1–1.

What is the weight in newtons of an object that has a mass of (a) 8 kg, (b) 0.04 kg, and (c) 760 Mg?

SOLUTION

(a) $W = 9.81(8) = 78.5\text{ N}$ **Ans.**

(b) $W = 9.81(0.04)\left(10^{-3}\right) = 3.92\left(10^{-4}\right)\text{ N} = 0.392\text{ mN}$ **Ans.**

(c) $W = 9.81(760)\left(10^{3}\right) = 7.46\left(10^{6}\right)\text{ N} = 7.46\text{ MN}$ **Ans.**

Ans:
$W = 78.5\text{ N}$
$W = 0.392\text{ mN}$
$W = 7.46\text{ MN}$

1–2.

Represent each of the following combinations of units in the correct SI form: (a) /KN/μs, (b) Mg/mN, and (c) MN/(kg · ms).

SOLUTION

(a) $\mathrm{kN}/\mu\mathrm{s} = 10^3\mathrm{N}/\left(10^{-6}\right)\mathrm{s} = \mathrm{GN/s}$ **Ans.**

(b) $\mathrm{Mg/mN} = 10^6\mathrm{g}/10^{-3}\,\mathrm{N} = \mathrm{Gg/N}$ **Ans.**

(c) $\mathrm{MN}/(\mathrm{kg}\cdot\mathrm{ms}) = 10^6\,\mathrm{N}/\mathrm{kg}(10^{-3}\,\mathrm{s}) = \mathrm{GN}/(\mathrm{kg}\cdot\mathrm{s})$ **Ans.**

Ans:
GN/s
Gg/N
GN/(kg · s)

1–3.

Represent each of the following combinations of unit in the correct SI form: (a) Mg/ms, (b) N/mm, (c) mN/(kg · μs).

SOLUTION

(a) $$\frac{\text{Mg}}{\text{ms}} = \frac{10^3 \text{ kg}}{10^{-3} \text{ s}} = 10^6 \text{ kg/s} = \text{Gg/s}$$ **Ans.**

(b) $$\frac{\text{N}}{\text{mm}} = \frac{1 \text{ N}}{10^{-3} \text{ m}} = 10^3 \text{ N/m} = \text{kN/m}$$ **Ans.**

(c) $$\frac{\text{mN}}{(\text{kg} \cdot \mu\text{s})} = \frac{10^{-3} \text{ N}}{10^{-6} \text{ kg} \cdot \text{s}} = \text{kN/(kg} \cdot \text{s)}$$ **Ans.**

Ans:
Gg/s
kN/m
kN/(kg · s)

***1–4.**

Convert: (a) 200 lb·ft to N·m, (b) 350 lb/ft^3 to kN/m^3, (c) 8 ft/h to mm/s. Express the result to three significant figures. Use an appropriate prefix.

SOLUTION

a) $(200 \text{ lb} \cdot \text{ft})\left(\dfrac{4.4482 \text{ N}}{1 \text{ lb}}\right)\left(\dfrac{0.3048 \text{ m}}{1 \text{ ft}}\right) = 271 \text{ N} \cdot \text{m}$ **Ans.**

b) $\left(\dfrac{350 \text{ lb}}{1 \text{ ft}^3}\right)\left(\dfrac{1 \text{ ft}}{0.3048 \text{ m}}\right)^3\left(\dfrac{4.4482 \text{ N}}{1 \text{ lb}}\right) = 55.0 \text{ kN/m}^3$ **Ans.**

c) $\left(\dfrac{8 \text{ ft}}{1 \text{ h}}\right)\left(\dfrac{1 \text{ h}}{3600 \text{ s}}\right)\left(\dfrac{0.3048 \text{ m}}{1 \text{ ft}}\right) = 0.677 \text{ mm/s}$ **Ans.**

Ans:
271 N·m
55.0 kN/m^3
0.677 mm/s

1–5.

Represent each of the following as a number between 0.1 and 1000 using an appropriate prefix: (a) 45 320 kN, (b) $568(10^5)$ mm, and (c) 0.00563 mg.

SOLUTION

(a) $45\,320 \text{ kN} = 45.3 \text{ MN}$ **Ans.**

(b) $568(10^5) \text{ mm} = 56.8 \text{ km}$ **Ans.**

(c) $0.00563 \text{ mg} = 5.63\ \mu\text{g}$ **Ans.**

Ans:
45.3 MN
56.8 km
5.63 μg

1–6.

Round off the following numbers to three significant figures: (a) 58 342 m, (b) 68.534 s, (c) 2553 N, and (d) 7555 kg.

SOLUTION

a) 58.3 km b) 68.5 s c) 2.55 kN d) 7.56 Mg **Ans.**

Ans:
58.3 km
68.5 s
2.55 kN
7.56 Mg

1–7.

Represent each of the following quantities in the correct SI form using an appropriate prefix: (a) 0.000 431 kg, (b) $35.3(10^3)$ N, and (c) 0.005 32 km.

SOLUTION

a) $0.000\,431 \text{ kg} = 0.000\,431\left(10^3\right) \text{ g} = 0.431 \text{ g}$ **Ans.**

b) $35.3\left(10^3\right) \text{ N} = 35.3 \text{ kN}$ **Ans.**

c) $0.005\,32 \text{ km} = 0.005\,32\left(10^3\right) \text{ m} = 5.32 \text{ m}$ **Ans.**

Ans:
0.431 g
35.3 kN
5.32 m

***1–8.**

Represent each of the following combinations of units in the correct SI form using an appropriate prefix: (a) Mg/mm, (b) mN/μs, (c) μm · Mg.

SOLUTION

a) $\text{Mg/mm} = \dfrac{10^3\text{ kg}}{10^{-3}\text{m}} = \dfrac{10^6\text{ kg}}{\text{m}} = \text{Gg/m}$ **Ans.**

b) $\text{mN}/\mu\text{s} = \dfrac{10^{-3}\text{ N}}{10^{-6}\text{ s}} = \dfrac{10^3\text{ N}}{\text{s}} = \text{kN/s}$ **Ans.**

c) $\mu\text{m}\cdot\text{Mg} = \left[10^{-6}\text{ m}\right]\cdot\left[10^3\text{ kg}\right] = (10)^{-3}\text{ m}\cdot\text{kg}$

$= \text{mm}\cdot\text{kg}$ **Ans.**

Ans:
Gg/m
kN/s
mm · kg

1–9.

Represent each of the following combinations of units in the correct SI form using an appropriate prefix: (a) m/ms, (b) μkm, (c) ks/mg, and (d) km $\cdot$ μN.

SOLUTION

a) $\text{m/ms} = \left(\dfrac{\text{m}}{(10)^{-3}\ \text{s}}\right) = \left(\dfrac{(10)^{3}\ \text{m}}{\text{s}}\right) = \text{km/s}$ **Ans.**

b) $\mu\text{km} = (10)^{-6}(10)^{3}\ \text{m} = (10)^{-3}\ \text{m} = \text{mm}$ **Ans.**

c) $\text{ks/mg} = \left(\dfrac{(10)^{3}\ \text{s}}{(10)^{-6}\ \text{kg}}\right) = \left(\dfrac{(10)^{9}\ \text{s}}{\text{kg}}\right) = \text{Gs/kg}$ **Ans.**

d) $\text{km} \cdot \mu\text{N} = [(10)^{3}\ \text{m}][(10)^{-6}\ \text{N}] = (10)^{-3}\ \text{m} \cdot \text{N} = \text{mm} \cdot \text{N}$ **Ans.**

Ans:
km/s
mm
Gs/kg
mm $\cdot$ N

1–10.

Represent each of the following combinations of units in the correct SI form: (a) $\mathrm{GN \cdot \mu m}$, (b) $\mathrm{kg/\mu m}$, (c) $\mathrm{N/ks^2}$, and (d) $\mathrm{KN/\mu s}$.

SOLUTION

(a) $\mathrm{GN \cdot \mu m} = 10^9(10^{-6})\ \mathrm{N \cdot m} = \mathrm{kN \cdot m}$ **Ans.**

(b) $\mathrm{kg/\mu m} = 10^3\ \mathrm{g}/10^{-6}\ \mathrm{m} = \mathrm{Gg/m}$ **Ans.**

(c) $\mathrm{N/ks^2} = \mathrm{N}/10^6\ \mathrm{s^2} = 10^{-6}\ \mathrm{N/s^2} = \mathrm{\mu N/s^2}$ **Ans.**

(d) $\mathrm{kN/\mu s} = 10^3\ \mathrm{N}/10^{-6}\ \mathrm{s} = 10^9\ \mathrm{N/s} = \mathrm{GN/s}$ **Ans.**

Ans:
$\mathrm{kN \cdot m}$
$\mathrm{Gg/m}$
$\mathrm{\mu N/s^2}$
$\mathrm{GN/s}$

1–11.

Represent each of the following with SI units having an appropriate prefix: (a) 8653 ms, (b) 8368 N, (c) 0.893 kg.

SOLUTION

a) $8653 \text{ ms} = 8.653(10)^3(10^{-3}) \text{ s} = 8.653 \text{ s}$ **Ans.**

b) $8368 \text{ N} = 8.368 \text{ kN}$ **Ans.**

c) $0.893 \text{ kg} = 893(10^{-3})(10^3) \text{ g} = 893 \text{ g}$ **Ans.**

Ans:
8.653 s
8.368 kN
893 g

***1–12.**

Evaluate each of the following to three significant figures and express each answer in SI units using an appropriate prefix: (a) $(684\ \mu\text{m})/(43\ \text{ms})$, (b) $(28\ \text{ms})(0.0458\ \text{Mm})/(348\ \text{mg})$, (c) (2.68 mm)(426 Mg).

SOLUTION

a) $(684\ \mu\text{m})/43\ \text{ms} = \dfrac{684(10^{-6})\ \text{m}}{43(10^{-3})\ \text{s}} = \dfrac{15.9(10^{-3})\ \text{m}}{\text{s}}$

$= 15.9\ \text{mm/s}$ **Ans.**

b) $(28\ \text{ms})(0.0458\ \text{Mm})/(348\ \text{mg}) = \dfrac{\left[28(10^{-3})\ \text{s}\right]\left[45.8(10^{-3})(10)^{6}\ \text{m}\right]}{348(10^{-3})(10^{-3})\ \text{kg}}$

$= \dfrac{3.69(10^{6})\ \text{m}\cdot\text{s}}{\text{kg}} = 3.69\ \text{Mm}\cdot\ \text{s/kg}$ **Ans.**

c) $(2.68\ \text{mm})(426\ \text{Mg}) = \left[2.68\left(10^{-3}\right)\ \text{m}\right]\left[426\left(10^{3}\right)\ \text{kg}\right]$

$= 1.14\left(10^{3}\right)\ \text{m}\cdot\ \text{kg} = 1.14\ \text{km}\cdot\text{kg}$ **Ans.**

Ans:
15.9 mm/s
3.69 Mm · s/kg
1.14 km · kg

1–13.

The density (mass/volume) of aluminum is 5.26 slug/ft^3. Determine its density in SI units. Use an appropriate prefix.

SOLUTION

$$5.26 \text{ slug/ft}^3 = \left(\frac{5.26 \text{ slug}}{\text{ft}^3}\right)\left(\frac{\text{ft}}{0.3048 \text{ m}}\right)^3\left(\frac{14.59 \text{ kg}}{1 \text{ slug}}\right)$$

$$= 2.71 \text{ Mg/m}^3 \qquad \textbf{Ans.}$$

Ans:
2.71 Mg/m^3

1–14.

Evaluate each of the following to three significant figures and express each answer in SI units using an appropriate prefix: (a) $(212\ \text{mN})^2$, (b) $(52800\ \text{ms})^2$, and (c) $[548(10^6)]^{1/2}$ ms.

SOLUTION

(a) $(212\ \text{mN})^2 = \left[212(10)^{-3}\ \text{N}\right]^2 = 0.0449\ \text{N}^2 = 44.9(10)^{-3}\ \text{N}^2$ **Ans.**

(b) $(52\,800\ \text{ms})^2 = \left[52\,800(10)^{-3}\right]^2\ \text{s}^2 = 2788\ \text{s}^2 = 2.79\left(10^3\right)\ \text{s}^2$ **Ans.**

(c) $\left[548(10)^6\right]^{\frac{1}{2}}\ \text{ms} = (23\,409)(10)^{-3}\ \text{s} = 23.4(10)^3(10)^{-3}\ \text{s} = 23.4\ \text{s}$ **Ans.**

Ans:
$44.9(10)^{-3}\ \text{N}^2$
$2.79\left(10^3\right)\ \text{s}^2$
$23.4\ \text{s}$

1–15.

Using the SI system of units, show that Eq. 1–2 is a dimensionally homogeneous equation which gives F in newtons. Determine to three significant figures the gravitational force acting between two spheres that are touching each other. The mass of each sphere is 200 kg and the radius is 300 mm.

SOLUTION

Using Eq. 1–2,

$$F = G\frac{m_1 m_2}{r^2}$$

$$\mathrm{N} = \left(\frac{\mathrm{m}^3}{\mathrm{kg}\cdot\mathrm{s}^2}\right)\left(\frac{\mathrm{kg}\cdot\mathrm{kg}}{\mathrm{m}^2}\right) = \frac{\mathrm{kg}\cdot\mathrm{m}}{\mathrm{s}^2} \qquad \textbf{\textit{(Q.E.D.)}}$$

$$F = G\frac{m_1 m_2}{r^2}$$

$$= 66.73\left(10^{-12}\right)\left[\frac{200(200)}{0.6^2}\right]$$

$$= 7.41\left(10^{-6}\right)\,\mathrm{N} = 7.41\ \mu\mathrm{N} \qquad \textbf{Ans.}$$

Ans:
7.41 μN

***1–16.**

The *pascal* (Pa) is actually a very small unit of pressure. To show this, convert $1 \text{ Pa} = 1 \text{ N/m}^2$ to lb/ft^2. Atmospheric pressure at sea level is 14.7 lb/in^2. How many pascals is this?

SOLUTION

Using Table 1–2, we have

$$1 \text{ Pa} = \frac{1 \text{ N}}{\text{m}^2}\left(\frac{1 \text{ lb}}{4.4482 \text{ N}}\right)\left(\frac{0.3048^2 \text{ m}^2}{1 \text{ ft}^2}\right) = 20.9\left(10^{-3}\right) \text{lb/ft}^2 \qquad \textbf{Ans.}$$

$$1 \text{ ATM} = \frac{14.7 \text{ lb}}{\text{in}^2}\left(\frac{4.448 \text{ N}}{1 \text{ lb}}\right)\left(\frac{144 \text{ in}^2}{1 \text{ ft}^2}\right)\left(\frac{1 \text{ ft}^2}{0.3048^2 \text{ m}^2}\right)$$

$$= 101.3\left(10^3\right) \text{N/m}^2$$

$$= 101 \text{ kPa} \qquad \textbf{Ans.}$$

Ans:
$20.9\left(10^{-3}\right) \text{lb/ft}^2$
101 kPa

1–17.

Water has a density of $1.94\ \text{slug/ft}^3$. What is the density expressed in SI units? Express the answer to three significant figures.

SOLUTION

Using Table 1–2, we have

$$\rho_w = \left(\frac{1.94\ \text{slug}}{\text{ft}^3}\right)\left(\frac{14.5938\ \text{kg}}{1\ \text{slug}}\right)\left(\frac{1\ \text{ft}^3}{0.3048^3\ \text{m}^3}\right)$$

$$= 999.8\ \text{kg/m}^3 = 1.00\ \text{Mg/m}^3 \qquad \textbf{Ans.}$$

Ans:
$1.00\ \text{Mg/m}^3$

1–18.

Evaluate each of the following to three significant figures and express each answer in SI units using an appropriate prefix: (a) 354 mg(45 km)/(0.0356 kN), (b) (0.004 53 Mg) (201 ms), and (c) 435 MN/23.2 mm.

SOLUTION

a) $$(354\text{ mg})(45\text{ km})/(0.0356\text{ kN}) = \frac{\left[354\left(10^{-3}\right)\text{ g}\right]\left[45\left(10^{3}\right)\text{ m}\right]}{0.0356\left(10^{3}\right)\text{ N}}$$

$$= \frac{0.447\left(10^{3}\right)\text{ g}\cdot\text{m}}{\text{N}}$$

$$= 0.447\text{ kg}\cdot\text{m/N}$$ **Ans.**

b) $$(0.00453\text{ Mg})(201\text{ ms}) = \left[4.53\left(10^{-3}\right)\left(10^{3}\right)\text{ kg}\right]\left[201\left(10^{-3}\right)\text{ s}\right]$$

$$= 0.911\text{ kg}\cdot\text{s}$$ **Ans.**

c) $$435\text{ MN}/23.2\text{ mm} = \frac{435\left(10^{6}\right)\text{ N}}{23.2\left(10^{-3}\right)\text{ m}} = \frac{18.75\left(10^{9}\right)\text{ N}}{\text{m}} = 18.8\text{ GN/m}$$ **Ans.**

Ans:
0.447 kg · m/N
0.911 kg · s
18.8 GN/m

1–19.

A concrete column has a diameter of 350 mm and a length of 2 m. If the density (mass/volume) of concrete is 2.45 Mg/m^3, determine the weight of the column in pounds.

SOLUTION

$$V = \pi r^2 h = \pi\left(\tfrac{0.35}{2}\text{ m}\right)^2 (2\text{ m}) = 0.1924\text{ m}^3$$

$$m = \rho V = \left(\frac{2.45(10^3)\text{kg}}{\text{m}^3}\right)\left(0.1924\text{ m}^3\right) = 471.44\text{ kg}$$

$$W = mg = (471.44\text{ kg})\left(9.81\text{ m/s}^2\right) = 4.6248\left(10^3\right)\text{ N}$$

$$W = \left[4.6248\left(10^3\right)\text{ N}\right]\left(\frac{1\text{ lb}}{4.4482\text{ N}}\right) = 1.04\text{ kip}$$ **Ans.**

Ans:
1.04 kip

***1–20.**

If a man weighs 155 lb on earth, specify (a) his mass in slugs, (b) his mass in kilograms, and (c) his weight in newtons. If the man is on the moon, where the acceleration due to gravity is $g_m = 5.30\ \mathrm{ft/s^2}$, determine (d) his weight in pounds, and (e) his mass in kilograms.

SOLUTION

a) $m = \dfrac{155}{32.2} = 4.81\ \text{slug}$ **Ans.**

b) $m = 155\left[\dfrac{14.59\ \text{kg}}{32.2}\right] = 70.2\ \text{kg}$ **Ans.**

c) $W = 155(4.4482) = 689\ \text{N}$ **Ans.**

d) $W = 155\left[\dfrac{5.30}{32.2}\right] = 25.5\ \text{lb}$ **Ans.**

e) $m = 155\left[\dfrac{14.59\ \text{kg}}{32.2}\right] = 70.2\ \text{kg}$ **Ans.**

Also,

$m = 25.5\left[\dfrac{14.59\ \text{kg}}{5.30}\right] = 70.2\ \text{kg}$ **Ans.**

Ans:
4.81 slug
70.2 kg
689 N
25.5 lb
70.2 kg

1–21.

Two particles have a mass of 8 kg and 12 kg, respectively. If they are 800 mm apart, determine the force of gravity acting between them. Compare this result with the weight of each particle.

SOLUTION

$$F = G\frac{m_1\, m_2}{r^2}$$

Where $G = 66.73\left(10^{-12}\right)\ \text{m}^3/(\text{kg}\cdot\text{s}^2)$

$$F = 66.73\left(10^{-12}\right)\left[\frac{8(12)}{(0.8)^2}\right] = 10.0\left(10^{-9}\right)\ \text{N} = 10.0\ \text{nN}$$ **Ans.**

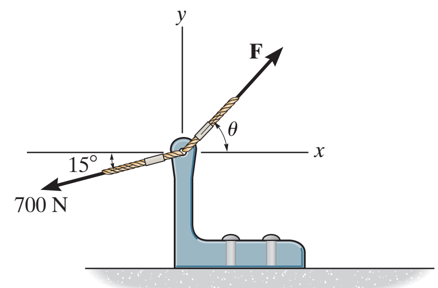
$$W_1 = 8(9.81) = 78.5\ \text{N}$$ **Ans.**

$$W_2 = 12(9.81) = 118\ \text{N}$$ **Ans.**

Ans:
$F = 10.0$ nN
$W_1 = 78.5$ N
$W_2 = 118$ N

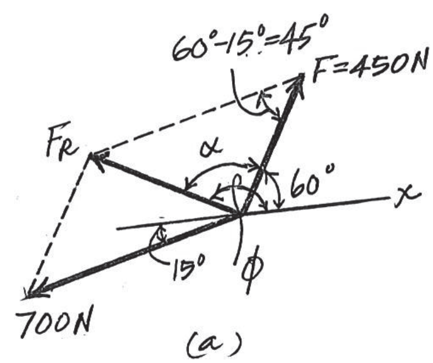
2–1.

If $\theta = 60°$ and $F = 450$ N, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



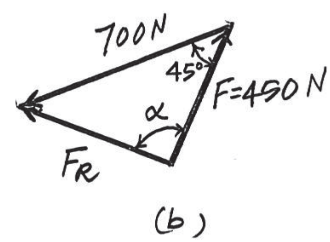
Applying the law of consines to Fig. *b*,

$$F_R = \sqrt{700^2 + 450^2 - 2(700)(450)\cos 45°}$$

$$= 497.01 \text{ N} = 497 \text{ N} \qquad \textbf{Ans.}$$

This yields

$$\frac{\sin\alpha}{700} = \frac{\sin 45°}{497.01} \qquad \alpha = 95.19°$$



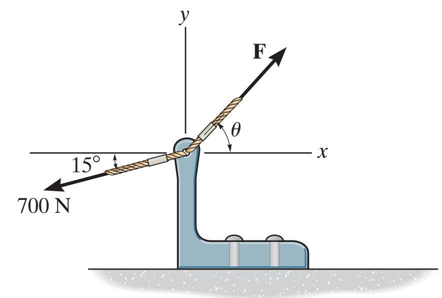
Thus, the direction of angle ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis, is

$$\phi = \alpha + 60° = 95.19° + 60° = 155° \qquad \textbf{Ans.}$$

Ans:
$F_R = 497$ N
$\phi = 155°$

2–2.

If the magnitude of the resultant force is to be 500 N, directed along the positive y axis, determine the magnitude of force **F** and its direction θ.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

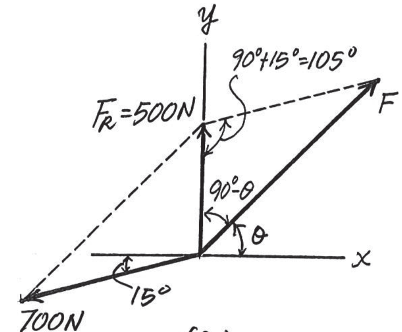
$$F = \sqrt{500^2 + 700^2 - 2(500)(700)\cos 105^\circ}$$

$$= 959.78 \text{ N} = 960 \text{ N} \qquad \textbf{Ans.}$$

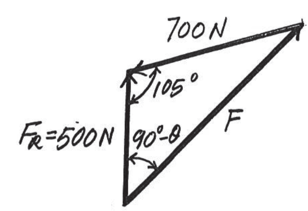
Applying the law of sines to Fig. *b*, and using this result, yields

$$\frac{\sin(90^\circ + \theta)}{700} = \frac{\sin 105^\circ}{959.78}$$

$$\theta = 45.2^\circ \qquad \textbf{Ans.}$$



(a)

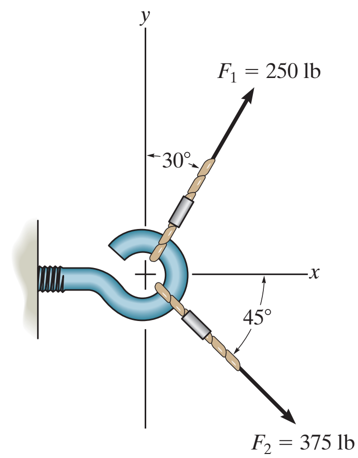


(b)

Ans:
$F = 960$ N
$\theta = 45.2^\circ$

2–3.

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured counterclockwise from the positive x axis.



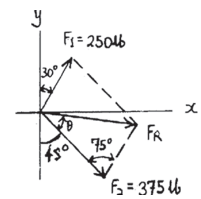
SOLUTION

$$F_R = \sqrt{(250)^2 + (375)^2 - 2(250)(375)\cos 75^\circ} = 393.2 = 393 \text{ lb} \qquad \textbf{Ans.}$$

$$\frac{393.2}{\sin 75^\circ} = \frac{250}{\sin \theta}$$

$$\theta = 37.89^\circ$$

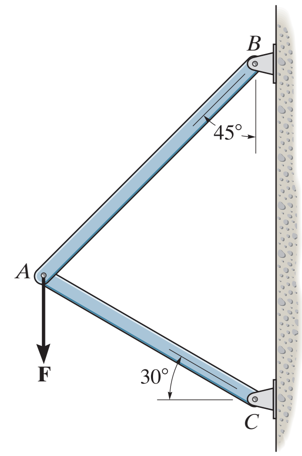
$$\phi = 360^\circ - 45^\circ + 37.89^\circ = 353^\circ \qquad \textbf{Ans.}$$



Ans:
$F_R = 393$ lb
$\phi = 353^\circ$

***2–4.**

The vertical force **F** acts downward at A on the two-membered frame. Determine the magnitudes of the two components of **F** directed along the axes of AB and AC. Set $F = 500$ N.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

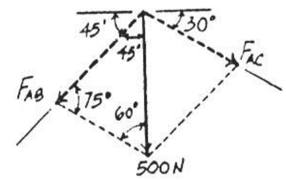
Trigonometry: Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60°} = \frac{500}{\sin 75°}$$

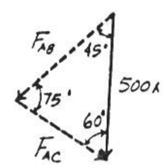
$$F_{AB} = 448 \text{ N} \qquad \textbf{Ans.}$$

$$\frac{F_{AC}}{\sin 45°} = \frac{500}{\sin 75°}$$

$$F_{AC} = 366 \text{ N} \qquad \textbf{Ans.}$$



(a)

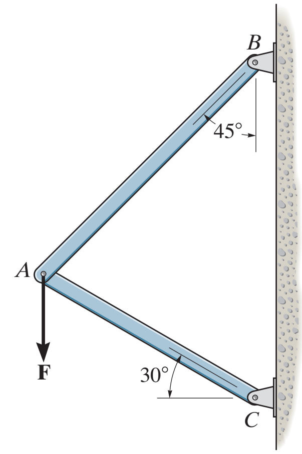


(b)

Ans:
$F_{AB} = 448$ N
$F_{AC} = 366$ N

2–5.

Solve Prob. 2-4 with $F = 350$ lb.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

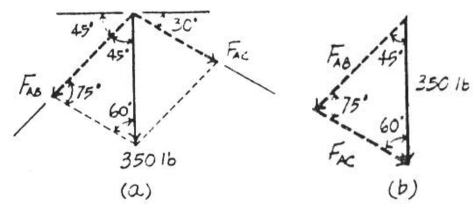
Trigonometry: Using the law of sines (Fig. *b*), we have

$$\frac{F_{AB}}{\sin 60°} = \frac{350}{\sin 75°}$$

$$F_{AB} = 314 \text{ lb}$$ **Ans.**

$$\frac{F_{AC}}{\sin 45°} = \frac{350}{\sin 75°}$$

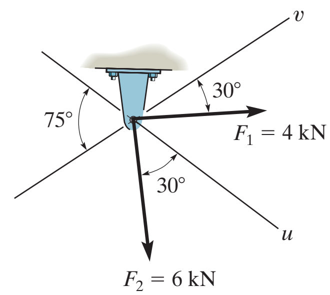
$$F_{AC} = 256 \text{ lb}$$ **Ans.**



Ans:
$F_{AB} = 314$ lb
$F_{AC} = 256$ lb

2–6.

Determine the magnitude of the resultant force $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$ and its direction, measured clockwise from the positive u axis.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. a,

Trigonometry. Applying Law of cosines by referring to Fig. b,

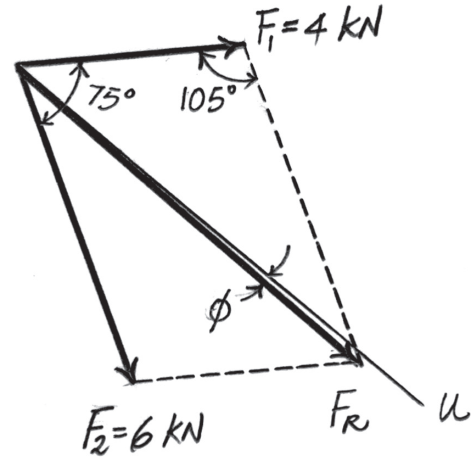
$$F_R = \sqrt{4^2 + 6^2 - 2(4)(6)\cos 105^\circ} = 8.026 \text{ kN} = 8.03 \text{ kN} \qquad \textbf{Ans.}$$

Using this result to apply Law of sines, Fig. b,

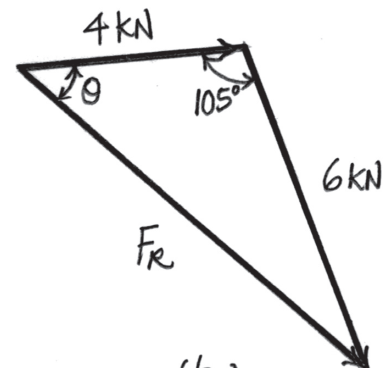
$$\frac{\sin\theta}{6} = \frac{\sin 105^\circ}{8.026}; \qquad \theta = 46.22^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured clockwise from the positive u axis is

$$\phi = 46.22^\circ - 45^\circ = 1.22^\circ \qquad \textbf{Ans.}$$



(a)

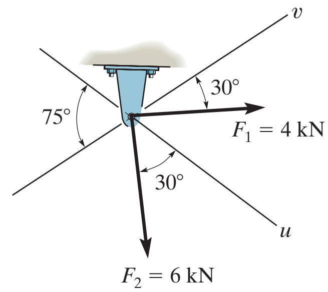


(b)

Ans:
$\phi = 1.22^\circ$

2–7.

Resolve the force $\mathbf{F}_1$ into components acting along the u and v axes and determine the magnitudes of the components.



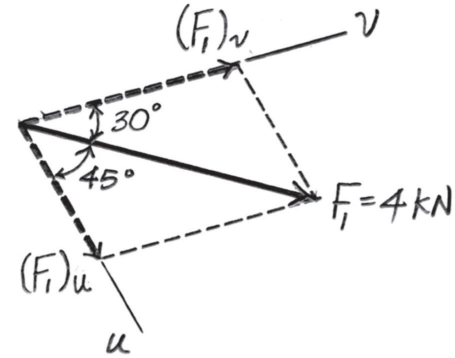
SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. a,

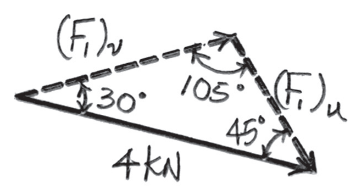
Trigonometry. Applying the sines law by referring to Fig. b.

$$\frac{(F_1)_v}{\sin 45^\circ} = \frac{4}{\sin 105^\circ}; \qquad (F_1)_v = 2.928 \text{ kN} = 2.93 \text{ kN} \qquad \textbf{Ans.}$$

$$\frac{(F_1)_u}{\sin 30^\circ} = \frac{4}{\sin 105^\circ}; \qquad (F_1)_u = 2.071 \text{ kN} = 2.07 \text{ kN} \qquad \textbf{Ans.}$$



(a)

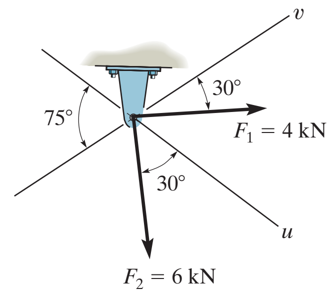


(b)

Ans:
$(F_1)_v = 2.93$ kN
$(F_1)_u = 2.07$ kN

***2–8.**

Resolve the force $\mathbf{F}_2$ into components acting along the u and v axes and determine the magnitudes of the components.

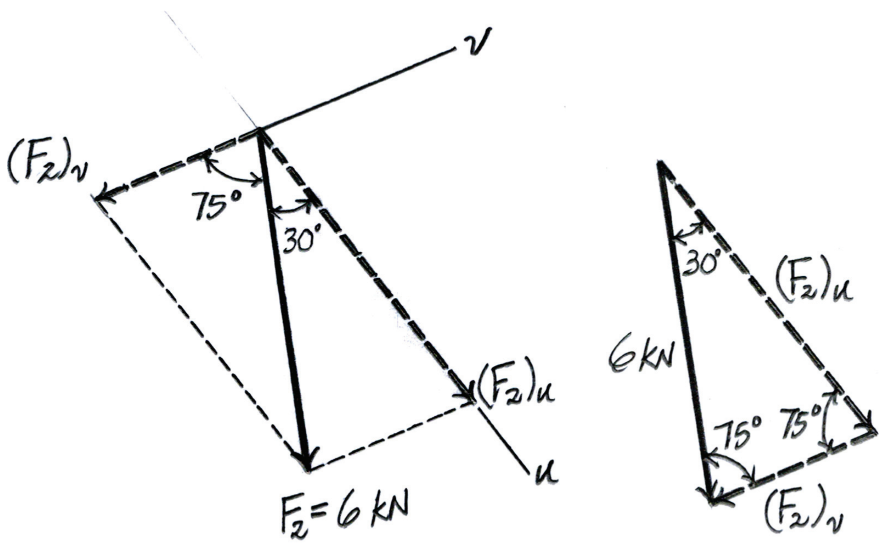


SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*,
Trigonometry. Applying the sines law of referring to Fig. *b*,

$$\frac{(F_2)_u}{\sin 75^\circ} = \frac{6}{\sin 75^\circ}; \qquad (F_2)_u = 6.00 \text{ kN} \qquad \textbf{Ans.}$$

$$\frac{(F_2)_v}{\sin 30^\circ} = \frac{6}{\sin 75^\circ}; \qquad (F_2)_v = 3.106 \text{ kN} = 3.11 \text{ kN} \qquad \textbf{Ans.}$$

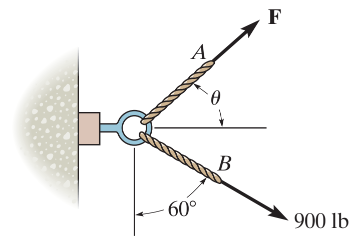


(a) (b)

Ans:
$(F_2)_u = 6.00$ kN
$(F_2)_v = 3.11$ kN

2–9.

If the resultant force acting on the support is to be 1200 lb, directed horizontally to the right, determine the force **F** in rope *A* and the corresponding angle θ.



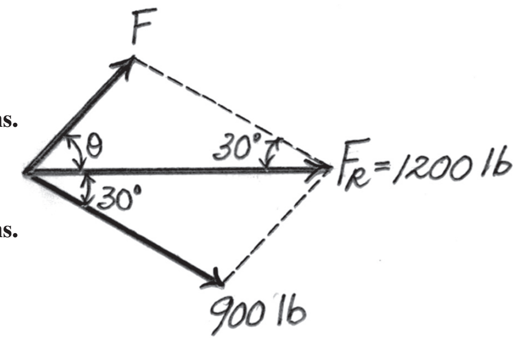
SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*,
Trigonometry. Applying the law of cosines by referring to Fig. *b*,

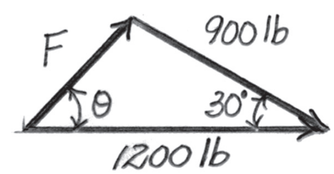
$$F = \sqrt{900^2 + 1200^2 - 2(900)(1200)\cos 30^\circ} = 615.94 \text{ lb} = 616 \text{ lb}$$ **Ans.**

Using this result to apply the sines law, Fig. *b*,

$$\frac{\sin\theta}{900} = \frac{\sin 30^\circ}{615.94}; \qquad \theta = 46.94^\circ = 46.9^\circ$$ **Ans.**



(a)

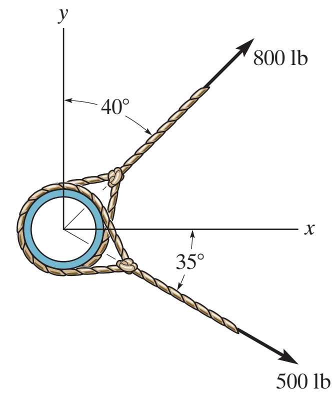


(b)

Ans:
$F = 616$ lb
$\theta = 46.9^\circ$

2–10.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*,
Trigonometry. Applying the law of cosines by referring to Fig. *b*,

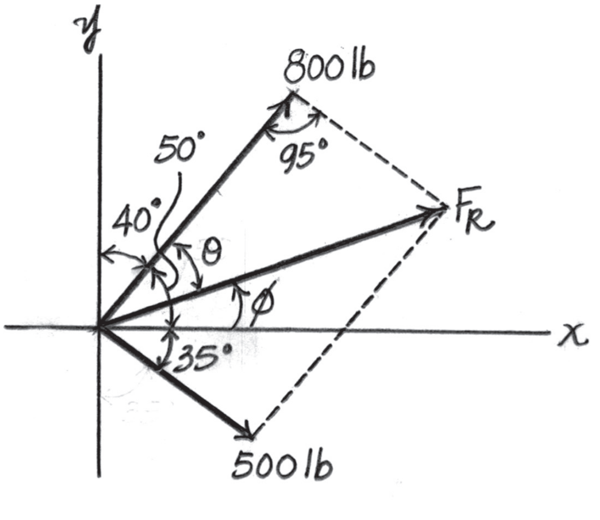
$$F_R = \sqrt{800^2 + 500^2 - 2(800)(500)\cos 95^\circ} = 979.66 \text{ lb} = 980 \text{ lb} \qquad \textbf{Ans.}$$

Using this result to apply the sines law, Fig. *b*,

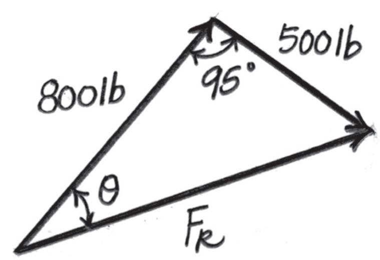
$$\frac{\sin\theta}{500} = \frac{\sin 95^\circ}{979.66}; \qquad \theta = 30.56^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

$$\phi = 50^\circ - 30.56^\circ = 19.44^\circ = 19.4^\circ \qquad \textbf{Ans.}$$



(a)

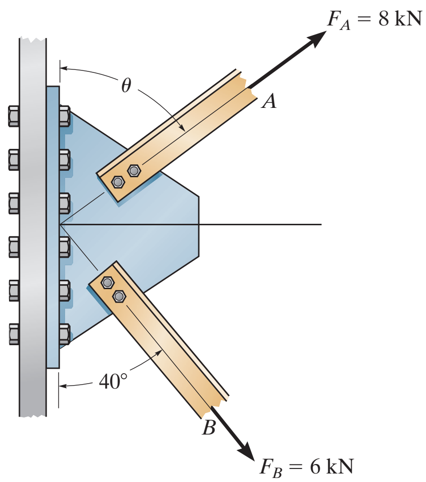


(b)

Ans:
$F_R = 980$ lb
$\phi = 19.4^\circ$

2–11.

The plate is subjected to the two forces at A and B as shown. If $\theta = 60^\circ$, determine the magnitude of the resultant of these two forces and its direction measured clockwise from the horizontal.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of cosines (Fig. *b*), we have

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 100^\circ}$$

$$= 10.80 \text{ kN} = 10.8 \text{ kN} \qquad \textbf{Ans.}$$

The angle θ can be determined using law of sines (Fig. b).

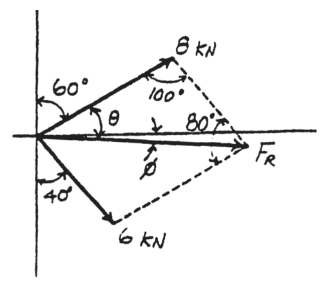
$$\frac{\sin\theta}{6} = \frac{\sin 100^\circ}{10.80}$$

$$\sin\theta = 0.5470$$

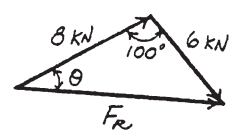
$$\theta = 33.16^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured from the x axis is

$$\phi = 33.16^\circ - 30^\circ = 3.16^\circ \qquad \textbf{Ans.}$$



(a)

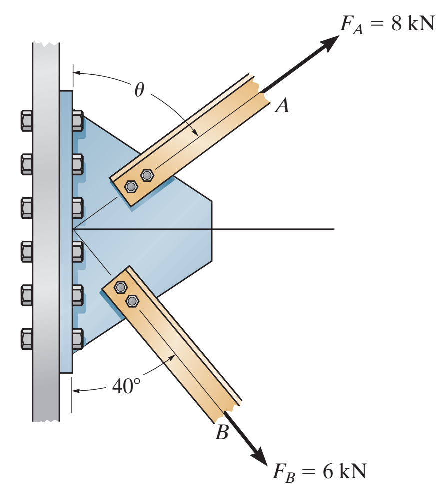


(b)

Ans:
$F_R = 10.8$ kN
$\phi = 3.16^\circ$

***2–12.**

Determine the angle of θ for connecting member A to the plate so that the resultant force of $\mathbf{F}_A$ and $\mathbf{F}_B$ is directed horizontally to the right. Also, what is the magnitude of the resultant force?



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig .*b*), we have

$$\frac{\sin(90^\circ - \theta)}{6} = \frac{\sin 50^\circ}{8}$$

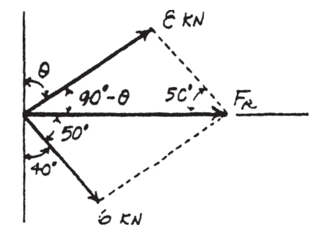
$$\sin(90^\circ - \theta) = 0.5745$$

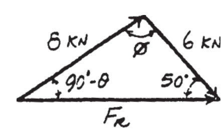
$$\theta = 54.93^\circ = 54.9^\circ \qquad \textbf{Ans.}$$

From the triangle, $\phi = 180^\circ - (90^\circ - 54.93^\circ) - 50^\circ = 94.93^\circ$. Thus, using law of cosines, the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{8^2 + 6^2 - 2(8)(6)\cos 94.93^\circ}$$

$$= 10.4 \text{ kN} \qquad \textbf{Ans.}$$



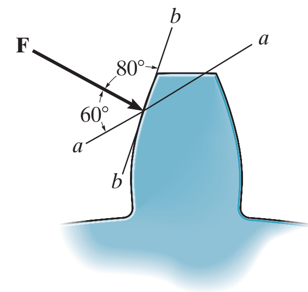


(b)

Ans:
$\theta = 54.9^\circ$
$F_R = 10.4$ kN

2–13.

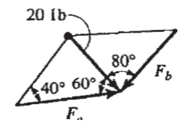
The force acting on the gear tooth is $F = 20$ lb. Resolve this force into two components acting along the lines *aa* and *bb*.



SOLUTION

$$\frac{20}{\sin 40^\circ} = \frac{F_a}{\sin 80^\circ}; \qquad F_a = 30.6 \text{ lb}$$ **Ans.**

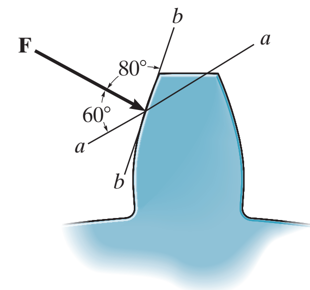
$$\frac{20}{\sin 40^\circ} = \frac{F_b}{\sin 60^\circ}; \qquad F_b = 26.9 \text{ lb}$$ **Ans.**



Ans:
$F_a = 30.6$ lb
$F_b = 26.9$ lb

2–14.

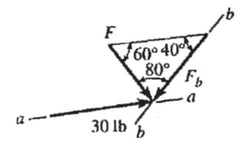
The component of force **F** acting along line *aa* is required to be 30 lb. Determine the magnitude of **F** and its component along line *bb*.



SOLUTION

$$\frac{30}{\sin 80^\circ} = \frac{F}{\sin 40^\circ}; \qquad F = 19.6 \text{ lb} \qquad \textbf{Ans.}$$

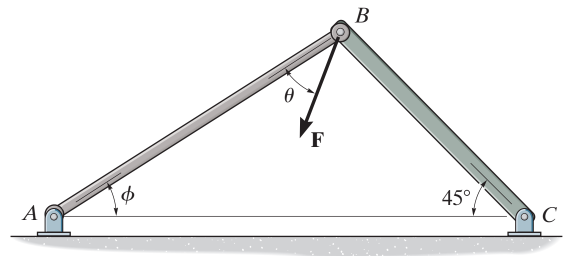
$$\frac{30}{\sin 80^\circ} = \frac{F_b}{\sin 60^\circ}; \qquad F_b = 26.4 \text{ lb} \qquad \textbf{Ans.}$$



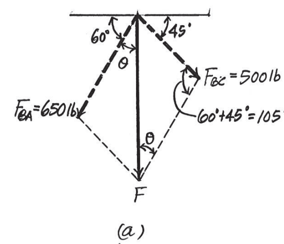
Ans:
$F = 19.6$ lb
$F_b = 26.4$ lb

2–15.

Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A, and the component acting along member BC is 500 lb, directed from B towards C. Determine the magnitude of **F** and its direction θ. Set $\phi = 60°$.



SOLUTION



The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively.

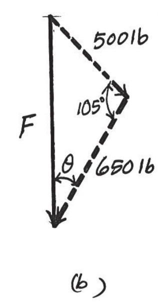
Applying the law of cosines to Fig. *b*,

$$F = \sqrt{500^2 + 650^2 - 2(500)(650)\cos 105°}$$

$$= 916.91 \text{ lb} = 917 \text{ lb}$$ **Ans.**

Using this result and applying the law of sines to Fig. *b*, yields

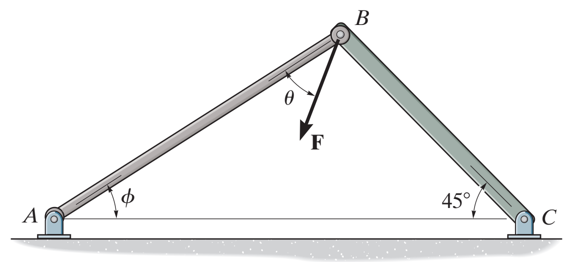
$$\frac{\sin\theta}{500} = \frac{\sin 105°}{916.91} \qquad \theta = 31.8°$$ **Ans.**



Ans:
$F = 917$ lb
$\theta = 31.8°$

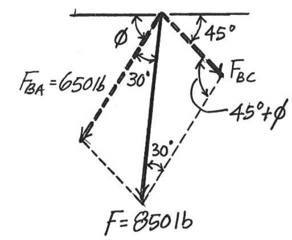
***2–16.**

Force **F** acts on the frame such that its component acting along member AB is 650 lb, directed from B towards A. Determine the required angle ϕ ($0° \le \phi \le 45°$) and the component acting along member BC. Set $F = 850$ lb and $\theta = 30°$.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



(a)

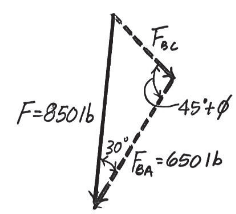
Applying the law of cosines to Fig. *b*,

$$F_{BC} = \sqrt{850^2 + 650^2 - 2(850)(650)\cos 30°}$$

$$= 433.64 \text{ lb} = 434 \text{ lb} \qquad \textbf{Ans.}$$

Using this result and applying the sine law to Fig. *b*, yields

$$\frac{\sin(45° + \phi)}{850} = \frac{\sin 30°}{433.64} \qquad \phi = 33.5° \qquad \textbf{Ans.}$$

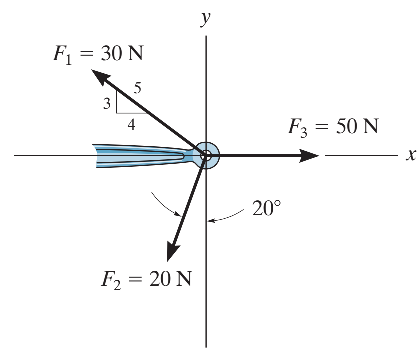


(b)

Ans:
$F_{BC} = 434$ lb
$\phi = 33.5°$

2–17.

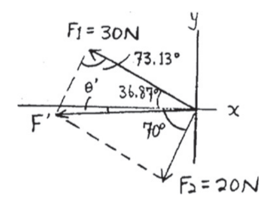
Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_2$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_3$.



SOLUTION

$$F' = \sqrt{(20)^2 + (30)^2 - 2(20)(30)\cos 73.13^\circ} = 30.85\text{ N}$$

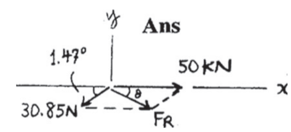
$$\frac{30.85}{\sin 73.13^\circ} = \frac{30}{\sin(70^\circ - \theta')}; \qquad \theta' = 1.47^\circ$$



$$F_R = \sqrt{(30.85)^2 + (50)^2 - 2(30.85)(50)\cos 1.47^\circ} = 19.18 = 19.2\text{ N} \qquad \textbf{Ans.}$$

$$\frac{19.18}{\sin 1.47^\circ} = \frac{30.85}{\sin\theta}; \qquad \theta = 2.37^\circ \ \searrow \qquad \textbf{Ans.}$$

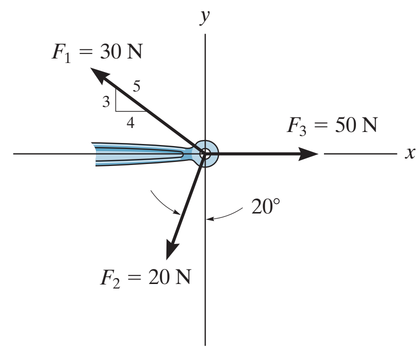
Ans



Ans:
$F_R = 19.2$ N
$\theta = 2.37^\circ \searrow$

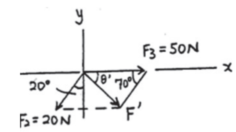
2–18.

Determine the magnitude and direction of the resultant $\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$ of the three forces by first finding the resultant $\mathbf{F}' = \mathbf{F}_2 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_1$.



SOLUTION

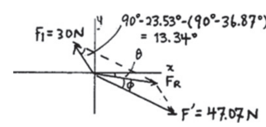
$$F' = \sqrt{(20)^2 + (50)^2 - 2(20)(50)\cos 70^\circ} = 47.07 \text{ N}$$



$$\frac{20}{\sin\theta'} = \frac{47.07}{\sin 70^\circ}; \qquad \theta' = 23.53^\circ$$

$$F_R = \sqrt{(47.07)^2 + (30)^2 - 2(47.07)(30)\cos 13.34^\circ} = 19.18 = 19.2 \text{ N} \qquad \textbf{Ans.}$$

$$\frac{19.18}{\sin 13.34^\circ} = \frac{30}{\sin\phi}; \qquad \phi = 21.15^\circ$$

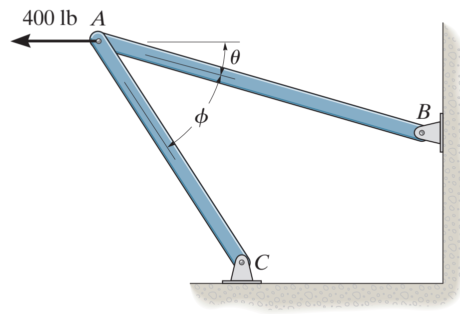


$$\theta = 23.53^\circ - 21.15^\circ = 2.37^\circ \measuredangle \qquad \textbf{Ans.}$$

Ans:
$F_R = 19.2$ N
$\theta = 2.37^\circ \measuredangle$

2–19.

Determine the design angle θ ($0° \leq \theta \leq 90°$) for strut AB so that the 400-lb horizontal force has a component of 500 lb directed from A towards C. What is the component of force acting along member AB? Take $\phi = 40°$.



SOLUTION

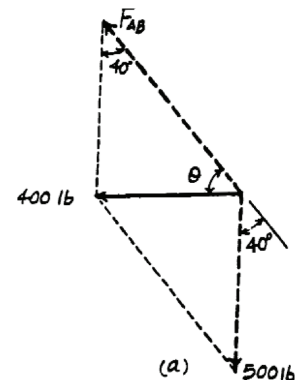
Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of sines (Fig. *b*), we have

$$\frac{\sin\theta}{500} = \frac{\sin 40°}{400}$$

$$\sin\theta = 0.8035$$

$$\theta = 53.46° = 53.5° \qquad \textbf{Ans.}$$



(a)

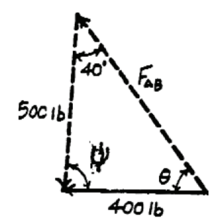
Thus,

$$\psi = 180° - 40° - 53.46° = 86.54°$$

Using law of sines (Fig. *b*)

$$\frac{F_{AB}}{\sin 86.54°} = \frac{400}{\sin 40°}$$

$$F_{AB} = 621 \text{ lb} \qquad \textbf{Ans.}$$

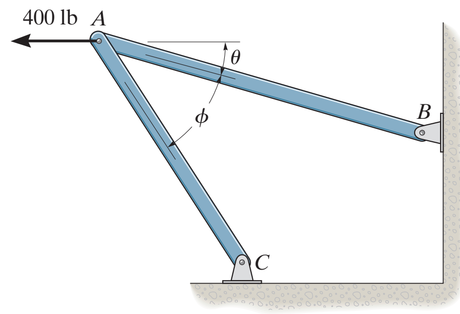


(b)

Ans:
$\theta = 53.5°$
$F_{AB} = 621$ lb

***2–20.**

Determine the design angle ϕ $(0° \leq \phi \leq 90°)$ between struts AB and AC so that the 400-lb horizontal force has a component of 600 lb which acts up to the left, in the same direction as from B towards A. Take $\theta = 30°$.



SOLUTION

Parallelogram Law: The parallelogram law of addition is shown in Fig. *a*.

Trigonometry: Using law of cosines (Fig. *b*), we have

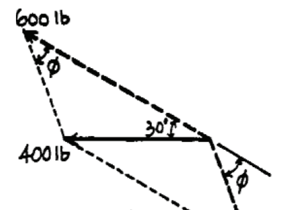
$$F_{AC} = \sqrt{400^2 + 600^2 - 2(400)(600)\cos 30°} = 322.97 \text{ lb}$$

The angle ϕ can be determined using law of sines (Fig. *b*).

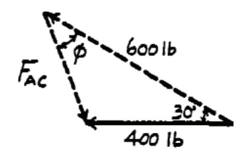
$$\frac{\sin \phi}{400} = \frac{\sin 30°}{322.97}$$

$$\sin \phi = 0.6193$$

$$\phi = 38.3° \qquad \textbf{Ans.}$$



(a)

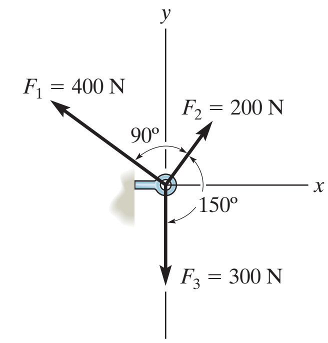


(b)

Ans:
$\phi = 38.3°$

2–21.

Determine the magnitude and direction of the resultant force, $\mathbf{F}_R$ measured counterclockwise from the positive x axis. Solve the problem by first finding the resultant $\mathbf{F}' = \mathbf{F}_1 + \mathbf{F}_2$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_3$.



SOLUTION

Parallelogram Law. The parallelogram law of addition for $\mathbf{F}_1$ and $\mathbf{F}_2$ and then their resultant $\mathbf{F}'$ and $\mathbf{F}_3$ are shown in Figs. *a* and *b*, respectively.
Trigonometry. Referring to Fig. *c*,

$$F' = \sqrt{200^2 + 400^2} = 447.21 \text{ N} \qquad \theta' = \tan^{-1}\left(\frac{200}{400}\right) = 26.57^\circ$$

Thus $\phi' = 90^\circ - 30^\circ - 26.57^\circ = 33.43^\circ$

Using these results to apply the law of cosines by referring to Fig. *d*,

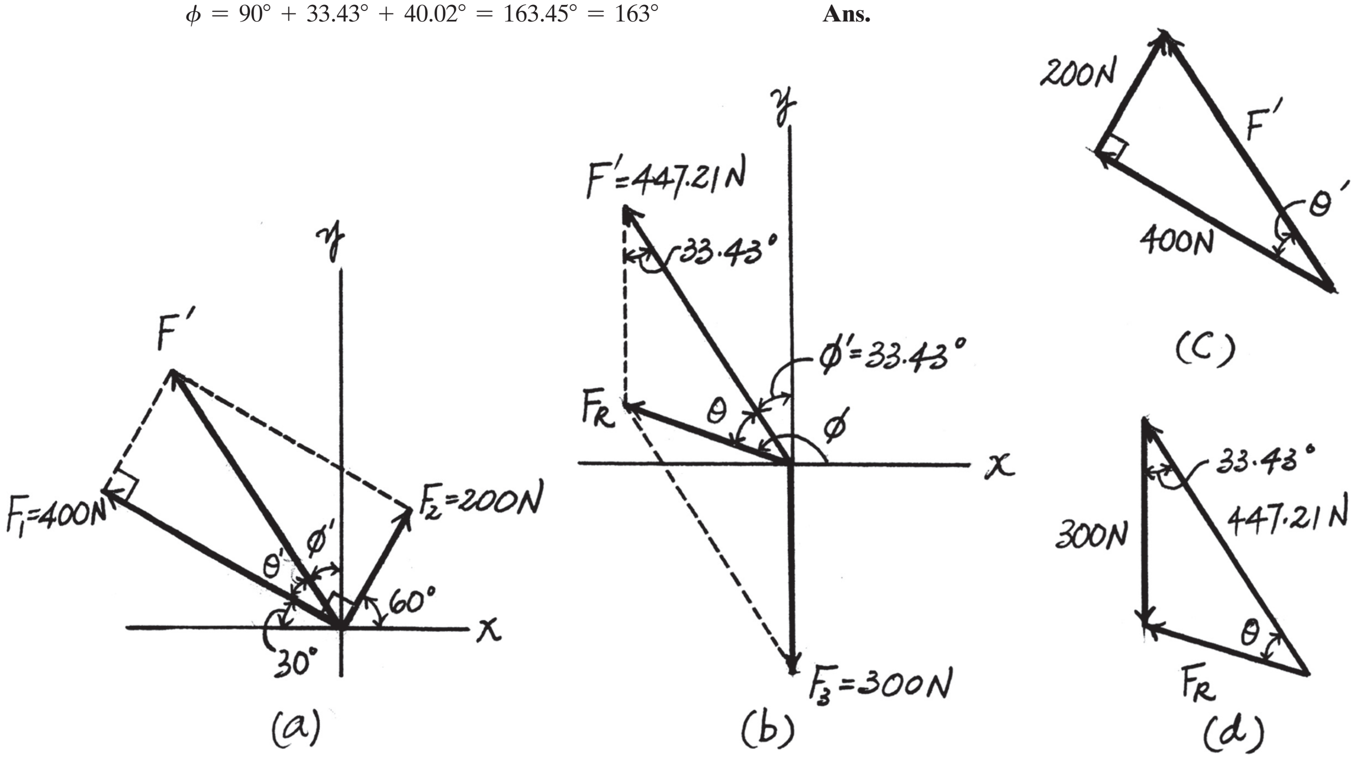
$$F_R = \sqrt{300^2 + 447.21^2 - 2(300)(447.21)\cos 33.43^\circ} = 257.05 \text{ N} = 257 \text{ kN} \quad \textbf{Ans.}$$

Then, apply the law of sines,

$$\frac{\sin\theta}{300} = \frac{\sin 33.43^\circ}{257.05}; \qquad \theta = 40.02^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

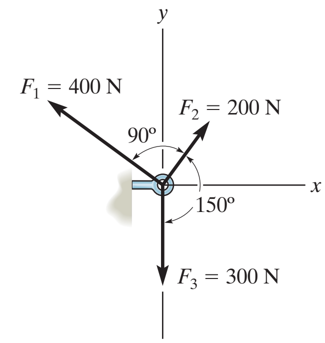
$$\phi = 90^\circ + 33.43^\circ + 40.02^\circ = 163.45^\circ = 163^\circ \quad \textbf{Ans.}$$



Ans:
$F_R = 257$ N
$\phi = 163^\circ$

2–22.

Determine the magnitude and direction of the resultant force, measured counterclockwise from the positive x axis. Solve l by first finding the resultant $\mathbf{F}' = \mathbf{F}_2 + \mathbf{F}_3$ and then forming $\mathbf{F}_R = \mathbf{F}' + \mathbf{F}_1$.



SOLUTION

Parallelogram Law. The parallelogram law of addition for $\mathbf{F}_2$ and $\mathbf{F}_3$ and then their resultant $\mathbf{F}'$ and $\mathbf{F}_1$ are shown in Figs. a and b, respectively.

Trigonometry. Applying the law of cosines by referring to Fig. c,

$$F' = \sqrt{200^2 + 300^2 - 2(200)(300)\cos 30^\circ} = 161.48 \text{ N} \qquad \textbf{Ans.}$$

Using this result to apply the sines law, Fig. c,

$$\frac{\sin\theta'}{200} = \frac{\sin 30^\circ}{161.48}; \qquad \theta' = 38.26^\circ$$

Using the results of $\mathbf{F}'$ and θ' to apply the law of cosines by referring to Fig. d,

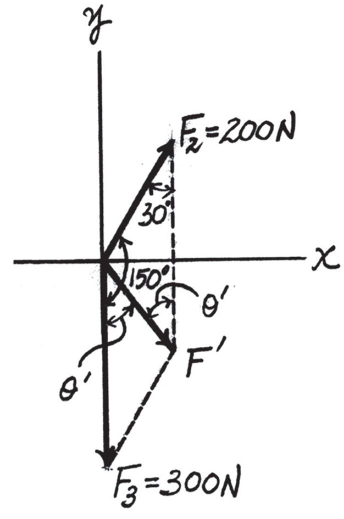
$$F_R = \sqrt{161.48^2 + 400^2 - 2(161.48)(400)\cos 21.74^\circ} = 257.05 \text{ N} = 257 \text{ N} \qquad \textbf{Ans.}$$

Then, apply the sines law,

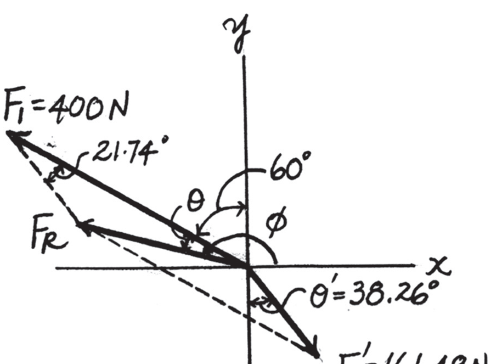
$$\frac{\sin\theta}{161.48} = \frac{\sin 21.74^\circ}{257.05}; \qquad \theta = 13.45^\circ$$

Thus, the direction ϕ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

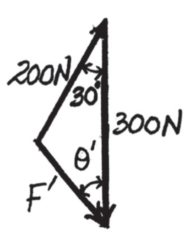
$$\phi = 90^\circ + 60^\circ + 13.45^\circ = 163.45^\circ = 163^\circ \qquad \textbf{Ans.}$$



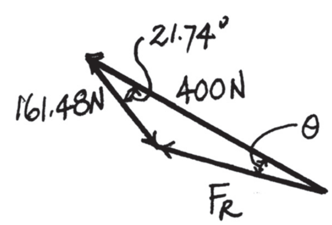
(a)



(b)



(c)

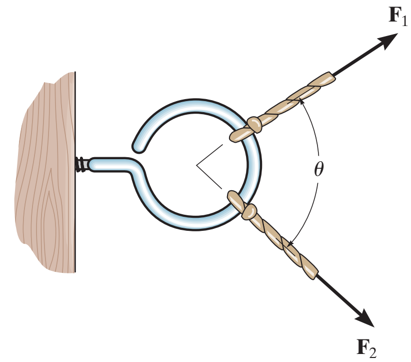


(d)

Ans:
$\phi = 163^\circ$
$F_R = 257$ N

2–23.

Two forces act on the screw eye. If $F_1 = 400$ N and $F_2 = 600$ N, determine the angle $\theta(0° \leq \theta \leq 180°)$ between them, so that the resultant force has a magnitude of $F_R = 800$ N.



SOLUTION

The parallelogram law of addition and triangular rule are shown in Figs. *a* and *b*, respectively. Applying law of cosines to Fig. *b*,

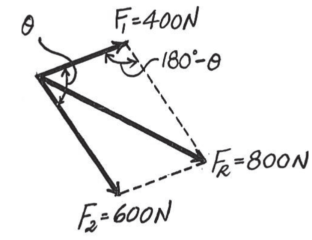
$$800 = \sqrt{400^2 + 600^2 - 2(400)(600)\cos(180° - \theta°)}$$

$$800^2 = 400^2 + 600^2 - 480000\cos(180° - \theta)$$

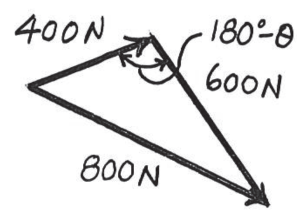
$$\cos(180° - \theta) = -0.25$$

$$180° - \theta = 104.48$$

$$\theta = 75.52° = 75.5° \qquad \textbf{Ans.}$$



(a)

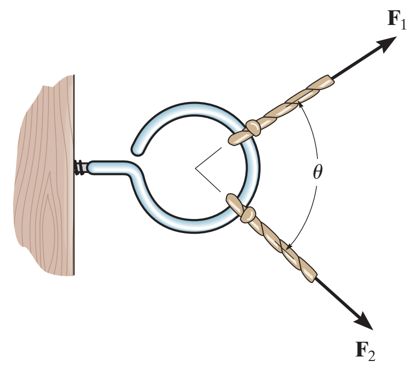


(b)

Ans:
$\theta = 75.5°$

***2–24.**

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the screw eye. If their lines of action are at an angle θ apart and the magnitude of each force is $F_1 = F_2 = F$, determine the magnitude of the resultant force $\mathbf{F}_R$ and the angle between $\mathbf{F}_R$ and $\mathbf{F}_1$.



SOLUTION

$$\frac{F}{\sin \phi} = \frac{F}{\sin (\theta - \phi)}$$

$$\sin (\theta - \phi) = \sin \phi$$

$$\theta - \phi = \phi$$

$$\phi = \frac{\theta}{2} \qquad \textbf{Ans.}$$

$$F_R = \sqrt{(F)^2 + (F)^2 - 2(F)(F)\cos(180° - \theta)}$$

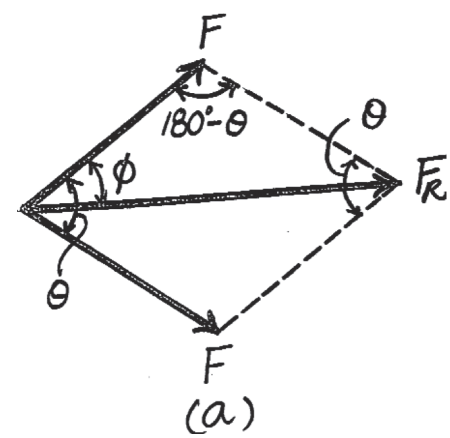
Since $\cos(180° - \theta) = -\cos\theta$

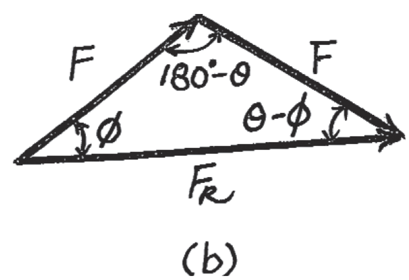
$$F_R = F\left(\sqrt{2}\right)\sqrt{1 + \cos\theta}$$

Since $\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1 + \cos\theta}{2}}$

Then

$$F_R = 2F\cos\left(\frac{\theta}{2}\right) \qquad \textbf{Ans.}$$





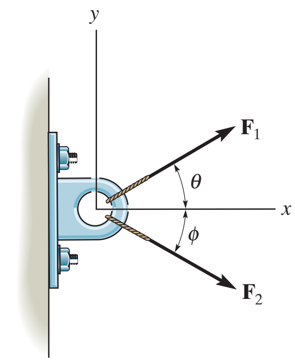
Ans:

$\phi = \frac{\theta}{2}$

$F_R = 2F\cos\left(\frac{\theta}{2}\right)$

2–25.

If $F_1 = 30$ lb and $F_2 = 40$ lb, determine the angles θ and ϕ so that the resultant force is directed along the positive x axis and has a magnitude of $F_R = 60$ lb.



SOLUTION

Parallelogram Law. The parallelogram law of addition is shown in Fig. *a*,
Trigonometry. Applying the law of cosine by referring to Fig. *b*,

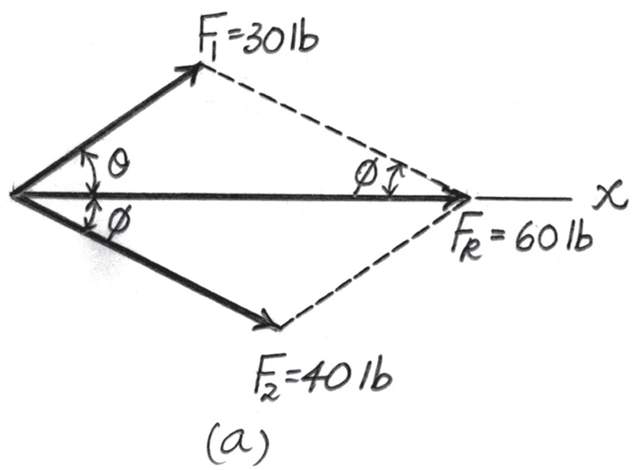
$$40^2 = 30^2 + 60^2 - 2(30)(60)\cos\theta$$

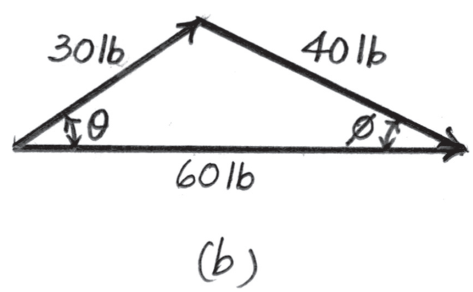
$$\theta = 36.34° = 36.3° \qquad \textbf{Ans.}$$

And

$$30^2 = 40^2 + 60^2 - 2(40)(60)\cos\phi$$

$$\phi = 26.38° = 26.4° \qquad \textbf{Ans.}$$

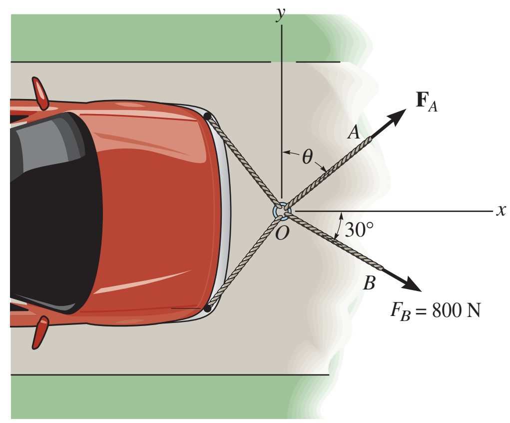




Ans:
$\theta = 36.3°$
$\phi = 26.4°$

2–26.

Determine the magnitude and direction θ of $\mathbf{F}_A$ so that the resultant force is directed along the positive x axis and has a magnitude of 1250 N.



SOLUTION

$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x;\qquad F_{R_x} = F_A \sin\theta + 800\cos 30^\circ = 1250$

$+\uparrow F_{R_y} = \Sigma F_y;\qquad F_{R_y} = F_A\cos\theta - 800\sin 30^\circ = 0$

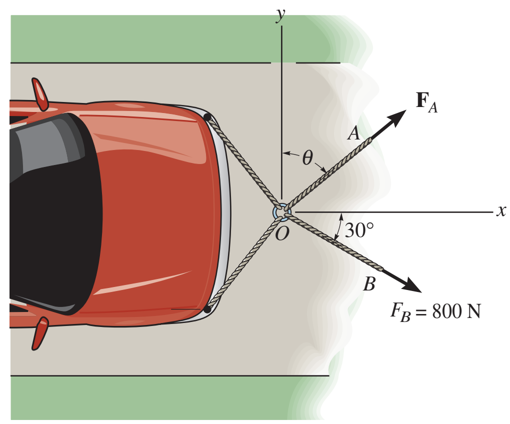
$\theta = 54.3^\circ$ **Ans.**

$F_A = 686\text{ N}$ **Ans.**

Ans:
$\theta = 54.3^\circ$
$F_A = 686\text{ N}$

2–27.

Determine the magnitude and direction, measured counterclockwise from the positive x axis, of the resultant force acting on the ring at O, if $F_A = 750$ N and $\theta = 45°$.



SOLUTION

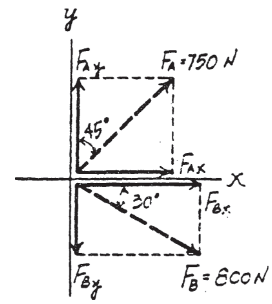
Scalar Notation: Suming the force components algebraically, we have

$$\xrightarrow{+} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 750 \sin 45° + 800 \cos 30°$$

$$= 1223.15 \text{ N} \rightarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = 750 \cos 45° - 800 \sin 30°$$

$$= 130.33 \text{ N} \uparrow$$



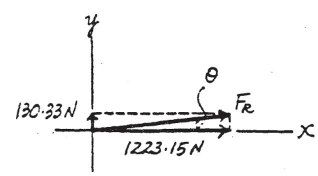
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2}$$

$$= \sqrt{1223.15^2 + 130.33^2} = 1230 \text{ N} = 1.23 \text{ kN} \qquad \textbf{Ans.}$$

The directional angle θ measured counterclockwise from positive x axis is

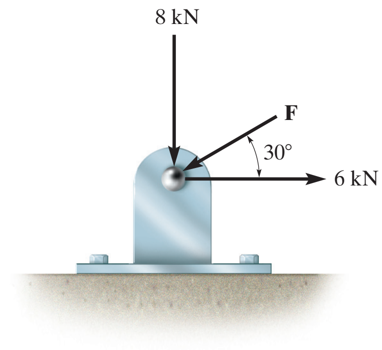
$$\theta = \tan^{-1}\frac{F_{R_y}}{F_{R_x}} = \tan^{-1}\left(\frac{130.33}{1223.15}\right) = 6.08° \qquad \textbf{Ans.}$$



Ans:
$F_R = 1.23$ kN
$\theta = 6.08°$

***2–28.**

Determine the magnitude of force **F** so that the resultant $\mathbf{F}_R$ of the three forces is as small as possible. What is the minimum magnitude of $\mathbf{F}_R$?.



SOLUTION

Parallelogram Law. The parallelogram laws of addition for 6 kN and 8 kN and then their resultant F' and F are shown in Figs. *a* and *b*, respectively. In order for F_R to be minimum, it must act perpendicular to **F**.

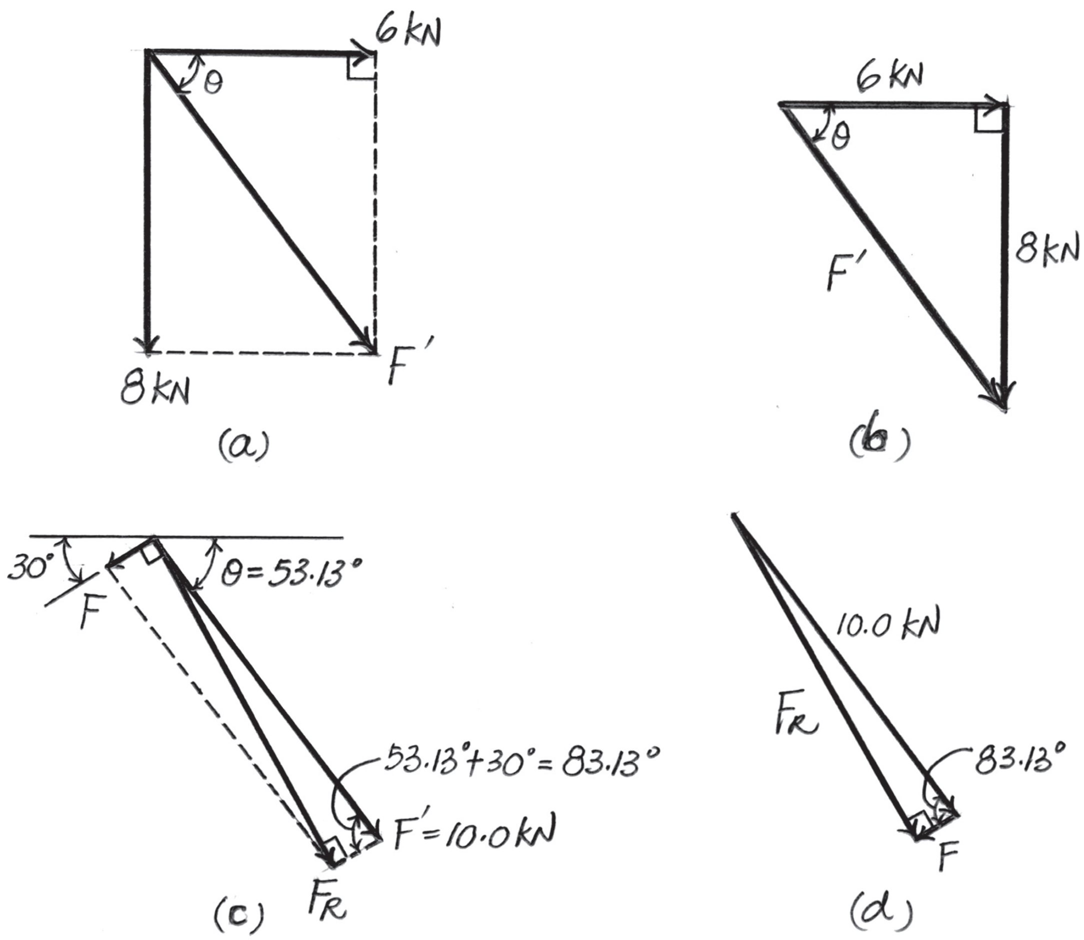
Trigonometry. Referring to Fig. *b*,

$$F' = \sqrt{6^2 + 8^2} = 10.0 \text{ kN} \qquad \theta = \tan^{-1}\left(\frac{8}{6}\right) = 53.13°.$$

Referring to Figs. *c* and *d*,

$$F_R = 10.0 \sin 83.13° = 9.928 \text{ kN} = 9.93 \text{ kN} \qquad \textbf{Ans.}$$

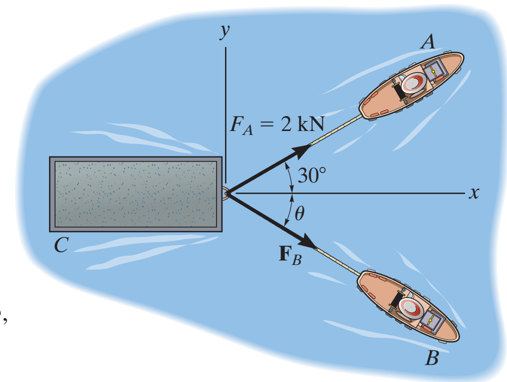
$$F = 10.0 \cos 83.13° = 1.196 \text{ kN} = 1.20 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$F_R = 9.93$ kN
$F = 1.20$ kN

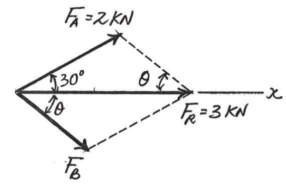
2–29.

If the resultant force of the two tugboats is 3 kN, directed along the positive x axis, determine the required magnitude of force $\mathbf{F}_B$ and its direction θ.

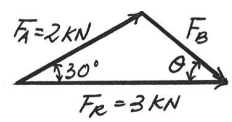


SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



(a)



(b)

Applying the law of cosines to Fig. *b*,

$$F_B = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 30^\circ}$$

$$= 1.615\text{kN} = 1.61 \text{ kN} \qquad \textbf{Ans.}$$

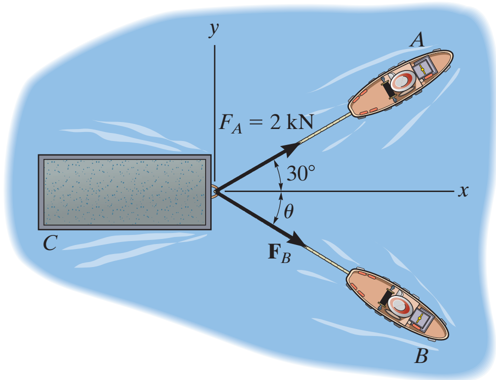
Using this result and applying the law of sines to Fig. *b*, yields

$$\frac{\sin\theta}{2} = \frac{\sin 30^\circ}{1.615} \qquad \theta = 38.3^\circ \qquad \textbf{Ans.}$$

Ans:
$F_B = 1.61$ kN
$\theta = 38.3^\circ$

2–30.

If $F_B = 3$ kN and $\theta = 45°$, determine the magnitude of the resultant force of the two tugboats and its direction measured clockwise from the positive x axis.



SOLUTION

The parallelogram law of addition and the triangular rule are shown in Figs. a and b, respectively.

Applying the law of cosines to Fig. b,

$$F_R = \sqrt{2^2 + 3^2 - 2(2)(3)\cos 105°}$$

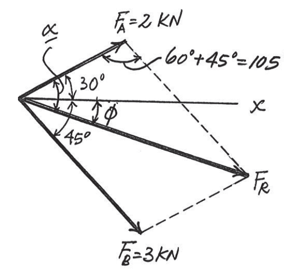
$$= 4.013 \text{ kN} = 4.01 \text{ kN} \qquad \textbf{Ans.}$$

Using this result and applying the law of sines to Fig. b, yields

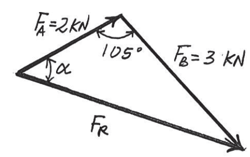
$$\frac{\sin\alpha}{3} = \frac{\sin 105°}{4.013} \qquad \alpha = 46.22°$$

Thus, the direction angle ϕ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

$$\phi = \alpha - 30° = 46.22° - 30° = 16.2° \qquad \textbf{Ans.}$$



(a)

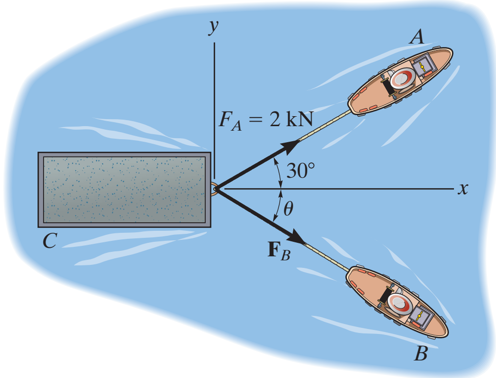


(b)

Ans:
$F_R = 4.01$ kN
$\phi = 16.2°$

2–31.

If the resultant force of the two tugboats is required to be directed towards the positive x axis, and F_B is to be a minimum, determine the magnitude of $\mathbf{F}_R$ and $\mathbf{F}_B$ and the angle θ.



SOLUTION

For $\mathbf{F}_B$ to be minimum, it has to be directed perpendicular to $\mathbf{F}_R$. Thus,

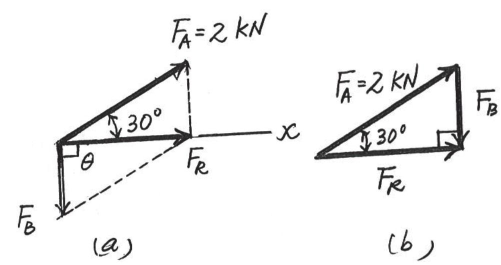
$$\theta = 90^\circ \qquad \textbf{Ans.}$$

The parallelogram law of addition and triangular rule are shown in Figs. a and b, respectively.

By applying simple trigonometry to Fig. b,

$$F_B = 2 \sin 30^\circ = 1 \text{ kN} \qquad \textbf{Ans.}$$

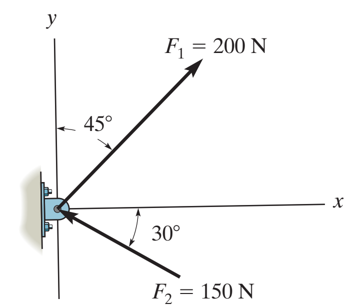
$$F_R = 2 \cos 30^\circ = 1.73 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$\theta = 90^\circ$
$F_B = 1$ kN
$F_R = 1.73$ kN

***2–32.**

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 200 \sin 45° - 150 \cos 30° = 11.518 \text{ N} \rightarrow$$

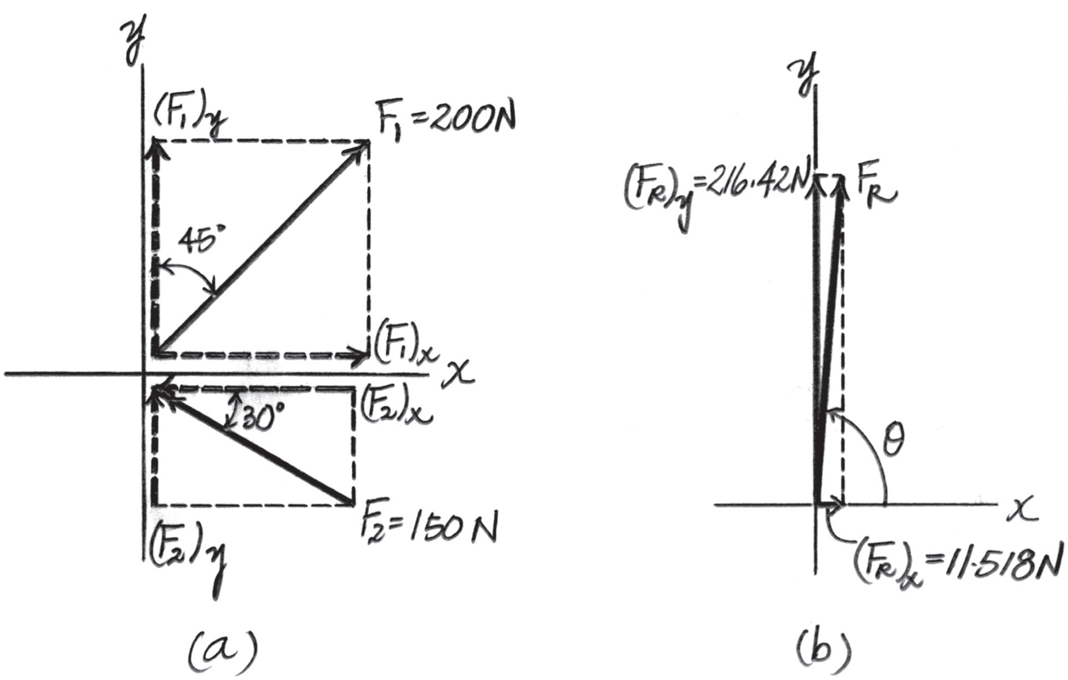
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 200 \cos 45° + 150 \sin 30° = 216.42 \text{ N} \uparrow$$

Referring to Fig. b, the magnitude of the resultant force F_R is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{11.518^2 + 216.42^2} = 216.73 \text{ N} = 217 \text{ N} \qquad \textbf{Ans.}$$

And the directional angle θ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

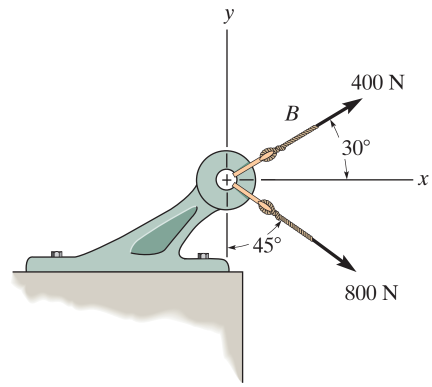
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{216.42}{11.518}\right) = 86.95° = 87.0° \qquad \textbf{Ans.}$$



Ans:
$F_R = 217$ N
$\theta = 87.0°$

2–33.

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes by referring to Fig. a,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 400 \cos 30° + 800 \sin 45° = 912.10 \text{ N} \rightarrow$$

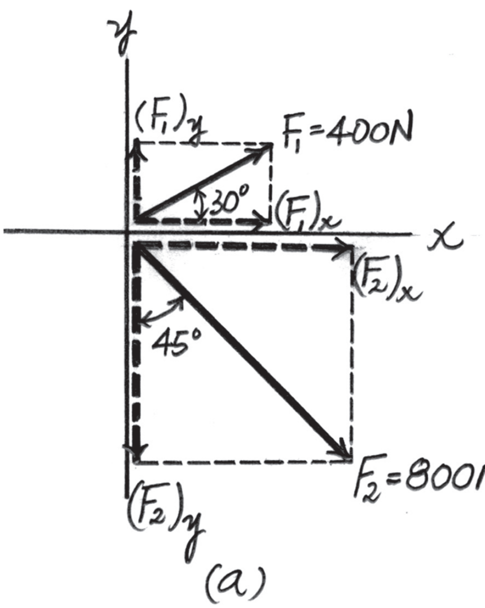
$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 400 \sin 30° - 800 \cos 45° = -365.69 \text{ N} = 365.69 \text{ N}\downarrow$$

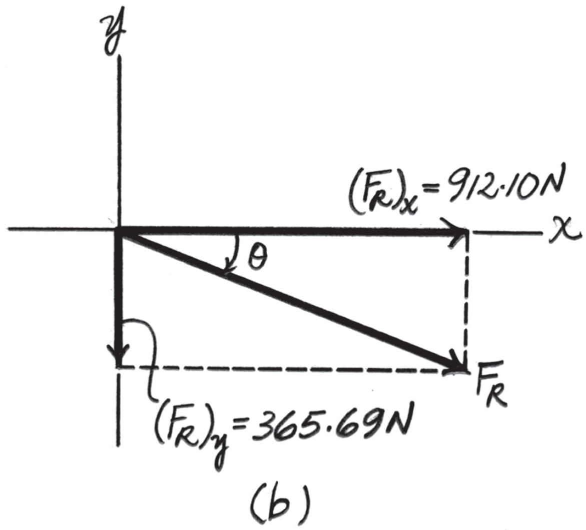
Referring to Fig. b, the magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{912.10^2 + 365.69^2} = 982.67 \text{ N} = 983 \text{ N} \qquad \textbf{Ans.}$$

And its directional angle θ measured clockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{365.69}{912.10}\right) = 21.84° = 21.8° \qquad \textbf{Ans.}$$

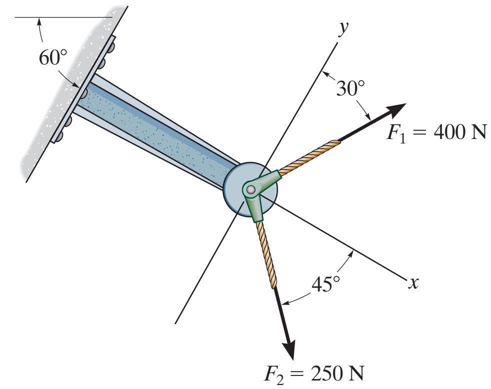




Ans:
$F_R = 983$ N
$\theta = 21.8°$

2–34.

Resolve $\mathbf{F}_1$ and $\mathbf{F}_2$ into their x and y components.



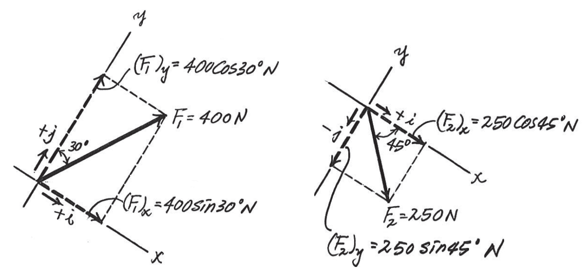
SOLUTION

$$\mathbf{F}_1 = \{400 \sin 30°(+\mathbf{i}) + 400 \cos 30°(+\mathbf{j})\} \text{ N}$$

$$= \{200\mathbf{i} + 346\mathbf{j}\} \text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = \{250 \cos 45°(+\mathbf{i}) + 250 \sin 45°(-\mathbf{j})\} \text{ N}$$

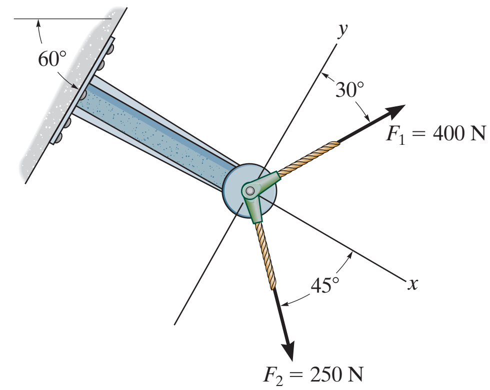
$$= \{177\mathbf{i} - 177\mathbf{j}\} \text{ N} \qquad \textbf{Ans.}$$



Ans:
$\mathbf{F}_1 = \{200\mathbf{i} + 346\mathbf{j}\}$ N
$\mathbf{F}_2 = \{177\mathbf{i} - 177\mathbf{j}\}$ N

2–35.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$ can be written as

$$(F_1)_x = 400 \sin 30° = 200 \text{ N} \qquad (F_1)_y = 400 \cos 30° = 346.41 \text{ N}$$

$$(F_2)_x = 250 \cos 45° = 176.78 \text{ N} \qquad (F_2)_y = 250 \sin 45° = 176.78 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 200 + 176.78 = 376.78 \text{ N} \qquad \textbf{Ans.}$$

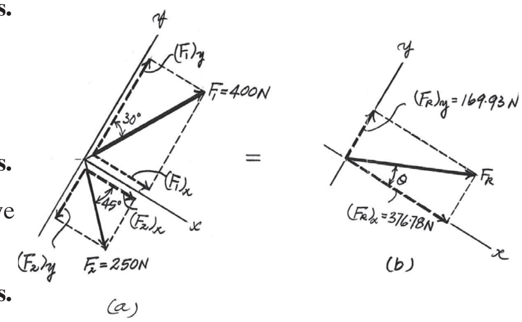
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 346.41 - 176.78 = 169.63 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{376.78^2 + 169.63^2} = 413 \text{ N} \qquad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, Fig. b, measured counterclockwise from the positive axis, is

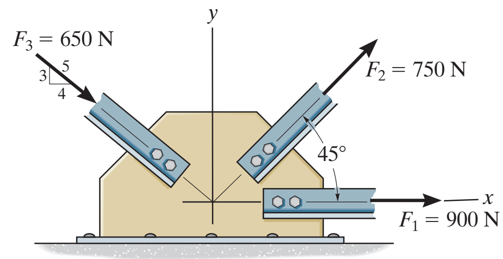
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{169.63}{376.78}\right) = 24.2° \qquad \textbf{Ans.}$$



Ans:
$F_R = 413$ N
$\theta = 24.2°$

***2–36.**

Resolve each force acting on the *gusset plate* into its x and y components, and express each force as a Cartesian vector.



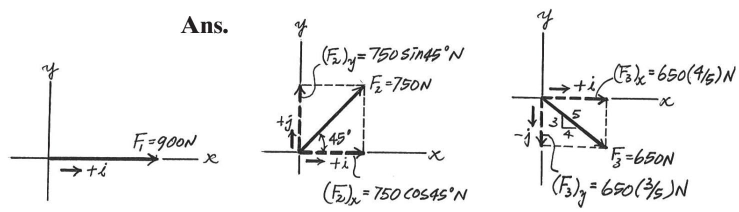
$\mathbf{F}_1 = \{900(+\mathbf{i})\} = \{900\mathbf{i}\}$ N **Ans.**

$\mathbf{F}_2 = \{750 \cos 45°(+\mathbf{i}) + 750 \sin 45°(+\mathbf{j})\}$ N

$= \{530\mathbf{i} + 530\mathbf{j}\}$ N **Ans.**

$$\mathbf{F}_3 = \left\{650\left(\frac{4}{5}\right)(+\mathbf{i}) + 650\left(\frac{3}{5}\right)(-\mathbf{j})\right\} \text{ N}$$

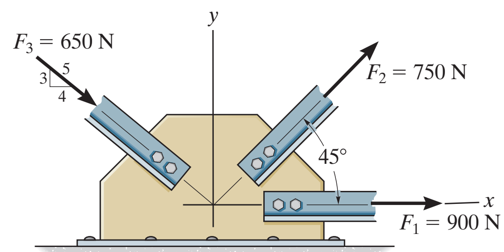
$= \{520\,\mathbf{i} - 390\mathbf{j})\}$ N **Ans.**



Ans:
$\mathbf{F}_1 = \{900\mathbf{i}\}$ N
$\mathbf{F}_2 = \{530\mathbf{i} + 530\mathbf{j}\}$ N
$\mathbf{F}_3 = \{520\mathbf{i} - 390\mathbf{j}\}$ N

2–37.

Determine the magnitude of the resultant force acting on the plate and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$$(F_1)_x = 900 \text{ N} \qquad (F_1)_y = 0$$

$$(F_2)_x = 750 \cos 45° = 530.33 \text{ N} \qquad (F_2)_y = 750 \sin 45° = 530.33 \text{ N}$$

$$(F_3)_x = 650\left(\frac{4}{5}\right) = 520 \text{ N} \qquad (F_3)_y = 650\left(\frac{3}{5}\right) = 390 \text{ N}$$

Resultant Force: Summing the force components algebraically along the x and y axes, we have

$$\xrightarrow{+} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900 + 530.33 + 520 = 1950.33 \text{ N} \rightarrow$$

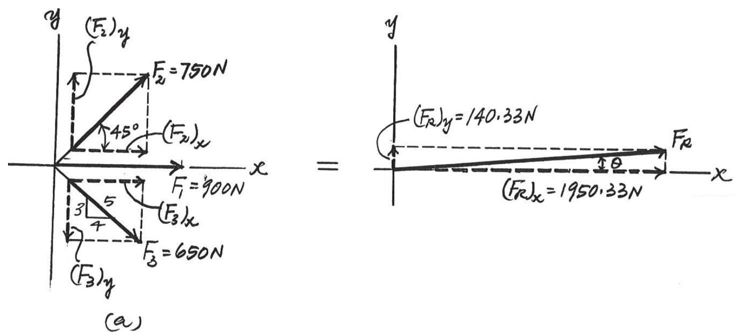
$$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = 530.33 - 390 = 140.33 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{1950.33^2 + 140.33^2} = 1955 \text{ N} = 1.96 \text{ kN} \quad \textbf{Ans.}$$

The direction angle θ of $\mathbf{F}_R$, measured clockwise from the positive x axis, is

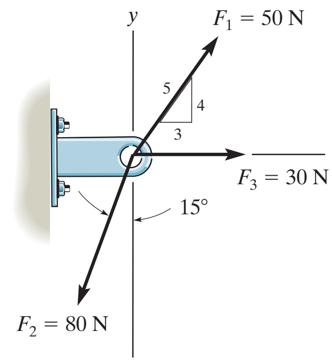
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{140.33}{1950.33}\right) = 4.12° \quad \textbf{Ans.}$$



Ans:
$F_R = 1.96$ kN
$\theta = 4.12°$

2–38.

Express each of the three forces acting on the support in Cartesian vector form and determine the magnitude of the resultant force and its direction, measured clockwise from positive x axis.



SOLUTION

Cartesian Notation. Referring to Fig. a,

$$\mathbf{F}_1 = (F_1)_x\,\mathbf{i} + (F_1)_y\,\mathbf{j} = 50\left(\frac{3}{5}\right)\mathbf{i} + 50\left(\frac{4}{5}\right)\mathbf{j} = \{30\,\mathbf{i} + 40\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = -(F_2)_x\,\mathbf{i} - (F_2)_y\,\mathbf{j} = -80\sin 15°\,\mathbf{i} - 80\cos 15°\,\mathbf{j}$$

$$= \{-20.71\,\mathbf{i} - 77.27\,\mathbf{j}\}\text{ N}$$

$$= \{-20.7\,\mathbf{i} - 77.3\,\mathbf{j}\}\text{ N} \qquad \textbf{Ans.}$$

$$F_3 = (F_3)_x\,\mathbf{i} = \{30\,\mathbf{i}\} \qquad \textbf{Ans.}$$

Thus, the resultant force is

$$\mathbf{F}_R = \Sigma\mathbf{F}\,; \qquad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= (30\mathbf{i} + 40\,\mathbf{j}) + (-20.71\mathbf{i} - 77.27\mathbf{j}) + 30\mathbf{i}$$

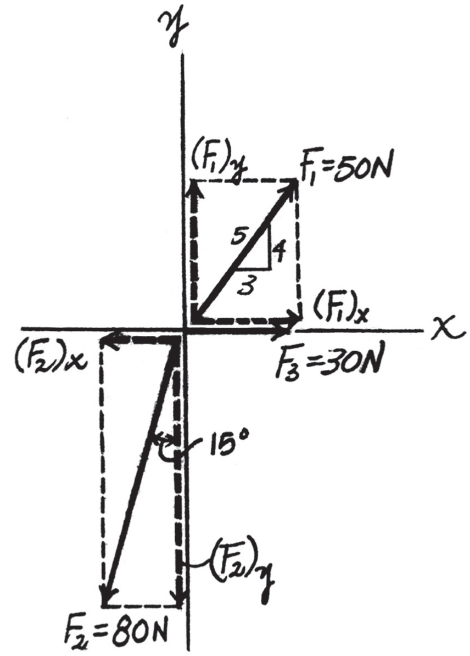
$$= \{39.29\,\mathbf{i} - 37.27\,\mathbf{j}\}\text{ N}$$

Referring to Fig. b, the magnitude of $\mathbf{F}_R$ is

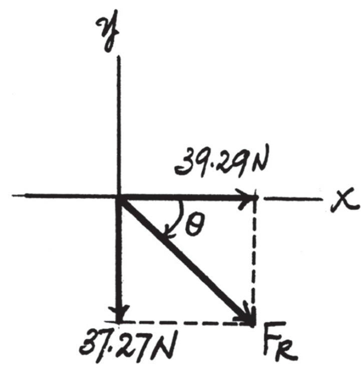
$$F_R = \sqrt{39.29^2 + 37.27^2} = 54.16\text{ N} = 54.2\text{ N} \qquad \textbf{Ans.}$$

And its directional angle θ measured clockwise from the positive x axis is

$$\theta = \tan^{-1}\left(\frac{37.27}{39.29}\right) = 43.49° - 43.5° \qquad \textbf{Ans.}$$



(a)

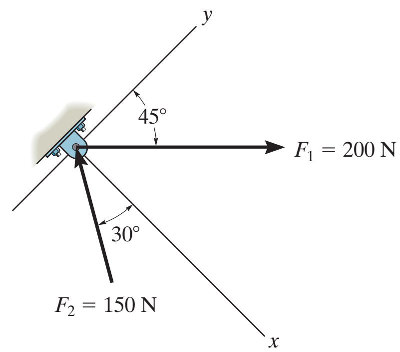


(b)

Ans:
$\mathbf{F}_1 = \{30\mathbf{i} + 40\mathbf{j}\}$ N
$\mathbf{F}_2 = \{-20.7\,\mathbf{i} - 77.3\,\mathbf{j}\}$ N
$F_3 = \{30\,\mathbf{i}\}$
$F_R = 54.2$ N
$\theta = 43.5°$

2–39.

Determine the x and y components of $\mathbf{F}_1$ and $\mathbf{F}_2$.



SOLUTION

$F_{1x} = 200 \sin 45° = 141\text{ N}$ **Ans.**

$F_{1y} = 200 \cos 45° = 141\text{ N}$ **Ans.**

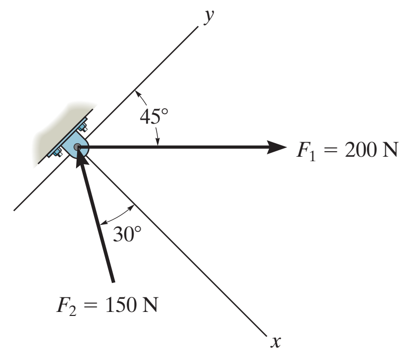
$F_{2x} = -150 \cos 30° = -130\text{ N}$ **Ans.**

$F_{2y} = 150 \sin 30° = 75\text{ N}$ **Ans.**

Ans:
$F_{1x} = 141$ N
$F_{1y} = 141$ N
$F_{2x} = -130$ N
$F_{2y} = 75$ N

***2–40.**

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

$+\searrow F_{Rx} = \Sigma F_x; \qquad F_{Rx} = -150 \cos 30° + 200 \sin 45° = 11.518 \text{ N}$

$\nearrow + F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 150 \sin 30° + 200 \cos 45° = 216.421 \text{ N}$

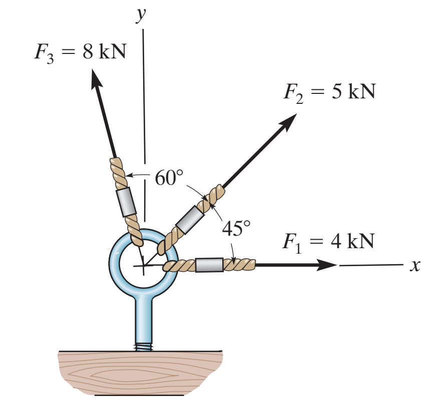
$F_R = \sqrt{(11.518)^2 + (216.421)^2} = 217 \text{ N}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{216.421}{11.518}\right) = 87.0°$ **Ans.**

Ans:
$F_R = 217$ N
$\theta = 87.0°$

2–41.

Determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 4 + 5 \cos 45° - 8 \sin 15° = 5.465 \text{ kN} \rightarrow$$

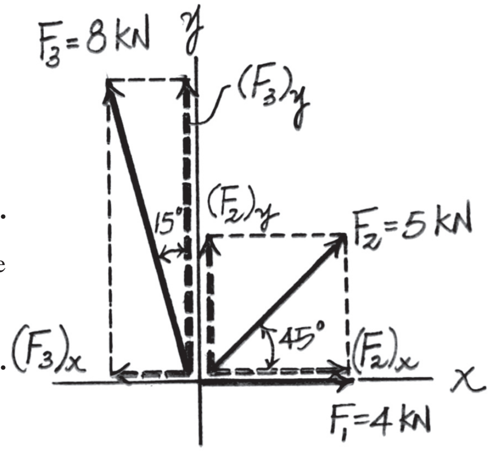
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 5 \sin 45° + 8 \cos 15° = 11.263 \text{ kN} \uparrow$$

By referring to Fig. b, the magnitude of the resultant force $\mathbf{F}_R$ is

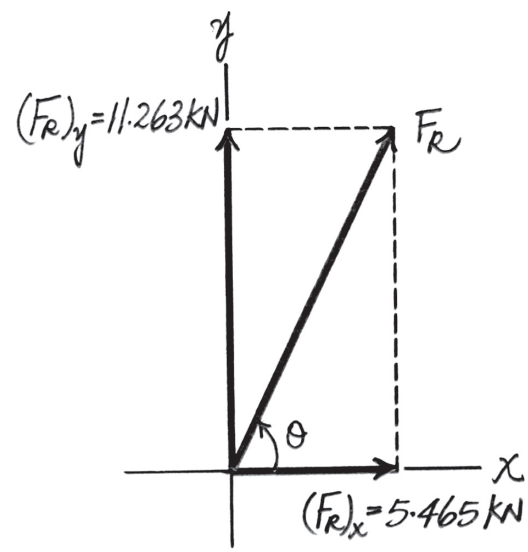
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{5.465^2 + 11.263^2} = 12.52 \text{ kN} = 12.5 \text{ kN} \qquad \textbf{Ans.}$$

And the directional angle θ of $\mathbf{F}_R$ measured counterclockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{11.263}{5.465}\right) = 64.12° = 64.1° \qquad \textbf{Ans.}$$



(a)

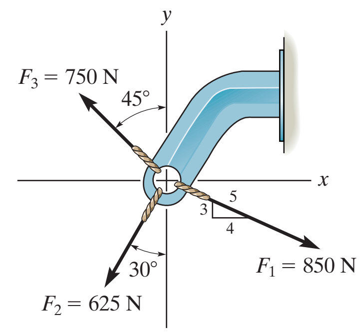


(b)

Ans:
$F_R = 12.5$ kN
$\theta = 64.1°$

2–42.

Express $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ as Cartesian vectors.



SOLUTION

$$\mathbf{F}_1 = \frac{4}{5}(850)\,\mathbf{i} - \frac{3}{5}(850)\,\mathbf{j}$$

$$= \{680\,\mathbf{i} - 510\,\mathbf{j}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_2 = -625\sin 30^\circ\,\mathbf{i} - 625\cos 30^\circ\,\mathbf{j}$$

$$= \{-312\,\mathbf{i} - 541\,\mathbf{j}\}\text{ N}$$ **Ans.**

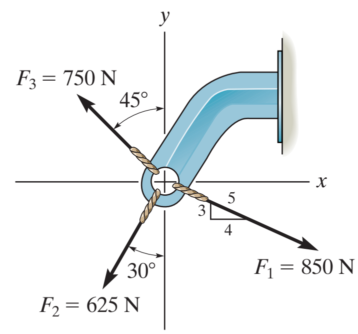
$$\mathbf{F}_3 = -750\sin 45^\circ\,\mathbf{i} + 750\cos 45^\circ\,\mathbf{j}$$

$$= \{-530\,\mathbf{i} + 530\,\mathbf{j}\}\text{ N}$$ **Ans.**

Ans:
$\mathbf{F}_1 = \{680\mathbf{i} - 510\mathbf{j}\}$ N
$\mathbf{F}_2 = \{-312\mathbf{i} - 541\mathbf{j}\}$ N
$\mathbf{F}_3 = \{-530\mathbf{i} + 530\mathbf{j}\}$ N

2–43.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

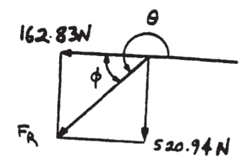
$$\xrightarrow{+} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = \frac{4}{5}(850) - 625 \sin 30° - 750 \sin 45° = -162.83 \text{ N}$$

$$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = -\frac{3}{5}(850) - 625 \cos 30° + 750 \cos 45° = -520.94 \text{ N}$$

$$F_R = \sqrt{(-162.83)^2 + (-520.94)^2} = 546 \text{ N}$$ **Ans.**

$$\phi = \tan^{-1}\left(\frac{520.94}{162.83}\right) = 72.64°$$

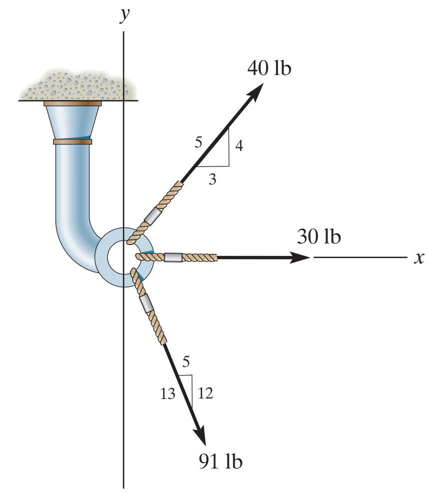
$$\theta = 180° + 72.64° = 253°$$ **Ans.**



Ans:
$F_R = 546$ N
$\theta = 253°$

***2–44.**

Determine the magnitude of the resultant force and its direction, measured clockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 40\left(\frac{3}{5}\right) + 91\left(\frac{5}{13}\right) + 30 = 89 \text{ lb} \rightarrow$$

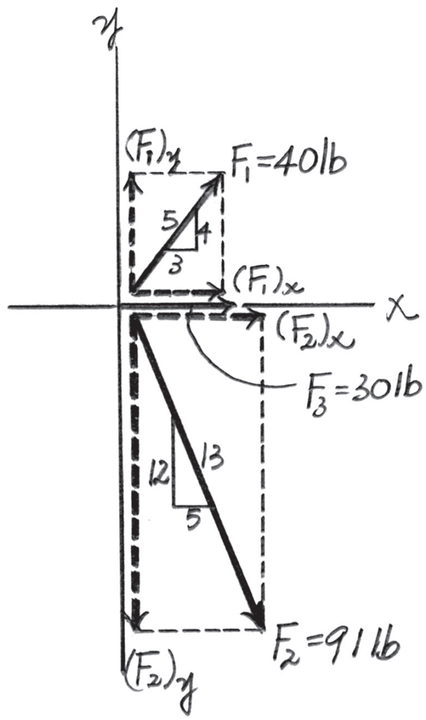
$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 40\left(\frac{4}{5}\right) - 91\left(\frac{12}{13}\right) = -52 \text{ lb} = 52 \text{ lb}\downarrow$$

By referring to Fig. b, the magnitude of resultant force is

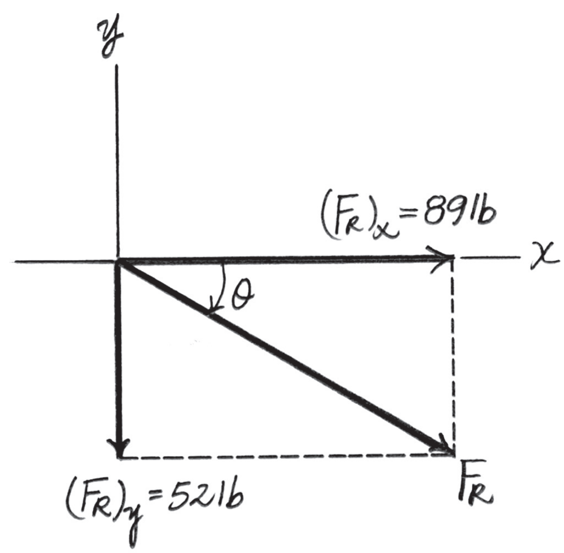
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{89^2 + 52^2} = 103.08 \text{ lb} = 103 \text{ lb} \qquad \textbf{Ans.}$$

And its directional angle θ measured clockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{52}{89}\right) = 30.30° = 30.3° \qquad \textbf{Ans.}$$



(a)



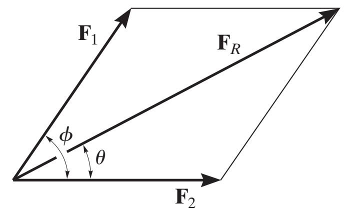
(b)

Ans:
$F_R = 103$ lb
$\theta = 30.3°$

2–45.

Determine the magnitude and direction θ of the resultant force $\mathbf{F}_R$. Express the result in terms of the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ and the angle ϕ.

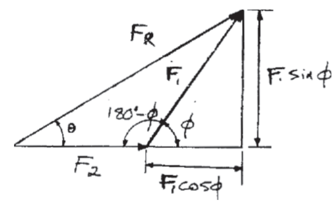
SOLUTION



$$F_R^2 = F_1^2 + F_2^2 - 2F_1F_2\cos(180° - \phi)$$

Since $\cos(180° - \phi) = -\cos\phi$,

$$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\phi} \qquad \textbf{Ans.}$$



From the figure,

$$\tan\theta = \frac{F_1\sin\phi}{F_2 + F_1\cos\phi}$$

$$\theta = \tan^{-1}\left(\frac{F_1\sin\phi}{F_2 + F_1\cos\phi}\right) \qquad \textbf{Ans.}$$

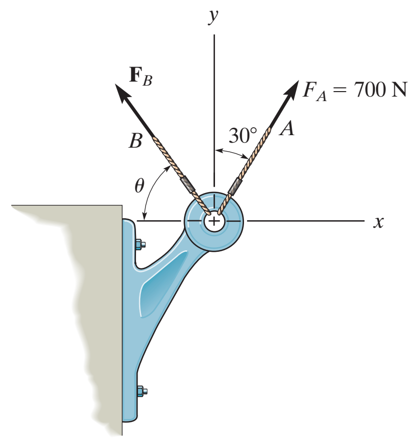
Ans:

$F_R = \sqrt{F_1^2 + F_2^2 + 2F_1F_2\cos\phi}$

$\theta = \tan^{-1}\left(\frac{F_1\sin\phi}{F_2 + F_1\cos\phi}\right)$

2–46.

Determine the magnitude and orientation θ of $\mathbf{F}_B$ so that the resultant force is directed along the positive y axis and has a magnitude of 1500 N.



SOLUTION

Scalar Notation: Suming the force components algebraically, we have

$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad 0 = 700 \sin 30° - F_B \cos\theta$

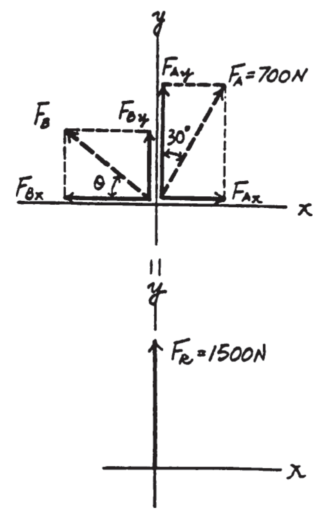
$$F_B \cos\theta = 350 \qquad \textbf{(1)}$$

$+\uparrow F_{R_y} = \Sigma F_y; \qquad 1500 = 700 \cos 30° + F_B \sin\theta$

$$F_B \sin\theta = 893.8 \qquad \textbf{(2)}$$

Solving Eq. (1) and (2) yields

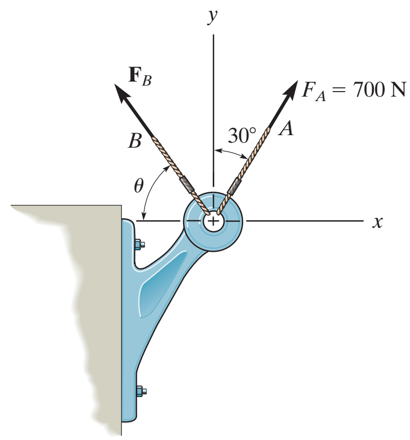
$$\theta = 68.6° \qquad F_B = 960 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$\theta = 68.6°$
$F_B = 960$ N

2–47.

Determine the magnitude and orientation, measured counterclockwise from the positive y axis, of the resultant force acting on the bracket, if $F_B = 600$ N and $\theta = 20°$.



SOLUTION

Scalar Notation: Suming the force components algebraically, we have

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \quad F_{R_x} = 700 \sin 30° - 600 \cos 20°$$

$$= -213.8 \text{ N} = 213.8 \text{ N} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \quad F_{R_y} = 700 \cos 30° + 600 \sin 20°$$

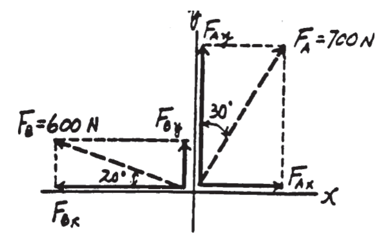
$$= 811.4 \text{ N} \uparrow$$

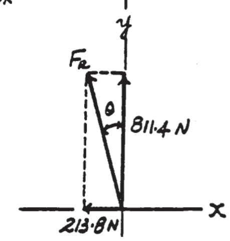
The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{213.8^2 + 811.4^2} = 839 \text{ N} \qquad \textbf{Ans.}$$

The directional angle θ measured counterclockwise from positive y axis is

$$\theta = \tan^{-1}\frac{F_{R_x}}{F_{R_y}} = \tan^{-1}\left(\frac{213.8}{811.4}\right) = 14.8° \qquad \textbf{Ans.}$$

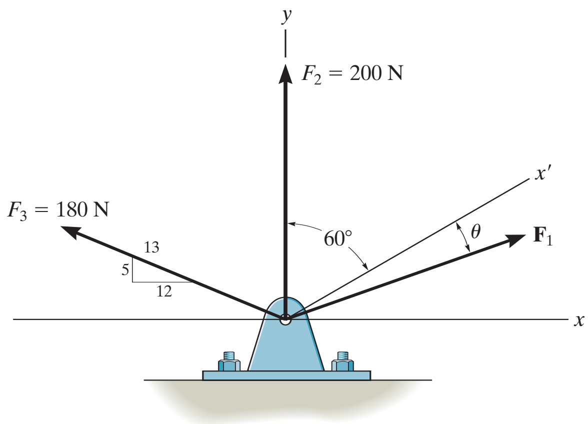




Ans:
$F_R = 839$ N
$\theta = 14.8°$

***2–48.**

Three forces act on the bracket. Determine the magnitude and direction θ of $\mathbf{F}_1$ so that the resultant force is directed along the positive x' axis and has a magnitude of 800 N.



SOLUTION

$$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x; \qquad 800 \sin 60° = F_1 \sin(60° + \theta) - \frac{12}{13}(180)$$

$$+\uparrow F_{Ry} = \Sigma F_y; \qquad 800 \cos 60° = F_1 \cos(60° + \theta) + 200 + \frac{5}{13}(180)$$

$$60° + \theta = 81.34°$$

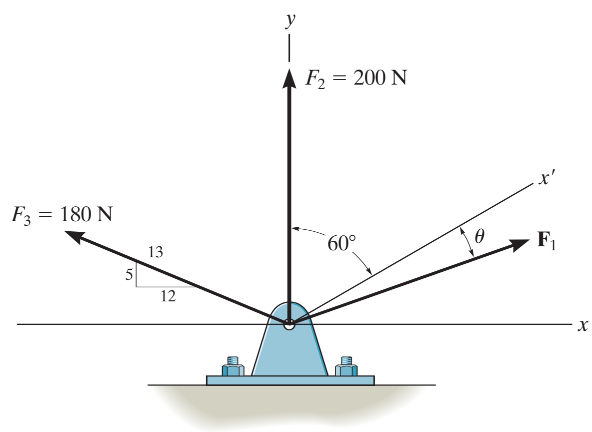
$\theta = 21.3°$ **Ans.**

$F_1 = 869$ N **Ans.**

Ans:
$\theta = 21.3°$
$F_1 = 869$ N

2–49.

If $F_1 = 300$ N and $\theta = 10°$, determine the magnitude and direction, measured counterclockwise from the positive x' axis, of the resultant force acting on the bracket.



SOLUTION

$\overset{+}{\rightarrow} F_{Rx} = \Sigma Fx;\qquad F_{Rx} = 300 \sin 70° - \frac{12}{13}(180) = 115.8 \text{ N}$

$+\uparrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = 300 \cos 70° + 200 + \frac{5}{13}(180) = 371.8 \text{ N}$

$$F_R = \sqrt{(115.8)^2 + (371.8)^2} = 389 \text{ N}$$ **Ans.**

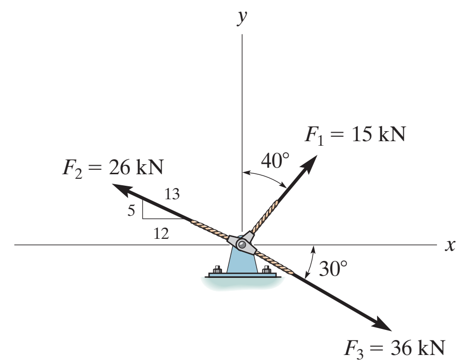
$$\phi = \tan^{-1}\left[\frac{371.8}{115.8}\right] = 72.71° \quad \measuredangle\theta$$

$$\phi' = 72.71° - 30° = 42.7°$$ **Ans.**

Ans:
$F_R = 389$ N
$\phi' = 42.7°$

2–50.

Express $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ as Cartesian vectors.



SOLUTION

$\mathbf{F}_1 = \{-200\,\mathbf{i}\}$ lb **Ans.**

$\mathbf{F}_2 = -250 \sin 30^\circ\,\mathbf{i} + 250 \cos 30^\circ\,\mathbf{j}$

$= \{-125\,\mathbf{i} + 217\,\mathbf{j}\}$ lb **Ans.**

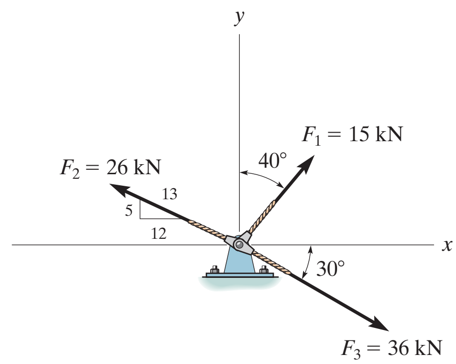
$\mathbf{F}_3 = 225 \cos 30^\circ\,\mathbf{i} + 225 \sin 30^\circ\,\mathbf{j}$

$= \{195\,\mathbf{i} + 112\,\mathbf{j}\}$ lb **Ans.**

Ans:
$\mathbf{F}_1 = \{-200\mathbf{i}\}$ lb
$\mathbf{F}_2 = \{-125\mathbf{i} + 217\mathbf{j}\}$ lb
$\mathbf{F}_3 = \{195\mathbf{i} + 112\mathbf{j}\}$ lb

2–51.

Determine the magnitude of the resultant force and its orientation measured counterclockwise from the positive x axis.



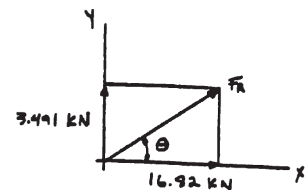
SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x;$ $\quad F_{Rx} = 15 \sin 40^\circ - \frac{12}{13}(26) + 36 \cos 30^\circ = 16.82 \text{ kN}$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = 15 \cos 40^\circ + \frac{5}{13}(26) - 36 \sin 30^\circ = 3.491 \text{ kN}$

$$F_R = \sqrt{(16.82)^2 + (3.491)^2} = 17.2 \text{ kN}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{3.491}{16.82}\right) = 11.7^\circ$$ **Ans.**



Also,

$\mathbf{F}_1 = \{15 \sin 40^\circ\, \mathbf{i} + 15 \cos 40^\circ\, \mathbf{j}\} \text{ kN} = \{9.64\mathbf{i} + 11.5\mathbf{j}\} \text{ kN}$

$\mathbf{F}_2 = \left\{-\frac{12}{13}(26)\mathbf{i} + \frac{5}{13}(26)\mathbf{j}\right\} \text{ kN} = \{-24\mathbf{i} + 10\mathbf{j}\} \text{ kN}$

$\mathbf{F}_3 = \{36 \cos 30^\circ\mathbf{i} - 36 \sin 30^\circ\mathbf{j}\} \text{ kN} = \{31.2\mathbf{i} - 18\mathbf{j}\} \text{ kN}$

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

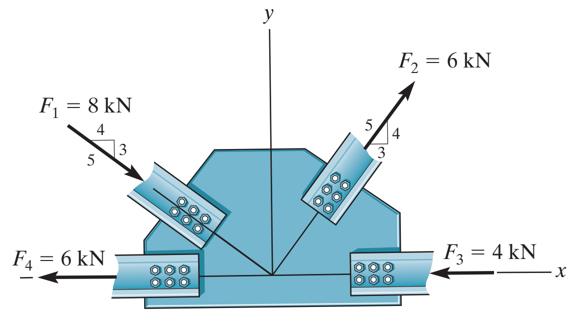
$\quad = \{9.64\mathbf{i} + 11.5\mathbf{j}\} + \{-24\mathbf{i} + 10\mathbf{j}\} + \{31.2\mathbf{i} - 18\mathbf{j}\}$

$\quad = \{16.8\mathbf{i} + 3.49\mathbf{j}\} \text{ kN}$

Ans:
$F_R = 17.2 \text{ kN}, \theta = 11.7^\circ$

***2–52.**

Determine the x and y components of each force acting on the *gusset plate* of a bridge truss. Show that the resultant force is zero.



SOLUTION

Scalar Notation. Referring to Fig. *a*, the x and y components of each forces are

$$(F_1)_x = 8\left(\frac{4}{5}\right) = 6.40 \text{ kN} \rightarrow \qquad \textbf{Ans.}$$

$$(F_1)_y = 8\left(\frac{3}{5}\right) = 4.80 \text{ kN} \downarrow \qquad \textbf{Ans.}$$

$$(F_2)_x = 6\left(\frac{3}{5}\right) = 3.60 \text{ kN} \rightarrow \qquad \textbf{Ans.}$$

$$(F_2)_y = 6\left(\frac{4}{5}\right) = 4.80 \text{ kN} \uparrow \qquad \textbf{Ans.}$$

$$(F_3)_x = 4 \text{ kN} \leftarrow \qquad \textbf{Ans.}$$

$$(F_3)_y = 0 \qquad \textbf{Ans.}$$

$$(F_4)_x = 6 \text{ kN} \leftarrow \qquad \textbf{Ans.}$$

$$(F_4)_y = 0 \qquad \textbf{Ans.}$$

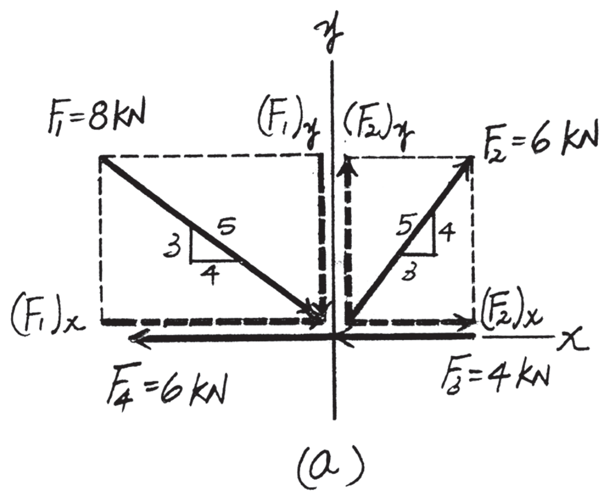
Summing these force components along x and y axes algebraically,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 6.40 + 3.60 - 4 - 6 = 0$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 4.80 - 4.80 = 0$$

Thus,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{O^2 + O^2} = O \qquad \textbf{(Q.E.D)}$$

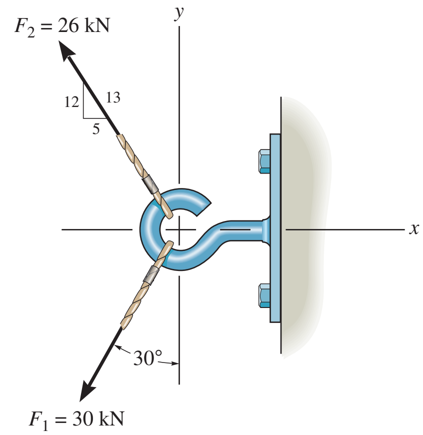


(a)

Ans:
$(F_1)_x = 6.40$ kN $\rightarrow$
$(F_1)_y = 4.80$ kN $\downarrow$
$(F_2)_x = 3.60$ kN $\rightarrow$
$(F_2)_y = 4.80$ kN $\uparrow$
$(F_3)_x = 4$ kN $\leftarrow$
$(F_3)_y = 0$
$(F_4)_x = 6$ kN $\leftarrow$
$(F_4)_y = 0$

2–53.

Express $\mathbf{F}_1$ and $\mathbf{F}_2$ as Cartesian vectors.



SOLUTION

$$\mathbf{F}_1 = -30 \sin 30° \, \mathbf{i} - 30 \cos 30° \, \mathbf{j}$$

$$= \{-15.0 \, \mathbf{i} - 26.0 \, \mathbf{j}\} \text{ kN} \qquad \textbf{Ans.}$$

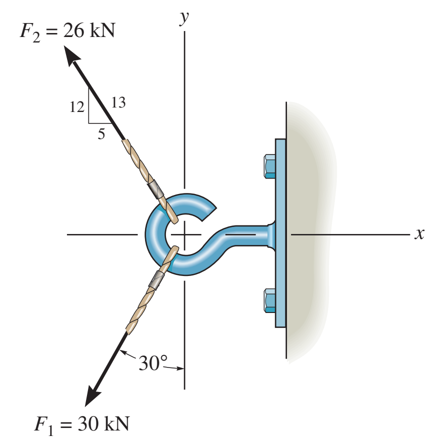
$$\mathbf{F}_2 = -\frac{5}{13}(26) \, \mathbf{i} + \frac{12}{13}(26) \, \mathbf{j}$$

$$= \{-10.0 \, \mathbf{i} + 24.0 \, \mathbf{j}\} \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_1 = \{-15.0\mathbf{i} - 26.0\mathbf{j}\}$ kN
$\mathbf{F}_2 = \{-10.0\mathbf{i} + 24.0\mathbf{j}\}$ kN

2–54.

Determine the magnitude of the resultant force and its direction measured counterclockwise from the positive x axis.



SOLUTION

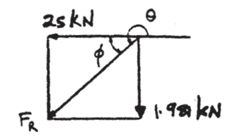
$\xrightarrow{+} F_{Rx} = \Sigma F_x;$ $\quad F_{Rx} = -30 \sin 30° - \frac{5}{13}(26) = -25 \text{ kN}$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = -30 \cos 30° + \frac{12}{13}(26) = -1.981 \text{ kN}$

$$F_R = \sqrt{(-25)^2 + (-1.981)^2} = 25.1 \text{ kN}$$ **Ans.**

$$\phi = \tan^{-1}\left(\frac{1.981}{25}\right) = 4.53°$$

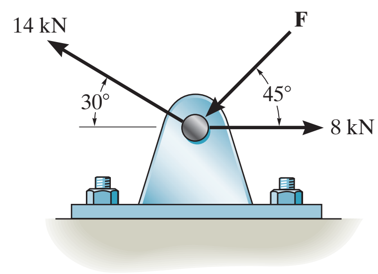
$$\theta = 180° + 4.53° = 185°$$ **Ans.**



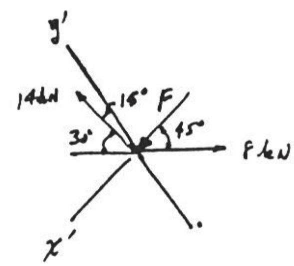
Ans:
$F_R = 25.1$ kN
$\theta = 185°$

2–55.

Determine the magnitude of force **F** so that the resultant force of the three forces is as small as possible. What is the magnitude of the resultant force?



SOLUTION



$\xrightarrow{+} F_{Rx} = \Sigma F_x;$ $\qquad F_{Rz} = 8 - F\cos 45° - 14\cos 30°$

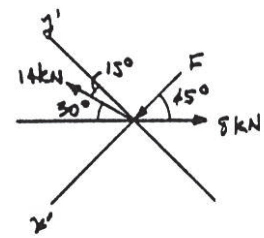
$$= -4.1244 - F\cos 45°$$

$+\uparrow F_{Ry} = \Sigma F_y;$ $\qquad F_{Ry} = -F\sin 45° + 14\sin 30°$

$$= 7 - F\sin 45°$$

$$F_R^2 = (-4.1244 - F\cos 45°)^2 + (7 - F\sin 45°)^2 \qquad \textbf{(1)}$$

$$2F_R\frac{dF_R}{dF} = 2(-4.1244 - F\cos 45°)(-\cos 45°) + 2(7 - F\sin 45°)(-\sin 45°) = 0$$



$F = 2.03$ kN **Ans.**

From Eq. (1); $\qquad F_R = 7.87$ kN **Ans.**

Also, from the figure require

$(F_R)_{x'} = 0 = \Sigma F_{x'};$ $\qquad F + 14\sin 15° - 8\cos 45° = 0$

$F = 2.03$ kN **Ans.**

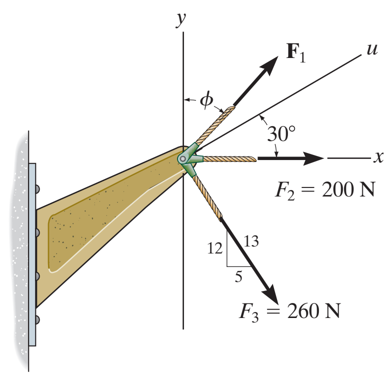
$(F_R)_{y'} = \Sigma F_{y'};$ $\qquad F_R = 14\cos 15° - 8\sin 45°$

$F_R = 7.87$ kN **Ans.**

Ans:
$F = 2.03$ kN
$F_R = 7.87$ kN

***2–56.**

If the magnitude of the resultant force acting on the bracket is to be 450 N directed along the positive u axis, determine the magnitude of $\mathbf{F}_1$ and its direction ϕ.



SOLUTION

Rectangular Components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, and $\mathbf{F}_R$ can be written as

$(F_1)_x = F_1 \sin\phi \qquad (F_1)_y = F_1 \cos\phi$

$(F_2)_x = 200 \text{ N} \qquad (F_2)_y = 0$

$(F_3)_x = 260\left(\dfrac{5}{13}\right) = 100 \text{ N} \qquad (F_3)_y = 260\left(\dfrac{12}{13}\right) = 240 \text{ N}$

$(F_R)_x = 450 \cos 30^\circ = 389.71 \text{ N} \qquad (F_R)_y = 450 \sin 30^\circ = 225 \text{ N}$

Resultant Force: Summing the force components algebraically along the x and y axes,

$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \quad 389.71 = F_1 \sin\phi + 200 + 100$

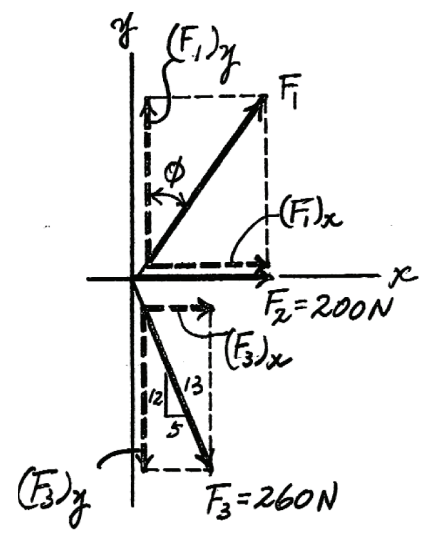
$$F_1 \sin\phi = 89.71 \qquad \textbf{(1)}$$

$+\uparrow \Sigma(F_R)_y = \Sigma F_y; \quad 225 = F_1 \cos\phi - 240$

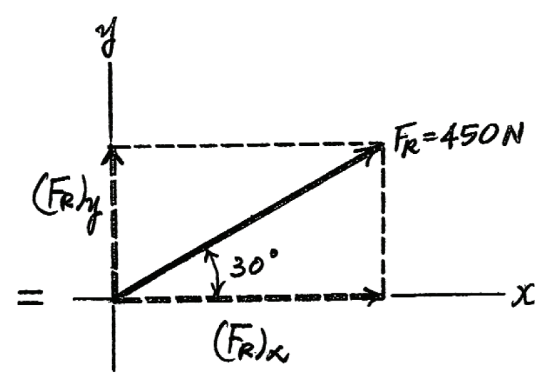
$$F_1 \cos\phi = 465 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$\phi = 10.9^\circ \qquad F_1 = 474 \text{ N}$ **Ans.**



(a)

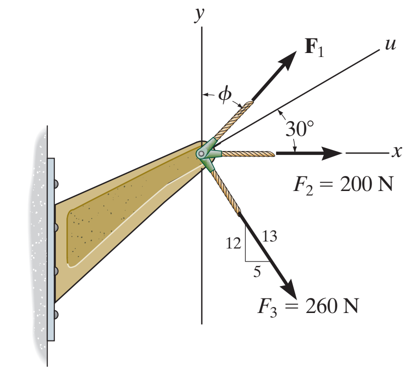


(b)

Ans:
$\phi = 10.9^\circ$
$F_1 = 474 \text{ N}$

2–57.

If the resultant force acting on the bracket is required to be a minimum, determine the magnitudes of $\mathbf{F}_1$ and the resultant force. Set $\phi = 30°$.



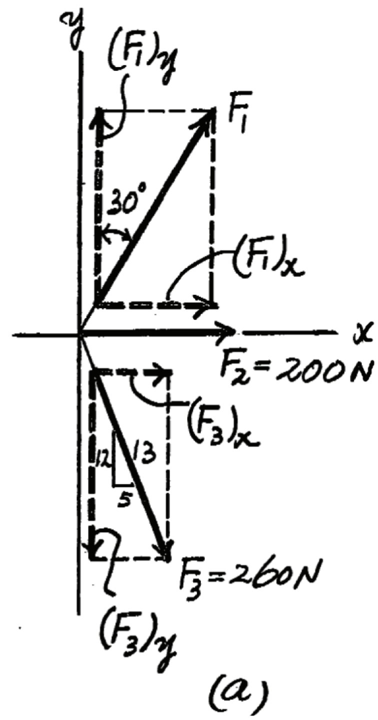
SOLUTION

Rectangular Components: By referring to Fig. *a*, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$(F_1)_x = F_1 \sin 30° = 0.5F_1$ $\qquad$ $(F_1)_y = F_1 \cos 30° = 0.8660F_1$

$(F_2)_x = 200 \text{ N}$ $\qquad$ $(F_2)_y = 0$

$(F_3)_x = 260\left(\frac{5}{13}\right) = 100 \text{ N}$ $\qquad$ $(F_3)_y = 260\left(\frac{12}{13}\right) = 240 \text{ N}$



Resultant Force: Summing the force components algebraically along the x and y axes,

$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x;\quad (F_R)_x = 0.5F_1 + 200 + 100 = 0.5F_1 + 300$

$+\uparrow \Sigma(F_R)_y = \Sigma F_y;\quad (F_R)_y = 0.8660F_1 - 240$

The magnitude of the resultant force $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2}$$

$$= \sqrt{(0.5F_1 + 300)^2 + (0.8660F_1 - 240)^2}$$

$$= \sqrt{F_1^2 - 115.69F_1 + 147\,600} \qquad \textbf{(1)}$$

Thus,

$$F_R^2 = F_1^2 - 115.69F_1 + 147\,600 \qquad \textbf{(2)}$$

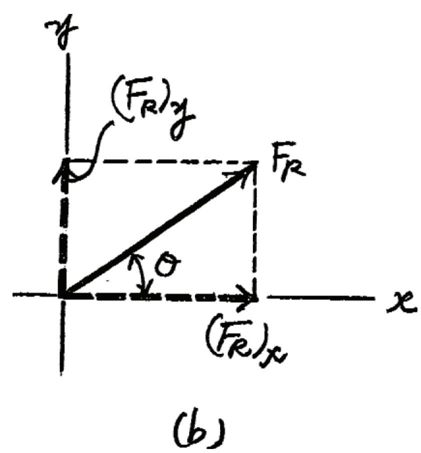
The first derivative of Eq. (2) is

$$2F_R\frac{dF_R}{dF_1} = 2F_1 - 115.69 \qquad \textbf{(3)}$$

For $\mathbf{F}_R$ to be minimum, $\frac{dF_R}{dF_1} = 0$. Thus, from Eq. (3)

$$2F_R\frac{dF_R}{dF_1} = 2F_1 - 115.69 = 0$$

$$F_1 = 57.846 \text{ N} = 57.8 \text{ N} \qquad \textbf{Ans.}$$



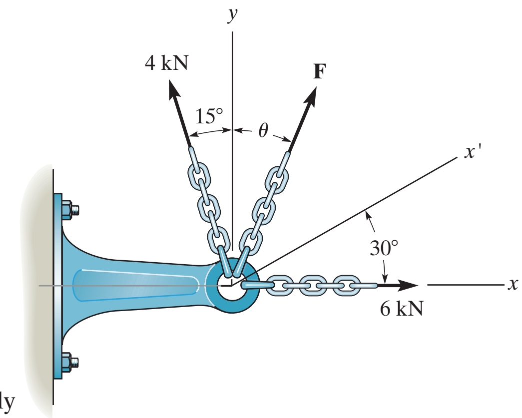
from Eq. (1),

$$F_R = \sqrt{(57.846)^2 - 115.69(57.846) + 147\,600} = 380 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$F_R = 380 \text{ N}$
$F_1 = 57.8 \text{ N}$

2–58.

Three forces act on the bracket. Determine the magnitude and direction θ of **F** so that the resultant force is directed along the positive x' axis and has a magnitude of 8 kN.



SOLUTION

Scalar Notation. Equating the force components along the x and y axes algebraically by referring to Fig. a,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad 8 \cos 30° = F \sin \theta + 6 - 4 \sin 15°$$

$$F \sin \theta = 1.9635 \qquad (1)$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad 8 \sin 30° = F \cos \theta + 4 \cos 15°$$

$$F \cos \theta = 0.1363 \qquad (2)$$

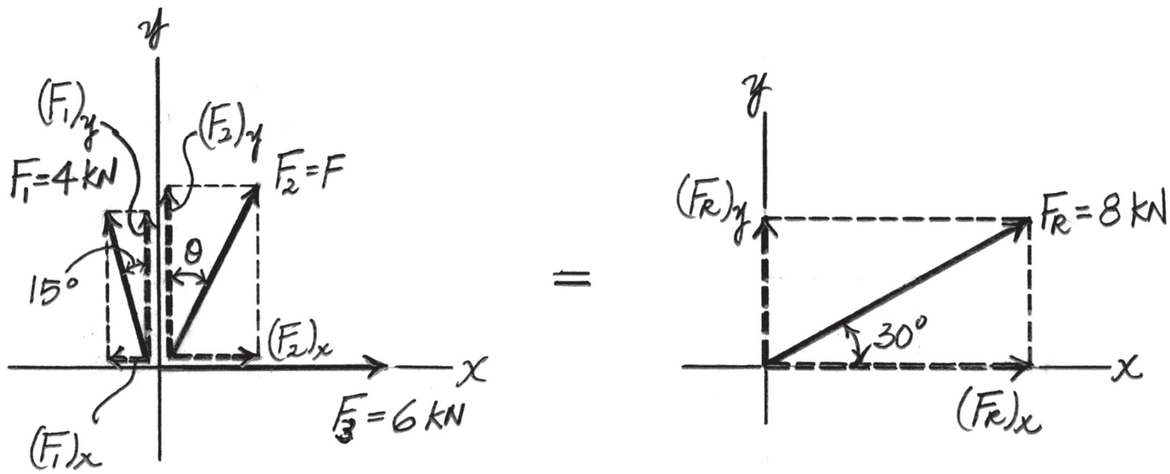
Divide Eq (1) by (2)

$$\tan \theta = 14.406 \qquad \theta = 86.03° = 86.0° \qquad \textbf{Ans.}$$

Substitute this result into Eq (1)

$$F \sin 86.03° = 1.9635$$

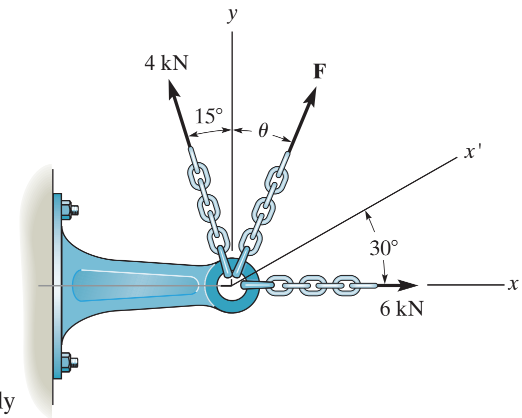
$$F = 1.968 \text{ kN} = 1.97 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$\theta = 86.0°$
$F = 1.97$ kN

2–59.

If $F = 5$ kN and $\theta = 30°$, determine the magnitude of the resultant force and its direction, measured counterclockwise from the positive x axis.



SOLUTION

Scalar Notation. Summing the force components along x and y axes algebraically by referring to Fig. a,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 5 \sin 30° + 6 - 4 \sin 15° = 7.465 \text{ kN} \rightarrow$$

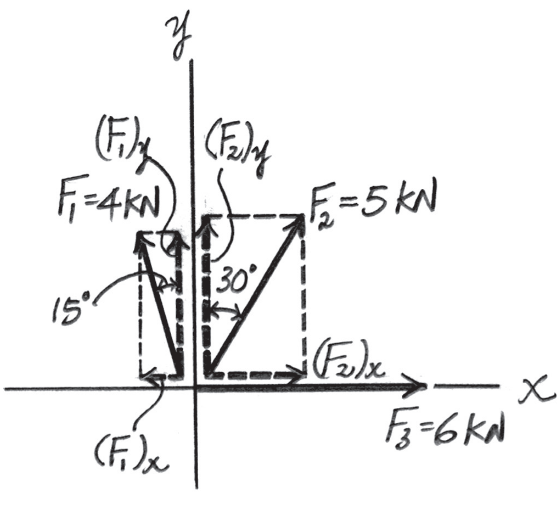
$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 4 \cos 15° + 5 \cos 30° = 8.194 \text{ kN} \uparrow$$

By referring to Fig. b, the magnitude of the resultant force is

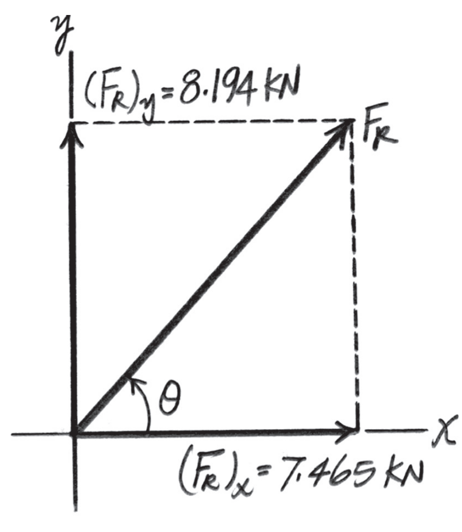
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{7.465^2 + 8.194^2} = 11.08 \text{ kN} = 11.1 \text{ kN} \quad \textbf{Ans.}$$

And its directional angle θ measured counterclockwise from the positive x axis is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{8.194}{7.465}\right) = 47.67° = 47.7° \quad \textbf{Ans.}$$



(a)

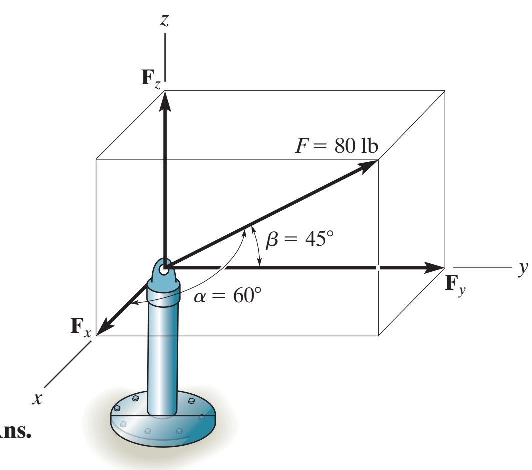


(b)

Ans:
$F_R = 11.1$ kN
$\theta = 47.7°$

***2–60.**

The force **F** has a magnitude of 80 lb and acts within the octant shown. Determine the magnitudes of the x, y, z components of **F**.



SOLUTION

$1 = \cos^2 60° + \cos^2 45° + \cos^2 \gamma$

Solving for the positive root, $\gamma = 60°$

$F_x = 80 \cos 60° = 40.0 \text{ lb}$ **Ans.**

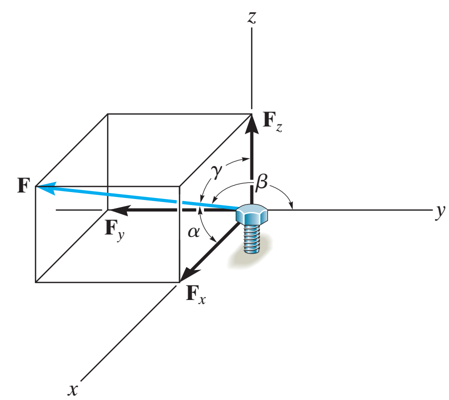
$F_y = 80 \cos 45° = 56.6 \text{ lb}$ **Ans.**

$F_z = 80 \cos 60° = 40.0 \text{ lb}$ **Ans.**

Ans:
$F_x = 40.0 \text{ lb}$
$F_y = 56.6 \text{ lb}$
$F_z = 40.0 \text{ lb}$

2–61.

The bolt is subjected to the force **F**, which has components acting along the *x, y, z* axes as shown. If the magnitude of **F** is 80 N, and $\alpha = 60°$ and $\gamma = 45°$, determine the magnitudes of its components.



SOLUTION

$$\cos\beta = \sqrt{1 - \cos^2\alpha - \cos^2\gamma}$$

$$= \sqrt{1 - \cos^2 60° - \cos^2 45°}$$

$$\beta = 120°$$

$$F_x = |80 \cos 60°| = 40 \text{ N}$$ **Ans.**

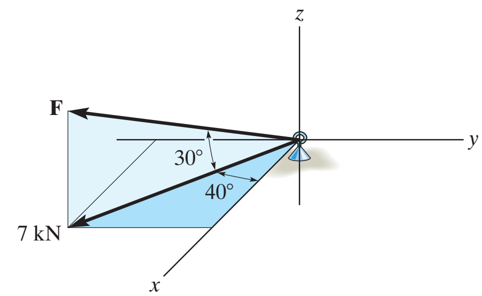
$$F_y = |80 \cos 120°| = 40 \text{ N}$$ **Ans.**

$$F_z = |80 \cos 45°| = 56.6 \text{ N}$$ **Ans.**

Ans:
$F_x = 40$ N
$F_y = 40$ N
$F_z = 56.6$ N

2–62.

Determine the magnitude and coordinate direction angles of the force **F** acting on the support. The component of **F** in the x–y plane is 7 kN.



SOLUTION

Coordinate Direction Angles. The unit vector of **F** is

$$\mathbf{u}_F = \cos 30° \cos 40° \mathbf{i} - \cos 30° \sin 40° \mathbf{j} + \sin 30° \, \mathbf{k}$$

$$= \{0.6634\mathbf{i} - 0.5567\mathbf{j} + 0.5\,\mathbf{k}\}$$

Thus,

$\cos \alpha = 0.6634; \quad \alpha = 48.44° = 48.4°$ **Ans.**

$\cos \beta = -0.5567; \quad \beta = 123.83° = 124°$ **Ans.**

$\cos \gamma = 0.5; \quad \gamma = 60°$ **Ans.**

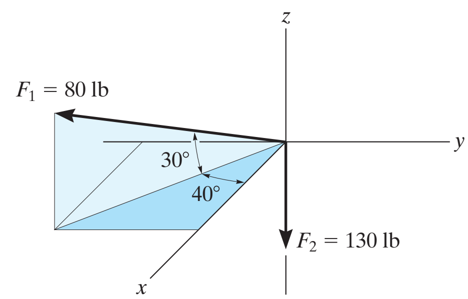
The magnitude of **F** can be determined from

$F \cos 30° = 7; \quad F = 8.083 \text{ kN} = 8.08 \text{ kN}$ **Ans.**

Ans:
$\alpha = 48.4°$
$\beta = 124°$
$\gamma = 60°$
$F = 8.08$ kN

2–63.

Determine the magnitude and coordinate direction angles of the resultant force and sketch this vector on the coordinate system.



SOLUTION

$\mathbf{F}_1 = \{80 \cos 30° \cos 40° \mathbf{i} - 80 \cos 30° \sin 40° \mathbf{j} + 80 \sin 30° \mathbf{k}\}$ lb

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb

$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

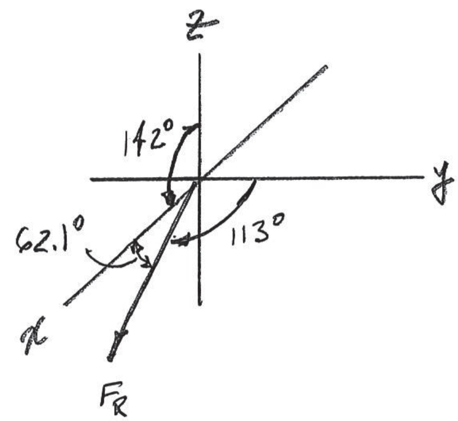
$\mathbf{F}_R = \{53.1\mathbf{i} - 44.5\mathbf{j} - 90.0\mathbf{k}\}$ lb

$F_R = \sqrt{(53.1)^2 + (-44.5)^2 + (-90.0)^2} = 114$ lb **Ans.**

$\alpha = \cos^{-1}\left(\dfrac{53.1}{113.6}\right) = 62.1°$ **Ans.**

$\beta = \cos^{-1}\left(\dfrac{-44.5}{113.6}\right) = 113°$ **Ans.**

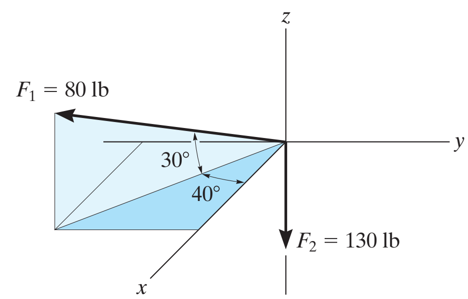
$\gamma = \cos^{-1}\left(\dfrac{-90.0}{113.6}\right) = 142°$ **Ans.**



Ans:
$F_R = 114$ lb
$\alpha = 62.1°$
$\beta = 113°$
$\gamma = 142°$

***2–64.**

Specify the coordinate direction angles of $\mathbf{F}_1$ and $\mathbf{F}_2$ and express each force as a Cartesian vector.



SOLUTION

$\mathbf{F}_1 = \{80 \cos 30° \cos 40° \mathbf{i} - 80 \cos 30° \sin 40° \mathbf{j} + 80 \sin 30° \mathbf{k}\}$ lb

$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb **Ans.**

$\alpha_1 = \cos^{-1}\left(\frac{53.1}{80}\right) = 48.4°$ **Ans.**

$\beta_1 = \cos^{-1}\left(\frac{-44.5}{80}\right) = 124°$ **Ans.**

$\gamma_1 = \cos^{-1}\left(\frac{40}{80}\right) = 60°$ **Ans.**

$\mathbf{F}_2 = \{-130\mathbf{k}\}$ lb **Ans.**

$\alpha_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$ **Ans.**

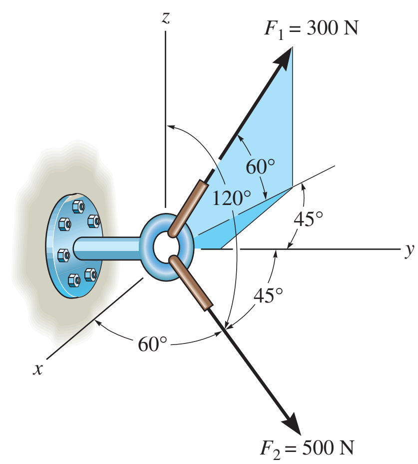
$\beta_2 = \cos^{-1}\left(\frac{0}{130}\right) = 90°$ **Ans.**

$\gamma_2 = \cos^{-1}\left(\frac{-130}{130}\right) = 180°$ **Ans.**

Ans:
$\mathbf{F}_1 = \{53.1\mathbf{i} - 44.5\mathbf{j} + 40\mathbf{k}\}$ lb
$\alpha_1 = 48.4°$
$\beta_1 = 124°$
$\gamma_1 = 60°$
$\mathbf{F}_2 = \{-130\text{k}\}$ lb
$\alpha_2 = 90°$
$\beta_2 = 90°$
$\gamma_2 = 180°$

2–65.

The screw eye is subjected to the two forces shown. Express each force in Cartesian vector form and then determine the resultant force. Find the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$\mathbf{F}_1 = 300(-\cos 60° \sin 45° \mathbf{i} + \cos 60° \cos 45° \mathbf{j} + \sin 60° \mathbf{k})$

$= \{-106.07\mathbf{i} + 106.07\mathbf{j} + 259.81\mathbf{k}\}$ N

$= \{-106\mathbf{i} + 106\mathbf{j} + 260\mathbf{k}\}$ N **Ans.**

$\mathbf{F}_2 = 500(\cos 60° \mathbf{i} + \cos 45° \mathbf{j} + \cos 120° \mathbf{k})$

$= \{250.0\mathbf{i} + 353.55\mathbf{j} - 250.0\mathbf{k}\}$ N

$= \{250\mathbf{i} + 354\mathbf{j} - 250\mathbf{k}\}$ N **Ans.**

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$

$= -106.07\mathbf{i} + 106.07\mathbf{j} + 259.81\mathbf{k} + 250.0\mathbf{i} + 353.55\mathbf{j} - 250.0\mathbf{k}$

$= 143.93\mathbf{i} + 459.62\mathbf{j} + 9.81\mathbf{k}$

$= \{144\mathbf{i} + 460\mathbf{j} + 9.81\mathbf{k}\}$ N **Ans.**

$F_R = \sqrt{143.93^2 + 459.62^2 + 9.81^2} = 481.73 \text{ N} = 482 \text{ N}$ **Ans.**

$$\mathbf{u}_{F_R} = \frac{\mathbf{F}_R}{F_R} = \frac{143.93\mathbf{i} + 459.62\mathbf{j} + 9.81\mathbf{k}}{481.73} = 0.2988\mathbf{i} + 0.9541\mathbf{j} + 0.02036\mathbf{k}$$

$\cos\alpha = 0.2988$ $\quad\alpha = 72.6°$ **Ans.**

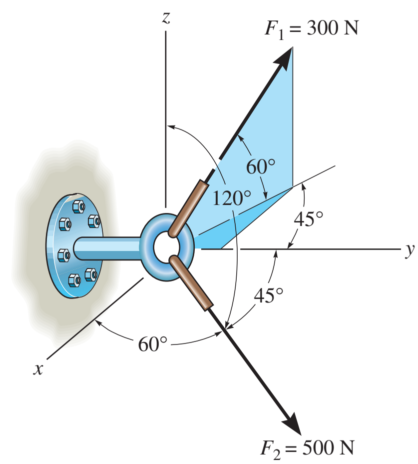
$\cos\beta = 0.9541$ $\quad\beta = 17.4°$ **Ans.**

$\cos\gamma = 0.02036$ $\quad\gamma = 88.8°$ **Ans.**

Ans:
$\mathbf{F}_1 = \{-106\mathbf{i} + 106\mathbf{j} + 260\mathbf{k}\}$ N
$\mathbf{F}_2 = \{250\mathbf{i} + 354\mathbf{j} - 250\mathbf{k}\}$ N
$\mathbf{F}_R = \{144\mathbf{i} + 460\mathbf{j} + 9.81\mathbf{k}\}$ N
$F_R = 482$ N
$\alpha = 72.6°$
$\beta = 17.4°$
$\gamma = 88.8°$

2–66.

Determine the coordinate direction angles of $\mathbf{F}_1$.



SOLUTION

$$\mathbf{F}_1 = 300(-\cos 60^\circ \sin 45^\circ \mathbf{i} + \cos 60^\circ \cos 45^\circ \mathbf{j} + \sin 60^\circ \mathbf{k})$$

$$= \{-106.07\mathbf{i} + 106.07\mathbf{j} + 259.81\mathbf{k}\}\text{ N}$$

$$= \{-106\mathbf{i} + 106\mathbf{j} + 260\mathbf{k}\}\text{ N}$$

$$\mathbf{u}_1 = \frac{\mathbf{F}_1}{300} = -0.3536\mathbf{i} + 0.3536\mathbf{j} + 0.8660\mathbf{k}$$

$\alpha_1 = \cos^{-1}(-0.3536) = 111^\circ$ **Ans.**

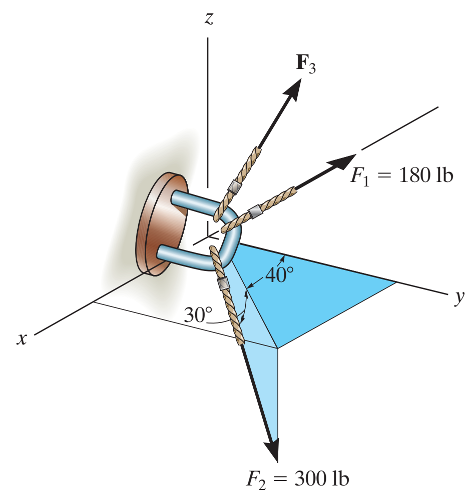
$\beta_1 = \cos^{-1}(0.3536) = 69.3^\circ$ **Ans.**

$\gamma_1 = \cos^{-1}(0.8660) = 30.0^\circ$ **Ans.**

Ans:
$\alpha_1 = 111^\circ$
$\beta_1 = 69.3^\circ$
$\gamma_1 = 30.0^\circ$

2–67.

Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces acts along the positive y axis and has a magnitude of 600 lb.



SOLUTION

$F_{Rx} = \Sigma F_x\;;\qquad 0 = -180 + 300 \cos 30° \sin 40° + F_3 \cos \alpha$

$F_{Ry} = \Sigma F_y\;;\qquad 600 = 300 \cos 30° \cos 40° + F_3 \cos \beta$

$F_{Rz} = \Sigma F_z\;;\qquad 0 = -300 \sin 30° + F_3 \cos \gamma$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Solving:

$F_3 = 428$ lb **Ans.**

$\alpha = 88.3°$ **Ans.**

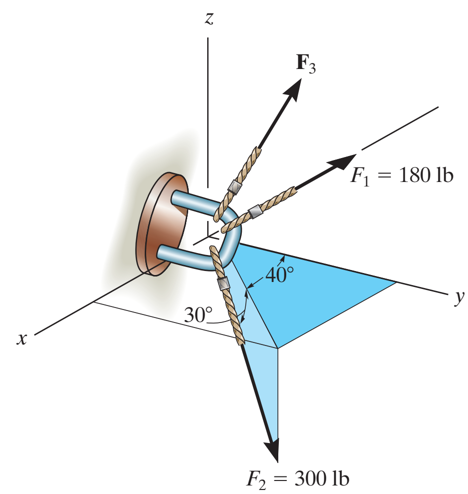
$\beta = 20.6°$ **Ans.**

$\gamma = 69.5°$ **Ans.**

Ans:
$F_3 = 428$ lb
$\alpha = 88.3°$
$\beta = 20.6°$
$\gamma = 69.5°$

***2–68.**

Determine the magnitude and coordinate direction angles of $\mathbf{F}_3$ so that the resultant of the three forces is zero.



SOLUTION

$F_{Rx} = \Sigma F_x; \qquad 0 = -180 + 300 \cos 30° \sin 40° + F_3 \cos \alpha$

$F_{Ry} = \Sigma F_y; \qquad 0 = 300 \cos 30° \cos 40° + F_3 \cos \beta$

$F_{Rz} = \Sigma F_z; \qquad 0 = -300 \sin 30° + F_3 \cos \gamma$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

Solving:

$F_3 = 250$ lb **Ans.**

$\alpha = 87.0°$ **Ans.**

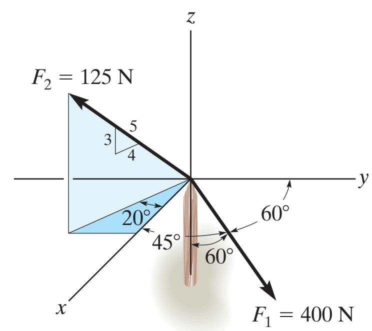
$\beta = 143°$ **Ans.**

$\gamma = 53.1°$ **Ans.**

Ans:
$F_3 = 250$ lb
$\alpha = 87.0°$
$\beta = 143°$
$\gamma = 53.1°$

2–69.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.

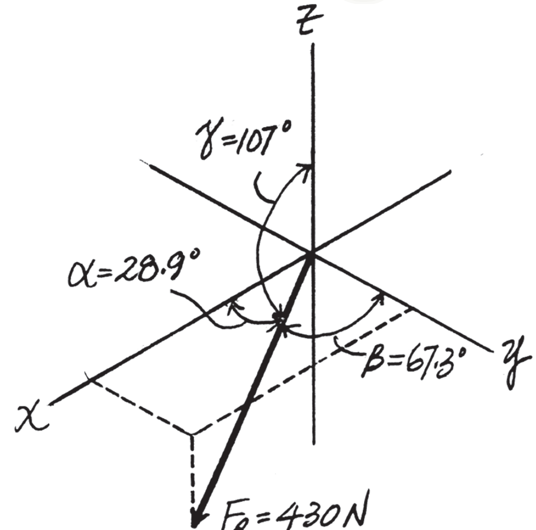


SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_1$ and $\mathbf{F}_2$,

$$\mathbf{F}_1 = 400\,(\cos 45°\mathbf{i} + \cos 60°\mathbf{j} - \cos 60°\mathbf{k}) = \{282.84\mathbf{i} + 200\mathbf{j} - 200\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 125\left[\frac{4}{5}(\cos 20°)\mathbf{i} - \frac{4}{5}(\sin 20°)\mathbf{j} + \frac{3}{5}\mathbf{k}\right] = \{93.97\mathbf{i} - 34.20\mathbf{j} + 75.0\mathbf{k}\}$$



Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= \{282.84\mathbf{i} + 200\mathbf{j} - 200\mathbf{k}\} + \{93.97\mathbf{i} - 34.20\mathbf{j} + 75.0\mathbf{k}\}$$
$$= \{376.81\mathbf{i} + 165.80\mathbf{j} - 125.00\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{376.81^2 + 165.80^2 + (-125.00)^2}$$
$$= 430.23\text{ N} = 430\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{376.81}{430.23}; \qquad \alpha = 28.86° = 28.9° \qquad \textbf{Ans.}$$

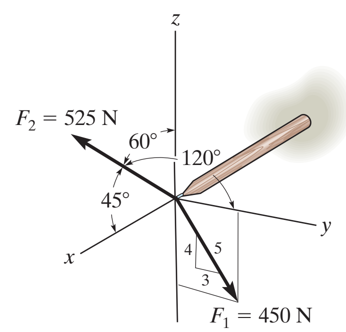
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{165.80}{430.23}; \qquad \beta = 67.33° = 67.3° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-125.00}{430.23}; \qquad \gamma = 106.89° = 107° \qquad \textbf{Ans.}$$

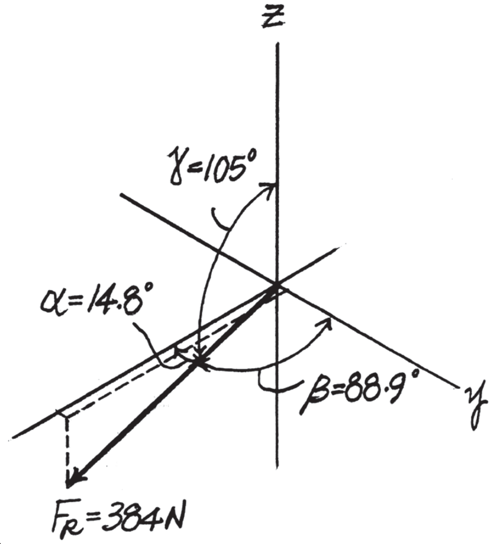
Ans:
$F_R = 430$ N
$\alpha = 28.9°$
$\beta = 67.3°$
$\gamma = 107°$

2–70.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION



Cartesian Vector Notation. For $\mathbf{F}_1$ and $\mathbf{F}_2$,

$$\mathbf{F}_1 = 450\left(\frac{3}{5}\mathbf{j} - \frac{4}{5}\mathbf{k}\right) = \{270\mathbf{j} - 360\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 525\,(\cos 45°\mathbf{i} + \cos 120°\mathbf{j} + \cos 60°\mathbf{k}) = \{371.23\mathbf{i} - 262.5\mathbf{j} + 262.5\mathbf{k}\}\text{ N}$$

Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$
$$= \{270\mathbf{j} - 360\mathbf{k}\} + \{371.23\mathbf{i} - 262.5\mathbf{j} + 262.5\mathbf{k}\}$$
$$= \{371.23\mathbf{i} + 7.50\mathbf{j} - 97.5\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{371.23^2 + 7.50^2 + (-97.5)^2}$$
$$= 383.89\text{ N} = 384\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{371.23}{383.89}; \qquad \alpha = 14.76° = 14.8° \qquad \textbf{Ans.}$$

$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{7.50}{383.89}; \qquad \beta = 88.88° = 88.9° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-97.5}{383.89}; \qquad \gamma = 104.71° = 105° \qquad \textbf{Ans.}$$

Ans:

$F_R = 384$ N

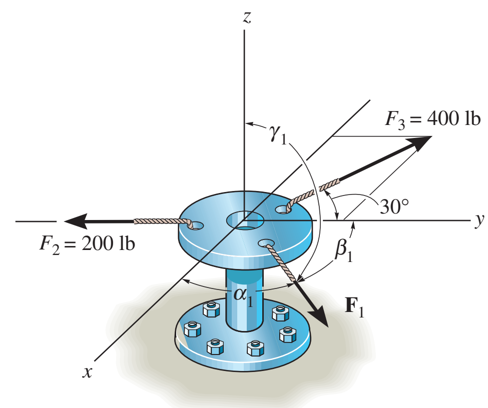
$\cos\alpha = \frac{371.23}{383.89}$; $\alpha = 14.8°$

$\cos\beta = \frac{7.50}{383.89}$; $\beta = 88.9°$

$\cos\gamma = \frac{-97.5}{383.89}$; $\gamma = 105°$

2–71.

Specify the magnitude and coordinate direction angles α_1, β_1, γ_1 of $\mathbf{F}_1$ so that the resultant of the three forces acting on the bracket is $\mathbf{F}_R = \{-350\mathbf{k}\}$ lb. Note that $\mathbf{F}_3$ lies in the x–y plane.



SOLUTION

$\mathbf{F}_1 = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k}$

$\mathbf{F}_2 = -200\,\mathbf{j}$

$\mathbf{F}_3 = -400\sin 30°\,\mathbf{i} + 400\cos 30°\,\mathbf{j}$

$\quad = -200\,\mathbf{i} + 346.4\,\mathbf{j}$

$\mathbf{F}_R = \Sigma\mathbf{F}$

$-350\,\mathbf{k} = F_x\,\mathbf{i} + F_y\,\mathbf{j} + F_z\,\mathbf{k} - 200\,\mathbf{j} - 200\,\mathbf{i} + 346.4\,\mathbf{j}$

$0 = F_x - 200\,; \qquad F_x = 200$ lb

$0 = F_y - 200 + 346.4\,; \qquad F_y = -146.4$ lb

$F_z = -350$ lb

$F_1 = \sqrt{(200)^2 + (-146.4)^2 + (-350)^2}$

$F_1 = 425.9 \text{ lb} = 429 \text{ lb}$ **Ans.**

$\alpha_1 = \cos^{-1}\left(\dfrac{200}{428.9}\right) = 62.2°$ **Ans.**

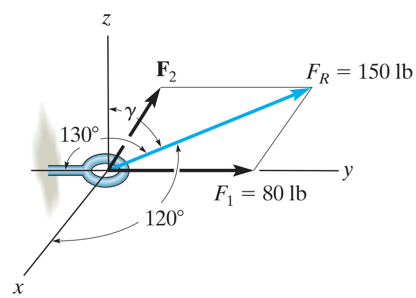
$\beta_1 = \cos^{-1}\left(\dfrac{-146.4}{428.9}\right) = 110°$ **Ans.**

$\gamma_1 = \cos^{-1}\left(\dfrac{-350}{428.9}\right) = 145°$ **Ans.**

Ans:
$F_1 = 429$ lb
$\alpha_1 = 62.2°$
$\beta_1 = 110°$
$\gamma_1 = 145°$

***2–72.**

Two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ act on the screw eye. If the resultant force $\mathbf{F}_R$ has a magnitude of 150 lb and the coordinate direction angles shown, determine the magnitude of $\mathbf{F}_2$ and its coordinate direction angles.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_R$, γ can be determined from

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\cos^2 120° + \cos^2 50° + \cos^2\gamma = 1$$

$$\cos\gamma = \pm 0.5804$$

Here $\gamma < 90°$, then

$$\gamma = 54.52°$$

Thus

$$\mathbf{F}_R = 150(\cos 120°\mathbf{i} + \cos 50°\mathbf{j} + \cos 54.52°\mathbf{k})$$

$$= \{-75.0\mathbf{i} + 96.42\mathbf{j} + 87.05\mathbf{k}\}\text{ lb}$$

Also

$$\mathbf{F}_1 = \{80\mathbf{j}\}\text{ lb}$$

Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\{-75.0\mathbf{i} + 96.42\mathbf{j} + 87.05\mathbf{k}\} = \{80\mathbf{j}\} + \mathbf{F}_2$$

$$F_2 = \{-75.0\mathbf{i} + 16.42\mathbf{j} + 87.05\mathbf{k}\}\text{ lb}$$

Thus, the magnitude of $\mathbf{F}_2$ is

$$F_2 = \sqrt{(F_2)_x + (F_2)_y + (F_2)_z} = \sqrt{(-75.0)^2 + 16.42^2 + 87.05^2}$$

$$= 116.07\text{ lb} = 116\text{ lb} \qquad \textbf{Ans.}$$

And its coordinate direction angles are

$$\cos\alpha_2 = \frac{(F_2)_x}{F_2} = \frac{-75.0}{116.07}; \qquad \alpha_2 = 130.25° = 130° \qquad \textbf{Ans.}$$

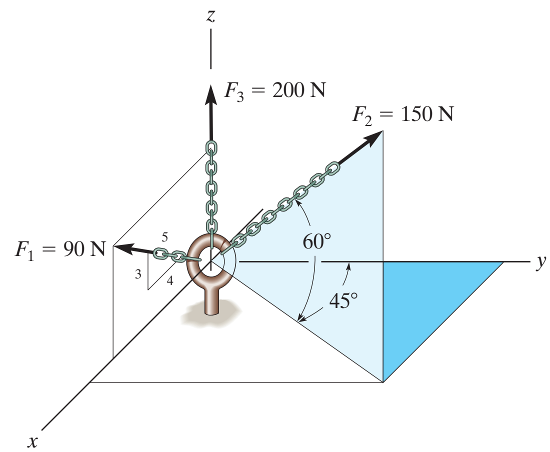
$$\cos\beta_2 = \frac{(F_2)_y}{F_2} = \frac{16.42}{116.07}; \qquad \beta_2 = 81.87° = 81.9° \qquad \textbf{Ans.}$$

$$\cos\gamma_2 = \frac{(F_2)_z}{F_2} = \frac{87.05}{116.07}; \qquad \gamma_2 = 41.41° = 41.4° \qquad \textbf{Ans.}$$

Ans:
$F_R = 116$ lb
$\cos\alpha_2 = 130°$
$\cos\beta_2 = 81.9°$
$\cos\gamma_2 = 41.4°$

2–73.

Express each force in Cartesian vector form.



SOLUTION

Cartesian Vector Notation. For $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$,

$$\mathbf{F}_1 = 90\left(\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right) = \{72.0\mathbf{i} + 54.0\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = 150\,(\cos 60°\sin 45°\mathbf{i} + \cos 60°\cos 45°\mathbf{j} + \sin 60°\mathbf{k})$$

$$= \{53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}\}\text{ N}$$

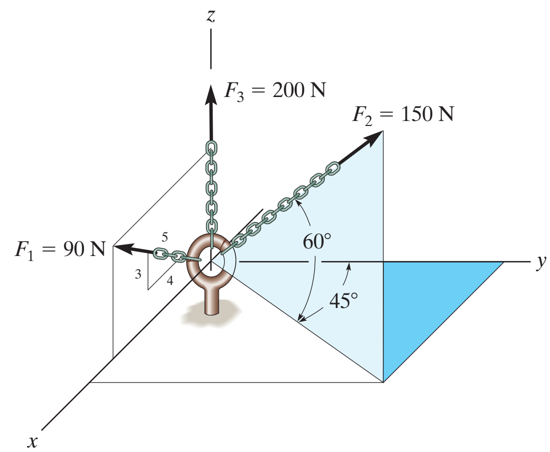
$$= \{53.0\mathbf{i} + 53.0\mathbf{j} + 130\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_3 = \{200\,\mathbf{k}\} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_1 = \{72.0\mathbf{i} + 54.0\mathbf{k}\}$ N
$\mathbf{F}_2 = \{53.0\mathbf{i} + 53.0\mathbf{j} + 130\mathbf{k}\}$ N
$\mathbf{F}_3 = \{200\,\mathbf{k}\}$

2–74.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

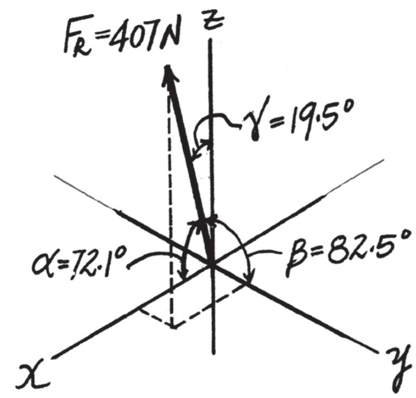
Cartesian Vector Notation. For $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$,

$$\mathbf{F}_1 = 90\left(\frac{4}{5}\mathbf{i} + \frac{3}{5}\mathbf{k}\right) = \{72.0\mathbf{i} + 54.0\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 150\,(\cos 60° \sin 45°\mathbf{i} + \cos 60° \cos 45°\mathbf{j} + \sin 60°\mathbf{k})$$

$$= \{53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_3 = \{200\text{ k}\}\text{ N}$$



Resultant Force.

$$\mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= (72.0\mathbf{i} + 54.0\mathbf{k}) + (53.03\mathbf{i} + 53.03\mathbf{j} + 129.90\mathbf{k}) + (200\mathbf{k})$$

$$= \{125.03\mathbf{i} + 53.03\mathbf{j} + 383.90\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{125.03^2 + 53.03^2 + 383.90^2}$$

$$= 407.22\text{ N} = 407\text{ N} \qquad \textbf{Ans.}$$

And the coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{125.03}{407.22}; \qquad \alpha = 72.12° = 72.1° \qquad \textbf{Ans.}$$

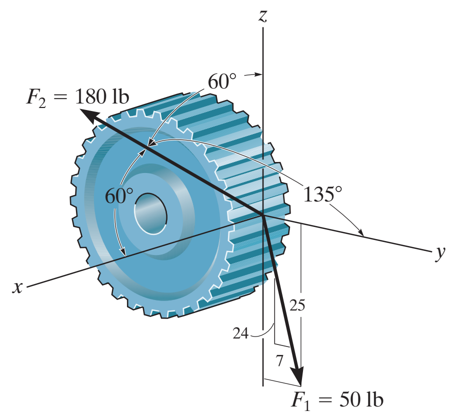
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{53.03}{407.22}; \qquad \beta = 82.52° = 82.5° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{383.90}{407.22}; \qquad \gamma = 19.48° = 19.5° \qquad \textbf{Ans.}$$

Ans:
$F_R = 407$ N
$\alpha = 72.1°$
$\beta = 82.5°$
$\gamma = 19.5°$

2–75.

The spur gear is subjected to the two forces caused by contact with other gears. Express each force as a Cartesian vector.



SOLUTION

$$\mathbf{F}_1 = \frac{7}{25}(50)\mathbf{j} - \frac{24}{25}(50)\mathbf{k} = \{14.0\mathbf{j} - 48.0\mathbf{k}\}\text{ lb}$$ **Ans.**

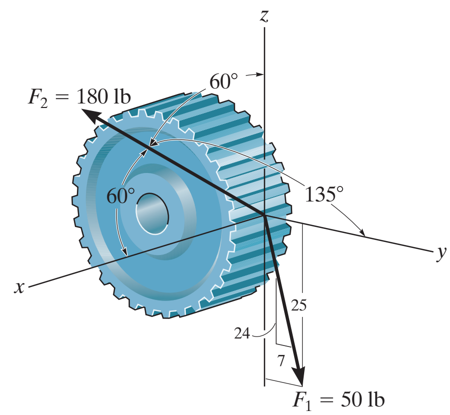
$$\mathbf{F}_2 = 180\cos 60°\mathbf{i} + 180\cos 135°\mathbf{j} + 180\cos 60°\mathbf{k}$$

$$= \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$\mathbf{F}_1 = \{14.0\mathbf{j} - 48.0\mathbf{k}\}$ lb
$\mathbf{F}_2 = \{90\mathbf{i} - 127\mathbf{j} + 90\mathbf{k}\}$ lb

***2–76.**

The spur gear is subjected to the two forces caused by contact with other gears. Determine the resultant of the two forces and express the result as a Cartesian vector.



SOLUTION

$$F_{Rx} = 180 \cos 60° = 90$$

$$F_{Ry} = \frac{7}{25}(50) + 180 \cos 135° = -113$$

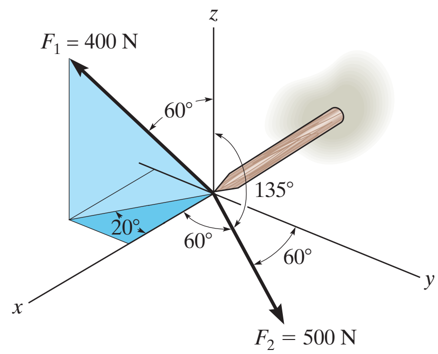
$$F_{Rz} = -\frac{24}{25}(50) + 180 \cos 60° = 42$$

$$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\} \text{ lb}$$ **Ans.**

Ans:
$F_{Rx} = 90$
$F_{Ry} = -113$
$F_{Rz} = 42$
$\mathbf{F}_R = \{90\mathbf{i} - 113\mathbf{j} + 42\mathbf{k}\}$ lb

2–77.

Determine the magnitude and coordinate direction angles of the resultant force, and sketch this vector on the coordinate system.



SOLUTION

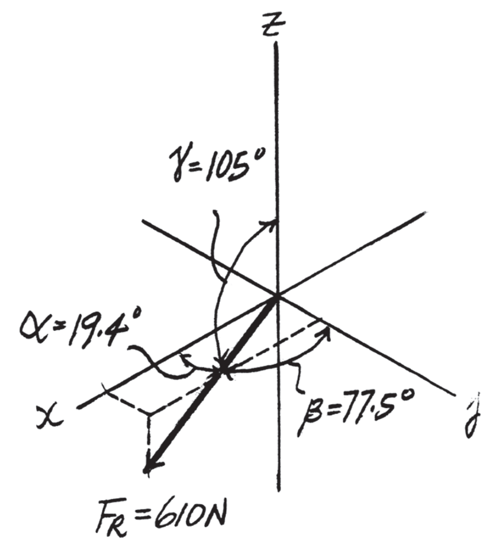
Cartesian Vector Notation. For $\mathbf{F}_1$ and $\mathbf{F}_2$,

$$\mathbf{F}_1 = 400\,(\sin 60^\circ \cos 20^\circ \mathbf{i} - \sin 60^\circ \sin 20^\circ \mathbf{j} + \cos 60^\circ \mathbf{k})$$

$$= \{325.52\mathbf{i} - 118.48\mathbf{j} + 200\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 500\,(\cos 60^\circ \mathbf{i} + \cos 60^\circ \mathbf{j} + \cos 135^\circ \mathbf{k})$$

$$= \{250\mathbf{i} + 250\mathbf{j} - 353.55\mathbf{k}\}\text{ N}$$



Resultant Force.

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (325.52\mathbf{i} - 118.48\mathbf{j} + 200\mathbf{k}) + (250\mathbf{i} + 250\mathbf{j} - 353.55\mathbf{k})$$

$$= \{575.52\mathbf{i} + 131.52\mathbf{j} - 153.55\,\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{575.52^2 + 131.52^2 + (-153.55)^2}$$

$$= 610.00\text{ N} = 610\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{575.52}{610.00} \qquad \alpha = 19.36^\circ = 19.4^\circ \qquad \textbf{Ans.}$$

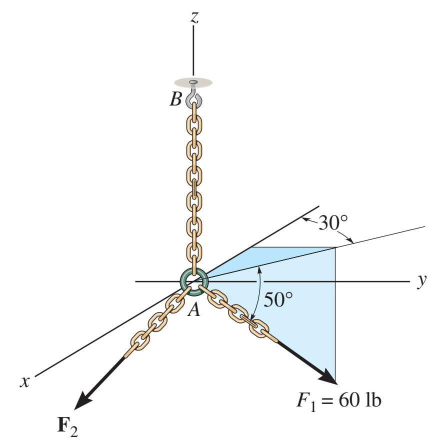
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{131.52}{610.00} \qquad \beta = 77.549^\circ = 77.5^\circ \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-153.55}{610.00} \qquad \gamma = 104.58^\circ = 105^\circ \qquad \textbf{Ans.}$$

Ans:
$F_R = 610$ N
$\alpha = 19.4^\circ$
$\beta = 77.5^\circ$
$\gamma = 105^\circ$

2–78.

The two forces $\mathbf{F}_1$ and $\mathbf{F}_2$ acting at A have a resultant force of $\mathbf{F}_R = \{-100\mathbf{k}\}$ lb. Determine the magnitude and coordinate direction angles of $\mathbf{F}_2$.



SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_R = \{-100\,\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_1 = 60\{-\cos 50^\circ \cos 30^\circ\,\mathbf{i} + \cos 50^\circ \sin 30^\circ\,\mathbf{j} - \sin 50^\circ\,\mathbf{k}\}\text{ lb}$$

$$= \{-33.40\,\mathbf{i} + 19.28\,\mathbf{j} - 45.96\,\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_2 = \{F_{2_x}\,\mathbf{i} + F_{2_y}\,\mathbf{j} + F_{2_z}\,\mathbf{k}\}\text{ lb}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$-100\mathbf{k} = \left\{\left(F_{2_x} - 33.40\right)\mathbf{i} + \left(F_{2_y} + 19.28\right)\mathbf{j} + \left(F_{2_z} - 45.96\right)\mathbf{k}\right\}$$

Equating **i, j** and **k** components, we have

$$F_{2_x} - 33.40 = 0 \qquad F_{2_x} = 33.40\text{ lb}$$

$$F_{2_y} + 19.28 = 0 \qquad F_{2_y} = -19.28\text{ lb}$$

$$F_{2_z} - 45.96 = -100 \qquad F_{2_z} = -54.04\text{ lb}$$

The magnitude of force $\mathbf{F}_2$ is

$$F_2 = \sqrt{F_{2_x}^2 + F_{2_y}^2 + F_{2_z}^2}$$

$$= \sqrt{33.40^2 + (-19.28)^2 + (-54.04)^2}$$

$$= 66.39\text{ lb} = 66.4\text{ lb} \qquad \textbf{Ans.}$$

The coordinate direction angles for $\mathbf{F}_2$ are

$$\cos\alpha = \frac{F_{2_x}}{F_2} = \frac{33.40}{66.39} \qquad \alpha = 59.8^\circ \qquad \textbf{Ans.}$$

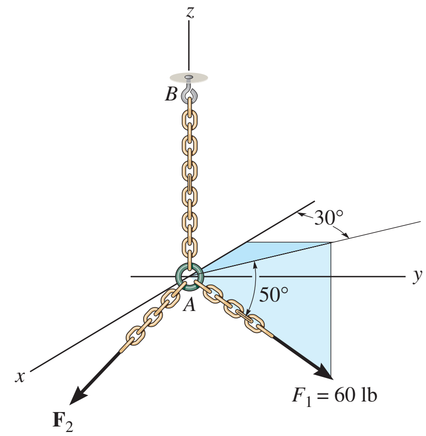
$$\cos\beta = \frac{F_{2_y}}{F_2} = \frac{-19.28}{66.39} \qquad \beta = 107^\circ \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{F_{2_z}}{F_2} = \frac{-54.04}{66.39} \qquad \gamma = 144^\circ \qquad \textbf{Ans.}$$

Ans:
$F_2 = 66.4$ lb
$\alpha = 59.8^\circ$
$\beta = 107^\circ$
$\gamma = 144^\circ$

2–79.

Determine the coordinate direction angles of the force $\mathbf{F}_1$ and indicate them on the figure.



SOLUTION

Unit Vector For Force $\mathbf{F}_1$:

$$\mathbf{u}_{F_1} = -\cos 50^\circ \cos 30^\circ\, \mathbf{i} + \cos 50^\circ \sin 30^\circ\, \mathbf{j} - \sin 50^\circ\, \mathbf{k}$$

$$= -0.5567\,\mathbf{i} + 0.3214\,\mathbf{j} - 0.7660\,\mathbf{k}$$

Coordinate Direction Angles: From the unit vector obtained above, we have

$$\cos\alpha = -0.5567 \qquad \alpha = 124^\circ \qquad \textbf{Ans.}$$

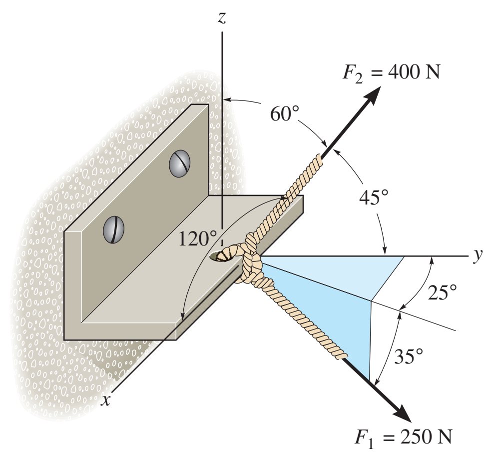
$$\cos\beta = 0.3214 \qquad \beta = 71.3^\circ \qquad \textbf{Ans.}$$

$$\cos\gamma = -0.7660 \qquad \gamma = 140^\circ \qquad \textbf{Ans.}$$

Ans:
$\alpha = 124^\circ$
$\beta = 71.3^\circ$
$\gamma = 140^\circ$

***2–80.**

The bracket is subjected to the two forces shown. Express each force in Cartesian vector form and then determine the resultant force $\mathbf{F}_R$. Find the magnitude and coordinate direction angles of the resultant force.



SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_1 = 250\{\cos 35^\circ \sin 25^\circ \mathbf{i} + \cos 35^\circ \cos 25^\circ \mathbf{j} - \sin 35^\circ \mathbf{k}\}\text{ N}$$

$$= \{86.55\mathbf{i} + 185.60\mathbf{j} - 143.39\mathbf{k}\}\text{ N}$$

$$= \{86.5\mathbf{i} + 186\mathbf{j} - 143\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_2 = 400\{\cos 120^\circ \mathbf{i} + \cos 45^\circ \mathbf{j} + \cos 60^\circ \mathbf{k}\}\text{ N}$$

$$= \{-200.0\mathbf{i} + 282.84\mathbf{j} + 200.0\mathbf{k}\}\text{ N}$$

$$= \{-200\mathbf{i} + 283\mathbf{j} + 200\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= \{(86.55 - 200.0)\mathbf{i} + (185.60 + 282.84)\mathbf{j} + (-143.39 + 200.0)\,\mathbf{k}\}$$

$$= \{-113.45\mathbf{i} + 468.44\mathbf{j} + 56.61\mathbf{k}\}\text{ N}$$

$$= \{-113\mathbf{i} + 468\mathbf{j} + 56.6\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

The magnitude of the resultant force is

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2 + F_{R_z}^2}$$

$$= \sqrt{(-113.45)^2 + 468.44^2 + 56.61^2}$$

$$= 485.30\text{ N} = 485\text{ N} \qquad \textbf{Ans.}$$

The coordinate direction angles are

$$\cos\alpha = \frac{F_{R_x}}{F_R} = \frac{-113.45}{485.30} \qquad \alpha = 104^\circ \qquad \textbf{Ans.}$$

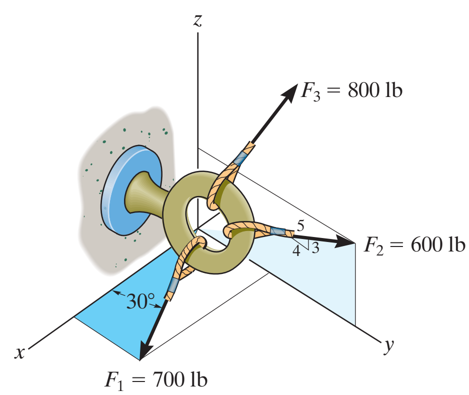
$$\cos\beta = \frac{F_{R_y}}{F_R} = \frac{468.44}{485.30} \qquad \beta = 15.1^\circ \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{F_{R_z}}{F_R} = \frac{56.61}{485.30} \qquad \gamma = 83.3^\circ \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_1 = \{86.5\mathbf{i} + 186\mathbf{j} - 143\mathbf{k}\}$ N
$\mathbf{F}_2 = \{-200\mathbf{i} + 283\mathbf{j} + 200\mathbf{k}\}$ N
$\mathbf{F}_R = \{-113\mathbf{i} + 468\mathbf{j} + 56.6\mathbf{k}\}$ N
$F_R = 485$ N
$\alpha = 104^\circ$
$\beta = 15.1^\circ$
$\gamma = 83.3^\circ$

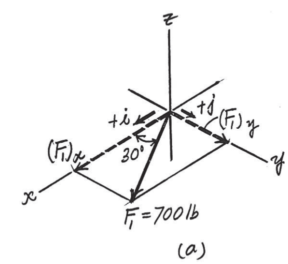
2–81.

If the coordinate direction angles for $\mathbf{F}_3$ are $\alpha_3 = 120°$, $\beta_3 = 60°$ and $\gamma_3 = 45°$, determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ into their x, y, and z components, as shown in Figs. a, b, and c, respectively, $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ can be expressed in Cartesian vector form as



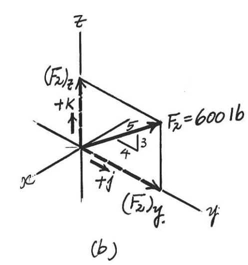
$$\mathbf{F}_1 = 700\cos 30°(+\mathbf{i}) + 700\sin 30°(+\mathbf{j}) = \{606.22\mathbf{i} + 350\mathbf{j}\}\text{ lb}$$

$$\mathbf{F}_2 = 0\mathbf{i} + 600\left(\frac{4}{5}\right)(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k}) = \{480\mathbf{j} + 360\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_3 = 800\cos 120°\mathbf{i} + 800\cos 60°\mathbf{j} + 800\cos 45°\mathbf{k} = [-400\mathbf{i} + 400\mathbf{j} + 565.69\mathbf{k}]\text{ lb}$$

Resultant Force: By adding $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\begin{aligned}\mathbf{F}_R &= \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 \\ &= (606.22\mathbf{i} + 350\mathbf{j}) + (480\mathbf{j} + 360\mathbf{k}) + (-400\mathbf{i} + 400\mathbf{j} + 565.69\mathbf{k}) \\ &= [206.22\mathbf{i} + 1230\mathbf{j} + 925.69\mathbf{k}]\text{ lb}\end{aligned}$$



The magnitude of $\mathbf{F}_R$ is

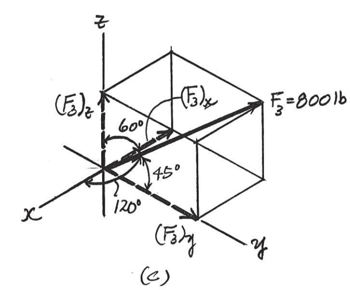
$$\begin{aligned}F_R &= \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2} \\ &= \sqrt{(206.22)^2 + (1230)^2 + (925.69)^2} = 1553.16\text{ lb} = 1.55\text{ kip} \qquad \textbf{Ans.}\end{aligned}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{206.22}{1553.16}\right) = 82.4° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{1230}{1553.16}\right) = 37.6° \qquad \textbf{Ans.}$$

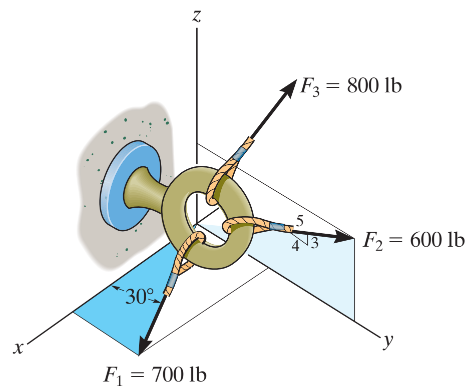
$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{925.69}{1553.16}\right) = 53.4° \qquad \textbf{Ans.}$$



Ans:
$F_R = 1.55$ kip
$\alpha = 82.4°$
$\beta = 37.6°$
$\gamma = 53.4°$

2–82.

If the coordinate direction angles for $\mathbf{F}_3$ are $\alpha_3 = 120°$, $\beta_3 = 45°$ and $\gamma_3 = 60°$, determine the magnitude and coordinate direction angles of the resultant force acting on the eyebolt.



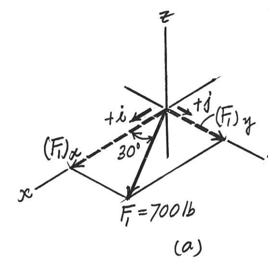
SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ into their x, y, and z components, as shown in Figs. a, b, and c, respectively, $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be expressed in Cartesian vector form as

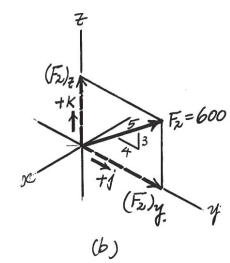
$$\mathbf{F}_1 = 700\cos 30°(+\mathbf{i}) + 700\sin 30°(+\mathbf{j}) = \{606.22\mathbf{i} + 350\mathbf{j}\}\text{ lb}$$

$$\mathbf{F}_2 = 0\mathbf{i} + 600\left(\frac{4}{5}\right)(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k}) = \{480\mathbf{j} + 360\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_3 = 800\cos 120°\mathbf{i} + 800\cos 45°\mathbf{j} + 800\cos 60°\mathbf{k} = \{-400\mathbf{i} + 565.69\mathbf{j} + 400\mathbf{k}\}\text{ lb}$$



$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$
$$= 606.22\mathbf{i} + 350\mathbf{j} + 480\mathbf{j} + 360\mathbf{k} - 400\mathbf{i} + 565.69\mathbf{j} + 400\mathbf{k}$$
$$= \{206.22\mathbf{i} + 1395.69\mathbf{j} + 760\mathbf{k}\}\text{ lb}$$

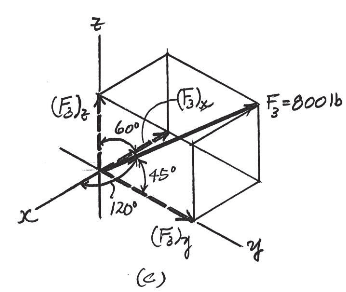


$$F_R = \sqrt{(206.22)^2 + (1395.69)^2 + (760)^2}$$
$$= 1602.52\text{ lb} = 1.60\text{ kip} \qquad \textbf{Ans.}$$

$$\alpha = \cos^{-1}\left(\frac{206.22}{1602.52}\right) = 82.6° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{1395.69}{1602.52}\right) = 29.4° \qquad \textbf{Ans.}$$

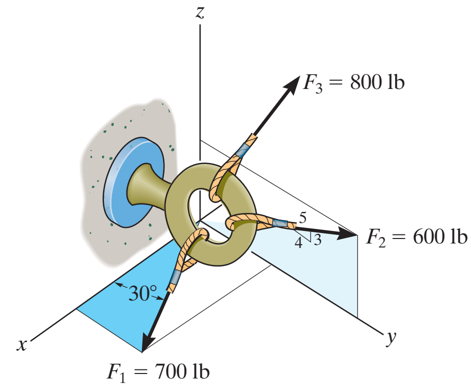
$$\gamma = \cos^{-1}\left(\frac{760}{1602.52}\right) = 61.7° \qquad \textbf{Ans.}$$



Ans:
$F_R = 1.60$ kip
$\alpha = 82.6°$
$\beta = 29.4°$
$\gamma = 61.7°$

2–83.

If the direction of the resultant force acting on the eyebolt is defined by the unit vector $\mathbf{u}_{F_R} = \cos 30°\mathbf{j} + \sin 30°\mathbf{k}$, determine the coordinate direction angles of $\mathbf{F}_3$ and the magnitude of $\mathbf{F}_R$.



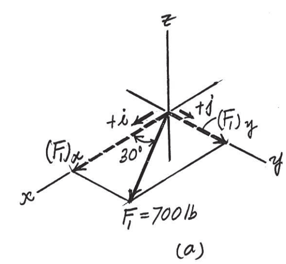
SOLUTION

Force Vectors: By resolving $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{F}_3$ into their x, y, and z components, as shown in Figs. *a*, *b*, and *c*, respectively, $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be expressed in Cartesian vector form as

$$\mathbf{F}_1 = 700\cos 30°(+\mathbf{i}) + 700\sin 30°(+\mathbf{j}) = \{606.22\mathbf{i} + 350\mathbf{j}\}\text{ lb}$$

$$\mathbf{F}_2 = 0\mathbf{i} + 600\left(\frac{4}{5}\right)(+\mathbf{j}) + 600\left(\frac{3}{5}\right)(+\mathbf{k}) = \{480\mathbf{j} + 360\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_3 = 800\cos\alpha_3\mathbf{i} + 800\cos\beta_3\mathbf{j} + 800\cos\gamma_3\mathbf{k}$$



Since the direction of $\mathbf{F}_R$ is defined by $\mathbf{u}_{F_R} = \cos 30°\mathbf{j} + \sin 30°\mathbf{k}$, it can be written in Cartesian vector form as

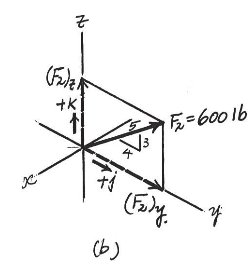
$$\mathbf{F}_R = F_R\mathbf{u}_{F_R} = F_R(\cos 30°\mathbf{j} + \sin 30°\mathbf{k}) = 0.8660F_R\mathbf{j} + 0.5F_R\mathbf{k}$$

Resultant Force: By adding $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ vectorally, we obtain $\mathbf{F}_R$. Thus,

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$0.8660F_R\mathbf{j} + 0.5F_R\mathbf{k} = (606.22\mathbf{i} + 350\mathbf{j}) + (480\mathbf{j} + 360\mathbf{k}) + (800\cos\alpha_3\mathbf{i} + 800\cos\beta_3\mathbf{j} + 800\cos\gamma_3\mathbf{k})$$

$$0.8660F_R\mathbf{j} + 0.5F_R\mathbf{k} = (606.22 + 800\cos\alpha_3)\mathbf{i} + (350 + 480 + 800\cos\beta_3)\mathbf{j} + (360 + 800\cos\gamma_3)\mathbf{k}$$

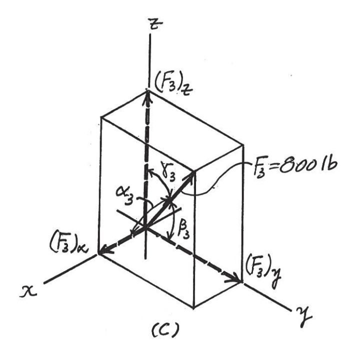


Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components, we have

$$0 = 606.22 + 800\cos\alpha_3$$
$$800\cos\alpha_3 = -606.22 \qquad \textbf{(1)}$$

$$0.8660F_R = 350 + 480 + 800\cos\beta_3$$
$$800\cos\beta_3 = 0.8660F_R - 830 \qquad \textbf{(2)}$$

$$0.5F_R = 360 + 800\cos\gamma_3$$
$$800\cos\gamma_3 = 0.5F_R - 360 \qquad \textbf{(3)}$$



Squaring and then adding Eqs. (1), (2), and (3), yields

$$800^2\left[\cos^2\alpha_3 + \cos^2\beta_3 + \cos^2\gamma_3\right] = F_R^{\,2} - 1797.60F_R + 1{,}186{,}000 \qquad \textbf{(4)}$$

However, $\cos^2\alpha_3 + \cos^2\beta_3 + \cos^2\gamma_3 = 1$. Thus, from Eq. (4)

$$F_R^{\,2} - 1797.60F_R + 546{,}000 = 0$$

Solving the above quadratic equation, we have two positive roots

$F_R = 387.09\text{ N} = 387\text{ N}$ **Ans.**

$F_R = 1410.51\text{ N} = 1.41\text{ kN}$ **Ans.**

From Eq. (1),

$\alpha_3 = 139°$ **Ans.**

Substituting $F_R = 387.09$ N into Eqs. (2), and (3), yields

$\beta_3 = 128°$ $\qquad \gamma_3 = 102°$ **Ans.**

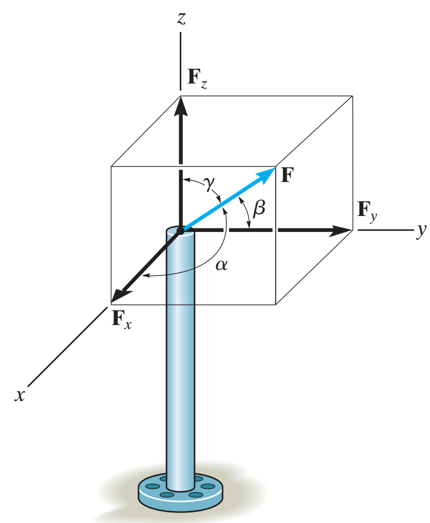
Substituting $F_R = 1410.51$ N into Eqs. (2), and (3), yields

$\beta_3 = 60.7°$ $\qquad \gamma_3 = 64.4°$ **Ans.**

Ans:
$\alpha_3 = 139°$
$\beta_3 = 128°, \gamma_3 = 102°, F_{R1} = 387\text{ N}$
$\beta_3 = 60.7°, \gamma_3 = 64.4°, F_{R2} = 1.41\text{ kN}$

***2–84.**

The pole is subjected to the force **F**, which has components acting along the *x, y, z* axes as shown. If the magnitude of **F** is 3 kN, $\beta = 30°$, and $\gamma = 75°$, determine the magnitudes of its three components.



SOLUTION

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

$$\cos^2 \alpha + \cos^2 30° + \cos^2 75° = 1$$

$$\alpha = 64.67°$$

$$F_x = 3 \cos 64.67° = 1.28 \text{ kN}$$ **Ans.**

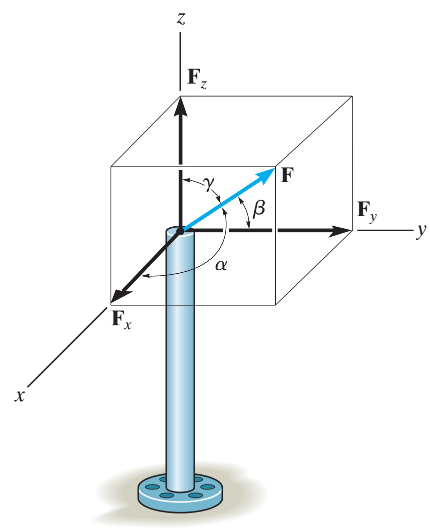
$$F_y = 3 \cos 30° = 2.60 \text{ kN}$$ **Ans.**

$$F_z = 3 \cos 75° = 0.776 \text{ kN}$$ **Ans.**

Ans:
$F_x = 1.28$ kN
$F_y = 2.60$ kN
$F_z = 0.776$ kN

2–85.

The pole is subjected to the force **F** which has components $F_x = 1.5$ kN and $F_z = 1.25$ kN. If $\beta = 75°$, determine the magnitudes of **F** and $\mathbf{F}_y$.



SOLUTION

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

$$\left(\frac{1.5}{F}\right)^2 + \cos^2 75° + \left(\frac{1.25}{F}\right)^2 = 1$$

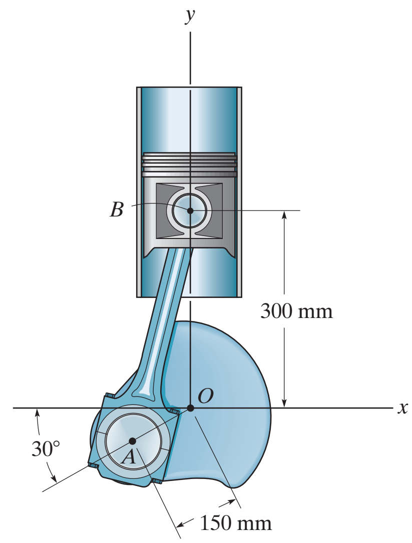
$$F = 2.02 \text{ kN}$$ **Ans.**

$$F_y = 2.02 \cos 75° = 0.523 \text{ kN}$$ **Ans.**

Ans:
$F = 2.02$ kN
$F_y = 0.523$ kN

2–86.

Determine the length of the connecting rod AB by first formulating a Cartesian position vector from A to B and then determining its magnitude.



SOLUTION

Position Vector. The coordinates of points A and B are $A(-150\cos 30°, -150\sin 30°)$ mm and $B(0, 300)$ mm respectively. Then

$$\mathbf{r}_{AB} = [0 - (-150\cos 30°)]\mathbf{i} + [300 - (-150\sin 30°)]\mathbf{j}$$

$$= \{129.90\mathbf{i} + 375\mathbf{j}\}\text{ mm}$$

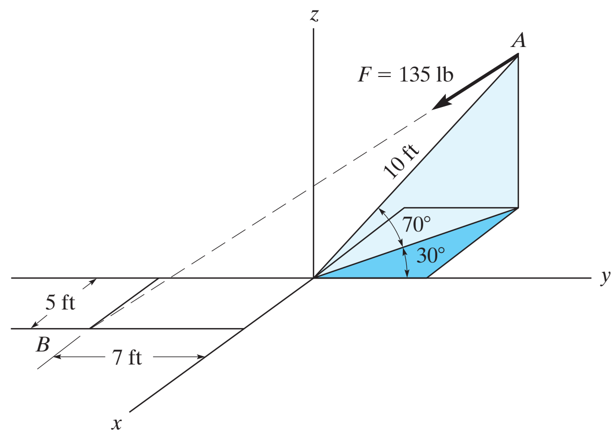
Thus, the magnitude of $\mathbf{r}_{AB}$ is

$$\mathbf{r}_{AB} = \sqrt{129.90^2 + 375^2} = 396.86\text{ mm} = 397\text{ mm} \qquad \textbf{Ans.}$$

Ans:
$r_{AB} = 397$ mm

2–87.

Express force **F** as a Cartesian vector; then determine its coordinate direction angles.



SOLUTION

$$\mathbf{r}_{AB} = (5 + 10 \cos 70° \sin 30°)\mathbf{i} + (-7 - 10 \cos 70° \cos 30°)\mathbf{j} - 10 \sin 70°\mathbf{k}$$

$$\mathbf{r}_{AB} = \{6.710\mathbf{i} - 9.962\mathbf{j} - 9.397\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{(6.710)^2 + (-9.962)^2 + (-9.397)^2} = 15.25$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = (0.4400\mathbf{i} - 0.6532\mathbf{j} - 0.6162\mathbf{k})$$

$$\mathbf{F} = 135\mathbf{u}_{AB} = (59.40\mathbf{i} - 88.18\mathbf{j} - 83.18\mathbf{k})$$

$$= \{59.4\mathbf{i} - 88.2\mathbf{j} - 83.2\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{59.40}{135}\right) = 63.9°$$ **Ans.**

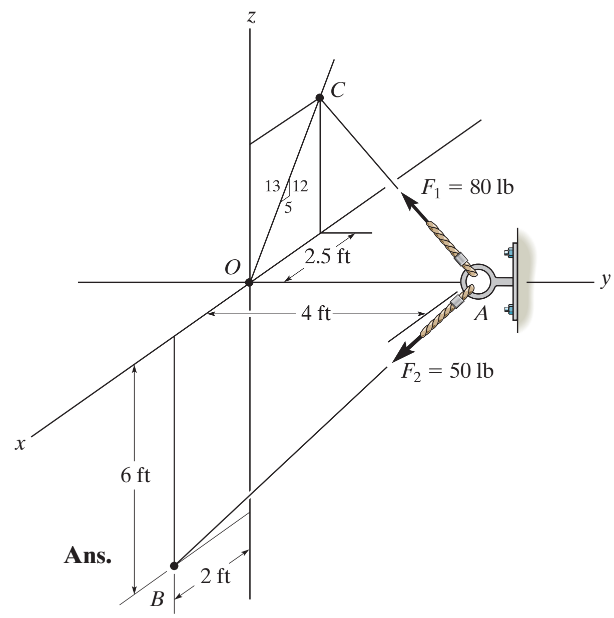
$$\beta = \cos^{-1}\left(\frac{-88.18}{135}\right) = 131°$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-83.18}{135}\right) = 128°$$ **Ans.**

Ans:
$\{59.4\mathbf{i} - 88.2\mathbf{j} - 83.2\mathbf{k}\}$ lb
$\alpha = 63.9°$
$\beta = 131°$
$\gamma = 128°$

***2–88.**

Express each of the forces in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$$\mathbf{r}_{AC} = \left\{-2.5\,\mathbf{i} - 4\,\mathbf{j} + \frac{12}{5}(2.5)\,\mathbf{k}\right\}\text{ ft}$$

$$\mathbf{F}_1 = 80\text{ lb}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = -26.20\,\mathbf{i} - 41.93\,\mathbf{j} + 62.89\,\mathbf{k}$$

$$= \{-26.2\,\mathbf{i} - 41.9\,\mathbf{j} + 62.9\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{r}_{AB} = \{2\,\mathbf{i} - 4\,\mathbf{j} - 6\,\mathbf{k}\}\text{ ft}$$

$$\mathbf{F}_2 = 50\text{ lb}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 13.36\,\mathbf{i} - 26.73\,\mathbf{j} - 40.09\,\mathbf{k}$$

$$= \{13.4\,\mathbf{i} - 26.7\,\mathbf{j} - 40.1\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= -12.84\,\mathbf{i} - 68.65\,\mathbf{j} + 22.80\,\mathbf{k}$$

$$= \{-12.8\,\mathbf{i} - 68.7\,\mathbf{j} + 22.8\,\mathbf{k}\}\text{ lb}$$

$$F_R = \sqrt{(-12.84)^2\,(-68.65)^2 + (22.80)^2} = 73.47 = 73.5\text{ lb}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{-12.84}{73.47}\right) = 100^\circ$$ **Ans.**

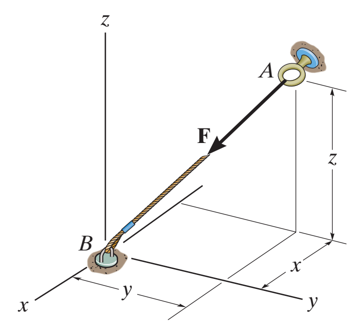
$$\beta = \cos^{-1}\left(\frac{-68.65}{73.47}\right) = 159^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{22.80}{73.47}\right) = 71.9^\circ$$ **Ans.**

Ans:
$\mathbf{F}_1 = \{-26.2\,\mathbf{i} - 41.9\,\mathbf{j} + 62.9\,\mathbf{k}\}$ lb
$\mathbf{F}_2 = \{13.4\,\mathbf{i} - 26.7\,\mathbf{j} - 40.1\,\mathbf{k}\}$ lb
$F_R = 73.5$ lb
$\alpha = 100^\circ$
$\beta = 159^\circ$
$\gamma = 71.9^\circ$

2–89.

If $\mathbf{F} = \{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}\}$ N and cable AB is 9 m long, determine the x, y, z coordinates of point A.



SOLUTION

Position Vector: The position vector $\mathbf{r}_{AB}$, directed from point A to point B, is given by

$$\mathbf{r}_{AB} = [0 - (-x)]\mathbf{i} + (0 - y)\mathbf{j} + (0 - z)\mathbf{k}$$

$$= x\mathbf{i} - y\mathbf{j} - z\mathbf{k}$$

Unit Vector: Knowing the magnitude of $\mathbf{r}_{AB}$ is 9 m, the unit vector for $\mathbf{r}_{AB}$ is given by

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9}$$

The unit vector for force $\mathbf{F}$ is

$$\mathbf{u}_F = \frac{\mathbf{F}}{F} = \frac{350\mathbf{i} - 250\mathbf{j} - 450\mathbf{k}}{\sqrt{350^2 + (-250)^2 + (-450)^2}} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Since force $\mathbf{F}$ is also directed from point A to point B, then

$$\mathbf{u}_{AB} = \mathbf{u}_F$$

$$\frac{x\mathbf{i} - y\mathbf{j} - z\mathbf{k}}{9} = 0.5623\mathbf{i} - 0.4016\mathbf{j} - 0.7229\mathbf{k}$$

Equating the $\mathbf{i}$, $\mathbf{j}$, and $\mathbf{k}$ components,

$$\frac{x}{9} = 0.5623 \qquad x = 5.06\text{ m} \qquad \textbf{Ans.}$$

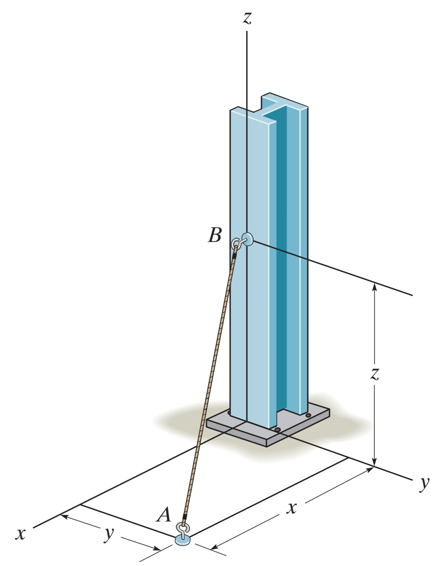
$$\frac{-y}{9} = -0.4016 \qquad y = 3.61\text{ m} \qquad \textbf{Ans.}$$

$$\frac{-z}{9} = 0.7229 \qquad z = 6.51\text{ m} \qquad \textbf{Ans.}$$

Ans:
$x = 5.06$ m
$y = 3.61$ m
$z = 6.51$ m

2–90.

The 8-m-long cable is anchored to the ground at A. If $x = 4$ m and $y = 2$ m, determine the coordinate z to the highest point of attachment along the column.



SOLUTION

$$\mathbf{r} = \{4\mathbf{i} + 2\mathbf{j} + z\mathbf{k}\}\ \text{m}$$

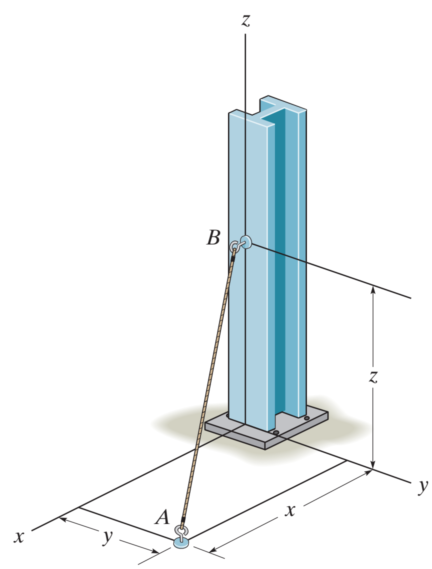
$$r = \sqrt{(4)^2 + (2)^2 + (z)^2} = 8$$

$$z = 6.63\ \text{m}$$ **Ans.**

Ans:
6.63 m

2–91.

The 8-m-long cable is anchored to the ground at A. If $z = 5$ m, determine the location $+x, +y$ of point A. Choose a value such that $x = y$.



SOLUTION

$\mathbf{r} = \{x\mathbf{i} + y\mathbf{j} + 5\mathbf{k}\}$ m

$r = \sqrt{(x)^2 + (y)^2 + (5)^2} = 8$

$x = y$, thus

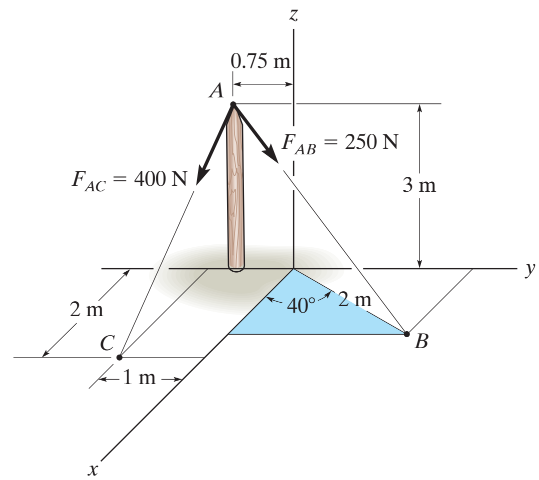
$2x^2 = 8^2 - 5^2$

$x = y = 4.42$ m **Ans.**

Ans:
4.42 m

***2–92.**

Express each of the forces in Cartesian vector form and determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

Unit Vectors. The coordinates for points A, B and C are $(0, -0.75, 3)$ m, $B(2\cos 40°, 2\sin 40°, 0)$ m and $C(2, -1, 0)$ m respectively.

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(2\cos 40° - 0)\mathbf{i} + [2\sin 40° - (-0.75)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(2\cos 40° - 0)^2 + [2\sin 40° - (-0.75)]^2 + (0-3)^2}}$$

$$= 0.3893\mathbf{i} + 0.5172\mathbf{j} - 0.7622\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{(2-0)\mathbf{i} + [-1-(-0.75)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(2-0)^2 + [-1-(-0.75)]^2 + (0-3)^2}}$$

$$= 0.5534\mathbf{i} - 0.0692\mathbf{j} - 0.8301\mathbf{k}$$

Force Vectors

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AB} = 250\,(0.3893\mathbf{i} + 0.5172\mathbf{j} - 0.7622\mathbf{k})$$

$$= \{97.32\mathbf{i} + 129.30\mathbf{j} - 190.56\mathbf{k}\}\text{ N}$$

$$= \{97.3\mathbf{i} + 129\mathbf{j} - 191\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{F}_{AC} = F_{AC}\,\mathbf{u}_{AC} = 400\,(0.5534\mathbf{i} - 0.06917\mathbf{j} - 0.8301\mathbf{k})$$

$$= \{221.35\mathbf{i} - 27.67\mathbf{j} - 332.02\mathbf{k}\}\text{ N}$$

$$= \{221\mathbf{i} - 27.7\mathbf{j} - 332\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

Resultant Force

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$$

$$= \{97.32\mathbf{i} + 129.30\mathbf{j} - 190.56\mathbf{k}\} + \{221.35\mathbf{i} - 27.67\mathbf{j} - 332.02\mathbf{k}\}$$

$$= \{318.67\mathbf{i} + 101.63\mathbf{j} - 522.58\,\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{318.67^2 + 101.63^2 + (-522.58)^2}$$

$$= 620.46\text{ N} = 620\text{ N}$$

And its coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{318.67}{620.46}; \qquad \alpha = 59.10° = 59.1° \qquad \textbf{Ans.}$$

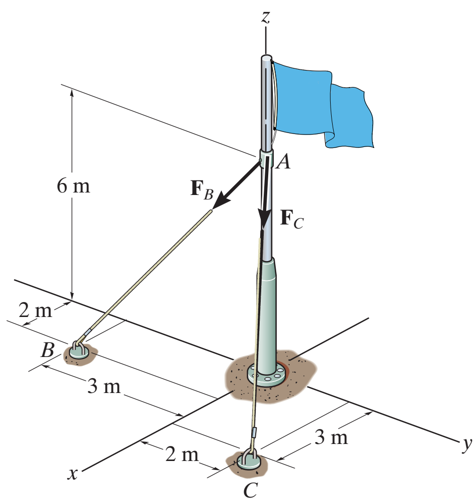
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{101.63}{620.46}; \qquad \beta = 80.57° = 80.6° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-522.58}{620.46}; \qquad \gamma = 147.38° = 147° \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_{AB} = \{97.3\mathbf{i} - 129\mathbf{j} - 191\mathbf{k}\}$ N
$\mathbf{F}_{AC} = \{221\mathbf{i} - 27.7\mathbf{j} - 332\mathbf{k}\}$ N
$F_R = 620$ N
$\cos\alpha = 59.1°$
$\cos\beta = 80.6°$
$\cos\gamma = 147°$

2–93.

If $F_B = 560$ N and $F_C = 700$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

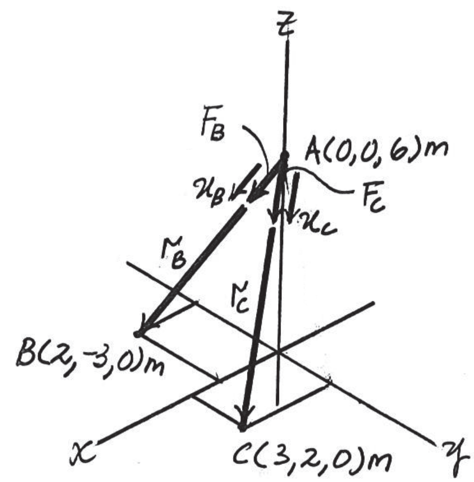
$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 560\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 700\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k}\}\text{ N}$$



(a)

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (160\mathbf{i} - 240\mathbf{j} - 480\mathbf{k}) + (300\mathbf{i} + 200\mathbf{j} - 600\mathbf{k})$$

$$= \{460\mathbf{i} - 40\mathbf{j} + 1080\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

$$= \sqrt{(460)^2 + (-40)^2 + (-1080)^2} = 1174.56\text{ N} = 1.17\text{ kN} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{460}{1174.56}\right) = 66.9^\circ \qquad \textbf{Ans.}$$

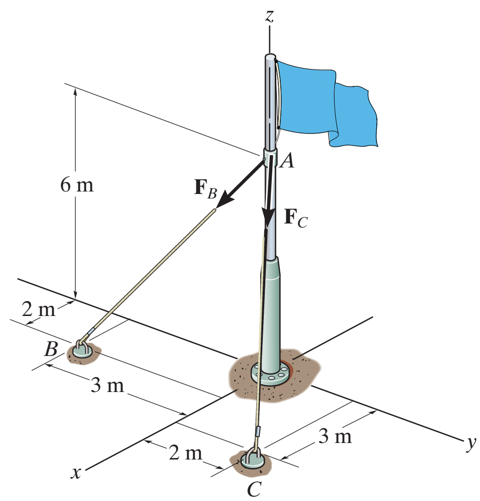
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-40}{1174.56}\right) = 92.0^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157^\circ \qquad \textbf{Ans.}$$

Ans:
$F_R = 1.17$ kN
$\alpha = 66.9^\circ$
$\beta = 92.0^\circ$
$\gamma = 157^\circ$

2–94.

If $F_B = 700$ N, and $F_C = 560$ N, determine the magnitude and coordinate direction angles of the resultant force acting on the flag pole.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_B$ and $\mathbf{u}_C$ of $\mathbf{F}_B$ and $\mathbf{F}_C$ must be determined first. From Fig. *a*

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(2-0)\mathbf{i} + (-3-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(2-0)^2 + (-3-0)^2 + (0-6)^2}} = \frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_B$ and $\mathbf{F}_C$ are given by

$$\mathbf{F}_B = F_B\mathbf{u}_B = 700\left(\frac{2}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 560\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = \{240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k}\}\text{ N}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = (200\mathbf{i} - 300\mathbf{j} - 600\mathbf{k}) + (240\mathbf{i} + 160\mathbf{j} - 480\mathbf{k})$$

$$= \{440\mathbf{i} - 140\mathbf{j} - 1080\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2 + (F_R)_z{}^2}$$

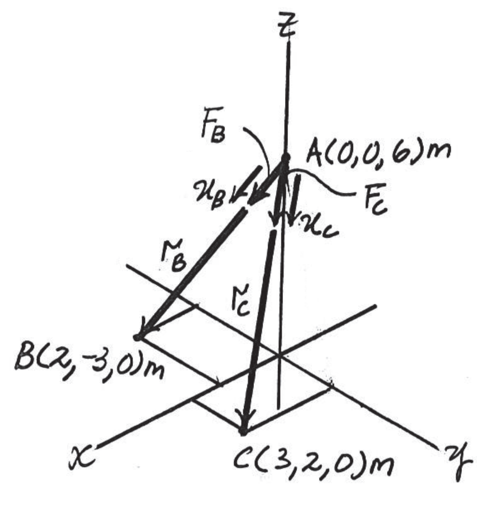
$$= \sqrt{(440)^2 + (-140)^2 + (-1080)^2} = 1174.56\text{ N} = 1.17\text{ kN} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{440}{1174.56}\right) = 68.0° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{-140}{1174.56}\right) = 96.8° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-1080}{1174.56}\right) = 157° \qquad \textbf{Ans.}$$

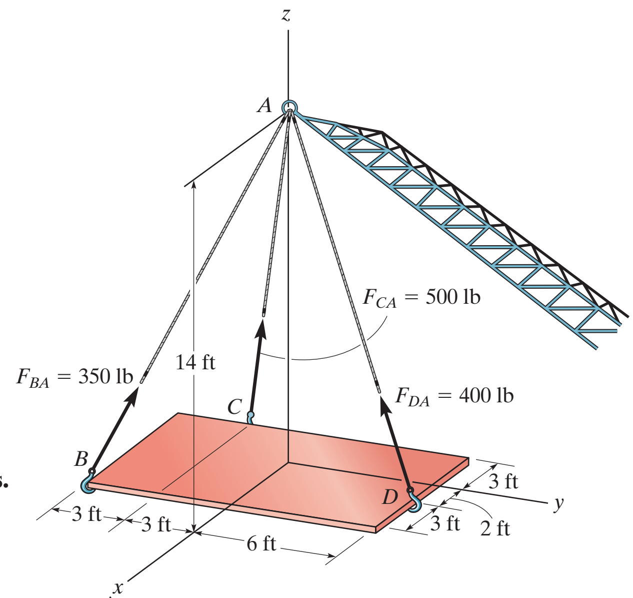


(a)

Ans:
$F_R = 1.17$ kN
$\alpha = 68.0°$
$\beta = 96.8°$
$\gamma = 157°$

2–95.

The plate is suspended using the three cables which exert the forces shown. Express each force as a Cartesian vector.



SOLUTION

$$\mathbf{F}_{BA} = 350\left(\frac{\mathbf{r}_{BA}}{r_{BA}}\right) = 350\left(-\frac{5}{16.031}\mathbf{i} + \frac{6}{16.031}\mathbf{j} + \frac{14}{16.031}\mathbf{k}\right)$$

$$= \{-109\,\mathbf{i} + 131\,\mathbf{j} + 306\,\mathbf{k}\}\text{ lb}$$ **Ans.**

$$\mathbf{F}_{CA} = 500\left(\frac{\mathbf{r}_{CA}}{r_{CA}}\right) = 500\left(\frac{3}{14.629}\mathbf{i} + \frac{3}{14.629}\mathbf{j} + \frac{14}{14.629}\mathbf{k}\right)$$

$$= \{103\,\mathbf{i} + 103\,\mathbf{j} + 479\,\mathbf{k}\}\text{ lb}$$ **Ans.**

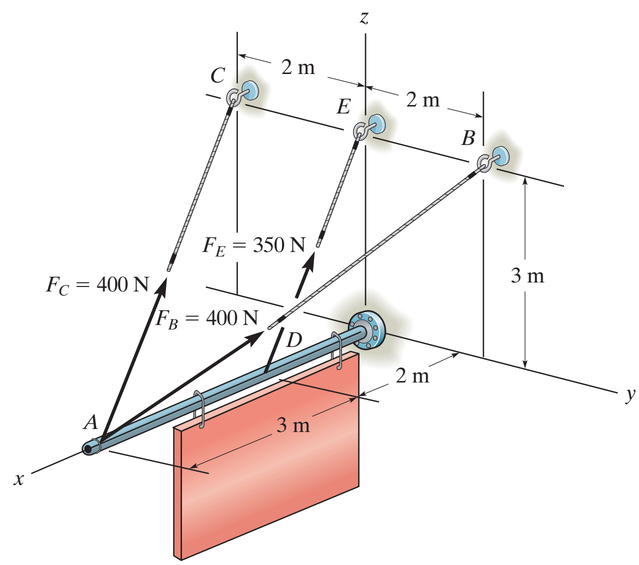
$$\mathbf{F}_{DA} = 400\left(\frac{\mathbf{r}_{DA}}{r_{DA}}\right) = 400\left(-\frac{2}{15.362}\mathbf{i} - \frac{6}{15.362}\mathbf{j} + \frac{14}{15.362}\mathbf{k}\right)$$

$$= \{-52.1\,\mathbf{i} - 156\,\mathbf{j} + 365\,\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$\mathbf{F}_{BA} = \{-109\,\mathbf{i} + 131\,\mathbf{j} + 306\,\mathbf{k}\}$ lb
$\mathbf{F}_{CA} = \{103\,\mathbf{i} + 103\,\mathbf{j} + 479\,\mathbf{k}\}$ lb
$\mathbf{F}_{DA} = \{-52.1\,\mathbf{i} - 156\,\mathbf{j} + 365\,\mathbf{k}\}$ lb

***2–96.**

The three supporting cables exert the forces shown on the sign. Represent each force as a Cartesian vector.



