

تقدم لجنة EiCoM الأكاديمية

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جزيل الشكر للطالب:
حمزة اسماعيل

* Digital Logic and Digital Electronics :-

* Number Systems :-

* Number Systems :-

1. Decimal System.

base (10) & element (0-9).

2. Binary System :-

base (2, B) & element (0, 1).

3. Octal System :-

base (8, O) & element (0-7).

4. Hexadecimal System :-

base (16, H) & element (0-9, A-F).

* Ex :-

$$\begin{aligned} 1. (2345)_{10} &= 2 \times 10^3 + 3 \times 10^2 + 4 \times 10^1 + 5 \times 10^0 \\ &= 2000 + 300 + 40 + 5 \\ &= (2345)_{10} \end{aligned}$$

$$\begin{aligned} 2. (0.1357)_{10} &= 1 \times 10^{-1} + 3 \times 10^{-2} + 5 \times 10^{-3} + 7 \times 10^{-4} \\ &= 0.1 + 0.03 + 0.005 + 0.0007 \\ &= (0.1357)_{10} \end{aligned}$$

1

* Number System :-

* Octal Conversion to Decimal :-

$$1. \overset{2}{2}\overset{1}{3}\overset{0}{7}_{(8)} = 2 \times 8^2 + 3 \times 8^1 + 7 \times 8^0 \\ = 2 \times 64 + 24 + 7 = 159_{(10)}$$

$$2. 0.\overset{-1}{2}\overset{-2}{6}_{(8)} = 2 \times 8^{-1} + 6 \times 8^{-2} \\ = 2 \times \frac{1}{8} + 6 \times \frac{1}{4 \times 8 \times 8} = \frac{11}{32}_{(10)}$$

$$3. \overset{1}{5}\overset{0}{1}.\overset{-1}{2}\overset{-2}{4}_{(8)} = 5 \times 8^1 + 1 \times 8^0 + 2 \times 8^{-1} + 4 \times 8^{-2} \\ = 40 + 1 + \frac{1}{4} + \frac{4}{2 \times 8 \times 8} = \left(41 \frac{5}{16}\right)_{(10)}$$

* Hexadecimal Conversion to Decimal :-

$$1. \overset{3}{A}\overset{2}{B}\overset{1}{E}\overset{0}{D}_{(16)} = A \times 16^3 + B \times 16^2 + E \times 16^1 + D \times 16^0 \\ = 10 \times 16^3 + 11 \times 16^2 + 14 \times 16^1 + 13 \times 16^0 \\ = 40960 + 2816 + 224 + 13 = 44013_{(10)}$$

$$2. 0.\overset{-1}{3}\overset{-2}{C}_{(16)} = 3 \times 16^{-1} + C \times 16^{-2} \\ = 3 \times \frac{1}{16} + 12 \times \frac{1}{4 \times 16 \times 16} = \frac{15}{64}_{(10)}$$

$$3. \overset{1}{2}\overset{0}{8}.\overset{-1}{8}\overset{-2}{2}_{(16)} = 2 \times 16^1 + 8 \times 16^0 + 8 \times 16^{-1} + 2 \times 16^{-2} \\ = 32 + 8 + \frac{1}{2} + \frac{1}{128} = \left(40 \frac{65}{128}\right)_{(10)}$$

$$4. \overset{1}{A}\overset{0}{0}_{(16)} = A \times 16^1 + 0 \times 16^0 = 160_{(10)}$$

2

* Number Systems :-

* Conversion from Decimal to Binary :-

$$1. 6_{(10)} = 110_{(2)}$$

6	
3	0 → LSB
1	1
0	1 → MSB

$$2. 11_{(10)} = 1011_{(2)}$$

11	
5	1 → LSB
2	1
1	0
0	1 → MSB

$$3. 100_{(10)} = 1100100_{(2)}$$

100	
50	0 → LSB
25	0
12	1
6	0
3	0
1	1
0	1 → MSB

$$4. 151_{(10)} = 10010111_{(2)}$$

151	
75	1 → LSB
37	1
18	1
9	0
4	1
2	0
1	0
0	1 → MSB

$$\begin{aligned}
 5. 0.C4_{(16)} &= C \times 16^{-1} + 4 \times 16^{-2} \\
 &= \frac{12 \times 1}{4 \times 16} + \frac{4 \times 1}{4 \times 16 \times 16} \\
 &= \left(\frac{69}{64} \right)_{(10)}
 \end{aligned}$$

3

* Number Systems:-

* Conversion from decimal to Binary:-

$$1. 0.125_{(10)} = 0.001_{(2)}$$

0.125	
0.25	0 → MSB
0.5	0
0	1 → LSB

$$2. 0.1_{(10)} \approx 0.0001_{(2)}$$

0.1	
0.2	0
0.4	0
0.8	0
0.1	1
0.2	1
0.4	0
0.8	0
:	:

$$3. 9.25_{(10)} = 1001.01_{(2)}$$

9		0.25	
4	1 → LSB	0.5	0 → MSB
2	0	0	1 → LSB
1	0		
0	1 → MSB		

* Conversion from decimal to Octal:-

$$1. 44_{(10)} = 54_{(8)}$$

44	
5	4 → LSB
0	5 → MSB

$$2. 812_{(10)} = 1154_{(8)}$$

812	
101	4 → LSB
12	2
1	4
0	1 → MSB

4

* Number Systems:-

* Conversion from decimal to Octal:

$$1. 0.0625 = 0.04_{(8)}$$

$$\begin{array}{r|l} 0.0625^{(10)} & \\ \hline 0.5 & 0 \rightarrow \text{MSD} \\ 0 & 4 \rightarrow \text{LSD} \end{array}$$

$$2. 17.1 = 21.263_{(8)}$$

$$\begin{array}{r|l} 17.1^{(10)} & \\ \hline 2 & 1 \rightarrow \text{LSB} \\ 0 & 2 \rightarrow \text{MSB} \end{array} \quad \begin{array}{r|l} 0.1^{(10)} & \\ \hline 0.8 & 0 \\ 0.4 & 6 \\ 0.2 & 3 \\ 0.06 & 1 \end{array}$$

* Conversion from decimal to Hexadecimal:-

$$1. 171 = AB_{(16)}$$

$$\begin{array}{r|l} 171^{(10)} & \\ \hline 10 & 11 = B \rightarrow \text{LSD} \\ 0 & 10 = A \rightarrow \text{MSD} \end{array}$$

$$2. 0.03125 = 0.08_{(16)}$$

$$\begin{array}{r|l} 0.03125^{(10)} & \\ \hline 0.5 & 0 \rightarrow \text{MSD} \\ 0 & 8 \rightarrow \text{LSD} \end{array}$$

$$3. 1.1 \approx 1.199_{(16)}$$

$$\begin{array}{r|l} 1^{(10)} & \\ \hline 0 & 1 \end{array} \quad \begin{array}{r|l} 0.1^{(10)} & \\ \hline 0.6 & 1 \rightarrow \text{MSD} \\ 0.6 & 9 \\ 0.6 & 9 \rightarrow \text{LSD} \end{array}$$

$$* \text{ Bit} = 0 \text{ or } 1$$

$$\text{Nibble} = 4 \text{ Bits}$$

$$\text{Byte} = 8 \text{ bits}$$

$$\text{Word} = 8 \text{ bits}$$

[5]

* Number Systems :-

* Conversion from Binary to Octal :-

$$1. \quad \begin{array}{ccc} 1 & 0 & 1110 \\ \hline & 421 & 421 \end{array} \begin{array}{c} (2) \\ 8 \end{array} = (56)_8$$

Binary	Octal
000	0
001	1
010	2
011	3
100	4
101	5
110	6
111	7

$$2. \quad 37_{(8)} = 011111_{(2)}$$

$\begin{array}{c} 9+1 \\ \swarrow \quad \searrow \\ 1-2+4 \end{array}$

$$3. \quad 63_{(8)} = 110011_{(2)}$$

$\begin{array}{c} 4+2 \\ \swarrow \quad \searrow \\ 2+1 \end{array}$

$$4. \quad 1.1_{(8)} = 001.001_{(2)}$$

$$5. \quad 1.4_{(8)} = 001.100_{(2)}$$

* Conversion from Hexadecimal to Binary and Vice Versa :-

Binary	Hexadecimal	Binary	Hexadecimal	Binary	Hex
0000	0	0110	6	1100	C
0001	1	0111	7	1101	D
0010	2	1000	8	1110	E
0011	3	1001	9	1111	F
0100	4	1010	A		
0101	5	1011	B		

* Number System :-

* Conversion from Hexadecimal to Binary and Vice Versa :-

$$1. \text{CAB.BEE}_{(16)} = 110010101011.101111101110_{(2)}$$

$$2. 1.1_{(H)} = 2001.0001_{(2)}$$

$$3. 1.4_{(16)} = 0001.0100_{(2)} = 1.01_{(2)}$$

* Conversion from Hexadecimal to Octal and Vice Versa :-

$$1. \text{ABED}_{(16)} = \underbrace{1010}_{(2)} \underbrace{1011}_{(2)} \underbrace{1110}_{(2)} \underbrace{1101}_{(2)} = 125755_{(8)}$$

$$2. 273_{(8)} = \underbrace{010}_{(2)} \underbrace{111011}_{(2)} = 3B_{(16)}$$

$$3. (B.A)_{(16)} = \underbrace{1011}_{(2)} \underbrace{1010}_{(2)} = 13.5_{(2)}$$

7

* Number System :-

* Binary Coded Decimal (BCD) :-

$$1. 29_{(10)} = \underbrace{0010}_{\text{BCD Digit}} \underbrace{1001}_{\text{(BCD)}}$$

$$2. 49_{(10)} = \underbrace{0100}_{\text{BCD Digit}} \underbrace{1001}_{\text{(BCD)}}$$

BCD	Decimal
0 0 0 0	0
0 0 0 1	1
0 0 1 0	2
0 0 1 1	3
0 1 0 0	4
0 1 0 1	5
0 1 1 0	6
0 1 1 1	7
1 0 0 0	8
1 0 0 1	9

* Gray Code :-

Gray Code	Decimal
0 0 0 0	0
0 0 0 1	1
0 0 1 1	2
0 0 1 0	3
0 1 1 0	4
0 1 1 1	5
0 1 0 1	6
0 1 0 0	7

* Convert

$$1. \underbrace{0010}_{\text{(Gray)}} \underbrace{0111}_{\text{(Gray)}} = 35_{(10)}$$

* Implement $42_{(10)}$ in Gray Code :-

$$42_{(10)} = \underbrace{0110}_{\text{Gray digit}} \underbrace{0011}_{\text{(Gray)}}$$

[8]

* Number Systems :-

* Complement :-

1. One's complement (1's comp) :-

$$\boxed{0 \rightarrow 1} \quad , \quad \boxed{1 \rightarrow 0}$$

$$\text{Ex: } 0110_{(2)} \xrightarrow{1's \text{ comp}} 1001_{(2)}$$

2. Two's complement (2's comp) :-

$$\boxed{2's \text{ comp} = 1's \text{ comp} + 1}$$

$$\text{Ex: } 001101 \rightarrow 2's \text{ comp.}$$

$$001101_{(2)} \xrightarrow{1's \text{ comp}} 110010_{(2)}$$

$$001101_{(2)} \xrightarrow{2's \text{ comp}} 110010_{(2)} + 1$$

$$001101_{(2)} \xrightarrow{2's \text{ comp}} 110011_{(2)}$$

$$\text{Ex: } 0011000_{(2)} \rightarrow 2's \text{ comp.}$$

$$0011000_{(2)} \xrightarrow{1's \text{ comp}} 1100111_{(2)}$$

$$0011000_{(2)} \xrightarrow{2's \text{ comp}} 1100111_{(2)} + 1$$

$$0011000_{(2)} \rightarrow 11001000_{(2)}$$

$$\text{مثال آخر: } 0011000_{(2)} \rightarrow 1101000_{(2)}$$

$$\text{Ex: } 1111 \xrightarrow{1's \text{ comp}} 0000 \xrightarrow{2's} 0001$$

$$1000 \xrightarrow{2's} 0111 \xrightarrow{2's} 1000$$

$$00110 \xrightarrow{1's} 11001 \xrightarrow{2's} 11010$$

[9]

* Number Systems :-

* Representation of number in binary :-

1. Sign-magnitude representation :-

* Ex: express the following

number in 4 bit-register :-

+2

0	0	1	0
---	---	---	---

-2

1	0	1	0
---	---	---	---

+3

0	0	1	1
---	---	---	---

-3

1	1	0	1
---	---	---	---

and in 8 bit register

+2

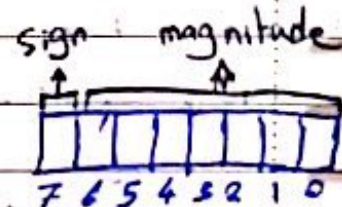
0	0	0	0	0	0	1	0
---	---	---	---	---	---	---	---

-2

1	0	0	0	0	0	1	0
---	---	---	---	---	---	---	---

+3

0	0	0	0	0	0	1	1
---	---	---	---	---	---	---	---



max/min number = $\pm(2^{n-1}-1)$
of bits

$n = 4 \text{ bits} \Rightarrow \text{max/min} = \pm(2^3-1) = \pm 7$

$n = 8 \text{ bits} \Rightarrow \text{max/min} = \pm(2^7-1) = \pm 127$

* Sign :-

Positive $\rightarrow 0$

Negative $\rightarrow 1$

2. One's Complement representation :-

Positive numbers stay the same.

Negative numbers you need 1's complement.

Ex: express the following numbers in 4 bit and 8 bits representation

+2

0	0	1	0
---	---	---	---

0	0	0	0	0	0	1	0
---	---	---	---	---	---	---	---

-2

1	1	0	1
---	---	---	---

1	1	1	1	1	1	1	0	1
---	---	---	---	---	---	---	---	---

+3

0	0	1	1
---	---	---	---

0	0	0	0	0	0	1	1
---	---	---	---	---	---	---	---

1's comp

10

* Number Systems :-

* Representation of number in binary :-

3. Two's Complement representation :-

Positive number $\xrightarrow{2's}$ no complement.

Negative number $\xrightarrow{2's}$ with complement.

* Ex :- Represent the following numbers in 2's complement representation for 4-bit data and 8-bit data.

