ElCoM
ELECTRICAL COMPUTER MECHATI

ميكانيكا هندسية

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للطالب المبدع

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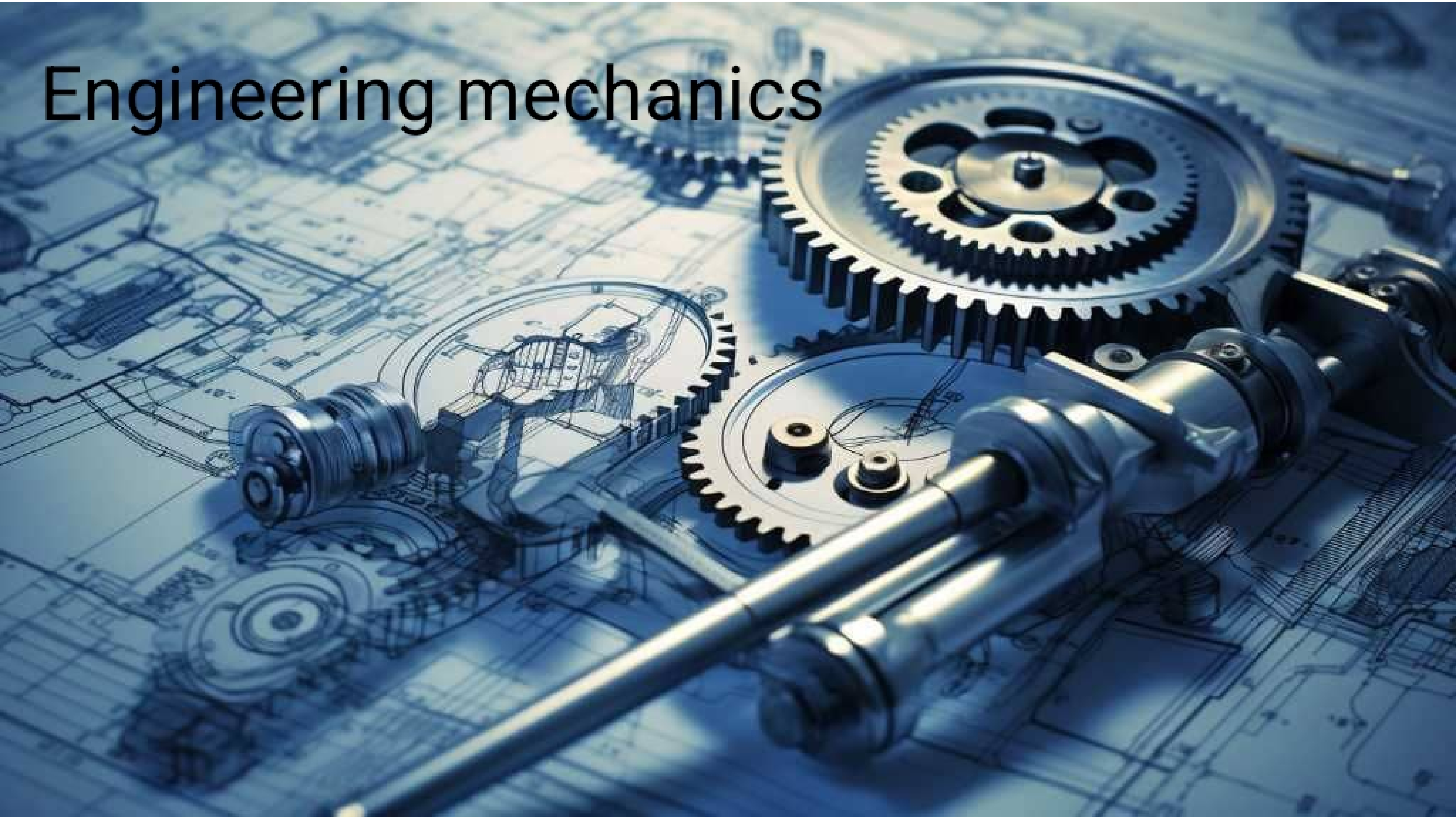
إرادة - ثقة - تغيير


Engineering mechanics

Made By :

KAREEM

كريم

Subject Introduction of mechanics Date 29/12/2024 No. Thursday

علم الميكانيكا علم يصف تنبأ

* mechanics: scince that describe and predicts the condition of rest or motion of bodies under the action of forces القوى

ظروف سكونه حركه أو الأجسام تحت تأثير

* هو

* Branches of mechanics: فروع الميكانيكا

1- rigid bodies الأجسام الصلبة
2- deformable bodies قابل للطي / الحركة
3- incompressible / compressible bodies (fluid / gases) غير قابل للضغط / قابل للضغط سائل / غاز
4- relativistic and quantum mechanics نسبي كمي

* rigid body (simple branch): consist of:

متوازن ساكن سرعة ثابتة

* statics: => equilibrium => rest and constant velocity
* dynamics => accelerated motion (a) Kinetics (b) Kinematics جسم متحرك

الزمن مستقل التسارع غير موجود

* (static are time-independent and (a) not present

* matirial properties: خصائص المواد
Yield strength, ultimate strength, young's module
toughiness, hardness, poisson's ratio stiffness

قوة الخضوع غير محدود قوة صلابة اقاس

* Geometric properties: خصائص هندسية
length, cross-section, boundary, holes

أسباب الفشل حمل زائد تشوه ضيق مكسر

* failure: over stress, deformation, distortion, cracking

Subject Date No.

الكميات الأساسية

* **Basic Quantities of mechanics:** يصف نقطة موجودة في الفراغ

فراغ - space: describe the position of a point in space

وقت - Time: succession of events تسلسل الوقت

مقاومة تغير سرعة

كتلة - mass: resistance and change (v) of bodies الأجسام

قوة - Force: action of one body on another تأثير قوة جسم على آخر

* **Fundimental principles** المبادئ الأساسية

1- parallelogram law for the addition of forces

2- transmissibility قابلية الانتقال

3- Newton's low of motion قوانين نيوتن الحركة

4- Newton's low of gravitation قوانين نيوتن الجاذبية

* **parallelogram law (addition of forces)**

قوى محصلة قوتين

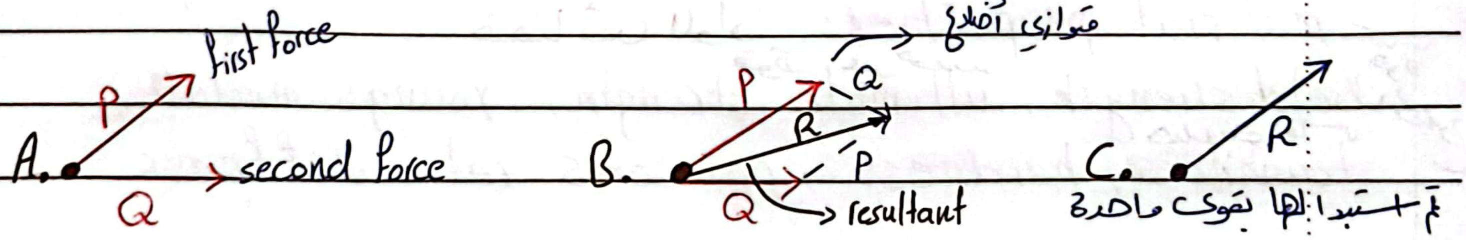
when we say a resultant force that means two

أو أكثر تؤثر نقطة واحدة

or more factors acting on as single point that

يمكن استبدالها

can be replaced by asingle force. بقوة واحدة



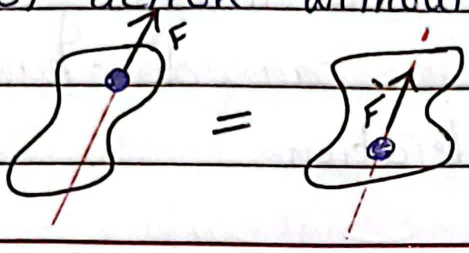
* principle of Transmissibility) قابلية الإنتقال

نقطة (A) يمكنه أن تتحرك في أي نقطة على طول

a point (A) can be moved any where along the

خط النقطة دون تغير في القوى المؤثرة

line of action without changing the force in body



* Newton's First law: قانون نيوتن الأول

* object at rest will remain at rest and object in motion will remain in motion with constant velocity unless acted by a net external force.

الجسم الساكن يبقى ساكن والجسم المتحرك يبقى متحرك بسرعة ثابتة مالم تؤثر عليها قوى تغير من حالته

* Newton's First law describe what happens in absence of a force not the net force also the (a) is zero

قانون نيوتن الأول يصف ما يحدث للجسم عند عدم وجود قوى مؤثرة

* والتسارع أيضاً يساوي صفر

* any isolated object move at constant velocity and at rest when force act cause a change in motion

أي جسم معزول يتحرك بسرعة ثابتة، وعند التأثير بقوى على جسم ساكنه يؤدي إلى تغير في حركة الجسم

* Newton's second law:

acceleration of an object is directly proportional To the net force acting on it and inversely proportional to its mass Force is causeof change in motion, as measured by acceleration

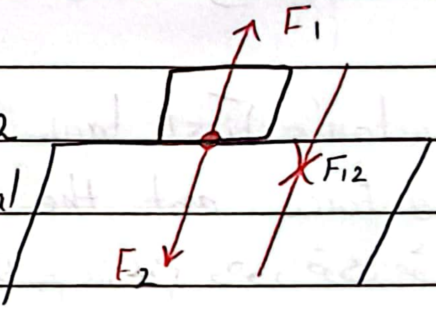
قانون نيوتن الثاني على أن تسارع الجسم يتناسب طردي مع القوة المؤثرة وعكسياً مع كتلته.

$$\Sigma \vec{F} = m \cdot \vec{a}$$

F: (Force) (vector) m: mass (scaler) a: acceleration (vector)

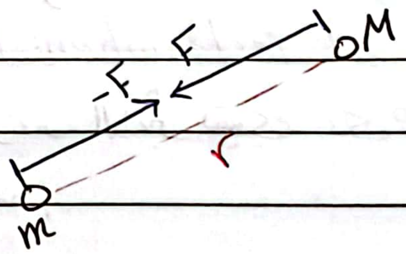
* Newton's third low:-

IF two objects interact the force F_{12} obserted by object 1 on 2 is equal to magnitude and oppisite direction of force F_{21} $F_{12} = -F_{21}$



* Newton's low graviation:

$$F = G \frac{mM}{r^2}$$



Two particals with masses with same forces but oppisite direction

G: Gravitional constant

Subject units and significant Figure Date No.

* we have 3 basic (kinetic) units:
  1 length 2 Time 3 mass
* SI (unit): L (meter) T (second) m (kilogram)
* US (unit): f (foot) t (second) lb (pound)
* The SI unit system include:
  1- SI base and derived units 2- SI prefixes

* SI units and English units:

Quantity	symbol	SI	English	convert
Force, weight	F, W	N	Ibf	4.448
pressure, stress	σ	MPa / N/mm^2	ksi	6.895
moment, Torque	M	N. m	ft. Ibf	1.356

* To convert from English to SI unit we multiply by
* SI prefixes:

prefix	symbol	abbreviation
1E+09	giga	G
1E+06	mega	M
1E+03	kilo	k
1E-03	milli	m
1E-06	micro	μ
1E-09	nano	n

Ex: convert 75 Ibf To N → 75 Ibf * 4.448 = 333.6 N

* rules of significant figure:

1- all nonzero digits are significant: 1.234 (4 S.f.)

2- zeroes between nonzero digits are S.f.: 3.07 (3.S.f.)

3- zero before first S.f. on left non S.f.: 0.012 (2 S.f.)

4- zero after last S.f. on the right are S.f.: 0.0230 (3.S.f.)

5- Ex: 0.0560

not S.f. not S.f. → there is 3 S.f.

كمية قياسية * scaler and vector quantities:

* scaler quantity: quantity have magnitude only)

كمية متدرجة ex: length, mass, time كمية محددة بدون متجه

* vector quantity: quantity have both magnitude and direction ($\vec{A}$) كمية تحدد بقيمة واتجاه

* components of a vector:

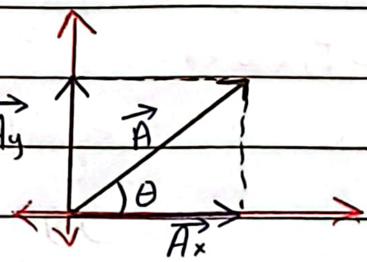
1- محصلة المتجه تكون بين المتجهين

2- (x, y) تقسيم المتجهات على المحاور

3- sin نأخذ (y) المحور و cos نأخذ (x) المحور

4- في حال الإتصال (cos) تكون القريبة من (θ)

5- نجد المحصلة

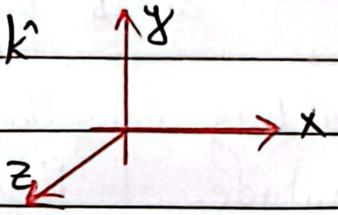


$$A = \sqrt{(A_x)^2 + (A_y)^2} \rightarrow \text{to find magnitude}$$

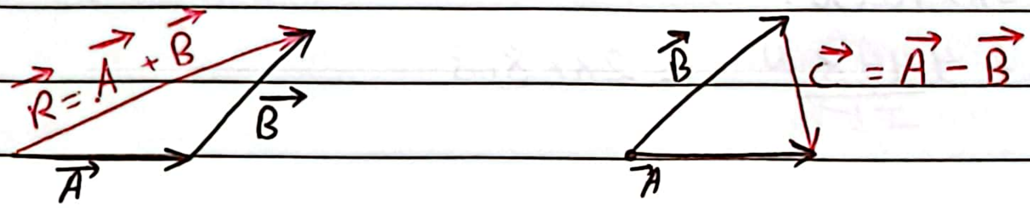
$$\theta = \tan^{-1}\left(\frac{A_y}{A_x}\right) \rightarrow \text{to find direction}$$

* **unit vectors:** (in this system we use $\hat{i}, \hat{j}, \hat{k}$)

$$A = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

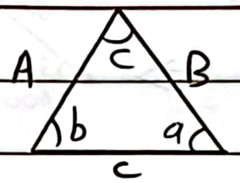


* **adding / subtracting Two vector graphically:**



* **component of a vector:**

From Trigonometry

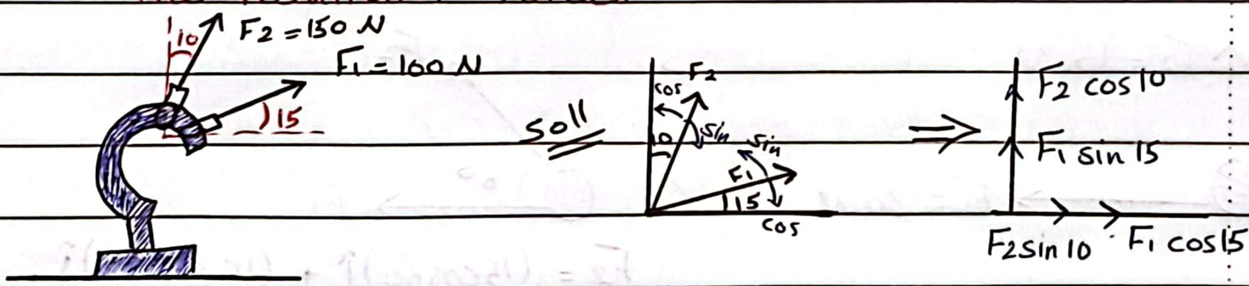


in this case we use:

cosine law: $C = \sqrt{A^2 + B^2 - 2AB\cos\theta}$

sine law: $A/\sin a = B/\sin b = C/\sin c$ (الضلع والزاوية المقابلة)

Ex: determine the magnitude and the direction of the resultant Force:



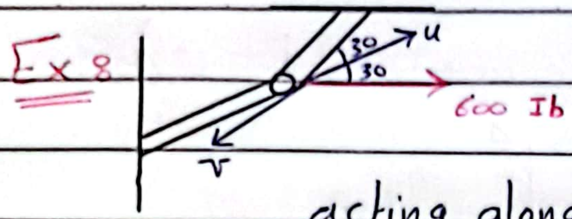
$$\sum F_x = (F_2 \sin 10 + F_1 \cos 15) = (150 \sin 10 + 100 \cos 15) = 122.6$$

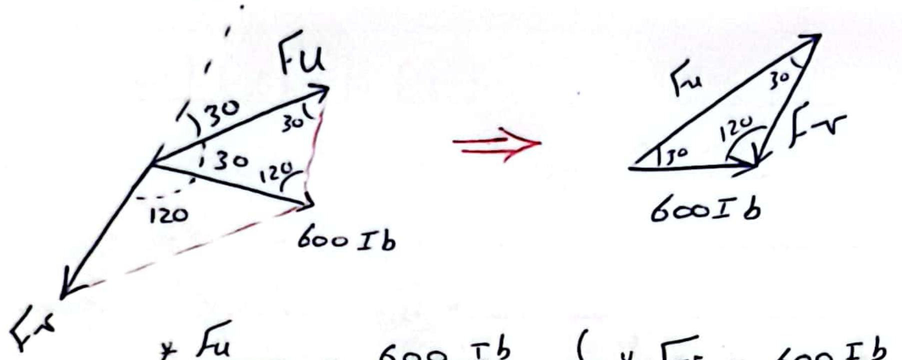
$$\sum F_y = (F_2 \cos 10 + F_1 \sin 15) = (150 \cos 10 + 100 \sin 15) = 173.6$$

$$F = \sqrt{(122.6)^2 + (173.6)^2} = 212.5$$

$$\theta = \tan^{-1}(173.6/122.6) = 54.7^\circ$$

Ex 8 resolve the horizontal 600 Ib Force in Figure into components acting along the u and v. and determine The magnitude of these components.



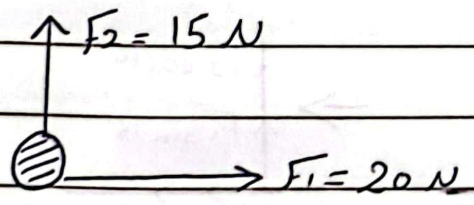


$$* \frac{F_u}{\sin 120} = \frac{600\ Ib}{\sin 30} \qquad * \frac{F_v}{\sin 30} = \frac{600\ Ib}{\sin 30}$$

$$F_u = 1039\ Ib \qquad F_v = 600\ Ib$$

Ex 8 Two Forces F_1 and F_2 act on a (5kg) mass IF $F_1 = 20N$ and $F_2 = 15N$ Find the acceleration in (a) and (b):

a.



sol

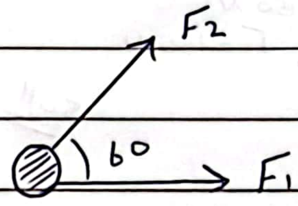
$\Sigma F = F_1 + F_2 = 20\hat{i} + 15\hat{j}$

$\Sigma F = ma = 5a$

$(20\hat{i} + 15\hat{j})/5 = a$

$a = (4\hat{i} + 3\hat{j})\ m/s^2$

b.



$F_2 = (15\cos 60)\hat{i} + (15\sin 60)\hat{j}$

$F_1 = 20$

$\Sigma F = (15\cos 60 + 20)\hat{i} + (15\sin 60)\hat{j}$

$\Sigma F = ma$

$a = ((15\cos 60 + 20)\hat{i} + (15\sin 60)\hat{j})/5$

Subject Date No.

Ex: Two vectors are given by $\vec{A} = 3\hat{i} - 2\hat{j}$ and $\vec{B} = -\hat{i} - 4\hat{j}$ calculate: a. $\vec{A}+\vec{B}$ b. $\vec{A}-\vec{B}$ c. $|\vec{A}+\vec{B}|$ d. $|\vec{A}-\vec{B}|$ e. The direction of $\vec{A}+\vec{B}$ and $|\vec{A}-\vec{B}|$.

solution:

a) $\vec{A}+\vec{B} = (3\hat{i}-2\hat{j}) + (-\hat{i}-4\hat{j}) = 2\hat{i} - 6\hat{j}$

b) $\vec{A}-\vec{B} = (3\hat{i}-2\hat{j}) - (-\hat{i}-4\hat{j}) = 4\hat{i} + 2\hat{j}$

c) $|\vec{A}+\vec{B}| = \sqrt{2^2+(-6)^2} = 6.32$

d) $|\vec{A}-\vec{B}| = \sqrt{4^2+2^2} = 4.47$

e) For $\vec{A}+\vec{B} = \theta = \tan^{-1}(-6/2) = -71.6 = 288°$

For $\vec{A}-\vec{B} = \theta = \tan^{-1}(2/4) = 26.6°$

* Dot and cross product:

A * when we talk about Dot product we have just magnitude and the direction we know it by (cosine angle)

Ex = $A \cdot B = A \cdot B \cdot \cos\theta$ (we have scaler quantity)

* If the vectors are perpendicular $A \cdot B = zero$

B cross product (magnitude vector and sine angle)

$A \times B - A \cdot B \cdot \sin\theta$ (we have vector quantity)

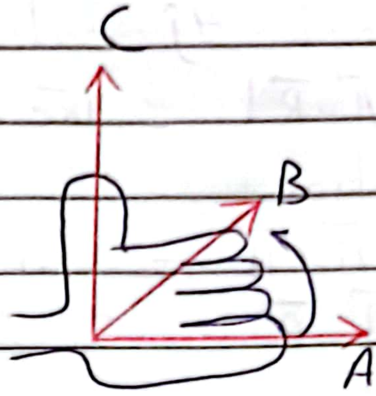
The direction give by right hand rule

* If the vectors are paralle to each other $A \times B = zero$

* Dot product

$$\vec{A} \cdot \vec{B} = |A| \cdot |B| \cos\theta$$

$$\theta = \cos^{-1}\left(\frac{|A \cdot B|}{|A| \cdot |B|}\right)$$



* cross product

$$\vec{A} \times \vec{B} = |A| \cdot |B| \cdot \sin\theta$$

$$\theta = \sin^{-1}\left(\frac{|\vec{A} \times \vec{B}|}{|A| \cdot |B|}\right)$$

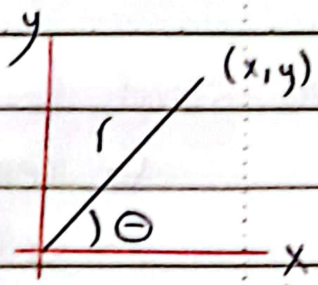
$\vec{A} \times \vec{B}$ we have a new product. $C = \vec{A} \times \vec{B}$

$$C_x - \begin{vmatrix} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} \qquad C_y = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$C_x = (A_y B_z - A_z B_y)$ $\qquad$ $C_y = -\hat{j}\,(A_z \times B_x - A_x B_z$

$$C_z \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$$

$$C_z = \hat{k}\,(A_x B_y - A_y B_x)$$

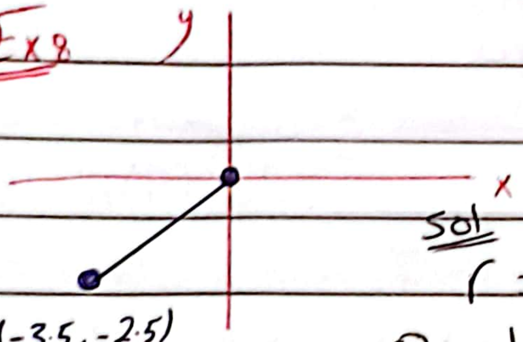


* Coordinate systems:

The most general coordinate system is:

1. The cartesian coordinate system (rectangular) with point (x, y)
2. polar coordinate system (r, θ)

Ex: Find the polar coordinate of the point.



Sol:

$r = \sqrt{x^2 + y^2} = \sqrt{(-3.5)^2 + (-2.5)^2} = 4.30\,m$

$\theta = \tan^{-1}\left(\frac{-2.5}{-3.5}\right) = 0.74$

$\theta = 216°$

* The cartesian system can be described into

1- local axes 2- Global axes

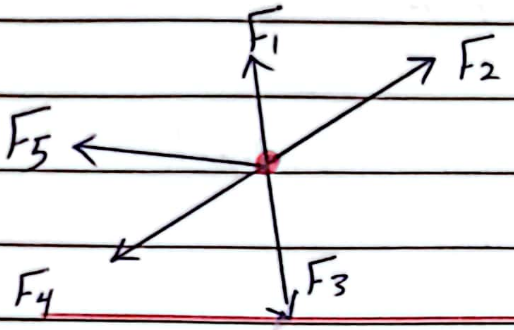
* static and rigid body in equilibrium:

* static equilibrium: its an object moves with constant ~~an~~ velocity and angular velocity

* rigid body equilibrium: particle move with constant velocity becouse the net force is zero

we said to particle it is equilibrium if:

1- remain at rest
2- originally at rest
3- constant velocity in motion

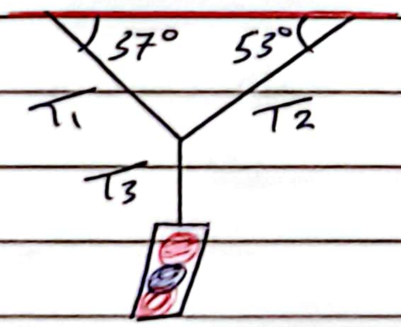


$\sum F = zero$

* Free body diagram:

* The most important step in solve mechanical problems
* we need the forces acting on the diagram
* start by modeling object (تحديد اتجاهات القوى)

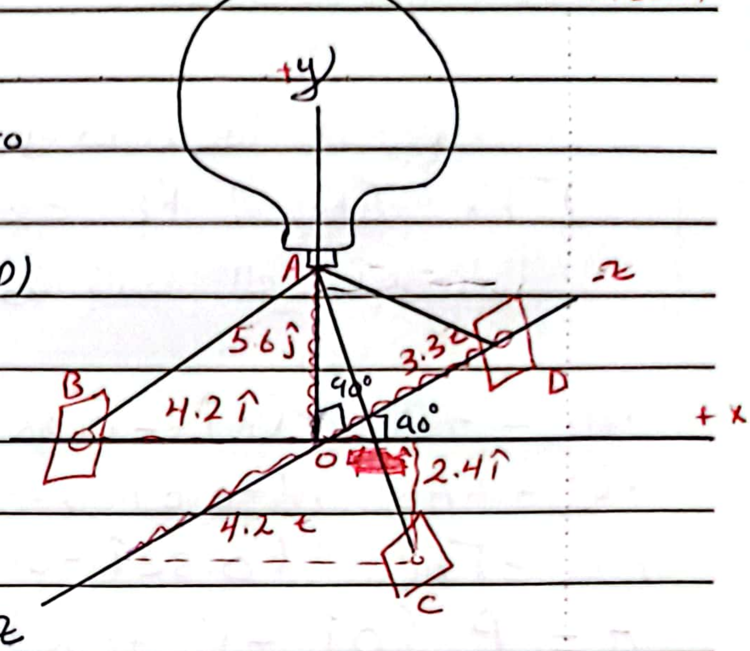
Ex: a traffic light weighing (122 N) hanges a cable with 2 other caples as shown if the tension in them exceeds (100 N) will the traffic light remain hanging in this situation:



يجب الأخذ بالإعتبار أن وزن
الإشارة يجب تحملها قوة الشد (٣)
لذا فهي متساوية

Subject equilibrium particles. Date Thursday No. 7/3/2024

Ex: Three cables are used to tether a ballon as shown. Determine the vertical force (P) exerted by the ballon at (A) knowing Tension in cable (AD) is 481 N.



sol: * ثلاث خيوط تربط بالون ثلاثي الأبعاد الموضح في الشكل

المطلوب من السؤال إيجاد القوى (P) عند النقطة A

١- في حل المثال (ثلاثي الأبعاد) نحدد بعد كل نقطة عن نقطة المرجع

D- $(0\hat{i}, 0\hat{j}, -3.3z)$ refrence

C- $(2.4\hat{i}, 0\hat{j}, 4.2z)$ A- $(0\hat{i}, 5.6\hat{j}, 0z)$

B- $(-4.2\hat{i}, 0\hat{j}, 0z)$

٢- نحط نطرح النقطة الأخيرة من نقطة (refrence)

AD $(0\hat{i}-0\hat{i}, 0\hat{j}-5.6\hat{j}, -3.3z-0z) = (0\hat{i}, -5.6\hat{j}, -3.3z)$

AC $(2.4-0, 0-5.6, 4.2-0) = (2.4\hat{i}, -5.6\hat{j}, 4.2z)$

AB $(-4.2-0, 0-5.6, 0-0) = (-4.2\hat{i}, -5.6\hat{j}, 0z)$

٣- نأتي بال (magnitude) لكل نقطة

AD: $\sqrt{0+5.6^2+3.3^2} = 6.5$

AC: $\sqrt{2.4^2+5.6^2+4.2^2} = 7.4$

AB: $\sqrt{4.2^2+5.6^2+0^2} = 7$

٤- نقسم كل نقطة (AD, AC, AB) على (magnitude) نفس النقطة

AD $\left(\frac{0}{6.5}, -\frac{5.6}{6.5}, -\frac{3.3}{6.5}\right) = (0\hat{i}, -0.861\hat{j}, -0.507z)$

AC $\left(\frac{2.4}{7.4}, \frac{-5.6}{7.4}, \frac{4.2}{7.4}\right) = (0.32\hat{i}, -0.756\hat{j}, 0.567z)$

A.B $\left(\frac{-4.2}{7}, \frac{-5.6}{7}, \frac{0}{7}\right) = (0.6\hat{i}, -0.8\hat{j}, 0z)$

مجموع مربع كل نقطة كاملة تساوي

NOTEBOOK

Subject Date No.