SOLUTION

$$\mathbf{r}_C = (0 - 5)\mathbf{i} + (-2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\}\text{ m}$$

$$r_C = \sqrt{(-5)^2 + (-2)^2 + 3^2} = \sqrt{38}\text{ m}$$

$$\mathbf{r}_B = (0 - 5)\mathbf{i} + (2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}\}\text{ m}$$

$$r_B = \sqrt{(-5)^2 + 2^2 + 3^2} = \sqrt{38}\text{ m}$$

$$\mathbf{r}_E = (0 - 2)\mathbf{i} + (0 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-2\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}\}\text{ m}$$

$$r_E = \sqrt{(-2)^2 + 0^2 + 3^2} = \sqrt{13}\text{ m}$$

$$\mathbf{F} = F_{\mathbf{u}} = F\left(\frac{\mathbf{r}}{r}\right)$$

$$\mathbf{F}_C = 400\left(\frac{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}}{\sqrt{38}}\right) = \{-324\mathbf{i} - 130\mathbf{j} + 195\mathbf{k}\}\text{ N}$$ **Ans.**

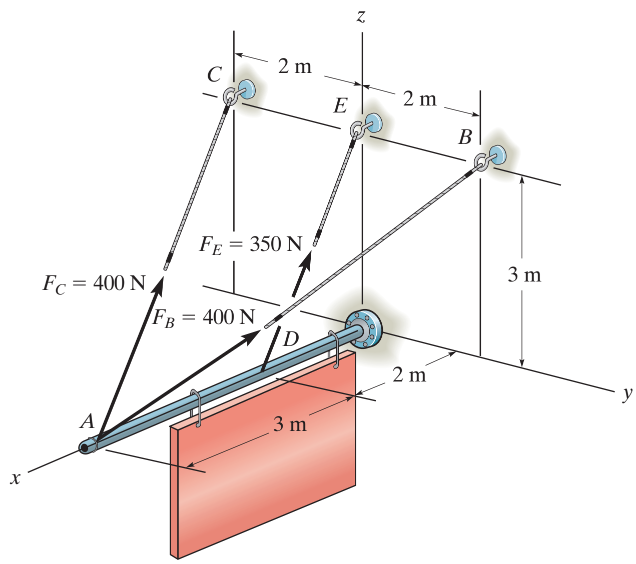
$$\mathbf{F}_B = 400\left(\frac{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}}{\sqrt{38}}\right) = \{-324\mathbf{i} + 130\mathbf{j} + 195\mathbf{k}\}\text{ N}$$ **Ans.**

$$\mathbf{F}_E = 350\left(\frac{-2\mathbf{i} + 0\mathbf{j} + 3\mathbf{k}}{\sqrt{13}}\right) = \{-194\mathbf{i} + 291\mathbf{k}\}\text{ N}$$ **Ans.**

Ans:
$\mathbf{F}_C = \{-324\mathbf{i} - 130\mathbf{j} + 195\mathbf{k}\}$ N
$\mathbf{F}_B = \{-324\mathbf{i} + 130\mathbf{j} + 195\mathbf{k}\}$ N
$\mathbf{F}_E = \{-194\mathbf{i} + 291\mathbf{k}\}$ N

2–97.

Determine the magnitude and coordinate direction angles of the resultant force of the two forces acting on the sign at point A.



SOLUTION

$$\mathbf{r}_C = (0 - 5)\mathbf{i} + (-2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}\}$$

$$r_C = \sqrt{(-5)^2 + (-2)^2 + (3)^2} = \sqrt{38}\text{ m}$$

$$\mathbf{F}_C = 400\left(\frac{\mathbf{r}_C}{r_C}\right) = 400\left(\frac{(-5\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})}{\sqrt{38}}\right)$$

$$\mathbf{F}_C = (-324.4428\mathbf{i} - 129.777\mathbf{j} + 194.666\mathbf{k})$$

$$\mathbf{r}_B = (0 - 5)\mathbf{i} + (2 - 0)\mathbf{j} + (3 - 0)\mathbf{k} = \{-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k}\}$$

$$r_B = \sqrt{(-5)^2 + 2^2 + 3^2} = \sqrt{38}\text{ m}$$

$$\mathbf{F}_B = 400\left(\frac{\mathbf{r}_B}{r_B}\right) = 400\left(\frac{(-5\mathbf{i} + 2\mathbf{j} + 3\mathbf{k})}{\sqrt{38}}\right)$$

$$\mathbf{F}_B = (-324.443\mathbf{i} + 129.777\mathbf{j} + 194.666\mathbf{k})$$

$$\mathbf{F}_R = \mathbf{F}_C + \mathbf{F}_B = (-648.89\mathbf{i} + 389.33\mathbf{k})$$

$$F_R = \sqrt{(-648.89)^2 + (389.33)^2 + 0^2} = 756.7242$$

$$F_R = 757\text{ N}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{-648.89}{756.7242}\right) = 149.03 = 149^\circ$$ **Ans.**

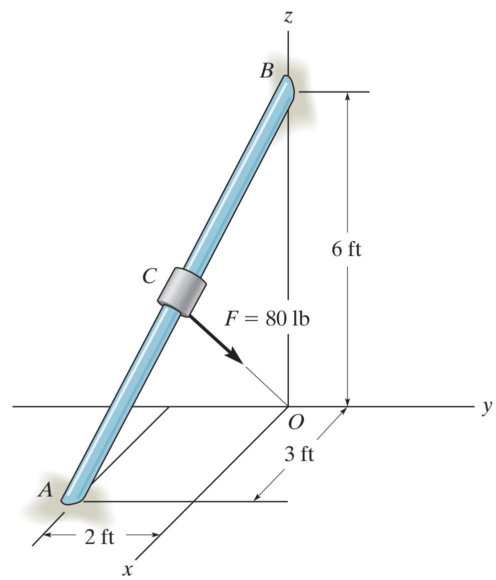
$$\beta = \cos^{-1}\left(\frac{0}{756.7242}\right) = 90.0^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{389.33}{756.7242}\right) = 59.036 = 59.0^\circ$$ **Ans.**

Ans:
$F_R = 757$ N
$\alpha = 149^\circ$
$\beta = 90.0^\circ$
$\gamma = 59.0^\circ$

2–98.

The force **F** has a magnitude of 80 lb and acts at the midpoint C of the thin rod. Express the force as a Cartesian vector.



SOLUTION

$\mathbf{r}_{AB} = (-3\mathbf{i} + 2\mathbf{j} + 6\mathbf{k})$

$\mathbf{r}_{CB} = \frac{1}{2}\mathbf{r}_{AB} = (-1.5\mathbf{i} + 1\mathbf{j} + 3\mathbf{k})$

$\mathbf{r}_{CO} = \mathbf{r}_{BO} + \mathbf{r}_{CB}$

$= -6\mathbf{k} - 1.5\mathbf{i} + 1\mathbf{j} + 3\mathbf{k}$

$= -1.5\mathbf{i} + 1\mathbf{j} - 3\mathbf{k}$

$r_{CO} = 3.5$

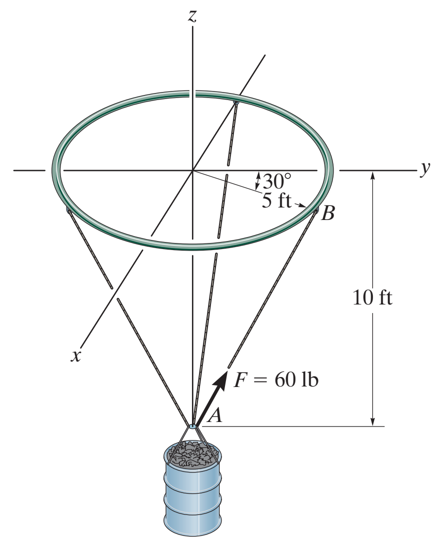
$F = 80\left(\frac{\mathbf{r}_{CO}}{r_{CO}}\right) = \{-34.3\mathbf{i} + 22.9\mathbf{j} - 68.6\mathbf{k}\}\text{ lb}$ **Ans.**

Ans:

$F = \{-34.3\mathbf{i} + 22.9\mathbf{j} - 68.6\mathbf{k}\}\text{ lb}$

2–99.

The load at A creates a force of 60 lb in wire AB. Express this force as a Cartesian vector acting on A and directed toward B as shown.



SOLUTION

Unit Vector: First determine the position vector $\mathbf{r}_{AB}$. The coordinates of point B are

$B\,(5 \sin 30°, 5 \cos 30°, 0)$ ft $= B\,(2.50, 4.330, 0)$ ft

Then

$$\mathbf{r}_{AB} = \{(2.50 - 0)\mathbf{i} + (4.330 - 0)\mathbf{j} + [0 - (-10)]\mathbf{k}\}\text{ ft}$$

$$= \{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{2.50^2 + 4.330^2 + 10.0^2} = 11.180\text{ ft}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2.50\mathbf{i} + 4.330\mathbf{j} + 10\mathbf{k}}{11.180}$$

$$= 0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}$$

Force Vector:

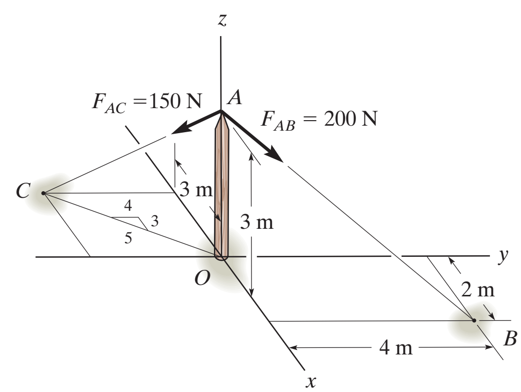
$$\mathbf{F} = F\mathbf{u}_{AB} = 60\,\{0.2236\mathbf{i} + 0.3873\mathbf{j} + 0.8944\mathbf{k}\}\text{ lb}$$

$$= \{13.4\mathbf{i} + 23.2\mathbf{j} + 53.7\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$\mathbf{F} = \{13.4\mathbf{i} + 23.2\mathbf{j} + 53.7\mathbf{k}\}$ lb

***2–100.**

Determine the magnitude and coordinate direction angles of the resultant force acting at point A on the post.



SOLUTION

Unit Vector. The coordinates for points A, B and C are $A(0, 0, 3)$ m, $B(2, 4, 0)$ m and $C(-3, -4, 0)$ m respectively

$$\mathbf{r}_{AB} = (2-0)\mathbf{i} + (4-0)\mathbf{j} + (0-3)\mathbf{k} = \{2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}\}\text{ m}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{2\mathbf{i} + 4\mathbf{j} - 3\mathbf{k}}{\sqrt{2^2 + 4^2 + (-3)^2}} = \frac{2}{\sqrt{29}}\mathbf{i} + \frac{4}{\sqrt{29}}\mathbf{j} - \frac{3}{\sqrt{29}}\mathbf{k}$$

$$\mathbf{r}_{AC} = (-3-0)\mathbf{i} + (-4-0)\mathbf{j} + (0-3)\mathbf{k} = \{-3\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}\}\text{ m}$$

$$\mathbf{u}_{AC} = \frac{\mathbf{r}_{AC}}{r_{AC}} = \frac{-3\mathbf{i} - 4\mathbf{j} - 3\mathbf{k}}{\sqrt{(-3)^2 + (-4)^2 + (-3)^2}} = -\frac{3}{\sqrt{34}}\mathbf{i} - \frac{4}{\sqrt{34}}\mathbf{j} - \frac{3}{\sqrt{34}}\mathbf{k}$$

Force Vectors

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AB} = 200\left(\frac{2}{\sqrt{29}}\mathbf{i} + \frac{4}{\sqrt{29}}\mathbf{j} - \frac{3}{\sqrt{29}}\mathbf{k}\right)$$

$$= \{74.28\mathbf{i} + 148.56\mathbf{j} - 111.42\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_{AC} = F_{AC}\,\mathbf{u}_{AC} = 150\left(-\frac{3}{\sqrt{34}}\mathbf{i} - \frac{4}{\sqrt{34}}\mathbf{j} - \frac{3}{\sqrt{34}}\mathbf{k}\right)$$

$$= \{-77.17\mathbf{i} - 102.90\mathbf{j} - 77.17\mathbf{k}\}\text{ N}$$

Resultant Force

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$$

$$= \{74.28\mathbf{i} + 148.56\mathbf{j} - 111.42\mathbf{k}\} + \{-77.17\mathbf{i} - 102.90\mathbf{j} - 77.17\mathbf{k}\}$$

$$= \{-2.896\mathbf{i} + 45.66\mathbf{j} - 188.59\,\mathbf{k}\}\text{ N}$$

The magnitude of the resultant force is

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2} = \sqrt{(-2.896)^2 + 45.66^2 + (-188.59)^2}$$

$$= 194.06\text{ N} = 194\text{ N} \qquad \textbf{Ans.}$$

And its coordinate direction angles are

$$\cos\alpha = \frac{(F_R)_x}{F_R} = \frac{-2.896}{194.06}; \qquad \alpha = 90.86^\circ = 90.9^\circ \qquad \textbf{Ans.}$$

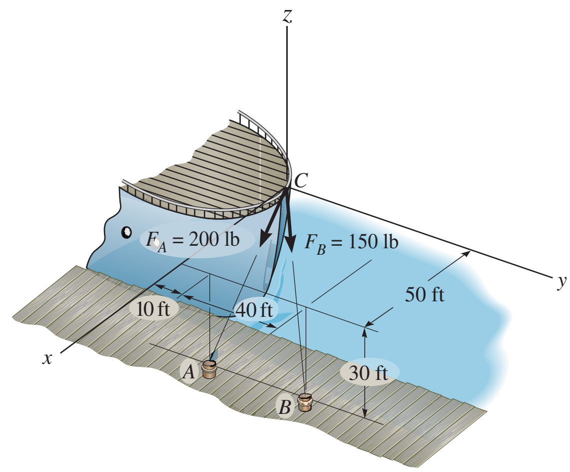
$$\cos\beta = \frac{(F_R)_y}{F_R} = \frac{45.66}{194.06}; \qquad \beta = 76.39^\circ = 76.4^\circ \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{(F_R)_z}{F_R} = \frac{-188.59}{194.06}; \qquad \gamma = 166.36^\circ = 166^\circ \qquad \textbf{Ans.}$$

Ans:
$F_R = 194$ N
$\cos\alpha = 90.9^\circ$
$\cos\beta = 76.4^\circ$
$\cos\gamma = 166^\circ$

2–101.

The two mooring cables exert forces on the stern of a ship as shown. Represent each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant.



SOLUTION

Unit Vector:

$$\mathbf{r}_{CA} = \{(50 - 0)\mathbf{i} + (10 - 0)\mathbf{j} + (-30 - 0)\mathbf{k}\}\text{ ft} = \{50\mathbf{i} + 10\mathbf{j} - 30\mathbf{k}\}\text{ ft}$$

$$r_{CA} = \sqrt{50^2 + 10^2 + (-30)^2} = 59.16\text{ ft}$$

$$\mathbf{u}_{CA} = \frac{\mathbf{r}_{CA}}{r_{CA}} = \frac{50\mathbf{i} + 10\mathbf{j} - 30\mathbf{k}}{59.16} = 0.8452\mathbf{i} + 0.1690\mathbf{j} - 0.5071\mathbf{k}$$

$$\mathbf{r}_{CB} = \{(50 - 0)\mathbf{i} + (50 - 0)\mathbf{j} + (-30 - 0)\mathbf{k}\}\text{ ft} = \{50\mathbf{i} + 50\mathbf{j} - 30\mathbf{k}\}\text{ ft}$$

$$r_{CB} = \sqrt{50^2 + 50^2 + (-30)^2} = 76.81\text{ ft}$$

$$\mathbf{u}_{CB} = \frac{\mathbf{r}_{CA}}{r_{CA}} = \frac{50\mathbf{i} + 50\mathbf{j} - 30\mathbf{k}}{76.81} = 0.6509\mathbf{i} + 0.6509\mathbf{j} - 0.3906\mathbf{k}$$

Force Vector:

$$\mathbf{F}_A = F_A\mathbf{u}_{CA} = 200\{0.8452\mathbf{i} + 0.1690\mathbf{j} - 0.5071\mathbf{k}\}\text{ lb}$$

$$= \{169.03\mathbf{i} + 33.81\mathbf{j} - 101.42\mathbf{k}\}\text{ lb}$$

$$= \{169\mathbf{i} + 33.8\mathbf{j} - 101\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_B = F_B\mathbf{u}_{CB} = 150\{0.6509\mathbf{i} + 0.6509\mathbf{j} - 0.3906\mathbf{k}\}\text{ lb}$$

$$= \{97.64\mathbf{i} + 97.64\mathbf{j} - 58.59\mathbf{k}\}\text{ lb}$$

$$= \{97.6\mathbf{i} + 97.6\mathbf{j} - 58.6\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B$$

$$= \{(169.03 + 97.64)\mathbf{i} + (33.81 + 97.64)\mathbf{j} + (-101.42 - 58.59)\mathbf{k}\}\text{ lb}$$

$$= \{266.67\mathbf{i} + 131.45\mathbf{j} - 160.00\mathbf{k}\}\text{ lb}$$

The magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{266.67^2 + 131.45^2 + (-160.00)^2}$$

$$= 337.63\text{ lb} = 338\text{ lb} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\cos\alpha = \frac{266.67}{337.63} \qquad \alpha = 37.8^\circ \qquad \textbf{Ans.}$$

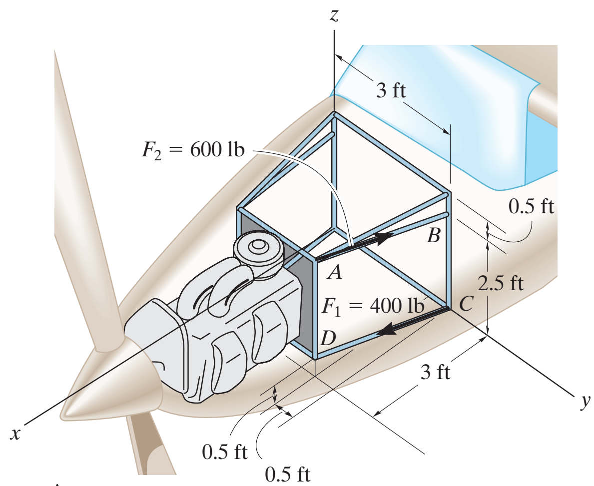
$$\cos\beta = \frac{131.45}{337.63} \qquad \beta = 67.1^\circ \qquad \textbf{Ans.}$$

$$\cos\gamma = -\frac{160.00}{337.63} \qquad \gamma = 118^\circ \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_A = \{169\mathbf{i} + 33.8\mathbf{j} - 101\mathbf{k}\}$ lb
$\mathbf{F}_B = \{97.6\mathbf{i} + 97.6\mathbf{j} - 58.6\mathbf{k}\}$ lb
$F_R = 338$ lb
$\alpha = 37.8^\circ$
$\beta = 67.1^\circ$
$\gamma = 118^\circ$

2–102.

The engine of the lightweight plane is supported by struts that are connected to the space truss that makes up the structure of the plane. The anticipated loading in two of the struts is shown. Express each of these forces as a Cartesian vector.



SOLUTION

$$\mathbf{F}_1 = 400\left(\frac{\mathbf{r}_{CD}}{r_{CD}}\right) = 400\left(\frac{3}{3.0822}\,\mathbf{i} - \frac{0.5}{3.0822}\,\mathbf{j} + \frac{0.5}{3.0822}\,\mathbf{k}\right)$$

$$= \{389\,\mathbf{i} - 64.9\,\mathbf{j} + 64.9\,\mathbf{k}\}\text{ lb}$$ **Ans.**

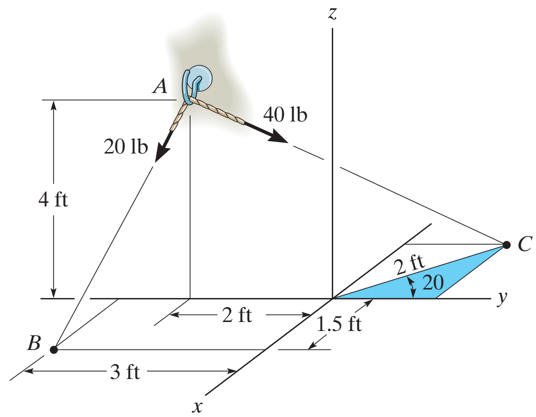
$$\mathbf{F}_2 = 600\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 600\left(-\frac{3}{3.0822}\,\mathbf{i} - \frac{0.5}{3.0822}\,\mathbf{j} + \frac{0.5}{3.0822}\,\mathbf{k}\right)$$

$$= \{-584\,\mathbf{i} + 97.3\,\mathbf{j} - 97.3\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
$\mathbf{F}_1 = \{389\mathbf{i} - 64.9\mathbf{j} + 64.9\mathbf{k}\}$ lb
$\mathbf{F}_2 = \{-584\mathbf{i} + 97.3\mathbf{j} - 97.3\mathbf{k}\}$ lb

2–103.

Determine the magnitude and coordinates on angles of the resultant force.



SOLUTION

$$\mathbf{r}_{AC} = \{-2\sin 20°\mathbf{i} + (2 + 2\cos 20°)\mathbf{j} - 4\,\mathbf{k}\}\text{ ft}$$

$$\mathbf{u}_{AC} = \left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = -0.1218\mathbf{i} + 0.6910\,\mathbf{j} - 0.7125\,\mathbf{k}$$

$$\mathbf{F}_{Ac} = 4\text{ lb}\mathbf{u}_{AC} = \{-4.874\mathbf{i} + 27.64\,\mathbf{j} - 28.50\,\mathbf{k}\}\text{ lb}$$

$$\mathbf{r}_{AB} = \{1.5\,\mathbf{i} - 1\,\mathbf{j} - 4\,\mathbf{k}\}\text{ ft}$$

$$\mathbf{u}_{AB} = \left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 0.3419\,\mathbf{i} + 0.2279\,\mathbf{j} - 0.9117\,\mathbf{k}$$

$$\mathbf{F}_{AB} = 20\text{ lb}\,\mathbf{u}_{AB} = \{6.838\mathbf{i} - 4.558\,\mathbf{j} - 18.23\,\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_R = \mathbf{F}_{AB} + \mathbf{F}_{AC}$$

$$\mathbf{F}_R = \{1.964\,\mathbf{i} + 23.08\,\mathbf{j} - 46.73\,\mathbf{k}\}\text{lb}$$

$$F_R = \sqrt{(1.964)^2 + (23.08)^2 + (-46.73)^2} = 52.16 = 52.2\text{ lb}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{1.964}{52.16}\right) = 87.8°$$ **Ans.**

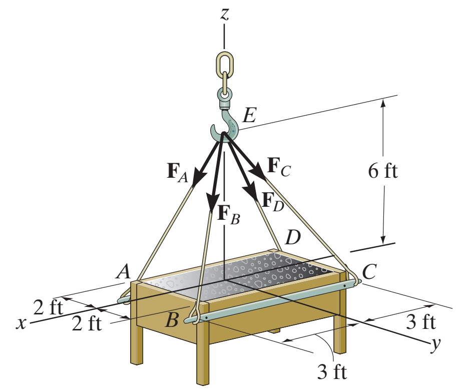
$$\beta = \cos^{-1}\left(\frac{23.08}{52.16}\right) = 63.7°$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-46.73}{52.16}\right) = 154°$$ **Ans.**

Ans:
$F_R = 52.2$ lb
$\alpha = 87.8°$
$\beta = 63.7°$
$\gamma = 154°$

***2–104.**

If the force in each cable tied to the bin is 70 lb, determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_A$, $\mathbf{u}_B$, $\mathbf{u}_C$, and $\mathbf{u}_D$ of $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ must be determined first. From Fig. *a*,

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{(3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (-2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(-3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (-2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Thus, the force vectors $\mathbf{F}_A$, $\mathbf{F}_B$, $\mathbf{F}_C$, and $\mathbf{F}_D$ are given by

$$\mathbf{F}_A = F_A\mathbf{u}_A = 70\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}] \text{ lb}$$

$$\mathbf{F}_B = F_B\mathbf{u}_B = 70\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}] \text{ lb}$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = 70\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [-30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}] \text{ lb}$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = 70\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) = [-30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}] \text{ lb}$$

Resultant Force:

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D = (30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k}) + (30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}) + (-30\mathbf{i} + 20\mathbf{j} - 60\mathbf{k}) + (-30\mathbf{i} - 20\mathbf{j} - 60\mathbf{k})$$
$$= \{-240\mathbf{k}\} \text{ N}$$

The magnitude of $\mathbf{F}_R$ is

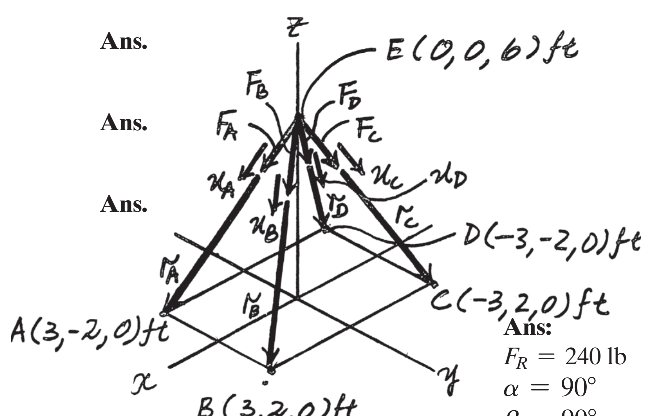
$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2 + (F_R)_z^2}$$
$$= \sqrt{0 + 0 + (-240)^2} = 240 \text{ lb} \qquad \textbf{Ans.}$$

The coordinate direction angles of $\mathbf{F}_R$ are

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{0}{240}\right) = 90° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{0}{240}\right) = 90° \qquad \textbf{Ans.}$$

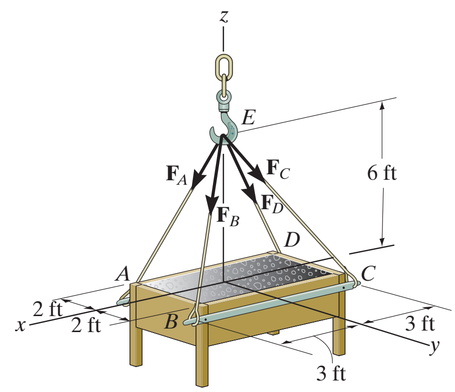
$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-240}{240}\right) = 180° \qquad \textbf{Ans.}$$



Ans:
$F_R = 240$ lb
$\alpha = 90°$
$\beta = 90°$
$\gamma = 180°$

2–105.

If the resultant of the four forces is $\mathbf{F}_R = \{-360\mathbf{k}\}$ lb, determine the tension developed in each cable. Due to symmetry, the tension in the four cables is the same.



SOLUTION

Force Vectors: The unit vectors $\mathbf{u}_A, \mathbf{u}_B, \mathbf{u}_C,$ and $\mathbf{u}_D$ of $\mathbf{F}_A, \mathbf{F}_B, \mathbf{F}_C,$ and $\mathbf{F}_D$ must be determined first. From Fig. *a*,

$$\mathbf{u}_A = \frac{\mathbf{r}_A}{r_A} = \frac{(3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (-2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(3-0)^2 + (2-0)^2 + (0-6)^2}} = \frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(-3-0)\mathbf{i} + (2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

$$\mathbf{u}_D = \frac{\mathbf{r}_D}{r_D} = \frac{(-3-0)\mathbf{i} + (-2-0)\mathbf{j} + (0-6)\mathbf{k}}{\sqrt{(-3-0)^2 + (-2-0)^2 + (0-6)^2}} = -\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}$$

Since the magnitudes of $\mathbf{F}_A, \mathbf{F}_B, \mathbf{F}_C,$ and $\mathbf{F}_D$ are the same and denoted as F, the four vectors or forces can be written as

$$\mathbf{F}_A = F_A\mathbf{u}_A = F\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_B = F_B\mathbf{u}_B = F\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_C = F_C\mathbf{u}_C = F\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

$$\mathbf{F}_D = F_D\mathbf{u}_D = F\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)$$

Resultant Force: The vector addition of $\mathbf{F}_A, \mathbf{F}_B, \mathbf{F}_C,$ and $\mathbf{F}_D$ is equal to $\mathbf{F}_R$. Thus,

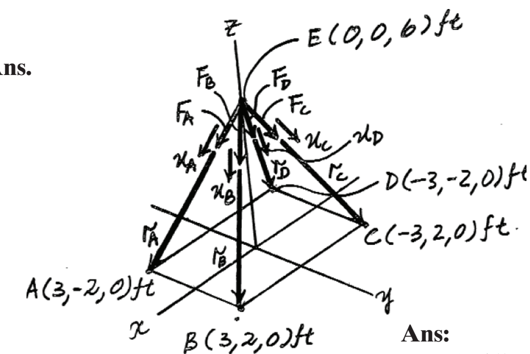
$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C + \mathbf{F}_D$$

$$\{-360\mathbf{k}\} = \left[F\left(\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] + \left[F\left(\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right] + \left[F\left(-\frac{3}{7}\mathbf{i} + \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right) + \left[F\left(-\frac{3}{7}\mathbf{i} - \frac{2}{7}\mathbf{j} - \frac{6}{7}\mathbf{k}\right)\right]\right.$$

$$-360\mathbf{k} = -\frac{24}{7}\mathbf{k}$$

Thus,

$$360 = \frac{24}{7}F \qquad F = 105 \text{ lb} \qquad \textbf{Ans.}$$

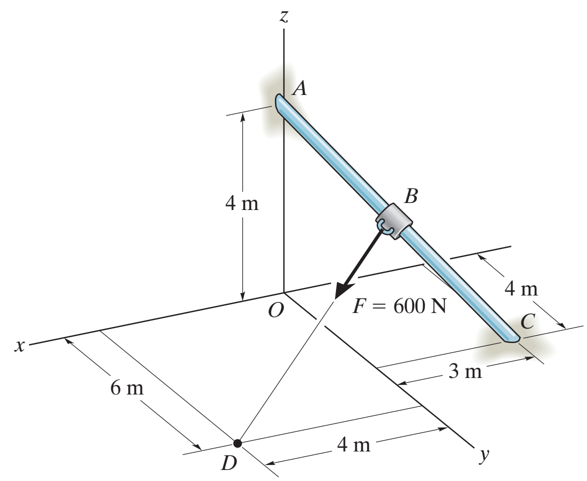


(a)

Ans:
$F = 105$ lb

2–106.

Express the force **F** in Cartesian vector form if it acts at the midpoint B of the rod.



SOLUTION

$$\mathbf{r}_{AB} = \frac{\mathbf{r}_{AC}}{2} = \frac{-3\mathbf{i} + 4\mathbf{j} - 4\mathbf{k}}{2} = -1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$$

$$\mathbf{r}_{AD} = \mathbf{r}_{AB} + \mathbf{r}_{BD}$$

$$\mathbf{r}_{BD} = \mathbf{r}_{AD} - \mathbf{r}_{AB}$$

$$= (4\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}) - (-1.5\mathbf{i} + 2\mathbf{j} - 2\mathbf{k})$$

$$= \{5.5\mathbf{i} + 4\mathbf{j} - 2\mathbf{k}\}\text{ m}$$

$$r_{BD} = \sqrt{(5.5)^2 + (4)^2 + (-2)^2} = 7.0887\text{ m}$$

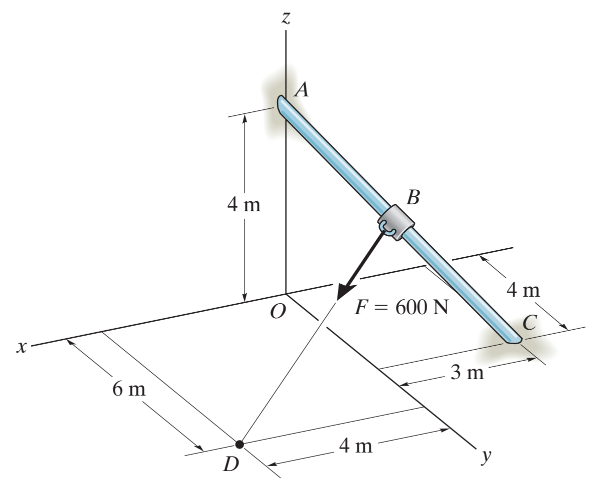
$$\mathbf{F} = 600\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = 465.528\mathbf{i} + 338.5659\mathbf{j} - 169.2829\mathbf{k}$$

$$\mathbf{F} = \{466\mathbf{i} + 339\mathbf{j} - 169\mathbf{k}\}\text{ N}$$ **Ans.**

Ans:
$\mathbf{F} = \{466\mathbf{i} + 339\mathbf{j} - 169\mathbf{k}\}$ N

2–107.

Express force **F** in Cartesian vector form if point *B* is located 3 m along the rod end *C*.



SOLUTION

$$\mathbf{r}_{CA} = 3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$$

$$r_{CA} = 6.403124$$

$$\mathbf{r}_{CB} = \frac{3}{6.403124}(\mathbf{r}_{CA}) = 1.4056\mathbf{i} - 1.8741\mathbf{j} + 1.8741\mathbf{k}$$

$$\mathbf{r}_{OB} = \mathbf{r}_{OC} + \mathbf{r}_{CB}$$

$$= -3\mathbf{i} + 4\mathbf{j} + \mathbf{r}_{CB}$$

$$= -1.59444\mathbf{i} + 2.1259\mathbf{j} + 1.874085\mathbf{k}$$

$$\mathbf{r}_{OD} = \mathbf{r}_{OB} + \mathbf{r}_{BD}$$

$$\mathbf{r}_{BD} = \mathbf{r}_{OD} - \mathbf{r}_{OB} = (4\mathbf{i} + 6\mathbf{j}) - \mathbf{r}_{OB}$$

$$= 5.5944\mathbf{i} + 3.8741\mathbf{j} - 1.874085\mathbf{k}$$

$$r_{BD} = \sqrt{(5.5914)^2 + (3.8741)^2 + (-1.874085)^2} = 7.0582$$

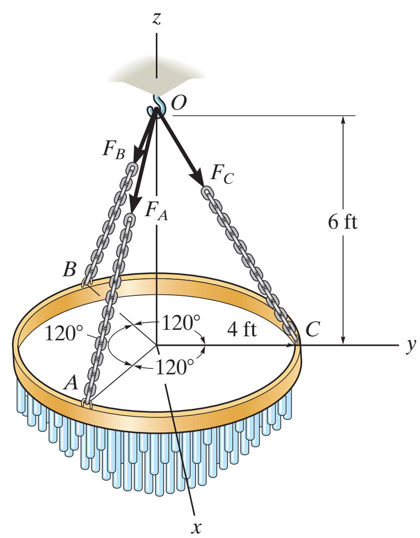
$$\mathbf{F} = 600\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = 475.568\mathbf{i} + 329.326\mathbf{j} - 159.311\mathbf{k}$$

$$\mathbf{F} = \{476\mathbf{i} + 329\mathbf{j} - 159\mathbf{k}\}\text{ N}$$ **Ans.**

Ans:
$\mathbf{F} = \{476\mathbf{i} + 329\mathbf{j} - 159\mathbf{k}\}$ N

***2–108.**

The chandelier is supported by three chains which are concurrent at point O. If the force in each chain has a magnitude of 60 lb, express each force as a Cartesian vector and determine the magnitude and coordinate direction angles of the resultant force.



SOLUTION

$$\mathbf{F}_A = 60\,\frac{(4\cos 30^\circ\,\mathbf{i} - 4\sin 30^\circ\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{(4\cos 30^\circ)^2 + (-4\sin 30^\circ)^2 + (-6)^2}}$$

$$= \{28.8\,\mathbf{i} - 16.6\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_B = 60\,\frac{(-4\cos 30^\circ\,\mathbf{i} - 4\sin 30^\circ\,\mathbf{j} - 6\mathbf{k})}{\sqrt{(-4\cos 30^\circ)^2 + (-4\sin 30^\circ)^2 + (-6)^2}}$$

$$= \{-28.8\,\mathbf{i} - 16.6\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_C = 60\,\frac{(4\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{(4)^2 + (-6)^2}}$$

$$= \{33.3\,\mathbf{j} - 49.9\,\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

$$\mathbf{F}_R = \mathbf{F}_A + \mathbf{F}_B + \mathbf{F}_C = \{-149.8\,\mathbf{k}\}\text{ lb}$$

$$F_R = 150\text{ lb} \qquad \textbf{Ans.}$$

$$\alpha = 90^\circ \qquad \textbf{Ans.}$$

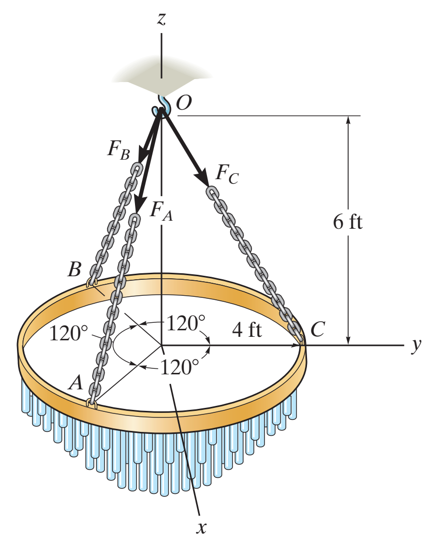
$$\beta = 90^\circ \qquad \textbf{Ans.}$$

$$\gamma = 180^\circ \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_A = \{28.8\mathbf{i} - 16.6\mathbf{j} - 49.9\mathbf{k}\}$ lb
$\mathbf{F}_B = \{-28.8\mathbf{i} - 16.6\mathbf{j} - 49.9\mathbf{k}\}$ lb
$\mathbf{F}_C = \{33.3\mathbf{j} - 49.9\mathbf{k}\}$ lb
$F_R = 150$ lb
$\alpha = 90^\circ$
$\beta = 90^\circ$
$\gamma = 180^\circ$

2–109.

The chandelier is supported by three chains which are concurrent at point O. If the resultant force at O has a magnitude of 130 lb and is directed along the negative z axis, determine the force in each chain.



SOLUTION

$$\mathbf{F}_C = F\frac{(4\,\mathbf{j} - 6\,\mathbf{k})}{\sqrt{4^2 + (-6)^2}} = 0.5547\,F\mathbf{j} - 0.8321\,F\mathbf{k}$$

$$F_A = F_B = F_C$$

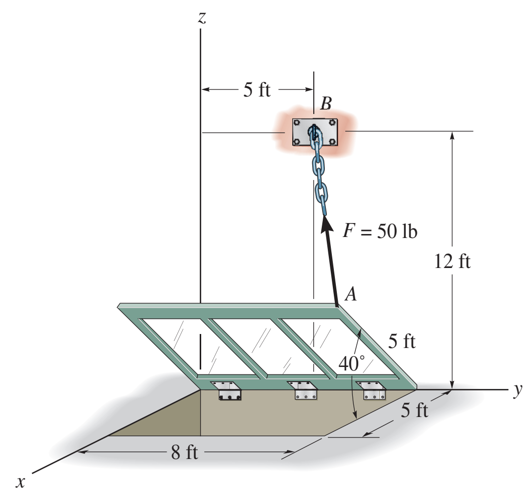
$$F_{Rz} = \Sigma F_z; \qquad 130 = 3(0.8321F)$$

$$F = 52.1 \text{ lb}$$ **Ans.**

Ans:
$F = 52.1$ lb

2–110.

The window is held open by chain AB. Determine the length of the chain, and express the 50-lb force acting at A along the chain as a Cartesian vector and determine its coordinate direction angles.



SOLUTION

Unit Vector: The coordinates of point A are

$$A(5\cos 40°, 8, 5\sin 40°)\text{ ft} = A(3.830, 8.00, 3.214)\text{ ft}$$

Then

$$\mathbf{r}_{AB} = \{(0 - 3.830)\mathbf{i} + (5 - 8.00)\mathbf{j} + (12 - 3.214)\mathbf{k}\}\text{ ft}$$

$$= \{-3.830\mathbf{i} - 3.00\mathbf{j} + 8.786\mathbf{k}\}\text{ ft}$$

$$r_{AB} = \sqrt{(-3.830)^2 + (-3.00)^2 + 8.786^2} = 10.043\text{ ft} = 10.0\text{ ft} \qquad \textbf{Ans.}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-3.830\mathbf{i} - 3.00\mathbf{j} + 8.786\mathbf{k}}{10.043}$$

$$= -0.3814\mathbf{i} - 0.2987\mathbf{j} + 0.8748\mathbf{k}$$

Force Vector:

$$\mathbf{F} = F\mathbf{u}_{AB} = 50\{-0.3814\mathbf{i} - 0.2987\mathbf{j} + 0.8748\mathbf{k}\}\text{ lb}$$

$$= \{-19.1\mathbf{i} - 14.9\mathbf{j} + 43.7\mathbf{k}\}\text{ lb} \qquad \textbf{Ans.}$$

Coordinate Direction Angles: From the unit vector $\mathbf{u}_{AB}$ obtained above, we have

$$\cos\alpha = -0.3814 \qquad \alpha = 112° \qquad \textbf{Ans.}$$

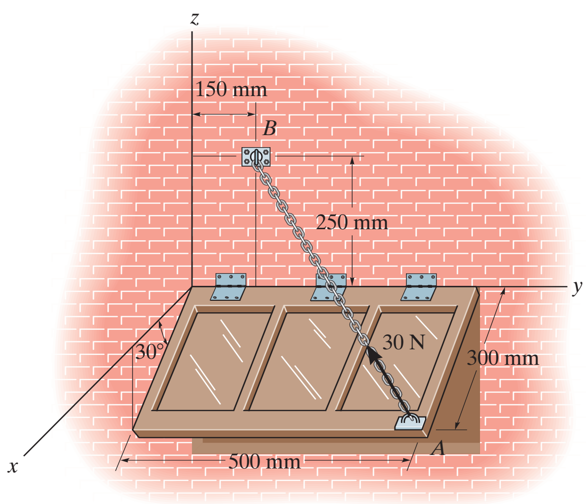
$$\cos\beta = -0.2987 \qquad \beta = 107° \qquad \textbf{Ans.}$$

$$\cos\gamma = 0.8748 \qquad \gamma = 29.0° \qquad \textbf{Ans.}$$

Ans:
$r_{AB} = 10.0$ ft
$\mathbf{F} = \{-19.1\mathbf{i} - 14.9\mathbf{j} + 43.7\mathbf{k}\}$ lb
$\alpha = 112°$
$\beta = 107°$
$\gamma = 29.0°$

2–111.

The window is held open by cable AB. Determine the length of the cable and express the 30-N force acting at A along the cable as a Cartesian vector.



SOLUTION

$$\mathbf{r}_{AB} = (0-300\cos 30°)\mathbf{i} + (150 - 500)\mathbf{j} + (250 + 300\sin 30°)\mathbf{k}$$

$$= -259.81\,\mathbf{i} - 350\,\mathbf{j} + 400\,\mathbf{k}$$

$$r_{AB} = \sqrt{(-259.81)^2 + (-350)^2 + (400)^2} = 591.61$$

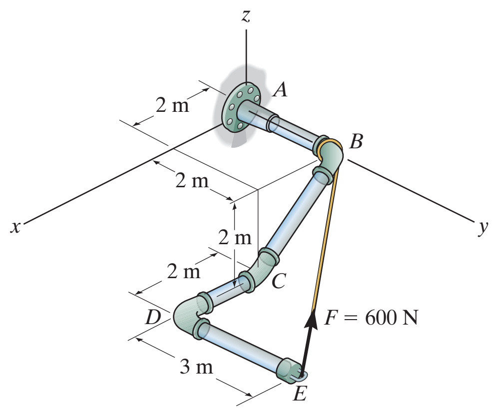
$$= 592 \text{ mm}$$ **Ans.**

$$\mathbf{F} = 30\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = \{-13.2\,\mathbf{i} - 17.7\,\mathbf{j} + 20.3\mathbf{k}\}\text{ N}$$ **Ans.**

Ans:
$r_{AB} = 592$ mm
$\mathbf{F} = \{-13.2\mathbf{i} - 17.7\mathbf{j} + 20.3\mathbf{k}\}$ N

***2–112.**

Given the three vectors **A**, **B**, and **D**, show that $\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D})$.



SOLUTION

Since the component of $(\mathbf{B}+\mathbf{D})$ is equal to the sum of the components of **B** and **D**, then

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = \mathbf{A}\cdot\mathbf{B} + \mathbf{A}\cdot\mathbf{D} \qquad \textbf{(QED)}$$

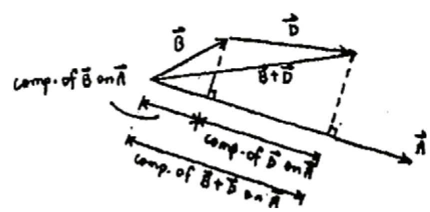
Also,

$$\mathbf{A}\cdot(\mathbf{B}+\mathbf{D}) = (A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\mathbf{k})\cdot[(B_x + D_x)\mathbf{i} + (B_y + D_y)\mathbf{j} + (B_z + D_z)\mathbf{k}]$$

$$= A_x(B_x + D_x) + A_y(B_y + D_y) + A_z(B_z + D_z)$$

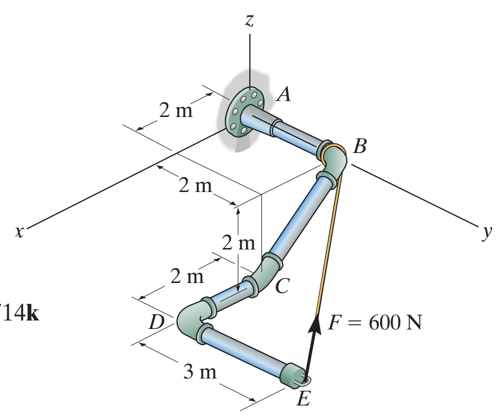
$$= (A_xB_x + A_yB_y + A_zB_z) + (A_xD_x + A_yD_y + A_zD_z)$$

$$= (\mathbf{A}\cdot\mathbf{B}) + (\mathbf{A}\cdot\mathbf{D}) \qquad \textbf{(QED)}$$



2–113.

Determine the magnitudes of the components of $F = 600$ N acting along and perpendicular to segment DE of the pipe assembly.



SOLUTION

Unit Vectors: The unit vectors $\mathbf{u}_{EB}$ and $\mathbf{u}_{ED}$ must be determined first. From Fig. *a*,

$$\mathbf{u}_{EB} = \frac{\mathbf{r}_{EB}}{r_{EB}} = \frac{(0-4)\mathbf{i} + (2-5)\mathbf{j} + [0-(-2)]\mathbf{k}}{\sqrt{(0-4)^2 + (2-5)^2 + [0-(-2)]^2}} = -0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}$$

$$\mathbf{u}_{ED} = -\mathbf{j}$$

Thus, the force vector $\mathbf{F}$ is given by

$$\mathbf{F} = F\mathbf{u}_{EB} = 600\left(-0.7428\mathbf{i} - 0.5571\mathbf{j} + 0.3714\mathbf{k}\right) = [-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}] \text{ N}$$

Vector Dot Product: The magnitude of the component of $\mathbf{F}$ *p*arallel to segment DE of the pipe assembly is

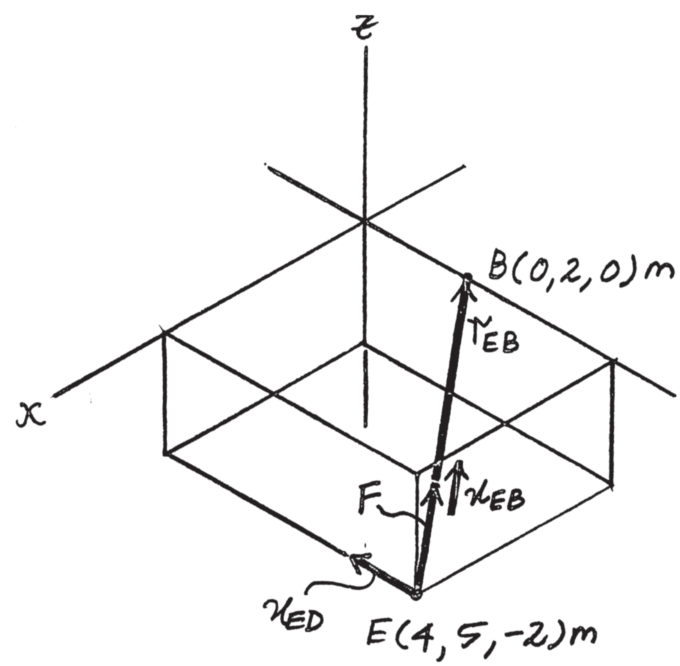
$$(F_{ED})_{\text{paral}} = \mathbf{F}\cdot\mathbf{u}_{ED} = \left(-445.66\mathbf{i} - 334.25\mathbf{j} + 222.83\mathbf{k}\right)\cdot\left(-\mathbf{j}\right)$$

$$= (-445.66)(0) + (-334.25)(-1) + (222.83)(0)$$

$$= 334.25 = 334 \text{ N}$$ **Ans.**

The component of $\mathbf{F}$ perpendicular to segment DE of the pipe assembly is

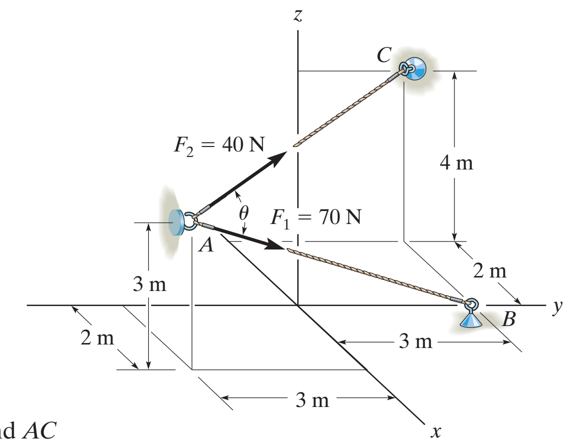
$$(F_{ED})_{\text{per}} = \sqrt{F^2 - (F_{ED})_{\text{paral}}{}^2} = \sqrt{600^2 - 334.25^2} = 498 \text{ N}$$ **Ans.**



Ans:
$(F_{ED})_{||} = 334$ N
$(F_{ED})_{\perp} = 498$ N

2–114.

Determine the angle θ between the two cables.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(2, -3, 3)$ m, $B(0, 3, 0)$ and $C(-2, 3, 4)$ m respectively. Thus, the unit vectors along AB and AC are

$$\mathbf{u}_{AB} = \frac{(0-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(0-2)^2 + [3-(-3)]^2 + (0-3)^2}} = -\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{(-2-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (4-3)\mathbf{k}}{\sqrt{(-2-2)^2 + [3-(-3)]^2 + (4-3)^2}} = -\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}$$

The Angle θ Between AB and AC.

$$\mathbf{u}_{AB}\cdot\mathbf{u}_{AC} = \left(-\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}\right)\cdot\left(-\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}\right)$$

$$= \left(-\frac{2}{7}\right)\left(-\frac{4}{\sqrt{53}}\right) + \frac{6}{7}\left(\frac{6}{\sqrt{53}}\right) + \left(-\frac{3}{7}\right)\left(\frac{1}{\sqrt{53}}\right)$$

$$= \frac{41}{7\sqrt{53}}$$

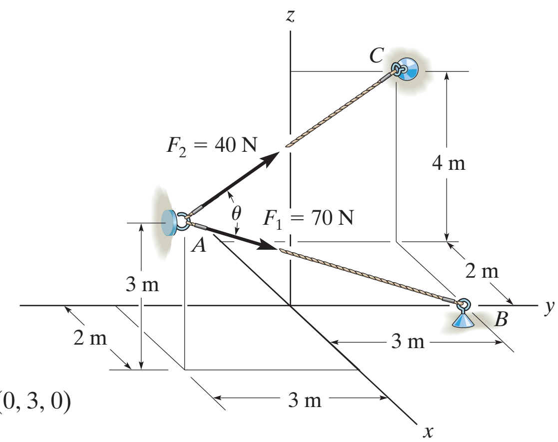
Then

$$\theta = \cos^{-1}\left(\mathbf{u}_{AB}\cdot\mathbf{u}_{AC}\right) = \cos^{-1}\left(\frac{41}{7\sqrt{53}}\right) = 36.43° = 36.4° \qquad \textbf{Ans.}$$

Ans:
$\theta = 36.4°$

2–115.

Determine the magnitude of the projection of the force $\mathbf{F}_1$ along cable AC.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(2, -3, 3)$m, $B(0, 3, 0)$ and $C(-2, 3, 4)$ m respectively. Thus, the unit vectors along AB and AC are

$$\mathbf{u}_{AB} = \frac{(0-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(0-2)^2 + [3-(-3)]^2 + (0-3)^2}} = -\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{(-2-2)\mathbf{i} + [3-(-3)]\mathbf{j} + (4-3)\mathbf{k}}{\sqrt{(-2-2)^2 + [3-(-3)]^2 + (4-3)^2}} = -\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}$$

Force Vector, For $\mathbf{F}_1$,

$$\mathbf{F}_1 = F_1\,\mathbf{u}_{AB} = 70\left(-\frac{2}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} - \frac{3}{7}\mathbf{k}\right) = \{-20\mathbf{i} + 60\mathbf{j} - 30\mathbf{k}\}\text{ N}$$

Projected Component of $\mathbf{F}_1$. Along AC, it is

$$(F_1)_{AC} = \mathbf{F}_1 \cdot \mathbf{u}_{AC} = (-20\mathbf{i} + 60\mathbf{j} - 30\mathbf{k}) \cdot \left(-\frac{4}{\sqrt{53}}\mathbf{i} + \frac{6}{\sqrt{53}}\mathbf{j} + \frac{1}{\sqrt{53}}\mathbf{k}\right)$$

$$= (-20)\left(-\frac{4}{\sqrt{53}}\right) + 60\left(\frac{6}{\sqrt{53}}\right) + (-30)\left(\frac{1}{\sqrt{53}}\right)$$

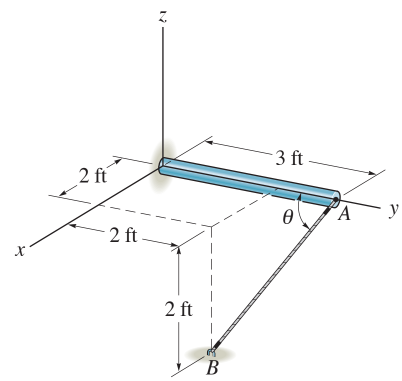
$$= 56.32\text{ N} = 56.3\text{ N}$$ **Ans.**

The positive sign indicates that this component points in the same direction as $\mathbf{u}_{AC}$.

Ans:
$(F_1)_{AC} = 56.3$ N

***2–116.**

Determine the angle θ between the y axis of the pole and the wire AB.



SOLUTION

Position Vector:

$$\mathbf{r}_{AC} = \{-3\mathbf{j}\} \text{ ft}$$

$$\mathbf{r}_{AB} = \{(2-0)\mathbf{i} + (2-3)\mathbf{j} + (-2-0)\mathbf{k}\} \text{ ft}$$

$$= \{2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k}\} \text{ ft}$$

The magnitudes of the position vectors are

$$r_{AC} = 3.00 \text{ ft} \qquad r_{AB} = \sqrt{2^2 + (-1)^2 + (-2)^2} = 3.00 \text{ ft}$$

The Angles Between Two Vectors θ: The dot product of two vectors must be determined first.

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AB} = (-3\mathbf{j}) \cdot (2\mathbf{i} - 1\mathbf{j} - 2\mathbf{k})$$

$$= 0(2) + (-3)(-1) + 0(-2)$$

$$= 3$$

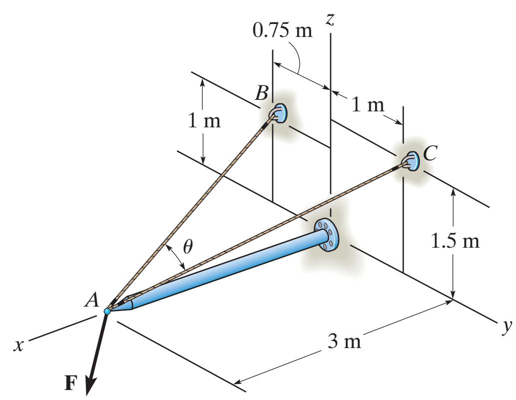
Then,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AO} \cdot \mathbf{r}_{AB}}{r_{AO} r_{AB}}\right) = \cos^{-1}\left[\frac{3}{3.00(3.00)}\right] = 70.5° \qquad \textbf{Ans.}$$

Ans:
$\theta = 70.5°$

2–117.

Determine the magnitudes of the projected components of the force $\mathbf{F} = [60\mathbf{i} + 12\mathbf{j} - 40\mathbf{k}]$ N along the cables AB and AC.



SOLUTION

$$\mathbf{F} = \{60\,\mathbf{i} + 12\,\mathbf{j} - 40\,\mathbf{k}\}\text{ N}$$

$$\mathbf{u}_{AB} = \frac{-3\,\mathbf{i} - 0.75\,\mathbf{j} + 1\,\mathbf{k}}{\sqrt{(-3)^2 + (-0.75)^2 + (1)^2}}$$

$$= -0.9231\,\mathbf{i} - 0.2308\,\mathbf{j} + 0.3077\,\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{-3\,\mathbf{i} + 1\,\mathbf{j} + 1.5\,\mathbf{k}}{\sqrt{(-3)^2 + (1)^2 + (1.5)^2}}$$

$$= -0.8571\,\mathbf{i} + 0.2857\,\mathbf{j} + 0.4286\,\mathbf{k}$$

$$\text{Proj } F_{AB} = \mathbf{F}\cdot\mathbf{u}_{AB} = (60)(-0.9231) + (12)(-0.2308) + (-40)(0.3077)$$

$$= -70.46\text{ N}$$

$$|\text{Proj } F_{AB}| = 70.5\text{ N} \qquad \textbf{Ans.}$$

$$\text{Proj } F_{AC} = \mathbf{F}\cdot\mathbf{u}_{AC} = (60)(-0.8571) + (12)(0.2857) + (-40)(0.4286)$$

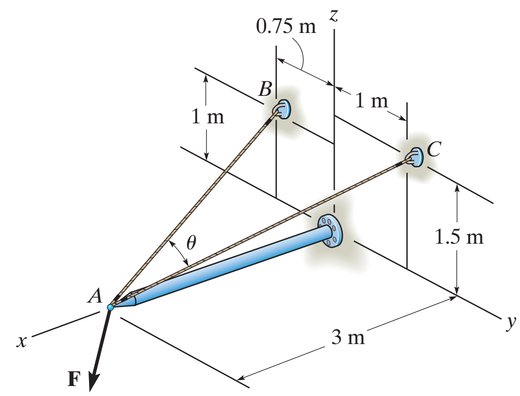
$$= -65.14\text{ N}$$

$$|\text{Proj } F_{AC}| = 65.1\text{ N} \qquad \textbf{Ans.}$$

Ans:
$|\text{Proj } F_{AB}| = 70.5$ N
$|\text{Proj } F_{AC}| = 65.1$ N

2–118.

Determine the angle θ between cables AB and AC.



SOLUTION

$\mathbf{r}_{AB} = \{-3\,\mathbf{i} - 0.75\,\mathbf{j} + 1\,\mathbf{k}\}$ m

$r_{AB} = \sqrt{(-3)^2 + (-0.75)^2 + (1)^2} = 3.25$ m

$\mathbf{r}_{AC} = \{-3\,\mathbf{i} + 1\,\mathbf{j} + 1.5\,\mathbf{k}\}$ m

$r_{AC} = \sqrt{(-3)^2 + (1)^2 + (1.5)^2} = 3.50$ m

$\mathbf{r}_{AB} \cdot \mathbf{r}_{AC} = (-3)(-3) + (-0.75)(1) + (1)(1.5) = 9.75$

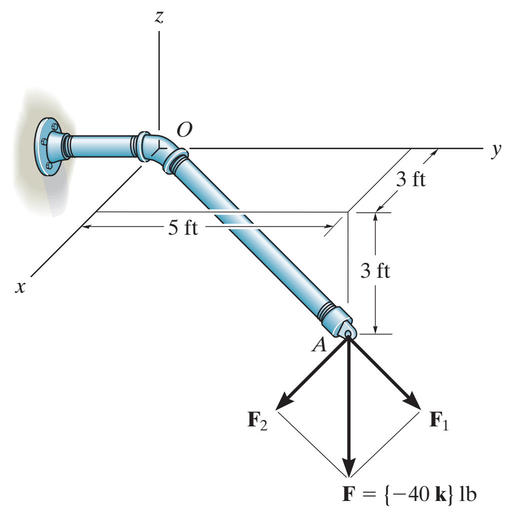
$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AC}}{r_{AB}\, r_{AC}}\right) = \cos^{-1}\left(\frac{9.75}{(3.25)(3.50)}\right)$$

$\theta = 31.0°$ **Ans.**

Ans:
$\theta = 31.0°$

2–119.

A force of $\mathbf{F} = \{-40\mathbf{k}\}$ lb acts at the end of the pipe. Determine the magnitudes of the components $\mathbf{F}_1$ and $\mathbf{F}_2$ which are directed along the pipe's axis and perpendicular to it.



SOLUTION

$$\mathbf{u}_{OA} = \frac{3\,\mathbf{i} + 5\,\mathbf{j} - 3\mathbf{k}}{\sqrt{3^2 + 5^2 + (-3)^2}} = \frac{3\,\mathbf{i} + 5\,\mathbf{j} - 3\,\mathbf{k}}{\sqrt{43}}$$

$$F_1 = \mathbf{F}\cdot\mathbf{u}_{OA} = (-40\,\mathbf{k})\cdot\left(\frac{3\mathbf{i} + 5\mathbf{j} - 3\mathbf{k}}{\sqrt{43}}\right)$$

$$= 18.3 \text{ lb}$$ **Ans.**

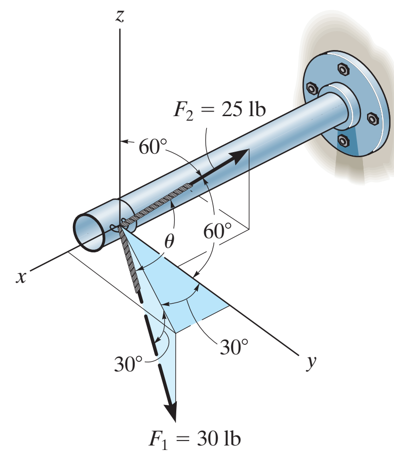
$$F_2 = \sqrt{F_2 - F_1^2}$$

$$F_2 = \sqrt{40^2 - 18.3^2} = 35.6 \text{ lb}$$ **Ans.**

Ans:
$F_1 = 18.3$ lb
$F_2 = 35.6$ lb

***2–120.**

Two cables exert forces on the pipe. Determine the magnitude of the projected component of $\mathbf{F}_1$ along the line of action of $\mathbf{F}_2$.



SOLUTION

Force Vector:

$$\mathbf{u}_{F_1} = \cos 30° \sin 30°\mathbf{i} + \cos 30° \cos 30°\mathbf{j} - \sin 30°\mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{F}_1 = F_R\mathbf{u}_{F_1} = 30(0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k})\text{ lb}$$

$$= \{12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}\}\text{ lb}$$

Unit Vector: One can obtain the angle $\alpha = 135°$ for $\mathbf{F}_2$ using Eq. 2–8. $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, with $\beta = 60°$ and $\gamma = 60°$. The unit vector along the line of action of $\mathbf{F}_2$ is

$$\mathbf{u}_{F_2} = \cos 135°\mathbf{i} + \cos 60°\mathbf{j} + \cos 60°\mathbf{k} = -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

***Projected Component of* $\mathbf{F}_1$ *Along the Line of Action of* $\mathbf{F}_2$:**

$$(F_1)_{F_2} = \mathbf{F}_1 \cdot \mathbf{u}_{F_2} = (12.990\mathbf{i} + 22.5\mathbf{j} - 15.0\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

$$= (12.990)(-0.7071) + (22.5)(0.5) + (-15.0)(0.5)$$

$$= -5.44\text{ lb}$$

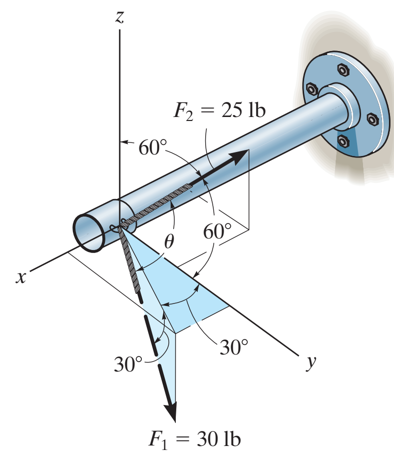
Negative sign indicates that the projected component of $(F_1)_{F_2}$ acts in the opposite sense of direction to that of $\mathbf{u}_{F_2}$.

The magnitude is $(F_1)_{F_2} = 5.44$ lb **Ans.**

Ans:
The magnitude is $(F_1)_{F_2} = 5.44$ lb

2–121.

Determine the angle θ between the two cables attached to the pipe.



SOLUTION

Unit Vectors:

$$\mathbf{u}_{F_1} = \cos 30° \sin 30°\mathbf{i} + \cos 30° \cos 30°\mathbf{j} - \sin 30°\mathbf{k}$$

$$= 0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}$$

$$\mathbf{u}_{F_2} = \cos 135°\mathbf{i} + \cos 60°\mathbf{j} + \cos 60°\mathbf{k}$$

$$= -0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k}$$

The Angles Between Two Vectors θ:

$$\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2} = (0.4330\mathbf{i} + 0.75\mathbf{j} - 0.5\mathbf{k}) \cdot (-0.7071\mathbf{i} + 0.5\mathbf{j} + 0.5\mathbf{k})$$

$$= 0.4330(-0.7071) + 0.75(0.5) + (-0.5)(0.5)$$

$$= -0.1812$$

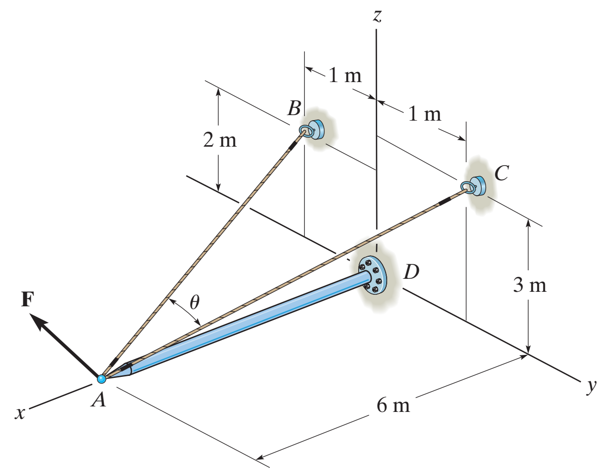
Then,

$$\theta = \cos^{-1}\left(\mathbf{u}_{F_1} \cdot \mathbf{u}_{F_2}\right) = \cos^{-1}(-0.1812) = 100° \qquad \textbf{Ans.}$$

Ans:
$\theta = 100°$

2–122.

Determine the angle θ between the cables AB and AC.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(6, 0, 0)$ m, $B(0, -1, 2)$ m and $C(0, 1, 3)$ respectively. Thus, the unit vectors along AB and AC are

$$\mathbf{u}_{AB} = \frac{(0-6)\mathbf{i} + (-1-0)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(0-6)^2 + (-1-0)^2 + (2-0)^2}} = -\frac{6}{\sqrt{41}}\mathbf{i} - \frac{1}{\sqrt{41}}\mathbf{j} + \frac{2}{\sqrt{41}}\mathbf{k}$$

$$\mathbf{u}_{AC} = \frac{(0-6)\mathbf{i} + (1-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-6)^2 + (1-0)^2 + (3-0)^2}} = -\frac{6}{\sqrt{46}}\mathbf{i} + \frac{1}{\sqrt{46}}\mathbf{j} + \frac{3}{\sqrt{46}}\mathbf{k}$$

The Angle θ Between AB and AC.

$$\mathbf{u}_{AB} \cdot \mathbf{u}_{AC} = \left(-\frac{6}{\sqrt{41}}\mathbf{i} - \frac{1}{\sqrt{41}}\mathbf{j} + \frac{2}{\sqrt{41}}\mathbf{k}\right)\cdot\left(-\frac{6}{\sqrt{46}}\mathbf{i} + \frac{1}{\sqrt{46}}\mathbf{j} + \frac{3}{\sqrt{46}}\mathbf{k}\right)$$

$$= \left(-\frac{6}{\sqrt{41}}\right)\left(-\frac{6}{\sqrt{46}}\right) + \left(-\frac{1}{\sqrt{41}}\right)\left(\frac{1}{\sqrt{46}}\right) + \frac{2}{\sqrt{41}}\left(\frac{3}{\sqrt{46}}\right)$$

$$= \frac{41}{\sqrt{1886}}$$

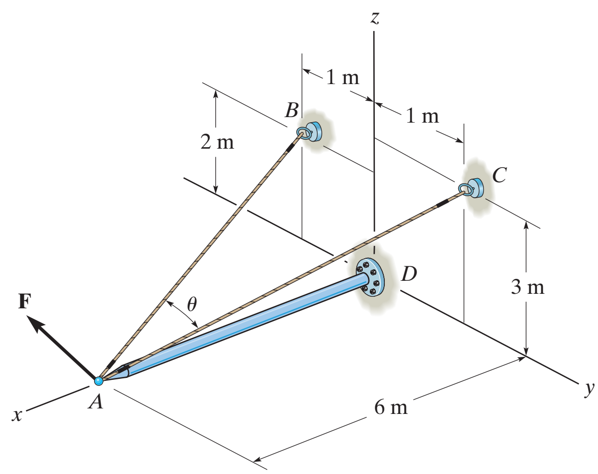
Then

$$\theta = \cos^{-1}\left(U_{AB} \cdot U_{AC}\right) = \cos^{-1}\left(\frac{41}{\sqrt{1886}}\right) = 19.24998^\circ = 19.2^\circ \qquad \textbf{Ans.}$$

Ans:
$\theta = 19.2^\circ$

2–123.

Determine the magnitude of the projected component of the force $\mathbf{F} = \{400\mathbf{i} - 200\mathbf{j} + 500\mathbf{k}\}$ N acting along the cable BA.



SOLUTION

Unit Vector. Here, the coordinates of points A and B are $A(6, 0, 0)$ m and $B(0, -1, 2)$ m respectively. Thus the unit vector along BA is

$$\mathbf{u}_{BA} = \frac{\mathbf{r}_{BA}}{r_{BA}} = \frac{(6-0)\mathbf{i} + [0-(-1)]\mathbf{j} + (0-2)\mathbf{k}}{\sqrt{(6-0)^2 + [0-(-1)]^2 + (0-2)^2}} = \frac{6}{\sqrt{41}}\mathbf{i} + \frac{1}{\sqrt{41}}\mathbf{j} - \frac{2}{\sqrt{41}}\mathbf{k}$$

Projected component of F. Along BA, it is

$$F_{BA} = \mathbf{F}\cdot\mathbf{u}_{BA} = (400\mathbf{i} - 200\mathbf{j} + 500\mathbf{k})\cdot\left(\frac{6}{\sqrt{41}}\mathbf{i} + \frac{1}{\sqrt{41}}\mathbf{j} - \frac{2}{\sqrt{41}}\mathbf{k}\right)$$

$$= 400\left(\frac{6}{\sqrt{41}}\right) + (-200)\left(\frac{1}{\sqrt{41}}\right) + 500\left(-\frac{2}{\sqrt{41}}\right)$$

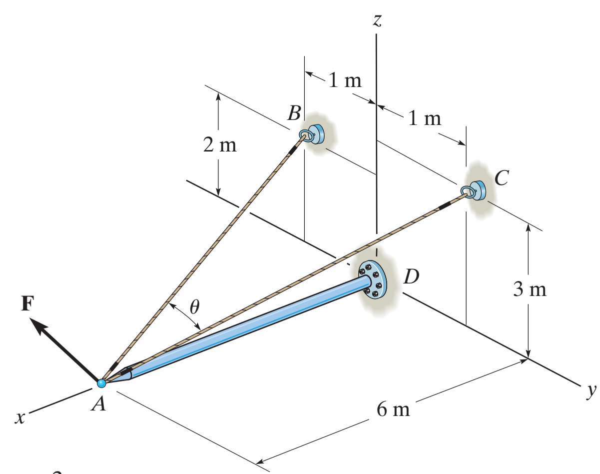
$$= 187.41 \text{ N} = 187 \text{ N} \qquad \textbf{Ans.}$$

The positive sign indicates that this component points in the same direction as $\mathbf{u}_{BA}$.

Ans:
$F_{BA} = 187$ N

***2–124.**

Determine the magnitude of the projected component of the force $\mathbf{F} = \{400\mathbf{i} - 200\mathbf{j} + 500\mathbf{k}\}$ N acting along the cable CA.



SOLUTION

Unit Vector. Here, the coordinates of points A and C are $A(6, 0, 0)$ m and $C(0, 1, 3)$ m respectively. Thus, the unit vector along CA is

$$\mathbf{u}_{CA} = \frac{\mathbf{r}_{CA}}{r_{CA}} = \frac{(6-0)\mathbf{i} + (0-1)\mathbf{j} + (0-3)\mathbf{k}}{\sqrt{(6-0)^2 + (0-1)^2 + (0-3)^2}} = \frac{6}{\sqrt{46}}\mathbf{i} - \frac{1}{\sqrt{46}}\mathbf{j} - \frac{3}{\sqrt{46}}\mathbf{k}$$

Projected component of F. Along CA, it is

$$\mathbf{F}_{CA} = \mathbf{F}\cdot\mathbf{u}_{CA} = (400\mathbf{i} - 200\mathbf{j} + 500\mathbf{k})\cdot\left(\frac{6}{\sqrt{46}}\mathbf{i} - \frac{1}{\sqrt{46}}\mathbf{j} - \frac{3}{\sqrt{46}}\mathbf{k}\right)$$

$$= 400\left(\frac{6}{\sqrt{46}}\right) + (-200)\left(-\frac{1}{\sqrt{46}}\right) + 500\left(-\frac{3}{\sqrt{46}}\right)$$

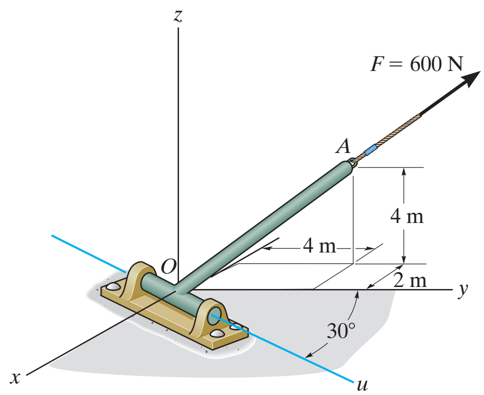
$$= 162.19 \text{ N} = 162 \text{ N} \qquad \textbf{Ans.}$$

The positive sign indicates that this component points in the same direction as $\mathbf{u}_{CA}$.

Ans:
$F_{CA} = 162$ N

2–125.

Determine the magnitude of the projection of force $F = 600$ N along the u axis.



SOLUTION

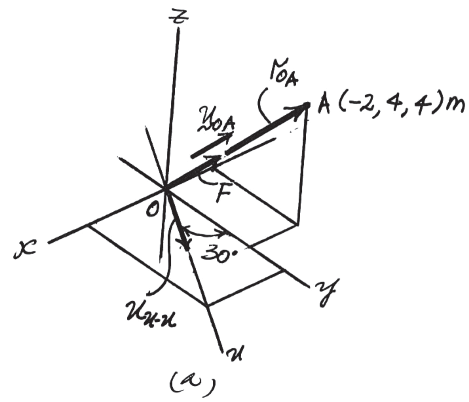
Unit Vectors: The unit vectors $\mathbf{u}_{OA}$ and $\mathbf{u}_u$ must be determined first. From Fig. a,

$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-2-0)\mathbf{i} + (4-0)\mathbf{j} + (4-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (4-0)^2 + (4-0)^2}} = -\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

$$\mathbf{u}_u = \sin 30°\mathbf{i} + \cos 30°\mathbf{j}$$

Thus, the force vectors $\mathbf{F}$ is given by

$$\mathbf{F} = F\,\mathbf{u}_{OA} = 600\left(-\frac{1}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right) = \{-200\mathbf{i} + 400\mathbf{j} + 400\mathbf{k}\}\text{ N}$$



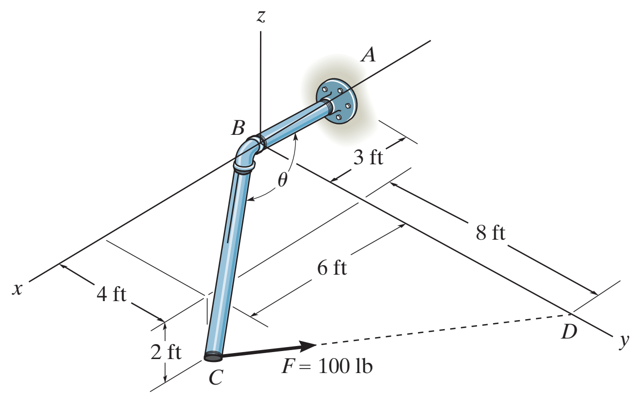
Vector Dot Product: The magnitude of the projected component of $\mathbf{F}$ along the u axis is

$$\begin{aligned} F_u = \mathbf{F}\cdot\mathbf{u}_u &= (-200\mathbf{i} + 400\mathbf{j} + 400\mathbf{k})\cdot(\sin 30°\mathbf{i} + \cos 30°\mathbf{j}) \\ &= (-200)(\sin 30°) + 400(\cos 30°) + 400(0) \\ &= 246\text{ N} \end{aligned}$$ **Ans.**

Ans:
$F_u = 246$ N

2–126.

Determine the magnitude of the projected component of the 100-lb force acting along the axis BC of the pipe.



SOLUTION

$$\vec{\gamma}_{BC} = \{6\hat{i} + 4\hat{j} - 2\hat{k}\}\text{ ft}$$

$$\vec{F} = 100\frac{\{-6\hat{i} + 8\hat{j} + 2\hat{k}\}}{\sqrt{(-6)^2 + 8^2 + 2^2}}$$

$$= \{-58.83\hat{i} + 78.45\hat{j} + 19.61\hat{k}\}\text{ lb}$$

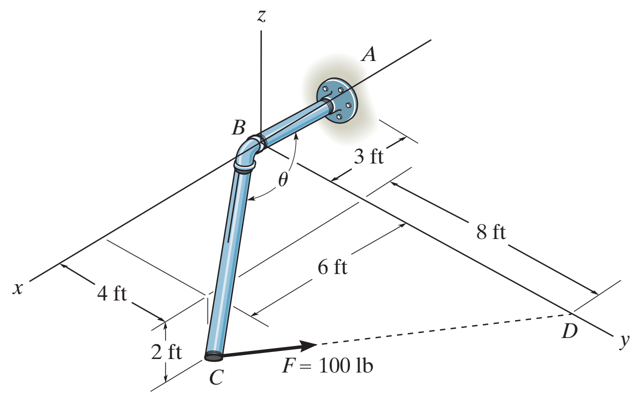
$$F_p = \vec{F}\cdot\vec{\mu}_{BC} = \vec{F}\cdot\frac{\vec{\gamma}_{BC}}{|\vec{\gamma}_{BC}|} = \frac{-78.45}{7.483} = -10.48$$

$F_p = 10.5$ lb **Ans.**

Ans:
$F_p = 10.5$ lb

2–127.

Determine the angle θ between pipe segments BA and BC.



SOLUTION

$\vec{\gamma}_{BC} = \{6\hat{\imath} + 4\hat{\jmath} - 2\hat{k}\}$ ft

$\vec{\gamma}_{BA} = \{-3\hat{\imath}\}$ ft

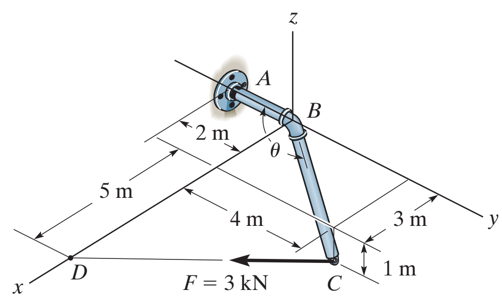
$$\theta = \cos^{-1}\left(\frac{\vec{\gamma}_{BC} \cdot \vec{\gamma}_{BA}}{|\vec{\gamma}_{BC}||\vec{\gamma}_{BA}|}\right) = \cos^{-1}\left(\frac{-18}{22.45}\right)$$

$\theta = 143°$ **Ans.**

Ans:
$\theta = 142°$

***2–128.**

Determine the angle θ between BA and BC.



SOLUTION

Unit Vectors. Here, the coordinates of points A, B and C are $A(0, -2, 0)$ m, $B(0, 0, 0)$ m and $C(3, 4, -1)$ m respectively. Thus, the unit vectors along BA and BC are

$$\mathbf{u}_{BA} = -\mathbf{j} \qquad \mathbf{u}_{BE} = \frac{(3-0)\,\mathbf{i} + (4-0)\,\mathbf{j} + (-1-0)\,\mathbf{k}}{\sqrt{(3-0)^2 + (4-0)^2 + (-1-0)^2}} = \frac{3}{\sqrt{26}}\,\mathbf{i} + \frac{\mathbf{4}}{\sqrt{\mathbf{26}}}\,\mathbf{j} - \frac{1}{\sqrt{26}}\,\mathbf{k}$$

The Angle θ Between BA and BC.

$$\mathbf{u}_{BA}\,\mathbf{u}_{BC} = (-\mathbf{j}) \cdot \left(\frac{3}{\sqrt{26}}\,\mathbf{i} + \frac{4}{\sqrt{26}}\,\mathbf{j} - \frac{1}{\sqrt{26}}\,\mathbf{k}\right)$$

$$= (-1)\left(\frac{4}{\sqrt{26}}\right) = -\frac{4}{\sqrt{26}}$$

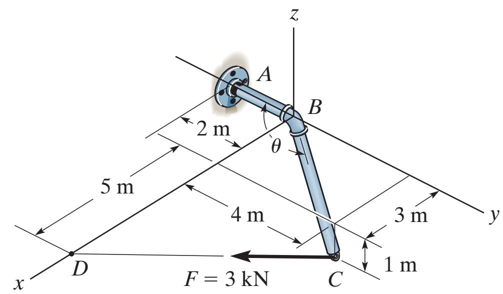
Then

$$\theta = \cos^{-1}(\mathbf{u}_{BA} \cdot \mathbf{u}_{BC}) = \cos^{-1}\left(-\frac{4}{\sqrt{26}}\right) = 141.67^\circ = 142^\circ \qquad \textbf{Ans.}$$

Ans:
$\theta = 142^\circ$

2–129.

Determine the magnitude of the projected component of the 3 kN force acting along the axis BC of the pipe.



SOLUTION

Unit Vectors. Here, the coordinates of points B, C and D are $B\,(0, 0, 0)$ m, $C(3, 4, -1)$ m and $D(8, 0, 0)$. Thus the unit vectors along BC and CD are

$$\mathbf{u}_{BC} = \frac{(3-0)\,\mathbf{i} + (4-0)\,\mathbf{j} + (-1-0)\,\mathbf{k}}{\sqrt{(3-0)^2 + (4-0)^2 + (-1-0)^2}} = \frac{3}{\sqrt{26}}\,\mathbf{i} + \frac{4}{\sqrt{26}}\,\mathbf{j} - \frac{1}{\sqrt{26}}\,\mathbf{k}$$

$$\mathbf{u}_{CD} = \frac{(8-3)\,\mathbf{i} + (0-4)\,\mathbf{j} + [0-(-1)]\,\mathbf{k}}{\sqrt{(8-3)^2 + (0-4)^2 + [0-(-1)]^2}} = \frac{5}{\sqrt{42}}\,\mathbf{i} - \frac{4}{\sqrt{42}}\,\mathbf{j} + \frac{1}{\sqrt{42}}\,\mathbf{k}$$

Force Vector. For **F**,

$$\mathbf{F} = F\mathbf{u}_{CD} = 3\left(\frac{5}{\sqrt{42}}\,\mathbf{i} - \frac{4}{\sqrt{42}}\,\mathbf{j} + \frac{1}{\sqrt{42}}\,\mathbf{k}\right)$$

$$= \left(\frac{15}{\sqrt{42}}\,\mathbf{i} - \frac{12}{\sqrt{42}}\,\mathbf{j} + \frac{3}{\sqrt{42}}\,\mathbf{k}\right)\text{ kN}$$

Projected Component of F. Along BC, it is

$$\left|(F_{BC})\right| = \left|\mathbf{F}\cdot\mathbf{u}_{BC}\right| = \left|\left(\frac{15}{\sqrt{42}}\mathbf{i} - \frac{12}{\sqrt{42}}\mathbf{j} + \frac{3}{\sqrt{42}}\mathbf{k}\right)\cdot\left(\frac{3}{\sqrt{26}}\mathbf{i} + \frac{4}{\sqrt{26}}\mathbf{j} - \frac{1}{\sqrt{26}}\mathbf{k}\right)\right|$$

$$= \left|\left(\frac{15}{\sqrt{42}}\right)\left(\frac{3}{\sqrt{26}}\right) + \left(-\frac{12}{\sqrt{42}}\right)\left(\frac{4}{\sqrt{26}}\right) + \frac{3}{\sqrt{42}}\left(-\frac{1}{\sqrt{26}}\right)\right|$$

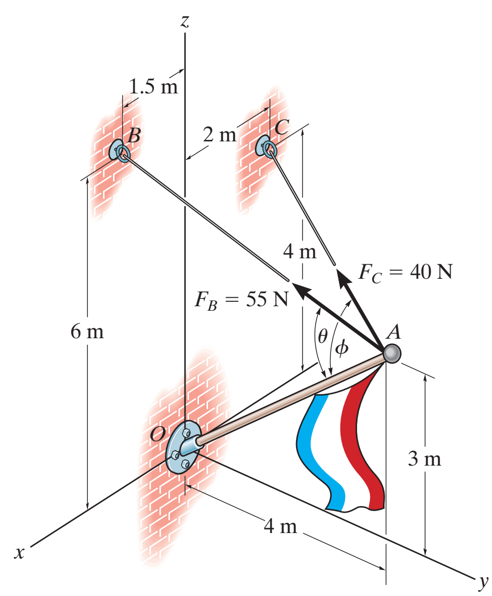
$$= \left|-\frac{6}{\sqrt{1092}}\right| = \left|-0.1816\text{ kN}\right| = 0.182\text{ kN}\qquad\textbf{Ans.}$$

The negative signs indicate that this component points in the direction opposite to that of $\mathbf{u}_{BC}$.

Ans:
0.182 kN

2–130.

Determine the angles θ and ϕ made between the axes OA of the flag pole and AB and AC, respectively, of each cable.



SOLUTION

$\mathbf{r}_{AC} = \{-2\mathbf{i} - 4\mathbf{j} + 1\mathbf{k}\}$ m ; $\quad r_{AC} = 4.58$ m

$\mathbf{r}_{AB} = \{1.5\mathbf{i} - 4\mathbf{j} + 3\mathbf{k}\}$ m; $\quad r_{AB} = 5.22$ m

$\mathbf{r}_{AO} = \{-4\mathbf{j} - 3\mathbf{k}\}$ m; $\quad r_{AO} = 5.00$ m

$$\mathbf{r}_{AB} \cdot \mathbf{r}_{AO} = (1.5)(0) + (-4)(-4) + (3)(-3) = 7$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB} \cdot \mathbf{r}_{AO}}{r_{AB}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{7}{5.22(5.00)}\right) = 74.4^\circ$$ **Ans.**

$$\mathbf{r}_{AC} \cdot \mathbf{r}_{AO} = (-2)(0) + (-4)(-4) + (1)(-3) = 13$$

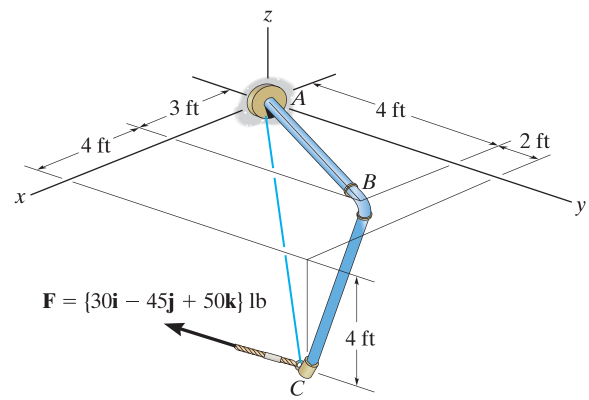
$$\phi = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AO}}{r_{AC}\, r_{AO}}\right)$$

$$= \cos^{-1}\left(\frac{13}{4.58(5.00)}\right) = 55.4^\circ$$ **Ans.**

Ans:
$\theta = 74.4^\circ$
$\phi = 55.4^\circ$

2–131.

Determine the magnitudes of the components of **F** acting along and perpendicular to segment BC of the pipe assembly.



SOLUTION

Unit Vector: The unit vector $\mathbf{u}_{CB}$ must be determined first. From Fig. *a*

$$\mathbf{u}_{CB} = \frac{\mathbf{r}_{CB}}{r_{CB}} = \frac{(3-7)\mathbf{i} + (4-6)\mathbf{j} + [0-(-4)]\mathbf{k}}{\sqrt{(3-7)^2 + (4-6)^2 + [0-(-4)]^2}} = -\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** parallel to segment BC of the pipe assembly is

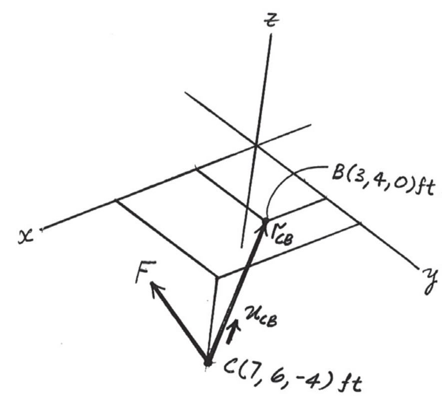
$$(F_{BC})_{\text{pa}} = \mathbf{F} \cdot \mathbf{u}_{CB} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k}) \cdot \left(-\frac{2}{3}\mathbf{i} - \frac{1}{3}\mathbf{j} + \frac{2}{3}\mathbf{k}\right)$$

$$= (30)\left(-\frac{2}{3}\right) + (-45)\left(-\frac{1}{3}\right) + 50\left(\frac{2}{3}\right)$$

$$= 28.33 \text{ lb} = 28.3 \text{ lb} \qquad \textbf{Ans.}$$

The magnitude of **F** is $F = \sqrt{30^2 + (-45)^2 + 50^2} = \sqrt{5425}$ lb. Thus, the magnitude of the component of **F** perpendicular to segment BC of the pipe assembly can be determined from

$$(F_{BC})_{\text{pr}} = \sqrt{F^2 - (F_{BC})_{\text{pa}}^{\,2}} = \sqrt{5425 - 28.33^2} = 68.0 \text{ lb} \qquad \textbf{Ans.}$$

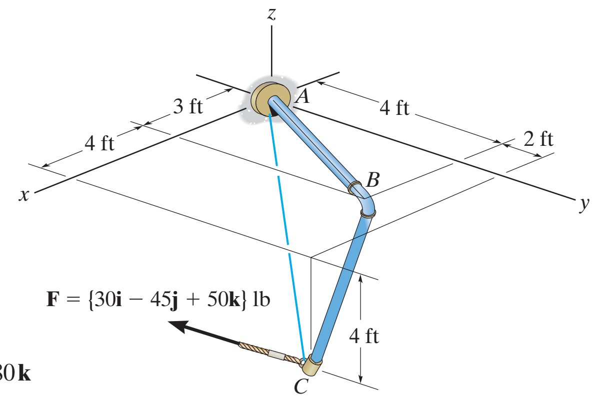


(a)

Ans:
$(F_{BC})_{\parallel} = 28.3$ lb
$(F_{BC})_{\perp} = 68.0$ lb

***2–132.**

Determine the magnitude of the projected component of **F** along *AC*. Express this component as a Cartesian vector.



SOLUTION

Unit Vector: The unit vector $\mathbf{u}_{AC}$ must be determined first. From Fig. *a*

$$\mathbf{u}_{AC} = \frac{(7-0)\mathbf{i} + (6-0)\mathbf{j} + (-4-0)\mathbf{k}}{\sqrt{(7-0)^2 + (6-0)^2 + (-4-0)^2}} = 0.6965\,\mathbf{i} + 0.5970\,\mathbf{j} - 0.3980\,\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of **F** along line *AC* is

$$F_{AC} = \mathbf{F}\cdot\mathbf{u}_{AC} = (30\mathbf{i} - 45\mathbf{j} + 50\mathbf{k})\cdot(0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

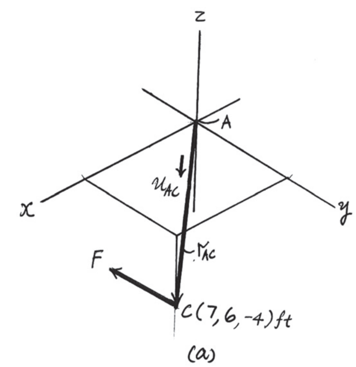
$$= (30)(0.6965) + (-45)(0.5970) + 50(-0.3980)$$

$$= 25.87 \text{ lb}$$ **Ans.**

Thus, $\mathbf{F}_{AC}$ expressed in Cartesian vector form is

$$\mathrm{F}_{AC} = F_{AC}\,\mathbf{u}_{AC} = -25.87(0.6965\mathbf{i} + 0.5970\mathbf{j} - 0.3980\mathbf{k})$$

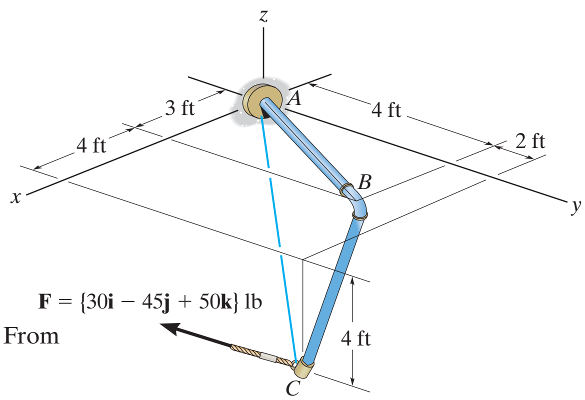
$$= \{-18.0\mathbf{i} - 15.4\mathbf{j} + 10.3\mathbf{k}\} \text{ lb}$$ **Ans.**



Ans:
$F_{AC} = 25.87$ lb
$F_{AC} = \{-18.0\mathbf{i} - 15.4\mathbf{j} + 10.3\mathbf{k}\}$ lb

2–133.

Determine the angle θ between the pipe segments BA and BC.

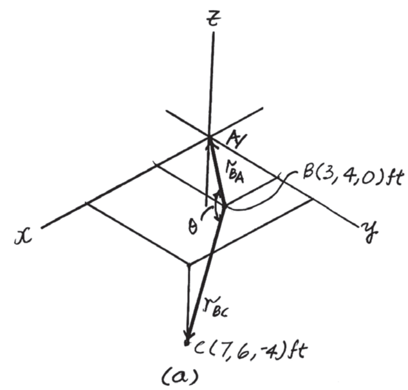


SOLUTION

Position Vectors: The position vectors $\mathbf{r}_{BA}$ and $\mathbf{r}_{BC}$ must be determined first. From Fig. *a*,

$$\mathbf{r}_{BA} = (0 - 3)\mathbf{i} + (0 - 4)\mathbf{j} + (0 - 0)\mathbf{k} = \{-3\mathbf{i} - 4\mathbf{j}\}\text{ ft}$$

$$\mathbf{r}_{BC} = (7 - 3)\mathbf{i} + (6 - 4)\mathbf{j} + (-4 - 0)\mathbf{k} = \{4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}\}\text{ ft}$$



The magnitude of $\mathbf{r}_{BA}$ and $\mathbf{r}_{BC}$ are

$$r_{BA} = \sqrt{(-3)^2 + (-4)^2} = 5\text{ ft}$$

$$r_{BC} = \sqrt{4^2 + 2^2 + (-4)^2} = 6\text{ ft}$$

Vector Dot Product:

$$\begin{aligned} \mathbf{r}_{BA} \cdot \mathbf{r}_{BC} &= (-3\mathbf{i} - 4\mathbf{j}) \cdot (4\mathbf{i} + 2\mathbf{j} - 4\mathbf{k}) \\ &= (-3)(4) + (-4)(2) + 0(-4) \\ &= -20\text{ ft}^2 \end{aligned}$$

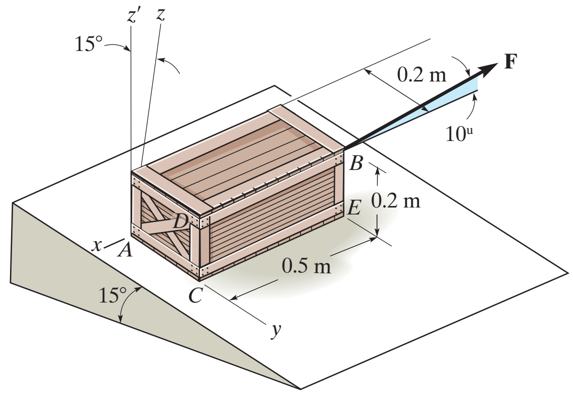
Thus,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{BA} \cdot \mathbf{r}_{BC}}{r_{BA}\, r_{BC}}\right) = \cos^{-1}\left[\frac{-20}{5(6)}\right] = 132° \qquad \textbf{Ans.}$$

Ans:
$\theta = 132°$

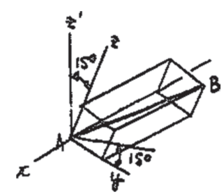
2–134.

If the force $F = 100$ N lies in the plane $DBEC$, which is parallel to the x–z plane, and makes an angle of 10° with the extended line DB as shown, determine the angle that $\mathbf{F}$ makes with the diagonal AB of the crate.



SOLUTION

Use the x, y, z axes.



$$\mathbf{u}_{AB} = \left(\frac{-0.5\mathbf{i} + 0.2\mathbf{j} + 0.2\mathbf{k}}{0.57446}\right)$$

$$= -0.8704\mathbf{i} + 0.3482\mathbf{j} + 0.3482\mathbf{k}$$

$$\mathbf{F} = -100 \cos 10°\mathbf{i} + 100 \sin 10°\mathbf{k}$$

$$\theta = \cos^{-1}\left(\frac{\mathbf{F} \cdot \mathbf{u}_{AB}}{F\,u_{AB}}\right)$$

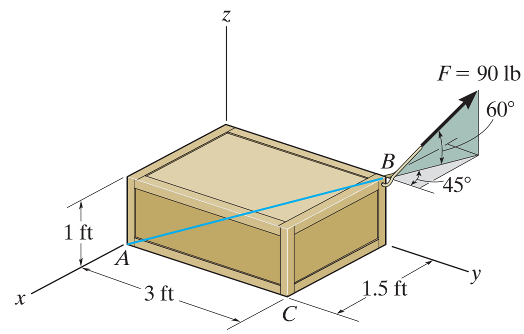
$$= \cos^{-1}\left(\frac{-100\,(\cos 10°)(-0.8704) + 0 + 100 \sin 10°\,(0.3482)}{100(1)}\right)$$

$$= \cos^{-1}(0.9176) = 23.4° \qquad \textbf{Ans.}$$

Ans:
$\theta = 23.4°$

2–135.

Determine the magnitudes of the components of force $F = 90$ lb acting parallel and perpendicular to diagonal AB of the crate.



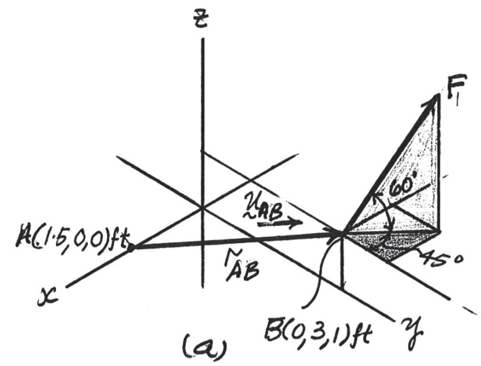
SOLUTION

Force and Unit Vector: The force vector **F** and unit vector $\mathbf{u}_{AB}$ must be determined first. From Fig. *a*

$$\mathbf{F} = 90(-\cos 60^\circ \sin 45^\circ \mathbf{i} + \cos 60^\circ \cos 45^\circ \mathbf{j} + \sin 60^\circ \mathbf{k})$$

$$= \{-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}\} \text{ lb}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{(0-1.5)\mathbf{i} + (3-0)\mathbf{j} + (1-0)\mathbf{k}}{\sqrt{(0-1.5)^2 + (3-0)^2 + (1-0)^2}} = -\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}$$



Vector Dot Product: The magnitude of the projected component of **F** parallel to the diagonal AB is

$$[(F)_{AB}]_{\text{pa}} = \mathbf{F} \cdot \mathbf{u}_{AB} = (-31.82\mathbf{i} + 31.82\mathbf{j} + 77.94\mathbf{k}) \cdot \left(-\frac{3}{7}\mathbf{i} + \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$$

$$= (-31.82)\left(-\frac{3}{7}\right) + 31.82\left(\frac{6}{7}\right) + 77.94\left(\frac{2}{7}\right)$$

$$= 63.18 \text{ lb} = 63.2 \text{ lb} \qquad \textbf{Ans.}$$

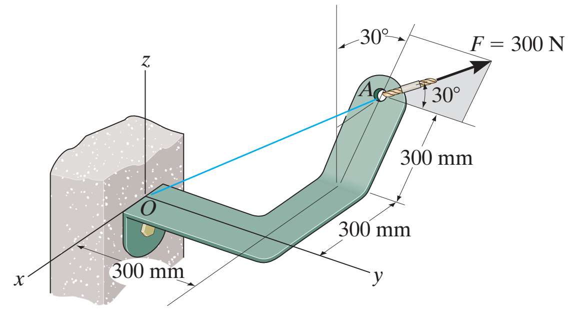
The magnitude of the component **F** perpendicular to the diagonal AB is

$$[(F)_{AB}]_{\text{pr}} = \sqrt{F^2 - [(F)_{AB}]_{\text{pa}}{}^2} = \sqrt{90^2 - 63.18^2} = 64.1 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$[(F)_{AB}]_{||} = 63.2$ lb
$[(F)_{AB}]_{\perp} = 64.1$ lb

***2–136.**

Determine the magnitudes of the projected components of the force $F = 300$ N acting along the x and y axes.



SOLUTION

Force Vector: The force vector **F** must be determined first. From Fig. *a*,

$$\mathbf{F} = -300 \sin 30° \sin 30° \mathbf{i} + 300 \cos 30° \mathbf{j} + 300 \sin 30° \cos 30° \mathbf{k}$$

$$= [-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}] \text{ N}$$

Vector Dot Product: The magnitudes of the projected component of **F** along the x and y axes are

$$F_x = \mathbf{F} \cdot \mathbf{i} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \mathbf{i}$$

$$= -75(1) + 259.81(0) + 129.90(0)$$

$$= -75 \text{ N}$$

$$F_y = \mathbf{F} \cdot \mathbf{j} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \mathbf{j}$$

$$= -75(0) + 259.81(1) + 129.90(0)$$

$$= 260 \text{ N}$$

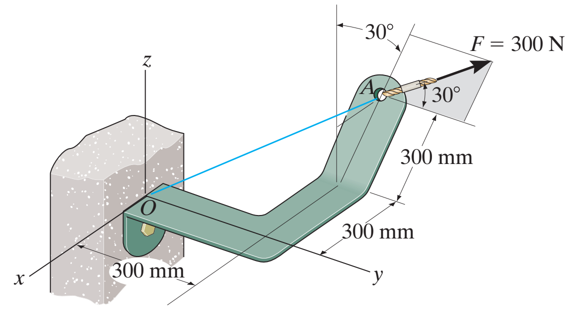
The negative sign indicates that $\mathbf{F}_x$ is directed towards the negative x axis. Thus

$$F_x = 75 \text{ N}, \qquad F_y = 260 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$F_x = 75$ N
$F_y = 260$ N

2–137.

Determine the magnitude of the projected component of the force $F = 300$ N acting along line OA.



SOLUTION

Force and Unit Vector: The force vector $\mathbf{F}$ and unit vector $\mathbf{u}_{OA}$ must be determined first. From Fig. a

$$\mathbf{F} = (-300 \sin 30° \sin 30°\mathbf{i} + 300 \cos 30°\mathbf{j} + 300 \sin 30° \cos 30°\mathbf{k})$$

$$= \{-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\} \text{ N}$$

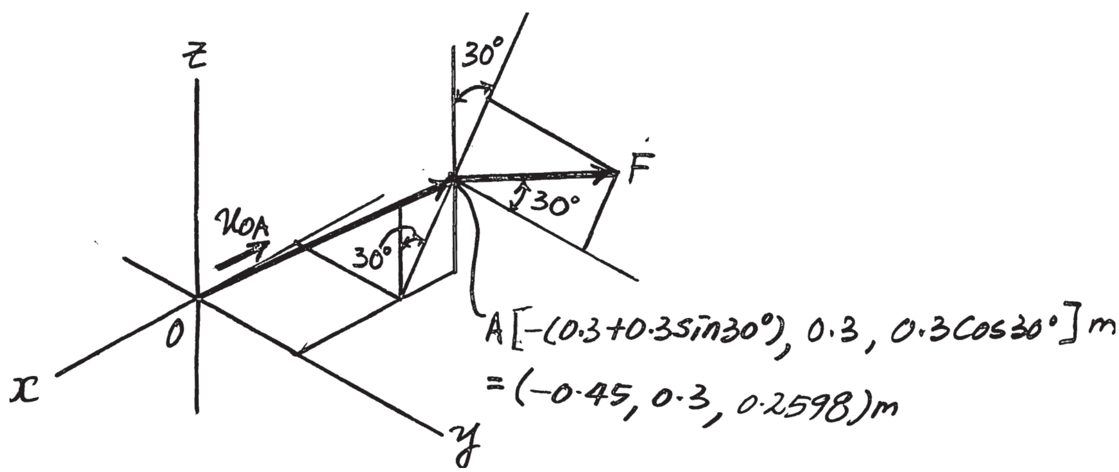
$$\mathbf{u}_{OA} = \frac{\mathbf{r}_{OA}}{r_{OA}} = \frac{(-0.45 - 0)\mathbf{i} + (0.3 - 0)\mathbf{j} + (0.2598 - 0)\mathbf{k}}{\sqrt{(-0.45 - 0)^2 + (0.3 - 0)^2 + (0.2598 - 0)^2}} = -0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}$$

Vector Dot Product: The magnitude of the projected component of $\mathbf{F}$ along line OA is

$$F_{OA} = \mathbf{F} \cdot \mathbf{u}_{OA} = \left(-75\mathbf{i} + 259.81\mathbf{j} + 129.90\mathbf{k}\right) \cdot \left(-0.75\mathbf{i} + 0.5\mathbf{j} + 0.4330\mathbf{k}\right)$$

$$= (-75)(-0.75) + 259.81(0.5) + 129.90(0.4330)$$

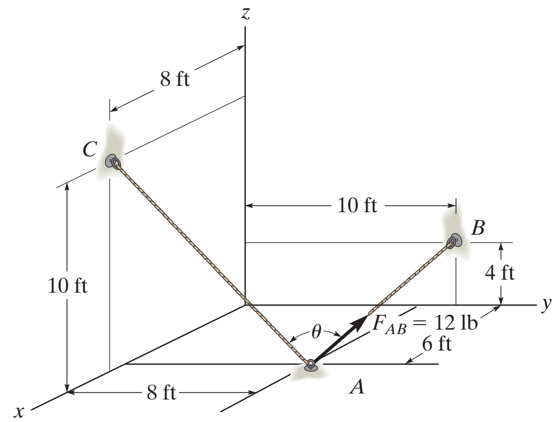
$$= 242 \text{ N}$$ **Ans.**



Ans:
$F_{OA} = 242$ N

2–138.

Determine the angle θ between the two cables.



SOLUTION

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AC} \cdot \mathbf{r}_{AB}}{r_{AC}\, r_{AB}}\right)$$

$$= \cos^{-1}\left[\frac{(2\,\mathbf{i} - 8\,\mathbf{j} + 10\,\mathbf{k})\cdot(-6\,\mathbf{i} + 2\,\mathbf{j} + 4\,\mathbf{k})}{\sqrt{2^2 + (-8)^2 + 10^2}\,\sqrt{(-6)^2 + 2^2 + 4^2}}\right]$$

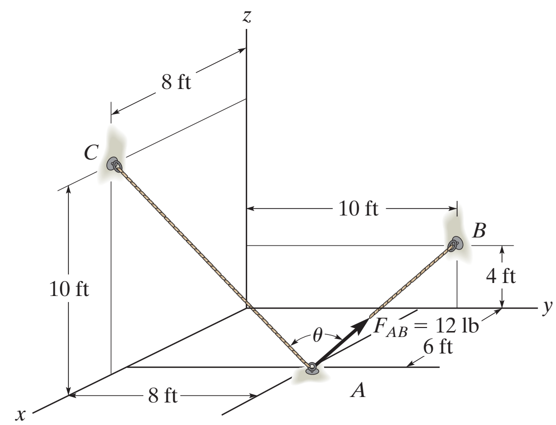
$$= \cos^{-1}\left(\frac{12}{96.99}\right)$$

$\theta = 82.9°$ **Ans.**

Ans:
$\theta = 82.9°$

2–139.

Determine the projected component of the force $F = 12$ lb acting in the direction of cable AC. Express the result as a Cartesian vector.



SOLUTION

$$\mathbf{r}_{AC} = \{2\,\mathbf{i} - 8\,\mathbf{j} + 10\,\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{AB} = \{-6\,\mathbf{i} + 2\,\mathbf{j} + 4\,\mathbf{k}\}\text{ ft}$$

$$\mathbf{F}_{AB} = 12\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = 12\left(-\frac{6}{7.483}\,\mathbf{i} + \frac{2}{7.483}\,\mathbf{j} + \frac{4}{7.483}\,\mathbf{k}\right)$$

$$\mathbf{F}_{AB} = \{-9.621\,\mathbf{i} + 3.207\,\mathbf{j} + 6.414\,\mathbf{k}\}\text{ lb}$$

$$\mathbf{u}_{AC} = \frac{2}{12.961}\,\mathbf{i} - \frac{8}{12.961}\,\mathbf{j} + \frac{10}{12.961}\,\mathbf{k}$$

$$\text{Proj } F_{AB} = \mathbf{F}_{AB}\cdot\mathbf{u}_{AC} = -9.621\left(\frac{2}{12.961}\right) + 3.207\left(-\frac{8}{12.961}\right) + 6.414\left(\frac{10}{12.961}\right)$$

$$= 1.4846$$

$$\text{Proj } \mathbf{F}_{AB} = F_{AB}\,\mathbf{u}_{AC}$$

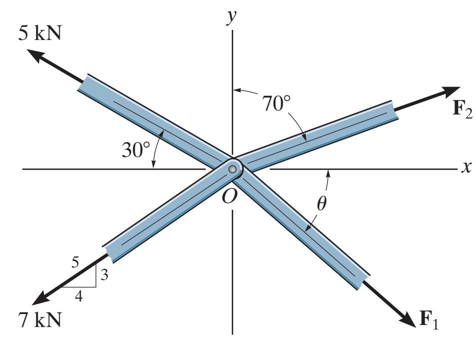
$$\text{Proj } \mathbf{F}_{AB} = (1.4846)\left[\frac{2}{12.962}\,\mathbf{i} - \frac{8}{12.962}\,\mathbf{j} + \frac{10}{12.962}\,\mathbf{k}\right]$$

$$\text{Proj } \mathbf{F}_{AB} = \{0.229\,\mathbf{i} - 0.916\,\mathbf{j} + 1.15\,\mathbf{k}\}\text{ lb}$$ **Ans.**

Ans:
Proj $\mathbf{F}_{AB} = \{0.229\,\mathbf{i} - 0.916\,\mathbf{j} + 1.15\,\mathbf{k}\}$ lb

3–1.

The members of a truss are pin connected at joint O. Determine the magnitudes of $\mathbf{F}_1$ and $\mathbf{F}_2$ for equilibrium. Set $\theta = 60^\circ$.



SOLUTION

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_2 \sin 70^\circ + F_1 \cos 60^\circ - 5 \cos 30^\circ - \frac{4}{5}(7) = 0$$

$$0.9397F_2 + 0.5F_1 = 9.930$$

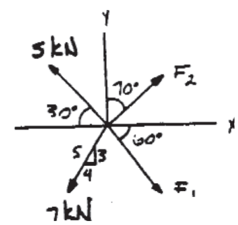
$$+\uparrow \Sigma F_y = 0; \qquad F_2 \cos 70^\circ + 5 \sin 30^\circ - F_1 \sin 60^\circ - \frac{3}{5}(7) = 0$$

$$0.3420F_2 - 0.8660F_1 = 1.7$$

Solving:

$F_2 = 9.60$ kN **Ans.**

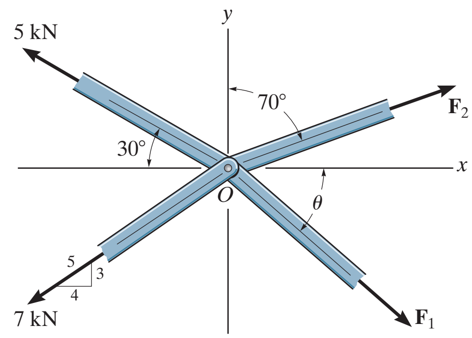
$F_1 = 1.83$ kN **Ans.**



Ans:
$F_2 = 9.60$ kN
$F_1 = 1.83$ kN

3–2.

The members of a truss are pin connected at joint O. Determine the magnitude of $\mathbf{F}_1$ and its angle θ for equilibrium. Set $F_2 = 6$ kN.



SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 6 \sin 70° + F_1 \cos \theta - 5 \cos 30° - \frac{4}{5}(7) = 0$

$F_1 \cos \theta = 4.2920$

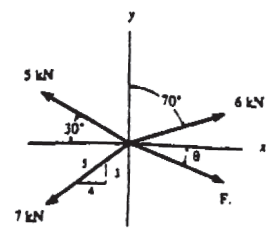
$+\uparrow \Sigma F_y = 0; \qquad 6 \cos 70° + 5 \sin 30° - F_1 \sin \theta - \frac{3}{5}(7) = 0$

$F_1 \sin \theta = 0.3521$

Solving:

$\theta = 4.69°$ **Ans.**

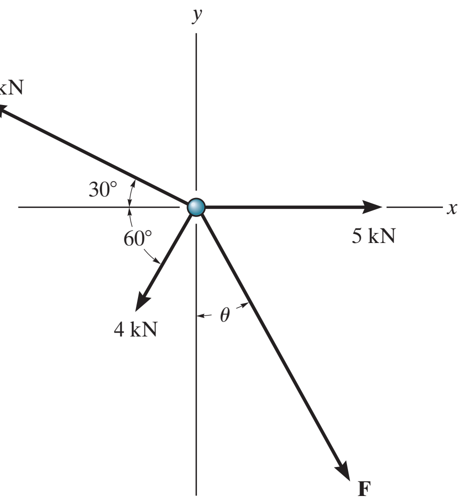
$F_1 = 4.31$ kN **Ans.**



Ans:
$\theta = 4.69°$
$F_1 = 4.31$ kN

3–3.

Determine the magnitude and direction θ of $\mathbf{F}$ so that the particle is in equilibrium.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* shown in Fig. *a*,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F\sin\theta + 5 - 4\cos 60^\circ - 8\cos 30^\circ = 0$$

$$F\sin\theta = 3.9282 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \qquad 8\sin 30^\circ - 4\sin 60^\circ - F\cos\theta = 0$$

$$F\cos\theta = 0.5359 \qquad \textbf{(2)}$$

Divide Eq (1) by (2),

$$\frac{\sin\theta}{\cos\theta} = 7.3301$$

Realizing that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$, then

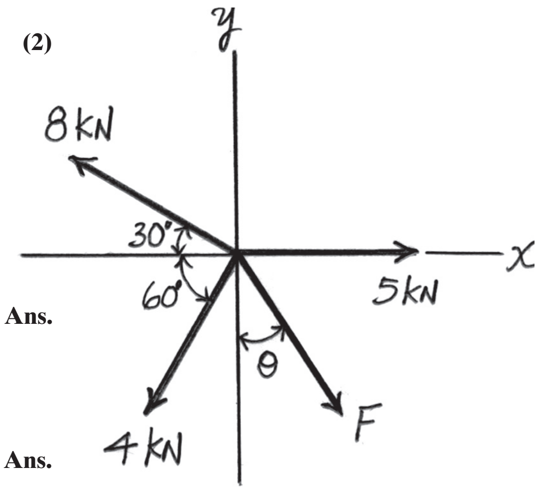
$$\tan\theta = 7.3301$$

$$\theta = 82.23^\circ = 82.2^\circ \qquad \textbf{Ans.}$$

Substitute this result into Eq. (1),

$$F\sin 82.23^\circ = 3.9282$$

$$F = 3.9646\text{ kN} = 3.96\text{ kN} \qquad \textbf{Ans.}$$

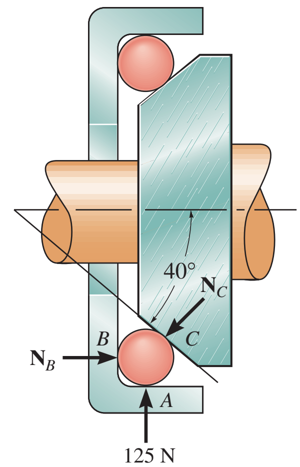


(a)

Ans:
$\theta = 82.2^\circ$
$F = 3.96$ kN

***3–4.**

The bearing consists of rollers, symmetrically confined within the housing. The bottom one is subjected to a 125-N force at its contact A due to the load on the shaft. Determine the normal reactions N_B and N_C on the bearing at its contact points B and C for equilibrium.



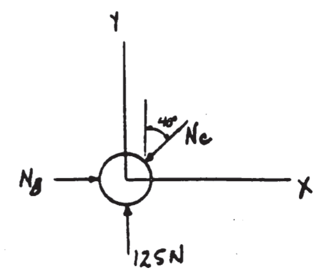
SOLUTION

$+\uparrow \Sigma F_y = 0; \quad 125 - N_C \cos 40° = 0$

$N_C = 163.176 = 163 \text{ N}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \quad N_B - 163.176 \sin 40° = 0$

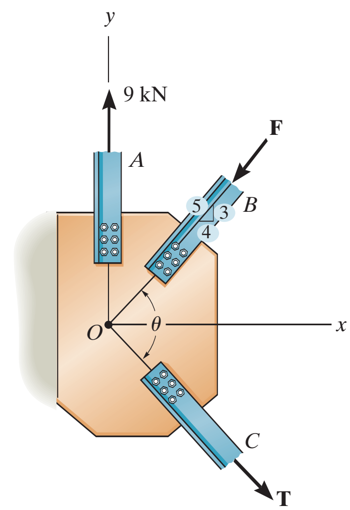
$N_B = 105 \text{ N}$ **Ans.**



Ans:
$N_C = 163 \text{ N}$
$N_B = 105 \text{ N}$

3–5.

The members of a truss are connected to the gusset plate. If the forces are concurrent at point O, determine the magnitudes of **F** and **T** for equilibrium. Take $\theta = 90°$.



SOLUTION

$$\phi = 90° - \tan^{-1}\left(\frac{3}{4}\right) = 53.13°$$

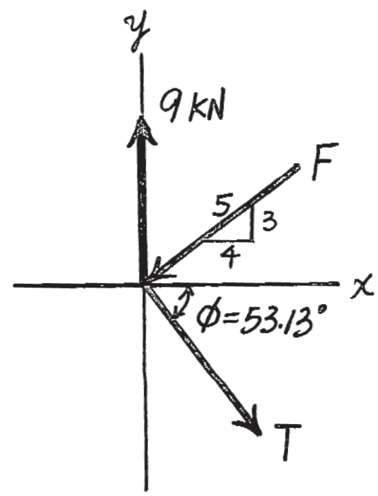
$$\xrightarrow{+} \Sigma F_x = 0; \qquad T \cos 53.13° - F\left(\frac{4}{5}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad 9 - T \sin 53.13° - F\left(\frac{3}{5}\right) = 0$$

Solving,

$$T = 7.20 \text{ kN} \qquad \textbf{Ans.}$$

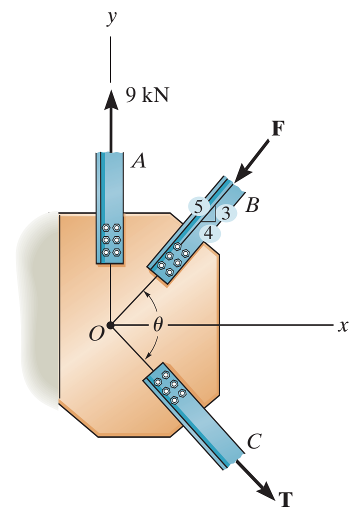
$$F = 5.40 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$T = 7.20$ kN
$F = 5.40$ kN

3–6.

The gusset plate is subjected to the forces of three members. Determine the tension force in member C and its angle θ for equilibrium. The forces are concurrent at point O. Take $F = 8$ kN.



SOLUTION

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T\cos\phi - 8\left(\frac{4}{5}\right) = 0 \qquad \textbf{(1)}$$

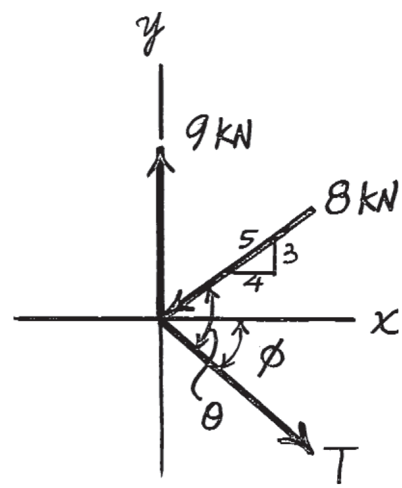
$$+\uparrow \Sigma F_y = 0; \qquad 9 - 8\left(\frac{3}{5}\right) - T\sin\phi = 0 \qquad \textbf{(2)}$$

Rearrange then divide Eq. (1) into Eq. (2):

$$\tan\phi = 0.656, \quad \phi = 33.27^\circ$$

$$T = 7.66 \text{ kN} \qquad \textbf{Ans.}$$

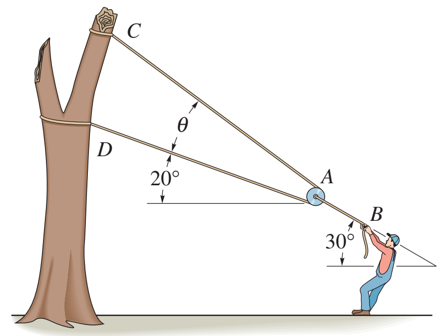
$$\theta = \phi + \tan^{-1}\left(\frac{3}{4}\right) = 70.1^\circ \qquad \textbf{Ans.}$$



Ans:
$T = 7.66$ kN
$\theta = 70.1^\circ$

3–7.

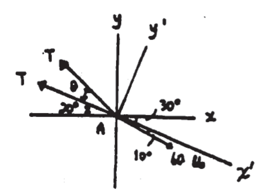
The man attempts to pull down the tree using the cable and *small* pulley arrangement shown. If the tension in AB is 60 lb, determine the tension in cable CAD and the angle θ which the cable makes at the pulley.



SOLUTION

$+\searrow \Sigma F_{x'} = 0; \qquad 60 \cos 10° - T - T \cos\theta = 0$

$+\nearrow \Sigma F_{y'} = 0; \qquad T \sin\theta - 60 \sin 10° = 0$



Thus,

$$T(1 + \cos\theta) = 60 \cos 10°$$

$$T\left(2\cos^2 \frac{\theta}{2}\right) = 60 \cos 10° \qquad \textbf{(1)}$$

$$2T \sin\frac{\theta}{2} \cos\frac{\theta}{2} = 60 \sin 10° \qquad \textbf{(2)}$$

Divide Eq.(2) by Eq.(1)

$$\tan\frac{\theta}{2} = \tan 10°$$

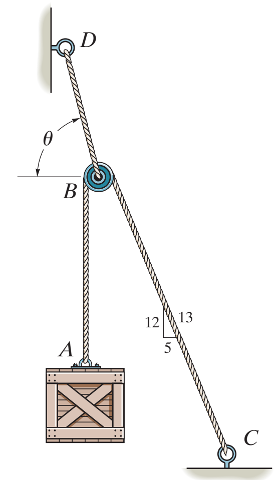
$$\theta = 20° \qquad \textbf{Ans.}$$

$$T = 30.5 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$\theta = 20°$
$T = 30.5$ lb

***3–8.**

The cords ABC and BD can each support a maximum load of 100 lb. Determine the maximum weight of the crate, and the angle θ for equilibrium.



SOLUTION

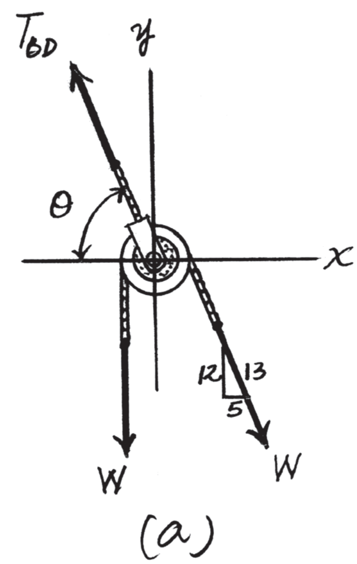
Equations of Equilibrium. Assume that for equilibrium, the tension along the length of rope ABC is constant. Assuming that the tension in cable BD reaches the limit first. Then, $T_{BD} = 100$ lb. Referring to the FBD shown in Fig. a,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad W\left(\frac{5}{13}\right) - 100\cos\theta = 0$$

$$100\cos\theta = \frac{5W}{13} \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \qquad 100\sin\theta - W - W\left(\frac{12}{13}\right) = 0$$

$$100\sin\theta = \frac{25}{13}W \qquad \textbf{(2)}$$



Divide Eq. (2) by (1),

$$\frac{\sin\theta}{\cos\theta} = 5$$

Realizing that $\tan\theta = \dfrac{\sin\theta}{\cos\theta}$,

$$\tan\theta = 5$$

$$\theta = 78.69° = 78.7° \qquad \textbf{Ans.}$$

Substitute this result into Eq. (1),

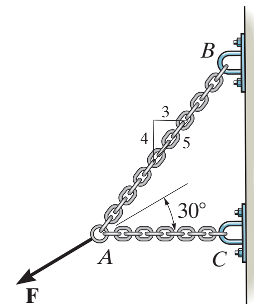
$$100\cos 78.69° = \frac{5}{13}W$$

$$W = 50.99\text{ lb} = 51.0\text{ lb} < 100\text{ lb} \ \textbf{(O.K)} \qquad \textbf{Ans.}$$

Ans:
$\theta = 78.7°$
$W = 51.0$ lb

3–9.

Determine the maximum force **F** that can be supported in the position shown if each chain can support a maximum tension of 600 lb before it fails.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* shown in Fig. *a*,

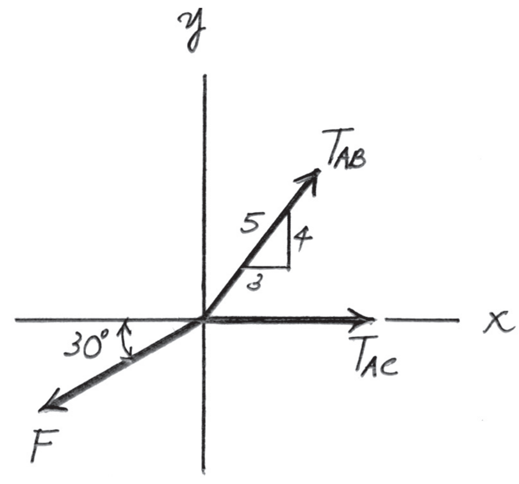
$$+\uparrow \Sigma F_y = 0; \qquad T_{AB}\left(\frac{4}{5}\right) - F \sin 30° = 0 \qquad T_{AB} = 0.625\,F$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad T_{AC} + 0.625\,F\left(\frac{3}{5}\right) - F \cos 30° = 0 \qquad T_{AC} = 0.4910\,F$$

Since chain AB is subjected to a higher tension, its tension will reach the limit first. Thus,

$$T_{AB} = 600; \qquad 0.625\,F = 600$$

$$F = 960 \text{ lb} \qquad \textbf{Ans.}$$

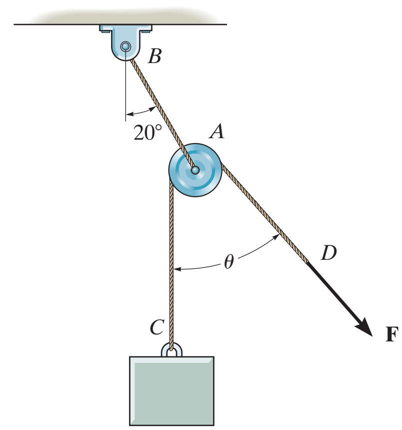


(a)

Ans:
$F = 960$ lb

3–10.

The block has a weight of 20 lb and is being hoisted at uniform velocity. Determine the angle θ for equilibrium and the force in cord AB.



SOLUTION

Equations of Equilibrium. Assume that for equilibrium, the tension along the length of cord CAD is constant. Thus, $F = 20$ lb. Referring to the FBD shown in Fig. a,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 20 \sin\theta - T_{AB} \sin 20° = 0$$

$$T_{AB} = \frac{20 \sin\theta}{\sin 20°} \tag{1}$$

$$+\uparrow\Sigma F_y = 0; \qquad T_{AB} \cos 20° - 20\cos\theta - 20 = 0 \tag{2}$$

Substitute Eq (1) into (2),

$$\frac{20 \sin\theta}{\sin 20°}\cos 20° - 20\cos\theta = 20$$

$$\sin\theta\cos 20° - \cos\theta \sin 20° = \sin 20°$$

Realizing that $\sin(\theta - 20°) = \sin\theta\cos 20° - \cos\theta\sin 20°$, then

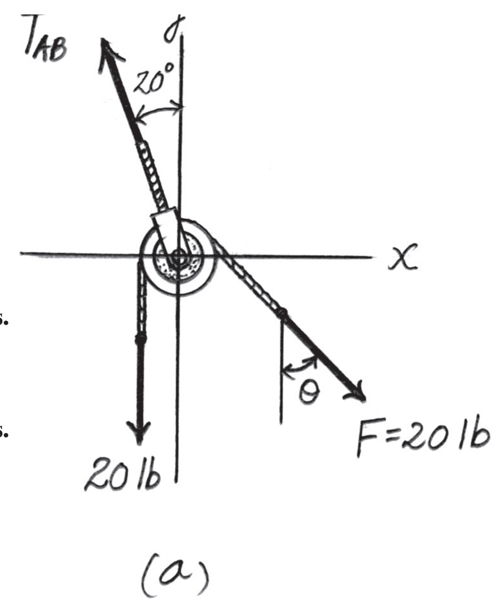
$$\sin(\theta - 20°) = \sin 20°$$

$$\theta - 20° = 20°$$

$$\theta = 40° \qquad \textbf{Ans.}$$

Substitute this result into Eq (1)

$$T_{AB} = \frac{20 \sin 40°}{\sin 20°} = 37.59 \text{ lb} = 37.6 \text{ lb} \qquad \textbf{Ans.}$$

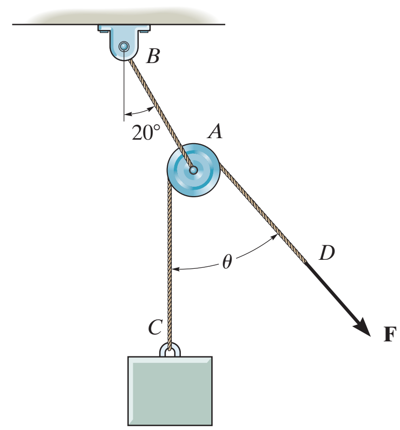


(a)

Ans:
$\theta = 40°$
$T_{AB} = 37.6$ lb

3–11.

Determine the maximum weight W of the block that can be suspended in the position shown if cords AB and CAD can each support a maximum tension of 80 lb. Also, what is the angle θ for equilibrium?



SOLUTION

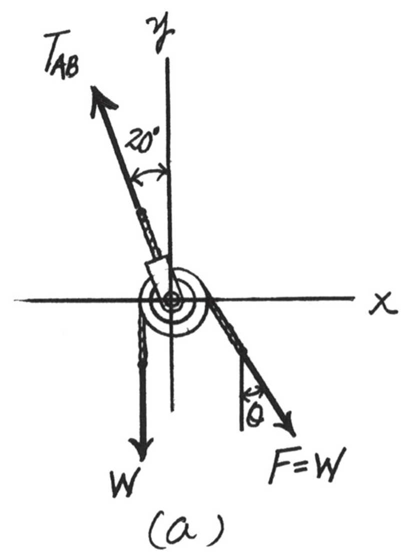
Equations of Equilibrium. Assume that for equilibrium, the tension along the length of cord CAD is constant. Thus, $F = W$. Assuming that the tension in cord AB reaches the limit first, then $T_{AB} = 80$ lb. Referring to the FBD shown in Fig. a,

$\xrightarrow{+} \Sigma F_x = 0;\qquad W \sin\theta - 80 \sin 20° = 0$

$$W = \frac{80 \sin 20°}{\sin\theta} \tag{1}$$

$+\uparrow \Sigma F_y = 0;\qquad 80 \cos 20° - W - W\cos\theta = 0$

$$W = \frac{80 \cos 20°}{1 + \cos\theta} \tag{2}$$



Equating Eqs (1) and (2),

$$\frac{80 \sin 20°}{\sin\theta} = \frac{80 \cos 20°}{1 + \cos\theta}$$

$$\sin\theta \cos 20° - \cos\theta \sin 20° = \sin 20°$$

Realizing then $\sin(\theta - 20°) = \sin\theta\cos 20° - \cos\theta \sin 20°$, then

$$\sin(\theta - 20°) = \sin 20°$$

$$\theta - 20° = 20°$$

$$\theta = 40° \qquad \textbf{Ans.}$$

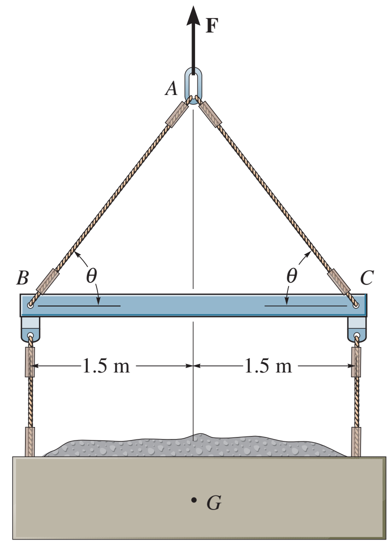
Substitute this result into Eq (1)

$$W = \frac{80 \sin 20°}{\sin 40°} = 42.56 \text{ lb} = 42.6 \text{ lb} < 80 \text{ lb} \quad \textbf{(O.K)} \qquad \textbf{Ans.}$$

Ans:
$\theta = 40°$
$W = 42.6$ lb

***3–12.**

The lift sling is used to hoist a container having a mass of 500 kg. Determine the force in each of the cables AB and AC as a function of θ. If the maximum tension allowed in each cable is 5 kN, determine the shortest lengths of cables AB and AC that can be used for the lift. The center of gravity of the container is located at G.



SOLUTION

Free-Body Diagram: By observation, the force $\mathbf{F}_1$ has to support the entire weight of the container. Thus, $F_1 = 500(9.81) = 4905$ N.

Equations of Equilibrium:

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AC}\cos\theta - F_{AB}\cos\theta = 0 \quad F_{AC} = F_{AB} = F$

$+\uparrow \Sigma F_y = 0; \qquad 4905 - 2F\sin\theta = 0 \quad F = \{2452.5\cos\theta\}\ \text{N}$

Thus,

$$F_{AC} = F_{AB} = F = \{2.45\cos\theta\}\ \text{kN} \qquad \textbf{Ans.}$$

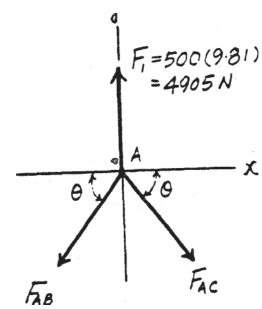
If the maximum allowable tension in the cable is 5 kN, then

$$2452.5\cos\theta = 5000$$

$$\theta = 29.37°$$

From the geometry, $l = \dfrac{1.5}{\cos\theta}$ and $\theta = 29.37°$. Therefore

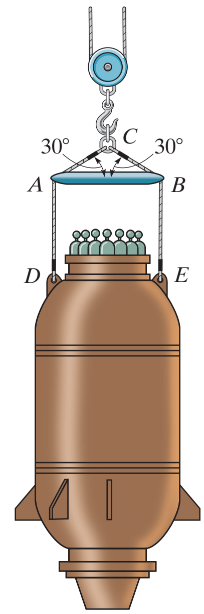
$$l = \frac{1.5}{\cos 29.37°} = 1.72\ \text{m} \qquad \textbf{Ans.}$$



Ans:
$F_{AC} = \{2.45\cos\theta\}$ kN
$l = 1.72$ m

3–13.

A nuclear-reactor vessel has a weight of $500(10^3)$ lb. Determine the horizontal compressive force that the spreader bar AB exerts on point A and the force that each cable segment CA and AD exert on this point while the vessel is hoisted upward at constant velocity.



SOLUTION

At point C:

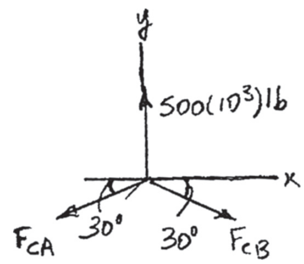
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{CB} \cos 30° - F_{CA} \cos 30° = 0$

$$F_{CB} = F_{CA}$$

$+\uparrow \Sigma F_y = 0;$ $\quad 500(10^3) - F_{CA} \sin 30° - F_{CB} \sin 30° = 0$

$$500(10^3) - 2F_{CA} \sin 30° = 0$$

$$F_{CA} = 500(10^3) \text{ lb} \qquad \textbf{Ans.}$$



At point A:

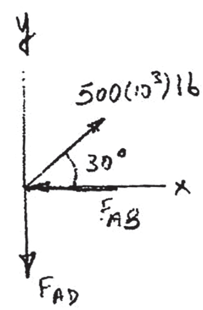
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 500(10^3) \cos 30° - F_{AB} = 0$

$$F_{AB} = 433(10^3) \text{ lb} \qquad \textbf{Ans.}$$

$+\uparrow \Sigma F_y = 0;$ $\quad 500(10^3) \sin 30° - F_{AD} = 0$

$$F_{AD} = 500(10^3) \sin 30°$$

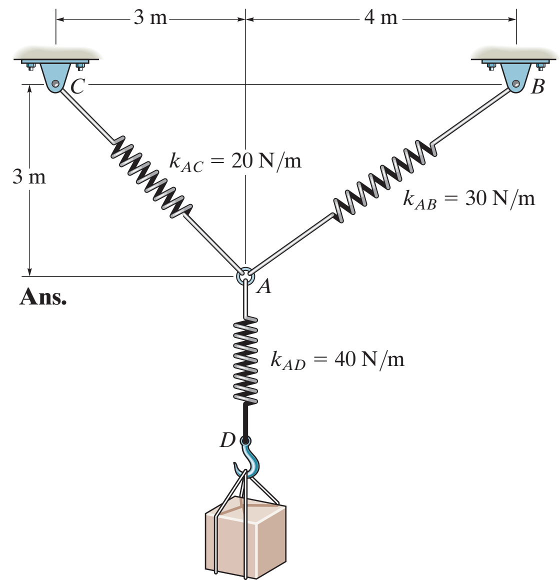
$$F_{AD} = 250(10^3) \text{ lb} \qquad \textbf{Ans.}$$



Ans:
$F_{CA} = 500(10^3)$ lb
$F_{AB} = 433(10^3)$ lb
$F_{AD} = 250(10^3)$ lb

3–14.

Determine the stretch in each spring for equlibrium of the 2-kg block. The springs are shown in the equilibrium position.



SOLUTION

$F_{AD} = 2(9.81) = x_{AD}(40) \quad x_{AD} = 0.4905$ m **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB}\left(\frac{4}{5}\right) - F_{AC}\left(\frac{1}{\sqrt{2}}\right) = 0$

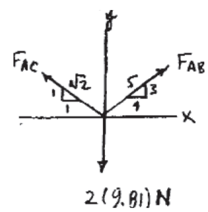
$+\uparrow \Sigma F_y = 0; \qquad F_{AC}\left(\frac{1}{\sqrt{2}}\right) + F_{AB}\left(\frac{3}{5}\right) - 2(9.81) = 0$

$F_{AC} = 15.86$ N

$x_{AC} = \frac{15.86}{20} = 0.793$ m **Ans.**

$F_{AB} = 14.01$ N

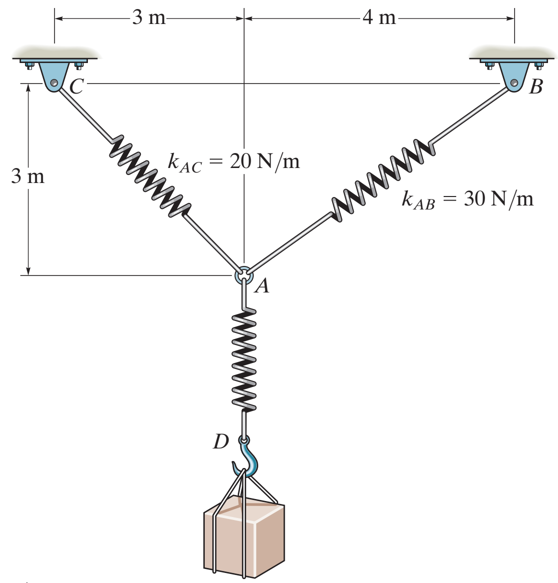
$x_{AB} = \frac{14.01}{30} = 0.467$ m **Ans.**



Ans:
$x_{AD} = 0.4905$ m
$x_{AC} = 0.793$ m
$x_{AB} = 0.467$ m

3–15.

The unstretched length of spring AB is 3 m. If the block is held in the equilibrium position shown, determine the mass of the block at D.



SOLUTION

$F = kx = 30(5 - 3) = 60 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad T \cos 45° - 60\left(\frac{4}{5}\right) = 0$

$T = 67.88 \text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad -W + 67.88 \sin 45° + 60\left(\frac{3}{5}\right) = 0$

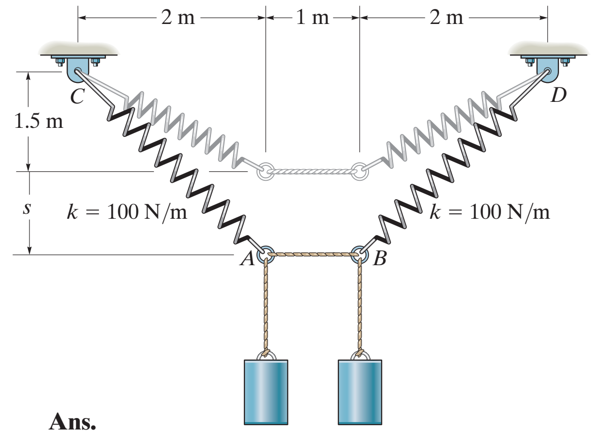
$W = 84 \text{ N}$

$m = \frac{84}{9.81} = 8.56 \text{ kg}$ **Ans.**

Ans:
$m = 8.56 \text{ kg}$

***3–16.**

Determine the mass of each of the two cylinders if they cause a sag of $s = 0.5$ m when suspended from the rings at A and B. Note that $s = 0$ when the cylinders are removed.

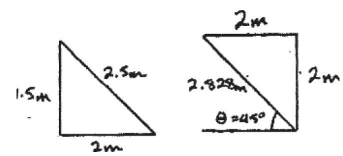


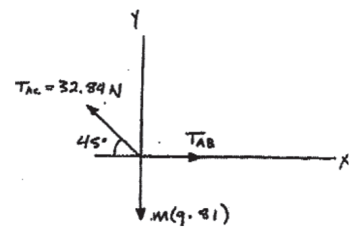
SOLUTION

$T_{AC} = 100 \text{ N/m } (2.828 - 2.5) = 32.84 \text{ N}$

$+\uparrow \Sigma F_y = 0; \qquad 32.84 \sin 45° - m(9.81) = 0$

$m = 2.37 \text{ kg}$ **Ans.**

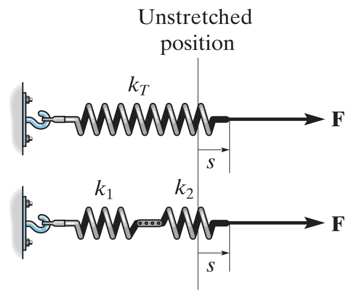




Ans:
$m = 2.37$ kg

3–17.

Determine the stiffness k_T of the single spring such that the force **F** will stretch it by the same amount s as the force **F** stretches the two springs. Express k_T in terms of stiffness k_1 and k_2 of the two springs.



SOLUTION

$$F = ks$$

$$s = s_1 + s_2$$

$$s = \frac{F}{k_T} = \frac{F}{k_1} + \frac{F}{k_2}$$

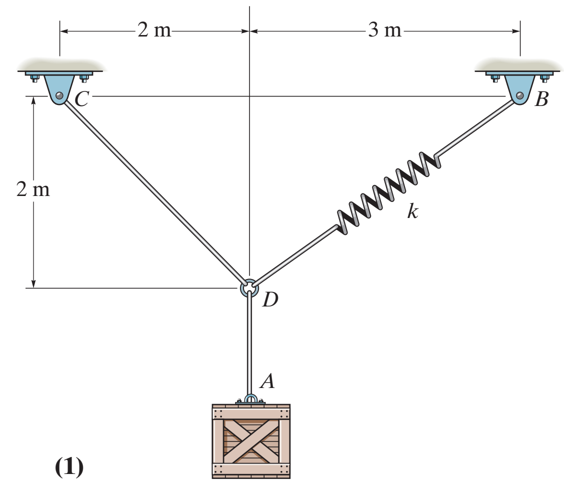
$$\frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2}$$ **Ans.**

Ans:

$$\frac{1}{k_T} = \frac{1}{k_1} + \frac{1}{k_2}$$

3–18.

If the spring DB has an unstretched length of 2 m, determine the stiffness of the spring to hold the 40-kg crate in the position shown.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* shown in Fig. *a*,

$$\xrightarrow{+}\Sigma F_x = 0; \qquad T_{BD}\left(\frac{3}{\sqrt{13}}\right) - T_{CD}\left(\frac{1}{\sqrt{2}}\right) = 0 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \qquad T_{BD}\left(\frac{2}{\sqrt{13}}\right) + T_{CD}\left(\frac{1}{\sqrt{2}}\right) - 40(9.81) = 0 \qquad \textbf{(2)}$$

Solving Eqs (1) and (2)

$$T_{BD} = 282.96 \text{ N} \qquad T_{CD} = 332.96 \text{ N}$$

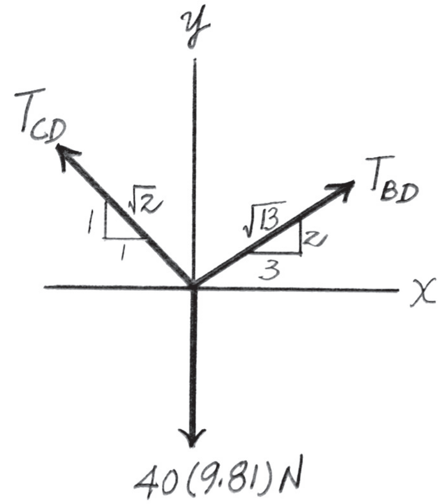
The stretched length of the spring is

$$l = \sqrt{3^2 + 2^2} = \sqrt{13} \text{ m}$$

Then, $x = l - l_0 = \left(\sqrt{13} - 2\right)$ m. Thus,

$$F_{sp} = kx; \qquad 282.96 = k\left(\sqrt{13} - 2\right)$$

$$k = 176.24 \text{ N/m} = 176 \text{ N/m} \qquad \textbf{Ans.}$$

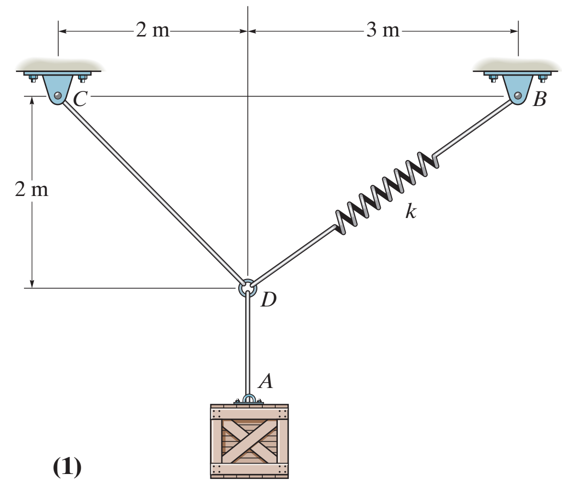


(a)

Ans:
$k = 176$ N/m

3–19.

Determine the unstretched length of DB to hold the 40-kg crate in the position shown. Take $k = 180\ \mathrm{N/m}$.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* shown in Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{BD}\left(\frac{3}{\sqrt{13}}\right) - T_{CD}\left(\frac{1}{\sqrt{2}}\right) = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad T_{BD}\left(\frac{2}{\sqrt{13}}\right) + T_{CD}\left(\frac{1}{\sqrt{2}}\right) - 40(9.81) = 0 \qquad \textbf{(2)}$$

Solving Eqs (1) and (2)

$$T_{BD} = 282.96\ \mathrm{N} \qquad T_{CD} = 332.96\ \mathrm{N}$$

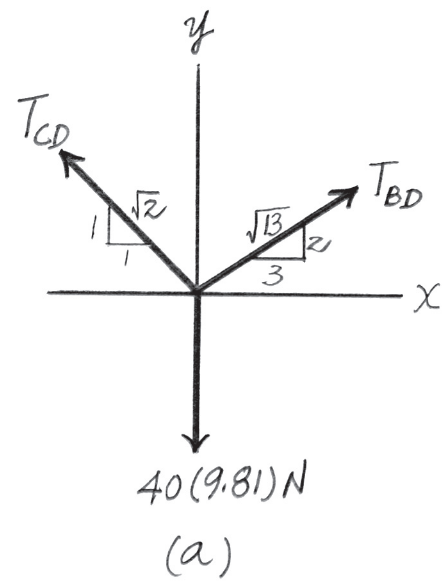
The stretched length of the spring is

$$l = \sqrt{3^2 + 2^2} = \sqrt{13}\ \mathrm{m}$$

Then, $x = l - l_0 = \sqrt{13} - l_0$. Thus

$$F_{sp} = kx; \qquad 282.96 = 180\left(\sqrt{13} - l_0\right)$$

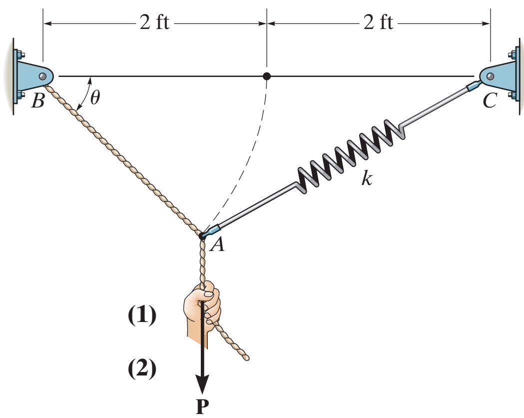
$$l_0 = 2.034\ \mathrm{m} = 2.03\ \mathrm{m} \qquad \textbf{Ans.}$$



Ans:
$l_0 = 2.03\ \mathrm{m}$

***3–20.**

A vertical force $P = 10$ lb is applied to the ends of the 2-ft cord AB and spring AC. If the spring has an unstretched length of 2 ft, determine the angle θ for equilibrium. Take $k = 15$ lb/ft.

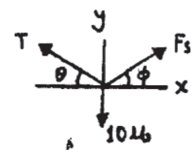


SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_s \cos\phi - T\cos\theta = 0 \qquad$ **(1)**

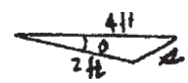
$+\uparrow \Sigma F_y = 0; \qquad T\sin\theta + F_s \sin\phi - 10 = 0 \qquad$ **(2)**

$$s = \sqrt{(4)^2 + (2)^2 - 2(4)(2)\cos\theta} - 2 = 2\sqrt{5 - 4\cos\theta} - 2$$



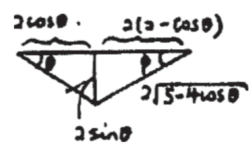
$$F_s = ks = 2k(\sqrt{5 - 4\cos\theta} - 1)$$

From Eq. (1): $T = F_s\left(\dfrac{\cos\phi}{\cos\theta}\right)$



$$T = 2k\left(\sqrt{5 - 4\cos\theta} - 1\right)\left(\frac{2 - \cos\theta}{\sqrt{5 - 4\cos\theta}}\right)\left(\frac{1}{\cos\theta}\right)$$

From Eq. (2):



$$\frac{2k\left(\sqrt{5-4\cos} - 1\right)(2-\cos\theta)}{\sqrt{5-4\cos\theta}}\tan\theta + \frac{2k\left(\sqrt{5-4\cos\theta} - 1\right)2\sin\theta}{2\sqrt{5-4\cos\theta}} = 10$$

$$\frac{\left(\sqrt{5-4\cos\theta} - 1\right)}{\sqrt{5-4\cos\theta}}(2\tan\theta - \sin\theta + \sin\theta) = \frac{10}{2k}$$

$$\frac{\tan\theta\left(\sqrt{5-4\cos\theta} - 1\right)}{\sqrt{5-4\cos\theta}} = \frac{10}{4k}$$

Set $k = 15$ lb/ft

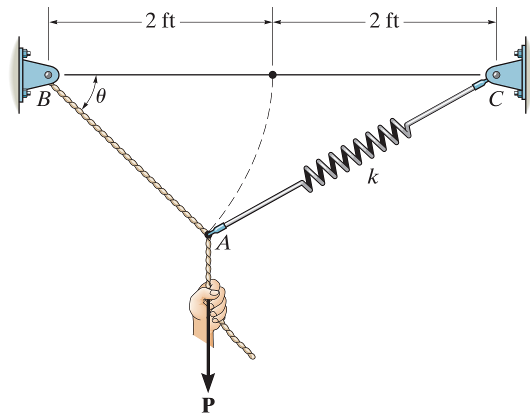
Solving for θ by trial and error,

$$\theta = 35.0^\circ \qquad \textbf{Ans.}$$

Ans:
$\theta = 35.0^\circ$

3–21.

Determine the unstretched length of spring AC if a force $P = 80$ lb causes the angle $\theta = 60°$ for equilibrium. Cord AB is 2 ft long. Take $k = 50$ lb/ft.



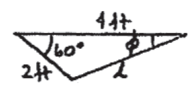
SOLUTION

$$l = \sqrt{4^2 + 2^2 - 2(2)(4)\cos 60°}$$

$$l = \sqrt{12}$$

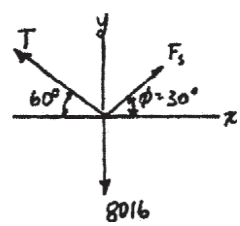
$$\frac{\sqrt{12}}{\sin 60°} = \frac{2}{\sin \phi}$$

$$\phi = \sin^{-1}\left(\frac{2\sin 60°}{\sqrt{12}}\right) = 30°$$



$+\uparrow \Sigma F_y = 0; \qquad T\sin 60° + F_s \sin 30° - 80 = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -T\cos 60° + F_s \cos 30° = 0$



Solving for F_s,

$F_s = 40$ lb

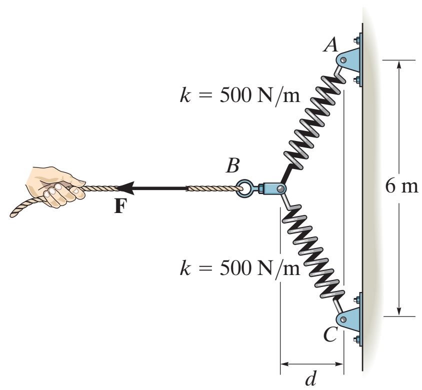
$F_s = kx$

$40 = 50(\sqrt{12} - l') \qquad l = \sqrt{12} - \frac{40}{50} = 2.66$ ft **Ans.**

Ans:
$l = 2.66$ ft

3–22.

The springs BA and BC each have a stiffness of 500 N/m and an unstretched length of 3 m. Determine the horizontal force **F** applied to the cord which is attached to the small ring B so that the displacement of the ring from the wall is $d = 1.5$ m.

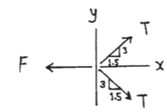


SOLUTION

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad \dfrac{1.5}{\sqrt{11.25}}(T)(2) - F = 0$

$T = ks = 500(\sqrt{3^2 + (1.5)^2} - 3) = 177.05 \text{ N}$

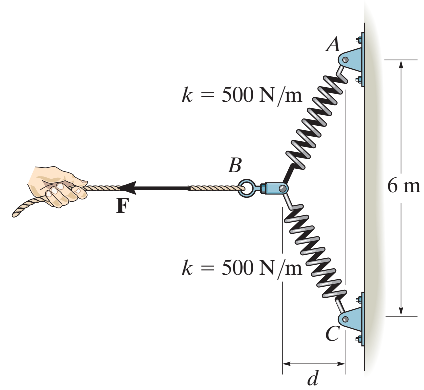
$F = 158 \text{ N}$ **Ans.**



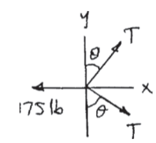
Ans:
$F = 158 \text{ N}$

3–23.

The springs BA and BC each have a stiffness of 500 N/m and an unstretched length of 3 m. Determine the displacement d of the cord from the wall when a force $F = 175$ N is applied to the cord.



SOLUTION



$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 175 = 2T \sin\theta$

$$T \sin\theta = 87.5$$

$$T\left[\frac{d}{\sqrt{3^2 + d^2}}\right] = 87.5$$

$$T = ks = 500(\sqrt{3^2 + d^2} - 3)$$

$$d\left(1 - \frac{3}{\sqrt{9 + d^2}}\right) = 0.175$$

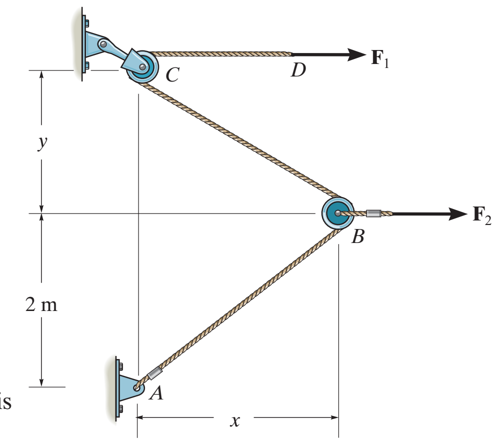
By trial and error:

$$d = 1.56 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$d = 1.56$ m

***3–24.**

Determine the distances x and y for equilibrium if $F_1 = 800$ N and $F_2 = 1000$ N.



SOLUTION

Equations of Equilibrium. The tension throughout rope $ABCD$ is constant, that is $F_1 = 800$ N. Referring to the FBD shown in Fig. a,

$$+\uparrow \Sigma F_y = 0; \qquad 800 \sin \phi - 800 \sin \theta = 0 \qquad \phi = 0$$

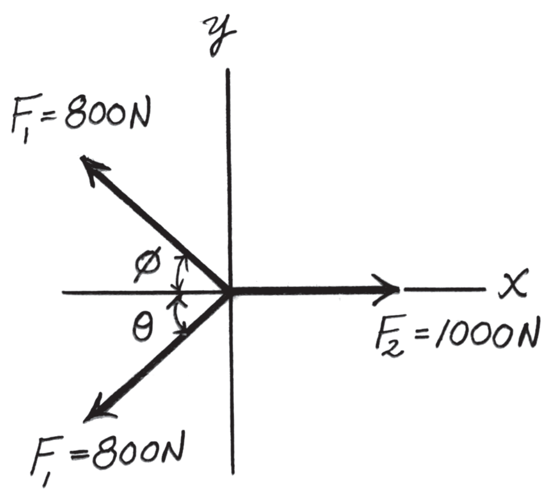
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 1000 - 2[800 \cos \theta] = 0 \qquad \theta = 51.32°$$

Referring to the geometry shown in Fig. b,

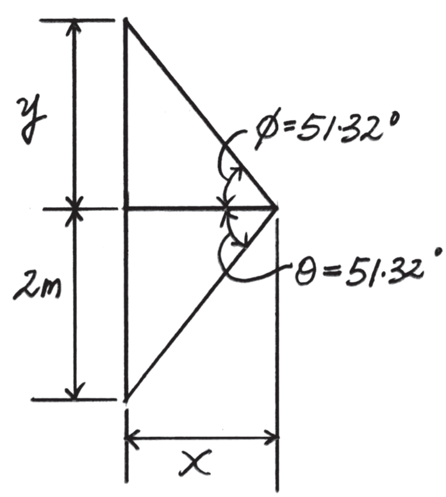
$$y = 2 \text{ m} \qquad \textbf{Ans.}$$

and

$$\frac{2}{x} = \tan 51.32°; \qquad x = 1.601 \text{ m} = 1.60 \text{ m} \qquad \textbf{Ans.}$$



(a)

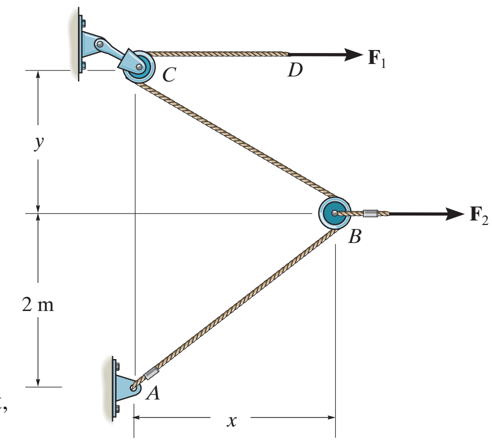


(b)

Ans:
$y = 2$ m
$x = 1.60$ m

3–25.

Determine the magnitude of F_1 and the distance y if $x = 1.5$ m and $F_2 = 1000$ N.



SOLUTION

Equations of Equilibrium. The tension throughout rope $ABCD$ is constant, that is $\mathbf{F}_1$. Referring to the *FBD* shown in Fig. *a*,

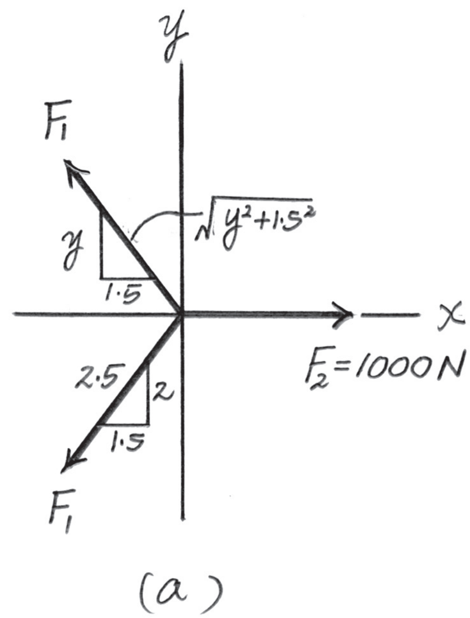
$$+\uparrow \Sigma F_y = 0; \qquad F_1\left(\frac{y}{\sqrt{y^2 + 1.5^2}}\right) - F_1\left(\frac{2}{2.5}\right) = 0$$

$$\frac{y}{\sqrt{y^2 + 1.5^2}} = \frac{2}{2.5}$$

$$y = 2 \text{ m} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 1000 - 2\left[F_1\left(\frac{1.5}{2.5}\right)\right] = 0$$

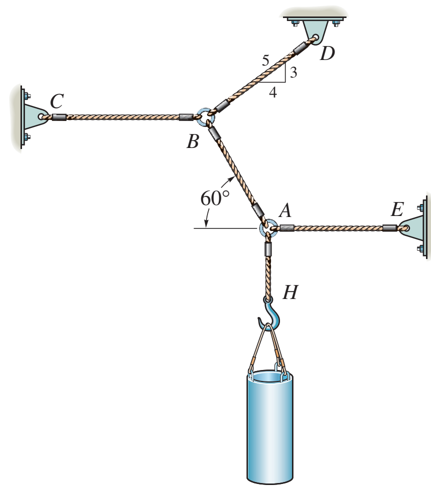
$$F_1 = 833.33 \text{ N} = 833 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$y = 2$ m
$F_1 = 833$ N

3–26.

The 30-kg pipe is supported at A by a system of five cords. Determine the force in each cord for equilibrium.



SOLUTION

At H:

$+\uparrow \Sigma F_y = 0; \qquad T_{HA} - 30(9.81) = 0$

$T_{HA} = 294 \text{ N}$ **Ans.**

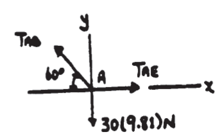
At A:

$+\uparrow \Sigma F_y = 0; \qquad T_{AB} \sin 60° - 30(9.81) = 0$

$T_{AB} = 339.83 = 340 \text{ N}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{AE} - 339.83 \cos 60° = 0$

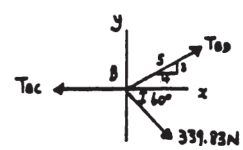
$T_{AE} = 170 \text{ N}$ **Ans.**



At B:

$+\uparrow \Sigma F_y = 0; \qquad T_{BD}\left(\frac{3}{5}\right) - 339.83 \sin 60° = 0$

$T_{BD} = 490.50 = 490 \text{ N}$ **Ans.**



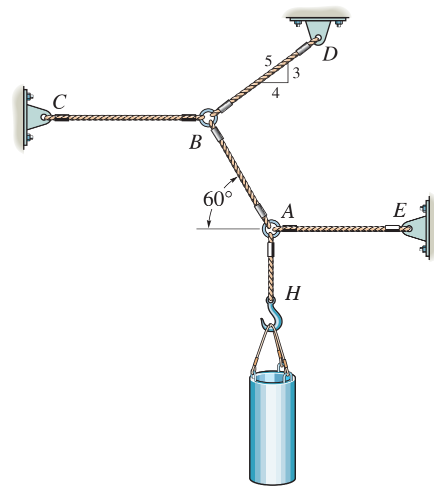
$\xrightarrow{+} \Sigma F_x = 0; \qquad 490.50\left(\frac{4}{5}\right) + 339.83 \cos 60° - T_{BC} = 0$

$T_{BC} = 562 \text{ N}$ **Ans.**

Ans:
$T_{HA} = 294 \text{ N}$
$T_{AB} = 340 \text{ N}$
$T_{AE} = 170 \text{ N}$
$T_{BD} = 490 \text{ N}$
$T_{BC} = 562 \text{ N}$

3–27.

Each cord can sustain a maximum tension of 500 N. Determine the largest mass of pipe that can be supported.

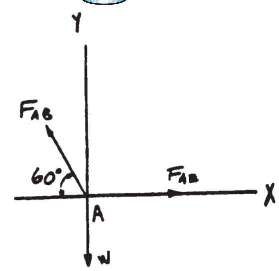


SOLUTION

At H:

$$+\uparrow \Sigma F_y = 0; \qquad F_{HA} = W$$

At A:



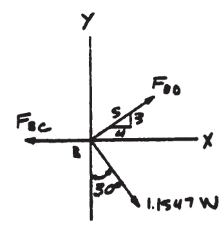
$$+\uparrow \Sigma F_y = 0; \qquad F_{AB} \sin 60^\circ - W = 0$$

$$F_{AB} = 1.1547\ W$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AE} - (1.1547\ W) \cos 60^\circ = 0$$

$$F_{AE} = 0.5774\ W$$

At B:



$$+\uparrow \Sigma F_y = 0; \qquad F_{BD}\left(\frac{3}{5}\right) - (1.1547 \cos 30^\circ)W = 0$$

$$F_{BD} = 1.667\ W$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_{BC} + 1.667\ W\left(\frac{4}{5}\right) + 1.1547 \sin 30^\circ = 0$$

$$F_{BC} = 1.9107\ W$$

By comparison, cord BC carries the largest load. Thus

$$500 = 1.9107\ W$$

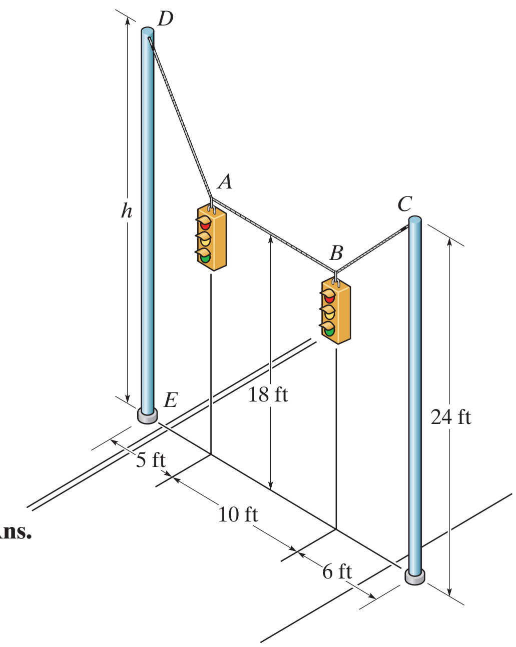
$$W = 261.69\ \text{N}$$

$$m = \frac{261.69}{9.81} = 26.7\ \text{kg} \qquad \textbf{Ans.}$$

Ans:
$m = 26.7$ kg

***3–28.**

The street-lights at A and B are suspended from the two poles as shown. If each light has a weight of 50 lb, determine the tension in each of the three supporting cables and the required height h of the pole DE so that cable AB is horizontal.



SOLUTION

At point B :

$+\uparrow \Sigma F_y = 0;$ $\quad \frac{1}{\sqrt{2}} F_{BC} - 50 = 0$

$F_{BC} = 70.71 = 70.7 \text{ lb}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad \frac{1}{\sqrt{2}} (70.71) - F_{AB} = 0$

$F_{AB} = 50 \text{ lb}$ **Ans.**

At point A :

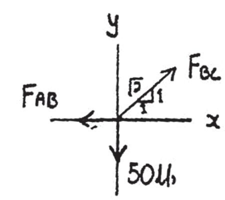
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 50 - F_{AD} \cos\theta = 0$ **Ans.**

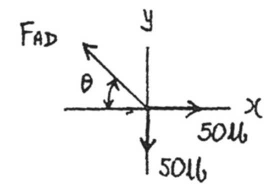
$+\uparrow \Sigma F_y = 0;$ $\quad F_{AD} \sin\theta - 50 = 0$

$\theta = 45^\circ$

$F_{AD} = 70.7 \text{ lb}$ **Ans.**

$h = 18 + 5 = 23 \text{ ft}$ **Ans.**

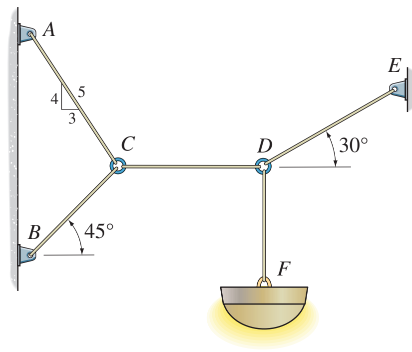




Ans:
$F_{AB} = 50 \text{ lb}$
$F_{AD} = 70.7 \text{ lb}$
$h = 23 \text{ ft}$

3–29.

Determine the tension developed in each cord required for equilibrium of the 20-kg lamp.

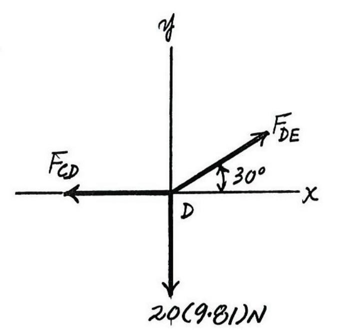


SOLUTION

Equations of Equilibrium: Applying the equations of equilibrium along the x and y axes to the free-body diagram of joint D shown in Fig. a, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{DE} \sin 30^\circ - 20(9.81) = 0 \quad F_{DE} = 392.4 \text{ N} = 392 \text{ N} \quad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad 392.4 \cos 30^\circ - F_{CD} = 0 \quad F_{CD} = 339.83 \text{ N} = 340 \text{ N} \quad \textbf{Ans.}$$



(a)

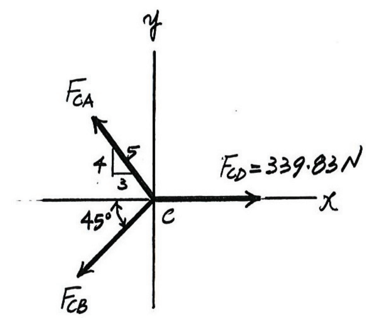
Using the result $F_{CD} = 339.83$ N and applying the equations of equilibrium along the x and y axes to the free-body diagram of joint D shown in Fig. b, we have

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 339.83 - F_{CA}\left(\frac{3}{5}\right) - F_{CD} \cos 45^\circ = 0 \quad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{CA}\left(\frac{4}{5}\right) - F_{CB} \sin 45^\circ = 0 \quad \textbf{(2)}$$

Solving Eqs. (1) and (2), yields

$F_{CB} = 275$ N $\qquad F_{CA} = 243$ N $\qquad$ **Ans.**

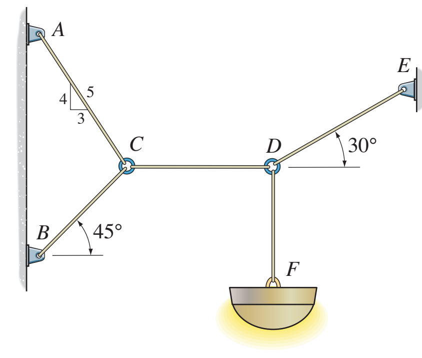


(b)

Ans:
$F_{DE} = 392$ N
$F_{CD} = 340$ N
$F_{CB} = 275$ N
$F_{CA} = 243$ N

3–30.

Determine the maximum mass of the lamp that the cord system can support so that no single cord develops a tension exceeding 400 N.

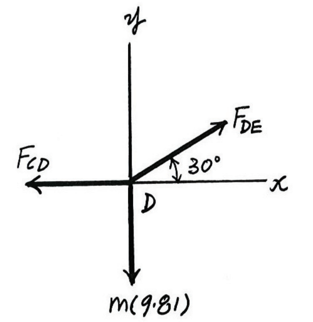


SOLUTION

Equations of Equilibrium: Applying the equations of equilibrium along the x and y axes to the free-body diagram of joint D shown in Fig. a, we have

$$+\uparrow \Sigma F_y = 0; \quad F_{DE} \sin 30° - m(9.81) = 0 \quad F_{DE} = 19.62m$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad 19.62m \cos 30° - F_{CD} = 0 \quad F_{CD} = 16.99m$$



(a)

Using the result $F_{CD} = 16.99m$ and applying the equations of equilibrium along the x and y axes to the free-body diagram of joint D shown in Fig. b, we have

$$\xrightarrow{+} \Sigma F_x = 0; \quad 16.99m - F_{CA}\left(\frac{3}{5}\right) - F_{CD} \cos 45° = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \quad F_{CA}\left(\frac{4}{5}\right) - F_{CB} \sin 45° = 0 \qquad \textbf{(2)}$$

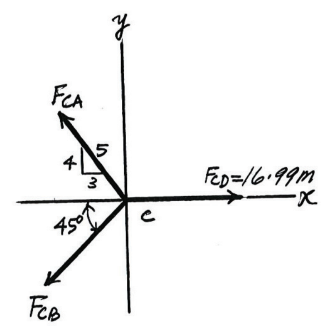
Solving Eqs. (1) and (2), yields

$F_{CB} = 13.73m$ $\qquad$ $F_{CA} = 12.14m$

Notice that cord DE is subjected to the greatest tensile force, and so it will achieve the maximum allowable tensile force first. Thus

$$F_{DE} = 400 = 19.62m$$

$$m = 20.4 \text{ kg} \qquad \textbf{Ans.}$$

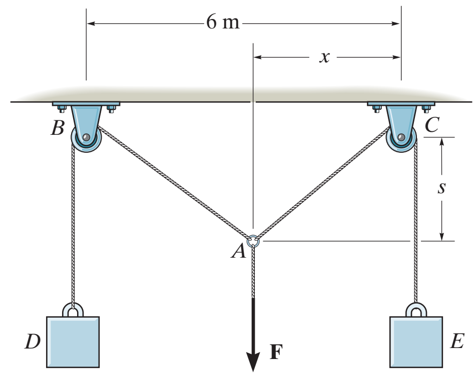


(b)

Ans:
$m = 20.4$ kg

3–31.

Blocks D and E have a mass of 4 kg and 6 kg, respectively. If $x = 2$ m determine the force **F** and the sag s for equilibrium.



SOLUTION

Equations of Equilibrium. Referring to the geometry shown in Fig. a,

$$\cos\phi = \frac{s}{\sqrt{s^2 + 2^2}} \qquad \sin\phi = \frac{2}{\sqrt{s^2 + 2^2}}$$

$$\cos\theta = \frac{s}{\sqrt{s^2 + 4^2}} \qquad \sin\theta = \frac{4}{\sqrt{s^2 + 4^2}}$$

Referring to the FBD shown in Fig. b,

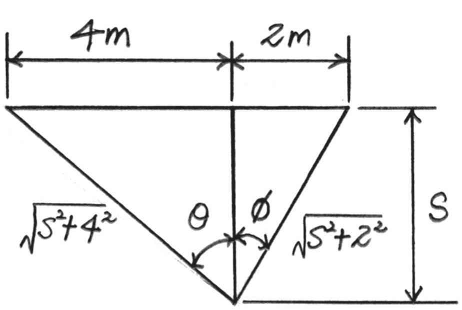
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 6(9.81)\left(\frac{2}{\sqrt{s^2 + 2^2}}\right) - 4(9.81)\left(\frac{4}{\sqrt{s^2 + 4^2}}\right) = 0$$

$$\frac{3}{\sqrt{s^2 + 2^2}} = \frac{4}{\sqrt{s^2 + 4^2}}$$

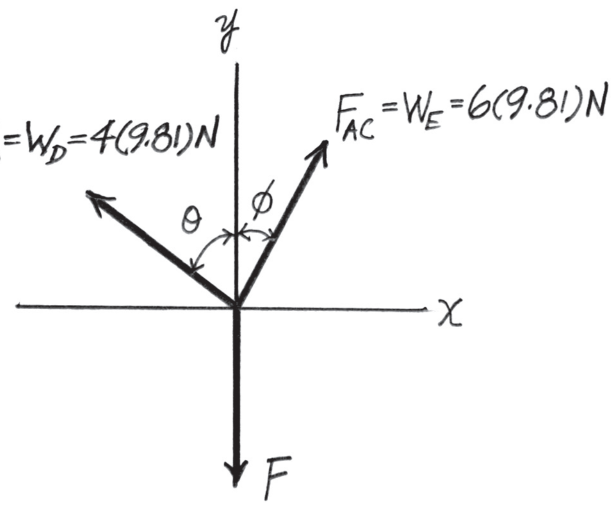
$$s = 3.381 \text{ m} = 3.38 \text{ m} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 6(9.81)\left(\frac{3.381}{\sqrt{3.381^2 + 2^2}}\right) + 4(9.81)\left(\frac{3.381}{\sqrt{3.381^2 + 4^2}}\right) - F = 0$$

$$F = 75.99 \text{ N} = 76.0 \text{ N} \qquad \textbf{Ans.}$$



(a)

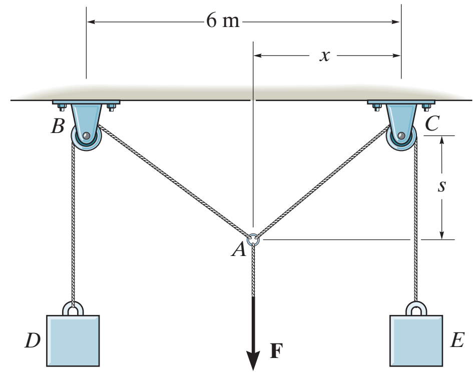


(b)

Ans:
$s = 3.38$ m
$F = 76.0$ N

***3–32.**

Blocks D and E have a mass of 4 kg and 6 kg, respectively. If $F = 80$ N, determine the sag s and distance x for equilibrium.



SOLUTION

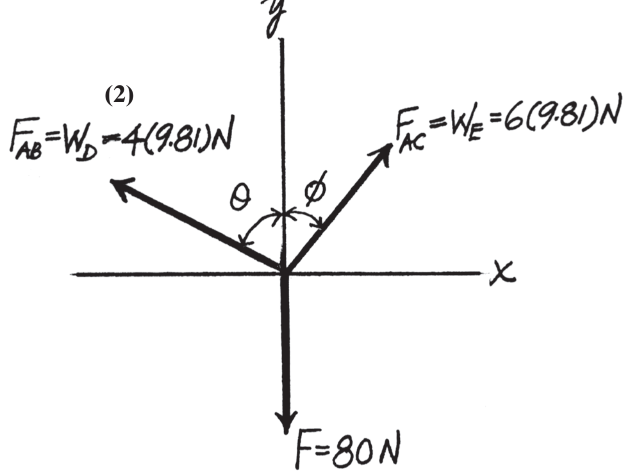
Equations of Equilibrium. Referring to the *FBD* shown in Fig. *a*,

$\xrightarrow{+} \Sigma F_x = 0; \qquad 6(9.81)\sin\phi - 4(9.81)\sin\theta = 0$

$$\sin\phi = \frac{2}{3}\sin\theta \qquad \textbf{(1)}$$

$+\uparrow \Sigma F_y = 0; \qquad 6(9.81)\cos\phi + 4(9.81)\cos\theta - 80 = 0$

$$3\cos\phi + 2\cos\theta = 4.0775 \qquad \textbf{(2)}$$



(a)

Using Eq (1), the geometry shown in Fig. *b* can be constructed. Thus

$$\cos\phi = \frac{\sqrt{9 - 4\sin^2\theta}}{3}$$

Substitute this result into Eq. (2),

$$3\left(\frac{\sqrt{9 - 4\sin^2\theta}}{3}\right) + 2\cos\theta = 4.0775$$

$$\sqrt{9 - 4\sin^2\theta} = 4.0775 - 2\cos\theta$$

$$9 - 4\sin^2\theta = 4\cos^2\theta - 16.310\cos\theta + 16.6258$$

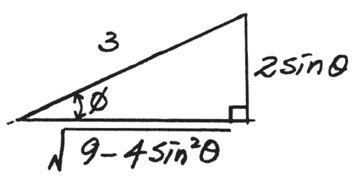
$$16.310\cos\theta = 4\left(\cos^2\theta + \sin^2\theta\right) + 7.6258$$

Here, $\cos^2\theta + \sin^2\theta = 1$. Then

$$\cos\theta = 0.7128 \qquad \theta = 44.54°$$

Substitute this result into Eq (1)

$$\sin\phi = \frac{2}{3}\sin 44.54° \qquad \phi = 27.88°$$



(b)

From Fig. *c*, $\dfrac{6-x}{s} = \tan 44.54°$ and $\dfrac{x}{s} = \tan 27.88°$.

So then,

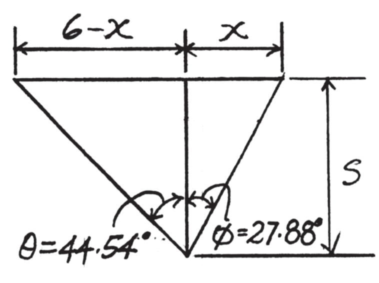
$$\frac{6-x}{s} + \frac{x}{s} = \tan 44.54° + \tan 27.88°$$

$$\frac{6}{s} = 1.5129$$

$$s = 3.9659 \text{ m} = 3.97 \text{ m} \qquad \textbf{Ans.}$$

$$x = 3.9659 \tan 27.88°$$

$$= 2.0978 \text{ m} = 2.10 \text{ m} \qquad \textbf{Ans.}$$

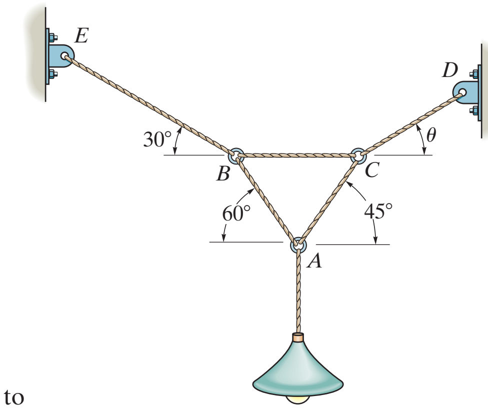


(c)

Ans:
$s = 3.97$ m
$x = 2.10$ m

3–33.

The lamp has a weight of 15 lb and is supported by the six cords connected together as shown. Determine the tension in each cord and the angle θ for equilibrium. Cord BC is horizontal.



SOLUTION

Equations of Equilibrium. Considering the equilibrium of Joint A by referring to its *FBD* shown in Fig. a,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad T_{AC}\cos 45^\circ - T_{AB}\cos 60^\circ = 0 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \qquad T_{AC}\sin 45^\circ + T_{AB}\sin 60^\circ - 15 = 0 \qquad \textbf{(2)}$$

Solving Eqs (1) and (2) yield

$$T_{AB} = 10.98 = 11.0\text{ lb} \qquad T_{AC} = 7.764\text{ lb} = 7.76\text{ lb} \qquad \textbf{Ans.}$$

Then, joint B by referring to its *FBD* shown in Fig. b

$$+\uparrow\Sigma F_y = 0; \quad T_{BE}\sin 30^\circ - 10.98\sin 60^\circ = 0 \quad T_{BE} = 19.02\text{ lb} = 19.0\text{ lb} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad T_{BC} + 10.98\cos 60^\circ - 19.02\cos 30^\circ = 0$$

$$T_{BC} = 10.98\text{ lb} = 11.0\text{ lb} \qquad \textbf{Ans.}$$

Finally joint C by referring to its *FBD* shown in Fig. c

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad T_{CD}\cos\theta - 10.98 - 7.764\cos 45^\circ = 0$$

$$T_{CD}\cos\theta = 16.4711 \qquad \textbf{(3)}$$

$$+\uparrow\Sigma F_y = 0; \qquad T_{CD}\sin\theta - 7.764\sin 45^\circ = 0$$

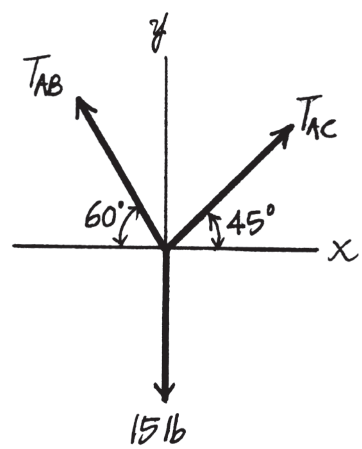
$$T_{CD}\sin\theta = 5.4904 \qquad \textbf{(4)}$$

Divided Eq (4) by (3)

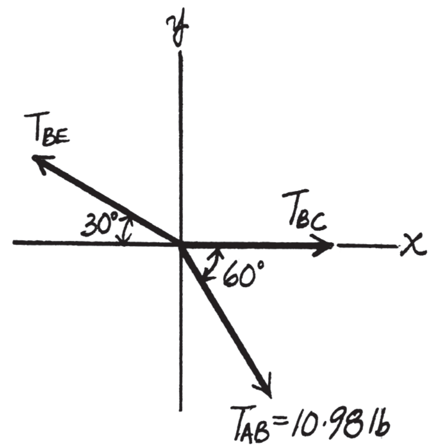
$$\tan\theta = 0.3333 \qquad \theta = 18.43^\circ = 18.4^\circ \qquad \textbf{Ans.}$$

Substitute this result into Eq (3)

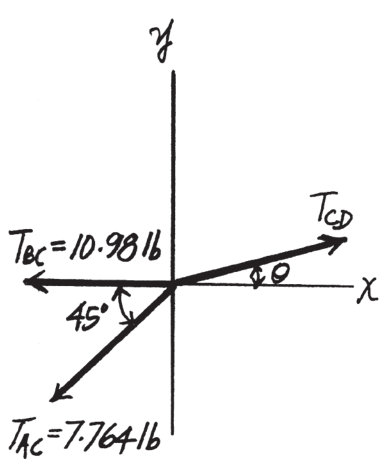
$$T_{CD}\cos 18.43^\circ = 16.4711 \qquad T_{CD} = 17.36\text{ lb} = 17.4\text{ lb} \qquad \textbf{Ans.}$$



(a)



(b)

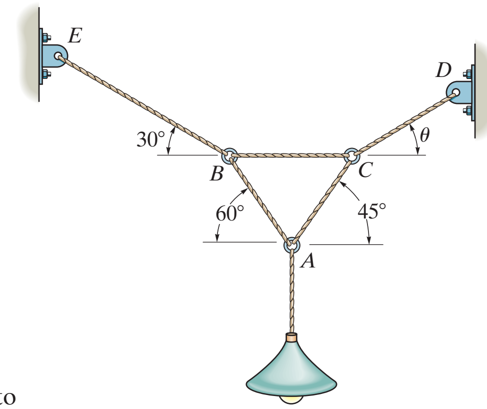


(c)

Ans:
$T_{AB} = 11.0$ lb
$T_{AC} = 7.76$ lb
$T_{BC} = 11.0$ lb
$T_{BE} = 19.0$ lb
$T_{CD} = 17.4$ lb
$\theta = 18.4^\circ$

3–34.

Each cord can sustain a maximum tension of 20 lb. Determine the largest weight of the lamp that can be supported. Also, determine θ of cord DC for equilibrium.



SOLUTION

Equations of Equilibrium. Considering the equilibrium of Joint A by referring to its FBD shown in Fig. a,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad T_{AC}\cos 45° - T_{AB}\cos 60° = 0 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \qquad T_{AC}\sin 45° - T_{AB}\sin 60° - W = 0 \qquad \textbf{(2)}$$

Solving Eqs (1) and (2) yield

$$T_{AB} = 0.7321\ W \qquad T_{AC} = 0.5176\ W$$

Then, joint B by referring to its FBD shown in Fig. b,

$$+\uparrow\Sigma F_y = 0; \quad T_{BE}\sin 30° - 0.7321W\sin 60° = 0 \quad T_{BE} = 1.2679\ W$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad T_{BC} + 0.7321\ W\cos 60° - 1.2679\ W\cos 30° = 0$$

$$T_{BC} = 0.7321\ W$$

Finally, joint C by referring to its FBD shown in Fig. c,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad T_{CD}\cos\theta - 0.7321\ W - 0.5176\ W\cos 45° = 0$$

$$T_{CD}\cos\theta = 1.0981\ W \qquad \textbf{(3)}$$

$$+\uparrow\Sigma F_y = 0; \qquad T_{CD}\sin\theta - 0.5176\ W\sin 45° = 0$$

$$T_{CD}\sin\theta = 0.3660\ W \qquad \textbf{(4)}$$

Divided Eq (4) by (3)

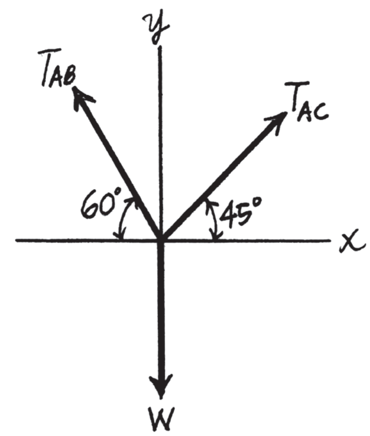
$$\tan\theta = 0.3333 \qquad \theta = 18.43° = 18.4° \qquad \textbf{Ans.}$$

Substitute this result into Eq (3),

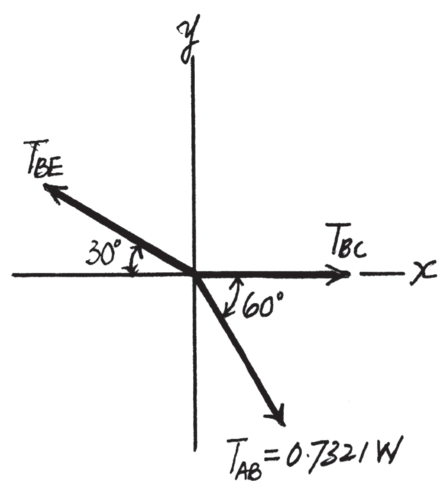
$$T_{CD}\cos 18.43° = 1.0981\ W \qquad T_{CD} = 1.1575\ W$$

Here cord BE is subjected to the largest tension. Therefore, its tension will reach the limit first, that is $T_{BE} = 20$ lb. Then

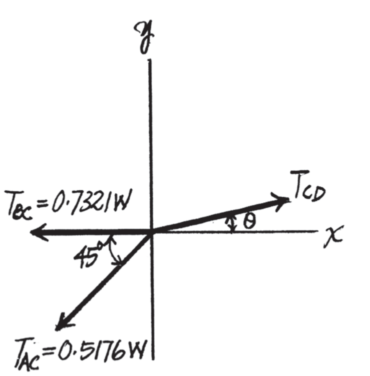
$$20 = 1.2679\ W; \qquad W = 15.77 \text{ lb} = 15.8 \text{ lb} \qquad \textbf{Ans.}$$



(a)



(b)

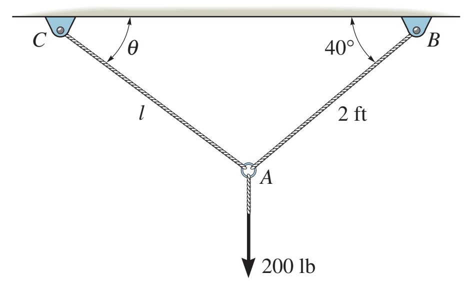


(c)

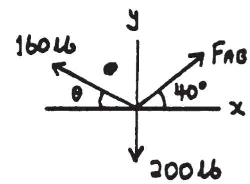
Ans:
$\theta = 18.4°$
$W = 15.8$ lb

3–35.

The ring of negligible size is subjected to a vertical force of 200 lb. Determine the required length l of cord AC such that the tension acting in AC is 160 lb. Also, what is the force in cord AB? *Hint:* Use the equilibrium condition to determine the required angle θ for attachment, then determine l using trigonometry applied to triangle ABC.



SOLUTION



$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad F_{AB}\cos 40^\circ - 160\cos\theta = 0$

$+\uparrow\Sigma F_y = 0;\qquad 160\sin\theta + F_{AB}\sin 40^\circ - 200 = 0$

Thus,

$$\sin\theta + 0.8391\cos\theta = 1.25$$

Solving by trial and error,

$$\theta = 33.25^\circ$$

$$F_{AB} = 175 \text{ lb} \qquad \textbf{Ans.}$$

$$\frac{2}{\sin 33.25^\circ} = \frac{l}{\sin 40^\circ}$$

$$l = 2.34 \text{ ft} \qquad \textbf{Ans.}$$

Also,

$$\theta = 66.75^\circ$$

$$F_{AB} = 82.4 \text{ lb} \qquad \textbf{Ans.}$$

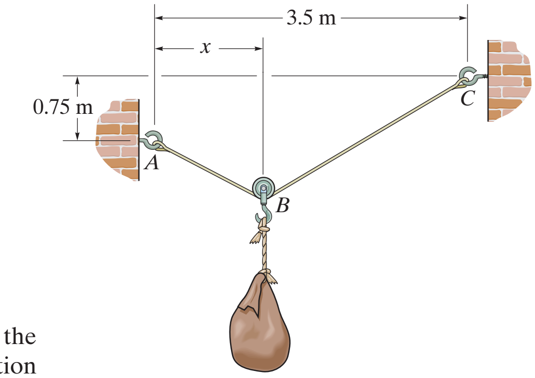
$$\frac{2}{\sin 66.75^\circ} = \frac{l}{\sin 40^\circ}$$

$$l = 1.40 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$F_{AB} = 175$ lb
$l = 2.34$ ft
$F_{AB} = 82.4$ lb
$l = 1.40$ ft

***3–36.**

Cable ABC has a length of 5 m. Determine the position x and the tension developed in ABC required for equilibrium of the 100-kg sack. Neglect the size of the pulley at B.



SOLUTION

Equations of Equilibrium: Since cable ABC passes over the smooth pulley at B, the tension in the cable is constant throughout its entire length. Applying the equation of equilibrium along the y axis to the free-body diagram in Fig. a, we have

$$+\uparrow \Sigma F_y = 0; \qquad 2T \sin\phi - 100(9.81) = 0 \qquad (1)$$

Geometry: Referring to Fig. b, we can write

$$\frac{3.5 - x}{\cos\phi} + \frac{x}{\cos\phi} = 5$$

$$\phi = \cos^{-1}\left(\frac{3.5}{5}\right) = 45.57°$$

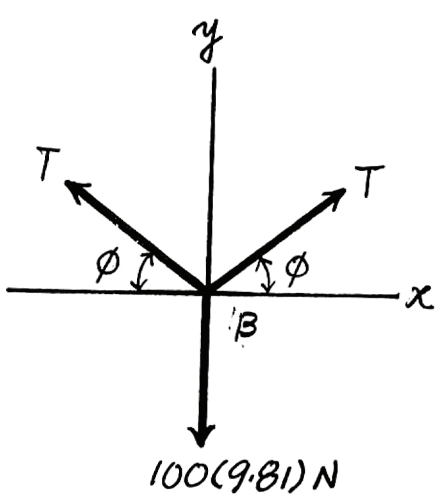
Also,

$$x \tan 45.57° + 0.75 = (3.5 - x)\tan 45.57°$$

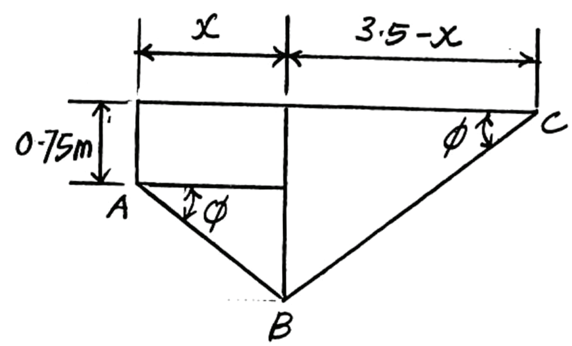
$$x = 1.38 \text{ m} \qquad \textbf{Ans.}$$

Substituting $\phi = 45.57°$ into Eq. (1), yields

$$T = 687 \text{ N} \qquad \textbf{Ans.}$$



(a)

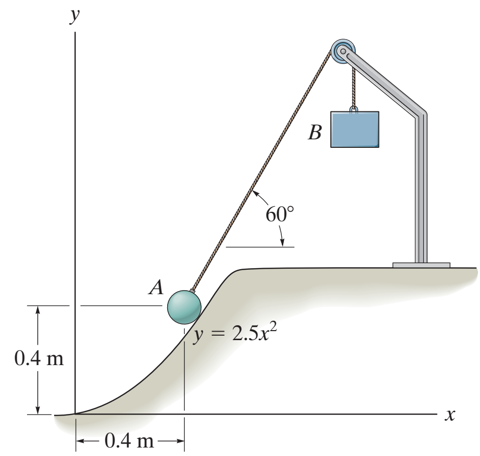


(b)

Ans:
$x = 1.38$ m
$T = 687$ N

3–37.

A 4-kg sphere rests on the smooth parabolic surface. Determine the normal force it exerts on the surface and the mass m_B of block B needed to hold it in the equilibrium position shown.

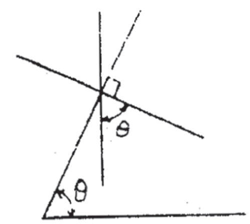


SOLUTION

Geometry: The angle θ which the surface make with the horizontal is to be determined first.

$$\tan\theta\Big|_{x=0.4\text{ m}} = \frac{dy}{dx}\Big|_{x=0.4\text{m}} = 5.0x\Big|_{x=0.4\text{ m}} = 2.00$$

$$\theta = 63.43^\circ$$



Free Body Diagram: The tension in the cord is the same throughout the cord and is equal to the weight of block B, $W_B = m_B\,(9.81)$.

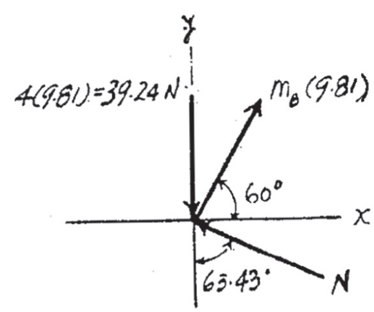
Equations of Equilibrium:

$\xrightarrow{+} \Sigma F_x = 0;\qquad m_B\,(9.81)\cos 60^\circ - N\sin 63.43^\circ = 0$

$$N = 5.4840 m_B \qquad \textbf{[1]}$$

$+\uparrow \Sigma F_y = 0;\qquad m_B\,(9.81)\sin 60^\circ + N\cos 63.43^\circ - 39.24 = 0$

$$8.4957 m_B + 0.4472 N = 39.24 \qquad \textbf{[2]}$$



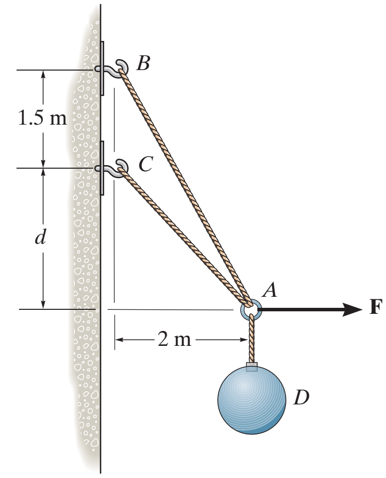
Solving Eqs. [1] and [2] yields

$$m_B = 3.58\text{ kg} \qquad N = 19.7\text{ N} \qquad \textbf{Ans.}$$

Ans:
$m_B = 3.58$ kg
$N = 19.7$ N

3–38.

Determine the forces in cables AC and AB needed to hold the 20-kg ball D in equilibrium. Take $F = 300$ N and $d = 1$ m.



SOLUTION

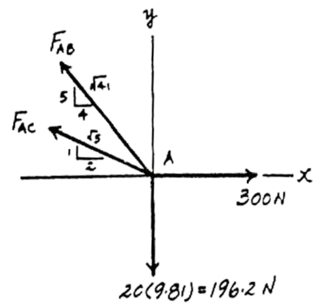
Equations of Equilibrium:

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 300 - F_{AB}\left(\frac{4}{\sqrt{41}}\right) - F_{AC}\left(\frac{2}{\sqrt{5}}\right) = 0$$

$$06247F_{AB} + 0.8944F_{AC} = 300 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\left(\frac{5}{\sqrt{41}}\right) + F_{AC}\left(\frac{1}{\sqrt{5}}\right) - 196.2 = 0$$

$$0.7809F_{AB} + 0.4472F_{AC} = 196.2 \qquad \textbf{(2)}$$



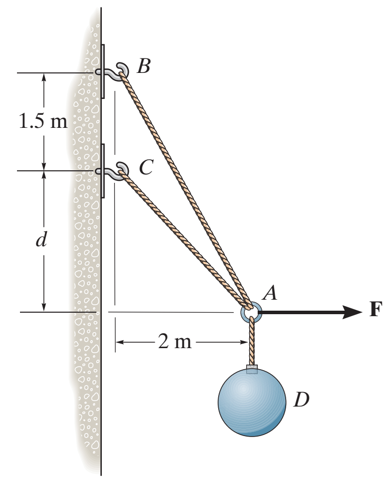
Solving Eqs. (1) and (2) yields

$F_{AB} = 98.6$ N $\qquad F_{AC} = 267$ N **Ans.**

Ans:
$F_{AB} = 98.6$ N
$F_{AC} = 267$ N

3–39.

The ball D has a mass of 20 kg. If a force of $F = 100$ N is applied horizontally to the ring at A, determine the largest dimension d so that the force in cable AC is zero.



SOLUTION

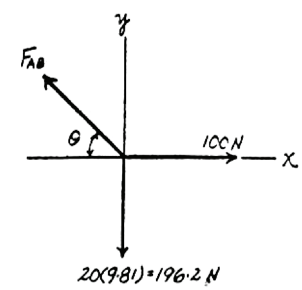
Equations of Equilibrium:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 100 - F_{AB} \cos\theta = 0 \qquad F_{AB} \cos\theta = 100 \qquad$ **(1)**

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} \sin\theta - 196.2 = 0 \qquad F_{AB} \sin\theta = 196.2 \qquad$ **(2)**

Solving Eqs. (1) and (2) yields

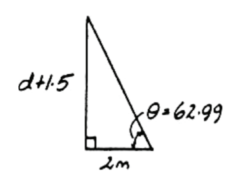
$$\theta = 62.99^\circ \qquad F_{AB} = 220.21 \text{ N}$$



From the geometry,

$$d + 1.5 = 2 \tan 62.99^\circ$$

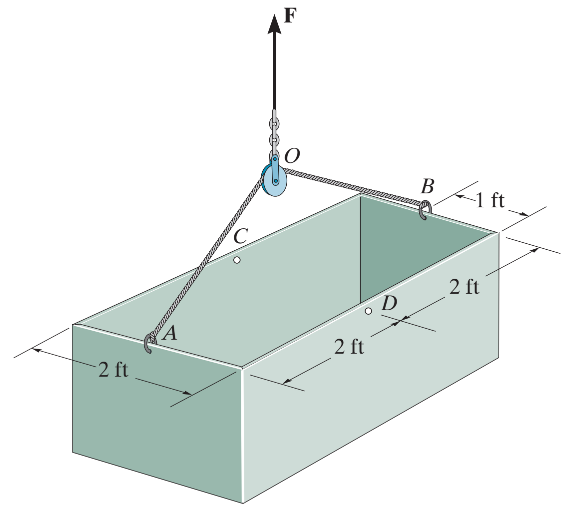
$$d = 2.42 \text{ m}$$ **Ans.**



Ans:
$d = 2.42$ m

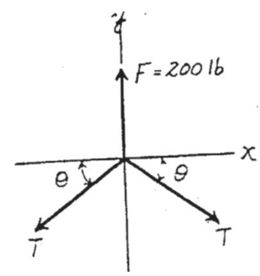
***3–40.**

The 200-lb uniform tank is suspended by means of a 6-ft-long cable, which is attached to the sides of the tank and passes over the small pulley located at O. If the cable can be attached at either points A and B, or C and D, determine which attachment produces the least amount of tension in the cable. What is this tension?



SOLUTION

Free-Body Diagram: By observation, the force **F** has to support the entire weight of the tank. Thus, $F = 200$ lb. The tension in cable AOB or COD is the same throughout the cable.

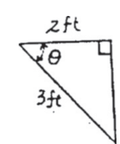


Equations of Equilibrium:

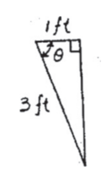
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad T\cos\theta - T\cos\theta = 0 \qquad \textbf{(}\textit{\textbf{Satisfied!}}\textbf{)}$$

$$+\uparrow \Sigma F_y = 0; \qquad 200 - 2T\sin\theta = 0 \qquad T = \frac{100}{\sin\theta} \qquad \textbf{(1)}$$

From the function obtained above, one realizes that in order to produce the least amount of tension in the cable, $\sin\theta$ hence θ must be as great as possible. Since the attachment of the cable to point C and D produces a greater θ $\left(\theta = \cos^{-1}\frac{1}{3} = 70.53°\right)$ as compared to the attachment of the cable to points A and B $\left(\theta = \cos^{-1}\frac{2}{3} = 48.19°\right)$,



the attachment of the cable to point C and D will produce the least amount of tension in the cable. **Ans.**



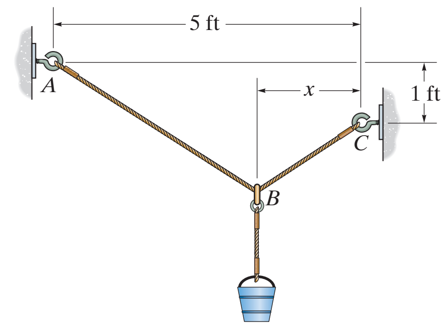
Thus,

$$T = \frac{100}{\sin 70.53°} = 106 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$T = 106$ lb

3–41.

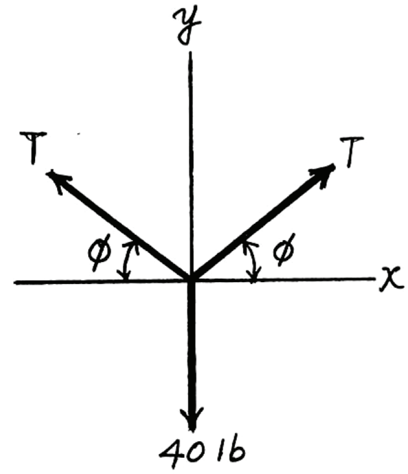
The single elastic cord ABC is used to support the 40-lb load. Determine the position x and the tension in the cord that is required for equilibrium. The cord passes through the smooth ring at B and has an unstretched length of 6 ft and stiffness of $k = 50$ lb/ft.



SOLUTION

Equations of Equilibrium: Since elastic cord ABC passes over the smooth ring at B, the tension in the cord is constant throughout its entire length. Applying the equation of equilibrium along the y axis to the free-body diagram in Fig. a, we have

$$+\uparrow \Sigma F_y = 0; \qquad 2T \sin \phi - 40 = 0 \qquad \textbf{(1)}$$



(a)

Geometry: Referring to Fig. (b), the stretched length of cord ABC is

$$l_{ABC} = \frac{x}{\cos \phi} + \frac{5 - x}{\cos \phi} = \frac{5}{\cos \phi} \qquad \textbf{(2)}$$

Also,

$$x \tan \phi + 1 = (5 - x) \tan \phi$$

$$x = \frac{5 \tan \phi - 1}{2 \tan \phi} \qquad \textbf{(3)}$$

Spring Force Formula: Applying the spring force formula using Eq. (2), we obtain

$$F_{sp} = k(l_{ABC} - l_0)$$

$$T = 50\left[\frac{5}{\cos \phi} - 6\right] \qquad \textbf{(4)}$$

Substituting Eq. (4) into Eq. (1) yields

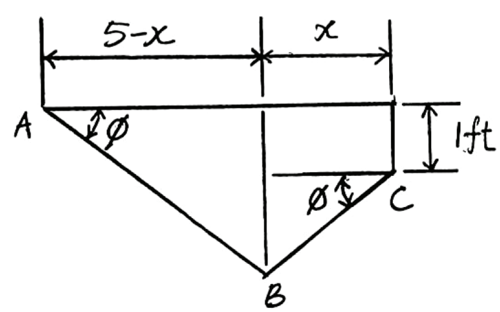
$$5 \tan \phi - 6 \sin \phi = 0.4$$

Solving the above equation by trial and error

$$\phi = 40.86°$$

Substituting $\phi = 40.86°$ into Eqs. (1) and (3) yields

$$T = 30.6 \text{ lb} \qquad x = 1.92 \text{ ft} \qquad \textbf{Ans.}$$

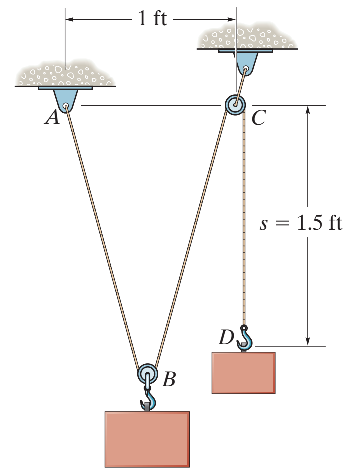


(b)

Ans:
$T = 30.6$ lb
$x = 1.92$ ft

3–42.

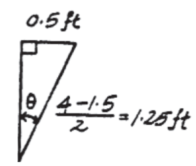
A "scale" is constructed with a 4-ft-long cord and the 10-lb block D. The cord is fixed to a pin at A and passes over two *small* pulleys at B and C. Determine the weight of the suspended block at B if the system is in equilibrium when $s = 1.5$ ft.



SOLUTION

Free-Body Diagram: The tension force in the cord is the same throughout the cord, that is, 10 lb. From the geometry,

$$\theta = \sin^{-1}\left(\frac{0.5}{1.25}\right) = 23.58^\circ$$

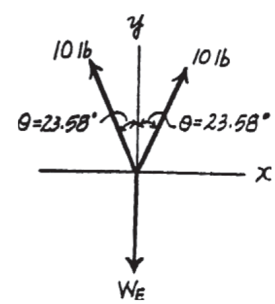


Equations of Equilibrium:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 10 \sin 23.58^\circ - 10 \sin 23.58^\circ = 0 \qquad \textbf{(Satisfied!)}$$

$$+\uparrow \Sigma F_y = 0; \qquad 2(10) \cos 23.58^\circ - W_B = 0$$

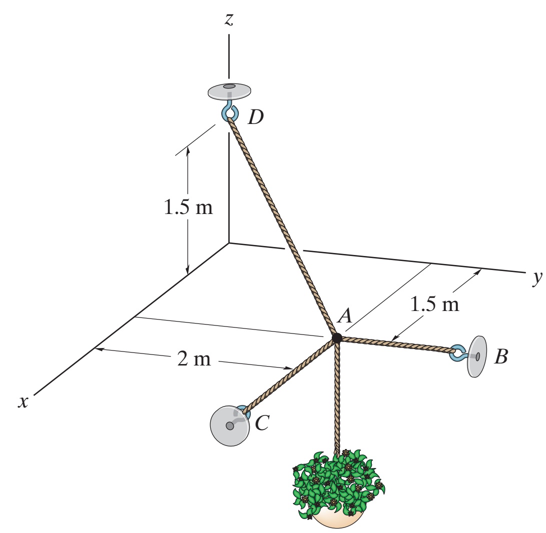
$$W_B = 18.3 \text{ lb} \qquad \textbf{Ans.}$$



Ans:
$W_B = 18.3$ lb

3–43.

The three cables are used to support the 40-kg flowerpot. Determine the force developed in each cable for equilibrium.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* shown in Fig. *a*,

$$\Sigma F_z = 0; \quad F_{AD}\left(\frac{1.5}{\sqrt{1.5^2 + 2^2 + 1.5^2}}\right) - 40(9.81) = 0$$

$$F_{AD} = 762.69 \text{ N} = 763 \text{ N} \qquad \textbf{Ans.}$$

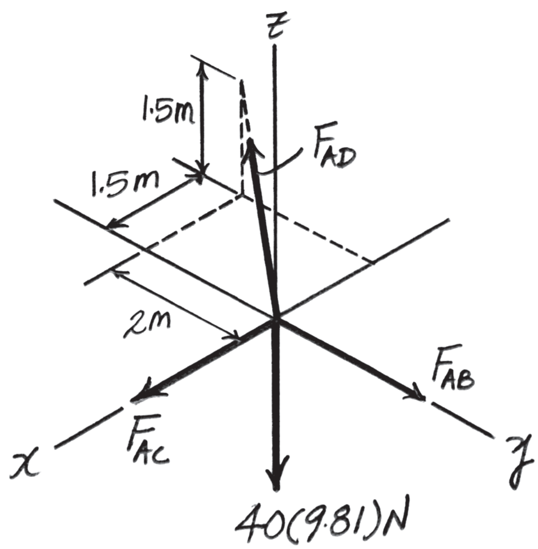
Using this result,

$$\Sigma F_x = 0; \quad F_{AC} - 762.69\left(\frac{1.5}{\sqrt{1.5^2 + 2^2 + 1.5^2}}\right) = 0$$

$$F_{AC} = 392.4 \text{ N} = 392 \text{ N} \qquad \textbf{Ans.}$$

$$\Sigma F_y = 0; \quad F_{AB} - 762.69\left(\frac{2}{\sqrt{1.5^2 + 2^2 + 1.5^2}}\right) = 0$$

$$F_{AB} = 523.2 \text{ N} = 523 \text{ N} \qquad \textbf{Ans.}$$

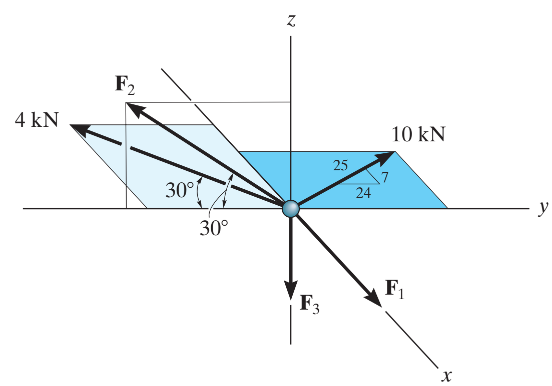


(a)

Ans:
$F_{AD} = 763$ N
$F_{AC} = 392$ N
$F_{AB} = 523$ N

***3–44.**

Determine the magnitudes of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ for equilibrium of the particle.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* shown,

$$\Sigma F_y = 0;\quad 10\left(\frac{24}{25}\right) - 4\cos 30° - F_2\cos 30° = 0 \quad F_2 = 7.085\text{ kN} = 7.09\text{ kN} \quad \textbf{Ans.}$$

$$\Sigma F_x = 0;\quad F_1 - 4\sin 30° - 10\left(\frac{7}{25}\right) = 0 \qquad F_1 = 4.80\text{ kN} \quad \textbf{Ans.}$$

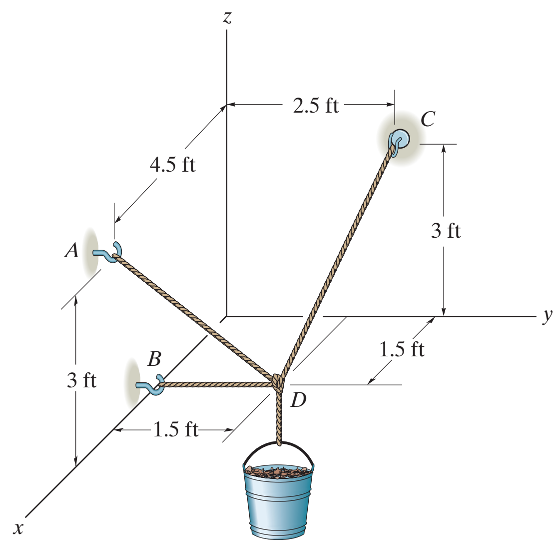
Using the result of $F_2 = 7.085$ kN,

$$\Sigma F_z = 0;\quad 7.085\sin 30° - F_3 = 0 \qquad F_3 = 3.543\text{ kN} = 3.54\text{ kN} \quad \textbf{Ans.}$$

Ans:
$F_2 = 7.09$ kN
$F_1 = 4.80$ kN
$F_3 = 3.54$ kN

3–45.

If the bucket and its contents have a total weight of 20 lb, determine the force in the supporting cables *DA, DB*, and *DC*.



SOLUTION

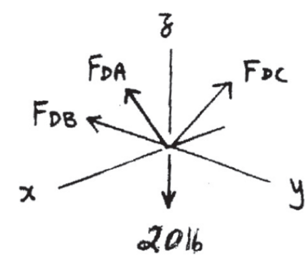
$$\mathbf{u}_{DA} = \{\frac{3}{4.5}\mathbf{i} - \frac{1.5}{4.5}\mathbf{j} + \frac{3}{4.5}\mathbf{k}\}$$

$$\mathbf{u}_{DC} = \{-\frac{1.5}{3.5}\mathbf{i} + \frac{1}{3.5}\mathbf{j} + \frac{3}{3.5}\mathbf{k}\}$$

$$\Sigma F_x = 0; \qquad \frac{3}{4.5}F_{DA} - \frac{1.5}{3.5}F_{DC} = 0$$

$$\Sigma F_y = 0; \qquad -\frac{1.5}{4.5}F_{DA} - F_{DB} + \frac{1}{3.5}F_{DC} = 0$$

$$\Sigma F_z = 0; \qquad \frac{3}{4.5}F_{DA} + \frac{3}{3.5}F_{DC} - 20 = 0$$



$F_{DA} = 10.0$ lb **Ans.**

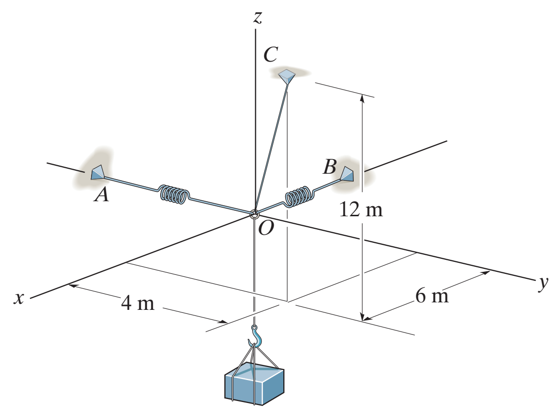
$F_{DB} = 1.11$ lb **Ans.**

$F_{DC} = 15.6$ lb **Ans.**

Ans:
$F_{DA} = 10.0$ lb
$F_{DB} = 1.11$ lb
$F_{DC} = 15.6$ lb

3–46.

Determine the stretch in each of the two springs required to hold the 20-kg crate in the equilibrium position shown. Each spring has an unstretched length of 2 m and a stiffness of $k = 360$ N/m.



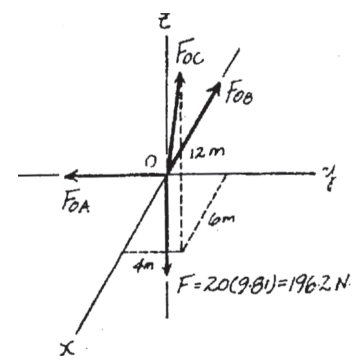
SOLUTION

Cartesian Vector Notation:

$$\mathbf{F}_{OC} = F_{OC}\left(\frac{6\mathbf{i} + 4\mathbf{j} + 12\mathbf{k}}{\sqrt{6^2 + 4^2 + 12^2}}\right) = \frac{3}{7}F_{OC}\mathbf{i} + \frac{2}{7}F_{OC}\mathbf{j} + \frac{6}{7}F_{OC}\mathbf{k}$$

$$\mathbf{F}_{OA} = -F_{OA}\mathbf{j} \qquad \mathbf{F}_{OB} = -F_{OB}\mathbf{i}$$

$$\mathbf{F} = \{-196.2\mathbf{k}\}\text{ N}$$



Equations of Equilibrium:

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{OC} + \mathbf{F}_{OA} + \mathbf{F}_{OB} + \mathbf{F} = \mathbf{0}$$

$$\left(\frac{3}{7}F_{OC} - F_{OB}\right)\mathbf{i} + \left(\frac{2}{7}F_{OC} - F_{OA}\right)\mathbf{j} + \left(\frac{6}{7}F_{OC} - 196.2\right)\mathbf{k} = \mathbf{0}$$

Equating **i**, **j**, and **k** components, we have

$$\frac{3}{7}F_{OC} - F_{OB} = 0 \qquad \textbf{(1)}$$

$$\frac{2}{7}F_{OC} - F_{OA} = 0 \qquad \textbf{(2)}$$

$$\frac{6}{7}F_{OC} - 196.2 = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2) and (3) yields

$$F_{OC} = 228.9\text{ N} \qquad F_{OB} = 98.1\text{ N} \qquad F_{OA} = 65.4\text{ N}$$

Spring Elongation: Using spring formula, Eq. 3–2, the spring elongation is $s = \dfrac{F}{k}$.

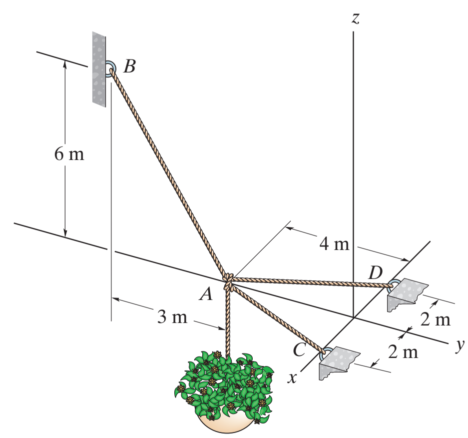
$$s_{OB} = \frac{98.1}{300} = 0.327\text{ m} = 327\text{ mm} \qquad \textbf{Ans.}$$

$$s_{OA} = \frac{65.4}{300} = 0.218\text{ m} = 218\text{ mm} \qquad \textbf{Ans.}$$

Ans:
$s_{OB} = 327$ mm
$s_{OA} = 218$ mm

3–47.

Determine the force in each cable needed to support the 20-kg flowerpot.



SOLUTION

Equations of Equilibrium.

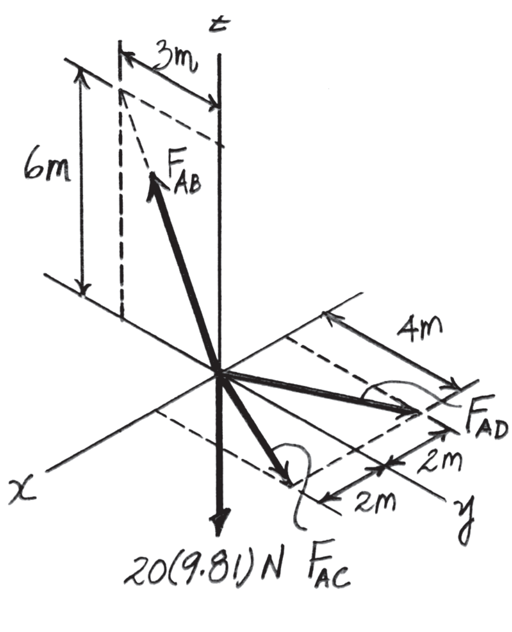
$\Sigma F_z = 0;\qquad F_{AB}\left(\dfrac{6}{\sqrt{45}}\right) - 20(9.81) = 0 \qquad F_{AB} = 219.36\text{ N} = 219\text{ N}$ **Ans.**

$\Sigma F_x = 0;\qquad F_{AC}\left(\dfrac{2}{\sqrt{20}}\right) - F_{AD}\left(\dfrac{2}{\sqrt{20}}\right) = 0 \qquad F_{AC} = F_{AD} = F$

Using the results of $F_{AB} = 219.36$ N and $F_{AC} = F_{AD} = F$,

$\Sigma F_y = 0;\qquad 2\left[F\left(\dfrac{4}{\sqrt{20}}\right)\right] - 219.36\left(\dfrac{3}{\sqrt{45}}\right) = 0$

$$F_{AC} = F_{AD} = F = 54.84\text{ N} = 54.8\text{ N} \qquad \textbf{Ans.}$$

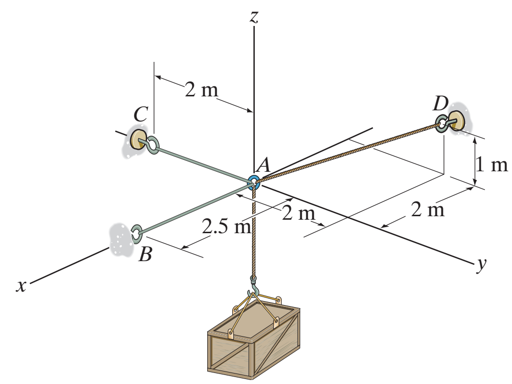


(a)

Ans:
$F_{AB} = 219$ N
$F_{AC} = F_{AD} = 54.8$ N

***3–48.**

Determine the tension in the cables in order to support the 100-kg crate in the equilibrium position shown.



SOLUTION

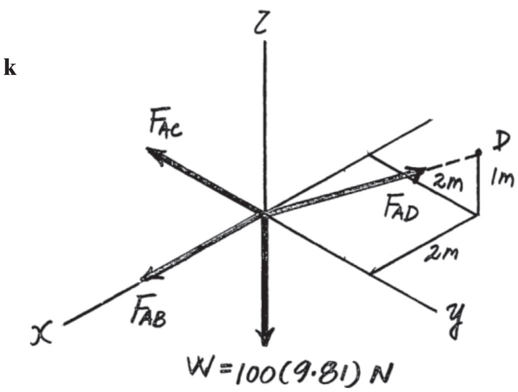
Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. (*a*) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{i}$$

$$\mathbf{F}_{AC} = -F_{AC}\,\mathbf{j}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(-2-0)\mathbf{i}+(2-0)\mathbf{j}+(1-0)\mathbf{k}}{\sqrt{(-2-0)^2+(2-0)^2+(1-0)^2}}\right] = -\frac{2}{3}F_{AD}\,\mathbf{i} + \frac{2}{3}F_{AD}\,\mathbf{j} + \frac{1}{3}F_{AD}\,\mathbf{k}$$

$$\mathbf{W} = [-100(9.81)\mathbf{k}]\text{N} = [-981\,\mathbf{k}]\text{N}$$



Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

$$F_{AB}\,\mathbf{i} + (-F_{AC}\,\mathbf{j}) + \left(-\frac{2}{3}F_{AD}\,\mathbf{i} + \frac{2}{3}F_{AD}\mathbf{j} + \frac{1}{3}F_{AD}\,\mathbf{k}\right) + (-981\mathbf{k}) = \mathbf{0}$$

$$\left(F_{AB} - \frac{2}{3}F_{AD}\right)\mathbf{i} + \left(-F_{AC} + \frac{2}{3}F_{AD}\right)\mathbf{j} + \left(\frac{1}{3}F_{AD} - 981\right)\mathbf{k} = 0$$

Equating the **i, j**, and **k** components yields

$$F_{AB} - \frac{2}{3}F_{AD} = 0 \qquad \textbf{(1)}$$

$$-F_{AC} + \frac{2}{3}F_{AD} = 0 \qquad \textbf{(2)}$$

$$\frac{1}{3}F_{AD} - 981 = 0 \qquad \textbf{(3)}$$

Solving Eqs. (1) through (3) yields

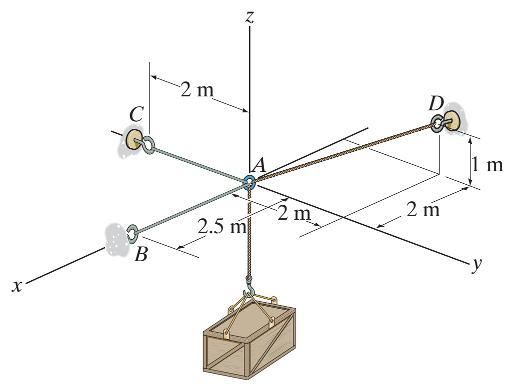
$$F_{AD} = 2943\text{ N} = 2.94\text{ kN} \qquad \textbf{Ans.}$$

$$F_{AB} = F_{AC} = 1962\text{ N} = 1.96\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F_{AD} = 2.94$ kN
$F_{AB} = 1.96$ kN

3–49.

Determine the maximum mass of the crate so that the tension developed in any cable does not exceeded 3 kN.



SOLUTION

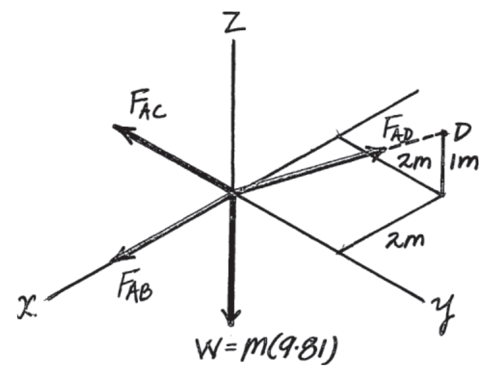
Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. (*a*) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\,\mathbf{i}$$

$$\mathbf{F}_{AC} = -F_{AC}\,\mathbf{j}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(-2-0)\mathbf{i}+(2-0)\mathbf{j}+(1-0)\mathbf{k}}{\sqrt{(-2-0)^2+(2-0)^2+(1-0)^2}}\right] = -\frac{2}{3}F_{AD}\,\mathbf{i} + \frac{2}{3}F_{AD}\,\mathbf{j} + \frac{1}{3}F_{AD}$$

$$\mathbf{W} = [-m(9.81)\mathbf{k}]$$



Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0};\qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

$$F_{AB}\,\mathbf{i} + (-F_{AC}\,\mathbf{j}) + \left(-\frac{2}{3}F_{AD}\,\mathbf{i} + \frac{2}{3}F_{AD}\,\mathbf{j} + \frac{1}{3}F_{AD}\,\mathbf{k}\right) + [-m(9.81)\mathbf{k}] = \mathbf{0}$$

$$\left(F_{AB} - \frac{2}{3}F_{AD}\right)\mathbf{i} + \left(-F_{AC} + \frac{2}{3}F_{AD}\right)\mathbf{j} + \left(\frac{1}{3}F_{AD} - 9.81m\right)\mathbf{k} = 0$$

Equating the **i, j**, and **k** components yields

$$F_{AB} - \frac{2}{3}F_{AD} = 0 \qquad \textbf{(1)}$$

$$-F_{AC} + \frac{2}{3}F_{AD} = 0 \qquad \textbf{(2)}$$

$$\frac{1}{3}F_{AD} - 9.81m = 0 \qquad \textbf{(3)}$$

When cable AD is subjected to maximum tension, $F_{AD} = 3000$ N. Thus, by substituting this value into Eqs. (1) through (3), we have

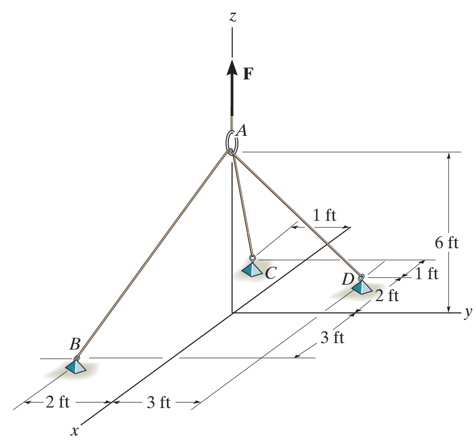
$$F_{AB} = F_{AC} = 2000\text{ N}$$

$$m = 102\text{ kg} \qquad \textbf{Ans.}$$

Ans:
$m = 102$ kg

3–50.

Determine the force in each cable if $F = 500$ lb.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* shown in Fig. *a*,

$$\Sigma F_x = 0;\quad F_{AB}\left(\frac{3}{7}\right) - F_{AC}\left(\frac{3}{\sqrt{46}}\right) - F_{AD}\left(\frac{2}{7}\right) = 0 \qquad \textbf{(1)}$$

$$\Sigma F_y = 0;\quad -F_{AB}\left(\frac{2}{7}\right) - F_{AC}\left(\frac{1}{\sqrt{46}}\right) + F_{AD}\left(\frac{3}{7}\right) = 0 \qquad \textbf{(2)}$$

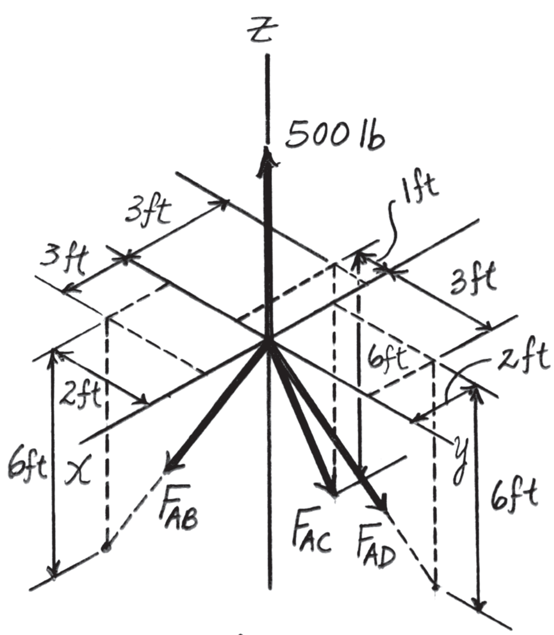
$$\Sigma F_z = 0;\quad -F_{AB}\left(\frac{6}{7}\right) - F_{AC}\left(\frac{6}{\sqrt{46}}\right) - F_{AD}\left(\frac{6}{7}\right) + 500 = 0 \qquad \textbf{(3)}$$

Solving Eqs (1), (2) and (3)

$$F_{AC} = 113.04 \text{ lb} = 113 \text{ lb} \qquad \textbf{Ans.}$$

$$F_{AB} = 256.67 \text{ lb} = 257 \text{ lb} \qquad \textbf{Ans.}$$

$$F_{AD} = 210 \text{ lb} \qquad \textbf{Ans.}$$

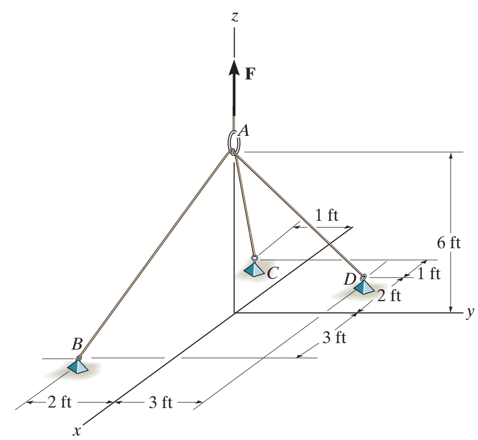


(a)

Ans:
$F_{AC} = 113$ lb
$F_{AB} = 257$ lb
$F_{AD} = 210$ lb

3–51.

