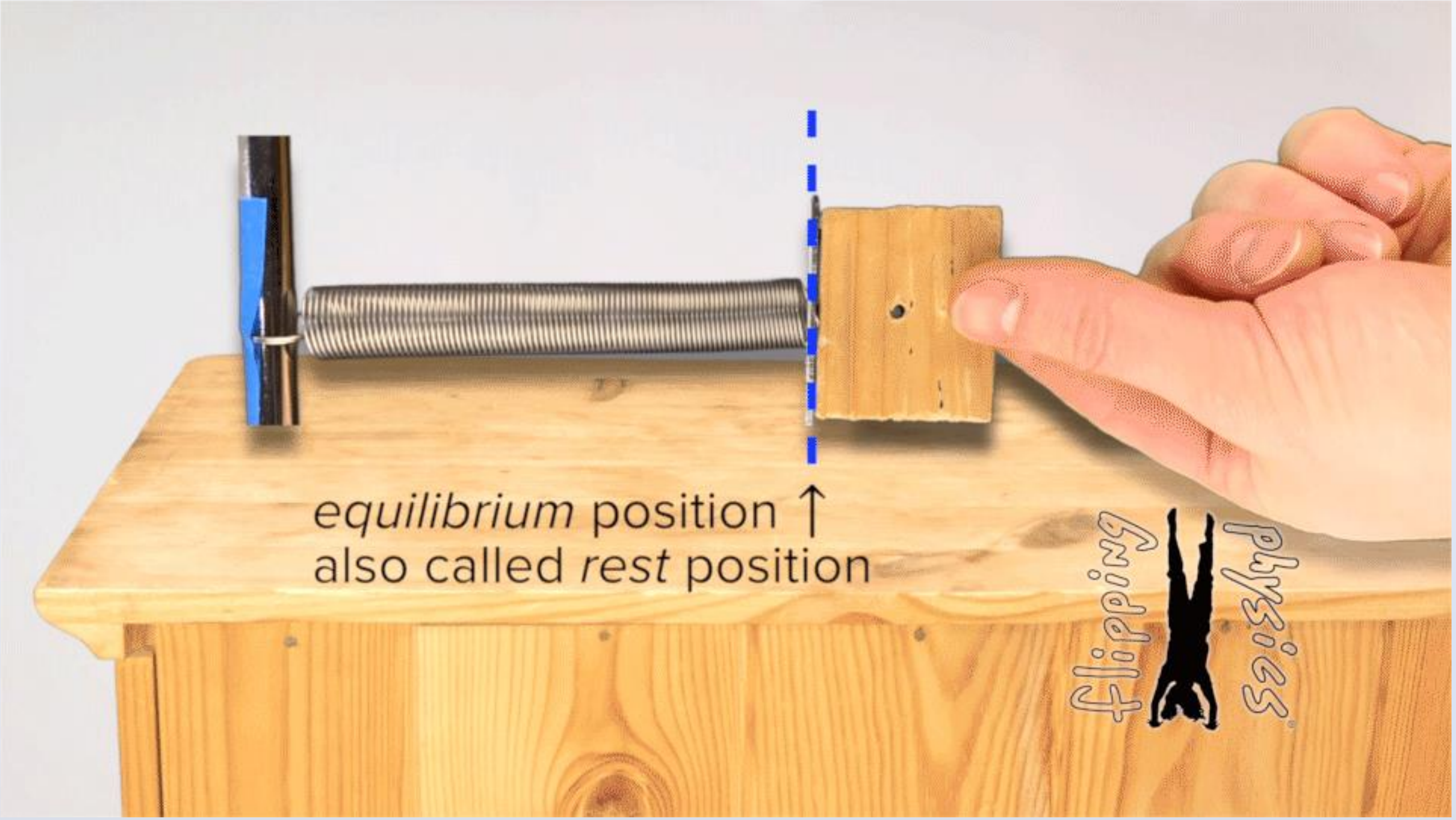
- The equation of motion is given as

$$F = k(x_1 - x_2)$$

- Where k is stiffness of spring expressed in N/m



D
s(t)
s0
0
m
m
m · g = D · s0
F_R


equilibrium position ↑
also called rest position
flipping physics

Translational Mass

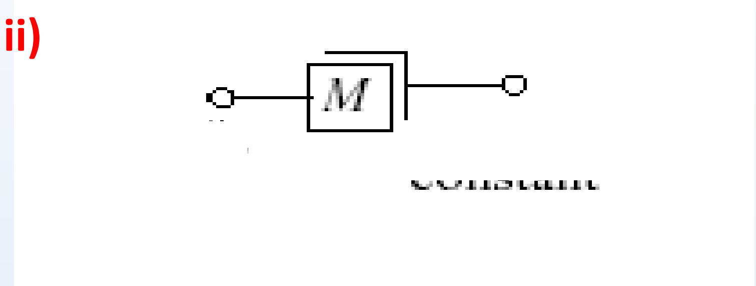
- Translational Mass is an inertia element.

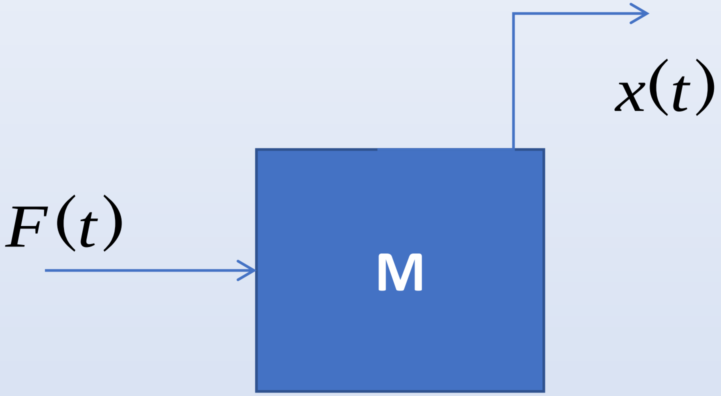
- A mechanical system without mass does not exist.

- If a force *F* is applied to a mass and it is displaced to *x* meters then the relation b/w force and displacements is given by Newton's law.

$$F = M\ddot{x}$$

Translational Mass





Translational Damper

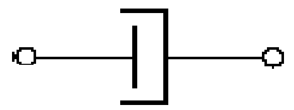
- When the viscosity or drag is not negligible in a system, we often model them with the damping force.

- All the materials exhibit the property of damping to some extent.

- If damping in the system is not enough then extra elements (e.g. Dashpot) are added to increase damping.

Translational Damper

iii)



Common Uses of Dashpots

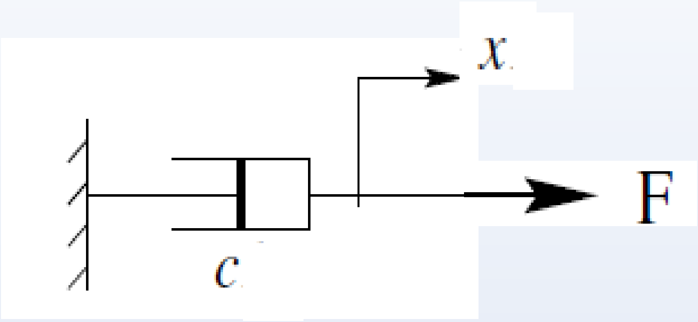
Door Stoppers

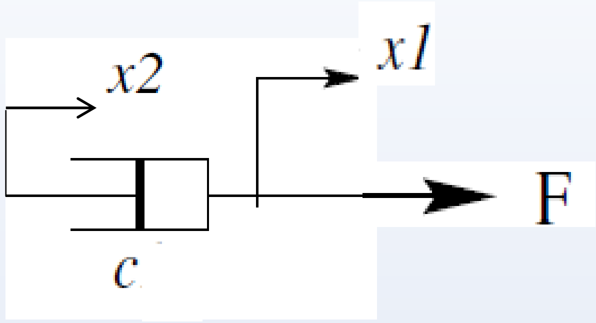
Vehicle Suspension

Bridge Suspension

Flyover Suspension

Translational Damper

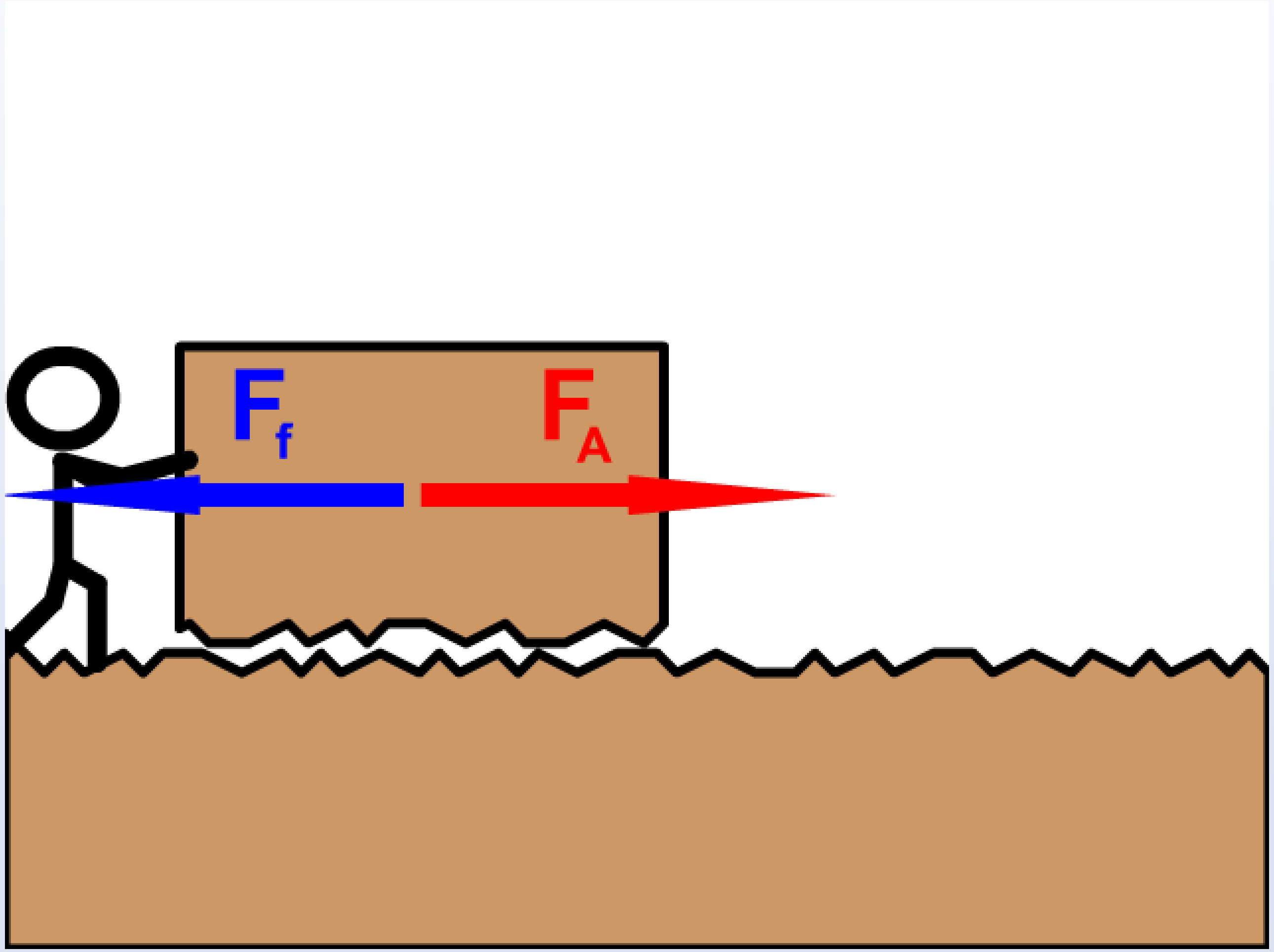




$$F = C\dot{x}$$

$$F = C(\dot{x}_1 - \dot{x}_2)$$

- Where *C* is damping coefficient (*N/ms*$^{-1}$).


F_f
F_A

Rotational motion

Rotational	Translational	Relationship
θ	x	$\theta = \frac{x}{r}$
ω	v	$\omega = \frac{v}{r}$
α	a	$\alpha = \frac{a_t}{r}$

Rotational systems

Analogous to linear mechanical systems

Torsional spring (resilient shaft)

$$\tau = k\theta$$

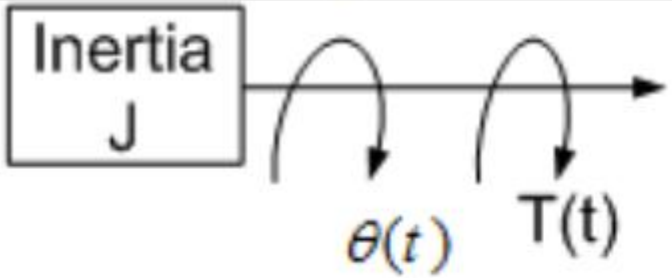
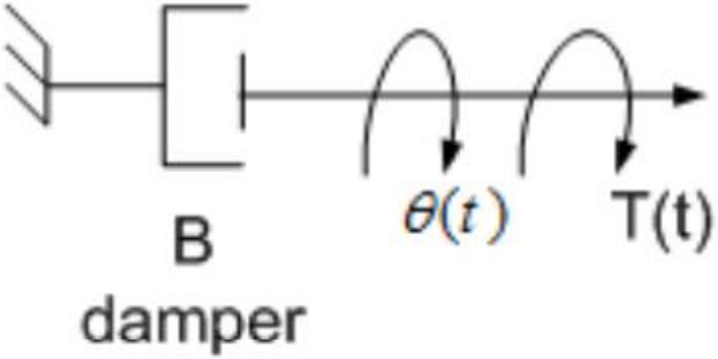
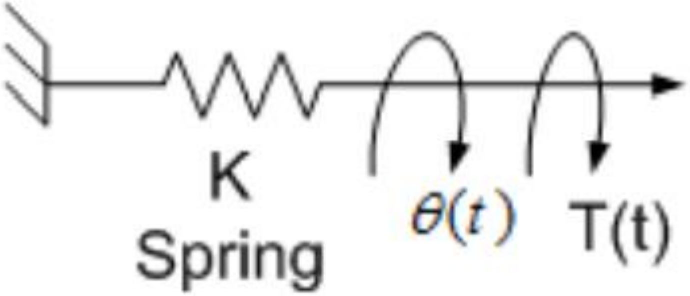
Torsional viscous damping

$$\tau = B\frac{d\theta}{dt} = B\omega$$

Rotating Inertia

$$\tau = J\frac{d^2\theta}{dt^2} = J\frac{d\omega}{dt}$$

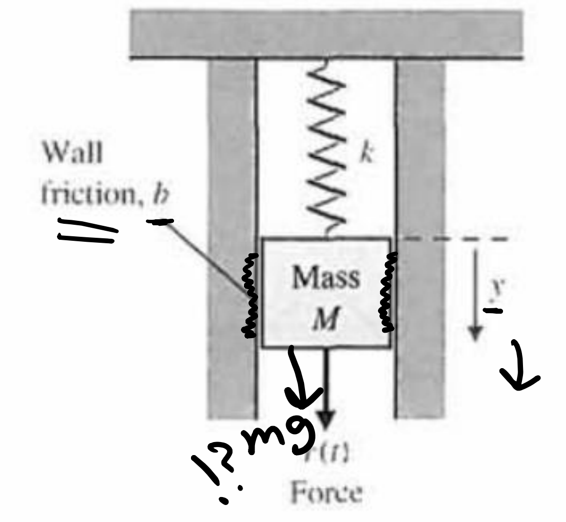
Mechanical Components: Rotational motion

Components	Time relation
	$T(t) = J\dfrac{d^2\theta(t)}{dt}$
	$T(t) = B\dfrac{d\ \theta(t)}{dt}$
	$T(t) = K\theta(t)$

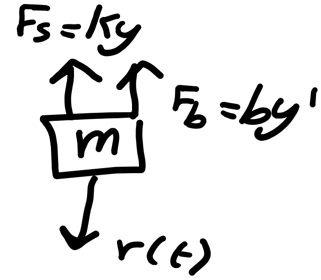
EXAMPLES

In a separate video

①



① Freebody diagram.

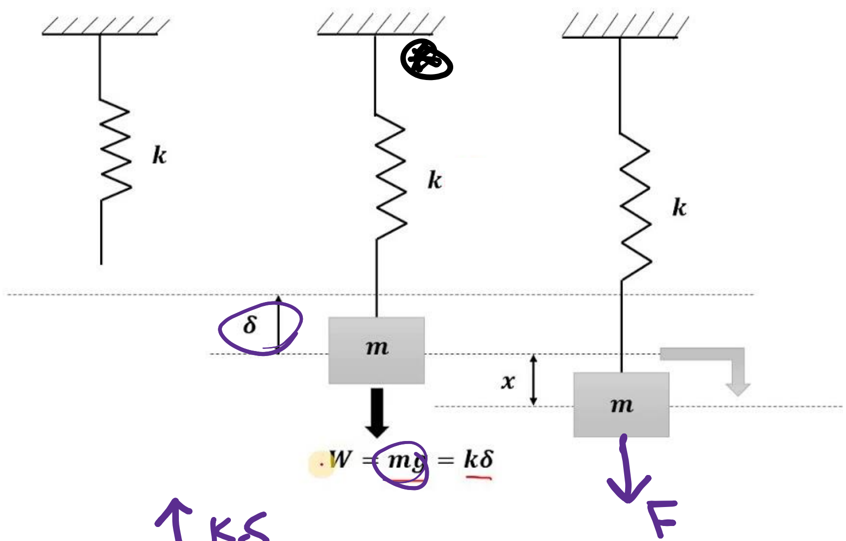


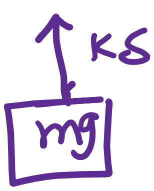
② $\Sigma F = ma$

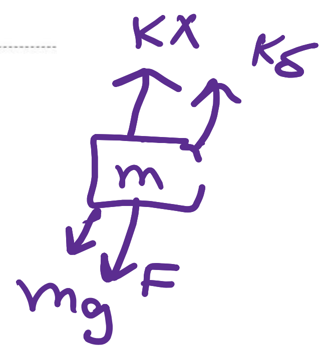
$$\underline{r(t)} - ky - by' = my''$$

$$my'' + by' + ky = r(t)$$

Spring-Mass system







$$\Sigma F = \overset{0}{ma}$$

$$mg - k\delta = 0$$

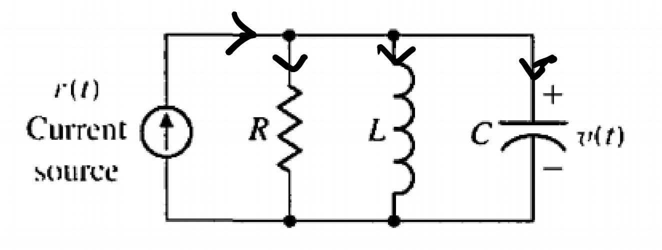
$$\boxed{mg = k\delta} \quad \cdots (1)$$

$$\Sigma F = ma$$

$$F + \cancel{mg} = kx - \cancel{k\delta} + m\ddot{x}$$

$$F - kx = mx''$$

$$\boxed{mx'' + kx = F}$$



KVL → $V_R = V_L = V_C = V$

KCL →

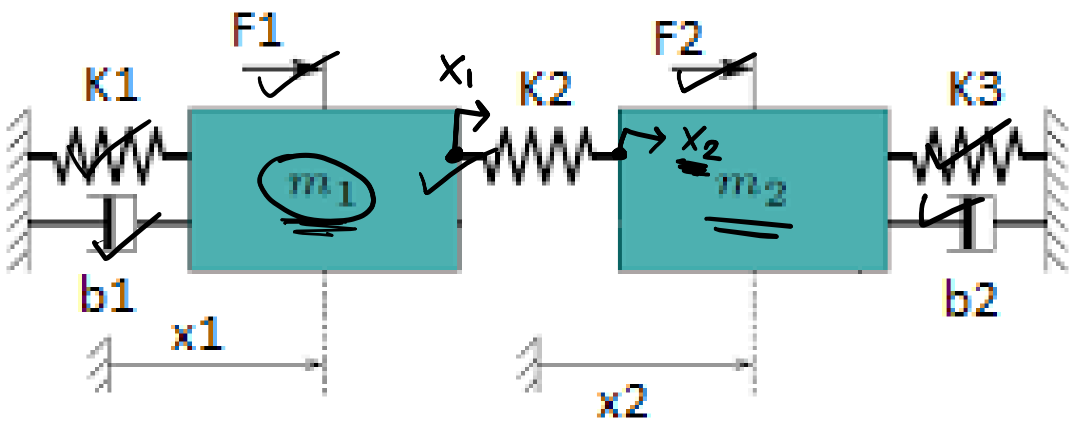
$$r(t) = I_R + I_L + I_C$$

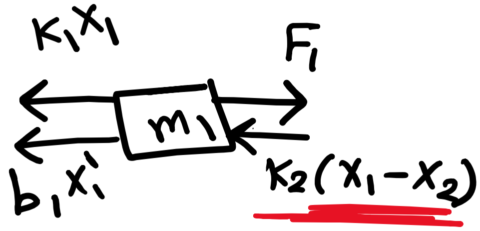
$$\frac{V_R}{R} + \frac{1}{L}\int V_L\,dt + C\frac{dV_C}{dt}$$

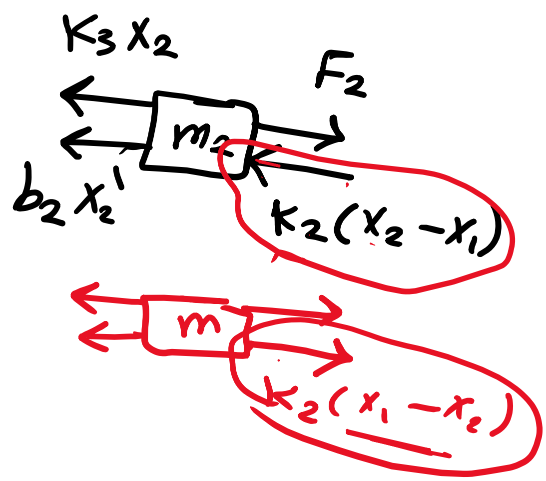
$$\leftarrow r(t) = \frac{V}{R} + \frac{1}{L}\int V\,dt + C\frac{dV}{dt} \Longrightarrow$$

intregrodiff. eq.

$$\boxed{r' = \frac{V'}{R} + \frac{1}{L}V + CV''}$$







$$\sum F = m_1 a_1$$

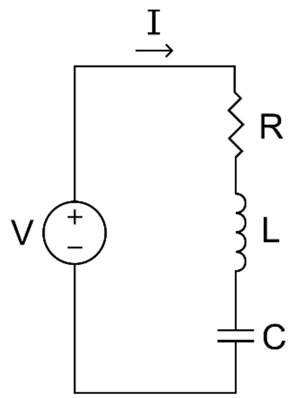
$$F_1 - K_1 x_1 - b_1 x_1' - K_2(x_1 - x_2) = m_1 x_1''$$

$$m_1 x_1'' + b_1 x_1' + K_2(x_1 - x_2) - K_1 x_1 = F_1 \quad \ldots\ldots 1$$

$$\sum F = m_2 a_2$$

$$F_2 - K_3 X_2 - b_2 x_2' - K_2(x_2 - x_1) = m_2 x_2''$$

$$m_2 x_2'' + b_2 x_2' + K_2(x_2 - x_1) + k_3 X_2 = F_2 \quad \ldots 2$$



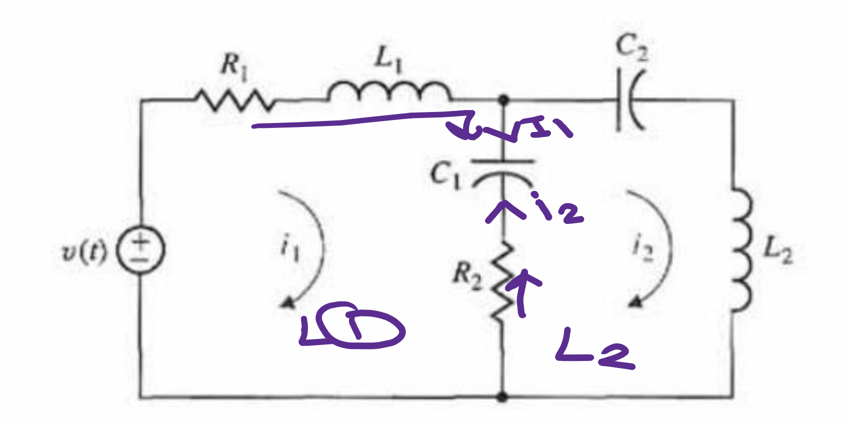
KCL → $I_R = I_L = I_c = I$

KVL =

$$V = V_R + V_L + V_C$$

$$V = I_R R + L\frac{dI_L}{dt} + \frac{1}{c}\int I_c\,dt$$

$$\boxed{V' = I'R + LI'' + \frac{1}{c}I}$$



KVL ⇒

Loop 1

$$V(t) = V_{R_1} + V_{L_1} + V_{C_1} + V_{R_2}$$

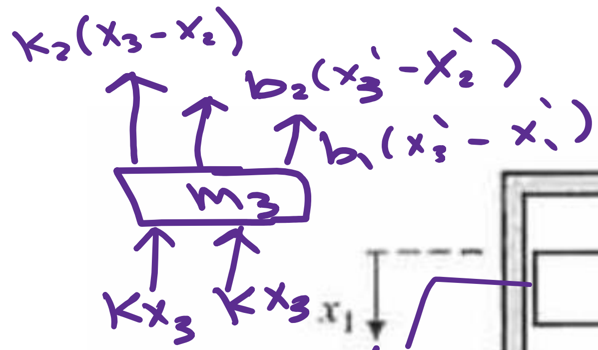
$$v(t) = R_1 i_1 + L_1 \frac{di_1}{dt} + \frac{1}{C_1}\int (I_1 - I_2) + R_2(I_1 - I_2)$$

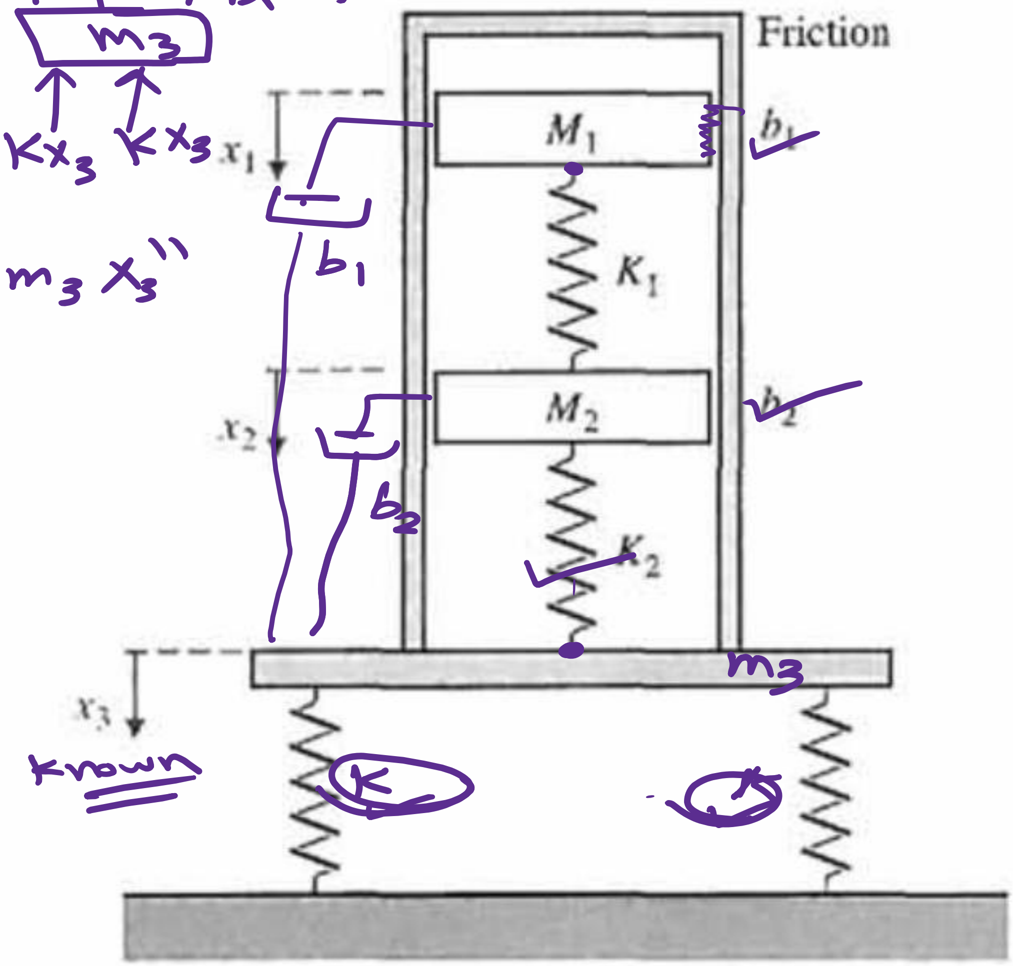
$$\frac{d}{dt}(\text{———})$$

Loop 2 :-

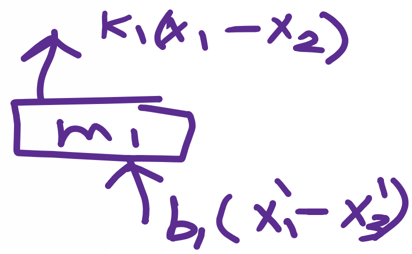
$$0 = \frac{1}{C_2}\int i_2 dt + L_2 \frac{di_2}{dt} + R_2(I_2 - I_1) + \frac{1}{C_1}\int i_2 - i_1$$

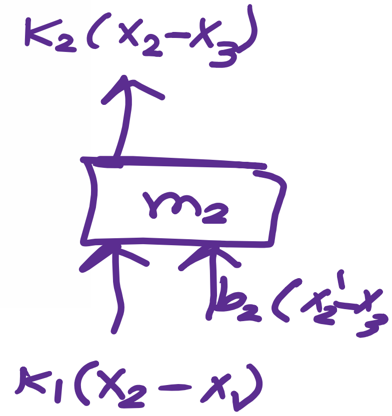
$$\frac{d}{dt}(\text{———})$$





$\Sigma F = m_3 x_3''$





m_1:-

$\Sigma F = ma$

$$-k_1(x_1-x_2)-b_1(x_1'-x_3') = m_1 x_1''$$

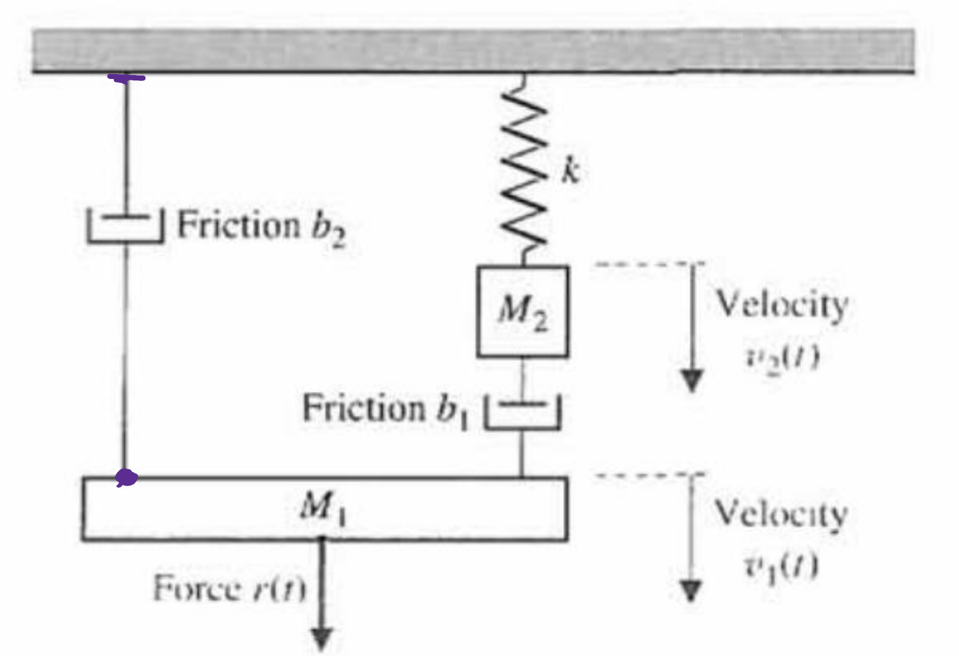
$$m_1 x_1'' + b_1(x_1'-x_3') + k_1(x_1-x_2) = 0 \quad \cdots (1)$$

m_2:-

$$-k_2(x_2-x_3) - k_1(x_2-x_1) - b_2(x_2'-x_3') = m_2 x_2''$$

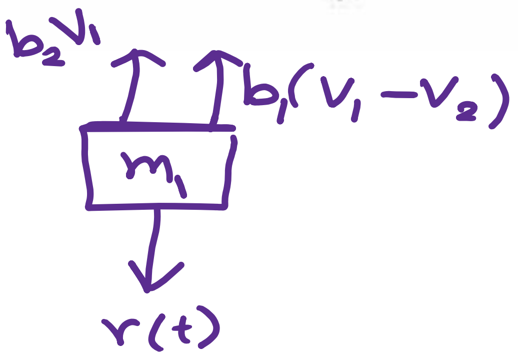
$$m_2 x_2'' + k_2(x_2-x_3) + k_1(x_2-x_1) + b_2(x_2'-x_3') = 0$$

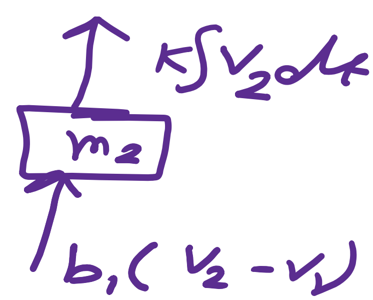
(2)



$V = x'$

$x = \int V$





$\sum F = ma$

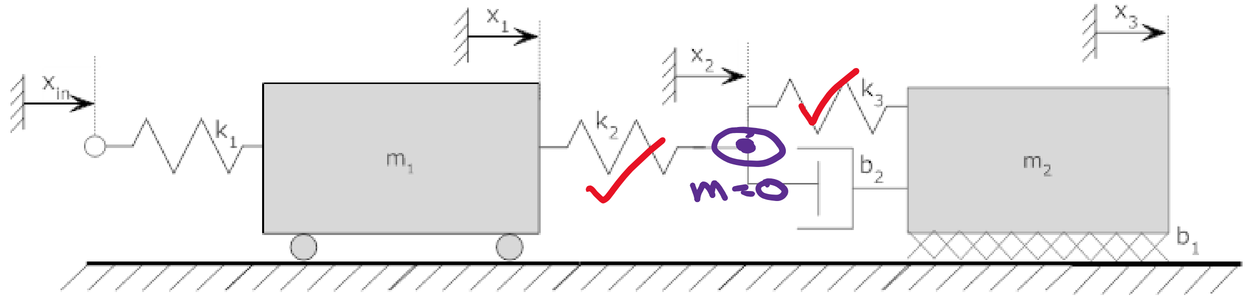
$r(t) - b_2 V_1 - b_1(V_1 - V_2) = m_1 V_1'$

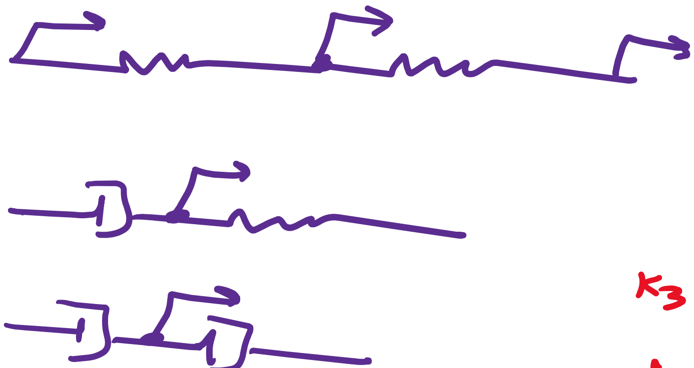
$m_1 V_1' + b_2 V_1 + b_1(V_1 - V_2) = r(t) \ldots$ ①

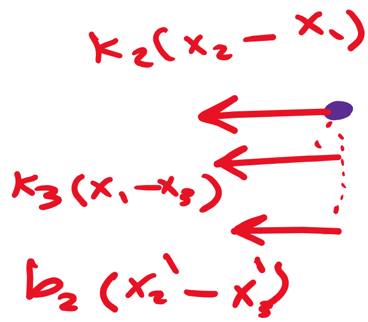
$m_2 \rightarrow$

$-k\int V_2 dt - b_1(V_2 - V_1) = m_2 V_2'$

$\frac{d}{dt}(\quad)$



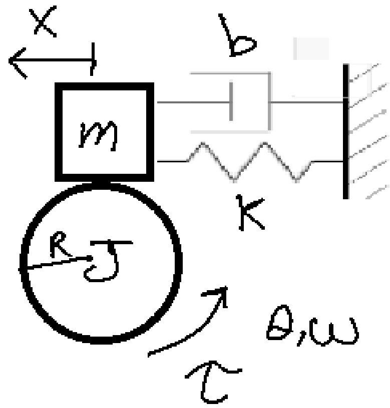




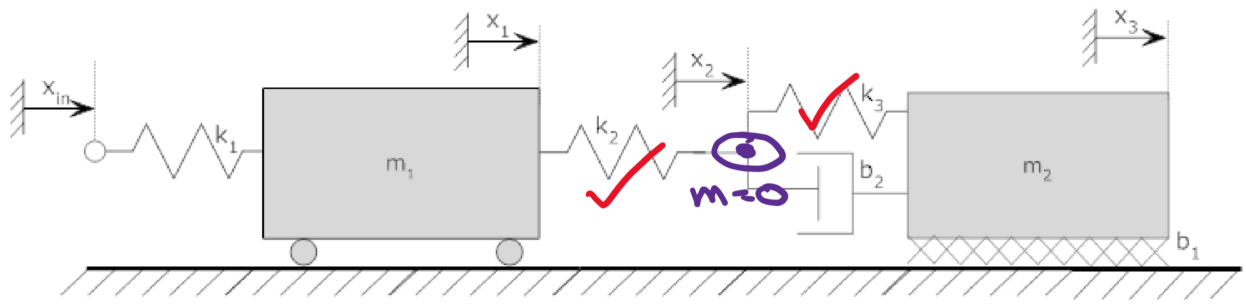
$$\sum F = m a = 0$$

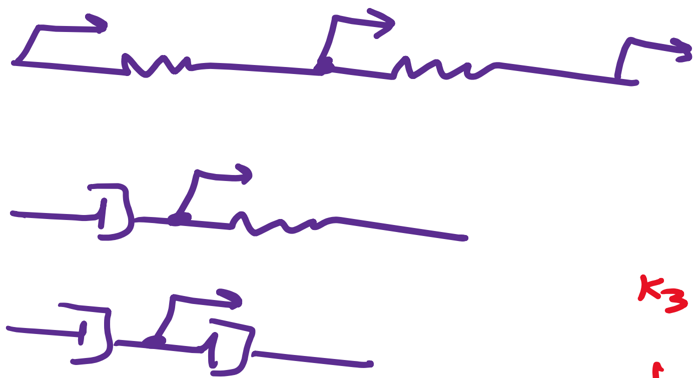
$$-k_2(x_2 - x_1) - k_3(x_1 - x_3)$$

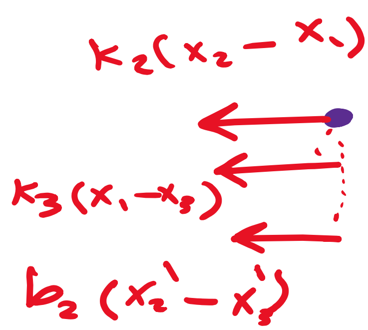
$$- b_2(x_2' - x_3') = 0$$



X
b
m
K
R
J
θ,ω
τ





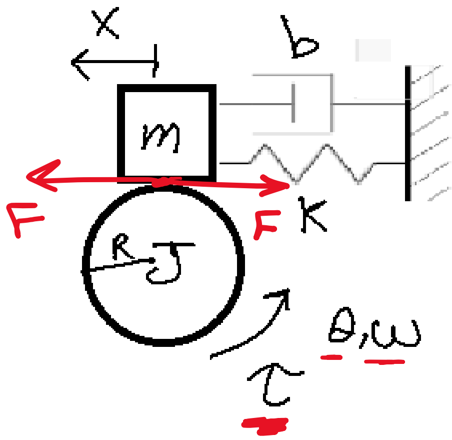


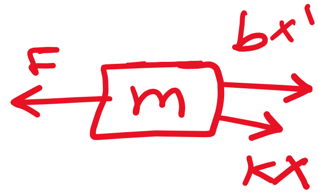
$$\sum F = m\overset{0}{a} = 0$$

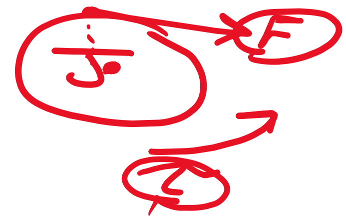
$$-k_2(x_2-x_1) - k_3(x_1-x_3)$$

$$-b_2(x_2'-x_3') = 0$$

$T = F \cdot R$







$\Sigma \tau = J\alpha$

$\tau - F.R = J\theta''$

$\tau = J\theta'' + F.R \quad \ldots (2)$

For mass m :-

$\Sigma F = ma$

$F - bx' - kx = mx''$

$F = mx'' + bx' + kx \quad \ldots (1)$

$\tau = J\theta'' + [mx'' + bx' + kx].R$

$x = R\theta$

$x' = R\theta'$

$x'' = R\theta''$

$\tau = J\frac{x''}{R} +$

τ I/P

x O/P

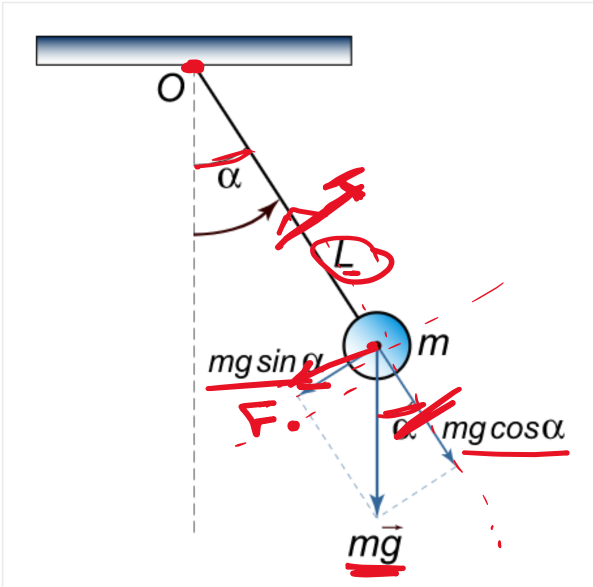


Figure 1.

ΣF at y axis $\Rightarrow$

$$T = mg\cos\alpha$$

$$T - mg\cos\alpha = m\overset{0}{a}$$

$$\cancel{T = mg\cos\alpha}$$

$\cancel{\Sigma F}$ at x axis :-

$\Sigma\tau$

$$\Sigma\tau = -mgL\sin\alpha = J\alpha''$$

$$J\alpha'' + mgL\sin\alpha = 0$$

- The minus sign indicates that at a positive angle of rotation α (counterclockwise), the torque of the forces causes rotation in the opposite direction.

Laplace Transform

Of linear Systems

A linear system satisfies the principle superposition

- In general, a necessary condition for a linear system can be determined in terms of an excitation *x(t) and a response y(t)*

  ⇒ *When the* system at rest is subjected to an excitation $x_1(t)$, it provides a response $y_1(t)$ and when the system is subjected to an excitation $x_2(t)$, *it provides a corresponding* response $y_2(t)$;

  ⇓

  For a linear system, it is necessary that the excitation $x_1(t) + x_2(t)$ *result in a response* $y_1(t) + y_2(t)$

  ***This is usually called the principle* of superposition**

- If the system is nonlinear a linear one can be obtained using Taylor series expansion around a known operating conditions

Taylor Series Expansion

Consider a system whose input variable is x(t) and output variable is y(t) where the relationship between them is nonlinear given by y=g(x); If the operation conditions corresponds to $(x, \bar{y})$, then a linear relationship around this point can be found using the Taylor series as follows:

$$y = g(x) = g(x_0) + \left.\frac{dg}{dx}\right|_{x=x_0} \frac{(x - x_0)}{1!} + \left.\frac{d^2g}{dx^2}\right|_{x=x_0} \frac{(x - x_0)^2}{2!} + \cdots. \quad (2.7)$$

Pendulum Example

$$T = MgL \sin\theta,$$

$$T - T_0 \cong MgL \frac{\partial \sin\theta}{\partial\theta}\bigg|_{\theta=\theta_0} (\theta - \theta_0),$$

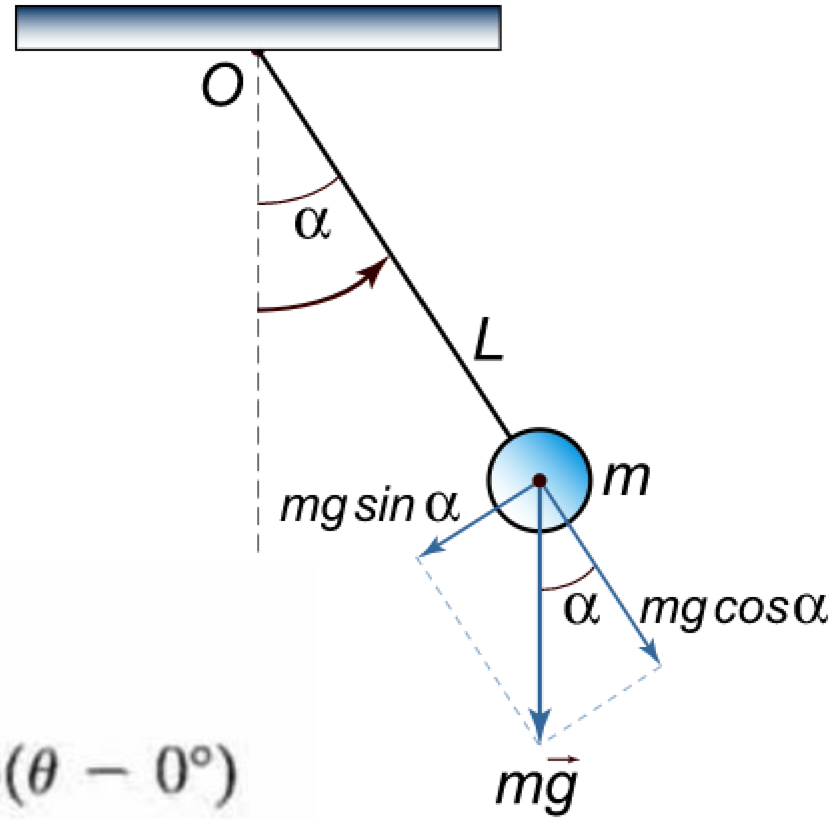


Figure 1.

where $T_0 = 0$. Then, we have

$$\begin{aligned} T &= MgL(\cos 0°)(\theta - 0°) \\ &= MgL\theta. \end{aligned}$$

Laplace Transforms

- The Laplace transform is a mathematical tool for solving linear time invariant differential equation.
- It allows a time domain differential equation model of a system to be transformed in to algebraic model

⇒ Therefore simplifying the analysis and design of a control system

- Definition

$$F(s) = L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt \quad \text{for} \quad f(t), t > 0$$

$$f(t) = L^{-1}\{F(s)\}$$

x(t) can be found by applying the inverse Laplace transform of X(s)

Laplace transform of the unit step (u(t)=1)

$$L[u(t)] = \int_0^\infty 1 e^{-st}\, dt = \frac{-1}{s} e^{-st} \Big|_0^\infty = \frac{1}{s}$$

Laplace transform of time Differentiation

$$L[\frac{df(t)}{dt}] = sF(s) - f(0)$$

We can extend the time differentiation to be:

$$L\left[\frac{df(t)^2}{dt^2}\right] = s^2 F(s) - sf(0) - f'(0)$$

$$L\left[\frac{df(t)^3}{dt^3}\right] = s^3 F(s) - s^2 f(0) - sf'(0) - f''(0)$$

general case

$$L\left[\frac{df(t)^n}{dt^n}\right] = s^n F(s) - s^{n-1} f(0) - s^{n-2} f'(0)$$

$$- \ldots - f^{(n-1)}(0)$$

	f(t)	$\mathcal{L}$(f)		f(t)	$\mathcal{L}$(f)
1	Unit-impulse $\delta(t)$	1	7	cos ωt	$\frac{s}{s^2+\omega^2}$
2	Unit-step 1	1/s	8	sin ωt	$\frac{\omega}{s^2+\omega^2}$
3	Unit-ramp t	$1/s^2$	9	cosh at	$\frac{s}{s^2-a^2}$
4	t^2	$2!/s^3$	10	t e^{at}	$\frac{1}{(s-a)^2}$
5	t^n (n is +ve integer)	$\frac{n!}{s^{n+1}}$	11	$e^{at}\cos\omega t$	$\frac{s-a}{(s-a)^2+\omega^2}$
6	e^{at}	$\frac{1}{s-a}$	12	$e^{at}\sin\omega t$	$\frac{\omega}{(s-a)^2+\omega^2}$

For most engineering purposes the inverse Laplace transformation can be accomplished simply by referring to Laplace transform tables

The Laplace Transform….cont

- The Laplace variable *s* can be considered to be the differential operator so that $\Longrightarrow$ $s = \dfrac{d}{dt}$

  And we also have the integral operator $\Longrightarrow$ $\dfrac{1}{s} = \int_0^{\infty} dt$

- *s*-operator is a complex quantity has a real and imaginary parts

  $\rightarrow s = a + jb$

- Initial Value Theorem

$$f(0) = \lim_{s \to \infty} sF(s) = \lim_{t \to 0} f(t)$$

- Final Value Theorem

$$f(\infty) = \lim_{s \to 0} sF(s) = \lim_{t \to \infty} f(t)$$

For function f(t)

The final value theorem is very useful for analysis and design of control systems, since it gives the final value of a time function **f(t)**

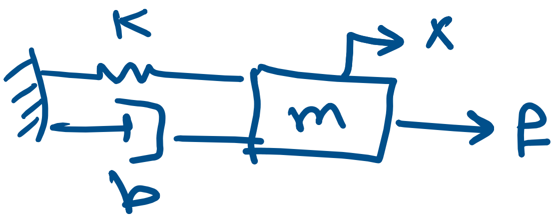
Examples

diff $\longrightarrow$ algebric eq. $\quad x^2+4x+5=0$

$\quad\hookrightarrow$ laplace transform ( ) ( ) ✓

$\downarrow$

$$x^{(n)}+3x^{(1)}+4x=f(t)$$



$$m\underline{x''}+\underbrace{bx'}+Kx=f(t)$$

$$m\underbrace{\left(s^2X(s)-s\cancel{x(0)}-\cancel{x'(0)}\right)}_{x''}+$$

$$b\underbrace{\left[sX(s)-\cancel{x(0)}\right]}_{x'}+K\,X(s)=F(s)$$

if initial cond $=0$

$x(0)=0$

$x'(0)=0$

$$ms^2X(s)+bsX(s)+KX(s)=F(s)$$

$$X(s)\left[ms^2+bs+K\right]=F(s)$$

$$\boxed{X(s)=\frac{\frac{F(s)}{m}}{\frac{ms^2}{m}+\frac{b}{m}s+\frac{K}{m}}}$$

if $\frac{f(t)}{m} = 1$

$\frac{b}{m} = 3 \qquad \frac{K}{m} = 2 \qquad \frac{F(s)}{m} = 1 \Rightarrow$?!

$X(s) = \frac{1}{s^2+3s+2} \rightsquigarrow x(t)$?!

$\mathcal{L}^{-1}[X(s)] \Rightarrow$

$F(s)$ $\qquad$ $f(t)$

$\frac{4}{s} \Rightarrow 4$

$\frac{2}{s+4} \Rightarrow 2e^{-4t}$

partial fraction:—

الحالات

1. real diff.
2. complex
3. real multi.

$$x(s) = \frac{1}{s^2 + 3s + 2}$$

$$\left(\frac{1}{(s+2)(s+1)} = \frac{A}{s+2} + \frac{B}{s+1}\right) * (s+2)(s+1)$$

$$1 = A(\cancel{s}+1) + \cdot \underbrace{B(s+2)}_{0}$$

$\Rightarrow$ if $s = -1$

$$\boxed{B = 1}$$

if $s = -2$

$$\boxed{A = -1}$$

$$x(s) = \frac{-1}{s+2} + \frac{1}{s+1}$$

$$\boxed{x(t) = -e^{-2t} + e^{-t}} \Rightarrow \frac{f(t)}{m} = 1$$

?!

Case 2 :- real multi.

$$Y(s) = \frac{2}{(s+4)(s+2)^2} = \frac{A}{s+4} + \frac{B}{s+2} + \frac{C}{(s+2)^2}$$

$* (s+4)(s+2)^2$

$s = -4, -2, -2$

$$2 = A(s+2)^2 + B(s+4)(s+2) + C(s+4)$$

$s = -4$

$2 = 4A \Rightarrow A = \frac{1}{2}$

$s = -2$

$2 = 2C \Rightarrow C = 1$

$s = 0$

$$2 = 4\overset{\frac{1}{2}}{A} + 8B + 4\overset{1}{C}$$

$$2 = 2 + 8B + 4$$

$$\frac{-4}{8} = B = -\frac{1}{2}$$

$$y(s) = \frac{1/2}{s+4} + \frac{-1/2}{s+2} + \frac{1}{(s+2)^2}$$

$$y(t) = \frac{1}{2} e^{-4t} - \frac{1}{2} e^{-2t} + t e^{-2t} \ldots\ldots$$

$$Y(s) = \left[\frac{3}{s(\underbrace{s^2+2s+5}_{\text{-ve complex}})} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+5}\right] * s(s^2+2s+5)$$

o

$$3 = A(s^2+2s+5) + (Bs+C)s$$

$$As^2 + 2As + 5A + Bs^2 + Cs$$

$$3 = (A+B)s^2 + (2A+C)s + 5A$$

$$A + B = 0 \Rightarrow B = -\frac{3}{5}$$

$$2A + C = 0 \Rightarrow C = -\frac{6}{5}$$

$$5A = 3 \Rightarrow A = \frac{3}{5}$$

$$Y(s) = \frac{3/5}{s} + \frac{-3/5\,s + {}^{-}6/5}{s^2+2s+5}$$

sin
cos.

$$\cos = \frac{s+a}{(s+a)^2+\omega^2}$$

$$\sin = \frac{\omega}{\quad}$$

$$\frac{3/5}{s} + \frac{-3}{5}\;\frac{s+2}{s^2+2s+5}$$

$$\frac{3/5}{s} + \frac{-3}{5}\;\frac{s+1+1}{\underbrace{s^2+2s+1}_{(s+1)^2}+4}$$

$$\frac{3/5}{s} + \frac{-3}{5}\left[\frac{s+1}{(s+1)^2+4} + \frac{1\times\frac{2}{2}}{(s+1)^2+4}\right]$$

$$\frac{3/5}{s} + \frac{-3}{5}\,\frac{s+1}{(s+1)^2+4} + \overset{\frac{-3}{5}\times\frac{1}{2}}{\frac{-3}{10}}\,\frac{2}{(\underline{s+1})^2+4}$$

$$\boxed{y(t) = \frac{3}{5} - \frac{3}{5}e^{-t}\cos 2t - \frac{3}{10}e^{-t}\sin 2t}$$

Transfer Function of linear systems

CH2

Eng. Fadwa Momani

The Transfer Function (T.F) of Linear Systems

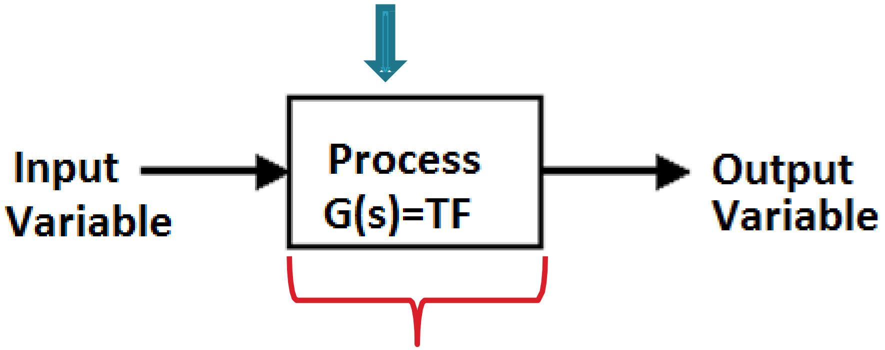
- The transfer function of a linear system is defined as the ratio of the Laplace transform of the output variable to the Laplace transform of the input variable, **with all initial conditions assumed to be zero.**

$$\frac{\text{output}}{\text{Input}}$$

- The transfer function of a system (or element) represents the relationship describing the dynamics of the system under consideration

$$T.F = \frac{output}{input} = G(s)$$

$T(s)$



Represents system dynamics in s-domain

$$T.F = \frac{output}{input} = G(s)$$

$$= \frac{p(s)}{q(s)}$$

- Where *p(s)* and *q(s)* are polynomials

zeros

- The roots of *p(s)* are called the zeros of the system where the roots of *q(s)* are called the poles of the system

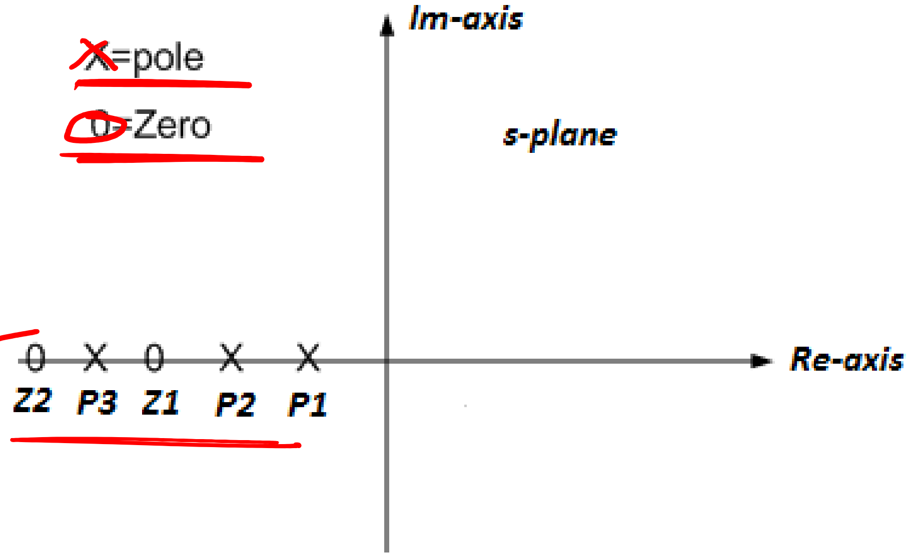
q(s) is also known as the characteristic equation of the systems

- The location of the roots of *q(s)* in s-plane gives a character to the system performance

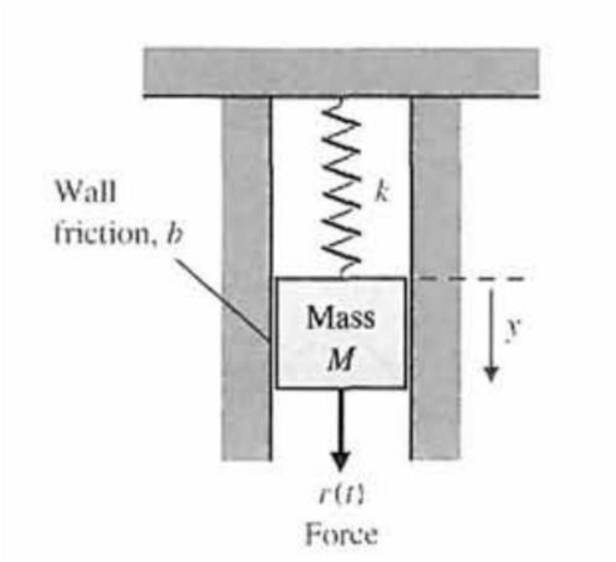
S-plane

$$G(s) = \frac{p(s)}{q(s)}$$

$$= \frac{(s+z_1)(s+z_2)}{(s+p_1)(s+p_2)(s+p_3)}$$



Examples



Input $\to R(s)$ $\qquad \dfrac{Y(s)}{R(s)}$

output $\rightsquigarrow$ position y

$$my'' + by' + Ky = r(t)$$

$$ms^2 Y(s) + bs\,Y(s) + K\,Y(s) = R(s)$$

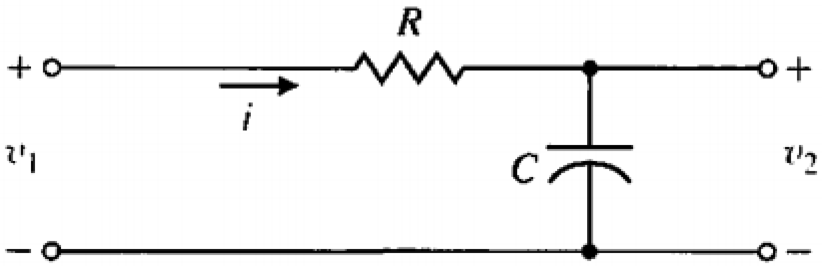
$$Y(s)\left[ms^2 + bs + K\right] = R(s)$$

$$\left\{ \frac{Y(s)}{R(s)} = \frac{1}{ms^2 + bs + K} \right\} \Rightarrow \text{transfer function.}$$

output $\to$ velocity ?!

$$mv' + bV + K\int V = r(t) \to \frac{K}{s} V(s)$$

$$\boxed{\frac{V(s)}{R(s)} = \frac{1}{ms + b + \frac{K}{s}}}$$



NOTE :- I/P V_1

O/P I

$$V_1(s) = I(s)R + \frac{1}{Cs} I(s)$$

$$V_1(s) = I(s)\left[R + \frac{1}{Cs}\right]$$

$$\frac{I(s)}{V_1(s)} = \frac{1}{R + \frac{1}{Cs}}$$

T.F

I/P → V_1

O/P → V_2

$$\frac{V_2(s)}{V_1(s)}$$

$$V_1 = V_R + V_c$$

$$V_2 = V_c = \frac{1}{C}\int i\,dt \Rightarrow V_2 = \frac{1}{C}\int I\,dt$$

$$V_1 = IR + \frac{1}{C}\int i\,dt$$

$$V_1(s) = CsV_2(s)R + V_2(s)$$

$$V_1(s) = V_2(s)\left[RCs + 1\right]$$

$$\boxed{\frac{V_2(s)}{V_1(s)} = \frac{1}{RCs + 1}}$$

I ? !

$$V_2(s) = \frac{1}{Cs} I(s)$$

$$I(s) = Cs\,V_2(s)$$

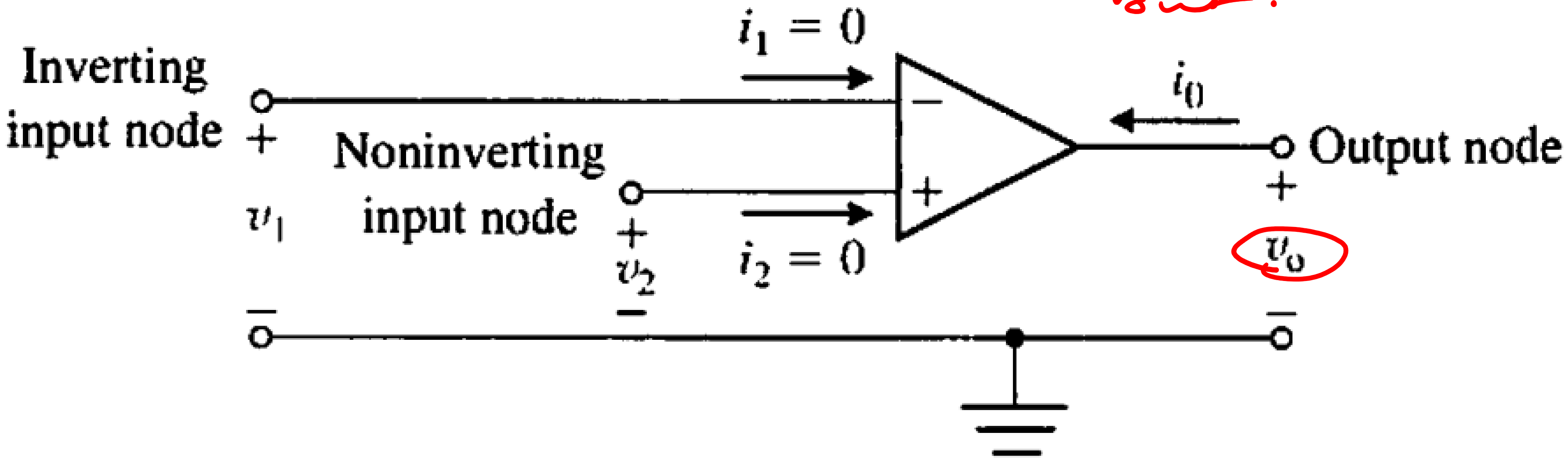
Operational Amplifier Transfer Function (Op-Amp)

- The ideal op amp

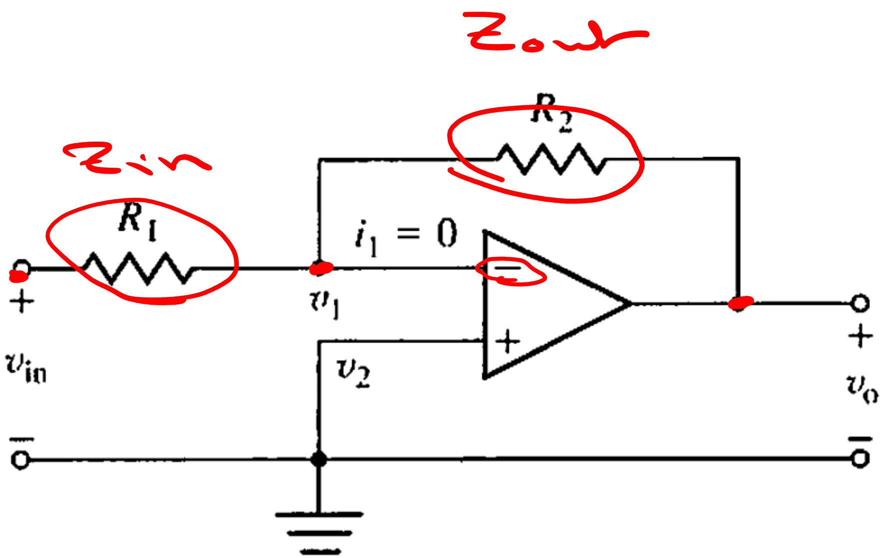
$I_1 = I_2 = 0$

$V_1 = V_2$

V_{out}.



Inverting amplifier



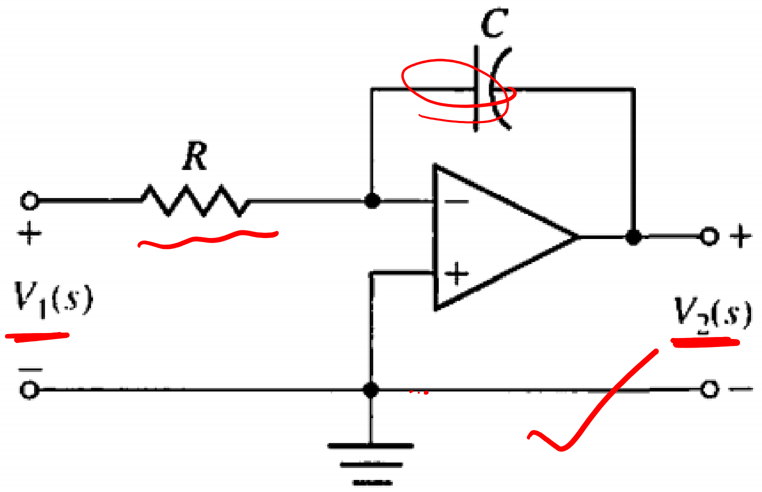
Just for inverting Amp -

$$T.F = - \frac{Z_{out}}{Z_{in}}$$

T.F

$$T(s) = \frac{V_o(s)}{V_i(s)} = \boxed{-\frac{R_2}{R_1}}$$

1. Integrating circuit, filter



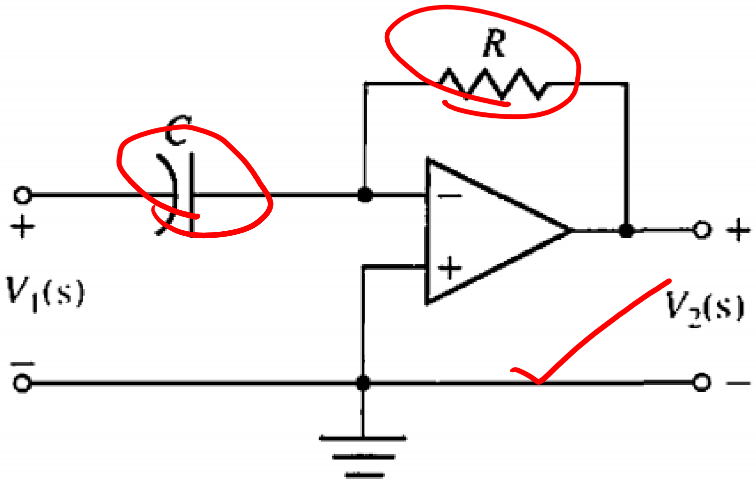
R	R
C	$\frac{1}{Cs}$
L	Ls

$$\frac{V_2(s)}{V_1(s)} = -\frac{Z_{out}}{Z_{in}} = -\frac{\frac{1}{Cs}}{R} = \boxed{-\frac{1}{RCs}}$$

$sy \rightarrow$

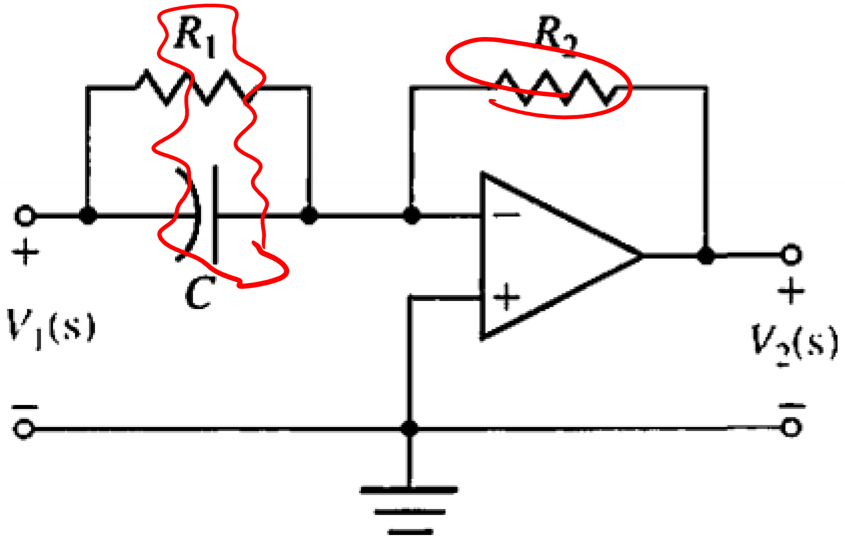
$\frac{1}{s} Y(s)$ Integrator

2. Differentiating circuit



$y' \rightarrow s Y(s)$

$$\frac{V_2(s)}{V_1(s)} = -\frac{Z_{out}}{Z_{in}} = -\frac{R}{1/Cs} = \boxed{-RCs}$$



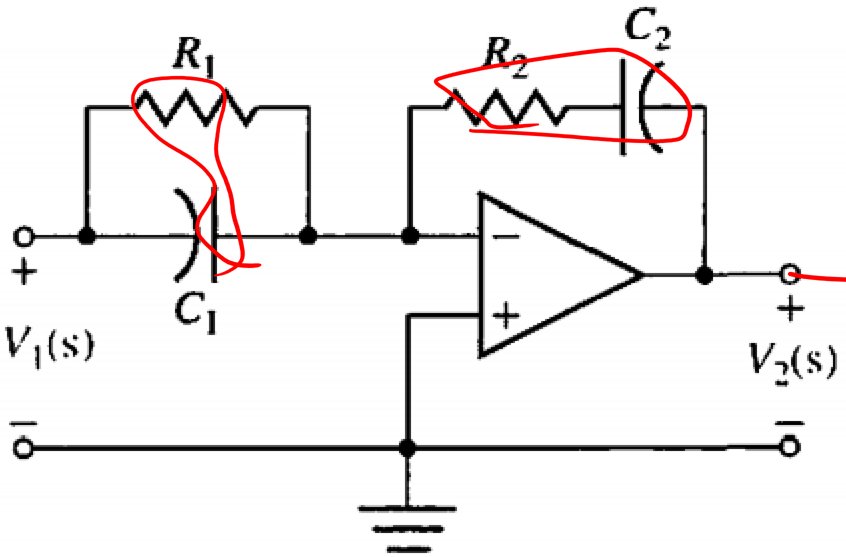
$$\frac{V_2(s)}{V_1(s)} = -\frac{R_2(R_1Cs+1)}{R_1}$$

$$\frac{V_2(s)}{V_1(s)} = -\frac{Z_{out}}{Z_{in}} = -\frac{R_2}{R_{eq}} \qquad R_{eq} \leftarrow R_1 // C$$

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{1/Cs} = \frac{1}{R_1} + \frac{CsR_1}{R_1} = \frac{1+CsR_1}{R_1}$$

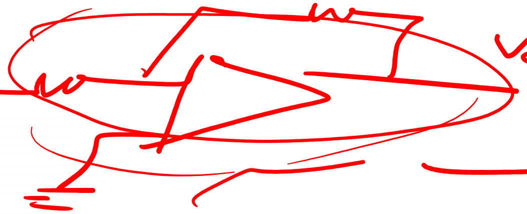
$$R_{eq} = \frac{R_1}{1+CsR_1}$$

$$\frac{V_2(s)}{V_1(s)} = -\frac{R_2}{\frac{R_1}{1+CsR_1}} = \frac{-R_2(1+CsR_1)}{R_1}$$



$$\frac{V_2(s)}{V_1(s)} = -\frac{(R_1C_1s + 1)(R_2C_2s + 1)}{R_1C_2s}$$

$$\frac{V_2(s)}{V_1(s)} = \frac{-Z_{out}}{Z_{in}} = \frac{R_2 + \frac{1}{Cs}}{\frac{R_1}{1 + R_1C_1S}}$$

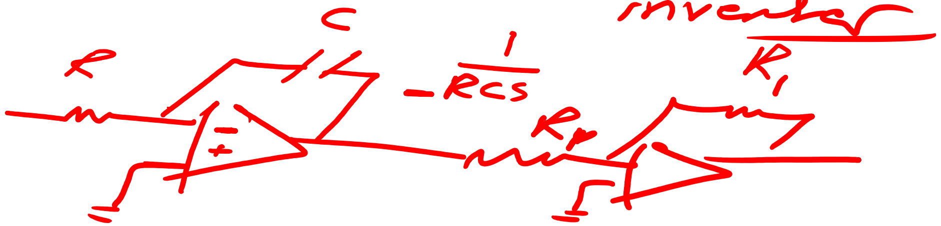


$$\frac{V_{out}}{V_{in}} = \frac{V_2}{V_1} * \frac{V_{out}}{\cancel{V_2}} = \frac{V_{out}}{V_1}$$

$$\frac{V_2}{V_1} = \frac{-s}{s}$$

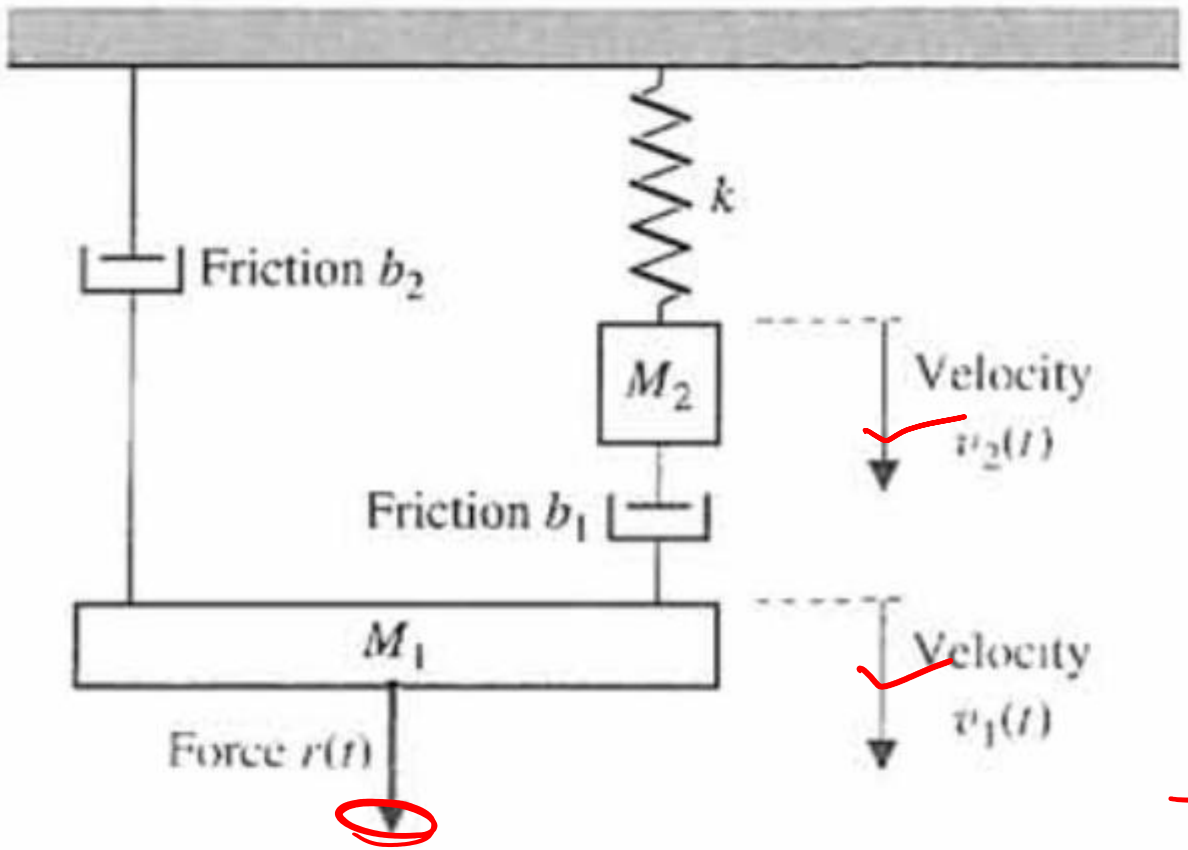
$$\frac{s}{s}$$

inverter



$$-\frac{1}{RCs} * -\frac{R_1}{R_1} = -1$$

$$= \frac{1}{RCs}$$



Total T.F

2 T.F

$\frac{V_2(s)}{R(s)}$

$\frac{V_1(s)}{R(s)}$

2.Input 2.output.

