Strength Of Materials Lab. 110402330

Dr. Hitham Tlilan

Tensile Test

Stress Strain Diagram

Tensile Test

Tensile test determines the strength of the material when subjected to a simple stretching operation. Typically, standard dimension test samples are pulled slowly and at uniform rate in a testing machine while the *strain* (the elongation of the sample) is defined as:

Engineering Strain

$$e = \frac{\ell - \ell_o}{\ell_o} = \frac{\ell}{\ell_o} - 1$$

and the *stress* (the applied force divided by the original cross-sectional area) is defined as:

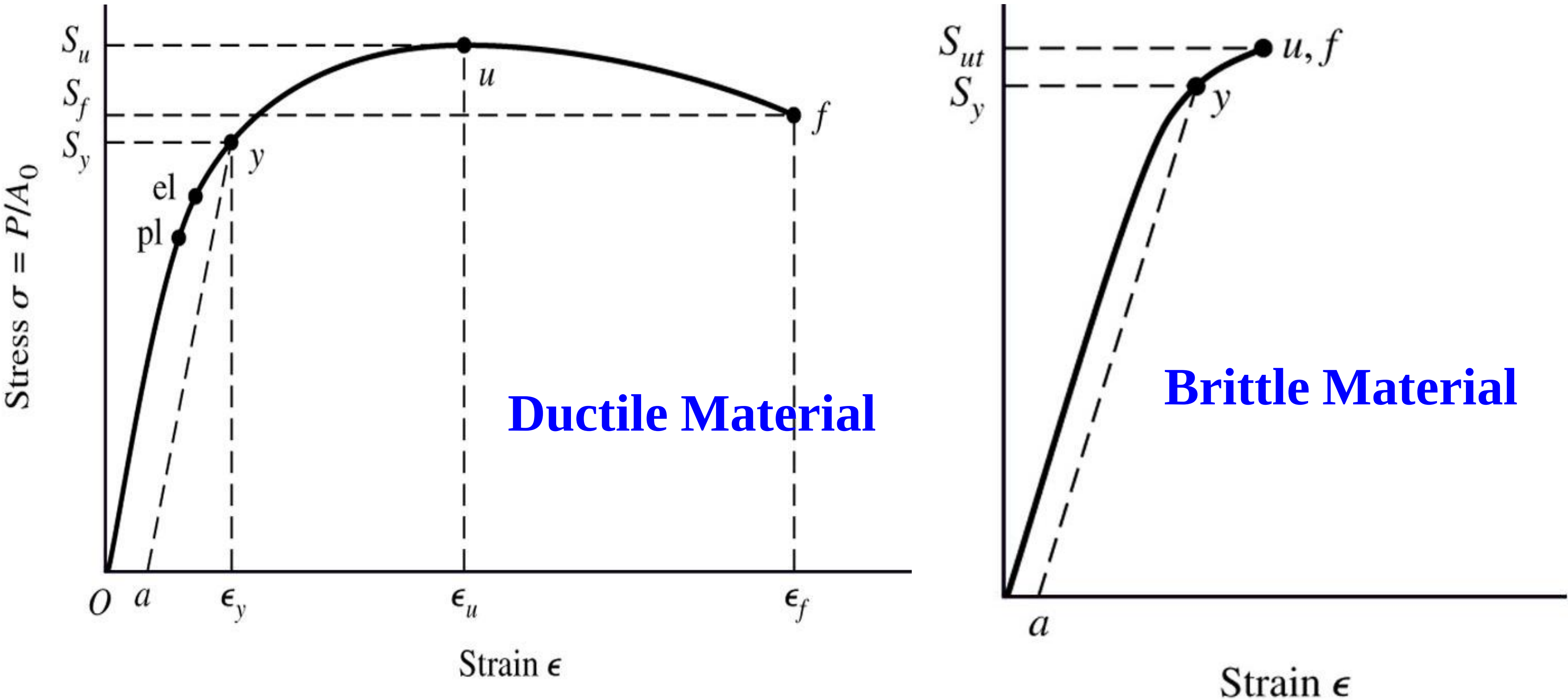
Engineering Stress

$$\sigma = \frac{P}{A_0}$$

Ductility and Brittleness

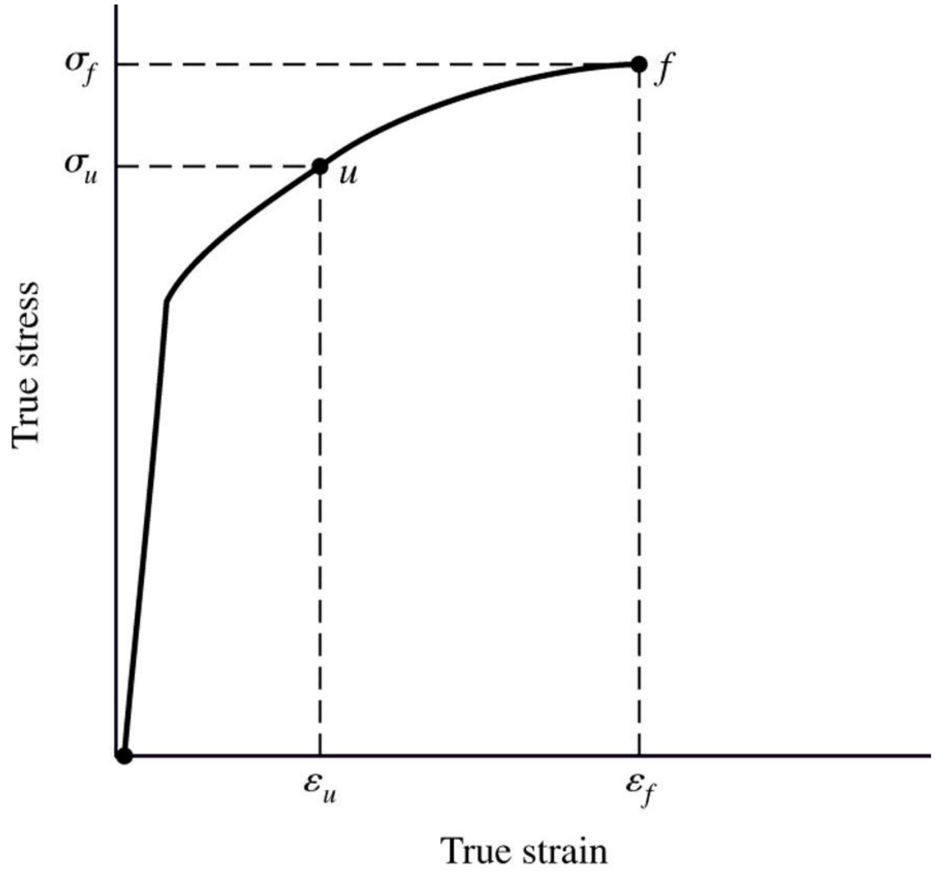
- **Ductility**: presence of significant plastic region
- **Brittleness:** no plastic region before failure

Necking of ductile material under tensile load



Engineering stress-strain curves

True stress-strain curve



True Strain

$$\varepsilon = \int_{\ell_o}^{\ell} \frac{d\ell}{\ell} = \ln \frac{\ell}{\ell_o} = \ln(e+1)$$

Constant volume;i.e.

$$A\ell = A_o\ell_o \Rightarrow \varepsilon = \ln \frac{A_o}{A}$$

True Stress σ_{true}

$$\sigma = \frac{P}{A_o} = P\frac{\ell_o}{A\ell} = \frac{P}{A}\frac{\ell_o}{\ell}$$

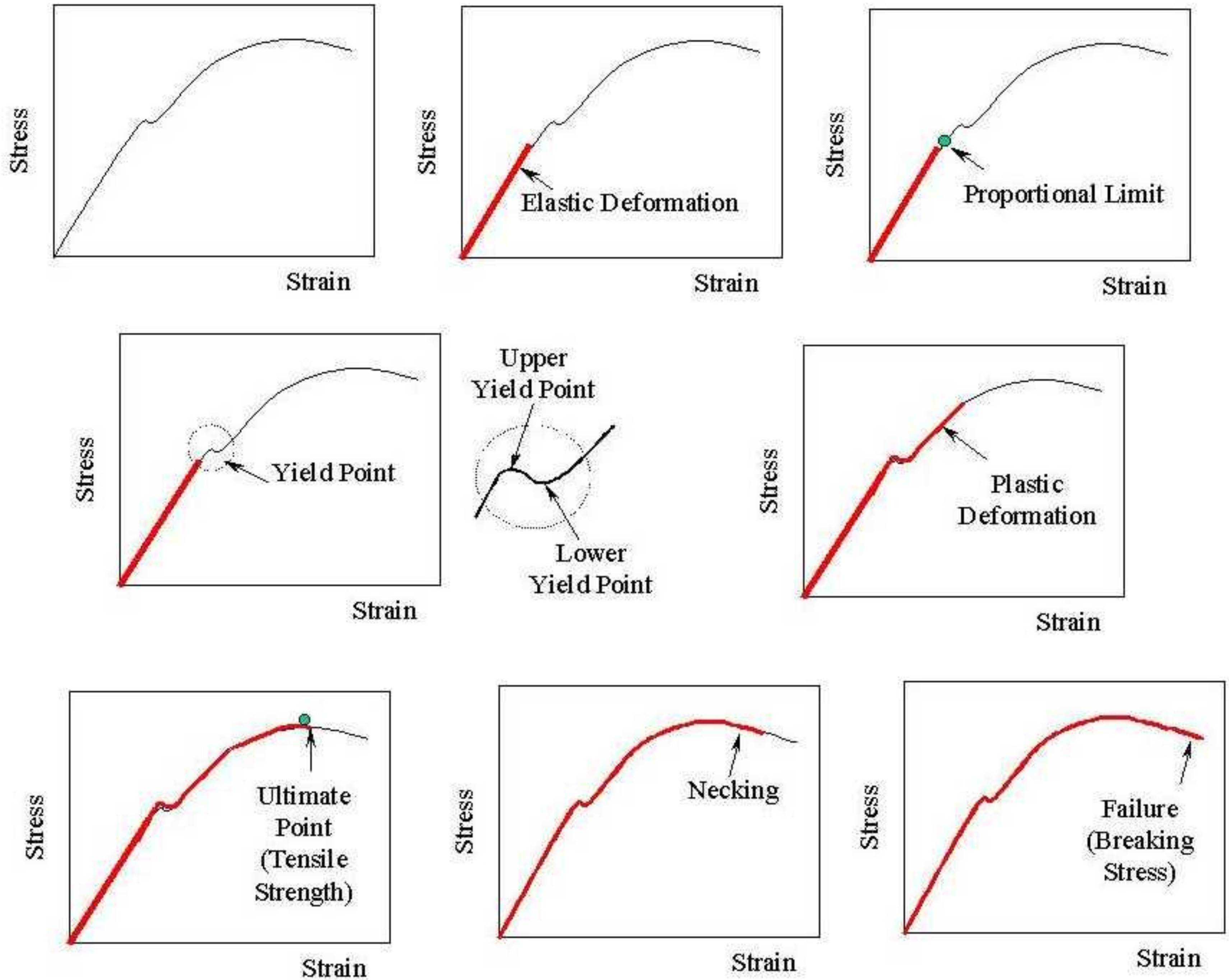
$$\sigma_{true} = \frac{P}{A} = \sigma\frac{\ell}{\ell_o}$$

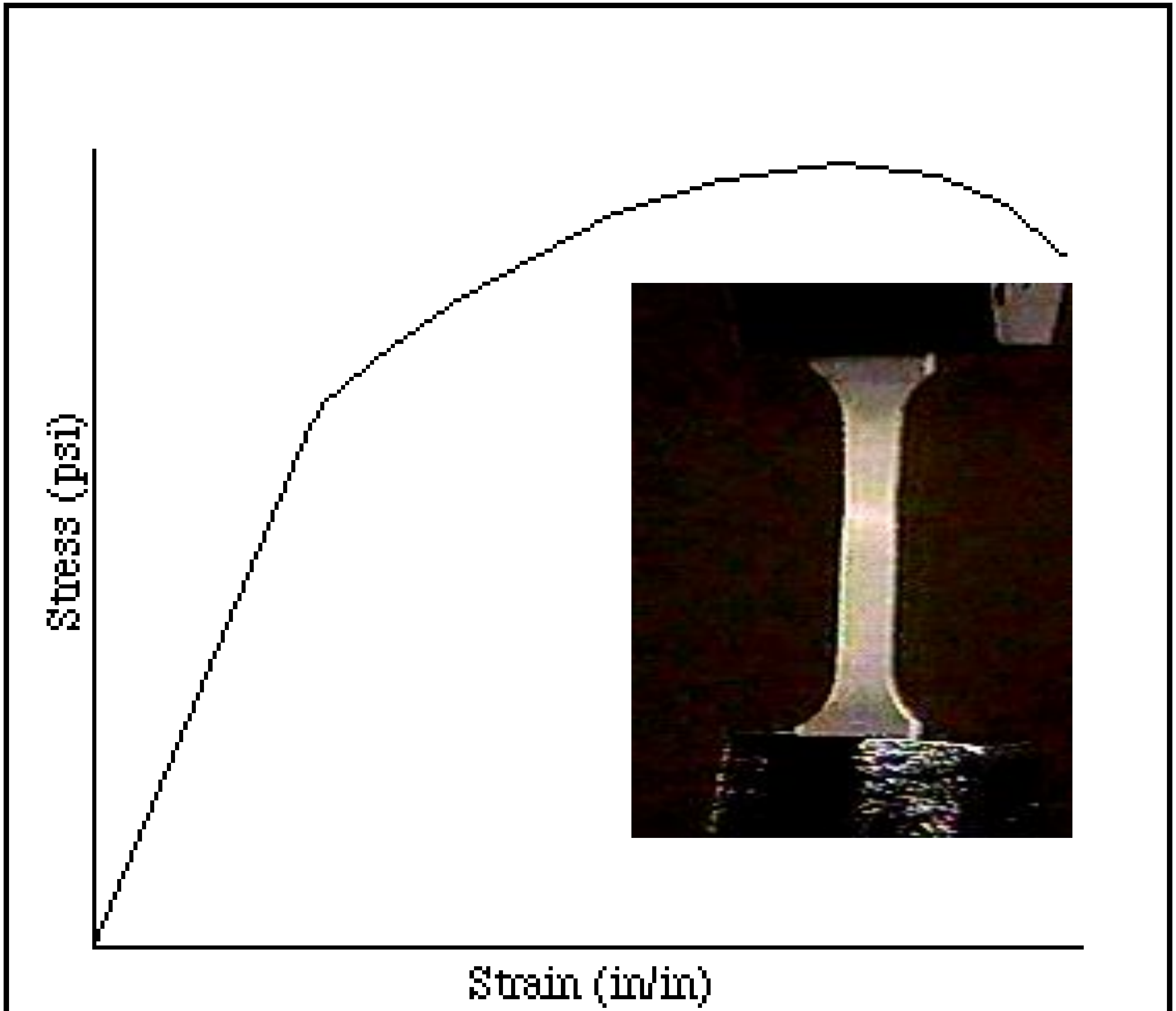
Substitution of the current area into the equation gives a larger stress (true stress) than the engineering stress. Note that engineering stress uses the initial area, regardless of the change in diameter during the tensile test.

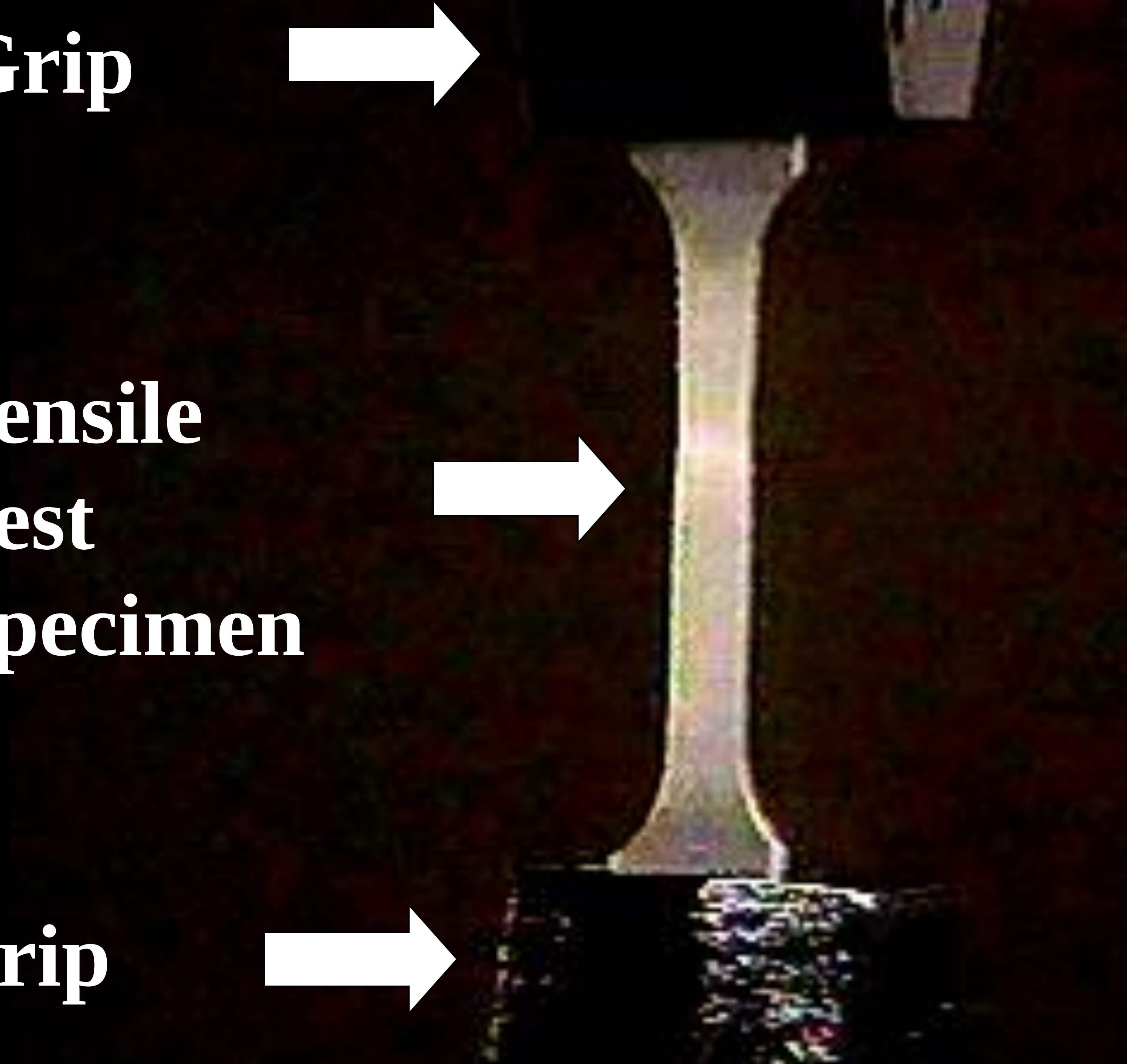
Tensile Test Stress Strain Diagram

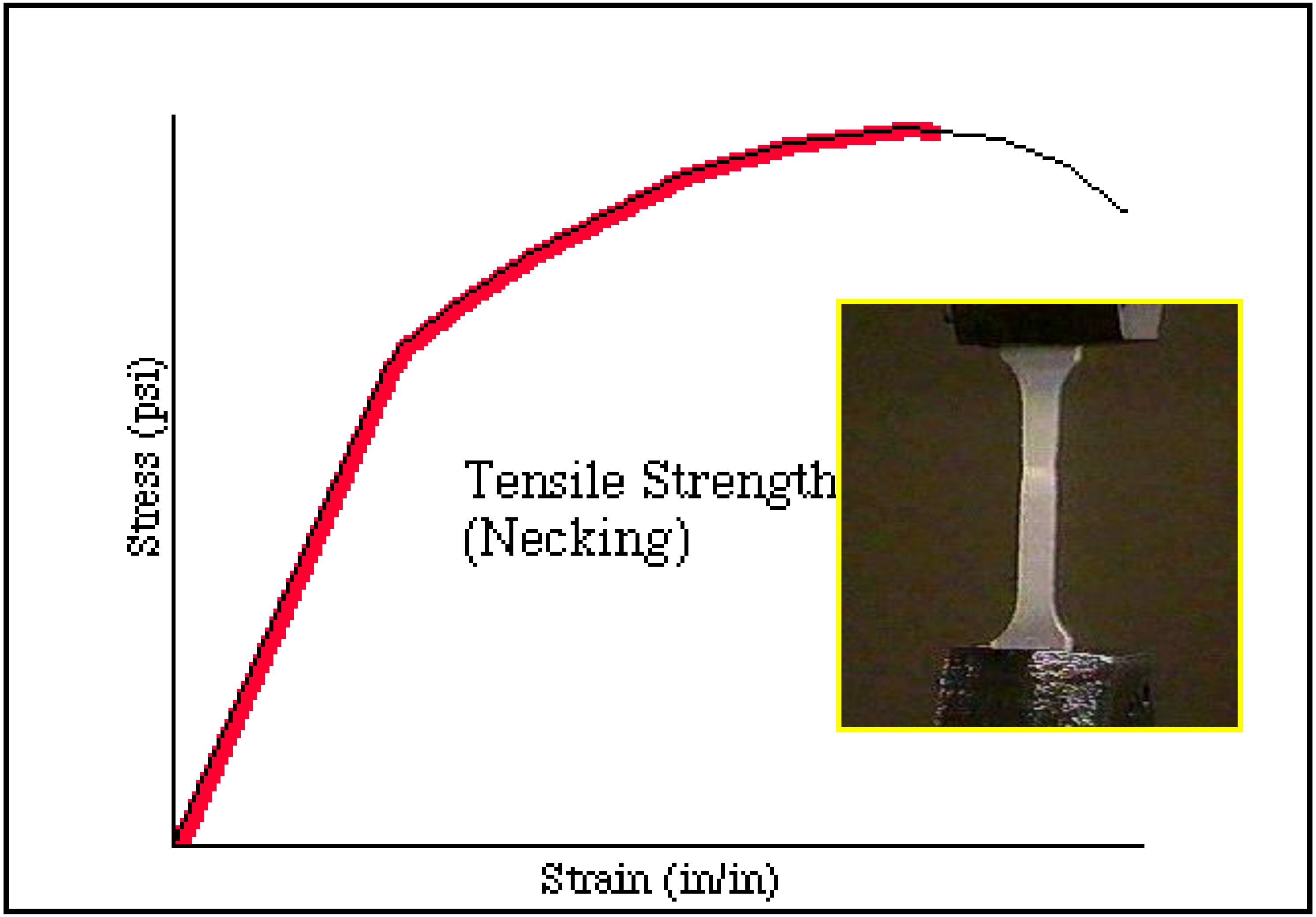
The applied stress versus the strain or elongation of the specimen shows the initial elastic response of the material, followed by yielding, plastic deformation and finally necking and failure. Several measurements are taken from the plot, called the Engineering Stress-Strain Diagram. These include:

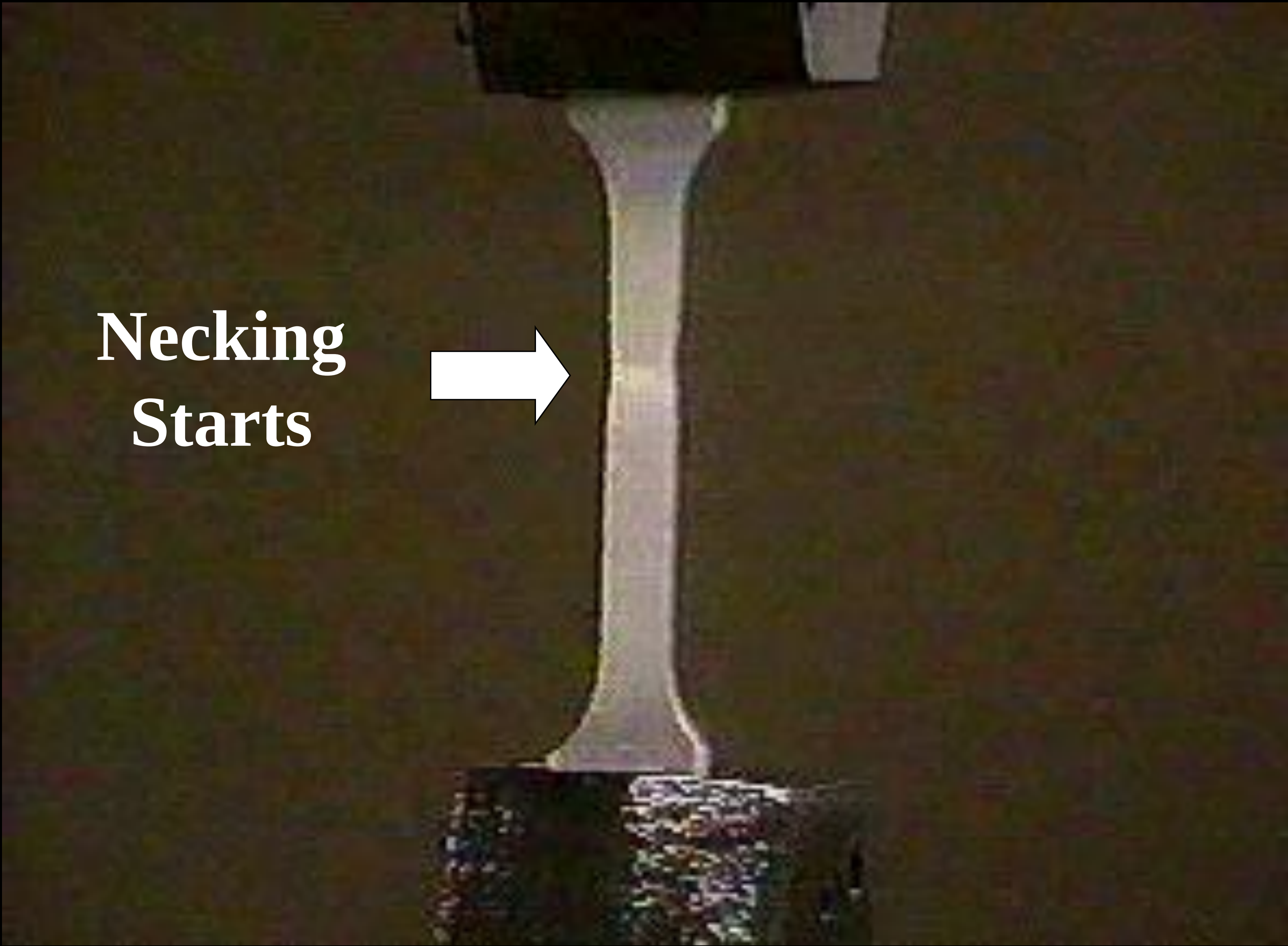
- **Modulus of elasticity**
- **Yield strength**
- **Tensile strength**
- **Modulus of resilience**
- **Failure stress**
- **Ductility**
- **Toughness**

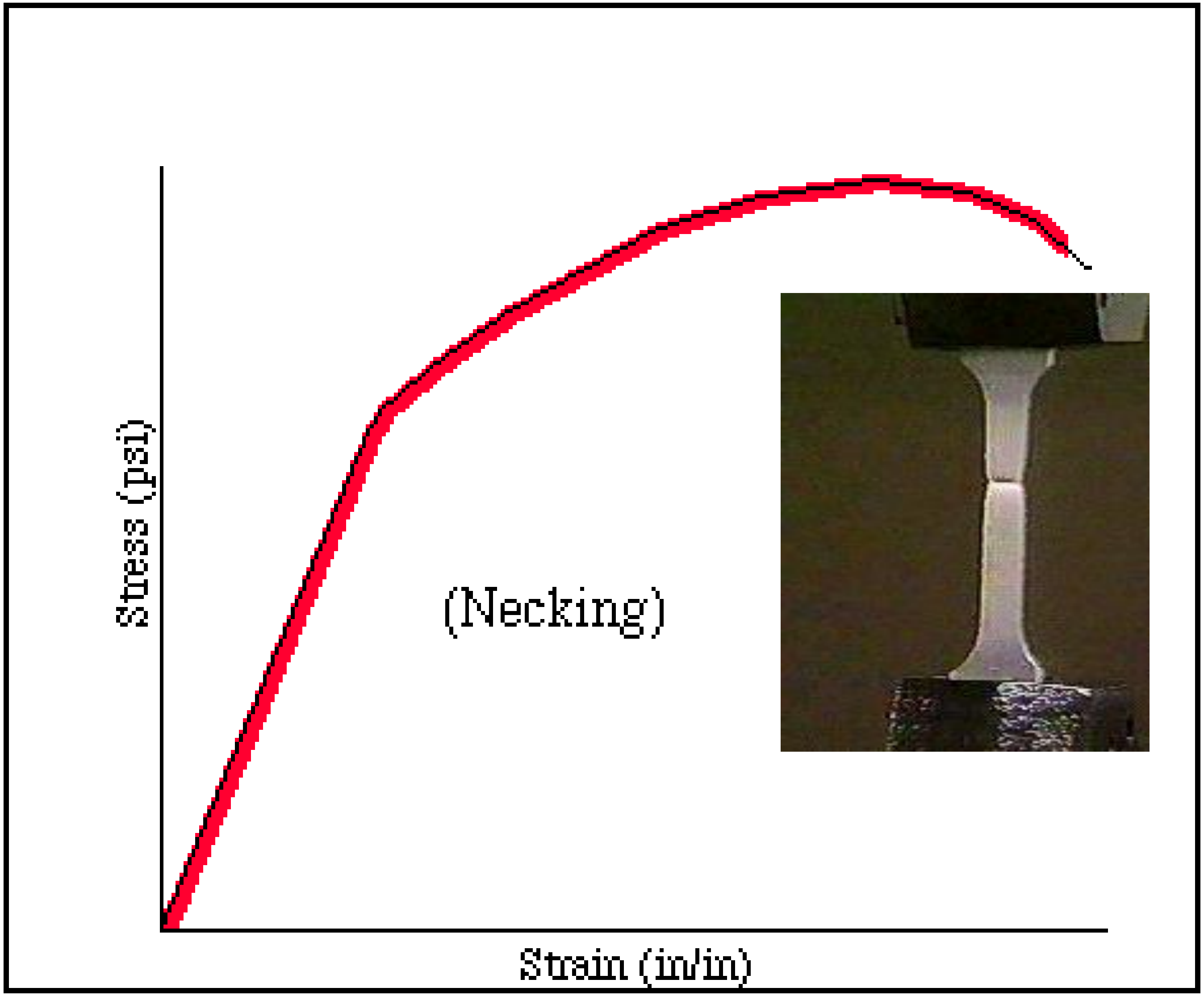

Stress
Strain
Elastic Deformation
Proportional Limit
Yield Point
Upper Yield Point
Lower Yield Point
Plastic Deformation
Ultimate Point (Tensile Strength)
Necking
Failure (Breaking Stress)


