

Question:

When are two vectors equal and when are they identical?

Answer:

Step 1

Consider a vector in cartesian coordinates is,

$$\mathbf{A} = \hat{\mathbf{a}}A = \hat{\mathbf{x}}A_x + \hat{\mathbf{y}}A_y + \hat{\mathbf{z}}A_z$$

$$\mathbf{B} = \hat{\mathbf{b}}B = \hat{\mathbf{x}}B_x + \hat{\mathbf{y}}B_y + \hat{\mathbf{z}}B_z$$

Step 2

The two vectors are equal; if they have **equal magnitude and identical vector units**. On the other hand, the equality of two vectors does not imply they are identical.

The two vectors are identical; when only if, they lie on top of one another.

Question:

When is the position vector of a point identical to the distance vector between two points?

Answer:

Step 1

The position vector of a point P in space is the vector from the origin to P . Now by assuming two points P_1 and P_2 are at (x_1, y_1, z_1) and (x_2, y_2, z_2) .
The origin O and now finding the position vectors,

$$\mathbf{R}_1 = \overrightarrow{OP_1} = \hat{x}x_1 + \hat{y}y_1 + \hat{z}z_1 \dots\dots (1)$$

$$\mathbf{R}_2 = \overrightarrow{OP_2} = \hat{x}x_2 + \hat{y}y_2 + \hat{z}z_2 \dots\dots (2)$$

Step 2

Calculate the distance vector from P_1 to P_2 .

$$\begin{aligned}\mathbf{R}_{12} &= \overrightarrow{P_1P_2} \\ &= \mathbf{R}_2 - \mathbf{R}_1 \\ &= \hat{x}(x_2 - x_1) + \hat{y}(y_2 - y_1) + \hat{z}(z_2 - z_1)\end{aligned}$$

Thus, the distance vector between two points P_1 and P_2 is,

$$\mathbf{R}_{12} = \hat{x}(x_2 - x_1) + \hat{y}(y_2 - y_1) + \hat{z}(z_2 - z_1) \dots\dots (3)$$

Step 3

By comparing equations (1) and (3), the position vector of a point P_1 identical to the distance vector, when

$$x_1 = x_2 - x_1; y_1 = y_2 - y_1; z_1 = z_2 - z_1$$

Thus, the condition for position vector of a point P_1 , identical to the distance vector is,

$$\boxed{x_1 = \frac{x_2}{2}; y_1 = \frac{y_2}{2}; z_1 = \frac{z_2}{2}}.$$

Step 4

Similarly, by comparing equations (2) and (3), the position vector of a point P_2 identical to the distance vector, when

$$x_2 = x_2 - x_1; y_2 = y_2 - y_1; z_2 = z_2 - z_1$$

Thus, the condition for position vector of a point P_2 identical to the distance vector is,

$$\boxed{x_1 = 0; x_2 = 0; x_3 = 0}.$$

Question:

If $\vec{A} \cdot \vec{B} = 0$, what is θ_{AB} ?

Answer:

Step 1

By the definition of the dot product, we have

$$\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta_{AB}$$

Here,

The vectors are \vec{A} and \vec{B} and

The angle between \vec{A} and \vec{B} is θ_{AB} .

Step 2

Consider $\vec{A} \cdot \vec{B} = 0$.

Determine the angle θ_{AB} .

$$\cos \theta_{AB} = 0$$

$$\begin{aligned} \theta_{AB} &= \cos^{-1}(0) \\ &= 90^\circ \end{aligned}$$

This means that, $\theta_{AB} = \arccos(0) = 90^\circ$ or we can say that the vectors \vec{A} and \vec{B} are perpendicular.

Hence, $\vec{A} \cdot \vec{B} = 0$ means vectors \vec{A} and \vec{B} are perpendicular to each other.

Question:

If $\vec{A} \times \vec{B} = 0$, what is θ_{AB} ?

