



تقدم لجنة EiCoM الاكاديمية

دفتر لمادة:

منطق رقمي والكترونيات رقمية

جزيل الشكر للطالبتين:

رؤى وماريا



First Logic:

بسم الله الرحمن الرحيم

①

RoAA IKparich

number system.

Decimal system (10)

Binary system (2)

Hexa system (16)

Octal system (8)

① Decimal system: (10)

0 1₍₁₀₎ 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20.....

② Binary system: (2)

0 1₍₂₎ 10 11 100 101 110 111 1000 1001 1010 1011 1100
1101 1110 1111

③ Octal system: (8)

0 1₍₈₎ 2 3 4 5 6 7 10 11 12 13 14 15 16 17 20 21 22 23
24 25 26 27 30 67 70 100

④ Hexa system: (16)

0 1₍₁₆₎ 2 3 4 5 6 7 8 9 A B C D E F 10 11 12 13 14 15 16 17 18
19 1A 1B 1C 1D 1E 1F 99 9A 9B 9C 9D 9E 9F 100

* 77 100

* Conversion From Hexa to decimal:

① $A0_{16}$: $160_{(10)}$

* Hexa \rightarrow decimal
(16) \rightarrow (10)

$$A_{(16)} = 16^1, \quad 0_{(16)} = 16^0$$

$$A_{(10)} = 10, \quad 0_{(10)} = 0$$

$$\Rightarrow 16^1 * 10 + 16^0 * 0 = 160$$

② $ABCD_{16}$: $43981_{(10)}$

$$A_{(16)} = 16^3, \quad B_{(16)} = 16^2$$

$$A_{(10)} = 10, \quad B_{(10)} = 11, \quad C_{(16)} = 16^1$$

$$D_{(16)} = 16^0$$

$$C_{(10)} = 12$$

$$D_{(10)} = 13$$

$$16^3 * 10 + 16^2 * 11 + 16^1 * 12 + 16^0 * 13 = 43981$$

3 0.A3(16) = 0.636(10)

A(16) = 16^-1, 3(16) = 16^-2 => 16^-1 * 10 + 16^-2 * 3 = 0.636
A(10) = 10, 3(10) = 3

4 2B.07(16) = 43.02(10)

2B
2(16) = 16^1, B(16) = 16^0 => 16^1 * 2 + 16^0 * 11 = 43
2(10) = 2, B(10) = 11

.07
0(16) = 16^-1, 7(16) = 16^-2 => 16^-1 * 0 + 16^-2 * 7 = .02
0(10) = 0, 7(10) = 7

Conversion From Binary to Decimal:

* Binary (2) -> Decimal (10)

1 10111(2) = 23(10)

1 * 2^0 + 1 * 2^1 + 1 * 2^2 + 0 * 2^3 + 1 * 2^4 = 23

2 110.0011(2) = 6.1875(10)

0 * 2^0 + 1 * 2^1 + 1 * 2^2 + 0 * 2^-1 + 0 * 2^-2 + 1 * 2^-3 + 1 * 2^-4 = 6.1875

Conversion From octal to Decimal:

* octal (8) -> Decimal (10)

1 36(8) = 30(10)

3(8) = 8^1, 6(8) = 8^0 => 8^1 * 3 + 8^0 * 6 = 30
3(10) = 3, 6(10) = 6

2 2.07(8) = 2.125(10)

2(8) = 8^0, 0(8) = 8^-1, 7(8) = 8^-2
2(10) = 2, 0(10) = 0, 7(10) = 7 => 8^0 * 2 + 8^-1 * 0 + 8^-2 * 7 = 2.125

3 38(8) =

لا يوجد لأنه داخل نظام ال Octal
لا يوجد له 8 ولا 9.

Conversion From Decimal To Binary:

(3)

① $6_{(10)} = 110_{(2)}$

6	
3	0 → Last significant bit
1	1
0	1 → most significant bit

* قسمة علی 2

Decimal → Binary
(10) → (2)

② $.25_{(10)} = .01_{(2)}$

.25	
.5	0 → most significant bit
1.0	1 → last significant bit

* ضرب ب 2

③ $.2_{(10)} = .0011_{(2)}$

.2	
.4	0 msb
.8	0
.6	1
.2	1 lsb

دور کیے
بوتق

④ $6.2_{(10)} = 110.0011_{(2)}$

$6_{(10)} = 110_{(2)}$

6	
3	0 lsb
1	1
0	1 msb

ظرب 2

$.2_{(10)} = .0011_{(2)}$

.2	
.4	0 msb
.8	0
.6	1
.2	1 lsb

تقسیم 2

Conversion From Decimal To Octal:-

① $83_{(10)} = 123_{(8)}$

83	
10	3 lsb
1	2
0	1 msb

تقسیم کی 8

Decimal → Octal
(10) → (8)

② $.0625_{(10)} = .04_{(8)}$

.0625	
.5	0 msb
0	4 lsb

ظرب ب 8

③ $83.0625_{(10)} = 123.04_{(8)}$

83	
10	3 lsb
1	2
0	1 msb

.0625	
.5	0 msb
0	4 lsb

Conversion: From Decimal to Hexa:

(4)

① $69_{(10)} = 45_{(16)}$

② $.01325_{(10)} = .08_{(16)}$

Decimal \rightarrow Hexa
(10) (16)

69	
4	5 Lsb \uparrow
0	4 msb

تقسیم سے 16

.01325	
.5	0 msb \downarrow
0	8 Lsb

\Rightarrow تقریباً 16

③ $69.01325_{(10)} = 45.08_{(16)}$

Octal to Binary

0	\rightarrow	000
1	\rightarrow	001
2	\rightarrow	010
3	\rightarrow	011
4	\rightarrow	100
5	\rightarrow	101
6	\rightarrow	110
7	\rightarrow	111

3 bit

Hexa to Binary

0	\rightarrow	0000
1	\rightarrow	0001
2	\rightarrow	0010
3	\rightarrow	0011
4	\rightarrow	0100
5	\rightarrow	0101
6	\rightarrow	0110
7	\rightarrow	0111
8	\rightarrow	1000
9	\rightarrow	1001
A	\rightarrow	1010
B	\rightarrow	1011
C	\rightarrow	1100
D	\rightarrow	1101
E	\rightarrow	1110
F	\rightarrow	1111

Note:-

Octal to Hexa
or Hexa to Octal:
Octal to binary to Hexa
Hexa to binary to Octal

4 bit

Conversion From Binary to Octal:

Binary \rightarrow Octal
(2) (8)

① $011011_2 = 33_8$

② $1.1_{(2)} = 1.4_{(8)}$

011	011
3	3

001	100
1	4

3 bit

Conversion From Octal to Binary:

Octal \rightarrow Binary
(8) (2)

① $7.3_8 = 111.011_2$

② $3.06_8 = 011.000110_2$

③ $12.17_8 = 001010.001111_2$

3 bit

Conversion From binary to Hexa:

① 0101: $11011000_{(2)} = 5.D8_{(16)}$

$$\begin{array}{ccc} \boxed{0101} & \boxed{1101} & \boxed{1000} \\ 5 & D & 8 \end{array}$$

binary \rightarrow Hexa (5)
(2) (16)

4 bit

② 0110: $0100_{(2)} = 6.4_{(16)}$

$$\begin{array}{cc} \boxed{0110} & \boxed{0100} \\ 6 & 4 \end{array}$$

Conversion From Hexa to binary:

① 5BC₍₁₆₎ = 0101 1011 1100₍₂₎

Hexa \rightarrow binary
(16) (2)

② 43.BF₍₁₆₎ = 0100 0011. 1011 1111₍₂₎

4 bit

Conversion From octal to Hexa:

~~Answer~~ $241.570_{(8)} = A1.BC_{(16)}$

$241.570_{(8)} = 010100001.10111000_{(2)}$

* octal \rightarrow binary \rightarrow Hexa
(8) (2) (16)

$010100001.10111000_{(2)} =$

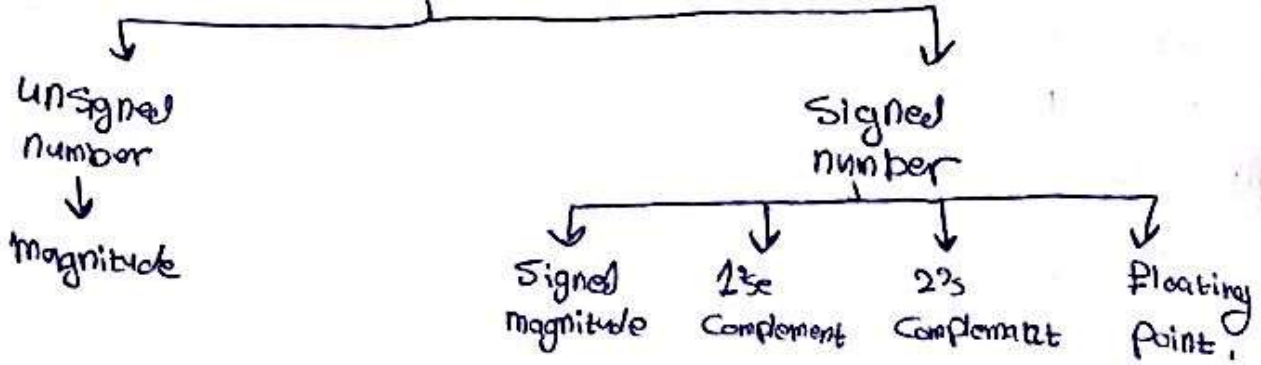
1010 = A

0001 = 1 \Rightarrow A1.BC₍₁₆₎

1011 = B

1100 = C

numbers



Complement:

1. First Complement: (1's Complement):

0	→	1
1	→	0

Example: Find the First Complement:

- 110 → 001
- 101 → 010
- 1010 → 0101
- 1111 → 0000

2. Second Complement: (2's Complement):

1's complement + 1 = 2's complement.

Example: Find 2's complement :-

1001 ^{1's} → 0110 + 1

$$\begin{array}{r} 0110 \\ + 1 \\ \hline 0111 \end{array}$$
 → 2's complement.

OR: Constant Leading number and 1's Complement:

Example:-

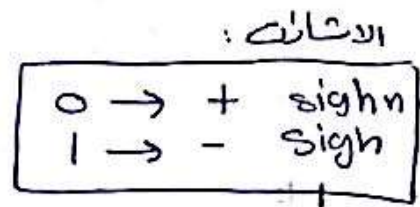
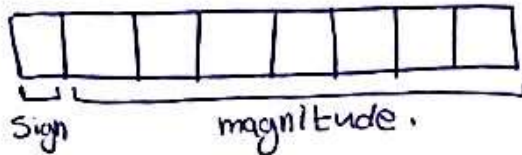
- 1101100[] → 0010011[]
- []0000 → []0000
- 1[]0000 → 0[]0000
- 1010[] → 0101[]
- 11[] → 00[]
- 10[] → 01[]

* The purpose of Complement is to Convert Subtraction operation to addition operation. (7)

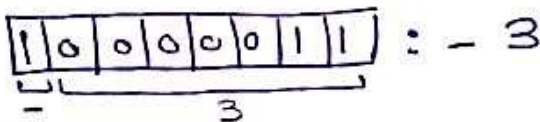
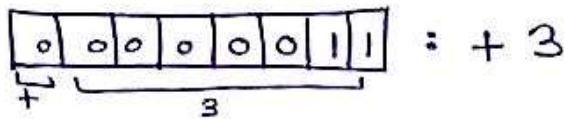
number representation for digital numbers:

① Sign-magnitude representation:-

8 bits



Example: represent [+3] and [-3] using 8 bits sign magnitude



Example: 4 Bits sign magnitude representation:-



(Note) for 4 Bit maximum number for magnitude 7 or -7

sign number Representation:-

1. sign magnitude Representation → Sign

2. 1's complement:-

Positive number → same as binary.

Negative number → First complement of binary number.

Example:- Sign Represent the following using First complement in 4Bit system:-

+3 : 0011

+4 : 0100

-3 : 1100

-4 : 1011

3. Second complement:-

Positive number → take binary only.

Negative number → 2's complement the binary number.

Example:- Representation the following number using

2's representation in 4bit and 8bit system:-

	4bit	8bit
+3	0011	00000011
-3	1101	11111101
+4	0100	00000100
-4	1000	11111100

Note: Computers use 2's complement for number representation to convert subtraction operation

Example: in 4bit find:-

$$\begin{array}{r}
 1. \quad \begin{array}{r} 4 \quad 0100 \\ + 3 \quad 0011 \\ \hline + 7 \quad 0111 \end{array}
 \end{array}$$

$$\begin{array}{r}
 2. \quad \begin{array}{r} +4 \quad 0100 \\ -3 \quad 1101 \\ \hline +1 \quad 0001 \end{array}
 \end{array}$$

$$\begin{array}{r}
 3. \quad \begin{array}{r} +3 \quad 0011 \\ -4 \quad 1100 \\ \hline -1 \quad 1111 \\ \downarrow \\ \text{Sign} = - \\ \text{Value} = 2's \text{ comple} \\ 1111 \rightarrow 0001 \end{array}
 \end{array}$$

Range of signal number:-

- ① sign magnitude
- ② First complement → $-(2^{n-1}-1)$, $+(2^{n-1}-1)$
- ③ Second complement → $-(2^{n-1})$, $+(2^{n-1}-1)$

Example: find the range of 4bit sign number for all 3 sys: sign magnitude and first complement;

$$-(2^{4-1}-1) / +(2^{4-1}-1) \Rightarrow \boxed{-7 \text{ to } 7} \quad n=4:$$

$$\text{Second complement } n=4: \\
 -2^{4-1} / +(2^{4-1}-1) \Rightarrow \boxed{-8 \text{ to } 7}$$

Example:- Represent the following in 4Bit system in at three system:-

9

Number	Sign	1's Complement	2's Complement
+3	0011	0011	0011
-3	1011	1100	0011 1101
0	0000	0000	0000
1	0001	0001	0001
-1	1001	1110	1111
-8	Not Valid	Not Valid	1000

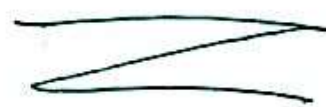
Code:-

Binary Coded decimal (BCD)

Decimal	BCD
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Digit ke Jai Bits *
Jai Bits ke Decimal Digital
Digit ke Decimal

Gray Code	Decimal equivalent
0	0000
1	0001
2	0011
3	0010
4	0110
5	0111
6	0101
7	0100
8	1100
9	1101

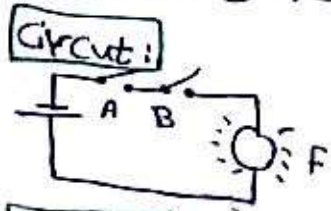


Example:- Express the following Decimal number in BCD.

Decimal	BCD	
3(10)	0011	1 Digit.
23(10)	0010 0011	2 Digit.
101(10)	001 0000 0011	3 Digit

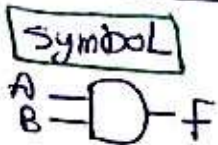
Boolean Gates:

① AND Gate:-



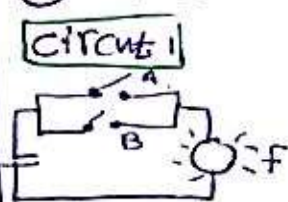
Truth table

A	B	F
0	0	0
0	1	0
1	0	0
1	1	1



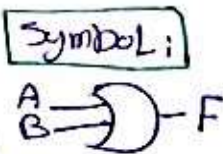
A, B : input
F : output

② OR Gate:



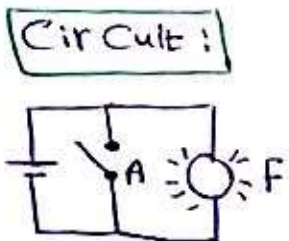
Truth table:-

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1



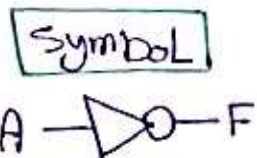
A, B : input
F : output.

③ Not Gate:-



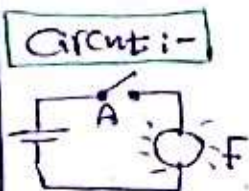
Truth table

A	F
0	1
1	0



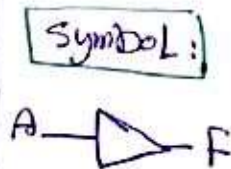
A : input
F : output

④ Buffer Gate:-



Truth table.

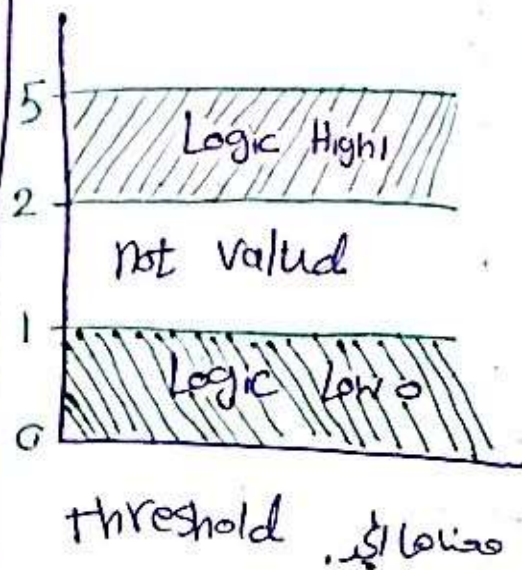
A	F
0	0
1	1



A : input
F : output.

Note:-

1. Following Follows \uparrow input \downarrow output.
2. bubble \rightarrow Not \leftarrow bubble
3. $F = A \cdot B$ \rightarrow AND operator
 $F = A + B$ \rightarrow OR operator.
 $F = A'$ \rightarrow Not operator.
 $F = A$ \rightarrow Buffer operator.



Boolean Theorems and Properties.

11

- 1) * closure with respect to the operator OR (+).
* closure with respect to the operator AND (·)

- 2) * An identity element with respect to +

$$X + 0 = 0 + X = X$$

- * An identity element with respect to ·

$$X \cdot 1 = 1 \cdot X = X$$

- 3) * Commutative with respect to +

$$X + Y = Y + X$$

- * Commutative with respect to ·

$$X \cdot Y = Y \cdot X$$

- 4) * · is distributive over +

$$X \cdot (Y + Z) = (X \cdot Y) + (X \cdot Z)$$

- * + is distributive over ·

$$X + (Y \cdot Z) = (X + Y) \cdot (X + Z)$$

- 5) * $X + X' = 1$

* $X \cdot X' = 0$

X' is a 2's complement

theorem 1 :-

$$\begin{aligned} \text{a) } X + X &= (X + X) \cdot 1 \\ &= (X + X)(X + X') \\ &= XX + XX' + XX + XX' \\ &= XX + XX' \\ &= X + 0 \\ &= \boxed{X} \end{aligned}$$

$$\begin{aligned} \text{b) } X \cdot X &= (X \cdot X) + 0 \\ &= (X \cdot X) + (X \cdot X') \\ &= X(X + X') \\ &= X \cdot 1 \\ &= \boxed{X} \end{aligned}$$

Theorem 2 :-

$$\begin{aligned}
 \text{a) } X + XY &= X \cdot 1 + (XY) \\
 &= (X \cdot (1 + Y)) + (XY) \\
 &= X(1 + Y) \\
 &= X \cdot 1 \\
 &= \boxed{X}
 \end{aligned}$$

$$\begin{aligned}
 \text{b) } X(X+Y) &= (X+0)(X+Y) \\
 &= XX + 0 \cdot X + XY + 0 \cdot Y \\
 &= XX + XY + \\
 &= \boxed{X}
 \end{aligned}$$

Theorem 3 :-

$$* X + (Y + Z) = (X + Y) + Z$$

$$* X + (Y + X) = X + (X + Y)$$

$$* X(YZ) = (XY)Z$$

Associative

$$* X(Y + Z) = XY + XZ$$

$$* X + (Y \cdot Z) = (X + Y) \cdot (X + Z)$$

* Demorgan theorem :-

$$* (X + Y)' = X' \cdot Y'$$

$$* (X \cdot Y)' = X' + Y'$$

Example: Apply Demorgan theorem on the following: (13)

$$\textcircled{a} \overline{(A \cdot B)} = A'' + B'' = A + B$$

$$\textcircled{b} \overline{(A \cdot B)} = A'' + B' = A + B'$$

$$\textcircled{c} \overline{A+B+C} = A' \cdot B' \cdot C'$$

$$\textcircled{d} \overline{ABC} = A' + B' + C'$$

$$\textcircled{e} \overline{A \cdot (B+C)} = A' + (B+C)' = A' + (B' \cdot C')$$

$$\begin{aligned} \textcircled{f} \overline{(A \cdot C) + (A \cdot B)} &= \overline{(A \cdot C)} \cdot \overline{(A \cdot B)} = (A' + C') \cdot (A' + B') \\ &= A' + A'B' + A'C' + B'C' = A' + B'C' \end{aligned}$$

$$\textcircled{g} \overline{AB} + \overline{CD} = A' + B' + C' + D'$$

$$\textcircled{h} \overline{AB+CD} = \overline{AB} \cdot \overline{CD} = (A' + B') \cdot (C' + D')$$

$$\begin{aligned} \textcircled{i} (A + \overline{B}) \cdot (\overline{C} + D) &= \overline{(A + \overline{B}) + (\overline{C} + D)} = \overline{(A' \cdot B'') + (C'' \cdot D)} \\ &= \overline{(A' \cdot B) + (C \cdot D')} \end{aligned}$$

$$\textcircled{o} \overline{A \cdot (B+C)} = A' + \overline{(B+C)} = A' + (B' \cdot C')$$

Example:

Example: - Simplify using Boolean properties and theorem. (14)

$$\boxed{1} \quad A(A+B) = AA + AB = A + AB = A(AB) = A(1 \cdot B) \\ A \cdot 1 = \boxed{A}$$

$$\boxed{2} \quad A(\bar{A} + AB) = A\bar{A} + AAB = 0 + AB = \boxed{AB}$$

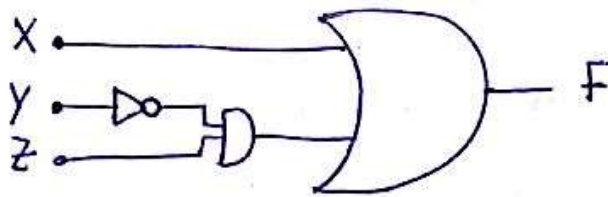
$$\boxed{3} \quad BC + B'C = C[B + B'] = C[1] = \boxed{C}$$

$$\boxed{4} \quad A(A + A'B) = AA + AA'B = A + 0 \cdot B = A + 0 = \boxed{A}$$

Truth table

$$2^3 = 8 =$$

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

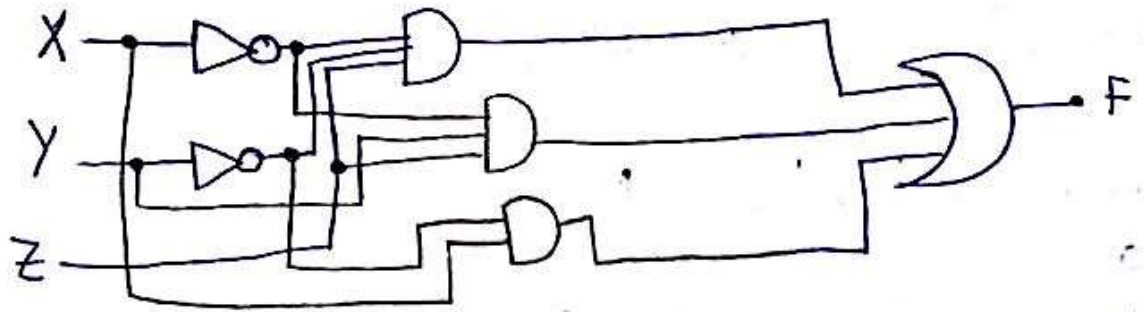


$$X + (Y' \cdot Z) = F(x, y, z)$$

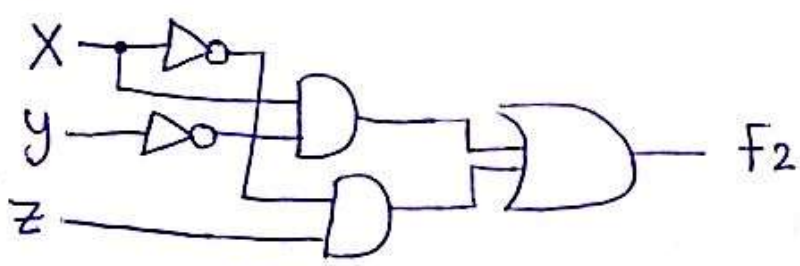
* $\overline{A \cdot (B+C)} =$
 $A' + \overline{(B+C)} =$
 $A' + (B' \cdot C')$

Example:-

$$F_2 = x'y'z + x'yz + xy'$$



$$\begin{aligned}
 F_2 &= x'y'z + x'yz + xy' = \\
 &= x'z(y'+y) + xy' = \\
 &= x'z \cdot 1 + xy' = \boxed{x'z + xy'} = F_2
 \end{aligned}$$



truth table:- $2^3 = 8$

X	Y	Z	F ₂	x'z	xy'
0	0	0	0	0	0
0	0	1	1	1	0
0	1	0	0	0	0
0	1	1	1	1	0
1	0	0	1	0	1
1	0	1	1	0	1
1	1	0	0	0	0
1	1	1	0	0	0

Minterms and Maxterms

Minterms $\Rightarrow m$

Maxterm $\Rightarrow M$

When $2^3 = 8$ The binary number forms to 8

Number	X	Y	Z	Minterm Term Designation	Max term Term Designation
0	0	0	0	$\bar{x}\bar{y}\bar{z}$ m_0	$x+y+z$ M_0
1	0	0	1	$\bar{x}\bar{y}z$ m_1	$x+y+\bar{z}$ M_1
2	0	1	0	$\bar{x}y\bar{z}$ m_2	$x+\bar{y}+z$ M_2
3	0	1	1	$\bar{x}yz$ m_3	$x+\bar{y}+\bar{z}$ M_3
4	1	0	0	$x\bar{y}\bar{z}$ m_4	$\bar{x}+y+z$ M_4
5	1	0	1	$x\bar{y}z$ m_5	$\bar{x}+y+\bar{z}$ M_5
6	1	1	0	$xy\bar{z}$ m_6	$\bar{x}+\bar{y}+z$ M_6
7	1	1	1	xyz m_7	$\bar{x}+\bar{y}+\bar{z}$ M_7

Example:-

	x	y	z	F ₁	F ₂
0	0	0	0	0	0
1	0	0	1	1	0
2	0	1	0	0	0
3	0	1	1	0	1
4	1	0	0	1	0
5	1	0	1	0	1
6	1	1	0	0	1
7	1	1	1	1	1

① minterm F₁

$$F_1 = \dot{x}\dot{y}z + x\dot{y}\dot{z} + xy\dot{z} = m_1 + m_4 + m_7$$

② minterm F₂

$$F_2 = \dot{x}y\dot{z} + x\dot{y}z + xy\dot{z} + xy\dot{z} = m_3 + m_5 + m_6 + m_7$$

③ Complement minterm F₁

$$F_1' = \dot{x}\dot{y}\dot{z} + \dot{x}y\dot{z} + \dot{x}y\dot{z} + x\dot{y}z + xy\dot{z} = m_0 + m_2 + m_3 + m_5 + m_6$$

④ Complement minterm F₂

$$F_2' = \dot{x}\dot{y}\dot{z} + \dot{x}\dot{y}z + \dot{x}y\dot{z} + x\dot{y}\dot{z} = m_0 + m_1 + m_2 + m_4$$

⑤ max term F₁ :-

$$F_1 = (x+y+z)(x+\dot{y}+\dot{z})(x+\dot{y}+\dot{z})(\dot{x}+\dot{y}+\dot{z})(\dot{y}+\dot{y}+z) = M_0 M_2 M_3 M_5 M_6$$

⑥ Complement max term F₁ :-

$$F_1' = (x+y+\dot{z})(\dot{x}+\dot{y}+\dot{z})(\dot{x}+\dot{y}+\dot{z}) = M_1 M_4 M_7$$

⑦ Max term F₂ :-

$$F_2 = (x+y+z)(x+y+\dot{z})(x+\dot{y}+z)(\dot{x}+\dot{y}+z) = M_0 M_1 M_2 M_4$$

⑧ Complement Max term F₂ :-

$$F_2' = (x+\dot{y}+\dot{z})(\dot{x}+\dot{y}+\dot{z})(\dot{x}+\dot{y}+\dot{z})(\dot{x}+\dot{y}+\dot{z}) = M_3 M_5 M_6 M_7$$

Example:- Express the Boolean Function (F) in a sum of \sum (18)
 min term the function has 3 variables A, B, C

$F = A + B'C$

$F = A(B+B') = AB + AB' + B'C$

$F = AB(C+C') + AB'(C+C') + B'C(A'+A)$

$= ABC + ABC' + AB'C + AB'C' + B'CA' + B'CA$

$F(ABC) = ABC + ABC' + AB'C + AB'C' + A'B'C + A'B'C$

$= 111 + 110 + 101 + 100 + 001 + 101$

$= m_7 + m_6 + m_5 + m_4 + m_1 + m_5$

$= m_1 + m_4 + m_5 + m_6 + m_7$

$= A'B'C + ABC' + AB'C + ABC + ABC$

Sum of min term $F(A,B,C) = \sum (m_1 + m_4 + m_5 + m_6 + m_7)$

m ₀	000
m ₁	001
m ₂	010
m ₃	011
m ₄	100
m ₅	101
m ₆	110
m ₇	111

Example:- $F = A + B'C$ ~~min term~~ in truth table \uparrow

~~min term~~

A	B	C	F	number
0	0	0	0	m ₀
0	0	1	1	m ₁
0	1	0	0	m ₂
0	1	1	0	m ₃
1	0	0	1	m ₄
1	0	1	1	m ₅
1	1	0	1	m ₆
1	1	1	1	m ₇