4 T.F

$\frac{O/P1}{I/P1}$ $\frac{OP2}{In1}$

$\frac{OP1}{IP1}$ $\frac{OP2}{In2}$

DC Motor Transfer Function

CH2- part 10

Eng. Fadwa Momani

How does DC motor work??

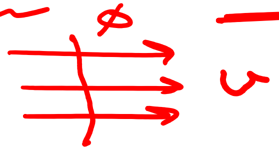
Actuator

- Important concepts:

1. Current in a conductor will produce electromagnetic flux (ϕ).

$$\phi \alpha I$$

2. A moving conductor in a flux will produce induced voltage(E)

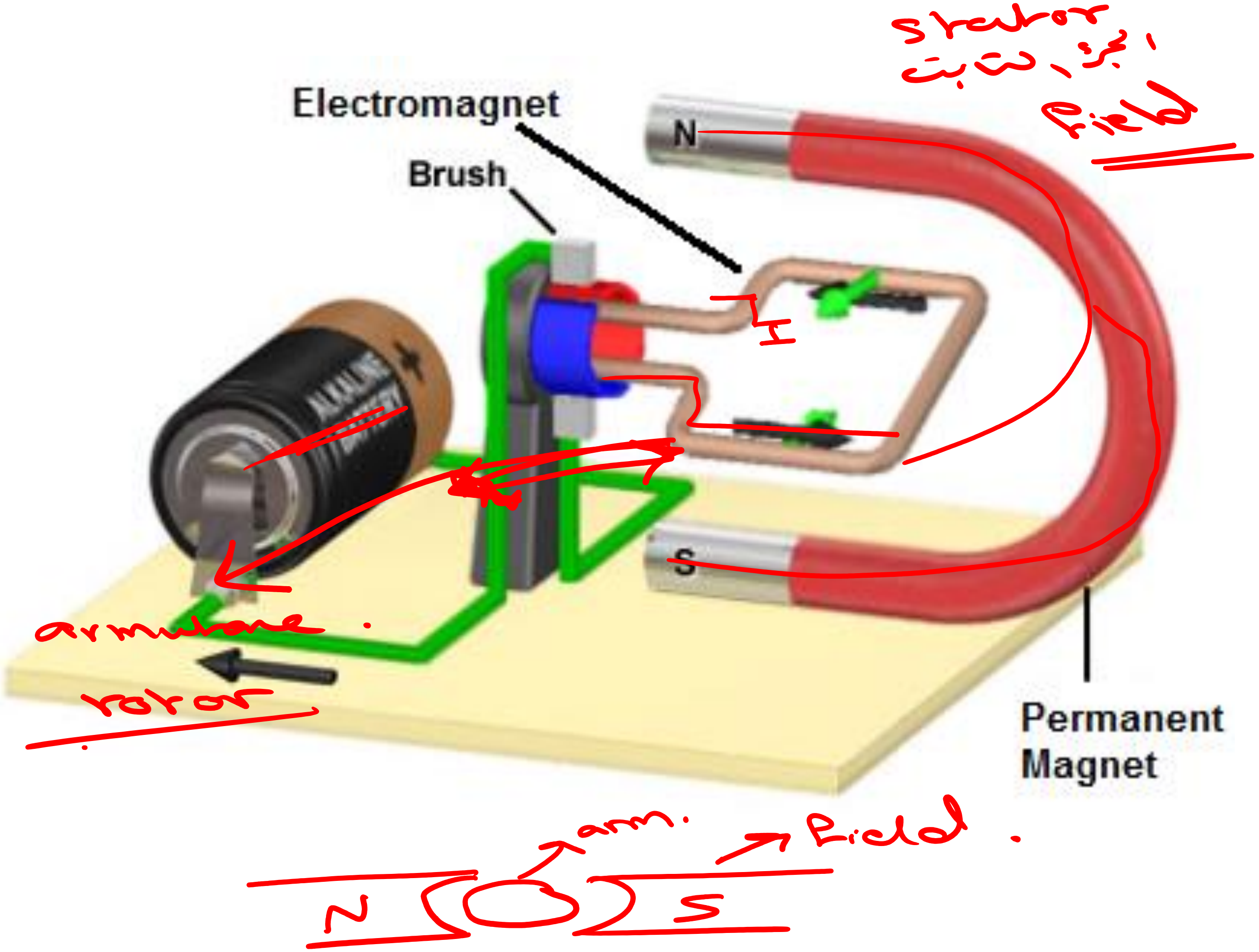


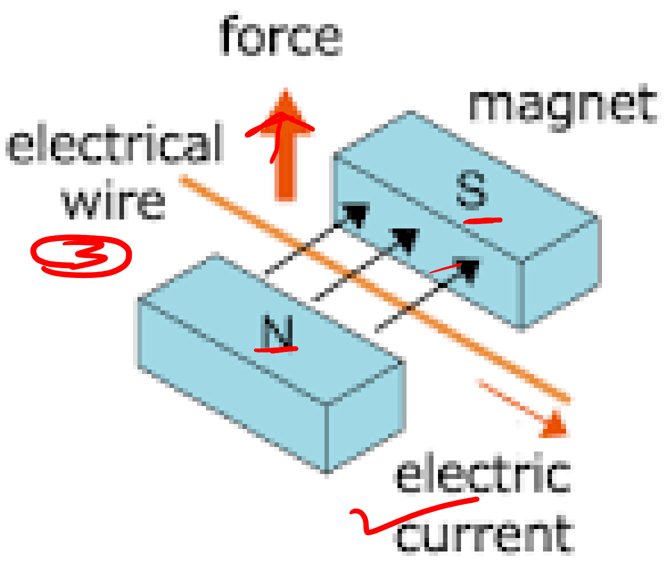
$$E \alpha \omega \phi$$

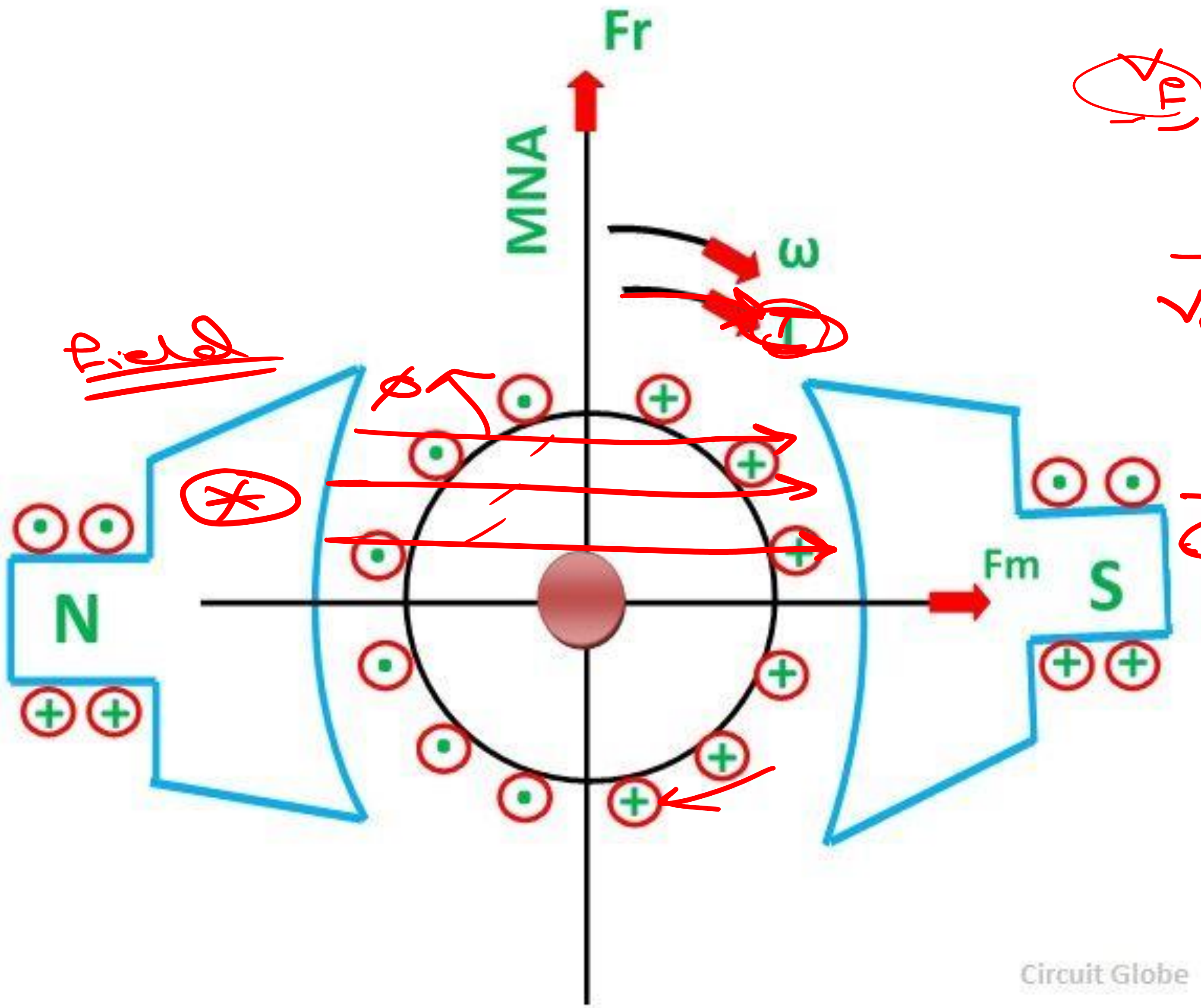
3. current holding conductor exists in a flux will be affected by force or torque.

$$T \alpha \phi I$$

magent
wire with I







V_f $I_f \rightarrow$ $\underline{L_f} + \underline{R_f}$

Field

V_a $I_a \rightarrow$ $\underline{L_a} + \underline{R_a}$

armature.

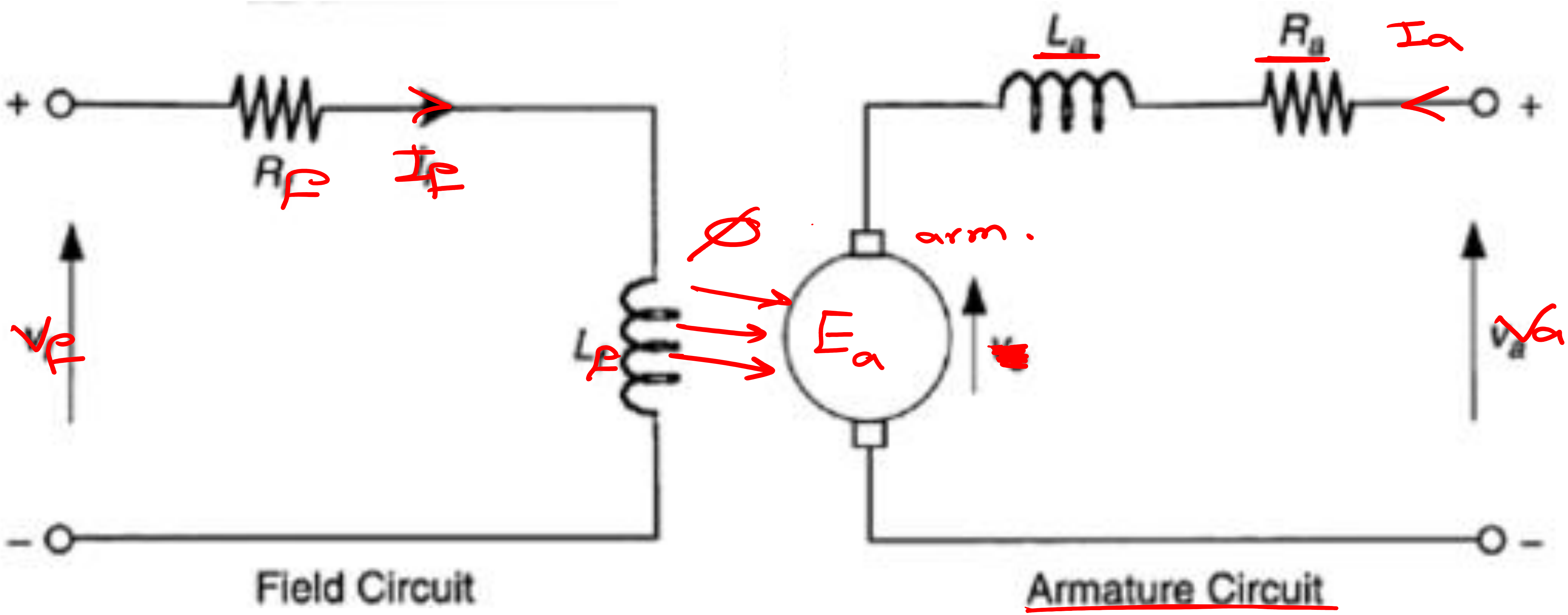
② $\underline{E_a}$

electromechanical sys.

$\phi = K_f I_f$

Electrical Part

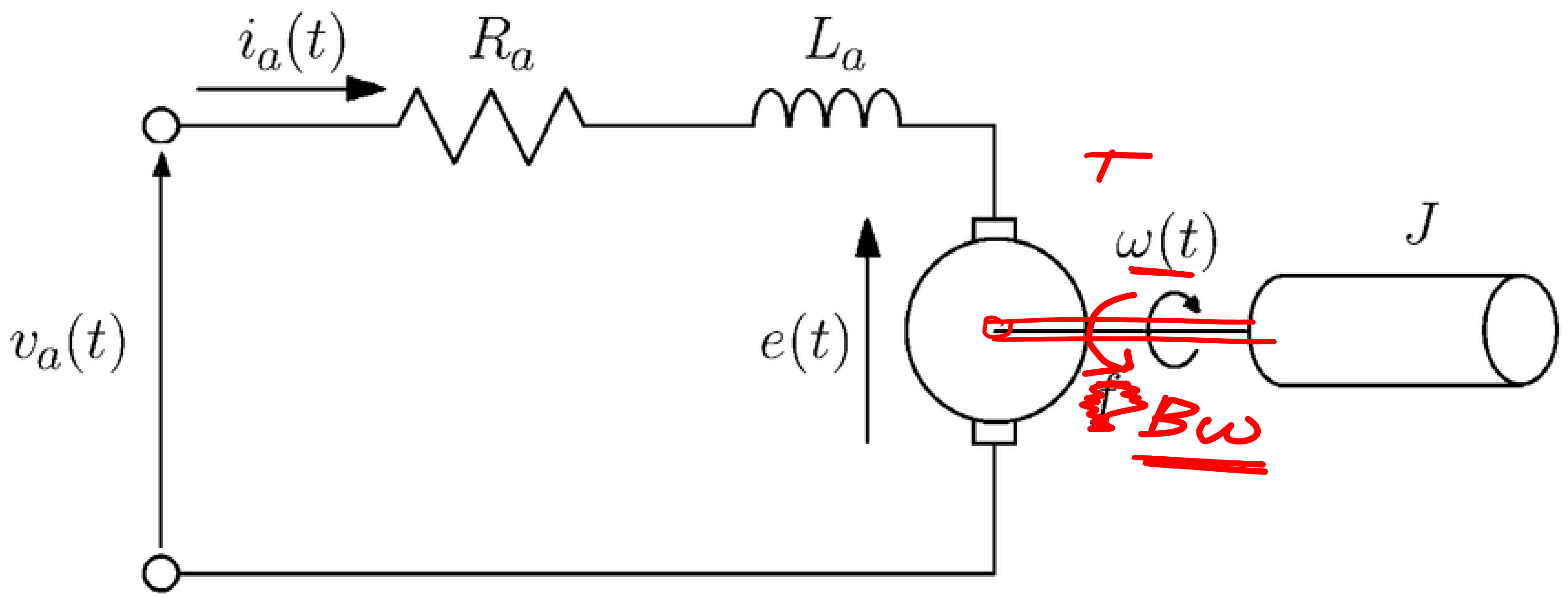
electrical $\longrightarrow$ mechanical



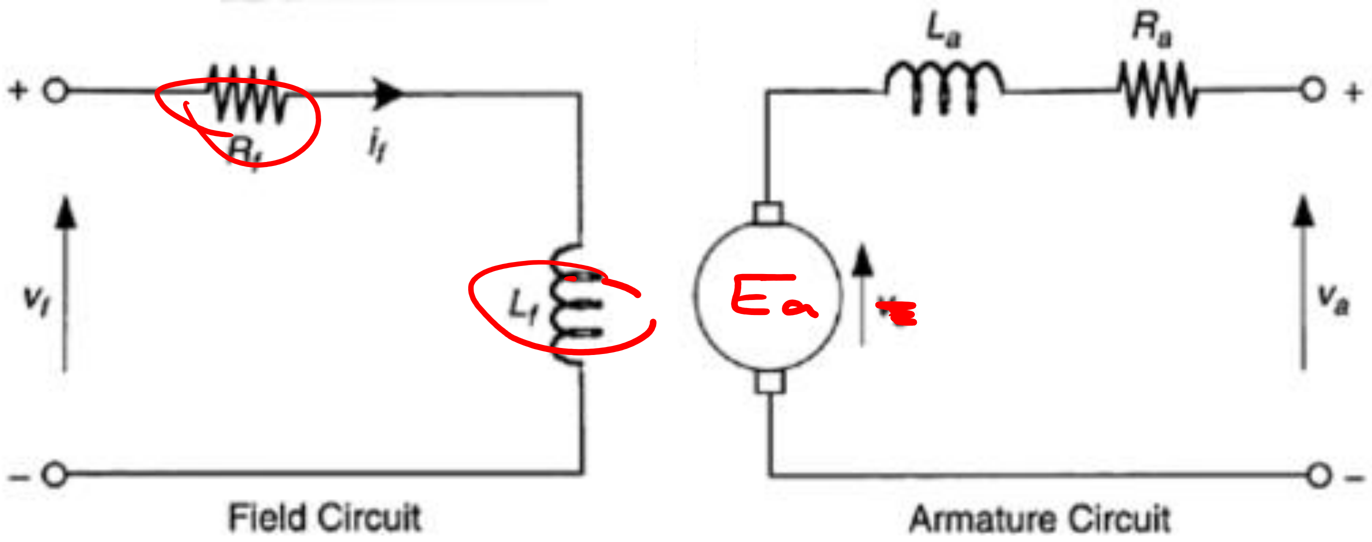
Electromechanical circuit

$T = K\phi I_a$

$E_a = K_b \phi \omega$



Mathematical model



$$V_f = I_f R_f + L_f \frac{dI_f}{dt} \quad (1)$$

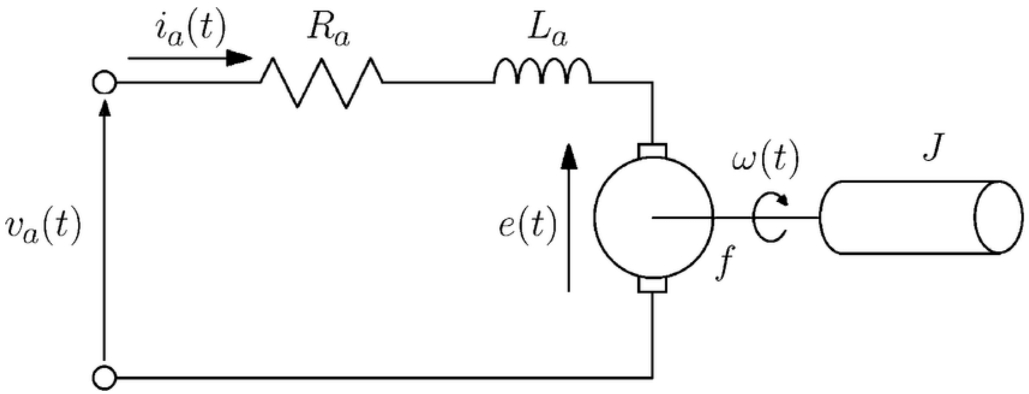
$$V_a = E_a + I_a R_a + L_a \frac{dI_a}{dt} \quad (2)$$

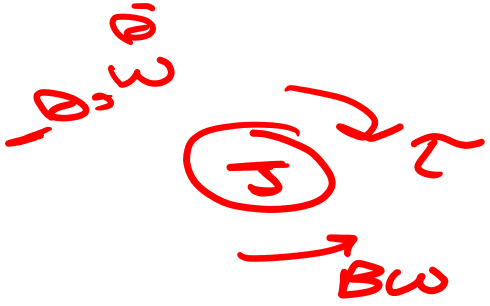
$$V_f(s) = I_f(s) R_f + L_f S I_f(s)$$

$$V_a(s) = E_a(s) + I_a(s)[R_a + sL_a]$$

$$V_f(s) = I_f(s)[R_f + sL_f]$$

$$I_f = V_f/(R_f + sL_f)$$





$\omega = \theta'$

$\alpha = \omega' = \theta''$

$$\Sigma T = J\omega' = J\theta''$$

$$\tau - B\omega = J\omega'$$

$$\tau - B\theta' = J\theta'' \quad \cdots\cdots (3)$$

$$T(s) - Bs\,\theta(s) = Js^2\theta(s)$$

$$T(s) = \theta(s)\left[Js^2 + Bs\right] \quad \cdots (3)$$

Transfer function of DC motor

1. What is the input? V_f I_f V_a I_a
2. What is the output? θ, ω,

$$\frac{\theta(s)}{V_f(s)} \Rightarrow \begin{array}{ll} I/P & V_f \\ O/P & \theta \end{array} \checkmark$$

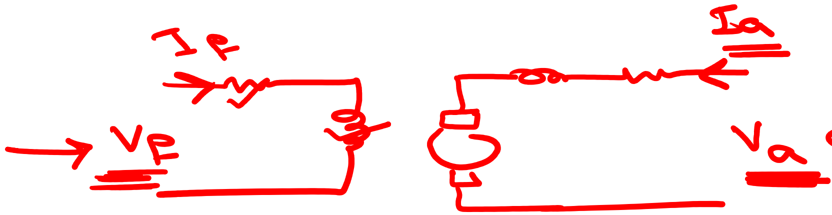
$$\frac{\theta(s)}{V_a(s)} \Rightarrow \begin{array}{ll} I/P & V_a \\ O/P & \theta \end{array} \checkmark$$

Field control motor

I_f

I/P V_f $\qquad \dfrac{\theta(s)}{V_f(s)}$

O/P θ



V_a constant.

$T_{elec} = T_{mech}$

$$K\phi(s) I_a(s) = (Js^2 + bs)\theta(s)$$

$$\underbrace{\underbrace{K K_f}\, I_f(s)\, \underbrace{I_a(s)}}_{K_1} = (Js^2+bs)\,\theta(s)$$

$$K_1 I_f(s) = (Js^2+bs)\,\theta(s)$$

$$K_1 \frac{V_f(s)}{sL_f + R_f} = (Js^2+bs)\,\theta(s)$$

note

$K_1 = K K_f I_a(s)$
const.

$$\frac{\theta(s)}{V_f(s)} = \frac{K_1}{(Js^2+bs)(sL_f+R_f)}$$

$\omega \Rightarrow$ output.

$(Js+b)\,\omega(s) \Rightarrow$

Armature control motor

Armature control motor

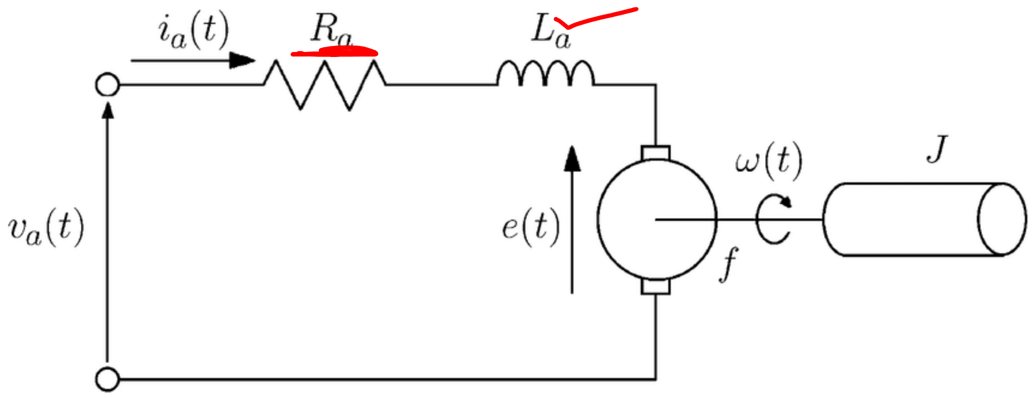
DC Motor Transfer Function

CH2- part 10

Eng. Fadwa Momani

Armature control motor

V_a I_a



$$V_a = E_a + I_a R_a + L_a \frac{dI_a}{dt}$$

$$V_a(s) = E_a(s) + I_a(s)\left[R_a + sL_a\right]$$

$T = K\phi I_a$

$\phi = K_f I_f$

$E_a = K_b \phi \omega$

$$I_a = \frac{V_a - E_a}{R_a + sL_a}$$

I/P V_a I_a

O/P θ ω $(Js+b)\omega(s)$

V_f, I_f constant.

$T_{elec} = T_{mech}$

$$K\phi(s) I_a(s) = (Js^2 + bs)\theta(s)$$

$$K K_f I_f I_a = (Js^2 + bs)\theta(s)$$

$$K_m I_a(s) = (Js^2 + bs)\theta(s)$$

$$\frac{K_m (V_a(s) - E_a(s))}{R_a + sL_a} = (Js^2 + bs)\theta(s)$$

$$\frac{K_m (V_a(s) - K_{b1}\theta(s))}{R_a + sL_a} = (Js^2 + bs)\theta(s)$$

Armature control motor

I/P $\rightarrow V_a \quad V_f \quad \underline{I_a} \quad I_f$

O/P $\rightarrow \theta, \underline{\omega}$

$E_a = K\phi\underline{\omega} = \underbrace{K\phi}\dot{\theta}$

$K_{b1}\dot{\theta}$

$E_a(s) = K_{b1}\theta(s) \quad \ldots \quad X$

$$\frac{\theta(s)}{V_a(s)} = \frac{K_m}{s[(R_a + sL_a)(Js^2 + bs) + K_m K_b]}$$

Block Diagram fundamentals & reduction techniques

CH2-12 AND 13

Introduction

Block diagram is a shorthand, graphical representation of a physical system, illustrating the functional relationships among its components.

OR

A Block Diagram is a shorthand pictorial representation of the cause-and-effect relationship of a system.

input output.

Introduction

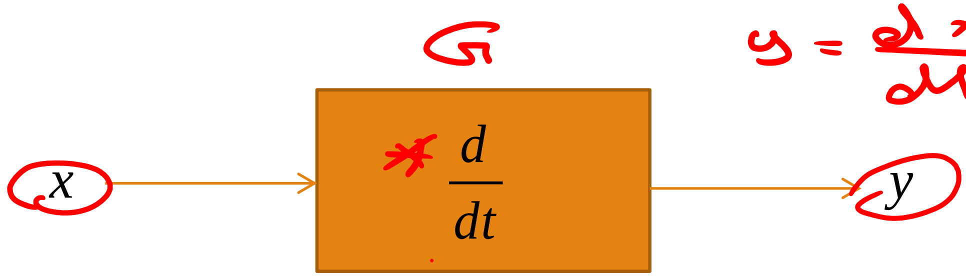
The simplest form of the block diagram is the single ***block, with one input and one output.***

The interior of the rectangle representing the block usually contains a description of or the name of the element, or the symbol for the mathematical operation to be performed on the input to yield the output.

The arrows represent the direction of information or signal flow.

$y = Gx$

$y = \frac{dx}{dt}$



Introduction

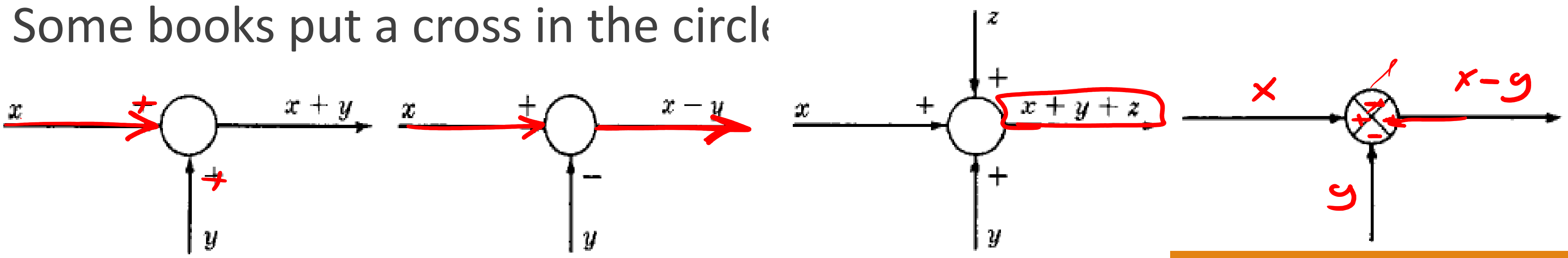
The operations of addition and subtraction have a special representation.

The block becomes a small circle, called a summing point, with the appropriate plus or minus sign associated with the arrows entering the circle.

Any number of inputs may enter a summing point.

The output is the algebraic sum of the inputs.

Some books put a cross in the circl

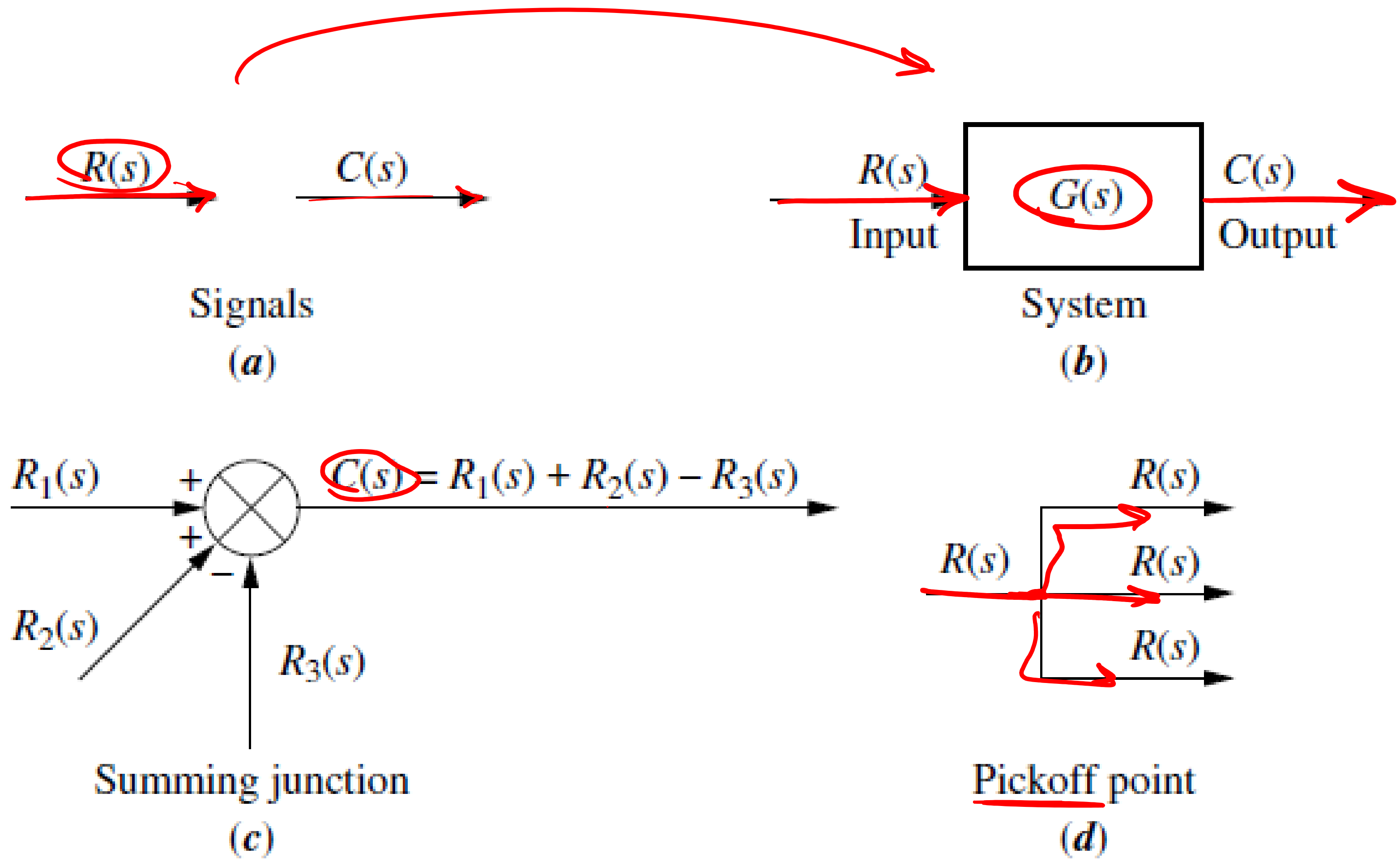


Components of a Block Diagram for a Linear Time Invariant System

System components are alternatively called elements of the system.

Block diagram has four components:

- ***Signals***
- ***System/ block***
- ***Summing junction***
- ***Pick-off/ Take-off point***

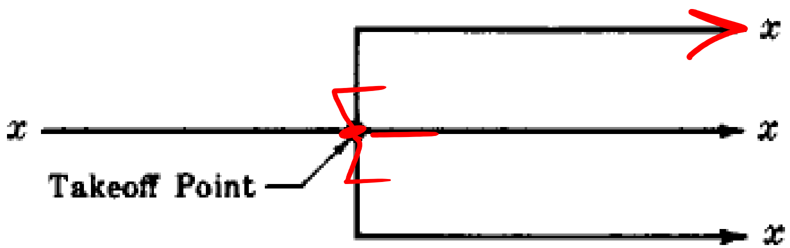


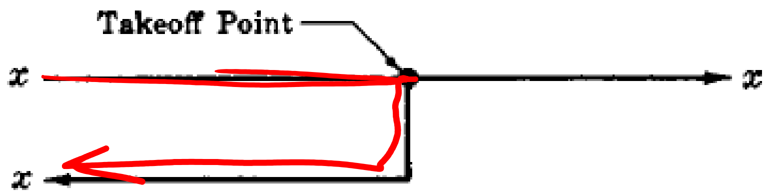
R(s)
C(s)
Signals
(a)
R(s)
Input
G(s)
C(s)
Output
System
(b)
R1(s)
+
+
−
C(s) = R1(s) + R2(s) − R3(s)
R2(s)
R3(s)
Summing junction
(c)
R(s)
R(s)
R(s)
R(s)
Pickoff point
(d)

In order to have the same signal or variable be an input to more than one block or summing point, a takeoff point is used.

Distributes the input signal, undiminished, to several output points.

This permits the signal to proceed unaltered along several different paths to several destinations.

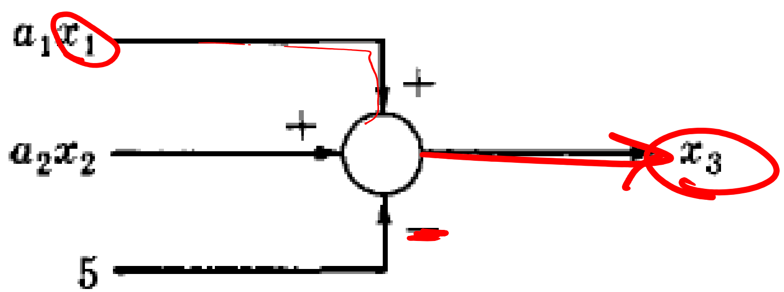


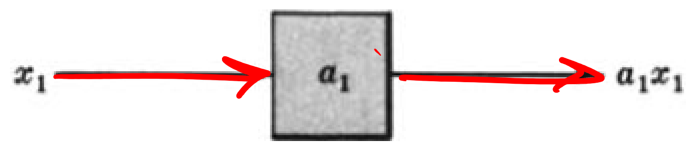


Example-1

Consider the following equations in which x_1, x_2, x_3, are variables, and a_1, a_2 *are* general coefficients or mathematical operators.

$$x_3 = a_1 x_1 + a_2 x_2 - 5$$

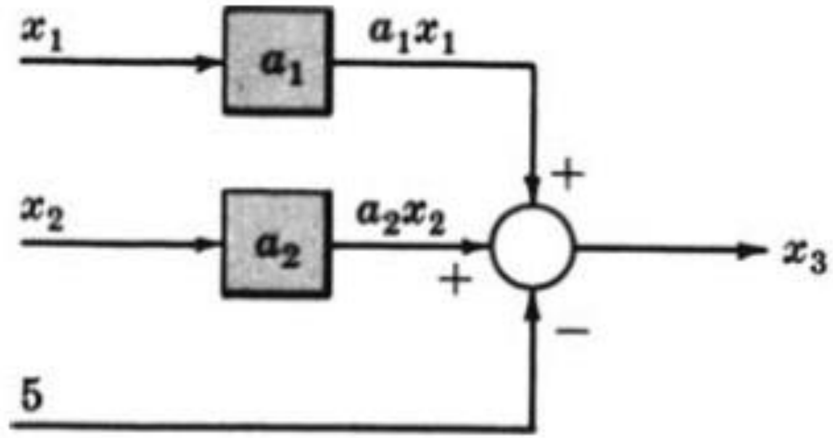




Example-1

Consider the following equations in which x_1, x_2, x_3, are variables, and a_1, a_2 *are* general coefficients or mathematical operators.

$$x_3 = a_1 x_1 + a_2 x_2 - 5$$



Topologies

We will now examine some common topologies for interconnecting subsystems and derive the single transfer function representation for each of them.

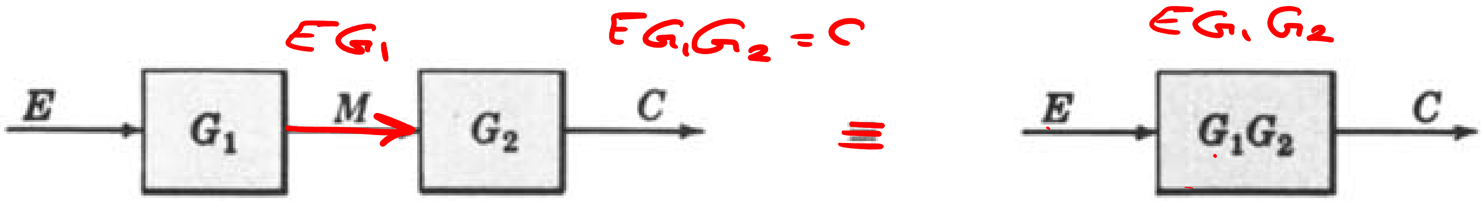
These common topologies will form the basis for reducing more complicated systems to a single block.

CASCADE

- Any finite number of blocks in series may be algebraically combined by multiplication of transfer functions.
- That is, *n components or blocks with transfer functions G_1, G_2, . . . , G_n, connected in cascade* are equivalent to a single element G with a transfer function given by

$$G = G_1 \cdot G_2 \cdot G_3 \cdots G_n = \prod_{i=1}^{n} G_i$$

Example



Multiplication of transfer functions is *commutative; that is,*

$$G_iG_j = G_jG_i$$

for any i or *j* .

Cascade:

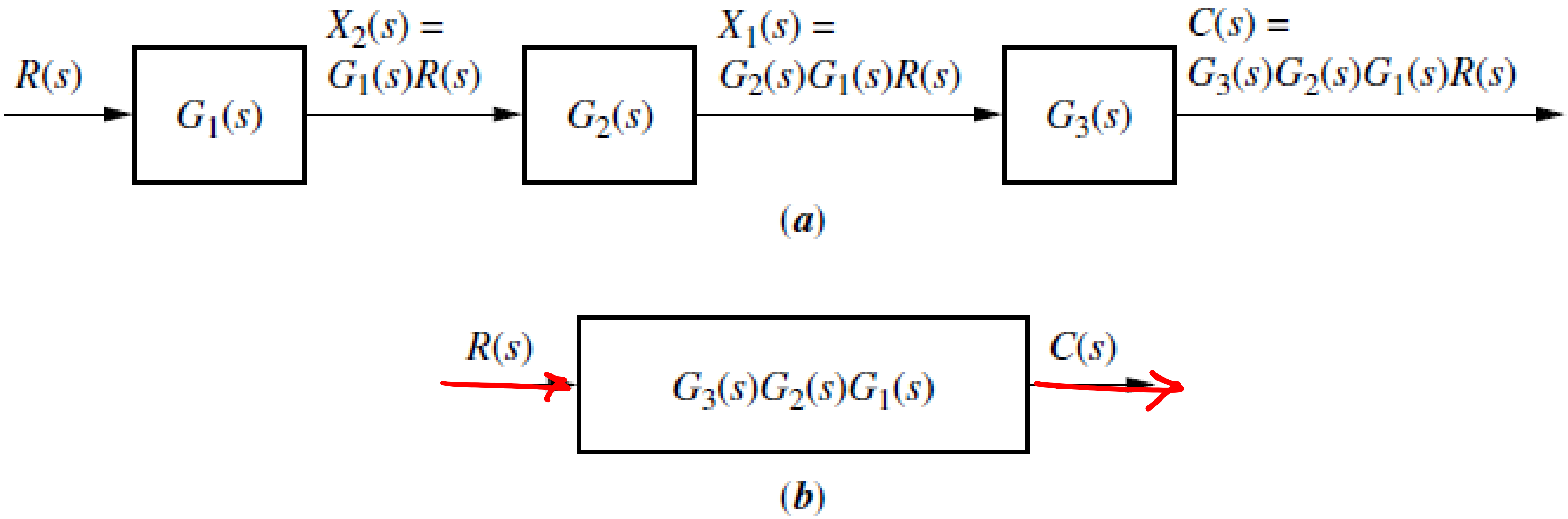


Figure:
a) Cascaded Subsystems.
b) Equivalent Transfer Function.

The equivalent transfer function is

$$G_e(s) = G_3(s)G_2(s)G_1(s)$$

Parallel Form:

Parallel subsystems have a common input and an output formed by the algebraic sum of the outputs from all of the subsystems.

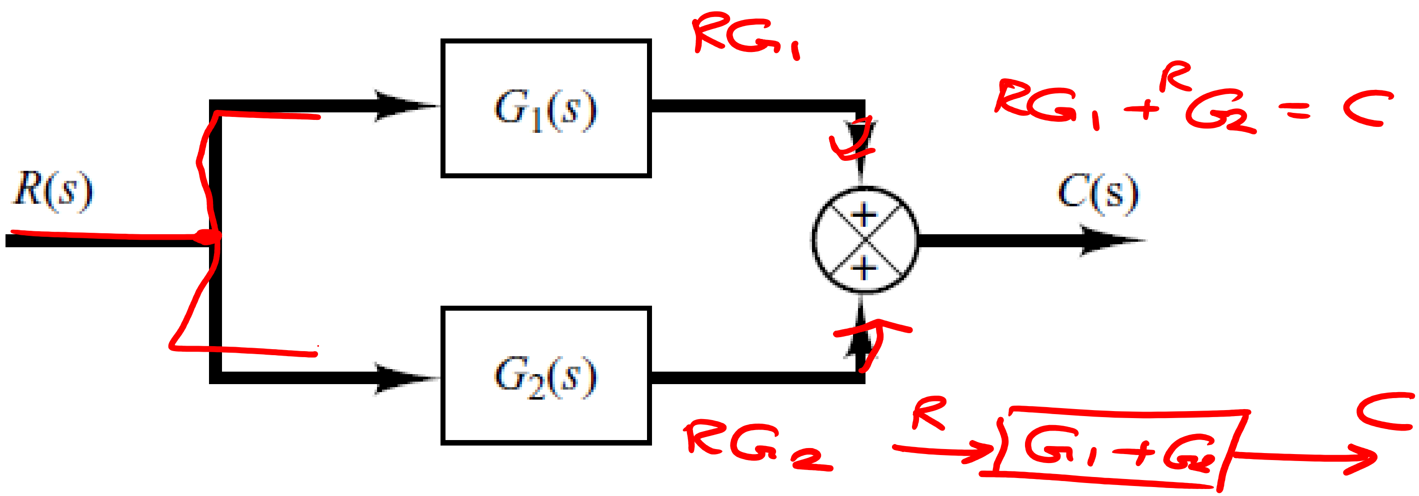


Figure: Parallel Subsystems.

Parallel Form:

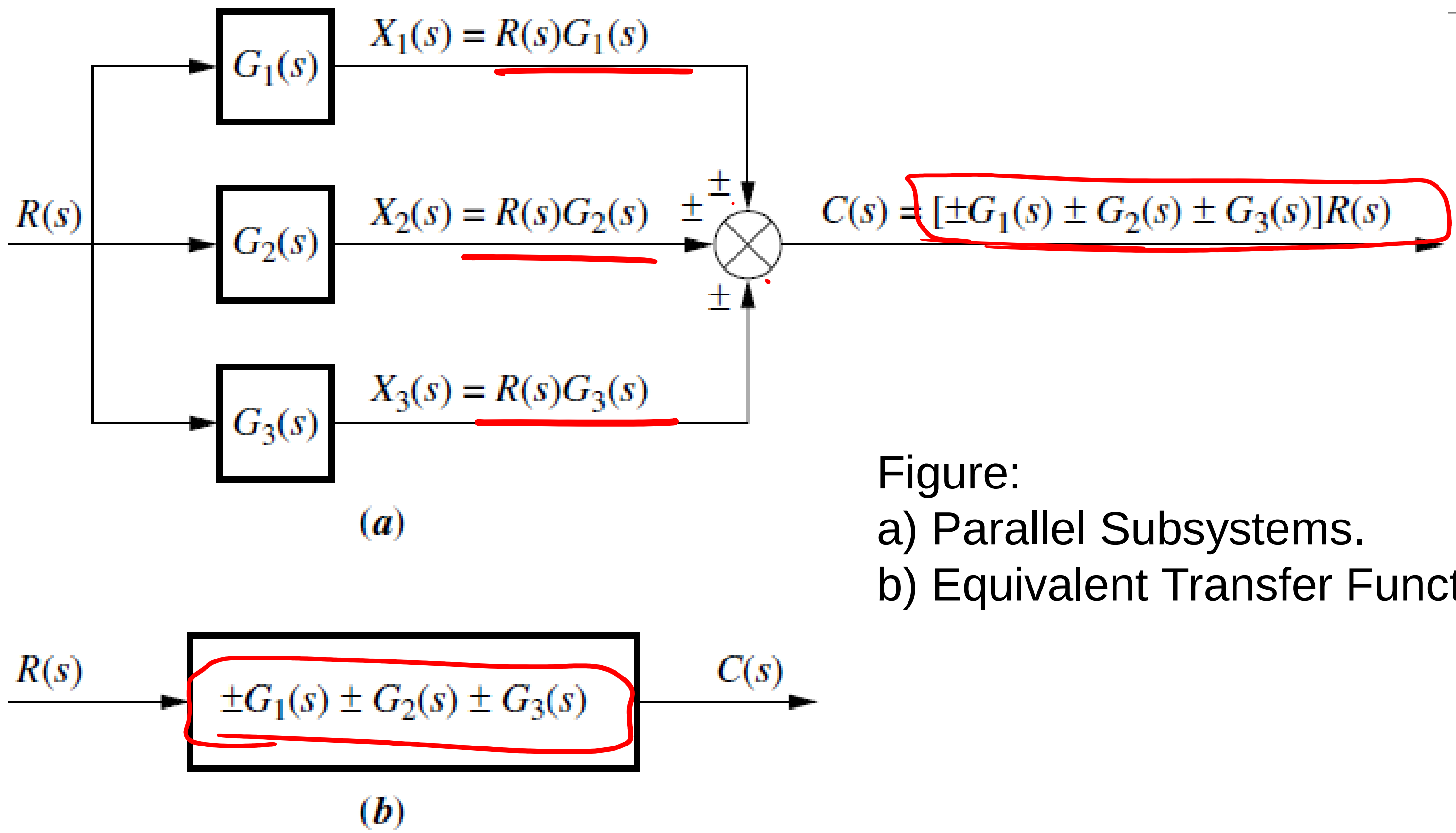
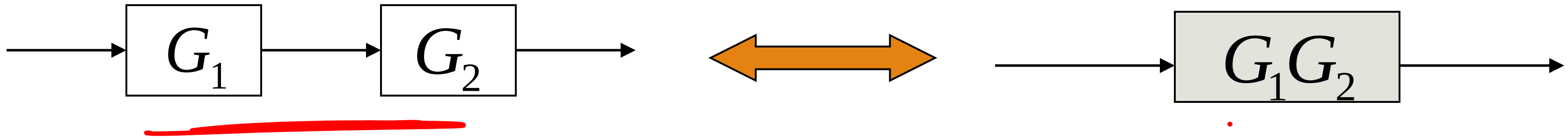


Figure:
a) Parallel Subsystems.
b) Equivalent Transfer Function.

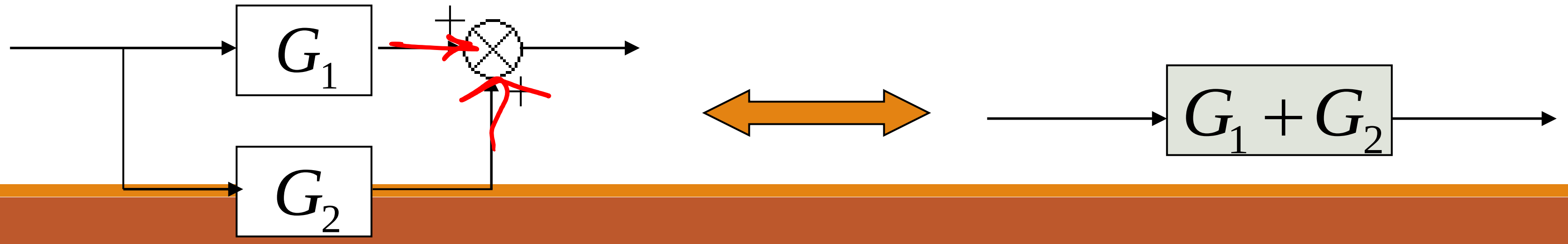
The equivalent transfer function is $G_e(s) = \pm G_1(s) \pm G_2(s) \pm G_3(s)$

Reduction techniques

1. Combining blocks in cascade

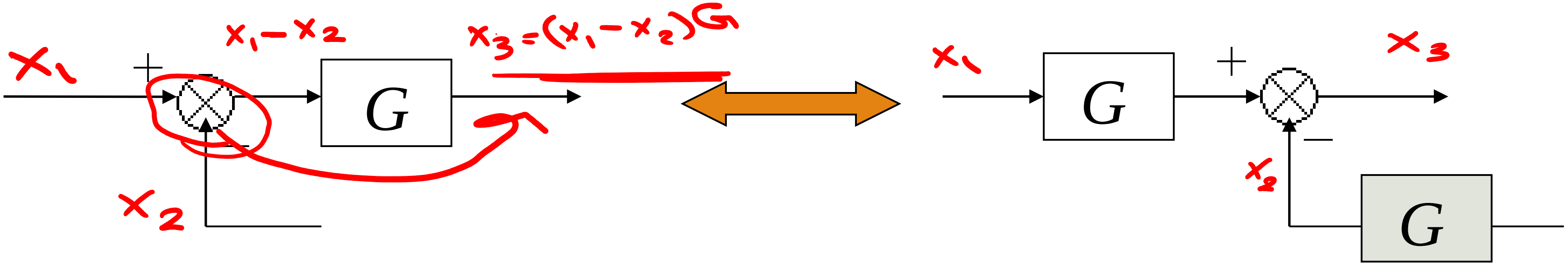


2. Combining blocks in parallel



Reduction techniques

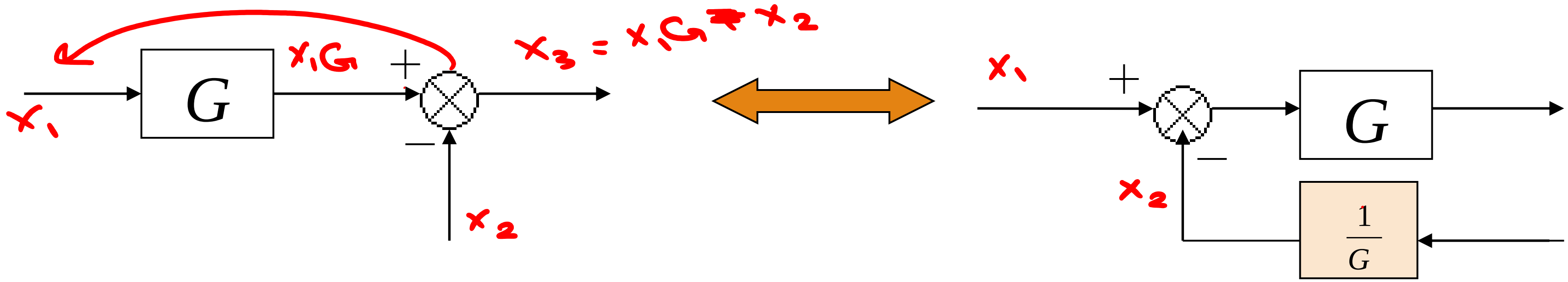
3. Moving a summing point behind a block



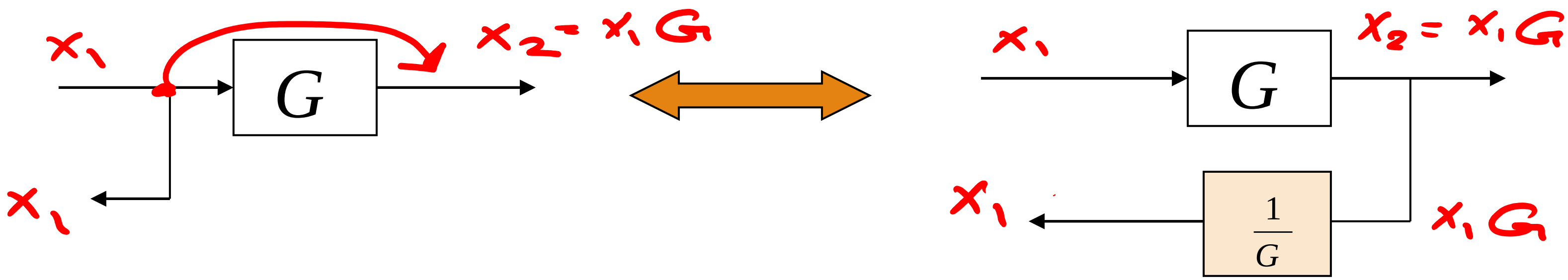
$x_3 = x_1 G + x_2 G$

Reduction techniques

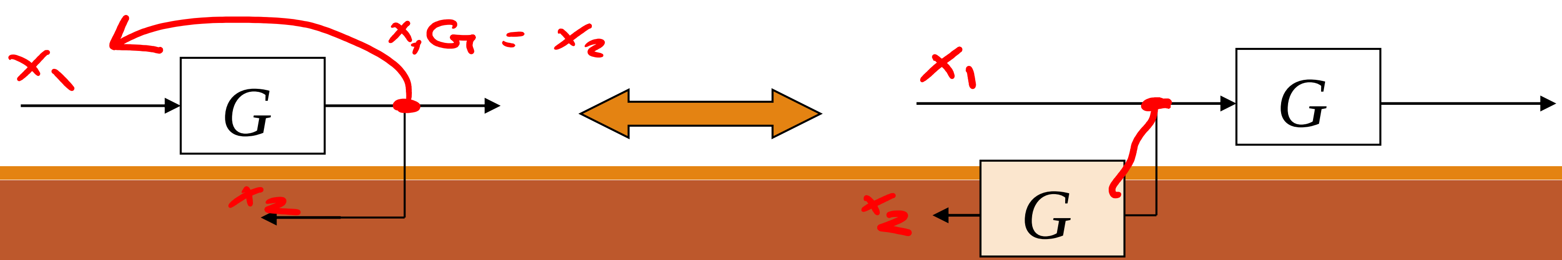
3. Moving a summing point ahead of a block



4. Moving a pickoff point behind a block

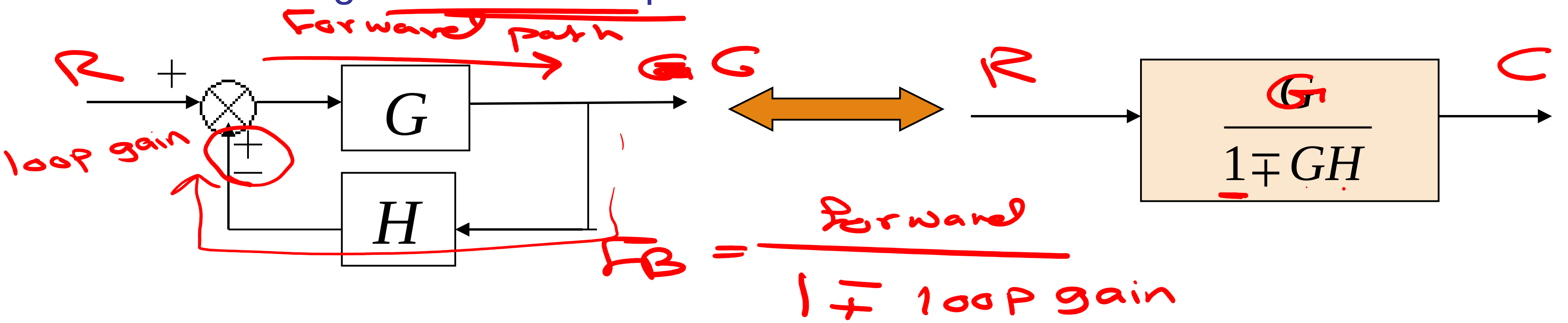


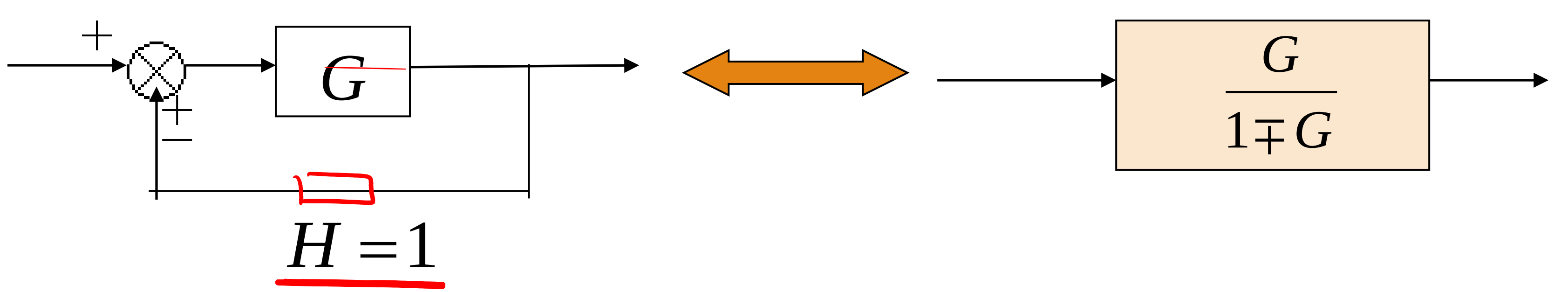
5. Moving a pickoff point ahead of a block



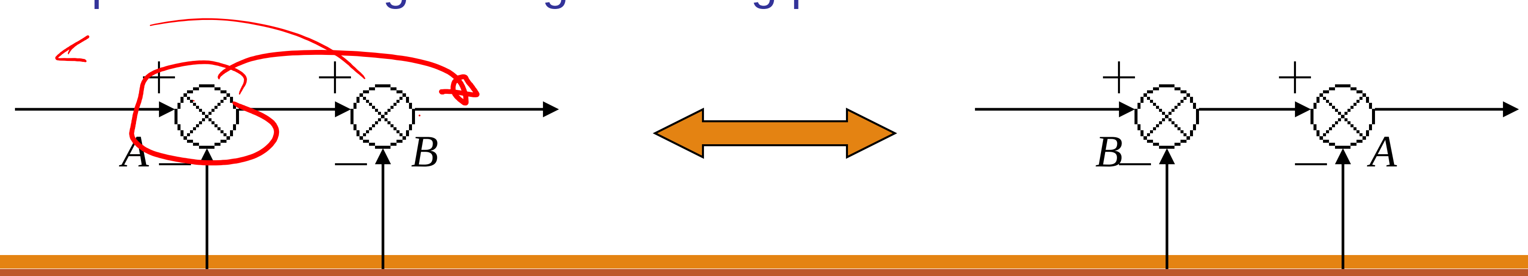
Reduction techniques

6. Eliminating a feedback loop





7. Swap with two neighboring summing points



Feedback Form:

The third topology is the feedback form. Let us derive the transfer function that represents the system from its input to its output. The typical feedback system, shown in figure:

$\frac{G_1}{1+G_1G_2}$

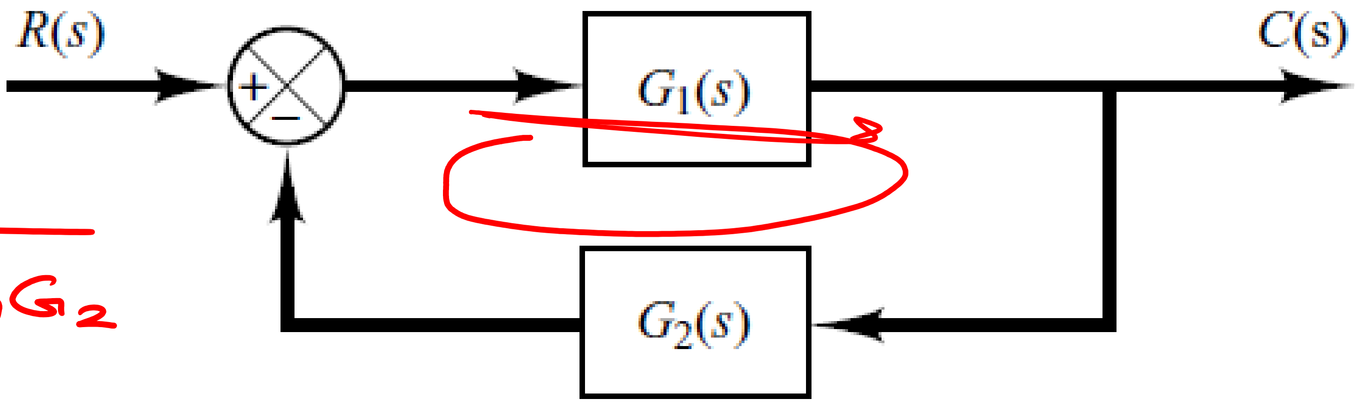
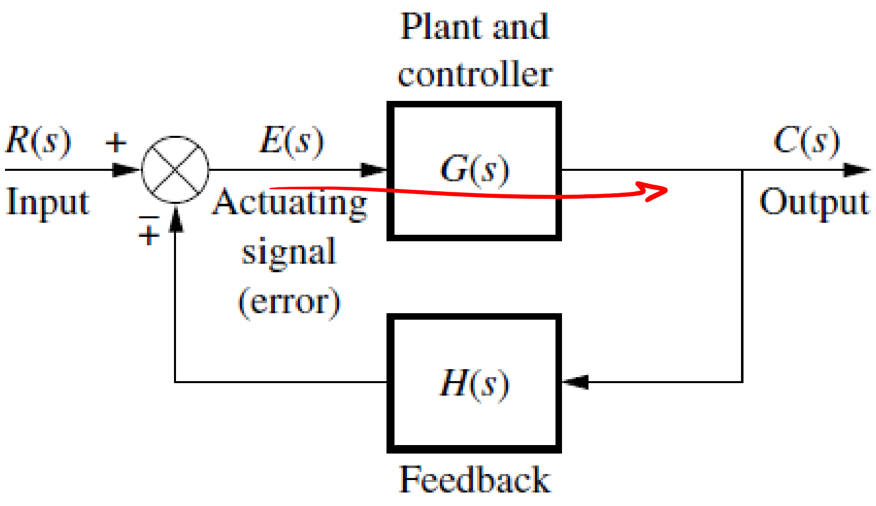


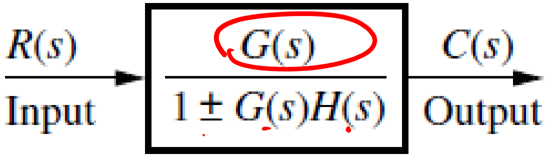
Figure: Feedback (Closed Loop) Control System.

The system is said to have negative feedback if the sign at the summing junction is negative and positive feedback if the sign is positive.

Feedback Form:



(b)

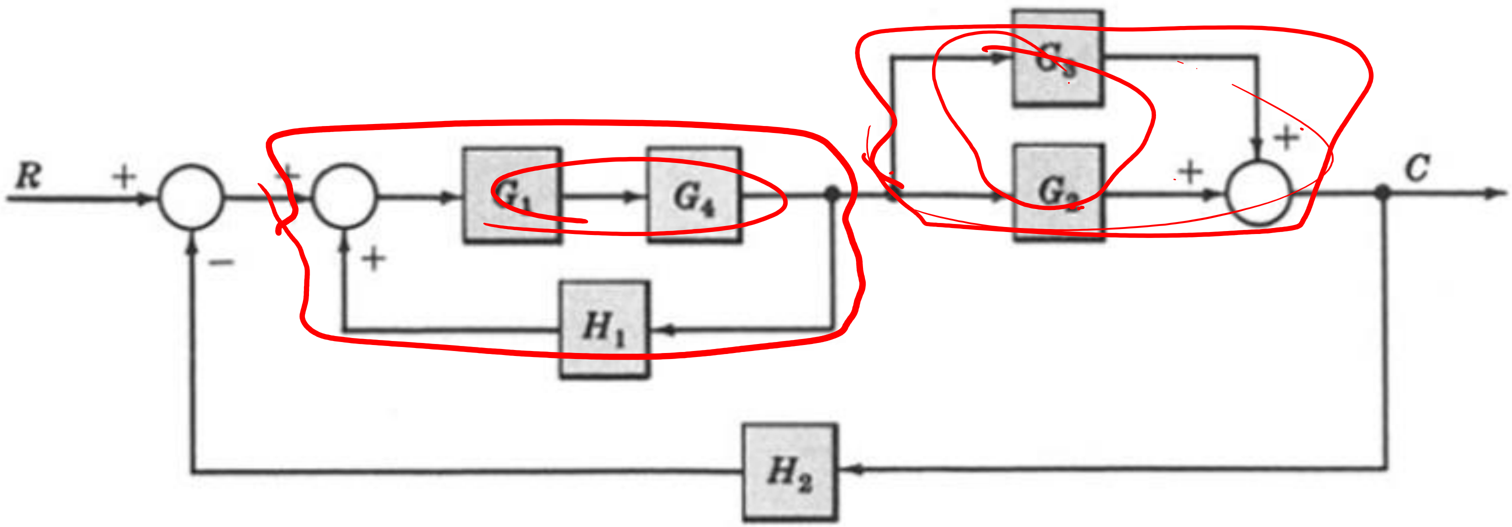


(c)

The equivalent or closed-loop transfer function is

$$G_e(s) = \frac{G(s)}{1 \pm G(s)H(s)}$$

Example-4: Reduce the Block Diagram to

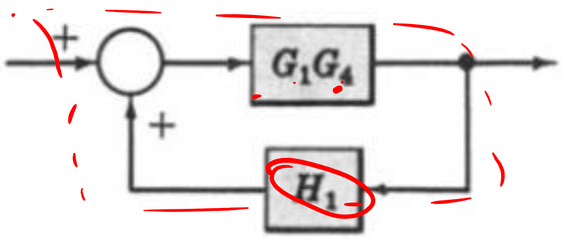


Step 1: Combine all cascade blocks using Transformation 1.

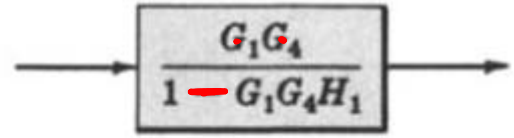
Step 2: Combine all parallel blocks using Transformation 2.

Example-4: Continue.

Step 3: Eliminate all minor feedback loops using Transformation 4.

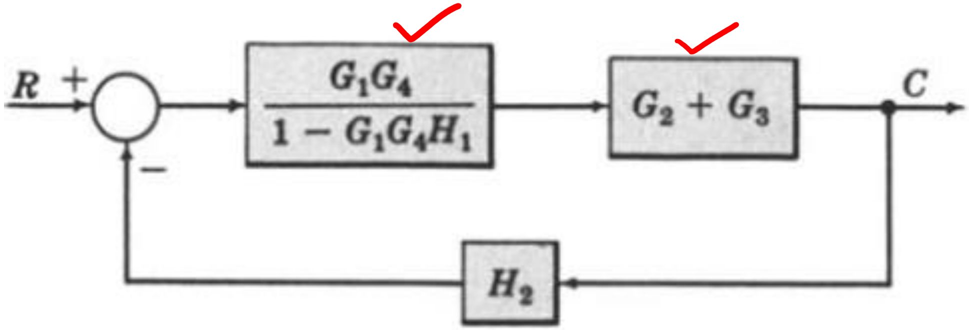


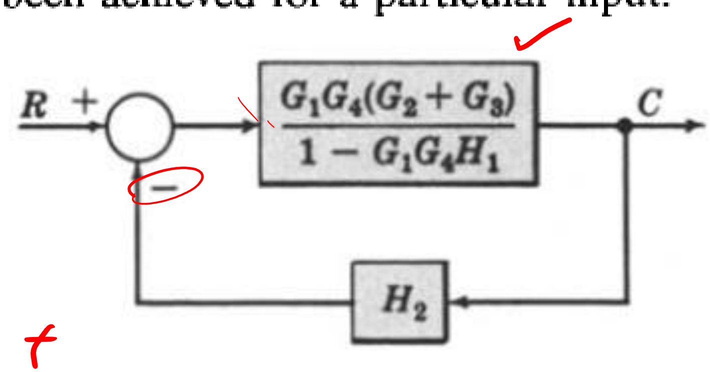
≡



Step 4: Shift summing points to the left and takeoff points to the right of the major loop, using Transformations 7, 10, and 12. However in this example step-4 does not apply.

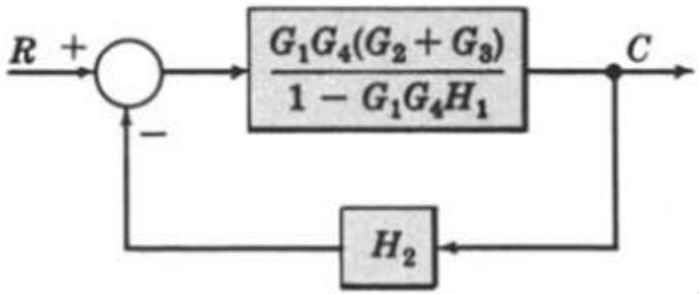
Step 5: Repeat Steps 1 to 4 until the canonical form has been achieved for a particular input.





Step 6: Repeat Steps 1 to 5 for each input, as required.

However in this example step-6 does not apply.



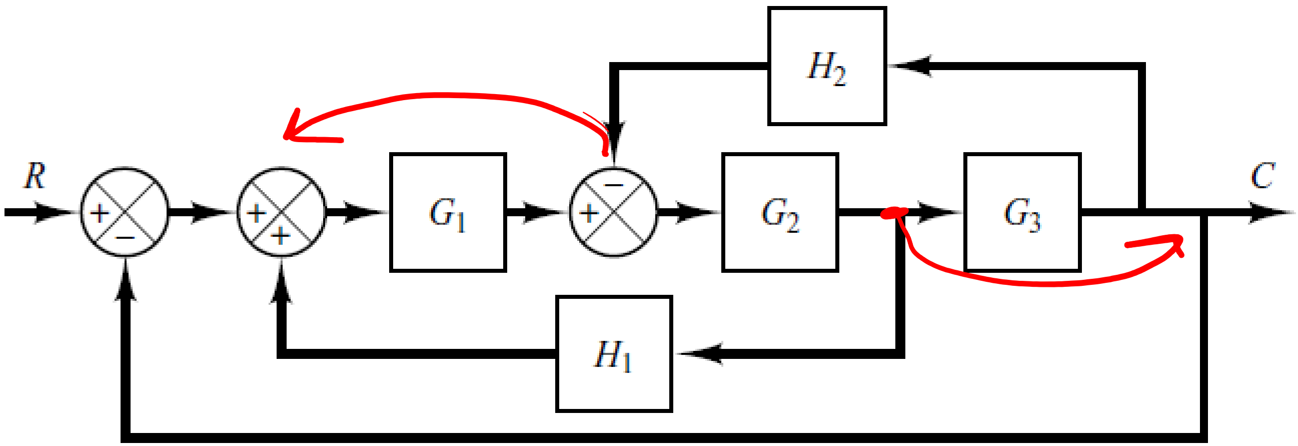
$$\frac{\dfrac{G_1G_4(G_2+G_3)}{1-\cancel{G_1}G_4H_1}}{1+\dfrac{G_1G_4(G_2+G_3)}{1-\cancel{G_1}G_4H_1} * H_2} =$$

R

$$= \boxed{\frac{G_1G_4(G_2+G_3)}{1-G_1G_4H_1+G_1G_4(G_2+G_3)H_2}}$$

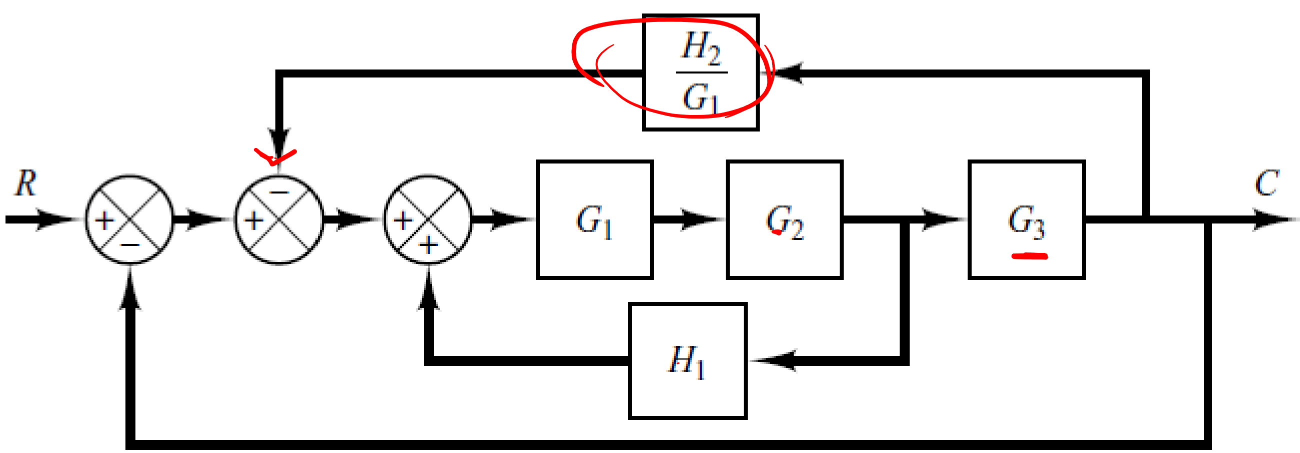
C

Example-5: Simplify the Block Diagram.



By moving the summing point of the negative feedback loop containing H_2 outside the positive feedback loop containing H_1, we obtain Figure

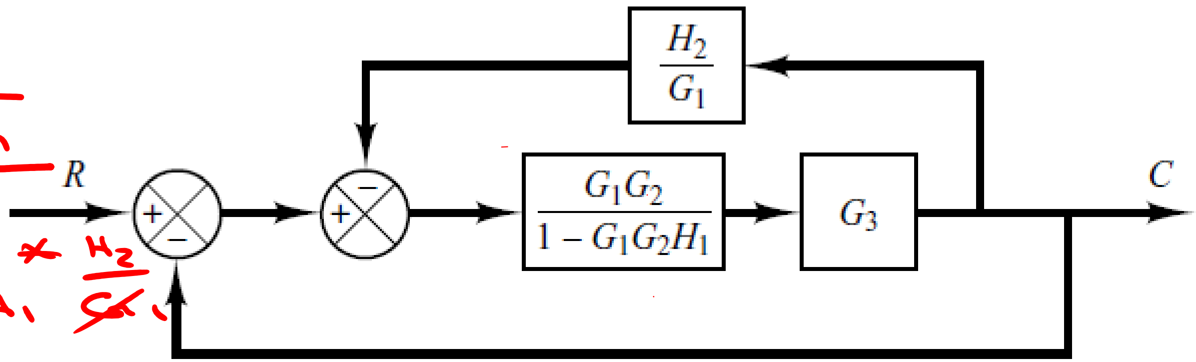
$$\frac{G_1G_2}{1-G_1G_2H_1}$$



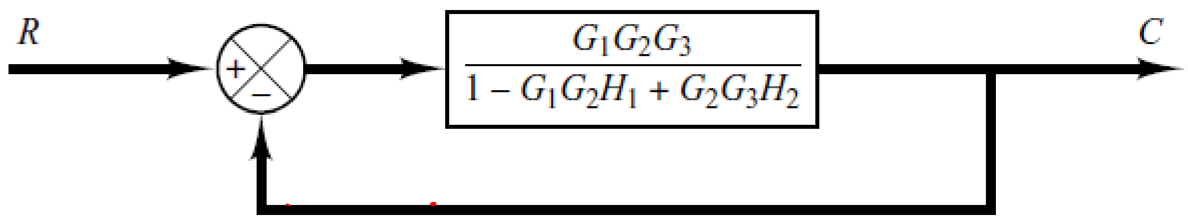
Example-5: Continue.