+2	$\boxed{0010}$	$\rightarrow 2's \text{ comp.}$	+2	$\boxed{00000010}$	$\rightarrow 2's \text{ comp.}$
-2	$\boxed{1110}$		-2	$\boxed{11111110}$	
+3	$\boxed{0011}$	$\rightarrow 2's \text{ comp.}$	+3	$\boxed{00000011}$	$\rightarrow 2's \text{ comp.}$
-3	$\boxed{1101}$		-3	$\boxed{11111101}$	

4. Floating Point Representation :- نقطة عائمة

مثال : 0.6×10^{21} (Decimal)

مثال : 0.53202×10^{12} (Decimal)

* Ranges :-

1. Sign - magnitude representation $\Rightarrow$ Positive $\Rightarrow \pm (2^{n-1} - 1)$
2. One's Complement representation $\Rightarrow$ Positive $\Rightarrow \pm (2^{n-1} - 1)$
3. Two's complement representation $\Rightarrow$ Positive $\Rightarrow \pm (2^{n-1} - 1)$
 $\Rightarrow$ Negative $\Rightarrow (-2^{n-1} \rightarrow 2^{n-1} - 1)$

* Number Systems:-

* Ex:-

$$2. \quad 33_{(10)} - 49_{(10)}$$

33		49	
16	1 → LSB	24	1 → LSB
8	0	12	0
4	0	6	0
2	0	3	0
1	0	1	1
0	1 → MSB	0	1 → MSB

$$33_{(10)} = 10000_{(2)} \xrightarrow{2^5} 0010000_{(2)}$$

$$\ominus 49_{(10)} = 11000_{(2)} \xrightarrow{2^5} 1100111_{(2)}$$

$$33_{(10)} + -49_{(10)}$$

$$00100001$$

$$11001111^+$$

$$11110000 \rightarrow 2's \text{ Comp.}$$

$$3. \quad 4_{(10)} - 6_{(10)}$$

$$4_{(10)} = 0100_{(2)} \xrightarrow{2^5} 0100_{(2)}$$

$$6_{(10)} = 0110_{(2)} \xrightarrow{2^5} 1010_{(2)}$$

$$0100$$

$$1010^+$$

$$1110 \rightarrow 2's \text{ Comp.}$$

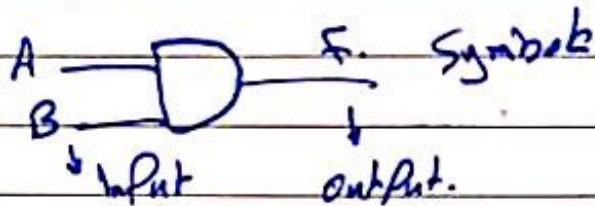
$$4_{(10)} - 6_{(10)} = -2_{(10)}$$

13

* Logic Gates :-

* Logic Gates :-

1- AND Gates :-



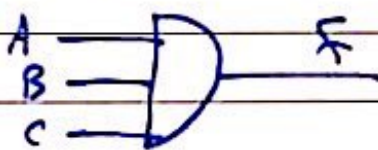
$$A \cdot B = F$$

AND operator

* Truth tables :-

A	B	F
0	0	0
0	1	0
1	0	0
1	1	1

* Three Input AND Gates :-

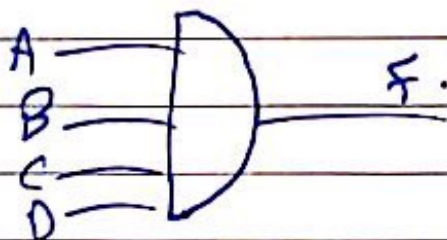


$$F = A \cdot B \cdot C$$

* Truth tables :-

A	B	C	F
0	0	0	0
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	1

* Four input AND



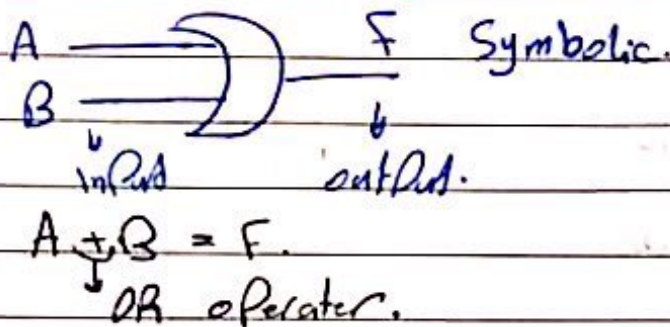
$$F = A \cdot B \cdot C \cdot D$$

14

* Logic Gates :-

* Logic gates :-

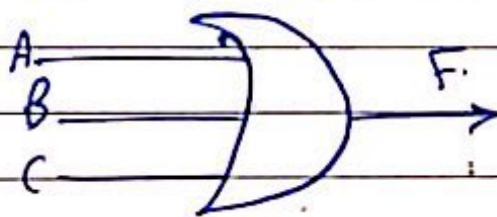
2. OR Gate :-



* Truth tables :-

A	B	F
0	0	0
0	1	1
1	0	1
1	1	1

* Three Input OR Gates :-

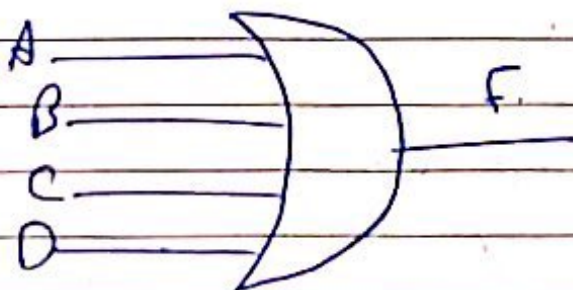


$$F = A \cup B \cup C$$

* Truth tables :-

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

* Four Input OR Gates :-



$$F = A \cup B \cup C \cup D$$

15

* Logic Gates :-

* Logic Gates :-

3- Inverter (NOT) Gates

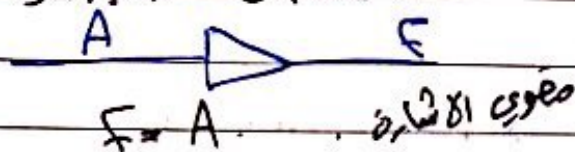


$$F = A' \rightarrow A \text{ complement}$$

* Truth tables :-

A	F
0	1
1	0

4- Buffer Gates :-



* Truth tables

A	F
0	0
1	1

* Boolean Theorems and Properties :-

1. Closure with respect to "+" operator.

Closure with respect to "." operator.

$$2. X + 0 = 0 + X = X \rightarrow \boxed{X + 1 = 1}$$

$$X \cdot 1 = 1 \cdot X = X \rightarrow \boxed{X \cdot 0 = 0}$$

$$3. \text{Commutative} \quad X + Y = Y + X.$$

$$X \cdot Y = Y \cdot X.$$

$$4. \text{Distributive} \quad X \cdot (Y + Z) = X \cdot Y + X \cdot Z$$

$$X + (Y \cdot Z) = X \cdot Y + X \cdot Z.$$

$$5. X + \bar{X} = 1$$

$$X \cdot \bar{X} = 0.$$

$$Y + \bar{Y} = 1$$

$$Y \cdot \bar{Y} = 0$$

16

* Logic Gates :-

* De Morgan Theorem :-

$$1. (X+Y)' = \overline{(X+Y)} = \bar{X} \cdot \bar{Y}$$

$$2. (X \cdot Y)' = \overline{(X \cdot Y)} = \bar{X} + \bar{Y}$$

* Ex :- $\overline{(XY + \bar{X}\bar{Y})}$

$$XY + \bar{X}\bar{Y} = (\overline{X \cdot Y}) \cdot (\overline{\bar{X} \bar{Y}})$$

$$= (\bar{X} + \bar{Y}) \cdot (\bar{\bar{X}} + \bar{\bar{Y}})$$

$$= (\bar{X} + \bar{Y}) \cdot (X + Y)$$

$$= \bar{X}X + \bar{X}Y + X\bar{Y} + XY \cdot 0$$

$$= \bar{X}Y + X\bar{Y} \quad \#$$

* Absorption Theorem :-

$$1. X + XY = X$$

$$2. X(X+Y) = X$$

Proof :-

$$- X + XY = X(1+Y)$$

$$= X(1) = X \quad \#$$

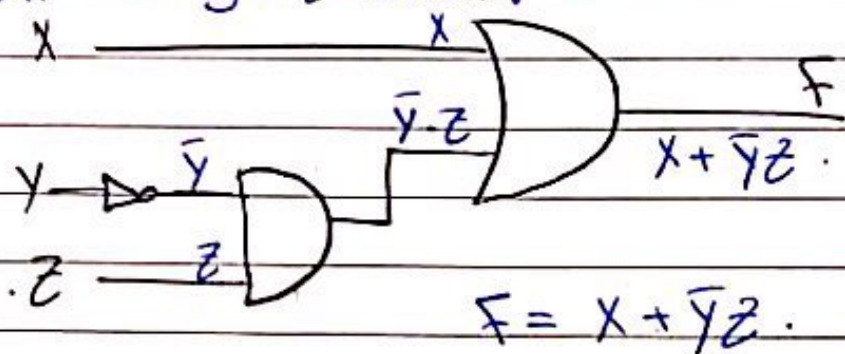
$$- X(\bar{X} + Y) = X + XY$$

$$X(1+Y) = X(1) = X \quad \#$$

17

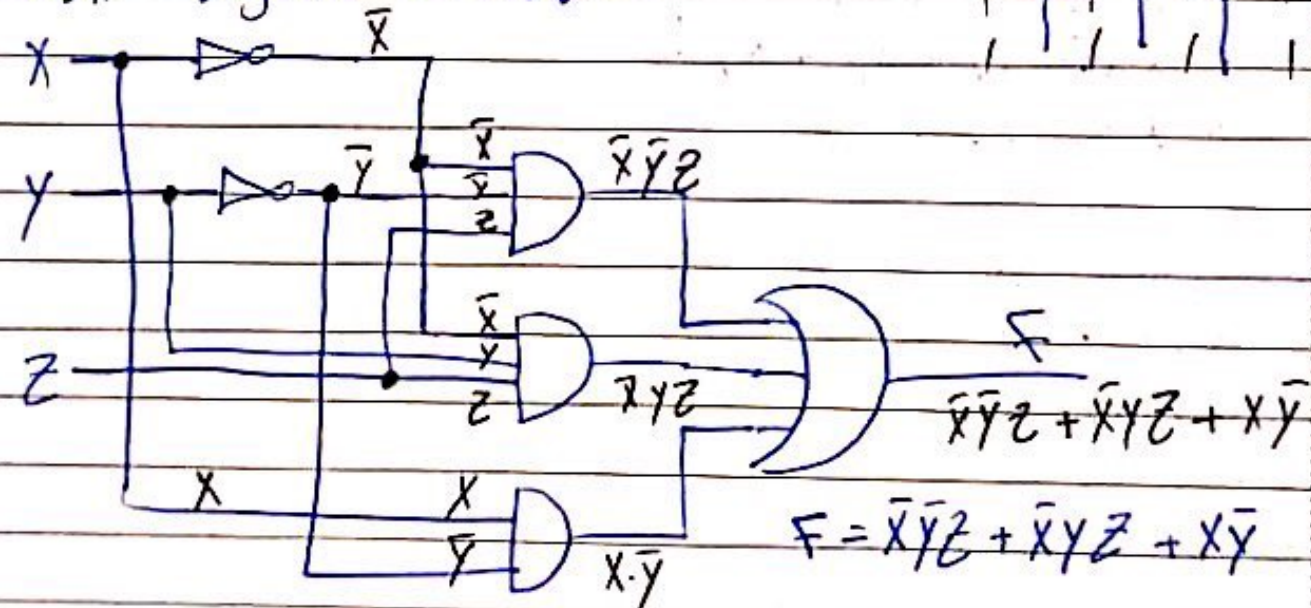
* Logic Gates :-

* Ex:- Digital Circuit :-



X	Y	Z	F
0	0	0	0
0	0	1	1
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	1

* Ex:- Digital Circuit :-

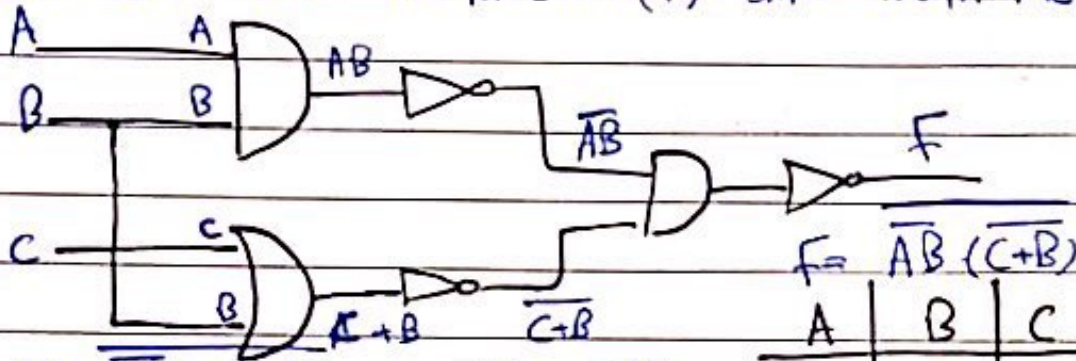


$$\begin{aligned}
 F &= \bar{X}\bar{Y}Z + \bar{X}YZ + X\bar{Y} \\
 &= \bar{X}(\bar{Y} + Y)Z + X\bar{Y} \\
 &= \bar{X}(1)Z + X\bar{Y} = \bar{X}Z + X\bar{Y}
 \end{aligned}$$

[18]

* Logic Gates :-

* Ex:- Write down the function (F) and truth table:-



$$F = (\overline{AB}) \cdot (\overline{C+B}) = AB + C + B$$

$$= B(A+1) + C = B + C$$



$$F = \overline{AB}(\overline{C+B})$$

A	B	C	F
0	0	0	0
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	0
1	0	1	1
1	1	0	1
1	1	1	1

* Use demorgan theorem to simplify:-

1. $A + B + \overline{ABC}$

$$= \overline{\overline{A} \cdot \overline{B} \cdot \overline{ABC}} = 0$$

2. $A + B + \overline{ABC}$

$$\overline{\overline{A} \cdot \overline{B} \cdot \overline{ABC}} = \overline{\overline{A} \cdot \overline{B} \cdot (\overline{A+B})C}$$

$$= \overline{\overline{A} \cdot \overline{B} \cdot (\overline{A+B})C} = \overline{\overline{A} \cdot \overline{B} \cdot \overline{C}}$$

* Logic Gates :-

* Use demorgan theorem to simplify :-

$$1. (A+B)(\bar{C}+D) \\ = (\overline{A+B}) + (\overline{\bar{C}+D}) = \bar{A}\bar{B} + C\bar{D}.$$

$$2. A(\bar{A}+AB) \\ = A\bar{A} + AAB = AB.$$

$$3. A(A+\bar{A}B) \quad [X + \bar{X}Y = X + Y] \\ = A(A+B) = A + AB = A.$$

$$4. A(A+AB) = A.A = A.$$

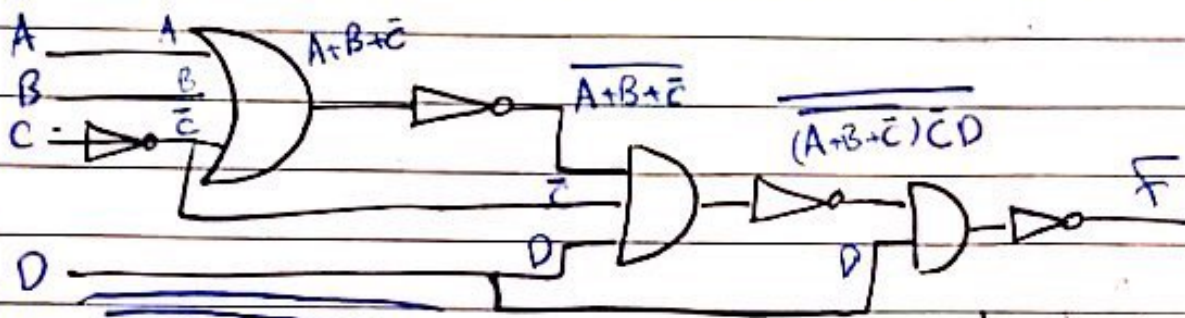
$$5. A + ABC + AB + ABCD + ABCDE = A.$$

$$6. \overline{ABCEFG} + \overline{ABCEFG} + \overline{AF} \\ = \overline{ABCEFG} \cdot \overline{ABCEFG} \cdot \overline{AF} \\ = (ABC + \bar{E}\bar{F}) \cdot (\bar{A}\bar{B}\bar{F}\bar{G} + CE) (AF) \\ = (ABC + \bar{E} + \bar{F}) (\bar{A} + \bar{B} + \bar{F} + \bar{G} + CE) (AF) \\ = AB\bar{F}\bar{G} + ABCEFG + A\bar{B}\bar{C}\bar{F} + A\bar{C}\bar{F}\bar{G} \\ + A\bar{B}\bar{E}\bar{F} + A\bar{E}\bar{G}\bar{F}.$$

[20]

* Logic Gates :-

* Ex: Write down the function F and truth table :-



$$F = \overline{(A+B+\bar{C})} \bar{C} \bar{D}$$

$$= (\overline{A+B+\bar{C}}) \bar{C} \bar{D} + \bar{D} = (\bar{A}\bar{B}\bar{C}) \bar{C} \bar{D} + \bar{D}$$

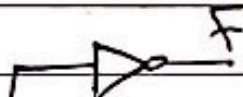
$$F = \bar{D}$$

A —

B —

C —

D —



$$F = \bar{D}$$

A	B	C	D	F
0	0	0	0	1
0	0	0	1	0
0	0	1	0	1
0	0	1	1	0
0	1	0	0	1
0	1	0	1	0
0	1	1	0	1
0	1	1	1	0
1	0	0	0	1
1	0	0	1	0
1	0	1	0	1
1	0	1	1	0
1	1	0	0	1
1	1	0	1	0
1	1	1	0	1
1	1	1	1	0

* Logic Gates :-

5- NAND Gates :-

NAND = AND Not Gates.