Stress (psi)
Strain (in/in)


Grip
Tensile
Test
Specimen
Grip

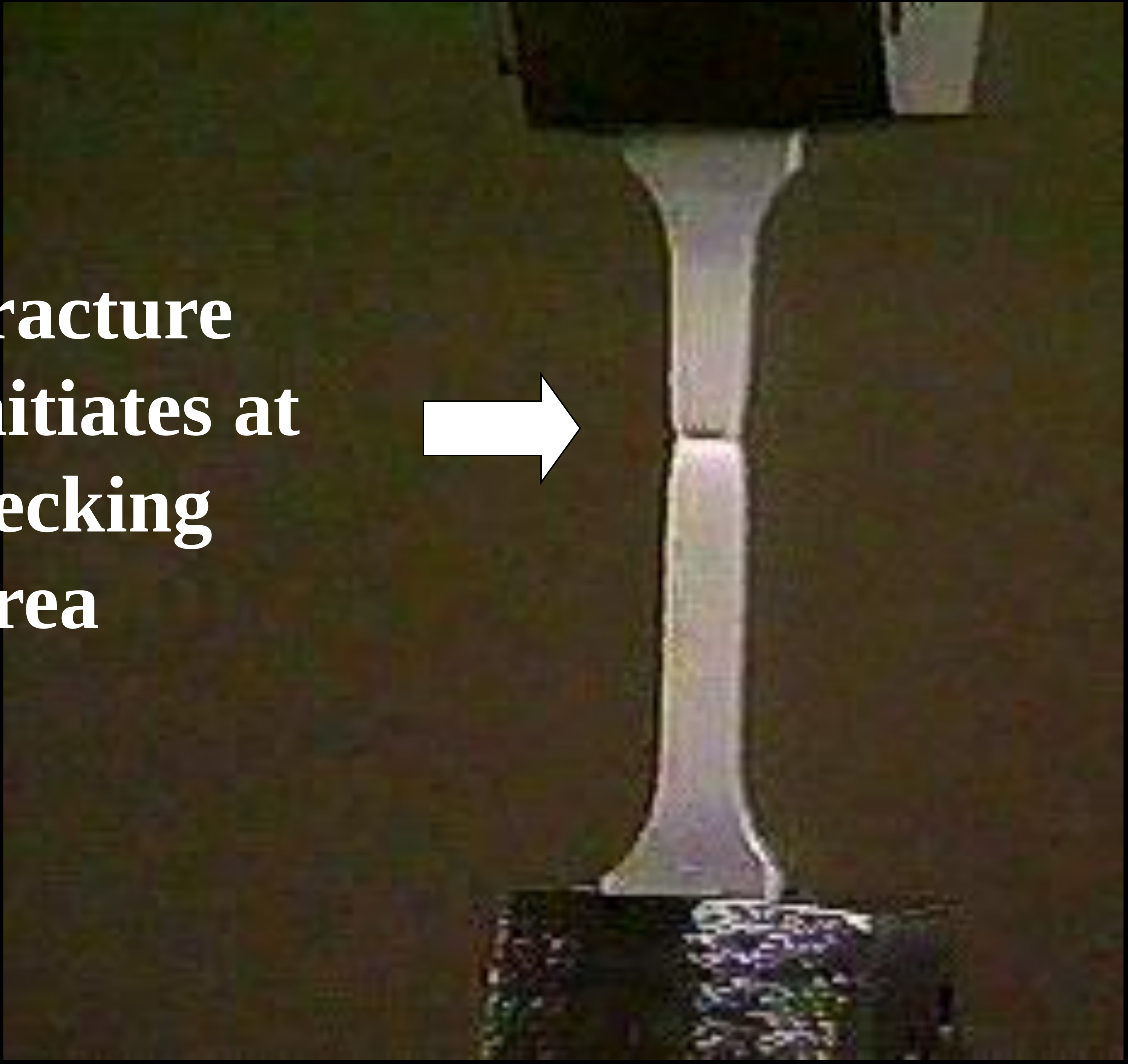

Stress (psi)
Tensile Strength
(Necking)
Strain (in/in)

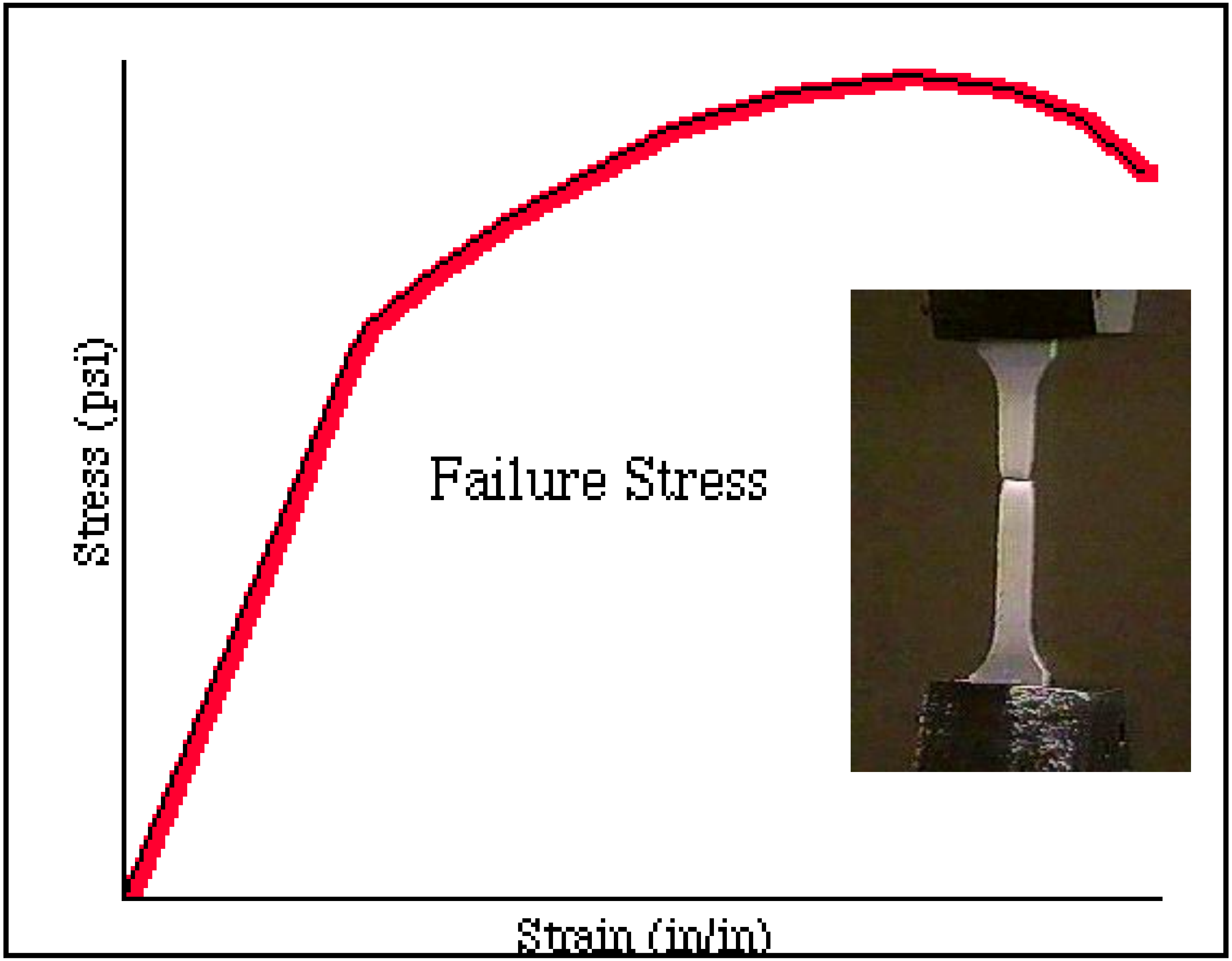

Necking
Starts

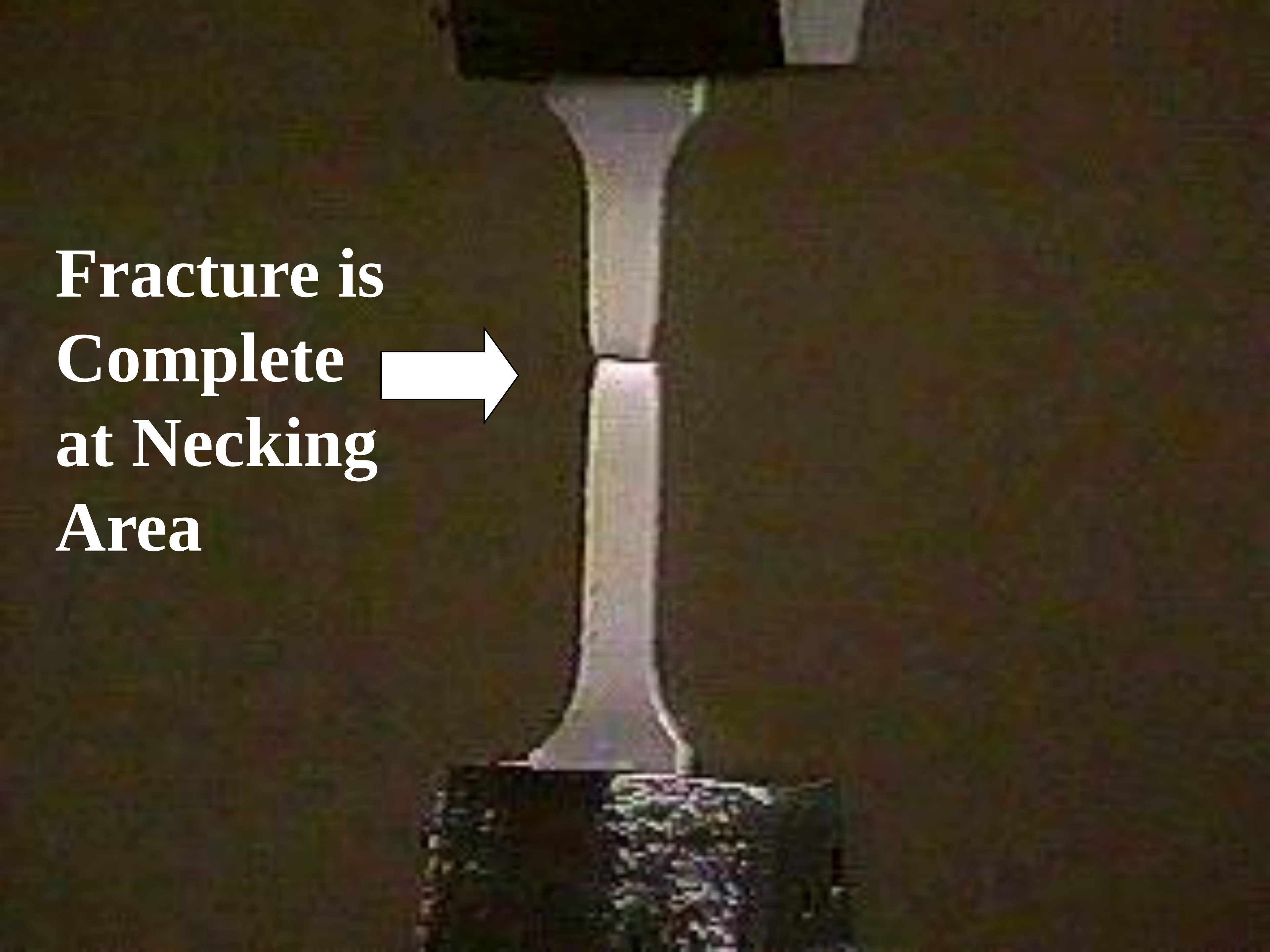


Stress (psi)
(Necking)
Strain (in/in)

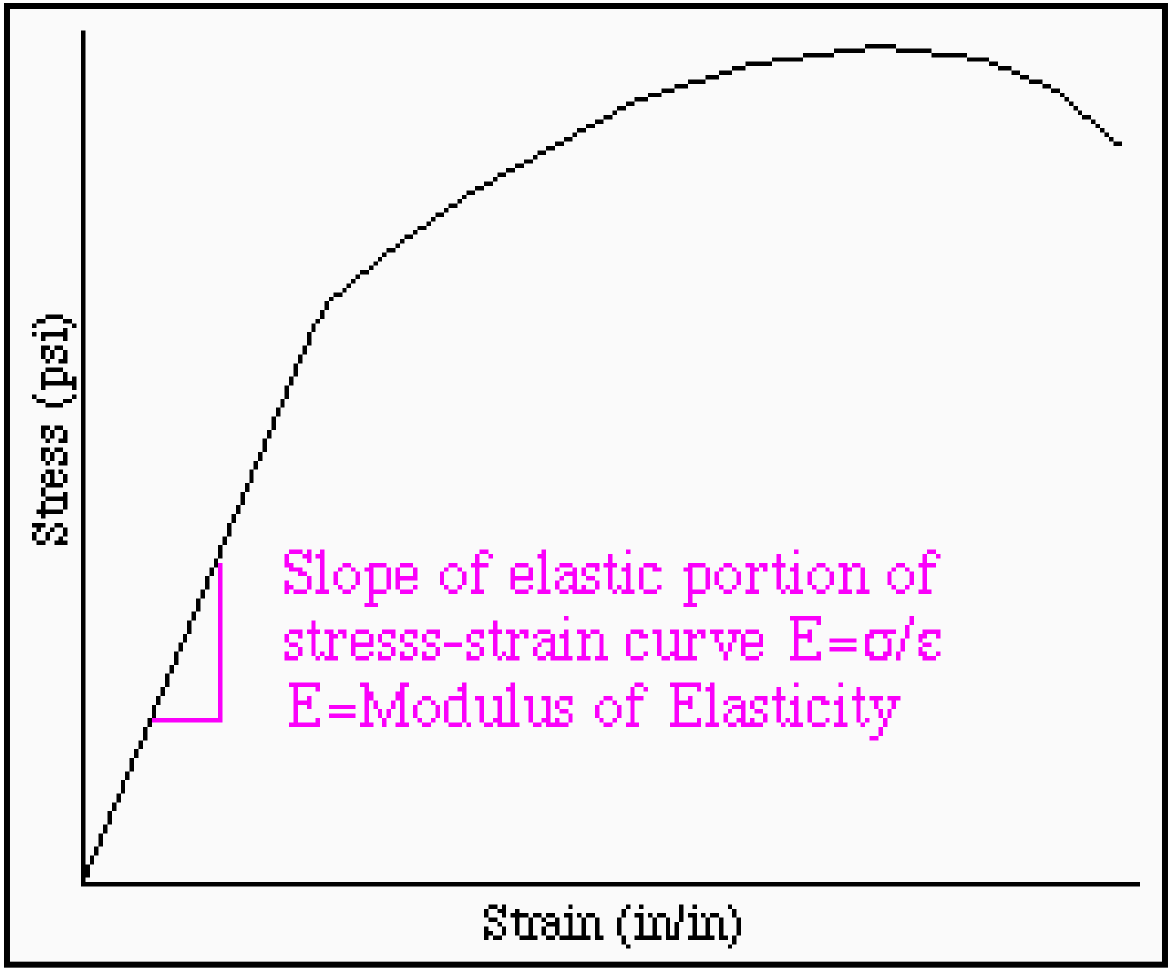
Fracture Initiates at Necking Area



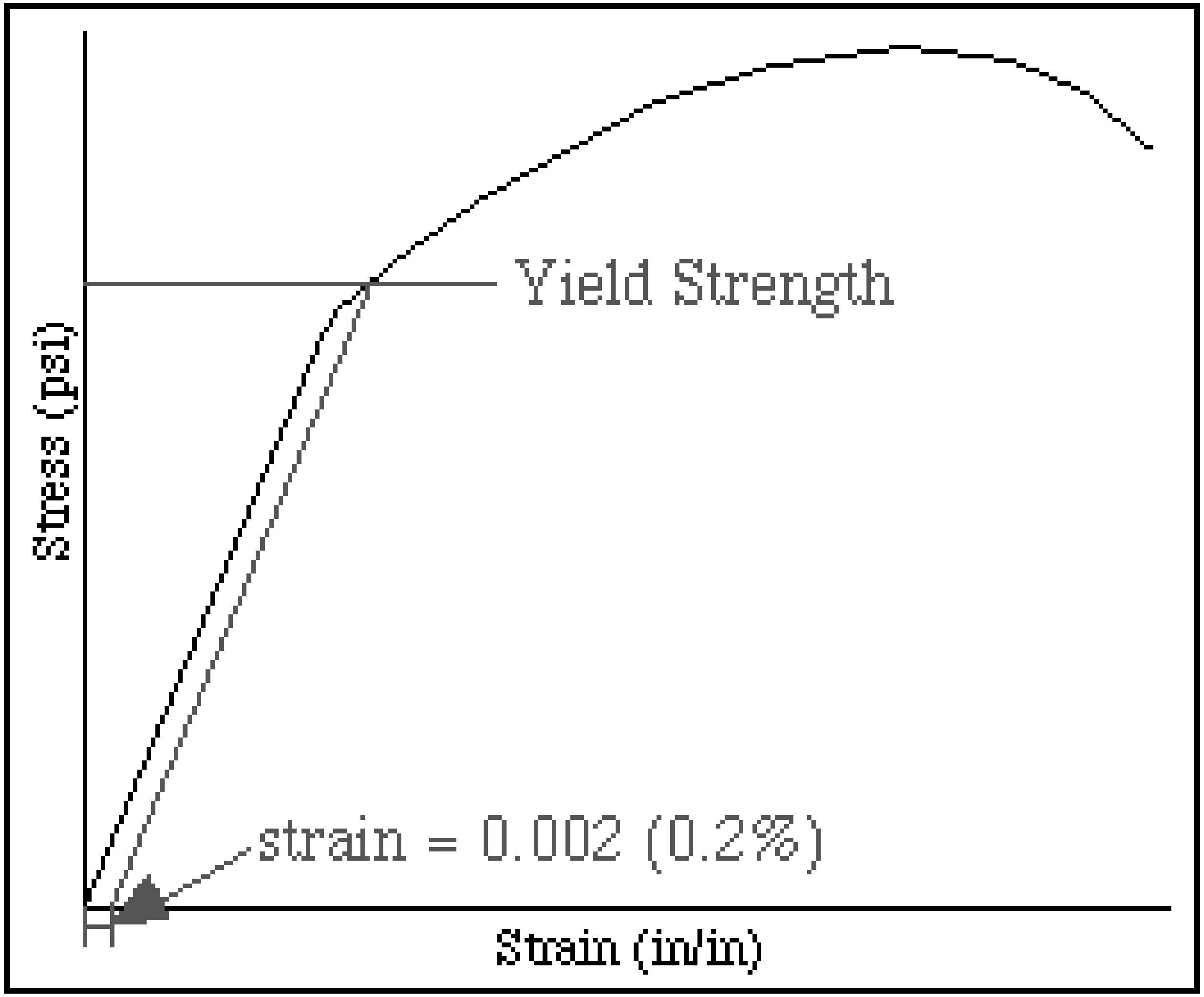

Stress (psi)
Failure Stress
Strain (in/in)


Fracture is Complete at Necking Area

Modulus of elasticity - **the initial slope of the curve, related directly to the strength of the atomic bonds.**



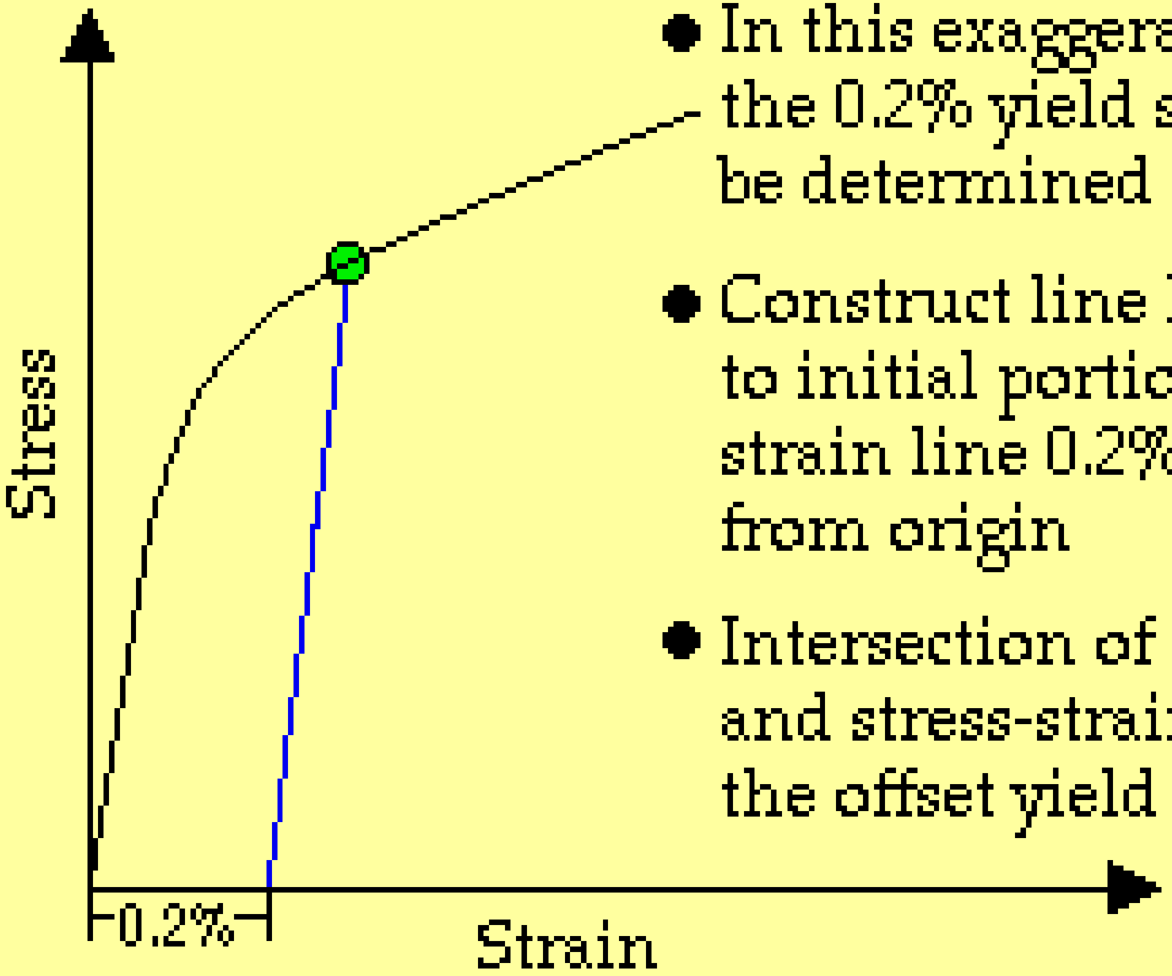
Yield strength, usually defined as the point at which a consistent and measureable amount of permanent strain remains in the specimen.



0.2 % Offset Yield StrengthOffset Yield Strength

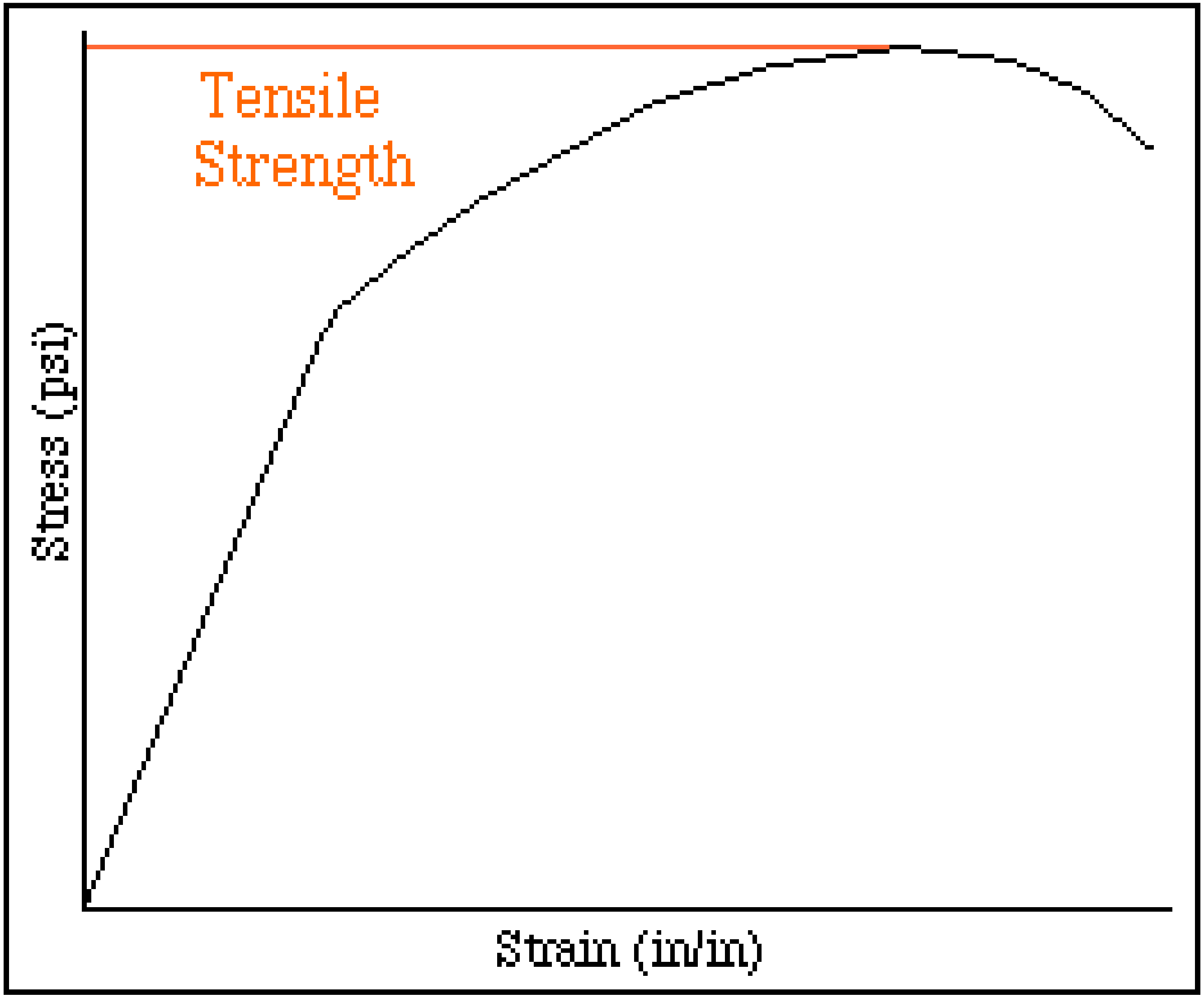
Defining the yield stress as the point separating elastic from plastic deformation is easier than determining that point. The elastic portion of the curve is not perfectly linear, and microscopic amounts of deformation can occur. As a matter of practical convenience, the yield strength is determined by constructing a line parallel to the initial portion of the stress-strain curve but offset by 0.2% from the origin. The intersection of this line and the measured stress-strain line is used as an approximation of the material's yield strength, called the 0.2% offset yield.