Determine the greatest force **F** that can be applied to the ring if each cable can support a maximum force of 800 lb.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* shown in Fig. *a*,

$$\Sigma F_x = 0; \quad F_{AB}\left(\frac{3}{7}\right) - F_{AC}\left(\frac{3}{\sqrt{46}}\right) - F_{AD}\left(\frac{2}{7}\right) = 0 \qquad \textbf{(1)}$$

$$\Sigma F_y = 0; \quad -F_{AB}\left(\frac{2}{7}\right) - F_{AC}\left(\frac{1}{\sqrt{46}}\right) + F_{AD}\left(\frac{3}{7}\right) = 0 \qquad \textbf{(2)}$$

$$\Sigma F_z = 0; \quad -F_{AB}\left(\frac{6}{7}\right) - F_{AC}\left(\frac{6}{\sqrt{46}}\right) - F_{AD}\left(\frac{6}{7}\right) + F = 0 \qquad \textbf{(3)}$$

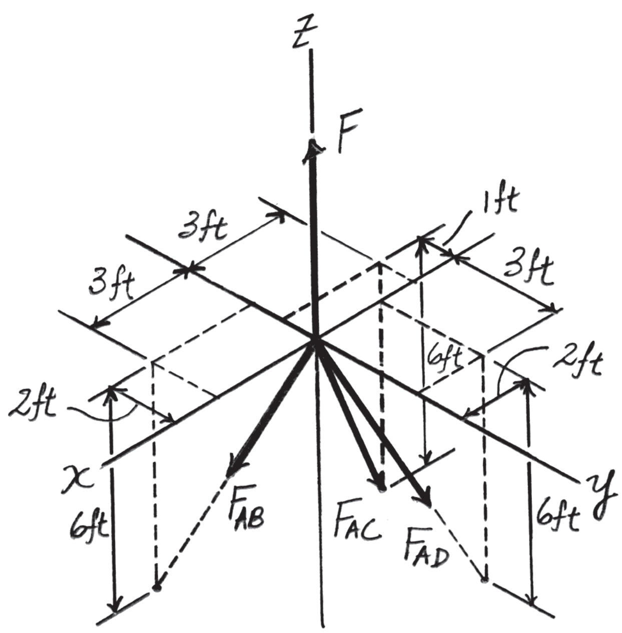
Solving Eqs (1), (2) and (3)

$$F_{AC} = 0.2261\,F \qquad F_{AB} = 0.5133\,F \qquad F_{AD} = 0.42\,F$$

Since cable AB is subjected to the greatest tension, its tension will reach the limit first that is $F_{AB} = 800$ lb. Then

$$800 = 0.5133\,F$$

$$F = 1558.44 \text{ lb} = 1558 \text{ lb} \qquad \textbf{Ans.}$$

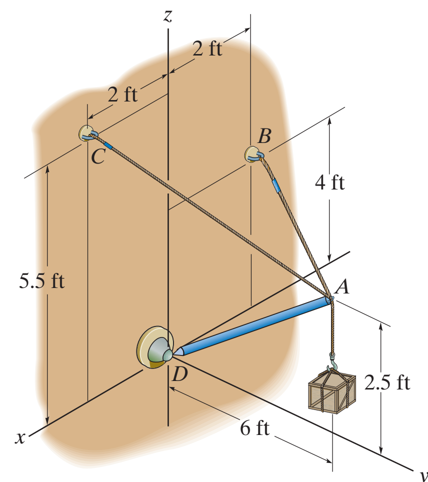


(a)

Ans:
$F = 1558$ lb

***3–52.**

Determine the tension developed in cables AB and AC and the force developed along strut AD for equilibrium of the 400-lb crate.



SOLUTION

Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. a in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-2-0)\mathbf{i} + (-6-0)\mathbf{j} + (1.5-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (-6-0)^2 + (1.5-0)^2}}\right] = -\frac{4}{13}F_{AB}\,\mathbf{i} - \frac{12}{13}F_{AB}\,\mathbf{j} + \frac{3}{13}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(2-0)\mathbf{i} + (-6-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(2-0)^2 + (-6-0)^2 + (3-0)^2}}\right] = \frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(0-0)\mathbf{i} + [0-(-6)]\mathbf{j} + [0-(-2.5)]\mathbf{k}}{\sqrt{(0-0)^2 + [0-(-6)]^2 + (0-(-2.5))^2}}\right] = \frac{12}{13}F_{AD}\,\mathbf{j} + \frac{5}{13}F_{AD}\,\mathbf{k}$$

$$\mathbf{W} = \{-400\mathbf{k}\}\text{ lb}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

$$\left(-\frac{4}{13}F_{AB}\,\mathbf{i} - \frac{12}{13}F_{AB}\,\mathbf{j} + \frac{3}{13}F_{AB}\,\mathbf{k}\right) + \left(\frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}\right) + \left(\frac{12}{13}F_{AD}\,\mathbf{j} + \frac{5}{13}F_{AD}\,\mathbf{k}\right) + (-400\mathbf{k}) = 0$$

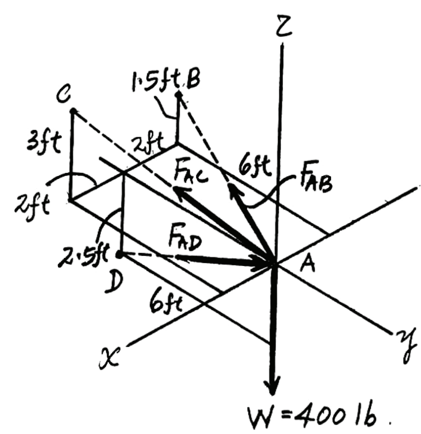
$$\left(-\frac{4}{13}F_{AB} + \frac{2}{7}F_{AC}\right)\mathbf{i} + \left(-\frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} + \frac{12}{13}F_{AD}\right)\mathbf{j} + \left(\frac{3}{13}F_{AB} + \frac{3}{7}F_{AC} + \frac{5}{13}F_{AD} - 400\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{4}{13}F_{AB} + \frac{2}{7}F_{AC} = 0 \qquad \textbf{(1)}$$

$$-\frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} + \frac{12}{13}F_{AD} = 0 \qquad \textbf{(2)}$$

$$\frac{3}{13}F_{AB} + \frac{3}{7}F_{AC} + \frac{5}{13}F_{AD} - 400 = 0 \qquad \textbf{(3)}$$



(a)

Solving Eqs. (1) through (3) yields

$F_{AB} = 274$ lb **Ans.**

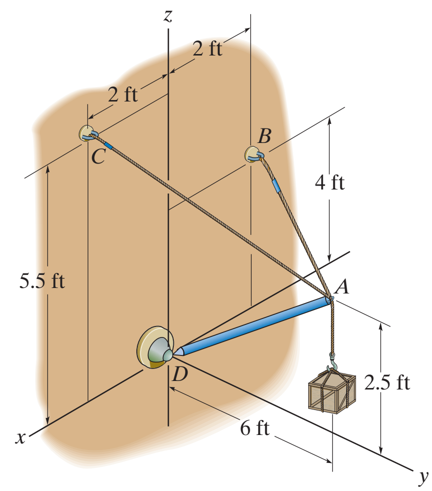
$F_{AC} = 295$ lb **Ans.**

$F_{AD} = 547$ lb **Ans.**

Ans:
$F_{AB} = 274$ lb
$F_{AC} = 295$ lb
$F_{AD} = 547$ lb

3–53.

If the tension developed in each of the cables cannot exceed 300 lb, determine the largest weight of the crate that can be supported. Also, what is the force developed along strut AD?



SOLUTION

Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. *a* in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-2-0)\mathbf{i} + (-6-0)\mathbf{j} + (1.5-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (-6-0)^2 + (1.5-0)^2}}\right] = -\frac{4}{13}F_{AB}\,\mathbf{i} - \frac{12}{13}F_{AB}\,\mathbf{j} + \frac{3}{13}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(2-0)\mathbf{i} + (-6-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(2-0)^2 + (-6-0)^2 + (3-0)^2}}\right] = \frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(0-0)\mathbf{i} + [0-(-6)]\mathbf{j} + [0-(-2.5)]\mathbf{k}}{\sqrt{(0-0)^2 + [0-(-6)]^2 + [0-(-2.5)]^2}}\right] = \frac{12}{13}F_{AD}\,\mathbf{j} + \frac{5}{13}F_{AD}\,\mathbf{k}$$

$$\mathbf{W} = -W\mathbf{k}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

$$\left(-\frac{4}{13}F_{AB}\,\mathbf{i} - \frac{12}{13}F_{AB}\,\mathbf{j} + \frac{3}{13}F_{AB}\,\mathbf{k}\right) + \left(\frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}\right) + \left(\frac{12}{13}F_{AD}\,\mathbf{j} + \frac{5}{13}F_{AD}\,\mathbf{k}\right) + (-W\mathbf{k}) = 0$$

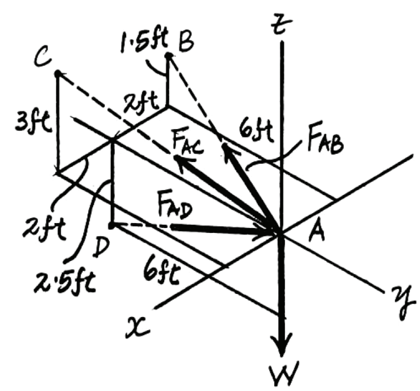
$$\left(-\frac{4}{13}F_{AB} + \frac{2}{7}F_{AC}\right)\mathbf{i} + \left(-\frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} + \frac{12}{13}F_{AD}\right)\mathbf{j} + \left(\frac{3}{13}F_{AB} + \frac{3}{7}F_{AC} + \frac{5}{13}F_{AD} - W\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{4}{13}F_{AB} + \frac{2}{7}F_{AC} = 0 \qquad (1)$$

$$-\frac{12}{13}F_{AB} - \frac{6}{7}F_{AC} + \frac{12}{13}F_{AD} = 0 \qquad (2)$$

$$\frac{3}{13}F_{AB} + \frac{3}{7}F_{AC} + \frac{5}{13}F_{AD} - W = 0 \qquad (3)$$



(a)

Let us assume that cable AC achieves maximum tension first. Substituting $F_{AC} = 300$ lb into Eqs. (1) through (3) and solving, yields

$F_{AB} = 278.57$ lb

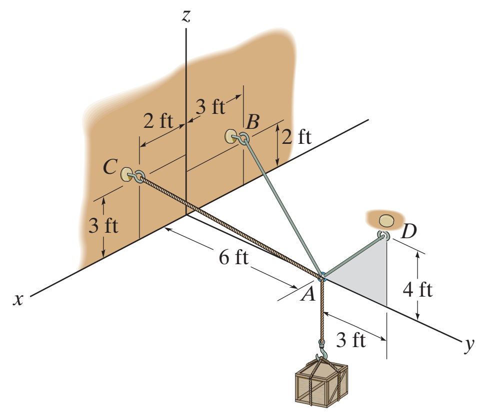
$F_{AD} = 557$ lb $\qquad W = 407$ lb **Ans.**

Since $F_{AB} = 278.57 \text{ lb} < 300 \text{ lb}$, our assumption is correct.

Ans:
$F_{AD} = 557$ lb
$W = 407$ lb

3–54.

Determine the tension developed in each cable for equilibrium of the 300-lb crate.



SOLUTION

Force Vectors: We can express each of the forces shown in Fig. *a* in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-3-0)\mathbf{i} + (-6-0)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(-3-0)^2 + (-6-0)^2 + (2-0)^2}}\right] = -\frac{3}{7}F_{AB}\,\mathbf{i} - \frac{6}{7}F_{AB}\,\mathbf{j} + \frac{2}{7}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(2-0)\mathbf{i} + (-6-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(2-0)^2 + (-6-0)^2 + (3-0)^2}}\right] = \frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(0-0)\mathbf{i} + (3-0)\mathbf{j} + (4-0)\mathbf{k}}{\sqrt{(0-0)^2 + (3-0)^2 + (4-0)^2}}\right] = \frac{3}{5}F_{AD}\,\mathbf{j} + \frac{4}{5}F_{AD}\,\mathbf{k}$$

$$\mathbf{W} = \{-300\mathbf{k}\}\text{ lb}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

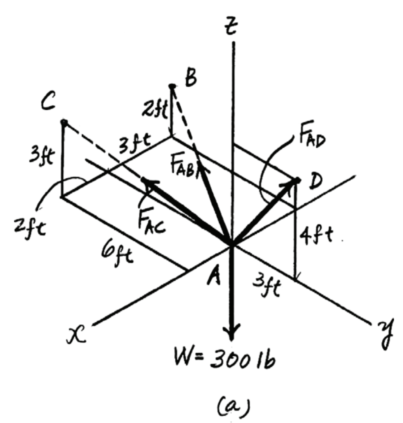
$$\left(-\frac{3}{7}F_{AB}\,\mathbf{i} - \frac{6}{7}F_{AB}\,\mathbf{j} + \frac{2}{7}F_{AB}\,\mathbf{k}\right) + \left(\frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}\right) + \left(\frac{3}{5}F_{AD}\,\mathbf{j} + \frac{4}{5}F_{AD}\,\mathbf{k}\right) + (-300\mathbf{k}) = 0$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{3}{7}F_{AB} + \frac{2}{7}F_{AC} = 0 \qquad \textbf{(1)}$$

$$-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} + \frac{3}{5}F_{AD} = 0 \qquad \textbf{(2)}$$

$$\frac{2}{7}F_{AB} + \frac{3}{7}F_{AC} + \frac{4}{5}F_{AD} - 300 = 0 \qquad \textbf{(3)}$$



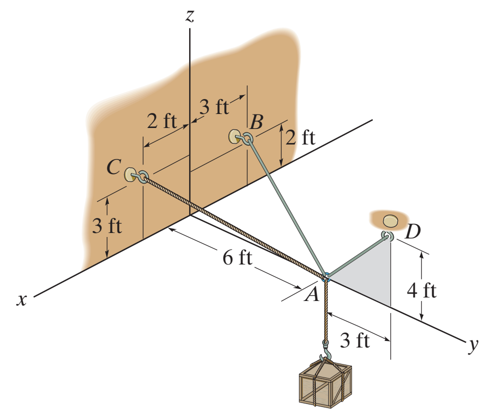
Solving Eqs. (1) through (3) yields

$F_{AB} = 79.2$ lb $\qquad F_{AC} = 119$ lb $\qquad F_{AD} = 283$ lb **Ans.**

Ans:
$F_{AB} = 79.2$ lb
$F_{AC} = 119$ lb
$F_{AD} = 283$ lb

3–55.

Determine the maximum weight of the crate that can be suspended from cables AB, AC, and AD so that the tension developed in any one of the cables does not exceed 250 lb.



SOLUTION

Force Vectors: We can express each of the forces shown in Fig. a in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-3-0)\mathbf{i} + (-6-0)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(-3-0)^2 + (-6-0)^2 + (2-0)^2}}\right] = -\frac{3}{7}F_{AB}\,\mathbf{i} - \frac{6}{7}F_{AB}\,\mathbf{j} + \frac{2}{7}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(2-0)\mathbf{i} + (-6-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(2-0)^2 + (-6-0)^2 + (3-0)^2}}\right] = \frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(0-0)\mathbf{i} + (3-0)\mathbf{j} + (4-0)\mathbf{k}}{\sqrt{(0-0)^2 + (3-0)^2 + (4-0)^2}}\right] = \frac{3}{5}F_{AD}\,\mathbf{j} + \frac{4}{5}F_{AD}\,\mathbf{k}$$

$$\mathbf{W} = -W_C\,\mathbf{k}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

$$\left(-\frac{3}{7}F_{AB}\,\mathbf{i} - \frac{6}{7}F_{AB}\,\mathbf{j} + \frac{2}{7}F_{AB}\,\mathbf{k}\right) + \left(\frac{2}{7}F_{AC}\,\mathbf{i} - \frac{6}{7}F_{AC}\,\mathbf{j} + \frac{3}{7}F_{AC}\,\mathbf{k}\right) + \left(\frac{3}{5}F_{AD}\,\mathbf{j} + \frac{4}{5}F_{AD}\,\mathbf{k}\right) + (-W_C\,\mathbf{k}) = 0$$

$$\left(-\frac{3}{7}F_{AB} + \frac{2}{7}F_{AC}\right)\mathbf{i} + \left(-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} + \frac{3}{5}F_{AD}\right)\mathbf{j} + \left(\frac{2}{7}F_{AB} + \frac{3}{7}F_{AC} + \frac{4}{5}F_{AD} - W_C\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{3}{7}F_{AB} + \frac{2}{7}F_{AC} = 0 \qquad \textbf{(1)}$$

$$-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} + \frac{3}{5}F_{AD} = 0 \qquad \textbf{(2)}$$

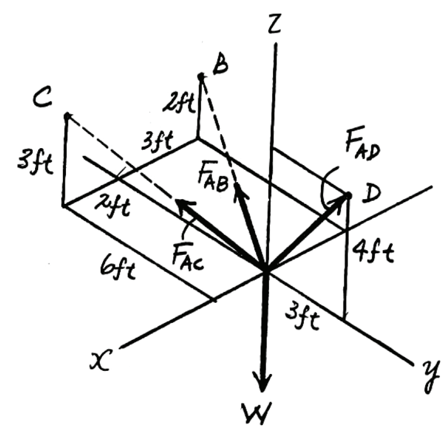
$$\frac{2}{7}F_{AB} + \frac{3}{7}F_{AC} + \frac{4}{5}F_{AD} - W_C = 0 \qquad \textbf{(3)}$$

Assuming that cable AD achieves maximum tension first, substituting $F_{AD} = 250$ lb into Eqs. (2) and (3), and solving Eqs. (1) through (3) yields

$F_{AB} = 70$ lb $\qquad F_{AC} = 105$ lb

$W_C = 265$ lb **Ans.**

Since $F_{AB} = 70\text{ lb} < 250$ lb and $F_{AC} = 105$ lb, the above assumption is correct.

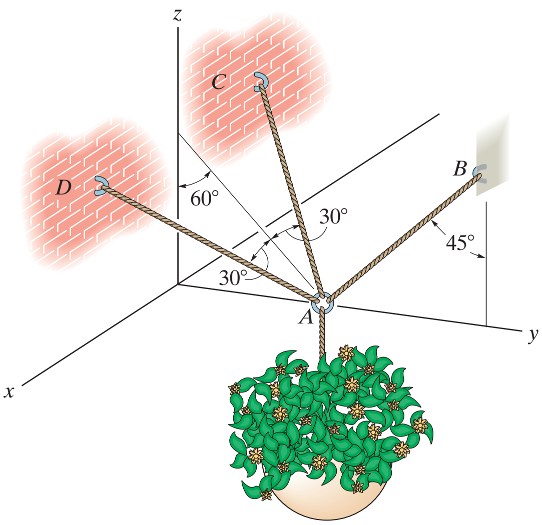


(a)

Ans.:
$W_C = 265$ lb

***3–56.**

The 25-kg flowerpot is supported at A by the three cords. Determine the force acting in each cord for equilibrium.



SOLUTION

$$\mathbf{F}_{AD} = F_{AD}(\sin 30°\mathbf{i} - \cos 30° \sin 60°\mathbf{j} + \cos 30° \cos 60°\mathbf{k})$$

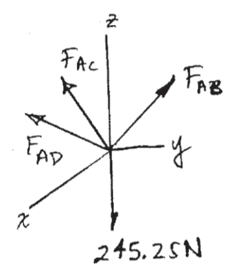
$$= 0.5F_{AD}\mathbf{i} - 0.75F_{AD}\mathbf{j} + 0.4330F_{AD}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}(-\sin 30°\mathbf{i} - \cos 30° \sin 60°\mathbf{j} + \cos 30° \cos 60°\mathbf{k})$$

$$= -0.5F_{AC}\mathbf{i} - 0.75F_{AC}\mathbf{j} + 0.4330F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AB} = F_{AB}(\sin 45°\mathbf{j} + \cos 45°\mathbf{k}) = 0.7071F_{AB}\mathbf{j} + 0.7071F_{AB}\mathbf{k}$$

$$\mathbf{F} = -25(9.81)\mathbf{k} = \{-245.25\mathbf{k}\}\text{ N}$$



$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AD} + \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F} = \mathbf{0}$$

$$(0.5F_{AD}\mathbf{i} - 0.75F_{AD}\mathbf{j}) + 0.4330F_{AD}\mathbf{k} + (0.7071F_{AB}\mathbf{j} + 0.7071F_{AB}\mathbf{k})$$
$$+(-0.5F_{AC}\mathbf{i} - 0.75F_{AC}\mathbf{j} + 0.4330F_{AC}\mathbf{k}) + (-245.25\mathbf{k}) = \mathbf{0}$$

$$(0.5F_{AD} - 0.5F_{AC})\mathbf{i} + (-0.75F_{AD} + 0.7071F_{AB} - 0.75F_{AC})\mathbf{j}$$
$$+(0.4330F_{AD} + 0.7071F_{AB} + 0.4330F_{AC} - 245.25)\mathbf{k} = \mathbf{0}$$

Thus,

$\Sigma F_x = 0; \qquad 0.5F_{AD} - 0.5F_{AC} = 0$ **[1]**

$\Sigma F_y = 0; \qquad -0.75F_{AD} + 0.7071F_{AB} - 0.75F_{AC} = 0$ **[2]**

$\Sigma F_z = 0; \qquad 0.4330F_{AD} + 0.7071F_{AB} + 0.4330F_{AC} - 245.25 = 0$ **[3]**

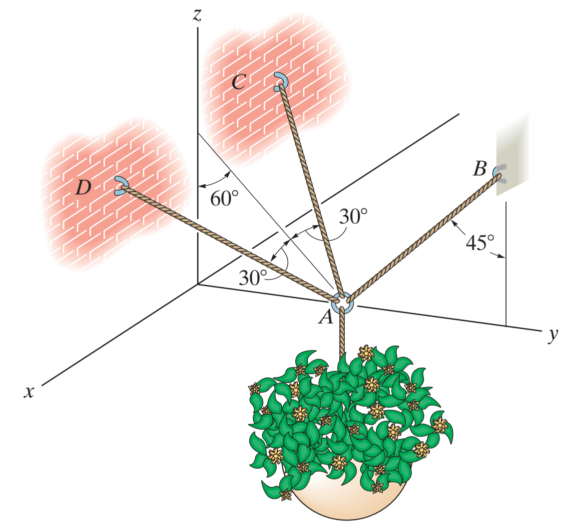
Solving Eqs. [1], [2], and [3] yields:

$F_{AD} = F_{AC} = 104 \text{ N} \qquad F_{AB} = 220 \text{ N}$ **Ans.**

Ans:
$F_{AD} = F_{AC} = 104$ N
$F_{AB} = 220$ N

3–57.

If each cord can sustain a maximum tension of 50 N before it fails, determine the greatest weight of the flowerpot the cords can support.



SOLUTION

$$\mathbf{F}_{AD} = F_{AD}(\sin 30°\mathbf{i} - \cos 30° \sin 60°\mathbf{j} + \cos 30° \cos 60°\mathbf{k})$$

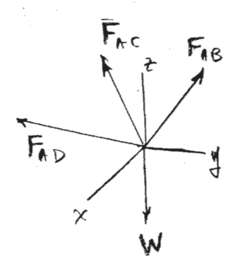
$$= 0.5F_{AD}\mathbf{i} - 0.75F_{AD}\mathbf{j} + 0.4330F_{AD}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}(-\sin 30°\mathbf{i} - \cos 30° \sin 60°\mathbf{j} + \cos 30° \cos 60°\mathbf{k})$$

$$= -0.5F_{AC}\mathbf{i} - 0.75F_{AC}\mathbf{j} + 0.4330F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AB} = F_{AB}(\sin 45°\mathbf{j} + \cos 45°\mathbf{k}) = 0.7071F_{AB}\mathbf{j} + 0.7071F_{AB}\mathbf{k}$$

$$\mathbf{W} = -W\mathbf{k}$$



$\Sigma F_x = 0;\quad 0.5F_{AD} - 0.5F_{AC} = 0$

$$F_{AD} = F_{AC} \tag{1}$$

$\Sigma F_y = 0;\quad -0.75F_{AD} + 0.7071F_{AB} - 0.75F_{AC} = 0$

$$0.7071F_{AB} = 1.5F_{AC} \tag{2}$$

$\Sigma F_z = 0;\quad 0.4330F_{AD} + 0.7071F_{AB} + 0.4330F_{AC} - W = 0$

$$0.8660F_{AC} + 1.5F_{AC} - W = 0$$

$$2.366F_{AC} = W$$

Assume $F_{AC} = 50$ N then

$$F_{AB} = \frac{1.5(50)}{0.7071} = 106.07 \text{ N} > 50 \text{ N } (\textbf{N.G!})$$

Assume $F_{AB} = 50$ N. Then

$$F_{AC} = \frac{0.7071(50)}{1.5} = 23.57 \text{ N} < 50 \text{ N } (\textbf{O.K!})$$

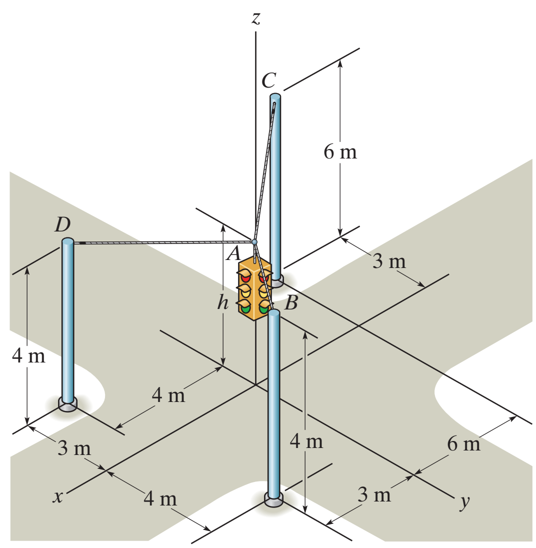
Thus,

$$W = 2.366(23.57) = 55.767 = 55.8 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$W = 55.8$ N

3–58.

Determine the tension developed in the three cables required to support the traffic light, which has a mass of 15 kg. Take $h = 4$ m.



SOLUTION

$$\mathbf{u}_{AB} = \left\{\frac{3}{5}\mathbf{i} + \frac{4}{5}\mathbf{j}\right\}$$

$$\mathbf{u}_{AC} = \left\{-\frac{6}{7}\mathbf{i} - \frac{3}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right\}$$

$$\mathbf{u}_{AD} = \left\{\frac{4}{5}\mathbf{i} - \frac{3}{5}\mathbf{j}\right\}$$

$$\Sigma F_x = 0; \qquad \frac{3}{5}F_{AB} - \frac{6}{7}F_{AC} + \frac{4}{5}F_{AD} = 0$$

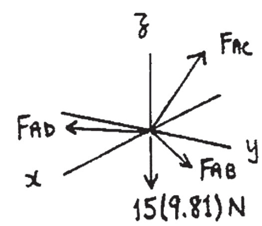
$$\Sigma F_y = 0; \qquad \frac{4}{5}F_{AB} - \frac{3}{7}F_{AC} - \frac{3}{5}F_{AD} = 0$$

$$\Sigma F_z = 0; \qquad \frac{2}{7}F_{AC} - 15(9.81) = 0$$

$F_{AB} = 441$ N **Ans.**

$F_{AC} = 515$ N **Ans.**

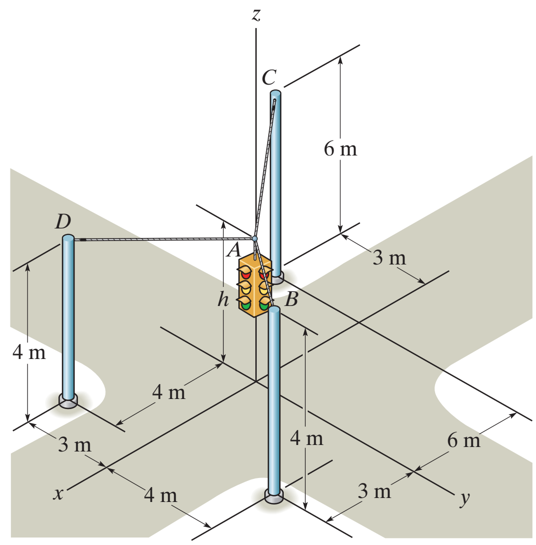
$F_{AD} = 221$ N **Ans.**



Ans:
$F_{AB} = 441$ N
$F_{AC} = 515$ N
$F_{AD} = 221$ N

3–59.

Determine the tension developed in the three cables required to support the traffic light, which has a mass of 20 kg. Take $h = 3.5$ m.



SOLUTION

$$\mathbf{u}_{AB} = \frac{3\mathbf{i} + 4\mathbf{j} + 0.5\mathbf{k}}{\sqrt{3^2 + 4^2 + (0.5)^2}} = \frac{3\mathbf{i} + 4\mathbf{j} + 0.5\mathbf{k}}{\sqrt{25.25}}$$

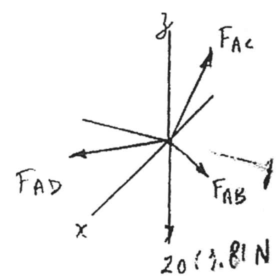
$$\mathbf{u}_{AC} = \frac{-6\mathbf{i} - 3\mathbf{j} + 2.5\mathbf{k}}{\sqrt{(-6)^2 + (-3)^2 + 2.5^2}} = \frac{-6\mathbf{i} - 3\mathbf{j} + 2.5\mathbf{k}}{\sqrt{51.25}}$$

$$\mathbf{u}_{AD} = \frac{4\mathbf{i} - 3\mathbf{j} + 0.5\mathbf{k}}{\sqrt{4^2 + (-3)^2 + 0.5^2}} = \frac{4\mathbf{i} - 3\mathbf{j} + 0.5\mathbf{k}}{\sqrt{25.25}}$$

$$\Sigma F_x = 0; \qquad \frac{3}{\sqrt{25.25}}F_{AB} - \frac{6}{\sqrt{51.25}}F_{AC} + \frac{4}{\sqrt{25.25}}F_{AD} = 0$$

$$\Sigma F_y = 0; \qquad \frac{4}{\sqrt{25.25}}F_{AB} - \frac{3}{\sqrt{51.25}}F_{AC} - \frac{3}{\sqrt{25.25}}F_{AD} = 0$$

$$\Sigma F_z = 0; \qquad \frac{0.5}{\sqrt{25.25}}F_{AB} + \frac{2.5}{\sqrt{51.25}}F_{AC} + \frac{0.5}{\sqrt{25.25}}F_{AD} - 20(9.81) = 0$$



Solving,

$F_{AB} = 348$ N **Ans.**

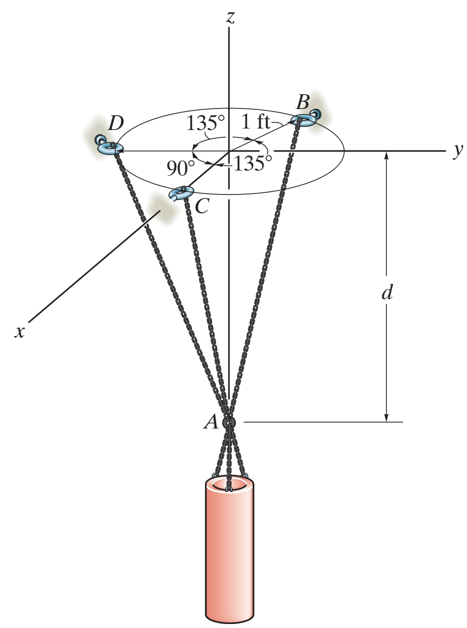
$F_{AC} = 413$ N **Ans.**

$F_{AD} = 174$ N **Ans.**

Ans:
$F_{AB} = 348$ N
$F_{AC} = 413$ N
$F_{AD} = 174$ N

***3–60.**

The 800-lb cylinder is supported by three chains as shown. Determine the force in each chain for equilibrium. Take $d = 1$ ft.



SOLUTION

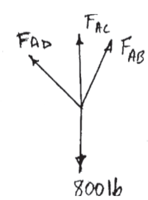
$$\mathbf{F}_{AD} = F_{AD}\left(\frac{-1\mathbf{j} + 1\mathbf{k}}{\sqrt{(-1)^2 + 1^2}}\right) = -0.7071F_{AD}\mathbf{j} + 0.7071F_{AD}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{1\mathbf{i} + 1\mathbf{k}}{\sqrt{1^2 + 1^2}}\right) = 0.7071F_{AC}\mathbf{i} + 0.7071F_{AC}\mathbf{k}$$

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{-0.7071\mathbf{i} + 0.7071\mathbf{j} + 1\mathbf{k}}{\sqrt{(-0.7071)^2 + 0.7071^2 + 1^2}}\right)$$

$$= -0.5F_{AB}\mathbf{i} + 0.5F_{AB}\mathbf{j} + 0.7071F_{AB}\mathbf{k}$$

$$\mathbf{F} = \{-800\mathbf{k}\}\text{ lb}$$



$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AD} + \mathbf{F}_{AC} + \mathbf{F}_{AB} + \mathbf{F} = \mathbf{0}$$

$$(-0.7071F_{AD}\mathbf{j} + 0.7071F_{AD}\mathbf{k}) + (0.7071F_{AC}\mathbf{i} + 0.7071F_{AC}\mathbf{k})$$
$$+ (-0.5F_{AB}\mathbf{i} + 0.5F_{AB}\mathbf{j} + 0.7071F_{AB}\mathbf{k}) + (-800\mathbf{k}) = \mathbf{0}$$

$$(0.7071F_{AC} - 0.5F_{AB})\mathbf{i} + (-0.7071F_{AD} + 0.5F_{AB})\mathbf{j}$$
$$+ (0.7071F_{AD} + 0.7071F_{AC} + 0.7071F_{AB} - 800)\mathbf{k} = \mathbf{0}$$

$$\Sigma F_x = 0; \qquad 0.7071F_{AC} - 0.5F_{AB} = 0 \qquad \textbf{(1)}$$

$$\Sigma F_y = 0; \qquad -0.7071F_{AD} + 0.5F_{AB} = 0 \qquad \textbf{(2)}$$

$$\Sigma F_z = 0; \qquad 0.7071F_{AD} + 0.7071F_{AC} + 0.7071F_{AB} - 800 = 0 \qquad \textbf{(3)}$$

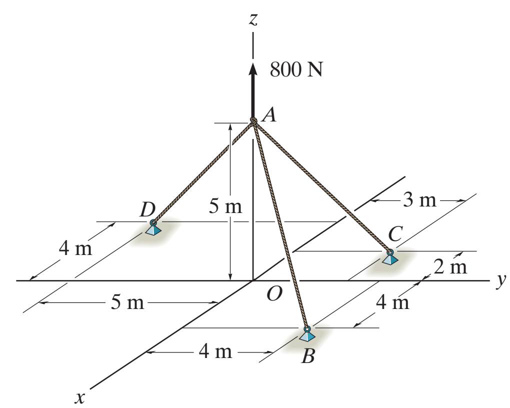
Solving Eqs. (1), (2), and (3) yields:

$F_{AB} = 469$ lb $\qquad F_{AC} = F_{AD} = 331$ lb **Ans.**

Ans:
$F_{AB} = 469$ lb
$F_{AC} = F_{AD} = 331$ lb

3–61.

Determine the tension in each cable for equilibrium.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* shown in Fig. *a*,

$$\Sigma F_x = 0;\quad F_{AB}\left(\frac{4}{\sqrt{57}}\right) - F_{AC}\left(\frac{2}{\sqrt{38}}\right) - F_{AD}\left(\frac{4}{\sqrt{66}}\right) = 0 \qquad \textbf{(1)}$$

$$\Sigma F_y = 0;\quad F_{AB}\left(\frac{4}{\sqrt{57}}\right) + F_{AC}\left(\frac{3}{\sqrt{38}}\right) - F_{AD}\left(\frac{5}{\sqrt{66}}\right) = 0 \qquad \textbf{(2)}$$

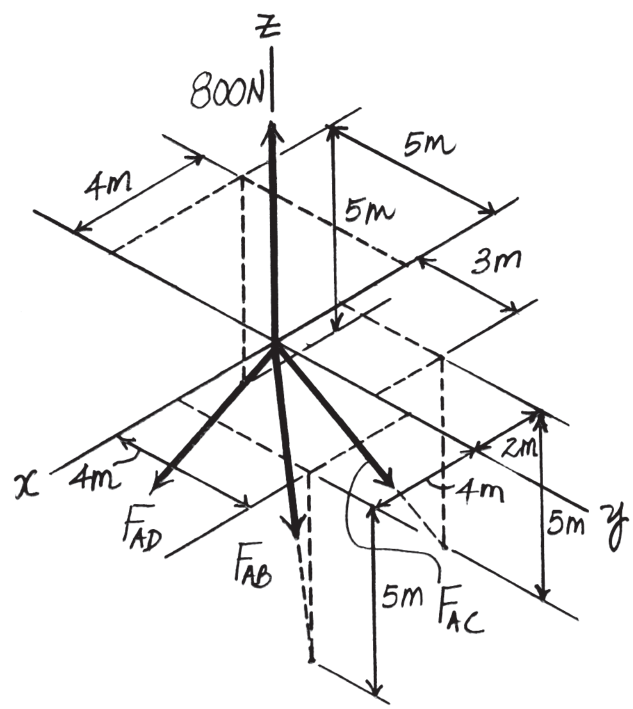
$$\Sigma F_z = 0;\quad -F_{AB}\left(\frac{5}{\sqrt{57}}\right) - F_{AC}\left(\frac{5}{\sqrt{38}}\right) - F_{AD}\left(\frac{5}{\sqrt{66}}\right) + 800 = 0 \qquad \textbf{(3)}$$

Solving Eqs (1), (2) and (3)

$F_{AC} = 85.77\text{ N} = 85.8\text{ N}$ **Ans.**

$F_{AB} = 577.73\text{ N} = 578\text{ N}$ **Ans.**

$F_{AD} = 565.15\text{ N} = 565\text{ N}$ **Ans.**

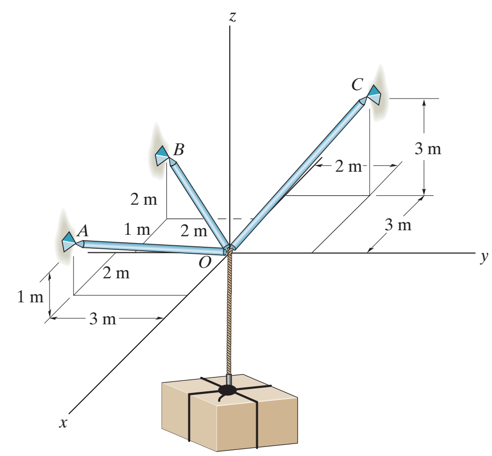


(a)

Ans:
$F_{AC} = 85.8\text{ N}$
$F_{AB} = 578\text{ N}$
$F_{AD} = 565\text{ N}$

3–62.

If the maximum force in each rod can not exceed 1500 N, determine the greatest mass of the crate that can be supported.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* shown in Fig. *a*,

$$\Sigma F_x = 0;\quad F_{OA}\left(\frac{2}{\sqrt{14}}\right) - F_{OC}\left(\frac{3}{\sqrt{22}}\right) + F_{OB}\left(\frac{1}{3}\right) = 0 \qquad \textbf{(1)}$$

$$\Sigma F_y = 0;\quad -F_{OA}\left(\frac{3}{\sqrt{14}}\right) + F_{OC}\left(\frac{2}{\sqrt{22}}\right) + F_{OB}\left(\frac{2}{3}\right) = 0 \qquad \textbf{(2)}$$

$$\Sigma F_z = 0;\quad F_{OA}\left(\frac{1}{\sqrt{14}}\right) + F_{OC}\left(\frac{3}{\sqrt{22}}\right) - F_{OB}\left(\frac{2}{3}\right) - \mathrm{m}(9.81) = 0 \qquad \textbf{(3)}$$

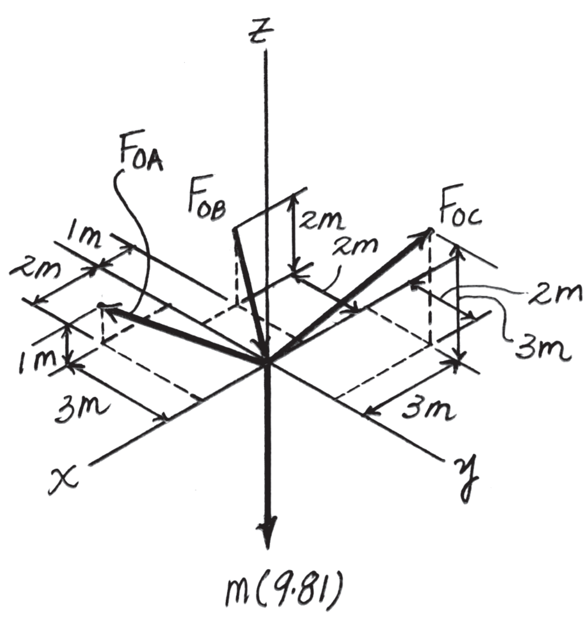
Solving Eqs (1), (2) and (3),

$$F_{OC} = 16.95m \qquad F_{OA} = 15.46m \qquad F_{OB} = 7.745m$$

Since link OC subjected to the greatest force, it will reach the limiting force first, that is $F_{OC} = 1500$ N. Then

$$1500 = 16.95\text{ m}$$

$$m = 88.48\text{ kg} = 88.5\text{ kg} \qquad \textbf{Ans.}$$

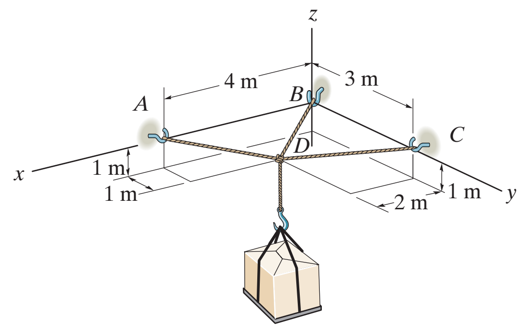


(a)

Ans:
$m = 88.5$ kg

3–63.

The crate has a mass of 130 kg. Determine the tension developed in each cable for equilibrium.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* shown in Fig. *a*,

$$\Sigma F_x = 0; \quad F_{AD}\left(\frac{2}{\sqrt{6}}\right) - F_{BD}\left(\frac{2}{\sqrt{6}}\right) - F_{CD}\left(\frac{2}{3}\right) = 0 \qquad \textbf{(1)}$$

$$\Sigma F_y = 0; \quad -F_{AD}\left(\frac{1}{\sqrt{6}}\right) - F_{BD}\left(\frac{1}{\sqrt{6}}\right) + F_{CD}\left(\frac{2}{3}\right) = 0 \qquad \textbf{(2)}$$

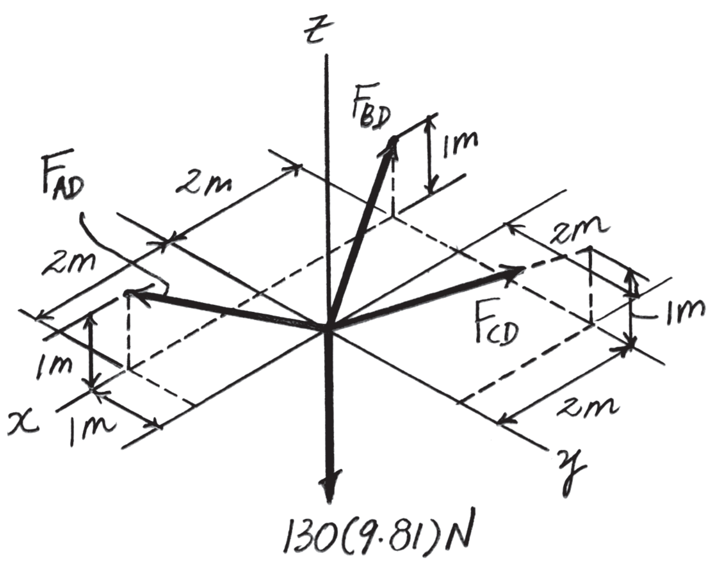
$$\Sigma F_z = 0; \quad F_{AD}\left(\frac{1}{\sqrt{6}}\right) + F_{BD}\left(\frac{1}{\sqrt{6}}\right) + F_{CD}\left(\frac{1}{3}\right) - 130(9.81) = 0 \qquad \textbf{(3)}$$

Solving Eqs (1), (2) and (3)

$F_{AD} = 1561.92 \text{ N} = 1.56 \text{ kN}$ **Ans.**

$F_{BD} = 520.64 \text{ N} = 521 \text{ N}$ **Ans.**

$F_{CD} = 1275.3 \text{ N} = 1.28 \text{ kN}$ **Ans.**

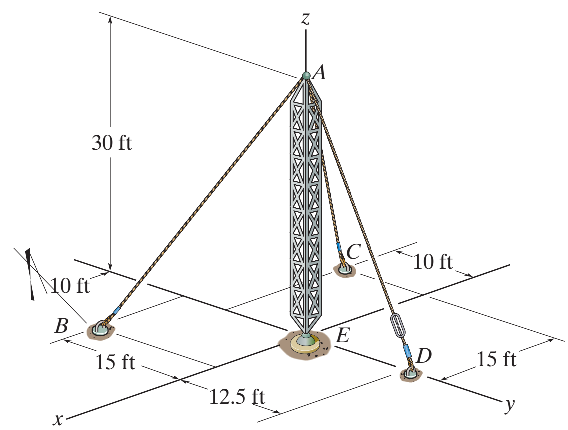


(a)

Ans:
$F_{AD} = 1.56 \text{ kN}$
$F_{BD} = 521 \text{ N}$
$F_{CD} = 1.28 \text{ kN}$

***3–64.**

If cable AD is tightened by a turnbuckle and develops a tension of 1300 lb, determine the tension developed in cables AB and AC and the force developed along the antenna tower AE at point A.



SOLUTION

Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. a in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(10-0)\mathbf{i} + (-15-0)\mathbf{j} + (-30-0)\mathbf{k}}{\sqrt{(10-0)^2 + (-15-0)^2 + (-30-0)^2}}\right] = \frac{2}{7}F_{AB}\,\mathbf{i} - \frac{3}{7}F_{AB}\,\mathbf{j} - \frac{6}{7}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(-15-0)\mathbf{i} + (-10-0)\mathbf{j} + (-30-0)\mathbf{k}}{\sqrt{(-15-0)^2 + (-10-0)^2 + (-30-0)^2}}\right] = -\frac{3}{7}F_{AC}\,\mathbf{i} - \frac{2}{7}F_{AC}\,\mathbf{j} - \frac{6}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\left[\frac{(0-0)\mathbf{i} + (12.5-0)\mathbf{j} + (-30-0)\mathbf{k}}{\sqrt{(0-0)^2 + (12.5-0)^2 + (-30-0)^2}}\right] = \{500\mathbf{j} - 1200\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_{AE} = F_{AE}\,\mathbf{k}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F}_{AE} = \mathbf{0}$$

$$\left(\frac{2}{7}F_{AB}\,\mathbf{i} - \frac{3}{7}F_{AB}\,\mathbf{j} - \frac{6}{7}F_{AB}\,\mathbf{k}\right) + \left(-\frac{3}{7}F_{AC}\,\mathbf{i} - \frac{2}{7}F_{AC}\,\mathbf{j} - \frac{6}{7}F_{AC}\,\mathbf{k}\right) + (500\mathbf{j} - 1200\mathbf{k}) + F_{AE}\,\mathbf{k} = 0$$

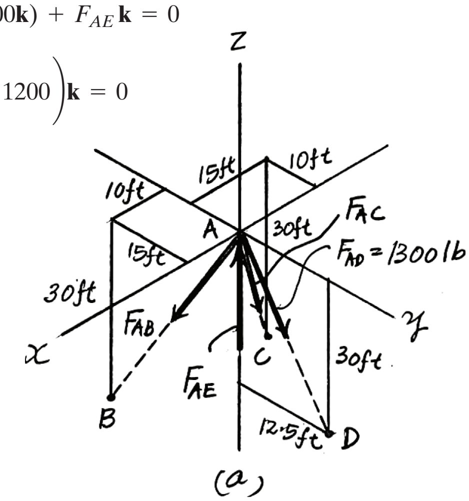
$$\left(\frac{2}{7}F_{AB} - \frac{3}{7}F_{AC}\right)\mathbf{i} + \left(-\frac{3}{7}F_{AB} - \frac{2}{7}F_{AC} + 500\right)\mathbf{j} + \left(-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} + F_{AE} - 1200\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$\frac{2}{7}F_{AB} - \frac{3}{7}F_{AC} = 0 \tag{1}$$

$$-\frac{3}{7}F_{AB} - \frac{2}{7}F_{AC} + 500 = 0 \tag{2}$$

$$-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} + F_{AE} - 1200 = 0 \tag{3}$$



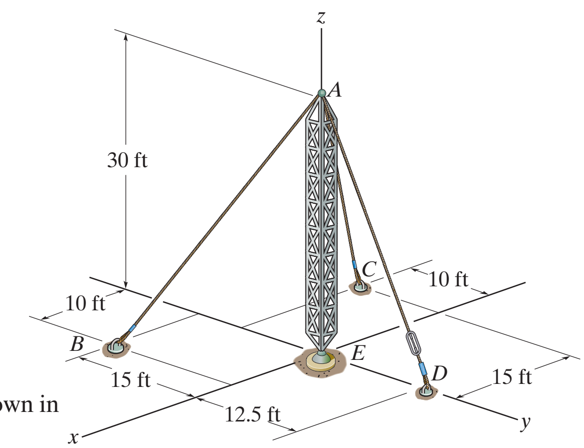
Solving Eqs. (1) through (3) yields

$F_{AB} = 808$ lb **Ans.**
$F_{AC} = 538$ lb **Ans.**
$F_{AE} = 2354$ lb $= 2.35$ kip **Ans.**

Ans:
$F_{AB} = 808$ lb
$F_{AC} = 538$ lb
$F_{AE} = 2.35$ kip

3–65.

If the tension developed in either cable AB or AC cannot exceed 1000 lb, determine the maximum tension that can be developed in cable AD when it is tightened by the turnbuckle. Also, what is the force developed along the antenna tower at point A?



SOLUTION

Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. a in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(10-0)\mathbf{i} + (-15-0)\mathbf{j} + (-30-0)\mathbf{k}}{\sqrt{(10-0)^2 + (-15-0)^2 + (-30-0)^2}}\right] = \frac{2}{7}F_{AB}\,\mathbf{i} - \frac{3}{7}F_{AB}\,\mathbf{j} - \frac{6}{7}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(-15-0)\mathbf{i} + (-10-0)\mathbf{j} + (-30-0)\mathbf{k}}{\sqrt{(-15-0)^2 + (-10-0)^2 + (-30-0)^2}}\right] = -\frac{3}{7}F_{AC}\,\mathbf{i} - \frac{2}{7}F_{AC}\,\mathbf{j} - \frac{6}{7}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F\left[\frac{(0-0)\mathbf{i} + (12.5-0)\mathbf{j} + (-30-0)\mathbf{k}}{\sqrt{(0-0)^2 + (12.5-0)^2 + (-30-0)^2}}\right] = \frac{5}{13}F\mathbf{j} - \frac{12}{13}F\,\mathbf{k}$$

$$\mathbf{F}_{AE} = F_{AE}\,\mathbf{k}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{F}_{AE} = \mathbf{0}$$

$$\left(\frac{2}{7}F_{AB}\mathbf{i} - \frac{3}{7}F_{AB}\mathbf{j} - \frac{6}{7}F_{AB}\mathbf{k}\right) + \left(-\frac{3}{7}F_{AC}\mathbf{i} - \frac{2}{7}F_{AC}\,\mathbf{j} - \frac{6}{7}F_{AC}\,\mathbf{k}\right) + \left(\frac{5}{13}F\mathbf{j} - \frac{12}{13}F\mathbf{k}\right) + F_{AE}\mathbf{k} = 0$$

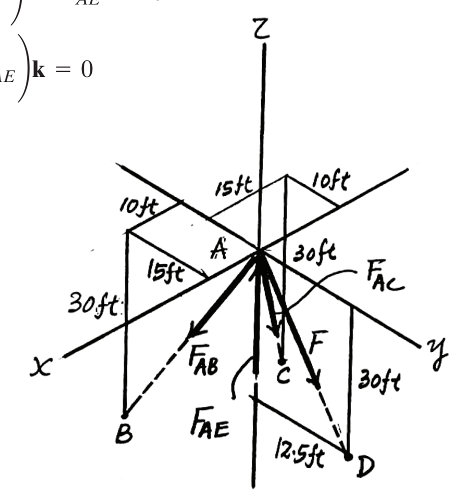
$$\left(\frac{2}{7}F_{AB} - \frac{3}{7}F_{AC}\right)\mathbf{i} + \left(-\frac{3}{7}F_{AB} - \frac{2}{7}F_{AC} + \frac{5}{13}F\right)\mathbf{j} + \left(-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} - \frac{12}{13}F + F_{AE}\right)\mathbf{k} = 0$$

Equating the **i**, **j**, and **k** components yields

$$\frac{2}{7}F_{AB} - \frac{3}{7}F_{AC} = 0 \qquad \textbf{(1)}$$

$$-\frac{3}{7}F_{AB} - \frac{2}{7}F_{AC} + \frac{5}{13}F = 0 \qquad \textbf{(2)}$$

$$-\frac{6}{7}F_{AB} - \frac{6}{7}F_{AC} - \frac{12}{13}F + F_{AE} = 0 \qquad \textbf{(3)}$$



(a)

Let us assume that cable AB achieves maximum tension first. Substituting $F_{AB} = 1000$ lb into Eqs. (1) through (3) and solving yields

$F_{AC} = 666.67$ lb

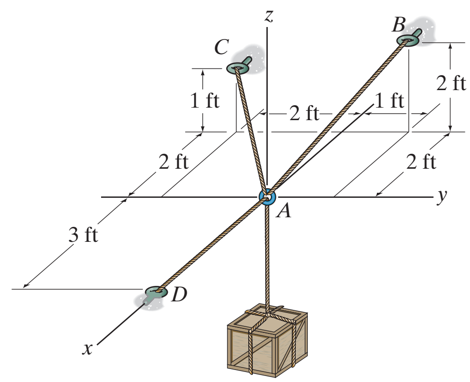
$F_{AE} = 2914$ lb $= 2.91$ kip $\qquad F = 1610$ lb $= 1.61$ kip **Ans.**

Since $F_{AC} = 666.67$ lb < 1000 lb, our assumption is correct.

Ans:
$F_{AE} = 2.91$ kip
$F = 1.61$ kip

3–66.

Determine the tension developed in cables AB, AC, and AD required for equilibrium of the 300-lb crate.



SOLUTION

Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. (*a*) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-2-0)\mathbf{i} + (1-0)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (1-0)^2 + (2-0)^2}}\right] = -\frac{2}{3}F_{AB}\,\mathbf{i} + \frac{1}{3}F_{AB}\,\mathbf{j} + \frac{2}{3}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(-2-0)\mathbf{i} + (-2-0)\mathbf{j} + (1-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (-2-0)^2 + (1-0)^2}}\right] = -\frac{2}{3}F_{AC}\,\mathbf{i} - \frac{2}{3}F_{AC}\,\mathbf{j} + \frac{1}{3}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\,\mathbf{i}$$

$$\mathbf{W} = [-300\mathbf{k}]\text{ lb}$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

$$\left(-\frac{2}{3}F_{AB}\,\mathbf{i} + \frac{1}{3}F_{AB}\,\mathbf{j} + \frac{2}{3}F_{AB}\,\mathbf{k}\right) + \left(-\frac{2}{3}F_{AC}\,\mathbf{i} - \frac{2}{3}F_{AC}\,\mathbf{j} + \frac{1}{3}F_{AC}\mathbf{k}\right) + F_{AD}\,\mathbf{i} + (-300\mathbf{k}) = \mathbf{0}$$

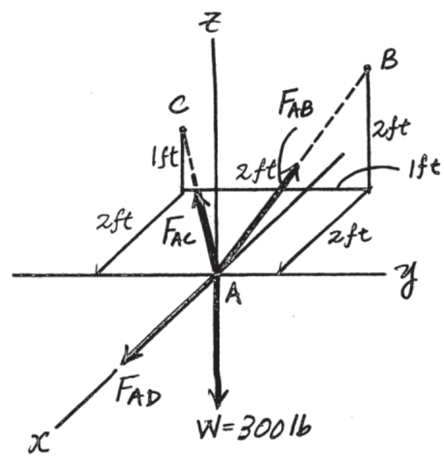
$$\left(-\frac{2}{3}F_{AB} - \frac{2}{3}F_{AC} + F_{AD}\right)\mathbf{i} + \left(\frac{1}{3}F_{AB} - \frac{2}{3}F_{AC}\right)\mathbf{j} + \left(\frac{2}{3}\mathrm{F}_{\mathrm{AB}} + \frac{1}{3}\mathrm{F}_{\mathrm{AC}} - 300\right)\mathbf{k} = \mathbf{0}$$

Equating the **i**, **j**, and **k** components yields

$$-\frac{2}{3}F_{AB} - \frac{2}{3}F_{AC} + F_{AD} = 0 \qquad \textbf{(1)}$$

$$\frac{1}{3}F_{AB} - \frac{2}{3}F_{AC} = 0 \qquad \textbf{(2)}$$

$$\frac{2}{3}F_{AB} + \frac{1}{3}F_{AC} - 300 = 0 \qquad \textbf{(3)}$$



Solving Eqs. (1) through (3) yields

$$F_{AB} = 360\text{ lb} \qquad \textbf{Ans.}$$

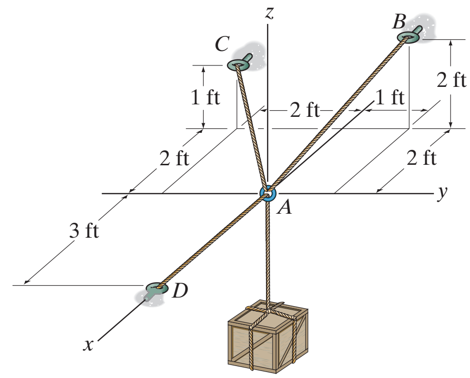
$$F_{AC} = 180\text{ lb} \qquad \textbf{Ans.}$$

$$F_{AD} = 360\text{ lb} \qquad \textbf{Ans.}$$

Ans:
$F_{AB} = 360$ lb
$F_{AC} = 180$ lb
$F_{AD} = 360$ lb

3–67.

Determine the maximum weight of the crate so that the tension developed in any cable does not exceed 450 lb.



SOLUTION

Force Vectors: We can express each of the forces on the free-body diagram shown in Fig. (*a*) in Cartesian vector form as

$$\mathbf{F}_{AB} = F_{AB}\left[\frac{(-2-0)\mathbf{i} + (1-0)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (1-0)^2 + (2-0)^2}}\right] = -\frac{2}{3}F_{AB}\,\mathbf{i} + \frac{1}{3}F_{AB}\,\mathbf{j} + \frac{2}{3}F_{AB}\,\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left[\frac{(-2-0)\mathbf{i} + (-2-0)\mathbf{j} + (1-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (-2-0)^2 + (1-0)^2}}\right] = -\frac{2}{3}F_{AC}\,\mathbf{i} - \frac{2}{3}F_{AC}\,\mathbf{j} + \frac{1}{3}F_{AC}\,\mathbf{k}$$

$$\mathbf{F}_{AD} = F_{AD}\,\mathbf{i}$$

$$\mathbf{W} = -Wk$$

Equations of Equilibrium: Equilibrium requires

$$\Sigma\mathbf{F} = \mathbf{0}; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}_{AD} + \mathbf{W} = \mathbf{0}$$

$$\left(-\frac{2}{3}F_{AB}\,\mathbf{i} + \frac{1}{3}F_{AB}\,\mathbf{j} + \frac{2}{3}F_{AB}\mathbf{k}\right) + \left(-\frac{2}{3}F_{AC}\,\mathbf{i} - \frac{2}{3}F_{AC}\,\mathbf{j} + \frac{1}{3}F_{AC}\mathbf{k}\right) + F_{AD}\,\mathbf{i} + (-W\mathbf{k}) = \mathbf{0}$$

$$\left(-\frac{2}{3}F_{AB} - \frac{2}{3}F_{AC} + F_{AD}\right)\mathbf{i} + \left(\frac{1}{3}F_{AB} - \frac{2}{3}F_{AC}\right)\mathbf{j} + \left(\frac{2}{3}F_{AB} + \frac{1}{3}F_{AC} - W\right)\mathbf{k} = \mathbf{0}$$

Equating the **i, j**, and **k** components yields

$$-\frac{2}{3}F_{AB} - \frac{2}{3}F_{AC} + F_{AD} = 0 \qquad \textbf{(1)}$$

$$\frac{1}{3}F_{AB} - \frac{2}{3}F_{AC} = 0 \qquad \textbf{(2)}$$

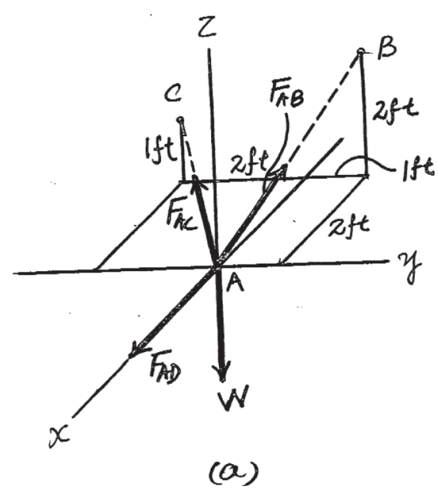
$$\frac{2}{3}F_{AB} + \frac{1}{3}F_{AC} - W = 0 \qquad \textbf{(3)}$$

Let us assume that cable AB achieves maximum tension first. Substituting $F_{AB} = 450$ lb into Eqs. (1) through (3) and solving, yields

$F_{AC} = 225$ lb $\qquad F_{AD} = 450$ lb

$W = 375$ lb **Ans.**

Since $F_{AC} = 225 \text{ lb} < 450 \text{ lb}$, our assumption is correct.



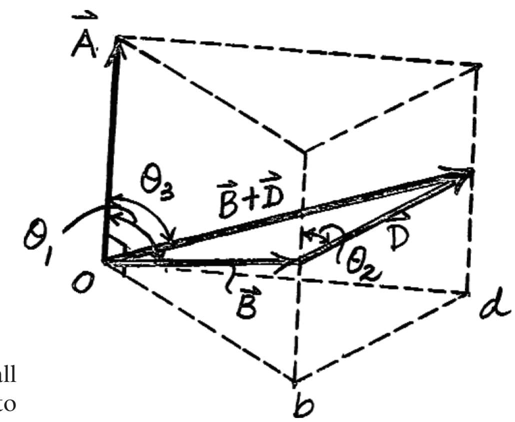
(a)

Ans:
$W = 375$ lb

4–1.

If **A**, **B**, and **D** are given vectors, prove the distributive law for the vector cross product, i.e., $\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D})$.

SOLUTION



Consider the three vectors; with **A** vertical.

Note obd is perpendicular to **A**.

$$od = |\mathbf{A} \times (\mathbf{B} + \mathbf{D})| = |\mathbf{A}||\mathbf{B} + \mathbf{D}| \sin \theta_3$$

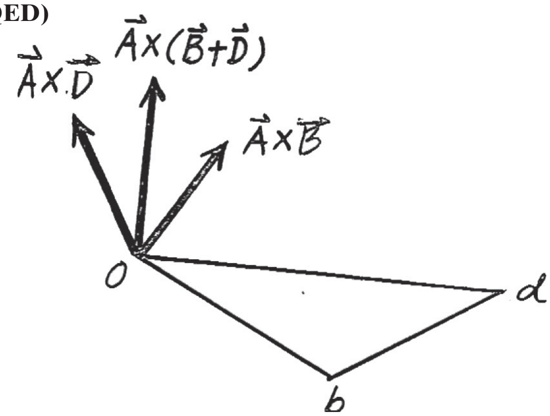
$$ob = |\mathbf{A} \times \mathbf{B}| = |\mathbf{A}||\mathbf{B}| \sin \theta_1$$

$$bd = |\mathbf{A} \times \mathbf{D}| = |\mathbf{A}||\mathbf{D}| \sin \theta_2$$

Also, these three cross products all lie in the plane obd since they are all perpendicular to **A**. As noted the magnitude of each cross product is proportional to the length of each side of the triangle.

The three vector cross products also form a closed triangle $o'b'd'$ which is similar to triangle obd. Thus from the figure,

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D}) \qquad \textbf{(QED)}$$



Note also,

$$\mathbf{A} = A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\,\mathbf{k}$$

$$\mathbf{B} = B_x\,\mathbf{i} + B_y\,\mathbf{j} + B_z\,\mathbf{k}$$

$$\mathbf{D} = D_x\,\mathbf{i} + D_y\,\mathbf{j} + D_z\,\mathbf{k}$$

$$\mathbf{A} \times (\mathbf{B} + \mathbf{D}) = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x + D_x & B_y + D_y & B_z + D_z \end{vmatrix}$$

$$= [A_y(B_z + D_z) - A_z(B_y + D_y)]\mathbf{i}$$

$$- [A_x(B_z + D_z) - A_z(B_x + D_x)]\mathbf{j}$$

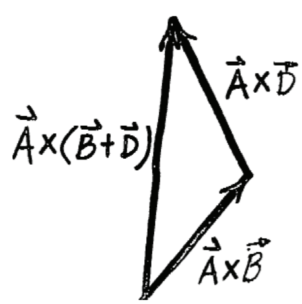
$$+ [A_x(B_y + D_y) - A_y(B_x + D_x)]\mathbf{k}$$

$$= [(A_y B_z - A_z B_y)\mathbf{i} - (A_x B_z - A_z B_x)]\mathbf{j} + (A_x B_y - A_y B_x)\mathbf{k}$$

$$+ [(A_y D_z - A_z D_y)\mathbf{i} - (A_x D_z - A_z D_x)\mathbf{j} + (A_x D_y - A_y D_x)\mathbf{k}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_x & A_y & A_z \\ D_x & D_y & D_z \end{vmatrix}$$

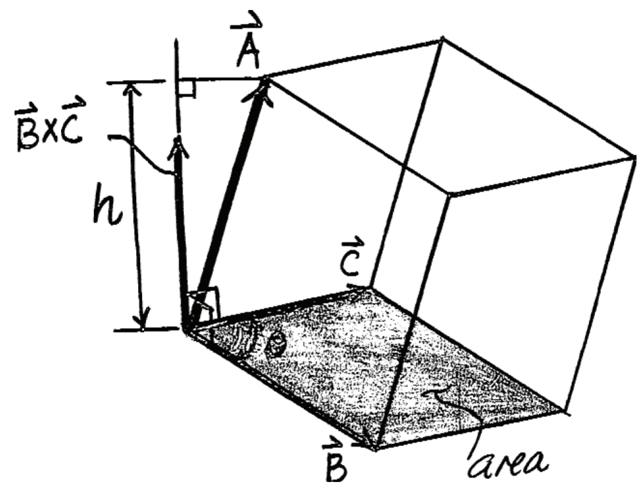
$$= (\mathbf{A} \times \mathbf{B}) + (\mathbf{A} \times \mathbf{D}) \qquad \textbf{(QED)}$$



4–2.

Prove the triple scalar product identity $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}$.

SOLUTION



As shown in the figure

$$\text{Area} = B(C\sin\theta) = |\mathbf{B}\times\mathbf{C}|$$

Thus,

$$\text{Volume of parallelepiped is } |\mathbf{B}\times\mathbf{C}||h|$$

But,

$$|h| = |\mathbf{A}\cdot\mathbf{u}_{(\mathbf{B}\times\mathbf{C})}| = \left|\mathbf{A}\cdot\left(\frac{\mathbf{B}\times\mathbf{C}}{|\mathbf{B}\times\mathbf{C}|}\right)\right|$$

Thus,

$$\text{Volume} = |\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})|$$

Since $|(\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}|$ represents this same volume then

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C} \qquad \textbf{(QED)}$$

Also,

$$LHS = \mathbf{A}\cdot(\mathbf{B}\times\mathbf{C})$$

$$= (A_x\,\mathbf{i} + A_y\,\mathbf{j} + A_z\,\mathbf{k})\cdot\begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ B_x & B_y & B_z\\ C_x & C_y & C_z\end{vmatrix}$$

$$= A_x(B_yC_z - B_zC_y) - A_y(B_xC_z - B_zC_x) + A_z(B_xC_y - B_yC_x)$$

$$= A_xB_yC_z - A_xB_zC_y - A_yB_xC_z + A_yB_zC_x + A_zB_xC_y - A_zB_yC_x$$

$$RHS = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C}$$

$$= \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\ A_x & A_y & A_z\\ B_x & B_y & B_z\end{vmatrix}\cdot(C_x\,\mathbf{i} + C_y\,\mathbf{j} + C_z\,\mathbf{k})$$

$$= C_x(A_y B_z - A_zB_y) - C_y(A_xB_z - A_zB_x) + C_z(A_xB_y - A_yB_x)$$

$$= A_xB_yC_z - A_xB_zC_y - A_yB_xC_z + A_yB_zC_x + A_zB_xC_y - A_zB_yC_x$$

Thus, $LHS = RHS$

$$\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = (\mathbf{A}\times\mathbf{B})\cdot\mathbf{C} \qquad \textbf{(QED)}$$

4–3.

Given the three nonzero vectors **A**, **B**, and **C**, show that if $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$, the three vectors *must* lie in the same plane.

SOLUTION

Consider,

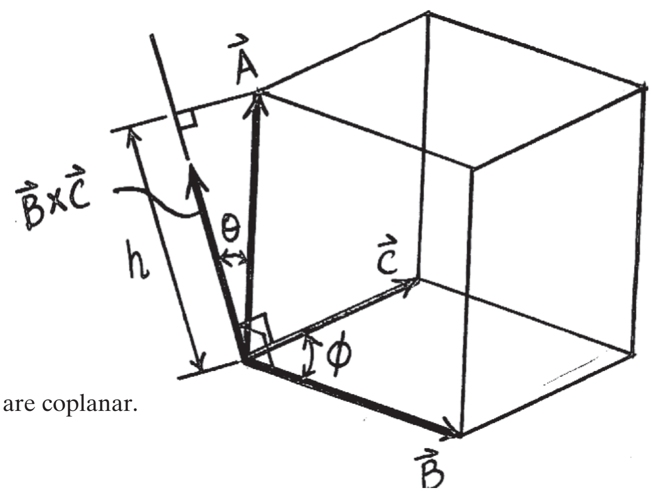
$$|\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C})| = |\mathbf{A}|\,|\mathbf{B} \times \mathbf{C}\,| \cos\theta$$

$$= (|\mathbf{A}| \cos\theta)|\mathbf{B} \times \mathbf{C}|$$

$$= |h|\,|\mathbf{B} \times \mathbf{C}|$$

$$= BC\,|h| \sin\phi$$

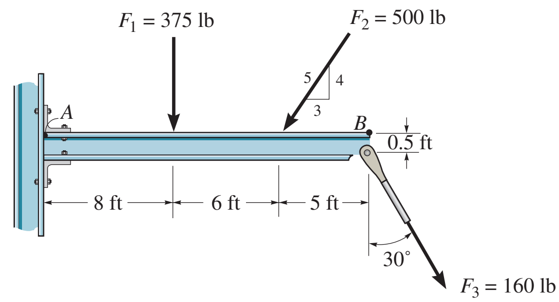
$$= \text{volume of parallelepiped.}$$



If $\mathbf{A} \cdot (\mathbf{B} \times \mathbf{C}) = 0$, then the volume equals zero, so that **A, B,** and **C** are coplanar.

***4–4.**

Determine the moment about point A of each of the three forces acting on the beam.



SOLUTION

$$\zeta + (M_{F_1})_A = -375(8)$$

$$= -3000 \text{ lb}\cdot\text{ft} = 3.00 \text{ kip}\cdot\text{ft} \ \textbf{\textit{(Clockwise)}}$$ **Ans.**

$$\zeta + (M_{F_2})_A = -500\left(\frac{4}{5}\right)(14)$$

$$= -5600 \text{ lb}\cdot\text{ft} = 5.60 \text{ kip}\cdot\text{ft} \ \textbf{\textit{(Clockwise)}}$$ **Ans.**

$$\zeta + (M_{F_3})_A = -160(\cos 30^\circ)(19) + 160 \sin 30^\circ(0.5)$$

$$= -2593 \text{ lb}\cdot\text{ft} = 2.59 \text{ kip}\cdot\text{ft} \ \textbf{\textit{(Clockwise)}}$$ **Ans.**

Ans:

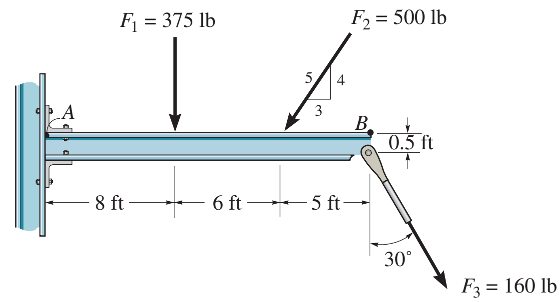
$(M_{F_1})_A = 3.00 \text{ kip}\cdot\text{ft}$ ***(Clockwise)***

$(M_{F_2})_A = 5.60 \text{ kip}\cdot\text{ft}$ ***(Clockwise)***

$(M_{F_3})_A = 2.59 \text{ kip}\cdot\text{ft}$ ***(Clockwise)***

4–5.

Determine the moment about point B of each of the three forces acting on the beam.



SOLUTION

$$\circlearrowleft +(M_{F_1})_B = 375(11)$$

$$= 4125 \text{ lb} \cdot \text{ft} = 4.125 \text{ kip} \cdot \text{ft} \quad \textit{(Counterclockwise)}$$ **Ans.**

$$\circlearrowleft +(M_{F_2})_B = 500\left(\frac{4}{5}\right)(5)$$

$$= 2000 \text{ lb} \cdot \text{ft} = 2.00 \text{ kip} \cdot \text{ft} \quad \textit{(Counterclockwise)}$$ **Ans.**

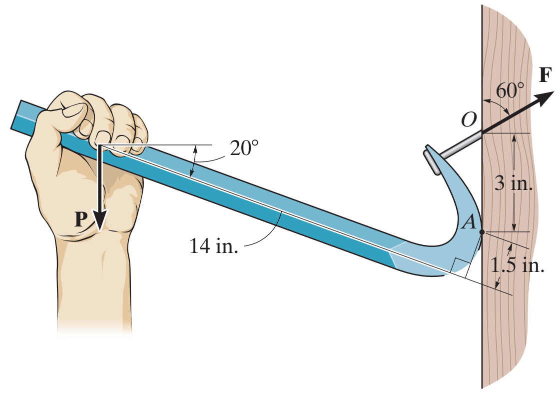
$$\circlearrowleft +(M_{F_3})_B = 160 \sin 30°(0.5) - 160 \cos 30°(0)$$

$$= 40.0 \text{ lb} \cdot \text{ft} \quad \textit{(Counterclockwise)}$$ **Ans.**

Ans:
$(M_{F_1})_B = 4.125 \text{ kip} \cdot \text{ft} \circlearrowleft$
$(M_{F_2})_B = 2.00 \text{ kip} \cdot \text{ft} \circlearrowleft$
$(M_{F_3})_B = 40.0 \text{ lb} \cdot \text{ft} \circlearrowleft$

4–6.

The crowbar is subjected to a vertical force of $P = 25$ lb at the grip, whereas it takes a force of $F = 155$ lb at the claw to pull the nail out. Find the moment of each force about point A and determine if **P** is sufficient to pull out the nail. The crowbar contacts the board at point A.



SOLUTION

$\circlearrowleft + M_P = 25(14 \cos 20° + 1.5 \sin 20°) = 341 \text{ in} \cdot \text{lb}$ ***(Counterclockwise)***

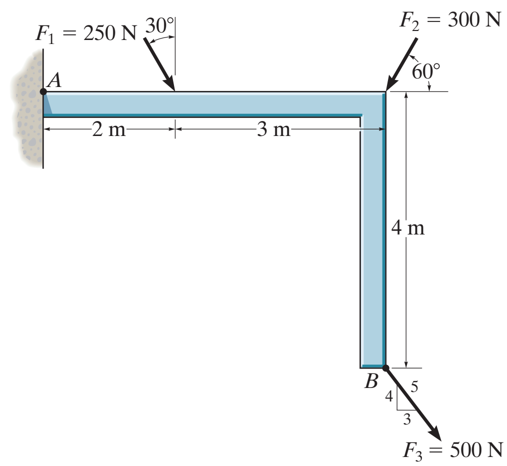
$\circlearrowright + M_F = 155 \sin 60°(3) = 403 \text{ in} \cdot \text{lb}$ ***(Clockwise)***

Since $M_F > M_P$, $P = 25$ lb is **not sufficient** to pull out the nail. **Ans.**

Ans:
$M_P = 341 \text{ in.} \cdot \text{lb} \circlearrowleft$
$M_F = 403 \text{ in.} \cdot \text{lb} \circlearrowright$
Not sufficient

4–7.

Determine the moment of each of the three forces about point A.



SOLUTION

The moment arm measured perpendicular to each force from point A is

$$d_1 = 2 \sin 60° = 1.732 \text{ m}$$

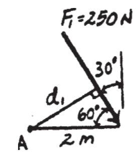
$$d_2 = 5 \sin 60° = 4.330 \text{ m}$$

$$d_3 = 2 \sin 53.13° = 1.60 \text{ m}$$

Using each force where $M_A = Fd$, we have

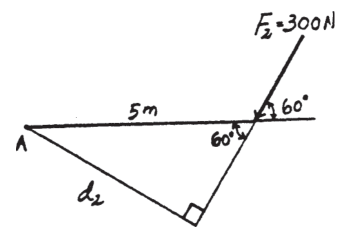
$\circlearrowleft + (M_{F_1})_A = -250(1.732)$

$= -433 \text{ N}\cdot\text{m} = 433 \text{ N}\cdot\text{m}$ ***(Clockwise)*** **Ans.**



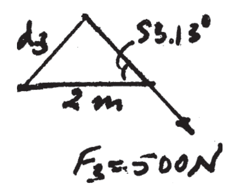
$\circlearrowleft + (M_{F_2})_A = -300(4.330)$

$= -1299 \text{ N}\cdot\text{m} = 1.30 \text{ kN}\cdot\text{m}$ ***(Clockwise)*** **Ans.**



$\circlearrowleft + (M_{F_3})_A = -500(1.60)$

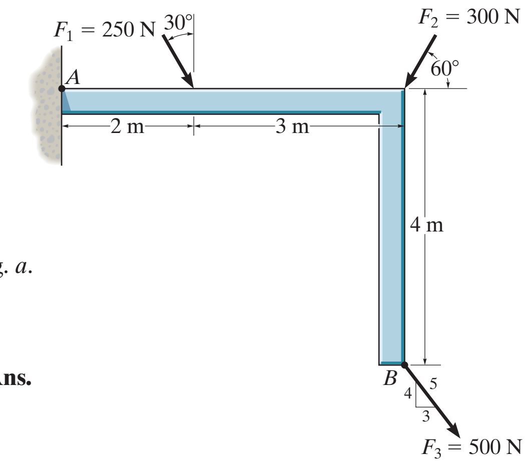
$= -800 \text{ N}\cdot\text{m} = 800 \text{ N}\cdot\text{m}$ ***(Clockwise)*** **Ans.**



Ans:
$(M_{F_1})_A = 433 \text{ N}\cdot\text{m} \circlearrowright$
$(M_{F_2})_A = 1.30 \text{ kN}\cdot\text{m} \circlearrowright$
$(M_{F_3})_A = 800 \text{ N}\cdot\text{m} \circlearrowright$

***4–8.**

Determine the moment of each of the three forces about point B.



SOLUTION

The forces are resolved into horizontal and vertical component as shown in Fig. *a*. For $\mathbf{F}_1$,

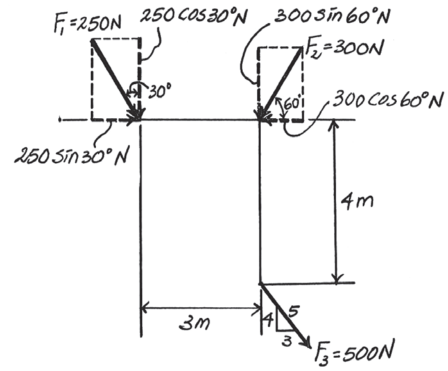
$$\circlearrowleft + M_B = 250 \cos 30°(3) - 250 \sin 30°(4)$$
$$= 149.51 \text{ N} \cdot \text{m} = 150 \text{ N} \cdot \text{m} \circlearrowleft \qquad \textbf{Ans.}$$

For $\mathbf{F}_2$,

$$\circlearrowleft + M_B = 300 \sin 60°(0) + 300 \cos 60°(4)$$
$$= 600 \text{ N} \cdot \text{m} \circlearrowleft \qquad \textbf{Ans.}$$

Since the line of action of $\mathbf{F}_3$ passes through B, its moment arm about point B is zero. Thus

$$M_B = 0 \qquad \textbf{Ans.}$$

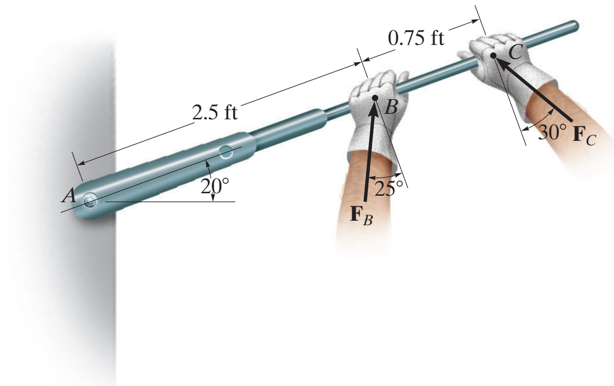


(a)

Ans:
$M_B = 150 \text{ N} \cdot \text{m} \circlearrowleft$
$M_B = 600 \text{ N} \cdot \text{m} \circlearrowleft$
$M_B = 0$

4–9.

Determine the moment of each force about the bolt located at A. Take $F_B = 40$ lb, $F_C = 50$ lb.



SOLUTION

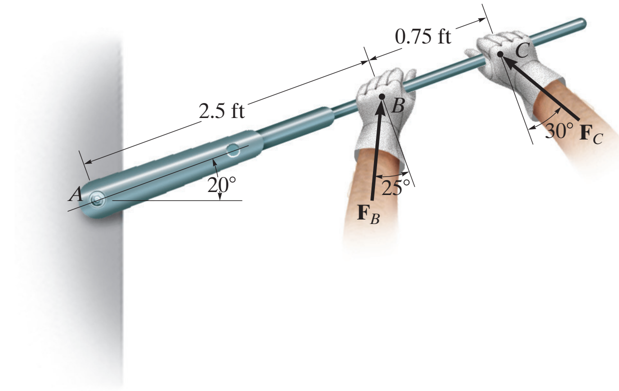
$$\circlearrowleft + M_B = 40 \cos 25°(2.5) = 90.6 \text{ lb} \cdot \text{ft} \circlearrowright$$ **Ans.**

$$\circlearrowleft + M_C = 50 \cos 30°(3.25) = 141 \text{ lb} \cdot \text{ft} \circlearrowright$$ **Ans.**

Ans:
$M_B = 90.6 \text{ lb} \cdot \text{ft} \circlearrowright$
$M_C = 141 \text{ lb} \cdot \text{ft} \circlearrowright$

4–10.

If $F_B = 30$ lb and $F_C = 45$ lb, determine the resultant moment about the bolt located at A.



SOLUTION

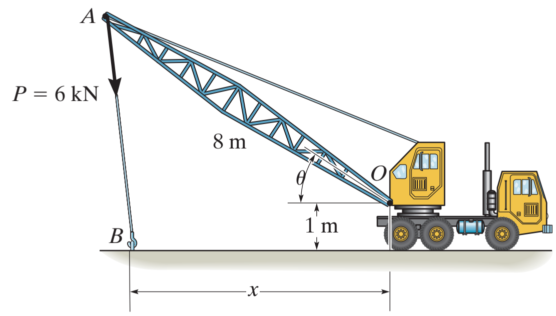
$$\circlearrowleft + M_A = 30 \cos 25°(2.5) + 45 \cos 30°(3.25)$$

$$= 195 \text{ lb} \cdot \text{ft} \circlearrowleft$$

Ans:
$M_A = 195 \text{ lb} \cdot \text{ft} \circlearrowleft$

4–11.

The towline exerts a force of $P = 6$ kN at the end of the 8-m-long crane boom. If $\theta = 30°$, determine the placement x of the hook at B so that this force creates a maximum moment about point O. What is this moment?



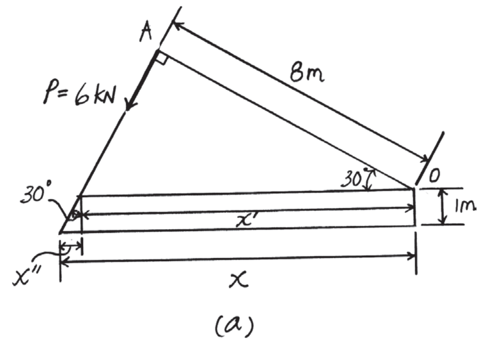
SOLUTION

In order to produce the maximum moment about point O, **P** must act perpendicular to the boom's axis OA as shown in Fig. a. Thus

$$\circlearrowleft + (M_O)_{max} = 6\,(8) = 48.0 \text{ kN} \cdot \text{m (counterclockwise)}$$ **Ans.**

Referring to the geometry of Fig. a,

$$x = x' + x'' = \frac{8}{\cos 30°} + \tan 30° = 9.814 \text{ m} = 9.81 \text{ m}$$ **Ans.**

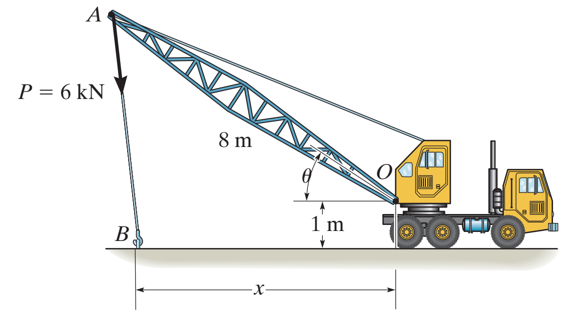


(a)

Ans:
$(M_O)_{max} = 48.0$ kN · m $\circlearrowleft$
$x = 9.81$ m

***4–12.**

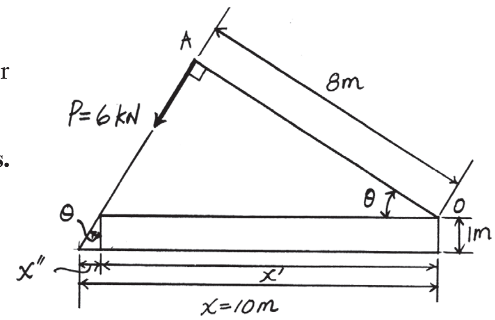
The towline exerts a force of $P = 6$ kN at the end of the 8-m-long crane boom. If $x = 10$ m, determine the position θ of the boom so that this force creates a maximum moment about point O. What is this moment?



SOLUTION

In order to produce the maximum moment about point O, **P** must act perpendicular to the boom's axis OA as shown in Fig. a. Thus,

$$\circlearrowleft + (M_O)_{max} = 6\,(8) = 48.0\ \text{kN}\cdot\text{m (counterclockwise)} \qquad \textbf{Ans.}$$



(a)

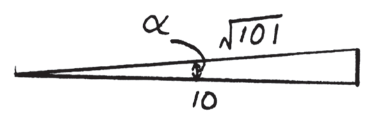
Referring to the geometry of Fig. a,

$$x = x' + x''; \qquad 10 = \frac{8}{\cos\theta} + \tan\theta$$

$$10 = \frac{8}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$$

$$10\cos\theta - \sin\theta = 8$$

$$\frac{10}{\sqrt{101}}\cos\theta - \frac{1}{\sqrt{101}}\sin\theta = \frac{8}{\sqrt{101}} \qquad \textbf{(1)}$$



(b)

From the geometry shown in Fig. b,

$$\alpha = \tan^{-1}\left(\frac{1}{10}\right) = 5.711^\circ$$

$$\sin\alpha = \frac{1}{\sqrt{101}} \qquad \cos\alpha = \frac{10}{\sqrt{101}}$$

Then Eq (1) becomes

$$\cos\theta\cos 5.711^\circ - \sin\theta\sin 5.711^\circ = \frac{8}{\sqrt{101}}$$

Referring that $\cos(\theta + 5.711^\circ) = \cos\theta\cos 5.711^\circ - \sin\theta\sin 5.711^\circ$

$$\cos(\theta + 5.711^\circ) = \frac{8}{\sqrt{101}}$$

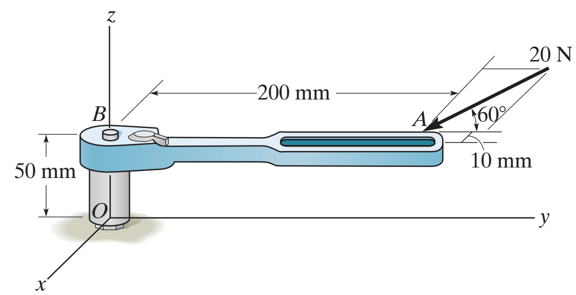
$$\theta + 5.711^\circ = 37.247^\circ$$

$$\theta = 31.54^\circ = 31.5^\circ \qquad \textbf{Ans.}$$

Ans:
$(M_O)_{max} = 48.0$ kN·m (counterclockwise)
$\theta = 31.5^\circ$

4–13.

The 20-N horizontal force acts on the handle of the socket wrench. What is the moment of this force about point B. Specify the coordinate direction angles α, β, γ of the moment axis.



SOLUTION

Force Vector And Position Vector. Referring to Fig. a,

$$F = 20\,(\sin 60°\mathbf{i} - \cos 60°\mathbf{j}) = \{17.32\mathbf{i} - 10\mathbf{j}\}\text{ N}$$

$$\mathbf{r}_{BA} = \{-0.01\mathbf{i} + 0.2\mathbf{j}\}\text{ m}$$

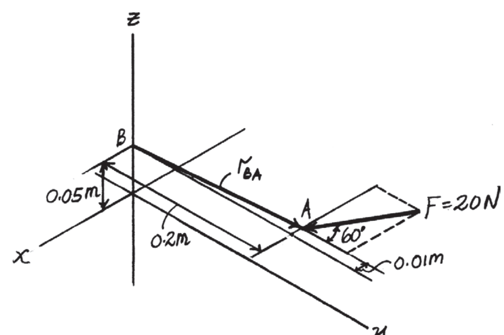
Moment of Force *F* about point *B*.

$$\mathbf{M}_B = \mathbf{r}_{BA} \times F$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.01 & 0.2 & 0 \\ 17.32 & -10 & 0 \end{vmatrix}$$

$$= \{-3.3641\,\mathbf{k}\}\text{ N}\cdot\text{m}$$

$$= \{-3.36\,\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

Here the unit vector for $\mathbf{M}_B$ is $\mathbf{u} = -\mathbf{k}$. Thus, the coordinate direction angles of $\mathbf{M}_B$ are

$$\alpha = \cos^{-1} 0 = 90° \qquad \textbf{Ans.}$$

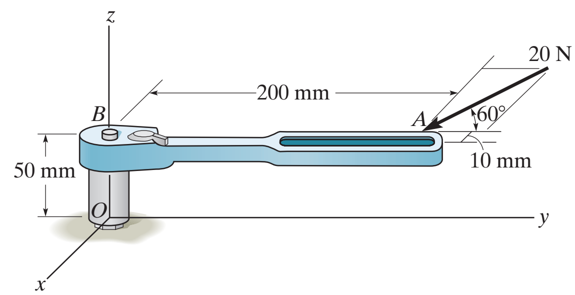
$$\beta = \cos^{-1} 0 = 90° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}(-1) = 108° \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_B = \{-3.36\,\mathbf{k}\}\text{ N}\cdot\text{m}$
$\alpha = 90°$
$\beta = 90°$
$\gamma = 180°$

4–14.

The 20-N horizontal force acts on the handle of the socket wrench. Determine the moment of this force about point O. Specify the coordinate direction angles α, β, γ of the moment axis.

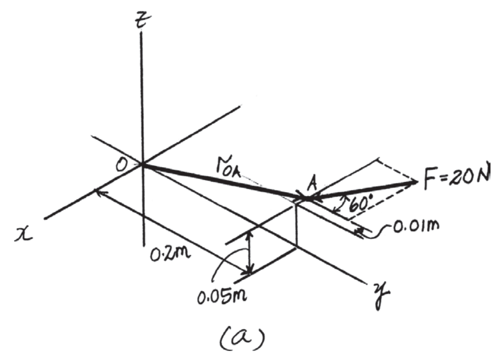


SOLUTION

Force Vector And Position Vector. Referring to Fig. *a*,

$$\mathbf{F} = 20\,(\sin 60°\mathbf{i} - \cos 60°\mathbf{j}) = \{17.32\mathbf{i} - 10\mathbf{j}\}\text{ N}$$

$$\mathbf{r}_{OA} = \{-0.01\mathbf{i} + 0.2\mathbf{j} + 0.05\mathbf{k}\}\text{ m}$$



Moment of *F* About point *O*.

$$\mathbf{M}_O = \mathbf{r}_{OA} \times F$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.01 & 0.2 & 0.05 \\ 17.32 & -10 & 0 \end{vmatrix}$$

$$= \{0.5\mathbf{i} + 0.8660\mathbf{j} - 3.3641\mathbf{k}\}\text{ N}\cdot\text{m}$$

$$= \{0.5\mathbf{i} + 0.866\mathbf{j} - 3.36\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

The magnitude of $\mathbf{M}_O$ is

$$M_O = \sqrt{(M_O)_x^2 + (M_O)_y^2 + (M_O)_z^2} = \sqrt{0.5^2 + 0.8660^2 + (-3.3641)^2}$$

$$= 3.5096\text{ N}\cdot\text{m}$$

Thus, the coordinate direction angles of $\mathbf{M}_O$ are

$$\alpha = \cos^{-1}\left[\frac{(M_O)_x}{M_O}\right] = \cos^{-1}\left(\frac{0.5}{3.5096}\right) = 81.81° = 81.8° \qquad \textbf{Ans.}$$

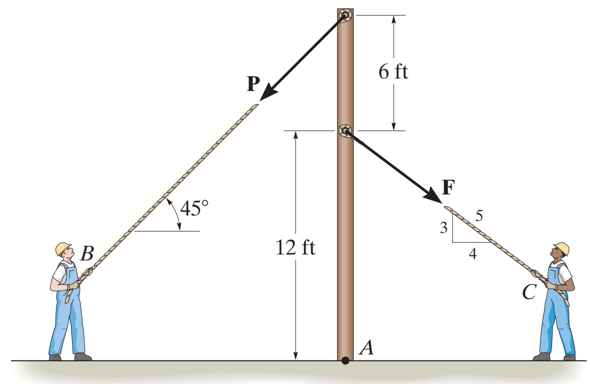
$$\beta = \cos^{-1}\left[\frac{(M_O)_y}{M_O}\right] = \cos^{-1}\left(\frac{0.8660}{3.5096}\right) = 75.71° = 75.7° \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left[\frac{(M_O)_z}{M_O}\right] = \cos^{-1}\left(\frac{-3.3641}{3.5096}\right) = 163.45° = 163° \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_O = \{0.5\mathbf{i} + 0.866\mathbf{j} - 3.36\mathbf{k}\}\text{ N}\cdot\text{m}$
$\alpha = 81.8°$
$\beta = 75.7°$
$\gamma = 163°$

4–15.

Two men exert forces of $F = 80$ lb and $P = 50$ lb on the ropes. Determine the moment of each force about A. Which way will the pole rotate, clockwise or counterclockwise?



SOLUTION

$$\circlearrowright + (M_A)_C = 80\left(\frac{4}{5}\right)(12) = 768 \text{ lb} \cdot \text{ft} \circlearrowright$$ **Ans.**

$$\circlearrowleft + (M_A)_B = 50\,(\cos 45°)(18) = 636 \text{ lb} \cdot \text{ft} \circlearrowleft$$ **Ans.**

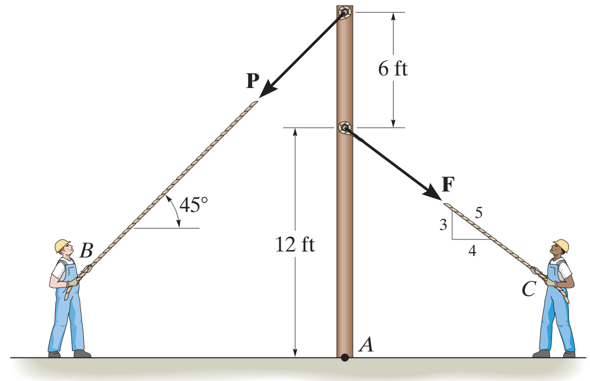
Since $(M_A)_C > (M_A)_B$

Clockwise **Ans.**

Ans:
$(M_A)_C = 768 \text{ lb} \cdot \text{ft} \circlearrowright$
$(M_A)_B = 636 \text{ lb} \cdot \text{ft} \circlearrowleft$
Clockwise

***4–16.**

If the man at B exerts a force of $P = 30$ lb on his rope, determine the magnitude of the force **F** the man at C must exert to prevent the pole from rotating, i.e., so the resultant moment about A of both forces is zero.



SOLUTION

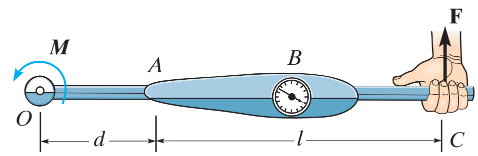
$$\circlearrowleft + \qquad 30\,(\cos 45°)(18) = F\left(\frac{4}{5}\right)(12) = 0$$

$$F = 39.8 \text{ lb}$$ **Ans.**

Ans:
$F = 39.8$ lb

4–17.

The torque wrench ABC is used to measure the moment or torque applied to a bolt when the bolt is located at A and a force is applied to the handle at C. The mechanic reads the torque on the scale at B. If an extension AO of length d is used on the wrench, determine the required scale reading if the desired torque on the bolt at O is to be M.



SOLUTION

Moment at $A = m = Fl$

Moment at $O = M = (d + l)F$

$$M = (d + l)\frac{m}{l}$$

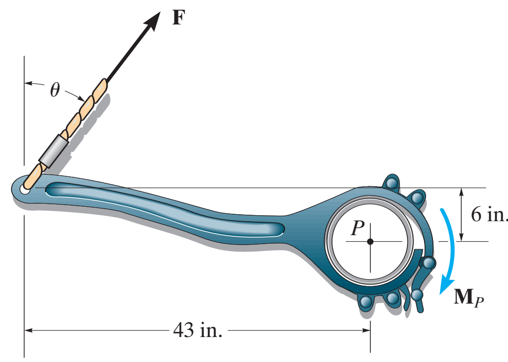
$$m = \left(\frac{l}{d + l}\right)M \qquad \textbf{Ans.}$$

Ans:

$$m = \left(\frac{l}{d + l}\right)M$$

4–18.

The tongs are used to grip the ends of the drilling pipe P. Determine the torque (moment) M_P that the applied force $F = 150$ lb exerts on the pipe about point P as a function of θ. Plot this moment M_P versus θ for $0 \leq \theta \leq 90°$.



SOLUTION

$$M_P = 150 \cos\theta(43) + 150 \sin\theta(6)$$

$$= (6450 \cos\theta + 900 \sin\theta) \text{ lb} \cdot \text{in.}$$

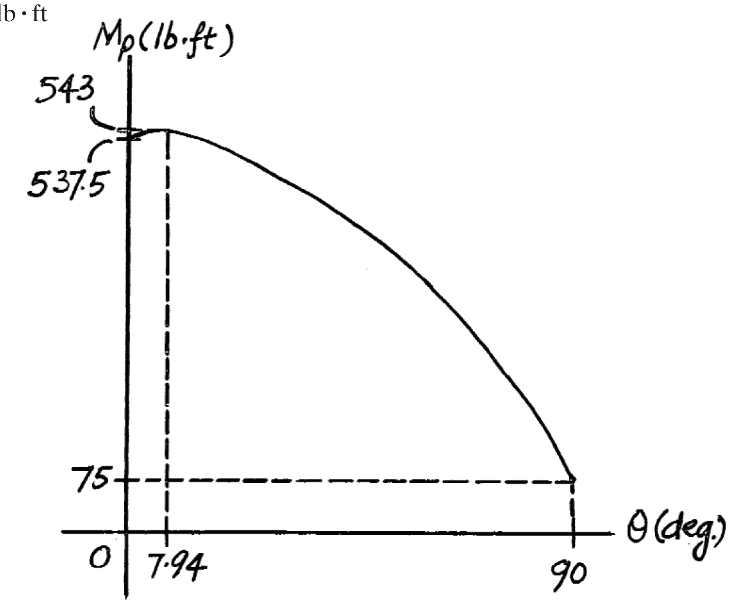
$$= (537.5 \cos\theta + 75 \sin\theta) \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$

$$\frac{dM_P}{d\theta} = -537.5 \sin\theta + 75 \cos\theta = 0 \qquad \tan\theta = \frac{75}{537.5} \qquad \theta = 7.943°$$

At $\theta = 7.943°$, M_P is maximum.

$$(M_P)_{max} = 538 \cos 7.943° + 75 \sin 7.943° = 543 \text{ lb} \cdot \text{ft}$$

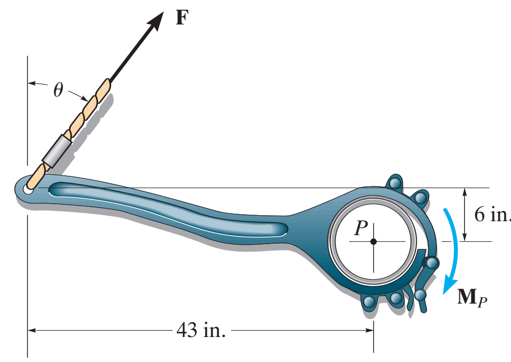
$$\text{Also } (M_P)_{max} = 150 \text{ lb}\left(\left(\frac{43}{12}\right)^2 + \left(\frac{6}{12}\right)^2\right)^{\frac{1}{2}} = 543 \text{ lb} \cdot \text{ft}$$



Ans:
$M_P = (537.5 \cos\theta + 75 \sin\theta) \text{ lb} \cdot \text{ft}$

4–19.

The tongs are used to grip the ends of the drilling pipe P. If a torque (moment) of $M_P = 800 \text{ lb} \cdot \text{ft}$ is needed at P to turn the pipe, determine the cable force F that must be applied to the tongs. Set $\theta = 30°$.



SOLUTION

$$M_P = F \cos 30°(43) + F \sin 30°(6) \qquad \text{Set } M_P = 800(12) \text{ lb} \cdot \text{in.}$$

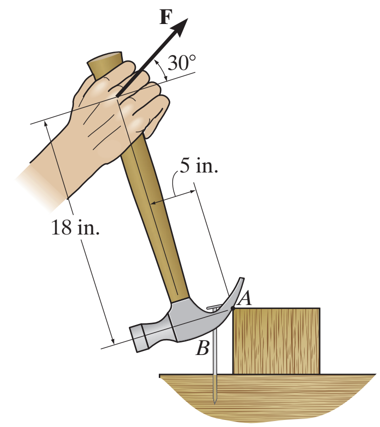
$$800(12) = F \cos 30°(43) + F \sin 30°(6)$$

$$F = 239 \text{ lb}$$ **Ans.**

Ans:
$F = 239$ lb

***4–20.**

The handle of the hammer is subjected to the force of $F = 20$ lb. Determine the moment of this force about the point A.

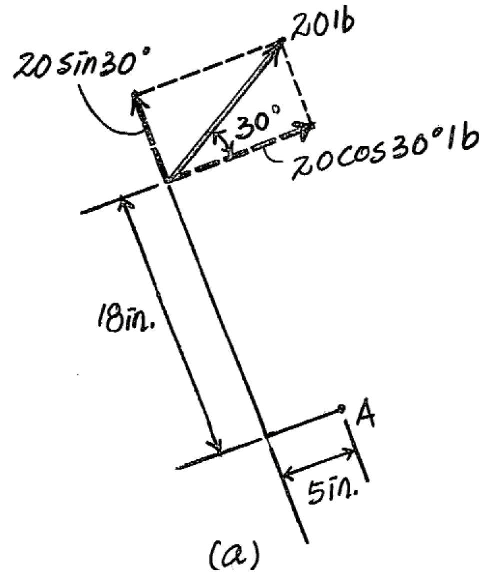


SOLUTION

Resolving the 20-lb force into components parallel and perpendicular to the hammer, Fig. a, and applying the principle of moments,

$$\circlearrowleft + M_A = -20\cos 30°(18) - 20\sin 30°(5)$$

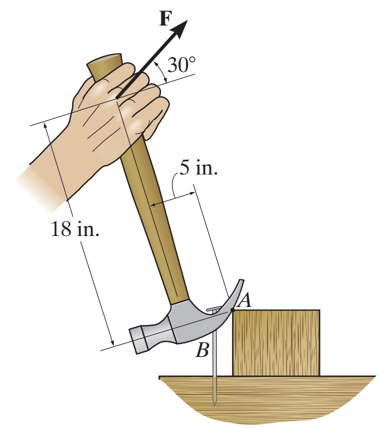
$$= -361.77 \text{ lb}\cdot\text{in} = 362 \text{ lb}\cdot\text{in} \quad \textbf{(\textit{Clockwise})} \qquad \textbf{Ans.}$$



Ans:
$M_A = 362 \text{ lb}\cdot\text{in}$ **(*Clockwise*)**

4–21.

In order to pull out the nail at B, the force **F** exerted on the handle of the hammer must produce a clockwise moment of 500 lb · in. about point A. Determine the required magnitude of force **F**.

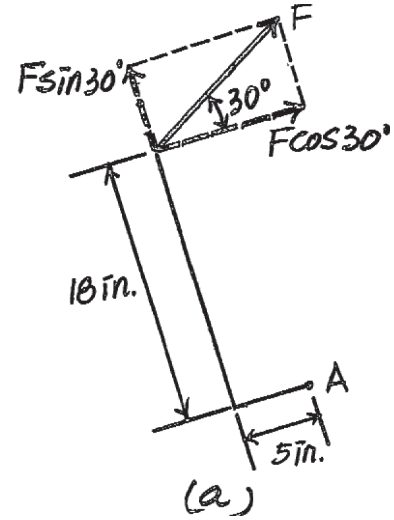


SOLUTION

Resolving force **F** into components parallel and perpendicular to the hammer, Fig. a, and applying the principle of moments,

$$\circlearrowleft +M_A = -500 = -F\cos 30°(18) - F\sin 30°(5)$$

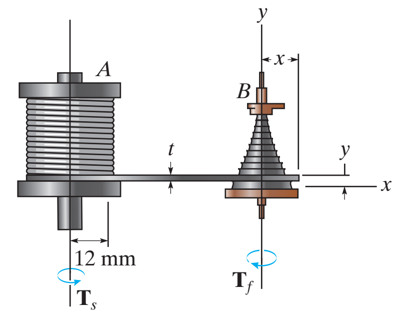
$$F = 27.6\text{ lb} \qquad \textbf{Ans.}$$



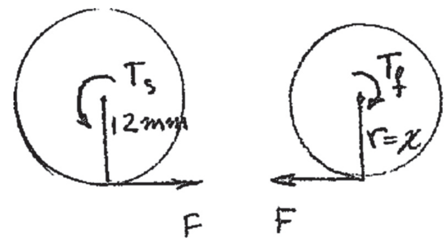
Ans:
$F = 27.6$ lb

4–22.

Old clocks were constructed using a *fusee* B to drive the gears and watch hands. The purpose of the fusee is to increase the leverage developed by the mainspring A as it uncoils and thereby loses some of its tension. The mainspring can develop a torque (moment) $T_s = k\theta$, where $k = 0.015\ \mathrm{N \cdot m/rad}$ is the torsional stiffness and θ is the angle of twist of the spring in radians. If the torque T_f developed by the fusee is to remain constant as the mainspring winds down, and $x = 10$ mm when $\theta = 4$ rad, determine the required radius of the fusee when $\theta = 3$ rad.



SOLUTION



When $\theta = 4$ rad, $r = 10$ mm

$T_s = 0.015(4) = 0.06\ \mathrm{N \cdot m}$

$F = \dfrac{0.06}{0.012} = 5\ \mathrm{N}$

$T_f = 5(0.010) = 0.05\ \mathrm{N \cdot m}$ (constant)

When $\theta = 3$ rad,

$T_s = 0.015(3) = 0.045\ \mathrm{N \cdot m}$

$F = \dfrac{0.045}{0.012} = 3.75\ \mathrm{N}$

For the fusee require

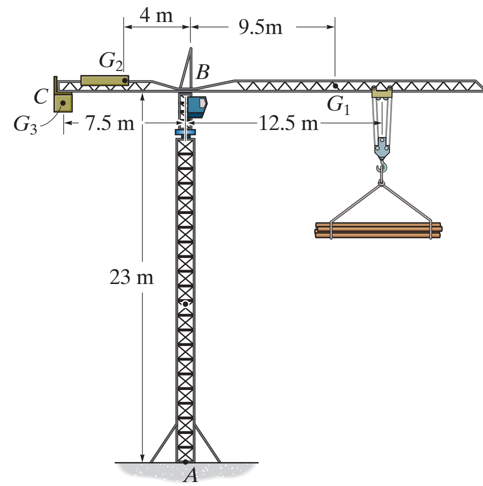
$0.05 = 3.75\, r$

$r = 0.0133\ \mathrm{m} = 13.3\ \mathrm{mm}$ **Ans.**

Ans:
$r = 13.3$ mm

4–23.

The tower crane is used to hoist the 2-Mg load upward at constant velocity. The 1.5-Mg jib BD, 0.5-Mg jib BC, and 6-Mg counterweight C have centers of mass at G_1, G_2, and G_3, respectively. Determine the resultant moment produced by the load and the weights of the tower crane jibs about point A and about point B.



SOLUTION

Since the moment arms of the weights and the load measured to points A and B are the same, the resultant moments produced by the load and the weight about points A and B are the same.

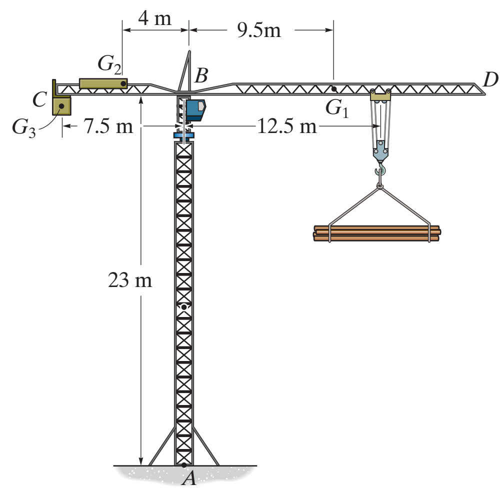
$$\circlearrowleft + (M_R)_A = (M_R)_B = \Sigma Fd; \qquad (M_R)_A = (M_R)_B = 6000(9.81)(7.5) + 500(9.81)(4) - 1500(9.81)(9.5)$$
$$- 2000(9.81)(12.5) = 76\,027.5 \text{ N}\cdot\text{m} = 76.0 \text{ kN}\cdot\text{m} \textbf{ (\textit{Counterclockwise})}$$

Ans.

Ans:
$(M_R)_A = (M_R)_B = 76.0 \text{ kN}\cdot\text{m} \circlearrowleft$

***4–24.**

The tower crane is used to hoist a 2-Mg load upward at constant velocity. The 1.5-Mg jib BD and 0.5-Mg jib BC have centers of mass at G_1 and G_2, respectively. Determine the required mass of the counterweight C so that the resultant moment produced by the load and the weight of the tower crane jibs about point A is zero. The center of mass for the counterweight is located at G_3.



SOLUTION

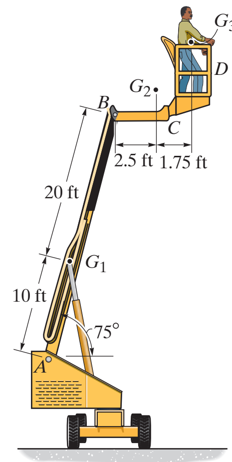
$\zeta + (M_R)_A = \Sigma Fd;\qquad 0 = M_C(9.81)(7.5) + 500(9.81)(4) - 1500(9.81)(9.5) - 2000(9.81)(12.5)$

$M_C = 4966.67 \text{ kg} = 4.97 \text{ Mg}$ **Ans.**

Ans:
$M_C = 4.97 \text{ Mg}$

4–25.

If the 1500-lb boom AB, the 200-lb cage BCD, and the 175-lb man have centers of gravity located at points G_1, G_2 and G_3, respectively, determine the resultant moment produced by each weight about point A.



SOLUTION

Moment of the weight of boom AB about point A:

$$\circlearrowleft + (M_{AB})_A = -1500(10\cos 75^\circ) = -3882.29 \text{ lb}\cdot\text{ft}$$

$$= 3.88 \text{ kip}\cdot\text{ft } \textbf{(\textit{Clockwise})} \qquad \textbf{Ans.}$$

Moment of the weight of cage BCD about point A:

$$\circlearrowleft + (M_{BCD})_A = -200(30\cos 75^\circ + 2.5) = -2052.91 \text{ lb}\cdot\text{ft}$$

$$= 2.05 \text{ kip}\cdot\text{ft } \textbf{(\textit{Clockwise})} \qquad \textbf{Ans.}$$

Moment of the weight of the man about point A:

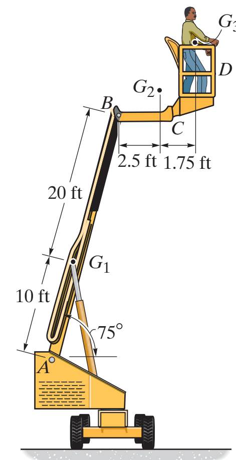
$$\circlearrowleft + (M_{\text{man}})_A = -175(30\cos 75^\circ + 4.25) = -2102.55 \text{ lb}\cdot\text{ft}$$

$$= 2.10 \text{ kip}\cdot\text{ft } \textbf{(\textit{Clockwise})} \qquad \textbf{Ans.}$$

Ans:
$(M_{AB})_A = 3.88 \text{ kip}\cdot\text{ft} \curvearrowright$
$(M_{BCD})_A = 2.05 \text{ kip}\cdot\text{ft} \curvearrowright$
$(M_{\text{man}})_A = 2.10 \text{ kip}\cdot\text{ft} \curvearrowright$

4–26.

If the 1500-lb boom AB, the 200-lb cage BCD, and the 175-lb man have centers of gravity located at points G_1, G_2 and G_3, respectively, determine the resultant moment produced by all the weights about point A.

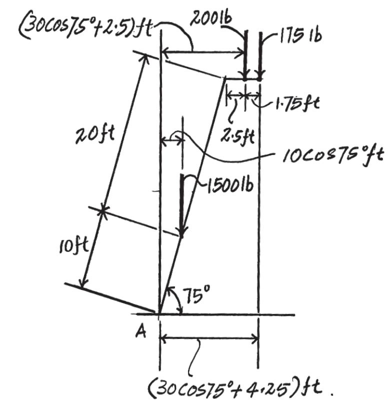


SOLUTION

Referring to Fig. a, the resultant moment of the weight about point A is given by

$$\circlearrowleft +(M_R)_A = \Sigma Fd; \qquad (M_R)_A = -1500(10\cos 75°) - 200(30\cos 75° + 2.5) - 175(30\cos 75° + 4.25)$$

$$= -8037.75 \text{ lb} \cdot \text{ft} = 8.04 \text{ kip} \cdot \text{ft } \textbf{(Clockwise)} \qquad \textbf{Ans.}$$

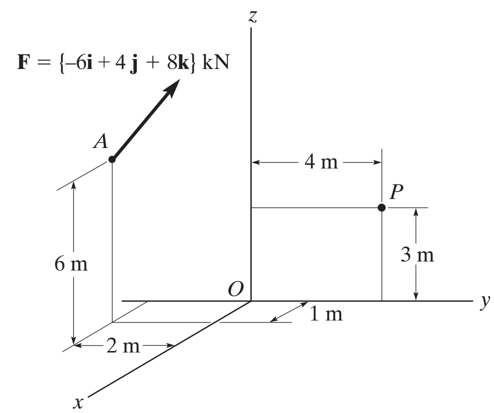


(a)

Ans:
$(M_R)_A = 8.04 \text{ kip} \cdot \text{ft} \circlearrowright$

4–27.

Determine the moment of the force **F** about point O. Express the result as a Cartesian vector.



SOLUTION

Position Vector. The coordinates of point A are $(1, -2, 6)$ m.

Thus,

$$\mathbf{r}_{OA} = \{\mathbf{i} - 2\mathbf{j} + 6\mathbf{k}\}\text{ m}$$

The moment of *F* About Point *O*.

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$$

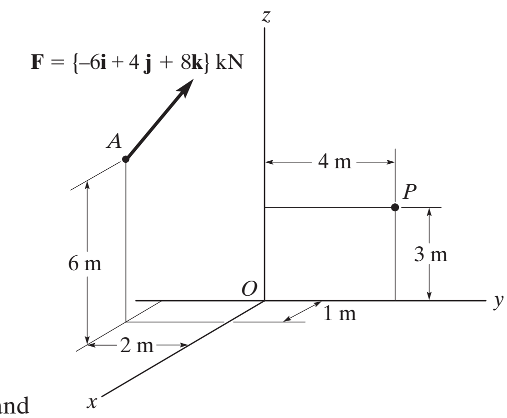
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -2 & 6 \\ -6 & 4 & 8 \end{vmatrix}$$

$$= \{-40\mathbf{i} - 44\mathbf{j} - 8\mathbf{k}\}\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_O = \{-40\mathbf{i} - 44\mathbf{j} - 8\mathbf{k}\}\text{ kN}\cdot\text{m}$

***4–28.**

Determine the moment of the force **F** about point P. Express the result as a Cartesian vector.



SOLUTION

Position Vector. The coordinates of points A and P are $A\,(1, -2, 6)$ m and $P\,(0, 4, 3)$ m, respectively. Thus

$$\mathbf{r}_{PA} = (1 - 0)\mathbf{i} + (-2 - 4)\mathbf{j} + (6 - 3)\mathbf{k}$$

$$= \{\mathbf{i} - 6\mathbf{j} + 3\mathbf{k}\}\text{ m}$$

The moment of *F* About Point *P*.

$$M_P = \mathbf{r}_{PA} \times \mathbf{F}$$

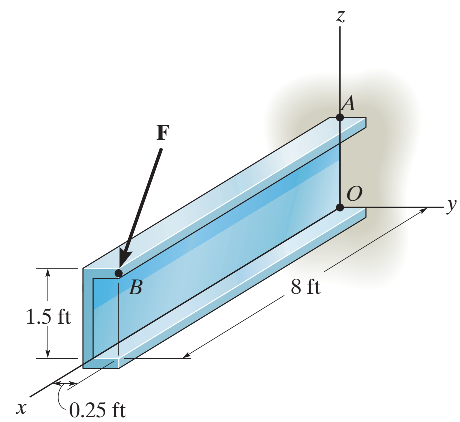
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & -6 & 3 \\ -6 & 4 & 8 \end{vmatrix}$$

$$= \{-60\mathbf{i} - 26\mathbf{j} - 32\mathbf{k}\}\text{ kN}\cdot\text{m}$$ **Ans.**

Ans:
$M_P = \{-60\mathbf{i} - 26\mathbf{j} - 32\mathbf{k}\}\text{ kN}\cdot\text{m}$

4–29.

The force $\mathbf{F} = \{400\mathbf{i} - 100\mathbf{j} - 700\mathbf{k}\}$ lb acts at the end of the beam. Determine the moment of this force about point O.



SOLUTION

Position Vector. The coordinates of point B are $B(8, 0.25, 1.5)$ ft.

Thus,

$$\mathbf{r}_{OB} = \{8\mathbf{i} + 0.25\mathbf{j} + 1.5\mathbf{k}\} \text{ ft}$$

Moments of *F* About Point *O*.

$$M_O = \mathbf{r}_{OB} \times \mathbf{F}$$

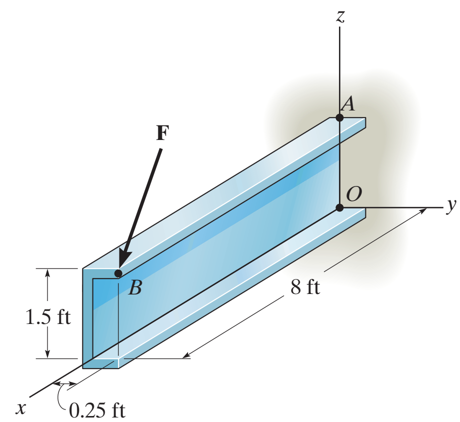
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0.25 & 1.5 \\ 400 & -100 & -700 \end{vmatrix}$$

$$= \{-25\mathbf{i} + 6200\mathbf{j} - 900\mathbf{k}\} \text{ lb} \cdot \text{ft}$$ **Ans.**

Ans:
$M_O = \{-25\mathbf{i} + 6200\mathbf{j} - 900\mathbf{k}\} \text{ lb} \cdot \text{ft}$

4–30.

The force $\mathbf{F} = \{400\mathbf{i} - 100\mathbf{j} - 700\mathbf{k}\}$ lb acts at the end of the beam. Determine the moment of this force about point A.



SOLUTION

Position Vector. The coordinates of points A and B are $A\,(0, 0, 1.5)$ ft and $B\,(8, 0.25, 1.5)$ ft, respectively. Thus,

$$\mathbf{r}_{AB} = (8 - 0)\mathbf{i} + (0.25 - 0)\mathbf{j} + (1.5 - 1.5)\mathbf{k}$$
$$= \{8\mathbf{i} + 0.25\mathbf{j}\}\text{ ft}$$

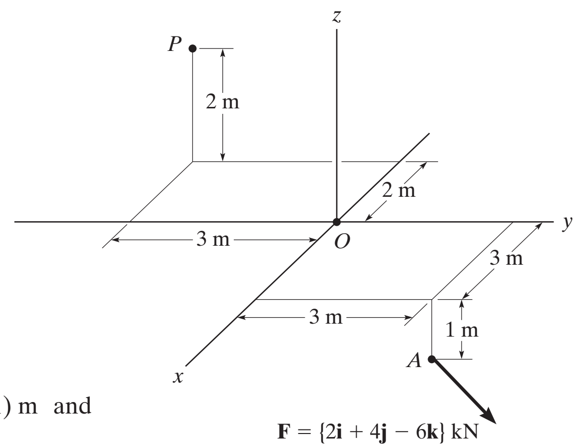
Moment of *F* About Point *A*.

$$M_A = \mathbf{r}_{AB} \times \mathbf{F}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 8 & 0.25 & 0 \\ 400 & -100 & -700 \end{vmatrix}$$
$$= \{-175\mathbf{i} + 5600\mathbf{j} - 900\mathbf{k}\}\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

Ans:
$M_A = \{-175\mathbf{i} + 5600\mathbf{j} - 900\mathbf{k}\}$ lb · ft

4–31.

Determine the moment of the force **F** about point P. Express the result as a Cartesian vector.



SOLUTION

Position Vector. The coordinates of points A and P are $A\,(3, 3, -1)$ m and $P\,(-2, -3, 2)$ m respectively. Thus,

$$\mathbf{r}_{PA} = [3 - (-2)]\mathbf{i} + [3 - (-3)]\mathbf{j} + (-1 - 2)\mathbf{k}$$

$$= \{5\mathbf{i} + 6\mathbf{j} - 3\mathbf{k}\}\text{ m}$$

Moment of *F* About Point *P*.

$$M_P = \mathbf{r}_{AP} \times \mathbf{F}$$

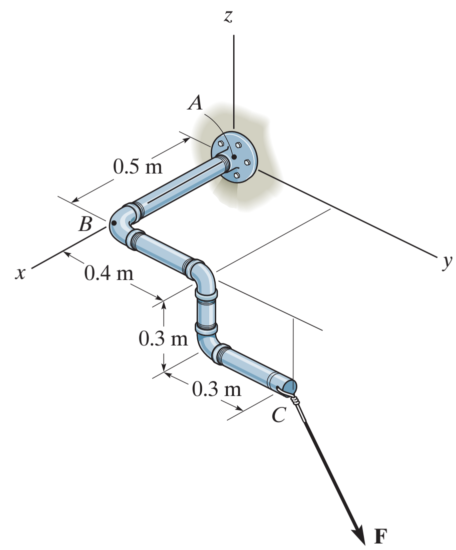
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 5 & 6 & -3 \\ 2 & 4 & -6 \end{vmatrix}$$

$$= \{-24\mathbf{i} + 24\mathbf{j} + 8\mathbf{k}\}\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$M_P = \{-24\mathbf{i} + 24\mathbf{j} + 8\mathbf{k}\}\text{ kN}\cdot\text{m}$

***4–32.**

The pipe assembly is subjected to the force of $\mathbf{F} = \{600\mathbf{i} + 800\mathbf{j} - 500\mathbf{k}\}$ N. Determine the moment of this force about point A.



SOLUTION

Position Vector. The coordinates of point C are C (0.5, 0.7, −0.3) m. Thus

$$\mathbf{r}_{AC} = \{0.5\mathbf{i} + 0.7\mathbf{j} - 0.3\mathbf{k}\} \text{ m}$$

Moment of Force *F* About Point *A*.

$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F}$$

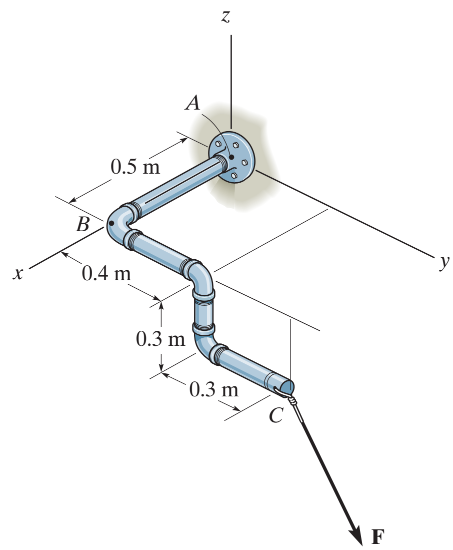
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.5 & 0.7 & -0.3 \\ 600 & 800 & -500 \end{vmatrix}$$

$$= \{-110\mathbf{i} + 70\mathbf{j} - 20\mathbf{k}\} \text{ N} \cdot \text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_A = \{-110\mathbf{i} + 70\mathbf{j} - 20\mathbf{k}\}$ N · m

4–33.

The pipe assembly is subjected to the force of $\mathbf{F} = \{600\mathbf{i} + 800\mathbf{j} - 500\mathbf{k}\}$ N. Determine the moment of this force about point B.



SOLUTION

Position Vector. The coordinates of points B and C are $B\,(0.5, 0, 0)$ m and $C\,(0.5, 0.7, -0.3)$ m, respectively. Thus,

$$\mathbf{r}_{BC} = (0.5 - 0.5)\mathbf{i} + (0.7 - 0)\mathbf{j} + (-0.3 - 0)\mathbf{k}$$

$$= \{0.7\mathbf{j} - 0.3\mathbf{k}\}\text{ m}$$

Moment of Force *F* About Point *B*. Applying Eq. 4

$$\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F}$$

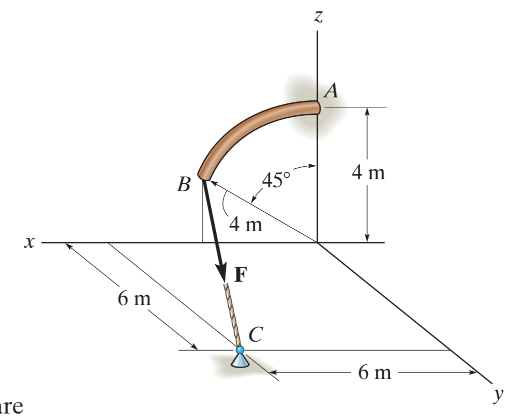
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.7 & -0.3 \\ 600 & 800 & -500 \end{vmatrix}$$

$$= \{-110\mathbf{i} - 180\mathbf{j} - 420\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_B = \{-110\mathbf{i} - 180\mathbf{j} - 420\mathbf{k}\}\text{ N}\cdot\text{m}$

4–34.

Determine the moment of the force of $F = 600$ N about point A.



SOLUTION

Position Vectors And Force Vector. The coordinates of points A, B and C are A (0, 0, 4) m, B (4 sin 45°, 0, 4 cos 45°) m and C (6, 6, 0) m, respectively. Thus

$$\mathbf{r}_{AB} = (4\sin 45^\circ - 0)\mathbf{i} + (0 - 0)\mathbf{j} + (4\cos 45^\circ - 4)\mathbf{k}$$

$$= \{2.8284\mathbf{i} - 1.1716\mathbf{k}\}\text{ m}$$

$$\mathbf{r}_{AC} = (6 - 0)\mathbf{i} + (6 - 0)\mathbf{j} + (0 - 4)\mathbf{k}$$

$$= \{6\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}\}\text{ m}$$

$$\mathbf{r}_{BC} = (6 - 4\sin 45^\circ)\mathbf{i} + (6 - 0)\mathbf{j} + (0 - 4\cos 45^\circ)\mathbf{k}$$

$$= \{3.1716\mathbf{i} + 6\mathbf{j} - 2.8284\mathbf{k}\}\text{ m}$$

$$\mathbf{F} = F\left(\frac{\mathbf{r}_{BC}}{r_{BC}}\right) = 600\left(\frac{3.1716\mathbf{i} + 6\mathbf{j} - 2.8284\mathbf{k}}{\sqrt{3.1716^2 + 6^2 + (-2.8284)^2}}\right)$$

$$= \{258.82\mathbf{i} + 489.63\mathbf{j} - 230.81\mathbf{k}\}\text{ N}$$

The Moment of Force *F* About Point *A*.

$$\mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.8284 & 0 & -1.1716 \\ 258.82 & 489.63 & -230.81 \end{vmatrix}$$

$$= \{573.64\mathbf{i} + 349.62\mathbf{j} + 1384.89\mathbf{k}\}\text{ N}\cdot\text{m}$$

$$= \{574\mathbf{i} + 350\mathbf{j} + 1385\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

OR

$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 6 & -4 \\ 258.82 & 489.63 & -230.81 \end{vmatrix}$$

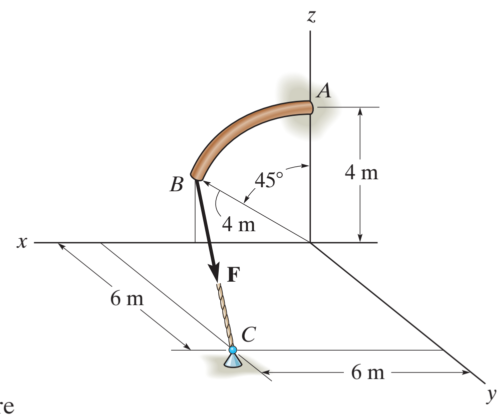
$$= \{573.64\mathbf{i} + 349.62\mathbf{j} + 1384.89\mathbf{k}\}\text{ N}\cdot\text{m}$$

$$= \{574\mathbf{i} + 350\mathbf{j} + 1385\mathbf{k}\} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_A = \{574\mathbf{i} + 350\mathbf{j} + 1385\mathbf{k}\}\text{ N}\cdot\text{m}$

4–35.

Determine the smallest force F that must be applied along the rope in order to cause the curved rod, which has a radius of 4 m, to fail at the support A. This requires a moment of $M = 1500\ \text{N}\cdot\text{m}$ to be developed at A.



SOLUTION

Position Vectors And Force Vector. The coordinates of points A, B and C are A (0, 0, 4) m, B (4 sin 45°, 0, 4 cos 45°) m and C (6, 6, 0) m, respectively.

Thus,

$$\mathbf{r}_{AB} = (4\sin 45^\circ - 0)\mathbf{i} + (0 - 0)\mathbf{j} + (4\cos 45^\circ - 4)\mathbf{k}$$

$$= \{2.8284\mathbf{i} - 1.1716\mathbf{k}\}\ \text{m}$$

$$\mathbf{r}_{AC} = (6 - 0)\mathbf{i} + (6 - 0)\mathbf{j} + (0 - 4)\mathbf{k}$$

$$= \{6\mathbf{i} + 6\mathbf{j} - 4\mathbf{k}\}\ \text{m}$$

$$\mathbf{r}_{BC} = (6 - 4\sin 45^\circ)\mathbf{i} + (6 - 0)\mathbf{j} + (0 - 4\cos 45^\circ)\mathbf{k}$$

$$= \{3.1716\mathbf{i} + 6\mathbf{j} - 2.8284\mathbf{k}\}\ \text{m}$$

$$\mathbf{F} = F\left(\frac{\mathbf{r}_{BC}}{r_{BC}}\right) = F\left(\frac{3.1716\mathbf{i} + 6\mathbf{j} - 2.8284\mathbf{k}}{\sqrt{3.1716^2 + 6^2 + (-2.8284)^2}}\right)$$

$$= 0.4314F\mathbf{i} + 0.8161F\mathbf{j} - 0.3847F\mathbf{k}$$

The Moment of Force F About Point A.

$$\mathbf{M}_A = \mathbf{r}_{AB} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2.8284 & 0 & -1.1716 \\ 0.4314F & 0.8161F & -0.3847F \end{vmatrix}$$

$$= 0.9561F\mathbf{i} + 0.5827F\mathbf{j} + 2.3081F\mathbf{k}$$

OR

$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 6 & -4 \\ 0.4314F & 0.8161F & -0.3847F \end{vmatrix}$$

$$= 0.9561F\mathbf{i} + 0.5827F\mathbf{j} + 2.3081F\mathbf{k}$$

The magnitude of $\mathbf{M}_A$ is

$$M_A = \sqrt{(M_A)_x^2 + (M_A)_y^2 + (M_A)_z^2} = \sqrt{(0.9561F)^2 + (0.5827F)^2 + (2.3081F)^2}$$

$$= 2.5654F$$

It is required that $M_A = 1500\ \text{N}\cdot\text{m}$, then

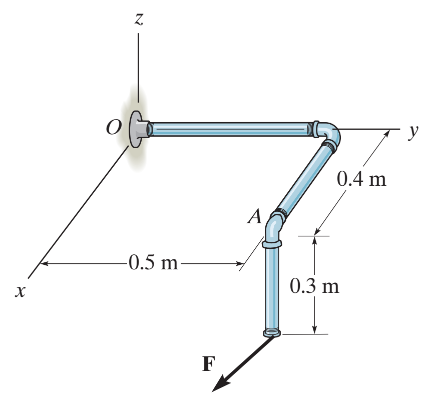
$$1500 = 2.5654F$$

$$F = 584.71\ \text{N} = 585\ \text{N}$$ **Ans.**

Ans:
$F = 585$ N

***4–36.**

Determine the coordinate direction angles α, β, γ of force **F**, so that the moment of **F** about O is zero.



SOLUTION

Position And Force Vectors. The coordinates of point A are A (0.4, 0.5, −0.3) m. Thus,

$$\mathbf{r}_{OA} = \{0.4\mathbf{i} + 0.5\mathbf{j} - 0.3\mathbf{k}\}\text{ m}$$

$$u_{OA} = \frac{\mathbf{r}_{OA}}{\mathbf{r}_{OA}} = \frac{0.4\mathbf{i} + 0.5\mathbf{j} - 0.3\mathbf{k}}{\sqrt{0.4^2 + 0.5^2 + (-0.3)^2}} = \frac{4}{\sqrt{50}}\mathbf{i} + \frac{5}{\sqrt{50}}\mathbf{j} - \frac{3}{\sqrt{50}}\mathbf{k}$$

$$r_{AO} = \{-0.4\mathbf{i} - 0.5\mathbf{j} + 0.3\mathbf{k}\}\text{ m}$$

$$u_{AO} = \frac{\mathbf{r}_{AO}}{\mathbf{r}_{AO}} = \frac{-0.4\mathbf{i} - 0.5\mathbf{j} + 0.3\mathbf{k}}{\sqrt{(-0.4)^2 + (-0.5)^2 + 0.3^2}} = -\frac{4}{\sqrt{50}}\mathbf{i} - \frac{5}{\sqrt{50}}\mathbf{j} + \frac{3}{\sqrt{50}}\mathbf{k}$$

Moment of *F* About Point *O*. To produce zero moment about point O, the line of action of **F** must pass through point O. Thus, **F** must directed from O to A (direction defined by $\mathbf{u}_{OA}$). Thus,

$$\cos\alpha = -\frac{4}{\sqrt{50}}; \qquad \alpha = 55.56° = 55.6° \qquad \textbf{Ans.}$$

$$\cos\beta = -\frac{5}{\sqrt{50}}; \qquad \beta = 45° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{-3}{\sqrt{50}}; \qquad \gamma = 115.10° = 115° \qquad \textbf{Ans.}$$

OR **F** must directed from A to O (direction defined by $\mathbf{u}_{AO}$). Thus

$$\cos\alpha = -\frac{4}{\sqrt{50}}; \qquad \alpha = 124.44° = 124° \qquad \textbf{Ans.}$$

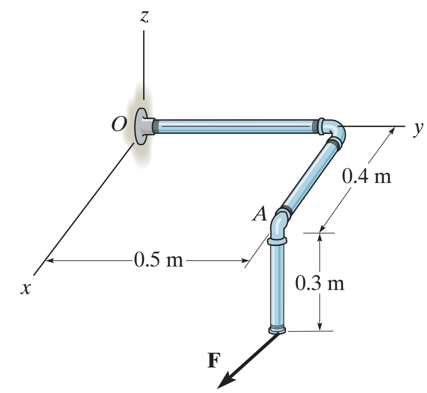
$$\cos\beta = -\frac{5}{\sqrt{50}}; \qquad \beta = 135° \qquad \textbf{Ans.}$$

$$\cos\gamma = \frac{3}{\sqrt{50}}; \qquad \gamma = 64.90° = 64.9° \qquad \textbf{Ans.}$$

Ans:
$\alpha = 55.6°$
$\beta = 45°$
$\gamma = 115°$
OR
$\alpha = 124°$
$\beta = 135°$
$\gamma = 64.9°$

4–37.

Determine the moment of force **F** about point O. The force has a magnitude of 800 N and coordinate direction angles of $\alpha = 60°$, $\beta = 120°$, $\gamma = 45°$. Express the result as a Cartesian vector.



SOLUTION

Position And Force Vectors. The coordinates of point A are $A\,(0.4, 0.5, -0.3)$ m. Thus

$$\mathbf{r}_{OA} = \{0.4\mathbf{i} + 0.5\mathbf{j} - 0.3\mathbf{k}\}\text{ m}$$

$$F = F\mathbf{u}_F = 800\,(\cos 60°\mathbf{i} + \cos 120°\mathbf{j} + \cos 45°\mathbf{k})$$

$$= \{400\mathbf{i} - 400\mathbf{j} + 565.69\mathbf{k}\}\text{ N}$$

Moment of *F* About Point *O*.

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.4 & 0.5 & -0.3 \\ 400 & -400 & 565.69 \end{vmatrix}$$

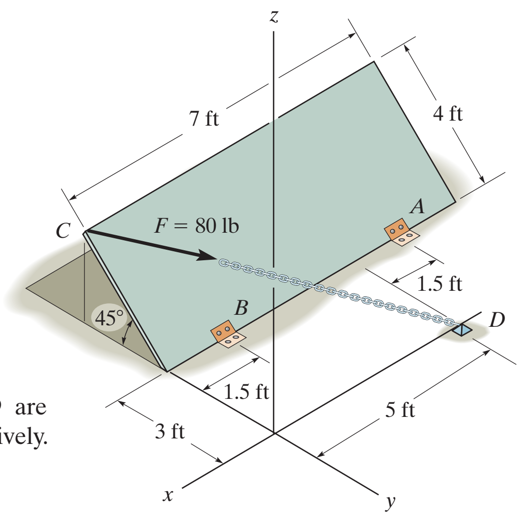
$$= \{162.84\mathbf{i} - 346.27\mathbf{j} - 360\mathbf{k}\}\text{ N}\cdot\text{m}$$

$$= \{163\mathbf{i} - 346\mathbf{j} - 360\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_O = \{163\mathbf{i} - 346\mathbf{j} - 360\mathbf{k}\}\text{ N}\cdot\text{m}$

4–38.

Determine the moment of the force **F** about the door hinge at A. Express the result as a Cartesian vector.



SOLUTION

Position Vectors And Force Vector. The coordinates of points A, C and D are $A\,(-6.5, -3, 0)$ ft, $C\,[0, -(3 + 4\cos 45°), 4\sin 45°]$ ft and $D\,(-5, 0, 0)$ ft, respectively. Thus,

$$\mathbf{r}_{AC} = [0 - (-6.5)]\mathbf{i} + [-(3 + 4\cos 45°) - (-3)]\mathbf{j} + (4\sin 45° - 0)\mathbf{k}$$

$$= \{6.5\mathbf{i} - 2.8284\mathbf{j} + 2.8284\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{AD} = [-5 - (-6.5)]\mathbf{i} + [0 - (-3)]\mathbf{j} + (0 - 0)\mathbf{k} = \{1.5\mathbf{i} + 3\mathbf{j}\}\text{ ft}$$

$$\mathbf{r}_{CD} = (-5 - 0)\mathbf{i} + \{0 - [-(3 + 4\cos 45°)]\}\mathbf{j} + (0 - 4\sin 45°)\mathbf{k}$$

$$= \{-5\mathbf{i} + 5.8284\mathbf{j} - 2.8284\mathbf{k}\}\text{ ft}$$

$$\mathbf{F} = F\left(\frac{\mathbf{r}_{CD}}{r_{CD}}\right) = 80\left(\frac{-5\mathbf{i} + 5.8284\mathbf{j} - 2.8284\mathbf{k}}{\sqrt{(-5)^2 + 5.8284^2 + (-2.8284)^2}}\right)$$

$$= \{-48.88\,\mathbf{i} + 56.98\mathbf{j} - 27.65\mathbf{k}\}\text{ lb}$$

Moment of F About Point A.

$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6.5 & -2.8284 & 2.8284 \\ -48.88 & 56.98 & -27.65 \end{vmatrix}$$

$$= \{-82.9496\mathbf{i} + 41.47\mathbf{j} + 232.10\mathbf{k}\}\text{ lb}\cdot\text{ft}$$

$$= \{-82.9\mathbf{i} + 41.5\mathbf{j} + 232\mathbf{k}\}\text{ lb}\cdot\text{ft}$$ **Ans.**

OR

$$\mathbf{M}_A = \mathbf{r}_{AD} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 3 & 0 \\ -48.88 & 56.98 & -27.65 \end{vmatrix}$$

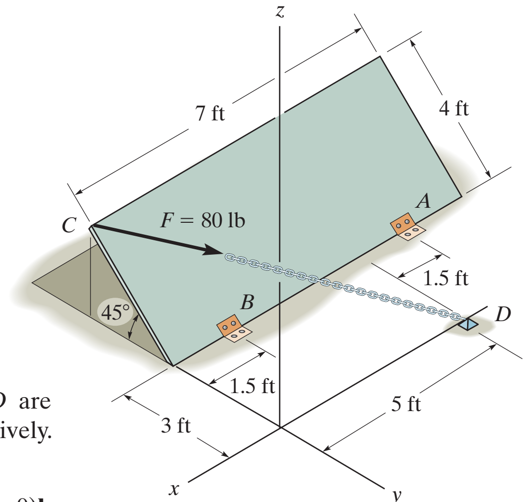
$$= \{-82.9496\mathbf{i} + 41.47\mathbf{j} + 232.10\mathbf{k}\}\text{ lb}\cdot\text{ft}$$

$$= \{-82.9\mathbf{i} + 41.5\mathbf{j} + 232\mathbf{k}\}\text{ lb}\cdot\text{ft}$$ **Ans.**

Ans:
$\mathbf{M}_A = \{-82.9\mathbf{i} + 41.5\mathbf{j} + 232\mathbf{k}\}\text{ lb}\cdot\text{ft}$

4–39.

Determine the moment of the force **F** about the door hinge at B. Express the result as a Cartesian vector.



SOLUTION

Position Vectors And Force Vector. The coordinates of points B, C and D are $B\,(-1.5, -3, 0)$ ft, $C\,[0, -(3 + 4\cos 45°), 4\sin 45°]$ ft and $D\,(-5, 0, 0)$ ft, respectively. Thus,

$$\mathbf{r}_{BC} = [0 - (-1.5)]\mathbf{i} + [-(3 + 4\cos 45°) - (-3)]\mathbf{j} + (4\sin 45° - 0)\mathbf{k}$$

$$= \{1.5\mathbf{i} - 2.8284\mathbf{j} + 2.8284\mathbf{k}\}\text{ ft}$$

$$\mathbf{r}_{BD} = [-5 - (-1.5)]\mathbf{i} + [0 - (-3)]\mathbf{j} + (0 - 0)\mathbf{k} = \{-3.5\mathbf{i} + 3\mathbf{j}\}\text{ ft}$$

$$\mathbf{r}_{CD} = (-5-0)\mathbf{i} + \{0 - [-(3 + 4\cos 45°)]\}\mathbf{j} + (0 - 4\sin 45°)\mathbf{k}$$

$$= \{-5\mathbf{i} + 5.8284\mathbf{j} - 2.8284\mathbf{k}\}\text{ ft}$$

$$\mathbf{F} = F\left(\frac{\mathbf{r}_{CD}}{r_{CD}}\right) = 80\left(\frac{-5\mathbf{i} + 5.8284\mathbf{j} - 2.8284\mathbf{k}}{\sqrt{(-5)^2 + 5.8284^2 + (-2.8284)^2}}\right)$$

$$= \{-48.88\mathbf{i} + 56.98\mathbf{j} - 27.65\mathbf{k}\}\text{ lb}$$

Moment of *F* About Point *B*.

$$\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & -2.8284 & 2.8284 \\ -48.88 & 56.98 & -27.65 \end{vmatrix}$$

$$= \{-82.9496\mathbf{i} - 96.77\mathbf{j} - 52.78\mathbf{k}\}\text{ lb}\cdot\text{ft}$$

$$= \{-82.9\mathbf{i} - 96.8\mathbf{j} - 52.8\mathbf{k}\}\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

or

$$\mathbf{M}_B = \mathbf{r}_{BD} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3.5 & 3 & 0 \\ -48.88 & 56.98 & -27.65 \end{vmatrix}$$

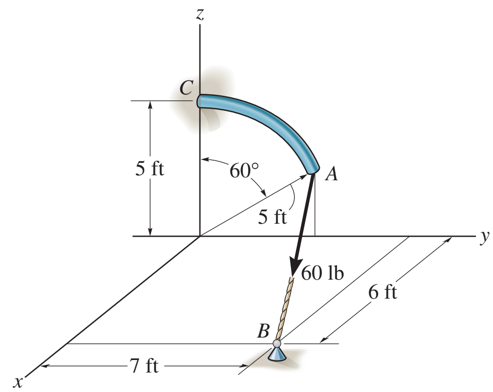
$$= \{-82.9496\mathbf{i} - 96.77\mathbf{j} - 52.78\mathbf{k}\}\text{ lb}\cdot\text{ft}$$

$$= \{-82.9\mathbf{i} - 96.8\mathbf{j} - 52.8\mathbf{k}\}\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_B = \{-82.9\mathbf{i} - 96.8\mathbf{j} - 52.8\mathbf{k}\}\text{ lb}\cdot\text{ft}$

***4–40.**

The curved rod has a radius of 5 ft. If a force of 60 lb acts at its end as shown, determine the moment of this force about point C.



SOLUTION

Position Vector and Force Vector:

$$\mathbf{r}_{CA} = \{(5 \sin 60° - 0)\mathbf{j} + (5 \cos 60° - 5)\mathbf{k}\} \text{ m}$$

$$= \{4.330\mathbf{j} - 2.50\mathbf{k}\} \text{ m}$$

$$\mathbf{F}_{AB} = 60\left(\frac{(6-0)\mathbf{i} + (7 - 5\sin 60°)\mathbf{j} + (0 - 5\cos 60°)\mathbf{k}}{\sqrt{(6-0)^2 + (7 - 5\sin 60°)^2 + (0 - 5\cos 60°)^2}}\right) \text{ lb}$$

$$= \{51.231\mathbf{i} + 22.797\mathbf{j} - 21.346\mathbf{k}\} \text{ lb}$$

Moment of Force $\mathbf{F}_{AB}$ About Point C: Applying Eq. 4–7, we have

$$\mathbf{M}_C = \mathbf{r}_{CA} \times \mathbf{F}_{AB}$$

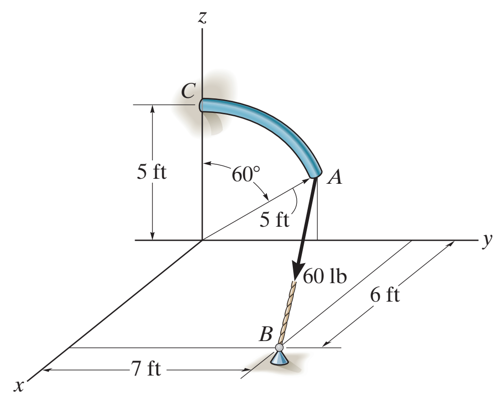
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4.330 & -2.50 \\ 51.231 & 22.797 & -21.346 \end{vmatrix}$$

$$= \{-35.4\mathbf{i} - 128\mathbf{j} - 222\mathbf{k}\} \text{ lb} \cdot \text{ft}$$ **Ans.**

Ans:
$\mathbf{M}_C = \{-35.4\mathbf{i} - 128\mathbf{j} - 222\mathbf{k}\}$ lb · ft

4–41.

Determine the smallest force F that must be applied along the rope in order to cause the curved rod, which has a radius of 5 ft, to fail at the support C. This requires a moment of $M = 80 \text{ lb} \cdot \text{ft}$ to be developed at C.



SOLUTION

Position Vector and Force Vector:

$$\mathbf{r}_{CA} = \{(5 \sin 60° - 0)\mathbf{j} + (5 \cos 60° - 5)\mathbf{k}\} \text{ m}$$
$$= \{4.330\mathbf{j} - 2.50\,\mathbf{k}\} \text{ m}$$

$$\mathbf{F}_{AB} = F\left(\frac{(6-0)\mathbf{i} + (7 - 5\sin 60°)\mathbf{j} + (0 - 5\cos 60°)\mathbf{k}}{\sqrt{(6-0)^2 + (7 - 5\sin 60°)^2 + (0 - 5\cos 60°)^2}}\right) \text{lb}$$
$$= 0.8539F_{\mathbf{i}} + 0.3799F_{\mathbf{j}} - 0.3558F_{\mathbf{k}}$$

Moment of Force $\mathbf{F}_{AB}$ About Point C:

$$\mathbf{M}_C = \mathbf{r}_{CA} \times \mathbf{F}_{AB}$$
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4.330 & -2.50 \\ 0.8539F & 0.3799F & -0.3558F \end{vmatrix}$$
$$= -0.5909F_{\mathbf{i}} - 2.135\mathbf{j} - 3.697\mathbf{k}$$

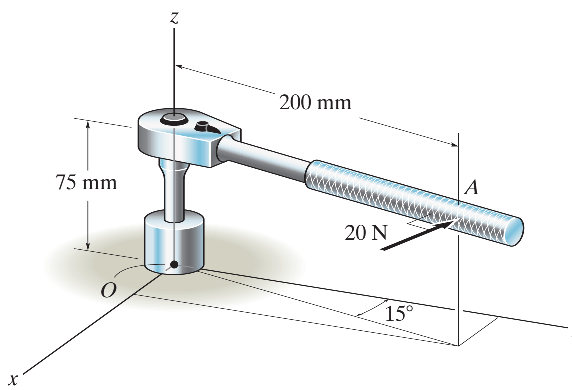
Require

$$80 = \sqrt{(0.5909)^2 + (-2.135)^2 + (-3.697)^2}\,F$$
$$F = 18.6 \text{ lb.} \qquad \textbf{Ans.}$$

Ans:
$F = 18.6$ lb

4–42.

A 20-N horizontal force is applied perpendicular to the handle of the socket wrench. Determine the magnitude and the coordinate direction angles of the moment created by this force about point O.



SOLUTION

$$\mathbf{r}_A = 0.2 \sin 15°\mathbf{i} + 0.2 \cos 15°\mathbf{j} + 0.075\mathbf{k}$$

$$= 0.05176\mathbf{i} + 0.1932\mathbf{j} + 0.075\mathbf{k}$$

$$\mathbf{F} = -20 \cos 15°\mathbf{i} + 20 \sin 15°\mathbf{j}$$

$$= -19.32\mathbf{i} + 5.176\mathbf{j}$$

$$M_O = \mathbf{r}_A \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.05176 & 0.1932 & 0.075 \\ -19.32 & 5.176 & 0 \end{vmatrix}$$

$$= \{-0.3882\mathbf{i} - 1.449\mathbf{j} + 4.00\mathbf{k}\} \text{ N} \cdot \text{m}$$

$$M_O = 4.272 = 4.27 \text{ N} \cdot \text{m}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{-0.3882}{4.272}\right) = 95.2°$$ **Ans.**

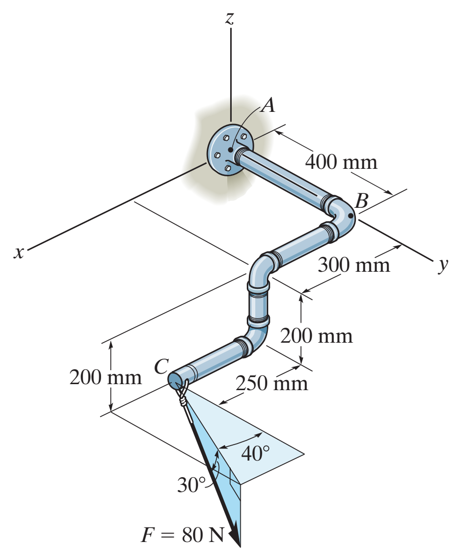
$$\beta = \cos^{-1}\left(\frac{-1.449}{4.272}\right) = 110°$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{4}{4.272}\right) = 20.6°$$ **Ans.**

Ans:
$M_O = 4.27 \text{ N} \cdot \text{m}$
$\alpha = 95.2°$
$\beta = 110°$
$\gamma = 20.6°$

4–43.

The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point A.



SOLUTION

Position Vector And Force Vector:

$$\mathbf{r}_{AC} = \{(0.55 - 0)\mathbf{i} + (0.4 - 0)\mathbf{j} + (-0.2 - 0)\mathbf{k}\}\text{ m}$$

$$= \{0.55\mathbf{i} + 0.4\mathbf{j} - 0.2\mathbf{k}\}\text{ m}$$

$$\mathbf{F} = 80(\cos 30° \sin 40°\mathbf{i} + \cos 30° \cos 40°\mathbf{j} - \sin 30°\mathbf{k})\text{ N}$$

$$= (44.53\mathbf{i} + 53.07\mathbf{j} - 40.0\mathbf{k}\}\text{ N}$$

Moment of Force* F *About Point A: Applying Eq. 4–7, we have

$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F}$$

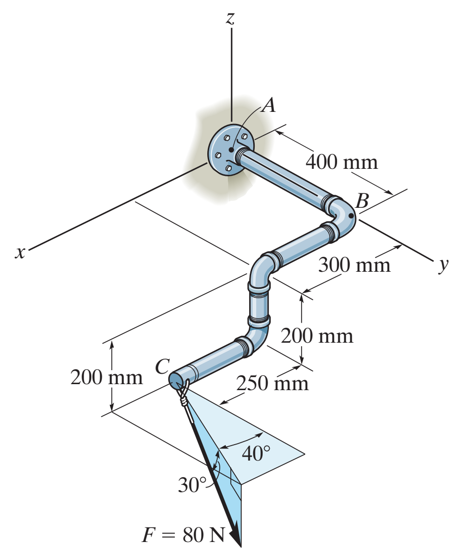
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0.4 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix}$$

$$= \{-5.39\mathbf{i} + 13.1\mathbf{j} + 11.4\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_A = \{-5.39\mathbf{i} + 13.1\mathbf{j} + 11.4\mathbf{k}\}\text{ N}\cdot\text{m}$

***4–44.**

The pipe assembly is subjected to the 80-N force. Determine the moment of this force about point B.



SOLUTION

Position Vector And Force Vector:

$$\mathbf{r}_{BC} = \{(0.55 - 0)\mathbf{i} + (0.4 - 0.4)\mathbf{j} + (-0.2 - 0)\mathbf{k}\}\text{ m}$$

$$= \{0.55\mathbf{i} - 0.2\mathbf{k}\}\text{ m}$$

$$\mathbf{F} = 80\,(\cos 30^\circ \sin 40^\circ\mathbf{i} + \cos 30^\circ \cos 40^\circ\mathbf{j} - \sin 30^\circ\mathbf{k})\text{ N}$$

$$= (44.53\mathbf{i} + 53.07\mathbf{j} - 40.0\mathbf{k}\}\text{ N}$$

Moment of Force* F *About Point B: Applying Eq. 4–7, we have

$$\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F}$$

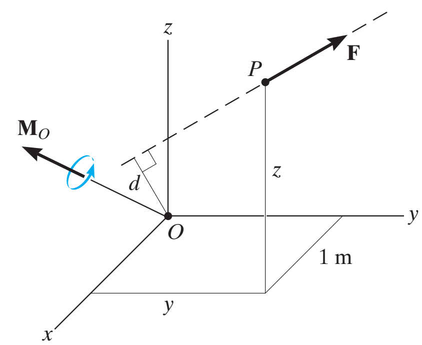
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0 & -0.2 \\ 44.53 & 53.07 & -40.0 \end{vmatrix}$$

$$= \{10.6\mathbf{i} + 13.1\mathbf{j} + 29.2\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_B = \{10.6\mathbf{i} + 13.1\mathbf{j} + 29.2\mathbf{k}\}\text{ N}\cdot\text{m}$

4–45.

A force of $\mathbf{F} = \{6\mathbf{i} - 2\mathbf{j} + 1\mathbf{k}\}$ kN produces a moment of $\mathbf{M}_O = \{4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k}\}$ kN·m about the origin of coordinates, point O. If the force acts at a point having an x coordinate of $x = 1$ m, determine the y and z coordinates. *Note:* The figure shows $\mathbf{F}$ and $\mathbf{M}_O$ in an arbitrary position.



SOLUTION

$$\mathbf{M}_O = \mathbf{r} \times \mathbf{F}$$

$$4\mathbf{i} + 5\mathbf{j} - 14\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & y & z \\ 6 & -2 & 1 \end{vmatrix}$$

$$4 = y + 2z$$

$$5 = -1 + 6z$$

$$-14 = -2 - 6y$$

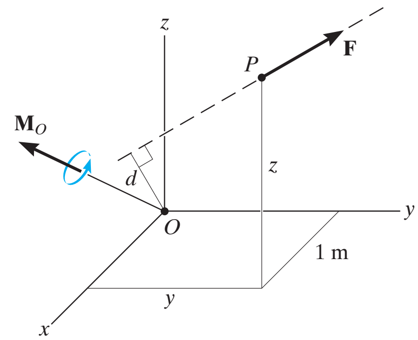
$$y = 2 \text{ m} \qquad \textbf{Ans.}$$

$$z = 1 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$y = 2$ m
$z = 1$ m

4–46.

The force $\mathbf{F} = \{6\mathbf{i} + 8\mathbf{j} + 10\mathbf{k}\}$ N creates a moment about point O of $\mathbf{M}_O = \{-14\mathbf{i} + 8\mathbf{j} + 2\mathbf{k}\}$ N·m. If the force passes through a point having an x coordinate of 1 m, determine the y and z coordinates of the point. Also, realizing that $M_O = Fd$, determine the perpendicular distance d from point O to the line of action of $\mathbf{F}$. *Note:* The figure shows $\mathbf{F}$ and $\mathbf{M}_O$ in an arbitrary position.



SOLUTION

$$-14\mathbf{i} + 8\mathbf{j} + 2\mathbf{k} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & y & z \\ 6 & 8 & 10 \end{vmatrix}$$

$$-14 = 10y - 8z$$

$$8 = -10 + 6z$$

$$2 = 8 - 6y$$

$$y = 1 \text{ m} \qquad \textbf{Ans.}$$

$$z = 3 \text{ m} \qquad \textbf{Ans.}$$

$$M_O = \sqrt{(-14)^2 + (8)^2 + (2)^2} = 16.25 \text{ N}\cdot\text{m}$$

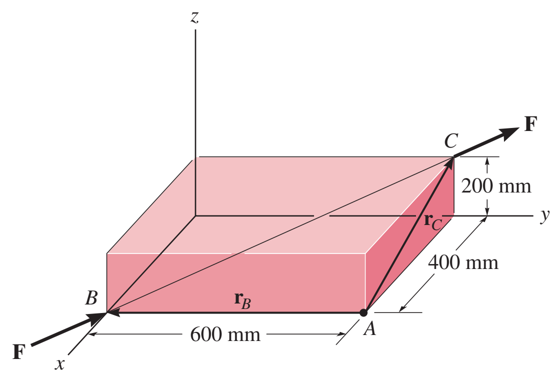
$$F = \sqrt{(6)^2 + (8)^2 + (10)^2} = 14.14 \text{ N}$$

$$d = \frac{16.25}{14.14} = 1.15 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$y = 1$ m
$z = 3$ m
$d = 1.15$ m

4–47.

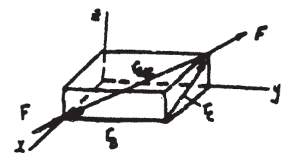
A force **F** having a magnitude of $F = 100$ N acts along the diagonal of the parallelepiped. Determine the moment of **F** about point A, using $\mathbf{M}_A = \mathbf{r}_B \times \mathbf{F}$ and $\mathbf{M}_A = \mathbf{r}_C \times \mathbf{F}$.



SOLUTION

$$\mathbf{F} = 100\left(\frac{-0.4\,\mathbf{i} + 0.6\,\mathbf{j} + 0.2\,\mathbf{k}}{0.7483}\right)$$

$$\mathbf{F} = \{-53.5\,\mathbf{i} + 80.2\,\mathbf{j} + 26.7\,\mathbf{k}\}\text{ N}$$



$$\mathbf{M}_A = \mathbf{r}_B \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -0.6 & 0 \\ -53.5 & 80.2 & 26.7 \end{vmatrix} = \{-16.0\,\mathbf{i} - 32.1\,\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

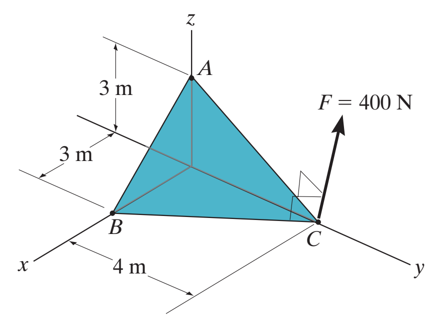
Also,

$$\mathbf{M}_A = \mathbf{r}_C \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.4 & 0 & 0.2 \\ -53.5 & 80.2 & 26.7 \end{vmatrix} = \{-16.0\,\mathbf{i} - 32.1\,\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_A = \{-16.0\mathbf{i} - 32.1\mathbf{k}\}\text{ N}\cdot\text{m}$

***4–48.**

Force **F** acts perpendicular to the inclined plane. Determine the moment produced by **F** about point A. Express the result as a Cartesian vector.

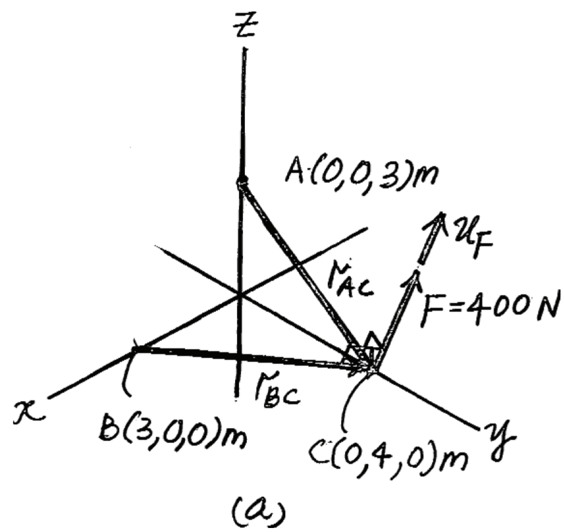


SOLUTION

Force Vector: Since force **F** is perpendicular to the inclined plane, its unit vector $\mathbf{u}_F$ is equal to the unit vector of the cross product, $\mathbf{b} = \mathbf{r}_{AC} \times \mathbf{r}_{BC}$, Fig. *a*. Here

$$\mathbf{r}_{AC} = (0-0)\mathbf{i} + (4-0)\mathbf{j} + (0-3)\mathbf{k} = [4\mathbf{j} - 3\mathbf{k}]\text{ m}$$

$$\mathbf{r}_{BC} = (0-3)\mathbf{i} + (4-0)\mathbf{j} + (0-0)\mathbf{k} = [-3\mathbf{i} + 4\mathbf{j}]\text{ m}$$



Thus,

$$\mathbf{b} = \mathbf{r}_{CA} \times \mathbf{r}_{CB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -3 \\ -3 & 4 & 0 \end{vmatrix}$$

$$= [12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}]\text{ m}^2$$

Then,

$$\mathbf{u}_F = \frac{\mathbf{b}}{b} = \frac{12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}}{\sqrt{12^2 + 9^2 + 12^2}} = 0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k}$$

And finally

$$\mathbf{F} = F\mathbf{u}_F = 400(0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k})$$

$$= [249.88\mathbf{i} + 187.41\mathbf{j} + 249.88\mathbf{k}]\text{ N}$$

Vector Cross Product: The moment of **F** about point A is

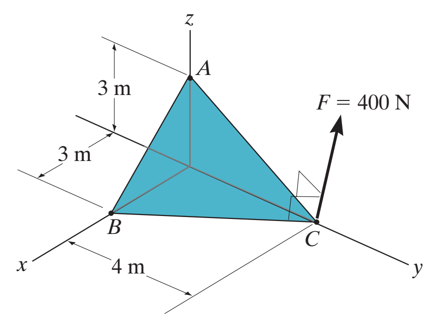
$$\mathbf{M}_A = \mathbf{r}_{AC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -3 \\ 249.88 & 187.41 & 249.88 \end{vmatrix}$$

$$= [1.56\mathbf{i} - 0.750\mathbf{j} - 1.00\mathbf{k}]\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_A = [1.56\mathbf{i} - 0.750\mathbf{j} - 1.00\mathbf{k}]\text{ kN}\cdot\text{m}$

4–49.

Force **F** acts perpendicular to the inclined plane. Determine the moment produced by **F** about point B. Express the result as a Cartesian vector.



SOLUTION

Force Vector: Since force **F** is perpendicular to the inclined plane, its unit vector $\mathbf{u}_F$ is equal to the unit vector of the cross product, $\mathbf{b} = \mathbf{r}_{AC} \times \mathbf{r}_{BC}$, Fig. a. Here

$$\mathbf{r}_{AC} = (0-0)\mathbf{i} + (4-0)\mathbf{j} + (0-3)\mathbf{k} = [4\mathbf{j} - 3\mathbf{k}]\text{ m}$$

$$\mathbf{r}_{BC} = (0-3)\mathbf{i} + (4-0)\mathbf{j} + (0-0)\mathbf{k} = [-3\mathbf{k} + 4\mathbf{j}]\text{ m}$$

Thus,

$$\mathbf{b} = \mathbf{r}_{CA} \times \mathbf{r}_{CB} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & -3 \\ -3 & 4 & 0 \end{vmatrix} = [12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}]\text{ m}^2$$

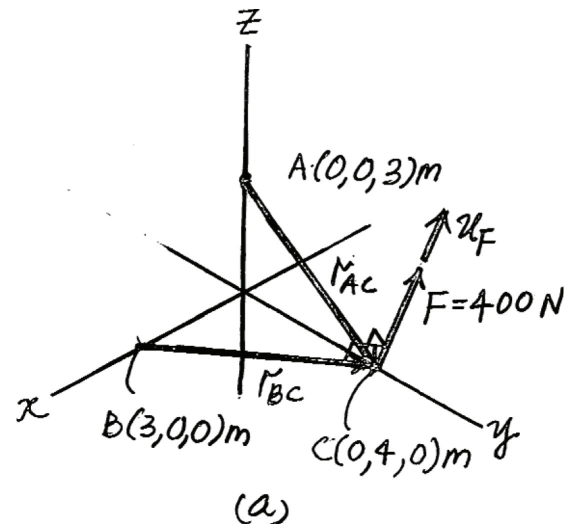
Then,

$$\mathbf{u}_F = \frac{\mathbf{b}}{b} = \frac{12\mathbf{i} + 9\mathbf{j} + 12\mathbf{k}}{\sqrt{12^2 + 9^2 + 12^2}} = 0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k}$$

And finally

$$\mathbf{F} = F\mathbf{u}_F = 400(0.6247\mathbf{i} + 0.4685\mathbf{j} + 0.6247\mathbf{k})$$

$$= [249.88\mathbf{i} + 187.41\mathbf{j} + 249.88\mathbf{k}]\text{ N}$$



Vector Cross Product: The moment of **F** about point B is

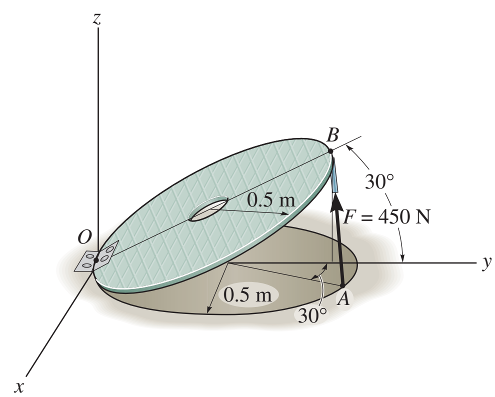
$$\mathbf{M}_B = \mathbf{r}_{BC} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & 4 & 0 \\ 249.88 & 187.41 & 249.88 \end{vmatrix}$$

$$= [1.00\mathbf{i} + 0.750\mathbf{j} - 1.56\mathbf{k}]\text{ kN}\cdot\text{m}$$ **Ans.**

Ans:
$\mathbf{M}_B = \{1.00\mathbf{i} + 0.750\mathbf{j} - 1.56\mathbf{k}\}\text{ kN}\cdot\text{m}$

4–50.

Strut AB of the 1-m-diameter hatch door exerts a force of 450 N on point B. Determine the moment of this force about point O.



SOLUTION

Position Vector And Force Vector:

$$\mathbf{r}_{OB} = \{(0 - 0)\mathbf{i} + (1\cos 30° - 0)\mathbf{j} + (1\sin 30° - 0)\mathbf{k}\}\text{ m}$$

$$= \{0.8660\mathbf{j} + 0.5\mathbf{k}\}\text{ m}$$

$$\mathbf{r}_{OA} = \{(0.5\sin 30° - 0)\mathbf{i} + (0.5 + 0.5\cos 30° - 0)\mathbf{j} + (0 - 0)\mathbf{k}\}\text{ m}$$

$$= \{0.250\mathbf{i} + 0.9330\mathbf{j}\}\text{ m}$$

$$\mathbf{F} = 450\left(\frac{(0 - 0.5\sin 30°)\mathbf{i} + [1\cos 30° - (0.5 + 0.5\cos 30°)]\mathbf{j} + (1\sin 30° - 0)\mathbf{k}}{\sqrt{(0 - 0.5\sin 30°)^2 + [1\cos 30° - (0.5 + 0.5\cos 30°)]^2 + (1\sin 30° - 0)^2}}\right)\text{N}$$

$$= \{-199.82\mathbf{i} - 53.54\mathbf{j} + 399.63\mathbf{k}\}\text{ N}$$

Moment of Force* F *About Point O: Applying Eq. 4–7, we have

$$\mathbf{M}_O = \mathbf{r}_{OB} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.8660 & 0.5 \\ -199.82 & -53.54 & 399.63 \end{vmatrix}$$

$$= \{373\mathbf{i} - 99.9\mathbf{j} + 173\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Or

$$\mathbf{M}_O = \mathbf{r}_{OA} \times \mathbf{F}$$

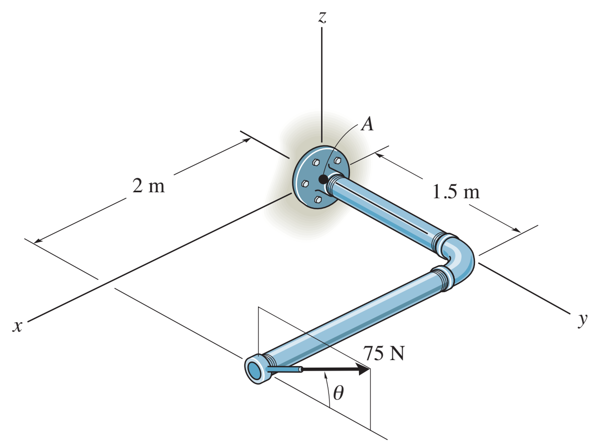
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.250 & 0.9330 & 0 \\ -199.82 & -53.54 & 399.63 \end{vmatrix}$$

$$= \{373\mathbf{i} - 99.9\mathbf{j} + 173\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_O = \{373\mathbf{i} - 99.9\mathbf{j} + 173\mathbf{k}\}\text{ N}\cdot\text{m}$

4–51.

Using a ring collar the 75-N force can act in the vertical plane at various angles θ. Determine the magnitude of the moment it produces about point A, plot the result of M (ordinate) versus θ (abscissa) for $0° \leq \theta \leq 180°$, and specify the angles that give the maximum and minimum moment.



SOLUTION

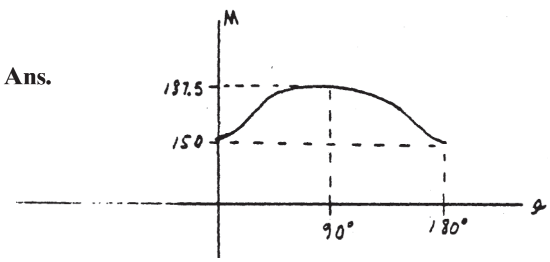
$$\mathbf{M}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 1.5 & 0 \\ 0 & 75\cos\theta & 75\sin\theta \end{vmatrix}$$

$$= 112.5\sin\theta\,\mathbf{i} - 150\sin\theta\,\mathbf{j} + 150\cos\theta\,\mathbf{k}$$

$$M_A = \sqrt{(112.5\sin\theta)^2 + (-150\sin\theta)^2 + (150\cos\theta)^2} = \sqrt{12\,656.25\sin^2\theta + 22\,500}$$

$$\frac{dM_A}{d\theta} = \frac{1}{2}(12\,656.25\sin^2\theta + 22\,500)^{-\frac{1}{2}}(12\,656.25)(2\sin\theta\cos\theta) = 0$$

$\sin\theta\cos\theta = 0; \quad \theta = 0°, 90°, 180°$ **Ans.**



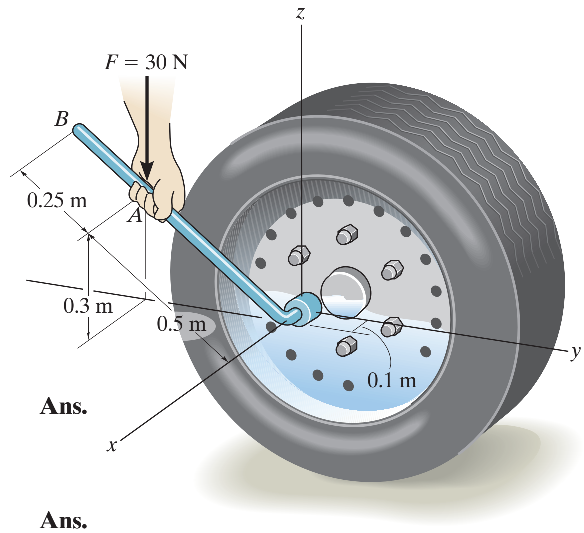
$\theta_{max} = 187.5\ \text{N}\cdot\text{m at } \theta = 90°$

$\theta_{min} = 150\ \text{N}\cdot\text{m at } \theta = 0°, 180°$

Ans:
$\theta_{max} = 90°$
$\theta_{min} = 0, 180°$

***4–52.**

The lug nut on the wheel of the automobile is to be removed using the wrench and applying the vertical force of $F = 30$ N at A. Determine if this force is adequate, provided 14 N·m of torque about the x axis is initially required to turn the nut. If the 30-N force can be applied at A in any other direction, will it be possible to turn the nut?



SOLUTION

$M_x = 30\left(\sqrt{(0.5)^2 - (0.3)^2}\right) = 12 \text{ N}\cdot\text{m} < 14 \text{ N}\cdot\text{m},$ No **Ans.**

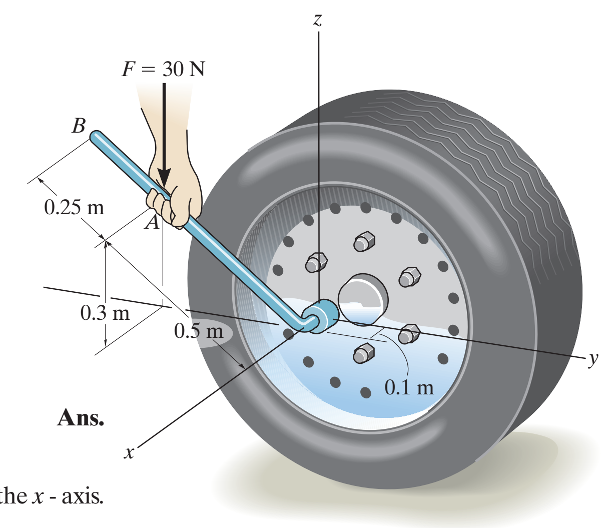
For $(M_x)_{max}$, apply force perpendicular to the handle and the x - axis.

$(M_x)_{max} = 30\,(0.5) = 15 \text{ N}\cdot\text{m} > 14 \text{ N}\cdot\text{m},$ Yes **Ans.**

Ans:
No
Yes

4–53.

Solve Prob. 4–52 if the cheater pipe AB is slipped over the handle of the wrench and the 30-N force can be applied at any point and in any direction on the assembly.



SOLUTION

$M_x = 30\,(0.75)\left(\frac{4}{5}\right) = 18\ \text{N}\cdot\text{m} > 14\ \text{N}\cdot\text{m},$ Yes **Ans.**

$(M_x)_{max}$ occurs when force is applied perpendicular to both the handle and the x - axis.

$(M_x)_{max} = 30(0.75) = 22.5\ \text{N}\cdot\text{m} > 14\text{N}\cdot\text{m},$ Yes **Ans.**

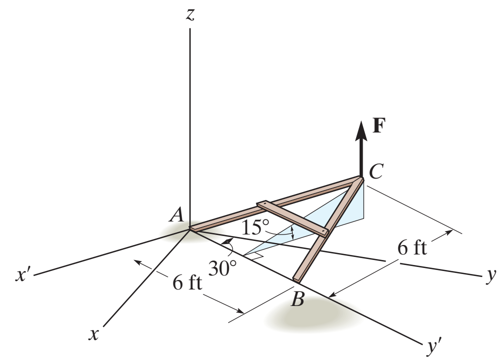
Ans:

Yes

Yes

4–54.

The A-frame is being hoisted into an upright position by the vertical force of $F = 80$ lb. Determine the moment of this force about the y' axis passing through points A and B when the frame is in the position shown.



SOLUTION

Scalar analysis :

$M_{y'} = 80\,(6 \cos 15°) = 464 \text{ lb} \cdot \text{ft}$

Vector analysis :

$\mathbf{u}_{AB} = \cos 60°\,\mathbf{i} + \cos 30°\,\mathbf{j}$

Coordinates of point C :

$x = 3 \sin 30° - 6 \cos 15° \cos 30° = -3.52$ ft

$y = 3 \cos 30° + 6 \cos 15° \sin 30° = 5.50$ ft

$z = 6 \sin 15° = 1.55$ ft

$\mathbf{r}_{AC} = -3.52\,\mathbf{i} + 5.50\,\mathbf{j} + 1.55\,\mathbf{k}$

$\mathbf{F} = 80$ k

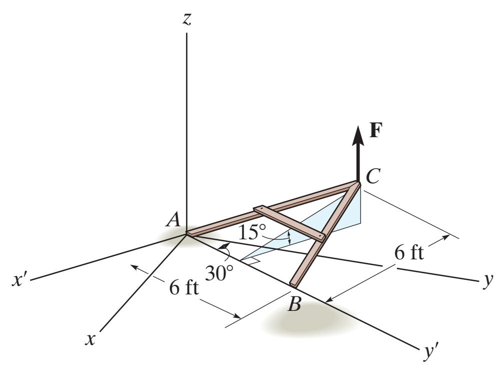
$$M_{y'} = \begin{vmatrix} \sin 30° & \cos 30° & 0 \\ -3.52 & 5.50 & 1.55 \\ 0 & 0 & 80 \end{vmatrix}$$

$M_{y'} = 464 \text{ lb} \cdot \text{ft}$ **Ans.**

Ans:
$M_{y'} = 464 \text{ lb} \cdot \text{ft}$

4–55.

The A-frame is being hoisted into an upright position by the vertical force of $F = 80$ lb. Determine the moment of this force about the x axis when the frame is in the position shown.



SOLUTION

Using x', y', z:

$\mathbf{u}_x = \cos 30° \, \mathbf{i}' + \sin 30° \, \mathbf{j}'$

$\mathbf{r}_{AC} = -6 \cos 15° \, \mathbf{i}' + 3 \, \mathbf{j}' + 6 \sin 15° \, \mathbf{k}$

$\mathrm{F} = 80 \, \mathbf{k}$

$$M_x = \begin{vmatrix} \cos 30° & \sin 30° & 0 \\ -6 \cos 15° & 3 & 6 \sin 15° \\ 0 & 0 & 80 \end{vmatrix} = 207.85 + 231.82 + 0$$

$M_x = 440 \text{ lb} \cdot \text{ft}$

Also, using x, y, z,

Coordinates of point C:

$x = 3 \sin 30° - 6 \cos 15° \cos 30° = -3.52 \text{ ft}$

$y = 3 \cos 30° + 6 \cos 15° \sin 30° = 5.50 \text{ ft}$

$z = 6 \sin 15° = 1.55 \text{ ft}$

$\mathbf{r}_{AC} = -3.52 \, \mathbf{i} + 5.50 \, \mathbf{j} + 1.55 \, \mathbf{k}$

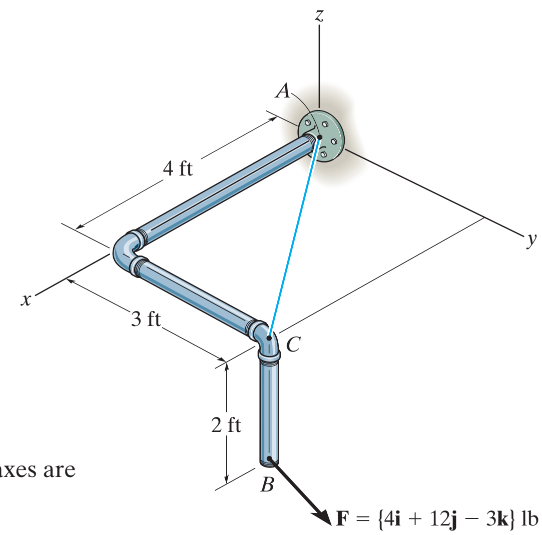
$\mathbf{F} = 80 \, \mathbf{k}$

$$M_x = \begin{vmatrix} 1 & 0 & 0 \\ -3.52 & 5.50 & 1.55 \\ 0 & 0 & 80 \end{vmatrix} = 440 \text{ lb} \cdot \text{ft}$$ **Ans.**

Ans:
$M_x = 440 \text{ lb} \cdot \text{ft}$

***4–56.**

Determine the magnitude of the moments of the force **F** about the x, y, and z axes. Solve the problem (a) using a Cartesian vector approach and (b) using a scalar approach.



SOLUTION

a) ***Vector Analysis***

Position Vector:

$$\mathbf{r}_{AB} = \{(4-0)\,\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k}\}\text{ ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}\text{ ft}$$

Moment of Force* F *About x, y, and z Axes: The unit vectors along x, y, and z axes are **i**, **j**, and **k** respectively. Applying Eq. 4–11, we have

$$M_x = \mathbf{i}\cdot(\mathbf{r}_{AB}\times\mathbf{F})$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 1[3(-3) - (12)(-2)] - 0 + 0 = 15.0\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

$$M_y = \mathbf{j}\cdot(\mathbf{r}_{AB}\times\mathbf{F})$$

$$= \begin{vmatrix} 0 & 1 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 1[4(-3) - (4)(-2)] + 0 = 4.00\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

$$M_z = \mathbf{k}\cdot(\mathbf{r}_{AB}\times\mathbf{F})$$

$$= \begin{vmatrix} 0 & 0 & 1 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0 - 0 + 1[4(12) - (4)(3)] = 36.0\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

b) ***Scalar Analysis***

$M_x = \Sigma M_x;\qquad M_x = 12(2) - 3(3) = 15.0\text{ lb}\cdot\text{ft}$ **Ans.**

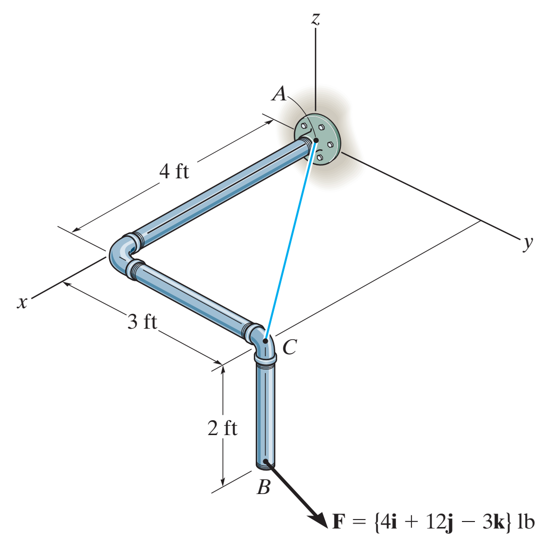
$M_y = \Sigma M_y;\qquad M_y = -4(2) + 3(4) = 4.00\text{ lb}\cdot\text{ft}$ **Ans.**

$M_z = \Sigma M_z;\qquad M_z = -4(3) + 12(4) = 36.0\text{ lb}\cdot\text{ft}$ **Ans.**

Ans:
$M_x = 15.0\text{ lb}\cdot\text{ft}$
$M_y = 4.00\text{ lb}\cdot\text{ft}$
$M_z = 36.0\text{ lb}\cdot\text{ft}$

4–57.

Determine the moment of the force **F** about an axis extending between A and C. Express the result as a Cartesian vector.



SOLUTION

Position Vector:

$$\mathbf{r}_{CB} = \{-2\mathbf{k}\}\ \text{ft}$$

$$\mathbf{r}_{AB} = \{(4-0)\mathbf{i} + (3-0)\mathbf{j} + (-2-0)\mathbf{k}\}\ \text{ft} = \{4\mathbf{i} + 3\mathbf{j} - 2\mathbf{k}\}\ \text{ft}$$

Unit Vector Along AC Axis:

$$\mathbf{u}_{AC} = \frac{(4-0)\mathbf{i} + (3-0)\mathbf{j}}{\sqrt{(4-0)^2 + (3-0)^2}} = 0.8\mathbf{i} + 0.6\mathbf{j}$$

Moment of Force* F *About AC Axis: With $\mathbf{F} = \{4\mathbf{i} + 12\mathbf{j} - 3\mathbf{k}\}$ lb, applying Eq. 4–7, we have

$$M_{AC} = \mathbf{u}_{AC} \cdot (\mathbf{r}_{CB} \times \mathbf{F})$$

$$= \begin{vmatrix} 0.8 & 0.6 & 0 \\ 0 & 0 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0.8[(0)(-3) - 12(-2)] - 0.6[0(-3) - 4(-2)] + 0$$

$$= 14.4\ \text{lb} \cdot \text{ft}$$

Or

$$M_{AC} = \mathbf{u}_{AC} \cdot (\mathbf{r}_{AB} \times \mathbf{F})$$

$$= \begin{vmatrix} 0.8 & 0.6 & 0 \\ 4 & 3 & -2 \\ 4 & 12 & -3 \end{vmatrix}$$

$$= 0.8[(3)(-3) - 12(-2)] - 0.6[4(-3) - 4(-2)] + 0$$

$$= 14.4\ \text{lb} \cdot \text{ft}$$

Expressing $\mathbf{M}_{AC}$ as a Cartesian vector yields

$$\mathbf{M}_{AC} = M_{AC}\,\mathbf{u}_{AC}$$

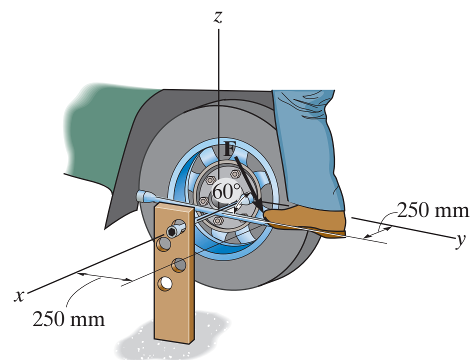
$$= 14.4(0.8\mathbf{i} + 0.6\mathbf{j})$$

$$= \{11.5\mathbf{i} + 8.64\mathbf{j}\}\ \text{lb} \cdot \text{ft} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_{AC} = \{11.5\mathbf{i} + 8.64\mathbf{j}\}\ \text{lb} \cdot \text{ft}$

4–58.

The board is used to hold the end of a four-way lug wrench in the position shown when the man applies a force of $F = 100$ N. Determine the magnitude of the moment produced by this force about the x axis. Force **F** lies in a vertical plane.



SOLUTION

Vector Analysis
Moment About the x Axis: The position vector $\mathbf{r}_{AB}$, Fig. *a*, will be used to determine the moment of **F** about the x axis.

$$\mathbf{r}_{AB} = (0.25 - 0.25)\mathbf{i} + (0.25 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = \{0.25\mathbf{j}\}\text{ m}$$

The force vector **F**, Fig. *a*, can be written as

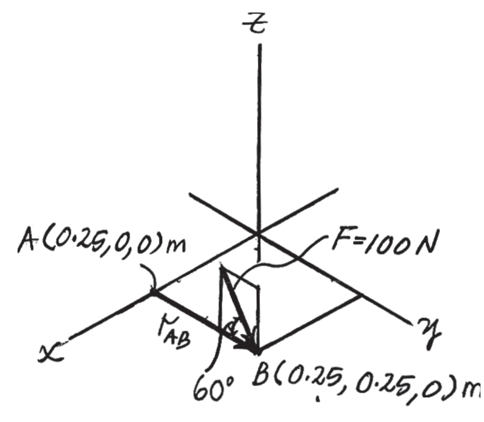
$$\mathbf{F} = 100(\cos 60°\mathbf{j} - \sin 60°\mathbf{k}) = \{50\mathbf{j} - 86.60\mathbf{k}\}\text{ N}$$

Knowing that the unit vector of the x axis is **i**, the magnitude of the moment of **F** about the x axis is given by

$$M_x = \mathbf{i} \cdot \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 50 & -86.60 \end{vmatrix}$$

$$= 1[0.25(-86.60) - 50(0)] + 0 + 0$$

$$= -21.7\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



(a)

The negative sign indicates that M_x is directed towards the negative x axis.

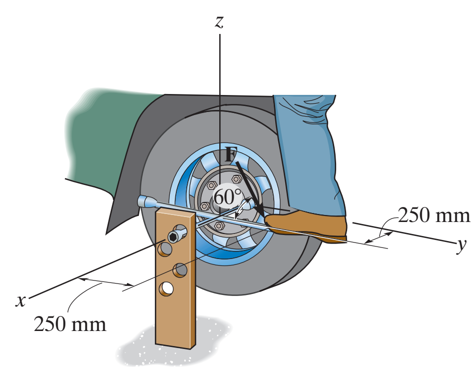
Scalar Analysis
This problem can be solved by summing the moment about the x axis

$$M_x = \Sigma M_x; \qquad M_x = -100\ \sin 60°(0.25) + 100\ \cos 60°(0)$$
$$= -21.7\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$M_x = 21.7\text{ N}\cdot\text{m}$

4–59.

The board is used to hold the end of a four-way lug wrench in position. If a torque of 30 N · m about the x axis is required to tighten the nut, determine the required magnitude of the force **F** that the man's foot must apply on the end of the wrench in order to turn it. Force **F** lies in a vertical plane.



SOLUTION

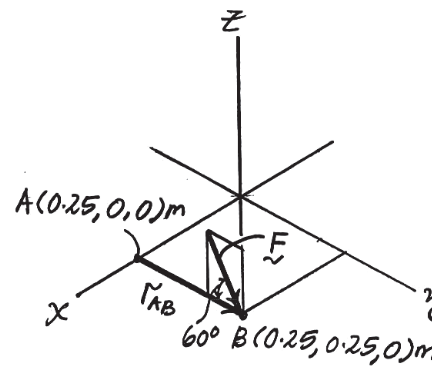
Vector Analysis

Moment About the x Axis: The position vector $\mathbf{r}_{AB}$, Fig. a, will be used to determine the moment of **F** about the x axis.

$$\mathbf{r}_{AB} = (0.25 - 0.25)\mathbf{i} + (0.25 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = \{0.25\mathbf{j}\}\text{ m}$$

The force vector **F**, Fig. a, can be written as

$$\mathbf{F} = F(\cos 60°\mathbf{j} - \sin 60°\mathbf{k}) = 0.5F\mathbf{j} - 0.8660F\mathbf{k}$$



(a)

Knowing that the unit vector of the x axis is **i**, the magnitude of the moment of **F** about the x axis is given by

$$M_x = \mathbf{i} \cdot \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0.5F & -0.8660F \end{vmatrix}$$

$$= 1[0.25(-0.8660F) - 0.5F(0)] + 0 + 0$$

$$= -0.2165F \qquad \textbf{Ans.}$$

The negative sign indicates that M_x is directed towards the negative x axis. The magnitude of **F** required to produce $M_x = 30$ N · m can be determined from

$$30 = 0.2165F$$

$$F = 139\text{ N} \qquad \textbf{Ans.}$$

Scalar Analysis

This problem can be solved by summing the moment about the x axis

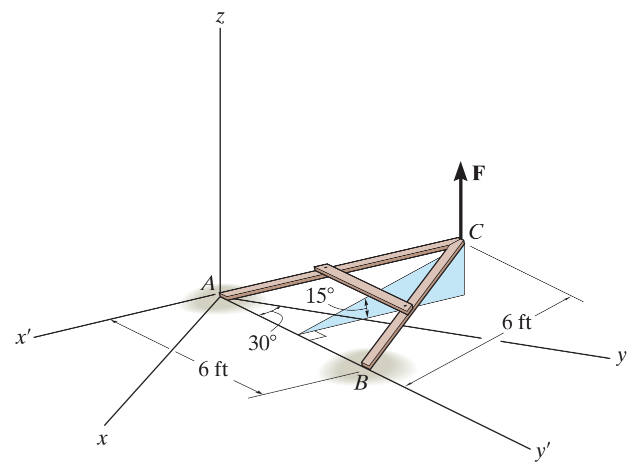
$$M_x = \Sigma M_x; \qquad -30 = -F\sin 60°(0.25) + F\cos 60°(0)$$

$$F = 139\text{ N} \qquad \textbf{Ans.}$$

Ans:
$F = 139$ N

***4–60.**

The A-frame is being hoisted into an upright position by the vertical force of $F = 80$ lb. Determine the moment of this force about the y axis when the frame is in the position shown.



SOLUTION

Using x', y', z:

$$\mathbf{u}_y = -\sin 30^\circ\,\mathbf{i}' + \cos 30^\circ\,\mathbf{j}'$$

$$\mathbf{r}_{AC} = -6\cos 15^\circ\mathbf{i}' + 3\,\mathbf{j}' + 6\sin 15^\circ\,\mathbf{k}$$

$$\mathbf{F} = 80\,\mathbf{k}$$

$$M_y = \begin{vmatrix} -\sin 30^\circ & \cos 30^\circ & 0 \\ -6\cos 15^\circ & 3 & 6\sin 15^\circ \\ 0 & 0 & 80 \end{vmatrix} = -120 + 401.53 + 0$$

$M_y = 282\ \text{lb}\cdot\text{ft}$ **Ans.**

Also, using x, y, z:

Coordinates of point C:

$$x = 3\sin 30^\circ - 6\cos 15^\circ \cos 30^\circ = -3.52\ \text{ft}$$

$$y = 3\cos 30^\circ + 6\cos 15^\circ \sin 30^\circ = 5.50\ \text{ft}$$

$$z = 6\sin 15^\circ = 1.55\ \text{ft}$$

$$\mathbf{r}_{AC} = -3.52\,\mathbf{i} + 5.50\,\mathbf{j} + 1.55\,\mathbf{k}$$

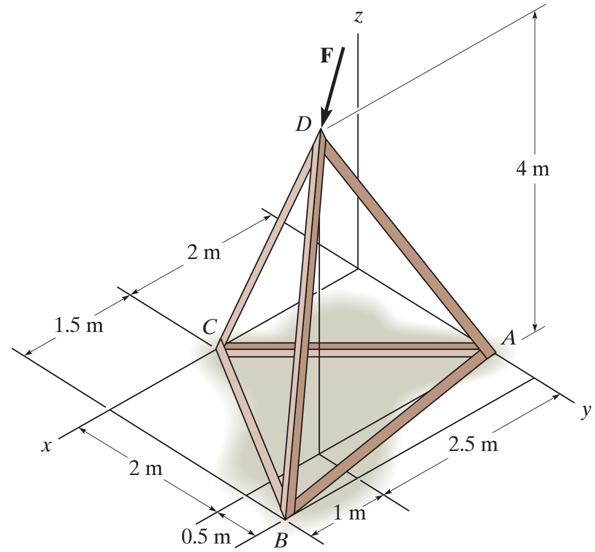
$$\mathbf{F} = 80\,\mathbf{k}$$

$$M_y = \begin{vmatrix} 0 & 1 & 0 \\ -3.52 & 5.50 & 1.55 \\ 0 & 0 & 80 \end{vmatrix} = 282\ \text{lb}\cdot\text{ft}$$ **Ans.**

Ans:
$M_y = 282\ \text{lb}\cdot\text{ft}$

4–61.

Determine the magnitude of the moment of the force $\mathbf{F} = \{50\mathbf{i} - 20\mathbf{j} - 80\mathbf{k}\}$ N about the base line AB of the tripod.



SOLUTION

$$\mathbf{u}_{AB} = \frac{\{3.5\mathbf{i} + 0.5\mathbf{j}\}}{\sqrt{(3.5)^2 + (0.5)^2}}$$

$$\mathbf{u}_{AB} = \{0.9899\mathbf{i} + 0.1414\mathbf{j}\}$$

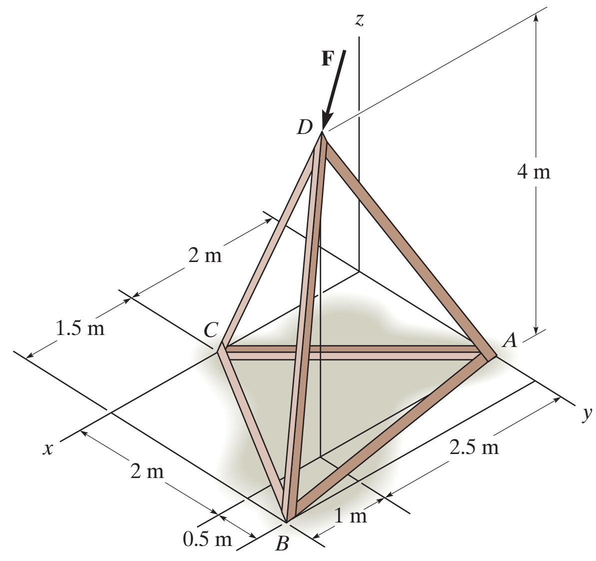
$$M_{AB} = \mathbf{u}_{AB} \cdot (\mathbf{r}_{AD} \times \mathbf{F}) = \begin{vmatrix} 0.9899 & 0.1414 & 0 \\ 2.5 & 0 & 4 \\ 50 & -20 & -80 \end{vmatrix}$$

$M_{AB} = 136 \text{ N} \cdot \text{m}$ **Ans.**

Ans:
$M_{AB} = 136 \text{ N} \cdot \text{m}$

4–62.

Determine the magnitude of the moment of the force $\mathbf{F} = \{50\mathbf{i} - 20\mathbf{j} - 80\mathbf{k}\}$ N about the base line BC of the tripod.



SOLUTION

$$\mathbf{u}_{BC} = \frac{\{-1.5\mathbf{i} - 2.5\mathbf{j}\}}{\sqrt{(-1.5)^2 + (-2.5)^2}}$$

$$\mathbf{u}_{BC} = \{-0.5145\mathbf{i} - 0.8575\mathbf{j}\}$$

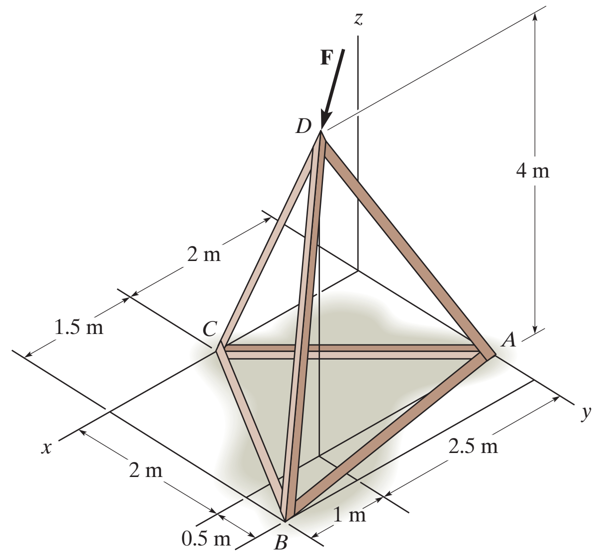
$$M_{BC} = \mathbf{u}_{BC} \cdot (\mathbf{r}_{CD} \times \mathbf{F}) = \begin{vmatrix} -0.5145 & -0.8575 & 0 \\ 0.5 & 2 & 4 \\ 50 & -20 & -80 \end{vmatrix}$$

$M_{BC} = 165 \text{ N} \cdot \text{m}$ **Ans.**

Ans:
$M_{BC} = 165 \text{ N} \cdot \text{m}$

4–63.

Determine the magnitude of the moment of the force $\mathbf{F} = \{50\mathbf{i} - 20\mathbf{j} - 80\mathbf{k}\}$ N about the base line CA of the tripod.



SOLUTION

$$\mathbf{u}_{CA} = \frac{\{-2\mathbf{i} + 2\mathbf{j}\}}{\sqrt{(-2)^2 + (2)^2}}$$

$$\mathbf{u}_{CA} = \{-0.707\mathbf{i} + 0.707\mathbf{j}\}$$

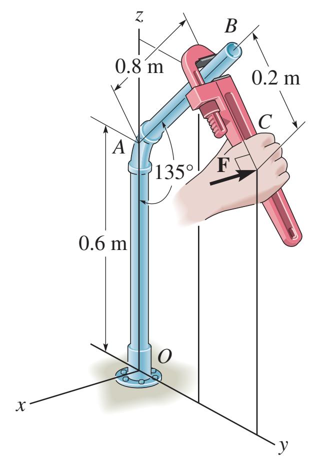
$$M_{CA} = \mathbf{u}_{CA} \cdot (\mathbf{r}_{AD} \times \mathbf{F}) = \begin{vmatrix} -0.707 & 0.707 & 0 \\ 2.5 & 0 & 4 \\ 50 & -20 & -80 \end{vmatrix}$$

$M_{CA} = 226\ \mathrm{N \cdot m}$ **Ans.**

Ans:
$M_{CA} = 226\ \mathrm{N \cdot m}$

***4–64.**

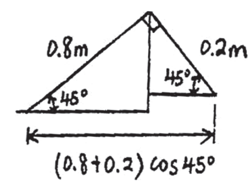
A horizontal force of $\mathbf{F} = \{-50\mathbf{i}\}$ N is applied perpendicular to the handle of the pipe wrench. Determine the moment that this force exerts along the axis OA (z axis) of the pipe assembly. Both the wrench and pipe assembly, $OABC$, lie in the y-z plane. *Suggestion:* Use a scalar analysis.



SOLUTION

$$M_z = 50(0.8 + 0.2)\cos 45° = 35.36\ \text{N} \cdot \text{m}$$

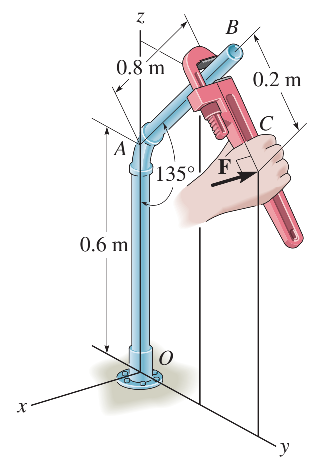
$$M_z = \{35.4\,\mathbf{k}\}\ \text{N} \cdot \text{m}$$ **Ans.**



Ans:
$M_z = \{35.4\,\mathbf{k}\}\ \text{N} \cdot \text{m}$

4–65.

Determine the magnitude of the horizontal force $\mathbf{F} = -F\mathbf{i}$ acting on the handle of the wrench so that this force produces a component of the moment along the OA axis (z axis) of the pipe assembly of $\mathbf{M}_z = \{4\mathbf{k}\}\ \mathrm{N \cdot m}$. Both the wrench and the pipe assembly, $OABC$, lie in the y-z plane. *Suggestion*: Use a scalar analysis.



SOLUTION

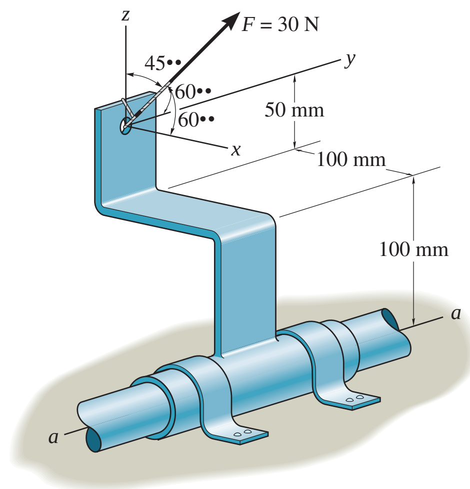
$M_z = F(0.8 + 0.2)\cos 45° = 4$

$F = 5.66\ \mathrm{N}$ **Ans.**

Ans:
$F = 5.66\ \mathrm{N}$

4–66.

The force of $F = 30\text{ N}$ acts on the bracket as shown. Determine the moment of the force about the $a-a$ axis of the pipe if $\alpha = 60°$, $\beta = 60°$, and $\gamma = 45°$. Also, determine the coordinate direction angles of F in order to produce the maximum moment about the a–a axis. What is this moment?



SOLUTION

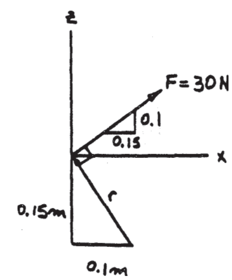
$$\mathbf{F} = 30\,(\cos 60°\,\mathbf{i} + \cos 60°\,\mathbf{j} + \cos 45°\,\mathbf{k})$$

$$= \{15\,\mathbf{i} + 15\,\mathbf{j} + 21.21\,\mathbf{k}\}\text{ N}$$

$$\mathbf{r} = \{-0.1\,\mathbf{i} + 0.15\,\mathbf{k}\}\text{ m}$$

$$\mathbf{u} = \mathbf{j}$$

$$M_a = \begin{vmatrix} 0 & 1 & 0 \\ -0.1 & 0 & 0.15 \\ 15 & 15 & 21.21 \end{vmatrix} = 4.37\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



F must be perpendicular to **u** and **r**.

$$\mathbf{u}_F = \frac{0.15}{0.1803}\mathbf{i} + \frac{0.1}{0.1803}\mathbf{k}$$

$$= 0.8321\mathbf{i} + 0.5547\mathbf{k}$$

$$\alpha = \cos^{-1} 0.8321 = 33.7° \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1} 0 = 90° \qquad \textbf{Ans.}$$

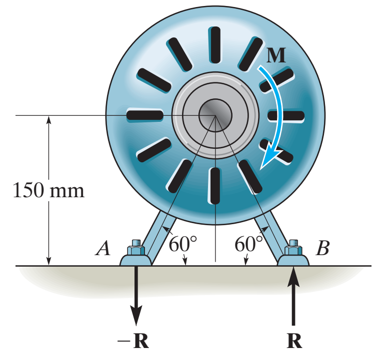
$$\gamma = \cos^{-1} 0.5547 = 56.3° \qquad \textbf{Ans.}$$

$$M = 30\,(0.1803) = 5.41\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$M_a = 4.37\text{ N}\cdot\text{m}$
$\alpha = 33.7°$
$\beta = 90°$
$\gamma = 56.3°$
$M = 5.41\text{ N}\cdot\text{m}$

4–67.

A clockwise couple $M = 5\ \text{N}\cdot\text{m}$ is resisted by the shaft of the electric motor. Determine the magnitude of the reactive forces $-\mathbf{R}$ and $\mathbf{R}$ which act at supports A and B so that the resultant of the two couples is zero.



SOLUTION

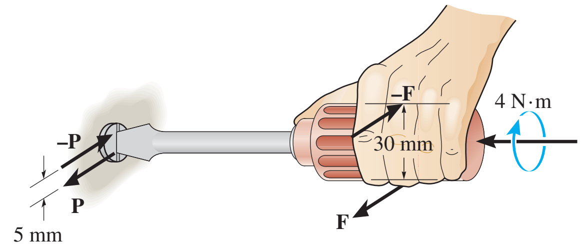
$\zeta + M_C = -5 + R\,(2(0.15)/\tan 60^\circ) = 0$

$R = 28.9\ \text{N}$ **Ans.**

Ans:
$R = 28.9\ \text{N}$

***4–68.**

A twist of 4 N · m is applied to the handle of the screwdriver. Resolve this couple moment into a pair of couple forces **F** exerted on the handle and **P** exerted on the blade.



SOLUTION

For the handle

$M_C = \Sigma M_x;$ $\quad F(0.03) = 4$

$F = 133\text{ N}$ **Ans.**

For the blade,

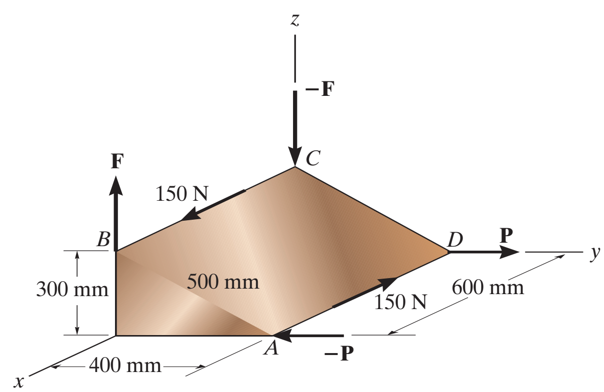
$M_C = \Sigma M_x;$ $\quad P(0.005) = 4$

$P = 800\text{ N}$ **Ans.**

Ans:
$F = 133\text{ N}$
$P = 800\text{ N}$

4–69.

If the resultant couple of the three couples acting on the triangular block is to be zero, determine the magnitude of forces **F** and **P**.



SOLUTION

$BA = 0.5\text{ m}$

The couple created by the 150 - N forces is

$M_{C1} = 150\,(0.5) = 75\text{ N}\cdot\text{m}$

Then

$$\mathbf{M}_{C1} = 75\left(\frac{3}{5}\right)\mathbf{j} + 75\left(\frac{4}{5}\right)\mathbf{k}$$

$$= 45\,\mathbf{j} + 60\,\mathbf{k}$$

$$\mathbf{M}_{C2} = -P\,(0.6)\,\mathbf{k}$$

$$\mathbf{M}_{C3} = -F\,(0.6)\,\mathbf{j}$$

Require

$$\mathbf{M}_{C1} + \mathbf{M}_{C2} + \mathbf{M}_{C3} = \mathbf{0}$$

$$45\,\mathbf{j} + 60\,\mathbf{k} - P\,(0.6)\,\mathbf{k} - F\,(0.6)\,\mathbf{j} = \mathbf{0}$$

Equate the **j** and **k** components

$45 - F\,(0.6) = 0$

$F = 75\text{ N}$ **Ans.**

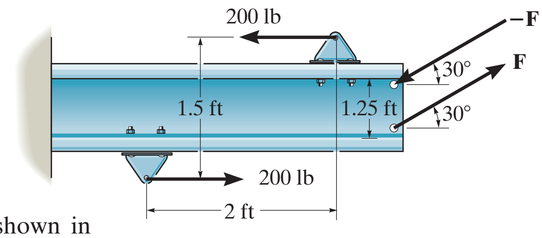
$60 - P\,(0.6) = 0$

$P = 100\text{ N}$ **Ans.**

Ans:
$F = 75\text{ N}$
$P = 100\text{ N}$

4–70.

Two couples act on the beam. If $F = 125$ lb , determine the resultant couple moment.

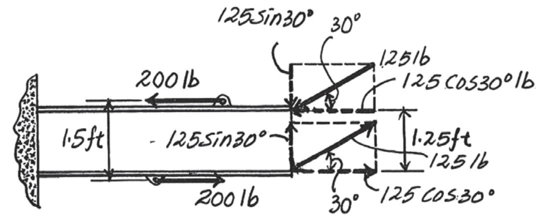


SOLUTION

125 lb couple is resolved in to their horizontal and vertical components as shown in Fig. *a*.

$$\circlearrowleft + (M_R)_C = 200(1.5) + 125\cos 30°(1.25)$$

$$= 435.32 \text{ lb} \cdot \text{ft} = 435 \text{ lb} \cdot \text{ft} \circlearrowleft \qquad \textbf{Ans.}$$

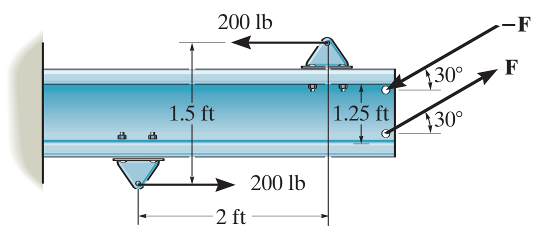


(a)

Ans:
$(M_R)_C = 435$ lb · ft $\circlearrowleft$

4–71.

Two couples act on the beam. Determine the magnitude of **F** so that the resultant couple moment is 450 lb · ft, counterclockwise. Where on the beam does the resultant couple moment act?



SOLUTION

$$\circlearrowleft +M_R = \Sigma M; \qquad 450 = 200(1.5) + F\cos 30°(1.25)$$

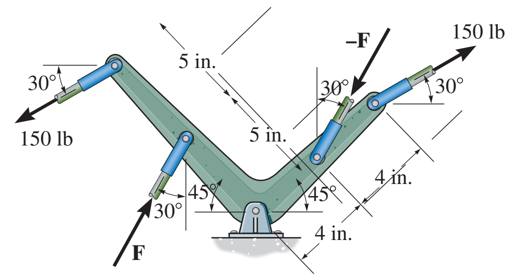
$$F = 139 \text{ lb} \qquad \textbf{Ans.}$$

The resultant couple moment is a free vector. It can act at any point on the beam.

Ans:
$F = 139$ lb

***4–72.**

Determine the magnitude of the couple force **F** so that the resultant couple moment on the crank is zero.



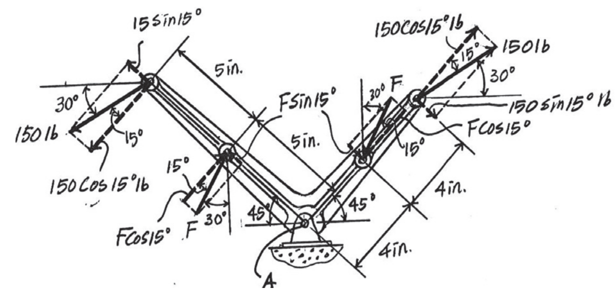
SOLUTION

By resolving **F** and the 150-lb couple into components parallel and perpendicular to the lever arm of the crank, Fig. *a*, and summing the moment of these two force components about point A, we have

$$\circlearrowleft +(M_C)_R = \Sigma M_A; \qquad 0 = 150 \cos 15°(10) - F \cos 15°(5) - F \sin 15°(4) - 150 \sin 15°(8)$$

$$F = 194 \text{ lb} \qquad \textbf{Ans.}$$

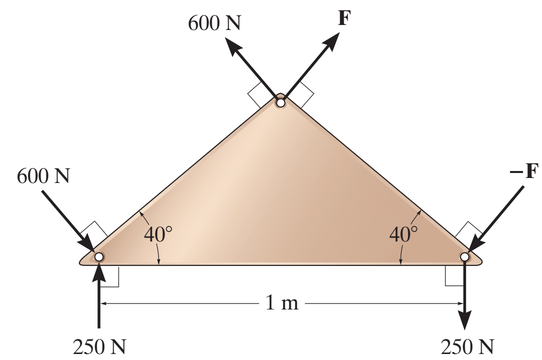
Note: Since the line of action of the force component parallel to the lever arm of the crank passes through point A, no moment is produced about this point.



Ans:
$F = 194$ lb

4–73.

The ends of the triangular plate are subjected to three couples. Determine the magnitude of the force **F** so that the resultant couple moment is 400 N · m clockwise.



SOLUTION

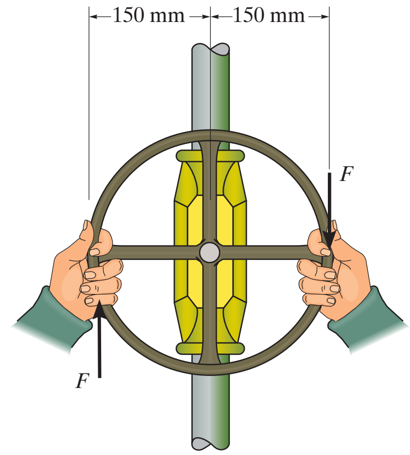
$$\circlearrowleft + M_R = \Sigma M; \qquad -400 = 600\left(\frac{0.5}{\cos 40^\circ}\right) - F\left(\frac{0.5}{\cos 40^\circ}\right) - 250(1)$$

$$F = 830 \text{ N}$$ **Ans.**

Ans:
$F = 830$ N

4–74.

The man tries to open the valve by applying the couple forces of $F = 75$ N to the wheel. Determine the couple moment produced.



SOLUTION

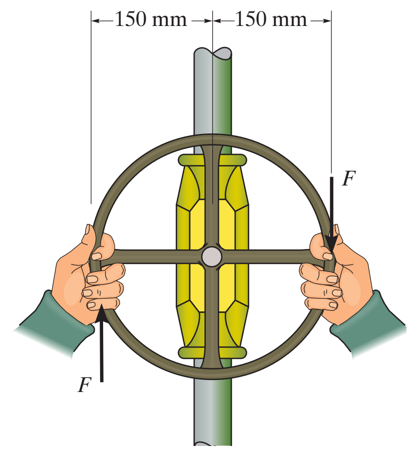
$\circlearrowleft + M_c = \Sigma M;$ $\quad M_c = -75(0.15 + 0.15)$

$= -22.5\ \text{N}\cdot\text{m} = 22.5\ \text{N}\cdot\text{m} \circlearrowright$ **Ans.**

Ans:
$M_C = 22.5\ \text{N}\cdot\text{m} \circlearrowright$

4–75.

If the valve can be opened with a couple moment of 25 N · m, determine the required magnitude of each couple force which must be applied to the wheel.



SOLUTION

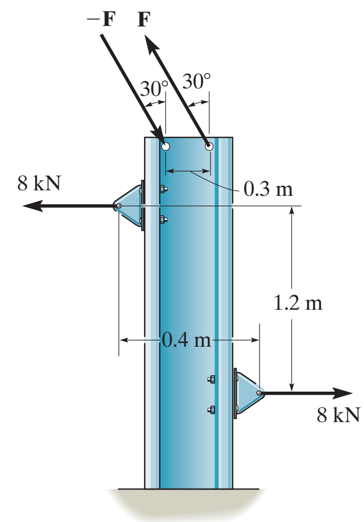
$\circlearrowleft + M_c = \Sigma M;$ $\quad -25 = -F(0.15 + 0.15)$

$F = 83.3\text{ N}$ **Ans.**

Ans:
$F = 83.3\text{ N}$

***4–76.**

Determine the magnitude of **F** so that the resultant couple moment is 12 kN · m, counterclockwise. Where on the beam does the resultant couple moment act?



SOLUTION

$\circlearrowleft + M_R = \Sigma M_C;\qquad 12 = (F\cos 30°)(0.3) + 8(1.2)$

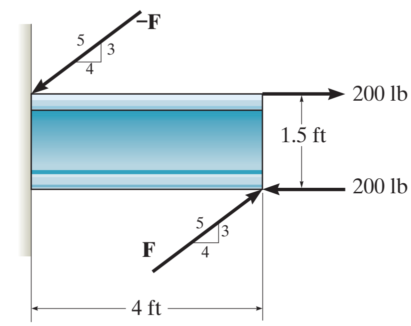
$F = 9.238\text{ kN} = 9.24\text{ kN}$ **Ans.**

Since the couple moment is a free vector, the resultant couple moment can act **at any point on or off the beam**.

Ans:
$F = 9.24\text{ kN}$

4–77.

Two couples act on the beam as shown. If $F = 150$ lb, determine the resultant couple moment.

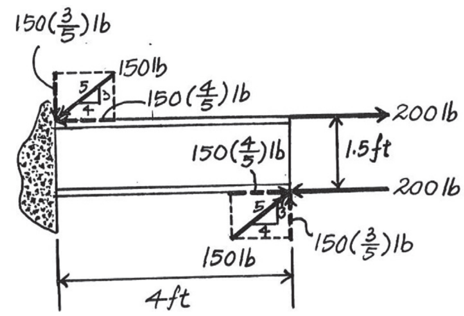


SOLUTION

150 lb couple is resolved into their horizontal and vertical components as shown in Fig. *a*

$$\circlearrowleft + (M_R)_c = 150\left(\frac{4}{5}\right)(1.5) + 150\left(\frac{3}{5}\right)(4) - 200(1.5)$$

$$= 240 \text{ lb} \cdot \text{ft} \circlearrowleft$$ **Ans.**

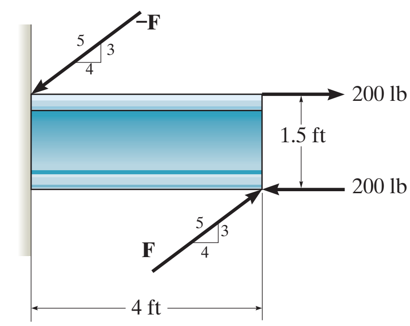


(a)

Ans:
$(M_R)_C = 240 \text{ lb} \cdot \text{ft} \circlearrowleft$

4–78.

Two couples act on the beam as shown. Determine the magnitude of **F** so that the resultant couple moment is 300 lb · ft counterclockwise. Where on the beam does the resultant couple act?



SOLUTION

$$\circlearrowleft + (M_C)_R = \frac{3}{5}F(4) + \frac{4}{5}F(1.5) - 200(1.5) = 300$$

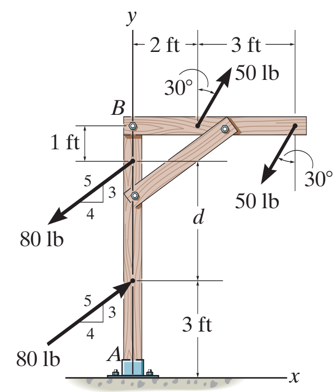
$$F = 167 \text{ lb}$$ **Ans.**

Resultant couple can act anywhere. **Ans.**

Ans:
$F = 167$ lb
Resultant couple can act anywhere.

4–79.

Two couples act on the frame. If the resultant couple moment is to be zero, determine the distance d between the 80-lb couple forces.



SOLUTION

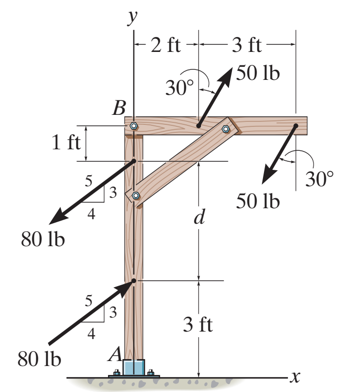
$$\zeta + M_C = -50 \cos 30°(3) + \frac{4}{5}(80)(d) = 0$$

$$d = 2.03 \text{ ft}$$ **Ans.**

Ans:
$d = 2.03$ ft

***4–80.**

Two couples act on the frame. If $d = 4$ ft, determine the resultant couple moment. Compute the result by resolving each force into x and y components and (a) finding the moment of each couple (Eq. 4–13) and (b) summing the moments of all the force components about point A.



SOLUTION

(a) $\mathbf{M}_C = \Sigma(\mathbf{r} \times \mathbf{F})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -50 \sin 30^\circ & -50 \cos 30^\circ & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 4 & 0 \\ -\frac{4}{5}(80) & -\frac{3}{5}(80) & 0 \end{vmatrix}$$

$\mathbf{M}_C = \{126\mathbf{k}\}\ \text{lb} \cdot \text{ft}$ **Ans.**

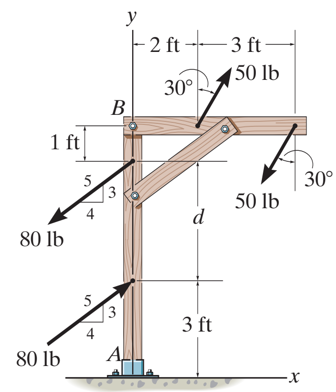
(b) $\zeta + M_C = -\frac{4}{5}(80)(3) + \frac{4}{5}(80)(7) + 50 \cos 30^\circ(2) - 50 \cos 30^\circ(5)$

$M_C = 126\ \text{lb} \cdot \text{ft}$ **Ans.**

Ans:
$M_C = 126\ \text{lb} \cdot \text{ft}$

4–81.

Two couples act on the frame. If $d = 4$ ft, determine the resultant couple moment. Compute the result by resolving each force into x and y components and (a) finding the moment of each couple (Eq. 4–13) and (b) summing the moments of all the force components about point B.



SOLUTION

(a) $\mathbf{M}_C = \Sigma(\mathbf{r} \times \mathbf{F})$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ -50\sin 30° & -50\cos 30° & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -4 & 0 \\ \frac{4}{5}(80) & \frac{3}{5}(80) & 0 \end{vmatrix}$$

$\mathbf{M}_C = \{126\mathbf{k}\}\ \text{lb}\cdot\text{ft}$ **Ans.**

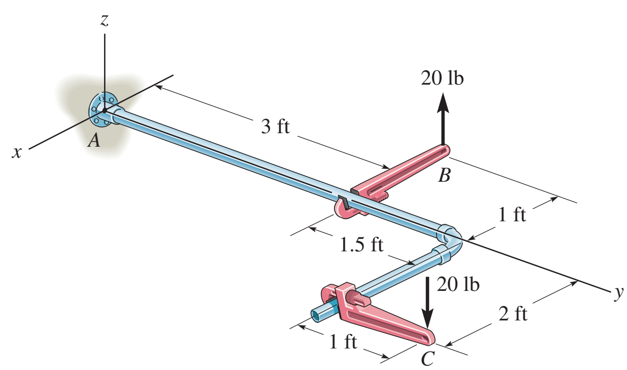
(b) $\circlearrowleft + M_C = 50\cos 30°(2) - 50\cos 30°(5) - \frac{4}{5}(80)(1) + \frac{4}{5}(80)(5)$

$M_C = 126\ \text{lb}\cdot\text{ft}$ **Ans.**

Ans:
$M_C = 126\ \text{lb}\cdot\text{ft} \circlearrowleft$

4–82.

Express the moment of the couple acting on the pipe assembly in Cartesian vector form. What is the magnitude of the couple moment?



SOLUTION

$$\mathbf{r}_{CB} = \{-3\mathbf{i} - 2.5\mathbf{j}\} \text{ ft}$$

$$\mathbf{M}_C = \mathbf{r}_{CB} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -3 & -2.5 & 0 \\ 0 & 0 & 20 \end{vmatrix}$$

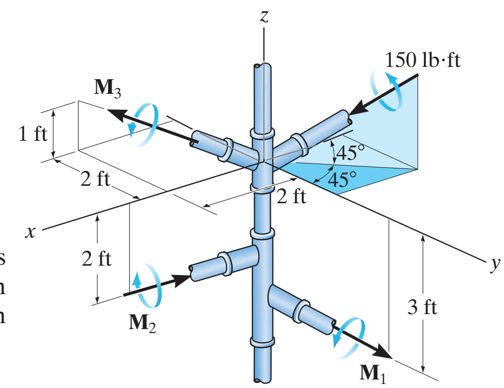
$$\mathbf{M}_C = \{-50\mathbf{i} + 60\mathbf{j}\} \text{ lb} \cdot \text{ft}$$ **Ans.**

$$M_C = \sqrt{(-50)^2 + (60)^2} = 78.1 \text{ lb} \cdot \text{ft}$$ **Ans.**

Ans:
$\mathbf{M}_C = 78.1 \text{ lb} \cdot \text{ft}$

4–83.

If $M_1 = 180$ lb·ft, $M_2 = 90$ lb·ft, and $M_3 = 120$ lb·ft, determine the magnitude and coordinate direction angles of the resultant couple moment.



SOLUTION

Since the couple moment is a free vector, it can act at any point without altering its effect. Thus, the couple moments $\mathbf{M}_1, \mathbf{M}_2, \mathbf{M}_3$, and $\mathbf{M}_4$ acting on the gear deducer can be simplified, as shown in Fig. *a*. Expressing each couple moment in Cartesian vector form,

$$\mathbf{M}_1 = [180\mathbf{j}]\text{lb}\cdot\text{ft}$$

$$\mathbf{M}_2 = [-90\mathbf{i}]\text{lb}\cdot\text{ft}$$

$$\mathbf{M}_3 = \mathrm{M}_3\mathbf{u} = 120\left[\frac{(2-0)\mathbf{i} + (-2-0)\mathbf{j} + (1+0)\mathbf{k}}{\sqrt{(2-0)^2 + (-2-0)^2 + (1-0)^2}}\right] = [80\mathbf{i} - 80\mathbf{j} + 40\mathbf{k}]\text{lb}\cdot\text{ft}$$

$$\mathbf{M}_4 = 150[\cos 45^\circ \sin 45^\circ\mathbf{i} - \cos 45^\circ \cos 45^\circ\mathbf{j} - \sin 45^\circ\mathbf{k}] = [75\mathbf{i} - 75\mathbf{j} - 106.07\mathbf{k}]\text{lb}\cdot\text{ft}$$

The resultant couple moment is given by

$$(\mathbf{M}_c)_R = \Sigma\mathbf{M}; \qquad (\mathbf{M}_c)_R = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4$$

$$= 180\mathbf{j} - 90\mathbf{i} + (80\mathbf{i} - 80\mathbf{j} + 40\mathbf{k}) + (75\mathbf{i} - 75\mathbf{j} - 106.07\mathbf{k})$$

$$= [65\mathbf{i} + 25\mathbf{j} - 66.07\mathbf{k}]\text{lb}\cdot\text{ft}$$

The magnitude of $(\mathbf{M}_c)_R$ is

$$(M_c)_R = \sqrt{[(M_c)_R]_x{}^2 + [(M_c)_R]_y{}^2 + [(M_c)_R]_z{}^2}$$

$$= \sqrt{(65)^2 + (25)^2 + (-66.07)^2}$$

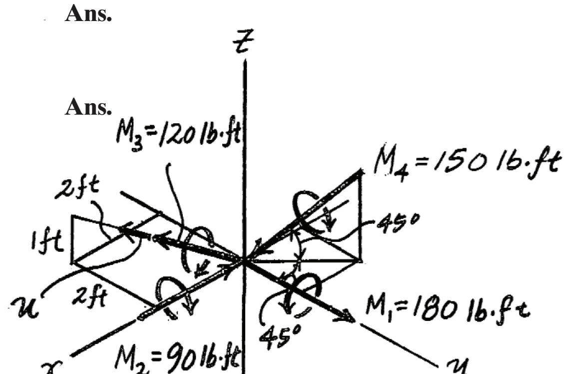
$$= 95.99\text{ lb}\cdot\text{ft} = 96.0\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

The coordinate angles of $(\mathbf{M}_c)_R$ are

$$\alpha = \cos^{-1}\left(\frac{[(M_c)_R]_x}{(M_c)_R}\right) = \cos\left(\frac{65}{95.99}\right) = 47.4^\circ \qquad \textbf{Ans.}$$

$$\beta = \cos^{-1}\left(\frac{[(M_c)_R]_y}{(M_c)_R}\right) = \cos\left(\frac{25}{95.99}\right) = 74.9^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{[(M_c)_R]_z}{(M_c)_R}\right) = \cos\left(\frac{-66.07}{95.99}\right) = 133^\circ \qquad \textbf{Ans.}$$

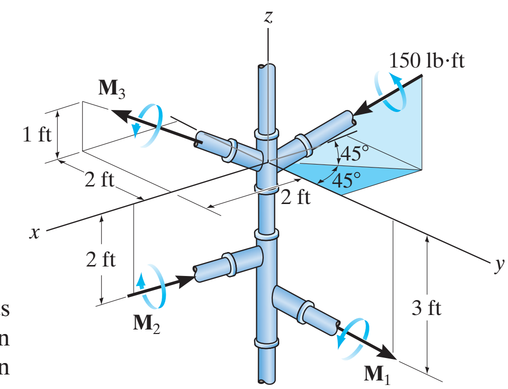


(a)

Ans:
$M_R = 96.0\text{lb}\cdot\text{ft}, \alpha = 47.4^\circ, \beta = 74.9^\circ, \gamma = 133^\circ$

***4–84.**

Determine the magnitudes of couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ so that the resultant couple moment is zero.



SOLUTION

Since the couple moment is a free vector, it can act at any point without altering its effect. Thus, the couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, $\mathbf{M}_3$, and $\mathbf{M}_4$ acting on the gear deducer can be simplified, as shown in Fig. *a*. Expressing each couple moment in Cartesian vector form,

$$\mathbf{M}_1 = M_1\mathbf{j}$$

$$\mathbf{M}_2 = -M_2\mathbf{i}$$

$$\mathbf{M}_3 = M_3\mathbf{u} = M_3\left[\frac{(2-0)\mathbf{i} + (-2-0)\mathbf{j} + (1+0)\mathbf{k}}{\sqrt{(2-0)^2 + (-2-0)^2 + (1-0)^2}}\right] = \frac{2}{3}M_3\mathbf{i} - \frac{2}{3}M_3\mathbf{j} + \frac{1}{3}M_3\mathbf{k}$$

$$\mathbf{M}_4 = 150[\cos 45° \sin 45°\mathbf{i} - \cos 45° \cos 45°\mathbf{j} - \sin 45°\mathbf{k}] = [75\mathbf{i} - 75\mathbf{j} - 106.07\mathbf{k}]\text{lb}\cdot\text{ft}$$

The resultant couple moment is required to be zero. Thus,

$$(\mathbf{M}_c)_R = \Sigma\mathbf{M}; \qquad 0 = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4$$

$$0 = M_1\mathbf{j} + (-M_2\mathbf{i}) + \left(\frac{2}{3}M_3\mathbf{i} - \frac{2}{3}M_3\mathbf{j} + \frac{1}{3}M_3\mathbf{k}\right) + (75\mathbf{i} - 75\mathbf{j} - 106.07\mathbf{k})$$

$$0 = \left(-M_2 + \frac{2}{3}M_3 + 75\right)\mathbf{i} + \left(M_1 - \frac{2}{3}M_3 - 75\right)\mathbf{j} + \left(\frac{1}{3}M_3 - 106.07\right)\mathbf{k}$$

Equating the **i, j,** and **k** components,

$$0 = -M_2 + \frac{2}{3}M_3 + 75 \qquad \textbf{(1)}$$

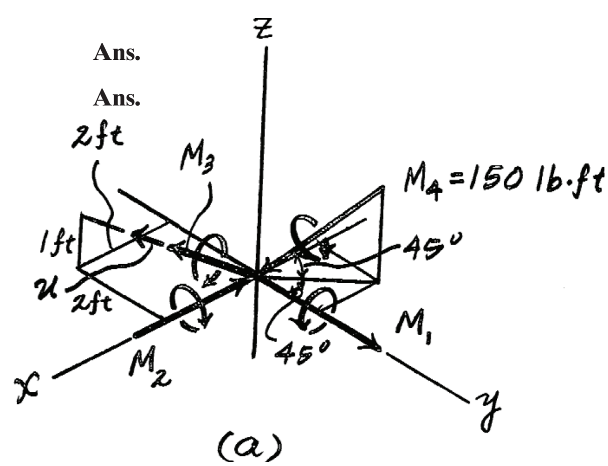
$$0 = M_1 - \frac{2}{3}M_3 - 75 \qquad \textbf{(2)}$$

$$0 = \frac{1}{3}M_3 - 106.07 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields

$$M_3 = 318 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

$$M_1 = M_2 = 287 \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

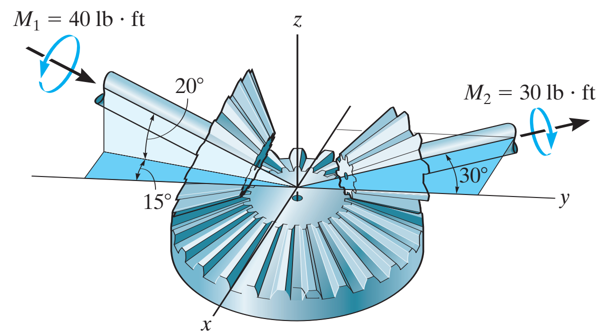


(a)

Ans:
$M_3 = 318 \text{ lb}\cdot\text{ft}$, $M_1 = M_2 = 287 \text{ lb}\cdot\text{ft}$

4–85.