A	B	F
0	0	1
0	1	1
1	0	1
1	1	0



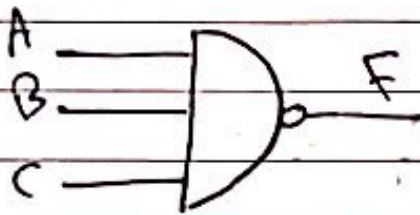
$$F = \overline{AB}$$

$$F = \bar{A} + \bar{B}$$

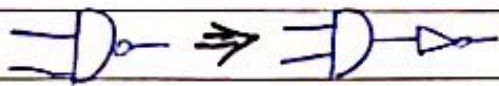
* Logic Gates :-

* Logic Gates :-

5. NAND Gates :-



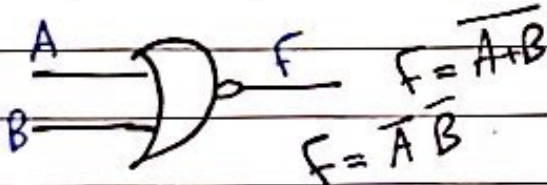
$$F = \overline{ABC} = \bar{A} + \bar{B} + \bar{C}$$



A	B	C	F
0	0	0	1
0	0	1	1
0	1	0	1
0	1	1	1
1	0	0	1
1	0	1	1
1	1	0	1
1	1	1	0

6. NOR Gates :-

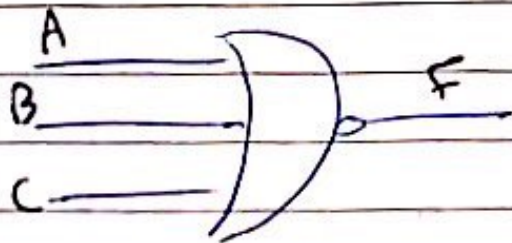
NOR = OR Not Gates.



$$F = \overline{A+B}$$

$$F = \bar{A}\bar{B}$$

A	B	F
0	0	1
0	1	0
1	0	0
1	1	0



A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0

A	B	C	F
1	0	0	0
1	0	1	0
1	1	0	0
1	1	1	0

* Logic Gates :-

* Logic Gates :-

7- Exclusive OR (XOR) :-



$$F = A \oplus B = A\bar{B} + \bar{A}B$$

A	B	F
0	0	0
0	1	1
1	0	1
1	1	0

No Three or more input for XOR Gates

Only two input.



$$F = (A \oplus B) \oplus C$$

$$F = (A\bar{B} + \bar{A}B) \oplus C$$

$$F = (A\bar{B} + \bar{A}B)\bar{C} + (\overline{A\bar{B} + \bar{A}B})C$$

$$F = (A\bar{B} + \bar{A}B)\bar{C} + (\overline{A\bar{B} + \bar{A}B})C$$

$$F = A\bar{B}\bar{C} + \bar{A}B\bar{C} + ((\overline{A\bar{B}}) \cdot (\overline{\bar{A}B}))C$$

$$F = A\bar{B}\bar{C} + \bar{A}B\bar{C} + ((\bar{A}+B)(A+\bar{B}))C$$

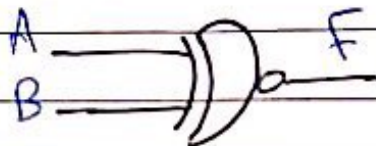
$$F = A\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C + AB\bar{C}$$

23

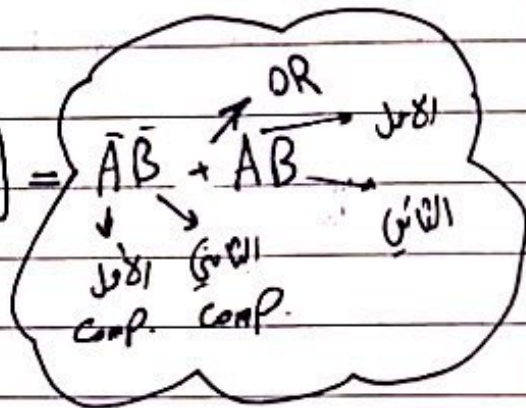
* Logic Gates :-

* Logic Gates :-

2- Exclusive NOR (XNOR)



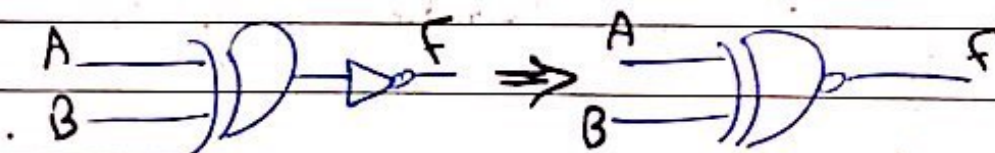
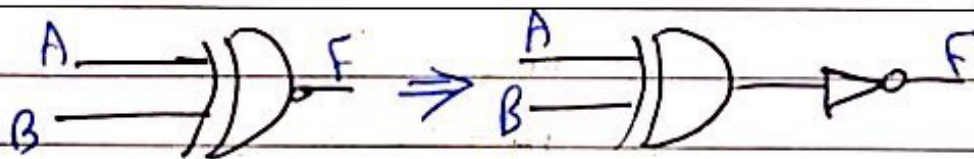
$$F = A \oplus B$$



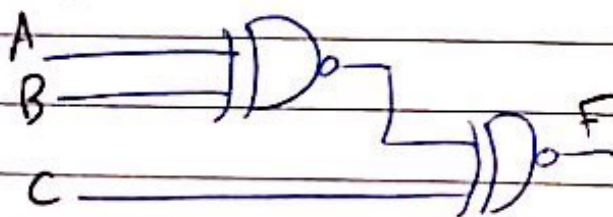
$$F = A \oplus B = \overline{A\bar{B}} + \overline{\bar{A}B}$$

$$F = (\overline{A\bar{B}}) \cdot (\overline{\bar{A}B}) = (\bar{A} + B)(A + \bar{B})$$

$$F = \bar{A}\bar{B} + AB$$



Only Two input



$$F = (A \oplus B) \oplus C$$

$$F = (\bar{A}\bar{B} + AB)\bar{C} + (\bar{A}\bar{B} + AB)C$$

$$F = (\bar{A}\bar{B} + \bar{A}B) \bar{C} + \bar{A}\bar{B}C + ABC$$

$$F = \bar{A}\bar{B}\bar{C} + \bar{A}B\bar{C} + \bar{A}\bar{B}C + ABC$$

$$F = (A \oplus B) \oplus C$$

$$= (\bar{A}\bar{B} + AB) \oplus C$$

$$= (\bar{A}\bar{B} + AB)\bar{C} + (\bar{A}\bar{B} + AB)C$$

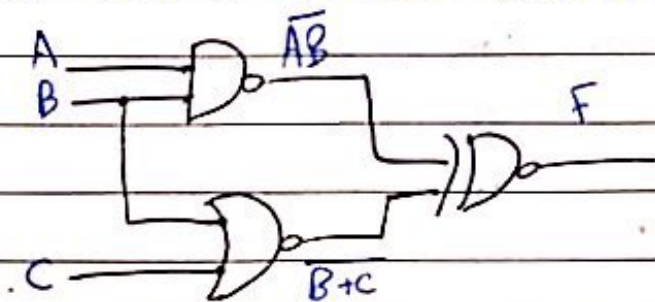
$$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$$

Comp. A̅B̅ Comp. A̅B̅ A̅B̅ A̅B̅

24

* Logic Gates :-

* Ex :- Find the function and the truth table :-



A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	0
0	1	1	0
1	0	0	1
1	0	1	0
1	1	0	1
1	1	1	1

$$F = \overline{AB} (\overline{B+C}) + \overline{AB} (B+C)$$

$$F = (AB)(B+C) + (\overline{A} + \overline{B})(\overline{B+C})$$

$$F = AB + ABC + \overline{A}\overline{B}\overline{C} + \overline{B}\overline{C}$$

$$F = AB(1+C) + \overline{B}\overline{C}(\overline{A}+1)$$

$$F = AB + \overline{B}\overline{C} \#$$

K-map :-

Number	x	y	z	Minterms Term Designation	Maxterms Term Designation
0	0	0	0	$\overline{x}\overline{y}\overline{z}$ ① m_0	$x+y+z$ ① M_0
1	0	0	1	$\overline{x}\overline{y}z$ ② m_1	$x+y+\overline{z}$ ② M_1
2	0	1	0	$\overline{x}y\overline{z}$ ③ m_2	$x+\overline{y}+z$ ③ M_2
3	0	1	1	$\overline{x}yz$ ④ m_3	$x+\overline{y}+\overline{z}$ ④ M_3
4	1	0	0	$x\overline{y}\overline{z}$ ⑤ m_4	$\overline{x}+y+z$ ⑤ M_4
5	1	0	1	$x\overline{y}z$ ⑥ m_5	$\overline{x}+y+\overline{z}$ ⑥ M_5
6	1	1	0	$xy\overline{z}$ ⑦ m_6	$\overline{x}+\overline{y}+z$ ⑦ M_6
7	1	1	1	xyz ⑧ m_7	$\overline{x}+\overline{y}+\overline{z}$ ⑧ M_7

Sum of Product (SOP)

$x, y, z \rightarrow 1$
 $\overline{x}, \overline{y}, \overline{z} \rightarrow 0$

25

$\overline{x}, \overline{y}, \overline{z} \rightarrow 1$
 $x, y, z \rightarrow 0$

Logic Gates :-

* Ex :- $F(x, y) = (\bar{x} + y) \cdot (\bar{x} + \bar{y})$
 $= M_2 \cdot M_3 = \prod (2, 3)$
 $= \prod (2, 3)$

	X	Y	F
m_0	0	0	1
m_1	0	1	1
m_2	1	0	0
m_3	1	1	0

* Ex: $F(A, B) = AB + \bar{A}\bar{B}$
 $= m_3 + m_0$
 $= \sum (0, 3)$

	A	B	F
m_0	0	0	1
m_1	0	1	0
m_2	1	0	0
m_3	1	1	1

K-map :-

number	A	B	miniterms	maxterms
0	0	0	$\bar{A}\bar{B} \quad m_0$	$A+B \quad M_0$
1	0	1	$\bar{A}B \quad m_1$	$A+\bar{B} \quad M_1$
2	1	0	$A\bar{B} \quad m_2$	$\bar{A}+B \quad M_2$
3	1	1	$AB \quad m_3$	$\bar{A}+\bar{B} \quad M_3$

$\bar{A}, \bar{B} \rightarrow 0$

$A, B \rightarrow 0$

$A, B \rightarrow 1$

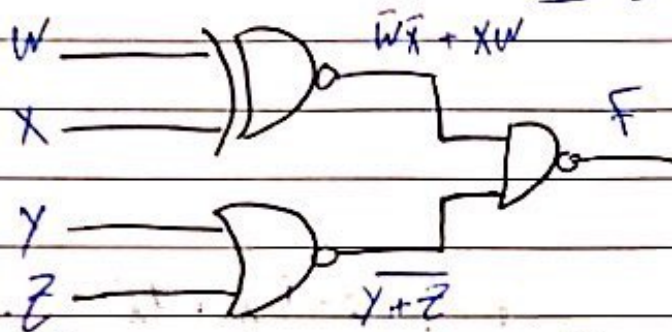
$\bar{A}, \bar{B} \rightarrow 1$

$\sum_{i=1}^n X_i$
 $= X_1 + X_2 + X_3$

26

* Logic Gates :-

* Ex: Represent the following system in all possible representation:-



$$F = (\overline{Y+Z})(\overline{W}X + WX)$$

$$F = (Y+Z) + (\overline{W}X + WX)$$

$$F = (Y+Z) + (\overline{W}X + WX)$$

$$F(W, X, Y, Z) = Y + Z + \overline{W}X + W\overline{X}$$

SOP →

$$F(W, X, Y, Z) = Y + Z + \overline{W}X + W\overline{X}$$

$$F = (W + \overline{W})Y(X + \overline{X})Y(Z + \overline{Z}) + (W + \overline{W})(X + \overline{X})(Y + \overline{Y})Z + \overline{W}X(Y + \overline{Y})(Z + \overline{Z}) + W\overline{X}(Y + \overline{Y})(Z + \overline{Z})$$

Standard Form.

Y			
W	X	Y	Z
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

Z			
W	X	Y	Z
0	0	0	0
0	0	0	1
0	0	1	0
0	0	1	1
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1

W̄X			
W	X	Y	Z
0	1	0	0
0	1	0	1
0	1	1	0
0	1	1	1
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

WX			
W	X	Y	Z
1	0	0	0
1	0	0	1
1	0	1	0
1	0	1	1
1	1	0	0
1	1	0	1
1	1	1	0
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

$$F = \sum (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15)$$

$$F = \prod (0, 12) = M_0 \cdot M_{12}$$

$$F = (W + X + Y + Z) \cdot (\overline{W} + \overline{X} + \overline{Y} + \overline{Z})$$

Pos.

