ElCoM
ELECTRICAL COMPUTER MECHATRONICS

تقدم لجنة ElCoM الاكاديمية

دفتر لمادة:

ميكانيكا هندسية

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Engineering mechanics

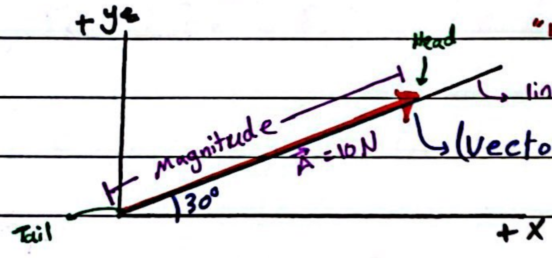
CH2: vectors in 2D

Vectors are quantities with magnitude and ~~dir~~ direction.

ex: Forces and displacement. ⇒ $\vec{F} = (30\hat{i} + 40\hat{j})N$

Scalar quantities have a magnitude only.

ex: Temp, time, distance, length. ⇒ $F = 30N$



"Head" also called ~~line of action~~ "Tip"

$|\vec{A}| = 10N$ $\theta = 30°$. from +x

or: $\theta = 30°$ ccw x

counter clock wise

International System of units (SI):

length → meters (m)		kilo	$\times 10^3$	K
Time → seconds (s)		Mega	$\times 10^6$	M
Mass → kilograms (kg)		Giga	$\times 10^9$	G

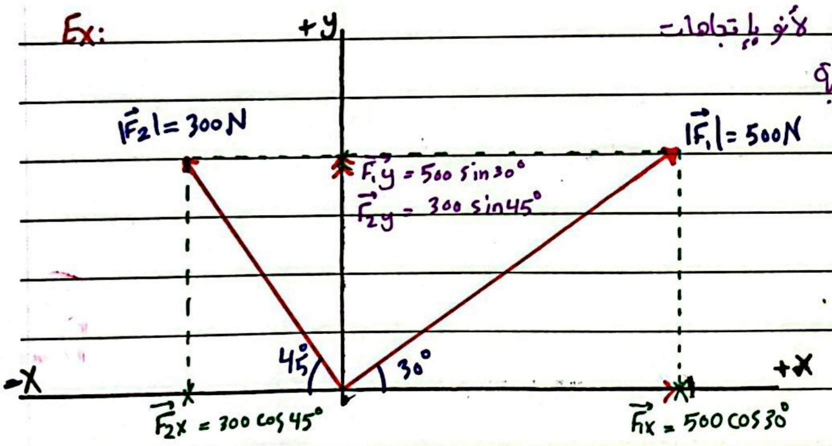
Force → Newtons (N) is derived from $kg.m/s^2$

2.2: Vector operations:

Resultant Force

components methods

Ex:



* بهاي الحالة ما بقدر اجمع الفيكتورز لأنو بإتجاهات مختلفة فلازم نحلل الفيكتور لمركبة سينية وصادية.

* الأقرب للزاوية بوخذ cos.

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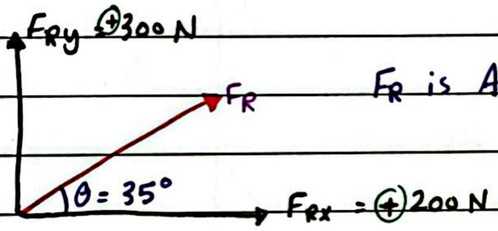
To calculate the Resultant Force of the previous example:

$F_{Rx} = F_{1x} + F_{2x} = 500\cos 30^\circ - 300\cos 45^\circ = +200\text{ N}$ } components
$F_{Ry} = F_{1y} + F_{2y} = 500\cos 30^\circ + 300\cos 45^\circ = +300\text{ N}$ } for the resultant.

$F_R = \sqrt{F_{Rx}^2 + F_{Ry}^2} = \sqrt{200^2 + 300^2} = 360.555$

$\theta = \tan^{-1}\left(\frac{300}{200}\right) = 56.3 \qquad \left(\tan\theta = \left|\frac{F_{Ry}}{F_{Rx}}\right|\right).$

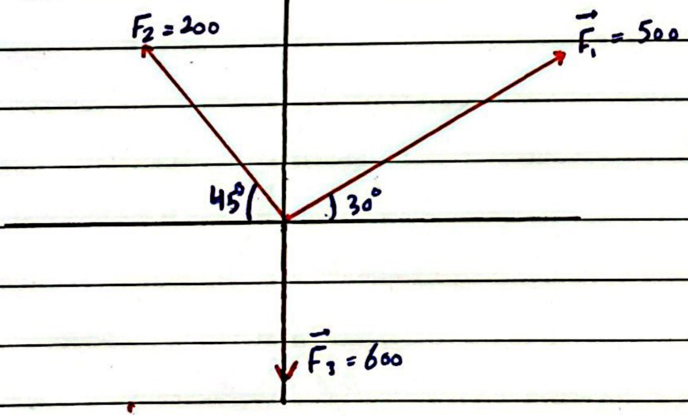
* المطلق لأنو ما بهمني الإشارات
لأنو رح اعرف المحصلة بأي ربع موجود من خلال الـ components للفيكتور.



F_R is ALWAYS between F_{Rx} and F_{Ry}.

$$\boxed{\begin{aligned} F_{Rx} &= \Sigma F_x \\ F_{Ry} &= \Sigma F_y \end{aligned}}$$

Ex:



$F_{Rx} = 500\cos 30^\circ - 200\cos 45^\circ$
$F_{Ry} = 200\sin 45^\circ + 500\sin 60^\circ - 600$

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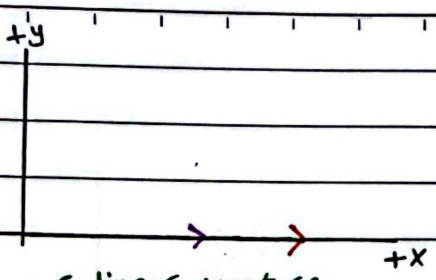
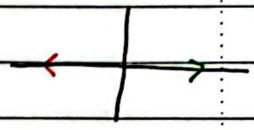
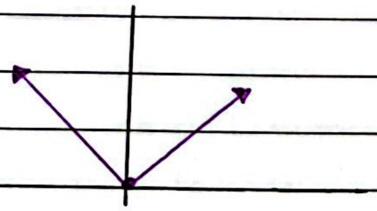


Colinear vectors

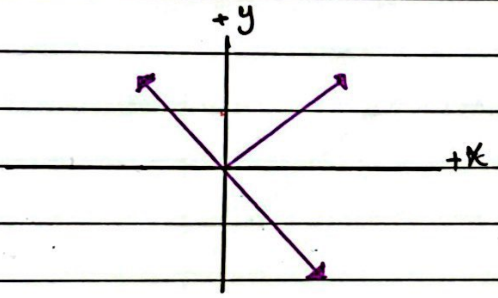
* Vectors on the same line : x axis (+x and or -x)





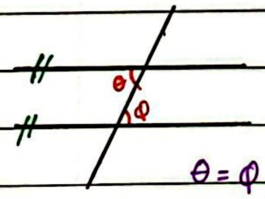
Concurrent Forces

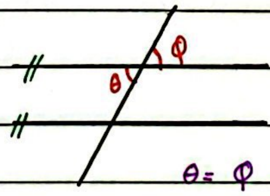
* A concurrent force system contains forces whose lines of action intersect at a point

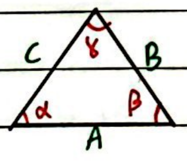


coplaner vectors
(vectors on the same plane).

To calculate the magnitude of a side and the magnitude of the othe 2 sides and the angle between them we can use the cosine law.







sine law

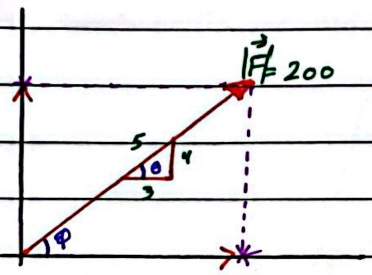
$$\frac{C}{\sin\beta} = \frac{B}{\sin\alpha} = \frac{A}{\sin\gamma}$$

cosine law

$$C = \sqrt{A^2 + B^2 - 2AB\cos\beta}$$

Five Apple

Ex:



$\cos\theta = \frac{3}{5}$ $\sin\theta = \frac{4}{5}$

$\theta = \phi$

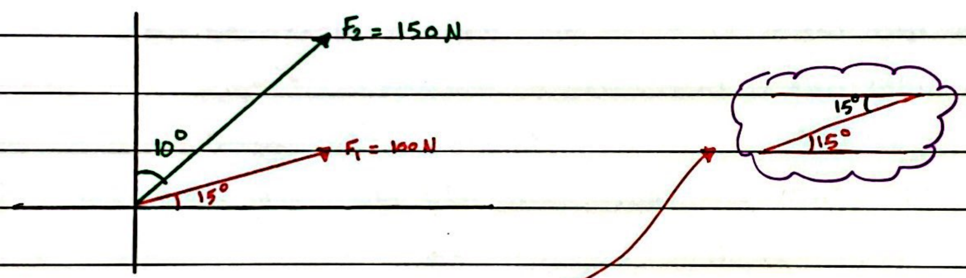
بالتالي

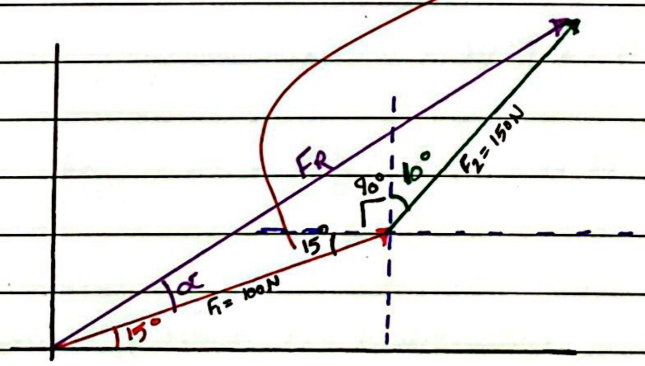
$F_x = |\vec{F}| \cos\phi = 200 \cos\phi = 200 * \frac{3}{5}$

$F_y = |\vec{F}| \sin\phi = 200 \sin\phi = 200 * \frac{4}{5}$

Graphical method

Ex:





* طلعت ال magnitude ل FR

بضل الزاوية كيف؟ (α)

بما انو عندي ضلعين وزاوية معلومين

بقدر استخدم ال sine law،

$\theta_{12} = 10° + 90° + 15° = 115°$

$|F_R| = \sqrt{100^2 + 150^2 - 2(100)(150)\cos 115°}$

$= 212.55\ N$

$\alpha \Rightarrow \frac{150}{\sin\alpha} = \frac{212.15}{\sin 115} \rightarrow \alpha = 39.85°$

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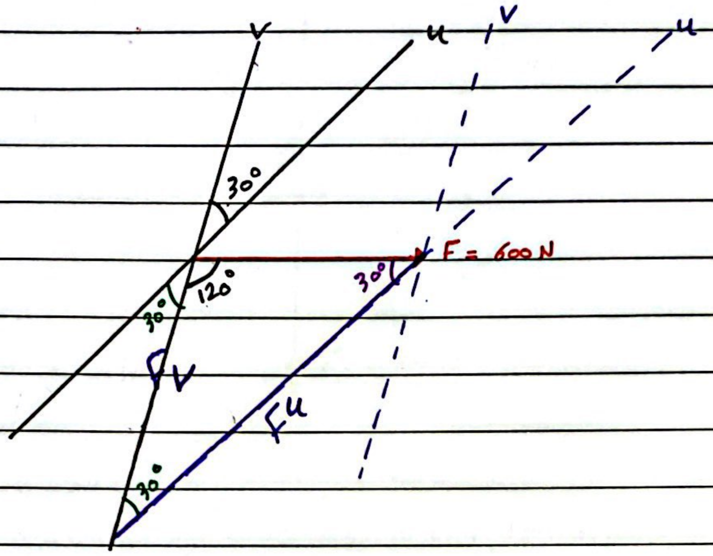
Ex: Find the components of the force a long u and v lines



ملاحظة :- بهاذ النوع من المسائل ما بعطيني x-axis و y-axis فما بزبط استخدم طرق التحليل زي cos و sin ، محورين الزاوية بيناتهم $\neq 90°$

عشان أحل برسم خط موازي لكل محور فبتكون مثلث

$30°$ و $120°$ معطى بالسؤال ، $30°$ تقابل بالرأس و $30°$ الثانية عشكل Z .

وصيك بقدر اطلع زاوية المثلث الأخيرة ($180° = \theta + 30° + 120°$ ، $\theta = 30°$).

$$\frac{600}{\sin 30} = \frac{F_u}{\sin 120} \rightarrow F_u = 1039.23 \text{ N}$$

$$\frac{600}{\sin 30} = \frac{F_v}{\sin 30} \rightarrow F_v = 600 \text{ N}$$

* This kind of questions can't be solved using the graphical method.

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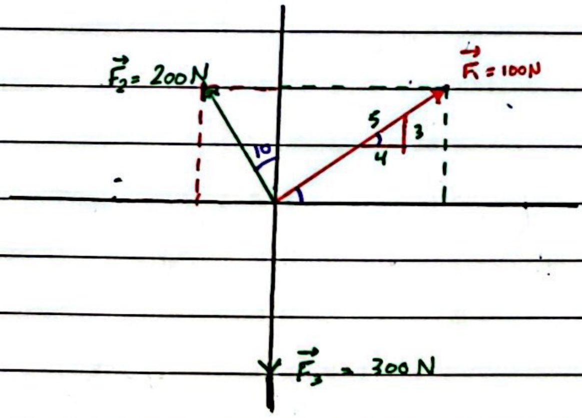
Ex:



$F_{1x} = 100\left(\frac{4}{5}\right)$

$F_{1y} = 100\left(\frac{3}{5}\right)$

$F_{2x} = -200\sin 10$

$F_{2y} = 200\cos 10$

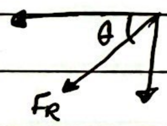
$\vec{F_1} = 80\hat{i} + 60\hat{j}$

$\vec{F_2} = -200\sin 10\,\hat{i} + 200\cos 10\,\hat{j}$

$\vec{F_3} = 0\,\hat{i} - 300\hat{j}$

* الأفضل احط الصفر عشان اعرف اني قاعد اجمع 3 components

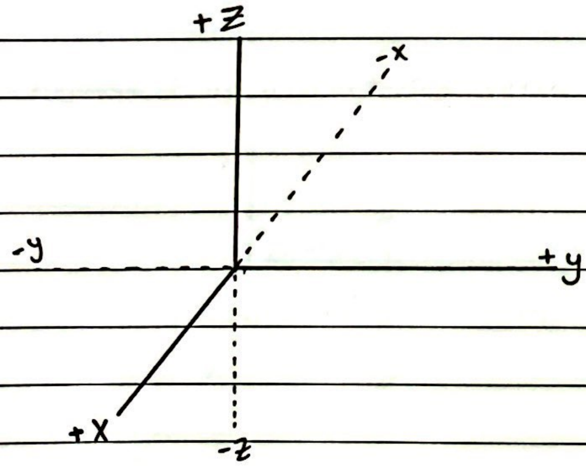
مهم اوضح الاتجاه من خلال إني ارسم



$\vec{F_R} = (80 - 200\sin 10 + 0)\,\hat{i} + (60 + 200\cos 10 - 300)\,\hat{j}$

$\vec{F_R} = -70\hat{i} - 30\hat{j}$ $\quad |\vec{F_R}| = \sqrt{70^2 + 30^2} = 80\,N$ $\quad \tan\theta = \left|\frac{F_{Ry}}{F_{Rx}}\right| = \frac{30}{70}$

Addition of Forces in 3D.



This will be the default location for the ~~x~~ axises.

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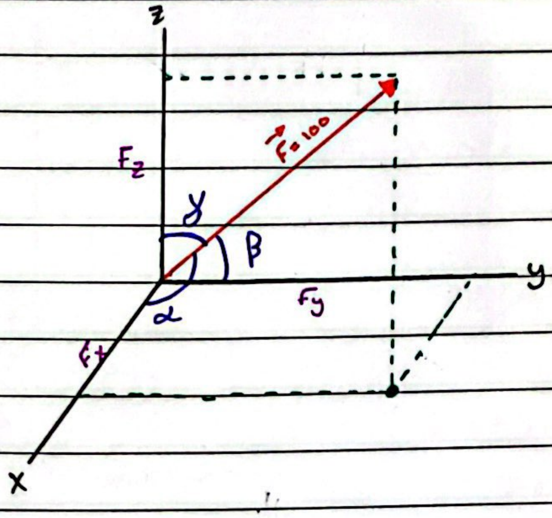
Ex:



$F_x = F\cos\alpha$ ألفا

$F_y = F\cos\beta$ بيتا

$F_z = F\cos\gamma$ غاما

$\vec{F} = F\cos\alpha\,\hat{i} + F\cos\beta\,\hat{j} + F\cos\gamma\,\hat{k}$

★ $\cos\alpha$, $\cos\beta$, $\cos\gamma$ = Direction cosine.

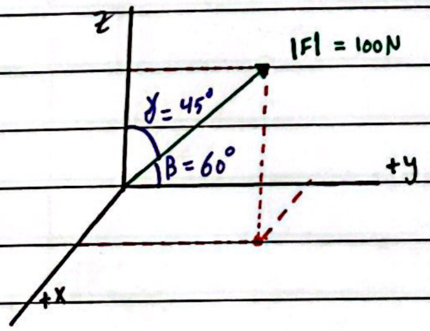
★ α , β , γ = Direction angles

$\Rightarrow \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Unit vector : $\vec{u}_F = \dfrac{\vec{F}}{|F|} = \dfrac{F\cos\alpha\,\hat{i} + F\cos\beta\,\hat{j} + F\cos\gamma\,\hat{k}}{F}$

$|u_F| = 1$ $\qquad = 1\cos\alpha\,\hat{i} + 1\cos\beta\,\hat{j} + 1\cos\gamma\,\hat{k}$.

Ex: Express F as a cartesian vector



$(\cos 45)^2 + (\cos 60)^2 + (\cos\alpha)^2 = 1$

$\cos\alpha = \sqrt{1 - \cos^2 45 - \cos^2 60} = \pm \frac{1}{2}$

لـ اخترنا الموجب بالحل لأنو واضح

ال Force موجوه نحو ال axis(+X).

$\vec{F} = 100\cos\alpha\,\hat{i} + 100\cos\beta\,\hat{j} + 100\cos\gamma\,\hat{k}$

$= 100\left(\frac{1}{2}\right)\hat{i} + 100\cos 60\,\hat{j} + 100\cos 45\,\hat{k}$

Five Apple

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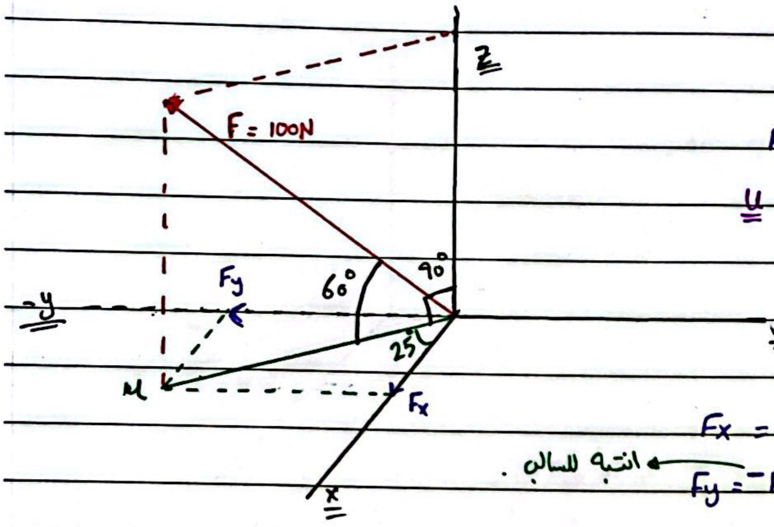
Ex: Express F a cartesian vector $(\vec{F} = F_x\,\hat{i} + F_y\,\hat{j} + F_z\,\hat{k})$.



$F_z = 100 \sin 60$

$F_u = 100 \cos 60$

* هسا صار عندي زي قوة جديدة على $\underline{u}$
بدي ارجع احللها.

$F_x = F_u \cos 25 = 100 \cos 60 \cos 25$

انتبه للسالب. ←

$F_y = -F_u \sin 25 = -100 \cos 60 \sin 25$

$$\vec{F} = 100 \cos 60 \cos 25\,\hat{i} + -100 \cos 60 \sin 25\,\hat{j} + 100 \sin 60\,\hat{k}$$

$$u_F = \frac{\vec{F}}{|F|} = \frac{\vec{F}}{100} = \underbrace{\cos 60 \cos 25}_{\cos\alpha}\,\hat{i} - \underbrace{\cos 60 \sin 25}_{\cos\beta}\,\hat{j} + \underbrace{\sin 60}_{\cos\gamma}\,\hat{k}$$

find the direction angles:

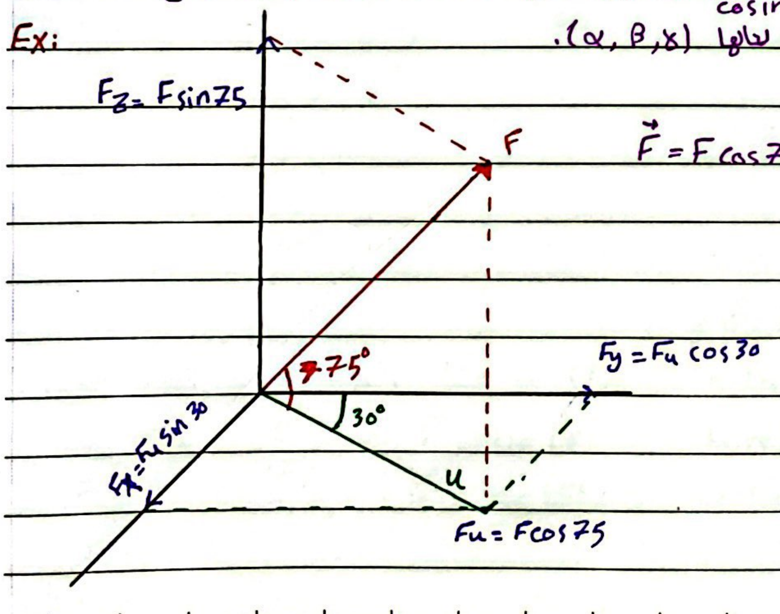
$$\vec{u}_F = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$$

$\cos\alpha = \cos 25 \cos 60$ $\quad\cos\beta = -\cos 60 \sin 25$ $\quad\cos\gamma = \sin 60$

لـ هاي ← Direction cosine ، لازم اطلع الـ $\cos^{-1}(\;)$ عشان اطلع

Ex:

الـ Direction angle تبعها (α, β, γ).



$$\vec{F} = F\cos 75 \sin 30\,\hat{i} + F\cos 75 \cos 30\,\hat{j} + F\sin 75\,\hat{k}$$

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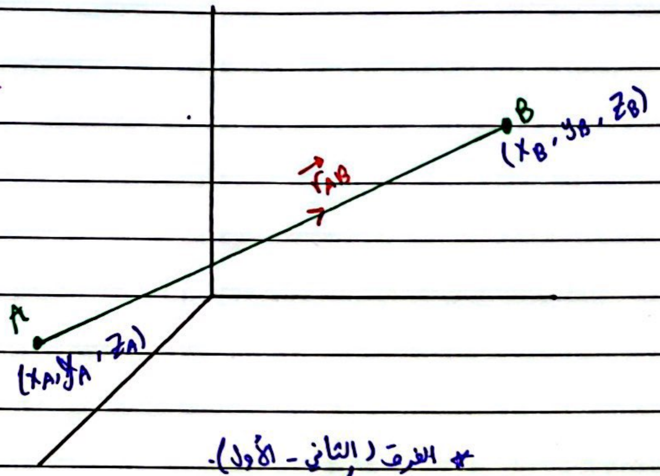
position vector



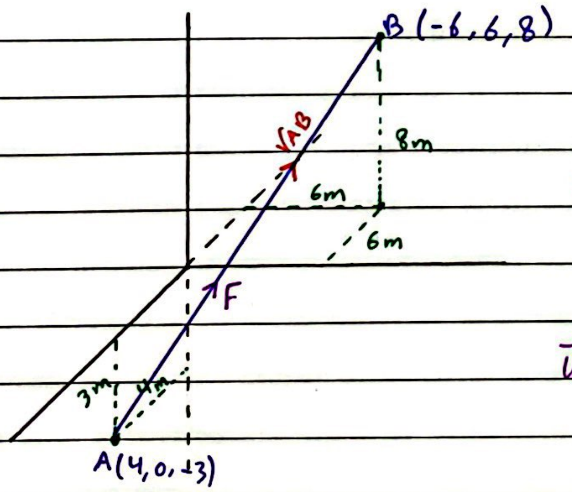
* الفرق (الثاني - الأول).

position vector : $\vec{r}_{AB} = \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$

$(x_B - x_A)\,\hat{i} + (y_B - y_A)\,\hat{j} + (z_B - z_A)\,\hat{k}$

$$\vec{U_r} = \frac{\vec{r}_{AB}}{|r_{AB}|} = \frac{\Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}}{\sqrt{(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2}}$$

Ex:



$\vec{r}_{AB} = (-6-4)\,\hat{i} + (6-0)\,\hat{j} + (8-3)\,\hat{k}$

$= -10\,\hat{i} + 6\,\hat{j} + 11\,\hat{k}$

$$\vec{U_r} = \frac{\vec{r}_{AB}}{|r_{AB}|} = \frac{-10\hat{i} + 6\hat{j} + 11\hat{k}}{\sqrt{10^2 + 6^2 + 11^2}}$$

* $|U| = 1 \rightsquigarrow U_F = U_r$

$U_F = \frac{\vec{F}}{|F|} \Rightarrow \vec{F} = U_F\,|F|$

$\vec{F} = U_r\,|F|$

* if i have a position vector and a force ON that vector then they have the same direction (unit vector).