The gears are subjected to the couple moments shown. Determine the magnitude and coordinate direction angles of the resultant couple moment.



SOLUTION

$$\mathbf{M}_1 = 40\cos 20^\circ \sin 15^\circ\,\mathbf{i} + 40\cos 20^\circ \cos 15^\circ\,\mathbf{j} - 40\sin 20^\circ\,\mathbf{k}$$

$$= 9.728\,\mathbf{i} + 36.307\,\mathbf{j} - 13.681\,\mathbf{k}$$

$$\mathbf{M}_2 = -30\sin 30^\circ\,\mathbf{i} + 30\cos 30^\circ\,\mathbf{j}$$

$$= -15\,\mathbf{i} + 25.981\,\mathbf{j}$$

$$\mathbf{M}_R = \mathbf{M}_1 + \mathbf{M}_2 = -5.272\,\mathbf{i} + 62.288\,\mathbf{j} - 13.681\,\mathbf{k}$$

$$M_R = \sqrt{(-5.272)^2 + (62.288)^2 + (-13.681)^2} = 63.990 = 64.0 \text{ lb}\cdot\text{ft}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{-5.272}{63.990}\right) = 94.7^\circ$$ **Ans.**

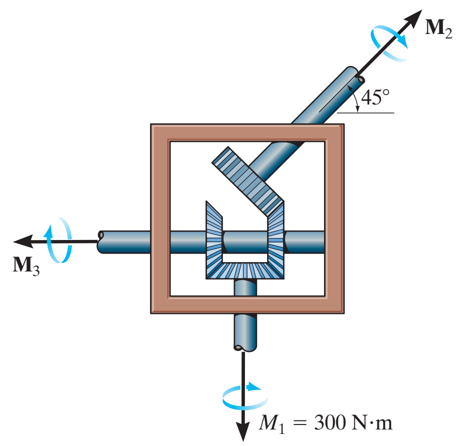
$$\beta = \cos^{-1}\left(\frac{62.288}{63.990}\right) = 13.2^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{-13.681}{63.990}\right) = 102^\circ$$ **Ans.**

Ans:
$M_R = 64.0$ lb · ft
$\alpha = 94.7^\circ$
$\beta = 13.2^\circ$
$\gamma = 102^\circ$

4–86.

Determine the required magnitude of the couple moments $\mathbf{M}_2$ and $\mathbf{M}_3$ so that the resultant couple moment is zero.



SOLUTION

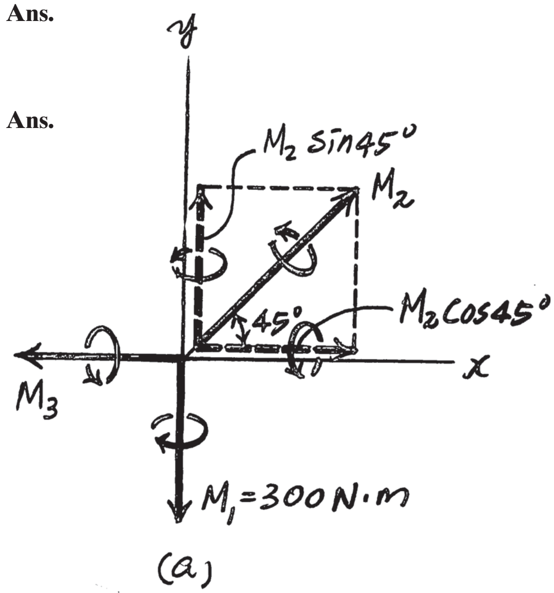
Since the couple moment is the free vector, it can act at any point without altering its effect. Thus, the couple moments $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ can be simplified as shown in Fig. *a*. Since the resultant of $\mathbf{M}_1$, $\mathbf{M}_2$, and $\mathbf{M}_3$ is required to be zero,

$(M_R)_y = \Sigma M_y$; $\quad 0 = M_2 \sin 45° - 300$

$M_2 = 424.26 \text{ N}\cdot\text{m} = 424 \text{ N}\cdot\text{m}$ **Ans.**

$(M_R)_x = \Sigma M_x$; $\quad 0 = 424.26 \cos 45° - M_3$

$M_3 = 300 \text{ N}\cdot\text{m}$ **Ans.**

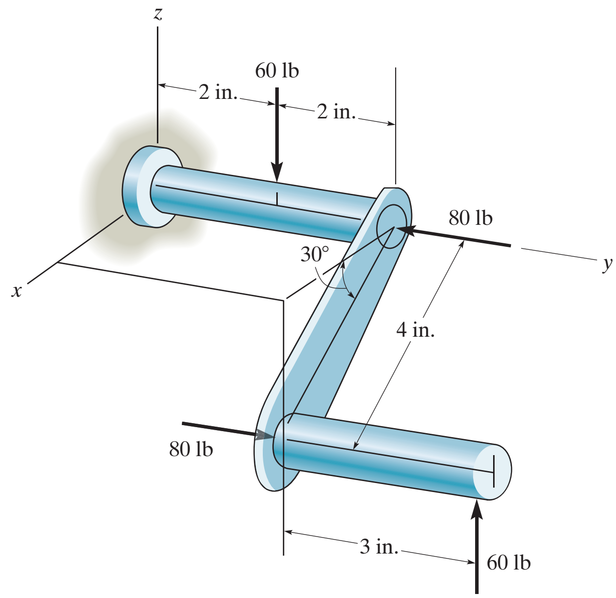


(a)

Ans:
$M_2 = 424 \text{ N}\cdot\text{m}$
$M_3 = 300 \text{ N}\cdot\text{m}$

4–87.

Determine the resultant couple moment of the two couples that act on the assembly. Specify its magnitude and coordinate direction angles.



SOLUTION

$$\mathbf{M}_R = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4\cos 30^\circ & 5 & -4\sin 30^\circ \\ 0 & 0 & 60 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4\cos 30^\circ & 0 & -4\sin 30^\circ \\ 0 & 80 & 0 \end{vmatrix}$$

$$= 300\,\mathbf{i} - 207.85\,\mathbf{j} + 160\,\mathbf{i} + 277.13\,\mathbf{k}$$

$$= \{460\,\mathbf{i} - 207.85\,\mathbf{j} + 277.13\,\mathbf{k}\}\ \text{lb}\cdot\text{in.}$$

$$M_R = \sqrt{(460)^2 + (-207.85)^2 + (277.13)^2} = 575.85 = 576\ \text{lb}\cdot\text{in.}$$ **Ans.**

$$\alpha = \cos^{-1}\left(\frac{460}{575.85}\right) = 37.0^\circ$$ **Ans.**

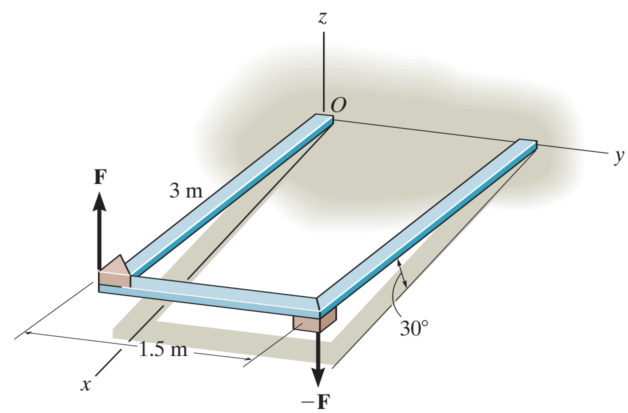
$$\beta = \cos^{-1}\left(\frac{-207.85}{575.85}\right) = 111^\circ$$ **Ans.**

$$\gamma = \cos^{-1}\left(\frac{277.13}{575.85}\right) = 61.2^\circ$$ **Ans.**

Ans:
$M_R = 576\ \text{lb}\cdot\text{in.}$
$\alpha = 37.0^\circ$
$\beta = 111^\circ$
$\gamma = 61.2^\circ$

***4–88.**

Express the moment of the couple acting on the frame in Cartesian vector form. The forces are applied perpendicular to the frame. What is the magnitude of the couple moment? Take $F = 50$ N.

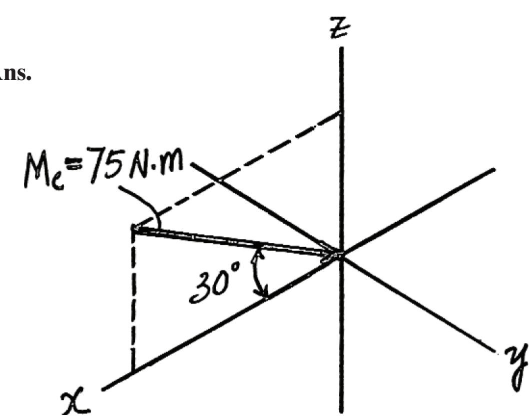


SOLUTION

$$M_C = 80(1.5) = 75 \text{ N} \cdot \text{m}$$ **Ans.**

$$M_C = -75(\cos 30^\circ\, \mathbf{i} + \cos 60^\circ\, \mathbf{k})$$

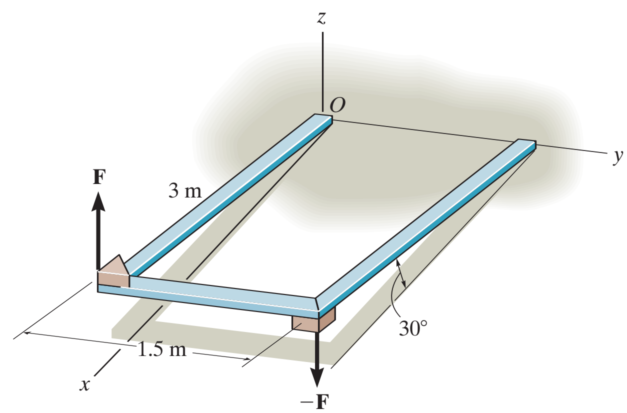
$$= \{-65.0\mathbf{i} - 37.5\mathbf{k}\} \text{ N} \cdot \text{m}$$ **Ans.**



Ans:
$M_C = \{-65.0\mathbf{i} - 37.5\mathbf{k}\}$ N · m

4–89.

In order to turn over the frame, a couple moment is applied as shown. If the component of this couple moment along the x axis is $\mathbf{M}_x = \{-20\mathbf{i}\}$ N·m, determine the magnitude F of the couple forces.



SOLUTION

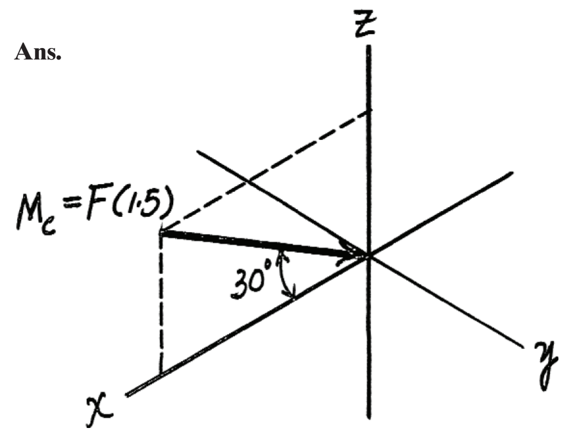
$M_C = F\,(1.5)$

Thus

$$20 = F\,(1.5)\cos 30°$$

$$F = 15.4\text{ N}$$

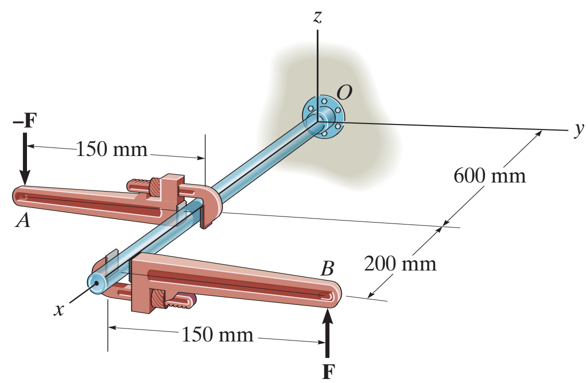
Ans.



Ans:
$F = 15.4$ N

4–90.

Express the moment of the couple acting on the pipe in Cartesian vector form. What is the magnitude of the couple moment? Take $F = 125$ N.



SOLUTION

$$\mathbf{M}_C = \mathbf{r}_{AB} \times (125\,\mathbf{k})$$

$$\mathbf{M}_C = (0.2\mathbf{i} + 0.3\mathbf{j}) \times (125\,\mathbf{k})$$

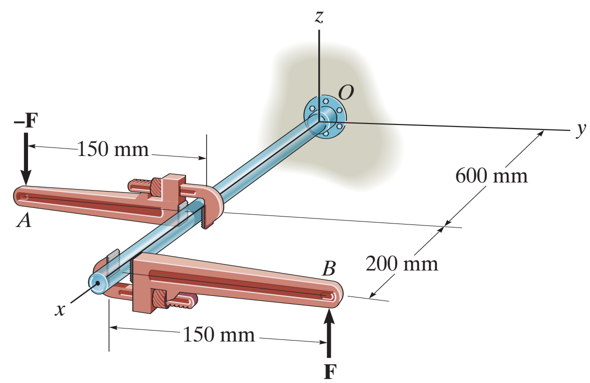
$$\mathbf{M}_C = \{37.5\mathbf{i} - 25\mathbf{j}\}\ \text{N}\cdot\text{m}$$

$$M_C = \sqrt{(37.5)^2 + (-25)^2} = 45.1\ \text{N}\cdot\text{m}$$ **Ans.**

Ans:
$M_C = 45.1\ \text{N}\cdot\text{m}$

4–91.

If the couple moment acting on the pipe has a magnitude of 300 N · m, determine the magnitude F of the forces applied to the wrenches.



SOLUTION

$$\mathbf{M}_C = \mathbf{r}_{AB} \times (F\,\mathbf{k})$$

$$= (0.2\mathbf{i} + 0.3\mathbf{j}) \times (F\,\mathbf{k})$$

$$= \{0.2F\mathbf{i} - 0.3F\mathbf{j}\}\ \text{N} \cdot \text{m}$$

$$M_C = F\sqrt{(0.2F)^2 + (-0.3F)} = 0.3606\ F$$

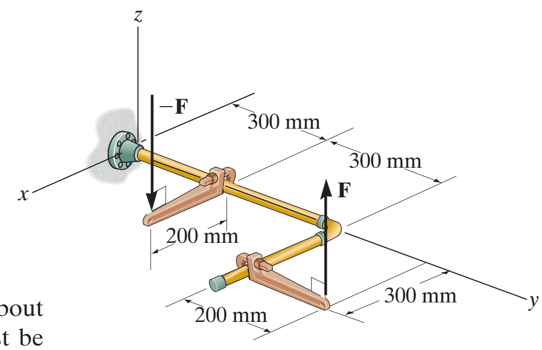
$$300 = 0.3606\ F$$

$$F = 832\ \text{N}$$ **Ans.**

Ans:
$F = 832$ N

***4–92.**

If $F = 80$ N, determine the magnitude and coordinate direction angles of the couple moment. The pipe assembly lies in the x–y plane.



SOLUTION

It is easiest to find the couple moment of **F** by taking the moment of **F** or –**F** about point A or B, respectively, Fig. a. Here the position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{BA}$ must be determined first.

$$\mathbf{r}_{AB} = (0.3 - 0.2)\mathbf{i} + (0.8 - 0.3)\mathbf{j} + (0 - 0)\mathbf{k} = [0.1\mathbf{i} + 0.5\mathbf{j}]\text{ m}$$

$$\mathbf{r}_{BA} = (0.2 - 0.3)\mathbf{i} + (0.3 - 0.8)\mathbf{j} + (0 - 0)\mathbf{k} = [-0.1\mathbf{i} - 0.5\mathbf{j}]\text{ m}$$

The force vectors **F** and –**F** can be written as

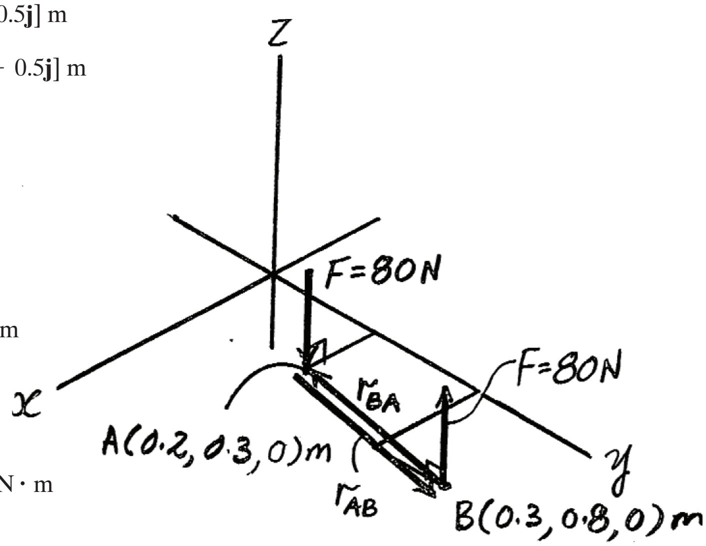
$$\mathbf{F} = \{80\,\mathbf{k}\}\text{ N and } -\mathbf{F} = [-80\,\mathbf{k}]\text{ N}$$

Thus, the couple moment of **F** can be determined from

$$\mathbf{M}_c = \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & 80 \end{vmatrix} = [40\mathbf{i} - 8\mathbf{j}]\text{ N}\cdot\text{m}$$

or

$$\mathbf{M}_c = \mathbf{r}_{BA} \times -\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.1 & -0.5 & 0 \\ 0 & 0 & -80 \end{vmatrix} = [40\mathbf{i} - 8\mathbf{j}]\text{ N}\cdot\text{m}$$



(a)

The magnitude of M_c is given by

$$M_c = \sqrt{M_x{}^2 + M_y{}^2 + M_z{}^2} = \sqrt{40^2 + (-8)^2 + 0^2} = 40.79\text{ N}\cdot\text{m} = 40.8\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

The coordinate angles of $\mathbf{M}_c$ are

$$\alpha = \cos^{-1}\left(\frac{M_x}{M}\right) = \cos\left(\frac{40}{40.79}\right) = 11.3^\circ \qquad \textbf{Ans.}$$

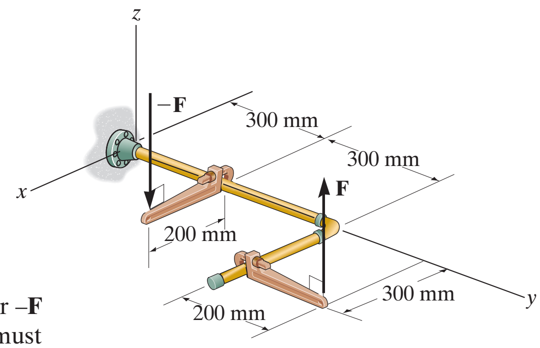
$$\beta = \cos^{-1}\left(\frac{M_y}{M}\right) = \cos\left(\frac{-8}{40.79}\right) = 101^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{M_z}{M}\right) = \cos\left(\frac{0}{40.79}\right) = 90^\circ \qquad \textbf{Ans.}$$

Ans:
$M_c = 40.8\text{ N}\cdot\text{m}$
$\alpha = 11.3^\circ$
$\beta = 101^\circ$
$\gamma = 90^\circ$

4–93.

If the magnitude of the couple moment acting on the pipe assembly is 50 N · m, determine the magnitude of the couple forces applied to each wrench. The pipe assembly lies in the x–y plane.



SOLUTION

It is easiest to find the couple moment of **F** by taking the moment of either **F** or –**F** about point A or B, respectively, Fig. a. Here the position vectors $\mathbf{r}_{AB}$ and $\mathbf{r}_{BA}$ must be determined first.

$$\mathbf{r}_{AB} = (0.3 - 0.2)\mathbf{i} + (0.8 - 0.3)\mathbf{j} + (0 - 0)\mathbf{k} = [0.1\mathbf{i} + 0.5\mathbf{j}]\text{ m}$$

$$\mathbf{r}_{BA} = (0.2 - 0.3)\mathbf{i} + (0.3 - 0.8)\mathbf{j} + (0 - 0)\mathbf{k} = [-0.1\mathbf{i} - 0.5\mathbf{j}]\text{ m}$$

The force vectors **F** and –**F** can be written as
$\mathbf{F} = \{F\mathbf{k}\}$ N and $-\mathbf{F} = [-F\mathbf{k}]$N

Thus, the couple moment of **F** can be determined from

$$\mathbf{M}_c = \mathbf{r}_{AB} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.1 & 0.5 & 0 \\ 0 & 0 & F \end{vmatrix} = 0.5F\mathbf{i} - 0.1F\mathbf{j}$$

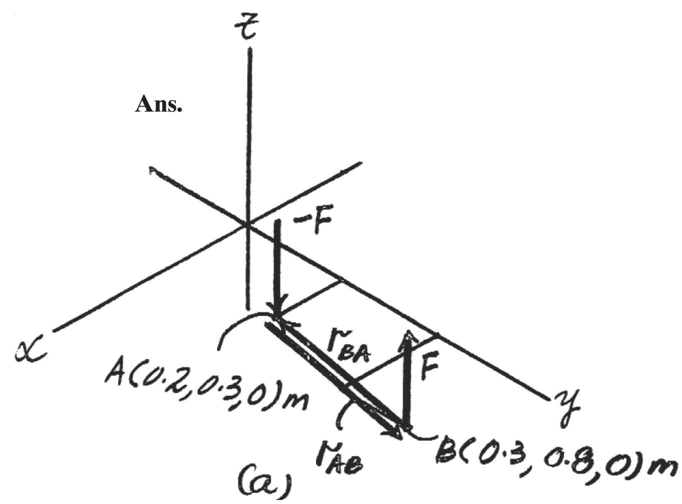
The magnitude of M_c is given by

$$M_c = \sqrt{M_x{}^2 + M_y{}^2 + M_z{}^2} = \sqrt{(0.5F)^2 + (0.1F)^2 + 0^2} = 0.5099F$$

Since M_c is required to equal 50 N · m,

$$50 = 0.5099F$$

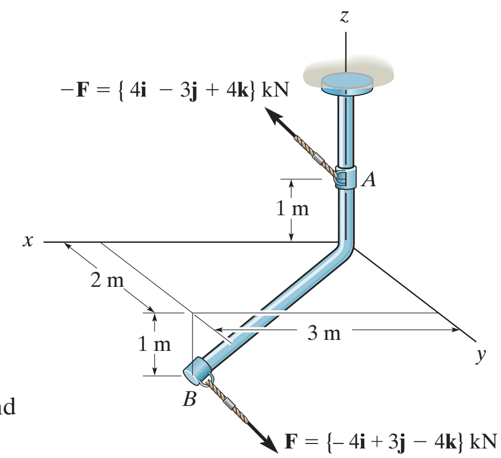
$$F = 98.1\text{ N} \qquad \textbf{Ans.}$$



Ans:
$F = 98.1$ N

4–94.

Express the moment of the couple acting on the rod in Cartesian vector form. What is the magnitude of the couple moment?



SOLUTION

Position Vector. The coordinates of points A and B are $A\,(0, 0, 1)$ m and $B\,(3, 2, -1)$ m, respectively. Thus,

$$\mathbf{r}_{AB} = (3 - 0)\mathbf{i} + (2 - 0)\mathbf{j} + (-1 - 1)\mathbf{k} = \{3\mathbf{i} + 2\mathbf{j} - 2\mathbf{k}\}\text{ m}$$

Couple Moment.

$$\mathbf{M}_C = \mathbf{r}_{AB} \times \mathbf{F}$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 2 & -2 \\ -4 & 3 & -4 \end{vmatrix}$$

$$= \{-2\mathbf{i} + 20\mathbf{j} + 17\mathbf{k}\}\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

The magnitude of $\mathbf{M}_C$ is

$$M_C = \sqrt{(M_C)_x^2 + (M_C)_y^2 + (M_C)_z^2}$$

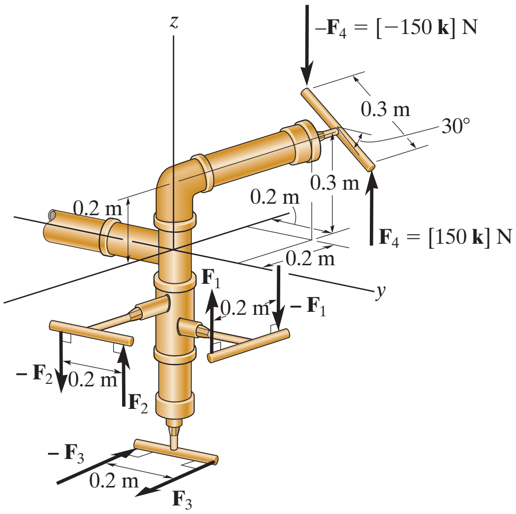
$$= \sqrt{(-2)^2 + 20^2 + 17^2}$$

$$= 26.32\text{ kN}\cdot\text{m} = 26.3\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{M}_C = \{-2\mathbf{i} + 20\mathbf{j} + 17\mathbf{k}\}\text{ kN}\cdot\text{m}$
$M_C = 26.3\text{ kN}\cdot\text{m}$

4–95.

If $F_1 = 100\text{ N}$, $F_2 = 120\text{ N}$ and $F_3 = 80\text{ N}$, determine the magnitude and coordinate direction angles of the resultant couple moment.



SOLUTION

Couple Moment: The position vectors $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, and $\mathbf{r}_4$, Fig. *a*, must be determined first.

$\mathbf{r}_1 = \{0.2\mathbf{i}\}\text{ m}$ $\qquad$ $\mathbf{r}_2 = \{0.2\mathbf{j}\}\text{ m}$ $\qquad$ $\mathbf{r}_3 = \{0.2\mathbf{j}\}\text{ m}$

From the geometry of Figs. *b* and *c*, we obtain

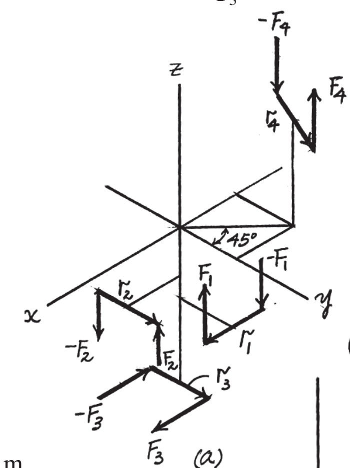
$$\mathbf{r}_4 = 0.3\cos 30^\circ \cos 45^\circ \mathbf{i} + 0.3\cos 30^\circ \sin 45^\circ \mathbf{j} - 0.3\sin 30^\circ \mathbf{k}$$
$$= \{0.1837\mathbf{i} + 0.1837\mathbf{j} - 0.15\mathbf{k}\}\text{ m}$$

The force vectors $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are given by

$\mathbf{F}_1 = \{100\mathbf{k}\}\text{ N}$ $\qquad$ $\mathbf{F}_2 = \{120\mathbf{k}\}\text{ N}$ $\qquad$ $\mathbf{F}_3 = \{80\mathbf{i}\}\text{ N}$

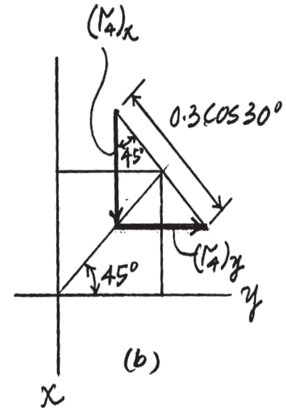
Thus,

$$\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1 = (0.2\mathbf{i}) \times (100\mathbf{k}) = \{-20\mathbf{j}\}\text{ N}\cdot\text{m}$$
$$\mathbf{M}_2 = \mathbf{r}_2 \times \mathbf{F}_2 = (0.2\mathbf{j}) \times (120\mathbf{k}) = \{24\mathbf{i}\}\text{ N}\cdot\text{m}$$
$$\mathbf{M}_3 = \mathbf{r}_3 \times \mathbf{F}_3 = (0.2\mathbf{j}) \times (80\mathbf{i}) = \{-16\mathbf{k}\}\text{ N}\cdot\text{m}$$
$$\mathbf{M}_4 = \mathbf{r}_4 \times \mathbf{F}_4 = (0.1837\mathbf{i} + 0.1837\mathbf{j} - 0.15\mathbf{k}) \times (150\mathbf{k}) = \{27.56\mathbf{i} - 27.56\mathbf{j}\}\text{ N}\cdot\text{m}$$



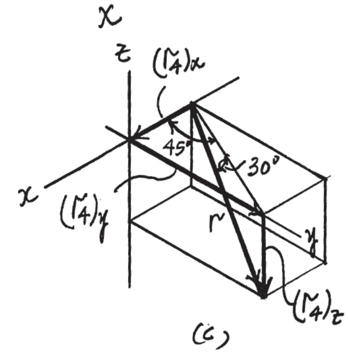
Resultant Moment: The resultant couple moment is given by

$$(\mathbf{M}_c)_R = \Sigma\mathbf{M}_c; \qquad (\mathbf{M}_c)_R = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4$$
$$= (-20\mathbf{j}) + (24\mathbf{i}) + (-16\mathbf{k}) + (27.56\mathbf{i} - 27.56\mathbf{j})$$
$$= \{51.56\mathbf{i} - 47.56\mathbf{j} - 16\mathbf{k}\}\text{ N}\cdot\text{m}$$



The magnitude of the couple moment is

$$(\mathbf{M}_c)_R = \sqrt{[(\mathbf{M}_c)_R]_x{}^2 + [(\mathbf{M}_c)_R]_y{}^2 + [(\mathbf{M}_c)_R]_z{}^2}$$
$$= \sqrt{(51.56)^2 + (-47.56)^2 + (-16)^2}$$
$$= 71.94\text{ N}\cdot\text{m} = 71.9\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



The coordinate angles of $(\mathbf{M}_c)_R$ are

$$\alpha = \cos^{-1}\left(\frac{[(M_c)_R]_x}{(M_c)_R}\right) = \cos\left(\frac{51.56}{71.94}\right) = 44.2^\circ \qquad \textbf{Ans.}$$

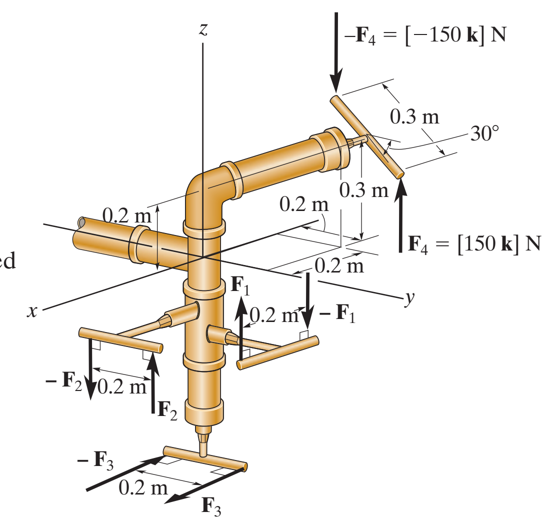
$$\beta = \cos^{-1}\left(\frac{[(M_c)_R]_y}{(M_c)_R}\right) = \cos\left(\frac{-47.56}{71.94}\right) = 131^\circ \qquad \textbf{Ans.}$$

$$\gamma = \cos^{-1}\left(\frac{[(M_c)_R]_z}{(M_c)_R}\right) = \cos\left(\frac{-16}{71.94}\right) = 103^\circ \qquad \textbf{Ans.}$$

Ans:
$(M_C)_R = 71.9\text{ N}\cdot\text{m}$
$\alpha = 44.2^\circ$
$\beta = 131^\circ$
$\gamma = 103^\circ$

***4–96.**

Determine the required magnitude of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ so that the resultant couple moment is $(M_c)_R = [50\,\mathbf{i} - 45\,\mathbf{j} - 20\,\mathbf{k}]\ \text{N}\cdot\text{m}$.



SOLUTION

Couple Moment: The position vectors $\mathbf{r}_1$, $\mathbf{r}_2$, $\mathbf{r}_3$, and $\mathbf{r}_4$, Fig. *a*, must be determined first.

$$\mathbf{r}_1 = \{0.2\mathbf{i}\}\ \text{m} \qquad \mathbf{r}_2 = \{0.2\mathbf{j}\}\ \text{m} \qquad \mathbf{r}_3 = \{0.2\mathbf{j}\}\ \text{m}$$

From the geometry of Figs. *b* and *c*, we obtain

$$\mathbf{r}_4 = 0.3\cos 30^\circ \cos 45^\circ\mathbf{i} + 0.3\cos 30^\circ \sin 45^\circ\mathbf{j} - 0.3\sin 30^\circ\mathbf{k}$$
$$= \{0.1837\mathbf{i} + 0.1837\mathbf{j} - 0.15\mathbf{k}\}\ \text{m}$$

The force vectors $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are given by

$$\mathbf{F}_1 = F_1\mathbf{k} \qquad \mathbf{F}_2 = F_2\mathbf{k} \qquad \mathbf{F}_3 = F_3\mathbf{i}$$

Thus,

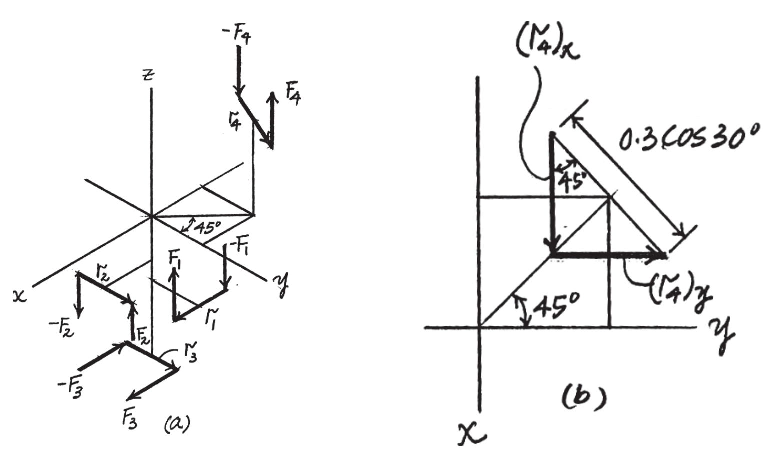
$$\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1 = (0.2\mathbf{i}) \times (F_1\mathbf{k}) = -0.2\,F_1\mathbf{j}$$
$$\mathbf{M}_2 = \mathbf{r}_2 \times \mathbf{F}_2 = (0.2\mathbf{j}) \times (F_2\mathbf{k}) = 0.2\,F_2\mathbf{i}$$
$$\mathbf{M}_3 = \mathbf{r}_3 \times \mathbf{F}_3 = (0.2\mathbf{j}) \times (F_3\mathbf{i}) = -0.2\,F_3\mathbf{k}$$
$$\mathbf{M}_4 = \mathbf{r}_4 \times \mathbf{F}_4 = (0.1837\mathbf{i} + 0.1837\mathbf{j} - 0.15\mathbf{k}) \times (150\mathbf{k}) = \{27.56\mathbf{i} - 27.56\mathbf{j}\}\ \text{N}\cdot\text{m}$$

Resultant Moment: The resultant couple moment required to equal $(\mathbf{M}_c)_R = \{50\mathbf{i} - 45\mathbf{j} - 20\mathbf{k}\}\ \text{N}\cdot\text{m}$. Thus,

$$(\mathbf{M}_c)_R = \Sigma\mathbf{M}_c; \qquad (\mathbf{M}_c)_R = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3 + \mathbf{M}_4$$
$$50\mathbf{i} - 45\mathbf{j} - 20\mathbf{k} = (-0.2F_1\mathbf{j}) + (0.2F_2\mathbf{i}) + (-0.2F_3\mathbf{k}) + (27.56\mathbf{i} - 27.56\mathbf{j})$$
$$50\mathbf{i} - 45\mathbf{j} - 20\mathbf{k} = (0.2F_2 + 27.56)\mathbf{i} + (-0.2F_1 - 27.56)\mathbf{j} - 0.2F_3\mathbf{k}$$

Equating the **i**, **j**, and **k** components yields

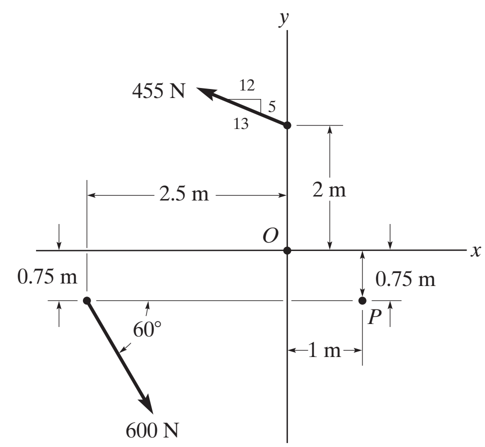
$$50 = 0.2F_2 + 27.56 \qquad F_2 = 112\ \text{N} \qquad \textbf{Ans.}$$
$$-45 = -0.2F_1 - 27.56 \qquad F_1 = 87.2\ \text{N} \qquad \textbf{Ans.}$$
$$-20 = -0.2F_3 \qquad F_3 = 100\ \text{N} \qquad \textbf{Ans.}$$



Ans:
$F_2 = 112\ \text{N}$
$F_1 = 87.2\ \text{N}$
$F_3 = 100\ \text{N}$

4–97.

Replace the force system by an equivalent resultant force and couple moment at point O.



SOLUTION

Equivalent Resultant Force And Couple Moment At *O*.

$$\xrightarrow{+} (F_R)_x = \Sigma F_x;\quad (F_R)_x = 600\cos 60^\circ - 455\left(\frac{12}{13}\right) = -120\text{ N} = 120\text{ N}\leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y;\quad (F_R)_y = 455\left(\frac{5}{13}\right) - 600\sin 60^\circ = -344.62\text{ N} = 344.62\text{ N}\downarrow$$

As indicated in Fig. *a*

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{120^2 + 344.62^2} = 364.91\text{ N} = 365\text{ N} \qquad \textbf{Ans.}$$

And

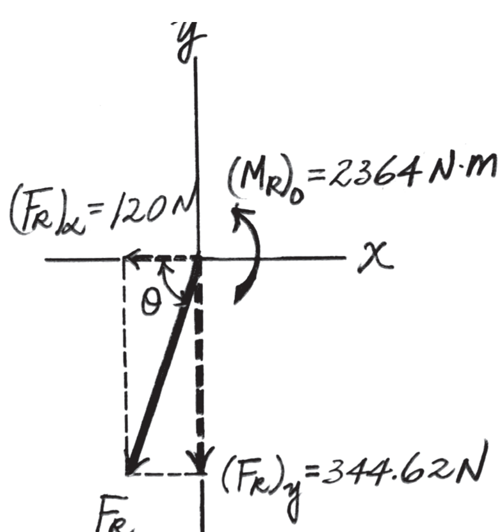
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{344.62}{120}\right) = 70.80^\circ = 70.8^\circ \qquad \textbf{Ans.}$$

Also,

$$\circlearrowleft +(M_R)_O = \Sigma M_O;\ (M_R)_O = 455\left(\frac{12}{13}\right)(2) + 600\cos 60^\circ\,(0.75) + 600\sin 60^\circ\,(2.5)$$

$$= 2364.04\text{ N}\cdot\text{m}$$

$$= 2364\text{ N}\cdot\text{m (\textbf{counterclockwise})} \qquad \textbf{Ans.}$$

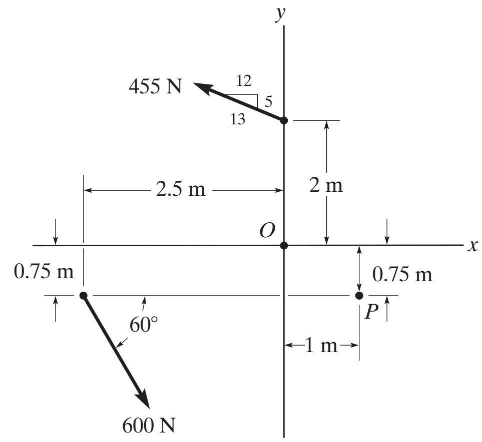


(a)

Ans:
$F_R = 365$ N
$\theta = 70.8^\circ$
$(M_R)_O = 2364$ N·m (**counterclockwise**)

4–98.

Replace the force system by an equivalent resultant force and couple moment at point *P*.



SOLUTION

Equivalent Resultant Force And Couple Moment At *P*.

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 600 \cos 60° - 455\left(\frac{12}{13}\right) = -120 \text{ N} = 120 \text{ N} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = 455\left(\frac{5}{13}\right) - 600 \sin 60° = -344.62 \text{ N} = 344.62 \text{ N} \downarrow$$

As indicated in Fig. *a*,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{120^2 + 344.62^2} = 364.91 \text{ N} = 365 \text{ N} \qquad \textbf{Ans.}$$

And

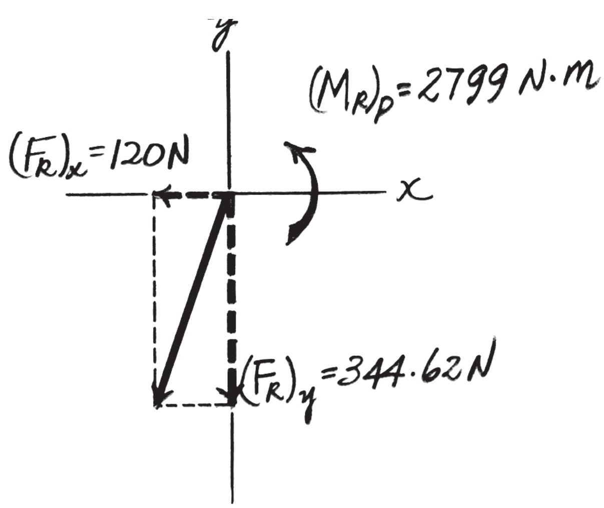
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{344.62}{120}\right) = 70.80° = 70.8° \measuredangle \qquad \textbf{Ans.}$$

Also,

$$\circlearrowleft +(M_R)_P = \Sigma M_P; \quad (M_R)_P = 455\left(\frac{12}{13}\right)(2.75) - 455\left(\frac{5}{13}\right)(1) + 600 \sin 60° \,(3.5)$$

$$= 2798.65 \text{ N} \cdot \text{m}$$

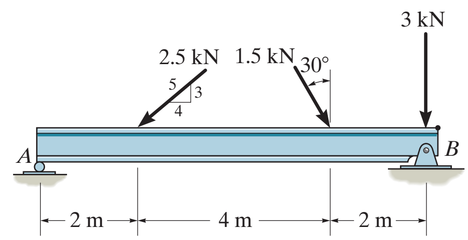
$$= 2799 \text{ N} \cdot \text{m (\textbf{counterclockwise})} \qquad \textbf{Ans.}$$



Ans:
$F_R = 365$ N
$\theta = 70.8° \measuredangle$
$(M_R)_P = 2799$ N·m (**counterclockwise**)

4–99.

Replace the force system acting on the beam by an equivalent force and couple moment at point A.



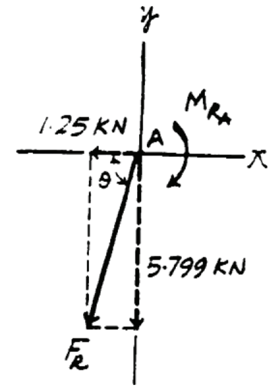
SOLUTION

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 1.5 \sin 30° - 2.5\left(\frac{4}{5}\right)$$

$$= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = -1.5 \cos 30° - 2.5\left(\frac{3}{5}\right) - 3$$

$$= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow$$



Thus,

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN} \qquad \textbf{Ans.}$$

and

$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8° \qquad \textbf{Ans.}$$

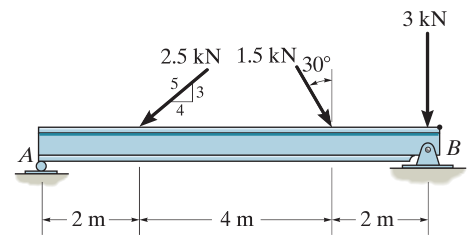
$$\circlearrowleft + M_{R_A} = \Sigma M_A; \qquad M_{R_A} = -2.5\left(\frac{3}{5}\right)(2) - 1.5 \cos 30°(6) - 3(8)$$

$$= -34.8 \text{ kN}\cdot\text{m} = 34.8 \text{ kN}\cdot\text{m} \ \textbf{(\textit{Clockwise})} \qquad \textbf{Ans.}$$

Ans:
$F_R = 5.93$ kN
$\theta = 77.8°$
$M_{R_A} = 34.8$ kN·m $\circlearrowright$

***4–100.**

Replace the force system acting on the beam by an equivalent force and couple moment at point B.



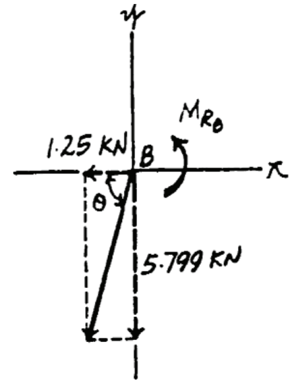
SOLUTION

$$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = 1.5 \sin 30° - 2.5\left(\frac{4}{5}\right)$$

$$= -1.25 \text{ kN} = 1.25 \text{ kN} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = -1.5 \cos 30° - 2.5\left(\frac{3}{5}\right) - 3$$

$$= -5.799 \text{ kN} = 5.799 \text{ kN} \downarrow$$



Thus,

$$F_R = \sqrt{F_{R_x}^2 + F_{R_y}^2} = \sqrt{1.25^2 + 5.799^2} = 5.93 \text{ kN} \qquad \textbf{Ans.}$$

and

$$\theta = \tan^{-1}\left(\frac{F_{R_y}}{F_{R_x}}\right) = \tan^{-1}\left(\frac{5.799}{1.25}\right) = 77.8° \qquad \textbf{Ans.}$$

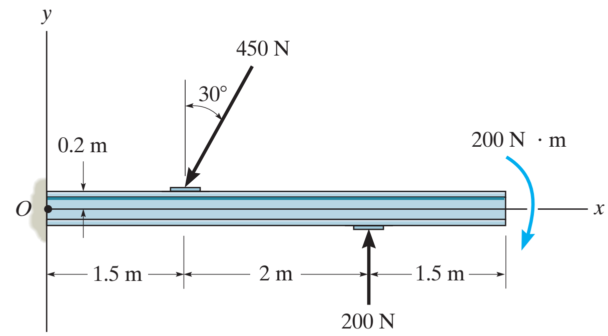
$$\circlearrowleft + M_{R_B} = \Sigma M_{R_B}; \qquad M_B = 1.5\cos 30°(2) + 2.5\left(\frac{3}{5}\right)(6)$$

$$= 11.6 \text{ kN} \cdot \text{m} \quad \textbf{(\textit{Counterclockwise})} \qquad \textbf{Ans.}$$

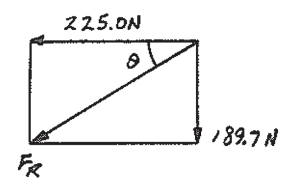
Ans:
$F_R = 5.93$ kN
$\theta = 77.8°$
$M_B = 11.6$ kN · m **(*Counterclockwise*)**

4–101.

Replace the loading system acting on the beam by an equivalent resultant force and couple moment at point O.



SOLUTION



$\overset{+}{\leftarrow} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 450 \sin 30° = 225.0$

$+\downarrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 450 \cos 30° - 200 = 189.7$

$F_R = \sqrt{(225)^2 + (189.7)^2} = 294\text{ N}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{189.7}{225}\right) = 40.1°$ **Ans.**

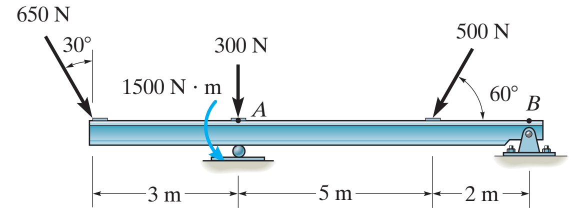
$\circlearrowleft + M_{RO} = \Sigma M_O; \quad M_{RO} = 450 \cos 30° (1.5) - 450 (\sin 30°)(0.2) - 200 (3.5) + 200$

$M_{RO} = 39.6\text{ N}\cdot\text{m} \circlearrowright$ **Ans.**

Ans:
$F_R = 294\text{ N}$
$\theta = 40.1°$
$M_{RO} = 39.6\text{ N}\cdot\text{m} \circlearrowright$

4–102.

Replace the loading system acting on the post by an equivalent resultant force and couple moment at point A.



SOLUTION

Equivalent Resultant Force And Couple Moment at Point *A*.

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 650 \sin 30° - 500 \cos 60° = 75 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = -650 \cos 30° - 300 - 500 \sin 60°$$

$$= -1295.93 \text{ N} = 1295.93 \text{ N}\downarrow$$

As indicated in Fig. a,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{75^2 + 1295.93^2} = 1298.10 \text{ N} = 1.30 \text{ kN} \quad \textbf{Ans.}$$

And

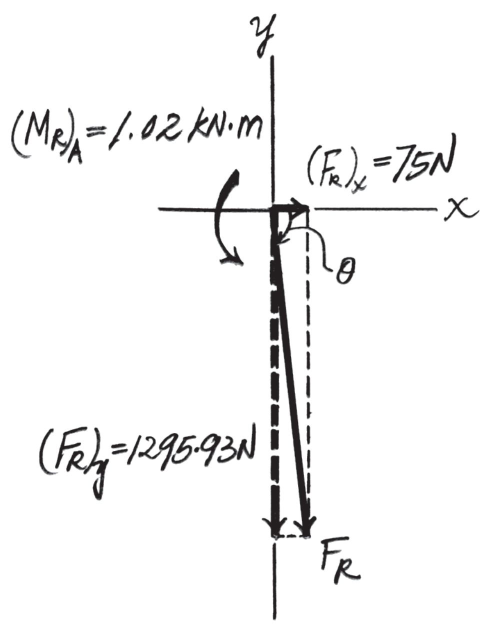
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{1295.93}{75}\right) = 86.69° = 86.7° \quad \textbf{Ans.}$$

Also,

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \quad (M_R)_A = 650 \cos 30° (3) + 1500 - 500 \sin 60° (5)$$

$$= 1023.69 \text{ N} \cdot \text{m}$$

$$= 1.02 \text{ kN} \cdot \text{m } (\textbf{counter clockwise}) \quad \textbf{Ans.}$$

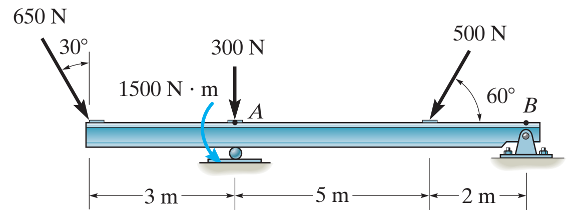


(a)

Ans:
$F_R = 1.30$ kN
$\theta = 86.7°$
$(M_R)_A = 1.02$ kN · m (**counterclockwise**)

4–103.

Replace the loading system acting on the post by an equivalent resultant force and couple moment at point B.



SOLUTION

Equivalent Resultant Force And Couple Moment At Point *B*.

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \quad (F_R)_x = 650 \sin 30° - 500 \cos 60° = 75 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \quad (F_R)_y = -650 \cos 30° - 300 - 500 \sin 60°$$

$$= -1295.93 \text{ N} = 1295.93 \text{ N}\downarrow$$

As indicated in Fig. a,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{75^2 + 1295.93^2} = 1298.10 \text{ N} = 1.30 \text{ kN} \quad \textbf{Ans.}$$

And

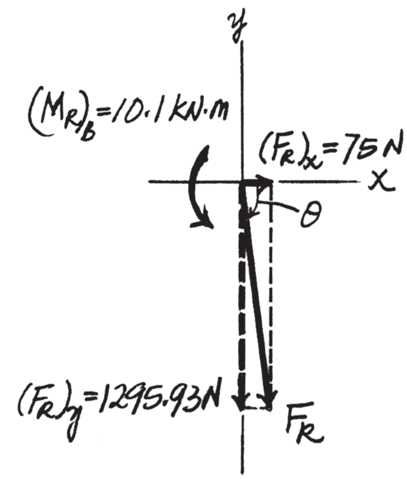
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{1295.93}{75}\right) = 86.69° = 86.7° \quad \textbf{Ans.}$$

Also,

$$\circlearrowleft + (M_R)_B = \Sigma M_B; \quad (M_R)_B = 650 \cos 30° (10) + 300(7) + 500 \sin 60°(2) + 1500$$

$$= 10{,}095.19 \text{ N} \cdot \text{m}$$

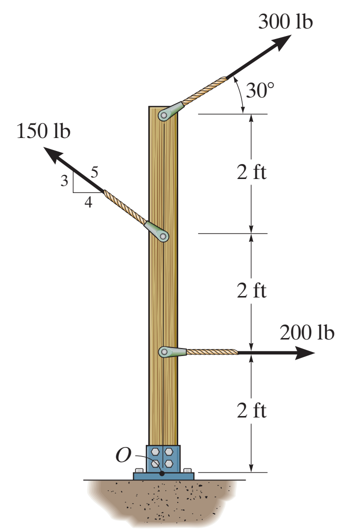
$$= 10.1 \text{ kN} \cdot \text{m } (\textbf{counterclockwise}) \quad \textbf{Ans.}$$



Ans:
$F_R = 1.30$ kN
$\theta = 86.7°$
$(M_R)_B = 1.01$ kN · m (**counterclockwise**)

***4–104.**

Replace the force system acting on the post by a resultant force and couple moment at point O.



SOLUTION

Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes, we have

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 300\cos 30^\circ - 150\left(\frac{4}{5}\right) + 200 = 339.81 \text{ lb} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 300\sin 30^\circ + 150\left(\frac{3}{5}\right) = 240 \text{ lb} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{339.81^2 + 240^2} = 416.02 \text{ lb} = 416 \text{ lb} \qquad \textbf{Ans.}$$

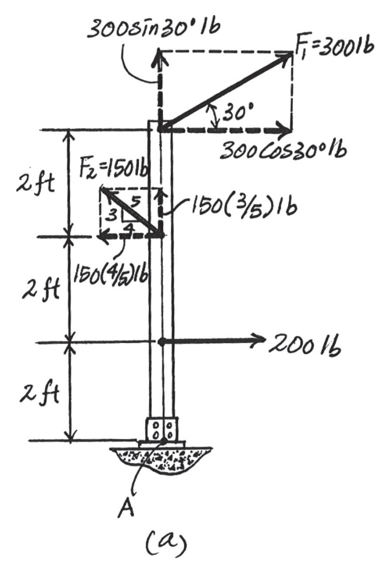
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{240}{339.81}\right] = 35.23^\circ = 35.2^\circ \measuredangle \qquad \textbf{Ans.}$$

Equivalent Resultant Couple Moment: Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point A, we can write

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad (M_R)_A = 150\left(\frac{4}{5}\right)(4) - 200(2) - 300\cos 30^\circ(6)$$

$$= -1478.85 \text{ lb}\cdot\text{ft} = 1.48 \text{ kip}\cdot\text{ft } \textbf{(\textit{Clockwise}) Ans.}$$

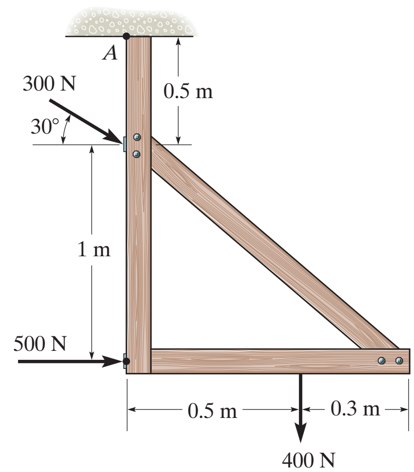


(a)

Ans:
$F_R = 416$ lb
$\theta = 35.2^\circ \measuredangle$
$(M_R)_A = 1.48$ kip·ft **(*Clockwise*)**

4–105.

Replace the force system acting on the frame by an equivalent resultant force and couple moment acting at point A.



SOLUTION

Equivalent Resultant Force And Couple Moment At *A*.

$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x;\qquad (F_R)_x = 300\cos 30^\circ + 500 = 759.81\text{ N}\quad\rightarrow$

$+\uparrow (F_R)_y = \Sigma F_y;\qquad (F_R)_y = -300\sin 30^\circ - 400 = -550\text{ N} = 550\text{ N}\quad\downarrow$

As indicated in Fig. *a*,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{759.81^2 + 550^2} = 937.98\text{ N} = 938\text{ N}\qquad\textbf{Ans.}$$

And

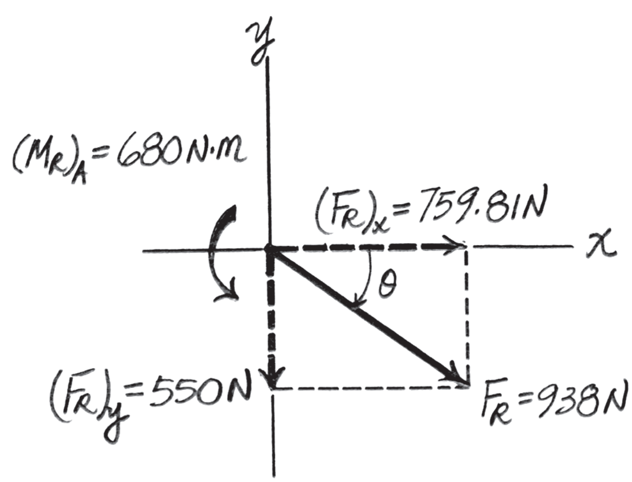
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{550}{759.81}\right) = 35.90^\circ = 35.9^\circ \qquad\textbf{Ans.}$$

Also;

$$\circlearrowleft +(M_R)_A = \Sigma M_A;\qquad (M_R)_A = 300\cos 30^\circ(0.5) + 500(1.5) - 400(0.5)$$

$$= 679.90\text{ N}\cdot\text{m}$$

$$= 680\text{ N}\cdot\text{m}\ (\textbf{counterclockwise})\qquad\textbf{Ans.}$$

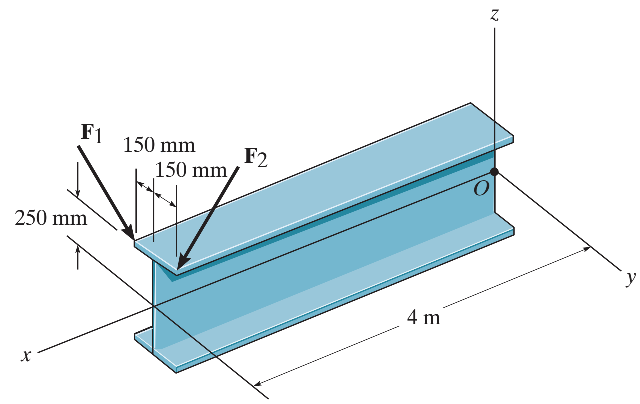


(a)

Ans:
$F_R = 938$ N
$\theta = 35.9^\circ$
$(M_R)_A = 680$ N·m (**counterclockwise**)

4–106.

The forces $\mathbf{F}_1 = \{-4\mathbf{i} + 2\mathbf{j} - 3\mathbf{k}\}$ kN and $\mathbf{F}_2 = \{3\mathbf{i} - 4\mathbf{j} - 2\mathbf{k}\}$ kN act on the end of the beam. Replace these forces by an equivalent force and couple moment acting at point O.



SOLUTION

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 = \{-1\mathbf{i} - 2\mathbf{j} - 5\mathbf{k}\}$ kN **Ans.**

$\mathbf{M}_{RO} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -0.15 & 0.25 \\ -4 & 2 & -3 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0.15 & 0.25 \\ 3 & -4 & -2 \end{vmatrix}$$

$$= (-0.05\mathbf{i} + 11\mathbf{j} + 7.4\mathbf{k}) + (0.7\mathbf{i} + 8.75\mathbf{j} - 16.45\mathbf{k})$$

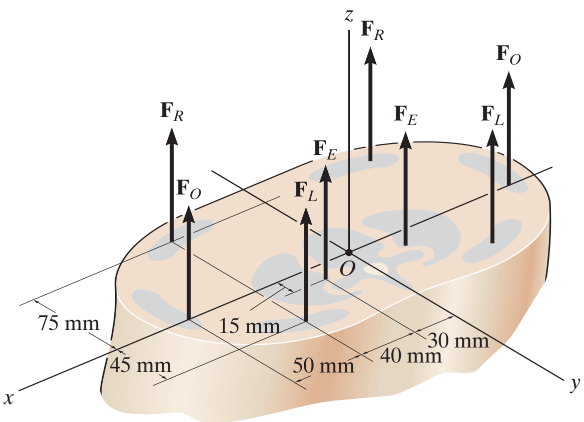
$$= (0.65\mathbf{i} + 19.75\mathbf{j} - 9.05\mathbf{k})$$

$\mathbf{M}_{RO} = \{0.650\mathbf{i} + 19.75\mathbf{j} - 9.05\mathbf{k}\}$ kN·m **Ans.**

Ans:
$\mathbf{M}_{RO} = \{0.650\mathbf{i} + 19.75\mathbf{j} - 9.05\mathbf{k}\}$ kN·m

4–107.

A biomechanical model of the lumbar region of the human trunk is shown. The forces acting in the four muscle groups consist of $F_R = 35$ N for the rectus, $F_O = 45$ N for the oblique, $F_L = 23$ N for the lumbar latissimus dorsi, and $F_E = 32$ N for the erector spinae. These loadings are symmetric with respect to the y–z plane. Replace this system of parallel forces by an equivalent force and couple moment acting at the spine, point O. Express the results in Cartesian vector form.



SOLUTION

$\mathbf{F}_R = \Sigma \mathbf{F}_z;\qquad \mathbf{F}_R = \{2(35 + 45 + 23 + 32)\mathbf{k}\} = \{270\mathbf{k}\}$ N **Ans.**

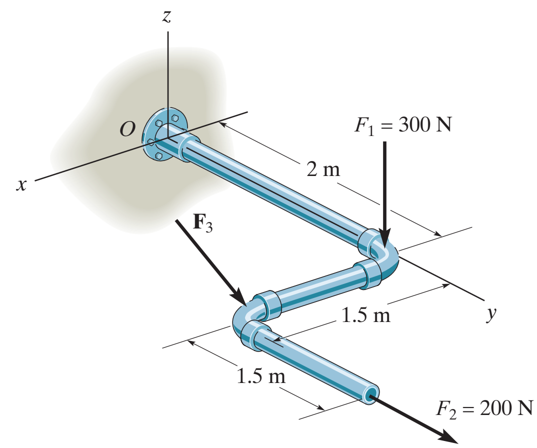
$\mathbf{M}_{RO_x} = \Sigma \mathbf{M}_{O_x};\qquad \mathbf{M}_{RO} = [-2(35)(0.075) + 2(32)(0.015) + 2(23)(0.045)]\mathbf{i}$

$\mathbf{M}_{RO} = \{-2.22\mathbf{i}\}\ \text{N}\cdot\text{m}$ **Ans.**

Ans:
$\mathbf{F}_R = \{270\mathbf{k}\}$ N
$\mathbf{M}_{RO} = \{-2.22\mathbf{i}\}\ \text{N}\cdot\text{m}$

***4–108.**

Replace the force system by an equivalent resultant force and couple moment at point O. Take $\mathbf{F}_3 = \{-200\mathbf{i} + 500\mathbf{j} - 300\mathbf{k}\}$ N.



SOLUTION

Position And Force Vectors.

$\mathbf{r}_1 = \{2\mathbf{j}\}$ m $\quad \mathbf{r}_2 = \{1.5\mathbf{i} + 3.5\mathbf{j}\} \quad \mathbf{r}_3 = \{1.5\mathbf{i} + 2\mathbf{j}\}$ m

$\mathbf{F}_1 = \{-300\mathbf{k}\}$ N $\quad \mathbf{F}_2 = \{200\mathbf{j}\}$ N $\quad \mathbf{F}_3 = \{-200\mathbf{i} + 500\mathbf{j} - 300\mathbf{k}\}$ N

Equivalent Resultant Force And Couple Moment At Point *O*.

$$\mathbf{F}_R = \Sigma F; \qquad F_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= (-300\mathbf{k}) + 200\mathbf{j} + (-200\mathbf{i} + 500\mathbf{j} - 300\mathbf{k})$$

$$= \{-200\mathbf{i} + 700\mathbf{j} - 600\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

$$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O; \quad (\mathbf{M}_R)_O = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{r}_3 \times \mathbf{F}_3$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 2 & 0 \\ 0 & 0 & -300 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 3.5 & 0 \\ 0 & 200 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1.5 & 2 & 0 \\ -200 & 500 & -300 \end{vmatrix}$$

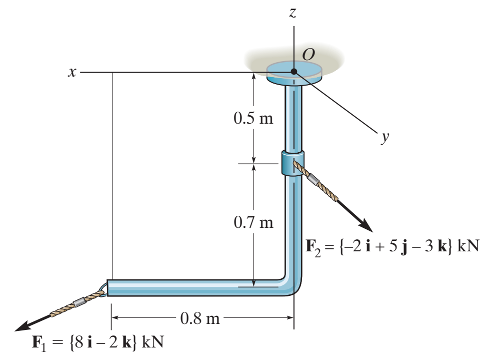
$$= (-600\mathbf{i}) + (300\mathbf{k}) + (-600\mathbf{i} + 450\mathbf{j} + 1150\mathbf{k})$$

$$= \{-1200\mathbf{i} + 450\mathbf{j} + 1450\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$F_R = \{-200\mathbf{i} + 700\mathbf{j} - 600\mathbf{k}\}$ N
$(\mathbf{M}_R)_O = \{-1200\mathbf{i} + 450\mathbf{j} + 1450\mathbf{k}\}$ N·m

4–109.

Replace the loading by an equivalent resultant force and couple moment at point O.



SOLUTION

Position Vectors. The required position vectors are

$$\mathbf{r}_1 = \{0.8\mathbf{i} - 1.2\mathbf{k}\}\text{ m} \qquad \mathbf{r}_2 = \{-0.5\mathbf{k}\}\text{ m}$$

Equivalent Resultant Force And Couple Moment At Point O.

$$\mathbf{F}_R = \Sigma\mathbf{F}; \qquad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (8\mathbf{i} - 2\mathbf{k}) + (-2\mathbf{i} + 5\mathbf{j} - 3\mathbf{k})$$

$$= \{6\mathbf{i} + 5\mathbf{j} - 5\mathbf{k}\}\text{ kN} \qquad \textbf{Ans.}$$

$$(\mathbf{M}_R)_O = \Sigma\mathbf{M}_O; \qquad (\mathbf{M}_R)_O = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.8 & 0 & -1.2 \\ 8 & 0 & -2 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0 & -0.5 \\ -2 & 5 & -3 \end{vmatrix}$$

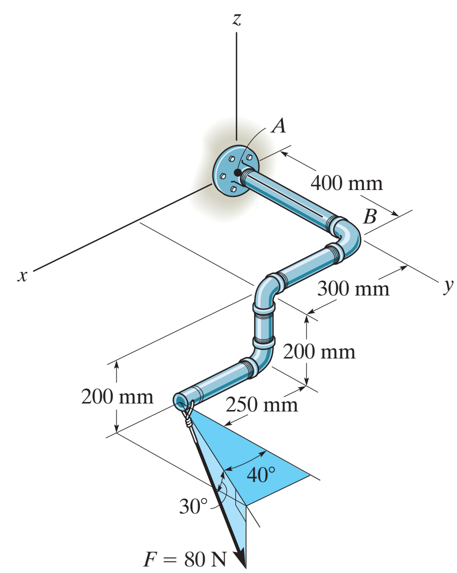
$$= (-8\mathbf{j}) + (2.5\mathbf{i} + \mathbf{j})$$

$$= \{2.5\mathbf{i} - 7\mathbf{j}\}\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_R = \{6\mathbf{i} + 5\mathbf{j} - 5\mathbf{k}\}$ kN
$(\mathbf{M}_R)_O = \{2.5\mathbf{i} - 7\mathbf{j}\}$ kN·m

4–110.

Replace the force of $F = 80$ N acting on the pipe assembly by an equivalent resultant force and couple moment at point A.



SOLUTION

$\mathbf{F}_R = \Sigma\mathbf{F}$;

$\mathbf{F}_R = 80\cos 30°\sin 40°\,\mathbf{i} + 80\cos 30°\cos 40°\,\mathbf{j} - 80\sin 30°\,\mathbf{k}$

$= 44.53\,\mathbf{i} + 53.07\,\mathbf{j} - 40\,\mathbf{k}$

$= \{44.5\,\mathbf{i} + 53.1\,\mathbf{j} - 40\,\mathbf{k}\}$ N **Ans.**

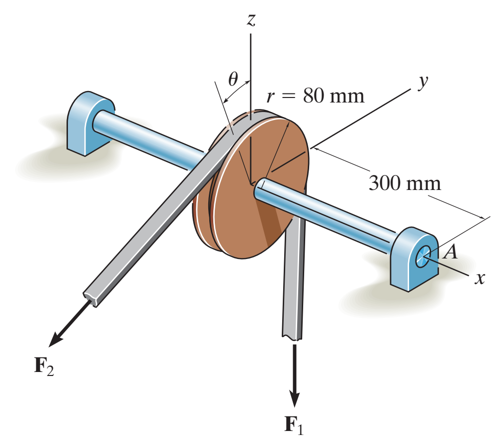
$$\mathbf{M}_{RA} = \Sigma\mathbf{M}_A;\quad \mathbf{M}_{RA} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.55 & 0.4 & -0.2 \\ 44.53 & 53.07 & -40 \end{vmatrix}$$

$= \{-5.39\,\mathbf{i} + 13.1\,\mathbf{j} + 11.4\,\mathbf{k}\}\ \text{N}\cdot\text{m}$ **Ans.**

Ans:
$\mathbf{F}_R = \{44.5\,\mathbf{i} + 53.1\,\mathbf{j} + 40\,\mathbf{k}\}$ N
$\mathbf{M}_{RA} = \{-5.39\,\mathbf{i} + 13.1\,\mathbf{j} + 11.4\,\mathbf{k}\}\ \text{N}\cdot\text{m}$

4–111.

The belt passing over the pulley is subjected to forces $\mathbf{F}_1$ and $\mathbf{F}_2$, each having a magnitude of 40 N. $\mathbf{F}_1$ acts in the $-\mathbf{k}$ direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in Cartesian vector form. Set $\theta = 0°$ so that $\mathbf{F}_2$ acts in the $-\mathbf{j}$ direction.



SOLUTION

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$\mathbf{F}_R = \{-40\mathbf{j} - 40\mathbf{k}\}\text{ N}$$ **Ans.**

$$\mathbf{M}_{RA} = \Sigma(\mathbf{r} \times \mathbf{F})$$

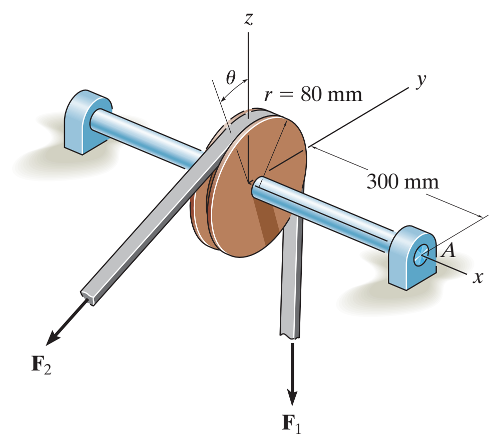
$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0 & 0.08 \\ 0 & -40 & 0 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0.08 & 0 \\ 0 & 0 & -40 \end{vmatrix}$$

$$\mathbf{M}_{RA} = \{-12\mathbf{j} + 12\mathbf{k}\}\text{ N}\cdot\text{m}$$ **Ans.**

Ans:
$\mathbf{F}_R = \{-40\mathbf{j} - 40\mathbf{k}\}$ N
$\mathbf{M}_{RA} = \{-12\mathbf{j} + 12\mathbf{k}\}$ N·m

***4–112.**

The belt passing over the pulley is subjected to two forces $\mathbf{F}_1$ and $\mathbf{F}_2$, each having a magnitude of 40 N. $\mathbf{F}_1$ acts in the $-\mathbf{k}$ direction. Replace these forces by an equivalent force and couple moment at point A. Express the result in Cartesian vector form. Take $\theta = 45°$.



SOLUTION

$$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

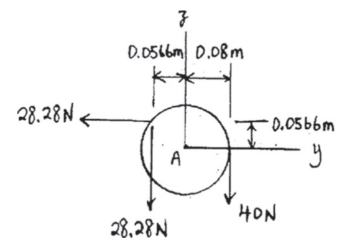
$$= -40\cos 45°\mathbf{j} + (-40 - 40\sin 45°)\mathbf{k}$$

$$\mathbf{F}_R = \{-28.3\mathbf{j} - 68.3\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

$$\mathbf{r}_{AF1} = \{-0.3\mathbf{i} + 0.08\mathbf{j}\}\text{ m}$$

$$\mathbf{r}_{AF2} = -0.3\mathbf{i} - 0.08\sin 45°\mathbf{j} + 0.08\cos 45°\mathbf{k}$$

$$= \{-0.3\mathbf{i} - 0.0566\mathbf{j} + 0.0566\mathbf{k}\}\text{ m}$$



$$\mathbf{M}_{RA} = (\mathbf{r}_{AF1} \times \mathbf{F}_1) + (\mathbf{r}_{AF2} \times \mathbf{F}_2)$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & 0.08 & 0 \\ 0 & 0 & -40 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.3 & -0.0566 & 0.0566 \\ 0 & -40\cos 45° & -40\sin 45° \end{vmatrix}$$

$$\mathbf{M}_{RA} = \{-20.5\mathbf{j} + 8.49\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Also,

$$M_{RA_x} = \Sigma M_{A_x}$$

$$M_{RA_x} = 28.28(0.0566) + 28.28(0.0566) - 40(0.08)$$

$$M_{RA_x} = 0$$

$$M_{RA_y} = \Sigma M_{A_y}$$

$$M_{RA_y} = -28.28(0.3) - 40(0.3)$$

$$M_{RA_y} = -20.5\text{ N}\cdot\text{m}$$

$$M_{RA_z} = \Sigma M_{A_z}$$

$$M_{RA_z} = 28.28(0.3)$$

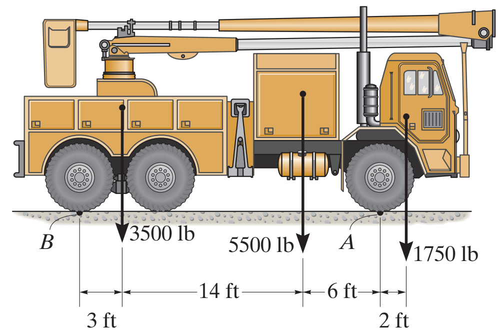
$$M_{RA_z} = 8.49\text{ N}\cdot\text{m}$$

$$\mathbf{M}_{RA} = \{-20.5\mathbf{j} + 8.49\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_R = \{-28.3\mathbf{j} - 68.3\mathbf{k}\}$ N
$\mathbf{M}_{RA} = \{-20.5\mathbf{j} + 8.49\mathbf{k}\}$ N·m

4–113.

The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from B.



SOLUTION

$$+\uparrow F_R = \Sigma F_y; \qquad F_R = -1750 - 5500 - 3500$$

$$= -10\,750 \text{ lb} = 10.75 \text{ kip}\downarrow \qquad \textbf{Ans.}$$

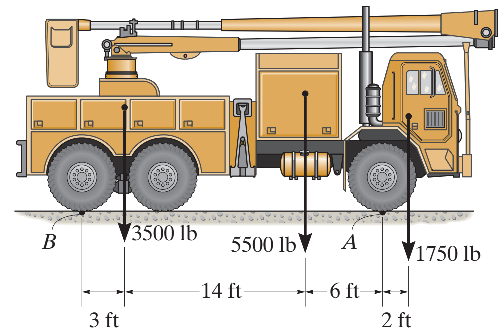
$$\circlearrowleft + M_{R_A} = \Sigma M_A; \quad -10\,750d = -3500(3) - 5500(17) - 1750(25)$$

$$d = 13.7 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$F_R = 10.75 \text{ kip}\downarrow$
$d = 13.7 \text{ ft}$

4–114.

The weights of the various components of the truck are shown. Replace this system of forces by an equivalent resultant force and specify its location measured from point A.

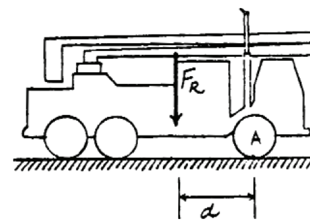


SOLUTION

Equivalent Force:

$$+\uparrow F_R = \Sigma F_y; \qquad F_R = -1750 - 5500 - 3500$$

$$= -10\,750 \text{ lb} = 10.75 \text{ kip}\downarrow \qquad \textbf{Ans.}$$



Location of Resultant Force From Point A:

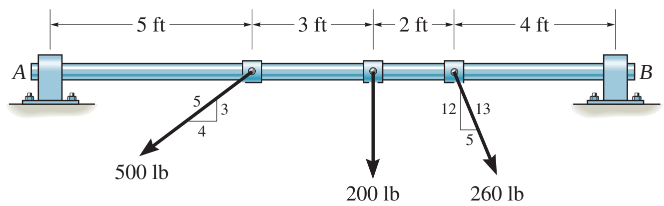
$$\circlearrowleft + M_{R_A} = \Sigma M_A; \qquad 10\,750(d) = 3500(20) + 5500(6) - 1750(2)$$

$$d = 9.26 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$F_R = 10.75 \text{ kip}\downarrow$
$d = 9.26$ ft

4–115.

Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end A.



SOLUTION

$\overset{+}{\rightarrow} F_{R_x} = \Sigma F_x; \qquad F_{R_x} = -500\left(\frac{4}{5}\right) + 260\left(\frac{5}{13}\right) = -300 \text{ lb} = 300 \text{ lb} \leftarrow$

$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = -500\left(\frac{3}{5}\right) - 200 - 260\left(\frac{12}{13}\right) = -740 \text{ lb} = 740 \text{ lb} \downarrow$

$F = \sqrt{(-300)^2 + (-740)^2} = 798 \text{ lb}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{740}{300}\right) = 67.9^\circ$ **Ans.**

$\circlearrowleft + M_{RA} = \Sigma M_A; \qquad 740(x) = 500\left(\frac{3}{5}\right)(5) + 200(8) + 260\left(\frac{12}{13}\right)(10)$

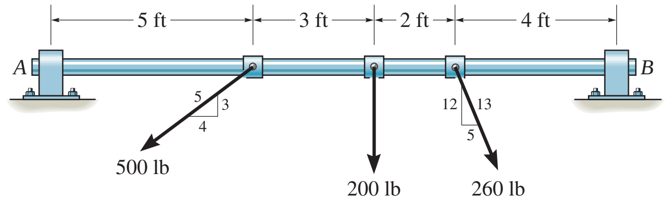
$740(x) = 5500$

$x = 7.43 \text{ ft}$ **Ans.**

Ans:
$F = 798 \text{ lb}$
67.9°
$x = 7.43 \text{ ft}$

***4–116.**

Replace the three forces acting on the shaft by a single resultant force. Specify where the force acts, measured from end B.



SOLUTION

$$\overset{+}{\rightarrow} \Sigma F_{R_x} = \Sigma F_x; \qquad F_{R_x} = -500\left(\frac{4}{5}\right) + 260\left(\frac{5}{13}\right) = -300 \text{ lb} = 300 \text{ lb} \leftarrow$$

$$+\uparrow F_{R_y} = \Sigma F_y; \qquad F_{R_y} = -500\left(\frac{3}{5}\right) - 200 - 260\left(\frac{12}{13}\right) = -740 \text{ lb} = 740 \text{ lb} \downarrow$$

$$F = \sqrt{(-300)^2 + (-740)^2} = 798 \text{ lb}$$ **Ans.**

$$\theta = \tan^{-1}\left(\frac{740}{300}\right) = 67.9^\circ$$ **Ans.**

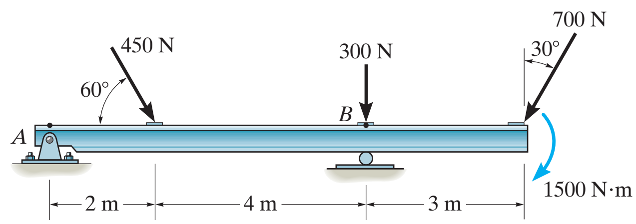
$$\circlearrowleft + M_{RB} = \Sigma M_B; \qquad 740(x) = 500\left(\frac{3}{5}\right)(9) + 200(6) + 260\left(\frac{12}{13}\right)(4)$$

$$x = 6.57 \text{ ft}$$ **Ans.**

Ans:
$F = 798$ lb
$\theta = 67.9^\circ$
$x = 6.57$ ft

4–117.

Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from end A.



SOLUTION

$\xrightarrow{+} F_{Rx} = \Sigma F_x;\quad F_{Rx} = 450 \cos 60° - 700 \sin 30° = -125 \text{ N} = 125 \text{ N} \leftarrow$

$+\uparrow F_{Ry} = \Sigma F_y;\quad F_{Ry} = -450 \sin 60° - 700 \cos 30° - 300 = -1296 \text{ N} = 1296 \text{ N} \downarrow$

$F = \sqrt{(-125)^2 + (-1296)^2} = 1302 \text{ N}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{1296}{125}\right) = 84.5°$ ⦣ **Ans.**

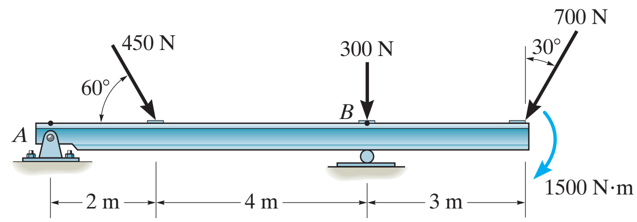
$\curvearrowright + M_{RA} = \Sigma M_A;\quad 1296(x) = 450 \sin 60°(2) + 300(6) + 700 \cos 30°(9) + 1500$

$x = 7.36 \text{ m}$ **Ans.**

Ans:
$F = 1302 \text{ N}$
$\theta = 84.5°$ ⦣
$x = 7.36 \text{ m}$

4–118.

Replace the loading acting on the beam by a single resultant force. Specify where the force acts, measured from B.



SOLUTION

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;\qquad F_{Rx} = 450 \cos 60° - 700 \sin 30° = -125 \text{ N} = 125 \text{ N} \leftarrow$

$+\uparrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = -450 \sin 60° - 700 \cos 30° - 300 = -1296 \text{ N} = 1296 \text{ N} \downarrow$

$F = \sqrt{(-125)^2 + (-1296)^2} = 1302 \text{ N}$ **Ans.**

$\theta = \tan^{-1}\left(\dfrac{1296}{125}\right) = 84.5°$ **Ans.**

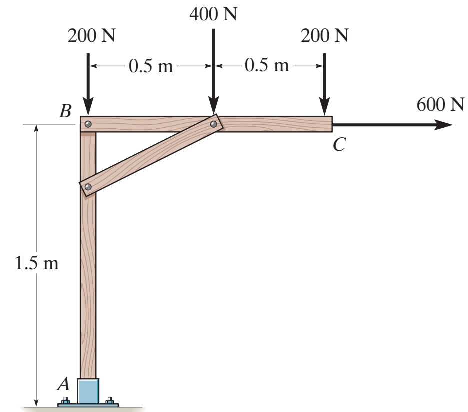
$\circlearrowleft + M_{RB} = \Sigma M_B;\qquad 1296(x) = -450 \sin 60°(4) + 700 \cos 30°(3) + 1500$

$x = 1.36 \text{ m (to the right)}$ **Ans.**

Ans:
$F = 1302 \text{ N}$
$\theta = 84.5°$
$x = 1.36 \text{ m (to the right)}$

4–119.

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a vertical line along member AB, measured from A.



SOLUTION

Equivalent Resultant Force. Referring to Fig. a,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 600 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -200 - 400 - 200 = -800 \text{ N} = 800 \text{ N}\downarrow$$

As indicated in Fig. a,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{600^2 + 800^2} = 1000 \text{ N} \qquad \textbf{Ans.}$$

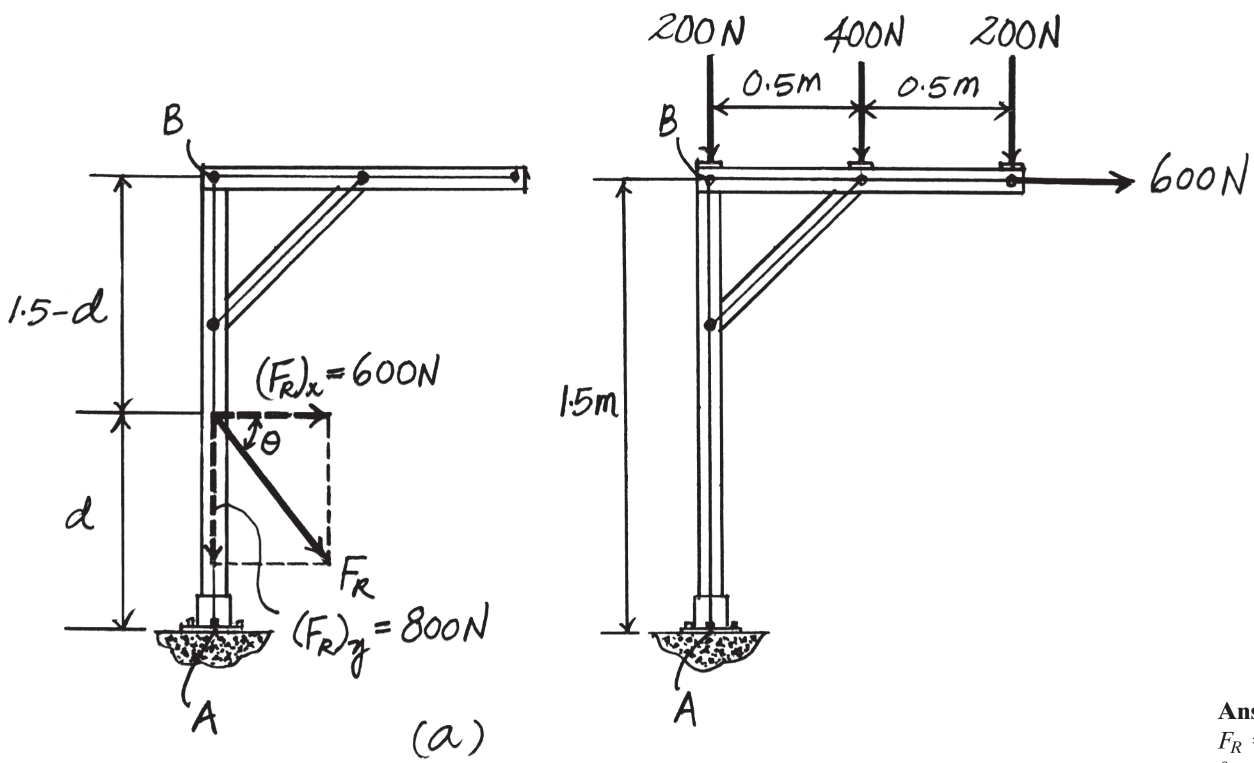
And

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{800}{600}\right) = 53.13° = 53.1° \;\triangleleft \qquad \textbf{Ans.}$$

Location of Resultant Force. Along AB,

$$\circlearrowleft + (M_R)_B = \Sigma M_B; \qquad 600(1.5 - d) = -400(0.5) - 200(1)$$

$$d = 2.1667 \text{ m} = 2.17 \text{ m} \qquad \textbf{Ans.}$$

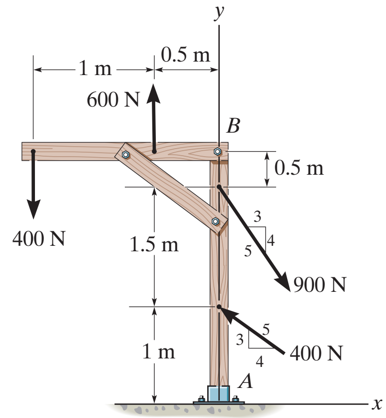


(a)

Ans:
$F_R = 1000$ N
$\theta = 53.1° \triangleleft$
$d = 2.17$ m

***4–120.**

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a vertical line along member AB, measured from A.



SOLUTION

Equivalent Resultant Force. Referring to Fig. *a*

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900\left(\frac{3}{5}\right) - 400\left(\frac{4}{5}\right) = 220 \text{ N} \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 600 + 400\left(\frac{3}{5}\right) - 400 - 900\left(\frac{4}{5}\right)$$

$$= -280 \text{ N} = 280 \text{ N} \downarrow$$

As indicated in Fig. *a*,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{220^2 + 280^2} = 356.09 \text{ N} = 356 \text{ N} \qquad \textbf{Ans.}$$

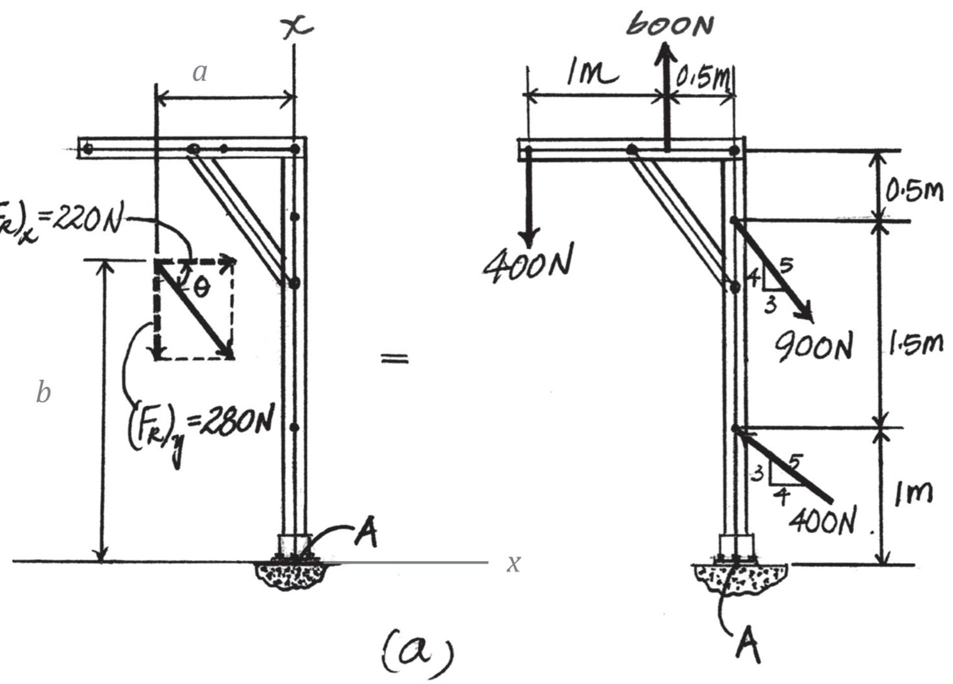
And

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{280}{220}\right) = 51.84° = 51.8° \qquad \textbf{Ans.}$$

Location of Resultant Force. Referring to Fig. *a*

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad 280\,a - 220\,b = 400(1.5) - 600(0.5) - 900\left(\frac{3}{5}\right)(2.5) + 400\left(\frac{4}{5}\right)(1)$$

$$220\,b - 280\,a = 730 \qquad \textbf{(1)}$$



***4–120. Continued**

Along AB, $a = 0$. Then Eq (1) becomes

$$220\,b - 280(0) = 730$$

$$b = 3.318 \text{ m}$$

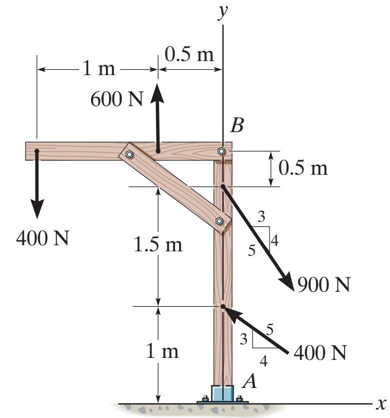
Thus, the intersection point of line of action of $\mathbf{F}_R$ on AB measured upward from point A is

$$d = b = 3.32 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 356$ N
$\theta = 51.8°$
$d = b = 3.32$ m

4–121.

Replace the loading on the frame by a single resultant force. Specify where its line of action intersects a horizontal line along member CB, measured from end C.



SOLUTION

Equivalent Resultant Force. Referring to Fig. *a*

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 900\left(\frac{3}{5}\right) - 400\left(\frac{4}{5}\right) = 220 \text{ N} \quad \rightarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 600 + 400\left(\frac{3}{5}\right) - 400 - 900\left(\frac{4}{5}\right)$$

$$= -280 \text{ N} = 280 \text{ N} \downarrow$$

As indicated in Fig. *a*,

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{220^2 + 280^2} = 356.09 \text{ N} = 356 \text{ N} \qquad \textbf{Ans.}$$

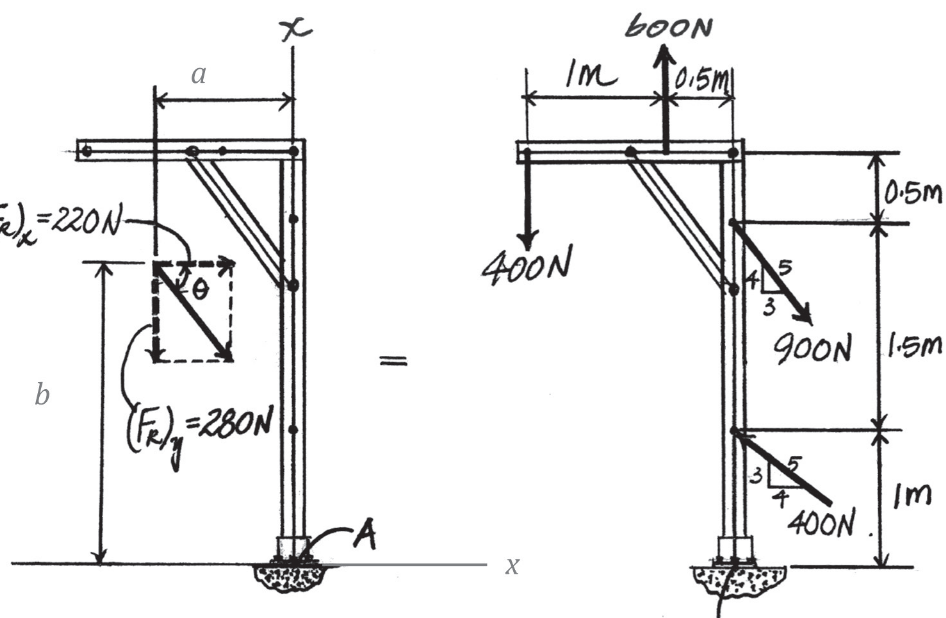
And

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{280}{220}\right) = 51.84° = 51.8° \qquad \textbf{Ans.}$$

Location of Resultant Force. Referring to Fig. *a*

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \qquad 280\,a - 220\,b = 400(1.5) - 600(0.5) - 900\left(\frac{3}{5}\right)(2.5) + 400\left(\frac{4}{5}\right)(1)$$

$$220\,b - 280\,a = 730 \qquad \textbf{(1)}$$



(a)

4–121. Continued

Along BC, $b = 3$ m. Then Eq (1) becomes

$$220(3) - 280\,a = 730$$

$$a = -0.25 \text{ m}$$

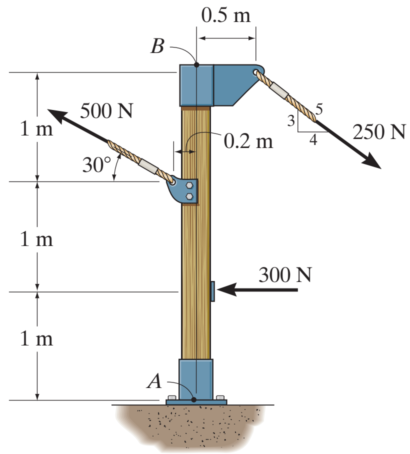
Thus, the intersection point of line of action of $\mathbf{F}_R$ on CB measured to the right of point C is

$$d = 1.5 - (-0.25) = 1.75 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 356$ N
$\theta = 51.8°$
$d = 1.75$ m

4–122.

Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post AB measured from point A.



SOLUTION

Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = 250\left(\frac{4}{5}\right) - 500\cos 30° - 300 = -533.01 \text{ N} = 533.01 \text{ N} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 500 \sin 30° - 250\left(\frac{3}{5}\right) = 100 \text{ N} \uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{533.01^2 + 100^2} = 542.31 \text{ N} = 542 \text{ N} \qquad \textbf{Ans.}$$

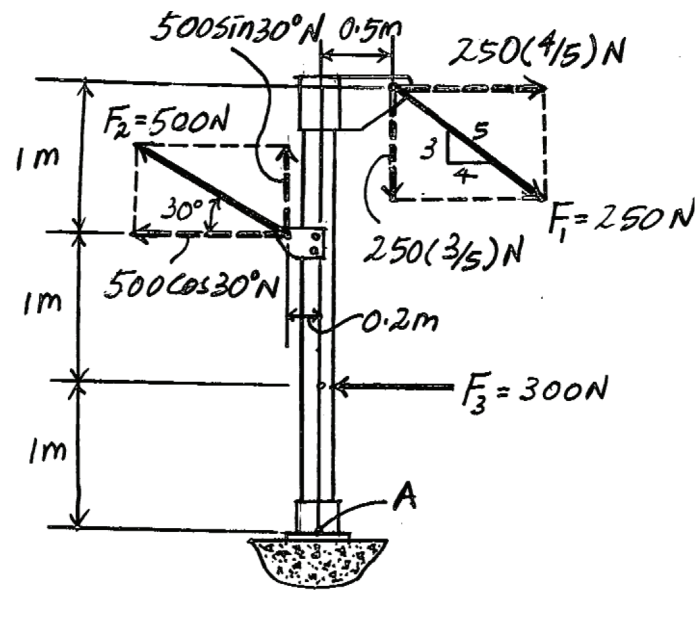
The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{100}{533.01}\right] = 10.63° = 10.6° \qquad \textbf{Ans.}$$

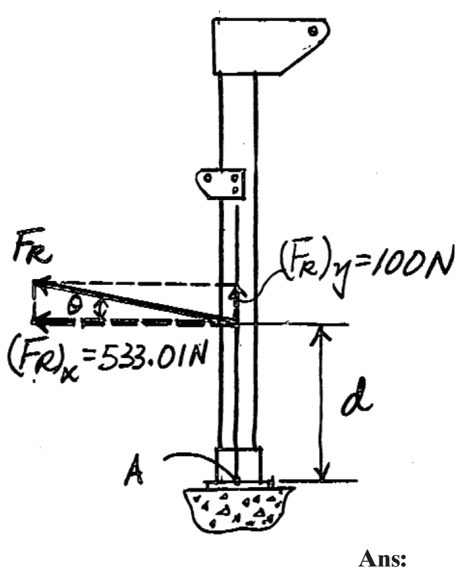
Location of the Resultant Force: Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point A,

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \qquad 533.01(d) = 500 \cos 30°(2) - 500 \sin 30°(0.2) - 250\left(\frac{3}{5}\right)(0.5) - 250\left(\frac{4}{5}\right)(3) + 300(1)$$

$$d = 0.8274 \text{ mm} = 827 \text{ mm} \qquad \textbf{Ans.}$$



(a)

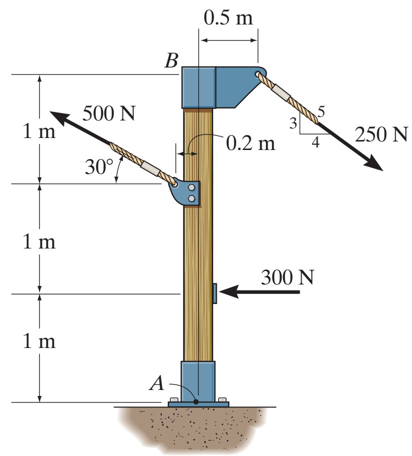


(b)

Ans:
$F_R = 542$ N
$\theta = 10.6°$
$d = 0.827$ m

4–123.

Replace the force system acting on the post by a resultant force, and specify where its line of action intersects the post AB measured from point B.



SOLUTION

Equivalent Resultant Force: Forces $\mathbf{F}_1$ and $\mathbf{F}_2$ are resolved into their x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow} \Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 250\left(\frac{4}{5}\right) - 500\cos 30° - 300 = -533.01\text{N} = 533.01\text{ N} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = 500\sin 30° - 250\left(\frac{3}{5}\right) = 100\text{ N}\uparrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x{}^2 + (F_R)_y{}^2} = \sqrt{533.01^2 + 100^2} = 542.31\text{ N} = 542\text{ N} \qquad \textbf{Ans.}$$

The angle θ of $\mathbf{F}_R$ is

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{100}{533.01}\right] = 10.63° = 10.6° \qquad \textbf{Ans.}$$

Location of the Resultant Force: Applying the principle of moments, Figs. a and b, and summing the moments of the force components algebraically about point B,

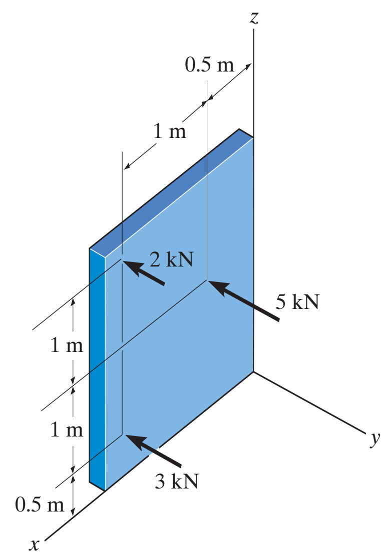
$$\circlearrowleft +(M_R)_B = \Sigma M_b; \qquad -533.01(d) = -500\cos 30°(1) - 500\sin 30°(0.2) - 250\left(\frac{3}{5}\right)(0.5) - 300(2)$$

$$d = 2.17\text{ m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 542$ N
$\theta = 10.6°$
$d = 2.17$ m

***4–124.**

Replace the parallel force system acting on the plate by a resultant force and specify its location on the x–z plane.



SOLUTION

Resultant Force: Summing the forces acting on the plate,

$(F_R)_y = \Sigma F_y;$ $\qquad F_R = -5\text{ kN} - 2\text{ kN} - 3\text{ kN}$

$= -10\text{ kN}$ **Ans.**

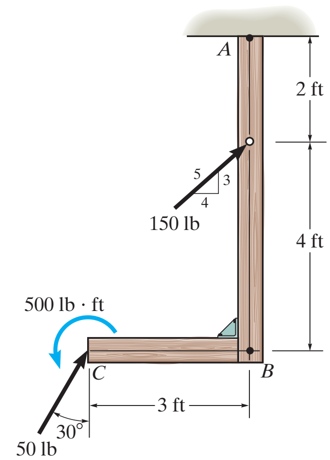
The negative sign indicates that $\mathbf{F}_R$ acts along the negative y axis.

Resultant Moment: Using the right-hand rule, and equating the moment of $\mathbf{F}_R$ to the sum of the moments of the force system about the x and z axes,

Ans:
$F_R = -10\text{ kN}$

4–125.

Replace the force and couple system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member AB, measured from A.



SOLUTION

$$\xrightarrow{+} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 150\left(\frac{4}{5}\right) + 50 \sin 30° = 145 \text{ lb}$$

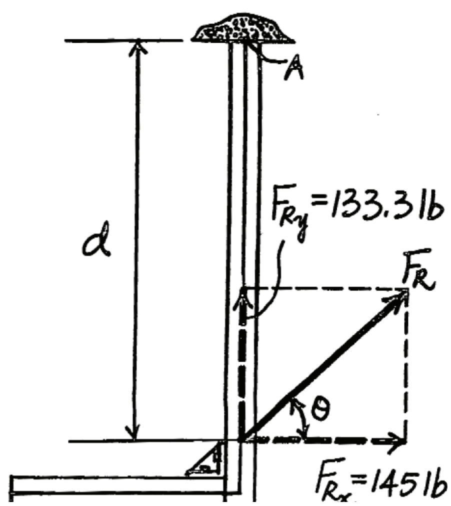
$$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 50 \cos 30° + 150\left(\frac{3}{5}\right) = 133.3 \text{ lb}$$

$$F_R = \sqrt{(145)^2 + (133.3)^2} = 197 \text{ lb} \qquad \textbf{Ans.}$$

$$\theta = \tan^{-1}\left(\frac{133.3}{145}\right) = 42.6° \qquad \textbf{Ans.}$$

$$\circlearrowleft + M_{RA} = \Sigma M_A; \qquad 145\,d = 150\left(\frac{4}{5}\right)(2) - 50 \cos 30°\,(3) + 50 \sin 30°\,(6) + 500$$

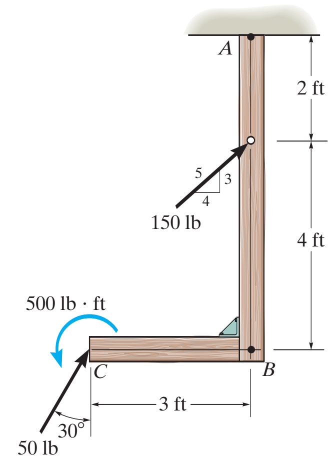
$$d = 5.24 \text{ ft} \qquad \textbf{Ans.}$$



Ans:
$F_R = 197$ lb
$\theta = 42.6°$
$d = 5.24$ ft

4–126.

Replace the force and couple system acting on the frame by an equivalent resultant force and specify where the resultant's line of action intersects member BC, measured from B.



SOLUTION

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x; \qquad F_{Rx} = 150\left(\frac{4}{5}\right) + 50 \sin 30° = 145 \text{ lb}$

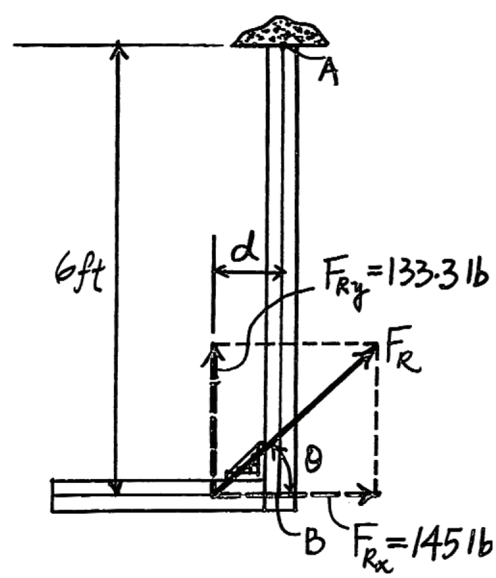
$+\uparrow F_{Ry} = \Sigma F_y; \qquad F_{Ry} = 50 \cos 30° + 150\left(\frac{3}{5}\right) = 133.3 \text{ lb}$

$F_R = \sqrt{(145)^2 + (133.3)^2} = 197 \text{ lb}$ **Ans.**

$\theta = \tan^{-1}\left(\frac{133.3}{145}\right) = 42.6°$ **Ans.**

$\circlearrowleft + M_{RA} = \Sigma M_A; \quad 145\,(6) - 133.3\,(d) = 150\left(\frac{4}{5}\right)(2) - 50 \cos 30°\,(3) + 50 \sin 30°\,(6) + 500$

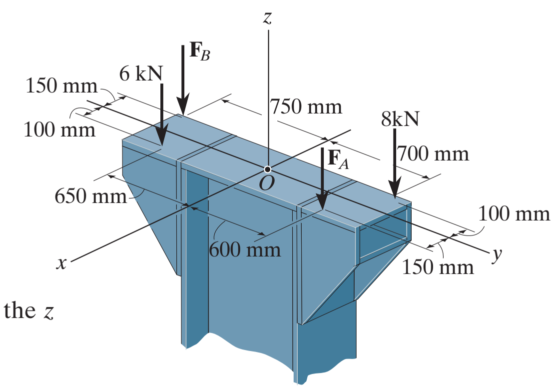
$d = 0.824 \text{ ft}$ **Ans.**



Ans:
$F_R = 197$ lb
$\theta = 42.6°$
$d = 0.824$ ft

4–127.

If $F_A = 7$ kN and $F_B = 5$ kN, represent the force system acting on the corbels by a resultant force, and specify its location on the x–y plane.



SOLUTION

Equivalent Resultant Force: By equating the sum of the forces in Fig. *a* along the z axis to the resultant force $\mathbf{F}_R$, Fig. *b*,

$$+\uparrow F_R = \Sigma F_z; \qquad -F_R = -6-5-7-8$$

$$F_R = 26 \text{ kN} \qquad \textbf{Ans.}$$

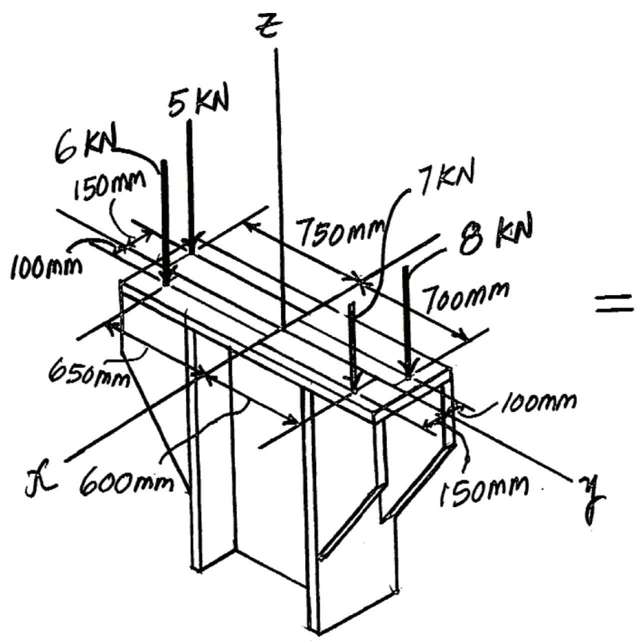
Point of Application: By equating the moment of the forces shown in Fig. *a* and $\mathbf{F}_R$, Fig. *b*, about the x and y axes,

$$(M_R)_x = \Sigma M_x; \qquad -26(y) = 6(650) + 5(750) - 7(600) - 8(700)$$

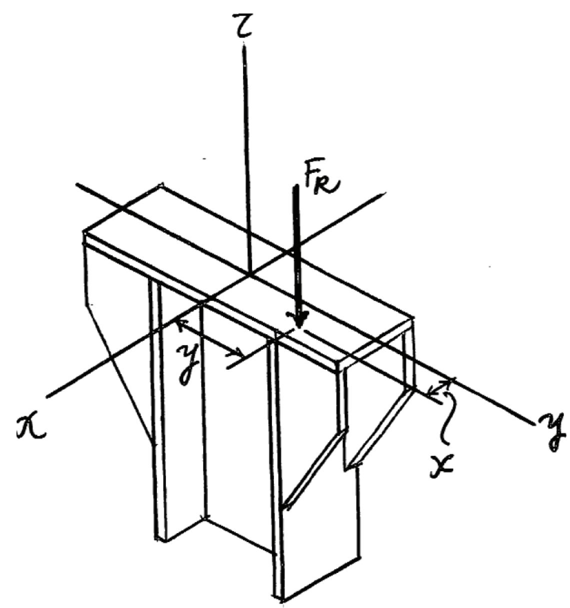
$$y = 82.7 \text{ mm} \qquad \textbf{Ans.}$$

$$(M_R)_y = \Sigma M_y; \qquad 26(x) = 6(100) + 7(150) - 5(150) - 8(100)$$

$$x = 3.85 \text{ mm} \qquad \textbf{Ans.}$$



(a)

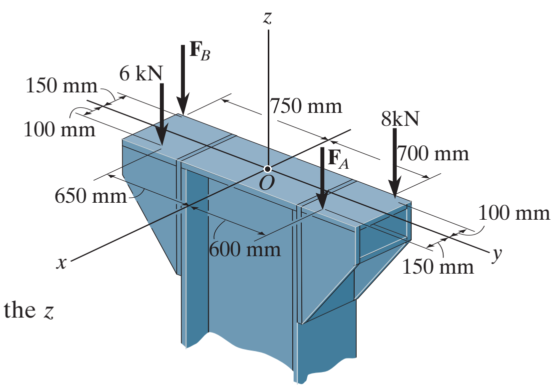


(b)

Ans:
$F_R = 26$ kN
$y = 82.7$ mm
$x = 3.85$ mm

***4–128.**

Determine the magnitudes of $\mathbf{F}_A$ and $\mathbf{F}_B$ so that the resultant force passes through point O of the column.



SOLUTION

Equivalent Resultant Force: By equating the sum of the forces in Fig. *a* along the z axis to the resultant force $\mathbf{F}_R$, Fig. b,

$$+\uparrow F_R = \Sigma F_z; \qquad -F_R = -F_A - F_B - 8 - 6$$

$$F_R = F_A + F_B + 14 \qquad \textbf{(1)}$$

Point of Application: Since $\mathbf{F}_R$ is required to pass through point O, the moment of $\mathbf{F}_R$ about the x and y axes are equal to zero. Thus,

$$(M_R)_x = \Sigma M_x; \qquad 0 = F_B(750) + 6(650) - F_A(600) - 8(700)$$

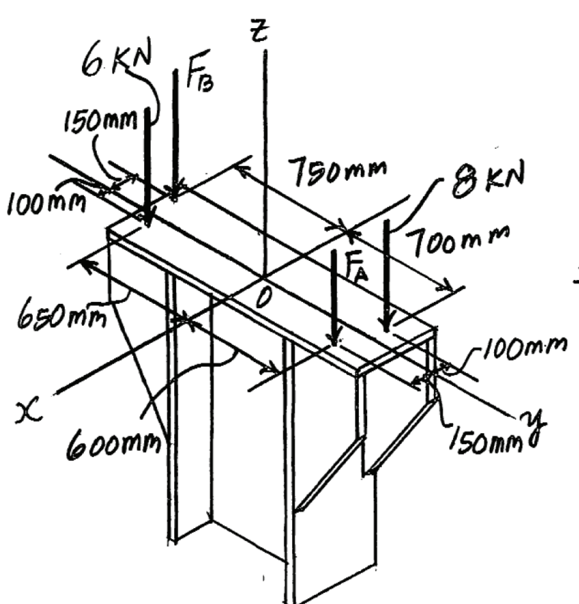
$$750F_B - 600F_A - 1700 = 0 \qquad \textbf{(2)}$$

$$(M_R)_y = \Sigma M_y; \qquad 0 = F_A(150) + 6(100) - F_B(150) - 8(100)$$

$$159F_A - 150F_B + 200 = 0 \qquad \textbf{(3)}$$

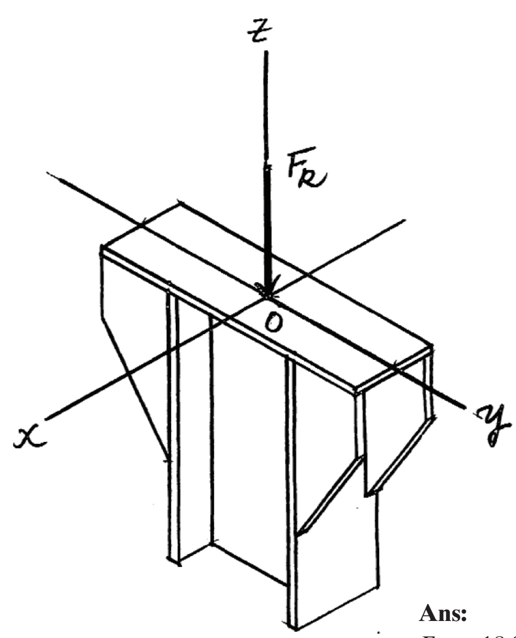
Solving Eqs. (1) through (3) yields

$F_A = 18.0\text{ kN}$ $\qquad$ $F_B = 16.7\text{ kN}$ $\qquad$ $F_R = 48.7\text{ kN}$ $\qquad$ **Ans.**



(a)

=

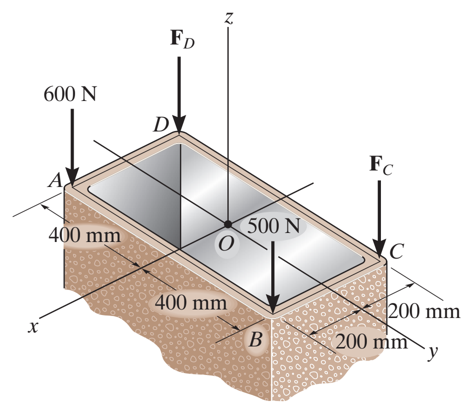


(b)

Ans:
$F_A = 18.0\text{ kN}$
$F_B = 16.7\text{ kN}$
$F_R = 48.7\text{ kN}$

4–129.

The tube supports the four parallel forces. Determine the magnitudes of forces $\mathbf{F}_C$ and $\mathbf{F}_D$ acting at C and D so that the equivalent resultant force of the force system acts through the midpoint O of the tube.



SOLUTION

Since the resultant force passes through point O, the resultant moment components about x and y axes are both zero.

$$\Sigma M_x = 0; \qquad F_D(0.4) + 600(0.4) - F_C(0.4) - 500(0.4) = 0$$

$$F_C - F_D = 100 \qquad \textbf{(1)}$$

$$\Sigma M_y = 0; \qquad 500(0.2) + 600(0.2) - F_C(0.2) - F_D(0.2) = 0$$

$$F_C + F_D = 1100 \qquad \textbf{(2)}$$

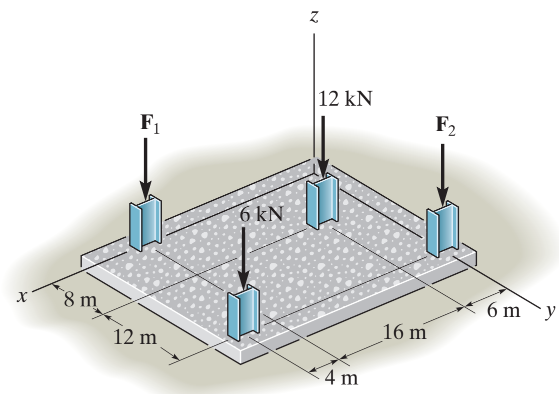
Solving Eqs. (1) and (2) yields:

$$F_C = 600 \text{ N} \qquad F_D = 500 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$F_C = 600$ N
$F_D = 500$ N

4–130.

The building slab is subjected to four parallel column loadings. Determine the equivalent resultant force and specify its location (x, y) on the slab. Take $F_1 = 8$ kN and $F_2 = 9$ kN.



SOLUTION

Equivalent Resultant Force. Sum the forces along z axis by referring to Fig. a

$+\uparrow (F_R)_z = \Sigma F_z; \qquad -F_R = -8 - 6 - 12 - 9 \qquad F_R = 35 \text{ kN}$ **Ans.**

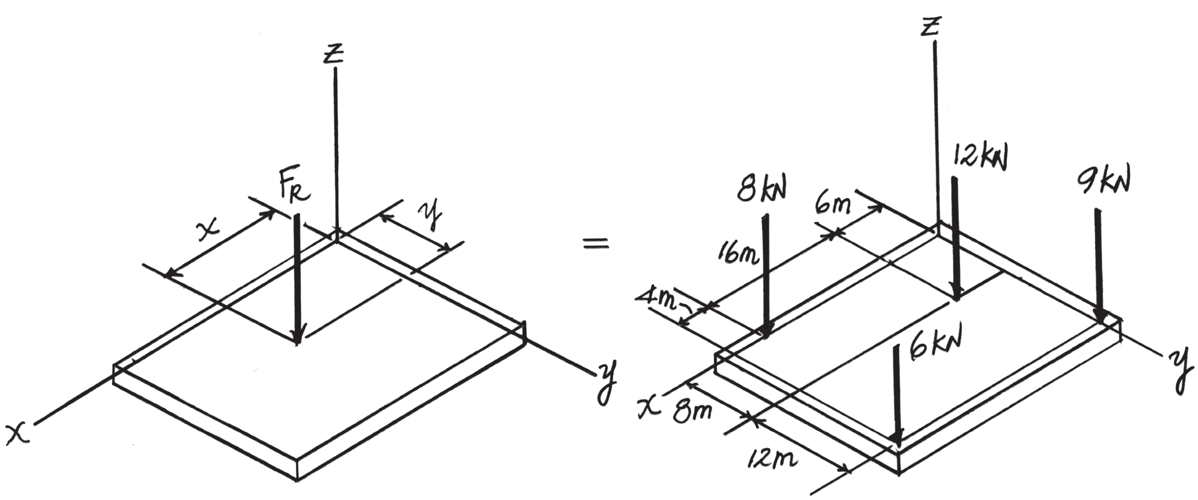
Location of the Resultant Force. Sum the moments about the x and y axes by referring to Fig. a,

$$(M_R)_x = \Sigma M_x; \qquad -35\,y = -12(8) - 6(20) - 9(20)$$

$$y = 11.31 \text{ m} = 11.3 \text{ m} \qquad \textbf{Ans.}$$

$$(M_R)_y = \Sigma M_y; \qquad 35\,x = 12(6) + 8(22) + 6(26)$$

$$x = 11.54 \text{ m} = 11.5 \text{ m} \qquad \textbf{Ans.}$$

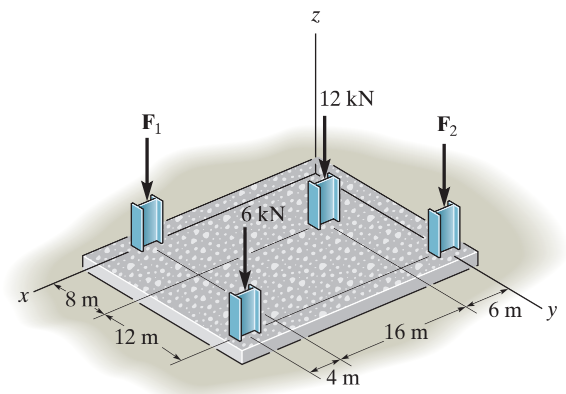


(a)

Ans:
$F_R = 35$ kN
$y = 11.3$ m
$x = 11.5$ m

4–131.

The building slab is subjected to four parallel column loadings. Determine $\mathbf{F}_1$ and $\mathbf{F}_2$ if the resultant force acts through point (12 m, 10 m).



SOLUTION

Equivalent Resultant Force. Sum the forces along z axis by referring to Fig. a,

$$+\uparrow (F_R)_z = \Sigma F_z; \qquad -F_R = -F_1 - F_2 - 12 - 6 \qquad F_R = F_1 + F_2 + 18$$

Location of the Resultant Force. Sum the moments about the x and y axes by referring to Fig. a,

$$(M_R)_x = \Sigma M_x; \qquad -(F_1 + F_2 + 18)(10) = -12(8) - 6(20) - F_2(20)$$

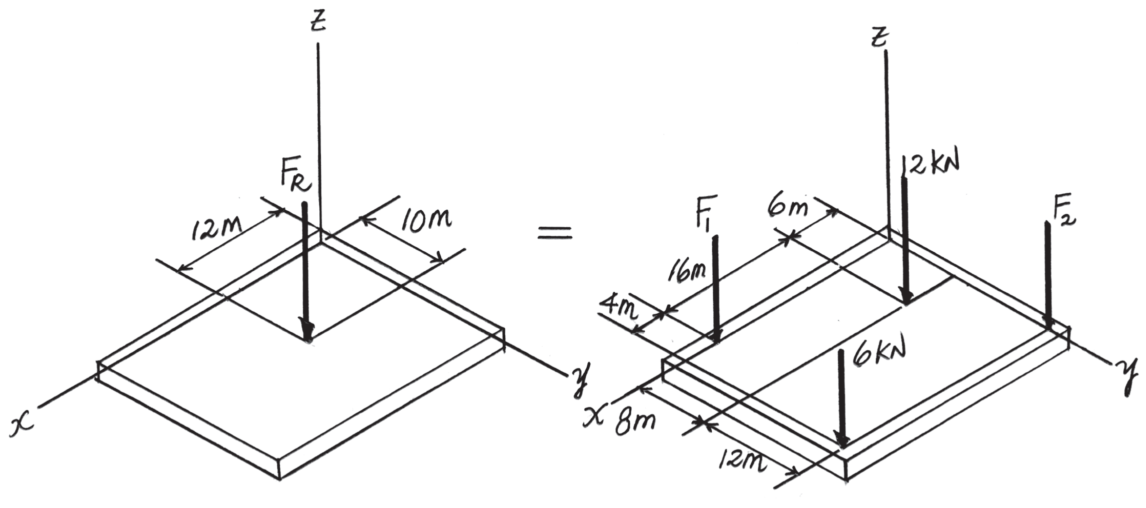
$$10F_1 - 10F_2 = 36 \qquad \textbf{(1)}$$

$$(M_R)_y = \Sigma M_y; \qquad (F_1 + F_2 + 18)(12) = 12(6) + 6(26) + F_1(22)$$

$$12F_2 - 10F_1 = 12 \qquad \textbf{(2)}$$

Solving Eqs (1) and (2),

$$F_1 = 27.6 \text{ kN} \qquad F_2 = 24.0 \text{ kN} \qquad \textbf{Ans.}$$

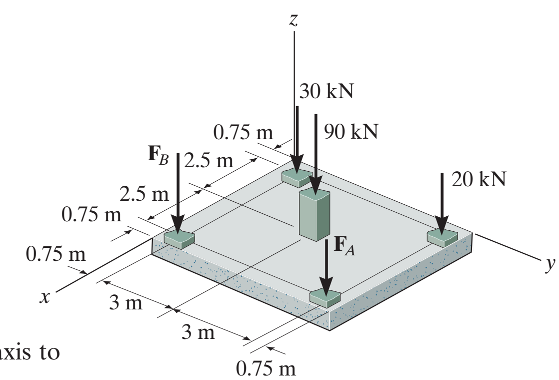


(a)

Ans:
$F_1 = 27.6$ kN
$F_2 = 24.0$ kN

***4–132.**

If $F_A = 40$ kN and $F_B = 35$ kN, determine the magnitude of the resultant force and specify the location of its point of application (x, y) on the slab.



SOLUTION

Equivalent Resultant Force: By equating the sum of the forces along the z axis to the resultant force $\mathbf{F}_R$, Fig. b,

$$+\uparrow F_R = \Sigma F_z; \qquad -F_R = -30 - 20 - 90 - 35 - 40$$

$$F_R = 215 \text{ kN} \qquad \textbf{Ans.}$$

Point of Application: By equating the moment of the forces and $\mathbf{F}_R$, about the x and y axes,

$$(M_R)_x = \Sigma M_x; \qquad -215(y) = -35(0.75) - 30(0.75) - 90(3.75) - 20(6.75) - 40(6.75)$$

$$y = 3.68 \text{ m} \qquad \textbf{Ans.}$$

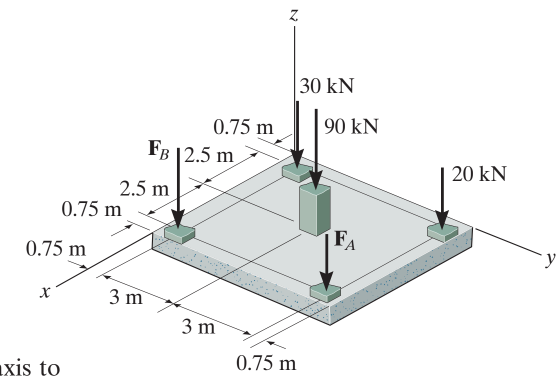
$$(M_R)_y = \Sigma M_y; \qquad 215(x) = 30(0.75) + 20(0.75) + 90(3.25) + 35(5.75) + 40(5.75)$$

$$x = 3.54 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 215$ kN
$y = 3.68$ m
$x = 3.54$ m

4–133.

If the resultant force is required to act at the center of the slab, determine the magnitude of the column loadings $\mathbf{F}_A$ and $\mathbf{F}_B$ and the magnitude of the resultant force.



SOLUTION

Equivalent Resultant Force: By equating the sum of the forces along the z axis to the resultant force $\mathbf{F}_R$,

$$+\uparrow F_R = \Sigma F_z; \qquad -F_R = -30 - 20 - 90 - F_A - F_B$$

$$F_R = 140 + F_A + F_B \qquad \textbf{(1)}$$

Point of Application: By equating the moment of the forces and $\mathbf{F}_R$, about the x and y axes,

$$(M_R)_x = \Sigma M_x; \qquad -F_R(3.75) = -F_B(0.75) - 30(0.75) - 90(3.75) - 20(6.75) - F_A(6.75)$$

$$F_R = 0.2F_B + 1.8F_A + 132 \qquad \textbf{(2)}$$

$$(M_R)_y = \Sigma M_y; \qquad F_R(3.25) = 30(0.75) + 20(0.75) + 90(3.25) + F_A(5.75) + F_B(5.75)$$

$$F_R = 1.769F_A + 1.769F_B + 101.54 \qquad \textbf{(3)}$$

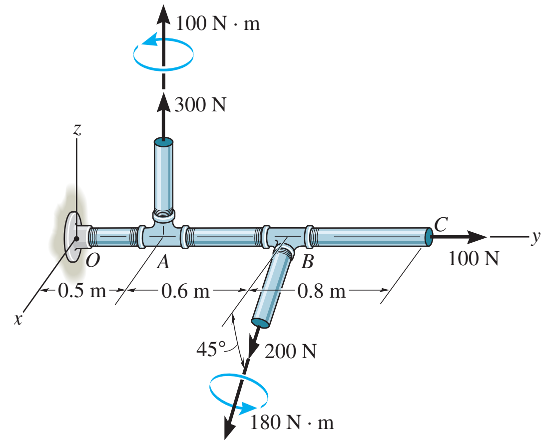
Solving Eqs.(1) through (3) yields

$F_A = 30\,\text{kN}$ $\qquad F_B = 20\text{ kN}$ $\qquad F_R = 190\,\text{kN}$ $\qquad$ **Ans.**

Ans:
$F_A = 30\text{ kN}$
$F_B = 20\text{ kN}$
$F_R = 190\text{ kN}$

4–134.

Replace the two wrenches and the force, acting on the pipe assembly, by an equivalent resultant force and couple moment at point O.



SOLUTION

Force And Moment Vectors:

$$\mathbf{F}_1 = \{300\mathbf{k}\}\text{ N} \qquad \mathbf{F}_3 = \{100\mathbf{j}\}\text{ N}$$

$$\mathbf{F}_2 = 200\{\cos 45°\mathbf{i} - \sin 45°\mathbf{k}\}\text{ N}$$

$$= \{141.42\mathbf{i} - 141.42\mathbf{k}\}\text{ N}$$

$$\mathbf{M}_1 = \{100\mathbf{k}\}\text{ N}\cdot\text{m}$$

$$\mathbf{M}_2 = 180\{\cos 45°\mathbf{i} - \sin 45°\mathbf{k}\}\text{ N}\cdot\text{m}$$

$$= \{127.28\mathbf{i} - 127.28\mathbf{k}\}\text{ N}\cdot\text{m}$$

Equivalent Force and Couple Moment At Point O:

$\mathbf{F}_R = \Sigma\mathbf{F};\qquad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$

$$= 141.42\mathbf{i} + 100.0\mathbf{j} + (300 - 141.42)\mathbf{k}$$

$$= \{141\mathbf{i} + 100\mathbf{j} + 159\mathbf{k}\}\text{ N} \qquad \textbf{Ans.}$$

The position vectors are $\mathbf{r}_1 = \{0.5\mathbf{j}\}$ m and $\mathbf{r}_2 = \{1.1\mathbf{j}\}$ m.

$\mathbf{M}_{R_O} = \Sigma\mathbf{M}_O;\qquad \mathbf{M}_{R_O} = \mathbf{r}_1 \times \mathbf{F}_1 + \mathbf{r}_2 \times \mathbf{F}_2 + \mathbf{M}_1 + \mathbf{M}_2$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.5 & 0 \\ 0 & 0 & 300 \end{vmatrix}$$

$$+ \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.1 & 0 \\ 141.42 & 0 & -141.42 \end{vmatrix}$$

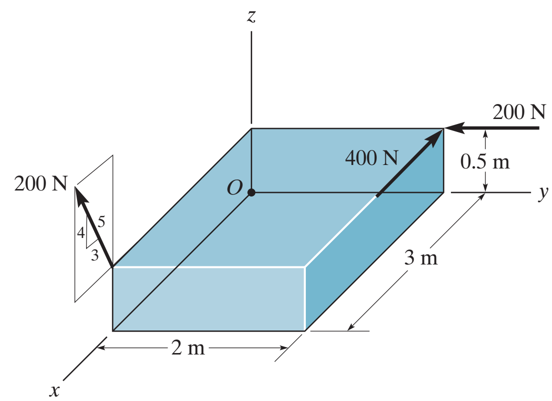
$$+ 100\mathbf{k} + 127.28\mathbf{i} - 127.28\mathbf{k}$$

$$= \{122\mathbf{i} - 183\mathbf{k}\}\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$\mathbf{F}_R = \{141\mathbf{i} + 100\mathbf{j} + 159\mathbf{k}\}$ N
$\mathbf{M}_{R_O} = \{122\mathbf{i} - 183\mathbf{k}\}$ N · m

4–135.

Replace the force system by a wrench and specify the magnitude of the force and couple moment of the wrench and the point where the wrench intersects the *x–z* plane.



SOLUTION

Resultant Force. Referring to Fig. *a*

$$\mathbf{F}_R = \left\{\left[200\left(\frac{3}{5}\right) - 400\right]\mathbf{i} - 200\mathbf{j} + 200\left(\frac{4}{5}\right)\mathbf{k}\right\}$$

$$= \{-280\mathbf{i} - 200\mathbf{j} + 160\mathbf{k}\}\text{ N}$$

The magnitude of $\mathbf{F}_R$ is

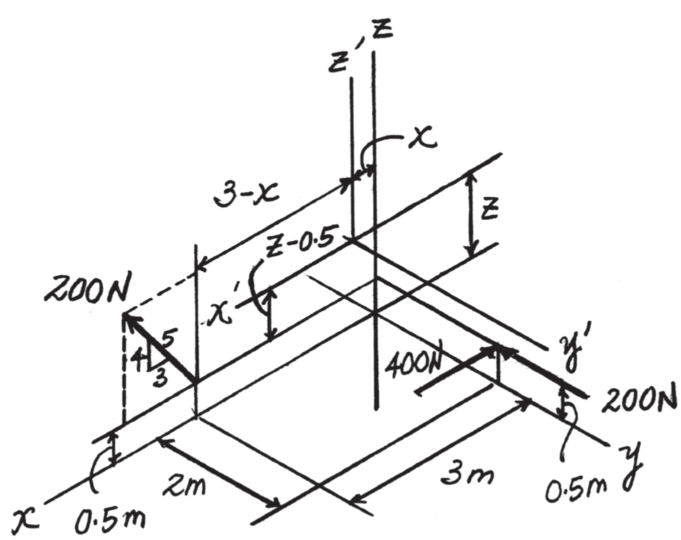
$$F_R = \sqrt{(-280)^2 + (-200)^2 + 160^2} = 379.47\text{ N} = 379\text{ N} \qquad \textbf{Ans.}$$

The direction of $\mathbf{F}_R$ is defined by

$$\mathbf{u}_{F_R} = \frac{\mathbf{F}_R}{F_R} = \frac{-280\mathbf{i} - 200\mathbf{j} + 160\mathbf{k}}{379.47} = -0.7379\mathbf{i} - 0.5270\mathbf{j} + 0.4216\mathbf{k}$$

Resultant Moment. The line of action of $\mathbf{M}_R$ of the wrench is parallel to that of $\mathbf{F}_R$. Also, assume that $\mathbf{M}_R$ and $\mathbf{F}_R$ have the same sense. Then

$$\mathbf{u}_{M_R} = -0.7379\mathbf{i} - 0.5270\mathbf{j} + 0.4216\mathbf{k}$$



(a)

4–135. Continued

Referring to Fig. *a*, where the origin of the x', y', z' axes is the point where the wrench intersects the xz plane,

$(M_R)_{x'} = \Sigma M_{x'};\ -0.7379\, M_R = -200(z - 0.5)$ **(1)**

$(M_R)_{y'} = \Sigma M_{y'};\ -0.5270\, M_R = -200\left(\frac{2}{5}\right)(z - 0.5) - 200\left(\frac{4}{5}\right)(3 - x) + 400(z - 0.5)$ **(2)**

$(M_R)_{z'} = \Sigma M_{z'};\ 0.4216\, M_R = 200x + 400(2)$ **(3)**

Solving Eqs (1), (2) and (3)

$M_R = 590.29\ \text{N}\cdot\text{m} = 590\ \text{N}\cdot\text{m}$ **Ans.**

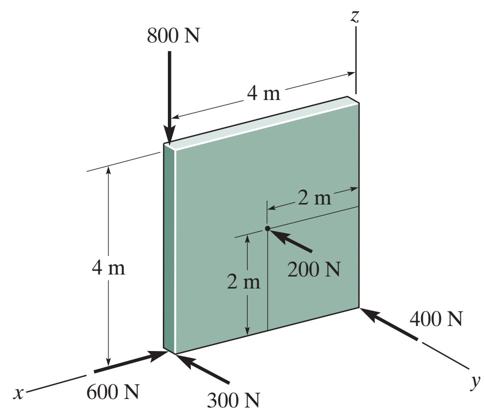
$z = 2.6778\ \text{m} = 2.68\ \text{m}$ **Ans.**

$x = -2.7556\ \text{m} = -2.76\ \text{m}$ **Ans.**

Ans:
$F_R = 379\ \text{N}$
$M_R = 590\ \text{N}\cdot\text{m}$
$z = 2.68\ \text{m}$
$x = -2.76\ \text{m}$

***4–136.**

Replace the five forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, z)$ where the wrench intersects the x–z plane.



SOLUTION

Resultant Force. Referring to Fig. *a*

$$\mathbf{F}_R = \{-600\mathbf{i} - (300 + 200 + 400)\mathbf{j} - 800\mathbf{k}\}\text{ N}$$

$$= \{-600\mathbf{i} - 900\mathbf{j} - 800\mathbf{k}\}\text{ N}$$

Then the magnitude of $\mathbf{F}_R$ is

$$F_R = \sqrt{(-600)^2 + (-900)^2 + (-800)^2} = 1345.36\text{ N} = 1.35\text{ kN} \qquad \textbf{Ans.}$$

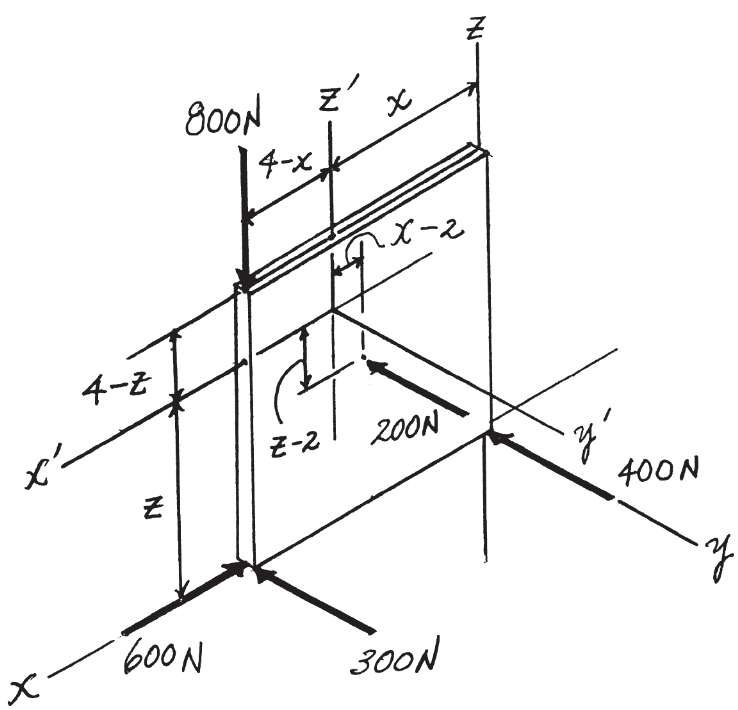
The direction of $\mathbf{F}_R$ is defined by

$$\mathbf{u}_{F_R} = \frac{\mathbf{F}_R}{F_R} = \frac{-600\mathbf{i} - 900\mathbf{j} - 800\mathbf{k}}{1345.36} = -0.4460\mathbf{i} - 0.6690\mathbf{j} - 0.5946\mathbf{k}$$

Resultant Moment.

The line of action of $\mathbf{M}_R$ of the wrench is parallel to that of $\mathbf{F}_R$. Also, assume that both $\mathbf{M}_R$ and $\mathbf{F}_R$ have the same sense. Then

$$\mathbf{u}_{M_R} = -0.4460\mathbf{i} - 0.6690\mathbf{j} - 0.5946\mathbf{k}$$



(a)

***4–136. Continued**

Referring to Fig. *a*,

$$(M_R)_{x'} = \Sigma M_{x'}; \quad -0.4460\,\mathbf{M}_R = -300z - 200(z - 2) - 400z \qquad \textbf{(1)}$$

$$(M_R)_{y'} = \Sigma M_{y'}; \quad -0.6690\,\mathbf{M}_R = 800(4 - x) + 600z \qquad \textbf{(2)}$$

$$(M_R)_{z'} = \Sigma M_{z'}; \quad -0.5946\,\mathbf{M}_R = 200(x - 2) + 400x - 300(4 - x) \qquad \textbf{(3)}$$

Solving Eqs (1), (2) and (3)

$$M_R = -1367.66\ \text{N}\cdot\text{m} = -1.37\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

$$x = 2.681\ \text{m} = 2.68\ \text{m} \qquad \textbf{Ans.}$$

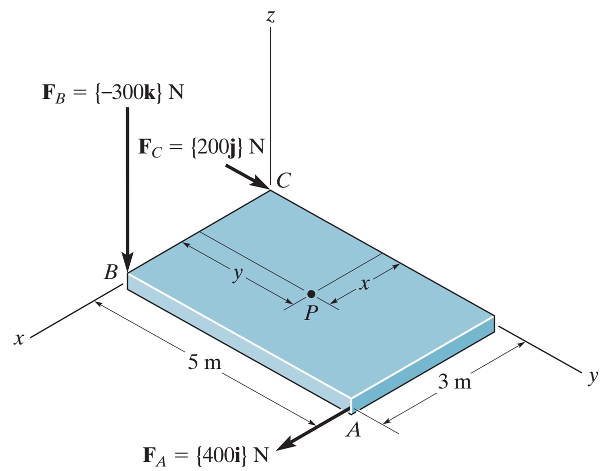
$$z = -0.2333\ \text{m} = -0.233\ \text{m} \qquad \textbf{Ans.}$$

The negative sign indicates that the line of action of $\mathbf{M}_R$ is directed in the opposite sense to that of $\mathbf{F}_R$.

Ans:
$M_R = -1.37\ \text{kN}\cdot\text{m}$
$x = 2.68\ \text{m}$
$z = -0.233\ \text{m}$

4–137.

Replace the three forces acting on the plate by a wrench. Specify the magnitude of the force and couple moment for the wrench and the point $P(x, y)$ where the wrench intersects the plate.



SOLUTION

Resultant Force. Referring to Fig. *a*,

$$F_R = \{400\mathbf{i} + 200\mathbf{j} - 300\mathbf{k}\}\text{ N}$$

Then, the magnitude of $\mathbf{F}_R$ is

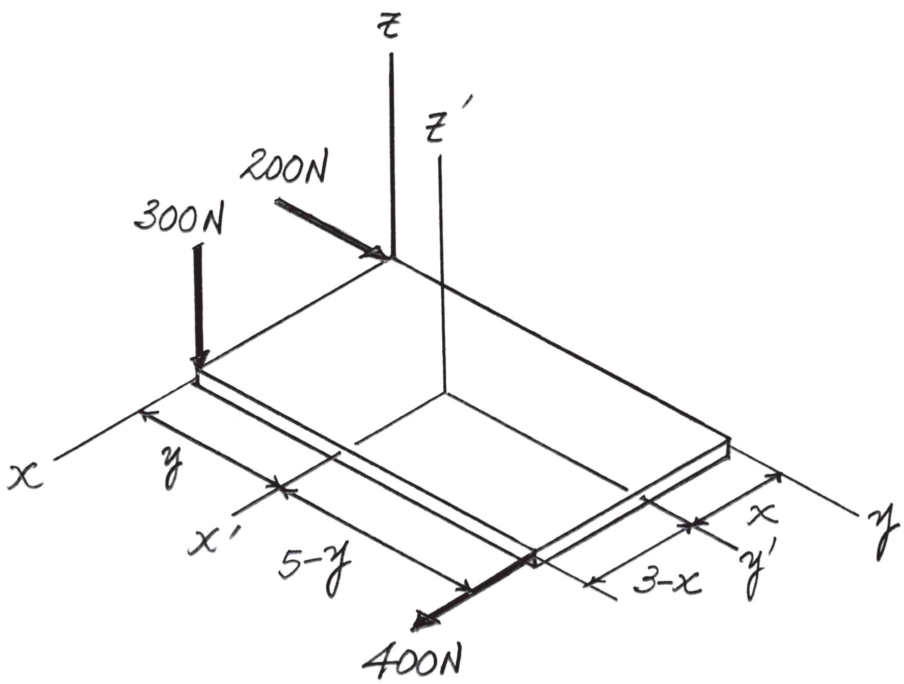
$$F_R = \sqrt{400^2 + 200^2 + (-300)^2} = 538.52\text{ N} = 539\text{ N} \qquad \textbf{Ans.}$$

The direction of $\mathbf{F}_R$ is defined by

$$\mathbf{u}_{F_R} = \frac{\mathbf{F}_R}{F_R} = \frac{400\mathbf{i} + 200\mathbf{j} - 300\mathbf{k}}{538.52} = 0.7428\mathbf{i} + 0.3714\mathbf{j} - 0.5571\mathbf{k}$$

Resultant Moment. The line of action of $\mathbf{M}_R$ of the wrench is parallel to that of $\mathbf{F}_R$. Also, assume that both $\mathbf{M}_R$ and $\mathbf{F}_R$ have the same sense. Then

$$\mathbf{u}_{M_R} = 0.7428\mathbf{i} + 0.3714\mathbf{j} - 0.5571\mathbf{k}$$



(a)

4–137. Continued

Referring to Fig. *a*,

$$(M_R)_{x'} = \Sigma M_{x'};\ 0.7428\ M_R = 300y \qquad \textbf{(1)}$$

$$(M_R)_{y'} = \Sigma M_{y'};\ 0.3714\ M_R = 300(3 - x) \qquad \textbf{(2)}$$

$$(M_R)_{z'} = \Sigma M_z;\ -0.5571\ M_R = -200x - 400(5 - y) \qquad \textbf{(3)}$$

Solving Eqs (1), (2) and (3)

$$M_R = 1448.42\ \text{N}\cdot\text{m} = 1.45\ \text{kN}\cdot\text{m} \qquad \textbf{Ans.}$$

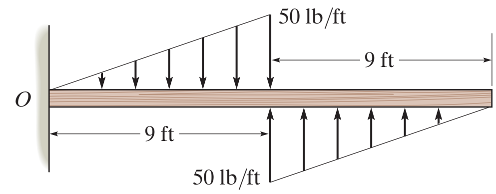
$$x = 1.2069\ \text{m} = 1.21\ \text{m} \qquad \textbf{Ans.}$$

$$y = 3.5862\ \text{m} = 3.59\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 539$ N
$M_R = 1.45$ kN·m
$x = 1.21$ m
$y = 3.59$ m

4–138.

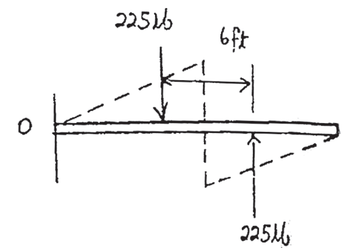
Replace the loading by an equivalent resultant force and couple moment acting at point O.



SOLUTION

$+\uparrow F_R = \Sigma F;$ $\quad F_R = 0$ **Ans.**

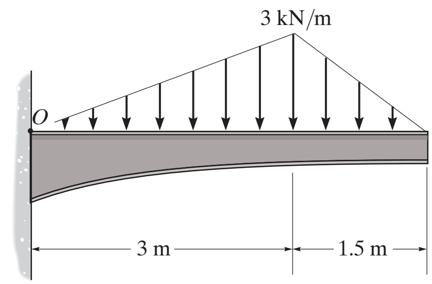
$\circlearrowleft + M_{RO} = \Sigma M_O;$ $\quad M_{RO} = 225(6) = 1350 \text{ lb} \cdot \text{ft} = 1.35 \text{ kip} \cdot \text{ft}$ **Ans.**



Ans:
$F_R = 0$
$M_{RO} = 1.35 \text{ kip} \cdot \text{ft}$

4–139.

Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point O.



SOLUTION

Loading: The distributed loading can be divided into two parts as shown in Fig. a.

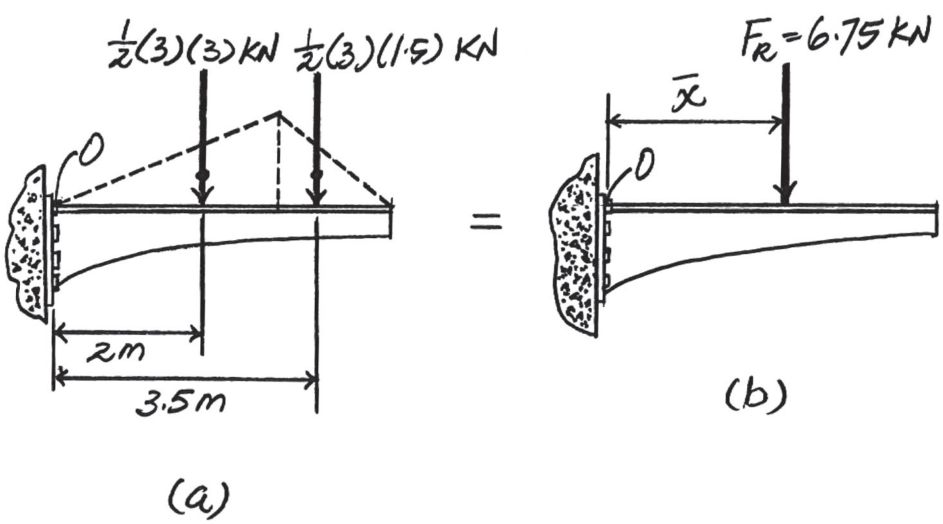
Equations of Equilibrium: Equating the forces along the y axis of Figs. a and b, we have

$+\downarrow F_R = \Sigma F;\qquad F_R = \frac{1}{2}(3)(3) + \frac{1}{2}(3)(1.5) = 6.75\text{ kN}\downarrow$ **Ans.**

If we equate the moment of F_R, Fig. b, to the sum of the moment of the forces in Fig. a about point O, we have

$\circlearrowleft + (M_R)_O = \Sigma M_O;\qquad -6.75(\bar{x}) = -\frac{1}{2}(3)(3)(2) - \frac{1}{2}(3)(1.5)(3.5)$

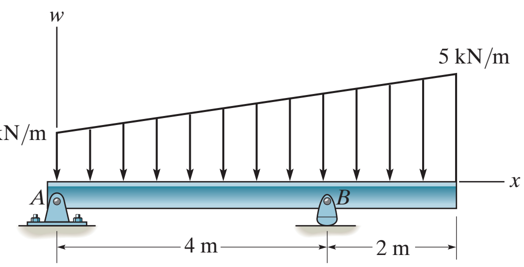
$\bar{x} = 2.5\text{ m}$ **Ans.**



Ans:
$F_R = 6.75\text{ kN}$
$\bar{x} = 2.5\text{ m}$

***4–140.**

Replace the loading by an equivalent resultant force and specify its location on the beam, measured from point A.



SOLUTION

Equivalent Resultant Force. Summing the forces along the y axis by referring to Fig. a

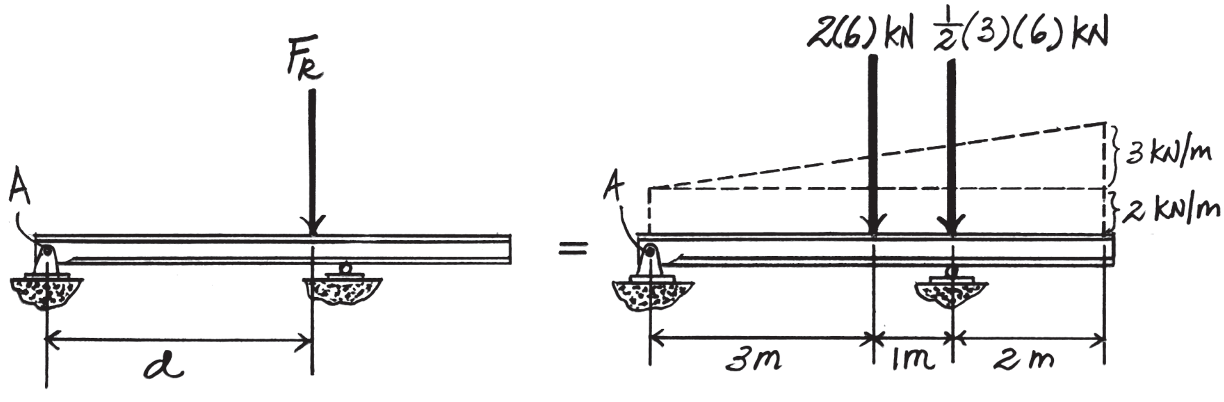
$$+\uparrow\ (F_R)_y = \Sigma F_y; \quad -F_R = -2(6) - \frac{1}{2}(3)(6)$$ **Ans.**

$$F_R = 21.0\ \text{kN}\downarrow$$ **Ans.**

Location of the Resultant Force. Summing the moments about point A,

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \quad -21.0(d) = -2(6)(3) - \frac{1}{2}(3)(6)(4)$$

$$d = 3.429\ \text{m} = 3.43\ \text{m}$$ **Ans.**

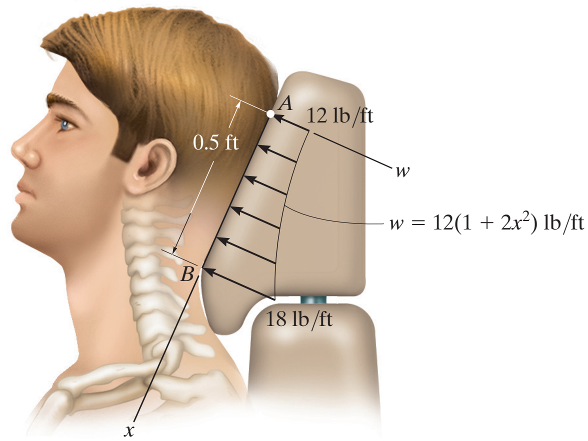


(a)

Ans:
$F_R = 21.0$ kN
$d = 3.43$ m

4–141.

Currently eighty-five percent of all neck injuries are caused by rear-end car collisions. To alleviate this problem, an automobile seat restraint has been developed that provides additional pressure contact with the cranium. During dynamic tests the distribution of load on the cranium has been plotted and shown to be parabolic. Determine the equivalent resultant force and its location, measured from point A.



SOLUTION

$$F_R = \int w(x)\,dx = \int_0^{0.5} 12\left(1 + 2x^2\right)dx = 12\left[x + \frac{2}{3}x^3\right]_0^{0.5} = 7\text{ lb} \qquad \textbf{Ans.}$$

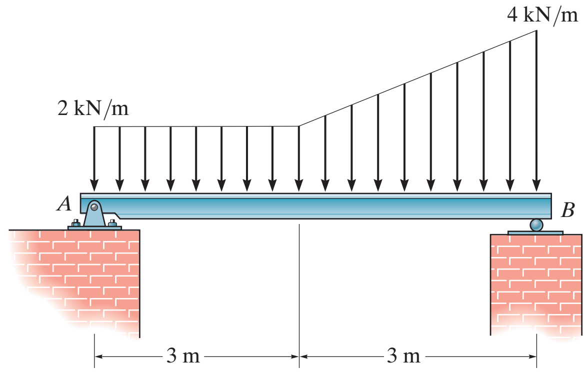
$$\bar{x} = \frac{\int x\,w(x)\,dx}{\int w(x)dx} = \frac{\int_0^{0.5} x(12)\left(1 + 2x^2\right)dx}{7} = \frac{12\left[\frac{x^2}{2} + (2)\frac{x^4}{4}\right]_0^{0.5}}{7}$$

$$\bar{x} = 0.268\text{ ft} \qquad \textbf{Ans.}$$

Ans:
$F_R = 7$ lb
$\bar{x} = 0.268$ ft

4–142.

Replace the distributed loading by an equivalent resultant force, and specify its location on the beam, measured from the pin at A.



SOLUTION

Equivalent Resultant Force. Summing the forces along the y axis by referring to Fig. a,

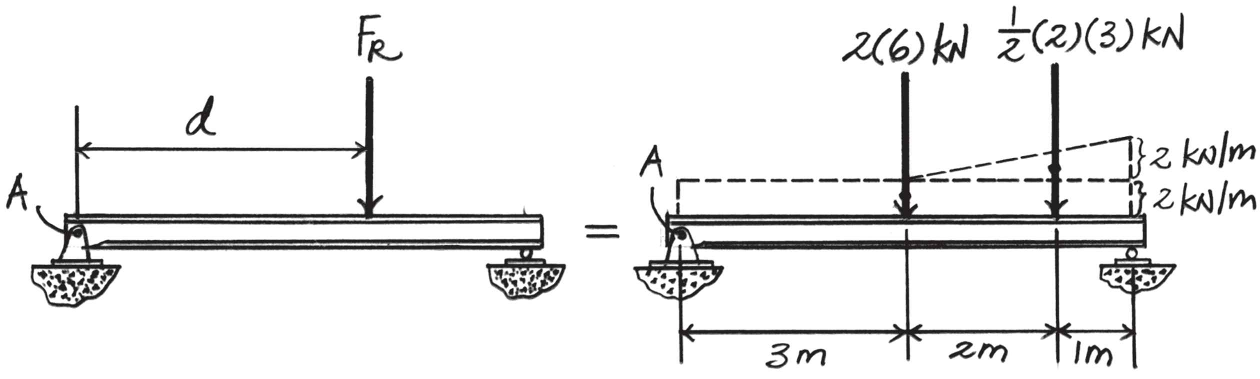
$$+\uparrow (F_R)_y = \Sigma F_y; \quad -F_R = -2(6) - \frac{1}{2}(2)(3)$$

$$F_R = 15.0 \text{ kN} \downarrow \qquad \textbf{Ans.}$$

Location of the Resultant Force. Summing the Moments about point A,

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \quad -15.0(d) = -2(6)(3) - \frac{1}{2}(2)(3)(5)$$

$$d = 3.40 \text{ m} \qquad \textbf{Ans.}$$

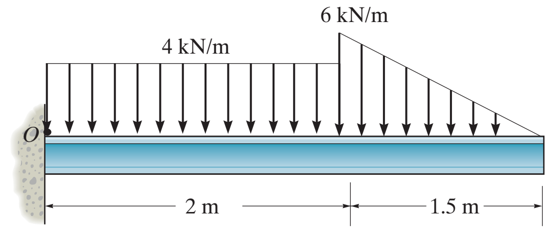


(a)

Ans:
$F_R = 15.0$ kN
$d = 3.40$ m

4–143.

Replace this loading by an equivalent resultant force and specify its location, measured from point O.



SOLUTION

Equivalent Resultant Force. Summing the forces along the y axis by referring to Fig. a,

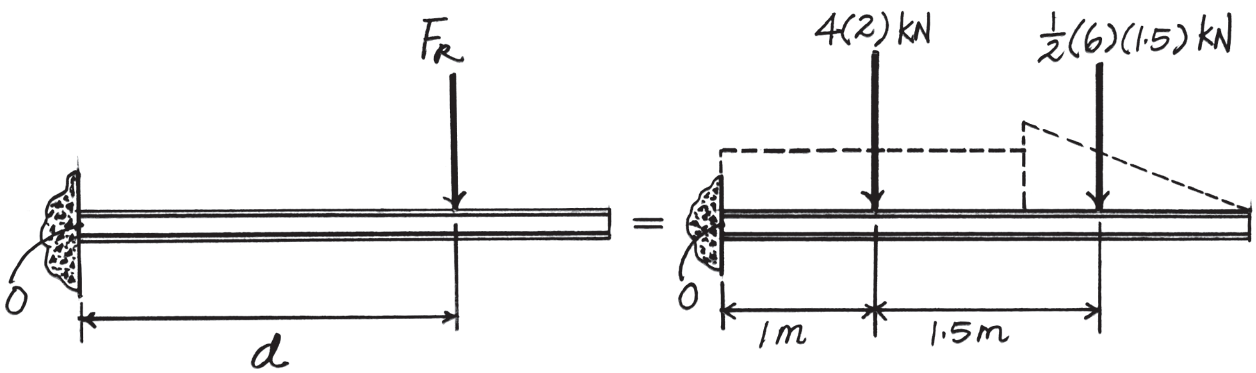
$$+\uparrow (F_R)_y = \Sigma F_y; \quad -F_R = -4(2) - \frac{1}{2}(6)(1.5)$$

$$F_R = 12.5 \text{ kN} \qquad \textbf{Ans.}$$

Location of the Resultant Force. Summing the Moment about point O,

$$\circlearrowleft + (M_R)_O = \Sigma M_O; \quad -12.5(d) = -4(2)(1) - \frac{1}{2}(6)(1.5)(2.5)$$

$$d = 1.54 \text{ m} \qquad \textbf{Ans.}$$

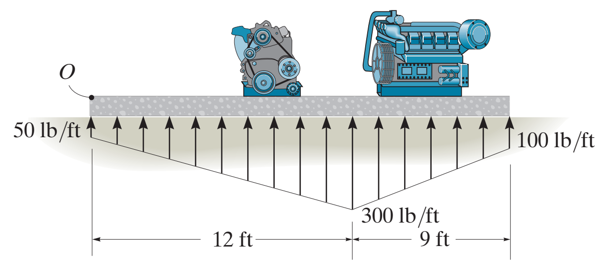


(a)

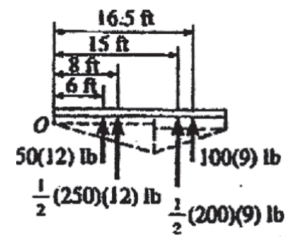
Ans:
$F_R = 12.5$ kN
$d = 1.54$ m

***4–144.**

The distribution of soil loading on the bottom of a building slab is shown. Replace this loading by an equivalent resultant force and specify its location, measured from point O.



SOLUTION



$$+\uparrow F_R = \Sigma F_y; \quad F_R = 50(12) + \tfrac{1}{2}(250)(12) + \tfrac{1}{2}(200)(9) + 100(9)$$

$$= 3900 \text{ lb} = 3.90 \text{ kip} \uparrow \qquad \textbf{Ans.}$$

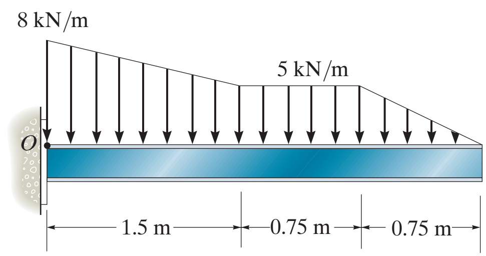
$$\zeta + M_{Ro} = \Sigma M_O; \quad 3900(d) = 50(12)(6) + \tfrac{1}{2}(250)(12)(8) + \tfrac{1}{2}(200)(9)(15) + 100(9)(16.5)$$

$$d = 11.3 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$F_R = 3.90$ kip $\uparrow$
$d = 11.3$ ft

4–145.

Replace the loading by an equivalent resultant force and couple moment acting at point O.



SOLUTION

Equivalent Resultant Force And Couple Moment About Point O. Summing the forces along the y axis by referring to Fig. a,

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad F_R = -\frac{1}{2}(3)(1.5) - 5(2.25) - \frac{1}{2}(5)(0.75)$$

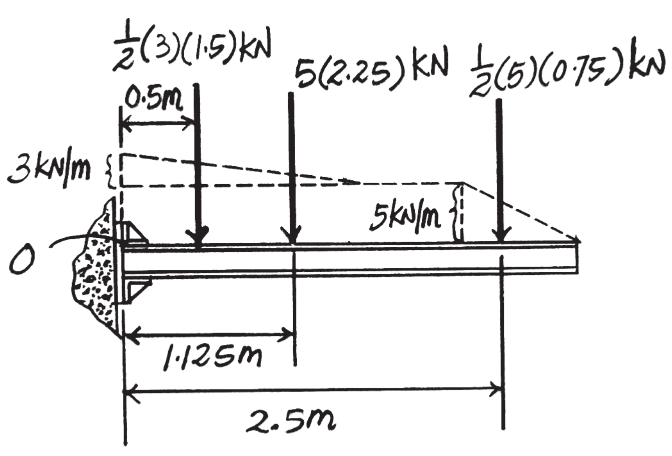
$$= -15.375 \text{ kN} = 15.4 \text{ kN} \downarrow \qquad \textbf{Ans.}$$

Summing the Moment about point O,

$$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad (M_R)_O = -\frac{1}{2}(3)(1.5)(0.5) - 5(2.25)(1.125)$$

$$-\frac{1}{2}(5)(0.75)(2.5)$$

$$= -18.46875 \text{ kN}\cdot\text{m} = 18.5 \text{ kN}\cdot\text{m} \ (\textbf{clockwise}) \quad \textbf{Ans.}$$

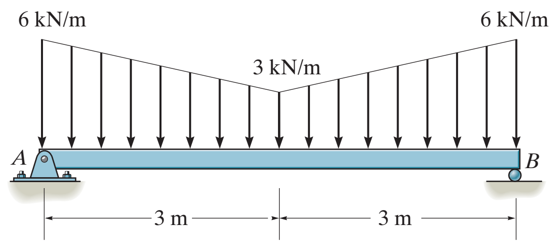


(a)

Ans:
$F_R = 15.4$ kN
$(M_R)_O = 18.5$ kN·m (**clockwise**)

4–146.

Replace the distributed loading by an equivalent resultant force and couple moment acting at point A.



SOLUTION

Equivalent Resultant Force And Couple Moment About Point *A*. Summing the forces along the y axis by referring to Fig. a,

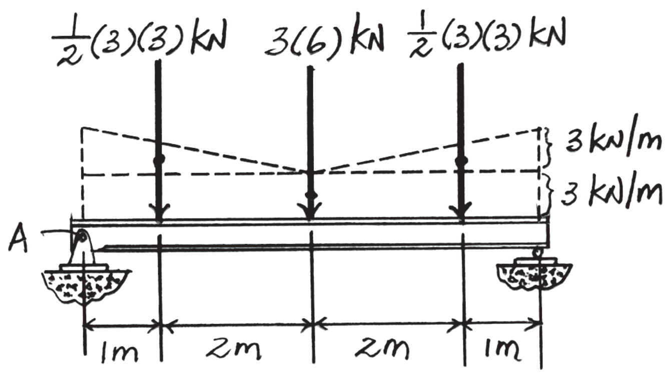
$$+\uparrow (F_R)_y = \Sigma F_y; \quad F_R = -\frac{1}{2}(3)(3) - 3(6) - \frac{1}{2}(3)(3)$$

$$= -27.0 \text{ kN} = 27.0 \text{ kN}\downarrow \qquad \textbf{Ans.}$$

Summing the moments about point A,

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \quad (M_R)_A = -\frac{1}{2}(3)(3)(1) - 3(6)(3) - \frac{1}{2}(3)(3)(5)$$

$$= -81.0 \text{ kN}\cdot\text{m} = 81.0 \text{ kN}\cdot\text{m (\textbf{clockwise})} \qquad \textbf{Ans.}$$

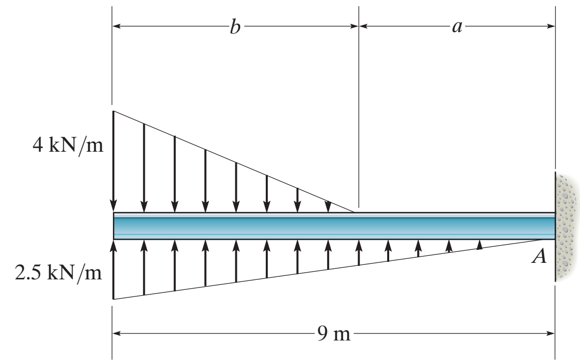


(a)

Ans:
$F_R = 27.0$ kN
$(M_R)_A = 81.0$ kN·m (**clockwise**)

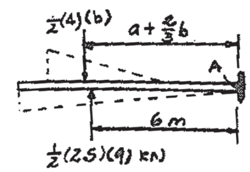
4–147.

Determine the length b of the triangular load and its position a on the beam such that the equivalent resultant force is zero and the resultant couple moment is 8 kN·m clockwise.



SOLUTION

$+\uparrow F_R = 0 = \Sigma F_y; \qquad 0 = \frac{1}{2}(2.5)(9) - \frac{1}{2}(4)(b) \qquad b = 5.625 \text{ m}$ **Ans.**



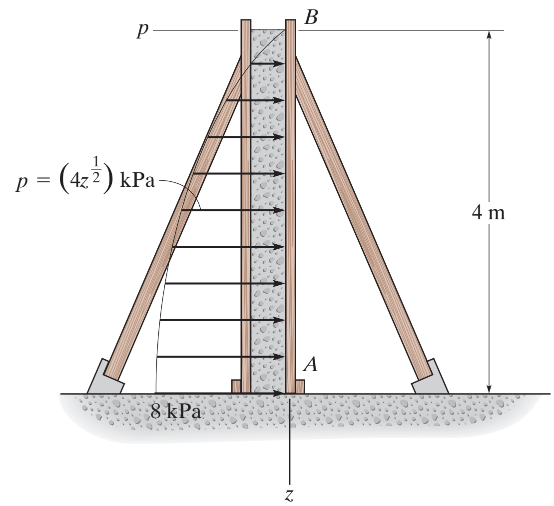
$\circlearrowleft + M_{RA} = \Sigma M_A; \qquad -8 = -\frac{1}{2}(2.5)(9)(6) + \frac{1}{2}(4)(5.625)\left(a + \frac{2}{3}(5.625)\right)$

$a = 1.54 \text{ m}$ **Ans.**

Ans:
$a = 1.54$ m

***4–148.**

The form is used to cast a concrete wall having a width of 5 m. Determine the equivalent resultant force the wet concrete exerts on the form AB if the pressure distribution due to the concrete can be approximated as shown. Specify the location of the resultant force, measured from point B.



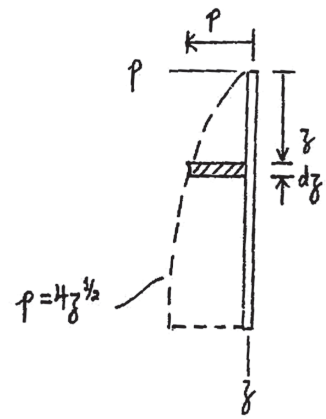
SOLUTION

$$\int dA = \int_0^4 4z^{\frac{1}{2}}\,dz$$

$$= \left[\frac{2}{3}(4)z^{\frac{3}{2}}\right]_0^4$$

$$= 21.33 \text{ kN/m}$$

$$F_R = 21.33(5) = 107 \text{ kN}$$ **Ans.**



$$\int \bar{z}\,dA = \int_0^4 4z^{\frac{3}{2}}\,dz$$

$$= \left[\frac{2}{5}(4)z^{\frac{5}{2}}\right]_0^4$$

$$= 51.2 \text{ kN}$$

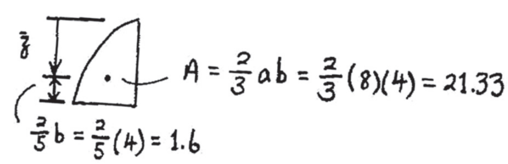
$$\bar{z} = \frac{51.2}{21.33} = 2.40 \text{ m}$$ **Ans.**

Also, from the back of the book,

$$A = \frac{2}{3}ab = \frac{2}{3}(8)(4) = 21.33$$

$$F_R = 21.33\,(5) = 107 \text{ kN}$$ **Ans.**

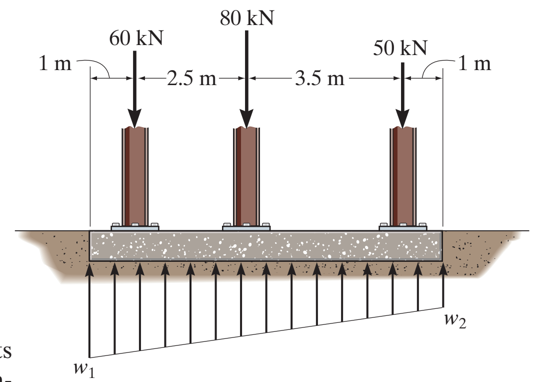
$$\bar{z} = 4 - 1.6 = 2.40 \text{ m}$$ **Ans.**



Ans:
$F_R = 107$ kN
$\bar{z} = 2.40$ m

4–149.

If the soil exerts a trapezoidal distribution of load on the bottom of the footing, determine the intensities w_1 and w_2 of this distribution needed to support the column loadings.



SOLUTION

Loading: The trapezoidal reactive distributed load can be divided into two parts as shown on the free-body diagram of the footing, Fig. *a*. The magnitude and location measured from point A of the resultant force of each part are also indicated in Fig. *a*.

Equations of Equilibrium: Writing the moment equation of equilibrium about point B, we have

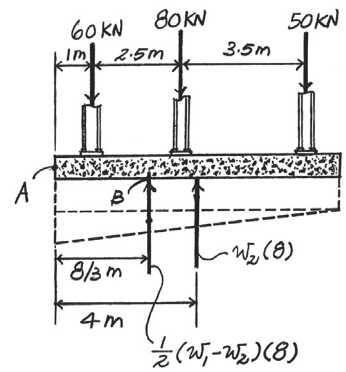
$$\circlearrowleft +\Sigma M_B = 0; \quad w_2(8)\left(4-\frac{8}{3}\right)+60\left(\frac{8}{3}-1\right)-80\left(3.5-\frac{8}{3}\right)-50\left(7-\frac{8}{3}\right)=0$$

$$w_2 = 17.1875 \text{ kN/m} = 17.2 \text{ kN/m} \qquad \textbf{Ans.}$$

Using the result of w_2 and writing the force equation of equilibrium along the y axis, we obtain

$$+\uparrow \Sigma F_y = 0; \quad \frac{1}{2}(w_1-17.1875)8+17.1875(8)-60-80-50=0$$

$$w_1 = 30.3125 \text{ kN/m} = 30.3 \text{ kN/m} \qquad \textbf{Ans.}$$

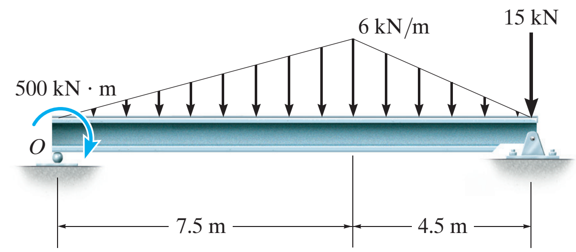


(a)

Ans:
$w_2 = 17.2$ kN/m
$w_1 = 30.3$ kN/m

4–150.

Replace the loading by an equivalent force and couple moment acting at point O.



SOLUTION

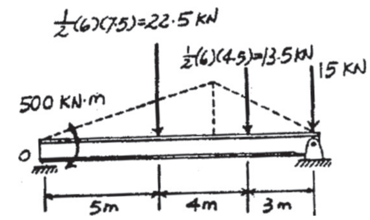
$+\uparrow F_R = \Sigma F_y;$ $\quad F_R = -22.5 - 13.5 - 15.0$

$= -51.0 \text{ kN} = 51.0 \text{ kN}\downarrow$ **Ans.**

$\circlearrowleft + M_{R_o} = \Sigma M_o;$ $\quad M_{R_o} = -500 - 22.5(5) - 13.5(9) - 15(12)$

$= -914 \text{ kN}\cdot\text{m}$

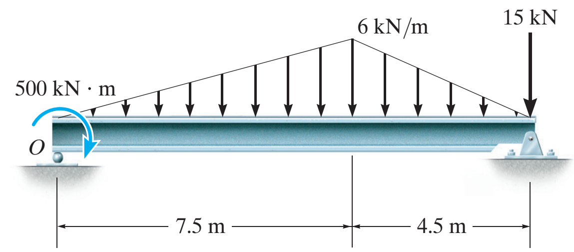
$= 914 \text{ kN}\cdot\text{m}$ **(*Clockwise*)** **Ans.**



Ans:
$F_R = 51.0 \text{ kN} \downarrow$
$M_{R_O} = 914 \text{ kN}\cdot\text{m} \circlearrowright$

4–151.

Replace the loading by a single resultant force, and specify the location of the force measured from point O.



SOLUTION

Equivalent Resultant Force:

$+\uparrow F_R = \Sigma F_y; \qquad -F_R = -22.5 - 13.5 - 15$

$F_R = 51.0 \text{ kN} \downarrow$ **Ans.**

Location of Equivalent Resultant Force:

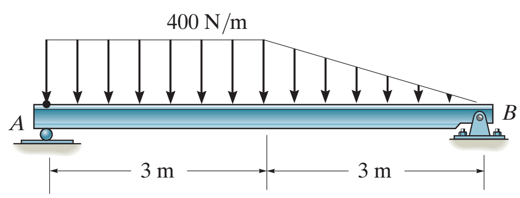
$\circlearrowleft + (M_R)_O = \Sigma M_O; \qquad -51.0(d) = -500 - 22.5(5) - 13.5(9) - 15(12)$

$d = 17.9 \text{ m}$ **Ans.**

Ans:
$F_R = 51.0 \text{ kN} \downarrow$
$d = 17.9 \text{ m}$

***4–152.**

Replace the loading by an equivalent resultant force and couple moment acting at point A.



SOLUTION

Equivalent Resultant Force And Couple Moment At Point *A*. Summing the forces along the y axis by referring to Fig. a,

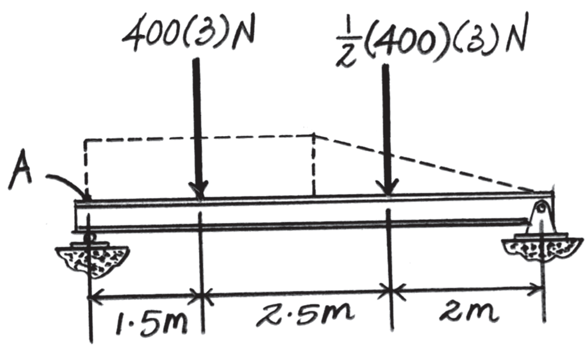
$$+\uparrow (F_R)_y = \Sigma F_y; \quad F_R = -400(3) - \frac{1}{2}(400)(3)$$

$$= -1800 \text{ N} = 1.80 \text{ kN}\downarrow \qquad \textbf{Ans.}$$

Summing the moment about point A,

$$\zeta+(M_R)_A = \Sigma M_A; \quad (M_R)_A = -400(3)(1.5) - \frac{1}{2}(400)(3)(4)$$

$$= -4200 \text{ N}\cdot\text{m} = 4.20 \text{ kN}\cdot\text{m (\textbf{clockwise})} \qquad \textbf{Ans.}$$

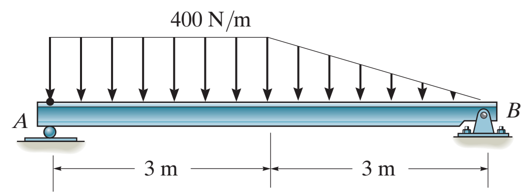


(a)

Ans:
$F_R = 1.80$ kN
$(M_R)_A = 4.20$ kN·m (**clockwise**)

4–153.

Replace the loading by a single resultant force, and specify its location on the beam measured from point A.



SOLUTION

Equivalent Resultant Force. Summing the forces along the y axis by referring to Fig. a,

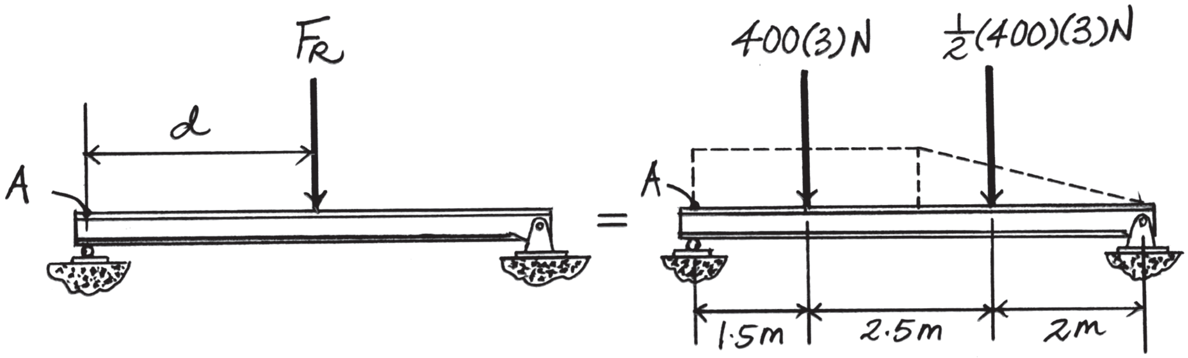
$$+\uparrow (F_R)_y = \Sigma F_y; \quad -F_R = -400(3) - \frac{1}{2}(400)(3)$$

$$F_R = 1800 \text{ N} = 1.80 \text{ kN} \downarrow \qquad \textbf{Ans.}$$

Location of Resultant Force. Summing the moment about point A by referring to Fig. a,

$$\circlearrowleft + (M_R)_A = \Sigma M_A; \quad -1800\,d = -400(3)(1.5) - \frac{1}{2}(400)(3)(4)$$

$$d = 2.333 \text{ m} = 2.33 \text{ m} \qquad \textbf{Ans.}$$

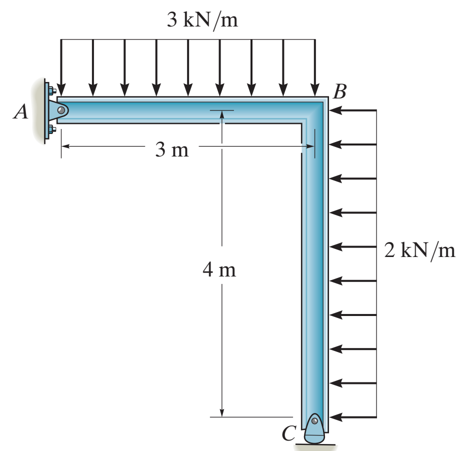


(a)

Ans:
$F_R = 1.80$ kN
$d = 2.33$ m

4–154.

Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects a horizontal line along member AB, measured from A.



SOLUTION

Equivalent Resultant Force. Summing the forces along the x and y axes by referring to Fig. a,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = -2(4) = -8 \text{ kN} = 8 \text{ kN} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -3(3) = -9 \text{ kN} = 9 \text{ kN} \downarrow$$

Then

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{8^2 + 9^2} = 12.04 \text{ kN} = 12.0 \text{ kN} \qquad \textbf{Ans.}$$

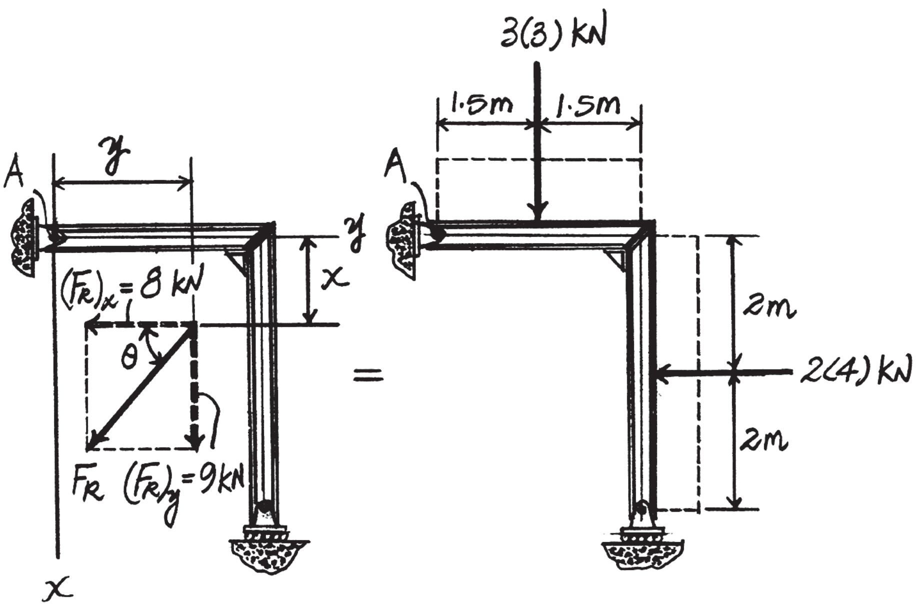
And

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{9}{8}\right) = 48.37° = 48.4° \qquad \textbf{Ans.}$$

Location of the Resultant Force. Summing the moments about point A, by referring to Fig. a,

$$\circlearrowleft +(M_R)_A = \Sigma M_A; \qquad -8x - 9y = -3(3)(1.5) - 2(4)(2)$$

$$8x + 9y = 29.5 \qquad (1)$$



(a)

4–154. Continued

Along AB, $x = 0$. Then Eq (1) becomes

$$8(0) + 9y = 29.5$$

$$y = 3.278 \text{ m}$$

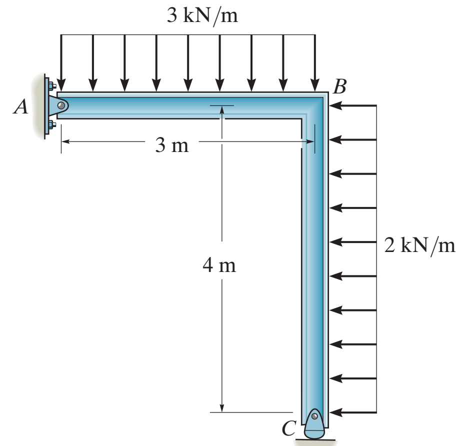
Thus, the inter section point of line of action of $\mathbf{F}_R$ on AB measured to the right from point A is

$$d = y = 3.28 \text{ m}$$ **Ans.**

Ans:
$F_R = 12.0$ kN
$\theta = 48.4°$
$d = 3.28$ m

4–155.

Replace the distributed loading by an equivalent resultant force and specify where its line of action intersects a vertical line along member BC, measured from C.



SOLUTION

Equivalent Resultant Force. Summing the forces along the x and y axes by referring to Fig. a,

$$\xrightarrow{+} (F_R)_x = \Sigma F_x; \qquad (F_R)_x = -2(4) = -8 \text{ kN} = 8 \text{ kN} \leftarrow$$

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad (F_R)_y = -3(3) = -9 \text{ kN} = 9 \text{ kN} \downarrow$$

Then

$$F_R = \sqrt{(F_R)_x^2 + (F_R)_y^2} = \sqrt{8^2 + 9^2} = 12.04 \text{ kN} = 12.0 \text{ kN} \qquad \textbf{Ans.}$$

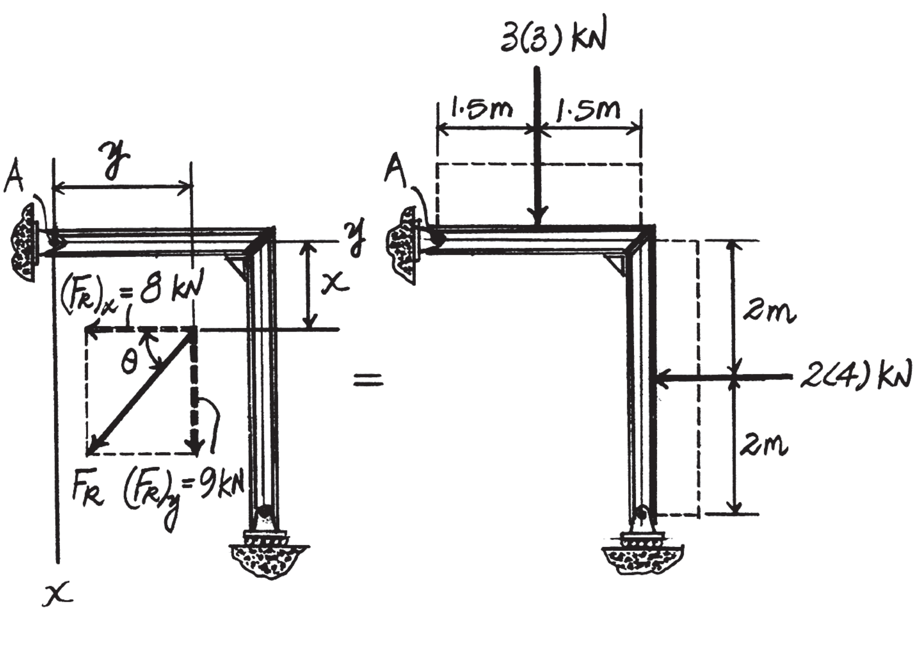
And

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{9}{8}\right) = 48.37° = 48.4° \qquad \textbf{Ans.}$$

Location of the Resultant Force. Summing the moments about point A, by referring to Fig. a,

$$\zeta+(M_R)_A = \Sigma M_A; \qquad -8x - 9y = -3(3)(1.5) - 2(4)(2)$$

$$8x + 9y = 29.5 \qquad (1)$$



(a)

4–155. Continued

Along BC, $y = 3$ m. Then Eq (1) becomes

$$8x + 9(3) = 29.5$$

$$x = 0.3125 \text{ m}$$

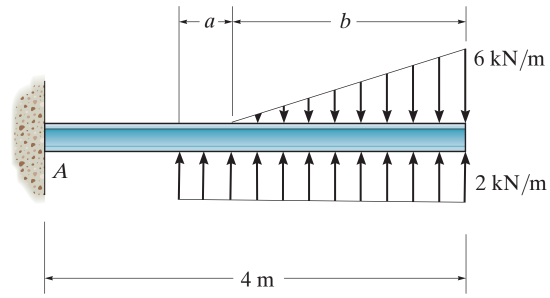
Thus, the intersection point of line of action of $\mathbf{F}_R$ on BC measured upward from point C is

$$d = 4 - x = 4 - 0.3125 = 3.6875 \text{ m} = 3.69 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 12.0$ kN
$\theta = 48.4°$
$d = 3.69$ m

***4–156.**

Determine the length b of the triangular load and its position a on the beam such that the equivalent resultant force is zero and the resultant couple moment is 8 kN · m clockwise.



SOLUTION

Equivalent Resultant Force And Couple Moment At Point *A*. Summing the forces along the y axis by referring to Fig. a, with the requirement that $\mathbf{F}_R = 0$,

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad 0 = 2(a+b) - \frac{1}{2}(6)(b)$$

$$2a - b = 0 \tag{1}$$

Summing the moments about point A, with the requirement that $(M_R)_A = 8\text{ kN}\cdot\text{m}$,

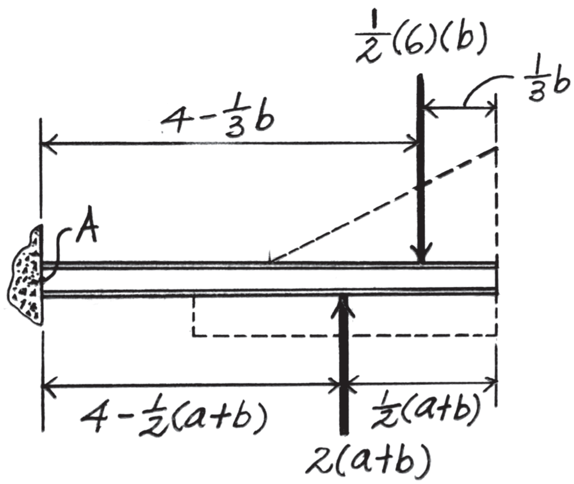
$$\circlearrowleft +(M_R)_A = \Sigma M_A; \; -8 = 2(a+b)\left[4 - \frac{1}{2}(a+b)\right] - \frac{1}{2}(6)(b)\left(4 - \frac{1}{3}b\right)$$

$$-8 = 8a - 4b - 2ab - a^2 \tag{2}$$

Solving Eqs (1) and (2),

$$a = 1.264\text{ m} = 1.26\text{ m} \qquad \textbf{Ans.}$$

$$b = 2.530\text{ m} = 2.53\text{ m} \qquad \textbf{Ans.}$$

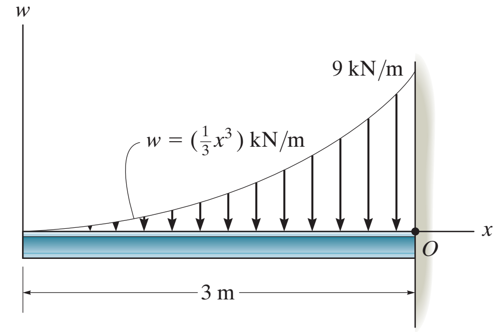


(a)

Ans:
$a = 1.26$ m
$b = 2.53$ m

4–157.

Determine the equivalent resultant force and couple moment at point O.



SOLUTION

Equivalent Resultant Force And Couple Moment About Point *O*. The differential force indicated in Fig. *a* is $dF_R = w\,dx = \dfrac{1}{3}x^3dx$. Thus, summing the forces along the y axis,

$$+\uparrow (F_R)_y = \Sigma F_y; \qquad F_R = -\int dF_R = -\int_0^{3\text{m}} \frac{1}{3}x^3dx$$

$$= -\frac{1}{12}x^4 \Big|_0^{3\text{m}}$$

$$= -6.75\text{ kN} = 6.75\text{ kN}\downarrow \qquad \textbf{Ans.}$$

Summing the moments about point O,

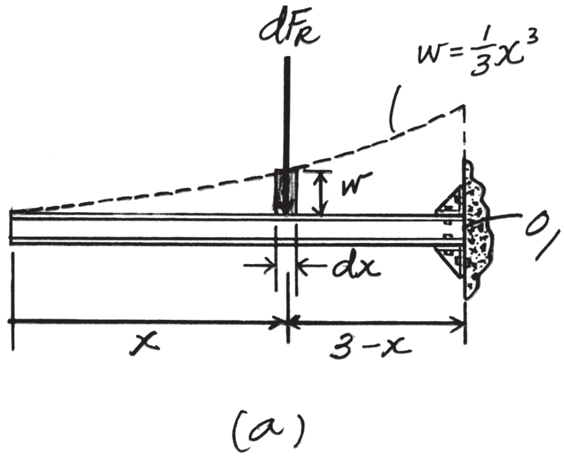
$$\circlearrowleft+(M_R)_O = \Sigma M_O; \quad (M_R)_O = \int (3-x)dF_R$$

$$= \int_0^{3\text{m}} (3-x)\left(\frac{1}{3}x^3dx\right)$$

$$= \int_0^{3\text{m}} \left(x^3 - \frac{1}{3}x^4\right)dx$$

$$= \left(\frac{x^4}{4} - \frac{1}{15}x^5\right)\Big|_0^{3\text{m}}$$

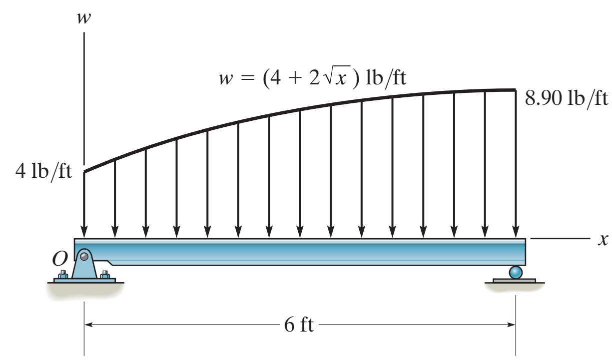
$$= 4.05\text{ kN}\cdot\text{m (\textbf{counterclockwise})} \qquad \textbf{Ans.}$$



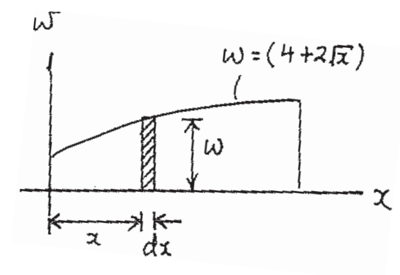
Ans:
$F_R = 6.75$ kN$\downarrow$
$(M_R)_O = 4.05$ kN · m (**counterclockwise**)

4–158.

Determine the magnitude of the equivalent resultant force and its location, measured from point O.



SOLUTION



$dA = w dx$

$$F_R = \int dA = \int_0^6 (4 + 2\sqrt{x})\, dx$$

$$= \left[4x + \frac{4}{3} x^{\frac{3}{2}} \right]_0^6$$

$$F_R = 43.6 \text{ lb}$$ **Ans.**

$$\int \bar{x} dF = \int_0^6 (4x + 2x^{\frac{3}{2}})\, dx$$

$$= \left[2x^2 + \frac{4}{5} x^{\frac{5}{2}} \right]_0^6$$

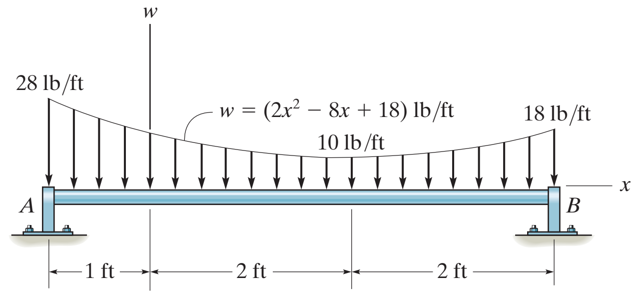
$$= 142.5 \text{ lb} \cdot \text{ft}$$

$$\bar{x} = \frac{142.5}{43.6} = 3.27 \text{ ft}$$ **Ans.**

Ans:
$F_R = 43.6$ lb
$\bar{x} = 3.27$ ft

4–159.

The distributed load acts on the shaft as shown. Determine the magnitude of the equivalent resultant force and specify its location, measured from the support, A.



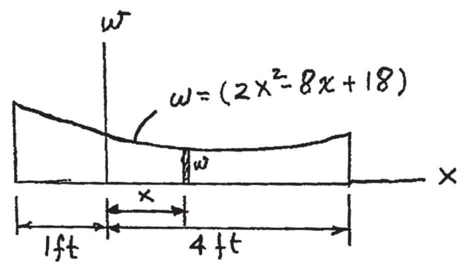
SOLUTION

$$F_R = \int_{-1}^{4} \left(2x^2 - 8x + 18\right) dx = \frac{2}{3}x^3 - \frac{8x^2}{2} + 18x \Big|_{-1}^{4} = 73.33 = 73.3 \text{ lb} \qquad \textbf{Ans.}$$

$$\int x\, dF = \int_{-1}^{4} \left(2x^3 - 8x^2 + 18x\right) dx = \frac{2}{4}x^4 - \frac{8}{3}x^3 + \frac{18}{2}x^2 \Big|_{-1}^{4} = 89.166 \text{ lb} \cdot \text{ft}$$

$$x = \frac{89.166}{73.3} = 1.22 \text{ ft}$$

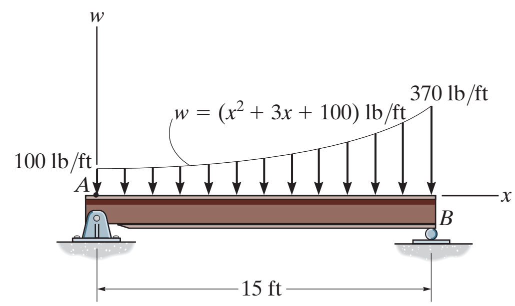
$$d = 1 + 1.22 = 2.22 \text{ ft} \qquad \textbf{Ans.}$$



Ans:
$d = 2.22$ ft

***4–160.**

Replace the distributed loading with an equivalent resultant force, and specify its location on the beam measured from point A.



SOLUTION

Resultant: The magnitude of the differential force $d\mathbf{F}_R$ is equal to the area of the element shown shaded in Fig. a. Thus,

$$dF_R = w\,dx = \left(x^2 + 3x + 100\right)dx$$

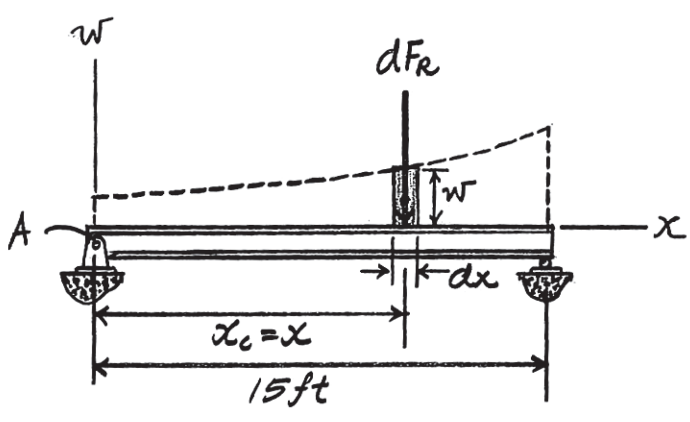
Integrating $d\mathbf{F}_R$ over the entire length of the beam gives the resultant force $\mathbf{F}_R$.

$$+\downarrow \qquad F_R = \int_L dF_R = \int_0^L \left(x^2 + 3x + 100\right)dx = \left(\frac{x^3}{3} + \frac{3x^2}{2} + 100x\right)\Bigg|_0^{15\text{ ft}}$$

$$= 2962.5\text{ lb} = 2.96\text{ kip} \qquad \textbf{Ans.}$$

Location: The location of $d\mathbf{F}_R$ on the beam is $x_c = x$ measured from point A. Thus, the location $\bar{x}$ of $\mathbf{F}_R$ measured from point A is given by

$$\bar{x} = \frac{\int_L x_c dF_R}{\int_L dF_R} = \frac{\int_0^{15\text{ ft}} x\left(x^2 + 3x + 100\right)dx}{2962.5} = \frac{\left(\frac{x^4}{4} + x^3 + 50x^2\right)\Big|_0^{15\text{ ft}}}{2962.5} = 9.21\text{ ft} \qquad \textbf{Ans.}$$

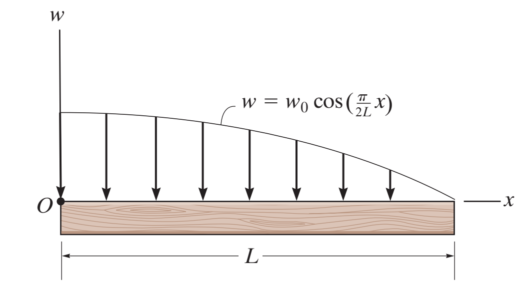


(a)

Ans:
$F_R = 2.96$ kip
$\bar{x} = 9.21$ ft

4–161.

Replace the loading by an equivalent resultant force and couple moment acting at point O.



SOLUTION

Equivalent Resultant Force And Couple Moment About Point *O*. The differential force indicated in Fig. *a* is $dF_R = w\,dx = \left(w_0 \cos \dfrac{\pi}{2L} x\right) dx$. Thus, summing the forces along the y axis,

$$+\uparrow (F_R)_y = \Sigma F_y; \quad F_R = -\int dF_R = -\int_O^L \left(w_0 \cos \frac{\pi}{2L} x\right) dx$$

$$= -\frac{2Lw_0}{\pi}\left(\sin \frac{\pi}{2L} x\right)\Big|_O^L$$

$$= -\frac{2Lw_0}{\pi} = \frac{2Lw_0}{\pi} \downarrow \qquad \textbf{Ans.}$$

Summing the moments about point O,

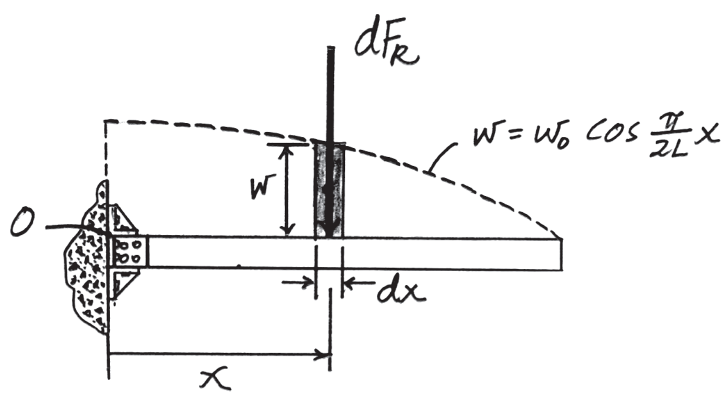
$$\circlearrowleft + (M_R)_O = \Sigma M_O; \quad (M_R)_O = -\int x\,dF_R$$

$$= -\int_O^L x\left(w_0 \cos \frac{\pi}{2L} x\right) dx$$

$$= -w_0\left(\frac{2L}{\pi} x \sin \frac{\pi}{2L} x + \frac{4L^2}{\pi^2} \cos \frac{\pi}{2L} x\right)\Big|_O^L$$

$$= -\left(\frac{2\pi - 4}{\pi^2}\right) w_0 L^2$$

$$= \left(\frac{2\pi - 4}{\pi^2}\right) w_0 L^2 \ \textbf{(clockwise)} \qquad \textbf{Ans.}$$



(a)

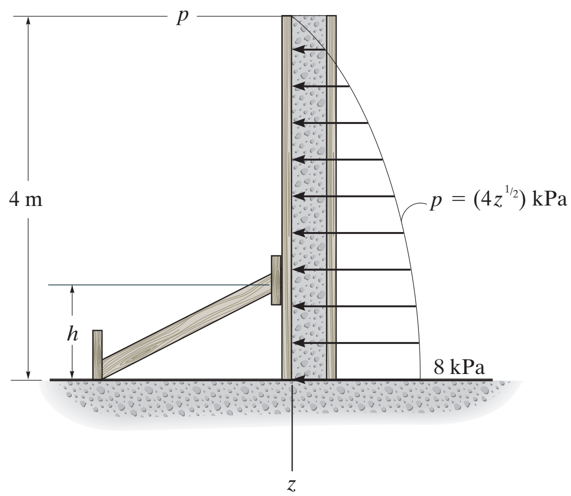
Ans:

$F_R = \dfrac{2Lw_0}{\pi}$

$(M_R)_O = \left(\dfrac{2\pi - 4}{\pi^2}\right) w_0 L^2$ **(clockwise)**

4–162.

Wet concrete exerts a pressure distribution along the wall of the form. Determine the resultant force of this distribution and specify the height h where the bracing strut should be placed so that it lies through the line of action of the resultant force. The wall has a width of 5 m.



SOLUTION

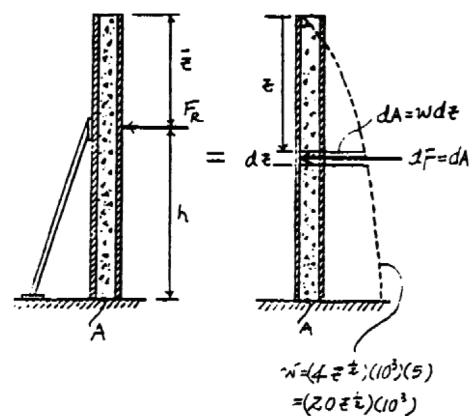
Equivalent Resultant Force:

$$\xrightarrow{+} F_R = \Sigma F_x; \qquad -F_R = -\int dA = -\int_0^z w\,dz$$

$$F_R = \int_0^{4\text{ m}} \left(20z^{\frac{1}{2}}\right)\left(10^3\right) dz$$

$$= 106.67\left(10^3\right) \text{ N} = 107 \text{ kN} \leftarrow$$

Ans.



Location of Equivalent Resultant Force:

$$\bar{z} = \frac{\int_A z\,dA}{\int_A dA} = \frac{\int_0^z zw\,dz}{\int_0^z w\,dz}$$

$$= \frac{\int_0^{4\text{ m}} z\left[\left(20z^{\frac{1}{2}}\right)(10^3)\right]dz}{\int_0^{4\text{ m}} \left(20z^{\frac{1}{2}}\right)(10^3)dz}$$

$$= \frac{\int_0^{4\text{ m}} \left[\left(20z^{\frac{3}{2}}\right)(10^3)\right]dz}{\int_0^{4\text{ m}} \left(20z^{\frac{1}{2}}\right)(10^3)dz}$$

$$= 2.40 \text{ m}$$

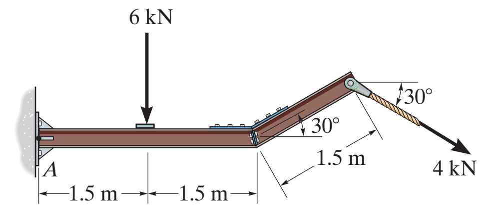
Thus, $\quad h = 4 - \bar{z} = 4 - 2.40 = 1.60$ m

Ans.

Ans:
$F_R = 107$ kN
$h = 1.60$ m

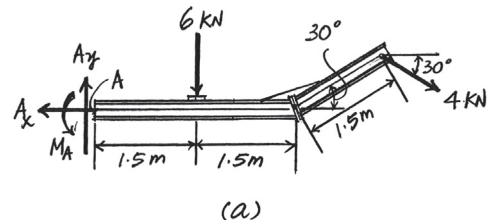
5–10.

Determine the components of the support reactions at the fixed support A on the cantilevered beam.



SOLUTION

Equations of Equilibrium: From the free-body diagram of the cantilever beam, Fig. *a*, A_x, A_y, and M_A can be obtained by writing the moment equation of equilibrium about point A.



(a)

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 4\cos 30° - A_x = 0$

$A_x = 3.46 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad A_y - 6 - 4\sin 30° = 0$

$A_y = 8 \text{ kN}$ **Ans.**

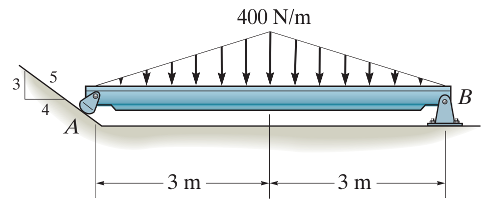
$\circlearrowleft + \Sigma M_A = 0; M_A - 6(1.5) - 4\cos 30°\,(1.5\sin 30°) - 4\sin 30°(3 + 1.5\cos 30°) = 0$

$M_A = 20.2 \text{ kN}\cdot\text{m}$ **Ans.**

Ans:
$A_x = 3.46$ kN
$A_y = 8$ kN
$M_A = 20.2$ kN·m

5–11.

Determine the reactions at the supports.



SOLUTION

Equations of Equilibrium. $\mathbf{N}_A$ and $\mathbf{B}_y$ can be determined directly by writing the moment equations of equilibrium about points B and A, respectively, by referring to the beam's *FBD* shown in Fig. *a*.

$$\circlearrowleft + \Sigma M_B = 0; \qquad \frac{1}{2}(400)(6)(3) - N_A\left(\frac{4}{5}\right)(6) = 0$$

$$N_A = 750 \text{ N} \qquad \textbf{Ans.}$$

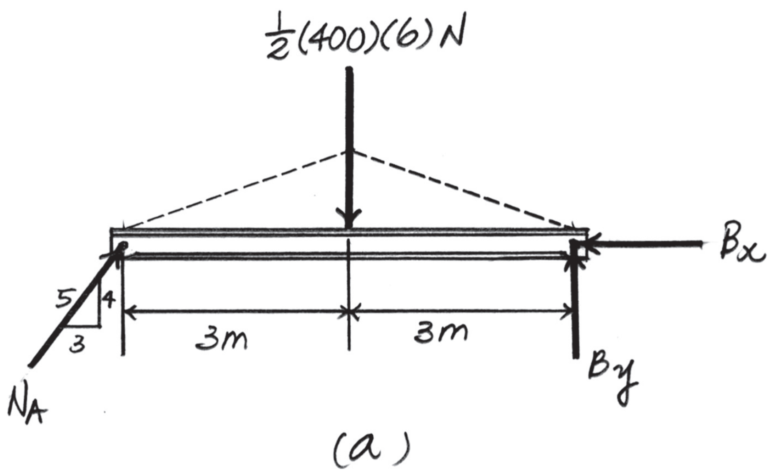
$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(6) - \frac{1}{2}(400)(6)(3) = 0$$

$$B_y = 600 \text{ N} \qquad \textbf{Ans.}$$

Using the result of $\mathbf{N}_A$ to write the force equation of equilibrium along the x axis,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 750\left(\frac{3}{5}\right) - B_x = 0$$

$$B_x = 450 \text{ N} \qquad \textbf{Ans.}$$

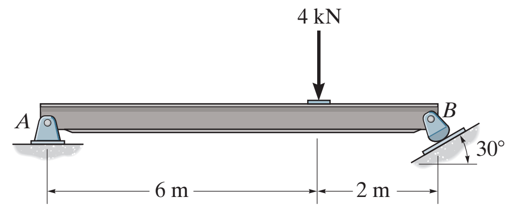


(a)

Ans:
$N_A = 750$ N
$B_y = 600$ N
$B_x = 450$ N

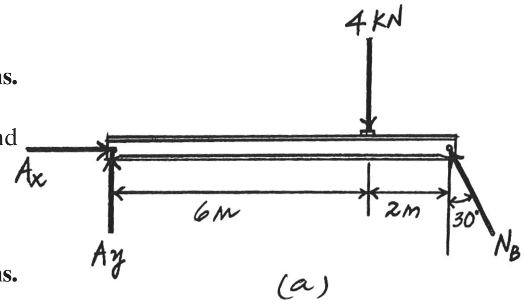
***5–12.**

Determine the horizontal and vertical components of reaction at the pin A and the reaction of the rocker B on the beam.



SOLUTION

Equations of Equilibrium: From the free-body diagram of the beam, Fig. a, N_B can be obtained by writing the moment equation of equilibrium about point A.



$\circlearrowleft \Sigma M_A = 0;$ $\quad N_B \cos 30°(8) - 4(6) = 0$

$N_B = 3.464 \text{ kN} = 3.46 \text{ kN}$ **Ans.**

Using this result and writing the force equations of equilibrium along the x and y axes, we have

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x - 3.464 \sin 30° = 0$

$A_x = 1.73 \text{ kN}$ **Ans.**

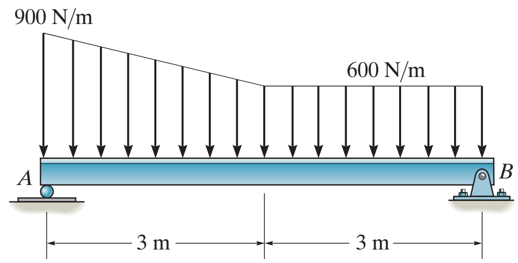
$+\uparrow \Sigma F_y = 0;$ $\quad A_y + 3.464 \cos 30° - 4 = 0$

$A_y = 1.00 \text{ kN}$ **Ans.**

Ans:
$N_B = 3.46$ kN
$A_x = 1.73$ kN
$A_y = 1.00$ kN

5–13.

Determine the reactions at the supports.



SOLUTION

Equations of Equilibrium. $\mathbf{N}_A$ and $\mathbf{B}_y$ can be determined directly by writing the moment equations of equilibrium about points B and A, respectively, by referring to the *FBD* of the beam shown in Fig. *a*.

$$\circlearrowleft+\Sigma M_B = 0; \qquad 600(6)(3) + \frac{1}{2}(300)(3)(5) - N_A(6) = 0$$

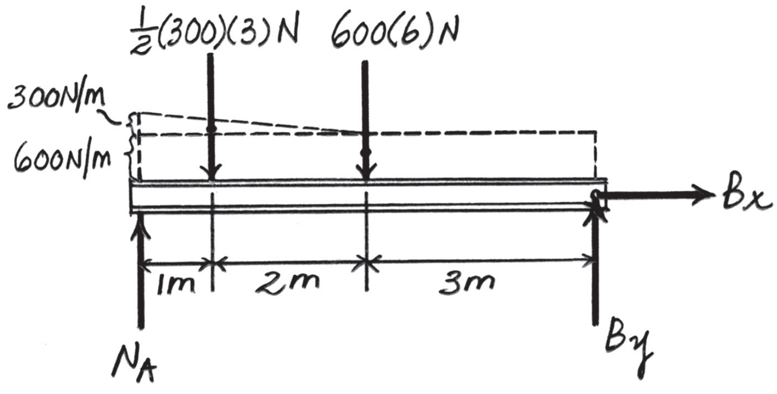
$$N_A = 2175 \text{ N} = 2.175 \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft+\Sigma M_A = 0; \qquad B_y(6) - \frac{1}{2}(300)(3)(1) - 600(6)(3) = 0$$

$$B_y = 1875 \text{ N} = 1.875 \text{ kN} \qquad \textbf{Ans.}$$

Also, $\mathbf{B}_x$ can be determined directly by writing the force equation of equilibrium along the x axis.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad B_x = 0 \qquad \textbf{Ans.}$$

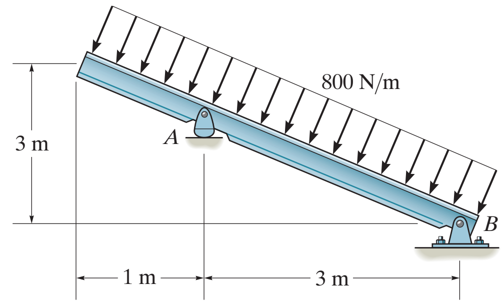


(a)

Ans:
$N_A = 2.175$ kN
$B_y = 1.875$ kN
$B_x = 0$

5–14.

Determine the reactions at the supports.



SOLUTION

Equations of Equilibrium. $\mathbf{N}_A$ can be determined directly by writing the moment equation of equilibrium about point B by referring to the *FBD* of the beam shown in Fig. *a*.

$$\circlearrowleft + \Sigma M_B = 0; \qquad 800(5)(2.5) - N_A(3) = 0$$

$$N_A = 3333.33 \text{ N} = 3.33 \text{ kN} \qquad \textbf{Ans.}$$

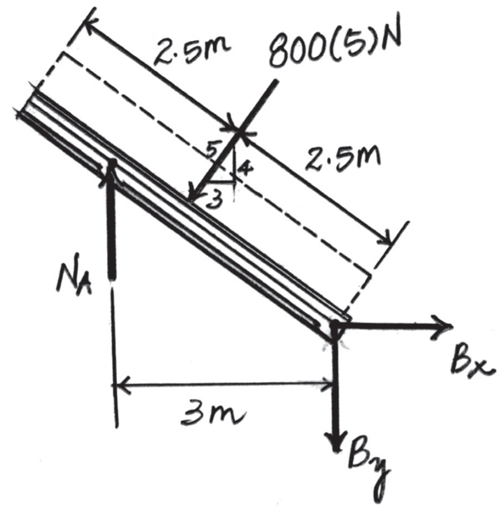
Using this result to write the force equations of equilibrium along the x and y axes,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x - 800(5)\left(\frac{3}{5}\right) = 0$$

$$B_x = 2400 \text{ N} = 2.40 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad 3333.33 - 800(5)\left(\frac{4}{5}\right) - B_y = 0$$

$$B_y = 133.33 \text{ N} = 133 \text{ N} \qquad \textbf{Ans.}$$

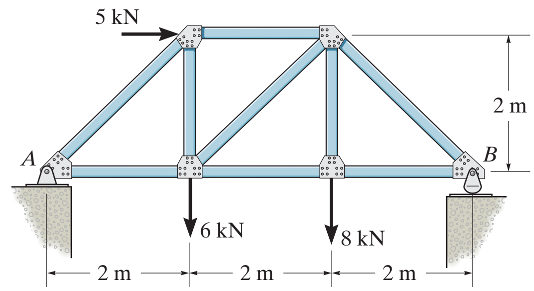


(a)

Ans:
$N_A = 3.33$ kN
$B_x = 2.40$ kN
$B_y = 133$ N

5–15.

Determine the reactions at the supports.



SOLUTION

Equations of Equilibrium. $\mathbf{A}_y$ and $\mathbf{N}_B$ can be determined by writing the moment equations of equilibrium about points B and A, respectively, by referring to the *FBD* of the truss shown in Fig. *a*.

$\zeta+\Sigma M_B = 0; \qquad 8(2) + 6(4) - 5(2) - A_y(6) = 0$

$$A_y = 5.00 \text{ kN} \qquad \textbf{Ans.}$$

$\zeta+\Sigma M_A = 0; \qquad N_B(6) - 8(4) - 6(2) - 5(2) = 0$

$$N_B = 9.00 \text{ kN} \qquad \textbf{Ans.}$$

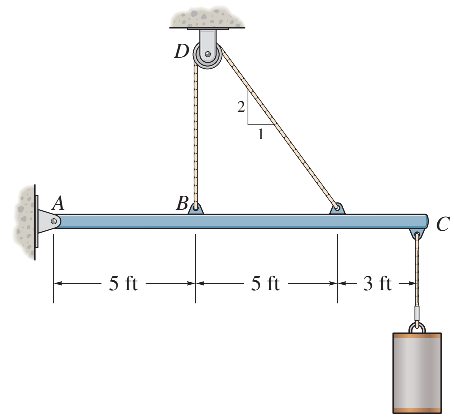
Also, $\mathbf{A}_x$ can be determined directly by writing the force equation of equilibrium along x axis.

$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 5 - A_x = 0 \qquad A_x = 5.00 \text{ kN} \qquad$ **Ans.**

Ans:
$A_y = 5.00$ kN
$N_B = 9.00$ kN
$A_x = 5.00$ kN

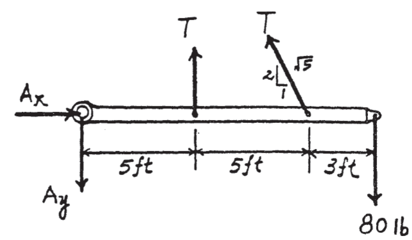
***5–16.**

Determine the tension in the cable and the horizontal and vertical components of reaction of the pin A. The pulley at D is frictionless and the cylinder weighs 80 lb.



SOLUTION

Equations of Equilibrium: The tension force developed in the cable is the same throughout the whole cable. The force in the cable can be obtained directly by summing moments about point A.



$$\circlearrowleft + \Sigma M_A = 0; \quad T(5) + T\left(\frac{2}{\sqrt{5}}\right)(10) - 80(13) = 0$$

$$T = 74.583 \text{ lb} = 74.6 \text{ lb} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 74.583\left(\frac{1}{\sqrt{5}}\right) = 0$$

$$A_x = 33.4 \text{ lb} \qquad \textbf{Ans.}$$

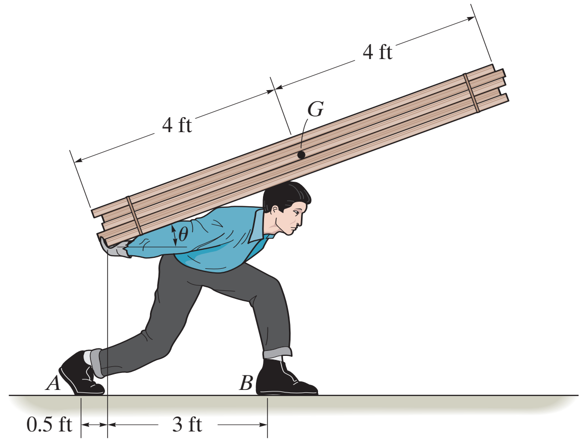
$$+\uparrow \Sigma F_y = 0; \quad 74.583 + 74.583\left(\frac{2}{\sqrt{5}}\right) - 80 - B_y = 0$$

$$A_y = 61.3 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$T = 74.6$ lb
$A_x = 33.4$ lb
$A_y = 61.3$ lb

5–17.

The man attempts to support the load of boards having a weight W and a center of gravity at G. If he is standing on a smooth floor, determine the smallest angle θ at which he can hold them up in the position shown. Neglect his weight.



SOLUTION

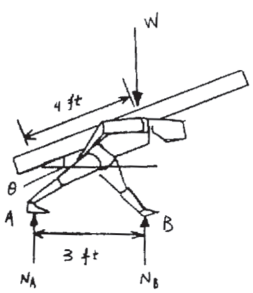
$\circlearrowleft + \Sigma M_B = 0; \qquad -N_A\,(3.5) + W(3 - 4\cos\theta) = 0$

As θ becomes smaller, $N_A \rightarrow 0$ so that,

$$W(3 - 4\cos\theta) = 0$$

$$\theta = 41.4^\circ$$

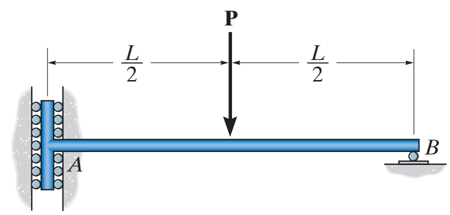
Ans.



Ans:
$\theta = 41.4^\circ$

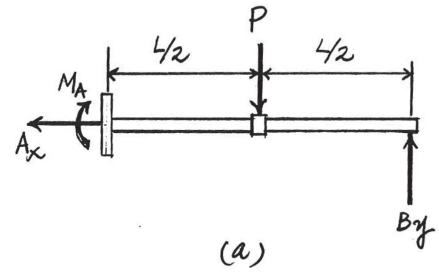
5–18.

Determine the components of reaction at the supports A and B on the rod.



SOLUTION

Equations of Equilibrium: Since the roller at A offers no resistance to vertical movement, the vertical component of reaction at support A is equal to zero. From the free-body diagram, A_x, B_y, and M_A can be obtained by writing the force equations of equilibrium along the x and y axes and the moment equation of equilibrium about point B, respectively.



$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad B_y - P = 0$

$$B_y = P$$ **Ans.**

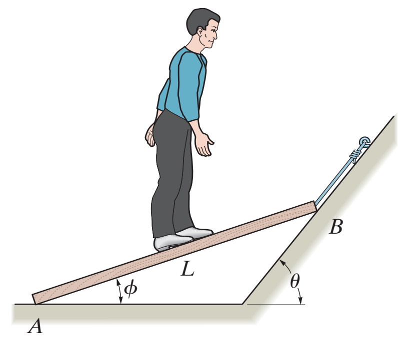
$\circlearrowleft + \Sigma M_B = 0;$ $\quad P\left(\frac{L}{2}\right) - M_A = 0$

$$M_A = \frac{PL}{2}$$ **Ans.**

Ans:
$A_x = 0$
$B_y = P$
$M_A = \frac{PL}{2}$

5–19.

The man has a weight W and stands at the center of the plank. If the planes at A and B are smooth, determine the tension in the cord in terms of W and θ.

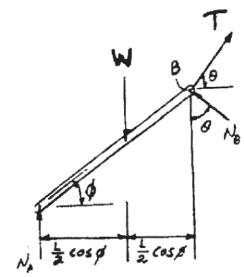


SOLUTION

$\zeta + \Sigma M_B = 0;$ $\quad W\left(\dfrac{L}{2}\cos\phi\right) - N_A(L\cos\phi) = 0 \qquad N_A = \dfrac{W}{2}$

$\stackrel{+}{\rightarrow} \Sigma F_x = 0;$ $\quad T\cos\theta - N_B\sin\theta = 0$ **(1)**

$+\uparrow \Sigma F_y = 0;$ $\quad T\sin\theta + N_B\cos\theta + \dfrac{W}{2} - W = 0$ **(2)**



Solving Eqs. (1) and (2) yields:

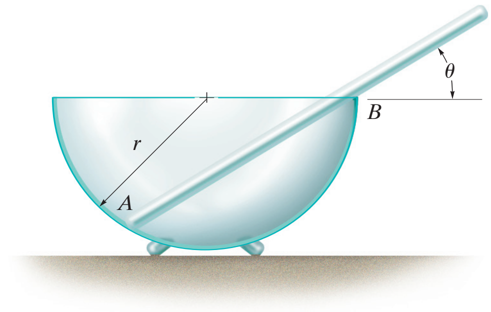
$$T = \frac{W}{2}\sin\theta$$ **Ans.**

$$N_B = \frac{W}{2}\cos\theta$$

Ans:
$T = \dfrac{W}{2}\sin\theta$

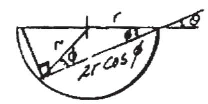
***5–20.**

A uniform glass rod having a length L is placed in the smooth hemispherical bowl having a radius r. Determine the angle of inclination θ for equilibrium.



SOLUTION

By observation $\phi = \theta$.



Equilibrium:

$$\circlearrowleft + \Sigma M_A = 0; \qquad N_B(2r\cos\theta) - W\left(\frac{L}{2}\cos\theta\right) = 0 \qquad N_B = \frac{WL}{4r}$$

$$+\nearrow \Sigma F_x = 0; \qquad N_A\cos\theta - W\sin\theta = 0 \qquad N_A = W\tan\theta$$

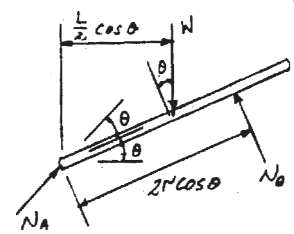
$$+\nwarrow \Sigma F_y = 0; \qquad (W\tan\theta)\sin\theta + \frac{WL}{4r} - W\cos\theta = 0$$

$$\sin^2\theta - \cos^2\theta + \frac{L}{4r}\cos\theta = 0$$

$$(1 - \cos^2\theta) - \cos^2\theta + \frac{L}{4r}\cos\theta = 0$$

$$2\cos^2\theta - \frac{L}{4r}\cos\theta - 1 = 0$$

$$\cos\theta = \frac{L \pm \sqrt{L^2 + 128r^2}}{16r}$$



Take the positive root

$$\cos\theta = \frac{L + \sqrt{L^2 + 128r^2}}{16r}$$

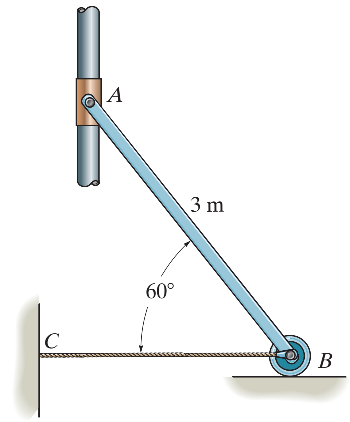
$$\theta = \cos^{-1}\left(\frac{L + \sqrt{L^2 + 128r^2}}{16r}\right) \qquad \textbf{Ans.}$$

Ans:

$$\theta = \cos^{-1}\left(\frac{L + \sqrt{L^2 + 12r^2}}{16r}\right)$$

5–21.

The uniform rod AB has a mass of 40 kg. Determine the force in the cable when the rod is in the position shown. There is a smooth collar at A.

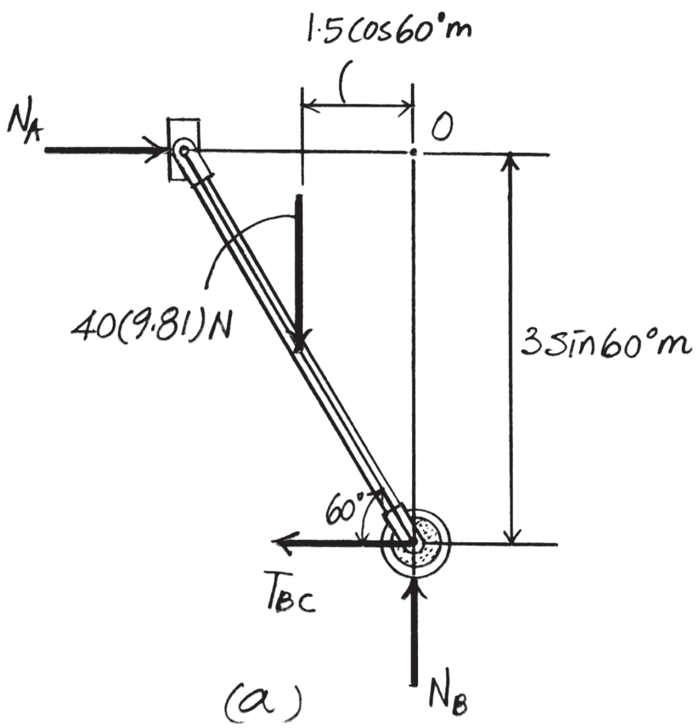


SOLUTION

Equations of Equilibrium. $\mathbf{T}_{BC}$ can be determined by writing the moment equation of equilibrium about point O by referring to the *FBD* of the rod shown in Fig. *a*.

$\circlearrowleft + \Sigma M_O = 0; \qquad 40(9.81)(1.5 \cos 600°) - T_{BC}(3 \sin 60°) = 0$

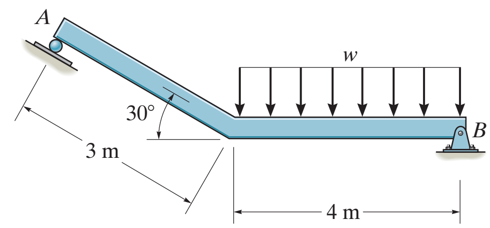
$$T_{BC} = 113.28 \text{ N} = 113 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$T_{BC} = 113$ N

5–22.

If the intensity of the distributed load acting on the beam is $w = 3$ kN/m, determine the reactions at the roller A and pin B.



SOLUTION

Equations of Equilibrium. $\mathbf{N}_A$ can be determined directly by writing the moment equation of equilibrium about point B by referring to the *FBD* of the beam shown in Fig. *a*.

$$\circlearrowleft + \Sigma M_B = 0; \qquad 3(4)(2) - N_A \sin 30° (3 \sin 30°) - N_A \cos 30° (3 \cos 30° + 4) = 0$$

$$N_A = 3.713 \text{ kN} = 3.71 \text{ kN} \qquad \textbf{Ans.}$$

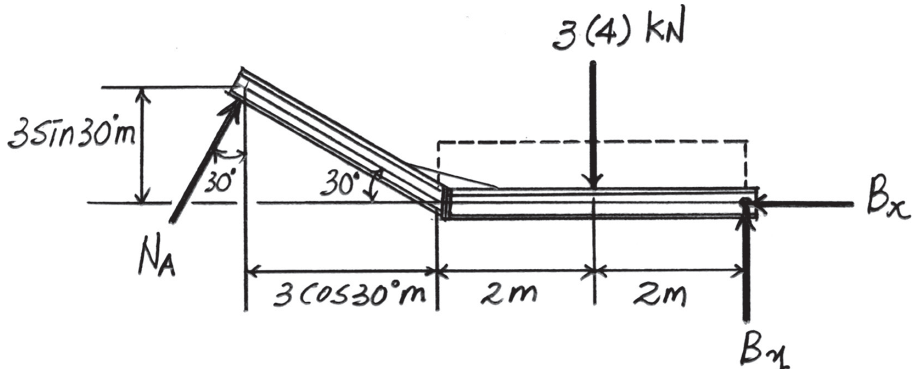
Using this result to write the force equation of equilibrium along the x and y axes,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 3.713 \sin 30° - B_x = 0$$

$$B_x = 1.856 \text{ kN} = 1.86 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y + 3.713 \cos 30° - 3(4) = 0$$

$$B_y = 8.7846 \text{ kN} = 8.78 \text{ kN} \qquad \textbf{Ans.}$$

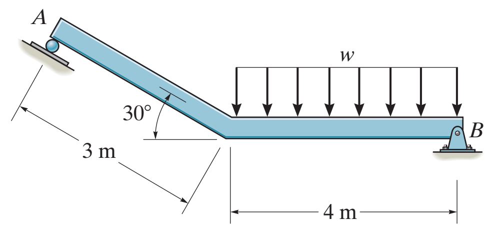


(a)

Ans:
$N_A = 3.71$ kN
$B_x = 1.86$ kN
$B_y = 8.78$ kN

5–23.

If the roller at A and the pin at B can support a load up to 4 kN and 8 kN, respectively, determine the maximum intensity of the distributed load w, measured in kN/m, so that failure of the supports does not occur.



SOLUTION

Equations of Equilibrium. $\mathbf{N}_A$ can be determined directly by writing the moment equation of equilibrium about point B by referring to the *FBD* of the beam shown in Fig. *a*.