Eliminating the positive feedback loop, we have

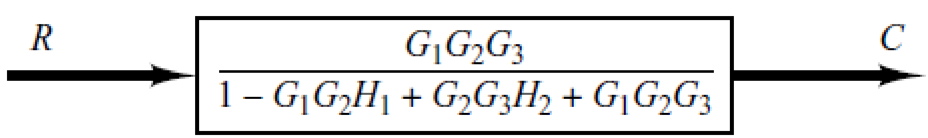
$$\frac{\dfrac{G_1G_2G_3}{1-G_1G_2H_1}}{1+\dfrac{\cancel{G_1}G_2G_3}{1-G_1G_2H_1}\times\dfrac{H_2}{\cancel{G_1}}}$$



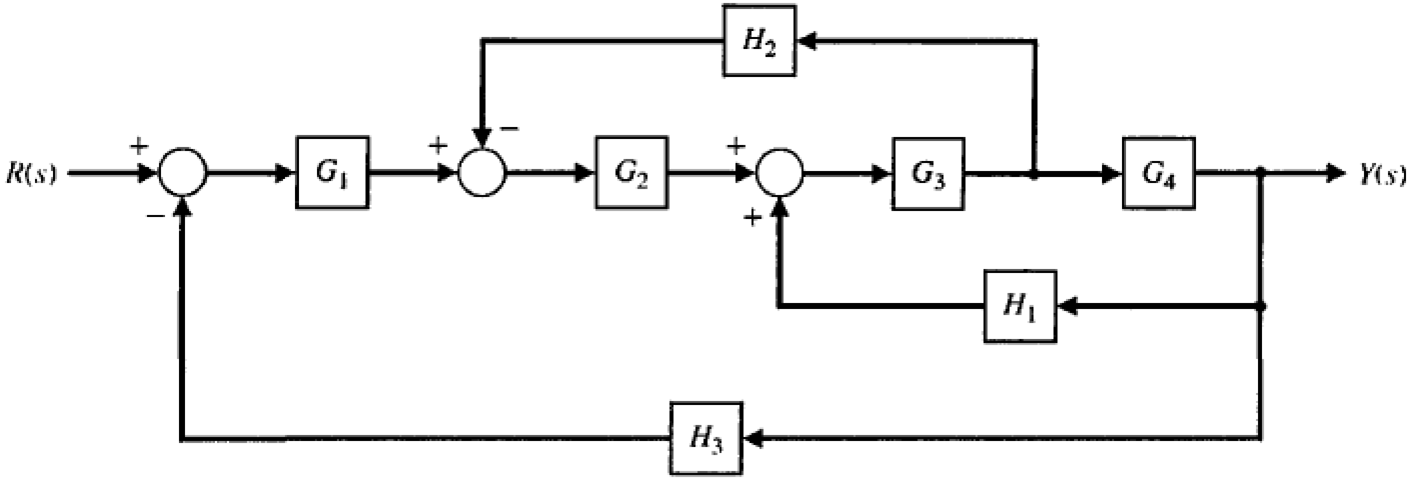
The elimination of the loop containing H_2/G_1 gives



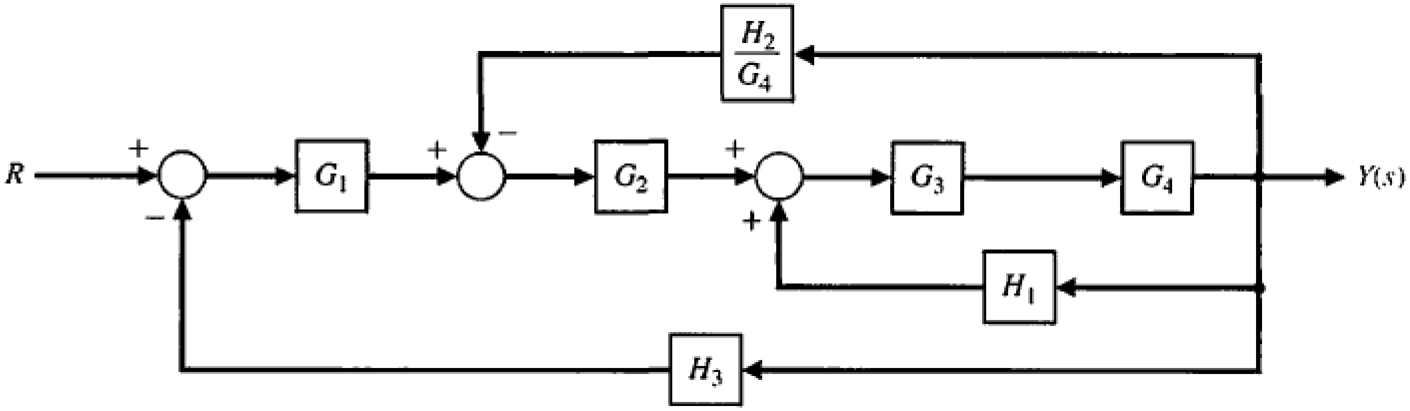
Finally, eliminating the feedback loop results in



Example-6: Reduce the Block Diagram.

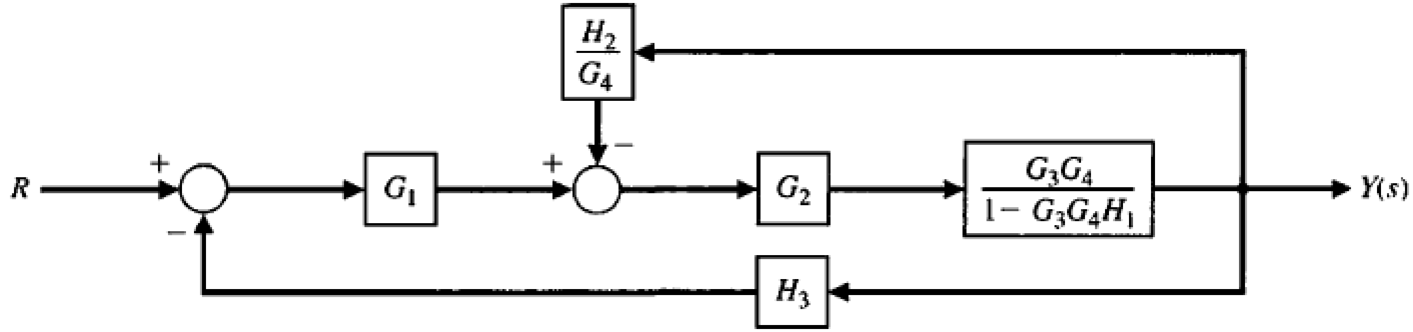


First, to eliminate the loop $G_3G_4H_1$, we move H_2 behind block G_4

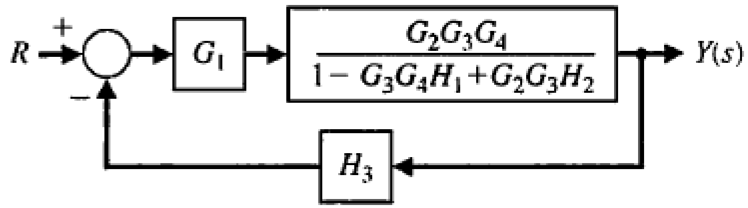


Eliminating the loop $G_3G_4H_1$ we obtain

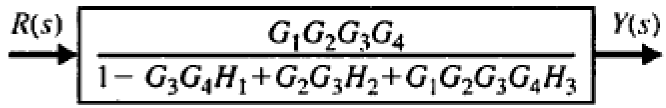
Example-6: Continue.



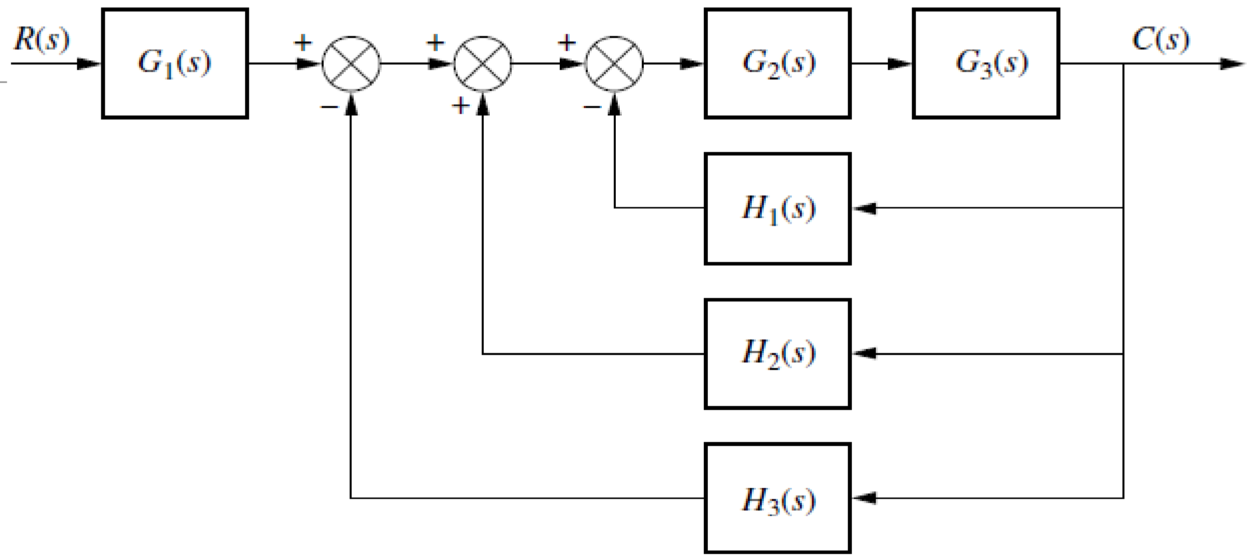
Then, eliminating the inner loop containing H_2/G_4, we obtain



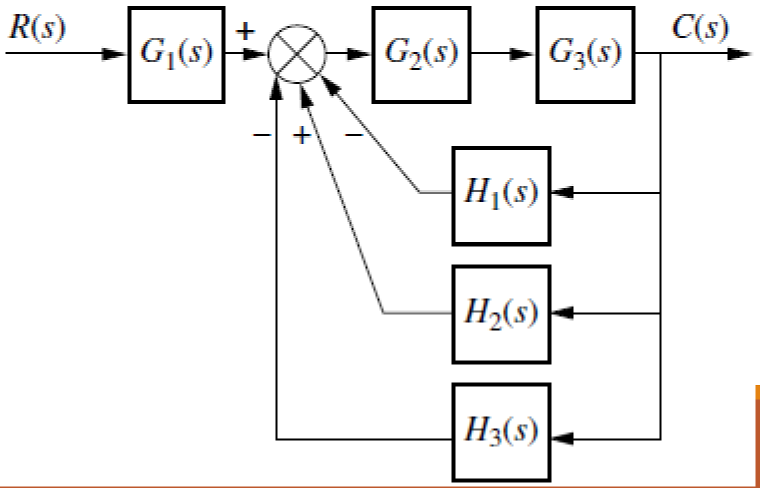
Finally, by reducing the loop containing H_3, we obtain



Example-7: Reduce the Block Diagram. (from Nise: page-242)

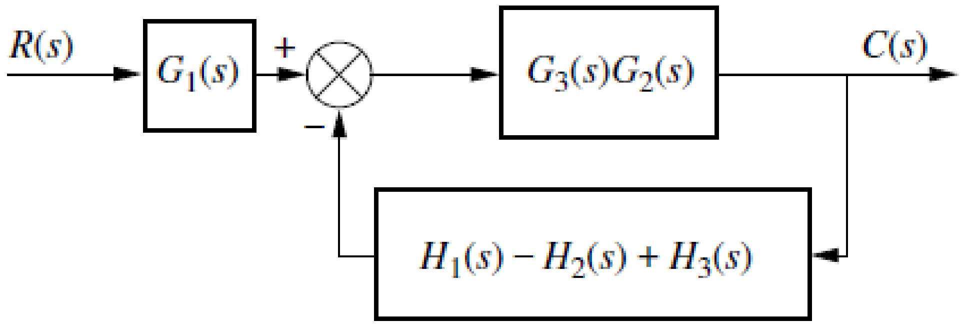


First, the three summing junctions can be collapsed into a single summing junction,

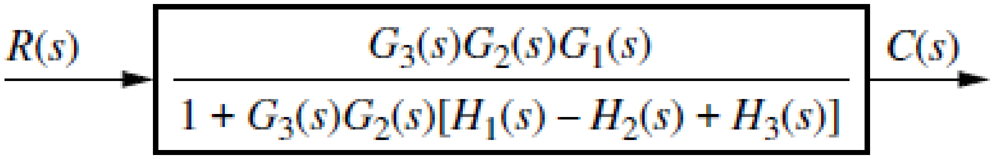


Example-7: Continue.

Second, recognize that the three feedback functions, $H_1(s)$, $H_2(s)$, and $H_3(s)$, are connected in parallel. They are fed from a common signal source, and their outputs are summed. Also recognize that $G_2(s)$ and $G_3(s)$ are connected in cascade.

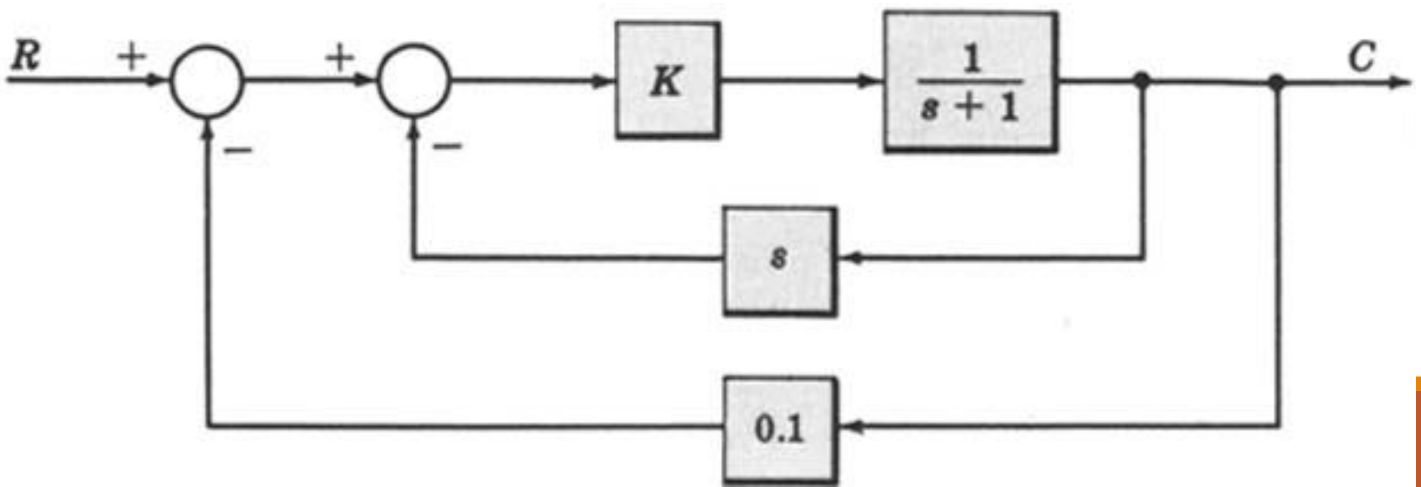


Finally, the feedback system is reduced and multiplied by $G_1(s)$ to yield the equivalent transfer function shown in Figure



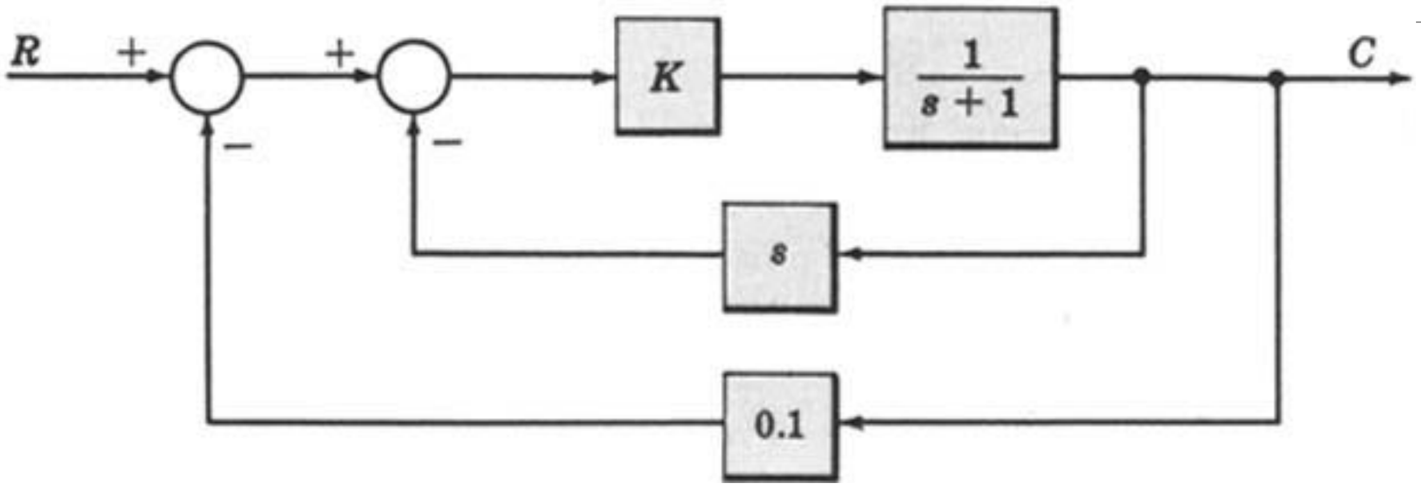
Example-8: For the system represented by the following block diagram determine:

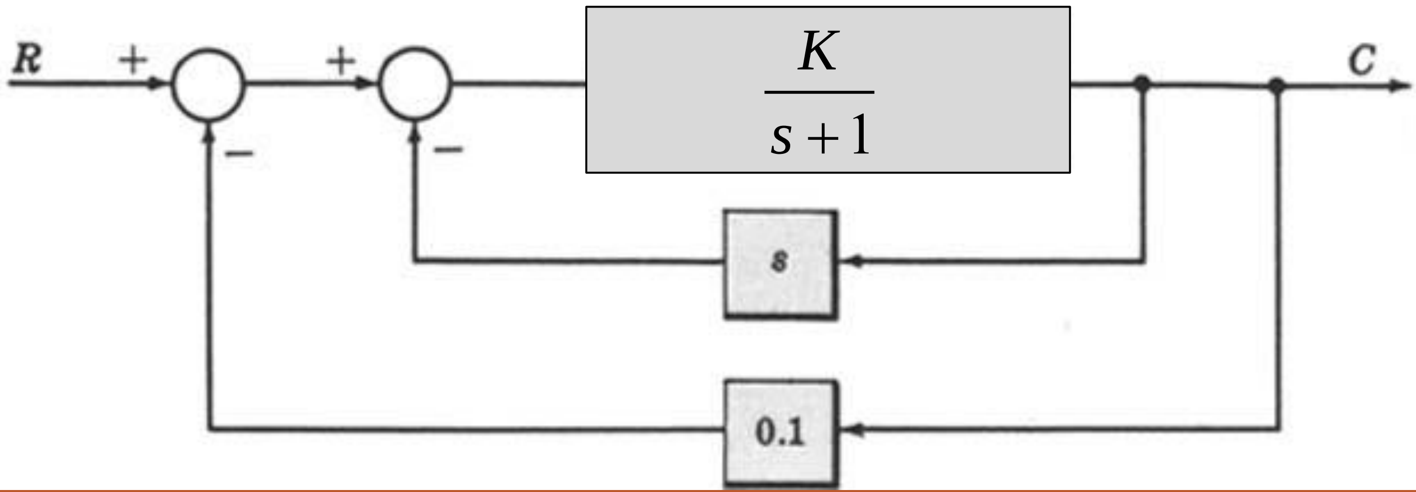
1. Open loop transfer function
2. Feed Forward Transfer function
3. control ratio
4. feedback ratio
5. error ratio
6. closed loop transfer function
7. characteristic equation
8. closed loop poles and zeros if K=10.



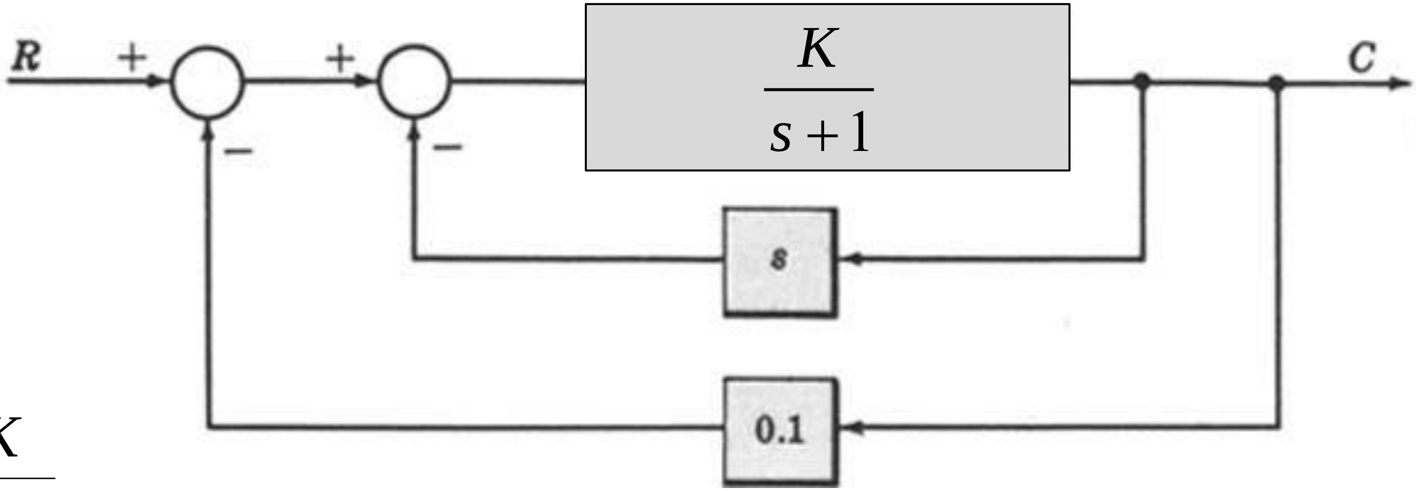
Example-8: Continue

- First we will reduce the given block diagram to canonical form

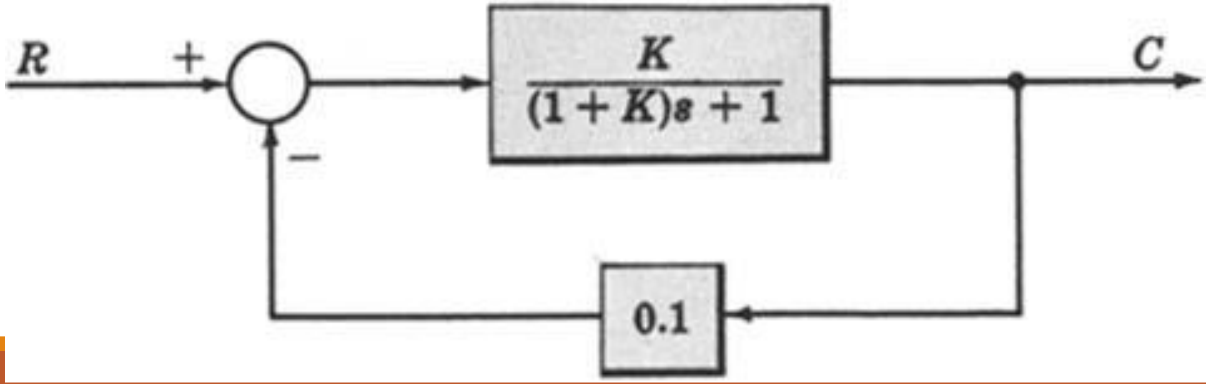




Example-8: Continue

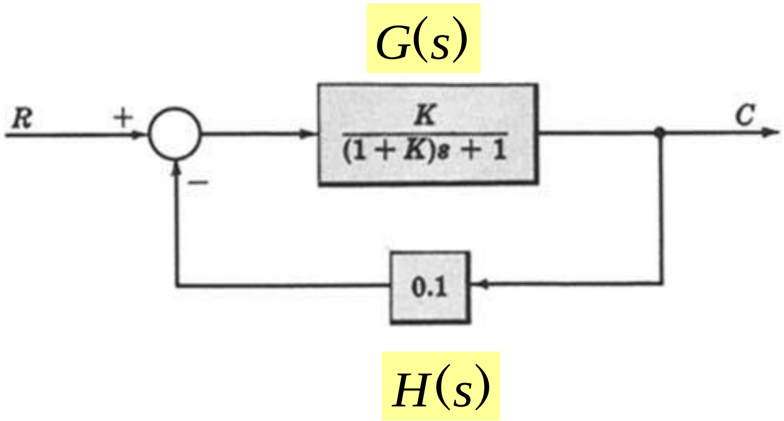


$$\frac{G}{1+GH}=\frac{\dfrac{K}{s+1}}{1+\dfrac{K}{s+1}s}$$



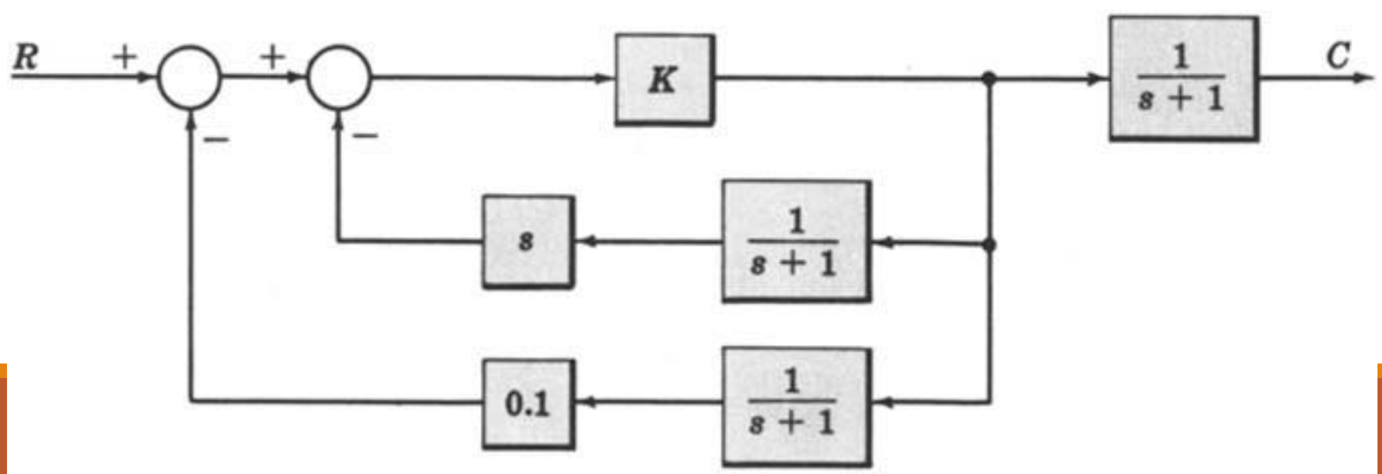
Example-8: Continue

1. Open loop transfer function $\dfrac{B(s)}{E(s)} = G(s)H(s)$
2. Feed Forward Transfer function $\dfrac{C(s)}{E(s)} = G(s)$
3. control ratio $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$
4. feedback ratio $\dfrac{B(s)}{R(s)} = \dfrac{G(s)H(s)}{1+G(s)H(s)}$
5. error ratio $\dfrac{E(s)}{R(s)} = \dfrac{1}{1+G(s)H(s)}$
6. closed loop transfer function $\dfrac{C(s)}{R(s)} = \dfrac{G(s)}{1+G(s)H(s)}$
7. characteristic equation $1+G(s)H(s) = 0$
8. closed loop poles and zeros if K=10.

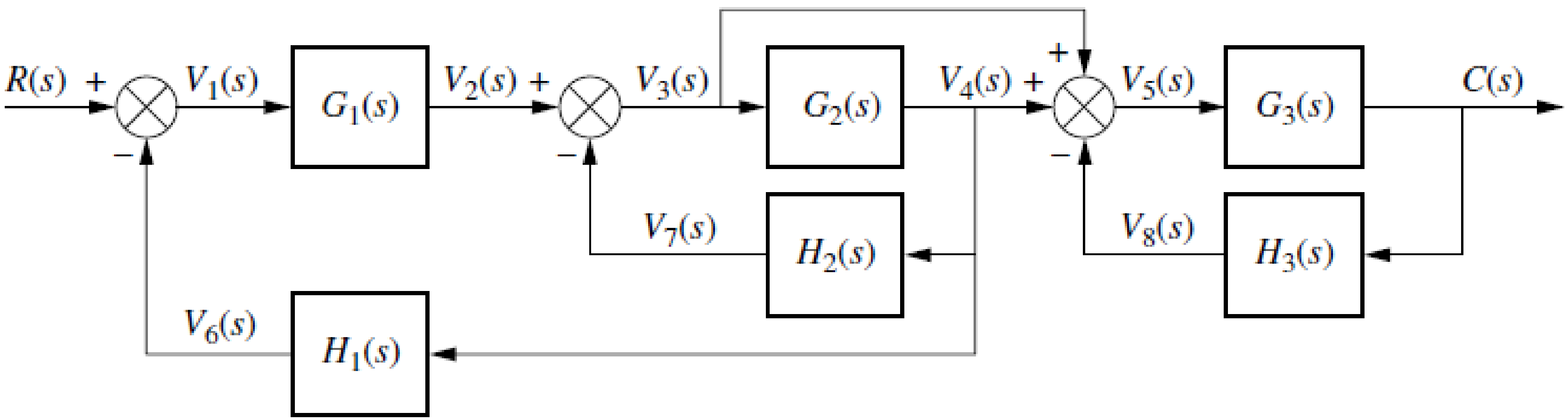


Example-9: For the system represented by the following block diagram determine:

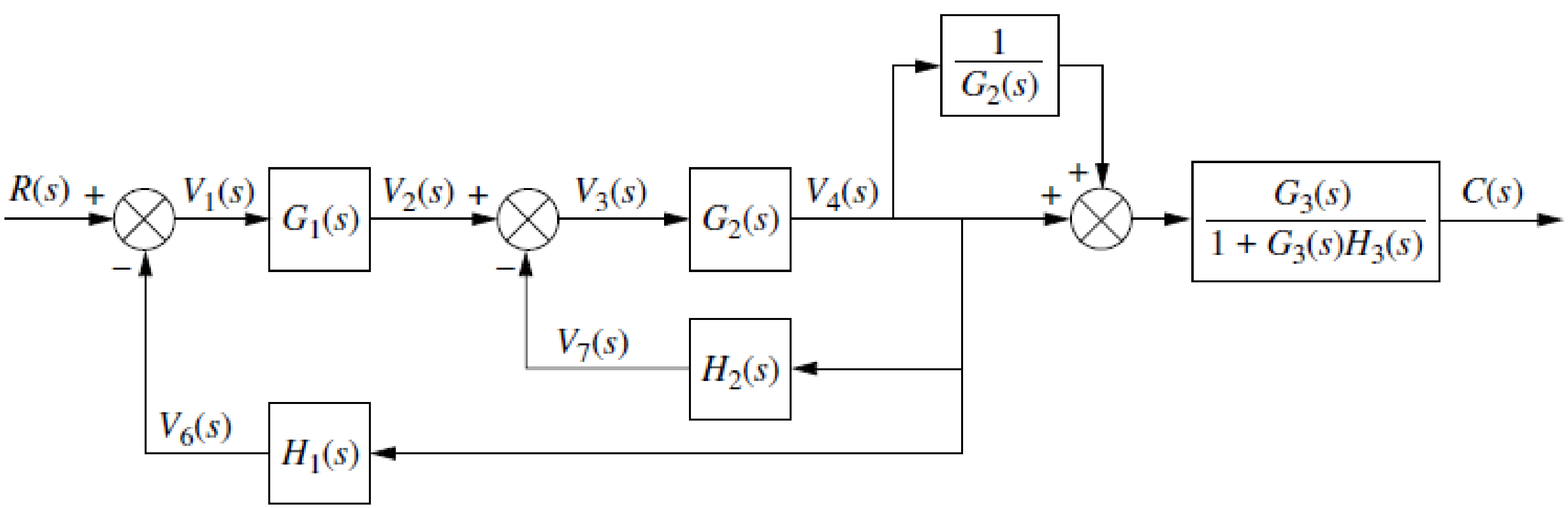
1. Open loop transfer function
2. Feed Forward Transfer function
3. control ratio
4. feedback ratio
5. error ratio
6. closed loop transfer function
7. characteristic equation
8. closed loop poles and zeros if K=100.



Example-10: Reduce the system to a single transfer function. (from Nise:page-243).

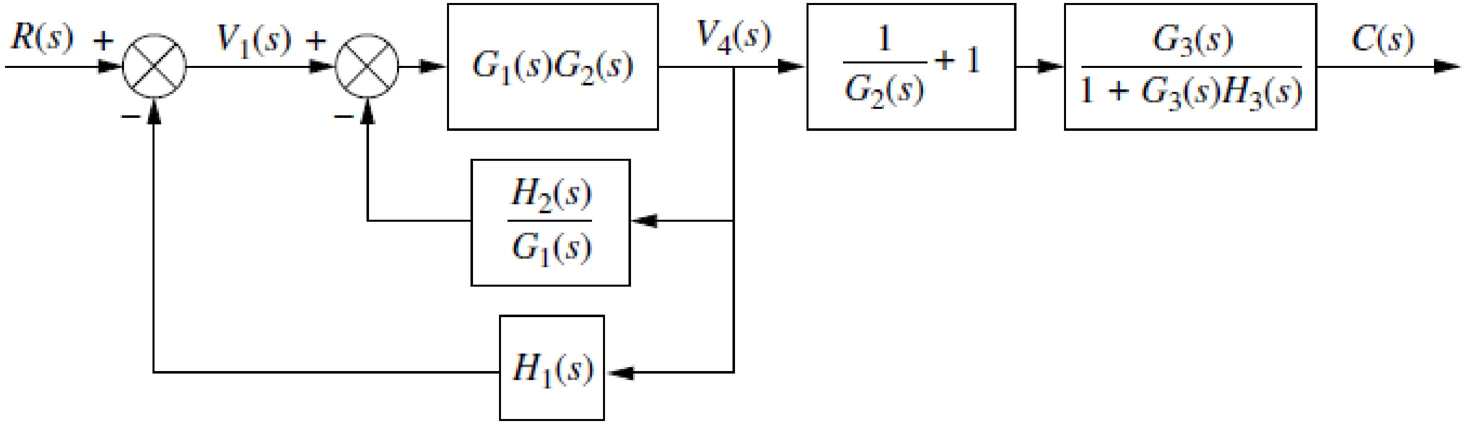


First, move $G_2(s)$ to the left past the pickoff point to create parallel subsystems, and reduce the feedback system consisting of $G_3(s)$ and $H_3(s)$.

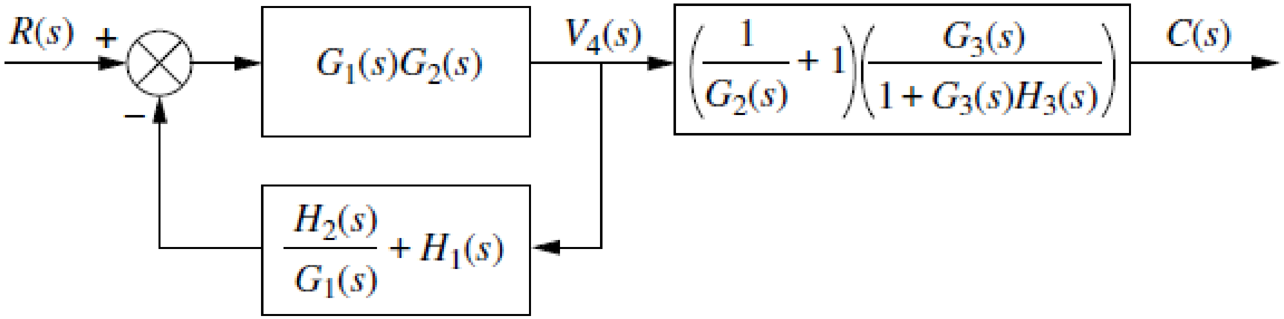


Example-10: Continue.

Second, reduce the parallel pair consisting of $1/G_2(s)$ and unity, and push $G_1(s)$ to the right past the summing junction, creating parallel subsystems in the feedback.



Third, collapse the summing junctions, add the two feedback elements together, and combine the last two cascaded blocks.



Example-10: Continue.

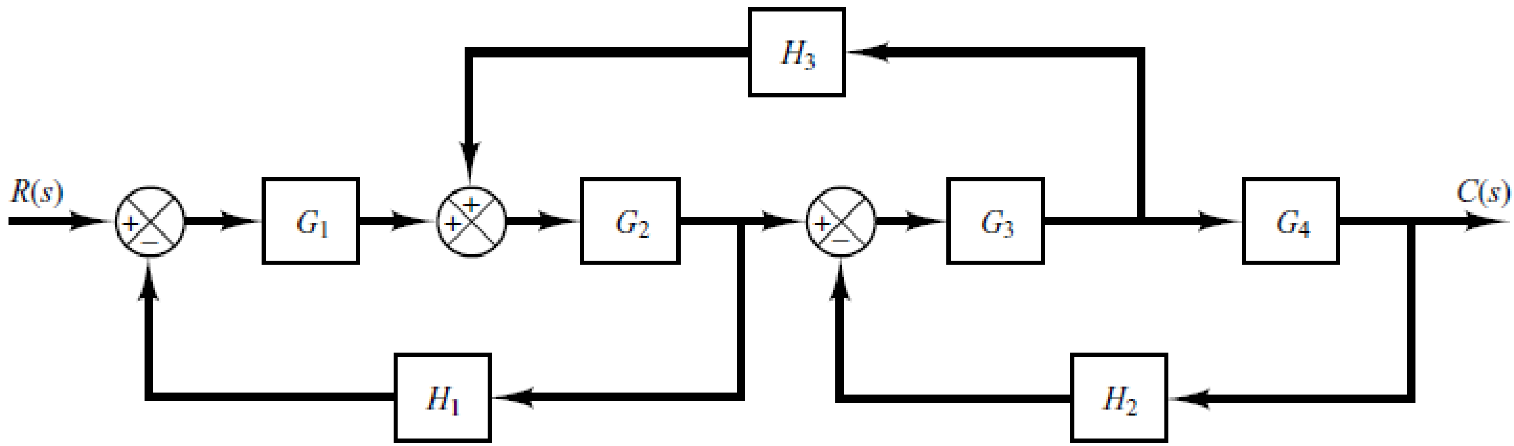
Fourth, use the feedback formula to obtain Figure

$$R(s) \longrightarrow \boxed{\dfrac{G_1(s)G_2(s)}{1 + G_2(s)H_2(s) + G_1(s)G_2(s)H_1(s)}} \xrightarrow{V_4(s)} \boxed{\left(\dfrac{1}{G_2(s)} + 1\right)\left(\dfrac{G_3(s)}{1 + G_3(s)H_3(s)}\right)} \longrightarrow C(s)$$

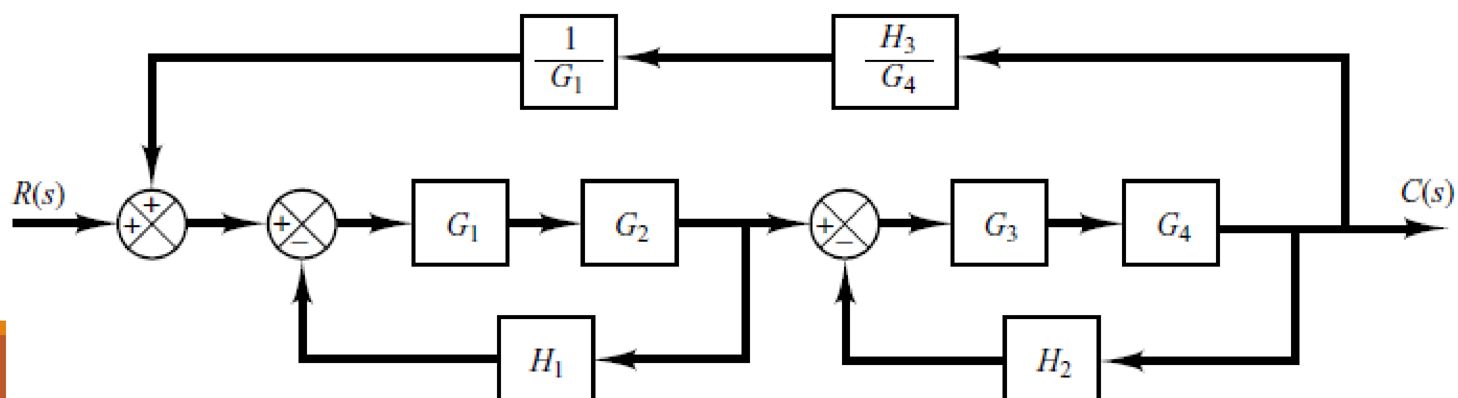
Finally, multiply the two cascaded blocks and obtain the final result,

$$R(s) \longrightarrow \boxed{\dfrac{G_1(s)G_3(s)[1 + G_2(s)]}{[1 + G_2(s)H_2(s) + G_1(s)G_2(s)H_1(s)][1 + G_3(s)H_3(s)]}} \longrightarrow C(s)$$

Example-11: Simplify the block diagram then obtain the close-loop transfer function C(S)/R(S). (from Ogata: Page-47)

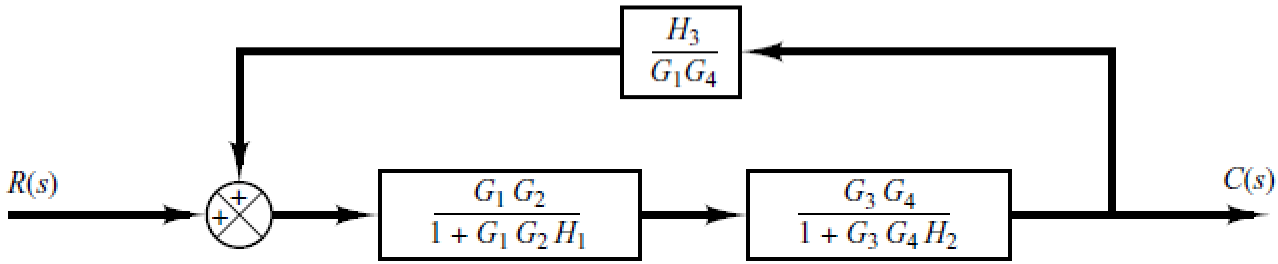


First move the branch point between G_3 and G_4 to the right-hand side of the loop containing G_3, G_4, and H_2. Then move the summing point between G_1 and G_2 to the left-hand side of the first summing point.

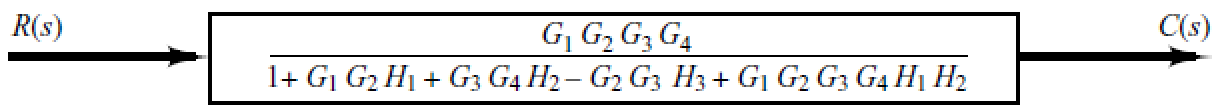


Example-11: Continue.

By simplifying each loop, the block diagram can be modified as



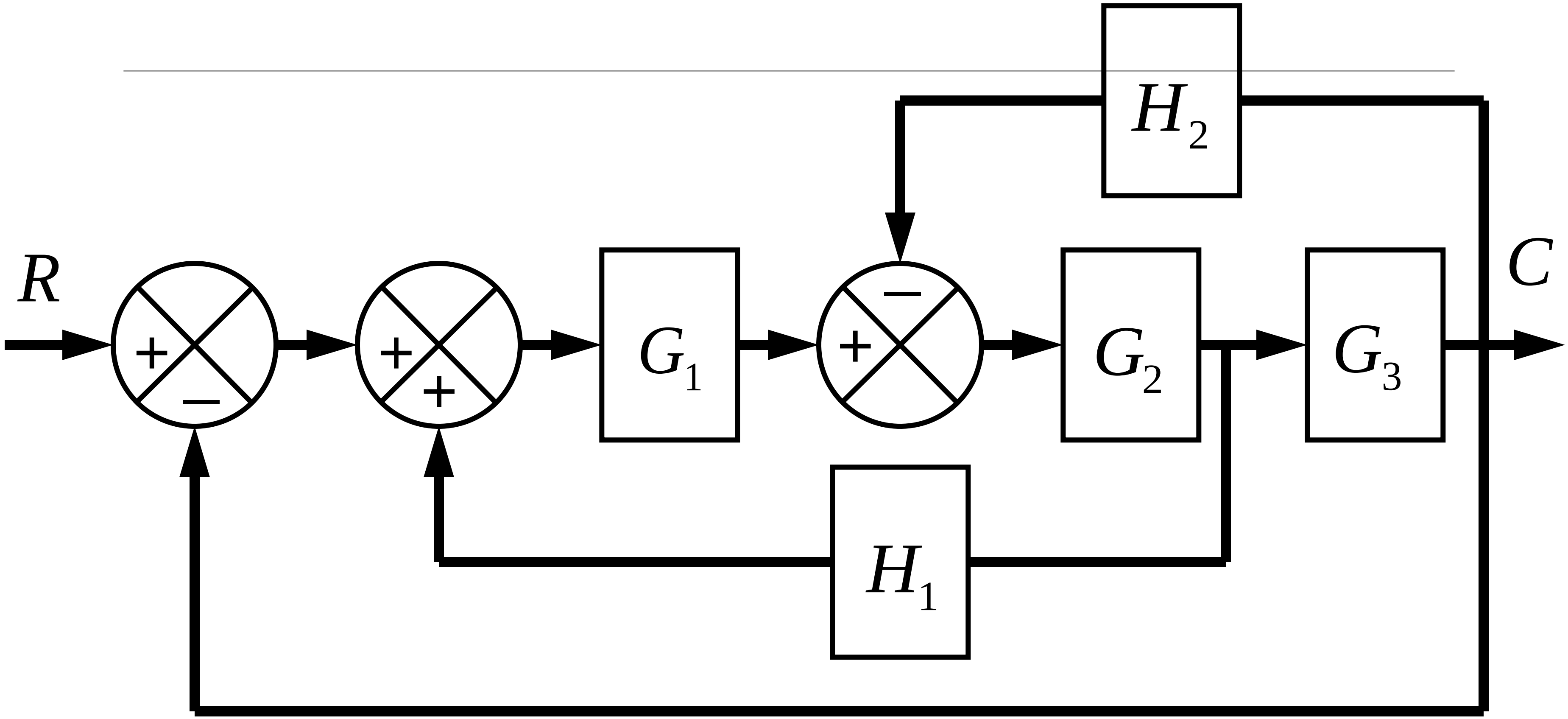
Further simplification results in



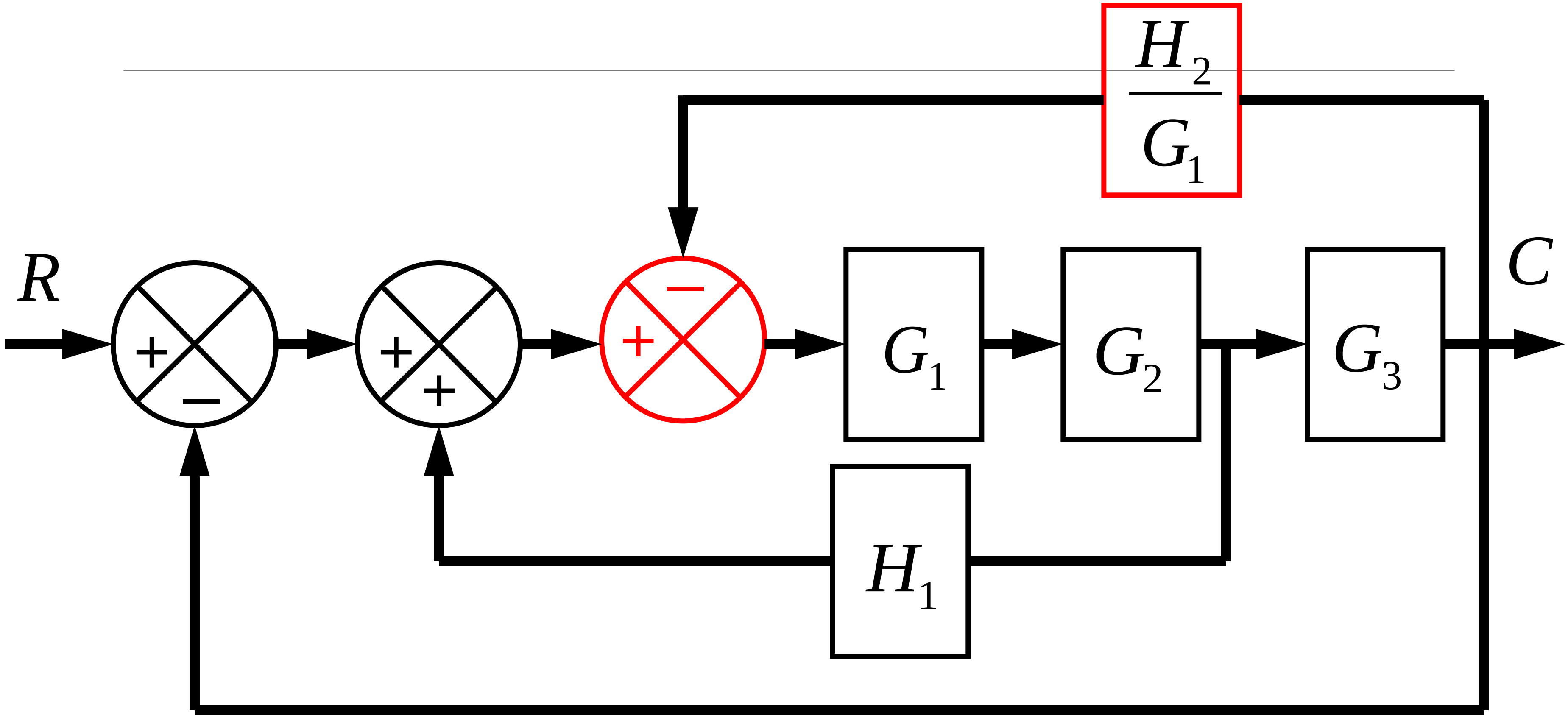
the closed-loop transfer function $C(s)/R(s)$ is obtained as

$$\frac{C(s)}{R(s)} = \frac{G_1 G_2 G_3 G_4}{1 + G_1 G_2 H_1 + G_3 G_4 H_2 - G_2 G_3 H_3 + G_1 G_2 G_3 G_4 H_1 H_2}$$

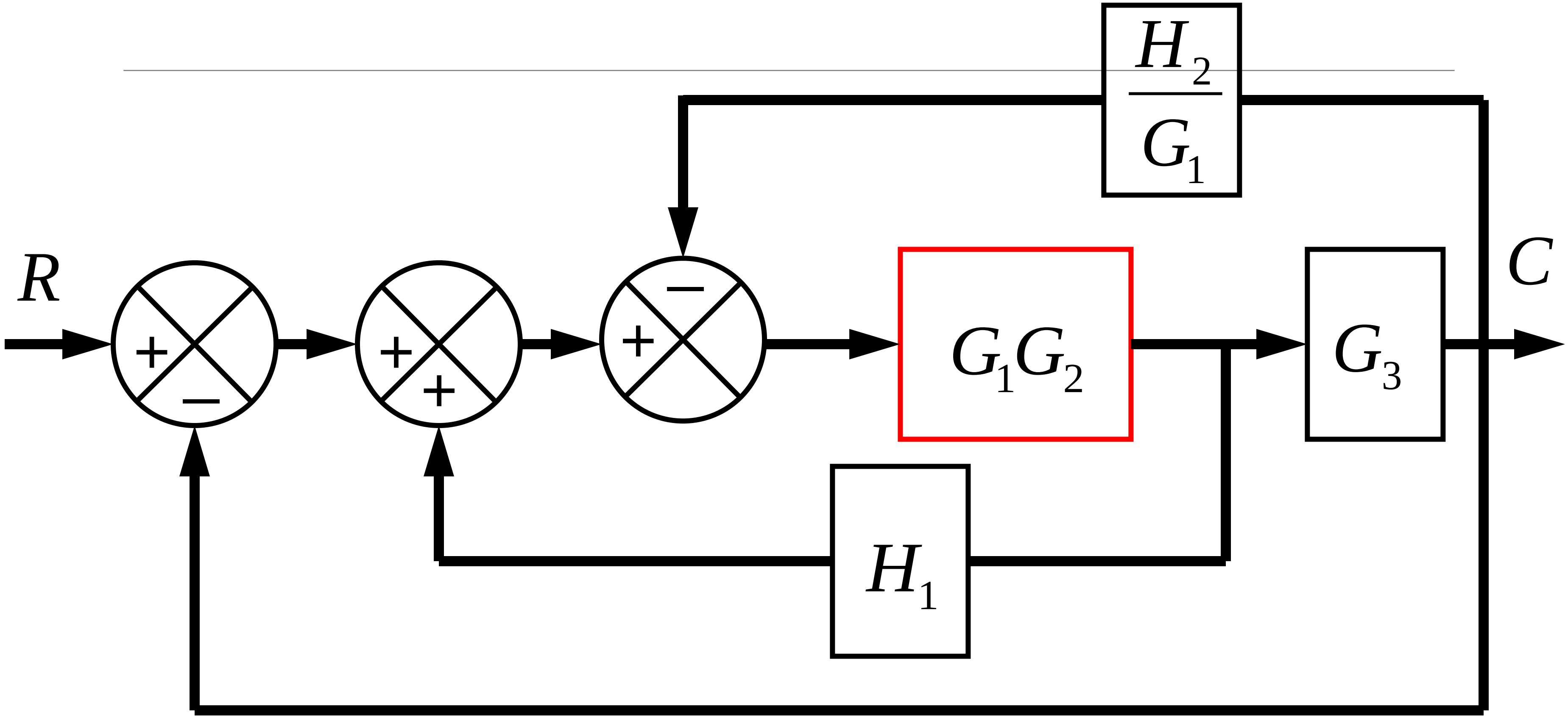
Example-12: Reduce the Block Diagram.



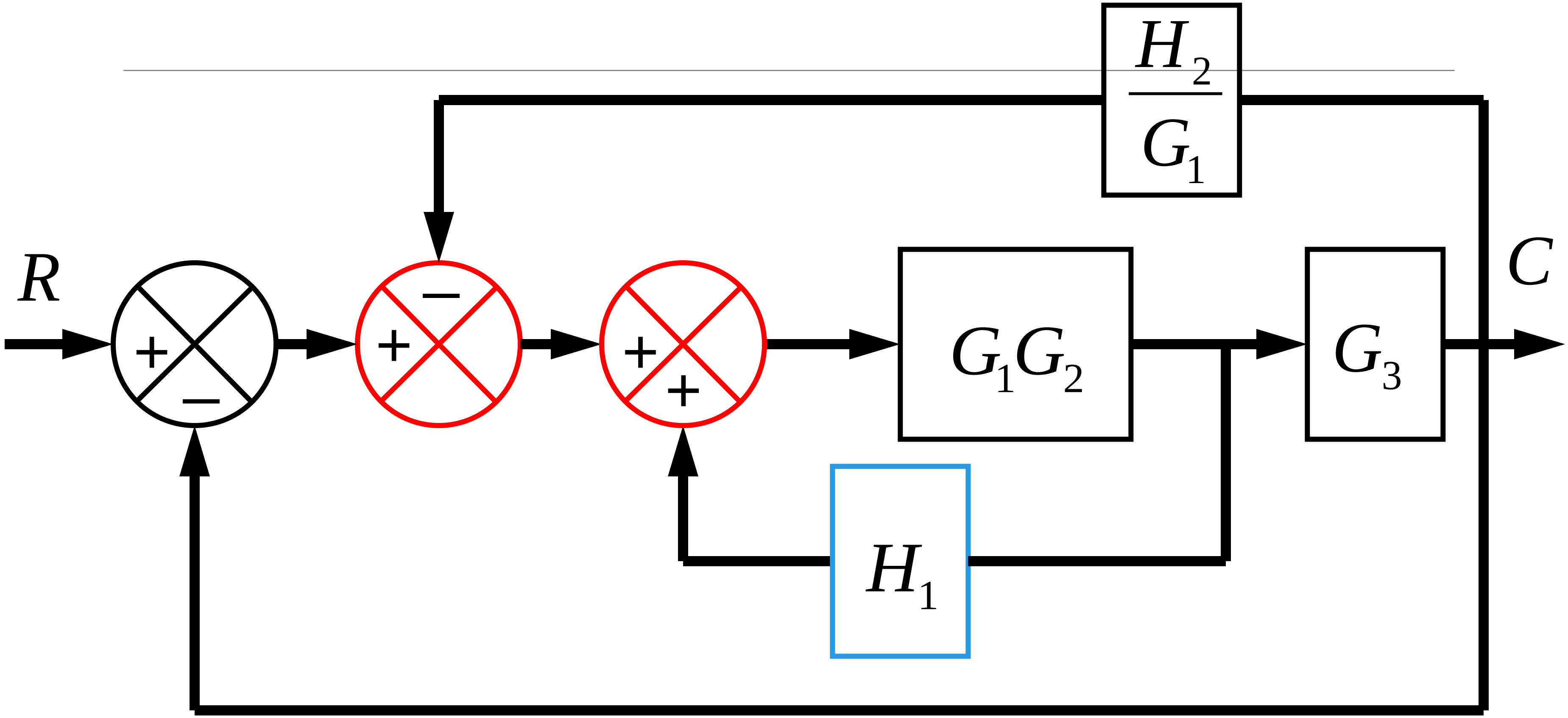
Example-12:



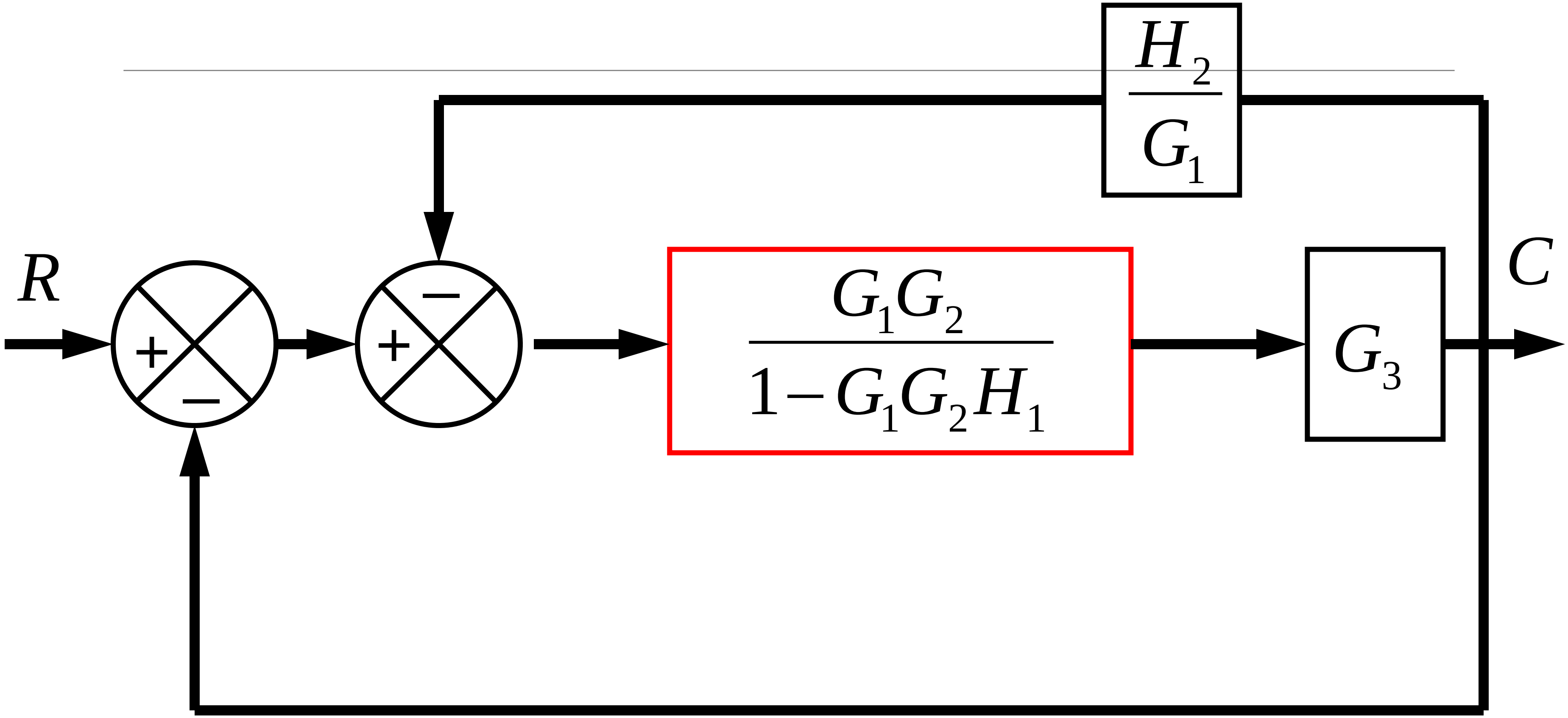
Example-12:



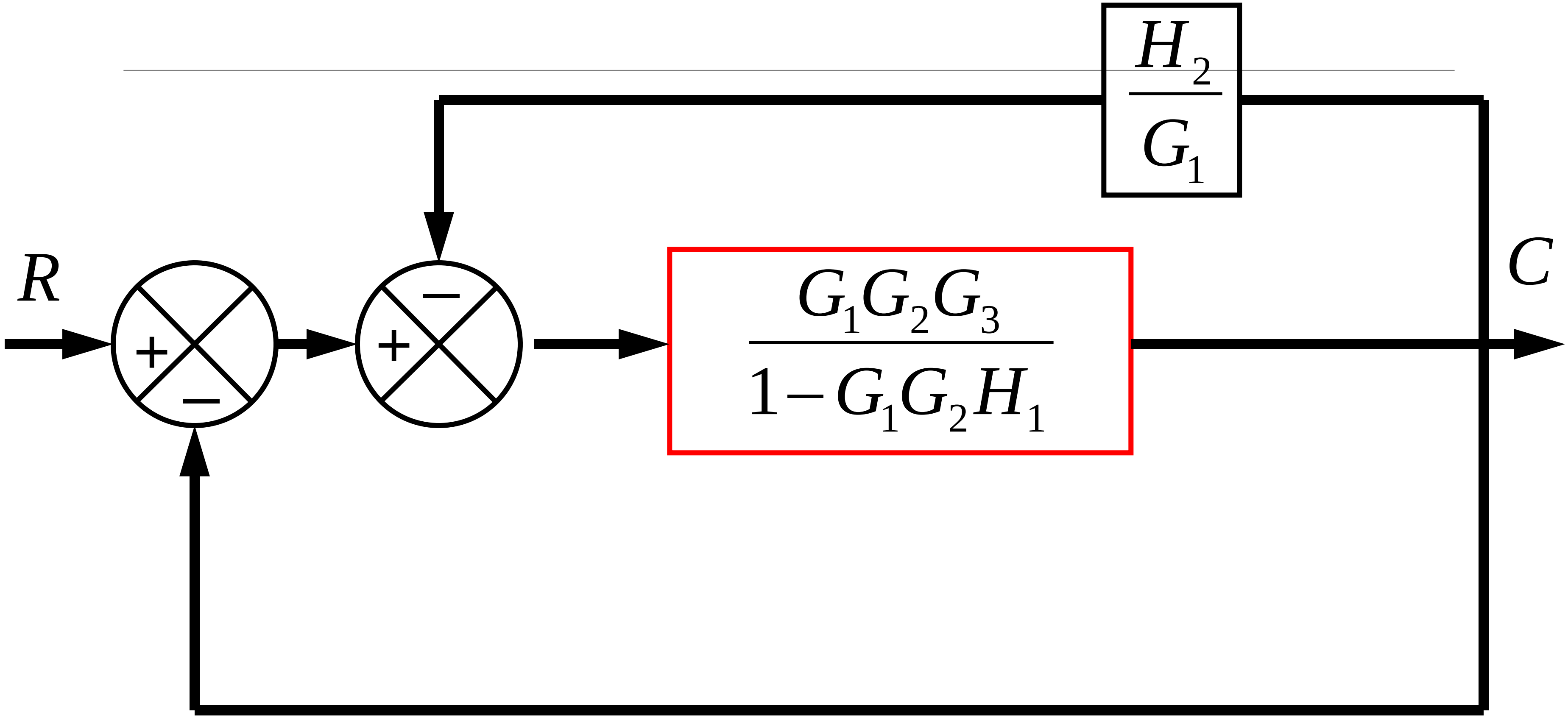
Example-12:



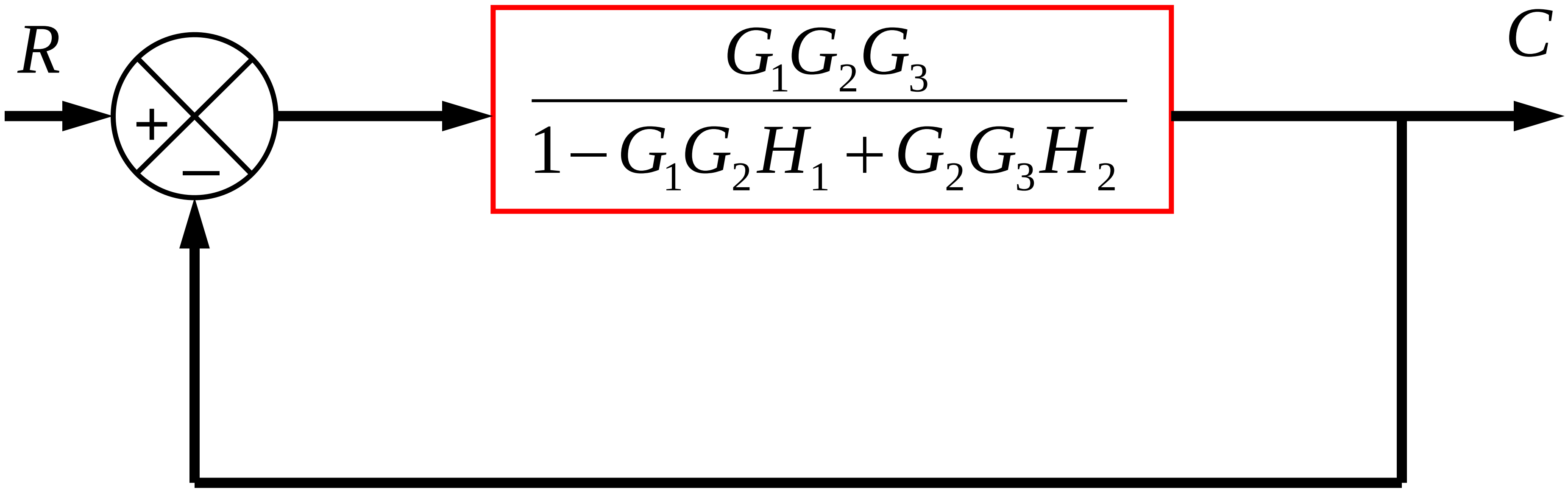
Example-12:



Example-12:



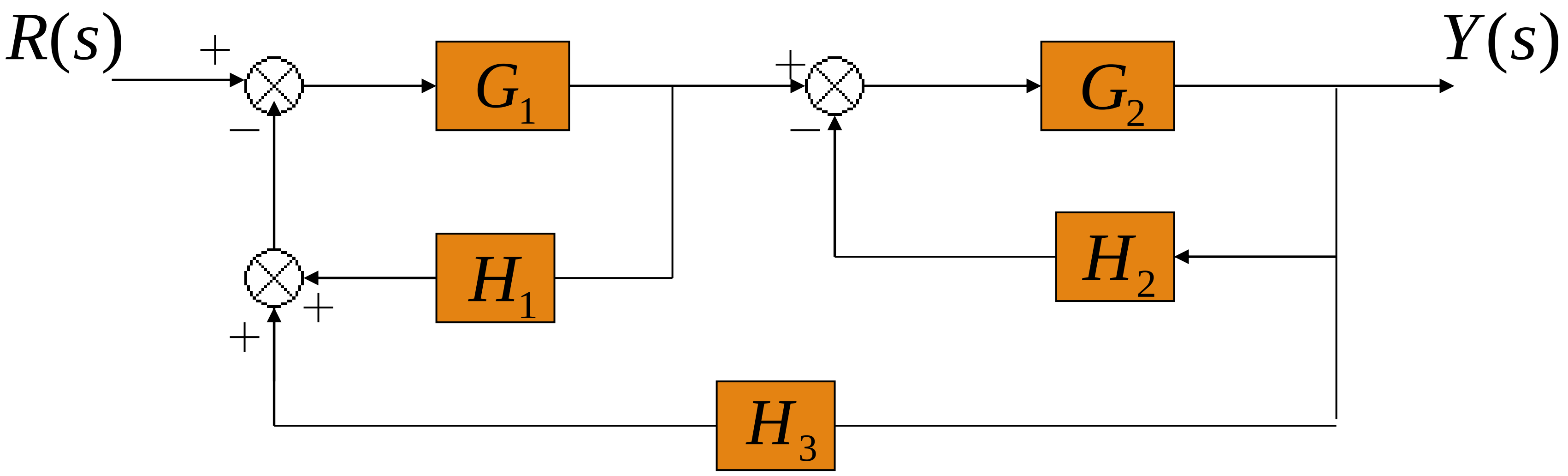
Example-12:



Example-12:

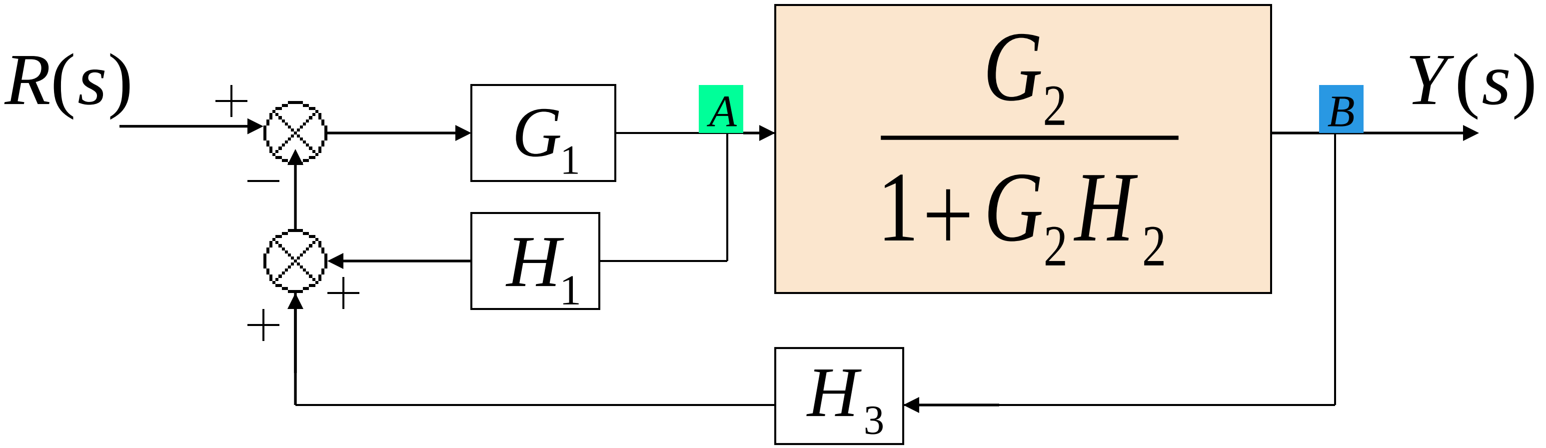
$$R \longrightarrow \boxed{\frac{G_1G_2G_3}{1-G_1G_2H_1+G_2G_3H_2+G_1G_2G_3}} \longrightarrow C$$

Example 13: Find the transfer function of the following block diagrams.

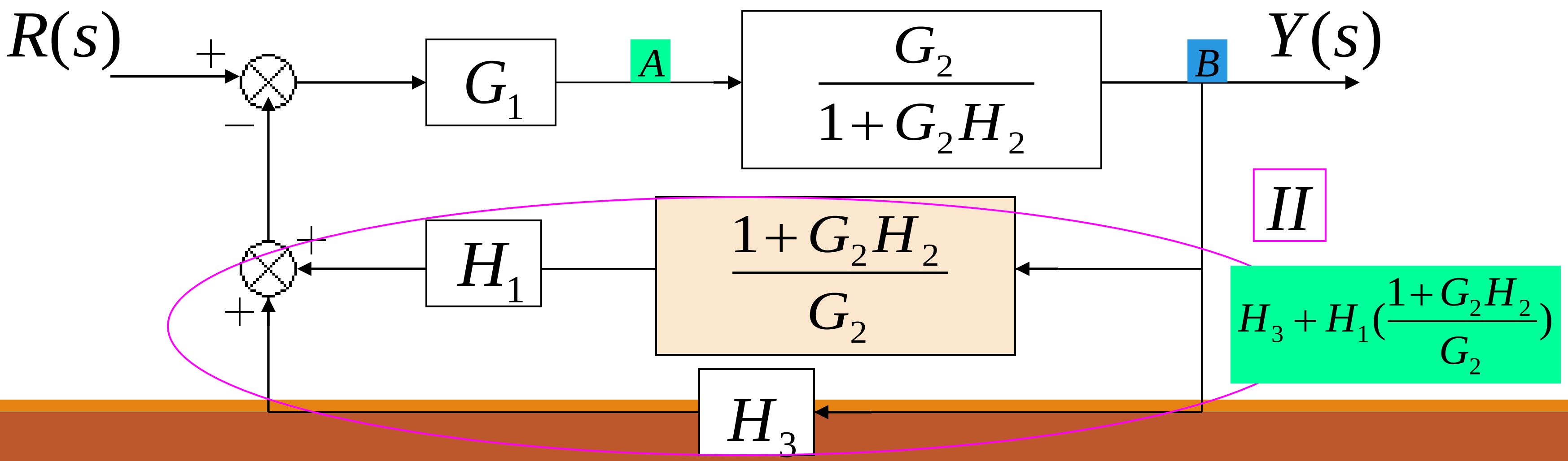


Solution:

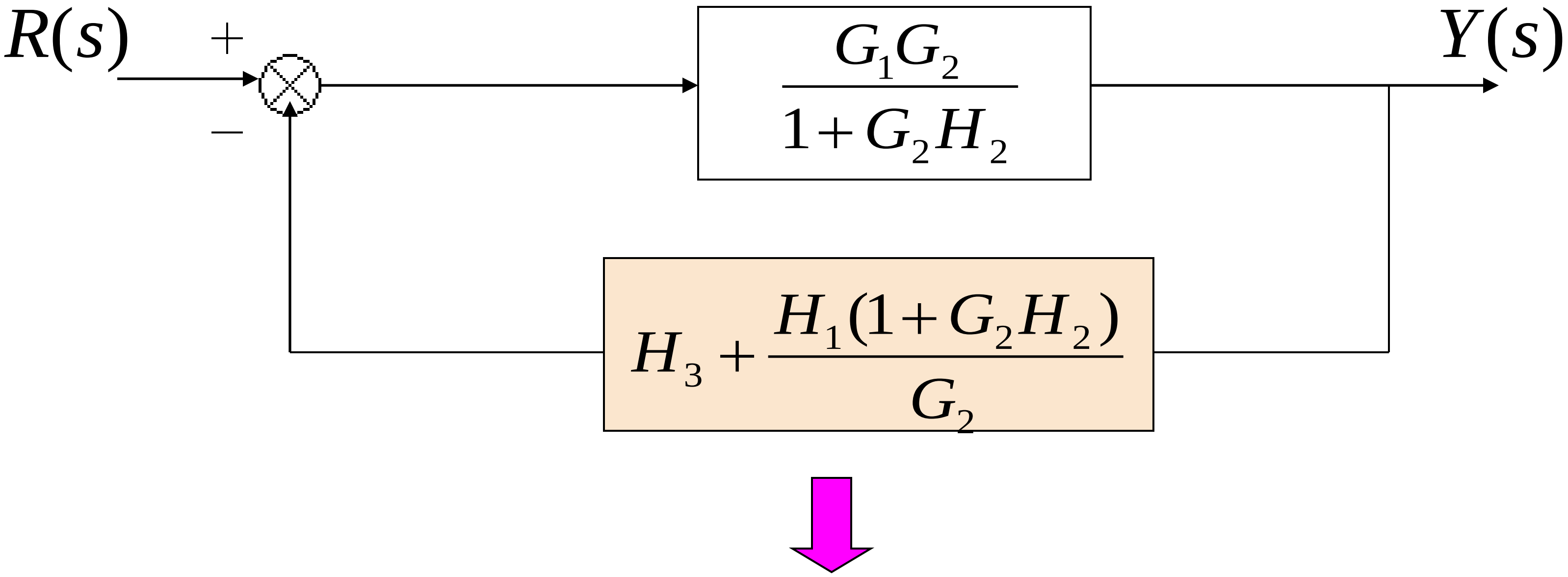
1. Eliminate loop I



2. Moving pickoff point A behind block $\frac{G_2}{1+G_2H_2}$



3. Eliminate loop II



$$\frac{Y(s)}{R(s)} = \frac{G_1G_2}{1+G_2H_2+G_1G_2H_3+G_1H_1+G_1G_2H_1H_2}$$

Superposition of Multiple Inputs

Step 1: Set all inputs except one equal to zero.

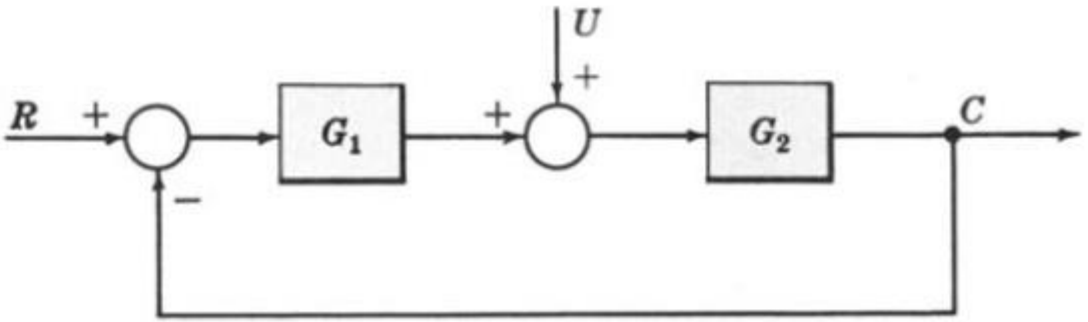
Step 2: Transform the block diagram to canonical form, using the transformations of Section 7.5.