0.2 % Offset Yield Strength

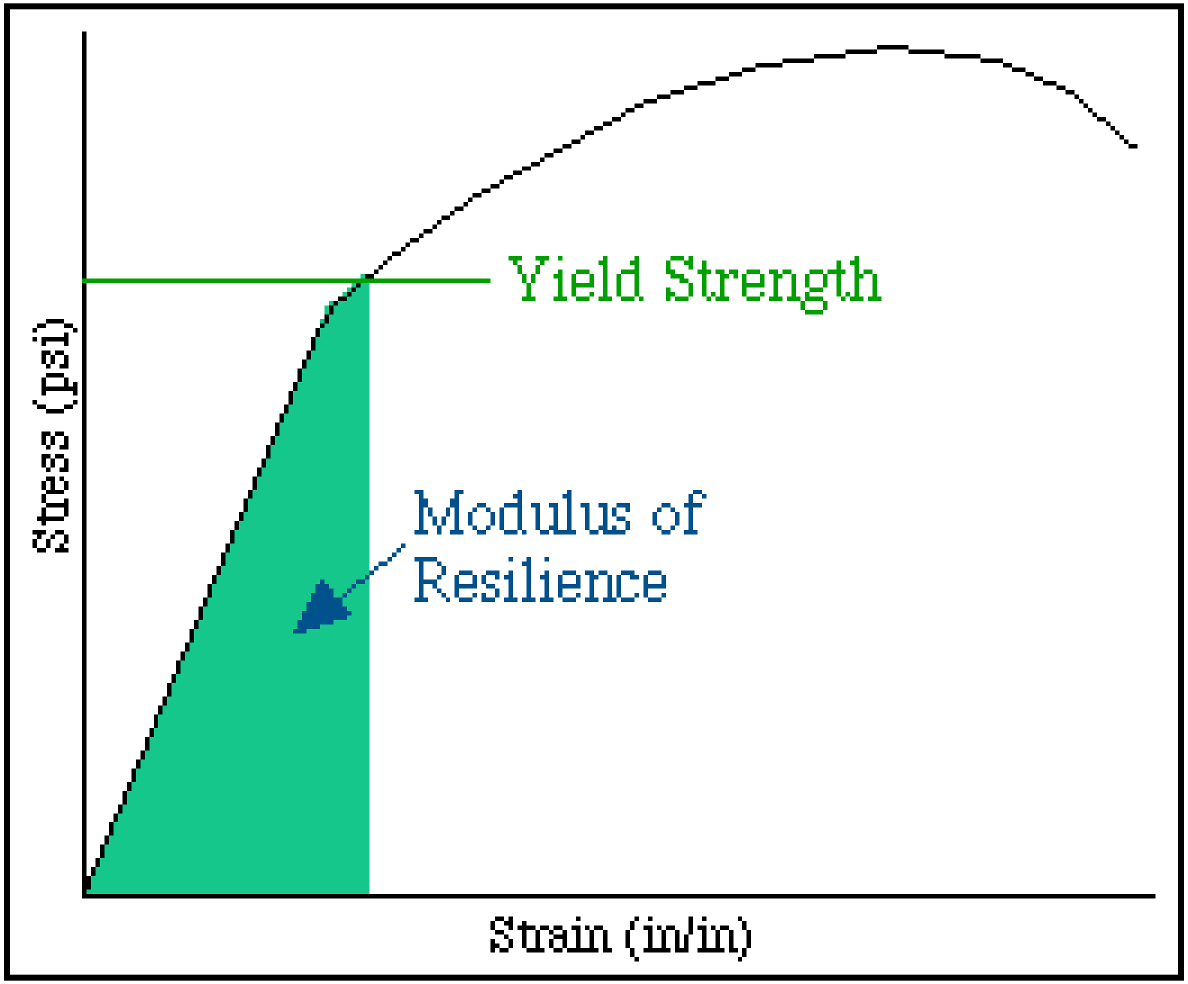


- In this exaggerated sketch, the 0.2% yield stress is to be determined
- Construct line Parallel to initial portion of stress-strain line 0.2% distance from origin
- Intersection of new line and stress-strain line is the offset yield strength

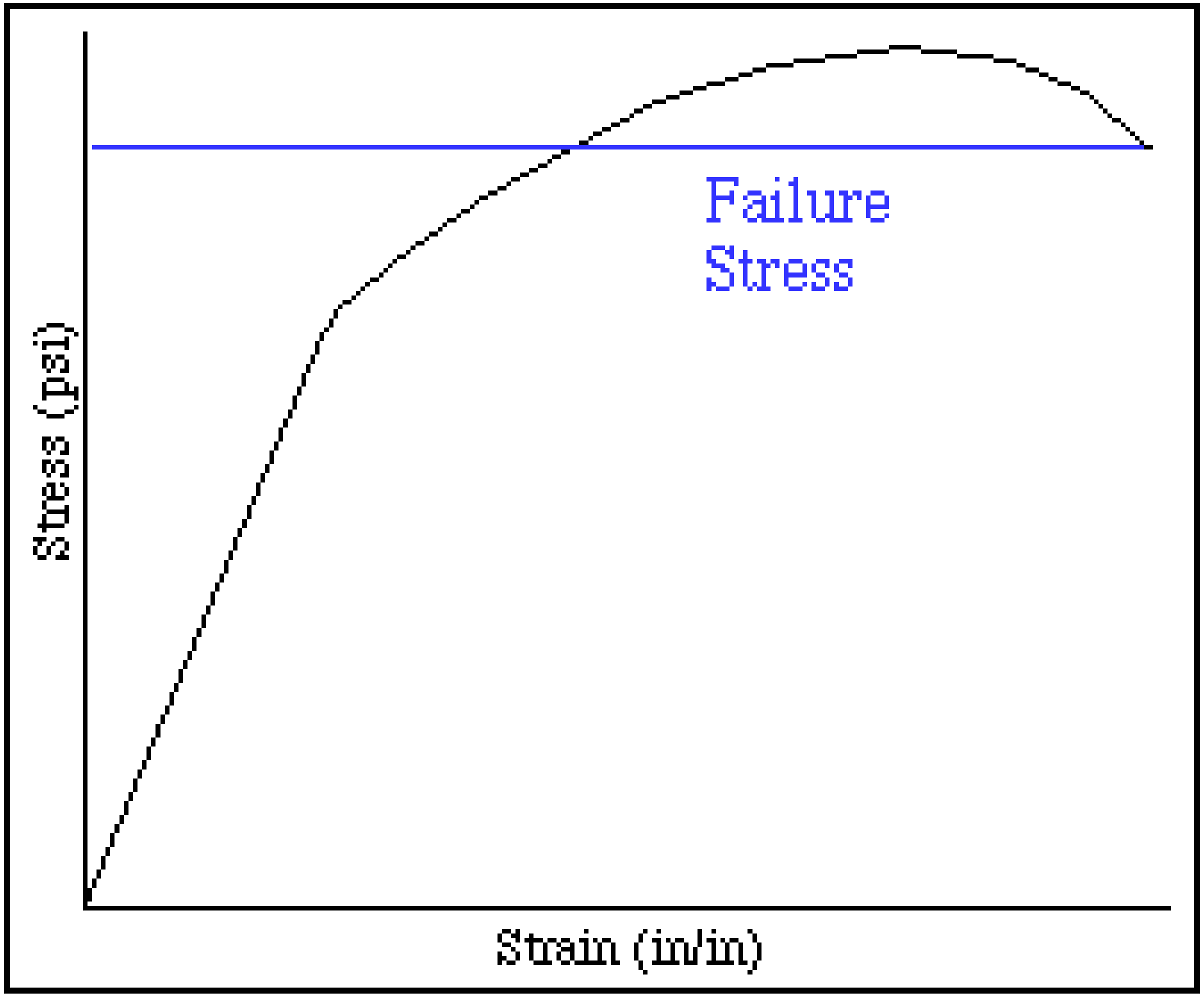
Tensile strength - the maximum stress applied to the specimen.



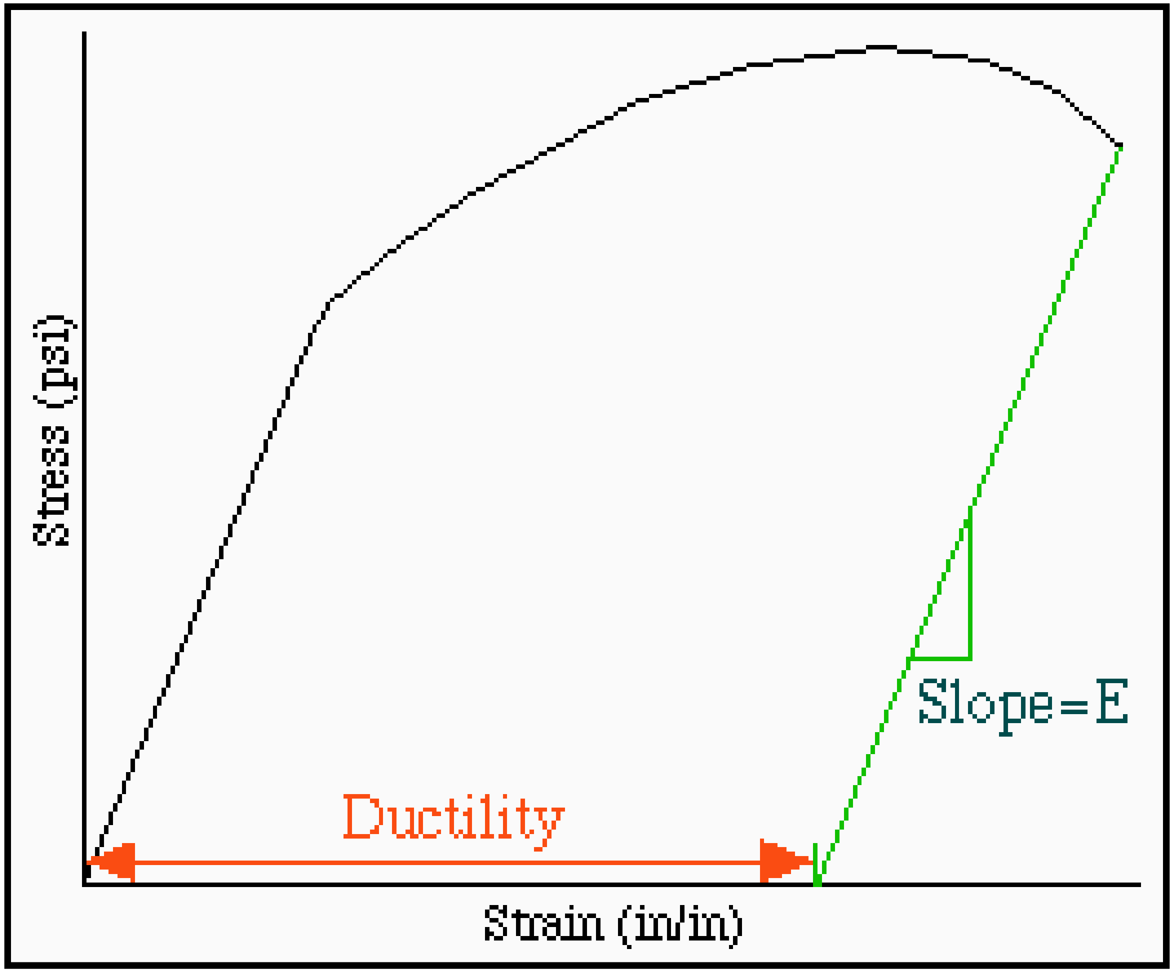
Modulus of resilience - the area under the linear part of the curve, measuring the stored elastic energy.



Failure stress - the stress applied to the specimen at failure (usually less than the maximum tensile strength because necking reduces the cross-sectional area)



Ductility - the total elongation of the specimen due to plastic deformation, neglecting the elastic stretching (the broken ends snap back and separate after failure).



Ductility

% Elongation:

% elongation is a measure of ductility, which is given by:

$$\% \text{ elongation} = 100 * (L_f - L_o)/ L_o$$

where,

L_o = Initial length

L_f = Final Length

Ductility

% Reduction in Area:

% reduction in area is a measure of ductility, which is given by:

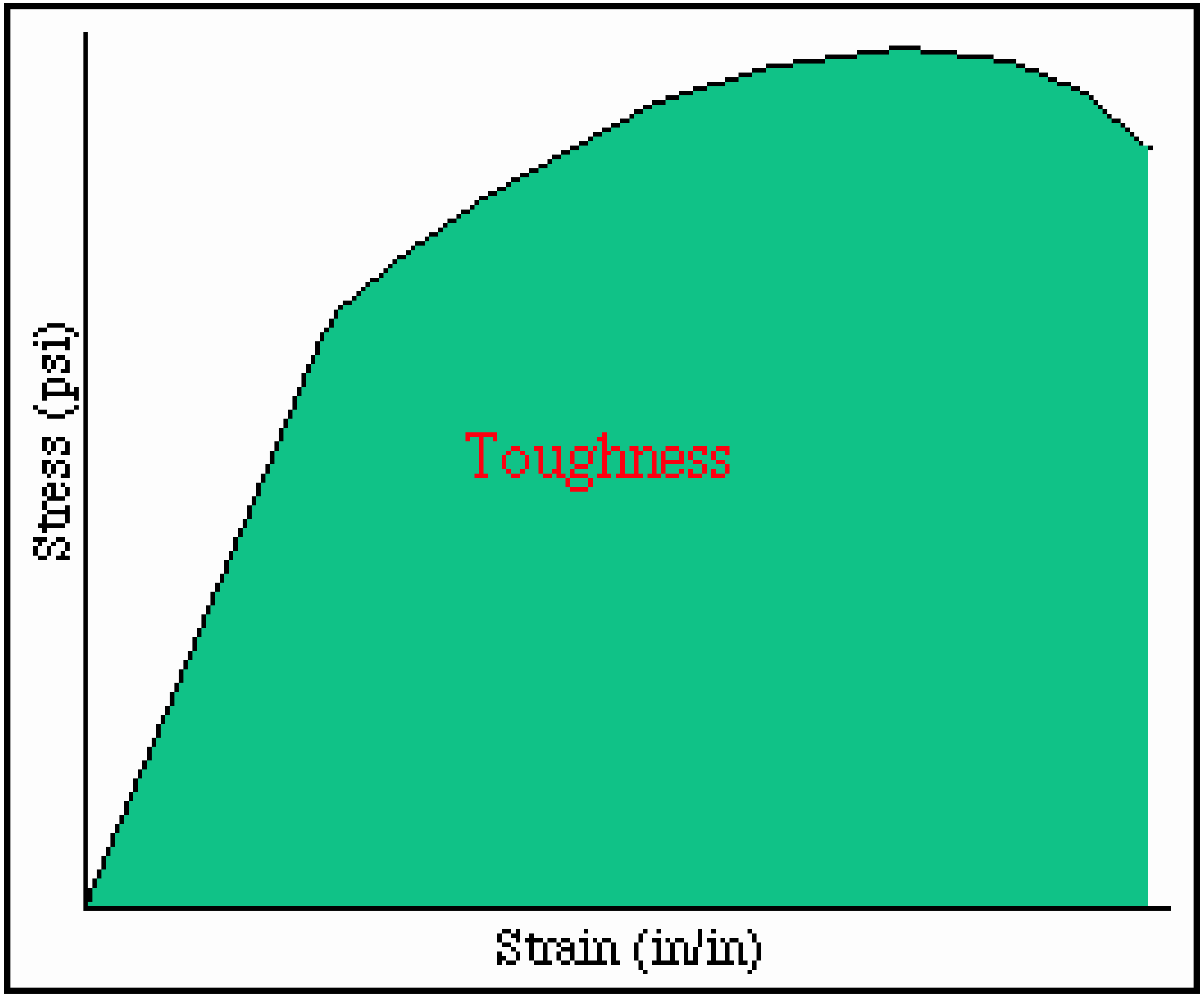
$$\% \text{ reduction in area} = 100 * (A_o - A_f)/ A_o$$

where,

A_o = Initial arae

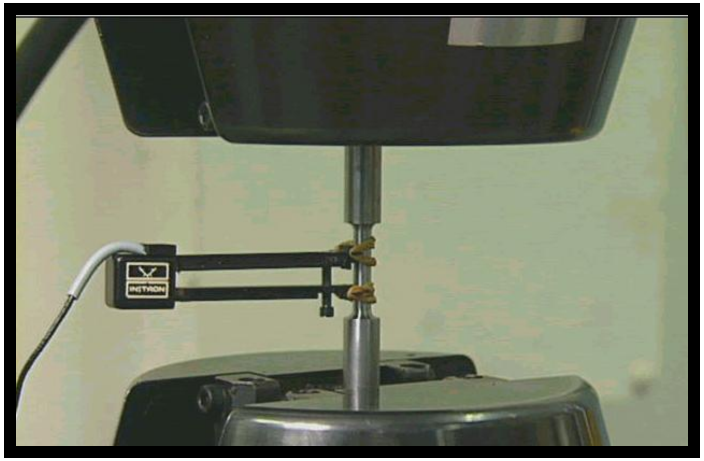
A_f = Final area

Toughness - the total area under the curve, which measures the energy absorbed by the specimen in the process of breaking.

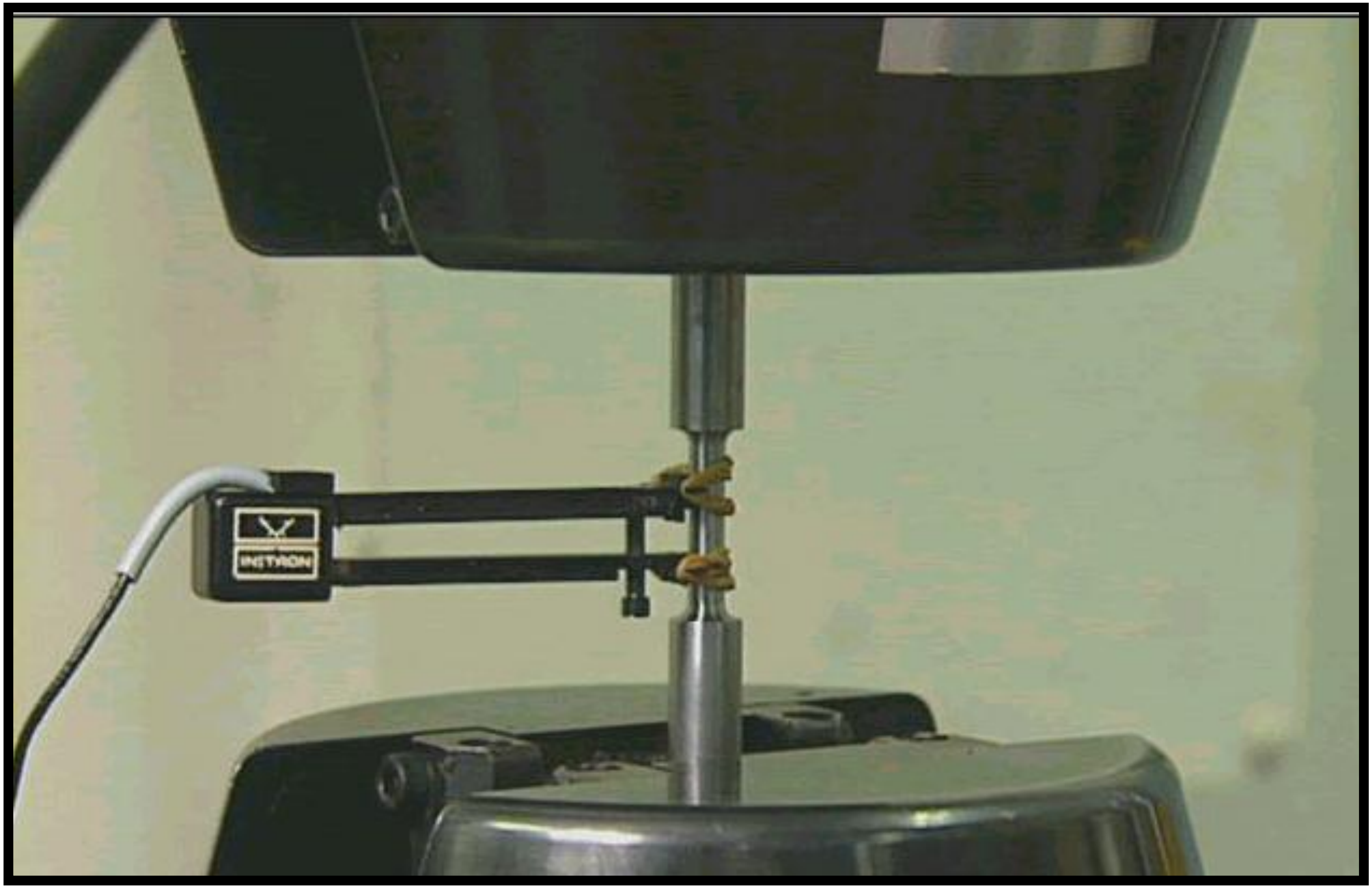


Tensile Test Equipment

This is an Instron tensile-test machine. The specimen to be tested is clamped at its two ends by two grips. In the current setting, the upper grip is fixed although its vertical position can be adjusted so as to accommodate specimens of different sizes. The lower grip is driven by a powerful hydraulic actuator. Once the specimen has been attached to the grips, the vertical movement of the lower grip generates the desired loading on the specimen.



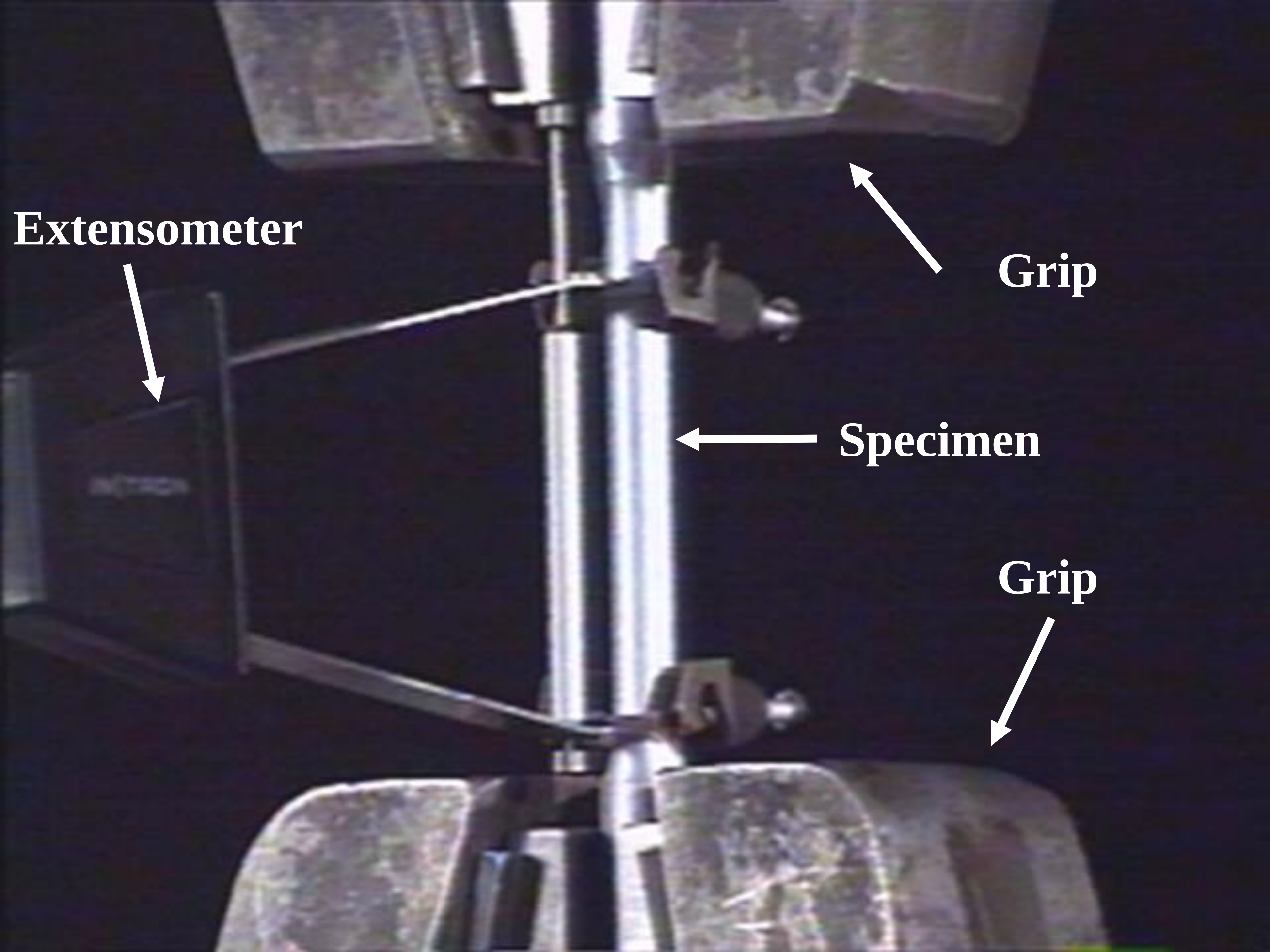
In a uniaxial tension test, the particular deformation of the specimen which is of greatest interest is its elongation. The Instron machine automatically measures the distance between the two grips, and if the specimen is uniform, this would be its elongation. The elongation can be measured by a device called extensometer. The extensometer is attached at two points to the central uniform part of the specimen, and the extensometer continually measures the length of this segment as the test proceeds. This transducer is very sensitive. Care must be taken to avoid damaging the transducer when the specimen breaks.



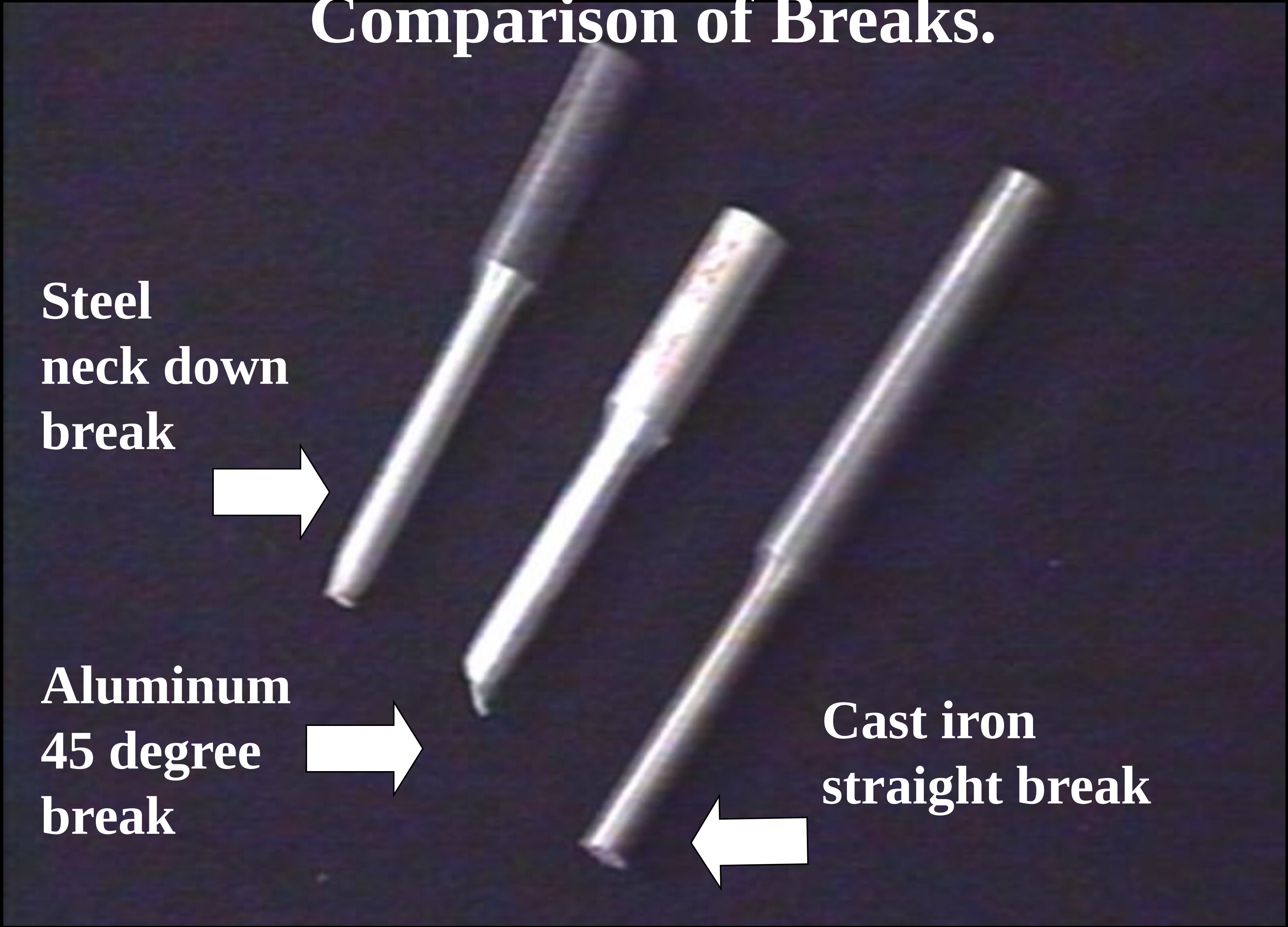
Electronic Extensometer

The mechanical properties of a material are obtained by subjecting a specimen to prescribed loads and then measuring the resulting deformation. Usually, the test is carried out on a special machine that is specifically designed for this purpose. The measurements of the load and of the deformation are carried out by transducers in conjunction with a computer data acquisition system.

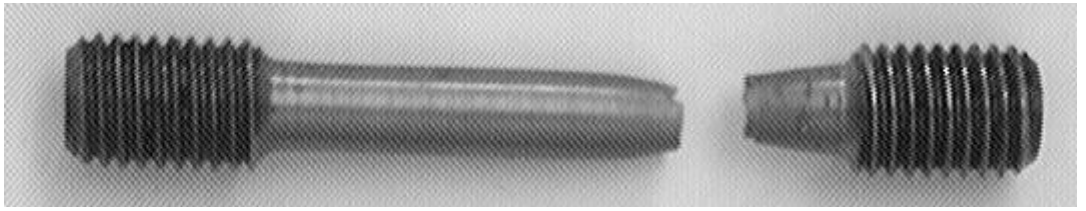
Tensile Test Machine (Instron)


Extensometer
Grip
Specimen
Grip

Comparison of Breaks.

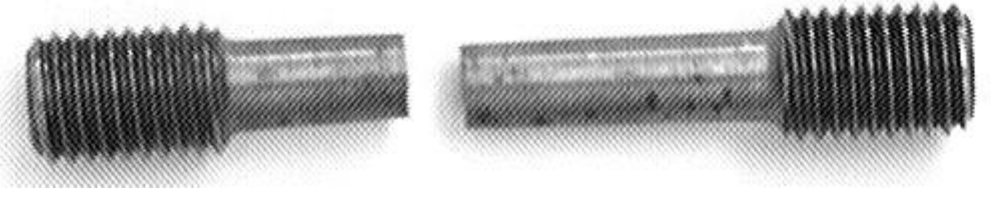


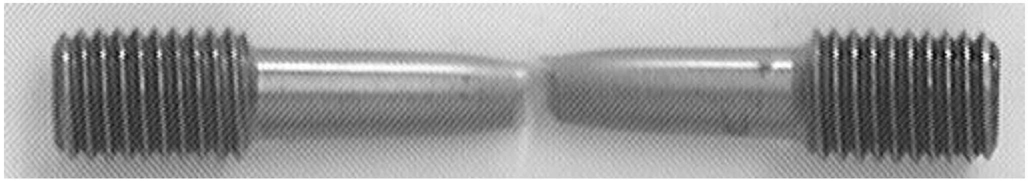
Characteristics of Tensile Test Breaks for various steel samples



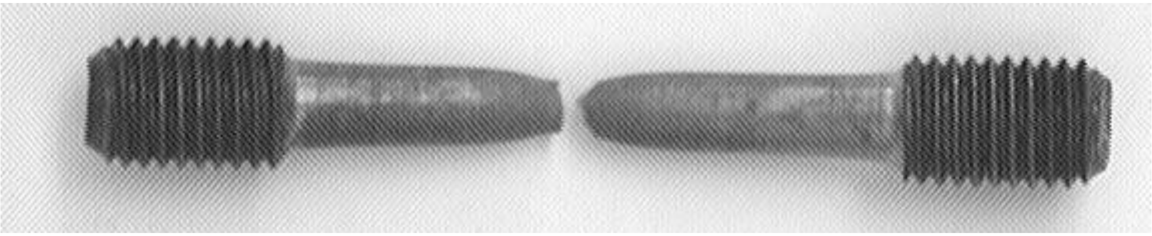
SAE1045 Hot rolled steel

SAE1090 Hot rolled steel



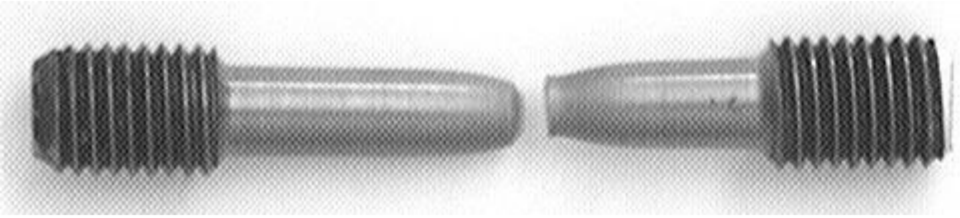


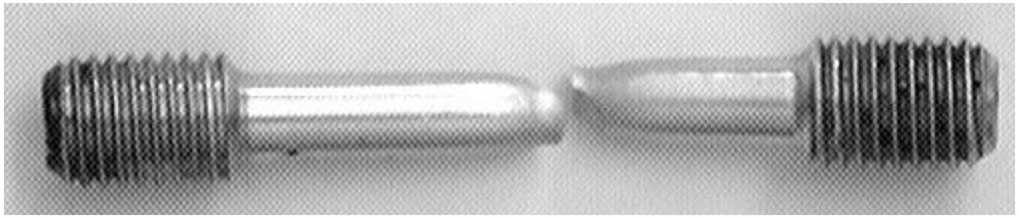
SAE1095 Spheroidized steel



Carbon Steel ASTM A36

SAE1018Cold Drawn steel





6016 Aluminum

What happened ?