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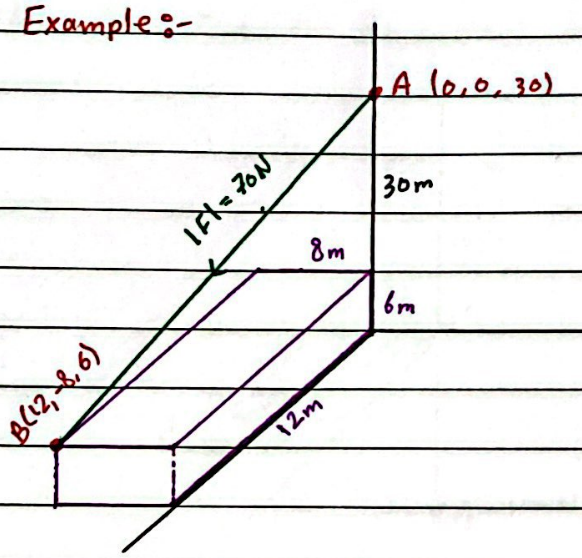
Example :-

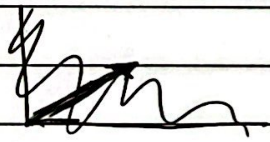


Express F as a cartesian vector:

$$\vec{r}_{AB} = \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$$
$$= (12-0)\hat{i} + (-8-0)\hat{j} + (6-30)\hat{k}$$
$$= 12\hat{i} - 8\hat{j} - 24\hat{k}$$

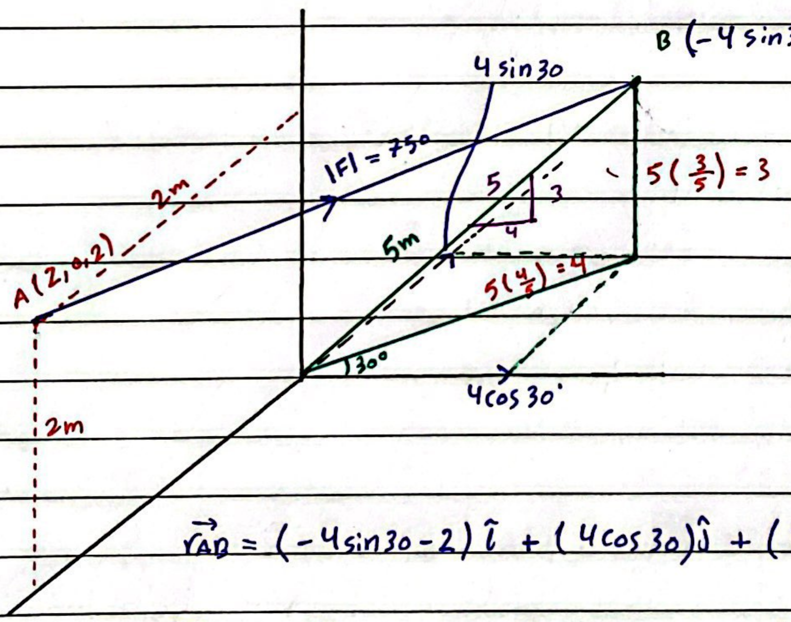
$$\vec{U}_{rAB} = \frac{\vec{r}_{AB}}{|r_{AB}|} = \frac{12\hat{i} - 8\hat{j} - 24k}{\sqrt{12^2 + 8^2 + 24^2}}$$

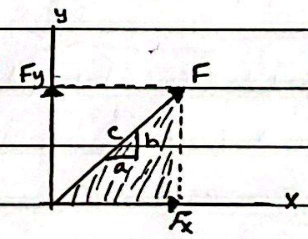
$$\vec{U}_{rAB} = \underbrace{\frac{12}{28}}_{\cos\alpha}\hat{i} - \underbrace{\frac{8}{28}}_{\cos\beta}\hat{j} - \underbrace{\frac{24}{28}}_{\cos\gamma}\hat{k}$$



$$\vec{F} = |F|\,U_{\vec{r}AB} = 70\left(\frac{12}{28}\hat{i} - \frac{8}{28}\hat{j} - \frac{24}{28}\hat{k}\right)$$
$$= 30\hat{i} - 20\hat{j} - 60\hat{k}$$

Example :-





$$F_x = F\left(\frac{a}{c}\right) \quad F_y = F\left(\frac{b}{c}\right)$$

$$\vec{r}_{AB} = (-4\sin 30 - 2)\hat{i} + (4\cos 30)\hat{j} + (3-2)\hat{k}$$

$$\vec{U}_{rAB} = \frac{\vec{r}_{AB}}{|r_{AB}|} = \frac{1.95\hat{i} + 0.617\hat{j} + 1\hat{k}}{\sqrt{1.95^2 + 0.617^2 + 1^2}} = 0.85\hat{i} + 0.271\hat{j} + 0.44\hat{k}$$

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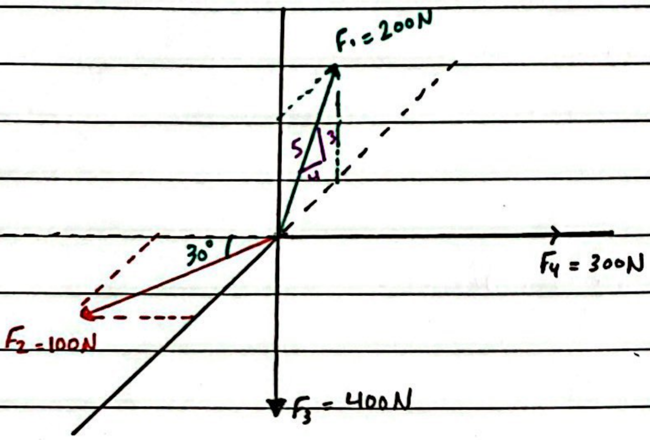
Example :- Find the Resaultant force:



$\vec{F_1} = -200\left(\frac{4}{5}\right)\hat{i} + 0\hat{j} + 200\left(\frac{3}{5}\right)\hat{k}$

$\vec{F_2} = 100 \sin 30\hat{i} - 100\cos 30\hat{j} + 0\hat{k}$

$\vec{F_3} = 0\hat{i} + 0\hat{j} - 400\hat{k}$

$\vec{F_4} = 0\hat{i} + 300\hat{j} + 0\hat{k}$

$\vec{U}_{F_R} = \frac{\vec{F_R}}{|F_R|}$

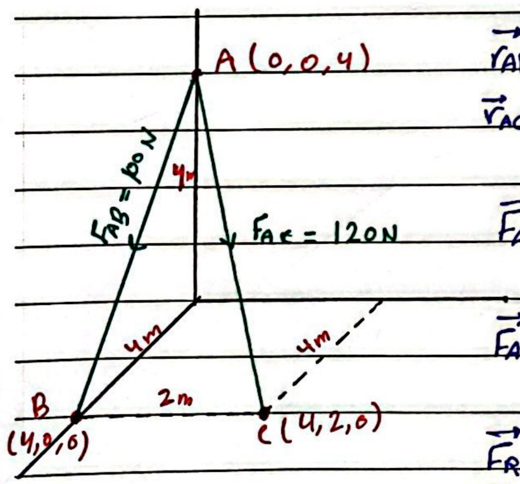
$\vec{F_R} = \left(-200\left(\frac{4}{5}\right) + 100\sin 30\right)\hat{i} + \left(300 - 100\cos 30\right)\hat{j} + \left(200\left(\frac{3}{5}\right) - 400\right)\hat{k}$

Example:- Express as a cartesian Vector:-

$\boxed{U_{r_{AB}} = U_{F_{AB}}}$

$\frac{\vec{r}}{|r|} = \frac{\vec{F}}{|F|}$

$\vec{F} = |F|\frac{\vec{r}}{|r|}$



$\vec{r}_{AB} = 4\hat{i} + 0\hat{j} - 4\hat{k}$

$\vec{r}_{AC} = 4\hat{i} + 2\hat{j} - 4\hat{k}$

$\vec{F}_{AB} = 100\, U_{r_{AB}} = 100\, \frac{4\hat{i} + 0\hat{j} - 4\hat{k}}{\sqrt{4^2 + 0^2 + 4^2}} = 70.7\hat{i} + 0\hat{j} - 70.7\hat{k}$

$\vec{F}_{AC} = 120\, U_{AC} = 120\, \frac{4\hat{i} + 2\hat{j} - 4\hat{k}}{\sqrt{4^2 + 2^2 + 4^2}} = 80\hat{i} + 40\hat{j} - 80\hat{k}$

$\vec{F_R} = (70.7 + 80)\hat{i} + (0 + 40)\hat{j} + (-70.7 - 80)\hat{k}$

$|\vec{F_R}| = \sqrt{\quad}$

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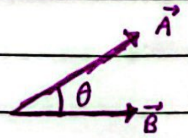
2.9 Dot product

$\vec{A} \cdot \vec{B} = \text{scalar}$

ex: $(1\hat{i} + 2\hat{j} + 3k) \cdot (1\hat{i} + 3\hat{j} + 5\hat{k}) = 1 + 6 + 15 = 22$

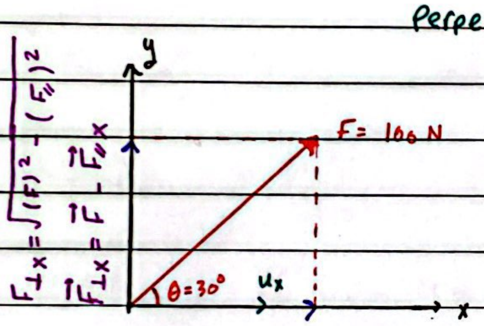
$\vec{A} \cdot \vec{B} = |A||B| \cos\theta$



$$\cos\theta = \frac{\vec{A} \cdot \vec{B}}{|A||B|}$$

component parallel

* اول ما اشوف Parallel أو Perpendecular بالسؤال بعرف انو Dot product.



$F_{\parallel x} = F_x = 100 \cos 30 = |F||u_x| \cos\theta$

$= \vec{F} \cdot \vec{u}_{axis}$

steps:-

1. unit vector for the axis.
2. Express the Force as a cartesian vector.
3. $F_{\parallel axis} = \vec{F} \cdot \vec{u}_{axis} \Rightarrow$ Scalar.
4. $\vec{F}_{\parallel axis} = |F_{\parallel axis}|\, u_{axis} \Rightarrow$ Vector.
5. $\vec{F}_{\perp axis} = \vec{F} - \vec{F}_{\parallel axis}$. $|F_{\perp}| = \sqrt{F^2 - F_{\parallel}^2}$

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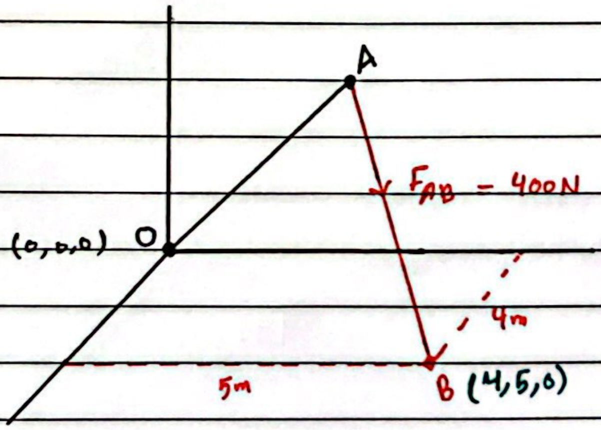
Example:- Find the magnitude of the projected component of F along Ao axis:

$\hookrightarrow (F_{\parallel Ao})$



① $\vec{U}_{Ao} = \vec{U}_{\vec{r}Ao} = \dfrac{0\hat{i} - 4\hat{j} - 6\hat{k}}{\sqrt{0^2 + 4^2 + 6^2}} = 0\hat{i} - 0.55\hat{j} - 0.83\hat{k}$

② $\vec{F}_{AB} = 400\, U_{\vec{r}AB} = 400\left(\dfrac{4\hat{i} + 1\hat{j} - 6\hat{k}}{\sqrt{4^2 + 1^2 + 6^2}}\right) = 219.6\hat{i} + 54.9\hat{j} - 329.4\hat{k}$

③ $\vec{F}_{\parallel Ao} = (219.6\hat{i} + 54.9\hat{j} - 329.4\hat{k}) \cdot (0\hat{i} - 0.55\hat{j} - 0.83\hat{k})$

$= 243.3N$

④ $\vec{F}_{\parallel Ao} = 243.3\, \vec{U}_{Ao} = 243.3\,(0\hat{i} - 0.55\hat{j} - 0.83k)$

$|F_{\perp Ao}| = \sqrt{F^2 - F_{\parallel Ao}^2} = \sqrt{400^2 - 243.3^2}$

$\vec{F}_{\perp Ao} = \vec{F} - \vec{F}_{\parallel Ao} = 219.6\hat{i} + 54.9\hat{j} - 329.4k - [243.3(0\hat{i} - 0.55\hat{j} - 0.83\hat{k})]$

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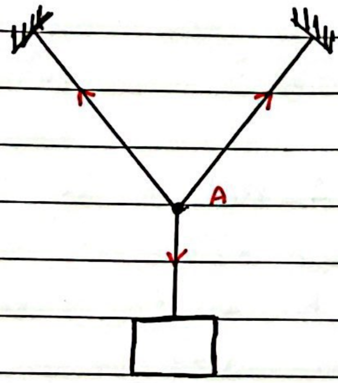
Chapter 3 Equilibrium of a particle



A is a particle (جسم مهمل الأبعاد).

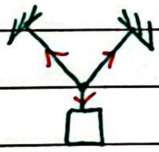
For A to be in equilibrium the net force on A must be zero ($\Sigma F = 0$).

* for any body equilibrium happens when $\Sigma F = 0$ ($\Sigma F_x = 0$, $\Sigma F_y = 0$).

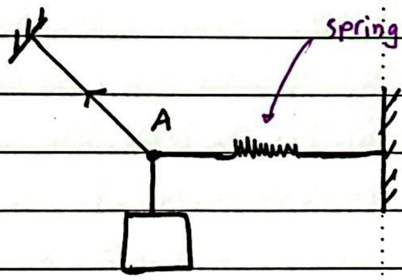
* if it was in 3D → $\Sigma F_x = 0$, $\Sigma F_y = 0$, $\Sigma F_z = 0$ ≡ equilibrium.

Free Body Diagram (FBD).

1. cable, string, chord (Tension force).



2. spring



K = spring constant (N/m)

$\boxed{F_{spring} = K\Delta}$ Hook's law

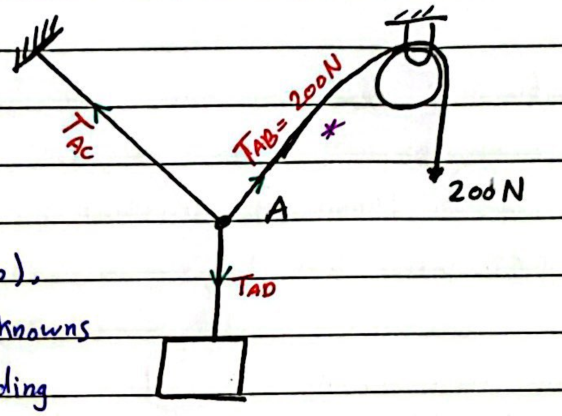
Δ = stretch, displacement, change in length...

$$\boxed{\Delta = l_f - l_i}$$

l_f = final length, stretched length, deformed length.
l_i = initial length, undeformed length, unstretched length.

③ pully

$\Sigma F_x = 0$, $\Sigma F_y = 0$

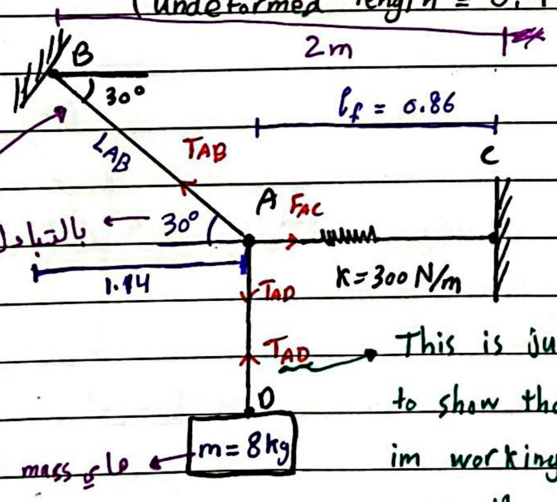


☆ in 2D we have 2 equations which are ($\Sigma F_x = 0$ and $\Sigma F_y = 0$). meaning we can find max 2 unknowns in 3D we have 3 equations adding $\Sigma F_z = 0$ to the previous two, meaning we can find 3 unknowns max.

* الـ force اللي بتحملها البكرة هي نفس الـ force على طول الحبل.

→ This is only for 1 particle, if we had 2 particles we would have 4 equations meaning we can solve 4 unkowns max.

Example: ① Find The Tension in the cable and the force in the spring (undeformed length = 0.4 m).



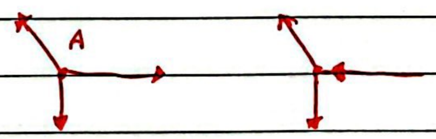
بالتبادل مع

T_{AD} → This is just to show that im working on the AD cable (nothing to do with direction).

(kg) mass على ← عشان احولها لـ (w) force بضرب بتسارع الجاذبية الأرضية (10)

$W = mg$
$= 8 \times 10$
$= 80\ N$

* in springs I consider them Tension forces and the direction i can choose whatever, and i will know if what i chose is ~~wat~~ Right or not, based on the sign of the answer (will be explained later on).



تابع ⇒

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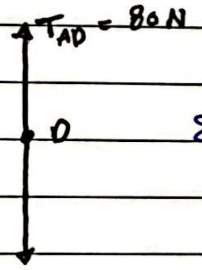
Solution:

in the question we have 3 unknowns and we have 2 equations ($\sum F_x=0$ and $\sum F_y=0$) so we have to make the unknowns at most 2 unknowns (because we have 2 equations) and we can, because we have a weight and using that we can find the Tension in ~~cable~~ cable AD.

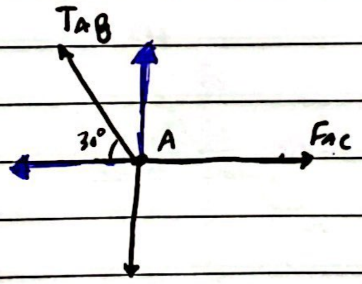
Draw the Free Body Diagram.



$\sum F_y = 0 \qquad T_{AD} = 80N$

$W = mg = 8 \times 10 = 80N$

now we have 2 unknowns left.



$T_{AD} = 80N.$

$\sum F_y = 0$

$-T_{AD} + T_{AB}\sin 30 = 0$

$-80 + T_{AB}\sin 30 = 0$

$T_{AB} = \frac{80}{\sin 30} = 160N$

$\sum F_x = 0$

$F_{AC} - T_{AB}\cos 30 = 0$

$F_{AC} - 160\cos 30 = 0$

$F_{AC} = 160\cos 30\ N$

$F_{AC} = 138.5\ N$

② Find the length AB

$F_{AC} = k\Delta$

$138.5 = 300(l_f - 0.4)$

$l_f = \frac{138.5}{300} + 0.4 = 0.86\ m$

$2 - 0.86 = 1.14$

$L_{AB}\cos 30 = 1.14$

$L_{AB} = \frac{1.14}{\cos 30} = 1.32\ m$

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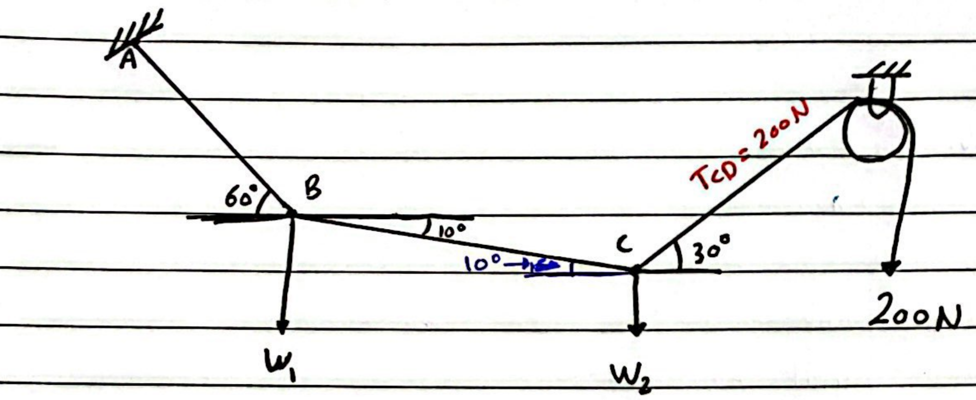
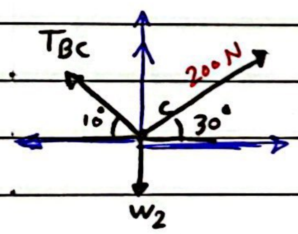
Example :- Find W_1 , W_2 and the Tension in each cable:



Here i have 2 particles (B and C) so 4 equations and 5 unknowns



FBD for particle C.

$\Sigma F_x = 0$

$\Sigma F_x = 200 \cos 30 - T_{BC} \cos 10 = 0$

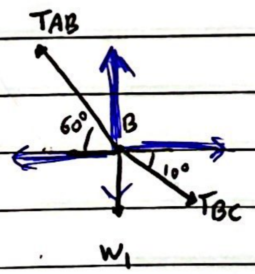
$\boxed{T_{BC} = \frac{200 \cos 30}{\cos 10} = 175.8\text{ N}}$

$\Sigma F_y = 0$

$200 \sin 30 + T_{BC} \sin 10 - W_2 = 0$

$W_2 = 200 \sin 30 + 175.8 \sin 10$

$\boxed{W_2 = 130.5\text{ N}}$



FBD for particle B

$\Sigma F_x = 0$

$T_{BC} \cos 10 - T_{BA} \cos 60 = 0$

$175.8 \cos 10 - T_{BA} \cos 60 = 0$

$\boxed{T_{BA} = 346.3\text{ N}}$

$\Sigma F_y = 0$

$-T_{BC} \sin 10 - W_1 + T_{BA} \sin 60 = 0$

$\boxed{W_1 = 269.4\text{ N}}$

☆ we know $\boxed{T_{CD} = 200\text{N}}$ from the Drawing itself.

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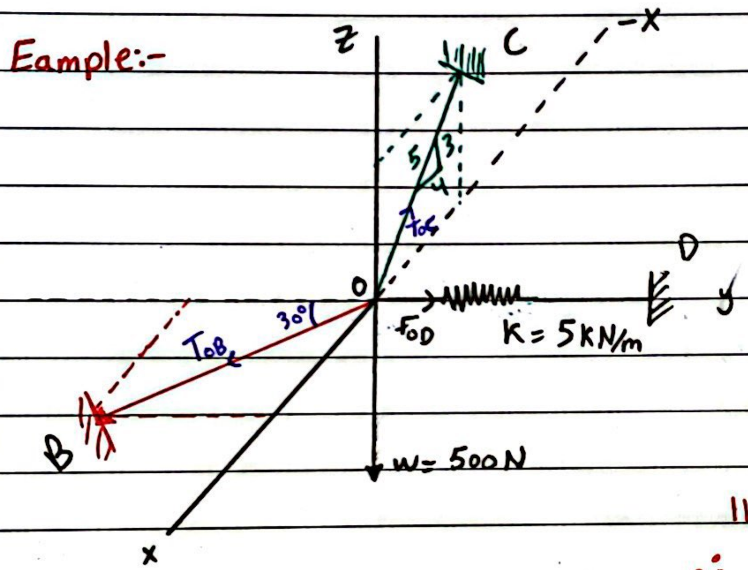
Equilibriam of a particle in 3D

Key :- Express each force as a cartesian vector

$\cos\alpha, \cos\beta, \cos\gamma$ angles   position vectors

$\Sigma F_x = 0 \qquad \Sigma F_y = 0 \qquad \Sigma F_z = 0$

Eample:-



Find the stretch in the spring $\Delta = ??$

$F_{OD} = K\Delta$

$1154.6 = 5000\,\Delta$

$\therefore \Delta = \frac{1154.6}{5000} = 0.23\,m$

$\vec{w} = 0\hat{i} + 0\hat{j} - 500\hat{k}$

$\vec{F_{OD}} = 0\hat{i} + F_{OD}\hat{j} + 0\hat{k}$

$\vec{T_{OC}} = -T_{OC}\left(\frac{4}{5}\right)\hat{i} + 0\hat{j} + T_{OC}\left(\frac{3}{5}\right)\hat{k}$

$\vec{T_{OB}} = T_{OB}\sin(30)\hat{i} - T_{OB}\cos(30)\hat{j} + 0\hat{k}$

$\Sigma F_x = 0 \rightsquigarrow 0 + 0 - T_{OC}\left(\frac{4}{5}\right) + T_{OB}\sin(30) = 0$ —— ①

$\Sigma F_y = 0 \rightsquigarrow 0 + F_{OD} + 0 - T_{OB}\cos(30) = 0$ —— ②

$\Sigma F_z = 0 \rightsquigarrow -500 + 0 + T_{OC}\left(\frac{3}{5}\right) + 0 = 0$ —— ③

from ③ $\longrightarrow T_{OC} = 833.3N$

from ① $\longrightarrow T_{OB} = 1333.3\,N$

$\boxed{\text{from ② } \longrightarrow F_{OD} = 1154.6}$

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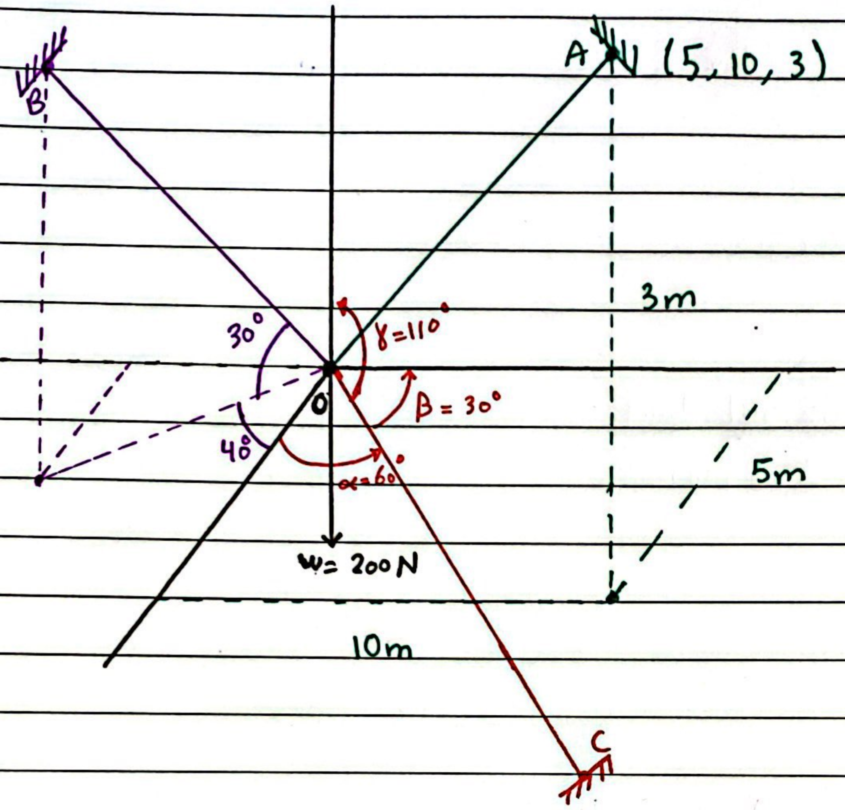
Example:- Find the Equations on equilibriam.



$$\vec{F}_{OB} = F_{OB} \cos 30 \cos 40 \, \hat{i} - F_{OB} \cos 30 \sin 40 \, \hat{j} + F_{OB} \sin 30 \, \hat{k}$$

$$\vec{F}_{OA} = F_{OA} \frac{\vec{r}_{OA}}{|r_{OA}|} = F_{OA} \left(\frac{5\hat{i} + 10\hat{j} + 3\hat{k}}{\sqrt{5^2 + 10^2 + 3^2}} \right)$$

$$\vec{T}_{OC} = T_{OC} \cos 60 \, \hat{i} + T_{OC} \cos 30 \, \hat{j} + T_{OC} \cos 110 \, \hat{k}$$

$$W = 0\hat{i} + 0\hat{j} - 200\hat{k}$$

$$\sum F_x = 0 \rightarrow F_{OB} \cos 30 \cos 40 + F_{OA} \left(\frac{5}{\sqrt{134}} \right) + T_{OC} \cos 60 + 0 = 0 \quad (1)$$

$$\sum F_y = 0 \rightarrow -F_{OB} \cos 30 \sin 40 + F_{OA} \left(\frac{10}{\sqrt{134}} \right) + T_{OC} \cos 30 + 0 = 0 \quad (2)$$

$$\sum F_z = 0 \rightarrow F_{OB} \sin 30 + F_{OA} \left(\frac{3}{\sqrt{134}} \right) + T_{OC} \cos 110 - 200 = 0 \quad (3)$$

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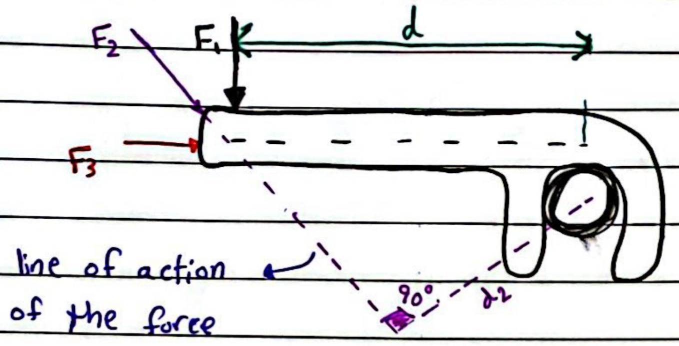
Chapter 4: Moment of a force



* The rotation or the tendency to rotate is some times called a torque, but most often it is called the Moment

$\boxed{M_o = F\,d}$ → وحدتها N.m

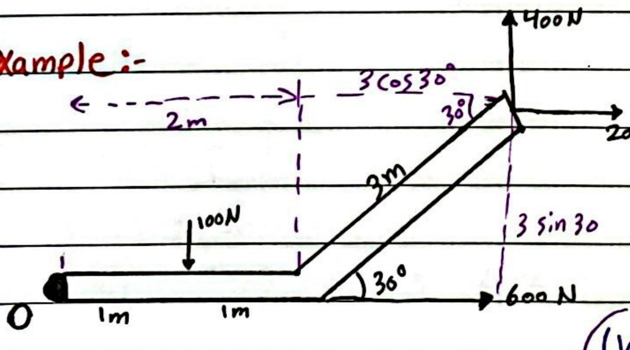
* d = perpendicular distance between F and the point of rotation.

← يعني المسافة العامودية بين الـ force ونقطة الدوران.

← يعني F_3 ما رح يكون الها moment $(M=0)$، ليش؟ ← لأنو ما في مسافة عامودية بينو وبين نقطة الدوران (على نفس الإستقامة).

← هسا F_2 اكيد ما رح تأثر عل الجسم وتعمل moment زي F_1 لأنو في زاوية عشان اعرف الـ moment بقدر احلل الـ force وبحلل زي ما رح نتعلم لاحقاً، أو انو نمد خط للـ force ومنو بطلع خط عامودي عليه لنقطة الدوران بعدين بنطلع d_2 وبنحل على القانون طبيعي.