Answer:

Step 1

By the definition of the cross product, we have

$$\vec{A} \times \vec{B} = \hat{n} |\vec{A}| |\vec{B}| \sin \theta_{AB}$$

Where,

The vectors are \vec{A} and \vec{B}

Unit vector normal to the plane containing \vec{A} and \vec{B} is \hat{n} and

The angle between \vec{A} and \vec{B} is θ_{AB} .

If $\vec{A} \times \vec{B} = 0$, then $\sin \theta_{AB} = 0$.

This means that $\theta_{AB} = \arcsin(0) = 0^\circ$ or we can say that the vectors \vec{A} and \vec{B} are parallel.

Question:

Is $\mathbf{A}(\mathbf{B} \cdot \mathbf{C})$ a vector triple product?

Answer:

Step 1

The vector triple product is the cross product of one vector with the cross product of the other two. For example the vector triple product of three vectors \mathbf{A} , \mathbf{B} , and \mathbf{C} is,

$$\mathbf{A} \times (\mathbf{B} \times \mathbf{C}) = (\mathbf{A} \cdot \mathbf{C})\mathbf{B} - (\mathbf{A} \cdot \mathbf{B})\mathbf{C}$$

Here, the dot product of vectors \mathbf{A} and \mathbf{C} produces the scalar as a output. While multiplying with the vector \mathbf{B} produces output as a vector.

Similarly, the given product is $\mathbf{A}(\mathbf{B} \cdot \mathbf{C})$. Here, the dot product of vectors \mathbf{B} and \mathbf{C} produces the scalar as a output. While multiplying with the vector \mathbf{A} produces output as a vector triple product.

Therefore, $\mathbf{A}(\mathbf{B} \cdot \mathbf{C})$ is a vector triple product.

Question:

If $\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{C}$, does it follow that $\mathbf{B} = \mathbf{C}$?

Answer:

Step 1

Let us consider vector be,

$$\vec{\mathbf{A}} = \hat{i}$$

then $\hat{i} \cdot \hat{j} = \hat{i} \cdot \hat{k} = 0$ even $\hat{j} \neq \hat{k}$

Similarly, $\mathbf{A} \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{C}$ yet $\mathbf{B} \neq \mathbf{C}$

Step 2

For example, the vectors are defined as,

$$\mathbf{A} = \hat{x}\mathbf{1}$$

$$\mathbf{B} = \hat{x}\mathbf{2} + \hat{y}\mathbf{1}$$

$$\mathbf{C} = \hat{x}\mathbf{2} + \hat{y}\mathbf{2}$$

Now calculate the dot product between $\mathbf{A} \cdot \mathbf{B}$ and $\mathbf{A} \cdot \mathbf{C}$

$$\begin{aligned}\mathbf{A} \cdot \mathbf{B} &= (\hat{x}\mathbf{1}) \cdot (\hat{x}\mathbf{2} + \hat{y}\mathbf{1}) \\ &= 2\end{aligned}$$

$$\begin{aligned}\mathbf{A} \cdot \mathbf{C} &= (\hat{x}\mathbf{1}) \cdot (\hat{x}\mathbf{2} + \hat{y}\mathbf{2}) \\ &= 2\end{aligned}$$

Although the dot product between both is the same still as the vectors \mathbf{B} and \mathbf{C} are different.

Question:

Why do we use more than one coordinate system?

Answer:

Step 1

There are so many co-ordinate systems that allow us to solve practical problems.

Step 2

The need for more than one co-ordinate system is, although a point in space or an object having the same location or direction still needs to solve practically with the right choice of geometry.
Hence, the choice of a co-ordinate system greatly facilitates the choices that best fits the geometry under consideration.

Question:

Why is it that the base vectors (x, y, z) are independent of the location of a point, but \hat{r} and $\hat{\phi}$ are not?

Answer:

Step 1

In Cartesian co-ordinate system, the base vectors \hat{x} , \hat{y} , and \hat{z} are independent of the location of point because these are unit vectors along x , y , and z directions respectively.