Preventing Failure

In service, under loading (mechanical, thermal)

- How to assure performance, safety and durability?
  - Avoid excess deformation that may deteriorate the functionality
  - Avoid cracking that may propagate to complete fracture
- The study of deformation and fracture in materials

  the response of materials to mechanical loads or deformation.

Deformation and Failure

- Deformation
  - Time independent
    - Elastic
    - Plastic
  - Time dependent
    - Creep
- Fracture
  - Static loading
    - Brittle: rapid run of cracks through a stressed material
    - Ductile
    - Environmental (combination of stress and chemical effects)
      - High-strength steel may crack in the presence of hydrogen gas
      - Creep rupture (creep deformation proceeding to the point of separation)
  - Fatigue/cycling loading
    - High cycle/low cycle
    - Fatigue crack growth
    - Corrosion fatigue

Types of Failure

- Fracture
  - Cracking to the extent that component to be separated into pieces
  - Steps in fracture:
    - crack formation
    - crack propagation

- Depending on the ability of material to undergo plastic deformation before the fracture two fracture modes can be defined - **ductile or brittle**
  - **Ductile fracture** - most metals (not too cold):
    - Extensive plastic deformation ahead of crack
    - Crack is "stable": resists further extension unless applied stress is increased
  - **Brittle fracture** - ceramics, ice, cold metals:
    - Relatively little plastic deformation
    - Crack is "unstable": propagates rapidly without increase in applied stress

Fracture of Materials

Crack formation mechanisms

Metals typically form cracks by the accumulation of dislocations at a crack nucleation site (grain boundaries, precipitate interface, free surface, etc.)

Ceramics, semiconductors, some plastics (hard and brittle, eg., thermosetting plastics) and intermetallic compounds form cracks by planar defects (grain boundaries, two-phase interfaces, etc.)

Soft plastics crack by the sliding of the long polymer chairs across each other by breaking the Van der Wall bonds.

Fracture of Materials

Fracture can be classified according to the path of crack propagation:

•Transgranular – the crack travels directly through the grains of the material (sometimes called cleavage because it occurs along certain crystallographic planes). It can be ductile or brittle.

•Intergranular – the crack propagates along grain boundaries. This is primarily brittle fracture.

A variety of Loading Conditions can lead to fracture:

•Static Overloading ($\sigma > \sigma_{yield}$) and ($\sigma >$ Tensile Strength)

•Dynamic Overloading (impacting)

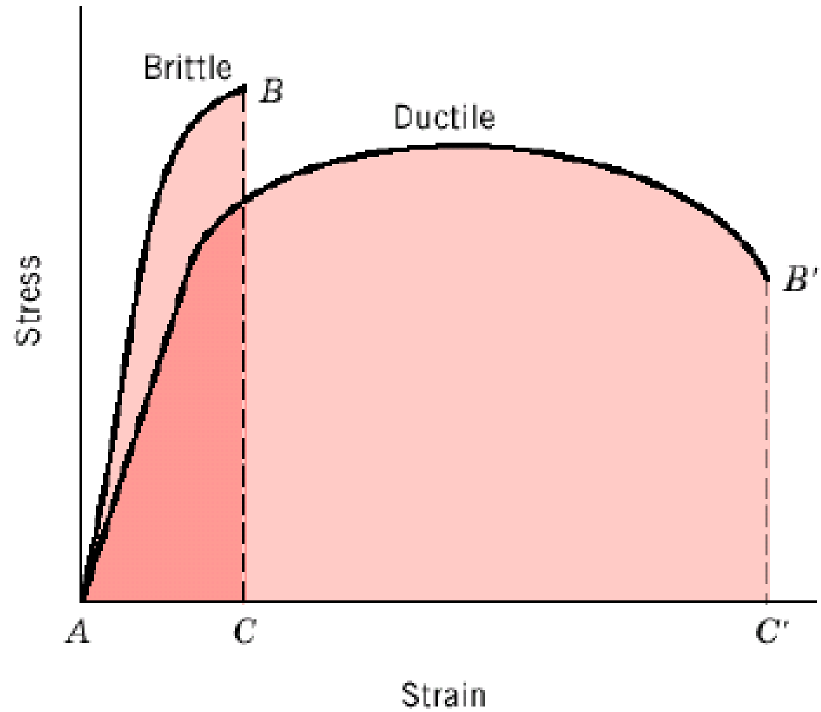
•Cyclic loading (fatigue)

•Loaded at elevated temperatures (creep)

•Loading at cryogenic temperatures (ductile to brittle transition)

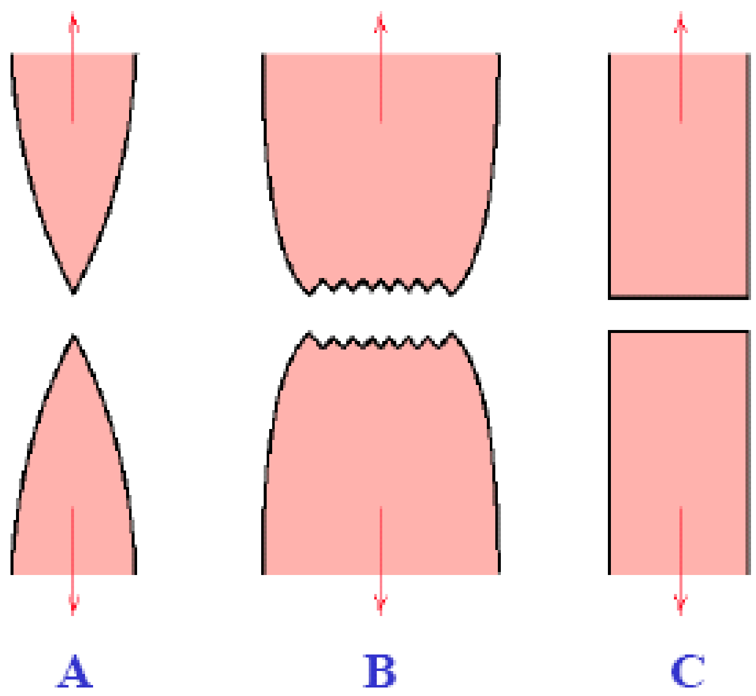
•Loading in a corrosive environment (stress corrosion)

Brittle vs. Ductile Fracture



- **Ductile materials** - extensive plastic deformation and energy absorption (“toughness”) before fracture
- **Brittle materials** - little plastic deformation and low energy absorption before fracture

Brittle vs. Ductile Fracture

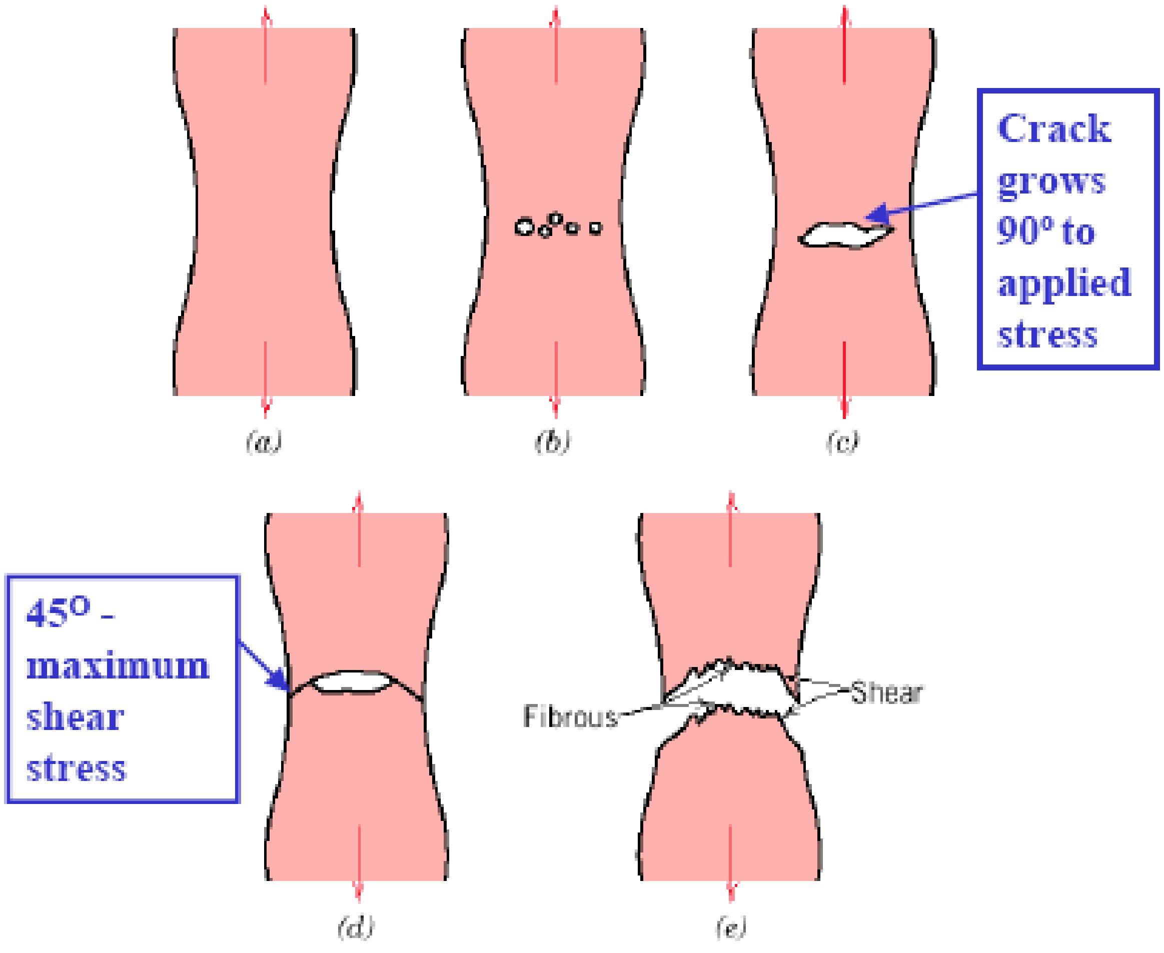


A. **Very ductile**, soft metals (e.g. Pb, Au) at room temperature, other metals, polymers, glasses at high temperature.

B. **Moderately ductile fracture**, typical for ductile metals

C. **Brittle fracture,** cold metals, ceramics.

Ductile Fracture

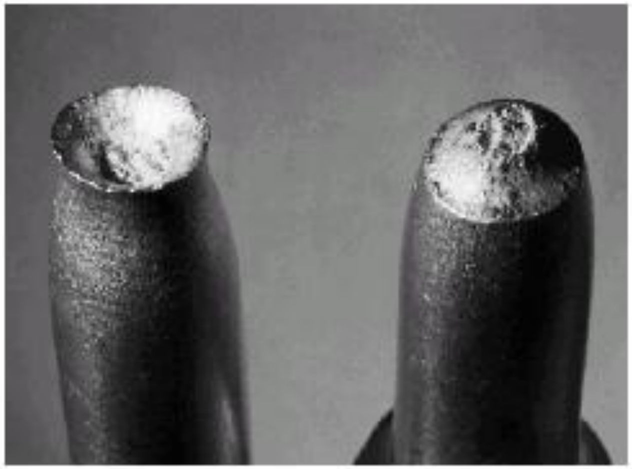


(a) Necking, (b) Cavity Formation,

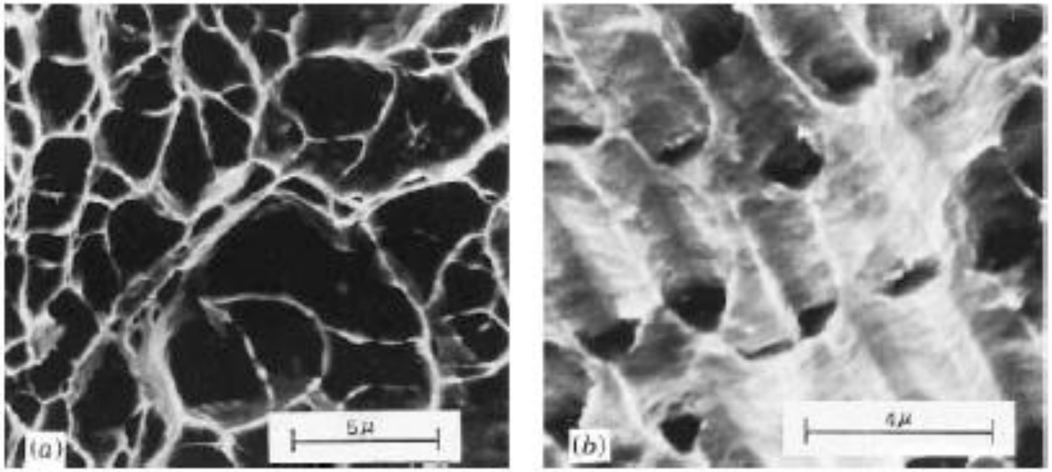
(c) Cavity coalescence to form a crack,

(d) Crack propagation, (e) Fracture

Ductile Fracture

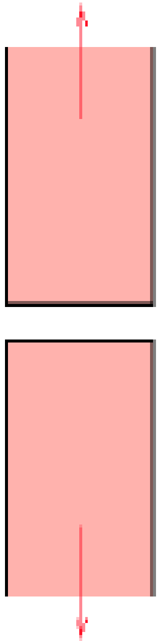


(Cap-and-cone fracture in A1



Scanning Electron Microscopy: *Fractographic* studies at high resolution. Spherical "dimples" correspond to micro-cavities that initiate crack formation.

Brittle Fracture



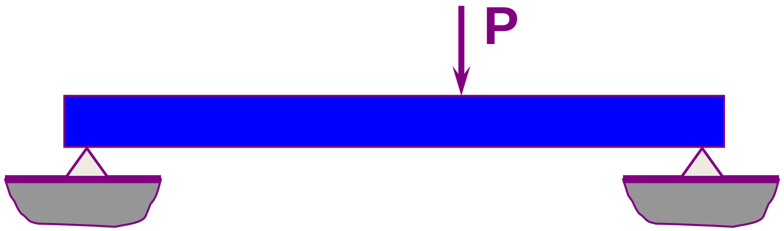
Brittle fracture in a mild steel

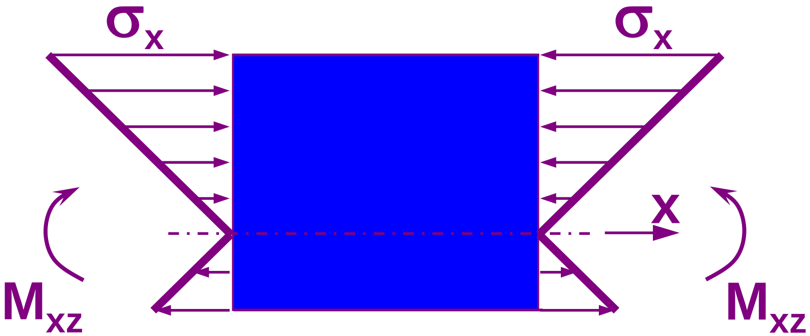
- No appreciable plastic deformation
- Crack propagation is very fast
- Crack propagates nearly perpendicular to the direction of the applied stress
- Crack often propagates by cleavage - breaking of atomic bonds along specific crystallographic planes (cleavage planes).

Bending of Beams

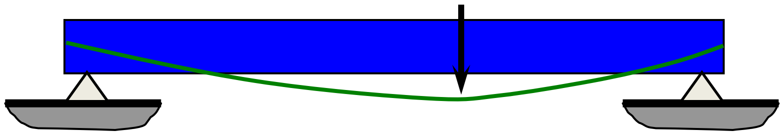
2.1 Revision – Bending Moments

2.2 Stresses in Beams





2.4 Deflections in Beams

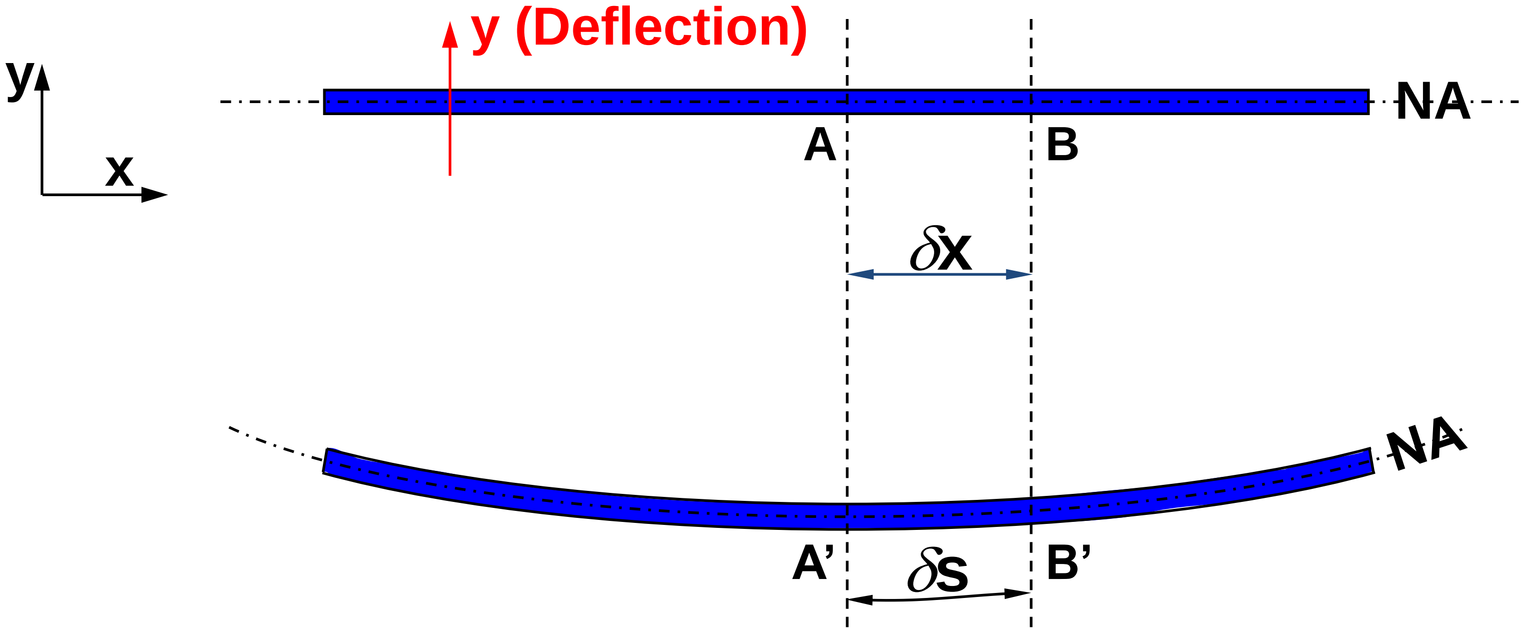


2.4 Beam Deflection

Recall: THE ENGINEERING BEAM THEORY

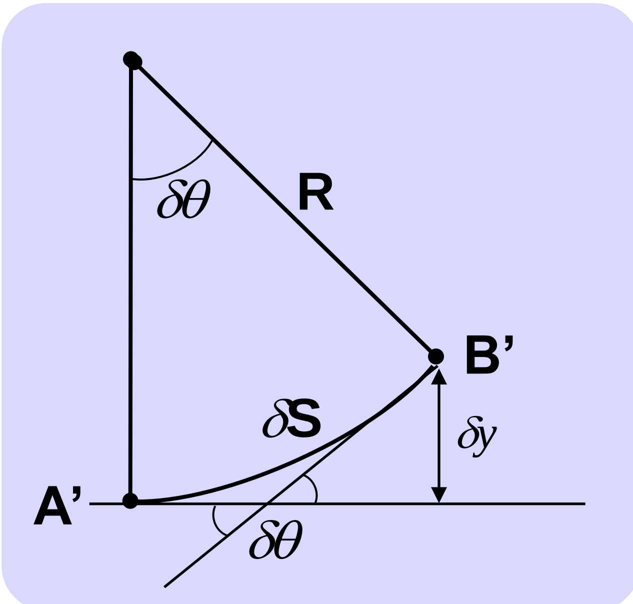
$$-\frac{\sigma_x}{y'} = \boxed{\frac{M_{xz}}{I_z} = \frac{E}{R}}$$

2.4.1 Moment-Curvature Equation



If deformation is small (i.e. slope is "flat"):

$$\delta s \approx \delta x$$

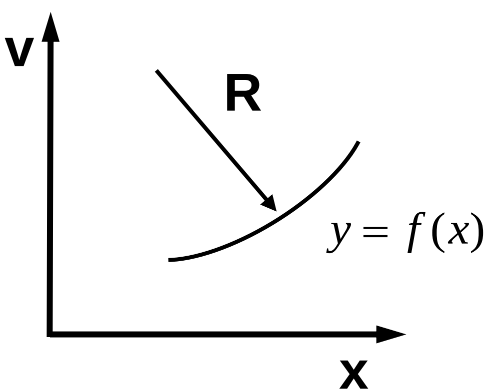


$$\mathbf{R}.\delta\theta = \delta\mathbf{S} \approx \delta\mathbf{x} \qquad \therefore \frac{\mathbf{I}}{\mathbf{R}} \approx \frac{\mathbf{d}\theta}{\mathbf{dx}}$$

and $\delta\theta \approx \frac{\delta y}{\delta x}$ **(slope is "flat")**

$$\Rightarrow \frac{I}{R} \approx \frac{d^2 y}{dx^2}$$

Alternatively: from Newton's Curvature Equation



$$\frac{I}{R} = \frac{\left(\frac{d^2 y}{dx^2}\right)}{\left(1+\left(\frac{dy}{dx}\right)^2\right)^{\frac{3}{2}}}$$

if $\left(\frac{dy}{dx}\right)^2 <<<< 1$

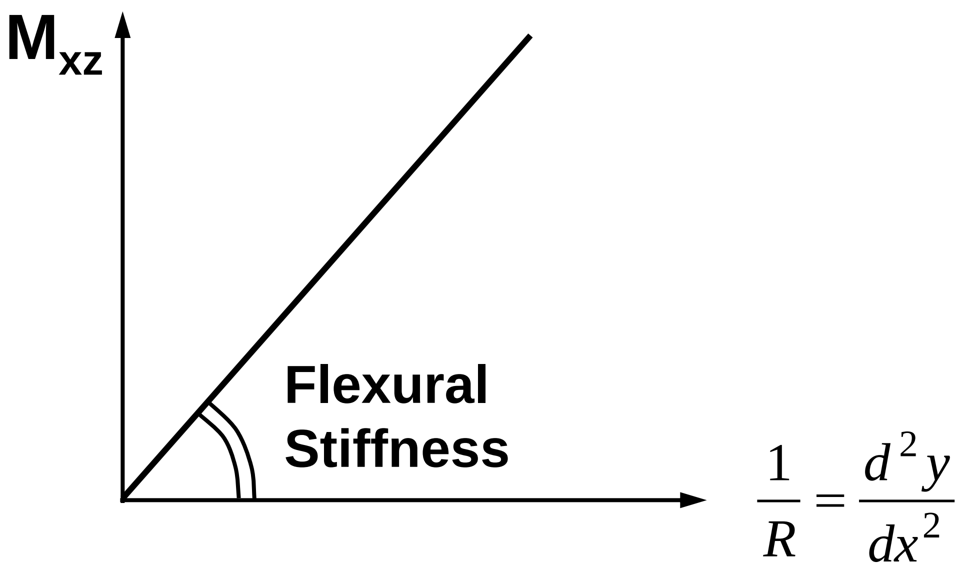
$$\Rightarrow \frac{I}{R} \approx \frac{d^2 y}{dx^2}$$

From the Engineering Beam Theory:

$$\frac{M_{xz}}{I_z} = \frac{E}{R} \qquad \frac{1}{R} = \frac{M_{xz}}{EI_z} = \frac{d^2y}{dx^2}$$

$$\Rightarrow \left(EI_z\right)\frac{d^2y}{dx^2} = M_{xz}$$

Flexural Stiffness — **Curvature** — **Bending Moment**



Recall, for Bars under axial loading:

$$\mathbf{K} \cdot \mathbf{u} = \mathbf{Load}$$

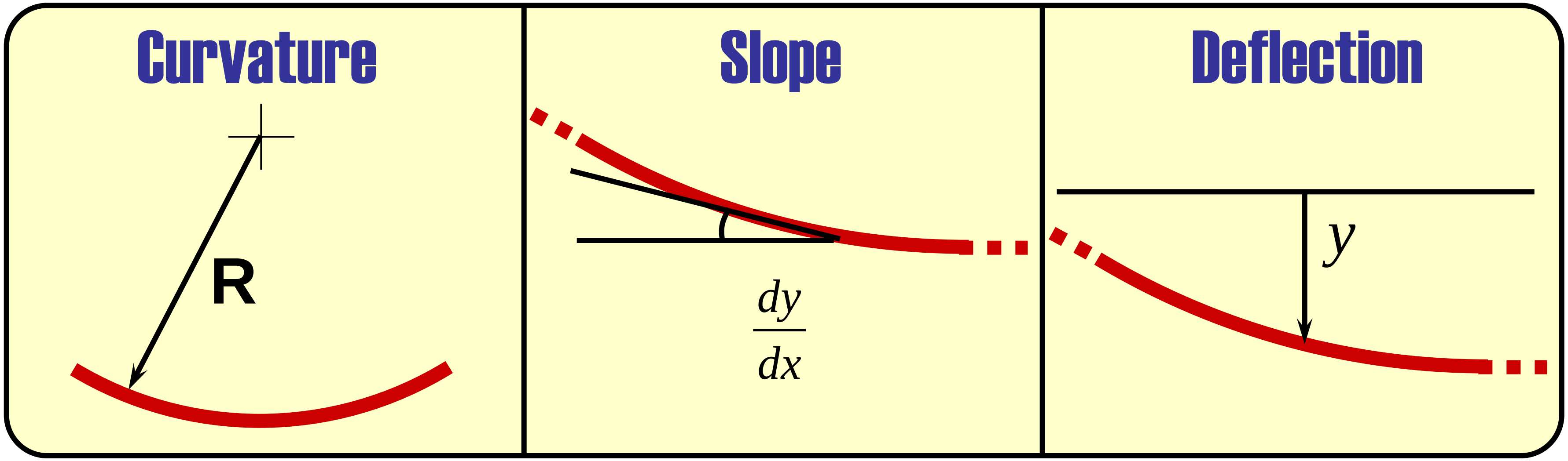
Axial Stiffness — **Extension**

Since, $$\frac{d^2y}{dx^2} = \left(\frac{1}{EI_z}\right)M_{xz}$$ ← **Curvature**

$$\Rightarrow \quad \frac{dy}{dx} = \left(\frac{1}{EI_z}\right)\int M_{xz}\cdot dx \; + \; C_1$$ ← **Slope**

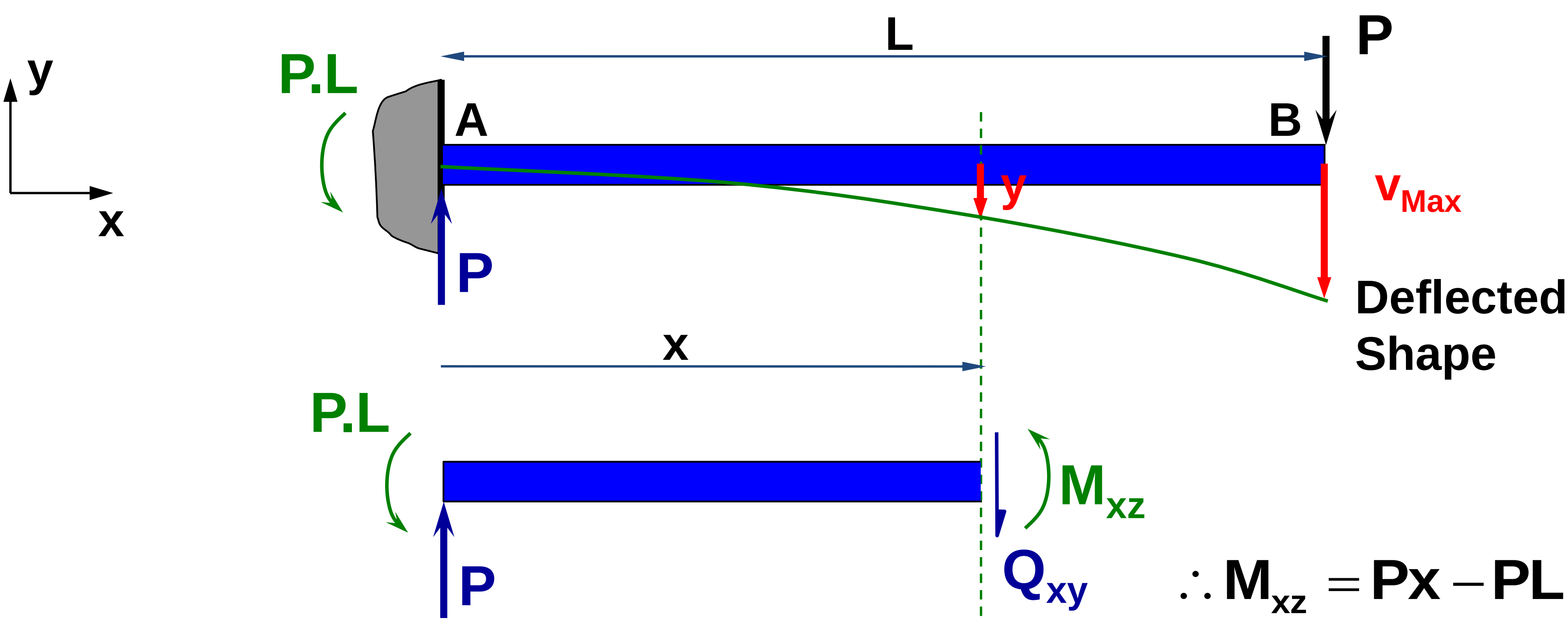
$$\Rightarrow \quad y = \left(\frac{1}{EI_z}\right)\int\int M_{xz}\cdot dx \cdot dx \; + \; \int C_1 \cdot dx \; + \; C_2$$ ← **Deflection**

Where C_1 and C_2 are found using the boundary conditions.