Example:-



* بعتبر شكل المجسم مهمل بما انو ما ذكرلي اياها، اما اذا بذكرلي اياها لازم ادخلها بالحسابات.

* طبعاً الـ moment عبارة عن متجه (Vector) والإتجاه رح احددو من خلال قاعدة اليد اليمنى يا مع أو عكس عقارب الساعة.

$\circlearrowleft +$ $M_o = -100(1) + 0 - 200(3\sin 30) + 400(2 + 3\cos 30) = 1439\ N.m$ CCW

لازم ابينها بالإمتحان.

لأنو فرضت الموجب هي CCW.

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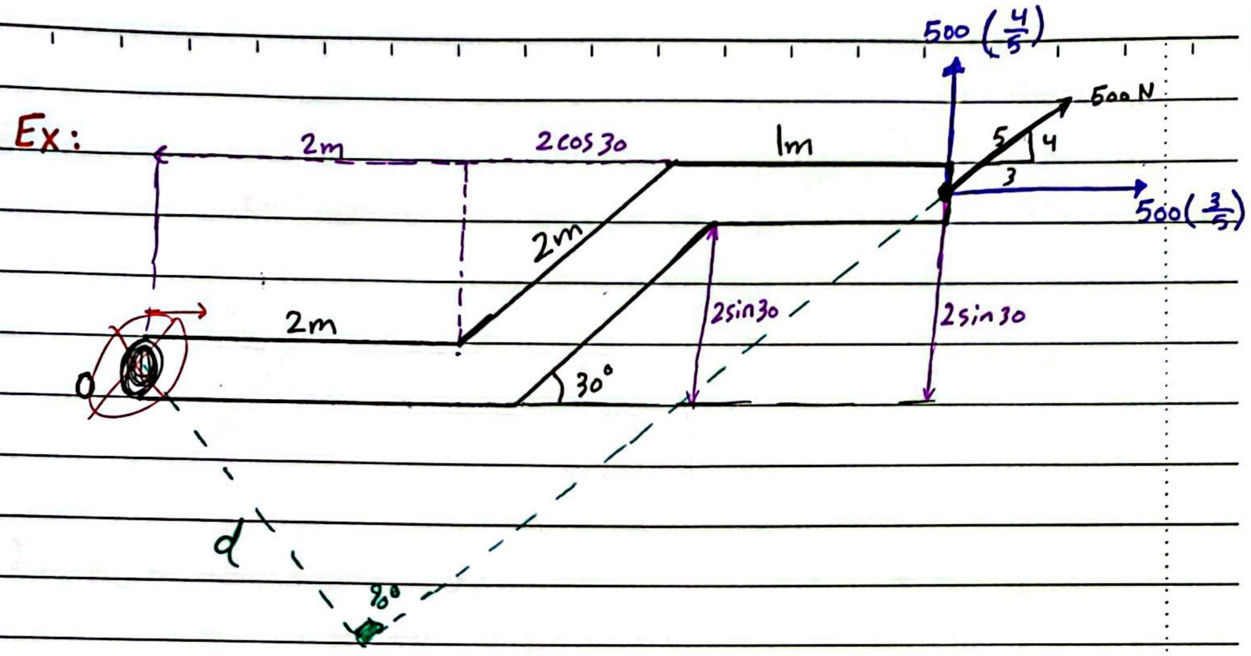
Ex:



$$\overset{+}{\circlearrowleft} M_o = -500\left(\frac{3}{5}\right)(2\sin 30) + 500\left(\frac{4}{5}\right)(1 + 2\cos 30 + 2)$$

$= 1593$ N.m

CCW

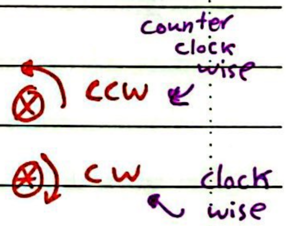
*

هون ، ايش المسافة العامودية على الـ X-axis اللي هي $500\left(\frac{3}{5}\right)$ ؟

اللي هي المسافة اللي على الـ y-axis : $2\sin30$.

* طب ليش سالب؟ من خلال تطبيقي لقاعدة اليد اليمين بشوف انو بلحق الـ force لما اكون امشي cw وانا فرضت الموجب هو ccw فعشان هيك بحط سالب.

∴ وعلى هاد المبدأ بكون الحل.



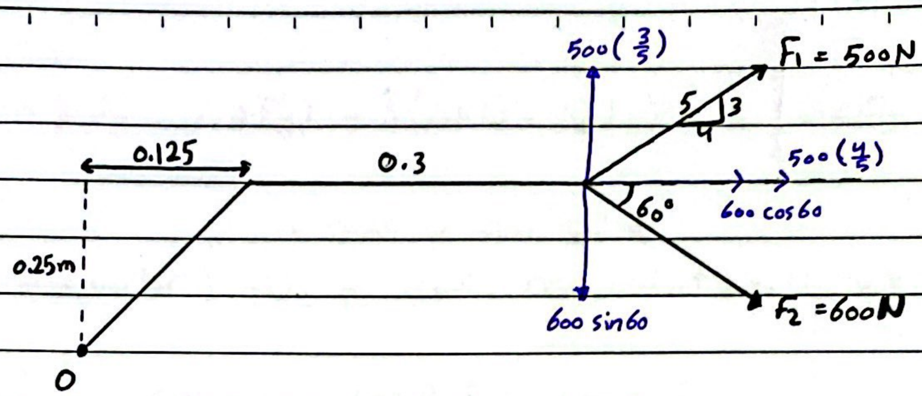
* find d ??

$M = F d$

$1593 = 500\ d$

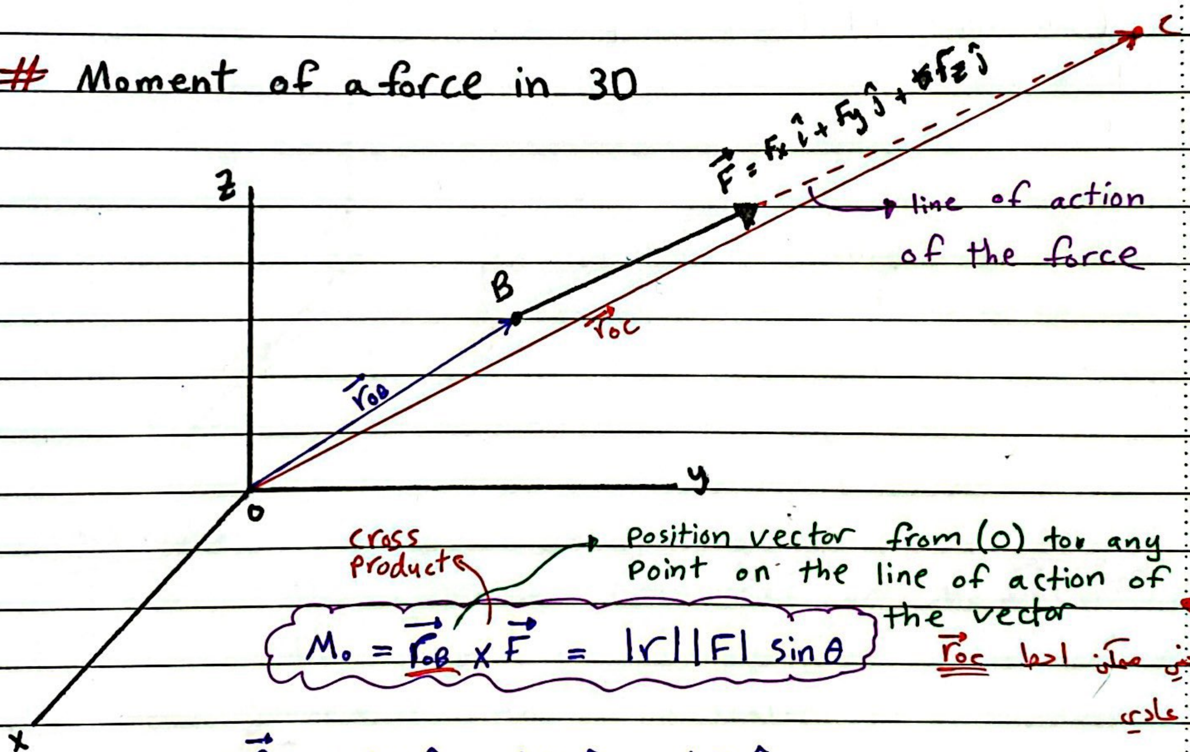
$\therefore d = \frac{1593}{500} \Rightarrow d = 3.2$ m

Ex:



$$\circlearrowleft^{+} M_o = -500\left(\tfrac{4}{5}\right)(0.25) - 600\cos 60\,(0.25) + 500\left(\tfrac{3}{5}\right)(0.3+0.125)$$
$$-\,600\sin 60\,(0.3+0.125) = \ldots$$

Moment of a force in 3D



cross product

position vector from (O) to any point on the line of action of the vector

$$\boxed{M_o = \vec{r}_{OB} \times \vec{F} = |r||F|\sin\theta}$$

* يعني ممكن احطها $\vec{r}_{OC}$ عادي

$\vec{r}_{OB} = \Delta x\,\hat{i} + \Delta y\,\hat{j} + \Delta z\,\hat{k}$

$\vec{F} = F_x\,\hat{i} + F_y\,\hat{j} + F_z\,\hat{k}$

cross product

$$M_o = \vec{r}_{OB} \times \vec{F} = \begin{array}{c} \\ \\ \\ \end{array}\begin{vmatrix} \overset{\oplus}{\hat{i}} & \overset{\ominus}{\hat{j}} & \overset{\oplus}{\hat{k}} \\ \Delta x & \Delta y & \Delta z \\ F_x & F_y & F_z \end{vmatrix}\begin{array}{l} \\ \to \vec{r}_{OB} \\ \to \vec{F} \end{array}$$

Determinent

* الترتيب مهم

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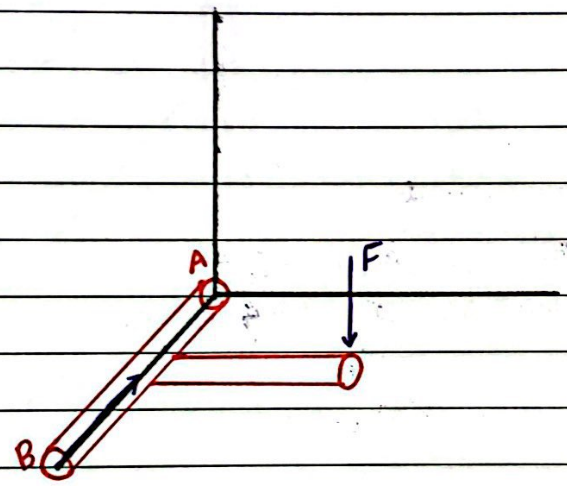
$$\vec{M} = \hat{i}\,(\Delta y\, F_z - \Delta z\, F_y) - \hat{j}\,(\Delta x\, F_z - \Delta z\, F_x) + \hat{k}\,(\Delta x\, F_y - \Delta y\, F_x)$$

* انتبه السالب من القانون *

* طبعًا لما نحل cross product الجواب رح يطلع (as a vector).

$$u_M = \frac{\vec{M}}{|M|} = \cos\alpha\,\hat{i} + \cos\beta\,\hat{j} + \cos\gamma\,\hat{k}$$

4.5 : Moment about specific axis



M_{AB} ?

The axis AB

خطوات الحل:

1. $M_{\text{any point on the axis}}$ $(M_A = \vec{r} \times \vec{F}$ or $M_B = \vec{r} \times \vec{F})$
2. $u_{axis} = u_{AB} = \dfrac{\vec{r}_{AB}}{|r_{AB}|}$
3. $M_{AB} = M_A \cdot u_{axis} \longrightarrow$ scalar
4. $\vec{M}_{AB} = M_{AB} * u_{axis} \longrightarrow$ To make the moment a vector.

OR i can use the triple dot product $\longrightarrow$

$$M_{AB} = \begin{vmatrix} u_x & u_y & u_z \\ r_x & r_y & r_z \\ F_x & F_y & F_z \end{vmatrix} \begin{array}{l} \rightarrow \text{axis} \\ \\ \rightarrow \text{force} \end{array}$$

any point on the axis to any point on the force

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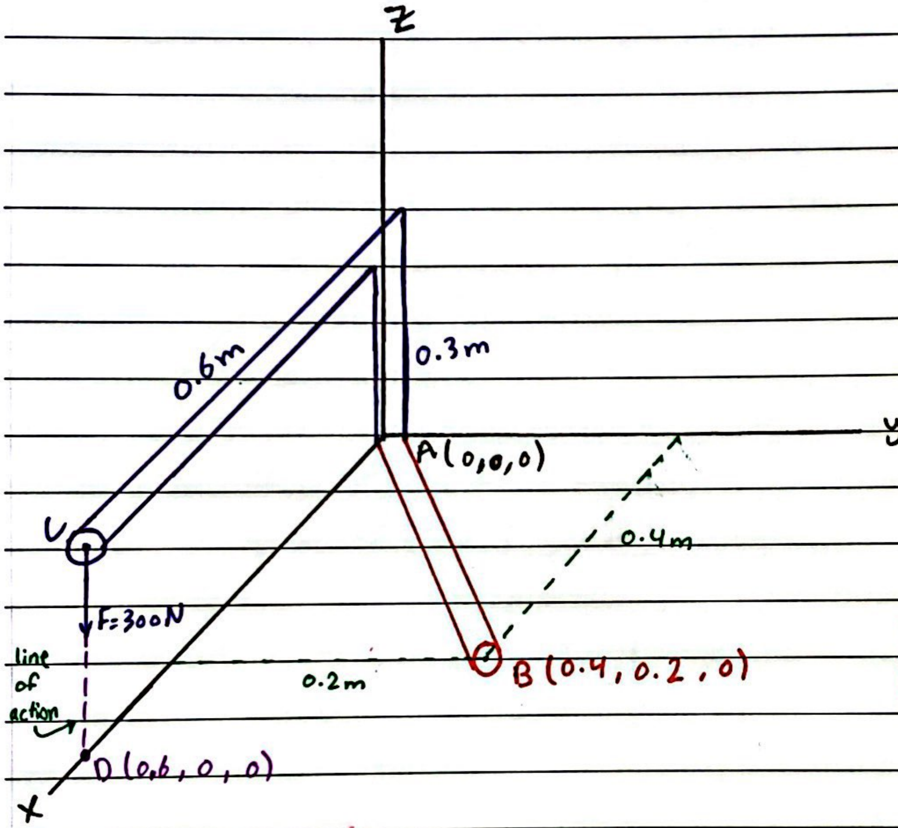
Example: Determine the moment produced by the Force about axis (AB).



$\vec{M}_{AB} = 80.5\,(0.894\hat{i} + 0.447\hat{j})$
$= 72\hat{i} + 36\hat{j} + 0\hat{k}$

$$U_{AB} = \frac{\vec{r}_{AB}}{|r_{AB}|} = \frac{0.4\hat{i} + 0.2\hat{j} + 0\hat{k}}{\sqrt{0.4^2 + 0.2^2 + 0^2}} = 0.894\hat{i} + 0.447\hat{j} + 0\hat{k}$$

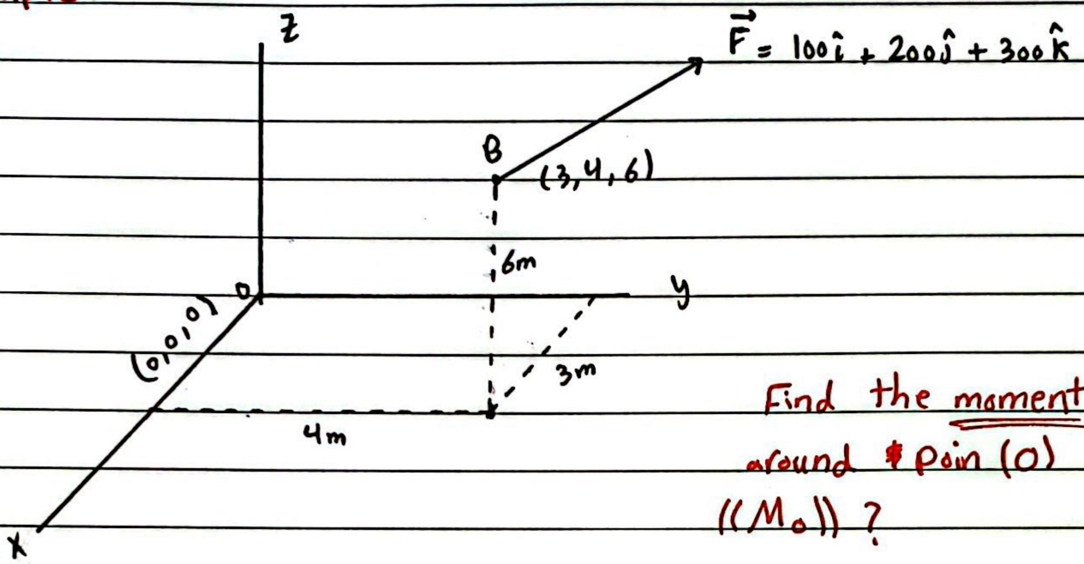
$$\vec{r}_{AD} = 0.6\hat{i} + 0\hat{j} + 0\hat{k}$$

* Now we want a position vector from any point on the axis AB to any point on the line of action of the force, we choose D because it has one component on the x-axis only, which will make solving the problem easier.

$$M_{AB} = \begin{array}{c} \begin{vmatrix} 0.894 & 0.447 & 0 \\ 0.6 & 0 & 0 \\ 0 & 0 & -300 \end{vmatrix} \end{array} \begin{array}{l} \rightarrow \vec{U}_{axis} \\ \rightarrow \vec{r}_{AD} \\ \rightarrow \vec{F} \end{array}$$

$= 0.894\,(0 * -300 - 0) - 0.447\,(0.6 * -300)$
$+ 0 = \boxed{80.5\ N.M}$ → we got a scalar because we used the triple dot product.

Example:



Find the moment around ~~#~~ poin (O) ((M_o)) ?

$$M_o = \vec{r}_{oB} \times \vec{F}$$

$$\vec{r}_{oB} = 3\hat{i} + 4\hat{j} + 6\hat{k}$$

$$M_o = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 4 & 6 \\ 100 & 200 & 300 \end{vmatrix} = \hat{i}(4*300 - 6*200) - \hat{j}(3*300 - 6*100) + \hat{k}(3*200 - 4*100)$$

$$= 0\hat{i} - 300\hat{j} + 200\hat{k}$$

$$|M_o| = \sqrt{0^2 + 300^2 + 200^2} = 360.56 \text{ N.m}$$

$$U_m = \frac{\vec{M}}{|M|} = \frac{0\hat{i} - 300\hat{j} + 200\hat{k}}{360.5}$$

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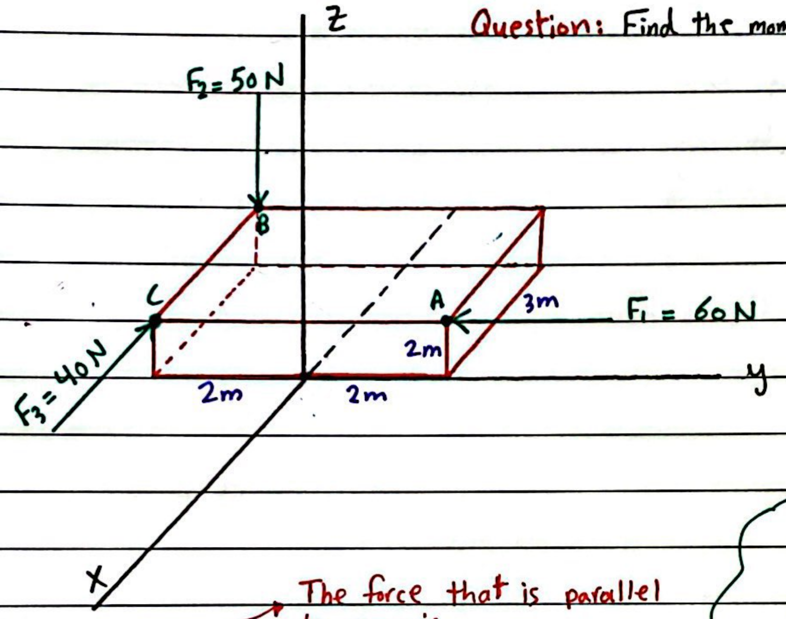
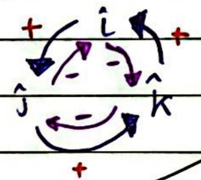
* عشان نطلع الـ moment about an axis كنا نطلعها باستخدام الـ Triple dot product

بس لما يكون عنا forces موازية للـ axis بقدر اسهل الحل كالآتي:

Question: Find the moment about x, y and z axises.





cross product

مثلاً

$\hat{i} \times \hat{j} = +\hat{k}$

$\hat{j} \times \hat{i} = -\hat{k}$

moment along the x-axis

The force that is parallel to y-axis

$M_x = F_y\, d_z$ → perpendecular distance to the y-axis on the z-axis

أو → $F_z\, d_y$

يعني → $M_{x_1} = \underset{F_y}{60} * \underset{d_z}{2}$

The force that is parallel to the y-axis * The distance that is required to reach the force on the z-axis (which is perpendecular to the y-axis).

* طبعاً في كمان force بتعمل moment على الـ x-axis

$M_{x_2} = \underset{F_z}{50} * \underset{d_y}{2}$

لاحظ أن → $M_x = F_y * d_z$

و → $M_x = F_z * d_y$

و بالمنطق ان الـ forces الموازية للـ x مبين من الرسمة انو ما رح تسبب بدوران الجسم حول الـ x-axis

فبهم كل الأكسيز فبعرف اني بشتغل صح.

هون لازم يكون في اشارة موجب او سالب و انا بحددها يا بالـ Right hand Rule أو باستخدام

$\hat{i} \times \hat{k} = \ldots$

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مثلاً لو بدي اطلع اتجاه ال moment بإستخدام ال Right hand Rule

$$M_x = +\overset{F_y}{60} * 2$$

①
الإبهام بحطو بإتجاه ال x-axis
وممكن بإتجاه ال (-x)-axis عادي.
واصابعي بحركها بإتجاه ال Fy
(بهاي الحالة) اذا اصابعي فعلاً
لحقو ال force اذا اتجاه الإبهام
اللي فرضتو هو الصح.
وهون رح يطلع + لأنو
فرضت ابهامي باتجاه ال +ve
.x-axis

نفس الاشي

②
اما الطريقة الثانية بعمل التالي:

$\boxed{d \text{ Direction} \times F \text{ Direction}}$

(طبعًا الترتيب مهم لأنو cross product)
فحلى نفس السؤال:

$$+\hat{k} \times -\hat{j} = +\hat{i}$$

* اتجاه المسافة اللي قطعتها
حتى وصلت ال force.

Sol: $M_x = \pm 60(2) \pm 50(2) = 220 \text{ N.m}$

$+\hat{k} \times -\hat{j} = \pm\hat{i} \qquad -\hat{j} \times -\hat{k} = +\hat{i}$

$M_y = -50(3) - 40(2) = -230 \text{ N.m}$

$F_z \, d_x \qquad F_x \, d_z$

$-\hat{i} \times -\hat{k} = -\hat{j} \qquad +\hat{k} \times -\hat{i} = -\hat{j}$

$M_z = 0 - 40(2) = -80 \text{ N.m}$

$F_y \, d_x \qquad F_x \, d_y$

$= -\hat{k}$

$-\hat{j} \times -\hat{i}$

$(d_y \times F_x)$

$+\hat{k}$

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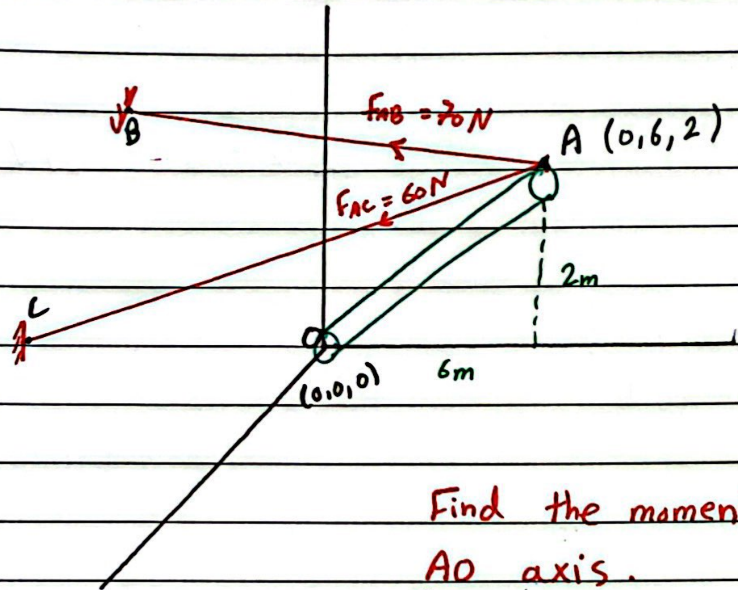

Find the moment about the AO axis.

not the same as (OA).

$$u_{AO} = \frac{0\hat{i} - 6\hat{j} - 2\hat{k}}{\sqrt{6^2 + 2^2}}$$

$$M_{AO} = \begin{vmatrix} u_{AO} \\ r_{OA} \\ \vec{F}_{AB} \end{vmatrix} + \begin{vmatrix} u_{AO} \\ r_{OA} \\ \vec{F}_{AC} \end{vmatrix} = 0$$

عرفت إنو صفر عطول لأنها ال forces تتقاطع مع ال axis
بشكل مش عامودي على ال axis

* هاي الصفحة كلها مش مهمة *

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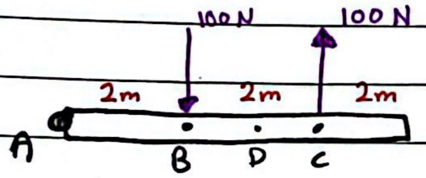
4.6 moment of a couple



$\circlearrowleft + \; M_A = -100(2) + 100(4) = 200\ N.m$

$\circlearrowleft + \; M_B = 0 + 100(2) = 200\ N.m$

$\circlearrowleft + \; M_C = 100(2) + 0 = 200\ N.m$

★ لاحظ الـ moment couple دايمًا متساوي على كل النقاط على الجسم.

★ for amoment to be called "moment of a couple" the forces mus be 1. Equel. 2. opposite. 3. non colinear.

لـ يعني مش على نفس الامتداد

يعني انا بدي احسب ★ الـ moment ويعرف انو عندي moment couple فخلص بحسب الـ moment على نقطة وحدة وهي نفس الـ moment إلي على كل الجسم.

$\circlearrowleft + \; M_B = F(d)$

$= +100(2)$ ccw

★ الأسهل إني اخذ الـ moment عند نقطة على امتداد الـ force وبهاي الحالة ما رح تعمل moment الا من الـ force الثانية، يعني الـ force إلي عند B ما رح تعمل moment. الـ moment رح يجي بس من الـ force إلي عند الـ C.

★ لو طلعت الـ moment عند الـ A رح احتاج احسب الـ force الأولى والثانية فممكن اصعبها على حالي هيك.

⇒ when there is a moment of a couple we calculate it using the Distance between the two forces not between the force and a certian point.

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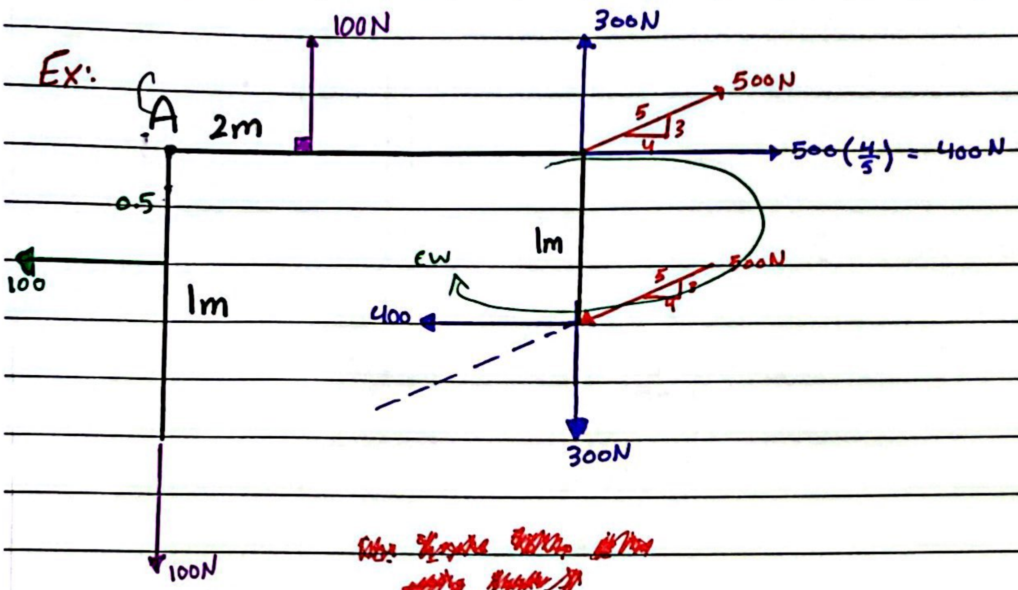
Ex:



$\overset{+}{\circlearrowleft} M_A = -400(1) + 0 + 100(2) - 100(0.5) = -250$ CW

لـ ال 300N calinear وما في

يعني مسافة عامودية

بينهم وغير هيك واضحة انو

رح يلغو بعض.