Step 3: Calculate the response due to the chosen input acting alone.

Step 4: Repeat Steps 1 to 3 for each of the remaining inputs.

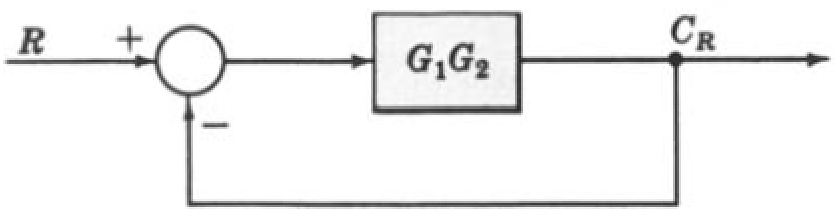
Step 5: Algebraically add all of the responses (outputs) determined in Steps 1 to 4. This sum is the total output of the system with all inputs acting simultaneously.

Example-14: Multiple Input System. Determine the output C due to inputs R and U using the Superposition Method.



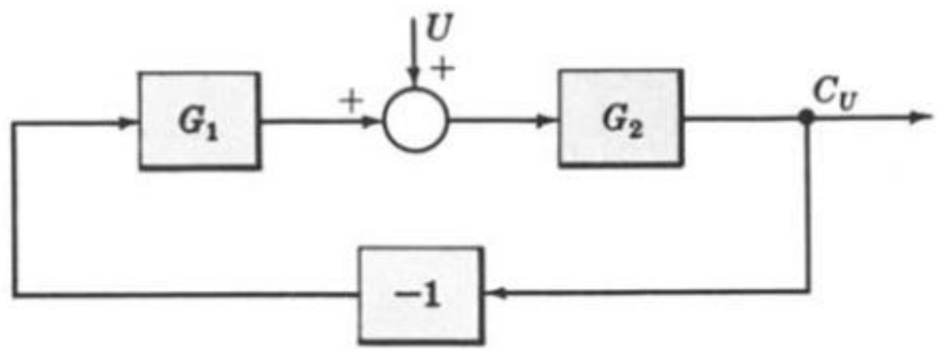
Step 1: Put $U \equiv 0$.

Step 2: The system reduces to



Step 3: the output C_R due to input R is $\boxed{C_R = [G_1G_2/(1 + G_1G_2)]R}$.

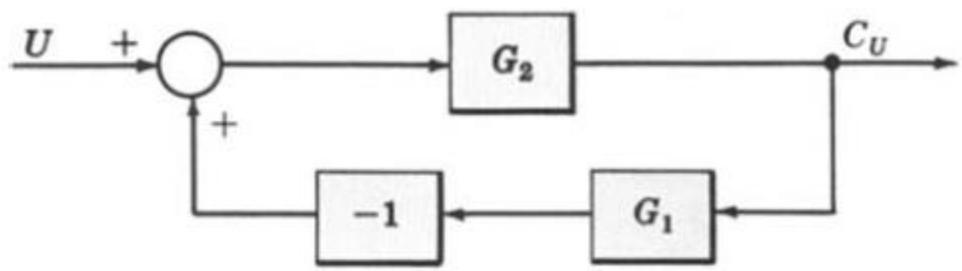
Example-14: Continue.



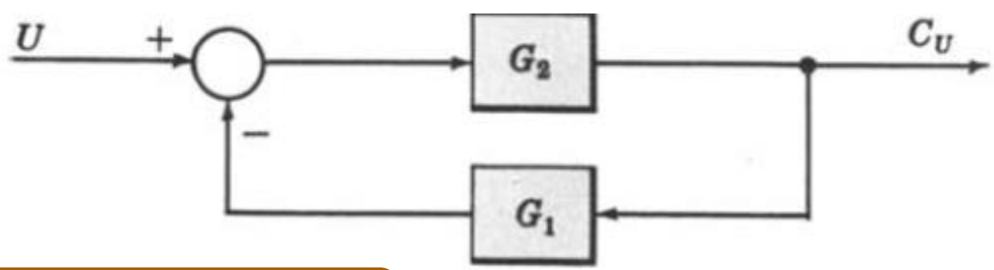
Step 4a: Put $R = 0$.

Step 4b: Put -1 into a block, representing the negative feedback effect:

Rearrange the block diagram:



Let the -1 block be absorbed into the summing point:



Step 4c: the output C_U due to input U is $\boxed{C_U = [G_2/(1 + G_1G_2)]U.}$

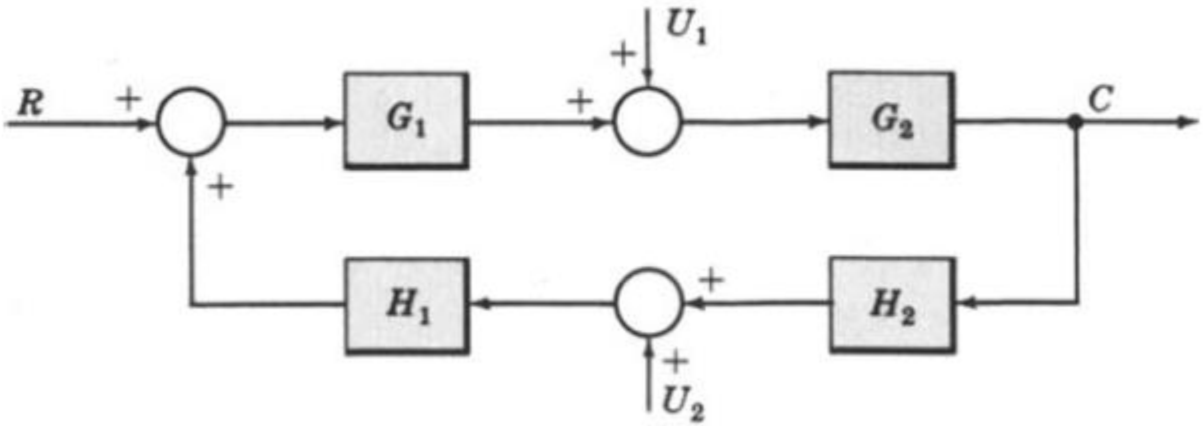
Example-14: Continue.

Step 5: The total output is $C = C_R + C_U$

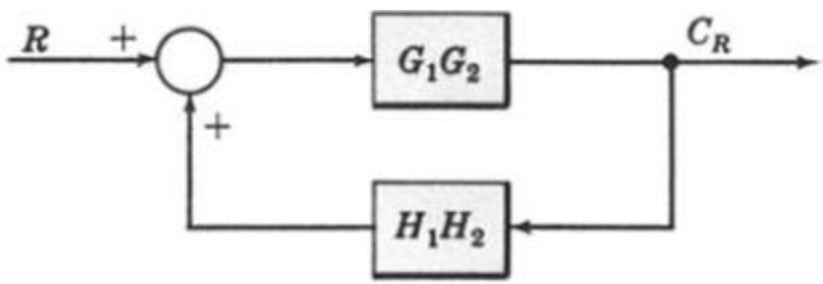
$$= \left[\frac{G_1G_2}{1 + G_1G_2}\right]R + \left[\frac{G_2}{1 + G_1G_2}\right]U$$

$$= \left[\frac{G_2}{1 + G_1G_2}\right][G_1R + U]$$

Example-15: Multiple-Input System. Determine the output C due to inputs R, U_1 and U_2 using the Superposition Method.



Let $U_1 = U_2 = 0$.

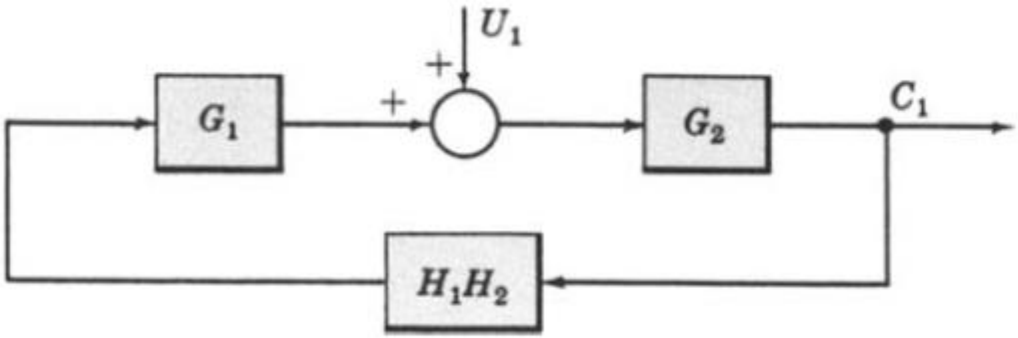


$$C_R = [G_1G_2/(1 - G_1G_2H_1H_2)]R$$

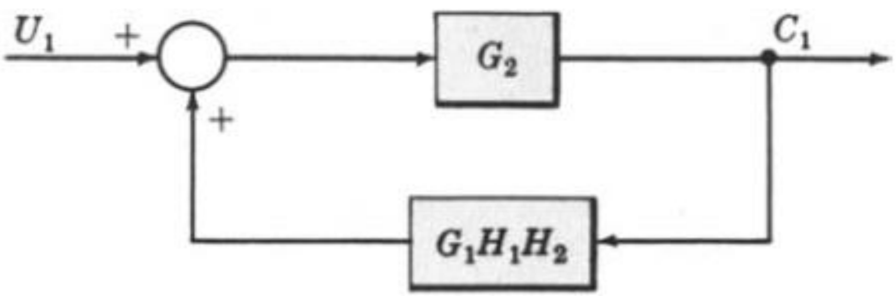
where C_R is the output due to R acting alone.

Example-15: Continue.

Now let $R = U_2 = 0$



Rearranging the blocks, we get

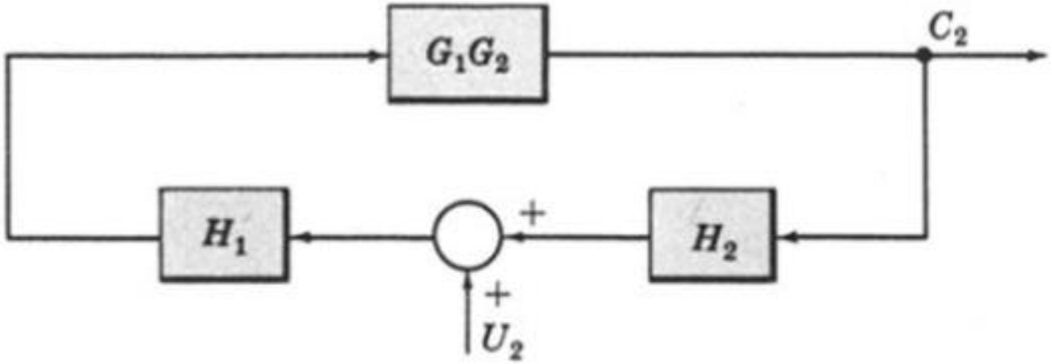


$$C_1 = [G_2/(1 - G_1G_2H_1H_2)]U_1$$

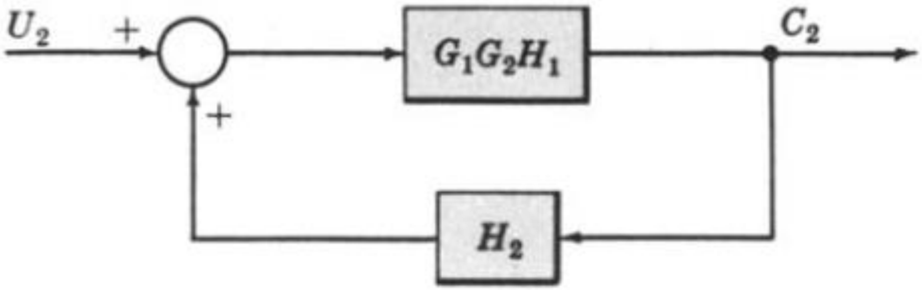
where C_1 is the response due to U_1 acting alone.

Example-15: Continue.

Finally, let $R = U_1 = 0$.



Rearranging the blocks, we get



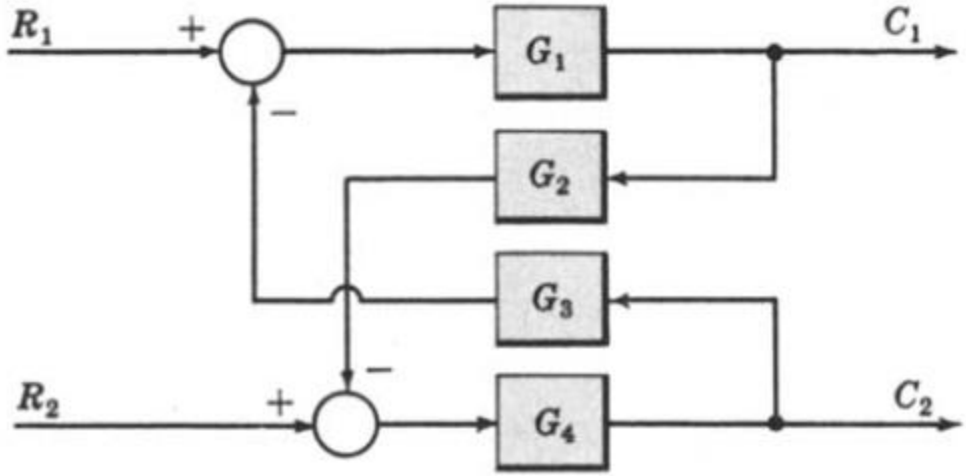
$$\boxed{C_2 = [G_1G_2H_1/(1 - G_1G_2H_1H_2)]U_2}$$

where C_2 is the response due to U_2 acting alone.

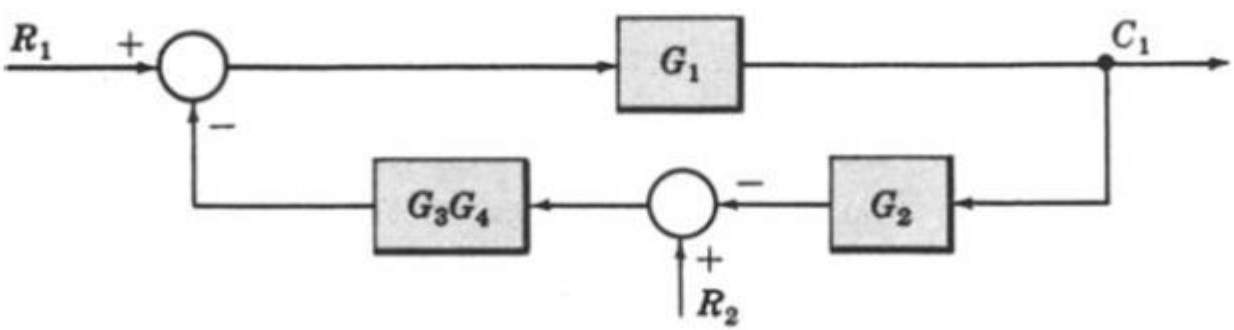
By superposition, the total output is

$$\boxed{C = C_R + C_1 + C_2 = \frac{G_1G_2R + G_2U_1 + G_1G_2H_1U_2}{1 - G_1G_2H_1H_2}}$$

Example-16: Multi-Input Multi-Output System. Determine C_1 and C_2 due to R_1 and R_2.

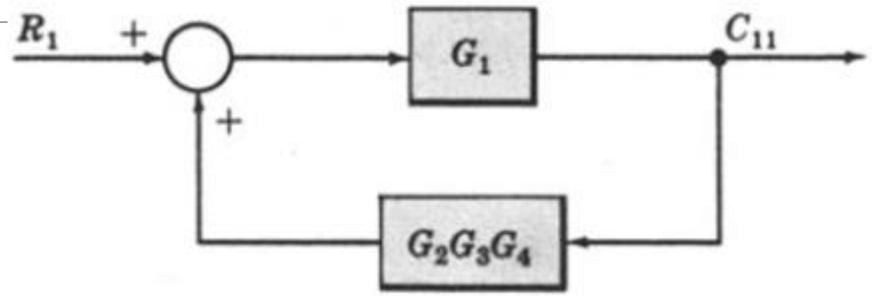


First ignoring the output C_2.



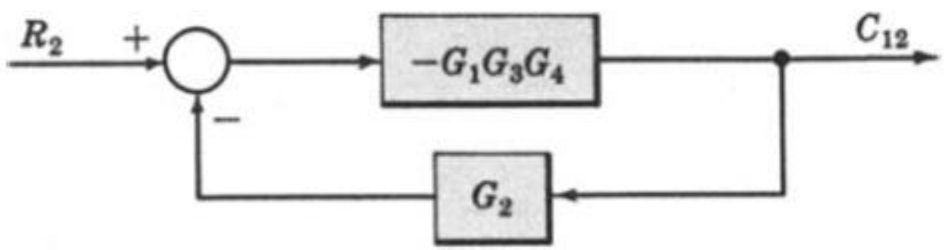
Example-16: Continue.

Letting $R_2 = 0$ and combining the summing points,



Hence C_{11}, the output at C_1 due to R_1 alone, is $C_{11} = G_1 R_1/(1 - G_1 G_2 G_3 G_4)$.

For $R_1 = 0$,

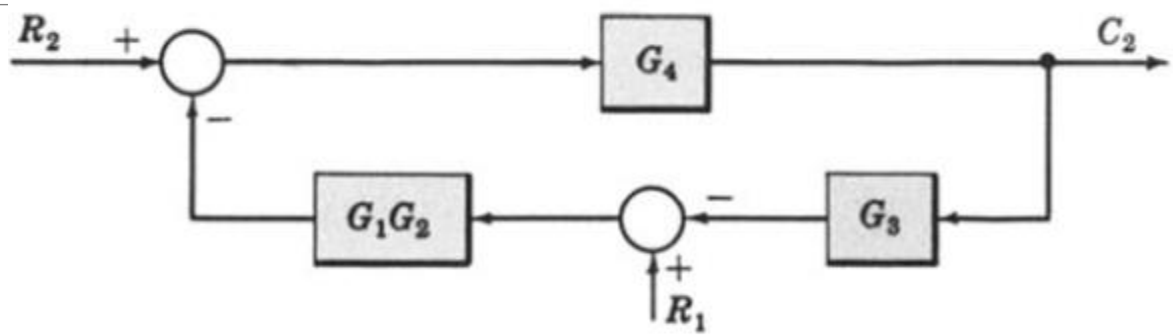


Hence $C_{12} = -G_1 G_3 G_4 R_2/(1 - G_1 G_2 G_3 G_4)$ is the output at C_1 due to R_2 alone.

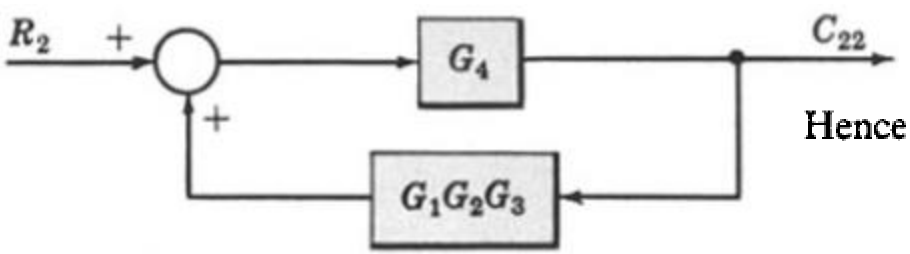
Thus $C_1 = C_{11} + C_{12} = (G_1 R_1 - G_1 G_3 G_4 R_2)/(1 - G_1 G_2 G_3 G_4)$

Example-16: Continue.

Now we reduce the original block diagram, ignoring output C_1.

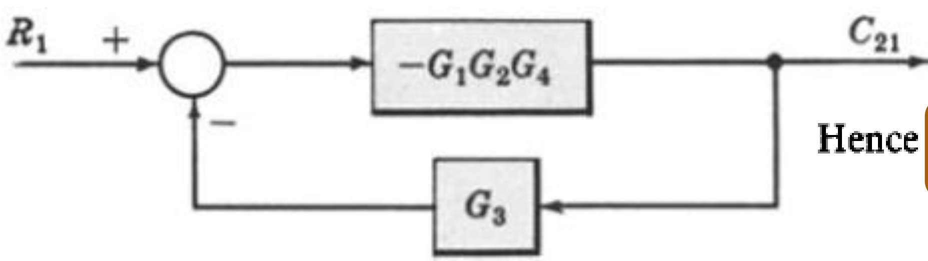


When R1 = 0,



Hence $C_{22} = G_4 R_2/(1 - G_1G_2G_3G_4)$

When R2 = 0,

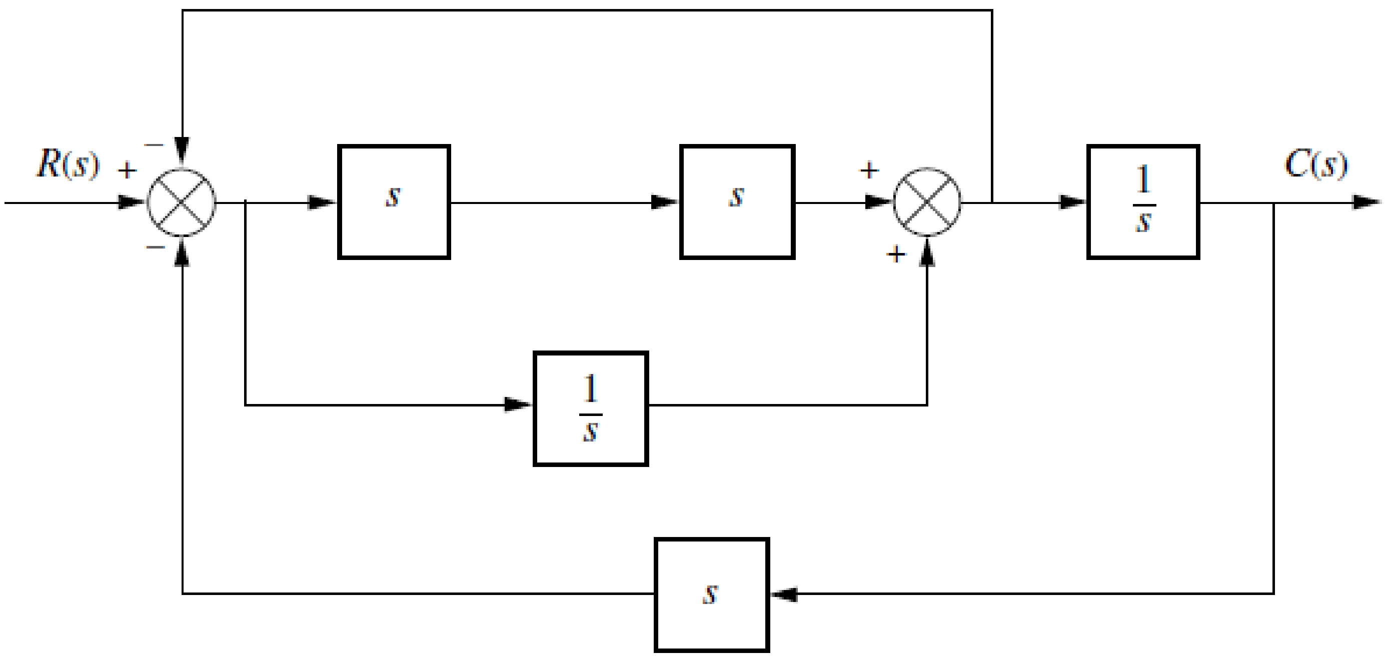


Hence $C_{21} = -G_1G_2G_4R_1/(1 - G_1G_2G_3G_4)$

Finally, $C_2 = C_{22} + C_{21} = (G_4R_2 - G_1G_2G_4R_1)/(1 - G_1G_2G_3G_4)$

Skill Assessment Exercise:

PROBLEM: Find the equivalent transfer function, $T(s) = C(s)/R(s)$, for the system



Answer of Skill Assessment Exercise:

ANSWER: $$T(s) = \frac{s^3 + 1}{2s^4 + s^2 + 2s}$$

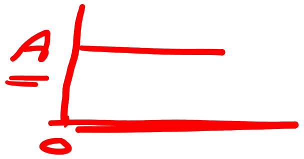
Final and initial values theorems

Examples

$G(s) = \frac{2}{S+5}$ input unit step input

step inp.

$R(s) = \frac{1}{S}$



unit step

$A = 1$

$G(s) = \frac{\text{output}}{\text{input}} = \frac{Y(s)}{R(s)}$

$Y(s) = G(s) * R(s)$

$Y(s) = \frac{2}{S+5} * \frac{1}{S} = \frac{2}{S(S+5)}$

Final value :- steady-state value

$Y_{s.s} = \lim_{s \to 0} S \cdot Y(s) = \lim_{s \to 0} \cancel{S} \cdot \frac{2}{\cancel{S}(S+5)} = \frac{2}{5}$ ✓

Initial value

$$\lim_{s\to\infty} s\cdot Y(s) = \lim_{s\to\infty} \cancel{s}\cdot \frac{2}{\cancel{s}(s+5)} = 0$$ ✓

$Y(s) \Rightarrow \underline{\underline{y(t)}}$

$$\frac{2}{s(s+5)} = \frac{\overset{2/5}{A}}{s} + \frac{\overset{-2/5}{B}}{s+5} \Rightarrow 2/5 - \frac{2}{5} e^{-5t} = y(t)$$

$s=0 \rightarrow \quad A = \frac{2}{5}$

$s=-5= \quad B = -\frac{2}{5}$

Final value

$$\lim_{t\to\infty} y(t) = \lim_{t\to\infty} 2/5 - \frac{2}{5} e^{-5t} = \frac{2}{5}$$ ✓

Initial value

$$\lim_{t \to 0} \left(\frac{2}{5} - \frac{2}{5} e^{-5t} \right) = 0 \checkmark$$

Signal Flow Graph

CH2-14

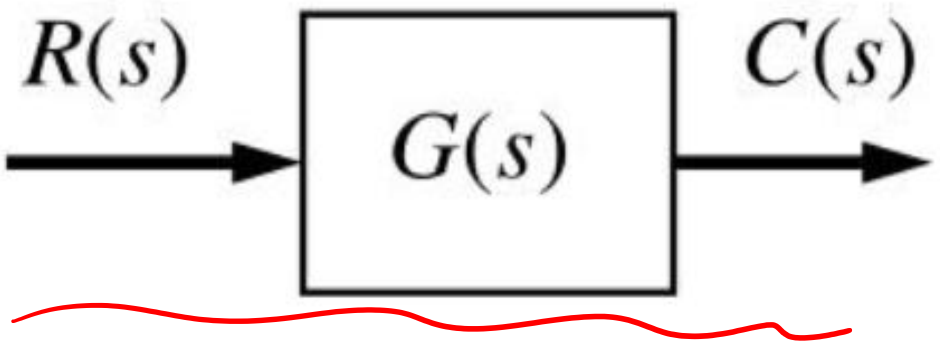
Eng. Fadwa Momani

What is Signal Flow Graph

- SFG is a diagram which represents a set of simultaneous equations.
- This method was developed by S.J.Mason. This method does n't require any reduction technique.
- It consists of nodes and these nodes are connected by a directed line called branches.
- Every branch has an arrow which represents the flow of signal.
- For complicated systems, when Block Diagram (BD) reduction method becomes tedious and time consuming then SFG is a good choice.

BD Vs SFG

block diagram:



In this case at each step block diagram is to be redrawn. That's why it is tedious method. So wastage of time and space.

signal flow graph:

Only one time SFG is to be drawn and then Mason's gain formula is to be evaluated.
So time and space is saved.

SFG

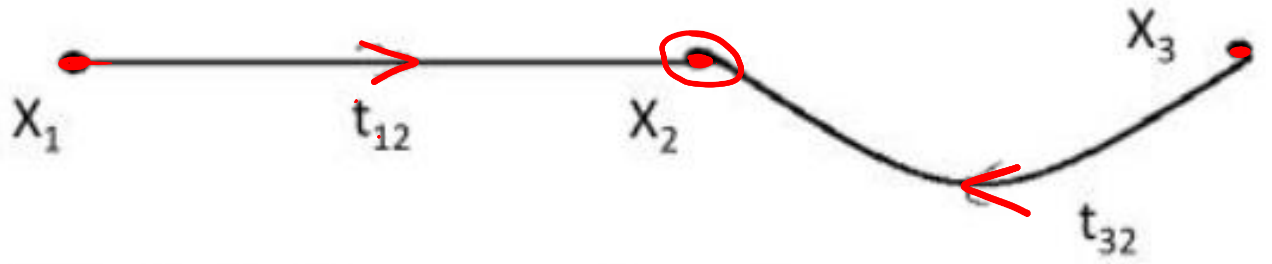
Alternative to block diagram;

Consists only **branches** (systems), and **nodes** (signals)

Node: It is a point representing a variable.

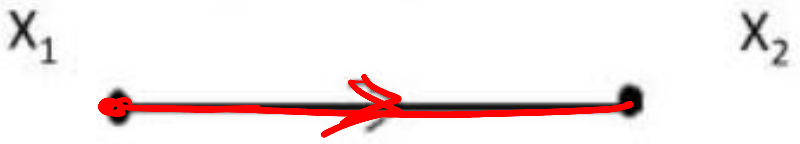
$$x_2 = t_{12}\,x_1 + t_{32}\,x_3$$

$$x_2 = x_1 t_{12} + x_3 t_{32}$$



In this SFG there are 3 nodes.

Branch : A line joining two nodes.



Input Node : Node which has only outgoing branches.

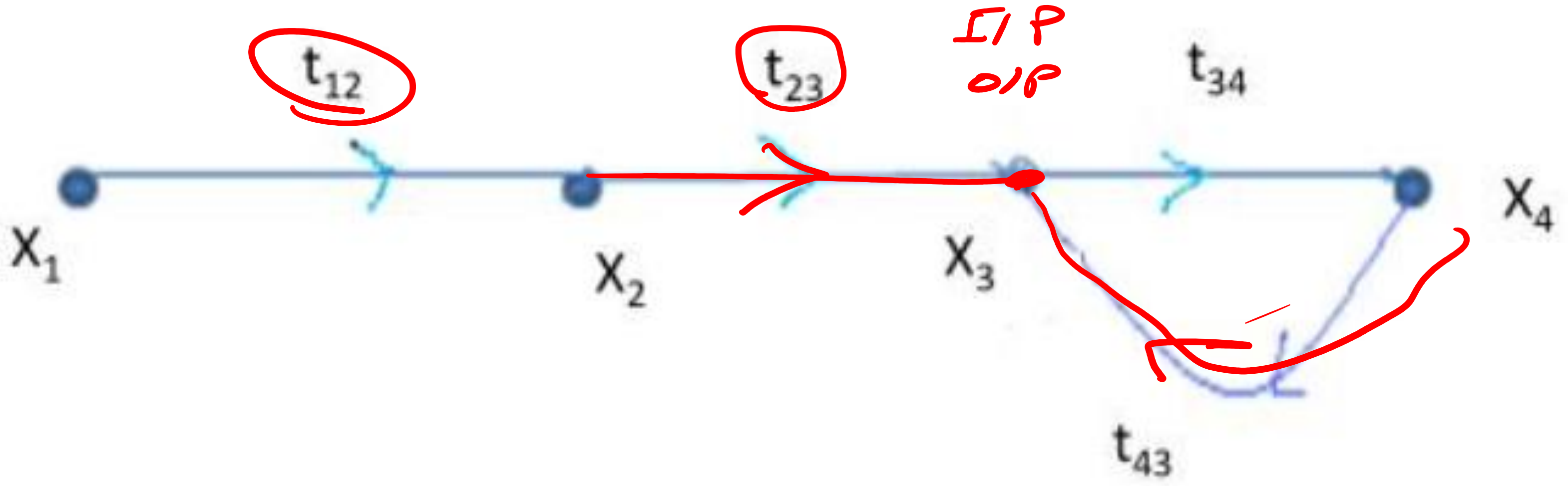
X_1 is input node.

Output node/ sink node: Only incoming branches.

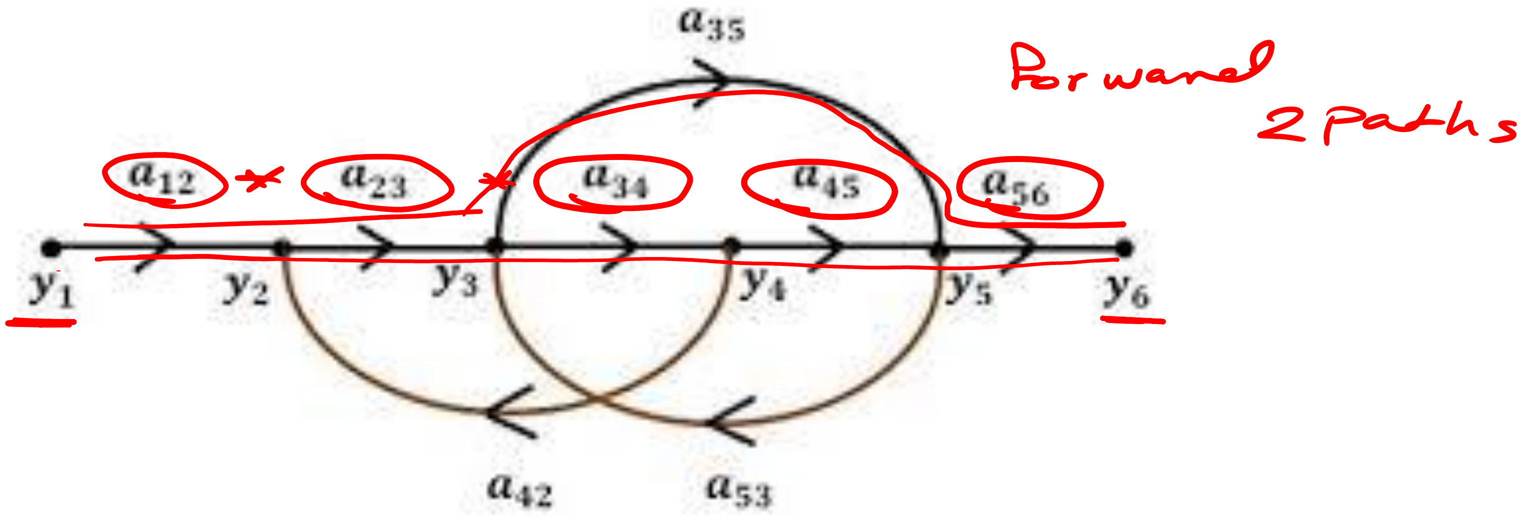
output node

Mixed nodes: Has both incoming and outgoing branches.

Transmittance : It is the gain between two nodes. It is generally written on the branch near the arrow.



- **Path** : It is the traversal of connected branches in the direction of branch arrows, such that no node is traversed more than once.
- **Forward path** : A path which originates from the input node and terminates at the output node and along which no node is traversed more than once.
- **Forward Path gain** : It is the product of branch transmittances of a forward path.



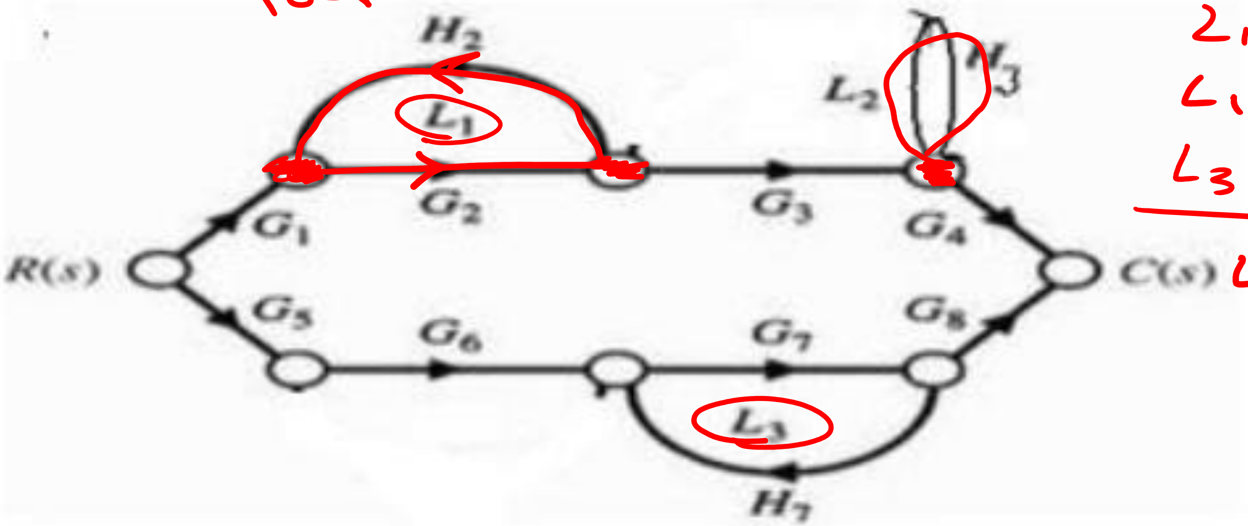
Loop : Path that originates and terminates at the same node and along which no other node is traversed more than once.

Self loop: Path that originates and terminates at the same node.

Loop gain: it is the product of branch transmittances of a loop.

Non-touching loops: Loops that don't have any common node or branch.

loop = G_2H_2



$L_1 L_2$ non-T.L

$L_1 L_3$ 2 =

$L_3 L_2$ 2 =

$L_1 L_2 L_3$ 3 n.T.L

Mason's Rule

$P_1 \rightarrow \Delta_1$

$P_2 \rightarrow \Delta_2$

$P_1 \Delta_1 + P_2 \Delta_2 \cdots$

- A technique to reduce a signal-flow graph to a single transfer function requires the application of one formula.
- The transfer function, C(s)/R(s), of a system represented by a signal-flow graph is

$$G(s) = \frac{C(s)}{R(s)} = \frac{\sum_k P_k \Delta_k}{\Delta} =$$

k = *number of forward path*

P_k = *the kth forward path gain*

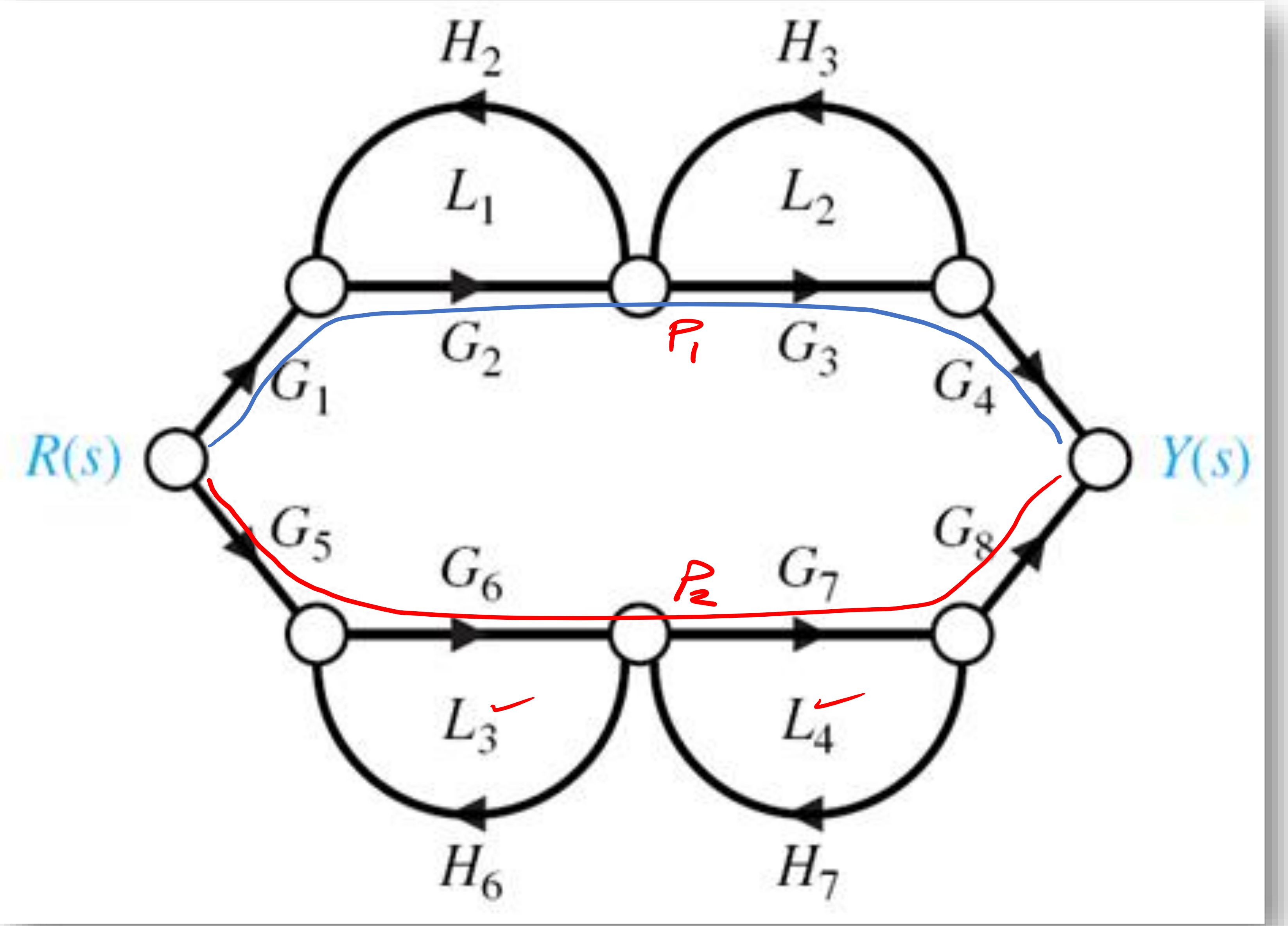
$L_1 L_2 + L_1 L_3$

$\Delta = 1 -$ (Σ loop gains) + (Σ non-touching loop gains taken two at a time) − (Σ non-touching loop gains taken three at a time)+ so on .

$\Delta_k = 1 - $ *(loop-gain which does not touch the forward path)*

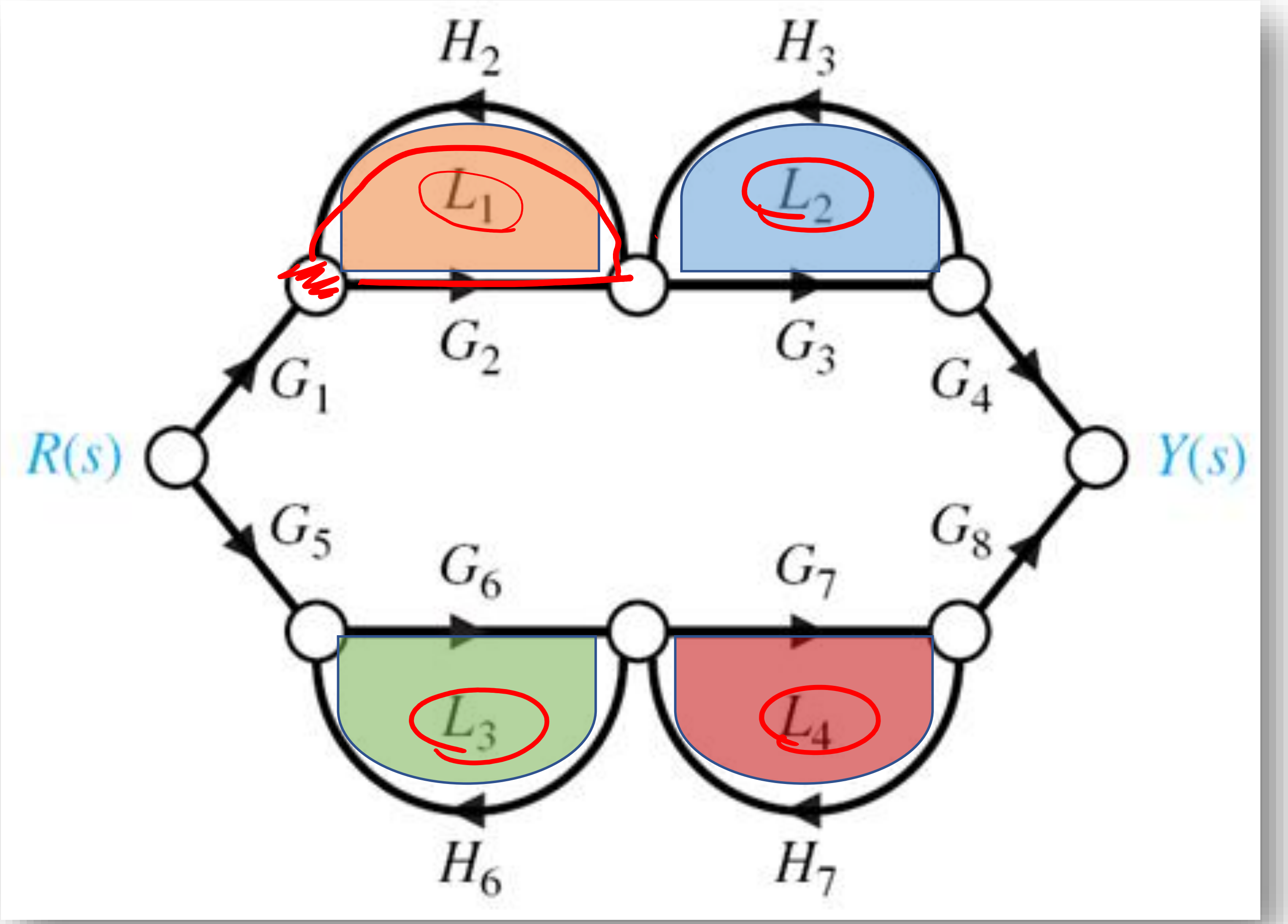
Example 1:

- P1=G1.G2.G3.G4
- P2=G5.G6.G7.G8



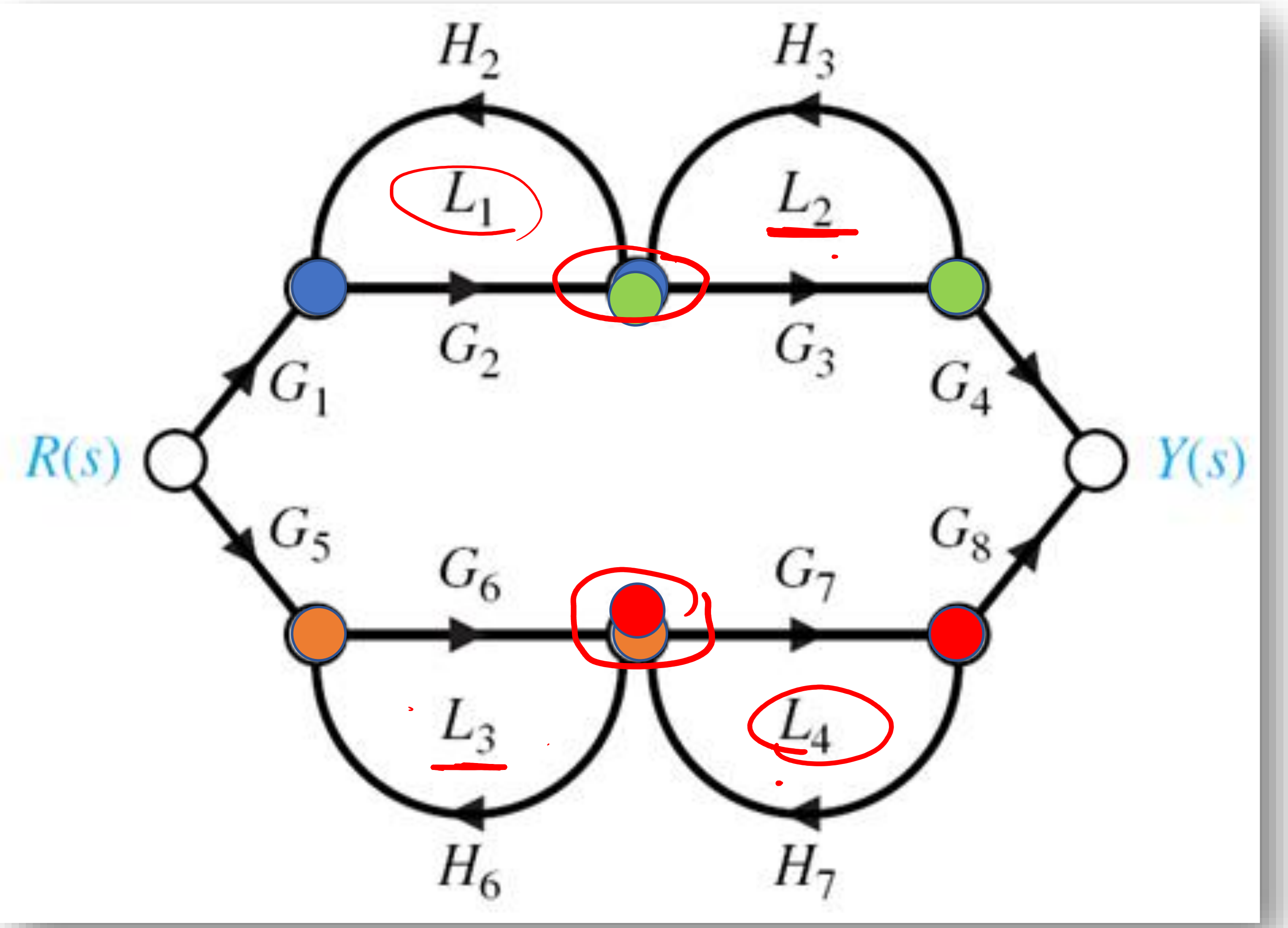
LOOPS

- L1=G2.H2
- L2=G3.H3
- L3=G6.H6
- L4=G7.H7



Two None touching Loops

- L1L3
- L1L4
- L2L3
- L2L4



Transfer Function using Mason's Rule

2 FG. 2 paths.

1 2

- Cofactors

Δ1=1-(L3+L4)

Δ2=1-(L1+L2)

- Δ= 1-(L1+L2+L3+L4)+(L1L3+L1L4+L2L3+L2L4)

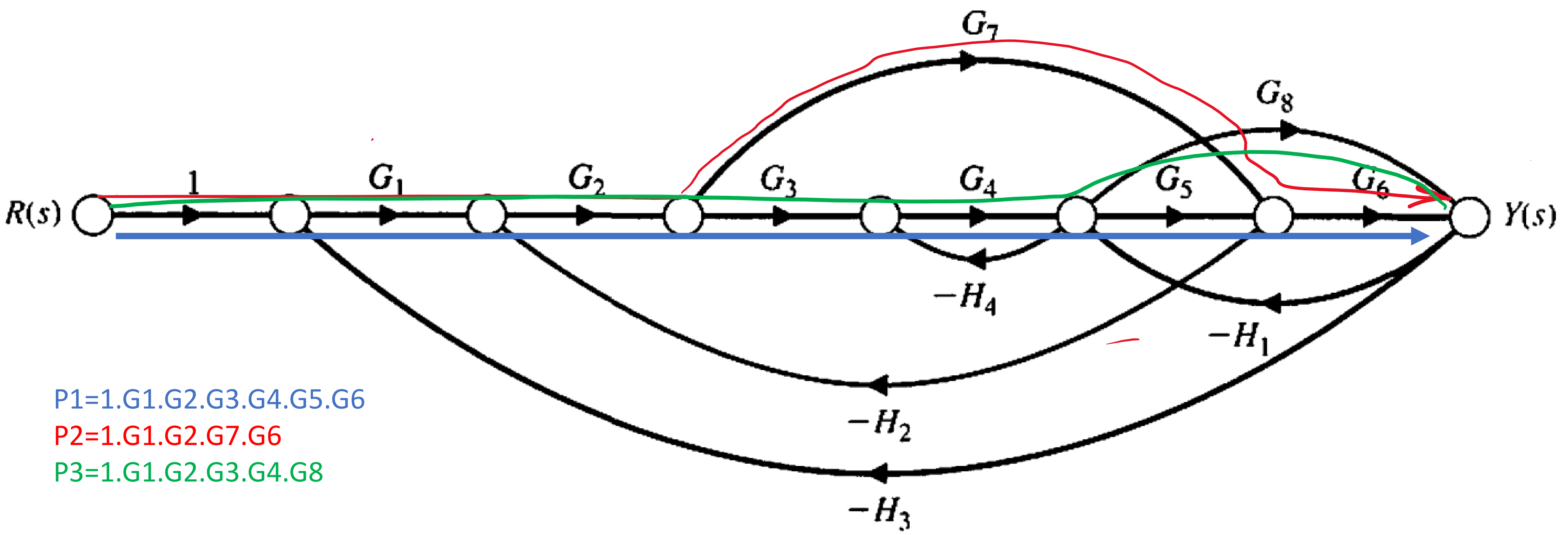
$$T.F=\frac{Y(s)}{R(S)}=\frac{(P1*\Delta 1)+(P2*\Delta 2)}{\Delta}$$

$$\frac{\sum P_K \Delta_K}{\Delta}$$

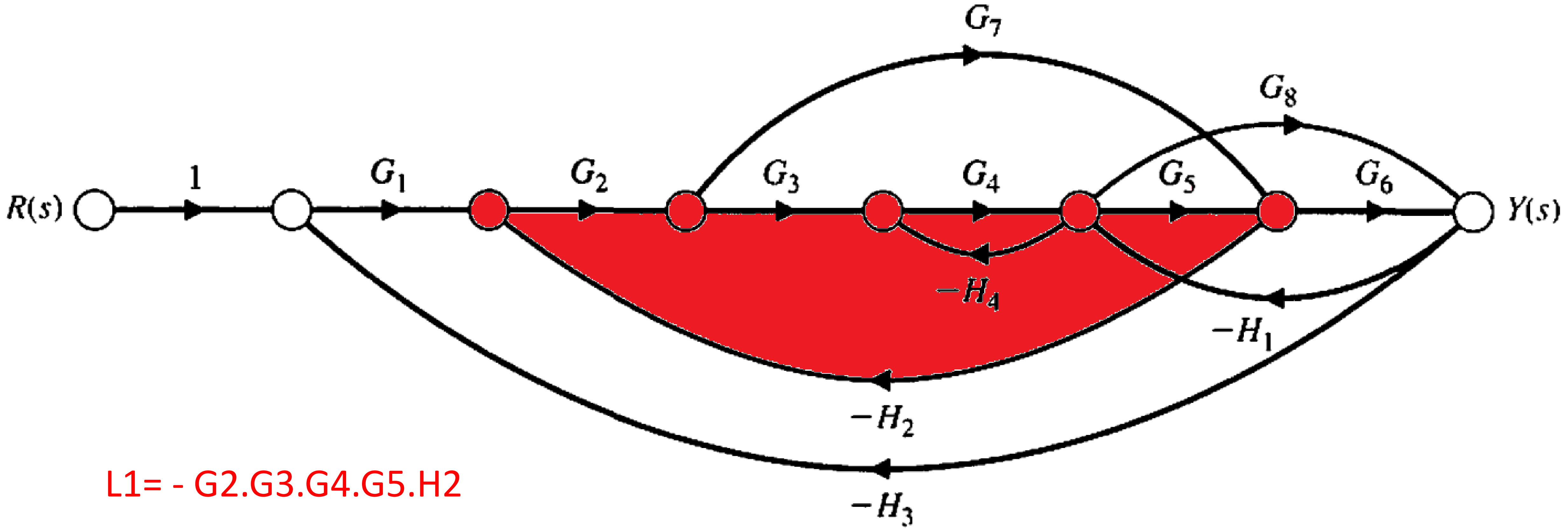
$$\frac{Y(s)}{R(s)} = \frac{\left[\overset{P_1}{G_1 \cdot G_2 \cdot G_3 \cdot G_4} \cdot \overset{\Delta_1}{\left(1 - L_3 - L_4\right)}\right] + \left[\overset{P_2}{G_5 \cdot G_6 \cdot G_7 \cdot G_8} \cdot \overset{\Delta_2}{\left(1 - L_1 - L_2\right)}\right]}{\underset{\Delta}{1 - L_1 - L_2 - L_3 - L_4 + L_1 \cdot L_3 + L_1 \cdot L_4 + L_2 \cdot L_3 + L_2 \cdot L_4}}$$

Example 2

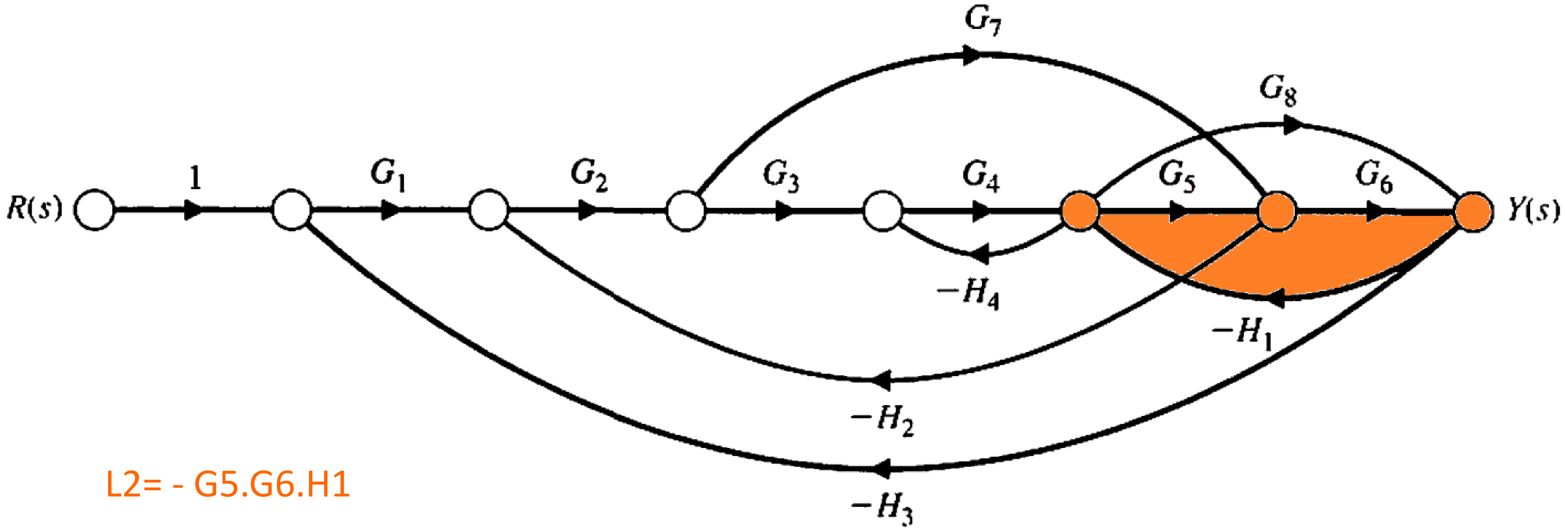
I/P R(s)
O/P Y(s)



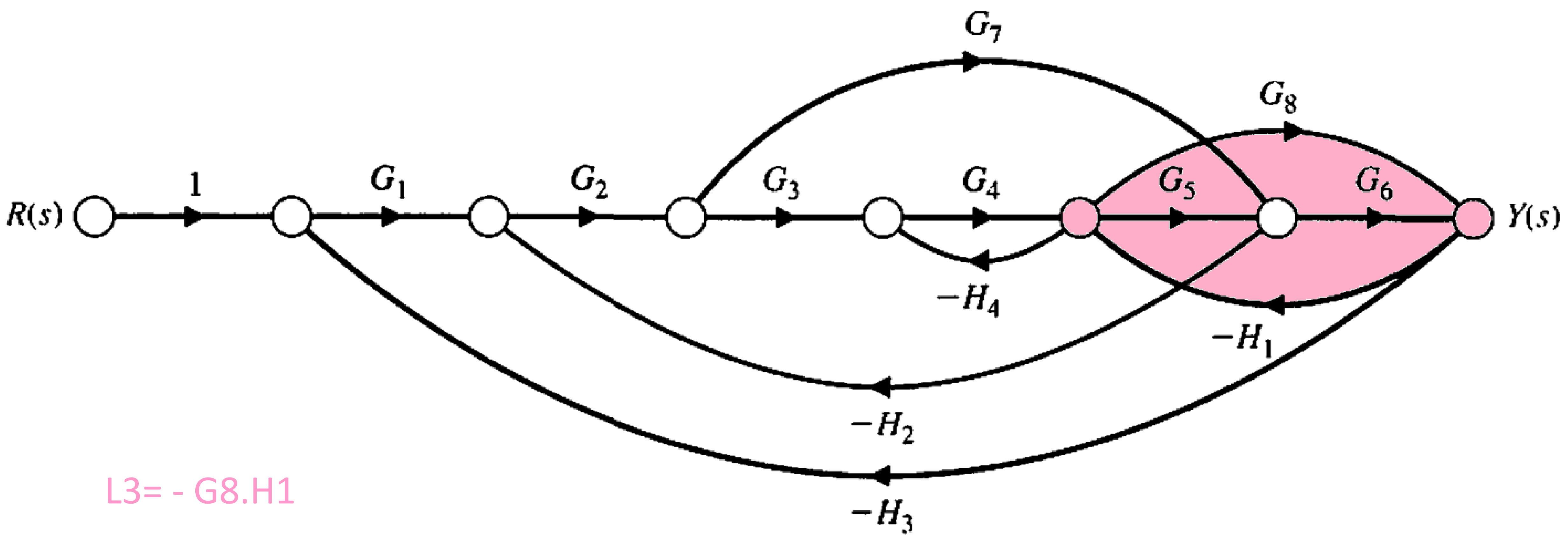
P1=1.G1.G2.G3.G4.G5.G6
P2=1.G1.G2.G7.G6
P3=1.G1.G2.G3.G4.G8



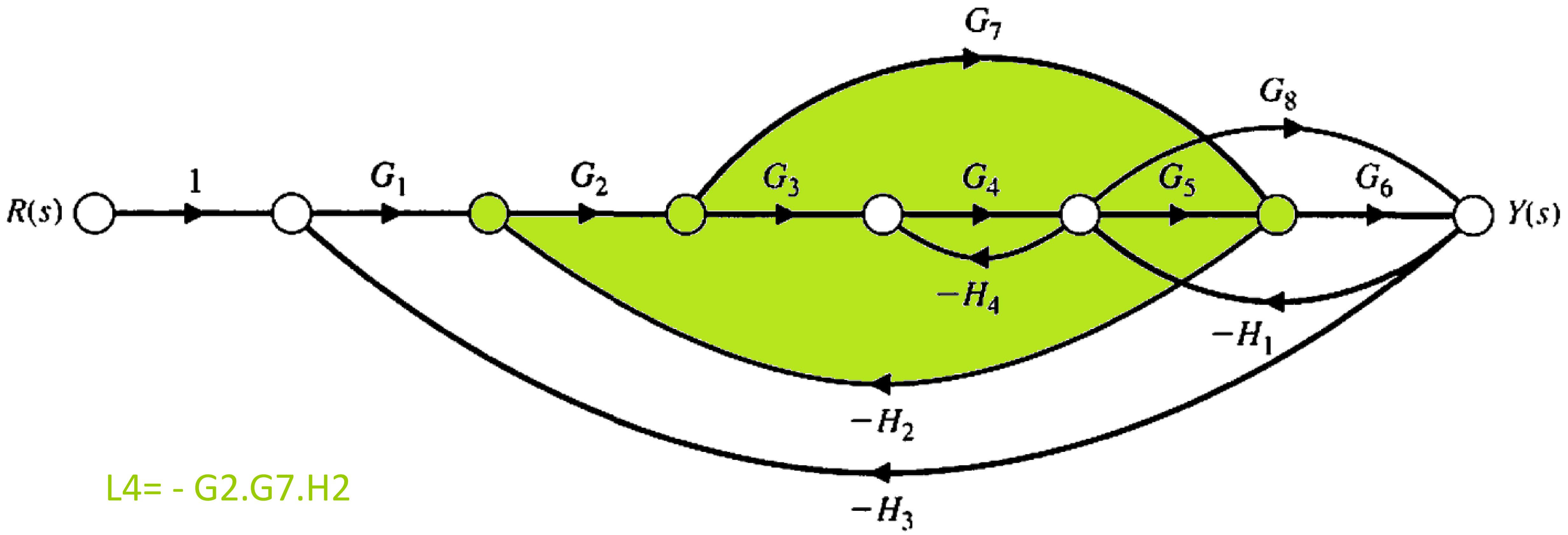
L1= - G2.G3.G4.G5.H2



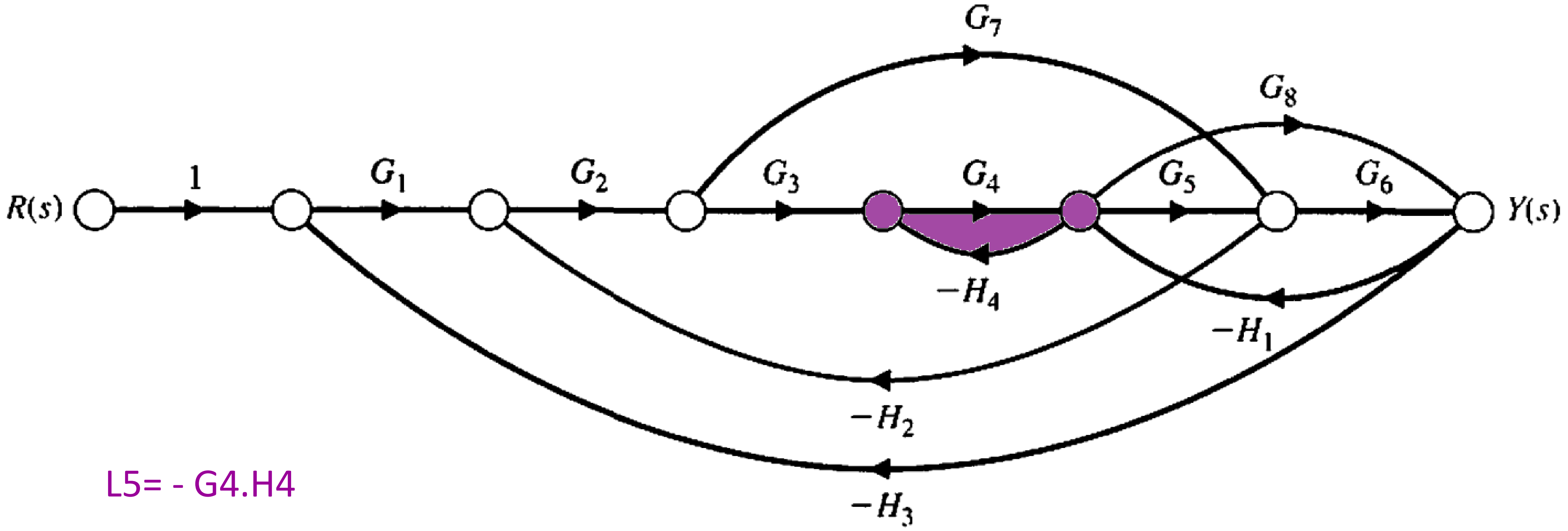
L2= - G5.G6.H1



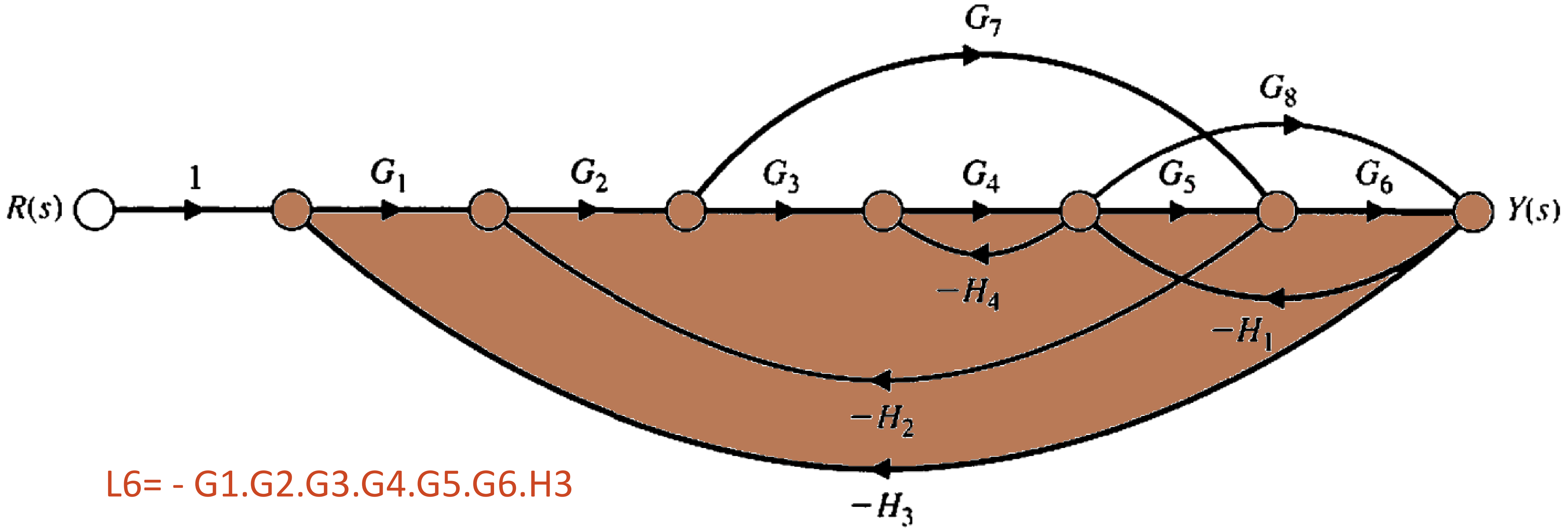
L3= - G8.H1



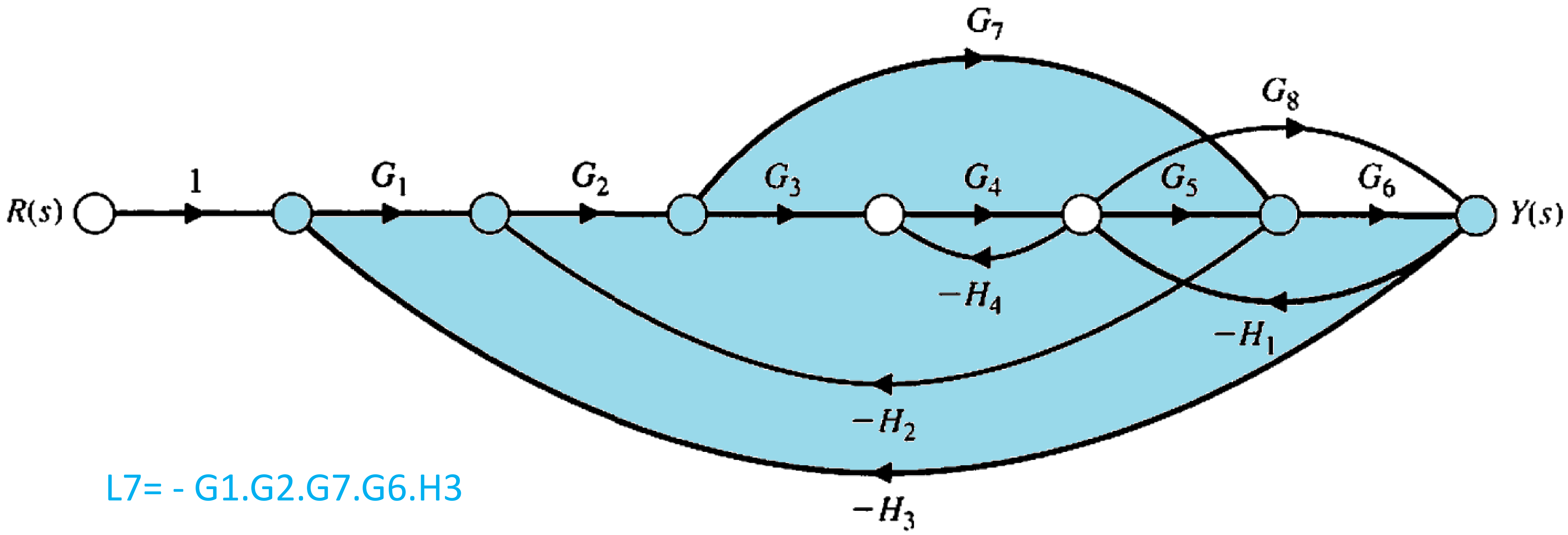
L4= - G2.G7.H2



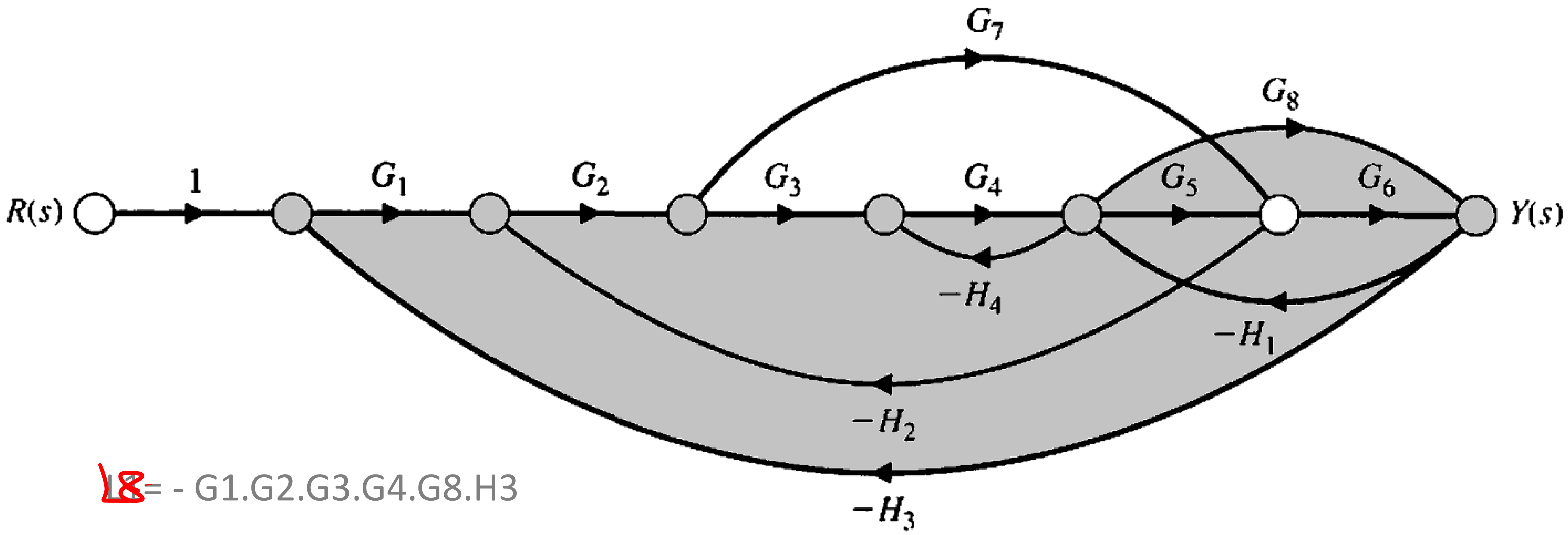
L5= - G4.H4



L6= - G1.G2.G3.G4.G5.G6.H3

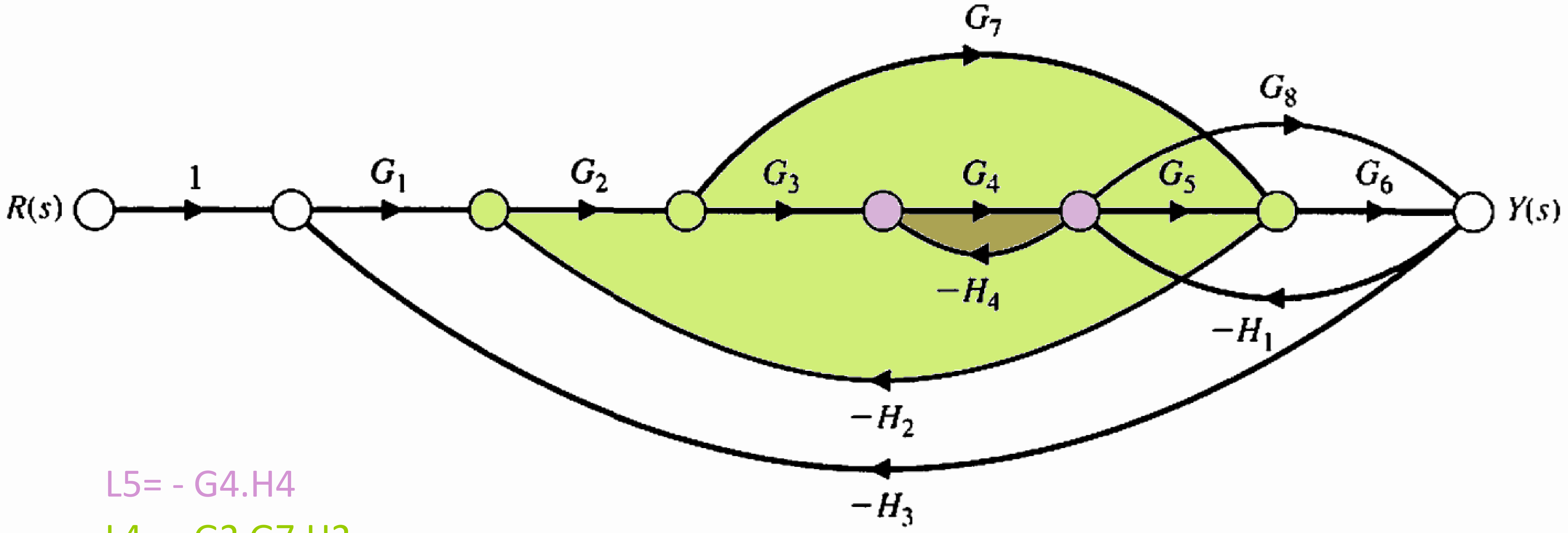


L7= - G1.G2.G7.G6.H3



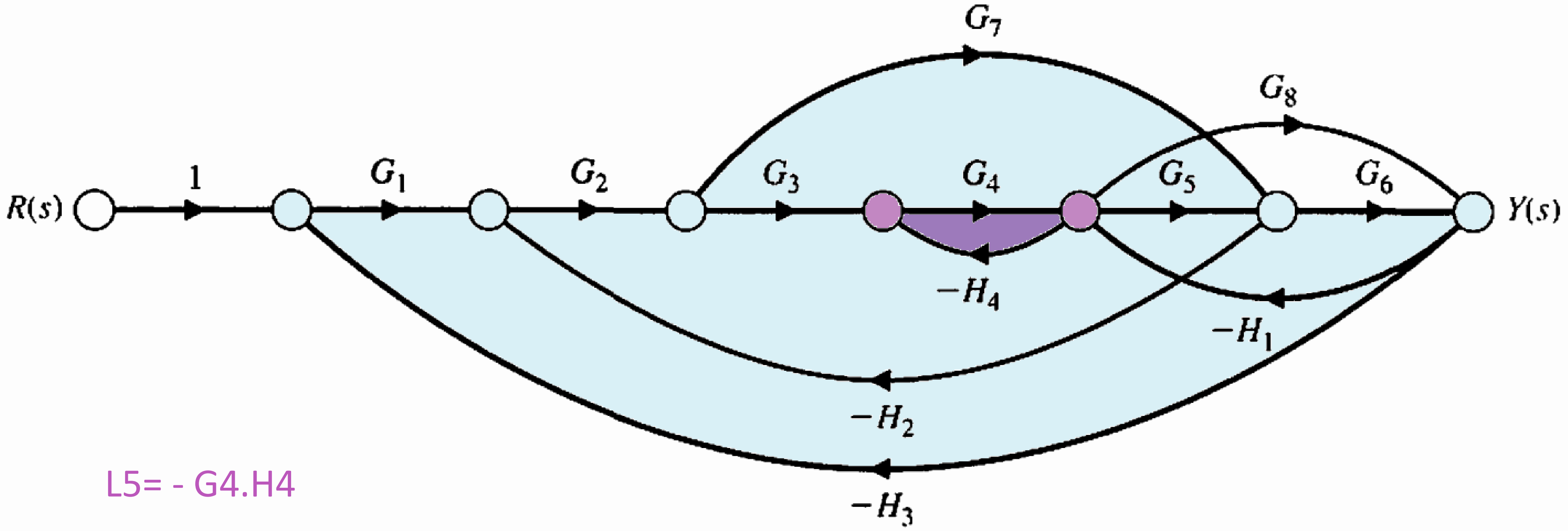
L8= - G1.G2.G3.G4.G8.H3

2.n.t.L:



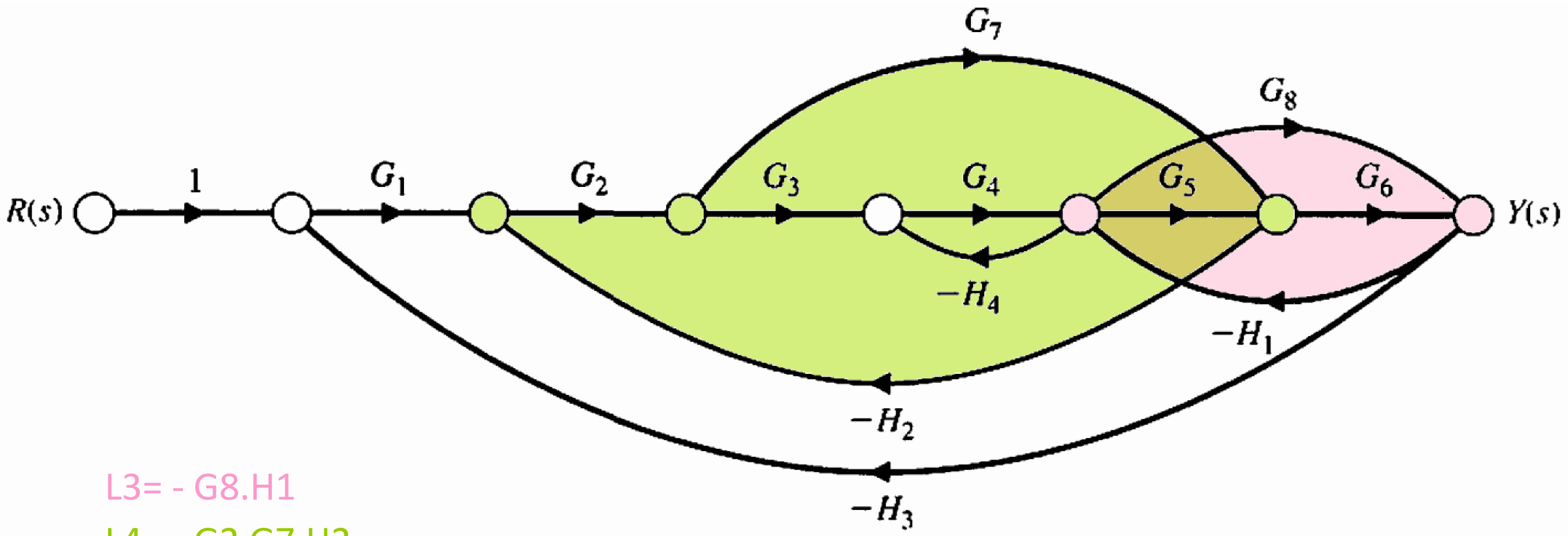
L5= - G4.H4

L4= - G2.G7.H2



L5= - G4.H4

L7= - G1.G2.G7.G6.H3



L3= - G8.H1

L4= - G2.G7.H2

- $\Delta 1=1$
- $\Delta 2=1-L5$
- $\Delta 3=1$

- $\Delta=1-(L1+L2+L3+L4+L5+L6+L7+L8)+(L5.L7+L5.L4+L3.L4)$

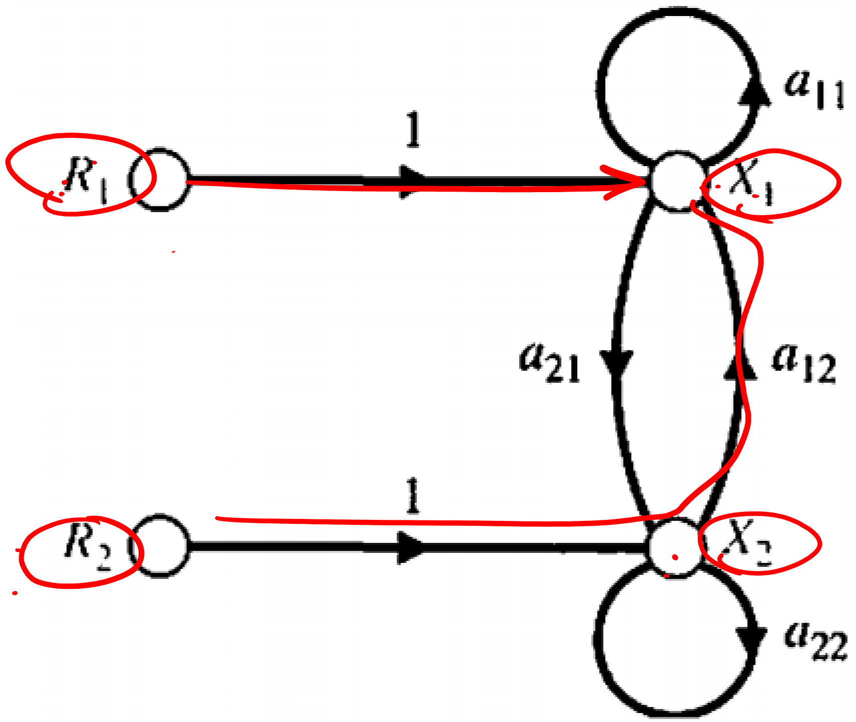
$$T.F=\frac{Y(s)}{R(S)}=\frac{(P1*\Delta 1)+(P2*\Delta 2)+(P3*\Delta 3)}{\Delta}$$

Example 3

$\frac{X_1}{R_1}$

- 4 transfer function

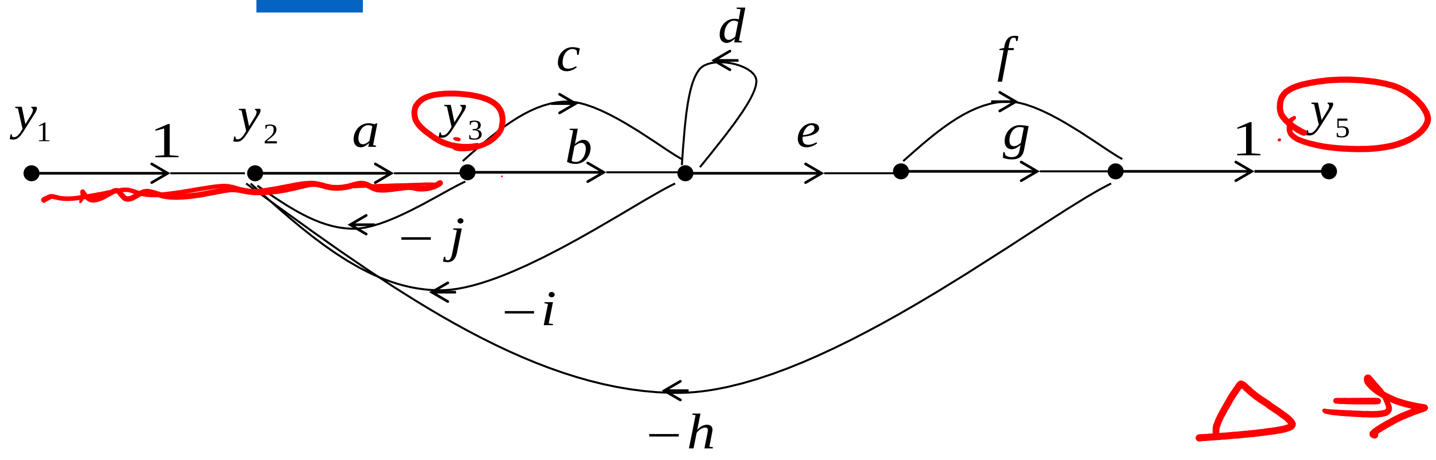
$$\frac{X1(s)}{R1(s)}, \frac{X2(s)}{R1(s)}, \frac{X1(s)}{R2(s)}, \frac{X2(s)}{R2(s)}$$



Example 4

find $\frac{y_5}{y_3}$

path, Δ_1



$\Delta \Rightarrow$

$L \Rightarrow$

4 paths

Feedback Control System Characteristics

closed loop system.