*Logic Gates :-

*Ex:- Truth table :-

$$F = Y + Z + \bar{W}X + W\bar{X}$$

W	X	Y	Z	F	$\bar{F}$
0	0	0	0	0	1
0	0	0	1	1	0
0	0	1	0	1	0
0	0	1	1	1	0
0	1	0	0	1	0
0	1	0	1	1	0
0	1	1	0	1	0
0	1	1	1	1	0
1	0	0	0	1	0
1	0	0	1	1	0
1	0	1	0	1	0
1	0	1	1	1	0
1	1	0	0	0	1
1	1	0	1	1	0
1	1	1	0	1	0
1	1	1	1	1	0

$$F = Y + Z + \bar{W}X + W\bar{X}$$

$$F = \Sigma(1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15)$$

$$F = m_1 + m_2 + m_3 + \dots + m_{15}$$

$$F = \Pi(0, 12)$$

$$= M_0 \cdot M_{12}$$

$$F = \Sigma(1, 2, 3, 4, \dots, 11, 13, 14, 15)$$

$$F = \bar{W}\bar{X}\bar{Y}Z + \bar{W}\bar{X}YZ + \dots$$

$$F = \Pi(0, 12) = M_0 \cdot M_{12}$$

$$F = (W \cdot X + Y + Z) \cdot (\bar{W} + \bar{X} + \bar{Y} + Z)$$

* Logic Gates :-

* Ex :-

$$F(A, B, C) = AB + BC' \cdot (A + C)$$

$$F = AB + BC'A + BC'C$$

$$F = AB + ABC'$$

$$F = AB(C + C') + ABC'$$

$$F = ABC + ABC' + ABC'$$

$$F = ABC + AB\bar{C}$$

$$F = m_7 + m_6$$

$$F = \Sigma(6, 7)$$

$$F = \Pi(0, 1, 2, 3, 4, 5)$$

A	B	C	F	$\bar{F}$
0	0	0	0	1
0	0	1	0	1
0	1	0	0	1
0	1	1	0	1
1	0	0	0	1
1	0	1	0	1
1	1	0	1	0
1	1	1	1	0

* Karnaugh map (K-map) :-

1- Two Variable (K-map) :-

A \ B	0	1
0	m_0	m_1
1	m_2	m_3

B	0	1
0	m_0	m_2
1	m_1	m_3

$$F(A, B) = A\bar{B} + AB$$

$$F(A, B) = m_0 + m_2$$

$$F(A, B) = \Sigma(0, 2)$$

B	0	1
0	0	0
1	1	1

$$F(A, B) = A$$

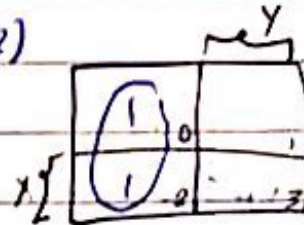
29

* Logic Gates :-

* Ex:- Simplify $F(x,y) = \sum(0,2)$

$$F(x,y) = \sum(0,2)$$

$$F(x,y) = \bar{y}$$



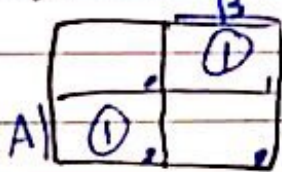
* Ex:- K-map.



$$F(A,B) = \bar{A}\bar{B} + AB \rightarrow \underline{\text{XNOR}}$$

$$F(A,B) = A \odot B$$

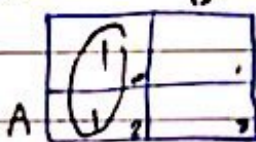
* Ex:- K-map:-



$$F(A,B) = A\bar{B} + \bar{A}B \rightarrow \underline{\text{XOR}}$$

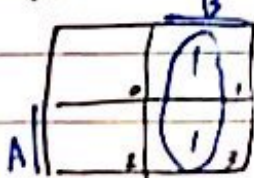
$$F(A,B) = A \oplus B$$

* Ex:- K-map:-



$$F(A,B) = \bar{B}$$

* Ex:- K-map:-

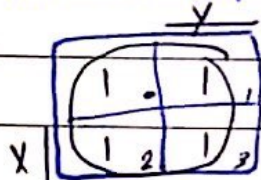


$$F(A,B) = B$$

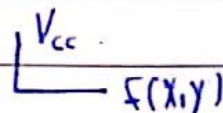
30

* Logic Gates :-

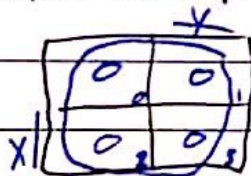
* Ex:- K-map :-



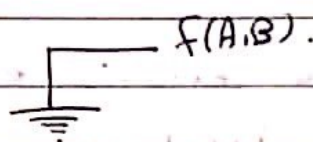
$$F(x,y) = 1$$



* Ex:- K-map :-



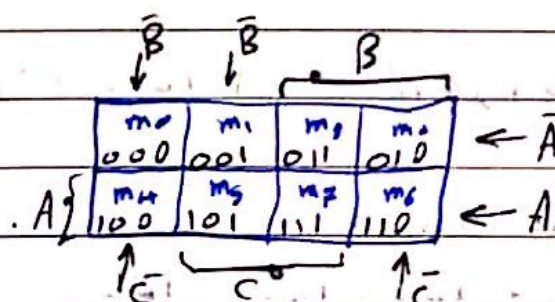
$$F(x,y) = 0$$



* K-map :-

2 Three Variable K-map :-

A	B	C	
0	0	0	m_0
0	0	1	m_1
0	1	0	m_2
0	1	1	m_3
1	0	0	m_4
1	0	1	m_5
1	1	0	m_6
1	1	1	m_7



* Three Variable K-map

K-map $\Rightarrow$ Cylinder

cilindro şeklinde K-map'dır.

(Three Variable K-map) N.

* Logic Gates :-

* Ex:- Simplify $F(A, B, C) = \sum(0, 1, 2, 6)$.

	<u>B</u>			
	0	1	2	3
A	4	5	6	7

$$F(A, B, C) = \bar{A}\bar{B} + B\bar{C}$$

* Ex:- Find the standard form for:-

$$F(A, B, C) = A + B + C$$

a) Using minterm.

b) using maxterm.

a)

	<u>B</u>			
	0	1	2	3
A	4	5	6	7

b)

	<u>B</u>			
	0	1	2	3
A	4	5	6	7

$$F(A, B, C) = \sum(1, 2, 3, 4, 5, 6, 7)$$

$$F(A, B, C) = \prod(0)$$

* Ex:- Simplify using K-map:-

$$F(A, B, C) = \bar{A}BC + A\bar{B}C + \bar{A}BC + ABC$$

$$= m_7 + m_5 + m_3 + m_6 = \sum(3, 5, 6, 7)$$

$$F(A, B, C) = AC + AB + BC$$

	<u>B</u>			
	0	1	2	3
A	4	5	6	7

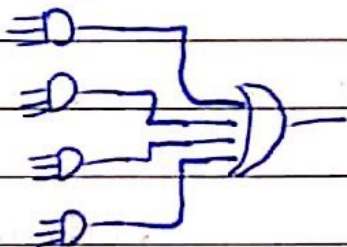
[32]

* Logic Gates :-

* Ex:-

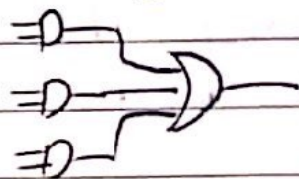
$$F(A, B, C) = ABC + A\bar{B}C + \bar{A}BC + A\bar{B}\bar{C}$$

as logic Gates



$$F(A, B, C) = AC + BC + AB$$

as logic Gates



* Rules for K-map simplification using minterms :-

1. Cover all ones and none of zeros.
2. Minimize the number of circles (term).
3. Maximize the size for every circles (variables).
4. Every circle should be included 1, 2, 4, 8, 16 ones.

* Four Variable K-map :-

		C			
		m ₀ 0000	m ₁ 0001	m ₃ 0011	m ₂ 0010
		m ₄ 0100	m ₅ 0101	m ₇ 0111	m ₆ 0110
		m ₁₂ 1100	m ₁₃ 1101	m ₁₅ 1111	m ₁₄ 1110
A	B	m ₈ 1000	m ₉ 1001	m ₁₁ 1011	m ₁₀ 1010

$$F(A, B, C, D)$$

W X Y Z

* Four Variable K-map

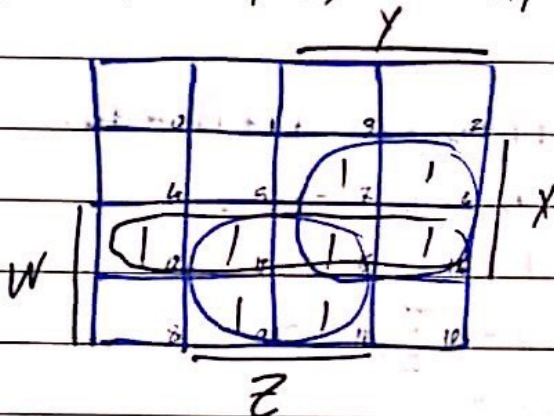
K-map $\Rightarrow$ sphere

في K-map على شكل

(Four Variable K-map) .

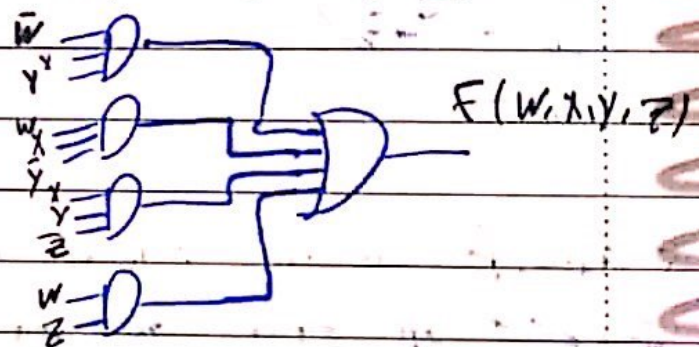
* Logic Gates :-

* Ex:- $F(W, X, Y, Z) = \bar{W}XY + W\bar{X}\bar{Y} + XY\bar{Z} + WZ$

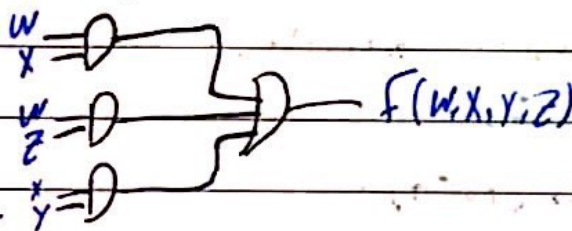


$$F(W, X, Y, Z) = W\bar{X} + WZ + XY$$

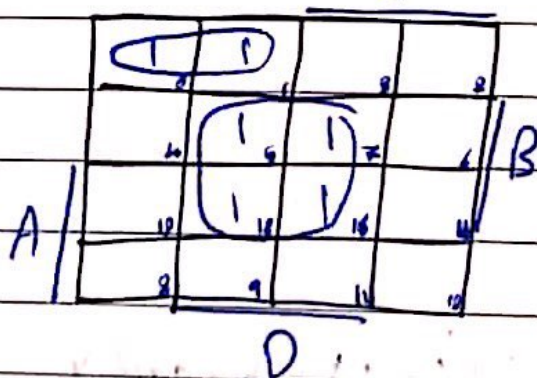
$$F(W, X, Y, Z) = \bar{W}XY + W\bar{X}\bar{Y} + XY\bar{Z} + WZ$$



$$F(W, X, Y, Z) = W\bar{X} + WZ + XY$$



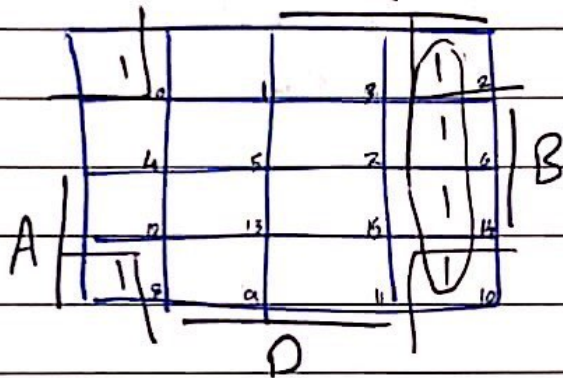
* Ex:- Simplify $F(A, B, C, D) = \sum(0, 1, 5, 7, 13, 15)$



$$F(A, B, C, D) = \bar{A}\bar{B}\bar{C} + BD$$

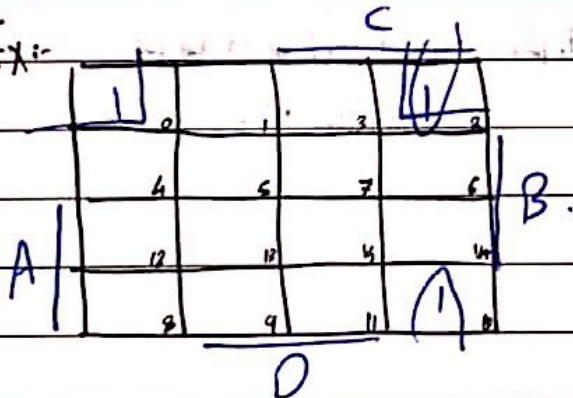
* Logic Gates :-

* Ex:- Find the function :-



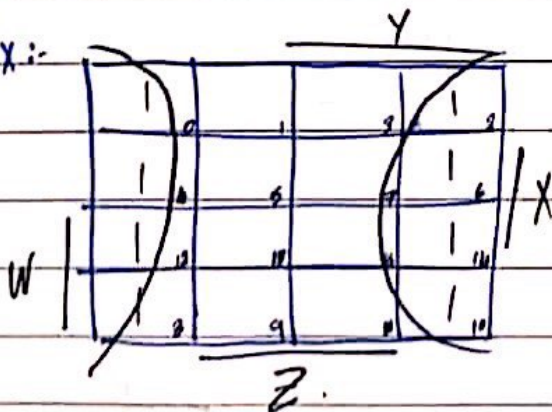
$$F(A, B, C, D) = C\bar{D} + \bar{B}\bar{D}$$

* Ex:-



$$F(A, B, C, D) = \bar{A}\bar{B}\bar{D} + \bar{B}C\bar{D}$$

* Ex:-

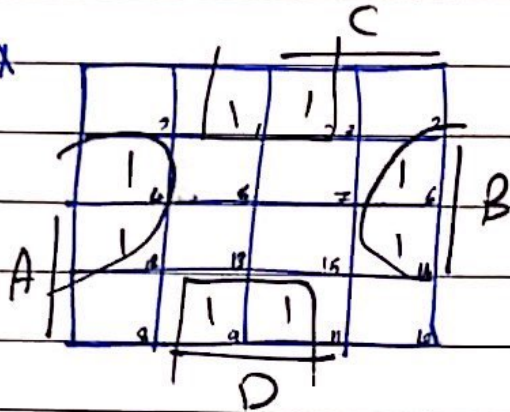


$$F(W, X, Y, Z) = \bar{Z}$$

35

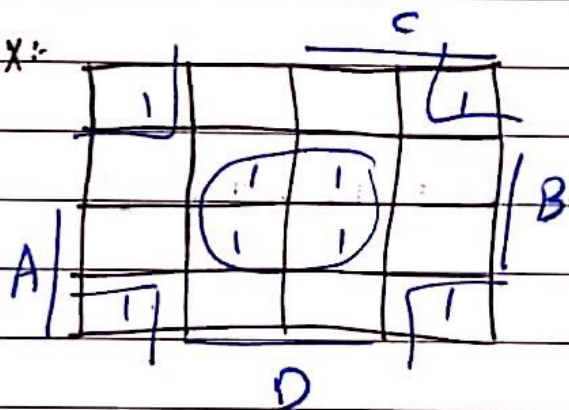
* Logic Gates :-

* Ex



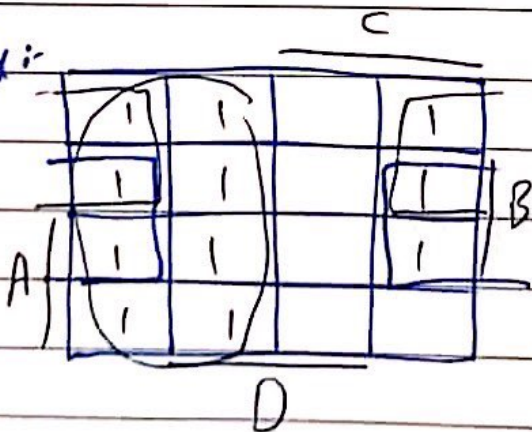
$$F(A, B, C, D) = B\bar{D} + \bar{B}D \\ = B \oplus D.$$

* Ex:-



$$F(A, B, C, D) = \bar{B}\bar{D} + B D \\ = B \odot D.$$

* Ex:-



$$F(A, B, C, D) =$$

36

* Logic Gates :-

* Ex :- $F(A, B, C, D)$ Simplify.