⇒ in this example we have 4 forces that make a 2 pair of a couple moment and a force that has a ⊥ distance (0.5m) with point A, The 500N forces when ~~...~~ broken up to its' components make up 300 N and 400 N, the 300N forces have <u>no</u> ⊥ distance <u>and</u> they're calinear meaning they will have no moment

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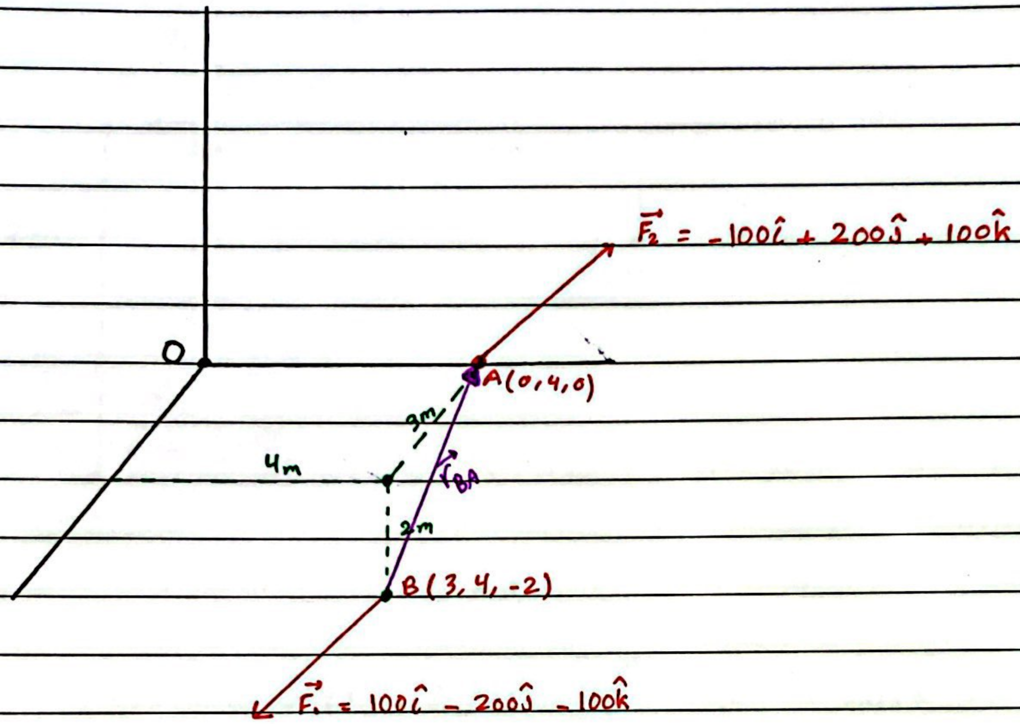

$M_B = \vec{r}_{BA} \times \vec{F_2}$

↳ The Distance between the point of rotation to any point on the line of action of the force vector

$$= \begin{array}{c} \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 0 & 2 \\ -100 & 200 & 100 \end{vmatrix} \end{array} \begin{array}{l} \\ \vec{r}_{BA} \\ \vec{F_2} \end{array}$$

$M_A = \vec{r}_{AB} \times \vec{F_1}$

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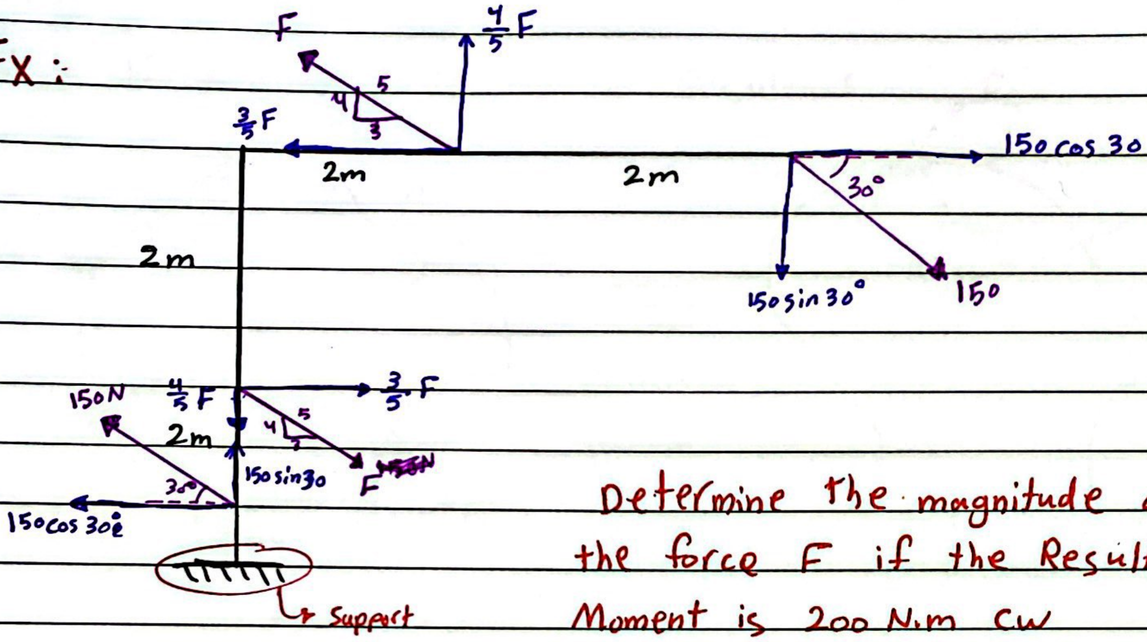
Ex :



Determine the magnitude of the force F if the Resultant Moment is 200 N.m cw

$$\circlearrowleft + M = -150\cos 30\,(4) - 150\sin(30)\,(4) + \frac{3}{5}F\,(2) + \frac{4}{5}F\,(2) = -200$$

بعوض ؟ دريغري

The question gave us 200 cw but my moment is ccw which is the opposite, that's why we put the (−).

$$\boxed{F = 221.3\ N}$$

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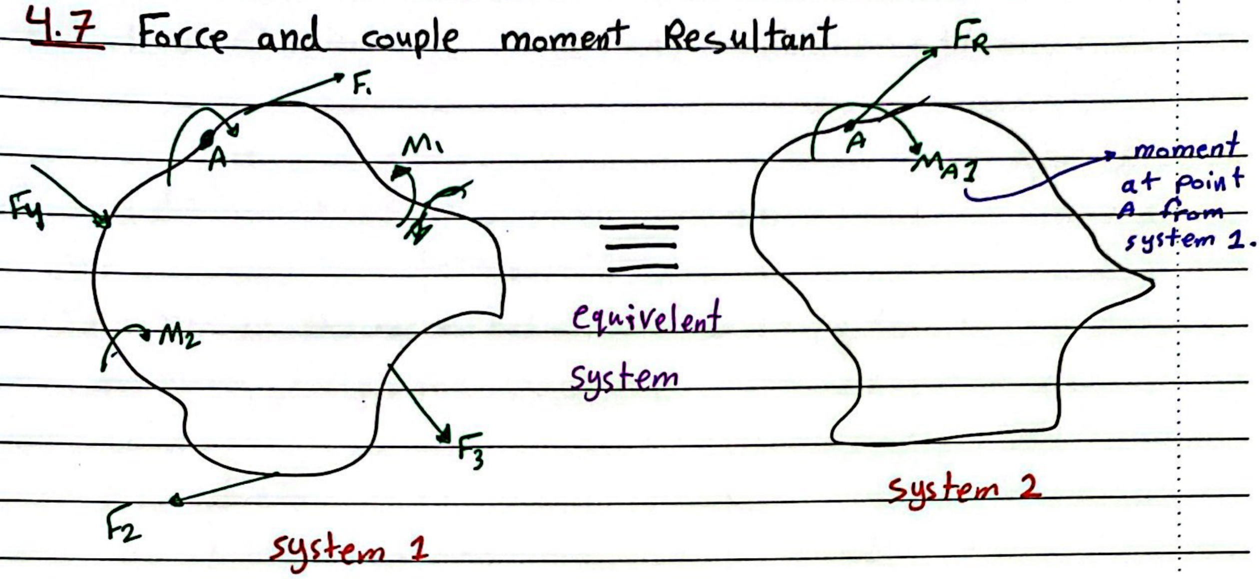
4.7 Force and couple moment Resultant



$$M_{A1} = M_1 - M_2 + \sum F * d$$

step 1:- F_R

$\Sigma F_x \quad \Sigma F_y \quad F_R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$

$\vec{F_R} = \Sigma F_x \, \hat{i} + \Sigma F_y \, \hat{j}$

$\theta = \tan^{-1} \left| \frac{F_{Ry}}{F_{Rx}} \right|$

step 2:- M_A

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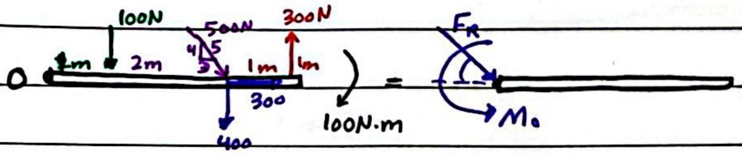
4.7 Simplification of a force and a couple system.



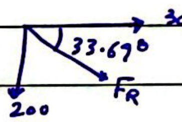
* شكل الجسم مُهمل

Ex: Replace the force and couple ~~moment~~ system by a single force and moment O

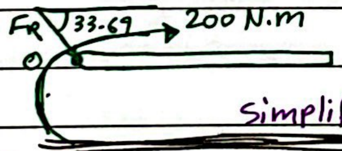
السؤال
غشمشني
شو بدو.

step 1: F_R

$$\begin{cases} \Sigma F_x = 300N \\ \Sigma F_y = -100 - 400 + 300 = -200\ N \\ F_R = \sqrt{300^2 + 200^2} = 360.5\ N \\ \tan\theta = \left|\frac{\Sigma F_y}{\Sigma F_x}\right| = \left|\frac{200}{300}\right| \Rightarrow \theta = 33.69^\circ \end{cases}$$

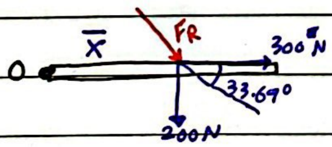


step 2: M_o $\left\{ (\circlearrowleft +)\ M_{o1} = -100(1) - 400(3) + 300(4) - 100 = -200\ N.m\ \text{CW} \right.$



simplified system

* Replace the force and ~~coup~~ couple system by a single force and Determine it's location

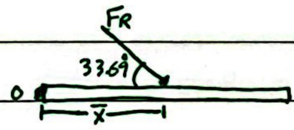


system 3

$M_{o1} = M_{o3}$

$-200 = -200\ \bar{X}$

$\bar{X} = 1m$



Further simplification.

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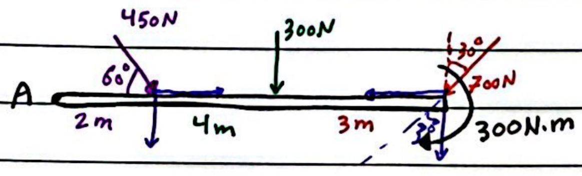
Ex: Replace the force system and couple system by a single force and where to be located from A

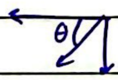


$\Sigma F_x = 450\cos 60 - 700 \sin 30 = -125 \text{ N}$

$\Sigma F_y = -450 \sin 60 - 700 \cos 30 - 300 = -1295.9 \text{ N}$

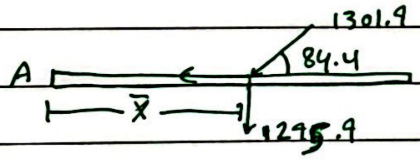
$F_R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = 1301.9 \text{ N}$

$\theta = \tan^{-1}\left| \frac{\Sigma F_y}{\Sigma F_x} \right| = 84.4$



$M_A = -450 \sin(60)(2) - 300(6) - 700\cos(30)(9) - 300$

$= -8335.4 \text{ N.m}$



$-8335.4 = -1295.9\,\bar{x}$

$\bar{x} = \frac{8335.4}{1295.9} = 6.43 \text{ m}$

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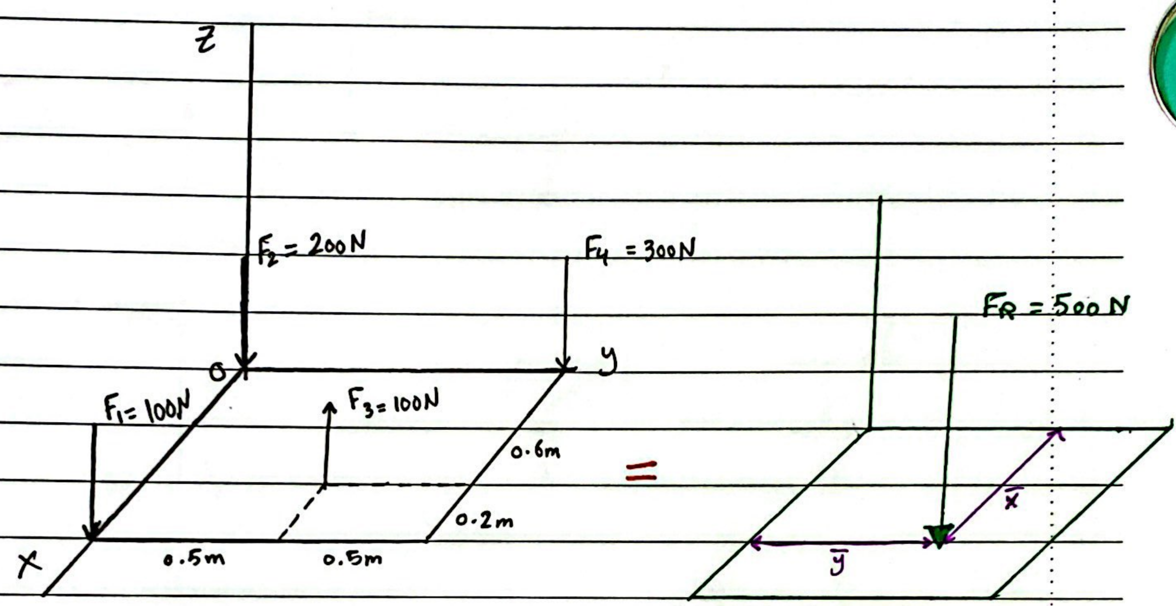
Example: replace the loading by a single force and determine its location.



$\vec{F_R} = -100\hat{k} + -200\hat{k} + 100\hat{k} + -300\hat{k}$ ← كل ال forces موازية لل
$= -500\hat{k}$ z-axis مغادي بقدر اجمعهم

① $M_{x1} = M_{x2}$
② $M_{y1} = M_{y2}$ ← كل ال forces موازية لل z-axis فما رح نعمل moment على ال z-axis ~~(M_z)~~

① $\overset{F_3}{+100(0.5)} - \overset{F_4}{300(1)} = -250 = -500\bar{y}$

$\bar{y} = \frac{-250}{-500} = 0.5m$

F_1 and F_2 won't make a moment on the x-axis since they intersect it.

② $100(0.8) - 100(0.6) = 20 = +500\bar{x}$

$\bar{x} = \frac{20}{500} = 0.04m$

F_2 and F_4 won't make a moment on the y-axis since they intersect it.

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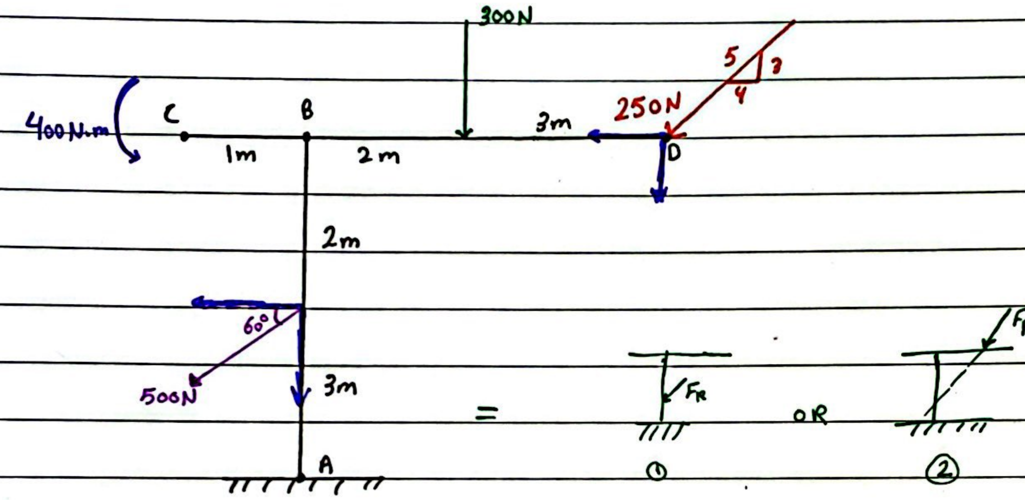
Eample:- Replace the loading by a single force and specify where its line of action intersects member CD measured from C.



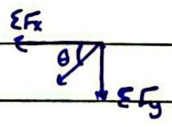
فعلياً ① و ② نفس الإشي فبدي اخُط الـ F_R زي ② لأنو طالب الحساب من عند ©.

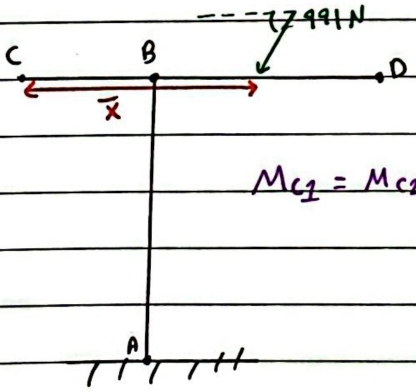
$$\Sigma F_x = -250\left(\frac{4}{5}\right) - 500\cos 60 = -450\,N$$

$$\Sigma F_y = -250\left(\frac{3}{5}\right) - 300 - 500\sin 60 = -883\,N$$

$$F_R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2} = \sqrt{450^2 + 883^2} = 991\,N$$

$$\tan\theta = \left|\frac{\Sigma F_{Ry}}{\Sigma F_x}\right| = \frac{883}{450} \qquad \theta = 62.9°$$





$M_{C1} = M_{C2}$

$$-300(3) + 400 - 250\left(\frac{3}{5}\right)(6) - 500\cos 60(2) - 500\sin(60)(1) = -883\,\bar{x}$$

$$\bar{x} = 2.64\,m$$

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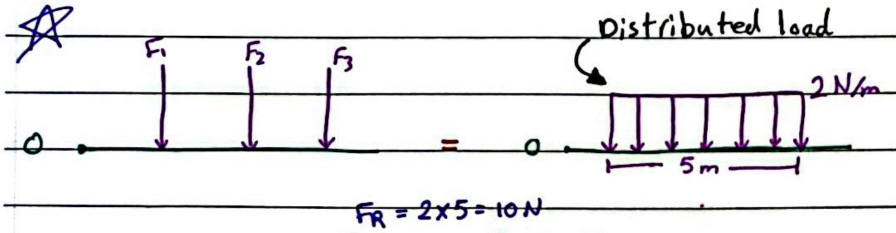
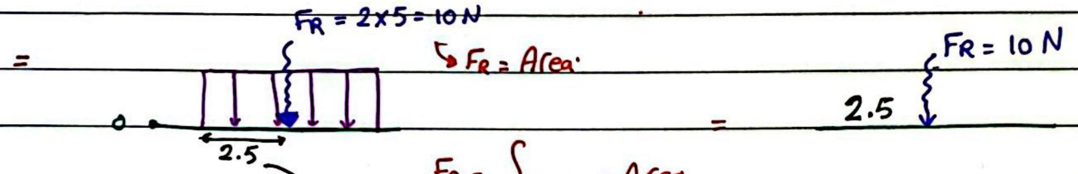
4.9 Reduction of a simple Distributed loading.





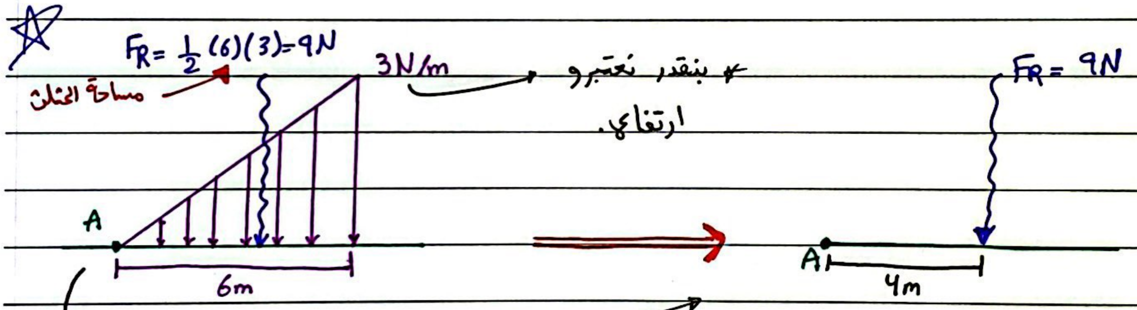
$\hookrightarrow F_R = Area$

$F_R = \int \dots = Area$

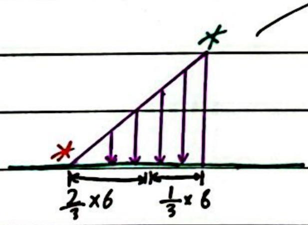
$F_R = \underline{2 \times 5} = 10N$

F_R بتكون بنص الـ Rectangular Distributed loading.

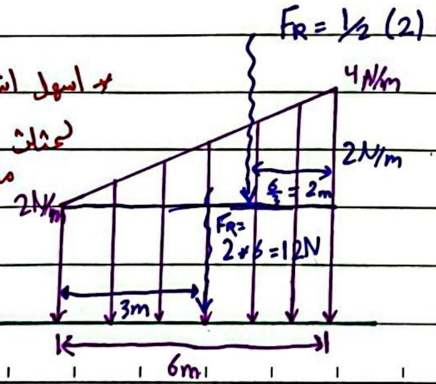
لأنو مستطيل بقدر اطلع المساحة بسهولة.



* بنقدر نعتبرو ارتفاع.



* في المثلث ما بقدر احط الـ F_R بالنص زي المستطيل، طب كيف؟ ← ببعد ثلث ($\frac{1}{3}$) المسافة عن الضلع الكبير و ثلثين ($\frac{2}{3}$) عن بداية المثلث.



* اسهل اشي اعتبرو لمثلث و مستطيل.

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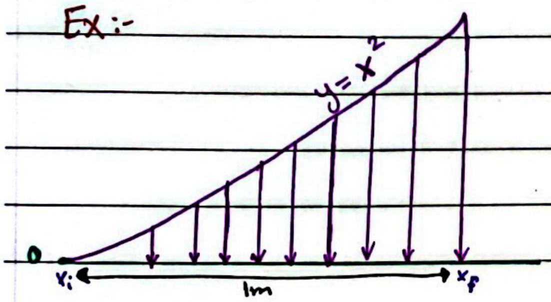
Ex:-



Determine the magnitude and location of the equivelent Force.

$$\boxed{F_R = \int_{x_i}^{x_f} y \, dx} = \int_0^1 x^2 \, dx = \frac{x^3}{3}\Big]_0^1 = \frac{1}{3} \text{ N}$$

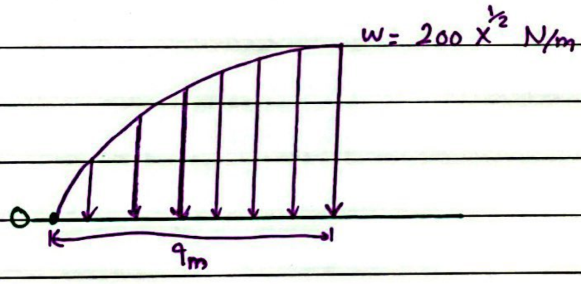
$$\boxed{\bar{x} = \frac{\int_{x_i}^{x_f} x \, y \, dx}{\int_{x_i}^{x_f} y \, dx \rightarrow F_R}}$$

$$\bar{x} = \frac{\int_0^1 x * x^2 \, dx}{1/3} = \frac{x^4/4\Big]_0^1}{1/3} = 3/4 \text{ m}$$

عشان اعرف موقع FR.

بالسؤال إللي فوق ما عنا جسم منتظم زي مثلث او مستطيل ، عشان هيك احتجنا إنو نستخدم قانون التكامل.

Ex:-



$$F_R = \int_0^9 200 x^{1/2} \, dx = 3600 \text{ N}$$

$$\bar{x} = \frac{\int_0^9 x \cdot 200 x^{1/2} \, dx}{\int_0^9 200 x^{1/2} \, dx} = 5.4 \text{ m}$$

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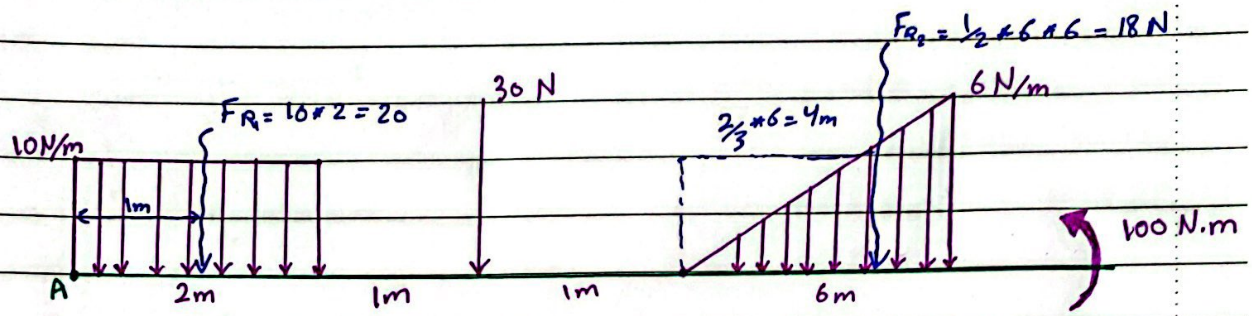
Ex:- Replace the loading by an equlvelent Force and Moment A.



$\Sigma F_x = 0$

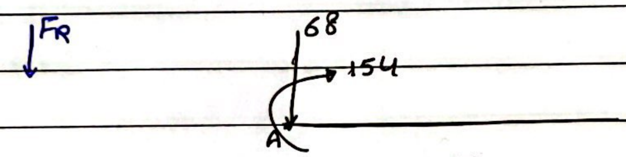
$\Sigma F_y = -20 - 30 - 18 = -68N$

$F_R = \sqrt{0^2 + 68^2} = 68^2$

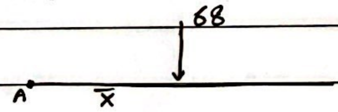
$\tan\theta = \left|\frac{68}{0}\right| \quad \theta = 90°$

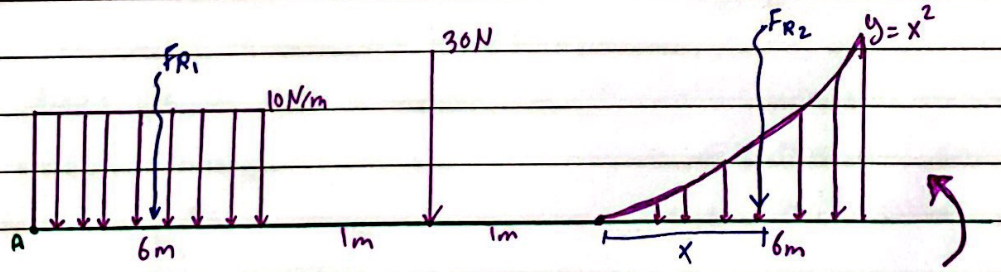
$\circlearrowleft + \ \Sigma M_A = -20(1) - 30(3) - 18(8) + 100$

$= -154 \text{ N.m}$



$-68\bar{x} = -154 \longrightarrow \bar{x} = \frac{154}{68}$





$F_{R_2} = \int_0^6 x^2\,dx = \frac{x^3}{3}\Big]_0^6 = 72$

$x = \frac{\int_0^6 x\, x^2\,dx}{\int_0^6 x^2\,dx} = \frac{\int_0^6 x\, x^2\,dx}{72}$

$= \frac{\frac{x^4}{4}\Big]_0^6}{72} = 4.9\text{ m}$

* دائمًا حدود التكامل بتكون من بداية لنهاية ال Distributed loading.