CH4

Open Loop Vs. Closed Loop

- An open-loop system operates without feedback and directly generates the output in response to an input signal.

- A closed-loop system uses a measurement of the output signal and a comparison with the desired output to generate an error signal that is used by the controller to adjust the actuator.

Open Loop Vs. Closed Loop

- Steady state error (Accuracy)
- Sensitivity → system sens.
- Disturbance rejection
- Noise Rejection sensor
- Transient Response →

Closed Loop System

- The two forms of control systems are shown in both block diagram and signal-flow graph form. Despite the cost and increased system complexity, closed-loop feedback control has the following advantages:

R.

- Decreased sensitivity of the system to variations in the parameters of the process.
- Improved rejection of the disturbances.
- Improved measurement noise $\boxed{\text{attenuation}}$,
- Improved reduction of the steady-state error of the system.
- Easy control and adjustment of the transient response of the system.

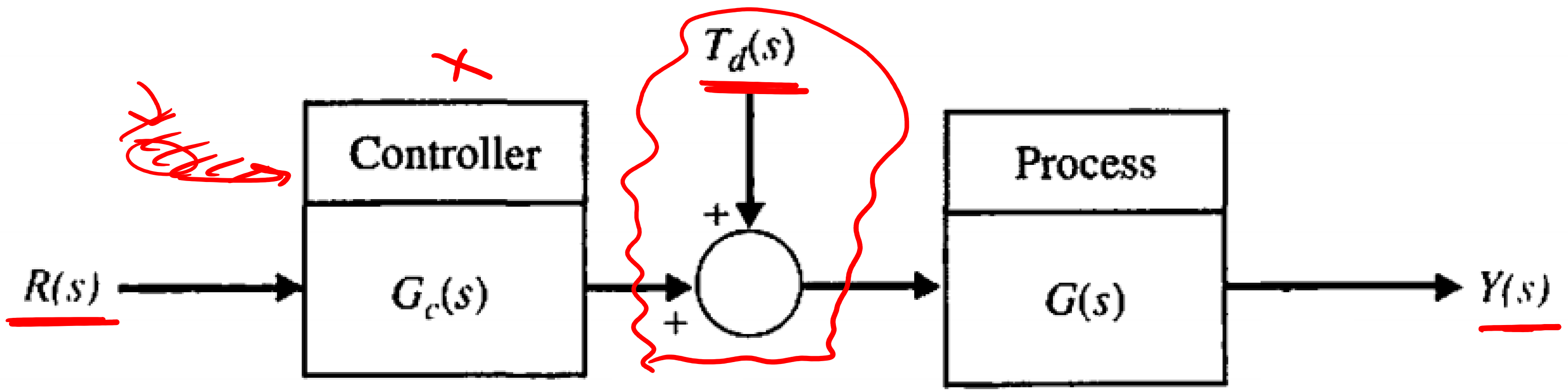
$y_{s.s} = \lim_{s \to 0} \, Y(s$

$E_{ss} = \lim_{s \to 0} s \, . \, E(s)$

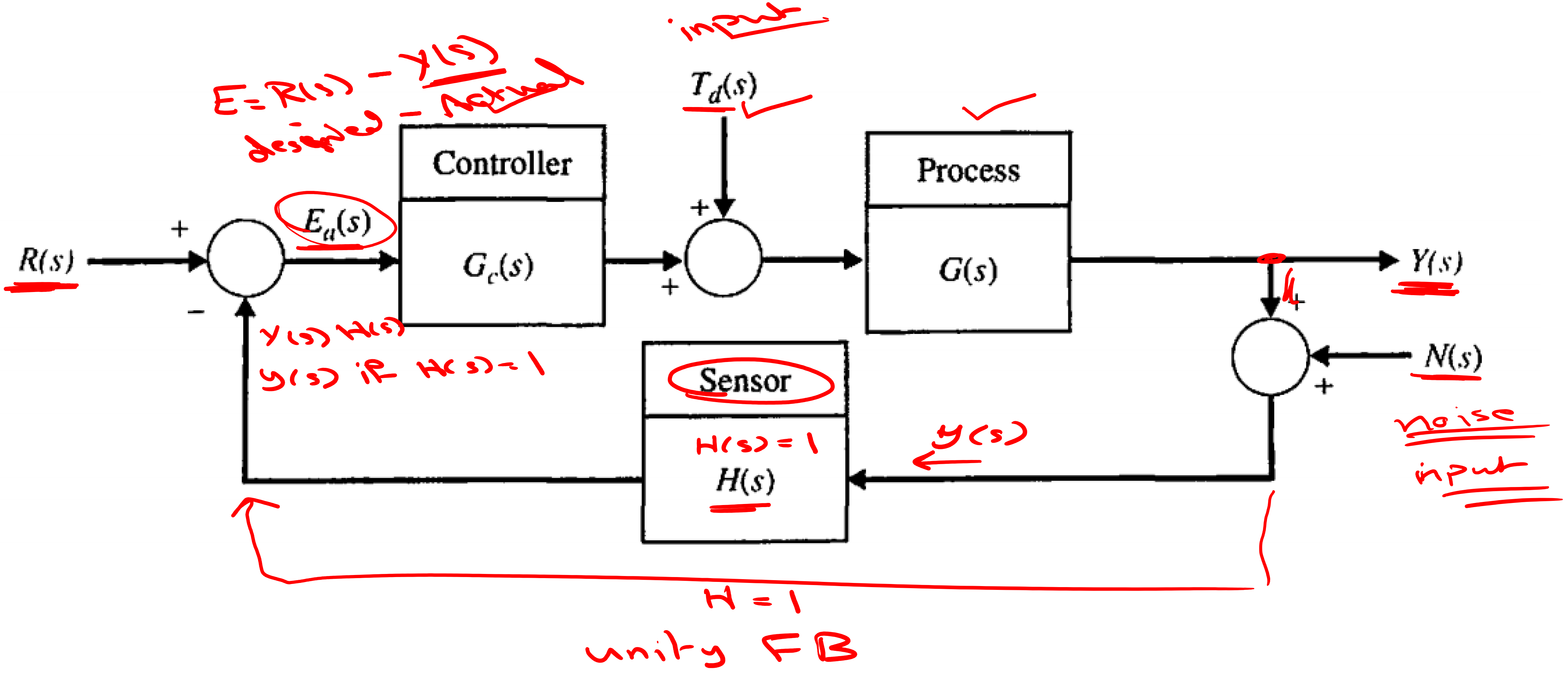
Open Loop System

general o.L.sys.

disturbance.



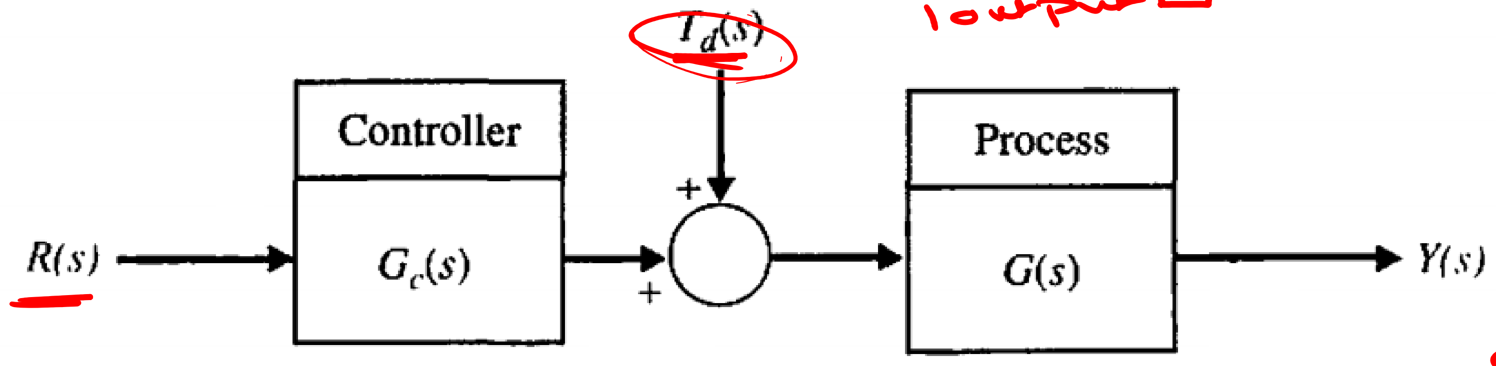
Closed Loop System



Transfer Function

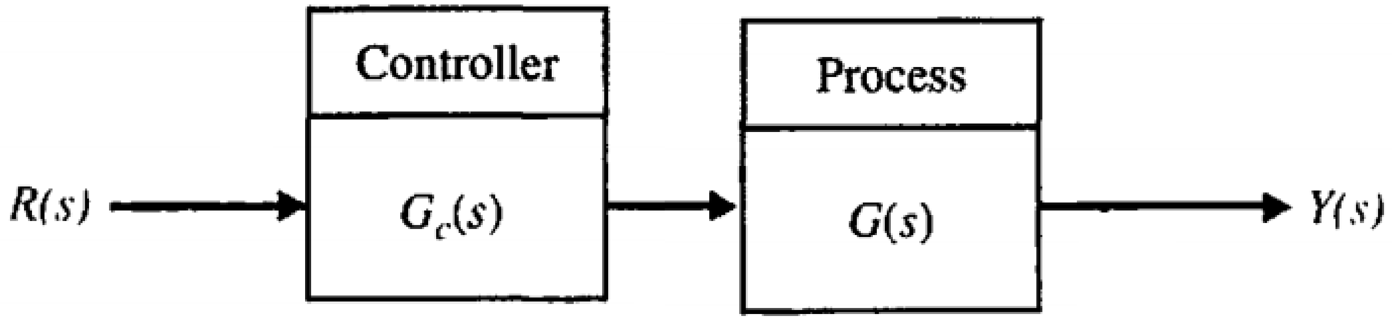
- Open Loop

2 inputs, 1 output $\rightarrow$ 2 T.F



$$\frac{Y(s)}{R(s)} \Rightarrow T_d(s) = 0$$

$$\frac{Y(s)}{T_d(s)} \Rightarrow R(s) = 0$$

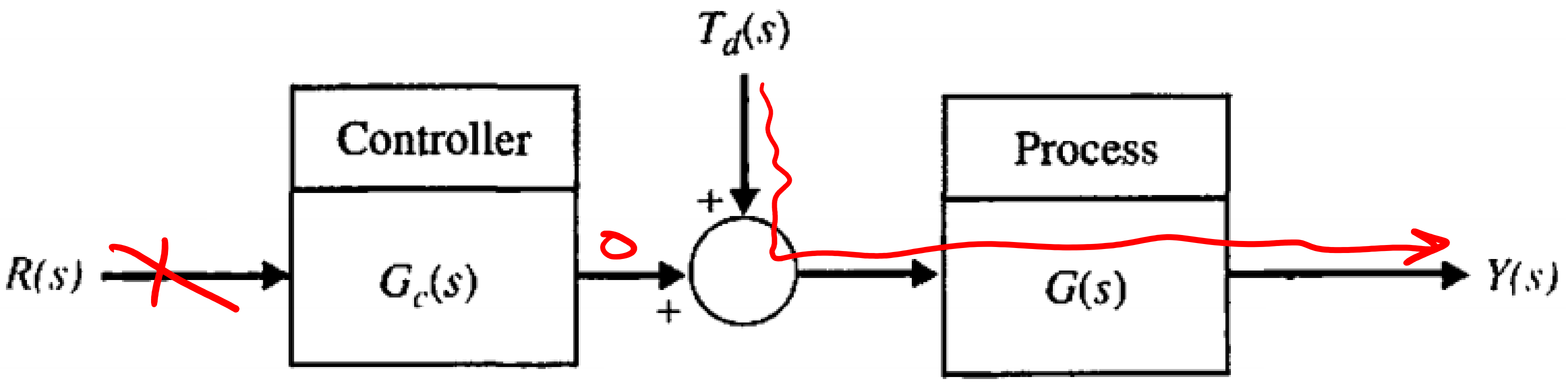


$$\boxed{\frac{Y(s)}{R(s)} = G_c(s)\, G(s)}$$

$$Y(s) = G_c G R(s)$$

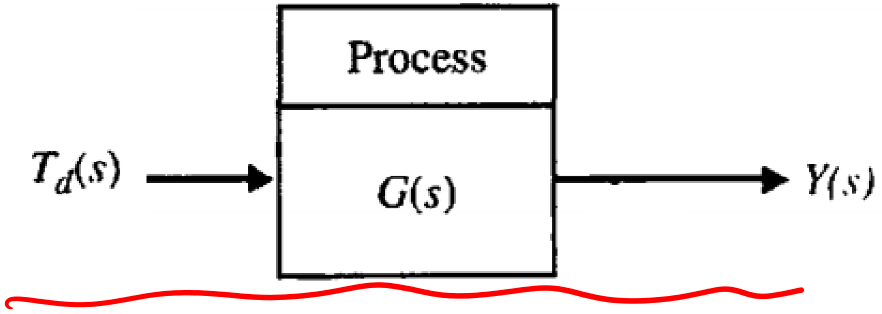
Transfer Function

- Open Loop



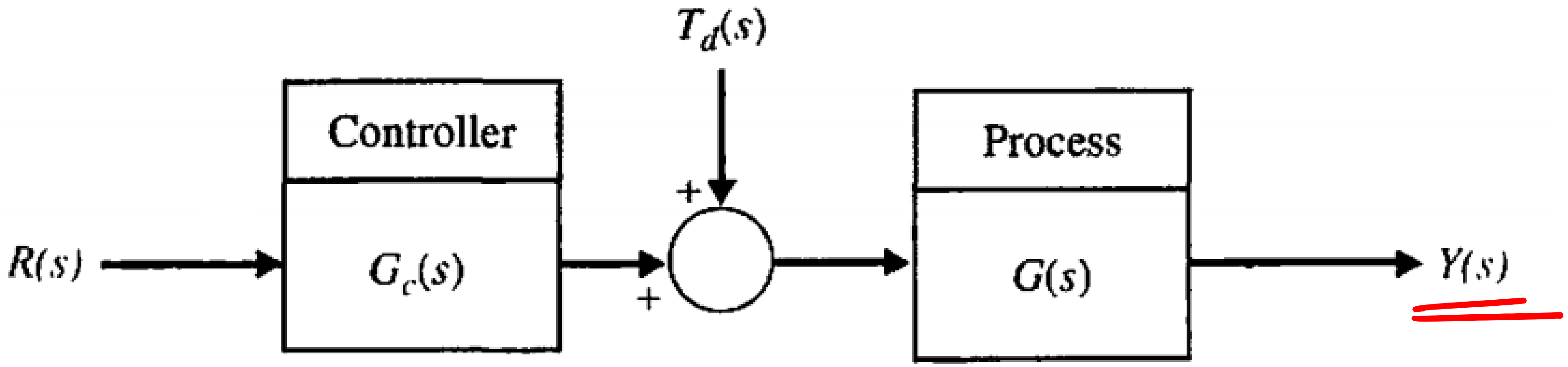
$$\boxed{\frac{Y(s)}{T_d(s)} = G(s)}$$

$$\boxed{Y(s) = T_d(s)\, G(s)}$$



Open Loop System

$\frac{Y(s)}{T_d(s)}$ $\frac{Y(s)}{R(s)}$



$Y(s) = G_c G R(s) + G T_d(s)$ total output

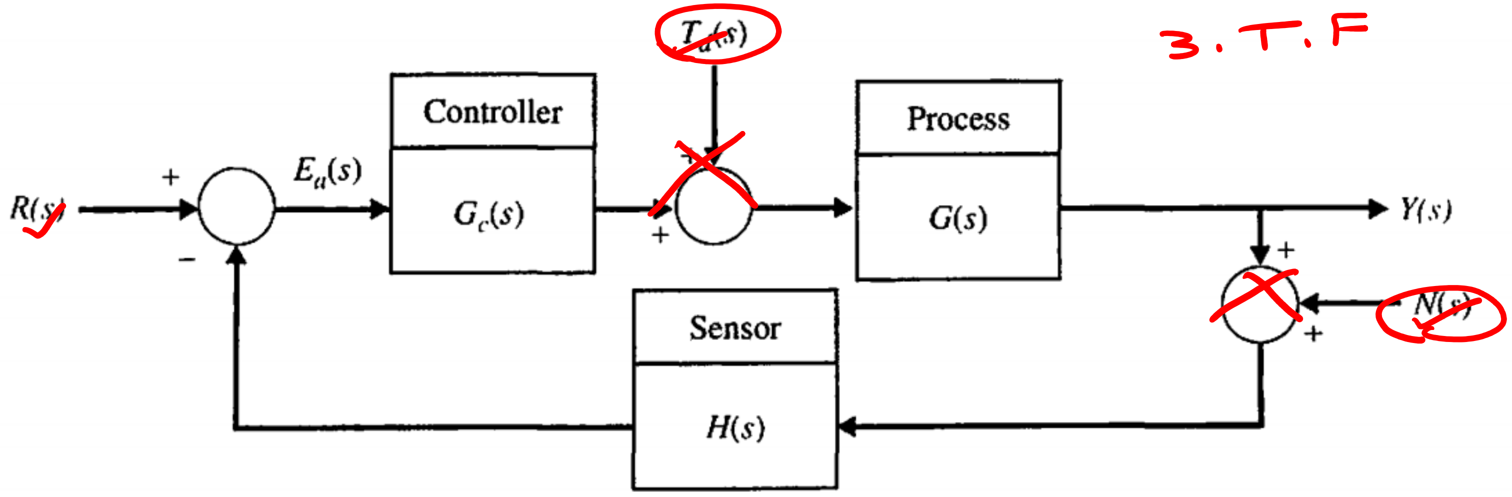
$y(t) =$

Closed Loop TF

inputs ⇒ 3 i/P

output ⇒ 1 o/P

3. T.F

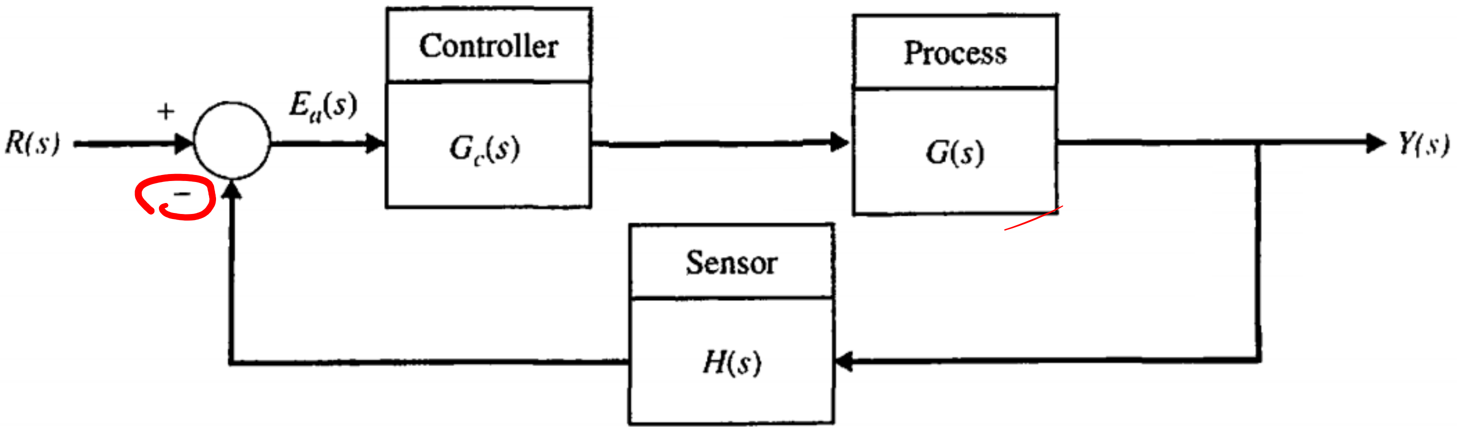


$$\frac{Y(s)}{R(s)} \Rightarrow T_d(s) = 0$$

$$N(s) = 0$$

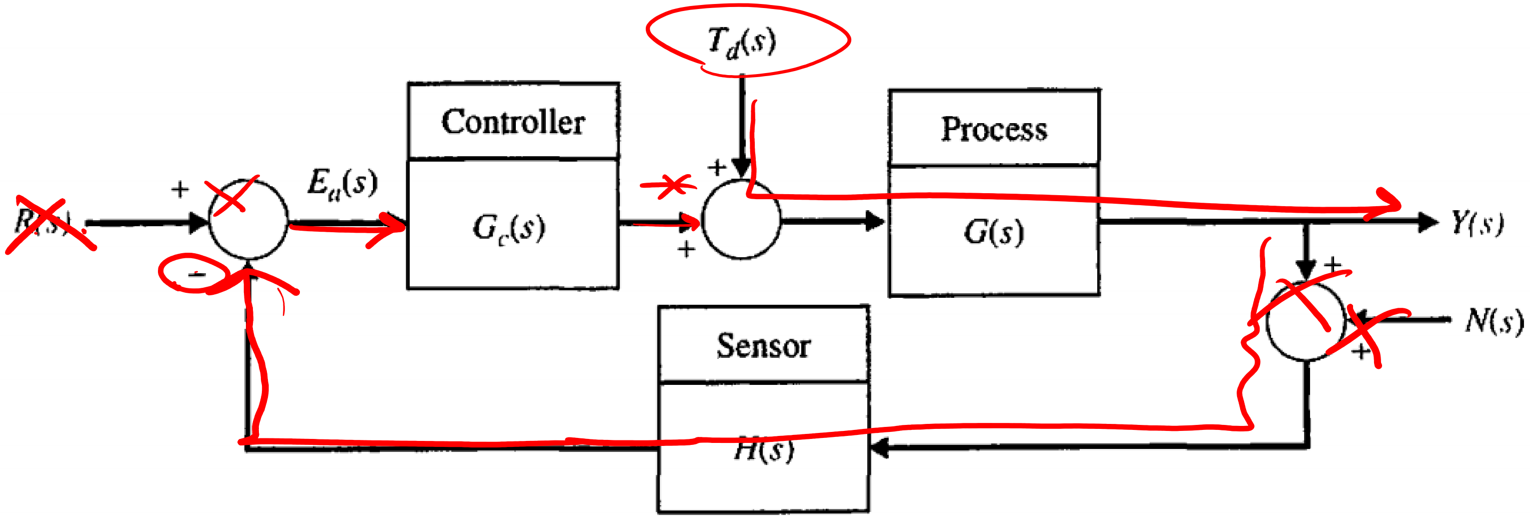
$$\frac{Y(s)}{R(s)} = \frac{G_c G}{1 + G_c G H}$$

$$Y(s) = \frac{G_c G}{1 + G_c G H} * R(s)$$



Closed Loop TF

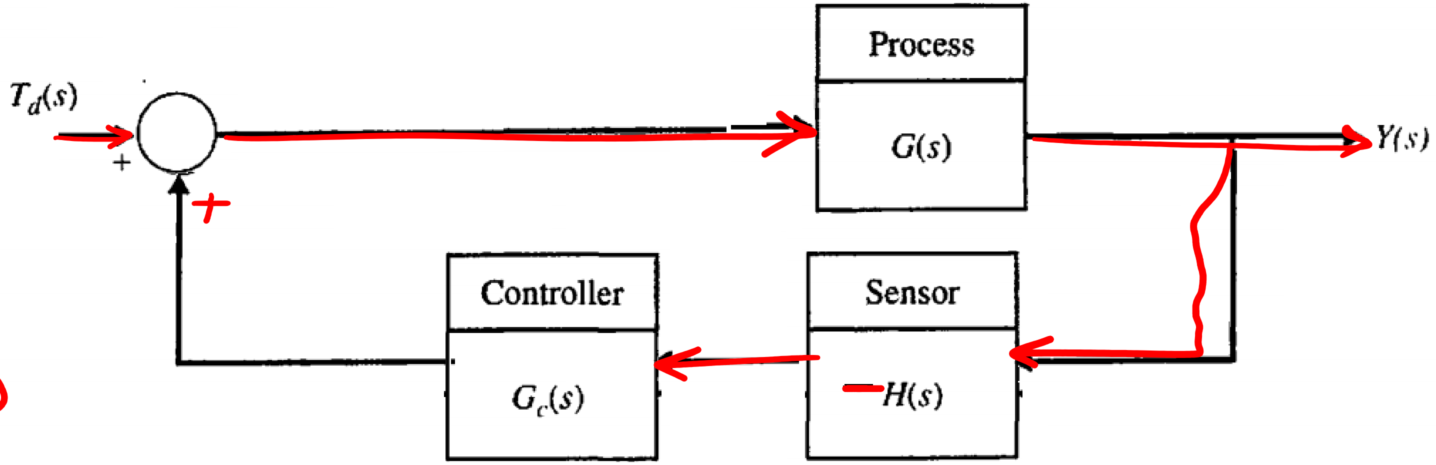
$$\frac{Y(s)}{T_d(s)} \Rightarrow \quad R(s)=0, \quad N(s)=0$$



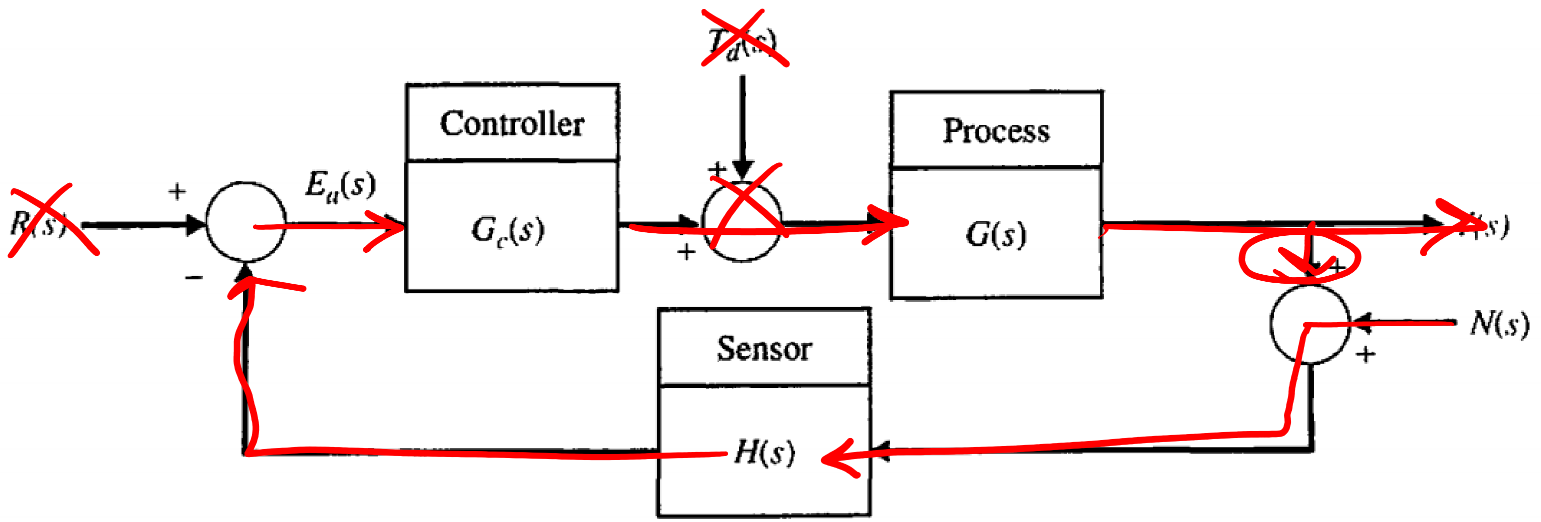
$$\frac{Y(s)}{T_d(s)} = \frac{G(s)}{1 - GG_c * -H}$$

$$\frac{Y(s)}{T_d(s)} = \frac{G}{1+GG_cH}$$

$$Y(s) = \frac{G}{1+GG_cH} * T_d(s)$$



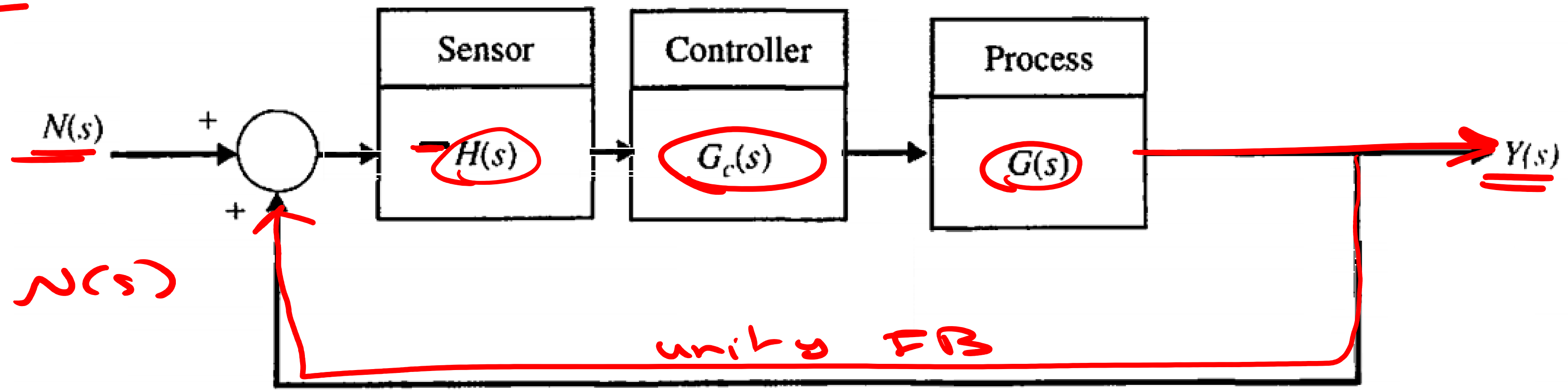
Closed Loop TF



$$\frac{Y(s)}{N(s)} \Rightarrow R(s)=0$$

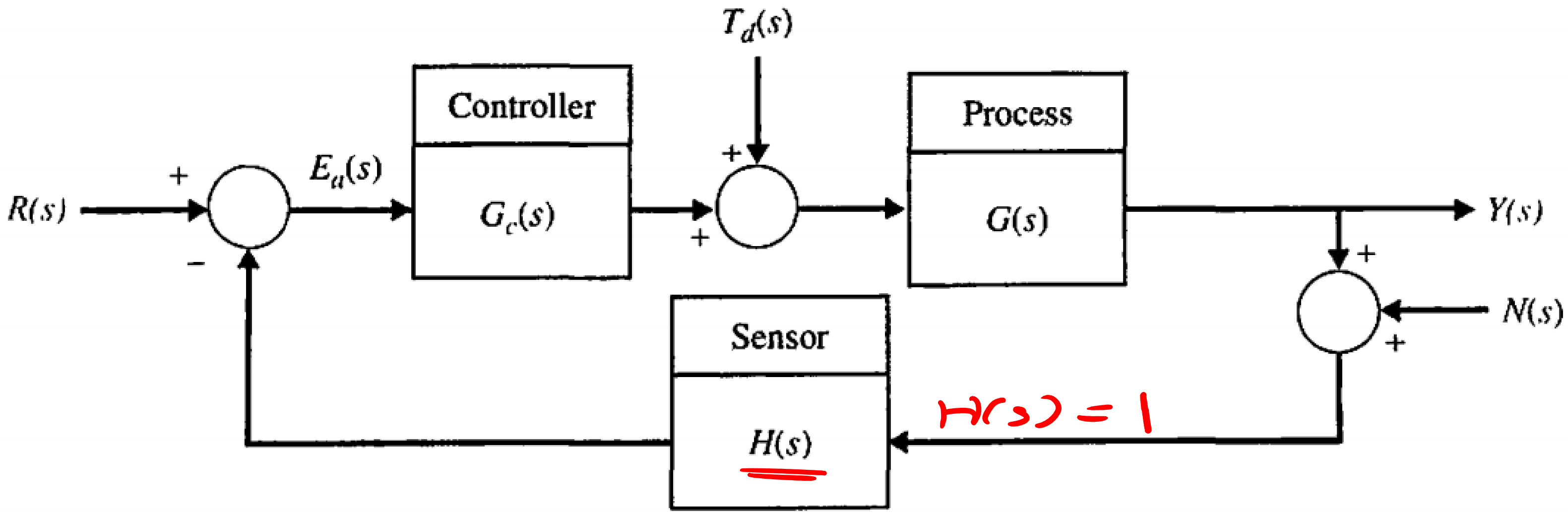
$$T_d(s)=0$$

$$\frac{Y(s)}{N(s)} = \frac{-GG_cH}{1+GG_cH}$$



$$Y(s) = \frac{-GG_cH}{1+GG_cH} * N(s)$$

Closed Loop System



$$Y(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}R(s) + \frac{G(s)}{1 + G_c(s)G(s)}T_d(s) - \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}N(s).$$

7 Characteristic equ.

Accuracy.

Error Analysis

$$E(s) = R(s) - Y(s).$$

Error = desired − Actual

Error due to $R(s) = \frac{1}{s}$ unit step

$T_d(s), N(s) = 0$

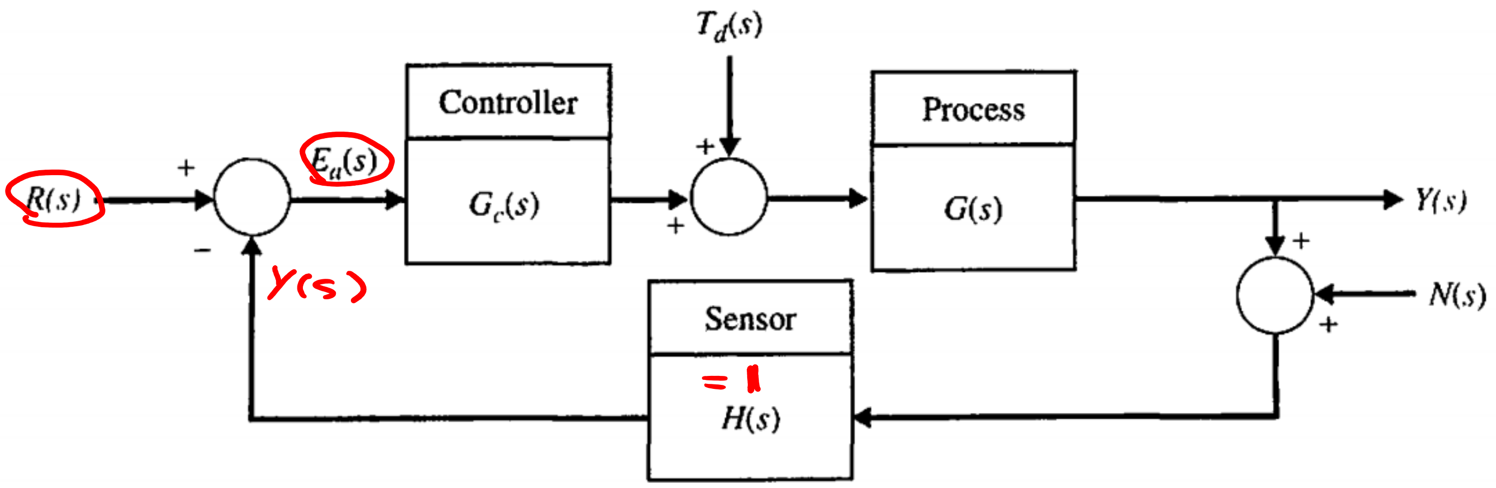
$$E(s) = \frac{R(s)}{1+G_cG} - \frac{G_cG}{1+G_cG}R(s)$$

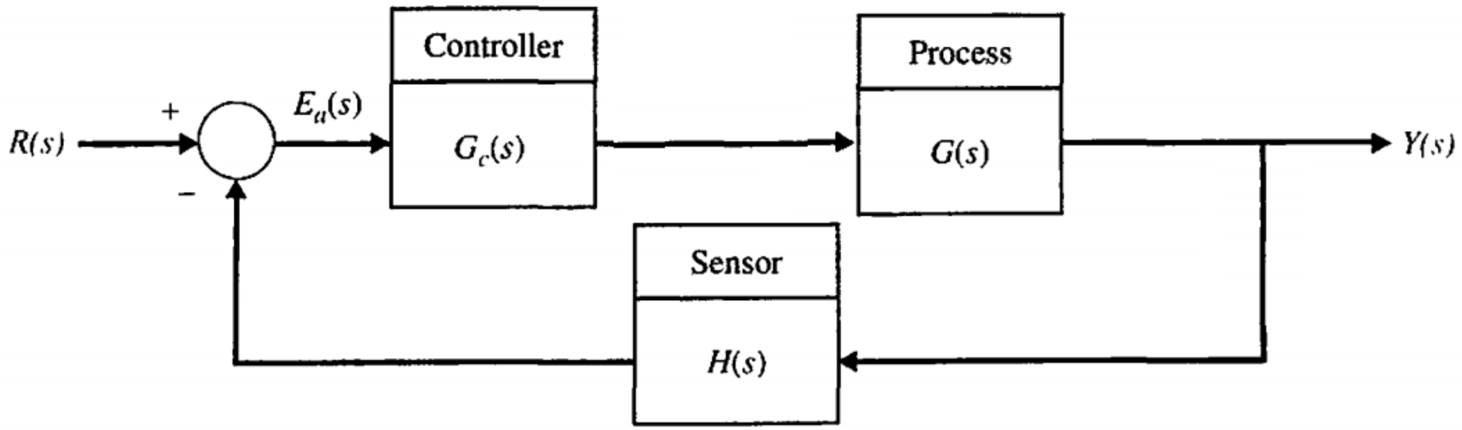
$$E(s) = R(s)\left[\frac{1}{1+G_cG}\right]$$

$$E_{ss} = \lim_{s\to 0} s\cdot E(s) = \lim_{s\to 0} \cancel{s}\cdot\frac{1}{\cancel{s}}\cdot\frac{1}{1+G_c(s)G(s)} = \frac{1}{1+G_c(0)G(0)} = e_{ss}$$

$G_c(0)G(0)$: DC gain

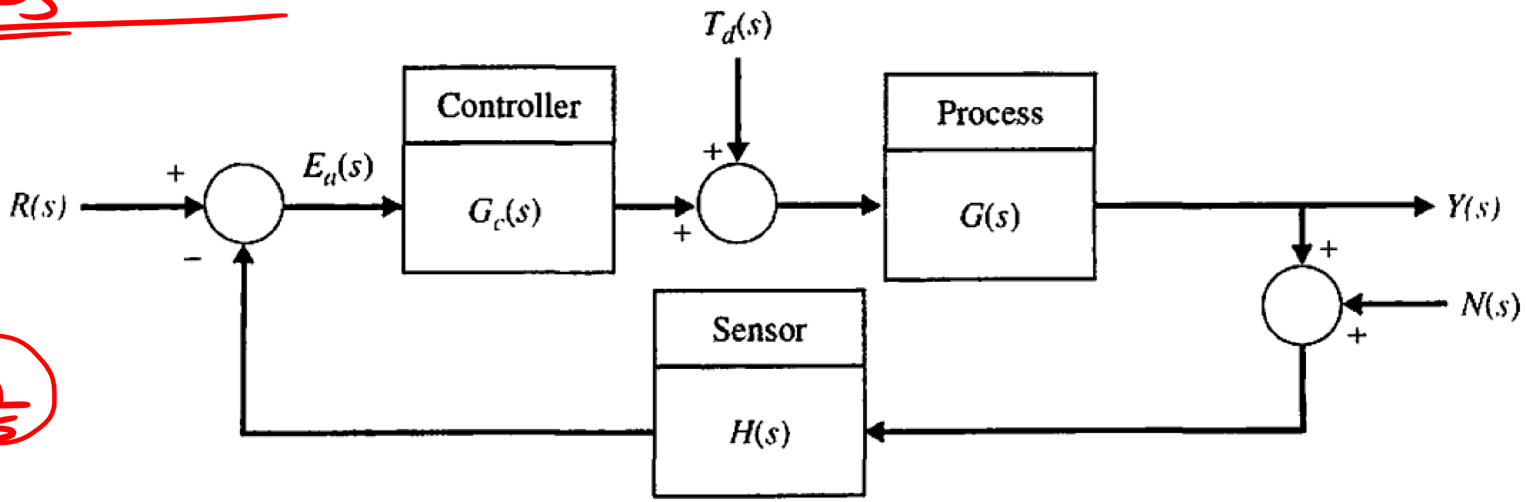
e_{ss} = steady state error





Disturbance rejection

Error Analysis



$$E(s) = R(s) - Y(s).$$

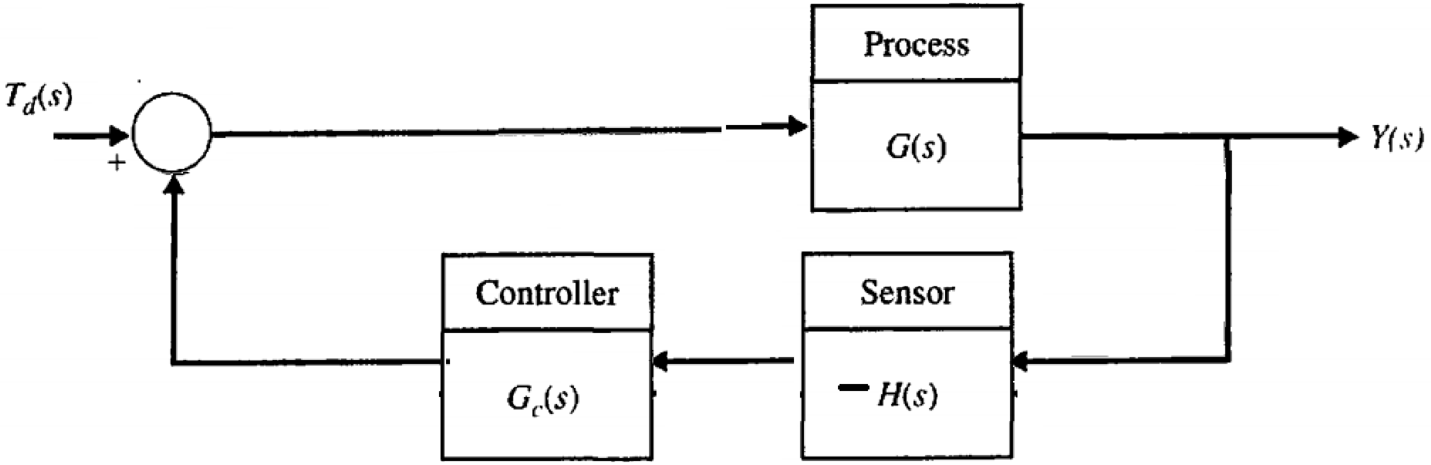
Find e_{ss} due to $T_d(s) = \frac{1}{s}$

$R(s), N(s) = 0$

$E(s) = R(s) - Y(s) \rightarrow$ desired - Actual.

$$E(s) = 0 - \frac{G}{1+G_cG} T_d(s)$$

$$= -\frac{G}{1+G_cG} T_d$$



$$E_{ss} = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \cdot -\frac{G}{1+G_cG}$$

$$E_{ss} = \frac{-G(0)}{1+G_c(0)G(0)}$$ DC gain.

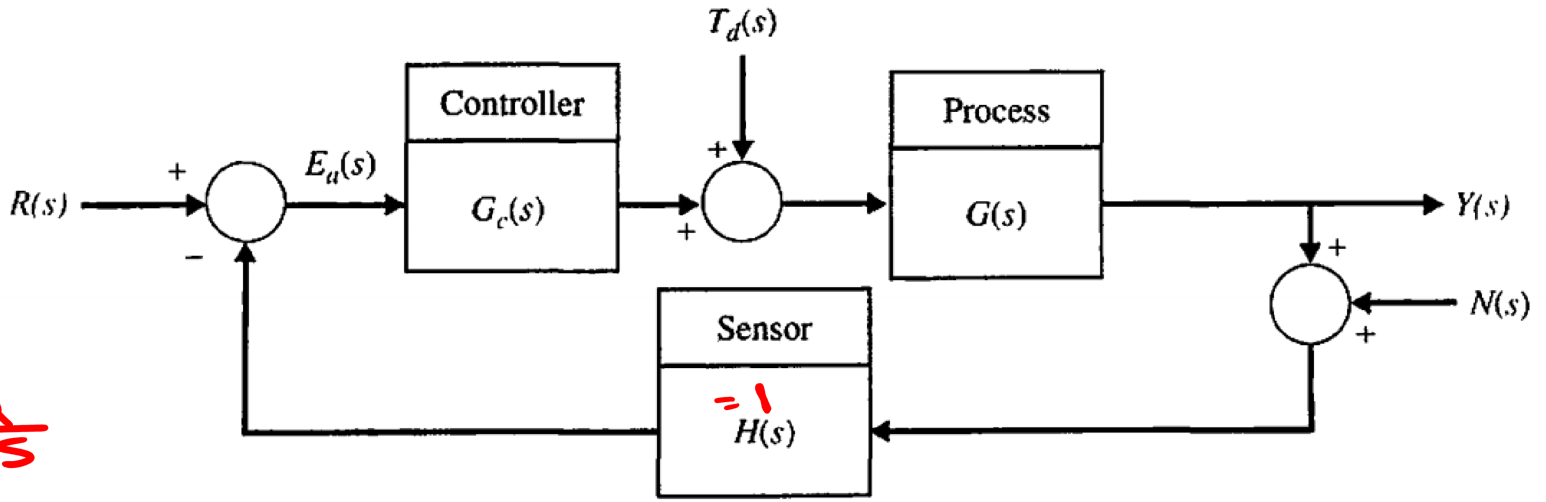
Noise rejection

Error Analysis

$$E(s) = R(s) - Y(s).$$

error due to $N(s) = \frac{1}{s}$

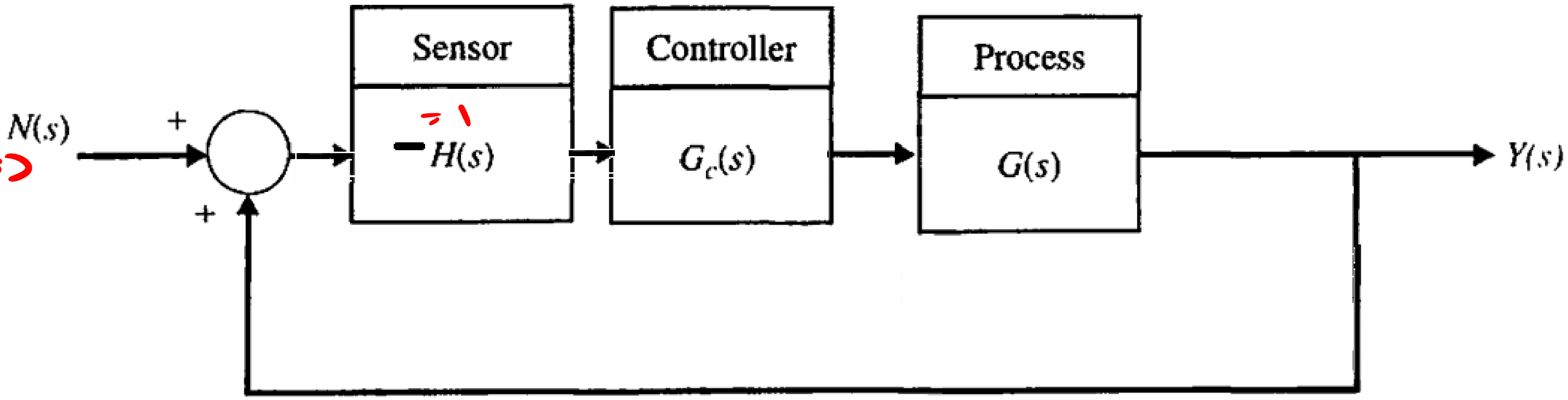
$R(s), T_d(s) = 0$



$$E(s) = \underline{R(s)} - Y(s)$$

$$E(s) = 0 - \frac{-G_cG}{1+G_cG} N(s)$$

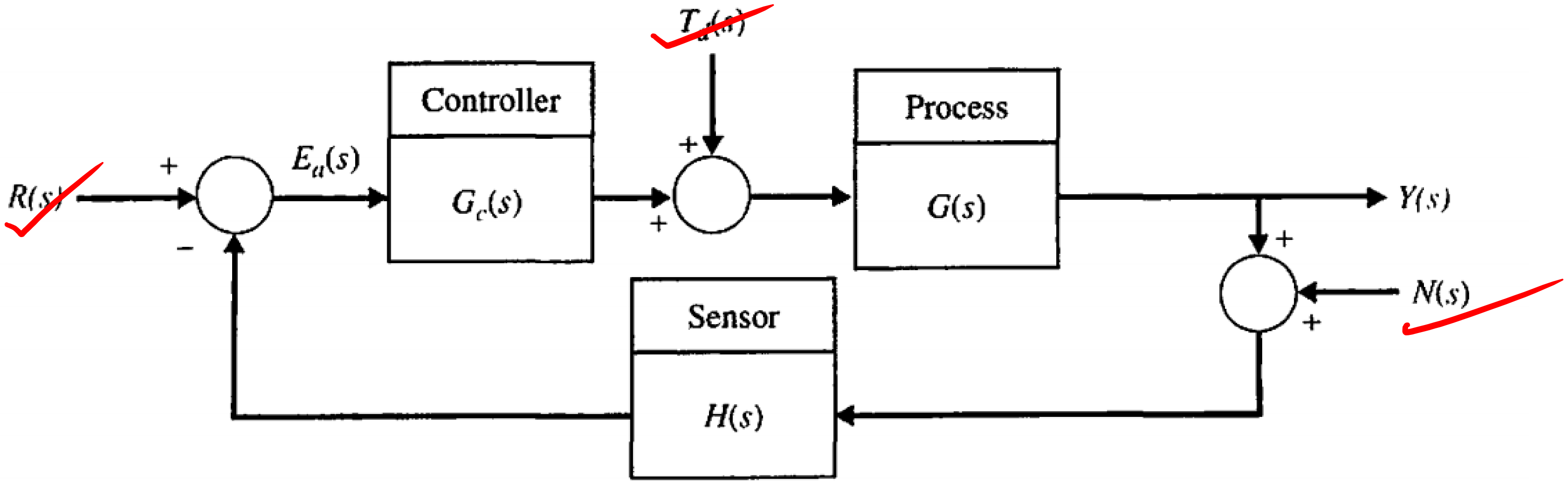
$$E(s) = \frac{G_cG}{1+G_cG} N(s)$$



$$E_{ss} = \lim_{s \to 0} \cancel{s} \frac{1}{\cancel{s}} \cdot \frac{G_cG}{1+G_cG} = \frac{G_c(0)G(0)}{1+\underline{G_c(0)G(0)}}$$

Error Analysis (Total Error)

$$E(s) = R(s) - Y(s).$$

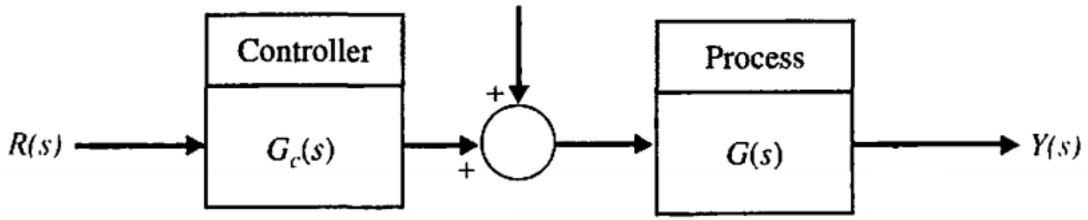


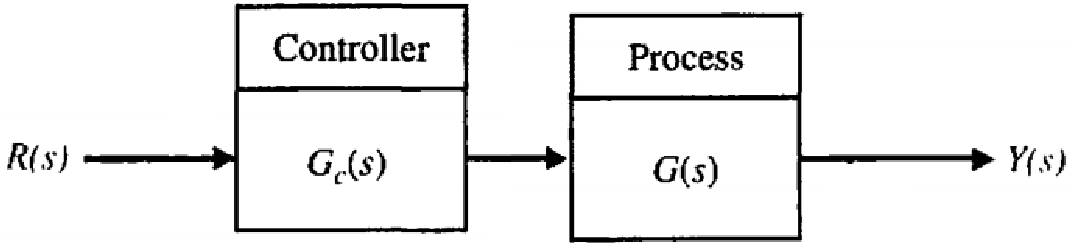
$$E(s) = \frac{1}{1 + G_c(s)G(s)}R(s) - \frac{G(s)}{1 + G_c(s)G(s)}T_d(s) + \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}N(s).$$

open loop sys.

Error Analysis

$$E(s) = R(s) - Y(s).$$ desired − Actual





due to $R(s) = \frac{1}{s}$

$T_d(s) = 0$

$E = R(s) - G_c G\, R(s)$

$E = R(s)\left[1 - G_c G\right)$

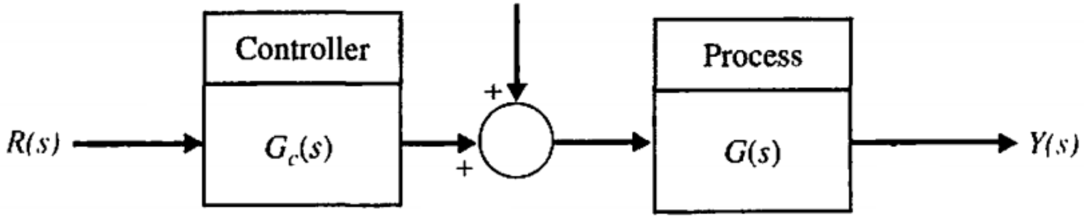
$E_{ss} = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} (1 - G_c G)$

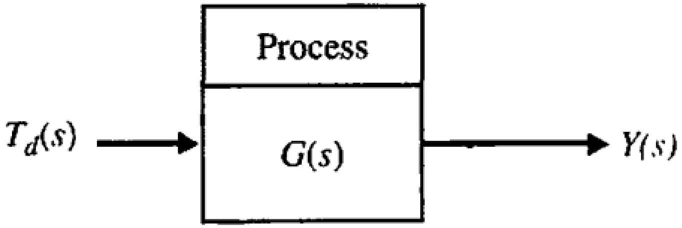
$$\boxed{E_{ss} = 1 - G_c(0)\, G(0)}$$

Error Analysis

dis. rejection.

$$E(s) = R(s) - Y(s).$$





Error due $T_d(s) = \frac{1}{s}$, $R(s) = 0$

$E(s) = R(s) - Y(s)$

$E(s) = 0 - G\,T_d(s)$

$- G\,T_d(s)$

$E_{ss} = \lim \cancel{s} \cdot \frac{1}{\cancel{s}} \cdot -G$

$$\boxed{E_{ss} = -G(0)}$$

Open Loop Vs. Closed Loop

Steady state error (e_{ss})

	Open Loop	Closed Loop
Error Due to R(s) Accuracy	$\left\| 1 - G_c(0)G(0) \right\|$	$\left\| \frac{1}{1+G_c(0)G(0)} \right\|$
Error Due to Td(s) Disturbance Rejection	$\left\| -G(0) \right\|$	$\left\| \frac{-G(0)}{1+G(0)G_c(0)} \right\|$
Error Due to N(s) Noise Rejection	—	$\frac{G_c(0)G(0)}{1+G_c(0)G(0)}$

$e_{ss\,o.L} > e_{ss\,c.L}$ ✓

$e_{ss\,o.L}\big|_{Td} > e_{ss\,c.L}\big|_{Td}$ ✓

System Sensitivity

- System sensitivity is the ratio of the change in the system transfer function to the change of a process transfer function (or parameter) for a small incremental change.

$$S = \frac{\Delta T(s)/T(s)}{\Delta G(s)/G(s)}.$$

$$S_G^T = \frac{\partial T}{\partial G} \cdot \frac{G}{T}$$

Feedback control system Characteristics

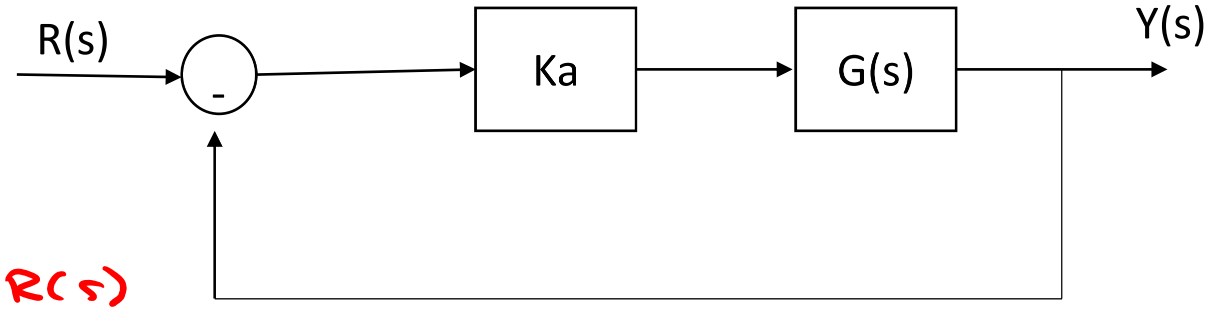
CH4

Part 2

Steady state error example

$\frac{1}{s}$

- If $G(s) = \dfrac{10}{s(0.001s+1)}$ find Ess if R(s) is a unit step input



$$T(s) = \frac{K_a G}{1 + K_a G} \Rightarrow = \frac{Y(s)}{R(s)}$$

$$E(s) = R(s) - Y(s) = R(s) - T(s)R(s)$$

$$E(s) = R(s)[1 - T(s)] = R(s)\left[1 - \frac{K_a G}{1 + K_a G}\right] = \left(\frac{1}{1 + K_a G}\right) R(s)$$

$$E_{ss} = \lim_{s \to 0} s \cdot \frac{1}{s} \cdot \left[1 - \frac{K_a G}{1 + K_a G}\right] = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \left[\frac{1}{1 + K_a\left[\frac{10}{\underbrace{s(0.001s+1)}_{0}}\right]}\right] = 0$$

H.W :- Find e_{ss} if the sys is <u>open loop</u>

System Sensitivity (0 – 100)%

- System sensitivity is the ratio of the change in the system transfer function to the change of a process transfer function (or parameter) for a small incremental change.

$\Delta T(s)$

ΔG

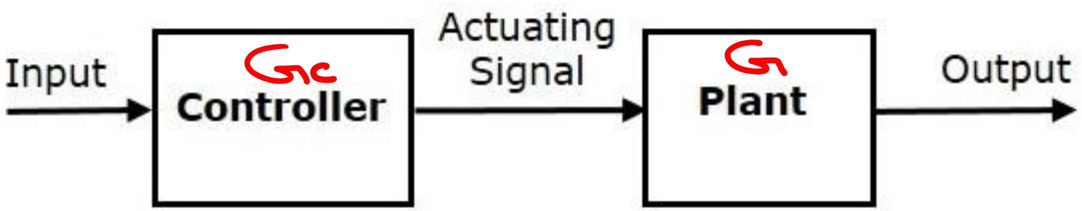
ΔR, K, B

$$S = \frac{\Delta T(s)/T(s)}{\Delta G(s)/G(s)}.$$

$$\boxed{S_G^T = \frac{\partial T}{\partial G} \cdot \frac{G}{T}}$$

S_G^T

Open Loop Sensitivity



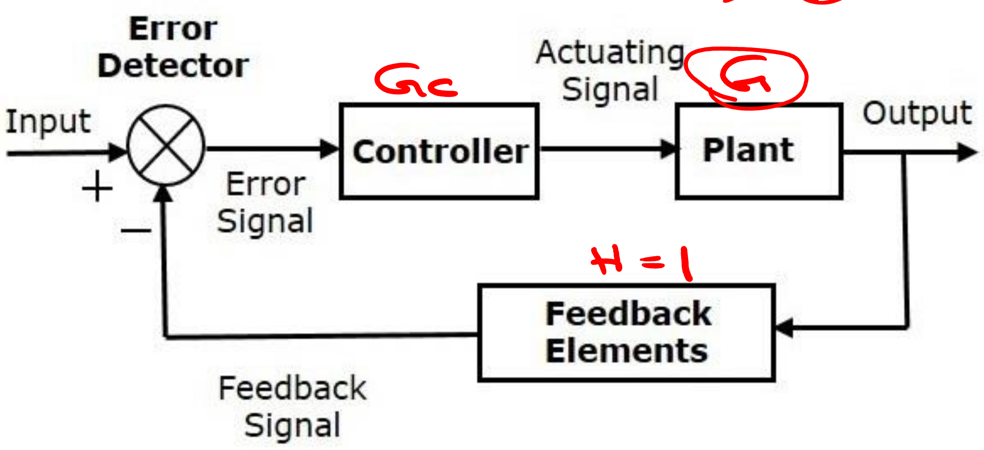
$$S_G^T = \frac{\partial T}{\partial G}\cdot\frac{G}{T}$$

$$T(s) = G_c G$$

$$S_G^T = \cancel{G_c} * \frac{\cancel{G}}{\cancel{G_c}\cancel{G}} = \boxed{1}$$

Closed Loop System Sensitivity

$\frac{2R}{3s+1}$



$$T(s) = \frac{G_c(s)G(s)}{1 + G_c(s)G(s)}.$$

Therefore, the sensitivity of the feedback system is

$$S_G^T = \frac{\partial T}{\partial G} \cdot \frac{G}{T} = \frac{G_c}{(1 + G_cG)^2} \cdot \frac{G}{GG_c/(1 + G_cG)}$$

S ↓

or

$$\boxed{S_G^T = \frac{1}{1 + G_c(s)G(s)}.}$$

$\Rightarrow < 1$

If we seek to determine S_α^T, where α is a parameter within G(s), using the chain rule gives

$$S_\alpha^T = S_G^T * S_\alpha^G$$

EXAMPLE 4.1 **Feedback amplifier**

T(s)

An amplifier used in many applications has a gain $-K_a$, as shown in Figure 4.4(a). The output voltage is

$$T(s) = \frac{v_0}{v_{in}} = -K_a \frac{v_{in}}{v_{in}}. \tag{4.17}$$

We often add feedback using a potentiometer R_p, as shown in Figure 4.4(b). The transfer function of the amplifier without feedback is

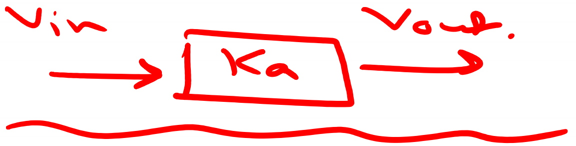
$$\boxed{T = -K_a,} \qquad S^T_{K_a} = -1 \times \frac{K_a}{-\cancel{K_a}} \tag{4.18}$$ $= 1$

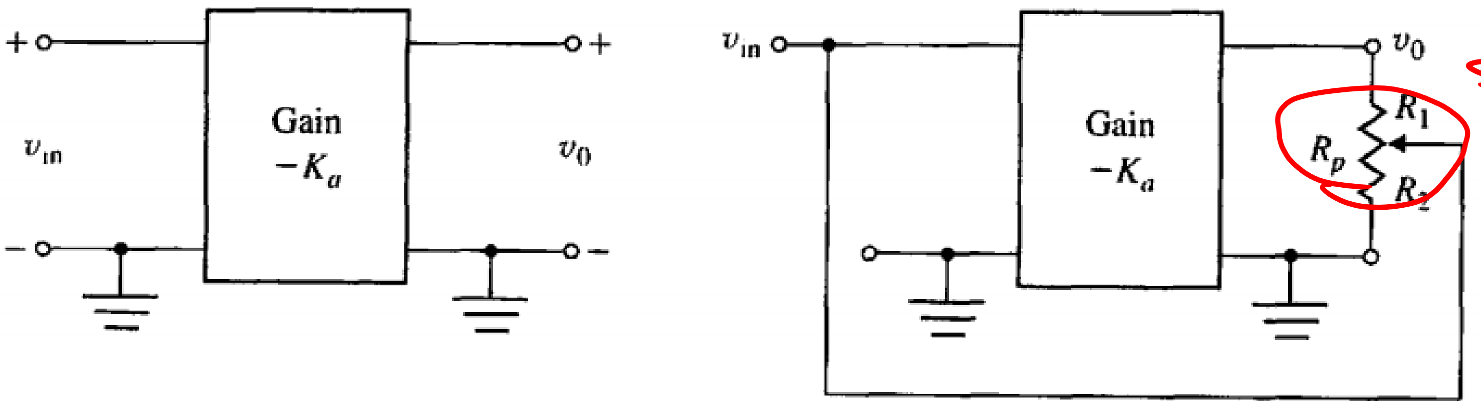
and the sensitivity to changes in the amplifier gain is

$$S^T_{K_a} = 1. \tag{4.19}$$

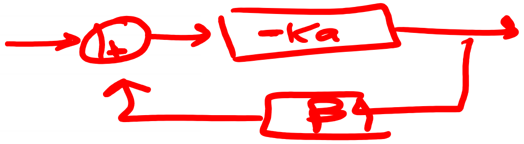
The block diagram model of the amplifier with feedback is shown in Figure 4.5, where

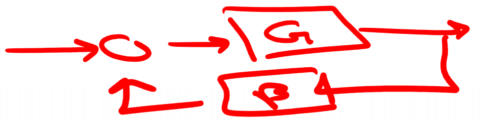
$$\beta = \frac{R_2}{R_1} \tag{4.20}$$





sensor.





$G = -K_a$

$$R_p = R_1 + R_2. \tag{4.21}$$

The closed-loop transfer function of the feedback amplifier is

$$\frac{-K_a}{1 - (-K_a\beta)}$$

$$\boxed{T = \frac{-K_a}{1 + K_a\beta}}. \tag{4.22}$$

$S^T_{K_a} =$

The sensitivity of the closed-loop feedback amplifier is

$$S^T_{K_a} = S^T_G S^G_{K_a} = \boxed{\frac{1}{1 + K_a\beta}}. \tag{4.23}$$

$K_a \uparrow$ $S \downarrow$ better sys

If K_a is large, the sensitivity is low. For example, if

$$K_a = 10^4$$

and

$$\beta = 0.1, \tag{4.24}$$

we have

$$S^T_{K_a} = \frac{1}{1 + 10^3}, \tag{4.25}$$

Note:

Uncertainties in the system model might come from : Aging , changing environment, and ignorance of exact values of the system parameters which all affect the control process.

- In open-loop : inaccurate output result from these effects.
- In closed-loop: the system attempts to compensate and correct for these effects by use of the controller

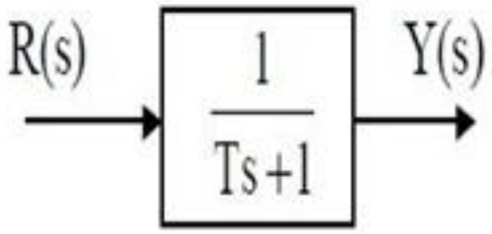
Sensitivity is reduced in closed-loop by increasing $G_C(s)G(s)$ of the system compared to the open-loop case where $S=1$ in the Case of $T(s)=G(s)$

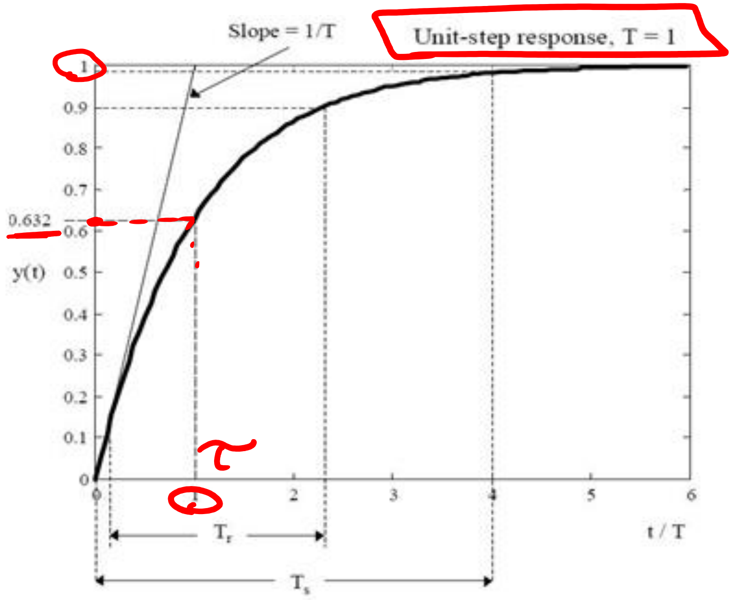
CONTROL OF THE TRANSIENT RESPONSE

$T(s) = \frac{1}{\tau s + 1} \Rightarrow$ First order sys.