A	B	C	D	F
---	---	---	---	---

0	0	0	0	0
---	---	---	---	---

 m_0

0	0	0	1	0
---	---	---	---	---

 m_1

0	0	1	0	1
---	---	---	---	---

 m_2

0	0	1	1	0
---	---	---	---	---

 m_3

0	1	0	0	1
---	---	---	---	---

 m_4

0	1	0	1	0
---	---	---	---	---

 m_5

0	1	1	0	1
---	---	---	---	---

 m_6

0	1	1	1	0
---	---	---	---	---

 m_7

1	0	0	0	0
---	---	---	---	---

 m_8

1	0	0	1	0
---	---	---	---	---

 m_9

1	0	1	0	1
---	---	---	---	---

 m_{10}

1	0	1	1	0
---	---	---	---	---

 m_{11}

1	1	0	0	1
---	---	---	---	---

 m_{12}

1	1	0	1	0
---	---	---	---	---

 m_{13}

1	1	1	0	1
---	---	---	---	---

 m_{14}

1	1	1	1	0
---	---	---	---	---

 m_{15}


$$F(A, B, C, D) = B\bar{D} + C\bar{D}$$

37

* Logic Gates :-

* Five - Variables K-map :-

D				D			
m_0	m_1	m_3	m_2	m_{16}	m_{17}	m_{19}	m_{18}
0000	0001	0011	0010	1000	1001	1011	1010
m_4	m_5	m_7	m_6	m_{20}	m_{21}	m_{23}	m_{22}
0010	0011	0011	0010	1010	1011	1011	1010
m_{12}	m_{13}	m_{15}	m_{14}	m_{28}	m_{29}	m_{31}	m_{30}
0100	0101	0111	0110	1100	1101	1111	1110
m_8	m_9	m_{11}	m_{10}	m_{24}	m_{25}	m_{27}	m_{26}
0100	0101	0101	0100	1100	1101	1101	1100

* Ex: Simplify :-

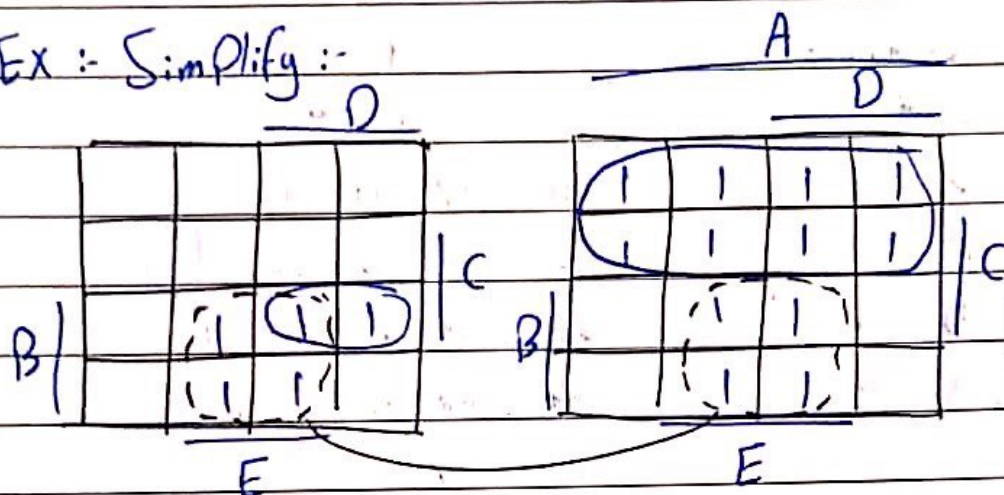
D				D			

$$F(A, B, C, D, E) = \bar{A}\bar{B}D + B\bar{C}\bar{E} + \bar{B}DE$$

38

* Logic Gates :-

* Ex :- Simplify :-

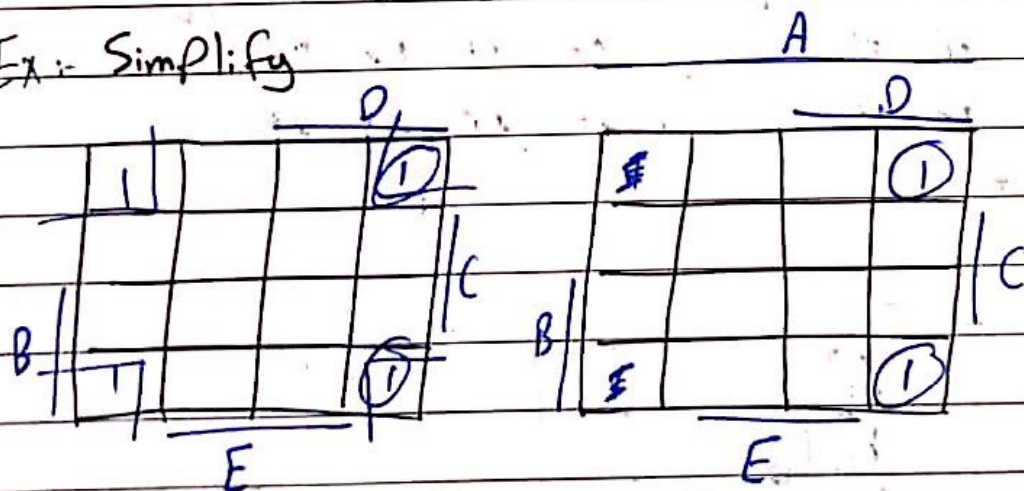


قل عدد
الدوائر
وزيد حجمها

$$F(A, B, C, D, E) = \bar{A}BCD + ABE + \bar{A}BE + A\bar{B}$$

$$\Rightarrow F(A, B, C, D, E) = BE + \bar{A}BCD + A\bar{B}$$

* Ex :- Simplify



$$F(A, B, C, D, E) = \bar{A}\bar{C}\bar{E} + \bar{C}D\bar{E}$$

39

* Logic Gates :-

* Six Variable K-map :-

		E				
		0	1	3	2	
		4	5	7	6	
C		12	13	15	14	D
		8	9	11	10	
		F				

		E				
		16	17	19	18	
		20	21	23	22	
C		28	29	31	30	D
		24	25	27	26	
		F				

		E				
		32	33	35	34	
		36	37	39	38	
B		44	45	47	46	D
		40	41	43	42	
		F				

		E				
		48	49	51	50	
		52	53	55	54	
C		60	61	63	62	D
		56	57	59	58	
		F				

* Ex :-

		C				
					1	
					1	
A		1				B
		1				
		D				

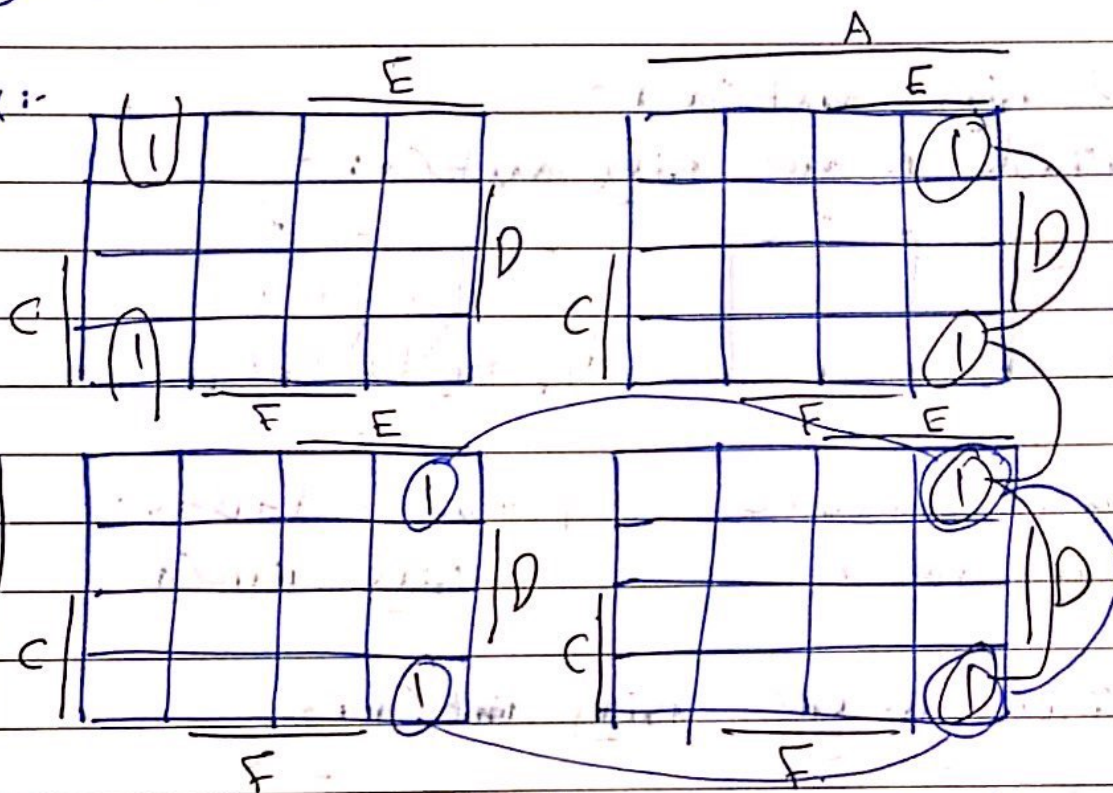
$$F(A,B,C,D) = A\bar{C}D' + BCD + A\bar{C}D$$

		C				
					1	
					1	
A		1				B
		1				
		D				

$$F(A,B,C,D) = A\bar{C}D' + ABD + A\bar{C}D$$

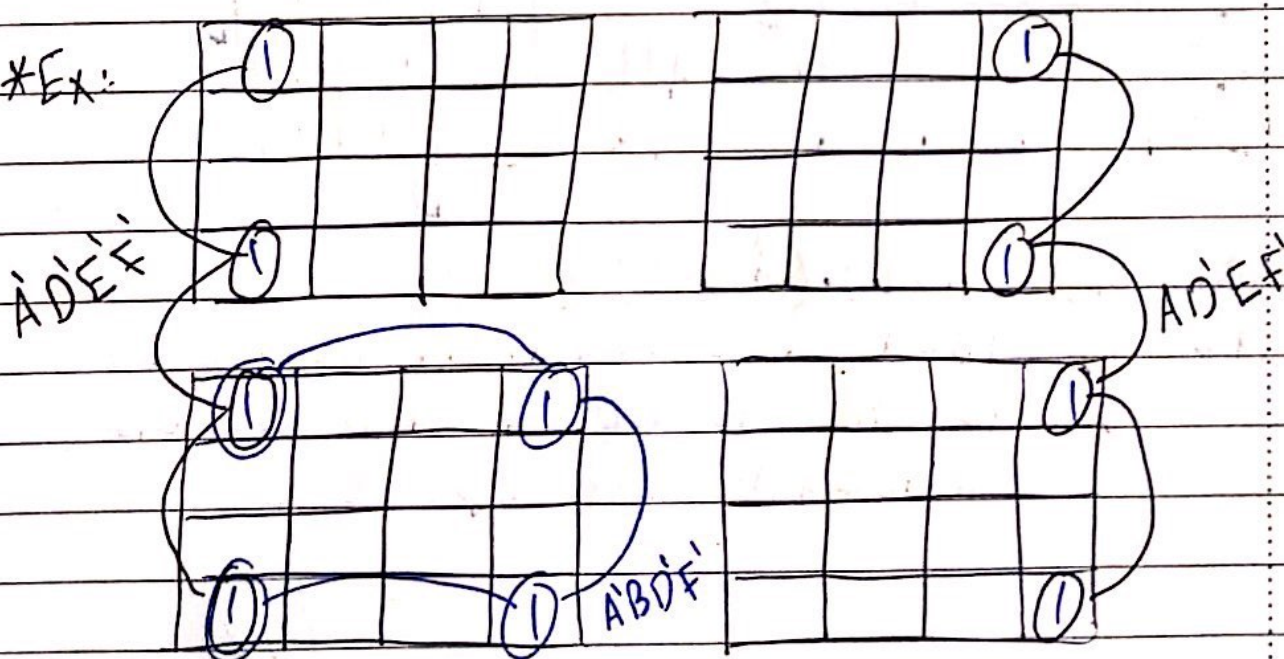
* Logic Gates :-

* Ex :-



$$F(A, B, C, D, E, F) = \bar{A}\bar{B}\bar{D}\bar{E}F' + B\bar{D}E\bar{F} + A\bar{D}EF'$$

* Ex :-



* Logic Gates :-

* K-map using maxterm :-

* 3 Variable K-map using maxterm :-

$$F(A, B, C) = A + B + C$$

using maxterm

$$F(A, B, C) = A + B + C$$

Using minterm

* 4 Variable K-map using maxterm :-

=

$$F(A, B, C, D) = (A + B + D) \cdot (\bar{A} + B + C) \cdot (C + D)$$

$$F(A, B, C, D) = (AB + AC + A'B + B + BC + A'D + BD + DC) \cdot (C + D)$$

$$= ABC + ABD + AC + ACD + A'B'C + A'B'D + BC + BD + A'DC + A'D + BCD + BD + DC + DC$$

42

* Logic Gates :-

* K-map using BCD :-

for BCD we have :- 2 or 1 or X

where X :: not used.

W	X	Y	Z	Decimal	F
0	0	0	0	0	0
0	0	0	1	1	1
0	0	1	0	2	0
0	0	1	1	3	1
0	1	0	0	4	0
0	1	0	1	5	1
0	1	1	0	6	0
0	1	1	1	7	1
1	0	0	0	8	0
1	0	0	1	9	1
1	0	1	0	X	X
1	0	1	1	X	X
1	1	0	0	X	X
1	1	0	1	X	X
1	1	1	0	X	X
1	1	1	1	X	X

C				B
0	1	1	2	
	1	1	1	X
X	X	X	X	
	1	1	1	X
				X

$$F(A, B, C, D) = D$$

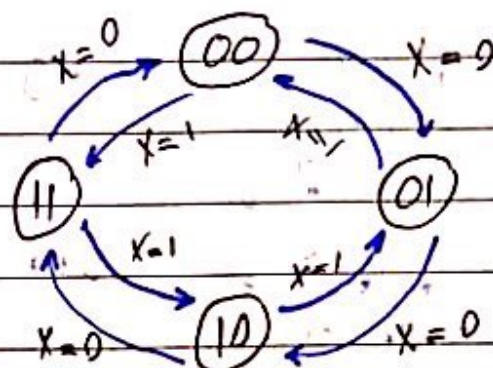
ال not used
ع حسابها (بجربا نفسا)
إذا بنقدر نوسع الدائرة

43

* Digital logic and Digital Electronics :-

* Design a 2-bit up/down Counter :-

A	B	X	F ₁	F ₂
0	0	0	0	1
0	0	1	1	1
0	1	0	1	0
0	1	1	0	0
1	0	0	1	1
1	0	1	0	1
1	1	0	0	0
1	1	1	1	0

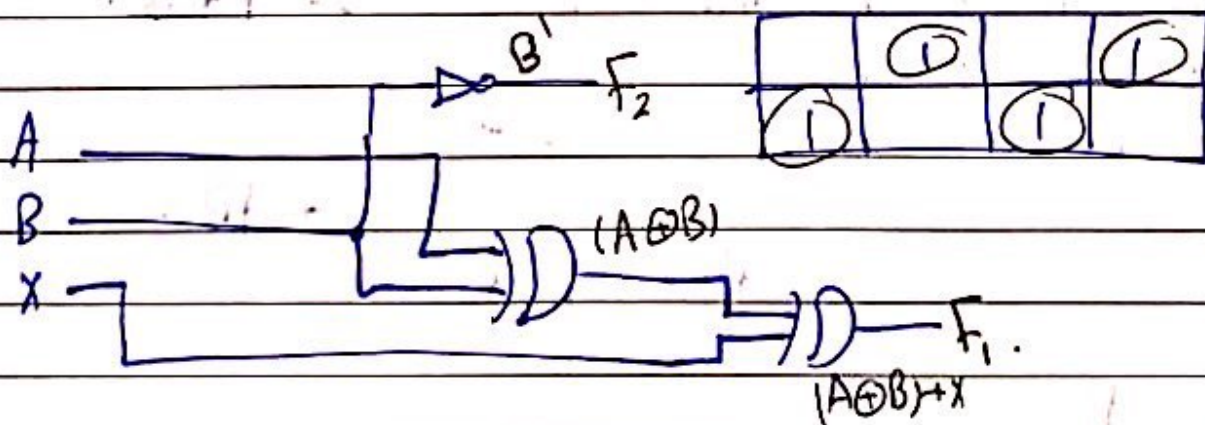


$$(A \oplus B) \oplus X$$

$$(A \oplus B) \bar{X} + (\overline{A \oplus B}) X$$

$$(A\bar{B} + \bar{A}B) \bar{X} + (A \oplus B) X$$

$$F_2 = B' \quad A\bar{B}\bar{X} + \bar{A}B\bar{X} + \bar{A}\bar{B}X + ABX$$



44

* Adder :-

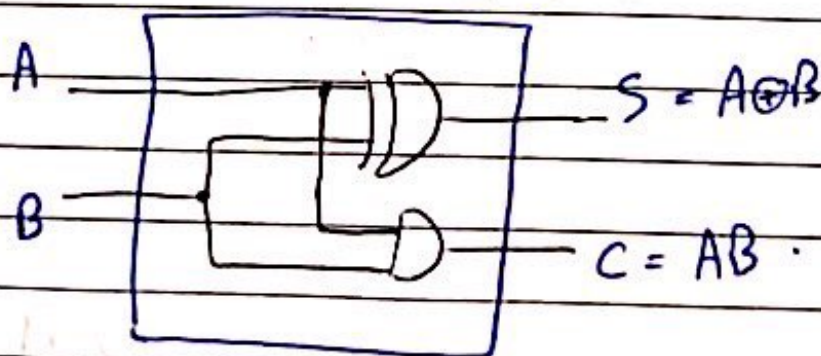
0	1	0		C_2	
0	0	1	1	C_1	Carry (C)
0	0	1	0	$\oplus$	B_2
0	1	0	1	S_2	Sum (S)
				S_1	