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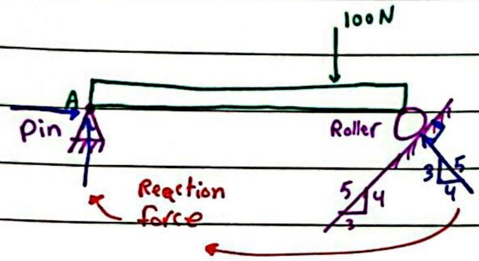
Chapter 5:- Equilibrium of a Rigid Body:



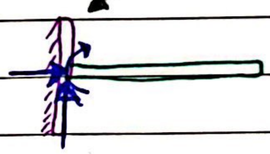
* Roller: makes a reaction that is perpendicular to the surface at the point of contact.

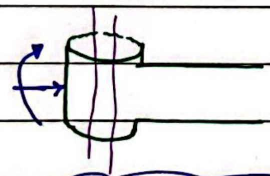
* Roller and pin are examples on support Reactions.
* support Reactions are the forces and moment that keeps the body from moving by creating opposite forces and moments that ~~keep the~~ prevents an object from moving or rotating.

* pin: makes reactions in the x and y directions, (so no vertical or horizantal movement).

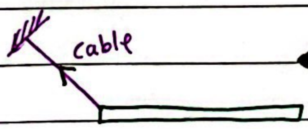
* in Roller and pin Rotation can still accure, However if iwant the object to not move or rotate then we can use a fixed support: makes reactions in the x and y direction and and opposite moment that prevents rotation.

* Roller and pin are examples on support reactions.

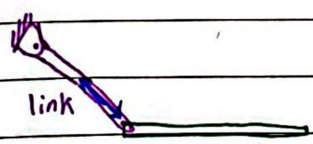




Guided Roller



in cable there is only tension.



in a link (or a 2-force-member) there can be tension or compression).

for a body to bee in equilibrium:-

$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M = 0$

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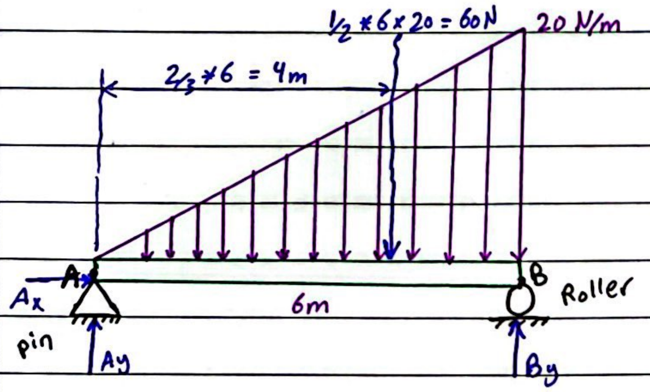
Example: Find the unknown Reactions:

→ A_x, A_y, B_y ?



$\Sigma F_x = 0$

$\Sigma F_y = 0$

$\Sigma M = 0$

* دايمًا أفضل إشي خذ معادلة الإتزان لل moment أول إشي ، وخذ النقطة إللي عليها أكثر عدد من ال Reactions (لأنو هذول ال Reaction ما رح يعلو على النقطة إللي هم عندها moment).

$\circlearrowleft + \Sigma M_A = 0$

$-60(4) + B_y(6) = 0$

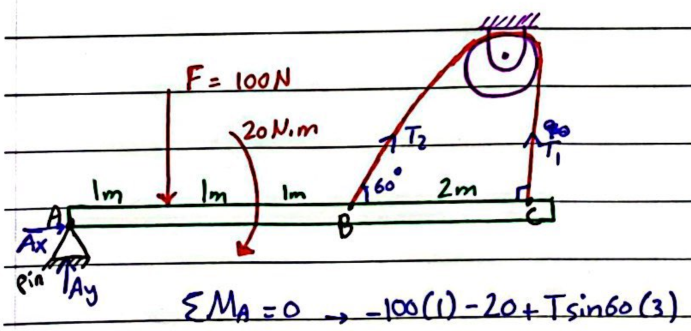
$B_y = 40N$

طلعت ال reaction معي موجبة هاذ يعني اتجاهها إللي أنا فرضتو صح (للأعلى) $\uparrow B_y$

$\Sigma F_x = 0$ $\quad\quad$ $\Sigma F_y = 0$

$A_x = 0$ $\quad\quad$ $A_y - 60 + B_y = 0 \longrightarrow A_y = +20\,N \uparrow A_y$

Example:- Find the unknown Reactions:



* في هاذ المثال ال Unknown reactions هم A_x ، A_y ، وقوة الشد بالحبل T_1 , T_2 يعني عندي 4 مجاهيل و 3 معادلات اتزان فما رح اقدر احل (3 معادلات لازم 3 مجاهيل أو أقل)، بس هون $T_2 = T_1$ لأنو نفس الحبل والإتجاه مش مهم انهم عكس بعض.

$\Sigma M_A = 0 \rightarrow -100(1) - 20 + T\sin 60(3) + T(5) = 0 \quad T = \ldots$

$\Sigma F_x = 0 \rightarrow A_x + T\cos 60 = 0$

$A_x = \ldots$

$\Sigma F_y = 0 \rightarrow A_y - 100 + T\sin 60 + T = 0$

بس شغل تعويض ارقام بتهفي

$A_y = \ldots$

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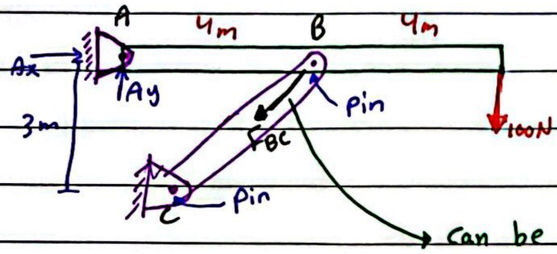
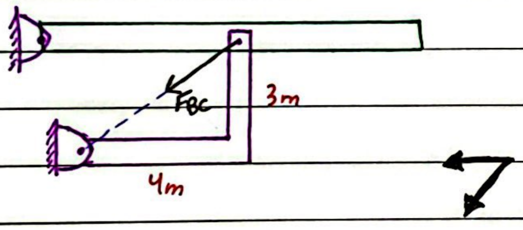
5.4 2-force members



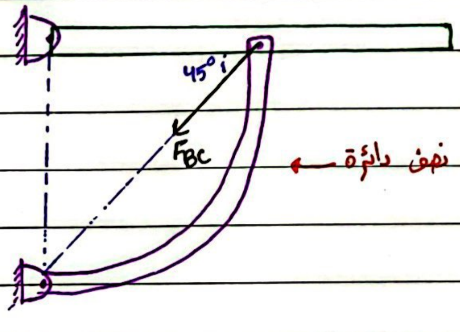
can be tension or compression.

① 2 ends (pins) ② No force → 2 force member (link)

→ 2 force member is a link support that has no force in it and its 2 ends are pins.



← هدول الرسمتين الحل عليهم
نفس الرسمة الأولى وكلهم
اسمهم 2-force members



Five Apple

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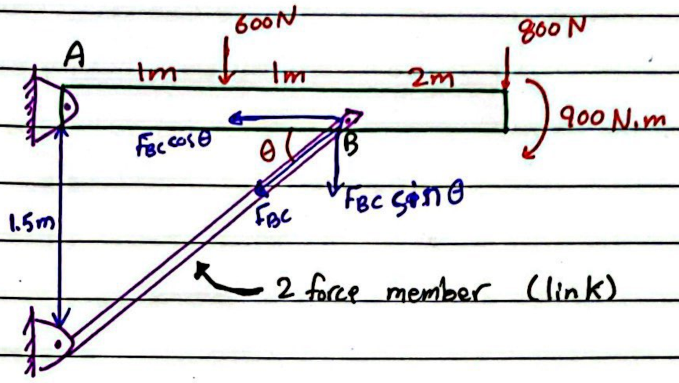
Example:



$\sum M_A = 0$

$-600(1) - 800(4) - 900 - F_{BC}\sin\theta(2) = 0$

$\underline{\underline{F_{BC}}} = \ldots$

$\sum F_x = 0$

$A_x - F_{BC}\cos\theta = 0$

$\underline{\underline{A_x}} = \ldots$

$\sum F_y = 0$

$A_y - F_{BC}\sin\theta - 600 - 800 = 0$

$\underline{\underline{A_y}} = \ldots$

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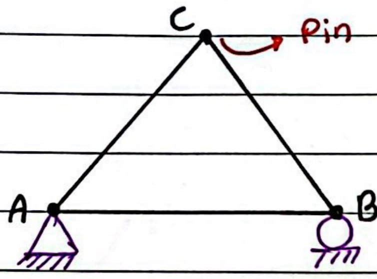
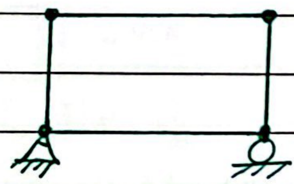
Chapter 6 : Structural Analysis

6.1 : Simple Trusses



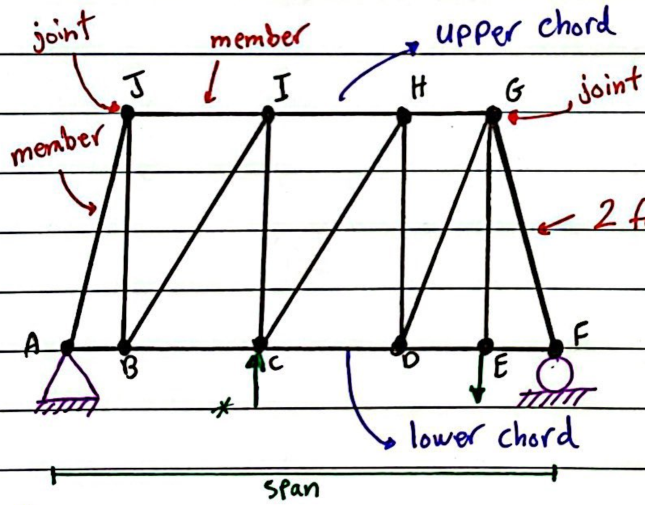
* Trusses are 3 "2-force member"s connected by pins
* pins are also called joints (because they join 2 members)

← ال pins ~~[illegible]~~ عبارة براغي أو مسامير.



≠ Truss

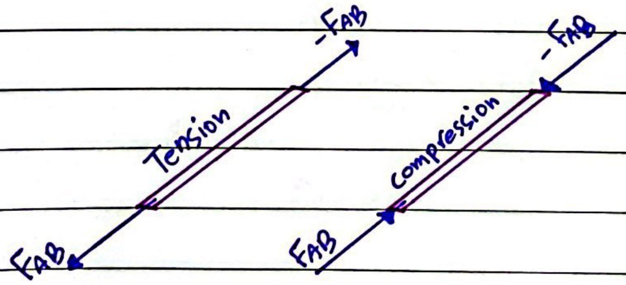
← هون مع كل member مرتبط ب joint بس في 4 members وهاد غلط، لازم 3 members عشان يكونو Truss.



Force in members are internal (T, C).

وهنا إللي رح نحسبو

find the internal force in each member, Tension (T) or Compression (C).



* دائمًا ال external forces بتكون على ال joint مش على ال members.

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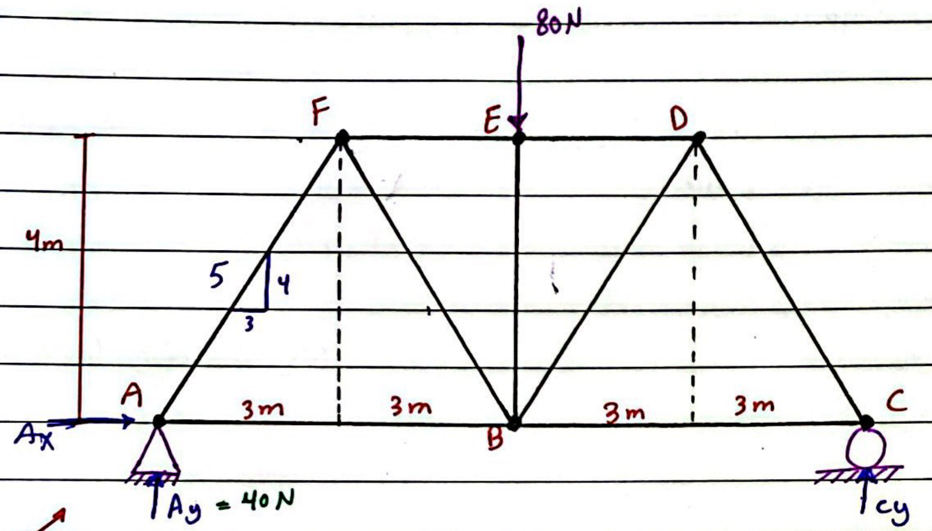
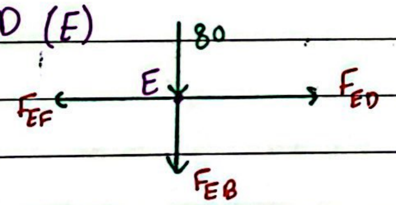
joint Method



Ex: Find the force in each member and determine if the force in T or C.

F.B.D (E)



$\Sigma F_x = 0$

$\Sigma F_y = 0$

* لاحظ هون ما عندي غير معادلتين اتزان و 3 مجاهيل فما بقدر احل فعشان هيك بلجأ للReactions

Reactions

$\Sigma M_A = 0$

$-80(6) + C_y(12) = 0$

$C_y = 40N$

$\Sigma F_y = 0$

$A_y + C_y - 80 = 0$

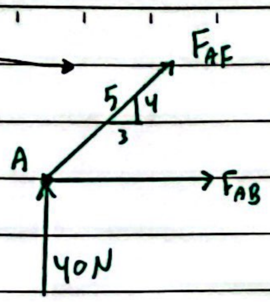
$A_y = 40N$

$\Sigma F_x = 0$

$A_x = 0$

* بلاحظ هون عند D في رح يكون 3 مجاهيل إللي هم الmembers و E ← 3 و B ← 5 و F ← 3 و ~~عند~~ A ← 4 (بس 2 منهم طلعتهم من الReactions) فبلش من A احسن لأنو أقل عدد من المجاهيل.

Five Apple



هون فرضت F_{AF} هي شد بس فعليا ممكن تطلع ضغط وبالحل بيبين.

F.B.D for A

دايمًا افرض شد بالأول احسنلي

$\Sigma F_x = 0$

$F_{AB} + F_{AF}\left(\frac{3}{5}\right) = 0 \quad \dots (1)$

$\Sigma F_y = 0$

$40 + F_{AF}\left(\frac{4}{5}\right) = 0$

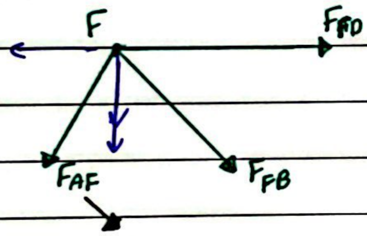
$F_{AF} = -50\text{ N}$ (C)

* هون الجواب طلع سالب والسالب معناه إنو عكس إلي فرضتو يعني F_{AF} هي بتطلع ضغط.

برجع بعوض ب (1) (مع الإشارة)

$F_{AB} + (-50)\left(\frac{3}{5}\right) = 0$

$F_{AB} = 30\text{N}$ (T)



* هاي عشان تكون شد من نقطة F لازم تكون F_{FA} بس لأنو طلعت F_{AF} ضغط بخليه هيك عشان ما اخربط حالي بالإشارات وهيك

$\Sigma F_x = 0$

$F_{FE} + F_{FB}\left(\frac{3}{5}\right) - F_{AF}\left(\frac{3}{5}\right) = 0 \quad \dots (1)$

$\Sigma F_y = 0$

$-F_{AF}\left(\frac{4}{5}\right) - F_{FB}\left(\frac{4}{5}\right) = 0$

$-(-50)\left(\frac{4}{5}\right) - F_{FB}\left(\frac{4}{5}\right) = 0$

$F_{FB} = 50\text{N}$ (T)

من خلال التعويض ب (1)

$F_{FE} + (50)\left(\frac{3}{5}\right) - (-50)\left(\frac{3}{5}\right) = 0$

$F_{FE} = -60\text{N}$ (C)

Five Apple

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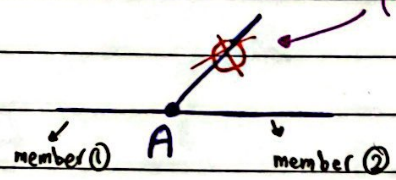
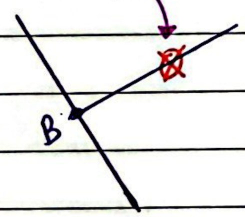
6.3 Zero-force members

3 members
no load, no reaction

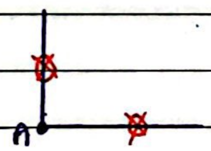
هاد صفر (بروح)

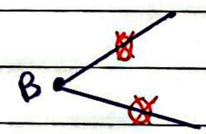




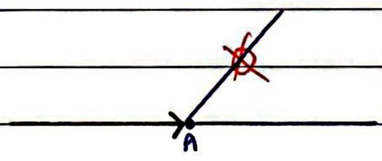
* member ① and member ②
بكونو على نفس الإستقامة.

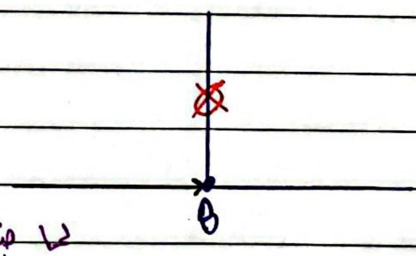
2 members
no load, no reaction





2 members <u>with</u>
Load OR Reactions.





لما ضفنا
هاد ال load أو ال Reaction
صار الشكل بشبه الحالة الأولى.

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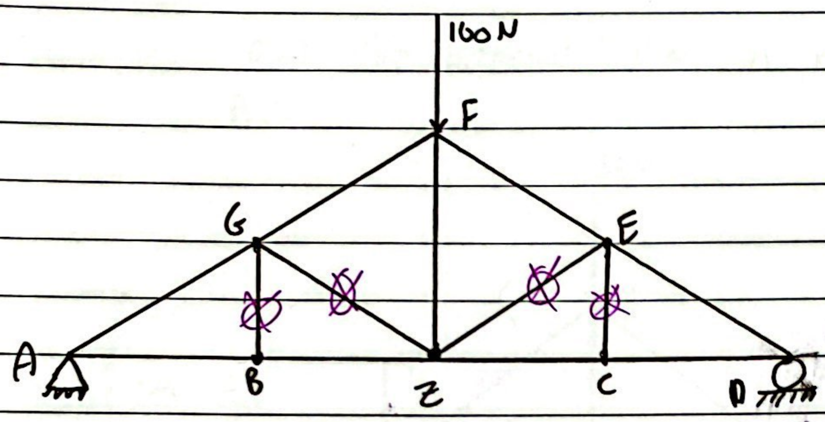
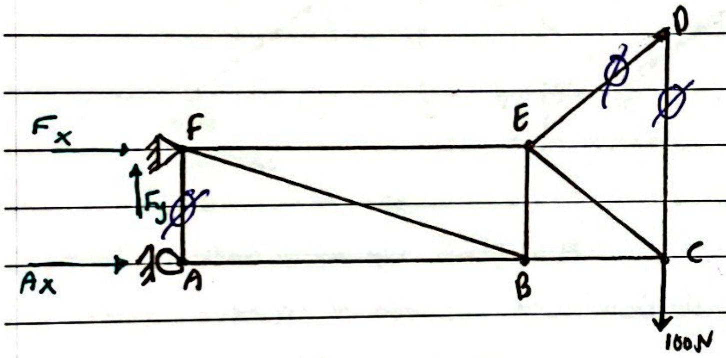
Example :- By inspection find zero force members:

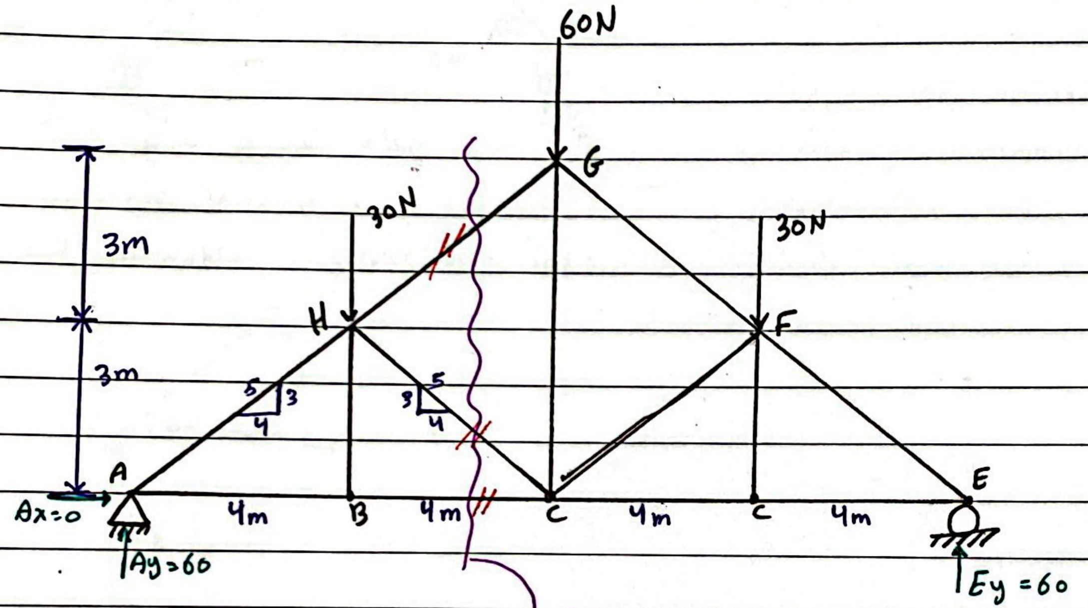


* CE مبين رح تروح فلما تروح عليك بقدر الاحظ ان EZ كمان بتروح وهكذا..



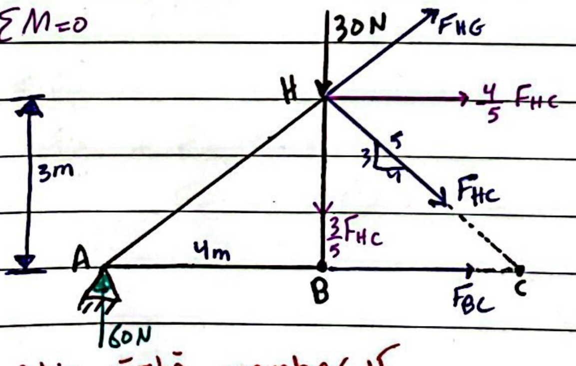
6.4 Method of section

Example:- Find the internal forces in members HG, HC, BC



بعمل خط قطع يمر بالـ members الي بدي اياهم و شرط خط القطع إنو يمر بس بـ 3 members لأنو بس عندي 3 معادلات اتزان $\Sigma F_y=0$, $\Sigma F_x=0$, $\Sigma M=0$

* هسا بدي ارجع ارسم الرسمة بعد القطع وبقدر اختار الجهة اليمين أو اليسار، حسب الأسهل



كل member قطعتو بخليه زي انو force.

$\Sigma M_H = 0$

$-60(4) + F_{BC}(3) = 0$

$F_{BC} = 80\ N$ (T)

$\Sigma M_A = 0$

$-30(4) - \frac{3}{5}F_{HC}(4) - \frac{4}{5}F_{HC}(3) = 0$

$F_{HC} = ...$

$\Sigma F_x = 0$

* بقدر اخذ اكثر من معادلة moment وحدة عادي.

Five Apple

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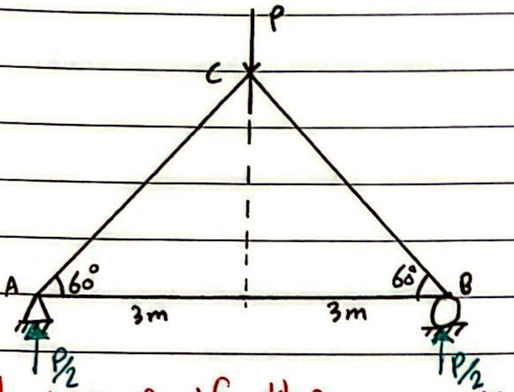
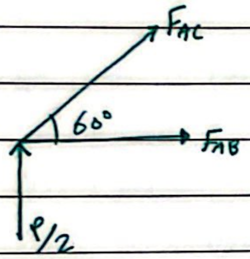


Find max P if the allowable tension is 200 N and allowable compression is 100 N.

عرفت انو $P/2$ لأنو الشكل
symmetric يعني الـ load (P)
رح يتنصف و كل Reaction رح
يوخذ النص عشان يظل متزن



$\Sigma F_x = 0$

$F_{AB} + F_{AC}\cos 60 = 0$

$F_{AB} = -F_{AC}\cos 60$

$F_{AB} = \dfrac{P}{2\sin 60}\cos 60 = 200$

$\boxed{P = 200 * 2 * \tan 60}$

smallest

$\Sigma F_y = 0$

$P/2 + F_{AC}\sin 60 = 0$

$F_{AC} = -P/2\sin 60 = -100$

$\boxed{P = 100 * 2 * \sin 60}$

باخذ القيمة الأقل و بتكون هي الجواب.

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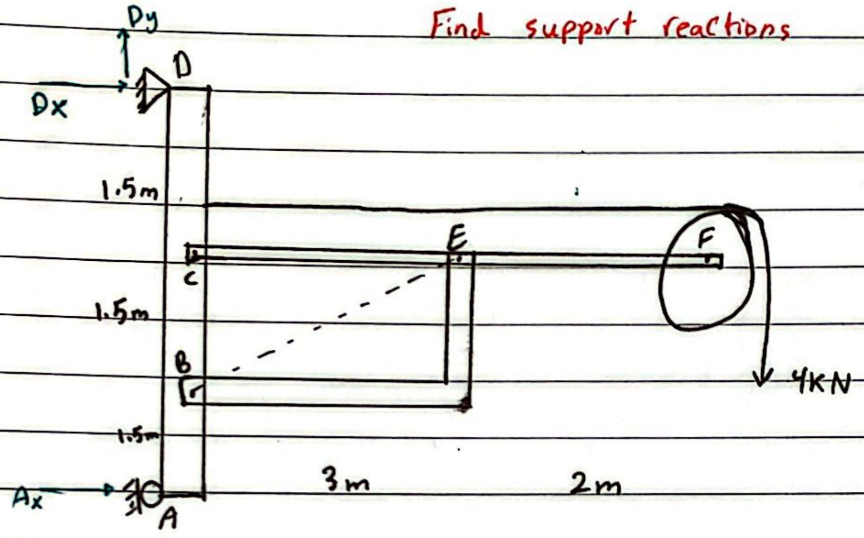
6.6 Frames and machines.

