

Question:

What are the major differences between the behaviors of the electric force F_e and the magnetic force F_m ?

Answer:

Step 1

The three major differences between the behaviors of the electric force F_e and the magnetic force F_m are:

1. The electric force is always in the direction of the electric field, but the magnetic force is always perpendicular to the magnetic field.
2. The electric force acts on a charged particle whether or not it is moving, the magnetic force acts on a charged particle, only when it is in motion.
3. The electric force expends energy in displacing a charged particle, the magnetic force does no work when a particle is displaced.

Question:

The ends of a 10-cm-long wire carrying a constant current I are anchored at two points on the x -axis, $x = 0$ and $x = 6$ cm. If the wire lies in the xy -plane in a magnetic field $\mathbf{B} = B_0 \hat{y}$, which of the following arrangements produces a greater magnetic force on the wire: (a) wire is V-shaped with corners at $(0, 0)$, $(3, 4)$, and $(6, 0)$, (b) wire is an open rectangle with corners at $(0, 0)$, $(0, 2)$, $(6, 2)$, and $(6, 0)$.

Answer:

Step 1

Determine the magnetic force, \mathbf{F}_m of the arrangements using the following formula:

$$\mathbf{F}_m = I \int_a^b d\mathbf{l} \times \mathbf{B}$$

Here,

I is the current flowing in the wire

$d\mathbf{l}$ is the differential length

\mathbf{B} is the magnetic field

(a)

The wire is V-shaped with corners at $(0, 0)$, $(3, 4)$ and $(6, 0)$ as shown in Figure 1.

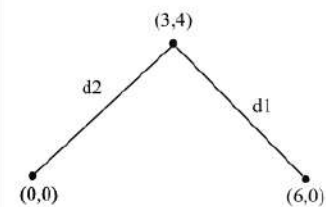


Figure 1

Step 2

Determine the distance, d_2 .

$$\begin{aligned}d_2 &= \sqrt{3^2 + 4^2} \\ &= 5 \text{ cm}\end{aligned}$$

Determine the unit vector along the direction of d_2 .

$$\frac{3\hat{x} + 4\hat{y}}{\sqrt{3^2 + 4^2}} = \frac{3\hat{x} + 4\hat{y}}{5}$$

As the length of the wire is 10 cm, the distance, d_1 is,

$$\begin{aligned}d_1 &= 10 - 5 \\ &= 5 \text{ cm}\end{aligned}$$

Determine the unit vector along the direction of d_1 .

$$\frac{3\hat{x} - 4\hat{y}}{\sqrt{3^2 + 4^2}} = \frac{3\hat{x} - 4\hat{y}}{5}$$

The magnetic field, $\mathbf{B} = \hat{y}B_0$

Determine the magnetic force, \mathbf{F}_m for the V-shaped wire.

$$\begin{aligned}\mathbf{F}_m &= I \int 5 \left(\frac{3\hat{x} + 4\hat{y}}{5} \right) \times \hat{y} B_0 + I \int 5 \left(\frac{3\hat{x} - 4\hat{y}}{5} \right) \times \hat{y} B_0 \\ &= B_0 I \int (3\hat{x} + 4\hat{y}) \times \hat{y} + B_0 I \int (3\hat{x} - 4\hat{y}) \times \hat{y} \\ &= 3B_0 I \hat{z} + 3B_0 I \hat{z} \\ &= 6B_0 I \hat{z}\end{aligned}$$

Thus, the magnetic force for the V-shaped wire is $6B_0 I \hat{z}$.

Step 3

(b)

The wire is an open rectangle with corners at $(0,0)$, $(0,2)$, $(6,2)$ and $(6,0)$ as shown in Figure 2.

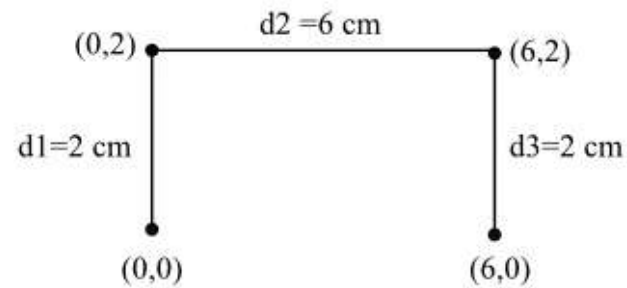


Figure 2

Step 4

The total length of the wire is 10 cm as shown in Figure 2.