τ = time constant.

$\Leftarrow 0.632 y_{ss}$



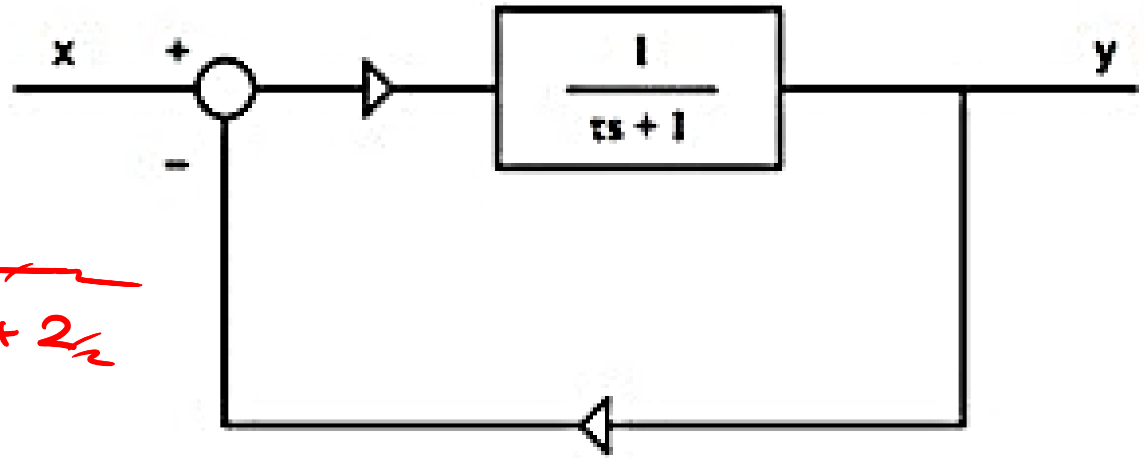


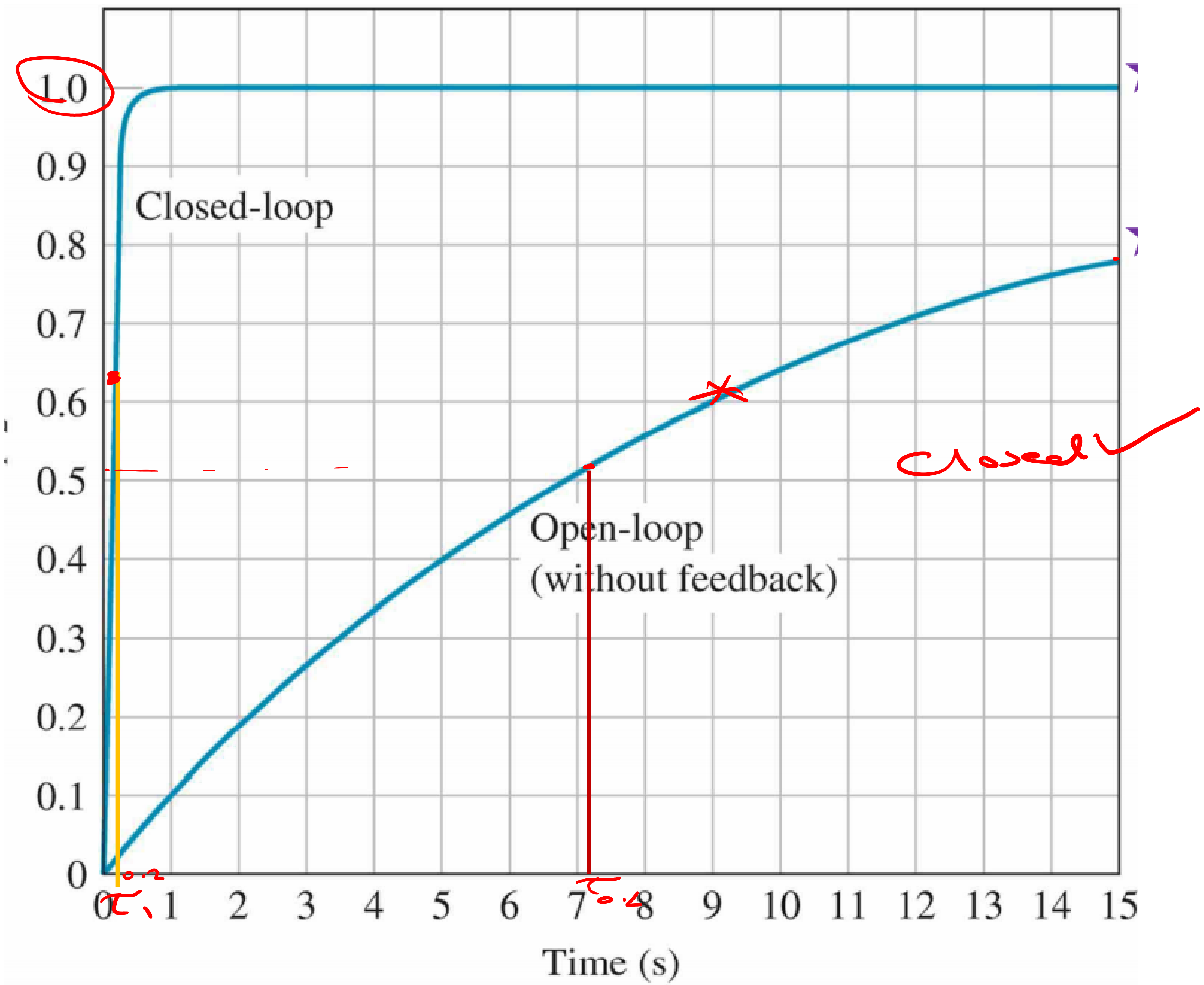
Closed loop

$$T(s) = \frac{\frac{1}{\tau s + 1}}{1 + \frac{1}{\tau s + 1}}$$

$$\frac{1}{\tau s + 1 + 1} = \frac{1}{\frac{\tau s}{2} + 2/2}$$

Time cons $= \frac{\tau}{2}$ ✓



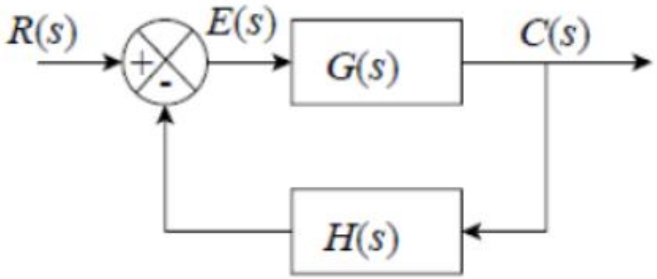


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
Closed-loop
Open-loop
(without feedback)
0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
Time (s)
Closed

1. Steady State Error
2. Stability

Steady-state Error Analysis

For the feedback system shown in block diagram below, the transfer function is given by:



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

The system error is given by:

$$\begin{aligned} E(s) &= R(s) - C(s)H(s) \\ &= \left[1 - \frac{G(s)H(s)}{1+G(s)H(s)}\right] R(s) \\ &= \frac{1}{1+G(s)H(s)} R(s) \end{aligned}$$

This last expression shows that the loop gain *G(s)H(s)* determine the amount and nature of the steady state error of a system.

The loop gain *G(s)H(s) can be expressed in the general form;*

$$G(s)H(s) = \frac{K(s+z_1)(s+z_2)(s+z_3)\cdots\cdots(s+z_m)}{s^N(s+p_1)(s+p_2)(s+p_3)\cdots\cdots(s+p_n)}$$

$$= \frac{K\prod_{i=1}^{i=m}(s+z_i)}{s^N \prod_{j=1}^{i=n}(s+p_j)}$$

The error in this case would be given by:

$$E(s) = \frac{1}{1+G(s)H(s)}\ R(s)$$

$$= \frac{s^N \prod_{j=1}^{i=n}(s+p_j)}{s^N \prod_{j=1}^{i=n}(s+p_j) + K\prod_{i=1}^{i=m}(s+z_i)}\ R(s)$$

The steady state error is calculated as follows:

$$e_{ss} = \lim_{s\to 0}\left[s \frac{s^N \prod_{j=1}^{i=n}(s+p_j)}{s^N \prod_{j=1}^{i=n}(s+p_j) + K\prod_{i=1}^{i=m}(s+z_i)} R(s)\right]$$

- When the standard test signals of a step *(A/s), a ramp (A/s2), and an acceleration (A/s3) are* used, the Laplace operator "*s" in the input test signal denominator will cancel or reduce* from the power of "*s" in the numerator of the expression above.

- The power of "*s" (the poles of the G(s)H(s) located on the origin of s-plane), i.e. N, determines the* steady state error response of the system when subjected to standard test signals, and is called the "**type number" of the system.**

- For *N = 0, the system is a type zero, for N = 1, the system is a type one, and so on.*

Type Zero System:

The steady state error for a step input; *A/s is given by*

$$e_{ss} = \lim_{s\to 0}\left[\cancel{s}\frac{\prod_{j=1}^{i=n}(s+p_j)}{\prod_{j=1}^{i=n}(s+p_j)+K\prod_{i=1}^{i=m}(s+z_i)}\frac{A}{\cancel{s}}\right]$$

$$= \frac{\prod_{j=1}^{i=n}p_j}{\prod_{j=1}^{i=n}p_j+K\prod_{i=1}^{i=m}z_i}A$$

$$= \frac{A}{1+K_p} \qquad \text{for } K_p = \frac{K\prod_{i=1}^{i=m}z_i}{\prod_{j=1}^{i=n}p_j}$$

Position error constant

$$K_p = \lim_{s\to 0} G(s)H(s)$$

The steady state error for a ramp input; *A/s2 is given by*

$$
\begin{aligned}
e_{ss} &= \lim_{s\to 0}\left[\cancel{s}\,\frac{\prod_{j=1}^{i=n}(s+p_j)}{\prod_{j=1}^{i=n}(s+p_j)+K\prod_{i=1}^{i=m}(s+z_i)}\;\frac{A}{s^{\cancel{2}}}\right] \\
&= \infty
\end{aligned}
$$

Type One System:

The steady state error for a step input; *A/s is given by*

$$
\begin{aligned}
e_{ss} &= \lim_{s\to 0}\left[\cancel{s}\,\frac{s\prod_{j=1}^{i=n}(s+p_j)}{s\prod_{j=1}^{i=n}(s+p_j)+K\prod_{i=1}^{i=m}(s+z_i)}\;\frac{A}{\cancel{s}}\right] \\
&= 0
\end{aligned}
$$

TYPE	STEP INPUT $R(s)=A/s$	RAMP INPUT $R(s)=A/s^2$	ACCELERATION INPUT $R(s)=A/s^3$
0	$\frac{A}{1+K_p}$	∞	∞
1			
2			

The steady state error for a ramp input; *A/s2 is given by*

$$e_{ss} = \lim_{s\to 0}\left[\cancel{s}\frac{\cancel{s}\prod_{j=1}^{i=n}(s+p_j)}{s\prod_{j=1}^{i=n}(s+p_j)+K\prod_{i=1}^{i=m}(s+z_i)}\frac{A}{\cancel{s^2}}\right]$$

$$= \frac{\prod_{j=1}^{i=n} p_j}{K\prod_{i=1}^{i=m} z_i} A$$

$$= \frac{A}{K_v} \qquad \text{for } K_v = \frac{K\prod_{i=1}^{i=m} z_i}{\prod_{j=1}^{i=n} p_j}$$

Velocity error constant

$$K_v = \lim_{s\to 0} sG(s)H(s)$$

The steady state error for an acceleration input; *A/s3 is given by*

$$e_{ss} = \lim_{s\to 0}\left[\cancel{s}\frac{\cancel{s}\prod_{j=1}^{i=n}(s+p_j)}{s\prod_{j=1}^{i=n}(s+p_j)+K\prod_{i=1}^{i=m}(s+z_i)}\frac{A}{\cancel{s^3}}\right]$$

$$= \infty$$

TYPE	STEP INPUT $R(s)=A/s$	RAMP INPUT $R(s)=A/s^2$	ACCELERATION INPUT $R(s)=A/s^3$
0	$\frac{A}{1+K_p}$	∞	∞
1	0	$\frac{A}{K_v}$	∞
2			

Type Two System:

The steady state error for a step input; *A/s is given by*

$$e_{ss} = \lim_{s\to 0}\left[\cancel{s}\,\frac{s^2\prod_{j=1}^{i=n}(s+p_j)}{s^2\prod_{j=1}^{i=n}(s+p_j)+K\prod_{i=1}^{i=m}(s+z_i)}\,\frac{A}{\cancel{s}}\right]$$

$$= 0$$

A/S^2 → ess=0

The steady state error for an acceleration input; *A/s3* is given by

$$e_{ss} = \lim_{s\to 0}\left[\cancel{s}\,\frac{\cancel{s^2}\prod_{j=1}^{i=n}(s+p_j)}{s^2\prod_{j=1}^{i=n}(s+p_j)+K\prod_{i=1}^{i=m}(s+z_i)}\,\frac{A}{\cancel{s^3}}\right]$$

$$= \frac{\prod_{j=1}^{i=n} p_j}{K\prod_{i=1}^{i=m} z_i}\,A$$

$$= \frac{A}{K_a} \qquad \text{for } K_a = \frac{K\prod_{i=1}^{i=m} z_i}{\prod_{j=1}^{i=n} p_j}$$

Acceleration error constant

$$K_a = \lim_{s\to 0} s^2 G(s)H(s)$$

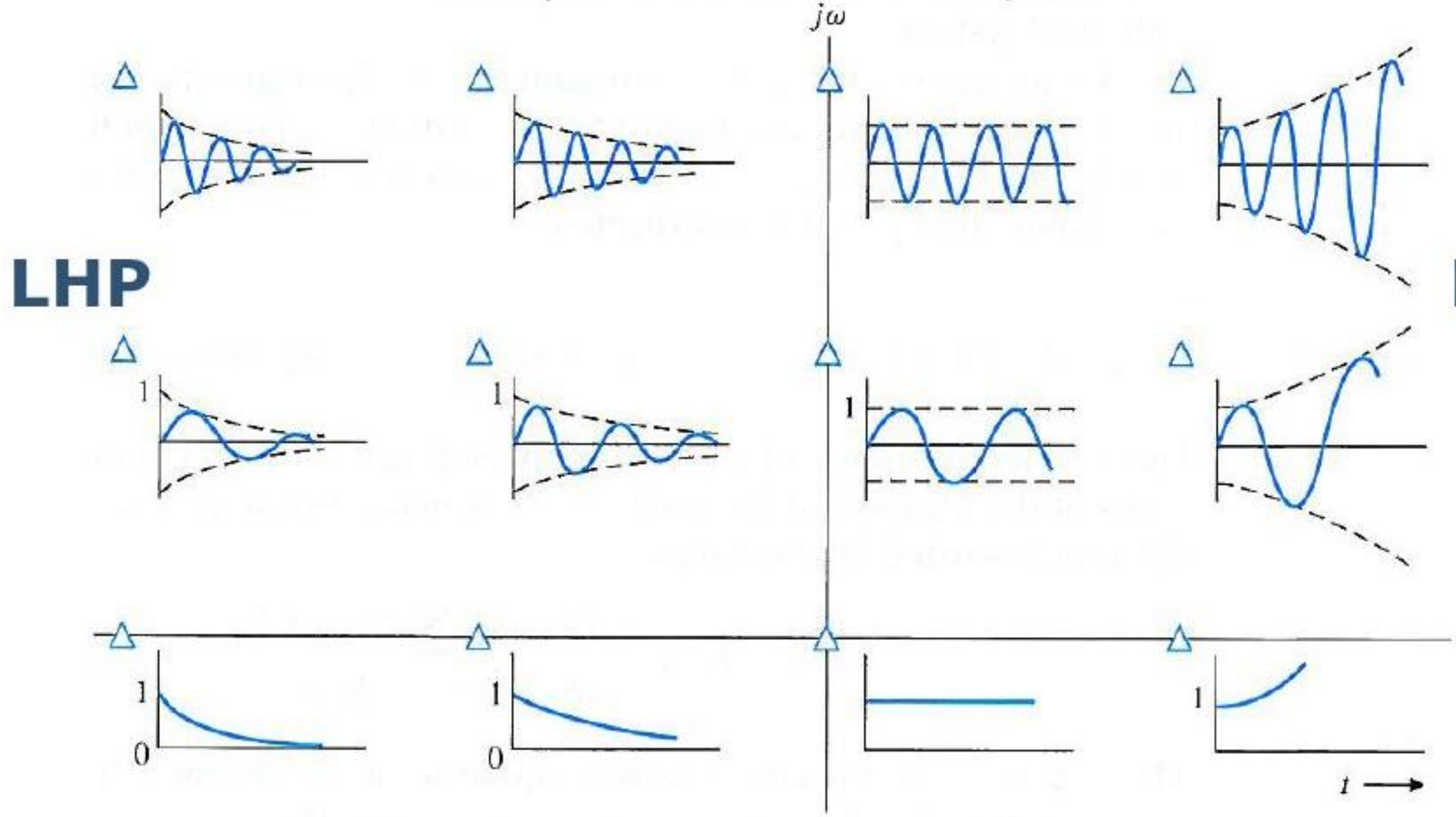
In summary;

$$\text{For } K_p = K_v = K_a = \frac{K \prod_{i=1}^{i=m} z_i}{\prod_{j=1}^{i=n} p_j}$$

TYPE	STEP INPUT $R(s)=A/s$	RAMP INPUT $R(s)=A/s^2$	ACCELERATION INPUT $R(s)=A/s^3$
0	$\frac{A}{1+K_p}$	∞	∞
1	0	$\frac{A}{K_v}$	∞
2	0	0	$\frac{A}{K_a}$

Effect of Pole Locations

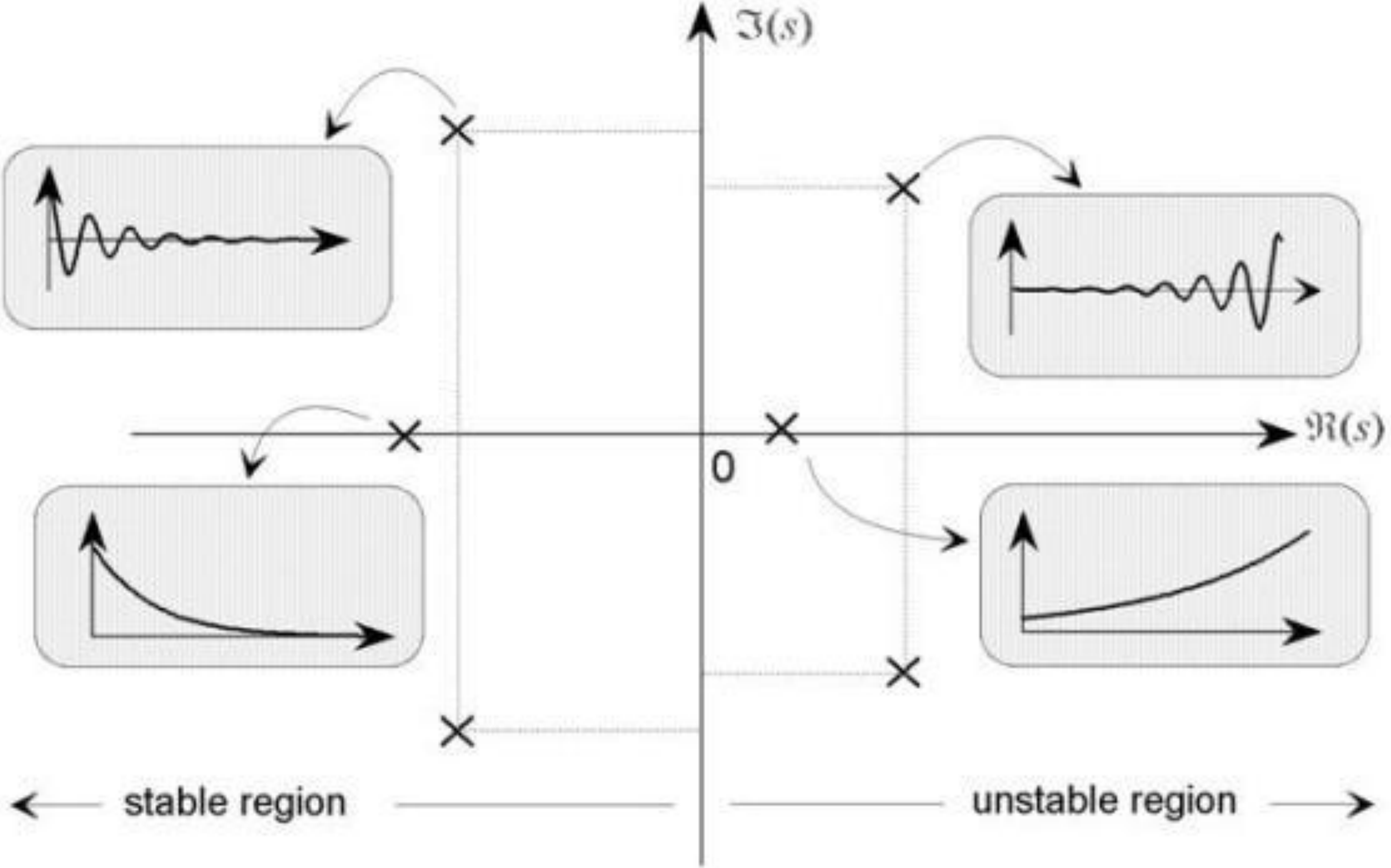
Time function of impulse response assosiated with the pole location in s-plane



LHP : left half-plane
RHP : right half-plane

Stability

- A BIBO (bounded-input bounded-output) stable system is a system for which the outputs will remain bounded for all time, for any finite initial condition and input. A continuous-time linear time-invariant system is BIBO stable if and only if all the poles of the system have real parts less than 0.



$\Im(s)$
$\Re(s)$
0
stable region
unstable region

Find if the systems stable or not

G(s)= $\frac{s+3}{(s+5)(s+2)}$

G(s)= $\frac{s+3}{(s+5)(s-2)}$

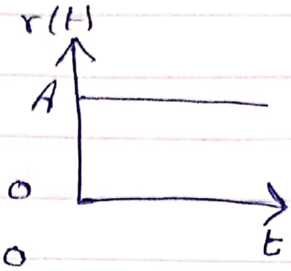
CH5: The performance of FB control systems:-

Test signals:-

① Step test signal:-

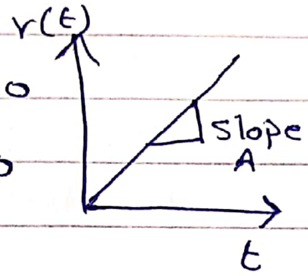
$$R(s) = \frac{A}{s}$$

$$r(t) = \begin{cases} A & t \geq 0 \\ 0 & t < 0 \end{cases}$$



② Ramp test signal

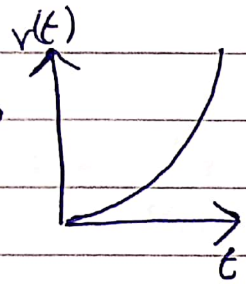
$$r(t) = \begin{cases} At & t > 0 \\ 0 & t < 0 \end{cases}$$



$$R(s) = \frac{A}{s^2}$$

③ parabolic T.S:-

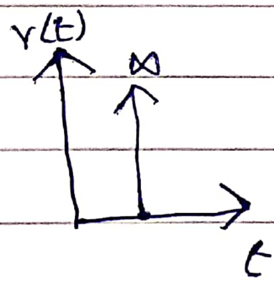
$$r(t) = \begin{cases} At^2 & t > 0 \\ 0 & t < 0 \end{cases}$$



$$R(s) = \frac{2A}{s^3}$$

④ Impulse T.S:-

$$R(s) = 1$$



Scanned with CamScanner

System order :-

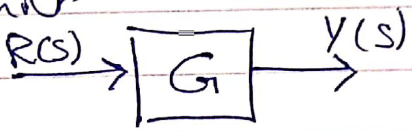
$$\frac{Y(s)}{R(s)} = \frac{A}{as^{2}+bs+c} \rightarrow \text{charac. eq.}$$

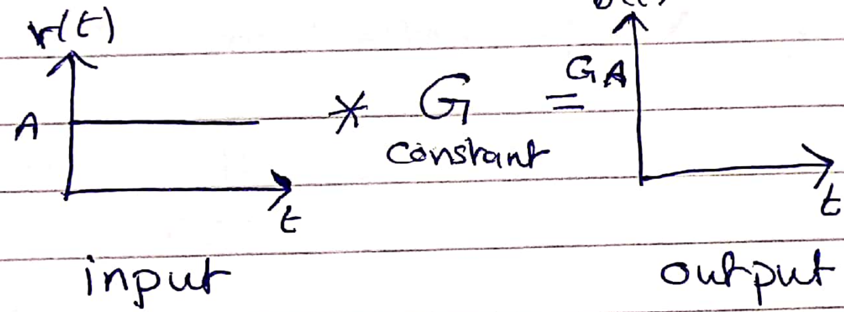
s^0

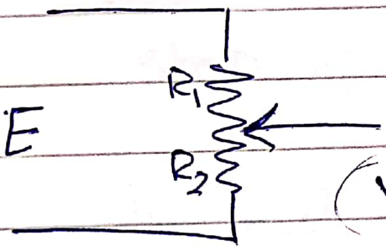
Zero order system

$GS^0 \Rightarrow$ constant

$$\frac{Y(s)}{R(s)} = G$$







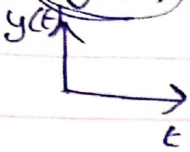
$$V_{out} = \frac{R_2}{R_1+R_2} E$$

$$\frac{V_{out}(s)}{E(s)} = \frac{R_2}{R_1+R_2} \quad X$$

s^0

First order system:-

general T.F
standard test signal
general $y(t)$



general T.F

$$\frac{Y(s)}{R(s)} = \frac{K}{\tau s+1}$$

τ: time constant $\Rightarrow 0.632\, y_{ss}$

$$Y(s) = \frac{K}{\tau s+1} * R(s)$$

$$y(t) = \mathcal{L}^{-1}(Y(s))$$

if $R(s)$ step input $R(s) = \frac{A}{s}$

$$Y(s) = \frac{K}{\tau s+1} * \frac{A}{s} = \frac{AK}{s(\tau s+1)} = \frac{A_1}{s} + \frac{A_2}{\tau s+1}$$

0 $\quad -\frac{1}{\tau}$

$$\boxed{y(t) = AK\left(1 - e^{-t/\tau}\right)}$$

general response first order sys
input $\frac{A}{s}$

general T.F 1^{st} order $\frac{Y(s)}{R(s)} = \frac{K}{\tau s+1}$ ✓

general response $\frac{A}{s}$ $\Rightarrow y(t) = AK(1-e^{-t/\tau})$ ✓
or ~~unit~~ step response

$$y_{ss} = \lim_{t\to\infty} AK(1-e^{-\infty/\tau}) = AK$$

$$\boxed{y_{ss} = AK}$$

$$y(\tau) = AK(1-e^{-\tau/\tau}) = 0.632\,AK = 0.632\,y_{ss}$$

$$y(4\tau) = AK(1-e^{-4\tau/\tau}) \simeq 0.98\,AK = 0.98\,y_{ss}$$

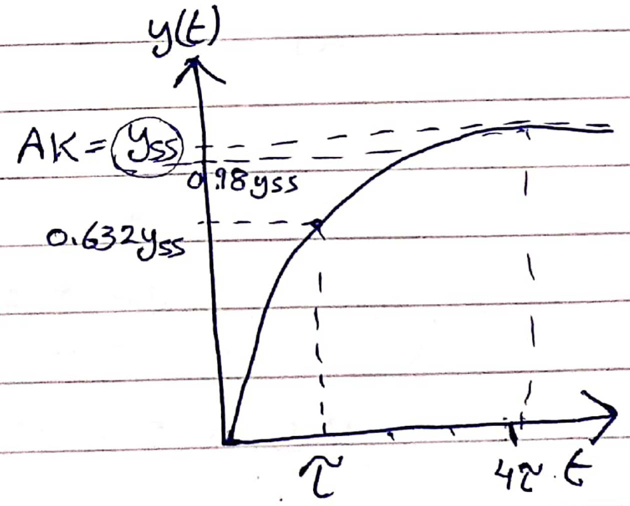
T_s : settling time

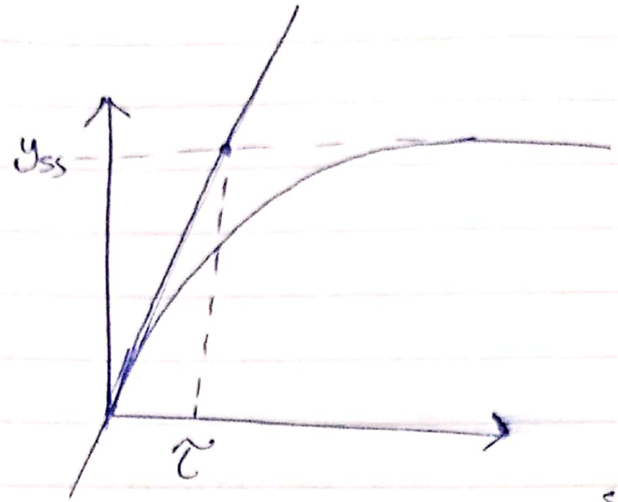
$T_s = 4\tau$ 2% error

$$E_{ss} = \underset{R_{ss}}{\cancel{y_{ss}}} - 0.98\,y_{ss} \Rightarrow 2\%$$

$T_s = 3\tau$ 5%

$T_s = 5\tau$ 1%





slope $\neq 0$

general step response of 1^{st} order sys.

$$\text{slope} = \frac{y_{ss} - 0}{\tau - 0} = \frac{y_{ss}}{\tau}$$

$R(s) = 1$

$y_{ss} = 1$

$$\text{slope} = \frac{1}{\tau} \neq 0$$

$$\frac{Y(s)}{R(s)} = \frac{2}{S+7} \cdot \frac{1}{S} \; ; \quad R(s) = \frac{1}{S}$$

$=1$

Find τ ?!

$$\frac{2/7}{\frac{S}{7} + \frac{7}{7}}$$

$$\tau = \frac{1}{7}$$

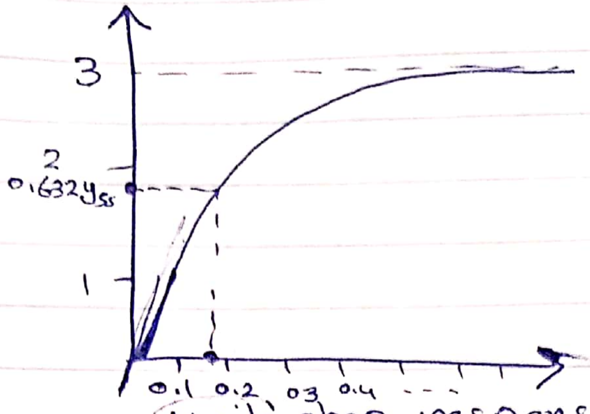
$y(t) = AK(1 - e^{-t/\tau})$

$1 * \frac{2}{7} (1 - e^{-t/\tau})$

$$y_{ss} = \lim_{S \to 0} \cancel{S} \cdot \frac{2}{S+7} \cdot \frac{1}{\cancel{S}} = \frac{2}{7}$$

$$y(\tau) = 0.632\, y_{ss} = 0.632 * \frac{2}{7}$$

$$T_s = 4\tau = 4 * \frac{1}{7}$$



unit step response
1^{st} order.

$\tau \Rightarrow$

$3 * 0.632 = 1.896$

$\tau = 0.18$ sec

T.F $\dfrac{K}{\tau s+1}$

$y_{ss} = A K$ $\quad R(s) = \dfrac{A}{s} \Rightarrow A = 1$

unit step

$y_{ss} = K$

$3 = K$

$$\frac{Y(s)}{R(s)} = \frac{3}{0.18s+1}$$

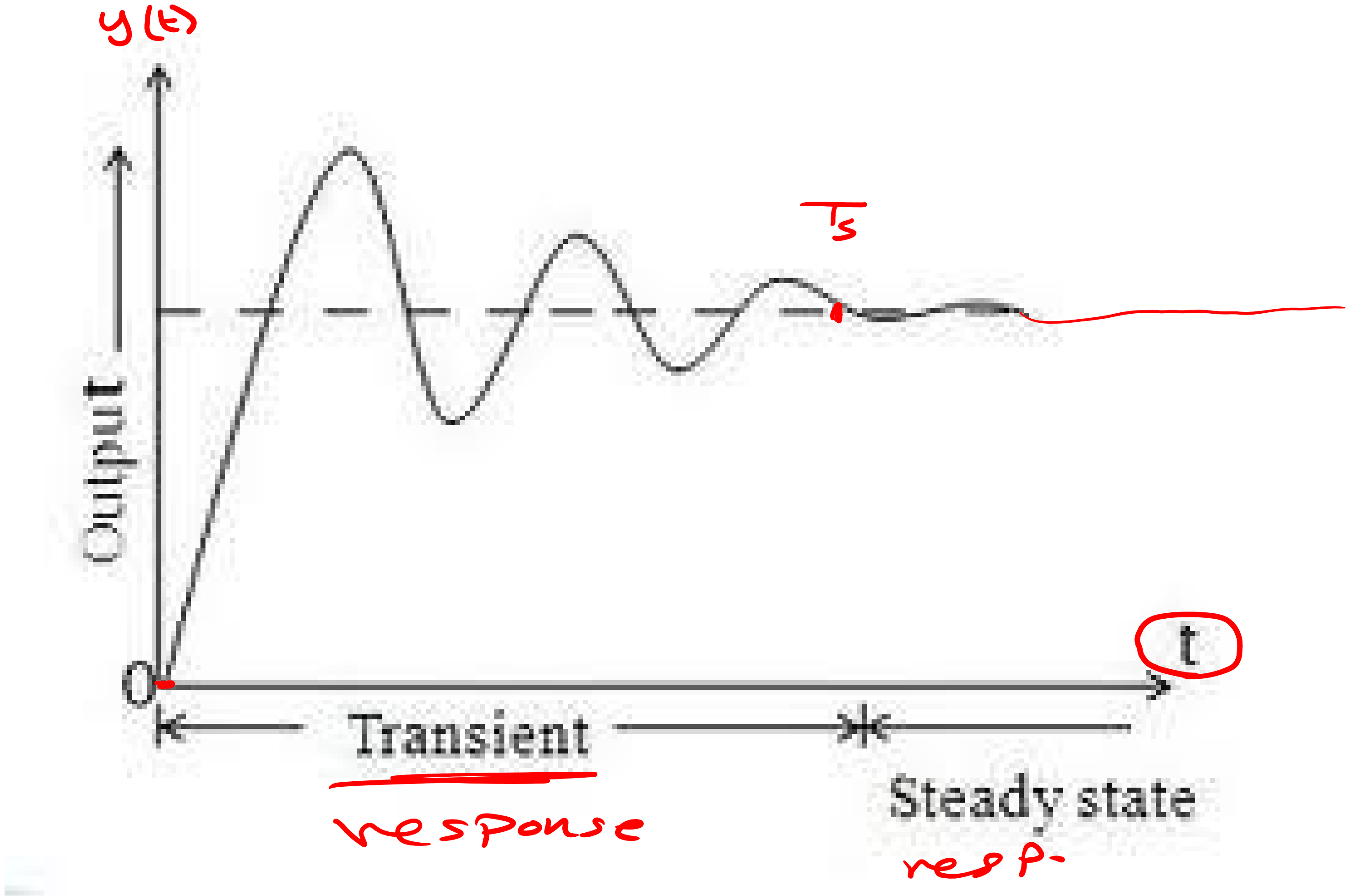
$$y(t) = 3\left(1 - e^{-t/0.18}\right)$$

$T_s = 4\tau = 4 * 0.18 =$ sec

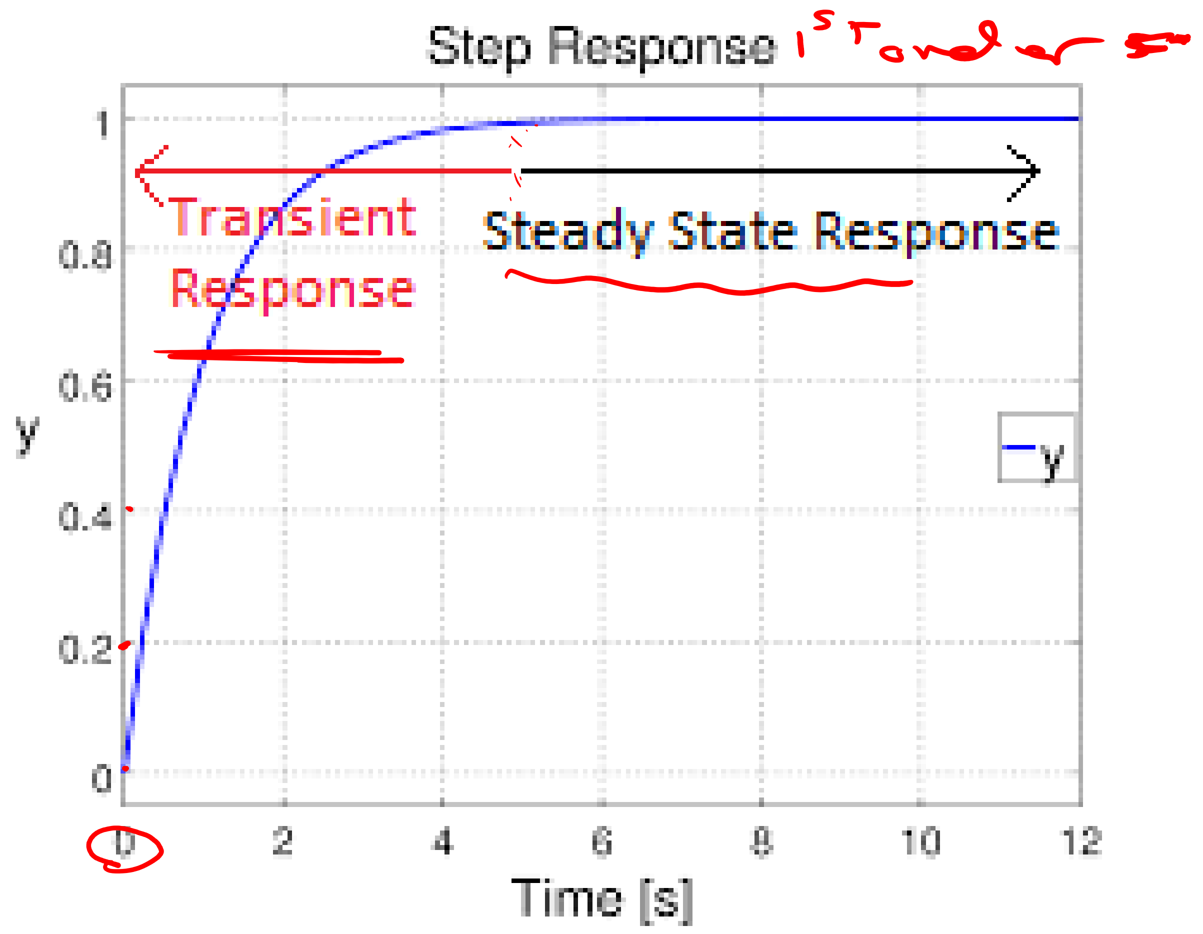
Second Order Response

Eng. Fadwa Momani

Review Of Transient and Steady state Response



Review Of Transient an Steady state Response



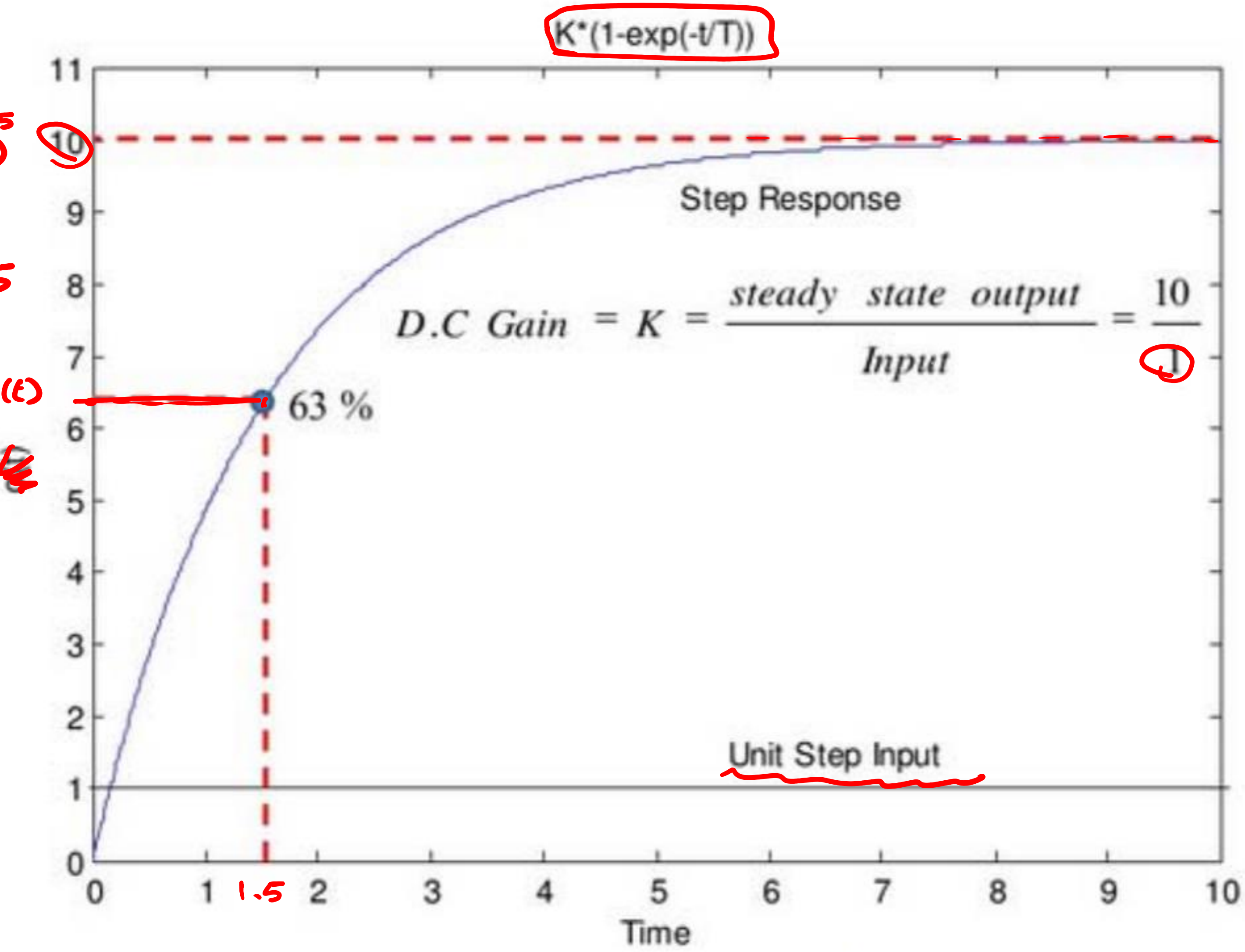
Review of First Order Response (unit Step Response)

$y(t) = AK(1 - e^{-t/\tau})$

$y(t) = 10(1 - e^{-t/1.5})$

$T_s = 4 * \tau = 4 * 1.5$

6 sec.



The step response specification of first order system

$\tau \rightarrow$

$y_{ss} * 0.632$

$y(\tau) = 6.32$

$10 * 0.632$

$\tau \simeq 1.5$ sec.

$K = \frac{y_{ss}}{R_{ss}} = \frac{10}{1}$

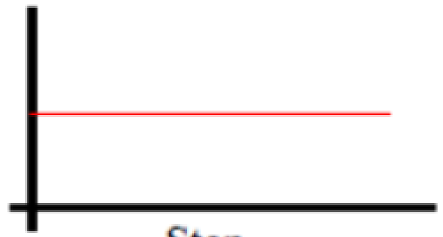
$= 10$

$T(s) = \frac{AK}{\tau s + 1}$

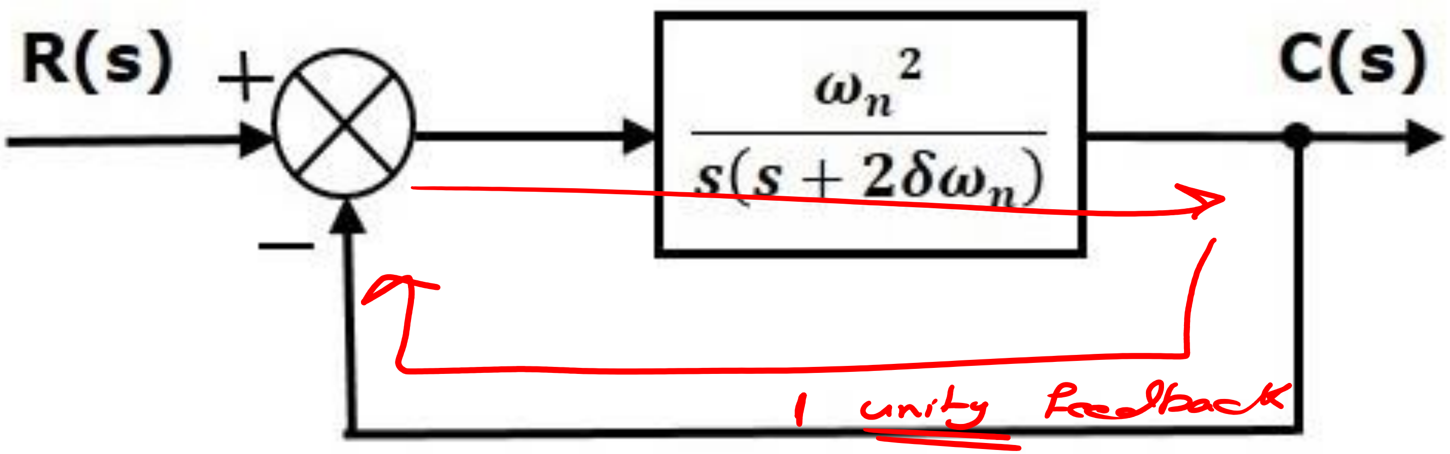
$T(s) = \frac{10}{1.5s + 1}$

First order system response for different input signals:

DIFFERENTIATION

GRAPHICAL REPRESENTATION	INPUT	OUTPUT
 unit ramp. Ramp	At $r(t) = t$	$c(t) = t - T + Te^{-t/T}$
 Step	$r(t) = 1$	$c(t) = 1 - e^{-t/T}$
 Impulse	$r(t) = \begin{cases} 1 & for\ t = 0 \\ 0 & for\ t \neq 0 \end{cases}$	$c(t) = \frac{1}{T} e^{-t/T}$

Second Order System (S^2 ...)



$$\frac{\frac{\omega_n^2}{s(s+2\xi\omega_n)}}{1+\frac{\omega_n^2}{s(s+2\xi\omega_n)}\times 1}$$

$$\boxed{G(s)=\frac{\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}}$$

general T.F 2^{nd} order sys.

ω_n : natural Freq.

ξ : damping ratio

Second Order System (S^2 …)

ω_n ζ $\boxed{\text{Input}}$

- Second-order systems exhibit a wide range of responses which must be analyzed and described.

τ

- For a first-order system, varying a single parameter changes the speed of response, Changes in the parameters of a second order system can change the form of the response not only the speed of the response.
- For example: a second-order system can display characteristics much like a first-order system or, depending on the system's parameters values, pure oscillations or damped behavior might result for its transient response.

Second Order System

general T.F

standard Input

general response

- General Transfer Function of 2nd Order System:

$$G(s) = \frac{c}{1 s^2 + ds + e} = K \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} \quad (*)$$

$c = K\omega_n^2 \Rightarrow K = \frac{c}{e}$

$\omega_n^2 = e$

$\omega_n = \sqrt{e}$

$d = 2\xi\omega_n$

$\xi = \frac{d}{2\sqrt{e}}$

Where:

$$\omega_n = \sqrt{e}, \quad \text{the natural (or resonant) frequency (rad/sec)},$$

$$\zeta = \frac{d}{2\sqrt{e}}, \quad \text{the damping ration (unitless), and}$$

$$K = \frac{c}{e}, \quad \text{the gain (same units as y/x)}.$$

Second Order System

- Characteristic Equation

2 poles

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

*

$$s_1, s_2 = \frac{-2\zeta\omega_n \pm \sqrt{(2\zeta\omega_n)^2 - 4\omega_n^2}}{2} = \boxed{-\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}}$$

$\varepsilon \Rightarrow$

Second Order System

The value of ζ determines 4 cases of interest that are given special names (whose origin will soon be apparent):

*

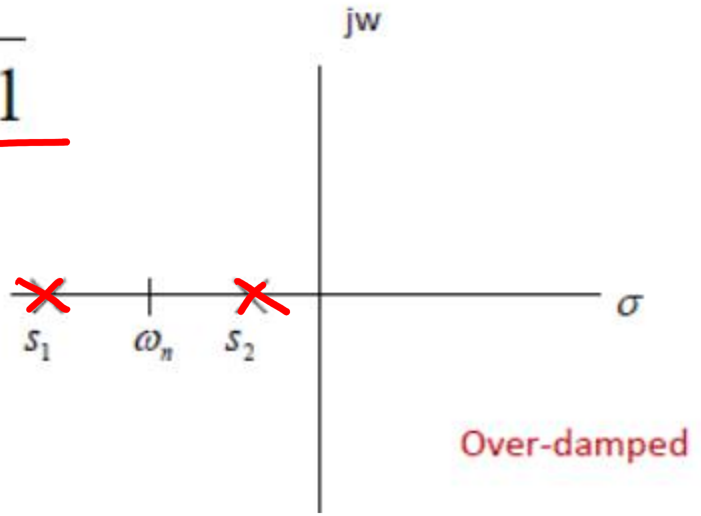
Name	Value of ζ	Roots of s	Characteristics of "s"
Overdamped	$\zeta>1$	$s=-\zeta\omega_0 \pm \omega_0\sqrt{\zeta^2-1}$	Two real and negative roots
Critically Damped	$\zeta=1$	$s = -\omega_0$	A single (repeated) negative root
Underdamped	$0<\zeta<1$	$s=-\zeta\omega_0 \pm j\omega_0\sqrt{1-\zeta^2}$	Complex conjugate ($j = \sqrt{-1}$);
Undamped	$\zeta=0$	$s = \pm j\omega_0$	Pure imaginary (no real part)

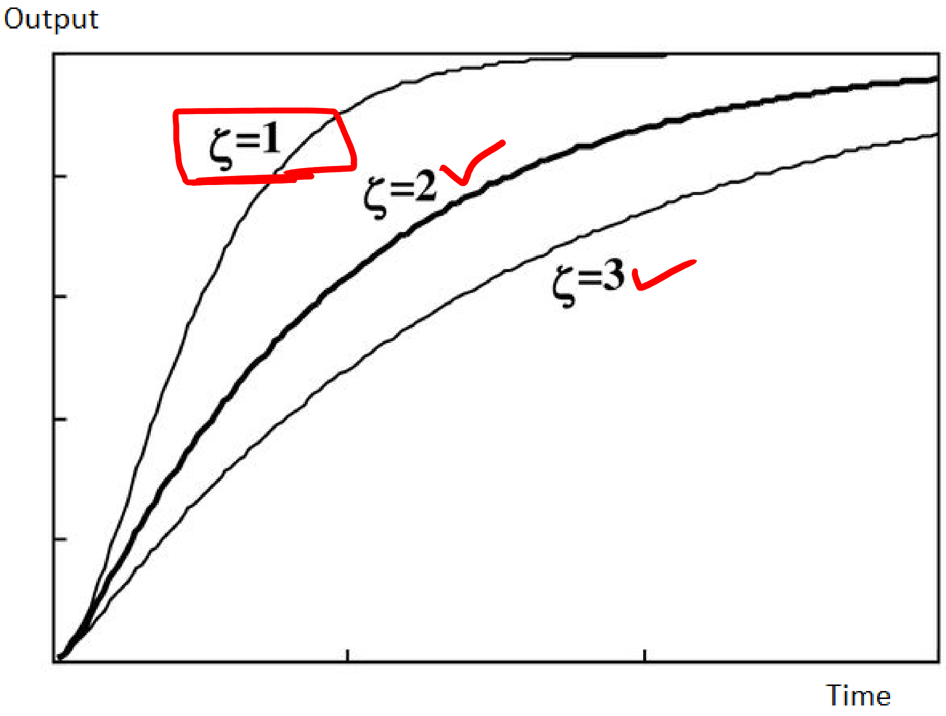
Case I: Over damped case (Stable) $\xi > 1$

Unit step input

Real different poles $s_{1,2} = -\xi\omega_n \pm \omega_n\sqrt{\xi^2 - 1}$

$$y(t) = 1 + \frac{\omega_n}{2\sqrt{\xi - 1}}\left(\frac{e^{-s_1 t}}{s_1} - \frac{e^{-s_2 t}}{s_2}\right)$$





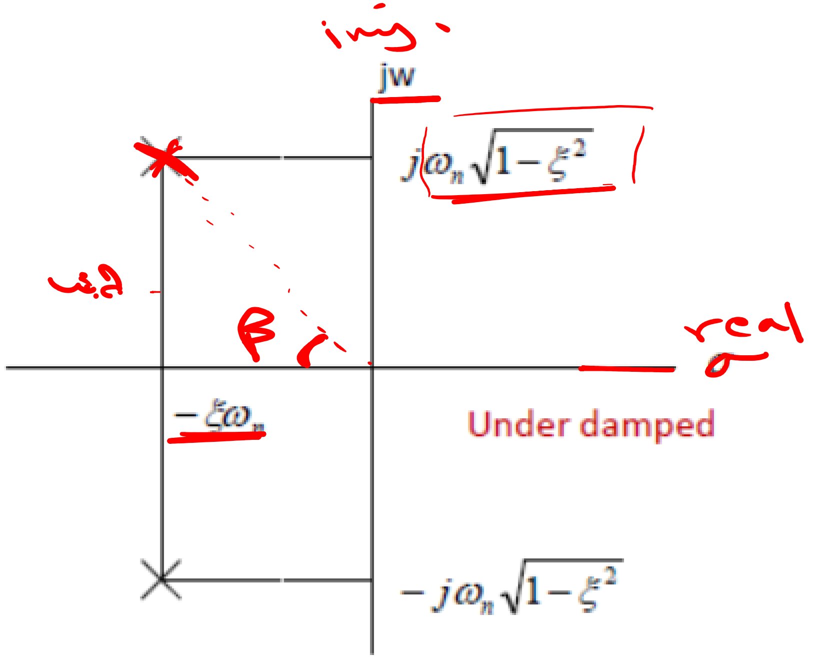
unit step response for 2nd order sys if $\xi > 1$

$-\xi\omega_n \mp \omega_n\sqrt{\xi^2-1}$

R(s)

Case II: Underdamped case (Stable) $0 < \xi < 1$ Unit step input

Complex poles $s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$



$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}}\sin(\omega_d t + \beta)$$

$$\text{where} \quad \beta = \tan^{-1}\frac{\omega_d}{\xi\omega_n} = \tan^{-1}\frac{\cancel{\omega_n}\sqrt{1-\xi^2}}{\xi\cancel{\omega_n}}$$

$\omega_d = \omega_n\sqrt{1-\xi^2}$: is the damped natural frequency

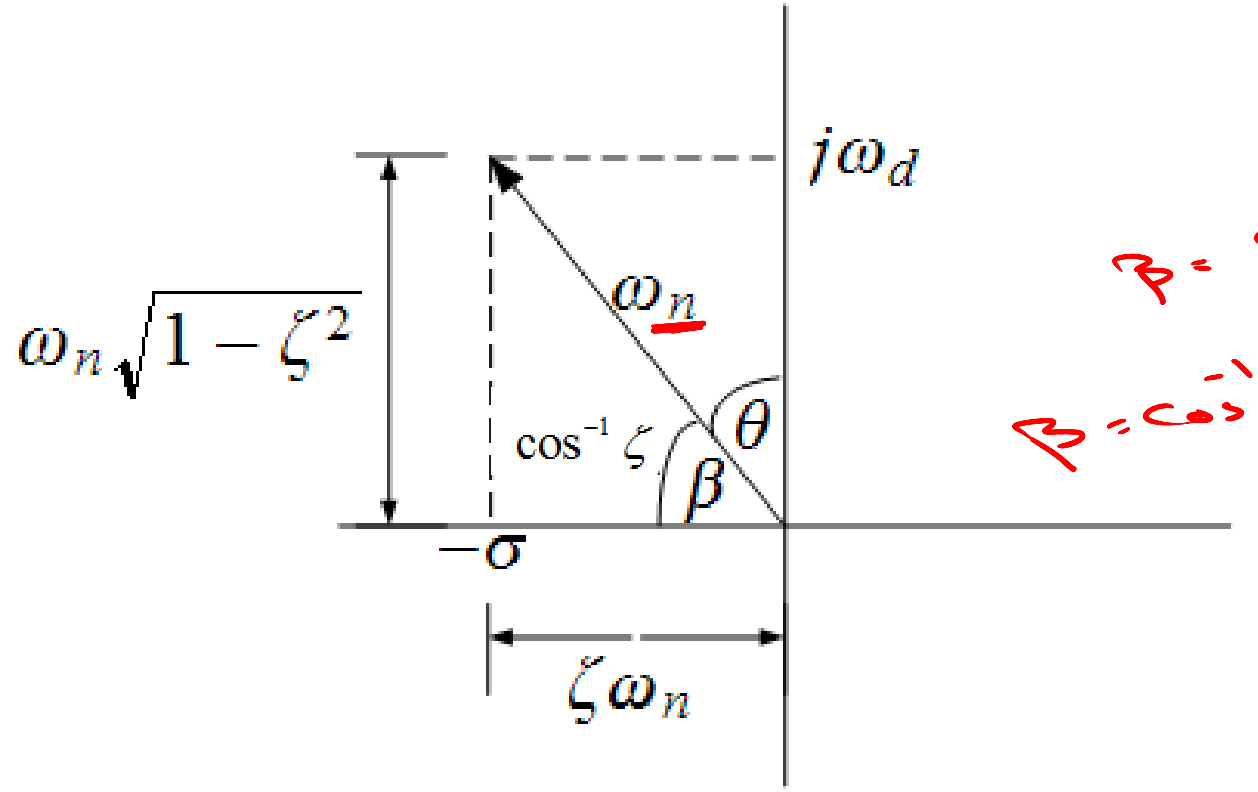
Eg

$$\beta = \tan^{-1}\frac{\sqrt{1-\xi^2}}{\xi}$$

$$s_{1,2} = -\xi\omega_n \pm j\omega_n\sqrt{1-\xi^2}$$

$$\zeta = \sin\theta$$

$$s = -\sigma + j\omega_d$$

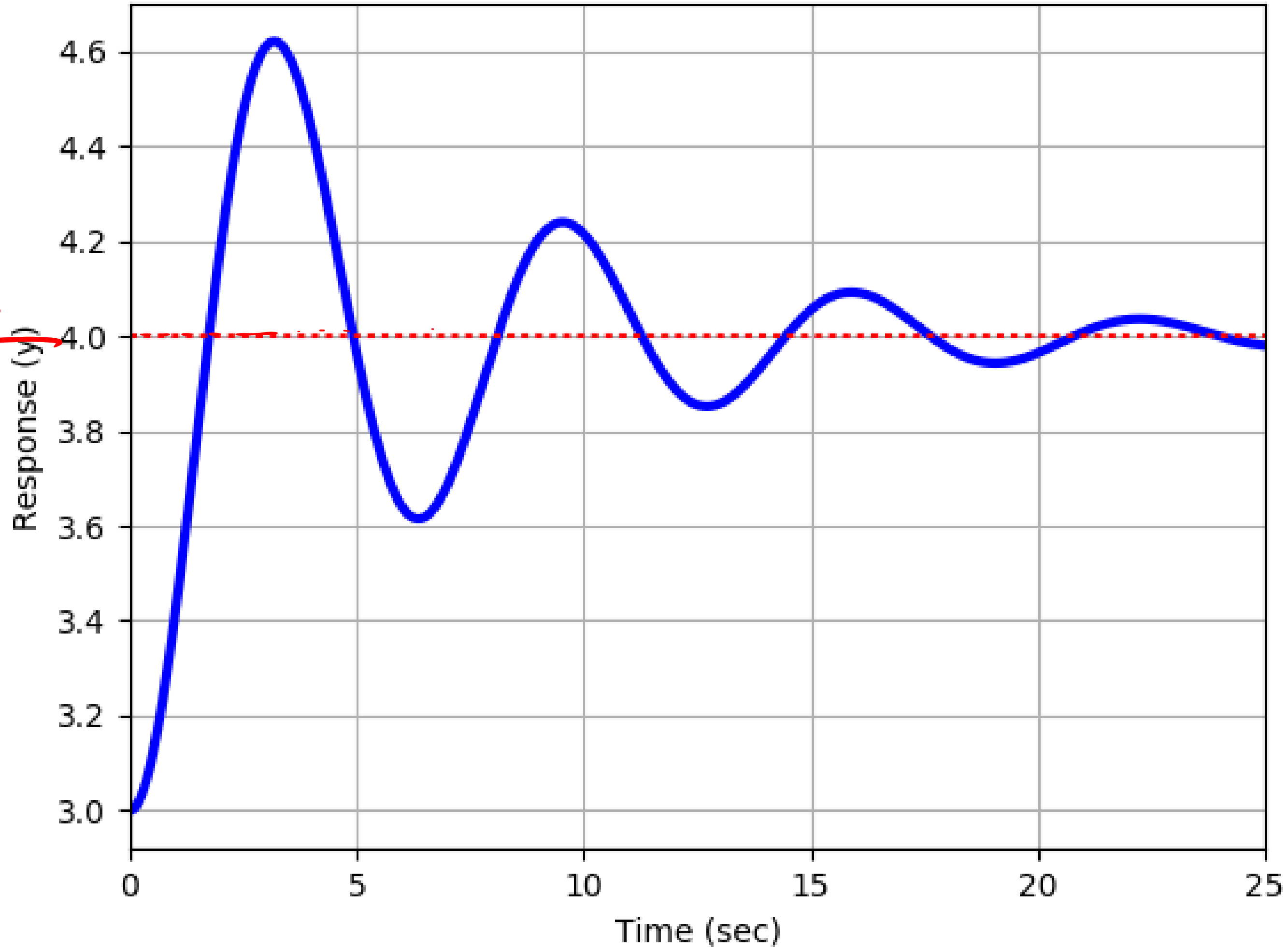


$\beta = \cos^{-1}\xi$

$\beta = \cos^{-1}\dfrac{\xi\omega_n}{\omega_n}$

Dominant pole parameters

$R(s) = \frac{4}{s}$



step response
2nd order sy

$0 < \varepsilon < 1$

underdamped
response.

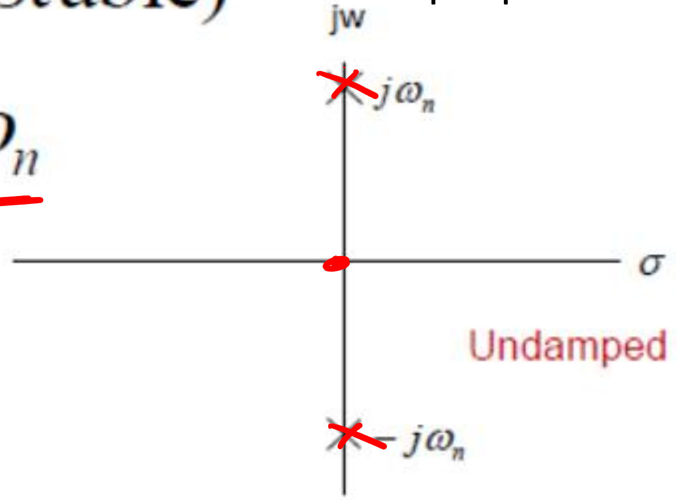
Case III:

$= -\xi\omega_n \pm \omega_n\sqrt{\xi^2-1} = \pm j\omega$

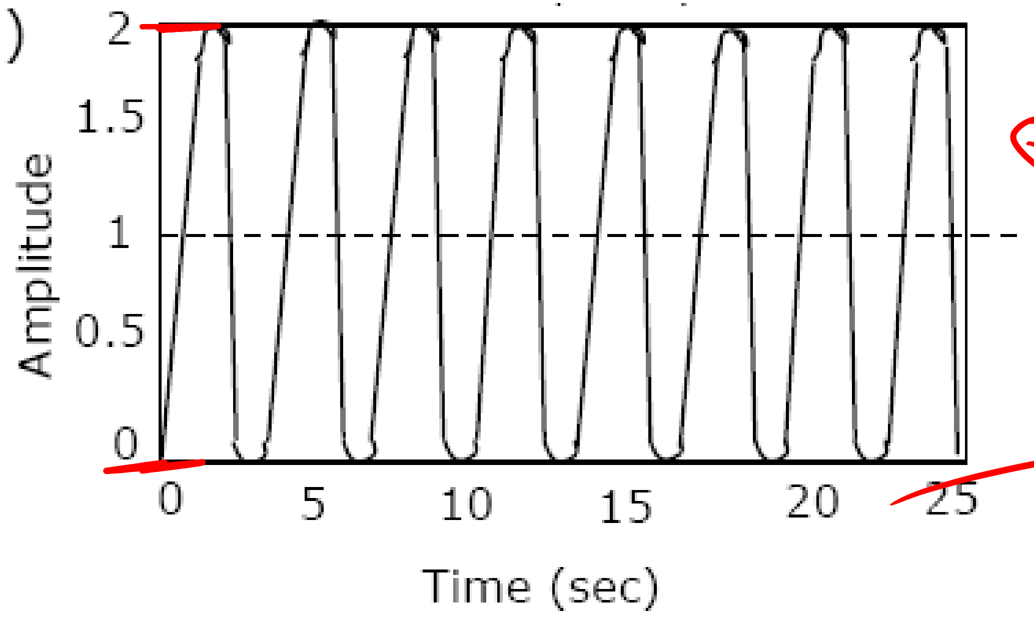
Zero damped (*Undamped*) case (Stable)

Unit step input

$\xi = 0$, Imaginary poles $s_{1,2} = \pm j\omega_n$



$y(t) = 1 - \cos(\omega_n t)$



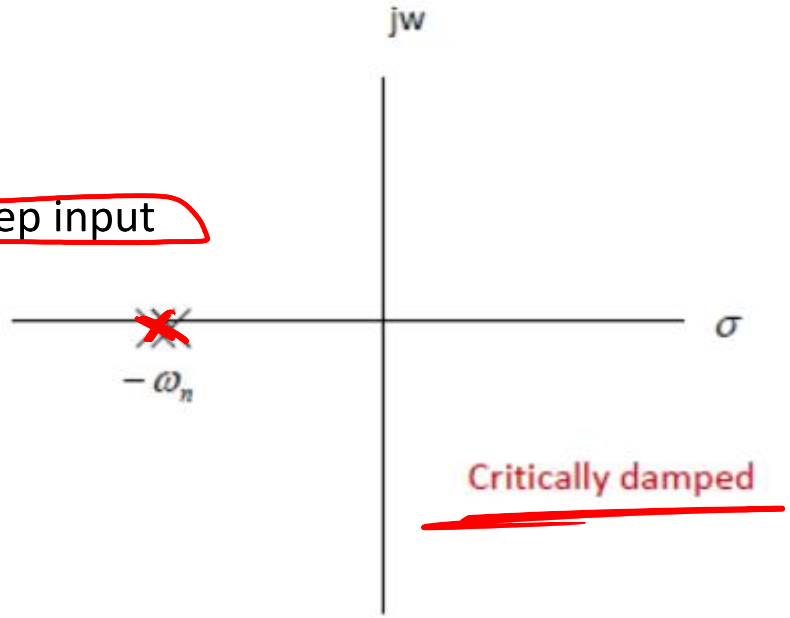
stable :- oscilatory

$s = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2-1}$

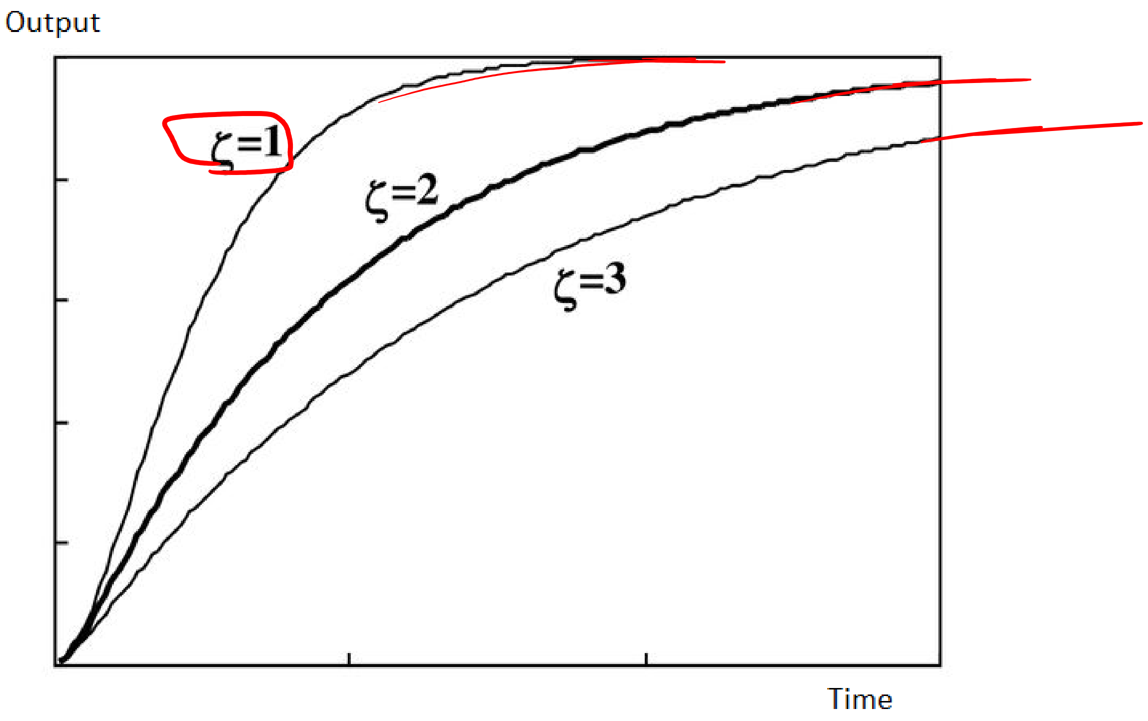
Case IV:

Critically damped (Stable) $\xi = 1$ Unit step input

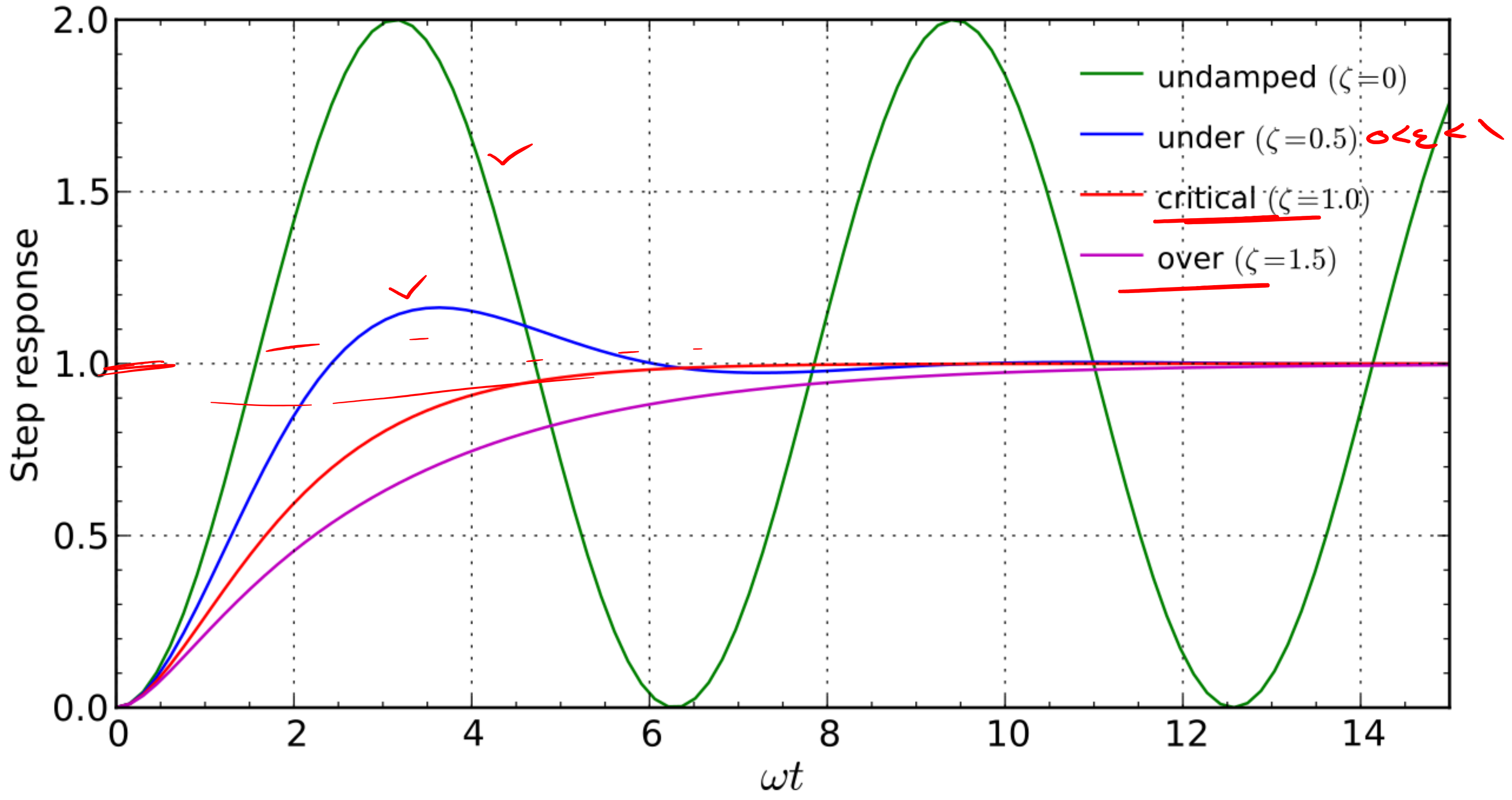
Real equall double pole $-\omega_n$



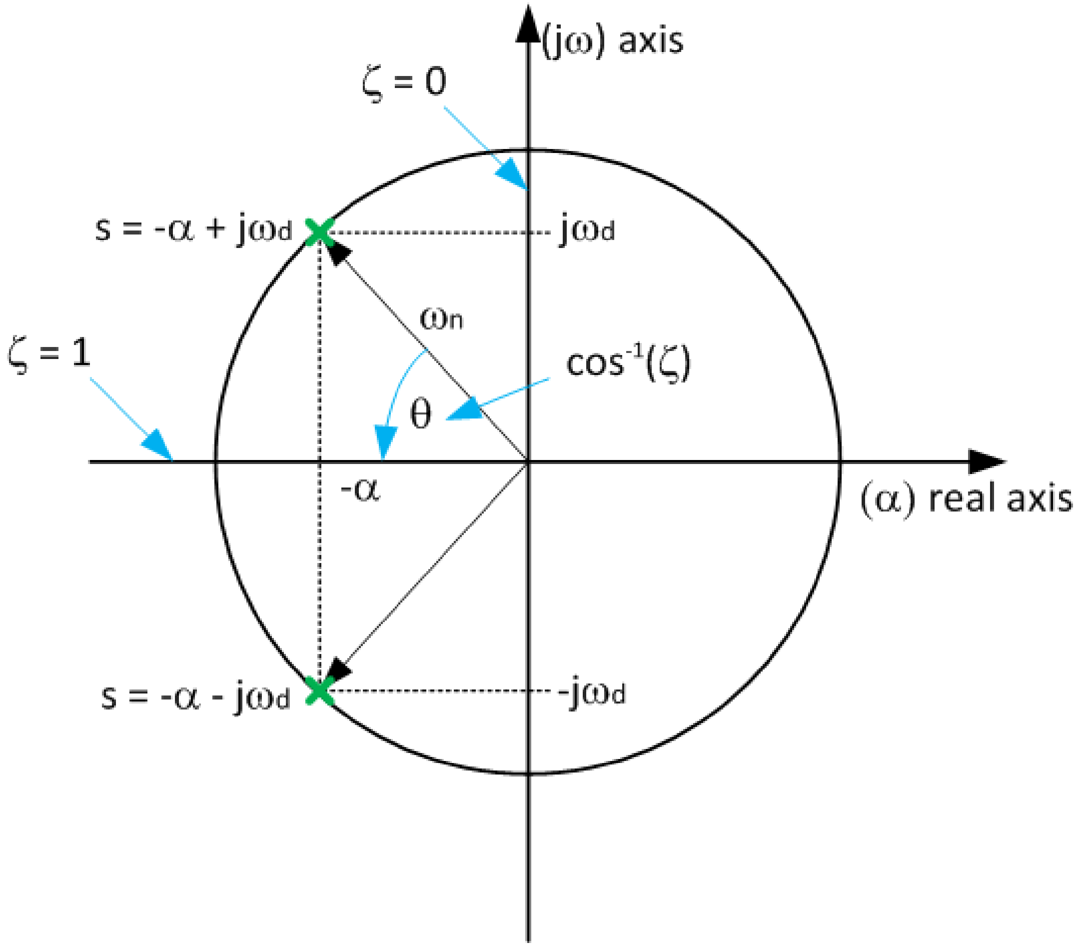
$$* \quad y(t) = 1 - e^{-\omega_n t}(1 + \omega_n t)$$



2^{nd} order , Step input, Different damping ratio (ξ)