Find support reactions



$\Sigma M_o = 0$

$A_x(4.5) - 4(5) = 0$

$A_x = 4.8\,KN$

$\Sigma F_x = 0$

$D_x + A_x = 0$

$D_x = -4.8\,KN$

$\Sigma F_y = 0$

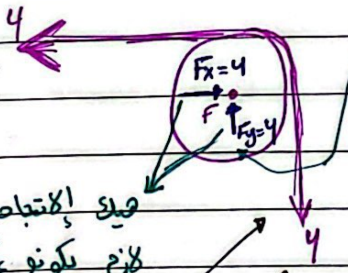
$D_y - 4 = 0$

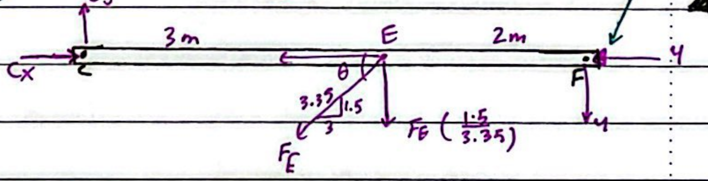
$D_y = 4\,KN$

هون فكيت الـ member FEC

Find the internal forces at the pins

هيك الإتجاه لأنو لازم يكون عكس اللي هون





هيك الاتجاه لأنو لازم يكونو عكس اتجاه هذول

هون فكيت البكرة وطلعت الريا كشنز عليها

$\boxed{\Sigma M_C = 0}$ $-F_E\left(\frac{1.5}{3.35}\right)(3) - 4(5) = 0$

$F_E = -16.4\,kN$

$\boxed{\Sigma F_x = 0}$ $C_x - F_E\left(\frac{3}{3.35}\right) - 4 = 0$

$C_x - (-16.4)\left(\frac{3}{3.35}\right) - 4 = 0$

$C_x = -10.7\,kN$

$\boxed{\Sigma F_y = 0}$ $C_y = 3.3\,kN$

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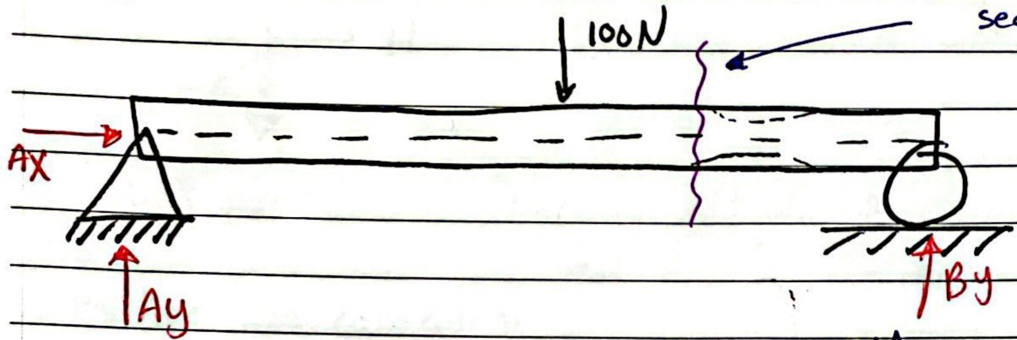
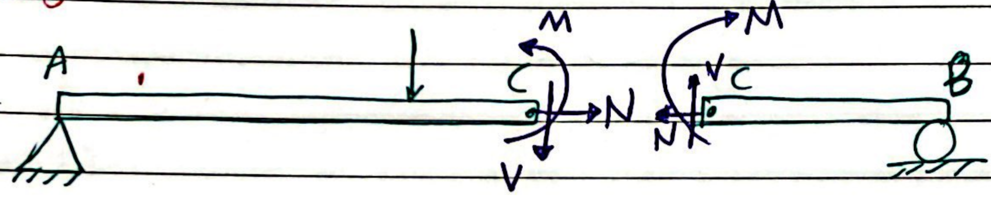
Chapter 7: Internal forces

* زي method of sections





internal forces:

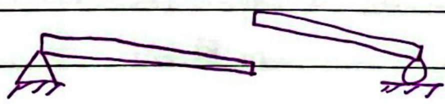
- M = Bending moment. ← دايمًا بكون في زي قوة بدها تطعج الجسم *
(bending) بتكون زي moment أو torque.
- V = shear force.
- N = axial force (Normal force).

→ The force that wants to make the body longer or shorter

← زي اكني بشد او بضغط بالجسم ، وهاي القوة بتكون عامودية على مكان القص أو القطع.

حرف (V) بشبه المقص وهيك

احفظها ← shear force ← قوة القص



زي اكنو صار في قص لفوق ولأعلى.

* احنا قبل كنا نطلع ال tension and compression forces in members

وهسا نفس الإشي.

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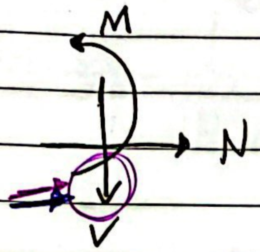
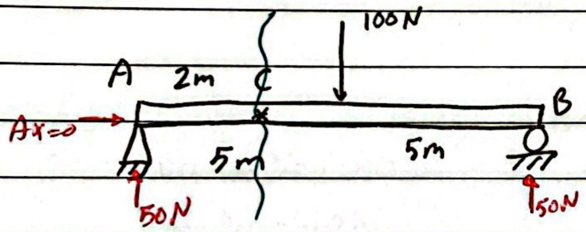


هيك دائمًا الإتجاه على الجزء الشمال من ال section.

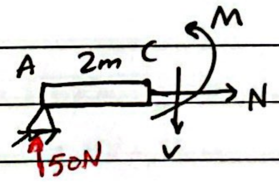
left part
positive sign convention

"Don't get left at wallmart or you'll feel down".

Example:-



* اول اشي لازم اطلع كل ال Reactions وهون طلعناه عن السريع لأنو symmetric.
* بعد ما أعمل section ممكن اخذ الجهة اليمين أو الشمال حسب الأسهل لأنو رح يطلع نفس الإجابة.



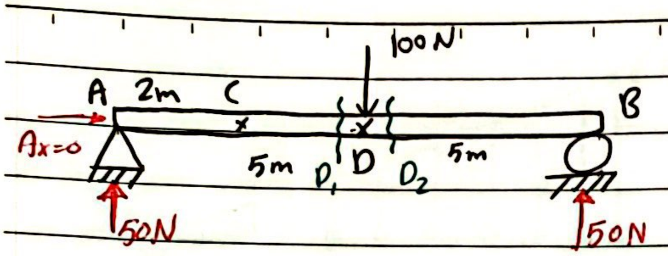
$\Sigma F_y = 0 \rightarrow V = 50N$

$\Sigma F_x = 0 \rightarrow N = 0N$

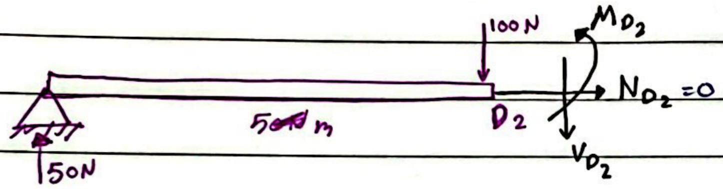
$\curvearrowleft + \Sigma M_A = 0 \rightarrow M_A = M - 50 * 2 = 0$

$M = 100 N.m$

Five Apple



* لو بدي اطلع ال internal forces على النقطة D ما رح يزبط اقطع بالزبط عند D لأنو في force (100N) و بالمنطق ما بصير اقسم ال force لقسمين ف اللي بنعملو إنو بنوخذ section. قبل مسافة كثير صغيرة من نقطة D يعني mm اشي يكاد أن يُهمل (أو بعد)



$\Sigma F_x=0$

$N_{D_2}=0$

$\Sigma F_y=0$

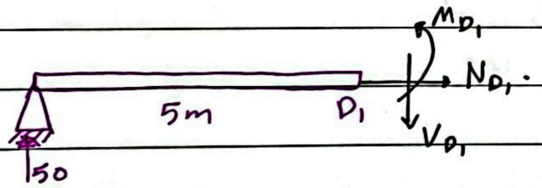
$50-100-V_{D_2}=0$

$V_{D_2}=-50N$

$\Sigma M_{D_2}=0$

$-50(5)+M_{D_2}=0$

$M_{D_2}=250 N\cdot m$



$\Sigma F_x=0$

~~$M_{D_2}=0$~~ $N_{D_1}=0$

$\Sigma F_y=0$

$50-V_{D_1}=0$

$V_{D_1}=50N$

$\Sigma M_{D_1}=0$

$-50(5)+M_{D_1}=0$

$M_{D_1}=250 N\cdot m$

Five Apple

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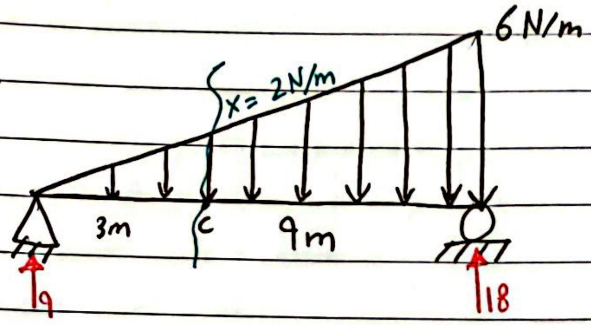
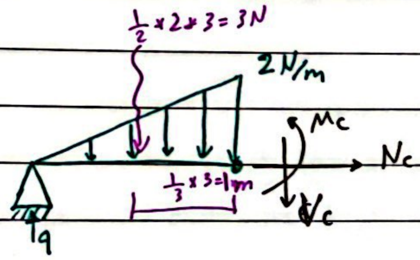


Find the Internal forces at point C

$$\frac{6}{9} = \frac{x}{3} \qquad \boxed{x = 2}$$

* نسبة وتناسب بين المثلث الكبير والمثلث الصغير



$\Sigma F_x = 0 \qquad N_c = 0$

$\Sigma F_y = 0 \qquad 9 - 3 - V_c = 0$

$V_c = 6N$

$\Sigma M_c = 0 \qquad -9(3) + 3(1) + M_c = 0$

$M_c = 24 \, N.m$

Five Apple

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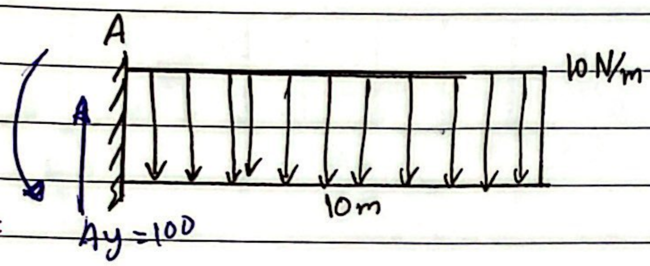
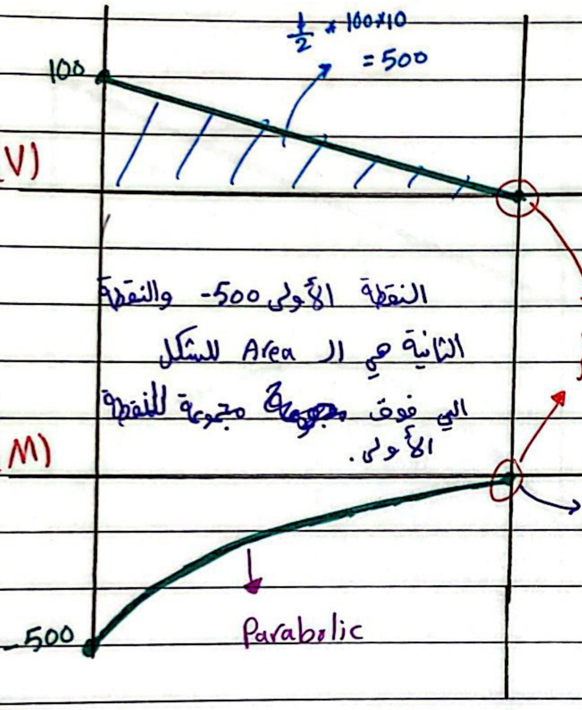
7.3 Shear and moment Diagram



load digram

تكامل

moment عندي يعني CCC بدها تكون -500 بال moment Diagram



shear diagram (V)

$\frac{1}{2} * 100 \times 10 = 500$

النقطة الأولى -500 والنقطة الثانية هي ال Area للشكل الي فوق مجموعة للنقطة الأولى.

لازم يسكر الشكل عند الصفر

تكامل

* دائمًا بيلش من اليسار لليمين.

moment (M) diagram

$-500 + 500 = 0$

CW +

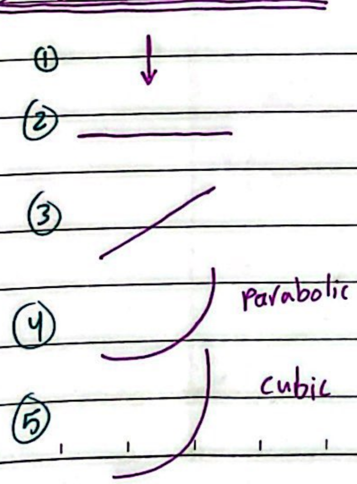
CCW −

نقطة البداية

Parabolic

* قبل كنا نطلع M و V عند نقطة معينة بس شو لو بدي قيمهم عند كل النقاط؟ ← بستخدم ال shear moment diagram

order of the lines



* لما يكون خط مستقيم بكون ثابت مثلاً 5 بعدين الخط الي بعدو رح يكون slope يعني صار 5x يعني $\int 5 \leftarrow 5x$ وهاد المقصود بالتكامل و $\int 5x \leftarrow \frac{5x^2}{2}$ صار parabolic وهكذا.

Vertical lines → forces and Reactions.

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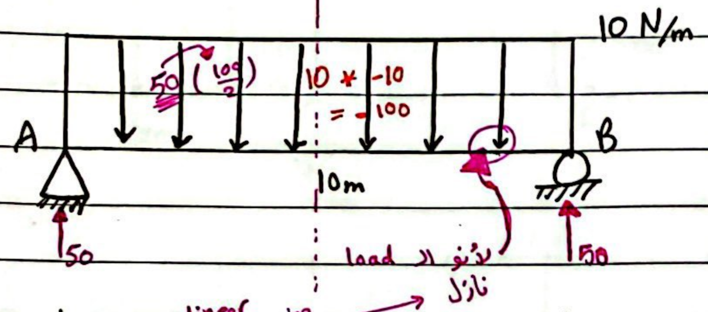
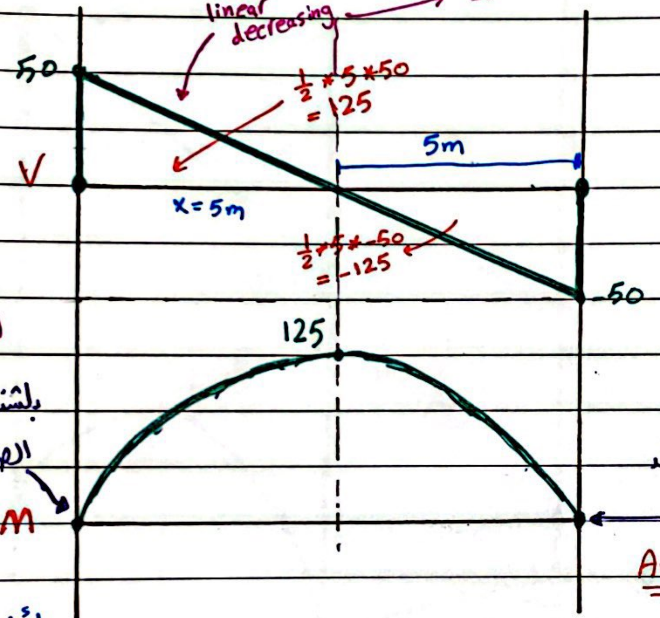
Example:-



M_{max} , V_{max} * بنوخذ القيمة المطلقة لأكبر قيمة



V= Area under Distributed load

$$\frac{50}{X} = \frac{100}{10}$$

$$500 = 100X$$

$$\boxed{X = 5m}$$

بنبلش من اليسار لليمين

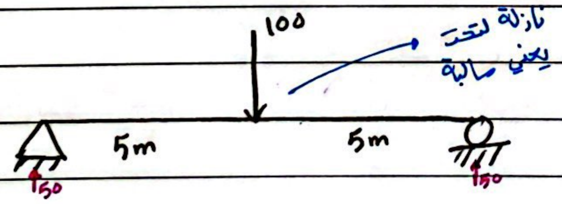
بلشنا من الصفر

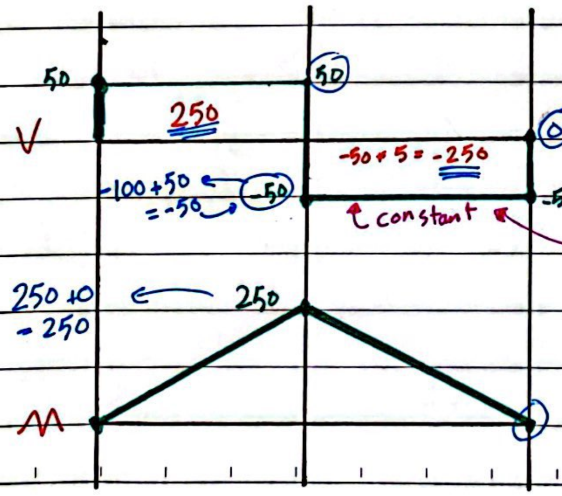
لأنو ما في moment عندي بالبداية

وخلصنا عند الصفر

Area + previous value

Example:-





* انا بظل بمشي من اليسار لليمين وبلاحظ إنو فش اشي قاعد بتغير يعني فش force أو Reaction عشان هيك constant

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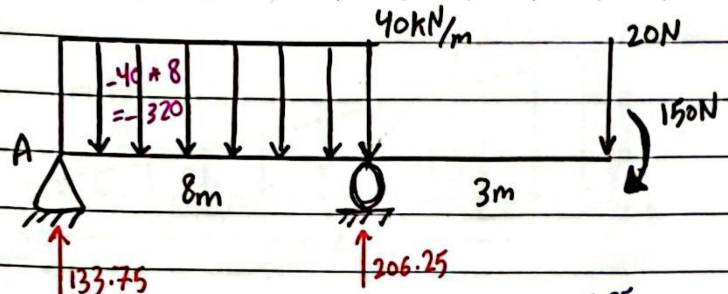
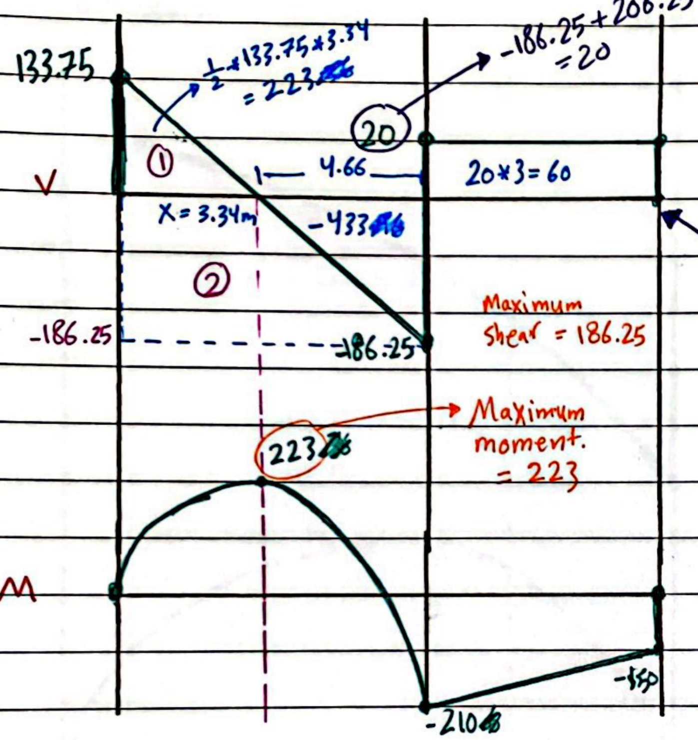
Example :-



نسبة وتناسب
بين مثلثين
② ①

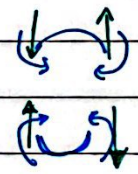
$$\frac{133.75 + 186.25}{8} = \frac{133.75}{x}$$

$x = 3.34\ m$



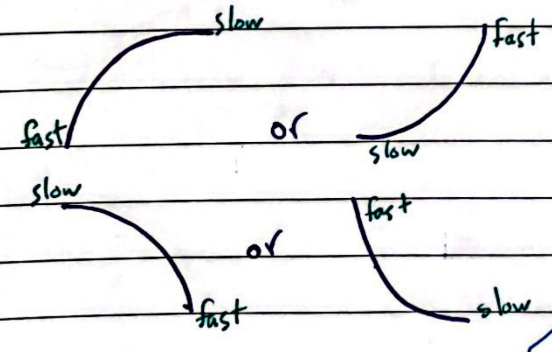
* انتهينا عند الصفر يعني صح ✓

Moment :
CW + ↑
CCW - ↓

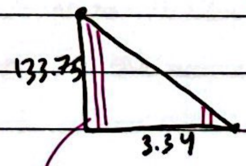


"In the kitchen, clock is above and the counter is below."

How do I choose ?

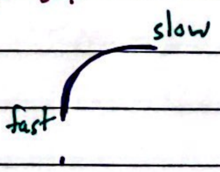


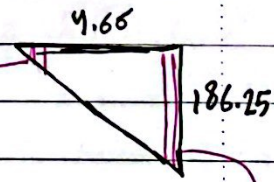
Example



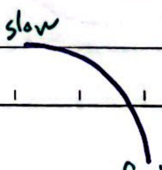
بلش كثير يعني سريع

خلص قليل يعني بطيء





بلش قليل يعني بطيء



سريع خلص كثير يعني

Five Apple

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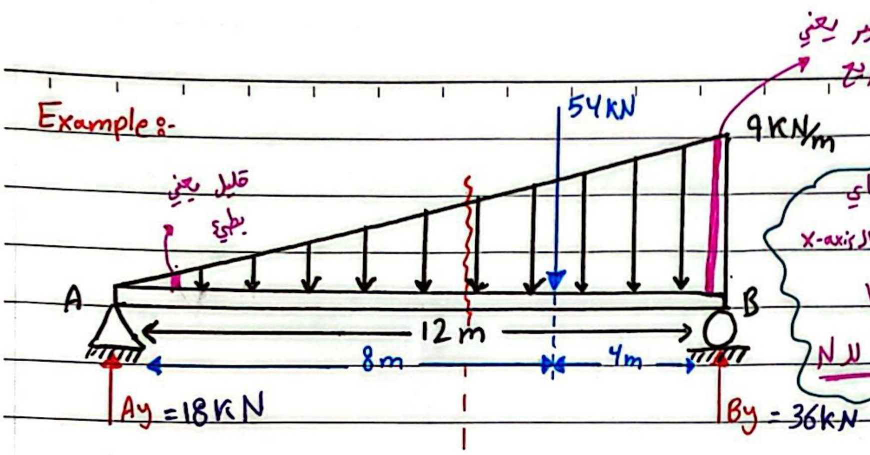
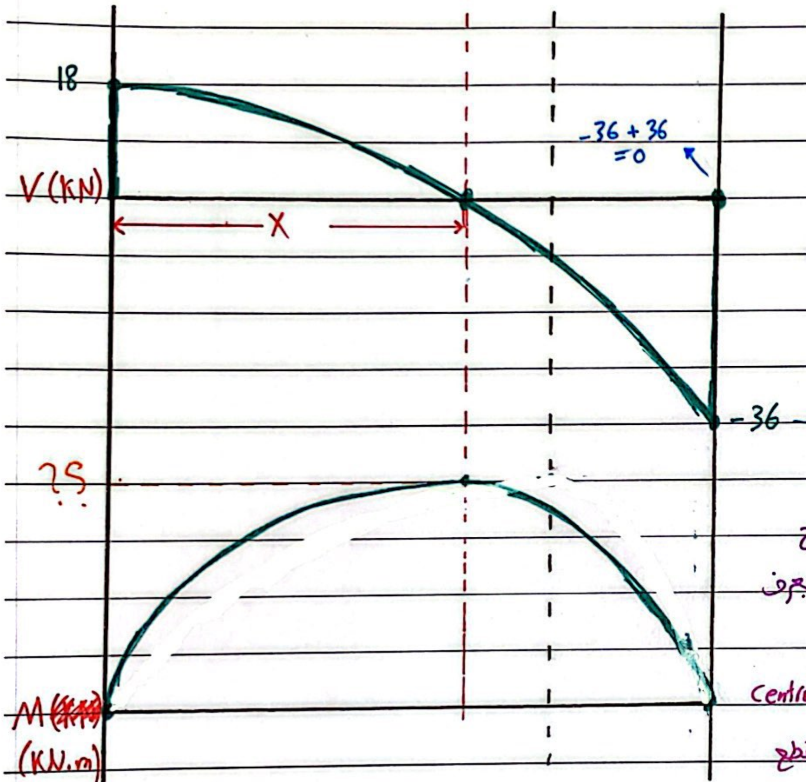
Example:-



* Note: الـ Ax ما بتهمني بهاي الأسئلة ولا اي قوة على الـ x-axis بتهمني لأنو ما في رسم الها عنا بس للـ V و M مش للـ N



$\sum M_A = 0$

$-54(8) + B_y(12) = 0$

$B_y = 36\ KN$

$\sum F_y = 0$

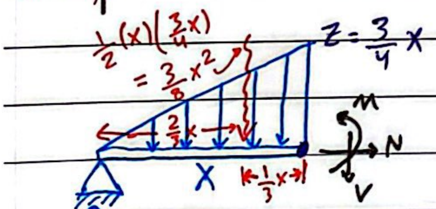
$36 + A_y = 54$

$A_y = 18\ KN$

$-36 \rightarrow 18 - 54 = -36$

* هسا انا بعرف هيك الرسمة رح تكون بس ما بعرف X ولا بعرف M_{max} وهو اهم إشي

* الخط المقطع بالأسود هو الـ Centroide للمثلث بس الـ V graph بقطع الـ x axis بنقطة غير يعني مش 8.



$\frac{9}{12} = \frac{Z}{X}$

$Z = \frac{3}{4} X$

To find $x \rightarrow V = 0$

(النقطة إلي بقطع فيها محور السينات).

I need an equation that represents V:

$\sum F_y = 0$

$18 - \frac{3}{8} x^2 - V = 0$

$V = 18 - \frac{3}{8} x^2$

$X = 6.93\ m$

$M_{max} = 83.14\ KN.m$

I need an equation that represents M:

$\circlearrowleft \sum M_x = 0$

$M - 18X + \frac{3}{8} X^2 \left(\frac{X}{3}\right) = 0$

$M = 18X - \frac{3}{24} X^3$

عشان اطلع الـ M_{max}

لاحظ إنو الـ M تكامل معادلة الـ V

Five Apple

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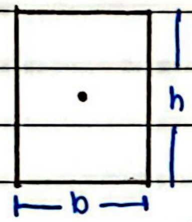
Chapter 9:- center of gravity and centroid

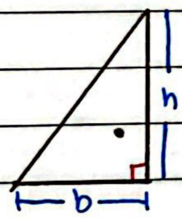
$$\bar{X} = \frac{\sum A_i X_i}{\sum A_i} \qquad \bar{Y} = \frac{\sum A_i y_i}{\sum A_i}$$

→ So, the center of gravity or centroid of a shape is at point $(\bar{x}, \bar{Y})$



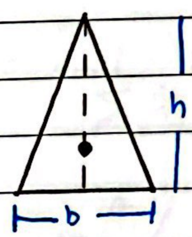
$\bar{x} = \frac{b}{2}$

$\bar{Y} = \frac{h}{2}$



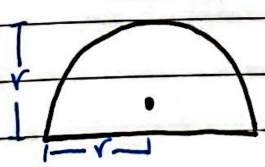
$\bar{x} = \frac{b}{3}$ $\qquad$ $\bar{x} = \frac{2b}{3}$

$\bar{Y} = \frac{h}{3}$ $\qquad$ $\bar{Y} = \frac{2h}{3}$



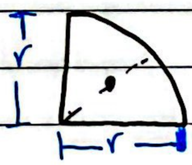
$\bar{x} = \frac{b}{2}$

$\bar{Y} = \frac{h}{3}$



$\bar{X} = r$

$\bar{Y} = \frac{4r}{3\pi}$



$\bar{X} = \frac{4r}{3\pi}$

$\bar{Y} = \frac{4r}{3\pi}$

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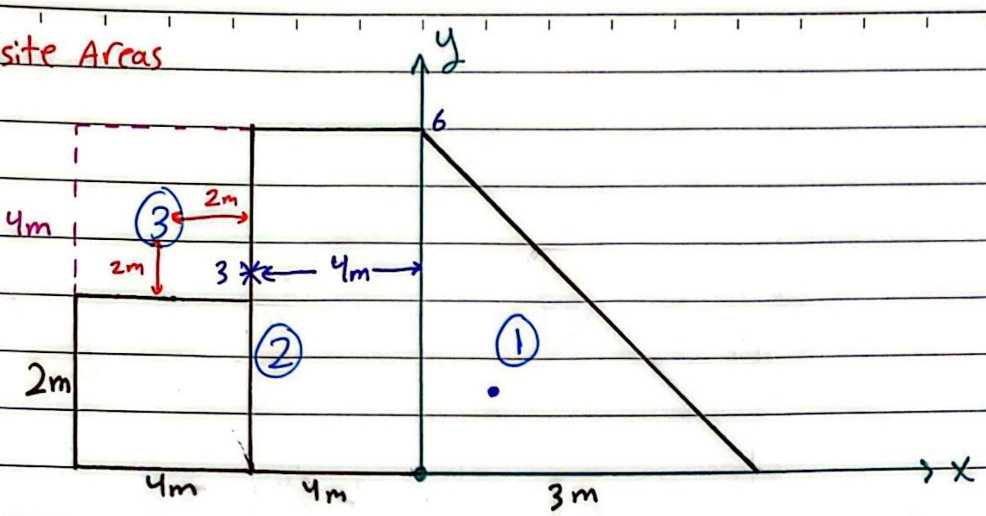
Composite Areas



* مثلا بعطيني شكل زي الي فوق وبحكيلي طلع ال centroid ف الي بعملو انو بطلع ال centroid لكل شكل لحال بعدين بنجمعهم ، الطريقة رح توضح بعدين.