Complement min term:-
of function:-

$A'B'C' + A'BC' + A'BC$
 $m_0 + m_2 + m_3$

Example: $F(A, B, C, D) = AB'C + ACD + A'B'D + A'D$ (19)

* $AB'C(C+D) = AB'CD + AB'CD' = 1011 + 1010$
 $\Rightarrow \boxed{m_{11} + m_{10}}$

* $A(B'+B)CD = AB'CD + ABCD = 1011 + 1111$
 $\Rightarrow \boxed{m_{11} + m_{15}}$

* $A'B'(C+D)D' = A'B'CD' + A'B'C'D' = 0010 + 0000$
 $\Rightarrow \boxed{m_2 + m_0}$

* $A'(B+B')(C+C')D = A'BCD + A'BC'D + A'B'CD + A'B'C'D$
 $= 0111 + 0101 + 0011 + 0001 \Rightarrow \boxed{m_7 + m_5 + m_3 + m_1}$

$F(A, B, C, D) = \sum (m_0 + m_1 + m_2 + m_3 + m_5 + m_7 + m_{10} + m_{11} + m_{15})$

Complement OR Max Term OR Product of sum

~~$F(A, B, C, D) = (M_4 \cdot M_6 \cdot M_8 \cdot M_9 \cdot M_{12} \cdot M_{13} \cdot M_{14})$~~
 $F(A, B, C, D) = \prod (4, 6, 8, 9, 12, 13, 14)$

~~$F = (A+B+C+D)(A+B+C+D)(A+B+C+D)(A+B+C+D)(A+B+C+D)(A+B+C+D)$~~
 $F = (A+B+C+D)(A+B+C+D)(A+B+C+D)(A+B+C+D)(A+B+C+D)(A+B+C+D)$

1	1	1	1	1	1	1	1	0	0	0	0	0	0	0	0	A
1	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	B
1	1	0	0	1	1	0	0	1	0	0	0	1	1	0	0	C
1	0	1	0	1	0	1	0	1	0	1	0	1	0	1	0	D
m_{15}	m_{14}	m_{13}	m_{12}	m_{11}	m_{10}	m_9	m_8	m_7	m_6	m_5	m_4	m_3	m_2	m_1	m_0	m
1	0	0	0	1	1	0	0	1	0	1	0	1	1	1	1	F

Example:- $F(A, B, C, D) = ABC$

$$ABC(D + D') = A\bar{B}CD + AB\bar{C}D'$$

$$= 1011 + 1010$$

$$= m_{10} + m_{11}$$

$$F(A, B, C, D) = \sum (m_{10}, m_{11})$$

$$F'(A, B, C, D) = \sum (m_0 + m_1 + m_2 + m_3 + m_4 + m_5 + m_6 + m_7 + m_8 + m_9 + m_{12} + m_{13} + m_{14} + m_{15})$$

$$F(A, B, C, D) = \prod (m_0, m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8, m_9, m_{12}, m_{13}, m_{14}, m_{15})$$

Example:- express the following function in all possible representation.

$$F(A, B, C) = (A + B + C') \cdot (A' + B) \cdot (A + C')$$

$$\# A' + B + C' \Rightarrow (101) \Rightarrow (M_5)$$

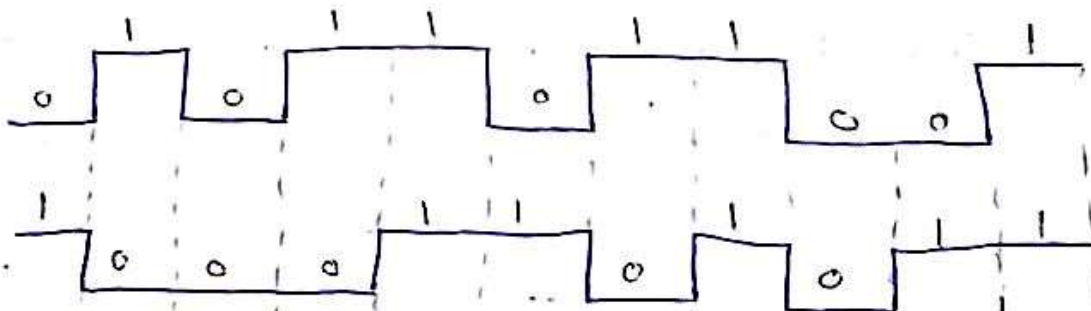
$$\# (A' + B + C)(A' + B + C') \Rightarrow (100)(101) \Rightarrow (M_4)(M_5)$$

$$\# (A + C') \Rightarrow (A + B + C')(A + B' + C') \Rightarrow (001)(011)$$

$$(M_1)(M_3)$$

$$F(A, B, C) = \prod (1, 4, 5, 3)$$

$$F'(A, B, C) = \sum (0, 2, 6, 7) = \prod (0, 2, 6, 7)$$



Example: $F = xy + x'z$... max term (21)

$$(x+x')(x+z)(y+x')(y+z) \Rightarrow (x+z)(y+x')(y+z)$$

$$\# (x+z) \Rightarrow (x+y+z)(x+y'+z) \Rightarrow (M_0)(M_2)$$

$$\# (x'+y) \Rightarrow (x'+y+z)(x'+y+z') \Rightarrow (M_4)(M_5)$$

$$\# (y+z) \Rightarrow (x+y+z)(x'+y+z) \Rightarrow (M_0)(M_4)$$

$$F(x,y,z) = \prod(0, 2, 4, 5)$$

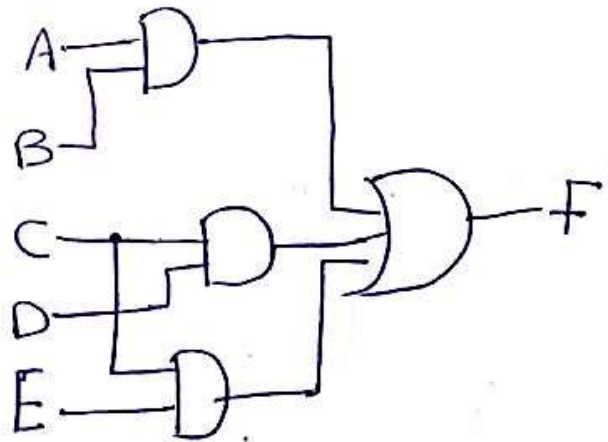
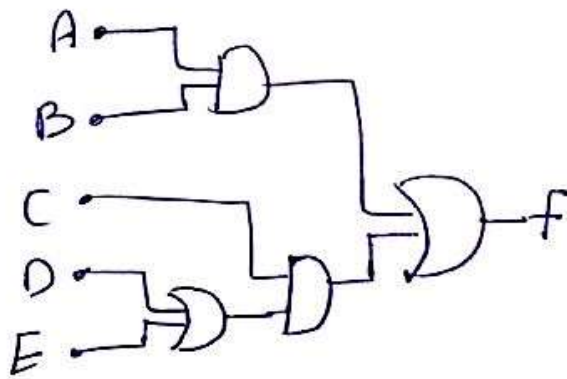
$$F'(x,y,z) = \prod(1, 3, 6, 7)$$

$$F(x,y,z) = \sum(1, 3, 6, 7)$$

$$F'(x,y,z) = \sum(0, 2, 4, 5)$$

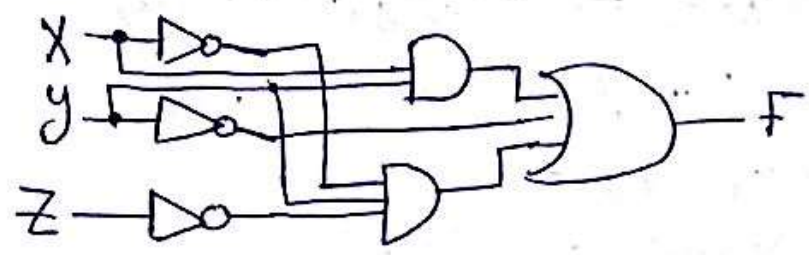
Example: $AB + C(D+E) = F$

$$AB + CD + CE = F$$



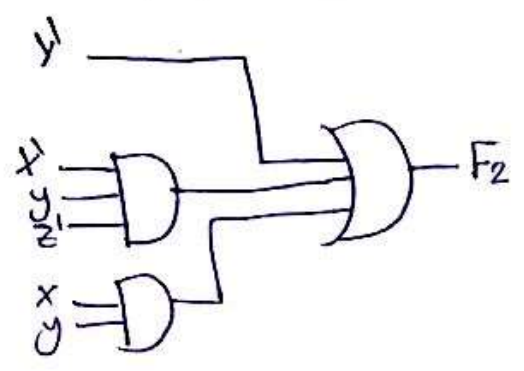
نفس الاجابة.

Exampler- $F = y' + xy + x'yz'$

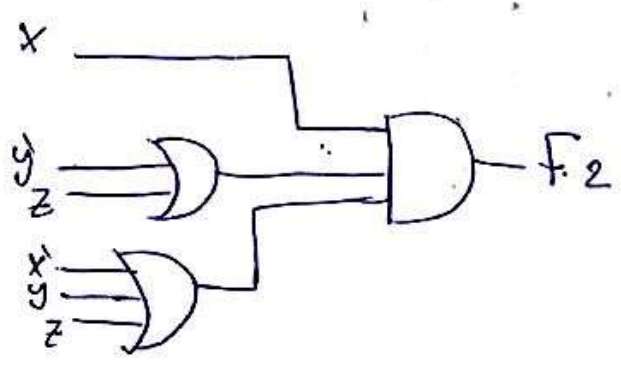



* $F_2 = X(x' + Z)(x' + y + z')$

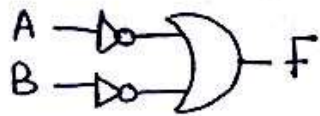
Sum of products. minterm.

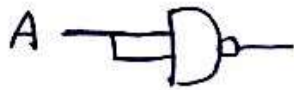
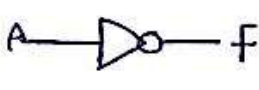



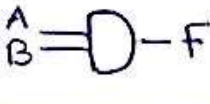
Products of sum max term.

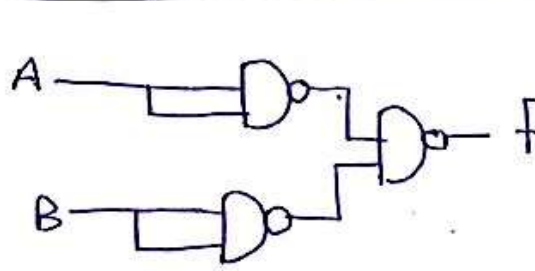
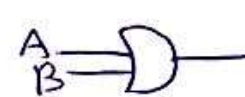



 $F = \overline{A \cdot B} = A' + B'$




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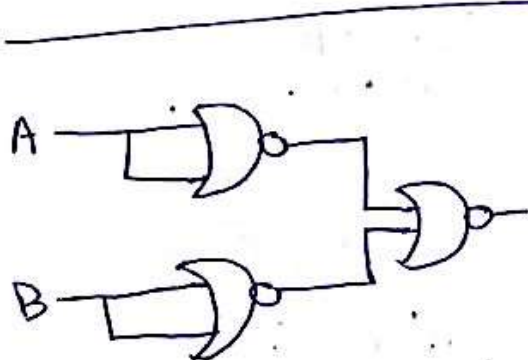
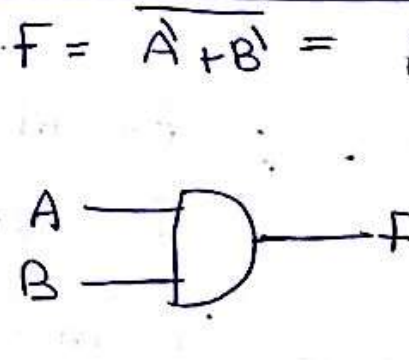

 $F = \overline{\overline{A \cdot B}} = A'' + B'' = A + B$
 \equiv



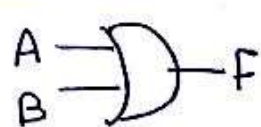
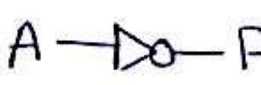
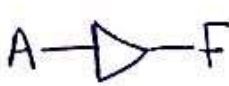

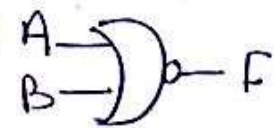
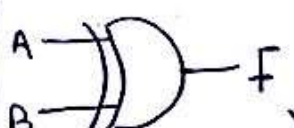
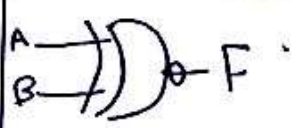

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$F = \overline{\overline{A + B}} = A'' \cdot B'' = \underline{\underline{A \cdot B}}$


 \equiv


Name	Gate	Function	truth table.															
AND		$F = A \cdot B$	<table border="1"> <tr><th>x</th><th>y</th><th>F</th></tr> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </table>	x	y	F	0	0	0	0	1	0	1	0	0	1	1	1
x	y	F																
0	0	0																
0	1	0																
1	0	0																
1	1	1																
OR		$F = A + B$	<table border="1"> <tr><th>A</th><th>B</th><th>F</th></tr> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </table>	A	B	F	0	0	0	0	1	1	1	0	1	1	1	1
A	B	F																
0	0	0																
0	1	1																
1	0	1																
1	1	1																
Inverter OR NOT		$F = A'$	<table border="1"> <tr><th>A</th><th>F</th></tr> <tr><td>0</td><td>1</td></tr> <tr><td>1</td><td>0</td></tr> </table>	A	F	0	1	1	0									
A	F																	
0	1																	
1	0																	
Buffer		$F = A$	<table border="1"> <tr><th>A</th><th>F</th></tr> <tr><td>1</td><td>1</td></tr> <tr><td>0</td><td>0</td></tr> </table>	A	F	1	1	0	0									
A	F																	
1	1																	
0	0																	
NAND		$F = A \cdot B$ $F = (A \cdot B)'$	<table border="1"> <tr><th>A</th><th>B</th><th>F</th></tr> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </table>	A	B	F	0	0	1	0	1	1	1	0	1	1	1	0
A	B	F																
0	0	1																
0	1	1																
1	0	1																
1	1	0																
NOR		$F = (A + B)'$	<table border="1"> <tr><th>A</th><th>B</th><th>F</th></tr> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </table>	A	B	F	0	0	1	0	1	0	1	0	0	1	1	0
A	B	F																
0	0	1																
0	1	0																
1	0	0																
1	1	0																
XOR		$F = AB' + BA'$ $F = A \oplus B$	<table border="1"> <tr><th>A</th><th>B</th><th>F</th></tr> <tr><td>0</td><td>0</td><td>0</td></tr> <tr><td>0</td><td>1</td><td>1</td></tr> <tr><td>1</td><td>0</td><td>1</td></tr> <tr><td>1</td><td>1</td><td>0</td></tr> </table>	A	B	F	0	0	0	0	1	1	1	0	1	1	1	0
A	B	F																
0	0	0																
0	1	1																
1	0	1																
1	1	0																
NXOR		$F = AB + A'B'$ $F = (A \oplus B)'$	<table border="1"> <tr><th>A</th><th>B</th><th>F</th></tr> <tr><td>0</td><td>0</td><td>1</td></tr> <tr><td>0</td><td>1</td><td>0</td></tr> <tr><td>1</td><td>0</td><td>0</td></tr> <tr><td>1</td><td>1</td><td>1</td></tr> </table>	A	B	F	0	0	1	0	1	0	1	0	0	1	1	1
A	B	F																
0	0	1																
0	1	0																
1	0	0																
1	1	1																

Sacred logic

K-map

□ two-variable-map $\Rightarrow 2^2 = 4$ جادين \neq *

	m_0	m_1
	m_2	m_3

	y'	y
x'	$x'y'$	$x'y$
x	xy'	xy

Example:-

A	B	F
0	0	0 m_0
0	1	0 m_1
1	0	1 m_2
1	1	1 m_3

	B'	B
A'		
A	1	1

$$F = AB' + AB$$

$$F = A(B' + B)$$

$$F = A \cdot 1$$

$$F = A$$

ف الجيران $F = A$

شبان موجودين بل ال 2

نفس الجواب

	y'	y
x'	1	
x	1	

$F = y'$

	y'	y
x'	1	1
x		

$F = x'$

	y	y'
x'		
x	1	1

$F = x$

	y	y'
x'		1
x		1

$F = y$

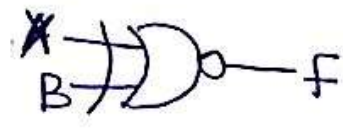
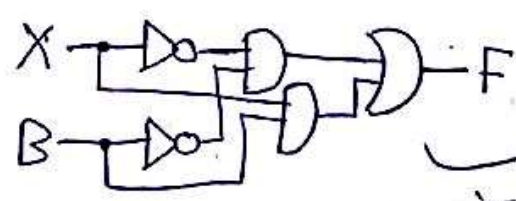
Example:-

A	B	F
0	0	1 m_0
0	1	0 m_1
1	0	0 m_2
1	1	1 m_3

	B
x'	1
x	1

$$F = x'B + xB$$

$$F = (x \oplus B)'$$



نفس الجواب

Three variable - K-map $\Rightarrow 2^3 = 8$

26
* ۳ متغیران کا ۱

m ₀	m ₁	m ₃	m ₂
m ₄	m ₅	m ₇	m ₆

	y'		y	
x'	$x'y'z'$	$x'y'z$	$x'yz'$	$x'yz$
x	$xy'z'$	$xy'z$	xyz'	xyz
	z'		z	

Example:-

X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	0
1	1	1	0

	y		
x'	1		1
x	1	1	
	z		

* عدد الٹو = تعداد المورڈ

$$f = xy' + y'z + x'yz$$

Example:-

	y		
x'	1		1
x			
	z		

$$F = x'z'$$

Example:

$$xy'z + xyz = F$$

$$F = xz(y' + y)$$

$$F = xz \cdot 1 \Rightarrow \boxed{xz}$$

Example

	y		
x'		1	1
x	1	1	
	z		

$$F(x,y,z) = \sum(2,3,4,5)$$

$$= xy' + x'y = (x \oplus y)$$

AND
 $F = xy$

	y	
x'		
x		1

Example:-

$$F(x,y,z) = \sum(3,4,6,7)$$

OR
 $F = x + y$

	y	
x'	1	1
x	1	1

	y		
x'		1	
x	1		1
	z		

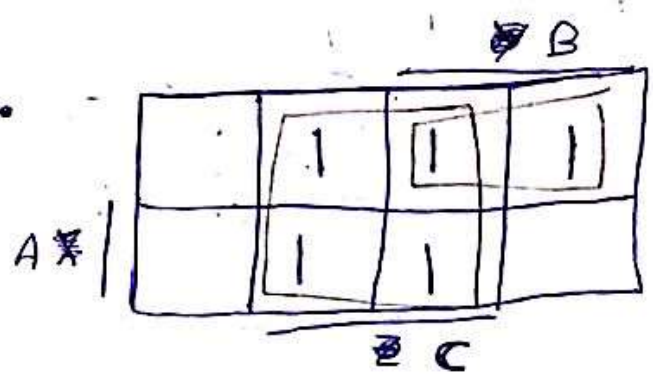
$$f = xz' + zy$$

Example:- $F = A'C + A'B + AB'C + BC$

- 1] sum of products
- 2] product of sums

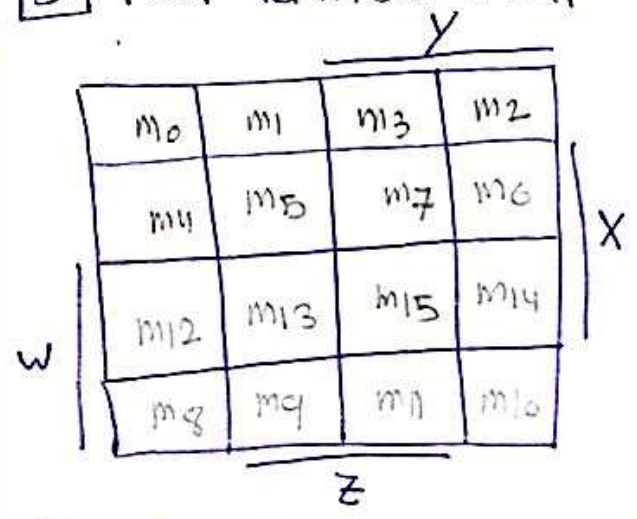
1] $F = \sum(1, 2, 3, 5, 7)$

2] $F' = \sum(0, 4, 6)$



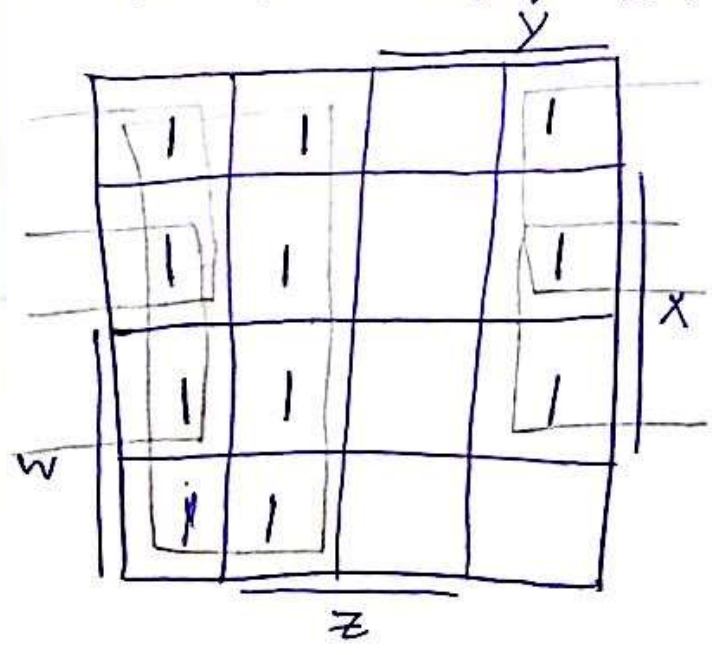
3] Four variable Kmap $\Rightarrow 2^4 = 16$

wxyz



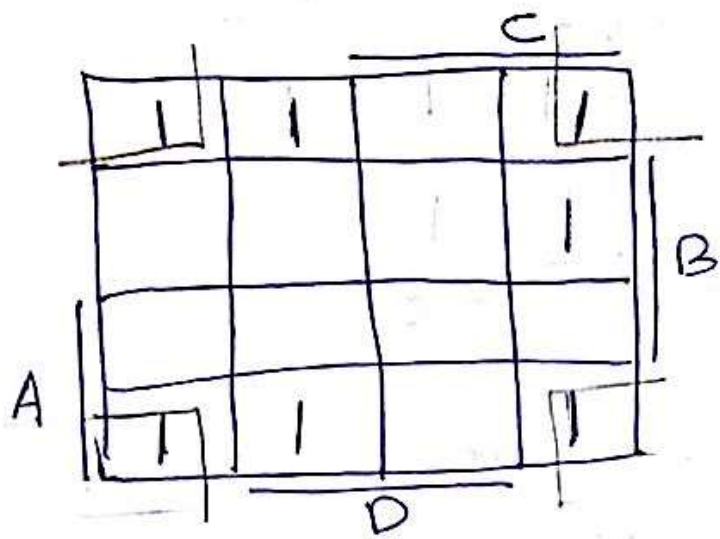
Example:- $F = (w, x, y, z) = \sum(0, 1, 2, 4, 5, 6, 8, 9, 12, 13, 14)$

$F = y' + w'z' + xz'$



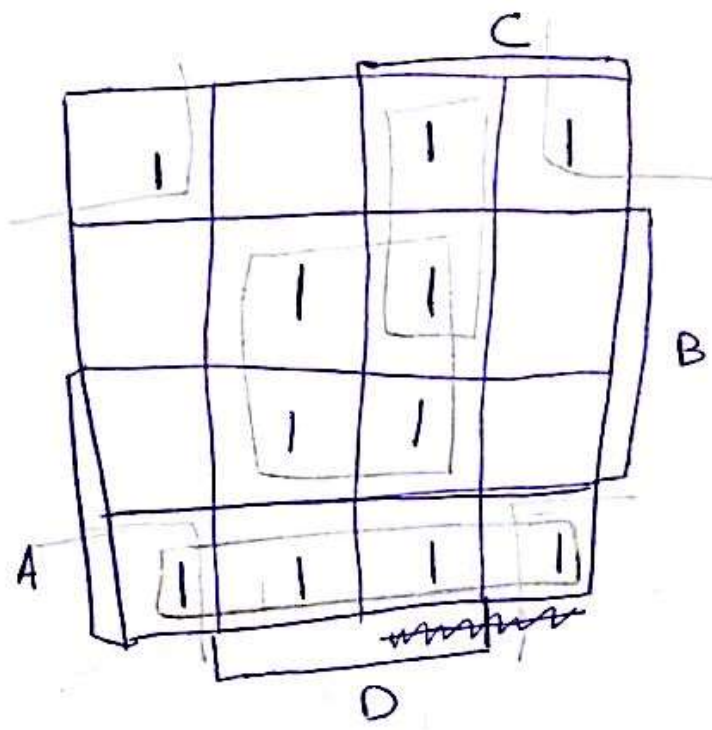
Example:- $F = A'B'C' + B'CD' + A'BCB' + AB'C'$ (28)

$$F = A'CD' + B'C' + B'D$$



Example:-

$$F(A, B, C, D) = \sum (0, 2, 3, 5, 7, 8, 9, 10, 11, 13, 15)$$



$$F = B'D' + AB' + A'CD + BD$$

Example:- minimise the following Function using K-map.

29

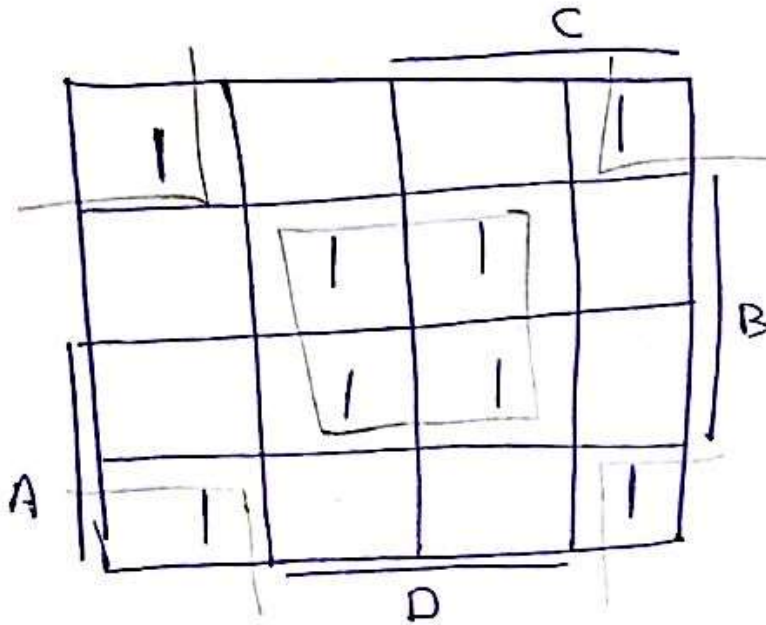
$$F(A, B, C, D) = A^3 B^3 C^3 D^3 + A^1 B^3 D^3 + A^1 B^3 C^0 D^1 + A^1 B^0 D + A^1 B^0 D^0$$

$$A^3 B^3 C^3 D^3 + A^1 B^3 C^3 D^3 + A^1 B^3 C^0 D^1 + A^1 B^3 C^0 D^0 + A^1 B^0 C^0 D^1 + A^1 B^0 C^0 D^0 + A^1 B^0 C^1 D^1 + A^1 B^0 C^1 D^0$$

$$1000 + 0000 + 0010 + 1010 + 1111 + 1101 + 0110 + 0100$$

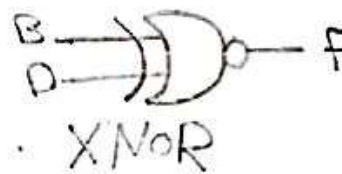
$$8 + 0 + 2 + 10 + 15 + 13 + 7 + 5$$

$$= \sum(0, 2, 5, 7, 8, 10, 13, 15)$$



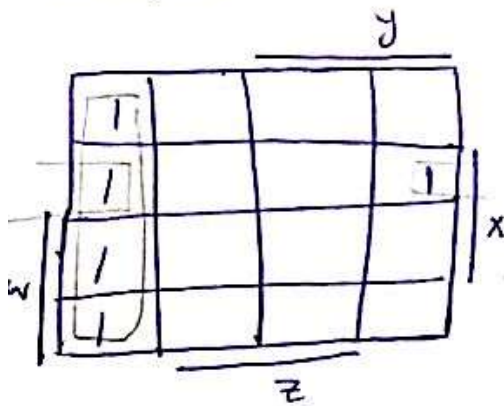
$$F = B^1 D^1 + B D$$

$$F = (B \oplus D)^1$$



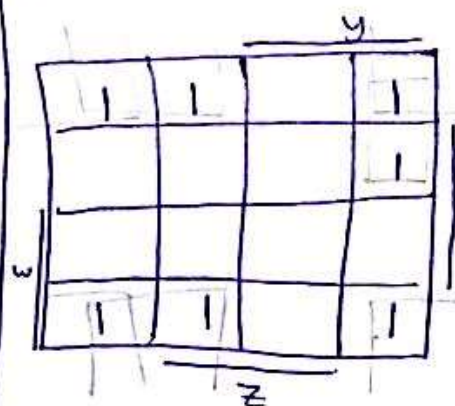
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001
10	1010
11	1011
12	1100
13	1101
14	1110
15	1111

Example:-



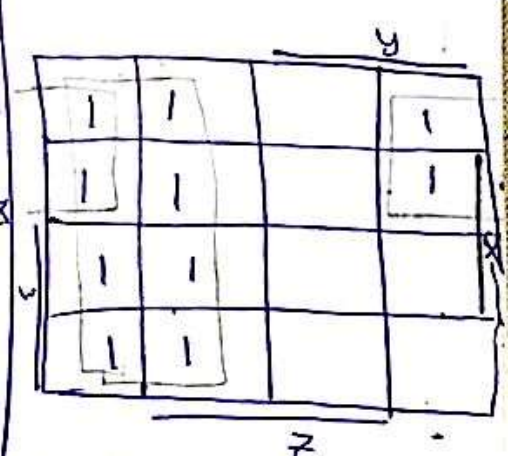
$$F = y^1 z^1 + w^1 x^1 z^1$$

Example:-



$$F = x^1 z^1 + w^1 y^1 z^1 + x^1 y^1$$

Example:-



$$F = w^1 z^1 + y^1$$

Max term:

min term class

نصف دوائر الـ 0 في خريطة الـ 0 map

Example:-

		C		
	1	1	0	1
	1	1	0	1
A	1	1	1	1
	1	1	1	1
	D			

$$F = \sum (0, 2, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15)$$

min term.

$$F = \sum (3, 7) \text{ max term.}$$

$$F = \prod (3, 7)$$

$$F = (A + C + D)$$

$$F = (A + C + D)$$

Example:-

		C		
			0	
			0	
A		0	0	
		0	0	
	D			

$$F = (A + D) \cdot (C + D)$$

$$F = (A + D) \cdot (C + D)$$

$$F = \prod (6, 7, 13, 15)$$

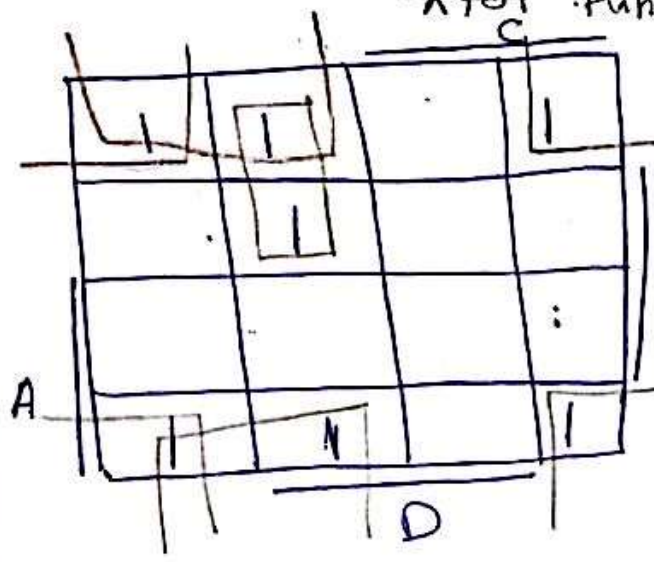
$$F = (A + B + D) \cdot (A + B + C)$$

		C		
		0	0	
A		0	0	
	D			

$$F = (A + B + D) \cdot (A + B + C)$$

Example:- Find \sum sum of products \cong products of sum.