A	B	S	C			<u>B</u>
0	0	0	0	m_0		1
0	1	1	0	m_1	A	1
1	0	1	0	m_2		
1	1	0	1	m_3		

A		<u>B</u>
		1

$$S = A \oplus B.$$

$$C = AB.$$



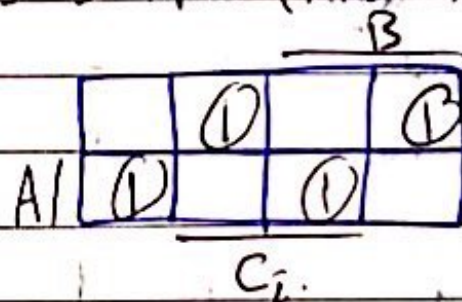
45

* Design a 2-bit up/down Counter:-

* Full-adder:-

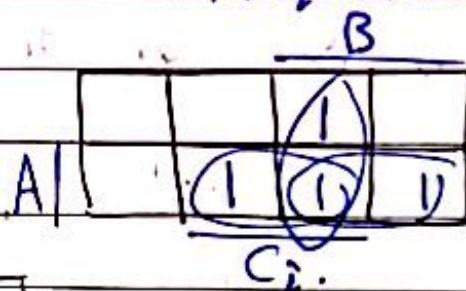
Design a full adder that accepts 3-input (A, B, C)

A	B	C _i	S	C _o	
0	0	0	0	0	m ₀
0	0	1	1	0	m ₁
0	1	0	1	0	m ₂
0	1	1	0	1	m ₃
1	0	0	1	0	m ₄
1	0	1	0	1	m ₅
1	1	0	0	1	m ₆
1	1	1	1	1	m ₇

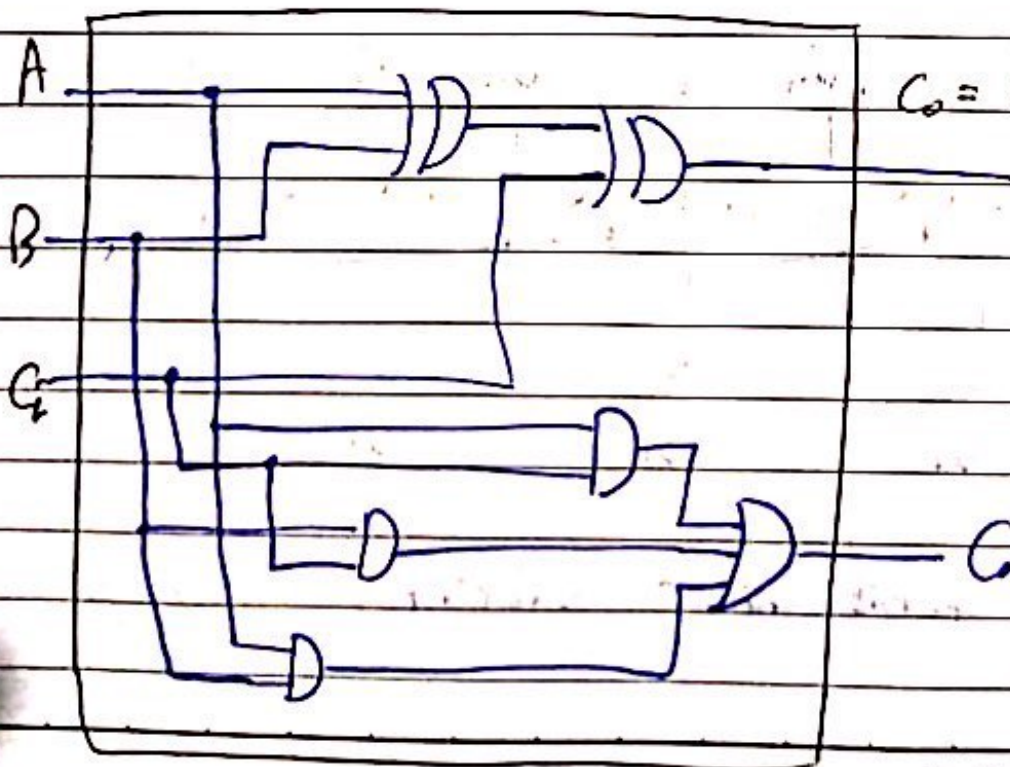


$$S = A \oplus B \oplus C_i$$

$$S = A\bar{B}\bar{C}_i + ABC_i + \bar{A}B\bar{C}_i + \bar{A}\bar{B}C_i$$



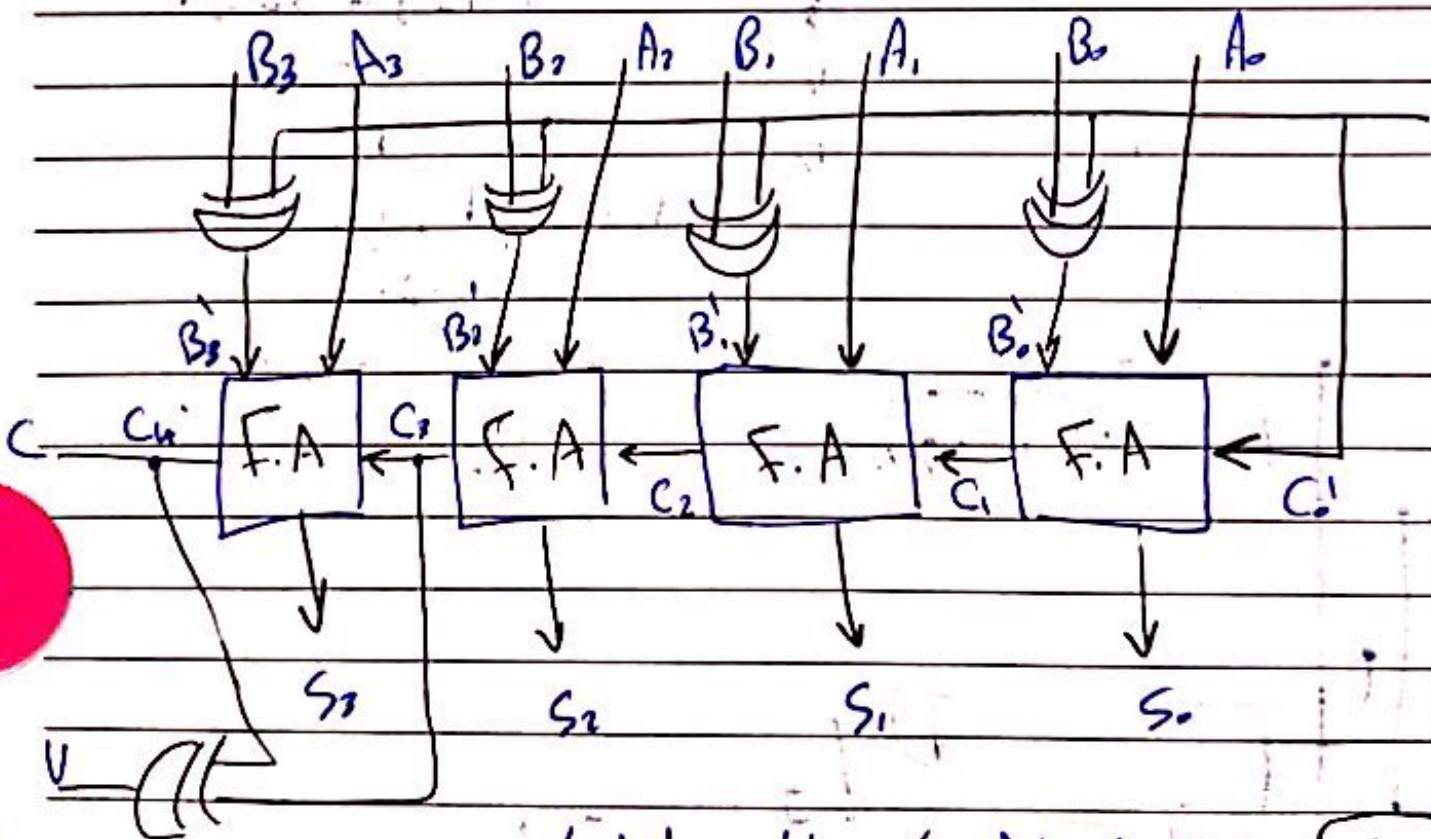
$$C_o = AC_i + BC_i + AB$$



46

* Design a 2-bit up/down Counter :-

Subscript	3	2	1	0	
Input + Carry	0	1	1	0	$C_i \rightarrow \text{Carry}$
Augend	1	0	1	1	$A_i \rightarrow \text{number}$
Addend	0	0	1	1	$B_i \rightarrow \text{number}$
Sum	1	1	1	0	S_i
	0	0	1	1	C_{i+1}



4-bit adder / subtractor

47

* BCD addition :-

$$1. 0011_{BCD} + 0100_{BCD} = 0111_{BCD}$$

$$\begin{array}{r} 0011 \\ + 0100 \\ \hline \end{array}$$

$$\begin{array}{r} 0111 \\ \hline \end{array} \rightarrow 7 \leq 9$$

This way is right until the number 9. after that it won't be true

$$2. 0111_{BCD} + 0101_{BCD} = 00010010$$

$$\begin{array}{r} 0111 \\ + 0101 \\ \hline \end{array}$$

$$\begin{array}{r} 1100 \\ \hline \end{array} \rightarrow 12_{10} = 00010010$$

گروپ میں ہوا
Connection factor
unsied bits 11

$$0110 = (6_{10}) \text{ گروپ}$$

$$\underline{00010010}$$

$$3. 01101000_{BCD} + 00110111_{BCD} = 000100000101_{BCD}$$

$$\begin{array}{r} 01101000 \\ + 00110111 \\ \hline 10011111 \\ + 0110 \\ \hline \end{array}$$

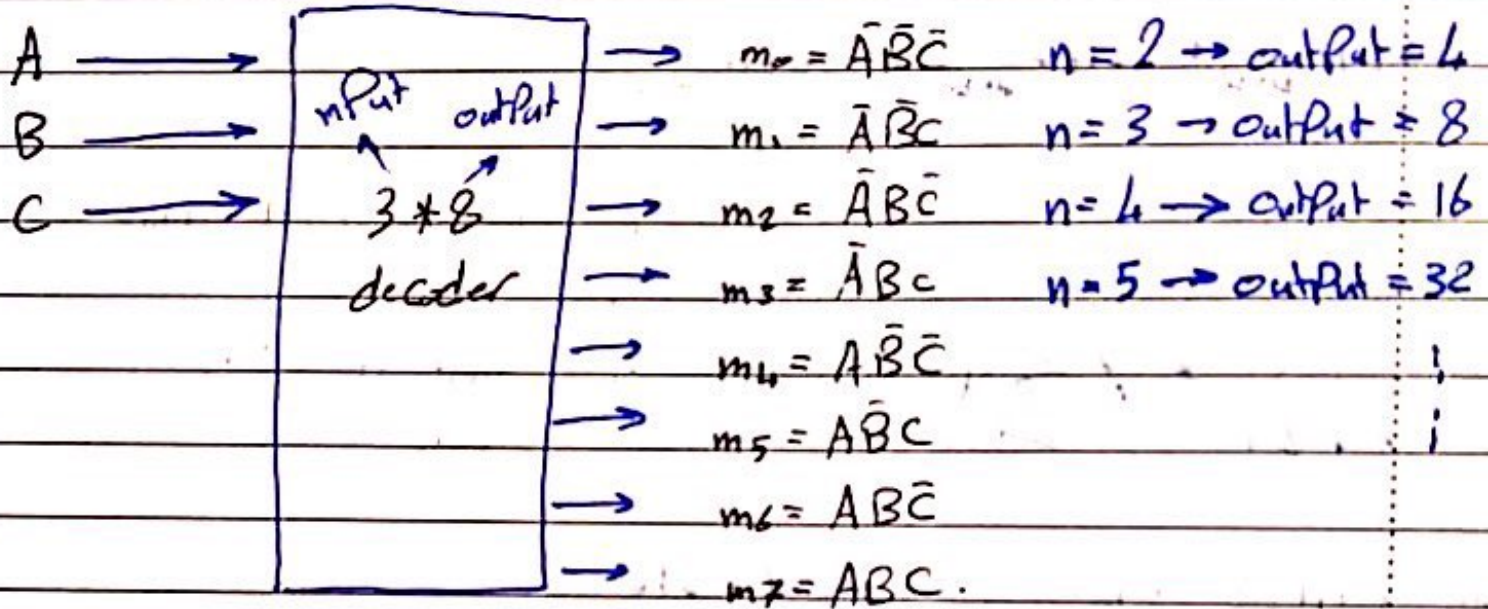
bigger than 15 we sum 6 = 0110

$$\begin{array}{r} 10100101 \\ + 0110 \\ \hline \end{array}$$

$$\underline{000100000101}$$

48

* Decoder :-



A	B	C		D_0	D_1	D_2	D_3	D_4	D_5	D_6	D_7
0	0	0	m_0	1	0	0	0	0	0	0	0
0	0	1	m_1	0	1	0	0	0	0	0	0
0	1	0	m_2	0	0	1	0	0	0	0	0
0	1	1	m_3	0	0	0	1	0	0	0	0
1	0	0	m_4	0	0	0	0	1	0	0	0
1	0	1	m_5	0	0	0	0	0	1	0	0
1	1	0	m_6	0	0	0	0	0	0	1	0
1	1	1	m_7	0	0	0	0	0	0	0	1

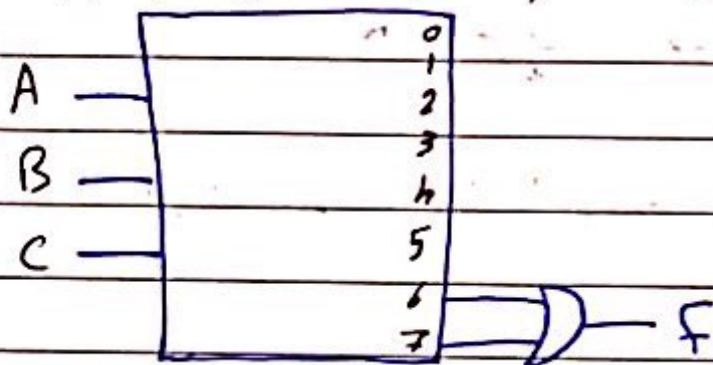
[49]

* Decoder :-

* Ex :- Design $F(A, B, C) = AB$ using decoder.