5- نجاح الدرس لمحصلة قوى كل نقطة على حدة أ ب

$\sum F_x , \sum F_y , \sum F_z = zero$

وذلك عن طريقة ضرب كل قوة بالنقطة التي أوضحناها

$F_{AD} = 481\,N\,(0\hat{i} - 0.86\hat{j} - 0.507\hat{k})$

$F_{AB} = F_{AB}\,(-0.6\hat{i} - 0.8\hat{j} + 0\hat{k})$

$F_{AC} = F_{AC}\,(0.32\hat{i} - 0.75\hat{j} + 0.56\hat{k})$

$A = P\,(0\hat{i} + 1\hat{j} + 0\hat{k})$

6- عن طريقة معادلات حذف أو تعويض لتحصيل المجاهيل

$\sum F = zero$

$* \; F_{AC} * 0.56 - 481\,(0.507) = zer$

$F_{AC} = 435\,N$

$* \; F_{AB} * (-0.6) + 432\,(-3.3) = zero$

$F_{AB} = 232\,N$

$* \; P = -481 * 0.6 + 232 * -0.8 + 435\,(-0.75) = 80$

$P = 800\,N$

Subject rotation, Torque and moment Date No.

هي عبارة عن قوة تسبب التسبب في دوران الجسم حول محور معين

Tendency of the force cause a rotation of a rigid body) which called moment.

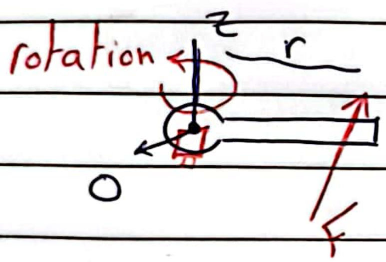
* To determine direction we use the right hand rule

$$\vec{M_z} = \vec{r} \times \vec{F}$$

حيث (r) تعبر عن (distance) و (F) تعبر عن Force

$$M_z = F \cdot r \cdot \sin\theta$$

*remark: إذا أردنا استخدام (cross product) القانون يجب اتباع ترتيب القوى r ثم F

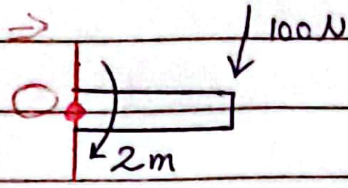


* when the moment is in counter clockwise it will be positive
but if the moment goes clock wise it will be negative

Subject: resultant moment   Date   No.

Ex: determine the moment Force about point O

(a)

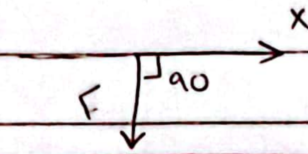


soll

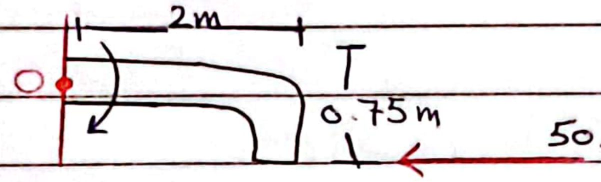
$m = r \times F \times \sin\theta$

$m = 2 * 100 * \sin 90$

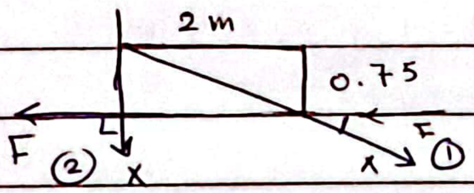
$m = -200$ (clock wise) (-)



(b)



soll



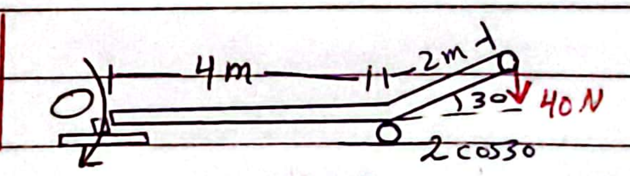
$m = r \times F \times \sin\theta$

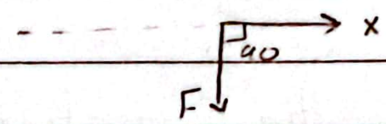
$m = 0.75 * 50 * \sin 90$

$m = -37.5$ (clock wise) (-)

(c)

H.w





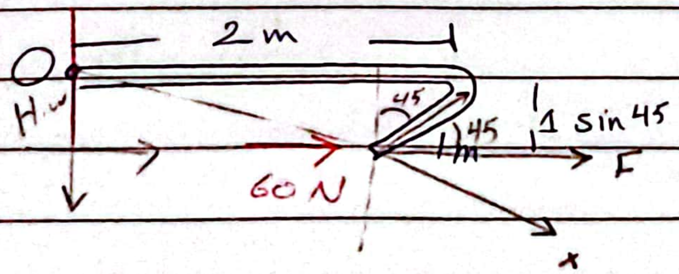
$x = 4 + 2\cos 30 = 5.7\,m$

$m = r \times F \times \sin\theta = 5.7 * 40 * \sin 90 = -216\,N$

(clock wise) (-)

(D)

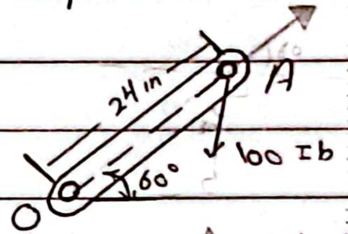
(*) H.w



$M = (1\sin 45) * 60 * \sin 90$

For a 100 Ib vertical force is applied to the end of a lever that is attached to a shaft at O, determine the following:

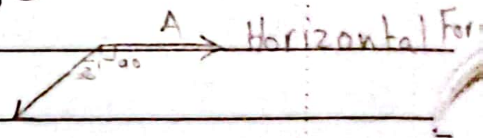
1- moment about O.
2- Horizontal force at A, which creat same moment
3- smallest force at A, which produce the same moment
4- location of 240 vertical force to produce the same momen



1- $M = r \times F \times \sin\theta = -24 \times 100 \times \sin 150$

$= -1200$ in. Ib

2- $M = r \times F \times \sin\theta =$



$1200 = 24 \times F \times \sin 120$

$F = \frac{100}{\sqrt{3}}$ Ib down

3- $1200 = 24 \times F \times \sin 90$

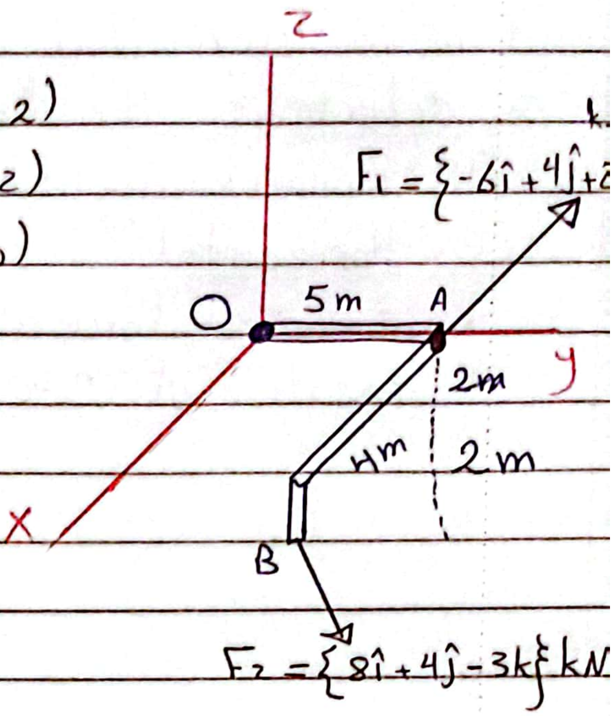
$x = 50$ in

4- $1200 = r \times 240$ Ib $\times \sin 150$

$x = 10$ in

Subject Date No.

Ex: Determine the moment of the force about O



* $\vec{M_R} = \vec{M_1} + \vec{M_2}$ O(0,0,2)

$r_1 = (A-O) = (0\hat{i}, 5\hat{j}, 0\hat{k})$ A(0,5,2)

$r_2 = (B-O) = (4\hat{i}+5\hat{j}-2\hat{k})$ B(4,5,0)

* $M_1 = \vec{r_1} \times \vec{F_1}$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 5 & 0 \\ -6 & 4 & 2 \end{vmatrix}$$

$M_1 = (10\hat{i} + 0\hat{j} + 30\hat{k})$

* $M_2 = \vec{r_2} \times \vec{F_2}$

$$\begin{vmatrix} \hat{i} & \hat{j} & k \\ 4 & 5 & -2 \\ 8 & 4 & -3 \end{vmatrix}$$

$M_2 = (-7\hat{i} - 4\hat{j} - 24\hat{k})$

$M_R = M_1 + M_2$

$\vec{M_R} = (3\hat{i} - 4\hat{j} + 6\hat{k})$

$M_R = \sqrt{9+16+36} = 7.8$ kN.m

Now we need direction

$\left(\cos^{-1}\frac{3}{7.8}\quad \cos^{-1}\frac{4}{7.8}\quad \cos^{-1}\frac{6}{7.8}\right)$

❖ Determine the resultant forces and moment on X, Y and Z axis for the system shown below.

$m_x = F_y \, r_z = 120$ N.m

$F_z \, r_y$ = zero N.m

$m_y = F_x \, r_z = -80$ N.m

$F_z \, r_x$ = zero N.m

$m_z = F_x \, r_y = -80$ N.m

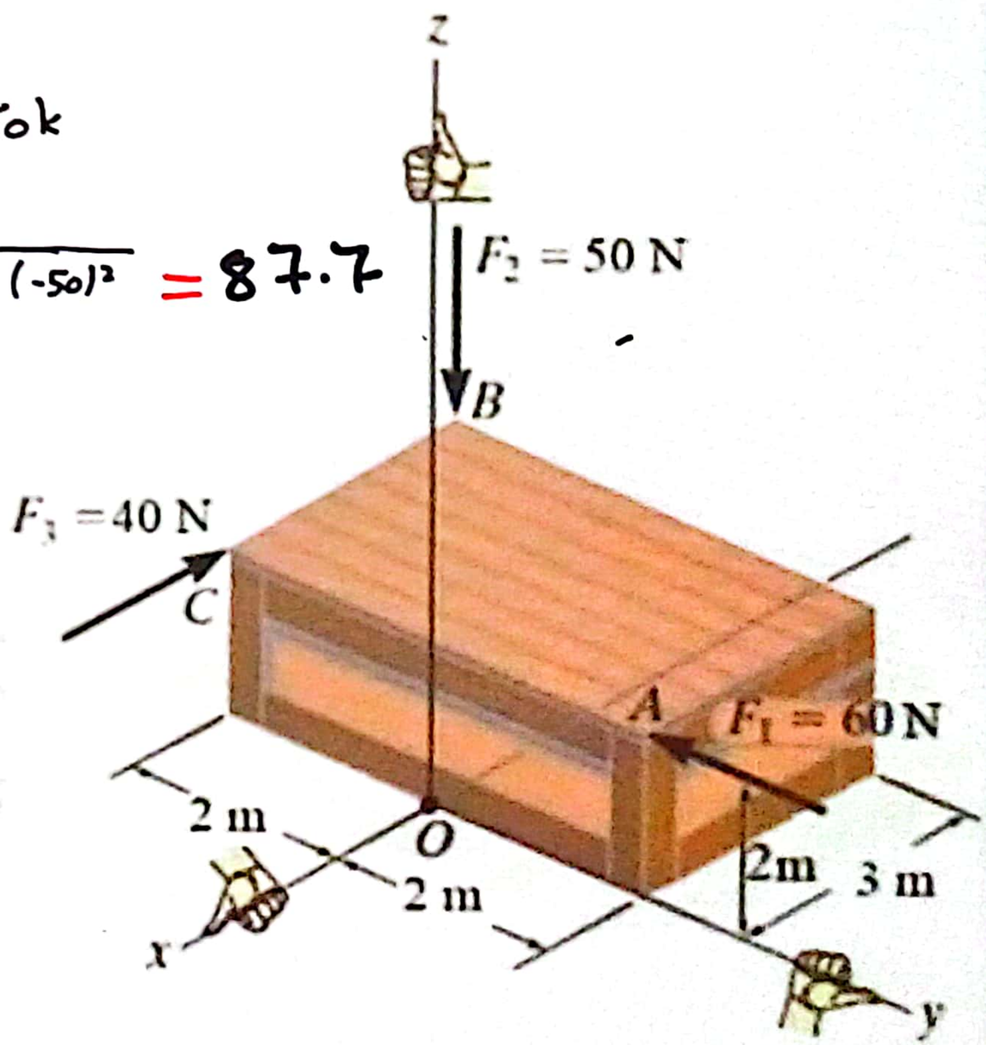
$F_y \, r_x$ = zero N.m

$M = 120\hat{i} - 80\hat{j} - 80\hat{k}$

⊙ Forces

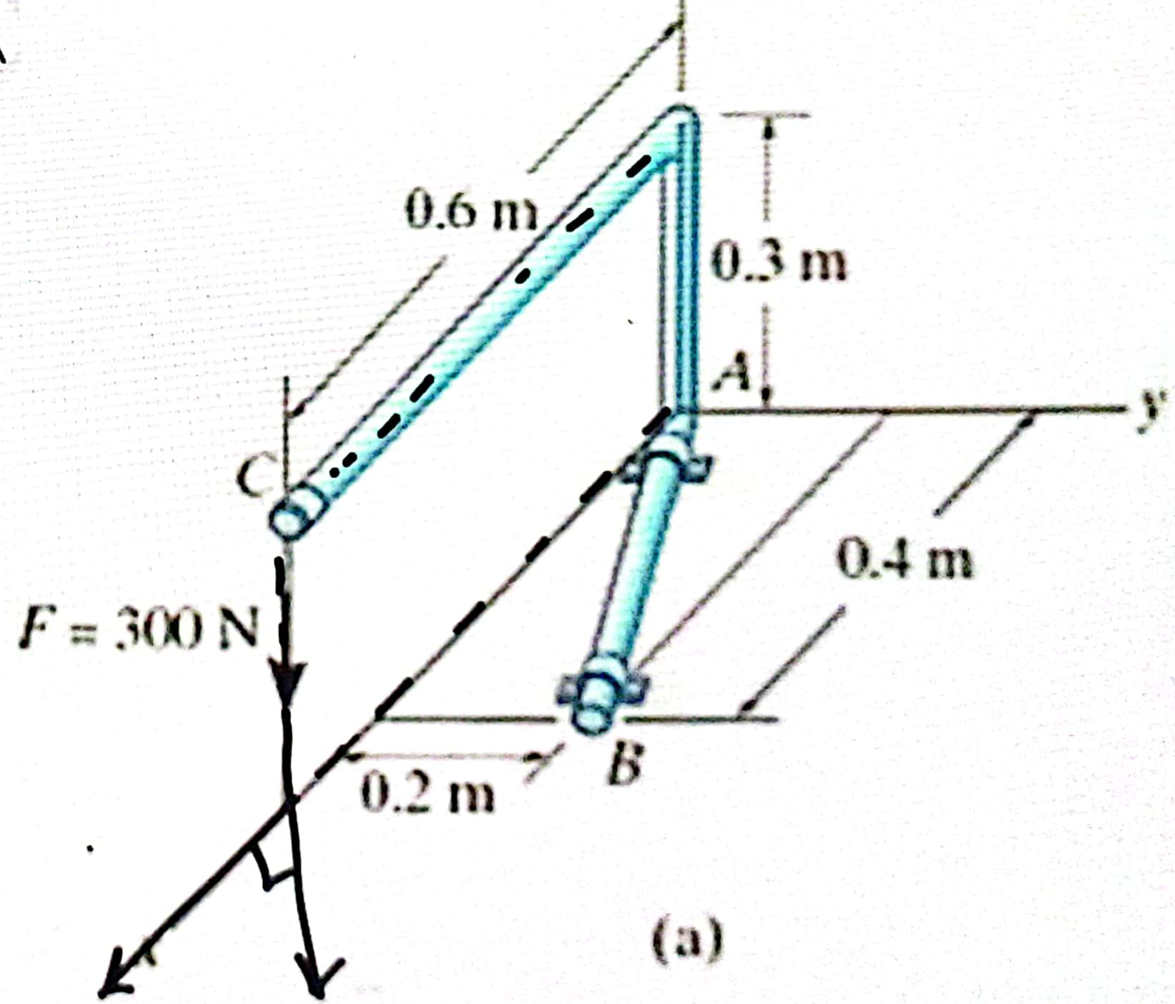
$F = -40\hat{i} - 60\hat{j} - 50k$

$F = \sqrt{(-40)^2 + (-60)^2 + (-50)^2} = 87.7$



❖ Determine the moment about point **A** (**M_A**) produced by force **F**

$M = r_y \times F_z \times \sin\theta = 300 * 0.6 * \sin 90 = \overset{+}{} 180\ N.m\ \hat{j}$



❖ Determine the moment about line **AB** (M_{AB}) produced by force ***F***

A, refrence point $(0\hat{i}, 0\hat{j}, 0\hat{k})$ } $r_{AC} = (0.6\hat{i}, 0\hat{j}, 0.3\hat{k})$

C, $(0.6\hat{i}, 0\hat{j}, 0.3\hat{k})$

F, $(0\hat{i}, 0\hat{j}, -300\hat{k})$

* $\vec{M} = \vec{r}_{AC} \times \vec{F}$

$$\begin{vmatrix} \hat{i} & -\hat{j} & \hat{k} \\ 0.6 & 0 & 0.3 \\ 0 & 0 & -300 \end{vmatrix}$$

$\vec{M} = 0\hat{i} + 180\hat{j} + 0k$

resultant moment

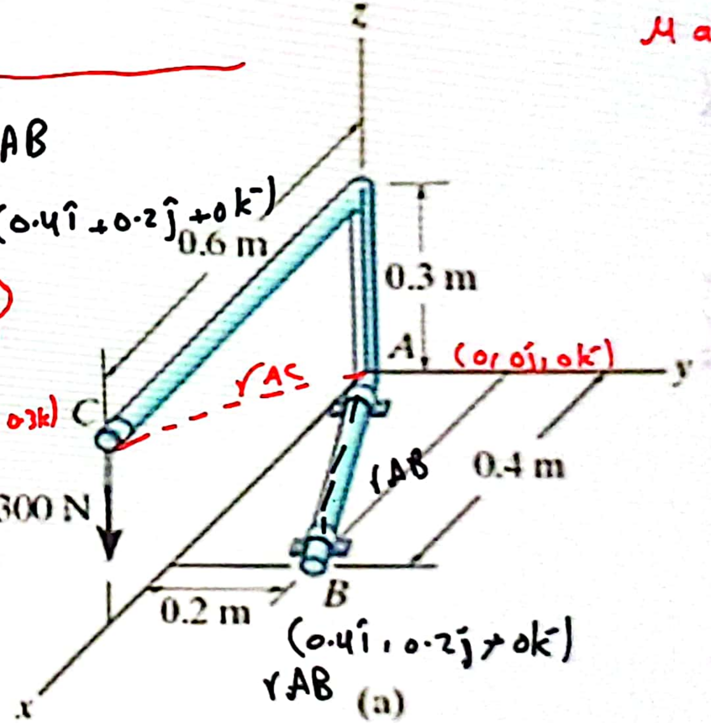
* $M_{AB} = M_R * r_{AB}$

$(0\hat{i} + 180\hat{j} + 0\hat{k}) * (0.4\hat{i} + 0.2\hat{j} + 0\hat{k})$

$= 0 + 36 + 0 = 36$

* في البداية نأتي M_R

* ثم $\hat{M}$ about axis



(a)

في الطريقتين يجب أن يكونو متساويات (هنا يوجد خطأ (غير مقصود) ولكنه للتوضيح K1 ?)

طريقة ② نأتي بـ (unit vector for axis AB)

* $r_{AB} = \sqrt{(0.4)^2 + (0.2)^2 + (0)^2} = 0.44 \Rightarrow \left(\frac{0.4}{0.44}\hat{i}, \frac{0.2}{0.44}, \frac{0}{0.44}\right) \Rightarrow (0.9\hat{i} + 0.45\hat{j} + 0\hat{k})$

* $M = r * F$

$$\begin{vmatrix} 0.9\hat{i} & 0.45\hat{j} & 0k \\ 0.6 & 0 & 0.3 \end{vmatrix} \Rightarrow 0 + (0.6 \times 300)(0.45\hat{j}) + 0\hat{k} = 180 \times 0.45$$

Subject **Moment at specific axis** Date No.

* in this case we use dotproduct

$$M = u_a (r \times F)$$

and then we have a scaler product (triple)

$M_o = u_a (\vec{r} \times \vec{F})$ u_a: is a vector extended between two points

❖ Determine the magnitude of the moment of force ***F*** about point ***O***

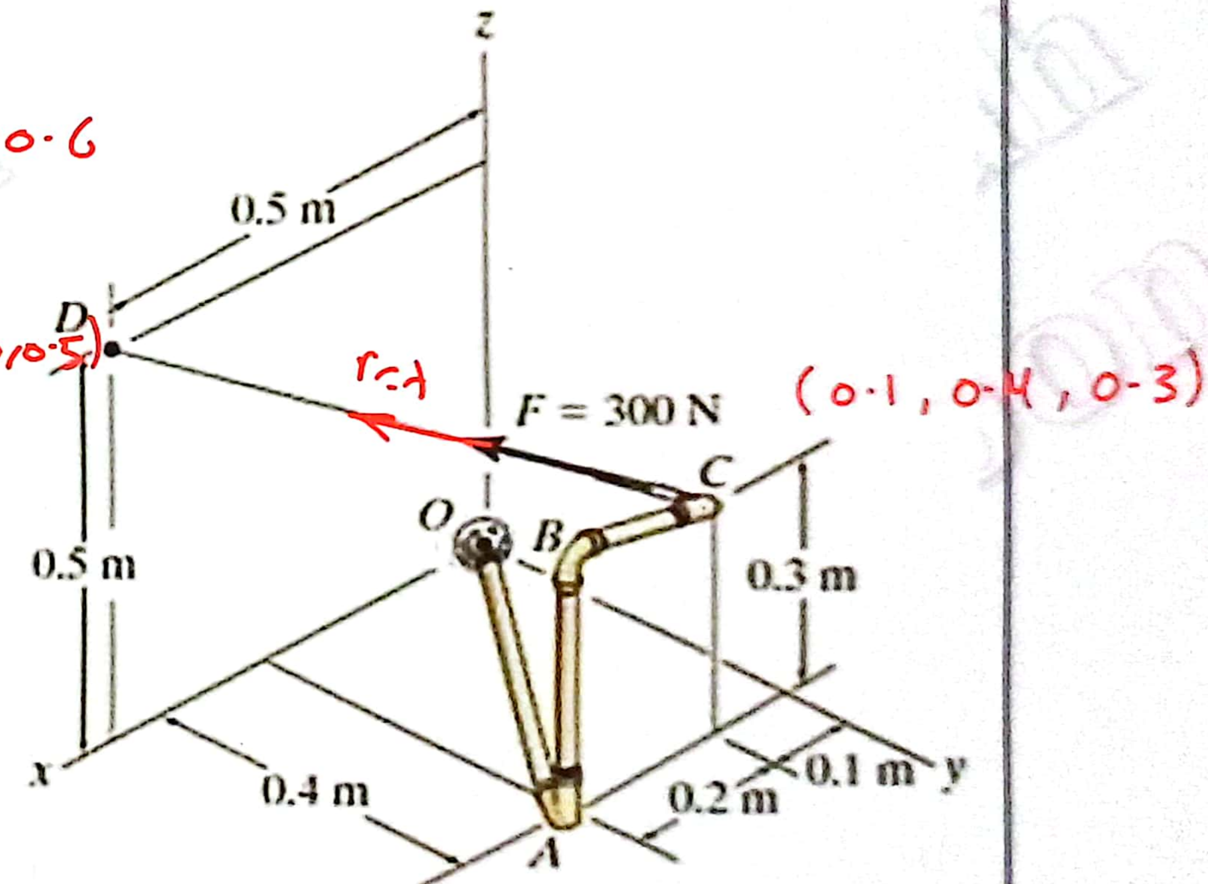
$*r_{CD} = (0.4\hat{i} - 0.4\hat{j} + 0.2k)$

$|r_{CD}| = \sqrt{(0.4)^2 + (0.4)^2 + (0.2)^2} = 0.6$

$\lambda_{CD} = 0.6\hat{i} - 0.6\hat{j} + 0.333k$

$F = 300(0.6\hat{i} - 0.6\hat{j} + 0.333k)$ (0.5, 0, 0.5)

$F = 180\hat{i} - 180\hat{j} + 100\hat{k}$



$M = r \times F$

$$\begin{array}{ccc} \hat{i} & \hat{j} & \hat{k} \\ 0.5 & 0 & 0.5 \\ 180 & -180 & 100 \end{array}$$

$M_R = 90\hat{i} + 40\hat{j} - 90\hat{k}$

❖ Determine the magnitude of the moment of force **F** about segment ***OA***

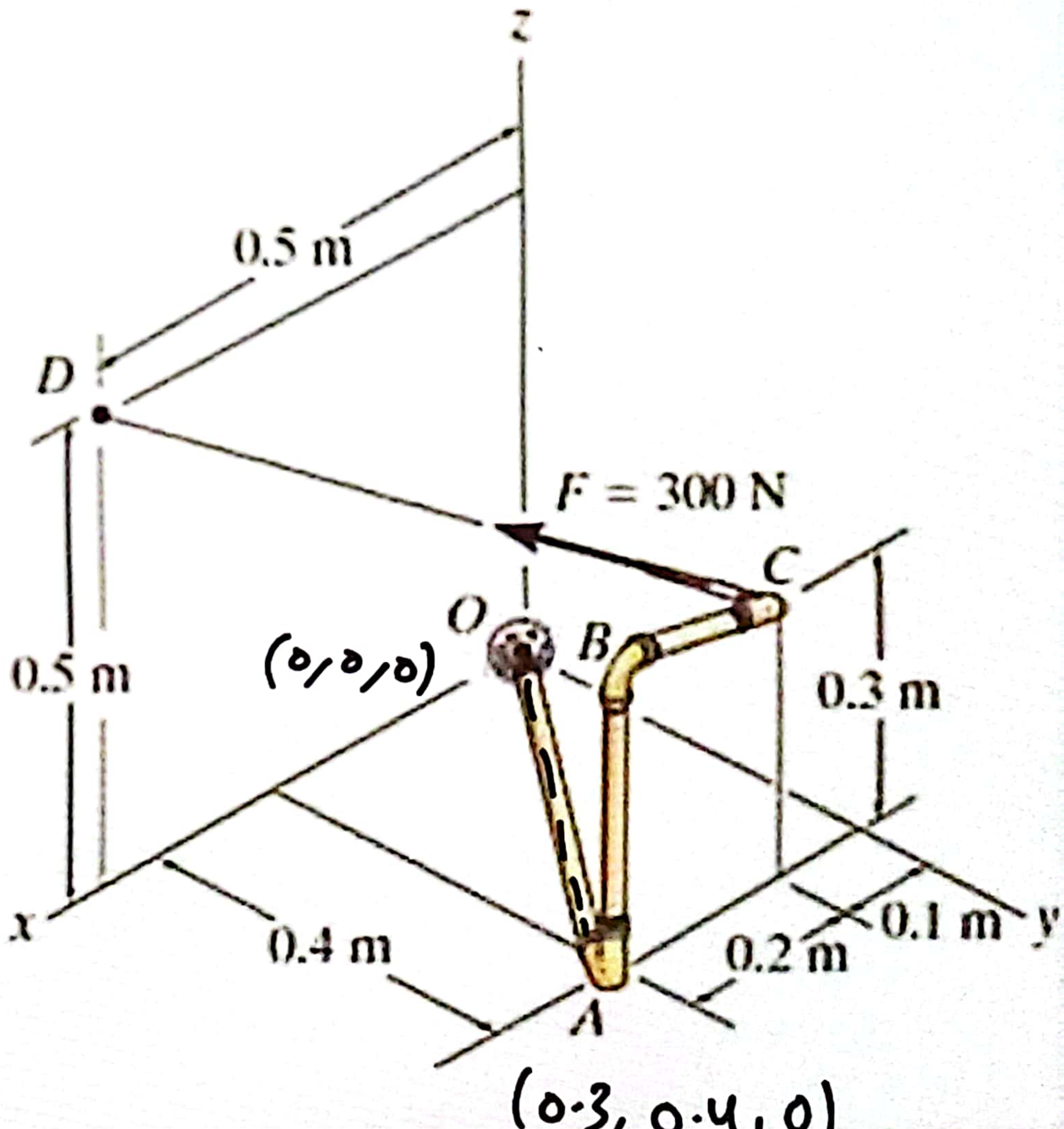
* $\lambda_{OA} = 0.6\hat{i} + 0.8\hat{j} + 0\hat{k}$

$M = r \times F$

$0.6\,\hat{i}$	$0.8\,\hat{j}$	$0\,\hat{k}$
0.5	0.5	0.5
180	-180	100

86 N.m

Final answer

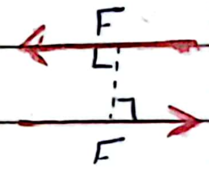


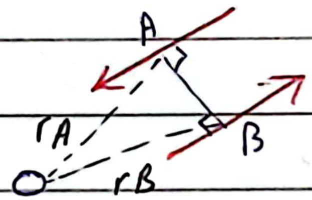
+ there is an exercises on PDF File.

⊛ moment of a couple.