* For function $F(A, B, C, D) = (0, 1, 2, 5, 8, 9, 10)$



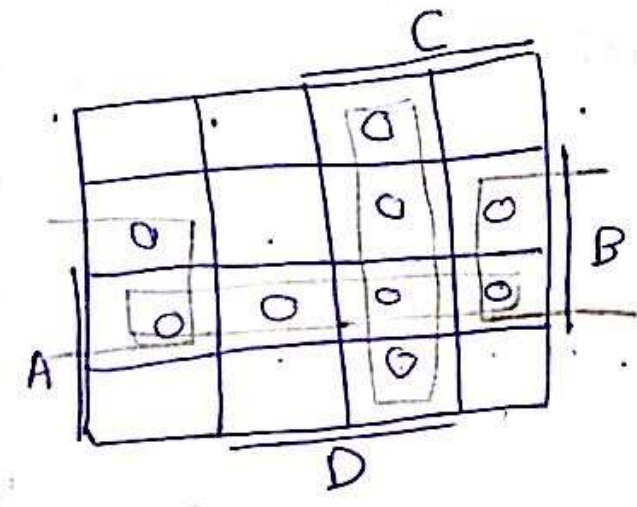
\sum sum of product : minterm.

$F = A'c'D + B'c' + B'D'$

\cong product of sum : Max term.

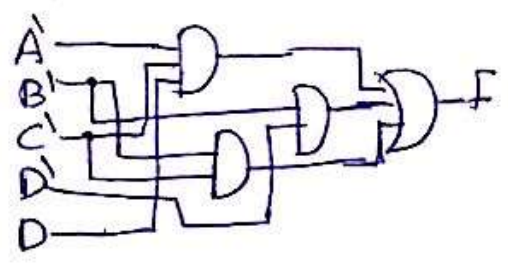
$F = (B+D) \cdot (C+D) \cdot (A+B)$

$F = (A'+B') \cdot (C'+D) \cdot (B'+D)$

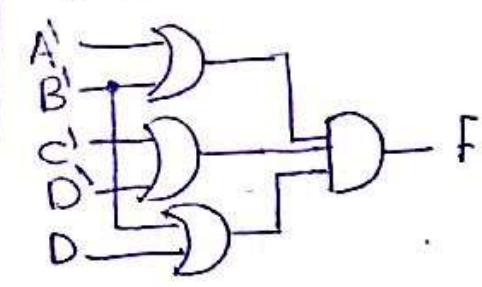


S of P

minterm



P of S max term.



Example:- $F = (A'+B'+C')(B+D)$

Find $F' = (A'+B'+C')(B+D)$

$= (A'+B'+C') + (B+D)$

$= (A''B''C'') + (B'D)$

$= (A \cdot B \cdot C) + (B' \cdot D')$

5 Five Variable - K-map

$2^5 = 32$

A (32) (32)

	D			
	0	1	3	2
	4	5	7	6
A	12	13	15	14
	8	9	11	10
	E			

	D			
	16	17	19	18
	20	21	23	22
A	28	29	31	30
	24	25	27	26
	E			

Example:-

	D			
			1	
		1		
B				
	E			

	D			
			1	
B				
	E			

$F = A' C D E + B' C D E$

Example:-

	D			
	1			1
B				
	1			1
	E			

	D			
	1			
B				
	1			
	E			

$F = C' D' E' + A' C' E'$

Example:- max term.

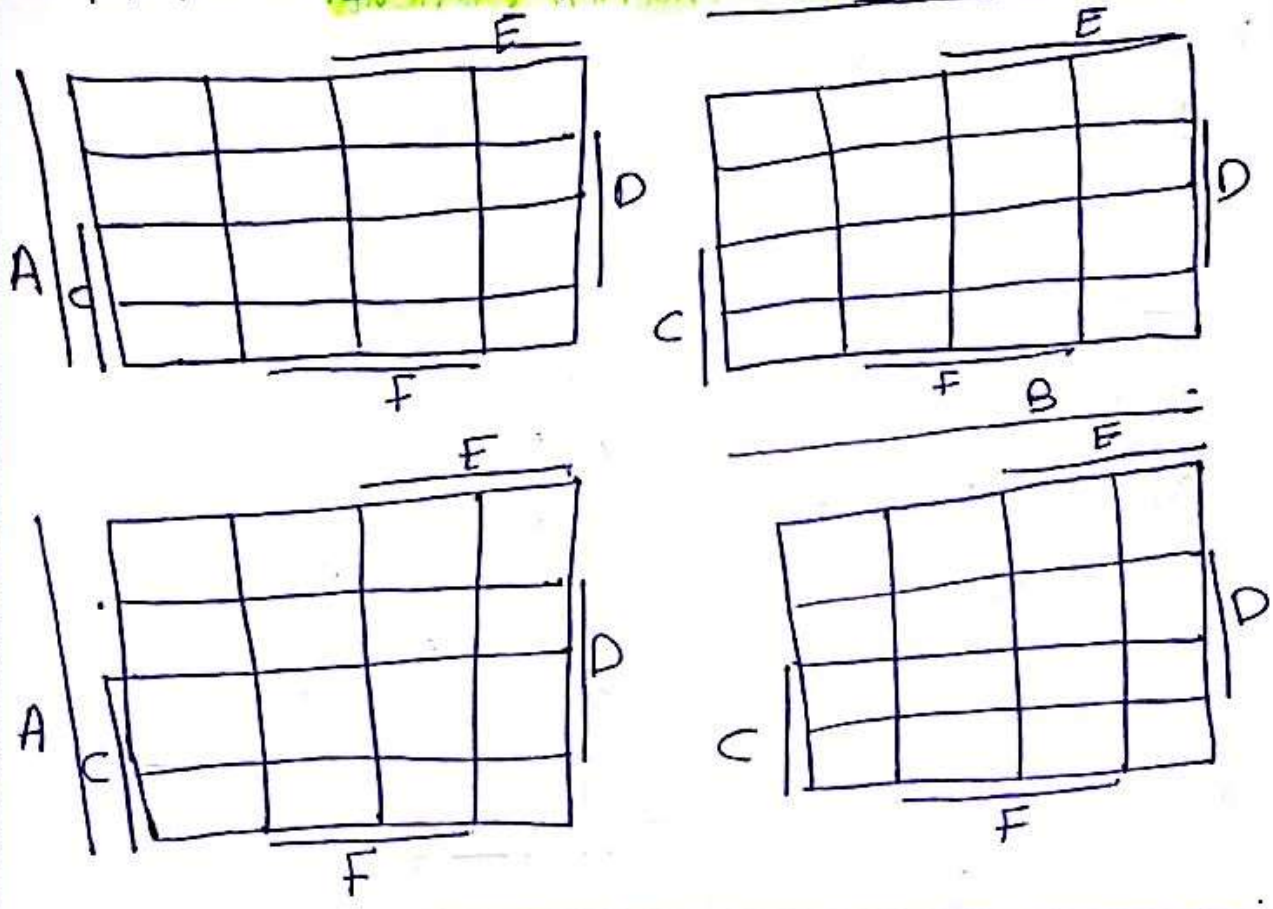
	D			
	0	0		
	0	0		
B	0	0		
	0	0		
	E			

	D			
	0	0		
B				
	0	0		
	E			

$F = (B' + C + E) \cdot (B + C' + E) \cdot (A' + E)$

$F = (B + C' + E') \cdot (B' + C + E) \cdot (A + E')$

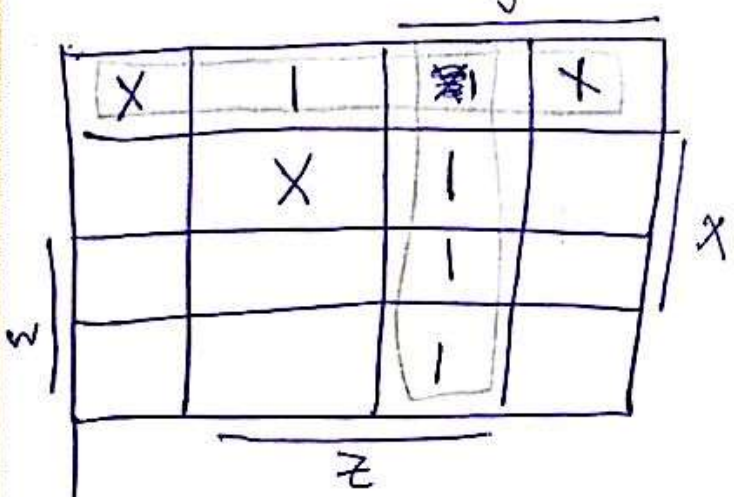
5 Six variable K-map $2^6 = 64$ B



Don't-Care

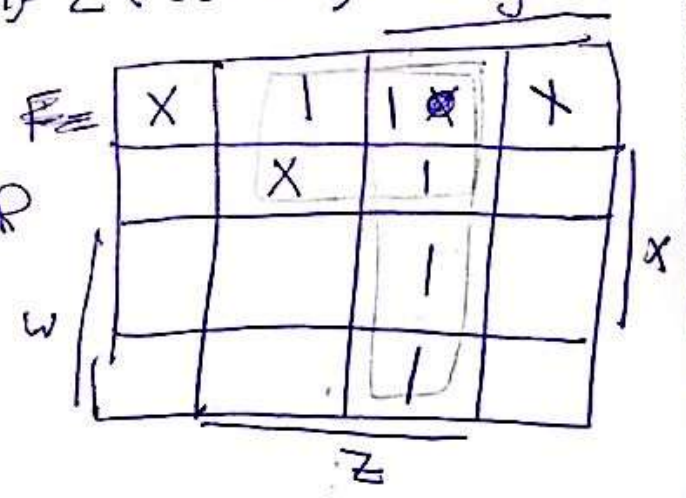
$$F(w, x, y, z) = \sum (1, 3, 7, 11, 15)$$

$$\text{Don't-Care } (w, x, y, z) = \sum (0, 2, 5)$$



$$F = w'x' + yz$$

$$F = (0, 1, 2, 3, 7, 11, 15)$$



$$F = w'z + yz$$

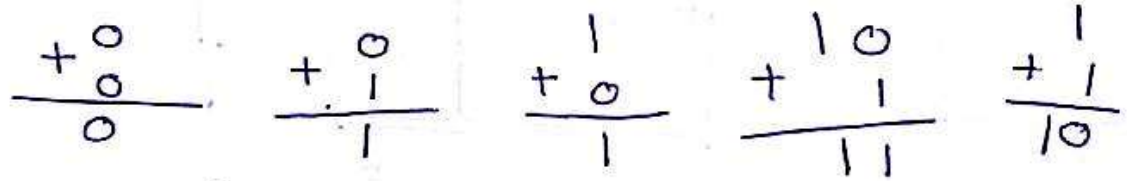
$$F = z(w' + y)$$

$$F = (1, 3, 5, 7, 11, 15)$$

Combination Circuit.

Combinational \rightarrow without memory.

Sequential \rightarrow with memory.



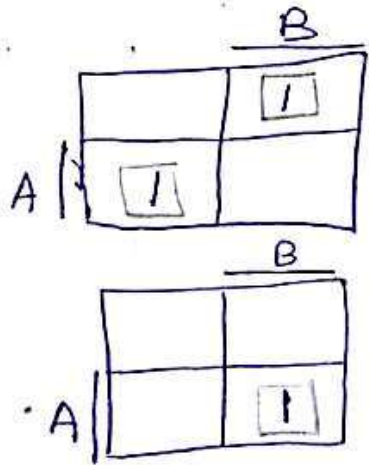
Type Combinational:

- 1] Half Adder [H.A]
- 2] Full Adder [F.A]

1] Half Adder

2 input
2 output

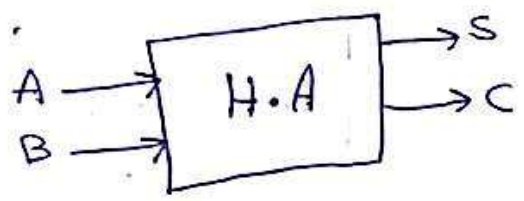
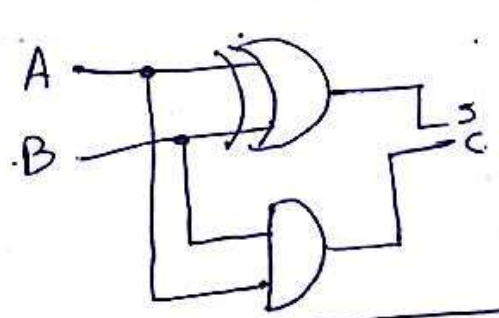
A	B	S	C
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1



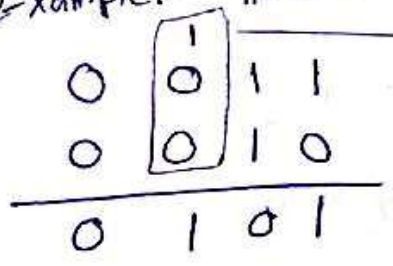
$$S = AB' + BA'$$

$$S = A \oplus B \text{ XOR}$$

$$C = AB$$



Example: In 4bit system



3 input \Rightarrow not Half Adder use Full Adder.

2 Full Adder

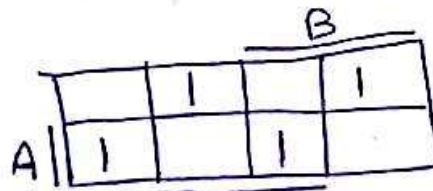
3 input
2 output.

35

es li gate dhas
output di se

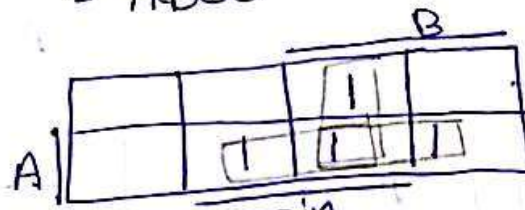


A	B	Cin	S	Cont
0	0	0	0	0
0	0	1	1	0
0	1	0	1	0
0	1	1	0	1
1	0	0	1	0
1	0	1	0	1
1	1	0	0	1
1	1	1	1	1

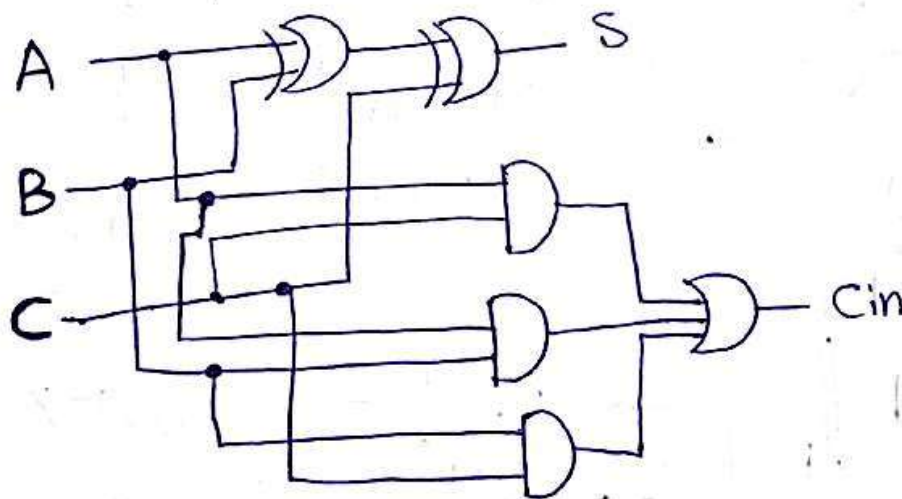


$$S = AB'C + A'B'C + ABC + A'BC$$

$$= A \oplus B \oplus C$$

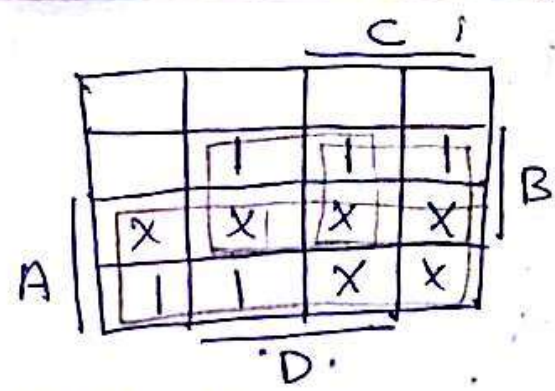


$$Cont = AC + AB + BC$$

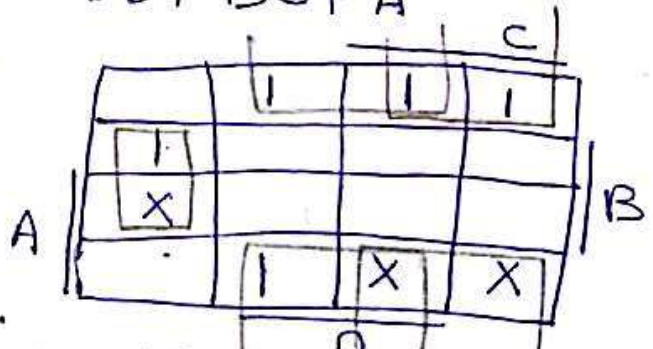


Exers 3 Code i-

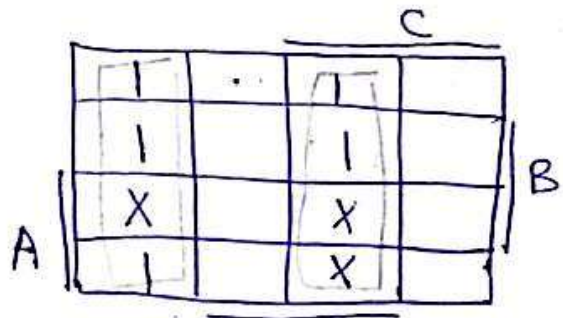
A	B	C	D	w	x	y	z
0	0	0	0	0	0	1	1
0	0	0	1	0	1	0	0
0	0	1	0	0	1	0	1
0	0	1	1	0	1	1	0
0	1	0	0	0	1	1	1
0	1	0	1	1	0	0	0
0	1	1	0	1	0	0	1
0	1	1	1	1	0	1	0
1	0	0	0	1	0	1	1
1	0	0	1	1	0	0	0
1	0	1	0	X	X	X	X
1	0	1	1	X	X	X	X
1	1	0	0	X	X	X	X
1	1	0	1	X	X	X	X
1	1	1	0	X	X	X	X
1	1	1	1	X	X	X	X



$W = BD + BC + A$

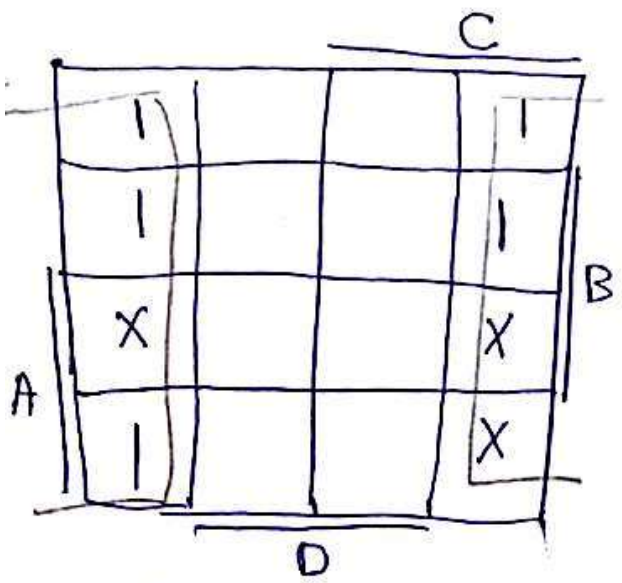


$X = BC'D + B'D + B'C$

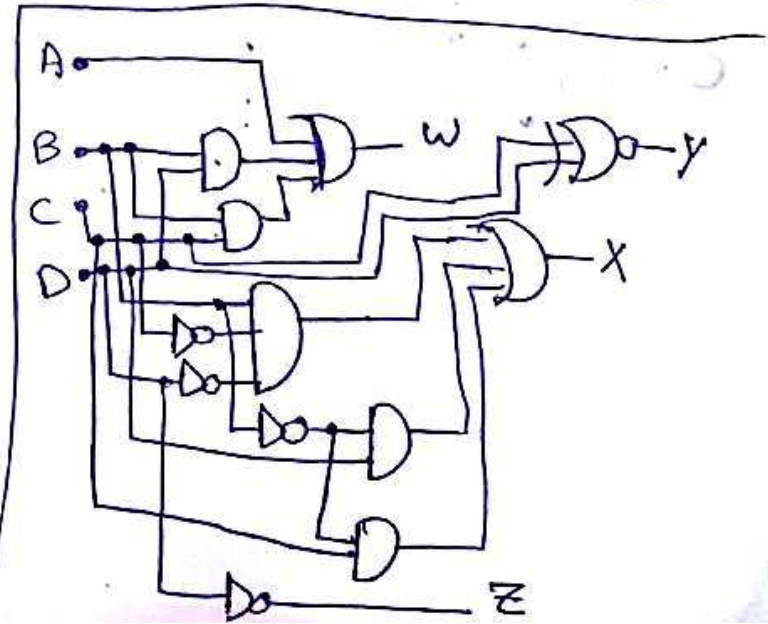


$Y = C'D + CD$

$Y = (C \oplus D)' = C \odot D$

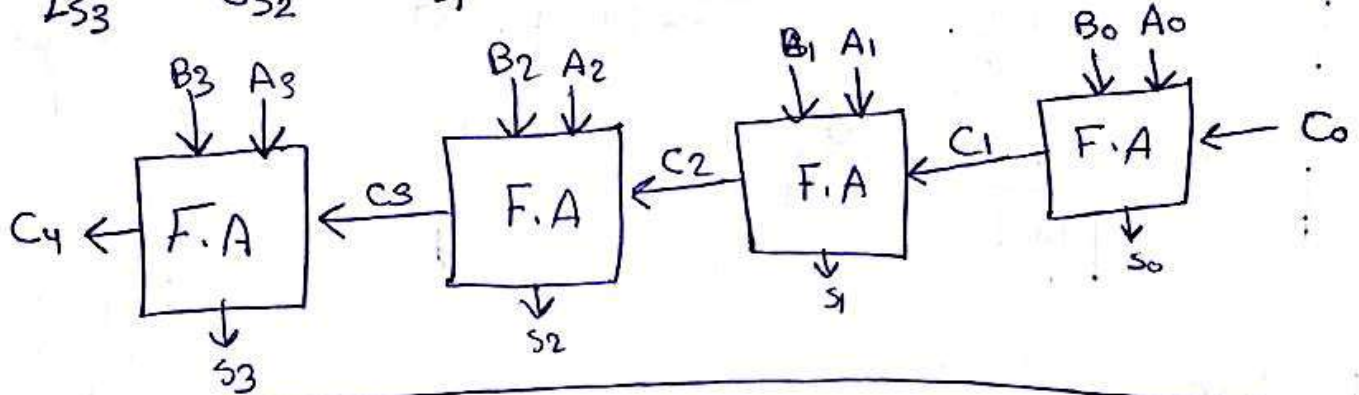
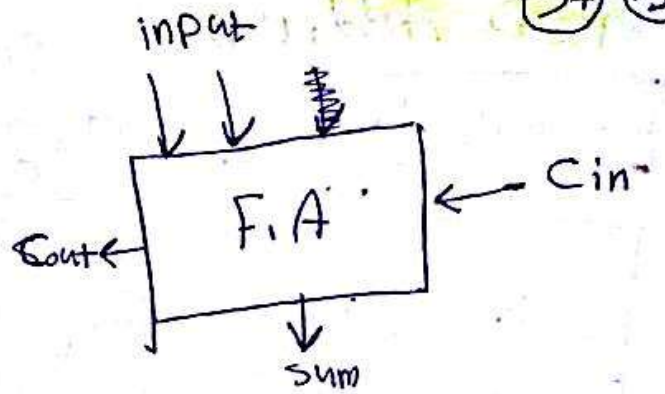
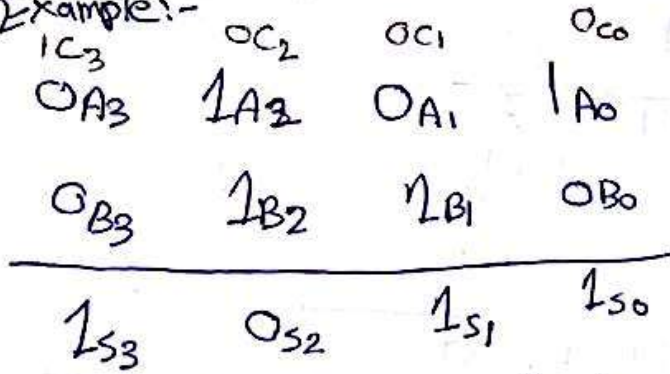


$Z = D'$



Binary Adder

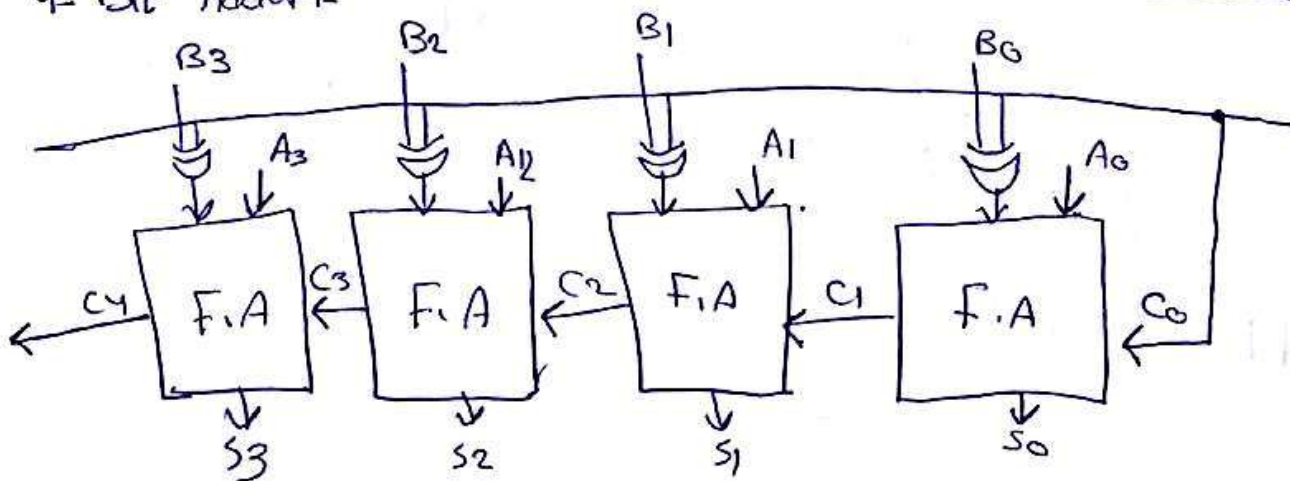
Example:-



A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

- $B \oplus 0 = C_0 B = 0$
- $B \oplus 1 = C_0 B' = 1$

4-Bit Adder:-



if $M=0 \Rightarrow$ 4 bit adder \oplus

if $M=1 \Rightarrow A + \underbrace{B+1}_{2's\ complement} \Rightarrow$ ~~4 bit adder~~ 4 bit subtraction \ominus