$$F(A, B, C) = AB(C + \bar{C}) = ABC + AB\bar{C}$$

$$F(A, B, C) = m_6 + m_7 = \Sigma(6, 7)$$

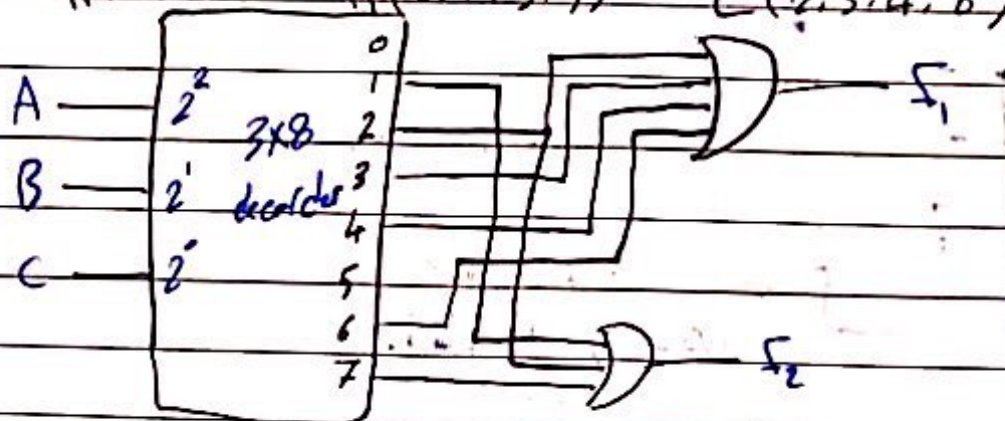


* Ex: Design the following two function using decoder

$$F_1(A, B, C) = (\bar{A} + \bar{B} + \bar{C})(\bar{A} + \bar{B} + C)(\bar{A} + B + \bar{C})(\bar{A} + B + C)$$

$$F_2(A, B, C) = \Sigma(1, 2, 7)$$

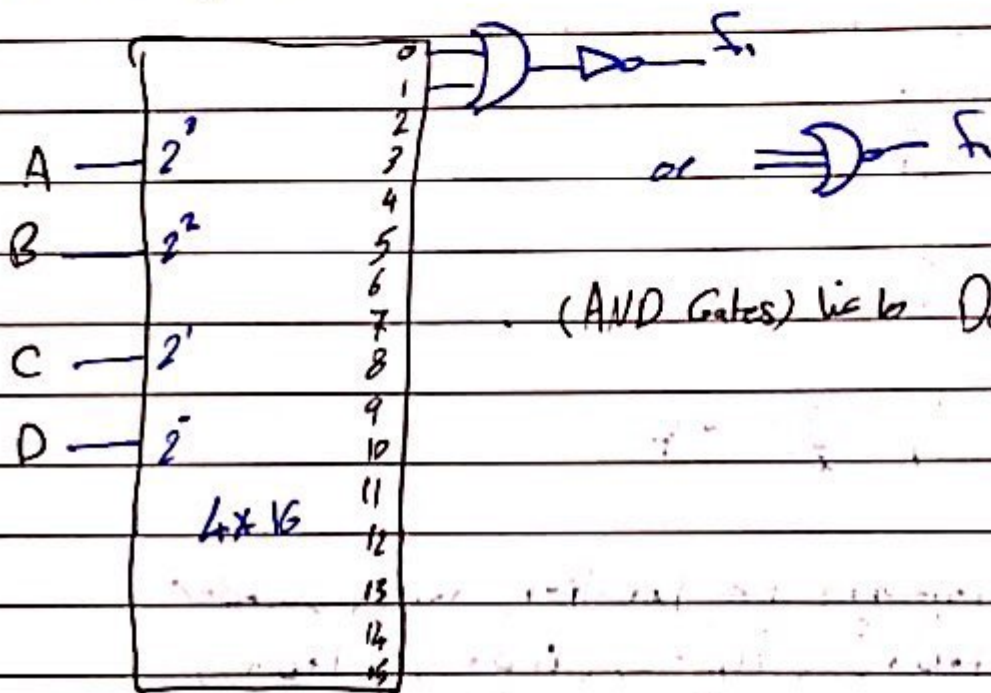
$$F(A, B, C) = \Pi(0, 1, 5, 7) = \Sigma(2, 3, 4, 6)$$



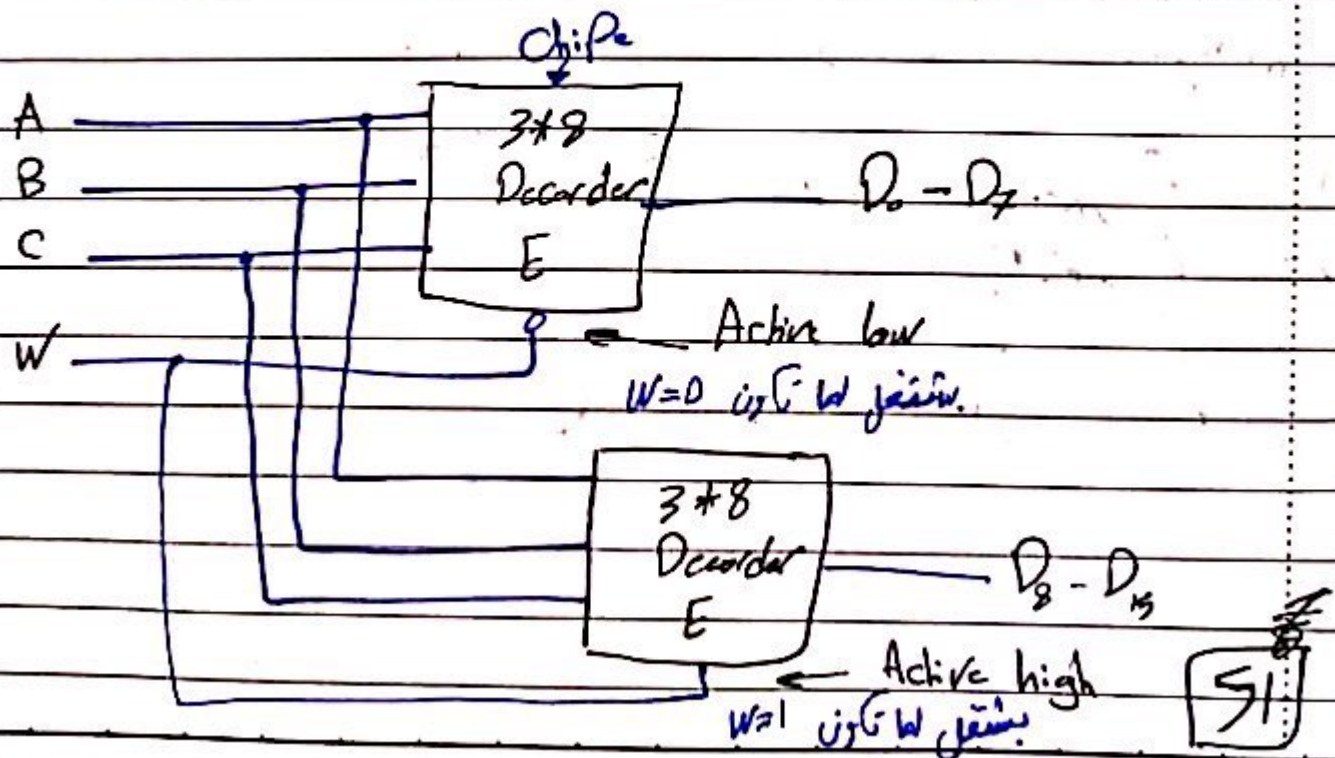
[50]

*Decoder:-

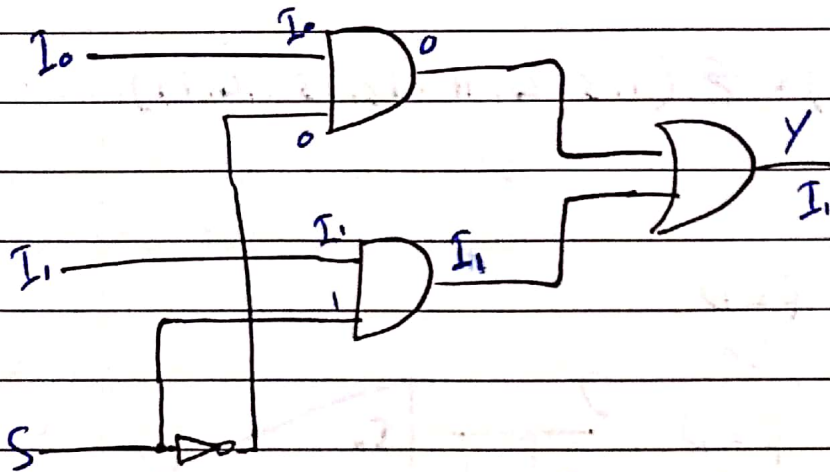
*Ex: Design $F_i(A, B, C, D) = \sum (2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$



(AND Gates) lie to Decoder 16.



* Design with Multiplexers (MUX) :-

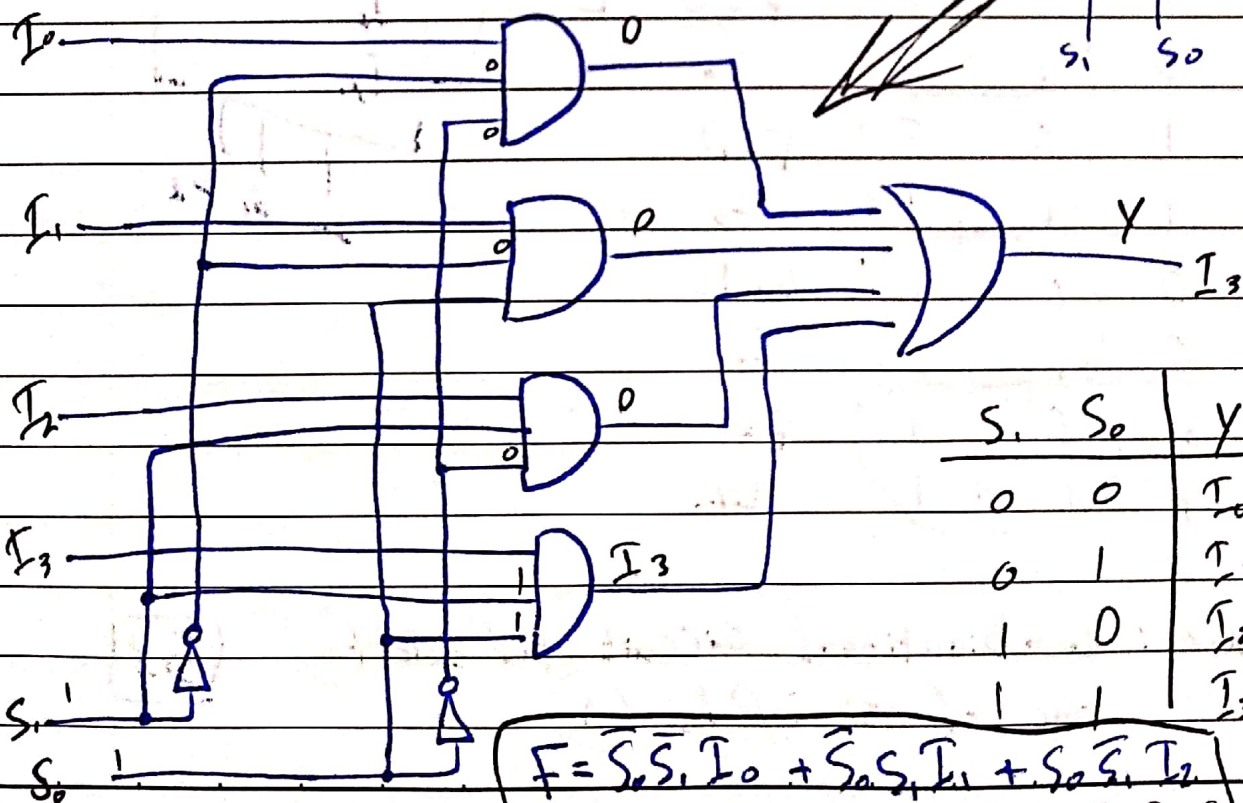
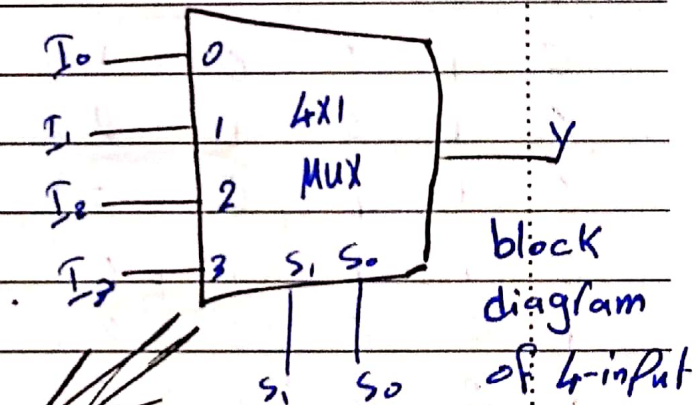
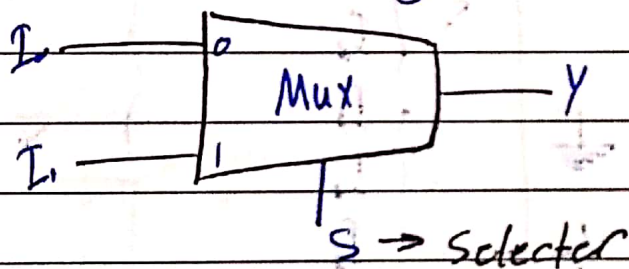


a) logic diagram
(2x1) MUX.

S	Y
0	I_0
1	I_1

$$Y = I_0 \cdot \bar{S} + I_1 \cdot S$$

b) block diagram of 2-input



S_1	S_0	Y
0	0	I_0
0	1	I_1
1	0	I_2
1	1	I_3

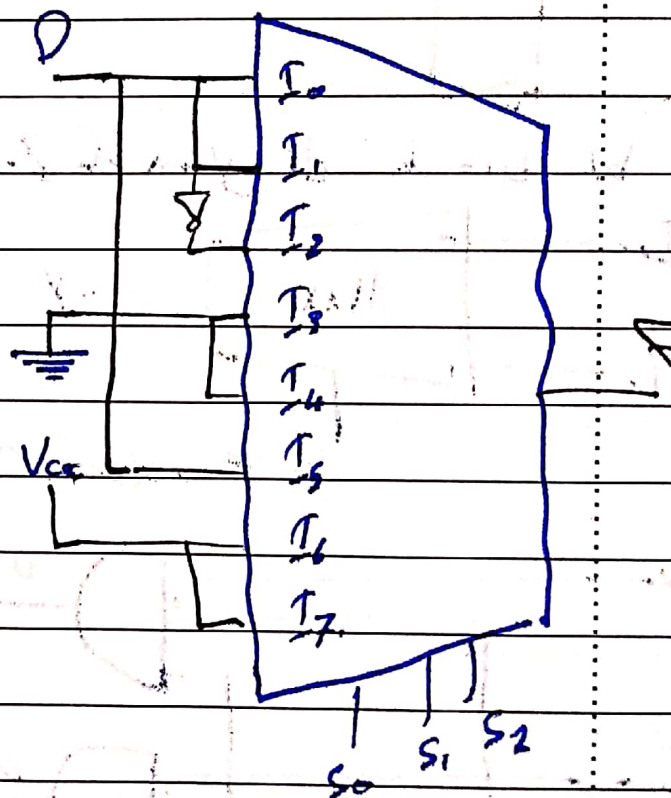
$$F = \bar{S}_0 \bar{S}_1 I_0 + \bar{S}_0 S_1 I_1 + S_0 \bar{S}_1 I_2 + S_0 S_1 I_3$$

*Design with Multiplexer (MUX) :-

*Ex :- Design $F(A, B, C, D) = \sum (1, 3, 4, 11, 12, 13, 14, 15)$

Using Mux Selection

A	B	C	D	F
0	0	0	0	0
0	0	0	1	1
0	0	1	0	0
0	0	1	1	1
0	1	0	0	1
0	1	0	1	0
0	1	1	0	0
0	1	1	1	0
1	0	0	0	0
1	0	0	1	0
1	0	1	0	0
1	0	1	1	1
1	1	0	0	1
1	1	0	1	1
1	1	1	0	1
1	1	1	1	1



Multiplexer :- Combinational Circuit that select one of the inputs $\left(\begin{smallmatrix} n \\ 1 \end{smallmatrix} \right)$.