⊛ a couple is defined as two parallel forces (have same magnitude but opposite direction and peripendecular distance)

قوتان متوازيتان متساويتان في المقدار
ومتعاكستان في الاتجاه
المسافة عمودية فيما بينهم

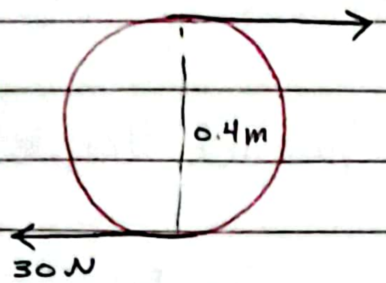




$$M = r_B \times F + r_A \times -F = (r_B - r_A) \times -F$$

$$\boxed{M = r \times F}$$

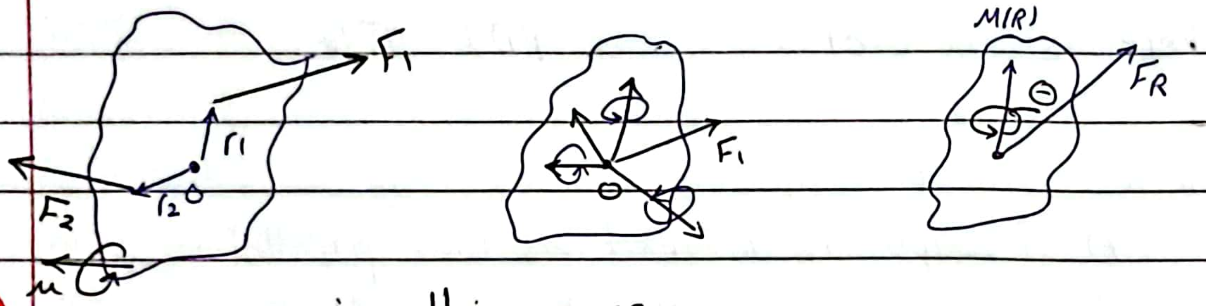
N O T E B O O K



two couples produce a moment with same magnitude and direction

قوة متساوية لإنشاء دوران

⊛ simplification of resultant force of couple moment



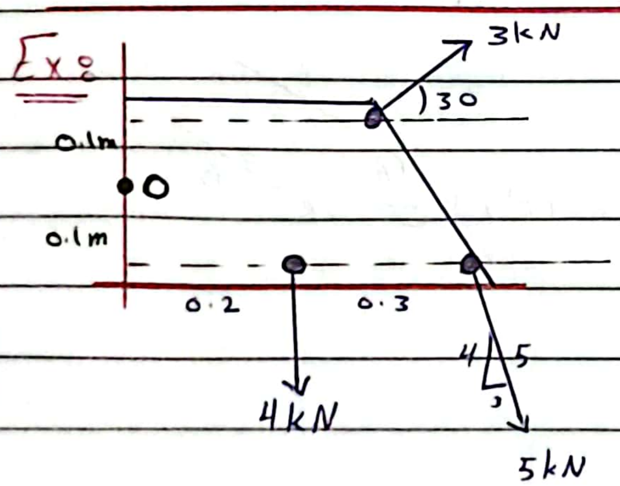
in this case :

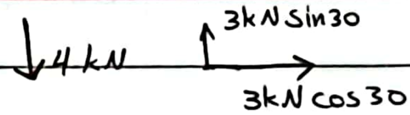
$F_{Rx} = \sum \vec{F_x}$ $\qquad$ $\vec{M_{Rx}} = \sum M_{ax} + \sum M_x$

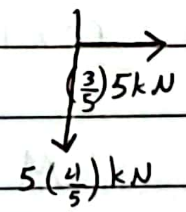
$F_{Ry} = \sum \vec{F_y}$ $\qquad$ $\vec{M_{Ry}} = \sum M_{ay} + \sum M_y$

$F_{Rz} = \sum \vec{F_z}$ $\qquad$ $\vec{M_{Rz}} = \sum M_{az} + \sum M_z$

Ex:







$\sum F_x = 5.598\ kN \rightarrow$

$\sum F_y = 6.5\ kN \downarrow$

$F_R = 8.58\ kN$

$\theta = 49.3$

Subject Date No.

$$M_R = (3kN)\sin 30\,(0.2) - (3kN)\cos 30\,(0.1) + \left(\tfrac{3}{5}\,5\right)(0.1) - \left(\tfrac{4}{5}\cdot 5\right)(0.5) - (4kN)(0.2) = -2.46\ kN.m$$

another example in pdf File

* concurrent Force systems: نظام القوة المتوازنة

* its line of action of all force about point and it didn't produce a moment in this point النقطة

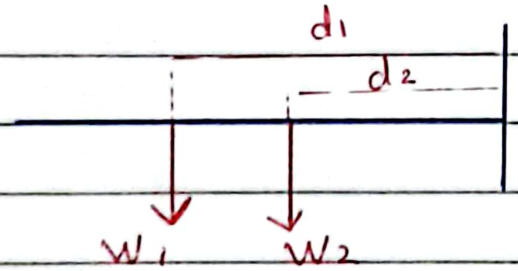
نقطة تتقاطع فيها جميع القوى ولكنها لا تنتج مومنت حول هذه

* coplanar Force: القوة المستوية

The resultant moment can be replaced by moving the resultant force or moment arm with distance D

$$D = \frac{M_R}{F_R}$$

Ex:



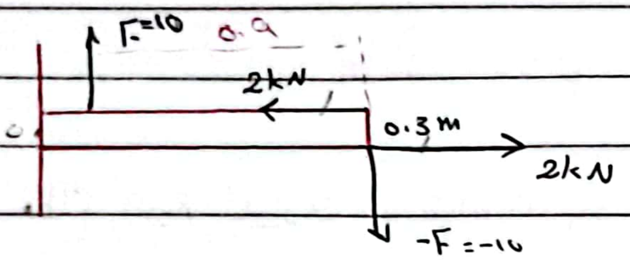
Find the equivalent force system at point O

$\Sigma F = F_{w1} + F_{w2}\ (-\hat{j})$

$M_R = (F_{w1}\, d_2 + F_{w1}\, d_1)\ (-\hat{k})$

N O T E B O O K

Ex: replace the force and couple moment acting on the beam about point O

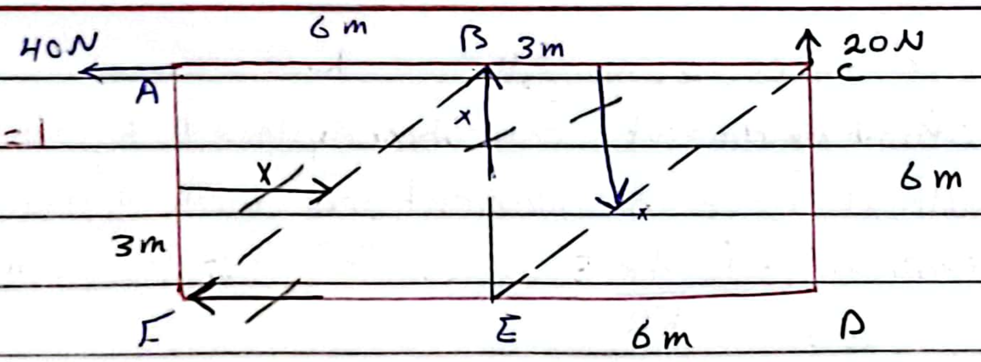


$\Sigma F = zero$

$M = {}^{+}2(0.3) + 10^{-}(0.9) + 2 * (0.3)$

$M = -7.8$

Ex:
For x=1

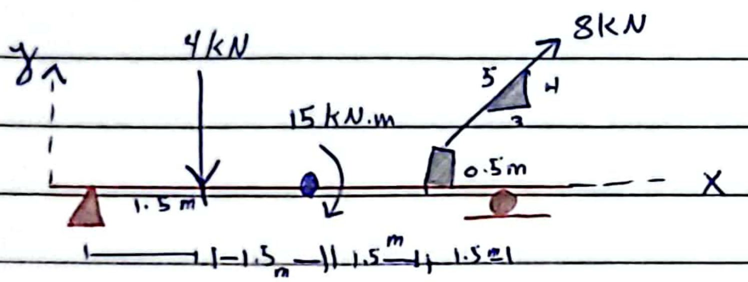


about point E. $\Sigma F = +20\hat{j} - 40\hat{i}$

$M = (40 * 6) + 20 * 6 - (1 * 3) - 3 * 1 = 354 \ N.m$

Subject Date No.

Ex: replace the force and couple moment system acting on the beam in figure with equivalent resultant force and find where its line of action intersects the beam from point O.



15 it is free moment it is not a force

① نحسب محصلات القوى على (x,y)

① $\Sigma F_x = (8kN\left(\frac{3}{5}\right) = \frac{24}{5} = 4.8\ kN.\ \hat{i}$

$\Sigma F_y = -4kN + 8kN\left(\frac{4}{5}\right) - 2.4\ k.N\ \hat{j}$

② $F_R = \sqrt{(4.8)^2 + (2.4)^2} = 5.3\ k.N$ ② الزاوية المحصلة

$\theta = \tan^{-1}\left(\frac{2.4}{4.8}\right) = 26.5$

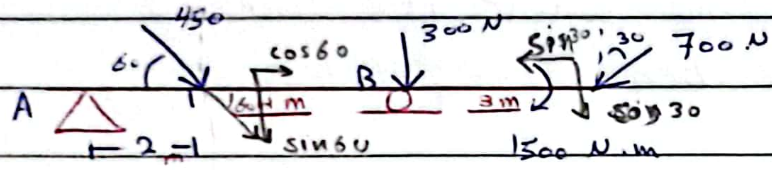
③ $M_x = (4.8kN * (0.5)m = +2.4\ kN.m$ ③ موقف على كل محور

$M_y = (-4kn.1.3) + (8kN * 4.5)m$

$= +30)kN.m$

$\Sigma M = 2.4 + 30 + 15 = 47.4\ kN.m$ ④ Mعلمه

Ex: Find resultant force and couple moment.



$\Sigma F_x = (450 \cos 60) - 700 \sin 30 =$

$\Sigma F_y = (-450 \sin 60) - 700 \sin 30 - 300 =$

$F_R = \sqrt{(F_x)^2 + (F_y)^2} =$

$M = M_x + M_y + M_F$

↳ Ther is no moment المحور (x) مع النقطة فهي صفر

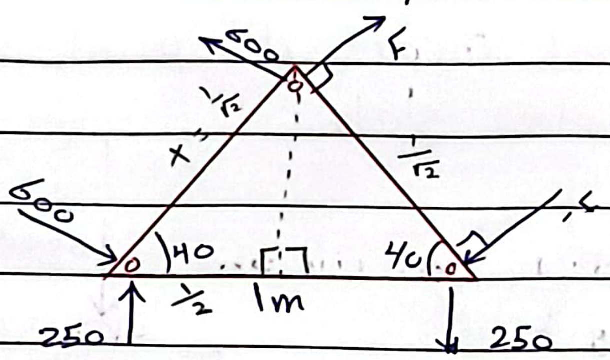
$M_y = (-450 \sin 60 * 2) - (300 * 6) - (700 \cos 30) =$

نجمع القوى

ثم نجمع ((Free moment)

Subject simplification of a resultant Force Date No.

Ex: The end of the triangle plate are subjected to three couples Determine the magnitude of the force (F) so that the resultant couple force 400 colock wise



* $\cos 40 = \dfrac{\frac{1}{2}}{x} \Longrightarrow x = \dfrac{1}{2\cos 40} = \left(\dfrac{1}{\sqrt{2}}\right)$

* The resultant moment (clock wise) عالى

$-400 = +600\left(\frac{1}{\sqrt{2}}\right) - (250)(1) - F\left(\frac{1}{\sqrt{2}}\right)$

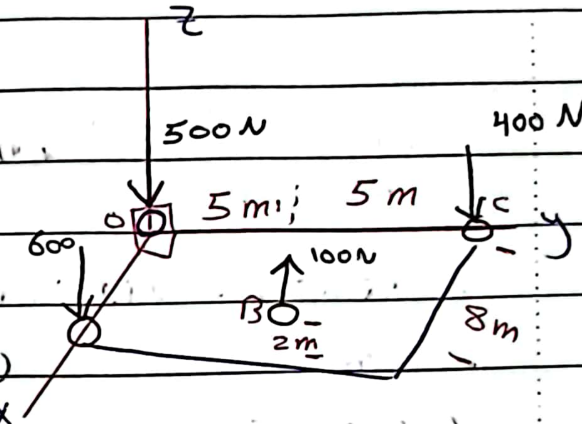
$-400 = 424.2 - 250 - F/\sqrt{2}$

$-400 = 174.2 - F/\sqrt{2}$

$-574 = -F\sqrt{2} \longrightarrow F = 405.8$

Subject Date No.

Ex: The slap system in the figure below is subjected to four parallel forces Determine the magnitude and direction of the resultant force system and locate its point of application on the ~~slap~~ slab:



The resultant force is determine by

$-500 - 400 + 100 - 600 = -1400\ N$

(A) moment $M_x = -400(10) + 0 + 0 + 100(5) = 3500\ N.m$

$M_y = +600(8) - 100(6) = 4200\ N.m$

موازيين للمحور $\leftarrow M_z = zero$

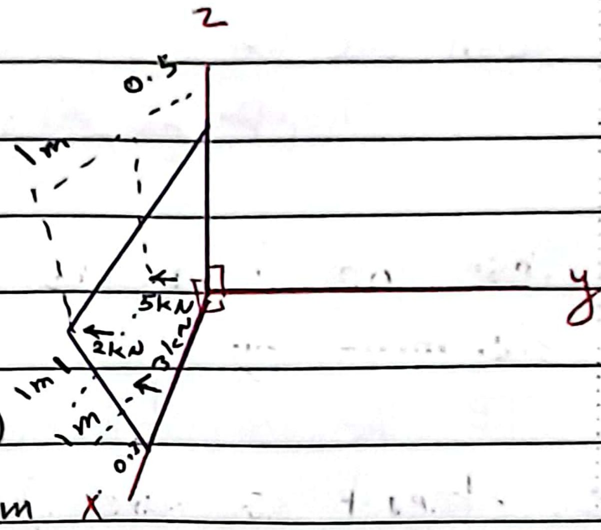
(*) $M = F \cdot d$

$d = \frac{M}{F}$

2D system $\begin{cases} d_x = \frac{M_y}{F_z} = 2.5\ \hat{i} \\ d_y = \frac{M_x}{F_z} = 3\ m\ \hat{j} \end{cases}$

Subject Date No.

Ex 8 replace the parallel force system acting on the plate by a resultant force and specify its location on x-z plane:



The resultant force determine by: $-5-2-3=-10\ kN$

$M_x = (5(1.5)) + (2(2.5)) + 3(0.5)$
$= {}^{+}14\ kN.m$

$M_y = zero$

$M_z = (3(1.5) + 5(0.5) + 2(1.5)) = 10\ kN\ m$ clock wise

$d_x = \frac{10}{10} = 1$ $\quad d_z = \frac{14}{10} = 1.4$

* particle mass with negligible size (single point) allows maximum simplification when shape and size don't matter

عبارة عن جسم مهمل الكتلة يمكن أن يعبر عنه بنقطة
عندما يكون الشكل والحجم غير مهم

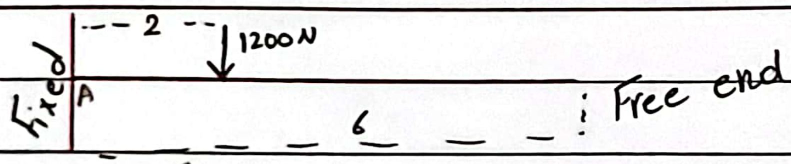
* The object in equilibrium:

1. If the acceleration of an object zero the object is said to be in equilibrium
$\Rightarrow$ The net force on the object zero

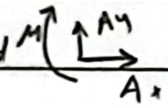
$$\Sigma F = \text{zero}$$

* $\Sigma F_{x,y,z} = \text{zero}$ $\qquad \Sigma M_{x,y,z} = \text{zero}$

Ex:



* Fixed

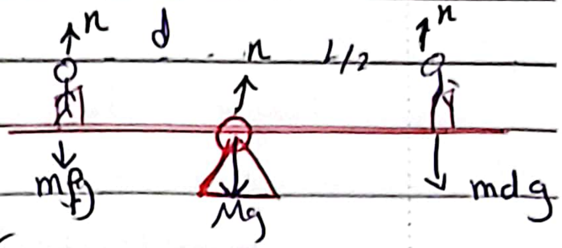


* $\Sigma F = 0$ $\quad \Sigma F_x = A_x = 0$

$\Sigma F_y = F_y - 1200 \longrightarrow A_y = 1200$

$\Sigma F_z = \text{zero}$

$M_z = -2400 \text{ N.m}$



Ex8 a sesaw consisting of a uniform board of mass M and length L support a father and daughter with mass mf and md respectivly and distanc d and L/2 from the center Determine the magnitude of the upward force n exerted by the support on the board.

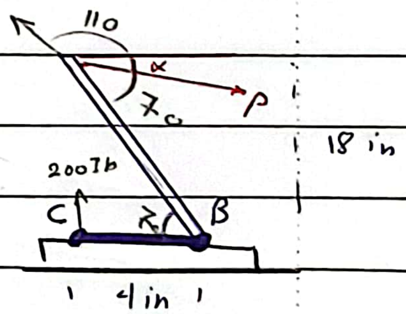
$\Sigma F_y = zero$ $\quad$ $n - m_f g - m_d g - Mg = zero$

$n = m_f g + m_d g + Mg$

$\Sigma F_x = zero$ $\quad$ * there is no forces on x →

Ex9 It is known that a vertical force of 200 Ib is required to move nail at C from the board As the nail start moving determine:

1. MB of the force on nail
2. The magnitude of the force P at α = 10 with same M
3. smallest force which creat the same moment



1. $M = rF = 200 * 4 = 800$ Ib.inch clock
2. $M = rF \sin\Theta \Rightarrow 800 = 18\ F \sin 110$

$F = 47.2$

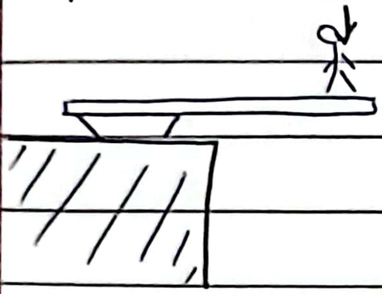
3. at $\Theta = 90$ $\quad$ $800 = F\ 18 \sin 90$

$F = 44.4$

we have two type of Forces

* concentrated force (point load يجمع بنقطة)

* Distributed Force (line load or pressure)

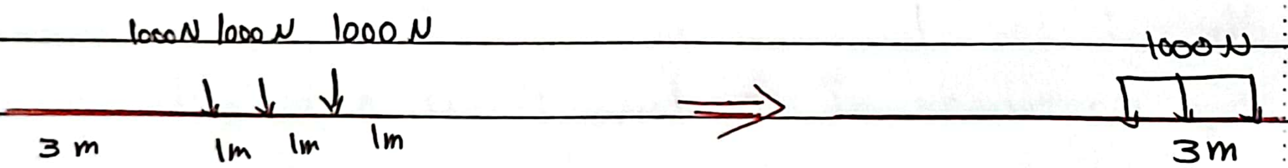


when we talk about distributed loading we talk about pressure $P = \frac{F}{A}$

$1\ Pa = 1\ N/m^2$

* large surface area of body may be subjected to distributed loading

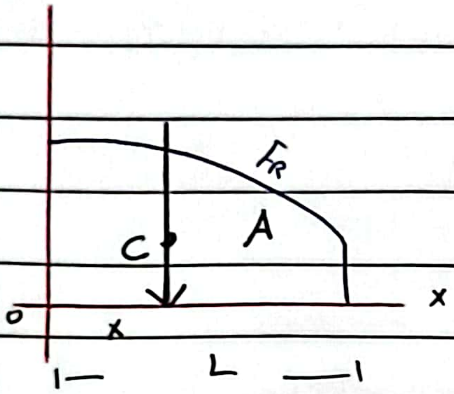
في الأجسام الكبيرة والثقيلة يكون الحمل موزع بانتظام (بالضغط) فما نعرف



Subject: magnitude of the resultant Force

① magnitude of the resultant force is equal to the total area under loading diagram

$$F_R = \int_A^B w(x)\,dx = \int_A^B \text{area of the object}$$



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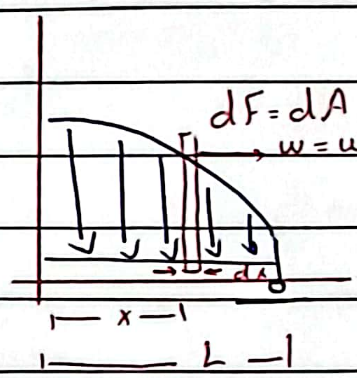
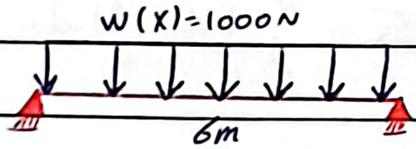


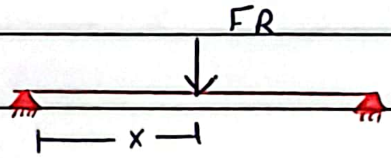
② location of resultant force

$$\bar{X} F_R = \frac{\int_A^B x\,w(x)}{F_R}$$

Ex:



$\longleftrightarrow$

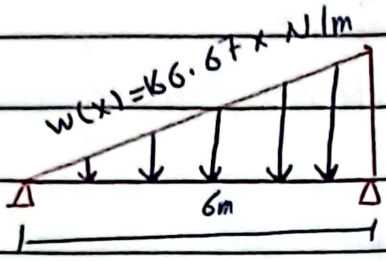


$$F_R = \int_0^6 1000\,N = 1000x\Big]_0^6 = 1000(6-0) = 6000\,N$$

$$\bar{X} = \frac{\int_0^6 1000x}{6000} = \frac{1000 \frac{x^2}{2}\Big]_0^6}{6000} = 3m$$

Subject Date No.

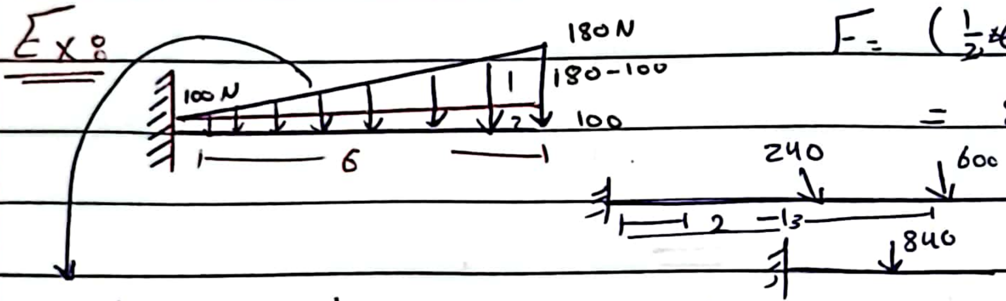
Ex:



$$F = \int_0^6 166.67x \, dx = \frac{166.67(6^2)}{2} =$$

$$\bar{x} = \frac{\int_0^6 166.67x^2 \, dx}{\int_0^6 166.67x \, dx} =$$

Ex:



$$F = \left(\tfrac{1}{2} \times 6^3 \times 80\right) + (100 \times 6)$$

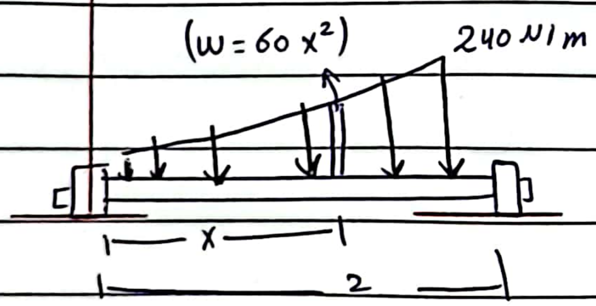
$$= 840$$

location $6 \times \frac{1}{3} = 2$

~~M = 2 × 240 = 480~~

~~M = 3 × 600 = 1800~~

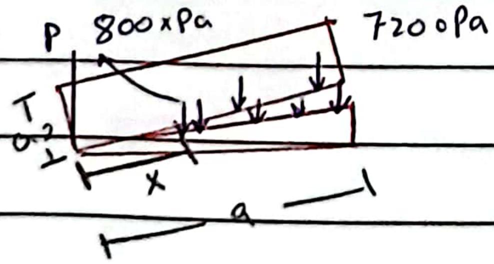
Ex:



$$F_R = \int_0^2 60x^2 = \frac{1}{3} 60 x^3 \Big]_0^2$$

$$\text{location } \bar{x} = \frac{\int_0^2 (60x^3) \, dx}{\int_0^2 60x^2 \, dx} =$$

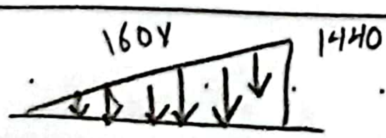
Ex:



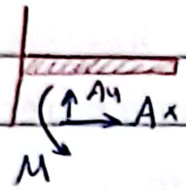
⊛ يفضل الرجوع إلى الركة لسلة بيان

ونحولها إلى (1D) عن طريقة ضرب

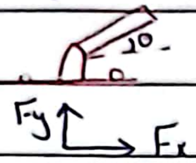
(w(x) ⟵ (السلاح)



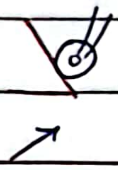
Subject support reaction Date No.



⊛ Fixed support with free end



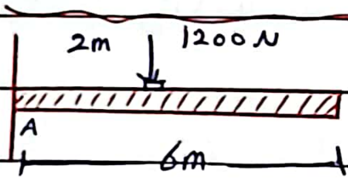
⊛ smooth pin or hinge



⊛ roller (Just normal force) (vertic.

⊛ There is 3 Tables in PDF File

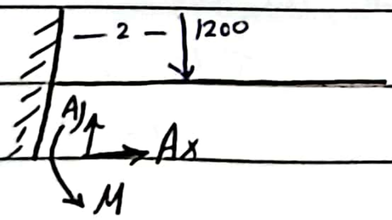
⊛ Equilibrium of rigid body:



⊛ Determine the horizontal vertical and moment reaction components on the beam:

⊛ we have fixed suppor with free end so we have 3 reactions

2D system



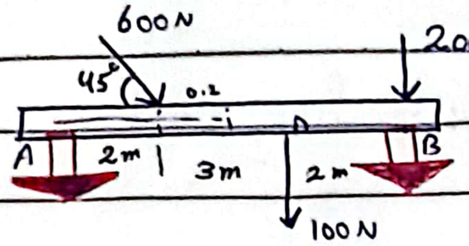
$\Sigma F_x = 0$ $\quad$ $\Sigma F_y = 0$ $\quad$ $\Sigma F_z = 0$

$A_x - 0$ $\quad$ $A_y - 1200 = 0$ $\quad$ $M_z - 1200(2) = 0$

$A_x = 0$ $\quad$ $A_y = 1200$ $\quad$ $M_z = 2400$

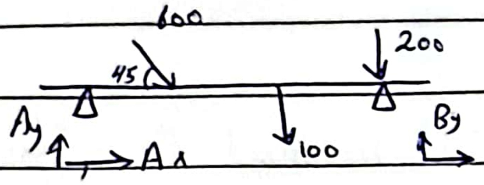
Subject Date No.

Ex:



Determine the horizontal vertical and moment reaction on the beam:

(*) we have two hinge



(*) $\Sigma F_x = 0$ $\quad +\uparrow \Sigma F_y = 0$ $\quad \circlearrowright \Sigma M = 0$

mathmetic calculation ←

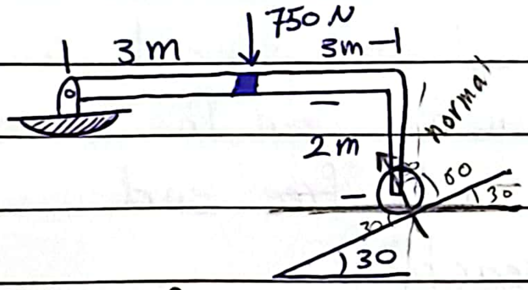
$\Sigma F_x = 0 \quad B_x + 600\cos(45) = 0$

$\Sigma F_y = 0 \quad A_y - 600\sin(45) - 200 - 100 + B_y = 0$

$\Sigma M = 0$

$100(2) + 600\sin(45)(2) - A_y(5) = 0$

Ex:



Determine the horizontal and vertical components of reaction on the member at A and B

(*) reaction for pin A_x A_y (*) reaction for roller $\uparrow$ (n)

(*) $\Sigma F_x = 0$ $\qquad \Sigma F_y = 0$ $\qquad \Sigma M = 0$

$A_x = B\sin 30$ $\qquad A_y + B\cos 30 - 750 = 0$

$-750(3) + B\sin(30)(2) + B\cos(30)(0) = 0$

* Truss system: structure composed of slender members joined togther at their ends points

* assumption for designs: (التصميم الإفتراضي)

  * we need first to determine the force developed in each member يجب أولاً تحديد القوى المؤثرة

  * To do this we make:

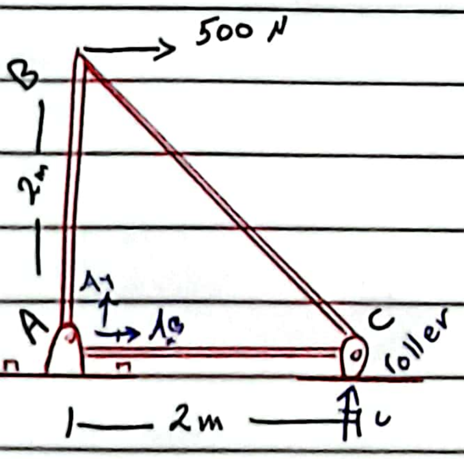
    ربط فيما بينهم 1. member joined by smooth pins

    تطبيق جميع القوى 2. all loading applied at joints

* $\Sigma F_x = zero$ $\Sigma F_y = zero$

* analysis method. (method of joints)

Ex: نختار نقطة تحتوي على أقل عدد من (unknowns)

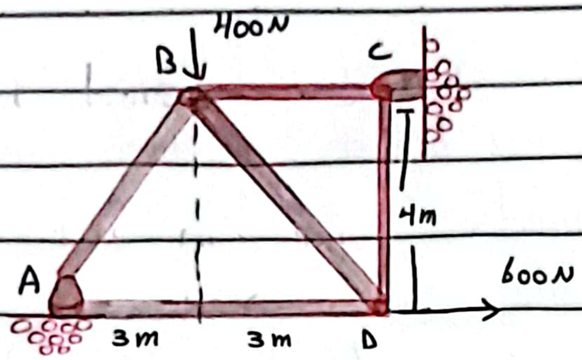


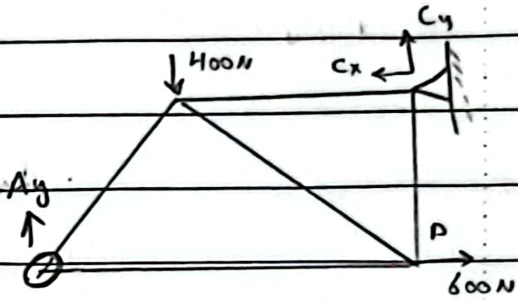
$\Sigma F_x = 0$

$\Sigma F_y = 0$

Subject Date No.