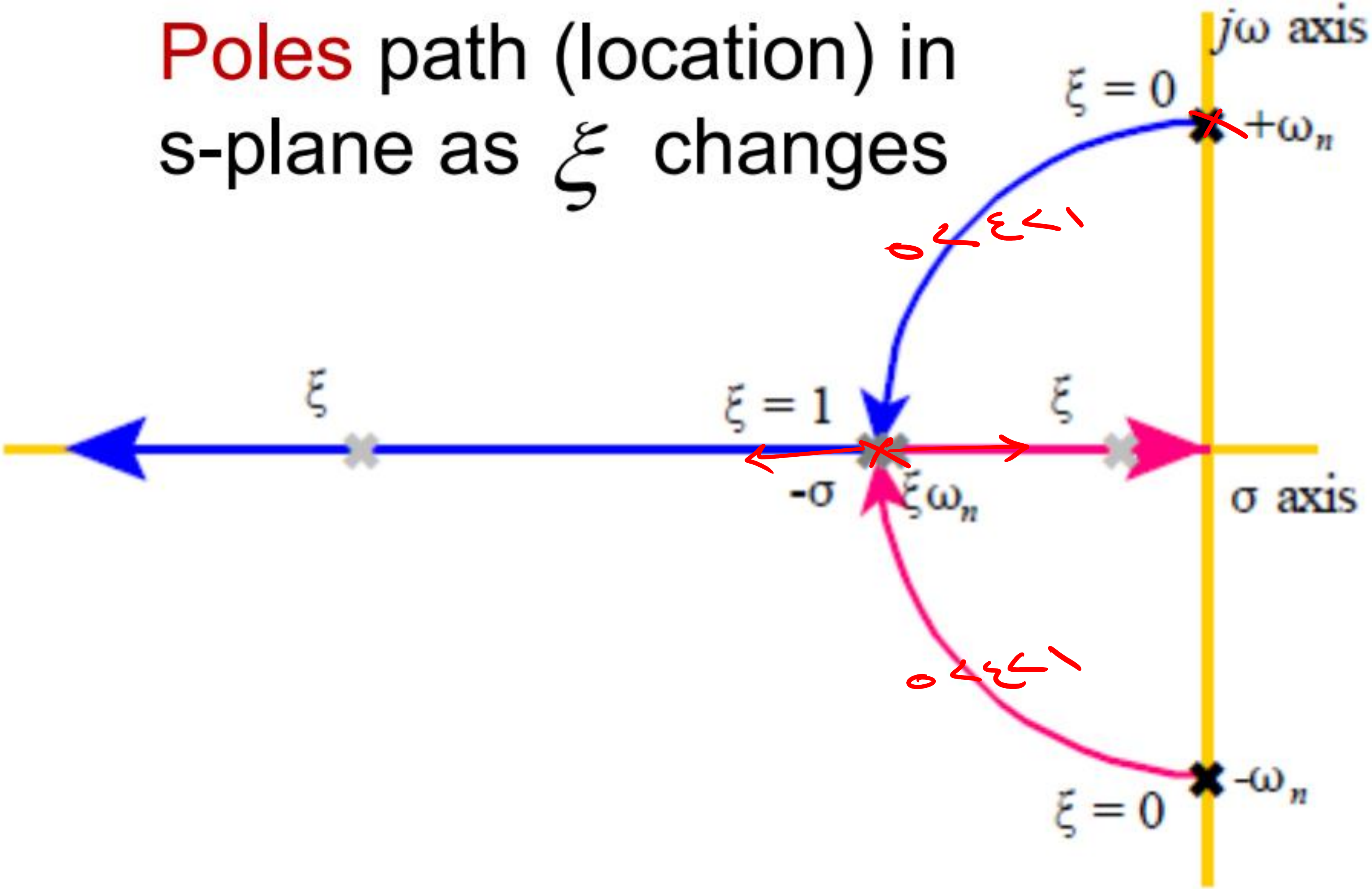


Pole Locations with different damping ratio (ξ)

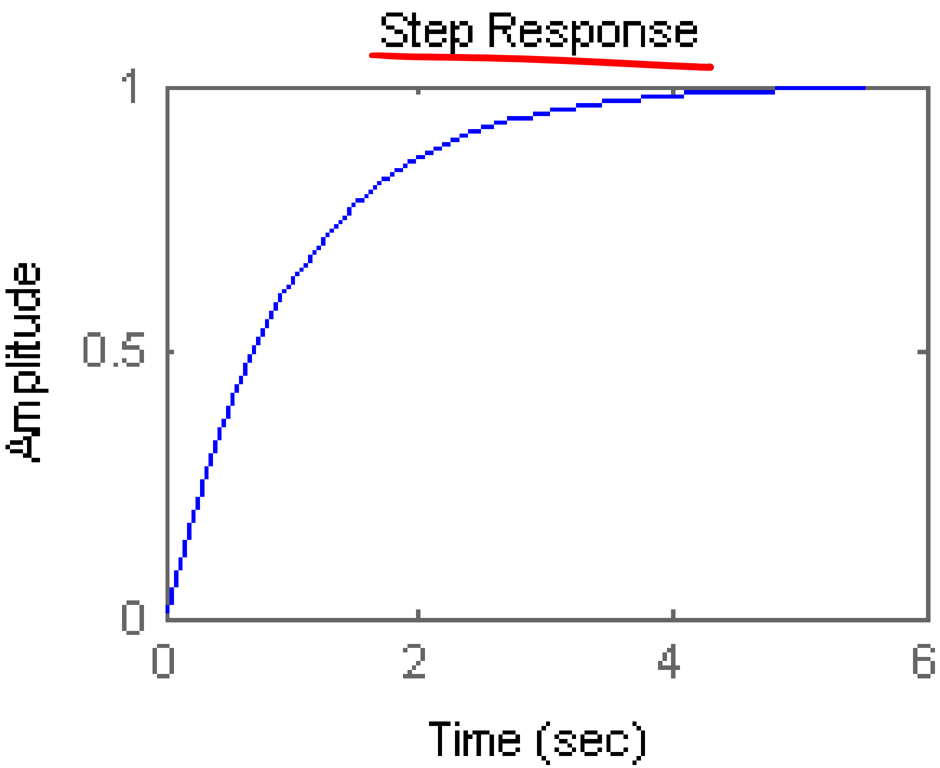


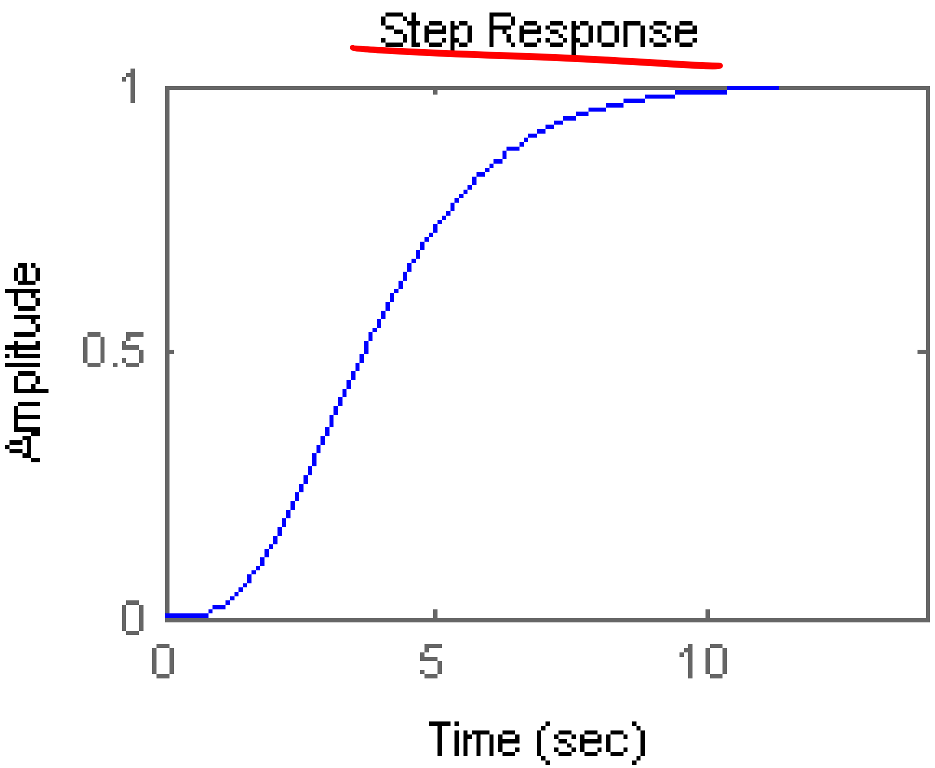
ξ	s
$\xi = 1$	$-\omega_n$
$0 < \xi < 1$	complex
$\xi = 0$	$\pm j\omega_n$
$\xi > 1$	real multi

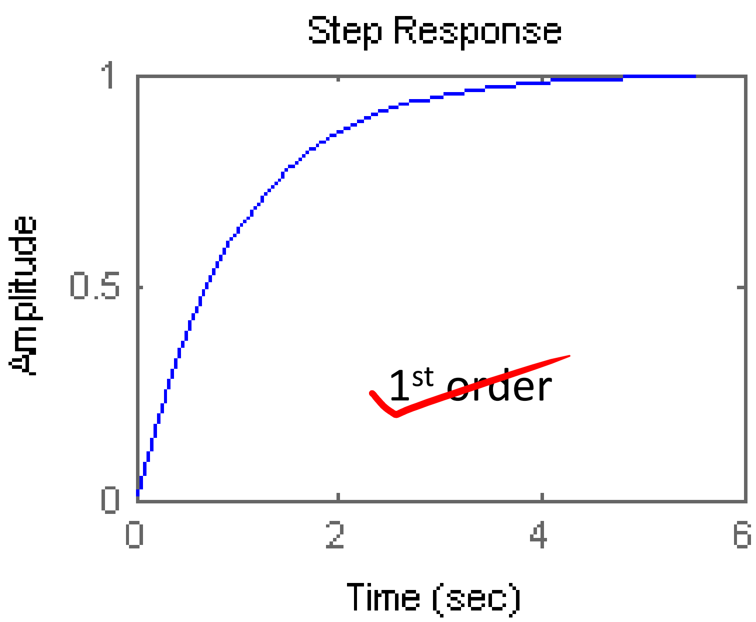
Poles path (location) in s-plane as ξ changes

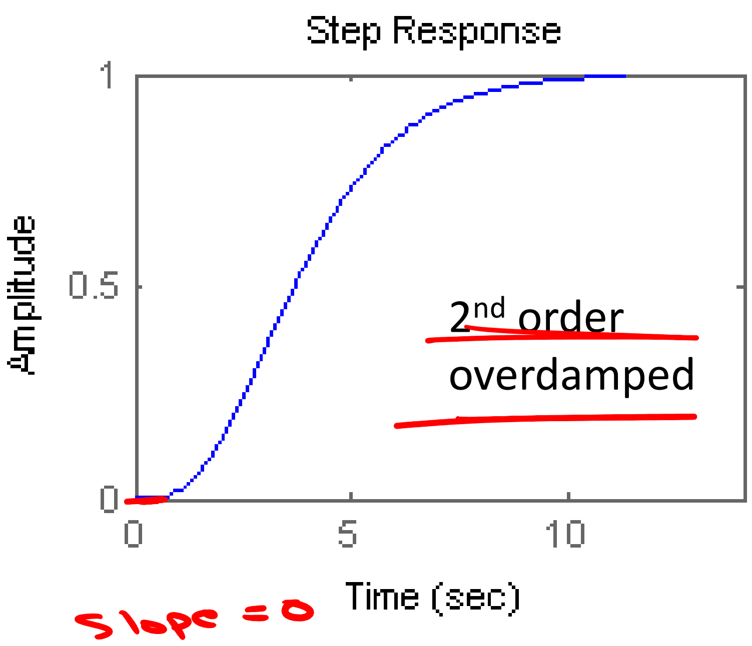


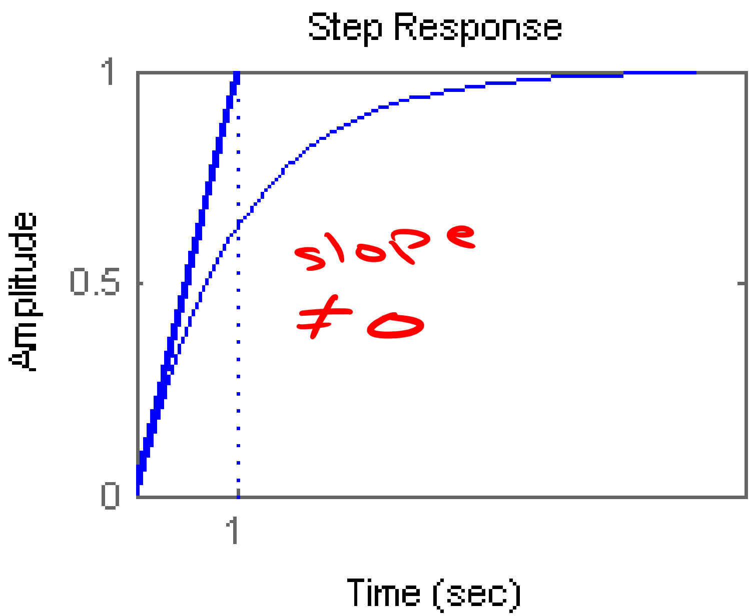
1st order
2nd "



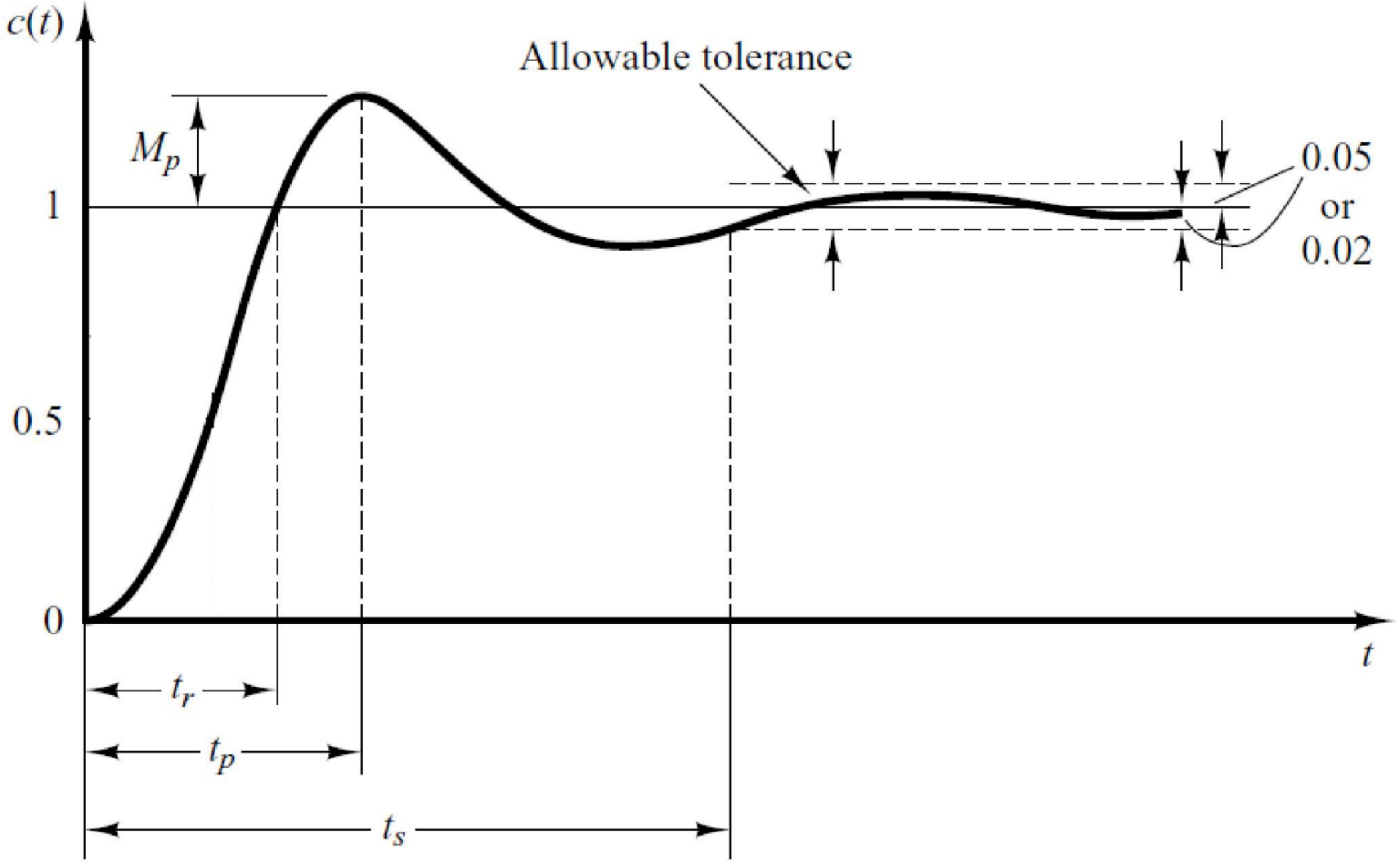



Step Response
Amplitude
Time (sec)
1
0.5
0
0
2
4
6
1st order


Step Response
Amplitude
Time (sec)
1
0.5
0
0
5
10
2nd order
overdamped
slope = 0


Step Response
Amplitude
Time (sec)
1
0.5
0
1
slope
≠ 0

Underdamped Second Order:



All performance specifications are derived from:

$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \beta)$$

1. Rise time(t_r) : rise time is the time required for the response to change from a lower prescribed value to a higher one.

$$t_r = \frac{\pi - \beta}{\omega_d} = \frac{\pi - \beta}{\omega_n \sqrt{1-\zeta^2}}$$

2. **Peak time**(t_p): the peak time is the time required for the response to reach the first peak

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n \sqrt{1-\varsigma^2}}$$

3. **Settling time** (t_s): the settling time is the time required for the amplitude of the sinusoid to decay to 2% or 5% of the steady-state value.

$$t_s = \frac{4}{\varsigma\omega_n}$$ 2% criterion

$$t_s = \frac{3}{\varsigma\omega_n}$$ 5% criterion

4. Maximum overshoot percentage: the percent overshoot is defined as the amount that the waveform at the peak time overshoots the steady-state value.

Maximum Peak Percentage MP%

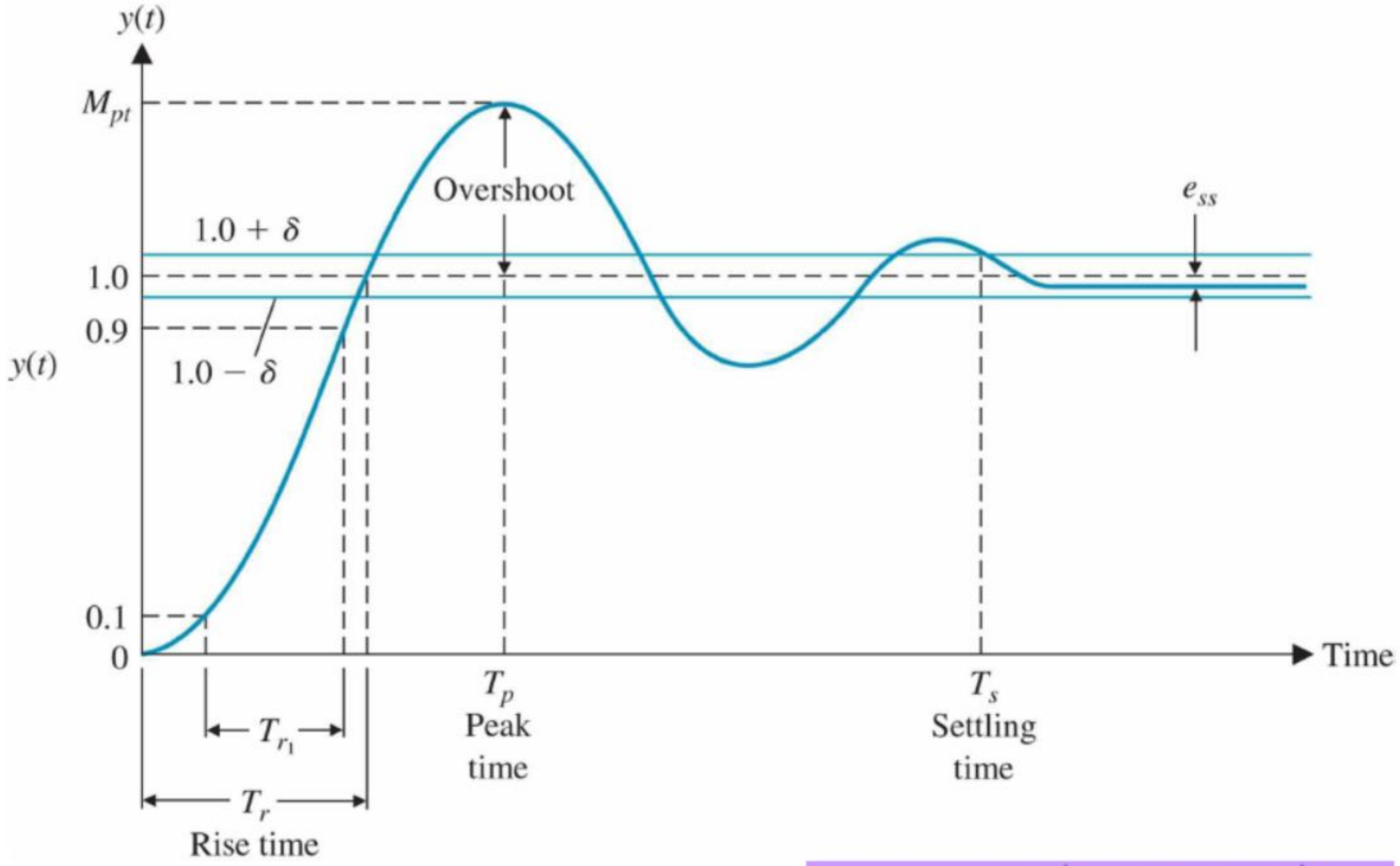
$$MP\% \equiv \frac{y(t_p) - y(\infty)}{y(\infty)} x100\%$$

OR

Overshoot Percentage OS%

$$OS\% = \frac{y\_\max - y\_final}{y\_final}\, x\ 100\%$$

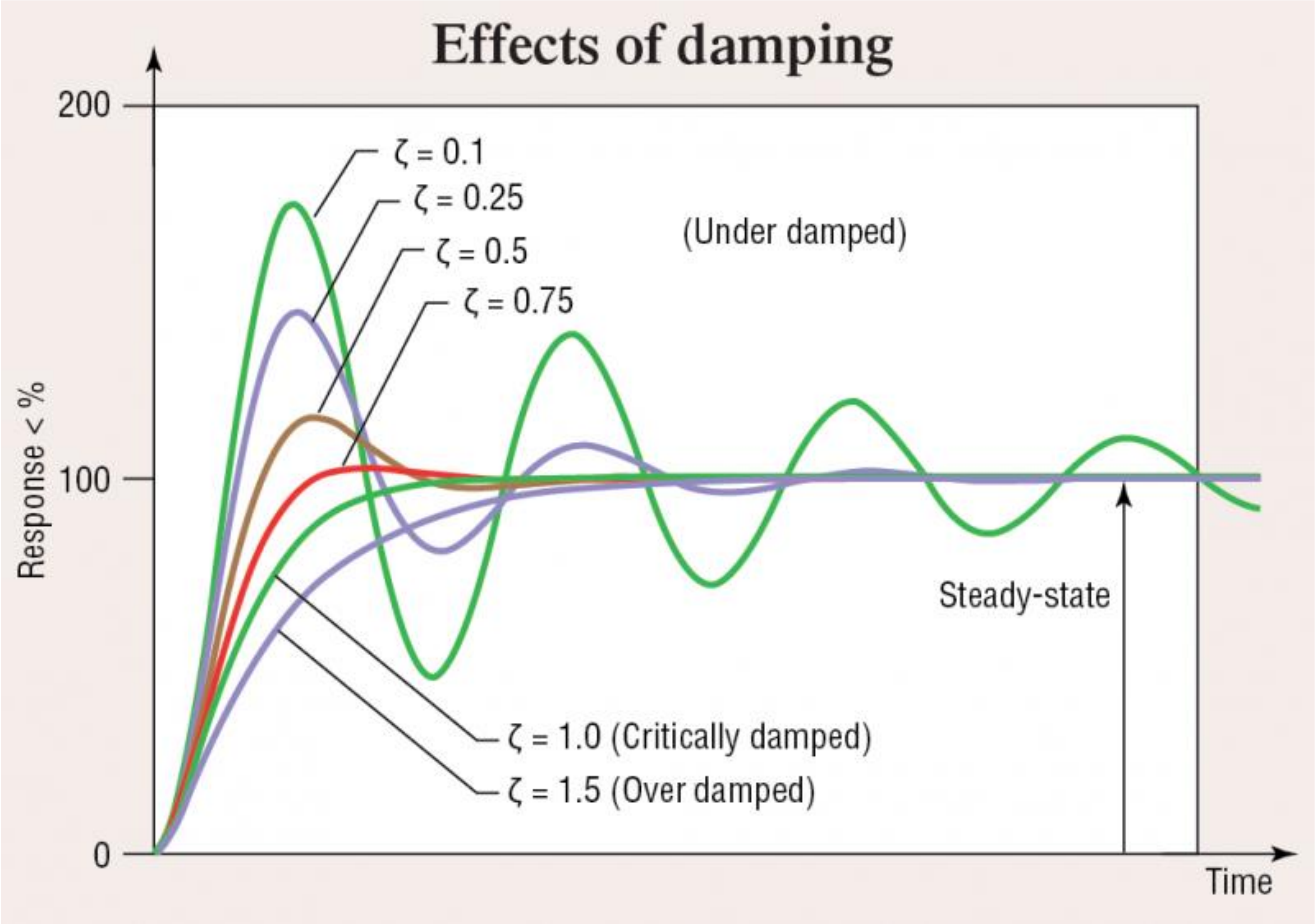
$$MP\% = 100e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$



For given *OS%*, the damping ratio can be solved from the OS% equation;

$$\varsigma = \frac{-\ln(\%MP/100)}{\sqrt{\pi^2 + \ln^2(\%MP/100)}}$$

O.S% is a function of ξ



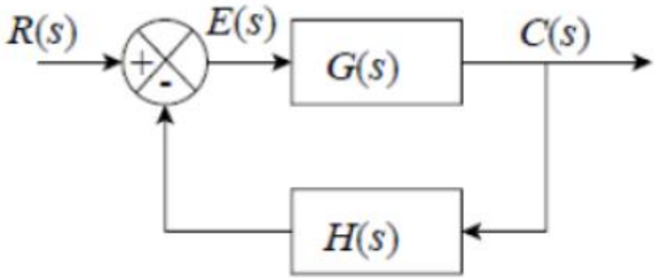
NOTE that ωn is constant for all

Examples will be discussed on a Video files

- If you have any questions you can contact me

Steady-state Error Analysis

For the feedback system shown in block diagram below, the transfer function is given by:



$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

The system error is given by:

$$\begin{aligned} E(s) &= R(s) - C(s)H(s) \\ &= \left[1 - \frac{G(s)H(s)}{1+G(s)H(s)}\right]R(s) \\ &= \frac{1}{1+G(s)H(s)}R(s) \end{aligned}$$

This last expression shows that the loop gain *G(s)H(s)* determine the amount and nature of the steady state error of a system.

The loop gain *G(s)H(s) can be expressed in the general form;*

$$G(s)H(s) = \frac{K(s+z_1)(s+z_2)(s+z_3)\cdots\cdots(s+z_m)}{s^N(s+p_1)(s+p_2)(s+p_3)\cdots\cdots(s+p_n)}$$

$$= \frac{K\prod_{i=1}^{i=m}(s+z_i)}{s^N \prod_{j=1}^{i=n}(s+p_j)}$$

The error in this case would be given by:

$$E(s) = \frac{1}{1+G(s)H(s)}\ R(s)$$

$$= \frac{s^N \prod_{j=1}^{i=n}(s+p_j)}{s^N \prod_{j=1}^{i=n}(s+p_j) + K\prod_{i=1}^{i=m}(s+z_i)}\ R(s)$$

The steady state error is calculated as follows:

$$e_{ss} = \lim_{s \to 0} \left[s \frac{s^N \prod_{j=1}^{i=n} (s+p_j)}{s^N \prod_{j=1}^{i=n} (s+p_j) + K \prod_{i=1}^{i=m} (s+z_i)} R(s) \right]$$

- When the standard test signals of a step *(A/s), a ramp (A/s2), and an acceleration (A/s3) are* used, the Laplace operator "*s" in the input test signal denominator will cancel or reduce* from the power of "*s" in the numerator of the expression above.

- The power of "*s" (the poles of the G(s)H(s) located on the origin of s-plane), i.e. N, determines the* steady state error response of the system when subjected to standard test signals, and is called the **"type number" of the system.**

- For *N = 0, the system is a type zero, for N = 1, the system is a type one, and so on.*

Type Zero System:

The steady state error for a step input; *A/s is given by*

$$e_{ss} = \lim_{s\to 0}\left[\cancel{s}\,\frac{\prod_{j=1}^{i=n}(s+p_j)}{\prod_{j=1}^{i=n}(s+p_j)+K\prod_{i=1}^{i=m}(s+z_i)}\,\frac{A}{\cancel{s}}\right]$$

$$= \frac{\prod_{j=1}^{i=n}p_j}{\prod_{j=1}^{i=n}p_j+K\prod_{i=1}^{i=m}z_i}\,A$$

$$= \frac{A}{1+K_p} \qquad \text{for } K_p = \frac{K\prod_{i=1}^{i=m}z_i}{\prod_{j=1}^{i=n}p_j}$$

Position error constant

$$K_p = \lim_{s\to 0} G(s)H(s)$$

The steady state error for a ramp input; *A/s2 is given by*

$$e_{ss} = \lim_{s\to 0}\left[\cancel{s}\,\frac{\prod_{j=1}^{i=n}(s+p_j)}{\prod_{j=1}^{i=n}(s+p_j)+K\prod_{i=1}^{i=m}(s+z_i)}\;\frac{A}{s^{\cancel{2}}}\right]$$

$$= \infty$$

Type One System:

The steady state error for a step input; *A/s is given by*

$$e_{ss} = \lim_{s\to 0}\left[\cancel{s}\,\frac{s\prod_{j=1}^{i=n}(s+p_j)}{s\prod_{j=1}^{i=n}(s+p_j)+K\prod_{i=1}^{i=m}(s+z_i)}\;\frac{A}{\cancel{s}}\right]$$

$$= 0$$

TYPE	STEP INPUT $R(s)=A/s$	RAMP INPUT $R(s)=A/s^2$	ACCELERATION INPUT $R(s)=A/s^3$
0	$\frac{A}{1+K_p}$	∞	∞
1			
2			

The steady state error for a ramp input; *A/s2 is given by*

$$e_{ss} = \lim_{s\to 0}\left[\cancel{s}\frac{\cancel{s}\prod_{j=1}^{i=n}(s+p_j)}{s\prod_{j=1}^{i=n}(s+p_j)+K\prod_{i=1}^{i=m}(s+z_i)}\frac{A}{\cancel{s^2}}\right]$$

$$= \frac{\prod_{j=1}^{i=n}p_j}{K\prod_{i=1}^{i=m}z_i}A$$

$$= \frac{A}{K_v} \qquad \text{for } K_v = \frac{K\prod_{i=1}^{i=m}z_i}{\prod_{j=1}^{i=n}p_j}$$

Velocity error constant

$$K_v = \lim_{s\to 0} sG(s)H(s)$$

The steady state error for an acceleration input; *A/s3 is given by*

$$e_{ss} = \lim_{s\to 0}\left[\cancel{s}\frac{\cancel{s}\prod_{j=1}^{i=n}(s+p_j)}{s\prod_{j=1}^{i=n}(s+p_j)+K\prod_{i=1}^{i=m}(s+z_i)}\frac{A}{s^{\cancel{3}}}\right]$$

$$= \infty$$

TYPE	STEP INPUT $R(s)=A/s$	RAMP INPUT $R(s)=A/s^2$	ACCELERATION INPUT $R(s)=A/s^3$
0	$\frac{A}{1+K_p}$	∞	∞
1	0	$\frac{A}{K_v}$	∞
2			

Type Two System:

The steady state error for a step input; *A/s is given by*

$$e_{ss} = \lim_{s \to 0} \left[\cancel{s} \frac{s^2 \prod_{j=1}^{i=n} (s+p_j)}{s^2 \prod_{j=1}^{i=n} (s+p_j) + K \prod_{i=1}^{i=m} (s+z_i)} \frac{A}{\cancel{s}} \right]$$

$$= 0$$

A/S^2 → ess=0

The steady state error for an acceleration input; *A/s3* is given by

$$e_{ss} = \lim_{s \to 0} \left[\cancel{s} \frac{\cancel{s^2} \prod_{j=1}^{i=n} (s+p_j)}{s^2 \prod_{j=1}^{i=n} (s+p_j) + K \prod_{i=1}^{i=m} (s+z_i)} \frac{A}{\cancel{s^3}} \right]$$

$$= \frac{\prod_{j=1}^{i=n} p_j}{K \prod_{i=1}^{i=m} z_i} A$$

$$= \frac{A}{K_a} \qquad \text{for } K_a = \frac{K \prod_{i=1}^{i=m} z_i}{\prod_{j=1}^{i=n} p_j}$$

Acceleration error constant

$$K_a = \lim_{s \to 0} s^2 G(s) H(s)$$

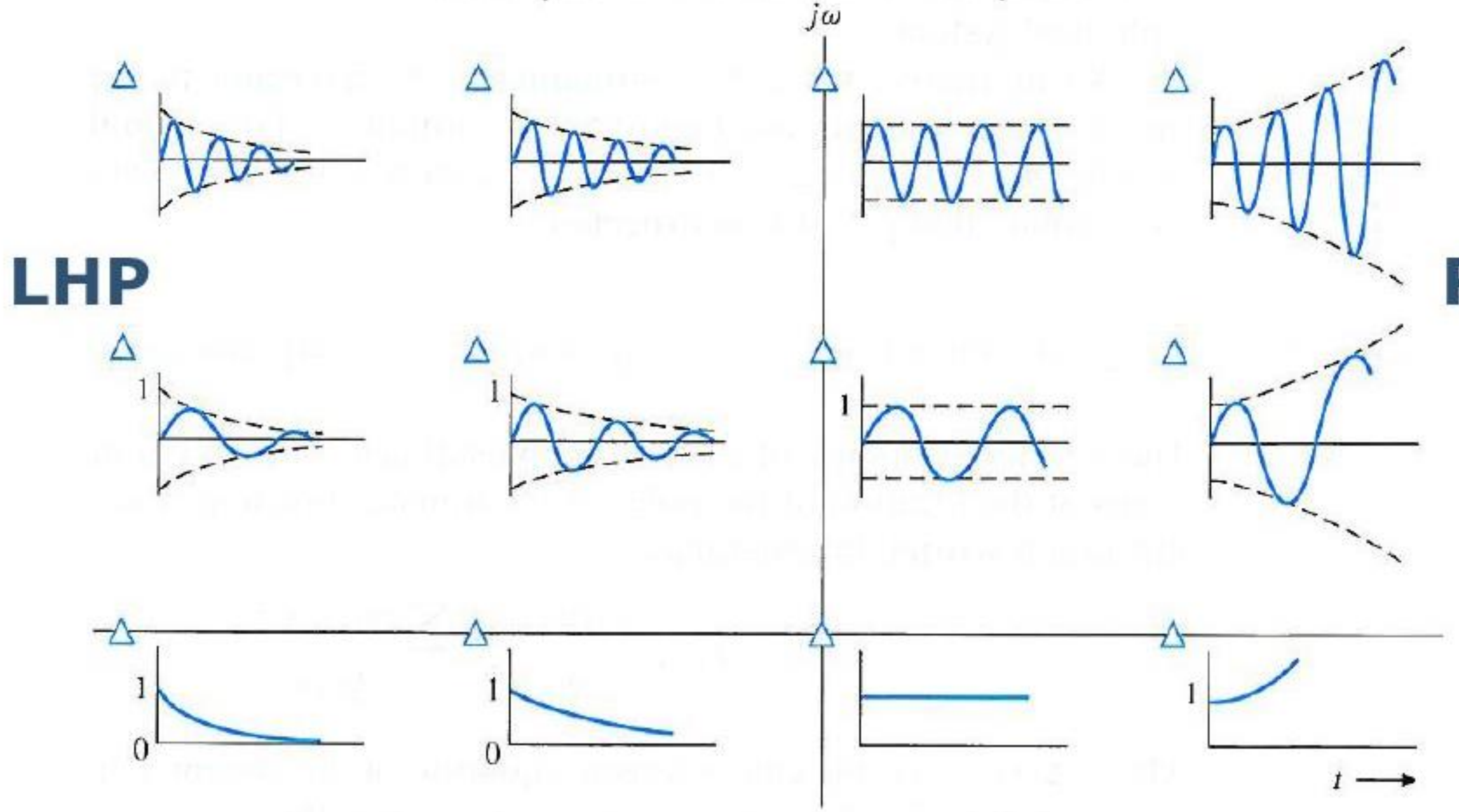
In summary;

$$\text{For } K_p = K_v = K_a = \frac{K \prod_{i=1}^{i=m} z_i}{\prod_{j=1}^{i=n} p_j}$$

TYPE	STEP INPUT $R(s)=A/s$	RAMP INPUT $R(s)=A/s^2$	ACCELERATION INPUT $R(s)=A/s^3$
0	$\frac{A}{1+K_p}$	∞	∞
1	0	$\frac{A}{K_v}$	∞
2	0	0	$\frac{A}{K_a}$

Effect of Pole Locations

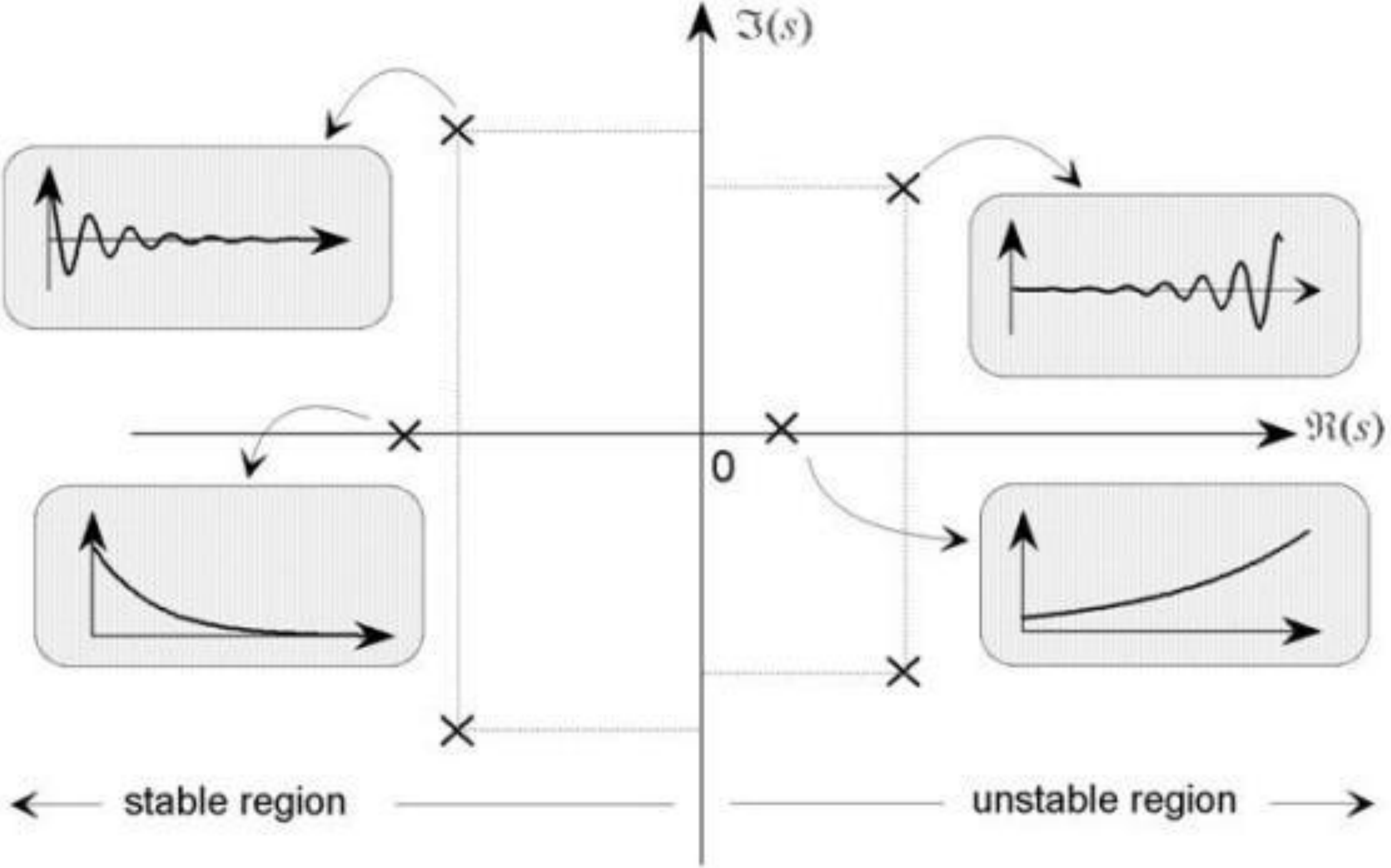
Time function of impulse response assosiated with the pole location in *s*-plane



LHP : **left half-plane**
RHP : **right half-plane**

Stability

- A BIBO (bounded-input bounded-output) stable system is a system for which the outputs will remain bounded for all time, for any finite initial condition and input. A continuous-time linear time-invariant system is BIBO stable if and only if all the poles of the system have real parts less than 0.



$\Im(s)$
$\Re(s)$
0
stable region
unstable region

Find if the systems stable or not

G(s)= $\frac{s+3}{(s+5)(s+2)}$

G(s)= $\frac{s+3}{(s+5)(s-2)}$

$$y(t) = 1 - \frac{e^{-\xi\omega_n t}}{\sqrt{1-\xi^2}} \sin(\omega_d t + \beta)$$

$$where \quad \beta = \tan^{-1}\frac{\omega_d}{\xi\omega_n} = \tan^{-1}\frac{\omega_n\sqrt{1-\xi^2}}{\xi\omega_n}$$

$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

$$t_r = \frac{\pi-\beta}{\omega_d} = \frac{\pi-\beta}{\omega_n\sqrt{1-\varsigma^2}}$$

$$t_p = \frac{\pi}{\omega_d} = \frac{\pi}{\omega_n\sqrt{1-\varsigma^2}}$$

$$t_s = \frac{4}{\varsigma\omega_n}$$ 2% criterion

$$MP\% \equiv \frac{y(t_p) - y(\infty)}{y(\infty)} x100\%$$

$$MP\% = 100e^{\frac{-\xi\pi}{\sqrt{1-\xi^2}}}$$

$$\varsigma = \frac{-\ln(\%MP/100)}{\sqrt{\pi^2 + \ln^2(\%MP/100)}}$$

$$G(s) = \frac{9}{s^2 + 2s + 9}$$ unit step input

Find $y(t)$, T_p, T_r, T_s, y_{ss}

ξ, ω_n $\qquad \frac{\omega_n^2}{s^2 + 2\xi\omega_n s + \omega_n^2}$

$2 = 2\xi\overset{3}{\omega_n} \Rightarrow \xi = \frac{1}{3}$ $\quad 0 < \xi < 1$ underdamped.

$9 = \omega_n^2 \Rightarrow \omega_n = 3$

$$\omega_d = \omega_n\sqrt{1-\xi^2} = 3\sqrt{1-\left(\frac{1}{3}\right)^2} = \sqrt{8} = 2\sqrt{2}$$

$$\beta = \cos^{-1}\xi = 70.5^\circ = 1.23$$

$$y(t) = 1 - \frac{e^{-\frac{1}{3}\times 3\times t}}{\sqrt{1-\left(\frac{1}{3}\right)^2}} \sin(2\sqrt{2}t + 70.5^\circ)$$

$$T_p = \frac{\pi}{\underset{\text{rad/sec}}{\omega_d}} = \frac{3.14}{2\sqrt{2}} = 1.11 \text{ sec}$$

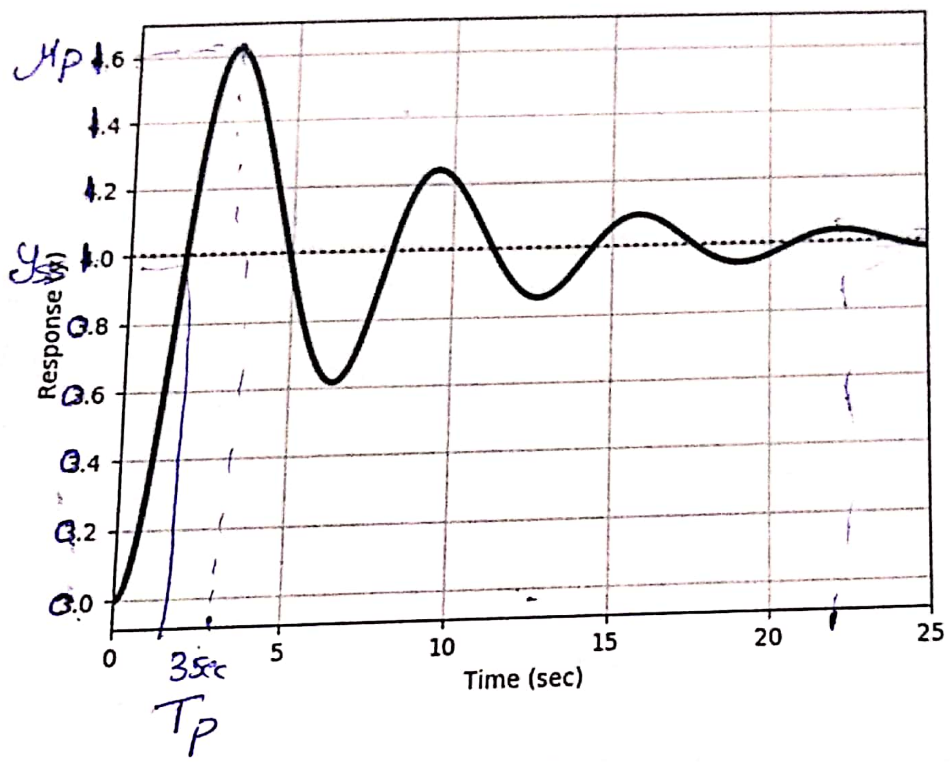
$$T_r = \frac{\pi - \beta}{\omega_d} = \frac{3.14 - 1.23}{2\sqrt{2}} = 0.6753 \text{ sec}$$

$$T_s = \frac{4}{\underset{\frac{1}{3}}{\xi}\underset{3}{\omega_n}} = 4 \text{ sec}$$

$$y_{ss} = \lim_{s\to 0} \cancel{s} \cdot \frac{1}{\cancel{s}} \cdot \frac{9}{s^2 + 2s + 9} = 1$$

Scanned with CamScanner

For the following unit step response find y(t), Ts,Tr,O.S%, damping ratio, natural frequency, ...



$\mathcal{E}$, $\omega_n \Rightarrow \omega_d$, β

$$0.S\% = \frac{Mp - y_{ss}}{y_{ss}} * 100\% = \frac{1.6 - 1}{1} * 100\% = 60\%$$

$$\boxed{\mathcal{E}} = \frac{-\ln(0.6)}{\sqrt{\pi^2 + \ln^2(0.6)}} = 0.16 \qquad 0 < \varepsilon < 1$$

$$T_p = 3\ sec = \frac{\pi}{\omega_n\sqrt{1-\varepsilon^2}} = \frac{\overset{3.14}{\pi}}{\underset{rad/sec}{\omega_n}\sqrt{1-(0.16)^2}} = 3\ sec$$

$$\boxed{\omega_n} = 1.061\ rad/sec.$$

$$\omega_d = \omega_n\sqrt{1-\varepsilon^2} = 1.061\sqrt{1-(0.16)^2} = 1.0473\ rad/sec$$

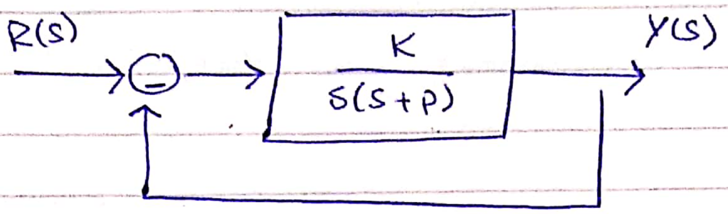
$$\beta = \cos^{-1}\varepsilon = 80.79^\circ = 1.41$$

$$y(t) = 1 - \frac{e^{-0.16*1.061t}}{\sqrt{1-(0.16)^2}}\ Sin(1.0473t + 80.79^\circ)$$

$$T_s = \frac{4}{\varepsilon\omega_n} = 23.578\ sec.$$

$$T_r = \frac{\pi - \beta}{\omega_d} = \frac{3.14 - 1.41}{1.0473} = 1.65\ sec$$

ex 5.1



$O.S\% = 4.3\%$

$T_s = 4\ sec \qquad 2\%\ cri. = \dfrac{4}{\xi \omega_n}$

$$\frac{Y(s)}{R(S)} = \frac{K}{S(S+P)+K} = \frac{K}{s^2+ps+K} \qquad \frac{\omega_n^2}{s^2+2\xi\omega_n s+\omega_n^2}$$

$$K = \omega_n^2 \Rightarrow \cancel{K = \sqrt{\omega_n}} \Rightarrow K = (\sqrt{2})^2 = 2$$

$$P = 2\xi\omega_n \qquad \Rightarrow P = 2 * \frac{1}{\sqrt{2}} * \sqrt{2} = 2$$

$$\bullet\ \xi = \frac{-\ln(0.043)}{\sqrt{\pi^2 + \ln^2(0.043)}} = \frac{1}{\sqrt{2}} \qquad 0 < \xi < 1$$

$$\omega_n \Rightarrow T_s = \frac{4}{\xi\omega_n} = 4\ sec.$$

$$\frac{4}{\frac{1}{\sqrt{2}}\omega_n} = 4 \Rightarrow \omega_n = \sqrt{2}$$

Routh-Herwitz Stability Criterion

Eng. Fadwa Momani

Routh-Hurwitz Stability Criterion

- It is a method for determining continuous system stability.

- The Routh-Hurwitz criterion states that “the number of roots of the characteristic equation with positive real parts is equal to the number of changes in sign of the first column of the Routh array”.

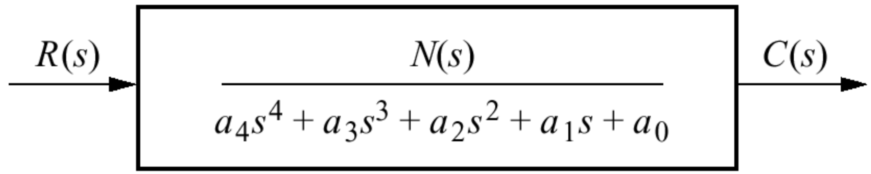
Routh-Hurwitz Stability Criterion

- This method yields stability information without the need to solve for the closed-loop system poles.
- Using this method, we can tell how many closed-loop system poles are in the left half-plane, in the right half-plane, and on the jw-axis. (Notice that we say how many, not where.)

- The method requires two steps:

1. Generate a data table called a Routh table.
2. interpret the Routh table to tell how many closed-loop system poles are in the LHP, the RHP, and on *the jw-axis*.

Example: Generating a basic Routh Table.



- Only the first 2 rows of the array are obtained from the characteristic eq. the remaining are calculated as follows;

s^4	a_4	a_2	a_0
s^3	a_3	a_1	0
s^2	$\dfrac{-\begin{vmatrix} a_4 & a_2 \\ a_3 & a_1 \end{vmatrix}}{a_3} = b_1$	$\dfrac{-\begin{vmatrix} a_4 & a_0 \\ a_3 & 0 \end{vmatrix}}{a_3} = b_2$	$\dfrac{-\begin{vmatrix} a_4 & 0 \\ a_3 & 0 \end{vmatrix}}{a_3} = 0$
s^1	$\dfrac{-\begin{vmatrix} a_3 & a_1 \\ b_1 & b_2 \end{vmatrix}}{b_1} = c_1$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$	$\dfrac{-\begin{vmatrix} a_3 & 0 \\ b_1 & 0 \end{vmatrix}}{b_1} = 0$
s^0	$\dfrac{-\begin{vmatrix} b_1 & b_2 \\ c_1 & 0 \end{vmatrix}}{c_1} = d_1$ a_0	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$	$\dfrac{-\begin{vmatrix} b_1 & 0 \\ c_1 & 0 \end{vmatrix}}{c_1} = 0$

Four Special Cases or Configurations in the First Column Array of the Routh’s Table:

1. **Case-I:** No element in the first column is zero.

2. **Case-II:** A zero in the first column but some other elements of the row containing the zero in the first column are nonzero.

3. **Case-III:** Entire Row is zero

Case-I: No element in the first column is zero.

Example-1: Find the stability of the continues system having the characteristic equation of

$$s^3 + 6s^2 + 12s + 8 = 0$$

The Routh table of the given system is computed as;

$$\begin{array}{c|ccc} s^3 & 1 & 12 & 0 \\ s^2 & 6 & 8 & 0 \\ s^1 & \frac{64}{6} & 0 & \\ s^0 & 8 & & \end{array}$$

$$b1 = \frac{(6*12)-(1*8)}{6} = \frac{64}{6}$$

- Since there are no sign changes in the first column of the Routh table, it means that all the roots of the characteristic equation have negative real parts and hence this system is stable.

Example-2: Find the stability of the continues system having the characteristic polynomial of a third order system is given below

$$s^3 + s^2 + 2s + 24$$

- The Routh array is

$$\begin{array}{c|cc} s^3 & 1 & 2 \\ s^2 & 1 & 24 \\ s^1 & -22 & 0 \\ s^0 & 24 & 0 \end{array}$$

$$b1 = \frac{(1*2)-(1*24)}{1} = -22$$

- Because TWO changes in sign appear in the first column, we find that two roots of the characteristic equation lie in the right hand side of the s-plane. Hence the system is unstable.

Example-3: Determine a rang of values of a system parameter K for which the system is stable.

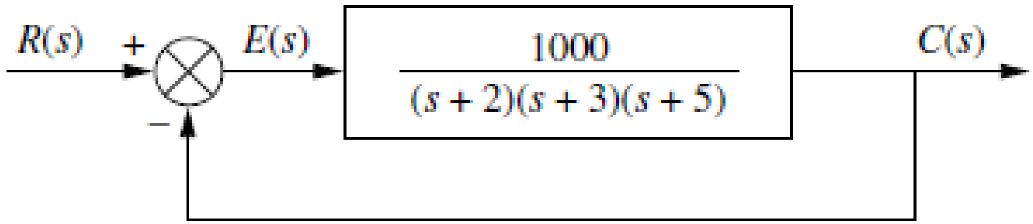
$$s^3 + 3s^2 + 3s + 1 + K = 0$$

- The Routh table of the given system is computed and shown is the table below;

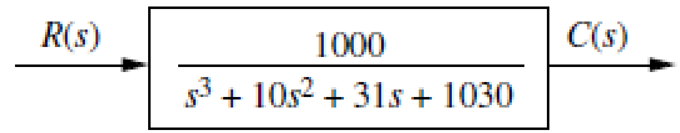
s^3	1	3	0
s^2	3	$1 + K$	0
s^1	$\dfrac{8 - K}{3}$	0	
s^0	$1 + K$		

- For system stability, it is necessary that the conditions $8 - k > 0$, and $1 + k > 0$, must be satisfied. Hence the rang of values of a system parameter k must be lies between -1 and 8 (i.e., $-1 < k < 8$).

Example-4: Find the stability of the system shown below using Routh criterion.



The close loop transfer function is shown in the figure



The Routh table of the system is shown in the table

s^3	1	31	0
s^2	$\cancel{10}\quad 1$	$\cancel{1030}\quad 103$	0
s^1	$\dfrac{-\begin{vmatrix} 1 & 31 \\ 1 & 103 \end{vmatrix}}{1} = -72$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix}}{1} = 0$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ 1 & 0 \end{vmatrix}}{1} = 0$
s^0	$\dfrac{-\begin{vmatrix} 1 & 103 \\ -72 & 0 \end{vmatrix}}{-72} = 103$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0$	$\dfrac{-\begin{vmatrix} 1 & 0 \\ -72 & 0 \end{vmatrix}}{-72} = 0$

Because TWO changes in sign appear in the first column, we find that two roots of the characteristic equation lie in the right hand side of the s-plane. Hence the system is unstable.

Example-5: Find the stability of the system shown below using Routh criterion.

$$2s^4 + s^3 + 3s^2 + 5s + 10 = 0$$

- The Routh table of the system is

$$\begin{array}{llll}
s^4 & 2 & 3 & 10 \\
s^3 & 1 & 5 & 0 \\
s^2 & \dfrac{(1)(3) - (2)(5)}{1} = -7 & 10 & 0 \\
s^1 & \dfrac{(-7)(5) - (1)(10)}{-7} = 6.43 & 0 & 0 \\
s^0 & 10 & 0 & 0
\end{array}$$

- System is unstable because there are two sign changes in the first column of the Routh's table. Hence the equation has two roots on the right half of the s-plane.

Case-II: A Zero Only in the First Column

Stability via Epsilon Method.

Case-II: Stability via Epsilon Method

- If the first element of a row is zero, division by zero would be required to form the next row.
- To avoid this phenomenon, an ***epsilon, ε,*** (a small positive number) is assigned to replace the zero in the first column.
- The value ε is then allowed to approach zero from either the positive or the negative side, after which the signs of the entries in the first column can be determined.

Case-II: Stability via Epsilon Method

Example-6: Determine the stability of the system having a characteristic equation given below;

$$q(s) = s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10$$

The Routh array is shown in the table;

$$\begin{array}{c|ccc} s^5 & 1 & 2 & 11 \\ s^4 & 2 & 4 & 10 \\ s^3 & \epsilon & 6 & 0 \\ s^2 & c_1 & 10 & 0 \\ s^1 & d_1 & 0 & 0 \\ s^0 & 10 & 0 & 0 \end{array}$$

Where

$$c_1 = \frac{4\epsilon - 12}{\epsilon} = \frac{-12}{\epsilon} \quad \text{and} \quad d_1 = \frac{6c_1 - 10\epsilon}{c_1} \to 6.$$

There are TWO sign changes due to the large negative number in the first column, $c_1 = -12/\epsilon.$
Therefore the system is unstable, and two roots of the equation lie in the right half of the s-plane.

Example-7: Determine the range of parameter *K* for which the system is stable.

$$q(s) = s^4 + s^3 + s^2 + s + K$$

The Routh array of the above characteristic equation is shown below;

$$\begin{array}{c|ccc} s^4 & 1 & 1 & K \\ s^3 & 1 & 1 & 0 \\ s^2 & \epsilon & K & 0, \\ s^1 & c_1 & 0 & 0 \\ s^0 & K & 0 & 0 \end{array}$$

Where

$$c_1 = \frac{\epsilon - K}{\epsilon} \longrightarrow \frac{-K}{\epsilon}$$

- Therefore, for any value of *K* greater than zero, the system is unstable.
- Also, because the last term in the first column is equal to K, a negative value of K will result in an unstable system.
- Consequently, the system is unstable for all values of gain K.

Example-8: Determine the stability of the closed-loop transfer function;

$$T(s) = \frac{10}{s^5 + 2s^4 + 3s^3 + 6s^2 + 5s + 3}$$

The complete Routh table is formed by using the denominator of the characteristic equation T(s).

s^5	1	3	5
s^4	2	6	3
s^3	$\cancel{0}\ \ \epsilon$	$\frac{7}{2}$	0
s^2	$\frac{6\epsilon - 7}{\epsilon}$	3	0
s^1	$\frac{42\epsilon - 49 - 6\epsilon^2}{12\epsilon - 14}$	0	0
s^0	3	0	0

- A zero appears only in the first column (the s3 row).
- Next replace the zero by a small number, ε, and complete the table.
- the sign in the first column of Routh table is changes twice.
- Hence, **the system is unstable and has two poles in the right half-plane**.

Case-III: Entire Row is Zero.

- Sometimes while making a Routh table, we find that an entire row consists of zeros.
- This happen because there is an even polynomial that is a factor of the original polynomial.
- This case must be handled differently from the case of a zero in only the first column of a row.

Example-9

- Determine the number of right-half-plane poles in the closed-loop transfer function.

$$T(s) = \frac{10}{s^5 + 7s^4 + 6s^3 + 42s^2 + 8s + 56}$$

s^5			1			6			8
s^4		$\cancel{7}$	1		$\cancel{42}$	6		$\cancel{56}$	8
s^3	$\cancel{0}$	$\cancel{4}$	1	$\cancel{0}$	$\cancel{12}$	3	$\cancel{0}$	$\cancel{0}$	0

- First we return to the row immediately above the row of zeros and form an auxiliary polynomial, using the entries in that row as coefficients.

$$P(s) = s^4 + 6s^2 + 8$$

- Next we differentiate the polynomial with respect to s and obtain

$$\frac{dP(s)}{ds} = 4s^3 + 12s + 0$$

- Finally, we use the coefficients of above equation to replace the row of zeros. Again, for convenience, the third row is multiplied by 1/4 after replacing the zeros.

Example-9

- The remainder of the table is formed in a straightforward manner by following the standard form .

s^5			1			6			8
s^4		$\cancel{7}$	1		$\cancel{42}$	6		$\cancel{56}$	8
s^3	$\cancel{0}$	$\cancel{4}$	1	$\cancel{0}$	$\cancel{12}$	3	$\cancel{0}$	$\cancel{0}$	0
s^2			3			8			0
s^1			$\frac{1}{3}$			0			0
s^0			8			0			0

- All the entries in the first column are positive. Hence, there are no right–half-plane poles.

Example-10: Determine the stability of the system.

The characteristic equation q(s) of the system is $q(s) = s^3 + 2s^2 + 4s + K$

Where ***K*** is an adjustable loop gain. The Routh array is then;

$$
\begin{array}{c|cc}
s^3 & 1 & 4 \\
s^2 & 2 & K \\
s^1 & \dfrac{8-K}{2} & 0 \\
s^0 & K & 0
\end{array}
$$

For a stable system, the value of ***K*** must be; $0 < K < 8$

When ***K*** = 8, the two roots exist on the *j*ω axis and the system will be marginally stable.

- Also, **when *K* = 8, we obtain a row of zeros (case-III).**
- The **auxiliary polynomial, *U(s)*,** is the equation of the row preceding the row of Zeros.
- The ***U(s)*** in this case, obtained from the **s^2** row.
- The order of the auxiliary polynomial is **always even** and indicates the number of **symmetrical root pairs**.

Example-11

- For the transfer function tell how many poles are in the right half-plane, in the left half-plane, and on the jw-axis.

$$T(s) = \frac{20}{s^8 + s^7 + 12s^6 + 22s^5 + 39s^4 + 59s^3 + 48s^2 + 38s + 20}$$

Example-11

$$P(s) = s^4 + 3s^2 + 2$$

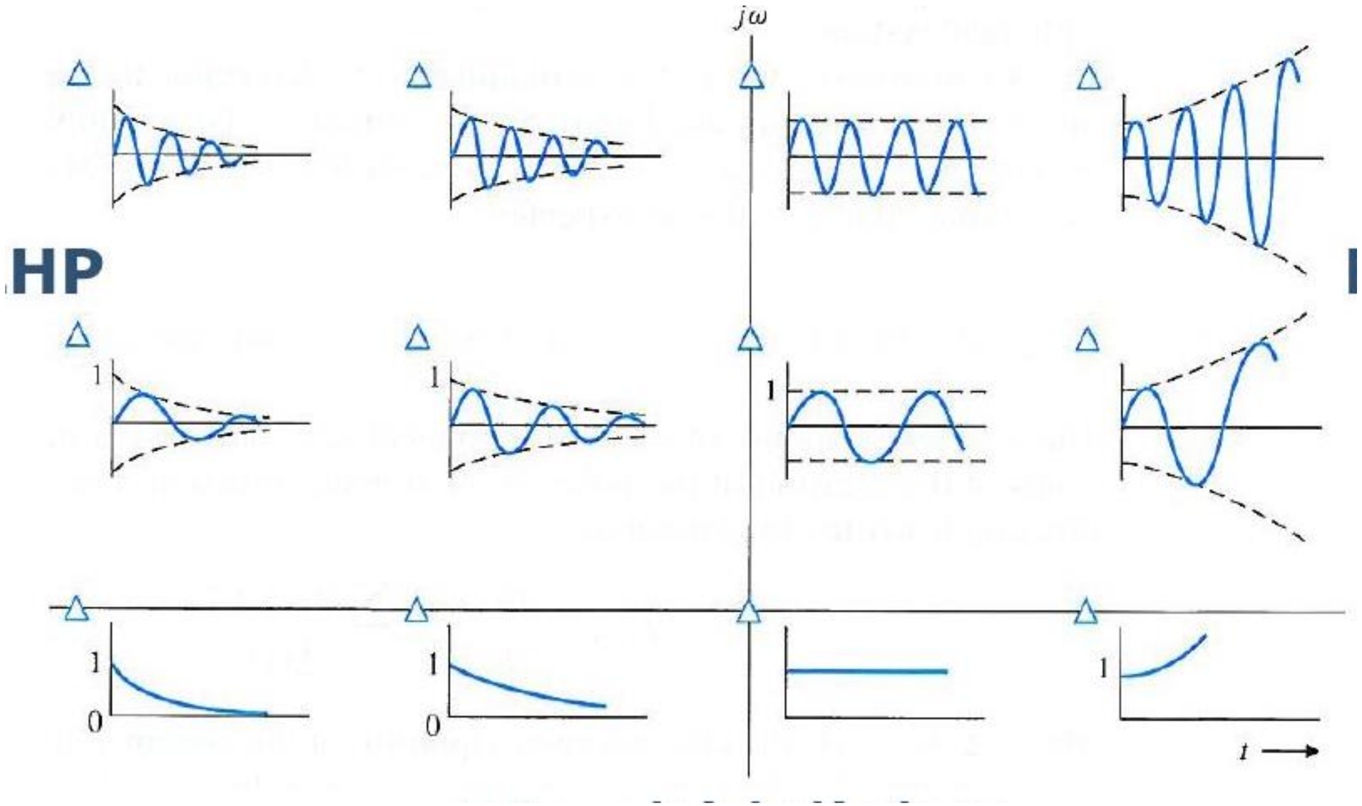
s^8	1	12	39	48	20
s^7	1	22	59	38	0
s^6	$-\cancel{10}\ -1$	$-\cancel{20}\ -2$	$\cancel{10}\ \ 1$	$\cancel{20}\ \ 2$	0
s^5	$\cancel{20}\ \ 1$	$\cancel{60}\ \ 3$	$\cancel{40}\ \ 2$	0	0
s^4	1	3	2	0	0
s^3	$\cancel{0}\ \ \cancel{4}\ \ 2$	$\cancel{0}\ \ \cancel{6}\ \ 3$	$\cancel{0}\ \ \cancel{0}\ \ 0$	0	0
s^2	$\frac{3}{\cancel{2}}\ \ 3$	$\cancel{2}\ \ 4$	0	0	0
s^1	$\frac{1}{3}$	0	0	0	0
s^0	4	0	0	0	0

Chapter 7: Root-Locus Method

Eng. Fadwa Momani

Introduction

- It is well known that the transient response of a feedback system is closely related to the locations of the closed-loop poles.



Introduction

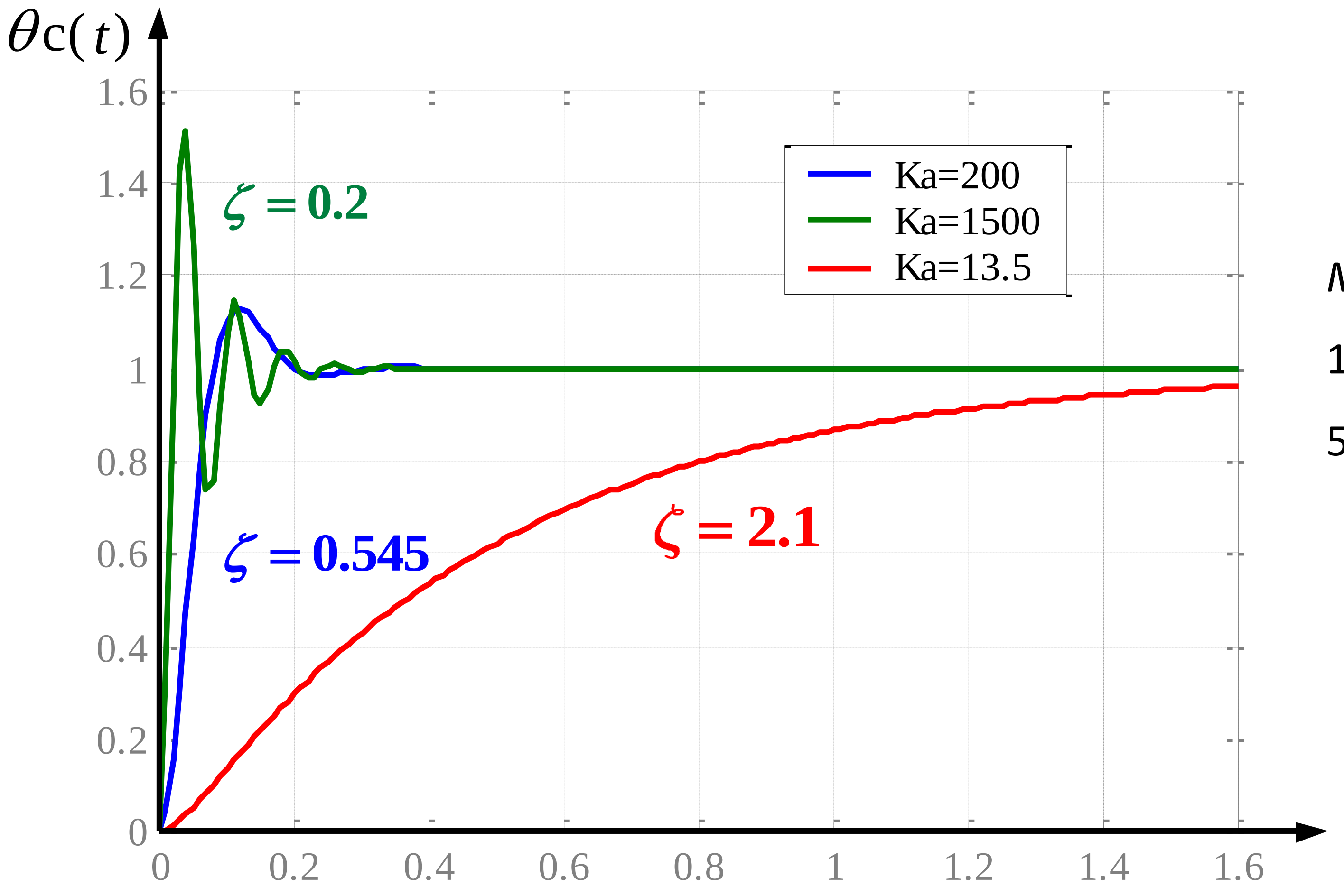
- From the design viewpoint, for some systems, simple gain adjustment may move the closed -loop poles to desired locations. For example, consider a unity-feedback system with open-loop transfer function

$$G(s) = \frac{5K_{\mathrm{A}}}{s(s+34.5)}$$

$$G(s) = \frac{5K_A}{s^2 + 34.5s + 5K_A}$$

HW1

For a 2nd order unit step response. Take open-loop gains with the values K_A=13.5, 200 and 1500, respectively, the system exhibits different responses:



M_p: 0,

13%,

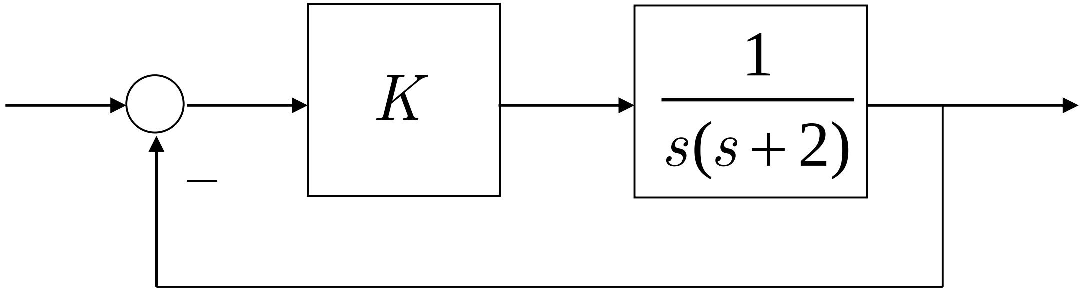
52.7%.

Therefore, it is important to determine how the roots of the characteristic equation move around the s-plane as we change the *open-loop gain*.

Definition: The root locus is the path of the roots of the characteristic equation traced out in the s-plane as a system parameter varies from 0 to $+\infty$.

Remark: The root locus analysis aims to investigate the closed-loop stability and the system controller design through *the open-loop transfer function* with variation of a certain system parameter, commonly the open-loop gain.

Example. Determine the closed-loop root loci when K varies from 0 to $+\infty$.



$$\boxed{G(s) = \frac{K}{s^2 + 2s + k}}$$

The closed-loop characteristic equation is:

$$s^2 + 2s + K = 0 \Rightarrow s_{1,2} = -1 \pm \sqrt{1 - K}$$

1) $K = 0 \quad s_1 = 0 \quad s_2 = -2$

2) $K = 1 \quad s_1 = s_2 = -1$

3) $K = 2 \quad s_{1,2} = -1 \pm j1$

4) $K \to \infty \quad s_{1,2} = -1 \pm j\infty$

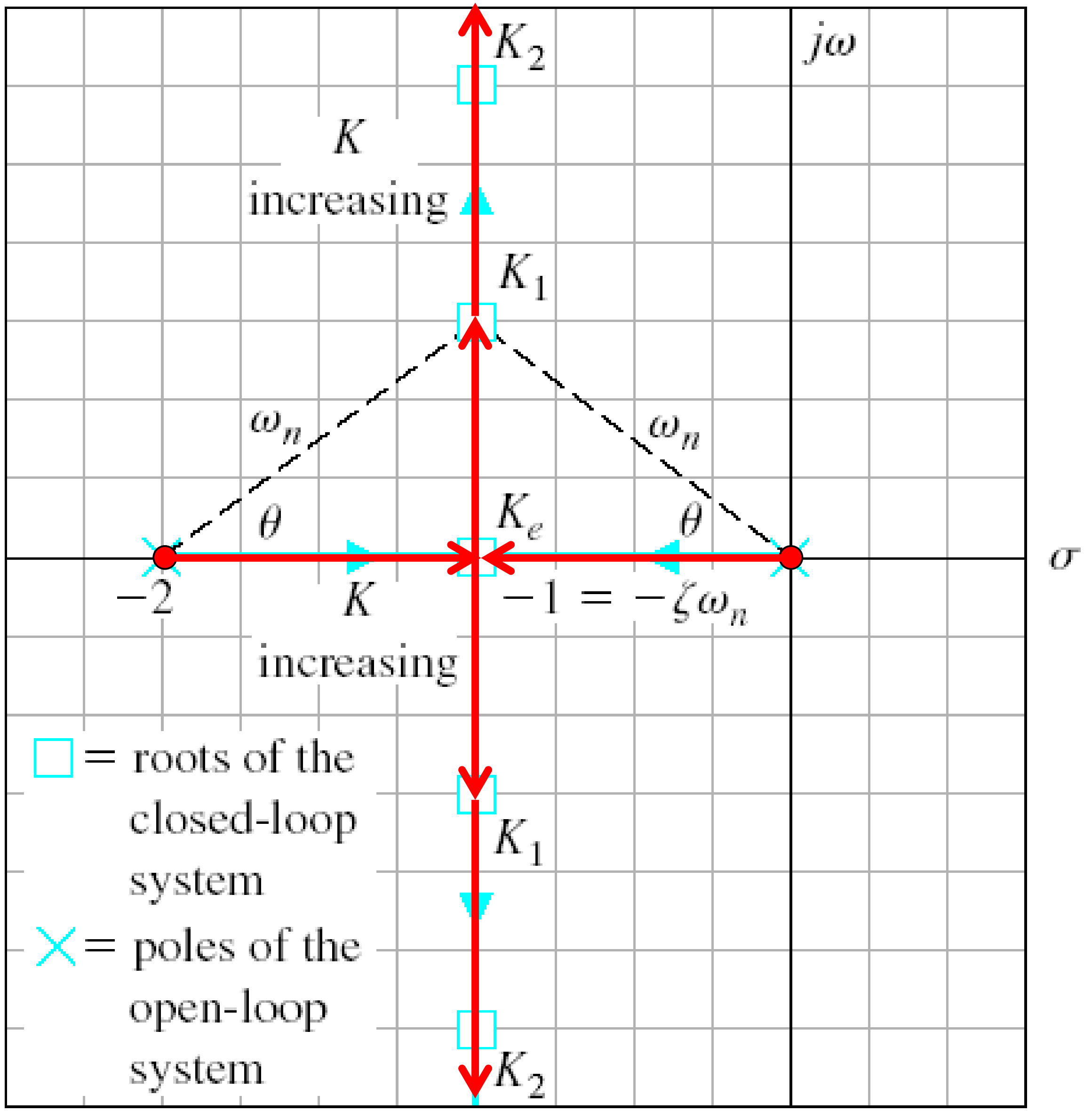
$$s_{1,2} = -1 \pm \sqrt{1-K}$$

$$1)\,K = 0 \quad s_1 = 0 \quad s_2 = -2$$

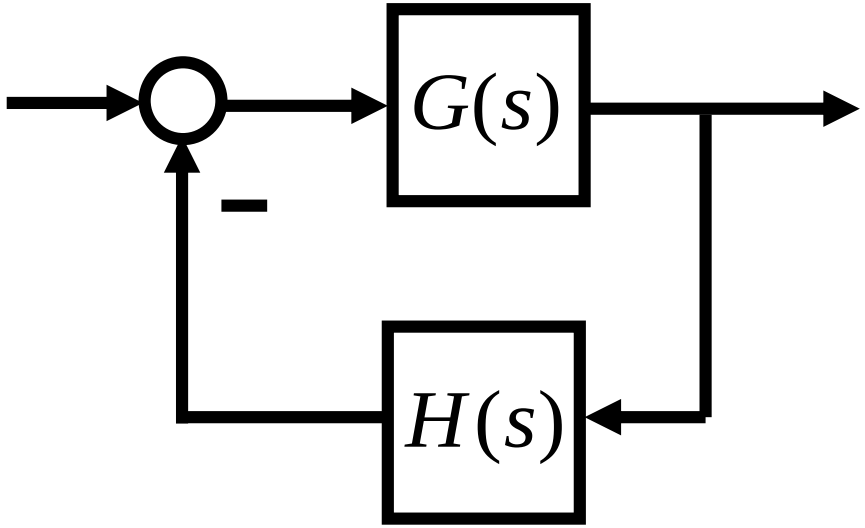
$$2)\,K = 1 \quad s_1 = s_2 = -1$$

$$3)\,K = 2 \quad s_{1,2} = -1 \pm j1$$

$$4)\,K \to \infty \quad s_{1,2} = -1 \pm j\infty$$



Root Locus Plots



Angle and Magnitude Conditions

$$\frac{C(s)}{R(s)} = \frac{G(s)}{1+G(s)H(s)}$$

Let us write GH as follows:

$$G(s)H(s) = \underbrace{\frac{K^*(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}}_{GH}$$

So the characteristic equation is equal to:

$$1+G(s)H(s)=0$$

$$G(s)H(s) = 1+ \underbrace{\frac{K^*(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}}_{GH} =0$$

The characteristic equation is defined as

$$1+G(s)H(s)=0$$

or

$$G(s)H(s)=-1$$

which can be split into two equations:

Angle Condition:

$$\angle G(s)H(s)=180^0(2k+1), \quad (k=0,\pm1,\pm2,\ldots)$$

Magnitude Condition:

$$\left|G(s)H(s)\right|=1$$

More precisely, we can write the characteristic equation as

$$G(s)H(s) = -1 \Rightarrow \underbrace{\frac{K^*(s+z_1)(s+z_2)\cdots(s+z_m)}{(s+p_1)(s+p_2)\cdots(s+p_n)}}_{GH} = -1$$

It is easy to see that *the magnitude condition can always be satisfied by a suitable* $K^* \geq 0$. Thus, the key is to find all those points that satisfy the angle
condition:

$$\angle G(s)H(s) = 180^0(2k+1)$$

As an example consider that the system in the previous figure have the following loop gain:

$$G(s)H(s) = \frac{K}{(s+1)(s+2)} = \frac{K}{s^2+3s+2}$$

The characteristic equation for the system is given by:

$$\Delta = 1 + G(s)H(s) = 1 + \frac{K}{s^2+3s+2} = \frac{\boxed{s^2+3s+(2+K)}}{s^2+3s+2} = 0$$

The numerator is in fact the denominator of the closed loop system TF. The roots of this polynomial are the closed loop poles of the system.