BCD Addition :-

Decimal	BCD
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111
8	1000
9	1001

Example:-

$$\begin{array}{r} \text{[1]} \quad + 0011 \quad 3 \\ \quad \quad 0100 \quad 4 \\ \hline \quad \quad 0111 \quad 7 \end{array}$$

$$\begin{array}{r} \text{[2]} \quad + 0010 \quad 2 \\ \quad \quad 0001 \quad 1 \\ \hline \quad \quad 0011 \quad 3 \end{array}$$

$$\begin{array}{r} \text{[3]} \quad + 1000 \quad 8 \\ \quad \quad 0001 \quad 1 \\ \hline \quad \quad 1001 \quad 9 \end{array}$$

اقلة فن 9

$$\begin{array}{r} 2 \quad 3 \\ 1 \quad 5 \\ \hline \end{array}$$

$$\boxed{3 \quad 8}$$

اقلة فن 9

$$\begin{array}{r} 8 \quad 6 \\ + 1 \quad 3 \\ \hline \end{array}$$

$$\boxed{9 \quad 9}$$

يا و 9
عادي خصل
كليك

$$\begin{array}{r} \text{[4]} \quad + 0100 \quad 4 \\ \quad \quad 0101 \quad 5 \\ \hline \quad \quad 1000 \quad 9 \end{array}$$

$$\begin{array}{r} 4 \quad 5 \quad 0 \\ 4 \quad 1 \quad 7 \\ \hline \end{array}$$

$$\boxed{8 \quad 6 \quad 7}$$

اقلة فن 9
عادي

Example:-

$$\begin{array}{r} + 0101 \quad 5 \\ \quad \quad 1001 \quad 9 \\ \hline \quad \quad 1010 \quad 10 \end{array}$$

$$\begin{array}{r} + 5 \quad 9 \quad 8 \\ \quad \quad 1 \quad 8 \quad 3 \\ \hline \quad \quad 7 \quad 8 \quad 1 \end{array}$$

اكبر فن 9
بتر 6

$$\begin{array}{r} 0010 \quad 2 \\ 0110 \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} 0111 \quad 7 \\ 1000 \quad 8 \\ 0001 \quad 1 \end{array}$$

Comparator

32456 . 701

32457 . 701 → أكبر

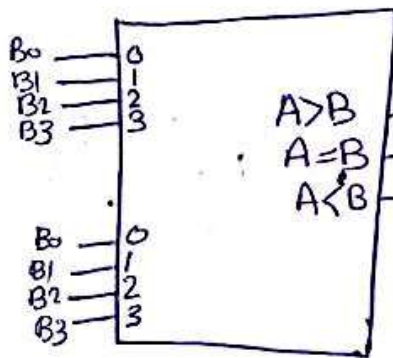
For Decimal

11011 . 1101

11001 . 1111 → أكبر

For Binary

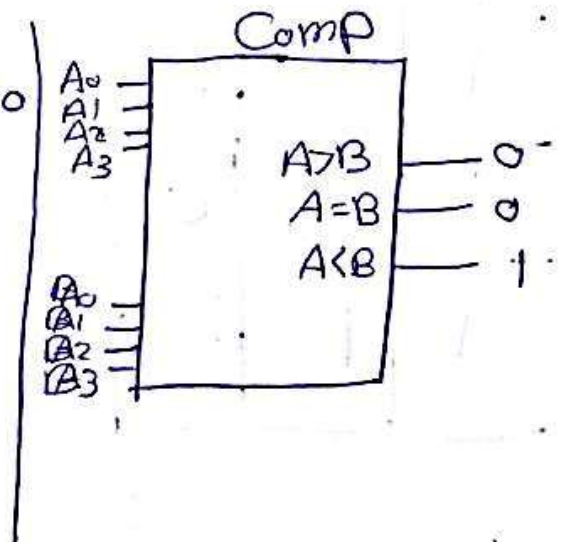
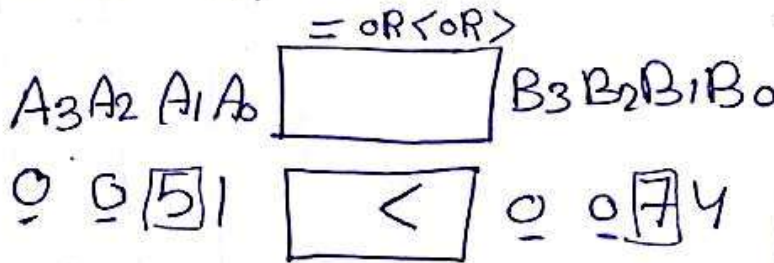
Comparator 4-Bit



نتيجة المقارنة
إذا تساوى شرط لخط ختمة
إذا لم يأتى الشرط
خطو zero

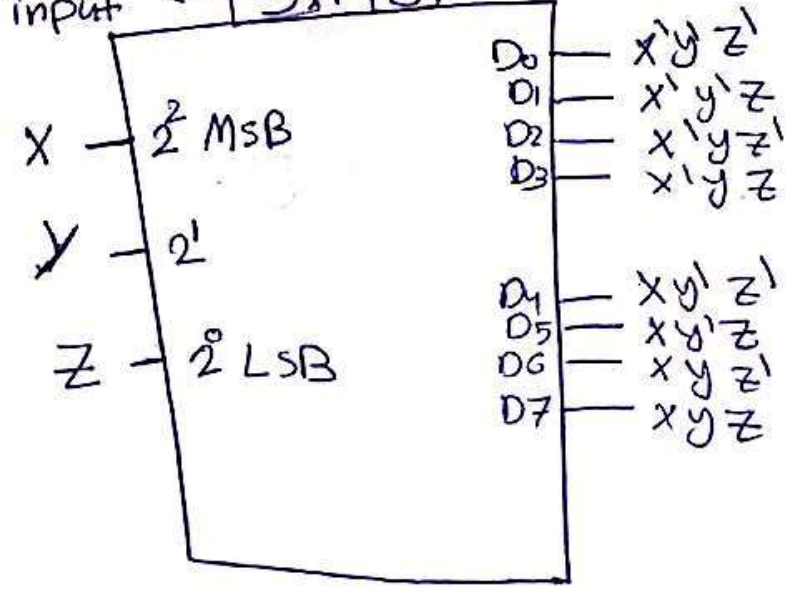
Example :-

$A_0=1$, $A_1=5$, $B_0=4$, $B_1=7$



دائرة التشفير de Coder

Num of input ← **3x8** → Num of output.



X	Y	Z	D0	D1	D2	D3	D4	D5	D6	D7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

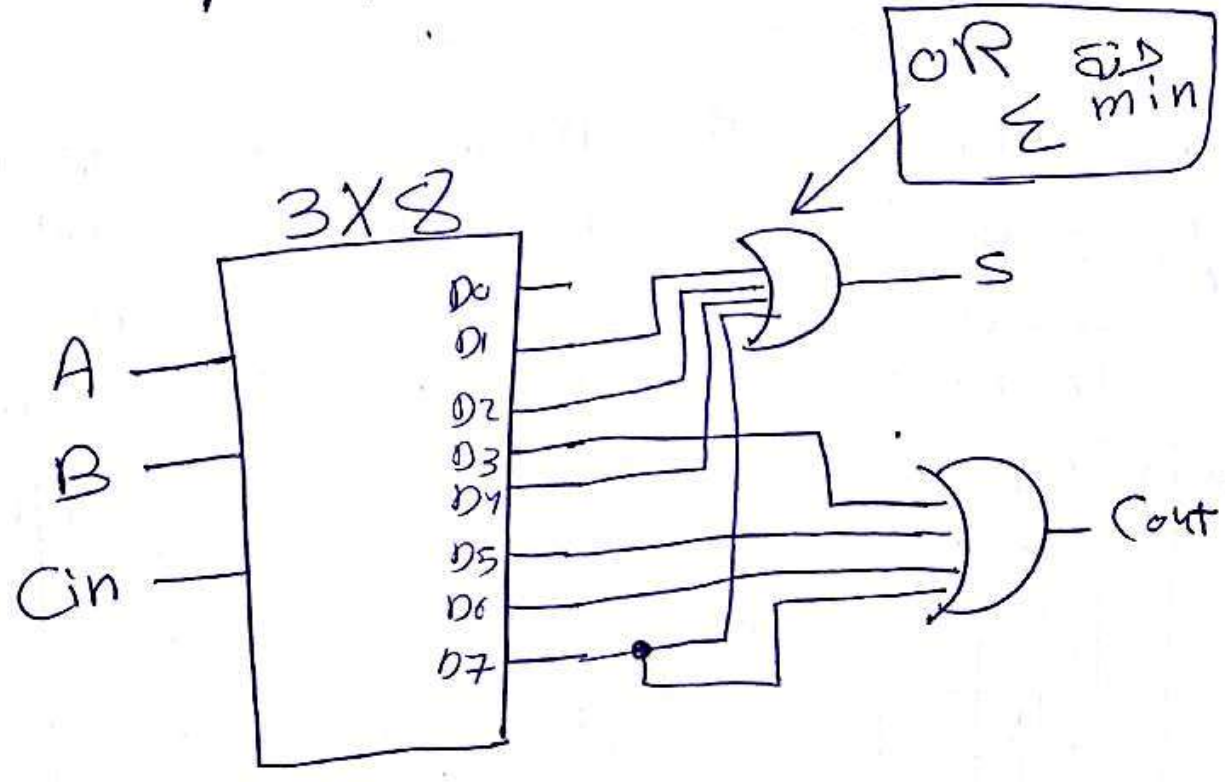
Example:- Implement a full adder using a decoder. 41

A	B	Cin	S	Cont
0	0	0	0	0 D ₀
0	0	1	1	0 D ₁
0	1	0	1	0 D ₂
0	1	1	0	1 D ₃
1	0	0	1	0 D ₄
1	0	1	0	1 D ₅
1	1	0	0	1 D ₆
1	1	1	1	1 D ₇

$$S = \Sigma(1, 2, 4, 7)$$

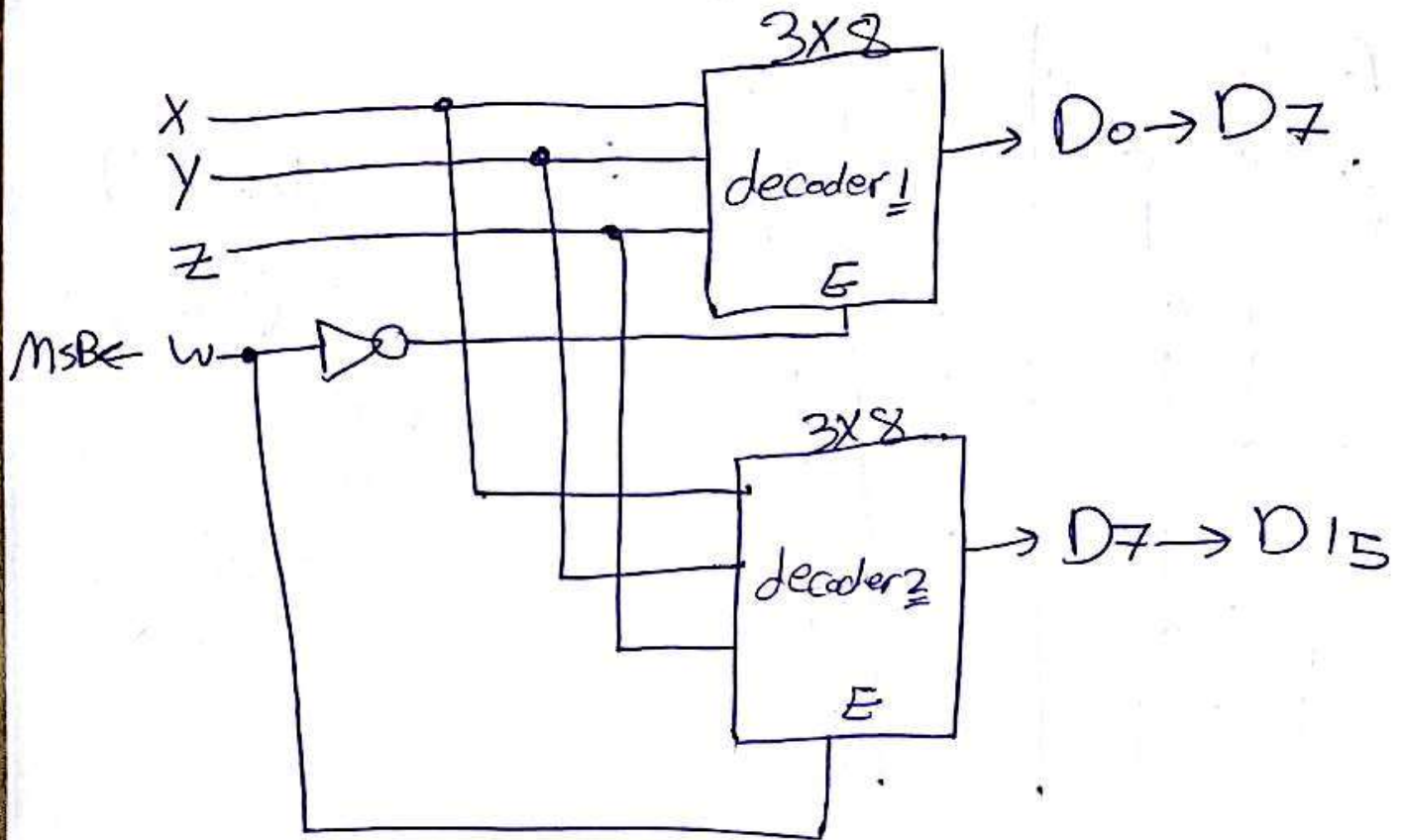
$$Cont = \Sigma(3, 5, 6, 7)$$

input $\Rightarrow 3$
 output = 2^{input}
 $= 2^3 = 8$
 $D_0 \rightarrow D_7$



Note:- 4 input 16 output use 2 decoder.

Example:-



E is Active High that means decoder work when $w=1$
 E is Active Low that means decoder work when $w=0$
 $w=0 \rightarrow$ decoder 1 work dec 2 not work
 $w=1 \rightarrow$ dec 1 not work dec 2 work.

w	X	Y	Z	D0	D1	D2	D3	D4	D5	D6	D7	D8	D9	D10	D11	D12	D13	D14	D15
0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0
0	0	1	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
0	1	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0
0	1	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0
0	1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
0	1	1	1	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0
1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0
1	0	0	1	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
1	0	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0
1	0	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0
1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1	1	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1
1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1

الترتيب

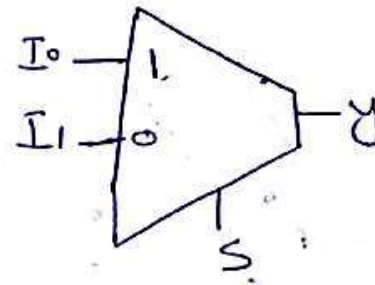
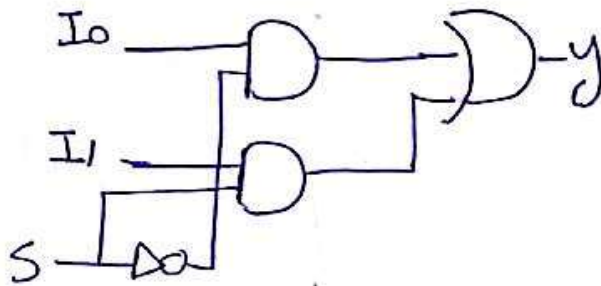
Multiplexer [MUX]

43

More than Input \rightarrow كير +
one output \rightarrow باء فقط

Type of Mux:-

1) 2 to 1 Line Multiplexer.

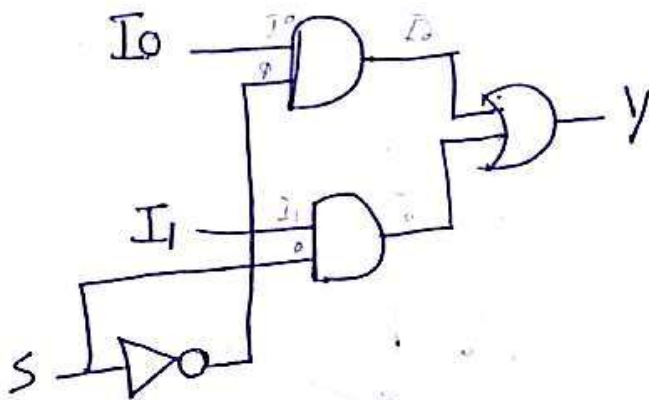


$I_0, I_1 \rightarrow$ Data

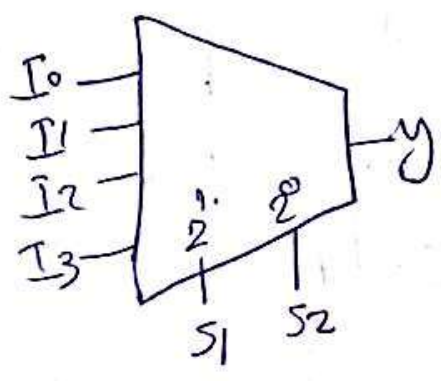
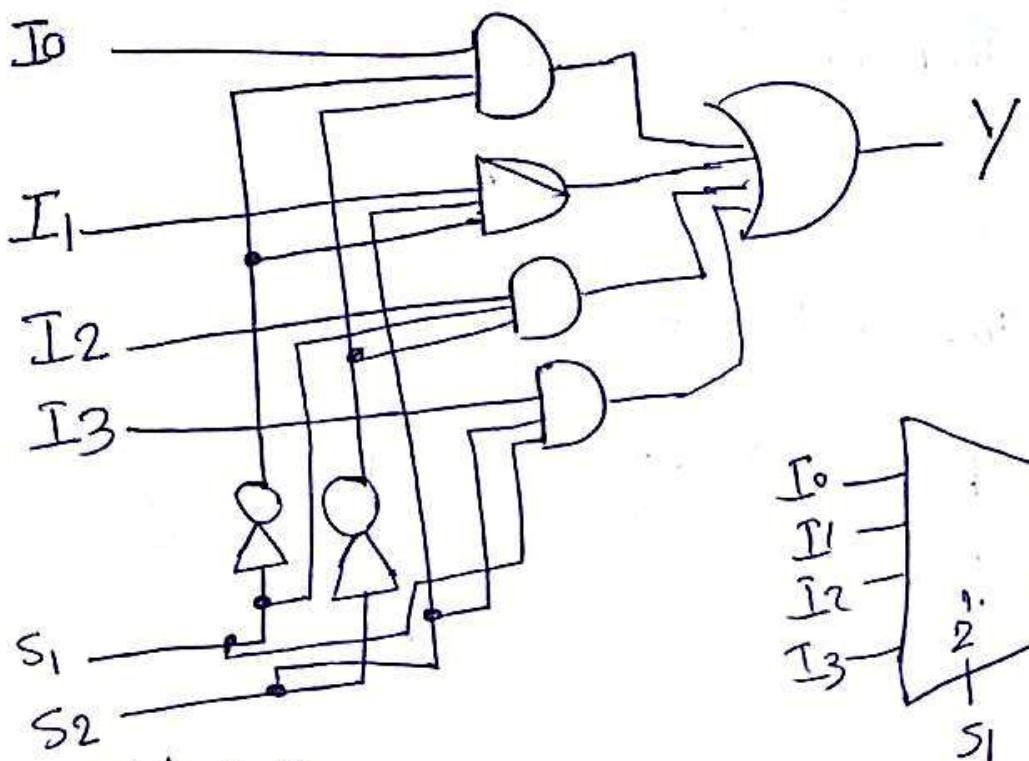
$S \rightarrow$ Control Bit

$X \rightarrow$ Address

S	
0	I_0
1	I_1



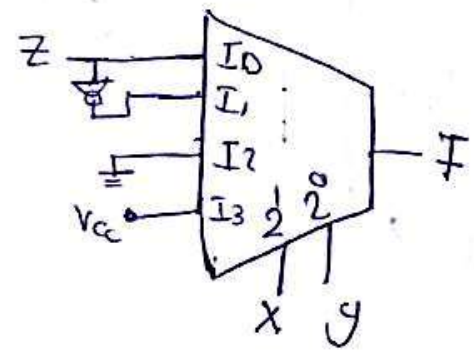
2) 4 to 1 → Line Mux



S1	S0	Y
0	0	I ₀
0	1	I ₁
1	0	I ₂
1	1	I ₃

	X	Y	Z	F
I ₀	0	0	0	0
	0	0	1	1
I ₁	0	1	0	1
	0	1	1	0
I ₂	1	0	0	0
	1	0	1	0
I ₃	1	1	0	1
	1	1	1	1

$F = Z$
 $F = Z'$
 $F = \text{zero}$
 $F = 1$



Example:- Implement function using max?

MS

$$F(x,y,z) = x'y'z + xy'z + z'$$

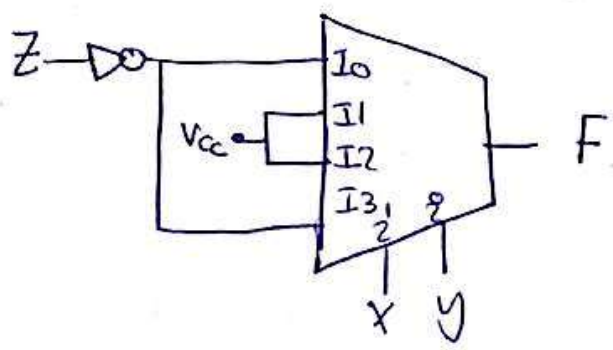
$$= x'y'z + xy'z + x'y'z + xy'z + x'y'z' + x'y'z'$$

$$= 011 + 101 + 110 + 100 + 000 + 010$$

	x	y	z	F
I ₀	0	0	0	0
	0	0	1	0
I ₁	0	1	0	1
	0	1	1	1
I ₂	1	0	0	1
	1	0	1	1
I ₃	1	1	0	1
	1	1	1	0

$$F = \sum (1, 2, 3, 4, 5, 6)$$

$$F = \prod (1, 7)$$



Minterm

- / → 0
- use → 1
- truth table =

Maxterm

- / → 1
- use → 0
- truth table =

Example:- Implement the following function using 46
MUX?

$$\boxed{1} F_1(A, B, C, D) = (A+B+C) \cdot (A+B) \cdot (A'+B'+D')$$

$$\boxed{2} F_2(A, B, C, D) = (A+B+D) \cdot (A+C)$$

Solution:-

$$\boxed{1} F_1(A, B, C, D) = (A+B+C) \cdot (A+D) \cdot (A'+B'+D')$$

$$(A+B+C+D)(A+B+C+D')(A+B+C+D)(A+B+C+D)(A+B'+C+D')(A+B'+C+D)$$

$$(A'+B'+C+D)(A'+B'+C'+D)$$

$$F_1 = \prod (1, 2, 3, 5, 7, 13, 15)$$

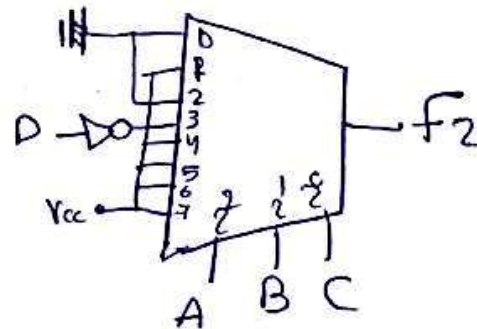
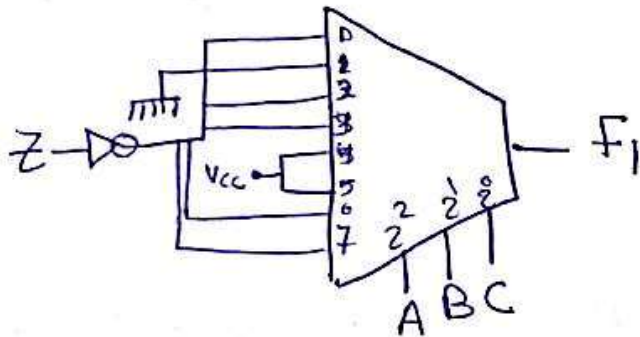
$$F_1 = \sum (0, 4, 6, 8, 9, 10, 11, 12, 14)$$

$$F_2 = (A+B+D) \cdot (A+C) \Rightarrow (A+B+C+D)(A+B'+C+D)(A+B+C+D)$$

$$F_2 = \prod (0, 1, 4, 5, 7)$$

$$F_2 = \sum (2, 3, 6, 8, 9, 10, 11, 12, 13, 14, 15)$$

A	B	C	D	F ₁	F ₂
0	0	0	0	0	0
0	0	0	1	0	0
0	0	1	0	0	0
0	0	1	1	0	0
0	1	0	0	0	0
0	1	0	1	0	0
0	1	1	0	0	0
0	1	1	1	0	0
1	0	0	0	0	0
1	0	0	1	0	0
1	0	1	0	0	0
1	0	1	1	0	0
1	1	0	0	0	0
1	1	0	1	0	0
1	1	1	0	0	0
1	1	1	1	0	0



~~Binary Logic~~

SEQUENTIAL CIRCUITS

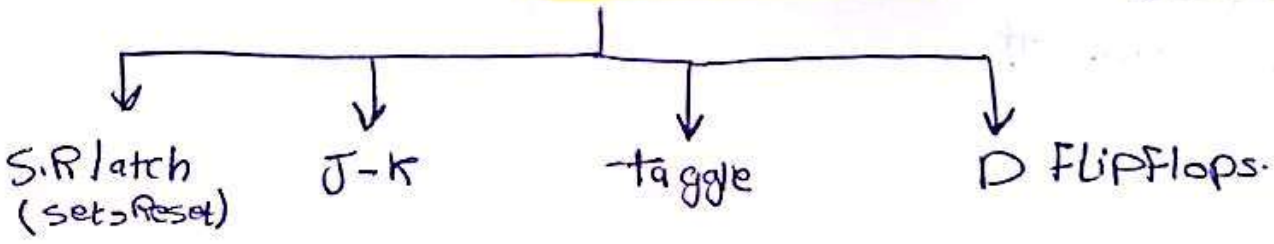
* Logic cct →

- ① Combinational cct : No memory. ← کل شیئہ مفید کان ...
- ② sequential cct : with memory. ← کل شیئہ بفر

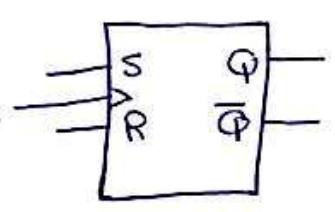
Note
 bit = 0 or 1
 byte = 8 bit
 word = 16 bit
 double word = 32 bit

*note.
 Set ● to 1 High
 Rest * to 0 Low

FLIP FLOPS

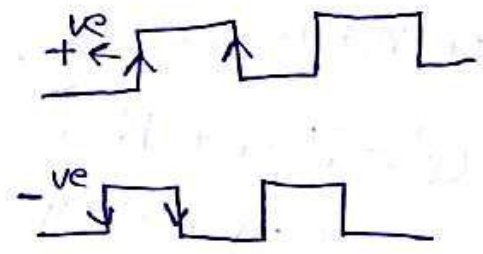
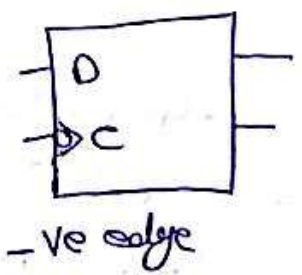
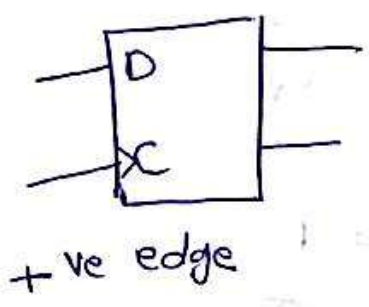


①

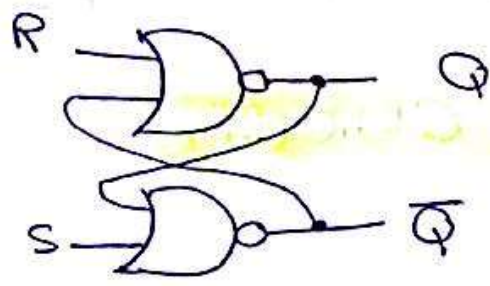


← output
 ← Complement output.

2 input
 2 output.



Negative clock. Positive clock



truth table

S	R	Q ⁺	next state (output)
0	0	Q	No change stable
0	1	0	Reset state
1	0	1	Set state
1	1	not valid	Δ or 0 I dont know??