$\circlearrowleft + \Sigma M_B = 0; \quad w(4)(2) - N_A \sin 30°\,(3 \sin 30°) - N_A \cos 30°\,(3 \cos 30° + 4) = 0$

$N_A = 1.2376\,w$

Using this result to write the force equation of equilibrium along x and y axes,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 1.2376w \sin 30° - B_x = 0 \qquad B_x = 0.6188w$

$+\uparrow \Sigma F_y = 0; \quad B_y + 1.2376w \cos 30° - w(4) = 0 \qquad B_y = 2.9282w$

Thus,

$$F_B = \sqrt{B_x^{\,2} + B_y^{\,2}} = \sqrt{(0.6188\,w)^2 + (2.9282\,w)^2} = 2.9929\,w$$

It is required that

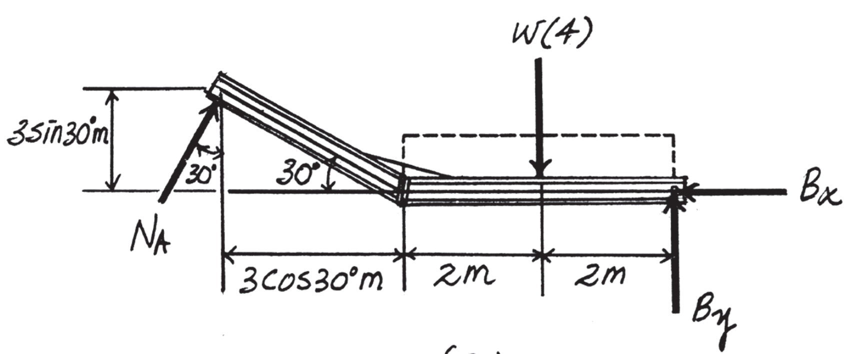
$F_B < 8\text{ kN}; \quad 2.9929\,w < 8 \qquad w < 2.673\text{ kN/m}$

And

$N_A < 4\text{ kN}; \quad 1.2376\,w < 4 \qquad w < 3.232\text{ kN/m}$

Thus, the maximum intensity of the distributed load is

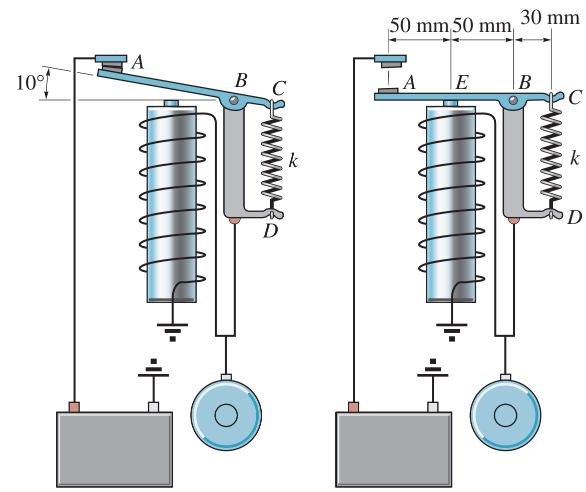
$$w = 2.673\text{ kN/m} = 2.67\text{ kN/m} \qquad \textbf{Ans.}$$



Ans:
$w = 2.67\text{ kN/m}$

***5–24.**

The relay regulates voltage and current. Determine the force in the spring CD, which has a stiffness of $k = 120$ N/m, so that it will allow the armature to make contact at A in figure (a) with a vertical force of 0.4 N. Also, determine the force in the spring when the coil is energized and attracts the armature to E, figure (b), thereby breaking contact at A.



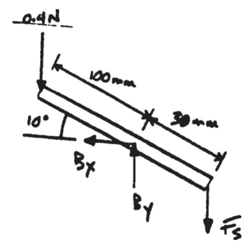
(a) (b)

SOLUTION

From Fig. (a):

$$\circlearrowleft + \Sigma M_B = 0; \qquad 0.4(100 \cos 10°) - F_s (30 \cos 10°) = 0$$

$$F_s = 1.333 \text{ N} = 1.33 \text{ N} \qquad \textbf{Ans.}$$



$$F_s = kx; \qquad 1.333 = 120\,x$$

$$x = 0.01111 \text{ m} = 11.11 \text{ mm}$$

From Fig (b), energizing the coil requires the spring to be stretched an additional amount

$$\Delta x = 30 \sin 10° = 5.209 \text{ mm.}$$

Thus

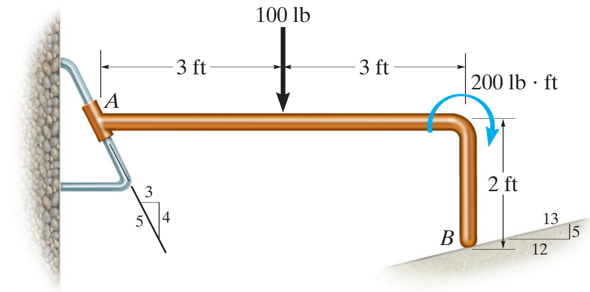
$$x' = 11.11 + 5.209 = 16.32 \text{ mm}$$

$$F_s = 120\,(0.01632) = 1.96 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$F_s = 1.33$ N
$F_s = 1.96$ N

5–25.

Determine the reactions on the bent rod which is supported by a smooth surface at B and by a collar at A, which is fixed to the rod and is free to slide over the fixed inclined rod.

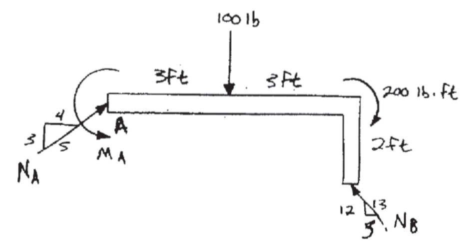


SOLUTION

$$\circlearrowleft + \Sigma M_A = 0; \quad M_A - 100\,(3) - 200 + N_B\left(\frac{12}{13}\right)(6) - N_B\left(\frac{5}{13}\right)(2) = 0$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_A\left(\frac{4}{5}\right) - N_B\left(\frac{5}{13}\right) = 0$$

$$+\uparrow \Sigma F_y = 0; \quad N_A\left(\frac{3}{5}\right) + N_B\left(\frac{12}{13}\right) - 100 = 0$$



Solving,

$N_A = 39.7$ lb **Ans.**

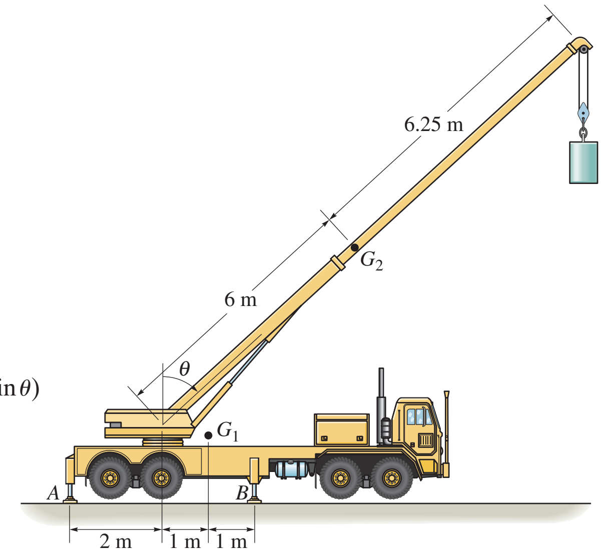
$N_B = 82.5$ lb **Ans.**

$M_A = 106$ lb · ft **Ans.**

Ans:
$N_A = 39.7$ lb
$N_B = 82.5$ lb
$M_A = 106$ lb · ft

5–26.

The mobile crane is symmetrically supported by two outriggers at A and two at B in order to relieve the suspension of the truck upon which it rests and to provide greater stability. If the crane and truck have a mass of 18 Mg and center of mass at G_1, and the boom has a mass of 1.8 Mg and a center of mass at G_2, determine the vertical reactions at each of the four outriggers as a function of the boom angle θ when the boom is supporting a load having a mass of 1.2 Mg. Plot the results measured from $\theta = 0°$ to the critical angle where tipping starts to occur.



SOLUTION

$$+\Sigma M_B = 0; \qquad -N_A(4) + 18(10^3)(9.81)(1) + 1.8(10^3)(9.81)(2 - 6\sin\theta)$$

$$+ 1.2(10^3)(9.81)(2 - 12.25\sin\theta) = 0$$

$$N_A = 58\,860 - 62\,539\sin\theta$$

Tipping occurs when $N_A = 0$, or

$$\theta = 70.3° \qquad \textbf{Ans.}$$

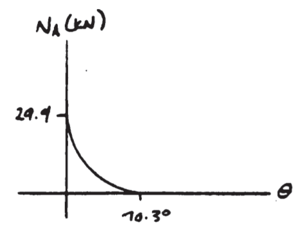
$$+\uparrow\Sigma F_y = 0; \qquad N_B + 58\,860 - 62\,539\sin\theta - (18 + 1.8 + 1.2)(10^3)(9.81) = 0$$

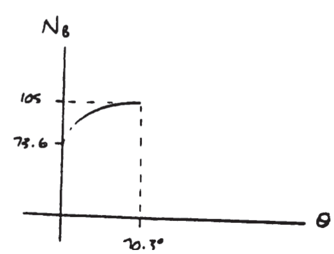
$$N_B = 147\,150 + 62\,539\sin\theta$$

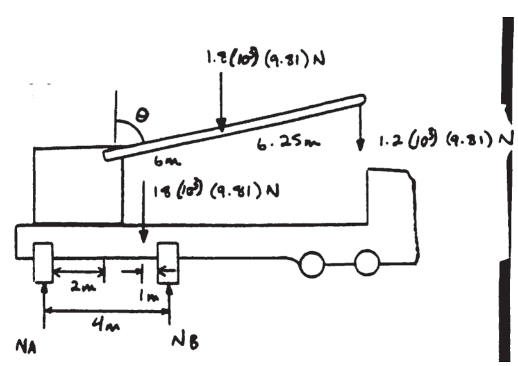
Since there are two outriggers on each side of the crane,

$$N'_A = \frac{N_A}{2} = (29.4 - 31.3\sin\theta)\text{ kN} \qquad \textbf{Ans.}$$

$$N'_B = \frac{N_B}{2} = (73.6 + 31.3\sin\theta)\text{ kN} \qquad \textbf{Ans.}$$



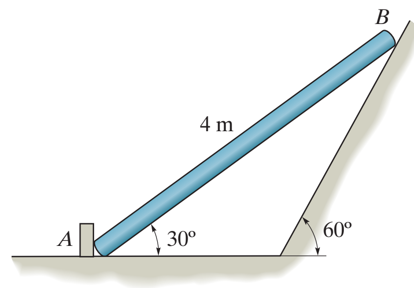




Ans:
$\theta = 70.3°$
$N'_A = (29.4 - 31.3\sin\theta)$ kN
$N'_B = (73.6 + 31.3\sin\theta)$ kN

5–27.

Determine the reactions acting on the smooth uniform bar, which has a mass of 20 kg.



SOLUTION

Equations of Equilibrium. $\mathbf{N}_B$ can be determined directly by writing the moment equation of equilibrium about point A by referring to the *FBD* of the bar shown in Fig. *a*.

$$\circlearrowleft+\Sigma M_A = 0; \qquad N_B \cos 30°(4) - 20(9.81)\cos 30°(2) = 0$$

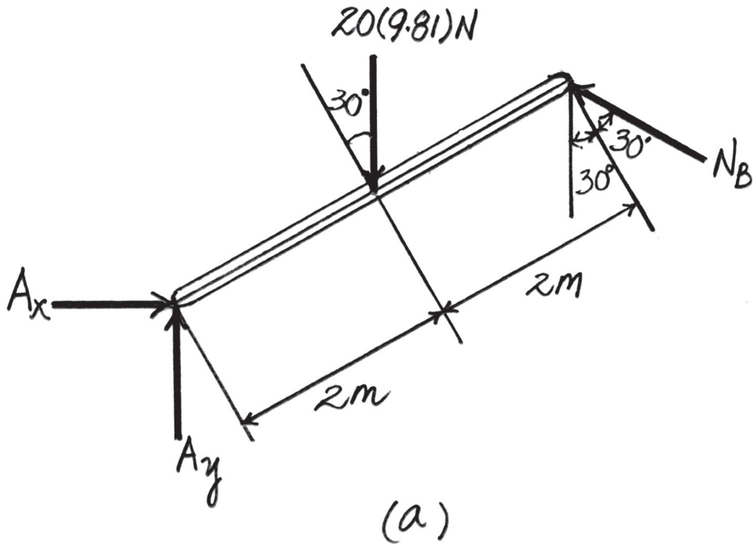
$$N_B = 98.1 \text{ N} \qquad \textbf{Ans.}$$

Using this result to write the force equation of equilibrium along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x - 98.1 \sin 60° = 0 \qquad A_x = 84.96 \text{ N} = 85.0 \text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \qquad A_y + 98.1 \cos 60° - 20(9.81) = 0$$

$$A_y = 147.15 \text{ N} = 147 \text{ N} \qquad \textbf{Ans.}$$

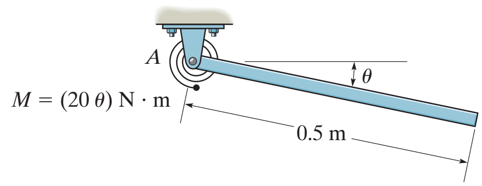


(a)

Ans:
$N_B = 98.1$ N
$A_x = 85.0$ N
$A_y = 147$ N

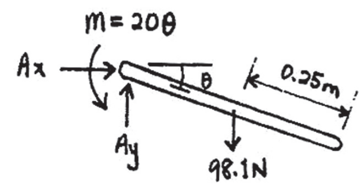
***5–28.**

A linear *torsional spring* deforms such that an applied couple moment M is related to the spring's rotation θ in radians by the equation $M = (20\,\theta)\ \mathrm{N \cdot m}$. If such a spring is attached to the end of a pin-connected uniform 10-kg rod, determine the angle θ for equilibrium. The spring is undeformed when $\theta = 0^\circ$.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \qquad -98.1\,(0.25\cos\theta) + 20(\theta) = 0$



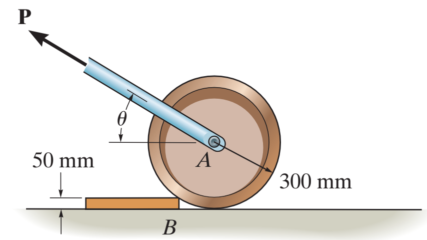
Solving for θ,

$$\theta = 47.5^\circ \qquad \textbf{Ans.}$$

Ans:
$\theta = 47.5^\circ$

5–29.

Determine the force P needed to pull the 50-kg roller over the smooth step. Take $\theta = 30^\circ$.

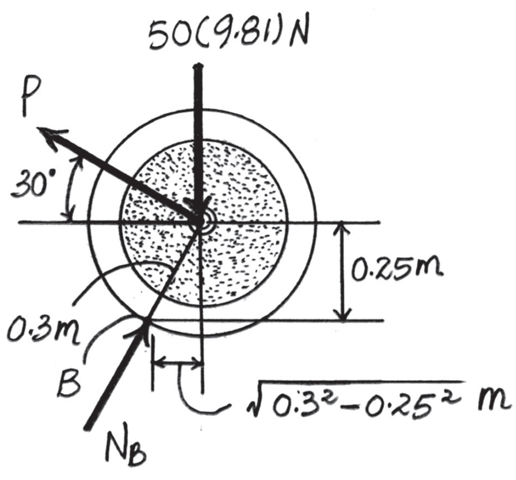


SOLUTION

Equations of Equilibrium. P can be determined directly by writing the moment equation of Equilibrium about point B, by referring to the *FBD* of the roller shown in Fig. *a*.

$$\zeta + \Sigma M_B = 0; \qquad P\cos 30^\circ(0.25) + P\sin 30^\circ\left(\sqrt{0.3^2 - 0.25^2}\right) - 50(9.81)\sqrt{0.3^2 - 0.25^2} = 0$$

$$P = 271.66\text{ N} = 272\text{ N} \qquad \textbf{Ans.}$$

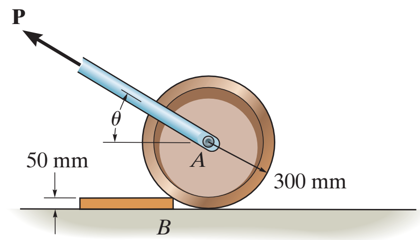


(a)

Ans:
$P = 272\text{ N}$

5–30.

Determine the magnitude and direction θ of the minimum force P needed to pull the 50-kg roller over the smooth step.



SOLUTION

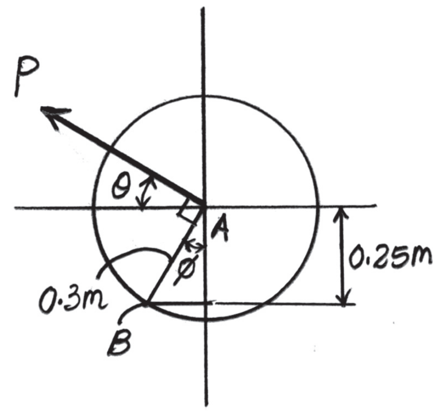
Equations of Equilibrium. P will be minimum if its orientation produces the greatest moment about point B. This happens when it acts perpendicular to AB as shown in Fig. a. Thus

$$\theta = \phi = \cos^{-1}\left(\frac{0.25}{0.3}\right) = 33.56° = 33.6° \qquad \textbf{Ans.}$$

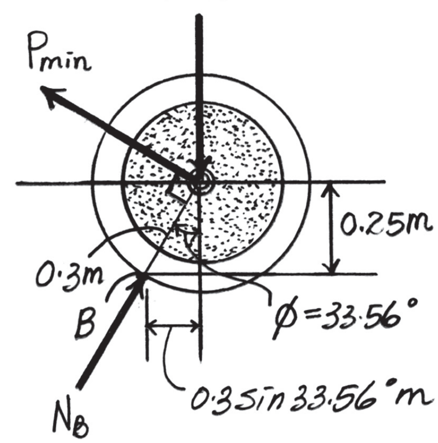
P_{min} can be determined by writing the moment equation of equilibrium about point B by referring to the FBD of the roller shown in Fig. b.

$$\circlearrowleft + \Sigma M_B = 0; \qquad P_{\text{min}}(0.3) - 50(9.81)(0.3 \sin 33.56°) = 0$$

$$P_{\text{min}} = 271.13 \text{ N} = 271 \text{ N} \qquad \textbf{Ans.}$$



(a)

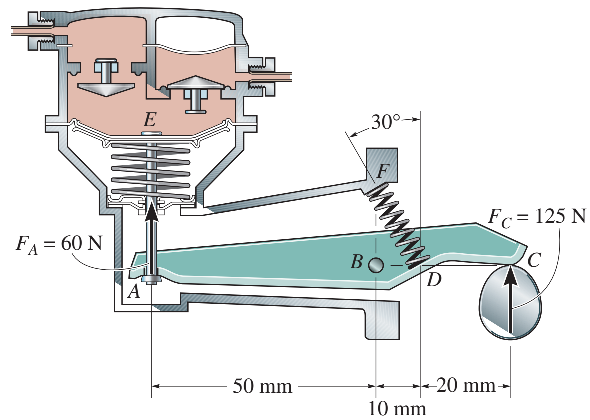


(b)

Ans:
$P_{\text{min}} = 271$ N

5–31.

The operation of the fuel pump for an automobile depends on the reciprocating action of the rocker arm ABC, which is pinned at B and is spring loaded at A and D. When the smooth cam C is in the position shown, determine the horizontal and vertical components of force at the pin and the force along the spring DF for equilibrium. The vertical force acting on the rocker arm at A is $F_A = 60$ N, and at C it is $F_C = 125$ N.



SOLUTION

$\circlearrowleft + \Sigma M_B = 0; \qquad -60(50) - F_B \cos 30°(10) + 125(30) = 0$

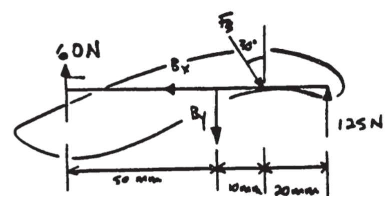
$$F_B = 86.6025 = 86.6 \text{ N} \qquad \textbf{Ans.}$$

$\xrightarrow{+} \Sigma F_x = 0; \qquad -B_x + 86.6025 \sin 30° = 0$

$$B_x = 43.3 \text{ N} \qquad \textbf{Ans.}$$

$+\uparrow \Sigma F_y = 0; \qquad 60 - B_y - 86.6025 \cos 30° + 125 = 0$

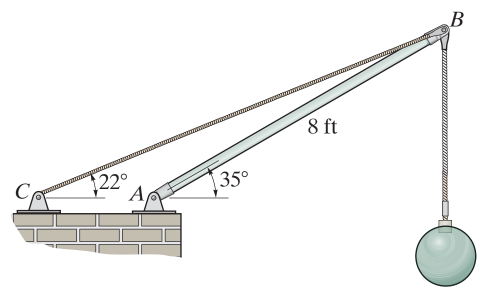
$$B_y = 110 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$F_B = 86.6$ N
$B_x = 43.3$ N
$B_y = 110$ N

***5–32.**

Determine the magnitude of force at the pin A and in the cable BC needed to support the 500-lb load. Neglect the weight of the boom AB.



SOLUTION

Equations of Equilibrium: The force in cable BC can be obtained directly by summing moments about point A.

$\circlearrowleft + \Sigma M_A = 0;$ $\quad F_{BC} \sin 13°(8) - 500 \cos 35°(8) = 0$

$F_{BC} = 1820.7 \text{ lb} = 1.82 \text{ kip}$ **Ans.**

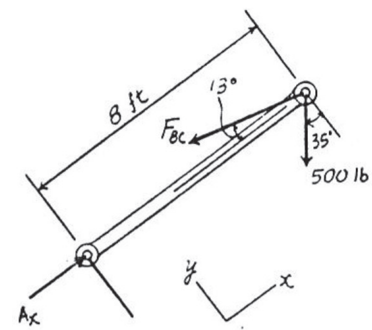
$\overset{+}{\nearrow} \Sigma F_x = 0;$ $\quad A_x - 1820.7 \cos 13° - 500 \sin 35° = 0$

$A_x = 2060.9 \text{ lb}$

$\nwarrow + \Sigma F_y = 0;$ $\quad A_y + 1820.7 \sin 13° - 500 \cos 35° = 0$

$A_y = 0$

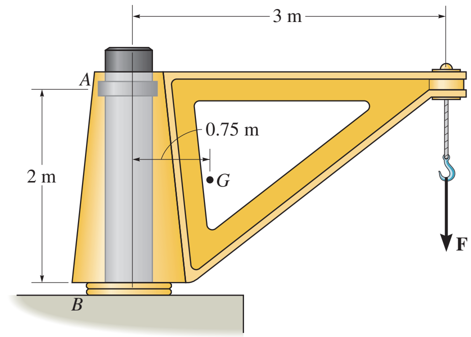
Thus, $\quad F_A = A_x = 2060.9 \text{ lb} = 2.06 \text{ kip}$ **Ans.**



Ans:
$F_{BC} = 1.82$ kip
$F_A = 2.06$ kip

5–33.

The dimensions of a jib crane, which is manufactured by the Basick Co., are given in the figure. If the crane has a mass of 800 kg and a center of mass at G, and the maximum rated force at its end is $F = 15$ kN, determine the reactions at its bearings. The bearing at A is a journal bearing and supports only a horizontal force, whereas the bearing at B is a thrust bearing that supports both horizontal and vertical components.



SOLUTION

$\zeta + \Sigma M_B = 0; \quad A_x(2) - 800\,(9.81)\,(0.75) - 15\,000(3) = 0$

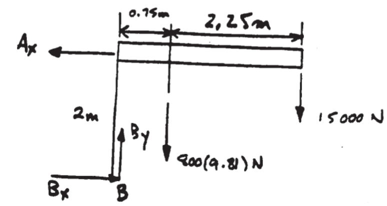
$A_x = 25.4$ kN **Ans.**

$+\uparrow \Sigma F_y = 0; \quad B_y - 800\,(9.81) - 15\,000 = 0$

$B_y = 22.8$ kN **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \quad B_x - 25.4 = 0$

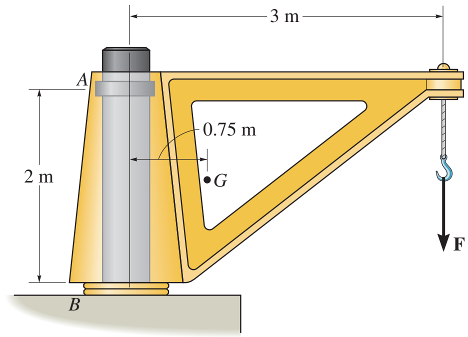
$B_x = 25.4$ kN **Ans.**



Ans:
$A_x = 25.4$ kN
$B_y = 22.8$ kN
$B_x = 25.4$ kN

5–34.

The dimensions of a jib crane, which is manufactured by the Basick Co., are given in the figure. The crane has a mass of 800 kg and a center of mass at G. The bearing at A is a journal bearing and can support a horizontal force, whereas the bearing at B is a thrust bearing that supports both horizontal and vertical components. Determine the maximum load F that can be suspended from its end if the selected bearings at A and B can sustain a maximum resultant load of 24 kN and 34 kN, respectively.

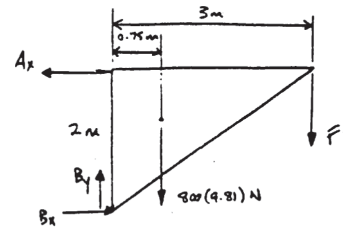


SOLUTION

$\circlearrowleft + \Sigma M_B = 0; \qquad A_x(2) - 800(9.81)(0.75) - F(3) = 0$

$+\uparrow \Sigma F_y = 0; \qquad B_y - 800(9.81) - F = 0$

$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x - A_x = 0$



Assume $A_x = 24\,000$ N.

Solving,

$$B_x = 24 \text{ kN}$$

$$B_y = 21.9 \text{ kN}$$

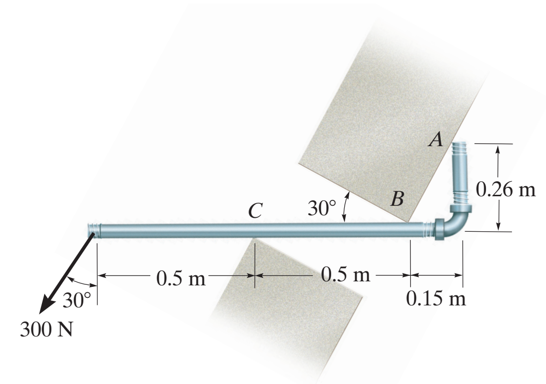
$$F = 14.0 \text{ kN} \qquad \textbf{Ans.}$$

$$F_B = \sqrt{(24)^2 + (21.9)^2} = 32.5 \text{ kN} < 34 \text{ kN} \qquad \textbf{OK}$$

Ans:
$F = 14.0$ kN

5–35.

The smooth pipe rests against the opening at the points of contact A, B, and C. Determine the reactions at these points needed to support the force of 300 N. Neglect the pipe's thickness in the calculation.



SOLUTION

Equations of Equilibrium. $\mathbf{N}_A$ can be determined directly by writing the force equation of equilibrium along the x axis by referring to the *FBD* of the pipe shown in Fig. *a*.

$\xrightarrow{+} \Sigma F_x = 0; \quad N_A \cos 30° - 300 \sin 30° = 0 \quad N_A = 173.21 \text{ N} = 173 \text{ N}$ **Ans.**

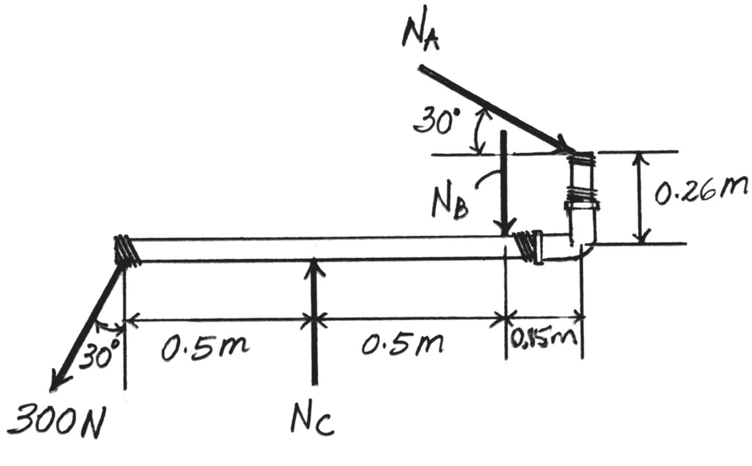
Using this result to write the moment equations of equilibrium about points B and C,

$\zeta + \Sigma M_B = 0; \quad 300 \cos 30°(1) - 173.21 \cos 30°(0.26) - 173.21 \sin 30°(0.15) - N_C(0.5) = 0$

$N_C = 415.63 \text{ N} = 416 \text{ N}$ **Ans.**

$\zeta + \Sigma M_C = 0; \quad 300 \cos 30°(0.5) - 173.21 \cos 30°(0.26) - 173.21 \sin 30°(0.65) - N_B(0.5) = 0$

$N_B = 69.22 \text{ N} = 69.2 \text{ N}$ **Ans.**

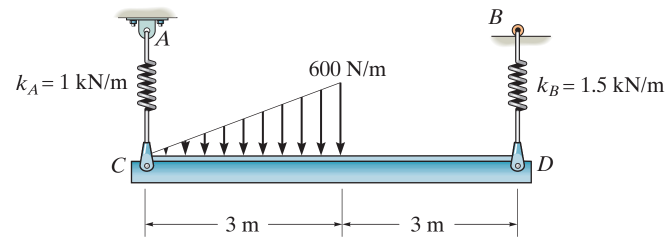


(a)

Ans:
$N_A = 173$ N
$N_C = 416$ N
$N_B = 69.2$ N

***5–36.**

The beam of negligible weight is supported horizontally by two springs. If the beam is horizontal and the springs are unstretched when the load is removed, determine the angle of tilt of the beam when the load is applied.



SOLUTION

Equations of Equilibrium. $\mathbf{F}_A$ and $\mathbf{F}_B$ can be determined directly by writing the moment equations of equilibrium about points B and A, respectively, by referring to the *FBD* of the beam shown in Fig. *a*.

Assuming that the angle of tilt is small,

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_B(6) - \frac{1}{2}(600)(3)(2) = 0 \quad F_B = 300 \text{ N}$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad \frac{1}{2}(600)(3)(4) - F_A(6) = 0 \quad F_A = 600 \text{ N}$$

Thus, the stretches of springs A and B can be determined from

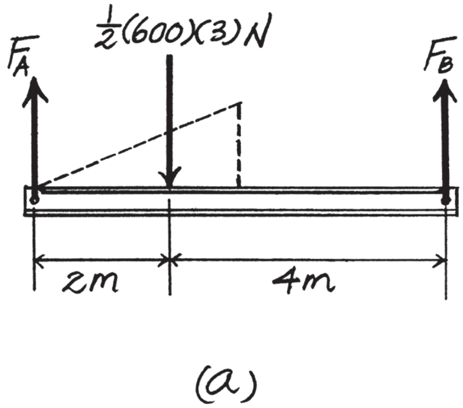
$$F_A = k_A x_A; \qquad 600 = 1000 x_A \qquad x_A = 0.6 \text{ m}$$

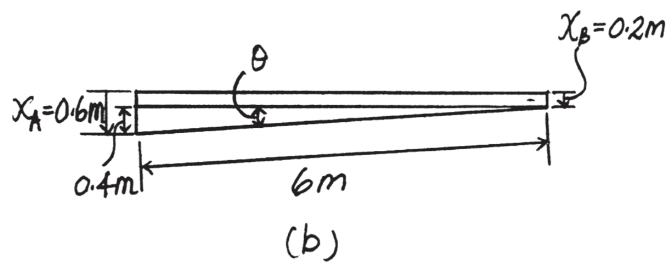
$$F_B = k_B x_B; \qquad 300 = 1500 x_B \qquad x_B = 0.2 \text{ m}$$

From the geometry shown in Fig. *b*

$$\theta = \sin^{-1}\left(\frac{0.4}{6}\right) = 3.82^\circ \qquad \textbf{Ans.}$$

The assumption of small θ is confirmed.

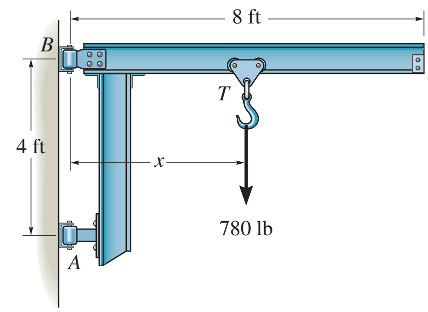




Ans:
$\theta = 3.82^\circ$

5–37.

The cantilevered jib crane is used to support the load of 780 lb. If $x = 5$ ft, determine the reactions at the supports. Note that the supports are collars that allow the crane to rotate freely about the vertical axis. The collar at B supports a force in the vertical direction, whereas the one at A does not.



SOLUTION

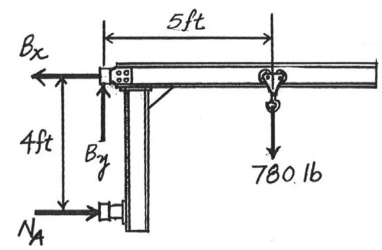
Equations of Equilibrium: Referring to the FBD of the jib crane shown in Fig. a, we notice that $\mathbf{N}_A$ and $\mathbf{B}_y$ can be obtained directly by writing the moment equation of equilibrium about point B and force equation of equilibrium along the y axis, respectively.

$\circlearrowleft + \Sigma M_B = 0; \quad N_A(4) - 780(5) = 0 \quad N_A = 975 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad B_y - 780 = 0 \quad B_y = 780$ **Ans.**

Using the result of $\mathbf{N}_A$ to write the force equation of equilibrium along x axis,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 975 - B_x = 0 \quad B_x = 975 \text{ lb}$ **Ans.**

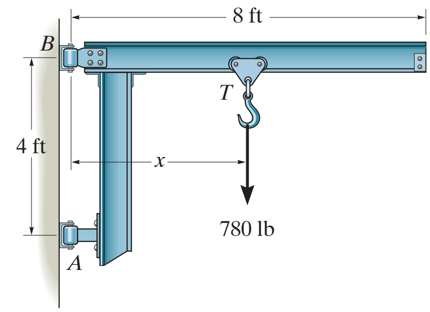


(a)

Ans:
$N_A = 975$ lb
$B_x = 975$ lb
$B_y = 780$ lb

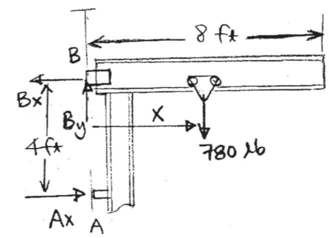
5–38.

The cantilevered jib crane is used to support the load of 780 lb. If the trolley T can be placed anywhere between 1.5 ft $\leq x \leq$ 7.5 ft, determine the maximum magnitude of reaction at the supports A and B. Note that the supports are collars that allow the crane to rotate freely about the vertical axis. The collar at B supports a force in the vertical direction, whereas the one at A does not.



SOLUTION

Require $x = 7.5$ ft



$\circlearrowleft + \Sigma M_A = 0; \qquad -780(7.5) + B_x(4) = 0$

$B_x = 1462.5$ lb

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - 1462.5 = 0$

$A_x = 1462.5 = 1462$ lb **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad B_y - 780 = 0$

$B_y = 780$ lb

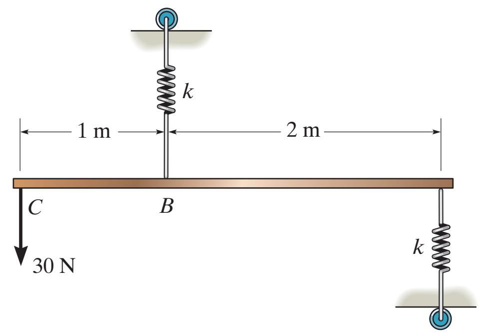
$F_B = \sqrt{(1462.5)^2 + (780)^2}$

$= 1657.5 \text{ lb} = 1.66 \text{ kip}$ **Ans.**

Ans:
$A_x = 1.46$ kip
$F_B = 1.66$ kip

5–39.

The bar of negligible weight is supported by two springs, each having a stiffness $k = 100$ N/m. If the springs are originally unstretched, and the force is vertical as shown, determine the angle θ the bar makes with the horizontal, when the 30-N force is applied to the bar.



SOLUTION

Equations of Equilibrium. $\mathbf{F}_A$ and $\mathbf{F}_B$ can be determined directly by writing the moment equation of equilibrium about points B and A respectively by referring to the *FBD* of the bar shown in Fig. *a*.

$\zeta + \Sigma M_B = 0; \quad 30(1) - F_A(2) = 0 \quad F_A = 15\text{ N}$

$\zeta + \Sigma M_A = 0; \quad 30(3) - F_B(2) = 0 \quad F_B = 45\text{ N}$

Thus, the stretches of springs A and B can be determined from

$F_A = kx_A; \quad 15 = 100x_A \quad x_A = 0.15\text{ m}$

$F_B = kx_B; \quad 45 = 100x_B \quad x_B = 0.45\text{ m}$

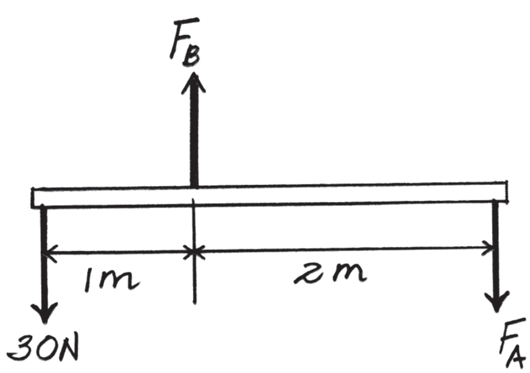
From the geometry shown in Fig. *b*,

$$\frac{d}{0.45} = \frac{2-d}{0.15}; \qquad d = 1.5\text{ m}$$

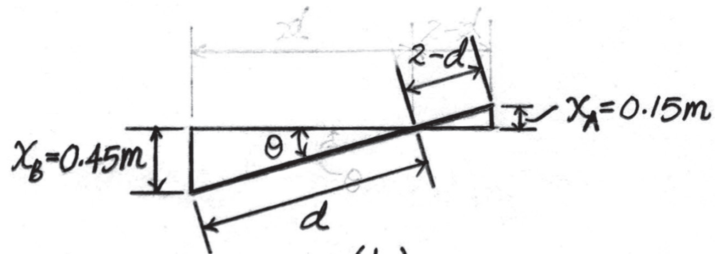
Thus

$$\theta = \sin^{-1}\left(\frac{0.45}{1.5}\right) = 17.46° = 17.5° \qquad \textbf{Ans.}$$

Note: The moment equations are set up assuming small θ, but even with non-small θ the reactions come out with the same F_A, F_B, and then the rest of the solution goes through as before.



(a)

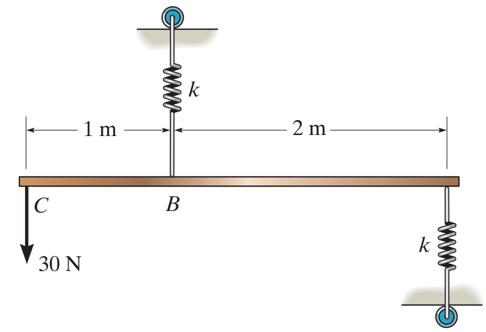


(b)

Ans:
$\theta = 17.5°$

***5–40.**

Determine the stiffness k of each spring so that the 30-N force causes the bar to tip $\theta = 15°$ when the force is applied. Originally the bar is horizontal and the springs are unstretched. Neglect the weight of the bar.



SOLUTION

Equations of Equilibrium. $\mathbf{F}_A$ and $\mathbf{F}_B$ can be determined directly by writing the moment equation of equilibrium about points B and A respectively by referring to the *FBD* of the bar shown in Fig. *a*.

$$\circlearrowleft + \Sigma M_B = 0; \qquad 30(1) - F_A(2) = 0 \qquad F_A = 15\text{ N}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad 30(3) - F_B(2) = 0 \qquad F_B = 45\text{ N}$$

Thus, the stretches of springs A and B can be determined from

$$F_A = kx_A; \qquad 15 = kx_A \qquad x_A = \frac{15}{k}$$

$$F_B = kx_B; \qquad 45 = kx_B \qquad x_B = \frac{45}{k}$$

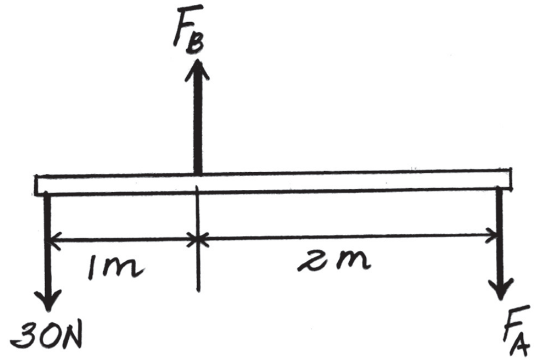
From the geometry shown in Fig. *b*

$$\frac{d}{45/k} = \frac{2-d}{15/k}; \qquad d = 1.5\text{ m}$$

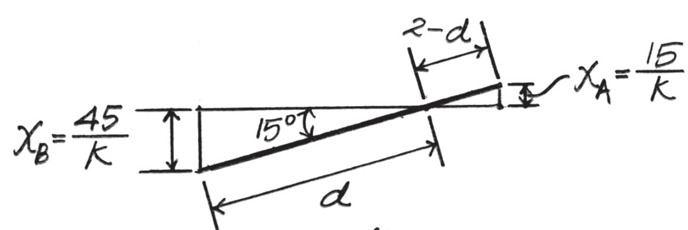
Thus,

$$\sin 15° = \frac{45/k}{1.5} \qquad k = 115.91\text{ N/m} = 116\text{ N/m} \qquad \textbf{Ans.}$$

Note: The moment equations are set up assuming small θ, but even with non-small θ the reactions come out with the same F_A, F_B, and then the rest of the solution goes through as before.



(a)

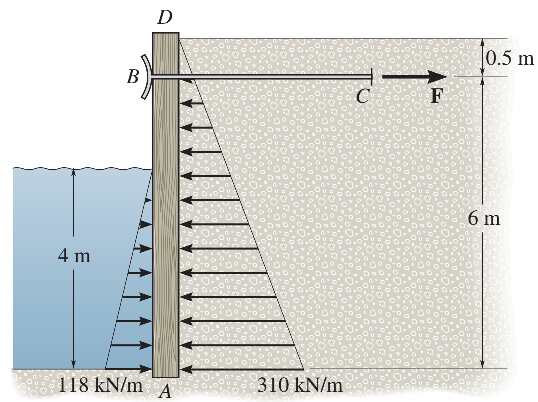


(b)

Ans:
$k = 116\text{ N/m}$

5–41.

The bulk head AD is subjected to both water and soil-backfill pressures. Assuming AD is "pinned" to the ground at A, determine the horizontal and vertical reactions there and also the required tension in the ground anchor BC necessary for equilibrium. The bulk head has a mass of 800 kg.



SOLUTION

Equations of Equilibrium: The force in ground anchor BC can be obtained directly by summing moments about point A.

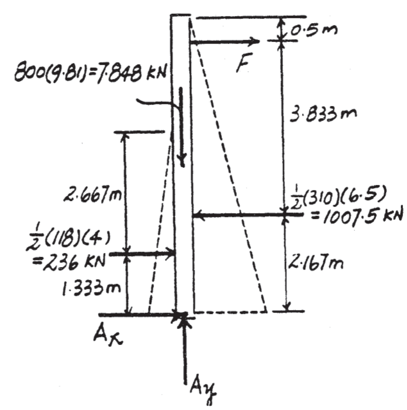
$$\circlearrowleft + \Sigma M_A = 0; \qquad 1007.5(2.167) - 236(1.333) - F(6) = 0$$

$$F = 311.375 \text{ kN} = 311 \text{ kN} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x + 311.375 + 236 - 1007.5 = 0$$

$$A_x = 460 \text{ kN} \qquad \textbf{Ans.}$$

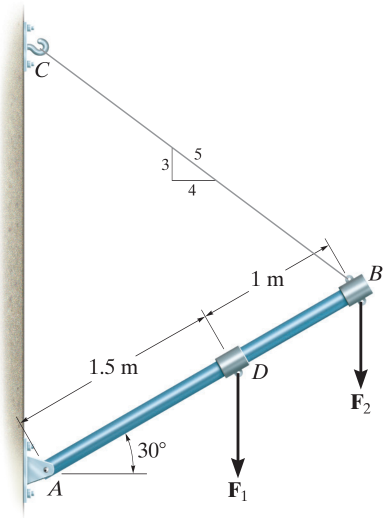
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 7.848 = 0 \qquad A_y = 7.85 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$F = 311$ kN
$A_x = 460$ kN
$A_y = 7.85$ kN

5–42.

The boom supports the two vertical loads. Neglect the size of the collars at D and B and the thickness of the boom, and compute the horizontal and vertical components of force at the pin A and the force in cable CB. Set $F_1 = 800$ N and $F_2 = 350$ N.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0;$ $\quad -800(1.5\cos 30°) - 350(2.5\cos 30°)$

$$+\frac{4}{5}F_{CB}(2.5\sin 30°) + \frac{3}{5}F_{CB}(2.5\cos 30°) = 0$$

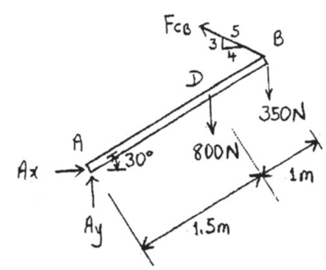
$F_{CB} = 781.6 = 782$ N **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x - \frac{4}{5}(781.6) = 0$

$A_x = 625$ N **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 800 - 350 + \frac{3}{5}(781.6) = 0$

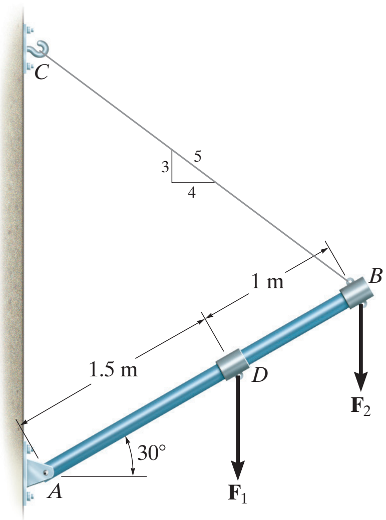
$A_y = 681$ N **Ans.**



Ans:
$F_{CB} = 782$ N
$A_x = 625$ N
$A_y = 681$ N

5–43.

The boom is intended to support two vertical loads, $\mathbf{F}_1$ and $\mathbf{F}_2$. If the cable CB can sustain a maximum load of 1500 N before it fails, determine the critical loads if $F_1 = 2F_2$. Also, what is the magnitude of the maximum reaction at pin A?



SOLUTION

$\circlearrowleft + \Sigma M_A = 0;$ $\quad -2F_2(1.5 \cos 30°) - F_2(2.5 \cos 30°)$

$$+ \frac{4}{5}(1500)(2.5 \sin 30°) + \frac{3}{5}(1500)(2.5 \cos 30°) = 0$$

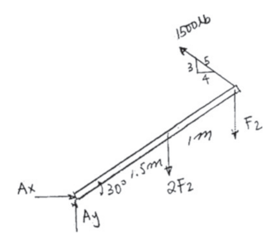
$F_2 = 724 \text{ N}$ **Ans.**

$F_1 = 2F_2 = 1448 \text{ N}$

$F_1 = 1.45 \text{ kN}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - \frac{4}{5}(1500) = 0$

$A_x = 1200 \text{ N}$



$+\uparrow \Sigma F_y = 0;$ $\quad A_y - 724 - 1448 + \frac{3}{5}(1500) = 0$

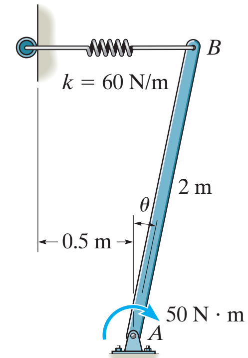
$A_y = 1272 \text{ N}$

$F_A = \sqrt{(1200)^2 + (1272)^2} = 1749 \text{ N} = 1.75 \text{ kN}$ **Ans.**

Ans:
$F_2 = 724 \text{ N}$
$F_1 = 1.45 \text{ kN}$
$F_A = 1.75 \text{ kN}$

***5–44.**

The 10-kg uniform rod is pinned at end A. If it is also subjected to a couple moment of 50 N·m, determine the smallest angle θ for equilibrium. The spring is unstretched when $\theta = 0$, and has a stiffness of $k = 60$ N/m.



SOLUTION

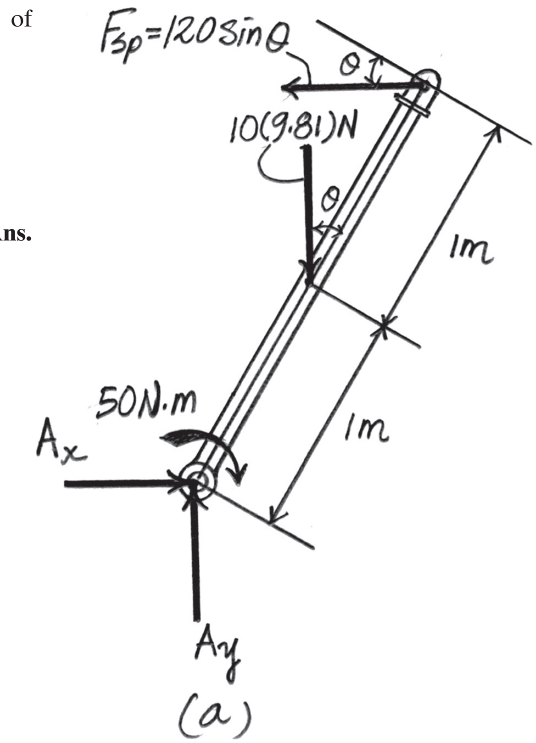
Equations of Equilibrium. Here the spring stretches $x = 2 \sin\theta$. The force in the spring is $F_{sp} = kx = 60\,(2\sin\theta) = 120\sin\theta$. Write the moment equation of equilibrium about point A by referring to the *FBD* of the rod shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad 120\sin\theta\cos\theta(2) - 10(9.81)\sin\theta\,(1) - 50 = 0$$

$$240\sin\theta\cos\theta - 98.1\sin\theta - 50 = 0$$

Solve numerically

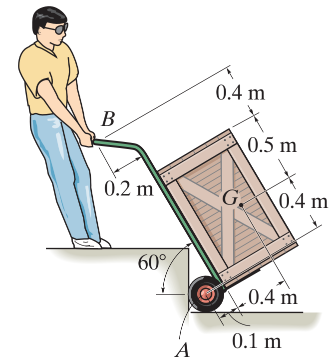
$$\theta = 24.598^\circ = 24.6^\circ \qquad \textbf{Ans.}$$



Ans:
$\theta = 24.6^\circ$

5–45.

The man uses the hand truck to move material up the step. If the truck and its contents have a mass of 50 kg with center of gravity at G, determine the normal reaction on both wheels and the magnitude and direction of the minimum force required at the grip B needed to lift the load.



SOLUTION

Equations of Equilibriums. $\mathbf{P}_y$ can be determined directly by writing the force equation of equilibrium along y axis by referring to the *FBD* of the hand truck shown in Fig. *a*.

$$+\uparrow \Sigma F_y = 0; \qquad P_y - 50(9.81) = 0 \qquad P_y = 490.5 \text{ N}$$

Using this result to write the moment equation of equilibrium about point A,

$$\circlearrowleft + \Sigma M_A = 0; \qquad P_x \sin 60°(1.3) - P_x \cos 60°(0.1) - 490.5 \cos 30°(0.1)$$
$$-490.5 \sin 30°(1.3) - 50(9.81) \sin 60°(0.5)$$
$$+50(9.81) \cos 60°(0.4) = 0$$
$$P_x = 442.07 \text{ N}$$

Thus, the magnitude of minimum force $\boldsymbol{P}$, Fig. b, is

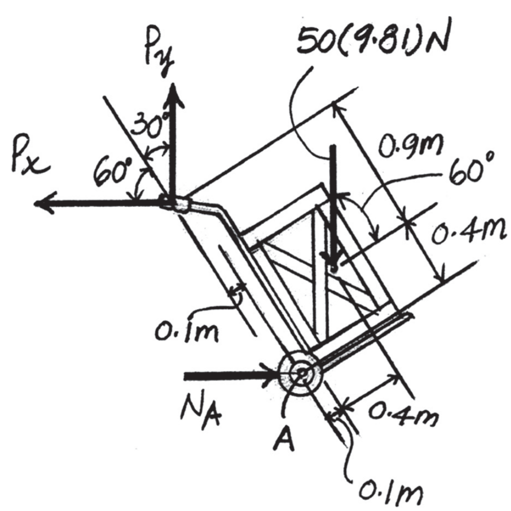
$$P = \sqrt{P_x^2 + P_y^2} = \sqrt{442.07^2 + 490.5^2} = 660.32 \text{ N} = 660 \text{ N} \qquad \textbf{Ans.}$$

and the angle is

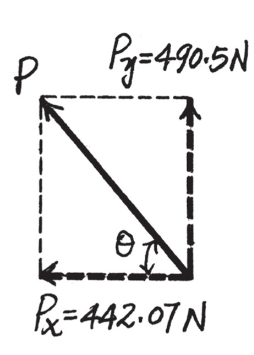
$$\theta = \tan^{-1}\left(\frac{490.5}{442.07}\right) = 47.97° = 48.0° \qquad \textbf{Ans.}$$

Write the force equation of equilibrium along x axis,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_A - 442.07 = 0 \qquad N_A = 442.07 \text{ N} = 442 \text{ N} \qquad \textbf{Ans.}$$



(a)

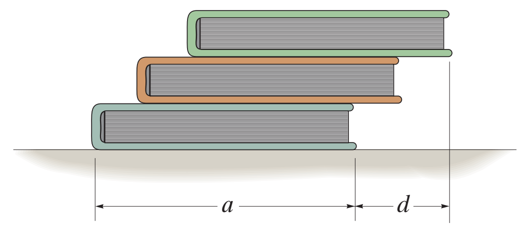


(b)

Ans:
$P = 660$ N
$N_A = 442$ N
$\theta = 48.0°$

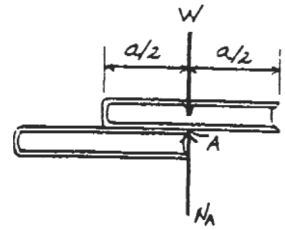
5–46.

Three uniform books, each having a weight W and length a, are stacked as shown. Determine the maximum distance d that the top book can extend out from the bottom one so the stack does not topple over.



SOLUTION

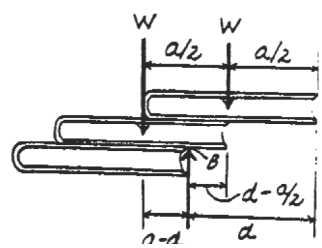
Equilibrium: For top two books, the upper book will topple when the center of gravity of this book is to the right of point A. Therefore, the maximum distance from the right edge of this book to point A is $a/2$.



Equation of Equilibrium: For the entire three books, the top two books will topple about point B.

$$\circlearrowleft +\Sigma M_B = 0; \qquad W(a-d)-W\left(d-\frac{a}{2}\right) = 0$$

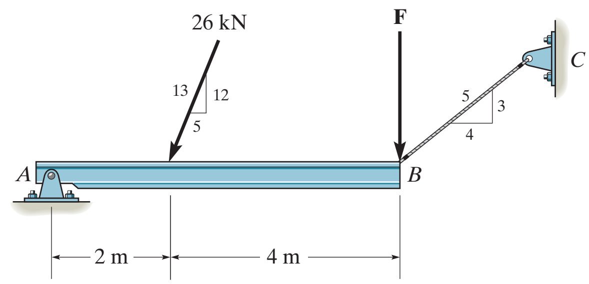
$$d = \frac{3a}{4} \qquad \textbf{Ans.}$$



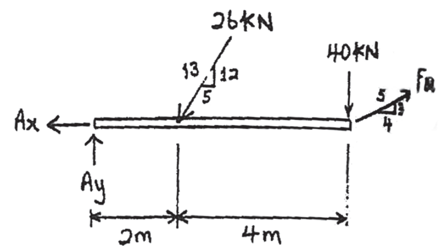
Ans:
$d = \dfrac{3a}{4}$

5–47.

Determine the reactions at the pin A and the tension in cord BC. Set $F = 40$ kN. Neglect the thickness of the beam.



SOLUTION



$$\circlearrowleft + \Sigma M_A = 0; \qquad -26\left(\frac{12}{13}\right)(2) - 40(6) + \frac{3}{5}F_{BC}(6) = 0$$

$$F_{BC} = 80 \text{ kN}$$ **Ans.**

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 80\left(\frac{4}{5}\right) - A_x - 26\left(\frac{5}{13}\right) = 0$$

$$A_x = 54 \text{ kN}$$ **Ans.**

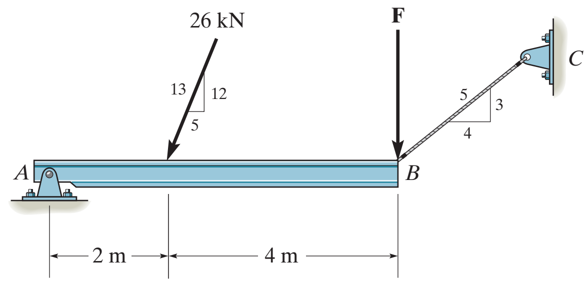
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 26\left(\frac{12}{13}\right) - 40 + 80\left(\frac{3}{5}\right) = 0$$

$$A_y = 16 \text{ kN}$$ **Ans.**

Ans:
$F_{BC} = 80$ kN
$A_x = 54$ kN
$A_y = 16$ kN

***5–48.**

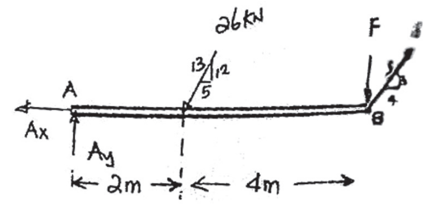
If rope BC will fail when the tension becomes 50 kN, determine the greatest vertical load F that can be applied to the beam at B. What is the magnitude of the reaction at A for this loading? Neglect the thickness of the beam.



SOLUTION

$\zeta + \Sigma M_A = 0; \qquad -26\left(\frac{12}{13}\right)(2) - F(6) + \frac{3}{5}(50)(6) = 0$

$F = 22 \text{ kN}$ **Ans.**



$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 50\left(\frac{4}{5}\right) - A_x - 26\left(\frac{5}{13}\right) = 0$

$A_x = 30 \text{ kN}$ **Ans.**

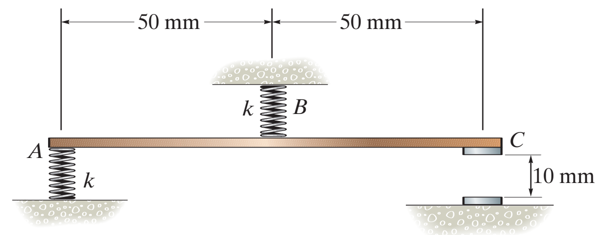
$+\uparrow \Sigma F_y = 0; \qquad A_y - 26\left(\frac{12}{13}\right) - 22 + 50\left(\frac{3}{5}\right) = 0$

$A_y = 16 \text{ kN}$ **Ans.**

Ans:
$F = 22$ kN
$A_x = 30$ kN
$A_y = 16$ kN

5–49.

The rigid metal strip of negligible weight is used as part of an electromagnetic switch. If the stiffness of the springs at A and B is $k = 5\text{ N/m}$, and the strip is originally horizontal when the springs are unstretched, determine the smallest force needed to close the contact gap at C.

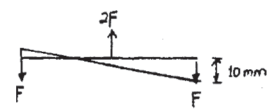


SOLUTION

$\Sigma M_B = 0; \qquad F_A = F_C = F$

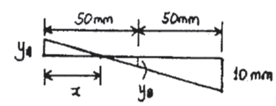
$\Sigma F_y = 0; \qquad F_B = 2F$

$$\frac{x}{y_A} = \frac{50-x}{y_B} \qquad (1)$$



$$\frac{2F}{F} = \frac{ky_B}{ky_A}$$

$$2y_A = y_B \qquad (2)$$



Substituting into Eq. (1):

$$\frac{x}{y_A} = \frac{50 - x}{2y_A}$$

$$2x = 50 - x$$

$$x = \frac{50}{3} = 16.67\text{ mm}$$

$$\frac{x}{y_A} = \frac{100-x}{10}$$

Set $x = 16.67$, then

$$y_A = 2\text{ mm}$$

From Eq. (2),

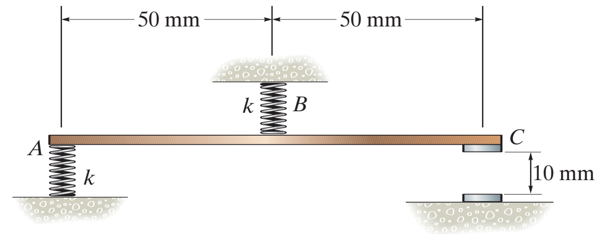
$$y_B = 4\text{ mm}$$

$$F_C = F_A = ky_A = (5)(0.002) = 10\text{ mN}$$ **Ans.**

Ans:
$F_C = 10\text{ mN}$

5–50.

The rigid metal strip of negligible weight is used as part of an electromagnetic switch. Determine the maximum stiffness k of the springs at A and B so that the contact at C closes when the vertical force developed there is 0.5 N. Originally the strip is horizontal as shown.

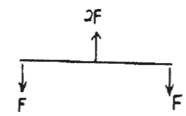


SOLUTION

$\Sigma M_B = 0; \qquad F_A = F_C = F$

$\Sigma F_y = 0; \qquad F_B = 2F$

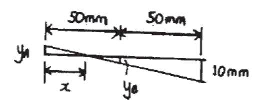
$$\frac{x}{y_A} = \frac{50 - x}{y_B} \qquad \textbf{(1)}$$



$$\frac{2F}{F} = \frac{k y_B}{k y_A}$$

$$2y_A = y_B \qquad \textbf{(2)}$$

Substituting into Eq. (1):



$$\frac{x}{y_A} = \frac{50 - x}{2y_A}$$

$$2x = 50 - x$$

$$x = \frac{50}{3} = 16.67 \text{ mm}$$

$$\frac{x}{y_A} = \frac{100 - x}{10}$$

Set $x = 16.67$, then

$$y_A = 2 \text{ mm}$$

From Eq. (2),

$$y_B = 4 \text{ mm}$$

$$F_C = F_A = k y_A$$

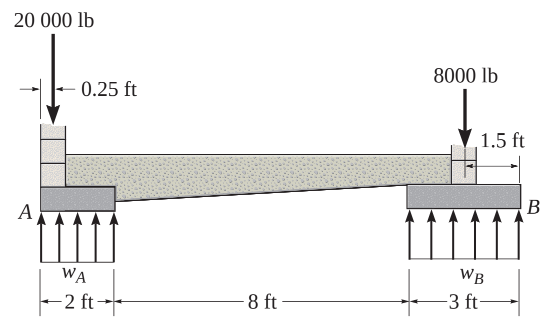
$$0.5 = k(0.002)$$

$$k = 250 \text{ N/m} \qquad \textbf{Ans.}$$

Ans:
$k = 250 \text{ N/m}$

5–51.

The cantilever footing is used to support a wall near its edge A so that it causes a uniform soil pressure under the footing. Determine the uniform distribution loads w_A and w_B, measured in lb/ft at pads A and B, necessary to support the wall forces of 8 000 lb and 20 000 lb.



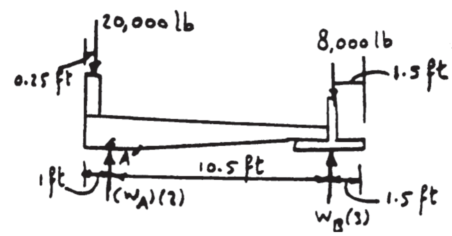
SOLUTION

$\zeta + \Sigma M_A = 0; \qquad -8000\,(10.5) + w_B\,(3)(10.5) + 20\,000\,(0.75) = 0$

$w_B = 2190.5 \text{ lb/ft} = 2.19 \text{ kip/ft}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad 2190.5\,(3) - 28\,000 + w_A\,(2) = 0$

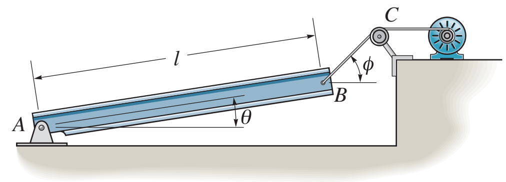
$w_A = 10.7 \text{ kip/ft}$ **Ans.**



Ans:
$w_B = 2.19$ kip/ft
$w_A = 10.7$ kip/ft

***5–52.**

The uniform beam has a weight W and length l and is supported by a pin at A and a cable BC. Determine the horizontal and vertical components of reaction at A and the tension in the cable necessary to hold the beam in the position shown.

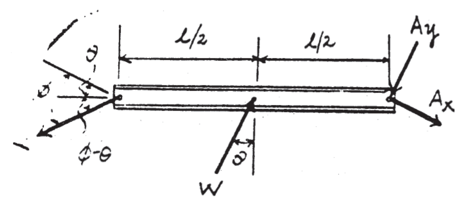


SOLUTION

Equations of Equilibrium: The tension in the cable can be obtained directly by summing moments about point A.

$$\circlearrowleft + \Sigma M_A = 0; \qquad T \sin(\phi - \theta) l - W \cos\theta\left(\frac{l}{2}\right) = 0$$

$$T = \frac{W\cos\theta}{2\sin(\phi - \theta)} \qquad \textbf{Ans.}$$



Using the result $T = \dfrac{W\cos\theta}{2\sin(\phi - \theta)}$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad \left(\frac{W\cos\theta}{2\sin(\phi - \theta)}\right)\cos\phi - A_x = 0$$

$$A_x = \frac{W\cos\phi\cos\theta}{2\sin(\phi - \theta)} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y + \left(\frac{W\cos\theta}{2\sin(\phi - \theta)}\right)\sin\phi - W = 0$$

$$A_y = \frac{W(\sin\phi\cos\theta - 2\cos\phi\sin\theta)}{2\sin(\phi - \theta)} \qquad \textbf{Ans.}$$

Ans:

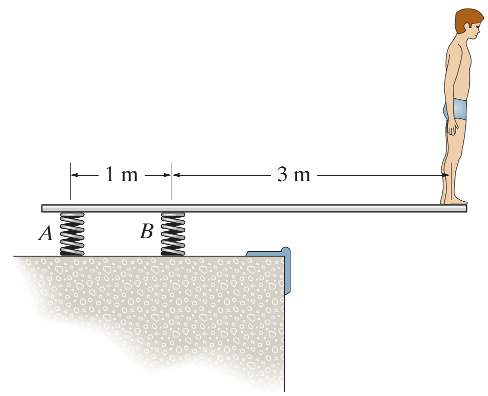
$$T = \frac{W\cos\theta}{2\sin(\phi - \theta)}$$

$$A_x = \frac{W\cos\phi\cos\theta}{2\sin(\phi - \theta)}$$

$$A_y = \frac{W(\sin\phi\cos\theta - 2\cos\phi\sin\theta)}{2\sin(\phi - \theta)}$$

5–53.

A boy stands out at the end of the diving board, which is supported by two springs A and B, each having a stiffness of $k = 15\text{kN/m}$. In the position shown the board is horizontal. If the boy has a mass of 40 kg, determine the angle of tilt which the board makes with the horizontal after he jumps off. Neglect the weight of the board and assume it is rigid.

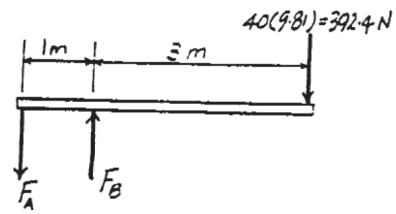


SOLUTION

Equations of Equilibrium: The spring force at A and B can be obtained directly by summing moments about points B and A, respectively.

$$\zeta + \Sigma M_B = 0; \qquad F_A(1) - 392.4(3) = 0 \qquad F_A = 1177.2\text{ N}$$

$$\zeta + \Sigma M_A = 0; \qquad F_B(1) - 392.4(4) = 0 \qquad F_B = 1569.6\text{ N}$$

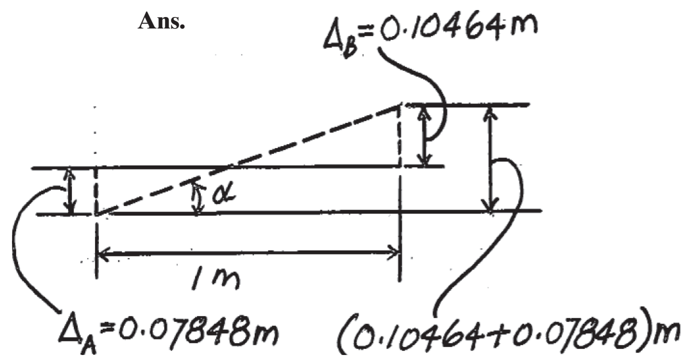


Spring Formula: Applying $\Delta = \dfrac{F}{k}$, we have

$$\Delta_A = \frac{1177.2}{15(10^3)} = 0.07848\text{ m} \qquad \Delta_B = \frac{1569.6}{15(10^3)} = 0.10464\text{ m}$$

Geometry: The angle of tilt α is

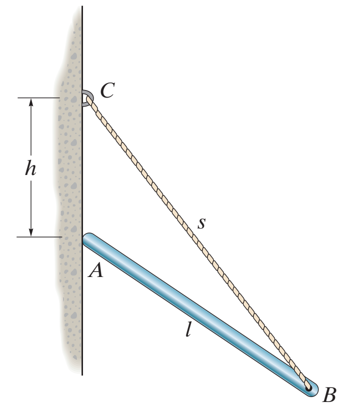
$$\alpha = \tan^{-1}\left(\frac{0.10464 + 0.07848}{1}\right) = 10.4° \qquad \textbf{Ans.}$$



Ans:
$\alpha = 10.4°$

5–54.

The 30-N uniform rod has a length of $l = 1$ m. If $s = 1.5$ m, determine the distance h of placement at the end A along the smooth wall for equilibrium.

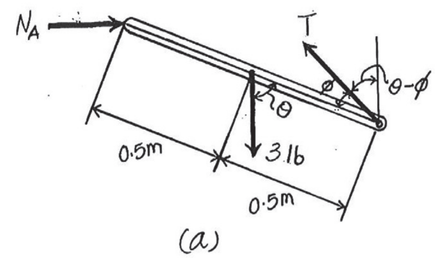


SOLUTION

Equations of Equilibrium: Referring to the FBD of the rod shown in Fig. *a*, write the moment equation of equilibrium about point *A*.

$$\circlearrowleft + \Sigma M_A = 0; \qquad T \sin \phi(1) - 3 \sin \theta(0.5) = 0$$

$$T = \frac{1.5 \sin \theta}{\sin \phi}$$



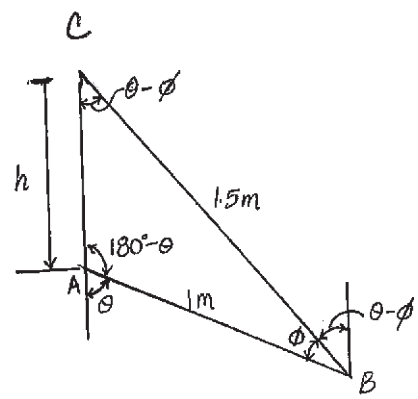
Using this result to write the force equation of equilibrium along *y* axis,

$$+\uparrow \Sigma F_y = 0; \qquad \left(\frac{15 \sin \theta}{\sin \phi}\right) \cos (\theta - \phi) - 3 = 0$$

$$\sin \theta \cos (\theta - \phi) - 2 \sin \phi = 0 \qquad \textbf{(1)}$$

Geometry: Applying the sine law with $\sin (180° - \theta) = \sin \theta$ by referring to Fig. *b*,

$$\frac{\sin \phi}{h} = \frac{\sin \theta}{1.5}; \qquad \sin \theta = \left(\frac{h}{1.5}\right) \sin \theta \qquad \textbf{(2)}$$



Substituting Eq. (2) into (1) yields

$$\sin \theta[\cos (\theta - \phi) - \frac{4}{3}h] = 0$$

since $\sin \theta \neq 0$, then

$$\cos (\theta - \phi) - (4/3)h \qquad \cos (\theta - \phi) = (4/3)h \qquad \textbf{(3)}$$

Again, applying law of cosine by referring to Fig. *b*,

$$l^2 = h^2 + 1.5^2 - 2(h)(1.5) \cos (\theta - \phi)$$

$$\cos (\theta - \phi) = \frac{h^2 + 1.25}{3h} \qquad \textbf{(4)}$$

Equating Eqs. (3) and (4) yields

$$\frac{4}{3}h = \frac{h^2 + 1.25}{3h}$$

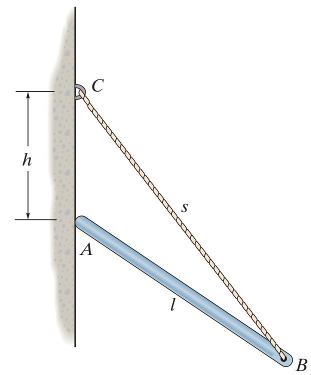
$$3h^2 = 1.25$$

$$h = 0.645 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$h = 0.645$ m

5–55.

The uniform rod has a length l and weight W. It is supported at one end A by a smooth wall and the other end by a cord of length s which is attached to the wall as shown. Determine the placement h for equilibrium.

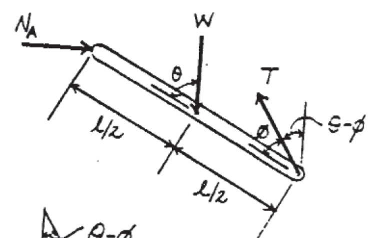


SOLUTION

Equations of Equilibrium: The tension in the cable can be obtained directly by summing moments about point A.

$$\zeta+\Sigma M_A = 0; \qquad T\sin\phi(l) - W\sin\theta\left(\frac{l}{2}\right) = 0$$

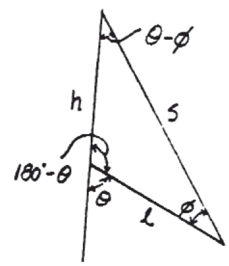
$$T = \frac{W\sin\theta}{2\sin\phi}$$



Using the result $T = \dfrac{W\sin\theta}{2\sin\phi}$,

$$+\uparrow\Sigma F_y = 0; \qquad \frac{W\sin\theta}{2\sin\phi}\cos(\theta-\phi) - W = 0$$

$$\sin\theta\cos(\theta-\phi) - 2\sin\phi = 0 \qquad \textbf{(1)}$$



Geometry: Applying the sine law with $\sin(180° - \theta) = \sin\theta$, we have

$$\frac{\sin\phi}{h} = \frac{\sin\theta}{s} \qquad \sin\phi = \frac{h}{s}\sin\theta \qquad \textbf{(2)}$$

Substituting Eq. (2) into (1) yields

$$\cos(\theta-\phi) = \frac{2h}{s} \qquad \textbf{(3)}$$

Using the cosine law,

$$l^2 = h^2 + s^2 - 2hs\cos(\theta-\phi)$$

$$\cos(\theta-\phi) = \frac{h^2+s^2-l^2}{2hs} \qquad \textbf{(4)}$$

Equating Eqs. (3) and (4) yields

$$\frac{2h}{s} = \frac{h^2+s^2-l^2}{2hs}$$

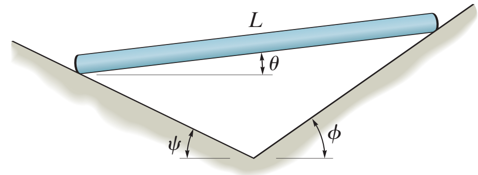
$$h = \sqrt{\frac{s^2-l^2}{3}} \qquad \textbf{Ans.}$$

Ans:

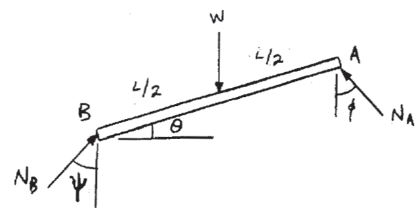
$$h = \sqrt{\frac{s^2-l^2}{3}}$$

***5–56.**

The uniform rod of length L and weight W is supported on the smooth planes. Determine its position θ for equilibrium. Neglect the thickness of the rod.



SOLUTION



$$\circlearrowleft + \Sigma M_B = 0; \quad -W\left(\frac{L}{2}\cos\theta\right) + N_A \cos\phi\,(L\cos\theta) + N_A \sin\phi\,(L\sin\theta) = 0$$

$$N_A = \frac{W\cos\theta}{2\cos(\phi-\theta)} \qquad \textbf{(1)}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad N_B \sin\psi - N_A \sin\phi = 0 \qquad \textbf{(2)}$$

$$+\uparrow \Sigma F_y = 0; \quad N_B \cos\psi + N_A\cos\phi - W = 0$$

$$N_B = \frac{W - N_A\cos\phi}{\cos\psi} \qquad \textbf{(3)}$$

Substituting Eqs. (1) and (3) into Eq. (2):

$$\left(W - \frac{W\cos\theta\cos\phi}{2\cos(\phi-\theta)}\right)\tan\psi - \frac{W\cos\theta\sin\phi}{2\cos(\phi-\theta)} = 0$$

$$2\cos(\phi-\theta)\tan\psi - \cos\theta\tan\psi\cos\phi - \cos\theta\sin\phi = 0$$

$$\sin\theta(2\sin\phi\tan\psi) - \cos\theta(\sin\phi - \cos\phi\tan\psi) = 0$$

$$\tan\theta = \frac{\sin\phi - \cos\phi\tan\psi}{2\sin\phi\tan\psi}$$

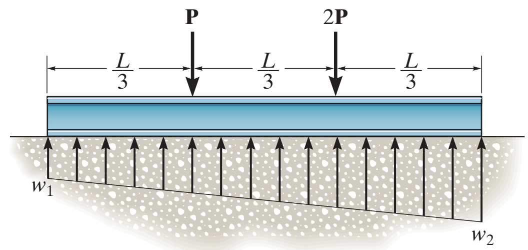
$$\theta = \tan^{-1}\left(\frac{1}{2}\cot\psi - \frac{1}{2}\cot\phi\right) \qquad \textbf{Ans.}$$

Ans:

$\theta = \tan^{-1}\left(\frac{1}{2}\cot\psi - \frac{1}{2}\cot\phi\right)$

5–57.

The beam is subjected to the two concentrated loads. Assuming that the foundation exerts a linearly varying load distribution on its bottom, determine the load intensities w_1 and w_2 for equilibrium if $P = 500$ lb and $L = 12$ ft.



SOLUTION

Equations of Equilibrium: Referring to the FBD of the beam shown in Fig. *a*, we notice that W_1 can be obtained directly by writing moment equations of equilibrium about point A.

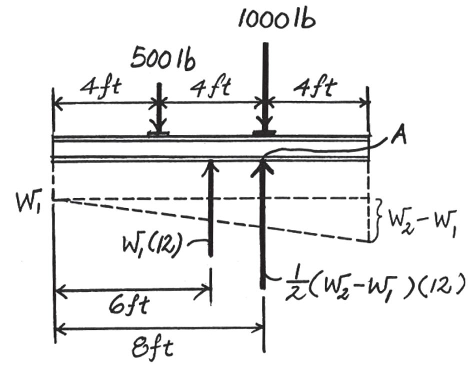
$\circlearrowleft \Sigma M_A = 0;$ $\quad 500(4) - W_1(12)(2) = 0$

$W_1 = 83.33 \text{ lb/ft} = 83.3 \text{ lb/ft}$ **Ans.**

Using this result to write the force equation of equilibrium along y axis,

$+\uparrow \Sigma F_y = 0;$ $\quad 83.33(12) + \frac{1}{2}(W_2 - 83.33)(12) - 500 - 1000 = 0$

$W_2 = 166.67 \text{ lb/ft} = 167 \text{ lb/ft}$ **Ans.**

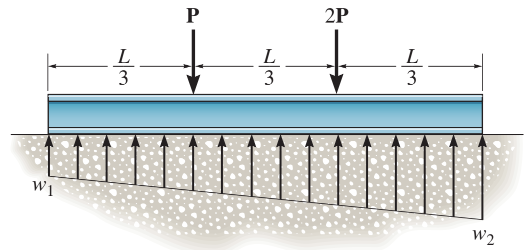


(a)

Ans:
$w_1 = 83.3$ lb/ft
$w_2 = 167$ lb/ft

5–58.

The beam is subjected to the two concentrated loads. Assuming that the foundation exerts a linearly varying load distribution on its bottom, determine the load intensities w_1 and w_2 for equilibrium in terms of the parameters shown.

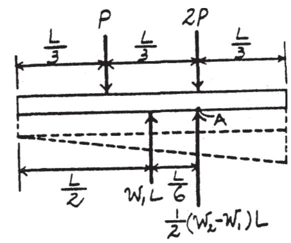


SOLUTION

Equations of Equilibrium: The load intensity w_1 can be determined directly by summing moments about point A.

$\zeta + \Sigma M_A = 0;$ $\quad P\left(\frac{L}{3}\right) - w_1 L\left(\frac{L}{6}\right) = 0$

$$w_1 = \frac{2P}{L}$$ **Ans.**



$+\uparrow \Sigma F_y = 0;$ $\quad \frac{1}{2}\left(w_2 - \frac{2P}{L}\right)L + \frac{2P}{L}(L) - 3P = 0$

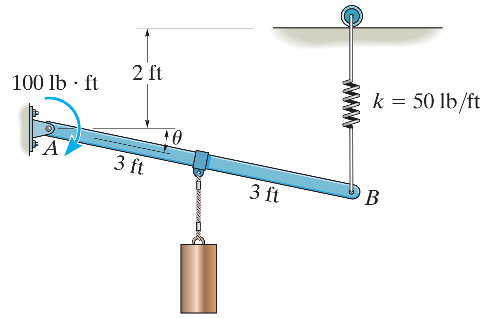
$$w_2 = \frac{4P}{L}$$ **Ans.**

Ans:

$w_1 = \frac{2P}{L}, w_2 = \frac{4P}{L}$

5–59.

The rod supports a weight of 200 lb and is pinned at its end A. If it is also subjected to a couple moment of 100 lb · ft, determine the angle θ for equilibrium. The spring has an unstretched length of 2 ft and a stiffness of $k = 50$ lb/ft.



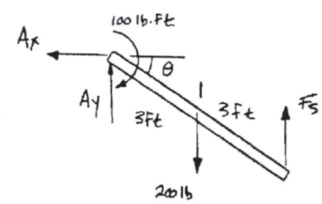
SOLUTION

$\zeta+\Sigma M_A = 0;\qquad 100 + 200\,(3\cos\theta) - F_s\,(6\cos\theta) = 0$

$F_s = kx;\qquad F_s = 50\,(6\sin\theta)$

$$100 + 600\cos\theta - 1800\sin\theta\cos\theta = 0$$

$$\cos\theta - 1.5\sin 2\theta + 0.1667 = 0$$



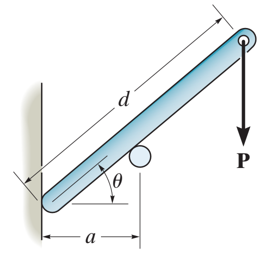
Solving by trial and error,

$$\theta = 23.2^\circ \text{ and } \theta = 85.2^\circ$$ **Ans.**

Ans:
$\theta = 23.2^\circ$
85.2°

***5–60.**

Determine the distance d for placement of the load **P** for equilibrium of the smooth bar in the position θ as shown. Neglect the weight of the bar.



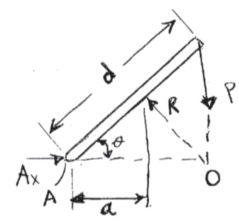
SOLUTION

$+\uparrow \Sigma F_y = 0;\qquad R\cos\theta - P = 0$

$\circlearrowleft + \Sigma M_A = 0;\qquad -P(d\cos\theta) + R\left(\dfrac{a}{\cos\theta}\right) = 0$

$$Rd\cos^2\theta = R\left(\frac{a}{\cos\theta}\right)$$

$$d = \frac{a}{\cos^3\theta}$$ **Ans.**



Also;

Require forces to be concurrent at point O.

$$AO = d\cos\theta = \frac{a/\cos\theta}{\cos\theta}$$

Thus,

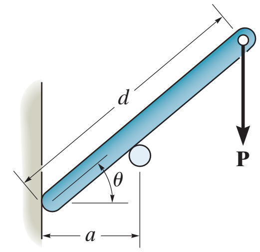
$$d = \frac{a}{\cos^3\theta}$$ **Ans.**

Ans:

$d = \dfrac{a}{\cos^3\theta}$

5–61.

If $d = 1$ m, and $\theta = 30°$, determine the normal reaction at the smooth supports and the required distance a for the placement of the roller if $P = 600$ N. Neglect the weight of the bar.



SOLUTION

Equations of Equilibrium: Referring to the FBD of the rod shown in Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad N_B = \left(\dfrac{a}{\cos 30°}\right) - 600 \cos 30°(1) = 0$

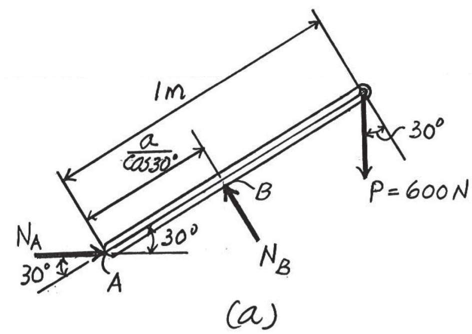
$$N_B = \frac{450}{a} \qquad \textbf{(1)}$$

$\nwarrow^{+} \Sigma F_{y'} = 0;$ $\quad N_B - N_A \sin 30° - 600 \cos 30° = 0$

$$N_B - 0.5N_A = 600 \cos 30° \qquad \textbf{(2)}$$

$\overset{+}{\nearrow} \Sigma F_{x'} = 0;$ $\quad N_A \cos 30° - 600 \sin 30° = 0$

$$N_A = 346.41 \text{ N} = 346 \text{ N} \qquad \textbf{Ans.}$$



Substitute this result into Eq (2),

$$N_B - 0.5(346.41) = 600 \cos 30°$$

$$N_B = 692.82 \qquad N = 693 \text{ N} \qquad \textbf{Ans.}$$

Substitute this result into Eq (1),

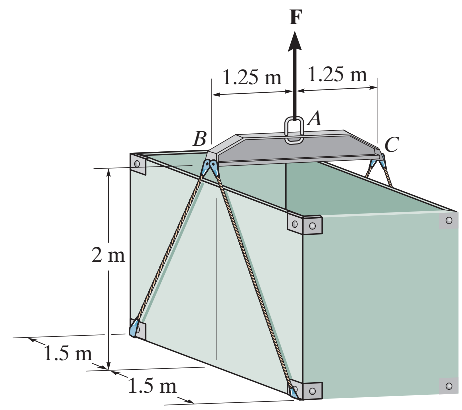
$$692.82 = \frac{450}{a}$$

$$a = 0.6495 \text{ m} \qquad a = 0.650 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$N_A = 346$ N
$N_B = 693$ N
$a = 0.650$ m

5–62.

The uniform load has a mass of 600 kg and is lifted using a uniform 30-kg strongback beam BAC and four wire ropes as shown. Determine the tension in each segment of rope and the force that must be applied to the sling at A.

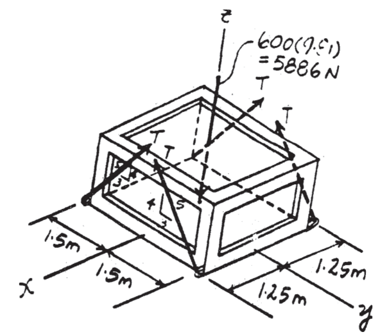


SOLUTION

Equations of Equilibrium: Due to symmetry, all wires are subjected to the same tension. This condition statisfies moment equilibrium about the x and y axes and force equilibrium along y axis.

$$\Sigma F_z = 0; \qquad 4T\left(\frac{4}{5}\right) - 5886 = 0$$

$$T = 1839.375 \text{ N} = 1.84 \text{ kN} \qquad \textbf{Ans.}$$



The force $\mathbf{F}$ applied to the sling A must support the weight of the load and strongback beam. Hence

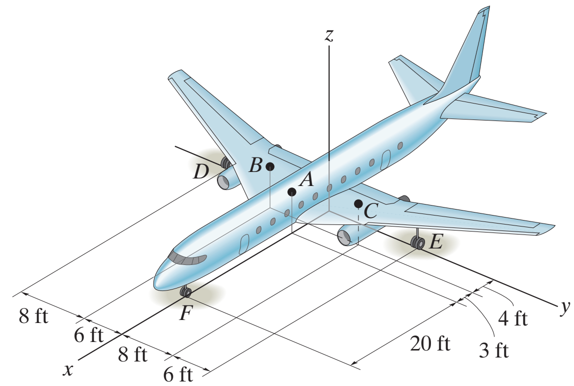
$$\Sigma F_z = 0; \qquad F - 600(9.81) - 30(9.81) = 0$$

$$F = 6180.3 \text{ N} = 6.18 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$T = 1.84$ kN
$F = 6.18$ kN

5–63.

Due to an unequal distribution of fuel in the wing tanks, the centers of gravity for the airplane fuselage A and wings B and C are located as shown. If these components have weights $W_A = 45\,000$ lb, $W_B = 8000$ lb, and $W_C = 6000$ lb, determine the normal reactions of the wheels D, E, and F on the ground.

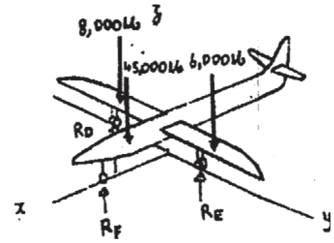


SOLUTION

$\Sigma M_x = 0;\qquad 8000(6) - R_D(14) - 6000(8) + R_E(14) = 0$

$\Sigma M_y = 0;\qquad 8000(4) + 45\,000(7) + 6000(4) - R_F(27) = 0$

$\Sigma F_z = 0;\qquad R_D + R_E + R_F - 8000 - 6000 - 45\,000 = 0$



Solving,

$R_D = 22.6$ kip **Ans.**

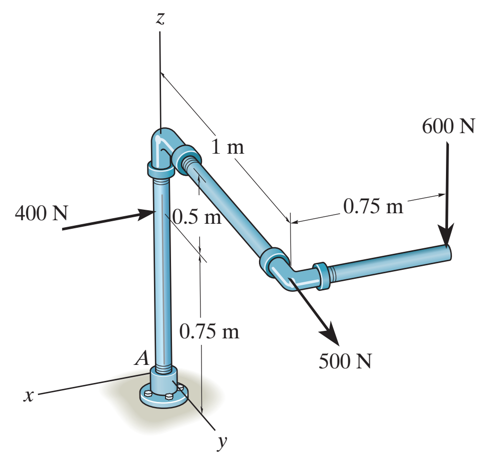
$R_E = 22.6$ kip **Ans.**

$R_F = 13.7$ kip **Ans.**

Ans:
$R_D = 22.6$ kip
$R_E = 22.6$ kip
$R_F = 13.7$ kip

***5–64.**

Determine the components of reaction at the fixed support A. The 400 N, 500 N, and 600 N forces are parallel to the x, y, and z axes, respectively.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the rod shown in Fig. *a*

$\Sigma F_x = 0; \quad A_x - 400 = 0 \quad A_x = 400\text{ N}$ **Ans.**

$\Sigma F_y = 0; \quad 500 - A_y = 0 \quad A_y = 500\text{ N}$ **Ans.**

$\Sigma F_z = 0; \quad A_z - 600 = 0 \quad A_z = 600\text{ N}$ **Ans.**

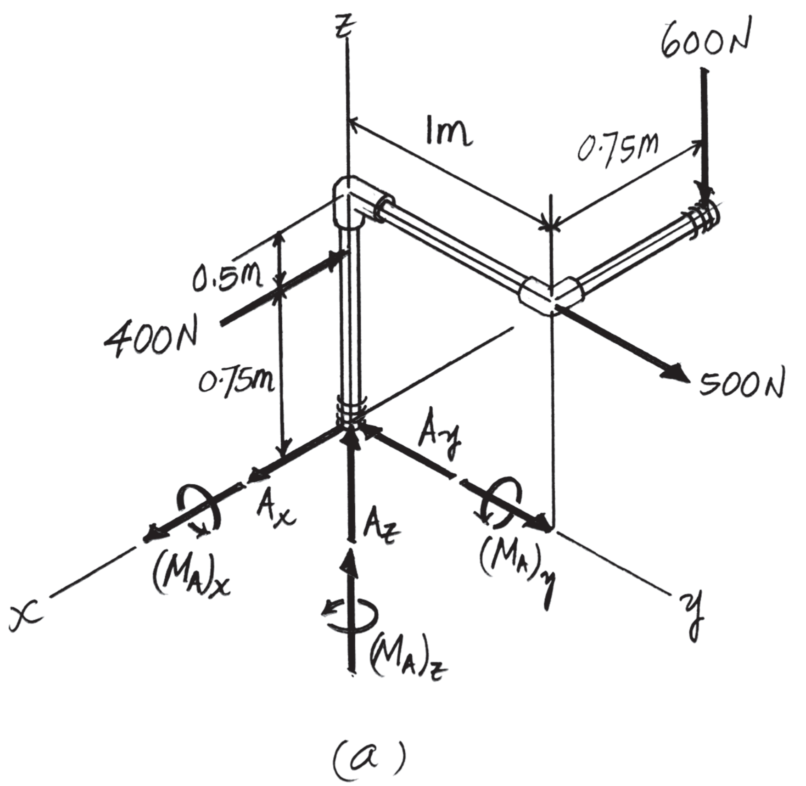
$\Sigma M_x = 0; \quad (M_A)_x - 500(1.25) - 600(1) = 0$

$(M_A)_x = 1225\text{ N}\cdot\text{m} = 1.225\text{ kN}\cdot\text{m}$ **Ans.**

$\Sigma M_y = 0; \quad (M_A)_y - 400(0.75) - 600(0.75) = 0$

$(M_A)_y = 750\text{ N}\cdot\text{m}$ **Ans.**

$\Sigma M_z = 0; \quad (M_A)_z = 0$ **Ans.**

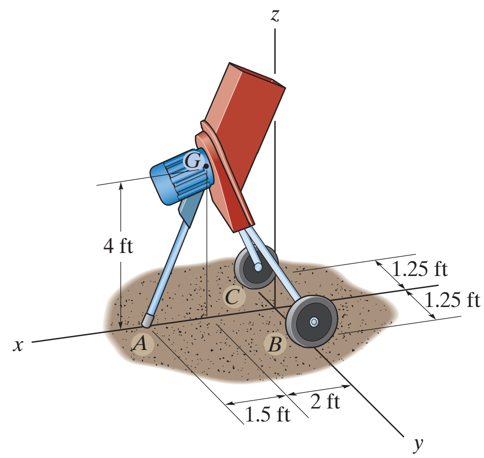


(a)

Ans:
$A_x = 400\text{ N}$
$A_y = 500\text{ N}$
$A_z = 600\text{ N}$
$(M_A)_x = 1.225\text{ kN}\cdot\text{m}$
$(M_A)_y = 750\text{ N}\cdot\text{m}$
$(M_A)_z = 0$

5–65.

The 50-lb mulching machine has a center of gravity at G. Determine the vertical reactions at the wheels C and B and the smooth contact point A.



SOLUTION

Equations of Equilibrium: From the free-body diagram of the mulching machine, Fig. a, N_A can be obtained by writing the moment equation of equilibrium about the y axis.

$$\Sigma M_y = 0; \quad 50(2) - N_A(1.5 + 2) = 0$$

$$N_A = 28.57 \text{ lb} = 28.6 \text{ lb} \qquad \textbf{Ans.}$$

Using the above result and writing the moment equation of equilibrium about the x axis and the force equation of equilibrium along the z axis, we have

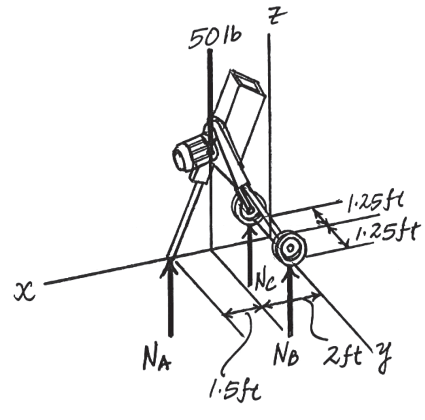
$$\Sigma M_x = 0; \quad N_B(1.25) - N_C(1.25) = 0 \qquad \textbf{(1)}$$

$$\Sigma F_z = 0; \quad N_B + N_C + 28.57 - 50 = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$N_B = N_C = 10.71 \text{ lb} = 10.7 \text{ lb} \qquad \textbf{Ans.}$$

Note: If we write the force equation of equilibrium $\Sigma F_x = 0$ and $\Sigma F_y = 0$ and the moment equation of equilibrium $\Sigma M_z = 0$. This indicates that equilibrium is satisfied.

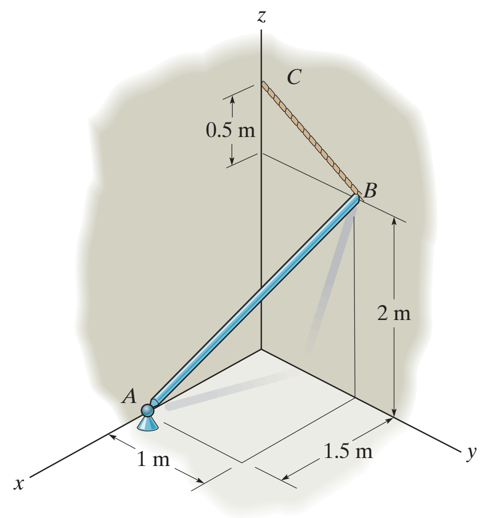


(a)

Ans:
$N_A = 28.6$ lb
$N_B = 10.7$ lb, $N_C = 10.7$ lb

5–66.

The smooth uniform rod AB is supported by a ball-and-socket joint at A, the wall at B, and cable BC. Determine the components of reaction at A, the tension in the cable, and the normal reaction at B if the rod has a mass of 20 kg.



SOLUTION

Force And Position Vectors. The coordinates of points A, B and G are $A(1.5, 0, 0)$ m, $B(0, 1, 2)$ m, $C(0, 0, 2.5)$ m and $G(0.75, 0.5, 1)$ m

$$F_A = -A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{T}_{BC} = T_{BC}\left(\frac{\mathbf{r}_{BC}}{r_{BC}}\right) = T_{BC}\left[\frac{(0-1)\mathbf{j} + (2.5-2)k}{\sqrt{(0-1)^2 + (2.5-2)^2}}\right] = -\frac{1}{\sqrt{1.25}}T_{BC}\mathbf{j} + \frac{0.5}{\sqrt{1.25}}T_{BC}\,\mathbf{k}$$

$$\mathbf{N}_B = N_B\mathbf{i}$$

$$\mathbf{W} = \{-20(9.81)\mathbf{k}\}\text{ N}$$

$$\mathbf{r}_{AG} = (0.75-1.5)\mathbf{i} + (0.5-0)\mathbf{j} + (1-0)\mathbf{k} = \{-0.75\mathbf{i} + 0.5\mathbf{j} + \mathbf{k}\}\text{ m}$$

$$\mathbf{r}_{AB} = (0-1.5)\mathbf{i} + (1-0)\mathbf{j} + (2-0)\mathbf{k} = \{-1.5\mathbf{i} + \mathbf{j} + 2\mathbf{k}\}\text{ m}$$

Equations of Equilibrium. Referring to the *FBD* of the rod shown in Fig. *a*, the force equation of equilibrium gives

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_A + \mathbf{T}_{BC} + \mathbf{N}_B + \mathbf{W} = 0$$

$$(-A_x + N_B)\mathbf{i} + \left(A_y - \frac{1}{\sqrt{1.25}}T_{BC}\right)\mathbf{j} + \left[A_z + \frac{0.5}{\sqrt{1.25}}T_{BC} - 20\,(9.81)\right]\mathbf{k} = 0$$

Equating **i**, **j** and **k** components,

$$-A_x + N_B = 0 \qquad \textbf{(1)}$$

$$A_y - \frac{1}{\sqrt{1.25}}T_{BC} = 0 \qquad \textbf{(2)}$$

$$A_z + \frac{0.5}{\sqrt{1.25}}T_{BC} - 20(9.81) = 0 \qquad \textbf{(3)}$$

The moment equation of equilibrium gives

$$\Sigma\mathbf{M}_A = 0; \qquad \mathbf{r}_{AG}\times\mathbf{W} + \mathbf{r}_{AB}\times(\mathbf{T}_{BC} + \mathbf{N}_B) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -0.75 & 0.5 & 1 \\ 0 & 0 & -20(9.81) \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ -1.5 & 1 & 2 \\ N_B & -\frac{1}{\sqrt{1.25}}T_{BC} & \frac{0.5}{\sqrt{1.25}}T_{BC} \end{vmatrix} = 0$$

$$\left(\frac{0.5}{\sqrt{1.25}}T_{BC} + \frac{2}{\sqrt{1.25}}T_{BC} - 98.1\right)\mathbf{i} + \left(\frac{0.75}{\sqrt{1.25}}T_{BC} + 2N_B - 147.15\right)\mathbf{j} + \left(\frac{1.5}{\sqrt{1.25}}T_{BC} - N_B\right)\mathbf{k} = 0$$

5–66. Continued

Equating **i**, **j** and **k** Components

$$\frac{0.5}{\sqrt{1.25}} T_{BC} + \frac{2}{\sqrt{1.25}} T_{BC} - 98.1 = 0 \quad \textbf{(4)}$$

$$\frac{0.75}{\sqrt{1.25}} T_{BC} + 2N_B - 147.15 = 0 \quad \textbf{(5)}$$

$$\frac{1.5}{\sqrt{1.25}} T_{BC} - N_B = 0 \quad \textbf{(6)}$$

Solving Eqs. (1) to (6)

$T_{BC} = 43.87\text{ N} = 43.9\text{ N}$ **Ans.**

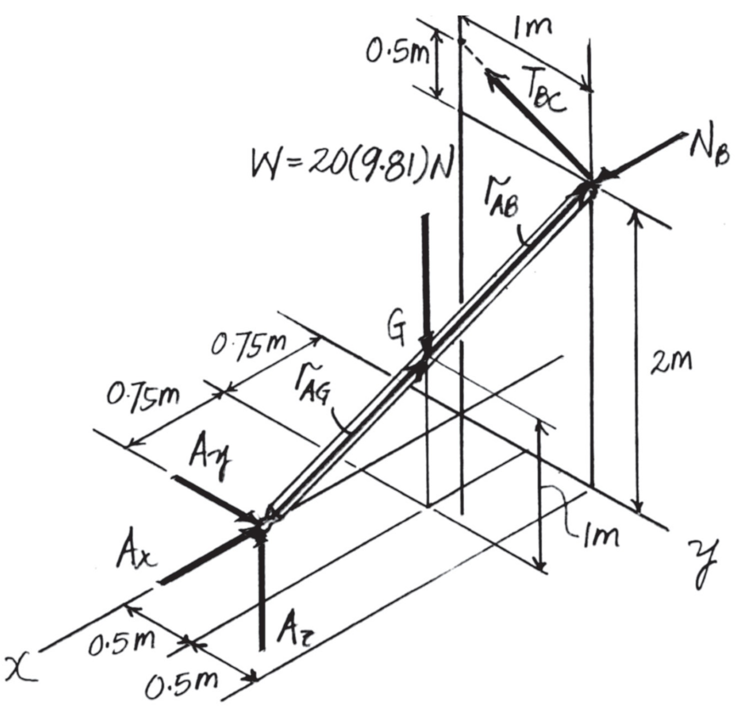
$N_B = 58.86\text{ N} = 58.9\text{ N}$ **Ans.**

$A_x = 58.86\text{ N} = 58.9\text{ N}$ **Ans.**

$A_y = 39.24\text{ N} = 39.2\text{ N}$ **Ans.**

$A_z = 176.58\text{ N} = 177\text{ N}$ **Ans.**

Note: One of the equations (4), (5) and (6) is redundant that will be satisfied automatically.

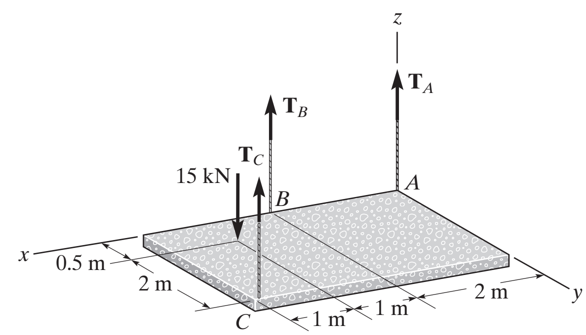


(a)

Ans:
$T_{BC} = 43.9\text{ N}$
$N_B = 58.9\text{ N}$
$A_x = 58.9\text{ N}$
$A_y = 39.2\text{ N}$
$A_z = 177\text{ N}$

5–67.

The uniform concrete slab has a mass of 2400 kg. Determine the tension in each of the three parallel supporting cables when the slab is held in the horizontal plane as shown.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the slab shown in Fig. *a*, we notice that T_C can be obtained directly by writing the moment equation of equilibrium about the x axis.

$\Sigma M_x = 0;\quad T_C(2.5) - 2400(9.81)(1.25) - 15(10^3)(0.5) = 0$

$T_C = 14{,}772\text{ N} = 14.8\text{ kN}$ **Ans.**

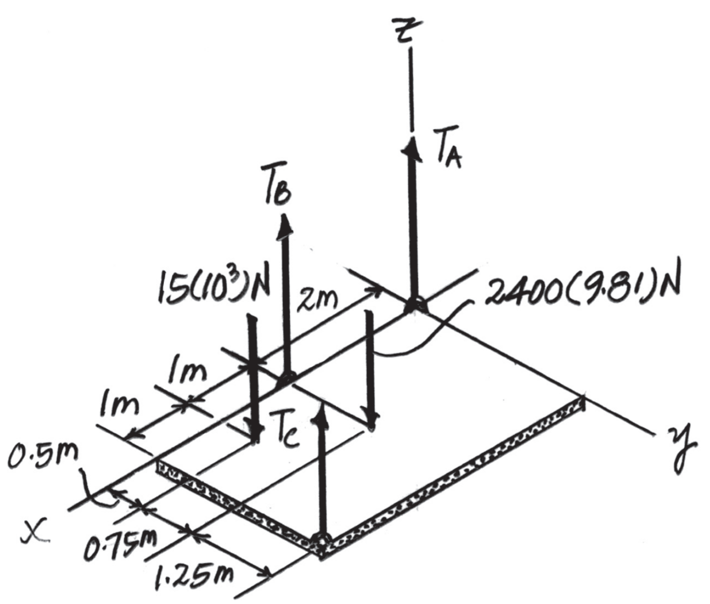
Using this result to write moment equation of equilibrium about y axis and force equation of equilibrium along z axis,

$\Sigma M_y = 0;\quad T_B(2) + 14{,}772(4) - 2400(9.81)(2) - 15(10^3)(3) = 0$

$T_B = 16{,}500\text{ N} = 16.5\text{ kN}$ **Ans.**

$\Sigma F_z = 0;\quad T_A + 16{,}500 + 14{,}772 - 2400(9.81) - 15(10^3) = 0$

$T_A = 7272\text{ N} = 7.27\text{ kN}$ **Ans.**

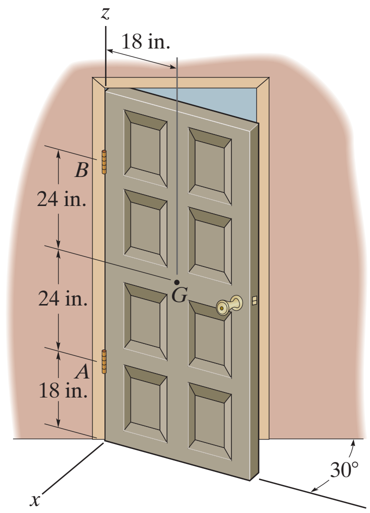


(a)

Ans:
$T_C = 14.8\text{ kN}$
$T_B = 16.5\text{ kN}$
$T_A = 7.27\text{ kN}$

***5–68.**

The 100-lb door has its center of gravity at G. Determine the components of reaction at hinges A and B if hinge B resists only forces in the x and y directions and A resists forces in the x, y, z directions.



SOLUTION

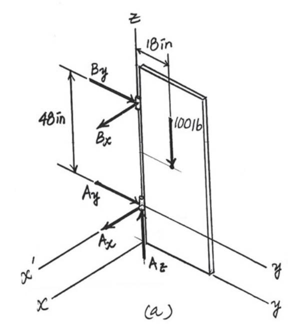
Equations of Equilibrium: From the free-body diagram of the door, Fig. *a*, $B_{y'}$, B_x, and A_z can be obtained by writing the moment equation of equilibrium about the x' and y' axes and the force equation of equilibrium along the z axis.

$\Sigma M_{x'} = 0;$ $\quad -B_y(48) - 100(18) = 0$

$B_y = -37.5 \text{ lb}$ **Ans.**

$\Sigma M_{y'} = 0;$ $\quad B_x = 0$ **Ans.**

$\Sigma F_z = 0;$ $\quad -100 + A_z = 0;$ $\quad A_z = 100 \text{ lb}$ **Ans.**



Using the above result and writing the force equations of equilibrium along the x and y axes, we have

$\Sigma F_x = 0;$ $\quad A_x = 0$ **Ans.**

$\Sigma F_y = 0;$ $\quad A_y + (-37.5) = 0$

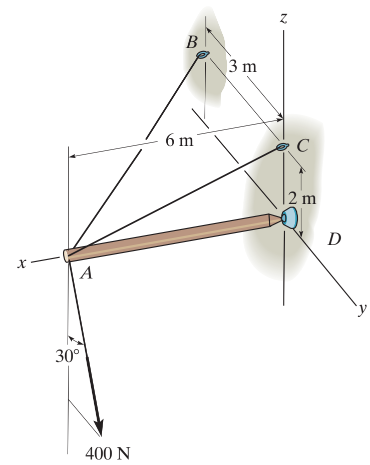
$A_y = 37.5 \text{ lb}$ **Ans.**

The negative sign indicates that $\mathbf{B}_y$ acts in the opposite sense to that shown on the free-body diagram. If we write the moment equation of equilibrium $\Sigma M_z = 0$, it shows that equilibrium is satisfied.

Ans:
$B_y = -37.5 \text{ lb}$
$B_x = 0$
$A_z = 100 \text{ lb}$
$A_x = 0$
$A_y = 37.5 \text{ lb}$

5–69.

Determine the tension in each cable and the components of reaction at D needed to support the load.



SOLUTION

Force And Position Vectors. The coordinates of points A, B, and C are $A(6, 0, 0)$ m, $B(0, -3, 2)$ m and $C(0, 0, 2)$ m respectively.

$$\mathbf{F}_{AB} = F_{AB}\left(\frac{\mathbf{r}_{AB}}{r_{AB}}\right) = F_{AB}\left[\frac{(0-6)\mathbf{i} + (-3-0)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(0-6)^2 + (-3-0)^2 + (2-0)^2}}\right] = -\frac{6}{7}F_{AB}\mathbf{i} - \frac{3}{7}F_{AB}\mathbf{j} + \frac{2}{7}F_{AB}\mathbf{k}$$

$$\mathbf{F}_{AC} = F_{AC}\left(\frac{\mathbf{r}_{AC}}{r_{AC}}\right) = F_{AC}\left[\frac{(0-6)\mathbf{i} + (2-0)\mathbf{k}}{\sqrt{(0-6)^2 + (2-0)^2}}\right] = -\frac{6}{\sqrt{40}}F_{AC}\mathbf{i} + \frac{2}{\sqrt{40}}F_{AC}\mathbf{k}$$

$$\mathbf{F} = 400\,(\sin 30°\mathbf{j} - \cos 30°\mathbf{k}) = \{200\mathbf{j} - 346.41\mathbf{k}\}\text{N}$$

$$F_D = D_x\mathbf{i} + D_y\mathbf{j} + D_z\mathbf{k}$$

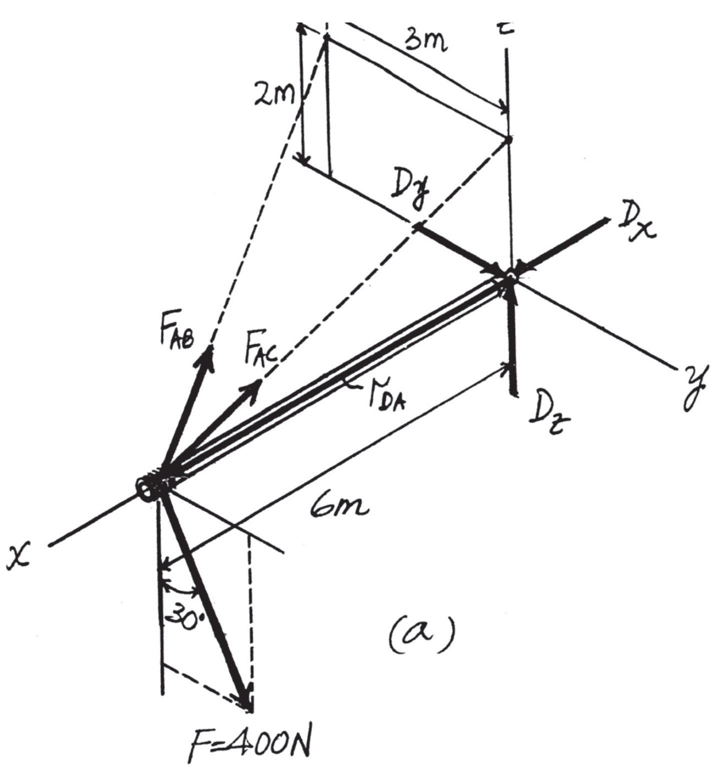
$$\mathbf{r}_{DA} = \{6\mathbf{i}\}\text{ m}$$

Referring to the *FBD* of the rod shown in Fig. *a*, the force equation of equilibrium gives

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F} + \mathbf{F}_D = 0$$

$$\left(-\frac{6}{7}F_{AB} - \frac{6}{\sqrt{40}}F_{AC} + D_x\right)\mathbf{i} + \left(-\frac{3}{7}F_{AB} + D_y + 200\right)\mathbf{j}$$

$$+\left(\frac{2}{7}F_{AB} + \frac{2}{\sqrt{40}}F_{AC} + D_z - 346.41\right)\mathbf{k} = 0$$



(a)

5–69. Continued

Equating **i**, **j** and **k** components,

$$-\frac{6}{7}F_{AB} - \frac{6}{\sqrt{40}}F_{AC} + D_x = 0 \qquad \textbf{(1)}$$

$$-\frac{3}{7}F_{AB} + D_y + 200 = 0 \qquad \textbf{(2)}$$

$$\frac{2}{7}F_{AB} + \frac{2}{\sqrt{40}}F_{AC} + D_z - 346.41 = 0 \qquad \textbf{(3)}$$

Moment equation of equilibrium gives

$\Sigma\mathbf{M}_D = 0; \qquad \mathbf{r}_{DA} \times (\mathbf{F}_{AB} + \mathbf{F}_{AC} + \mathbf{F}) = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0 & 0 \\ \left(-\frac{6}{7}F_{AB} - \frac{6}{\sqrt{40}}F_{AC}\right) & \left(-\frac{3}{7}F_{AB} + 200\right) & \left(\frac{2}{7}F_{AB} + \frac{2}{\sqrt{40}}F_{AC} - 346.41\right) \end{vmatrix} = 0$$

$$-6\left(\frac{2}{7}F_{AB} + \frac{2}{\sqrt{40}}F_{AC} - 346.41\right)\mathbf{j} + 6\left(-\frac{3}{7}F_{AB} + 200\right)\mathbf{k} = 0$$

Equating **j** and **k** Components,

$$-6\left(\frac{2}{7}F_{AB} + \frac{2}{\sqrt{40}}F_{AC} - 346.41\right) = 0 \qquad \textbf{(4)}$$

$$6\left(-\frac{3}{7}F_{AB} + 200\right) = 0 \qquad \textbf{(5)}$$

Solving Eqs. (1) to (5)

$F_{AB} = 466.67 \text{ N} = 467 \text{ N}$ **Ans.**

$F_{AC} = 673.81 \text{ N} = 674 \text{ N}$ **Ans.**

$D_x = 1039.23 \text{ N} = 1.04 \text{ kN}$ **Ans.**

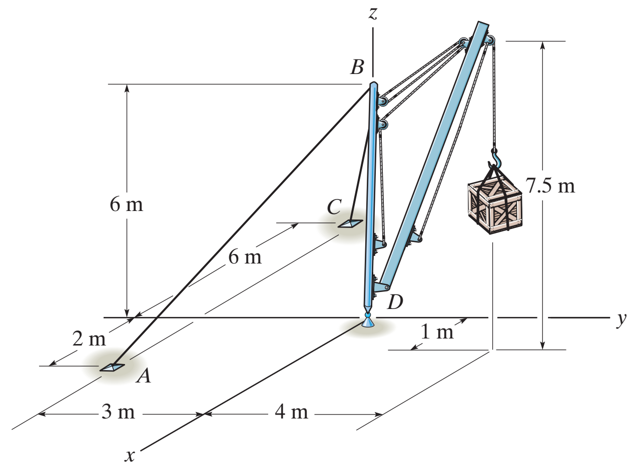
$D_y = 0$ **Ans.**

$D_z = 0$ **Ans.**

Ans:
$F_{AB} = 467$ N
$F_{AC} = 674$ N
$D_x = 1.04$ kN
$D_y = 0$
$D_z = 0$

5–70.

The stiff-leg derrick used on ships is supported by a ball-and-socket joint at D and two cables BA and BC. The cables are attached to a smooth collar ring at B, which allows rotation of the derrick about z axis. If the derrick supports a crate having a mass of 200 kg, determine the tension in the cables and the x, y, z components of reaction at D.



SOLUTION

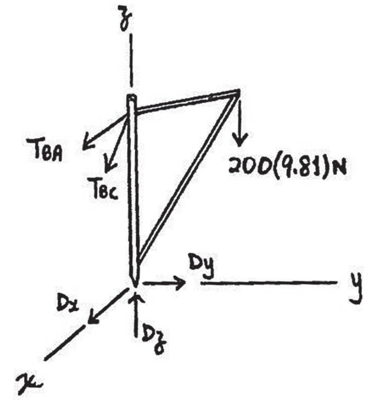
$\Sigma F_x = 0; \qquad D_x + \frac{2}{7}T_{BA} - \frac{6}{9}T_{BC} = 0$

$\Sigma F_y = 0; \qquad D_y - \frac{3}{7}T_{BA} - \frac{3}{9}T_{BC} = 0$

$\Sigma F_z = 0; \qquad D_z - \frac{6}{7}T_{BA} - \frac{6}{9}T_{BC} - 200(9.81) = 0$

$\Sigma M_x = 0; \qquad \frac{3}{7}T_{BA}(6) + \frac{3}{9}T_{BC}(6) - 200(9.81)(4) = 0$

$\Sigma M_y = 0; \qquad \frac{2}{7}T_{BA}(6) - \frac{6}{9}T_{BC}(6) + 200(9.81)(1) = 0$



$T_{BA} = 2.00 \text{ kN}$ **Ans.**

$T_{BC} = 1.35 \text{ kN}$ **Ans.**

$D_x = 0.327 \text{ kN}$ **Ans.**

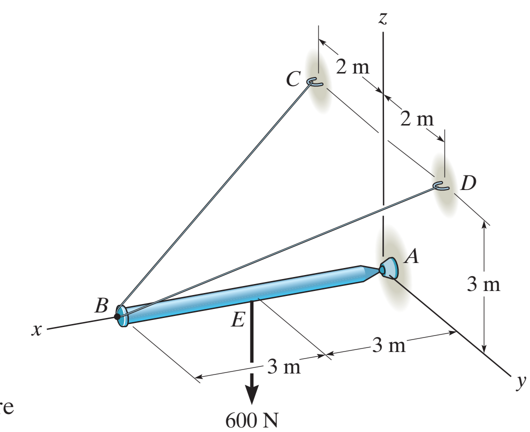
$D_y = 1.31 \text{ kN}$ **Ans.**

$D_z = 4.58 \text{ kN}$ **Ans.**

Ans:
$T_{BA} = 2.00 \text{ kN}$
$T_{BC} = 1.35 \text{ kN}$
$D_x = 0.327 \text{ kN}$
$D_y = 1.31 \text{ kN}$
$D_z = 4.58 \text{ kN}$

5–71.

Determine the components of reaction at the ball-and-socket joint A and the tension in each cable necessary for equilibrium of the rod.



SOLUTION

Force And Position Vectors. The coordinates of points A, B, C, D and E are $A(0, 0, 0)$, $B(6, 0, 0)$, $C(0, -2, 3)$ m, $D(0, 2, 3)$ m and $E(3, 0, 0)$ m respectively.

$$\mathbf{F}_{BC} = F_{BC}\left(\frac{\mathbf{r}_{BC}}{r_{BC}}\right) = F_{BC}\left[\frac{(0-6)\mathbf{i} + (-2-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-6)^2 + (-2-0)^2 + (3-0)^2}}\right] = -\frac{6}{7}F_{BC}\mathbf{i} - \frac{2}{7}F_{BC}\mathbf{j} + \frac{3}{7}F_{BC}\mathbf{k}$$

$$\mathbf{F}_{BD} = F_{BD}\left(\frac{\mathbf{r}_{BD}}{r_{BD}}\right) = F_{BD}\left[\frac{(0-6)\mathbf{i} + (2-0)\mathbf{j} + (3-0)\mathbf{k}}{\sqrt{(0-6)^2 + (2-0)^2 + (3-0)^2}}\right] = -\frac{6}{7}F_{BD}\mathbf{i} + \frac{2}{7}F_{BD}\mathbf{j} + \frac{3}{7}F_{BD}\mathbf{k}$$

$$F_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

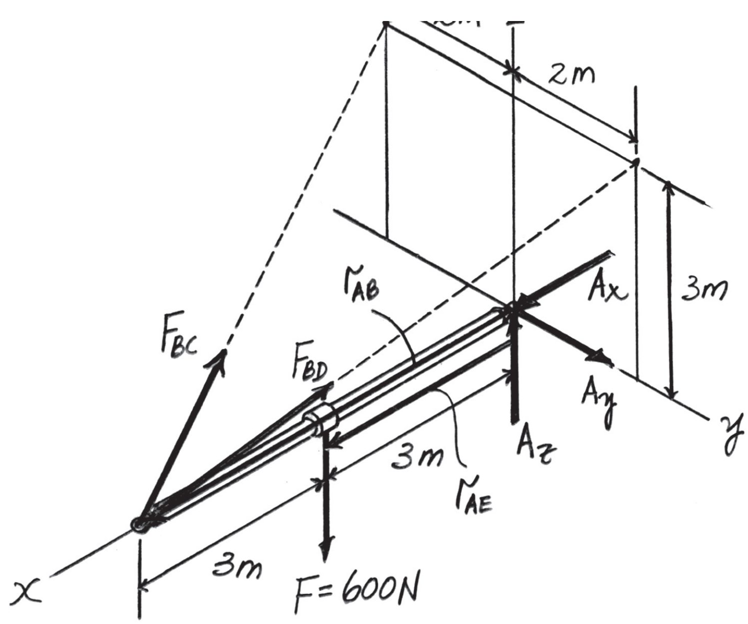
$$F = \{-600\mathbf{k}\}\text{ N}$$

$$r_{AB} = \{6\mathbf{i}\}\text{ m} \qquad r_{AE} = \{3\mathbf{i}\}\text{ m}$$

Equations of Equilibrium. Referring to the *FBD* of the rod shown in Fig. *a*, the force equation of equilibrium gives

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{BC} + \mathbf{F}_{BD} + \mathbf{F}_A + \mathbf{F} = 0$$

$$\left(-\frac{6}{7}F_{BC} - \frac{6}{7}F_{BD} + A_x\right)\mathbf{i} + \left(\frac{2}{7}F_{BD} - \frac{2}{7}F_{BC} + A_y\right)\mathbf{j} + \left(\frac{3}{7}F_{BC} + \frac{3}{7}F_{BD} + A_z - 600\right)\mathbf{k} = 0$$



(a)

5–71. Continued

Equating **i**, **j** and **k** components,

$$-\frac{6}{7}F_{BC} - \frac{6}{7}F_{BD} + A_x = 0 \quad \textbf{(1)}$$

$$\frac{2}{7}F_{BD} - \frac{2}{7}F_{BC} + A_y = 0 \quad \textbf{(2)}$$

$$\frac{3}{7}F_{BC} + \frac{3}{7}F_{BD} + A_z - 600 = 0 \quad \textbf{(3)}$$

The moment equation of equilibrium gives

$\Sigma \mathbf{M}_A = 0; \qquad \mathbf{r}_{AE} \times \mathbf{F} + \mathbf{r}_{AB} \times (\mathbf{F}_{BC} + \mathbf{F}_{BD}) = 0$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ 0 & 0 & -600 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 6 & 0 & 0 \\ -\frac{6}{7}(F_{BC} + F_{BD}) & \frac{2}{7}(F_{BD} - F_{BC}) & \frac{3}{7}(F_{BC} + F_{BD}) \end{vmatrix} = 0$$

$$\left[1800 - \frac{18}{7}(F_{BC} + F_{BD})\right]\mathbf{j} + \frac{12}{7}(F_{BD} - F_{BC})\mathbf{k} = 0$$

Equating **j** and **k** components,

$$1800 - \frac{18}{7}(F_{BC} + F_{BD}) = 0 \quad \textbf{(4)}$$

$$\frac{12}{7}(F_{BD} - F_{BC}) = 0 \quad \textbf{(5)}$$

Solving Eqs. (1) to (5),

$F_{BD} = F_{BC} = 350\text{ N}$ **Ans.**

$A_x = 600\text{ N}$ **Ans.**

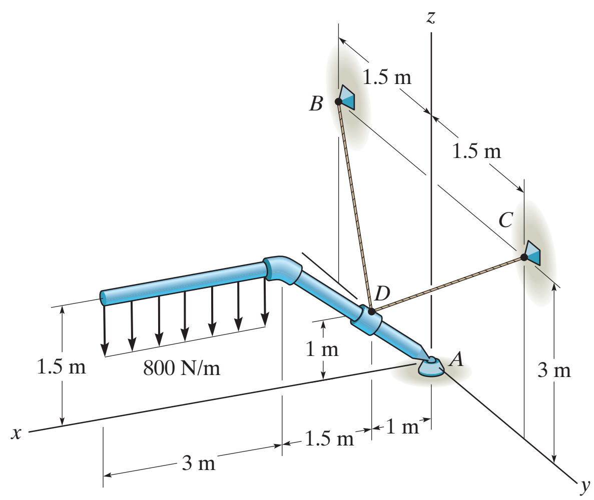
$A_y = 0$ **Ans.**

$A_z = 300\text{ N}$ **Ans.**

Ans:
$F_{BD} = F_{BC} = 350\text{ N}$
$A_x = 600\text{ N}$
$A_y = 0$
$A_z = 300\text{ N}$

***5–72.**

Determine the components of reaction at the ball-and-socket joint A and the tension in the supporting cables DB and DC.



SOLUTION

Force And Position Vectors. The coordinates of points A, B, C, and D are $A(0, 0, 0)$, $B(0, -1.5, 3)$ m, $C(0, 1.5, 3)$ m and $D(1, 0, 1)$ m, respectively.

$$\mathbf{F}_{DC} = F_{DC}\left(\frac{\mathbf{r}_{DC}}{r_{DC}}\right) = F_{DC}\left[\frac{(0-1)\mathbf{i} + (1.5-0)\mathbf{j} + (3-1)\mathbf{k}}{\sqrt{(0-1)^2 + (1.5-0)^2 + (3-1)^2}}\right]$$

$$= -\frac{1}{\sqrt{7.25}}F_{CD}\mathbf{i} + \frac{1.5}{\sqrt{7.25}}F_{DC}\mathbf{j} + \frac{2}{\sqrt{7.25}}F_{DC}\mathbf{k}$$

$$\mathbf{F}_{DB} = F_{DB}\left(\frac{\mathbf{r}_{DB}}{r_{DB}}\right) = F_{DB}\left[\frac{(0-1)\mathbf{i} + (-1.5-0)\mathbf{j} + (3-1)\mathbf{k}}{\sqrt{(0-1)^2 + (-1.5-0)^2 + (3-1)^2}}\right]$$

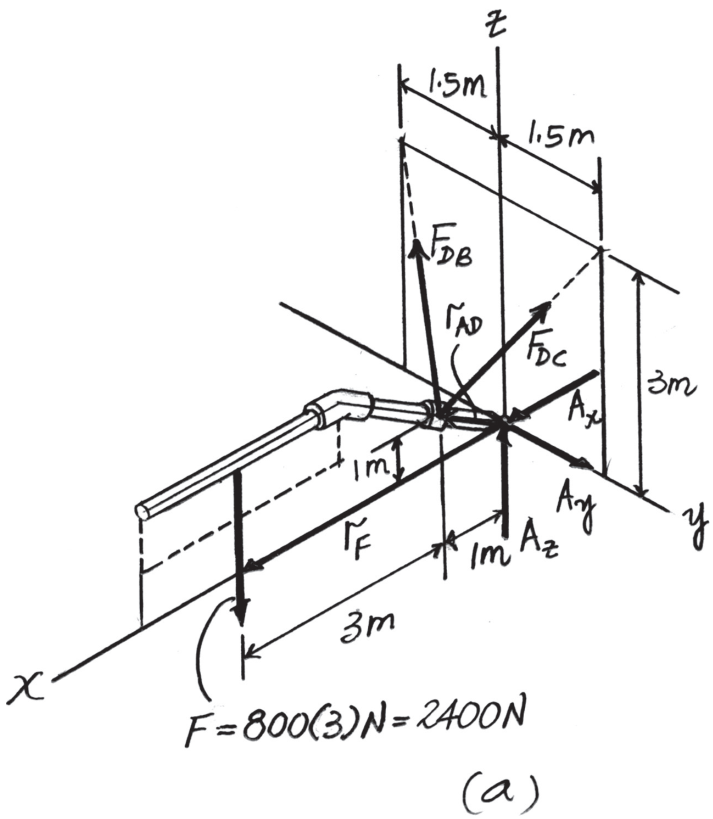
$$= -\frac{1}{\sqrt{7.25}}F_{DB}\mathbf{i} + \frac{1.5}{\sqrt{7.25}}F_{DB}\mathbf{j} + \frac{2}{\sqrt{7.25}}F_{DB}\mathbf{k}$$

$$F_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$F = \{-2400\mathbf{k}\}\text{ N}$$

$$\mathbf{r}_{AD} = (1-0)\mathbf{i} + (1-0)\mathbf{k} = \{\mathbf{i} + \mathbf{k}\}\text{ m}$$

$$\mathbf{r}_F = \{4\mathbf{i}\}\text{ m}$$



***5–72. Continued**

Equations of Equilibrium. Referring to the *FBD* of the assembly shown in Fig. *a*. Force equation of equilibrium gives

$$\Sigma \mathbf{F} = 0; \qquad \mathbf{F}_{DC} + \mathbf{F}_{DB} + \mathbf{F}_A + \mathbf{F} = 0$$

$$\left(-\frac{1}{\sqrt{7.25}}F_{DC} - \frac{1}{\sqrt{7.25}}F_{DB} + A_x\right)\mathbf{i} + \left(\frac{1.5}{\sqrt{7.25}}F_{DC} - \frac{1.5}{\sqrt{7.25}}F_{DB} + A_y\right)\mathbf{j}$$

$$+\left(\frac{2}{\sqrt{7.25}}F_{DC} + \frac{2}{\sqrt{7.25}}F_{DB} + A_z - 2400\right)\mathbf{k} = 0$$

Equating **i**, **j** and **k** components,

$$-\frac{1}{\sqrt{7.25}}F_{DC} - \frac{1}{\sqrt{7.25}}F_{DB} + A_x = 0 \qquad \textbf{(1)}$$

$$\frac{1.5}{\sqrt{7.25}}F_{DC} - \frac{1.5}{7.25}F_{DB} + A_y = 0 \qquad \textbf{(2)}$$

$$\frac{2}{\sqrt{7.25}}F_{DC} + \frac{2}{\sqrt{7.25}}F_{DB} + A_z - 2400 = 0 \qquad \textbf{(3)}$$

Moment equation of equilibrium gives

$$\Sigma M_A = 0; \qquad \mathbf{r}_F \times \mathbf{F} + \mathbf{r}_{AD} \times (\mathbf{F}_{DB} + \mathbf{F}_{DC}) = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 0 & 0 \\ 0 & 0 & -2400 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 1 & 0 & 1 \\ -\frac{1}{\sqrt{7.25}}(F_{DB} + F_{DC}) & \frac{1.5}{\sqrt{7.25}}(F_{DC} - F_{DB}) & \frac{2}{\sqrt{7.25}}(F_{DC} + F_{DB}) \end{vmatrix} = 0$$

$$-\frac{1.5}{\sqrt{7.25}}(F_{DC} - F_{DB})\mathbf{i} + \left[9600 - \frac{3}{\sqrt{7.25}}(F_{DC} + F_{DB})\right]\mathbf{j} + \frac{1.5}{\sqrt{7.25}}(F_{DC} + F_{DB})\mathbf{k} = 0$$

Equating **i**, **j** and **k** Components

$$-\frac{1.5}{\sqrt{7.25}}(F_{DC} - F_{DB}) = 0 \qquad \textbf{(4)}$$

$$9600 - \frac{3}{\sqrt{7.25}}(F_{DC} + F_{DB}) = 0 \qquad \textbf{(5)}$$

$$\frac{1.5}{\sqrt{7.25}}(F_{DC} - F_{DB}) = 0 \qquad \textbf{(6)}$$

Solving Eqs. (1) to (6)

$F_{DC} = F_{DB} = 4308.13 \text{ N} = 4.31 \text{ kN}$ **Ans.**

$A_x = 3200 \text{ N} = 3.20 \text{ kN}$ **Ans.**

$A_y = 0$ **Ans.**

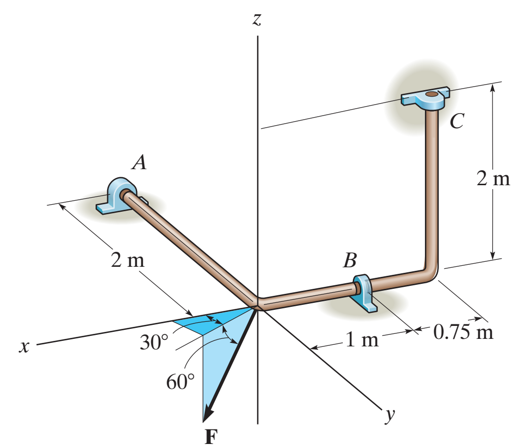
$A_z = -4000 \text{ N} = -4 \text{ kN}$ **Ans.**

Negative sign indicates that A_z directed in the sense opposite to that shown in *FBD*.

Ans:
$F_{DC} = F_{DB} = 4.31 \text{ kN}$
$A_x = 3.20 \text{ kN}$
$A_y = 0$
$A_z = -4 \text{ kN}$

5–73.

The bent rod is supported at A, B, and C by smooth journal bearings. Determine the components of reaction at the bearings if the rod is subjected to the force $F = 800$ N. The bearings are in proper alignment and exert only force reactions on the rod.



SOLUTION

Equations of Equilibrium. The x, y and z components of force F are

$F_x = 800 \cos 60° \cos 30° = 346.41$ N

$F_y = 800 \cos 60° \sin 30° = 200$ N

$F_z = 800 \sin 60° = 692.82$ N

Referring to the *FBD* of the bent rod shown in Fig. a,

$\Sigma M_x = 0; \quad -C_y(2) + B_z(2) - 692.82\,(2) = 0$ **(1)**

$\Sigma M_y = 0; \quad B_z(1) + C_x(2) = 0$ **(2)**

$\Sigma M_z = 0; \quad -C_y(1.75) - C_x(2) - B_y(1) - 346.41(2) = 0$ **(3)**

$\Sigma F_x = 0; \quad A_x + C_x + 346.41 = 0$ **(4)**

$\Sigma F_y = 0; \quad 200 + B_y + C_y = 0$ **(5)**

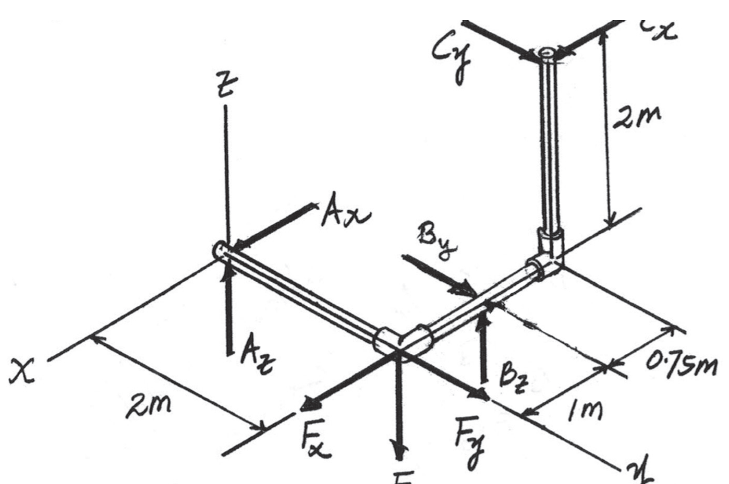
$\Sigma F_z = 0; \quad A_z + B_z - 692.82 = 0$ **(6)**

Solving Eqs. (1) to (6)

$C_y = 800$ N $\quad B_z = -107.18$ N $= 107$ N $\quad B_y = 600$ N **Ans.**

$C_x = 53.59$ N $= 53.6$ N $\quad A_x = 400$ N $\quad A_z = 800$ N **Ans.**

The negative signs indicate that $\mathbf{C}_y$, $\mathbf{B}_z$ and $\mathbf{A}_z$ are directed in the senses opposite to those shown in *FBD*.

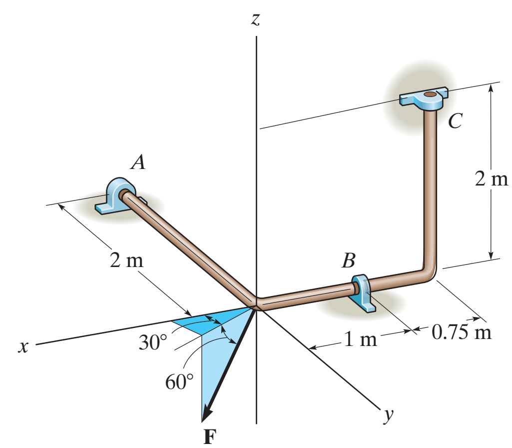


(a)

Ans:
$C_y = 800$ N
$B_z = 107$ N
$B_y = 600$ N
$C_x = 53.6$ N
$A_x = 400$ N
$A_z = 800$ N

5–74.

The bent rod is supported at A, B, and C by smooth journal bearings. Determine the magnitude of F which will cause the positive x component of reaction at the bearing C to be $C_x = 50$ N. The bearings are in proper alignment and exert only force reactions on the rod.



SOLUTION

Equations of Equilibrium. The x, y and z components of force **F** are

$$F_x = F\cos 60°\cos 30° = 0.4330\,F$$

$$F_y = F\cos 60°\sin 30° = 0.25\,F$$

$$F_z = F\sin 60° = 0.8660\,F$$

Here, it is required that $C_x = 50$. Thus, by referring to the *FBD* of the beat rod shown in Fig. *a*,

$\Sigma M_x = 0;\quad -C_y(2) + B_z(2) - 0.8660\,F(2) = 0$ **(1)**

$\Sigma M_y = 0;\quad B_z(1) + 50(2) = 0$ **(2)**

$\Sigma M_z = 0;\quad -C_y(1.75) - 50(2) - B_y(1) - 0.4330\,F(2) = 0$ **(3)**

$\Sigma F_y = 0;\quad 0.25\,F + B_y + C_y = 0$ **(4)**

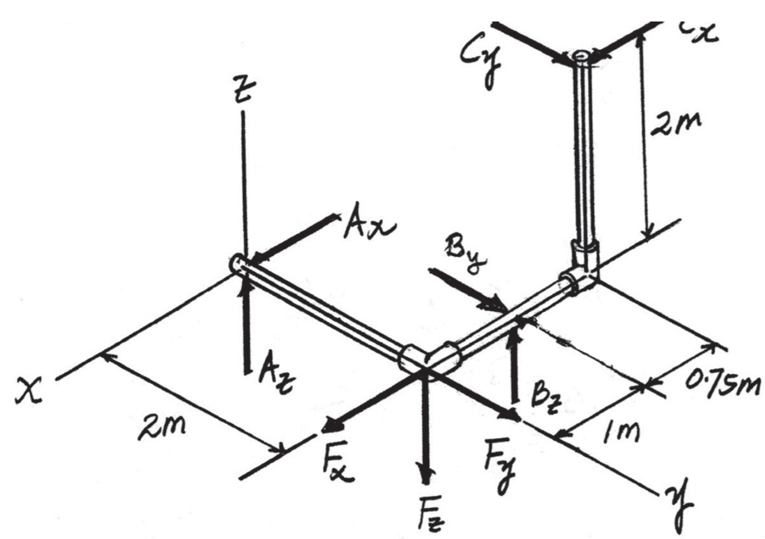
Solving Eqs. (1) to (4)

$F = 746.41\text{ N} = 746\text{ N}$ **Ans.**

$C_y = -746.41$ N

$B_z = -100$ N

$B_y = 559.81$ N

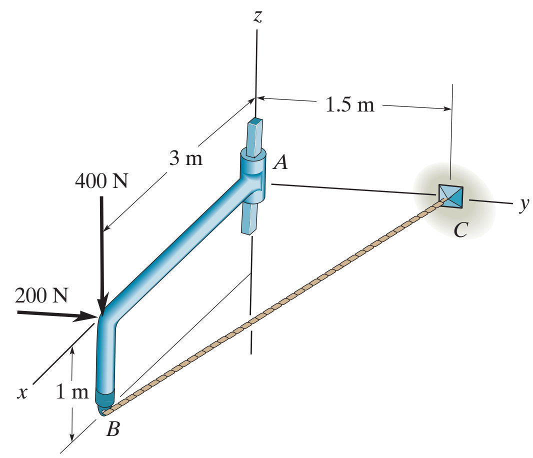


(a)

Ans:
$F = 746$ N

5–75.

Member AB is supported by a cable BC and at A by a *square* rod which fits loosely through the square hole in the collar fixed to the member as shown. Determine the components of reaction at A and the tension in the cable needed to hold the rod in equilibrium.



SOLUTION

Force And Position Vectors. The coordinates of points B and C are $B(3, 0, -1)$ m $C(0, 1.5, 0)$ m, respectively.

$$\mathbf{T}_{BC} = \mathbf{T}_{BC}\left(\frac{\mathbf{r}_{BC}}{\mathbf{r}_{BC}}\right) = T_{BC}\left\{\frac{(0-3)\mathbf{i} + (1.5-0)\mathbf{j} + [0-(-1)]\mathbf{k}}{\sqrt{(0-3)^2 + (1.5-0)^2 + [0-(-1)]^2}}\right\}$$

$$= -\frac{6}{7}T_{BC}\mathbf{i} + \frac{3}{7}T_{BC}\mathbf{j} + \frac{2}{7}T_{BC}\mathbf{k}$$

$$\mathbf{F} = \{200\mathbf{j} - 400\mathbf{k}\}\text{ N}$$

$$F_A = A_x\mathbf{i} + A_y\mathbf{j}$$

$$\mathbf{M}_A = (M_A)_x\mathbf{i} + (M_A)_y\mathbf{j} + (M_A)_z\mathbf{k}$$

$\mathbf{r}_1\{3\,\mathbf{i}\}$ m $\quad r_2 = \{1.5\,\mathbf{j}\}$ m

Equations of Equilibrium. Referring to the *FBD* of member AB shown in Fig. *a*, the force equation of equilibrium gives

$$\Sigma\mathbf{F} = 0; \quad \mathbf{T}_{BC} + \mathbf{F} + \mathbf{F}_A = 0$$

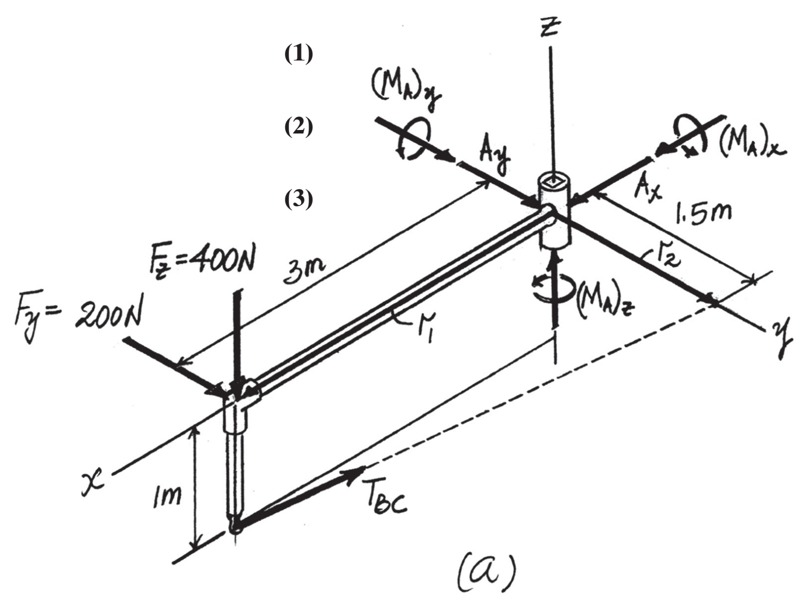
$$\left(-\frac{6}{7}T_{BC} + A_x\right)\mathbf{i} + \left(\frac{3}{7}T_{BC} + 200 + A_y\right)\mathbf{j} + \left(\frac{2}{7}T_{BC} - 400\right)\mathbf{k} = 0$$

Equating **i**, **j** and **k** components

$$-\frac{6}{7}T_{BC} + A_x = 0 \quad \textbf{(1)}$$

$$\frac{3}{7}T_{BC} + 200 + A_y = 0 \quad \textbf{(2)}$$

$$\frac{2}{7}T_{BC} - 400 = 0 \quad \textbf{(3)}$$



5–75. Continued

The moment equation of equilibrium gives

$\Sigma M_A = \mathbf{O};\qquad \mathbf{M}_A + \mathbf{r}_1 \times \mathbf{F} + r_2 \times \mathbf{T}_{BC} = 0$

$$(M_A)_x\mathbf{i} + (M_A)_y\mathbf{j} + (M_A)_z\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & 0 \\ 0 & 200 & -400 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 1.5 & 0 \\ -\frac{6}{7}T_{BC} & \frac{3}{7}T_{BC} & \frac{2}{7}T_{BC} \end{vmatrix} = 0$$

$$\left[(M_A)_x + \frac{3}{7}T_{BC}\right]\mathbf{i} + \left[(M_A)_y + 1200\right]\mathbf{j} + \left[(M_A)_z + \frac{9}{7}T_{BC} + 600\right]\mathbf{k} = 0$$

Equating **i**, **j**, and **k** components,

$$(M_A)_x + \frac{3}{7}T_{BC} = 0 \qquad \textbf{(4)}$$

$$(M_A)_y + 1200 = 0 \qquad \textbf{(5)}$$

$$(M_A)_z + \frac{9}{7}T_{BC} + 600 = 0 \qquad \textbf{(6)}$$

Solving Eqs. (1) to (6),

$T_{BC} = 1400\text{ N} = 1.40\text{ kN}$ **Ans.**

$A_y = 800\text{ N}$ **Ans.**

$A_x = 1200\text{ N} = 1.20\text{ kN}$ **Ans.**

$(M_A)_x = 600\text{ N}\cdot\text{m}$ **Ans.**

$(M_A)_y = -1200\text{ N}\cdot\text{m} = 1.20\text{ kN}\cdot\text{m}$ **Ans.**

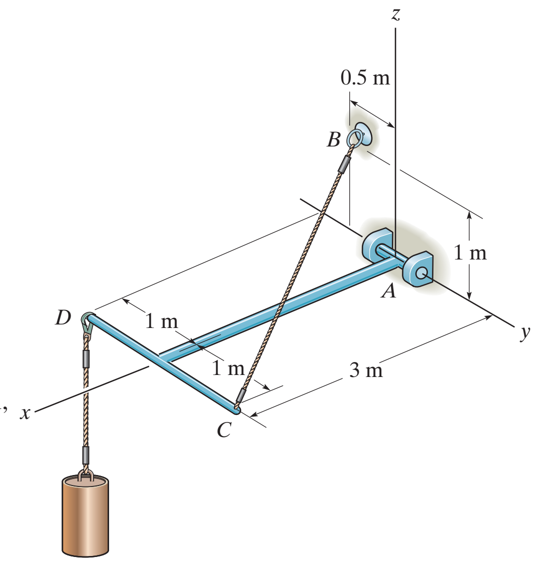
$(M_A)_z = -2400\text{ N}\cdot\text{m} = 2.40\text{ kN}\cdot\text{m}$ **Ans.**

The negative signs indicate that $\mathbf{A}_y$, $(\mathbf{M}_A)_x$, $(\mathbf{M}_A)_y$ and $(\mathbf{M}_A)_z$ are directed in sense opposite to those shown in *FBD*.

Ans:
$T_{BC} = 1.40\text{ kN}$
$A_y = 800\text{ N}$
$A_x = 1.20\text{ kN}$
$(M_A)_x = 600\text{ N}\cdot\text{m}$
$(M_A)_y = 1.20\text{ kN}\cdot\text{m}$
$(M_A)_z = 2.40\text{ kN}\cdot\text{m}$

***5–76.**

The member is supported by a pin at A and cable BC. Determine the components of reaction at these supports if the cylinder has a mass of 40 kg.



SOLUTION

Force And Position Vectors. The coordinates of points B, C and D are $B(0, -0.5, 1)$ m, $C(3, 1, 0)$ m and $D(3, -1, 0)$ m, respectively.

$$\mathbf{F}_{CB} = F_{CB}\left(\frac{\mathbf{r}_{CB}}{\mathbf{r}_{CB}}\right) = F_{CB}\left[\frac{(0-3)\mathbf{i} + (-0.5-1)\mathbf{j} + (1-0)\mathbf{k}}{\sqrt{(0-3)^2 + (-0.5-1)^2 + (1-0)^2}}\right]$$

$$= -\frac{6}{7}F_{CB}\mathbf{i} - \frac{3}{7}F_{CB}\mathbf{j} + \frac{2}{7}F_{CB}\mathbf{k}$$

$$\mathbf{W} = \{-40(9.81)\mathbf{k}\}\text{ N} = \{-392.4\mathbf{k}\}\text{ N.}$$

$$F_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{M}_A = (M_A)_x\mathbf{i} + (M_A)_z\mathbf{k}$$

$$\mathbf{r}_{AC} = \{3\mathbf{i} + \mathbf{j}\}\text{ m} \quad \mathbf{r}_{AD} = \{3\mathbf{i} - \mathbf{j}\}\text{ m}$$

Equations of Equilibrium. Referring to the *FBD* of the assembly shown in Fig. *a*. the force equation of equilibrium gives

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_{CB} + \mathbf{W} + \mathbf{F}_A = 0;$$

$$\left(-\frac{6}{7}F_{CB} + A_x\right)\mathbf{i} + \left(-\frac{3}{7}F_{CB} + A_y\right)\mathbf{j} + \left(\frac{2}{7}F_{CB} + A_z - 392.4\right)\mathbf{k} = 0$$

Equating **i**, **j** and **k** components

$$-\frac{6}{7}F_{CB} + A_x = 0 \tag{1}$$

$$-\frac{3}{7}F_{CB} + A_y = 0 \tag{2}$$

$$\frac{2}{7}F_{CB} + A_z - 392.4 = 0 \tag{3}$$

The moment equation of equilibrium gives

$$\Sigma\mathbf{M}_A = 0; \qquad \mathbf{r}_{AC} \times F_{CB} + \mathbf{r}_{AD} \times \mathbf{W} + \mathbf{M}_A = 0$$

$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 1 & 0 \\ -\frac{6}{7}F_{CB} & -\frac{3}{7}F_{CB} & \frac{2}{7}F_{CB} \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & -1 & 0 \\ 0 & 0 & -392.4 \end{vmatrix} + (M_A)_x\mathbf{i} + (M_A)_Z\mathbf{k} = 0$$

$$\left[\frac{2}{7}F_{CB} + 392.4 + (M_A)_x\right]\mathbf{i} + \left(-\frac{6}{7}\mathbf{F_{CB}} + \mathbf{1177.2}\right)\mathbf{j} + \left[-\frac{9}{7}F_{CB} + \frac{6}{7}F_{CB} + (M_A)_z\right]\mathbf{k} = 0$$

***5–76. Continued**

Equating **i**, **j** and **k** components,

$$\frac{2}{7}F_{CB} + 392.4 + (M_A)_x = 0 \qquad \textbf{(4)}$$

$$-\frac{6}{7}F_{CB} + 1177.2 = 0 \qquad \textbf{(5)}$$

$$-\frac{9}{7}F_{CB} + \frac{6}{7}F_{CB} + (M_A)_z = 0 \qquad \textbf{(6)}$$

Solving Eqs (1) to (6),

$F_{CB} = 1373.4 \text{ N} = 1.37 \text{ kN}$ **Ans.**

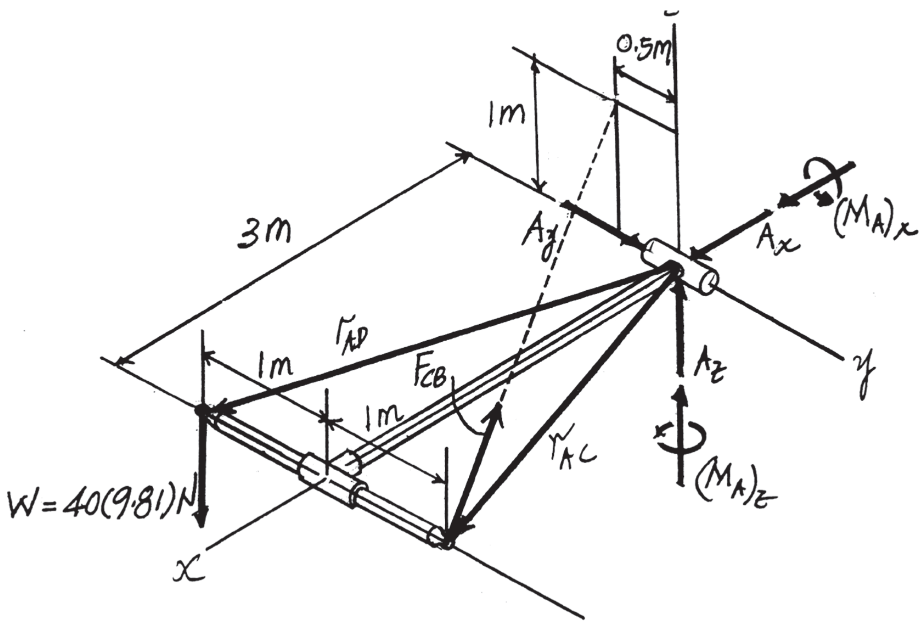
$(M_A)_x = -784.8 \text{ N} \cdot \text{m} = 785 \text{ N} \cdot \text{m}$ **Ans.**

$(M_A)_z = 588.6 \text{ N} \cdot \text{m} = 589 \text{ N} \cdot \text{m}$ **Ans.**

$A_x = 1177.2 \text{ N} = 1.18 \text{ kN}$ **Ans.**

$A_y = 588.6 \text{ N} = 589 \text{ N}$ **Ans.**

$A_z = 0$ **Ans.**

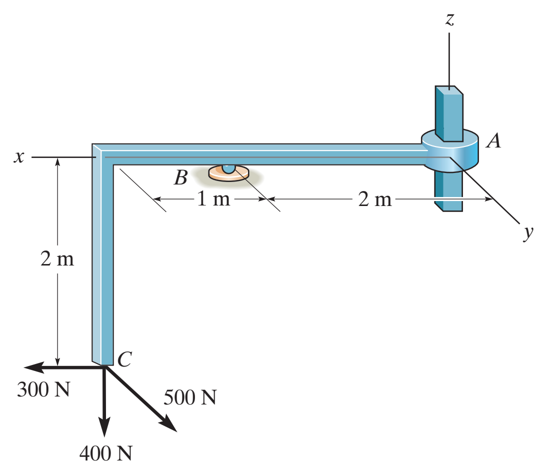


(a)

Ans:
$F_{CB} = 1.37 \text{ kN}$
$(M_A)_x = 785 \text{ N} \cdot \text{m}$
$(M_A)_z = 589 \text{ N} \cdot \text{m}$
$A_x = 1.18 \text{ kN}$
$A_y = 589 \text{ N}$
$A_z = 0$

5–77.

The member is supported by a square rod which fits loosely through the smooth square hole of the attached collar at A and by a roller at B. Determine the components of reaction at these supports when the member is subjected to the loading shown.



SOLUTION

Force And Position Vectors. The coordinates of points B and C are $B(2,0,0)$ m and $C(3,0,-2)$ m.

$$\mathbf{F}_A = -A_x\,\mathbf{i} - A_y\,\mathbf{j}$$

$$F = \{300\mathbf{i} + 500\mathbf{j} - 400\mathbf{k}\}\text{ N}$$

$$\mathbf{N}_B = N_B\,\mathbf{k}$$

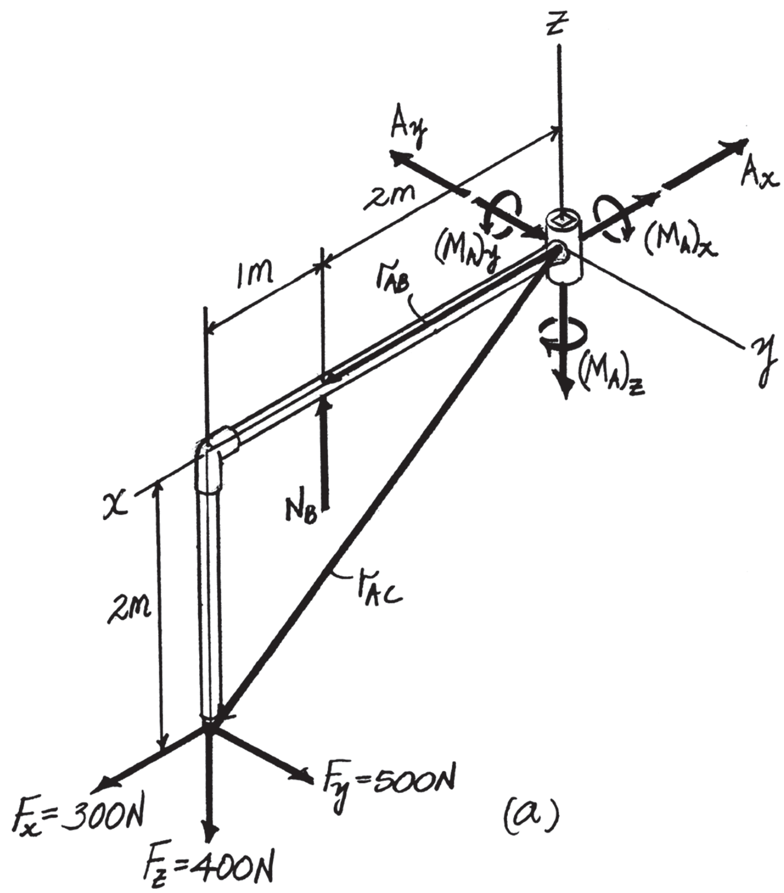
$$\mathbf{M}_A = -(M_A)_x\mathbf{i} + (M_A)_y\mathbf{j} - (M_A)_z\mathbf{k}$$

$$\mathbf{r}_{AB} = \{2\mathbf{i}\}\text{ m} \qquad \mathbf{r}_{AC} = \{3\mathbf{i} - 2\mathbf{k}\}\text{ m}$$

Equations of Equilibrium. Referring to the *FBD* of the member shown in Fig. *a*, the force equation of equilibrium gives

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_A + \mathbf{F} + \mathbf{N}_B = 0$$

$$(300 - A_x)\mathbf{i} + (500 - A_y)\mathbf{j} + (N_B - 400)\mathbf{k} = 0$$



5–77. Continued

Equating **i**, **j** and **k** components,

$300 - A_x = 0 \qquad A_x = 300\text{ N}$ **Ans.**

$500 - A_y = 0 \qquad A_y = 500\text{ N}$ **Ans.**

$N_B - 400 = 0 \qquad N_B = 400\text{ N}$ **Ans.**

The moment equation of equilibrium gives

$$\Sigma\mathbf{M}_A = 0; \qquad \mathbf{M}_A + \mathbf{r}_{AB} \times \mathbf{N}_B + \mathbf{r}_{AC} \times \mathbf{F} = 0$$

$$-(M_A)_x\mathbf{i} + (M_A)_y\mathbf{j} - (M_A)_z\mathbf{k} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 2 & 0 & 0 \\ 0 & 0 & 400 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 3 & 0 & -2 \\ 300 & 500 & -400 \end{vmatrix} = 0$$

$$\left[1000 - (M_A)_x\right]\mathbf{i} + \left[(M_A)_y - 200\right]\mathbf{j} + \left[1500 - (M_A)_z\right]\mathbf{k} = 0$$

Equating **i**, **j** and **k** components,

$1000 - (M_A)_x = 0 \qquad (M_A)_x = 1000\text{ N}\cdot\text{m} = 1.00\text{ kN}\cdot\text{m}$ **Ans.**

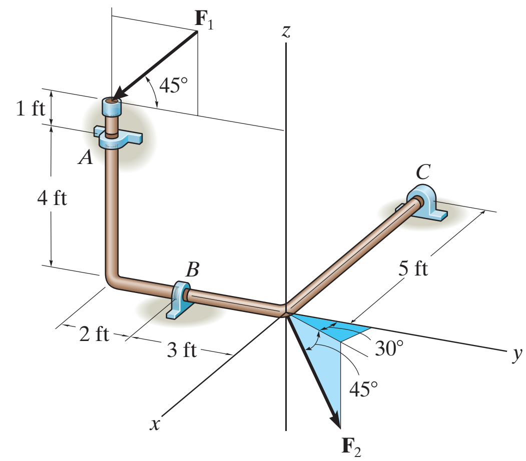
$(M_A)_y - 200 = 0 \qquad (M_A)_y = 200\text{ N}\cdot\text{m}$ **Ans.**

$1500 - (M_A)_z = 0 \qquad (M_A)_z = 1500\text{ N}\cdot\text{m} = 1.50\text{ kN}\cdot\text{m}$ **Ans.**

Ans:
$A_x = 300\text{ N}$
$A_y = 500\text{ N}$
$N_B = 400\text{ N}$
$(M_A)_x = 1.00\text{ kN}\cdot\text{m}$
$(M_A)_y = 200\text{ N}\cdot\text{m}$
$(M_A)_z = 1.50\text{ kN}\cdot\text{m}$

5–78.

The bent rod is supported at A, B, and C by smooth journal bearings. Compute the x, y, z components of reaction at the bearings if the rod is subjected to forces $F_1 = 300$ lb and $F_2 = 250$ lb. F_1 lies in the y–z plane. The bearings are in proper alignment and exert only force reactions on the rod.



SOLUTION

$$\mathbf{F}_1 = (-300\cos 45°\mathbf{j} - 300\sin 45°\mathbf{k})$$

$$= \{-212.1\mathbf{j} - 212.1\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_2 = (250\cos 45°\sin 30°\mathbf{i} + 250\cos 45°\cos 30°\mathbf{j} - 250\sin 45°\mathbf{k})$$

$$= \{88.39\mathbf{i} + 153.1\mathbf{j} - 176.8\mathbf{k}\}\text{ lb}$$

$\Sigma F_x = 0;\quad A_x + B_x + 88.39 = 0$

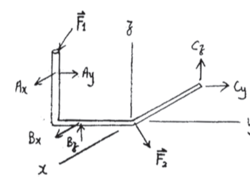
$\Sigma F_y = 0;\quad A_y + C_y - 212.1 + 153.1 = 0$

$\Sigma F_z = 0;\quad B_z + C_z - 212.1 - 176.8 = 0$

$\Sigma M_x = 0;\quad -B_z(3) - A_y(4) + 212.1(5) + 212.1(5) = 0$

$\Sigma M_y = 0;\quad C_z(5) + A_x(4) = 0$

$\Sigma M_z = 0;\quad A_x(5) + B_x(3) - C_y(5) = 0$



$A_x = 633$ lb **Ans.**

$A_y = -141$ lb **Ans.**

$B_x = -721$ lb **Ans.**

$B_z = 895$ lb **Ans.**

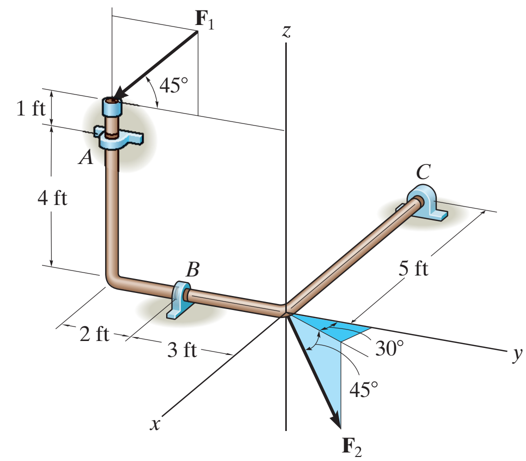
$C_y = 200$ lb **Ans.**

$C_z = -506$ lb **Ans.**

Ans:
$A_x = 633$ lb
$A_y = -141$ lb
$B_x = -721$ lb
$B_z = 895$ lb
$C_y = 200$ lb
$C_z = -506$ lb

5–79.

The bent rod is supported at A, B, and C by smooth journal bearings. Determine the magnitude of $\mathbf{F}_2$ which will cause the reaction $\mathbf{C}_y$ at the bearing C to be equal to zero. The bearings are in proper alignment and exert only force reactions on the rod. Set $F_1 = 300$ lb.



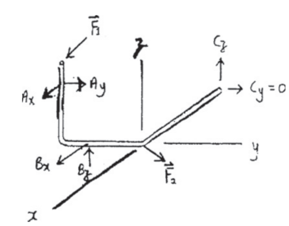
SOLUTION

$$\mathbf{F}_1 = (-300 \cos 45°\mathbf{j} - 300 \sin 45°\mathbf{k})$$

$$= \{-212.1\mathbf{j} - 212.1\mathbf{k}\}\text{ lb}$$

$$\mathbf{F}_2 = (F_2 \cos 45° \sin 30°\mathbf{i} + F_2 \cos 45° \cos 30°\mathbf{j} - F_2 \sin 45°\mathbf{k})$$

$$= \{0.3536F_2\mathbf{i} + 0.6124F_2\mathbf{j} - 0.7071F_2\mathbf{k}\}\text{ lb}$$



$\Sigma F_x = 0;\qquad A_x + B_x + 0.3536F_2 = 0$

$\Sigma F_y = 0;\qquad A_y + 0.6124F_2 - 212.1 = 0$

$\Sigma F_z = 0;\qquad B_z + C_z - 0.7071F_2 - 212.1 = 0$

$\Sigma M_x = 0;\qquad -B_z(3) - A_y(4) + 212.1(5) + 212.1(5) = 0$

$\Sigma M_y = 0;\qquad C_z(5) + A_x(4) = 0$

$\Sigma M_z = 0;\qquad A_x(5) + B_x(3) = 0$

$A_x = 357$ lb

$A_y = -200$ lb

$B_x = -595$ lb

$B_z = 974$ lb

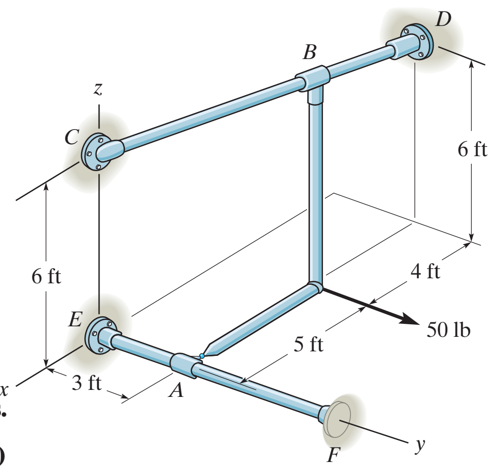
$C_z = -286$ lb

$F_2 = 674$ lb **Ans.**

Ans:
$F_2 = 674$ lb

***5–80.**

The bar AB is supported by two smooth collars. At A the connection is with a ball-and-socket joint and at B it is a rigid attachment. If a 50-lb load is applied to the bar, determine the x, y, z components of reaction at A and B.



SOLUTION

$A_x + B_x = 0$ **(1)**

$B_y + 50 = 0$

$B_y = -50 \text{ lb}$ **Ans.**

$A_z + B_z = 0$ **(2)**

$M_{B_z} = 0$ **Ans.**

$M_{B_x} + 50(6) = 0$

$M_{B_x} = -300 \text{ lb} \cdot \text{ft}$ **Ans.**

$B_{C_D} = -9\mathbf{i} + 3\mathbf{j}$

$B_{C_D} = -0.94868\mathbf{i} + 0.316228\mathbf{j}$

Require

$\mathbf{F}_B \cdot \mathbf{u}_{CD} = 0$

$(B_x\mathbf{i} - 50\mathbf{j} + B_z\mathbf{k}) \cdot (-0.94868\mathbf{i} + 0.316228\mathbf{j}) = 0$

$-0.94868B_x - 50(0.316228) = 0$

$B_x = -16.667 = -16.7 \text{ lb}$ **Ans.**

From Eq. (1);

$A_x = 16.7 \text{ lb}$ **Ans.**

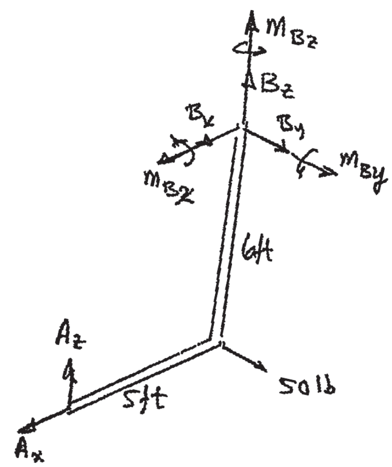
Require

$\mathbf{M}_B \cdot \mathbf{u}_{CD} = 0$

$(-300\mathbf{i} + M_{By}\mathbf{j}) \cdot (-0.94868\mathbf{i} + 0.316228\mathbf{j}) = 0$

$300(0.94868) + M_{By}(0.316228) = 0$

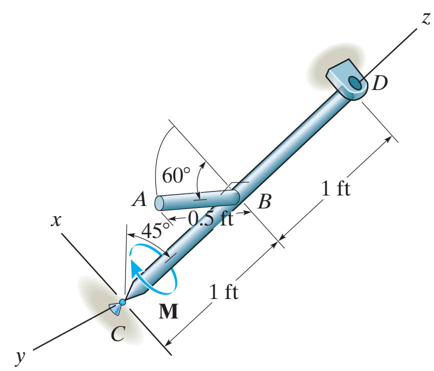
$M_{B_y} = -900 \text{ lb} \cdot \text{ft}$ **Ans.**



Ans:
$B_y = -50 \text{ lb}$
$M_{B_z} = 0$
$M_{B_x} = -300 \text{ lb} \cdot \text{ft}$
$B_x = -16.7 \text{ lb}$
$A_x = 16.7 \text{ lb}$

5–81.

The rod has a weight of 6 lb/ft. If it is supported by a ball-and-socket joint at C and a journal bearing at D, determine the x, y, z components of reaction at these supports and the moment M that must be applied along the axis of the rod to hold it in the position shown.



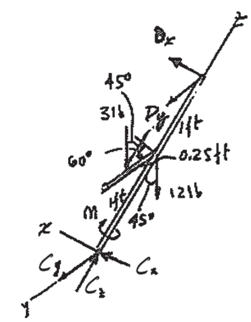
SOLUTION

$\Sigma F_x = 0;\qquad C_x + D_x - 15 \sin 45^\circ = 0 \qquad$ **(1)**

$\Sigma F_y = 0;\qquad C_y + D_y = 0 \qquad$ **(2)**

$\Sigma F_z = 0;\qquad C_z - 15 \cos 45^\circ = 0$

$C_z = 10.6 \text{ lb}$ **Ans.**



$\Sigma M_x = 0;\qquad -3 \cos 45^\circ(0.25 \sin 60^\circ) - D_y(2) = 0$

$D_y = -0.230 \text{ lb}$ **Ans.**

From Eq. (2);

$C_y = 0.230 \text{ lb}$ **Ans.**

$\Sigma M_y = 0;\qquad -(12 \sin 45^\circ)(1) - (3 \sin 45^\circ)(1) + (3 \cos 45^\circ)(0.25 \cos 60^\circ) + D_x(2) = 0$

$D_x = 5.17 \text{ lb}$ **Ans.**

From Eq. (1);

$C_x = 5.44 \text{ lb}$ **Ans.**

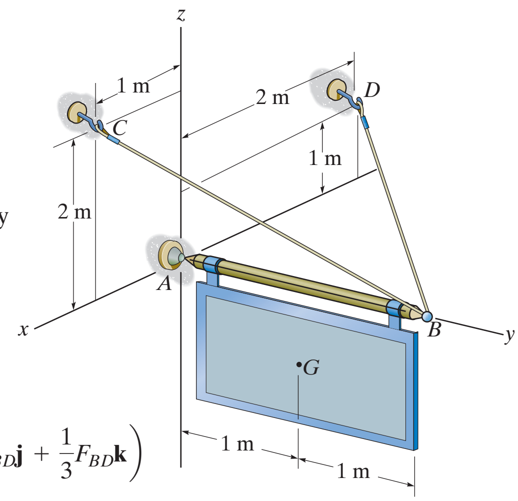
$\Sigma M_z = 0;\qquad -M + (3 \sin 45^\circ)(0.25 \sin 60^\circ) = 0$

$M = 0.459 \text{ lb} \cdot \text{ft}$ **Ans.**

Ans:
$C_z = 10.6$ lb
$D_y = -0.230$ lb
$C_y = 0.230$ lb
$D_x = 5.17$ lb
$C_x = 5.44$ lb
$M = 0.459$ lb · ft

5–82.

The sign has a mass of 100 kg with center of mass at G. Determine the x, y, z components of reaction at the ball-and-socket joint A and the tension in wires BC and BD.



SOLUTION

Equations of Equilibrium: Expressing the forces indicated on the free-body diagram, Fig. a, in Cartesian vector form, we have

$$\mathbf{F}_A = A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}$$

$$\mathbf{W} = \{-100(9.81)\mathbf{k}\}\text{ N} = \{-981\text{k}\}\text{ N}$$

$$\mathbf{F}_{BD} = F_{BD}\mathbf{u}_{BD} = F_{BD}\left[\frac{(-2-0)\mathbf{i} + (0-2)\mathbf{j} + (1-0)\mathbf{k}}{\sqrt{(-2-0)^2 + (0-2)^2 + (1-0)^2}}\right] = \left(-\frac{2}{3}F_{BD}\mathbf{i} - \frac{2}{3}F_{BD}\mathbf{j} + \frac{1}{3}F_{BD}\mathbf{k}\right)$$

$$\mathbf{F}_{BC} = F_{BC}\mathbf{u}_{BC} = F_{BC}\left[\frac{(1-0)\mathbf{i} + (0-2)\mathbf{j} + (2-0)\mathbf{k}}{\sqrt{(1-0)^2 + (0-2)^2 + (2-0)^2}}\right] = \left(\frac{1}{3}F_{BC}\mathbf{i} - \frac{2}{3}F_{BC}\mathbf{j} + \frac{2}{3}F_{BC}\mathbf{k}\right)$$

Applying the forces equation of equilibrium, we have

$$\Sigma\mathbf{F} = 0; \qquad \mathbf{F}_A + \mathbf{F}_{BD} + \mathbf{F}_{BC} + \mathbf{W} = 0$$

$$(A_x\mathbf{i} + A_y\mathbf{j} + A_z\mathbf{k}) + \left(-\frac{2}{3}F_{BD}\mathbf{i} - \frac{2}{3}F_{BD}\mathbf{j} + \frac{1}{3}F_{BD}\mathbf{k}\right) + \left(\frac{1}{3}F_{BC}\mathbf{i} - \frac{2}{3}F_{BC}\mathbf{j} + \frac{2}{3}F_{BC}\mathbf{k}\right) + (-981\,k) = 0$$

$$\left(A_x - \frac{2}{3}F_{BD} + \frac{1}{3}F_{BC}\right)\mathbf{i} + \left(A_y - \frac{2}{3}F_{BD} - \frac{2}{3}F_{BC}\right)\mathbf{j} + \left(A_z + \frac{1}{3}F_{BD} + \frac{2}{3}F_{BC} - 981\right)\mathbf{k} = 0$$

Equating **i**, **j**, and **k** components, we have

$$A_x - \frac{2}{3}F_{BD} + \frac{1}{3}F_{BC} = 0 \qquad \textbf{(1)}$$

$$A_y - \frac{2}{3}F_{BD} - \frac{2}{3}F_{BC} = 0 \qquad \textbf{(2)}$$

$$A_z + \frac{1}{3}F_{BD} + \frac{2}{3}F_{BC} - 981 = 0 \qquad \textbf{(3)}$$

In order to write the moment equation of equilibrium about point A, the position vectors $\mathbf{r}_{AG}$ and $\mathbf{r}_{AB}$ must be determined first.

$$\mathbf{r}_{AG} = \{1\mathbf{j}\}\text{ m}$$

$$\mathbf{r}_{AB} = \{2\mathbf{j}\}\text{ m}$$

5–82. Continued

Thus,

$$\Sigma \mathbf{M}_A = 0; \mathbf{r}_{AB} \times (\mathbf{F}_{BC} + \mathbf{F}_{BD}) + (\mathbf{r}_{AG} \times \mathbf{W}) = 0$$

$$(2\mathbf{j}) \times \left[\left(\frac{1}{3}F_{BC} - \frac{2}{3}F_{BD}\right)\mathbf{i} - \left(\frac{2}{3}F_{BC} + \frac{2}{3}F_{BD}\right)\mathbf{j} + \left(\frac{2}{3}F_{BC} + \frac{1}{3}F_{BD}\right)\mathbf{k}\right] + (1\mathbf{j}) \times (-981\mathbf{k}) = 0$$

$$\left(\frac{4}{3}F_{BC} + \frac{2}{3}F_{BD} - 981\right)\mathbf{i} + \left(\frac{4}{3}F_{BD} - \frac{2}{3}F_{BC}\right)\mathbf{k} = 0$$

Equating **i**, **j**, and **k** components we have

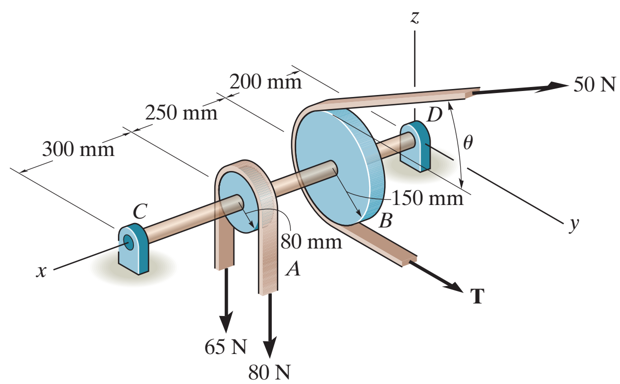
$$\frac{4}{3}F_{BC} + \frac{2}{3}F_{BC} - 981 = 0 \qquad \textbf{(4)}$$

$$\frac{4}{3}F_{BC} - \frac{2}{3}F_{BC} = 0 \qquad \textbf{(5)}$$

Ans:
$F_{BD} = 294 \text{ N}$
$F_{BC} = 589 \text{ N}$
$A_x = 0$
$A_y = 589 \text{ N}$
$A_z = 490.5 \text{ N}$

5–83.

Both pulleys are fixed to the shaft and as the shaft turns with constant angular velocity, the power of pulley A is transmitted to pulley B. Determine the horizontal tension $\mathbf{T}$ in the belt on pulley B and the x, y, z components of reaction at the journal bearing C and thrust bearing D if $\theta = 0°$. The bearings are in proper alignment and exert only force reactions on the shaft.



SOLUTION

Equations of Equilibrium:

$\Sigma M_x = 0;\quad 65(0.08) - 80(0.08) + T(0.15) - 50(0.15) = 0$

$T = 58.0\text{ N}$ **Ans.**

$\Sigma M_y = 0;\quad (65 + 80)(0.45) - C_z(0.75) = 0$

$C_z = 87.0\text{ N}$ **Ans.**

$\Sigma M_z = 0;\quad (50 + 58.0)(0.2) - C_y(0.75) = 0$

$C_y = 28.8\text{ N}$ **Ans.**

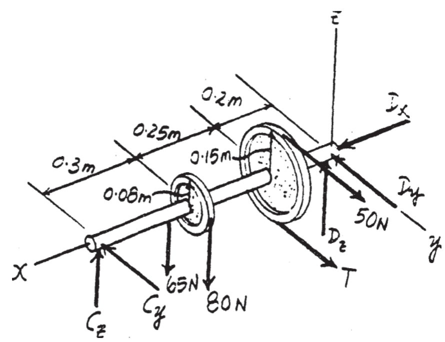
$\Sigma F_x = 0;\quad D_x = 0$ **Ans.**

$\Sigma F_y = 0;\quad D_y + 28.8 - 50 - 58.0 = 0$

$D_y = 79.2\text{ N}$ **Ans.**

$\Sigma F_z = 0;\quad D_z + 87.0 - 80 - 65 = 0$

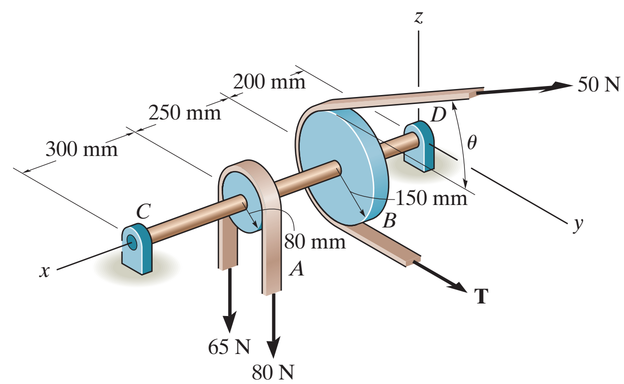
$D_z = 58.0\text{ N}$ **Ans.**



Ans:
$T = 58.0\text{ N}$
$C_z = 87.0\text{ N}$
$C_y = 28.8\text{ N}$
$D_x = 0$
$D_y = 79.2\text{ N}$
$D_z = 58.0\text{ N}$

***5–84.**

Both pulleys are fixed to the shaft and as the shaft turns with constant angular velocity, the power of pulley A is transmitted to pulley B. Determine the horizontal tension **T** in the belt on pulley B and the x, y, z components of reaction at the journal bearing C and thrust bearing D if $\theta = 45°$. The bearings are in proper alignment and exert only force reactions on the shaft.



SOLUTION

Equations of Equilibrium:

$\Sigma M_x = 0;\qquad 65(0.08) - 80(0.08) + T(0.15) - 50(0.15) = 0$

$T = 58.0\text{ N}$ **Ans.**

$\Sigma M_y = 0;\qquad (65 + 80)(0.45) - 50 \sin 45°(0.2) - C_z\,(0.75) = 0$

$C_z = 77.57\text{ N} = 77.6\text{ N}$ **Ans.**

$\Sigma M_z = 0;\qquad 58.0(0.2) + 50 \cos 45°(0.2) - C_y\,(0.75) = 0$

$C_y = 24.89\text{ N} = 24.9\text{ N}$ **Ans.**

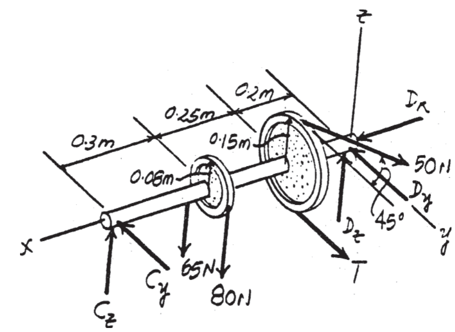
$\Sigma F_x = 0;\qquad D_x = 0$ **Ans.**

$\Sigma F_y = 0;\qquad D_y + 24.89 - 50 \cos 45° - 58.0 = 0$

$D_y = 68.5\text{ N}$ **Ans.**

$\Sigma F_z = 0;\qquad D_z + 77.57 + 50 \sin 45° - 80 - 65 = 0$

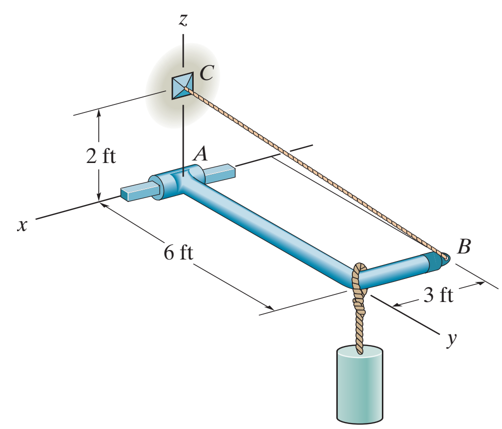
$D_z = 32.1\text{ N}$ **Ans.**



Ans:
$T = 58.0\text{ N}$
$C_z = 77.6\text{ N}$
$C_y = 24.9\text{ N}$
$D_y = 68.5\text{ N}$
$D_z = 32.1\text{ N}$

5–85.

Member AB is supported by a cable BC and at A by a *square* rod which fits loosely through the square hole at the end joint of the member as shown. Determine the components of reaction at A and the tension in the cable needed to hold the 800-lb cylinder in equilibrium.



SOLUTION

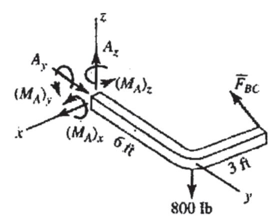
$$\mathbf{F}_{BC} = F_{BC}\left(\frac{3}{7}\mathbf{i} - \frac{6}{7}\mathbf{j} + \frac{2}{7}\mathbf{k}\right)$$

$\Sigma F_x = 0;\qquad F_{BC}\left(\frac{3}{7}\right) = 0$

$F_{BC} = 0$ **Ans.**

$\Sigma F_y = 0;\qquad A_y = 0$ **Ans.**

$\Sigma F_z = 0;\qquad A_z = 800\text{ lb}$ **Ans.**



$\Sigma M_x = 0;\qquad (M_A)_x - 800(6) = 0$

$(M_A)_x = 4.80\text{ kip}\cdot\text{ft}$ **Ans.**

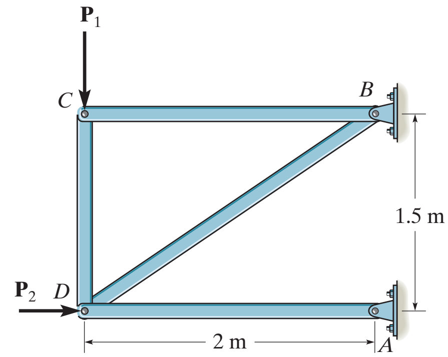
$\Sigma M_y = 0;\qquad (M_A)_y = 0$ **Ans.**

$\Sigma M_z = 0;\qquad (M_A)_z = 0$ **Ans.**

Ans:
$F_{BC} = 0$
$A_y = 0$
$A_z = 800\text{ lb}$
$(M_A)_x = 4.80\text{ kip}\cdot\text{ft}$
$(M_A)_y = 0$
$(M_A)_z = 0$

6–1.

Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 20$ kN, $P_2 = 10$ kN.



SOLUTION

Method of Joints. Start at joint C and then proceed to join D.

Joint *C*. Fig. *a*

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{CB} = 0$ **Ans.**

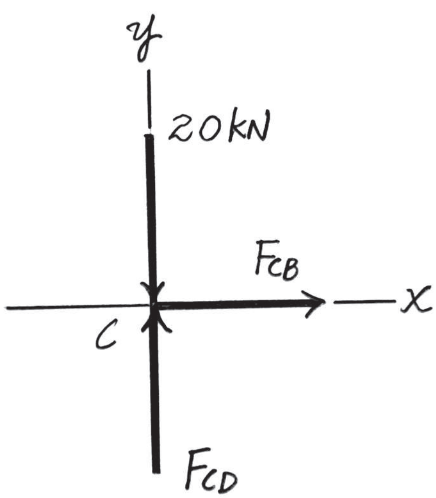
$+\uparrow \Sigma F_y = 0; \qquad F_{CD} - 20 = 0 \qquad F_{CD} = 20.0 \text{ kN (C)}$ **Ans.**

Joint *D*. Fig. *b*

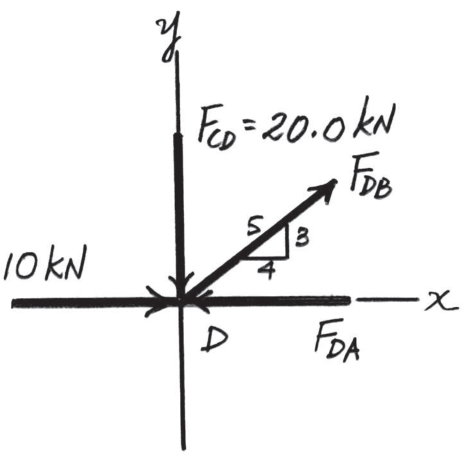
$+\uparrow \Sigma F_y = 0; \qquad F_{DB}\left(\frac{3}{5}\right) - 20.0 = 0 \qquad F_{DB} = 33.33 \text{ kN (T)} = 33.3 \text{ kN (T)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \qquad 10 + 33.33\left(\frac{4}{5}\right) - F_{DA} = 0$

$F_{DA} = 36.67 \text{ kN (C)} = 36.7 \text{ kN (C)}$ **Ans.**



(a)

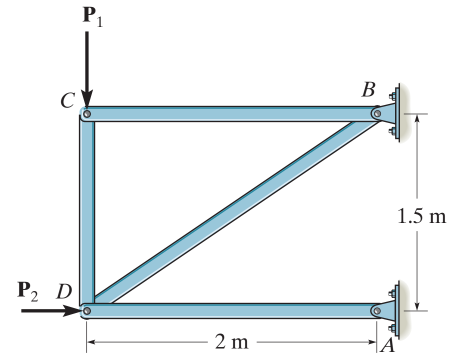


(b)

Ans:
$F_{CB} = 0$
$F_{CD} = 20.0$ kN (C)
$F_{DB} = 33.3$ kN (T)
$F_{DA} = 36.7$ kN (C)

6–2.

Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 45$ kN, $P_2 = 30$ kN.



SOLUTION

Method of Joints. Start at joint *C* and then proceed to joint *D*.

Joint *C*. Fig. *a*

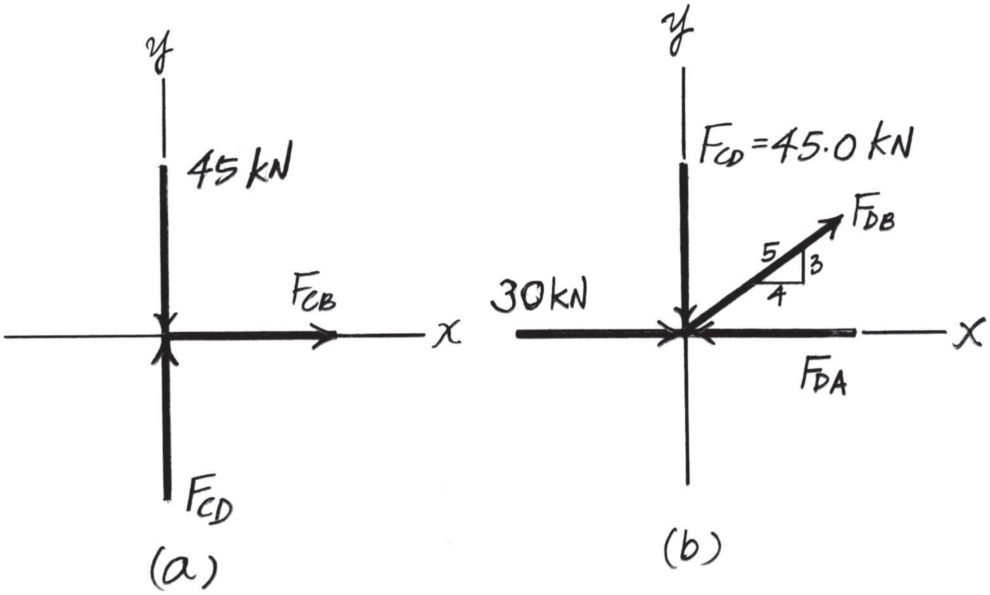
$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad F_{CB} = 0$ **Ans.**

$+\uparrow\Sigma F_y = 0; \quad F_{CD} - 45 = 0 \quad F_{CD} = 45.0 \text{ kN } (C)$ **Ans.**

Joint *D*. Fig. *b*

$+\uparrow\Sigma F_y = 0; \quad F_{DB}\left(\frac{3}{5}\right) - 45.0 = 0 \quad F_{DB} = 75.0 \text{ kN } (T)$ **Ans.**

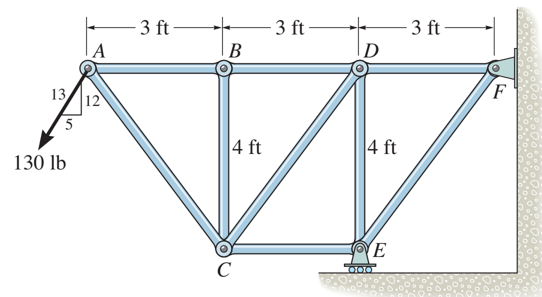
$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 30 + 75.0\left(\frac{4}{5}\right) - F_{DA} = 0 \quad F_{DA} = 90.0 \text{ kN } (C)$ **Ans.**



Ans:
$F_{CB} = 0$
$F_{CD} = 45.0$ kN (C)
$F_{DB} = 75.0$ kN (T)
$F_{DA} = 90.0$ kN (C)

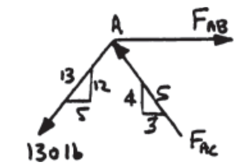
6–3.

Determine the force in each member of the truss. State if the members are in tension or compression.



SOLUTION

Joint A:

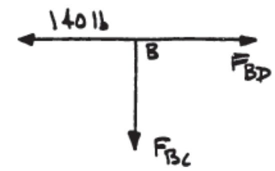


$$+\uparrow \Sigma F_y = 0; \qquad \frac{4}{5}(F_{AC}) - \frac{12}{13}(130) = 0$$

$$F_{AC} = 150 \text{ lb (C)} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB} - \frac{3}{5}(150) - \frac{5}{13}(130) = 0$$

$$F_{AB} = 140 \text{ lb (T)} \qquad \textbf{Ans.}$$

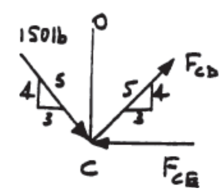


Joint B:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{BD} - 140 = 0$$

$$F_{BD} = 140 \text{ lb (T)} \qquad \textbf{Ans.}$$

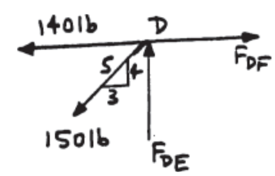
$$+\uparrow \Sigma F_y = 0; \qquad F_{BC} = 0 \qquad \textbf{Ans.}$$



Joint C:

$$+\uparrow \Sigma F_y = 0; \qquad \left(\frac{4}{5}\right) F_{CD} - \left(\frac{4}{5}\right) 150 = 0$$

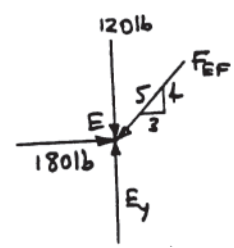
$$F_{CD} = 150 \text{ lb (T)} \qquad \textbf{Ans.}$$



$$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_{CE} + \frac{3}{5}(150) + \frac{3}{5}(150) = 0$$

$$F_{CE} = 180 \text{ lb (C)} \qquad \textbf{Ans.}$$

Joint D:



$$+\uparrow \Sigma F_y = 0; \qquad F_{DE} - \frac{4}{5}(150) = 0$$

$$F_{DE} = 120 \text{ lb (C)} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{DF} - 140 - \frac{3}{5}(150) = 0$$

$$F_{DF} = 230 \text{ lb (T)} \qquad \textbf{Ans.}$$

Joint E:

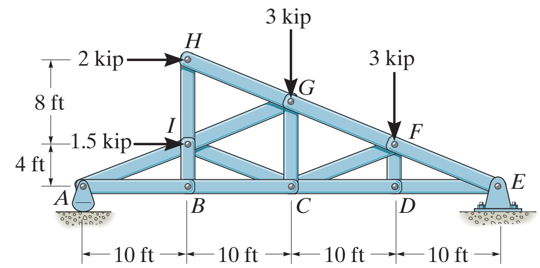
$$\xrightarrow{+} \Sigma F_x = 0; \qquad 180 - \frac{3}{5}(F_{EF}) = 0$$

$$F_{EF} = 300 \text{ lb (C)} \qquad \textbf{Ans.}$$

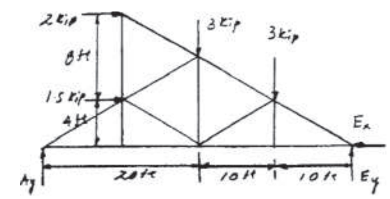
Ans:
$F_{AC} = 150$ lb (C)
$F_{AB} = 140$ lb (T)
$F_{BD} = 140$ lb (T)
$F_{BC} = 0$
$F_{CD} = 150$ lb (T)
$F_{CE} = 180$ lb (C)
$F_{DE} = 120$ lb (C)
$F_{DF} = 230$ lb (T)
$F_{EF} = 300$ lb (C)

***6–4.**

Determine the force in each member of the truss and state if the members are in tension or compression.



SOLUTION

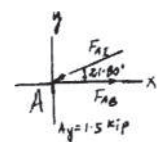


$\circlearrowleft + \Sigma M_E = 0; \quad A_y(40) + 1.5(4) + 2(12) - 3(10) - 3(20) = 0 \quad A_y = 1.5 \text{ kip}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 1.5 + 2 - E_x = 0 \quad E_x = 3.5 \text{ kip}$

$+\uparrow \Sigma F_y = 0; \quad E_y + 1.5 - 3 - 3 = 0 \quad E_y = 4.5 \text{ kip}$

Joint A:

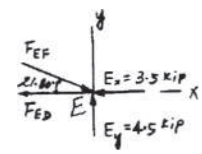


$+\uparrow \Sigma F_y = 0; \quad 1.5 - F_{AI} \sin 21.80° = 0$

$F_{AI} = 4.039 \text{ kip} = 4.04 \text{ kip (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{AB} - 4.039 \cos 21.80° = 0 \quad F_{AB} = 3.75 \text{ kip (T)}$ **Ans.**

Joint E:



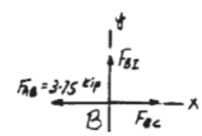
$+\uparrow \Sigma F_y = 0; \quad 4.5 - F_{EF} \sin 21.80° = 0$

$F_{EF} = 12.12 \text{ kip} = 12.1 \text{ kip (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_{ED} - 3.5 + 12.12 \cos 21.80° = 0$

$F_{ED} = 7.75 \text{ kip (T)}$ **Ans.**

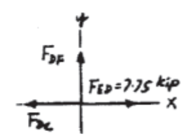
Joint B:



$+\uparrow \Sigma F_y = 0; \quad F_{BI} = 0$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{BC} - 3.75 = 0 \quad F_{BC} = 3.75 \text{ kip (T)}$ **Ans.**

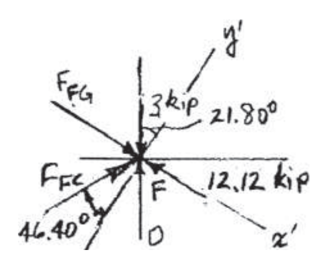
Joint D:



$+\uparrow \Sigma F_y = 0; \quad F_{DF} = 0$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -F_{DC} + 7.75 = 0 \quad F_{DC} = 7.75 \text{ kip (T)}$ **Ans.**

Joint F:



$+\nearrow \Sigma F_{y'} = 0; \quad F_{FC} \cos 46.40° - 3 \cos 21.80° = 0$

$F_{FC} = 4.039 \text{ kip} = 4.04 \text{ kip (C)}$ **Ans.**

$+\searrow \Sigma F_{x'} = 0; \quad F_{FG} + 3 \sin 21.80° + 4.039 \sin 46.40° - 12.12 = 0$

$F_{FG} = 8.078 \text{ kip} = 8.08 \text{ kip (C)}$ **Ans.**

Joint H:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 2 - F_{HG} \cos 21.80° = 0$

$F_{HG} = 2.154 \text{ kip} = 2.15 \text{ kip (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 2.154 \sin 21.80° - F_{HI} = 0 \quad F_{HI} = 0.8 \text{ kip (T)}$ **Ans.**

***6–4. Continued**

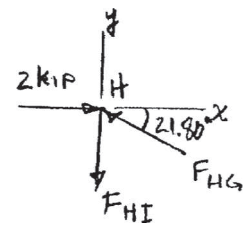
Joint C:

$\xrightarrow{+} \Sigma F_x = 0; \qquad -F_{CI} \cos 21.80° - 4.039 \cos 21.80° - 3.75 + 7.75 = 0$

$F_{CI} = 0.2692 \text{ kip} = 0.269 \text{ kip (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad F_{CG} + 0.2692 \sin 21.80° - 4.039 \sin 21.80° = 0$

$F_{CG} = 1.40 \text{ kip (T)}$ **Ans.**



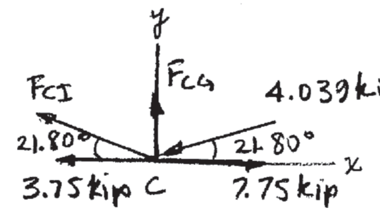
Joint G:

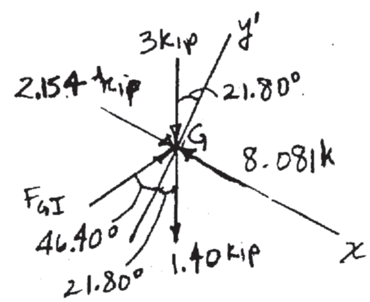
$+\nearrow \Sigma F_{y'} = 0; \qquad F_{GI} \cos 46.40° - 3 \cos 21.80° - 1.40 \cos 21.80° = 0$

$F_{GI} = 5.924 \text{ kip} = 5.92 \text{ kip (C)}$ **Ans.**

$+\searrow \Sigma F_{x'} = 0; \qquad 2.154 + 3 \sin 21.80° + 5.924 \sin 46.40°$

$+ 1.40 \sin 21.80° - 8.081 = 0 \text{ (Check)}$

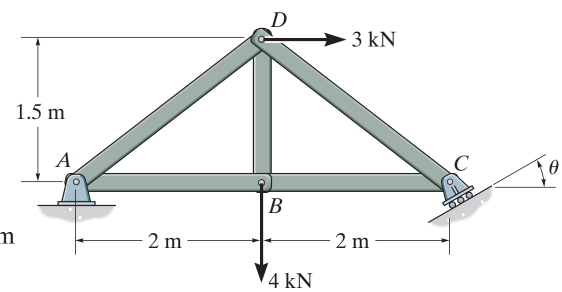




Ans:
$F_{AI} = 4.04$ kip (C)
$F_{AB} = 3.75$ kip (T)
$F_{EF} = 12.1$ kip (C)
$F_{ED} = 7.75$ kip (T)
$F_{BI} = 0$
$F_{BC} = 3.75$ kip (T)
$F_{DF} = 0$
$F_{DC} = 7.75$ kip (T)
$F_{FC} = 4.04$ kip (C)
$F_{FG} = 8.08$ kip (C)
$F_{HG} = 2.15$ kip (C)
$F_{HI} = 0.8$ kip (T)
$F_{CI} = 0.269$ kip (T)
$F_{CG} = 1.40$ kip (T)
$F_{GI} = 5.92$ kip (C)

6–5.

Determine the force in each member of the truss, and state if the members are in tension or compression. Set $\theta = 0°$.



SOLUTION

Support Reactions: Applying the equations of equilibrium to the free-body diagram of the entire truss, Fig. *a*, we have

$\circlearrowleft + \Sigma M_A = 0;$ $\quad N_C(2 + 2) - 4(2) - 3(1.5) = 0$

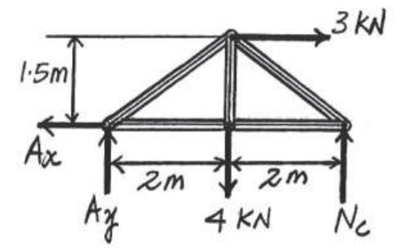
$N_C = 3.125 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 3 - A_x = 0$

$A_x = 3 \text{ kN}$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y + 3.125 - 4 = 0$

$A_y = 0.875 \text{ kN}$



(a)

Method of Joints: We will use the above result to analyze the equilibrium of joints C and A, and then proceed to analyze of joint B.

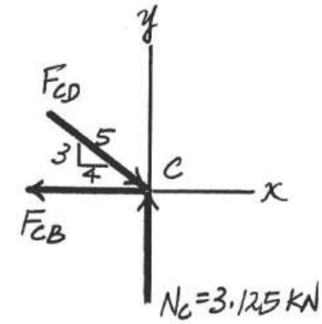
Joint C: From the free-body diagram in Fig. *b,* we can write

$+\uparrow \Sigma F_y = 0;$ $\quad 3.125 - F_{CD}\left(\frac{3}{5}\right) = 0$

$F_{CD} = 5.208 \text{ kN} = 5.21 \text{ kN (C)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 5.208\left(\frac{4}{5}\right) - F_{CB} = 0$

$F_{CB} = 4.167 \text{ kN} = 4.17 \text{ kN (T)}$ **Ans.**



(b)

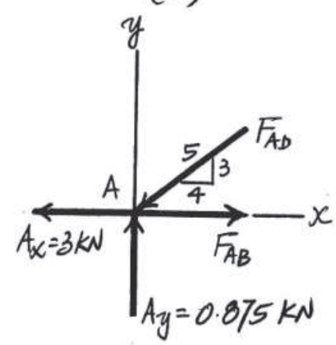
Joint A: From the free-body diagram in Fig. *c,* we can write

$+\uparrow \Sigma F_y = 0;$ $\quad 0.875 - F_{AD}\left(\frac{3}{5}\right) = 0$

$F_{AD} = 1.458 \text{ kN} = 1.46 \text{ kN (C)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{AB} - 3 - 1.458\left(\frac{4}{5}\right) = 0$

$F_{AB} = 4.167 \text{ kN} = 4.17 \text{ kN (T)}$ **Ans.**



(c)

Joint B: From the free-body diagram in Fig. *d,* we can write

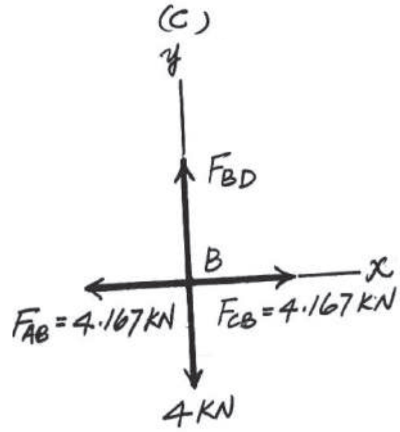
$+\uparrow \Sigma F_y = 0;$ $\quad F_{BD} - 4 = 0$

$F_{BD} = 4 \text{ kN (T)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 4.167 - 4.167 = 0 \quad \text{(check!)}$

Note: The equilibrium analysis of joint D can be used to check the accuracy of the solution obtained above.

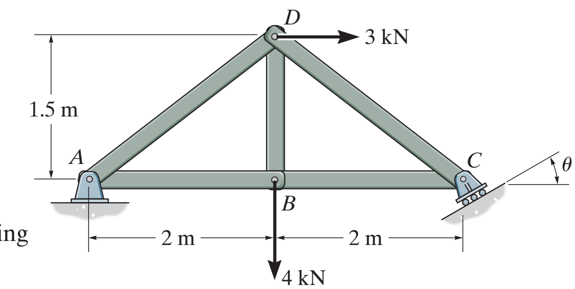
Ans:
$F_{CD} = 5.21 \text{ kN (C)}$
$F_{CB} = 4.17 \text{ kN (T)}$
$F_{AD} = 1.46 \text{ kN (C)}$
$F_{AB} = 4.17 \text{ kN (T)}$
$F_{BD} = 4 \text{ kN (T)}$



(d)

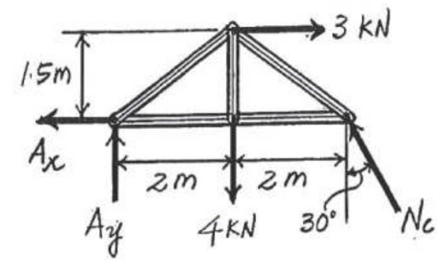
6–6.

Determine the force in each member of the truss, and state if the members are in tension or compression. Set $\theta = 30°$.



SOLUTION

Support Reactions: From the free-body diagram of the truss, Fig. *a*, and applying the equations of equilibrium, we have



(a)

$\circlearrowleft + \Sigma M_A = 0;$ $\quad N_C \cos 30°(2 + 2) - 3(1.5) - 4(2) = 0$

$$N_C = 3.608 \text{ kN}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 3 - 3.608 \sin 30° - A_x = 0$

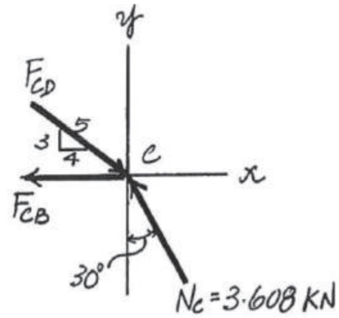
$$A_x = 1.196 \text{ kN}$$

$+\uparrow \Sigma F_y = 0;$ $\quad A_y + 3.608 \cos 30° - 4 = 0$

$$A_y = 0.875 \text{ kN}$$

Method of Joints: We will use the above result to analyze the equilibrium of joints C and A, and then proceed to analyze of joint B.

Joint C: From the free-body diagram in Fig. *b,* we can write



(b)

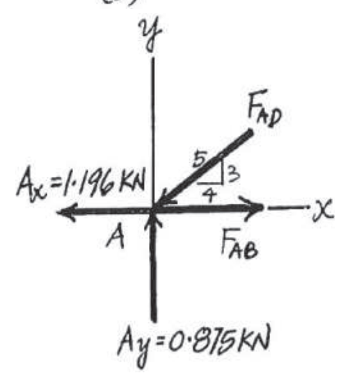
$+\uparrow \Sigma F_y = 0;$ $\quad 3.608 \cos 30° - F_{CD}\left(\frac{3}{5}\right) = 0$

$F_{CD} = 5.208 \text{ kN} = 5.21 \text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 5.208\left(\frac{4}{5}\right) - 3.608 \sin 30° - F_{CB} = 0$

$F_{CB} = 2.362 \text{ kN} = 2.36 \text{ kN (T)}$ **Ans.**

Joint A: From the free-body diagram in Fig. *c,* we can write



(c)

$+\uparrow \Sigma F_y = 0;$ $\quad 0.875 - F_{AD}\left(\frac{3}{5}\right) = 0$

$F_{AD} = 1.458 \text{ kN} = 1.46 \text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{AB} - 1.458\left(\frac{4}{5}\right) - 1.196 = 0$

$F_{AB} = 2.362 \text{ kN} = 2.36 \text{ kN (T)}$ **Ans.**

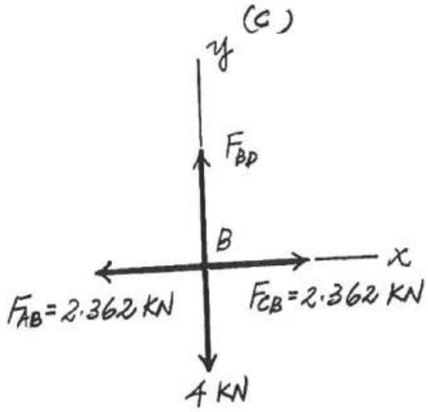
Joint B: From the free-body diagram in Fig. *d,* we can write

$+\uparrow \Sigma F_y = 0;$ $\quad F_{BD} - 4 = 0$

$F_{BD} = 4 \text{ kN (T)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 2.362 - 2.362 = 0 \quad$ (check!)

Note: The equilibrium analysis of joint D can be used to check the accuracy of the solution obtained above.

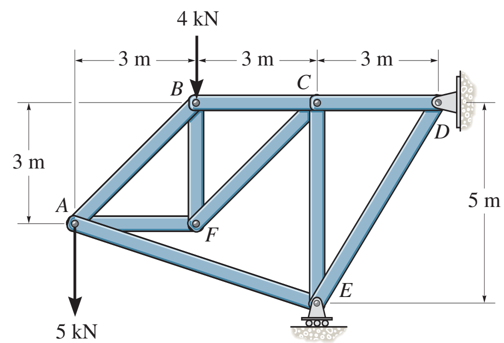


(d)

Ans:
$F_{CD} = 5.21 \text{ kN (C)}$
$F_{CB} = 2.36 \text{ kN (T)}$
$F_{AD} = 1.46 \text{ kN (C)}$
$F_{AB} = 2.36 \text{ kN (T)}$
$F_{BD} = 4 \text{ kN (T)}$

6–7.

Determine the force in each member of the truss and state if the members are in tension or compression.



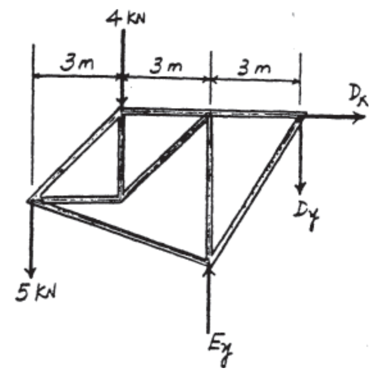
SOLUTION

Support Reactions:

$\zeta + \Sigma M_D = 0;\qquad 4(6) + 5(9) - E_y(3) = 0 \qquad E_y = 23.0 \text{ kN}$

$+\uparrow \Sigma F_y = 0;\qquad 23.0 - 4 - 5 - D_y = 0 \qquad D_y = 14.0 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0 \qquad D_x = 0$



Method of Joints:

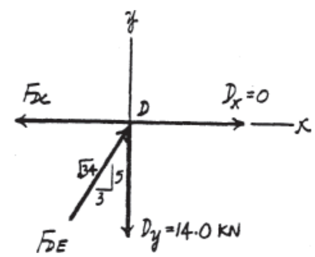
Joint D:

$+\uparrow \Sigma F_y = 0;\qquad F_{DE}\left(\frac{5}{\sqrt{34}}\right) - 14.0 = 0$

$F_{DE} = 16.33 \text{ kN (C)} = 16.3 \text{ kN (C)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;\qquad 16.33\left(\frac{3}{\sqrt{34}}\right) - F_{DC} = 0$

$F_{DC} = 8.40 \text{ kN (T)}$ **Ans.**



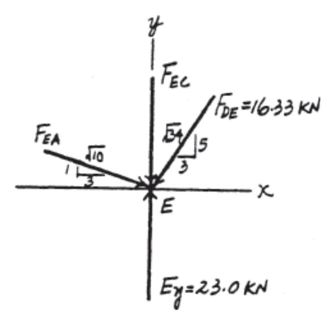
Joint E:

$\xrightarrow{+} \Sigma F_x = 0;\qquad F_{EA}\left(\frac{3}{\sqrt{10}}\right) - 16.33\left(\frac{3}{\sqrt{34}}\right) = 0$

$F_{EA} = 8.854 \text{ kN (C)} = 8.85 \text{ kN (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad 23.0 - 16.33\left(\frac{5}{\sqrt{34}}\right) - 8.854\left(\frac{1}{\sqrt{10}}\right) - F_{EC} = 0$

$F_{EC} = 6.20 \text{ kN (C)}$ **Ans.**



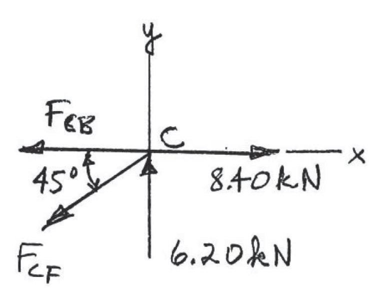
Joint C:

$+\uparrow \Sigma F_y = 0;\qquad 6.20 - F_{CF}\sin 45° = 0$

$F_{CF} = 8.768 \text{ kN (T)} = 8.77 \text{ kN (T)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;\qquad 8.40 - 8.768\cos 45° - F_{CB} = 0$

$F_{CB} = 2.20 \text{ kN (T)}$ **Ans.**



6–7. Continued

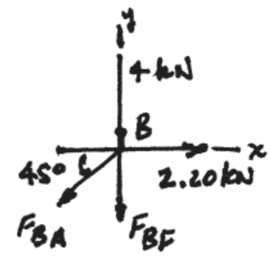
Joint B:

$\xrightarrow{+} \Sigma F_x = 0;\qquad 2.20 - F_{BA}\cos 45^\circ = 0$

$F_{BA} = 3.111 \text{ kN (T)} = 3.11 \text{ kN (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad F_{BF} - 4 - 3.111 \sin 45^\circ = 0$

$F_{BF} = 6.20 \text{ kN (C)}$ **Ans.**

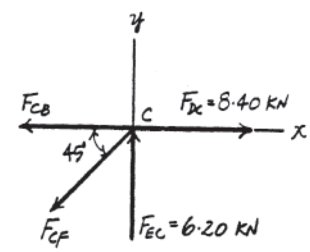


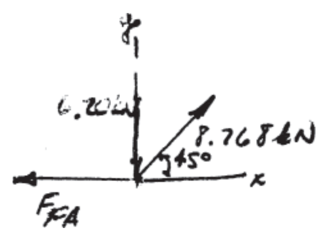
Joint F:

$+\uparrow \Sigma F_y = 0;\qquad 8.768 \sin 45^\circ - 6.20 = 0$ **(Check!)**

$\xrightarrow{+} \Sigma F_x = 0;\qquad 8.768 \cos 45^\circ - F_{FA} = 0$

$F_{FA} = 6.20 \text{ kN (T)}$ **Ans.**

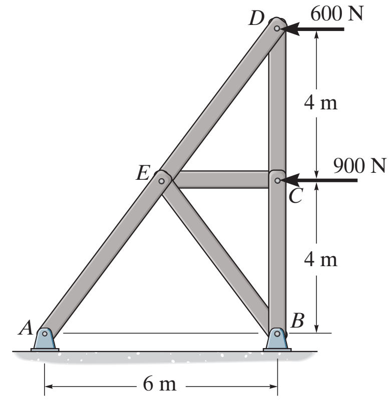




Ans:
$F_{DE} = 16.3$ kN (C)
$F_{DC} = 8.40$ kN (T)
$F_{EA} = 8.85$ kN (C)
$F_{EC} = 6.20$ kN (C)
$F_{CF} = 8.77$ kN (T)
$F_{CB} = 2.20$ kN (T)
$F_{BA} = 3.11$ kN (T)
$F_{BF} = 6.20$ kN (C)
$F_{FA} = 6.20$ kN (T)

***6–8.**

Determine the force in each member of the truss, and state if the members are in tension or compression.



SOLUTION

Method of Joints: We will begin by analyzing the equilibrium of joint D, and then proceed to analyze joints C and E.

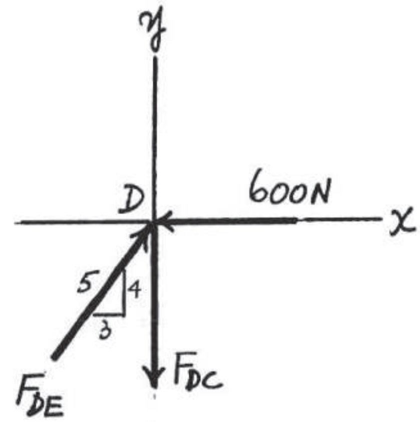
Joint D: From the free-body diagram in Fig. *a,*

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{DE}\left(\frac{3}{5}\right) - 600 = 0$

$F_{DE} = 1000 \text{ N} = 1.00 \text{ kN (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 1000\left(\frac{4}{5}\right) - F_{DC} = 0$

$F_{DC} = 800 \text{ N (T)}$ **Ans.**



(a)

Joint C: From the free-body diagram in Fig. *b,*

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{CE} - 900 = 0$

$F_{CE} = 900 \text{ N (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 800 - F_{CB} = 0$

$F_{CB} = 800 \text{ N (T)}$ **Ans.**

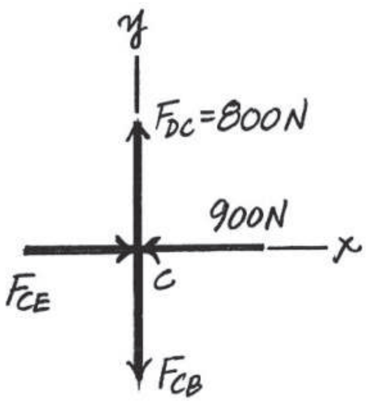
Joint E: From the free-body diagram in Fig. *c,*

$\searrow + \Sigma F_{x'} = 0;$ $\quad -900 \cos 36.87° + F_{EB} \sin 73.74° = 0$

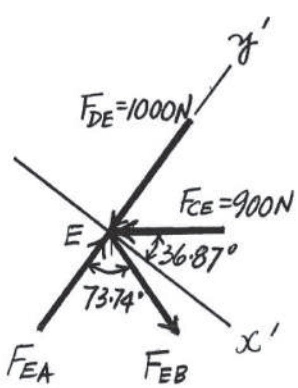
$F_{EB} = 750 \text{ N (T)}$ **Ans.**

$\nearrow + \Sigma F_{y'} = 0;$ $\quad F_{EA} - 1000 - 900 \sin 36.87° - 750 \cos 73.74° = 0$

$F_{EA} = 1750 \text{ N} = 1.75 \text{ kN (C)}$ **Ans.**



(b)

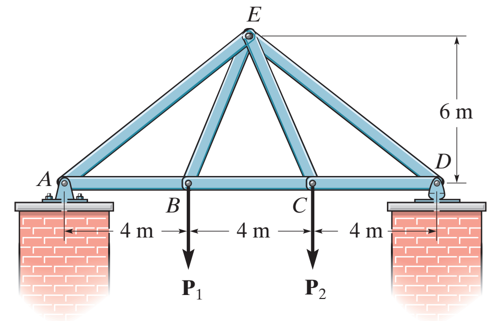


(c)

Ans:
$F_{DE} = 1.00 \text{ kN (C)}$
$F_{DC} = 800 \text{ N (T)}$
$F_{CE} = 900 \text{ N (C)}$
$F_{CB} = 800 \text{ N (T)}$
$F_{EB} = 750 \text{ N (T)}$
$F_{EA} = 1.75 \text{ kN (C)}$

6–9.

Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 3$ kN, $P_2 = 6$ kN.



SOLUTION

Support Reactions. Referring to the FBD of the entire truss shown in Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0; \quad N_D(12) - 3(4) - 6(8) = 0 \quad N_D = 5.00 \text{ kN}$

$\circlearrowleft + \Sigma M_D = 0; \quad 6(4) + 3(8) - A_y(12) = 0 \quad A_y = 4.00 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 0$

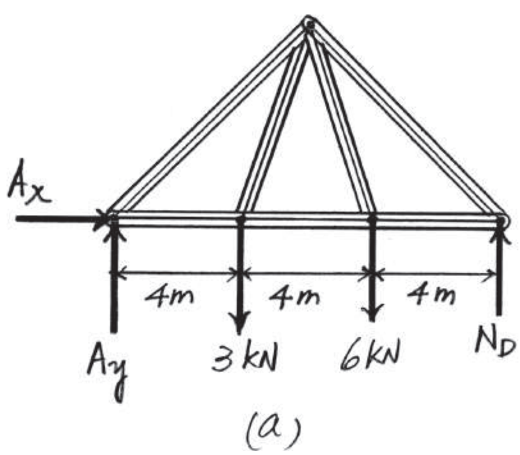
Method of Joints. We will carry out the analysis of joint equilibrium according to the sequence of joints A, D, B and C.

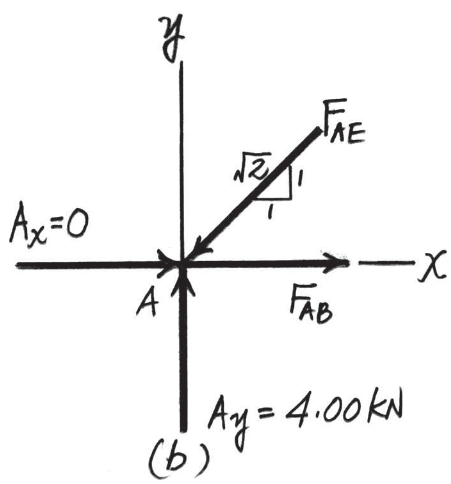
Joint *A*. Fig. *b*

$+\uparrow \Sigma F_y = 0; \quad 4.00 - F_{AE}\left(\dfrac{1}{\sqrt{2}}\right) = 0$

$F_{AE} = 4\sqrt{2} \text{ kN (C)} = 5.66 \text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{AB} - 4\sqrt{2}\left(\dfrac{1}{\sqrt{2}}\right) = 0 \quad F_{AB} = 4.00 \text{ kN (T)}$ **Ans.**





6–9. Continued

Joint *D*. Fig. *c*

$$+\uparrow \Sigma F_y = 0; \quad 5.00 - F_{DE}\left(\frac{1}{\sqrt{2}}\right) = 0 \quad F_{DE} = 5\sqrt{2}\text{ kN (C)} = 7.07\text{ kN (C)} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 5\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) - F_{DC} = 0 \quad F_{DC} = 5.00\text{ kN (T)} \qquad \textbf{Ans.}$$

Joint *B*. Fig. *d*

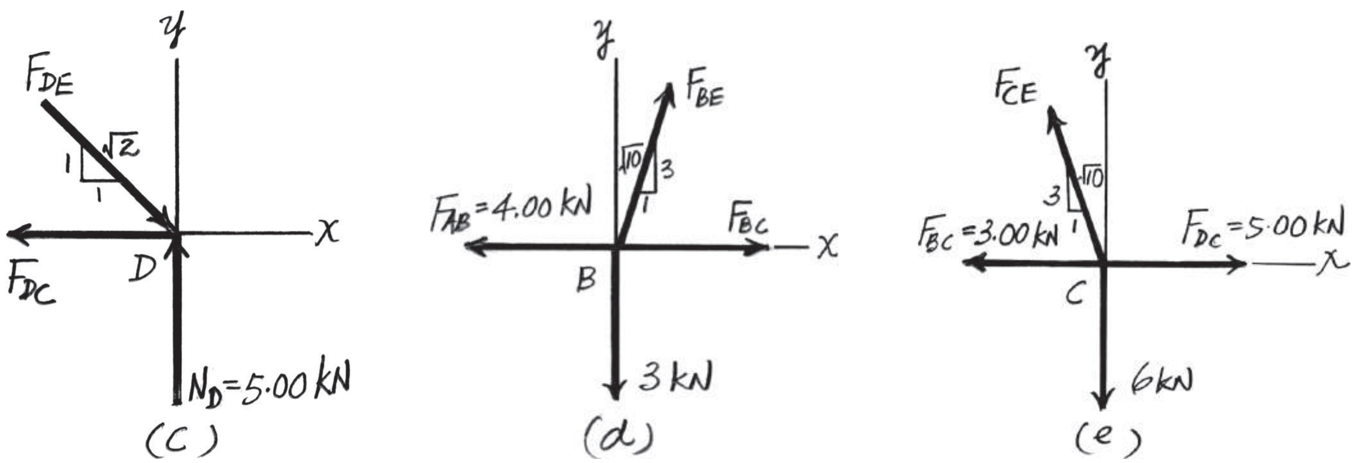
$$+\uparrow \Sigma F_y = 0; \quad F_{BE}\left(\frac{3}{\sqrt{10}}\right) - 3 = 0 \quad F_{BE} = \sqrt{10}\text{ kN (T)} = 3.16\text{ kN (T)} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad F_{BC} + \sqrt{10}\left(\frac{1}{\sqrt{10}}\right) - 4.00 = 0 \quad F_{BC} = 3.00\text{ kN (T)} \qquad \textbf{Ans.}$$

Joint *C*. Fig. *e*

$$+\uparrow \Sigma F_y = 0; \quad F_{CE}\left(\frac{3}{\sqrt{10}}\right) - 6 = 0 \quad F_{CE} = 2\sqrt{10}\text{ kN }(T) = 6.32\text{ kN (T)} \qquad \textbf{Ans.}$$

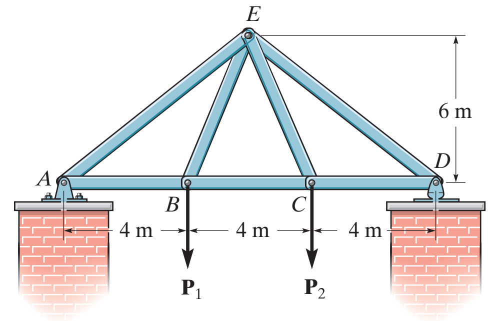
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 5.00 - 3.00 - \left(2\sqrt{10}\right)\left(\frac{1}{\sqrt{10}}\right) = 0 \qquad \text{(Check!!)}$$



Ans:
$F_{AE} = 5.66$ kN (C)
$F_{AB} = 4.00$ kN (T)
$F_{DE} = 7.07$ kN (C)
$F_{DC} = 5.00$ kN (T)
$F_{BE} = 3.16$ kN (T)
$F_{BC} = 3.00$ kN (T)
$F_{CE} = 6.32$ kN (T)

6–10.

Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 6$ kN, $P_2 = 9$ kN.



SOLUTION

Support Reactions. Referring to the FBD of the entire truss shown in Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0; \quad N_D(12) - 6(4) - 9(8) = 0 \qquad N_D = 8.00 \text{ kN}$

$\circlearrowleft + \Sigma M_D = 0; \quad 9(4) + 6(8) - A_y(12) = 0 \qquad A_y = 7.00 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$

Method of Joints. We will carry out the analysis of joint equilibrium according to the sequence of joints A, D, B and C.

Joint *A*. Fig. *a*

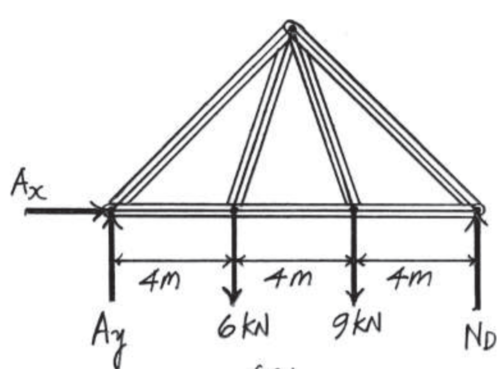
$+\uparrow \Sigma F_y = 0; \quad 7.00 - F_{AE}\left(\frac{1}{\sqrt{2}}\right) = 0 \quad F_{AE} = 7\sqrt{2} \text{ kN (C)} = 9.90 \text{ kN (C)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AB} - 7\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) = 0 \quad F_{AB} = 7.00 \text{ kN (T)}$ **Ans.**

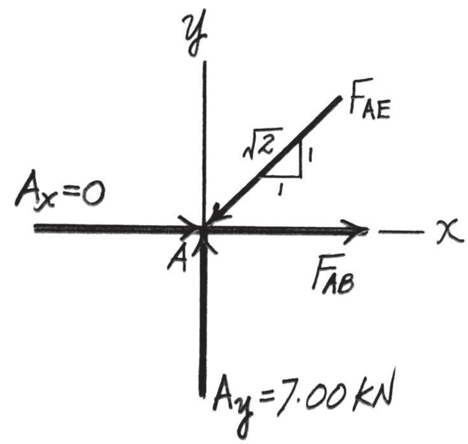
Joint *D*. Fig. *c*

$+\uparrow \Sigma F_y = 0; \quad 8.00 - F_{DE}\left(\frac{1}{\sqrt{2}}\right) = 0 \quad F_{DE} = 8\sqrt{2} \text{ kN (C)} = 11.3 \text{ kN (C)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \quad 8\sqrt{2}\left(\frac{1}{\sqrt{2}}\right) - F_{DC} = 0 \quad F_{DC} = 8.00 \text{ kN (T)}$ **Ans.**



(a)



(b)

6–10. Continued

Joint *B*. Fig. *d*

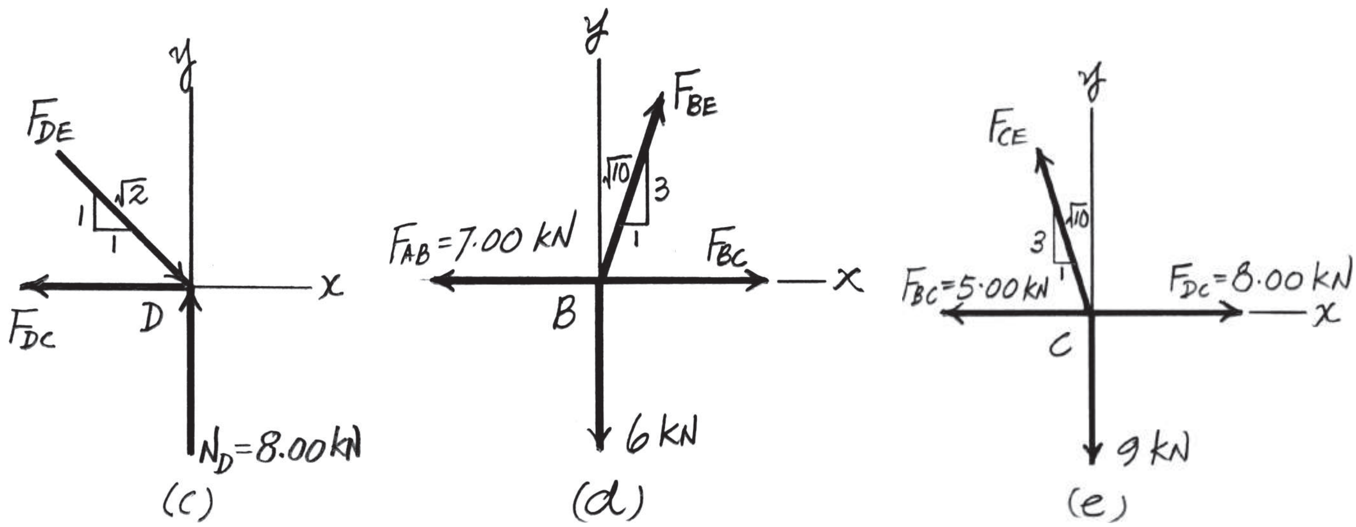
$+\uparrow\Sigma F_y = 0;\quad F_{BE}\left(\frac{3}{\sqrt{10}}\right) - 6 = 0 \quad F_{BE} = 2\sqrt{10}\text{ kN (T)} = 6.32\text{ kN (T)}$ **Ans.**

$\xrightarrow{+}\Sigma F_x = 0;\quad F_{BC} - 7.00 + \left(2\sqrt{10}\right)\left(\frac{1}{\sqrt{10}}\right) = 0 \quad F_{BC} = 5.00\text{ kN (T)}$ **Ans.**

Joint C. Fig. *e*

$+\uparrow\Sigma F_y = 0;\quad F_{CE}\left(\frac{3}{\sqrt{10}}\right) - 9 = 0 \quad F_{CE} = 3\sqrt{10}\text{ kN} = 9.49\text{ kN (T)}$ **Ans.**

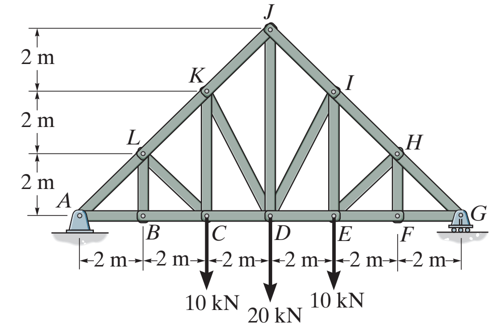
$\xrightarrow{+}\Sigma F_x = 0;\quad 8.00 - 5.00 - \left(3\sqrt{10}\right)\left(\frac{1}{\sqrt{10}}\right) = 0$ (Check!!)