* ملاحظة : ال centroid بكون من نقطة ال origin فإنتبه.

	A_i	X_i	Y_i	A_iX_i	A_iY_i
1	$\frac{1}{2} * 3 * 6 = 9$	1	2	9	18
2	$8 * 6 = 48$	-4	3	-192	144
3	-16	-6	4	96	-64
Σ	41			-87	98

$$\bar{X} = \frac{\sum A_i X_i}{\sum A_i} = \frac{-87}{41} = -2.12 \text{ m}$$

$$\bar{Y} = \frac{\sum A_i \bar{Y}}{\sum A_i} = \frac{98}{41} = 2.4 \text{ m}$$

centroid → (-2.12 , 2.4)

سالب لأنو المربع الصغير مش موجود ، زي الني بدي انقص المساحة الصغيرة من الكبيرة.

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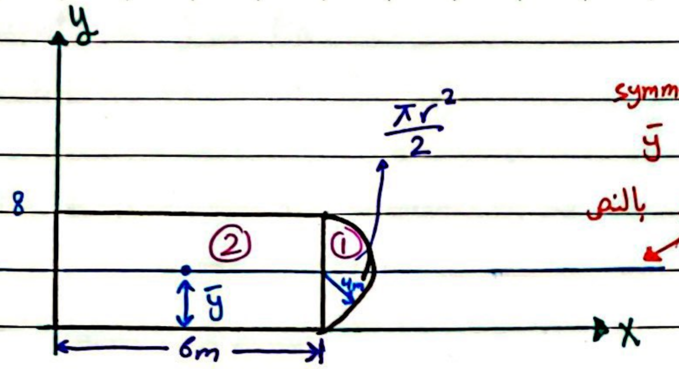
Examples:-

$\bar{y} = 4m$



* الشكل symmetric فأكيد ال $\bar{y}$ رح يكون بالنص للشكلين

	A_i	x_i	y_i	$A_i x_i$	$A_i y_i$
①	$\frac{\pi(4)^2}{2}$	$6+\frac{4r}{3\pi}$	4	193.46	
②	$8*6$	3	4	144	
Σ	73.13			337.5	

* ما رح احتاجو لأنو انا هيك هيك عارف شو ال $\bar{Y}$.

$$\bar{X} = \frac{\sum A_i x_i}{\sum A_i} = \frac{337.5}{73.13} = 4.6$$

Centroid → (4.6 , 4)

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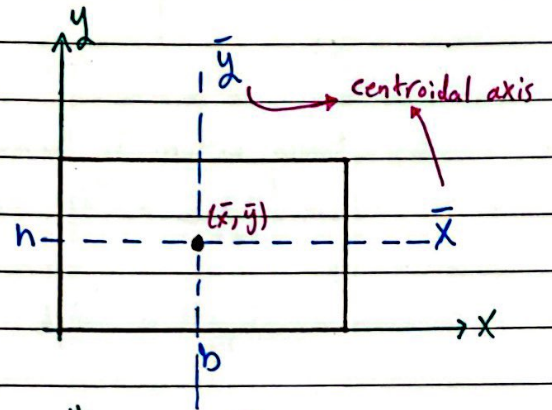
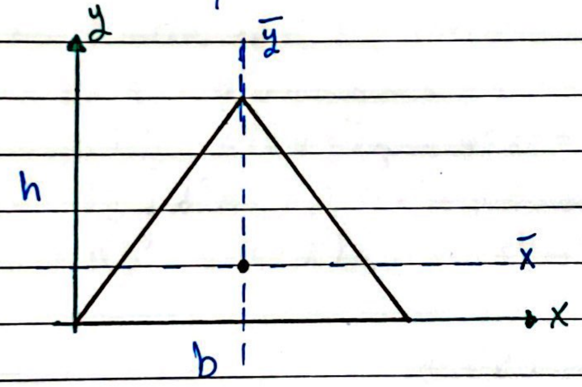
Chapter 10 :- moment of Inertia.
(The flexibility of Beams)

Moment of Inertia = I = Geometric property of the cross-section of a beam.



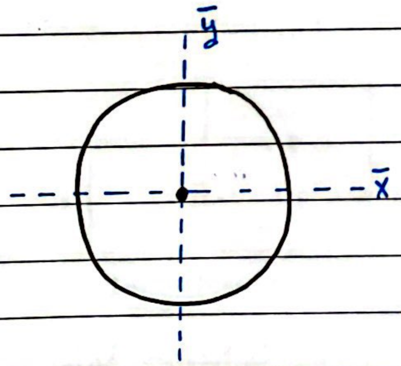
$I_{\bar{x}} = \frac{b h^3}{12}$

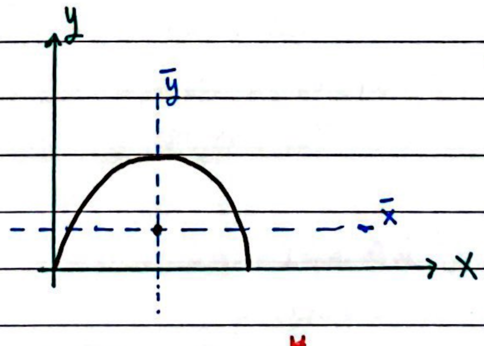
$I_{\bar{y}} = \frac{h b^3}{12}$



$I_{\bar{x}} = \frac{b h^3}{36}$

$I_{\bar{y}} = \frac{h b^3}{36}$





$I_{\bar{x}} = \frac{1}{8} \pi r^4$

$I_{\bar{x}} = \frac{\pi r^4}{4}$

$I_{\bar{y}} = \frac{\pi r^4}{4}$

Five Apple

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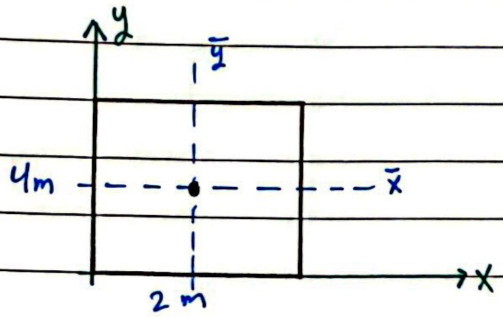
Example:-



* Parallel axis theorim

$I_x = I_{\bar{x}} + Ad^2$

$I_y = I_{\bar{y}} + Ad^2$

find moment of inirtia about centroidal Axis

$$I_{\bar{x}} = \frac{bh^3}{12} = \frac{2 * 4^3}{12} = 10.667\ m^4$$

$$I_{\bar{y}} = \frac{hb^3}{12} = \frac{4 * 2^3}{12} = 2.667\ m^4$$

find moment of inirtia about x, y axis :-

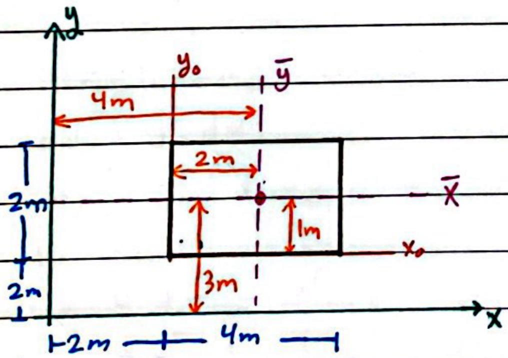
$$I_x = I_{\bar{x}} + d^2A = 10.667 + 2^2 * (2*4) = 42.667\ m^4$$

قانون ثاني ← $= \frac{bh^3}{3} = (2*4^3)/3 = 42.667\ m^4$ ✓

$$I_y = I_{\bar{y}} + d^2A = 2.667 + 8*1 = 10.667\ m^4$$

قانون ثاني ← $= \frac{hb^3}{3} = (4*2^3)/3 = 10.667\ m^4$ ✓

Example:-



find moment of inirtia about x, y axis

$I_x = I_{\bar{x}} + Ad^2$

$= 2.667 + (4*2) * 3^2$

$= 74.667\ m^4$

$I_y = I_{\bar{y}} + Ad^2$

$= 10.667 + (4*2) * 4^2$

$= 138.667\ m^4$

find moment of inirtia about centroidal axis

$I_{\bar{x}} = \frac{4 * 2^3}{12} = 2.667\ m^4$ $\quad I_{\bar{y}} = 10.667\ m^4$

find moment of inirtia about x_o

$I_{x_o} = I_{\bar{x}} + Ad^2$

$= 2.667 + (4*2) * 1^2$

$= 10.667\ m^4$

* d^2 هي المسافة بين ال centroid والمحور

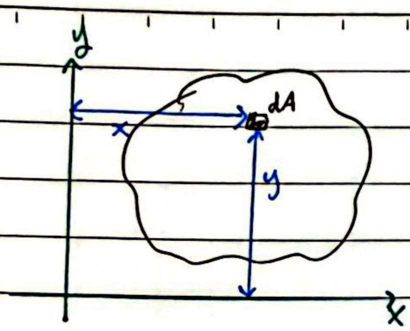
مثلاً : $\bar{x}$ ← centroid

x_o ← محور , المسافة بيناتهم 1m

بعدين بربعها.

42.667

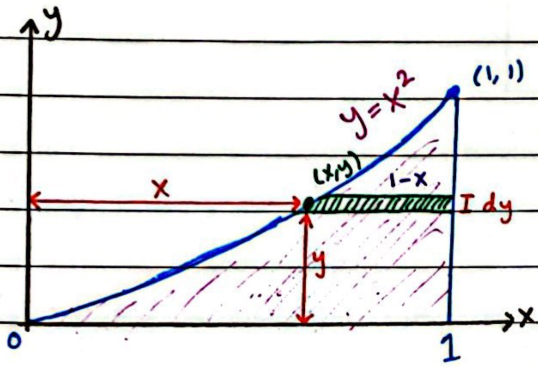
Five Apple

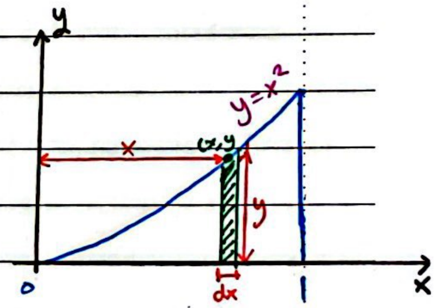


$$I_x = \int y_i^2 \, dA$$

$$I_y = \int x_i^2 \, dA$$

Example :-





I_x, I_y, I_o ?

I_x → Horizontal strip

I_y → Vertical strip

① $dA = (1-x)\,dy = (1-\sqrt{y})\,dy$

$I_x = \int y^2 \, dA$

$= \int_0^1 y^2 (1-\sqrt{y})\,dy = 0.0476 \ m^4$

② $dA = y\,dx = x^2 dx$

$I_y = \int_0^1 x^2 dA = \int_0^1 x^2 x^2 dx = 0.2 \ m^4$

③ $I_o = J = I_x + I_y$

$= 0.2476 \ m^4$

J → polar moment of inirtia

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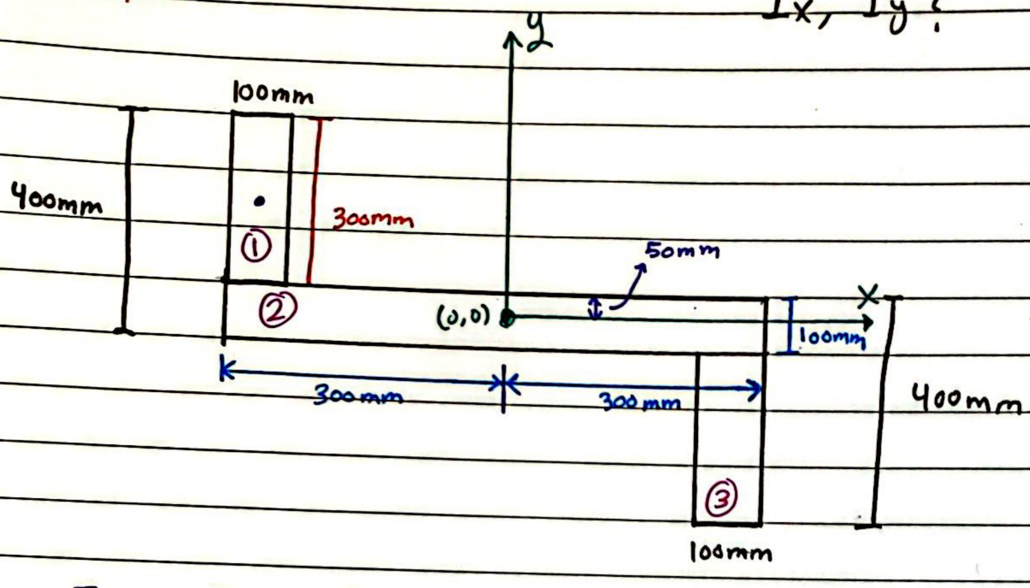
Example :-

I_x, I_y ?



$$I_x = I_{x_1} + I_{x_2} + I_{x_3} \qquad I_{\bar{x}} = \frac{b\,h^3}{12}$$

$$= \left(\frac{100 * 300^3}{12} + (100 * 300) * 200^2\right) + \left(\frac{600 * 100^3}{12} + (600 * 100)\,(0)^2\right)$$

$$+ \left(\frac{100 * 300^3}{12} + (100 * 300) * (-200)^2\right) = \ldots\ m^4$$

$$I_y = I_{y_1} + I_{y_2} + I_{y_3} \qquad I_{\bar{y}} = \frac{h\,b^3}{12}$$

$$= \left(\frac{300 * 100^3}{12} + (300 * 100) * (-250)^2\right) + \left(\frac{100 * 600^3}{12} + (100 * 600) * (0)^2\right)$$

$$+ \left(\frac{300 * 100^3}{12} + (300 * 100) * 250^2\right) = \ldots\ m^4$$

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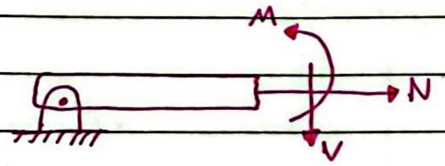
Part II :- Mechanics of Materials :-

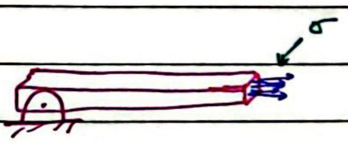
Stress

Normal stress

stress :- measure of intensity of a force over a given area.



Normal force is what's responsable for the stress.
(Normal)



$$\text{stress} = \sigma = \frac{P}{A} = \frac{N}{A} = \frac{\text{Normal force}}{\text{Area}}$$

$$\sigma = \frac{N}{m^2} = Pa$$

وحدة ال stress
(باسكال)

or : $\sigma = \frac{N}{mm^2} = MPa$

Normal stress > Normal strength ⟹ Failure.
600 MPa > 420

stress Units

$1\ KPa = 10^3\ Pa = 10^3\ N/m^2$
$1\ MPa = 10^6\ Pa = 10^6\ N/m^2$
$1\ GPa = 10^9\ Pa = 10^9\ N/m^2$

load
↓
internal forces
↓
stress
↓
strength

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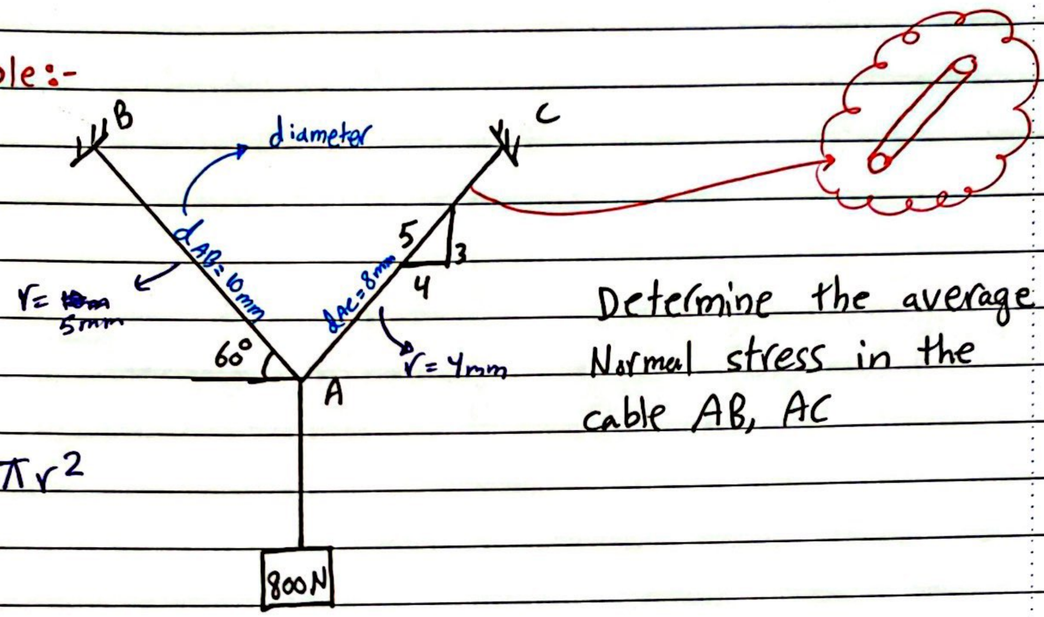
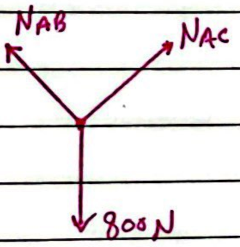
Example:-



Determine the average Normal stress in the cable AB, AC

$A = \pi r^2$



$\Sigma F_x = 0 \qquad \Sigma F_y = 0$

$N_{AB} = 632.4N$

$N_{AC} = 395.2N$

$$\sigma_{AB} = \frac{N_{AB}}{A} = \frac{632.4}{\pi (5)^2} = 8.052 \text{ MPa}$$

$$\sigma_{AC} = \frac{N_{AC}}{A} = \frac{395.2}{\underbrace{\pi (4)^2}_{mm^2}} = 7.862 \text{ MPa}$$

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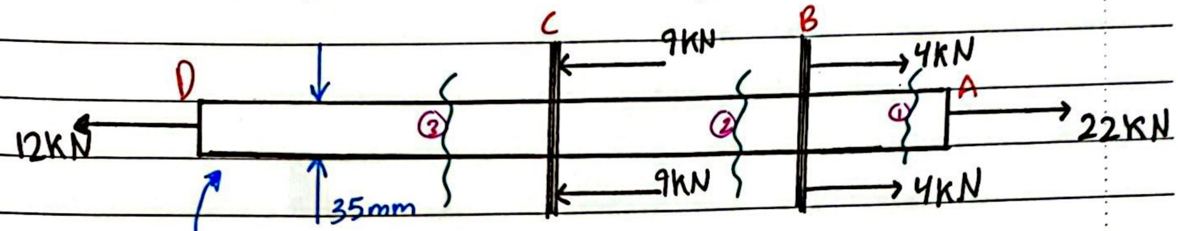
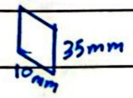
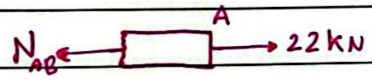
Example 1.5 :- a bar has a constant width of 35 mm and a thickness of 10 mm, Determine the maximum average normal stress in the bar when it is subjected to the loading shown.





* لأنو قلي بدو الـ Normal stress بالـ Bar كامل بعمل section بعد كل load.

①

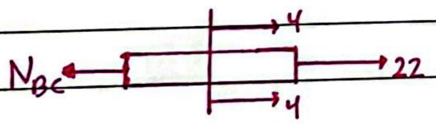


$\Sigma F_x = 0 \qquad 22\,kN - N_{AB} = 0$

$N_{AB} = 22\,kN$

$$\sigma_{AB} = \frac{N_{AB}}{A} = \frac{22\,kN}{(35 * 10)\,mm^2} = 0.0628\ GPa$$

②

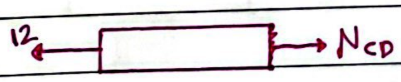


$\Sigma F_x = 0 \qquad 22 + 4 + 4 - N_{BC} = 0$

$N_{BC} = 30\,kN$

$$\sigma_{BC} = \frac{N_{BC}}{A} = \frac{30\,kN}{(35 * 10)\,mm^2} = 0.0857\ GPa$$

③

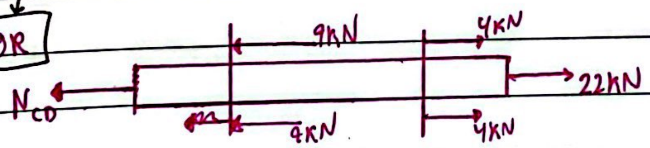


$\Sigma F_x = 0 \qquad N_{CD} = 12\,kN$

$$\sigma_{CD} = \frac{N_{CD}}{A} = \frac{12\,kN}{(35 * 10)\,mm^2} = 0.0343\ GPa$$

* ما بتفرق اذا اخترت الجهة الشمال او اليمين من الsection تذكر ال internal forces.

OR



$\Sigma F_x = 0 \qquad 22 + 4 + 4 - 9 - 9 - N_{CD}$

$N_{CD} = 12\,kN$

$$\sigma_{CD} = \frac{N_{CD}}{A} = \frac{12\,kN}{(35 * 10)\,mm^2} = 0.0343\ GPa$$

نفس الإشي

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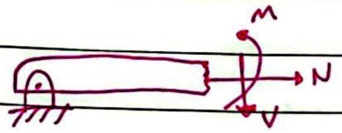
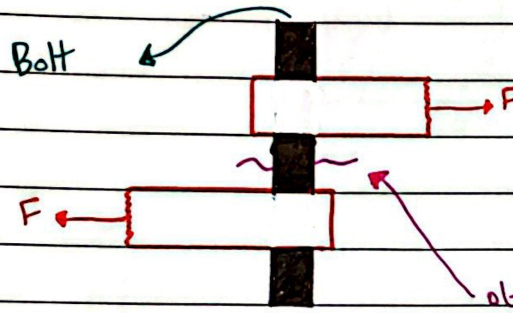
Shear stress



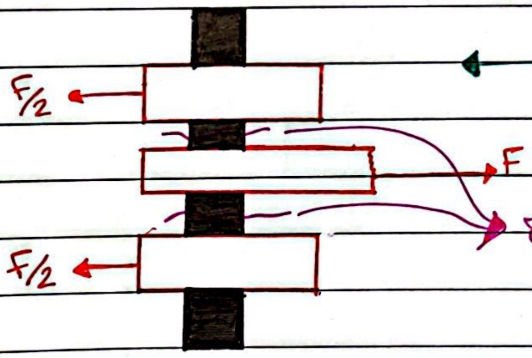
shear force is what's responsable for <u>shear</u> stress.



Two force members connected with a <u>bolt</u>

* عشان احمل shear هيك لازم يكون الإتجاه (ال shear force هي الي <u>بتقطع</u> يعني قوة بتكون موازية لل Area).

هون عشان يفشل بس احتجت انو ينقطع من مكان واحد عشان هيك بسميه
single shear.



A member connected to a pin connection

* عشان اعمل shear لازم يصير في قطع في مكانين عشان يفشل
و لهذا السبب بنسميه
Double shear.

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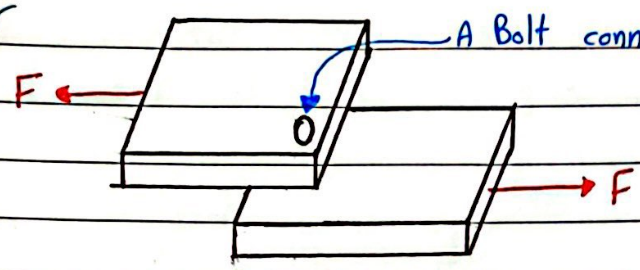
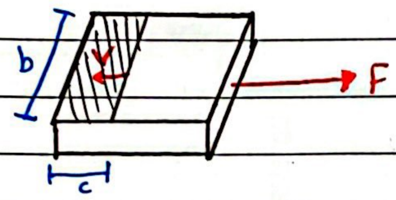
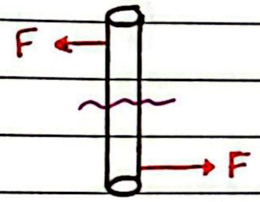
Single Shear



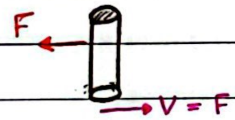
A Bolt connecting both pieces.



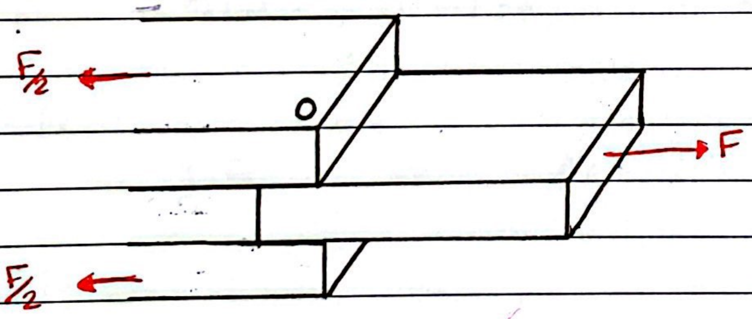
Area // V

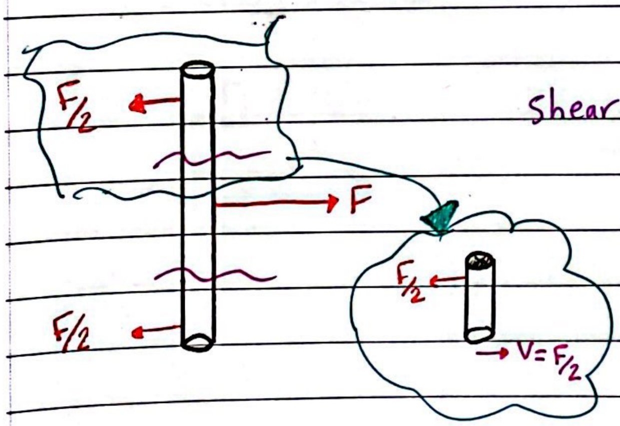


shear stress $\equiv \tau = \dfrac{V}{A} = \dfrac{F}{A} = \dfrac{\text{shear force}}{A}$



Double shear





shear stress $= \tau = \dfrac{F}{2A}$

oR $\tau = \dfrac{F/2}{A}$

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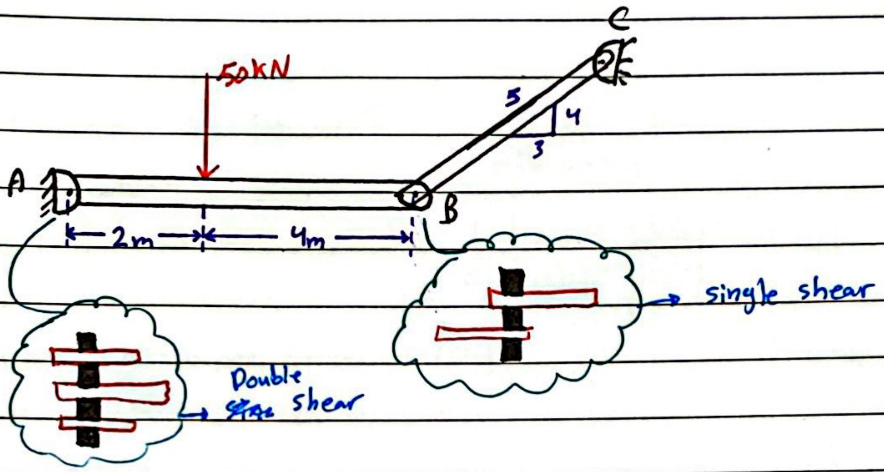
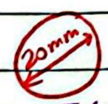
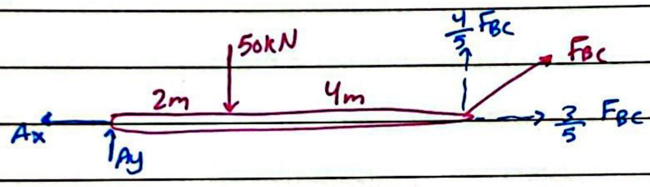
Example :- Find the average shear stress on the 20 mm pins at A and B for the given load.