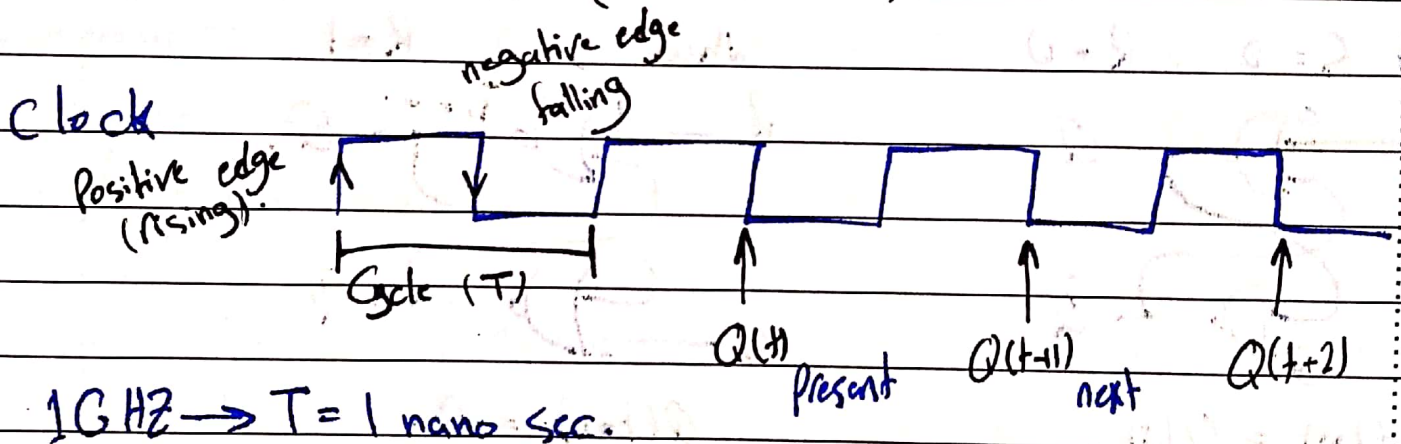
* Digital Circuits :-

* Digital Circuits :-

1. Combinational Circuits $\rightarrow$ (Mux) .
2. Sequential Circuits . دوائر متسلسلة

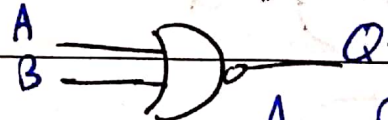
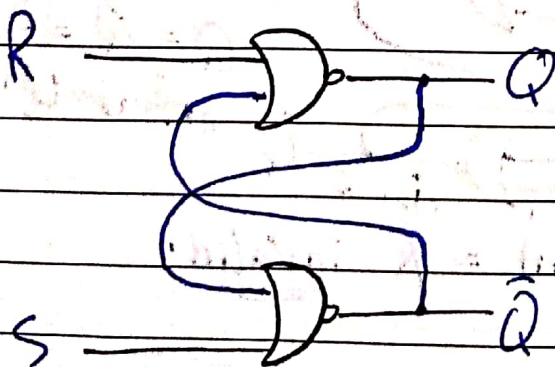
* Sequential Circuits :-

1. latch Circuits (Without clock) .
2. flip / flop Circuits (with clock) .



* Set / Reset Flip / Flop :-

Set to high (1) . Reset to low (0) .

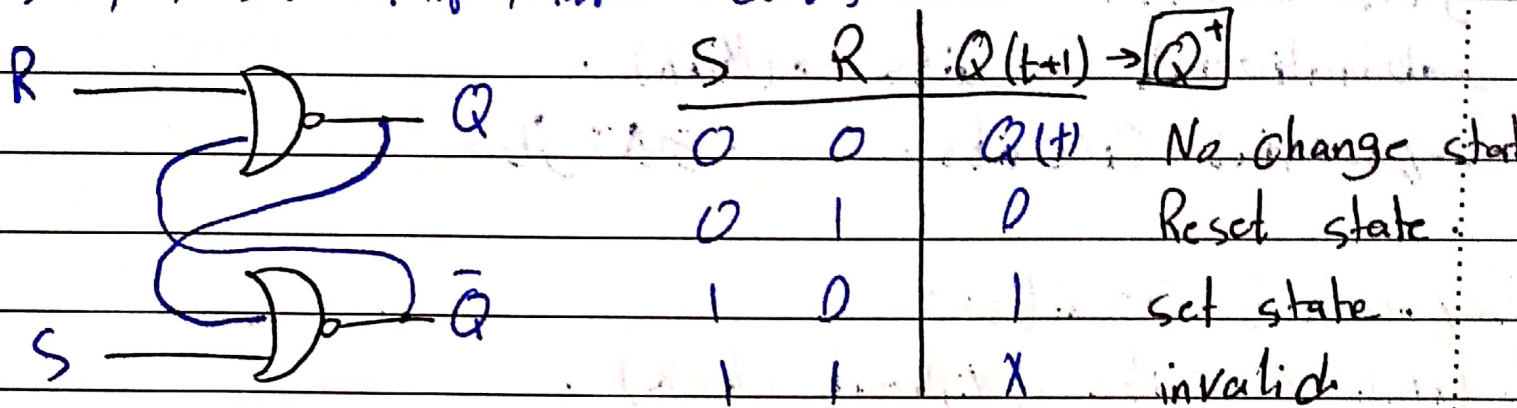


A	B	Q
0	0	1
0	1	0
1	0	0
1	1	0

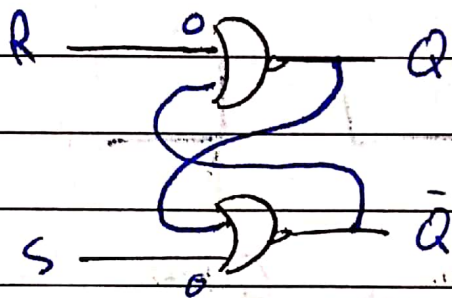
54

* Digital Circuits :-

* Set / Reset Flip / Flop circuits :-

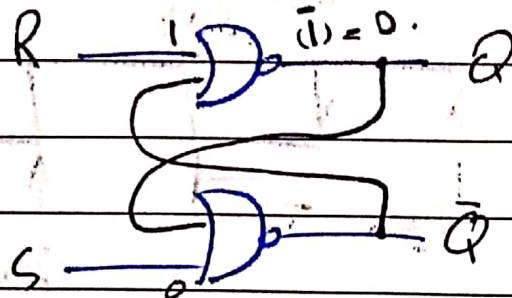


When $S = 0$, $R = 0$.



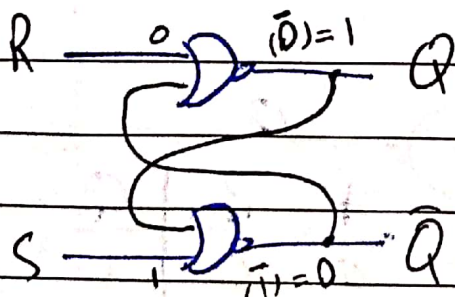
$$Q(t+1) = Q(t)$$

When $S = 0$, $R = 1$



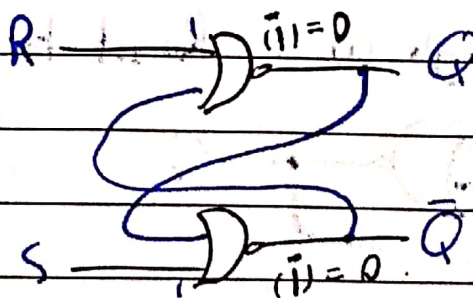
$$Q(t+1) = 0$$

When $S = 1$, $R = 0$.



$$Q(t+1) = 1$$

When $S = 1$, $R = 1$.



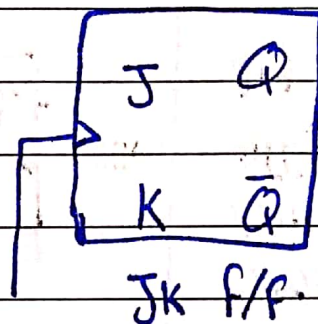
$$Q(t+1) = X \text{ invalid.}$$

$Q = \bar{Q} = 0$
التي هي من مقبول

* Digital Circuits :-

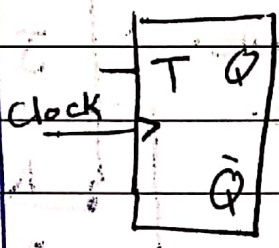
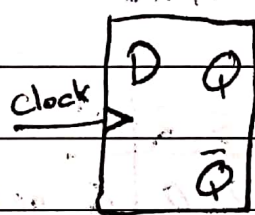
* JK Flip/Flop Circuits (JK F/F) :-

J	K	$Q(t+1) \rightarrow Q^+$	
0	0	$Q(t)$	no change state.
0	1	0	Reset state.
1	0	1	set state.
1	1	$\bar{Q}(t)$	toggle state.

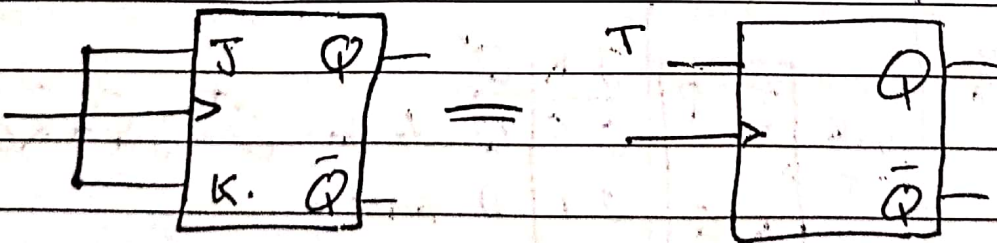


* number	Name	Symbol	state table	Characteristic equ																				
1	SR F/f		<table border="1"> <thead> <tr> <th>S</th> <th>R</th> <th>Q^+</th> <th></th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>Q</td> <td>no cha</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> <td>Reset</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> <td>set</td> </tr> <tr> <td>1</td> <td>1</td> <td>X</td> <td>invalid</td> </tr> </tbody> </table>	S	R	Q^+		0	0	Q	no cha	0	1	0	Reset	1	0	1	set	1	1	X	invalid	X
S	R	Q^+																						
0	0	Q	no cha																					
0	1	0	Reset																					
1	0	1	set																					
1	1	X	invalid																					
2	JK F/f		<table border="1"> <thead> <tr> <th>J</th> <th>K</th> <th>Q^+</th> <th></th> </tr> </thead> <tbody> <tr> <td>0</td> <td>0</td> <td>Q</td> <td>No cha</td> </tr> <tr> <td>0</td> <td>1</td> <td>0</td> <td>Reset</td> </tr> <tr> <td>1</td> <td>0</td> <td>1</td> <td>set</td> </tr> <tr> <td>1</td> <td>1</td> <td>$\bar{Q}$</td> <td>toggle</td> </tr> </tbody> </table>	J	K	Q^+		0	0	Q	No cha	0	1	0	Reset	1	0	1	set	1	1	$\bar{Q}$	toggle	$Q^+ = J\bar{Q} + KQ$
J	K	Q^+																						
0	0	Q	No cha																					
0	1	0	Reset																					
1	0	1	set																					
1	1	$\bar{Q}$	toggle																					

* Digital Circuits :-

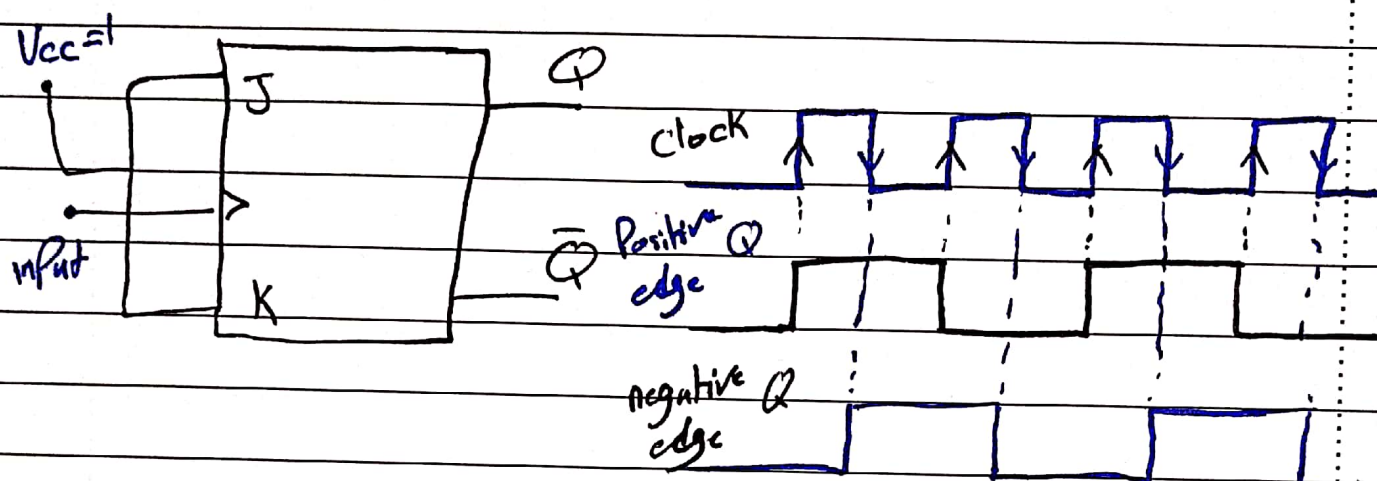
* Number	Name	Symbol	state table	characteristic eqn
3	T f/f Toggle Flip / flop		$\begin{array}{c c} T & Q^+ \\ \hline 0 & Q \text{ No change} \\ 1 & \bar{Q} \text{ Toggle} \end{array}$	$Q^+ = T \oplus Q$ $= T \cdot \bar{Q} + \bar{T} \cdot Q$
4	D f/f Data Flip / flop المسجل البيانات		$\begin{array}{c c} D & Q^+ \\ \hline 0 & 0 \\ 1 & 1 \end{array}$	$Q^+ = D$

* Ex - how do you make a T f/f out of JK f/f.

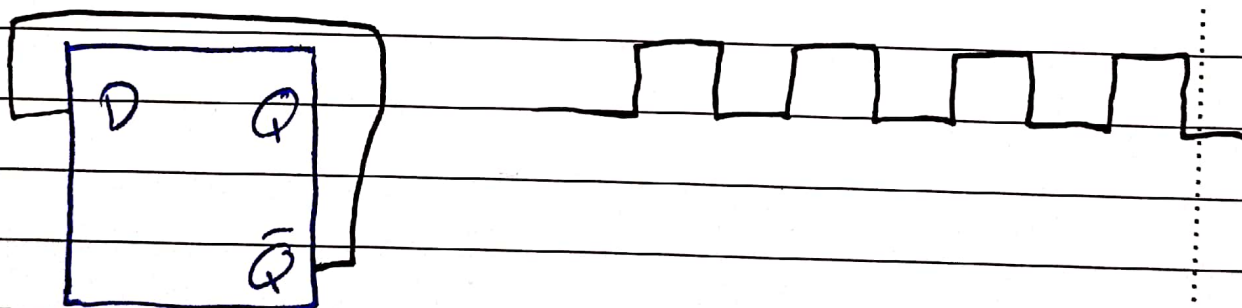


* Digital Circuits :-

* JK f/f :-



* D f/f :-



$$Q^+ = D$$

$$D = \bar{Q}$$