Ans:
$F_{AE} = 9.90$ kN (C)
$F_{AB} = 7.00$ kN (T)
$F_{DE} = 11.3$ kN (C)
$F_{DC} = 8.00$ kN (T)
$F_{BE} = 6.32$ kN (T)
$F_{BC} = 5.00$ kN (T)
$F_{CE} = 9.49$ kN (T)

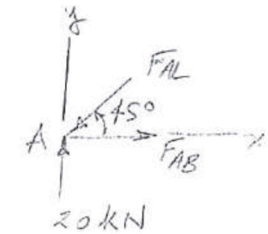
6–11.

Determine the force in each member of the *Pratt truss*, and state if the members are in tension or compression.



SOLUTION

Joint A:



$+\uparrow \Sigma F_y = 0;\qquad 20 - F_{AL} \sin 45° = 0$

$F_{AL} = 28.28 \text{ kN (C)}$

$\xrightarrow{+} \Sigma F_x = 0;\qquad F_{AB} - 28.28 \cos 45° = 0$

$F_{AB} = 20 \text{ kN (T)}$

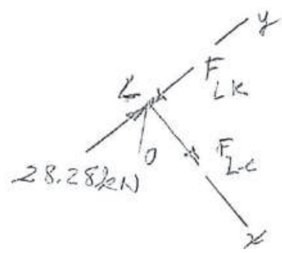
Joint B:

$\xrightarrow{+} \Sigma F_x = 0;\qquad F_{BC} - 20 = 0$

$F_{BC} = 20 \text{ kN (T)}$

$+\uparrow \Sigma F_y = 0;\qquad F_{BL} = 0$

Joint L:

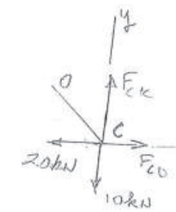


$\searrow\!+ \Sigma F_x = 0;\qquad F_{LC} = 0$

$+\nearrow \Sigma F_y = 0;\qquad 28.28 - F_{LK} = 0$

$F_{LK} = 28.28 \text{ kN (C)}$

Joint C:



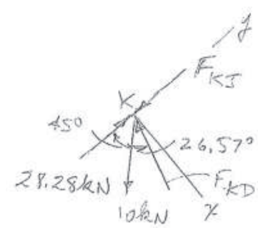
$\xrightarrow{+} \Sigma F_x = 0;\qquad F_{CD} - 20 = 0$

$F_{CD} = 20 \text{ kN (T)}$

$+\uparrow \Sigma F_y = 0;\qquad F_{CK} - 10 = 0$

$F_{CK} = 10 \text{ kN (T)}$

Joint K:



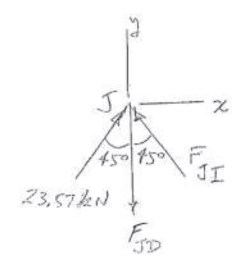
$\searrow\!+ \Sigma F_x - 0;\qquad 10 \sin 45° - F_{KD} \cos (45° - 26.57°) = 0$

$F_{KD} = 7.454 \text{ kN (L)}$

$+\nearrow \Sigma F_y = 0;\qquad 28.28 - 10 \cos 45° + 7.454 \sin (45° - 26.57°) - F_{KJ} = 0$

$F_{KJ} = 23.57 \text{ kN (C)}$

Joint J:



$\xrightarrow{+} \Sigma F_x = 0;\qquad 23.57 \sin 45° - F_{JI} \sin 45° = 0$

$F_{JI} = 23.57 \text{ kN (L)}$

$+\uparrow \Sigma F_y = 0;\qquad 2\,(23.57 \cos 45°) - F_{JD} = 0$

$F_{JD} = 33.3 \text{ kN (T)}$ **Ans.**

Due to Symmetry

$F_{AL} = F_{GH} = F_{LK} = F_{HI} = 28.3 \text{ kN (C)}$ **Ans.**

$F_{AB} = F_{GF} = F_{BC} = F_{FE} = F_{CD} = F_{ED} = 20 \text{ kN (T)}$ **Ans.**

$F_{BL} = F_{FH} = F_{LC} = F_{HE} = 0$ **Ans.**

$F_{CK} = F_{EI} = 10 \text{ kN (T)}$ **Ans.**

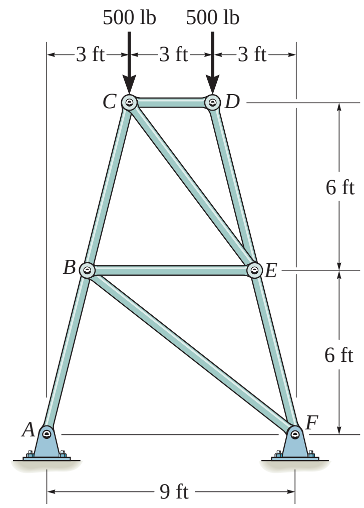
$F_{KJ} = F_{IJ} = 23.6 \text{ kN (C)}$ **Ans.**

$F_{KD} = F_{ID} = 7.45 \text{ kN (C)}$ **Ans.**

Ans:
$F_{JD} = 33.3 \text{ kN (T)}$
$F_{AL} = F_{GH} = F_{LK} = F_{HI} = 28.3 \text{ kN (C)}$
$F_{AB} = F_{GF} = F_{BC} = F_{FE} = F_{CD} = F_{ED} = 20 \text{ kN (T)}$
$F_{BL} = F_{FH} = F_{LC} = F_{HE} = 0$
$F_{CK} = F_{EI} = 10 \text{ kN (T)}$
$F_{KJ} = F_{IJ} = 23.6 \text{ kN (C)}$
$F_{KD} = F_{ID} = 7.45 \text{ kN (C)}$

***6–12.**

Determine the force in each member of the truss and state if the members are in tension or compression.



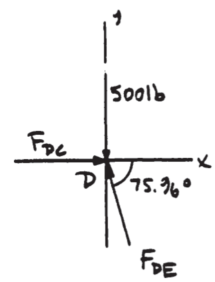
SOLUTION

Joint *D*:

$+\uparrow \Sigma F_y = 0; \qquad F_{DE} \sin 75.96° - 500 = 0$

$F_{DE} = 515.39 \text{ lb} = 515 \text{ lb (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{CD} - 515.39 \cos 75.96° = 0 \qquad F_{CD} = 125 \text{ lb (C)}$ **Ans.**



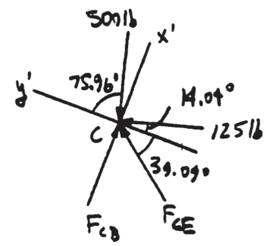
Joint *C*:

$+\nwarrow \Sigma F_{y'} = 0; \qquad F_{CE} \cos 39.09° + 125 \cos 14.04° - 500 \cos 75.96° = 0$

$F_{CE} = 0$ **Ans.**

$+\nearrow \Sigma F_{x'} = 0; \qquad F_{CB} - 500 \sin 75.96° - 125 \sin 14.04° = 0$

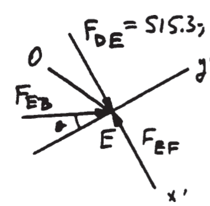
$F_{CB} = 515.39 \text{ lb} = 515 \text{ lb (C)}$ **Ans.**



Joint *E*:

$+\nearrow \Sigma F_{y'} = 0; \qquad F_{EB} \cos\theta = 0 \qquad F_{EB} = 0$ **Ans.**

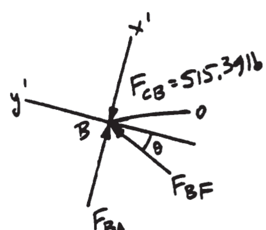
$+\searrow \Sigma F_{x'} = 0; \qquad 515.39 - F_{EF} = 0 \qquad F_{EF} = 515 \text{ lb (C)}$ **Ans.**



Joint *B*:

$+\nwarrow \Sigma F_{y'} = 0; \qquad F_{BF} \cos\theta = 0 \qquad F_{BF} = 0$ **Ans.**

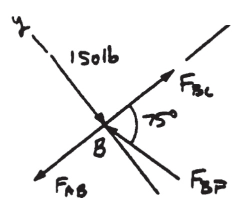
$+\nearrow \Sigma F_{x'} = 0; \qquad F_{BA} - 515.39 = 0 \qquad F_{BA} = 515 \text{ lb (C)}$ **Ans.**



Joint *B*:

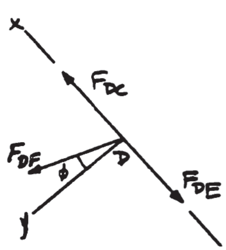
$+\nwarrow \Sigma F_y = 0; \qquad F_{BF} \sin 75° - 150 = 0$

$F_{BF} = 155 \text{ lb (C)}$ **Ans.**



Joint *D*:

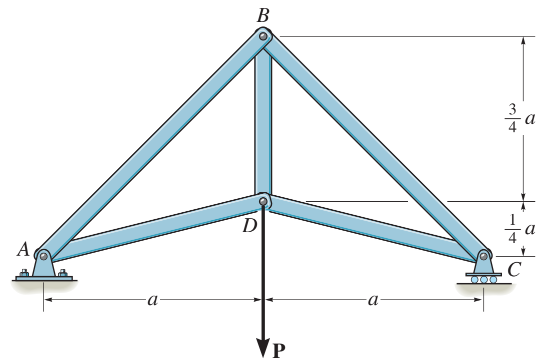
$+\swarrow \Sigma F_y = 0; \qquad F_{DF} = 0$ **Ans.**



Ans:
$F_{DE} = 515 \text{ lb (C)}$
$F_{CD} = 125 \text{ lb (C)}$
$F_{CE} = 0$
$F_{CB} = 515 \text{ lb (C)}$
$F_{EB} = 0$
$F_{EF} = 515 \text{ lb (C)}$
$F_{BF} = 0$
$F_{BA} = 515 \text{ lb (C)}$
$F_{BF} = 155 \text{ lb (C)}$
$F_{DF} = 0$

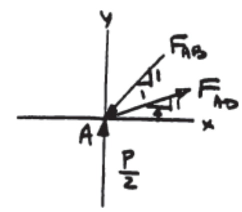
6–13.

Determine the force in each member of the truss in terms of the load P and state if the members are in tension or compression.



SOLUTION

Joint A:



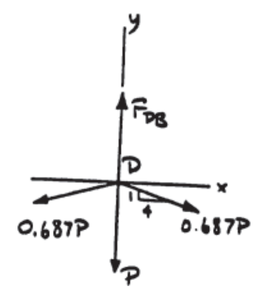
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad \frac{4}{\sqrt{14}}(F_{AD}) - \frac{1}{\sqrt{2}}F_{AB} = 0$$

$$+\uparrow\Sigma F_y = 0; \qquad \frac{P}{2} - \frac{1}{\sqrt{2}}(F_{AB}) + \frac{1}{\sqrt{17}}(F_{AD}) = 0$$

$$F_{CD} = F_{AD} = 0.687\,P\text{ (T)}$$ **Ans.**

$$F_{CB} = F_{AB} = 0.943\,P\text{ (C)}$$ **Ans.**

Joint D:



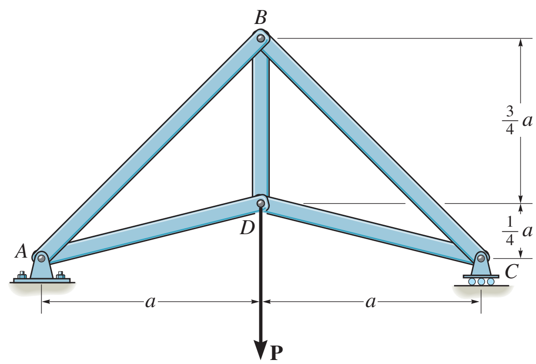
$$+\uparrow\Sigma F_y = 0; \qquad F_{DB} - 0.687\,P\left(\frac{1}{\sqrt{17}}\right) - \frac{1}{\sqrt{17}}(0.687\,P) - P = 0$$

$$F_{DB} = 1.33\,P\text{ (T)}$$ **Ans.**

Ans:
$F_{CD} = F_{AD} = 0.687P$ (T)
$F_{CB} = F_{AB} = 0.943P$ (C)
$F_{DB} = 1.33P$ (T)

6–14.

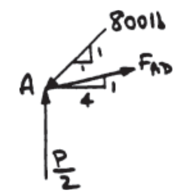
Members AB and BC can each support a maximum compressive force of 800 lb, and members AD, DC, and BD can support a maximum tensile force of 1500 lb. If $a = 10$ ft, determine the greatest load P the truss can support.



SOLUTION

Assume $F_{AB} = 800$ lb (C)

Joint A:



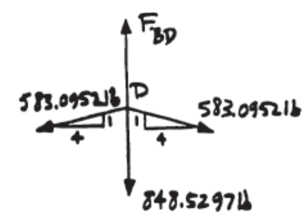
$\xrightarrow{+} \Sigma F_x = 0;\qquad -800\left(\frac{1}{\sqrt{2}}\right) + F_{AD}\left(\frac{4}{\sqrt{17}}\right) = 0$

$F_{AD} = 583.0952 \text{ lb} < 1500 \text{ lb}$ **OK**

$+\uparrow \Sigma F_y = 0;\qquad \frac{P}{2} - \frac{1}{\sqrt{2}}(800) + \frac{1}{\sqrt{17}}(583.0952) = 0$

$P = 848.5297$ lb

Joint D:



$+\uparrow \Sigma F_y = 0;\qquad -848.5297 - 583.0952(2)\left(\frac{1}{\sqrt{17}}\right) + F_{DB} = 0$

$F_{BD} = 1131.3724 \text{ lb} < 1500 \text{ lb}$ **OK**

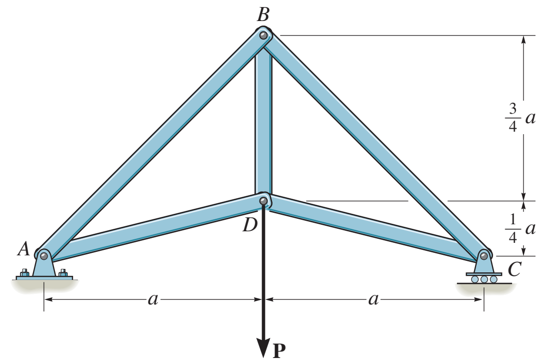
Thus,

$P_{max} = 849$ lb **Ans.**

Ans:
$P_{\max} = 849$ lb

6–15.

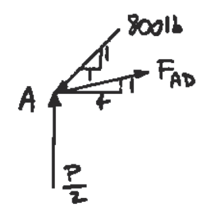
Members AB and BC can each support a maximum compressive force of 800 lb, and members AD, DC, and BD can support a maximum tensile force of 2000 lb. If $a = 6$ ft, determine the greatest load P the truss can support.



SOLUTION

1) Assume $F_{AB} = 800$ lb (C)

Joint A:



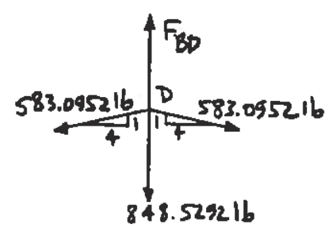
$\xrightarrow{+}\Sigma F_x = 0;\quad -800\left(\frac{1}{\sqrt{2}}\right) + F_{AD}\left(\frac{4}{\sqrt{17}}\right) = 0$

$F_{AD} = 583.0952 \text{ lb} < 2000 \text{ lb}$ **OK**

$+\uparrow\Sigma F_y = 0;\quad \frac{P}{2} - \frac{1}{\sqrt{2}}(800) + \frac{1}{\sqrt{17}}(583.0952) = 0$

$P = 848.5297$ lb **OK**

Joint D:



$+\uparrow\Sigma F_y = 0;\quad -848.5297 - 583.0952(2)\left(\frac{1}{\sqrt{17}}\right) + F_{DB} = 0$

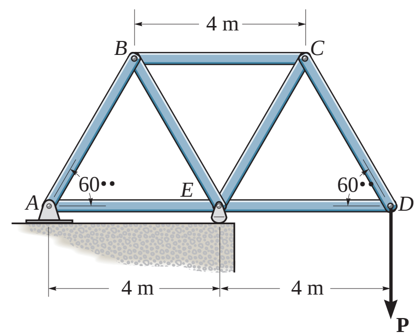
$F_{BD} = 1131.3724 \text{ lb} < 2000 \text{ lb}$ **OK**

Thus, $P_{max} = 849$ lb **Ans.**

Ans:
$P_{\max} = 849$ lb

***6–16.**

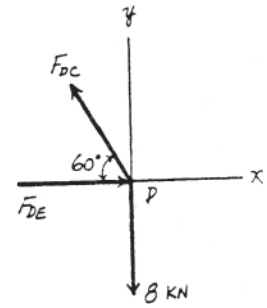
Determine the force in each member of the truss. State whether the members are in tension or compression. Set $P = 8$ kN.



SOLUTION

Method of Joints: In this case, the support reactions are not required for determining the member forces.

Joint D:



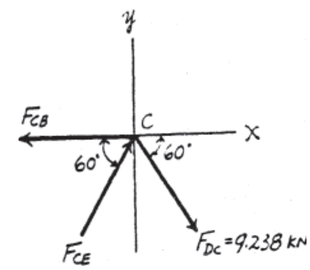
$+\uparrow \Sigma F_y = 0;\qquad F_{DC} \sin 60° - 8 = 0$

$$F_{DC} = 9.238 \text{ kN (T)} = 9.24 \text{ kN (T)} \qquad \textbf{Ans.}$$

$\xrightarrow{+} \Sigma F_x = 0;\qquad F_{DE} - 9.238 \cos 60° = 0$

$$F_{DE} = 4.619 \text{ kN (C)} = 4.62 \text{ kN (C)} \qquad \textbf{Ans.}$$

Joint C:



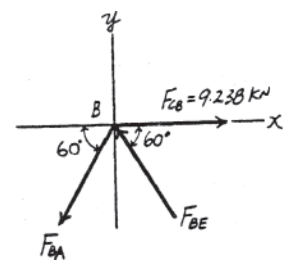
$+\uparrow \Sigma F_y = 0;\qquad F_{CE} \sin 60° - 9.238 \sin 60° = 0$

$$F_{CE} = 9.238 \text{ kN (C)} = 9.24 \text{ kN (C)} \qquad \textbf{Ans.}$$

$\xrightarrow{+} \Sigma F_x = 0;\qquad 2(9.238 \cos 60°) - F_{CB} = 0$

$$F_{CB} = 9.238 \text{ kN (T)} = 9.24 \text{ kN (T)} \qquad \textbf{Ans.}$$

Joint B:



$+\uparrow \Sigma F_y = 0;\qquad F_{BE} \sin 60° - F_{BA} \sin 60° = 0$

$$F_{BE} = F_{BA} = F$$

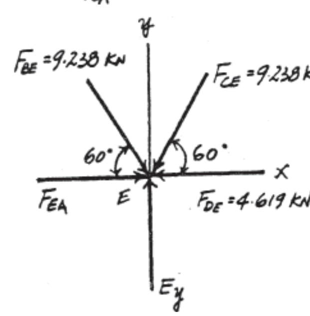
$\xrightarrow{+} \Sigma F_x = 0;\qquad 9.238 - 2F \cos 60° = 0$

$$F = 9.238 \text{ kN}$$

Thus,

$$F_{BE} = 9.24 \text{ kN (C)} \qquad F_{BA} = 9.24 \text{ kN (T)} \qquad \textbf{Ans.}$$

Joint E:



$+\uparrow \Sigma F_y = 0;\qquad E_y - 2(9.238 \sin 60°) = 0 \qquad E_y = 16.0 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0;\qquad F_{EA} + 9.238 \cos 60° - 9.238 \cos 60° - 4.619 = 0$

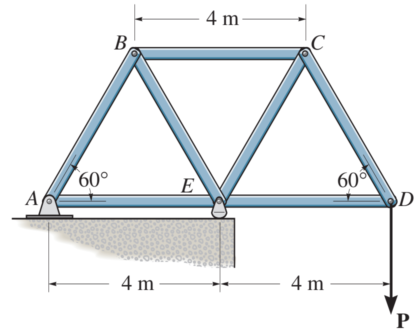
$$F_{EA} = 4.62 \text{ kN (C)} \qquad \textbf{Ans.}$$

Note: The support reactions A_x and A_y can be determinedd by analyzing Joint A using the results obtained above.

Ans:
$F_{DC} = 9.24$ kN (T)
$F_{DE} = 4.62$ kN (C)
$F_{CE} = 9.24$ kN (C)
$F_{CB} = 9.24$ kN (T)
$F_{BA} = 9.24$ kN (T)
$F_{EA} = 4.62$ kN (C)

6–17.

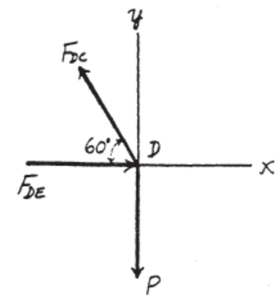
If the maximum force that any member can support is 8 kN in tension and 6 kN in compression, determine the maximum force P that can be supported at joint D.



SOLUTION

Method of Joints: In this case, the support reactions are not required for determining the member forces.

Joint D:



$+\uparrow \Sigma F_y = 0;$ $\quad F_{DC} \sin 60° - P = 0 \quad F_{DC} = 1.1547P$ (T)

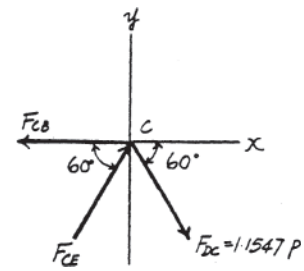
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{DE} - 1.1547P \cos 60° = 0 \quad F_{DE} = 0.57735P$ (C)

Joint C:

$+\uparrow \Sigma F_y = 0;$ $\quad F_{CE} \sin 60° - 1.1547P \sin 60° = 0$

$$F_{CE} = 1.1547P \text{ (C)}$$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 2(1.1547P \cos 60°) - F_{CB} = 0 \quad F_{CB} = 1.1547P$ (T)



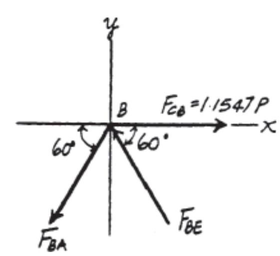
Joint B:

$+\uparrow \Sigma F_y = 0;$ $\quad F_{BE} \sin 60° - F_{BA} \sin 60° = 0 \quad F_{BE} = F_{BA} = F$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 1.1547P - 2F \cos 60° = 0 \quad F = 1.1547P$

Thus,

$$F_{BE} = 1.1547P \text{ (C)} \qquad F_{BA} = 1.1547P \text{ (T)}$$



Joint E:

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{EA} + 1.1547P \cos 60° - 1.1547P \cos 60°$

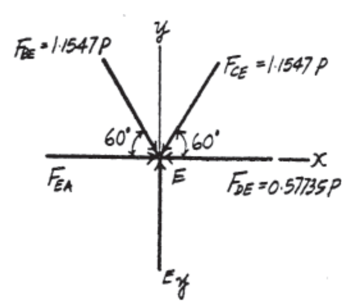
$$-0.57735P = 0$$

$$F_{EA} = 0.57735P \text{ (C)}$$

From the above analysis, the maximum compression and tension in the truss member is $1.1547P$. For this case, compression controls which requires

$$1.1547P = 6$$

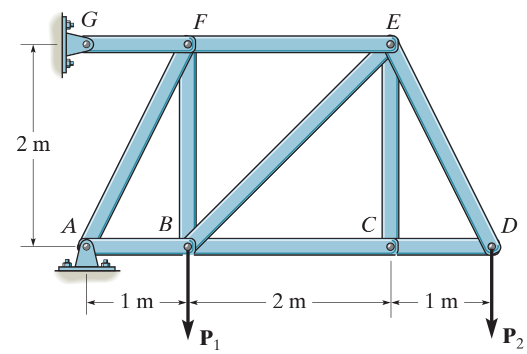
$$P = 5.20 \text{ kN} \qquad \textbf{Ans.}$$



Ans:
$P = 5.20$ kN

6–18.

Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 10$ kN, $P_2 = 8$ kN.



SOLUTION

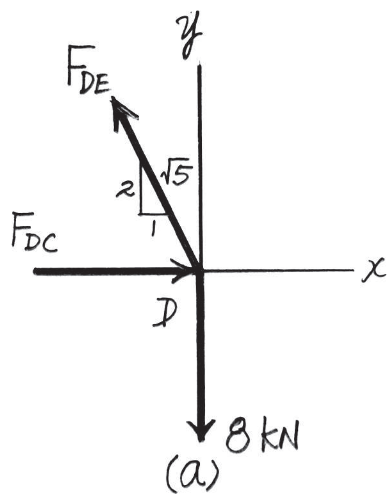
Support Reactions. Not required.

Method of Joints. We will perform the joint equilibrium according to the sequence of joints D, C, E, B and F.

Joint *D*. Fig. *a*

$+\uparrow \Sigma F_y = 0; \quad F_{DE}\left(\frac{2}{\sqrt{5}}\right) - 8 = 0 \quad F_{DE} = 4\sqrt{5}\text{ kN (T)} = 8.94\text{ kN (T)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{DC} - \left(4\sqrt{5}\right)\left(\frac{1}{\sqrt{5}}\right) = 0 \quad F_{DC} = 4.00\text{ kN (C)}$ **Ans.**



Joint *C*. Fig. *b*

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{CB} - 4.00 = 0 \quad F_{CB} = 4.00\text{ kN(C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad F_{CE} = 0$ **Ans.**

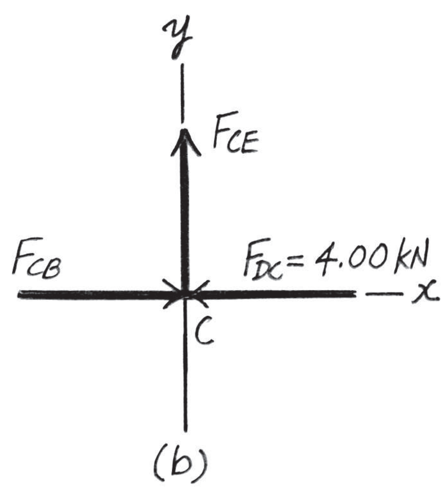
Joint *E*. Fig. *c*

$+\uparrow \Sigma F_y = 0; \quad F_{EB}\left(\frac{1}{\sqrt{2}}\right) - \left(4\sqrt{5}\right)\left(\frac{2}{\sqrt{5}}\right) = 0$

$F_{EB} = 8\sqrt{2}\text{ kN (C)} = 11.3\text{ kN (C)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \quad \left(8\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(4\sqrt{5}\right)\left(\frac{1}{\sqrt{5}}\right) - F_{EF} = 0$

$F_{EF} = 12.0\text{ kN (T)}$ **Ans.**



6–18. Continued

Joint *B*. Fig. *d*

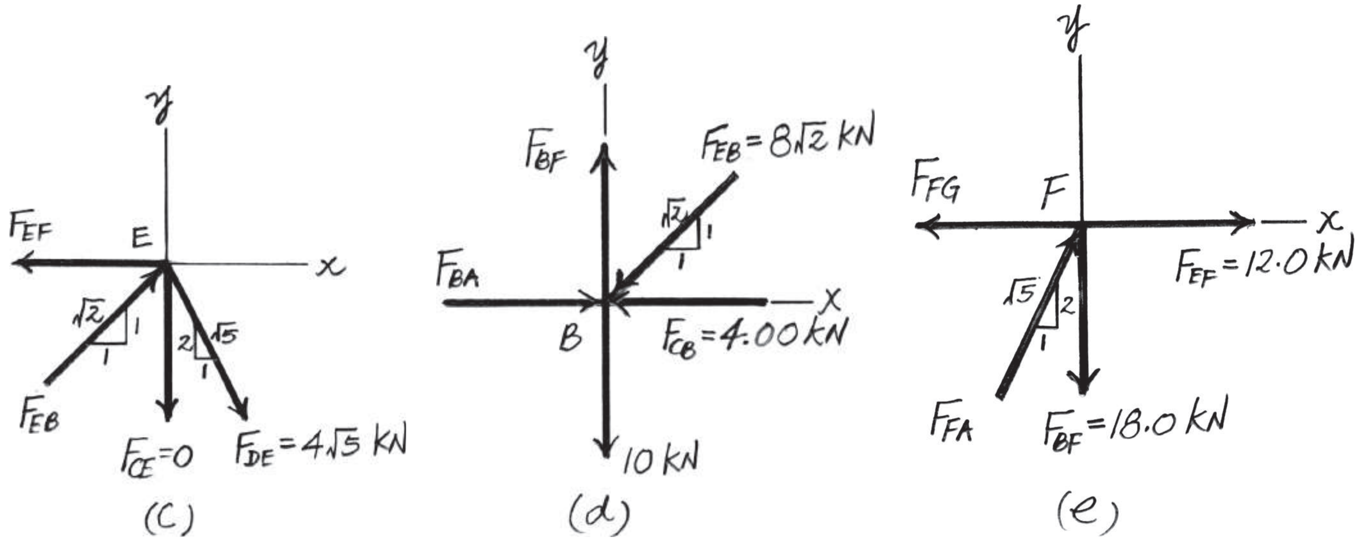
$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BA} - 4.00 - \left(8\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}\right) = 0 \quad F_{BA} = 12.0 \text{ kN (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad F_{BF} - 10 - \left(8\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}\right) = 0 \quad F_{BF} = 18.0 \text{ kN (T)}$ **Ans.**

Joint *F*. Fig. *e*

$+\uparrow \Sigma F_y = 0; \quad F_{FA}\left(\frac{2}{\sqrt{5}}\right) - 18.0 = 0 \quad F_{FA} = 9\sqrt{5} \text{ kN (C)} = 20.1 \text{ kN (C)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \quad 12.0 + \left(9\sqrt{5}\right)\left(\frac{1}{\sqrt{5}}\right) - F_{FG} = 0 \quad F_{FG} = 21.0 \text{ kN (T)}$ **Ans.**

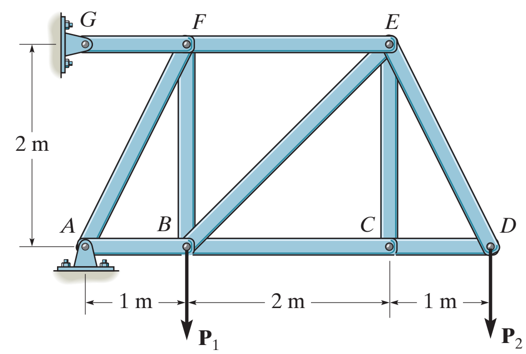


Ans:

$F_{DE} = 8.94$ kN (T)
$F_{DC} = 4.00$ kN (C)
$F_{CB} = 4.00$ kN (C)
$F_{CE} = 0$
$F_{EB} = 11.3$ kN (C)
$F_{EF} = 12.0$ kN (T)
$F_{BA} = 12.0$ kN (C)
$F_{BF} = 18.0$ kN (T)
$F_{FA} = 9\sqrt{5}$ kN (C) $= 20.1$ kN (C)
$F_{FG} = 21.0$ kN (T)

6–19.

Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 8$ kN, $P_2 = 12$ kN.



SOLUTION

Support Reactions. Not required.

Method of Joints. We will perform the joint equilibrium according to the sequence of joints D, C, E, B and F.

Joint *D*. Fig. *a*

$+\uparrow\Sigma F_y = 0;\quad F_{DE}\left(\frac{2}{\sqrt{5}}\right) - 12 = 0 \quad F_{DE} = 6\sqrt{5}\text{ kN (T)} = 13.4\text{ kN (T)}$ **Ans.**

$\xrightarrow{+}\Sigma F_x = 0;\quad F_{DC} - \left(6\sqrt{5}\right)\left(\frac{1}{\sqrt{5}}\right) = 0 \quad F_{DC} = 6.00\text{ kN (C)}$ **Ans.**

Joint *C*. Fig. *b*

$\xrightarrow{+}\Sigma F_x = 0;\quad F_{CB} - 6.00 = 0 \quad F_{CB} = 6.00\text{ kN (C)}$ **Ans.**

$+\uparrow\Sigma F_y = 0;\quad F_{CE} = 0$ **Ans.**

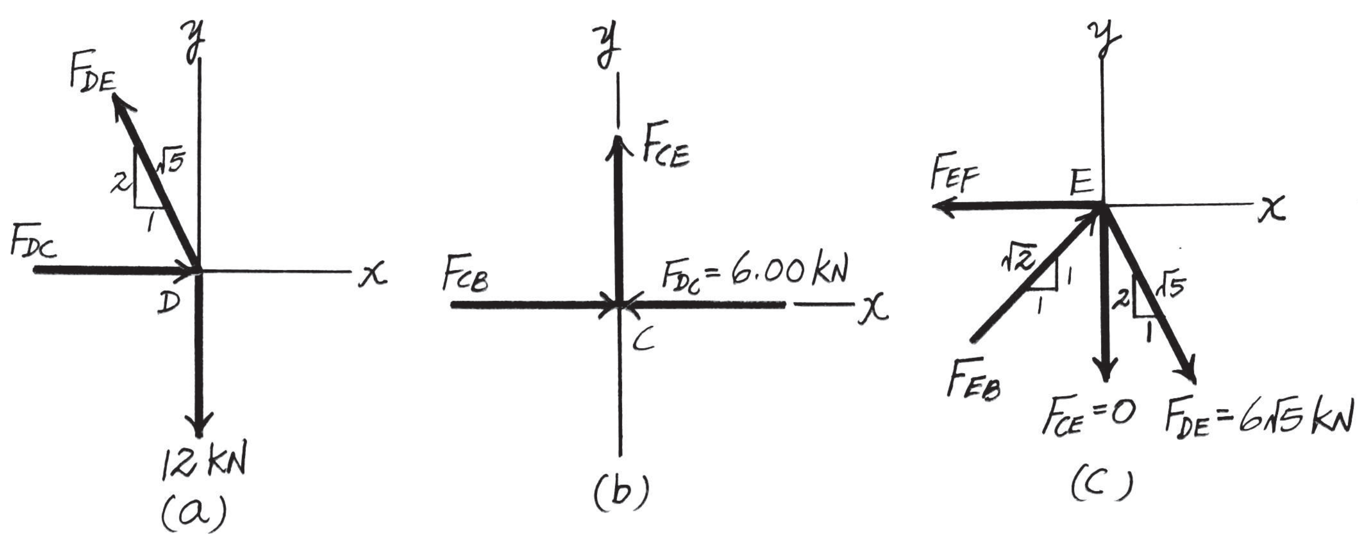
Joint *E*. Fig. *c*

$+\uparrow\Sigma F_y = 0;\quad F_{EB}\left(\frac{1}{\sqrt{2}}\right) - \left(6\sqrt{5}\right)\left(\frac{2}{\sqrt{5}}\right) = 0$

$F_{EB} = 12\sqrt{2}\text{ kN (C)} = 17.0\text{ kN (C)}$ **Ans.**

$\xrightarrow{+}\Sigma F_x = 0;\quad \left(12\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}\right) + \left(6\sqrt{5}\right)\left(\frac{1}{\sqrt{5}}\right) - F_{EF} = 0$

$F_{EF} = 18.0\text{ kN (T)}$ **Ans.**



6–19. Continued

Joint *B*. Fig. *d*

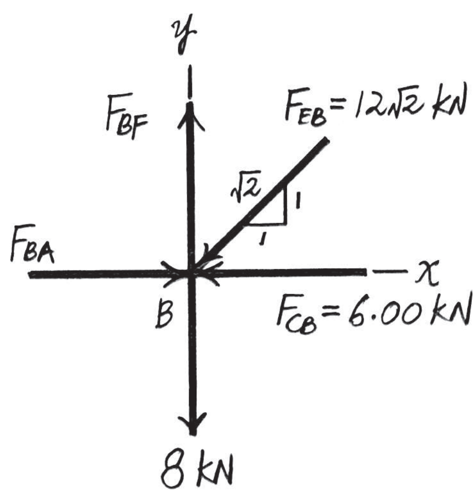
$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BA} - 6.00 - \left(12\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}\right) = 0 \quad F_{BA} = 18.0 \text{ kN (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad F_{BF} - 8 - \left(12\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}\right) = 0 \quad F_{BF} = 20.0 \text{ kN (T)}$ **Ans.**

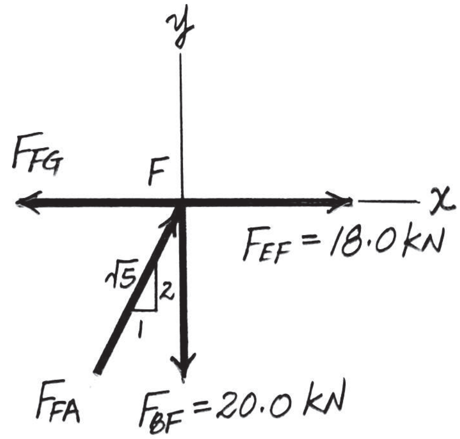
Joint *F*. Fig. *e*

$+\uparrow \Sigma F_y = 0; \quad F_{FA}\left(\frac{2}{\sqrt{5}}\right) - 20.0 = 0 \quad F_{FA} = 10\sqrt{5} \text{ kN (C)} = 22.4 \text{ kN (C)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \quad \left(10\sqrt{5}\right)\left(\frac{1}{\sqrt{5}}\right) + 18.0 - F_{FG} = 0 \quad F_{FG} = 28.0 \text{ kN(T)}$ **Ans.**



(d)

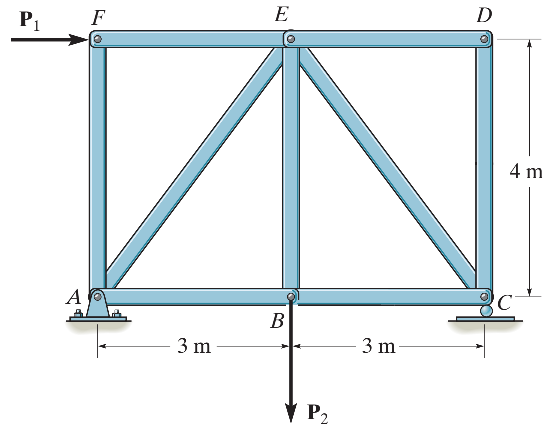


(e)

Ans:
$F_{DE} = 13.4$ kN (T)
$F_{DC} = 6.00$ kN (C)
$F_{CB} = 6.00$ kN (C)
$F_{CE} = 0$
$F_{EB} = 17.0$ kN (C)
$F_{EF} = 18.0$ kN (T)
$F_{BA} = 18.0$ kN (C)
$F_{BF} = 20.0$ kN (T)
$F_{FA} = 22.4$ kN (C)
$F_{FG} = 28.0$ kN (T)

***6–20.**

Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 9$ kN, $P_2 = 15$ kN.



SOLUTION

Support Reactions. Referring to the FBD of the entire truss shown in Fig. *a*

$\circlearrowleft + \Sigma M_A = 0; \quad N_C(6) - 15(3) - 9(4) = 0 \qquad N_C = 13.5 \text{ kN}$

$\circlearrowleft + \Sigma M_C = 0; \quad 15(3) - 9(4) - A_y(6) = 0 \qquad A_y = 1.50 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 9 - A_x = 0 \qquad A_x = 9.00 \text{ kN}$

Method of Joints: By inspecting joints D and F, we notice that members DE, DC and FA are zero force members. Thus

$F_{DE} = F_{DC} = F_{FA} = 0$ **Ans.**

We will perform the joint equilibrium analysis by following the sequence of joints C, B, A and F.

Joint *C*. Fig. *b*

$+\uparrow \Sigma F_y = 0; \quad 13.5 - F_{CE}\left(\frac{4}{5}\right) = 0$

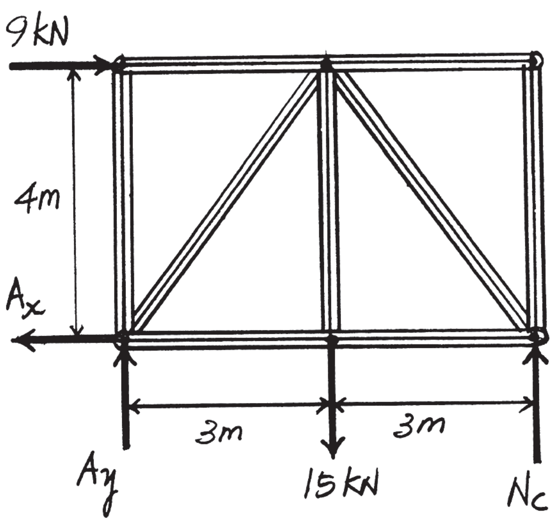
$F_{CE} = 16.875 \text{ kN (C)} = 16.9 \text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 16.875\left(\frac{3}{5}\right) - F_{CB} = 0 \quad F_{CB} = 10.125 \text{ kN (T)} = 10.1 \text{ kN (T)}$ **Ans.**

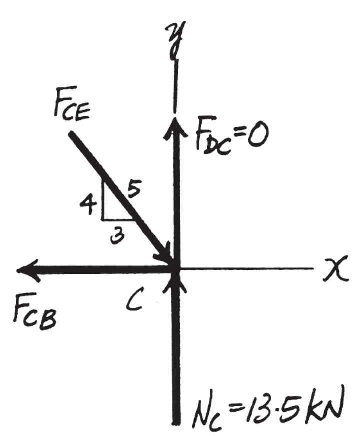
Joint *B*. Fig. *c*

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 10.125 - F_{BA} = 0 \qquad F_{BA} = 10.125 \text{ kN (T)} = 10.1 \text{ kN (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad F_{BE} - 15 = 0 \qquad F_{BE} = 15.0 \text{ kN (T)}$ **Ans.**



(a)



(b)

***6–20. Continued**

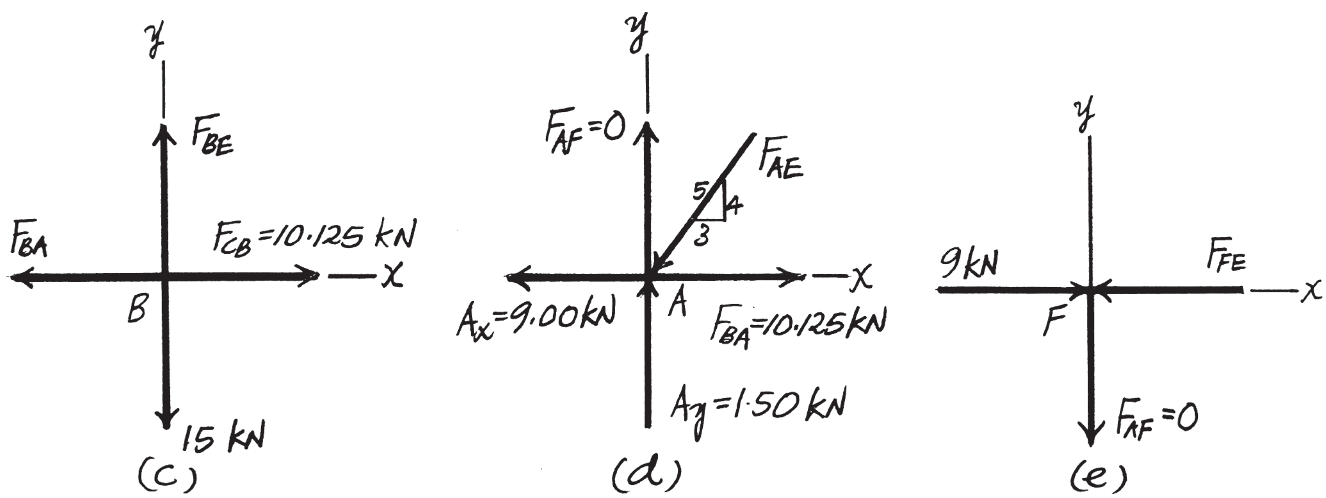
Joint *A*. Fig. *d*

$+\uparrow \Sigma F_y = 0; \qquad 1.50 - F_{AE}\left(\frac{4}{5}\right) = 0 \qquad F_{AE} = 1.875 \text{ kN (C)}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \qquad 10.125 - 1.875\left(\frac{3}{5}\right) - 9.00 = 0$ (Check!!)

Joint *F*. Fig. *e*

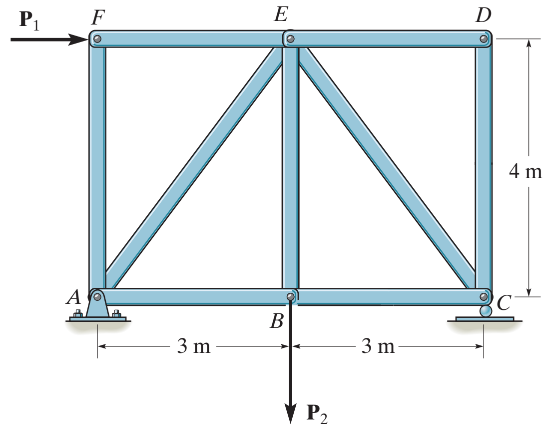
$\xrightarrow{+} \Sigma F_x = 0; \qquad 9 - F_{FE} = 0 \qquad F_{FE} = 9.00 \text{ kN (C)}$ **Ans.**



Ans:
$F_{CE} = 16.9$ kN (C)
$F_{CB} = 10.1$ kN (T)
$F_{BA} = 10.1$ kN (T)
$F_{BE} = 15.0$ kN (T)
$F_{AE} = 1.875$ kN (C)
$F_{FE} = 9.00$ kN (C)

6–21.

Determine the force in each member of the truss and state if the members are in tension or compression. Set $P_1 = 30$ kN, $P_2 = 15$ kN.



SOLUTION

Support Reactions.

$\circlearrowleft + \Sigma M_A = 0; \quad N_C(6) - 15(3) - 30(4) = 0 \qquad N_C = 27.5 \text{ kN}$

Method of Joints: By inspecting joints D and F, we notice that members DE, DC and FA are zero force members. Thus

$$F_{DE} = F_{DC} = F_{FA} = 0 \qquad \textbf{Ans.}$$

We will perform the joint equilibrium analysis by following the sequence of joints C, B, F and E.

Joint *C*. Fig. *b*

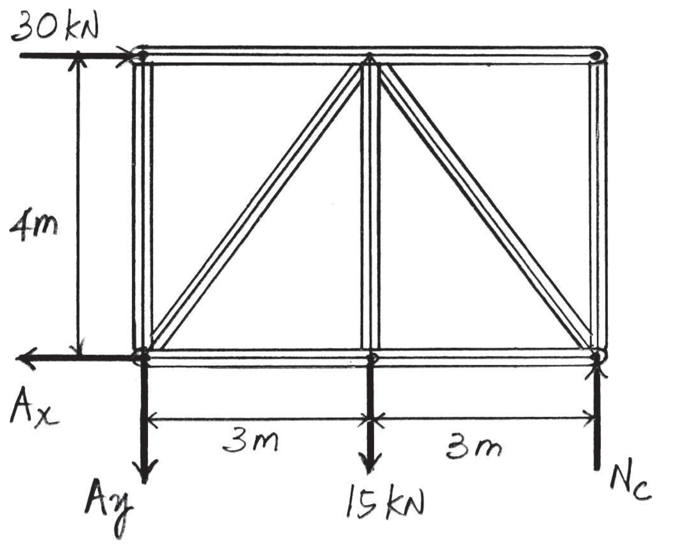
$+\uparrow \Sigma F_y = 0; \quad 27.5 - F_{CE}\left(\frac{4}{5}\right) = 0 \quad F_{CE} = 34.375 \text{ kN (C)} = 34.4 \text{ kN (C)} \qquad$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 34.375\left(\frac{3}{5}\right) - F_{CB} = 0 \quad F_{CB} = 20.625 \text{ kN (T)} = 20.6 \text{ kN (T)} \qquad$ **Ans.**

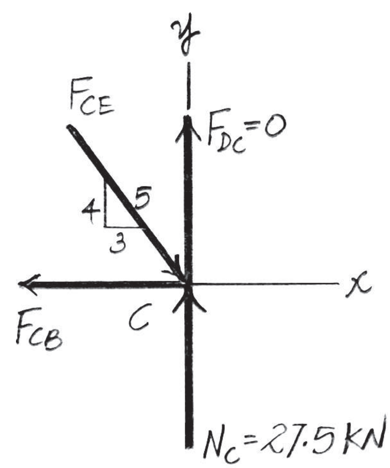
Joint *B*. Fig. *c*

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 20.625 - F_{BA} = 0 \quad F_{BA} = 20.625 \text{ kN (T)} = 20.6 \text{ kN (T)} \qquad$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad F_{BE} - 15 = 0 \quad F_{BE} = 15.0 \text{ kN (T)} \qquad$ **Ans.**



(a)



(b)

6–21. Continued

Joint *F*. Fig. *d*

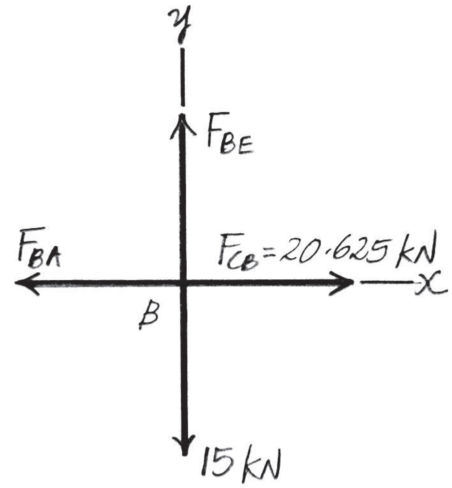
$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad 30 - F_{FE} = 0 \qquad F_{FE} = 30.0\text{ kN (C)}$ **Ans.**

Joint *E*. Fig. *e*

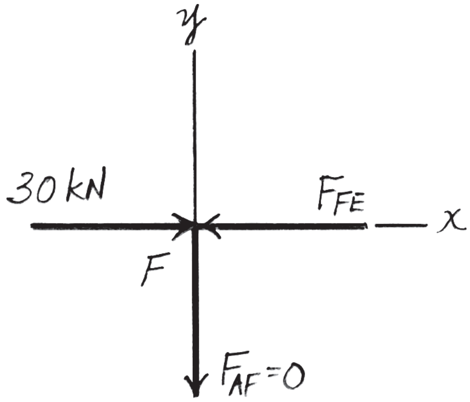
$+\uparrow\Sigma F_y = 0;\qquad 34.375\left(\frac{4}{5}\right) - 15.0 - F_{EA}\left(\frac{4}{5}\right) = 0$

$F_{EA} = 15.625\text{ kN (T)} = 15.6\text{ kN (T)}$ **Ans.**

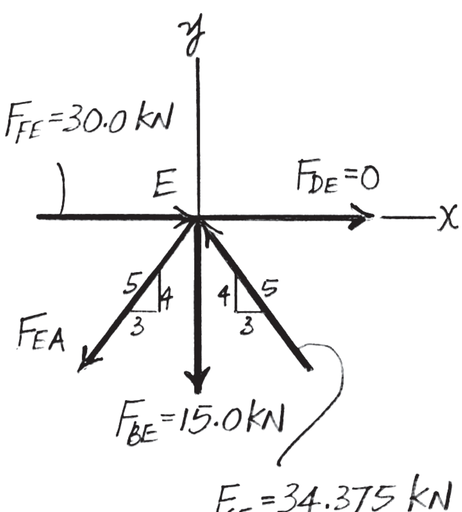
$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad 30.0 - 15.625\left(\frac{3}{5}\right) - 34.375\left(\frac{3}{5}\right) = 0\ \text{(Check!!)}$



(c)



(d)

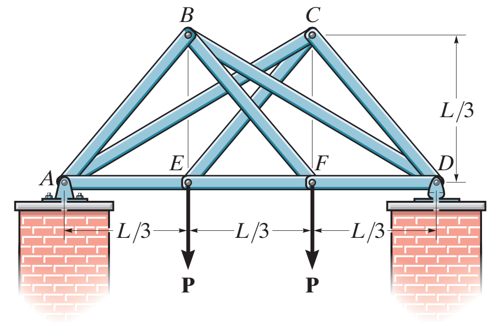


(e)

Ans:
$F_{DE} = F_{DC} = F_{FA} = 0$
$F_{CE} = 34.4\text{ kN (C)}$
$F_{CB} = 20.6\text{ kN (T)}$
$F_{BA} = 20.6\text{ kN (T)}$
$F_{BE} = 15.0\text{ kN (T)}$
$F_{FE} = 30.0\text{ kN (C)}$
$F_{EA} = 15.6\text{ kN (T)}$

6–22.

Determine the force in each member of the double scissors truss in terms of the load P and state if the members are in tension or compression.

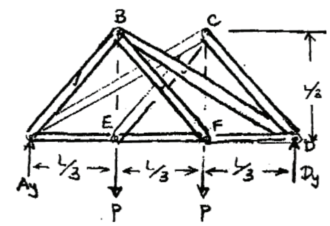


SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \quad P\left(\frac{L}{3}\right) + P\left(\frac{2L}{3}\right) - (D_y)(L) = 0$

$D_y = P$

$+\uparrow \Sigma F_y = 0; \quad A_y = P$



Joint F:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{FD} - F_{FE} - F_{FB}\left(\frac{1}{\sqrt{2}}\right) = 0 \qquad \textbf{(1)}$

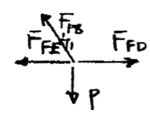
$F_{FD} - F_{FE} = P$

$\overset{+}{\rightarrow} \Sigma F_y = 0; \quad F_{FB}\left(\frac{1}{\sqrt{2}}\right) - P = 0$

$F_{FB} = \sqrt{2}P = 1.41\,P\text{ (T)}$

Similarly,

$F_{EC} = \sqrt{2}P$



Joint C:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{CA}\left(\frac{2}{\sqrt{5}}\right) - \sqrt{2}P\left(\frac{1}{\sqrt{2}}\right) - F_{CD}\left(\frac{1}{\sqrt{2}}\right) = 0$

$\frac{2}{\sqrt{5}}F_{CA} - \frac{1}{\sqrt{2}}F_{CD} = P$

$+\uparrow \Sigma F_y = 0; \quad F_{CA}\frac{1}{\sqrt{5}} - \sqrt{2}P\frac{1}{\sqrt{2}} + F_{CD}\frac{1}{\sqrt{2}} = 0$

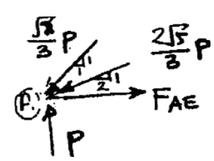
$F_{CA} = \frac{2\sqrt{5}}{3}P = 1.4907P = 1.49P\text{ (C)}$

$F_{CD} = \frac{\sqrt{2}}{3}P = 0.4714P = 0.471P\text{ (C)}$

Joint A:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_{AE} - \frac{\sqrt{2}}{3}P\left(\frac{1}{\sqrt{2}}\right) - \frac{2\sqrt{5}}{3}P\left(\frac{2}{\sqrt{5}}\right) = 0$

$F_{AE} = \frac{5}{3}P = 1.67\,P\text{ (T)}$



Similarly,

$F_{FD} = 1.67\,P\text{ (T)}$

From Eq.(1), and Symmetry,

$F_{FE} = 0.667\,P\text{ (T)}$ **Ans.**

$F_{FD} = 1.67\,P\text{ (T)}$ **Ans.**

$F_{AB} = 0.471\,P\text{ (C)}$ **Ans.**

$F_{AE} = 1.67\,P\text{ (T)}$ **Ans.**

$F_{AC} = 1.49\,P\text{ (C)}$ **Ans.**

$F_{BF} = 1.41\,P\text{ (T)}$ **Ans.**

$F_{BD} = 1.49\,P\text{ (C)}$ **Ans.**

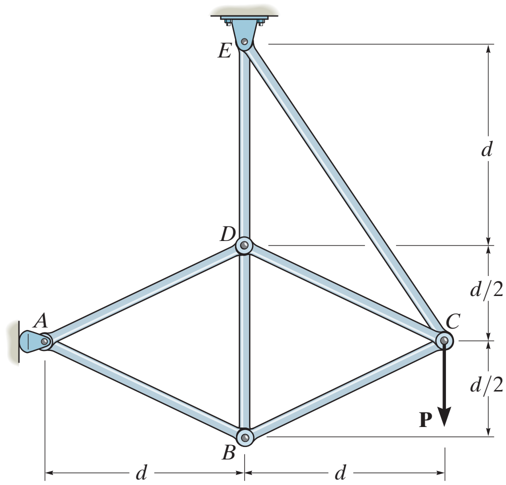
$F_{EC} = 1.41\,P\text{ (T)}$ **Ans.**

$F_{CD} = 0.471\,P\text{ (C)}$ **Ans.**

Ans:
$F_{FE} = 0.667P\text{ (T)}$
$F_{FD} = 1.67P\text{ (T)}$
$F_{AB} = 0.471P\text{ (C)}$
$F_{AE} = 1.67P\text{ (T)}$
$F_{AC} = 1.49P\text{ (C)}$
$F_{BF} = 1.41P\text{ (T)}$
$F_{BD} = 1.49P\text{ (C)}$
$F_{EC} = 1.41P\text{ (T)}$
$F_{CD} = 0.471P\text{ (C)}$

6–23.

Determine the force in each member of the truss in terms of the load P and state if the members are in tension or compression.



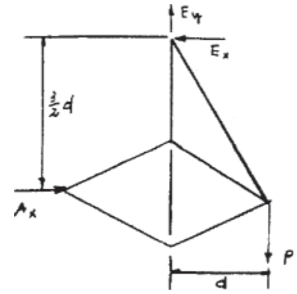
SOLUTION

Support reactions:

$\circlearrowleft + \Sigma M_E = 0;\quad A_x\left(\frac{3}{2}d\right) - Pd = 0 \qquad A_x = 0.6667P$

$\xrightarrow{+} \Sigma F_x = 0;\quad \frac{2}{3}P - E_x = 0 \qquad E_x = 0.6667P$

$+\uparrow \Sigma F_y = 0;\quad E_y - P = 0 \qquad E_y = P$

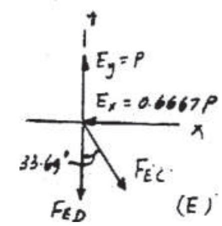


Joint E:

$\xrightarrow{+} \Sigma F_x = 0;\quad F_{EC}\sin 33.69^\circ - 0.6667P = 0$

$F_{EC} = 1.202P = 1.20P$ (T) **Ans.**

$+\uparrow \Sigma F_y = 0;\quad P - F_{ED} - 1.202P\cos 33.69^\circ = 0 \qquad F_{ED} = 0$ **Ans.**

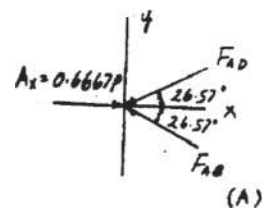


Joint A:

$+\uparrow \Sigma F_y = 0;\quad F_{AB}\sin 26.57^\circ - F_{AD}\sin 26.57^\circ = 0 \qquad F_{AB} = F_{AD} = F$

$\xrightarrow{+} \Sigma F_x = 0;\quad 0.6667P - 2F\cos 26.57^\circ = 0 \qquad F = 0.3727P$

$F_{AB} = F_{AD} = F = 0.373P$ (C) **Ans.**



Joint D:

$\xrightarrow{+} \Sigma F_x = 0;\quad 0.3727P\cos 26.57^\circ - F_{DC}\cos 26.57^\circ = 0$

$F_{DC} = 0.3727P = 0.373P$ (C) **Ans.**

$+\uparrow \Sigma F_y = 0;\quad 2(0.3727P\sin 26.57^\circ) - F_{DB} = 0$

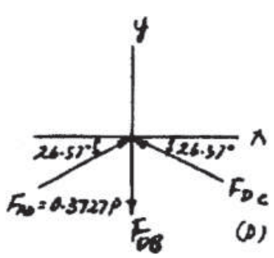
$F_{DB} = 0.3333P = 0.333P$ (T) **Ans.**

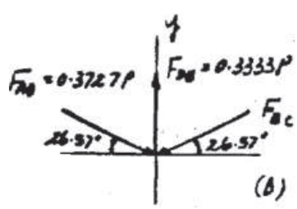
Joint B:

$\xrightarrow{+} \Sigma F_x = 0;\quad 0.3727P\cos 26.57^\circ - F_{BC}\cos 26.57^\circ = 0$

$F_{BC} = 0.3727P = 0.373P$ (C) **Ans.**

$+\uparrow \Sigma F_y = 0;\quad 0.3333P - 2(0.3727\sin 26.57^\circ) = 0$ (Check)

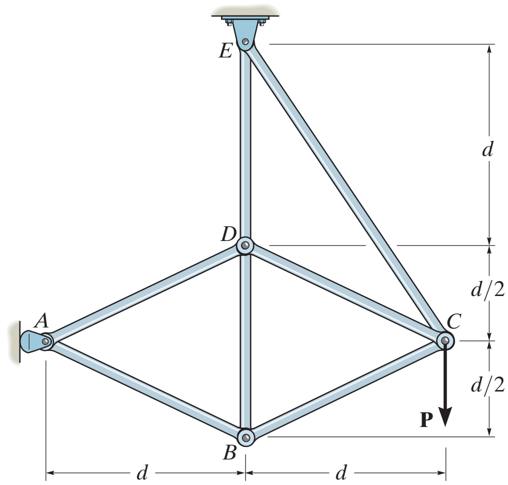




Ans:
$F_{EC} = 1.20P$ (T)
$F_{ED} = 0$
$F_{AB} = F_{AD} = 0.373P$ (C)
$F_{DC} = 0.373P$ (C)
$F_{DB} = 0.333P$ (T)
$F_{BC} = 0.373P$ (C)

***6–24.**

The maximum allowable tensile force in the members of the truss is $(F_t)_{max} = 5\text{ kN}$, and the maximum allowable compressive force is $(F_c)_{max} = 3\text{ kN}$. Determine the maximum magnitude of the load **P** that can be applied to the truss. Take $d = 2\text{ m}$.



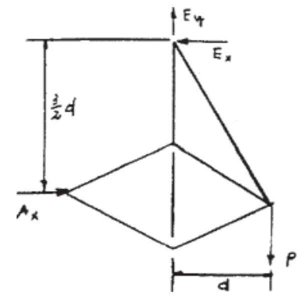
SOLUTION

Support reactions:

$\circlearrowleft + \Sigma M_E = 0; \qquad A_x\left(\frac{3}{2}d\right) - Pd = 0 \qquad A_x = 0.6667P$

$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{2}{3}P - E_x = 0 \qquad E_x = 0.6667P$

$+\uparrow \Sigma F_y = 0; \qquad E_y - P = 0 \qquad E_y = P$

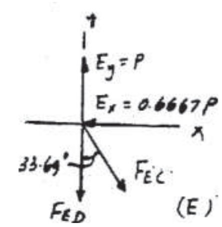


Joint E:

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{EC} \sin 33.69^\circ - 0.6667P = 0$

$$F_{EC} = 1.202P = 1.20P\ (\text{T})$$

$+\uparrow \Sigma F_y = 0; \qquad P - F_{ED} - 1.202P \cos 33.69^\circ = 0 \qquad F_{ED} = 0$

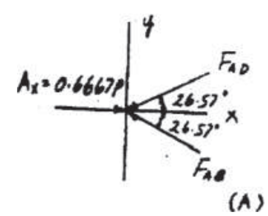


Joint A:

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} \sin 26.57^\circ - F_{AD} \sin 26.57^\circ = 0 \qquad F_{AB} = F_{AD} = F$

$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.6667P - 2F \cos 26.57^\circ = 0 \qquad F = 0.3727P$

$$F_{AB} = F_{AD} = F = 0.373P\ (\text{C})$$



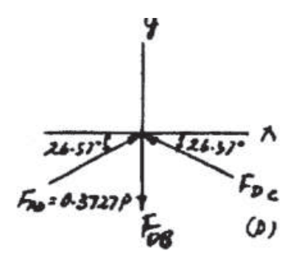
Joint D:

$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.3727P \cos 26.57^\circ - F_{DC} \cos 26.57^\circ = 0$

$$F_{DC} = 0.3727P = 0.373P\ (\text{C})$$

$+\uparrow \Sigma F_y = 0; \qquad 2(0.3727P \sin 26.57^\circ) - F_{DB} = 0$

$$F_{DB} = 0.3333P = 0.333P\ (\text{T})$$

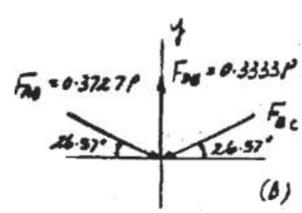


Joint B:

$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.3727P \cos 26.57^\circ - F_{BC} \cos 26.57^\circ = 0$

$$F_{BC} = 0.3727P = 0.373P\ (\text{C})$$

$+\uparrow \Sigma F_y = 0; \qquad 0.3333P - 2(0.3727 \sin 26.57^\circ) = 0\ (\text{Check})$



Maximum tension is in member EC.

$$F_{EC} = 1.202\,P = 5$$

$$P = 4.16\text{ kN}$$

Maximum compression is in members AB, AD, DC, and BC.

$$F = 0.3727\,P = 3$$

$$P = 8.05\text{ kN}$$

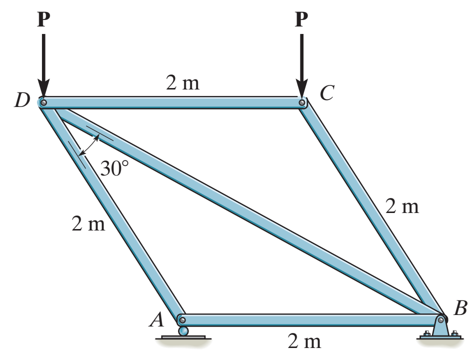
Thus, the maximum allowable load is

$$P = 4.16\text{ kN}$$ **Ans.**

Ans:
$P = 4.16\text{ kN}$

6–25.

Determine the force in each member of the truss in terms of the external loading and state if the members are in tension or compression. Take $P = 2$ kN.



SOLUTION

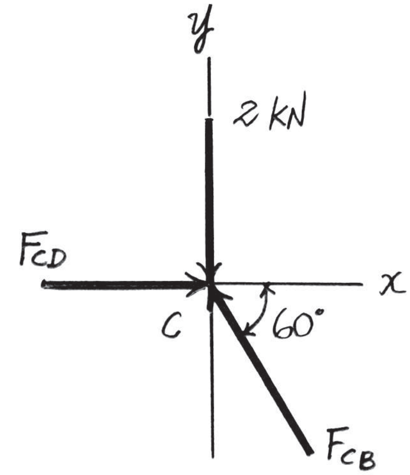
Support Reactions. Not required.

Method of Joints: We will start the joint equilibrium analysis at joint C, followed by joint D and finally joint A.

Joint *C*. Fig. *a*

$+\uparrow \Sigma F_y = 0;\ F_{CB} \sin 60° - 2 = 0 \qquad F_{CB} = 2.309 \text{ kN (C)} = 2.31 \text{ kN (C)}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\ F_{CD} - 2.309 \cos 60° = 0 \quad F_{CD} = 1.1547 \text{ kN (C)} = 1.15 \text{ kN (C)}$ **Ans.**



(a)

Joint *D*. Fig. *b*

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{DB} \cos 30° - F_{DA} \sin 30° - 1.1547 = 0$ **(1)**

$+\uparrow \Sigma F_y = 0; \qquad F_{DA} \cos 30° - F_{DB} \sin 30° - 2 = 0$ **(2)**

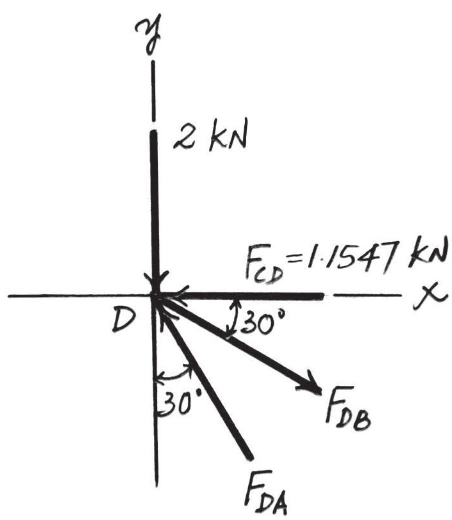
Solving Eqs. (1) and (2)

$F_{DB} = 4.00$ kN (T) $\qquad F_{DA} = 4.6188$ kN (C) $= 4.62$ kN (C) **Ans.**

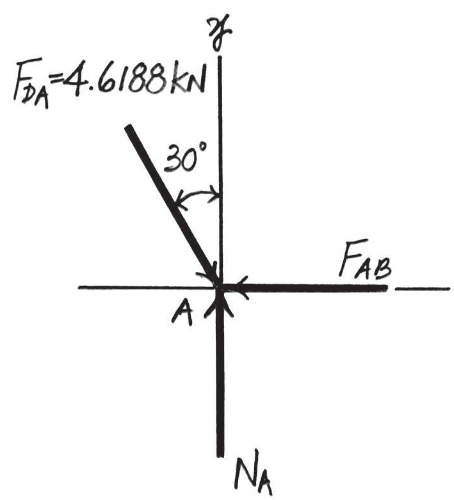
Joint *A*. Fig. *c*

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad 4.6188 \sin 30° - F_{AB} = 0 \quad F_{AB} = 2.3094 \text{ kN (C)} = 2.31 \text{ kN (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad -4.6188 \cos 30° + N_A = 0 \quad N_A = 4.00 \text{ kN}$



(b)

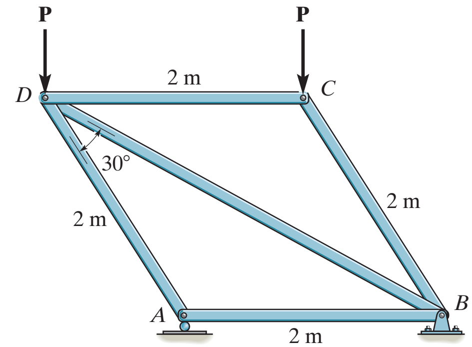


(c)

Ans:
$F_{CB} = 2.31$ kN (C)
$F_{CD} = 1.15$ kN (C)
$F_{DB} = 4.00$ kN (T)
$F_{DA} = 4.62$ kN (C)
$F_{AB} = 2.31$ kN (C)

6–26.

The maximum allowable tensile force in the members of the truss is $(F_t)_{max} = 5$ kN, and the maximum allowable compressive force is $(F_c)_{max} = 3$ kN. Determine the maximum magnitude P of the two loads that can be applied to the truss.



SOLUTION

Support Reactions. Not required.

Method of Joints: We will start the joint equilibrium analysis at joint C, followed by joint D and finally joint A.

Joint *C*. Fig. *a*

$+\uparrow \Sigma F_y = 0; \quad F_{CB}\sin 60° - P = 0 \qquad F_{CB} = 1.1547P\ (C)$

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{CD} - 1.1547P\cos 60° = 0 \quad F_{CD} = 0.5774P\ (C)$

Joint *D*. Fig. *b*

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{DB}\cos 30° - F_{DA}\sin 30° - 0.5774P = 0 \qquad$ **(1)**

$+\uparrow \Sigma F_y = 0; \quad F_{DA}\cos 30° - F_{DB}\sin 30° - P = 0 \qquad$ **(2)**

Solving Eqs. (1) and (2),

$$F_{DA} = 2.3094P\ (C) \qquad F_{DB} = 2.00P\ (T)$$

Joint *C*. Fig. *c*

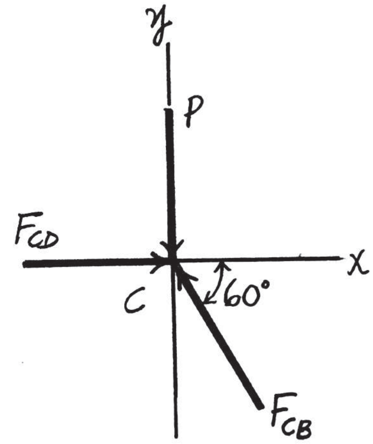
$\xrightarrow{+} \Sigma F_x = 0; \quad 2.3094\,P\sin 30° - F_{AB} = 0 \qquad F_{AB} = 1.1547P\ (C)$

$+\uparrow \Sigma F_y = 0; \quad N_A - 2.3094\,p\cos 30° = 0 \qquad N_A = 2.00P$

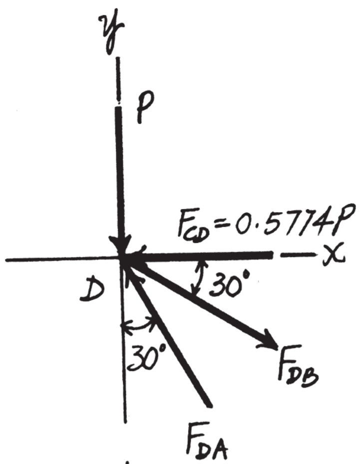
By observation, members DA and DB are subjected to maximum compression and tension, respectively. Thus, they will reach the limit first.

$F_{DA} = (F_C)_{max}; \quad 2.3094P = 3 \quad P = 1.299\text{ kN} = 1.30\text{ kN (Control!)}$ **Ans.**

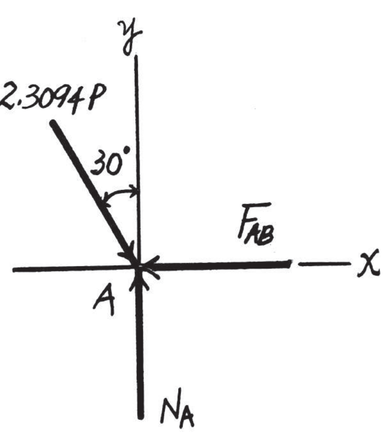
$F_{DB} = (F_C)_{max}; \quad 2.00P = 5 \quad P = 2.50\text{ kN}$



(a)



(b)

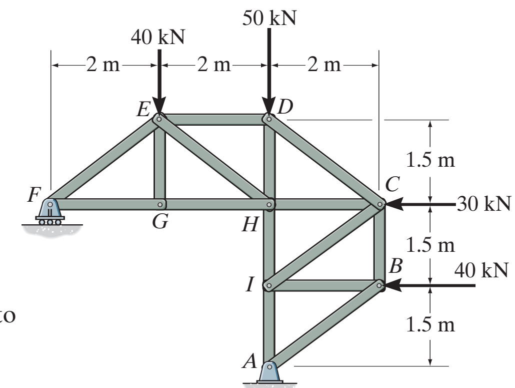


(c)

Ans:
$P = 1.30$ kN

6–27.

Determine the force in members DC, HC, and HI of the truss, and state if the members are in tension or compression.



SOLUTION

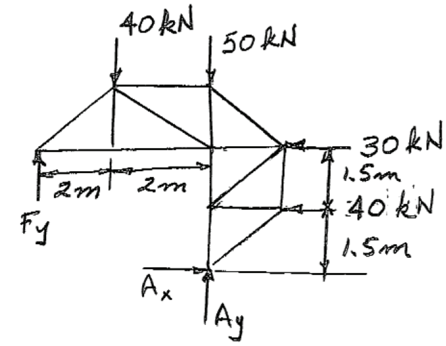
Support Reactions: Applying the moment equation of equilibrium about point A to the free - body diagram of the truss, Fig. a,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 40(1.5) + 30(3) + 40(2) - F_y(4) = 0$

$F_y = 57.5 \text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x - 30 - 40 = 0;$ $\quad A_x = 70 \text{ kN}$

$+\uparrow \Sigma F_y = 0;$ $\quad 57.5 - 40 - 50 + A_y = 0;$ $\quad A_y = 32.5 \text{ kN}$



(a)

Method of Sections: Using the bottom portion of the free - body diagram, Fig. b.

$\circlearrowleft + \Sigma M_C = 0;$ $\quad 70(3) - 32.5(2) - 40(1.5) - F_{HI}(2) = 0$

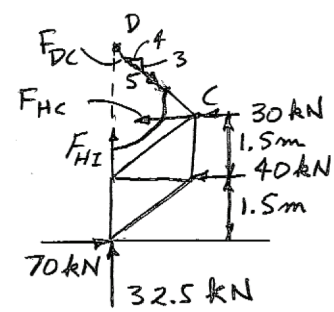
$F_{HI} = 42.5 \text{ kN (T)}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0;$ $\quad 70(4.5) - 40(3) - 30(1.5) - F_{HC}(1.5) = 0$

$F_{HC} = 100 \text{ kN (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 32.5 + 42.5 - F_{DC}\left(\frac{3}{5}\right) = 0$

$F_{DC} = 125 \text{ kN (C)}$ **Ans.**

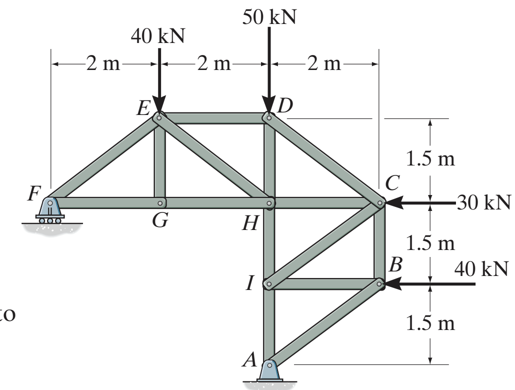


(b)

Ans:
$F_{HI} = 42.5 \text{ kN (T)}$
$F_{HC} = 100 \text{ kN (T)}$
$F_{DC} = 125 \text{ kN (C)}$

***6–28.**

Determine the force in members *ED*, *EH*, and *GH* of the truss, and state if the members are in tension or compression.



SOLUTION

Support Reactions: Applying the moment equation of equilibrium about point A to the free - body diagram of the truss, Fig. *a*,

$\zeta + \Sigma M_A = 0;$ $\quad 40(1.5) + 30(3) + 40(2) - F_y(4) = 0$

$$F_y = 57.5 \text{ kN}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad A_x - 30 - 40 = 0;$ $\quad A_x = 70 \text{ kN}$

$+\uparrow \Sigma F_y = 0;$ $\quad 57.5 - 40 - 50 + A_y = 0;$ $\quad A_y = 32.5 \text{ kN}$

Method of Sections: Using the left portion of the free - body diagram, Fig. *b*.

$\zeta + \Sigma M_E = 0;$ $\quad -57.5(2) + F_{GH}(1.5) = 0$

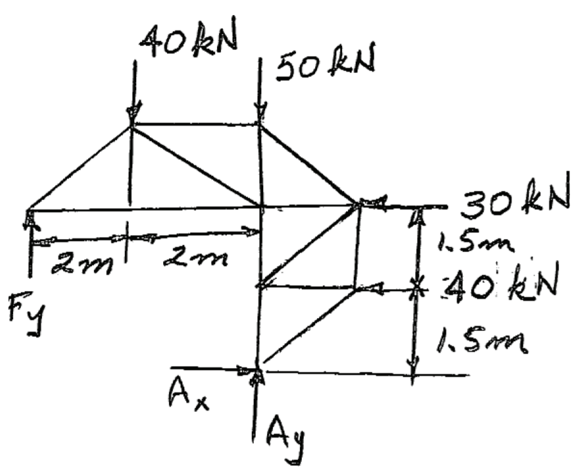
$F_{GH} = 76.7 \text{ kN (T)}$ **Ans.**

$\zeta + \Sigma M_H = 0;$ $\quad -57.5(4) + F_{ED}(1.5) + 40(2) = 0$

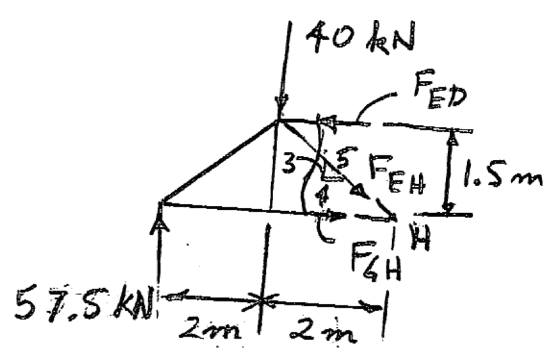
$F_{ED} = 100 \text{ kN (C)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 57.5 - F_{EH}\left(\frac{3}{5}\right) - 40 = 0$

$F_{EH} = 29.2 \text{ kN (T)}$ **Ans.**



(a)

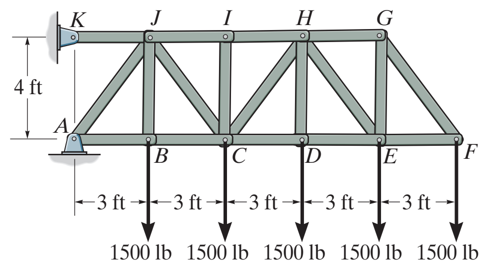


(b)

Ans:
$F_{GH} = 76.7 \text{ kN (T)}$
$F_{ED} = 100 \text{ kN (C)}$
$F_{EH} = 29.2 \text{ kN (T)}$

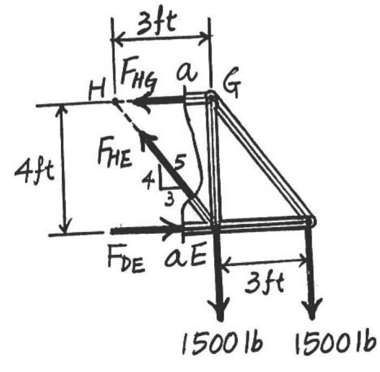
6–29.

Determine the force in members HG, HE, and DE of the truss, and state if the members are in tension or compression.



SOLUTION

Method of Sections: The forces in members HG, HE, and DE are exposed by cutting the truss into two portions through section a–a and using the upper portion of the free-body diagram, Fig. a. From this free-body diagram, F_{HG} and F_{DE} can be obtained by writing the moment equations of equilibrium about points E and H, respectively. $\mathbf{F}_{HE}$ can be obtained by writing the force equation of equilibrium along the y axis.



(a)

Joint D: From the free-body diagram in Fig. a,

$$\circlearrowleft + \Sigma M_E = 0; \qquad F_{HG}(4) - 1500(3) = 0$$

$$F_{HG} = 1125 \text{ lb (T)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_H = 0; \qquad F_{DE}(4) - 1500(6) - 1500(3) = 0$$

$$F_{DE} = 3375 \text{ lb (C)} \qquad \textbf{Ans.}$$

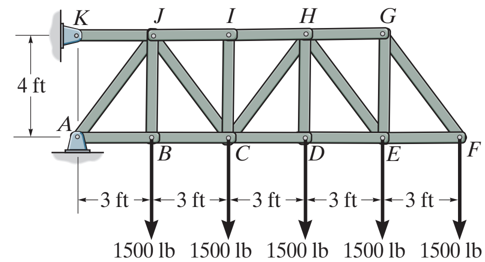
$$+\uparrow \Sigma F_y = 0; \qquad F_{HE}\left(\frac{4}{5}\right) - 1500 - 1500 = 0$$

$$F_{EH} = 3750 \text{ lb (T)} \qquad \textbf{Ans.}$$

Ans:
$F_{HG} = 1125$ lb (T)
$F_{DE} = 3375$ lb (C)
$F_{EH} = 3750$ lb (T)

6–30.

Determine the force in members *CD, HI*, and *CJ* of the truss, and state if the members are in tension or compression.



SOLUTION

Method of Sections: The forces in members *HI*, *CH*, and *CD* are exposed by cutting the truss into two portions through section *b–b* on the right portion of the free-body diagram, Fig. *a*. From this free-body diagram, F_{CD} and F_{HI} can be obtained by writing the moment equations of equilibrium about points *H* and *C*, respectively. F_{CH} can be obtained by writing the force equation of equilibrium along the *y* axis.

$\circlearrowleft + \Sigma M_H = 0;$ $\quad F_{CD}(4) - 1500(6) - 1500(3) = 0$

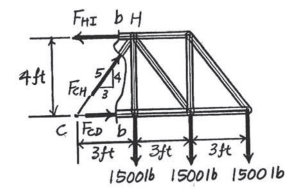
$F_{CD} = 3375 \text{ lb (C)}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;$ $\quad F_{HI}(4) - 1500(3) - 1500(6) - 1500(9) = 0$

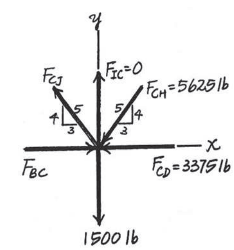
$F_{HI} = 6750 \text{ lb (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad F_{CH}\left(\frac{4}{5}\right) - 1500 - 1500 = 0$

$F_{CH} = 5625 \text{ lb (C)}$ **Ans.**



(a)

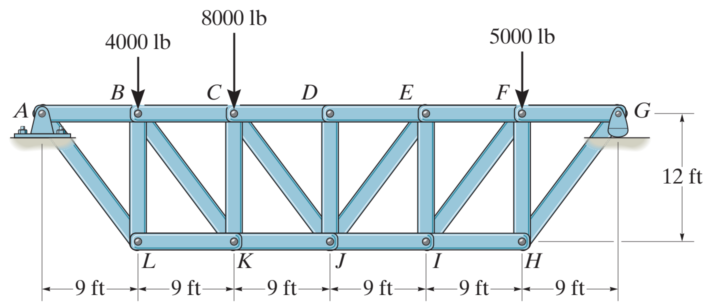


(b)

Ans:
$F_{CD} = 3375 \text{ lb (C)}$
$F_{HI} = 6750 \text{ lb (T)}$
$F_{CH} = 5625 \text{ lb (C)}$

6–31.

Determine the force in members *CD, CJ, KJ*, and *DJ* of the truss which serves to support the deck of a bridge. State if these members are in tension or compression.



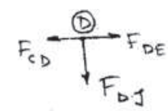
SOLUTION

$\zeta + \Sigma M_C = 0;$ $\quad -9500(18) + 4000(9) + F_{KJ}(12) = 0$

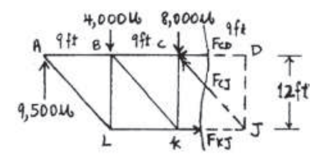
$F_{KJ} = 11\,250 \text{ lb} = 11.2 \text{ kip (T)}$ **Ans.**

$\zeta + \Sigma M_J = 0;$ $\quad -9500(27) + 4000(18) + 8000(9) + F_{CD}(12) = 0$

$F_{CD} = 9375 \text{ lb} = 9.38 \text{ kip (C)}$ **Ans.**



$\xrightarrow{+} \Sigma F_x = 0;$ $\quad -9375 + 11\,250 - \frac{3}{5}F_{CJ} = 0$



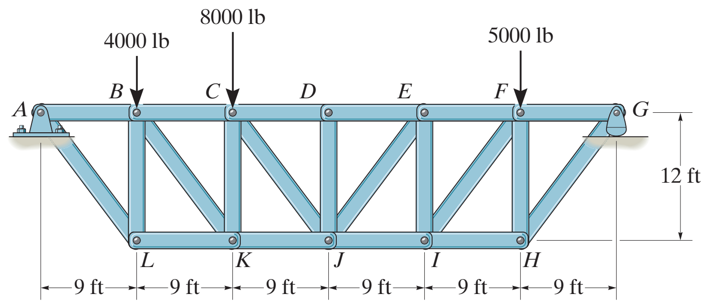
$F_{CJ} = 3125 \text{ lb} = 3.12 \text{ kip (C)}$ **Ans.**

Joint *D*: $\quad F_{DJ} = 0$ **Ans.**

Ans:
$F_{KJ} = 11.25$ kip (T)
$F_{CD} = 9.375$ kip (C)
$F_{CJ} = 3.125$ kip (C)
$F_{DJ} = 0$

***6–32.**

Determine the force in members EI and JI of the truss which serves to support the deck of a bridge. State if these members are in tension or compression.



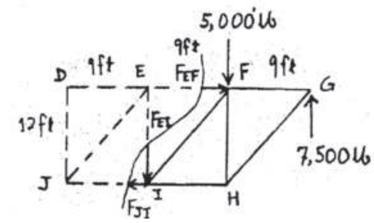
SOLUTION

$\zeta + \Sigma M_E = 0;$ $\quad -5000(9) + 7500(18) - F_{JI}(12) = 0$

$F_{JI} = 7500 \text{ lb} = 7.50 \text{ kip (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad 7500 - 5000 - F_{EI} = 0$

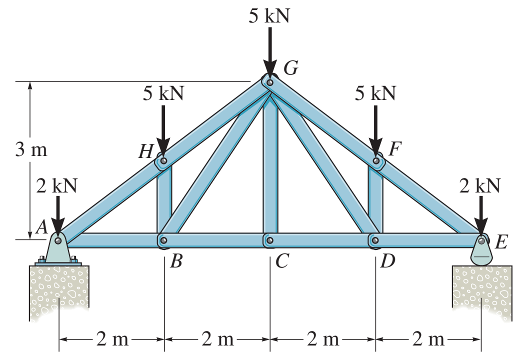
$F_{EI} = 2500 \text{ lb} = 2.50 \text{ kip (C)}$ **Ans.**



Ans:
$F_{JI} = 7.50 \text{ kip (T)}$
$F_{EI} = 2.50 \text{ kip (C)}$

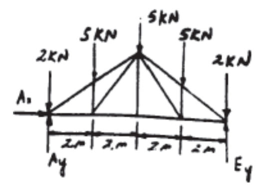
6–33.

The *Howe truss* is subjected to the loading shown. Determine the force in members *GF*, *CD*, and *GC*, and state if the members are in tension or compression.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \qquad E_y(8) - 2(8) - 5(6) - 5(4) - 5(2) = 0 \qquad E_y = 9.5\text{kN}$

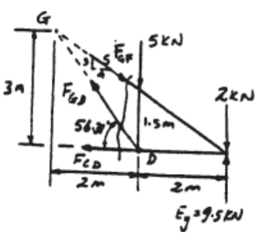


$\circlearrowleft + \Sigma M_D = 0; \qquad -\frac{4}{5}F_{GF}(1.5) - 2(2) + 9.5(2) = 0$

$F_{GF} = 12.5 \text{ kN (C)}$ **Ans.**

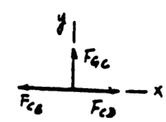
$\circlearrowleft + \Sigma M_G = 0; \qquad 9.5(4) - 2(4) - 5(2) - F_{CD}(3) = 0$

$F_{CD} = 6.67 \text{ kN (T)}$ **Ans.**



Joint C:

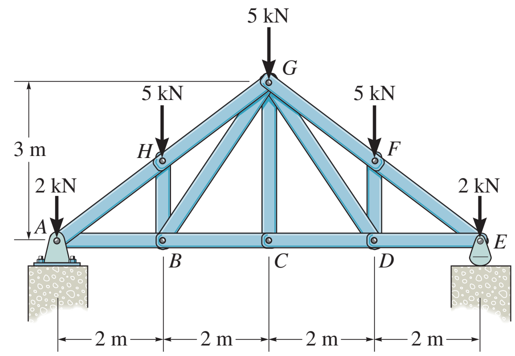
$+\uparrow \Sigma F_y = 0; \qquad F_{GC} = 0$ **Ans.**



Ans:
$F_{GF} = 12.5 \text{ kN (C)}$
$F_{CD} = 6.67 \text{ kN (T)}$
$F_{GC} = 0$

6–34.

The *Howe truss* is subjected to the loading shown. Determine the force in members GH, BC, and BG of the truss and state if the members are in tension or compression.



SOLUTION

$\zeta + \Sigma M_B = 0; \qquad -7.5(2) + F_{GH} \sin 36.87°(2) = 0$

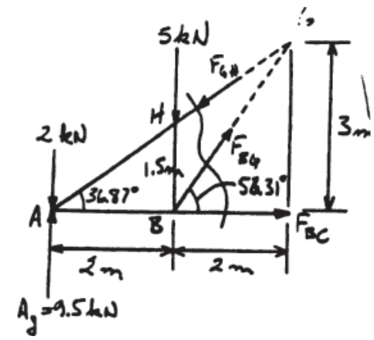
$F_{GH} = 12.5 \text{ kN (C)}$ **Ans.**

$\zeta + \Sigma M_A = 0; \qquad -5\,(2) + F_{BG} \sin 56.31°(2) = 0$

$F_{BG} = 6.01 \text{ kN (T)}$ **Ans.**

$\zeta + \Sigma M_H = 0; \qquad -7.5(4) + 5(2) + F_{BC}(3) = 0$

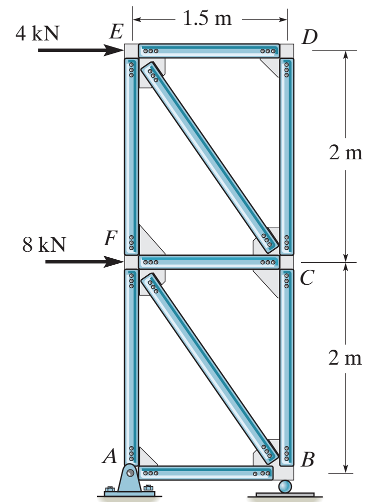
$F_{BC} = 6.67 \text{ kN (T)}$ **Ans.**



Ans:
$F_{GH} = 12.5 \text{ kN (C)}$
$F_{BG} = 6.01 \text{ kN (T)}$
$F_{BC} = 6.67 \text{ kN (T)}$

6–35.

Determine the force in members *EF*, *CF*, and *BC*, and state if the members are in tension or compression.



SOLUTION

Support Reactions. Not required.

Method of Sections. $\mathbf{F}_{BC}$ and $\mathbf{F}_{EF}$ can be determined directly by writing the moment equations of equilibrium about points *F* and *C*, respectively, by referring to the *FBD* of the upper portion of the truss section through a–a shown in Fig. *a*.

$$\circlearrowleft + \Sigma M_F = 0; \qquad F_{BC}(1.5) - 4(2) = 0$$

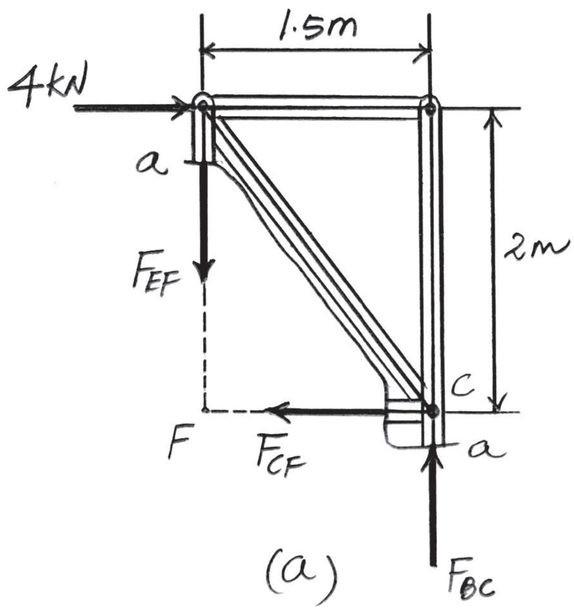
$$F_{BC} = 5.333 \text{ kN (C)} = 5.33 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{EF}(1.5) - 4(2) = 0$$

$$F_{EF} = 5.333 \text{ kN (T)} = 5.33 \text{ kN (T)} \qquad \textbf{Ans.}$$

Also, by writing the force equation of equilibrium along the *x* axis,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 4 - F_{CF} = 0 \qquad F_{CF} = 4.00 \text{ kN (T)} \qquad \textbf{Ans.}$$

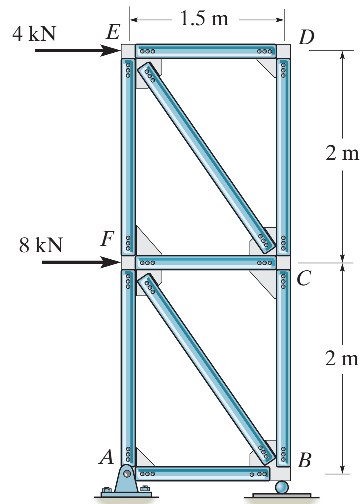


(a)

Ans:
$F_{BC} = 5.33$ kN (C)
$F_{EF} = 5.33$ kN (T)
$F_{CF} = 4.00$ kN (T)

***6–36.**

Determine the force in members AF, BF, and BC, and state if the members are in tension or compression.



SOLUTION

Support Reactions. Not required.

Method of Sections: Referring to the *FBD* of the upper portion of the truss section through *a–a* shown in Fig. *a*, $\mathbf{F}_{AF}$ and $\mathbf{F}_{BC}$ can be determined directly by writing the moment equations of equilibrium about points B and F, respectively.

$$\circlearrowleft + \Sigma M_B = 0; \qquad F_{AF}(1.5) - 8(2) - 4(4) = 0$$

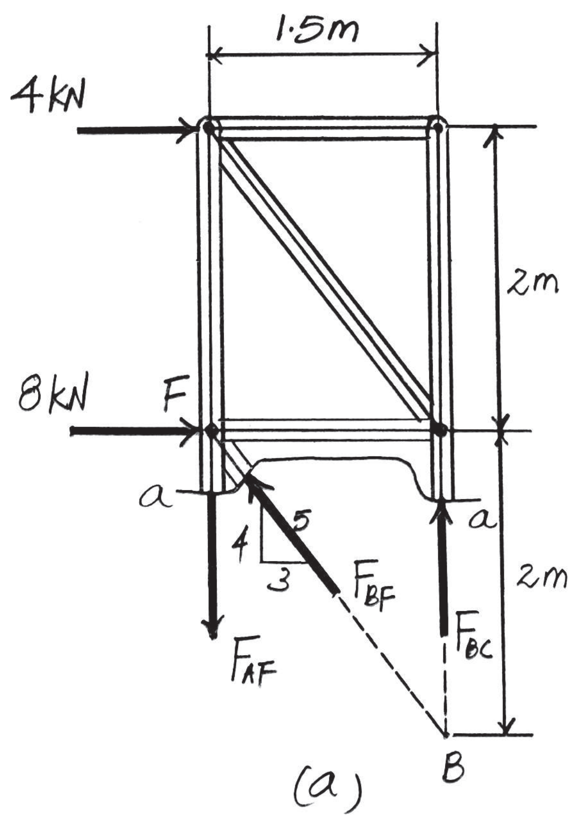
$$F_{AF} = 21.33 \text{ kN (T)} = 21.3 \text{ kN (T)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_F = 0; \qquad F_{BC}(1.5) - 4(2) = 0$$

$$F_{BC} = 5.333 \text{ kN (C)} = 5.33 \text{ kN (C)} \qquad \textbf{Ans.}$$

Also, write the force equation of equilibrium along the x axis, we can obtain $\mathbf{F}_{BF}$ directly.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 4 + 8 - F_{BF}\left(\frac{3}{5}\right) = 0 \qquad F_{BF} = 20.0 \text{ kN (C)} \qquad \textbf{Ans.}$$

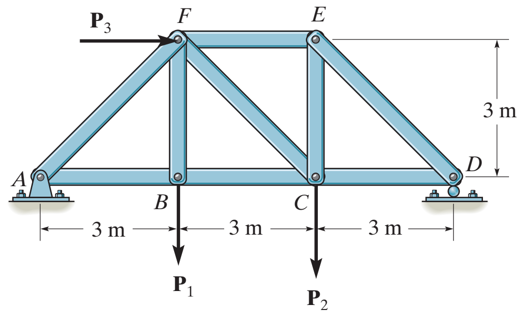


(a)

Ans:
$F_{AF} = 21.3$ kN (T)
$F_{BC} = 5.33$ kN (C)
$F_{BF} = 20.0$ kN (C)

6–37.

Determine the force in members EF, BE, BC and BF of the truss and state if these members are in tension or compression. Set $P_1 = 9$ kN, $P_2 = 12$ kN, and $P_3 = 6$ kN.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire truss shown in Fig. *a*, $\mathbf{N}_D$ can be determined directly by writing the moment equation of equilibrium about point A.

$\zeta + \Sigma M_A = 0; \quad N_D(9) - 12(6) - 9(3) - 6(3) = 0 \quad N_D = 13.0 \text{ kN}$

Method of Sections. Referring to the *FBD* of the right portion of the truss sectioned through *a–a* shown in Fig. *b*, $\mathbf{F}_{EF}$ and $\mathbf{F}_{BC}$ can be determined directly by writing the moment equation of equilibrium about points B and E, respectively.

$\zeta + \Sigma M_B = 0; \quad 13.0(6) - 12(3) - F_{EF}(3) = 0 \quad F_{EF} = 14.0 \text{ kN (C)}$ **Ans.**

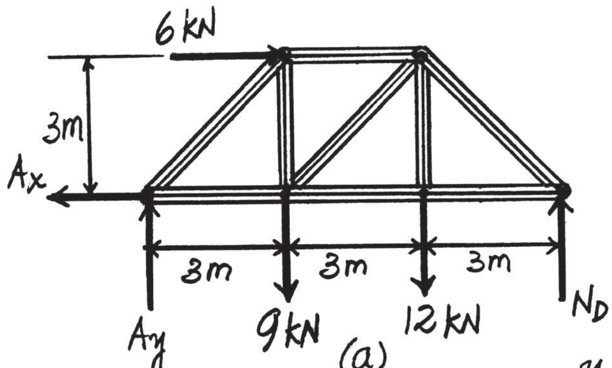
$\zeta + \Sigma M_E = 0; \quad 13.0(3) - F_{BC}(3) = 0 \quad F_{BC} = 13.0 \text{ kN (T)}$ **Ans.**

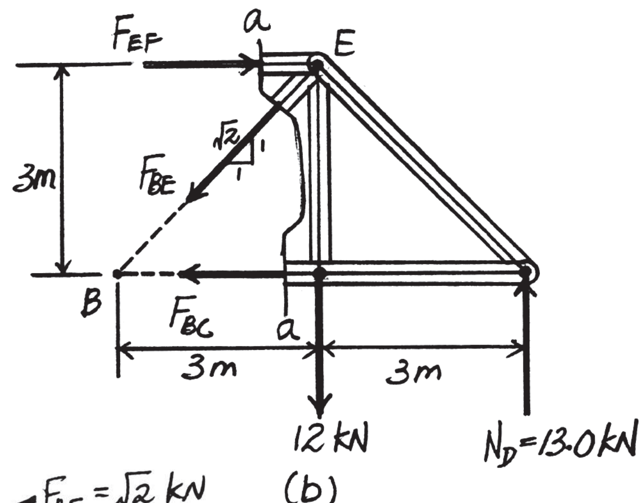
Also, $\mathbf{F}_{BE}$ can be determined by writing the force equation of equilibrium along the y axis.

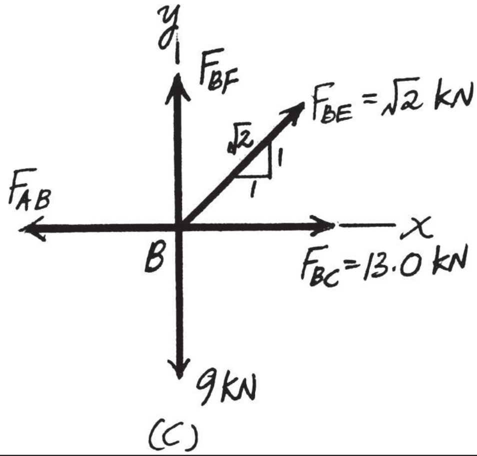
$+\uparrow \Sigma F_y = 0; \; 13.0 - 12 - F_{BE}\left(\frac{1}{\sqrt{2}}\right) = 0 \; F_{BE} = \sqrt{2} \text{ kN (T)} = 1.41 \text{ kN (T)}$ **Ans.**

Method of Joints. Using the result of $\mathbf{F}_{BE}$, the equilibrium of joint B, Fig. *c*, requires

$+\uparrow \Sigma F_y = 0; \quad F_{BF} + \left(\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}\right) - 9 = 0 \quad F_{BF} = 8.00 \text{ kN (T)}$ **Ans.**



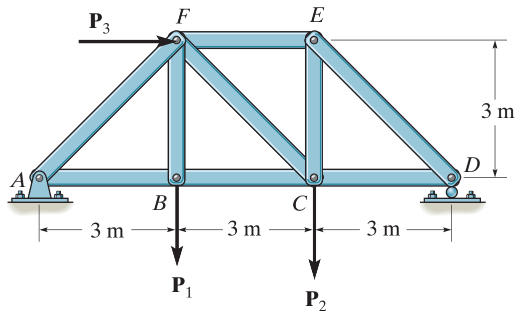




Ans:
$F_{EF} = 14.0$ kN (C)
$F_{BC} = 13.0$ kN (T)
$F_{BE} = 1.41$ kN (T)
$F_{BF} = 8.00$ kN (T)

6–38.

Determine the force in members BC, BE, and EF of the truss and state if these members are in tension or compression. Set $P_1 = 6$ kN, $P_2 = 9$ kN, and $P_3 = 12$ kN.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire truss shown in Fig. *a*, $\mathbf{N}_D$ can be determined directly by writing the moment equation of equilibrium about point A.

$\zeta+\Sigma M_A = 0;\quad N_D(9) - 12(3) - 6(3) - 9(6) = 0 \qquad N_D = 12.0 \text{ kN}$

Method of Sections. Referring to the *FBD* of the right portion of the truss sectioned through *a–a* shown in Fig. *b*, $\mathbf{F}_{EF}$ and $\mathbf{F}_{BC}$ can be determined directly by writing the moment equation of equilibrium about points B and E, respectively.

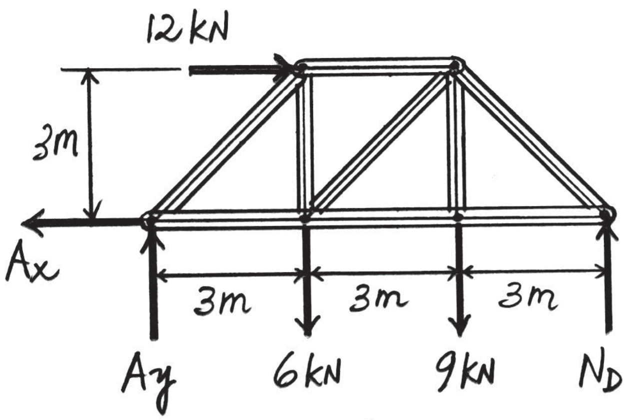
$\zeta+\Sigma M_B = 0;\quad 12.0(6) - 9(3) - F_{EF}(3) = 0 \qquad F_{EF} = 15.0 \text{ kN (C)}$ **Ans.**

$\zeta+\Sigma M_E = 0;\quad 12.0(3) - F_{BC}(3) = 0 \qquad F_{BC} = 12.0 \text{ kN (T)}$ **Ans.**

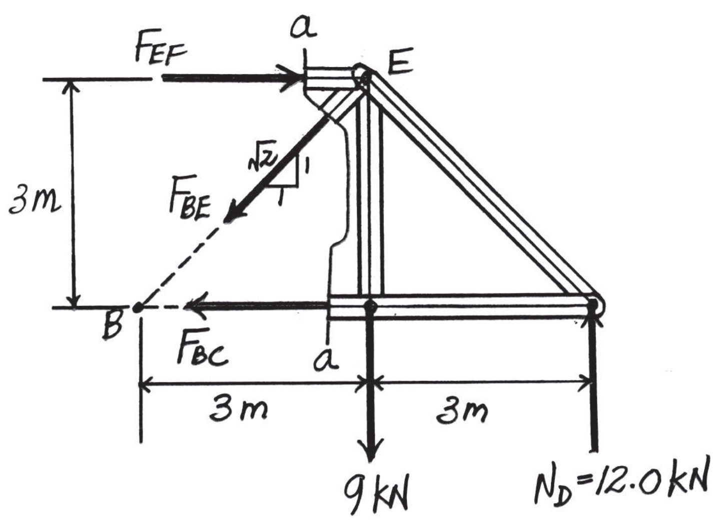
Also, $\mathbf{F}_{BE}$ can be obtained directly by writing the force equation of equilibrium along the y axis

$$+\uparrow\Sigma F_y = 0;\quad 12.0 - 9 - F_{BE}\left(\frac{1}{\sqrt{2}}\right) = 0$$

$$F_{BE} = 3\sqrt{2} \text{ kN} = 4.24 \text{ kN (T)}$$ **Ans.**



(a)

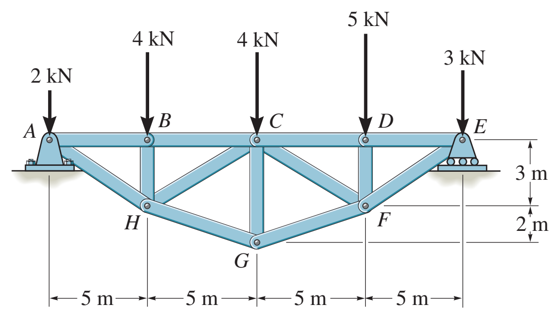


(b)

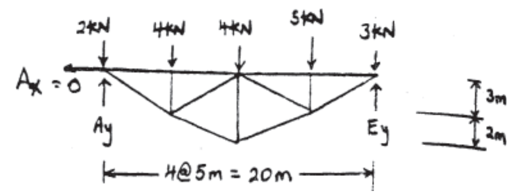
Ans:
$F_{EF} = 15.0$ kN (C)
$F_{BC} = 12.0$ kN (T)
$F_{BE} = 4.24$ kN (T)

6–39.

Determine the force in members *BC*, *HC*, and *HG*. After the truss is sectioned use a single equation of equilibrium for the calculation of each force. State if these members are in tension or compression.



SOLUTION

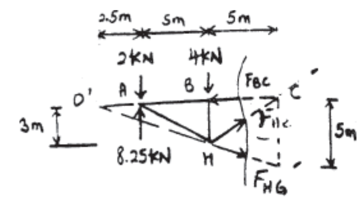


$\zeta + \Sigma M_E = 0; \quad -A_y(20) + 2(20) + 4(15) + 4(10) + 5(5) = 0$

$A_y = 8.25 \text{ kN}$

$\zeta + \Sigma M_H = 0; \quad -8.25(5) + 2(5) + F_{BC}(3) = 0$

$F_{BC} = 10.4 \text{ kN (C)}$ **Ans.**



$$\zeta + \Sigma M_C = 0; \quad -8.25(10) + 2(10) + 4(5) + \frac{5}{\sqrt{29}}F_{HG}(5) = 0$$

$F_{HG} = 9.155 = 9.16 \text{ kN (T)}$ **Ans.**

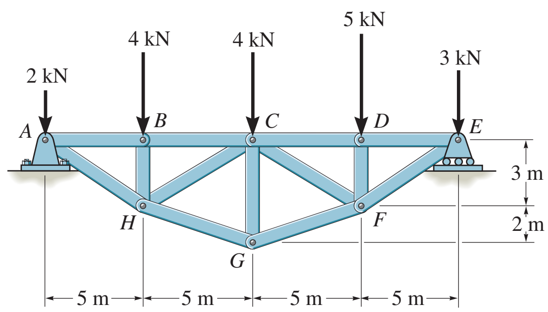
$$\zeta + \Sigma M_{O'} = 0; \quad -2(2.5) + 8.25(2.5) - 4(7.5) + \frac{3}{\sqrt{34}}F_{HC}(12.5) = 0$$

$F_{HC} = 2.24 \text{ kN (T)}$ **Ans.**

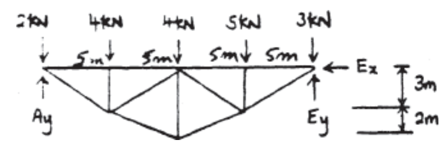
Ans:
$F_{BC} = 10.4 \text{ kN (C)}$
$F_{HG} = 9.16 \text{ kN (T)}$
$F_{HC} = 2.24 \text{ kN (T)}$

***6–40.**

Determine the force in members *CD*, *CF*, and *CG* and state if these members are in tension or compression.



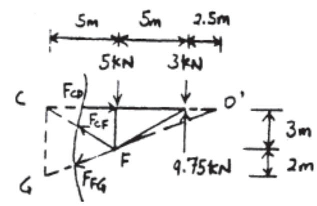
SOLUTION



$\xrightarrow{+} \Sigma F_x = 0; \qquad E_x = 0$

$\circlearrowleft + \Sigma M_A = 0; \qquad -4(5) - 4(10) - 5(15) - 3(20) + E_y(20) = 0$

$E_y = 9.75\text{ kN}$



$\circlearrowleft + \Sigma M_C = 0; \qquad -5(5) - 3(10) + 9.75(10) - \dfrac{5}{\sqrt{29}} F_{FG}(5) = 0$

$F_{FG} = 9.155\text{ kN (T)}$

$\circlearrowleft + \Sigma M_F = 0; \qquad -3(5) + 9.75(5) - F_{CD}(3) = 0$

$F_{CD} = 11.25 = 11.2\text{ kN (C)}$ **Ans.**

$\circlearrowleft + \Sigma M_{O'} = 0; \qquad -9.75(2.5) + 5(7.5) + 3(2.5) - \dfrac{3}{\sqrt{34}} F_{CF}(12.5) = 0$

$F_{CF} = 3.21\text{ kN (T)}$ **Ans.**

Joint *G*:

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{GH} = 9.155\text{ kN (T)}$

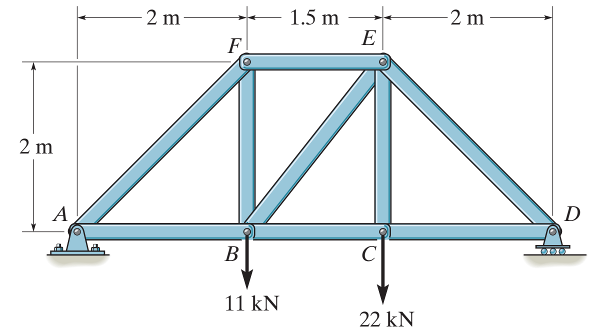
$+\uparrow \Sigma F_y = 0; \qquad \dfrac{2}{\sqrt{29}}(9.155)(2) - F_{CG} = 0$

$F_{CG} = 6.80\text{ kN (C)}$ **Ans.**

Ans:
$F_{CD} = 11.2\text{ kN (C)}$
$F_{CF} = 3.21\text{ kN (T)}$
$F_{CG} = 6.80\text{ kN (C)}$

6–41.

Determine the force developed in members FE, EB, and BC of the truss and state if these members are in tension or compression.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire truss shown in Fig. *a*, $\mathbf{N}_D$ can be determined directly by writing the moment equation of equilibrium about point A.

$\circlearrowleft + \Sigma M_A = 0;\quad N_D(5.5) - 11(2) - 22(3.5) = 0 \qquad N_D = 18.0 \text{ kN}$

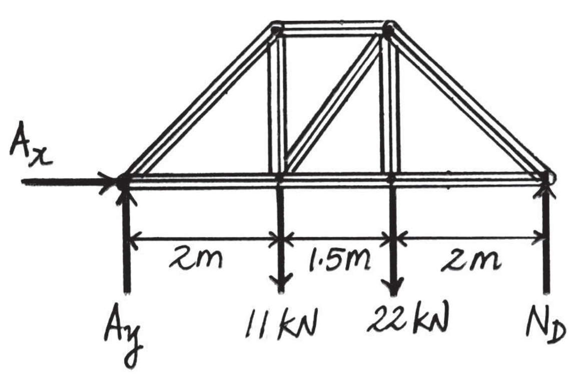
Method of Sections. Referring to the *FBD* of the right portion of the truss sectioned through *a–a* shown in Fig. *b*, $\mathbf{F}_{BC}$ and $\mathbf{F}_{FE}$ can be determined directly by writing the moment equations of equilibrium about point E and B, respectively.

$\circlearrowleft + \Sigma M_E = 0;\quad 18.0(2) - F_{BC}(2) = 0 \qquad F_{BC} = 18.0 \text{ kN (T)}$ **Ans.**

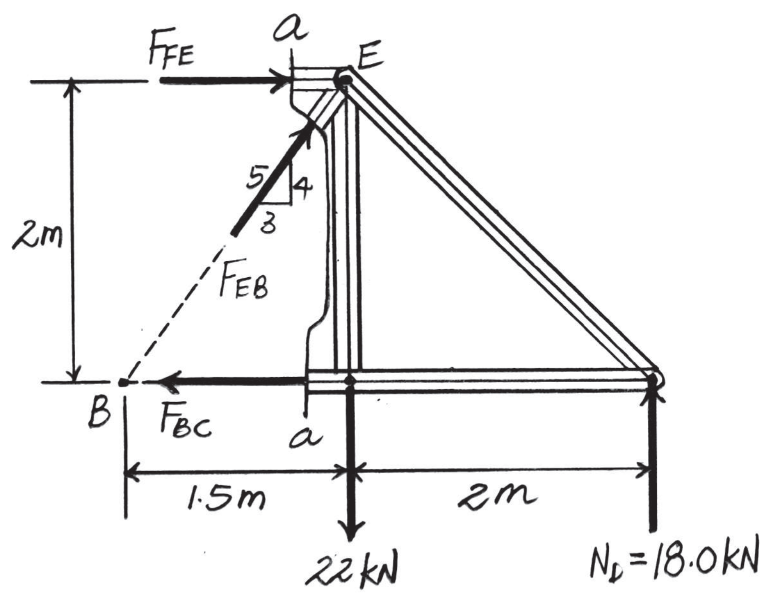
$\circlearrowleft + \Sigma M_B = 0;\quad 18.0(3.5) - 22(1.5) - F_{FE}(2) = 0 \qquad F_{FE} = 15.0 \text{ kN (C)}$ **Ans.**

Also, $\mathbf{F}_{EB}$ can be obtained directly by writing force equation of equilibrium along the y axis

$+\uparrow \Sigma F_y = 0;\quad F_{EB}\left(\frac{4}{5}\right) + 18.0 - 22 = 0 \qquad F_{EB} = 5.00 \text{ kN (C)}$ **Ans.**



(a)

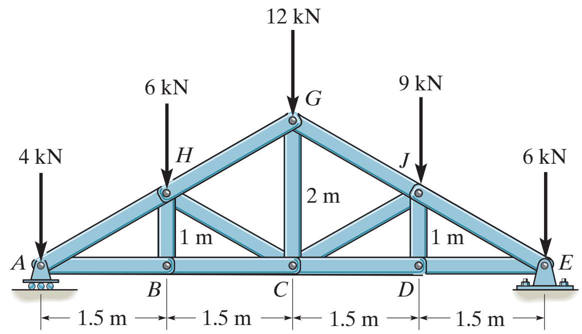


(b)

Ans:
$F_{BC} = 18.0 \text{ kN (T)}$
$F_{FE} = 15.0 \text{ kN (C)}$
$F_{EB} = 5.00 \text{ kN (C)}$

6–42.

Determine the force in members *BC*, *HC*, and *HG*. State if these members are in tension or compression.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire truss shown in Fig. *a*, $\mathbf{N}_A$ can be determined directly by writing the moment equation of equilibrium about point *E*.

$$\circlearrowleft + \Sigma M_E = 0; \quad 9(1.5) + 12(3) + 6(4.5) + 4(6) - N_A(6) = 0 \quad N_A = 16.75 \text{ kN}$$

Method of Sections. Referring to the *FBD* of the left portion of the truss sectioned through *a–a* shown in Fig. *b*, $\mathbf{F}_{HG}$, $\mathbf{F}_{HC}$ and $\mathbf{F}_{BC}$ can be determined directly by writing the moment equations of equilibrium about points *C*, *A*, and *H*, respectively.

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{HG}\left(\frac{2}{\sqrt{13}}\right)(3) + 6(1.5) + 4(3) - 16.75(3) = 0$$

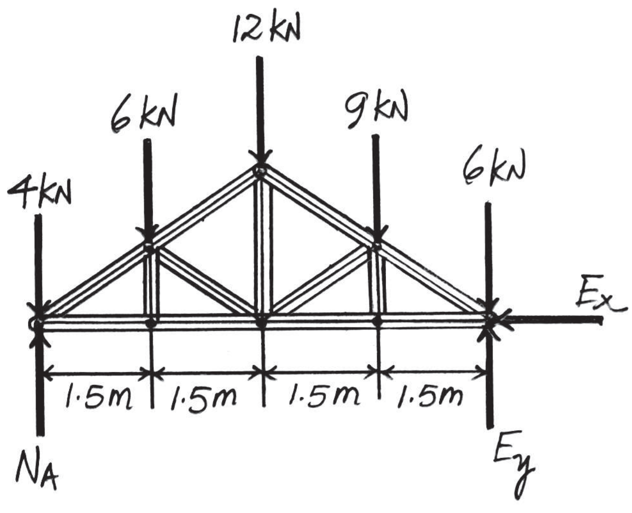
$$F_{HG} = 4.875\sqrt{13} \text{ kN (C)} = 17.6 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{HC}\left(\frac{2}{\sqrt{13}}\right)(3) - 6(1.5) = 0$$

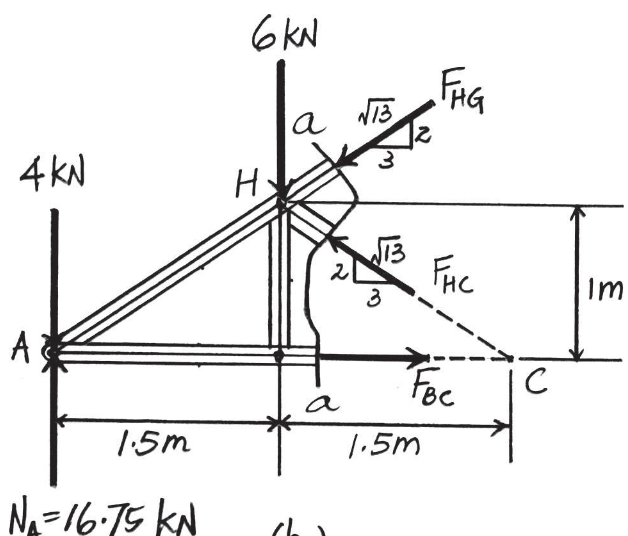
$$F_{HC} = 1.5\sqrt{13} \text{ kN (C)} = 5.41 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_H = 0; \qquad F_{BC}(1) + 4(1.5) - 16.75(1.5) = 0$$

$$F_{BC} = 19.125 \text{ kN (T)} = 19.1 \text{ kN (T)} \qquad \textbf{Ans.}$$



(a)

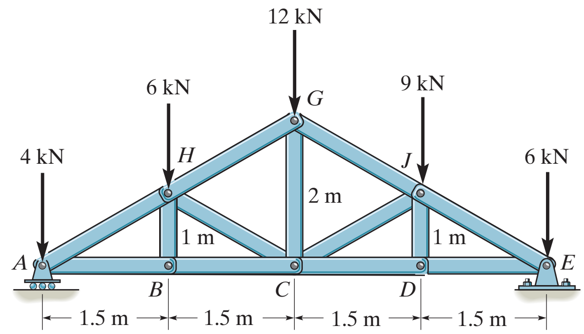


(b)

Ans:
$F_{HG} = 17.6$ kN (C)
$F_{HC} = 5.41$ kN (C)
$F_{BC} = 19.1$ kN (T)

6–43.

Determine the force in members *CD*, *CJ*, *GJ*, and *CG* and state if these members are in tension or compression.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire truss shown in Fig. *a*, $\mathbf{E}_y$ can be determined directly by writing the moment equation of equilibrium about point *A*.

$$\circlearrowleft +\Sigma M_A = 0; \quad E_y(6) - 6(6) - 9(4.5) - 12(3) - 6(1.5) = 0 \quad E_y = 20.25 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad E_x = 0$$

Method of Sections. Referring to the *FBD* of the right portion of the truss sectioned through *a–a* shown in Fig. *b*, $\mathbf{F}_{GJ}$, $\mathbf{F}_{CJ}$ and $\mathbf{F}_{CD}$ can be determined directly by writing moment equations of equilibrium about point *C*, *E* and *J*, respectively.

$$\circlearrowleft +\Sigma M_C = 0; \quad 20.25(3) - 6(3) - 9(1.5) - F_{GJ}\left(\frac{2}{\sqrt{13}}\right)(3) = 0$$

$$F_{GJ} = 4.875\sqrt{13} \text{ kN (C)} = 17.6 \text{ kN (C)} \qquad \textbf{Ans.}$$

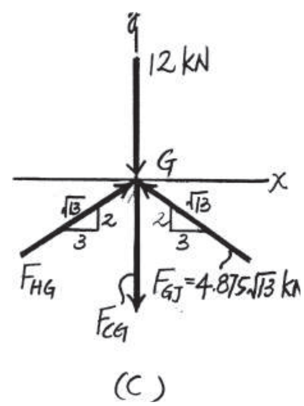
$$\circlearrowleft +\Sigma M_E = 0; \quad 9(1.5) - F_{CJ}\left(\frac{2}{\sqrt{13}}\right)(3) = 0$$

$$F_{CJ} = 2.25\sqrt{13} \text{ kN (C)} = 8.11 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$\circlearrowleft +\Sigma M_J = 0; \quad 20.25(1.5) - 6(1.5) - F_{CD}(1) = 0$$

$$F_{CD} = 21.375 \text{ kN (T)} = 21.4 \text{ kN (T)} \qquad \textbf{Ans.}$$

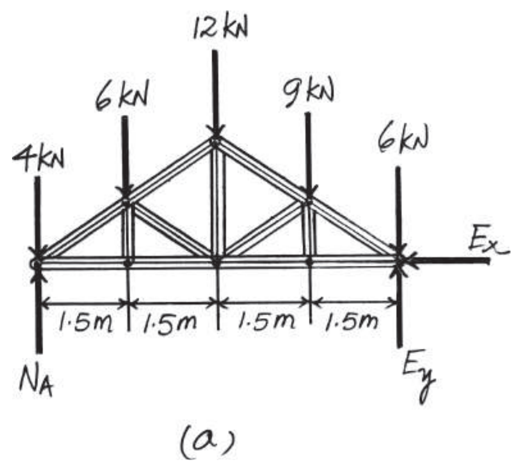
Method of Joints. Using the result of $\mathbf{F}_{GJ}$ to consider the equilibrium of joint *G*, Fig. *c*,

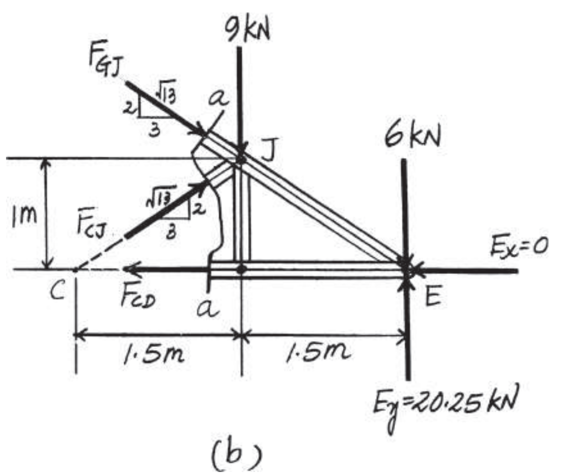


$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{HG}\left(\frac{3}{\sqrt{13}}\right) - (4.875\sqrt{13})\left(\frac{3}{\sqrt{13}}\right) = 0 \quad F_{HG} = 4.875\sqrt{13} \text{ kN (C)}$$

$$+\uparrow \Sigma F_y = 0; \quad 2\left(4.875\sqrt{13}\right)\left(\frac{2}{\sqrt{13}}\right) - 12 - F_{CG} = 0 \qquad F_{CG} = 7.50 \text{ kN (T)}$$

Ans.

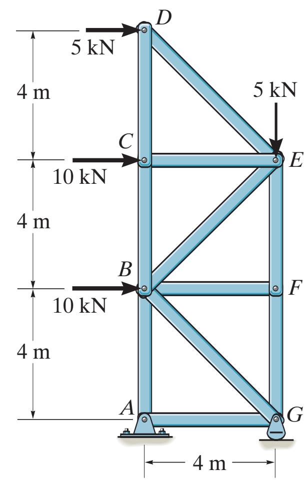




Ans:
$F_{GJ} = 17.6$ kN (C)
$F_{CJ} = 8.11$ kN (C)
$F_{CD} = 21.4$ kN (T)
$F_{CG} = 7.50$ kN (T)

***6–44.**

Determine the force in members *BE, EF*, and *CB*, and state if the members are in tension or compression.



SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \qquad 5 + 10 - F_{BE} \cos 45° = 0$

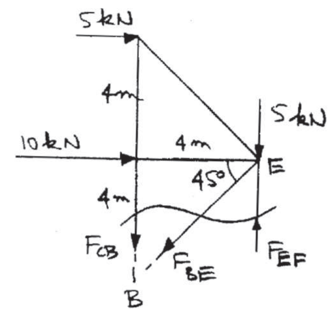
$F_{BE} = 21.2 \text{ kN (T)}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0; \qquad -5(4) + F_{CB}(4) = 0$

$F_{CB} = 5 \text{ kN (T)}$ **Ans.**

$\circlearrowleft + \Sigma M_B = 0; \qquad -5(8) - 10(4) - 5(4) + F_{EF}(4) = 0$

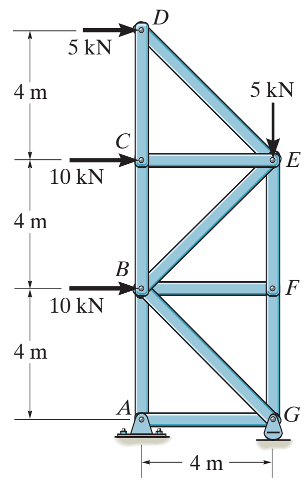
$F_{EF} = 25 \text{ kN (C)}$ **Ans.**



Ans:
$F_{BE} = 21.2 \text{ kN (T)}$
$F_{CB} = 5 \text{ kN (T)}$
$F_{EF} = 25 \text{ kN (C)}$

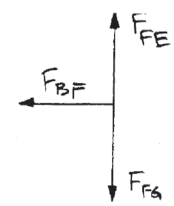
6–45.

Determine the force in members *BF*, *BG*, and *AB*, and state if the members are in tension or compression.



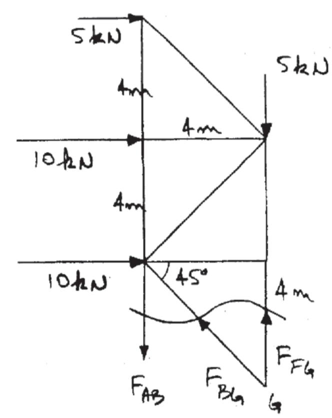
SOLUTION

Joint *F*:



$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{BF} = 0$ **Ans.**

Section:



$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 5 + 10 + 10 - F_{BG} \cos 45° = 0$

$F_{BG} = 35.4 \text{ kN (C)}$ **Ans.**

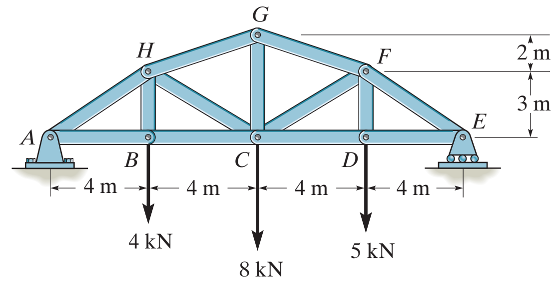
$\circlearrowleft + \Sigma M_G = 0;$ $\quad F_{AB}(4) - 10(4) - 10(8) - 5(12) = 0$

$F_{AB} = 45 \text{ kN (T)}$ **Ans.**

Ans:
$F_{BF} = 0$
$F_{BG} = 35.4 \text{ kN (C)}$
$F_{AB} = 45 \text{ kN (T)}$

6–46.

Determine the force in members BC, CH, GH, and CG of the truss and state if the members are in tension or compression.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire truss shown in Fig. *a*, $\mathbf{A}_y$ can be determined directly by writing the moment equation of equilibrium about point E.

$$\zeta+\Sigma M_E = 0; \quad 5(4) + 8(8) + 4(12) - A_y(16) = 0 \qquad A_y = 8.25 \text{ kN}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad A_x = 0$$

Method of Sections. Referring to the *FBD* of the left portion of the truss section through *a–a* shown in Fig. *a*, $\mathbf{F}_{BC}$, $\mathbf{F}_{GH}$ and $\mathbf{F}_{CH}$ can be determined directly by writing the moment equations of equilibrium about points H, C, and O, respectively,

$$\zeta+\Sigma M_H = 0; \quad F_{BC}(3) - 8.25(4) = 0 \quad F_{BC} = 11.0 \text{ kN (T)} \qquad \textbf{Ans.}$$

$$\zeta+\Sigma M_C = 0; \quad F_{GH}\left(\frac{1}{\sqrt{5}}\right)(10) + (4)(4) - 8.25(8) = 0$$

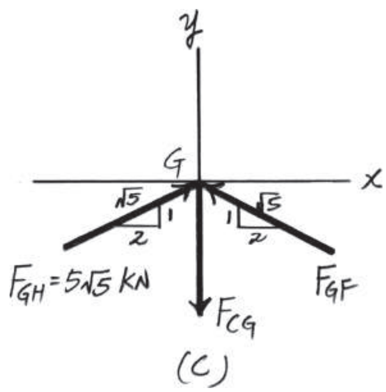
$$F_{GH} = 5\sqrt{5} \text{ kN (C)} = 11.2 \text{ kN (C)} \qquad \textbf{Ans.}$$

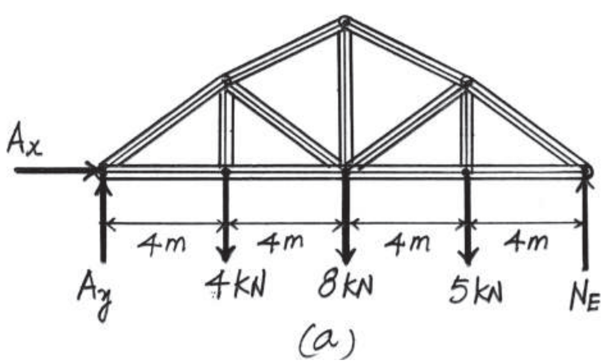
$$\zeta+\Sigma M_O = 0; \quad F_{CH}\left(\frac{3}{5}\right)(10) + (8.25)(2) - 4(6) = 0 \quad F_{CH} = 1.25 \text{ kN (C)} \qquad \textbf{Ans.}$$

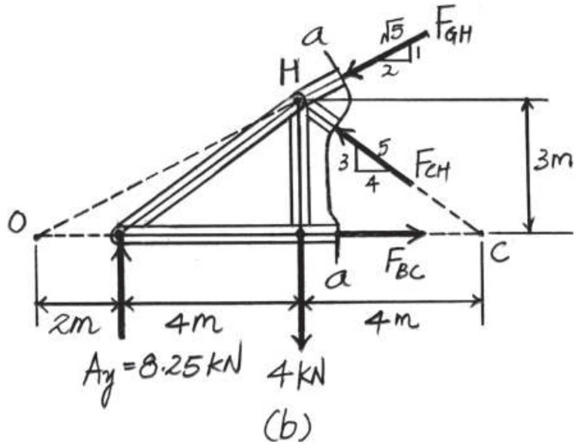
Method of Joints. Using the result of $\mathbf{F}_{GH}$, equilibrium of joint G, Fig. *c*, requires

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad \left(5\sqrt{5}\right)\left(\frac{2}{\sqrt{5}}\right) - F_{GF}\left(\frac{2}{\sqrt{5}}\right) = 0 \quad F_{GF} = \left(5\sqrt{5}\right) \text{kN (C)}$$

$$+\uparrow\Sigma F_y = 0; \quad 2\left(5\sqrt{5}\right)\left(\frac{1}{\sqrt{5}}\right) - F_{CG} = 0 \quad F_{CG} = 10.0 \text{ kN (T)} \qquad \textbf{Ans.}$$



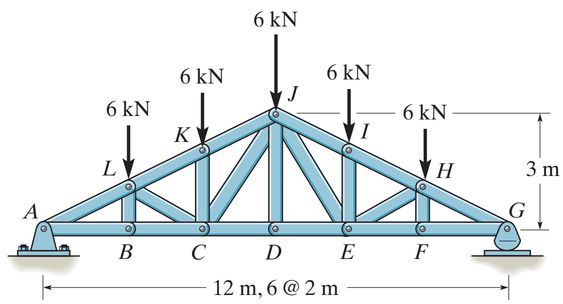




Ans:
$F_{BC} = 11.0$ kN(T)
$F_{GH} = 11.2$ kN(C)
$F_{CH} = 1.25$ kN(C)
$F_{CG} = 10.0$ kN(T)

6–47.

Determine the force in members *CD*, *CJ*, and *KJ* and state if these members are in tension or compression.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire truss shown in Fig. *a*, $\mathbf{A}_y$ can be determined directly by writing the moment equation of equilibrium about point *G*.

$$\circlearrowleft + \Sigma M_G = 0; \qquad 6(2) + 6(4) + 6(6) + 6(8) + 6(10) - A_y(12) = 0 \quad A_y = 15.0\text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$$

Method of Sections. Referring to the *FBD* of the left portion of the truss sectioned through $a - a$ shown in Fig. *b*, $\mathbf{F}_{CD}$, $\mathbf{F}_{CJ}$ and $\mathbf{F}_{KJ}$ can be determined directly by writing the moment equations of equilibrium about points *J*, *A* and *C*, respectively.

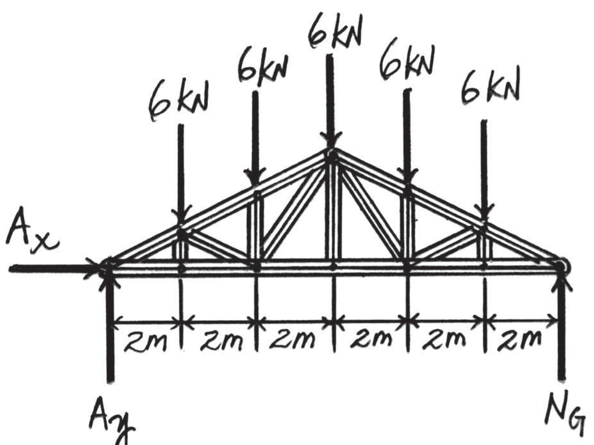
$$\circlearrowleft + \Sigma M_J = 0; \qquad F_{CD}(3) + 6(2) + 6(4) - 15.0(6) = 0 \qquad F_{CD} = 18.0\text{ kN (T)} \quad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{CJ}\left(\frac{3}{\sqrt{13}}\right)(4) - 6(4) - 6(2) = 0$$

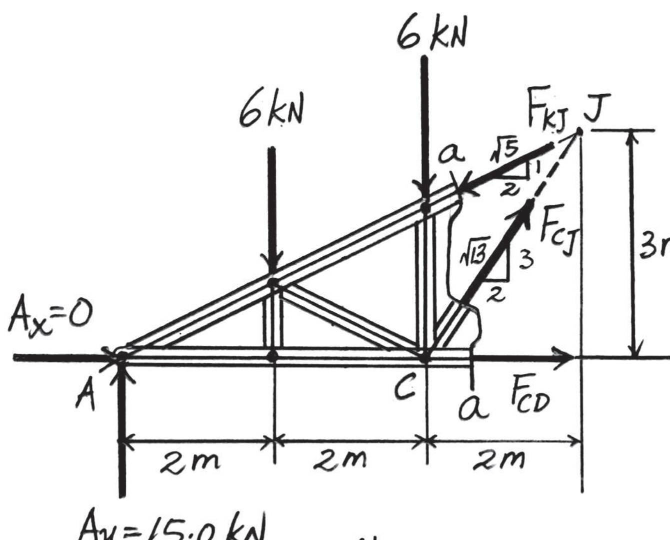
$$F_{CJ} = 3\sqrt{13}\text{ kN (T)} = 10.8\text{ kN (T)} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{KJ}\left(\frac{1}{\sqrt{5}}\right)(4) + 6(2) - 15.0\,(4) = 0$$

$$F_{KJ} = 12\sqrt{5}\text{ kN (C)} = 26.8\text{ kN (C)} \qquad \textbf{Ans.}$$



(a)

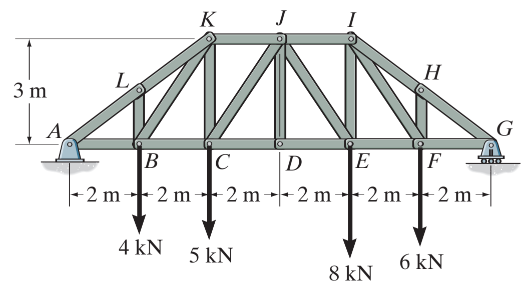


(b)

Ans:
$F_{CD} = 18.0$ kN (T)
$F_{CJ} = 10.8$ kN (T)
$F_{KJ} = 26.8$ kN (T)

***6–48.**

Determine the force in members JK, CJ, and CD of the truss, and state if the members are in tension or compression.



SOLUTION

Method of Joints: Applying the equations of equilibrium to the free - body diagram of the truss, Fig. a,

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$

$\circlearrowleft + \Sigma M_G = 0 \qquad 6(2) + 8(4) + 5(8) + 4(10) - A_y(12) = 0$

$A_y = 10.33 \text{ kN}$

Method of Sections: Using the left portion of the free - body diagram, Fig. a.

$\circlearrowleft + \Sigma M_C = 0; \qquad F_{JK}(3) + 4(2) - 10.33(4) = 0$

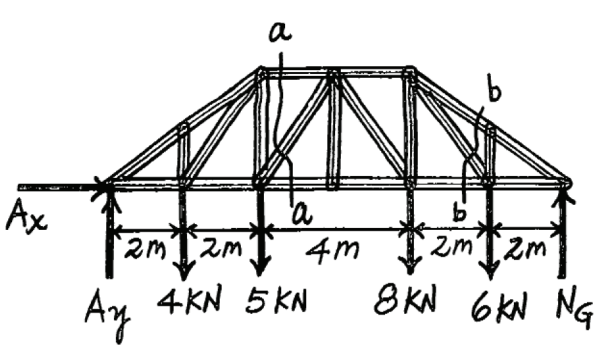
$F_{JK} = 11.111 \text{ kN} = 11.1 \text{ kN (C)}$ **Ans.**

$\circlearrowleft + \Sigma M_J = 0; \qquad F_{CD}(3) + 5(2) + 4(4) - 10.33(6) = 0$

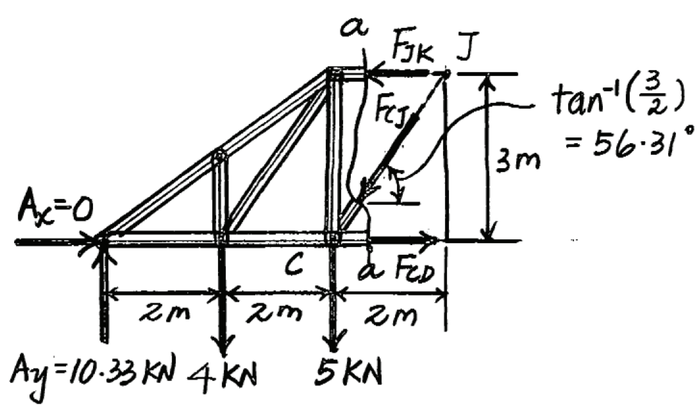
$F_{CD} = 12 \text{ kN (T)}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad 10.33 - 4 - 5 - F_{CJ} \sin 56.31° = 0$

$F_{CJ} = 1.602 \text{ kN} = 1.60 \text{ kN (C)}$ **Ans.**



(a)

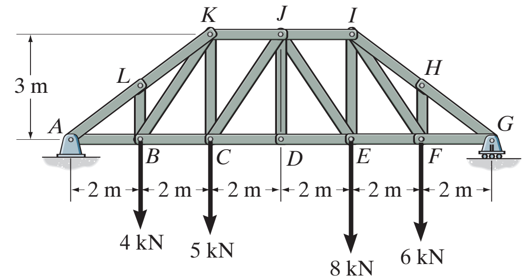


(b)

Ans:
$F_{JK} = 11.1 \text{ kN (C)}$
$F_{CD} = 12 \text{ kN (T)}$
$F_{CJ} = 1.60 \text{ kN (C)}$

6–49.

Determine the force in members HI, FI, and EF of the truss, and state if the members are in tension or compression.



SOLUTION

Support Reactions: Applying the moment equation of equilibrium about point A to the free - body diagram of the truss, Fig. a,

$$\zeta + \Sigma M_A = 0; \qquad N_G(2) - 4(2) - 5(4) - 8(8) - 6(10) = 0$$

$$N_G = 12.67 \text{ kN}$$

Method of Sections: Using the right portion of the free - body diagram, Fig. b.

$$\zeta + \Sigma M_I = 0; \qquad 12.67(4) - 6(2) - F_{EF}(3) = 0$$

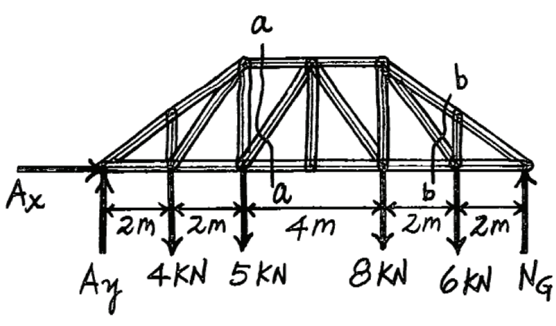
$$F_{EF} = 12.89 \text{ kN} = 12.9 \text{ kN (T)} \qquad \textbf{Ans.}$$

$$\zeta + \Sigma M_G = 0; \qquad -F_{FI} \sin 56.31°(2) + 6(2) = 0$$

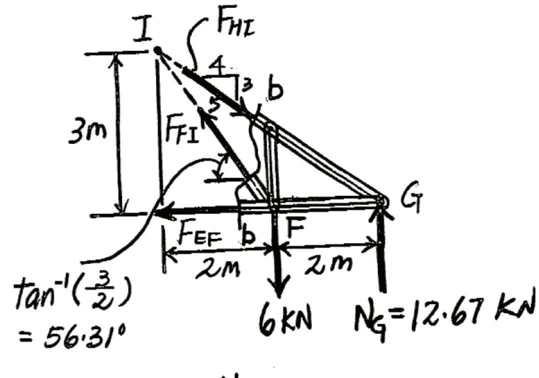
$$F_{FI} = 7.211 \text{ kN} = 7.21 \text{ kN (T)} \qquad \textbf{Ans.}$$

$$\zeta + \Sigma M_F = 0; \qquad 12.67(2) - F_{HI}\left(\frac{3}{5}\right)(2) = 0$$

$$F_{HI} = 21.11 \text{ kN} = 21.1 \text{ kN (C)} \qquad \textbf{Ans.}$$



(a)

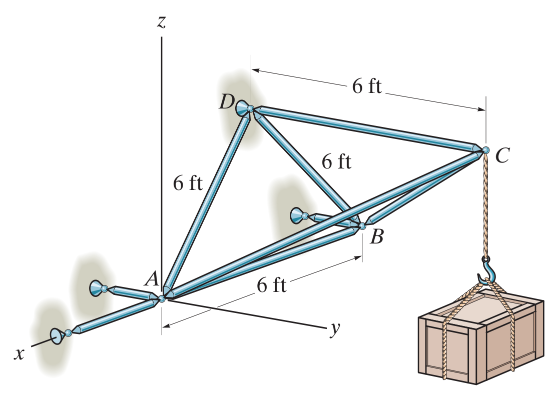


(b)

Ans:
$F_{EF} = 12.9$ kN (T)
$F_{FI} = 7.21$ kN (T)
$F_{HI} = 21.1$ kN (C)

6–50.

Determine the force developed in each member of the space truss and state if the members are in tension or compression. The crate has a weight of 150 lb.



SOLUTION

$$\mathbf{F}_{CA} = F_{CA}\left[\frac{-1\mathbf{i} + 2\mathbf{j} + 2\sin 60°\mathbf{k}}{\sqrt{8}}\right]$$

$$= -0.354\,F_{CA}\,\mathbf{i} + 0.707\,F_{CA}\mathbf{j} + 0.612\,F_{CA}\,\mathbf{k}$$

$$\mathbf{F}_{CB} = 0.354F_{CB}\,\mathbf{i} + 0.707F_{CB}\,\mathbf{j} + 0.612F_{CB}\,\mathbf{k}$$

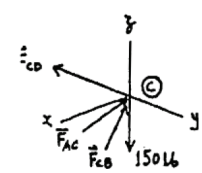
$$\mathbf{F}_{CD} = -F_{CD}\mathbf{j}$$

$$\mathbf{W} = -150\,\mathbf{k}$$

$\Sigma F_x = 0;\qquad -0.354F_{CA} + 0.354F_{CB} = 0$

$\Sigma F_y = 0;\qquad 0.707F_{CA} + 0.707F_{CB} - F_{CD} = 0$

$\Sigma F_z = 0;\qquad 0.612F_{CA} + 0.612F_{CB} - 150 = 0$



Solving:

$F_{CA} = F_{CB} = 122.5\text{ lb} = 122\text{ lb (C)}$ **Ans.**

$F_{CD} = 173\text{ lb (T)}$ **Ans.**

$$\mathbf{F}_{BA} = F_{BA}\,\mathbf{i}$$

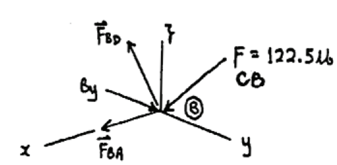
$$\mathbf{F}_{BD} = F_{BD}\cos 60°\mathbf{i} + F_{BD}\sin 60°\,\mathbf{k}$$

$$\mathbf{F}_{CB} = 122.5\,(-0.354\mathbf{i} - 0.707\,\mathbf{j} - 0.612\mathbf{k})$$

$$= -43.3\mathbf{i} - 86.6\mathbf{j} - 75.0\mathbf{k}$$

$\Sigma F_x = 0;\qquad F_{BA} + F_{BD}\cos 60° - 43.3 = 0$

$\Sigma F_z = 0;\qquad F_{BD}\sin 60° - 75 = 0$



Solving:

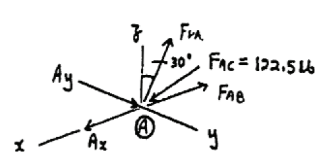
$F_{BD} = 86.6\text{ lb (T)}$ **Ans.**

$F_{BA} = 0$ **Ans.**

$$F_{AC} = 122.5(0.354\,\mathbf{i} - 0.707\mathbf{j} - 0.612\,\mathbf{k})$$

$\Sigma F_z = 0;\qquad F_{DA}\cos 30° - 0.612(122.5) = 0$

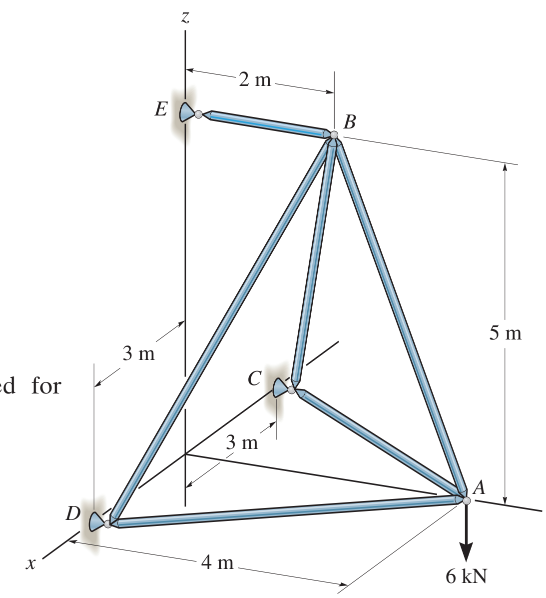
$F_{DA} = 86.6\text{ lb (T)}$ **Ans.**



Ans:
$F_{CA} = F_{CB} = 122\text{ lb (C)}$
$F_{CD} = 173\text{ lb (T)}$
$F_{BD} = 86.6\text{ lb (T)}$
$F_{BA} = 0$
$F_{DA} = 86.6\text{ lb (T)}$

6–51.

Determine the force in each member of the space truss and state if the members are in tension or compression. *Hint:* The support reaction at E acts along member EB. Why?



SOLUTION

Method of Joints: In this case, the support reactions are not required for determining the member forces.

Joint A:

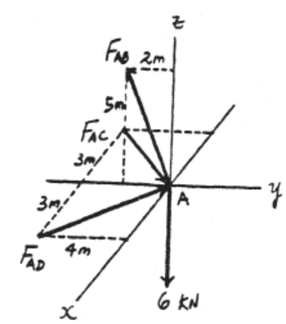
$$F_{AB}\left(\frac{5}{\sqrt{29}}\right)$$

$$F_{AB} = 6.462 \text{ kN (T)} = 6.46 \text{ kN (T)} \qquad \textbf{Ans.}$$

$$\Sigma F_x = 0; \qquad F_{AC}\left(\frac{3}{5}\right) - F_{AD}\left(\frac{3}{5}\right) = 0 \qquad F_{AC} = F_{AD} \qquad \textbf{(1)}$$

$$\Sigma F_y = 0; \qquad F_{AC}\left(\frac{4}{5}\right) + F_{AD}\left(\frac{4}{5}\right) - 6.462\left(\frac{2}{\sqrt{29}}\right) = 0$$

$$F_{AC} + F_{AD} = 3.00 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

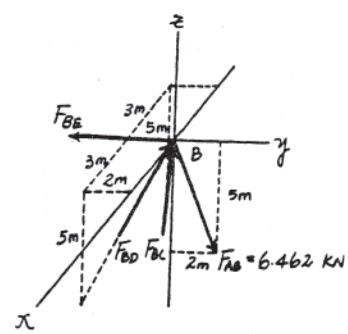
$$F_{AC} = F_{AD} = 1.50 \text{ kN (C)} \qquad \textbf{Ans.}$$

Joint B:

$$\Sigma F_x = 0; \qquad F_{BC}\left(\frac{3}{\sqrt{38}}\right) - F_{BD}\left(\frac{3}{\sqrt{38}}\right) = 0 \qquad F_{BC} = F_{BD} \qquad \textbf{(1)}$$

$$\Sigma F_z = 0; \qquad F_{BC}\left(\frac{5}{\sqrt{38}}\right) + F_{BD}\left(\frac{5}{\sqrt{38}}\right) - 6.462\left(\frac{5}{\sqrt{29}}\right) = 0$$

$$F_{BC} + F_{BD} = 7.397 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$$F_{BC} = F_{BD} = 3.699 \text{ kN (C)} = 3.70 \text{ kN (C)} \qquad \textbf{Ans.}$$

$$\Sigma F_y = 0; \qquad 2\left[3.699\left(\frac{2}{\sqrt{38}}\right)\right] + 6.462\left(\frac{2}{\sqrt{29}}\right) - F_{BE} = 0$$

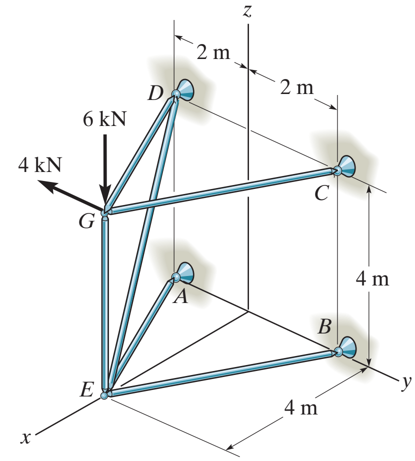
$$F_{BE} = 4.80 \text{ kN (T)} \qquad \textbf{Ans.}$$

Note: The support reactions at supports C and D can be determined by analyzing joints C and D, respectively using the results obtained above.

Ans:
$F_{AB} = 6.46$ kN (T)
$F_{AC} = F_{AD} = 1.50$ kN (C)
$F_{BC} = F_{BD} = 3.70$ kN (C)
$F_{BE} = 4.80$ kN (T)

***6–52.**

Determine the force in each member of the space truss and state if the members are in tension or compression. The truss is supported by ball-and-socket joints at A, B, C, and D.



SOLUTION

Support Reactions. Not required

Method of Joints. Perform the joint equilibrium analysis at joint G first and then proceed to joint E.

Joint *G*. Fig. *a*

$$\Sigma F_x = 0; \quad F_{GD}\left(\frac{2}{\sqrt{5}}\right) - F_{GC}\left(\frac{2}{\sqrt{5}}\right) = 0 \qquad F_{GD} = F_{GC} = F$$

$$\Sigma F_y = 0; \quad F\left(\frac{1}{\sqrt{5}}\right) + F\left(\frac{1}{\sqrt{5}}\right) - 4 = 0 \qquad F = 2\sqrt{5}\text{ kN}$$

Thus,

$$F_{GC} = 2\sqrt{5}\text{ kN (T)} = 4.47\text{ kN (T)} \qquad \textbf{Ans.}$$

$$F_{GD} = 2\sqrt{5}\text{ kN (C)} = 4.47\text{ kN (C)} \qquad \textbf{Ans.}$$

$$\Sigma F_z = 0; \quad F_{GE} - 6 = 0 \qquad F_{GE} = 6.00\text{ kN (C)} \qquad \textbf{Ans.}$$

Joint *E*. Fig. *b*

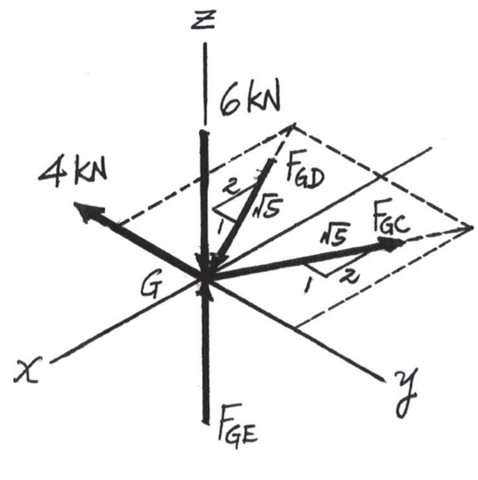
$$\Sigma F_z = 0; \quad F_{ED}\left(\frac{2}{3}\right) - 6.00 = 0 \qquad F_{ED} = 9.00\text{ kN (T)} \qquad \textbf{Ans.}$$

$$\Sigma F_x = 0; \quad F_{EA}\left(\frac{2}{\sqrt{5}}\right) - F_{EB}\left(\frac{2}{\sqrt{5}}\right) - 9.00\left(\frac{2}{3}\right) = 0 \qquad \textbf{(1)}$$

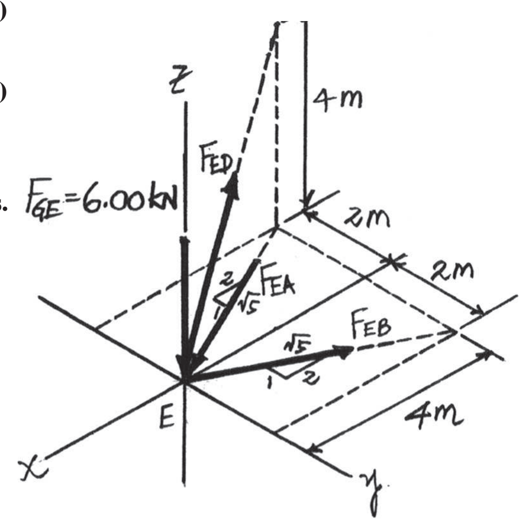
$$\Sigma F_y = 0; \quad F_{EA}\left(\frac{1}{\sqrt{5}}\right) + F_{EB}\left(\frac{1}{\sqrt{5}}\right) - 9.00\left(\frac{1}{3}\right) = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$F_{EA} = 3\sqrt{5}\text{ kN (C)} = 6.71\text{ kN (C)} \qquad F_{EB} = 0 \qquad \textbf{Ans.}$$



(a)

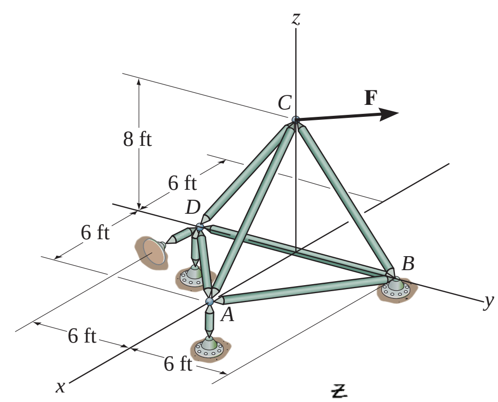


(b)

Ans:
$F_{GC} = 4.47$ kN (T)
$F_{GD} = 4.47$ kN (C)
$F_{GE} = 6.00$ kN (C)
$F_{ED} = 9.00$ kN (T)
$F_{EA} = 6.71$ kN (C)
$F_{EB} = 0$

6–53.

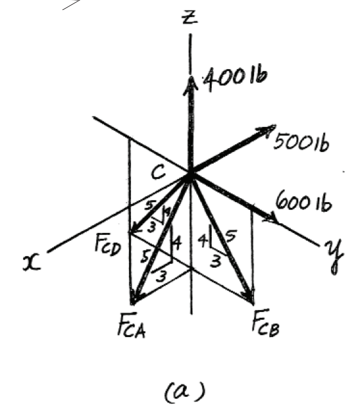
The space truss supports a force $\mathbf{F} = \{-500\mathbf{i} + 600\mathbf{j} + 400\mathbf{k}\}$ lb. Determine the force in each member, and state if the members are in tension or compression.



SOLUTION

Method of Joints: In this case, there is no need to compute the support reactions. We will begin by analyzing the equilibrium of joint C, and then that of joints A and D.

Joint C: From the free - body diagram, Fig. a,



$$\Sigma F_x = 0; \quad F_{CA}\left(\frac{3}{5}\right) - 500 = 0$$

$$F_{CA} = 833.33\text{ lb} = 833\text{ lb(T)} \qquad \textbf{Ans.}$$

$$\Sigma F_y = 0; \quad F_{CB}\left(\frac{3}{5}\right) - F_{CD}\left(\frac{3}{5}\right) + 600 = 0 \qquad \textbf{(1)}$$

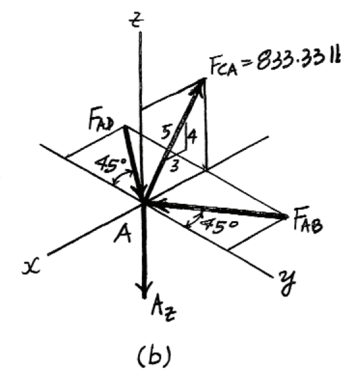
$$\Sigma F_z = 0; \quad 400 - 833.33\left(\frac{4}{5}\right) - F_{CD}\left(\frac{4}{5}\right) - F_{CB}\left(\frac{4}{5}\right) = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$F_{CB} = -666.67\text{ lb} = 667\text{ lb(C)} \qquad \textbf{Ans.}$$

$$F_{CD} = 333.33\text{ lb} = 333\text{ lb(T)} \qquad \textbf{Ans.}$$

Joint A: From the free - body diagram, Fig. b,



$$\Sigma F_x = 0; \quad F_{AD}\cos 45° - F_{AB}\cos 45° = 0$$

$$F_{AD} = F_{AB} = F$$

$$\Sigma F_y = 0; \quad F\sin 45° + F\sin 45° - 833.33\left(\frac{3}{5}\right) = 0$$

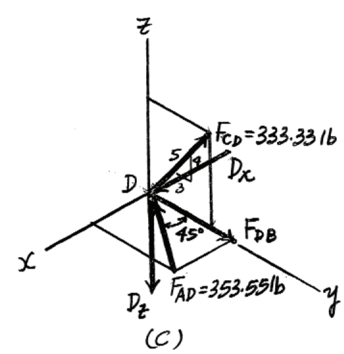
$$F = 353.55\text{ lb}$$

Thus, $F_{AD} = F_{AB} = 353.55\text{ lb} = 354\text{ lb(C)}$ **Ans.**

$$\Sigma F_z = 0; \quad 833.33\left(\frac{4}{5}\right) - A_z = 0$$

$$A_z = 666.67\text{ lb}$$

Joint D: From the free - body diagram, Fig. c,



$$\Sigma F_y = 0; \quad F_{DB} + 333.33\left(\frac{3}{5}\right) - 353.55\cos 45° = 0$$

$$F_{DB} = 50\text{ lb(T)} \qquad \textbf{Ans.}$$

$$\Sigma F_x = 0; \quad D_x - 353.55\sin 45° = 0$$

$$D_x = 250\text{ lb}$$

$$\Sigma F_z = 0; \quad 333.33\left(\frac{4}{5}\right) - D_z = 0$$

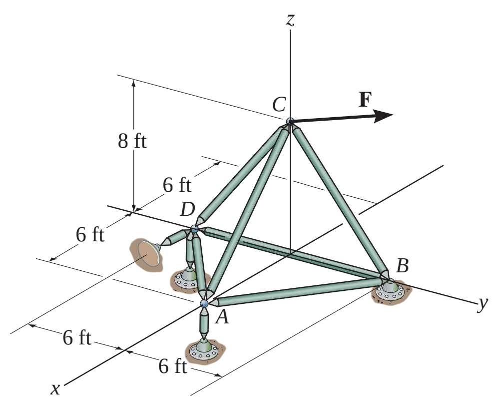
$$D_z = 266.67\text{ lb}$$

Note: The equilibrium analysis of joint B can be used to determine the components of support reaction of the ball and socket support at B.

Ans:
$F_{CA} = 833$ lb (T)
$F_{CB} = 667$ lb (C)
$F_{CD} = 333$ lb (T)
$F_{AD} = F_{AB} = 354$ lb (C)
$F_{DB} = 50$ lb (T)

6–54.

The space truss supports a force $\mathbf{F} = \{600\mathbf{i} + 450\mathbf{j} - 750\mathbf{k}\}$ lb. Determine the force in each member, and state if the members are in tension or compression.



SOLUTION

Method of Joints: In this case, there is no need to compute the support reactions. We will begin by analyzing the equilibrium of joint C, and then that of joints A and D.

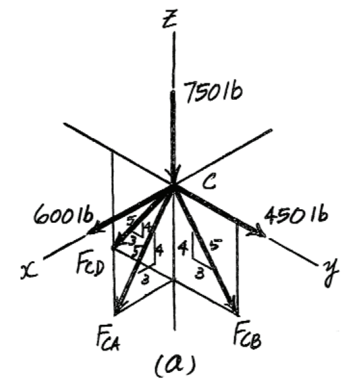
Joint C: From the free - body diagram, Fig. a,

$$\Sigma F_x = 0; \quad 600 + F_{CA}\left(\frac{3}{5}\right) = 0$$

$$F_{CA} = -1000 \text{ lb} = 1000 \text{ lb(C)} \qquad \textbf{Ans.}$$

$$\Sigma F_y = 0; \quad F_{CB}\left(\frac{3}{5}\right) - F_{CD}\left(\frac{3}{5}\right) + 450 = 0 \qquad \textbf{(1)}$$

$$\Sigma F_z = 0; \quad -F_{CB}\left(\frac{4}{5}\right) - F_{CD}\left(\frac{4}{5}\right) - (-1000)\left(\frac{4}{5}\right) - 750 = 0 \qquad \textbf{(2)}$$



Solving Eqs. (1) and (2) yields

$$F_{CD} = 406.25 \text{ lb} = 406 \text{ lb(T)} \qquad \textbf{Ans.}$$

$$F_{CB} = -343.75 \text{ lb} = 344 \text{ lb(C)} \qquad \textbf{Ans.}$$

Joint A: From the free - body diagram, Fig. b,

$$\Sigma F_y = 0; \quad F_{AB} \cos 45° - F_{AD} \cos 45° = 0$$

$$F_{AB} = F_{AD} = F$$

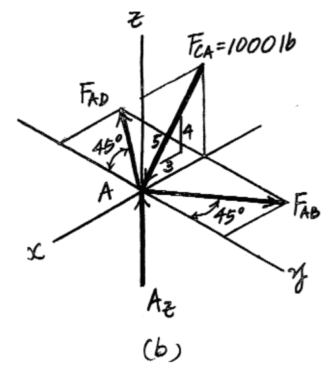
$$\Sigma F_x = 0; \quad 1000\left(\frac{3}{5}\right) - F \sin 45° - F \sin 45° = 0$$

$$F = 424.26 \text{ lb}$$

Thus, $F_{AB} = F_{AD} = 424.26 \text{ lb} = 424 \text{ lb (T)}$ **Ans.**

$$\Sigma F_z = 0; \quad A_z - 1000\left(\frac{4}{5}\right) = 0$$

$$A_z = 800 \text{ lb}$$



Joint D: From the free - body diagram, Fig. c,

$$\Sigma F_y = 0; \quad 406.25\left(\frac{3}{5}\right) + 406.25 \cos 45° - F_{DB} = 0$$

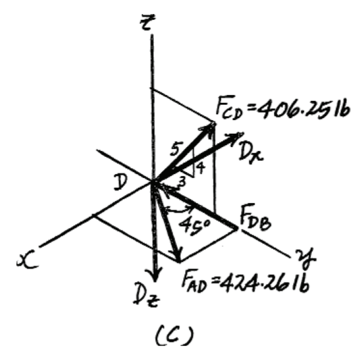
$$F_{DB} = 543.75 \text{ lb} = 544 \text{ lb(C)} \qquad \textbf{Ans.}$$

$$\Sigma F_x = 0; \quad 424.26 \sin 45° - D_x = 0$$

$$Dx = 300 \text{ lb}$$

$$\Sigma F_z = 0; \quad 406.25\left(\frac{4}{5}\right) - D_z = 0$$

$$D_z = 325 \text{ lb}$$

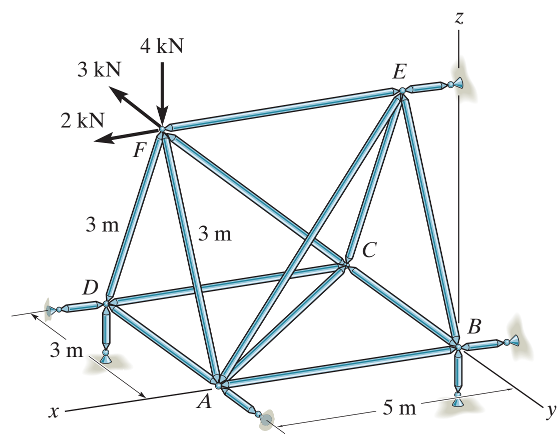


Note: The equilibrium analysis of joint B can be used to determine the components of support reaction of the ball and socket support at B.

Ans:
$F_{CA} = 1000$ lb (C)
$F_{CD} = 406$ lb (T)
$F_{CB} = 344$ lb (C)
$F_{AB} = F_{AD} = 424$ lb (T)
$F_{DB} = 544$ lb (C)

6–55.

Determine the force in members EF, AF, and DF of the space truss and state if the members are in tension or compression. The truss is supported by short links at A, B, D, and E.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire truss shown in Fig. *a*,

$\Sigma F_x = 0; \quad D_x - B_x - E_x + 2 = 0$ **(1)**

$\Sigma F_y = 0; \quad A_y - 3 = 0$ **(2)**

$\Sigma F_z = 0; \quad D_z + B_z - 4 = 0$ **(3)**

$\Sigma M_x = 0; \quad 4(1.5) + 3(3 \sin 60°) - D_z(3) = 0$ **(4)**

$\Sigma M_y = 0; \quad 2(3 \sin 60°) + 4(5) - D_z(5) - E_x(3 \sin 60°) = 0$ **(5)**

$\Sigma M_z = 0; \quad D_x(3) + A_y(5) - E_x(1.5) + 2(1.5) - 3(5) = 0$ **(6)**

Solving Eqs. (1) to (6)

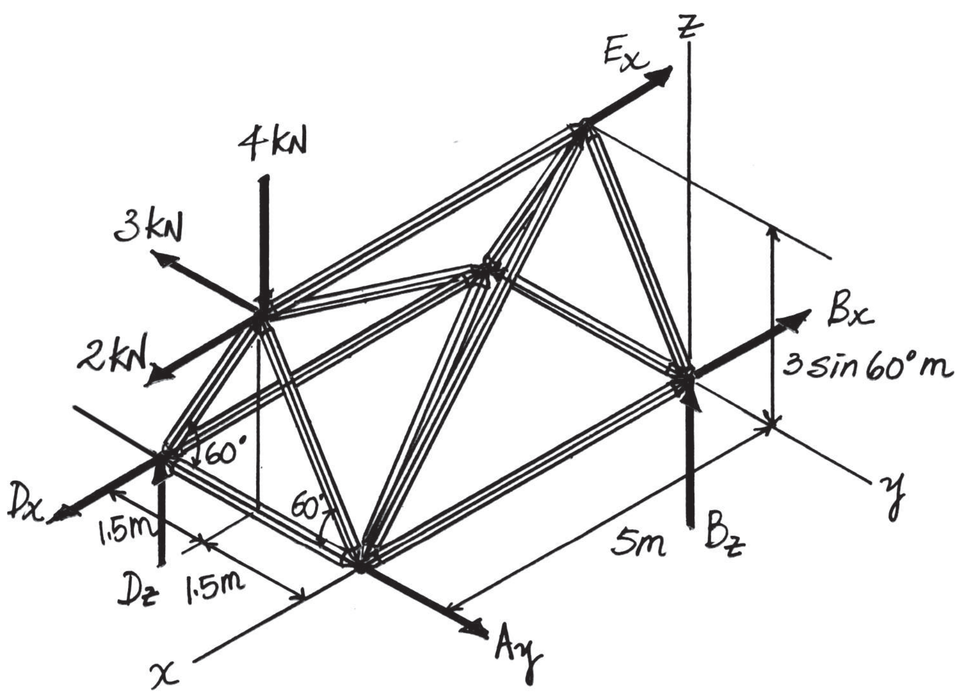
$A_y = 3.00 \text{ kN} \quad D_z = 4.5981 \text{ kN} \quad B_z = -0.5981 \text{ kN}$

$E_x = 0.8490 \text{ kN} \quad D_x = -0.5755 \text{ kN} \quad B_x = 0.5755 \text{ kN}$

Method of Joints. We will analyse the equilibrium of the joint at joint D first and then proceed to joint F.

Joint D. Fig. *b*

$+\uparrow \Sigma F_z = 0; \quad 4.5981 - F_{DF} \sin 60° = 0 \quad F_{DF} = 5.3094 \text{ kN (C)} = 5.31 \text{ kN (C)}$ **Ans.**



(a)

6–55. Continued

Joint *F*. Fig. *c*

$$\Sigma F_x = 0; \qquad 2 - F_{EF} - \frac{5}{\sqrt{34}} F_{CF} = 0 \qquad \textbf{(7)}$$

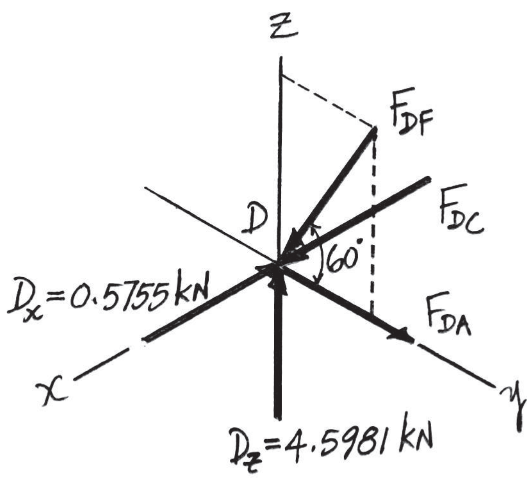
$$\Sigma F_y = 0; \qquad F_{AF} \cos 60° + 5.3094 \cos 60° - 3 - F_{CF}\left(\frac{1.5}{\sqrt{34}}\right) = 0 \qquad \textbf{(8)}$$

$$\Sigma F_z = 0; \qquad 5.3094 \sin 60° - 4 - F_{AF} \sin 60° - F_{CF}\left(\frac{3 \sin 60°}{\sqrt{34}}\right) = 0 \qquad \textbf{(9)}$$

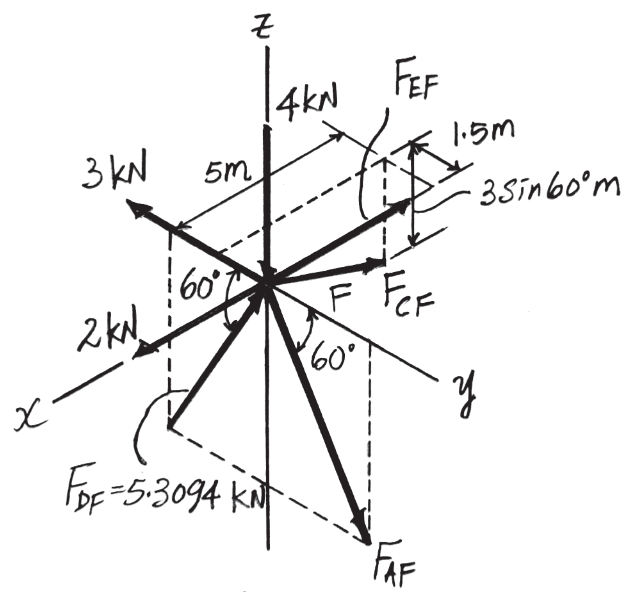
Solving Eqs. (7), (8) and (9)

$F_{CF} = 0 \qquad F_{EF} = 2.00 \text{ kN (T)}$ **Ans.**

$F_{AF} = 0.6906 \text{ kN (T)} = 0.691 \text{ kN (T)}$ **Ans.**



(b)

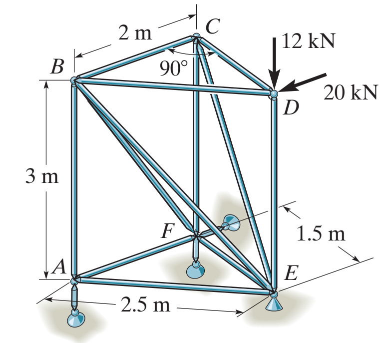


(c)

Ans:
$F_{DF} = 5.31$ kN (C)
$F_{EF} = 2.00$ kN (T)
$F_{AF} = 0.691$ kN (T)

***6–56.**

The space truss is used to support the forces at joints B and D. Determine the force in each member and state if the members are in tension or compression.

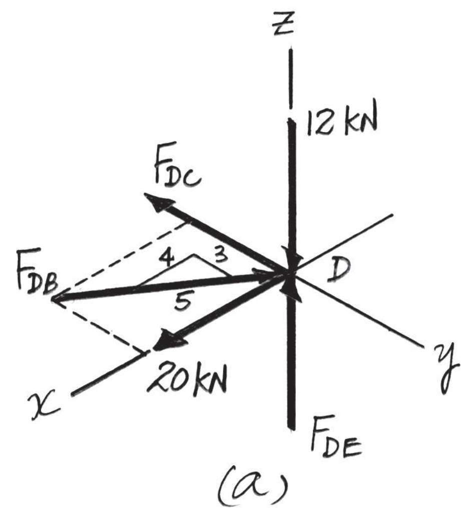


SOLUTION

Support Reactions. Not required

Method of Joints. Analysis of joint equilibrium will be in the sequence of joints D, C, B, A and F.

Joint *D*. Fig. *a*



$\Sigma F_x = 0;\quad 20 - F_{DB}\left(\frac{4}{5}\right) = 0\quad F_{DB} = 25.0 \text{ kN (T)}$ **Ans.**

$\Sigma F_y = 0;\quad 25.0\left(\frac{3}{5}\right) - F_{DC} = 0\quad F_{DC} = 15.0 \text{ kN (T)}$ **Ans.**

$\Sigma F_z = 0;\quad F_{DE} - 12 = 0\quad F_{DE} = 12.0 \text{ kN (C)}$ **Ans.**

Joint *C*. Fig. *b*

$\Sigma F_x = 0;\quad F_{CB} = 0$ **Ans.**

$\Sigma F_y = 0;\quad F_{CE}\left(\frac{1}{\sqrt{5}}\right) + 15.0 = 0\quad F_{CE} = -15\sqrt{5} \text{ kN} = 33.5 \text{ kN (C)}$ **Ans.**

$\Sigma F_z = 0;\quad -F_{CF} - \left(-15\sqrt{5}\right)\left(\frac{2}{\sqrt{5}}\right) = 0\quad F_{CF} = 30.0 \text{ kN (T)}$ **Ans.**

Joint *B*. Fig. *c*

$\Sigma F_y = 0;\quad F_{BE}\left(\frac{1.5}{\sqrt{15.25}}\right) - 25.0\left(\frac{3}{5}\right) = 0$

$F_{BE} = 10\sqrt{15.25} \text{ kN (T)} = 39.1 \text{ kN (T)}$ **Ans.**

$\Sigma F_x = 0;\quad 25.0\left(\frac{4}{5}\right) - \left(10\sqrt{15.25}\right)\left(\frac{2}{\sqrt{15.25}}\right) - F_{BF}\left(\frac{2}{\sqrt{13}}\right) = 0$

$F_{BF} = 0$ **Ans.**

$\Sigma F_z = 0;\quad -F_{BA} - \left(10\sqrt{15.25}\right)\left(\frac{3}{\sqrt{15.25}}\right) = 0$

$F_{BA} = -30.0 \text{ kN} = 30.0 \text{ kN (C)}$ **Ans.**

***6–56. Continued**

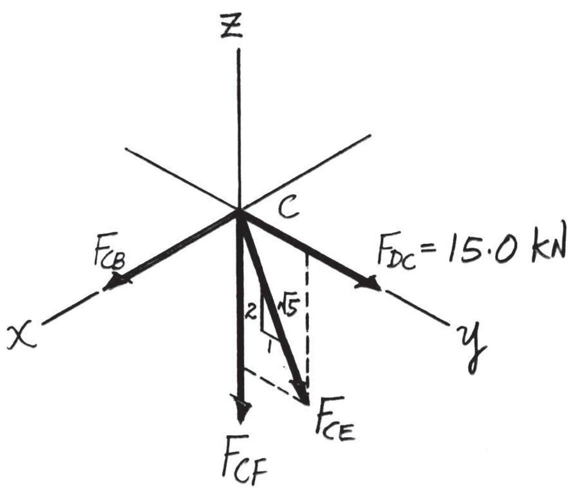
Joint *A*. Fig. *d*

$\Sigma F_y = 0; \quad F_{AE} = 0$ **Ans.**

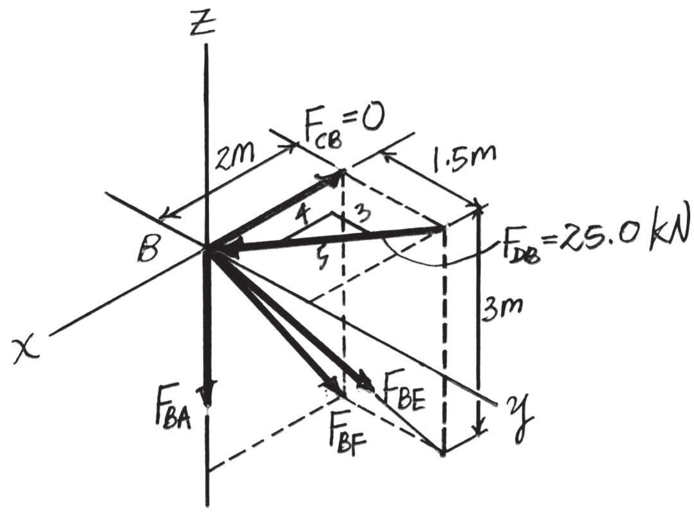
$\Sigma F_x = 0; \quad F_{AF} = 0$ **Ans.**

Joint *F*. Fig. *e*

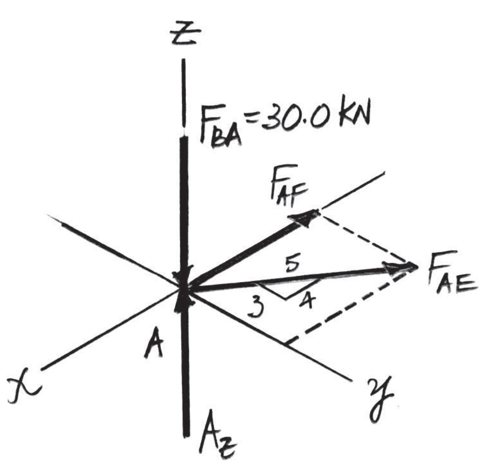
$\Sigma F_y = 0; \quad F_{FE} = 0$ **Ans.**



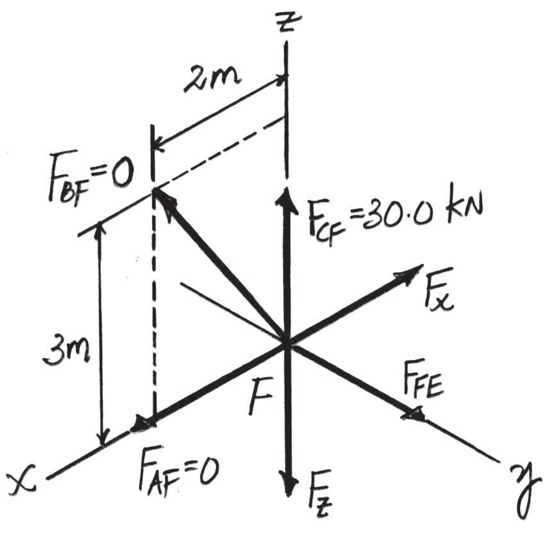
(b)



(C)



(d)

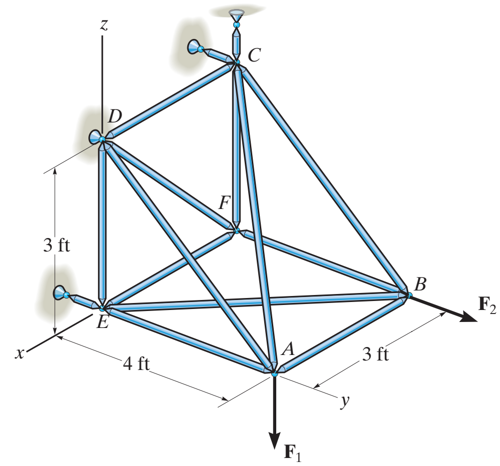


(e)

Ans:
$F_{DB} = 25.0$ kN (C)
$F_{DC} = 15.0$ kN (T)
$F_{DE} = 12.0$ kN (C)
$F_{CE} = 33.5$ kN (C)
$F_{CF} = 30.0$ kN (T)
$F_{BE} = 39.1$ kN (T)
$F_{BF} = 0$
$F_{BA} = 30.0$ kN (C)
$F_{AE} = 0$
$F_{AF} = 0$
$F_{FE} = 0$

6–57.

The space truss is supported by a ball-and-socket joint at D and short links at C and E. Determine the force in each member and state if the members are in tension or compression. Take $\mathbf{F}_1 = \{-500\mathbf{k}\}$ lb and $\mathbf{F}_2 = \{400\mathbf{j}\}$ lb.



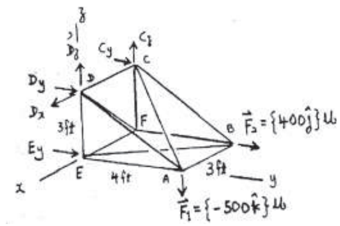
SOLUTION

$\Sigma M_z = 0; \quad -C_y(3) - 400(3) = 0$

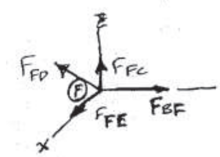
$C_y = -400$ lb

$\Sigma F_x = 0; \quad D_x = 0$

$\Sigma M_y = 0; \quad C_z = 0$



Joint F: $\quad \Sigma F_y = 0; \quad F_{BF} = 0$ **Ans.**



Joint B:

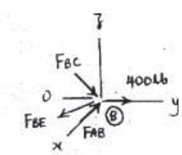
$\Sigma F_z = 0; \quad F_{BC} = 0$ **Ans.**

$\Sigma F_y = 0; \quad 400 - \frac{4}{5}F_{BE} = 0$

$F_{BE} = 500$ lb (T) **Ans.**

$\Sigma F_x = 0; \quad F_{AB} - \frac{3}{5}(500) = 0$

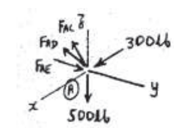
$F_{AB} = 300$ lb (C) **Ans.**



Joint A:

$\Sigma F_x = 0; \quad 300 - \frac{3}{\sqrt{34}}F_{AC} = 0$

$F_{AC} = 583.1 = 583$ lb (T) **Ans.**

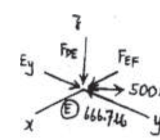


$\Sigma F_z = 0; \quad \frac{3}{\sqrt{34}}(583.1) - 500 + \frac{3}{5}F_{AD} = 0$

$F_{AD} = 333$ lb (T) **Ans.**

$\Sigma F_y = 0; \quad F_{AE} - \frac{4}{5}(333.3) - \frac{4}{\sqrt{34}}(583.1) = 0$

$F_{AE} = 667$ lb (C) **Ans.**



Joint E:

$\Sigma F_z = 0; \quad F_{DE} = 0$ **Ans.**

$\Sigma F_x = 0; \quad F_{EF} - \frac{3}{5}(500) = 0$

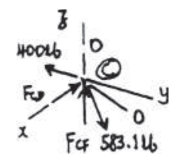
$F_{EF} = 300$ lb (C) **Ans.**

6–57. Continued

Joint C:

$$\Sigma F_x = 0; \qquad \frac{3}{\sqrt{34}}(583.1) - F_{CD} = 0$$

$$F_{CD} = 300 \text{ lb (C)}$$ **Ans.**



$$\Sigma F_z = 0; \qquad F_{CF} - \frac{3}{\sqrt{34}}(583.1) = 0$$

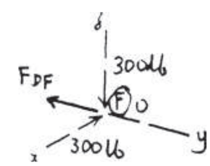
$$F_{CF} = 300 \text{ lb (C)}$$ **Ans.**

$$\Sigma F_y = 0; \qquad \frac{4}{\sqrt{34}}(583.1) - 400 = 0$$ Check!

Joint F:

$$\Sigma F_x = 0; \qquad \frac{3}{\sqrt{18}}F_{DF} - 300 = 0$$

$$F_{DF} = 424 \text{ lb (T)}$$ **Ans.**

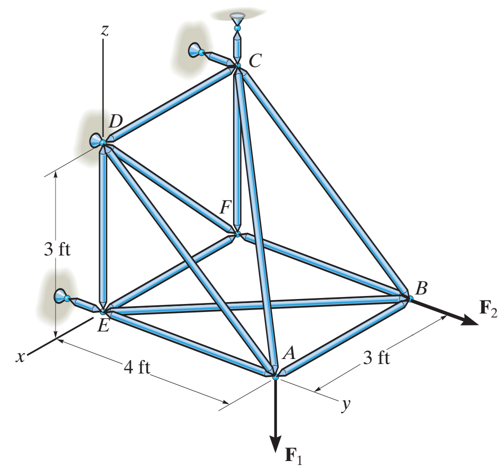


$$\Sigma F_z = 0; \qquad \frac{3}{\sqrt{18}}(424.3) - 300 = 0$$ Check!

Ans:
$F_{BF} = 0$
$F_{BC} = 0$
$F_{BE} = 500$ lb (T)
$F_{AB} = 300$ lb (C)
$F_{AC} = 583$ lb (T)
$F_{AD} = 333$ lb (T)
$F_{AE} = 667$ lb (C)
$F_{DE} = 0$
$F_{EF} = 300$ lb (C)
$F_{CD} = 300$ lb (C)
$F_{CF} = 300$ lb (C)
$F_{DF} = 424$ lb (T)

6–58.

The space truss is supported by a ball-and-socket joint at D and short links at C and E. Determine the force in each member and state if the members are in tension or compression. Take $\mathbf{F}_1 = \{200\mathbf{i} + 300\mathbf{j} - 500\mathbf{k}\}$ lb and $\mathbf{F}_2 = \{400\mathbf{j}\}$ lb.



SOLUTION

$\Sigma F_x = 0;$ $\quad D_x + 200 = 0$

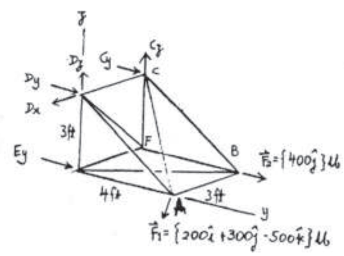
$D_x = -200$ lb

$\Sigma M_z = 0;$ $\quad -C_y(3) - 400(3) - 200(4) = 0$

$C_y = -666.7$ lb

$\Sigma M_y = 0;$ $\quad C_z(3) - 200(3) = 0$

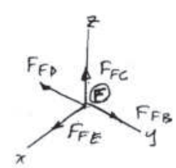
$C_z = 200$ lb



Joint F: $\quad F_{BF} = 0$ **Ans.**

Joint B:

$\Sigma F_z = 0;$ $\quad F_{BC} = 0$ **Ans.**

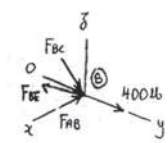


$\Sigma F_y = 0;$ $\quad 400 - \frac{4}{5}F_{BE} = 0$

$F_{BE} = 500$ lb (T) **Ans.**

$\Sigma F_x = 0;$ $\quad F_{AB} - \frac{3}{5}(500) = 0$

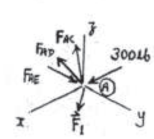
$F_{AB} = 300$ lb (C) **Ans.**



Joint A:

$\Sigma F_x = 0;$ $\quad 300 + 200 - \frac{3}{\sqrt{34}}F_{AC} = 0$

$F_{AC} = 971.8 = 972$ lb (T) **Ans.**

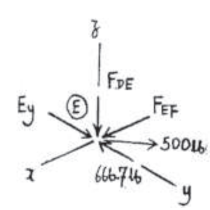


$\Sigma F_z = 0;$ $\quad \frac{3}{\sqrt{34}}(971.8) - 500 + \frac{3}{5}F_{AD} = 0$

$F_{AD} = 0$ **Ans.**

$\Sigma F_y = 0;$ $\quad F_{AE} + 300 - \frac{4}{\sqrt{34}}(971.8) = 0$

$F_{AE} = 367$ lb (C) **Ans.**



Joint E:

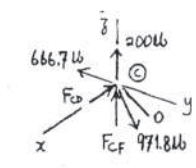
$\Sigma F_z = 0;$ $\quad F_{DE} = 0$ **Ans.**

$\Sigma F_x = 0;$ $\quad F_{EF} - \frac{3}{5}(500) = 0$

$F_{EF} = 300$ lb (C) **Ans.**

6–58. Continued

Joint C:

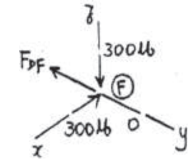


$\Sigma F_x = 0;$ $\quad \frac{3}{\sqrt{34}}(971.8) - F_{CD} = 0$

$F_{CD} = 500 \text{ lb (C)}$ **Ans.**

$\Sigma F_z = 0;$ $\quad F_{CF} - \frac{3}{\sqrt{34}}(971.8) + 200 = 0$

$F_{CF} = 300 \text{ lb (C)}$ **Ans.**



$\Sigma F_y = 0;$ $\quad \frac{4}{\sqrt{34}}(971.8) - 666.7 = 0$ Check!

Joint F:

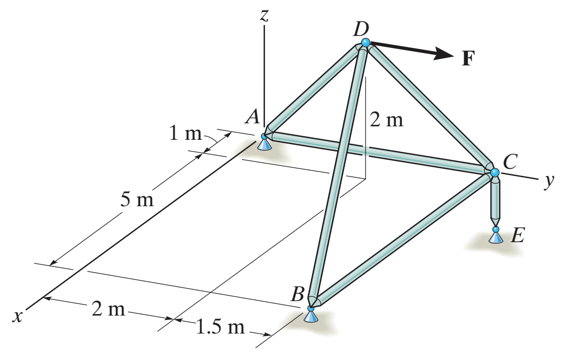
$\Sigma F_x = 0;$ $\quad \frac{3}{\sqrt{18}} F_{DF} - 300 = 0$

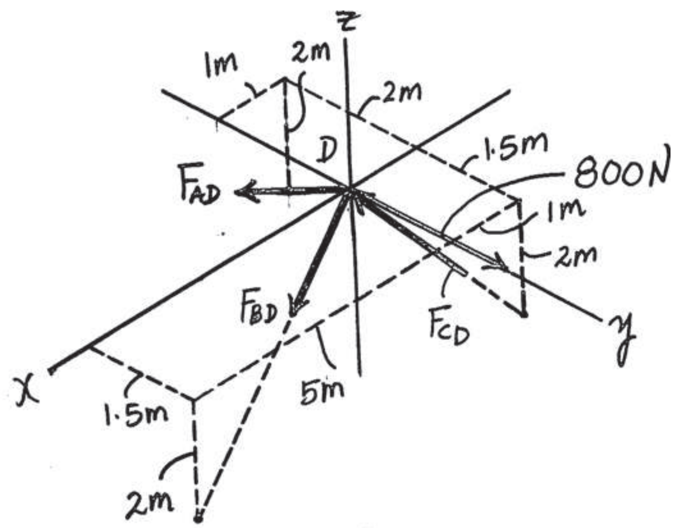
$F_{DF} = 424 \text{ lb (T)}$ **Ans.**

Ans:
$F_{BF} = 0$
$F_{BC} = 0$
$F_{BE} = 500 \text{ lb (T)}$
$F_{AB} = 300 \text{ lb (C)}$
$F_{AC} = 972 \text{ lb (T)}$
$F_{AD} = 0$
$F_{AE} = 367 \text{ lb (C)}$
$F_{DE} = 0$
$F_{EF} = 300 \text{ lb (C)}$
$F_{CD} = 500 \text{ lb (C)}$
$F_{CF} = 300 \text{ lb (C)}$
$F_{DF} = 424 \text{ lb (T)}$

6–59.

Determine the force in each member of the space truss and state if the members are in tension or compression. The The truss is supported by ball-and-socket joints at A, B, and E. Set $\mathbf{F} = \{800\mathbf{j}\}$ N. *Hint:* The support reaction at E acts along member EC. Why?





Joint D:

$\Sigma F_x = 0;$ $\quad -\frac{1}{3}F_{AD} + \frac{5}{\sqrt{31.25}}F_{BD} + \frac{1}{\sqrt{7.25}}F_{CD} = 0$

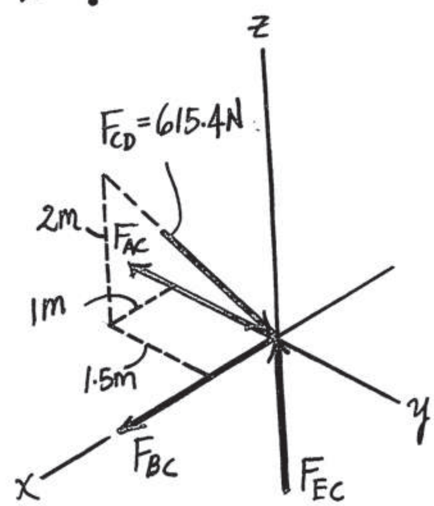
$\Sigma F_y = 0;$ $\quad -\frac{2}{3}F_{AD} + \frac{1.5}{\sqrt{31.25}}F_{BD} - \frac{1.5}{\sqrt{7.25}}F_{CD} + 800 = 0$

$\Sigma F_z = 0;$ $\quad -\frac{2}{3}F_{AD} - \frac{2}{\sqrt{31.25}}F_{BD} + \frac{2}{\sqrt{7.25}}F_{CD} = 0$

$F_{AD} = 686$ N (T) Ans

$F_{BD} = 0$ Ans

$F_{CD} = 615.4 = 615$ N (C) Ans



Joint C:

$\Sigma F_x = 0;$ $\quad F_{BC} - \frac{1}{\sqrt{7.25}}(615.4) = 0$

$F_{BC} = 229$ N (T) Ans

$\Sigma F_y = 0;$ $\quad \frac{1.5}{\sqrt{7.25}}(615.4) - F_{AC} = 0$

$F_{AC} = 343$ N (T) Ans

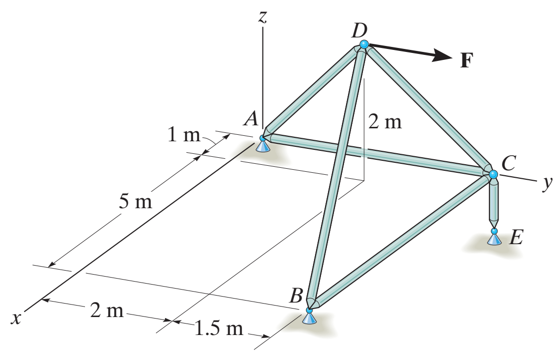
$\Sigma F_z = 0;$ $\quad F_{EC} - \frac{2}{\sqrt{7.25}}(615.4) = 0$

$F_{EC} = 457$ N (C) Ans

Ans:
$F_{AD} = 686$ N (T)
$F_{BD} = 0$
$F_{CD} = 615$ N (C)
$F_{BC} = 229$ N (T)
$F_{AC} = 343$ N (T)
$F_{EC} = 457$ N (C)

***6–60.**

Determine the force in each member of the space truss and state if the members are in tension or compression. The truss is supported by ball-and-socket joints at A, B, and E. Set $\mathbf{F} = \{-200\mathbf{i} + 400\mathbf{j}\}$ N. *Hint:* The support reaction at E acts along member EC. Why?



SOLUTION

Joint D:

$\Sigma F_x = 0;\qquad -\frac{1}{3}F_{AD} + \frac{5}{\sqrt{31.25}}F_{BD} + \frac{1}{\sqrt{7.25}}F_{CD} - 200 = 0$

$\Sigma F_y = 0;\qquad -\frac{2}{3}F_{AD} + \frac{1.5}{\sqrt{31.25}}F_{BD} - \frac{1.5}{\sqrt{7.25}}F_{CD} + 400 = 0$

$\Sigma F_z = 0;\qquad -\frac{2}{3}F_{AD} - \frac{2}{\sqrt{31.25}}F_{BD} + \frac{2}{\sqrt{7.25}}F_{CD} = 0$

$F_{AD} = 343 \text{ N (T)}$ **Ans.**

$F_{BD} = 186 \text{ N (T)}$ **Ans.**

$F_{CD} = 397.5 = 397 \text{ N (C)}$ **Ans.**

Joint C:

$\Sigma F_x = 0;\qquad F_{BC} - \frac{1}{\sqrt{7.25}}(397.5) = 0$

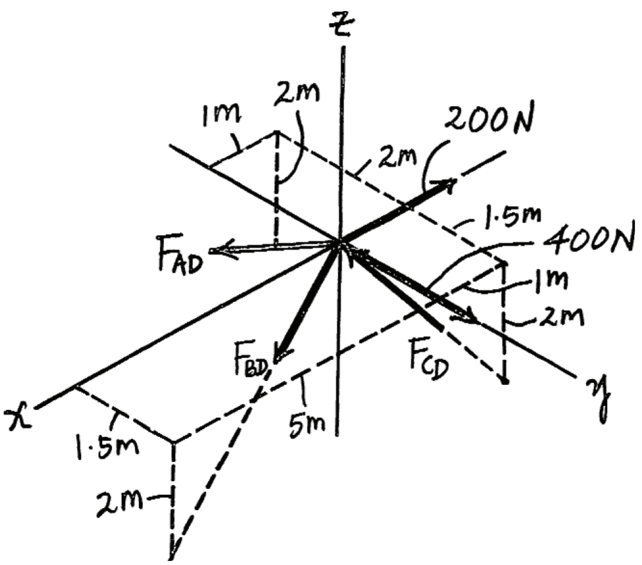
$F_{BC} = 148 \text{ N (T)}$ **Ans.**

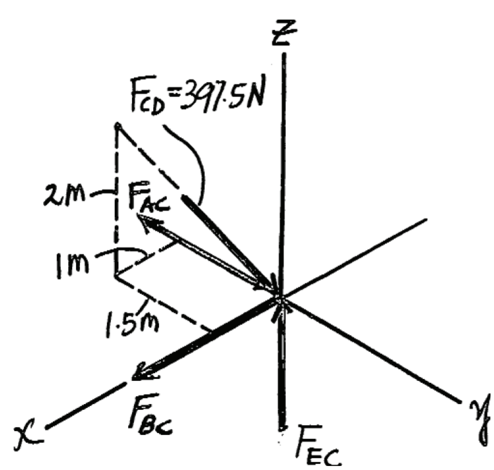
$\Sigma F_y = 0;\qquad \frac{1.5}{\sqrt{7.25}}(397.5) - F_{AC} = 0$

$F_{AC} = 221 \text{ N (T)}$ **Ans.**

$\Sigma F_z = 0;\qquad F_{EC} - \frac{2}{\sqrt{7.25}}(397.5) = 0$

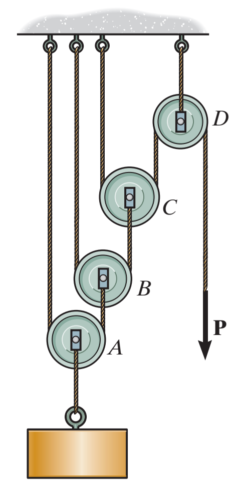
$F_{EC} = 295 \text{ N (C)}$ **Ans.**





6–61.

Determine the force **P** required to hold the 100-lb weight in equilibrium.



SOLUTION

Equations of Equilibrium: Applying the force equation of equilibrium along the y axis of pulley A on the free - body diagram, Fig. a,

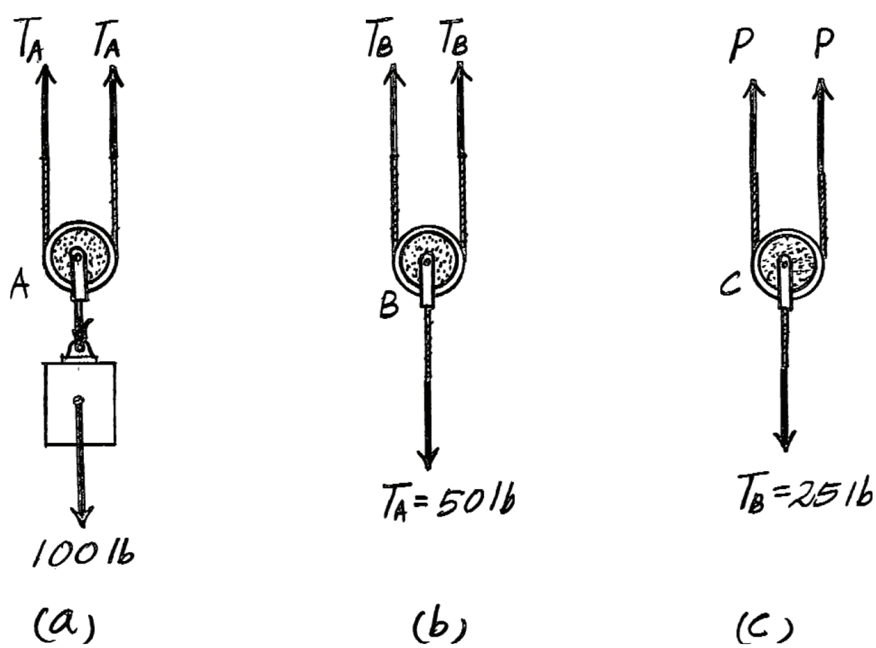
$+\uparrow \Sigma F_y = 0; \qquad 2T_A - 100 = 0 \qquad T_A = 50 \text{ lb}$

Applying $\Sigma F_y = 0$ to the free - body diagram of pulley B, Fig. b,

$+\uparrow \Sigma F_y = 0; \qquad 2T_B - 50 = 0 \qquad T_B = 25 \text{ lb}$

From the free - body diagram of pulley C, Fig. c,

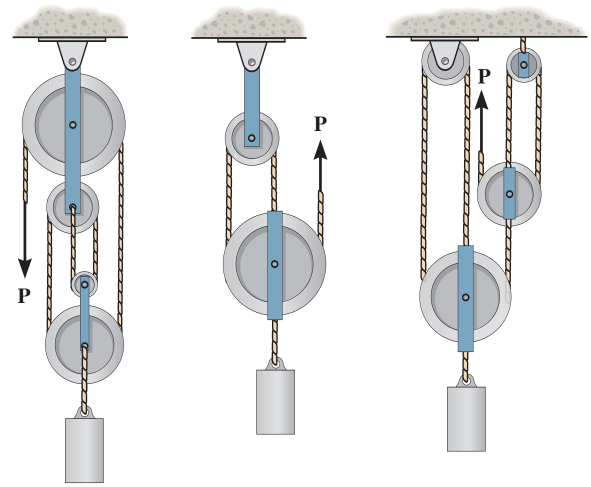
$+\uparrow \Sigma F_y = 0; \qquad 2P - 25 = 0 \qquad P = 12.5 \text{ lb}$ **Ans.**



Ans:
$P = 12.5$ lb

6–62.

In each case, determine the force **P** required to maintain equilibrium. The block weighs 100 lb.



(a) (b) (c)

SOLUTION

Equations of Equilibrium:

a) $+\uparrow \Sigma F_y = 0;\qquad 4P - 100 = 0$

$$P = 25.0 \text{ lb}$$ **Ans.**

b) $+\uparrow \Sigma F_y = 0;\qquad 3P - 100 = 0$

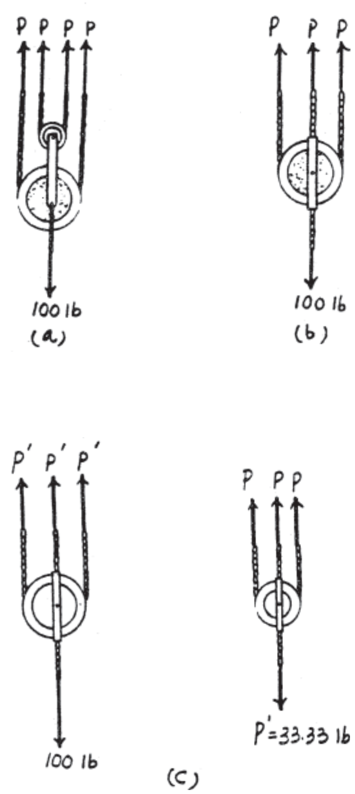
$$P = 33.3 \text{ lb}$$ **Ans.**

c) $+\uparrow \Sigma F_y = 0;\qquad 3P' - 100 = 0$

$$P' = 33.33 \text{ lb}$$

$+\uparrow \Sigma F_y = 0;\qquad 3P - 33.33 = 0$

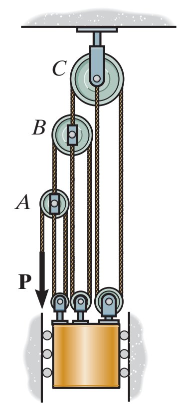
$$P = 11.1 \text{ lb}$$ **Ans.**



Ans:
$P = 25.0$ lb
$P = 33.3$ lb
$P = 11.1$ lb

6–63.

Determine the force **P** required to hold the 50-kg mass in equilibrium.



SOLUTION

Equations of Equilibrium: Applying the force equation of equilibrium along the y axis of each pulley.

$+\uparrow \Sigma F_y = 0; \qquad R - 3P = 0; \qquad R = 3P$

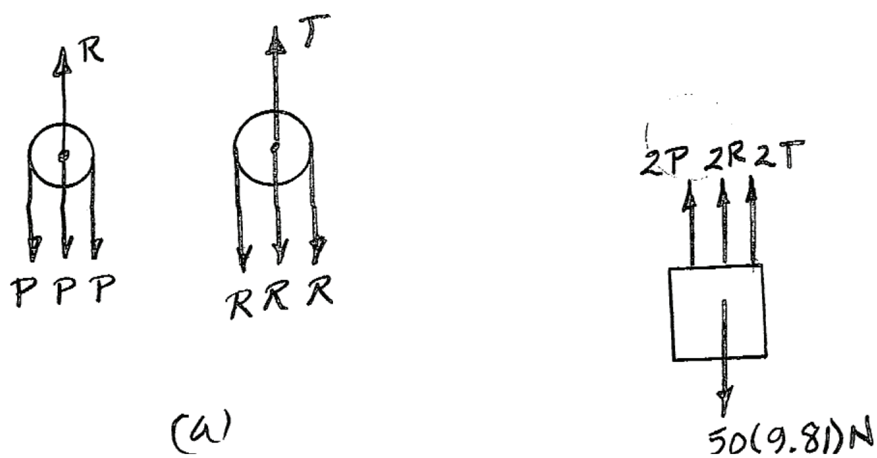
$+\uparrow \Sigma F_y = 0; \qquad T - 3R = 0; \qquad T = 3R = 9P$

$+\uparrow \Sigma F_y = 0; \qquad 2P + 2R + 2T - 50(9.81) = 0$ **Ans.**

Substituting Eqs.(1) and (2) into Eq.(3) and solving for P,

$$2P + 2(3P) + 2(9P) = 50(9.81)$$

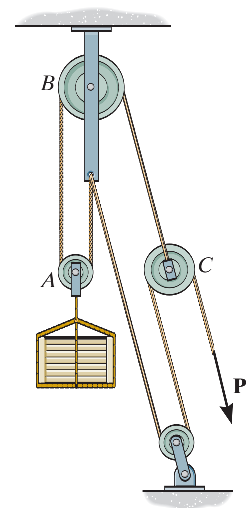
$$P = 18.9 \text{ N}$$ **Ans.**



Ans:
$P = 18.9$ N

***6–64.**

Determine the force **P** required to hold the 150-kg crate in equilibrium.



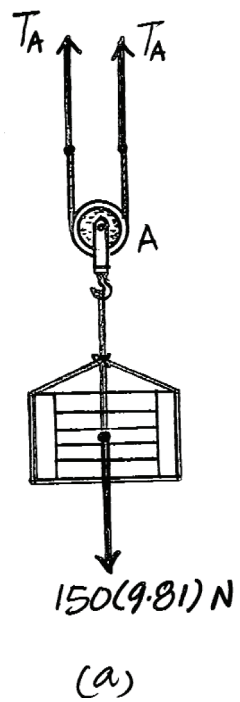
SOLUTION

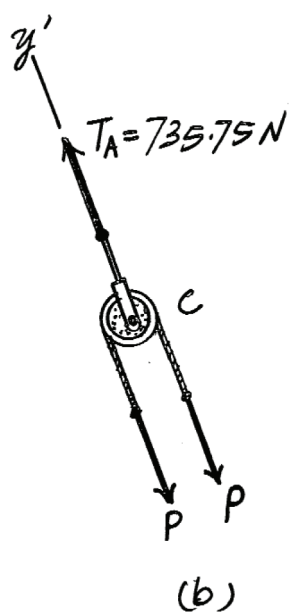
Equations of Equilibrium: Applying the force equation of equilibrium along the y axis of pulley A on the free - body diagram, Fig. a,

$+\uparrow \Sigma F_y = 0; \qquad 2T_A - 150(9.81) = 0 \quad T_A = 735.75\text{ N}$

Using the above result and writing the force equation of equilibrium along the y' axis of pulley C on the free - body diagram in Fig. b,

$\Sigma F_{y'} = 0; \quad 735.75 - 2P = 0 \qquad P = 367.88\text{ N} = 368\text{ N}$ **Ans.**

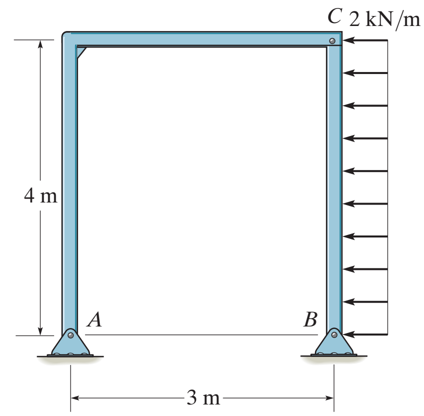




Ans:
$P = 368\text{ N}$

6–65.

Determine the horizontal and vertical components of force that pins A and B exert on the frame.



SOLUTION

Free Body Diagram. The frame will be dismembered into members BC and AC. The solution will be very much simplified if one recognizes that member AC is a two force member. The *FBDs* of member BC and pin A are shown in Figs. *a* and *b*, respectively.

Equations of Equilibrium. Consider the equilibrium of member BC, Fig. *a*,

$$\circlearrowleft + \Sigma M_B = 0; \qquad 2(4)(2) - F_{AC}\left(\frac{3}{5}\right)(4) = 0 \quad F_{AC} = 6.6667 \text{ kN}$$

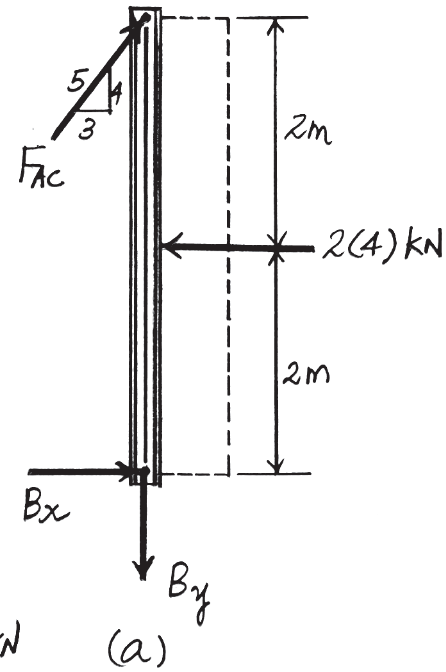
$$\circlearrowleft + \Sigma M_C = 0; \qquad B_x(4) - 2(4)(2) = 0 \quad B_x = 4.00 \text{ kN} \qquad \textbf{Ans.}$$

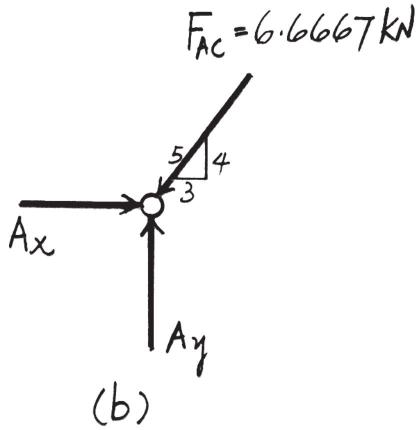
$$+\uparrow \Sigma F_y = 0; \qquad 6.6667\left(\frac{4}{5}\right) - B_y = 0 \quad B_y = 5.333 \text{ kN} = 5.33 \text{ kN} \qquad \textbf{Ans.}$$

Then, the equilibrium of pin A gives

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - 6.6667\left(\frac{3}{5}\right) = 0 \quad A_x = 4.00 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 6.6667\left(\frac{4}{5}\right) = 0 \quad A_y = 5.333 \text{ kN} = 5.33 \text{ kN} \qquad \textbf{Ans.}$$

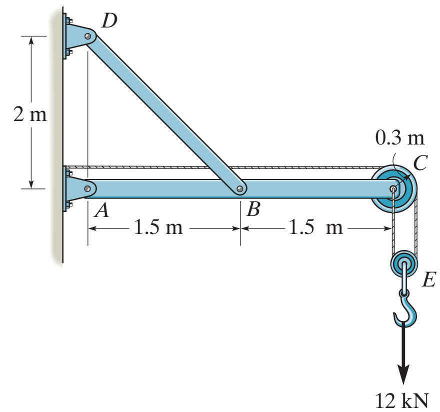




Ans:
$B_x = 4.00$ kN
$B_y = 5.33$ kN
$A_x = 4.00$ kN
$A_y = 5.33$ kN

6–66.

Determine the horizontal and vertical components of force at pins A and D.



SOLUTION

Free Body Diagram. The assembly will be dismembered into member AC, BD and pulley E. The solution will be very much simplified if one recognizes that member BD is a two force member. The FBD of pulley E and member AC are shown in Fig. a and b respectively.

Equations of Equilibrium. Consider the equilibrium of pulley E, Fig. a,

$$+\uparrow\Sigma F_y = 0; \qquad 2T - 12 = 0 \quad T = 6.00 \text{ kN}$$

Then, the equilibrium of member AC gives

$$\circlearrowleft +\Sigma M_A = 0; \qquad F_{BD}\left(\frac{4}{5}\right)(1.5) + 6(0.3) - 6(3) - 6(3.3) = 0$$

$$F_{BD} = 30.0 \text{ kN}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x - 30.0\left(\frac{3}{5}\right) - 6 = 0 \qquad A_x = 24.0 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \qquad 30.0\left(\frac{4}{5}\right) - 6 - 6 - A_y = 0 \quad A_y = 12.0 \text{ kN} \qquad \textbf{Ans.}$$

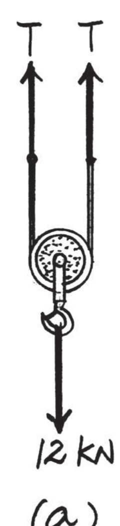
Thus,

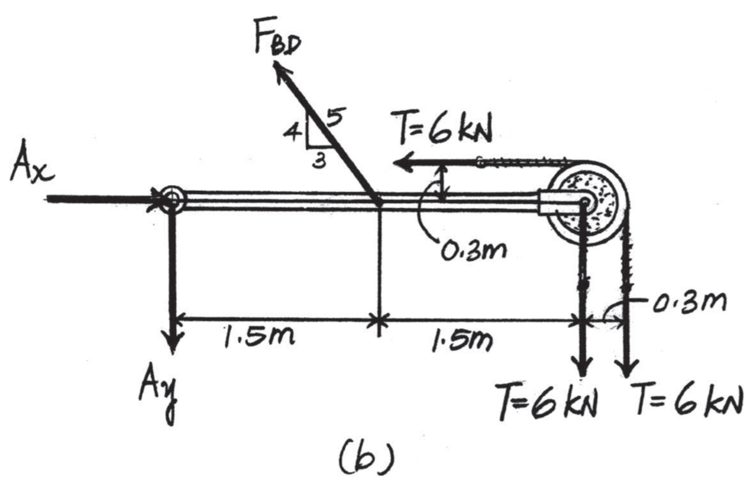
$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{24.0^2 + 12.0^2} = 26.83 \text{ kN} = 26.8 \text{ kN}$$

$$F_B = F_{BD} = 30.0 \text{ kN}$$

$$D_x = \frac{3}{5}(30) = 18.0 \text{ kN} \qquad \textbf{Ans.}$$

$$D_y = \frac{4}{5}(30) = 24.0 \text{ kN} \qquad \textbf{Ans.}$$

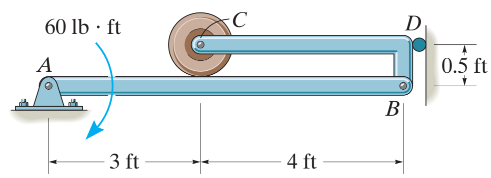




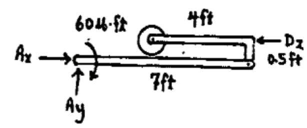
Ans:
$A_x = 24.0$ kN
$A_y = 12.0$ kN
$D_x = 18.0$ kN
$D_y = 24.0$ kN

6–67.

Determine the force that the smooth roller C exerts on member AB. Also, what are the horizontal and vertical components of reaction at pin A? Neglect the weight of the frame and roller.



SOLUTION

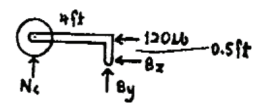


$\zeta + \Sigma M_A = 0; \quad -60 + D_x(0.5) = 0$

$D_x = 120 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 120 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad A_y = 0$ **Ans.**



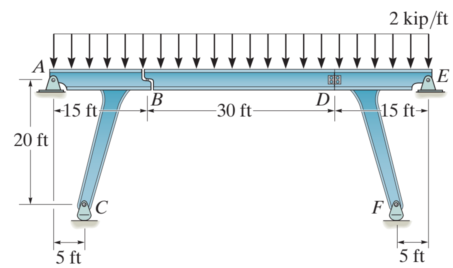
$\zeta + \Sigma M_B = 0; \quad -N_C(4) + 120(0.5) = 0$

$N_C = 15.0 \text{ lb}$ **Ans.**

Ans:
$A_x = 120 \text{ lb}$
$A_y = 0$
$N_C = 15.0 \text{ lb}$

***6–68.**

The bridge frame consists of three segments which can be considered pinned at A, D, and E, rocker supported at C and F, and roller supported at B. Determine the horizontal and vertical components of reaction at all these supports due to the loading shown.



SOLUTION

For segment BD:

$\circlearrowleft + \Sigma M_D = 0; \quad 2(30)(15) - B_y(30) = 0 \qquad B_y = 30 \text{ kip}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad D_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad D_y + 30 - 2(30) = 0 \qquad D_y = 30 \text{ kip}$ **Ans.**

For segment ABC:

$\circlearrowleft + \Sigma M_A = 0; \quad C_y(5) - 2(15)(7.5) - 30(15) = 0 \qquad C_y = 135\text{kip}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0$ **Ans.**

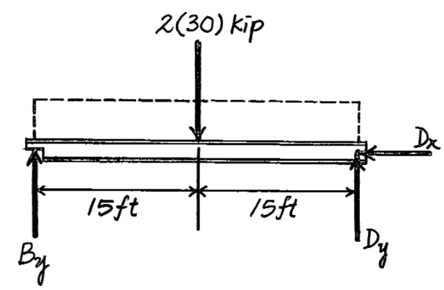
$+\uparrow \Sigma F_y = 0; \quad -A_y + 135 - 2(15) - 30 = 0 \qquad A_y = 75 \text{ kip}$ **Ans.**

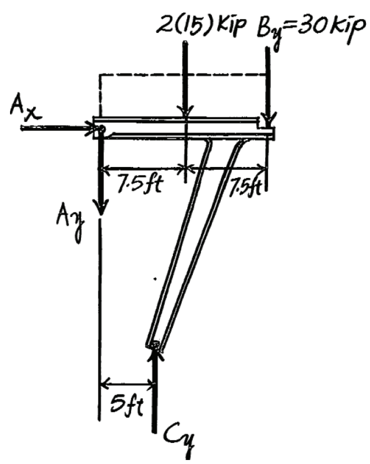
For segment DEF:

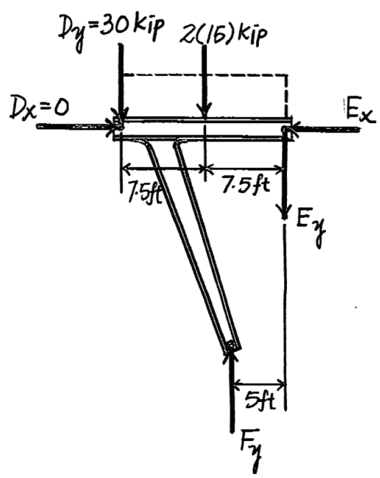
$\circlearrowleft + \Sigma M_g = 0; \quad -F_y(5) + 2(15)(7.5) + 30(15) = 0 \qquad F_y = 135 \text{ kip}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad E_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad -E_y + 135 - 2(15) - 30 = 0 \qquad E_y = 75 \text{ kip}$ **Ans.**



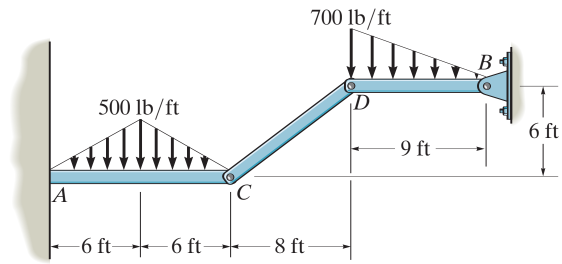




Ans:
$B_y = 30$ kip
$D_x = 0$
$D_y = 30$ kip
$C_y = 135$ kip
$A_y = 75$ kip
$F_y = 135$ kip
$E_x = 0$
$E_y = 75$ kip

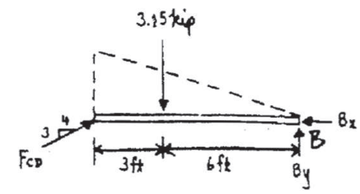
6–69.

Determine the reactions at supports A and B.



SOLUTION

Member DB:



$$\zeta + \Sigma M_B = 0; \qquad 3.15\,(6) - F_{CD}\left(\frac{3}{5}\right)(9) = 0$$

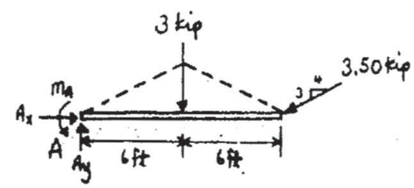
$$F_{CD} = 3.50 \text{ kip}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -B_x + 3.50\left(\frac{4}{5}\right) = 0$$

$$B_x = 2.80 \text{ kip} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y - 3.15 + 3.50\left(\frac{3}{5}\right) = 0$$

$$B_y = 1.05 \text{ kip} \qquad \textbf{Ans.}$$



Member AC:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - 3.50\left(\frac{4}{5}\right) = 0$$

$$A_x = 2.80 \text{ kip} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 3 - 3.50\left(\frac{3}{5}\right) = 0$$

$$A_y = 5.10 \text{ kip} \qquad \textbf{Ans.}$$

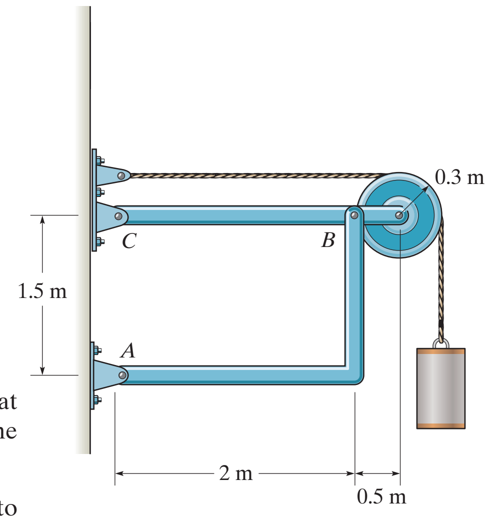
$$\zeta + \Sigma M_A = 0; \qquad M_A - 3(6) - 3.50\left(\frac{3}{5}\right)(12) = 0$$

$$M_A = 43.2 \text{ kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

Ans:
$B_x = 2.80$ kip
$B_y = 1.05$ kip
$A_x = 2.80$ kip
$A_y = 5.10$ kip
$M_A = 43.2$ kip · ft

6–70.

Determine the horizontal and vertical components of force at pins B and C. The suspended cylinder has a mass of 75 kg.



SOLUTION

Free Body Diagram. The solution will be very much simplified if one realizes that member AB is a two force member. Also, the tension in the cable is equal to the weight of the cylinder and is constant throughout the cable.

Equations of Equilibrium. Consider the equilibrium of member BC by referring to its FBD, Fig. a,

$$\zeta+\Sigma M_C = 0; \qquad F_{AB}\left(\frac{3}{5}\right)(2) + 75(9.81)(0.3) - 75(9.81)(2.8) = 0$$

$$F_{AB} = 1532.81 \text{ N}$$

$$\zeta+\Sigma M_B = 0; \qquad C_y(2) + 75(9.81)(0.3) - 75(9.81)(0.8) = 0$$

$$C_y = 183.94 \text{ N} = 184 \text{ N} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 1532.81\left(\frac{4}{5}\right) - 75(9.81) - C_x = 0$$

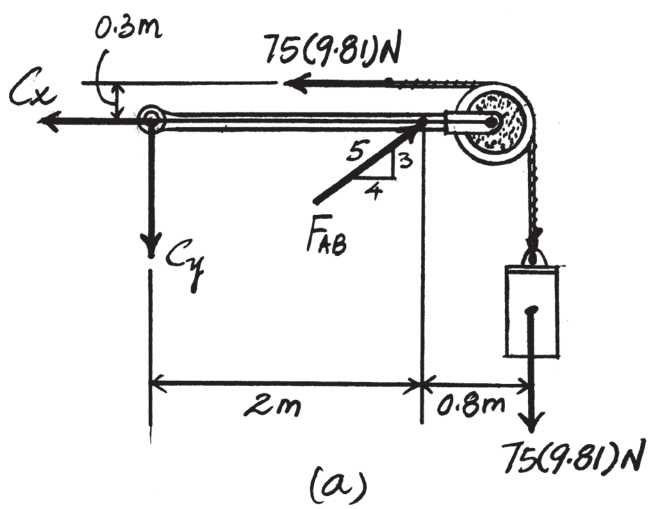
$$C_x = 490.5 \text{ N} \qquad \textbf{Ans.}$$

Thus,

$$F_B = F_{AB} = 1532.81 \text{ N}$$

$$B_x = \frac{4}{5}(1532.81) = 1226.25 \text{ N} = 1.23 \text{ kN} \qquad \textbf{Ans.}$$

$$B_y = \frac{3}{5}(1532.81) = 919.69 \text{ N} = 920 \text{ kN} \qquad \textbf{Ans.}$$

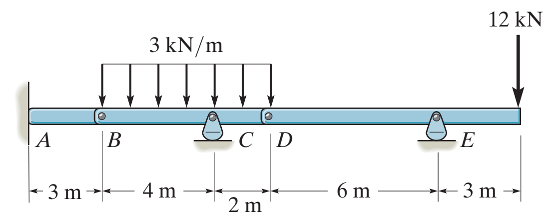


(a)

Ans:
$C_y = 184$ N
$C_x = 490.5$ N
$B_x = 1.23$ kN
$B_y = 920$ kN

6–71.

Determine the reactions at the supports A, C, and E of the compound beam.



SOLUTION

Free Body Diagram. The compound beam is being dismembered into members AB, BD and DE of which their respective $FBDs$ are shown in Fig. a, b and c.

Equations of Equilibrium. Equilibrium of member DE will be considered first by referring to Fig. c.

$\circlearrowleft + \Sigma M_D = 0;\quad N_E(6) - 12(9) = 0 \quad N_E = 18.0\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0;\quad D_y(6) - 12(3) = 0 \quad D_y = 6.00\text{ kN}$

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad D_x = 0$

Next, member BD, Fig. b.

$\circlearrowleft + \Sigma M_C = 0;\quad 6.00(2) + 3(6)(1) - B_y(4) = 0 \quad B_y = 7.50\text{ kN}$

$\circlearrowleft + \Sigma M_B = 0;\quad N_C(4) + 6.00(6) - 3(6)(3) = 0 \quad N_C = 4.50\text{ kN}$ **Ans.**

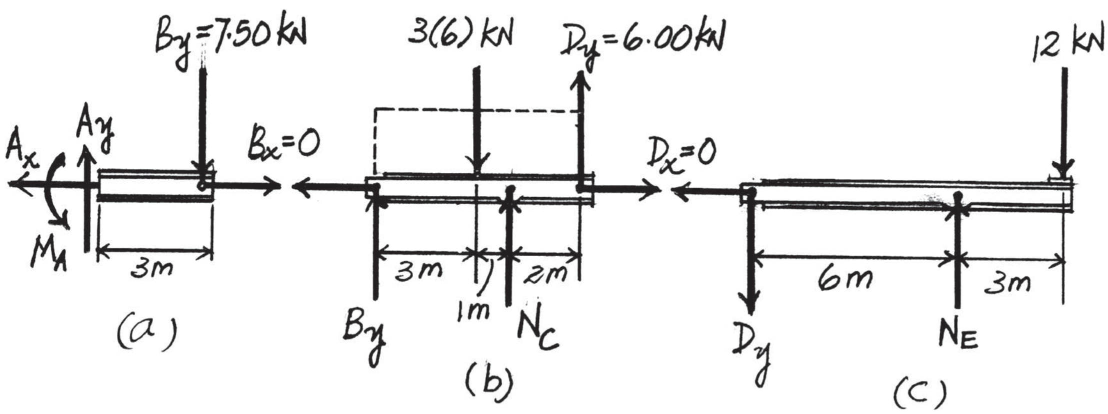
$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad B_x = 0$

Finally, member AB, Fig. a

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad A_x = 0$ **Ans.**

$+\uparrow\Sigma F_y = 0;\quad A_y - 7 - 50 = 0 \quad A_y = 7.50\text{ kN}$ **Ans.**

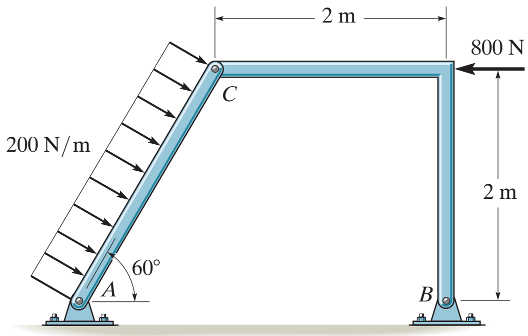
$\circlearrowleft + \Sigma M_A = 0;\quad M_A - 7.50(3) = 0 \quad M_A = 22.5\text{ kN}\cdot\text{m}$ **Ans.**



Ans:
$N_E = 18.0\text{ kN}$
$N_C = 4.50\text{ kN}$
$A_x = 0$
$A_y = 7.50\text{ kN}$
$M_A = 22.5\text{ kN}\cdot\text{m}$

***6–72.**

Determine the resultant force at pins A, B, and C on the three-member frame.



SOLUTION

Free Body Diagram. The frame is being dismembered into members AC and BC of which their respective $FBDs$ are shown in Fig. a and b.