Q(t) → present state

Q(t+1) next state.

No characteristic eq because 1 equal not valid

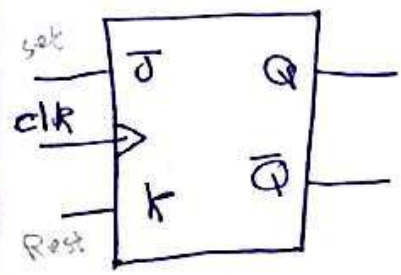
② J-K- Flibe Flop → Reset و set في valid ال صيغة ال

Latch → with out CLK

Flip Flop → with CLK

input اي تغيرات يعطيني تغيرات output (حاس)

يكون عندي تغيرات 0 و 1



truth table.

J	K	Q ⁺
0	0	Q
0	1	0
1	0	1
1	1	Q-bar → toggle (complement Q)

characteristic equation →

$$Q^+ = J\bar{Q} + KQ$$

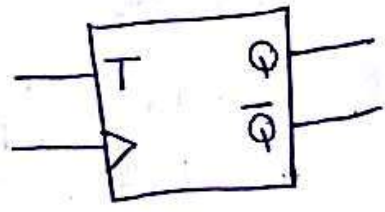
$$0\bar{Q} + 0Q \rightarrow 0\bar{Q} + 1Q = Q$$

$$0\bar{Q} + 1Q \rightarrow 0\bar{Q} + 0Q = 0$$

$$1\bar{Q} + 0Q \rightarrow 1\bar{Q} + 1Q = 1$$

$$1\bar{Q} + 1Q \rightarrow 1\bar{Q} + 0Q = \bar{Q}$$

③ Toggle Flip Flop :
(one terminal)



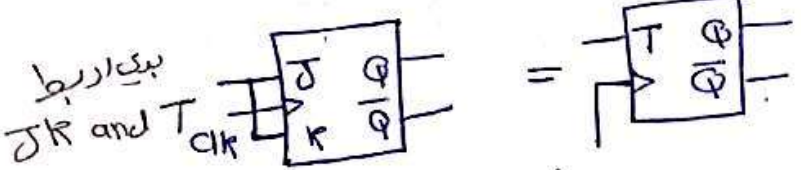
T	Q ⁺
0	Q
1	\bar{Q}

Characteristic equation :

$$Q^+ = T \cdot \bar{Q} + \bar{T} \cdot Q \Rightarrow \boxed{T \oplus Q = Q^+}$$

$$0 \cdot \bar{Q} + 1 \cdot Q = Q$$

$$1 \cdot \bar{Q} + 0 \cdot Q = \bar{Q}$$

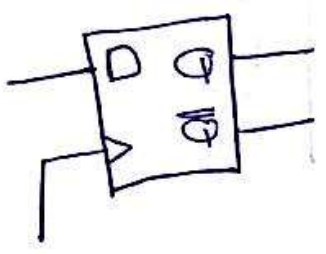


Characteristic equation in truth table

J	K	Q ⁺
0	0	Q
1	0	1
0	1	0
1	1	\bar{Q}

note:
بعض حالات میں
JK و T کی نفس
بعض لئے کسی بالورق
نفس ال input.

④ Data Flip Flop:-



truth table

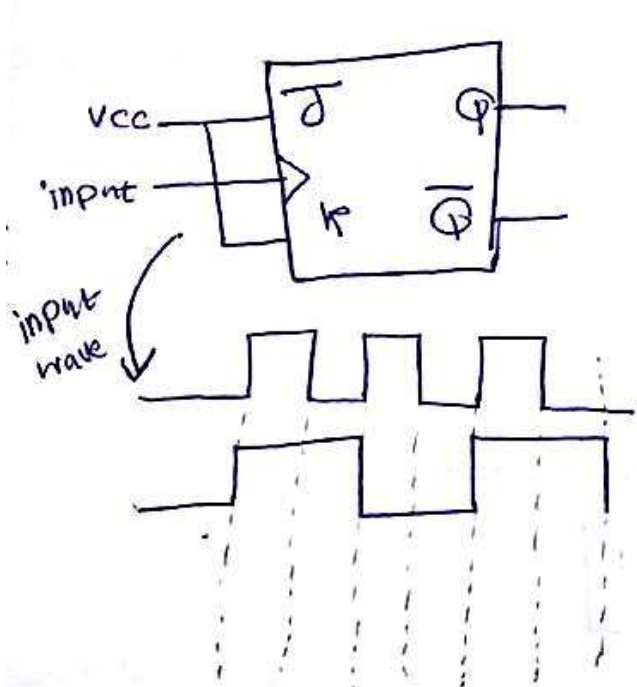
D	Q ⁺
0	0
1	1

Characteristic equation =

$$\boxed{D = Q^+}$$

F/F	Symbol	truth table	Characteristic equation.															
R.S		<table border="1"> <thead> <tr> <th>S</th> <th>R</th> <th>Q⁺</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>Q</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>not valid</td> </tr> </tbody> </table>	S	R	Q ⁺	0	0	Q	0	1	0	1	0	1	1	1	not valid	<p>Not eq</p> <p>Not eq</p> <p>because 11 → not valid.</p>
S	R	Q ⁺																
0	0	Q																
0	1	0																
1	0	1																
1	1	not valid																
J.K		<table border="1"> <thead> <tr> <th>J</th> <th>K</th> <th>Q⁺</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>Q</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> </tr> <tr> <td>1</td> <td>1</td> <td>Q-bar</td> </tr> </tbody> </table>	J	K	Q ⁺	0	0	Q	0	1	0	1	0	1	1	1	Q-bar	$Q^+ = J\bar{Q} + \bar{K}Q$
J	K	Q ⁺																
0	0	Q																
0	1	0																
1	0	1																
1	1	Q-bar																
T Toggle		<table border="1"> <thead> <tr> <th>T</th> <th>Q⁺</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>Q (No change)</td> </tr> <tr> <td>1</td> <td>Q-bar (Toggle)</td> </tr> </tbody> </table>	T	Q ⁺	0	Q (No change)	1	Q-bar (Toggle)	$Q^+ = T \oplus Q$ <p>1 و 0 و 0 و 1</p>									
T	Q ⁺																	
0	Q (No change)																	
1	Q-bar (Toggle)																	
Data D		<table border="1"> <thead> <tr> <th>D</th> <th>Q⁺</th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> </tr> <tr> <td>1</td> <td>1</td> </tr> </tbody> </table>	D	Q ⁺	0	0	1	1	$D = Q^+$ <p>Input = output</p>									
D	Q ⁺																	
0	0																	
1	1																	

* Analoge and design.



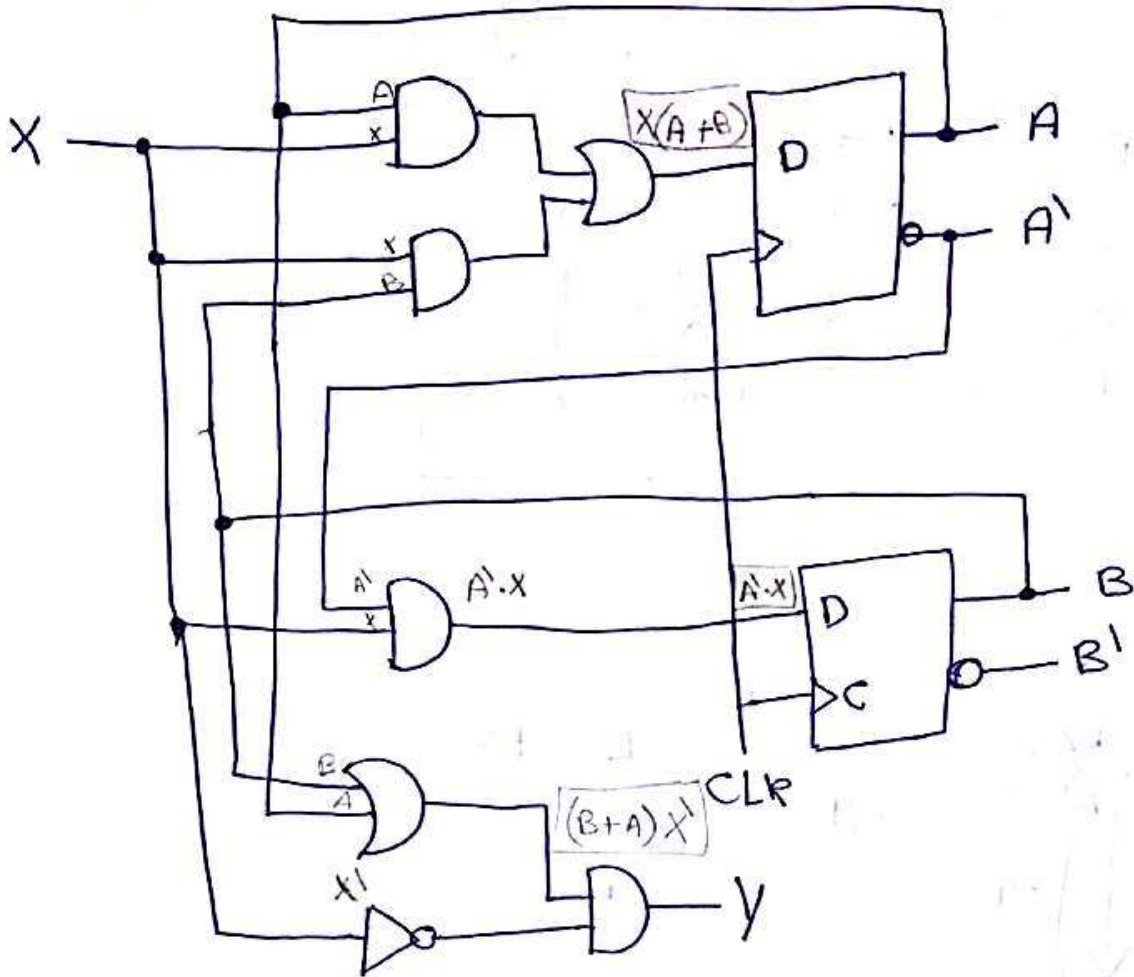
J	K	Q ⁺
0	0	Q
0	1	0
1	0	1
1	1	Q-bar

as 01 is -ve edge

as 11 is +ve edge

Example:-

(51)



input Combinational = 1 / output Combinational = 1
 input sequential = 2 / output sequential = 2

$$D_A = A^+ = ((A+B)X)$$

$$D_B = B^+ = (A' \cdot X)$$

$$Y = ((B+A)X')$$

Data Flip Flop
 $D = Q^+$

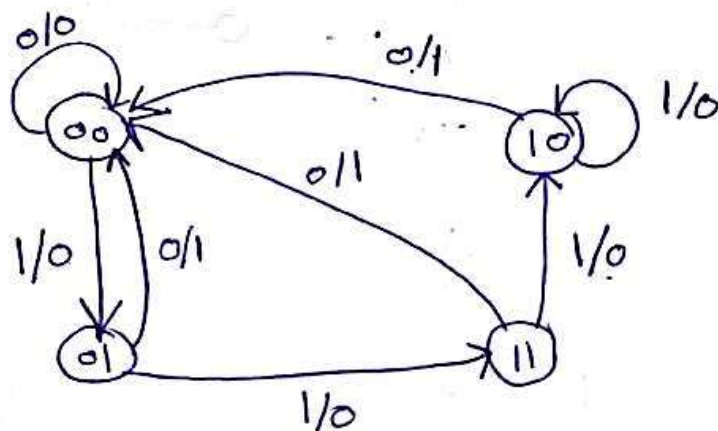
truth table.

A	B	X	A ⁺	B ⁺	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0



truth table design state diagram

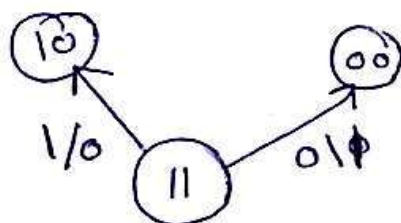
Present		Next state		y output	
A	B	X=0	X=1	X=0	X=1
0	0	0	0	0	0
0	1	0	1	1	0
1	0	0	1	1	0
1	1	0	1	1	0



Example : Taking First Line in the table state 00 input 0 Next state 00 output $y=0$



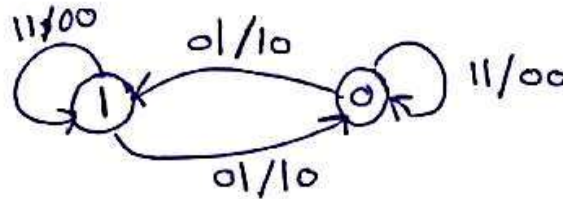
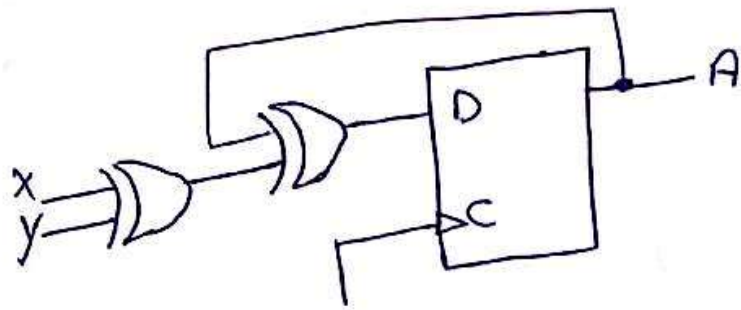
Example - Taking State 11 input can be 0 or 1 output is 1 or 0 next state is 00 or 10 (From table)



Analysis with D Flip-Flops:-

$$D_A = A \oplus X \oplus Y$$

A	X	Y	A ⁺
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1



Analysis with JK

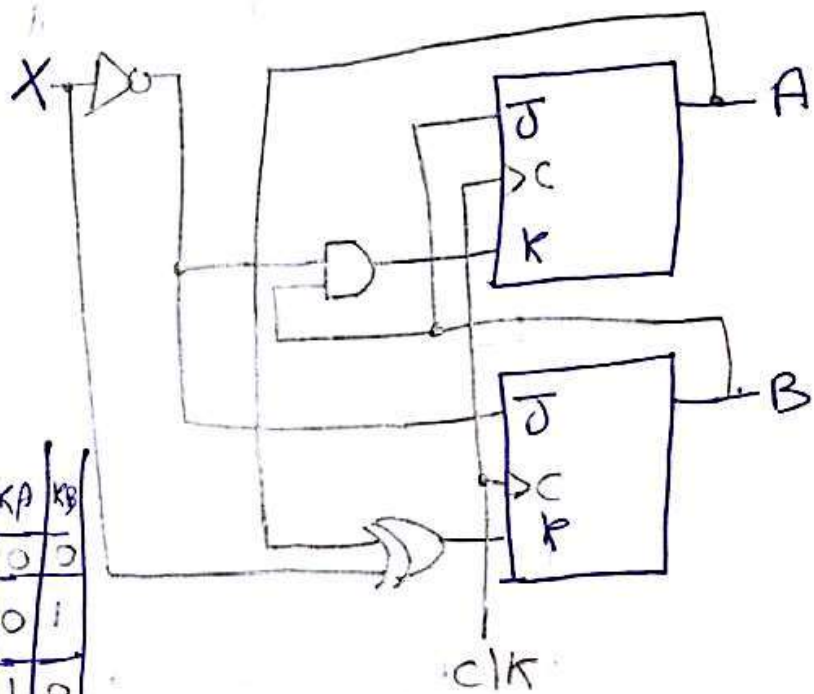
$$J_A = B$$

$$\bar{J}_B = X^1$$

$$K_A = B^1$$

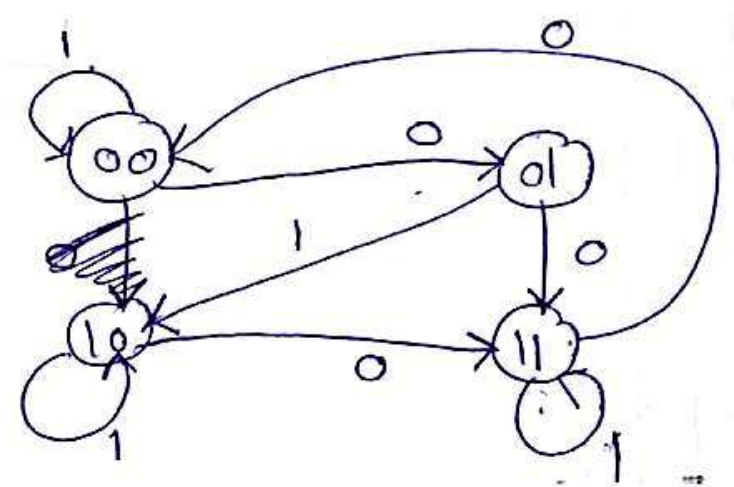
$$K_B = A^1X + AX^1 = A \oplus X$$

A	B	X	A ⁺	B ⁺	J _A	J _B	K _A	K _B
0	0	0			0	1	0	0
0	0	1			0	0	0	1
0	1	0			1	1	1	0
0	1	1			1	0	0	1
1	0	0			0	1	0	1
1	0	1			0	0	0	0
1	1	0			1	1	1	1
1	1	1			1	0	0	0



$$(\overline{J}_A \cdot A) + (K_A \cdot A) = A^+$$

$$(\overline{J}_B \cdot B) + (K_B \cdot B) = B^+$$



at K-map

$$A^+ = \bar{J}A \cdot A' + \bar{K}B \cdot A$$

$$\bar{J}A = B$$

$$\bar{J}B = X'$$

$$KA = BX'$$

$$KB = A \oplus X$$

$$(B \cdot A') + (BX')' \cdot A = A^+$$

$$A^+ = BA' + B'A + X \cdot A$$

$$B^+ = \bar{J}B \cdot \bar{B} + \bar{K}B \cdot B$$

$$B^+ = (X' \cdot B') + (ABX) + (A \cdot X')$$

	BX	00	01	11	10
A	0			1	1
	1	1	1	1	

$$A'B + AB' + X \cdot A$$

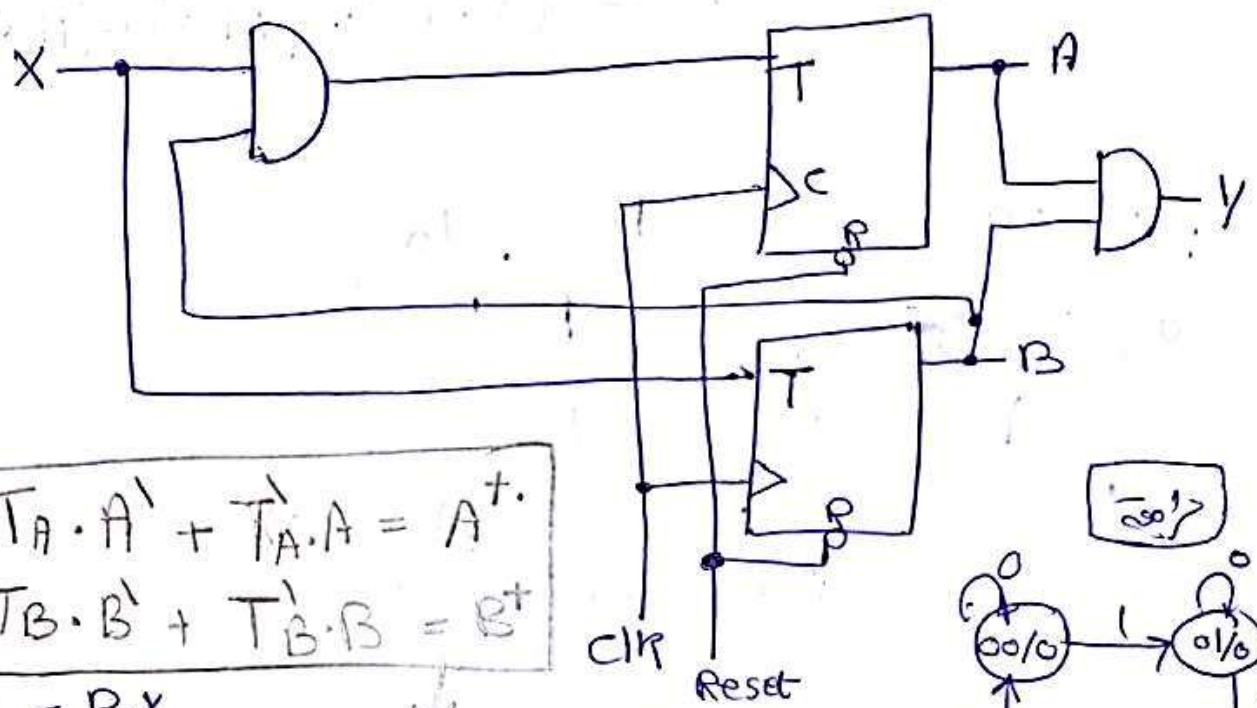
	BX				
A	1	0	0	1	
	1	0	1	0	

$$B'X' + ABX + AX'$$

Example:
 $D = T \oplus Q$

$$Q(t+1) = TQ + T'Q'$$

Analysis with T Flip Flop:



$$T_A \cdot A' + T_A' \cdot A = A^+$$

$$T_B \cdot B' + T_B' \cdot B = B^+$$

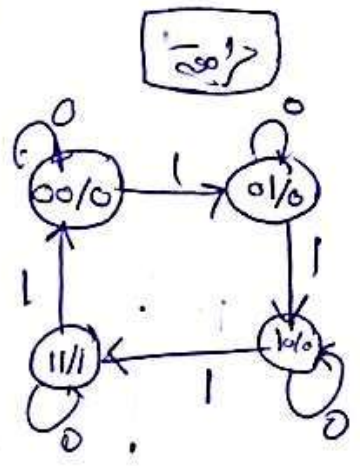
$$T_A = BX$$

$$T_B = X$$

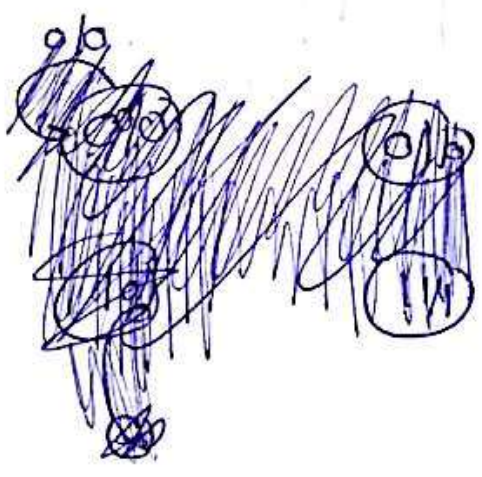
$$Y = AB$$

$$B \cdot X \cdot A' + B' \cdot A + X \cdot A$$

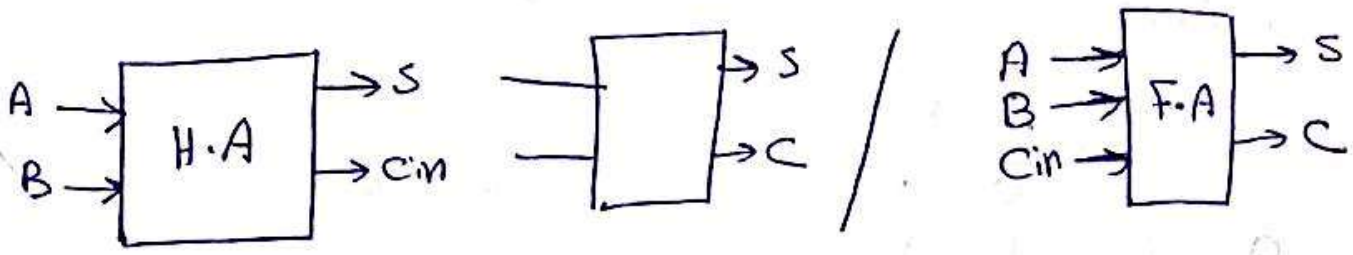
$$X \cdot B' + X' \cdot B$$



A	B	X	A ⁺	B ⁺	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	1

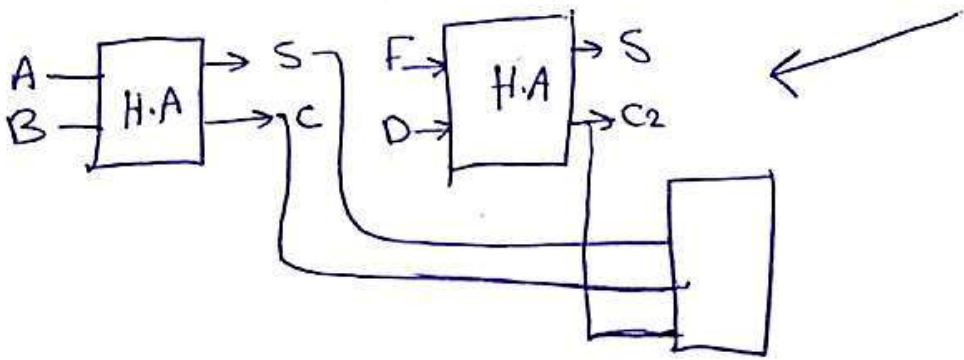


How to make a full adder using half adders? (57)

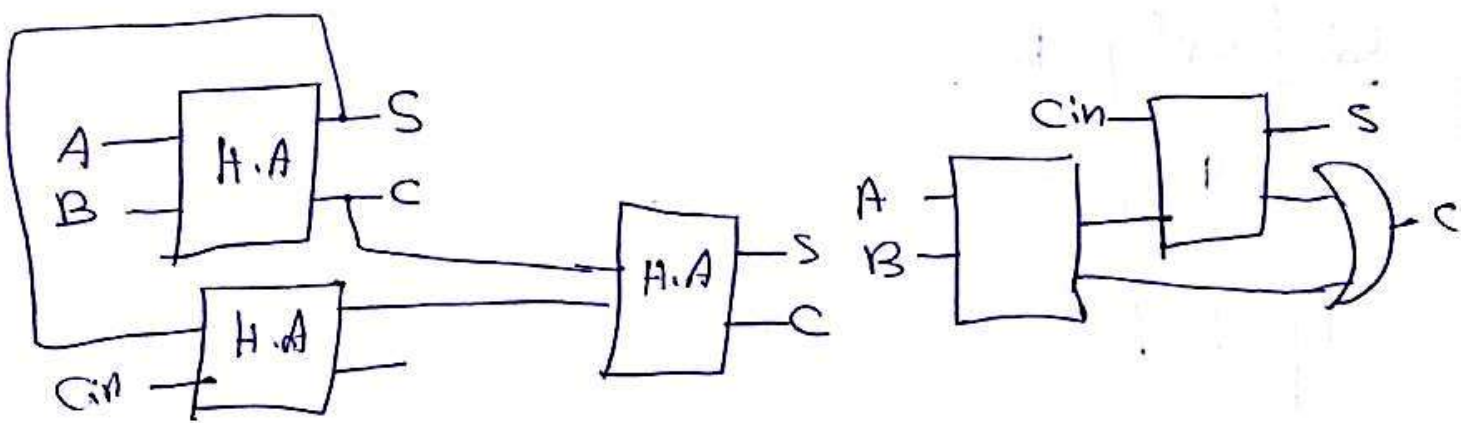


A	B	S	C _{in}
0	0	0	0
0	1	1	0
1	0	1	0
1	1	0	1

AB	C _{in}	S	C _o	C _{in}
00	0	0	0	0
00	1	1	0	0
01	0	1	0	0
01	1	0	1	0
10	0	1	0	0
10	1	0	1	0
11	0	0	1	0
11	1	1	1	0



Three half adders:



Excitation table:

① JK / F / F

Q	Q ⁺	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

$$J\bar{Q} + \bar{K}Q =$$

② T F/F

Q	Q ⁺	T
0	0	0
0	1	1
1	0	1
1	1	0

$$D = T = Q \oplus Q^+$$

T	Q ⁺
0	Q
1	Q'

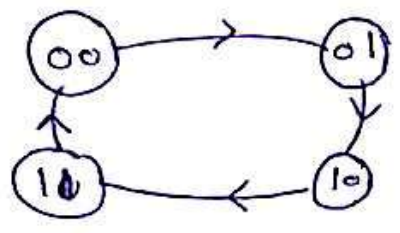
③ D F/F

Q	Q ⁺	D
0	0	0
0	1	1
1	0	1
1	1	0

$$D = Q^+$$

Example:- Design a 2 bit up Counter using

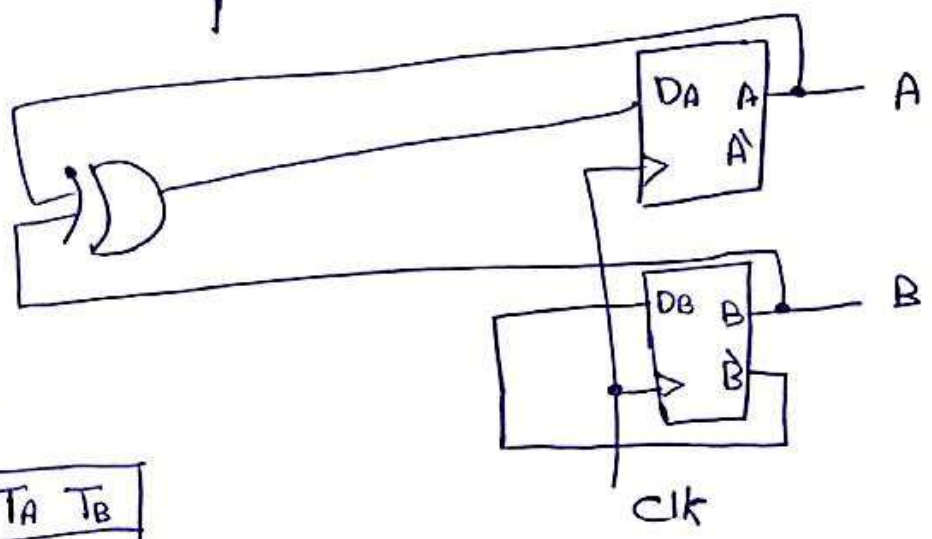
- ① D F/F ② T F/F ③ JK F/F



$D = Q^+$

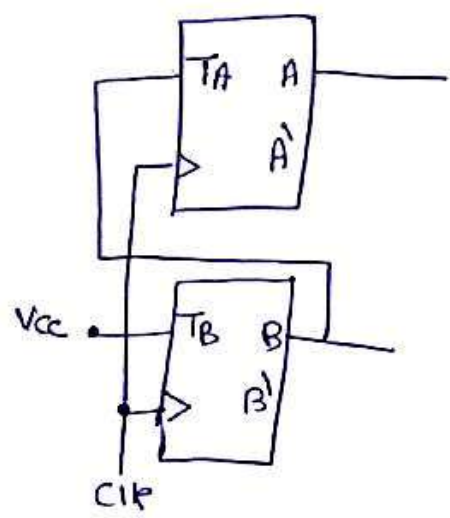
D F/F

input		output		D_A	D_B
A	B	A^+	B^+		
0	0	0	1	0	1
0	1	1	0	1	0
1	0	1	1	1	0
1	1	0	0	0	0