Cantilever Beam

v = Deflection



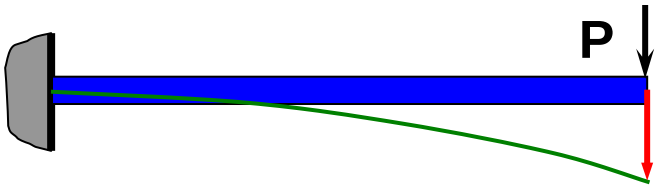
$$\therefore M_{xz} = Px - PL$$

$$\left(EI_z\right)\frac{d^2y}{dx^2} = M_{xz} = Px - PL$$

$$\Rightarrow \quad \left(EI_z\right)\frac{dy}{dx} = P\frac{x^2}{2} - PLx + C_1$$

$$\Rightarrow \quad \left(EI_z\right)y = P\frac{x^3}{6} - \frac{PLx^2}{2} + C_1x + C_2$$

$$\Rightarrow \quad (EI_z)y = P\frac{x^3}{6} - \frac{PLx^2}{2} + C_1x + C_2$$



<u>To find C_1 and C_2:</u>

Boundary conditions:

(i) @ x=0 $\quad \frac{dy}{dx} = 0$

(ii) @ x=0 $\quad y = 0$

$$\therefore \mathbf{C_1 = 0 \quad \& \quad C_2 = 0}$$

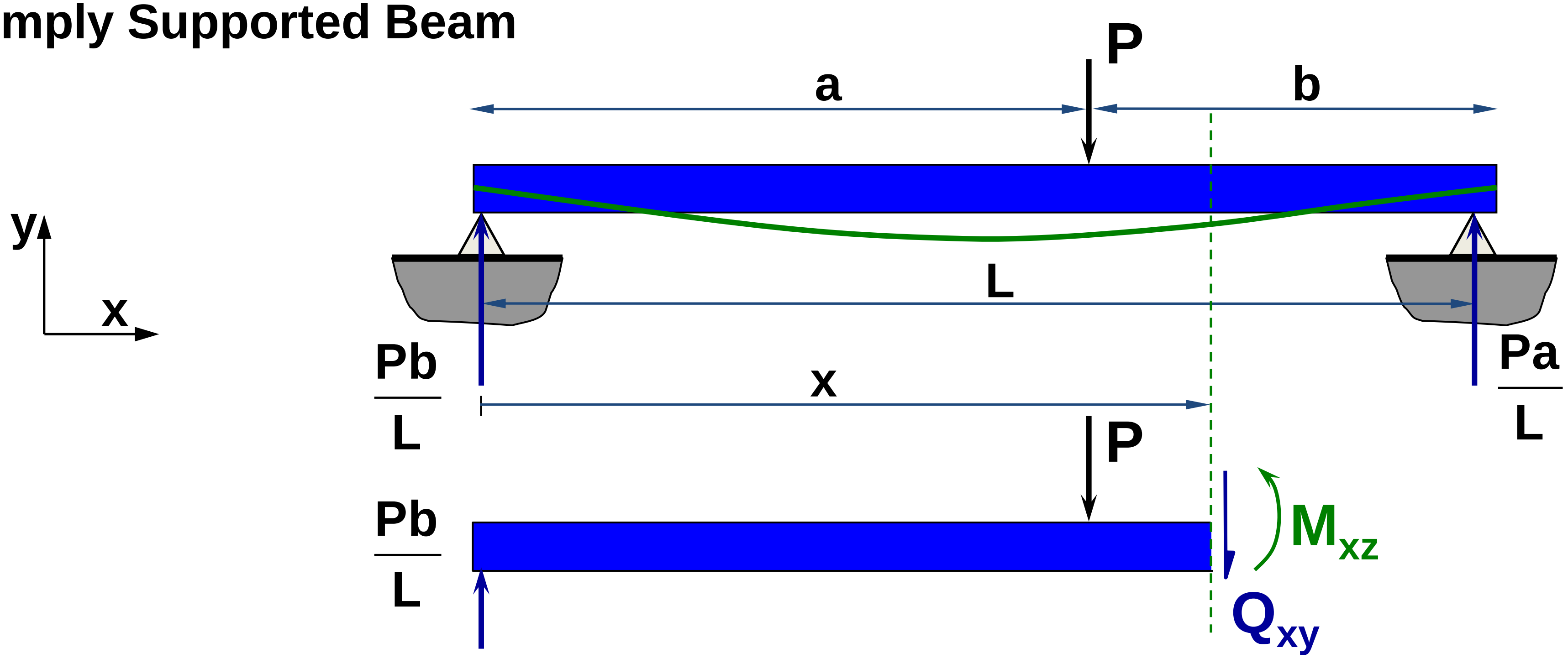
Equation of the deflected shape is:

$$(EI_z)y = P\frac{x^3}{6} - \frac{PLx^2}{2}$$

y_{Max} occurs at x=L

$$y_{Max} = -\frac{1}{3}\frac{PL^3}{EI_z}$$

Simply Supported Beam



$$\therefore \mathbf{M_{xz}} = \mathbf{Pb}/\mathbf{L}\,(\mathbf{x}) - \mathbf{P}\langle(\mathbf{x}-\mathbf{a})\rangle$$

$$\Rightarrow \quad (EI_z)\frac{d^2y}{dx^2} = M_{xz} \qquad = Pb/L\,(x) - P\langle(x-a)\rangle$$

$$\Rightarrow \quad (EI_z)\frac{dy}{dx} = Pb/L\left(\frac{x^2}{2}\right) - P/2\left\langle(x-a)^2\right\rangle + C_1$$

$$\Rightarrow \quad (EI_z)y = Pb/6L\left(x^3\right) - P/6\left\langle(x-a)^3\right\rangle + C_1(x) + C_2$$

$$\Rightarrow \quad (EI_z)y = \frac{Pb}{6L}\left(x^3\right) - \frac{P}{6}\left\langle (x-a)^3 \right\rangle + C_1(x) + C_2$$

Boundary conditions: **(i) @ x=0** $y = 0$

(ii) @ x=L $y = 0$

From (i): $\mathbf{C_2 = 0}$

From (ii): $$\mathbf{0 = \frac{Pb}{6L}\left(L^3\right) - \frac{P}{6}\left\langle (L-a)^3 \right\rangle + C_1(L)}$$

$$\mathbf{\therefore C_1 = \frac{Pb}{6L}\left(b^2 - L^2\right)}$$ **Since (L-a)=b**

Equation of the deflected shape is:

$$\Rightarrow \quad y = \frac{1}{EI_z}\left[\frac{Pb}{6L}\left(x^3\right) - \frac{P}{6}\left\langle (x-a)^3 \right\rangle + \frac{Pb}{6L}\left(b^2 - L^2\right)(x)\right]$$

To find v_{Max}:

y_{Max} **occurs where** $\frac{dy}{dx} = 0$ **(i.e. slope=0)**

$$\text{i.e.} \quad (EI_z)(0) = \frac{Pb}{L}\left(\frac{x^2}{2}\right) - \frac{P}{2}\left\langle (x-a)^2 \right\rangle + \frac{Pb}{6L}\left(b^2 - L^2\right)$$

Assuming v_{Max} will be at x<a, i.e. $\left\langle (x-a)^2 \right\rangle = 0$

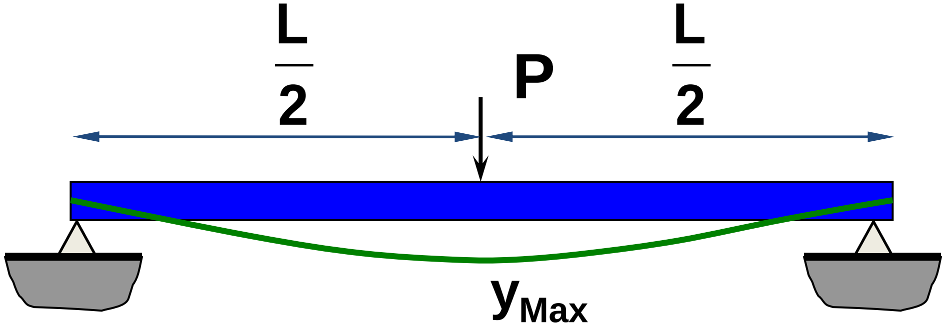
$$\therefore \frac{dy}{dx} = 0 \quad \text{when} \quad x^2 = -\frac{1}{3}\left(b^2 - L^2\right) = \frac{1}{3}\left(L^2 - b^2\right)$$

This value of x is then substituted into the above equation of the deflected shape in order to obtain v_{Max}.

Note:

$$\text{if} \quad a = b = \frac{L}{2}$$

$$\therefore y_{Max} = -\frac{PL^3}{48EI_z}$$



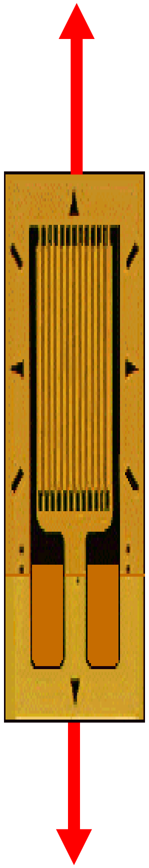
2.4.3 Summary

After considering stress caused by bending, we have now looked at the deflections generated. Keep in mind the relationships between Curvature, Slope, and Deflection, and understand what they are:

- **Curvature** $$\frac{d^2y}{dx^2} = \frac{1}{EI_z} M_{xz} \approx \frac{I}{R}$$
- **Slope** $$\frac{dy}{dx}$$
- **Deflection** $$y$$

Strain Gauge

Tension



$l \uparrow$

$R \uparrow$

$$R = \rho \frac{l}{A} \quad \Rightarrow \quad R \propto l$$

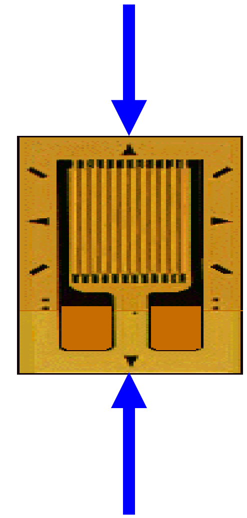
$R =$ Resistance

$\rho =$ Property of material

$l =$ Length of wire

$A =$ Effective cross sectional area of wire

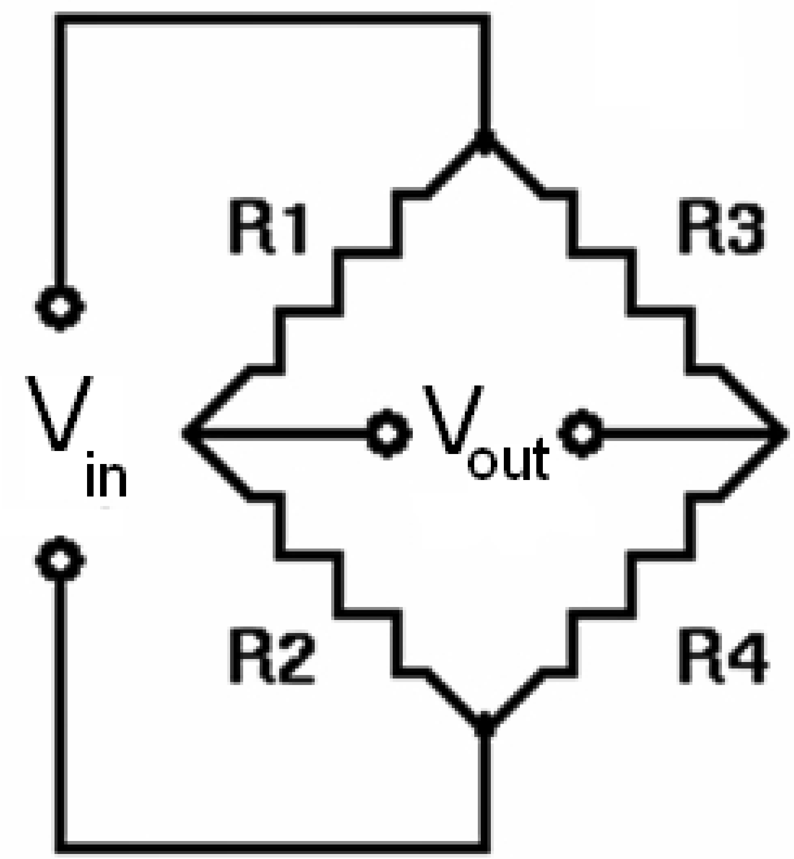
Compression



$l \downarrow$

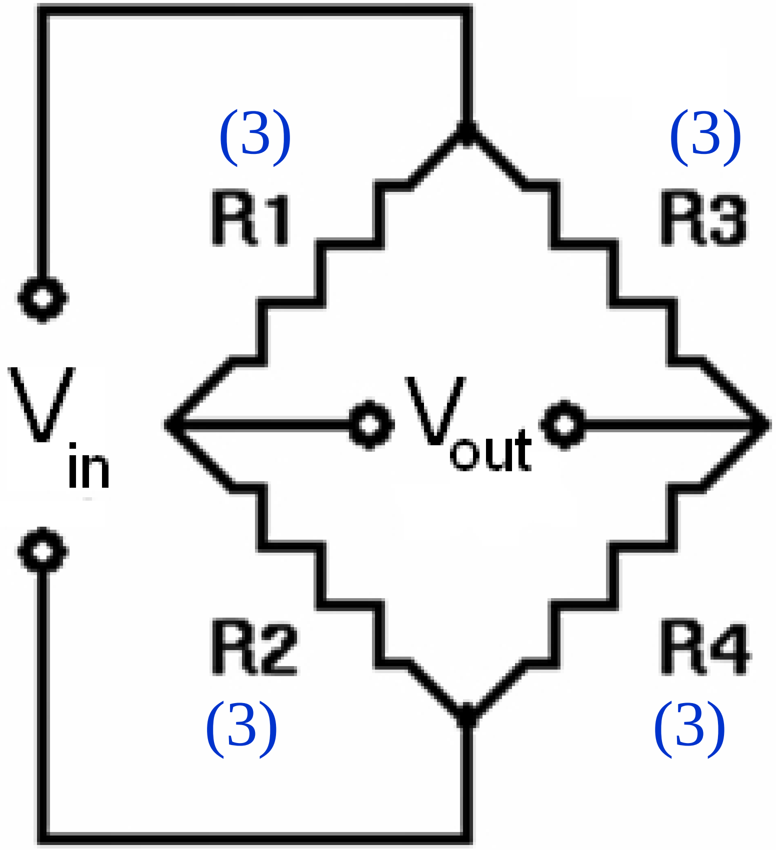
$R \downarrow$

Wheatstone Bridge



$$V_{out} = \left(\frac{R_4}{R_3 + R_4} - \frac{R_2}{R_1 + R_2} \right) V_{in}$$

Wheatstone Bridge



$$say,$$

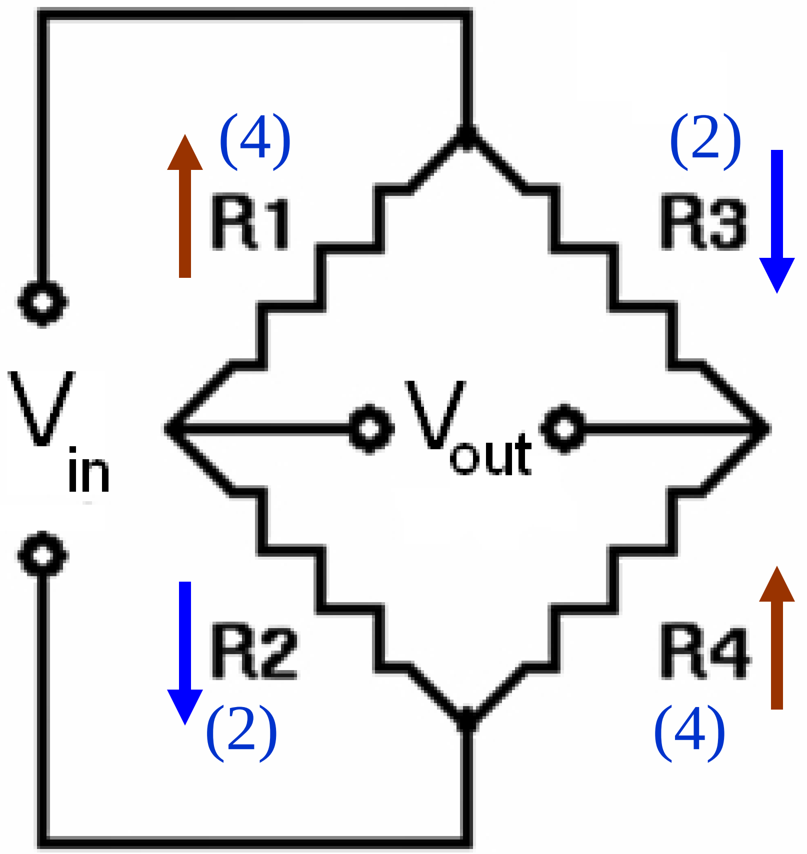
$$V_{in} = 5.00 volts$$

$$V_{out} = \left(\frac{R_4}{R_3 + R_4} - \frac{R_2}{R_1 + R_2} \right) V_{in}$$

$$V_{out} = \left(\frac{3}{3+3} - \frac{3}{3+3} \right) 5.0$$

$$V_{out} = 0$$

Wheatstone Bridge



say,

$$V_{in} = 5.00 volts$$

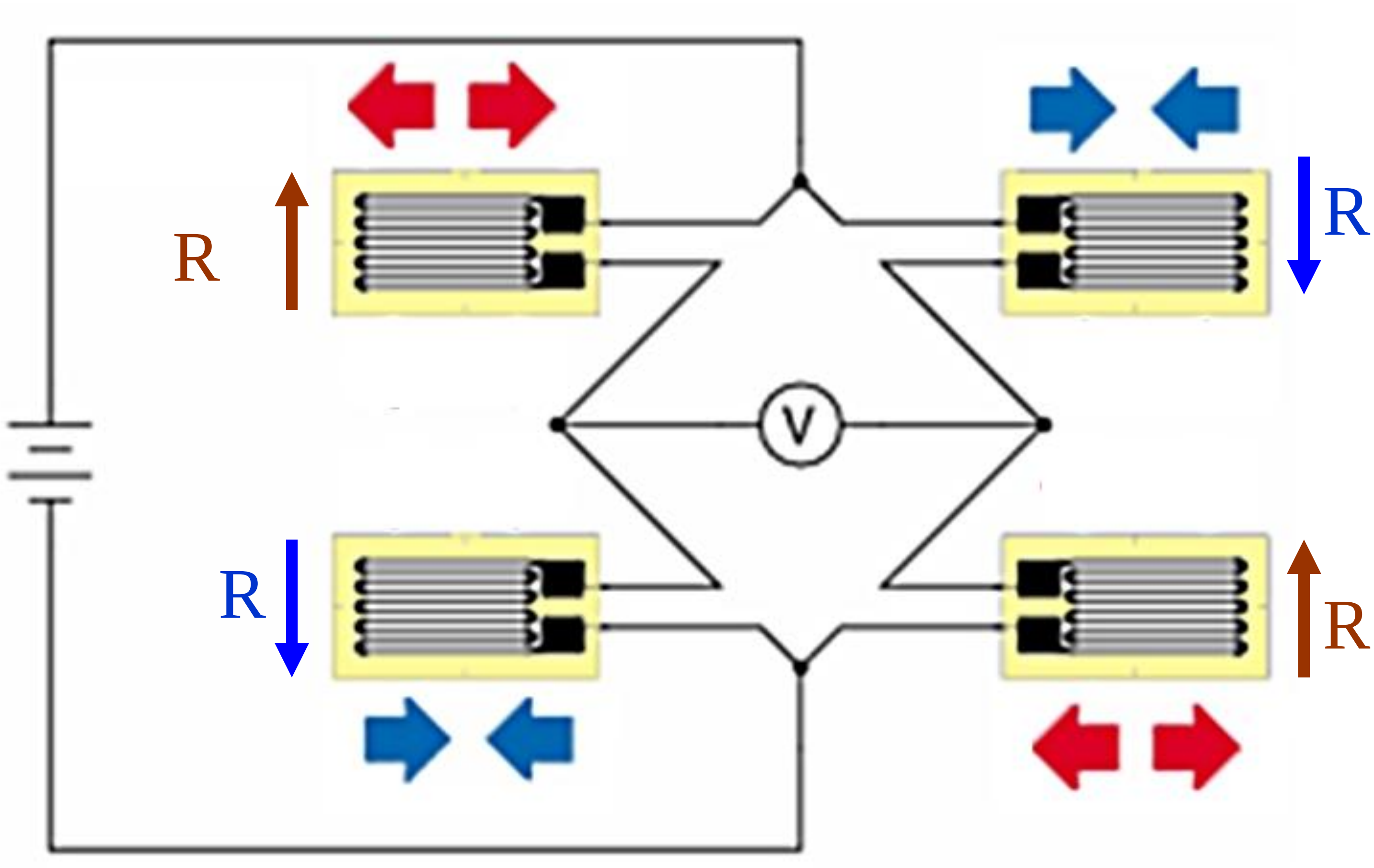
$$V_{out} = \left(\frac{R_4}{R_3 + R_4} - \frac{R_2}{R_1 + R_2} \right) V_{in}$$

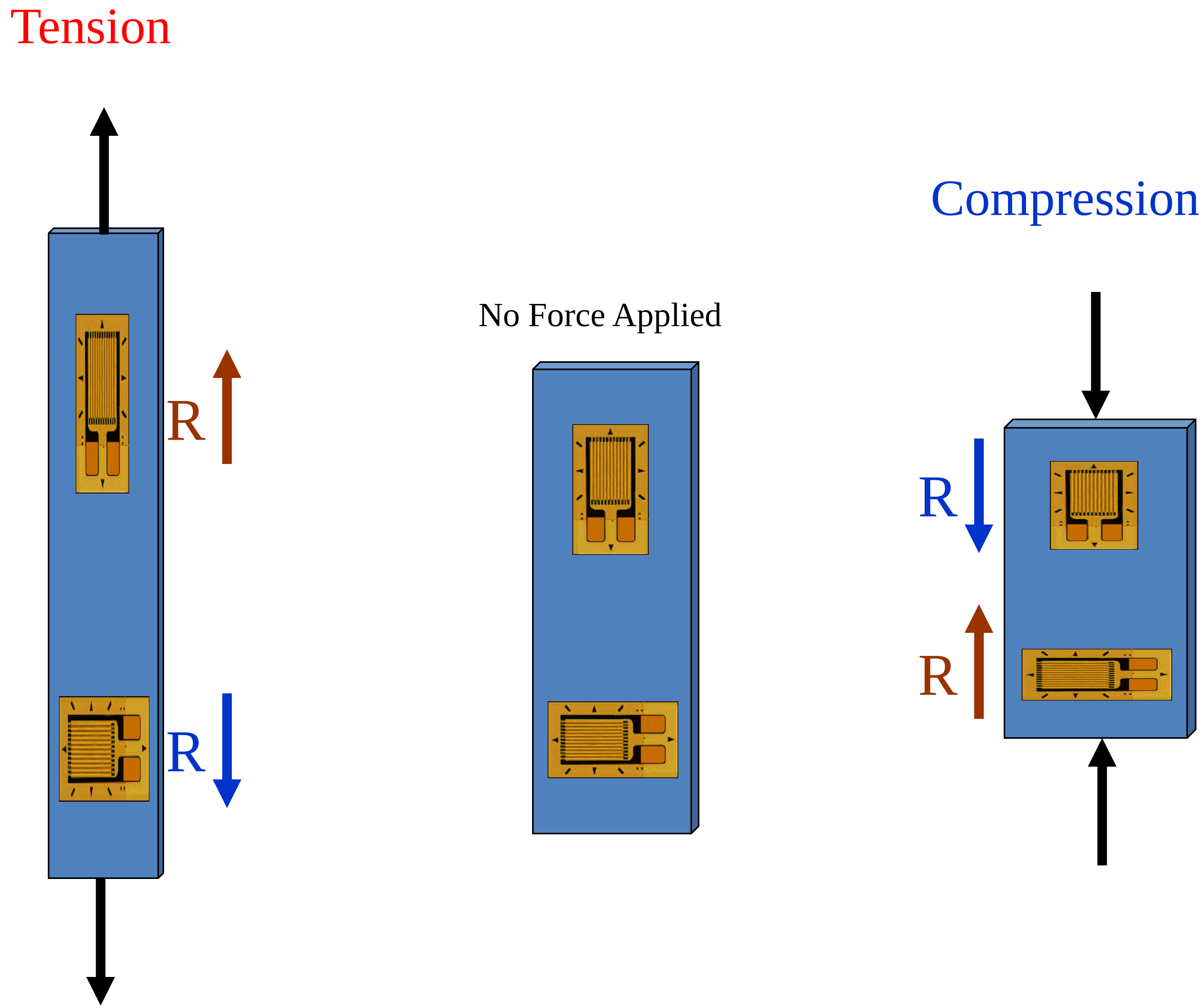
$$V_{out} = \left(\frac{4}{2+4} - \frac{2}{4+2} \right) 5.0$$