Equations of Equilibrium. Write the moment equation of equilibrium about point A for member AC, Fig. a and point B for member BC, Fig. b.

$$\circlearrowleft + \Sigma M_A = 0; \quad C_y\left(\frac{2}{\tan 60^\circ}\right) + C_x(2) - 200\left(\frac{2}{\sin 60^\circ}\right)\left(\frac{1}{\sin 60^\circ}\right) = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_B = 0; \quad C_y(2) - C_x(2) + 800(2) = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$C_y = -338.12 \text{ N} \quad C_x = 461.88 \text{ N}$$

The negative sign indicates that $\mathbf{C}_y$ acts in the sense opposite to that shown in the FBD write the force equation of equilibrium for member AC, Fig. a,

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x + 200\left(\frac{2}{\sin 60^\circ}\right)\sin 60^\circ - 461.88 = 0 \qquad A_x = 61.88 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad A_y + (-338.12) - 200\left(\frac{2}{\sin 60^\circ}\right)\cos 60^\circ = 0 \quad A_y = 569.06 \text{ N}$$

Also, for member BC, Fig. b

$$\xrightarrow{+} \Sigma F_x = 0; \quad B_x + 461.88 - 800 = 0 \quad B_x = 338.12 \text{ N}$$

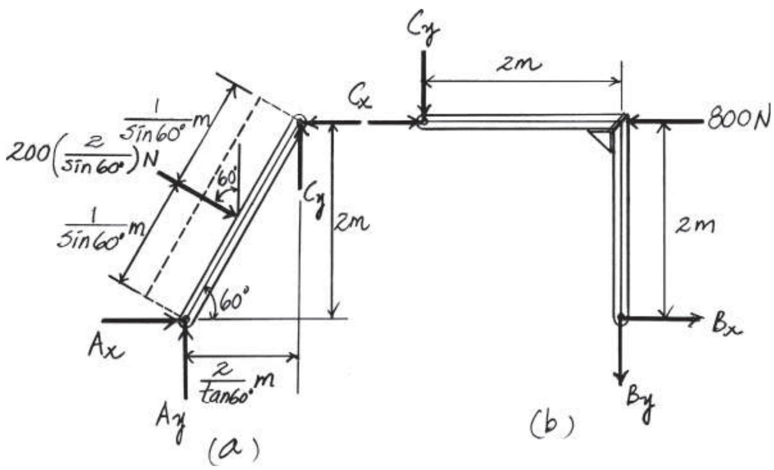
$$+\uparrow \Sigma F_y = 0; \quad -B_y - (-338.12) = 0 \quad B_y = 338.12 \text{ N}$$

Thus,

$$F_C = \sqrt{C_x^2 + C_y^2} = \sqrt{461.88^2 + (-338.12)^2} = 572.41 \text{ N} = 572 \text{ N} \qquad \textbf{Ans.}$$

$$F_A = \sqrt{A_x^2 + A_y^2} = \sqrt{61.88^2 + 569.06^2} = 572.41 \text{ N} = 572 \text{ N} \qquad \textbf{Ans.}$$

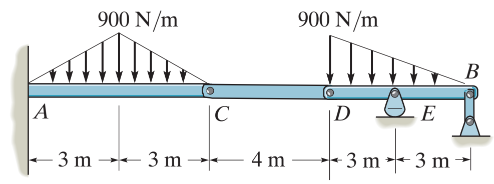
$$F_B = \sqrt{B_x^2 + B_y^2} = \sqrt{338.12^2 + 338.12^2} = 478.17 \text{ N} = 478 \text{ N} \qquad \textbf{Ans.}$$



Ans:
$F_C = 572$ N
$F_A = 572$ N
$F_B = 478$ N

6–73.

Determine the reactions at the supports at A, E, and B of the compound beam.



SOLUTION

Free Body Diagram. The solution will be very much simplified if one realizes that member CD is a two force member.

Equation of Equilibrium. Consider the equilibrium of member BD, Fig. b

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{CD} = 0$

$\circlearrowleft + \Sigma M_B = 0; \quad \frac{1}{2}(900)(6)(4) - N_E(3) = 0 \quad N_E = 3600 \text{ N} = 3.60 \text{ kN}$ **Ans.**

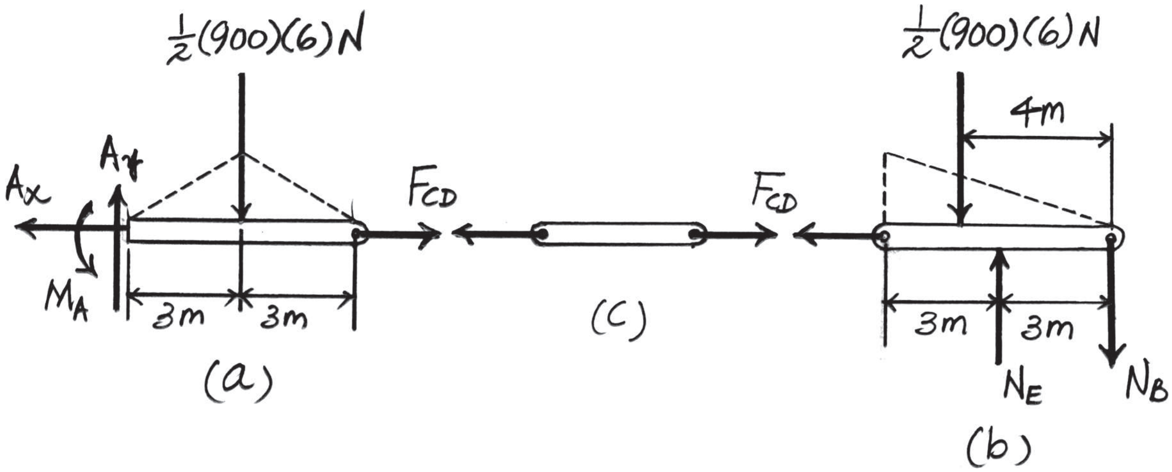
$\circlearrowleft + \Sigma M_E = 0; \quad \frac{1}{2}(900)(6)(1) - N_B(3) = 0 \quad N_B = 900 \text{ N}$ **Ans.**

Then the equilibrium of member AC gives

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad A_y - \frac{1}{2}(900)(6) = 0 \quad A_y = 2700 \text{ N} = 2.70 \text{ kN}$ **Ans.**

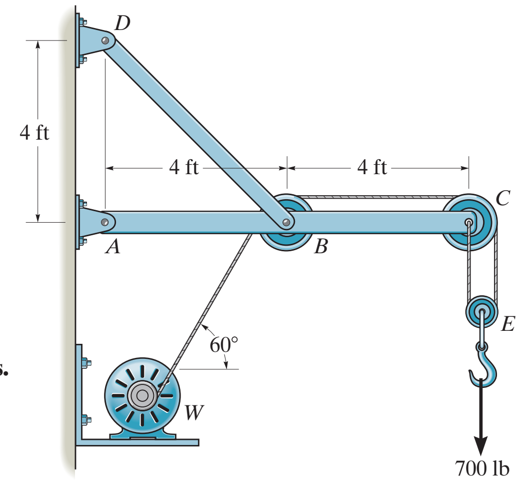
$\circlearrowleft + \Sigma M_A = 0; \quad M_A - \frac{1}{2}(900)(6)(3) = 0 \quad M_A = 8100 \text{ N} \cdot \text{m} = 8.10 \text{ kN} \cdot \text{m}$ **Ans.**



Ans:
$N_E = 3.60$ kN
$N_B = 900$ N
$A_x = 0$
$A_y = 2.70$ kN
$M_A = 8.10$ kN $\cdot$ m

6–74.

The wall crane supports a load of 700 lb. Determine the horizontal and vertical components of reaction at the pins A and D. Also, what is the force in the cable at the winch W?

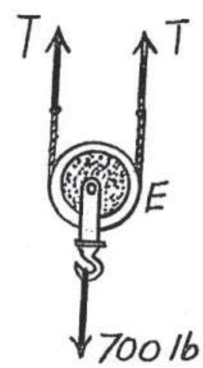


SOLUTION

Pulley E:

$+\uparrow \Sigma F_y = 0;$ $\quad 2T - 700 = 0$

$T = 350 \text{ lb}$ **Ans.**



Member ABC:

$\circlearrowleft + \Sigma M_A = 0;$ $\quad T_{BD} \sin 45°(4) - 350 \sin 60°(4) \cdot 700\,(8) = 0$

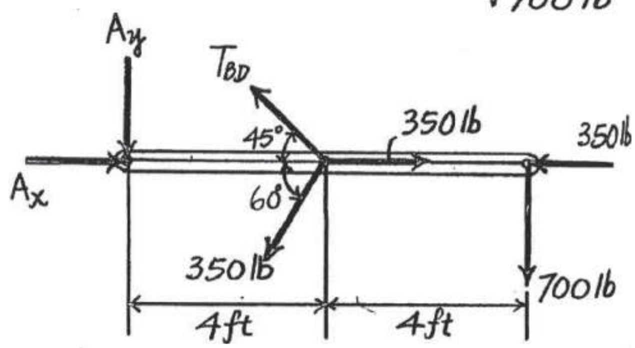
$T_{BD} = 2409 \text{ lb}$

$+\uparrow \Sigma F_y = 0;$ $\quad -A_y + 2409 \sin 45° - 350 \sin 60° - 700 = 0$

$A_y = 700 \text{ lb}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad -A_y - 2409 \cos 45° - 350 \cos 60° + 350 - 350 = 0$

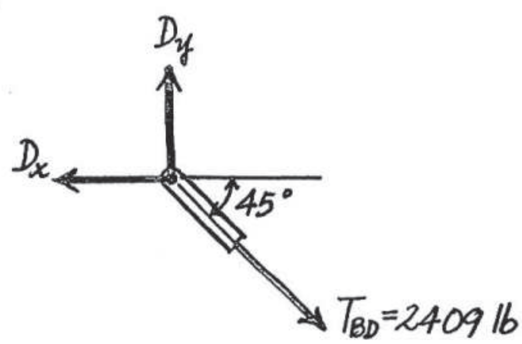
$A_x = 1.88 \text{ kip}$ **Ans.**



At D:

$D_x = 2409 \cos 45° = 1703.1 \text{ lb} = 1.70 \text{ kip}$ **Ans.**

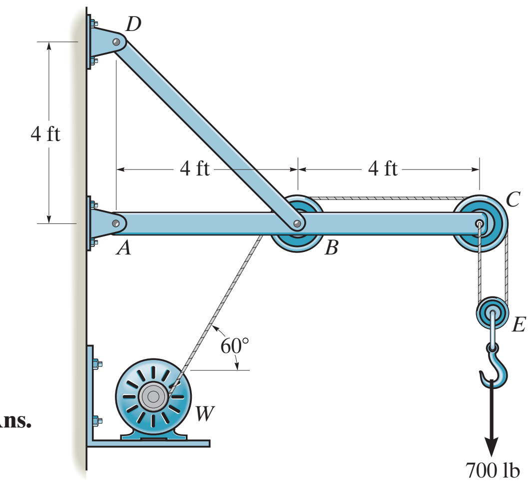
$D_y = 2409 \sin 45° = 1.70 \text{ kip}$ **Ans.**



Ans:
$T = 350 \text{ lb}$
$A_y = 700 \text{ lb}$
$A_x = 1.88 \text{ kip}$
$D_x = 1.70 \text{ kip}$
$D_y = 1.70 \text{ kip}$

6–75.

The wall crane supports a load of 700 lb. Determine the horizontal and vertical components of reaction at the pins A and D. Also, what is the force in the cable at the winch W? The jib ABC has a weight of 100 lb and member BD has a weight of 40 lb. Each member is uniform and has a center of gravity at its center.



SOLUTION

Pulley E:

$+\uparrow \Sigma F_y = 0;$ $\quad 2T - 700 = 0$

$T = 350 \text{ lb}$ **Ans.**

Member ABC:

$\circlearrowleft + \Sigma M_A = 0;$ $\quad B_y(4) - 700(8) - 100(4) - 350 \sin 60° (4) = 0$

$B_y = 1803.1 \text{ lb}$

$+\uparrow \Sigma F_y = 0;$ $\quad -A_y - 350 \sin 60° - 100 - 700 + 1803.1 = 0$

$A_y = 700 \text{ lb}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x - 350 \cos 60° - B_x + 350 - 350 = 0$

$A_x = B_x + 175$ **(1)**

Member DB:

$\circlearrowleft + \Sigma M_D = 0;$ $\quad -40(2) - 1803.1(4) + B_x(4) = 0$

$B_x = 1823.1 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad -D_x + 1823.1 = 0$

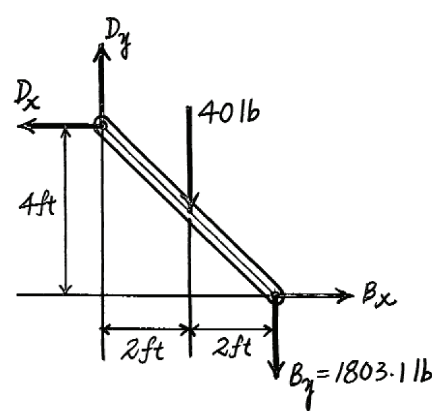
$D_x = 1.82 \text{ kip}$ **Ans.**

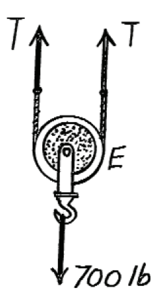
$+\uparrow \Sigma F_y = 0;$ $\quad D_y - 40 - 1803.1 = 0$

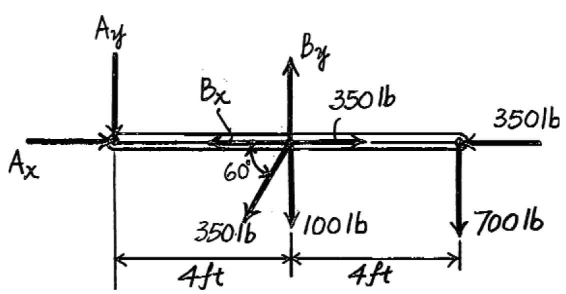
$D_y = 1843.1 = 1.84 \text{ kip}$ **Ans.**

From Eq. (1)

$A_x = 2.00 \text{ kip}$ **Ans.**



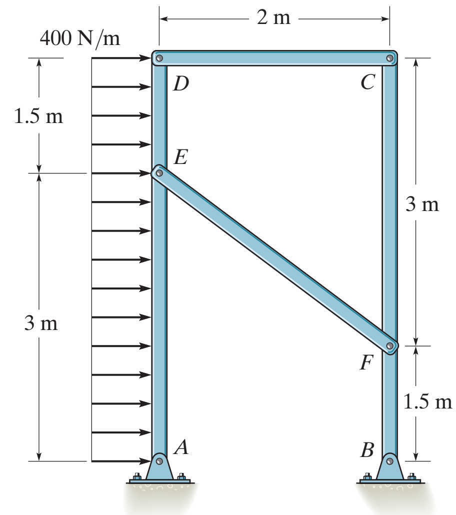




Ans:
$T = 350 \text{ lb}$
$A_y = 700 \text{ lb}$
$D_x = 1.82 \text{ kip}$
$D_y = 1.84 \text{ kip}$
$A_x = 2.00 \text{ kip}$

***6–76.**

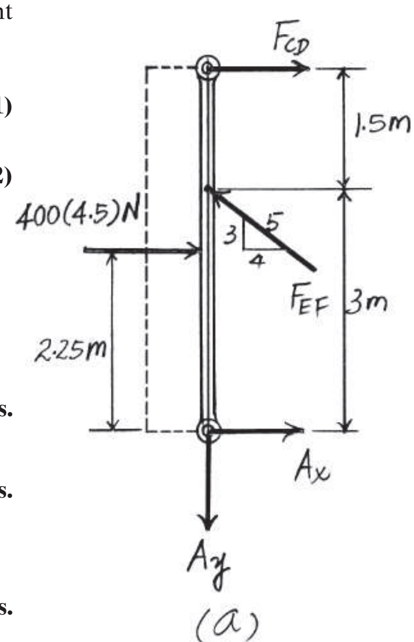
Determine the horizontal and vertical components of force which the pins at A and B exert on the frame.



SOLUTION

Free Body Diagram. The frame will be dismembered into members AD, EF, CD and BC. The solution will be very much simplified if one realizes that members CD and EF are two force member. Therefore, only the FBD of members AD and BC, Fig. a and b respectively, need to be drawn

Equations of Equilibrium. Write the moment equations of equilibrium about point A for member AD, Fig. a, and point B for member BC, Fig. b.



$\zeta+\Sigma M_A = 0;\quad F_{EF}\left(\frac{4}{5}\right)(3) - F_{CD}(4.5) - 400(4.5)(2.25) = 0$ **(1)**

$\zeta+\Sigma M_B = 0;\quad -F_{EF}\left(\frac{4}{5}\right)(1.5) + F_{CD}(4.5) = 0$ **(2)**

Solving Eqs. (1) and (2)

$$F_{EF} = 3375\text{ N} \quad F_{CD} = 900\text{ N}$$

Write the force equation of equilibrium for member AD, Fig. a,

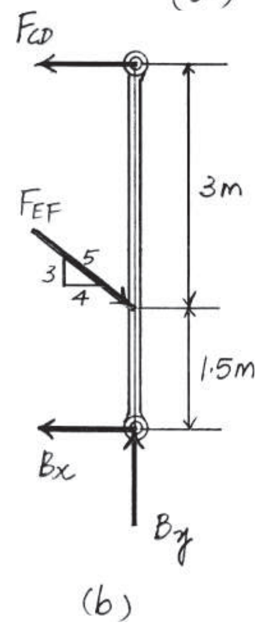
$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad A_x + 400(4.5) + 900 - 3375\left(\frac{4}{5}\right) = 0 \quad A_x = 0$ **Ans.**

$+\uparrow\Sigma F_y = 0;\quad 3375\left(\frac{3}{5}\right) - A_y = 0 \quad A_y = 2025\text{ N} = 2.025\text{ kN}$ **Ans.**

Also, for member BC, Fig. b

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad 3375\left(\frac{4}{5}\right) - 900 - B_x = 0 \quad B_x = 1800\text{ N} = 1.80\text{ kN}$ **Ans.**

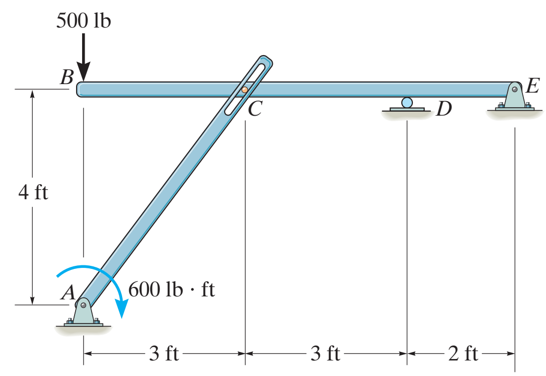
$+\uparrow\Sigma F_y = 0;\quad B_y - 3375\left(\frac{3}{5}\right) = 0 \quad B_y = 2025\text{ N} = 2.025\text{ kN}$ **Ans.**



Ans:
$A_x = 0$
$A_y = 2.025$ kN
$B_x = 1.80$ kN
$B_y = 2.025$ kN

6–77.

The two-member structure is connected at C by a pin, which is fixed to BDE and passes through the smooth slot in member AC. Determine the horizontal and vertical components of reaction at the supports.



SOLUTION

Member AC:

$\circlearrowleft + \Sigma M_A = 0; \qquad N_C\,(5) - 600 = 0$

$N_C = 120 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - 120\left(\frac{4}{5}\right) = 0$

$A_x = 96 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad -A_y + 120\left(\frac{3}{5}\right) = 0$

$A_y = 72 \text{ lb}$ **Ans.**

Member BDE:

$\circlearrowleft + \Sigma M_g = 0; \qquad 500\,(8) + 120\left(\frac{3}{5}\right)(5) - D_y\,(2) = 0$

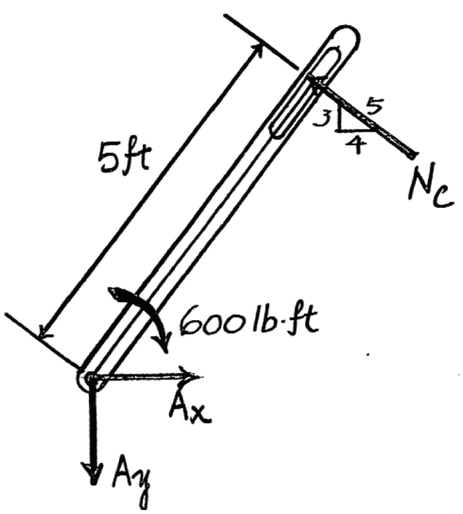
$D_y = 2180 \text{ lb} = 2.18 \text{ kip}$ **Ans.**

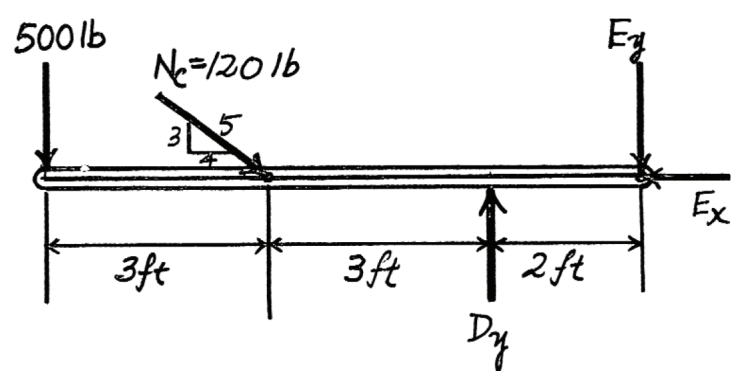
$\xrightarrow{+} \Sigma F_x = 0; \qquad -E_x + 120\left(\frac{4}{5}\right) = 0$

$E_x = 96 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad -500 - 120\left(\frac{3}{5}\right) + 2180 - E_y = 0$

$E_y = 1608 \text{ lb} = 1.61 \text{ kip}$ **Ans.**

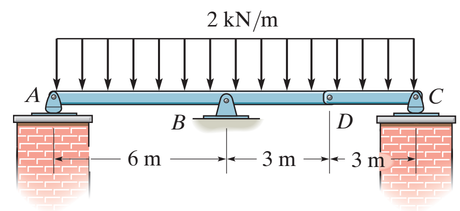




Ans:
$A_x = 96 \text{ lb}$
$A_y = 72 \text{ lb}$
$D_y = 2.18 \text{ kip}$
$E_x = 96.0 \text{ lb}$
$E_y = 1.61 \text{ kip}$

6–78.

The compound beam is pin supported at B and supported by rockers at A and C. There is a hinge (pin) at D. Determine the reactions at the supports.



SOLUTION

Free Body Diagram. The compound beam will be dismembered into members ABD and CD of which their respective FBD are shown in Fig. a and b.

Equations of Equilibrium. First, consider the equilibrium of member CD, Fig. b,

$\circlearrowleft + \Sigma M_D = 0;\quad N_C(3) - 2(3)(1.5) = 0 \quad N_C = 3.00\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;\quad 2(3)(1.5) - D_y(3) = 0 \quad D_y = 3.00\text{ kN}$ **Ans.**

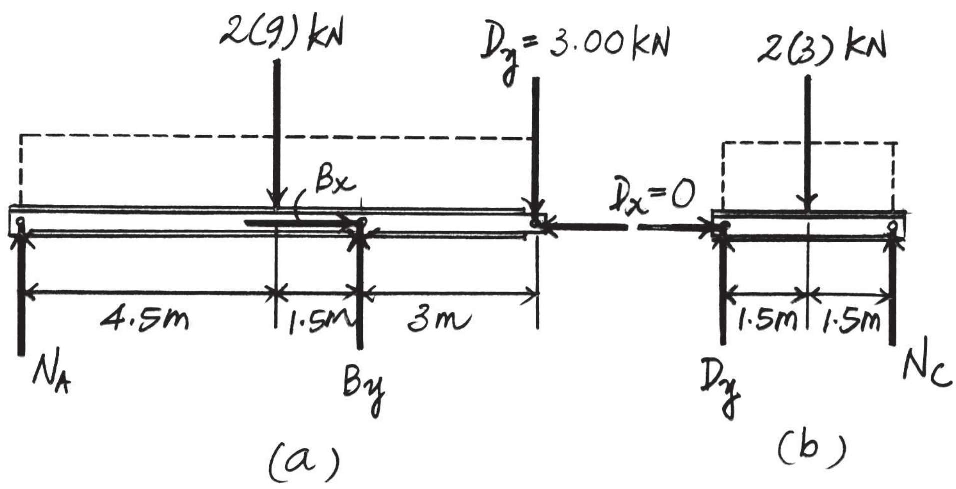
$\xrightarrow{+} \Sigma F_x = 0;\quad D_x = 0$

Next, the equilibrium of member ABD gives,

$\circlearrowleft + \Sigma M_B = 0;\quad 2(9)(1.5) - 3.00(3) - N_A(6) = 0 \quad N_A = 3.00\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0;\quad B_y(6) - 2(9)(4.5) - 3.00(9) = 0 \quad B_y = 18.0\text{ kN}$ **Ans.**

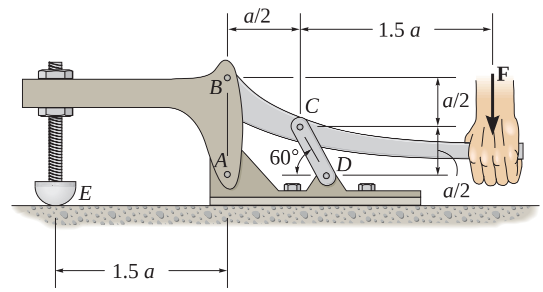
$\xrightarrow{+} \Sigma F_x = 0;\quad B_x = 0$ **Ans.**



Ans:
$N_C = 3.00\text{ kN}$
$N_A = 3.00\text{ kN}$
$B_y = 18.0\text{ kN}$
$B_x = 0$

6–79.

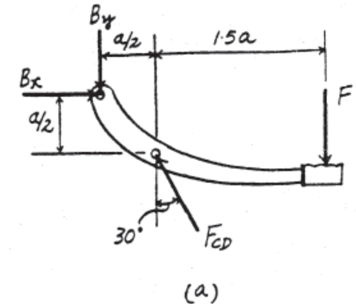
The toggle clamp is subjected to a force **F** at the handle. Determine the vertical clamping force acting at E.



SOLUTION

Free Body Diagram: The solution for this problem will be simplified if one realizes that member CD is a two force member.

Equations of Equilibrium: From FBD (a),



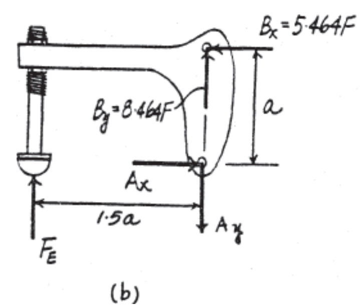
$$\circlearrowleft + \Sigma M_B = 0; \qquad F_{CD}\cos 30°\left(\frac{a}{2}\right) - F_{CD}\sin 30°\left(\frac{a}{2}\right) - F(2a) = 0$$

$$F_{CD} = 10.93F$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x - 10.93 \sin 30° = 0$$

$$B_x = 5.464F$$

From (b),



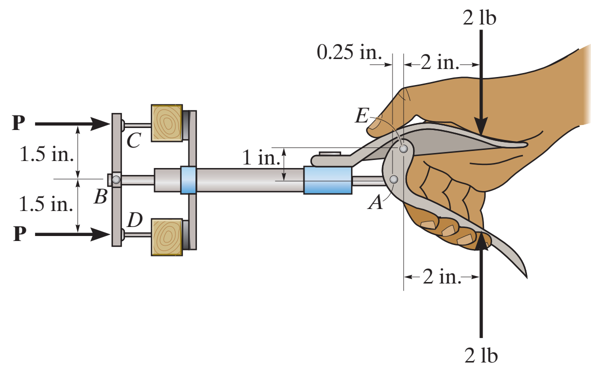
$$\circlearrowleft + \Sigma M_A = 0; \qquad 5.464F(a) - F_E\,(1.5a) = 0$$

$$F_E = 3.64F \qquad \textbf{Ans.}$$

Ans:
$N_C = N_D = 2$ lb

***6–80.**

When a force of 2 lb is applied to the handles of the brad squeezer, it pulls in the smooth rod AB. Determine the force $\mathbf{P}$ exerted on each of the smooth brads at C and D.

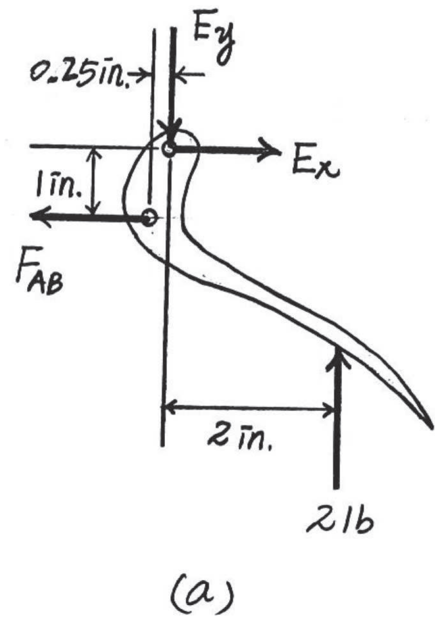


SOLUTION

Equations of Equilibrium: Applying the moment equation of equilibrium about point E to the free-body diagram of the lower handle in Fig. *a*, we have

$$+\Sigma M_E = 0; \qquad 2(2) - F_{AB}(1) = 0$$

$$F_{AB} = 4 \text{ lb}$$



Using the result of F_{AB} and considering the free-body diagram in Fig. *b*,

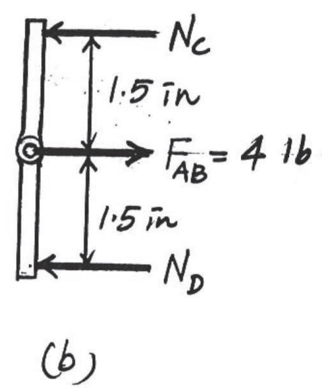
$$+\Sigma M_B = 0; \qquad N_C(1.5) - N_D(1.5) = 0$$

$$N_C = N_D \qquad \textbf{(1)}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 4 - N_C - N_D = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

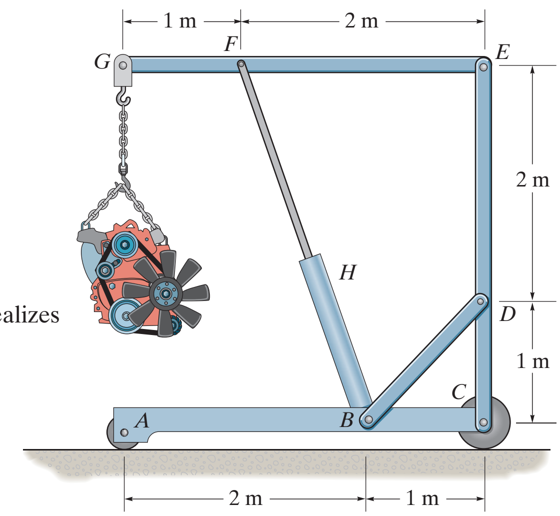
$$N_C = N_D = 2 \text{ lb} \qquad \textbf{Ans.}$$



Ans:
$N_C = N_D = 2 \text{ 1b}$

6–81.

The hoist supports the 125-kg engine. Determine the force the load creates in member DB and in member FB, which contains the hydraulic cylinder H.



SOLUTION

Free Body Diagrams: The solution for this problem will be simplified if one realizes that members FB and DB are two-force members.

Equations of Equilibrium: For FBD(a),

$\circlearrowleft + \Sigma M_E = 0;$ $$1226.25(3) - F_{FB}\left(\frac{3}{\sqrt{10}}\right)(2) = 0$$

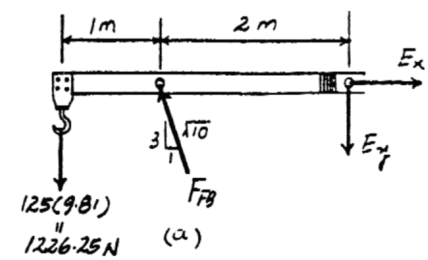
$$F_{FB} = 1938.87 \text{ N} = 1.94 \text{ kN}$$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $$1938.87\left(\frac{3}{\sqrt{10}}\right) - 1226.25 - E_y = 0$$

$$E_y = 613.125\text{N}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $$E_x - 1938.87\left(\frac{1}{\sqrt{10}}\right) = 0$$

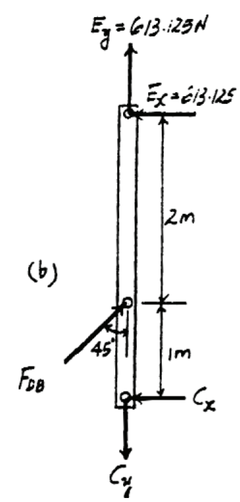
$$E_x = 613.125 \text{ N}$$



From FBD (b),

$\circlearrowleft + \Sigma M_C = 0;$ $$613.125(3) - F_{BD}\sin 45°(1) = 0$$

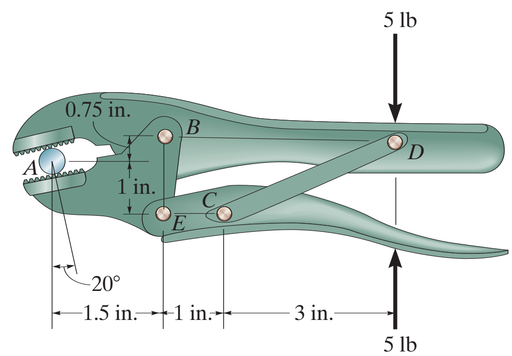
$$F_{BD} = 2601.27 \text{ N} = 2.60 \text{ kN}$$ **Ans.**



Ans:
$F_{FB} = 1.94$ kN
$F_{BD} = 2.60$ kN

6–82.

A 5-lb force is applied to the handles of the vise grip. Determine the compressive force developed on the smooth bolt shank A at the jaws.

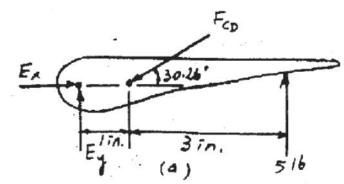


SOLUTION

From FBD (a)

$\circlearrowleft + \Sigma M_E = 0;$ $\quad 5(4) - F_{CD} \sin 30.26°(1) = 0 \quad F_{CD} = 39.693 \text{ lb}$

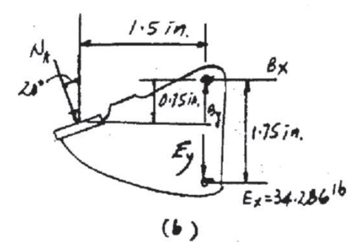
$\xrightarrow{+} \Sigma F_x = 0;$ $\quad E_x - 39.693 \cos 30.26° = 0 \quad E_x = 34.286 \text{ lb}$



From FBD (b)

$\circlearrowleft + \Sigma M_B = 0;$ $\quad N_A \sin 20° (0.75) + N_A \cos 20° (1.5) - 34.286(1.75) = 0$

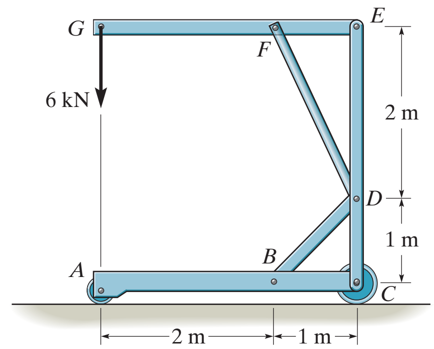
$$N_A = 36.0 \text{ lb}$$ **Ans.**



Ans:
$N_A = 36.0 \text{ lb}$

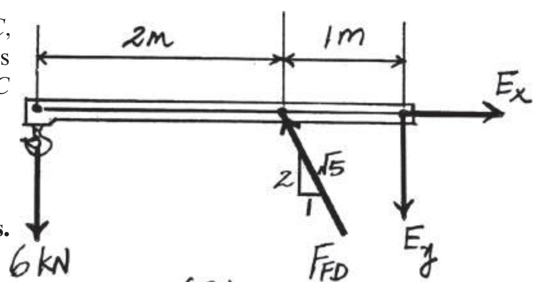
6–83.

Determine the force in members *FD* and *DB* of the frame. Also, find the horizontal and vertical components of reaction the pin at *C* exerts on member *ABC* and member *EDC*.



SOLUTION

Free Body Diagram. The assembly will be dismembered into members *GFE*, *EDC*, *FD*, *BD* and *ABC*. The solution will be very much simplified if one recognizes members *FD* and *BD* are two force members. The *FBDs* of members *GFE*, *EDC* and *ABC* are shown in Figs. *a*, *b* and *c* respectively.



(a)

Equations of Equilibrium. First, consider the equilibrium of member *GFE*, Fig. *a*,

$\circlearrowleft + \Sigma M_E = 0; \quad 6(3) - F_{FD}\left(\frac{2}{\sqrt{5}}\right)(1) = 0 \quad F_{FD} = 9\sqrt{5}\text{ kN} = 20.1\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_F = 0; \quad 6(2) - E_y(1) = 0 \quad E_y = 12.0\text{ kN}$

$\xrightarrow{+} \Sigma F_x = 0; \quad E_x - \left(9\sqrt{5}\right)\left(\frac{1}{\sqrt{5}}\right) = 0 \quad E_x = 9.00\text{ kN}$

Next, for member *EDC*, Fig. *b*,

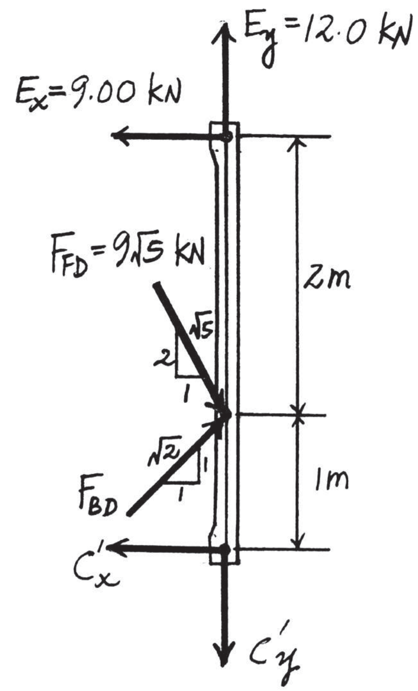
$\circlearrowleft + \Sigma M_C = 0; \quad 9.00(3) - \left(9\sqrt{5}\right)\left(\frac{1}{\sqrt{5}}\right)(1) - F_{BD}\left(\frac{1}{\sqrt{2}}\right)(1) = 0$

$F_{BD} = 18\sqrt{2}\text{ kN} = 25.5\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0; \quad 9.00(2) - C_x'(1) = 0 \quad C_x' = 18.0\text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 12.0 + \left(18\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}\right) - \left(9\sqrt{5}\right)\left(\frac{2}{\sqrt{5}}\right) - C_y' = 0$

$C_y' = 12.0\text{ kN}$ **Ans.**

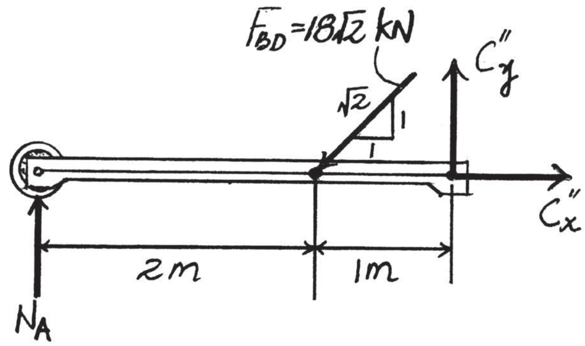


(b)

Finally, for member *ABC*, Fig. *c*

$\circlearrowleft + \Sigma M_A = 0; \quad C_y''(3) - \left(18\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}\right)(2) = 0 \quad C_y'' = 12.0\text{ kN}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \quad C_x'' - \left(18\sqrt{2}\right)\left(\frac{1}{\sqrt{2}}\right) = 0 \quad C_x'' = 18.0\text{ kN}$ **Ans.**

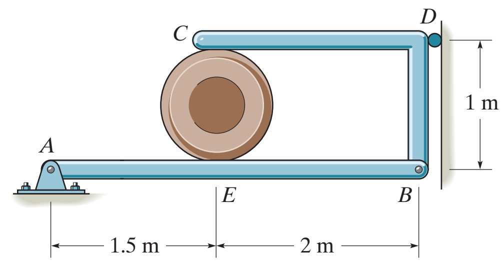


(c)

Ans:
$F_{FD} = 20.1$ kN
$F_{BD} = 25.5$ kN
$C_x'' = 18.0$ kN
$C_y'' = 12.0$ kN

***6–84.**

Determine the force that the smooth 20-kg cylinder exerts on members AB and CDB. Also, what are the horizontal and vertical components of reaction at pin A?



SOLUTION

Free Body Diagram. The *FBDs* of the entire assembly, member CDB and the cylinder are shown in Figs. a, b and c, respectively.

Equations of Equilibrium. First consider the equilibrium of the entire assembly, Fig. a,

$\circlearrowleft + \Sigma M_A = 0; \quad N_D(1) - 20(9.81)(1.5) = 0 \quad N_D = 294.3 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x - 294.3 = 0 \quad A_x = 294.3 \text{ N} = 294 \text{ N}$ **Ans.**

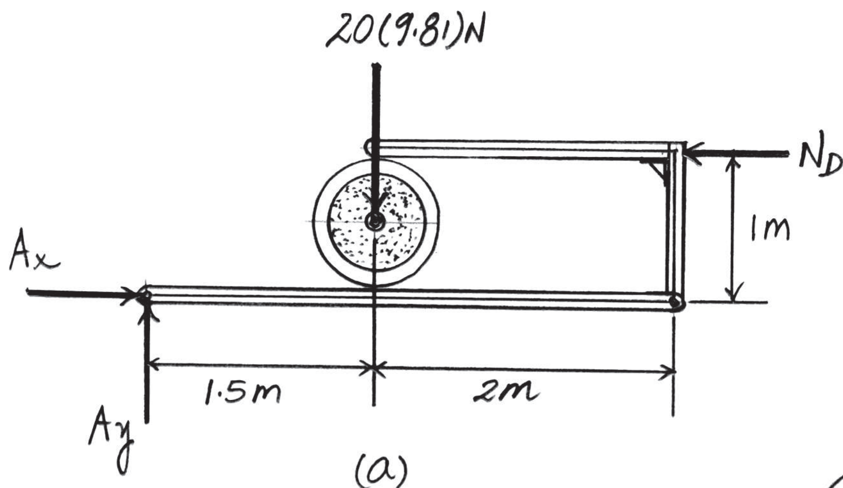
$+\uparrow \Sigma F_y = 0; \quad A_y - 20(9.81) = 0 \quad A_y = 196.2 \text{ N} = 196 \text{ N}$ **Ans.**

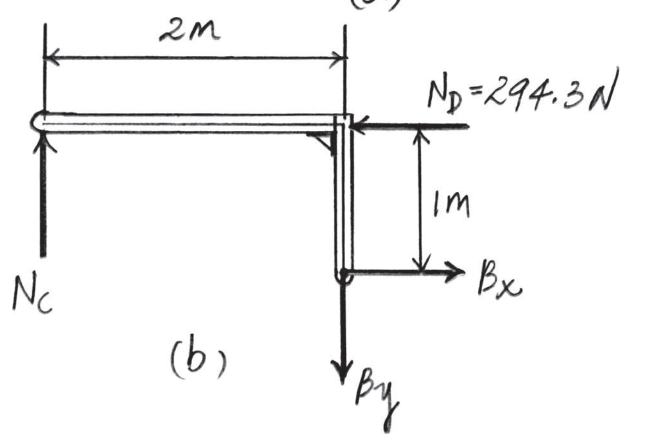
Next, for member CDB, Fig. b

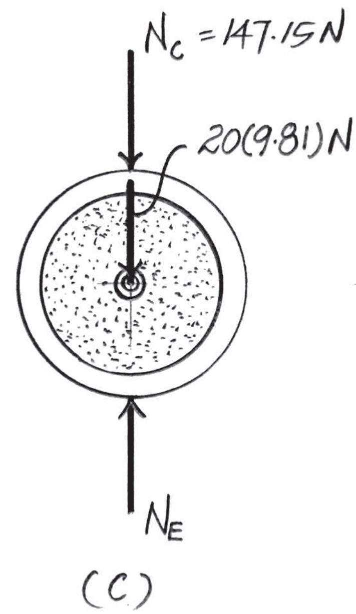
$\circlearrowleft + \Sigma M_B = 0; \quad 294.3(1) - N_C(2) = 0 \quad N_C = 147.15 \text{ N} = 147 \text{ N}$ **Ans.**

Finally for the cylinder, Fig. c

$+\uparrow \Sigma F_y = 0; \quad N_E - 20(9.81) - 147.15 = 0 \quad N_E = 343.35 \text{ N} = 343 \text{ N}$ **Ans.**



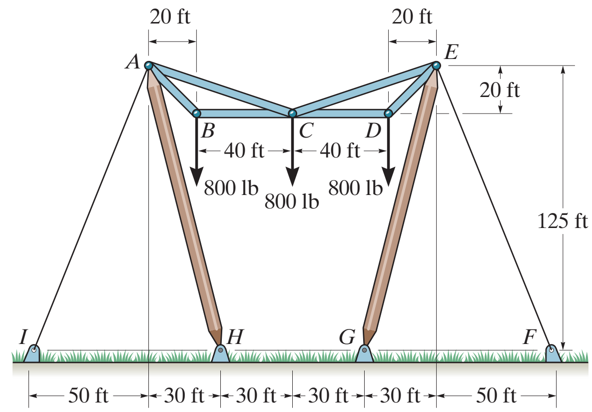




Ans:
$A_x = 294$ N
$A_y = 196$ N
$N_C = 147$ N
$N_E = 343$ N

6–85.

The three power lines exert the forces shown on the truss joints, which in turn are pin-connected to the poles AH and EG. Determine the force in the guy cable AI and the pin reaction at the support H.



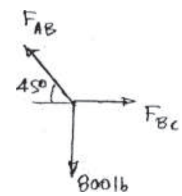
SOLUTION

AH is a two - force member.

Joint B:

$+\uparrow \Sigma F_y = 0; \qquad F_{AB} \sin 45° - 800 = 0$

$F_{AB} = 1131.37 \text{ lb}$



Joint C:

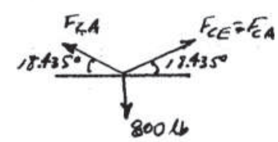
$+\uparrow \Sigma F_y = 0; \qquad 2F_{CA} \sin 18.435° - 800 = 0$

$F_{CA} = 1264.91 \text{ lb}$

Joint A:

$\xrightarrow{+} \Sigma F_x = 0; \qquad -T_{AI} \sin 21.801° - F_H \cos 76.504° + 1264.91 \cos 18.435° + 1131.37 \cos 45° = 0$

$+\uparrow \Sigma F_y = 0; \qquad -T_{AI} \cos 21.801° + F_H \sin 76.504° - 1131.37 \sin 45° - 1264.91 \sin 18.435° = 0$



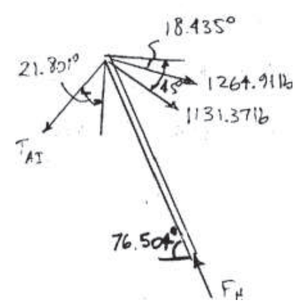
$T_{AI}(0.3714) + F_H(0.2334) = 2000$

$-T_{AI}(0.9285) + F_H(0.97239) = 1200$

Solving,

$T_{AI} = T_{EF} = 2.88 \text{ kip}$ **Ans.**

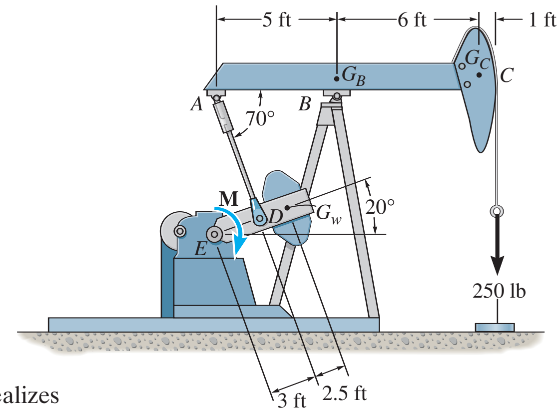
$F_H = F_G = 3.99 \text{ kip}$ **Ans.**



Ans:
$T_{AI} = 2.88 \text{ kip}$
$F_H = 3.99 \text{ kip}$

6–86.

The pumping unit is used to recover oil. When the walking beam ABC is horizontal, the force acting in the wireline at the well head is 250 lb. Determine the torque **M** which must be exerted by the motor in order to overcome this load. The horse-head C weighs 60 lb and has a center of gravity at G_C. The walking beam ABC has a weight of 130 lb and a center of gravity at G_B, and the counterweight has a weight of 200 lb and a center of gravity at G_W. The pitman, AD, is pin connected at its ends and has negligible weight.



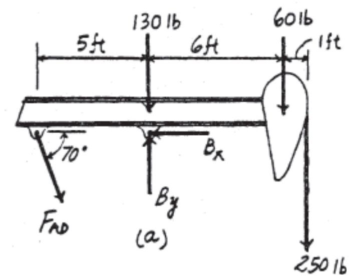
SOLUTION

Free-Body Diagram: The solution for this problem will be simplified if one realizes that the pitman AD is a two force member.

Equations of Equilibrium: From FBD (a),

$$\circlearrowleft + \Sigma M_B = 0; \qquad F_{AD} \sin 70°(5) - 60(6) - 250(7) = 0$$

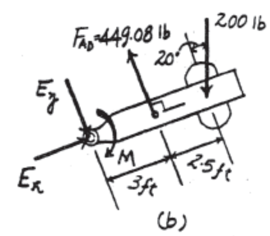
$$F_{AD} = 449.08 \text{ lb}$$



From (b),

$$\circlearrowleft + \Sigma M_E = 0; \qquad 449.08(3) - 200 \cos 20°(5.5) - M = 0$$

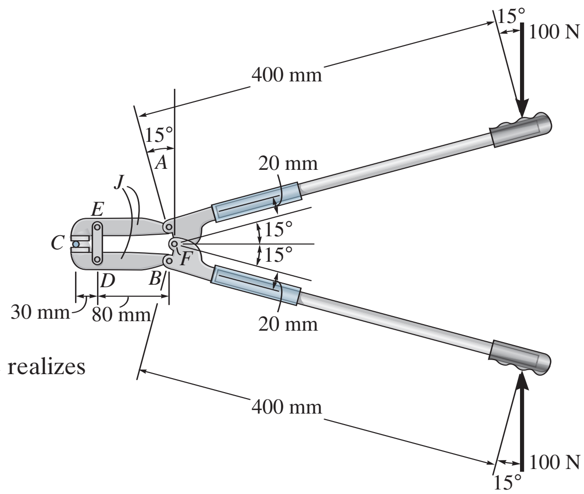
$$M = 314 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$



Ans:
$M = 314 \text{ lb} \cdot \text{ft}$

6–87.

Determine the force that the jaws J of the metal cutters exert on the smooth cable C if 100-N forces are applied to the handles. The jaws are pinned at E and A, and D and B. There is also a pin at F.



SOLUTION

Free Body Diagram: The solution for this problem will be simplified if one realizes that member ED is a two force member.

Equations of Equilibrium: From FBD (b),

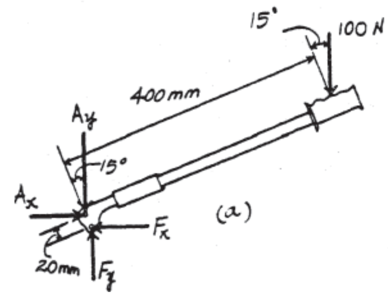
$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$

From (a),

$\circlearrowleft + \Sigma M_F = 0; \qquad A_y \sin 15°(20) + 100 \sin 15°(20)$

$$- 100 \cos 15°(400) = 0$$

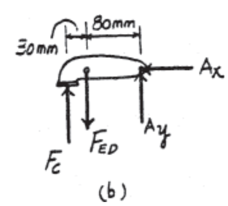
$$A_y = 7364.10 \text{ N}$$



From FBD (b),

$\circlearrowleft + \Sigma M_E = 0; \qquad 7364.10(80) - F_C(30) = 0$

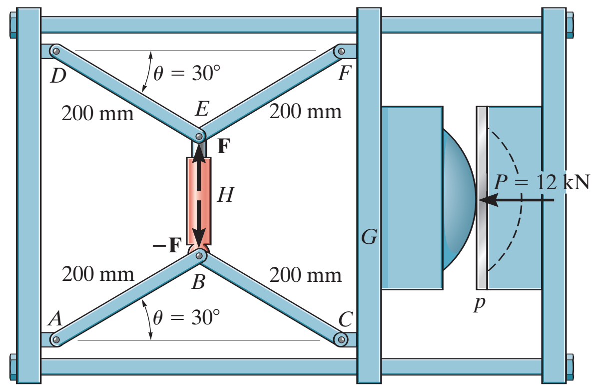
$$F_C = 19637.60 \text{ N} = 19.6 \text{ kN}$$ **Ans.**



Ans:
$F_C = 19.6$ kN

***6–88.**

The machine shown is used for forming metal plates. It consists of two toggles ABC and DEF, which are operated by the hydraulic cylinder H. The toggles push the movable bar G forward, pressing the plate p into the cavity. If the force which the plate exerts on the head is $P = 12$ kN, determine the force F in the hydraulic cylinder when $\theta = 30°$.

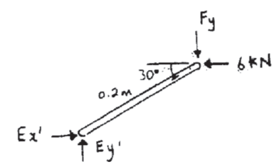


SOLUTION

Member EF:

$\circlearrowleft + \Sigma M_E = 0; \qquad -F_y(0.2\cos 30°) + 6\,(0.2\sin 30°) = 0$

$F_y = 3.464$ kN



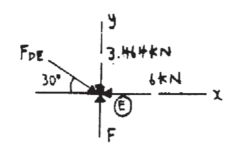
Joint E:

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{DE}\cos 30° - 6 = 0$

$F_{DE} = 6.928$ kN

$+\uparrow \Sigma F_y = 0; \qquad F - 3.464 - 6.928\sin 30° = 0$

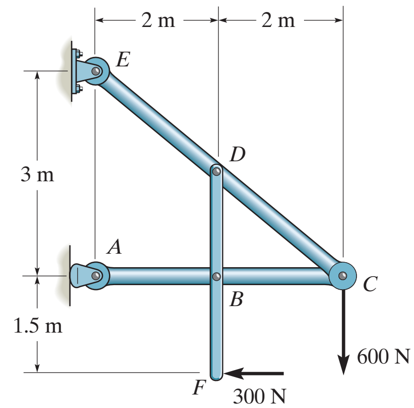
$F = 6.93$ kN **Ans.**



Ans:
$F = 6.93$ kN

6–89.

Determine the horizontal and vertical components of force which pin C exerts on member CDE. The 600-N load is applied to the pin.



SOLUTION

Free Body Diagram. The solution will be very much simplified if one determines the support reactions first and then considers the equilibrium of two of its three members after they are dismembered. The *FBDs* of the entire assembly, member DBF and member ABC are shown in Figs. a, b and c, respectively.

Equations of Equilibrium. Consider the equilibrium of the entire assembly, Fig. a,

$\circlearrowleft + \Sigma M_E = 0; \qquad N_A(3) - 300(4.5) - 600(4) = 0 \quad N_A = 1250\text{ N}$

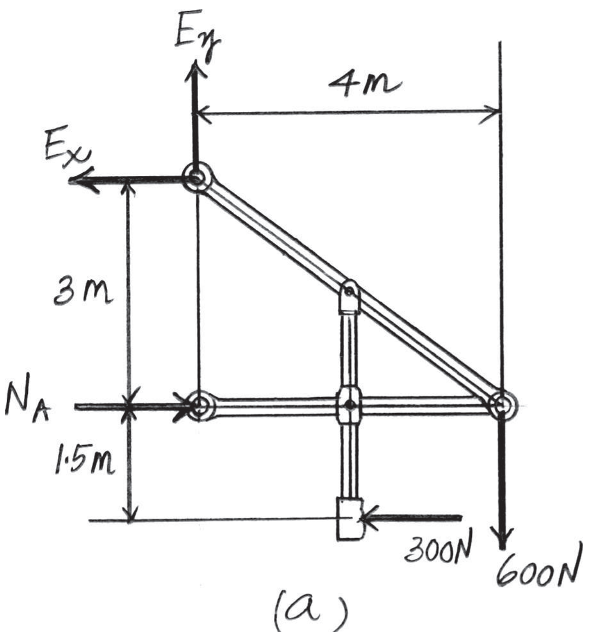
Next, write the moment equation of equilibrium about point D for member DBF, Fig. b.

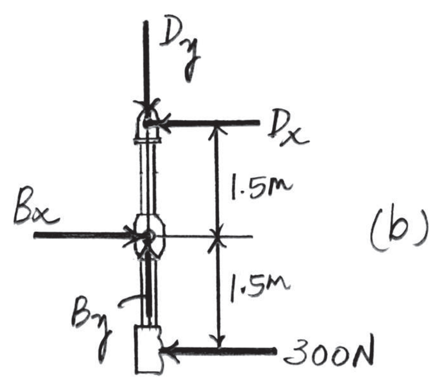
$\circlearrowleft + \Sigma M_D = 0; \qquad B_x(1.5) - 300(3) = 0 \quad B_x = 600\text{ N}$

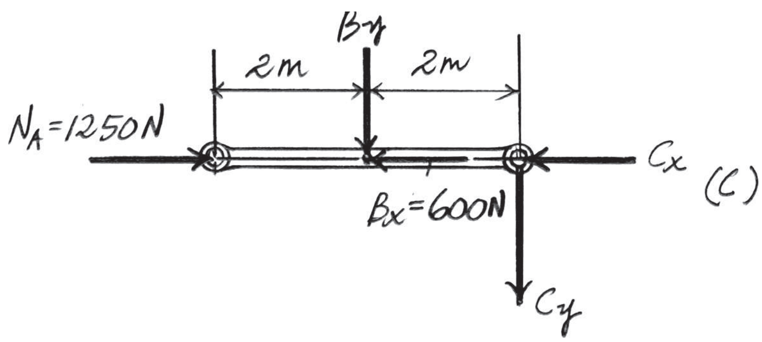
Finally, consider the equilibrium of member ABC, Fig. c

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 1250 - 600 - C_x = 0 \quad C_x = 650\text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_B = 0; \qquad C_y = 0$ **Ans.**



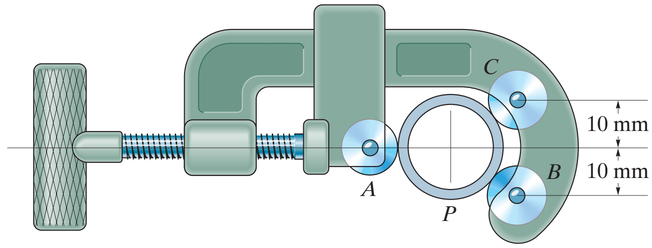




Ans:
$C_x = 650\text{ N}$
$C_y = 0$

6–90.

The pipe cutter is clamped around the pipe P. If the wheel at A exerts a normal force of $F_A = 80\text{ N}$ on the pipe, determine the normal forces of wheels B and C on the pipe. The three wheels each have a radius of 7 mm and the pipe has an outer radius of 10 mm.



SOLUTION

$$\theta = \sin^{-1}\left(\frac{10}{17}\right) = 36.03^\circ$$

Equations of Equilibrium:

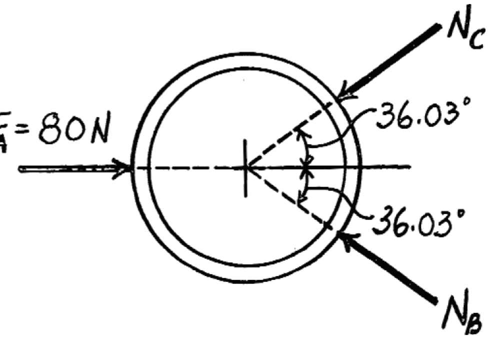
$+\uparrow \Sigma F_y = 0;\qquad N_B \sin 36.03^\circ - N_C \sin 36.03^\circ = 0$

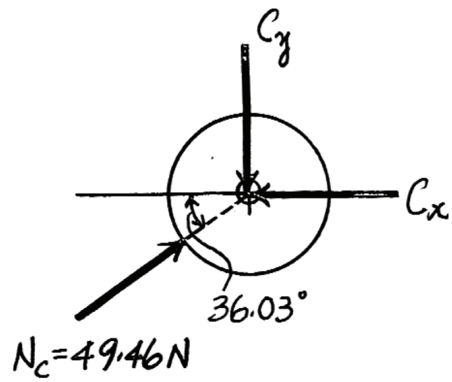
$$N_B = N_C$$

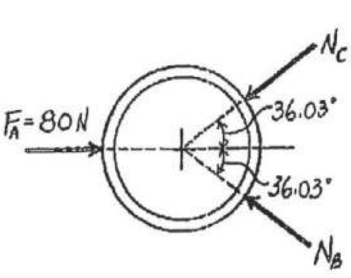
$\xrightarrow{+} \Sigma F_x = 0;\qquad 80 - N_C \cos 36.03^\circ - N_C \cos 36.03^\circ = 0$

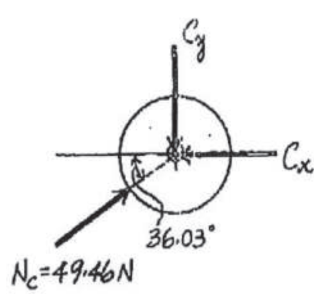
$$N_B = N_C = 49.5\text{ N}$$

Ans.





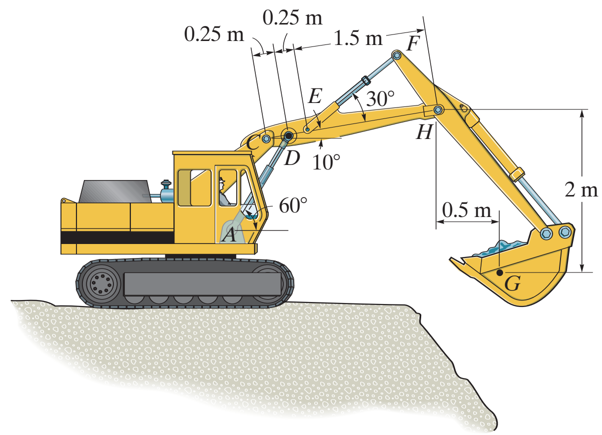




Ans:
$N_B = N_C = 49.5\text{ N}$

6–91.

Determine the force created in the hydraulic cylinders EF and AD in order to hold the shovel in equilibrium. The shovel load has a mass of 1.25 Mg and a center of gravity at G. All joints are pin connected.

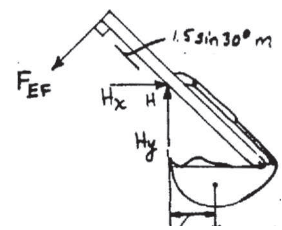


SOLUTION

Assembly FHG:

$\circlearrowleft + \Sigma M_H = 0; \qquad -1250(9.81)(0.5) + F_{EF}(1.5 \sin 30°) = 0$

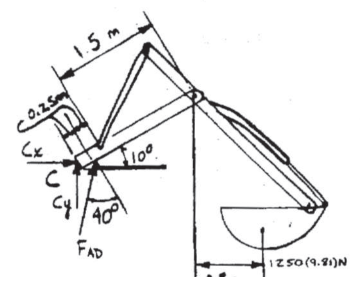
$F_{EF} = 8175 \text{ N} = 8.18 \text{ kN (T)}$ **Ans.**



Assembly $CEFHG$:

$\circlearrowleft + \Sigma M_C = 0; \qquad F_{AD} \cos 40° (0.25) - 1250(9.81)(2 \cos 10° + 0.5) = 0$

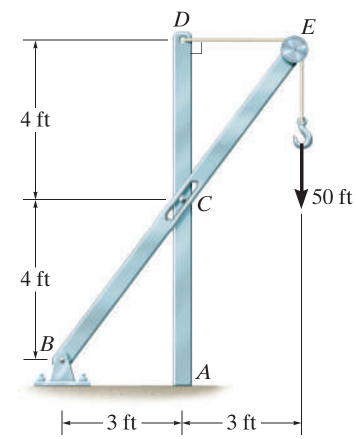
$F_{AD} = 158\,130 \text{ N} = 158 \text{ kN (C)}$ **Ans.**



Ans:
$F_{EF} = 8.18 \text{ kN (T)}$
$F_{AD} = 158 \text{ kN (C)}$

***6–92.**

Determine the horizontal and vertical components of force at pin B and the normal force the pin at C exerts on the smooth slot. Also, determine the moment and horizontal and vertical reactions of force at A. There is a pulley at E.



SOLUTION

BCE:

$\curvearrowleft + \Sigma M_B = 0; \quad -50(6) - N_C(5) + 50(8) = 0$

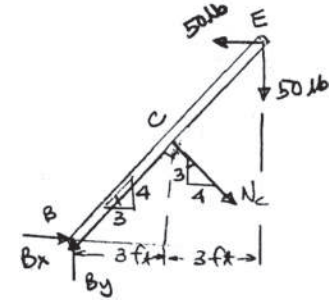
$N_C = 20 \text{ lb}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \quad B_x + 20\left(\frac{4}{5}\right) - 50 = 0$

$B_x = 34 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad B_y - 20\left(\frac{3}{5}\right) - 50 = 0$

$B_y = 62 \text{ lb}$ **Ans.**



ACD:

$\xrightarrow{+} \Sigma F_x = 0; \quad -A_x - 20\left(\frac{4}{5}\right) + 50 = 0$

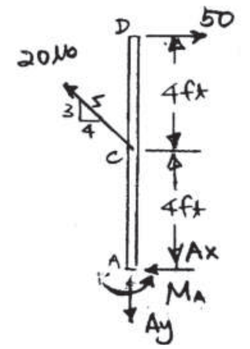
$A_x = 34 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad -A_y + 20\left(\frac{3}{5}\right) = 0$

$A_y = 12 \text{ lb}$ **Ans.**

$\curvearrowleft + \Sigma M_A = 0; \quad M_A + 20\left(\frac{4}{5}\right)(4) - 50(8) = 0$

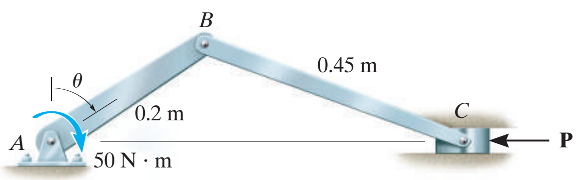
$M_A = 336 \text{ lb} \cdot \text{ft}$ **Ans.**



Ans:
$N_c = 20 \text{ lb}$
$B_x = 34 \text{ lb}$
$B_y = 62 \text{ lb}$
$A_x = 34 \text{ lb}$
$A_y = 12 \text{ lb}$
$M_A = 336 \text{ lb} \cdot \text{ft}$

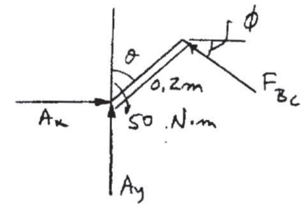
6–93.

The constant moment of $50\ \mathrm{N \cdot m}$ is applied to the crank shaft. Determine the compressive force P that is exerted on the piston for equilibrium as a function of θ. Plot the results of P (ordinate) versus θ (abscissa) for $0° \leq \theta \leq 90°$.



SOLUTION

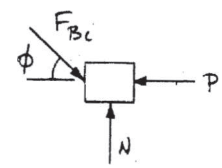
Member AB:



$\circlearrowleft + \Sigma M_A = 0;\qquad F_{BC} \sin\phi(0.2\sin\theta) + F_{BC}\cos\phi(0.2\cos\phi) - 50 = 0$

$$F_{BC} = \frac{250}{(\sin\phi\sin\theta + \cos\phi\cos\theta)}$$

Piston:

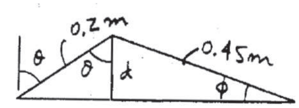


$\xrightarrow{+} \Sigma F_x = 0;\qquad F_{BC}\cos\phi - P = 0$

$$P = \frac{250\cos\phi}{(\sin\phi\sin\theta + \cos\phi\cos\theta)} \qquad \textbf{(1)}$$

$$d = 0.2\cos\theta = 0.45\sin\phi$$

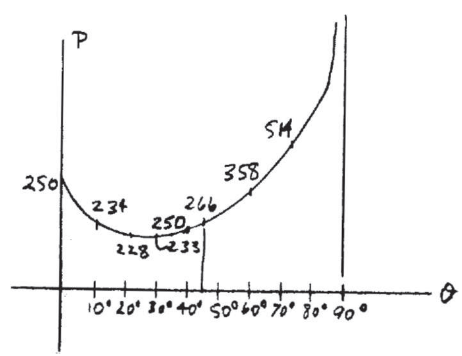
$$\phi = \sin^{-1}\left(\frac{\cos\theta}{2.25}\right) \qquad \textbf{(2)}$$



Select θ, solve for ϕ in Eq. (2), then P in Eq. (1).

$$P(\theta) = \frac{250\cos\left[\sin^{-1}\left(\frac{\cos\theta}{2.25}\right)\right]}{\frac{\sin\theta\cos\theta}{2.25} + cos\left[\sin^{-1}\left(\frac{\cos\theta}{2.25}\right)\right]\cdot\cos\theta}$$

$$= \frac{250\sqrt{2.25^2 - \cos^2\theta}}{\sin\theta\cos\theta + \sqrt{2.25^2 - \cos^2\theta}\cdot\cos\theta} \qquad \textbf{Ans.}$$

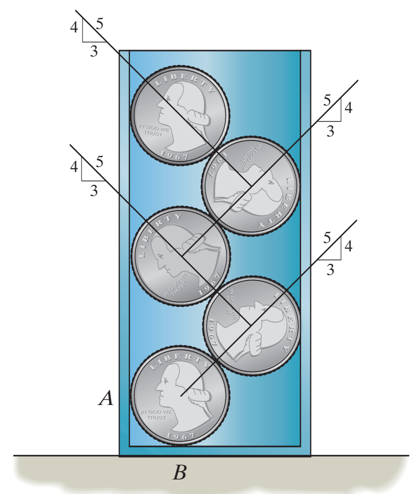


Ans:

$$P(\theta) = \frac{250\sqrt{2.25^2 - \cos^2\theta}}{\sin\theta\cos\theta + \sqrt{2.25^2 - \cos^2\theta}\cdot\cos\theta}$$

6–94.

Five coins are stacked in the smooth plastic container shown. If each coin weighs 0.0235 lb, determine the normal reactions of the bottom coin on the container at points A and B.

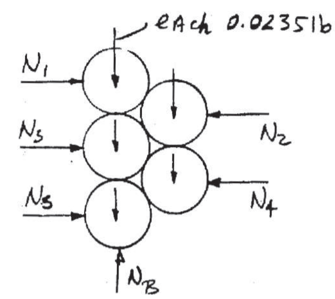


SOLUTION

All coins:

$$+\uparrow \Sigma F_y = 0; \qquad N_B - 5\,(0.0235) = 0$$

$$N_B = 0.1175 \text{ lb} \qquad \textbf{Ans.}$$



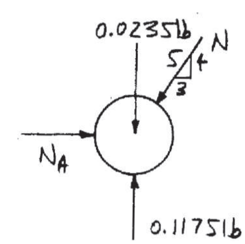
Bottom coin:

$$+\uparrow \Sigma F_y = 0; \qquad 0.1175 - 0.0235 - N\left(\frac{4}{5}\right) = 0$$

$$N = 0.1175 \text{ lb}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_A - 0.1175\left(\frac{3}{5}\right) = 0$$

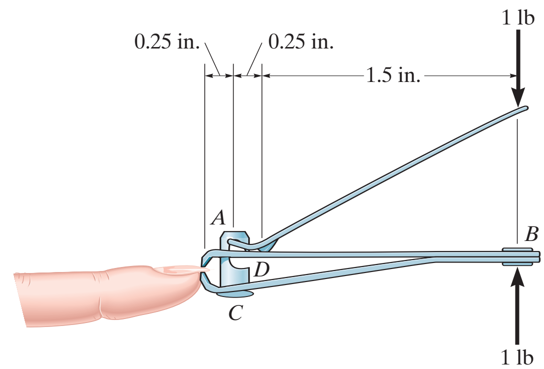
$$N_A = 0.0705 \text{ lb} \qquad \textbf{Ans.}$$



Ans:
$N_B = 0.1175$ lb
$N_A = 0.0705$ lb

6–95.

The nail cutter consists of the handle and the two cutting blades. Assuming the blades are pin connected at B and the surface at D is smooth, determine the normal force on the fingernail when a force of 1 lb is applied to the handles as shown. The pin AC slides through a smooth hole at A and is attached to the bottom member at C.



SOLUTION

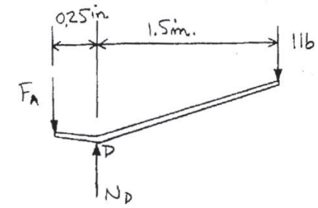
Handle:

$\circlearrowleft + \Sigma M_D = 0; \qquad F_A(0.25) - 1(1.5) = 0$

$F_A = 6 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad N_D - 1 - 6 = 0$

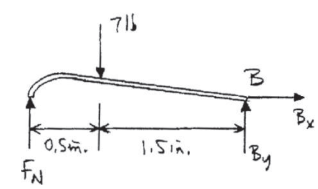
$N_D = 7 \text{ lb}$



Top blade:

$\circlearrowleft + \Sigma M_B = 0; \qquad 7(1.5) - F_N(2) = 0$

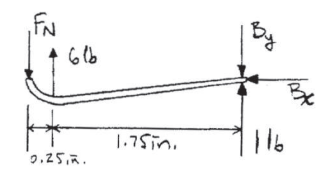
$F_N = 5.25 \text{ lb}$ **Ans.**



Or bottom blade:

$\circlearrowleft + \Sigma M_B = 0; \qquad F_N(2) - 6\,(1.75) = 0$

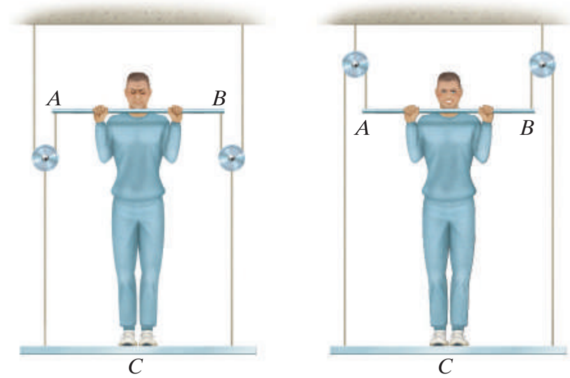
$F_N = 5.25 \text{ lb}$ **Ans.**



Ans:
$F_N = 5.25 \text{ lb}$

***6–96.**

A man having a weight of 175 lb attempts to hold himself using one of the two methods shown. Determine the total force he must exert on bar AB in each case and the normal reaction he exerts on the platform at C. Neglect the weight of the platform.

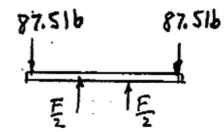


(a) (b)

SOLUTION

(a)

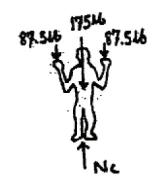
Bar:



$+\uparrow\Sigma F_y = 0;\qquad 2(F/2) - 2(87.5) = 0$

$F = 175\text{ lb}$ **Ans.**

Man:

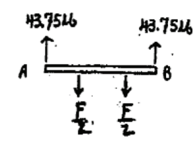


$+\uparrow\Sigma F_y = 0;\qquad N_C - 175 - 2(87.5) = 0$

$N_C = 350\text{ lb}$ **Ans.**

(b)

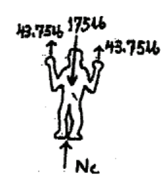
Bar:



$+\uparrow\Sigma F_y = 0;\qquad 2(43.75) - 2(F/2) = 0$

$F = 87.5\text{ lb}$ **Ans.**

Man:



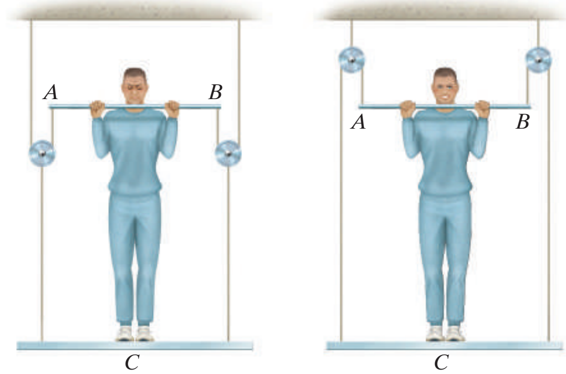
$+\uparrow\Sigma F_y = 0;\qquad N_C - 175 + 2(43.75) = 0$

$N_C = 87.5\text{ lb}$ **Ans.**

Ans:
$F = 175\text{ lb}$
$N_c = 350\text{ lb}$
$F = 87.5\text{ lb}$
$N_c = 87.5\text{ lb}$

6–97.

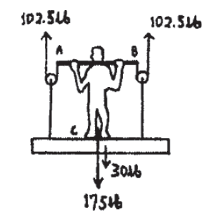
A man having a weight of 175 lb attempts to hold himself using one of the two methods shown. Determine the total force he must exert on bar AB in each case and the normal reaction he exerts on the platform at C. The platform has a weight of 30 lb.



(a) (b)

SOLUTION

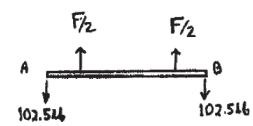
(a)



Bar:

$+\uparrow \Sigma F_y = 0; \qquad 2(F/2) - 102.5 - 102.5 = 0$

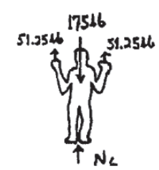
$F = 205 \text{ lb}$ **Ans.**



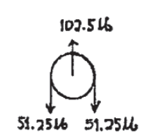
Man:

$+\uparrow \Sigma F_y = 0; \qquad N_C - 175 - 102.5 - 102.5 = 0$

$N_C = 380 \text{ lb}$ **Ans.**



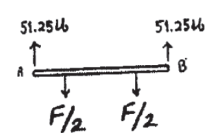
(b)



Bar:

$+\uparrow \Sigma F_y = 0; \qquad 2(F/2) - 51.25 - 51.25 = 0$

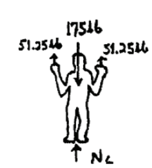
$F = 102 \text{ lb}$ **Ans.**



Man:

$+\uparrow \Sigma F_y = 0; \qquad N_C - 175 + 51.25 + 51.25 = 0$

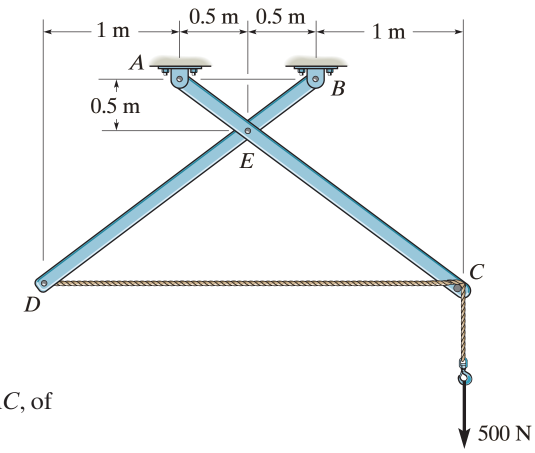
$N_C = 72.5 \text{ lb}$ **Ans.**



Ans:
$F = 205 \text{ lb}$
$N_C = 380 \text{ lb}$
$F = 102 \text{ lb}$
$N_C = 72.5 \text{ lb}$

6–98.

The two-member frame is pin connected at E. The cable is attached to D, passes over the smooth peg at C, and supports the 500-N load. Determine the horizontal and vertical reactions at each pin.



SOLUTION

Free Body Diagram. The frame will be dismembered into members BD and AC, of which their respective *FBDs* are shown in Figs. *a* and *b*.

Equations of Equilibrium. Write the moment equation of equilibrium about point B for member BD, Fig. *a*, and about point A for member AC, Fig. *b*,

$$\circlearrowleft+\Sigma M_B = 0; \quad E_y(0.5) - E_x(0.5) + 500(2) = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft+\Sigma M_A = 0; \quad E_y(0.5) + E_x(0.5) - 500(2) - 500(2) = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2),

$$E_y = 1000 \text{ N} = 1.00 \text{ kN} \quad E_x = 3000 \text{ N} = 3.00 \text{ kN} \qquad \textbf{Ans.}$$

Write the force equations of equilibrium for member BD, Fig. *a*.

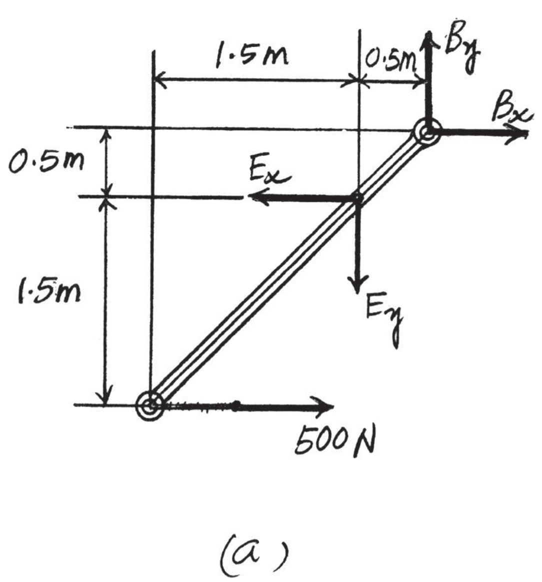
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad B_x + 500 - 3000 = 0 \quad B_x = 2500 \text{ N} = 2.50 \text{ kN} \qquad \textbf{Ans.}$$

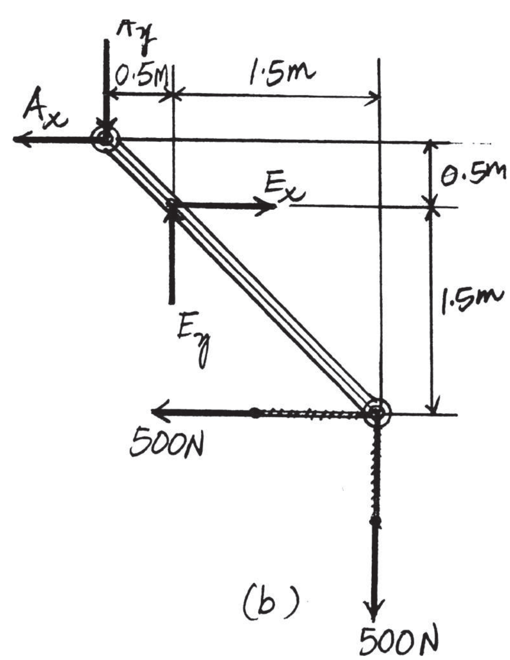
$$+\uparrow\Sigma F_y = 0; \quad B_y - 1000 = 0 \quad B_y = 1000 \text{ N} = 1.00 \text{ kN} \qquad \textbf{Ans.}$$

Also, for member AC, Fig. *b*

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 3000 - 500 - A_x = 0 \quad A_x = 2500 \text{ N} = 2.50 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \quad 1000 - 500 - A_y = 0 \quad A_y = 500 \text{ N} \qquad \textbf{Ans.}$$

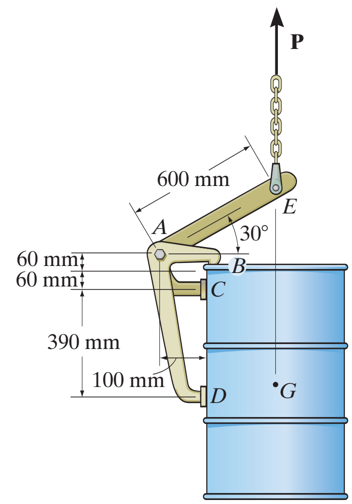




Ans:
$E_y = 1.00$ kN
$E_x = 3.00$ kN
$B_x = 2.50$ kN
$B_y = 1.00$ kN
$A_x = 2.50$ kN
$A_y = 500$ N

6–99.

If the 300-kg drum has a center of mass at point G, determine the horizontal and vertical components of force acting at pin A and the reactions on the smooth pads C and D. The grip at B on member DAB resists both horizontal and vertical components of force at the rim of the drum.



SOLUTION

Equations of Equilibrium: From the free - body diagram of segment CAE in Fig. a,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 300(9.81)(600 \cos 30°) - N_C(120) = 0$

$N_C = 12\,743.56 \text{ N} = 12.7 \text{ kN}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad A_x - 12\,743.56 = 0$

$A_x = 12\,743.56 \text{ N} = 12.7 \text{ kN}$ **Ans.**

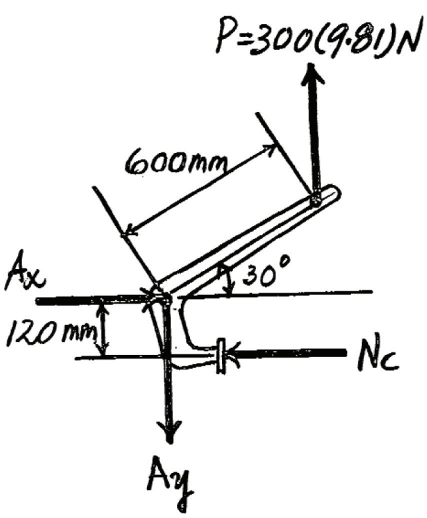
$+\uparrow \Sigma F_y = 0;$ $\quad 300(9.81) - A_y = 0$

$A_y = 2943 \text{ N} = 2.94 \text{ kN}$ **Ans.**

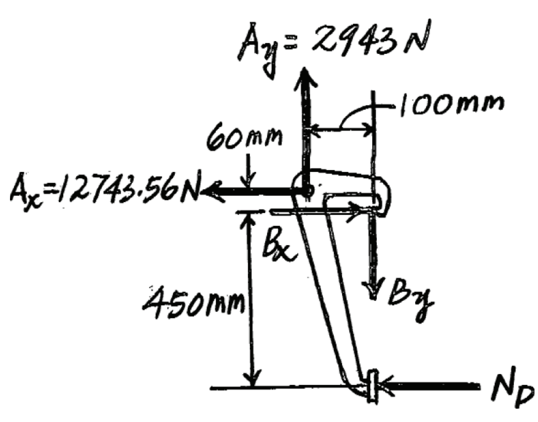
Using the results for A_x and A_y obtained above and applying the moment equation of equilibrium about point B on the free - body diagram of segment BAD, Fig. b,

$\circlearrowleft + \Sigma M_B = 0;$ $\quad 12\,743.56(60) - 2943(100) - N_D(450) = 0$

$N_D = 1045.14 \text{ N} = 1.05 \text{ kN}$ **Ans.**



(a)

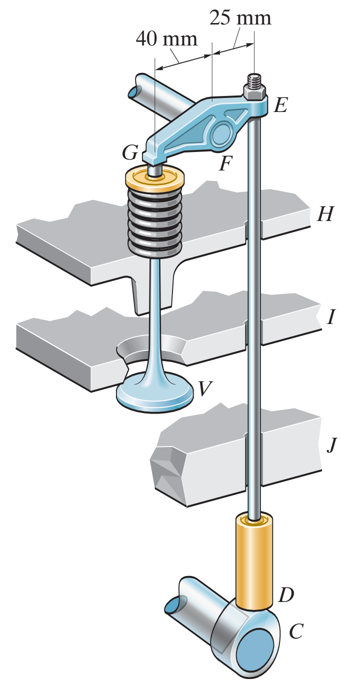


(b)

Ans:
$N_C = 12.7 \text{ kN}$
$A_x = 12.7 \text{ kN}$
$A_y = 2.94 \text{ kN}$
$N_D = 1.05 \text{ kN}$

***6–100.**

Operation of exhaust and intake valves in an automobile engine consists of the cam C, push rod DE, rocker arm EFG which is pinned at F, and a spring and valve, V. If the compression in the spring is 20 mm when the valve is open as shown, determine the normal force acting on the cam lobe at C. Assume the cam and bearings at H, I, and J are smooth. The spring has a stiffness of 300 N/m.



SOLUTION

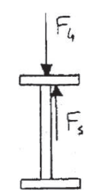
$F_s = kx; \qquad F_s = 300\,(0.02) = 6\text{ N}$

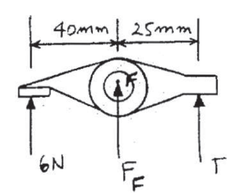
$+\uparrow \Sigma F_y = 0; \qquad -F_G + 6 = 0$

$F_G = 6\text{ N}$

$\circlearrowleft + \Sigma M_F = 0; \qquad -6(40) + T(25) = 0$

$T = 9.60\text{ N}$ **Ans.**

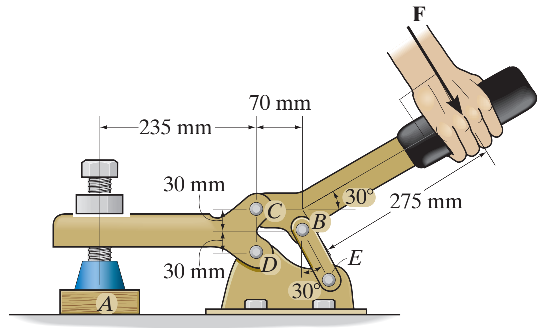




Ans:
$T = 9.60\text{ N}$

6–101.

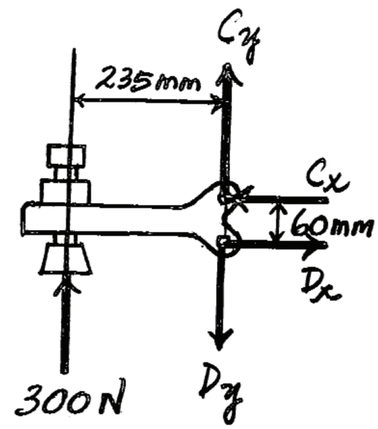
If a clamping force of 300 N is required at A, determine the amount of force **F** that must be applied to the handle of the toggle clamp.



SOLUTION

Equations of Equilibrium: First, we will consider the free-body diagram of the clamp in Fig. *a*. Writing the moment equation of equilibrium about point D,

$$\circlearrowleft + \Sigma M_D = 0; \qquad C_x(60) - 300(235) = 0$$
$$C_x = 1175 \text{ N}$$



(a)

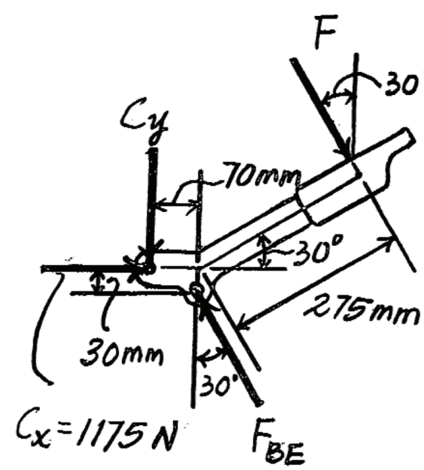
Subsequently, the free - body diagram of the handle in Fig. *b* will be considered.

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{BE}\cos 30°(70) - F_{BE}\sin 30°(30) - F\cos 30°(275\cos 30° + 70)$$
$$-F\sin 30°(275\sin 30°) = 0$$
$$45.62F_{BE} - 335.62F = 0 \qquad \textbf{(1)}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 1175 + F\sin 30° - F_{BE}\sin 30° = 0$$
$$0.5F_{BE} - 0.5F = 1175 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2) yields

$$F = 369.69 \text{ N} = 370 \text{ N} \qquad \textbf{Ans.}$$
$$F_{BE} = 2719.69\text{N}$$

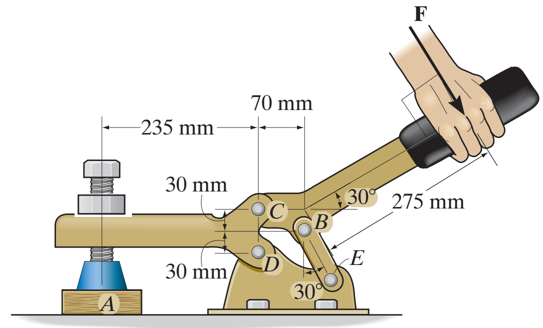


(b)

Ans:
$F = 370$ N

6–102.

If a force of $F = 350$ N is applied to the handle of the toggle clamp, determine the resulting clamping force at A.

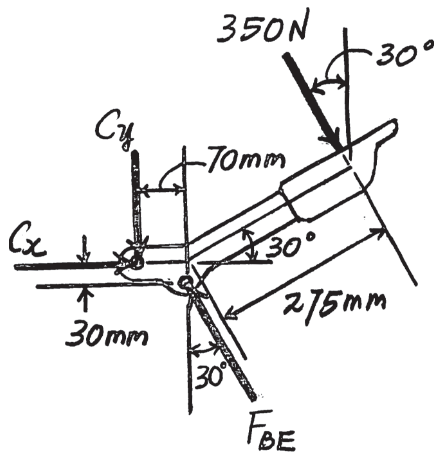


SOLUTION

Equations of Equilibrium: First, we will consider the free-body diagram of the handle in Fig. *a*.

$$\zeta + \Sigma M_C = 0; \qquad F_{BE}\cos 30°(70) - F_{BE}\sin 30°(30) - 350\cos 30°(275\cos 30° + 70)$$
$$-350\sin 30°(275\sin 30°) = 0$$
$$F_{BE} = 2574.81 \text{ N}$$

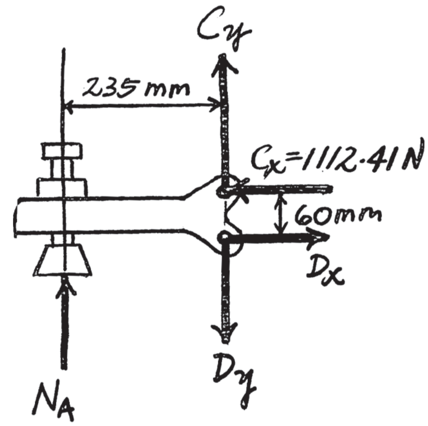
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad C_x - 2574.81\sin 30° + 350\sin 30° = 0$$
$$C_x = 1112.41 \text{ N}$$



(a)

Subsequently, the free-body diagram of the clamp in Fig. b will be considered. Using the result of C_x and writing the moment equation of equilibrium about point D,

$$\zeta + \Sigma M_D = 0; \qquad 1112.41(60) - N_A(235) = 0$$
$$N_A = 284.01 \text{ N} = 284 \text{ N} \qquad \textbf{Ans.}$$

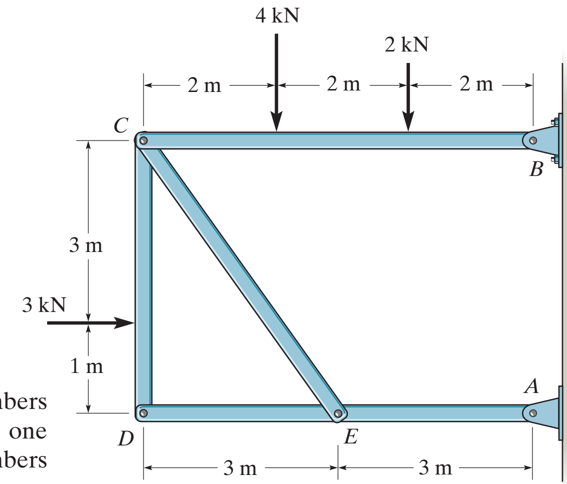


(b)

Ans:
$N_A = 284$ N

6–103.

Determine the horizontal and vertical components of force that the pins at A and B exert on the frame.



SOLUTION

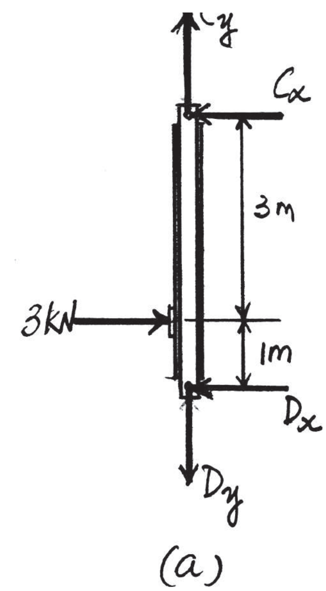
Free Body Diagram. The assembly will be dismembered into members BC, CD, CE and AD. The solution will be very much simplified if one recognizes member CE is a two force member. The *FBDs* of members CD, BC and AD are shown in Figs. *a*, *b* and *c*, respectively.

Equations of Equilibrium. First, consider the equilibrium of member CD, Fig. *a*.

$\zeta+\Sigma M_C = 0; \quad 3(3) - D_x(4) = 0 \quad D_x = 2.25\text{ kN}$

$\zeta+\Sigma M_D = 0; \quad C_x(4) - 3(1) = 0 \quad C_x = 0.75\text{ kN}$

$+\uparrow\Sigma F_y = 0; \quad C_y - D_y = 0 \quad C_y = D_y$



Next write the moment equation of equilibrium about point B for member BC, Fig. *b* and about point A for member AD, Fig. *c*,

$$\zeta+\Sigma M_B = 0; \quad 2(2) + 4(4) + D_y(6) - F_{CE}\left(\frac{4}{5}\right)(6) = 0 \qquad \textbf{(1)}$$

$$\zeta+\Sigma M_A = 0; \quad F_{CE}\left(\frac{4}{5}\right)(3) - D_y(6) = 0 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

$$F_{CE} = 8.3333\text{ kN} \quad D_y = 3.3333\text{ kN}$$

Finally, write the force equation of equilibrium for member BC, Fig. *b*,

$$+\uparrow\Sigma F_y = 0; \quad B_y + 8.3333\left(\frac{4}{5}\right) - 3.3333 - 4 - 2 = 0$$

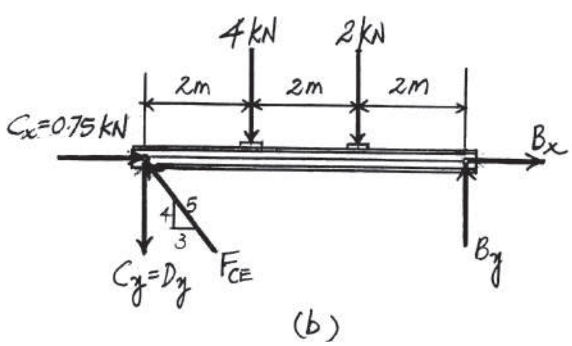
$$B_y = 2.6667\text{ kN} = 2.67\text{ kN} \qquad \textbf{Ans.}$$

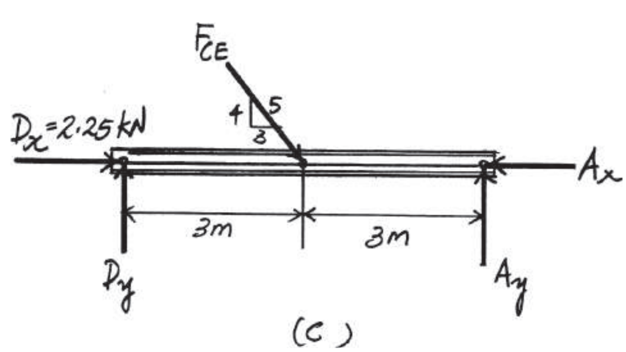
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad B_x + 0.75 - 8.3333\left(\frac{3}{5}\right) = 0 \quad B_x = 4.25\text{ kN} \qquad \textbf{Ans.}$$

Also, for member AD, Fig. *c*.

$$+\uparrow\Sigma F_y = 0; \quad A_y + 3.3333 - 8.3333\left(\frac{4}{5}\right) = 0 \quad A_y = 3.3333\text{ kN} = 3.33\text{ kN} \qquad \textbf{Ans.}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 2.25 + 8.3333\left(\frac{3}{5}\right) - A_x = 0 \quad A_x = 7.25\text{ kN} \qquad \textbf{Ans.}$$

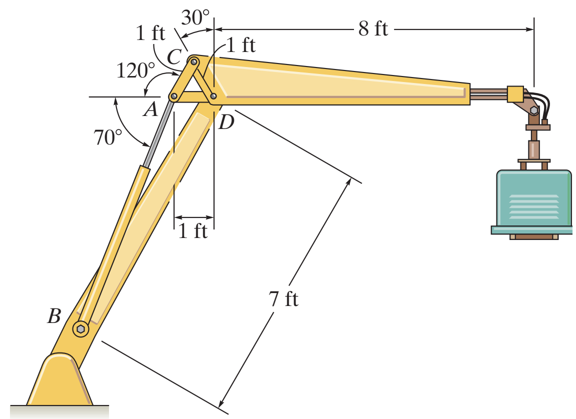




Ans:
$B_y = 2.67$ kN
$B_x = 4.25$ kN
$A_y = 3.33$ kN
$A_x = 7.25$ kN

***6–104.**

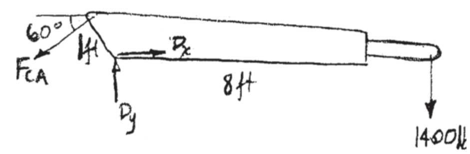
The hydraulic crane is used to lift the 1400-lb load. Determine the force in the hydraulic cylinder AB and the force in links AC and AD when the load is held in the position shown.



SOLUTION

$\circlearrowleft + \Sigma M_D = 0;$ $\quad F_{CA}(\sin 60°)(1) - 1400(8) = 0$

$F_{CA} = 12\,932.65 \text{ lb} = 12.9 \text{ kip}$ **Ans.**

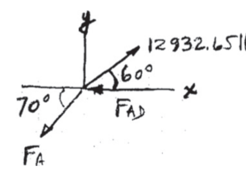


$+\uparrow \Sigma F_y = 0;$ $\quad 12\,932.65 \sin 60° - F_{AB} \sin 70° = 0$

$F_{AB} = 11\,918.79 \text{ lb} = 11.9 \text{ kip}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad -11\,918.79 \cos 70° + 12\,932.65 \cos 60° - F_{AD} = 0$

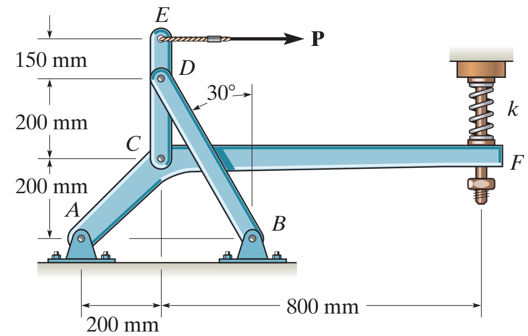
$F_{AD} = 2389.86 \text{ lb} = 2.39 \text{ kip}$ **Ans.**



Ans:
$F_{CA} = 12.9$ kip
$F_{AB} = 11.9$ kip
$F_{AD} = 2.39$ kip

6–105.

Determine force **P** on the cable if the spring is compressed 0.025 m when the mechanism is in the position shown. The spring has a stiffness of $k = 6$ kN/m.



SOLUTION

Free Body Diagram. The assembly will be dismembered into members ACF, CDE and BD. The solution will be very much simplified if one recognizes member BD is a two force member. Here, the spring force is $F_{SP} = kx = 600(0.025) = 150$ N. The *FBDs* of members ACF and CDE are shown in Figs. *a* and *b*, respectively.

Equations of Equilibrium. Write the moment equation of equilibrium about point A for member ACF, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad C_y(0.2) + C_x(0.2) - 150(1) = 0 \qquad \textbf{(1)}$$

Next, consider the equilibrium of member CDE

$$\circlearrowleft + \Sigma M_C = 0; \qquad F_{BD}\sin 30°(0.2) - P(0.35) = 0 \qquad \textbf{(2)}$$

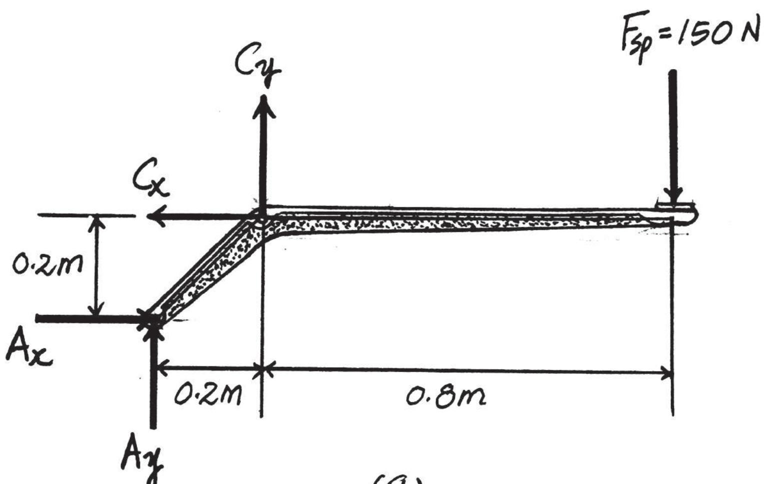
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad C_x + P - F_{BD}\sin 30° = 0 \qquad \textbf{(3)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BD}\cos 30° - C_y = 0 \qquad \textbf{(4)}$$

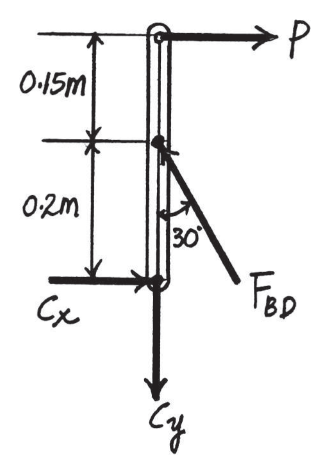
Solving Eqs. (1) to (4),

$$C_x = 148.77\text{ N} \quad C_y = 601.23\text{ N} \quad F_{BD} = 694.24\text{ N}$$

$$P = 198.36\text{ N} = 198\text{ N} \qquad \textbf{Ans.}$$



(a)

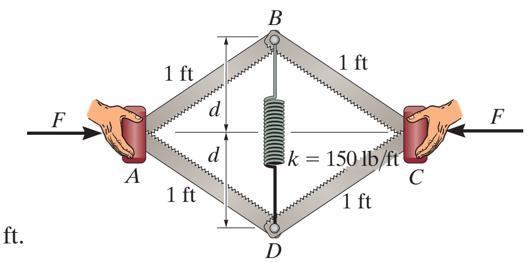


(b)

Ans:
$P = 198$ N

6–106.

If $d = 0.75$ ft and the spring has an unstretched length of 1 ft, determine the force F required for equilibrium.



SOLUTION

Spring Force Formula: The elongation of the spring is $x = 2(0.75) - 1 = 0.5$ ft. Thus, the force in the spring is given by

$$F_{sp} = kx = 150(0.5) = 75 \text{ lb}$$

Equations of Equilibrium: First, we will analyze the equilibrium of joint B. From the free-body diagram in Fig. a,

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad F_{AB} \cos 48.59° - F_{BC} \cos 48.59° = 0$

$$F_{AB} = F_{BC} = F'$$

$+\uparrow \Sigma F_y = 0;$ $\quad 2F' \sin 48.59° - 75 = 0$

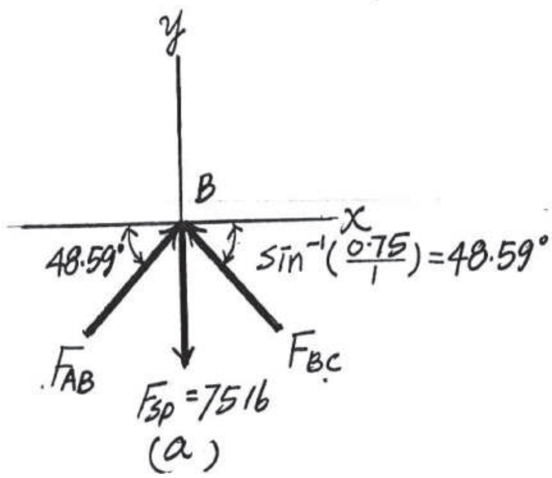
$$F' = 50 \text{ lb}$$

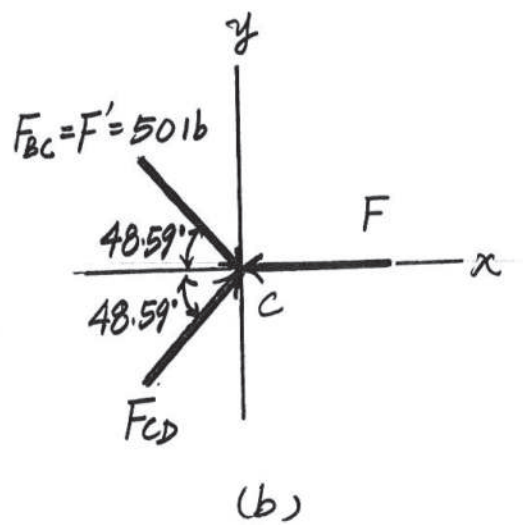
From the free-body diagram in Fig. b, using the result $F_{BC} = F' = 50$ lb, and analyzing the equilibrium of joint C, we have

$+\uparrow \Sigma F_y = 0;$ $\quad F_{CD} \sin 48.59° - 50 \sin 48.59° = 0 \qquad F_{CD} = 50 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 2(50 \cos 48.59°) - F = 0$

$$F = 66.14 \text{ lb} = 66.1 \text{ lb}$$ **Ans.**

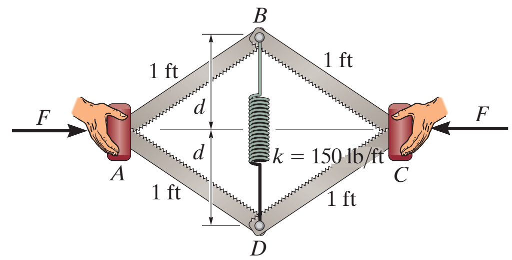




Ans:
$F = 66.1$ lb

6–107.

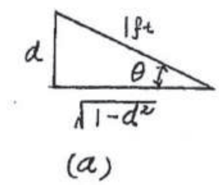
If a force of $F = 50$ lb is applied to the pads at A and C, determine the smallest dimension d required for equilibrium if the spring has an unstretched length of 1 ft.



SOLUTION

Geometry: From the geometry shown in Fig. *a*, we can write

$$\sin\theta = d \qquad \cos\theta = \sqrt{1 - d^2}$$



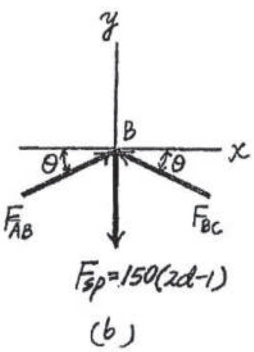
Spring Force Formula: The elongation of the spring is $x = 2d - 1$. Thus, the force in the spring is given by

$$F_{sp} = kx = 150(2d - 1)$$

***Equations of Equilibrium*:** First, we will analyze the equilibrium of joint B. From the free-body diagram in Fig. *b*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB}\cos\theta - F_{BC}\cos\theta = 0 \qquad F_{AB} = F_{BC} = F'$$

$$+\uparrow \Sigma F_y = 0; \qquad 2F'(d) - 150(2d - 1) = 0 \qquad F' = \frac{150d - 75}{d}$$

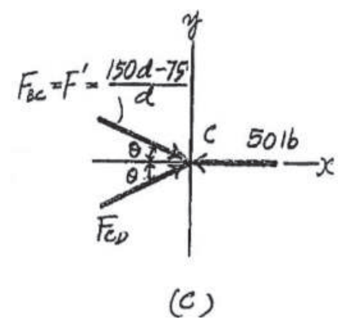


From the free-body diagram in Fig. c, using the result $F_{BC} = F' = \dfrac{150d - 75}{d}$, and analyzing the equilibrium of joint C, we have

$$+\uparrow \Sigma F_y = 0; \qquad F_{CD}\sin\theta - \left(\frac{150d - 75}{d}\right)\sin\theta = 0 \qquad F_{CD} = \frac{150d - 75}{d}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 2\left[\left(\frac{150d - 75}{d}\right)\left(\sqrt{1-d^2}\right)\right] - 50 = 0$$

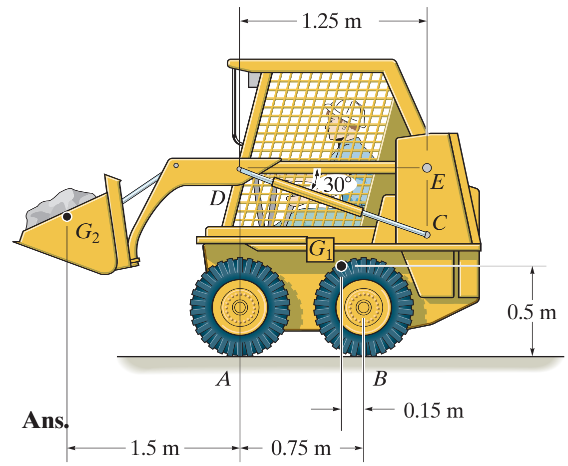
Solving the above equation using a graphing utility, we obtain **Ans.**
$d = 0.6381 \text{ ft} = 0.638 \text{ ft}$ or $d = 0.9334 \text{ ft} = 0.933 \text{ ft}$



Ans:
$d = 0.638$ ft

***6–108.**

The skid steer loader has a mass of 1.18 Mg, and in the position shown the center of mass is at G_1. If there is a 300-kg stone in the bucket, with center of mass at G_2, determine the reactions of each pair of wheels A and B on the ground and the force in the hydraulic cylinder CD and at the pin E. There is a similar linkage on each side of the loader.



SOLUTION

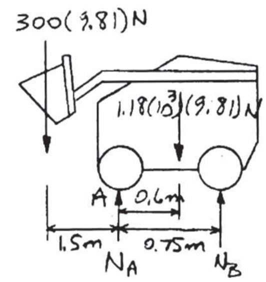
Entire system:

$\circlearrowleft + \Sigma M_A = 0;\quad 300(9.81)(1.5) - 1.18(10^3)(9.81)(0.6) + N_B(0.75) = 0$

$N_B = 3374.6\text{ N} = 3.37\text{ kN}$ (Both wheels) **Ans.**

$+\uparrow \Sigma F_y = 0;\quad 3374.6 - 300(9.81) - 1.18(10^3)(9.81) + N_A = 0$

$N_A = 11.1\text{ kN}$ (Both wheels) **Ans.**



Upper member:

$\circlearrowleft + \Sigma M_E = 0;\quad 300(9.81)(2.75) - F_{CD}\sin 30°(1.25) = 0$

$F_{CD} = 12\,949\text{ N} = 12.9\text{ kN}$

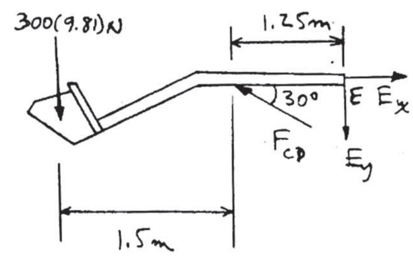
$F'_{CD} = \dfrac{F_{CD}}{2} = \dfrac{12\,949}{2} = 6.47\text{ kN}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad E_x - 12\,949\cos 30° = 0$

$E_x = 11\,214\text{ N}$

$+\uparrow \Sigma F_y = 0;\quad -E_y - 300(9.81) + 12\,949\sin 30° = 0$

$E_y = 3532\text{ N}$



$F_E = \sqrt{(11\,214)^2 + (3532)^2} = 11\,757\text{ N}$

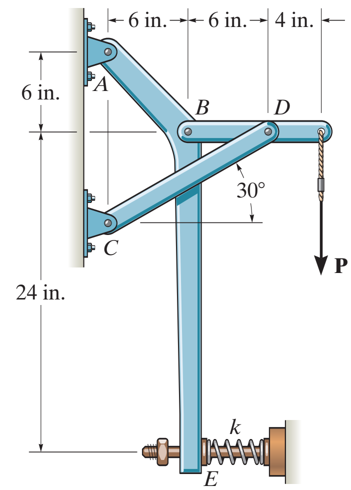
Since there are two members,

$F'_E = \dfrac{F_E}{2} = \dfrac{11\,757}{2} = 5.88\text{ kN}$ **Ans.**

Ans:
$N_A = 11.1\text{ kN}$ (Both wheels)
$F'_{CD} = 6.47\text{ kN}$
$F'_E = 5.88\text{ kN}$

6–109.

Determine the force P on the cable if the spring is compressed 0.5 in. when the mechanism is in the position shown. The spring has a stiffness of $k = 800$ lb/ft.



SOLUTION

$$F_E = ks = 800\left(\frac{0.5}{12}\right) = 33.33 \text{ lb}$$

$\circlearrowleft + \Sigma M_A = 0;\quad B_x(6) + B_y(6) - 33.33(30) = 0$

$B_x + B_y = 166.67 \text{ lb}$ **(1)**

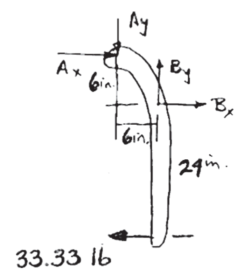
$\circlearrowleft + \Sigma M_D = 0;\quad B_y(6) - P(4) = 0$

$B_y = 0.6667P$ **(2)**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad -B_x + F_{CD}\cos 30° = 0$ **(3)**

$\circlearrowleft + \Sigma M_B = 0;\quad F_{CD}\sin 30°(6) - P(10) = 0$

$F_{CD} = 3.333\,P$



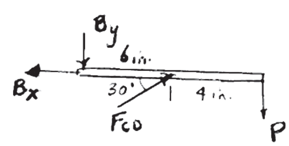
Thus from Eq. (3)

$B_x = 2.8867\,P$

Using Eqs. (1) and (2):

$2.8867\,P + 0.6667\,P = 166.67$

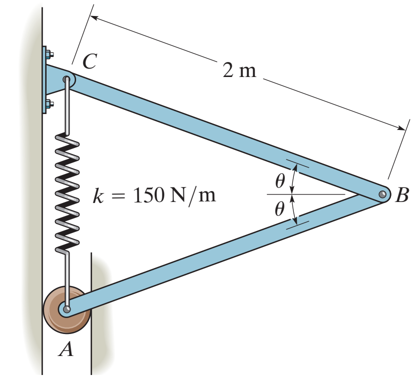
$P = 46.9 \text{ lb}$ **Ans.**



Ans:
$P = 46.9$ lb

6–110.

The spring has an unstretched length of 0.3 m. Determine the angle θ for equilibrium if the uniform bars each have a mass of 20 kg.



SOLUTION

Free Body Diagram. The assembly is being dismembered into members AB and BC of which their respective FBD are shown in Fig. b and a. Here, the spring stretches $x = 2(2\sin\theta) - 0.3 = 4\sin\theta - 0.3$. Thus, $F_{SP} = kx = 150\,(4\sin\theta - 0.3) = 600\sin\theta - 45$.

Equations of Equilibrium. Considered the equilibrium of member BC, Fig. a,

$$\circlearrowleft + \Sigma M_C = 0; \qquad B_y(2\cos\theta) - B_x(2\sin\theta) - 20(9.81)\cos\theta = 0 \qquad \textbf{(1)}$$

Also, member AB, Fig. b

$$\circlearrowleft + \Sigma M_A = 0; \qquad -B_y(2\cos\theta) - B_x(2\sin\theta) - 20(9.81)\cos\theta = 0 \qquad \textbf{(2)}$$

$$+\uparrow \Sigma F_y = 0; \qquad (600\sin\theta - 45) - 20(9.81) - B_y = 0 \qquad \textbf{(3)}$$

Solving Eq. (1) and (2)

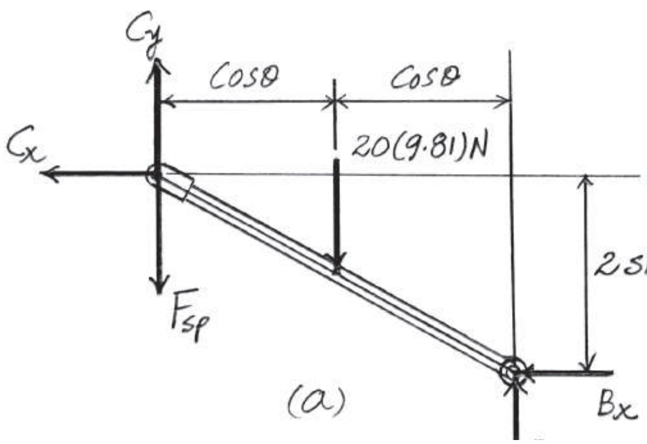
$$B_y = 0 \quad B_x = -\frac{9.81\cos\theta}{\sin\theta}$$

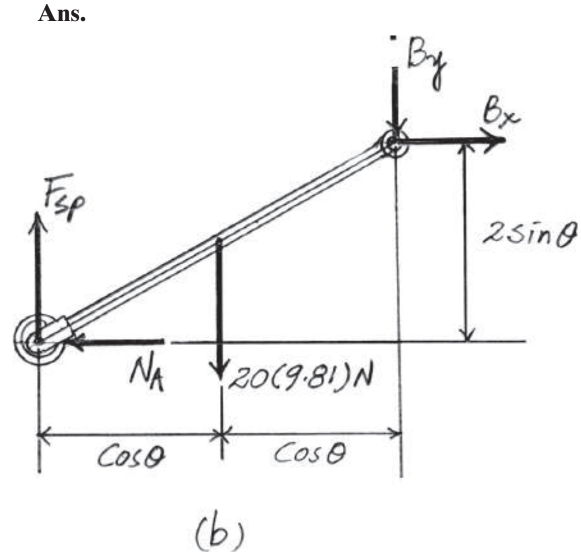
Substitute the result of $B_y = 0$ into Eq. (3)

$$600\sin\theta - 45 - 20(9.81) = 0$$

$$\sin\theta = 0.402$$

$$\theta = 23.7^\circ \qquad \textbf{Ans.}$$

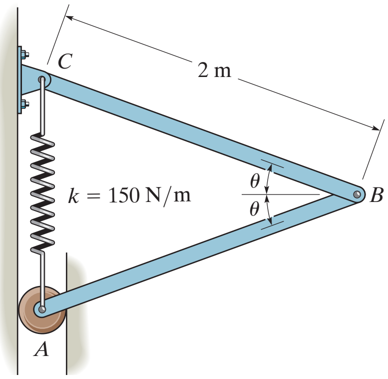




Ans:
$\theta = 23.7^\circ$

6–111.

The spring has an unstretched length of 0.3 m. Determine the mass m of each uniform bar if each angle $\theta = 30°$ for equilibrium.



SOLUTION

Free Body Diagram. The assembly is being dismembered into members AB and BC of which their respective FBD are shown in Fig. b and a. Here, the spring stretches $x = 2(2 \sin 30°) - 0.3 = 1.7$ m. Thus, $F_{SP} = kx = 150(1.7) = 255$ N.

Equations of Equilibrium. Consider the equilibrium of member BC, Fig. a,

$$\circlearrowleft + \Sigma M_C = 0; \qquad B_x(2 \sin 30°) + B_y(2 \cos 30°) - \text{m}(9.81) \cos 30° = 0 \qquad \textbf{(1)}$$

Also, member AB, Fig. b

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_x(2 \sin 30°) - B_y(2 \cos 30°) - \text{m}(9.81) \cos 30° = 0 \qquad \textbf{(2)}$$

$$+\uparrow \Sigma F_y = 0; \qquad 255 - \text{m}(9.81) - B_y = 0 \qquad \textbf{(3)}$$

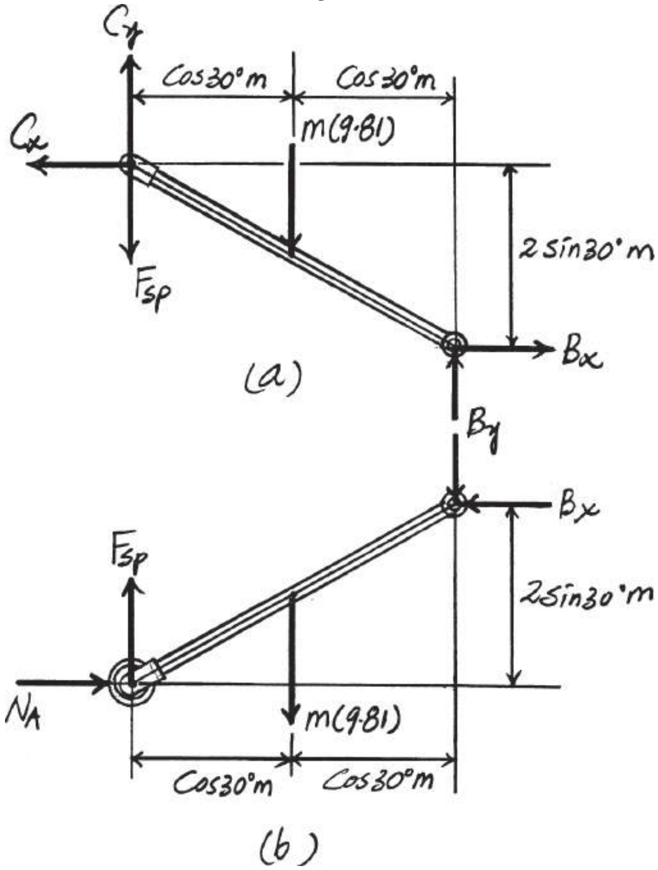
Solving Eqs. (1) and (2)

$$B_x = 8.4957 \text{ m} \quad B_y = 0$$

Substitute the result of $B_y = 0$ into Eq. (3)

$$255 - m(9.81) = 0$$

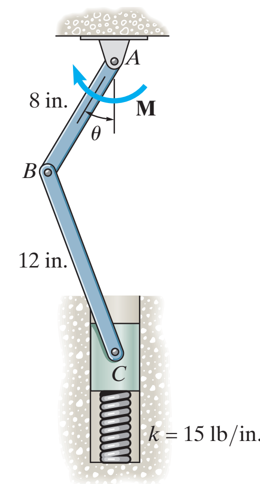
$$m = 26.0 \text{ kg} \qquad \textbf{Ans.}$$



Ans:
$m = 26.0$ kg

***6–112.**

The piston C moves vertically between the two smooth walls. If the spring has a stiffness of $k = 15$ lb/in., and is unstretched when $\theta = 0°$, determine the couple **M** that must be applied to AB to hold the mechanism in equilibrium when $\theta = 30°$.



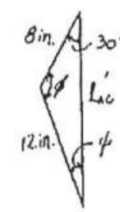
SOLUTION

Geometry:

$$\frac{\sin\psi}{8} = \frac{\sin 30°}{12} \qquad \psi = 19.47°$$

$$\phi = 180° - 30° - 19.47 = 130.53°$$

$$\frac{l'_{AC}}{\sin 130.53°} = \frac{12}{\sin 30°} \qquad l'_{AC} = 18.242 \text{ in.}$$

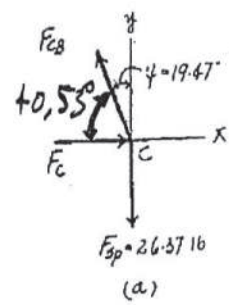


Free Body Diagram: The solution for this problem will be simplified if one realizes that member CB is a two force member. Since the spring stretches $x = l_{AC} - l'_{AC} = 20 - 18.242 = 1.758$ in. the spring force is $F_{sp} = kx = 15\,(1.758) = 26.37$ lb.

Equations of Equilibrium: Using the method of joints, [FBD (a)],

$$+\uparrow \Sigma F_y = 0; \qquad F_{CB}\cos 19.47° - 26.37 = 0$$

$$F_{CB} = 27.97 \text{ lb}$$



From FBD (b),

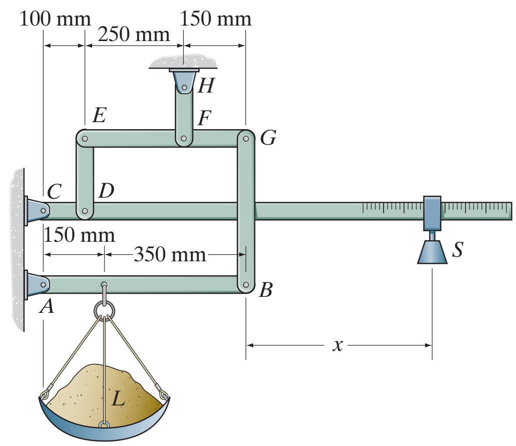
$$\circlearrowleft + \Sigma M_A = 0; \qquad 27.97\cos 40.53°\,(8) - M = 0$$

$$M = 170.08 \text{ lb}\cdot\text{in} = 14.2 \text{ lb}\cdot\text{ft}$$ **Ans.**

Ans:
$M = 14.2$ lb · ft

6–113.

The platform scale consists of a combination of third and first class levers so that the load on one lever becomes the effort that moves the next lever. Through this arrangement, a small weight can balance a massive object. If $x = 450$ mm, determine the required mass of the counterweight S required to balance a 90-kg load, L.



SOLUTION

Equations of Equilibrium: Applying the moment equation of equilibrium about point A to the free - body diagram of member AB in Fig. a,

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BG}(500) - 90(9.81)(150) = 0$$

$$F_{BG} = 264.87 \text{ N}$$

Using the result of F_{BG} and writing the moment equation of equilibrium about point F on the free - body diagram of member EFG in Fig. b,

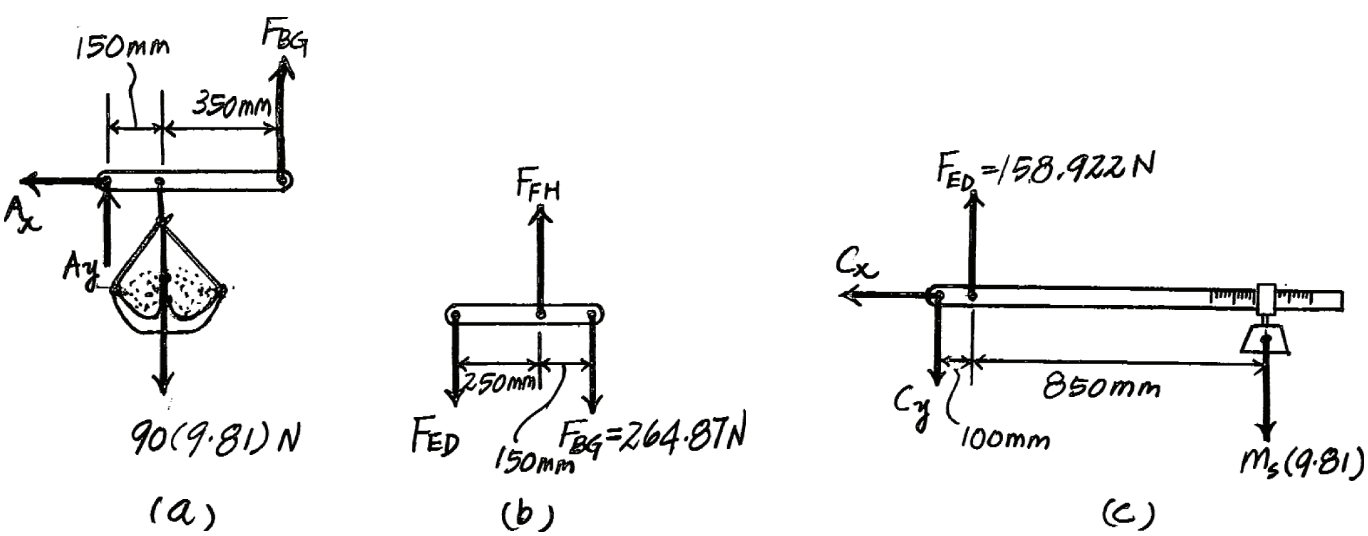
$$\circlearrowleft + \Sigma M_F = 0; \qquad F_{ED}(250) - 264.87(150) = 0$$

$$F_{ED} = 158.922 \text{ N}$$

Using the result of F_{ED} and writing the moment equation of equilibrium about point C on the free - body diagram of member CDI in Fig. c,

$$\circlearrowleft + \Sigma M_C = 0; \qquad 158.922(100) - m_S(9.81)(950) = 0$$

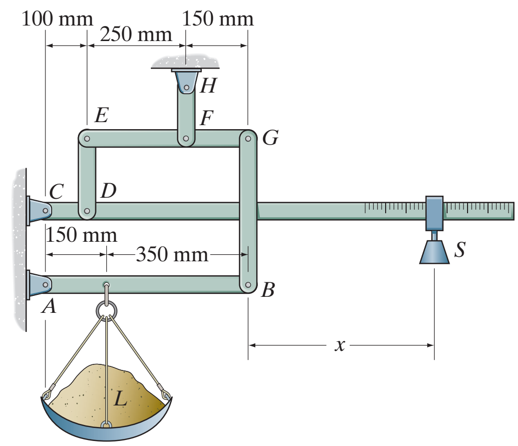
$$m_S = 1.705 \text{ kg} = 1.71 \text{ kg} \qquad \textbf{Ans.}$$



Ans:
$m_S = 1.71$ kg

6–114.

The platform scale consists of a combination of third and first class levers so that the load on one lever becomes the effort that moves the next lever. Through this arrangement, a small weight can balance a massive object. If $x = 450$ mm and, the mass of the counterweight S is 2 kg, determine the mass of the load L required to maintain the balance.



SOLUTION

Equations of Equilibrium: Applying the moment equation of equilibrium about point A to the free - body diagram of member AB in Fig. a,

$\circlearrowleft + \Sigma M_A = 0;$ $\quad F_{BG}(500) - M_L(9.81)(150) = 0$

$F_{BG} = 2.943$ lb

Using the result of F_{BG} and writing the moment equation of equilibrium about point F on the free - body diagram of member EFG in Fig. b,

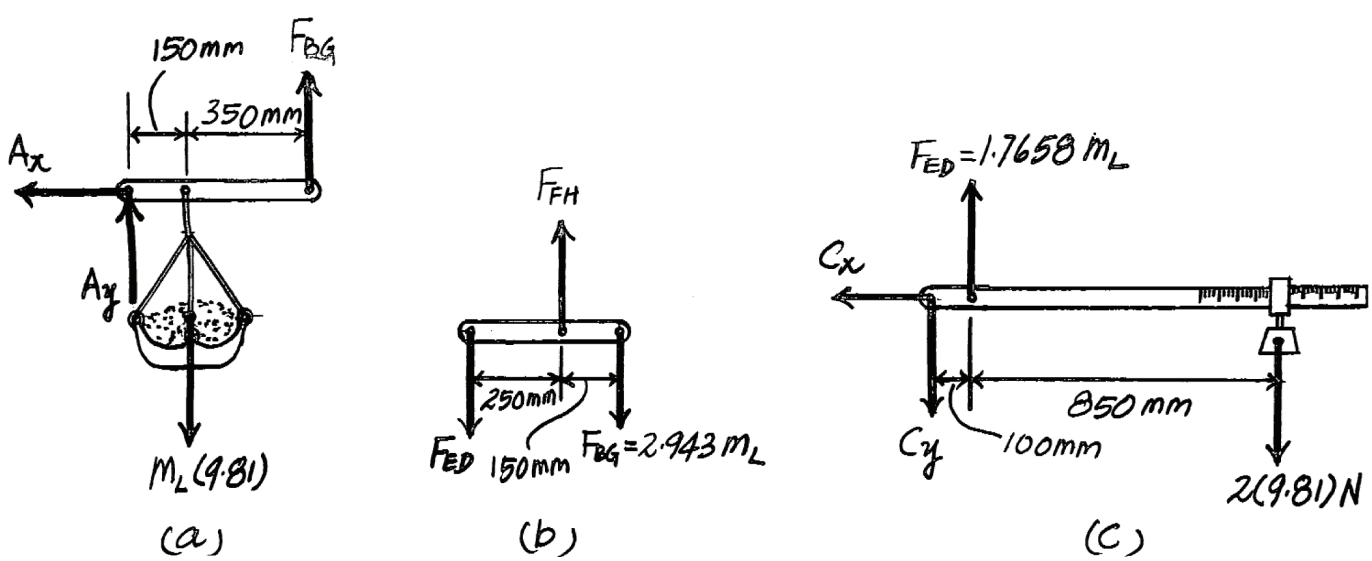
$\circlearrowleft + \Sigma M_F = 0;$ $\quad F_{ED}(250) - 2.943m_L(150) = 0$

$F_{ED} = 1.7658m_L$

Using the result of F_{ED} and writing the moment equation of equilibrium about point C on the free - body diagram of member CDI in Fig. c,

$\circlearrowleft + \Sigma M_C = 0;$ $\quad 1.7658m_L(100) - 2(9.81)(950) = 0$

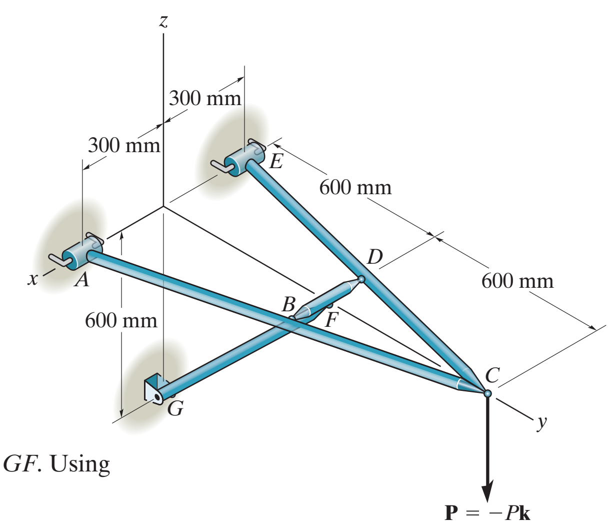
$m_L = 105.56 \text{ kg} = 106 \text{ kg}$ **Ans.**



Ans:
$m_L = 106$ kg

6–115.

The four-member “A” frame is supported at A and E by smooth collars and at G by a pin. All the other joints are ball-and-sockets. If the pin at G will fail when the resultant force there is 800 N, determine the largest vertical force P that can be supported by the frame. Also, what are the x, y, z force components which member BD exerts on members EDC and ABC? The collars at A and E and the pin at G only exert force components on the frame.



SOLUTION

GF is a two - force member, so the 800 - N force acts along the axis of GF. Using FBD (a),

$\Sigma M_x = 0;\qquad -P(1.2) + 800 \sin 45°(0.6) = 0$

$P = 283 \text{ N}$ **Ans.**

$\Sigma M_z = 0;\qquad -A_y(0.3) + E_y(0.3) = 0$

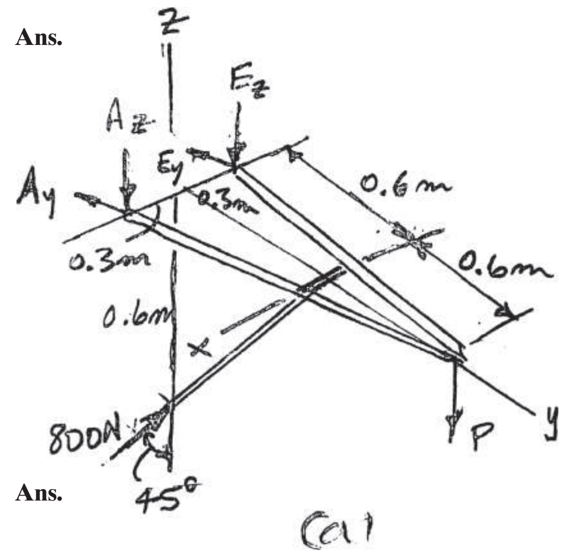
$\Sigma F_y = 0;\qquad -A_y - E_y + 800 \sin 45° = 0$

$A_y = E_y = 283 \text{ N}$

$\Sigma M_x = 0;\qquad A_z(0.6) + E_z(0.6) - 283((0.6) = 0$

$\Sigma M_y = 0;\qquad A_z(0.3) - E_z(0.3) = 0$

$A_z = E_z = 118 \text{ N}$



Using FBD (b),

$\Sigma F_y = 0;\qquad -B_y - D_y + 800 \sin 45° = 0$

$\Sigma M_z = 0;\qquad D_y(0.3) - B_y(0.3) = 0$

$B_y = D_y = 283 \text{ N}$ **Ans.**

$\Sigma F_z = 0;\qquad -B_z - D_z + 800 \cos 45° = 0$

$\Sigma M_y = 0;\qquad -D_z(0.3) + B_z(0.3) = 0$

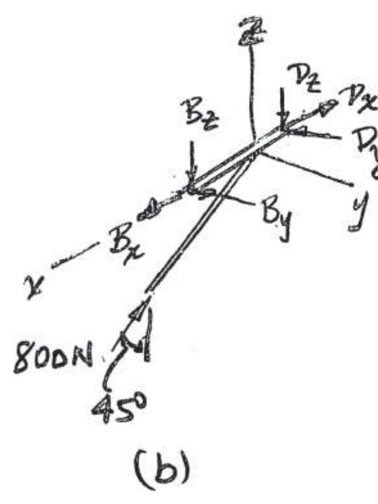
$B_z = D_z = 283 \text{ N}$ **Ans.**

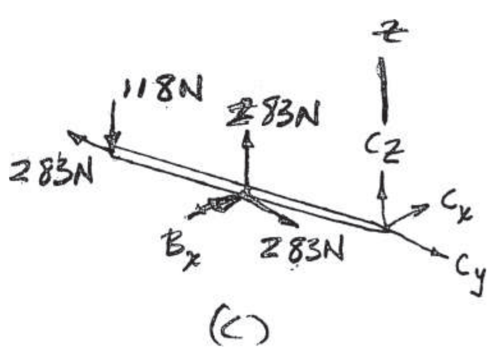
$\Sigma F_x = 0;\qquad -B_x + D_x = 0$

Using FBD (c),

$\Sigma M_z = 0;\qquad -B_y(0.6) + 283(0.15) - 283(0.3) = 0$

$B_x = D_x = 42.5 \text{ N}$ **Ans.**

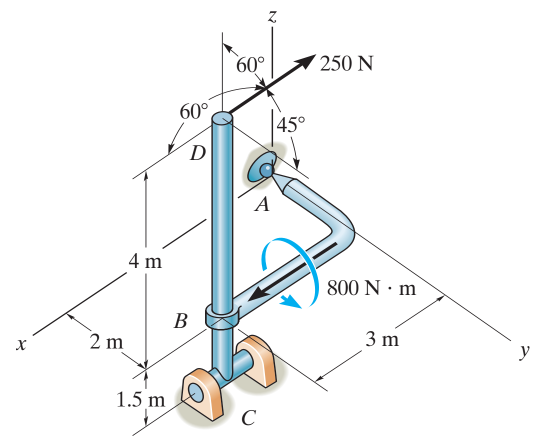




Ans:
$P = 283 \text{ N}$
$B_x = D_x = 42.5 \text{ N}$
$B_y = D_y = 283 \text{ N}$
$B_z = D_z = 283 \text{ N}$

***6–116.**

The structure is subjected to the loadings shown. Member AB is supported by a ball-and-socket at A and smooth collar at B. Member CD is supported by a pin at C. Determine the x, y, z components of reaction at A and C.



SOLUTION

From FBD (a)

$\Sigma M_y = 0;\qquad M_{By} = 0$

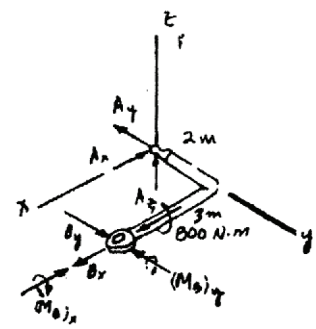
$\Sigma M_x = 0;\qquad -M_{Bx} + 800 = 0 \qquad M_{Bx} = 800\ \text{N}\cdot\text{m}$

$\Sigma M_z = 0;\qquad B_y(3) - B_x(2) = 0$ **(1)**

$\Sigma F_z = 0;\qquad A_z = 0$ **Ans.**

$\Sigma F_x = 0;\qquad -A_x + B_x = 0$ **(2)**

$\Sigma F_y = 0;\qquad -A_y + B_y = 0$ **(3)**



(a)

From FBD (b)

$\Sigma M_g = 0;\qquad B_y(1.5) + 800 - 250\cos 45°(5.5) = 0 \qquad B_y = 114.85\ \text{N}$

From Eq.(1) $\quad 114.85(3) - B_x(2) = 0 \qquad B_x = 172.27\ \text{N}$

From Eq.(2) $\quad A_x = 172\ \text{N}$ **Ans.**

From Eq.(3) $\quad A_y = 115\ \text{N}$ **Ans.**

$\Sigma F_x = 0;\qquad C_x + 250\cos 60° - 172.27 = 0 \qquad C_x = 47.3\ \text{N}$ **Ans.**

$\Sigma F_y = 0;\qquad 250\cos 45° - 114.85 - C_y = 0 \qquad C_y = 61.9\text{N}$ **Ans.**

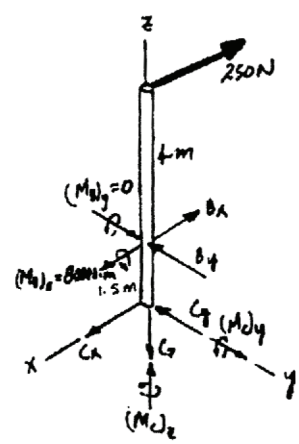
$\Sigma F_z = 0;\qquad 250\cos 60° - C_z = 0 \qquad C_z = 125\ \text{N}$ **Ans.**

$\Sigma M_y = 0;\qquad M_{Cy} - 172.27(1.5) + 250\cos 60°(5.5) = 0$

$\qquad M_{Cy} = -429\ \text{N}\cdot\text{m}$ **Ans.**

$\Sigma M_z = 0;\qquad M_{Cz} = 0$ **Ans.**

Negative sign indicates that M_{Cy} acts in the opposite sense to that shown on FBD.

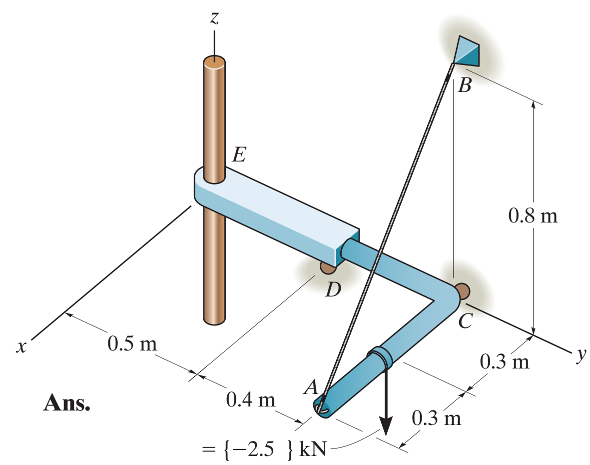


(b)

Ans:
$A_z = 0$
$A_x = 172\ \text{N}$
$A_y = 115\ \text{N}$
$C_x = 47.3\ \text{N}$
$C_y = 61.9\ \text{N}$
$C_z = 125\ \text{N}$
$M_{Cy} = -429\ \text{N}\cdot\text{m}$
$M_{Cz} = 0$

6–117.

The structure is subjected to the loading shown. Member AD is supported by a cable AB and roller at C and fits through a smooth circular hole at D. Member ED is supported by a roller at D and a pole that fits in a smooth snug circular hole at E. Determine the x, y, z components of reaction at E and the tension in cable AB.



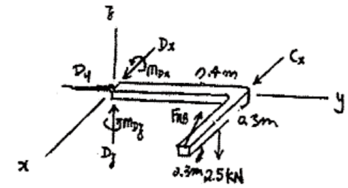
SOLUTION

$\Sigma M_y = 0;\quad -\frac{4}{5}F_{AB}(0.6) + 2.5(0.3) = 0$

$F_{AB} = 1.5625 = 1.56\text{ kN}$ **Ans.**

$\Sigma F_z = 0;\quad \frac{4}{5}(1.5625) - 2.5 + D_z = 0$

$D_z = 1.25\text{ kN}$



$\Sigma F_y = 0;\quad D_y = 0$

$\Sigma F_x = 0;\quad D_x + C_x - \frac{3}{5}(1.5625) = 0$ **(1)**

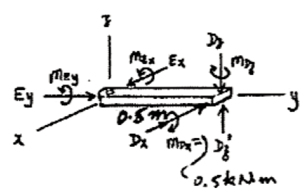
$\Sigma M_x = 0;\quad M_{Dx} + \frac{4}{5}(1.5625)(0.4) - 2.5(0.4) = 0$

$M_{Dx} = 0.5\text{ kN}\cdot\text{m}$

$\Sigma M_z = 0;\quad M_{Dz} + \frac{3}{5}(1.5625)(0.4) - C_x(0.4) = 0$ **(2)**

$\Sigma F_z = 0;\quad D_{z'} = 1.25\text{ kN}$

$\Sigma M_x = 0;\quad M_{Ex} = 0.5\text{ kN}\cdot\text{m}$ **Ans.**



$\Sigma M_y = 0;\quad M_{Ey} = 0$ **Ans.**

$\Sigma F_y = 0;\quad E_y = 0$ **Ans.**

$\Sigma M_z = 0;\quad E_x(0.5) - M_{Dz} = 0$ **(3)**

Solving Eqs. (1), (2) and (3):

$C_x = 0.938\text{ kN}$

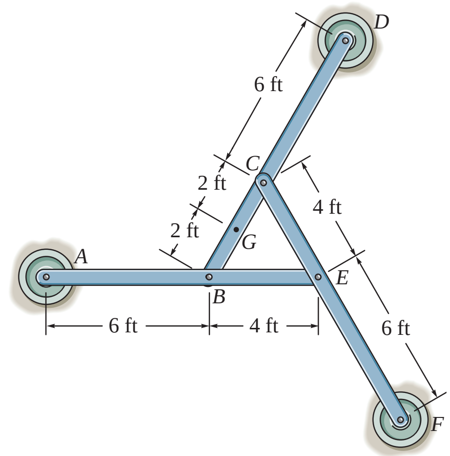
$M_{Dz} = 0$

$E_x = 0$ **Ans.**

Ans:
$M_{Ex} = 0.5\text{ kN}\cdot\text{m}$
$M_{Ey} = 0$
$E_y = 0$
$E_x = 0$

6–118.

The three pin-connected members shown in the *top view* support a downward force of 60 lb at G. If only vertical forces are supported at the connections B, C, E and pad supports A, D, F, determine the reactions at each pad.

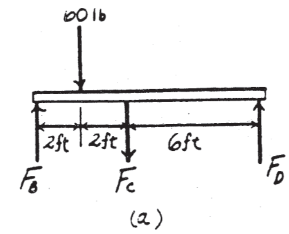


SOLUTION

Equations of Equilibrium : From FBD (a),

$\circlearrowleft +\Sigma M_D = 0;$ $\quad 60(8) + F_C(6) - F_B(10) = 0$ **(1)**

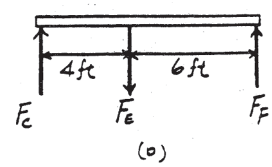
$+\uparrow \Sigma F_y = 0;$ $\quad F_B + F_D - F_C - 60 = 0$ **(2)**



From FBD (b),

$\circlearrowleft +\Sigma M_F = 0;$ $\quad F_E(6) - F_C(10) = 0$ **(3)**

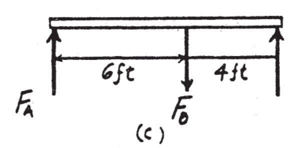
$+\uparrow \Sigma F_y = 0;$ $\quad F_C + F_F - F_E = 0$ **(4)**



From FBD (c),

$\circlearrowleft +\Sigma M_A = 0;$ $\quad F_E(10) - F_B(6) = 0$ **(5)**

$+\uparrow \Sigma F_y = 0;$ $\quad F_A + F_E - F_B = 0$ **(6)**



Solving Eqs. (1), (2), (3), (4), (5) and (6) yields,

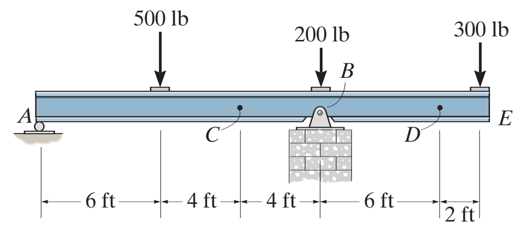
$F_E = 36.73 \text{ lb} \qquad F_C = 22.04 \text{ lb} \qquad F_B = 61.22 \text{ lb}$

$F_D = 20.8 \text{ lb} \qquad F_F = 14.7 \text{ lb} \qquad F_A = 24.5 \text{ lb}$ **Ans.**

Ans:
$F_D = 20.8$ lb
$F_F = 14.7$ lb
$F_A = 24.5$ lb

7–1.

Determine the shear force and moment at points C and D.



SOLUTION

Support Reactions: FBD (a).

$\zeta + \Sigma M_B = 0; \qquad 500(8) - 300(8) - A_y(14) = 0$

$A_y = 114.29 \text{ lb}$

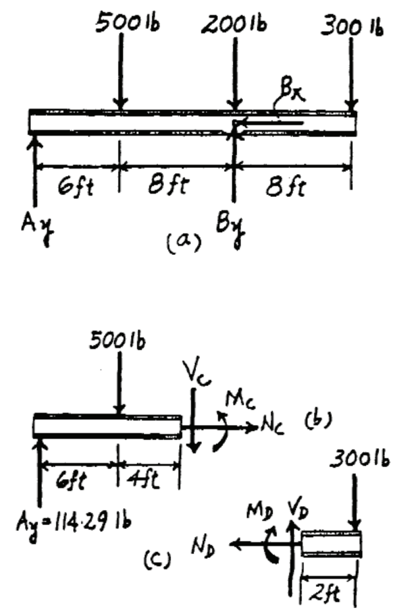
Internal Forces: Applying the equations of equilibrium to segment AC [FBD (b)], we have

$\xrightarrow{+} \Sigma F_x = 0 \qquad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad 114.29 - 500 - V_C = 0 \qquad V_C = -386 \text{ lb}$ **Ans.**

$\zeta + \Sigma M_C = 0; \qquad M_C + 500(4) - 114.29(10) = 0$

$M_C = -857 \text{ lb} \cdot \text{ft}$ **Ans.**



Applying the equations of equilibrium to segment ED [FBD (c)], we have

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_D = 0$ **Ans.**

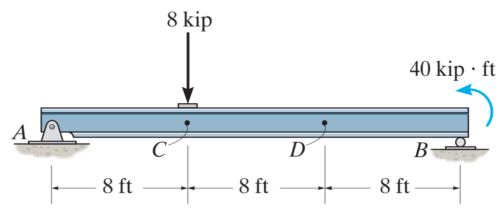
$+\uparrow \Sigma F_y = 0; \qquad V_D - 300 = 0 \qquad V_D = 300 \text{ lb}$ **Ans.**

$\zeta + \Sigma M_D = 0; \qquad -M_D - 300(2) = 0 \qquad M_D = -600 \text{ lb} \cdot \text{ft}$ **Ans.**

Ans:
$N_C = 0$
$V_C = -386 \text{ lb}$
$M_C = -857 \text{ lb} \cdot \text{ft}$
$N_D = 0$
$V_D = 300 \text{ lb}$
$M_D = -600 \text{ lb} \cdot \text{ft}$

7–2.

Determine the internal normal force and shear force, and the bending moment in the beam at points C and D. Assume the support at B is a roller. Point C is located just to the right of the 8-kip load.



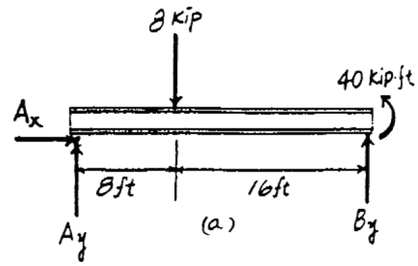
SOLUTION

Support Reactions: FBD (a).

$\circlearrowleft + \Sigma M_A = 0; \quad B_y(24) + 40 - 8(8) = 0 \quad B_y = 1.00 \text{ kip}$

$+\uparrow \Sigma F_y = 0; \quad A_y + 1.00 - 8 = 0 \quad A_y = 7.00 \text{ kip}$

$\xrightarrow{+} \Sigma F_x = 0 \quad A_x = 0$

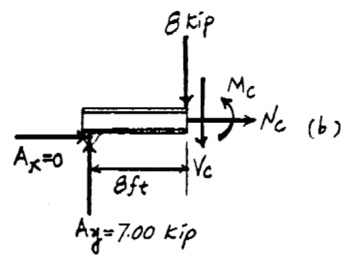


Internal Forces: Applying the equations of equilibrium to segment AC [FBD (b)], we have

$\xrightarrow{+} \Sigma F_x = 0 \quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 7.00 - 8 - V_C = 0 \quad V_C = -1.00 \text{ kip}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \quad M_C - 7.00(8) = 0 \quad M_C = 56.0 \text{ kip} \cdot \text{ft}$ **Ans.**



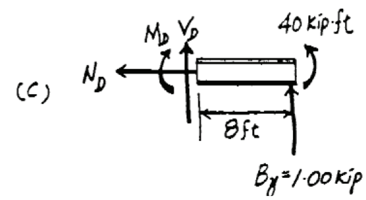
Applying the equations of equilibrium to segment BD [FBD (c)], we have

$\xrightarrow{+} \Sigma F_x = 0; \quad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_D + 1.00 = 0 \quad V_D = -1.00 \text{ kip}$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0; \quad 1.00(8) + 40 - M_D = 0$

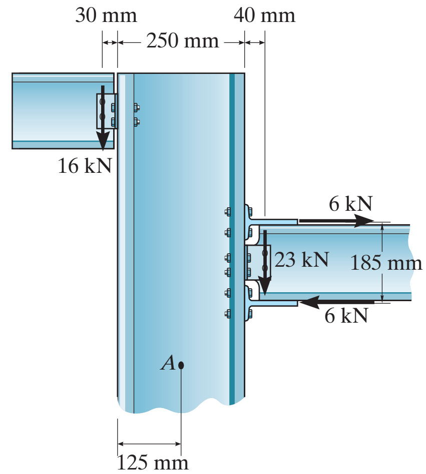
$M_D = 48.0 \text{ kip} \cdot \text{ft}$ **Ans.**



Ans:
$N_C = 0$
$V_C = -1.00 \text{ kip}$
$M_C = 56.0 \text{ kip} \cdot \text{ft}$
$N_D = 0$
$V_D = -1.00 \text{ kip}$
$M_D = 48.0 \text{ kip} \cdot \text{ft}$

7–3.

Two beams are attached to the column such that structural connections transmit the loads shown. Determine the internal normal force, shear force, and moment acting in the column at a section passing horizontally through point A.



SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \qquad 6 - 6 - V_A = 0$

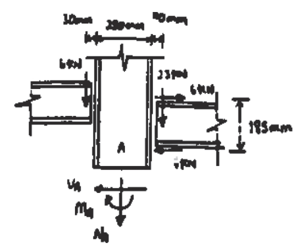
$V_A = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad -N_A - 16 - 23 = 0$

$N_A = -39\ \text{kN}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0; \qquad -M_A + 16(0.155) - 23(0.165) - 6(0.185) = 0$

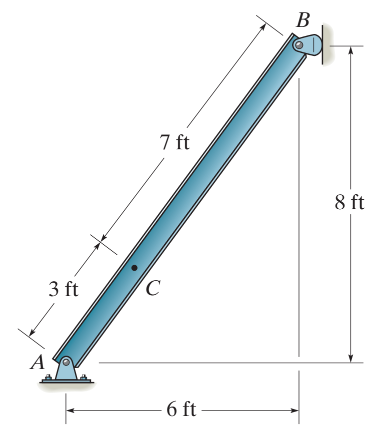
$M_A = -2.42\ \text{kN}\cdot\text{m}$ **Ans.**



Ans:
$V_A = 0$
$N_A = -39\ \text{kN}$
$M_A = -2.425\ \text{kN}\cdot\text{m}$

***7–4.**

The beam weighs 280 lb/ft. Determine the internal normal force, shear force, and moment at point C.



SOLUTION

Entire beam :

$\circlearrowleft + \Sigma M_A = 0;\quad -2.8\,(3) + B_x\,(8) = 0$

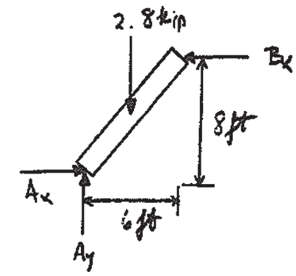
$B_x = 1.05 \text{ kip}$

$\xrightarrow{+} \Sigma F_x = 0;\quad A_x - 1.05 = 0$

$A_x = 1.05 \text{ kip}$

$+\uparrow \Sigma F_y = 0;\quad A_y - 2.8 = 0$

$A_y = 2.80 \text{ kip}$



Segment AC :

$+\nearrow \Sigma F_x = 0;\quad -N_C + 1.05 \cos 53.13° + 2.80 \sin 53.13° - 0.84 \sin 53.13° = 0$

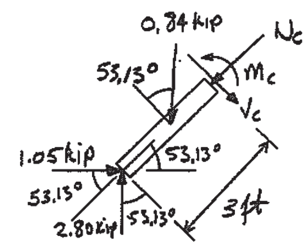
$N_C = 2.20 \text{ kip}$ **Ans.**

$\nwarrow + \Sigma F_y = 0;\quad -V_C - 0.84 \cos 53.13° + 2.80 \cos 53.13° - 1.05 \sin 53.13° = 0$

$V_C = 0.336 \text{ kip}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;\quad -2.80 \cos 53.13°\,(3) + 1.05 \sin 53.13°\,(3) + 0.84 \cos 53.13°\,(1.5) + M_C = 0$

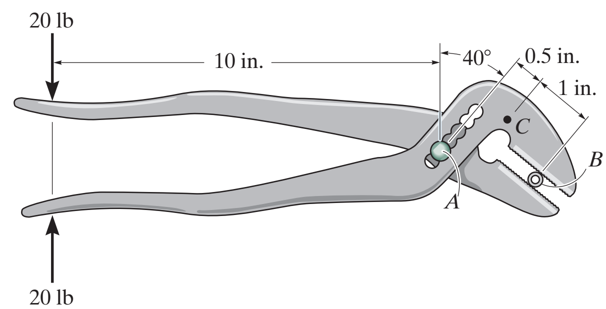
$M_C = 1.76 \text{ kip} \cdot \text{ft}$ **Ans.**



Ans:
$N_C = 2.20 \text{ kip}$
$V_C = 0.336 \text{ kip}$
$M_C = 1.76 \text{ kip} \cdot \text{ft}$

7–5.

The pliers are used to grip the tube at B. If a force of 20 lb is applied to the handles, determine the internal shear force and moment at point C. Assume the jaws of the pliers exert only normal forces on the tube.



SOLUTION

$+\Sigma M_A = 0;\qquad -20(10) + R_B(1.5) = 0$

$R_B = 133.3 \text{ lb}$

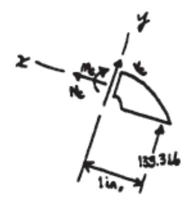
Segment BC:

$+\nearrow\Sigma F_y = 0;\qquad V_C + 133.3 = 0$

$V_C = -133 \text{ lb}$ **Ans.**

$\circlearrowleft +\Sigma M_C = 0;\qquad -M_C + 133.3\,(1) = 0$

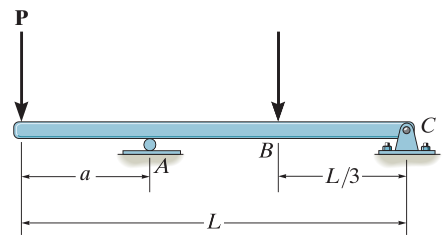
$M_C = 133 \text{ lb} \cdot \text{in.}$ **Ans.**



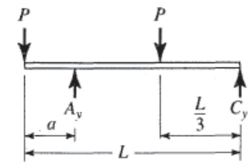
Ans:
$V_C = -133 \text{ lb}$
$M_C = 133 \text{ lb} \cdot \text{in.}$

7–6.

Determine the distance a as a fraction of the beam's length L for locating the roller support so that the moment in the beam at B is zero.



SOLUTION



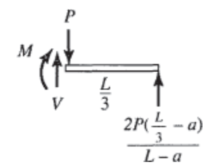
$$\zeta + \Sigma M_A = 0; \quad -P\left(\frac{2L}{3} - a\right) + C_y(L - a) + Pa = 0$$

$$C_y = \frac{2P\left(\frac{L}{3} - a\right)}{L - a}$$

$$\zeta + \Sigma M = 0; \quad M = \frac{2P\left(\frac{L}{3} - a\right)}{L - a}\left(\frac{L}{3}\right) = 0$$

$$2PL\left(\frac{L}{3} - a\right) = 0$$

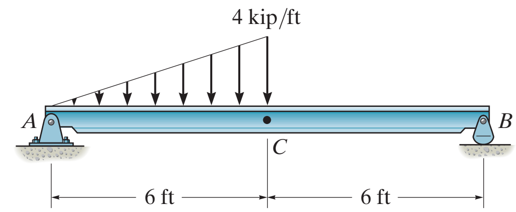
$$a = \frac{L}{3} \qquad \textbf{Ans.}$$



Ans:
$a = \dfrac{L}{3}$

7–7.

Determine the internal shear force and moment acting at point C in the beam.



SOLUTION

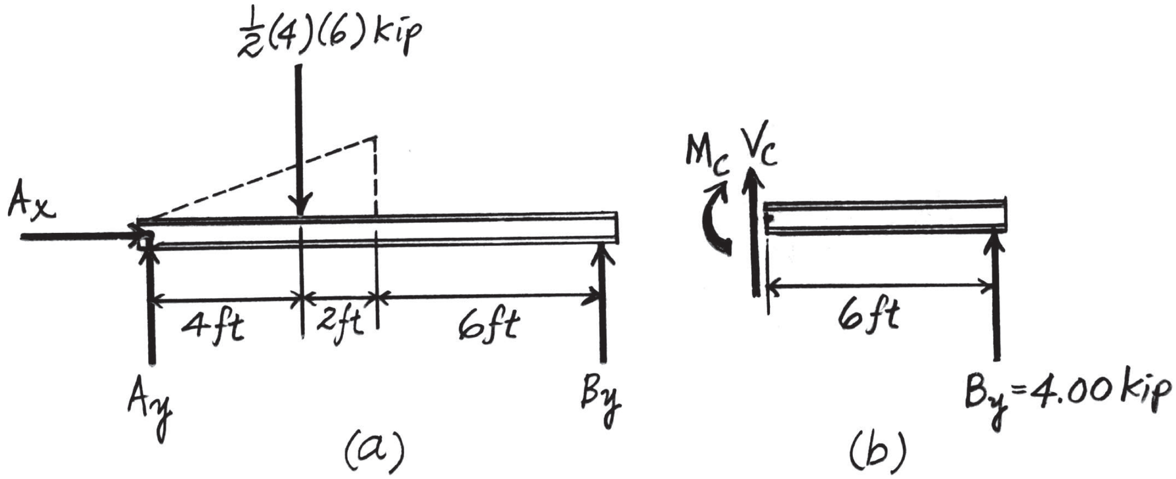
Support Reactions. Referring to the *FBD* of the entire beam shown in Fig. *a*,

$\zeta + \Sigma M_A = 0; \quad B_y(12) - \frac{1}{2}(4)(6)(4) = 0 \quad B_y = 4.00 \text{ kip}$

Internal Loadings. Referring to the *FBD* of the right segment of the beam sectioned through C, Fig. b,

$+\uparrow \Sigma F_y = 0; \quad V_C + 4.00 = 0 \quad V_C = -4.00 \text{ kip}$ **Ans.**

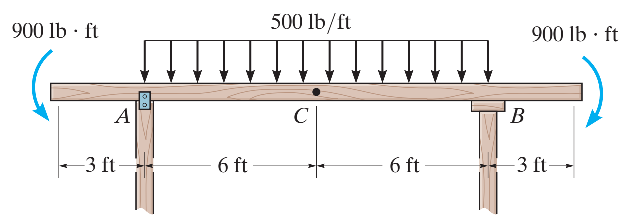
$\zeta + \Sigma M_C = 0; \quad 4.00(6) - M_C = 0 \quad M_C = 24.0 \text{ kip} \cdot \text{ft}$ **Ans.**



Ans:
$V_C = -4.00$ kip
$M_C = 24.0$ kip · ft

***7–8.**

Determine the internal shear force and moment acting at point C in the beam.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire beam shown in Fig. *a*

$\circlearrowleft + \Sigma M_B = 0;\quad 500(12)(6) + 900 - 900 - A_y(12) = 0 \quad A_y = 3000 \text{ lb}$

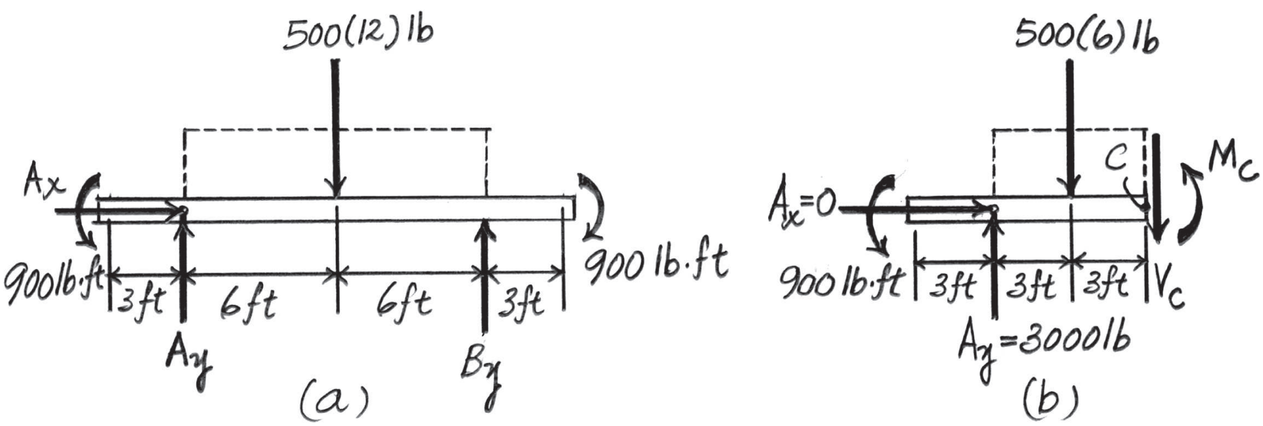
$\xrightarrow{+} \Sigma F_x = 0;\quad A_x = 0$

Internal Loadings. Referring to the *FBD* of the left segment of beam sectioned through C, Fig. *b*,

$+\uparrow \Sigma F_y = 0;\quad 3000 - 500(6) - V_C = 0 \quad V_C = 0$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;\quad M_C + 500(6)(3) + 900 - 3000(6) = 0$

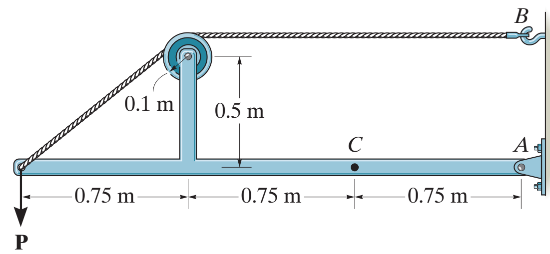
$M_C = 8100 \text{ lb} \cdot \text{ft} = 8.10 \text{ kip} \cdot \text{ft}$ **Ans.**



Ans:
$V_C = 0$
$M_C = 8.10 \text{ kip} \cdot \text{ft}$

7–9.

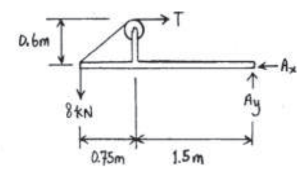
Determine the normal force, shear force, and moment at a section passing through point C. Take $P = 8$ kN.



SOLUTION

$\zeta + \Sigma M_A = 0; \quad -T(0.6) + 8(2.25) = 0$

$T = 30$ kN



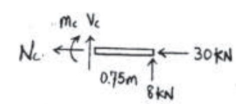
$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 30$ kN

$+\uparrow \Sigma F_y = 0; \quad A_y = 8$ kN

$\xrightarrow{+} \Sigma F_x = 0; \quad -N_C - 30 = 0$

$N_C = -30$ kN **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_C + 8 = 0$



$V_C = -8$ kN **Ans.**

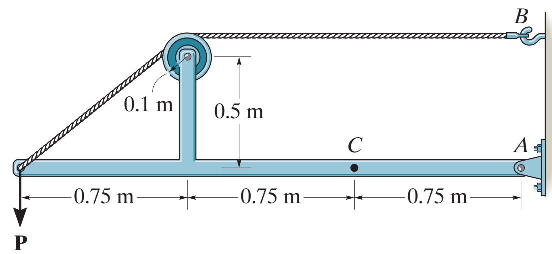
$\zeta + \Sigma M_C = 0; \quad -M_C + 8(0.75) = 0$

$M_C = 6 \text{ kN} \cdot \text{m}$ **Ans.**

Ans:
$N_C = -30$ kN
$V_C = -8$ kN
$M_C = 6 \text{ kN} \cdot \text{m}$

7–10.

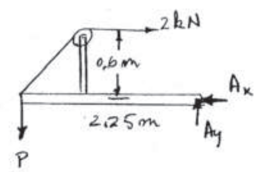
The cable will fail when subjected to a tension of 2 kN. Determine the largest vertical load P the frame will support and calculate the internal normal force, shear force, and moment at a section passing through point C for this loading.



SOLUTION

$\zeta + \Sigma M_A = 0; \quad -2(0.6) + P(2.25) = 0$

$P = 0.533 \text{ kN}$ **Ans.**



$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 2 \text{ kN}$

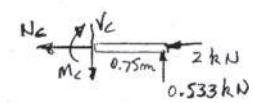
$+\uparrow \Sigma F_y = 0; \quad A_y = 0.533 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad -N_C - 2 = 0$

$N_C = -2 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_C - 0.533 = 0$

$V_C = -0.533 \text{ kN}$ **Ans.**



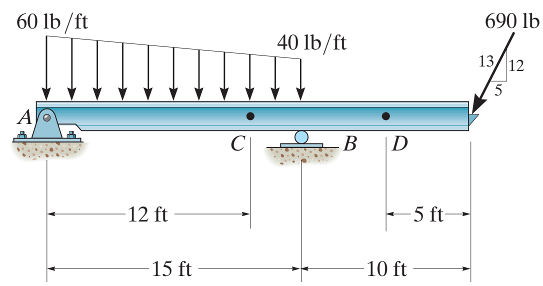
$\zeta + \Sigma M_C = 0; \quad -M_C + 0.533(0.75) = 0$

$M_C = 0.400 \text{ kN} \cdot \text{m}$ **Ans.**

Ans:
$P = 0.533$ kN
$N_C = -2$ kN
$V_C = -0.533$ kN
$M_C = 0.400$ kN·m

7–11.

Determine the internal normal force, shear force, and moment at points C and D of the beam.

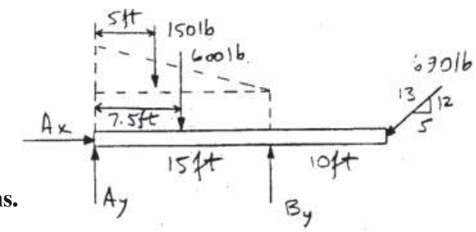


SOLUTION

Entire beam:

$\zeta + \Sigma M_A = 0;$ $\quad -150(5) - 600(7.5) + B_y(15) - \frac{12}{13}(690)(25) = 0$

$B_y = 1411.54 \text{ lb}$



Segment CBD:

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad -N_C - \frac{5}{13}(690) = 0$

$N_C = -265 \text{ lb}$ **Ans.**

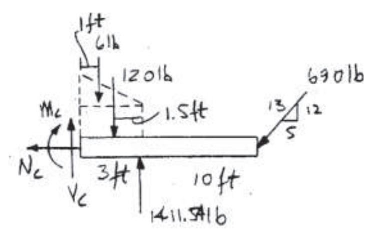
$+\uparrow \Sigma F_y = 0;$ $\quad V_C - 6 - 120 + 1411.54 - 690\left(\frac{12}{13}\right) = 0$

$V_C = -648.62 = -649 \text{ lb}$ **Ans.**

$\zeta + \Sigma M_C = 0;$ $\quad -6(1) - 120(1.5) + 1411.54(3)$

$- \frac{12}{13}(690)(13) - M_C = 0$

$M_C = -4231.38 \text{ lb} \cdot \text{ft} = -4.23 \text{ kip} \cdot \text{ft}$ **Ans.**



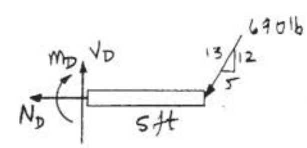
Segment D:

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad -N_D - \frac{5}{13}(690) = 0$

$N_D = -265 \text{ lb}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad V_D - \frac{12}{13}(690) = 0$

$V_D = 637 \text{ lb}$ **Ans.**



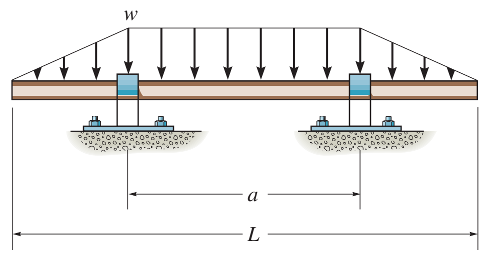
$\zeta + \Sigma M_D = 0;$ $\quad -M_D - 690\left(\frac{12}{13}\right)(5) = 0$

$M_D = -3.18 \text{ kip} \cdot \text{ft}$ **Ans.**

Ans:
$N_C = 265 \text{ lb}$
$V_C = -649 \text{ lb}$
$M_C = -4.23 \text{ kip} \cdot \text{ft}$
$N_D = -265 \text{ lb}$
$V_D = 637 \text{ lb}$
$M_D = -3.18 \text{ kip} \cdot \text{ft}$

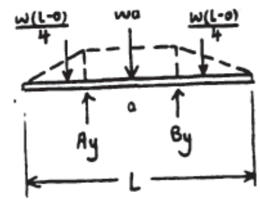
***7–12.**

Determine the distance a between the bearings in terms of the shaft's length L so that the moment in the *symmetric* shaft is zero at its center.



SOLUTION

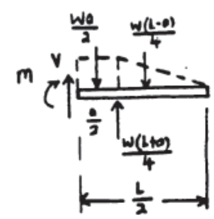
Due to symmetry, $A_y = B_y$



$$+\uparrow \Sigma F_y = 0; \qquad A_y + B_y - \frac{w(L-a)}{4} - wa - \frac{w(L-a)}{4} = 0$$

$$A_y = B_y = \frac{w}{4}(L + a)$$

$$\zeta + \Sigma M = 0; \qquad -M - \frac{wa}{2}\left(\frac{a}{4}\right) - \frac{w(La)}{4}\left(\frac{a}{2} + \frac{L}{6} - \frac{a}{6}\right) + \frac{w}{4}(L + a)\left(\frac{a}{2}\right) = 0$$



Since $M = 0$;

$$3a^2 + (L - a)(L + 2a) - 3a\,(L + a) = 0$$

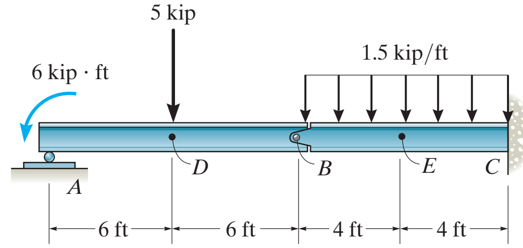
$$2a^2 + 2a\,L - L^2 = 0$$

$$a = 0.366\,L \qquad \textbf{Ans.}$$

Ans:
$a = 0.366\,L$

7–13.

Determine the internal normal force, shear force, and moment in the beam at sections passing through points D and E. Point D is located just to the left of the 5-kip load.



SOLUTION

Support Reaction. Referring to the *FBD* of member AB shown in Fig. a

$\zeta+\Sigma M_B = 0;\quad 5(6) + 6 - A_y(12) = 0 \qquad A_y = 3.00\text{ kip}$

$\zeta+\Sigma M_A = 0;\quad B_y(12) - 5(6) + 6 = 0 \qquad B_y = 2.00\text{ kip}$

$\overset{+}{\rightarrow}\Sigma F_x = 0 \qquad B_x = 0$

Internal Loading. Referring to the left segment of member AB sectioned through D, Fig. b,

$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad N_D = 0$ **Ans.**

$+\uparrow\Sigma F_y = 0;\quad 3.00 - V_D = 0 \qquad V_D = 3.00\text{ kip}$ **Ans.**

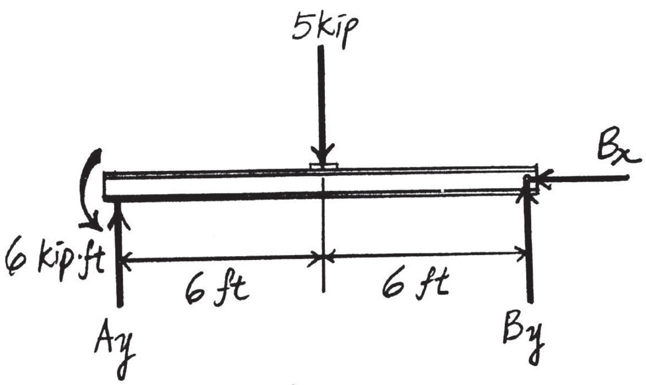
$\zeta+\Sigma M_D = 0;\quad M_D + 6 - 3.00(6) = 0 \qquad M_D = 12.0\text{ kip}\cdot\text{ft}$ **Ans.**

Referring to the left segment of member BC sectioned through E, Fig. c,

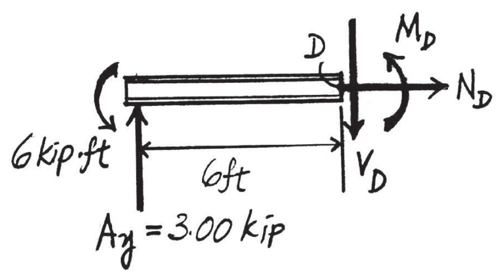
$\overset{+}{\rightarrow}\Sigma F_x = 0;\quad N_E = 0$ **Ans.**

$+\uparrow\Sigma F_y = 0;\quad -V_E - 1.5(4) - 2.00 = 0 \qquad V_E = -8.00\text{ kip}$ **Ans.**

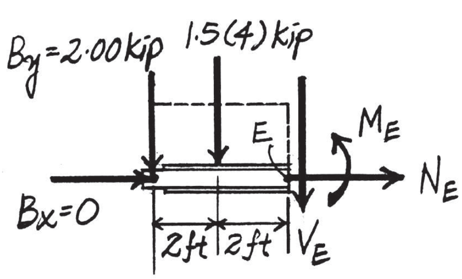
$\zeta+\Sigma M_E = 0;\quad M_E + 1.5(4)(2) + 2.00(4) = 0 \qquad M_E = -20.0\text{ kip}\cdot\text{ft}$ **Ans.**



(a)



(b)

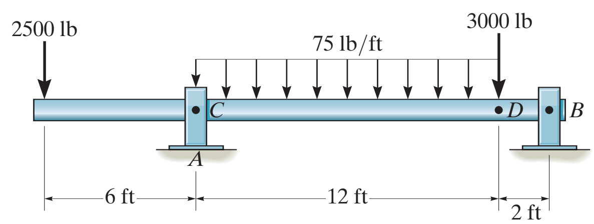


(c)

Ans:
$N_D = 0$
$V_D = 3.00\text{ kip}$
$M_D = 12.0\text{ kip}\cdot\text{ft}$
$N_E = 0$
$V_E = -8.00\text{ kip}$
$M_E = -20.0\text{ kip}\cdot\text{ft}$

7–14.

The shaft is supported by a journal bearing at A and a thrust bearing at B. Determine the normal force, shear force, and moment at a section passing through (a) point C, which is just to the right of the bearing at A, and (b) point D, which is just to the left of the 3000-lb force.



SOLUTION

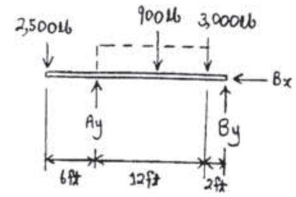
$\circlearrowleft + \Sigma M_B = 0; \qquad -A_y(14) + 2500(20) + 900(8) + 3000(2) = 0$

$A_y = 4514 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$

$+\uparrow \Sigma F_y = 0; \qquad 4514 - 2500 - 900 - 3000 + B_y = 0$

$B_y = 1886 \text{ lb}$



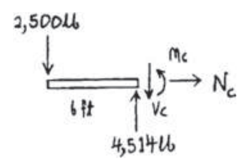
$\circlearrowleft + \Sigma M_C = 0; \qquad 2500(6) + M_C = 0$

$M_C = -15\,000 \text{ lb} \cdot \text{ft} = -15.0 \text{ kip} \cdot \text{ft}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad -2500 + 4514 - V_C = 0$

$V_C = 2014 \text{ lb} = 2.01 \text{ kip}$ **Ans.**



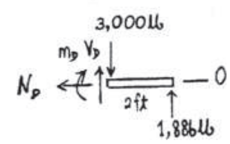
$\circlearrowleft + \Sigma M_D = 0; \qquad -M_D + 1886(2) = 0$

$M_D = 3771 \text{ lb} \cdot \text{ft} = 3.77 \text{ kip} \cdot \text{ft}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_D - 3000 + 1886 = 0$

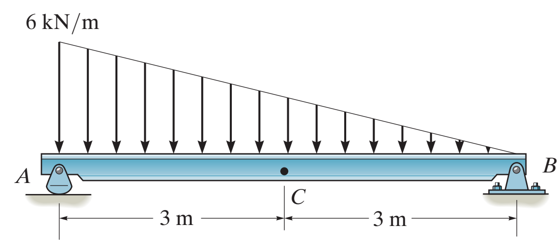
$V_D = 1114 \text{ lb} = 1.11 \text{ kip}$ **Ans.**



Ans:
$M_C = -15.0 \text{ kip} \cdot \text{ft}$
$N_C = 0$
$V_C = 2.01 \text{ kip}$
$M_D = 3.77 \text{ kip} \cdot \text{ft}$
$N_D = 0$
$V_D = 1.11 \text{ kip}$

7–15.

Determine the internal normal force, shear force, and moment at point C.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire beam shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(6) - \frac{1}{2}(6)(6)(2) = 0 \qquad B_y = 6.00 \text{ kN}$$

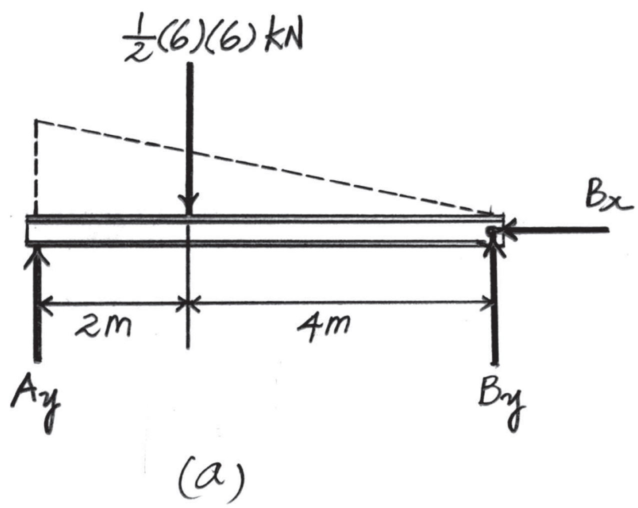
$$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$$

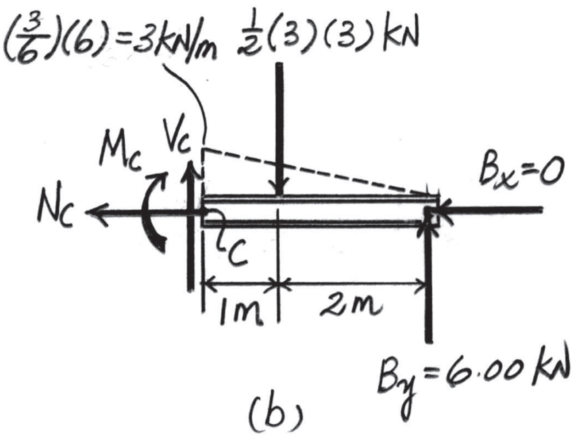
Internal Loadings. Referring to the *FBD* of right segment of the beam sectioned through C, Fig. b

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_C = 0 \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad V_C + 6.00 - \frac{1}{2}(3)(3) = 0 \qquad V_C = -1.50 \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_C = 0; \quad 6.00(3) - \frac{1}{2}(3)(3)(1) - M_C = 0 \quad M_C = 13.5 \text{ kN} \cdot \text{m} \qquad \textbf{Ans.}$$

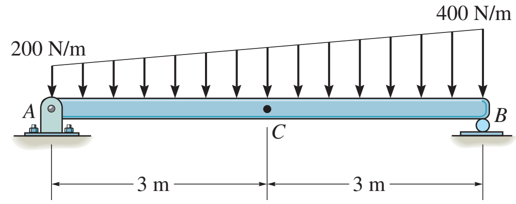




Ans:
$N_C = 0$
$V_C = -1.50$ kN
$M_C = 13.5$ kN $\cdot$ m

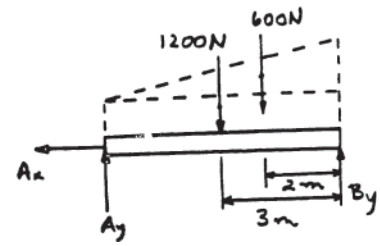
***7–16.**

Determine the internal normal force, shear force, and moment at point C of the beam.



SOLUTION

Beam:

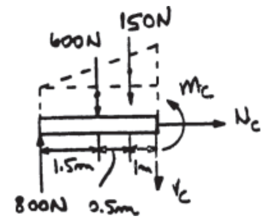


$\zeta + \Sigma M_B = 0; \quad 600\,(2) + 1200\,(3) - A_y\,(6) = 0$

$A_y = 800 \text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$

Segment AC:



$\xrightarrow{+} \Sigma F_x = 0; \quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 800 - 600 - 150 - V_C = 0$

$V_C = 50 \text{ N}$ **Ans.**

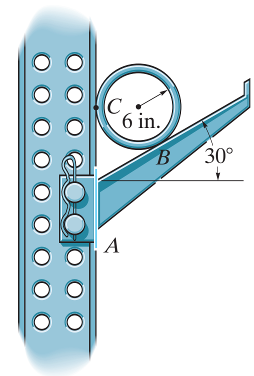
$\zeta + \Sigma M_C = 0; \quad -800\,(3) + 600\,(1.5) + 150\,(1) + M_C = 0$

$M_C = 1350 \text{ N} \cdot \text{m} = 1.35 \text{ kN} \cdot \text{m}$ **Ans.**

Ans:
$N_C = 0$
$V_C = 50 \text{ N}$
$M_C = 1.35 \text{ kN} \cdot \text{m}$

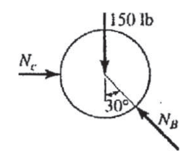
7–17.

The cantilevered rack is used to support each end of a smooth pipe that has a total weight of 300 lb. Determine the normal force, shear force, and moment that act in the arm at its fixed support A along a vertical section.



SOLUTION

Pipe:



$+\uparrow \Sigma F_y = 0;\quad N_B \cos 30° - 150 = 0$

$N_B = 173.205$ lb

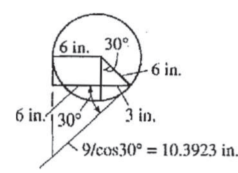
Rack:

$\xrightarrow{+} \Sigma F_x = 0;\quad -N_A + 173.205 \sin 30° = 0$

$N_A = 86.6$ lb **Ans.**

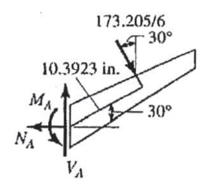
$+\uparrow \Sigma F_y = 0;\quad V_A - 173.205 \cos 30° = 0$

$V_A = 150$ lb **Ans.**



$\circlearrowleft + \Sigma M_A = 0;\quad M_A - 173.205(10.3923) = 0$

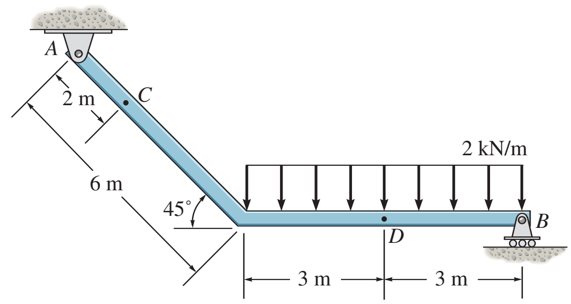
$M_A = 1.80$ lb · in. **Ans.**



Ans:
$N_A = 86.6$ lb
$V_A = 150$ lb
$M_A = 1.80$ kip · in.

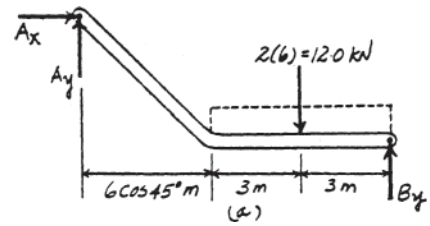
7–18.

Determine the internal normal force, shear force, and the moment at points C and D.



SOLUTION

Support Reactions: FBD (a).



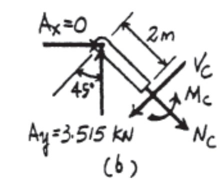
$\circlearrowleft + \Sigma M_A = 0;\qquad B_y(6 + 6\cos 45°) - 12.0(3 + 6\cos 45°) = 0$

$$B_y = 8.485 \text{ kN}$$

$+\uparrow \Sigma F_y = 0;\qquad A_y + 8.485 - 12.0 = 0 \qquad A_y = 3.515 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad A_x = 0$

Internal Forces: Applying the equations of equilibrium to segment AC [FBD (b)], we have



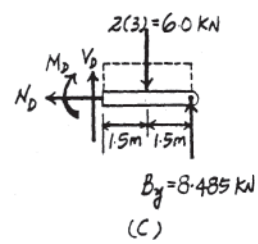
$\nearrow + \Sigma F_{x'} = 0;\qquad 3.515\cos 45° - V_C = 0 \qquad V_C = 2.49 \text{ kN}$ **Ans.**

$\nwarrow + \Sigma F_{y'} = 0;\qquad 3.515\sin 45° - N_C = 0 \qquad N_C = 2.49 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0;\qquad M_C - 3.515\cos 45°(2) = 0$

$M_C = 4.97 \text{ kN}\cdot\text{m}$ **Ans.**

Applying the equations of equilibrium to segment BD [FBD (c)], we have



$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\qquad V_D + 8.485 - 6.00 = 0 \qquad V_D = -2.49 \text{ kN}$ **Ans.**

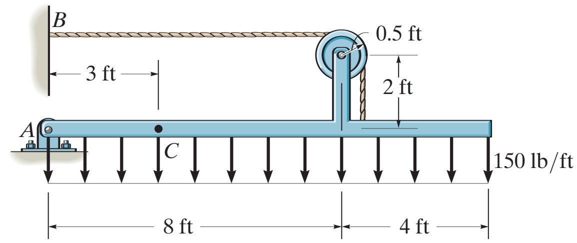
$\circlearrowleft + \Sigma M_D = 0;\qquad 8.485(3) - 6(1.5) - M_D = 0$

$M_D = 16.5 \text{ kN}\cdot\text{m}$ **Ans.**

Ans:
$V_C = 2.49$ kN
$N_C = 2.49$ kN
$M_C = 4.97$ kN·m
$N_D = 0$
$V_D = -2.49$ kN
$M_D = 16.5$ kN·m

7–19.

Determine the internal normal force, shear force, and moment at point C.

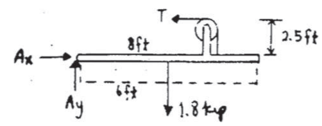


SOLUTION

Entire beam:

$\circlearrowleft + \Sigma M_A = 0; \qquad -1.8\,(6) + T\,(2.5) = 0$

$T = 4.32 \text{ kip}$



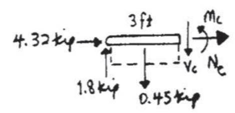
$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x - 4.32 = 0$

$A_x = 4.32 \text{ kip}$

$+\uparrow \Sigma F_y = 0; \qquad A_y - 1.8 = 0$

$A_y = 1.8 \text{ kip}$

Segment AC:



$\xrightarrow{+} \Sigma F_x = 0; \qquad 4.32 + N_C = 0$

$N_C = -4.32 \text{ kip}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad 1.8 - 0.45 - V_C = 0$

$V_C = 1.35 \text{ kip}$ **Ans.**

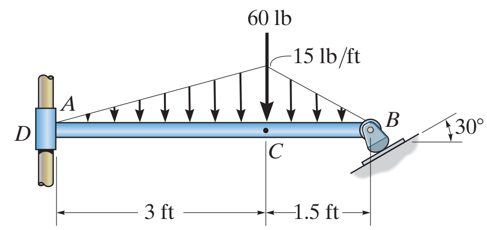
$\circlearrowleft + \Sigma M_C = 0; \qquad -1.8\,(3) + 0.45\,(1.5) + M_C = 0$

$M_C = 4.72 \text{ kip} \cdot \text{ft}$ **Ans.**

Ans:
$N_C = -4.32$ kip
$V_C = 1.35$ kip
$M_C = 4.72$ kip · ft

***7–20.**

Rod AB is fixed to a smooth collar D, which slides freely along the vertical guide. Determine the internal normal force, shear force, and moment at point C. which is located just to the left of the 60-lb concentrated load.



SOLUTION

With reference to Fig. *a*, we obtain

$$+\uparrow \Sigma F_y = 0; \qquad F_B \cos 30^\circ - \frac{1}{2}(15)(3) - 60 - \frac{1}{2}(15)(1.5) = 0 \qquad F_B = 108.25 \text{ lb}$$

Using this result and referring to Fig. *b*, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad -N_C - 108.25 \sin 30^\circ = 0 \qquad N_C = -54.1 \text{ lb} \qquad \textbf{Ans.}$$

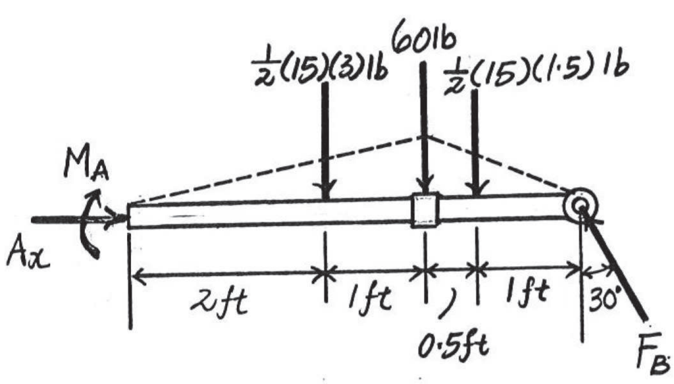
$$+\uparrow \Sigma F_y = 0; \qquad V_C - 60 - \frac{1}{2}(15)(1.5) + 108.25 \cos 30^\circ = 0$$

$$V_C = -22.5 \text{ lb} \qquad \textbf{Ans.}$$

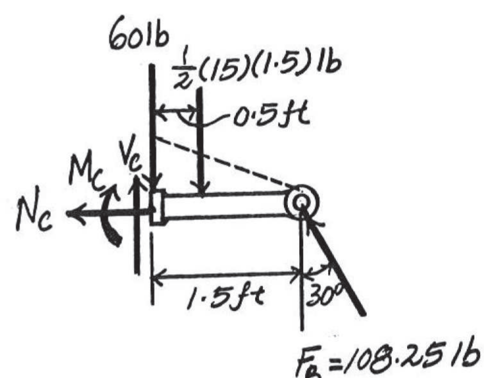
$$\circlearrowleft + \Sigma M_C = 0; \qquad 108.25 \cos 30^\circ(1.5) - \frac{1}{2}(15)(1.5)(0.5) - M_C = 0$$

$$M_C = 135 \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$

The negative signs indicates that $\mathbf{N}_C$ and $\mathbf{V}_C$ act in the opposite sense to that shown on the free-body diagram.



(a)

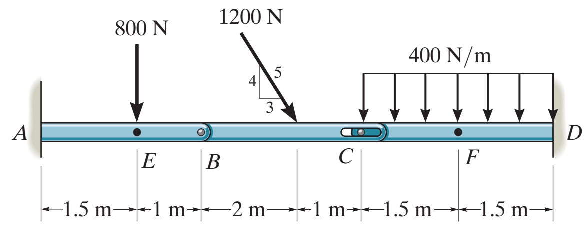


(b)

Ans:
$N_C = -54.1$ lb
$V_C = -22.5$ lb
$M_C = 135$ lb · ft

7–21.

Determine the internal normal force, shear force, and moment at points E and F of the compound beam. Point E is located just to the left of 800 N force.



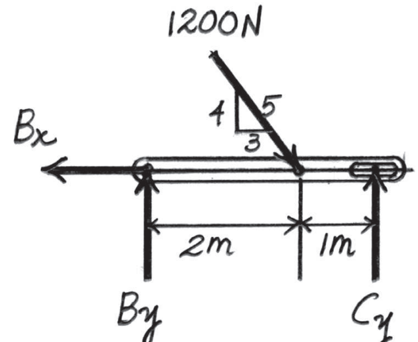
SOLUTION

Support Reactions. Referring to the *FBD* of member BC shown in Fig. a,

$$\zeta+\Sigma M_B = 0; \quad C_y(3) - 1200\left(\frac{4}{5}\right)(2) = 0 \qquad C_y = 640\text{ N}$$

$$\zeta+\Sigma M_C = 0; \quad 1200\left(\frac{4}{5}\right)(1) - B_y(3) = 0 \qquad B_y = 320\text{ N}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 1200\left(\frac{3}{5}\right) - B_x = 0 \qquad B_x = 720\text{ N}$$



(a)

Internal Loadings. Referring to the right segment of member AB sectioned through E, Fig. b

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad 720 - N_E = 0 \qquad N_E = 720\text{ N} \qquad \textbf{Ans.}$$

$$+\uparrow\Sigma F_y = 0; \quad V_E - 800 - 320 = 0 \qquad V_E = 1120\text{ N} = 1.12\text{ kN} \qquad \textbf{Ans.}$$

$$\zeta+\Sigma M_E = 0; \quad -M_E - 320(1) = 0 \qquad M_E = -320\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

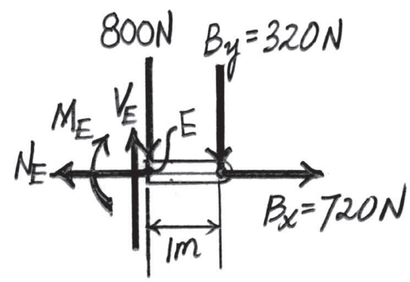
Referring to the left segment of member CD sectioned through F, Fig. c,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad N_F = 0 \qquad \textbf{Ans.}$$

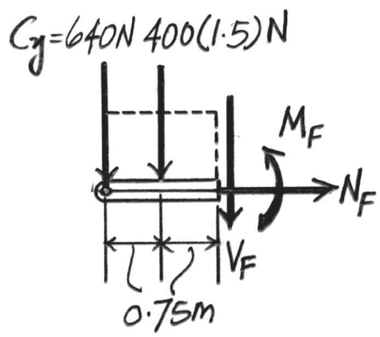
$$+\uparrow\Sigma F_y = 0; \quad -V_F - 640 - 400(1.5) = 0 \quad V_F = -1240\text{ N} = -1.24\text{ kN} \qquad \textbf{Ans.}$$

$$\zeta+\Sigma M_F = 0; \quad M_F + 400(1.5)(0.75) + 640(1.5) = 0$$

$$M_F = -1410\text{ N}\cdot\text{m} = -1.41\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



(b)

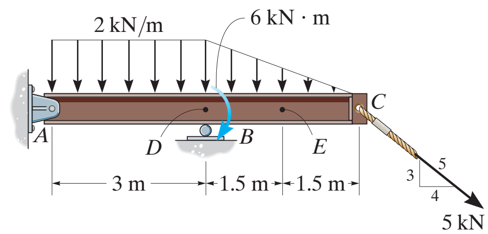


(c)

Ans:
$N_E = 720\text{ N}$
$V_E = 1.12\text{ kN}$
$M_E = -320\text{ N}\cdot\text{m}$
$N_F = 0$
$V_F = -1.24\text{ kN}$
$M_F = -1.41\text{ kN}\cdot\text{m}$

7–22.

Determine the internal normal force, shear force, and moment at points D and E in the overhang beam. Point D is located just to the left of the roller support at B, where the couple moment acts.



SOLUTION

The intensity of the triangular distributed load at E can be found using the similar triangles in Fig. b. With reference to Fig. a,

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(3) - 2(3)(1.5) - 6 - \frac{1}{2}(2)(3)(4) - 5\left(\frac{3}{5}\right)(6) = 0$$

$$B_y = 15 \text{ kN}$$

Using this result and referring to Fig. c,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 5\left(\frac{4}{5}\right) - N_D = 0 \qquad N_D = 4 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad V_D + 15 - \frac{1}{2}(2)(3) - 5\left(\frac{3}{5}\right) = 0 \qquad V_D = -9 \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_D = 0; \qquad -M_D - 6 - \frac{1}{2}(2)(3)(1) - 5\left(\frac{3}{5}\right)(3) = 0 \qquad M_D = -18 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

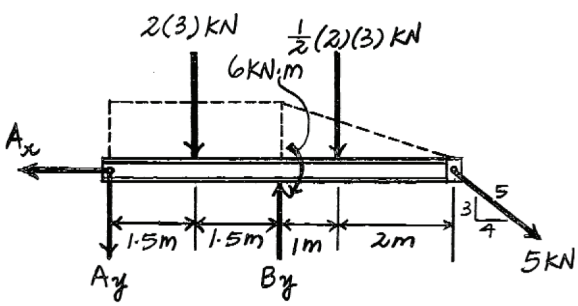
Also, by referring to Fig. d, we can write

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 5\left(\frac{4}{5}\right) - N_E = 0 \qquad N_E = 4 \text{ kN} \qquad \textbf{Ans.}$$

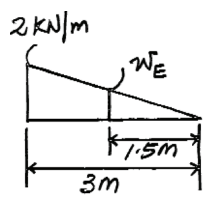
$$+\uparrow \Sigma F_y = 0; \qquad V_E - \frac{1}{2}(1)(1.5) - 5\left(\frac{3}{5}\right) = 0 \qquad V_E = 3.75 \text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_E = 0; \qquad -M_E - \frac{1}{2}(1)(1.5)(0.5) - 5\left(\frac{3}{5}\right)(1.5) = 0 \qquad M_E = -4.875 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$

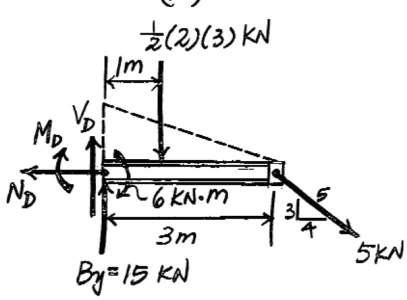
The negative sign indicates that $\mathbf{V}_D$, $\mathbf{M}_D$, and $\mathbf{M}_E$ act in the opposite sense to that shown on the free-body diagram.



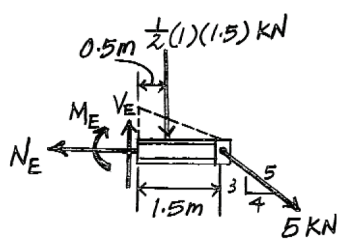
(a)



(b)



(c)

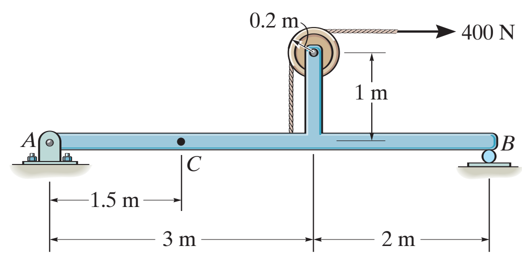


(d)

Ans:
$N_D = 4$ kN
$V_D = -9$ kN
$M_D = -18$ kN·m
$N_E = 4$ kN
$V_E = 3.75$ kN
$M_E = -4.875$ kN·m

7–23.

Determine the internal normal force, shear force, and moment at point C.



SOLUTION

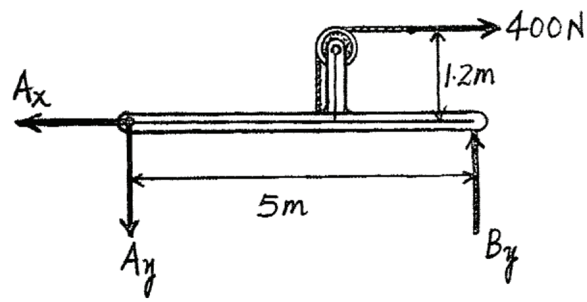
Beam:

$\xrightarrow{+} \Sigma F_x = 0; \qquad -A_x + 400 = 0$

$A_x = 400 \text{ N}$

$\circlearrowleft + \Sigma M_B = 0; \qquad A_y(5) - 400(1.2) = 0$

$A_y = 96 \text{ N}$



Segment *AC*:

$\xrightarrow{+} \Sigma F_x = 0; \qquad N_C - 400 = 0$

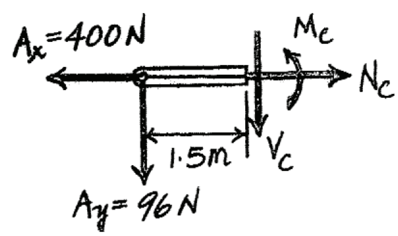
$N_C = 400 \text{ N}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad -96 - V_C = 0$

$V_C = -96 \text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \qquad M_C + 96(1.5) = 0$

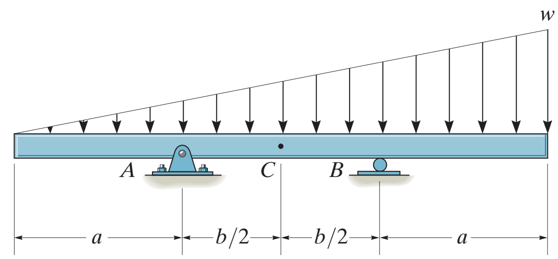
$M_C = -144 \text{ N} \cdot \text{m}$ **Ans.**



Ans:
$N_C = 400 \text{ N}$
$V_C = -96 \text{ N}$
$M_C = -144 \text{ N} \cdot \text{m}$

***7–24.**

Determine the ratio of a/b for which the shear force will be zero at the midpoint C of the beam.



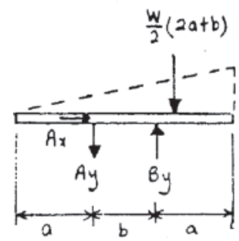
SOLUTION

$\zeta+\Sigma M_B = 0;$ $$-\frac{w}{2}(2a+b)\left[\frac{2}{3}(2a+b)-(a+b)\right]+A_y(b)=0$$

$$A_y=\frac{w}{6b}(2a+b)(a-b)$$

$\xrightarrow{+}\Sigma F_x = 0;$ $$A_x = 0$$

$+\uparrow\Sigma F_y = 0;$ $$-\frac{w}{6b}(2a+b)(a-b)-\frac{w}{4}\left(a+\frac{b}{2}\right)-V_C=0$$



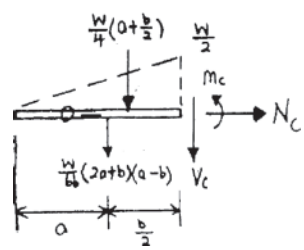
Since $V_C = 0$,

$$-\frac{1}{6b}(2a+b)(a-b)=\frac{1}{4}(2a+b)\left(\frac{1}{2}\right)$$

$$-\frac{1}{6b}(a-b)=\frac{1}{8}$$

$$-a+b=\frac{3}{4}b$$

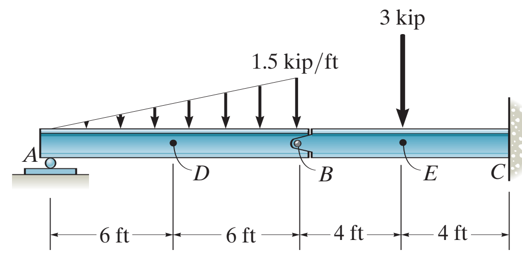
$$\frac{a}{b}=\frac{1}{4}$$ **Ans.**



Ans:
$\frac{a}{b}=\frac{1}{4}$

7–25.

Determine the normal force, shear force, and moment in the beam at sections passing through points D and E. Point E is just to the right of the 3-kip load.



SOLUTION

$\zeta + \Sigma M_B = 0;\quad \frac{1}{2}(1.5)(12)(4) - A_y(12) = 0$

$A_y = 3\text{ kip}$

$\xrightarrow{+} \Sigma F_x = 0;\quad B_x = 0$

$+\uparrow \Sigma F_y = 0;\quad B_y + 3 - \frac{1}{2}(1.5)(12) = 0$

$B_y = 6\text{ kip}$

$\xrightarrow{+} \Sigma F_x = 0;\quad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad 3 - \frac{1}{2}(0.75)(6) - V_D = 0$

$V_D = 0.75\text{ kip}$ **Ans.**

$\zeta \Sigma M_D = 0;\quad M_D + \frac{1}{2}(0.75)(6)(2) - 3(6) = 0$

$M_D = 13.5\text{ kip}\cdot\text{ft}$ **Ans.**

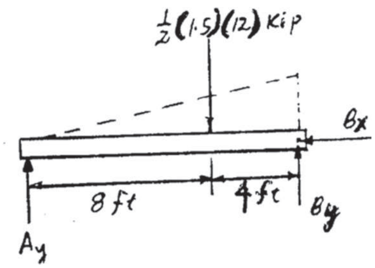
$\xrightarrow{+} \Sigma F_x = 0;\quad N_E = 0$ **Ans.**

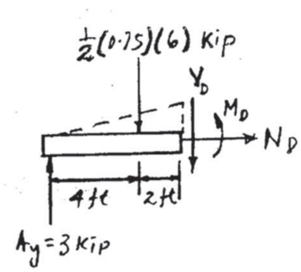
$+\uparrow \Sigma F_y = 0;\quad -V_E - 3 - 6 = 0$

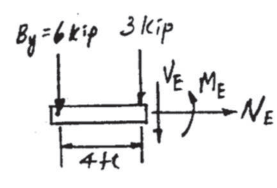
$V_E = -9\text{ kip}$ **Ans.**

$\Sigma M_E = 0;\quad M_E + 6(4) = 0$

$M_E = -24.0\text{ kip}\cdot\text{ft}$ **Ans.**



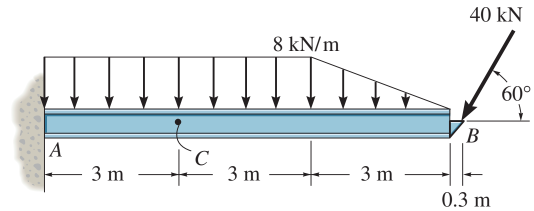




Ans:
$N_D = 0$
$V_D = 0.75\text{ kip}$
$M_D = 13.5\text{ kip}\cdot\text{ft}$
$N_E = 0$
$V_E = -9\text{ kip}$
$M_E = -24.0\text{ kip}\cdot\text{ft}$

7–26.

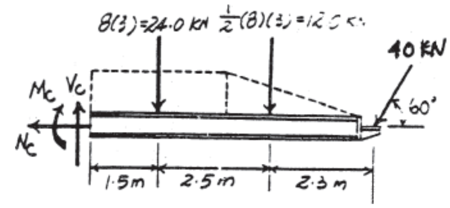
Determine the internal normal force, shear force, and bending moment at point C.



SOLUTION

Free body Diagram: The support reactions at A need not be computed.

Internal Forces: Applying equations of equilibrium to segment BC, we have



$\xrightarrow{+} \Sigma F_x = 0; \qquad -40 \cos 60° - N_C = 0 \qquad N_C = -20.0 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad V_C - 24.0 - 12.0 - 40 \sin 60° = 0$

$$V_C = 70.6 \text{ kN}$$ **Ans.**

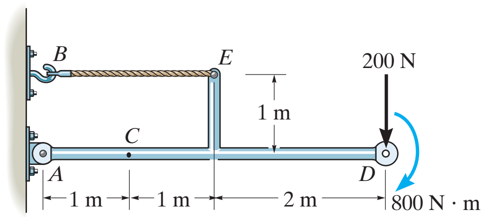
$\circlearrowleft + \Sigma M_C = 0; \qquad -24.0(1.5) - 12.0(4) - 40 \sin 60°(6.3) - M_C = 0$

$$M_C = -302 \text{ kN} \cdot \text{m}$$ **Ans.**

Ans:
$N_C = -20.0 \text{ kN}$
$V_C = 70.6 \text{ kN}$
$M_C = -302 \text{ kN} \cdot \text{m}$

7–27.

Determine the internal normal force, shear force, and moment at point C.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire assembly shown in Fig. *a*,

$\circlearrowleft + \Sigma M_A = 0;\quad F_{BE}(1) - 200(4) - 800 = 0 \quad F_{BE} = 1600\text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad A_x - 1600 = 0 \quad A_x = 1600\text{ N}$

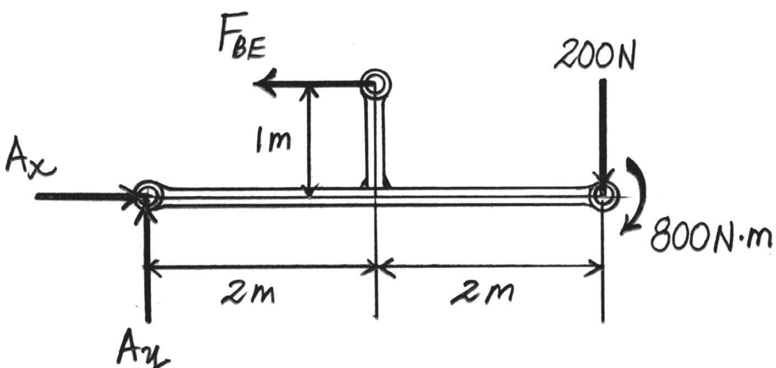
$+\uparrow \Sigma F_y = 0;\quad A_y - 200 = 0 \quad A_y = 200\text{ N}$

Internal Loading. Referring to the *FBD* of the left segment of the assembly sectioned through C, Fig. b,

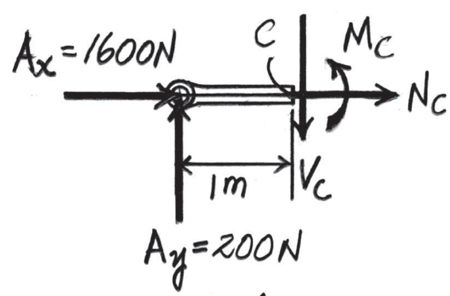
$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 1600 + N_C = 0 \quad N_C = -1600\text{ N} = -1.60\text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;\quad 200 - V_C = 0 \quad V_C = 200\text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0;\quad M_C - 200(1) = 0 \quad M_C = 200\text{ N}\cdot\text{m}$ **Ans.**



(a)

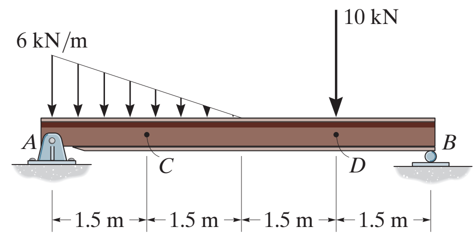


(b)

Ans:
$N_C = -1.60\text{ kN}$
$V_C = 200\text{ N}$
$M_C = 200\text{ N}\cdot\text{m}$

***7–28.**

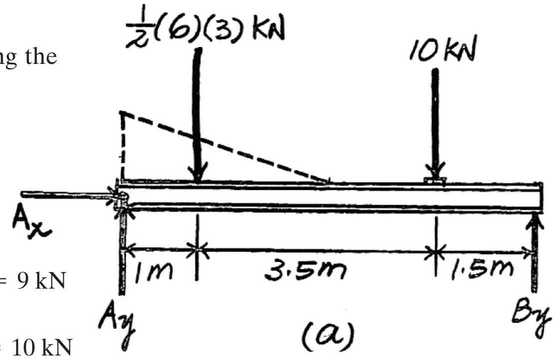
Determine the internal normal force, shear force, and moment at points C and D in the simply supported beam. Point D is located just to the left of the 10-kN concentrated load.



SOLUTION

The intensity of the triangular distributed loading at C can be computed using the similar triangles shown in Fig. b,

$$\frac{w_C}{1.5} = \frac{6}{3} \text{ or } w_C = 3 \text{ kN/m}$$



With reference to Fig. a,

$\zeta + \Sigma M_A = 0; \quad B_y(6) - 10(4.5) - \frac{1}{2}(6)(3)(1) = 0 \qquad B_y = 9 \text{ kN}$

$\zeta + \Sigma M_B = 0; \quad \frac{1}{2}(6)(3)(5) + 10(1.5) - A_y(6) = 0 \qquad A_y = 10 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad A_x = 0$

Using these results and referring to Fig. c,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_C = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad 10 - \frac{1}{2}(3)(1.5) - 3(1.5) - V_C = 0 \qquad V_C = 3.25 \text{ kN}$ **Ans.**

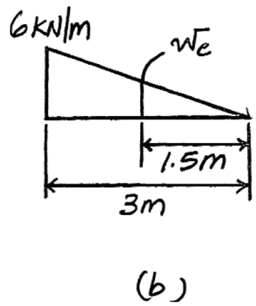
$\zeta + \Sigma M_C = 0; \quad M_C + 3(1.5)(0.75) + \frac{1}{2}(3)(1.5)(1) - 10(1.5) = 0 \qquad M_C = 9.375 \text{ kN} \cdot \text{m}$ **Ans.**

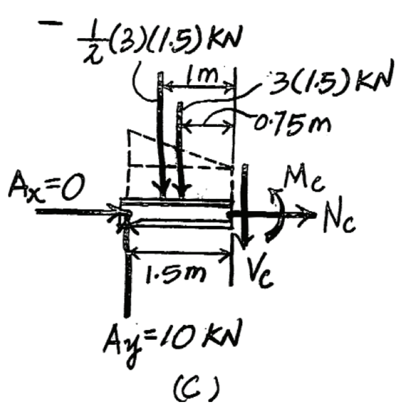
Also, by referring to Fig. d,

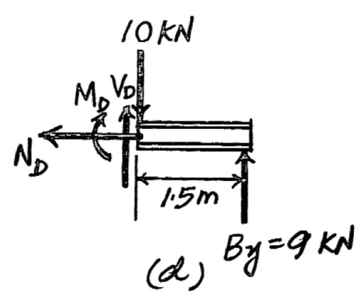
$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad N_D = 0$ **Ans.**

$+\uparrow \Sigma F_y = 0; \quad V_D + 9 - 10 = 0 \qquad V_D = 1 \text{ kN}$ **Ans.**

$\zeta + \Sigma M_D = 0; \quad 9(1.5) - M_D = 0 \qquad M_D = 13.5 \text{ kN} \cdot \text{m}$ **Ans.**



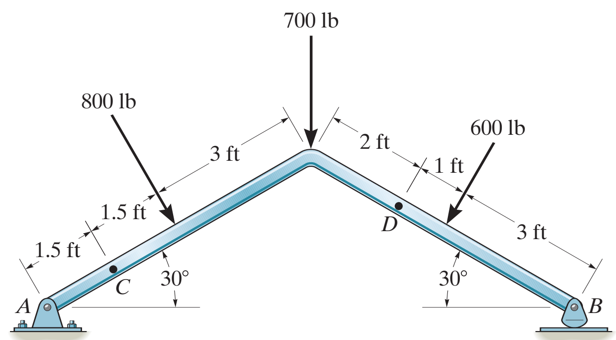




Ans:
$N_C = 0$
$V_C = 3.25 \text{ kN}$
$M_C = 9.375 \text{ kN} \cdot \text{m}$
$N_D = 0$
$V_D = 1 \text{ kN}$
$M_D = 13.5 \text{ kN} \cdot \text{m}$

7–29.

Determine the normal force, shear force, and moment acting at a section passing through point *C*.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \quad -800\,(3) - 700(6\cos 30^\circ) - 600\cos 30^\circ(6\cos 30^\circ + 3\cos 30^\circ)$

$+ 600\sin 30^\circ(3\sin 30^\circ) + B_y\,(6\cos 30^\circ + 6\cos 30^\circ) = 0$

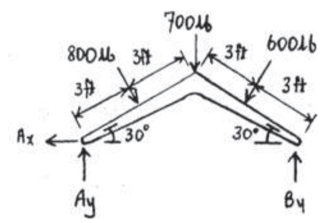
$B_y = 927.4\text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad 800\sin 30^\circ - 600\sin 30^\circ - A_x = 0$

$A_x = 100\text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad A_y - 800\cos 30^\circ - 700 - 600\cos 30^\circ + 927.4 = 0$

$A_y = 985.1\text{ lb}$



$\nearrow + \Sigma F_x = 0; \quad N_C - 100\cos 30^\circ + 985.1\sin 30^\circ = 0$

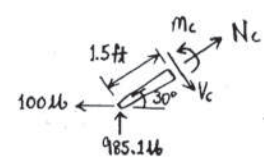
$N_C = -406\text{ lb}$ **Ans.**

$+\nwarrow \Sigma F_y = 0; \quad 100\sin 30^\circ + 985.1\cos 30^\circ - V_C = 0$

$V_C = 903\text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M_C = 0; \quad -985.1(1.5\cos 30^\circ) - 100(1.5\sin 30^\circ) + M_C = 0$

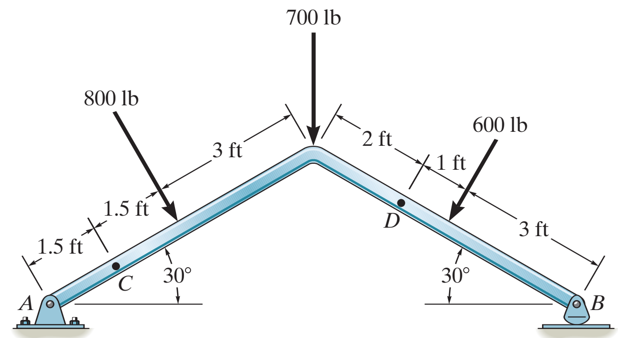
$M_C = 1355\text{ lb}\cdot\text{ft} = 1.35\text{ kip}\cdot\text{ft}$ **Ans.**



Ans:
$N_C = -406\text{ lb}$
$V_C = 903\text{ lb}$
$M_C = 1.35\text{ kip}\cdot\text{ft}$

7–30.

Determine the normal force, shear force, and moment acting at a section passing through point D.



SOLUTION

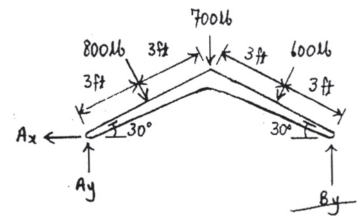
$\circlearrowleft + \Sigma M_A = 0;\quad -800(3) - 700(6\cos 30°) - 600\cos 30°(6\cos 30° + 3\cos 30°)$

$+ 600\sin 30°(3\sin 30°) + B_y(6\cos 30° + 6\cos 30°) = 0$

$B_y = 927.4\text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad 800\sin 30° - 600\sin 30° - A_x = 0$

$A_x = 100\text{ lb}$



$+\uparrow \Sigma F_y = 0;\quad A_y - 800\cos 30° - 700 - 600\cos 30° + 927.4 = 0$

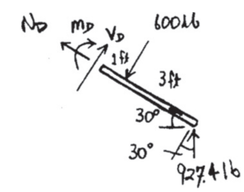
$A_y = 985.1\text{ lb}$

$+\nwarrow \Sigma F_x = 0;\quad N_D - 927.4\sin 30° = 0$

$N_D = -464\text{ lb}$ **Ans.**

$\nearrow + \Sigma F_y = 0;\quad V_D - 600 + 927.4\cos 30° = 0$

$V_D = -203\text{ lb}$ **Ans.**



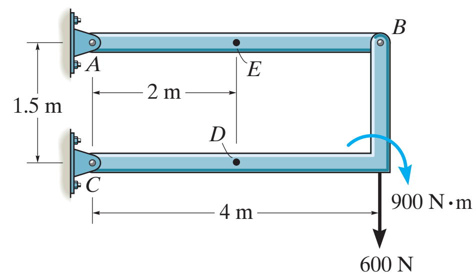
$\circlearrowleft + \Sigma M_D = 0;\quad -M_D - 600(1) + 927.4(4\cos 30°) = 0$

$M_D = 2612\text{ lb}\cdot\text{ft} = 2.61\text{ kip}\cdot\text{ft}$ **Ans.**

Ans:
$N_D = -464\text{ lb}$
$V_D = -203\text{ lb}$
$M_D = 2.61\text{ kip}\cdot\text{ft}$

7–31.

Determine the internal normal force, shear force, and moment acting at points D and E of the frame.



SOLUTION

Support Reactions. Notice that member AB is a two force member. Referring to the FBD of member BC,

$$\circlearrowleft + \Sigma M_C = 0; \quad F_{AB}(1.5) - 900 - 600(4) = 0 \quad F_{AB} = 2200 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad C_x - 2200 = 0 \quad C_x = 2200 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \quad C_y - 600 = 0 \quad C_y = 600 \text{ N}$$

Internal Loadings. Referring to the left segment of member AB sectioned through E, Fig. b,

$$\xrightarrow{+} \Sigma F_x = 0; \quad N_E - 2200 = 0 \quad N_E = 2200 \text{ N} = 2.20 \text{ kN} \quad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \quad V_E = 0 \quad \textbf{Ans.}$$

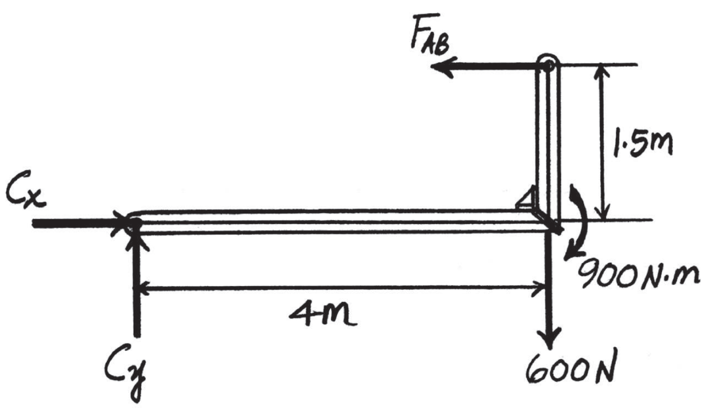
$$\circlearrowleft + \Sigma M_E = 0; \quad M_E = 0 \quad \textbf{Ans.}$$

Referring to the left segment of member BC sectioned through D, Fig. c

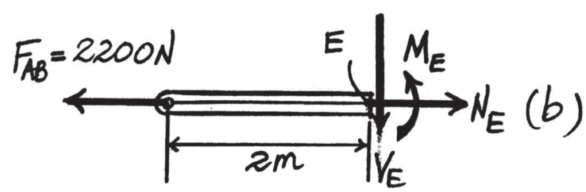
$$\xrightarrow{+} \Sigma F_x = 0; \quad N_D + 2200 = 0 \quad N_D = -2200 \text{ N} = -2.20 \text{ kN} \quad \textbf{Ans.}$$

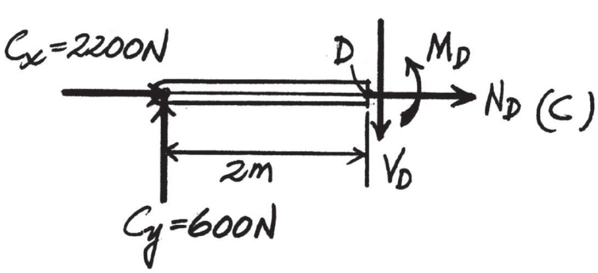
$$+\uparrow \Sigma F_y = 0; \quad 600 - V_D = 0 \quad V_D = 600 \text{ N} \quad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_D = 0; \quad M_D - 600(2) = 0 \quad M_D = 1200 \text{ N}\cdot\text{m} = 1.20 \text{ kN}\cdot\text{m} \quad \textbf{Ans.}$$



(a)

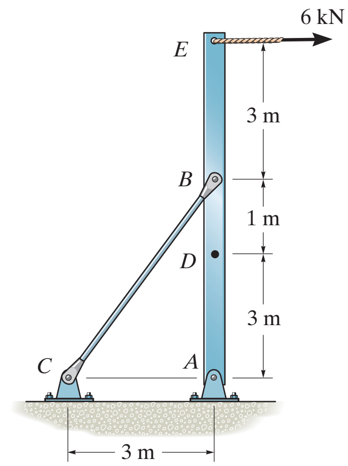




Ans:
$N_E = 2.20$ kN
$V_E = 0$
$M_E = 0$
$N_D = -2.20$ kN
$V_D = 600$ N
$M_D = 1.20$ kN·m

***7–32.**

Determine the internal normal force, shear force, and moment at point D.



SOLUTION

Support Reactions. Notice that member BC is a two force member. Referring to the FBD of member ABE shown in Fig. a,

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BC}\left(\frac{3}{5}\right)(4) - 6(7) = 0 \qquad F_{BC} = 17.5 \text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x - 17.5\left(\frac{3}{5}\right) + 6 = 0 \qquad A_x = 4.50 \text{ kN}$$

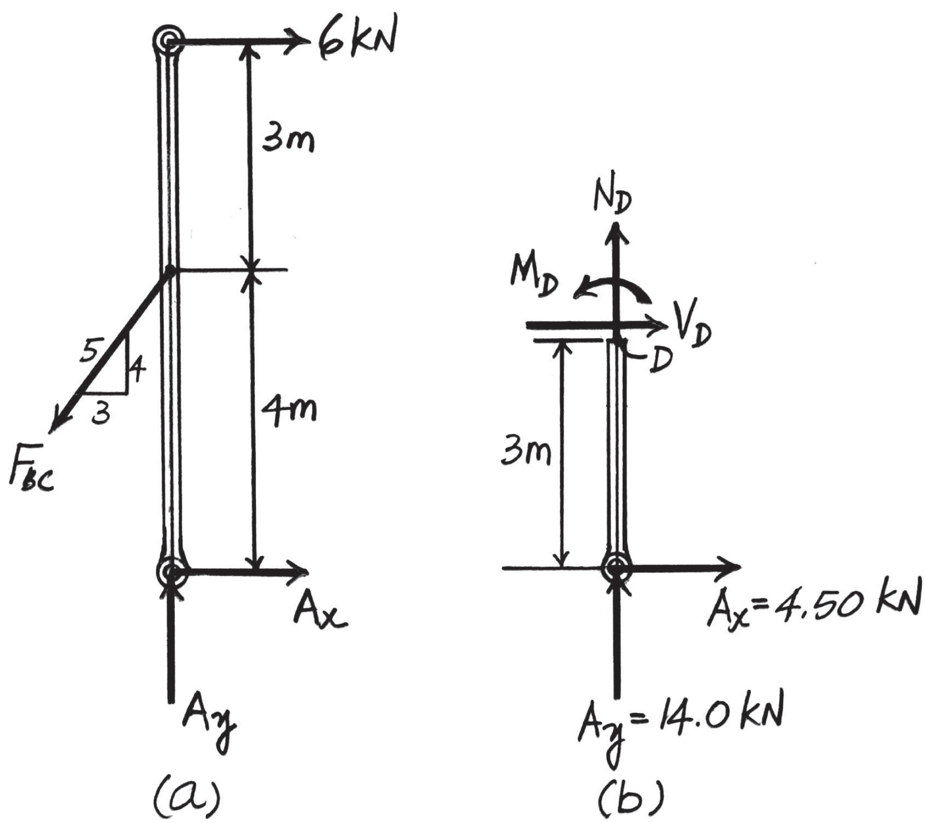
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 17.5\left(\frac{4}{5}\right) = 0 \qquad A_y = 14.0 \text{ kN}$$

Internal Loadings. Referring to the FBD of the lower segment of member ABE sectioned through D, Fig. b,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 4.50 + V_D = 0 \qquad V_D = -4.50 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_D + 14.0 = 0 \qquad N_D = -14.0 \text{ kN} \qquad \textbf{Ans.}$$

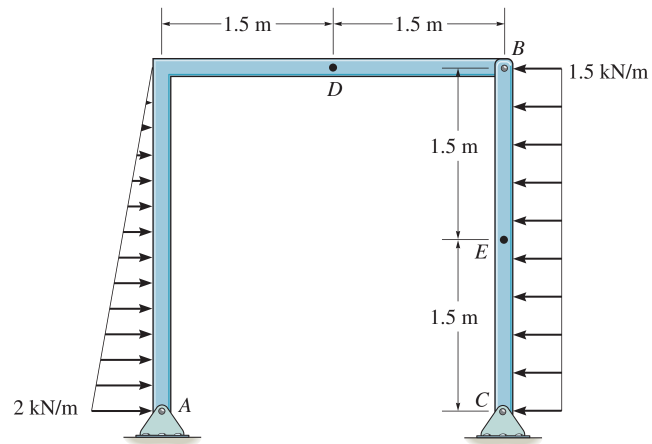
$$\circlearrowleft + \Sigma M_D = 0; \qquad M_D + 4.50(3) = 0 \qquad M_D = -13.5 \text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



Ans:
$V_D = -4.50$ kN
$N_D = -14.0$ kN
$M_D = -13.5$ kN·m

7–33.