② using T F/F

A	B	B^+	A^+	T_A	T_B
0	0	0	1	0	1
0	1	1	0	1	1
1	0	1	1	0	1
1	1	0	0	1	1



①

Logic final

* Logical circuits *

① Combinational circuit \Rightarrow No memory من الذاكرة

② Sequential circuit \Rightarrow with memory.

bit \rightarrow 0, 1

byte \rightarrow 8 bit

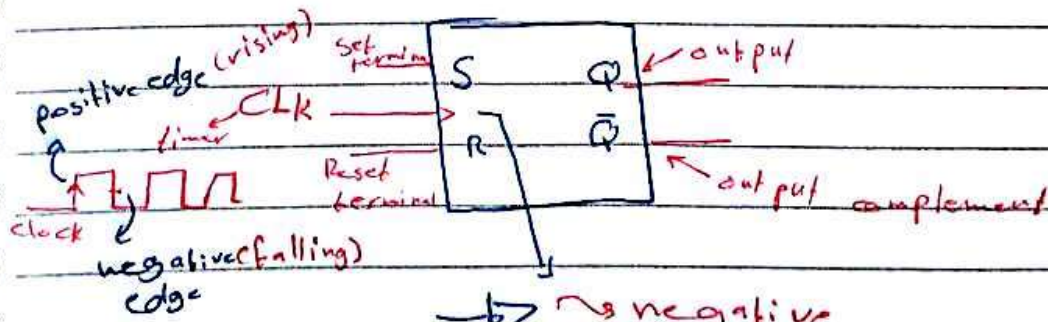
word \rightarrow 16 bit

double word \rightarrow 32 bit

* Set reset flip flop *

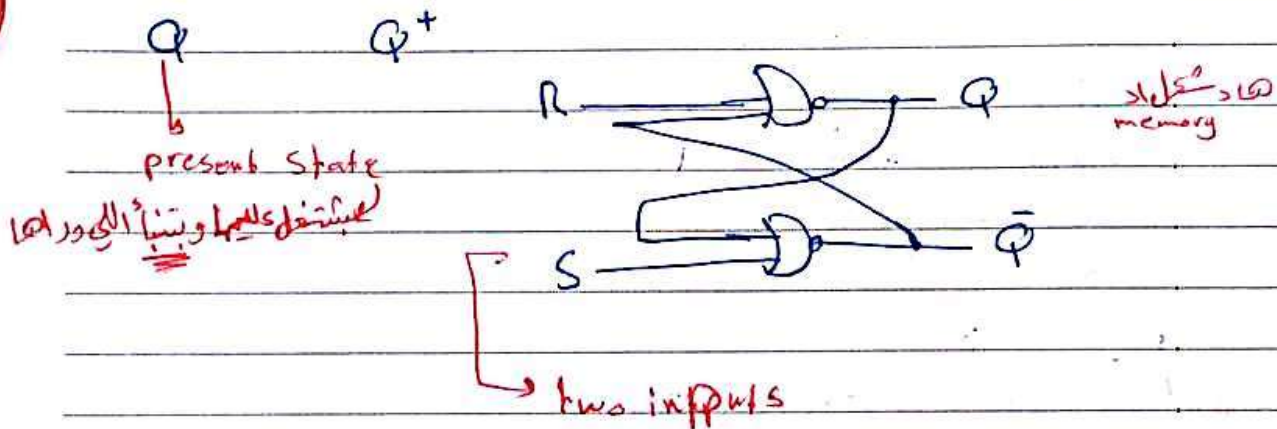
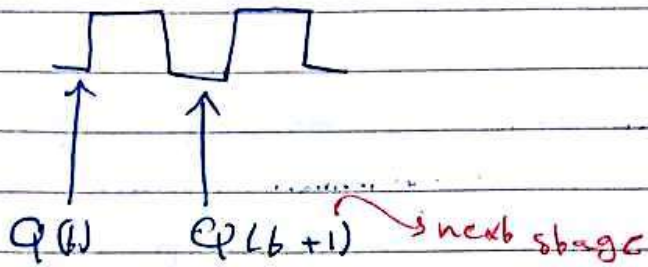
Set to one
high } Reset to zero
low

* Set Reset flip flop *

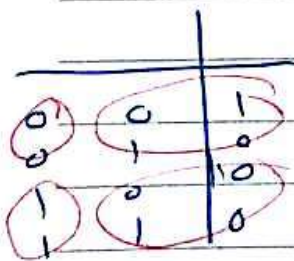


(من الذاكرة \rightarrow negative U positive U)

2



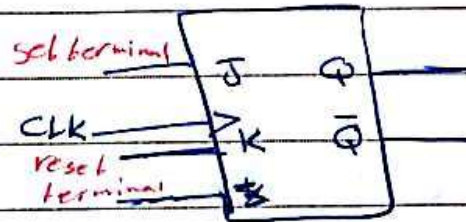
S	R	Q ⁺ next state	
0	0	Q	no change state
0	1	0	Reset state
1	0	1	set state
1	1		Not valid → onek berak احسن لبيرو



3

② JK Flipflop (JK F/F)

Latch \rightarrow without CLK \rightarrow بتغیر علی آئی تغیر
Flipflop \rightarrow with CLK \rightarrow بتغیر علی آئی تغیر



* Characteristic Table \rightarrow

set		reset		Q^+	next state
J	K	J	K		
0	0	0	0	Q	no change
0	1	0	1	0	reset
1	0	1	0	1	set
1	1	1	1	\bar{Q}	toggle (complement)

* Characteristic equation \rightarrow Table

$$Q^+ = J\bar{Q} + \bar{K}Q$$

(4)

(3) Toggle flip flop: T. f/f
↳ one terminal

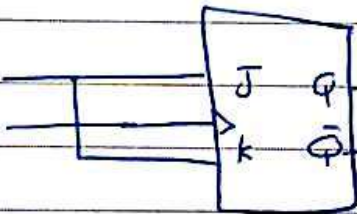


T	Q ⁺
0	Q
1	Q̄

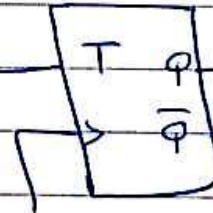
$$Q^+ = T\bar{Q} + \bar{T}Q$$

$$= T \oplus Q$$

* How you do make a T f/f from JK f/f??



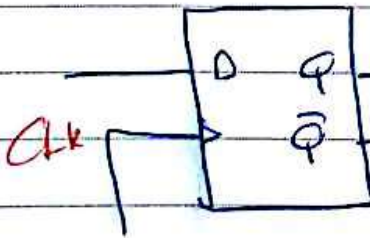
≡



J	K	Q ⁺
0	0	Q
0	1	Q
1	0	Q̄
1	1	Q̄

داده

(4) Data flip flop D f/f



D	Q ⁺
0	0
1	1

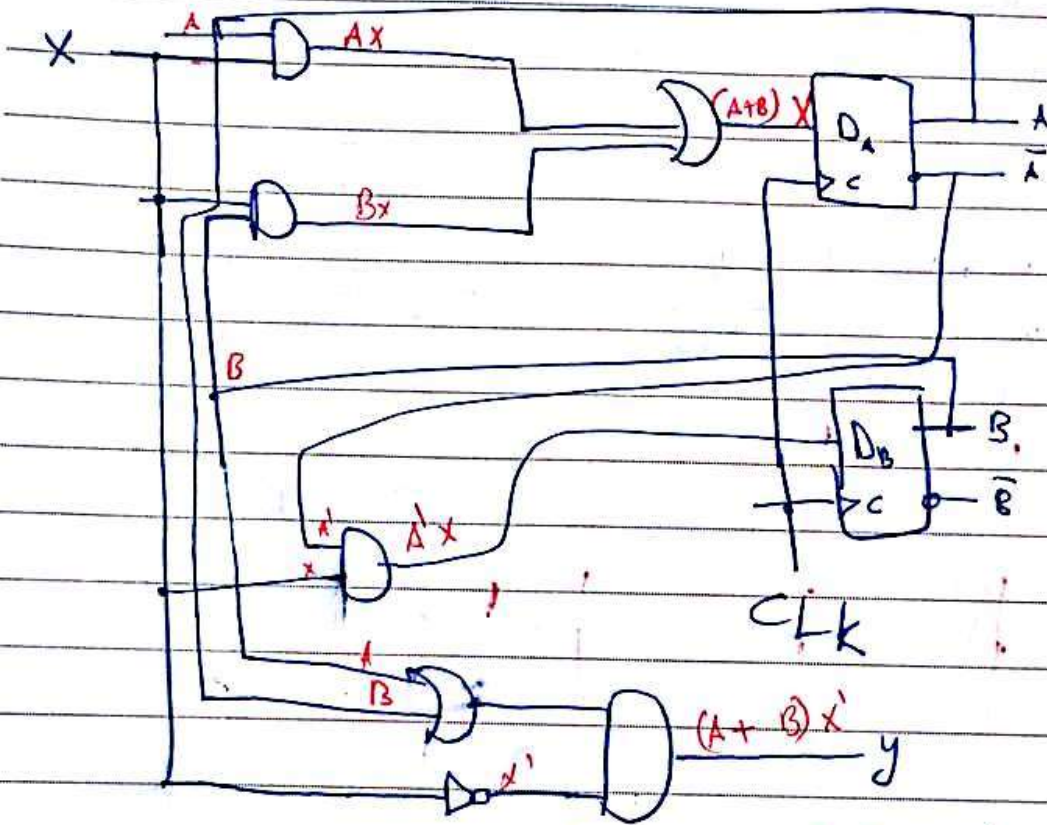
$Q^+ = D$

Current state = next state
الحالي = التالي

6

Analysis using D/F/f

فلج بالترتیب



⇒ One input (combinational)
two sequential inputs

$$D = Q^+$$

$$D_A = A^+$$

$$D_B = B^+$$

$$A^+ = D_A = (A+B)X$$

$$B^+ = D_B = A'X$$

$$y = (A+B)X'$$

⇒ two sequential outputs
one combinational

(7)

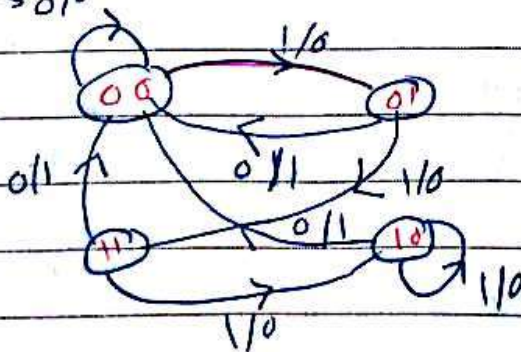
Truth table \Rightarrow

A	B	X	A'	B'	Y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	0	1
0	1	1	1	1	0
1	0	0	0	0	1
1	0	1	1	0	0
1	1	0	0	0	1
1	1	1	1	0	0

State table

Present state		next state		output \textcircled{y}	
A	B	X=0	X=1	X=0	X=1
				Y	Y
0	0	00	01	0	0
0	1	00	11	1	0
1	0	00	10	1	0
1	1	00	10	1	0

State diagram \rightarrow مخطط الحالات \rightarrow ترتيب بياني من الحالات التي قد تكون

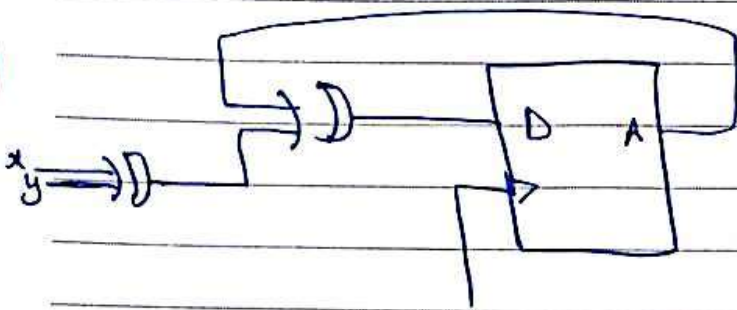


design \rightarrow 10

9

ملاحظة: يمكن ان يكون في input/output و يمكن ان يكون output و يمكن ان يكون input
 ↳ Two inputs

+ Analysis using D/F +



one sequential output
3 inputs

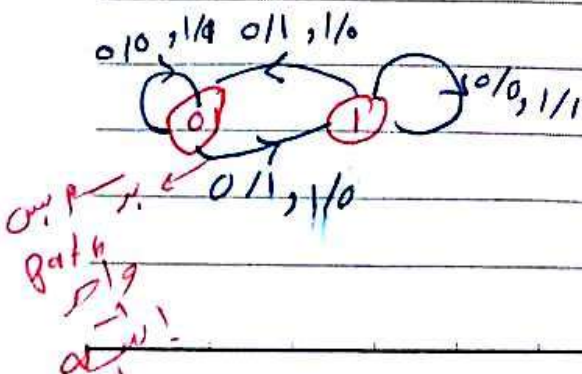
A	x	y	$x \oplus y$	A^+
0	0	0	0	0
0	0	1	1	1
0	1	0	1	1
0	1	1	0	0
1	0	0	0	1
1	0	1	1	0
1	1	0	1	0
1	1	1	0	1

$$A^+ = D = x \oplus y$$

x	y	$x \oplus y$
0	0	0
0	1	1
1	0	1
1	1	0

State diagram

+ عدد الدوائر يعتمد على عدد الـ sequential output



$$2^1 = 2 \text{ of } 1$$

A and B two sequential outputs.

Comb. outputs 6b

X → Combinational input

$$\begin{matrix} J_A = B \\ K_A = X'B \end{matrix} \Rightarrow A^+ = J A' + K' A$$

$$\begin{aligned} &= B A' + X' B \cdot A \\ &= A' B + A (X + B') \\ &= A' B + A X + A B' \\ &= A \oplus B + A X \end{aligned}$$

تقسيم المسألة للحل

2³ → 8

Table

A	B	X	Next A	Next B	J _A	K _A	J _B	K _B
0	0	0	0	1	0	0	1	0
0	0	1	0	0	0	0	0	1
0	1	0	1	1	1	1	1	0
0	1	1	1	0	1	0	0	1
1	0	0	1	1	0	0	1	0
1	0	1	1	0	0	0	0	1
1	1	0	0	0	1	1	1	0
1	1	1	1	1	1	0	0	1

J	K	Q ⁺
0	0	Q
0	1	0
1	0	1
1	1	Q

$$A^+ = J_A \bar{A} + K_A' A$$

$$= B \bar{A} + (X' B)' A$$

$$= A' B + (C X + B') A$$

$$= A' B + A B' + A X$$

$$\begin{matrix} 0 & 1 & 0 & 1 & 0 & 1 \\ & & 1 & 0 & 0 & 1 \\ & & & & 1 & 0 & 1 \end{matrix}$$

A^+ (S) \bar{A}
Table \bar{A} (S) \bar{A}

$$\rightarrow m_2 + m_3 + m_4 + m_5 + m_6 + m_7$$

$$B^+ = J_B B' + K_B' B$$

$$= X' B' + (\overline{X \oplus A}) B$$

$$= B' X' + (X \odot A) B$$

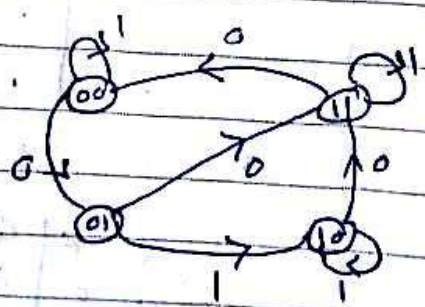
$$= B' X' + A B X + A' B X'$$

$$\begin{matrix} 0 & 0 & 0 & 0 \\ & & 1 & 1 & 1 & 0 & 1 & 0 \end{matrix}$$

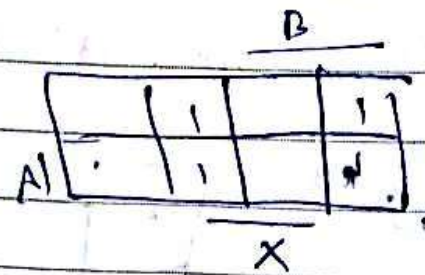
$$m_0 + m_4 + m_7 + m_2 \rightarrow B^+$$

two flipflops $\rightarrow 2^2 \rightarrow$ circles

state diagram \rightarrow

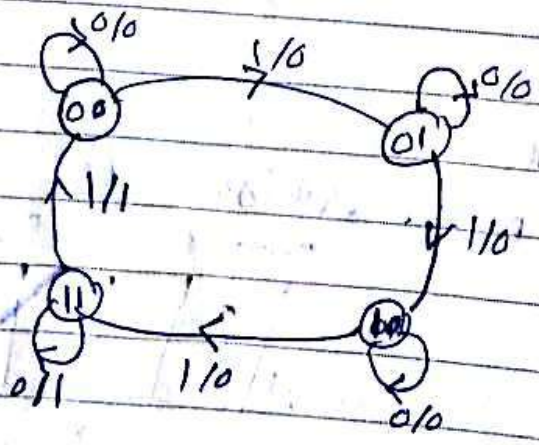


$$\begin{aligned}
 B^T &= T_B \oplus B \\
 &= X \oplus B \\
 &= XB' + X'B
 \end{aligned}$$



$$m_1 + m_2 + m_3 + m_5$$

A	B	X	next		
A	B	X	A	B	y
0	0	0	0	0	0
0	0	1	0	1	0
0	1	0	0	1	0
0	1	1	1	0	0
1	0	0	1	0	0
1	0	1	1	1	0
1	1	0	1	1	1
1	1	1	0	0	1



1.3

* Excitation table *

(prev. slide)

① JK f/f

Q	Q ⁺	J	K
0	0	0	X
0	1	1	X
1	0	X	1
1	1	X	0

→ don't care (0 or 1)

J	K	Q ⁺
0	0	Q
0	1	0
1	0	1
1	1	Q'

② T f/f

→ $T = Q \oplus Q^+$

Q	Q ⁺	T
0	0	0
0	1	1
1	0	1
1	1	0

no change

Toggle

T	Q ⁺
0	Q
1	Q'

③ D f/f

Q	Q ⁺	D
0	0	0
0	1	1
1	0	0
1	1	1

$$D = Q^+$$

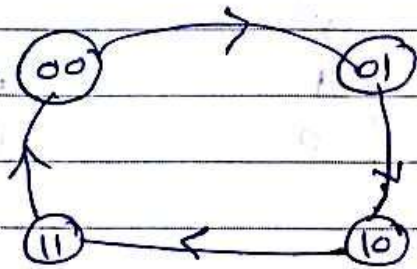
RS) لا يمكن
تغييره
Toggle

بِسْمِ اللَّهِ الرَّحْمَنِ الرَّحِيمِ
 (14)

* Ex 0 → Design a 2 bits up counter using
 ① D f/f ② T f/f ③ JK f/f

(بھی ارجح رہے گی)
 (حرف خطوات)

①



②

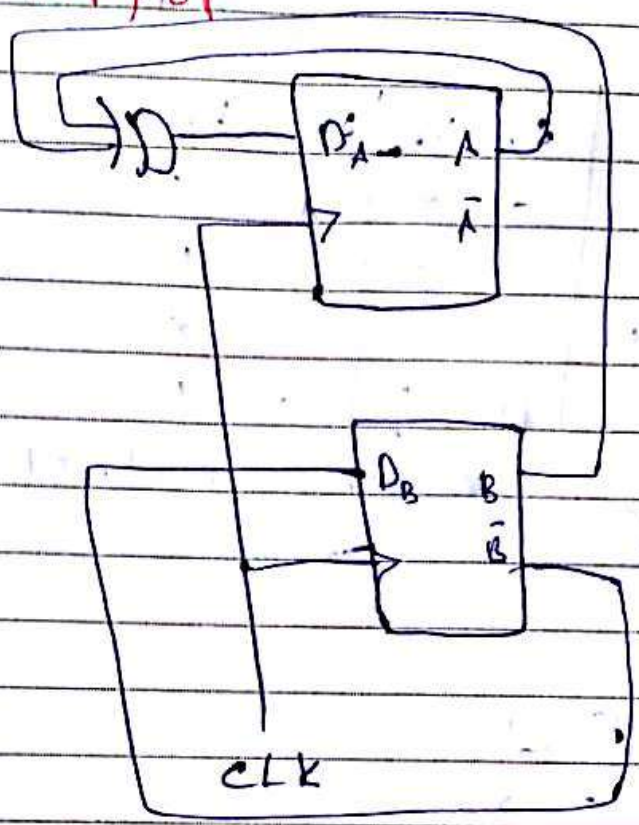
inputs		outputs		$D = Q^+$	
A	B	A^+	B^+	D_A	D_B
0	0	0	1	0	1
0	1	1	0	1	0
1	0	1	1	1	1
1	1	0	0	0	0

$$D_A = A^+$$

$$D_B = B^+ = \bar{B}$$

$A \oplus B$ B'
 k-map سے فی

two flip flop

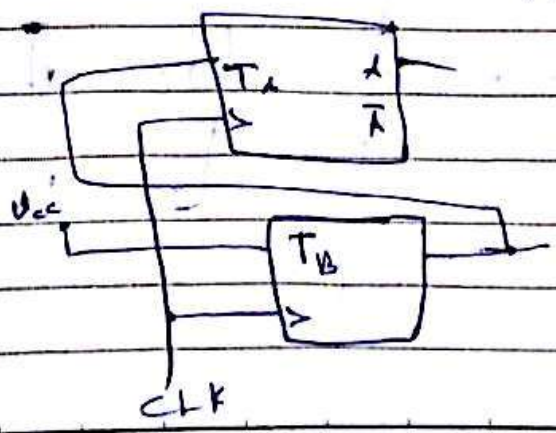


$$D_A = A^{\bar{}}$$

② using T f/f

A	B	$A^{\bar{}}$	$B^{\bar{}}$	T_A	T_B
0	0	1	1	0	1
0	1	1	0	1	1
1	0	0	1	0	1
1	1	0	0	1	1

$T_A = B$ $T_B = 1$ $\frac{dV}{V_{CC}}$



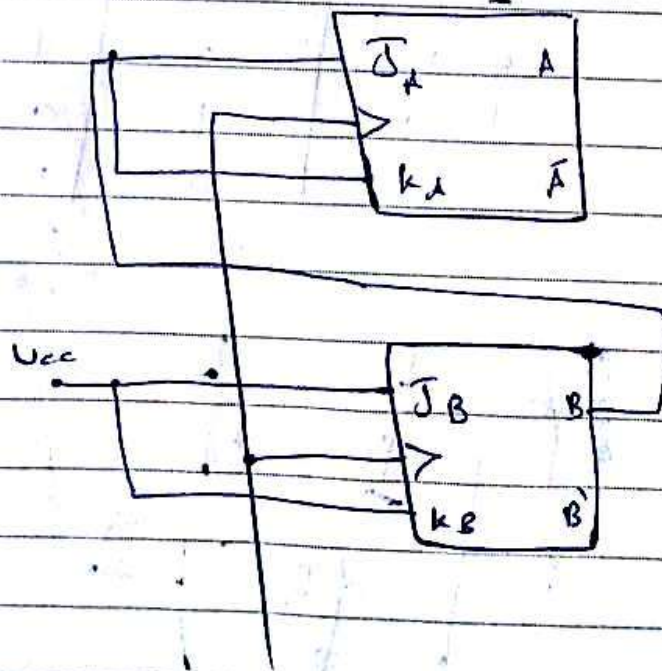
③ using JK f/f

		Next		J_A K_A		J_B K_B	
A	B	A	B	J_A	K_A	J_B	K_B
0	0	0	1	0	x	1	x
0	1	1	0	1	x	x	1
1	0	0	0	x	0	1	x
1	1	0	0	x	1	x	1

$J_A = K_A = B$ $J_B = K_B = 1$

L → Recall

Q	Q^+	J	K
0	0	0	x
0	1	1	x
1	0	x	1
1	1	x	0

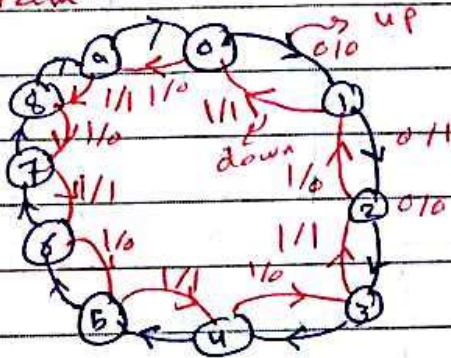


(17)

+ Example \Rightarrow Design a 4-bit BCD digit up/down counter such that it will give an indication when the output count is even number using $\Rightarrow D, T, JK, FF$

decimal (0-9) \leftarrow BCD \leftarrow 4-bit
input \leftarrow up/down \leftarrow \odot
output \leftarrow even \leftarrow \odot

① State diagram



① let input $X=0$ up
 $X=1$ down

$y=0$ odd

$y=1$ even

coding \leftarrow Table

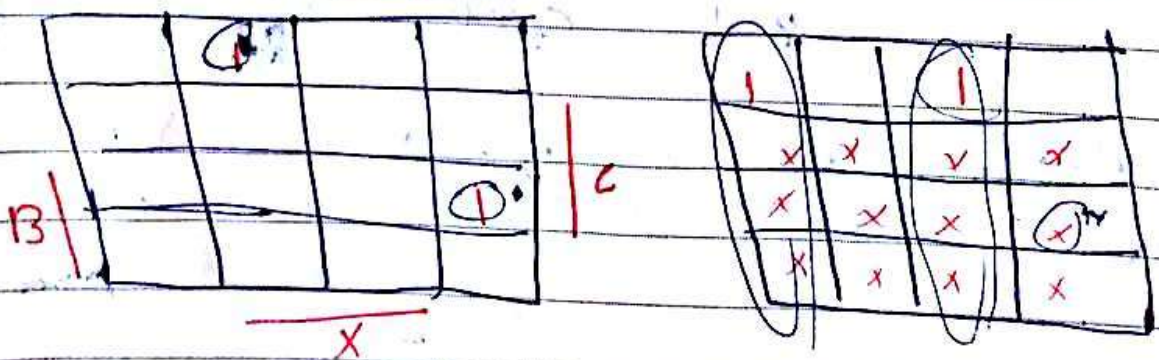
present

Next

A	B	C	D	X	A	B	C	D	y
0	0	0	0	0	0	0	0	0	0
0	0	0	0	1	0	0	0	0	0
0	0	0	0	2	0	0	0	0	0
0	0	0	0	3	0	0	0	0	0
0	0	0	0	4	0	0	0	0	0
0	0	0	0	5	0	0	0	0	0
0	0	0	0	6	0	0	0	0	0
0	0	0	0	7	0	0	0	0	0
0	0	0	0	8	0	0	0	0	0
0	0	0	0	9	0	0	0	0	0
0	0	0	0	10	0	0	0	0	0
0	0	0	0	11	0	0	0	0	0
0	0	0	0	12	0	0	0	0	0
0	0	0	0	13	0	0	0	0	0
0	0	0	0	14	0	0	0	0	0
0	0	0	0	15	0	0	0	0	0
0	0	0	0	16	0	0	0	0	0
0	0	0	0	17	0	0	0	0	0
0	0	0	0	18	0	0	0	0	0
0	0	0	0	19	0	0	0	0	0
0	0	0	0	20	0	0	0	0	0
0	0	0	0	21	0	0	0	0	0
0	0	0	0	22	0	0	0	0	0
0	0	0	0	23	0	0	0	0	0
0	0	0	0	24	0	0	0	0	0
0	0	0	0	25	0	0	0	0	0
0	0	0	0	26	0	0	0	0	0
0	0	0	0	27	0	0	0	0	0
0	0	0	0	28	0	0	0	0	0
0	0	0	0	29	0	0	0	0	0
0	0	0	0	30	0	0	0	0	0
0	0	0	0	31	0	0	0	0	0