Ex 8





① $\Sigma F_x = 0$ $C_x + 600 = \text{zero}$

$\Sigma F_y = 0$ $C_y + A_y - 400 = 0$

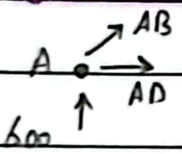
$\circlearrowleft \Sigma M = 0$ $A_y(6) + 600(4) + 400(3) = 0$

② $A_y = 600$ ④ $C_y = -200$ ③ $C_x = -600$

① we first get the reactions

② second we determine the method joint.

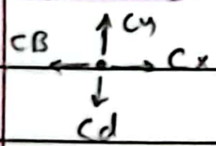
Joint A



$\Sigma F_x = \text{zero}$ $AB(\frac{3}{5}) + AD = 0$ $\rightarrow$ $AD = 450$

$\Sigma F_y = \text{zero}$ $AB(\frac{4}{5}) + 600 = 0$ $\rightarrow$ $AB = -750$

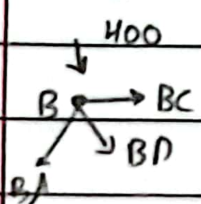
Joint C



$\Sigma F_x = 0$ $C_x - CB = \text{zero}$ $CB = -600$

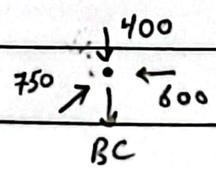
$\Sigma F_y = 0$ $-200 - CD = 0$ $CD = -200$

Joint B



$\Rightarrow$ BC = 250

point B



$\Sigma F_x = -600 + 750(\frac{3}{5}) + BC(\frac{3}{5}) = 0$

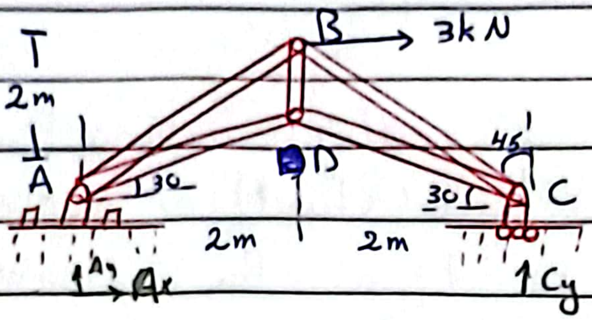
Ex:

(*) First we find the reactions:

the simplest step

$\circlearrowleft + \sum M = 0$

we choose point (C)

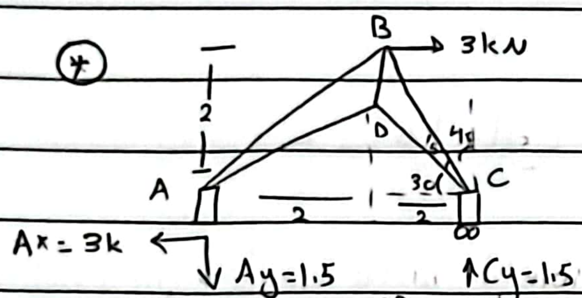


at point C $\circlearrowleft + \sum M = 0$

$-Ay(4) - 3(2) = 0 \Rightarrow Ay = -\frac{6}{4} = -1.5\ N$

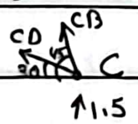
so $\Rightarrow$ $Cy = +1.5\ N$

(*)



Now we need to find method of Joints

Joint C



$\sum Fx = zero$

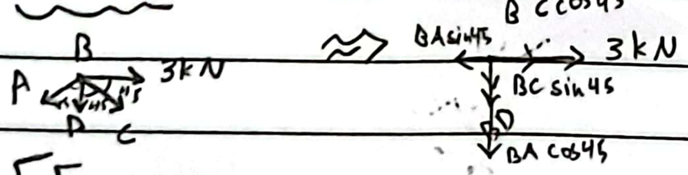
$-CD\cos 30 + CB\cos 45 = 0 \Rightarrow \frac{CB}{\sqrt{2}} = -CD\cos 30$ $\quad CB = +5.02\ kN$ (T)

$\sum Fy = zero$

$1.5 + CD\sin 30 + CB\sin 45 = 0 \Rightarrow 1.5 + CD\sin 30 - CD\cos 30 = 0$

$\therefore CD = 4.1\ kN$ (T)

Joint B:



$\rightarrow \sum Fx = zero$

$BC\cos 45 + 3 - BA\sin 45 = 0 \Rightarrow -5.02\cos 45 + 3 - (BA)\sin 45 = 0$

$\sum Fy = zero$ $\quad \boxed{BA = -0.77}$ (C)

$BC\sin 45 + BD + BA\cos 45 = 0 \Rightarrow BD = +4.09$ (T)

calculate DA it will be (T)

Subject zero Force element Date No.

its an element has a zero Force

(*) Case 1

If only two non-collinear members form a truss joints and no external forces (load) or support reaction is applied to the joint, the two members must be zero-force members.

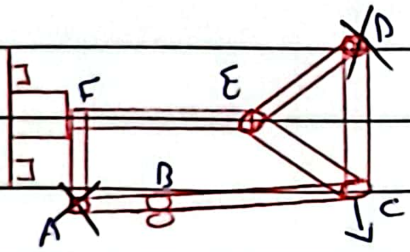
(*) إذا كان عندي (joint) ①: بيضم زاوية

② ضلعين غير متوازيين

③ لا يوجد عليهم قوة أو حمل

⇐ عندها نلغي هنا (joint)

(*)



D & A => (joint) with angle non collinear an with no force

we can't put out point (c) becouse there is an external load

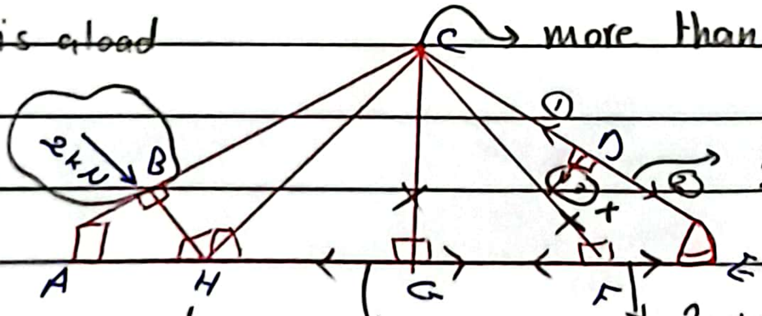
(*) case 2

Three members (2 in same line)

The third line will be zero

Ex: ~~there is a load~~

C → more than 3 member



2 in same line

2 in same line

more than 3 members

2 in same line

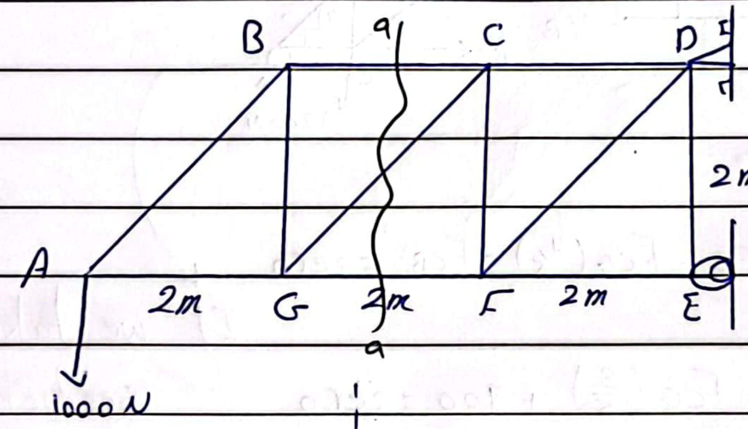
Kareem NOTEBOOK

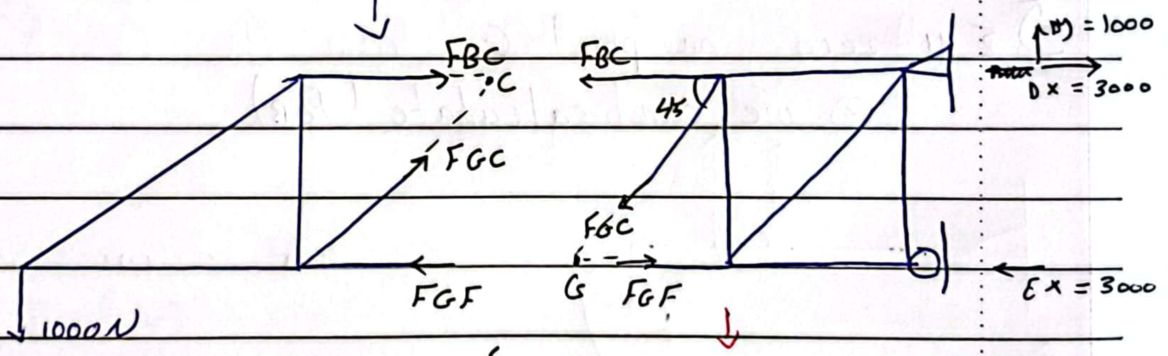
⊛ method of sections:

when we need to find a force on specific axis

we use method of sections, the we use equations of equilibrium

⊛ عندما نريد تحديد قوى (load) معين نستخدم قطع محدد ثم نطبق معادلات الإتزان





$\Sigma F_x = zero \quad -D_x - E_x = 0 \quad D_x = -3000$

$\Sigma F_y = zero \quad -1000 + D_y = 0 \quad D_y = 1000$

$\Sigma M = zero \quad 1000(6) - E(x)2 = 0$

$\boxed{E_x = 3000\,N}$

If we take this section we get

$\Sigma F_x = 0 \rightarrow -F_{CB} - F_{CG}\cos(45) - F_{FG} = 0$

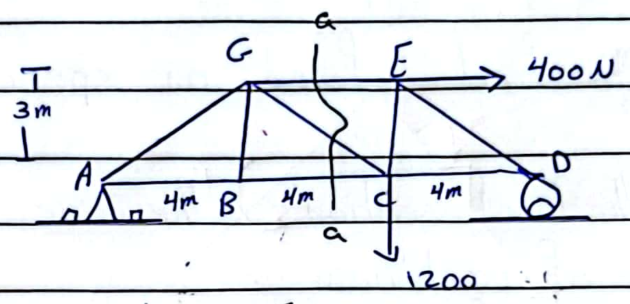
$\Sigma F_y = 0 \rightarrow -F_{CG}\sin 45 + 1000 = zero \quad \boxed{F_{CG} = \frac{1000}{\sin 45}}$

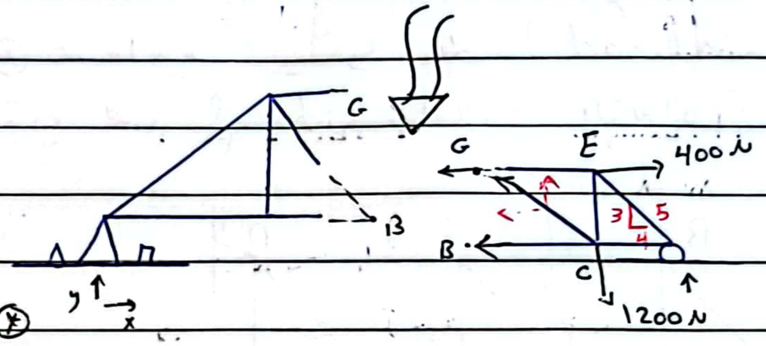
$\Sigma M = zero$ on G

$F_{CB} = 1000\,N\ (T)$

Subject Date No.

Ex: Determine The Force in membere GE, GC, BC





If we take this section

$\Sigma F_x = zero$

$\xrightarrow{+}$ $400 - F_{EG} - F_{CG}(4/5) - F_{CB} = zero$

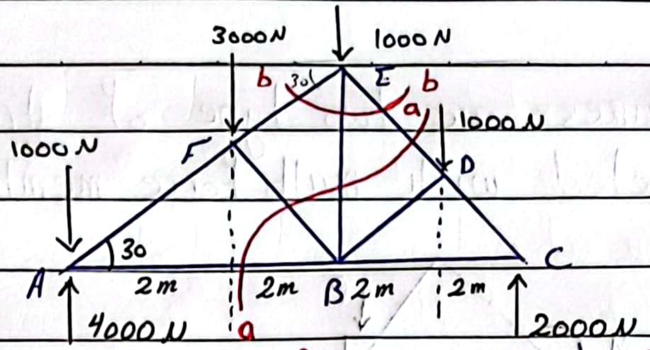
$+\uparrow$ $\Sigma F_y = zero$

$-1200 + F_{CG}\left(\frac{3}{5}\right) + 900 = zero$

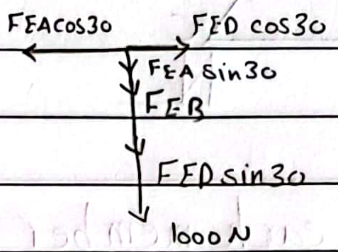
$+\circlearrowleft$ $\Sigma M = zero$ on point G

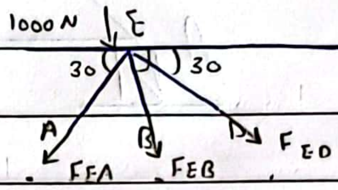
* we can calculate (F_{CB})

Ex 8



⊛ Determine the force on EB of the roof truss

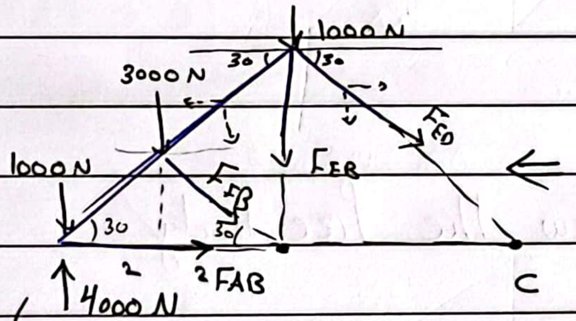


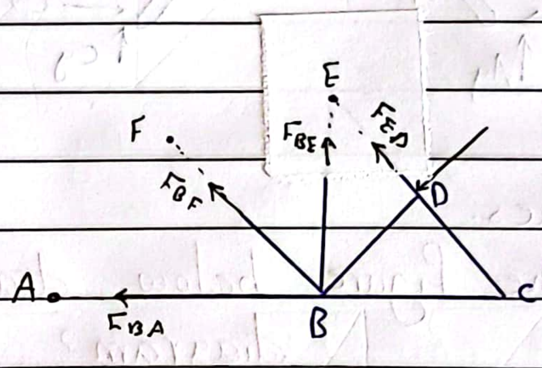


⊛ $\Sigma F_x = zero \Rightarrow (F_{ED}\cos 30 - F_{EA}\cos 30) = zero$ $\boxed{F_{EF} = -1500\ C}$

⊛ $\Sigma F_y = zero \Rightarrow -(F_{EA}\sin 30 + F_{EB} + F_{ED}\sin 30 + 1000) = zero$ $\boxed{F_{EB} = 500}$

⊛ $\Sigma M = zero \Rightarrow$





If we take this section.

$\Sigma F_x = zero$

$F_{ED}\cos 30 + F_{FB}\left(\frac{2}{12.9}\right) + F_{AB} = 0$ $\boxed{F_{AB} = 1428.2\ T}$

$\Sigma F_y = zero$

$-1000 - 3000 - 1000 + 4000 - F_{EB} - F_{FB}\left(\frac{12.7}{12.9}\right) - F_{EB}\sin 30 = 0$

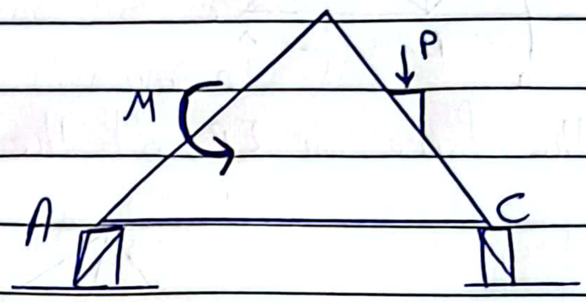
$\Sigma M = zero$ $\boxed{F_{FB} = -833.3\ C}$

$-F_{ED}(4) + 3000(2) + 1000(4) - 4000(4) = 0$

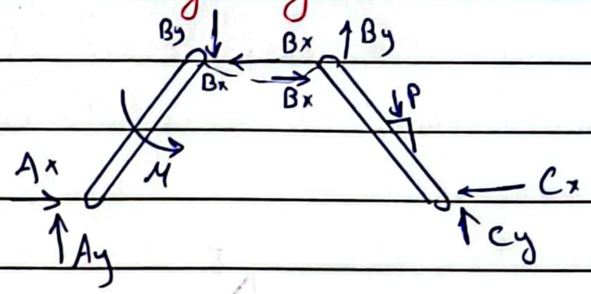
$-F_{ED}(4) = 6000 \Rightarrow \boxed{F_{ED} = -1500\ C}$

Subject Frames   Date   No.

Frames and machines are two type of structure which connected with multi force membere



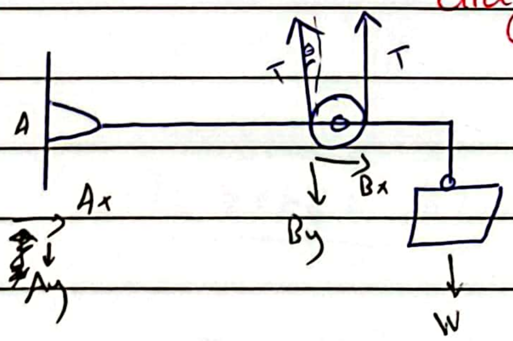
⊛ Free body diagram of each member:



صورة 3 ملف (PDF)

⊛ Frames:

in the figure below draw the free body diagram?



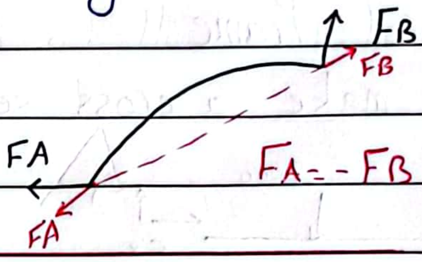
we take a cross section

$\Sigma F_x = zero \quad A_x + B_x$

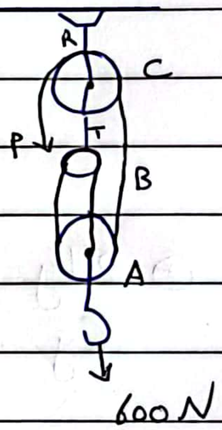
$\Sigma F_y = zero \quad - A_y - B_y - w = zero$

⊛ remark:

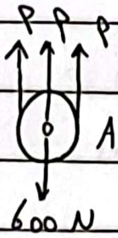
If we have two forces on members that regardless of their shape they have equal but opposite collinear forces acting on their ends



Ex: Determine the tension in the cables and also the force P to support the 600 N force using the frictionless pulley shown in figure below



Sol

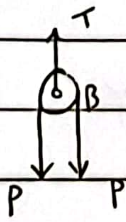


$\Sigma F_y = zero$

$3P = 600$

$\boxed{P = 200}$

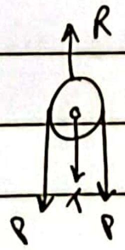
⊛



$\Sigma F_y = zero$

$\boxed{T = 400}$

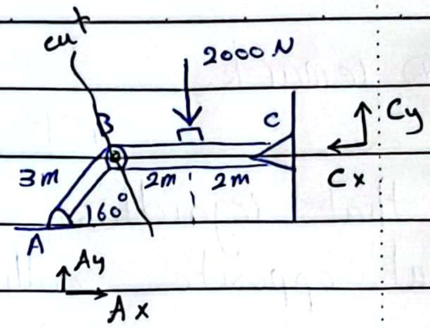
⊛



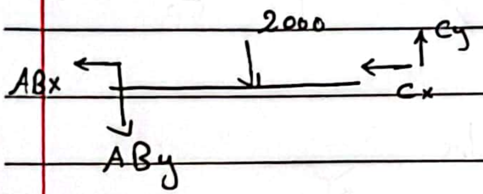
$\Sigma F_y = zero$

$\boxed{R = 800}$

Ex: Determine the horizontal and vertical components of force which the pin at C exerts on member BC of the Frame:



we make a cross section on B



⊛ $\sum M = zero$

$\boxed{Cy = 1000}$

⊛ $\sum Fx = zero$

$-Cx - ABx = zero \Rightarrow -Cx - 577.3 = 0$

نعكس اتجاه CX $\boxed{Cx = -577.3}$

⊛ $\sum Fy = zero$

$Cy - 2000 - ABy = zero$

$\boxed{ABy = -1000}$

نعكس اتجاه ABy

نصبح ↑

⊛ $FAB \sin 60 = FABy$

$\boxed{FAB = 1154.7}$

$FAB \cos 60 = FABx$

$\boxed{FABx = 577.3}$

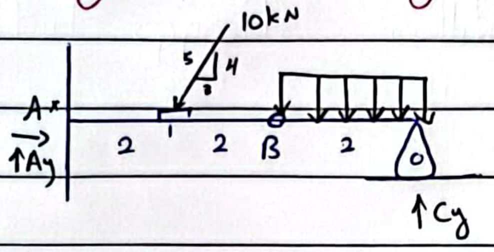
⊛ $Ay = 2000 - Cy$

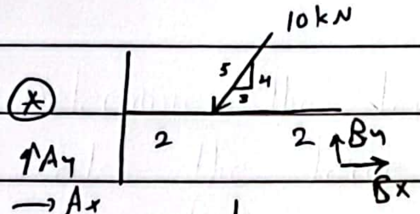
$\boxed{Ay = 1000}$

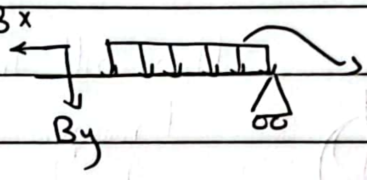
⊛ $Ax = Cx$ نعكس اتجاه

Subject Date No.

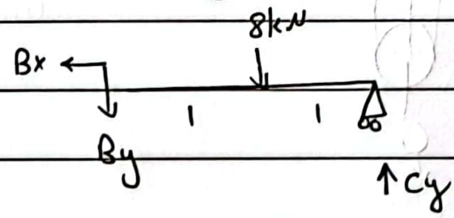
Ex: The compound beam shown in Figure is pin connected at B. Determine the components of reactions. Neglect the weight and thickness







value $= \int_0^2 4 \cdot dx = 8$



ΣM on A ? $B_y = \square$

$\Sigma F_y = \text{zero}$ $A_y = \square$

$\Sigma F_x = \text{zero}$ $A_x = \square$

(*) ΣM on B = zero

$C_y(2) - 8(1) = \text{zero}$ $\boxed{C_y = 4\ kN}$

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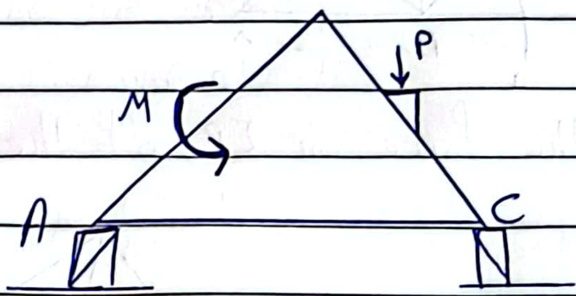
$C_y - B_y - 8 = \text{zero} \rightarrow 4 - 8 - B_y = \text{zero}$

$\boxed{B_y = -4}$ kN نعكس اتجاه By

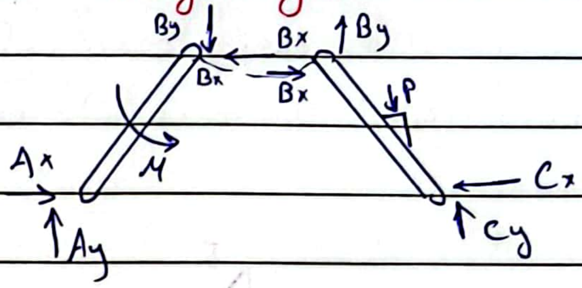
(*) $\Sigma F_x = \text{zer}$

$\boxed{B_x = \text{zero}}$

Frames and machines are two type of structure which connected with multi force membere



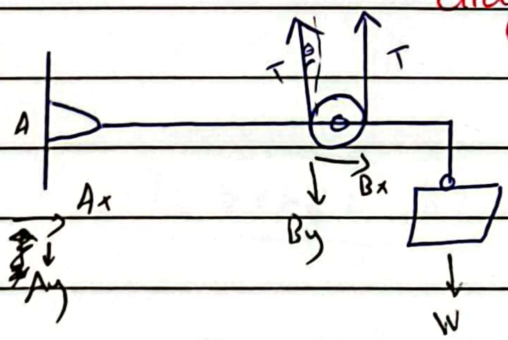
⊛ Free body diagram of each member:



(PDF) مسودة ملف

⊛ Frames:

in the figure below draw the free body diagram?



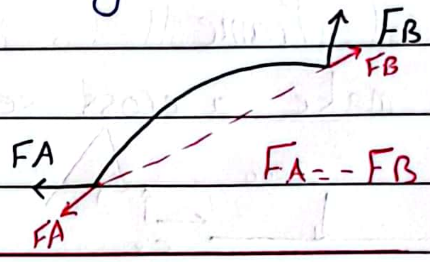
we take a cross section

$\sum F_x = ze \quad A_x + B_x$

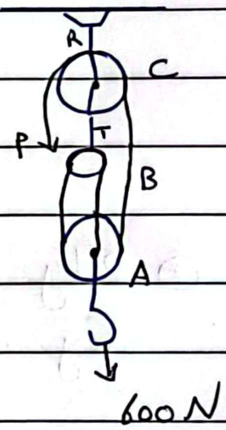
$\sum F_y = zero \quad - A_y - B_y - w = zero$

(*) remark:

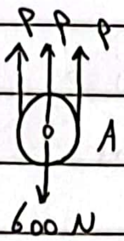
If we have two forces on members that regardless of their shape they have equal but opposite collinear forces acting on their ends



Ex: Determine the tension in the cables and also the force P to support the 600 N force using the frictionless pulley shown in figure below



Sol

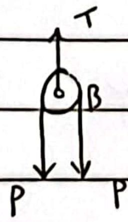


$\Sigma F_y = \text{zero}$

$3P = 600$

$\boxed{P = 200}$

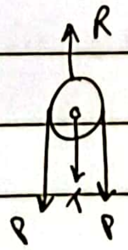
(*)



$\Sigma F_y = \text{zero}$

$\boxed{T = 400}$

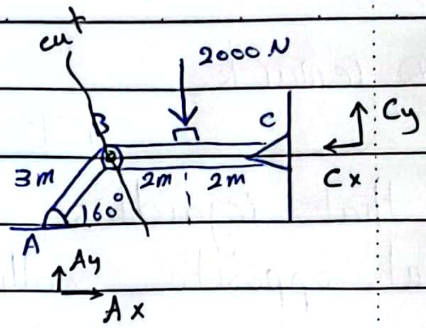
(*)



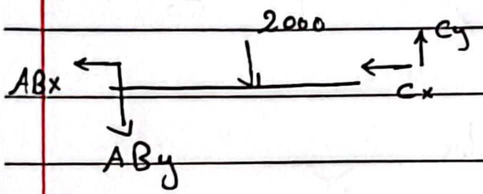
$\Sigma F_y = \text{zero}$

$\boxed{R = 800}$

Ex: Determine the horizontal and vertical components of force which the pin at C exerts on member BC of the Frame:



we make a cross section on B



(*) $\sum M = zero$

$\boxed{Cy = 1000}$

(*) $\sum Fx = zero$

$-Cx - ABx = zero \Rightarrow -Cx - 577.3 = 0$

CX نعكس اتجاه $\boxed{Cx = -577.3}$

(*) $\sum Fy = zero$

$Cy - 2000 - ABy = zero$

$\boxed{ABy = -1000}$ ABy نعكس اتجاه

نصبح ↑

(*) $FAB \sin 60 = FABy$

$\boxed{FAB = 1154.7}$

$FAB \cos 60 = FABx$

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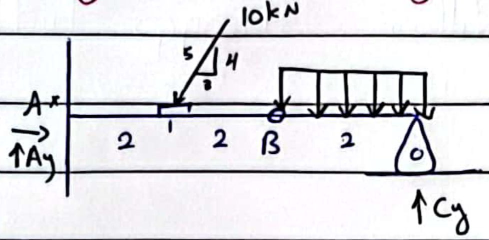
(*) $Ay = 2000 - Cy$

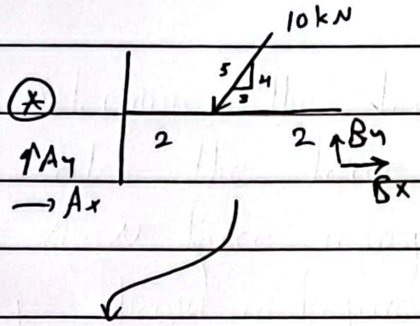
$\boxed{Ay = 1000}$

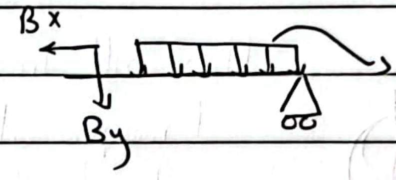
(*) $Ax = Cx$ نعكس اتجاه

Subject Date No.