$$V_{out} = \left(\frac{4}{6} - \frac{2}{6} \right) 5.0$$

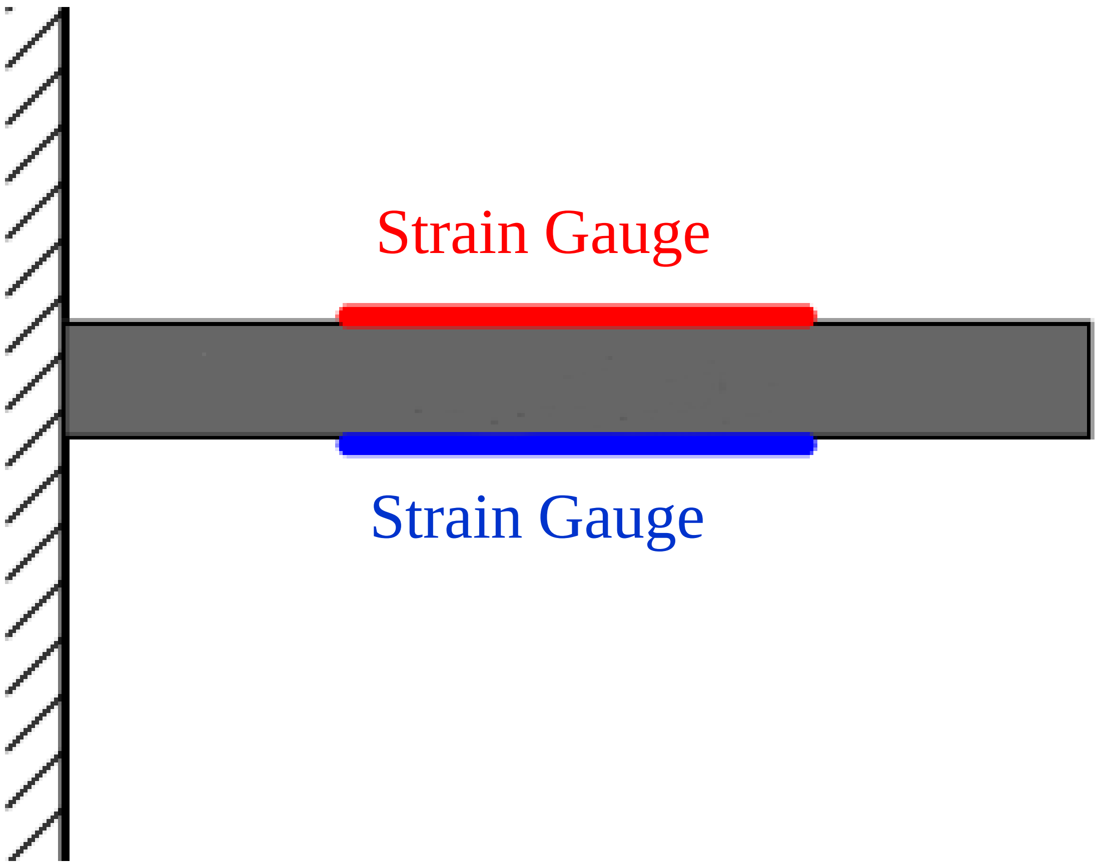
$$V_{out} = 1.667 volts$$

Full Bridge Strain Gauge

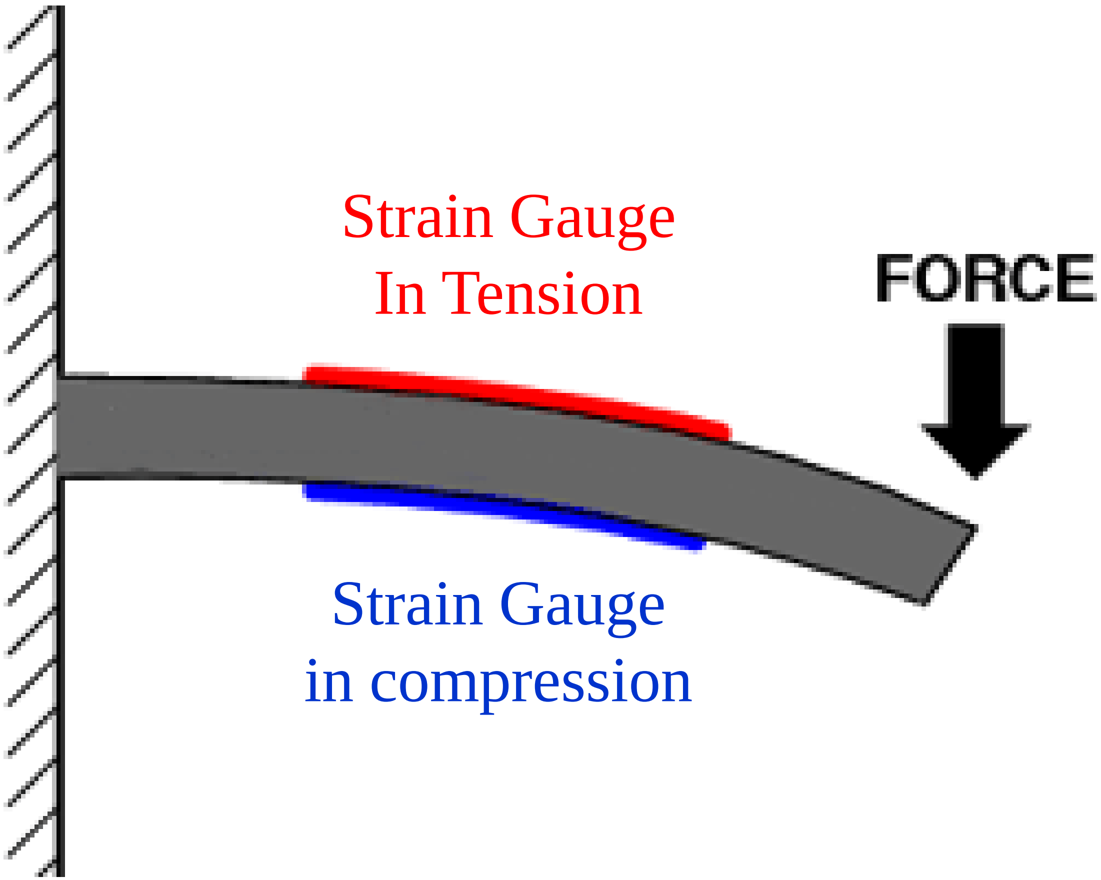



Tension
R
R
No Force Applied
Compression
R
R

Bending Beam Load Cell

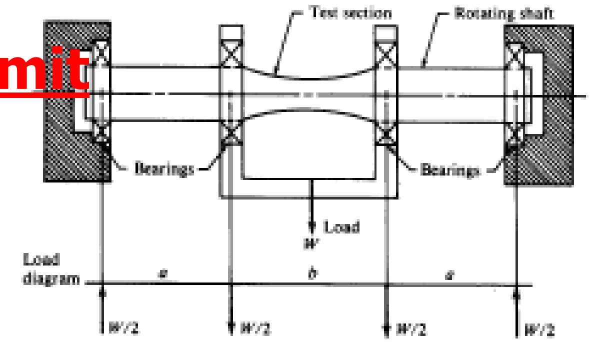


Bending Beam Load Cell

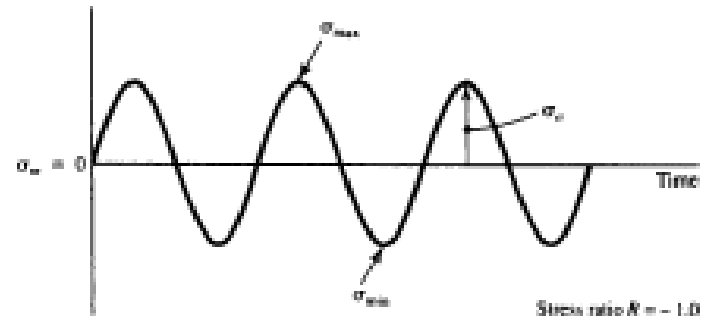


Fatigue Test

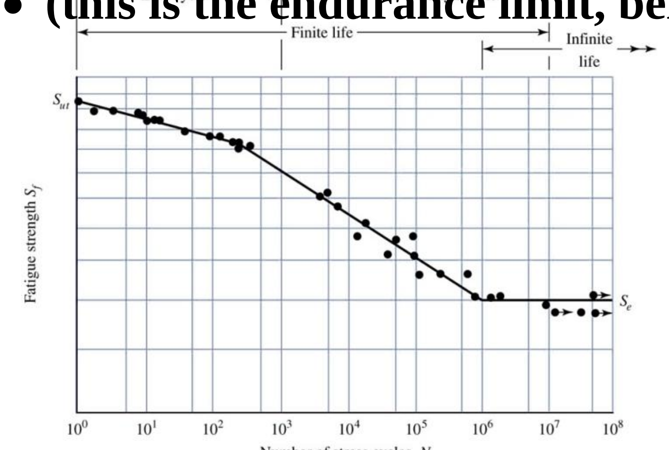
Fatigue Strength and Endurance Limit



- Most machine parts have time varying loads that produce time varying stresses
- Materials behave differently under this cyclic loading
- (Fatigue loads) – need to know fatigue strength
- Each point on the surface goes from maximum tension to 0 stress to maximum compression
- Run the test until failure
- Count the cycles and evaluate the stress level
- Repeat the test for different loads (stress levels)
- Generate the stress-life diagram



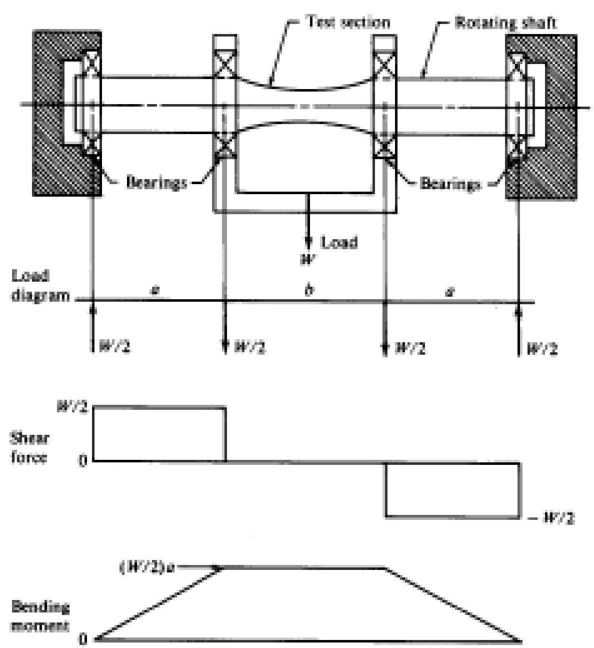
- Fatigue strength for 0 cycle is the same as S_{ut}
- The strength the decreases with increasing number of cycles
- The curve levels off at 10^6 cycles for steel and some other metals
- (this is the endurance limit, below which will give infinite life)

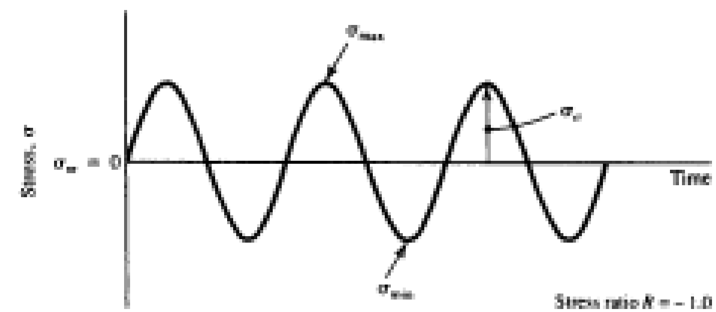


◆**Fatigue is a Damage accumulated through the application of repeated stress cycles**

◆**Variable amplitude loadings cause different levels of fatigue**

◆**Fatigue is cumulative through the life of an engineering element**





Factors Affecting Fatigue Life

- ***Loading Conditions***
  - Type of stress
  - Stress amplitude, mean value
- ***Condition of Specimen/Structural Member***
  - Stress concentrations
  - Surface finish
- ***Material***
  - Thermal history (e.g. grain size in metals)
- ***Environmental conditions***
  - Temperature
  - Corrosion effects

Fatigue-Life Methods

⌑ Fatigue Regimes

Based on the number of cycles (of stress/strain) that the part must undergo in its lifetime, we consider:

❖ **Low-Cycle Fatigue (LCF)**

$$1 \leq N \leq 10^3$$

❖ **High-Cycle Fatigue (HCF)**

$$N > 10^3$$

❖Stress-Life Approach

***Stress-based approach is most useful for* HCF.**

- Works best when stress cycles are regular.

- Seek the fatigue strength and endurance limit.

- Perform design based on factor of safety applied to the fatigue strength

❖ Strain-Life Approach

-Strain-based approach is most useful for describing the crack initiation stage.

- Most often applied to LCF.

❖ Linear Elastic Fracture Mechanics (LEFM)

- LEFM provides the best model for crack propagation stage.

- Most often applied to LCF.

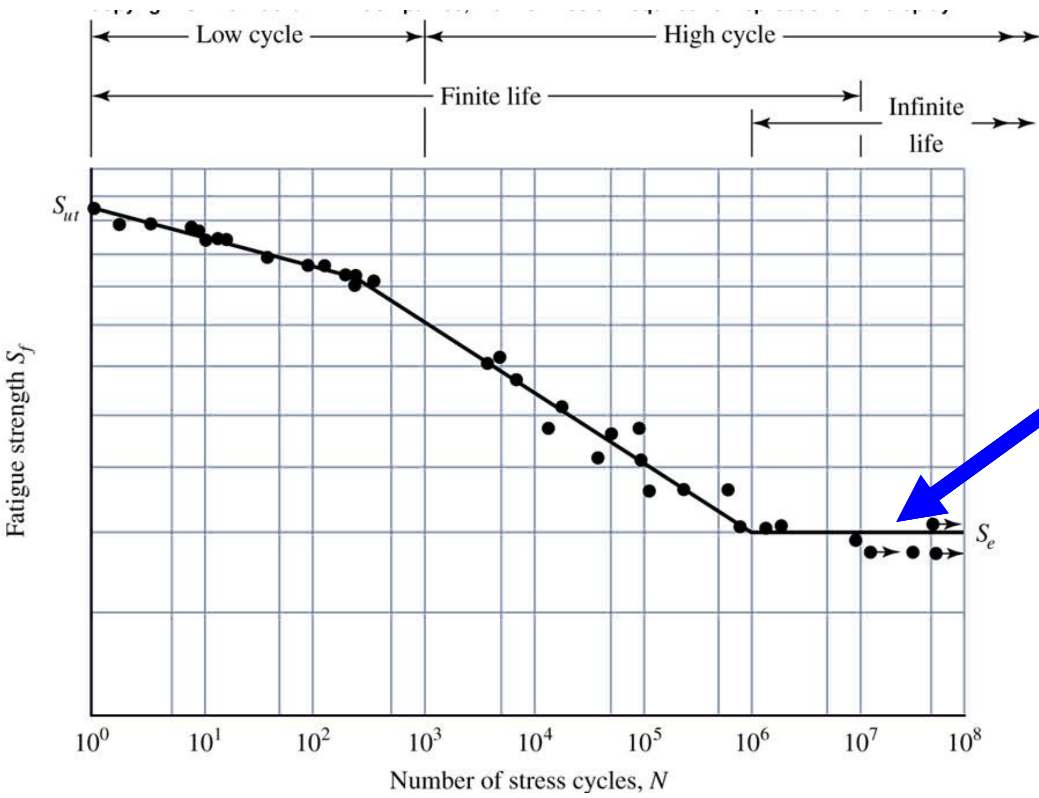
- Especially useful for predicting life of parts with known cracks.

The Stress-Life Method

Fatigue Strength

The Fatigue Strength, S_f (N), is the stress level that a material can endure for N cycles.

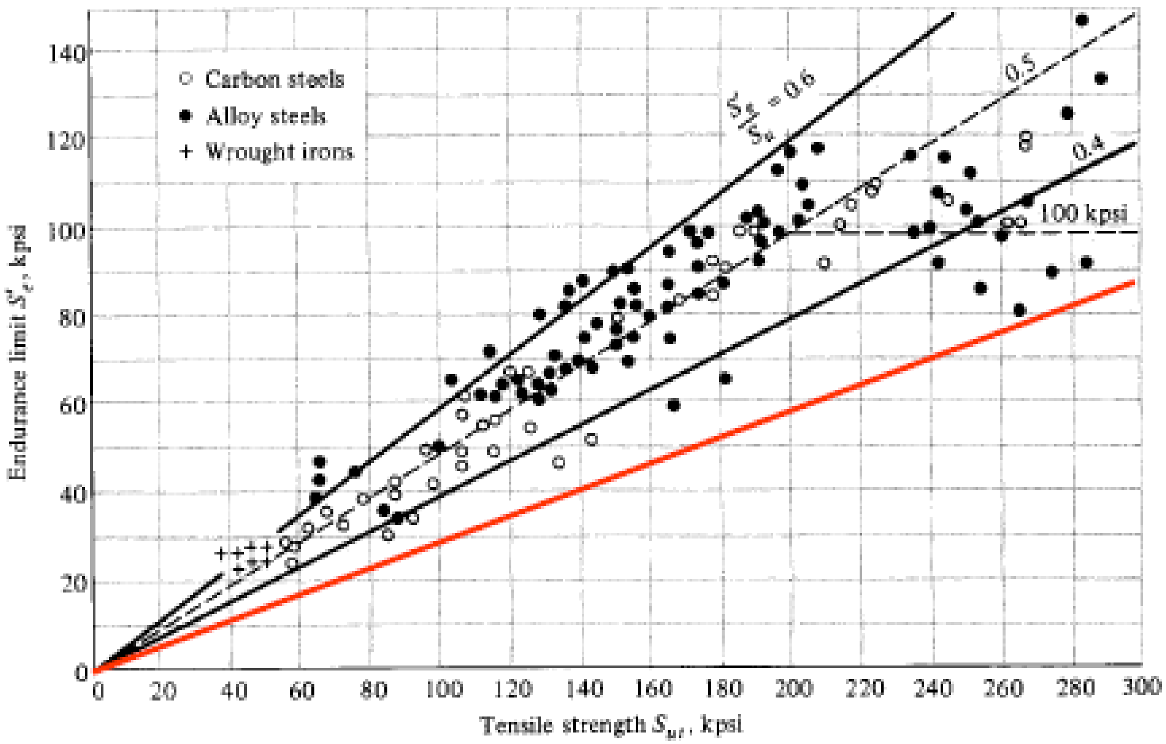
The stress level at which the material can withstand an infinite number of cycles is call the Endurance Limit.



The Endurance Limit is observed as a horizontal line on the S-N curve.

Endurance Limit

Endurance Limit Vs Tensile Strength



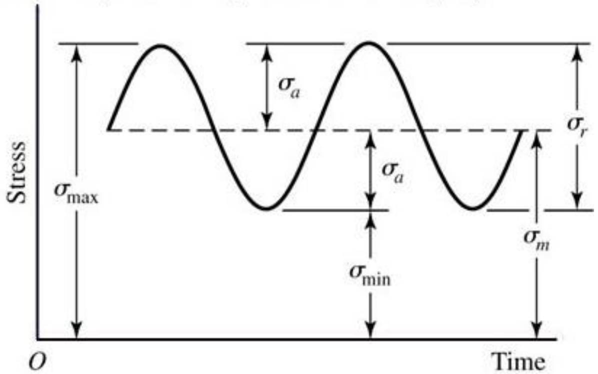
Conservative Lower Bound for Ferrous Materials

$$S'_e = 0.3 S_{ut}$$

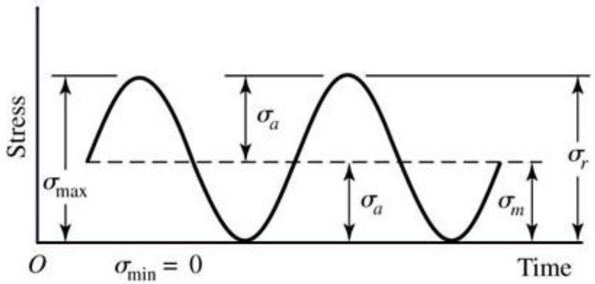
$S'_e \equiv$ Endurance Limit of Test Specimen

$S_{ut} \equiv$ Tensile Strength of Test Specimen

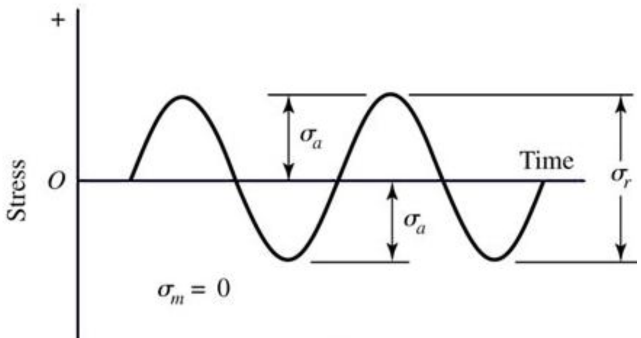
❖ Definitions



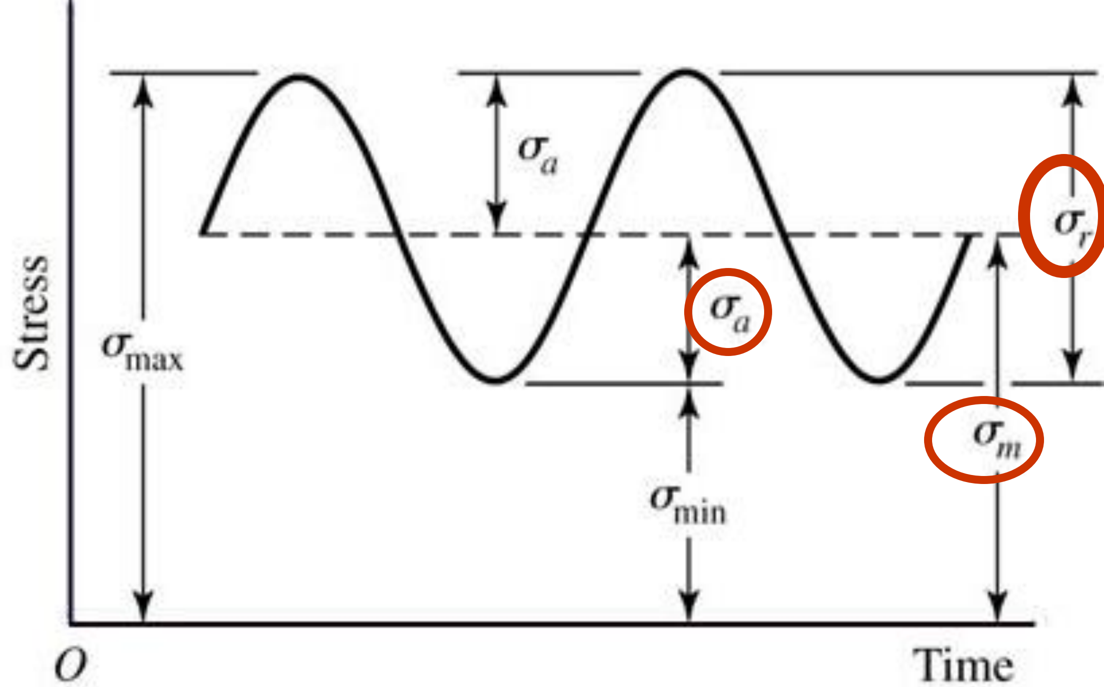
(d)



(e)



(f)



Mean Stress

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

Alternating Stress

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

Stress Range

$$\sigma_r = \sigma_{max} - \sigma_{min}$$

Note that R=-1 for a completely reversed stress state with zero mean stress.

Stress Ratio

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

Amplitude Ratio

$$A = \frac{\sigma_a}{\sigma_m}$$

Long Column with Central Loading

Introduction

Buckling is a mode of failure generally resulting from structural instability due to *compressive* action on the structural member or element involved.

Examples

- **Overloaded metal building columns.**
- **Compressive members in bridges.**
- **Roof trusses.**
- **Hull of submarine.**
- **Metal skin on aircraft fuselages or wings with excessive torsional and/or compressive loading.**
- **Any thin-walled torque tube.**
- **The thin web of an I-beam with excessive shear load**
- **A thin flange of an I-beam subjected to excessive compressive bending effects.**

The Nature of Buckling

– The failure (buckling) load bears no unique relationship to the stress and deformation at failure.

– Our usual approach of deriving a load stress and load-deformation relations cannot be used here, instead, the approach to find an expression for the buckling load P_{cr}.

– Buckling is unique from our other structural-element considerations in that it results from a state of unstable equilibrium.

– For example, buckling of a long column is not caused by failure of the material of which the column is composed, but by determination of what was a stable state of equilibrium to an unstable one.

Definition

"Buckling can be defined as the sudden large deformation of structure due to a slight increase of an existing load under which the structure had exhibited little, if any, deformation before the load was increased."

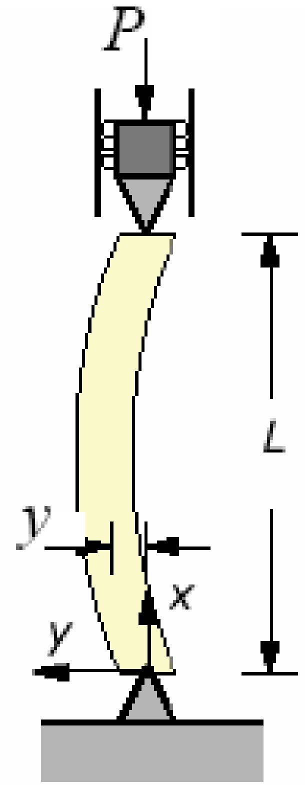
Critical Buckling Load

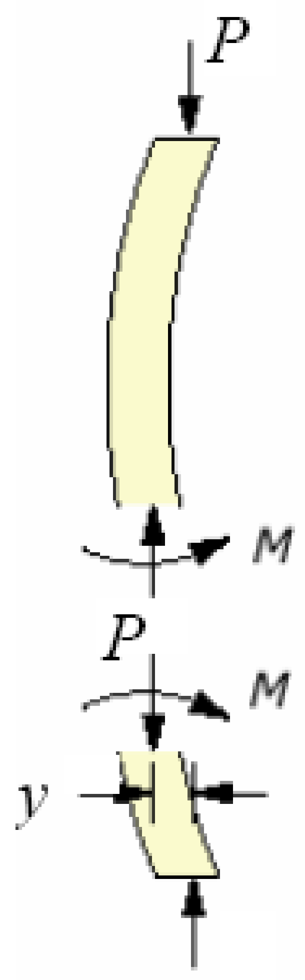
– The purpose of this analysis is to determine the minimum axial compressive load for which a column will experience lateral deflection.

Governing Differential Equation:

- Consider a buckled simply-supported column of length L under an external axial compression force P. The transverse displacement of the buckled column is represented by y.

$$Py + M = 0$$





Recall the relationship between the moment *M* and the transverse displacement *y* for the elastic curve,

$$EI\frac{dy^2}{dx^2} = M \quad \Rightarrow \quad \frac{d^2y}{dx^2} + \frac{P}{EI}y = 0$$

Buckling Solution:

• **The governing equation is a second order homogeneous ordinary differential equation with constant coefficients and can be solved by the method of characteristic equations. The solution is found to be,**

$$y(x) = A\sin px + B\cos px$$

- Where $p^2 = P/EI$. The coefficients *A* and *B* can be determined by the two boundary conditions, $y(0) = 0$ and $y(L) = 0$, which yields,

$$B = 0$$

$$A\sin pL = 0$$

$$\sin pL = 0$$

$$\Rightarrow pL = 0, \pi, 2\pi, 3\pi, \cdots, n\pi$$

or

$$p = 0, \frac{\pi}{L}, \frac{2\pi}{L}, \frac{3\pi}{L}, \cdots, \frac{n\pi}{L}$$

- Since $p^2 = P/EI$, therefore,

$$P = 0, \frac{\pi^2 EI}{L^2}, \frac{(2)^2 \pi^2 EI}{L^2}, \frac{(3)^2 \pi^2 EI}{L^2}, \cdots, \frac{n^2 \pi^2 EI}{L^2}$$

- Or

$$P = EI\left(\frac{n\pi}{L}\right)^2 \quad \text{for } n = 0, 1, 2, 3 \cdots$$

• The lowest load that causes buckling is called critical load (*n* = 1).

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

$$\textbf{Since } I = Ak^2, \xrightarrow{\text{then}} \frac{P_{cr}}{A} = \frac{\pi^2 E}{(L/k)^2}$$

L/k **: is the slenderness ratio**

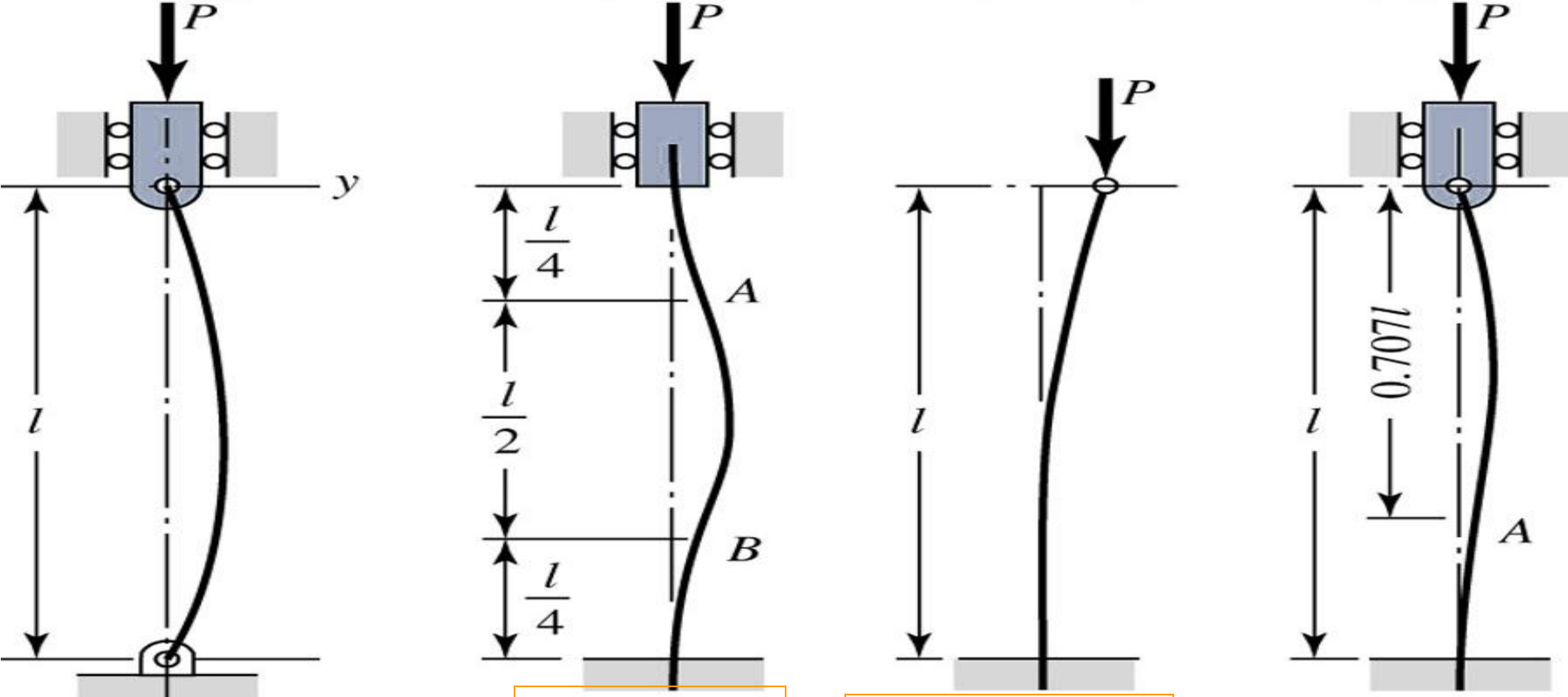
$\frac{P_{cr}}{A}$ **: is the cririca l unit load**

$$P_{cr} = \frac{\pi^2 EI}{L_{eff.}^2} = \frac{\pi^2 EI}{K^2 L^2} = \frac{C\pi^2 EI}{L^2}$$

where; $L_{eff.}$ = the buckled length = KL

K = end condition constant

$$C = \frac{1}{K^2}$$



Pinned – Pinned

$K = 1$

$L_{eff.} = L$

$$P_{cr} = \frac{\pi^2 EI}{L^2}$$

Fixed – Fixed

$K = 0.5$

$L_{eff.} = 0.5L$

$$P_{cr} = \frac{\pi^2 EI}{0.25L^2} = \frac{4\pi^2 EI}{L^2}$$

Free - Fixed

$K = 2$

$L_{eff.} = 2L$

$$P_{cr} = \frac{\pi^2 EI}{4L^2} = \frac{0.25\pi^2 EI}{L^2}$$

$$\mathbf{L_{eff.} = 0.707L}$$

$$\mathbf{P_{cr} = \frac{\pi^2 EI}{0.499L^2} = \frac{2\pi^2 EI}{L^2}}$$

Thin Walled Pressure Vessels

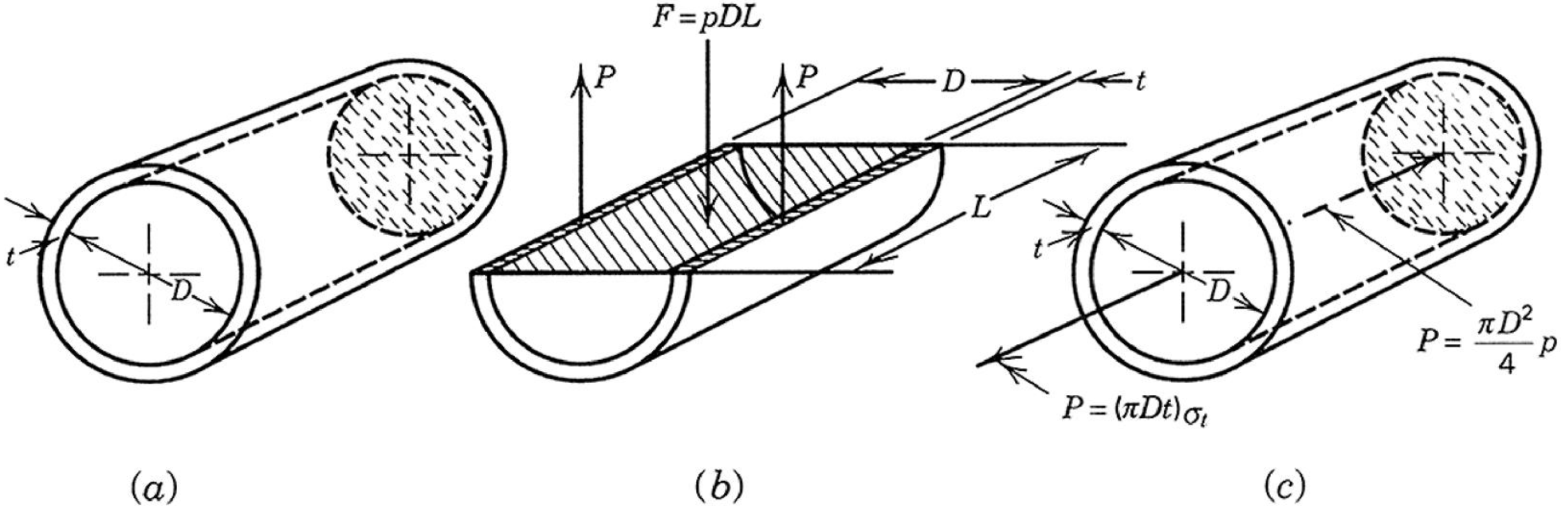


Figure 6-1. Thin Walled Pressure Vessels

Consider a cylindrical vessel section of:

L = Length

D = Internal diameter

t = Wall thickness

p = fluid pressure inside the vessel.