The distance vector, $\mathbf{d1} = 2\hat{y}$

The distance vector, $\mathbf{d2} = 6\hat{x}$

The distance vector, $\mathbf{d3} = -2\hat{y}$

Determine the magnetic force, \mathbf{F}_m for the rectangular shaped wire.

$$\begin{aligned}\mathbf{F}_m &= I \int 2\hat{y} \times \hat{y}B_0 + I \int 6\hat{x} \times \hat{y}B_0 + I \int -2\hat{y} \times \hat{y}B_0 \\ &= 6B_0 I \hat{z}\end{aligned}$$

Thus, the magnetic force for an open rectangle shaped wire is equal to $6B_0 I \hat{z}$.

Hence, both arrangements produce the same magnetic force.

Question:

How is the direction of the magnetic moment of a loop defined?

Answer:

Step 1

The expression for magnetic moment of a loop is:

$$\begin{aligned}\mathbf{m} &= (\hat{\mathbf{n}})(NIA) \text{ A} \cdot \text{m}^2 \\ &= (\hat{\mathbf{n}})(m) \text{ A} \cdot \text{m}^2\end{aligned}$$

Where, $\hat{\mathbf{n}}$ is the surface normal of the loop and governed by the right hand rule: when the four fingers of the right hand advance in the direction of the current, the direction of the thumb specifies the direction of $\hat{\mathbf{n}}$

Thus the direction of the magnetic moment of the loop is defined by **the surface normal vector $\hat{\mathbf{n}}$**

Question:

If one of two wires of equal length is formed into a closed square loop and the other into a closed circular loop, and if both wires are carrying equal currents and both loops have their planes parallel to a uniform magnetic field, which loop would experience the greater torque?

Answer:

Step 1

Refer to Figure 5-6, in the textbook, for a loop whose plane is parallel to the magnetic field.

The **magnitude** of the net torque exerted by a parallel magnetic field about the axis of rotation is, $T = IAB_0$.

The current and the magnetic field are the same for both loops.

Thus, only the area is the parameter of concern.

The torque is directly proportional to the area enclosed by the loop.

The loop with the highest enclosed area experiences the maximum torque.

Step 2

Both loops are formed from the line of the same length.

Since the square loop is formed from a wire of length, l .

The side of the square is, $\frac{l}{4}$.

$$A_{\text{square}} = \frac{l^2}{16}$$

Thus, the area of the loop is, $= 0.0625$.

Step 3

The other loop is a circle.

The perimeter of the circle is the length of the wire.

The perimeter of the circle is,

$$2\pi r$$

It is equal to the length of the wire.

$$2\pi r = l$$

$$r = \frac{l}{2\pi}$$

Step 4

The area of circle is, πr^2

Substitute $\frac{l}{2\pi}$ for r

$$\begin{aligned} A_{\text{circle}} &= \pi \left(\frac{l}{2\pi} \right)^2 \\ &= \frac{l^2}{4\pi} \\ &= 0.079l^2 \end{aligned}$$

Step 5

The area of the circle is greater than that of the square loop and the torque is directly proportional to the area.

The circular loop experiences the maximum torque.

Question:

Two infinitely long parallel wires carry currents of equal magnitude. What is the resultant magnetic field due to the two wires at a point midway between the wires, compared with the magnetic field due to one of them alone, if the currents are

(a) in the same direction and

(b) in opposite directions?

Answer:

Step 1

It is given that, two infinitely long parallel wires carry current of equal magnitude.

(a) We need to determine the resultant magnetic field due to the two wires at a point midway between the wires, compared with the magnetic field due to one of them alone if the currents are in the same direction.

As the currents are in the same direction, between the wires the field will be in opposite direction that is given by,

$$\begin{aligned}\mathbf{B} &= \mathbf{B}_1 - \mathbf{B}_2 \\ &= \hat{\phi} \frac{\mu_0 I_1}{2\pi r} - \hat{\phi} \frac{\mu_0 I_2}{2\pi r} \\ &= \hat{\phi} \frac{\mu_0}{2\pi r} (I_1 - I_2)\end{aligned}$$

Hence the resultant magnetic field is: $\boxed{\mathbf{B} = \hat{\phi} \frac{\mu_0}{2\pi r} (I_1 - I_2)}$.