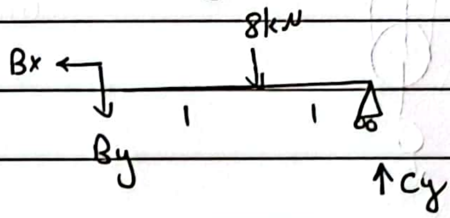
Ex: The compound beam shown in Figure is pin connected at B. Determine the components of reactions. Neglect the weight and thickness







value $= \int_0^2 4 \cdot dx = 8$



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$\boxed{B_y = -4}$ kN عكس اتجاه By

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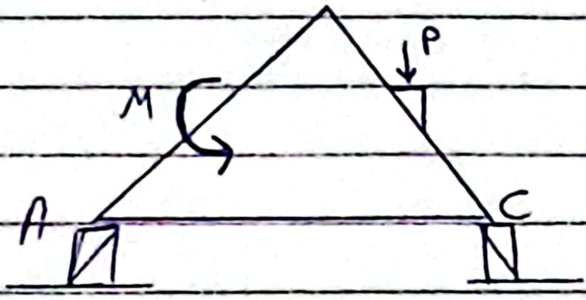
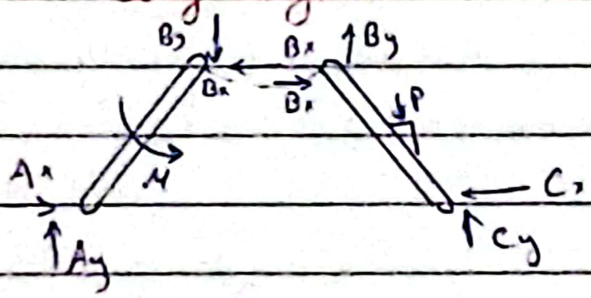
N O T E B O O K

Frames and machines are two type of structure which connected with multi force membere



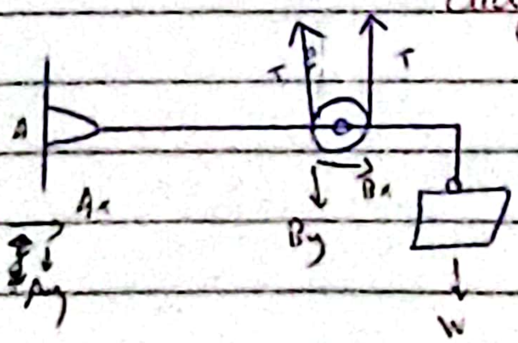
① Free body diagram of each member



(FBD)

② Frames:

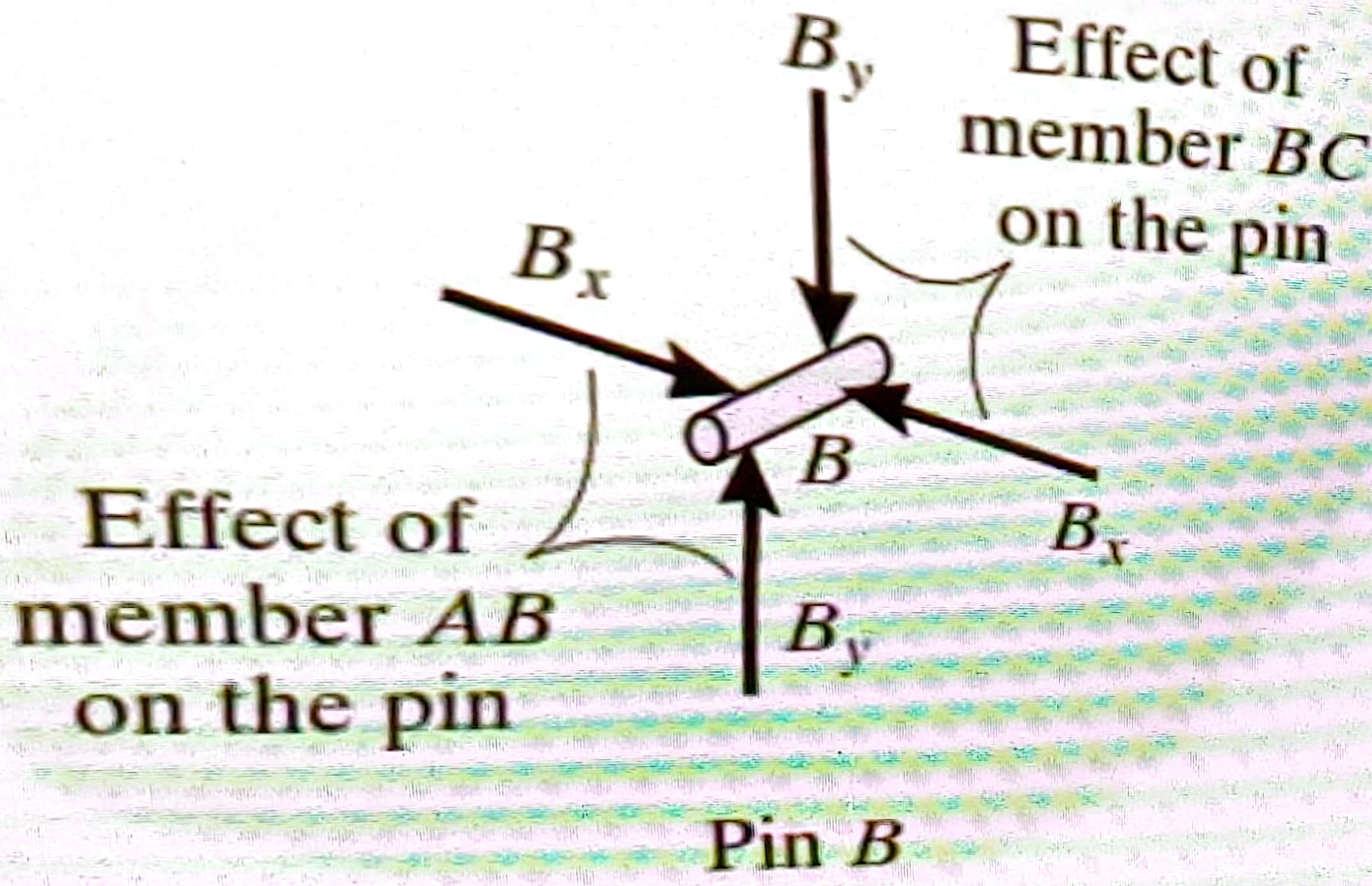
in the figure below draw the free body diagram?



we take a cross section

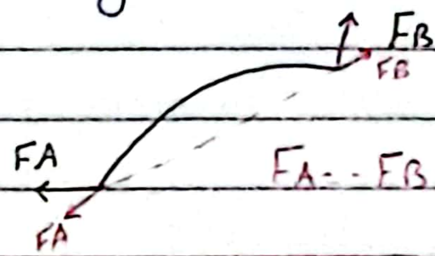
$\Sigma F_x = zero \quad A_x + B_x$

$\Sigma F_y = zero \quad - A_y - B_y - W + T = zero$


B_y
Effect of
member BC
on the pin
B_x
B
Effect of
member AB
on the pin
B_x
B_y
Pin B

⊛ remark

If we have two forces on members that regardless of their shape they have equal but opposite collinear forces acting on their ends



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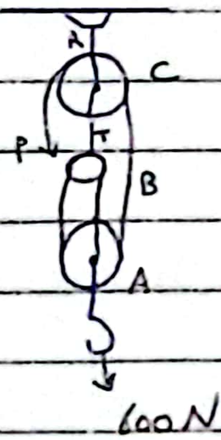
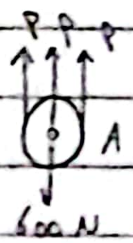


sol



$\Sigma F_y = zero$

$3P = 600$

$\boxed{P = 200}$

⊛

$\Sigma F_y = zero$

$\boxed{T = 400}$

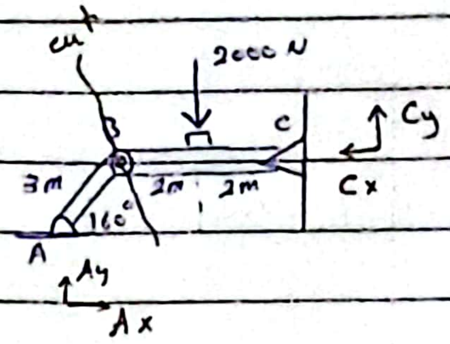
⊛

$\Sigma F_y = zero$

$\boxed{R = 800}$

Subject   Date   No.

Ex: Determine the horizontal and vertical components of force which the pin at C exerts on member BC of the frame:



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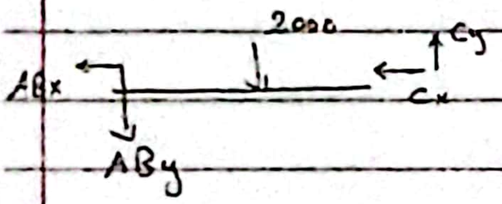


⊛ $\Sigma M = zero$

$\boxed{C_y = 1000}$

⊛ $\Sigma F_x = zero$

$-C_x - AB_x = zero \Rightarrow -C_x - 577.3 = 0$

نعكس اتجاه Cx $\boxed{C_x = -577.3}$

⊛ $\Sigma F_y = zero$

$C_y - 2000 - AB_y = zero$

$\boxed{AB_y = -1000}$ نعكس اتجاه ABy تصبح ↑

⊛ $F_{AB} \sin 60 = F_{ABy}$

$\boxed{F_{AB} = 1154.7}$

$F_{AB} \cos 60 = F_{ABx}$

$\boxed{F_{ABx} = 577.3}$

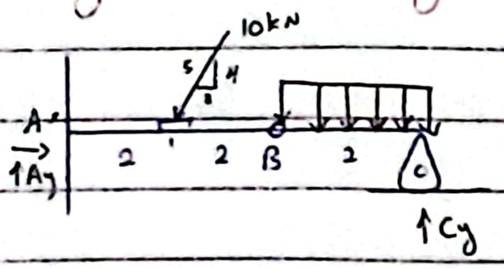
⊛ $A_y = 2000 - C_y$

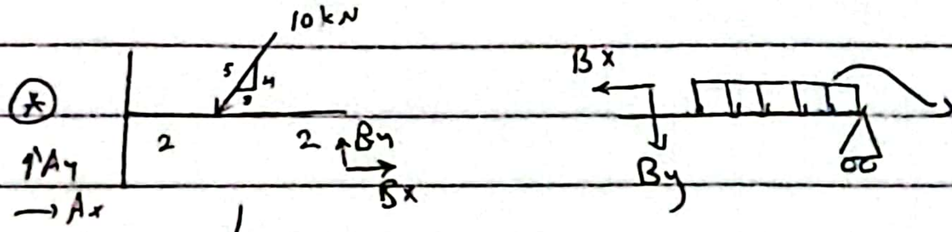
$\boxed{A_y = 1000}$

⊛ $A_x = C_x$ معكس اتجاه

NOTEBOOK

Ex: The compound beam shown in Figure is pin connected at B Determine the components of reactions Neglect the weight and thickness





value = $\int_0^2 4\,dx = 8$

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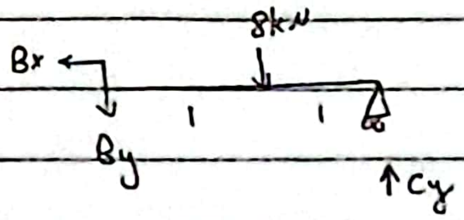

ΣM on B = zero

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$\Sigma F_y = \text{zero}$

$C_y - B_y - 8 = \text{zero} \longrightarrow 4 - 8 - B_y = \text{zero}$

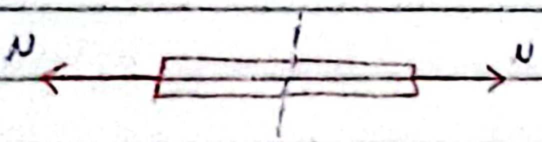
$\boxed{B_y = -4}$ kN عكس اتجاه B_y

$\Sigma F_x = \text{zer}$

$\boxed{B_x = \text{zero}}$

(Mechanics of material)

Subject Internal Force (moment, shear) Date No

* 

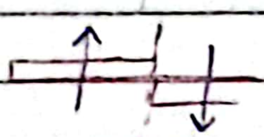
This is a tension force

* 

compression force

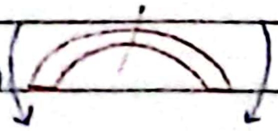
* 

negative shear force

* 
positive sheare force

* 

positive bending moment

* 
negative bending moment

* stress: force intensity or force per unit area

$$\text{stress} \leftarrow \sigma = \frac{P}{A}$$

$P \rightarrow$ external load

$A \rightarrow$ area of section

* unit N/m^2 or Pa

we may use

KPa 10^3 Pa

MPa 10^6 Pa

GPa 10^9 Pa

N/mm^2 it may be used

Types of stresses:
Ⓐ Normal stress
Ⓑ shear stress

(*) Normal stress (Direct stress): force per unit area if stress are normal to area

أحادي (uni , bi , tri) axial state

ثنائي ثلاثي

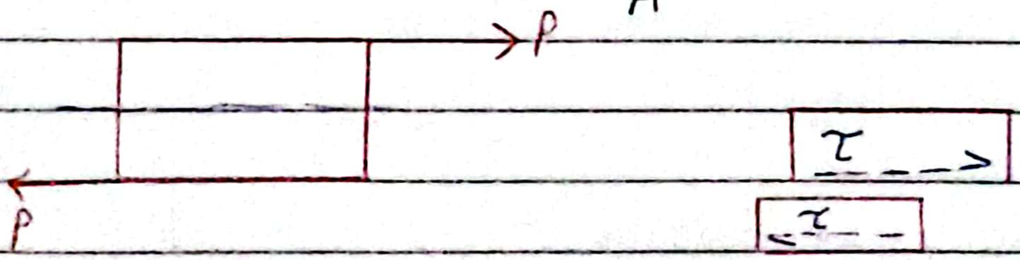
(*) Normal stress maybe tensile (شد) or compressive (ضغط)

tensile

compressive

(*) shear stress two equal and opposite forces acting tangentially across the resisting section.

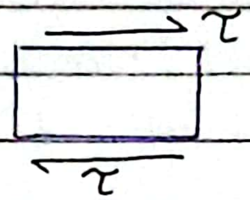
$$\tau = \frac{P}{A}$$



⊛ complementary shear :

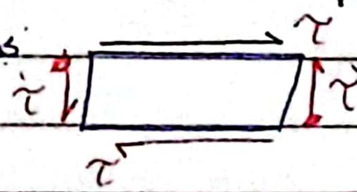
* when a body influenced by a shear stress

← عندما يتأثر الجسم بقص
← لا يبقى في حالة غير متزنة



If we need to balanced the body it balanced by two shears with same vale and oppisite direction and called complementary shear stress

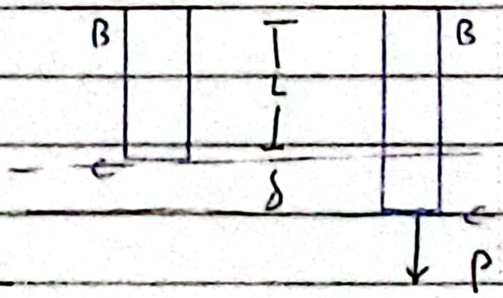
⊛ إذا أردنا موازنة جسم
يجب أن يكون هناك توازن
في الـ stress في جميع الاتجاهات



⊛ strain :

$$\text{strain } (\varepsilon) = \frac{\text{change in length}}{\text{original length}} = \frac{\Delta l}{L_o}$$

strain is subjected to a direct load and hence a stress the bar change in lenght



عندما يتعرض الجسم إلى ضغط زائد
وبالتالي يؤدي إلى تغير في طول الجسم

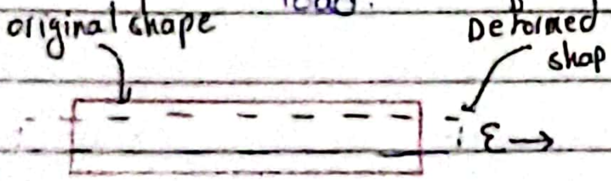
Types of strain:

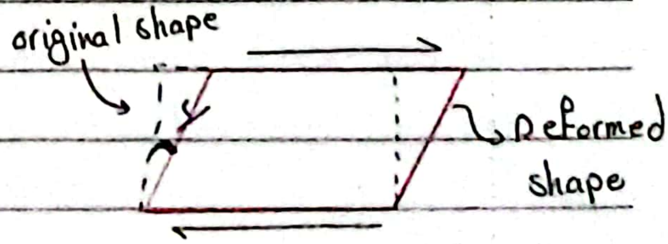
we have two basic stresses exist:

- normal strain: (Direct strain)
  direct stress to the load and must be tensile or compressive according to type of load

عبارة عن قوة الضغط المباشر على الجسم
يجب أن تكون مقابلة للشد أو الإنضغاط

- shear strain:
  strain accompanying a shear stress
- the angle in radian measure through which the body get distorted when subjected to external load.





الإتجاهات

- sign convention: for stress and strain as like tensile is positive and compressive is negative

Subject: Simple stress and strains   Date   No.

* relation between stress and strain:
when a load is applied to an element of any material, there exist a relation ship between the load and small change in the element and if the change dissapeared (the element retain to origin

* when the load removed this state called Elastic behaviour and the material be elastic under the load action.

* عندما تطبق قوة على جسم من أي عنصر هناك علاقة بين الحمل والتغير الطفيف الذي يحدث على الجسم
وإذا اختفى التأثير يعود الجسم (إلى الوضع الطبيعي)
عند إزالة التأثير تسمى هذه الحالة (السلوك المرن) تكون المادة مرنة

* Experiment has shown that the stress is directly proportional to the strain within straight line region. The relation is expressed by Robert hock → Hooke's law

straess α strain within straight line

* Hooke's law state: when a material is loaded within its elastic state the stress is directly proportional to the strain

stress ← $\sigma \propto \varepsilon$ → strain

Subject Date No.

(*) $\sigma = E\,\varepsilon$

σ → stress

E → Young's modulus of elasticity

ε → strain

Unit: N/mm^2

(*) $\tau = G\,\gamma$

τ → shear stress

G → modulus of rigidity

γ → shear strain

Unit: N/mm^2

(*) relation between load and deformation:

(1) If body of length L and x-section A subjected to an axial load P if the modulus of elasticity for the material of the body is E.

(*) إذا تعرض جسم بطول (L) ومقطع (A) لحمل P

إذا كان معامل المرونة (E) ⇐ نريد حساب (deformation)

$$\sigma = \frac{P}{A} \qquad \sigma = E\,\varepsilon \qquad \varepsilon = \frac{\Delta}{L}$$

$$\Delta = \frac{L\,P}{E \cdot A}$$

Subject Date No.

Example: a steel rod of 1m length and square x-section 20mm x 20mm subjected to a tensile force of 40 kN If modulus of elasticity E is 200 GPa (kN/mm²) Find:

1- σ

2- extension δ due to applied load

3- ε

1 $\sigma = \frac{P}{A} = 10 \times 10^{7}\ N/m^2$

3 $\varepsilon = \frac{\sigma}{E} = 5 \times 10^{-5}$

2. $\delta L = \varepsilon \times L_o = 5 \times 10^{-5}$

Example: a load of 5kN is carried by steel wire if the stress to be carried by wire is 100 MPa, Find the minimum diameter of steel wire

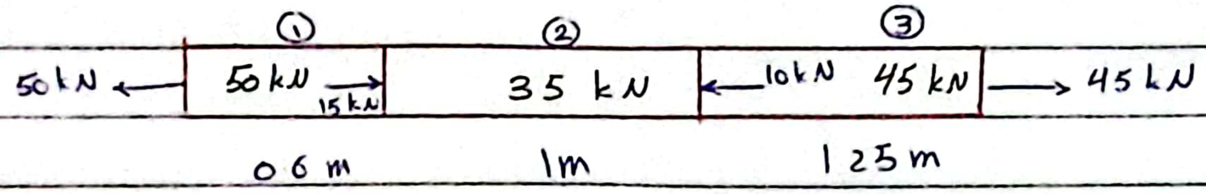
$\sigma = \frac{P}{A} \longrightarrow A = 5 \times 10^{-5}\ m^2$

$A = \pi \left(\frac{d}{2}\right)^2 \longrightarrow d = 7.97 \times 10^{-3}$

NOTEBOOK

Subject Date No.

Ex: steel bar of cross section 500 mm^2 is acted upon by the forces shown in figure Determine the total elongation of the bar For steel consider $E = 200$ GPa



$$\Delta = \frac{P \cdot L}{E\,A}$$

$A = 500\ mm^2$

$L = 600\ mm,\ 1000,\ 1250$

$E = 200\,000$

$$\Delta_1 = \frac{50000\ N \cdot 600\ mm}{200000\ \frac{N}{mm^2} \cdot 500\ mm} = 3 * 10^{-7}$$

$$\Delta_2 = \frac{35000 \cdot 1000}{200000 \cdot 500} = 3.5 * 10^{-7}$$

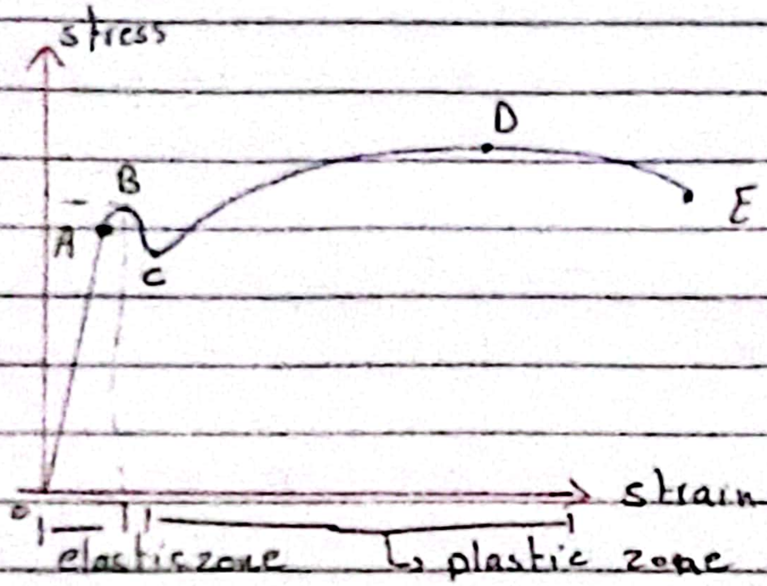
$$\Delta_3 = \frac{45000 \cdot 1250}{200000 \cdot 500} = 5.6 * 10^{-7}$$

$$\boxed{\Delta = 12.125\ m}$$

N O T E B O O K

Subject Date No.

Tensile test



A. Limite prapotionality
B. Yield stress (σ_y)
C.
D. ultimate stress
E. Fracture point

① There are two main types of material

① Ductile material ② Brittle material

①: Ductile material

مواد قادرة على (load) كبير قبل الفشل

1. materials that are cabable of undergoing large strains before ~~fau~~ failure (at normal temprature)

2. advantage of ductile material is that visible distortions may occur before failuer من عيوب المواد المرنة تحدث تشوهات قبل الفشل

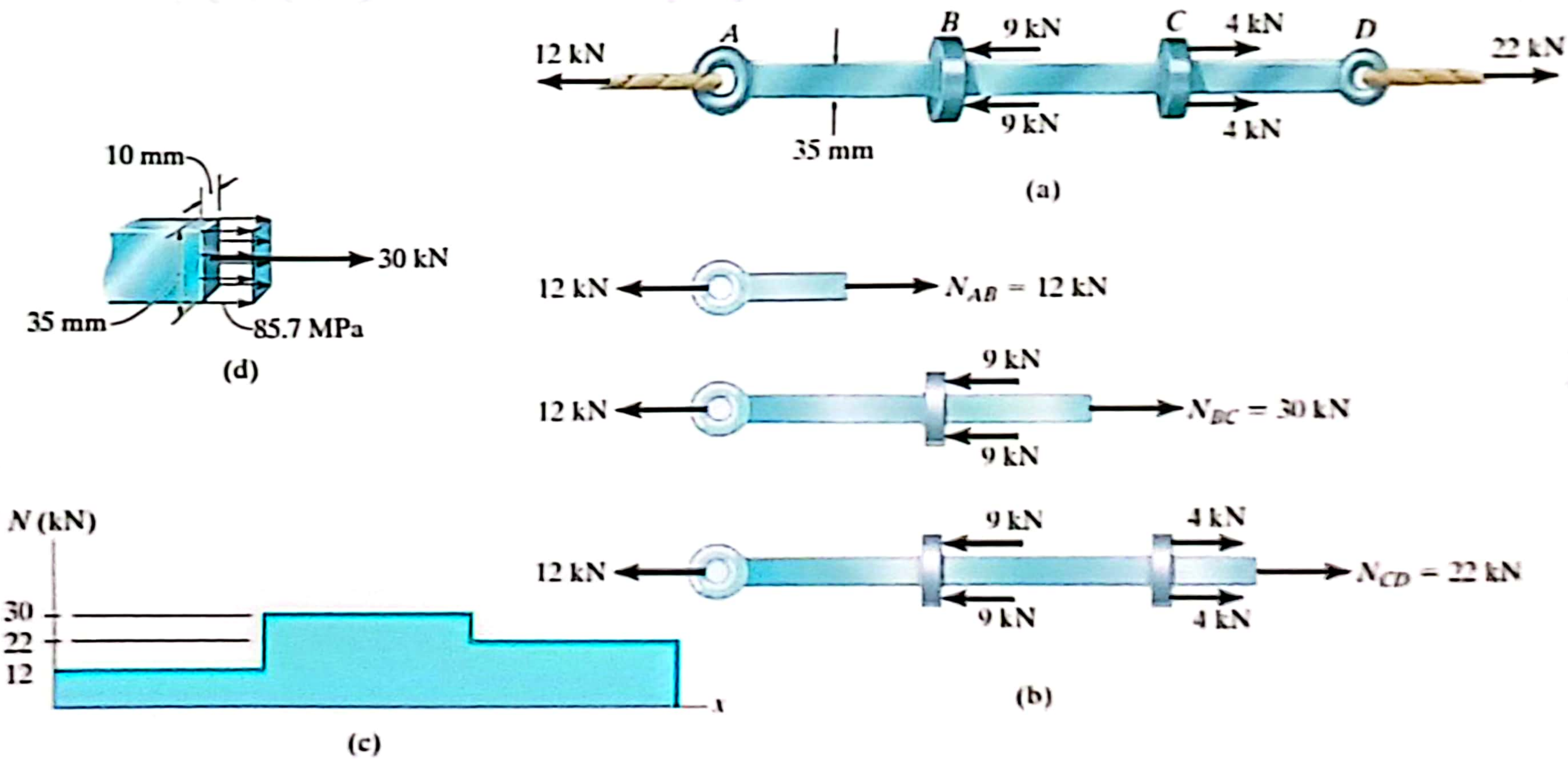
3. high resistance to tensile and compression stress

مقاومة عالية للشد والضغط

4. cabable of absorbing large amount of energy prior to failure قادرة على امتصاص كمية كبيرة من الطاقة قبل الفشل

Example

❖ The bar in the figure below has a constant width of 35 *mm* and a thickness of 10 *mm*. Determine the maximum average normal stress in the bar when it is subjected to the loading shown.



5. The failure in ductile material not happens suddenly but take time and generally happens by yielding or buckling.

"فشل التجربة لا تحدث مباشرة أو بشكل مفاجئ لكن تستغرق وقتا"

6. as example of ductile material: mild steel aluminum, copper, magnesium, nickle, etc.

② Brittle material:

1. material that exhibite very little inelastic deformation — تشوهات غير مرنة

2. as an advantage, have a large amount of resistance to tensile stress — تحتوي على مقاومة كبيرة جدًا

3. The failure in brittle material occur suddenly and generally happens by crushing

يحدث الفشل في التجربة بشكل مفاجئ:

4. example: concrete, stone, cast iron, glass

① Elasticity: The ability of material to return to its original dimensions

plasticity: The property which permits material to undegro a permenant chang in shape without fractu

Ductility: ability of material to stand large plastic deformation in tension

strength: resistance of material to any applied forces

stiffness: property of the material to resist any sort of deformation

stiff material: material has a high unit stress with relatively small unit deformation.