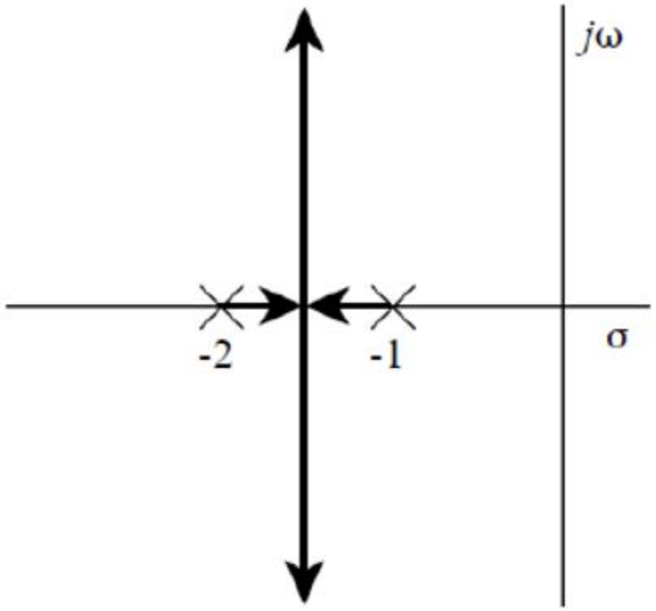
As K varies between zero and infinity, the closed loop poles of the system are changing and can be calculated as follows:

K	s_1	s_2
0	-2	-1
0.25	- 1.5	- 1.5
0.5	$-1.5 - j\,0.5$	$-1.5 + j\,0.5$
1	$-1.5 - j\,0.866$	$-1.5 + j\,0.866$
2	$-1.5 - j\,1.3229$	$-1.5 + j\,1.3229$
4	$-1.5 - j\,1.9365$	$-1.5 + j\,1.9365$
10	$-1.5 - j\,3.1225$	$-1.5 + j\,3.1225$
100	$-1.5 - j\,9.9875$	$-1.5 + j\,9.9875$
1000	$-1.5 - j\,31.6188$	$-1.5 + j\,31.6188$

$$\Delta = s^2 + 3s + (2 + K) = 0$$

The plot of system poles as they move throughout the s-domain when K varies between zero and infinity are as shown below:

The root locus is nothing but the path of the system closed loop poles as the gain of the system K varies between zero and infinity.



Since the root locus represent the path of the roots of the characteristic equation as the gain varies from zero to infinity, it follows that every point on the root locus must satisfy the characteristic equation, namely;

$$\Delta = 1 + G(s)H(s) = 0$$

I. Magnitude Condition:

$$|G(s)H(s)| = 1$$

$$\frac{K|(s+z_1)||(s+z_2)|\cdots\cdots|(s+z_m)|}{|(s+p_1)||(s+p_2)|\cdots\cdots|(s+p_n)|} = 1$$

For a given point on Root locii,
s=a+jb

$$\therefore \left.\frac{|(s+p_1)||(s+p_2)|\cdots\cdots|(s+p_n)|}{|(s+z_1)||(s+z_2)|\cdots\cdots|(s+z_m)|}\right|_{s=a+jb} = K$$

II. Angle Condition:

$$\angle G(s)H(s) = -1$$
$$= \pm 180^\circ (2k+1) \qquad \text{For } k = 0, 1, 2, \cdots$$

This equation can be rewritten as:

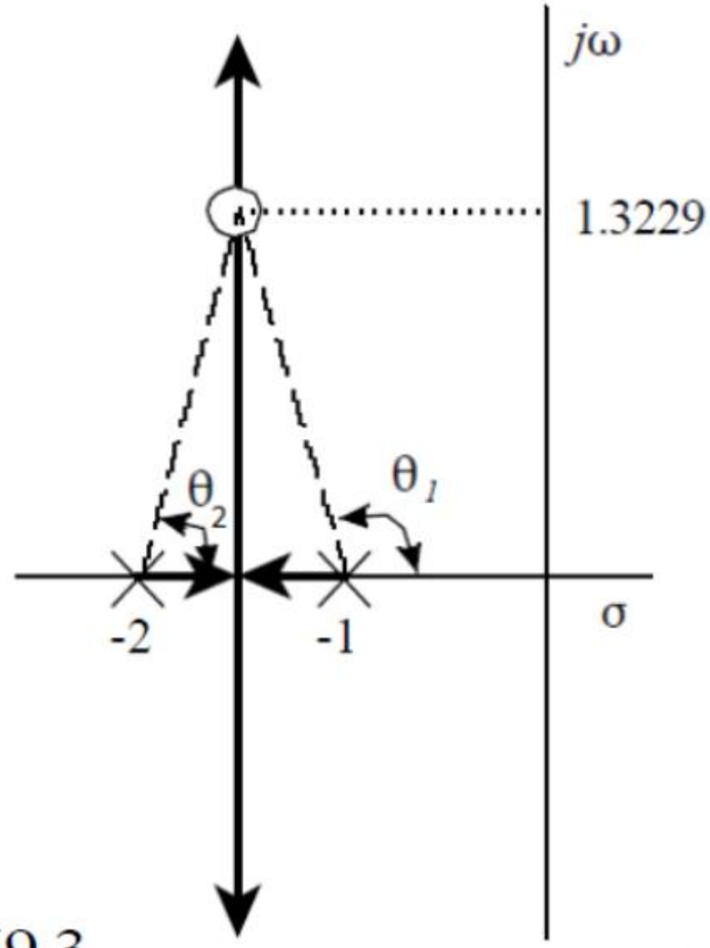
$$\angle G(s)H(s) = \angle s+z_1 + \angle s+z_2 + \angle s+z_3 + \cdots\cdots + \angle s+z_m - \angle s+p_1 - \angle s+p_2 - \cdots\cdots - \angle s+p_n$$
$$= \pm 180^\circ (2k+1)$$

For a given point on Root locii,
s=a+jb

For the system analyzed before and shown below at the point given by -1.5 ± *j 1.3229, the* angles of the vectors from that point to the two poles are calculated as follows:

$$\theta_1 = \tan^{-1}\frac{1.3229}{-1.5+1} = 110.70446^\circ$$

$$\theta_2 = \tan^{-1}\frac{1.3229}{-1.5+2} = 69.29554$$



Angle Condition,

$$\angle G(s)H(s) = \angle zeros - \angle poles$$
$$= 0 - \angle s+1 - \angle s+2$$
$$= -\theta_1 - \theta_2 = -110.7 - 69.3$$
$$= -180$$

Magnitude Condition,

$$\left|G(s)H(s)\right|_{-1.5+j1.3229} = 1$$

$$\left|\frac{K(s+z_1)(s+z_2)\ldots\ldots}{(s+p_1)(s+p_2)\ldots\ldots}\right| = 1$$

$$\therefore K = \left|(s+1)(s+2)\right|_{-1.5+j1.3229}$$

$$= 2$$

The point -1.5 ± *j 1.3229* satisfied the both conditions since it is on root locus

Before starting the steps of sketching Root locus, we have to know

1. **The Start and End Points of a Root Locus**
2. **Number of segments (branches) of root locus**
3. **Location of root locus segments on real axis**

The characteristic equation can be rearranged as follows:

$$(s+p_1)(s+p_2)\cdots\cdots(s+p_n) = K(s+z_1)(s+z_2)\cdots\cdots(s+z_m)$$

When $K = 0$, the last equation becomes;

$$(s+p_1)(s+p_2)\cdots\cdots(s+p_n) = 0$$

This indicates that the root locii starts at the poles of the system when $K = 0$.

The characteristic equation can also be arranged as follows:

$$\frac{(s+p_1)(s+p_2)\cdots\cdots(s+p_n)}{K} = (s+z_1)(s+z_2)\cdots\cdots(s+z_m)$$

When *K is at infinity* the above equation becomes;

$$0 = (s+z_1)(s+z_2)\cdots\cdots(s+z_m)$$

This indicates that the root locii ends at the zeros of the system when $K=\infty$

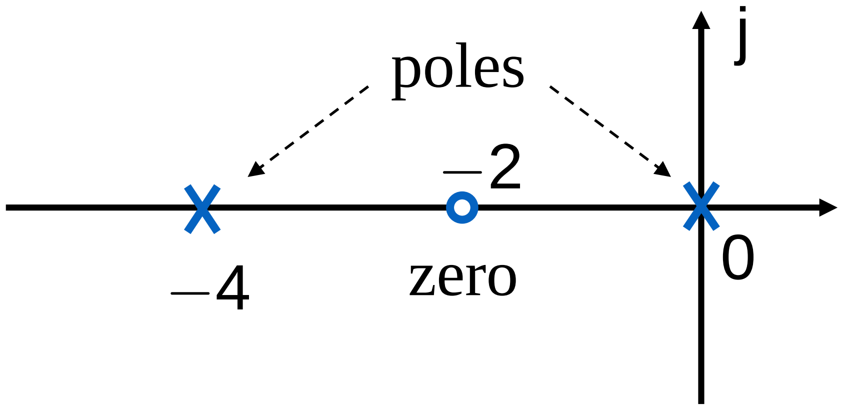
Root locii starts at the system poles (when $K=0$ *) and ends at the system zeros* (when $K=\infty$ *).*

Example. Second-order system

$$\frac{Zeros}{Poles}$$

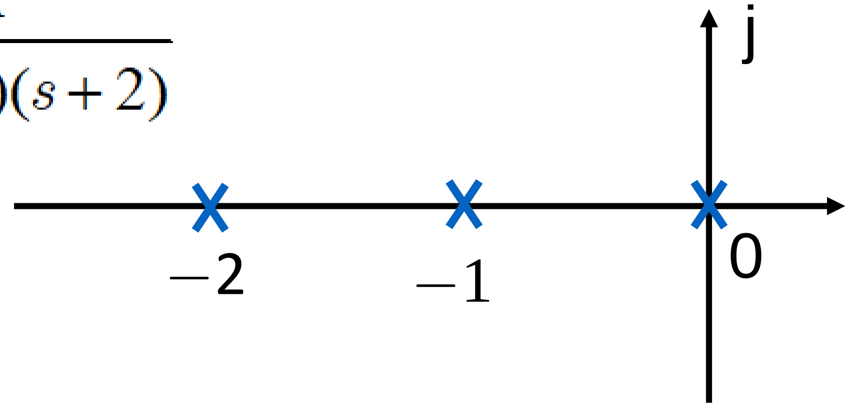
$$Zeros \rightarrow 0$$
$$Poles \rightarrow x$$

$$G(s)H(s) = \frac{K^*(s+2)}{s(s+4)}$$



Example. For a unity FB system The open-loop transfer function is

$$G(s) = \frac{K^*}{s(s+1)(s+2)}$$



Root Loci Construction Rules

If the number of poles in the loop gain equation is larger than the number of zeros, this means that a number of root locii segments equal to the number of poles and zeros will be ending at infinity.

Root Loci Construction Rules

Note that for root loci, the following facts are true:

- The number of root locus branches is equal to the order of the characteristic equation → # of poles.

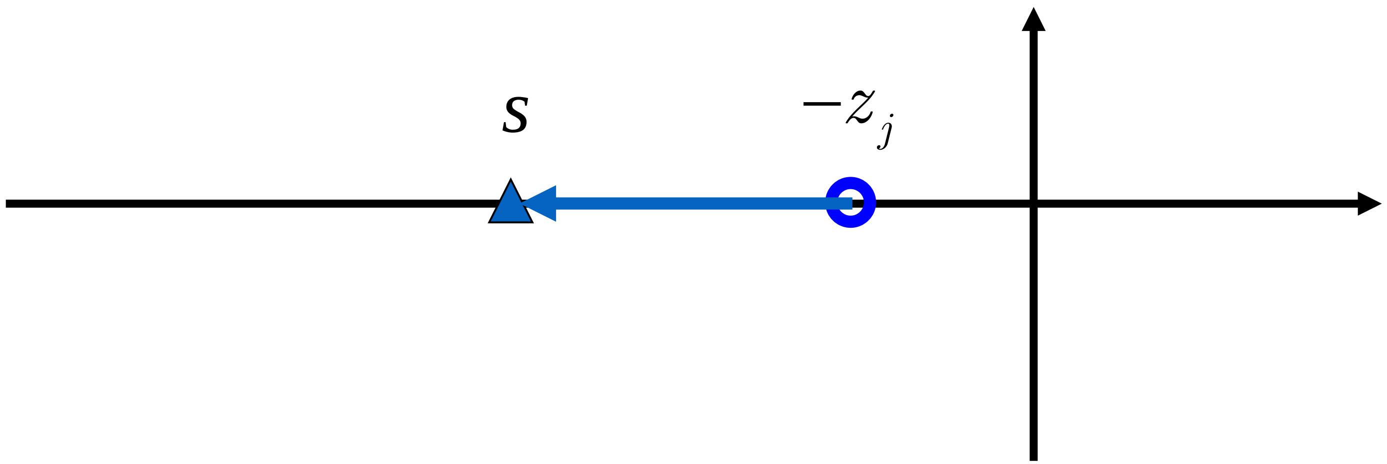
- The loci are symmetrical about the real axis.

The root locus is symmetrical about the real axis since the roots of $1+G(s)H(s)=0$ must either be real or appear as complex conjugates. Therefore, we only need to construct the upper half of the root loci and draw the mirror image of the upper half in the lower-half s-plane.

Root locii segments on the real axis can be found by applying the angle criteria as follows:

Let s be a test point on the real axis as shown below. Since zero $-z_j$ of $G(s)H(s)$ lies to the right of s, it follows that

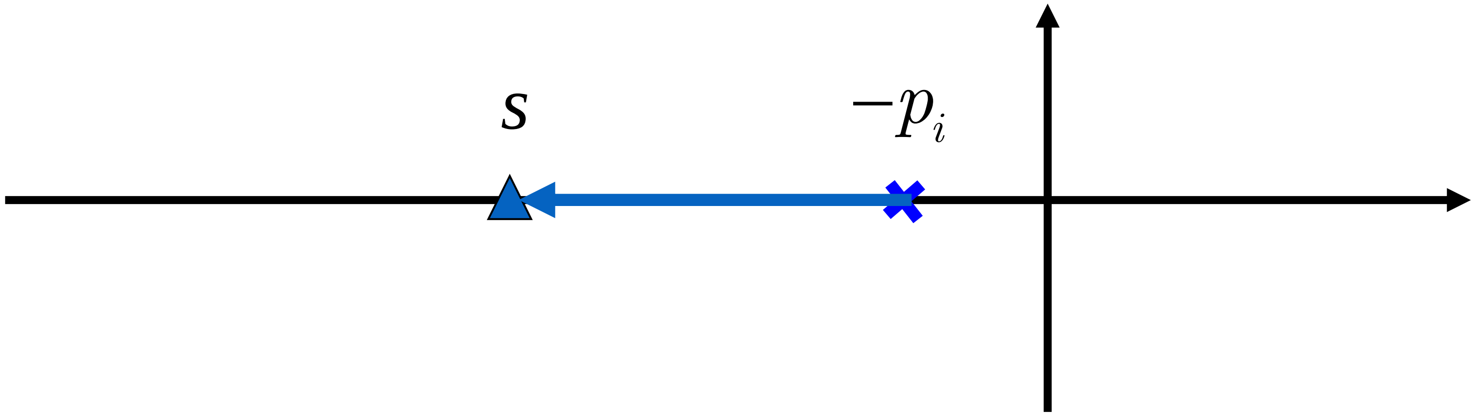
$$\angle(s+z_j)=180^0$$



Therefore, root locus exists on $(-\infty, -z_j]$.

Let s be a test point on the real axis as shown below. Since the pole $-p_i$ of $G(s)H(s)$ lies to the right of s, it follows that

$$\angle(s+p_i) = 180^0$$

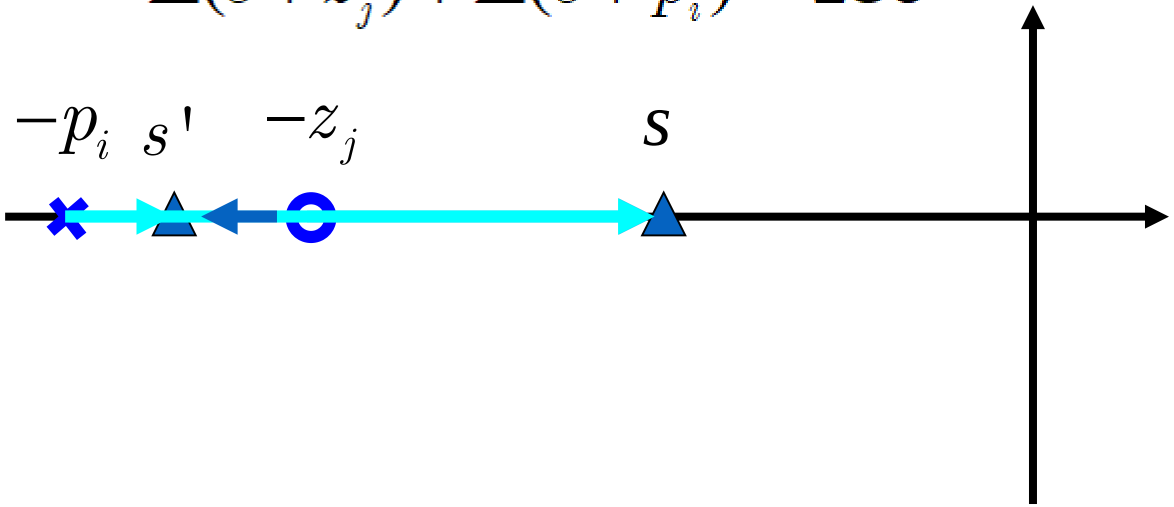


Therefore, root locus exists on $(-\infty, -p_i]$.

Whereas, let *s* be a test point as shown below. (the pole $-p_i$ and zero $-z_j$ lies to the left of *s*, it follows that

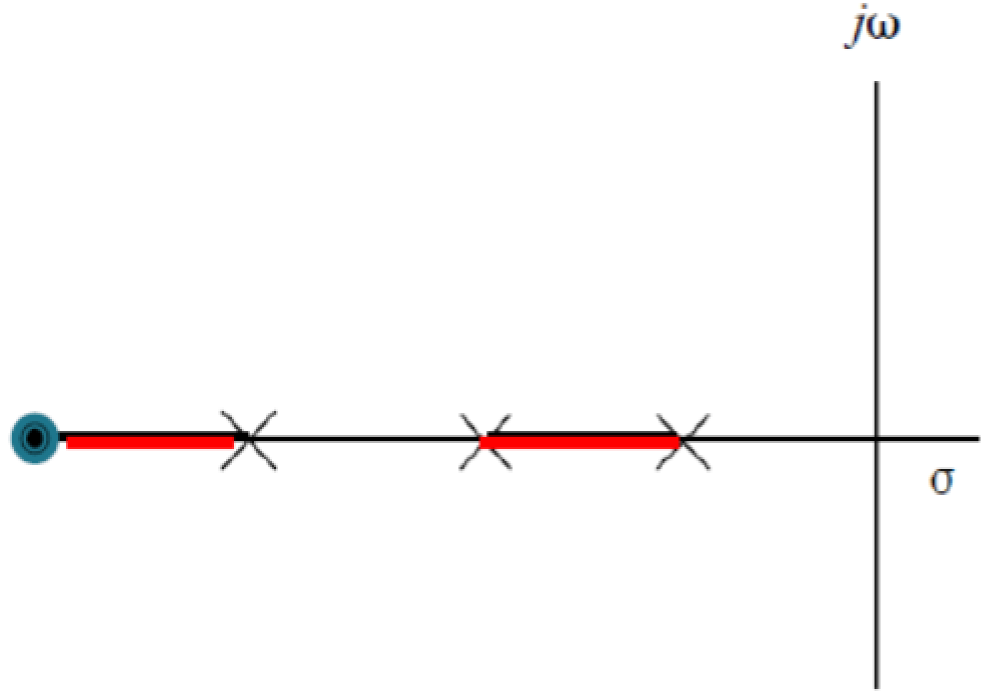
$$\angle(s+z_j)+\angle(s+p_i)=0^0$$

$$\angle(s+z_j)+\angle(s+p_i)=180$$



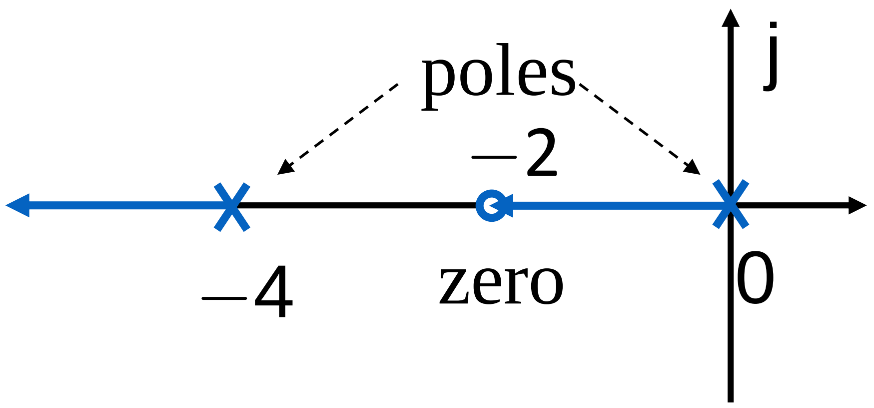
Therefore, no root locus exists on $[-z_j, +\infty)$. However, by the rule, root locus exists on $[-p_i, -z_j]$ since for the test point *s*', angle condition holds.

Root locii on the real axis will occur to the left of odd number of poles and zeros .



Example. Second-order system:

$$G(s)H(s) = \frac{K^*(s+2)}{s(s+4)}$$



Steps of Constructing Root Locus of a System:

1- Write the characteristic equation of the system in the following standard form

$$\Delta = 1 + K\frac{(s+z_1)(s+z_2).....(s+z_m)}{(s+p_1)(s+p_2).....(s+p_n)} = 0$$

Where *K* might be a controller gain (or system gain) and is the parameter of interest

2- Locate all poles p_1 , $p_2 , ... p_n$ and zeros z_1 , $z_2 , ... z_m$ in s-plane.

3- Determine the root locus segments on the real axis

Rule 4. Asymptotes of root loci: The loci proceed to the zeros at infinity along asymptotes.

These linear asymptotes are centered at a point on the real axis given by (Number of Asym. Lines = n-m)

$$\sigma_a = \frac{\sum_{j=1}^{n}(-p_j) - \sum_{i=1}^{m}(-z_i)}{n-m} = \frac{\sum \text{poles of } GH - \sum \text{zeros of } GH}{n-m}$$

The angle of the asymptotes with respect to the real axis is

$$\varphi_a = 180^0 \times \frac{(2k+1)}{n-m} \qquad (k = 0, \pm 1, \cdots, \pm n-m-1)$$

$$\alpha_a = \frac{\pm 180(2k+1)}{n-m},$$

Example: An open-loop transfer function of a unity-feedback system is

$$G(s) = \frac{K^*(s+2)}{s(s+1)(s+3)} \qquad 1 + G(s) = 1 + \frac{K^*(s+2)}{s(s+1)(s+3)}$$

Sketch the root locus plot.

→Zeros ; -2 m=1

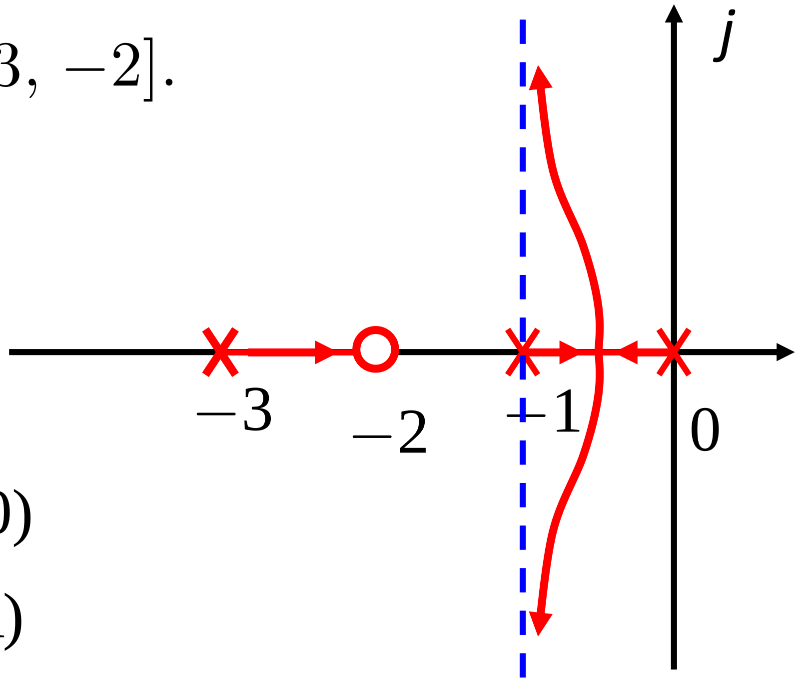
→Poles ; 0, -1, -3 n=3

•root loci exist on $[-1, 0]$ and $[-3, -2]$.

for the asymptotes:

$$\sigma_a = \frac{(-1-3)-(-2)}{3-1} = -1$$

$$\varphi_a = 180^0 \times \frac{2k+1}{3-1} = \begin{cases} 90^0 & (k=0) \\ 270^0 & (k=1) \end{cases}$$



Example: A unity-feedback system with open-loop transfer function as

$$G(s) = \frac{K^*}{s(s+1)(s+2)}$$

Sketch the root locus plot.

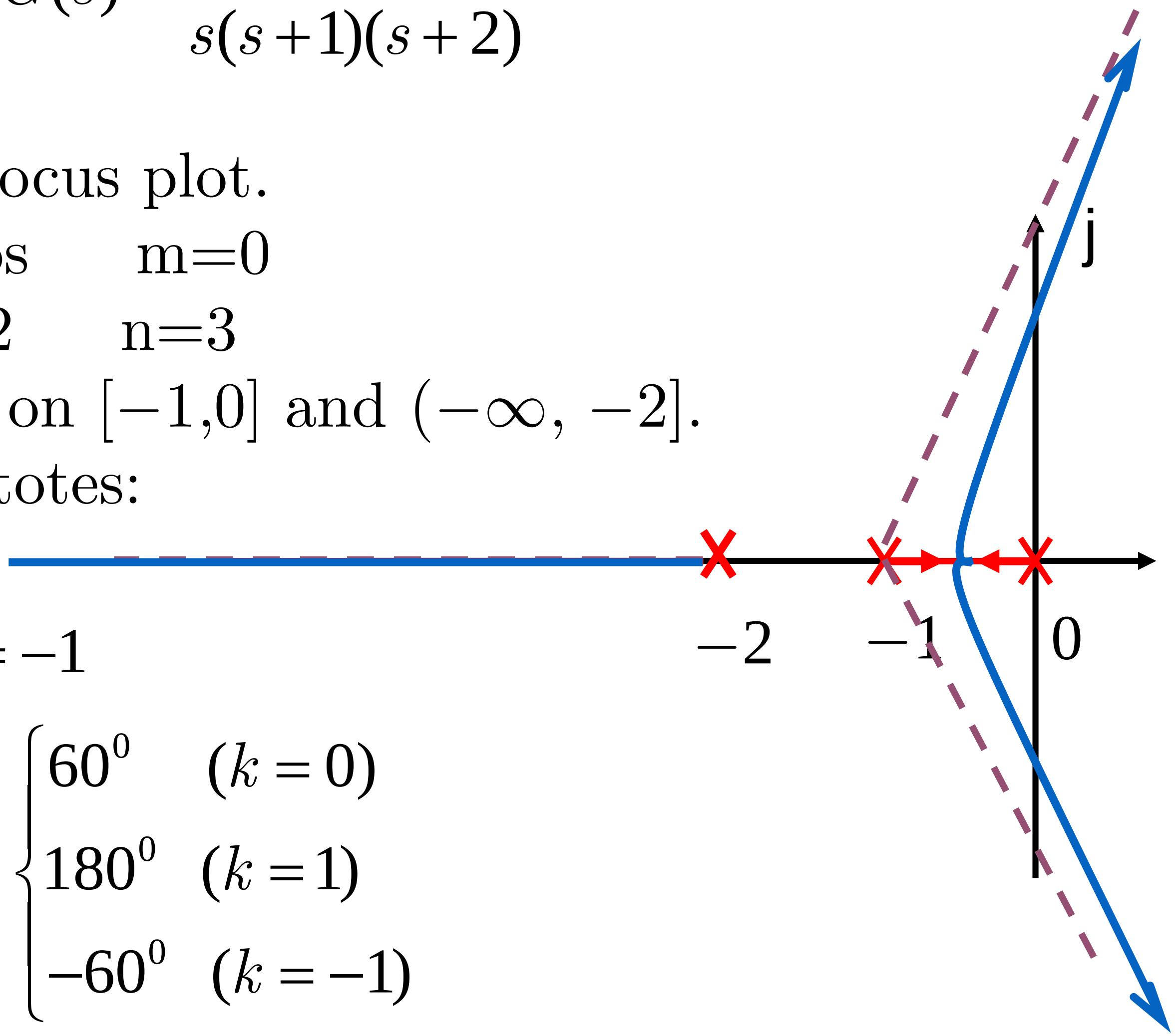
→Zeros ; no zeros m=0

→Poles ; 0, -1, -2 n=3

- root loci exist on $[-1,0]$ and $(-\infty, -2]$.
- for the asymptotes:

$$\sigma_a = \frac{(-1-2)-0}{3} = -1$$

$$\varphi_a = 180^0 \times \frac{2k+1}{3} = \begin{cases} 60^0 & (k=0) \\ 180^0 & (k=1) \\ -60^0 & (k=-1) \end{cases}$$



Example. Consider the open-loop transfer function

$$G(s)=\frac{K^*}{s(s+4)(s^2+4s+20)}$$

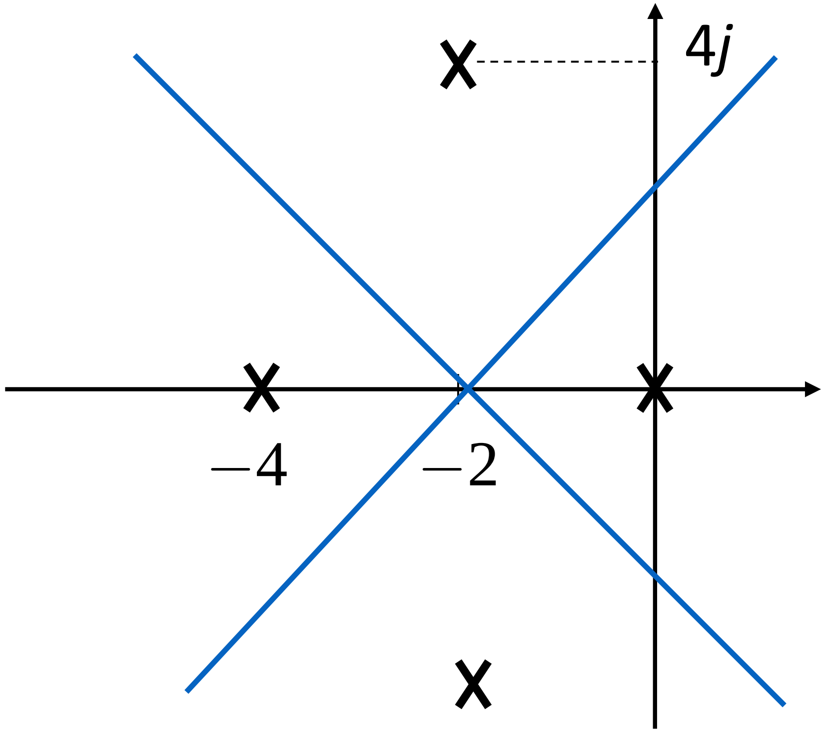
four open-loop poles are: $0,\ -4,\ -2\pm 4j$

Sketch its root locus plot.

- root locus exists on $[-4,\ 0]$.
- we obtain the asymptotes:

$$\sigma_a=\frac{-4-2+4j-2-4j-0}{4}=-2$$

$$\varphi_a=180^0\times\frac{2k+1}{4}=\begin{cases}45^0 & (k=0)\\ 135^0 & (k=1)\\ -45^0 & (k=-1)\\ -135^0 & (k=-2)\end{cases}$$



Rule 5. Breakaway (break in) point on the root loci.

Rearrange the characteristic equation then find the break away/in points that should result from

$$\frac{dK}{ds} = 0$$ Why ???!!

Example. Again, consider the open-loop transfer function

$$G(s) = \frac{K^*}{s(s+1)(s+2)}$$

→ Zeros ; no zeros m=0

→ Poles ; 0, -1, -2 n=3

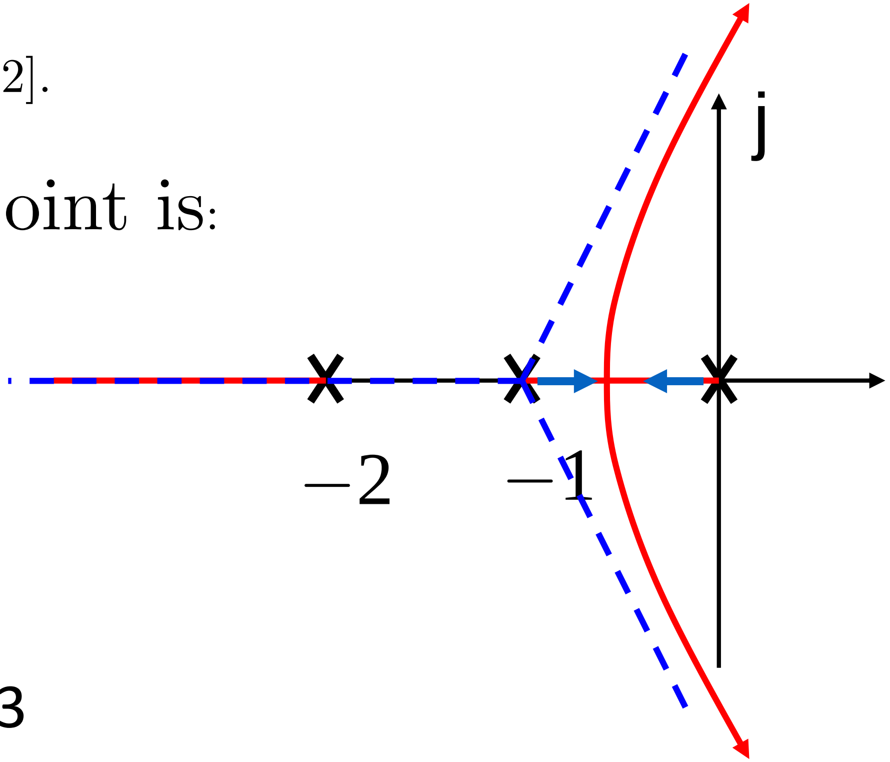
- root loci exist on $[-1, 0]$ and $(-\infty, -2]$.
- we obtain the asymptotes.
- By rule 4, Break away point is:

$$K = -[s(s+1)(s+2)]$$

$$K = -[s^3 + 3s^2 + 2s]$$

$$\frac{dK}{ds} = -[3s^2 + 6s + 2]$$

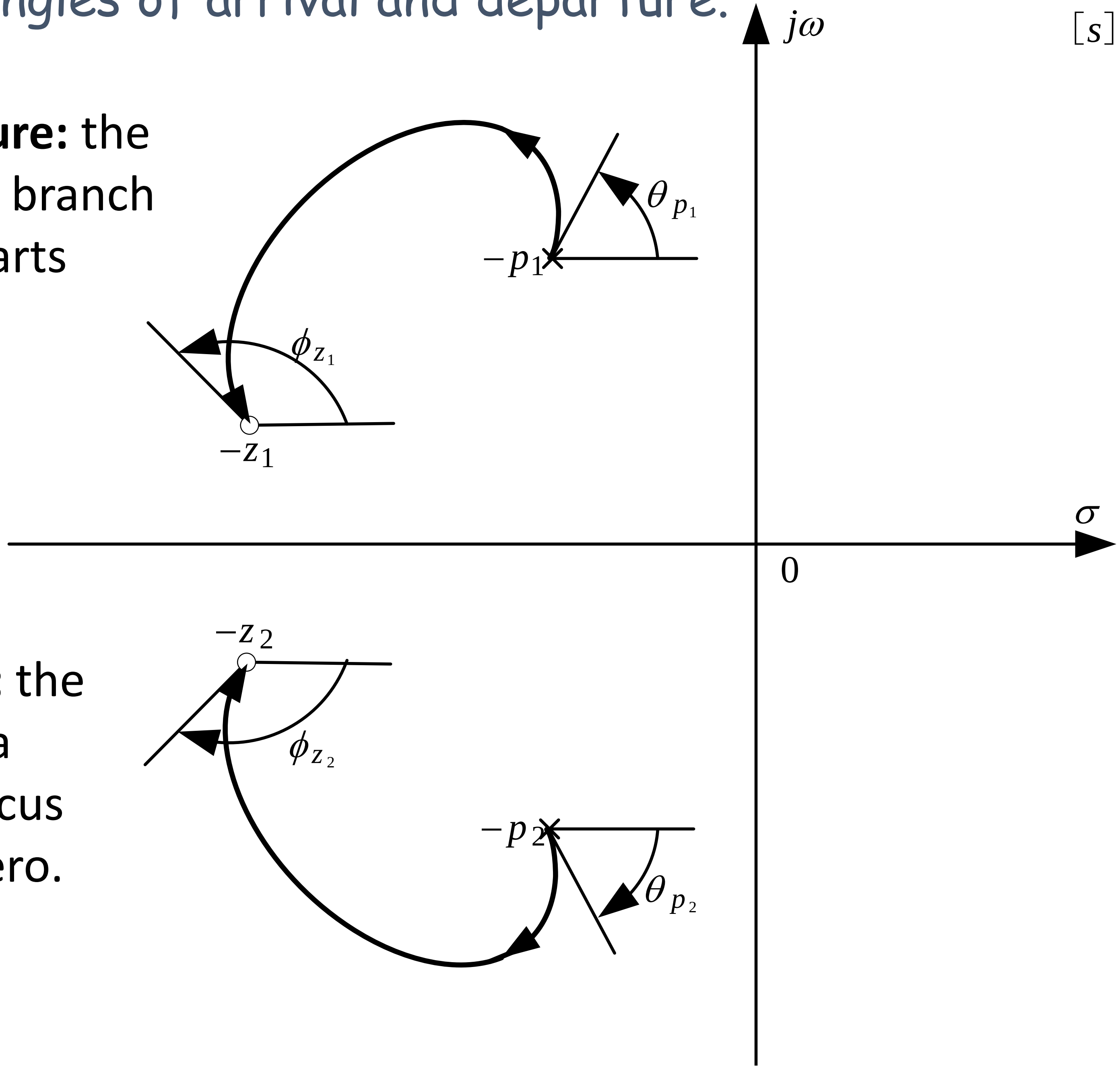
$$\frac{dK}{ds} = 0 \rightarrow s = -0.4226 , -1.5773$$



Rule 5. The angles of arrival and departure.

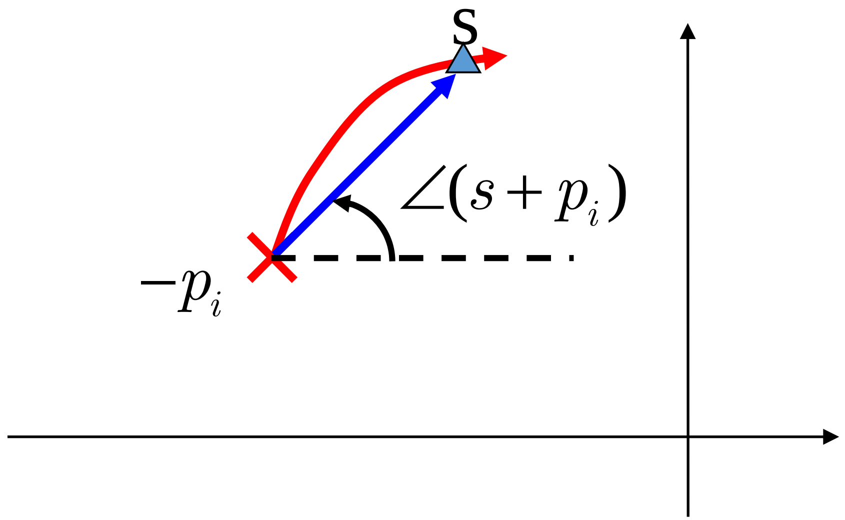
Angle of departure: the angle by which a branch of the locus departs from one pole.

Angle of arrival: the angle by which a branch of the locus arrives at one zero.



1) Angle of departure

Choose a test point *s* and move it in the very vicinity of $-p_i$. Then, if *s* is on the root locus, the angle condition must be satisfied:



$$\sum_{j=1}^{m}\angle(s+z_j)-\sum_{k=1}^{n}\angle(s+p_k)=180^0\times(2k+1)$$

Example. The open-loop transfer function

$$G(s)H(s) = \frac{K^*}{s(s+1+j)(s+i-j)}$$

Therefore,

$$-p_1 = 0, \quad -p_2 = -1+j, \quad -p_3 = -1-j$$

By using the departure angle formula,

$$Tan^{-1}(\frac{1}{1}) = 45^{\circ}$$

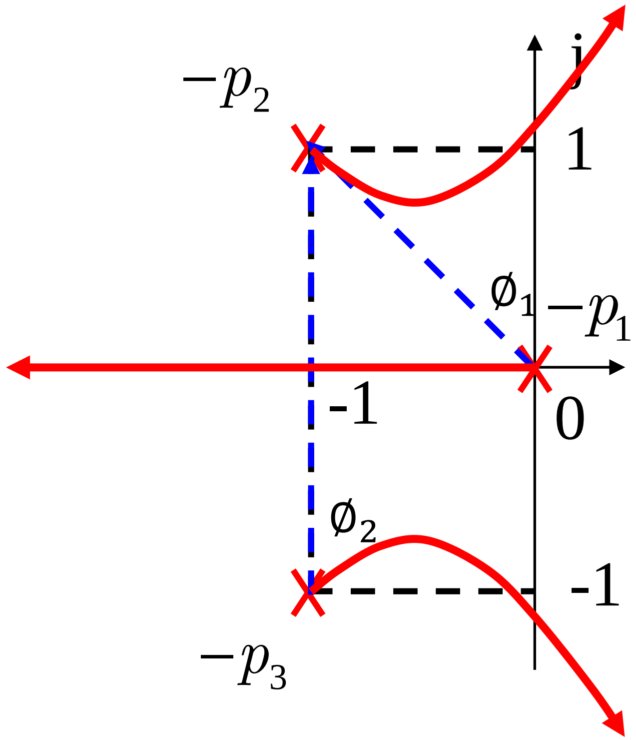
$$\emptyset_1 = 180^{\circ} - 45^{\circ} = 135^{\circ}$$

$$\emptyset_2 = 90^{\circ}$$

$$\theta_{p_2} = 180^0 - 135^0 - 90^0 = -45^0$$

Due to the symmetry property of the root locus,

$$\theta_{p_3} = 45^0$$



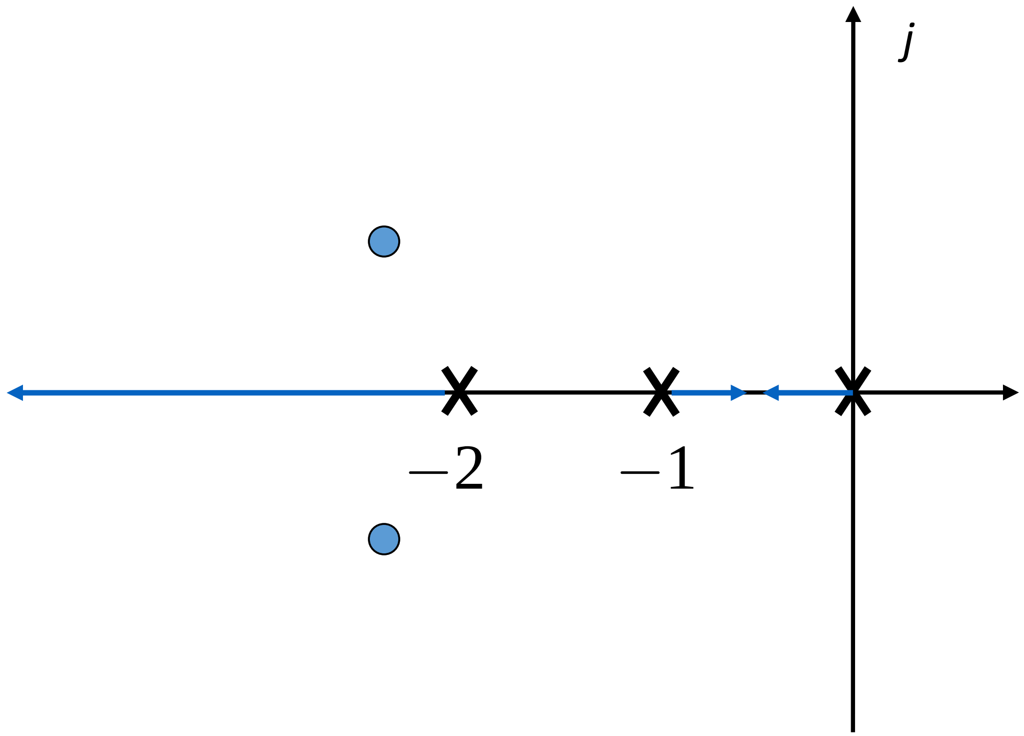
SELF STUDY ANGLE OF ARRIVAL

Example. The open-loop transfer function

$$G(s)H(s) = \frac{K^*(s^2 + 4.5s + 5.625)}{s(s+1)(s+2)}$$

$z_1 = -2.25 + j0.75$, $z_2 = -2.25 - j0.75$

- root loci exist on $[-1, 0]$, and $(-\infty, -2]$. By rule 1, the root locus from -2 to $-\infty$ can be determined.
- the breakaway point is (Find it) HW.2

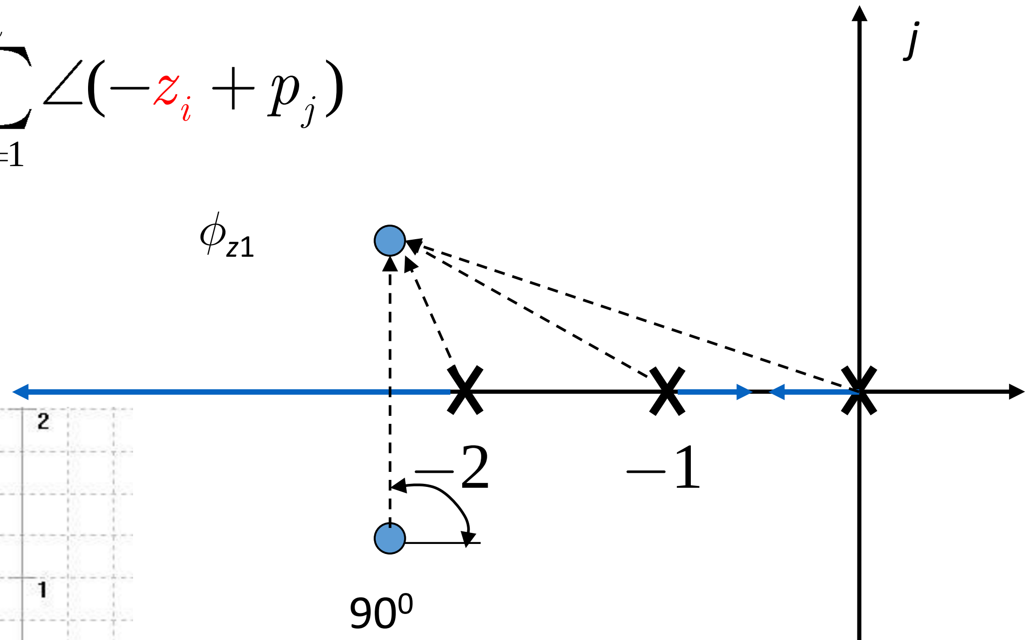


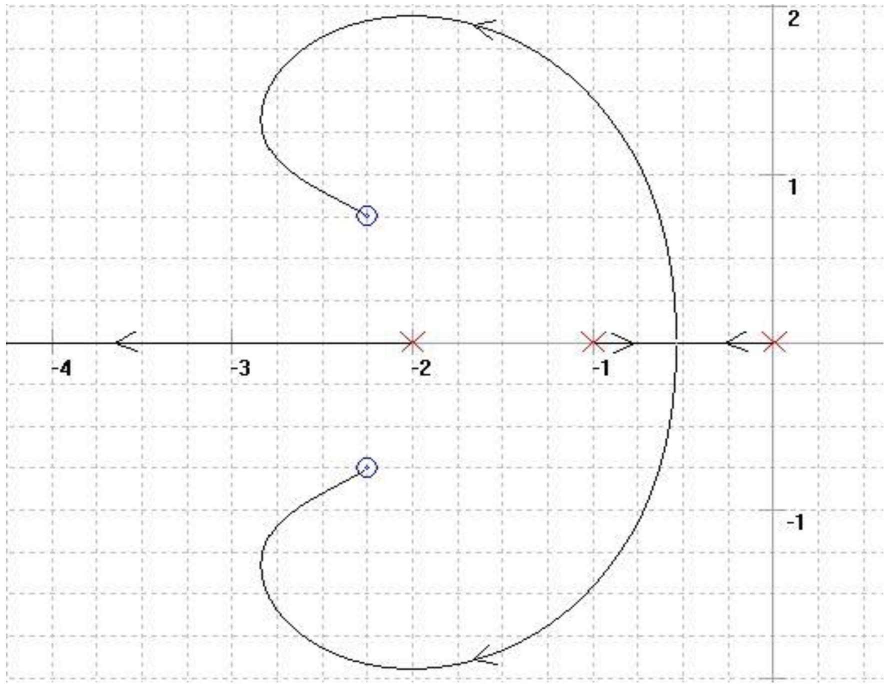
The two points that breakaway at 90^0.

By rule 5, the angle of arrival is

$$\phi_{z_1} = 180^0 - \sum_{\substack{j=1 \\ j \neq i}}^{m} \angle(-z_i + z_j) + \sum_{j=1}^{n} \angle(-z_i + p_j)$$

$$= 180^0 - 90^0 + \theta_1 + \theta_2 + \theta_3$$





Rule 6. Intersection of the root loci with the imaginary axis.

$$G(s) = \frac{K^*}{s(s+1)(s+2)}$$

The closed-loop characteristic equation is (Tell me how did we find this equ. H.W. 3)

$$s^3 + 3s^2 + 2s + K^* = 0$$

Using Routh-Herwitz Criteria

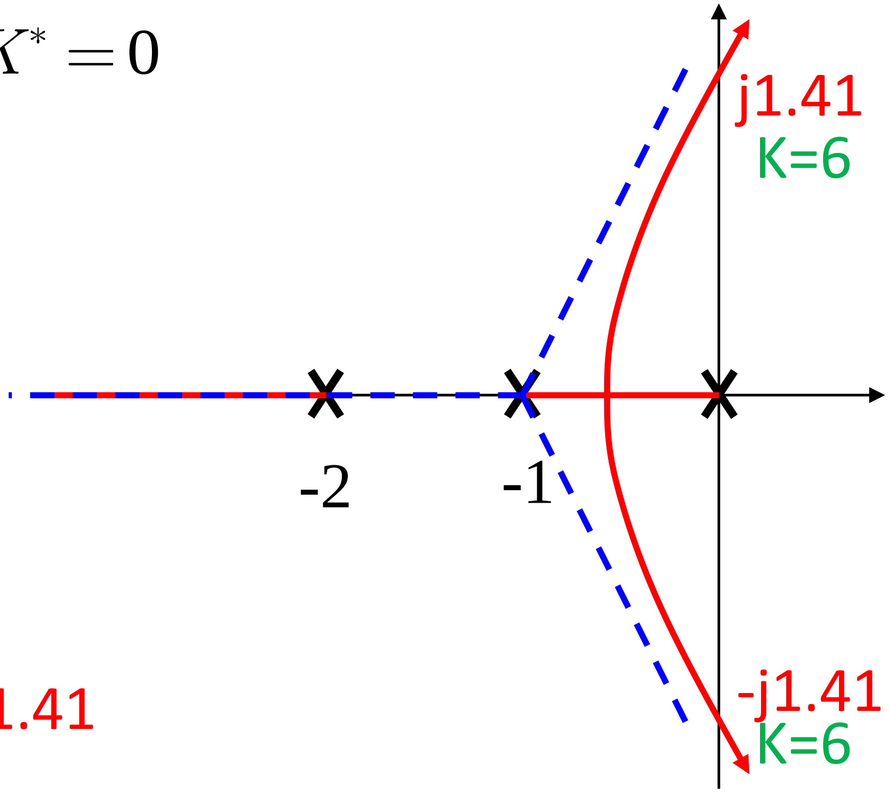
S^3	1	2
S^2	3	K
S^1	(6-K)/3	0
S^0	K	0

For marginally Stable → Poles on Jw axis

(6-K)/3 = 0 → K=6

$$s^3 + 3s^2 + 2s + 6 = 0$$

→ S= -3 ±j1.41

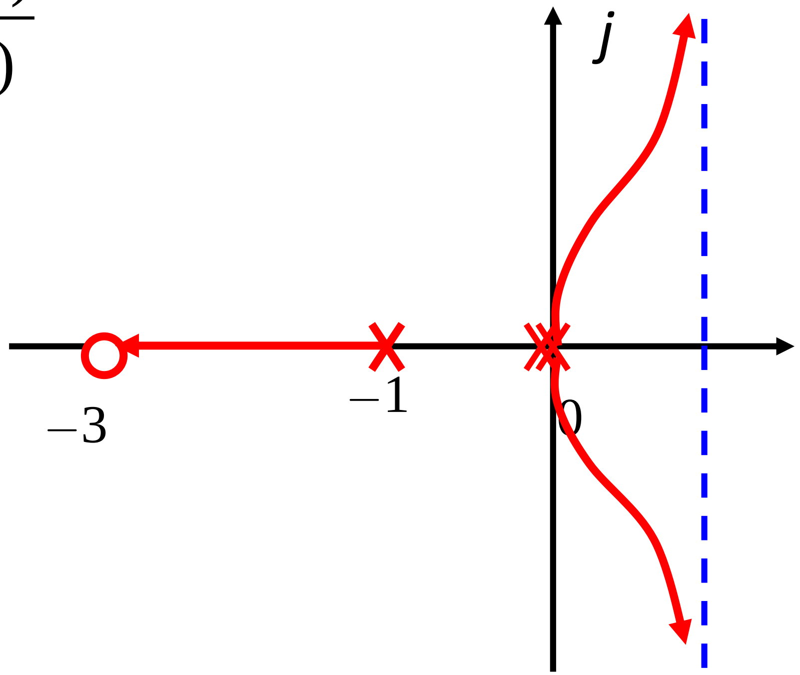


Example. An open-loop transfer function of a unity-feedback system is given below:

$$G(s) = \frac{K^*(s+3)}{s^2(s+1)}$$

Sketch the root locus plot.

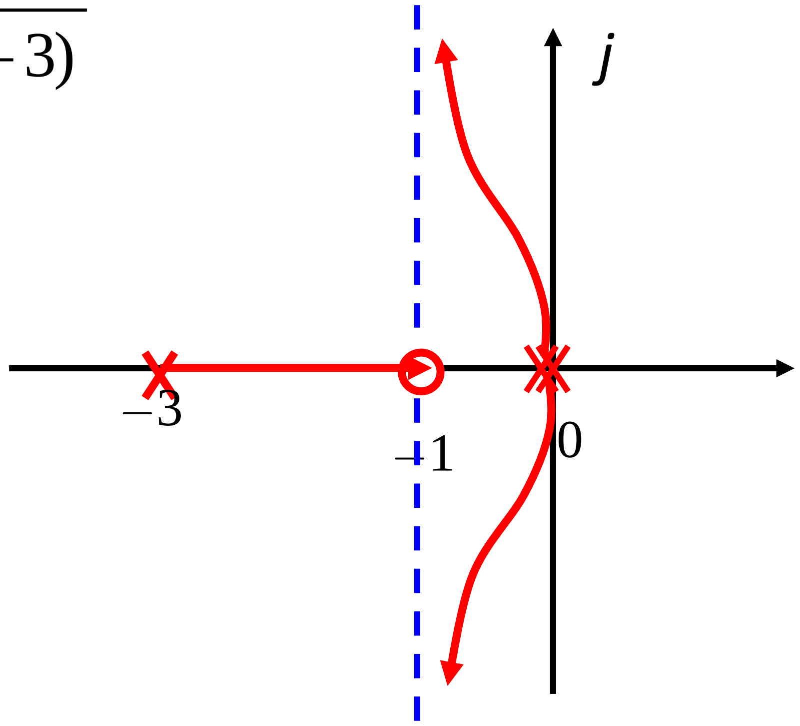
- root locus exists on $[-3, -1]$.
- for the asymptotes,



$$\sigma_a = \frac{-1+3}{2} = 1 \qquad \varphi_a = 180^0 \times \frac{2k+1}{2} = \begin{cases} 90^0 & (k=0) \\ -90^0 & (k=-1) \end{cases}$$

Example. An open-loop transfer function of a unity-feedback system is given below:

$$G(s) = \frac{K^*(s+1)}{s^2(s+3)}$$



Sketch the root locus plot.

- root locus exists on $[-3, -1]$.
- for the asymptotes,

$$\sigma_a = \frac{-3+1}{2} = -1 \qquad \varphi_a = 180^0 \times \frac{2k+1}{2} = \begin{cases} 90^0 & (k=0) \\ -90^0 & (k=-1) \end{cases}$$

Table 7.2 Seven Steps for Sketching a Root Locus

Step	Related Equation or Rule
1. Prepare the root locus sketch.	
(a) Write the characteristic equation so that the parameter of interest, K, appears as a multiplier.	$1 + KP(s) = 0.$
(b) Factor $P(s)$ in terms of n poles and M zeros.	$1 + K\dfrac{\prod_{i=1}^{M}(s + z_i)}{\prod_{j=1}^{n}(s + p_j)} = 0.$
(c) Locate the open-loop poles and zeros of $P(s)$ in the s-plane with selected symbols.	$\times$ = poles, $\bigcirc$ = zeros Locus begins at a pole and ends at a zero.
(d) Determine the number of separate loci, SL.	$SL = n$ when $n \geqslant M$; n = number of finite poles, M = number of finite zeros.
(e) The root loci are symmetrical with respect to the horizontal real axis.	
2. Locate the segments of the real axis that are root loci.	Locus lies to the left of an odd number of poles and zeros.
3. The loci proceed to the zeros at infinity along asymptotes centered at σ_A and with angles ϕ_A.	$\sigma_A = \dfrac{\sum(-p_j) - \sum(-z_i)}{n - M}.$ $\phi_A = \dfrac{2k + 1}{n - M}180^\circ, k = 0, 1, 2, \ldots (n - M - 1).$
4. Determine the points at which the locus crosses the imaginary axis (if it does so).	Use Routh–Hurwitz criterion (see Section 6.2).
5. Determine the breakaway point on the real axis (if any).	a) Set $K = p(s)$. b) Determine roots of $dp(s)/ds = 0$ or use graphical method to find maximum of $p(s)$.
6. Determine the angle of locus departure from complex poles and the angle of locus arrival at complex zeros, using the phase criterion.	$\angle P(s) = 180^\circ + k360^\circ$ at $s = -p_j$ or $-z_i$.
7. Complete the root locus sketch.	

$$1 + \frac{K}{s^4 + 12s^3 + 64s^2 + 128s} = 0.$$

$$1 + \frac{K}{s(s+4)(s+4+j4)(s+4-j4)} = 0$$

No Zeros, Poles = 0,-4,-4±j4

$$\sigma_A = \frac{-4-4-4}{4} = -3.$$

$$\phi_A = \frac{(2k+1)}{4}180°, \qquad k = 0, 1, 2, 3;$$

$$\phi_A = +45°, 135°, 225°, 315°.$$

Break away point

$$K = p(s) = -s(s+4)(s+4+j4)(s+4-j4) \quad s = -1.577,$$

Intersection with Jw

$$s(s+4)(s^2+8s+32) + K = s^4 + 12s^3 + 64s^2 + 128s + K = 0.$$

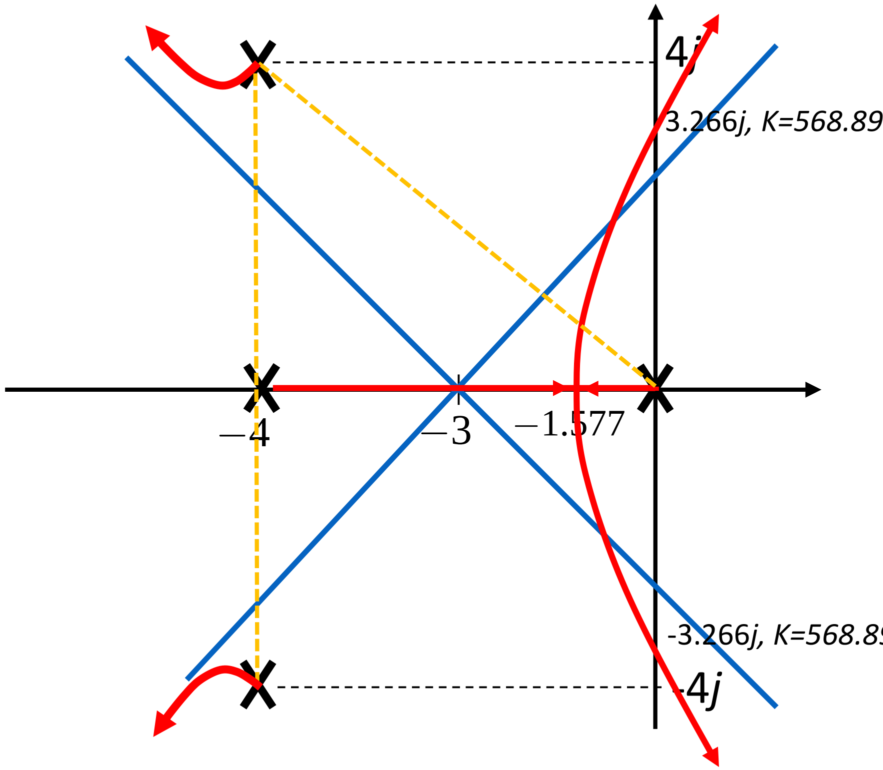
$$\begin{array}{c|ccc} s^4 & 1 & 64 & K \\ s^3 & 12 & 128 & \\ s^2 & b_1 & K & \\ s^1 & c_1 & & \\ s^0 & K & & \end{array} \qquad b_1 = \frac{12(64) - 128}{12} = 53.33 \quad \text{and} \quad c_1 = \frac{53.33(128) - 12K}{53.33}.$$

$$K = 568.89, \quad s = \pm j3.266.$$

Angle of departure

$$\theta_1 + 90° + 90° + \theta_3 = 180° + k360°. \quad \text{Since } \theta_3 = 135°,$$

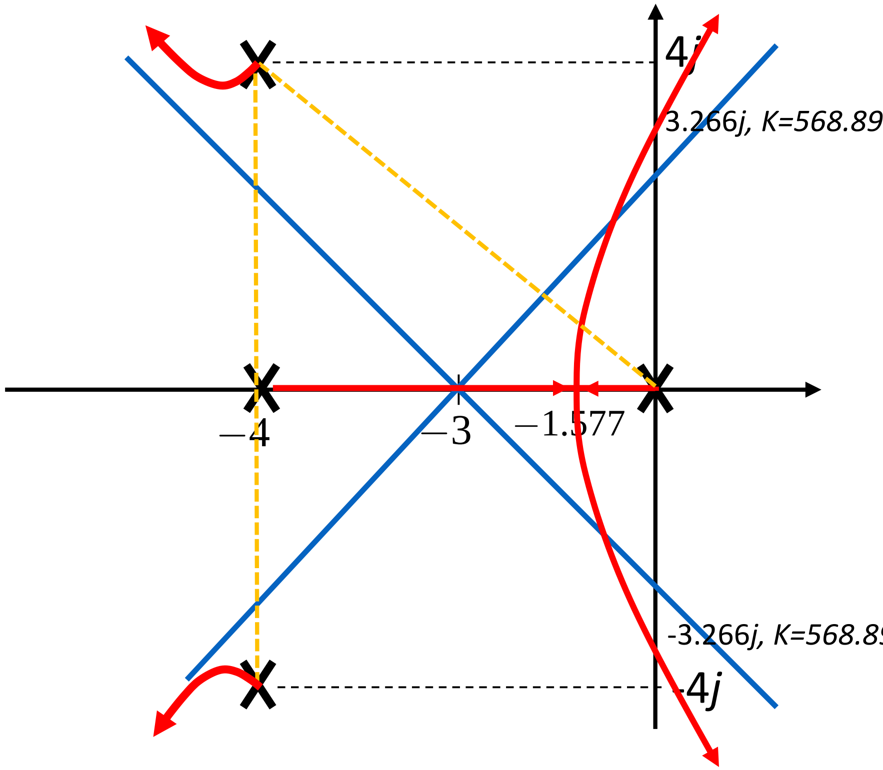
$$\theta_1 = -135° \equiv +225°,$$



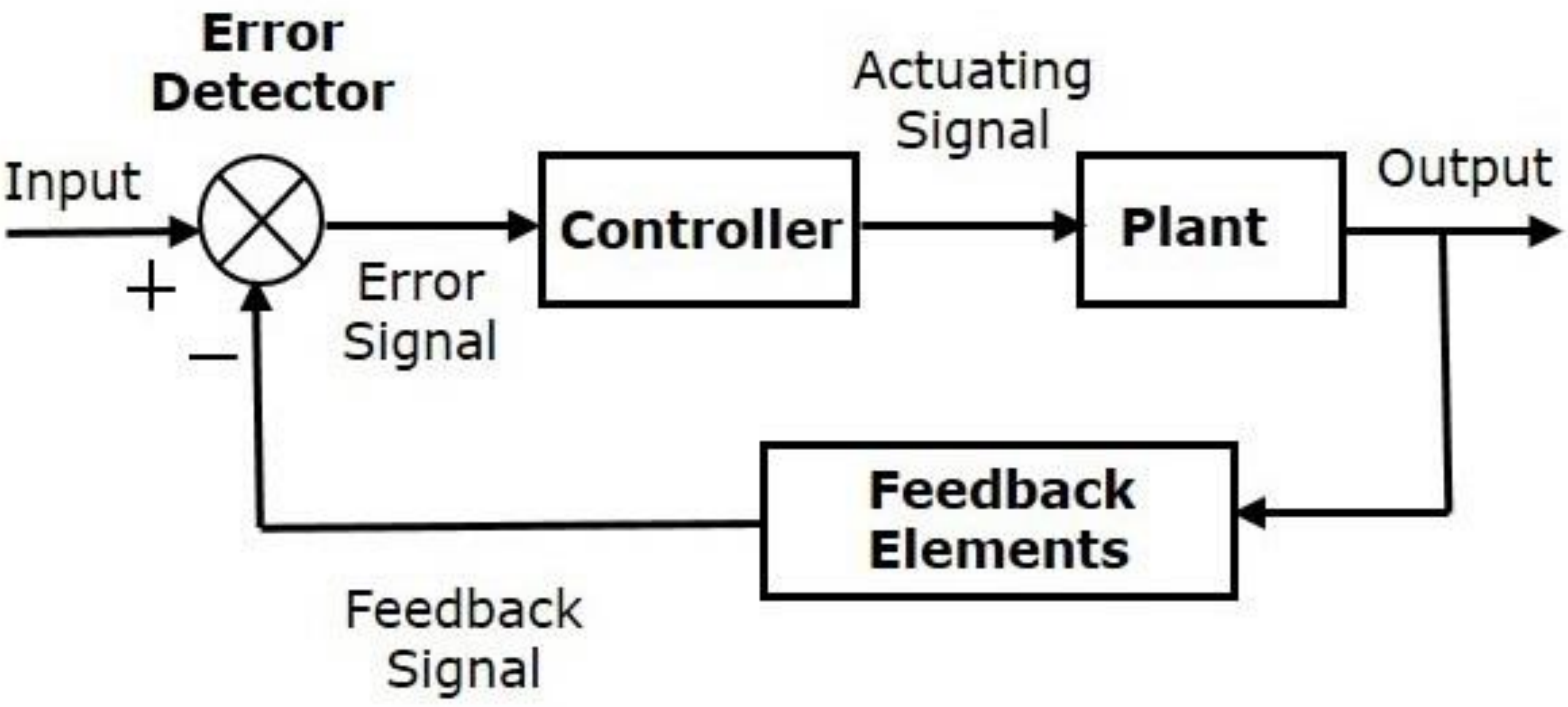
For the following R.L answer the following :

1. Order of the system?
2. Type of the system?
3. Breakaway point?
4. Center of asymp.?
5. Intersection with Jw?
6. Value of K at intersection with Jw?
7. K at S=-4?
8. K at s=-1.577?
9. K at s= -3.266J
10. K at s= -4+4j
11. The characteristic equation is?

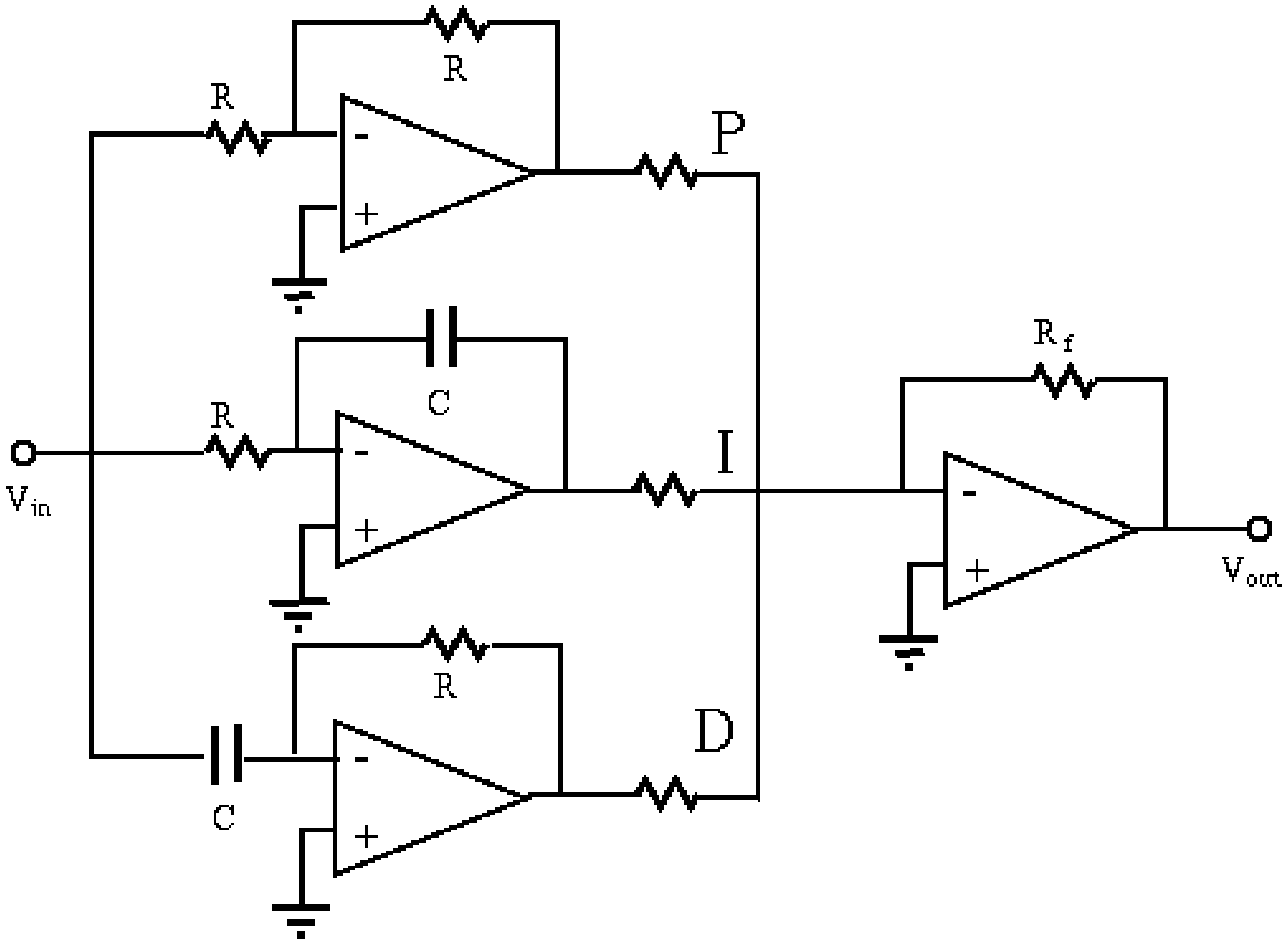
H.W.4



PID Controller



PID Controller

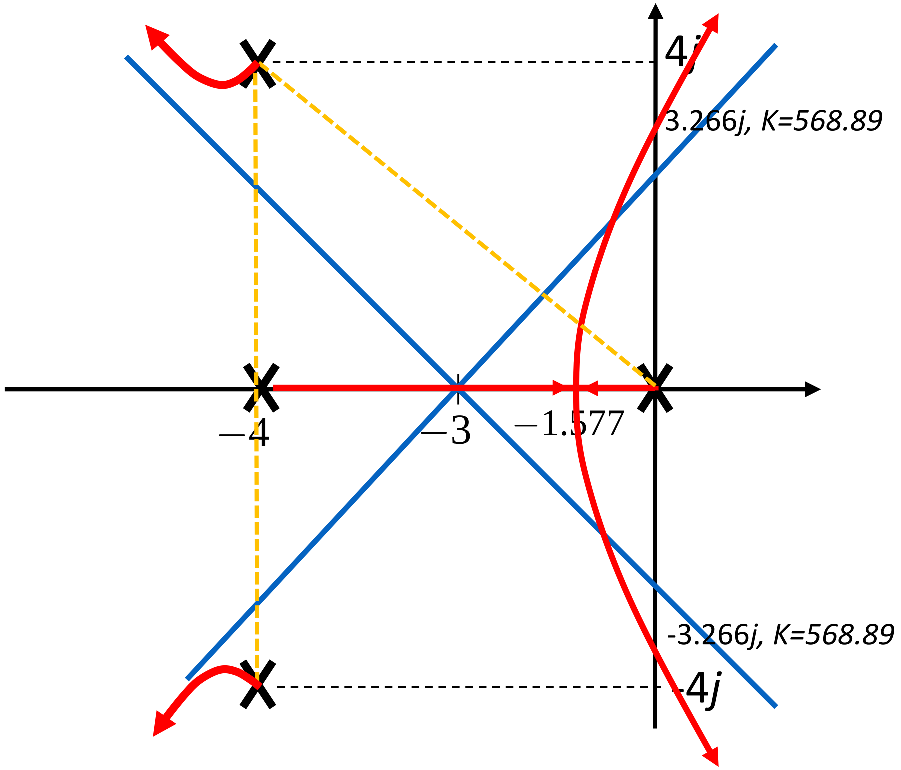


$$\boxed{G_c(s) = K_p + \frac{K_I}{s} + K_D s.}$$

$$= \frac{K_D s^2 + K_P s + K_I}{s}$$

Table 7.6 Effect of Increasing the PID Gains K_p, K_D, and K_I on the Step Response

PID Gain	Percent Overshoot	Settling Time	Steady-State Error
Increasing K_P	Increases	Minimal impact	Decreases
Increasing K_I	Increases	Increases	Zero steady-state error
Increasing K_D	Decreases	Decreases	No impact


4j
3.266j, K=568.89
−4
−3
−1.577
-3.266j, K=568.89
-4j