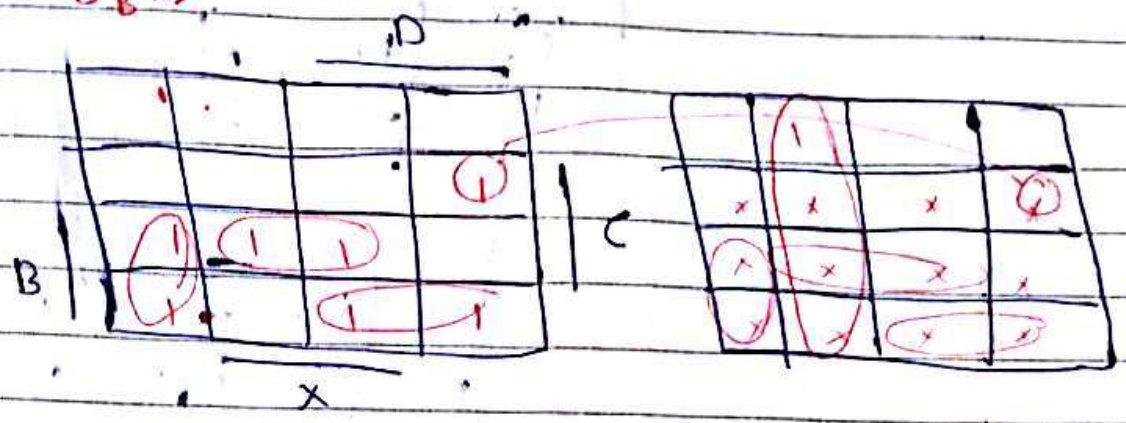
منه موجود
رقم 10
فقط
don't care

$D^* = D' // y = D \rightarrow$ الكومبلاiment
for $D_A \Rightarrow p$ A



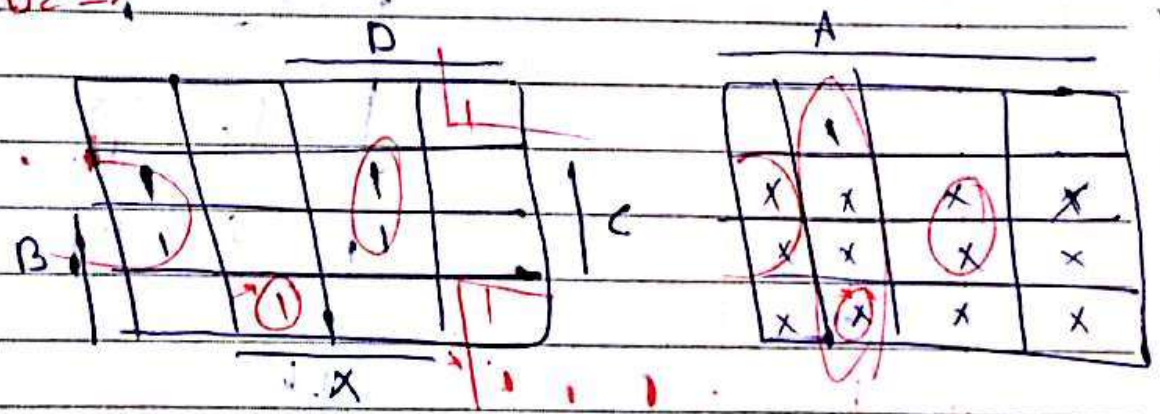
$$D_A = A'B'C'D'X + AD'X' + ADX + BCD'A'$$

for $D_B \Rightarrow$



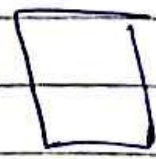
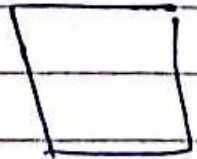
$$D_B = B^T = B D^T X^T + B C^T D + B^T C D X^T + A D^T X$$

for $D_C \Rightarrow$

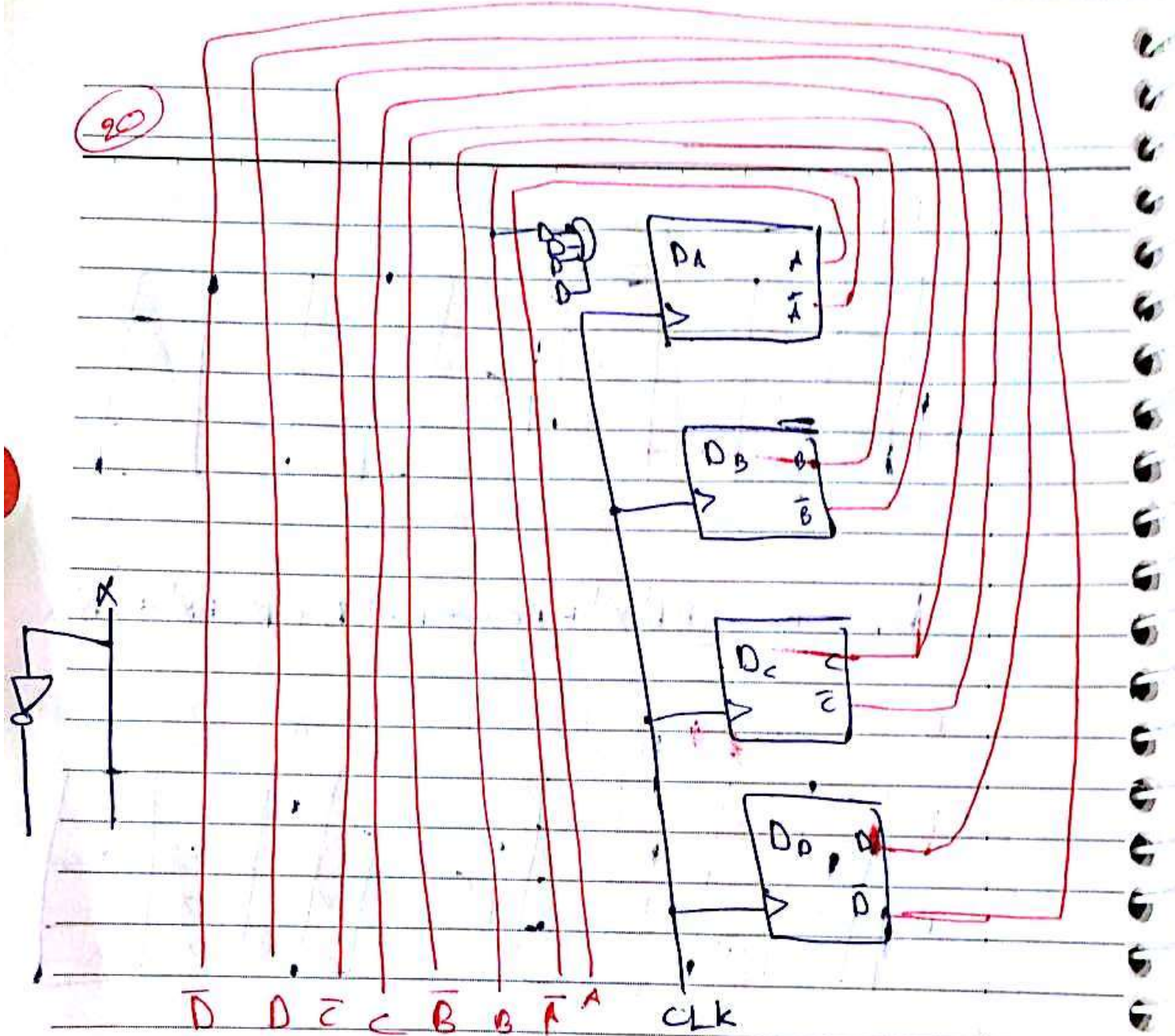


$$D_C = C^T = C D^T X^T + B C^T D^T X + C D X + A^T C^T D X^T + A D^T X$$

for implementation \Rightarrow

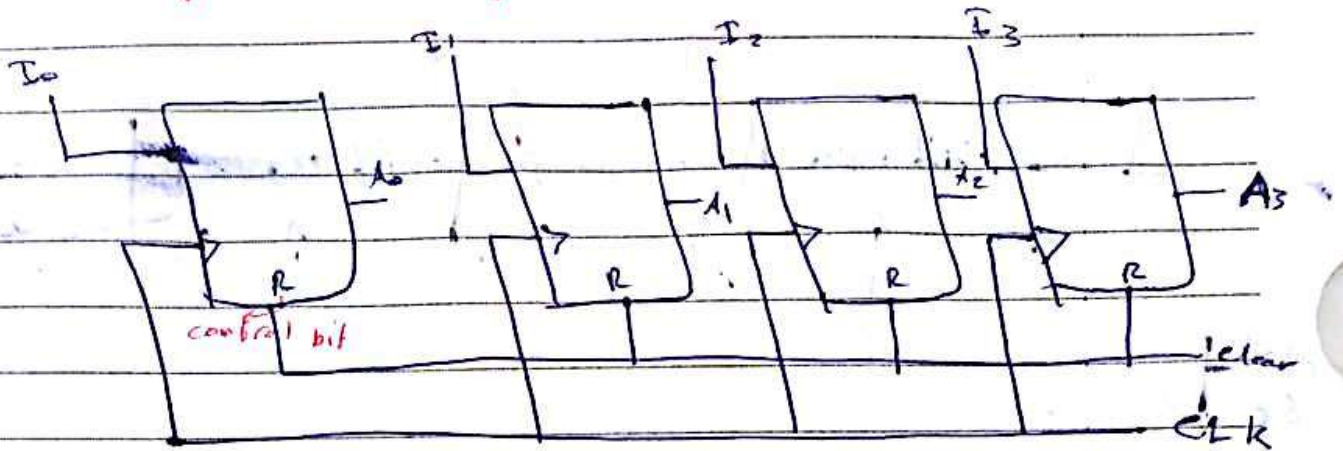


20



من دون ريسر كل لا اعتقاد على الدورية والد اسه

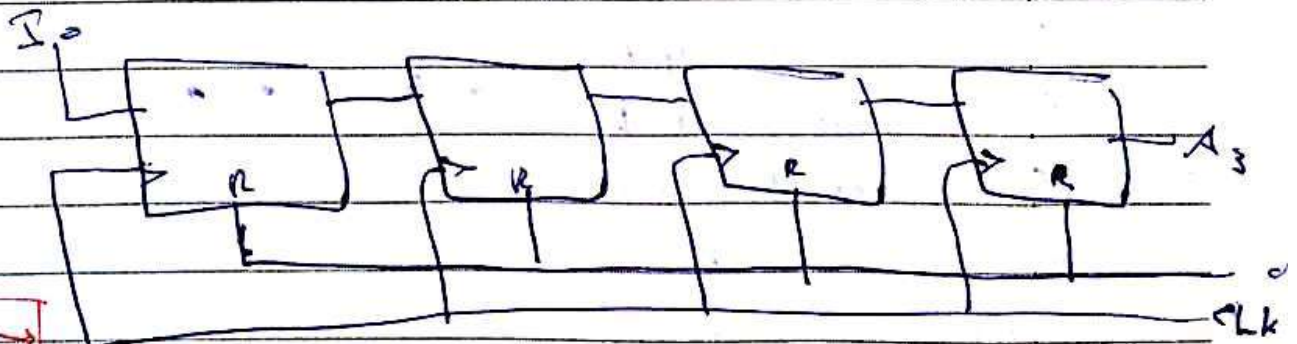
* 4-bit parallel in parallel out register



Parallel → Channel by channel
 Serial → bit by bit

sequential في cycle one data في cycle
 → present / next

* Serial in / serial out



Shift Right →
 Shift Left ←

1101 ← data في بي

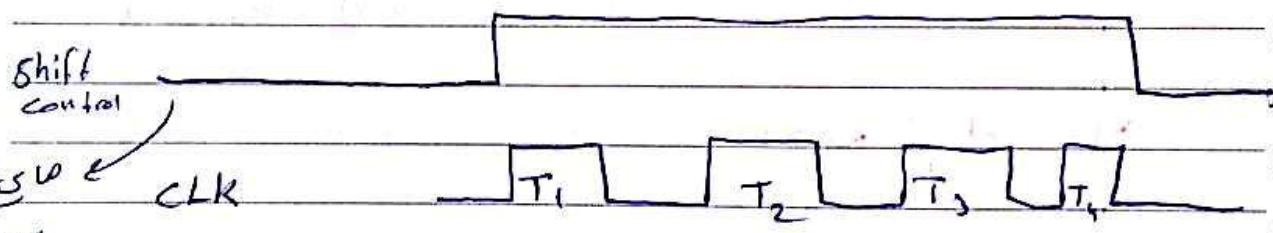
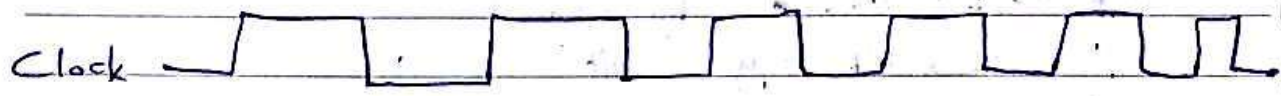
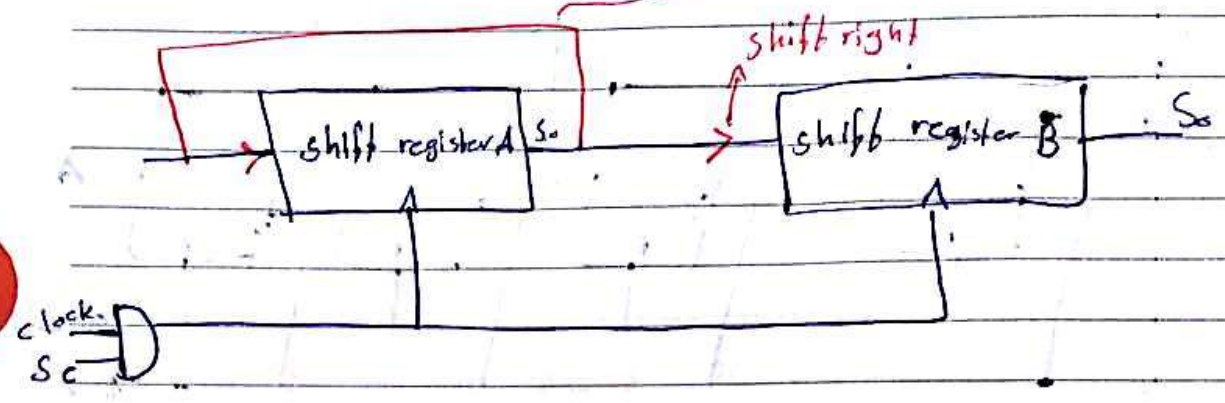
1	X	X	X	X
2	1	X	X	X
3	0	1	X	X
4	1	0	1	X
5	1	1	0	1

shift right

في cycle
 data في cycle
 في بي في cycle
 على ان امر في

(22)

* Serial Transfer * shift right rotate



masking

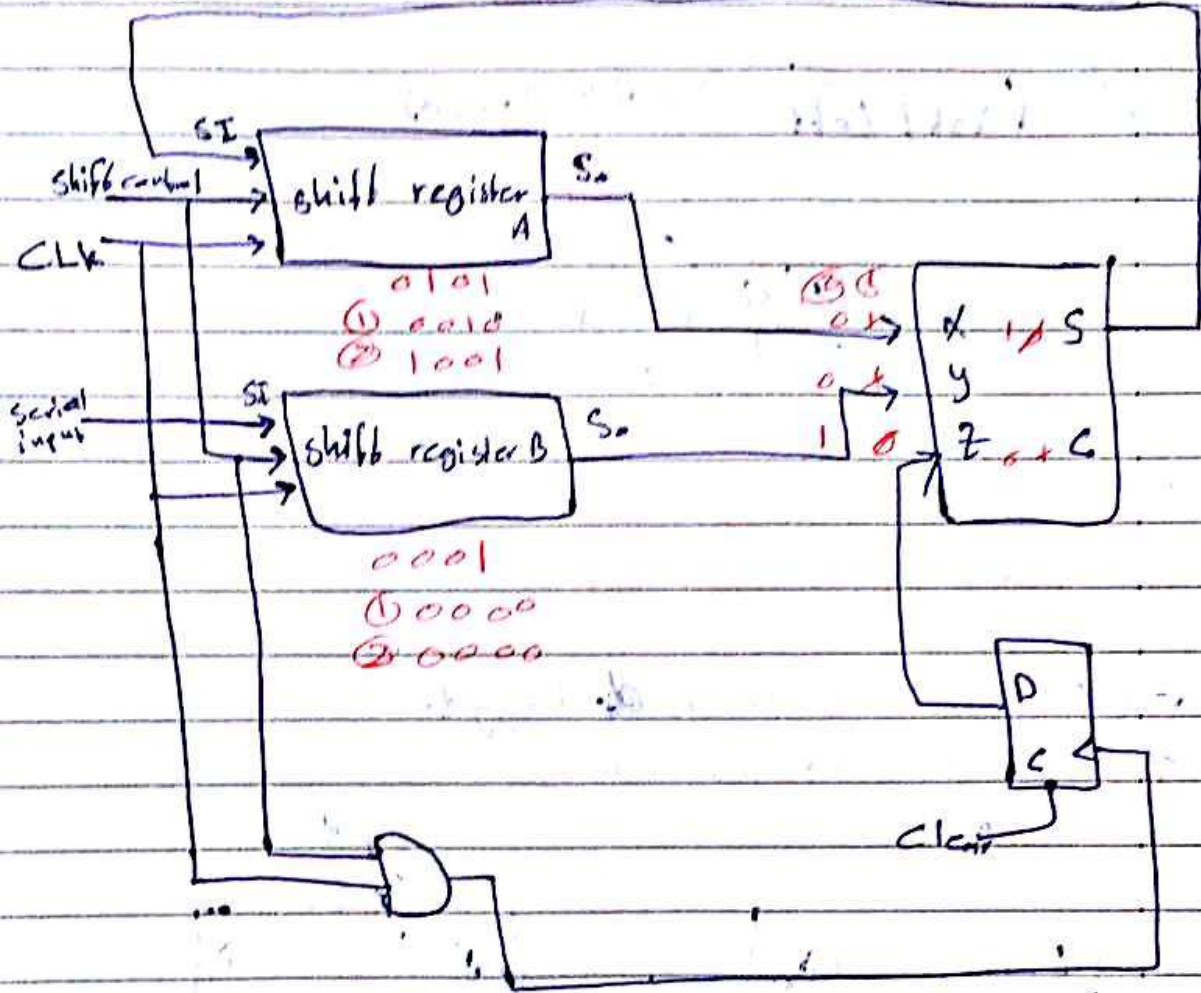
So = serial output
 Sc = shift control
 Si = serial input

Zero rotate cycle
 1st cycle
 2nd cycle

	A	B
0	0100	1110
①	0010	0111
②	0001	0011
	1000	1001
	0100	0100

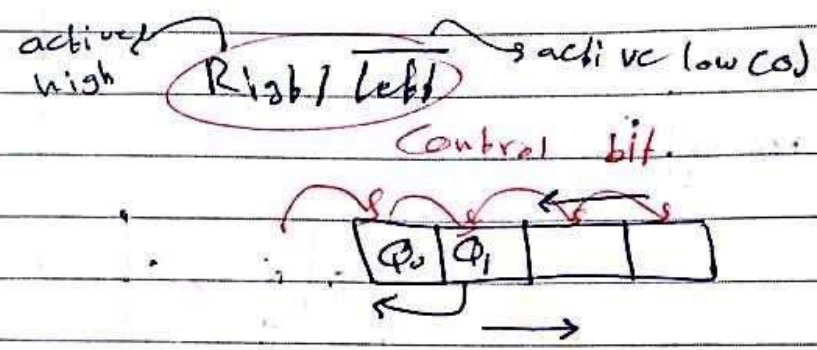
Handwritten notes in Arabic script at the bottom left of the page.

* Serial adder *



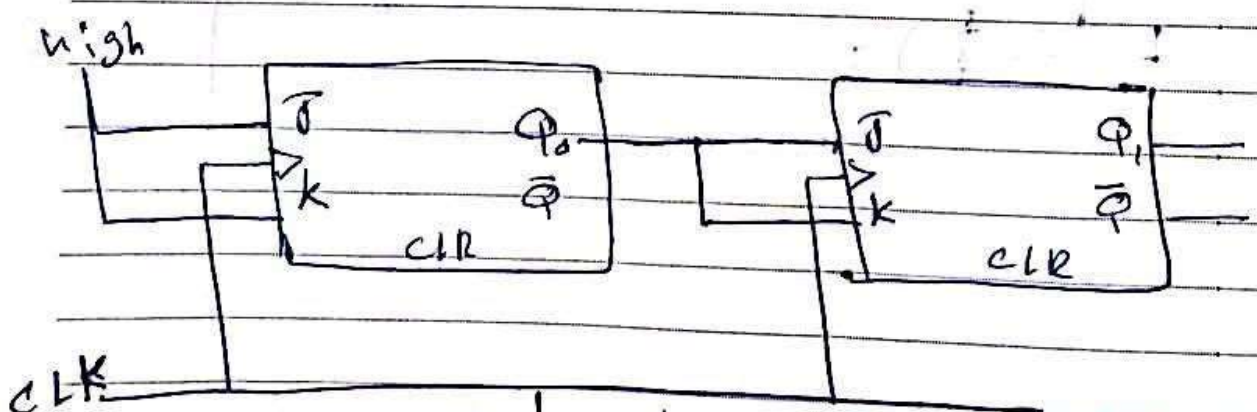
87.4 Right Left

4-bit shift left/right register



Toggle \leftarrow V_{CC} \rightarrow T_k \leftarrow Q_0, Q_1
 T flip \leftarrow Q_0, Q_1 \rightarrow Q_0, Q_1

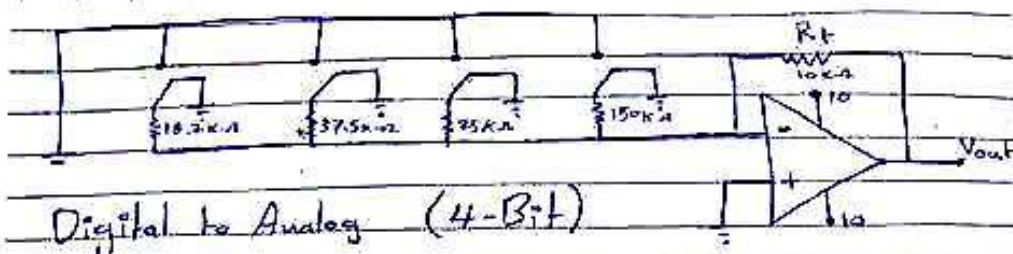
T flip \rightarrow ones \rightarrow always toggle
 0 \rightarrow zero \rightarrow no change



Q ₀	Q ₁	Q ₀ ⁺	Q ₁ ⁺	Notes
0	0	1	0	no change
0	1	1	1	no change
1	0	0	1	Toggle
1	1	0	0	Toggle

Q₀ \rightarrow zero \rightarrow no change
 Q₁ \rightarrow one \rightarrow Toggle

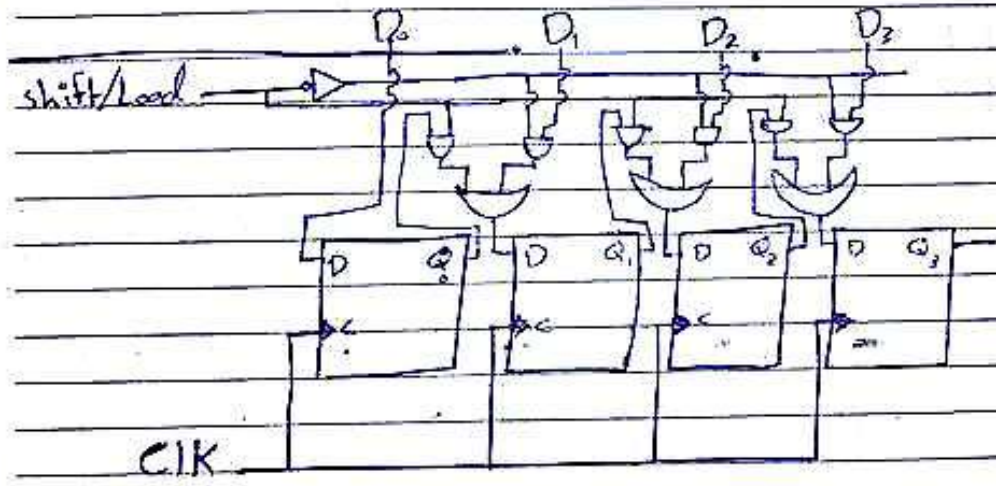
Region	$V_{CE} (V)$	$V_{CE} (V)$	current relation
Cut-off	< 0.6	0.6	$I_B = I_C = 0$
Active	$0.6 - 0.7$	> 0.8	$I_C = h_{ff} I_B$
Saturation	$0.7 - 0.8$	0.2	$I_B \geq \frac{I_C}{h_{ff}}$



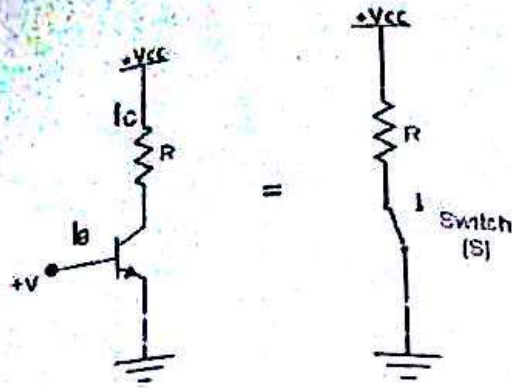
Digital to Analog (4-Bit)

DAC

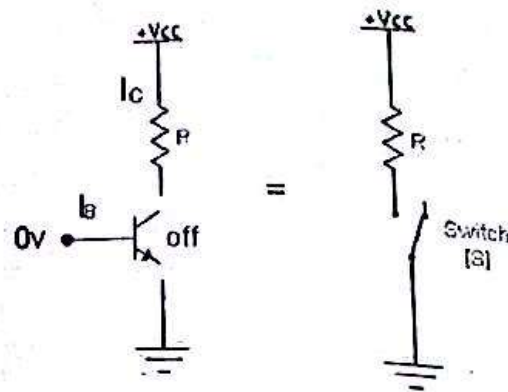
- Most bit \rightarrow 3V
- Bipolar \rightarrow 5V



Parallel in Serial out shift Register

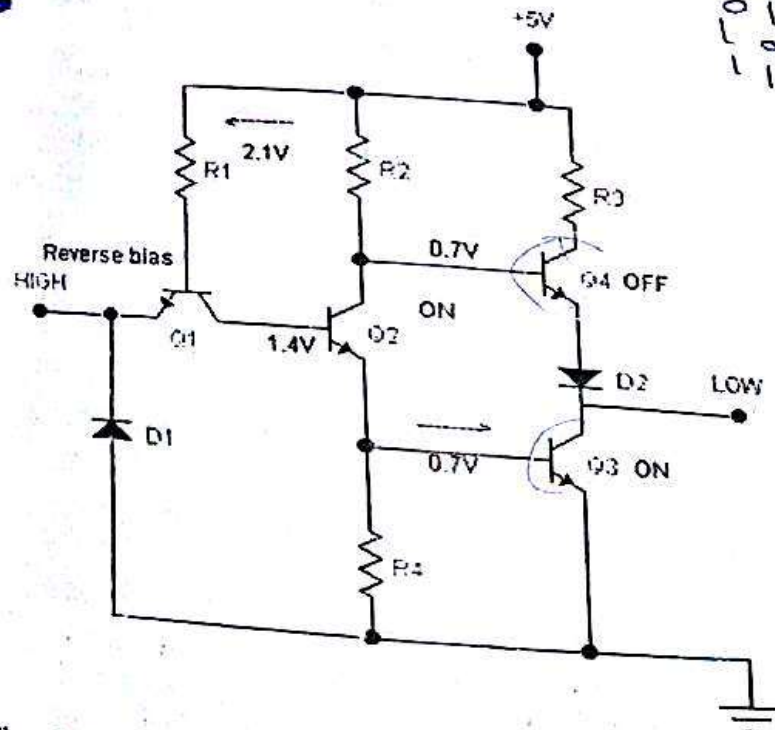


Saturated (ON) Transistor closed switch, ideal switch equivalent



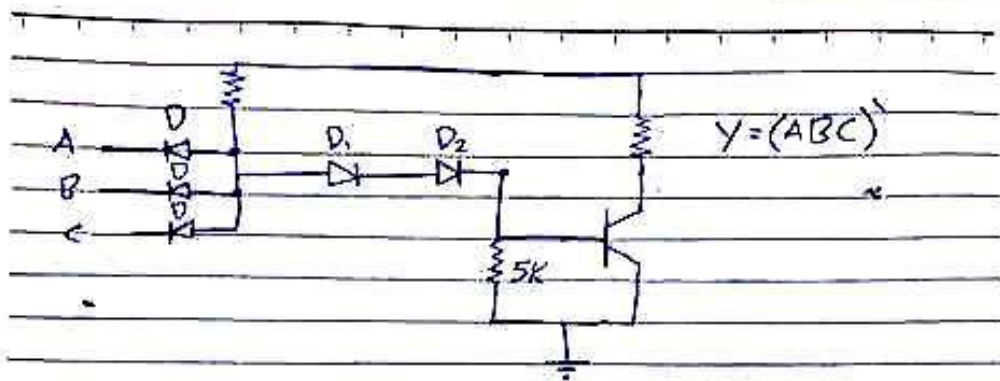
Off Transistor, ideal switch equivalent

TTL inverter

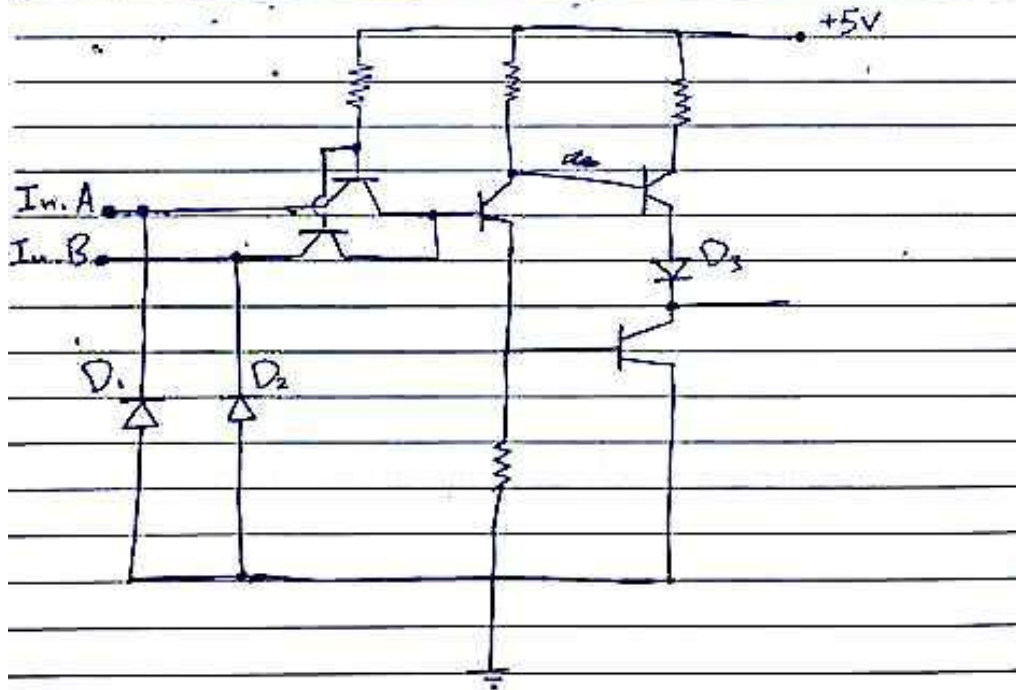


A	B	output
0	0	1
0	1	1
1	0	1
1	1	0

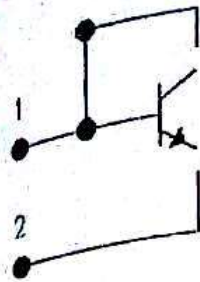
The above arrangement is called totem - pole arrangement.