Hardness: The ability of the material to resist scratching, abrasion, cutting, indentation

Durability: ability of material to resist the internal or external destructive condition over time

~~Hardness:~~ resilience: capacity of material to store mechanical energy.

Subject Date No.

Isotropic : material with elastic properties

anisotropic: material whose properties depend upon direction

$E = \frac{\sigma}{\varepsilon}$

Homogenous: material has same composition at every point in the body

* poisson's ratio (ν):

$$\nu = - \frac{\text{lateral strain}}{\text{longitudinal strain}}$$

minus sign: longitudinal strain is always opposite to lateral strain

* modulus of rigidity or shear modulus (G)

$$\boxed{G = \frac{\tau}{\gamma}}$$

* Bulk modulus (K)

$$K = \frac{\text{Direct pressure}}{\text{volumetric strain}}$$

* volumetric strain = $\varepsilon_x + \varepsilon_y + \varepsilon_z$

N O T E B O O K

Subject Date No.

relation between elastic constant

A. relation between Youngus and shear modulus

$$E = 2G(1+v)$$

B. relation between Youngus and Bulk modulus

$$E = 3k(1-2v)$$

C. Youngus, shear, Bulk modulus

$$E = \frac{9GK}{G+3k}$$

Ex: a rod made of a homogenus and isotropic material have 500 mm length and 16 mm diameter The rod is observed to increase in length 300 μm to decrease in diameter by 2.4 μm when subjected to an axial load of 12 kN Determine the modulus of elastisity and poissons ratio.

$L = 500mm$ $P = 12kN$ $d = 16mm$ $\Delta le = 300\ \mu m$

$\Delta lc = -2.4\ \mu m$

$A = 201.1\ mm^2$ $\sigma = \frac{P}{A} = 59.7\ N/mm^2$

$\varepsilon_{le} = \frac{\Delta le}{L} = 0.6 \times 10^{-3}$ $\varepsilon_{lc} = -0.15 \times 10^{-3}$

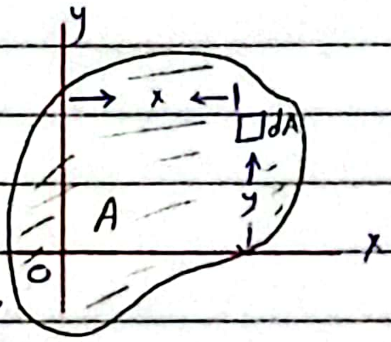
$V = 0.25$ $E = 99.5 \times 10^3\ N/mm^2$

Subject center of gravity, centroid. Date No.

⊛ Def: centroid:
geometric center which represent a point in the plane about which the area of the cross section is equally distributed
نقطة في الشكل الهندسي توزع عليها المساحة بالتساوي

⊛ center of gravity (CG): نقطة تحدد موقع الجاذبية
point which locate the gravity or weight of the body

⊛ consider an area A located in the xy plane in the figure shown below
Denoting by x and y the coordinate of an element of area dA

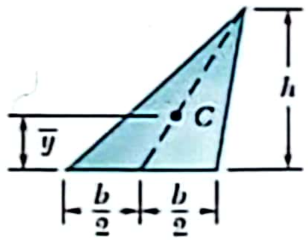
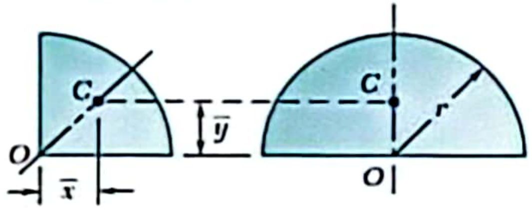
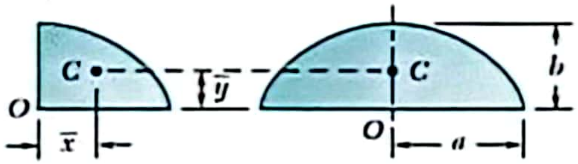
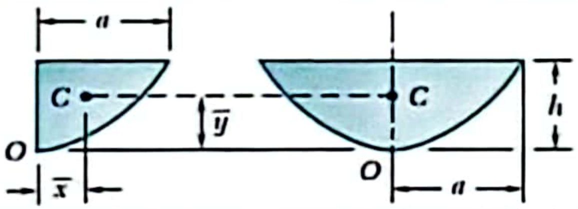
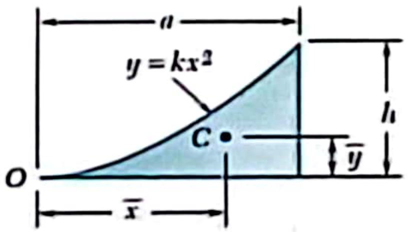


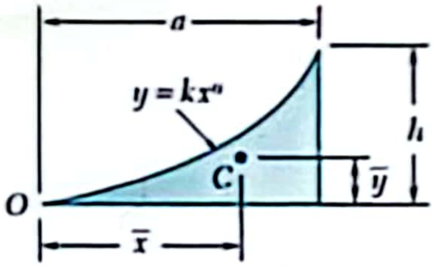
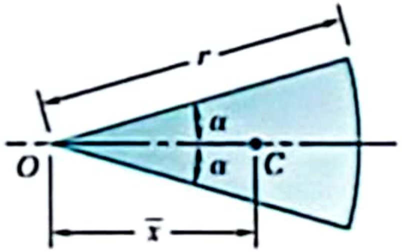
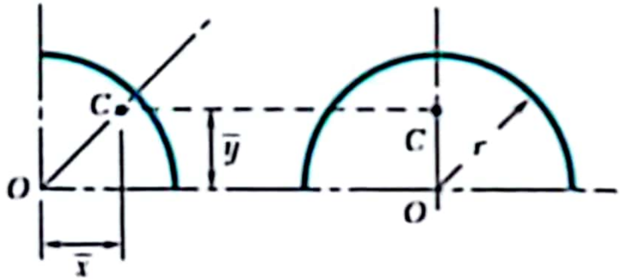
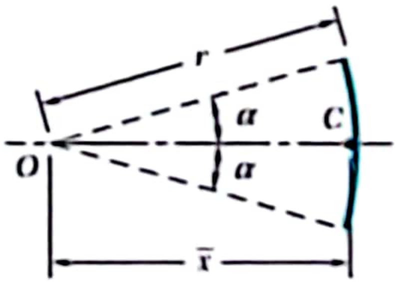
dA نقطة مساحة محددة بأبعاد (x, y)

⊛ $\bar{x} = \frac{\int x \cdot dA}{A}$ $\bar{y} = \frac{\int y \cdot dA}{A}$

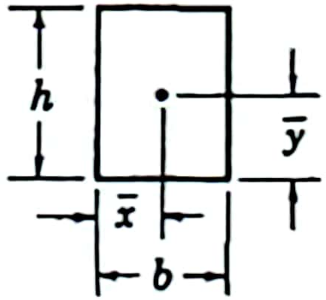
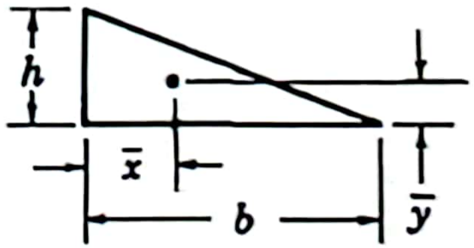
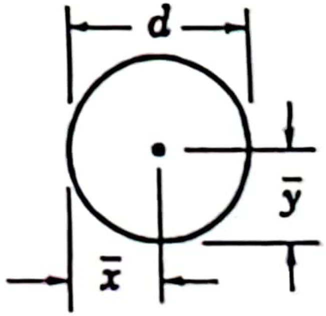
⊛ ⟶ If ~~two~~ homogenous plate, centroid with center of gravity

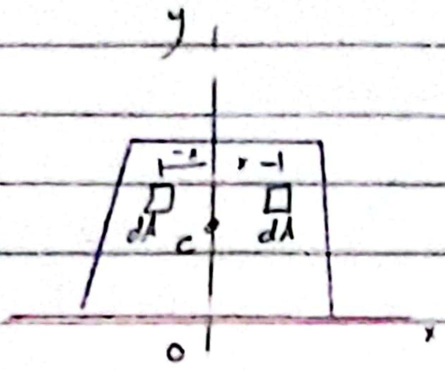
N O T E B O O K

Shape		$\bar{x}$	$\bar{y}$	Area
Triangular area			$\frac{h}{3}$	$\frac{bh}{2}$
Quarter-circular area		$\frac{4r}{3\pi}$	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{4}$
Semicircular area		0	$\frac{4r}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter-elliptical area		$\frac{4a}{3\pi}$	$\frac{4b}{3\pi}$	$\frac{\pi ab}{4}$
Semielliptical area		0	$\frac{4b}{3\pi}$	$\frac{\pi ab}{2}$
Semiparabolic area		$\frac{3a}{8}$	$\frac{3h}{5}$	$\frac{2ah}{3}$
Parabolic area		0	$\frac{3h}{5}$	$\frac{4ah}{3}$
Parabolic spandrel		$\frac{3a}{4}$	$\frac{3h}{10}$	$\frac{ah}{3}$

General spandrel		$\frac{n+1}{n+2}a$	$\frac{n+1}{4n+2}h$	$\frac{ah}{n+1}$
Circular sector		$\frac{2r\sin\alpha}{3\alpha}$	0	αr^2
Shape		$\bar{x}$	$\bar{y}$	Length
Quarter-circular arc		$\frac{2r}{\pi}$	$\frac{2r}{\pi}$	$\frac{\pi r}{2}$
Semicircular arc		0	$\frac{2r}{\pi}$	πr
Arc of circle		$\frac{r\sin\alpha}{\alpha}$	0	$2\alpha r$

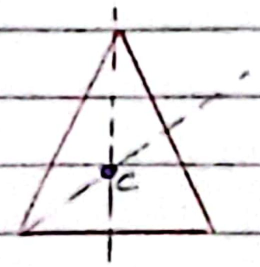
ery Important!

Shape	Area	$\bar{x}$	$\bar{y}$	
Rectangle	bh	$\frac{b}{2}$	$\frac{h}{2}$	
Triangle	$\frac{bh}{2}$	$\frac{b}{3}$	$\frac{h}{3}$	
Circle	$\frac{\pi d^2}{4}$	$\frac{d}{2}$	$\frac{d}{2}$	



① If an area or line possesses an axis of symmetry its centroid located on that axis

إذا كان الخط الفاصل محور تماثل فإن المركز يقع على نفس الخط

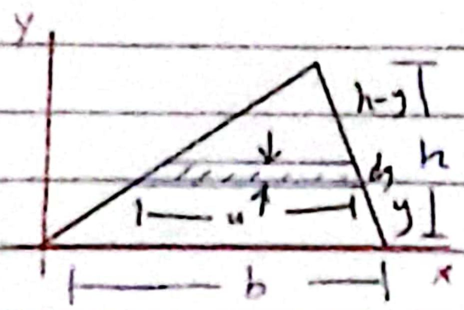


② If the area or line has two axes of symmetry the centroid located at

عند تقاطع خطين بين الخاصر فإن مركز الجسم في نقطة التقاطع

Ex: For triangle shown in figure below find

1. First moment Q_x of the area with respect to x-axis
2. The coordinate



$$\frac{u}{b} = \frac{h-y}{h}$$

$$u = \left(\frac{h-y}{h}\right)b$$

$$\text{area } dA = u\,dy = \left(\frac{h-y}{h}\right)b\,dy$$

$$Q_x = \int_A y\,dA = \int_0^h y\left(\frac{h-y}{h}\right)b\,dy$$

$$Q_x = \frac{b}{h}\left[\frac{y^2 h}{2} - \frac{y^3}{3}\right]_0^h = \frac{bh^2}{6}$$

$$\Rightarrow \bar{y} = \frac{Q_x}{A}$$

$$A = \frac{bh}{2}$$

$$\bar{y} = \frac{1}{3}h$$

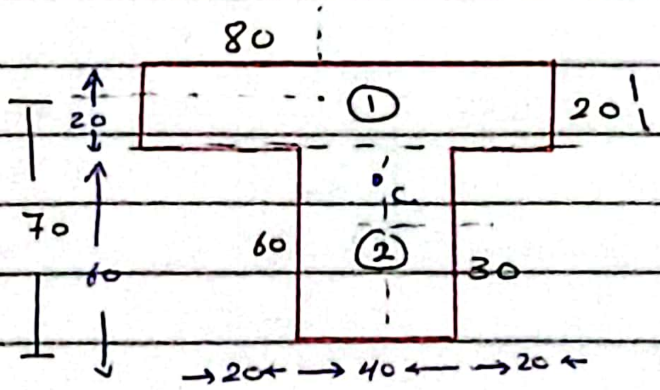
Subject center of complex shape Date No.

$$Q_x = \sum A_i \bar{y}_i \qquad Q_y = \sum A_i \bar{x}_i$$

$$\bar{y} = \frac{\sum A_i \bar{y}_i}{\sum A_i} \qquad \bar{x} = \frac{\sum A_i \bar{x}_i}{\sum A_i}$$

* area is of more complex shape a simple method of determining the location of centroid cabe used which divide complex shape into smaller simple geometric shape.

Ex:

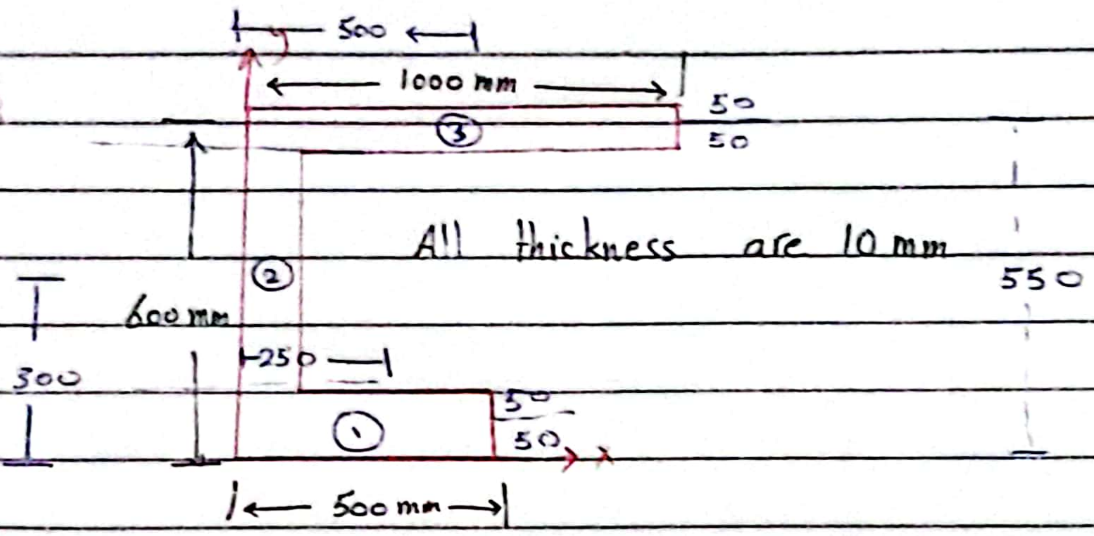


* تقسيم الشكل إلى نصفين

#	A	$\bar{y}$	$A_i \bar{y}$
1	80*60	70	80*60*70
2	40*60	30	40 * 60 * 70
$\sum$	$A_1 + A_2$		$A_1\bar{y}_1 + A_2\bar{y}_2$

$$\bar{x} = \frac{\sum A_i \bar{y}}{\sum A} = 48$$

Ex:



(1) locate the centroid shown in Figure below

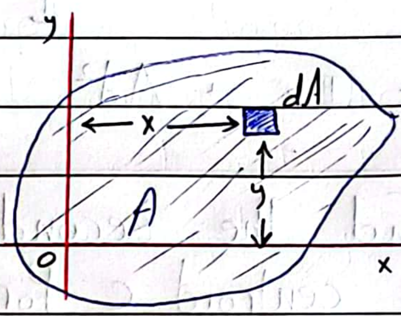
#	A_i	$\bar{y}$	$\bar{x}$	$A_i \bar{y}$	$A_i \bar{x}$
1	1000 * 100	550	250	(1000)(100)(550)	
2	400 * 100	300	50	(400)(100)(30)	
3	500 * 100	550	50	(500)(100)(50)	
Σ					

$$\bar{x} = \frac{A_i \bar{y}}{\Sigma A}$$

*

The second moment of area (second moment inertial)

used to predict the resistance of beam to bend and deflect

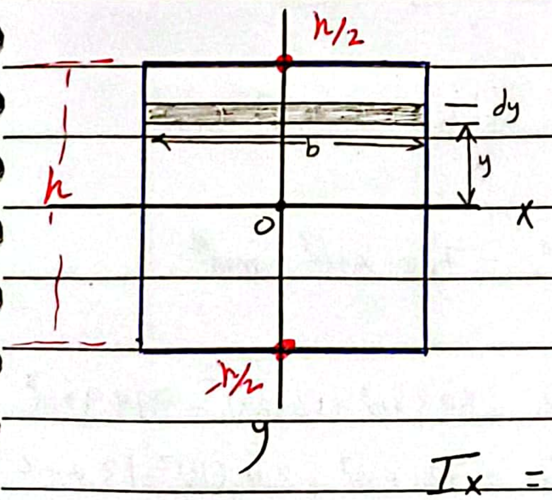


The moment of inertia of area A with respect to x and y axis are defined

$$I_x = \int_A y^2 \cdot dA \qquad I_y = \int_A x^2 \cdot dA$$

about x-axis about y axis

Ex: Find the moment of inertia and the radius of gyration about the x-axis for the rectangular shown



$$\circledast \quad I_x = \int_A y^2 \cdot dA$$

(الاستفادة)

مساحة مستطيل

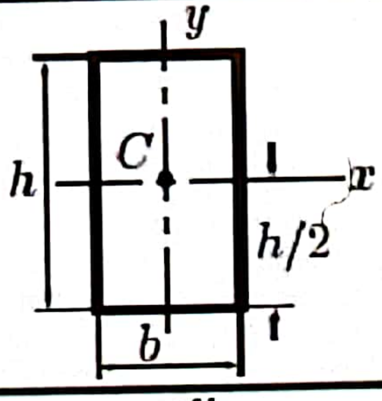
$dA = b \cdot dy$ (integration between $(-\frac{h}{2} / \frac{h}{2})$)

$$I_x = \int_{-h/2}^{h/2} y^2 \, b \cdot dy =$$

$$I_x = \left[b \, \frac{y^3}{3} \right]_{-h/2}^{h/2} = \frac{b}{3} \left[\left(\frac{h}{2}\right)^3 - \left(-\frac{h}{2}\right)^3 \right]$$

N O T E B O O K

$$I_x = \frac{bh^3}{3} \left(\frac{1}{8} + \frac{1}{8} \right) = \frac{bh^3}{12}$$

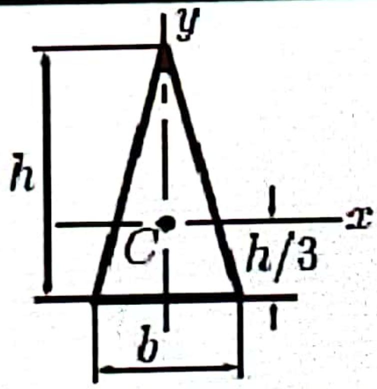


$$A = bh$$

$$I_{xx} = \frac{bh^3}{12}$$

$$I_{yy} = \frac{b^3h}{12}$$

$$I_C = \frac{bh}{12}(b^2 + h^2)$$

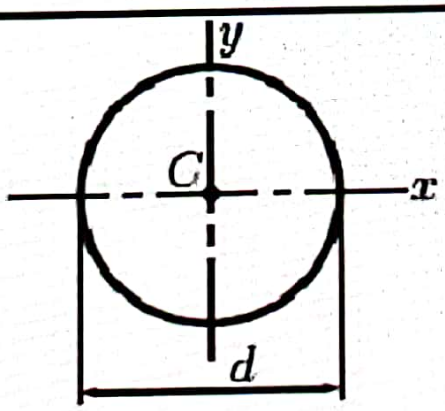


$$A = \frac{bh}{2}$$

$$I_{xx} = \frac{bh^3}{36}$$

$$I_{yy} = \frac{b^3h}{36}$$

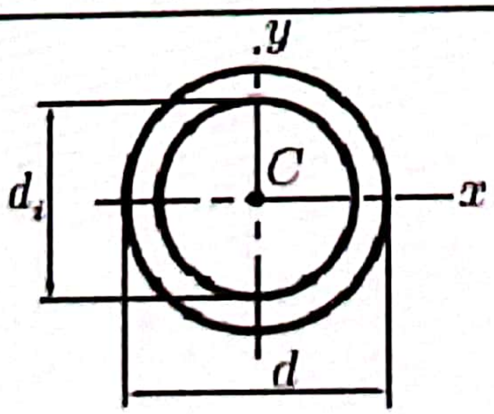
$$I_C = \frac{bh}{36}(b^2 + h^2)$$



$$A = \frac{\pi d^2}{4}$$

$$I_{xx} = I_{yy} = \frac{\pi d^4}{64}$$

$$I_C = \frac{\pi d^4}{32}$$

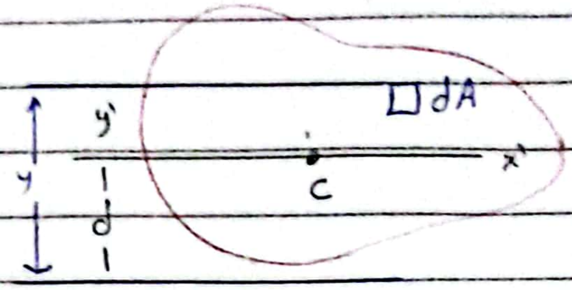


$$A = \frac{\pi}{4}(d^2 - d_i^2)$$

$$I_{xx} = I_{yy} = \frac{\pi}{64}(d^4 - d_i^4)$$

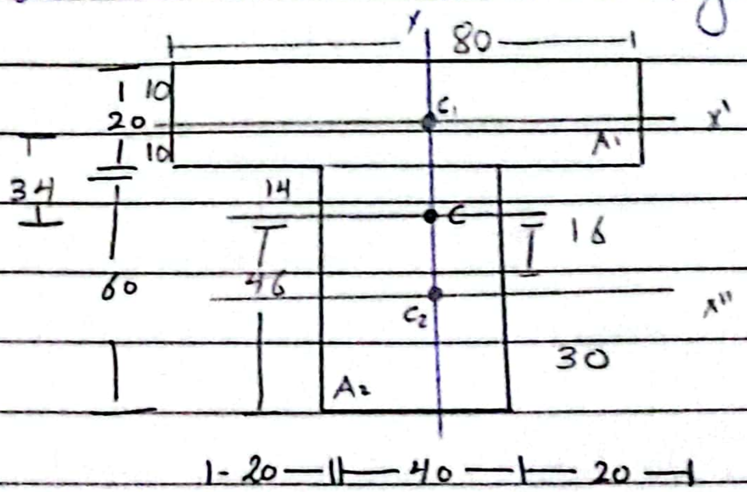
$$I_C = \frac{\pi}{32}(d^4 - d_i^4)$$

Subject: Parallel axis theorem



$I_x = I_{x'} + Ad^2$

Ex: Find the second moment of inertia about the centroid C for the section shown in figure



الأس يتبع
المحور العمودي على
$\left(\frac{b(h)^3}{12}\right)$ (القانون) الشكل عبارة عن مستطيل

For A_1 = $I_{x'} = \frac{80(20)^3}{12} = 53.3 \times 10^3 \ mm^4$

For A_2 = $I_{x''} = \frac{40(60)^3}{12} = 720 \times 10^3 \ mm^4$

$I_x = I_{x'} + Ad^2$ — For A_1: $I_{x_1} = 53.3 \times 10^3 + 1.6(24)^2 = 974.9$

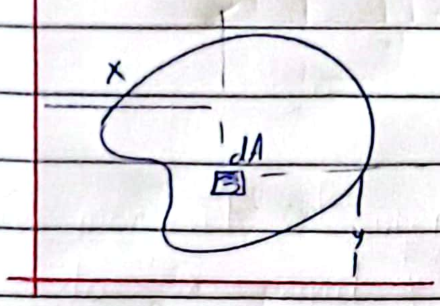
For A_2: $I_{x_2} = 720 \times 10^3 + 2.4\,(16)^2 = 1.3$

$I_x = 2.3 \times 10^6$

property of solid body that describes its resistance to torsional deformation

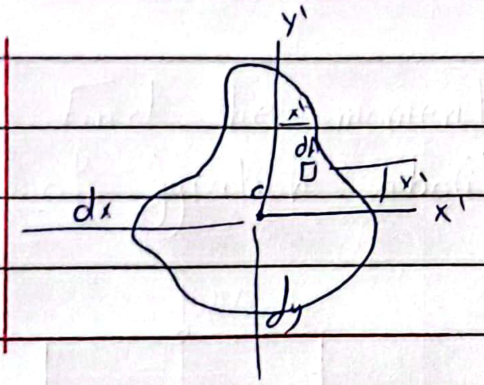
$$J_o = I_u + I_v = I_x + I_y$$

⊛ product of Inertial for an area



$$I_{xy} = \int_A xy \, dA$$

⊛ parallel axis theorem:

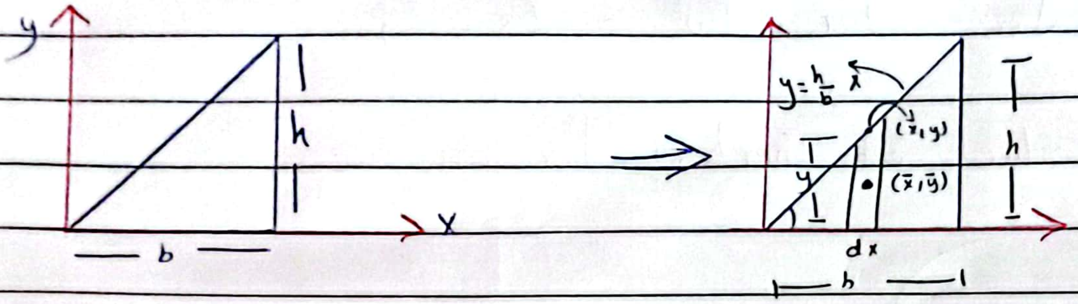


⊛ through the centroid of axis

$$I_{xy} = I_{x'y'} + A d_x d_y$$

Subject Date No.

Ex 8 Determine the product of inertia I_{xy} for triangle shown in figure



$$dI_{xy} = d\bar{I}_{x'y'} + dA\ \tilde{x}\tilde{y}$$

$$dI_{xy} = \text{zero} + y \cdot dx \left(x \cdot \frac{y}{2}\right)$$

$\circledast$ $\frac{y}{x} = \frac{h}{b}$

$y = x\frac{h}{b}$

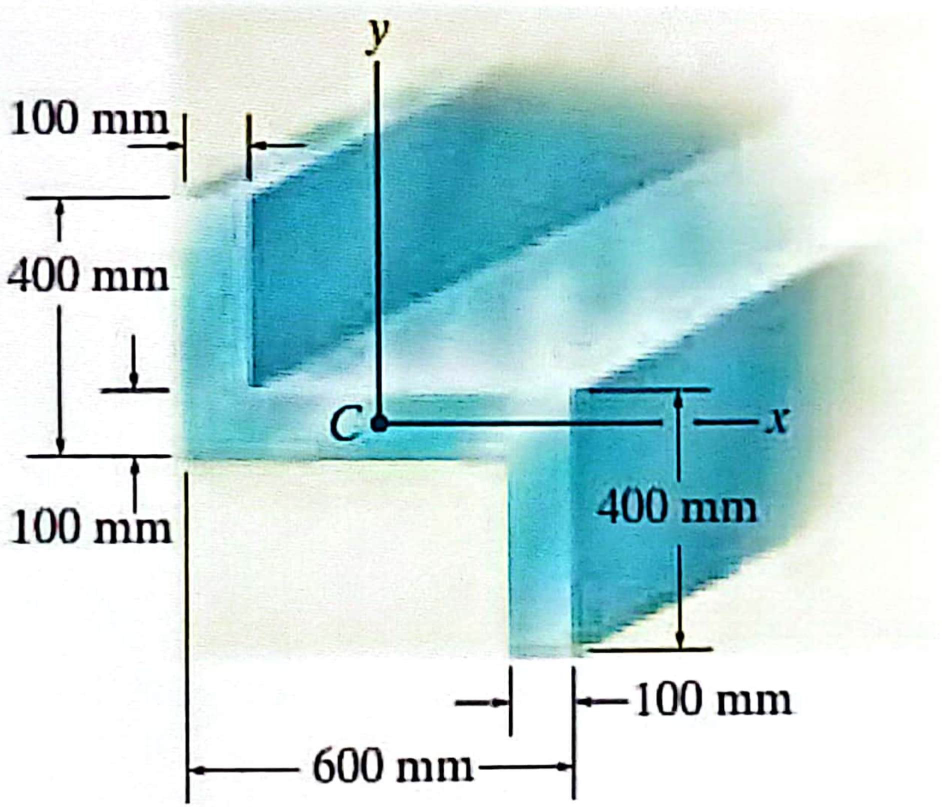
$$I_{xy} = \int x\frac{h}{b}\left(\frac{x}{2} \cdot x\frac{h}{b}\right) \cdot dx$$

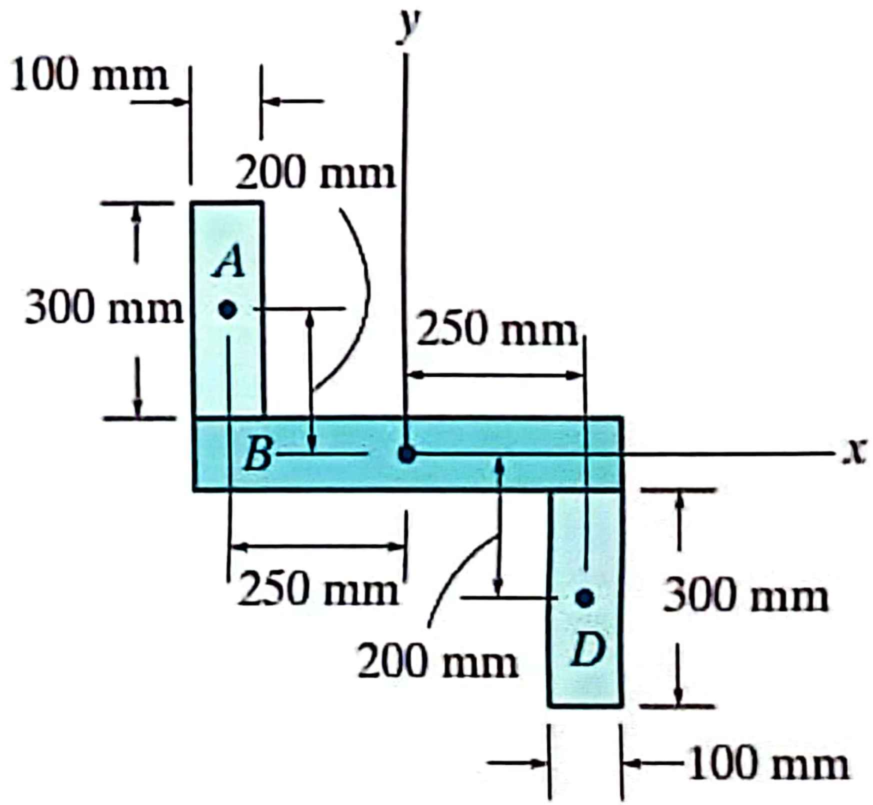
$$I_{xy} = \frac{b^2h^2}{8}$$

N O T E B O O K

Example

- Determine the product of inertia for the cross-sectional area of the member shown about the **x** and **y** centroidal axes.