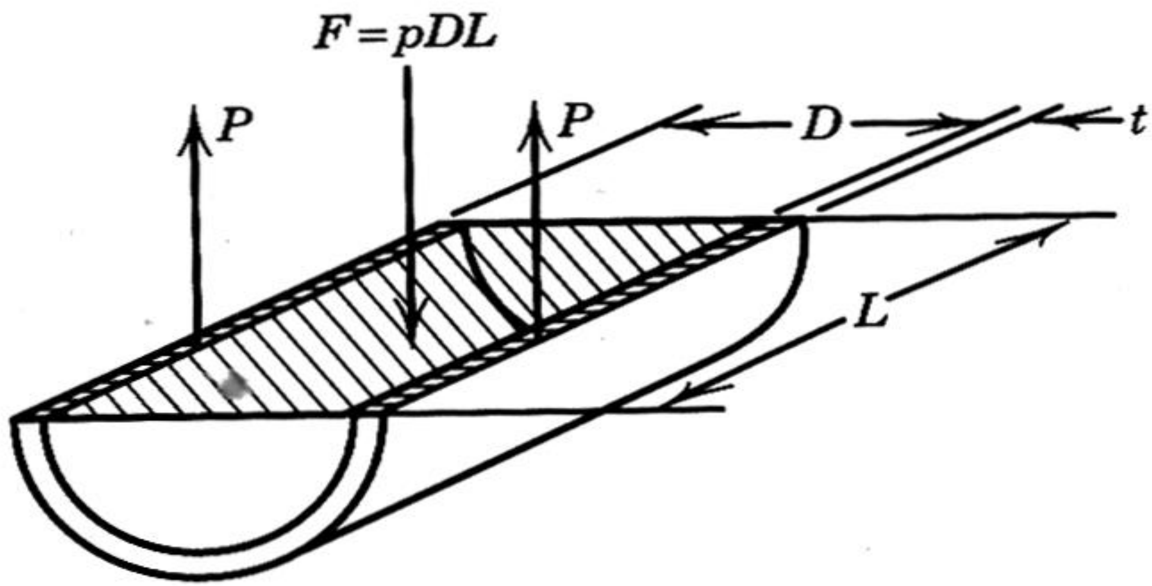


Figure 6-1b

By examining the free-body diagram of the lower half of the cylinder (Fig. 6-1b), one sees that the summation of forces acting normal to the mid-plane is given by :

$$[\Sigma F = 0] \quad F = pDL = 2P \qquad \text{(A6.1)}$$

or

$$P = \frac{pDL}{2} \qquad \text{(A6.2)}$$

The tangential or “hoop” stress, σ_t, acting on the wall thickness is then found to be:

$$\sigma_t = \frac{P}{A} = \frac{pDL}{2Lt} = \frac{pD}{2t} \qquad \text{(A6.3)}$$

or

$$\sigma_t = \frac{pr}{t} \qquad \text{(A6.4)}$$

where *r* is the radius of the vessel.

For the case of the thin-walled cylinders, where $r/t \geq 10$, Eq. 6-4 describes the hoop stress at all locations through the wall thickness. The vessel can be considered as **thick walled cylinder**.

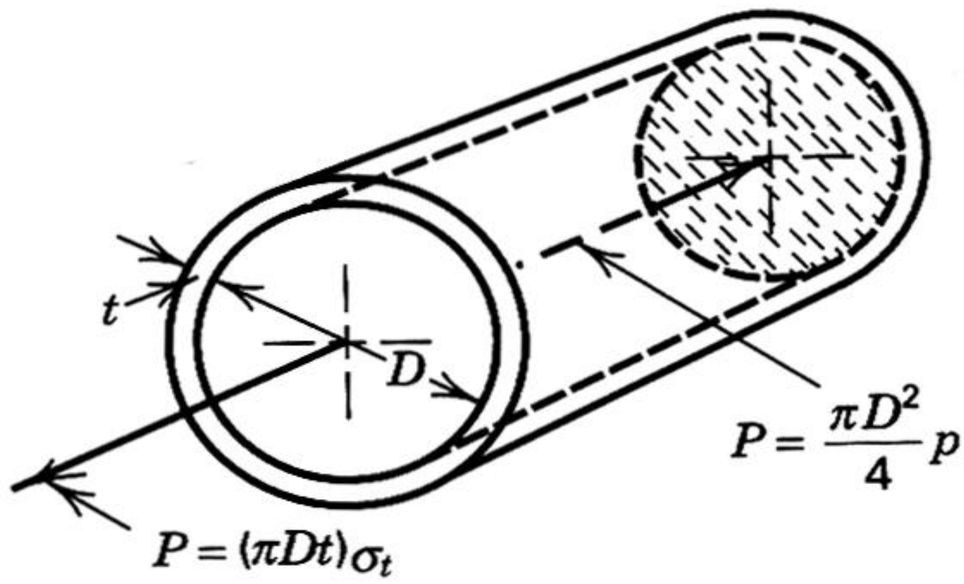


Figure 6-1c

- Fig. 6-1c shows a free-body diagram to account for cylindrical stresses in the longitudinal direction. The sum of forces acting along the axis of the cylinder is:

$$\frac{\pi D^2 p}{4} = P \qquad \text{(A6.5)}$$

- The cross-sectional area of the cylinder wall is characterized by the product of its wall thickness and the mean circumference.

i.e., $\pi(D+t)t$

- For the thin-wall pressure vessels where $D >> t$, the cylindrical cross-section area may be approximated by πDt.
- Therefore, the longitudinal stress in the cylinder is given by:

$$\sigma_l = \frac{P}{A} = \frac{\pi D^2 p}{4\pi Dt} = \frac{pD}{4t} \qquad \text{(A6.6)}$$

- By comparing Eq 6-3 and 6-6, one finds that the tangential or hoop stress is twice that in the longitudinal direction.
- Therefore, **thin vessel failure** is likely to occur along a longitudinal plane oriented normal to the transverse or **hoop stress direction**.

Generalized Hooke’s Law

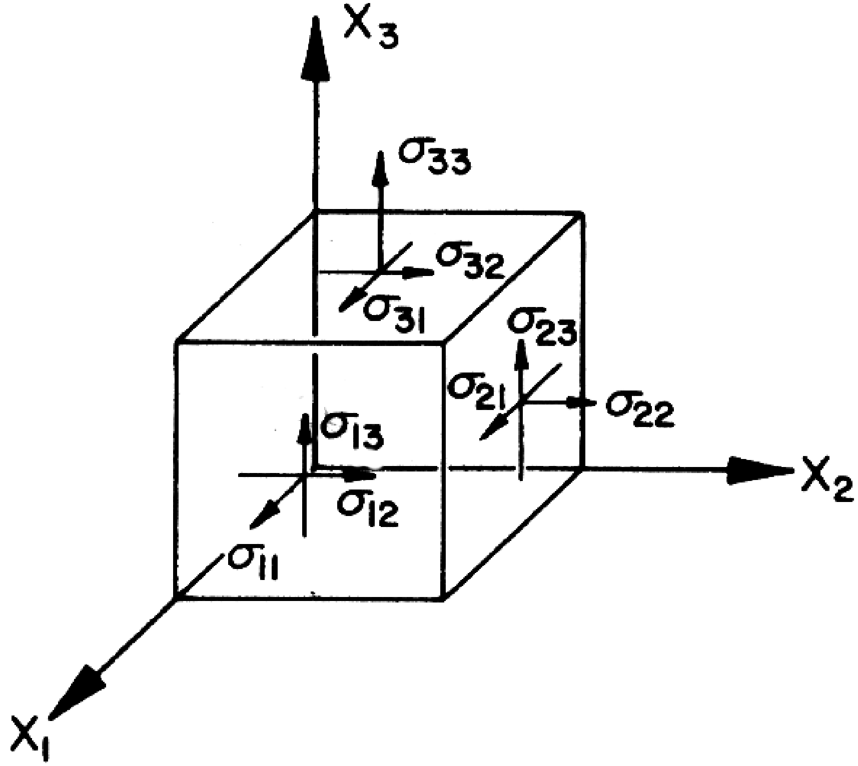


Figure 6-1.

- A complete description of the general state of stress at a point consists of:
  - normal stresses in three directions, σ_x (or σ_{11}), σ_y (or σ_{22}) and σ_z (or σ_{33}),
  - shear stresses on three planes, τ_x (or σ_{12} ...), τ_y (or σ_{23} .....), and τ_z (or σ_{31} ......).

- The stress, σ_x in the x-direction produces 3 strains:
  - longitudinal strain (extension) along the **x-axis** of:

$$\varepsilon_x = \frac{\sigma_x}{E} \qquad (6.7)$$

  - transverse strains (contraction) along the **y and z -axes**, which are related to the Poisson's ratio:

$$\varepsilon_y = \varepsilon_z = -\nu\varepsilon_x = -\frac{\nu\sigma_x}{E} \qquad (6.8)$$

Properties of ν

- Absolute values of ν are used in calculations.
- The value of ν is about:
  - 0.25 for a perfectly isotropic elastic materials.
  - 0.33 for most metals.

- In order to determine the total strain produced along a particular direction, we can apply the principle of superposition.
- For Example, the resultant strain along the x-axis, comes from the **strain contribution** due to the application of σ_x, σ_y and σ_z.
  - σ_x causes: $\frac{\sigma_x}{E}$ in the x-direction
  - σ_y causes: $-\frac{\nu\sigma_y}{E}$ in the x-direction
  - σ_z causes: $-\frac{\nu\sigma_z}{E}$ in the x-direction
  - Applying the principle of superposition (x-axis):

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu(\sigma_y + \sigma_z)\right] \qquad (6.9a)$$

The situation can be summarized by the following table:

Table 6 -1 Strain Contribution Due to Stresses

Stress	Strain in the x direction	Strain in the y direction	Strain in the z direction
σ_x	$\varepsilon_x = \frac{\sigma_x}{E}$	$\varepsilon_y = -\frac{v\sigma_x}{E}$	$\varepsilon_z = -\frac{v\sigma_x}{E}$
σ_y	$\varepsilon_x = -\frac{v\sigma_y}{E}$	$\varepsilon_y = \frac{\sigma_y}{E}$	$\varepsilon_z = -\frac{v\sigma_y}{E}$
σ_z	$\varepsilon_x = -\frac{v\sigma_z}{E}$	$\varepsilon_y = -\frac{v\sigma_z}{E}$	$\varepsilon_z = \frac{\sigma_z}{E}$

By superposition of the components of strain in the x, y, and z directions, the strain along each axis can be written as:

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - v\left(\sigma_y + \sigma_z\right)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - v\left(\sigma_z + \sigma_x\right)\right] \qquad (6.9)$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - v\left(\sigma_x + \sigma_y\right)\right]$$

The shearing stresses acting on the unit cube produce shearing strains.

$$\tau_{xy} = G\gamma_{xy}$$

$$\tau_{yz} = G\gamma_{yz} \qquad (6.10)$$

$$\tau_{xz} = G\gamma_{xz}$$

The proportionality constant G is the modulus of elasticity in shear, or the modulus of rigidity. Values of G are usually determined from a torsion test. See Table 6-2.

Table 6-2 Typical Room-Temperature values of elastic constants for isotropic materials.

Material	**Modulus of Elasticity, 10^-6 psi (GPa)**	**Shear Modulus 10^-6 psi (GPa)**	**Poisson's ratio, ν**
Aluminum alloys	10.5(72.4)	4.0(27.5)	0.31
Copper	16.0(110)	6.0(41.4)	0.33
Steel(plain carbon and low-alloy)	29.0(200)	11.0(75.8)	0.33
Stainless Steel	28.0(193)	9.5(65.6)	0.28
Titanium	17.0(117)	6.5(44.8)	0.31
Tungsten	58.0(400)	22.8(157)	0.27

The volume strain Δ, or cubical dilation, is the change in volume per unit volume.

Consider a rectangular parallelepiped with edges *dx*, *dy* and *dz*.

The volume in the strained condition is:

$$(1 + \varepsilon_x)(1 + \varepsilon_y)(1 + \varepsilon_z)\, dx\, dy\, dz$$

The dilation (or volume strain) Δ is given as:

$$\Delta = \frac{(1+\varepsilon_x)(1+\varepsilon_x)(1+\varepsilon_x)dx\,dy\,dz - dx\,dy\,dz}{dx\,dy\,dz}$$

$$= (1+\varepsilon_x)(1+\varepsilon_x)(1+\varepsilon_x) - 1$$

For small strains,

$$\Delta = \varepsilon_x + \varepsilon_y + \varepsilon_z \qquad (5.24b)$$

Aanother elastic constant is the *bulk modulus* or the *volumetric modulus* of elasticity K. The bulk modulus is the ratio of the hydrostatic pressure to the dilation that it produces.

$$K = \frac{\sigma_m}{\Delta} = \frac{-p}{\Delta} = \frac{1}{\beta} \qquad (6.11)$$

Where -p is the hydrostatic pressure, and β is the compressibility.

Many useful relationships may be derived between the elastic constants *E*, *G*, *v*, *K*. For example, if we add up the three equations (6.9).

$$\varepsilon_x + \varepsilon_y + \varepsilon_z = \frac{1-2v}{E}\left(\sigma_x + \sigma_y + \sigma_z\right)$$

$$\Delta = \frac{1-2v}{E} 3\sigma_m$$

or

$$K = \frac{\sigma_m}{\Delta} = \frac{E}{3(1-2v)} \quad (6.12)$$

Another important relationship is the expression relating E, G, and v. This equation is usually developed in a first course in strength of materials.

$$G = \frac{E}{2(1+v)} \quad (6.13)$$

- Equations 6-9 and 6-10 can be expressed in tensor notation as one equation:

$$* \ \varepsilon_{ij} = \frac{1+\nu}{E}\sigma_{ij} - \frac{\nu}{E}\sigma_{kk}\delta_{ij} \ * \qquad \textbf{*(6.14)}$$

Example, if i = j = x,

$$\varepsilon_{xx} = \frac{1+\nu}{E}\sigma_{xx} - \frac{\nu}{E}\left(\sigma_{xx} + \sigma_{yy} + \sigma_{zz}\right)(1)$$

$$\varepsilon_{xx} = \frac{1}{E}\left(\sigma_{xx} - \nu\left(\sigma_{yy} + \sigma_{zz}\right)\right)$$

If i = x and j = y,

$$\varepsilon_{xy} = \frac{1+\nu}{E}\tau_{xy} - \frac{\nu}{E}\sigma_{kk}(0)$$

Recall Eq. 6-13, and the shear strain relation between ε and γ:

$$\frac{1+\nu}{E} = \frac{1}{2G}; \ \varepsilon_{xy} = \frac{\gamma_{xy}}{2}$$

Therefore,

$$\gamma_{xy} = \frac{1}{G}\tau_{xy}$$

Special Cases

- **Plane Stress** ($\sigma_3 = 0$): This exists typically in:
  - a thin sheet loaded in the plane of the sheet, or
  - a thin wall tube loaded by internal pressure where there is no stress normal to a free surface.

  - Recall Eqs. 6-9, and set $\sigma_z = \sigma_3 = 0$.

Therefore,

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu\sigma_2\right] \qquad (6.15a)$$

$$\varepsilon_2 = \frac{1}{E}\left[\sigma_2 - \nu\sigma_3\right] \qquad (6.15b)$$

$$\varepsilon_3 = -\frac{1}{E}\nu\left[\sigma_1 + \sigma_2\right] \qquad (6.15c)$$

From Eqs. 6-9a and 6-9b, we have,

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu\left(\varepsilon_2 E + \nu\sigma_1\right)\right]$$

$$= \frac{1}{E}\left[\sigma_1\left(1 - \nu^2\right)\right] - \frac{1}{E}\left(\nu\varepsilon_2 E\right) \qquad (6.16)$$

Therefore,

$$\varepsilon_1 + \nu\varepsilon_2 = \frac{1 - \nu^2}{E}\sigma_1$$

Then,

$$\sigma_1 = \frac{E}{1 - \nu^2}\left[\varepsilon_1 + \nu\varepsilon_2\right]$$

Similarly,

$$\sigma_2 = \frac{E}{1 - \nu^2}\left[\varepsilon_2 + \nu\varepsilon_1\right] \qquad (6.17)$$

- **Plane Strain** ($\varepsilon_3 = 0$): This occurs typically when
  - One dimension is much greater than the other two

  Examples are a long rod or a cylinder with restrained ends.
  - Recall Eqs. 6-9,

$$\varepsilon_3 = \frac{1}{E}\left[\sigma_3 - \nu\left(\sigma_1 + \sigma_2\right)\right] = 0 \qquad (6.18)$$

but

$$\sigma_3 = \nu\left[\sigma_1 + \sigma_2\right] \qquad (6.19)$$

This shows that a stress exists along direction-3 (z-axis) even though the strain is zero.

- Substitute Eqs. 6-18 and 6-19 into Eq. 6-9, we have

$$\varepsilon_1 = \frac{1}{E}\left[\left(1 - \nu^2\right)\sigma_1 - \nu\left(1 + \nu\right)\sigma_2\right]$$

$$\varepsilon_2 = \frac{1}{E}\left[\left(1 - \nu^2\right)\sigma_2 - \nu\left(1 + \nu\right)\sigma_1\right]$$

$$\varepsilon_3 = 0$$

Example 1

A steel specimen is subjected to elastic stresses represented by the matrix

$$\sigma_{ij} = \begin{pmatrix} 2 & -3 & 1 \\ -3 & 4 & 5 \\ 1 & 5 & -1 \end{pmatrix} MPa$$

Calculate the corresponding strains.

Solution

Invoke Hooke's Law, Eqs. 6-9 and 6-10. Use the values of E, G and ν for steel in Table 6-2

$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\left(\sigma_y + \sigma_z\right)\right]$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\left(\sigma_x + \sigma_z\right)\right]$$

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(\sigma_x + \sigma_y\right)\right]$$

$$\tau_{xy} = G\gamma_{xy} \qquad \tau_{yz} = G\gamma_{yz} \qquad \tau_{xz} = G\gamma_{xz}$$

Substitute values of E, G and ν into the above equations.

Example 2

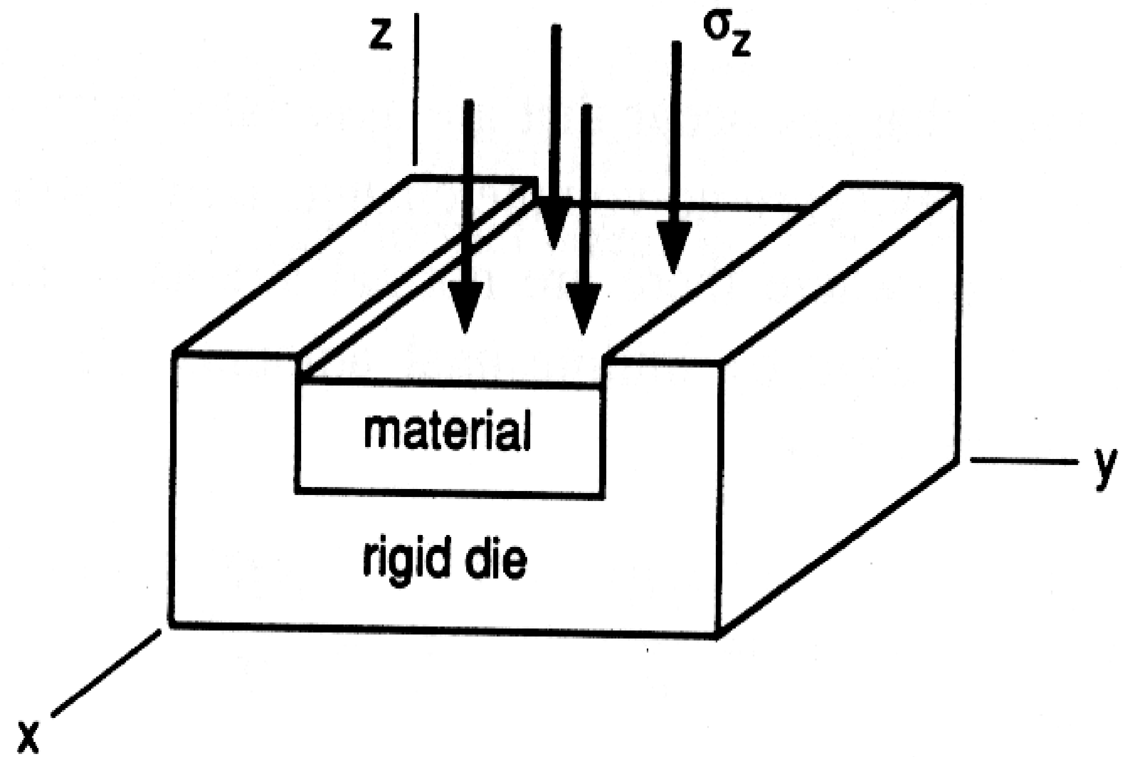
A sample of material subjected to a compressive stress σ_z is confined so that it cannot deform in the y-dir., but deformation is permitted in the x-dir. Assume that the material is isotropic and exhibits linear-elastic behavior. Determine the following in terms of σ_z and the elastic constant of the material:

(a) The stress that develops in the y-dir.

(b) The strain in the z-dir.

(c) The strain in the x-dir.

(d) The stiffness $E' = \sigma_z / \varepsilon_z$ in the z-dir. Is this apparent modulus equal to the elastic modulus E from the uniaxial test on the material? Why or why not?


z
σz
material
rigid die
y
x

Solution

Invoke Hooke's Law, Eq. 6-9

The situation posed requires that - $\varepsilon_y = 0$, $\sigma_x = 0$.

We also treat σ_z as a known quantity.

(a) The stress in the y-direction is obtained as:

$$\varepsilon_y = 0 = \frac{1}{E}\left[\sigma_y - \nu(0 + \sigma_z)\right]$$

$$\therefore \quad \sigma_y = \nu\sigma_z$$

(b) The stress in the z-direction is obtained by substituting σ_y into Eq. 6-9.

$$\varepsilon_z = \frac{1}{E}\left[\sigma_z - \nu\left(0 + \nu\sigma_z\right)\right]$$

$$\therefore \quad \varepsilon_z = \frac{1 - \nu^2}{E}\sigma_z$$

(c) The strain in the x-direction is given by Eq. 6-9, with σ_y from above substituted.

$$\varepsilon_x = \frac{1}{E}\left[0 - \nu\left(\nu\sigma_z + \sigma_z\right)\right]$$

$$\therefore \quad \varepsilon_x = -\frac{\nu\left(1 + \nu\right)}{E}\sigma_z$$

(d) The apparent stiffness in the z-direction is obtained immediately from the equation for ε_z.

$$E' = \frac{\sigma_z}{\varepsilon_z} = \frac{E}{1 - \nu^2} = 1.10E$$

$$(\mathit{for\ a\ typical\ value\ of}\ \nu = 0.3)$$

Obviously, this value is larger than the actual E

- The **value of E** is the **ratio of stress to strain** only for the **uniaxial** deformation.
- For any other case, such **ratios** are determined by the **behavior of the material** according to the three-dimensional form of **Hooke's Law**.

Example 3

Consider a plate under uniaxial tension that is prevented from contracting in the transverse direction. Find the effective modulus along the loading direction under this condition of plane strain.

Solution

Let, ν = Poisson's ratio

E = Young's Modulus,

Loading → Direction 1

Transverse → Direction 2

No stress normal to the free surface, → $\sigma_3 = 0$

Although the applied stress is uniaxial, the constraint on contraction in direction 2 results in a stress in direction 2.

The strain in direction 2 can be written in terms of Hooke's Law (ref. Eq. 6-9) as

$$\varepsilon_2 = 0 = \frac{1}{E}[\sigma_2 - \nu\sigma_1]$$

$$\therefore \quad \sigma_2 = \nu\sigma_1$$

In direction 1, we can write the strain as:

$$\varepsilon_1 = \frac{1}{E}\left[\sigma_1 - \nu\sigma_2\right] = \frac{1}{E}\left[\sigma_1 - \nu^2\sigma_1\right]$$

$$\therefore \qquad \varepsilon_1 = \frac{\sigma_1}{E}\left(1 - \nu^2\right)$$

Hence the plane strain modulus in direction 1 is given as

$$E' = \left(\sigma_1 / \varepsilon_1\right) = \frac{E}{1 - \nu^2}$$

If we take ν = 0.33, then the plane strain modulus

E′ = 1.12E

Example (4)

A cylinder pressure vessel 10 m long has closed ends, a wall thickness of 5 mm, and a diameter at mid-thickness of 3 mm. If the vessel is filled with air to a pressure of 2 MPa, how much do the length, diameter, and wall thickness change, and in each case state whether the change is an increase or a decrease. The vessel is made of a steel having elastic modulus E = 200,000 MPa and the Poisson's ratio ν = 0.3. Neglect any effects associated with the details of how the ends are attached.

Attach a coordinate system to the surface of the pressure vessel as shown below, such that the z-axis is normal to the surface.

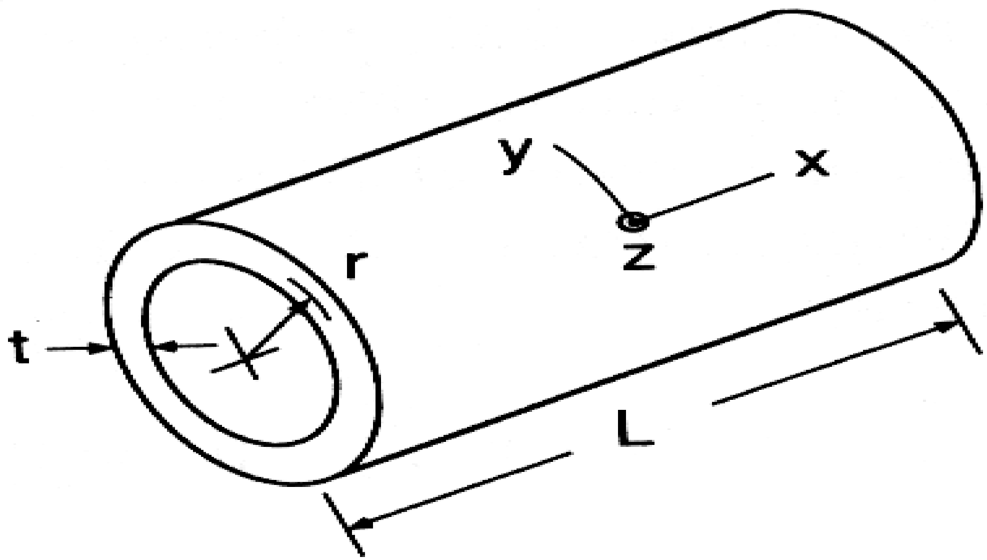


Figure E4.1

The ratio of radius to thickness, r/t, is such that it is reasonable to employ the thinwalled tube assumption, and the resulting stress equations A6-1 to A6-6.

Denoting the pressure as *p*, we have

$$\sigma_x = \frac{pr}{2t} = \frac{(2MPa)(1500mm)}{2(5mm)} = 300MPa$$

$$\sigma_y = \frac{pr}{t} = \frac{(2MPa)(1500mm)}{5mm} = 600MPa$$

The value of σ_z varies from -*p* on the inside wall to zero on the outside, and for a thinwalled tube is everywhere sufficiently small that $\sigma_z \approx 0$ can be used. Substitute these stresses, and the known *E* and *v* into Hooke's Law, Eqs.6-9 and 6-10, which gives

$$\varepsilon_x = 6.00*10^{-4} \qquad \varepsilon_y = 2.55*10^{-3} \qquad \varepsilon_z = -1.35*10^{-3}$$

These strains are related to the changes in length ΔL, circumference $\Delta(\pi d)$, diameter Δd, and thickness Δt, as follows:

$$\varepsilon_x = \frac{\Delta L}{L} \qquad \varepsilon_y = \frac{\Delta(\pi d)}{\pi d} = \frac{\Delta d}{d} \qquad \varepsilon_z = \frac{\Delta t}{t}$$

Substituting the strains from above and the known dimensions gives

$$\Delta L = 6mm \qquad \Delta d = 7.65mm \qquad \Delta t = -6.75*10^{-3}$$

Thus, there are small increases in length and diameter, and a tiny decrease in the wall thickness.