Step 2

(b) We need to determine the resultant magnetic field due to the two wires at a point midway between the wires, compared with the magnetic field due to one of them alone if the currents are in the opposite directions.

The magnetic field due to both the wires at a point midway is:

$$\begin{aligned} \mathbf{B}_1 &= \mathbf{B}_2 \\ &= \hat{\phi} \frac{\mu_0 I}{2\pi r} \\ &= \hat{\phi} \frac{\mu_0 I}{\pi (2r)} \\ &= \hat{\phi} \frac{\mu_0 I}{\pi d} \end{aligned}$$

The total field due to the two wires midway is:

$$\begin{aligned} \mathbf{B} &= \mathbf{B}_1 + \mathbf{B}_2 \\ &= \hat{\phi} \frac{\mu_0 I}{\pi d} + \hat{\phi} \frac{\mu_0 I}{\pi d} \\ &= \hat{\phi} \frac{2\mu_0 I}{\pi d} \end{aligned}$$

$$\mathbf{B} = \hat{\phi} \frac{2\mu_0 I}{\pi d}$$

Hence the resultant magnetic field is:

$$\mathbf{B} = \hat{\phi} \frac{2\mu_0 I}{\pi d}$$

Thus, the resultant magnetic field due to the two wires at a point midway between the wires is:

$$\mathbf{B} = \hat{\phi} \frac{\mu_0 I}{\pi d}$$

Thus, the resultant magnetic field due to one wire alone is:

Question:

Devise a right-hand rule for the direction of the magnetic field due to a linear current carrying conductor.

Answer:

Step 1

For a linear current carrying conductor the direction of magnetic field by applying the right hand thumb rule is, when we hold the straight wire with the right hand, the **extended thumb** points in the direction of the current, and the **curl of the fingers** gives the direction of the magnetic field around the straight wire.

Question:

What is a magnetic dipole? Describe its magnetic field distribution.

Answer:

Step 1

Magnetic dipole:

A current loop with dimensions much smaller than the distance between the loop and the observation point is called a magnetic dipole. This is because the pattern of its magnetic field lines is very similar to that of a permanent magnet as well as the pattern of the electric field lines of the electric dipole.

Question:

What are the fundamental differences between electric and magnetic fields?

Answer:

Step 1

The fundamental difference between electric and magnetic fields is given by Gauss's Law that can be elucidated in terms of field lines.

Electric field lines originate from positive electric charges and terminate on negative ones. So the electric field through a closed surface, surrounding one of its charges is nonzero.

In contrast, the magnetic field lines always form continuous closed loops, where the magnetic flux through a closed surface is always zero.

Question:

If the line integral of \mathbf{H} over a closed contour is zero, does it follow that $\mathbf{H} = 0$ at every point on the contour? If not, what then does it imply?

Answer:

Step 1

The Ampere circuit law states that the line integral of \mathbf{H} around a closed path is equal to the current traversing the surface bounded by that path, and the Ampere law can only be applied to current flowing through a closed path. But if the current is not enclosed in the closed path, the line integral of \mathbf{H} along it vanishes.

Question:

Compare the utility of applying the Biot–Savart law versus applying Ampère’s law for computing the magnetic field due to current-carrying conductors.

Answer:

Step 1

The biot savart’s law is applicable for calculating the magnetic field in all cases of current carrying conductors.

The direction of the magnetic field is determined directly in the calculation of magnetic field using Biot’s savarts law.

Step 2

The Ampere’s law exploits the symmetry of the magnetic field to determine the value of the magnetic field.

It simplifies the calculation of the magnetic field, when symmetry is present.

It cannot be used to calculate the magnetic field of non-symmetric magnetic fields.

Question:

What is a toroid? What is the magnetic field outside the toroid?

Answer:

Step 1

A toroidal coil or toroid is a doughnut shaped structure called the core that is wrapped in closely spaced turns of wire. The magnetic field \mathbf{H} outside a toroid is zero as the net current flowing through its surface is zero because an equal number of current coils cross the surface in both directions.

Thus the magnetic field outside the toroid is: $\mathbf{H} = 0$.

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