Rectangle A

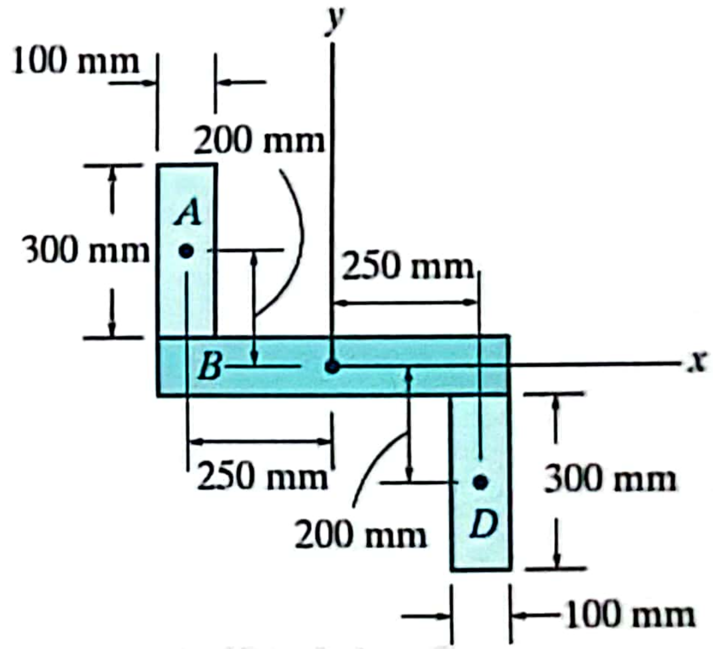
$$I_{xy} = \bar{I}_{x'y'} + A d_x d_y$$
$$= 0 + (300)(100)(-250)(200) = -1.50(10^9)\text{ mm}^4$$

Rectangle B

$$I_{xy} = \bar{I}_{x'y'} + A d_x d_y$$
$$= 0 + 0 = 0$$

Rectangle D

$$I_{xy} = \bar{I}_{x'y'} + A d_x d_y$$
$$= 0 + (300)(100)(250)(-200) = -1.50(10^9)\text{ mm}^4$$



The product of inertia for the entire cross section is therefore

$$I_{xy} = -1.50(10^9) + 0 - 1.50(10^9) = -3.00(10^9)\text{ mm}^4$$

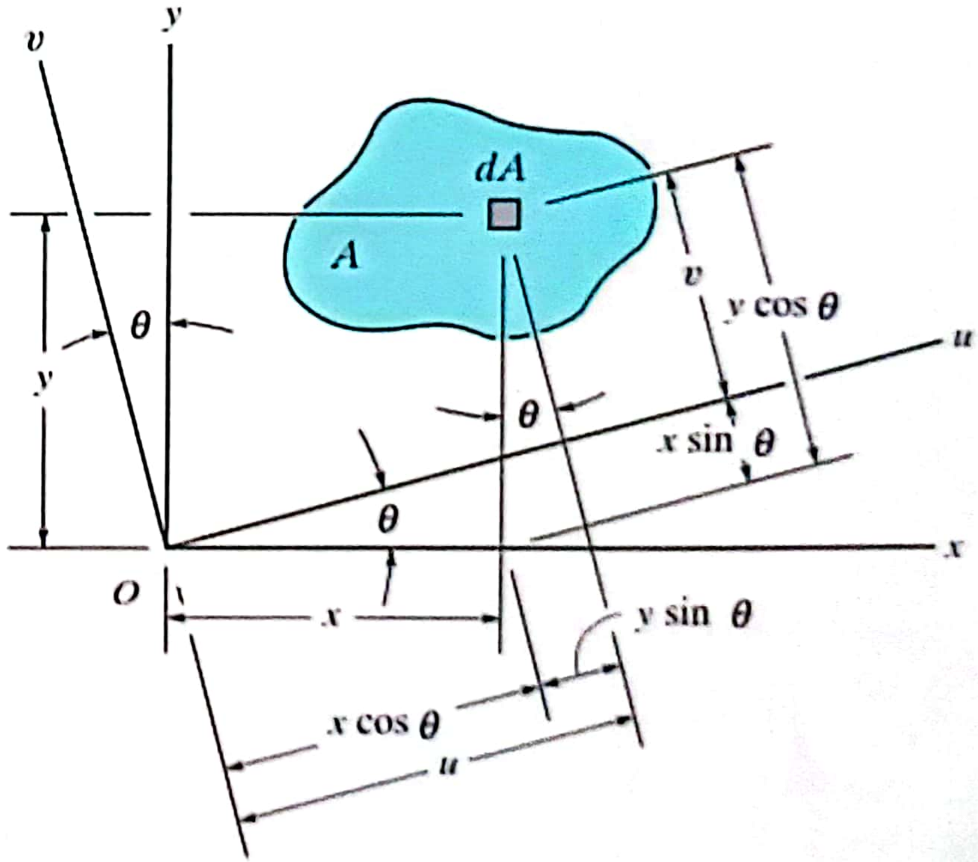
❖ In structural and mechanical design, it is sometimes necessary to calculate the moments and product of inertia ***I*** for an area with respect to a set of inclined I_u, I_v and I_{uv} axes when the values for ϕ, I_x, I_y, I_{xy} are known.

❖ To do this, we will use transformation equations that relate the ***x, y*** and ***u, v*** coordinates.

$$I_u = \frac{I_x + I_y}{2} + \frac{I_x - I_y}{2}\cos 2\theta - I_{xy}\sin 2\theta$$

$$I_v = \frac{I_x + I_y}{2} - \frac{I_x - I_y}{2}\cos 2\theta + I_{xy}\sin 2\theta$$

$$I_{uv} = \frac{I_x - I_y}{2}\sin 2\theta + I_{xy}\cos 2\theta$$



Subject Date No.

principal moment of Inertial

orientation about which the moment of inertia for the area are maximum and minimum (principal axis area)

* The angle which define the orientation of principal axe:

$$\tan 2\theta_p = \frac{-I_{xy}}{(I_x - I_y)/2}$$

* maximum and minimum moment of inertial

$$I_{\substack{max \\ min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I^2_{xy}}$$

Example

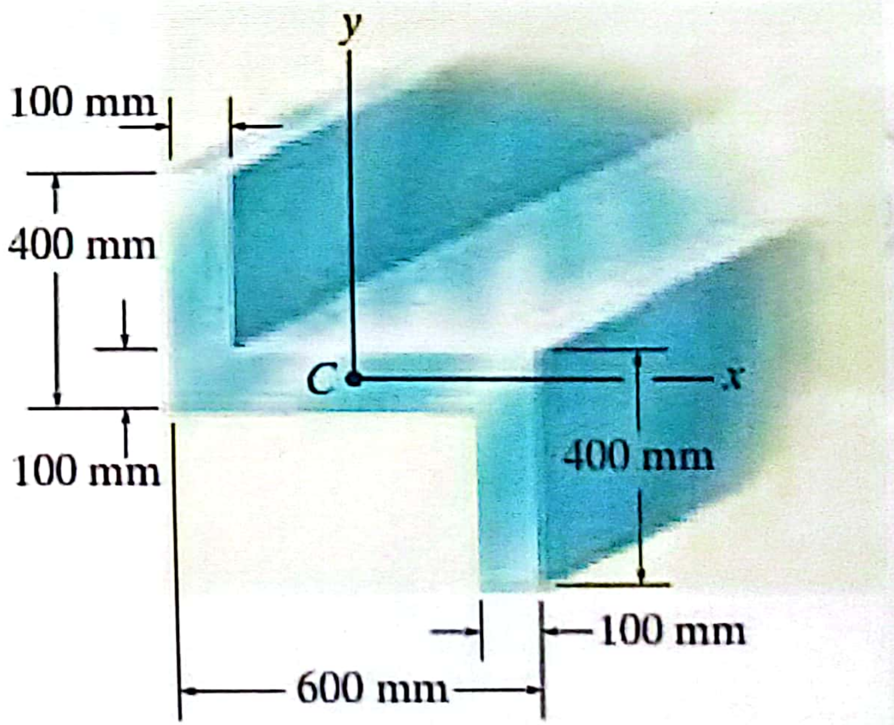
❖ Determine the principal moments of inertia and the orientation of the principal axes for the cross-sectional area of the member shown with respect to an axis passing through the centroid.

$$I_x = 2.90(10^9)\ \text{mm}^4 \quad I_y = 5.60(10^9)\ \text{mm}^4 \quad I_{xy} = -3.00(10^9)\ \text{mm}^4$$

$$\tan 2\theta_p = \frac{-I_{xy}}{(I_x - I_y)/2} = \frac{-[-3.00(10^9)]}{[2.90(10^9) - 5.60(10^9)]/2} = -2.22$$

$$2\theta_p = -65.8° \text{ and } 114.2°$$

$$\theta_{p_2} = -32.9° \quad \text{and} \quad \theta_{p_1} = 57.1°$$



$$I_{\substack{\max \\ \min}} = \frac{I_x + I_y}{2} \pm \sqrt{\left(\frac{I_x - I_y}{2}\right)^2 + I_{xy}^2}$$

$$= \frac{2.90(10^9) + 5.60(10^9)}{2}$$

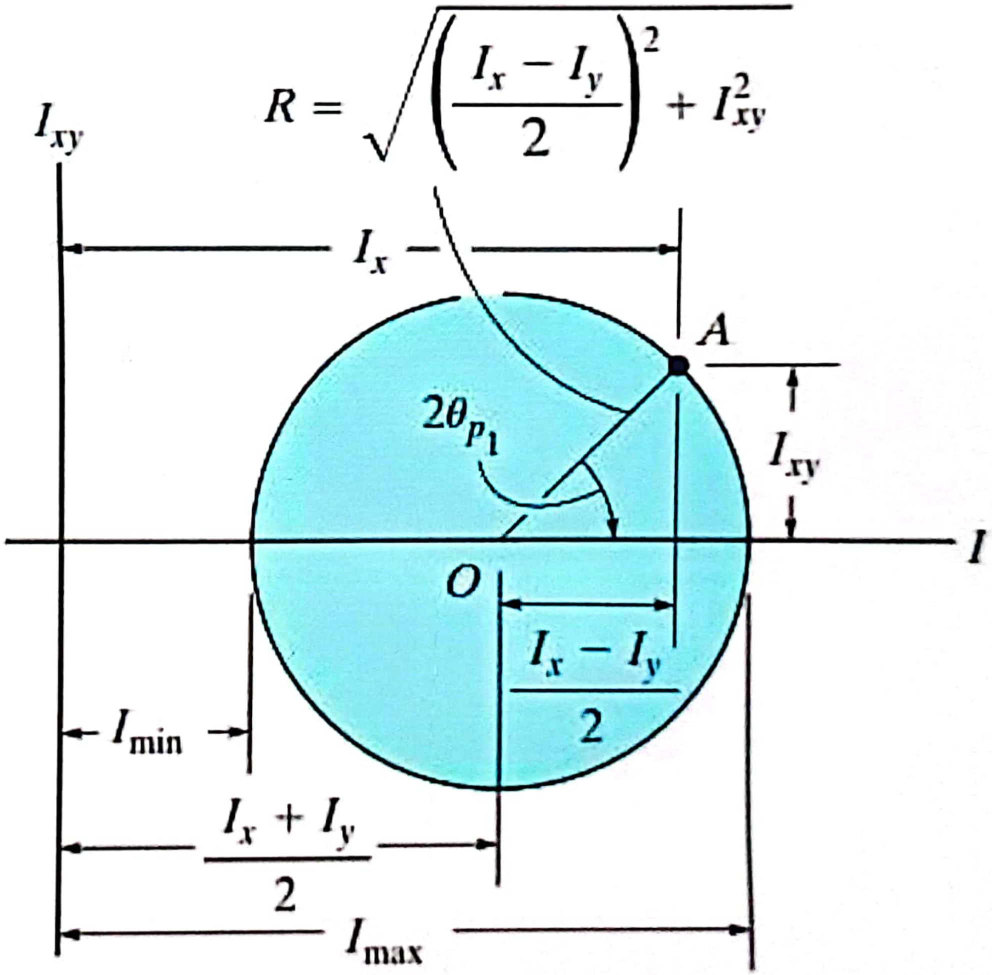
$$\pm \sqrt{\left[\frac{2.90(10^9) - 5.60(10^9)}{2}\right]^2 + [-3.00(10^9)]^2}$$

$$I_{\substack{\max \\ \min}} = 4.25(10^9) \pm 3.29(10^9)$$

or

$$I_{\max} = 7.54(10^9)\ \text{mm}^4 \quad I_{\min} = 0.960(10^9)\ \text{mm}^4$$

❖ The main purpose in using Mohr's circle here is to have a convenient means for finding the principal moments of inertia for an area. Also, have a graphical solution that is convenient to use and generally easy to remember



Subject Date No.

⊛ stresses in beams :

⊛ The theory of simple bending was developed by bernoulli

1. material of the beam is assumed to be homog.

2. all transverse bending , remain plane after bending

3. The radius of curvature of the beam before bending is very large

4. resultant pull or push across any transverse section is zero

5. (E) modulus of elastisity same in (T) and (C)

6. stresses within the proportional limite.

Subject　　　　　　　　　　　　　　　　Date　　　　　　No.

stresses due to bending moment

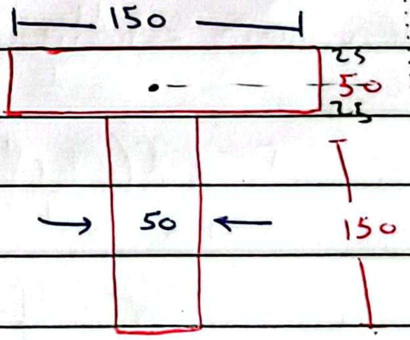
$$\sigma = \frac{M y}{I}$$

σ → stress; M → moment; y → distance from neutral axis; I → moment of inertia

Ex: Two wooden planks 150mm * 50mm, each connected to form a T-section of a beam. If a moment of 6.4 kN.m is applied around the horizontal neutral which inducing tension below the neutral axis. Find the bending stress at both the extreme of the cross section

$$\sigma = \frac{M\, y}{I}$$

$M = 6.4 * 10^6$, $y \to ?\ 0.462 * 10^3$, $I \to ?$



$$\bar{Y} = \frac{\Sigma A y}{\Sigma A} = \frac{(1.31 + 5.62) * 10^6}{15000}$$

$$\bar{Y} = 0.462 * 10^3$$

#	A	$\bar{y}$	Ay
1	150 * 50	175	$1.31 * 10^6$
2	150 * 50	75	$5.62 * 10^6$
	ΣA 15000	$\Sigma\bar{y} = 250$	

$$I = I_i + A_i d^2$$

$$= \frac{(50)^3 \cdot 150}{12} + 150(50)(125-175) + \frac{(150)^3 (50)}{12} + 150(50)(125-75) = \boxed{}$$

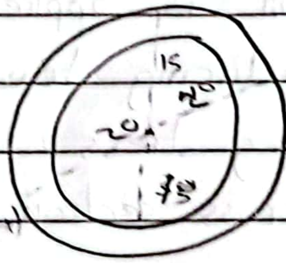
$$\boxed{\sigma = }$$

N O T E B O O K

Subject Date No.

Ex 8 a beam has a hollow circular cross section $40\ mm^3$ outer diameter and 30 mm iner diameter, It is made from metal with modulus elastisity of 205 GPa the maximum stress wass not exceed 350 MPa calculate:

1. The maximum bending moment
2. radius of curvature



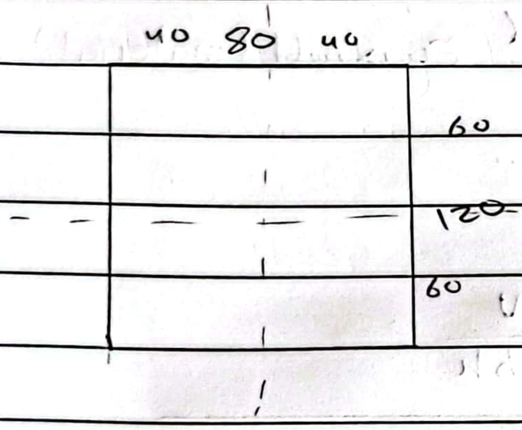
$$\overset{350}{\sigma} = \frac{\overset{?}{M}\overset{20\,(mm)}{y}}{I}$$

$$I = \left(\frac{\pi r^4}{4} - \frac{\pi r^4}{4}\right) = 1.5 * 10^6\ N.m$$

$$\sigma = \boxed{}$$

Subject Date No.

Ex: a beam has a rectangular cross section 80 mm wide and 120 mm deep. It is subjected to a bending moment of $10 * 10^6$ N.m at a certain point along its length calculate the maximum stress in section.



$$\sigma = \frac{\overset{10*10^6}{M}\ \overset{60}{y}}{I} =$$

$$I = \frac{(120)^3}{12} \times 80$$

* shear stress in beams

$$\tau = \frac{V\,Q}{I\,b}$$

τ: shear stress

Q: (مساحة * مسافة) الشكل اي فوق النقطة المطلوبة

I: second moment inertia

b: طبقة (material) المستخدمة) القطع العرضي

* maximum shear stress:

$$\tau = \frac{6V}{bh^3}\left(\frac{h}{4}\right) = 1.5\,\frac{V}{bh}$$

* Ex: a timber beam 10 cm wide by 15 cm deep carries a uniformly distributed load over a span of 2m If the permissible bending stress is 28 N/mm^2 and shear stress is 2 N/mm^2 calculate the maximum load which can be carried by the beam

يتبع

Subject Date No.

* Given data: $b = 10\text{ cm}$ (100 mm) $d = 15\text{ cm} = 150\text{ mm}$

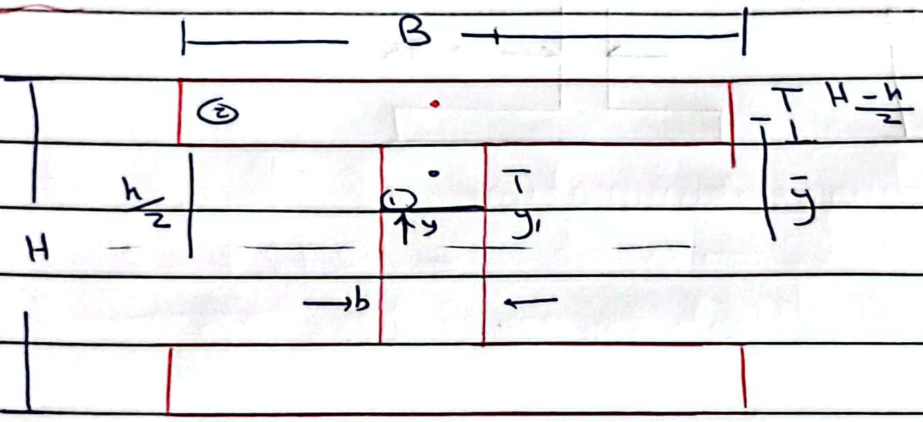
$L = 2\text{ m} = 2000\text{ mm}$ $\sigma = 28\text{ N/mm}^2$

$\tau = 2\text{ N/mm}^2$

* by calculations maximum load will be 20000 N/m

* shear stress in beam

* section



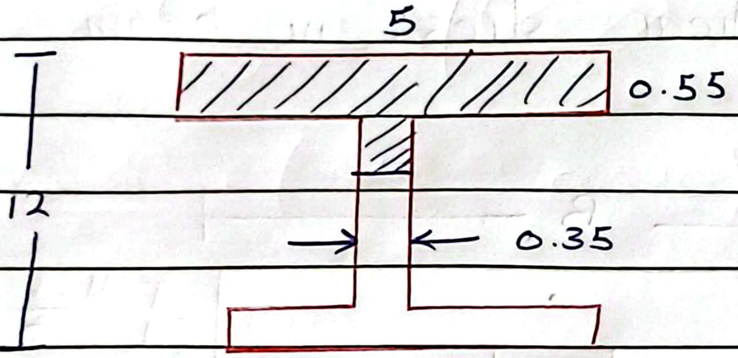
$$\tau_{max} = \frac{V}{8Is}\left[bh + B(H-h)\right]$$

NOTEBOOK

Ex: a 12 cm by 5cm I beam is subjected to a shearing force of 10 kN calculate the value of the transverse shear stress at the nutral axis at the top of the web

Take $I = 220\ cm^4$ Area $= 9.4\ cm^2$

web thickness $= 0.35$ cm Flang thickness



$$\tau = \frac{\overset{10\times10^3}{V}\ \overset{21\ cm^3}{Q}}{\underset{220\ cm^3}{I}\ \underset{0.35}{b}} = \boxed{2727.3\ \frac{N}{cm^2}}$$

$$I = \frac{((12-0.55)*2)^3\, 0.35}{12} + 0 + \left(\frac{(0.35)^3(5)}{12} + 5*0.55\left(\frac{12-0.55}{2}\right)^3\right.$$

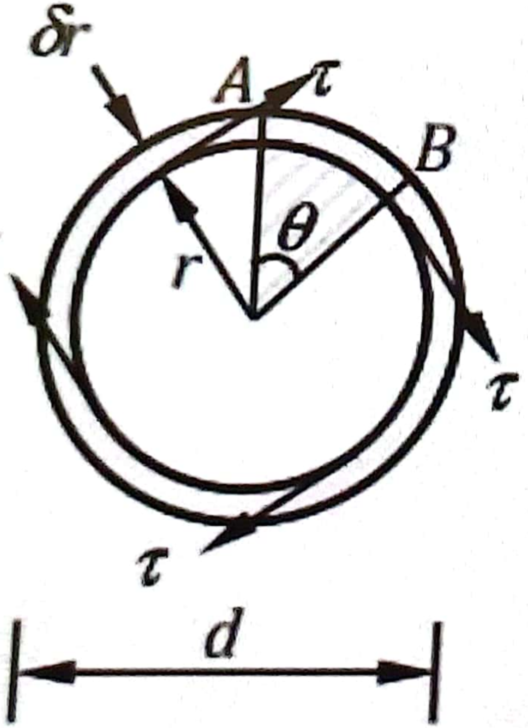
$$Q = \frac{(6-0.55)^2}{2}(0.35) + (5*0.35)\left(\frac{6-0.35}{2}\right)$$

Subject Torsion Date No.

a uniform circular shaft with radius (r)
and length L subjected to a torque
* subjected with pure shear
⇒ moment developed by shear.

* importance of Torsion.

1. (uniform elastic homogenous)
2. The material is elastic, following hockes law
with shear stress
3. doesnot exceed the elastic limite proportionalit
4. circular section remain circular
5. cross section remain plane
6. rotate if rigid


δr
A
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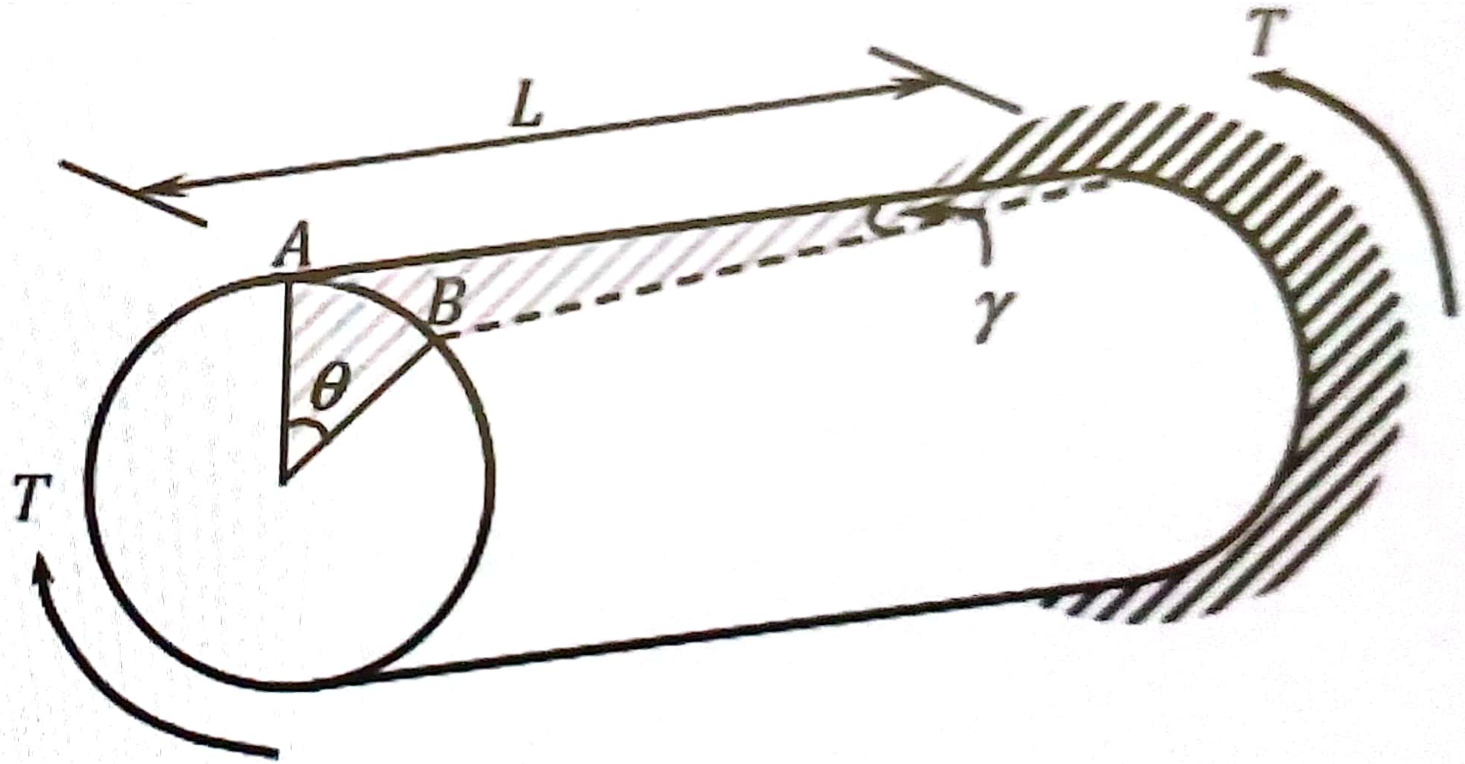
r
τ
τ
d


L
T
A
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γ
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T

* rules:

$$\gamma = \frac{\tau}{G} \qquad r\theta = \frac{L\tau}{G}$$

$$T = \text{force} * r$$

$$\left\{ \frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{L} \right\} \qquad k = \frac{T}{\theta} = \frac{GJ}{L}$$

$k \rightarrow$ Torsion stiffness

CamScanner

Example الأمثلة. جميعها عبارة عن تطبيق مباشر على القوانين

❖ If a twisting moment of 1100 ***N.m*** is impressed upon a 4.4 ***cm*** diameter shaft, what is the maximum shearing stress developed? Also, what is the angle of twist in a 150 ***cm*** length of the shaft? The material is steel, for which G = 85 ***GPa***.