In cylindrical co-ordinates the mutually perpendicular base vectors \hat{r} , $\hat{\phi}$ and \hat{z} are such that \hat{r} points away from the origin along radial distance r , and $\hat{\phi}$ points in a direction tangential to the cylindrical surface and \hat{z} points in the vertical direction. So, \hat{r} and $\hat{\phi}$ are dependent of the location of point.

Question:

What are the cyclic relations for the base vectors in (a) Cartesian coordinates, (b) cylindrical coordinates, and (c) spherical coordinates?

Answer:

Step 1

(a) In Cartesian co-ordinate system, the cyclic relation with the base vectors x , y and z is

$$\begin{array}{l} d\mathbf{s}_x = \hat{\mathbf{x}} \, dy \, dz \quad (y-z \text{ plane}) \\ d\mathbf{s}_y = \hat{\mathbf{y}} \, dx \, dz \quad (x-z \text{ plane}) \\ d\mathbf{s}_z = \hat{\mathbf{z}} \, dx \, dy \quad (x-y \text{ plane}) \end{array}.$$

Step 2

(a) In Cylindrical co-ordinate system, the cyclic relation with the base vectors \hat{r} , $\hat{\phi}$, and \hat{z} is,

$$\begin{array}{l} \hat{r} \times \hat{\phi} = \hat{z} \\ \hat{\phi} \times \hat{z} = \hat{r} \\ \hat{z} \times \hat{r} = \hat{\phi} \end{array}.$$

Step 3

(b) In the spherical co-ordinate system, the cyclic relation with the base vectors \hat{R} , $\hat{\theta}$ and $\hat{\phi}$ is,

$$\begin{array}{l} \hat{R} \times \hat{\theta} = \hat{\phi} \\ \hat{\theta} \times \hat{\phi} = \hat{R} \\ \hat{\phi} \times \hat{R} = \hat{\theta} \end{array}.$$

Question:

How is the position vector of a point in cylindrical coordinates related to its position vector in spherical coordinates?

Answer:

Step 1

In the cylindrical co-ordinate system the position vector \overrightarrow{OP} has components along r and z only. Thus,

$$\begin{aligned}\mathbf{R}_1 &= \overrightarrow{OP} \\ &= \hat{\mathbf{r}}r_1 + \hat{\mathbf{z}}z_1\end{aligned}$$

The dependence of \mathbf{R}_1 on ϕ_1 is implicit through the dependence of $\hat{\mathbf{r}}$ on ϕ_1 .

In the spherical co-ordinate system the position vector of point $P = (R_1, \theta_1, \phi_1)$ is given by,

$$\begin{aligned}\mathbf{R}_1 &= \overrightarrow{OP} \\ &= \hat{\mathbf{R}}R_1\end{aligned}$$

Here $\hat{\mathbf{R}}$ is implicitly dependent on θ_1 and ϕ_1 .

Hence from above we understand that the position vector in both the co-ordinate system have a common relation where both are dependent on ϕ_1 .

Question:

what do the magnitude and direction of the gradient of a scalar quantity represent?

Answer:

Step 1

The gradient of a scalar function V is defined as follows.

$$\nabla V = \hat{\mathbf{x}} \frac{\partial V}{\partial x} + \hat{\mathbf{y}} \frac{\partial V}{\partial y} + \hat{\mathbf{z}} \frac{\partial V}{\partial z}$$

The magnitude of gradient is equal to the maximum rate of change of the physical quantity per unit distance, and its direction is along the direction of maximum increase.

Question:

When is a vector field "conservative"?

Answer:

Step 1

Stoke's theorem converts the surface integral of the curl of a vector over an open surface S into a line integral of the vector along the contour C bounding the surface S .

That is,

$$\int_S (\nabla \times \mathbf{B}) \cdot d\mathbf{s} = \oint_C \mathbf{B} \cdot d\mathbf{l}$$

If $\nabla \times \mathbf{B} = \mathbf{0}$, the field \mathbf{B} is said to be conservative because its circulation, represented by the right hand side expression equals zero.

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