Impact Fracture Testing

Fracture behavior depends on many external factors:

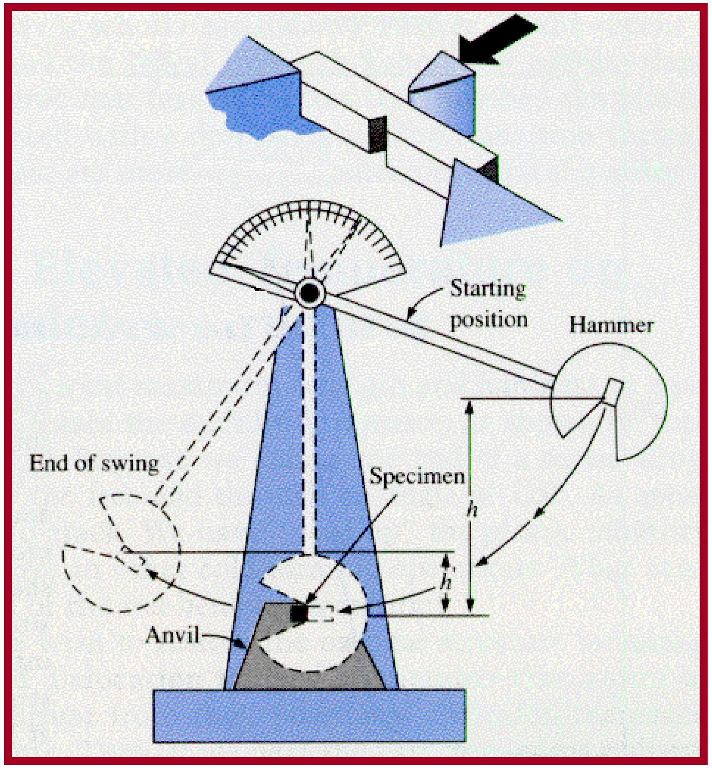
- **Strain rate**
- **Temperature**
- **Stress rate**

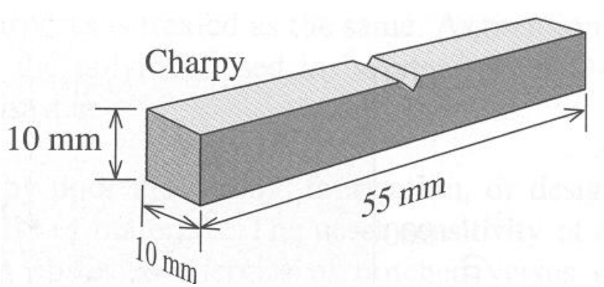
Impact testing is used to ascertain the fracture characteristics of materials at a high strain rate and a triaxial stress state.

In an impact test, a notched specimen is fractured by an impact blow, and the energy absorbed during the fracture is measured.

There are two types of tests – Charpy impact test and Izod impact test.

Impact Test: The Charpy Test





The ability of a material to withstand an impact blow is referred to as notch toughness.

The energy absorbed is the difference in height between initial and final position of the hammer. The material fractures at the notch and the structure of the cracked surface will help indicate whether it was a brittle or ductile fracture.

Impact Test (Charpy) Data for some of the Alloys

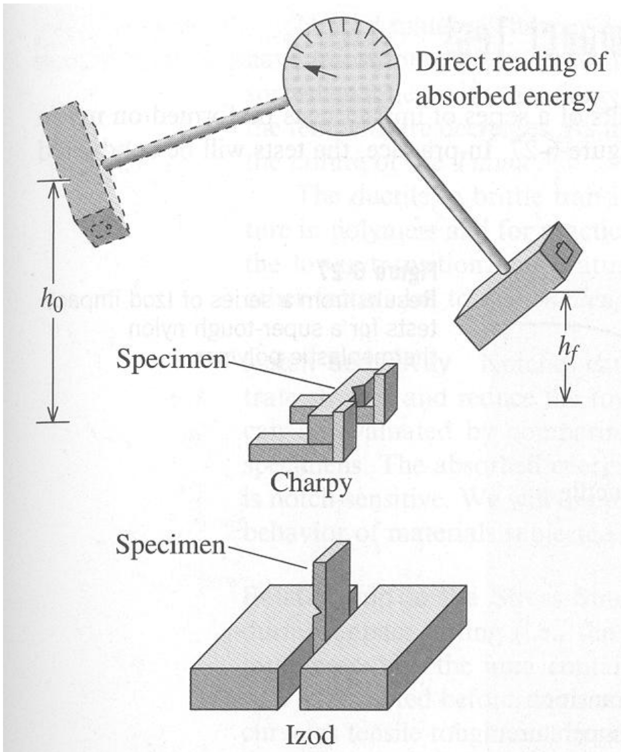
	Alloy	Impact energy [J (ft·lb)]
1.	1040 carbon steel	180 (133)
2.	8630 low-alloy steel	55 (41)
3.	c. 410 stainless steel	34 (25)
4.	L2 tool steel	26 (19)
5.	Ferrous superalloy (410)	34 (25)
6.	a. Ductile iron, quench	9 (7)
7.	b. 2048, plate aluminum	10.3 (7.6)
8.	a. AZ31B magnesium	4.3 (3.2)
	b. AM100A casting magnesium	0.8 (0.6)
9.	a. Ti–5Al–2.5Sn	23 (17)
10.	Aluminum bronze, 9% (copper alloy)	48 (35)
11.	Monel 400 (nickel alloy)	298 (220)
13.	50:50 solder (lead alloy)	21.6 (15.9)
14.	Nb–1 Zr (refractory metal)	174 (128)

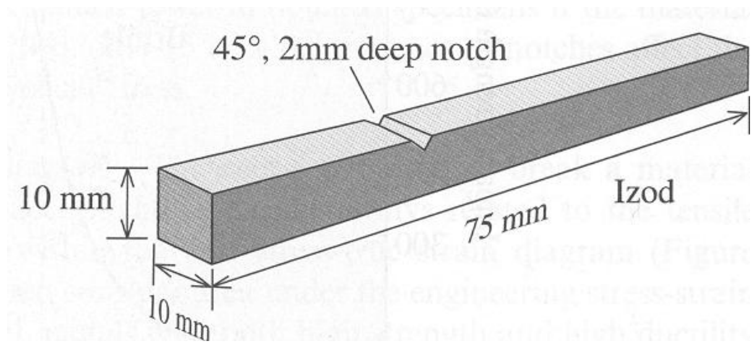
Table 8.1

Impact Test (Charpy) Data for Some of the Alloys of Table 6.1.

In effect, the Charpy test takes the tensile test to completion very rapidly. The impact energy from the Charpy test correlates with the area under the total stress-strain curve (toughness)

Impact Test: The Izod Test





Generally used for polymers. Izod test is different from the Charpy test in terms of the configuration of the notched test specimen

Impact Test (Izod) Data for various polymers

Polymer	Impact energy [J (ft·lb)]
General-use polymers	
Polyethylene	
High-density	1.4–16 (1–12)
Low-density	22 (16)
Polyvinylchloride	1.4 (1)
Polypropylene	1.4–15 (1–11)
Polystyrene	0.4 (0.3)
Polyesters	1.4 (1)
Acrylics (Lucite)	0.7 (0.5)
Polyamides (nylon 66)	1.4 (1)
Cellulosics	3–11 (2–8)
Engineering polymers	
ABS	1.4–14 (1–10)
Polycarbonates	19 (14)
Acetals	3 (2)
Polytetrafluoroethylene (Teflon)	5 (4)
Thermosets	
Phenolics (phenolformaldehyde)	0.4 (0.3)
Urea-melamine	0.4 (0.3)
Polyesters	0.5 (0.4)
Epoxies	1.1 (0.8)

Source: From data collections in R. A. Flinn and P. K. Trojan, *Engineering Materials and Their Applications*, 2nd ed., Houghton Mifflin Company, Boston, MA, 1981; M. F. Ashby and D. R. H. Jones, *Engineering Materials*, Pergamon Press, Inc., Elmsford, NY, 1980; and *Design Handbook for Du Pont Engineering Plastics.*

Table 8.2

Impact Test (Izod) Data for Various Polymers.

Impact Tests: Test conditions

- The impact data are sensitive to test conditions. Increasingly sharp notches can give lower impact-energy values due to the stress concentration effect at the notch tip
- The FCC alloys→ generally ductile fracture mode
- The HCP alloys→ generally brittle fracture mode
- Temperature is important
- The BCC alloys→ brittle modes at relatively low temperatures and ductile mode at relatively high temperature

Hardness Test

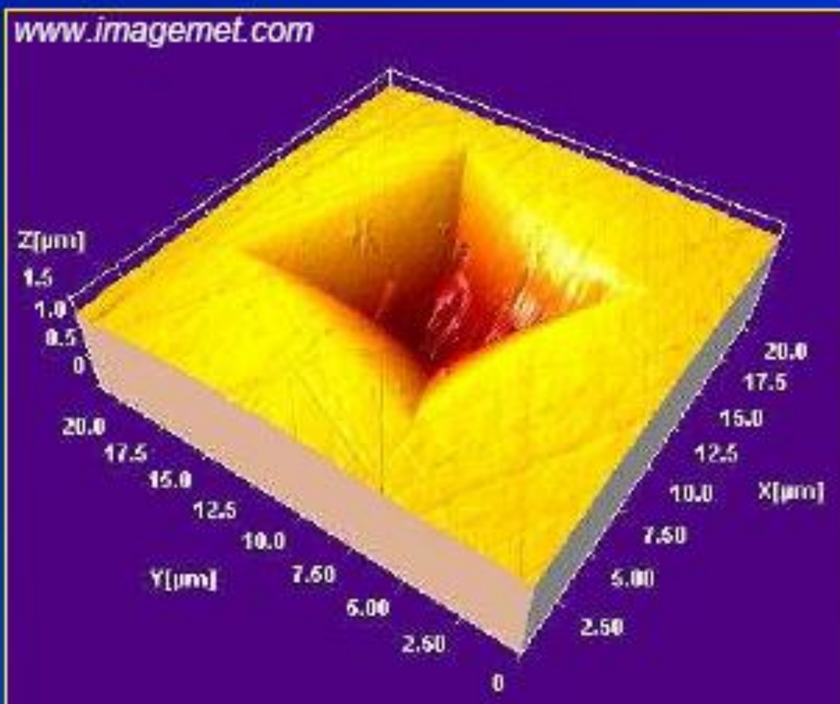
Subjects of interest

- *Introduction/objectives*
- *Brinell hardness*
- *Meyer hardness*
- *Vickers hardness*
- *Rockwell hardness*
- *Microhardness tests*
- *Relationship between hardness and the flow curve*
- *Hardness-conversion relationships*
- *Hardness at elevated temperatures*

Introduction

Definition ***Hardness is a resistance to deformation.***

(for people who are concerned with mechanics of materials, hardness is more likely to mean the resistance to indentation)



Hardness impression

Deeper or larger impression

↓

Softer materials

Introduction

There are three general types of hardness measurements

1) *Scratch hardness*

- The ability of material to scratch on one another
- Important to mineralogists, using ***Mohs'scale*** 1= talc, 10 = diamond
- Not suited for metal → annealed copper = 3, martensite = 7.

2) *Indentation hardness*

- Major important engineering interest for metals.
- Different types : Brinell, Meyer, Vickers, Rockwell hardness tests.

3) *Rebound or dynamic hardness*

- The indentor is dropped onto the metal surface and the hardness is expressed as the energy of impact.

Brinell hardness

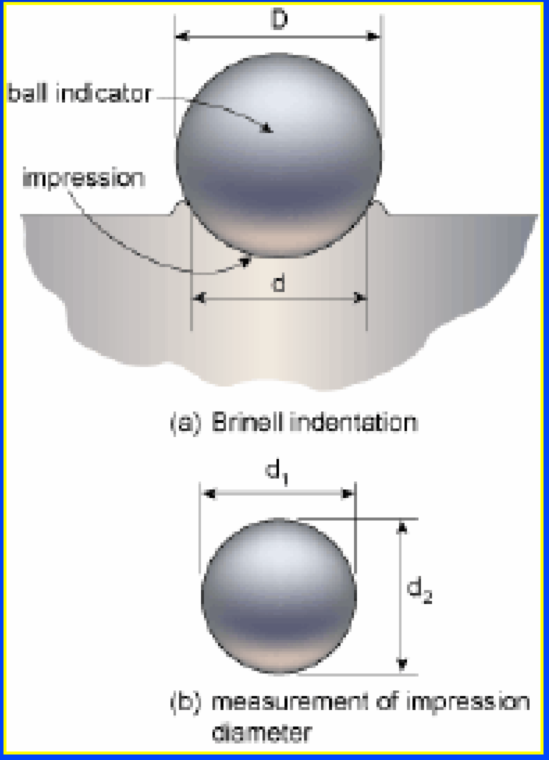
• ***J.A. Brinell*** introduced the ***first standardised indentation-hardness*** test in 1900. The ***Brinell hardness test*** consists in indenting the metal surface with a ***10-mm diameter steel ball*** at a load range of 500-3000 kg, depending of hardness of particular materials.

• The load is applied for a standard time (~30 s), and the ***diameter of the indentation is measured***.
→giving an average value of two readings of the diameter of the indentation at right angle.

• The ***Brinell hardness number (BHN or H_B)*** is expressed as the load ***P*** divided by surface area of the indentation.

$$BHN = \frac{P}{(\pi D/2)\left(D - \sqrt{D^2 - d^2}\right)} = \frac{P}{\pi D t} \qquad Eq.1$$

Where ***P*** is applied load, kg
D is diameter of ball, mm
d is diameter of indentation, mm
t is depth of the impression, mm



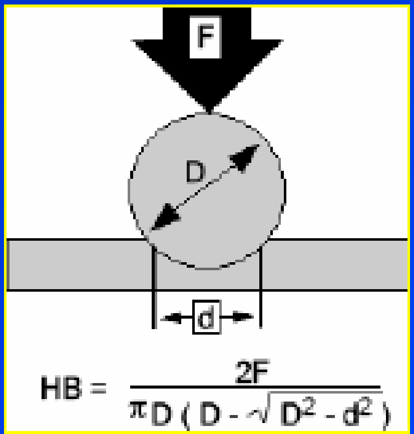
(a) Brinell indentation
(b) measurement of impression diameter

Unit kgf mm^{-2} =9.8 MPa

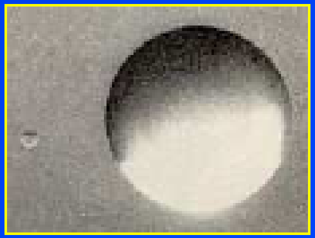
Advantages and disadvantages of Brinell hardness test

- Large indentation averages out ***local heterogeneities of microstructure***.
- Different loads are used to cover a wide rage of hardness of commercial metals.
- Brinell hardness test is ***less influenced by surface scratches and roughness*** than other hardness tests.
- The test has ***limitations*** on ***small specimens*** or in ***critically stressed parts*** where indentation could be a possible site of failure.

www.instron.com



www.alexdenouden.nl

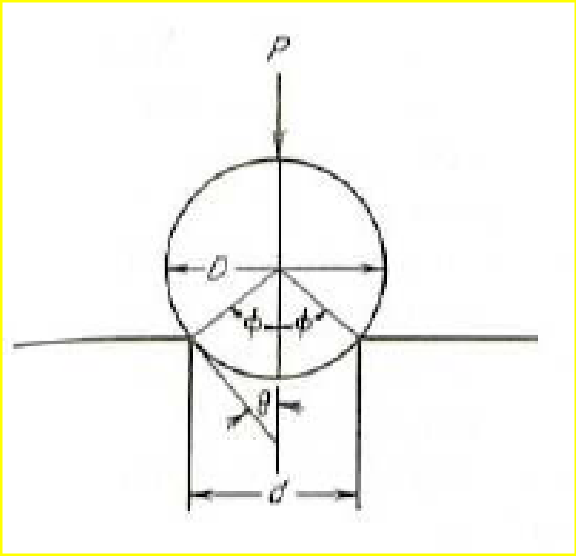


Brinell hardness impression

Brinell hardness test with nonstandard load or ball diameter

- From fig, ***d = Dsinϕ***, giving the alternative expression of ***Brinell hardness number*** as

$$BHN = \frac{P}{(\pi/2)D^2(1-\cos\phi)} \qquad Eq.2$$



Basic parameter in Brinell test

- In order to obtain the same ***BHN*** with a non-standard load or ball diameter, it is necessary to produce a ***geometrical similar indentations***.

- The included angle ***2ϕ*** should remain constant and the ***load and the ball diameter*** must be varied in the ratio

$$\frac{P_1}{D_1^2} = \frac{P_2}{D_2^2} = \frac{P_3}{D_3^3} \qquad Eq.3$$

Meyer hardness

- ***Meyer*** suggested that hardness should be expressed in terms of the ***mean pressure between the surface of the indenter and the indentation***, which is equal to the load divided by the projected area of the indentation.

$$p_m = \frac{P}{\pi r^2} \qquad Eq.4$$

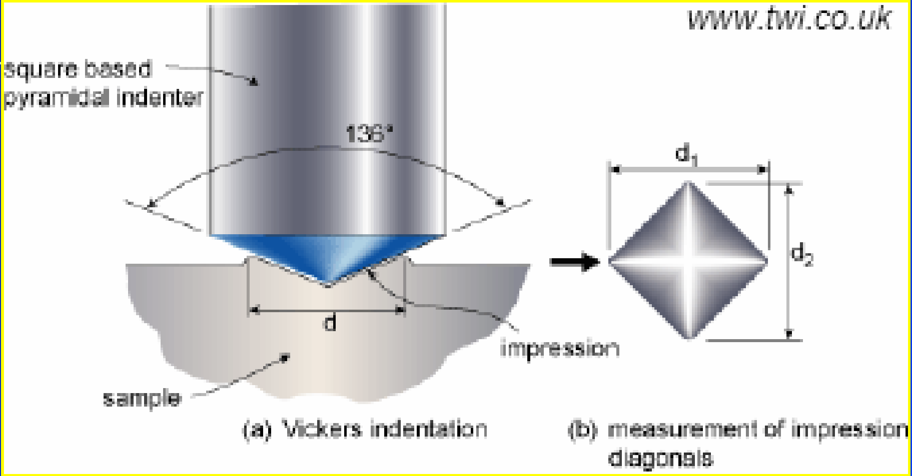
- ***Meyer hardness*** is therefore expressed as follows;

$$Meyer\ \ hardness = \frac{4P}{\pi d^2} \qquad Eq.5$$

Note: - *Meyer hardness is less sensitive to the applied load than Brinell hardness.*
- *Meyer hardness is a more fundamental measure of indentation hardness but it is rarely used for practical hardness measurement.*

Vickers hardness

- ***Vickers hardness test*** uses a ***square-base diamond pyramid*** as the indenter with the included angle between opposite faces of the pyramid of 136°.
- The ***Vickers hardness number (VHN)*** is defined as the load divided by the surface area of the indentation.



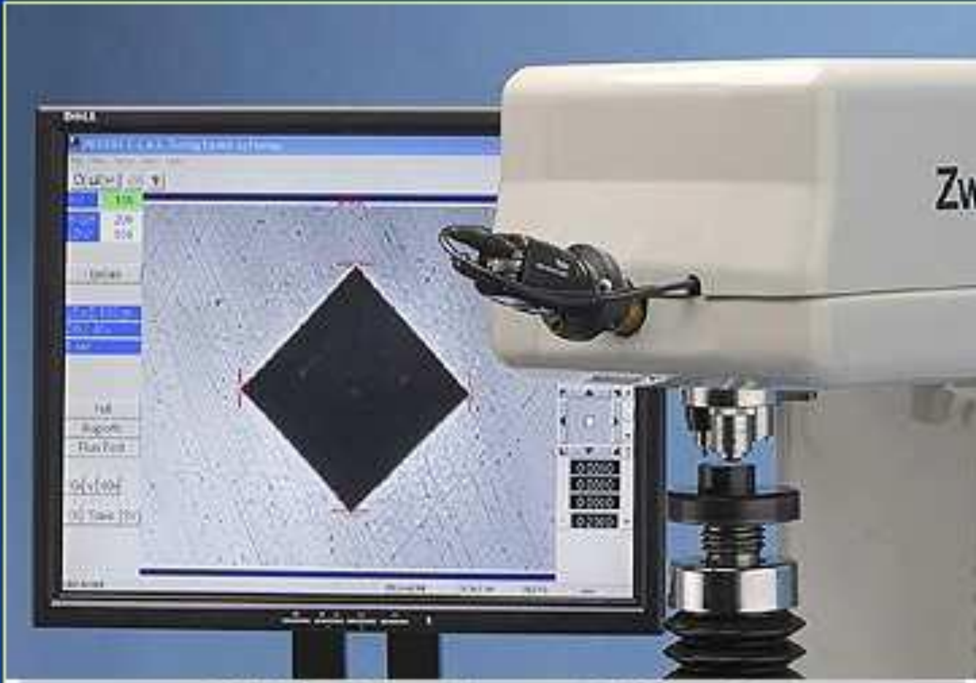
Note: not widely used for routine check due to a slower process and requires careful surface preparation.

$$VHN = \frac{2P\sin(\theta/2)}{L^2} = \frac{1.854P}{L^2} \qquad \text{Eq.6}$$

Where **P** is the applied load, kg
L is the average length of diagonals, mm
θ is the angle between opposite faces of diamond = 136°.

Vickers hardness

- *Vickers hardness test* uses the loads ranging from 1-120 kgf, applied for between 10 and 15 seconds.
- Provide a fairly ***wide acceptance for research work*** because it provides a continuous scale of hardness, for a given load.
- *VHN* = 5-1,500 can be obtained at the same load level→ ***easy for comparison***).



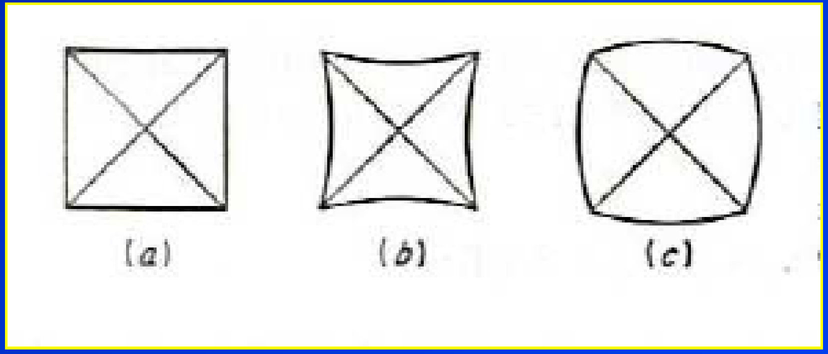
ZHV30 micro and macro Vickers with automatic impression measurement

Impressions made by Vickers hardness

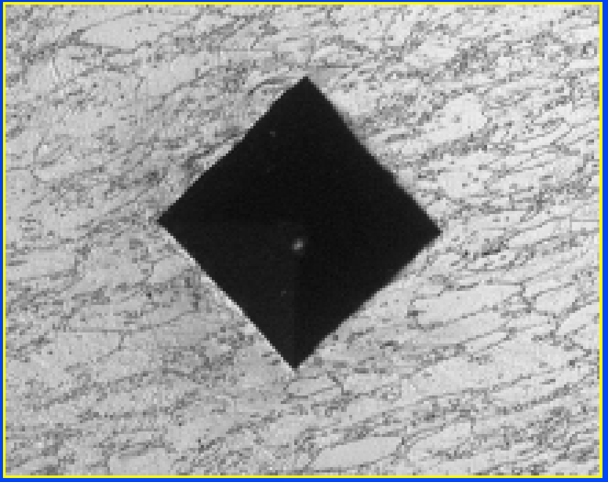
• ***A perfect square indentation (a)*** made with a perfect diamond-pyramid indenter would be a ***square***.

• ***The pincushion indentation (b)*** is the result of sinking in of the metal around the flat faces of the pyramid. This gives an overestimate of the diagonal length (observed in ***annealed metals***).

• ***The barrel-shaped indentation (c)*** is found in ***cold-worked metals***, resulting from ridging or piling up of the metal around the faces of the indenter. Produce a low value of contact area→ ***giving too high value***.



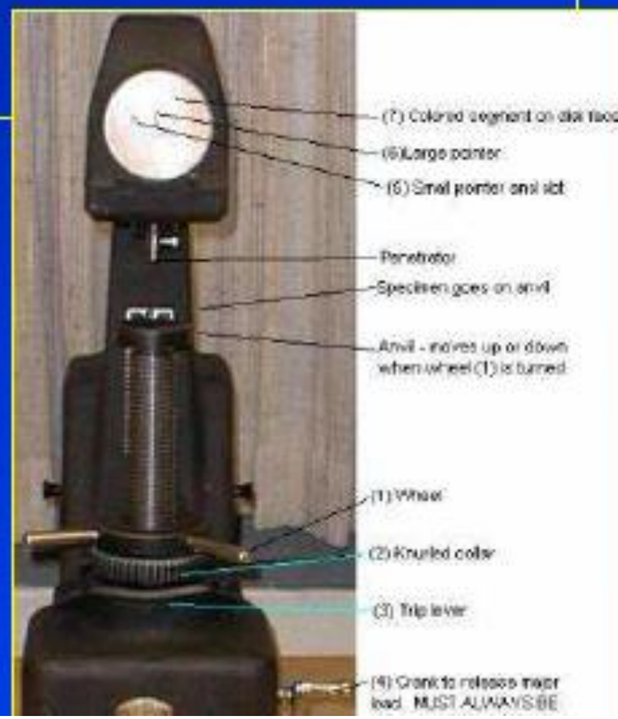
Types of diamond-pyramid indentation (a) perfect indentation (b) pincushion indentation due to sinking in (c) barrelled indentation due to ridging.



Rockwell hardness

- ***The most widely used hardness test in the US and generally accepted due to***
  1) Its speed
  2) Freedom from personal error.
  3) Ability to distinguish small hardness difference
  4) Small size of indentation.

- The hardness is measured according to the ***depth of indentation***, under a constant load.



8150 LK Combined Rockwell
Rockwell suprficial

Rockwell hardness scale

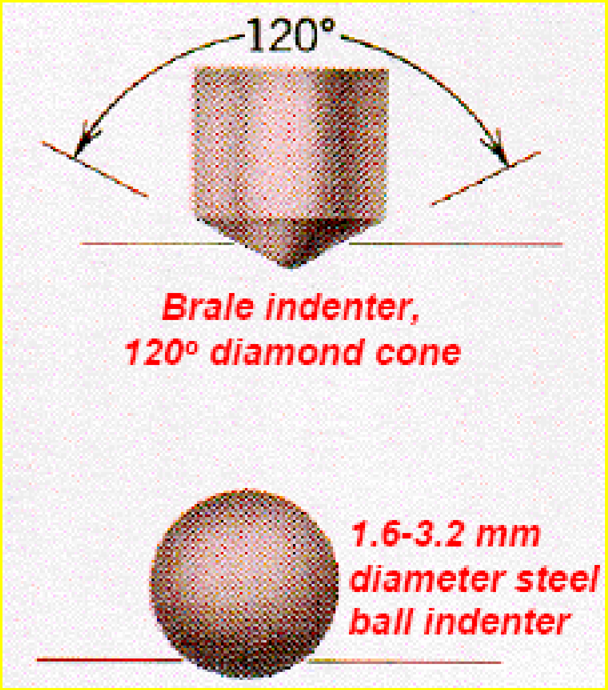
• ***Rockwell hardness number (RHN)*** represents in different scale, A, B, C,.. depending on types of indenters and major loads used.

EX:

Scale	Indenter	Load (kg.f)	Scale
A	Brale	60	HRA
B	1/16'' steel ball	100	HRB
C	Brale	150	HRC

• The Hardened steel is tested on the ***C scale*** with R_c *20-70*.

• Softer materials are tested on the ***B scale*** with R_b *30-100*.

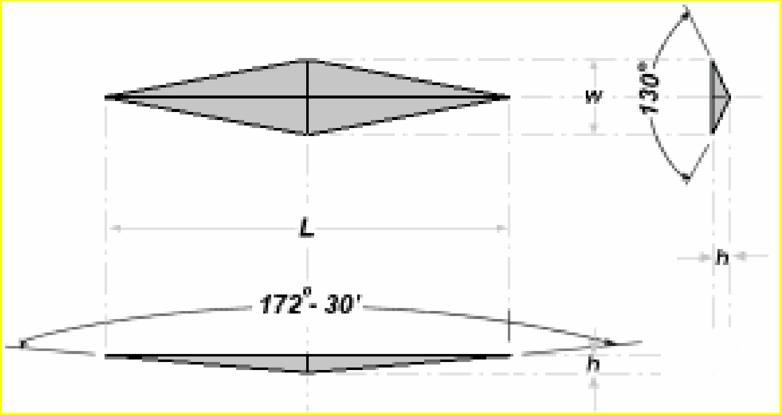


Microhardness

- Determination of hardness over very small areas for example individual constituents, phases, requires ***hardness testing machines in micro or sub-micro scales***.

- ***Vickers hardness*** can also be measured in a microscale, which is based on the same fundamental method as in a macroscale.

- The ***Knoop indenter*** (diamond-shape) is used for measuring in a small area, such as at the cross section of the heat-treated metal surface.



- The ***Knoop hardness number (KHN)*** is the applied load divided by the unrecovered projected area of the indentation.

$$KHN = \frac{P}{A_p} = \frac{P}{L^2 C} \qquad \text{Eq.7}$$

Where **P** = applied load, kg

A_p = unrecovered projected area of indentation, mm^2

L = length of long diagonal, mm

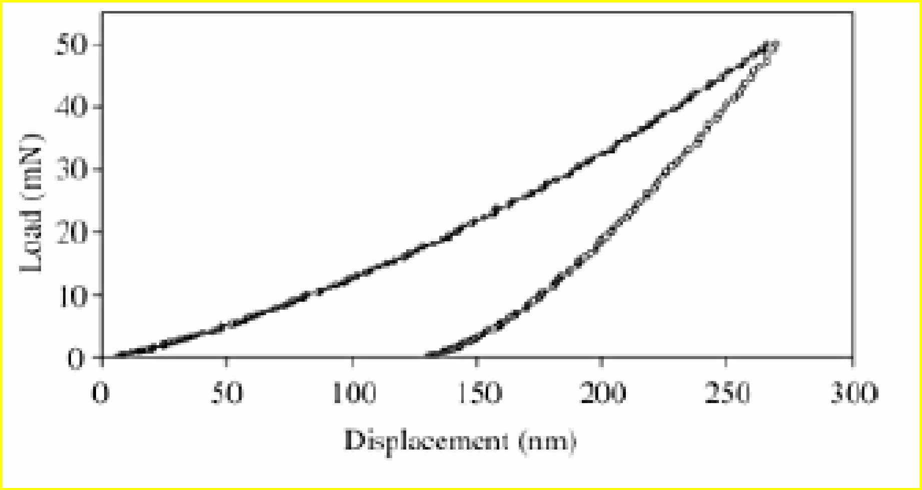
C = a constant for each indenter supplied by manufacturer.

Relationship between hardness and the flow curve

• For ***Brinell hardness***, a very useful correlation has been used for heat-treated plain-carbon and medium-alloy steels as follows:

$$UTS(MPa) = 3.4(BHN) \qquad Eq.9$$

• Furthermore, ***Young's modulus*** can also be given from the ***nano-hardness test***.



Load displacement curve obtained from hardness test