$T = 1100\ N\ m = 1.1x10^6\ N\ mm$

$D = 4.4\ cm = 44\ mm$

$L = 150\ cm = 1500\ mm$

$G = 85\ GPa = 85x10^3\ N/mm^2$

For the shear stress and from the general Equation of torsion:

$$\frac{T}{J} = \frac{\tau}{r}$$

$$\tau = \frac{T}{J} r$$

$$J = \frac{\pi D^4}{32} = \frac{\pi (44)^4}{32} = 368x10^3\ mm^4$$

$$r = \frac{D}{2} = \frac{44}{2} = 22\ mm$$

$$\tau = \frac{1.1x10^6}{368x10^3}(22) = 65.8\ N/mm^2$$

$$\frac{T}{J} = \frac{G\theta}{L} \qquad \theta = \frac{TL}{JG}$$

$$\theta = \frac{1.1x10^6(1{,}500)}{368x10^3(85x10^3)} = 0.0527\ rad$$

$$\theta = 0.0527\left(\frac{180}{\pi}\right) = 3°$$

Example

❖ A hollow 3.0 *m* long steel shaft must transmit a torque of 25 ***kN.m***. The total angle of twist in this length is not to exceed 2.5° and the allowable shearing stress is 90 MPa. Determine the inside and outside diameter

$T = 25\ kN\ m = 25x10^6\ N\ mm$

$L = 3\ m = 3000\ mm$

$$\theta = 2.5° = \frac{2.5(\pi)}{180} = 0.0436\ rad$$

$\tau = 90\ MPa = 90\ N/mm^2$

$G = 85\ GPa = 85x10^3\ N/mm^2$

$$J = \frac{TL}{G\theta}$$

$$J = \frac{25x10^6(3000)}{85x10^3\ (0.0436)} = 20.24x10^6\ mm^4$$

$$J = \frac{\pi\left(D^4 - d^4\right)}{32} = 20.24x10^6 \qquad (1)$$

$$\frac{T}{J} = \frac{\tau}{r} \qquad \frac{\pi(D^4 - d^4)}{32} = \frac{25x10^6 D}{2(90)} \qquad (2)$$

$$D = \frac{206.2x10^6}{1.415x10^6} = 145.7\ mm$$

$$d = \sqrt[4]{244.4x10^6} = 125\ mm$$

ϕ

Example

- The working conditions to be satisfied by a shaft of diameter 165 *mm* transmitting power are:

  1. That the shaft must not twist more than 1 degree on a length of 2.5 *m*
  2. The shear stress must not exceed 55 ***kN/mm²***

If G =80,000 ***N/mm²***, calculate the torque which can be transmitted for a given diameter according to the previous conditions.

$$\theta = 1° = \frac{1(\pi)}{180}\,rad = 0.0174\,rad$$

$L = 2.5\ m = 2500\ mm,\ D = 165\ mm$

$\tau = 55\ N/mm^2$

$G = 80{,}000\ N/mm^2$

$$J = \frac{\pi D^4}{32} = \frac{\pi(165)^4}{32} = 72.8x10^6\ mm^4$$

$$\frac{T}{J} = \frac{G\theta}{L}$$

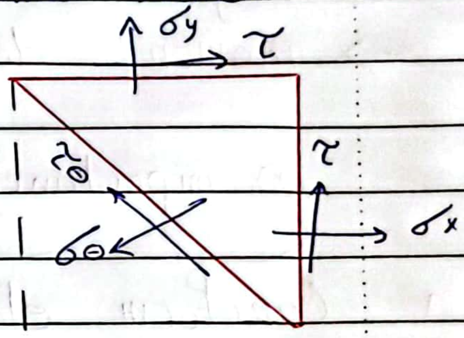
$$T = 40.5\ kN.m$$

$$\frac{T}{J} = \frac{\tau}{r}$$

$$T = 48.5\ kN.m$$

Subject complex (compound) stress Date No.

materiales in a stressed components often have direct and shear stress acting in two or more directions at the same time.



✱ by finding summation of forces: at (σ_θ)

$$\cancel{\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \left(\frac{(\sigma_x - \sigma_y)}{2}\right.}$$

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \left(\frac{\sigma_x - \sigma_y}{2}\cos 2\theta\right) + \tau \sin 2\theta$$

✱ by finding summation of forces at τ_θ

$$\tau_\theta = \frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta - \tau \cos 2\theta$$

✱ يوجد ٣ أسئلة عادية عبارة عن تعويض مباشر للقانون فقط

❖ If the stresses on two perpendicular planes through a point are 60 ***N/mm²*** tension applied horizontally, 40 ***N/mm²*** compression applied vertically and 30 ***N/mm²*** shear, find the stress components on a plane inclined by 60° to the vertical.

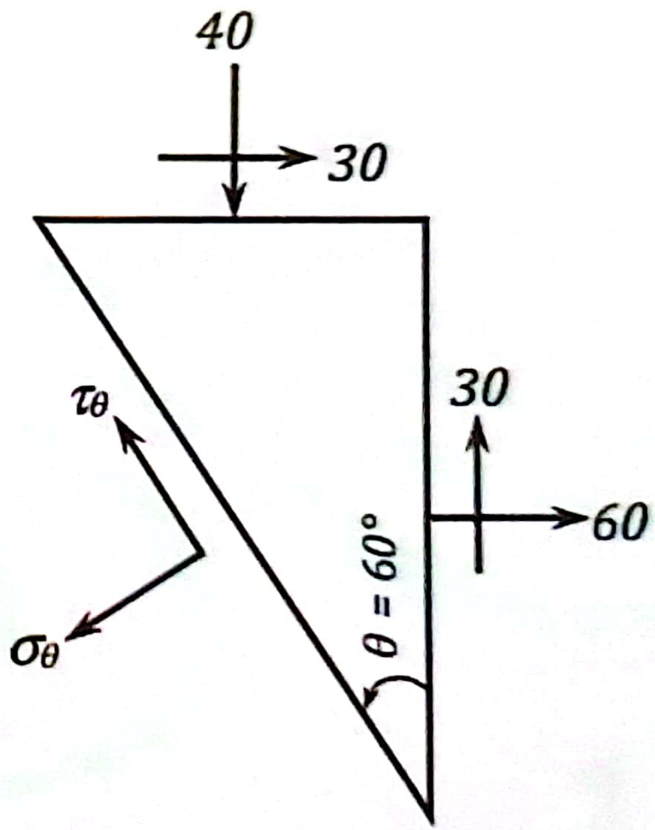
Given Data:

$\sigma_x = 60\ N/mm^2,\ \sigma_y = -40\ N/mm^2,\ \tau = 30\ N/mm^2,$

Calculate the stresses from the following equations:

$$\sigma_\theta = \frac{(\sigma_x + \sigma_y)}{2} + \frac{(\sigma_x - \sigma_y)}{2}\cos 2\theta + \tau \sin 2\theta$$

$$\tau_\theta = \frac{(\sigma_x - \sigma_y)}{2}\sin 2\theta - \tau \cos 2\theta$$



All data in N/mm²

$$\sigma_\theta = \frac{(60 + (-40))}{2} + \frac{(60 - (-40))}{2}\cos(120) + 30\ sin\ (120)$$

$$\sigma_\theta = 10 + 50\ (-0.5) + 30(0.866) = 11\ N/mm$$

$$\tau_\theta = \frac{(60 - (-40))}{2}\sin 2(60) - 30\ cos\ 2(60)$$

$$\tau_\theta = 43.3 + 15 = 58.3\ N/mm$$

⊛ complex compound stress:

$$\sigma = \frac{(\sigma_x + \sigma_y)}{2} \pm \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$\tan 2\theta = \frac{2\tau}{(\sigma_x - \sigma_y)}$$

$$\tau_{max} = \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma_t)$$

$\sigma_1 = \sigma$ with +

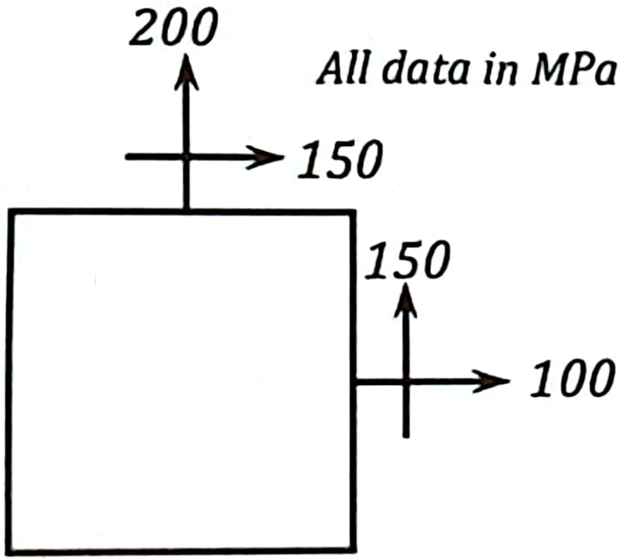
$\sigma = \sigma$ with −

$$\tan 2\theta = \frac{(\sigma_x - \sigma_y)}{2\tau}$$

Example

❖ For the stresses shown in Figure, find the following:

1. The principal stresses.
2. The position of the principal planes.
3. The maximum shear stresses.



All data in MPa

Solution:

Given data:

$\sigma_x = 100\ N/mm^2,\ \sigma_y = 200\ N/mm2,\ \tau = 150\ N/mm^2$

(Note that Pa = 1 N/m², MPa = 1 N/mm²)

1. The principal stresses:

$$\sigma_1 = \frac{(\sigma_x + \sigma_y)}{2} + \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$\sigma = \frac{(\sigma_x + \sigma_y)}{2} - \frac{1}{2}\sqrt{4\tau^2 + (\sigma_x - \sigma_y)^2}$$

$$\sigma_1 = \frac{(100 + 200)}{2} + \frac{1}{2}\sqrt{4(150)^2 + (100 - 200)^2}$$

$$\sigma_1 = 150 + 158.1 = 308.1\ N/mm$$

$$\sigma = 150 - 158.1 = -8.1\ N/mm$$

Example

2. *The position of the principal planes:*

$$\tan 2\theta = \frac{2\tau}{(\sigma_x - \sigma_y)}$$

$$\tan 2\theta = \frac{2(150)}{(100 - 200)} = -3$$

$$2\theta = \tan^{-1}(-3)$$

$$2\theta = -71.6°$$

$$\theta = -71.6/2 = -35.8°$$

or:

$$\theta = -35.8 + 90 = 54.2°$$

3. *The maximum shear stress:*

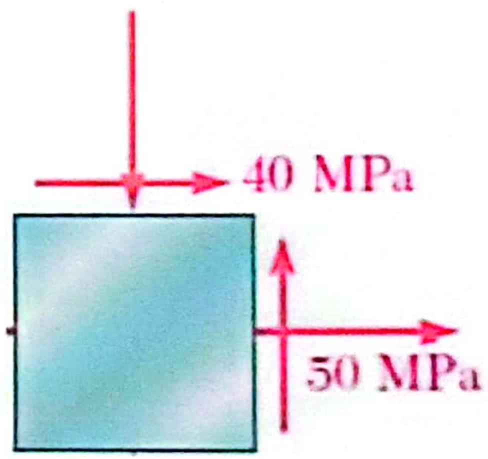
$$\tau_{max} = \frac{1}{2}(\sigma_1 - \sigma\ )$$

$$\tau_{max} = \frac{1}{2}(308.1 - (-8.1)) = 158.1\ N/mm$$

Example

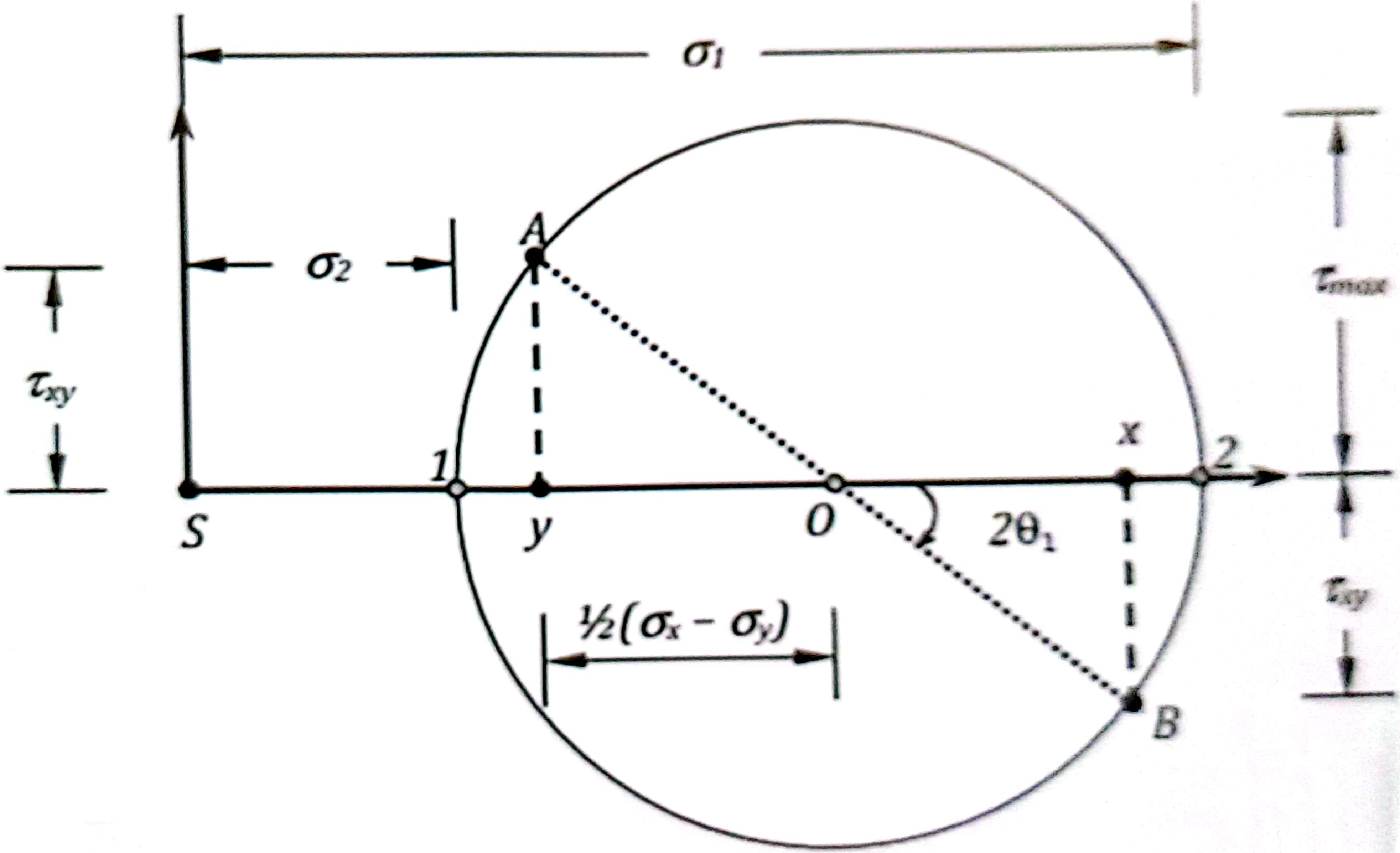
❖ For the state of plane stress shown in Figure below, determine:

1. The principal planes.
2. The principal stresses.
3. The maximum shear stresses.



(a) $26.6°, 116.6°$
(b) $\sigma_1 = 70\ MPa$, $\sigma_2 = -30\ MPa$
(c) $\tau_{max} = 20\ MPa$

Mohr's Stress Circle



* procedure for analysis

① support reactions:

find reactions before take sections

② Free body diagram:

you have to keep all distributed loading couple moment and forces acting on memeber

③ take section

at point we need to determine internal force take section and draw the free body diagram

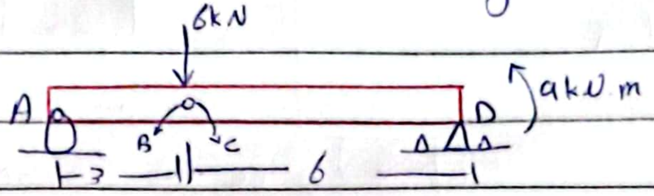
④ equation of equilibrium:

by $\Sigma M = zero$ $\Sigma F = zero$

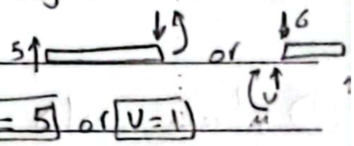
⑤ sign convention:

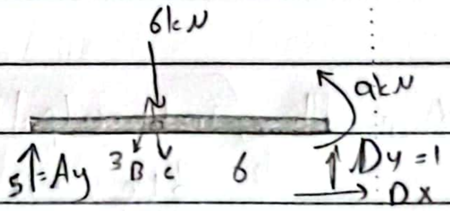
* إذا طلعت الإجابة سالبة نعكس الاتجاه

Example: Determine the normal force, shear force and bending moment acting just into left point B, and just to the right, point C of the 6-kN force on the beam in the figure below:



Sol: we need to find:

1. support reactions ✓ $A_y = 5$ $D_y = 1$ $D_x = \text{zero}$
2. Free body diagram (section) ✓ 
3. equation of equilibrium $\boxed{M = 15}$ $\boxed{V = 5}$ or $\boxed{V = 1}$
4. plot diagram



* ΣM on D:

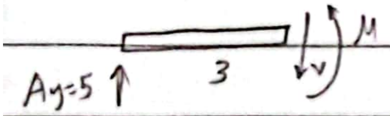
$6(6) - A_y(9) + 9 = \text{zero}$

$\boxed{A_y = 5\,kN}$

* $\Sigma F = \text{zero}$

$\boxed{D_y = 1\,kN}$ $\boxed{D_x = \text{zero}}$

* take section at point we need



نفرض (V، M) أي اتجاه إذا طلعت بالسالب بنعكسها

$\Sigma F = \text{zero}$

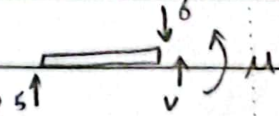
$\boxed{V = 5\,kN}$

$\Sigma M = \text{zero}$

$-5(3) + M = 0$

$\boxed{M = 15\,kN.m}$

or



$\boxed{V = 1}$ $\boxed{M = 15}$

Subject Date No.

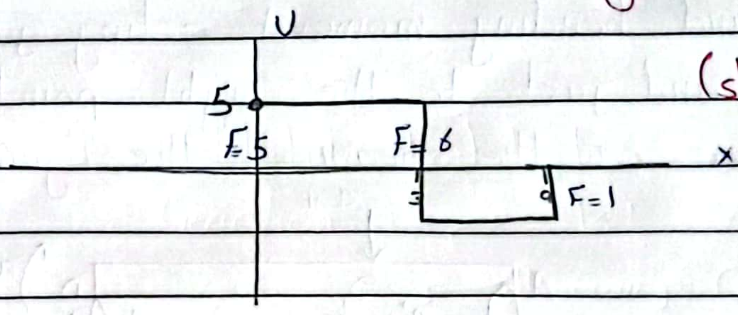
Now we need to plot the diagram

$A_y = 5$ $M_y = \text{zero}$

$F = 6$ $M = 15$

$D_y = 1$ $M = 9$

(shear diagram)



* طريقة الرسم

* ببدأ بـ (5) عند النقطة صفر

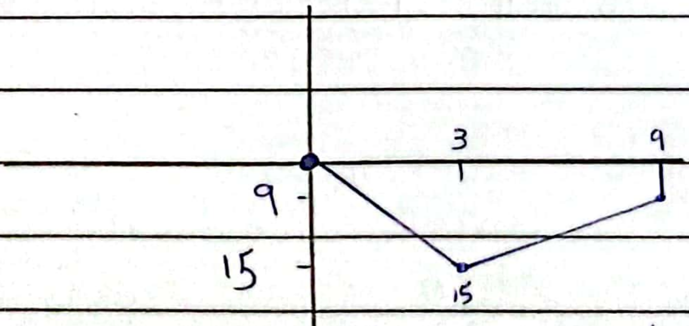
* خليك ثابت لوصلت النقطة ③ نزلت مقدار ⑥
الـمهم ما أمشي لتحت

* خليك ماشي لوصلت النقطة ⑨ طلعت مقدار ①
الـمهم ما أمشي لفوقه

* ملاحظة مهمة جداً: (مهم جداً في النهاية يسكر الشير عالصفر)
إذا ما سكر معك غلط

(Moment diagram)

(على نفس مستوى رسمة الشير)

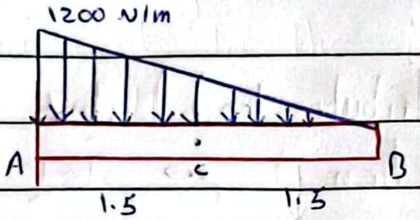


* عند (M) صفر (reactions) (انتبه) نزلت 15 وعند مومنت 9

سكر الشي للصفر

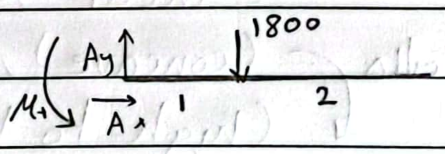
Subject Date No.

Ex: Determine the normal force, and bending moment at C of the beam in Figure below



- ⊛ support reactions $A_y = 1800$ $A_x = \text{zero}$ $M = 1800$
- ⊛ Free-body diagram (section at C) الجزء الصعب يجب أن يأخذ اليسار.
- ⊛ equation of equilibrium
- ⊛ plot shear and moment diagram.

Frist we have fixed support and free end and dist. load.



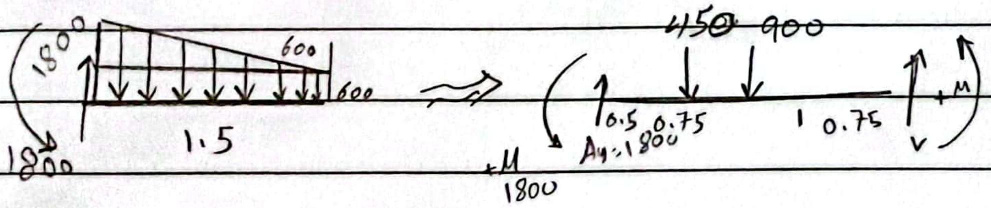
$\Sigma F = \text{zero}$

$A_y = 1800\,N$ $A_x = \text{zero}$

$\Sigma M = \text{zero}$

$M = 1800\ N.m$

⊛ when we take section at (C):



$\Sigma M = \text{zero}$ $\Sigma V = \text{zero}$

$\boxed{M = 675}$ $\boxed{V = 450}$

Subject relation between distributed load Date and shear No.

⊛ important points:

⊛ تحويل (distribute load) إلى (concentrated)

⊛ إيجاد قيم (reaction)

⊛ رسمة الشير:

ننزل خطوط عند (reaction)

(concentarted load)

خط من أول وخط من آخر (distributed load)

⊛ ننزل بمقدار المساحة

⊛ رسمة (M) عبارة عن مساحات رسمة الشير

⊛ v (contant) ⟶ M (linear)

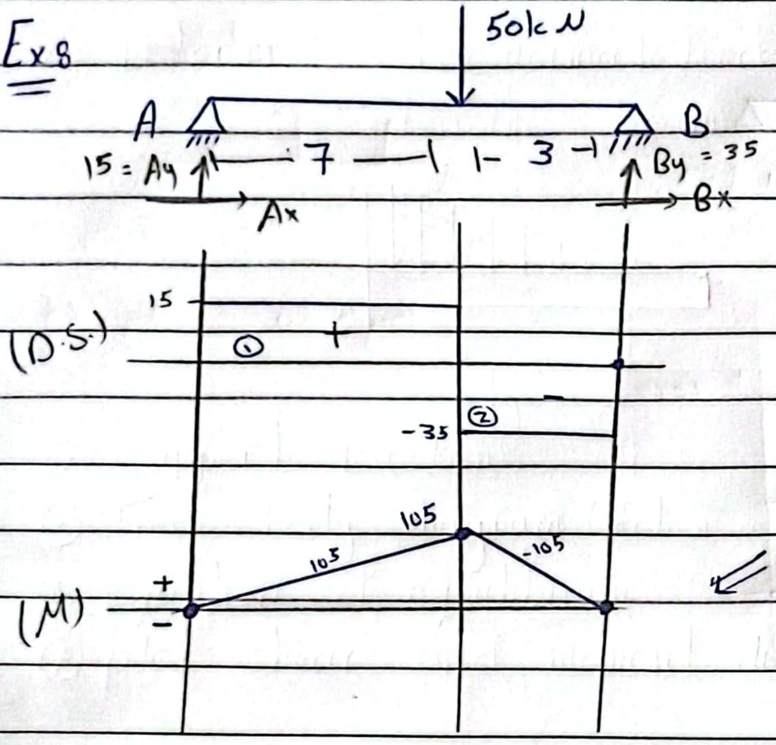
v (linear) ⟶ M (concave) حالة concave

⊛ concave down (high to low)

concave up (low to high)

Subject Date No.

Ex8



ΣM on A = zero

By = 35 kN

Ay = 15 kN

الشير بيسكر عصفر ⇐

بيسكر عند الصفر

(M = 0) pins

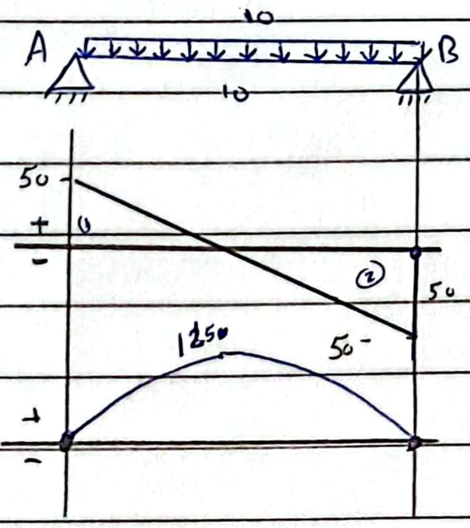
$A_1 = 15 * 7 = 105$

$A_2 = -35 * 3$

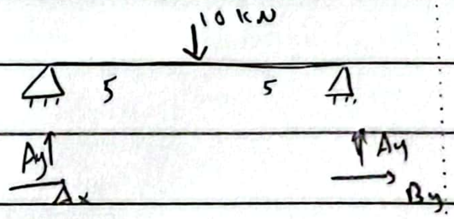
shear is constant

M must be linear

Ex



for reactions



Ay = 50 By = 50

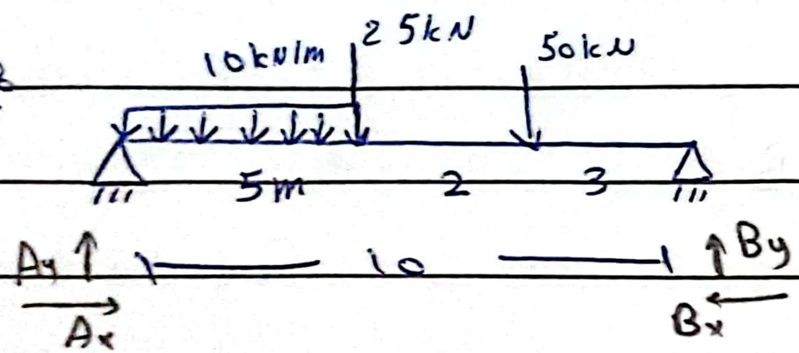
$\boxed{A_1 = 125}$

$\boxed{A_2 = -125}$

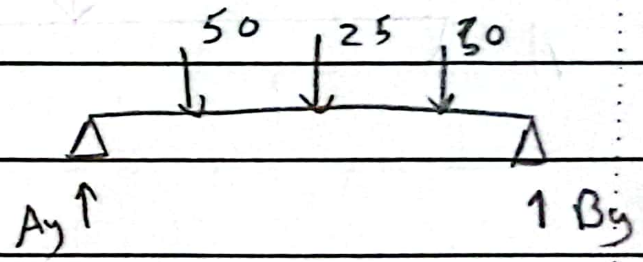
N O T E B O O K

Subject Date No.

Ex 8

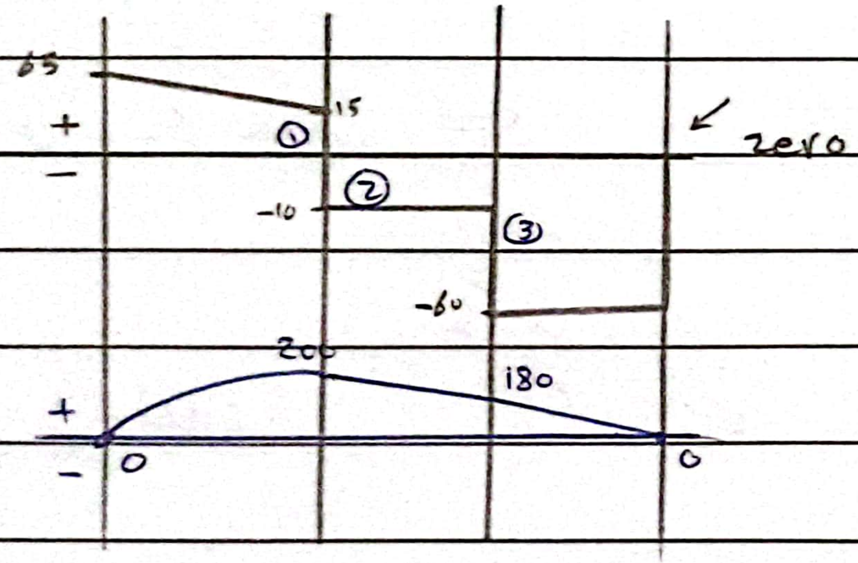


For reactions



By = 60 Ay = 65

D.S



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