$A = \pi (10)^2$

$= 314.16\ mm^2$



$\circlearrowleft + \sum M_A = 0$

$-50(2) + \frac{4}{5} F_{BC}(6) = 0 \qquad F_{BC} = 20.833\ kN$

$\sum F_x = 0$

$\frac{3}{5} F_{BC} - A_x = 0 \qquad A_x = 12.5\ kN$

$\sum F_y = 0$

$-50 + A_y + \frac{4}{5} F_{BC} = 0 \qquad A_y = 33.33\ KN$

$\tau_A = \frac{\sqrt{F_x^2 + F_y^2}}{2A} = \frac{35.6\ kN}{2 \times 314.16\ mm^2} = 0.05665\ GPa$

في 2 ~~areas~~ forces فلازم
اطلع ال Resultant force .

$\tau_B = \frac{20833\ N}{314.16\ mm^2} = 66.313\ MPa$

Double shear لأنو 2

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Allowable stress
Factor of safety #

* لازم ال load يكون اقل من اكبر load الجسم ممكن يتحمله.

Factor of safety = F.S = $\dfrac{\text{Ultimate load}}{\text{allowable load}}$

$= \dfrac{\text{Ultimate stress}}{\text{allowable stress}}$

τ_{ult} = Ultimate shear strength

σ_{ult} = Ultimate Normal stess.

$\dfrac{\text{Ultimate stress}}{F.S}$ = allowable stress.

$\sigma > \sigma_{all} \longrightarrow$ failure.

$\sigma < \sigma_{all} \longrightarrow$ safe.

لازم نقارن مع ال allowable stress.

$\sigma > \dfrac{\text{ultimate stress}}{F.S} \rightarrow$ failure.

$\sigma < \dfrac{\text{ultimate stress}}{F.S} \longrightarrow$ safe.

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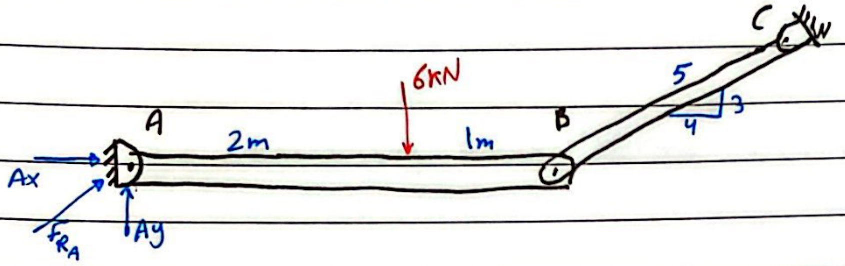
Example:- find minimum diameter of pin A and pin B if the pins have an allowable shear stress of 90 MPa and allowable Normal stress in member BC $\sigma_{allow(BC)} = 115\,MPa$ and find diameter of Rod BC ?



$\circlearrowleft + \sum M_A = 0$ $\quad F_{BC}\left(\frac{3}{5}\right)(3) - 6(2) = 0$

$F_{BC} = 6.67\,kN$

$\sum F_x = 0$ $\quad A_x = 5.32\,kN$ $\quad$ نفسها ال shear

$\sum F_y = 0$ $\quad A_y = 2\,kN$

$F_A = \sqrt{A_x^2 + A_y^2} = 5.68\,kN$

$\tau_A = \tau_{allowable} = \frac{V}{A} = \frac{5.68 * 10^3\,N}{\frac{\pi}{4} d^2 * 2} = 90\,\frac{N}{mm^2}$

2 لأنو في Double shear على A

$d_A = 6.3\,mm \approx 7\,mm$

$\tau_B = \tau_{all} = \frac{V}{A} = \frac{6.67 * 10^3\,N}{\frac{\pi}{4} d^2} = 90 * \cancel{10}\,\frac{N}{mm^2}$

$d_B = 9.7\,mm \approx 10\,mm$

$\sigma_{BC} = \sigma_{all} = 115 \quad \frac{6.67 * 10^3}{\frac{\pi}{4} d^2}$

$d_{BC} = 8.6\,mm \approx 9\,mm$

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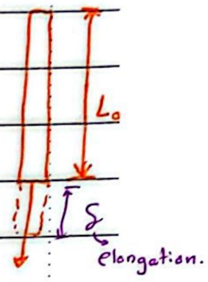
Chapter 2 :- strain

Strain :- measured By experiment.
(measure of deformation).

The unit

epsilon

$$\varepsilon = \frac{\text{Length}_{new} - \text{Length}_{original}}{\text{Length}_{original}} = \frac{\Delta L}{L_o} \qquad \frac{mm}{mm} \text{ or } \frac{m}{m}$$



Engineering strain / Normal strain

$\Delta L = \delta$

Hook's law $\longrightarrow$ $\boxed{\sigma = E\,\varepsilon}$

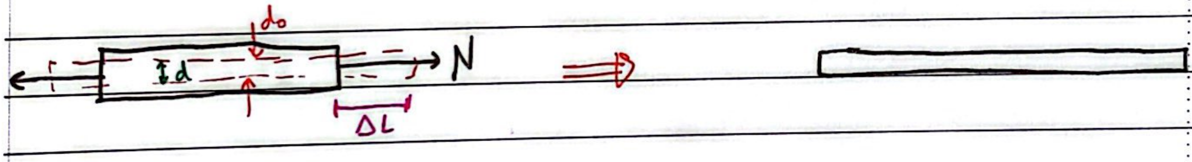
Modulus of elasticity (constant)

Normal strain

poisson effect. : How much the material will deform in the lateral directions.

عرض

It's like stretching a Rubber band, the length increase but the width decrease.



$$\nu = \frac{-\varepsilon_{\text{lateral strain}}}{\varepsilon_{\text{longitudinal strain}}}$$

poisson's Ratio

$$\varepsilon_{lateral} = \frac{\Delta d}{d_o}$$

$$\varepsilon_{long} = \frac{\Delta L}{L_o}$$

* we call the direction along which the load is applied the longitudinal direction.

* and we call the perpendicular directions the lateral directions.

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* السالب بقانون Poisson's ratio موجود لأنو انا لما اشد من جهة اصطلاحًا بكون موجب ومن خلال ال possion effect عرفنا انو رح يصير ضغط او تقلص من جهة ثانية وهاذ اصطلاحًا بكون سالب

$- \leftarrow \varepsilon_{lateral} \quad / \quad + \leftarrow \varepsilon_{long}$

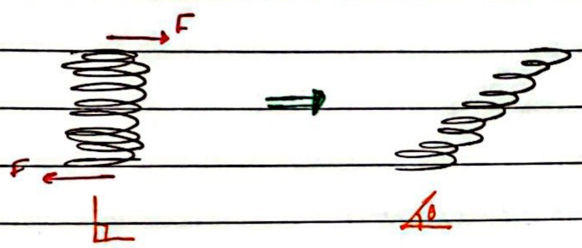
فالسالب اللي بالقانون موجود عشان القيمة السالبة تتحول لموجبة وهي Poisson's Ratio دائمًا يكون موجب.

Shear strain

* ممكن الإستطالة او ال strain ما تكون بشكل مستقيم ممكن تكون بزاوية اللي هو مبدأ ال shear strain.

يعني ال force بتكون موازية لل crossectional area.

$A \parallel F$



$\gamma \equiv$ Shear strain $= \frac{\pi}{2} - \theta$

تغير بالزاوية

Radians!

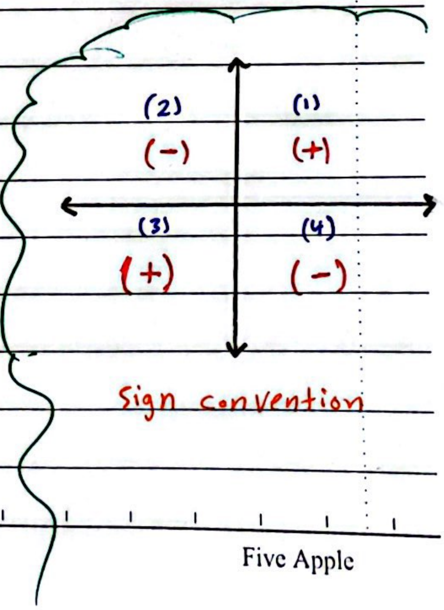
Like Hook's law we can put a relationship between shear stress and shear strain

Normal $\Longrightarrow \sigma = E\varepsilon$

Shear $\Longrightarrow \tau = G\gamma$

Modulus of rigidity

shear strain



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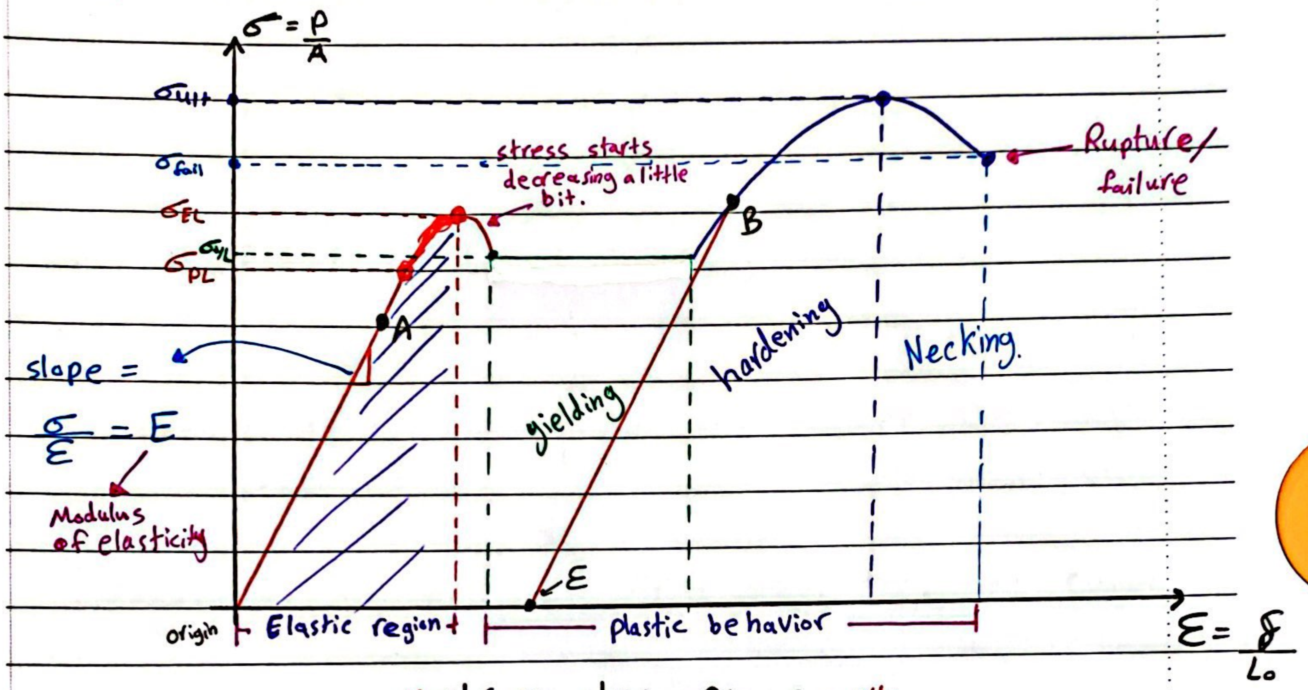
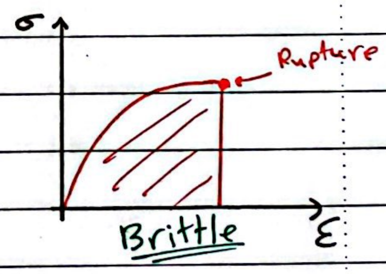
Chapter 3:- Mechanical properties of materials:-



stress strain Diagram
for Ductile material.

* A ductile material is a material that can be bent and formed and doesn't break easily.



σ_{PL} : Proportional limit.

σ_{EL} : Elastic limit. /upper yield point.

σ_{YL} : stress at which the elongation increases without any change in stress. /Lower yield point.

σ_{fail}: failure stress.

σ_{ult}: Ultimate stress/Ultimate strength.

* in the yielding region the material is permanently deformed.
* when we stop adding stress at point A the material will go back to the origin (original length), but at point B it doesn't since it's permenantly deformed and we can find the final length (ε) a parall line to the E slope that intersects the x-axis.

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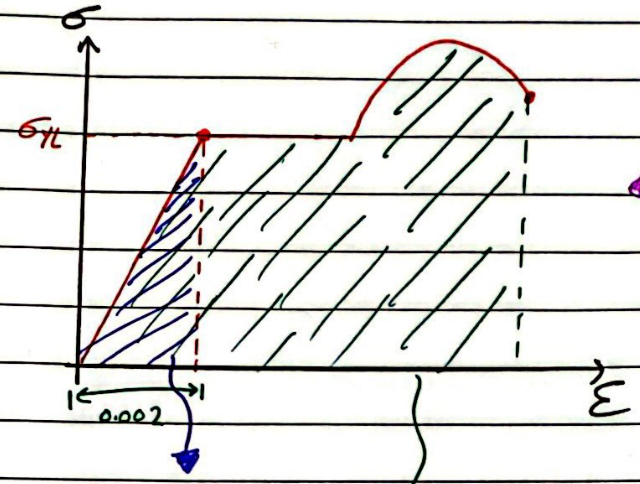
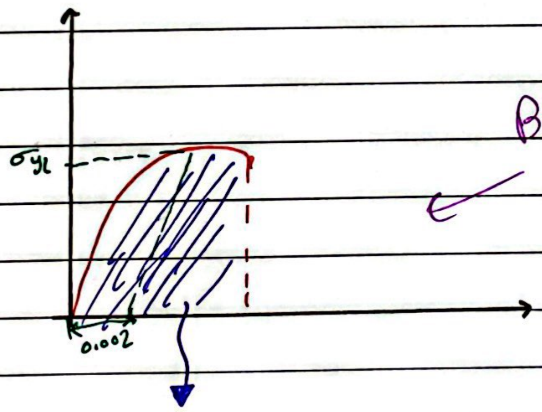


Ductile

Modulus of Resilience (energy) = Area under the curve (of the elastic region).

Modulus of toughness = Area under the entire curve (including the elastic region).

The total energy a material can take before it fails.



Brittle

Area under the curve = energy

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Chapter 4 :- Axial Load

we can find the displacement / deformation / chang in Length δ

$$\sigma = \frac{P}{A} \Rightarrow \sigma = E\varepsilon \Rightarrow \sigma = E\frac{\Delta L}{L} \Rightarrow$$

$$\frac{P}{A} = E\frac{\Delta L}{L}$$

force along the axis (Normal force).

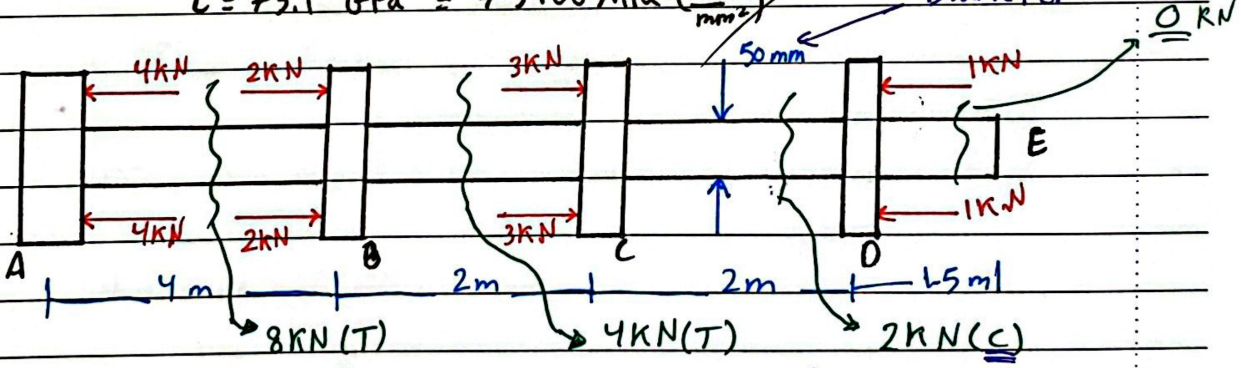
length

$$\boxed{\Delta L = \delta = \frac{PL}{AE}}$$

"دلتا"   Area cross sectional   Modulus of elasticity

Example :- find the change in length of the bar looking @ A w/ respect to E.

$E = 73.1\ GPa = 73100\ MPa\ (\frac{N}{mm^2})$



بطلع ال forces بعد كل load كما تعلمنا سابقاً

$$\delta_{tot} = \underbrace{\frac{(8000\ N)(4000mm)}{\pi(25\ mm)^2\ 73100\ (\frac{N}{mm^2})}}_{\delta_{AB}} + \underbrace{\frac{(4000)(2000)}{\pi(25)(73100)}}_{\delta_{BC}} - \underbrace{\frac{(2000)(2000)}{\pi(25)^2(73100)}}_{\delta_{CD}}$$

because of compression.

$$\delta_{tot} = 0.251\ mm$$

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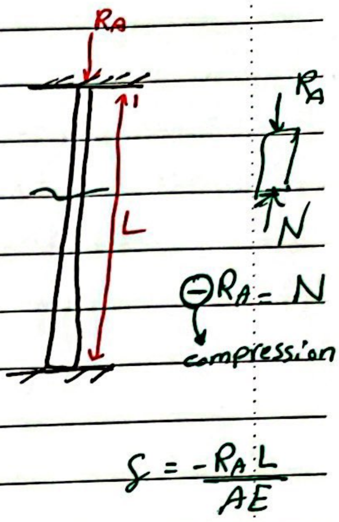
4.6: Thermal stress

$$\delta_{Thermal} = \alpha \Delta T L$$

change in temperature.

coefficient of thermal expansion



$$\Delta L = \alpha \Delta T L \qquad \sigma = E \varepsilon$$

$$\varepsilon = \frac{\Delta L}{L} = \frac{\alpha \Delta T \cancel{L}}{\cancel{L}}$$

$$\sigma_{Thermal} = \alpha \Delta T E$$

compatability

$$\delta_{Thermal} + \delta_{Reaction} = 0$$

$$\alpha \Delta T L - \frac{R_A L}{AE} = 0$$

$$\sigma_{Thermal} = \alpha \Delta T E$$

$$R_A = \alpha \Delta T A E$$

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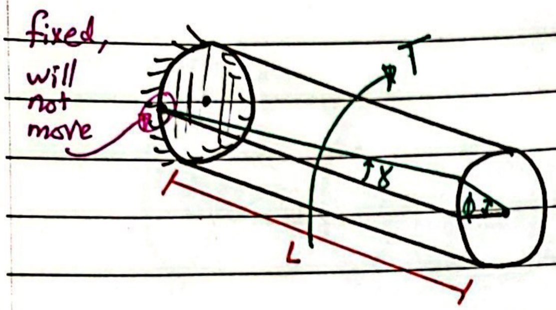
Chapter 5:- Torsion



* After applying a torque on a shaft the black line shifted and the new position is the green line.

ϕ = angle of twist.

γ = shear strain.

$\gamma = \dfrac{\rho \phi}{L}$; ρ = distance from the center.

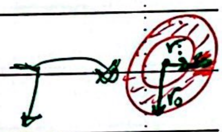
$\tau = \dfrac{T\rho}{J}$; J = polar moment of inirtia

$\quad \hookrightarrow I_x + I_y \Rightarrow$ chapter 10 → statics.

$\tau = G\gamma$; G = Modulus of rigidity / shear modulus.

$\phi = \dfrac{TL}{GJ}$; T = amount of Torque.

solid Bar $J = \dfrac{\pi r^4}{2}$ Hollow bar $J = \dfrac{\pi (r_o^4 - r_i^4)}{2}$



$J = I_o = I_y + I_x = \dfrac{\pi r^4}{4} + \dfrac{\pi}{4} r^2 = \dfrac{\pi r^4}{2}$

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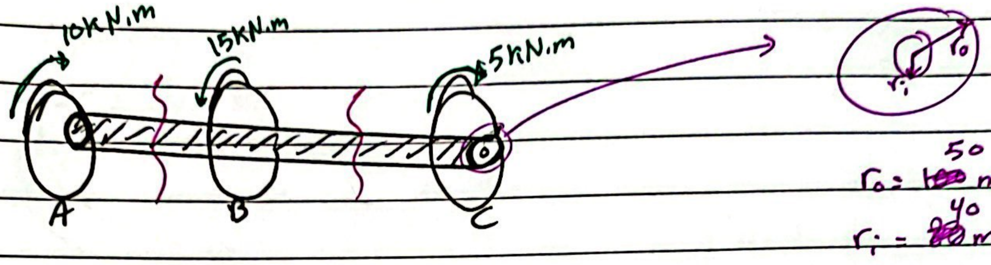
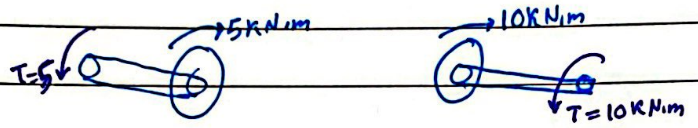
Example:- find the shear stress in all the sections of the hollow shaft, inner Diameter = 80 mm and outer Diameter = 100 mm



$r_o = \cancel{100}\ 50$ mm

$r_i = \cancel{80}\ 40$ mm



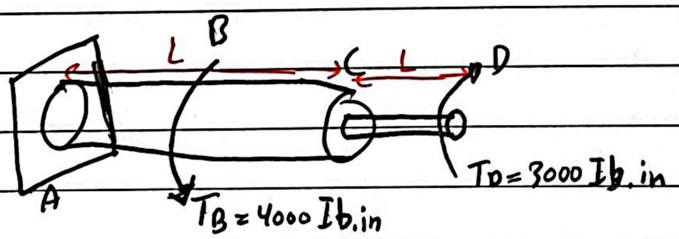
$$J = \frac{\pi}{2}(50^4 - 40^4)$$

$$\tau_{AB_{max}} = \frac{T\rho}{J} = \frac{10000\ N.m\ (50)}{\frac{\pi}{2}(50^4 - 40^4)}$$

$$\tau_{BC_{max}} = \frac{5000\ (50)}{\frac{\pi}{2}(50^4 - 40^4)}$$

Example:- Determine the twist of point B & D Relative to the wall at point A. $J_{AC} = 4 in^4$ $J_{CD} = 2 in^4$ $L = 10$ in

$G_{AC} = G_{CD} = G = 10 * 10^6$ psi



$$\phi = \phi_{AB} + \phi_{BC} + \phi_{CD}$$

$$= \left(\frac{T_{AB}\ L_{AB}}{J_{AB}\ G}\right) + \left(\frac{T_{BC}\ L_{BC}}{J_{BC}\ G}\right) + \left(\frac{T_{CD}\ L_{CD}}{J_{CD}\ G}\right)$$

$$= \frac{(-1000)(10 in)}{(4 in^4)(10*10^6 psi)} + \frac{3000(10)}{4(10*10^6)} + \frac{3000(10)}{2(10*10^6)}$$

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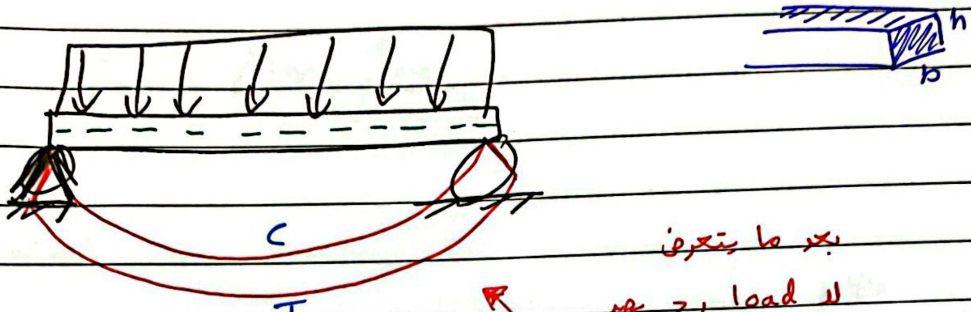
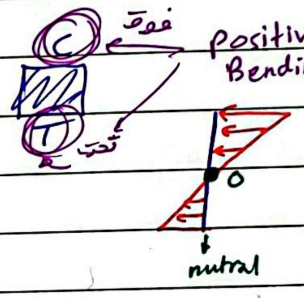
chapter 6:- Bending.

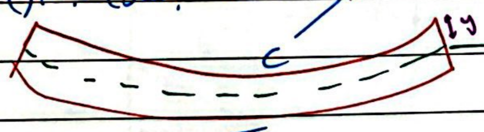
* we will use shear moment diagram from chapter 7 in statics so review that.



بعد ما يتعرض
لا load رح يصير
هيك
نتيجة moment

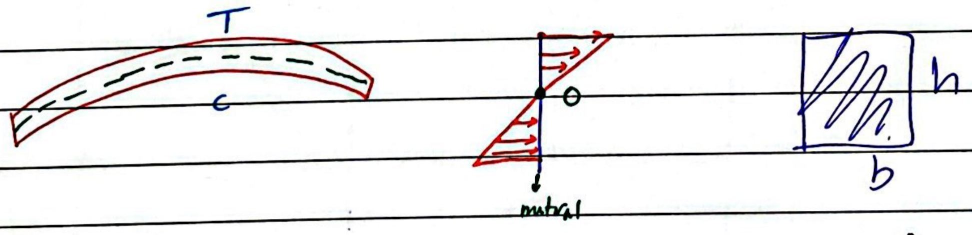


من الشكل مبين انو بتنضغط
(المسافة بتقل) (بتقصر)



nutral line (NA)
هو نفسو ال centroid
وما بتتغير في الطول
بعد ما يتعرض لا load

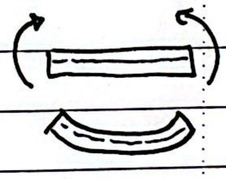
لأنو قاعد
يتمدد (بطول)



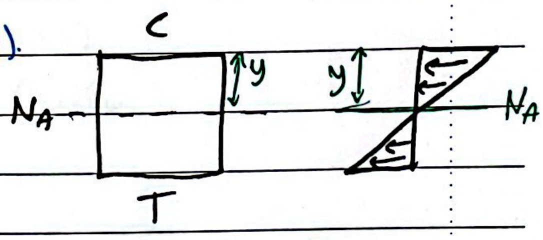
$$\sigma_{flexural} = \frac{M y}{I}$$

y ← location from centroid

I ← moment of inertia ↳ (Resistance to bending).



* from moment Diagram if M is positive then positive Bending and if M is Negative the Negative Bending.



Positive Bending

Five Apple

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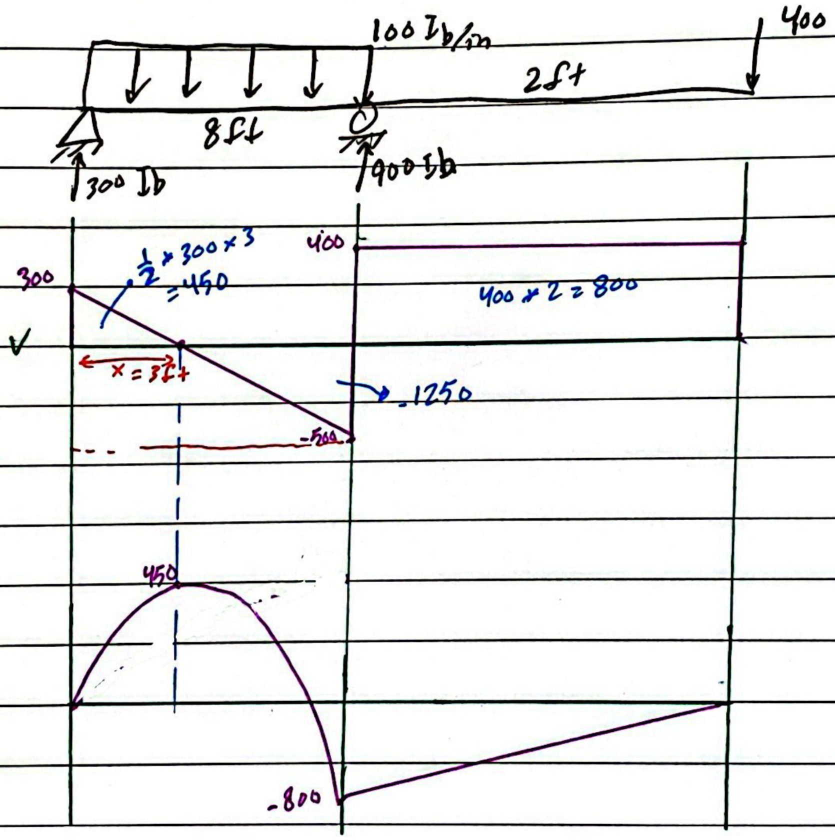
* To find maximum Flexural stress we need to find max moment (from shear moment diagram), and maximum distance from the centroid.

$$\sigma_{max} = \frac{M_{max}\, y_{max}}{I}$$

Example 8



$$\frac{800}{8} = \frac{300}{x}$$

$x = 3\ ft$

max moment is 800 and it's negative so negative bending → Tension (Top).

compression (bottom).

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