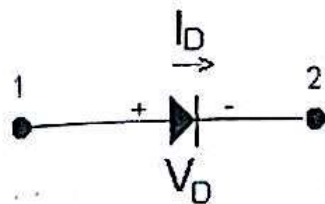
DTL NAND Gate



TTL NAND Gate



Transistor adapted for use as a diode

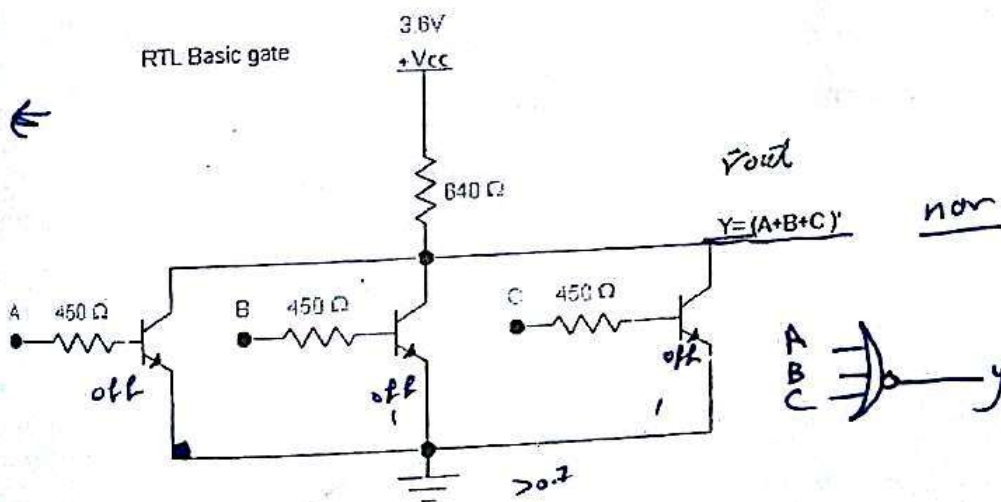


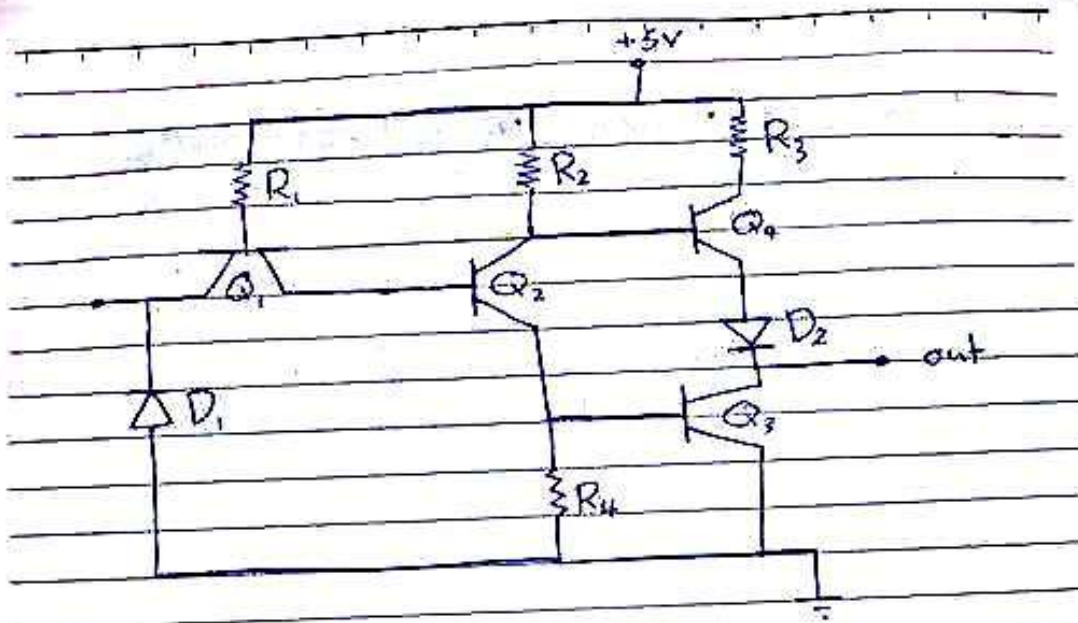
Diode graphical symbol

The diode is off and non-conducting when its forward voltage V_D is less than 0.6 V.
 When diode conducts $V_D = 0.7V$ and I_D flows in the direction of the arrow.

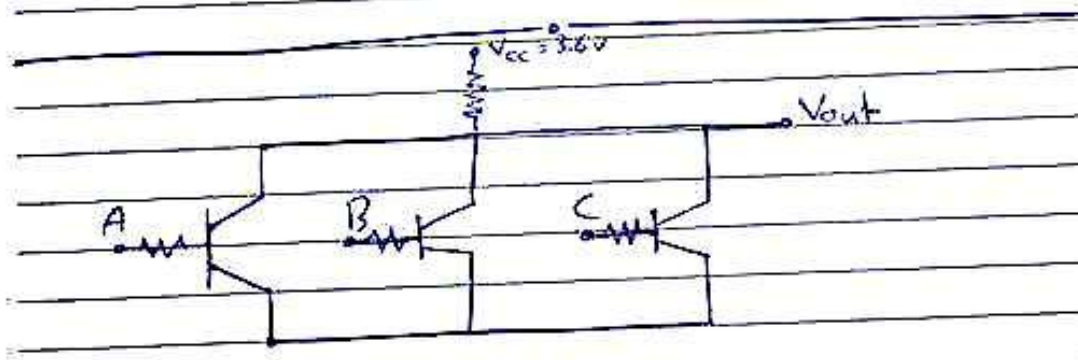
A diode or "Rectifier" is any device through which electricity can flow in one direction only.

RTL Basic Gate *Resistor-Transistor Logic*





TTL

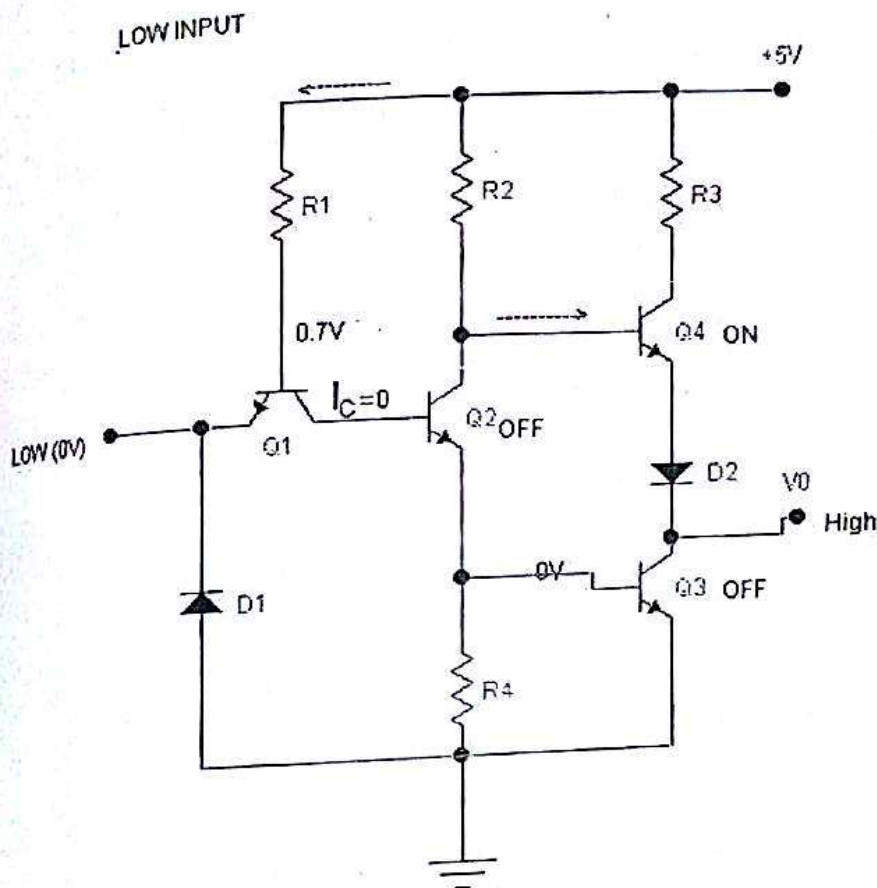


RTL (NOR)

When the input is high the base emitter junction of Q1 is reverse biased and the base - collector junction is forward biased. This condition permits current through R1 and the base - collector junction of Q1 into the base of Q2 thus driving Q2 into saturation as a result Q3 is turned on by Q2 and its collector voltage which is the output is near ground potential.

We therefore have a low output for a high input, at the same time, the collector of Q2 is at sufficiently low voltage level to keep Q4 off.

Low input



When the input is LOW, the base - emitter junction of Q1 is forward biased and the base - collector junction is reverse biased. There is current through R1 and the base - emitter junction of Q1 to the LOW input. A LOW provides a path to ground for the current. There is no current into the base of Q2, so it is off. The collector of Q2 is high

Digital Integrated Circuits

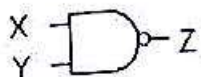
The IC digital logic families to be considered here are:

- RTL Resistor Transistor Logic
- DTL Diode Transistor Logic
- TTL Transistor – Transistor Logic
- ECL Emitter – Coupled Logic
- MOS Metal – Oxide semiconductors
- CMOS Complementary Metal Oxide Semiconductor

RTL & DTL have only historical significance since they are no longer used in digital systems. RTL was the first commercial family to be used extensively.

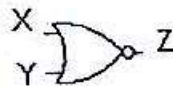
DTL have been replaced by TTL, In fact TTL is a modification of DTL gate.

The basic circuit in each IC digital logic family is either a NAND or NOR gate. This basic circuit is the primary building block from which all other more complex digital components are obtained.



X	Y	Z
L	L	H
L	H	H
H	L	H
H	H	L

Positive Logic Nand Gate



X	Y	Z
L	L	H
L	H	L
H	L	L
H	H	L

Positive Logic Nor Gate

A Bipolar junction transistor (BJT) can be either an npn or pnp junction transistor. Field-effect transistors are said to be Unipolar. The operation of a bipolar transistor depends on the flow of two types of carriers: electrons and holes. A unipolar transistor depends on the flow of only one type of majority carrier, which may be electrons (n-channel) or holes (p-channel). RTL, DTL, TTL, ECL are Bipolar Transistors.

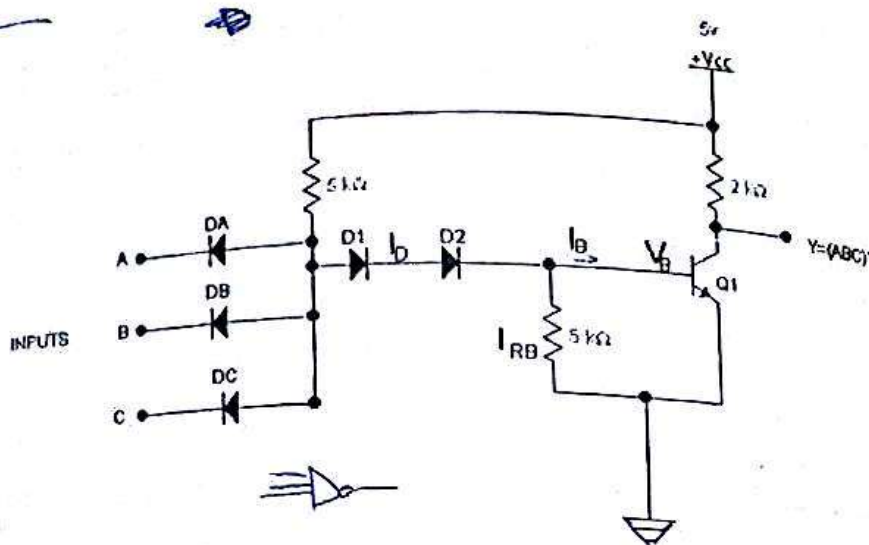
MOS and CMOS employ a type of unipolar transistor called metal-oxide-semiconductor field-effect transistor (MOSFET).

If any input of the RTL gate is High (A, B, C) the corresponding transistor is driven into saturation. This causes the output to be low, regardless of the states of the other transistor. If all inputs are low (0.2 V) all transistors are cut off because $V_{BE} < 0.6V$. This causes the output to be high, approaching the value of supply voltage V_{CC} .

DTL Basic gates

The basic circuit in the DTL logic family is the NAND gate shown below.

A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	0
1	1	1	0



The transistor serves as a current amplifier while inverting the digital signal. If any input is low (0.2 V) the corresponding input diode conducts current. This means that the diode is forward biased and here is a voltage drop of 0.75 Volts. But at output there is 0.2V the means that point P is $0.75 + 0.2 = 0.95V$.

0.95 volts is not enough to forward bias D1, D2.

Therefore Q1 is cut off and $Y = V_{CC}$ High.

$$V_B = V_P - V_{D1} - V_{D2} = 0.95 - 0.65 - 0.65 = -0.35V$$

∴

$$I_{RB} = \frac{V_{B2} - 0}{5 \times 10^{-3}} = \frac{0.35}{5 \times 10^{-3}} = 0.07mA$$

If all inputs are simultaneously at logic level 0 the current through resistor R will divide among the diodes D_A, D_B, D_C . Thus V_P decreases slightly and Q_1 is forced further into cutoff. Hence V_O remains at logic 1.

h_{FE} = Transistor parameter called dc current gain

It is important to remember that I_C maximum does not depend on I_B . Looking at load line V_{CE} is maximum when $V_{CE} = 0$ this gives

$$I_C = \frac{V_{CC}}{R_C}$$

saturation

The condition for saturation is determined from the relationship

$$I_B \geq \frac{I_C}{h_{FE}}$$

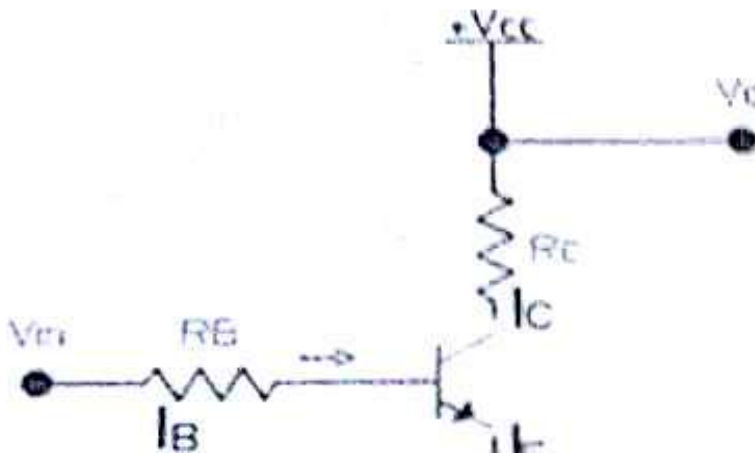
During saturation V_{CE} is not exactly 0 but almost equal to 0.2 V. V_{BE} hardly changes.

Typical npn silicon Transistor Parameters

Region	V_{BE} (Volts)	V_{CE} (volts)	Current Relationship
Cut-off	< 0.6	Open circuit	$I_B = I_C = 0$
Active	$0.6-0.7$	> 0.8	$I_C = h_{FE} I_B$
Saturation	$0.7-0.8$	0.2	$I_B \geq I_C / h_{FE}$



c



V_{BE} is less than 0.6V, transistor cut-off $I_B = 0, I_C = 0$, the collector to emitter circuit behaves like an open circuit (i.e. switch off).

Active region

V_{CE} may be any where between 0.8V up to V_{CC}

$$I_C = I_B h_{FE}$$

h_{FE} = Transistor parameter called dc current gain

It is important to remember that I_C maximum does not depend on I_B . looking at diagram line V_{CC} is maximum when $V_{CE} = 0$ this gives

$$I_C = \frac{V_{CC}}{R_C}$$

Saturation

The condition for saturation is determined from the relationship

$$I_B \geq \frac{I_C}{h_{FE}}$$

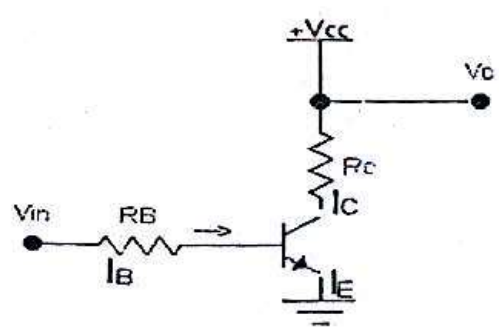
During saturation V_{CE} is not exactly 0 but almost equal to 0.2 V. V_{BE} hardly changes.

Typical npn silicon Transistor Parameters

Region	V_{BE} (Volts)	V_{CE} (volts)	Current Relationship
off Cut-off	<0.6	Open circuit	$I_B = I_C = 0$
Active	0.6-0.7	>0.8	$I_C = h_{FE} I_B$
on Short ckt Saturation	0.7-0.8	0.2	$I_B \geq \frac{I_C}{h_{FE}}$

$V_B > 0.7$

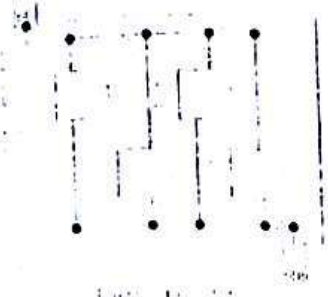
Example



Transistor-Transistor logic (TTL)

The usefulness of a DTL gate is limited by its speed of operation.

DC supply: The nominal value of the dc supply voltage for TTL (transistor-transistor logic) and CMOS (complementary metal oxide semiconductor) devices is +5V connected to V_{CC} or V_{DD} pins.



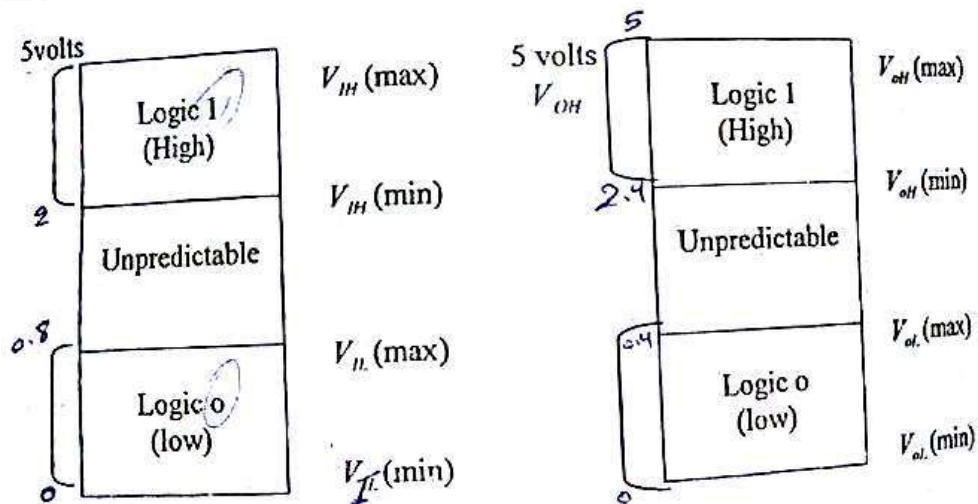
TTL logic levels

Input low = 0V to 0.8 V
 High = 2 V to V_{CC} (5V)

When an input voltage is in this range (0.8 – 2 V) it can be interpreted as either high or low by the logic circuit thus given unpredictable performance. Therefore TTL gates cannot be operated reliably when input voltages are in this range.

The output voltages of TTL

Logic low = 0 – 0.4 volts, Logic high = 2.4 – 5 volts, Unpredictable = 0.4 – 2.4 volts



$$I_{OH} = 400 \mu A$$

$$I_{IH} = 40 \mu A$$

$$I_{OL} = 16 mA$$

$$I_{IL} = 1.6 mA$$

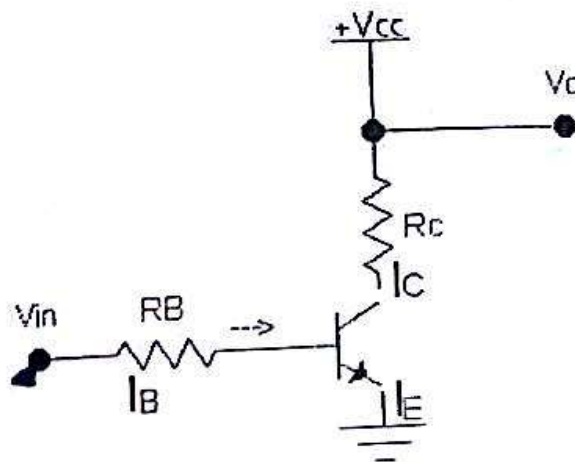
The two ratios give the same number in this case

$$\frac{400 \mu A}{40 \mu A} = \frac{16 mA}{1.6 mA} = 10$$

Therefore the fan-out of standard TTL is 10. This means that the output of a TTL gate can be connected to no more than ten inputs of other gates in the same logic family.

CMOS
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Bipolar-Transistor Characteristics



Common-Emitter npn silicon Transistor connected between V_{CC} and ground, V_i is input voltage

I_C = Collector current, it flows through R_C and the collector of the transistor

I_B = Base Current, Flows through R_B and base of the transistor

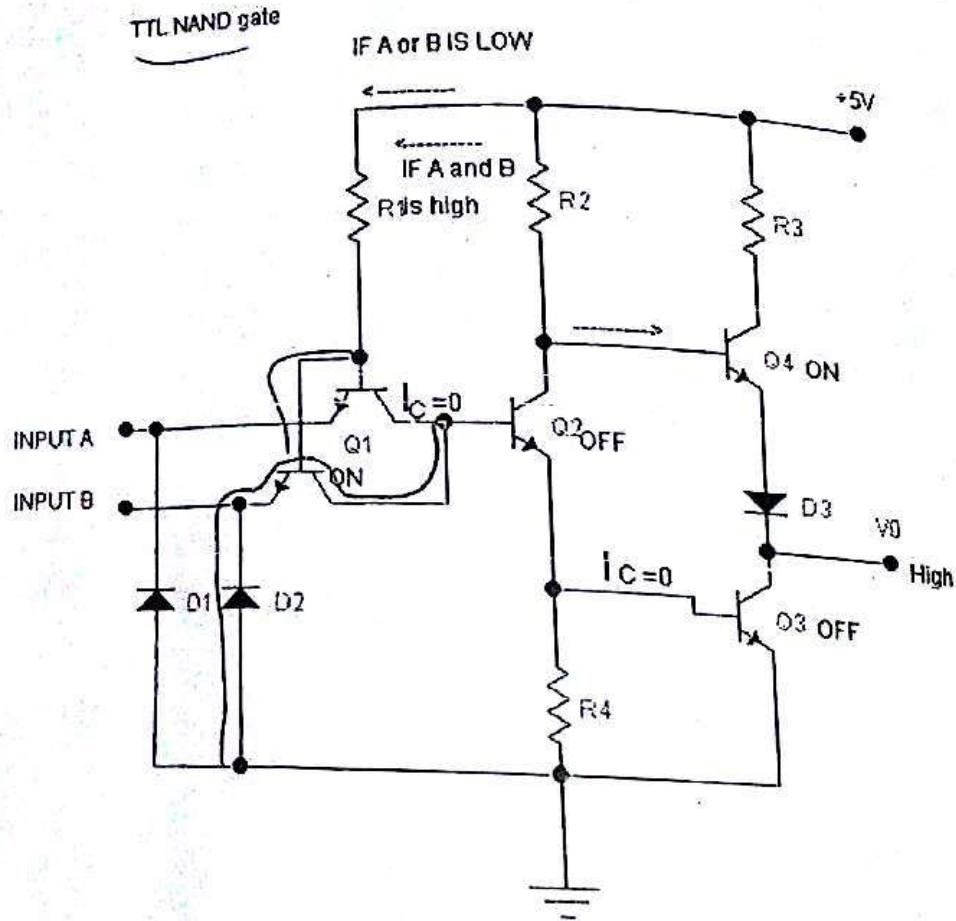
$I_E = I_C + I_B$ Emitter connected to ground, V_{CC} is supply voltage

thus turning Q4 on. A saturated Q4 provides a low resistance path from Vcc to the output. We therefore have a HIGH on the output for a low on the input. At the same time the emitter of Q2 is at ground potential keeping Q3 off.

Diode D1 is called the input clamp diode. D1 in the TTL circuit prevents negative spikes of voltage on the input from damaging Q1, Diode D2 ensures that Q4 will turn off when Q2 is on (HIGH input).

TTL NAND gate

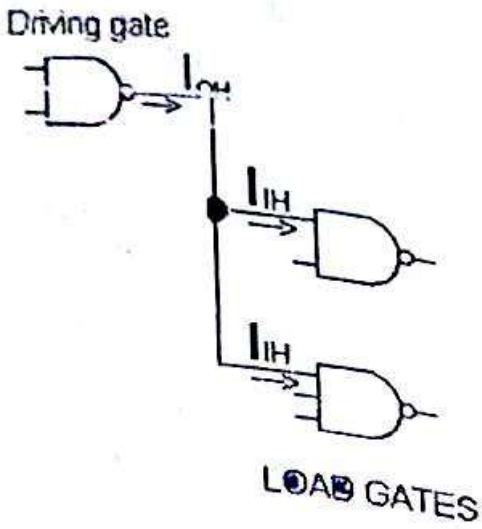
Inverter



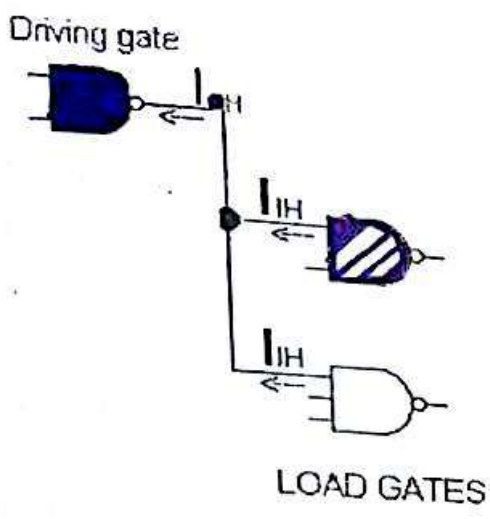
Basically TTL NAND is the same as the inverter circuit except for the additional input emitter of Q1. In TTL technology multiple-emitter transistors are used for the input devices. These multiple emitter transistors can be compared to the diode arrangement shown below.

to operate properly and is said to be overloaded. circuit above which it ceases

I_{OH}
current
output high



The fan - out is calculated from the amount of current available in the output of a gate and the amount of current needed in each input of a gate.



The Fan - out of the gate is calculated from the ratio I_{OH}/I_{IH} or I_{OL}/I_{IL} which ever is smaller. For example the standard TTL gates have the following values.

* Noise Immunity

Noise is unwanted voltage that is induced in electrical circuits and can present a threat to the proper operation of the circuit. Wires and other conductors within a system can pickup stray high - frequency electromagnetic radiation from adjacent conductors.

In order not to be adversely affected by noise, a logic circuit must have a certain amount of noise immunity. This is the ability to tolerate a certain amount of unwanted voltage fluctuation on its inputs without changing its output state.



If excessive noise causes input to go below $2v$, the gate may "think" that there is A low on its input and respond accordingly.

If excessive noise causes input to go above $0.8v$, the gate may "think" that there is A HIGH on its input and respond accordingly.

$V_{IL(max)}$

V_{IL}

▶ Noise riding on V_{IL} LEVEL