Determine the internal normal force, shear force, and moment at point D of the two-member frame.



SOLUTION

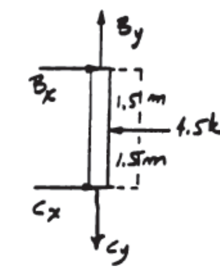
Member BC:

$\zeta + \Sigma M_C = 0;$ $\quad 4.5\,(1.5) - B_x\,(3) = 0$

$B_x = 2.25\text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad 2.25 + C_x - 4.5 = 0$

$C_x = 2.25\text{ kN}$



Member AB:

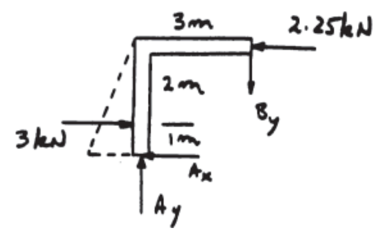
$\zeta + \Sigma M_A = 0;$ $\quad 2.25\,(3) - 3\,(1) - B_y\,(3) = 0$

$B_y = 1.25\text{ kN}$

Segment DB:

$\overset{+}{\rightarrow} \Sigma F_x = 0;$ $\quad -N_D - 2.25 = 0$

$N_D = -2.25\text{ kN}$ **Ans.**

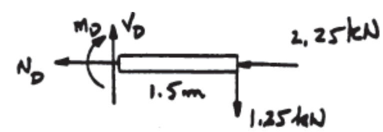


$+\uparrow \Sigma F_y = 0;$ $\quad V_D - 1.25 = 0$

$V_D = 1.25\text{ kN}$ **Ans.**

$\zeta + \Sigma M_D = 0;$ $\quad -M_D - 1.25\,(1.5) = 0$

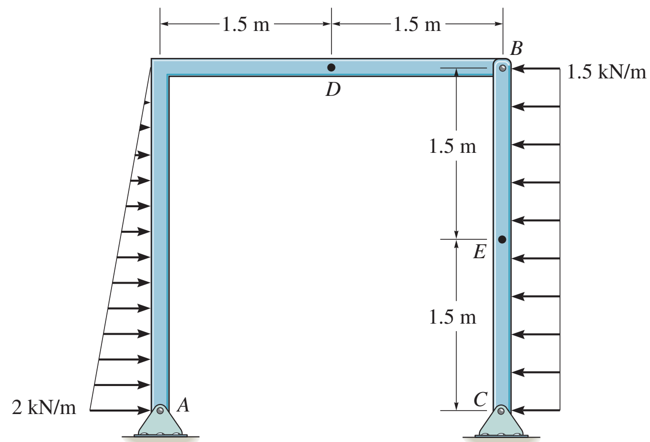
$M_D = -1.88\text{ kN}\cdot\text{m}$ **Ans.**



Ans:
$N_D = -2.25\text{ kN}$
$V_D = 1.25\text{ kN}$
$-1.88\text{ kN}\cdot\text{m}$

7–34.

Determine the internal normal force, shear force, and moment at point E.



SOLUTION

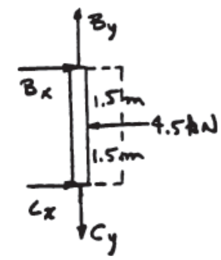
Member BC:

$$\zeta + \Sigma M_C = 0; \qquad 4.5\,(1.5) - B_x\,(3) = 0$$

$$B_x = 2.25\text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 2.25 + C_x - 4.5 = 0$$

$$C_x = 2.25\text{ kN}$$



Member AB:

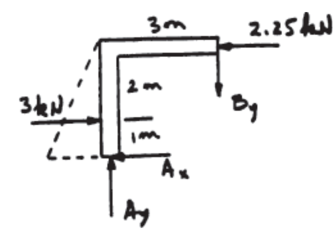
$$\zeta + \Sigma M_A = 0; \qquad 2.25\,(3) - 3\,(1) - B_y\,(3) = 0$$

$$B_y = 1.25\text{ kN}$$

Segment BE:

$$+\uparrow \Sigma F_y = 0; \qquad 1.25 - N_E = 0$$

$$N_E = 1.25\text{ kN} \qquad \textbf{Ans.}$$

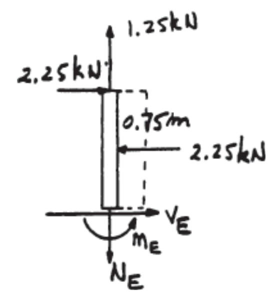


$$\xrightarrow{+} \Sigma F_x = 0; \qquad V_E + 2.25 - 2.25 = 0$$

$$V_E = 0 \qquad \textbf{Ans.}$$

$$+\Sigma M_g = 0; \qquad M_g - 2.25\,(0.75) = 0$$

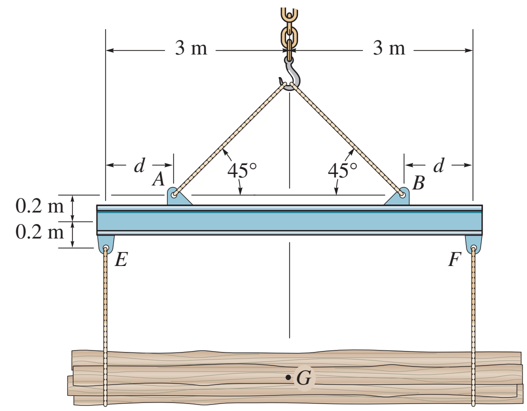
$$M_g = 1.6875\text{ kN}\cdot\text{m} = 1.69\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$



Ans:
$N_E = 1.25\text{ kN}$
$V_E = 0$
$M_B = 1.69\text{ kN}\cdot\text{m}$

7–35.

The strongback or lifting beam is used for materials handling. If the suspended load has a weight of 2 kN and a center of gravity of G, determine the placement d of the padeyes on the top of the beam so that there is no moment developed within the length AB of the beam. The lifting bridle has two legs that are positioned at 45°, as shown.

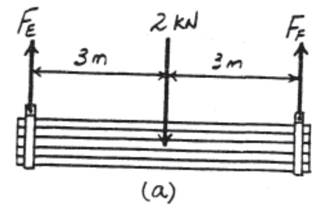


SOLUTION

Support Reactions: From FBD (a),

$\zeta + \Sigma M_E = 0; \quad F_F(6) - 2(3) = 0 \quad F_E = 1.00 \text{ kN}$

$+\uparrow \Sigma F_y = 0; \quad F_F + 1.00 - 2 = 0 \quad F_F = 1.00 \text{ kN}$

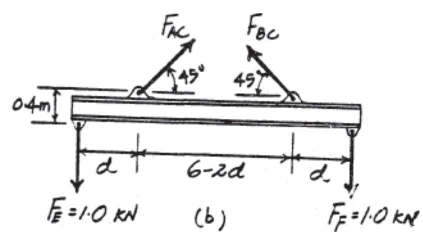


From FBD (b),

$\xrightarrow{+} \Sigma F_x = 0; \quad F_{AC} \cos 45° - F_{BC} \cos 45° = 0 \quad F_{AC} = F_{BC} = F$

$+\uparrow \Sigma F_y = 0; \quad 2F \sin 45° - 1.00 - 1.00 = 0$

$F_{AC} = F_{BC} = F = 1.414 \text{ kN}$

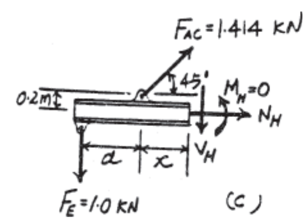


Internal Forces: This problem requires $M_H = 0$. Summing moments about point H of segment EH[FBD (c)], we have

$\zeta + \Sigma M_H = 0; \quad 1.00(d + x) - 1.414 \sin 45°(x)$

$- 1.414 \cos 45°(0.2) = 0$

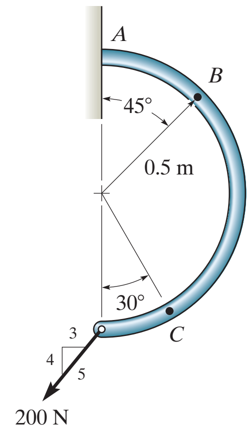
$d = 0.200 \text{ m}$ **Ans.**



Ans:
$d = 0.200$ m

***7–36.**

Determine the internal normal force, shear force, and moment acting at points B and C on the curved rod.



SOLUTION

Support Reactions. Not required

Internal Loadings. Referring to the *FBD* of bottom segment of the curved rod sectioned through C, Fig. *a*

$+\nearrow\Sigma F_x = 0;\ N_C - 200\sin(36.87° + 30°) = 0 \qquad N_C = 183.92\text{ N} = 184\text{ N}$ **Ans.**

$+\nwarrow\Sigma F_y = 0;\ -V_C - 200\cos(36.87° + 30°) \quad V_C = -78.56\text{ N} = -78.6\text{ N}$ **Ans.**

$$\circlearrowleft + \Sigma M_C = 0;\quad 200\left(\frac{4}{5}\right)(0.5\sin 30°) - 200\left(\frac{3}{5}\right)[0.5(1 - \cos 30°)] + M_C = 0$$

$$M_C = -31.96\text{ N}\cdot\text{m} = -32.0\text{ N}\cdot\text{m}$$ **Ans.**

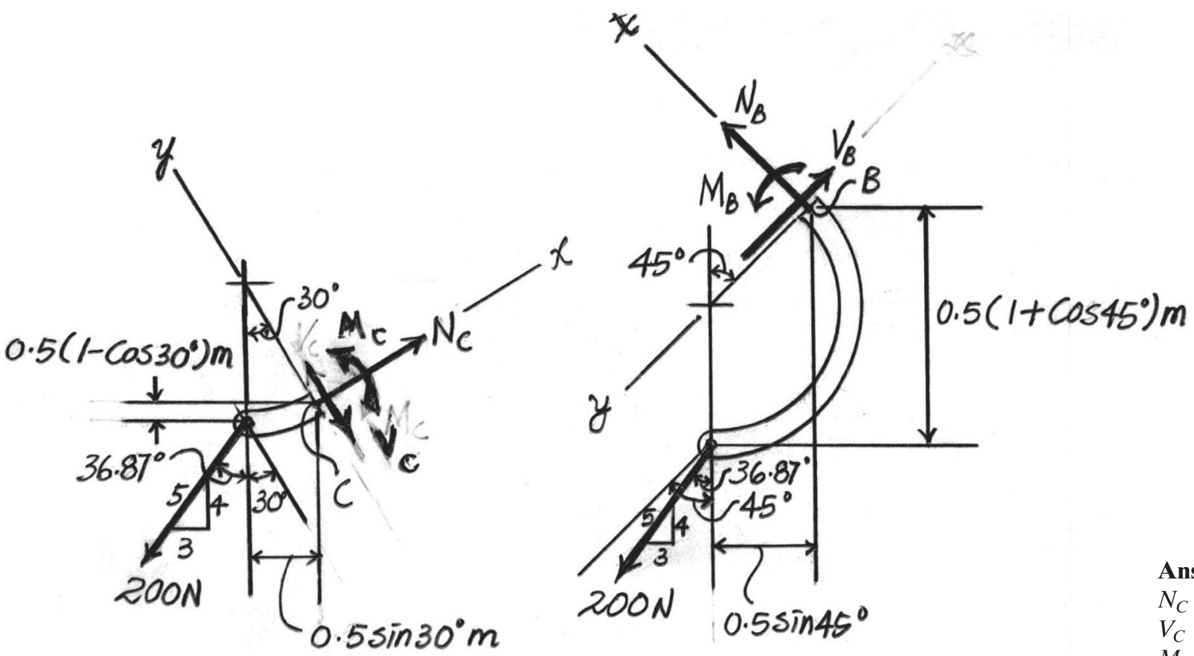
Referring to the *FBD* of bottom segment of the curved rod sectioned through B, Fig. *b*

$\nwarrow + \Sigma F_x = 0;\quad N_B - 200\sin(45° - 36.87°) = 0 \qquad N_B = 28.28\text{ N} = 28.3\text{ N}$ **Ans.**

$+\swarrow\Sigma F_y = 0;\quad -V_B + 200\cos(45° - 36.87°) = 0 \quad V_B = 197.99\text{ N} = 198\text{ N}$ **Ans.**

$$\circlearrowleft + \Sigma M_B = 0;\quad M_B + 200\left(\frac{4}{5}\right)(0.5\sin 45°) - 200\left(\frac{3}{5}\right)[0.5(1 + \cos 45°] = 0$$

$$M_B = 45.86\text{ N}\cdot\text{m} = 45.9\text{ N}\cdot\text{m}$$ **Ans.**

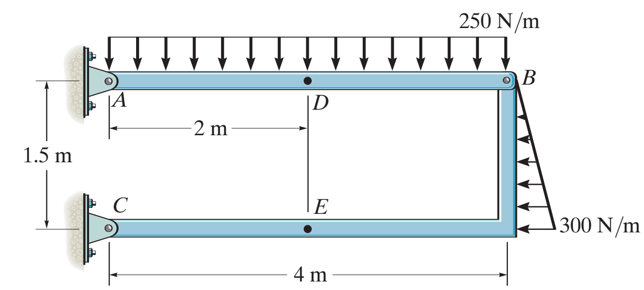


(a) (b)

Ans:
$N_C = 184$ N
$V_C = -78.6$ N
$M_C = -32.0\text{ N}\cdot\text{m}$
$N_B = 28.3$ N
$V_B = 198$ N
$M_B = 45.9\text{ N}\cdot\text{m}$

7–37.

Determine the internal normal force, shear force, and moment at point D of the two-member frame.

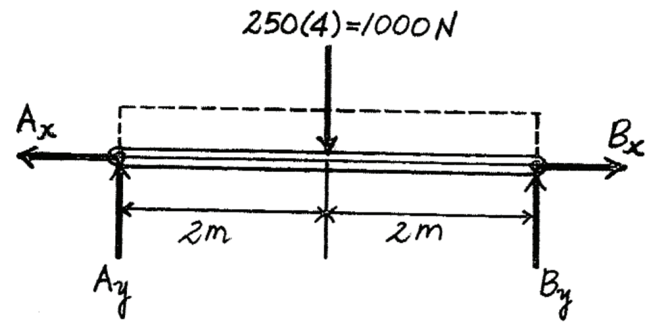


SOLUTION

Member *AB*:

$\circlearrowleft + \Sigma M_A = 0; \qquad B_y\,(4) - 1000\,(2) = 0$

$B_y = 500 \text{ N}$



Member *BC*:

$\circlearrowleft + \Sigma M_C = 0; \qquad -500\,(4) + 225\,(0.5) + B_x\,(1.5) = 0$

$B_x = 1258.33 \text{ N}$

Segment *DB*:

$\xrightarrow{+} \Sigma F_x = 0; \qquad -N_D + 1258.33 = 0$

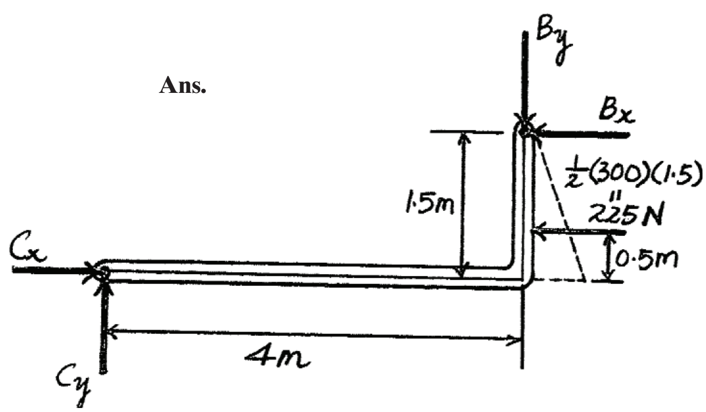
$N_D = 1.26 \text{ kN}$ **Ans.**

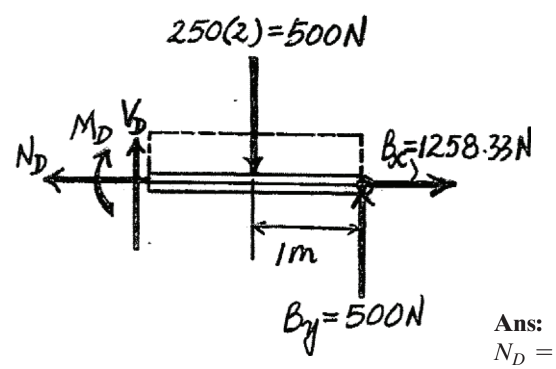
$+\uparrow \Sigma F_y = 0; \qquad V_D - 500 + 500 = 0$

$V_D = 0$ **Ans.**

$\circlearrowleft + \Sigma M_D = 0; \qquad -M_D + 500\,(1) = 0$

$M_D = 500 \text{ N} \cdot \text{m}$ **Ans.**

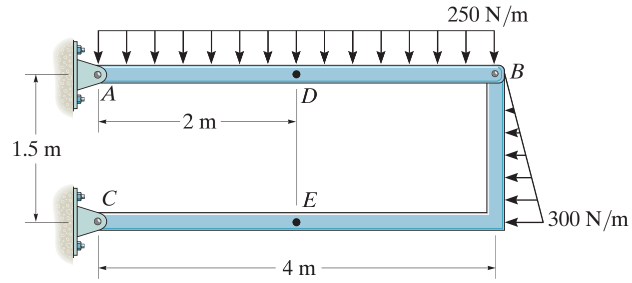




Ans:
$N_D = 1.26 \text{ kN}$
$V_D = 0$
$M_D = 500 \text{ N} \cdot \text{m}$

7–38.

Determine the internal normal force, shear force, and moment at point E of the two-member frame.

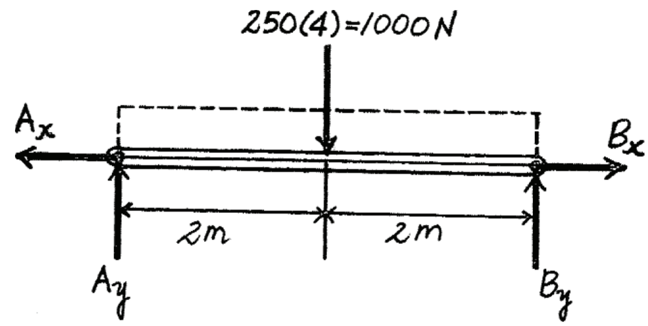


SOLUTION

Member *AB*:

$\circlearrowleft + \Sigma M_A = 0; \quad B_y(4) - 1000(2) = 0$

$B_y = 500 \text{ N}$



Member *BC*:

$\circlearrowleft + \Sigma M_C = 0; \quad -500(4) + 225(0.5) + B_x(1.5) = 0$

$B_x = 1258.33 \text{ N}$

Segment *EB*:

$\xrightarrow{+} \Sigma F_x = 0; \quad -N_E - 1258.33 - 225 = 0$

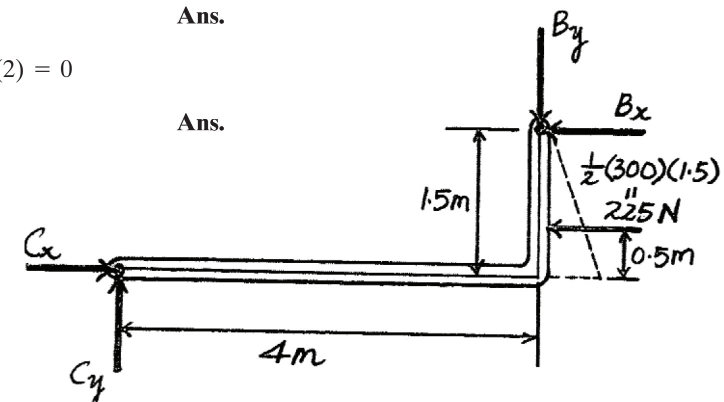
$N_E = -1.48 \text{ kN}$ **Ans.**

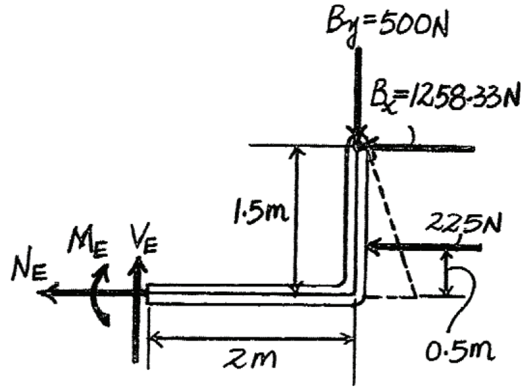
$+\uparrow \Sigma F_y = 0; \quad V_E - 500 = 0$

$V_E = 500 \text{ N}$ **Ans.**

$\circlearrowleft + \Sigma M_E = 0; \quad -M_E + 225(0.5) + 1258.33(1.5) - 500(2) = 0$

$M_E = 1000 \text{ N} \cdot \text{m}$ **Ans.**

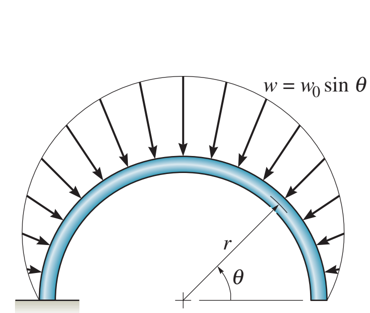




Ans:
$N_E = -1.48 \text{ kN}$
$V_E = 500 \text{ N}$
$M_E = 1000 \text{ N} \cdot \text{m}$

7–39.

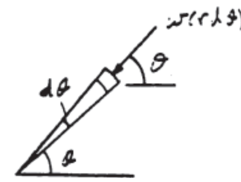
The distributed loading $w = w_0 \sin\theta$, measured per unit length, acts on the curved rod. Determine the internal normal force, shear force, and moment in the rod at $\theta = 45°$.



SOLUTION

$w = w_0 \sin\theta$

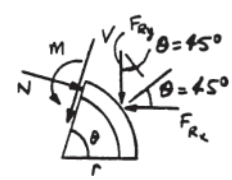
Resultants of distributed loading:



$$F_{Rx} = \int_0^{\theta} w_0 \sin\theta (r\,d\theta)\cos\theta = r w_0 \int_0^{\theta} \sin\theta\cos\theta\,d\theta = \frac{1}{2} r\,w_0 \sin^2\theta$$

$$F_{Ry} = \int_0^{\theta} w_0 \sin\theta (r\,d\theta)\sin\theta = r w_0 \int_0^{\theta} \sin^2\theta\,d\theta = r w_0\left[\frac{1}{2}\theta - \frac{1}{4}\sin 2\theta\right]$$

$\nearrow^{+}\Sigma F_x = 0; \qquad -V + F_{Rx}\cos 45° + F_{Ry}\sin 45° = 0$



$$V = \left(\frac{1}{2} r\,w_0 \sin^2 45°\right)\cos 45° + w_0\left(\frac{1}{2}\,\frac{\pi}{4} - \frac{1}{4}\sin 90°\right)\sin 45°$$

$V = 0.278\,w_0 r$ **Ans.**

$^{+}\nwarrow\Sigma F_y = 0; \qquad -N - F_{Ry}\cos 45° + F_{Rx}\sin 45° = 0$

$$N = -r\,w_0\left[\frac{1}{2}\left(\frac{\pi}{4}\right) - \frac{1}{4}\sin 90°\right]\cos 45° + \left(\frac{1}{2} r\,w_0 \sin^2 45°\right)\sin 45°$$

$N = 0.0759\,w_0\,r$ **Ans.**

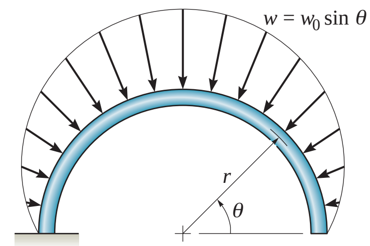
$\circlearrowleft + \Sigma M_O = 0; \qquad M - (0.0759\,r\,w_0)(r) = 0$

$M = 0.0759 \quad w_0\,r^2$ **Ans.**

Ans:
$V = 0.278\,w_0 r$
$N = 0.0759\,w_0 r$
$M = 0.0759\,w_0 r^2$

***7–40.**

Solve Prob. 7–39 for $\theta = 120°$.

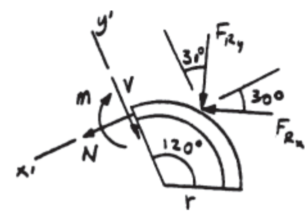


SOLUTION

Resultants of distributed load:

$$F_{Rx} = \int_0^\theta w_0 \sin\theta(r\,d\theta)\cos\theta = rw_0\int_0^\theta \sin\theta\cos\theta = \frac{1}{2}rw_0\sin^2\theta$$

$$F_{Ry} = \int_0^\theta w_0\sin\theta(r\,d\theta)\sin\theta = rw_0\int_0^\theta \sin^2\theta\,d\theta = rw_0\left[\frac{1}{2}\theta\frac{1}{4}\sin 2\theta\right]rw_0(\sin\theta)\Big|_0^\theta = r\,w_0(\sin\theta)$$



$$F_{Rx} = \tfrac{1}{2}r\,w_0\sin^2 120° = 0.375\,r\,w_0$$

$$F_{Ry} = r\,w_0\left[\frac{1}{2}(\pi)\left(\frac{120°}{180°}\right) - \frac{1}{4}\sin 240°\right] = 1.2637\,r\,w_0$$

$\overset{+}{\swarrow}\Sigma F_{x'} = 0;\qquad N + 0.375\,rw_0\cos 30° + 1.2637\,r\,w_0\sin 30° = 0$

$N = -0.957\,r\,w_0$ **Ans.**

$_{+}\nwarrow\Sigma F_{y'} = 0;\qquad -V + 0.375\,rw_0\sin 30° - 1.2637\,r\,w_0\cos 30° = 0$

$V = -0.907\,rw_0$ **Ans.**

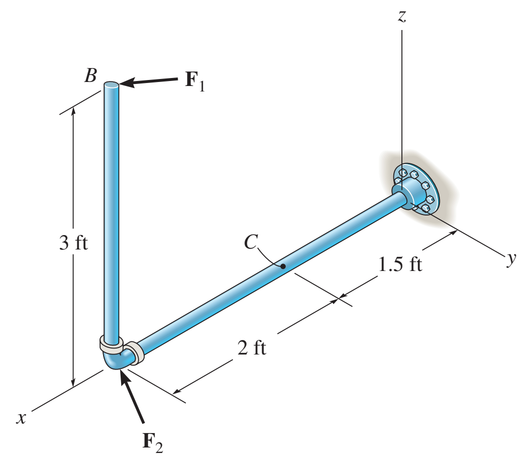
$\circlearrowleft + \Sigma M_O = 0;\qquad -M - 0.957\,r\,w_0(r) = 0$

$M = -0.957\,r^2 w_0$ **Ans.**

Ans:
$N = -0.957\,r\,w_0$
$V = -0.907\,rw_0$
$M = -0.957\,r^2 w_0$

7–41.

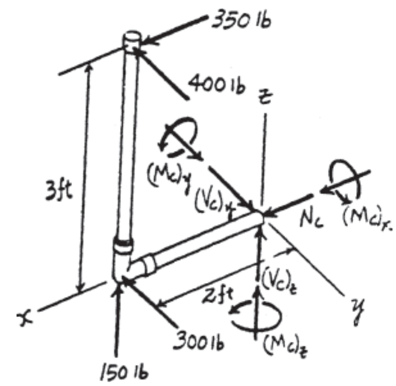
Determine the x, y, z components of force and moment at point C in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F}_1 = \{350\mathbf{i} - 400\mathbf{j}\}$ lb and $\mathbf{F}_2 = \{-300\mathbf{j} + 150\mathbf{k}\}$ lb.



SOLUTION

Free body Diagram: The support reactions need not be computed.

Internal Forces: Applying the equations of equilibrium to segment BC, we have



$\Sigma F_x = 0; \quad N_C + 350 = 0 \quad N_C = -350 \text{ lb}$ **Ans.**

$\Sigma F_y = 0; \quad (V_C)_y - 400 - 300 = 0 \quad (V_C)_y = 700 \text{ lb}$ **Ans.**

$\Sigma F_z = 0; \quad (V_C)_z + 150 = 0 \quad (V_C)_z = -150 \text{ lb}$ **Ans.**

$\Sigma M_x = 0; \quad (M_C)_x + 400(3) = 0$

$(M_C)_x = -1200 \text{ lb}\cdot\text{ft} = -1.20 \text{ kip}\cdot\text{ft}$ **Ans.**

$\Sigma M_y = 0; \quad (M_C)_y + 350(3) - 150(2) = 0$

$(M_C)_y = -750 \text{ lb}\cdot\text{ft}$ **Ans.**

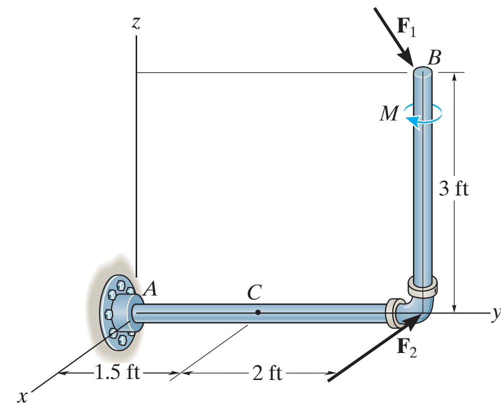
$\Sigma M_z = 0; \quad (M_C)_z - 300(2) - 400(2) = 0$

$(M_C)_z = 1400 \text{ lb}\cdot\text{ft} = 1.40 \text{ kip}\cdot\text{ft}$ **Ans.**

Ans:
$N_C = -350$ lb
$(V_C)_y = 700$ lb
$(V_C)_z = -150$ lb
$(M_C)_x = -1.20$ kip·ft
$(M_C)_y = -750$ lb·ft
$(M_C)_z = 1.40$ kip·ft

7–42.

Determine the x, y, z components of force and moment at point C in the pipe assembly. Neglect the weight of the pipe. The load acting at (0, 3.5 ft, 3 ft) is $\mathbf{F}_1 = \{-24\mathbf{i} - 10\mathbf{k}\}$ lb and $\mathbf{M} = \{-30\mathbf{k}\}$ lb · ft and at point (0, 3.5 ft, 0) $\mathbf{F}_2 = \{-80\mathbf{i}\}$ lb.



SOLUTION

Free body Diagram: The support reactions need not be computed.

Internal Forces: Applying the equations of equilibrium to segment BC, we have

$\Sigma F_x = 0;\quad (V_C)_x - 24 - 80 = 0 \quad (V_C)_x = 104\text{ lb}$ **Ans.**

$\Sigma F_y = 0;\quad N_C = 0$ **Ans.**

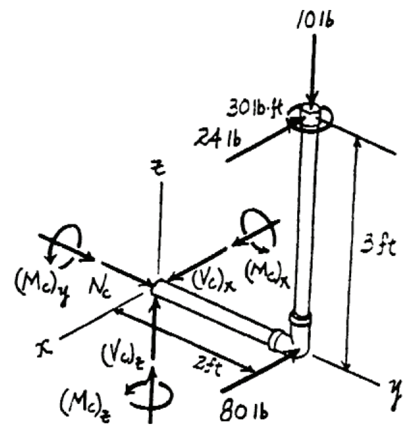
$\Sigma F_z = 0;\quad (V_C)_z - 10 = 0 \quad (V_C)_z = 10.0\text{ lb}$ **Ans.**

$\Sigma M_x = 0;\quad (M_C)_x - 10(2) = 0 \quad (M_C)_x = 20.0\text{ lb}\cdot\text{ft}$ **Ans.**

$\Sigma M_y = 0;\quad (M_C)_y - 24\,(3) = 0 \quad (M_C)_y = 72.0\text{ lb}\cdot\text{ft}$ **Ans.**

$\Sigma M_z = 0;\quad (M_C)_z + 24\,(2) + 80\,(2) - 30 = 0$

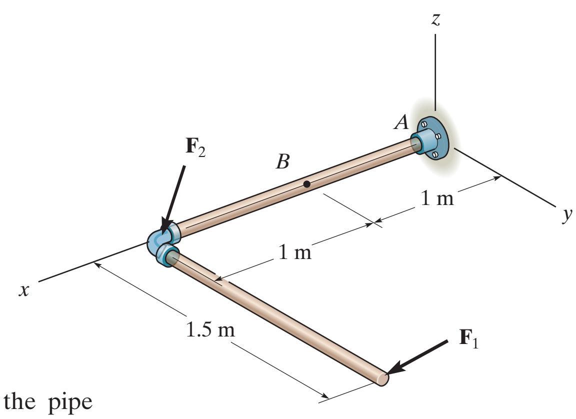
$(M_C)_z = -178\text{ lb}\cdot\text{ft}$ **Ans.**



Ans:
$(V_C)_x = 104$ lb
$N_C = 0$
$(V_C)_z = 10$ lb
$(M_C)_x = 20$ lb · ft
$(M_C)_y = 72$ lb · ft
$(M_C)_z = -178$ lb · ft

7–43.

Determine the *x, y, z* components of internal loading at a section passing through point *B* in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F}_1 = \{200\mathbf{i} - 100\mathbf{j} - 400\mathbf{k}\}$ N and $\mathbf{F}_2 = \{300\mathbf{i} - 500\mathbf{k}\}$ N.

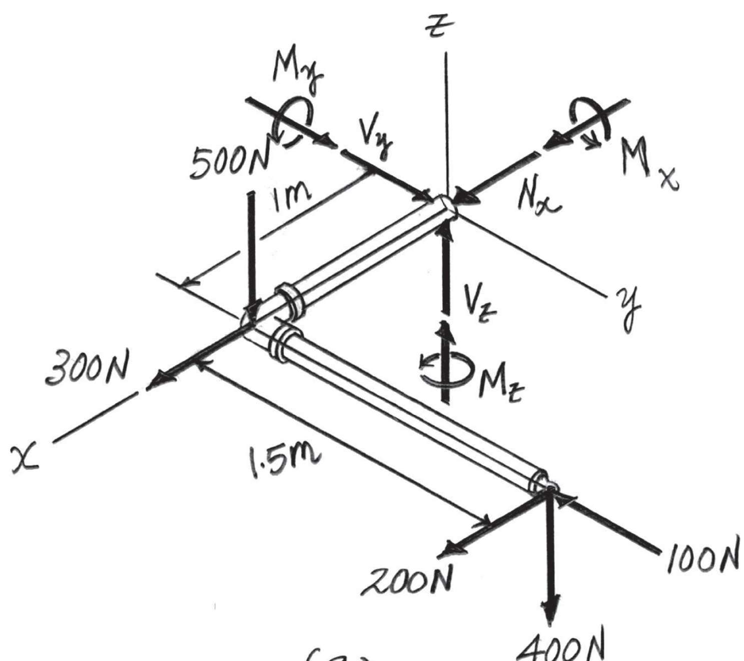


SOLUTION

Internal Loadings. Referring to the *FBD* of the free end segment of the pipe assembly sectioned through *B*, Fig. *a*,

$\Sigma F_x = 0;$	$N_x + 300 + 200 = 0$	$N_x = -500$ N	**Ans.**
$\Sigma F_y = 0;$	$V_y - 100 = 0$	$V_y = 100$ N	**Ans.**
$\Sigma F_z = 0;$	$V_z - 500 - 400 = 0$	$V_z = 900$ N	**Ans.**
$\Sigma M_x = 0;$	$M_x - 400(1.5) = 0$	$M_x = 600\ \text{N}\cdot\text{m}$	**Ans.**
$\Sigma M_y = 0;$	$M_y + 500(1) + 400(1) = 0$	$M_y = -900\ \text{N}\cdot\text{m}$	**Ans.**
$\Sigma M_z = 0$	$M_z - 200(1.5) - 100(1) = 0$	$M_z = 400\ \text{N}\cdot\text{m}$	**Ans.**

The negative signs indicate that $\mathbf{N}_x$ and $\mathbf{M}_y$ act in the opposite sense to those shown in *FBD*.

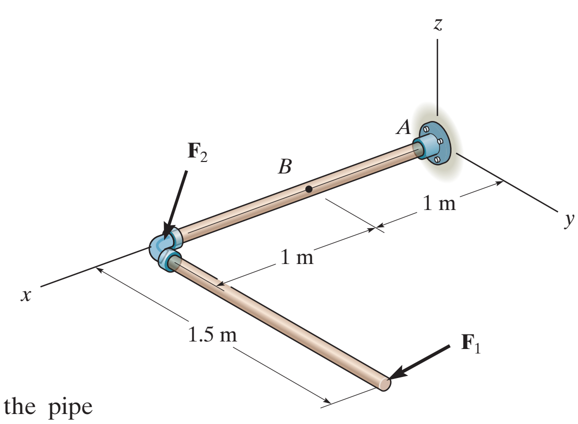


(a)

Ans:
$N_x = -500$ N
$V_y = 100$ N
$V_z = 900$ N
$M_x = 600\ \text{N}\cdot\text{m}$
$M_y = -900\ \text{N}\cdot\text{m}$
$M_z = 400\ \text{N}\cdot\text{m}$

***7–44.**

Determine the *x, y, z* components of internal loading at a section passing through point *B* in the pipe assembly. Neglect the weight of the pipe. Take $\mathbf{F}_1 = \{100\mathbf{i} - 200\mathbf{j} - 300\mathbf{k}\}$ N and $\mathbf{F}_2 = \{100\mathbf{i} + 500\mathbf{j}\}$ N.



SOLUTION

Internal Loadings. Referring to the *FBD* of the free end segment of the pipe assembly sectioned through *B*, Fig. *a*

$\Sigma F_x = 0;\quad N_x + 100 + 100 = 0 \qquad N_x = -200\text{ N}$ **Ans.**

$\Sigma F_y = 0;\quad V_y + 500 - 200 = 0 \qquad V_y = -300\text{ N}$ **Ans.**

$\Sigma F_z = 0;\quad V_z - 300 = 0 \qquad V_z = 300\text{ N}$ **Ans.**

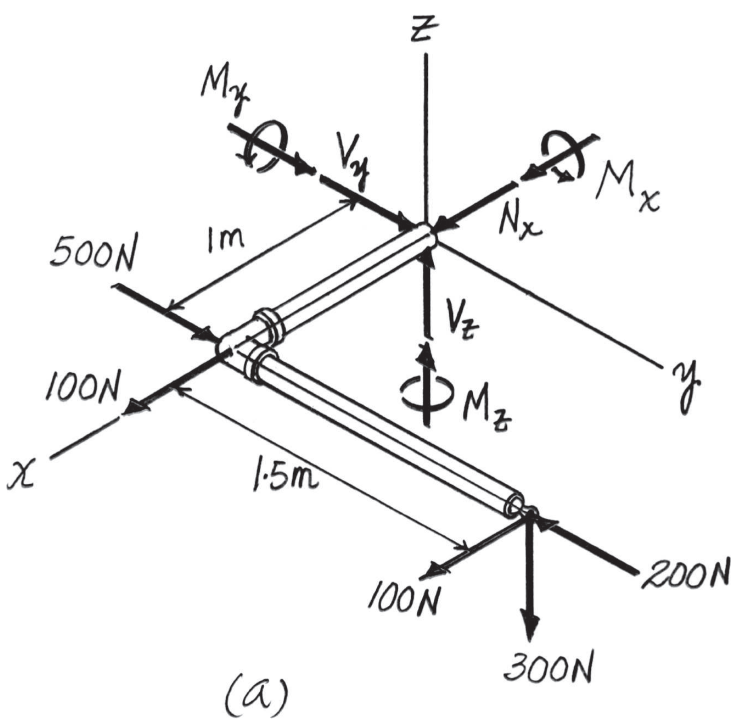
$\Sigma M_x = 0;\quad M_x - 300(1.5) = 0 \qquad M_x = 450\text{ N}\cdot\text{m}$ **Ans.**

$\Sigma M_y = 0;\quad M_y + 300(1) = 0 \qquad M_y = -300\text{ N}\cdot\text{m}$ **Ans.**

$\Sigma M_z = 0;\quad M_z + 500(1) - 100(1.5) - 200(1) = 0$

$M_z = -150\text{ N}\cdot\text{m}$ **Ans.**

The negative signs indicates that $\mathbf{N}_x$, $\mathbf{V}_y$, $\mathbf{M}_y$ and $\mathbf{M}_z$ act in the senses opposite to those shown in *FBD*.

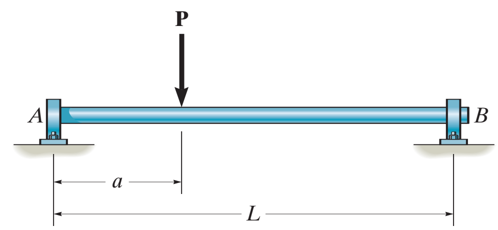


(a)

Ans:
$N_x = -200$ N
$V_y = -300$ N
$V_z = 300$ N
$M_x = 450$ N·m
$M_y = -300$ N·m
$M_z = -150$ N·m

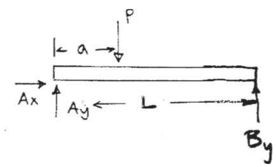
7–45.

Draw the shear and moment diagrams for the shaft (a) in terms of the parameters shown; (b) set $P = 9$ kN, $a = 2$ m, $L = 6$ m. There is a thrust bearing at A and a journal bearing at B.



SOLUTION

(a)



$\circlearrowleft + \Sigma M_B = 0;\qquad (A_y)(L) - P(L - a) = 0$

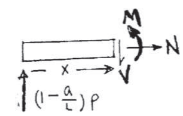
$$A_y = \left(\frac{L - a}{L}\right)P$$

$$A_y = \left(1 - \frac{a}{L}\right)P$$

$+\uparrow \Sigma F_y = 0;\qquad A_y + B_y - P = 0$

$$B_y = P - A_y = \left(\frac{a}{L}\right)P$$

$\xrightarrow{+} \Sigma F_x = 0;\qquad A_x = 0$

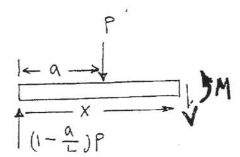


For $0 \le x \le a$

$+\uparrow \Sigma F_y = 0;\qquad \left(1 - \frac{a}{L}\right)P - V = 0$

$$V = \left(1 - \frac{a}{L}\right)P \qquad \textbf{Ans.}$$

$\xrightarrow{+} \Sigma F_x = 0;\qquad A = 0$



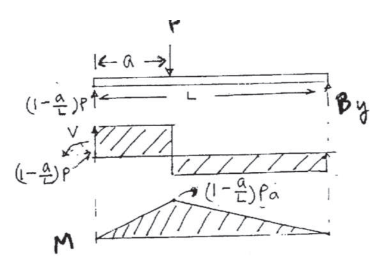
$\circlearrowleft + \Sigma M = 0;\qquad \left(1 - \frac{a}{L}\right)Px - M = 0$

$$M = \left(1 - \frac{a}{L}\right)Px \qquad \textbf{Ans.}$$

For $a < x < L$

$+\uparrow \Sigma F_y = 0;\qquad \left(1 - \frac{a}{L}\right)P - P - V = 0$

$$V = -\left(\frac{a}{L}\right)P \qquad \textbf{Ans.}$$



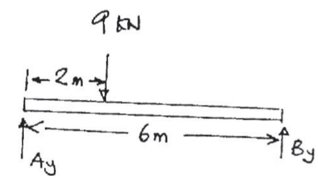
$\circlearrowleft + \Sigma M = 0;\qquad \left(1 - \frac{a}{L}\right)Px - P(x - a) - M = 0$

$$M = Px - \left(\frac{a}{L}\right)Px - Px + Pa$$

7–45. Continued

(b)

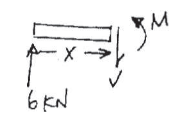
$$M = P\left(a - \frac{a}{L}x\right)$$ **Ans.**



$\circlearrowleft + \Sigma M_B = 0;$ $A_y(6) - 9(4) = 0$

$A_y = 6\text{ kN}$

$+\uparrow \Sigma F_y = 0;$ $B_y = 3\text{ kN}$

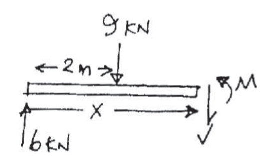


For $0 \leq x \leq 2\text{ m}$

$+\uparrow \Sigma F_y = 0;$ $6 - V = 0$

$V = 6\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M = 0;$ $6x - M = 0$



$M = 6x\text{ kN}\cdot\text{m}$ **Ans.**

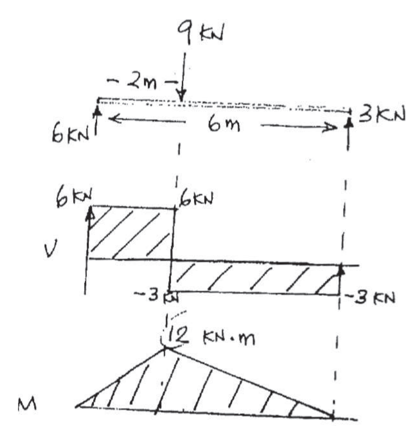
For $2\text{ m} < x \leq 6\text{ m}$

$+\uparrow \Sigma F_y = 0;$ $6 - 9 - V = 0$

$V = -3\text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M = 0;$ $6x - 9(x - 2) - M = 0$

$M = 18 - 3x\text{ kN}\cdot\text{m}$ **Ans.**



Ans:

$0 \leq x < a$: $V = \left(1 - \frac{a}{L}\right)P$

$M = \left(1 - \frac{a}{L}\right)Px$

$a < x \leq L$: $V = -\left(\frac{a}{L}\right)P$

$M = P\left(a - \frac{a}{L}x\right)$

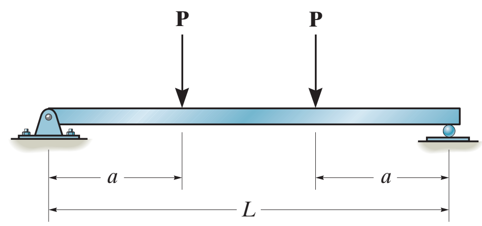
$0 \leq x < 2\text{ m}$: $V = 6\text{ kN}$, $M = \{6x\}\text{ kN}\cdot\text{m}$

$2\text{ m} < x \leq 6\text{ m}$: $V = -3\text{ kN}$

$M = \{18 - 3x\}\text{ kN}\cdot\text{m}$

7–46.

Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $P = 800$ lb, $a = 5$ ft, $L = 12$ ft.



SOLUTION

(a) For $0 \leq x < a$

$+\uparrow \Sigma F_y = 0; \quad V = P$ **Ans.**

$\zeta + \Sigma M = 0; \quad M = Px$ **Ans.**

For $a < x < L-a$

$+\uparrow \Sigma F_y = 0; \quad V = 0$ **Ans.**

$\zeta + \Sigma M = 0; \quad -Px + P(x - a) + M = 0$

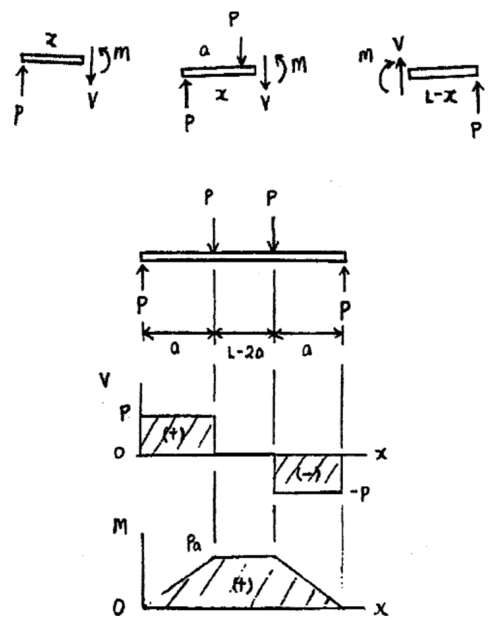
$M = Pa$ **Ans.**

For $L-a < x \leq L$

$+\uparrow \Sigma F_y = 0; \quad V = -P$ **Ans.**

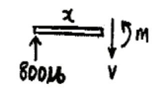
$\zeta + \Sigma M = 0; \quad -M + P(L - x) = 0$

$M = P(L - x)$ **Ans.**



7–46. Continued

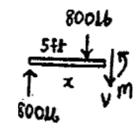
(b) Set $P = 800$ lb, $a = 5$ ft, $L = 12$ ft



For $0 \leq x < 5$ ft

$+\uparrow \Sigma F_y = 0; \quad V = 800$ lb **Ans.**

$\circlearrowleft + \Sigma M = 0; \quad M = 800x$ lb·ft **Ans.**

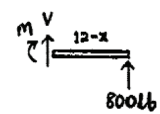


For 5 ft $< x < 7$ ft

$+\uparrow \Sigma F_y = 0; \quad V = 0$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \quad -800x + 800(x - 5) + M = 0$

$M = 4000$ lb·ft **Ans.**

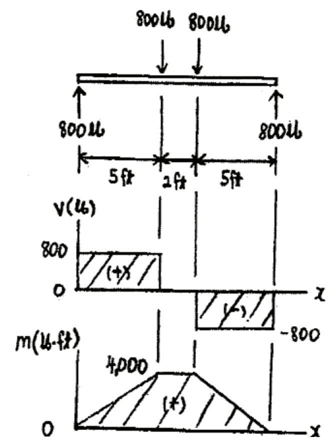


For 7 ft $< x \leq 12$ ft

$+\uparrow \Sigma F_y = 0; \quad V = -800$ lb **Ans.**

$\circlearrowleft + \Sigma M = 0; \quad -M + 800(12 - x) = 0$

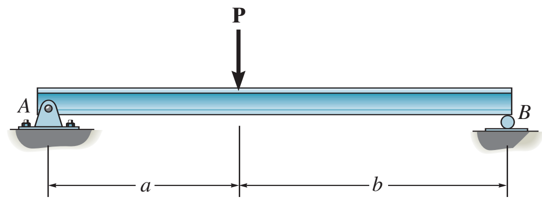
$M = (9600 - 800x)$ lb·ft **Ans.**



Ans:
For $0 \leq x < a$, $V = P$, $M = Px$
For $a < x < L - a$, $V = 0$, $M = Pa$
For $L - a < x \leq L$, $V = -P$, $M = P(L - x)$
For $0 \leq x < 5$ ft, $V = 800$ lb
$M = 800x$ lb·ft
For 5 ft $< x < 7$ ft, $V = 0$
$M = 4000$ lb·ft
For 7 ft $< x \leq 12$ ft, $V = -800$ lb
$M = (9600 - 800x)$ lb·ft

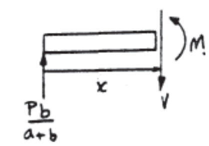
7–47.

Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $P = 600$ lb, $a = 5$ ft, $b = 7$ ft.



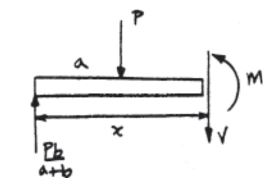
SOLUTION

(a) For $0 \le x < a$



$+\uparrow \Sigma F_y = 0;$ $$\frac{P\,b}{a+b} - V = 0$$

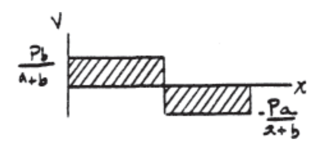
$$V = \frac{P\,b}{a+b}$$ **Ans.**



$\zeta + \Sigma M = 0;$ $$M - \frac{P\,b}{a+b}x = 0$$

$$M = \frac{P\,b}{a+b}x$$ **Ans.**

For $a < x \le (a+b)$



$+\uparrow \Sigma F_y = 0;$ $$\frac{P\,b}{a+b} - P - V = 0$$

$$V = -\frac{P\,a}{a+b}$$ **Ans.**

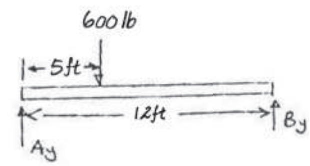
$\zeta + \Sigma M = 0;$ $$-\frac{P\,b}{a+b}x + P(x-a) + M = 0$$

$$M = P\,a - \frac{P\,a}{a+b}x$$ **Ans.**

(b) For $P = 600$ lb, $a = 5$ ft, $b = 7$ ft

7–47. Continued

(b)



$\circlearrowleft + \Sigma M_B = 0;$ $\quad A_y(12) - 600(7) = 0$

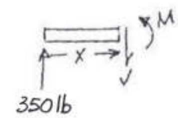
$A_y = 350 \text{ lb}$

$+\uparrow \Sigma F_y = 0;$ $\quad B_y = 250 \text{ lb}$

For $0 \leq x \leq 5$ ft

$+\uparrow \Sigma F_y = 0;$ $\quad 350 - V = 0$

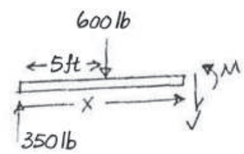
$V = 350 \text{ lb}$ **Ans.**



$\circlearrowleft + \Sigma M = 0;$ $\quad 350x - M = 0$

$M = 350x \text{ lb} \cdot \text{ft}$ **Ans.**

For $5 \text{ ft} < x \leq 12 \text{ ft}$

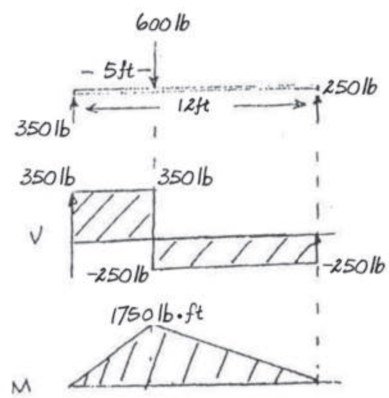


$+\uparrow \Sigma F_y = 0;$ $\quad 350 - 600 - V = 0$

$V = -250 \text{ lb}$ **Ans.**

$\circlearrowleft + \Sigma M = 0;$ $\quad 350x - 600(x - 5) - M = 0$

$M = \{3000 - 250x\} \text{ lb} \cdot \text{ft}$ **Ans.**



Ans:

For $0 \leq x < a$, $V = \dfrac{Pb}{a + b}$, $M = \dfrac{Pb}{a + b}x$

For $a < x \leq a + b$, $V = -\dfrac{Pa}{a + b}$

$M = Pa - \dfrac{Pa}{a + b}x$

For $0 \leq x < 5$ ft, $V = 350$ lb

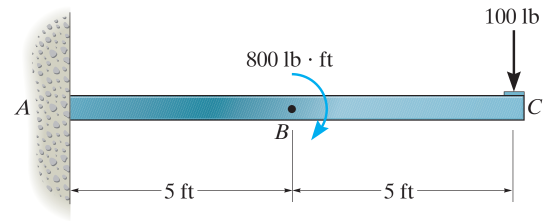
$M = 350x \text{ lb} \cdot \text{ft}$

For $5 \text{ ft} < x \leq 12$ ft, $V = -250$ lb

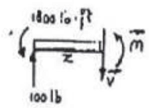
$M = \{3000 - 250x\} \text{ lb} \cdot \text{ft}$

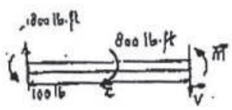
***7–48.**

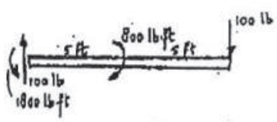
Draw the shear and moment diagrams for the cantilevered beam.

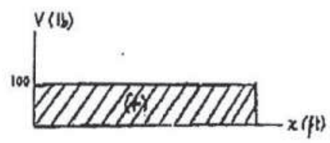


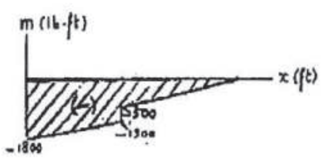
SOLUTION







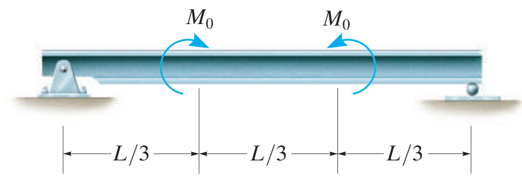




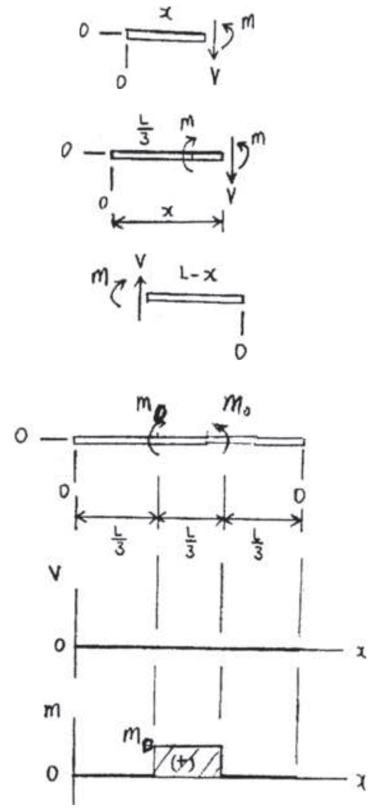
Ans:
$V = 100 \text{ lb}$
$M_{\max} = -1800 \text{ lb} \cdot \text{ft}$

7–49.

Draw the shear and moment diagrams for the beam (a) in terms of the parameters shown; (b) set $M_0 = 500\ \text{N}\cdot\text{m}$, $L = 8$ m.



SOLUTION



(a)

For $0 \le x \le \dfrac{L}{3}$

$+\uparrow \Sigma F_y = 0;\qquad V = 0$ **Ans.**

$\zeta + \Sigma M = 0;\qquad M = 0$ **Ans.**

For $\dfrac{L}{3} < x < \dfrac{2L}{3}$

$+\uparrow \Sigma F_y = 0;\qquad V = 0$ **Ans.**

$\zeta + \Sigma M = 0;\qquad M = M_0$ **Ans.**

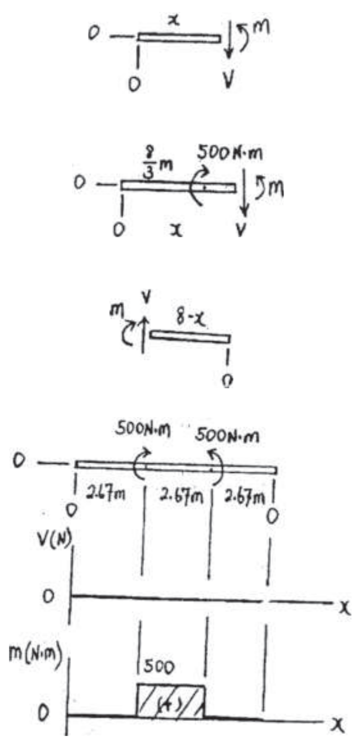
For $\dfrac{2L}{3} < x \le L$

$+\uparrow \Sigma F_y = 0;\qquad V = 0$ **Ans.**

$\zeta + \Sigma M = 0;\qquad M = 0$ **Ans.**

(b)

Set $M_0 = 500\ \text{N}\cdot\text{m}, L = 8$ m



For $0 \le x < \dfrac{8}{3}$ m

$+\uparrow \Sigma F_y = 0;\qquad V = 0$ **Ans.**

$\circlearrowright + \Sigma M = 0;\qquad M = 0$ **Ans.**

For $\dfrac{8}{3}$ m $< x < \dfrac{16}{3}$ m

$+\uparrow \Sigma F_y = 0;\qquad V = 0$ **Ans.**

$\circlearrowright + \Sigma M = 0;\qquad M = 500\ \text{N}\cdot\text{m}$ **Ans.**

For $\dfrac{16}{3}$ m $< x \le 8$ m

$+\uparrow \Sigma F_y = 0;\qquad V = 0$ **Ans.**

$\circlearrowright + \Sigma M = 0;\qquad M = 0$ **Ans.**

Ans:

$0 \le x < \dfrac{L}{3}$: $V = 0, M = 0$

$\dfrac{L}{3} < x < \dfrac{2L}{3}$: $V = 0, M = M_0$

$\dfrac{2L}{3} < x \le L$: $V = 0, M = 0$

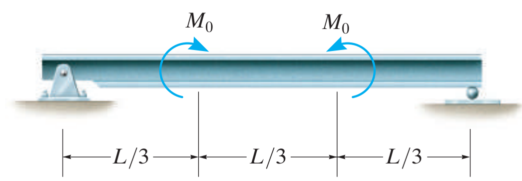
$0 \le x < \dfrac{8}{3}$ m: $V = 0, M = 0$

$\dfrac{8}{3}$ m $< x < \dfrac{16}{3}$ m: $V = 0, M = 500\ \text{N}\cdot\text{m}$

$\dfrac{16}{3}$ m $< x \le 8$ m: $V = 0, M = 0$

7–50.

If $L = 9$ m, the beam will fail when the maximum shear force is $V_{max} = 5$ kN or the maximum bending moment is $M_{max} = 2$ kN·m. Determine the magnitude M_0 of the largest couple moments it will support.



SOLUTION

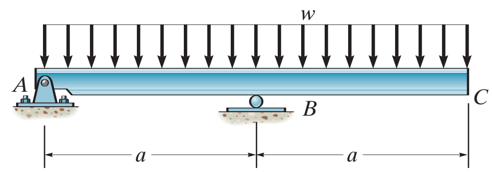
See solution to Prob. 7–48 a.

$M_{max} = M_0 = 2$ kN·m **Ans.**

Ans:
$M_{max} = 2$ kN·m

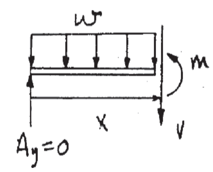
7–51.

Draw the shear and moment diagrams for the beam.



SOLUTION

$0 \leq x < a$



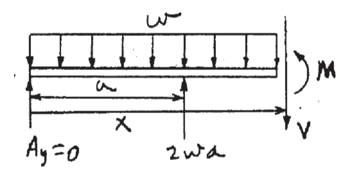
$+\uparrow \Sigma F_y = 0; \qquad -V - wx = 0$

$V = -wx$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad M + wx\left(\frac{x}{2}\right) = 0$

$M = -\frac{w}{2}x^2$ **Ans.**

$a < x \leq 2a$

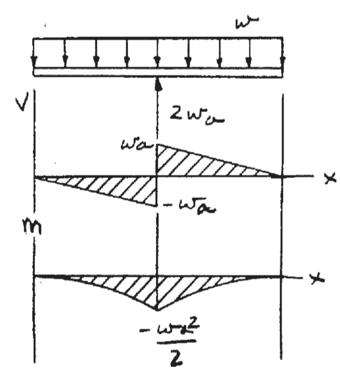


$+\uparrow \Sigma F_y = 0; \qquad -V + 2\,wa - wx = 0$

$V = w\,(2a - x)$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad M + wx\left(\frac{x}{2}\right) - 2\,wa\,(x - a) = 0$

$M = 2\,wax - 2\,wa^2 - \frac{w}{2}x^2$ **Ans.**



Ans:

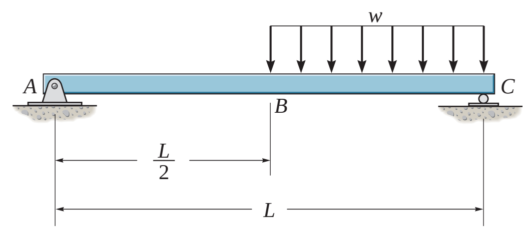
$0 \leq x < a$: $V = -wx$, $M = -\frac{w}{2}x^2$

$a < x \leq 2a$: $V = w(2a - x)$

$M = 2wax - 2wa^2 - \frac{w}{2}x^2$

***7–52.**

Draw the shear and moment diagrams for the beam.

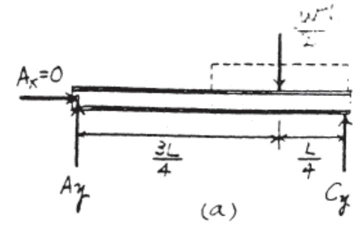


SOLUTION

Support Reactions: From FBD (a),

$$\circlearrowleft + \Sigma M_A = 0; \quad C_y(L) - \frac{wL}{2}\left(\frac{3L}{4}\right) = 0 \qquad C_y = \frac{3wL}{8}$$

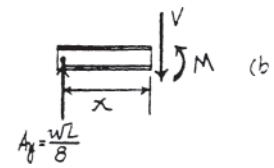
$$+\uparrow \Sigma F_y = 0; \quad A_y + \frac{3wL}{8} - \frac{wL}{2} = 0 \qquad A_y = \frac{wL}{8}$$



Shear and Moment Functions: For $\mathbf{0 \le x < \frac{L}{2}}$ [FBD (b)],

$$+\uparrow \Sigma F_y = 0; \quad \frac{wL}{8} - V = 0 \qquad V = \frac{wL}{8}$$ **Ans.**

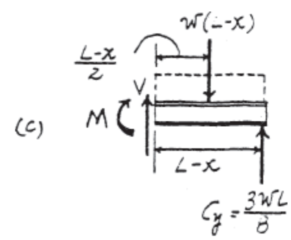
$$\circlearrowleft + \Sigma M = 0; \quad M - \frac{wL}{8}(x) = 0 \qquad M = \frac{wL}{8}x$$ **Ans.**



For $\mathbf{\frac{L}{2} < x \le L}$ [FBD (c)],

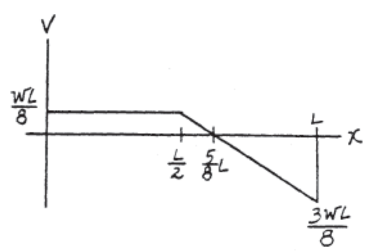
$$+\uparrow \Sigma F_y = 0; \quad V + \frac{3wL}{8} - w(L - x) = 0$$

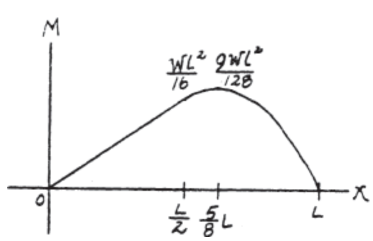
$$V = \frac{w}{8}(5L - 8x)$$ **Ans.**



$$\circlearrowleft + \Sigma M = 0; \quad \frac{3wL}{8}(L - x) - w(L - x)\left(\frac{L - x}{2}\right) - M = 0$$

$$M = \frac{w}{8}(-L^2 + 5Lx - 4x^2)$$ **Ans.**





Ans:

$V = \frac{wL}{8}$

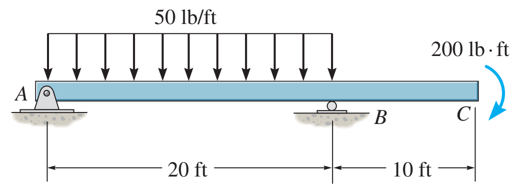
$M = \frac{wL}{8}x$

$V = \frac{w}{8}(5L - 8x)$

$M = \frac{w}{8}\left(-L^2 + 5Lx - 4x^2\right)$

7–53.

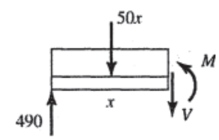
Draw the shear and bending-moment diagrams for the beam.



SOLUTION

Support Reactions:

$\circlearrowleft + \Sigma M_B = 0; \qquad 1000(10) - 200 - A_y(20) = 0 \qquad A_y = 490 \text{ lb}$



(a)

Shear and Moment Functions: For $0 \leq x < 20$ ft [FBD (a)],

$+\uparrow \Sigma F_y = 0; \qquad 490 - 50x - V = 0$

$V = \{490 - 50.0x\}$ lb **Ans.**

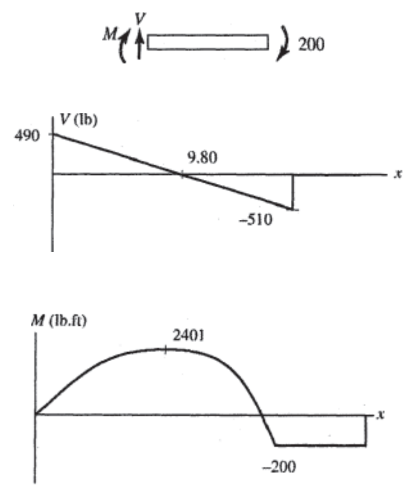
$\circlearrowleft + \Sigma M = 0; \qquad M + 50x\left(\frac{x}{2}\right) - 490x = 0$

$M = \{490x - 25.0x^2\}$ lb · ft **Ans.**

For 20 ft $< x \leq 30$ ft [FBD (b)],

$+\uparrow \Sigma F_y = 0; \qquad V = 0$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad -200 - M = 0 \quad M = -200$ lb · ft **Ans.**

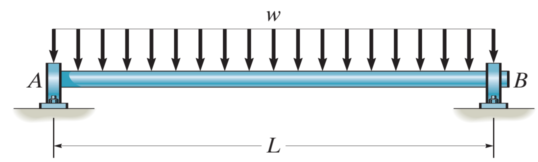


(b)

Ans:
For $0 \leq x < 20$ ft
$V = \{490 - 50.0x\}$ lb
$M = \{490x - 25.0x^2\}$ lb · ft
For 20 ft $< x \leq 30$ ft
$V = 0$
$M = -200$ lb · ft

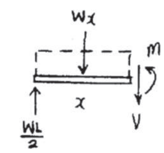
7–54.

The shaft is supported by a thrust bearing at A and a journal bearing at B. Draw the shear and moment diagrams for the shaft (a) in terms of the parameters shown; (b) set $w = 500$ lb/ft, $L = 10$ ft.



SOLUTION

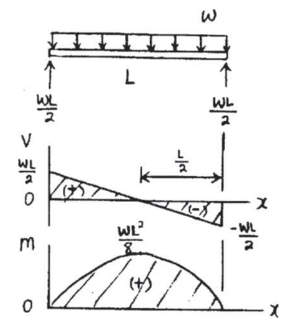
(a) For $0 \le x \le L$



$+\uparrow \Sigma F_y = 0;\qquad \dfrac{wL}{2} - wx - V = 0$

$$V = -wx + \frac{wL}{2}$$

$$V = \frac{w}{2}(L - 2x) \qquad \textbf{Ans.}$$

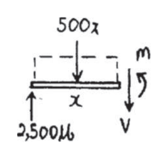


$\zeta + \Sigma M = 0;\qquad -\dfrac{wL}{2}x + wx\left(\dfrac{x}{2}\right) + M = 0$

$$M = \frac{wL}{2}x - \frac{wx^2}{2}$$

$$M = \frac{w}{2}\left(Lx - x^2\right) \qquad \textbf{Ans.}$$

(b) Set $w = 500$ lb/ft, $L = 10$ ft



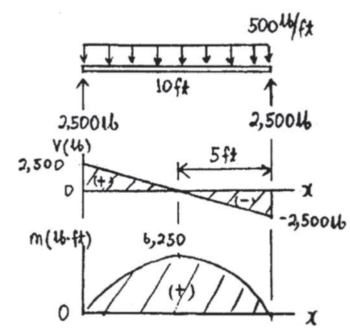
For $0 \le x \le 10$ ft

$+\uparrow \Sigma F_y = 0;\qquad 2500 - 500x - V = 0$

$$V = (2500 - 500x) \text{ lb} \qquad \textbf{Ans.}$$

$\zeta + \Sigma M = 0;\qquad -2500x + 500\dfrac{x^2}{2} + M = 0$

$$M = (2500x - 250x^2) \text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$



Ans:

$V = \dfrac{w}{2}(L - 2x)$

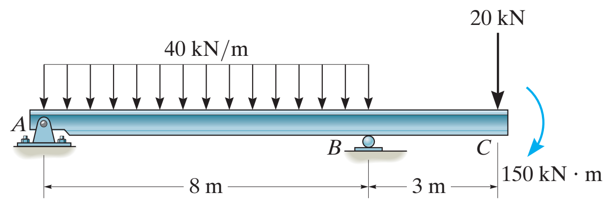
$M = \dfrac{w}{2}(Lx - x^2)$

$V = (2500 - 500x)$ lb

$M = (2500x - 250x^2)$ lb · ft

7–55.

Draw the shear and moment diagrams for the beam.



SOLUTION

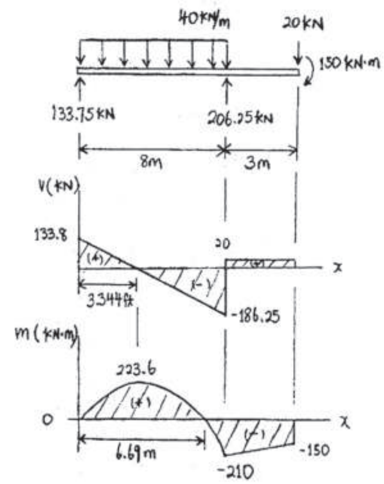
$0 \le x < 8$

$+\uparrow \Sigma F_y = 0; \qquad 133.75 - 40x - V = 0$

$V = 133.75 - 40x$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad M + 40x\left(\dfrac{x}{2}\right) - 133.75x = 0$

$M = 133.75x - 20x^2$ **Ans.**



$8 < x \le 11$

$+\uparrow \Sigma F_y = 0; \qquad V - 20 = 0$

$V = 20$ **Ans.**

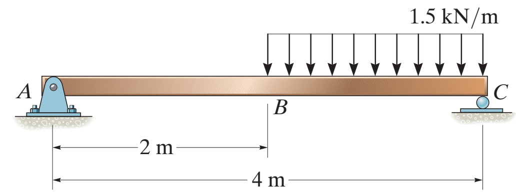
$\circlearrowleft + \Sigma M = 0; \qquad M + 20(11 - x) + 150 = 0$

$M = 20x - 370$ **Ans.**

Ans:
For $0 \le x < 8\text{ m}$
$V = (133.75 - 40x)\text{ kN}$
$M = \left(133.75x - 20x^2\right)\text{ kN}\cdot\text{m}$
For $8\text{ m} < x \le 11\text{ m}$
$V = 20\text{ kN}$
$M = (20x - 370)\text{ kN}\cdot\text{m}$

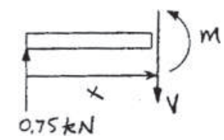
***7–56.**

Draw the shear and moment diagrams for the beam.



SOLUTION

$0 \le x \le 2$ m:



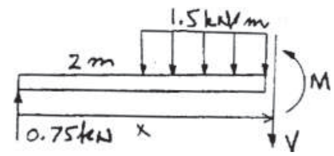
$+\uparrow \Sigma F_y = 0; \qquad 0.75 - V = 0$

$V = 0.75 \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad M - 0.75x = 0$

$M = 0.75\,x \text{ kN} \cdot \text{m}$ **Ans.**

$2 \text{ m} < x < 4$ m:

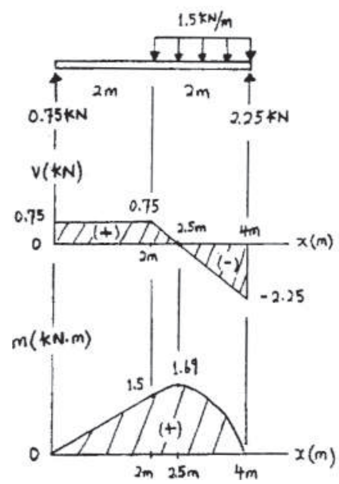


$+\uparrow \Sigma F_y = 0; \qquad 0.75 - 1.5\,(x - 2) - V = 0$

$V = 3.75 - 1.5\,x \text{ kN}$ **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad M + \dfrac{1.5}{2}(x - 2)^2 - 0.75\,x = 0$

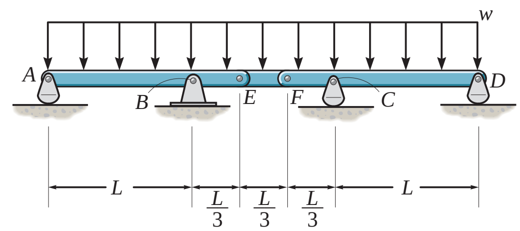
$M = -0.75\,x^2 + 3.75\,x - 3 \text{ kN} \cdot \text{m}$ **Ans.**



Ans:
$V = 0.75$ kN
$M = 0.75\,x$ kN · m
$V = 3.75 - 1.5\,x$ kN
$M = -0.75\,x^2 + 3.75\,x - 3$ kN · m

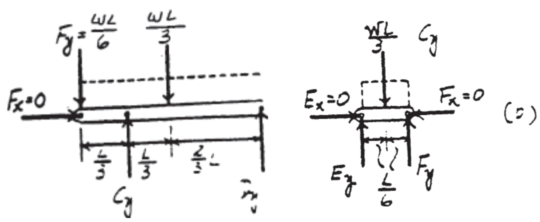
7–57.

Draw the shear and moment diagrams for the compound beam. The beam is pin-connected at E and F.



SOLUTION

Support Reactions: From FBD (b),



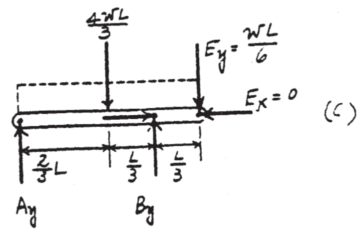
$$\circlearrowleft +\Sigma M_E = 0; \qquad F_y\left(\frac{L}{3}\right) - \frac{wL}{3}\left(\frac{L}{6}\right) = 0 \qquad F_y = \frac{wL}{6}$$

$$+\uparrow \Sigma F_y = 0; \qquad E_y + \frac{wL}{6} - \frac{wL}{3} = 0 \qquad E_y = \frac{wL}{6}$$

From FBD (a),

$$\circlearrowleft + \Sigma M_C = 0; \qquad D_y\,(L) + \frac{wL}{6}\left(\frac{L}{3}\right) - \frac{4wL}{3}\left(\frac{L}{3}\right) = 0 \qquad D_y = \frac{7wL}{18}$$

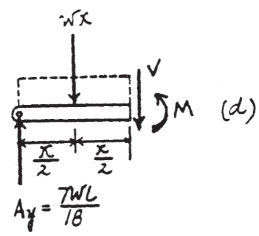
From FBD (c),

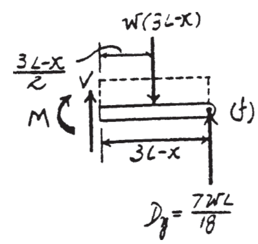


$$\circlearrowleft +\Sigma M_B = 0; \qquad \frac{4wL}{3}\left(\frac{L}{3}\right) - \frac{wL}{6}\left(\frac{L}{3}\right) - A_y\,(L) = 0 \qquad A_y = \frac{7wL}{18}$$

$$+\uparrow \Sigma F_y = 0; \qquad B_y + \frac{7wL}{18} - \frac{4wL}{3} - \frac{wL}{6} = 0 \qquad B_y = \frac{10wL}{9}$$

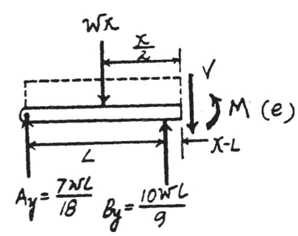
Shear and Moment Functions: For $\mathbf{0 \le x < L}$ [FBD (d)],





$$+\uparrow \Sigma F_y = 0; \qquad \frac{7wL}{18} - wx - V = 0$$

$$V = \frac{w}{18}(7L - 18x) \qquad \textbf{Ans.}$$



$$\circlearrowleft +\Sigma M = 0; \qquad M + wx\left(\frac{x}{2}\right) - \frac{7wL}{18}x = 0$$

$$M = \frac{w}{18}(7Lx - 9x^2) \qquad \textbf{Ans.}$$

7–57. Continued

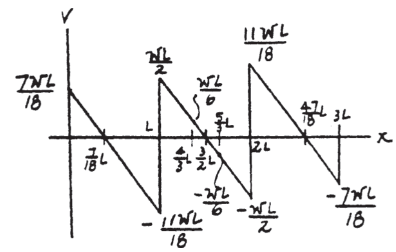
For $L \leq x < 2L$ [FBD (e)],

$$+\uparrow \Sigma F_y = 0; \qquad \frac{7wL}{18} + \frac{10wL}{9} - wx - V = 0$$

$$V = \frac{w}{2}(3L - 2x) \qquad \textbf{Ans.}$$

$$\zeta + \Sigma M = 0; \qquad M + wx\left(\frac{x}{2}\right) - \frac{7wL}{18}x - \frac{10wL}{9}(x - L) = 0$$

$$M = \frac{w}{18}(27Lx - 20L^2 - 9x^2) \qquad \textbf{Ans.}$$



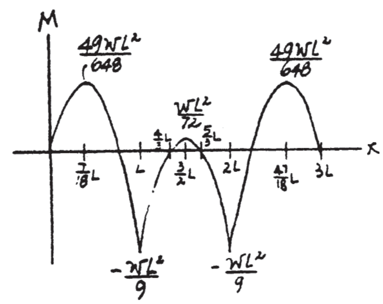
For $2L < x \leq 3L$ [FBD (f)],

$$+\uparrow \Sigma F_y = 0; \qquad V + \frac{7wL}{18} - w(3L - x) = 0$$

$$V = \frac{w}{18}(47L - 18x) \qquad \textbf{Ans.}$$

$$\zeta + \Sigma M = 0; \quad \frac{7wL}{18}(3L - x) - w(3L - x)\left(\frac{3L - x}{2}\right) - M = 0$$

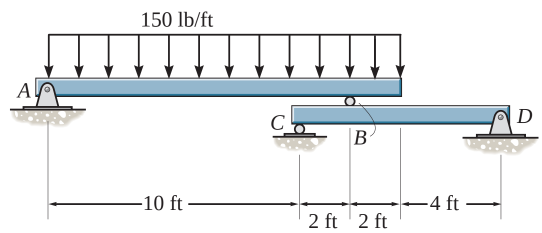
$$M = \frac{w}{18}(47Lx - 9x^2 - 60L^2) \qquad \textbf{Ans.}$$



Ans:
For $0 \leq x < L$
$V = \frac{w}{18}(7L - 18x)$
$M = \frac{w}{18}\left(7Lx - 9x^2\right)$
For $L < x < 2L$
$V = \frac{w}{2}(3L - 2x)$
$M = \frac{w}{18}\left(27Lx - 20L^2 - 9x^2\right)$
For $2L < x \leq 3L$
$V = \frac{w}{18}(47L - 18x)$
$M = \frac{w}{18}\left(47Lx - 9x^2 - 60L^2\right)$

7–58.

Draw the shear and bending-moment diagrams for each of the two segments of the compound beam.



SOLUTION

Support Reactions: From FBD (a),

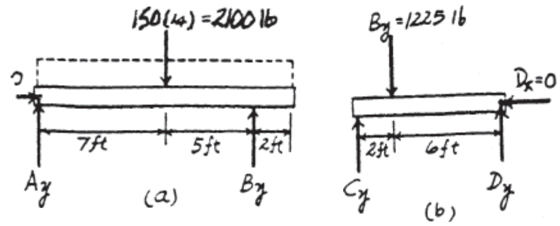
$\circlearrowleft +\Sigma M_A = 0;\quad B_y\,(12) - 2100(7) = 0 \qquad B_y = 1225 \text{ lb}$

$+\uparrow \Sigma F_y = 0;\quad A_y + 1225 - 2100 = 0 \qquad A_y = 875 \text{ lb}$

From FBD (b),

$\circlearrowleft +\Sigma M_D = 0;\quad 1225(6) - C_y(8) = 0 \qquad C_y = 918.75 \text{ lb}$

$+\uparrow \Sigma F_y = 0;\quad D_y + 918.75 - 1225 = 0 \qquad D_y = 306.25 \text{ lb}$



Shear and Moment Functions: Member AB.

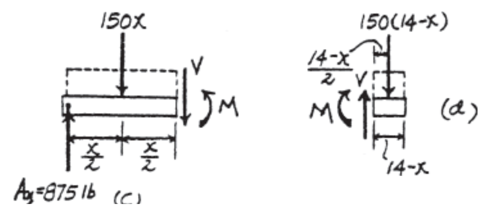
For $\mathbf{0 \le x < 12}$ **ft** [FBD (c)],

$+\uparrow \Sigma F_y = 0;\qquad 875 - 150x - V = 0$

$$V = \{875 - 150x\} \text{ lb} \qquad \textbf{Ans.}$$

$\circlearrowleft +\Sigma M = 0;\qquad M + 150x\left(\frac{x}{2}\right) - 875x = 0$

$$M = \{875x - 75.0x^2\} \text{ lb} \cdot \text{ft} \qquad \textbf{Ans.}$$

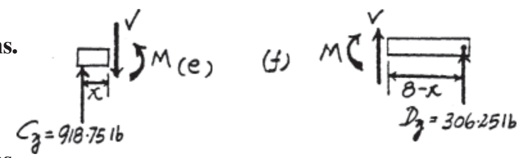


7–58. Continued

For **12 ft < x ≤ 14 ft** [FBD (d)],

$+\uparrow \Sigma F_y = 0; \qquad V - 150(14 - x) = 0$

$V = \{2100 - 150x\}$ lb **Ans.**



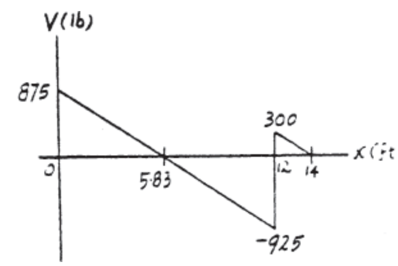
$\circlearrowleft + \Sigma M = 0; \qquad -150(14 - x)\left(\dfrac{14 - x}{2}\right) - M = 0$

$M = \{-75.0x^2 + 2100x - 14700\}$ lb · ft **Ans.**

For member CBD, **0 ≤ x < 2 ft** [FBD (e)],

$+\uparrow \Sigma F_y = 0; \qquad 918.75 - V = 0 \qquad V = 919$ lb **Ans.**

$\circlearrowleft + \Sigma M = 0; \qquad 918.75x - M = 0 \qquad M = \{919x\}$ lb · ft **Ans.**

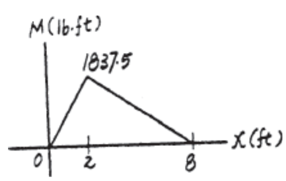


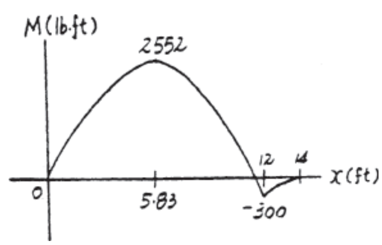
For **2 ft < x ≤ 8 ft** [FBD (f)],

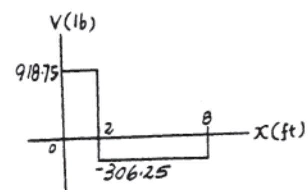
$+\uparrow \Sigma F_y = 0; \qquad V + 306.25 = 0 \qquad V = 306$ lb **Ans.**

$+\Sigma M = 0; \qquad 306.25(8 - x) - M = 0$

$M = \{2450 - 306x\}$ lb · ft **Ans.**



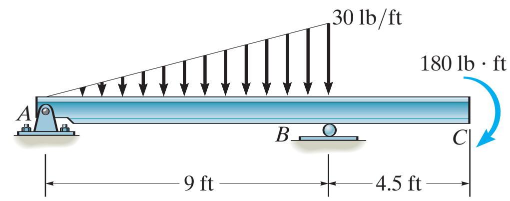




Ans:
Member AB: For $0 \le x < 12$ ft
$V = \{875 - 150x\}$ lb
$M = \{875x - 75.0x^2\}$ lb · ft
For 12 ft $< x \le 14$ ft
$V = \{2100 - 150x\}$ lb
$M = \{-75.0x^2 + 2100x - 14700\}$ lb · ft
Member CBD: For $0 \le x < 2$ ft
$V = 919$ lb
$M = \{919x\}$ lb · ft
For 2 ft $< x \le 8$ ft
$V = -306$ lb
$M = \{2450 - 306x\}$ lb · ft

7–59.

Draw the shear and moment diagrams for the beam.

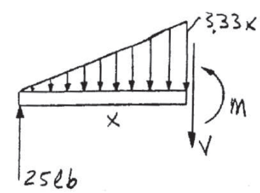


SOLUTION

$0 \le x < 9$ ft:

$+\uparrow \Sigma F_y = 0; \qquad 25 - \frac{1}{2}(3.33\,x)(x) - V = 0$

$V = 25 - 1.667\,x^2$ **Ans.**



$V = 0 = 25 - 1.667\,x^2$

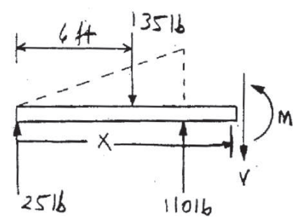
$x = 3.87$ ft

$\zeta + \Sigma M = 0; \qquad M + \frac{1}{2}(3.33\,x)(x)\left(\frac{x}{3}\right) - 25\,x = 0$

$M = 25\,x - 0.5556\,x^3$ **Ans.**

$M_{max} = 25\,(3.87) - 0.5556\,(3.87)^3 = 64.5$ lb · ft

$9 \text{ ft} < x < 13.5$ ft:

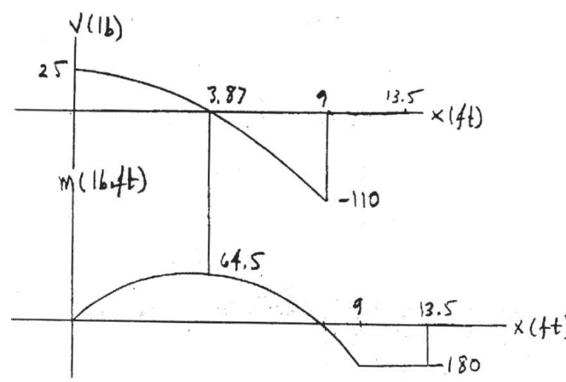


$+\uparrow \Sigma F_y = 0; \qquad 25 - 135 + 110 - V = 0$

$V = 0$ **Ans.**

$\zeta + \Sigma M = 0; \qquad -25\,x + 135\,(x - 6) - 110\,(x - 9) + M = 0$

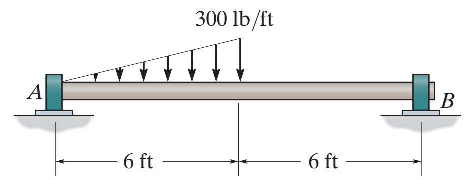
$M = -180$ **Ans.**



Ans:
For $0 \le x < 9$ ft
$V = 25 - 1.67x^2$
$M = 25x - 0.556x^3$
For $9 \text{ ft} < x \le 13.5$ ft
$V = 0, M = -180$

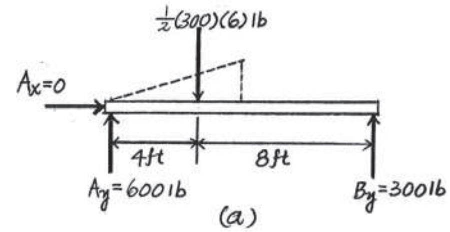
***7–60.**

The shaft is supported by a smooth thrust bearing at A and a smooth journal bearing at B. Draw the shear and moment diagrams for the shaft.



SOLUTION

Since the loading is discontinuous at the midspan, the shear and moment equations must be written for regions $0 \le x < 6$ ft and $6\text{ ft} < x \le 12$ ft of the beam. The free-body diagram of the beam's segment sectioned through the arbitrary points within these two regions are shown in Figs. *b* and *c*.

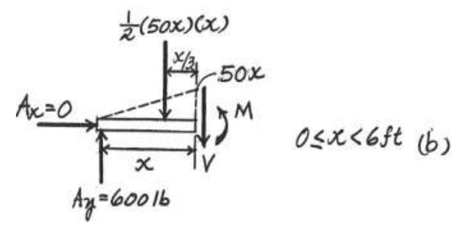


Region $0 \le x < 6$ ft, Fig. *b*

$$+\uparrow \Sigma F_y = 0; \qquad 600 - \frac{1}{2}(50x)(x) - V = 0 \qquad V = \{600 - 25x^2\}\text{ lb} \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M = 0; \qquad M + \frac{1}{2}(50x)(x)\left(\frac{x}{3}\right) - 600(x) = 0$$

$$M = \{600x - 8.333x^3\}\text{ lb}\cdot\text{ft} \qquad \textbf{(2)}$$



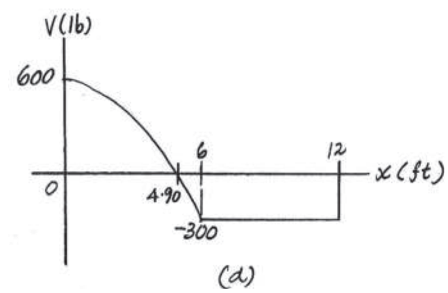
Region $6\text{ ft} < x \le 12$ ft, Fig. *c*

$$+\uparrow \Sigma F_y = 0; \qquad V + 300 = 0 \qquad V = -300\text{ lb} \qquad \textbf{(3)}$$

$$\circlearrowleft + \Sigma M = 0; \qquad 300(12 - x) - M = 0 \qquad M = \{300(12 - x)\}\text{ lb}\cdot\text{ft} \qquad \textbf{(4)}$$

The shear diagram shown in Fig. *d* is plotted using Eqs. (1) and (3). The location at which the shear is equal to zero is obtained by setting $V = 0$ in Eq. (1).

$$0 = 600 - 25x^2 \qquad x = 4.90\text{ ft}$$

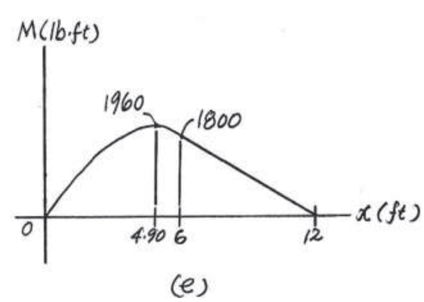


The moment diagram shown in Fig. *e* is plotted using Eqs. (2) and (4). The value of the moment at $x = 4.90$ ft ($V = 0$) is evaluated using Eq. (2).

$$M\big|_{x=4.90\text{ ft}} = 600(4.90) - 8.333(4.90^3) = 1960\text{ lb}\cdot\text{ft}$$

The value of the moment at $x = 6$ ft is evaluated using either Eq. (2) or Eq. (4).

$$M\big|_{x=6\text{ ft}} = 300(12 - 6) = 1800\text{ lb}\cdot\text{ft}$$



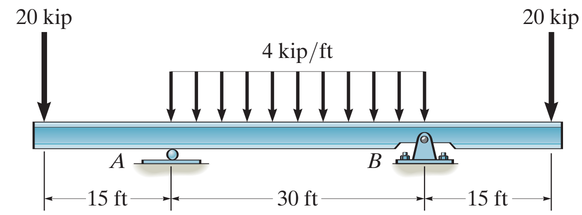
Ans:

$M\big|_{x=4.90\text{ ft}} = 1960\text{ lb}\cdot\text{ft}$

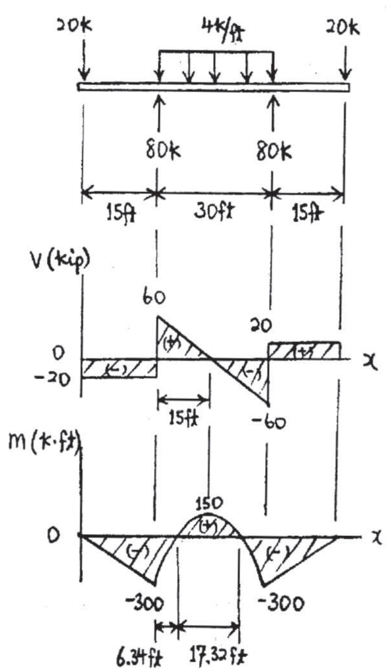
$M\big|_{x=6\text{ ft}} = 1800\text{ lb}\cdot\text{ft}$

7–61.

Draw the shear and moment diagrams for the beam.



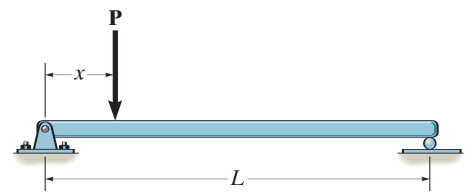
SOLUTION



Ans:
$x = 15^-$
$V = -20$
$M = -300$
$x = 30^+$
$V = 0$
$M = 150$
$x = 45^-$
$V = -60$
$M = -300$

7–62.

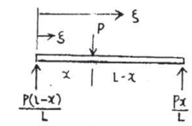
The beam will fail when the maximum internal moment is M_{max}. Determine the position x of the concentrated force **P** and its smallest magnitude that will cause failure.



SOLUTION

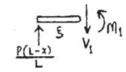
For $\xi < x$,

$$M_1 = \frac{P\xi(L - x)}{L}$$



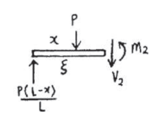
For $\xi > x$,

$$M_2 = -\frac{Px}{L}(L - \xi)$$



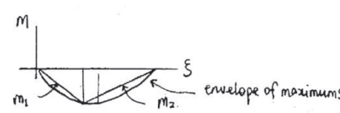
Note that $M_1 = M_2$ when $x = \xi$

$$M_{max} = M_1 = M_2 = \frac{Px}{L}(L - x)$$



$$\frac{dM_{max}}{dx} = \frac{P}{L}(L - 2x) = 0$$

$$x = \frac{L}{2}$$ **Ans.**



Thus, $M_{max} = \frac{P}{L}\left(\frac{L}{2}\right)\left(L - \frac{L}{2}\right) = \frac{P}{2}\left(\frac{L}{2}\right)$

$$P = \frac{4M_{max}}{L}$$ **Ans.**

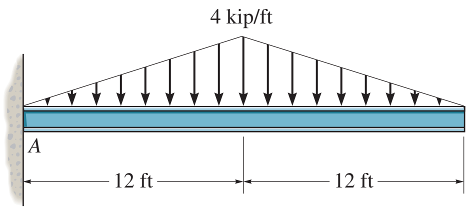
Ans:

$x = \frac{L}{2}$

$P = \frac{4M_{\text{max}}}{L}$

7–63.

Draw the shear and moment diagrams for the beam.

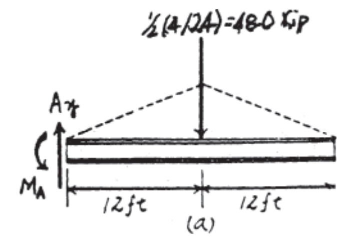


SOLUTION

Support Reactions: From FBD (a),

$$\circlearrowleft + \Sigma M_A = 0; \qquad M_A - 48.0(12) = 0 \qquad M_A = 576 \text{ kip} \cdot \text{ft}$$

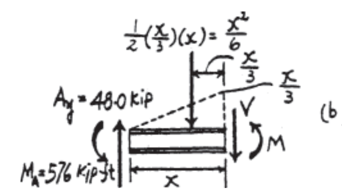
$$+\uparrow \Sigma F_y = 0; \qquad A_y - 48.0 = 0 \qquad A_y = 48.0 \text{ kip}$$



Shear and Moment Functions: For $\mathbf{0 \le x < 12}$ **ft** [FBD (b)],

$$+\uparrow \Sigma F_y = 0; \qquad 48.0 - \frac{x^2}{6} - V = 0$$

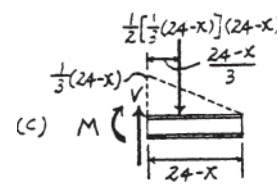
$$V = \left\{48.0 - \frac{x^2}{6}\right\} \text{kip} \qquad \textbf{Ans.}$$



$$\circlearrowleft + \Sigma M = 0; \qquad M + \frac{x^2}{6}\left(\frac{x}{3}\right) + 576 - 48.0x = 0$$

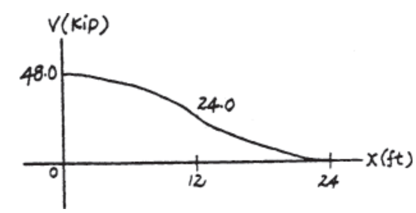
$$M = \left\{48.0x - \frac{x^3}{18} - 576\right\} \text{kip} \cdot \text{ft} \qquad \textbf{Ans.}$$

For **12 ft** $<$ $\boldsymbol{x} \le$ **24 ft** [FBD (c)],



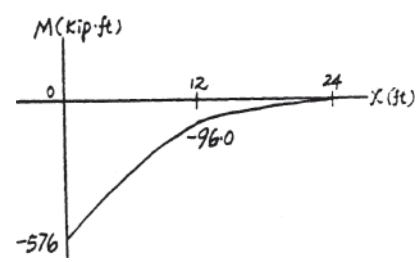
$$+\uparrow \Sigma F_y = 0; \qquad V - \frac{1}{2}\left[\frac{1}{3}(24 - x)\right](24 - x) = 0$$

$$V = \left\{\frac{1}{6}(24 - x)^2\right\} \text{kip} \qquad \textbf{Ans.}$$



$$\circlearrowleft + \Sigma M = 0; \qquad -\frac{1}{2}\left[\frac{1}{3}(24 - x)\right](24 - x)\left(\frac{24 - x}{3}\right) - M = 0$$

$$M = \left\{-\frac{1}{18}(24 - x)^3\right\} \text{kip} \cdot \text{ft} \qquad \textbf{Ans.}$$



Ans:

$0 \le x \le 12$ ft: $V = \left\{48.0 - \frac{x^2}{6}\right\}$ kip

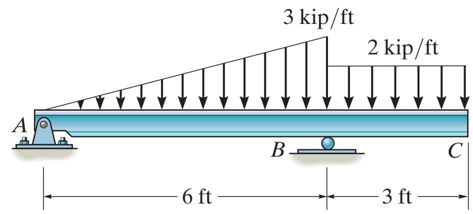
$M = \left\{48.0x - \frac{x^3}{18} - 576\right\}$ kip · ft

$12 < x \le 24$ ft: $V = \left\{\frac{1}{6}(24 - x)^2\right\}$ kip

$M = \left\{-\frac{1}{18}(24 - x)^3\right\}$ kip · ft

***7–64.**

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire beam shown in Fig. *a*,

$$\zeta + \Sigma M_A = 0; \quad N_B(6) - \frac{1}{2}(3)(6)(4) - 2(3)(7.5) = 0 \quad N_B = 13.5 \text{ kip}$$

$$\zeta + \Sigma M_B = 0; \quad \frac{1}{2}(3)(6)(2) - 2(3)(1.5) - A_y(6) = 0 \quad A_y = 1.50 \text{ kip}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$$

Shear And Moment Functions. The beam will be sectioned at two arbitrary distance x in region AB $(0 \leq x < 6\text{ ft})$ and region BC $(6\text{ ft} < x \leq 9\text{ ft})$. For region $(0 < x \leq 6\text{ ft})$, Fig. *b*

$$+\uparrow \Sigma F_y = 0; \quad 1.50 - \frac{1}{2}\left(\frac{1}{2}x\right)(x) - V = 0 \quad V = \left\{1.50 - \frac{1}{4}x^2\right\} \text{kip} \quad \textbf{Ans.}$$

$$\zeta + \Sigma M_O = 0; \quad M + \frac{1}{2}\left(\frac{1}{2}x\right)(x)\left(\frac{x}{3}\right) - 1.50x = 0$$

$$M = \left\{1.50x - \frac{1}{12}x^3\right\} \text{kip} \cdot \text{ft} \quad \textbf{Ans.}$$

Set $V = 0$, we obtain

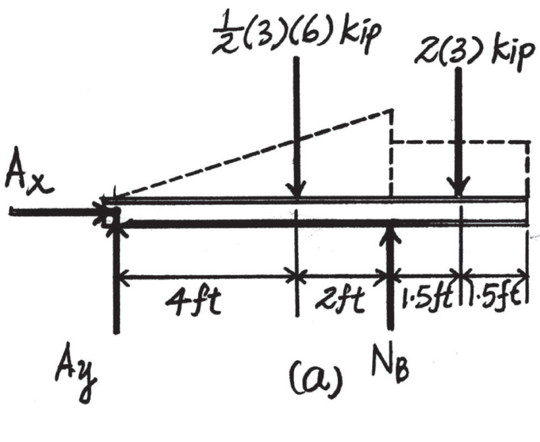
$$0 = 1.5 - \frac{1}{4}x^2 \qquad x = \sqrt{6}\text{ ft}$$

The corresponding Internal Moment is

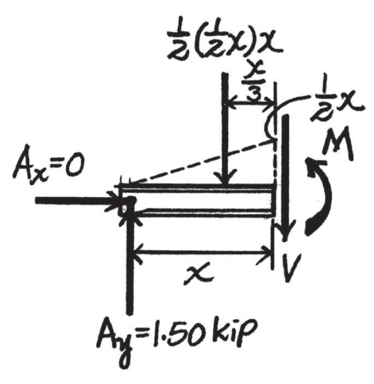
$$M = 1.50(\sqrt{6}) - \frac{1}{12}(\sqrt{6})^3 = 2.4495 \text{ kip} \cdot \text{ft} = 2.45 \text{ kip} \cdot \text{ft}$$

At $x = 6$ ft,

$$M = 1.50(6) - \frac{1}{12}(6^3) = -9.00 \text{ kip} \cdot \text{ft}$$



(a)



(b)

***7–64. Continued**

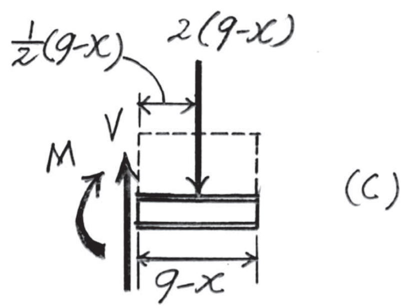
For Region 6 ft $< x \leq 9$ ft, Fig. *c*

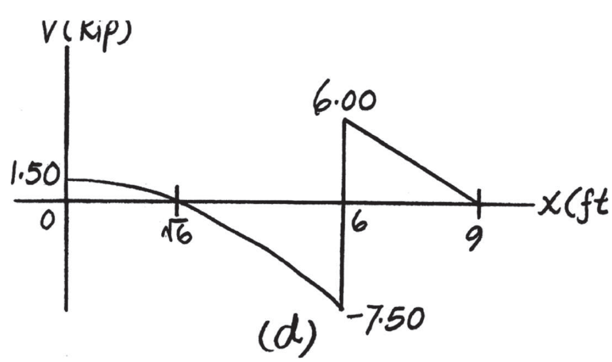
$+\uparrow \Sigma F_y = 0; \qquad V - 2(9 - x) = 0 \qquad V = \{18.0 - 2x\}\text{ kip}$ **Ans.**

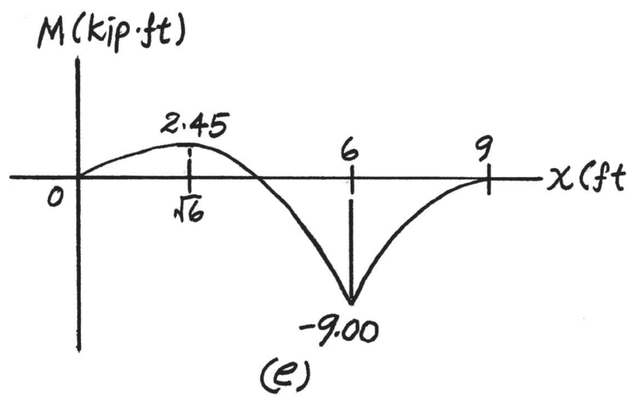
$\zeta + \Sigma M_O = 0; \qquad -M - 2(9 - x)\left[\frac{1}{2}(9 - x)\right] = 0$

$$M = \{-x^2 + 18x - 81\}\text{ kip}\cdot\text{ft}$$ **Ans.**

Plotting the shear and moment functions, the shear and moment diagram shown in Fig. *d* and *e* resulted.



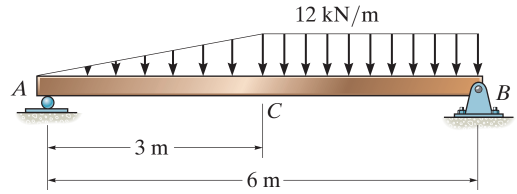




Ans:
For 6 ft $< x \leq 9$ ft,
$V = \{18.0 - 2x\}$ kip
$M = \{-x^2 + 18x - 81\}$ kip·ft

7–65.

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions. Referring to the *FBD* of the entire beam shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_B = 0; \qquad 12(3)(1.5) + \frac{1}{2}(12)(3)(4) - A_y(6) = 0 \qquad A_y = 21.0\text{ kN}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(6) - \frac{1}{2}(12)(3)(2) - 12(3)(4.5) = 0 \qquad B_y = 33.0\text{ kN}$$

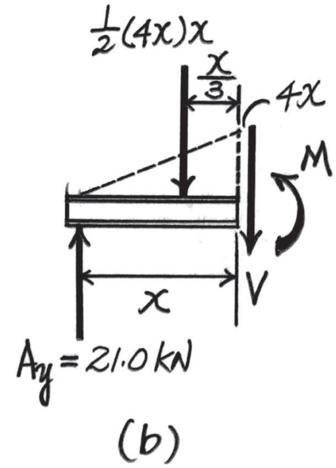
$$\xrightarrow{+} \Sigma F_x = 0; \qquad B_x = 0$$

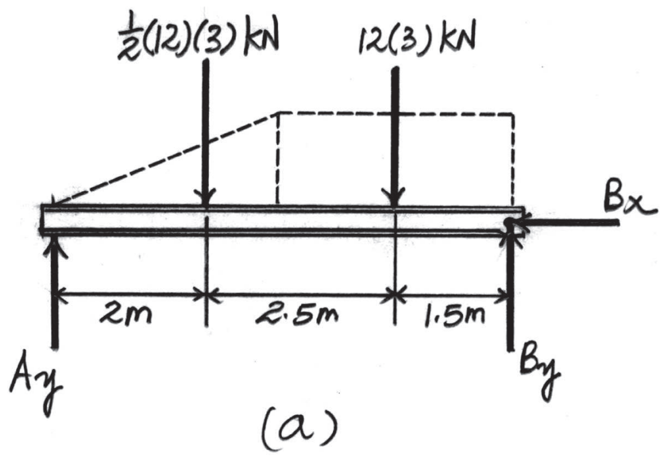
Shear And Moment Functions. The beam will be sectioned at two arbitrary distances x in region AC ($0 \le x$ 3 m) and region CB (3 m $< x \le 6$ m).
For region $0 \le x < 3$ m, Fig. *b*

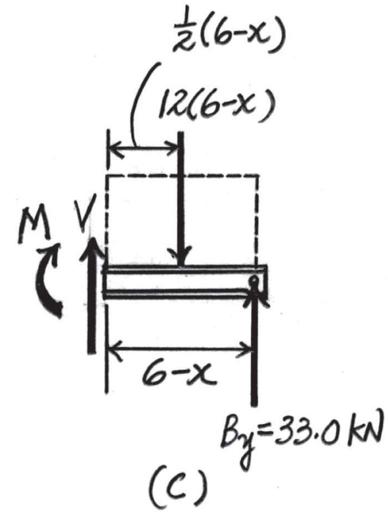
$$+\uparrow \Sigma F_y = 0; \qquad 21.0 - \frac{1}{2}(4x)(x) - V = 0 \quad V = \left\{21.0 - 2x^2\right\}\text{ kN} \qquad \textbf{Ans.}$$

$$\circlearrowleft + \Sigma M_O = 0 \qquad M + \left[\frac{1}{2}(4x)(x)\right]\left(\frac{x}{3}\right) - 21.0x = 0$$

$$M = \left\{21.0x - \frac{2}{3}x^3\right\}\text{ kN}\cdot\text{m} \qquad \textbf{Ans.}$$







7–65. Continued

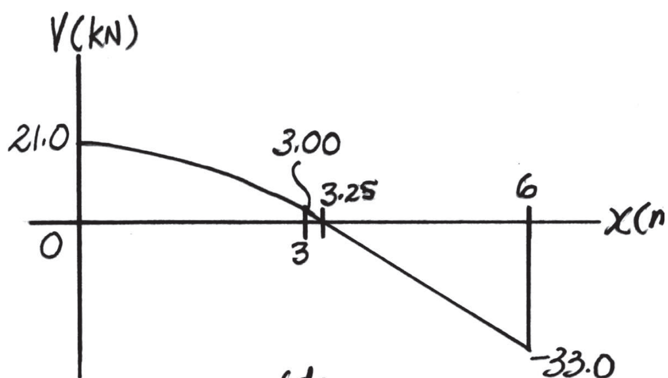
For region $3\text{ m} < x \leq 6\text{ m}$, Fig. *c*

$+\uparrow \Sigma F_y = 0; \qquad V + 33.0 - 12(6 - x) = 0 \qquad V = \{39.0 - 12x\}\text{ kN}$ **Ans.**

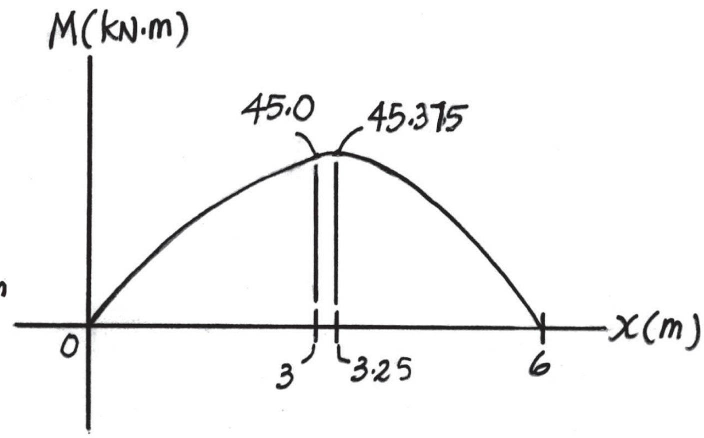
$\circlearrowleft + \Sigma M_O = 0 \qquad 33.0(6 - x) - [12(6 - x)]\left[\frac{1}{2}(6 - x)\right] - M = 0$

$$M = \left\{-6x^2 + 39x - 18\right\}\text{ kN}\cdot\text{m}$$ **Ans.**

Plotting the shear and moment functions obtained, the shear and moment diagram shown in Fig. *d* and *e* resulted.



(d)

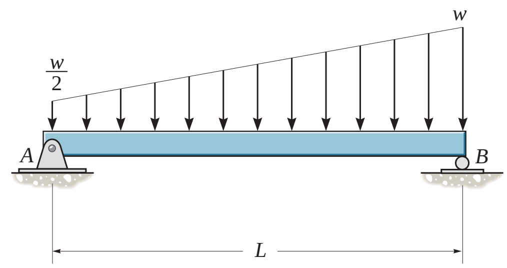


(e)

Ans:
For $0 \leq x < 3\text{ m}$
$V = \left\{21.0 - 2x^2\right\}\text{ kN}$
$M = \left\{21.0x - \frac{2}{3}x^3\right\}\text{ kN}\cdot\text{m}$
For $3\text{ m} < x \leq 6\text{ m}$
$V = \{39.0 - 12x\}\text{ kN}$
$M = \left\{-6x^2 + 39x - 18\right\}\text{ kN}\cdot\text{m}$

7–66.

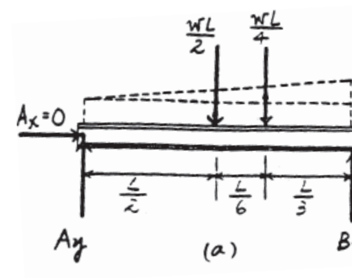
Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions: From FBD (a),

$$\circlearrowleft + \Sigma M_B = 0; \quad \frac{wL}{4}\left(\frac{L}{3}\right) + \frac{wL}{2}\left(\frac{L}{2}\right) - A_y\,(L) = 0 \qquad A_y = \frac{wL}{3}$$



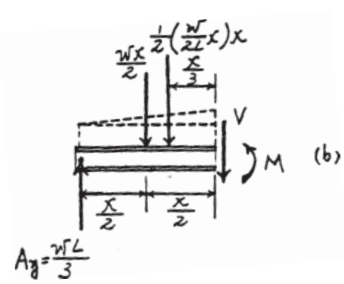
Shear and Moment Functions: For $0 \le x \le L$ [FBD (b)],

$$+\uparrow \Sigma F_y = 0; \qquad \frac{wL}{3} - \frac{w}{2}x - \frac{1}{2}\left(\frac{w}{2L}x\right)x - V = 0$$

$$V = \frac{w}{12L}(4L^2 - 6Lx - 3x^2) \qquad \textbf{Ans.}$$

The maximum moment occurs when $V = 0$, then

$$0 = 4L^2 - 6Lx - 3x^2 \qquad x = 0.5275L$$



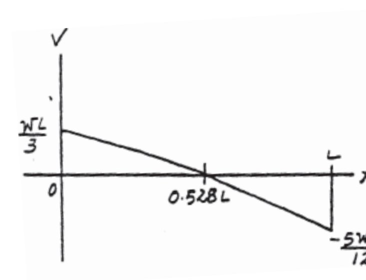
$$\circlearrowleft + \Sigma M = 0; \qquad M + \frac{1}{2}\left(\frac{w}{2L}x\right)x\left(\frac{x}{3}\right) + \frac{wx}{2}\left(\frac{x}{2}\right) - \frac{wL}{3}(x) = 0$$

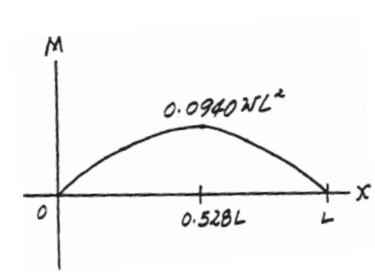
$$M = \frac{w}{12L}(4L^2x - 3Lx^2 - x^3) \qquad \textbf{Ans.}$$

Thus,

$$M_{\max} = \frac{w}{12L}[4L^2(0.5275L) - 3L(0.5275L)^2 - (0.5275L)^3]$$

$$= 0.0940wL^2 \qquad \textbf{Ans.}$$





Ans:

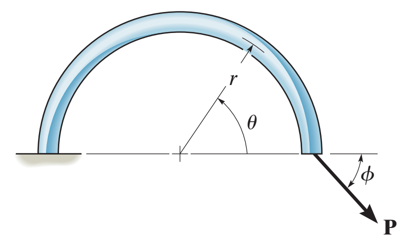
$V = \frac{w}{12L}(4L^2 - 6Lx - 3x^2)$

$M = \frac{w}{12L}(4L^2x - 3Lx^2 - x^3)$

$M_{\max} = 0.0940\, wL^2$

7–67.

Determine the internal normal force, shear force, and moment in the curved rod as a function of θ. The force **P** acts at the constant angle ϕ.



SOLUTION

Support Reactions. Not required

Internal Loadings. Referring to the *FBD* of the free and segment of the sectioned curved rod, Fig. *a*,

$+\nwarrow\Sigma F_y = 0;\quad N - P\sin(\theta + \phi) = 0 \qquad N = P\sin(\theta + \phi)$ **Ans.**

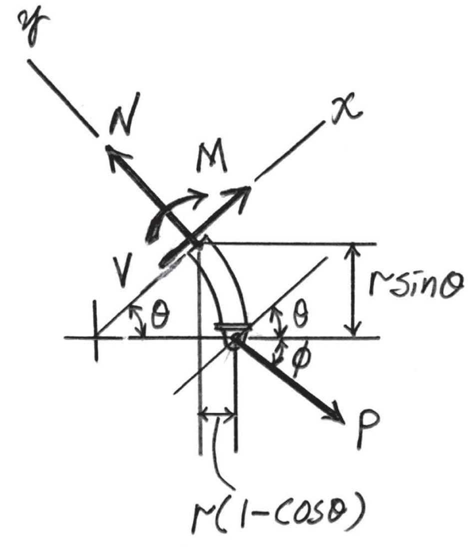
$+\nearrow\Sigma F_x = 0;\quad P\cos(\theta + \phi) + V = 0 \qquad V = -P\cos(\theta + \phi)$ **Ans.**

$\circlearrowleft+\Sigma M_O = 0;\quad P\cos\phi(r\sin\theta) - P\sin\phi[r(1 - \cos\theta)] - M = 0$

$$M = Pr(\sin\theta\cos\phi + \cos\theta\sin\phi - \sin\phi)$$

Using the identity $\sin(\theta + \phi) = \sin\theta\cos\phi + \cos\theta\sin\phi$,

$$M = Pr[\sin(\theta + \phi) - \sin\phi]$$ **Ans.**

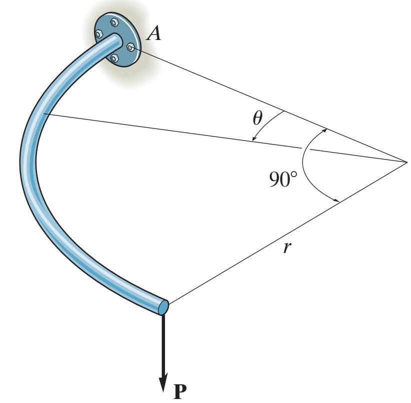


(a)

Ans:
$N = P\sin(\theta + \phi)$
$V = -P\cos(\theta + \phi)$
$M = Pr[\sin(\theta + \phi) - \sin\phi]$

***7–68.**

The quarter circular rod lies in the horizontal plane and supports a vertical force **P** at its end. Determine the magnitudes of the components of the internal shear force, moment, and torque acting in the rod as a function of the angle θ.



SOLUTION

$\Sigma F_z = 0;\qquad V = |P|$ **Ans.**

$\Sigma M_x = 0;\qquad M + P\,(r \sin(90° - \theta)) = 0$

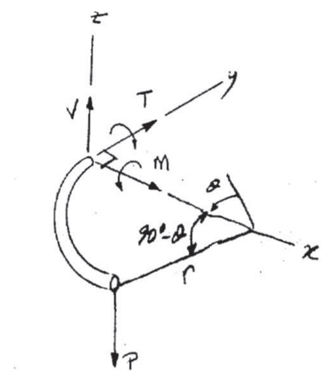
$M = -P\,r\cos\theta$

$M = |P\,r\cos\theta|$ **Ans.**

$\Sigma M_y = 0;\qquad T + P\,r\,(1 - \cos(90° - \theta)) = 0$

$T = -P\,r\,(1 - \sin\theta)$

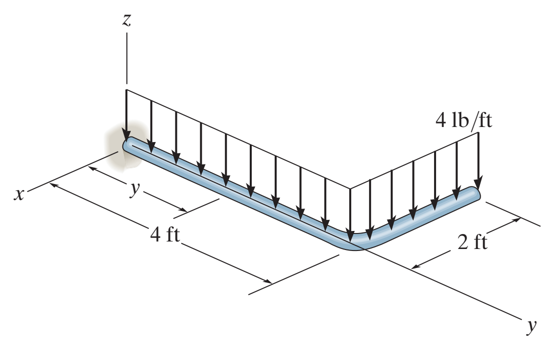
$T = |P\,r\,(1 - \sin\theta)|$ **Ans.**



Ans:
$V = |P|$
$M = |P\,r\cos\theta|$
$T = |P\,r\,(1 - \sin\theta)|$

7–69.

Express the internal shear and moment components acting in the rod as a function of y, where $0 \leq y \leq 4$ ft.



SOLUTION

Shear and Moment Functions:

$\Sigma F_x = 0;\qquad V_x = 0$ **Ans.**

$\Sigma F_z = 0;\qquad V_z - 4(4 - y) - 8.00 = 0$

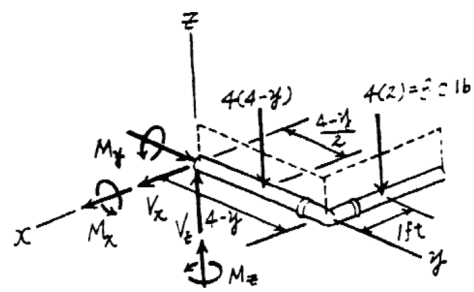
$$V_z = \{24.0 - 4y\}\text{ lb}$$ **Ans.**

$\Sigma M_x = 0;\qquad M_x - 4(4 - y)\left(\dfrac{4 - y}{2}\right) - 8.00(4 - y) = 0$

$$M_x = \{2y^2 - 24y + 64.0\}\text{ lb}\cdot\text{ft}$$ **Ans.**

$\Sigma M_y = 0;\qquad M_y - 8.00(1) = 0 \qquad M_y = 8.00\text{ lb}\cdot\text{ft}$ **Ans.**

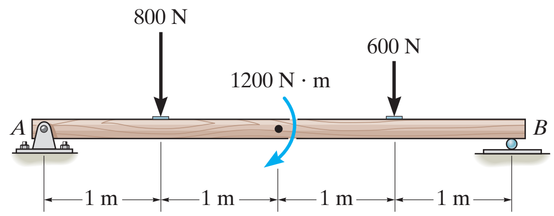
$\Sigma M_z = 0;\qquad M_z = 0$ **Ans.**



Ans:
$V_x = 0$
$V_z = \{24.0 - 4y\}$ lb
$M_x = \{2y^2 - 24y + 64.0\}$ lb·ft
$M_y = 8.00$ lb·ft
$M_z = 0$

7–70.

Draw the shear and moment diagrams for the beam.



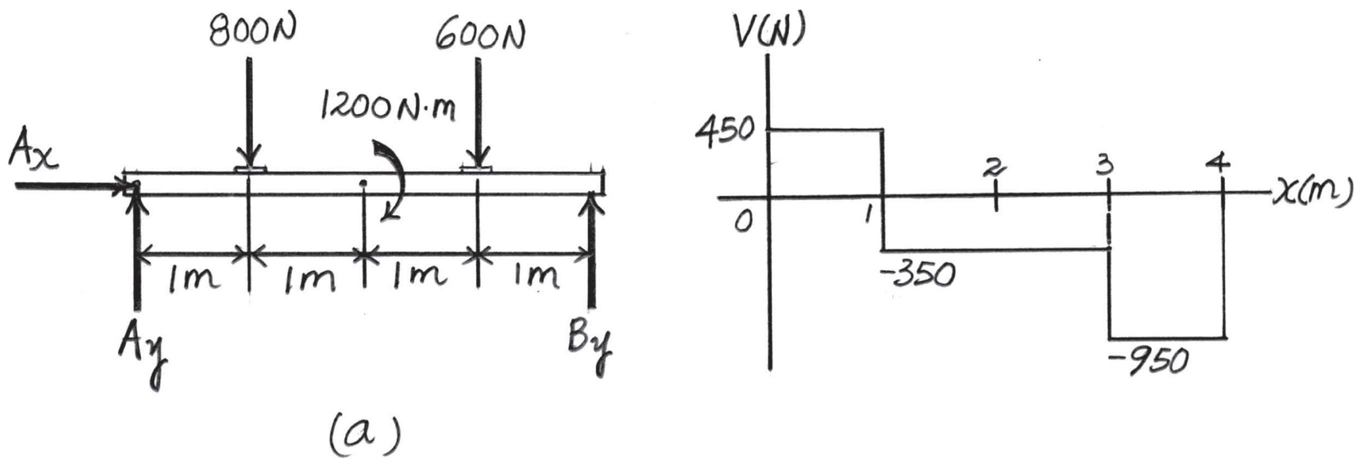
SOLUTION

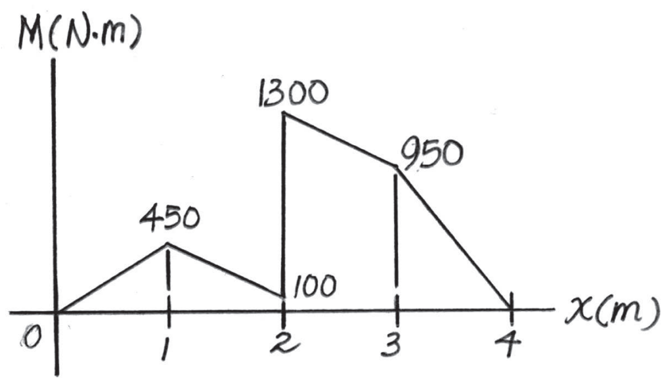
Support Reactions. Referring to the *FBD* of the simply supported beam shown in Fig. *a*

$\circlearrowleft + \Sigma M_A = 0; \quad B_y(4) - 800(1) - 600(3) - 1200 = 0 \quad B_y = 950\text{ N}$

$\circlearrowleft + \Sigma M_B = 0; \quad 600(1) + 800(3) - 1200 - A_y(4) = 0 \quad A_y = 450\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$

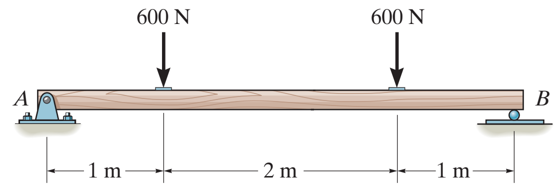




Ans:
$x = 1^-$
$V = 450\text{ N}$
$M = 450\text{ N}\cdot\text{m}$
$x = 3^+$
$V = -950\text{ N}$
$M = 950\text{ N}\cdot\text{m}$

7–71.

Draw the shear and moment diagrams for the beam.



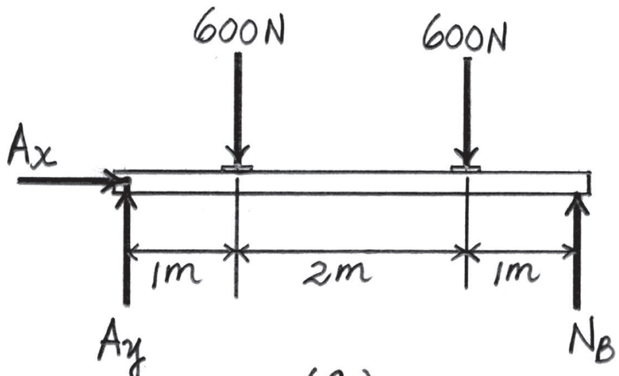
SOLUTION

Support Reactions. Referring to the *FBD* of the beam shown in Fig. *a*

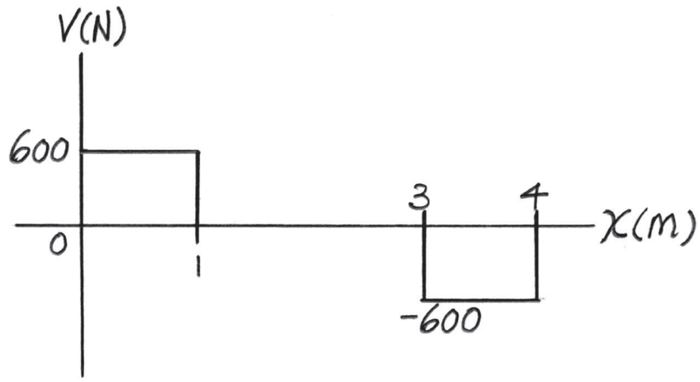
$\circlearrowleft + \Sigma M_A = 0; \quad N_B(4) - 600(3) - 600(1) = 0 \quad N_B = 600 \text{ N}$

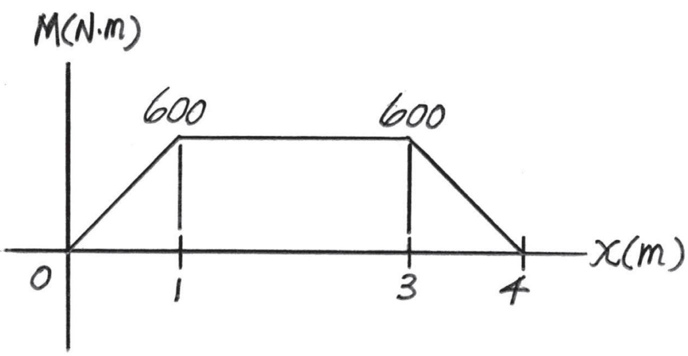
$\circlearrowleft + \Sigma M_B = 0; \quad 600(1) + 600(3) - A_y(4) = 0 \quad A_y = 600 \text{ N}$

$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad A_x = 0$



(a)

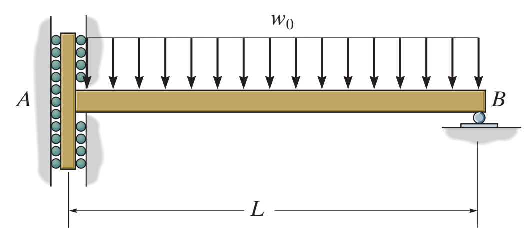




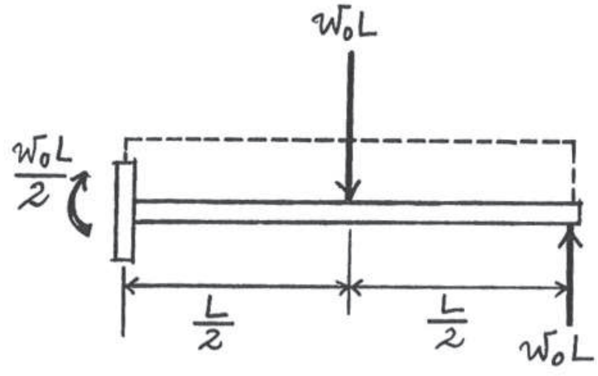
Ans:
$x = 1^-$
$V = 600 \text{ N}$
$M = 600 \text{ N} \cdot \text{m}$

***7–72.**

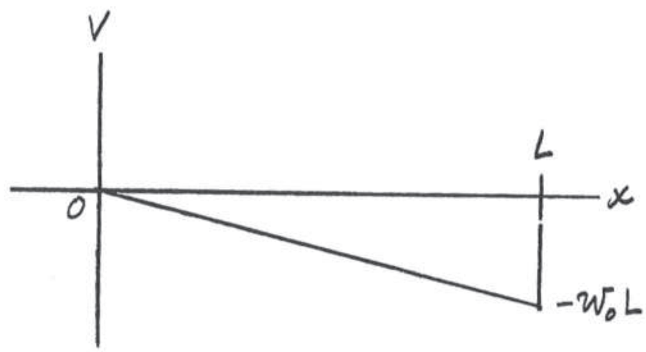
Draw the shear and moment diagrams for the beam. The support at A offers no resistance to vertical load.



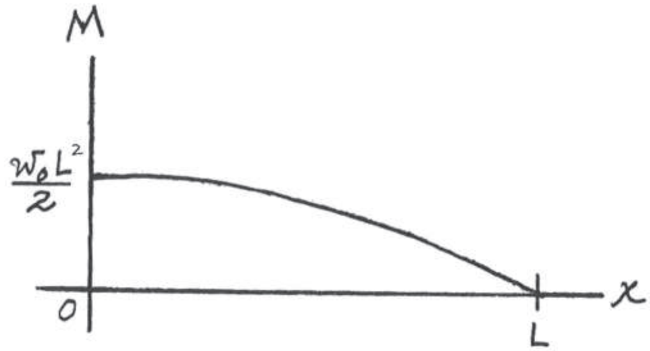
SOLUTION



(a)



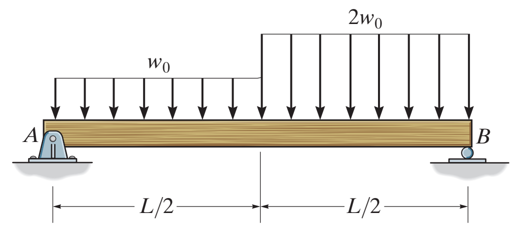
Shear diagram
(b)



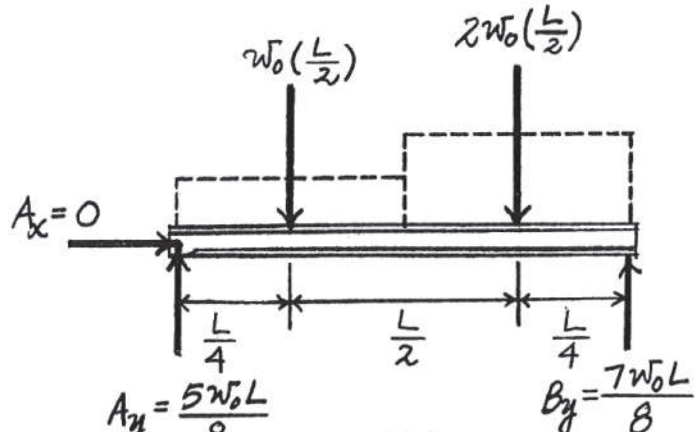
Moment diagram
(c)

7–73.

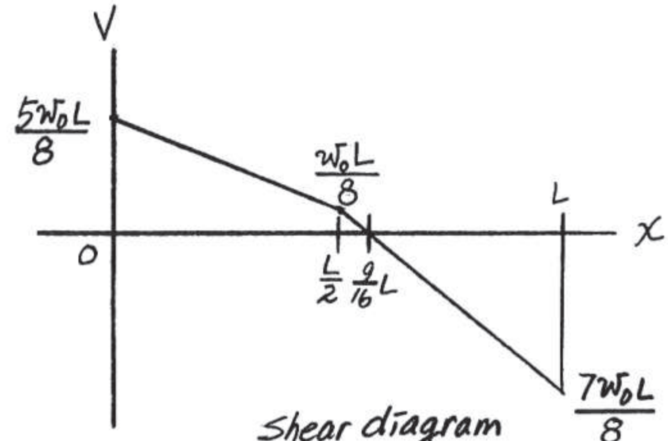
Draw the shear and moment diagrams for the simply-supported beam.



SOLUTION

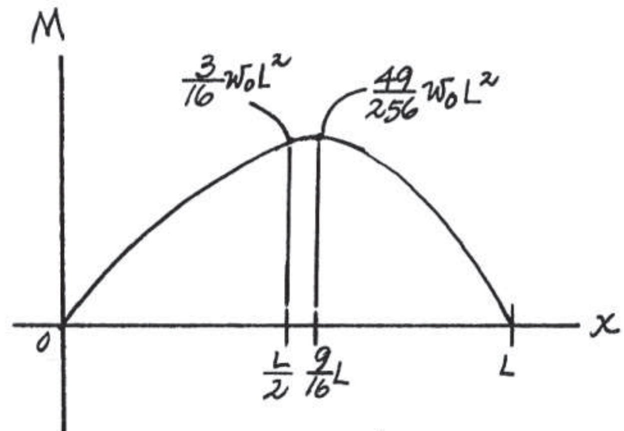


(a)



Shear diagram

(b)

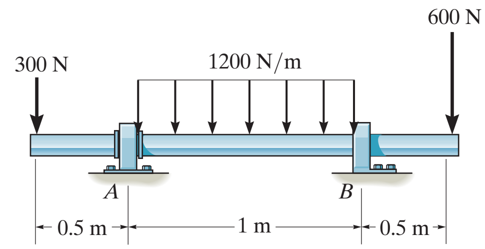


Moment diagram

(c)

7–74.

Draw the shear and moment diagrams for the beam. The supports at A and B are a thrust bearing and journal bearing, respectively.



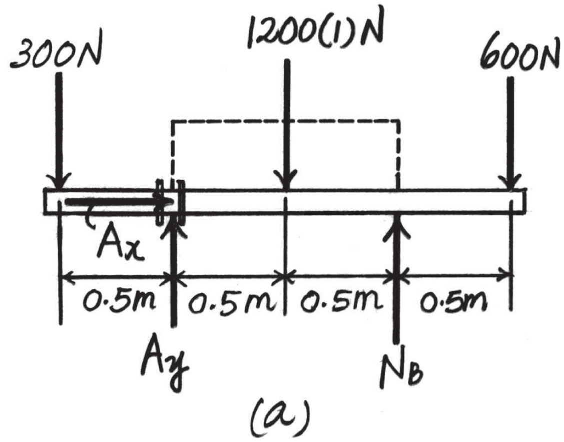
SOLUTION

Support Reactions. Referring to the *FBD* of the shaft shown in Fig. *a*,

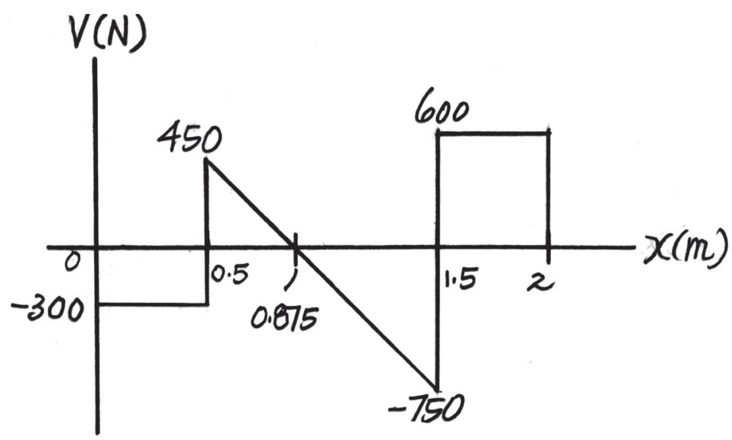
$$\circlearrowleft + \Sigma M_A = 0; \quad N_B(1) + 300(0.5) - 1200(1)(0.5) - 600(1.5) = 0 \quad N_B = 1350 \text{ N}$$

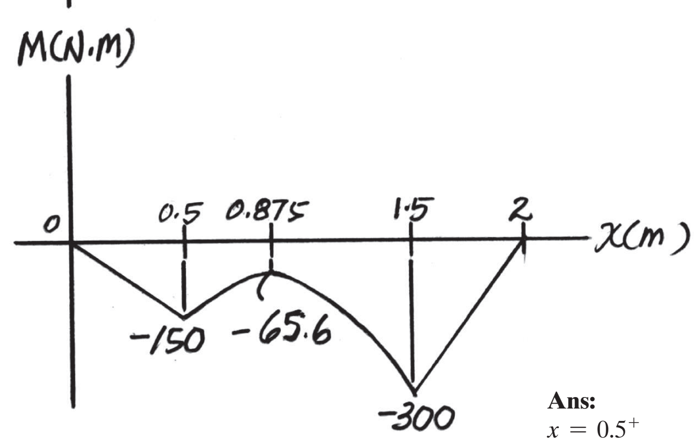
$$\circlearrowleft + \Sigma M_B = 0; \quad 1200(1)(0.5) + 300(1.5) - 600(0.5) - A_y(1) = 0 \quad A_y = 750 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad A_x = 0$$



(a)

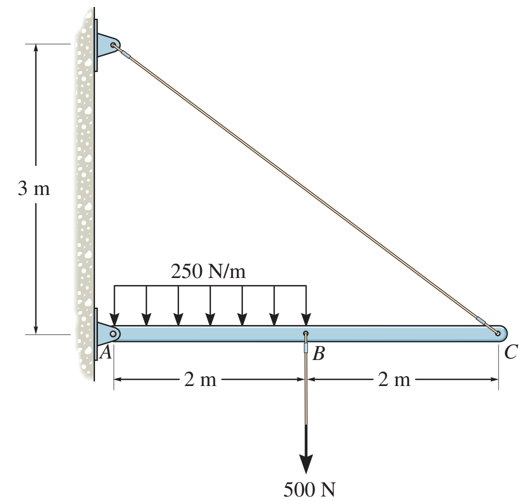




Ans:
$x = 0.5^+$
$V = 450 \text{ N}$
$M = -150 \text{ N}\cdot\text{m}$
$x = 1.5^-$
$V = -750 \text{ N}$
$M = -300 \text{ N}\cdot\text{m}$

7–75.

Draw the shear and moment diagrams for the beam.

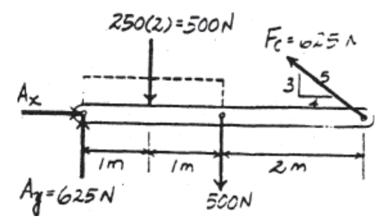


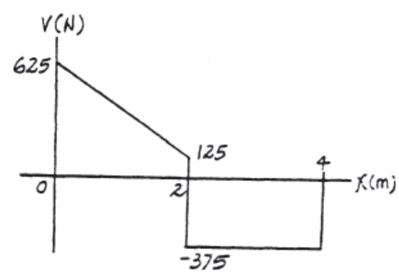
SOLUTION

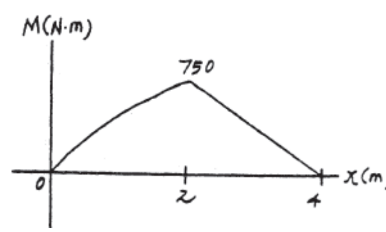
Support Reactions:

$\zeta + \Sigma M_A = 0;\qquad F_C\left(\frac{3}{5}\right)(4) - 500(2) - 500(1) = 0 \qquad F_C = 625\text{ N}$

$+\uparrow \Sigma F_y = 0;\qquad A_y + 625\left(\frac{3}{5}\right) - 500 - 500 = 0 \qquad A_y = 625\text{ N}$



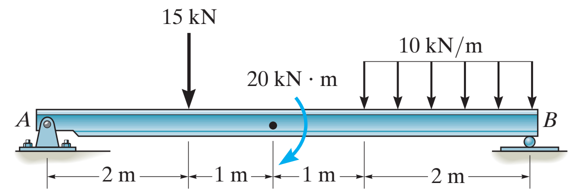




Ans:
$x = 2^+$
$V = -375\text{ N}$
$M = 750\text{ N}\cdot\text{m}$

***7–76.**

Draw the shear and moment diagrams for the beam.



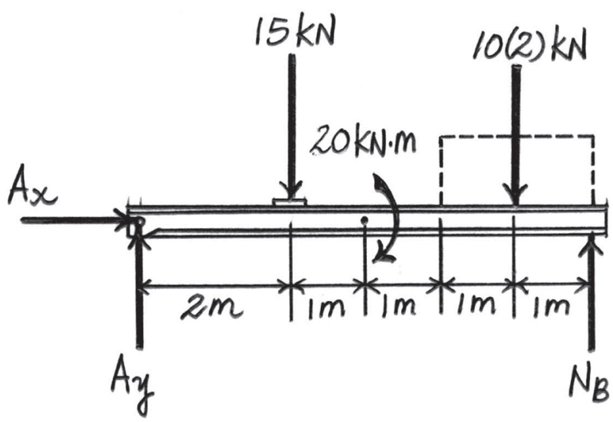
SOLUTION

Support Reaction. Referring to the *FBD* of the beam shown in Fig. *a*,

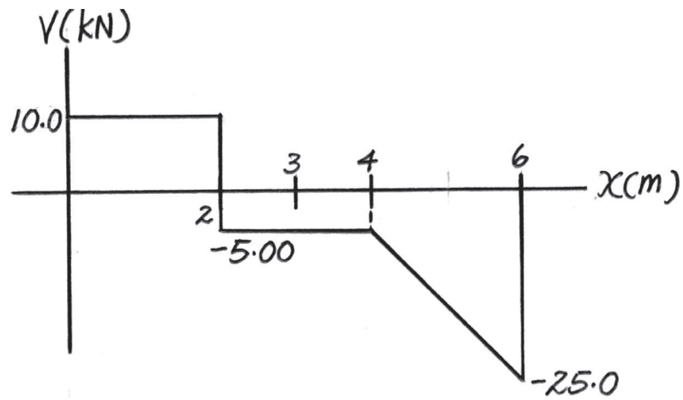
$\zeta + \Sigma M_A = 0;\quad N_B(6) - 15(2) - 20 - 10(2)(5) = 0 \qquad N_B = 25.0 \text{ kN}$

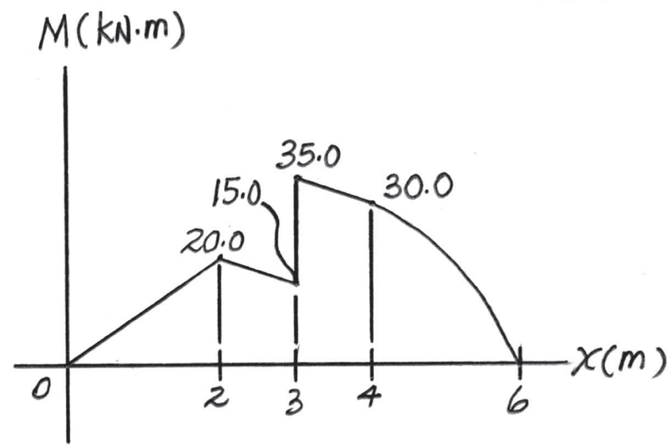
$\zeta + \Sigma M_B = 0;\quad 10(2)(1) + 15(4) - 20 - A_y(6) = 0 \qquad A_y = 10.0 \text{ kN}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad A_x = 0$



(a)

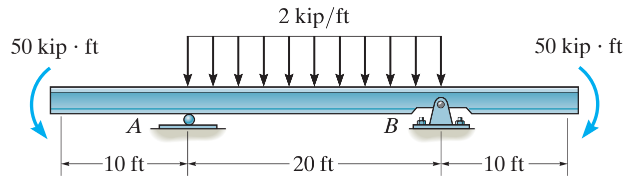




Ans:
$x = 2^-$
$V = 10.0 \text{ kN}$
$M = 20.0 \text{ kN} \cdot \text{m}$
$x = 3^+$
$V = -5.00 \text{ kN}$
$M = 35.0 \text{ kN} \cdot \text{m}$

7–77.

Draw the shear and moment diagrams for the beam.



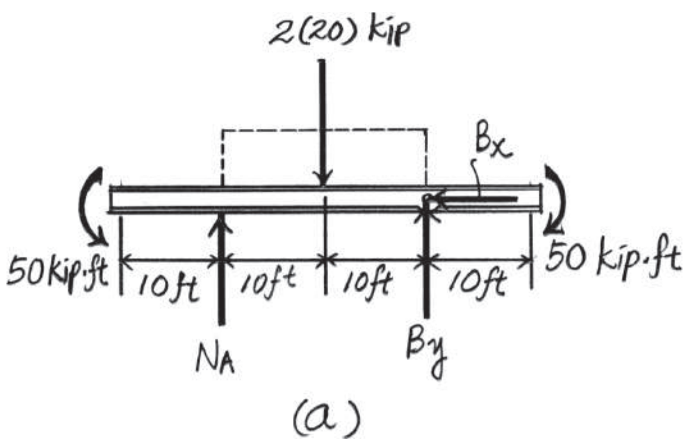
SOLUTION

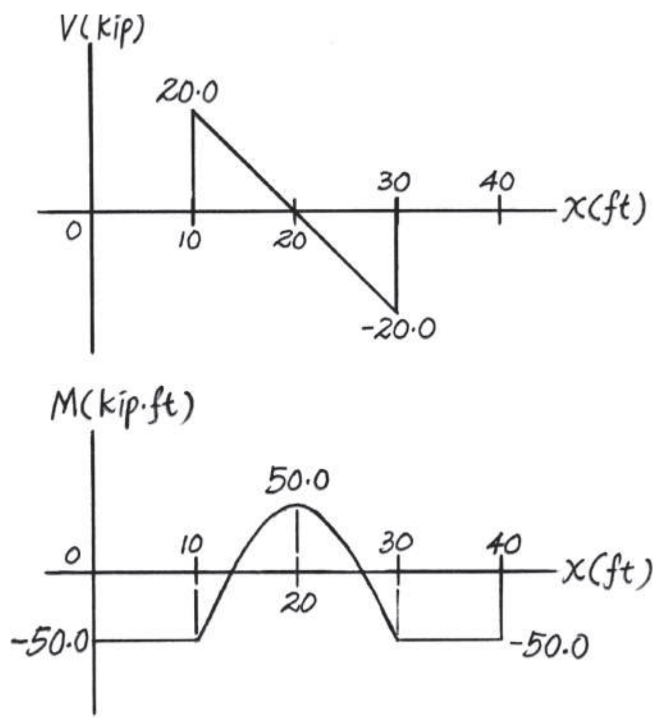
Support Reactions. Referring to the *FBD* of the beam shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad B_y(20) + 50 - 2(20)(10) - 50 = 0 \qquad B_y = 20.0 \text{ kip}$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad 2(20)(10) + 50 - 50 - N_A(20) = 0 \qquad N_A = 20.0 \text{ kip}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad B_x = 0$$

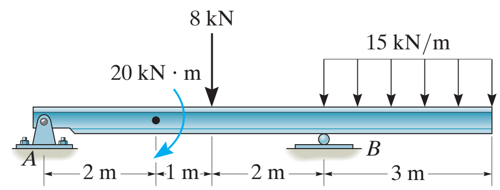




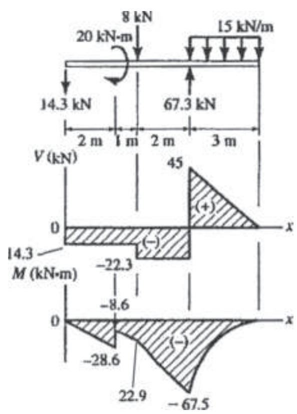
Ans:
$x = 10^+$
$V = 20.0$ kip
$M = -50.0$ kip · ft

7–78.

Draw the shear and moment diagrams for the beam.



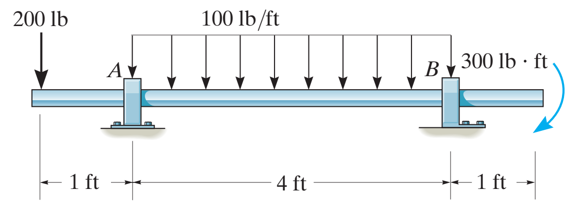
SOLUTION



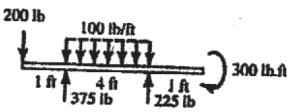
Ans:
$x = 2^+$
$V = -14.3$
$M = -8.6$

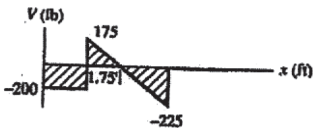
7–79.

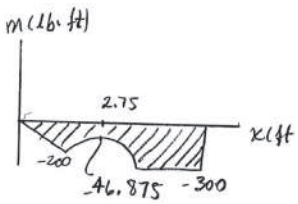
Draw the shear and moment diagrams for the shaft. The support at A is a journal bearing and at B it is a thrust bearing.



SOLUTION



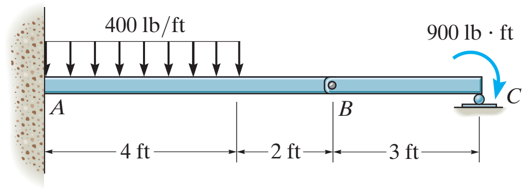




Ans:
$x = 1^+$
$V = 175$
$M = -200$
$x = 5^-$
$V = -225$
$M = -300$

***7–80.**

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions. Referring to the *FBD* of member *BC* shown in Fig. *a*,

$\circlearrowleft + \Sigma M_B = 0; \quad N_C(3) - 900 = 0 \quad N_C = 300 \text{ lb}$

$\circlearrowleft + \Sigma M_C = 0; \quad B_y(3) - 900 = 0 \quad B_y = 300 \text{ lb}$

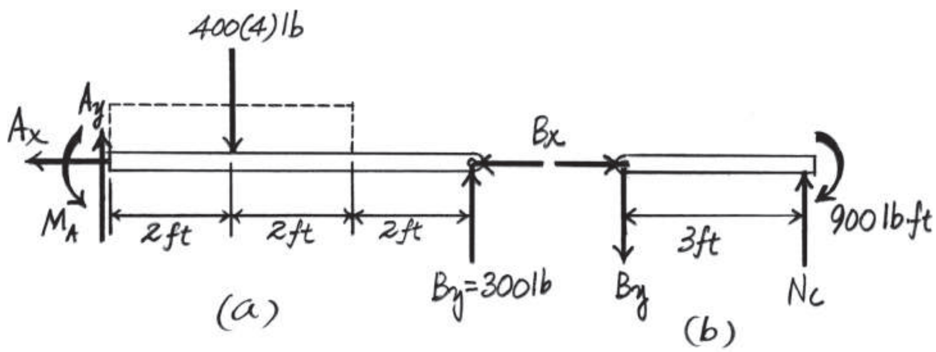
$\overset{+}{\rightarrow} \Sigma F_x = 0 \quad B_x = 0$

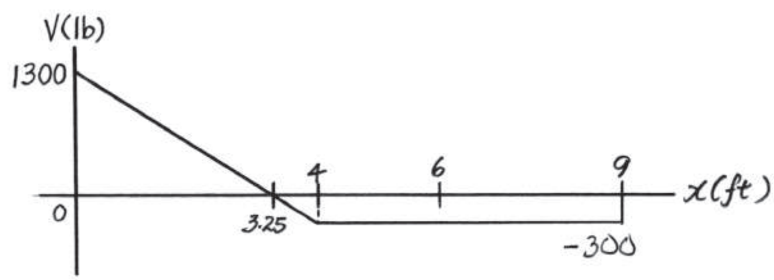
Also, for member *AB*,

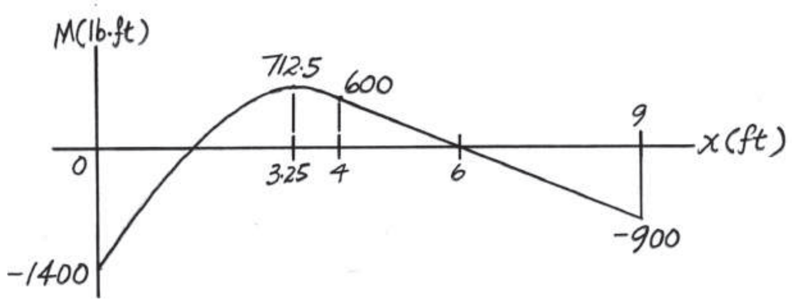
$\circlearrowleft + \Sigma M_A = 0; \quad M_A + 300(6) - 400(4)(2) = 0 \quad M_A = 1400 \text{ lb} \cdot \text{ft}$

$+\uparrow \Sigma F_y = 0; \quad A_y + 300 - 400(4) = 0 \quad A_y = 1300 \text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad A_x = 0$



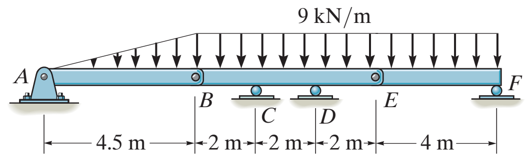




Ans:
$x = 3.25$
$V = 0$
$M = 712.5 \text{ lb} \cdot \text{ft}$
$x = 6$
$V = -300 \text{ lb}$
$M = 0$

7–81.

The beam consists of three segments pin connected at B and E. Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions. Referring to the *FBD* of member *EF*, Fig. *c*

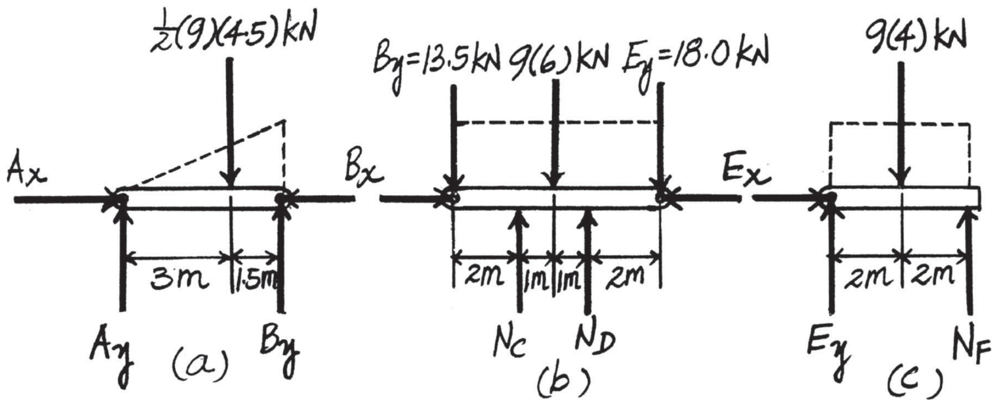
$\circlearrowleft + \Sigma M_E = 0; \quad N_F(4) - 9(4)(2) = 0 \qquad N_F = 18.0 \text{ kN}$

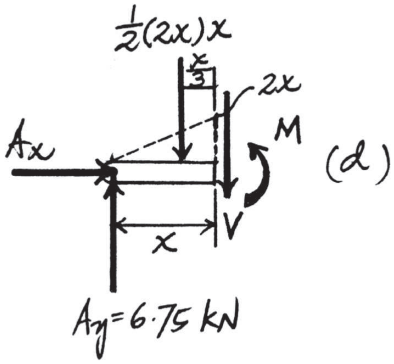
$\circlearrowleft + \Sigma M_F = 0; \quad 9(4)(2) - E_y(4) = 0 \qquad E_y = 18.0 \text{ kN}$

Also, for member AB, Fig. a

$\circlearrowleft + \Sigma M_A = 0; \quad B_y(4.5) - \frac{1}{2}(9)(4.5)(3) = 0 \qquad B_y = 13.5 \text{ kN}$

$\circlearrowleft + \Sigma M_B = 0; \quad \frac{1}{2}(9)(4.5)(1.5) - A_y(4.5) = 0 \qquad A_y = 6.75 \text{ kN}$





7–81. Continued

Finally, for member $BCDE$, Fig. b

$$\zeta + \Sigma M_C = 0; \quad N_D(2) + 13.5(2) - 9(6)(1) - 18(4) = 0 \qquad N_D = 49.5 \text{ kN}$$

$$\zeta + \Sigma M_D = 0; \quad 9(6)(1) + 13.5(4) - 18.0(2) - N_C(2) = 0 \quad N_C = 36.0 \text{ kN}$$

Shear And Moment Functions. Referring to the FBD of the left segment of member AB sectioned at an distance x, Fig. d,

$$+\uparrow \Sigma F_y = 0; \qquad 6.75 - \frac{1}{2}(2x)(x) - V = 0 \qquad V = \{6.75 - x^2\} \text{ kN}$$

$$\zeta + \Sigma M_O = 0; \quad M + \left[\frac{1}{2}(2x)(x)\right]\left(\frac{x}{3}\right) - 6.75x = 0$$

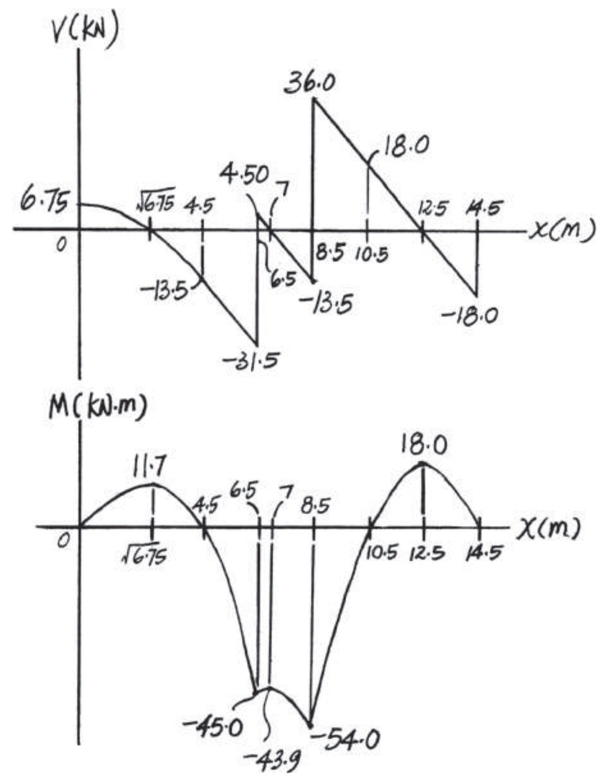
$$M = \left\{6.75x - \frac{1}{3}x^3\right\} \text{kN} \cdot \text{m}$$

Set $V = 0$, then

$$0 = 6.75 - x^2 \quad x = \sqrt{6.75} \text{ m}$$

The corresponding moment is

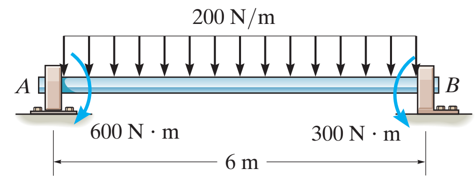
$$M = 6.75(\sqrt{6.75}) - \frac{1}{3}(\sqrt{6.75})^3 = 11.69 \text{ kN} \cdot \text{m} = 11.7 \text{ kN} \cdot \text{m}$$



Ans:
$x = 6.5^-$
$V = -31.5$ kN
$M = -45.0$ kN · m
$x = 8.5^+$
$V = 36.0$ kN
$M = -54.0$ kN · m

7–82.

Draw the shear and moment diagrams for the beam. The supports at A and B are a thrust and journal bearing, respectively.



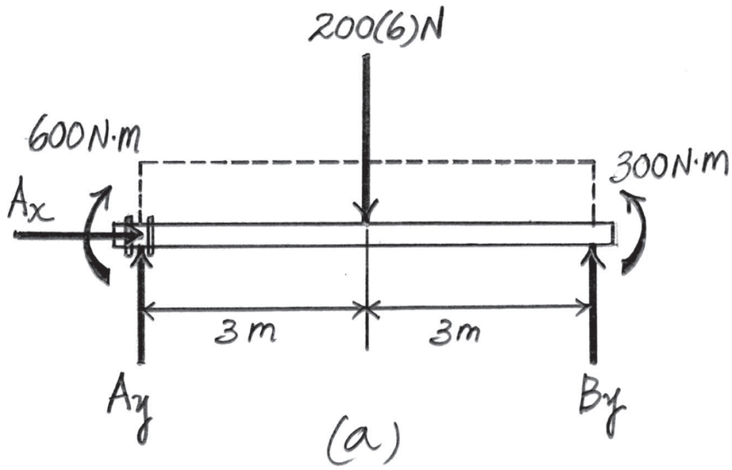
SOLUTION

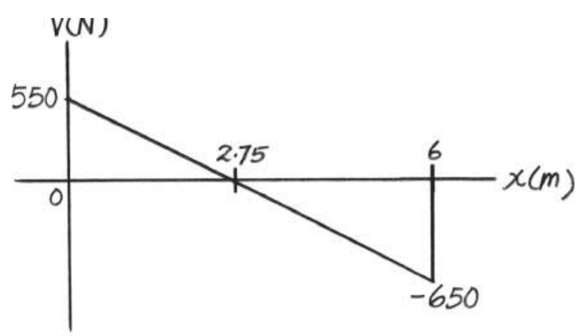
Support Reactions. Referring to the *FBD* of the shaft shown in Fig. *a*,

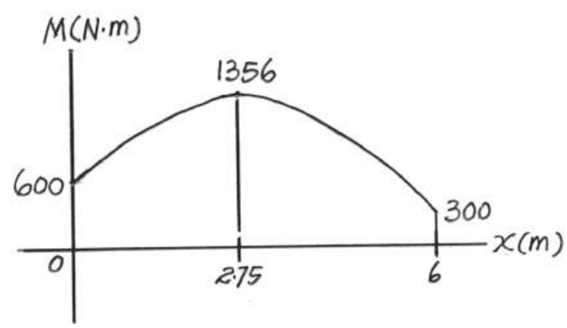
$$\circlearrowleft+\Sigma M_A = 0; \quad B_y(6) + 300 - 200(6)(3) - 600 = 0 \quad B_y = 650\text{ N}$$

$$\circlearrowleft+\Sigma M_B = 0; \quad 200(6)(3) + 300 - 600 - A_y(6) = 0 \quad A_y = 550\text{ N}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad A_x = 0$$



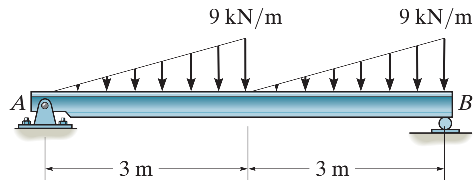




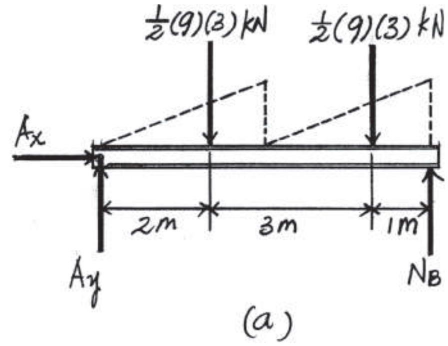
Ans:
$x = 2.75$
$V = 0$
$M = 1356\text{ N}\cdot\text{m}$

7–83.

Draw the shear and moment diagrams for the beam.



SOLUTION



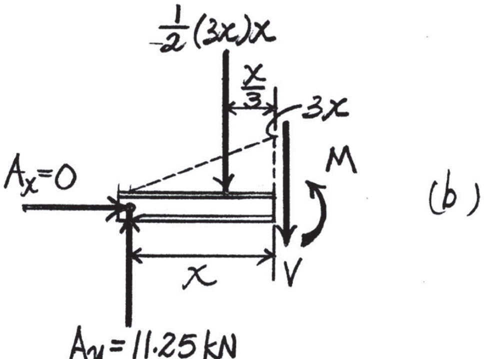
Support Reactions. Referring to the *FBD* of the simply supported beam shown in Fig. *a*,

$$\zeta+\Sigma M_A = 0; \qquad N_B(6) - \frac{1}{2}(9)(3)(2) - \frac{1}{2}(9)(3)(5) = 0 \qquad N_B = 15.75 \text{ kN}$$

$$\zeta+\Sigma M_B = 0; \qquad \frac{1}{2}(9)(3)(1) + \frac{1}{2}(9)(3)(4) - A_y(6) = 0 \qquad A_y = 11.25 \text{ kN}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0$$

Shear And Moment Functions. Referring to the *FBD* of the left segment of the beam sectioned at a distance x within region $0 < x \leq 3$ m.



$$+\uparrow\Sigma F_y = 0; \qquad 11.25 - \frac{1}{2}(3x)(x) - V = 0 \qquad V = \left\{11.25 - \frac{3}{2}x^2\right\} \text{kN}$$

$$\zeta+\Sigma M_O = 0; \qquad M + \left[\frac{1}{2}(3x)(x)\right]\left(\frac{x}{3}\right) - 11.25x = 0$$

$$M = \left\{11.25x - \frac{1}{2}x^3\right\} \text{kN}\cdot\text{m}$$

Set $V = 0$;

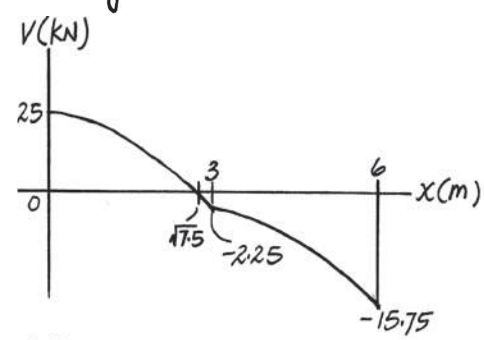
$$0 = 11.25 - \frac{3}{2}x^2 \qquad x = \sqrt{7.5} \text{ m}$$

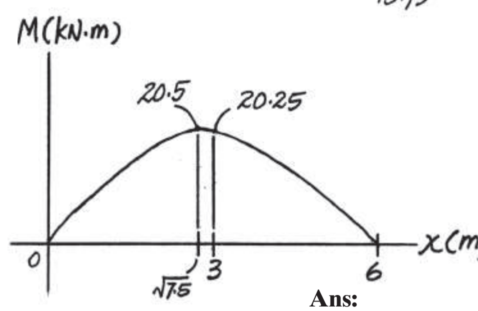
The corresponding moment is

$$M = 11.25\left(\sqrt{7.5}\right) - \frac{1}{2}\left(\sqrt{7.5}\right)^3 = 20.5 \text{ kN}\cdot\text{m}$$

The moment at $x = 3$ m is

$$M = 11.25(3) - \frac{1}{2}(3^3) = 20.25 \text{ kN}\cdot\text{m}$$

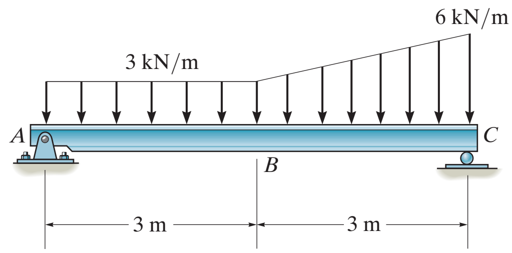




Ans:
$x = 3$
$V = -2.25$ kN
$M = 20.25$ kN·m

***7–84.**

Draw the shear and moment diagrams for the beam.



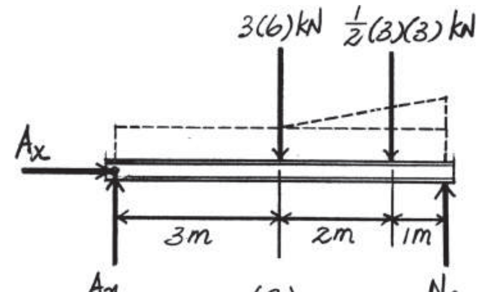
SOLUTION

Support Reactions. Referring to the *FBD* of the simply supported beam shown in Fig. *a*,

$$\zeta+\Sigma M_A = 0; \quad N_C(6) - 3(6)(3) - \frac{1}{2}(3)(3)(5) = 0 \qquad N_C = 12.75\text{ kN}$$

$$\zeta+\Sigma M_C = 0; \quad \frac{1}{2}(3)(3)(1) + 3(6)(3) - A_y(6) = 0 \qquad A_y = 9.75\text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$$



(a)

Shear And Moment Functions. Referring to the *FBD* of the left segment of the beam sectioned at a distance x within region BC (3 m $< x \leq$ 6 m),

$$+\uparrow\Sigma F_y = 0; \quad 9.75 - 3x - \frac{1}{2}(x-3)(x-3) - V = 0 \quad V = \left\{5.25 = \frac{1}{2}x^2\right\}\text{ kN}$$

$$\zeta+\Sigma M_O = 0; \quad M + \frac{1}{2}(x-3)(x-3)\left[\frac{1}{3}(x-3)\right] + 3x\left(\frac{x}{2}\right) - 9.75x = 0$$

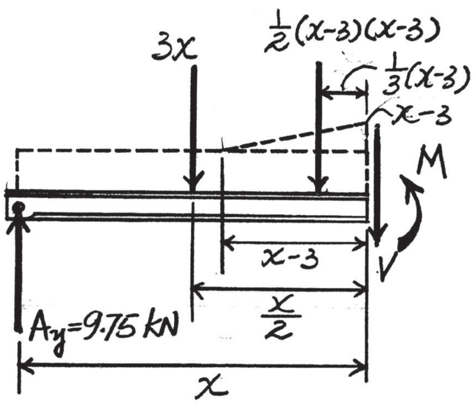
$$M = \left\{-\frac{1}{6}x^3 + 5.25x + 4.50\right\}\text{ kN}\cdot\text{m}$$

Set $V = 0$, we obtain

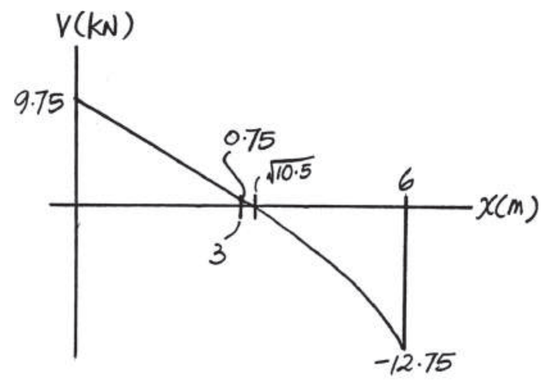
$$0 = 5.25 - \frac{1}{2}x^2 \qquad x = \sqrt{10.5}\text{ m}$$

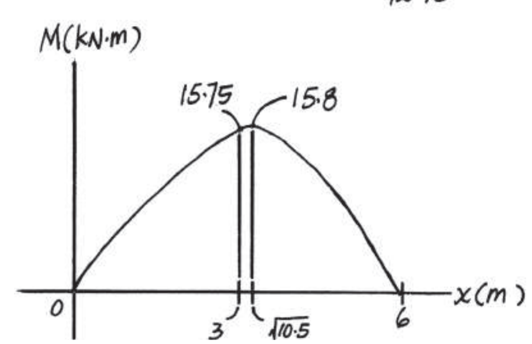
The corresponding moment is

$$M = -\frac{1}{6}\left(\sqrt{10.5}\right)^3 + 5.25\sqrt{10.5} + 4.50 = 15.8\text{ kN}\cdot\text{m}$$



(b)

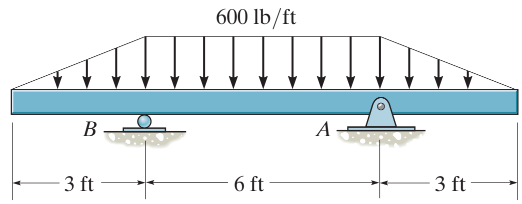




Ans:
$x = \sqrt{10.5}$
$V = 0$
$M = 15.8$ kN · m

7–85.

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions. Referring to the *FBD* of the beam shown in Fig. *b*,

$$\circlearrowleft + \Sigma M_A = 0; \quad 600(6)(3) + \frac{1}{2}(600)(3)(7) - \frac{1}{2}(600)(3)(1) - N_B(6) = 0$$

$$N_B = 2700 \text{ lb}$$

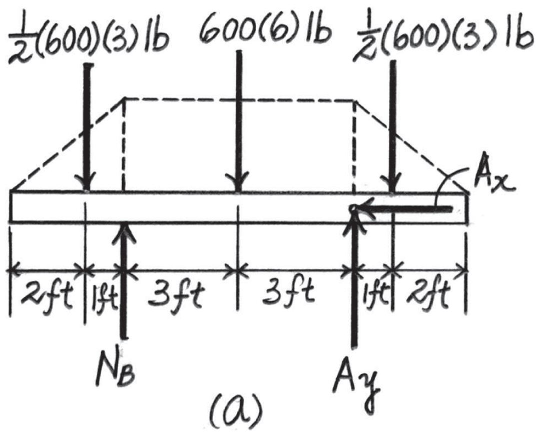
$$\circlearrowleft + \Sigma M_B = 0; \quad A_y(6) + \frac{1}{2}(600)(3)(1) - 600(6)(3) - \frac{1}{2}(600)(3)(7) = 0$$

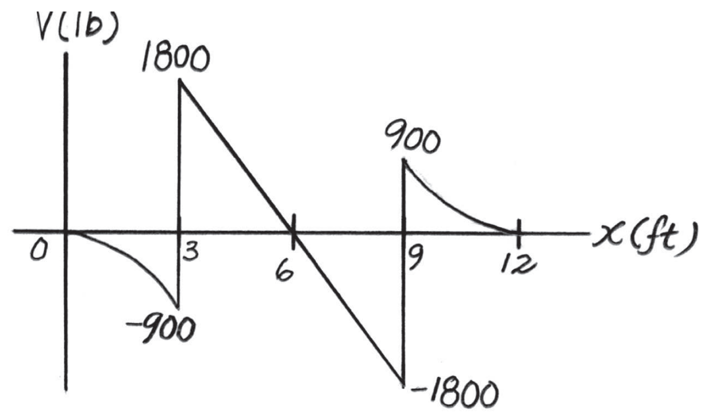
$$A_y = 2700 \text{ lb}$$

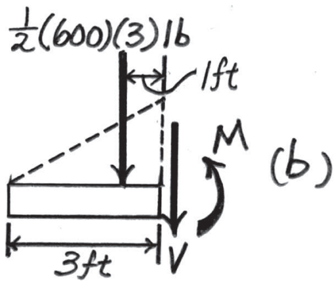
$$\xrightarrow{+} \Sigma F_x = 0; \qquad A_x = 0$$

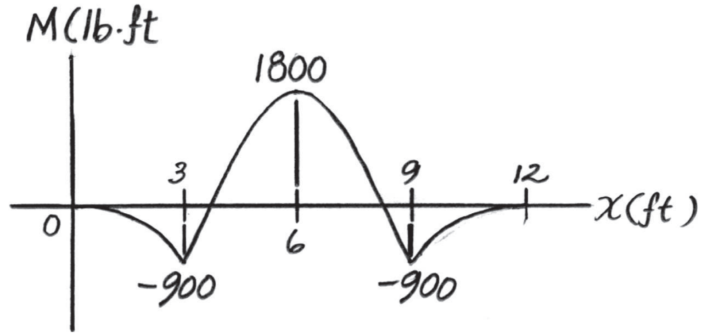
Internal Loadings. Referring to the *FBD* of the left segment of the beam sectioned at $x = 3$ ft shown in Fig. *a*, the internal moment at $x = 3$ ft is

$$\circlearrowleft + \Sigma M_O = 0; \qquad M + \frac{1}{2}(600)(3)(1) = 0 \qquad M = -900 \text{ lb} \cdot \text{ft}$$





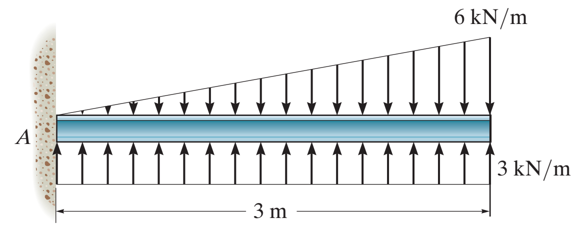




Ans:
$x = 3^+$
$V = 1800$ lb
$M = -900$ lb · ft
$x = 6$
$V = 0$
$M = 1800$ lb · ft

7–86.

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions. The *FBD* of the beam acted upon the equivalent loading (by superposition) is shown in Fig. *a*. Equilibrium gives

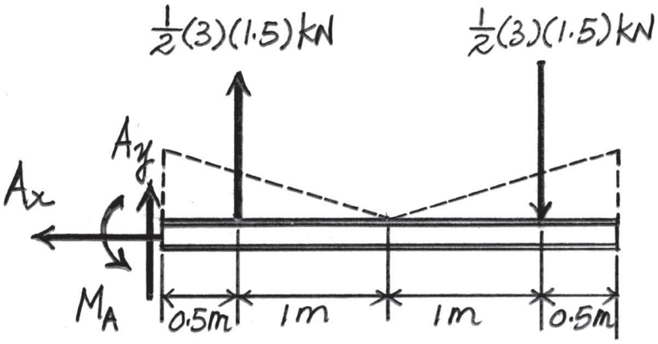
$$\circlearrowleft + \Sigma M_A = 0; \quad M_A + \frac{1}{2}(3)(1.5)(0.5) - \frac{1}{2}(3)(1.5)(2.5) = 0 \quad M_A = 4.50 \text{ kN}\cdot\text{m}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y = 0$$

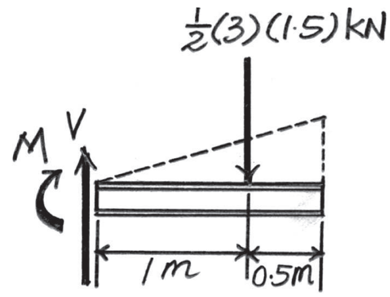
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0$$

Internal Loadings. Referring to the *FBD* of the right segment of the beam sectioned at $x = 1.5$ m, the internal moment at this section is

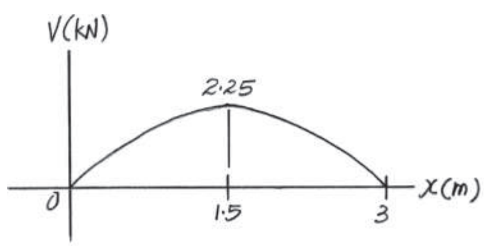
$$\circlearrowleft + \Sigma M_O = 0; \qquad -M - \frac{1}{2}(3)(1.5)(1) = 0 \qquad M = -2.25 \text{ kN}\cdot\text{m}$$

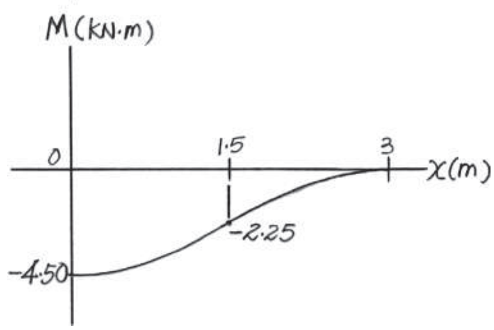


(a)



(b)

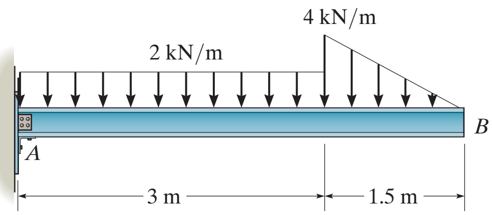




Ans:
$x = 1.5$
$V = 2.25$ kN
$M = -2.25$ kN·m

7–87.

Draw the shear and moment diagrams for the beam.

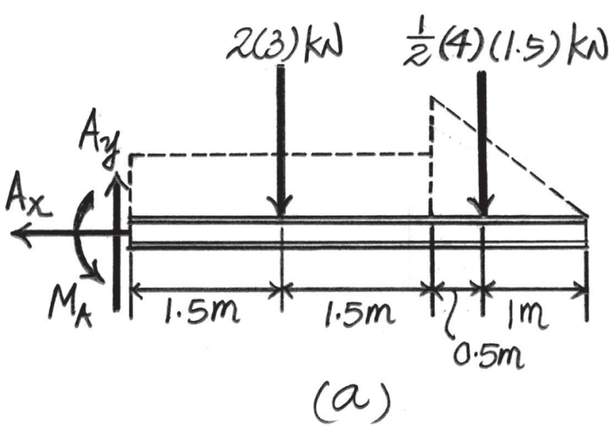


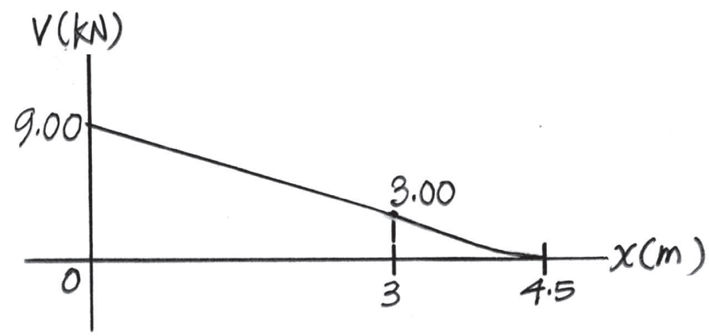
SOLUTION

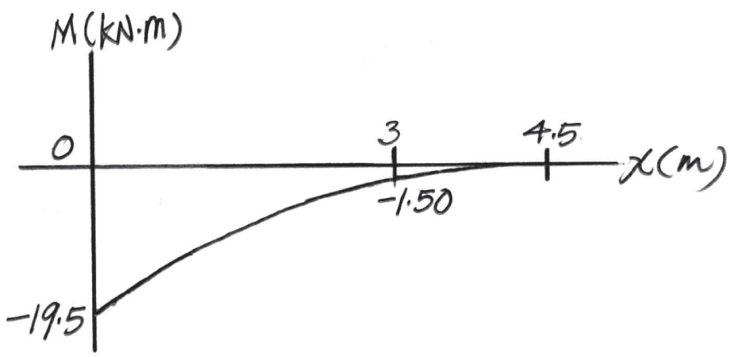
Support Reactions. Referring to the *FBD* of the cantilevered beam shown in Fig. *a*.

$$\zeta+\Sigma M_A = 0; \qquad M_A - 2(3)(1.5) - \frac{1}{2}(4)(1.5)(3.5) = 0 \qquad M_A = 19.5 \text{ kN} \cdot \text{m}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 2(3) - \frac{1}{2}(4)(1.5) = 0 \qquad A_y = 9.00 \text{ kN}$$



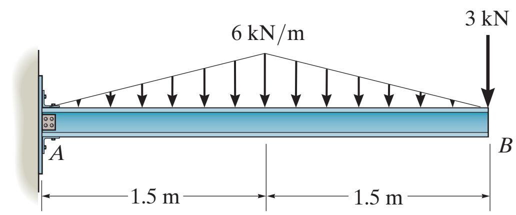




Ans:
$x = 3$
$V = 3.00$ kN
$M = -1.50$ kN · m

***7–88.**

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions. Referring to the *FBD* of the cantilevered beam shown in Fig. *a*.

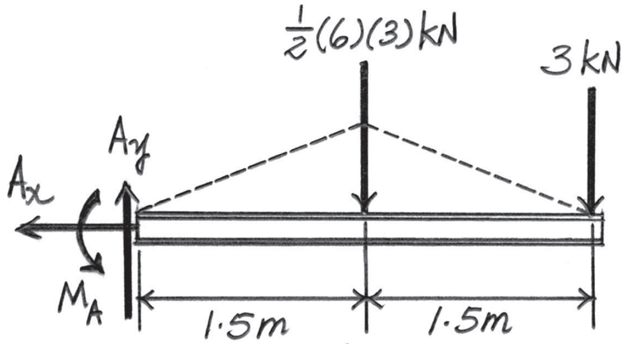
$$\circlearrowleft + \Sigma M_A = 0; \qquad M_A - \frac{1}{2}(6)(3)(1.5) - 3(3) = 0 \qquad M_A = 22.5 \text{ kN} \cdot \text{m}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - \frac{1}{2}(6)(3) - 3 = 0 \qquad A_y = 12.0 \text{ kN}$$

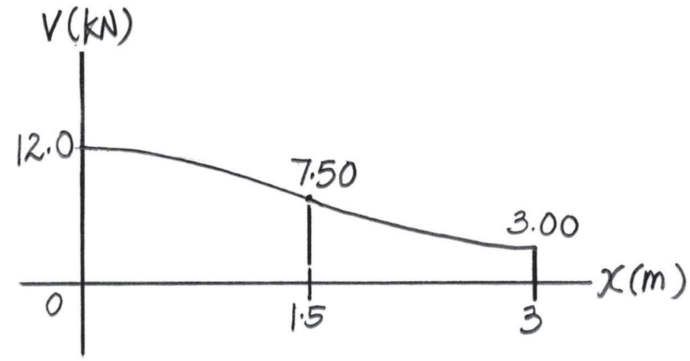
$$\overset{+}{\rightarrow} \Sigma F_x = 0 \qquad A_x = 0$$

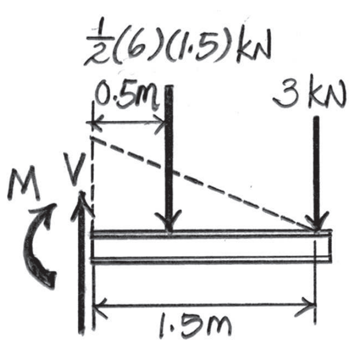
Internal Loadings. Referring to the *FBD* of the right segment of the beam sectioned at $x = 1.5$ m, the internal moment at this section is

$$\circlearrowleft + \Sigma M_O = 0; \qquad -M - \frac{1}{2}(6)(1.5)(0.5) - 3(1.5) = 0 \qquad M = -6.75 \text{ kN} \cdot \text{m}$$

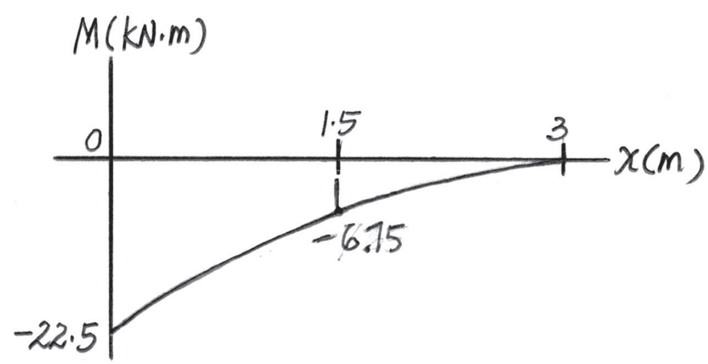


(a)





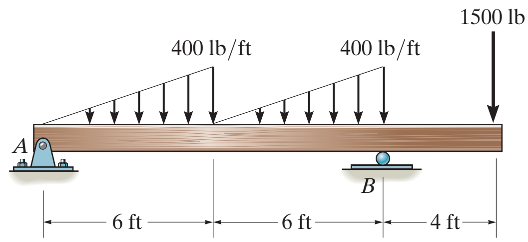
(b)



Ans:
$x = 1.5$
$V = 7.50$ kN
$M = -6.75$ kN·m

7–89.

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions. Referring to the *FBD* of the overhang beam shown in Fig. *a*

$$\zeta+\Sigma M_A = 0; \qquad N_B(12) - \frac{1}{2}(400)(6)(4) - \frac{1}{2}(400)(6)(10) - 1500(16) = 0$$

$$N_B = 3400 \text{ lb}$$

$$\zeta+\Sigma M_B = 0; \qquad \frac{1}{2}(400)(6)(2) + \frac{1}{2}(400)(6)(8) - 1500(4) - A_y(12) = 0$$

$$A_y = 500 \text{ lb}$$

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0$$

Shear And Moment Functions. Referring to the *FBD* of the left segment of the beam sectioned at a distance x within region $0 \leq x < 6$ ft, Fig. *b*

$$+\uparrow \Sigma F_y = 0; \qquad 500 - \frac{1}{2}\left(\frac{200}{3}x\right)x - V = 0 \qquad V = \left\{500 - \frac{100}{3}x^2\right\} \text{lb}$$

$$\zeta+\Sigma M_O = 0; \qquad M + \left[\frac{1}{2}\left(\frac{200}{3}x\right)x\right]\left(\frac{x}{3}\right) - 500x = 0$$

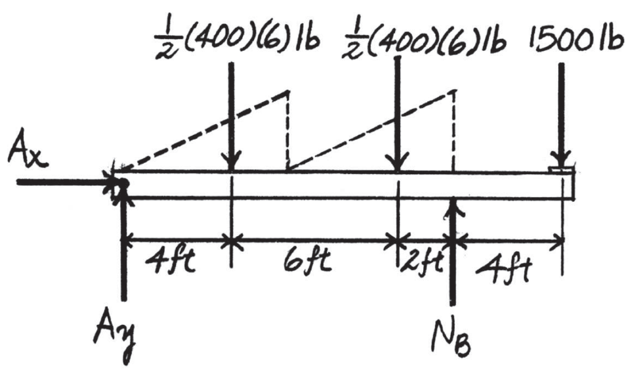
$$M = \left\{500x - \frac{100}{9}x^3\right\} \text{lb} \cdot \text{ft}$$

Set $V = 0$,

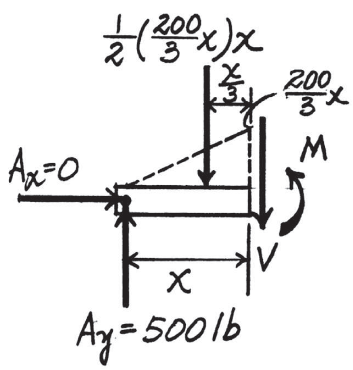
$$0 = 500 - \frac{100}{3}x^2 \qquad x = \sqrt{15} \text{ ft}$$

The corresponding moment is

$$M = 500\sqrt{15} - \frac{100}{9}\left(\sqrt{15}\right)^3 = 1291 \text{ lb} \cdot \text{ft}$$



(a)



(b)

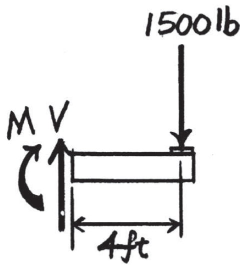
7–89. Continued

The moment at $x = 6$ ft is

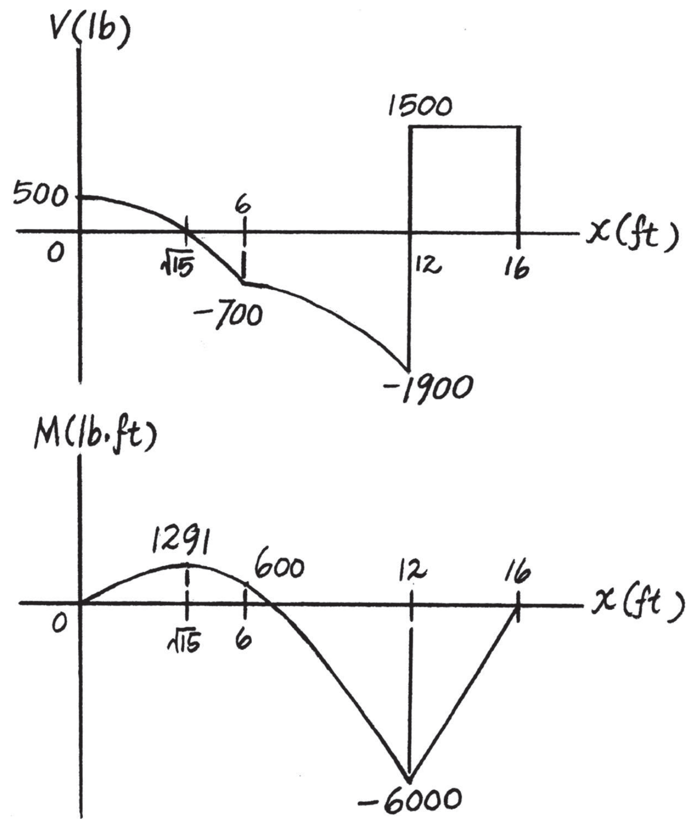
$$M = 500(6) - \left(\frac{100}{9}\right)(6^3) = 600 \text{ lb} \cdot \text{ft}$$

Referring to the *FBD* diagram of the right segment of the beam sectioned just to the right of support B, Fig. c, the moment at $x = 12$ ft is

$$\circlearrowleft + \Sigma M_O = 0; \qquad -M - 1500(4) = 0 \qquad M = -6000 \text{ lb} \cdot \text{ft}$$



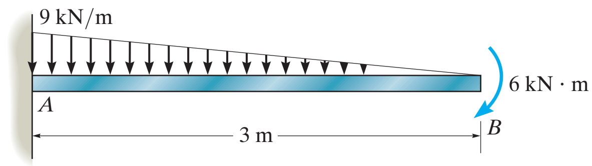
(C)



Ans:
$x = \sqrt{15}$
$V = 0$
$M = 1291 \text{ lb} \cdot \text{ft}$
$x = 12^-$
$V = -1900 \text{ lb}$
$M = -6000 \text{ lb} \cdot \text{ft}$

7–90.

Draw the shear and moment diagrams for the beam.



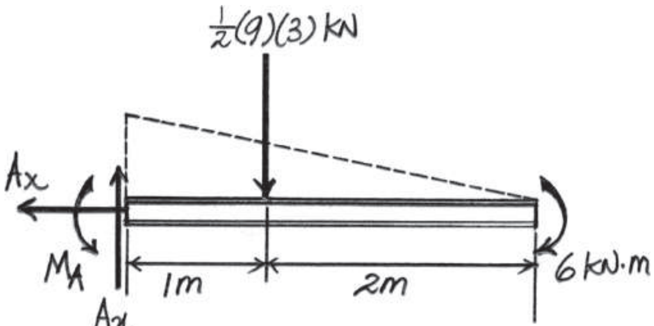
SOLUTION

Support Reactions. Referring to the *FBD* of the cantilivered beam shown in Fig. *a*,

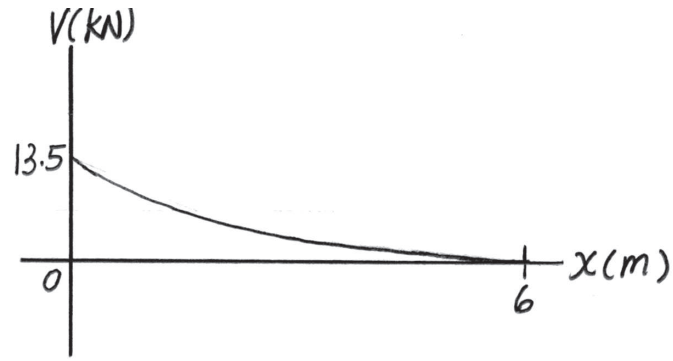
$$\circlearrowleft + \Sigma M_A = 0; \qquad M_A - \frac{1}{2}(9)(3)(1) - 6 = 0 \qquad M_A = 19.5 \text{ kN} \cdot \text{m}$$

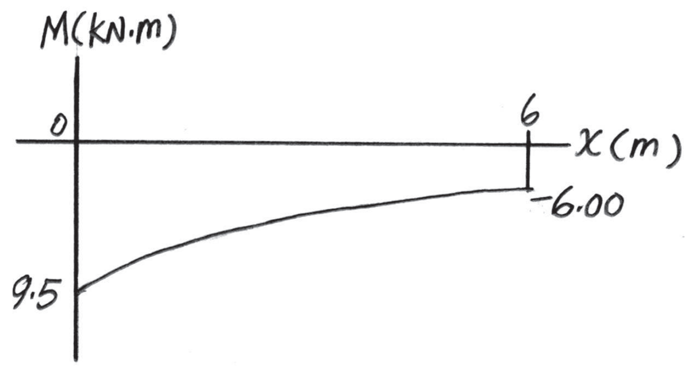
$$+\uparrow \Sigma F_y = 0; \qquad A_y - \frac{1}{2}(9)(3) = 0 \qquad A_y = 13.5 \text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0$$



(a)

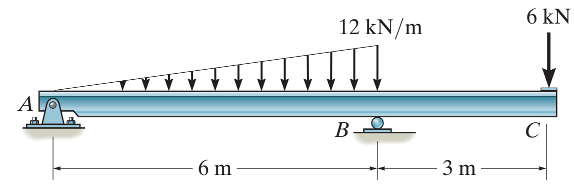




Ans:
$x = 0$
$V = 13.5$ kN
$M = -9.5$ kN·m

7–91.

Draw the shear and moment diagrams for the beam.



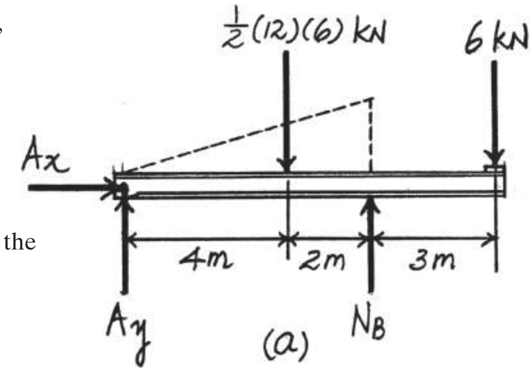
SOLUTION

Support Reactions. Referring to the *FBD* of the overhang beam shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \qquad N_B(6) - \frac{1}{2}(12)(6)(4) - 6(9) = 0 \qquad N_B = 33.0 \text{ kN}$$

$$\circlearrowleft + \Sigma M_B = 0; \qquad \frac{1}{2}(12)(6)(2) - 6(3) - A_y(6) = 0 \qquad A_y = 9.00 \text{ kN}$$

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad A_x = 0$$



Shear And Moment Functions. Referring to the *FBD* of the left segment of the beam sectioned at a distance x, Fig. *b*

$$+\uparrow \Sigma F_y = 0; \qquad 9.00 - \frac{1}{2}(2x)(x) - V = 0 \qquad V = \{9.00 - x^2\} \text{ kN}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad M + \left[\frac{1}{2}(2x)(x)\right]\left(\frac{x}{3}\right) - 9.00x = 0$$

$$M = \left\{9.00x - \frac{x^3}{3}\right\} \text{ kN} \cdot \text{m}$$

Set $V = 0$,

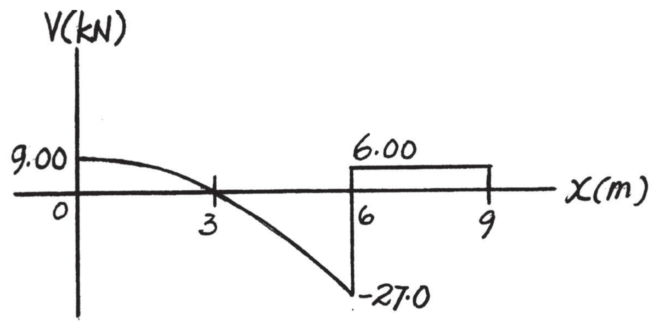
$$0 = 9.00 - x^2 \qquad x = 3 \text{ ft}$$

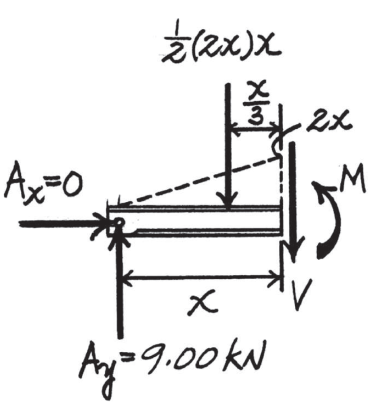
The corresponding moment is

$$M = 9.00(3) - \frac{3^3}{3} = 18.0 \text{ kN} \cdot \text{m}$$

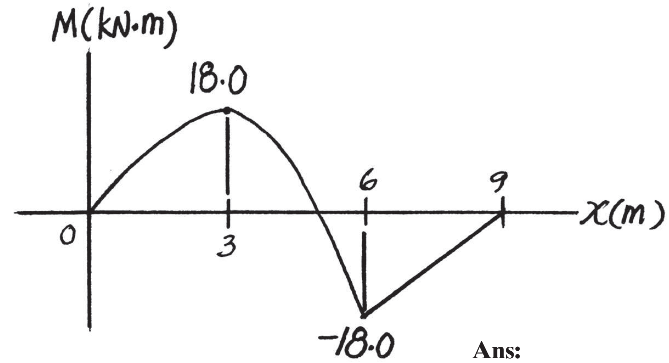
The moment at $x = 6$ m is

$$M = 9.00(6) - \frac{6^3}{3} = -18.0 \text{ kN} \cdot \text{m}$$





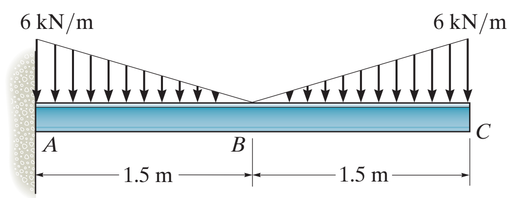
(b)



Ans:
$x = 3$
$V = 0$
$M = 18.0 \text{ kN} \cdot \text{m}$
$x = 6^-$
$V = -27.0 \text{ kN}$
$M = -18.0 \text{ kN} \cdot \text{m}$

***7–92.**

Draw the shear and moment diagrams for the beam.



SOLUTION

Support Reactions. Referring to the *FBD* of the cantilevered beam shown in Fig. *a*

$$\zeta+\Sigma M_A = 0; \qquad M_A - \frac{1}{2}(6)(1.5)(0.5) - \frac{1}{2}(6)(1.5)(2.5) = 0$$

$$M_A = 13.5\text{ kN}\cdot\text{m}$$

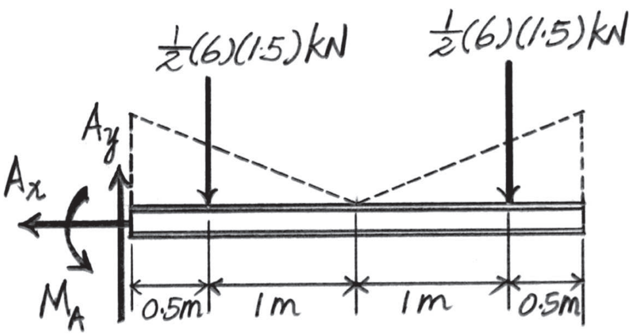
$$+\uparrow \Sigma F_y = 0; \qquad A_y - \frac{1}{2}(6)(1.5) - \frac{1}{2}(6)(1.5) = 0$$

$$A_y = 9.00\text{ kN}$$

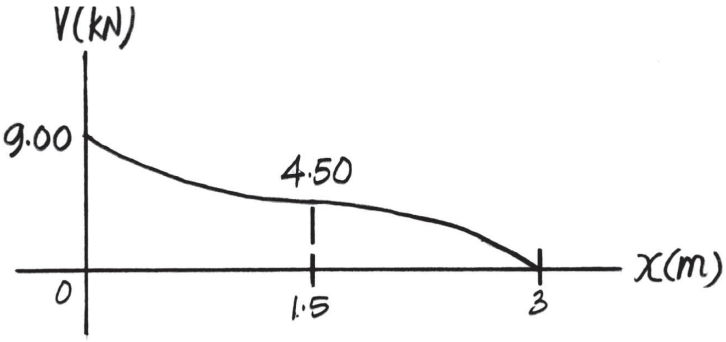
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad A_x = 0$$

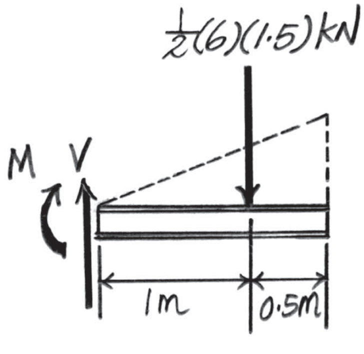
Internal loadings. Referring to the *FBD* of the right segment of the beam sectioned at $x = 1.5$ m,

$$\zeta+\Sigma M_O = 0; \qquad -M - \frac{1}{2}(6)(1.5)(1) = 0 \qquad M = 4.50\text{ kN}\cdot\text{m}$$

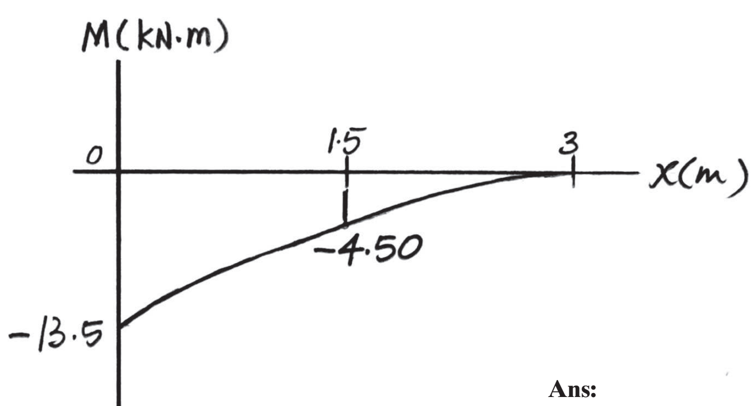


(a)





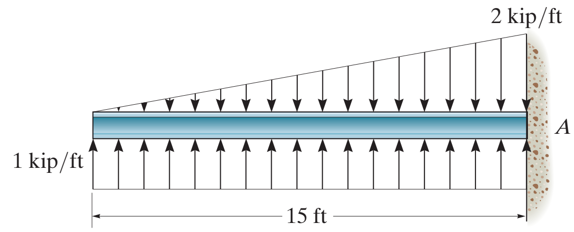
(b)



Ans:
$x = 1.5$
$V = 4.50$ kN
$M = -4.50$ kN·m

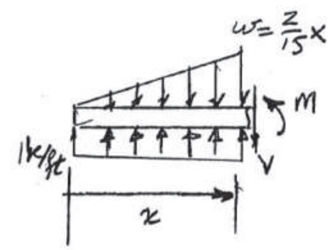
7–93.

Draw the shear and moment diagrams for the beam.



SOLUTION

Shear and Moment Functions: For $0 \leq x < 15$ ft

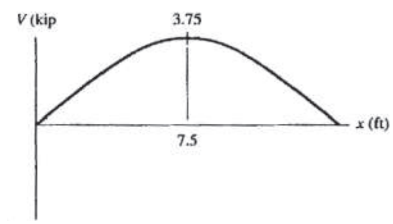


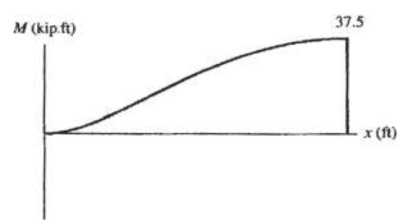
$+\uparrow \Sigma F_y = 0; \qquad 1x - x^2/15 - V = 0$

$$V = \{x - x^2/15\}\ \text{N} \qquad \textbf{Ans.}$$

$\circlearrowleft + \Sigma M = 0; \qquad M + (x^2/15)\left(\frac{x}{3}\right) - 1x(x/2) = 0$

$$M = \{x^2/2 - x^3/45\}\ \text{N} \cdot \text{m} \qquad \textbf{Ans.}$$

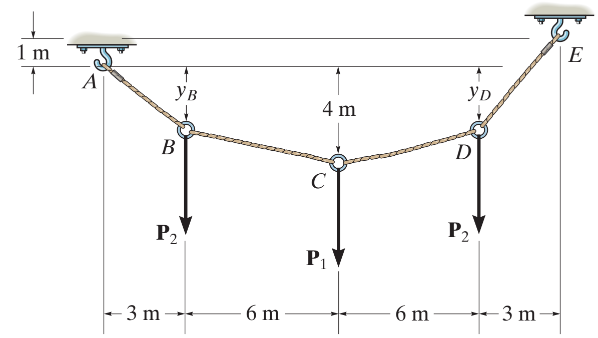




Ans:
$x = 15$
$V = 0$
$M = 37.5$ kip $\cdot$ ft

7–94.

The cable supports the three loads shown. Determine the sags y_B and y_D of B and D. Take $P_1 = 800$ N, $P_2 = 500$ N.



SOLUTION

Support Reactions. Referring to the *FBD* of the cable system sectioned through cable AB shown in Fig. *a*,

$$\zeta+\Sigma M_E = 0; \quad 500(3) + 800(9) + 500(15) - F_{AB}\left(\frac{y_B}{\sqrt{y_B^2 + 9}}\right)(15)$$

$$- F_{AB}\left(\frac{3}{\sqrt{y_B^2 + 9}}\right)(y_B + 1) = 0$$

$$F_{AB}\left(\frac{18y_B + 3}{\sqrt{y_B^2 + 9}}\right) = 16200 \qquad \textbf{(1)}$$

Also, referring to the *FBD* of the cable segment sectioned through cables AB and CD, shown Fig. *b*,

$$\zeta+\Sigma M_C = 0; \quad 500(6) + F_{AB}\left(\frac{3}{\sqrt{y_B^2 + 9}}\right)(4 - y_B) - F_{AB}\left(\frac{y_B}{\sqrt{y_B^2 + 9}}\right)(6) = 0$$

$$F_{AB}\left(\frac{9y_B - 12}{\sqrt{y_B^2 + 9}}\right) = 3000 \qquad \textbf{(2)}$$

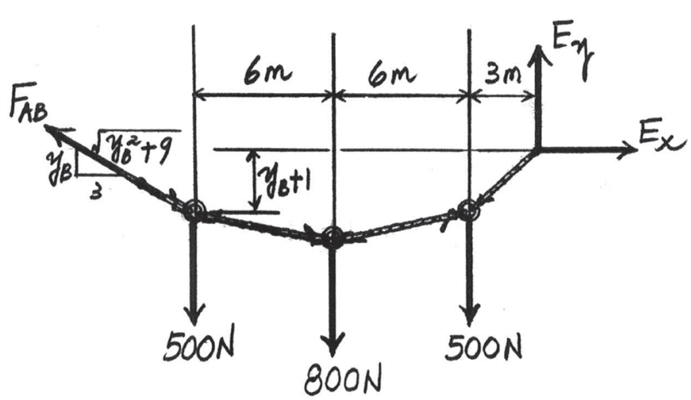
Divide Eq. (1) by (2)

$$y_B = 2.216 \text{ m} = 2.22 \text{ m} \qquad \textbf{Ans.}$$

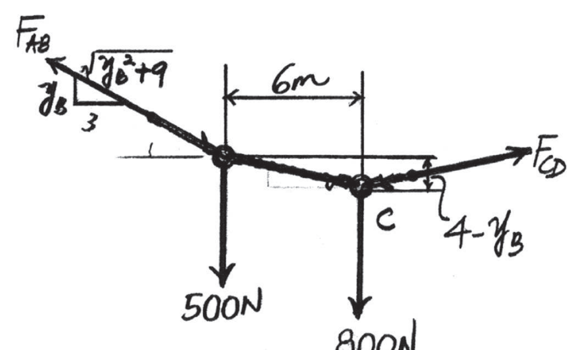
Substituting this result into Fig. (1)

$$F_{AB}\left[\frac{18(2.216) + 3}{\sqrt{2.216^2 + 9}}\right] = 16200$$

$$F_{AB} = 1408.93 \text{ N}$$



(a)



(b)

7–94. Continued

Method of joints. Perform the joint equilibrium analysis first for joint *B* and then joint *C*.

Joint *B*. Fig. *c*

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{BC} \cos 16.56° - 1408.93 \cos 36.45° = 0$

$$F_{BC} = 1182.39 \text{ N}$$

$+\uparrow \Sigma F_y = 0;$ $\quad 1408.93 \sin 36.45 - 1182.39 \sin 16.56° - 500 = 0$ **(Check!)**

Joint *C*. Fig. *d*

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad F_{CD}\left(\frac{6}{\sqrt{(4 - y_D)^2 + 36}}\right) - 1182.39 \cos 16.56° = 0$

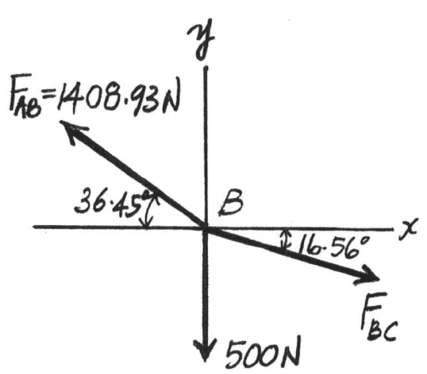
$$\frac{6}{\sqrt{(4 - y_D)^2 + 36}} F_{CD} = 1133.33 \qquad \textbf{(3)}$$

$+\uparrow \Sigma F_y = 0;$ $\quad F_{CD}\left(\frac{4 - y_D}{\sqrt{(4 - y_D)^2 + 36}}\right) + 1182.39 \sin 16.56° - 800 = 0$

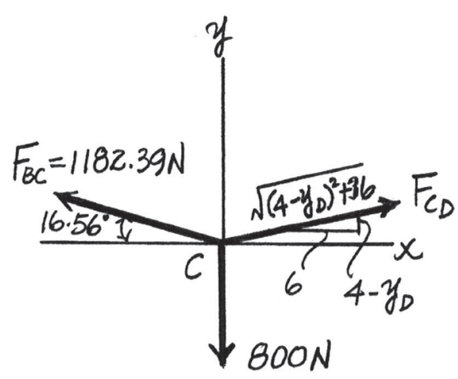
$$\frac{4 - y_D}{\sqrt{(4 - y_D)^2 + 36}} F_{CD} = 462.96 \qquad \textbf{(4)}$$

Divide Eq (4) by (3)

$$\frac{4 - y_D}{6} = 0.4085 \qquad y_D = 1.549 \text{ m} = 1.55 \text{ m}$$ **Ans.**



(c)

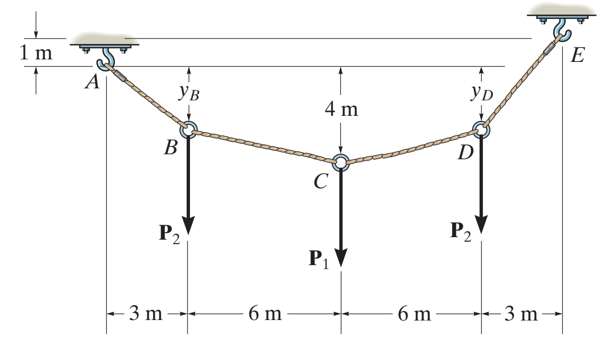


(d)

Ans:
$y_B = 2.22$ m
$y_D = 1.55$ m

7–95.

The cable supports the three loads shown. Determine the magnitude of $\mathbf{P}_1$ if $P_2 = 600$ N and $y_B = 3$ m. Also find sag y_D.



SOLUTION

Support Reactions. Referring to the *FBD* of the cable system sectioned through cable *BC*, Fig. *a*

$$\circlearrowleft + \Sigma M_E = 0; \qquad 600(3) + P_1(9) - F_{BC}\left(\frac{6}{\sqrt{37}}\right)(5) - F_{BC}\left(\frac{1}{\sqrt{37}}\right)(9) = 0$$

$$\frac{39}{\sqrt{37}}F_{BC} - 9P_1 = 1800 \qquad \textbf{(1)}$$

Method of Joints. Perform the joint equilibrium analysis for joint *B* first, Fig. *b*,

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{BC}\left(\frac{6}{\sqrt{37}}\right) - F_{AB}\left(\frac{1}{\sqrt{2}}\right) = 0 \qquad \textbf{(2)}$$

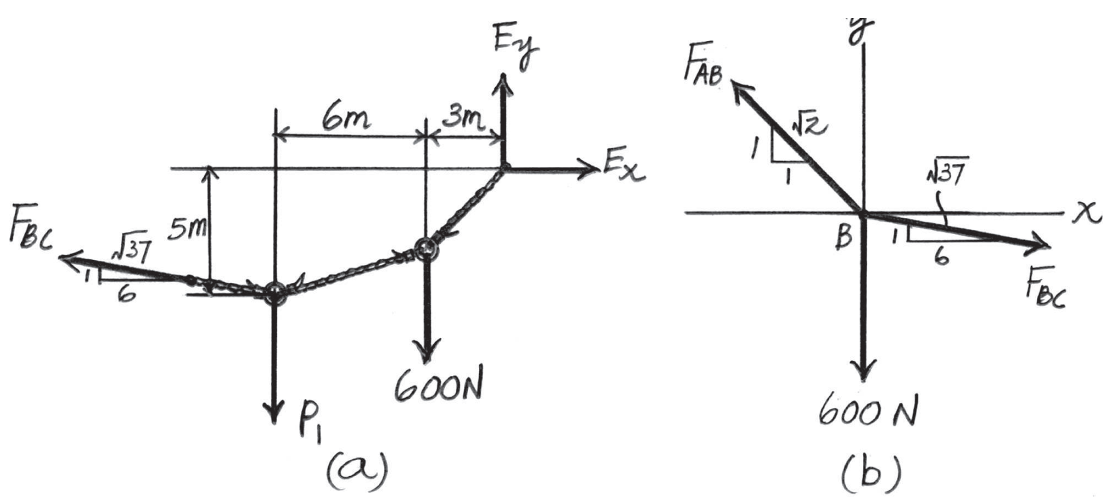
$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\left(\frac{1}{\sqrt{2}}\right) - F_{BC}\left(\frac{1}{\sqrt{37}}\right) - 600 = 0 \qquad \textbf{(3)}$$

Solving Eqs (2) and (3),

$$F_{BC} = 120\sqrt{37}\text{ N} \qquad F_{AB} = 720\sqrt{2}\text{ N}$$

Substitute the result of *FBC* into Fig. (1),

$$P_1 = 320\text{ N} \qquad \textbf{Ans.}$$



7–95. Continued

Next. Consider the equilibrium of joint C, Fig. c,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_{CD}\left(\frac{6}{\sqrt{(4-y_D)^2+36}}\right) - \left(120\sqrt{37}\right)\left(\frac{6}{\sqrt{37}}\right) = 0$$

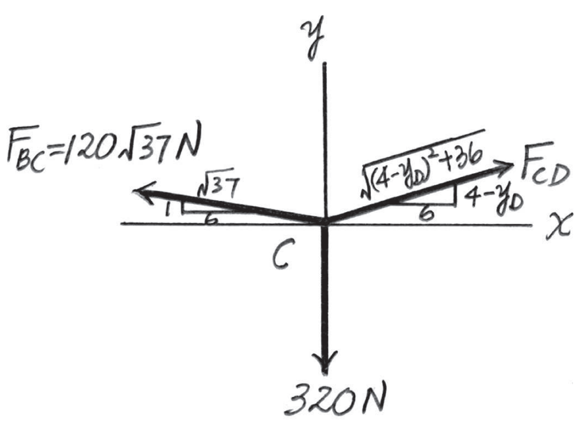
$$\frac{6}{\sqrt{(4-y_D)^2+36}}F_{CD} = 720 \qquad \textbf{(4)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{CD}\left(\frac{4-y_D}{\sqrt{(4-y_D)^2+36}}\right) + \left(120\sqrt{37}\right)\left(\frac{1}{\sqrt{37}}\right) - 320 = 0$$

$$\frac{4-y_D}{\sqrt{(4-y_D)^2+36}}F_{CD} = 200 \qquad \textbf{(5)}$$

Divide Eq (5) by (4)

$$\frac{4-y_D}{6} = \frac{5}{18} \qquad y_D = 2.3333 \text{ m} = 2.33 \text{ m} \qquad \textbf{Ans.}$$

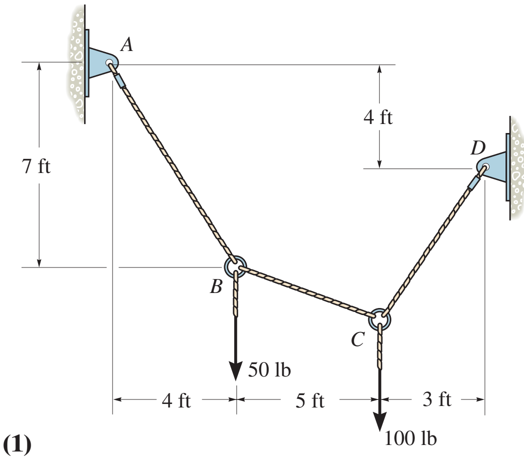


(C)

Ans:
$P_1 = 320$ N
$y_D = 2.33$ m

***7–96.**

Determine the tension in each segment of the cable and the cable's total length.



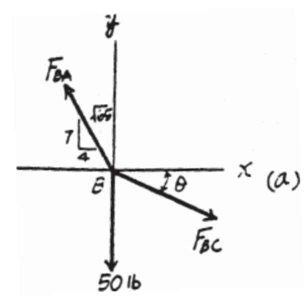
SOLUTION

Equations of Equilibrium: Applying method of joints, we have

Joint B

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{BC}\cos\theta - F_{BA}\left(\frac{4}{\sqrt{65}}\right) = 0 \qquad \textbf{(1)}$$

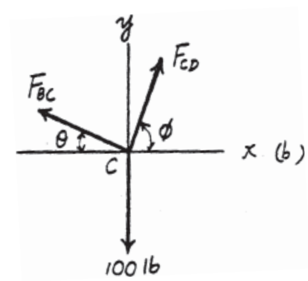
$$+\uparrow \Sigma F_y = 0; \qquad F_{BA}\left(\frac{7}{\sqrt{65}}\right) - F_{BC}\sin\theta - 50 = 0 \qquad \textbf{(2)}$$



Joint C

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{CD}\cos\phi - F_{BC}\cos\theta = 0 \qquad \textbf{(3)}$$

$$+\uparrow \Sigma F_y = 0; \qquad F_{BC}\sin\theta + F_{CD}\sin\phi - 100 = 0 \qquad \textbf{(4)}$$



Geometry:

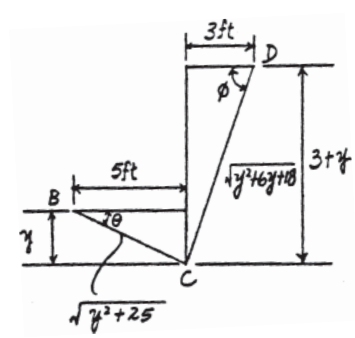
$$\sin\theta = \frac{y}{\sqrt{y^2 + 25}} \qquad \cos\theta = \frac{5}{\sqrt{y^2 + 25}}$$

$$\sin\phi = \frac{3 + y}{\sqrt{y^2 + 6y + 18}} \qquad \cos\phi = \frac{3}{\sqrt{y^2 + 6y + 18}}$$

Substitute the above results into Eqs. (1), (2), (3) and (4) and solve. We have

$$F_{BC} = 46.7 \text{ lb} \qquad F_{BA} = 83.0 \text{ lb} \qquad F_{CD} = 88.1 \text{ lb} \qquad \textbf{Ans.}$$

$$y = 2.679 \text{ ft}$$



The total length of the cable is

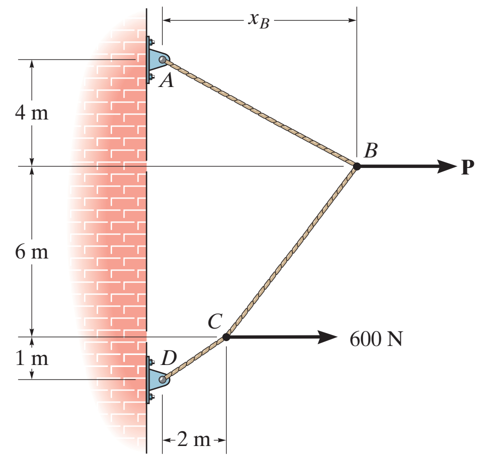
$$l = \sqrt{7^2 + 4^2} + \sqrt{5^2 + 2.679^2} + \sqrt{3^2 + (2.679 + 3)^2}$$

$$= 20.2 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$F_{BC} = 46.7$ lb
$F_{BA} = 83.0$ lb
$F_{CD} = 88.1$ lb
$l = 20.2$ ft

7–97.

The cable supports the loading shown. Determine the distance x_B the force at B acts from A. Set $P = 800$ N.



SOLUTION

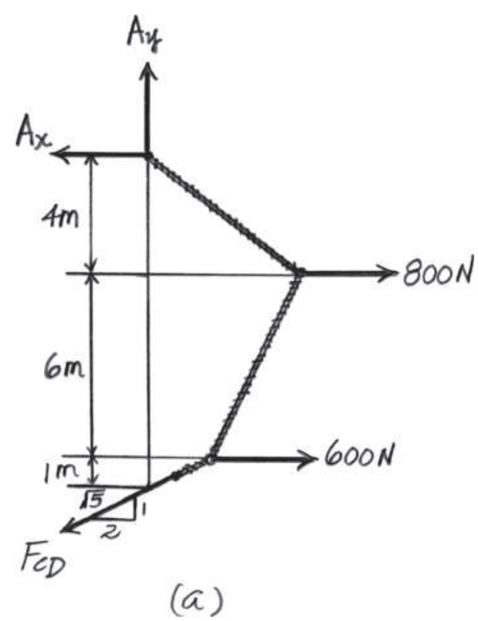
Support Reactions. Referring to the *FBD* of the cable system sectioned through cable *CD*, Fig. *a*

$$\circlearrowleft + \Sigma M_A = 0; \qquad 800(4) + 600(10) - F_{CD}\left(\frac{2}{\sqrt{5}}\right)(11) = 0$$

$$F_{CD} = 935.08 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 800 + 600 - 935.08\left(\frac{2}{\sqrt{5}}\right) - A_x = 0 \qquad A_x = 563.64 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad A_y - 935.08\left(\frac{1}{\sqrt{5}}\right) = 0 \qquad A_y = 418.18 \text{ N}$$



(a)

Method of Joints. Consider the equilibrium of joint *A*, Fig. *b*

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{AB}\left(\frac{x_B}{\sqrt{x_B^2 + 16}}\right) - 563.64 = 0$$

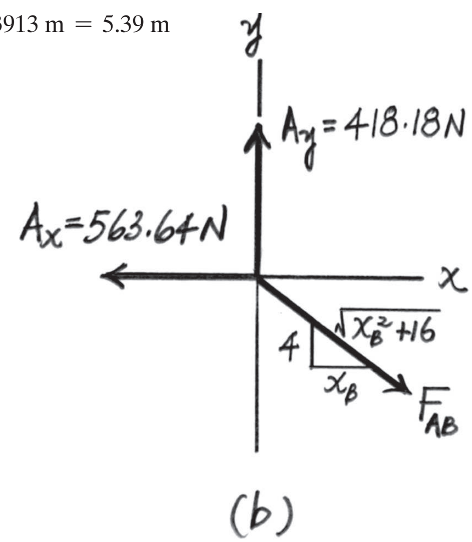
$$F_{AB}\left(\frac{x_B}{\sqrt{x_B^2 + 16}}\right) = 563.64 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad 418.18 - F_{AB}\left(\frac{4}{\sqrt{x_B^2 + 16}}\right) = 0$$

$$F_{AB}\left(\frac{4}{\sqrt{x_B^2 + 16}}\right) = 418.18 \qquad \textbf{(2)}$$

Divide Eq (1) by (2)

$$x_B = 5.3913 \text{ m} = 5.39 \text{ m} \qquad \textbf{Ans.}$$

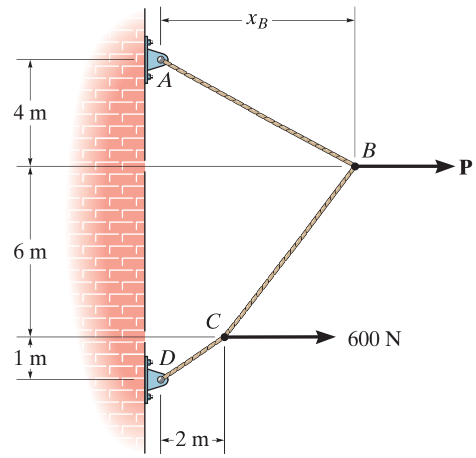


(b)

Ans:
$x_B = 5.39$ m

7–98.

The cable supports the loading shown. Determine the magnitude of the horizontal force **P** so that $x_B = 5$ m.



SOLUTION

Support Reactions. Referring to the *FBD* of the cable system sectioned through cable *AB*, Fig. *a*

$$\zeta + \Sigma M_D = 0; \qquad F_{AB} = \left(\frac{5}{\sqrt{41}}\right)(11) - P(7) - 600(1) = 0$$

$$\frac{55}{\sqrt{41}} F_{AB} - 7P = 600 \qquad \textbf{(1)}$$

Method of Joints. Consider the equilibrium of joint *B*, Fig. *b*,

$$+\uparrow \Sigma F_y = 0; \qquad F_{AB}\left(\frac{4}{\sqrt{41}}\right) - F_{BC}\left(\frac{2}{\sqrt{5}}\right) = 0 \qquad F_{BC} = \frac{2\sqrt{5}}{\sqrt{41}} F_{AB}$$

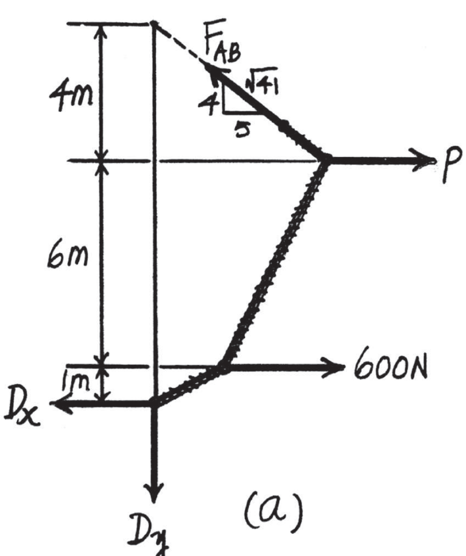
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad P - F_{AB}\left(\frac{5}{\sqrt{41}}\right) - \left(\frac{2\sqrt{5}}{\sqrt{41}} F_{AB}\right)\left(\frac{1}{\sqrt{5}}\right) = 0$$

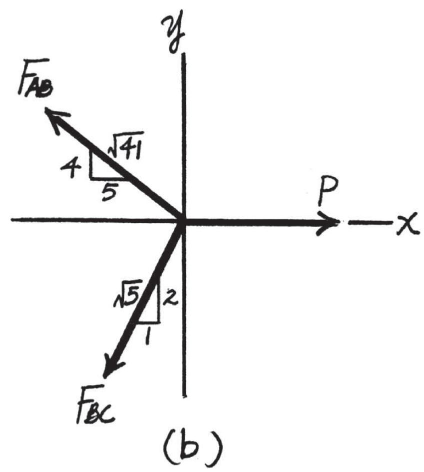
$$F_{AB} = \frac{\sqrt{41}}{7} P \qquad \textbf{(2)}$$

Substituting Eq. (2) into (1)

$$\frac{55}{\sqrt{41}}\left(\frac{\sqrt{41}}{7} P\right) - 7P = 600$$

$$P = 700 \text{ N} \qquad \textbf{Ans.}$$

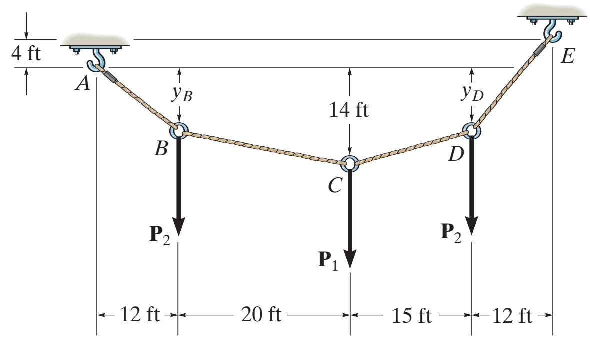


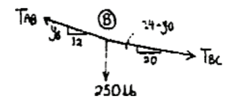


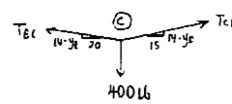
Ans:
$P = 700$ N

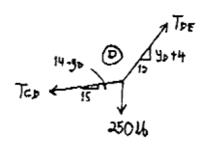
7–99.

The cable supports the three loads shown. Determine the sags y_B and y_D of points B and D. Take $P_1 = 400$ lb, $P_2 = 250$ lb.









$+\uparrow \Sigma F_y = 0;\qquad \dfrac{14 - y_D}{\sqrt{(14 - y_D)^2 + 225}} T_{CD} + \dfrac{14 - y_B}{\sqrt{(14 - y_B)^2 + 400}} T_{BC} - 400 = 0$

$$\frac{-20y_D + 490 - 15y_B}{\sqrt{(14 - y_B)^2 + 400}} T_{BC} = 6000 \qquad \textbf{(2)}$$

$$\frac{-20y_D + 490 - 15y_B}{\sqrt{(14 - y_D)^2 + 225}} T_{CD} = 8000 \qquad \textbf{(3)}$$

At D

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad \dfrac{12}{\sqrt{(4 + y_D)^2 + 144}} T_{DB} - \dfrac{15}{\sqrt{(14 - y_D)^2 + 225}} T_{CD} = 0$

$+\uparrow \Sigma F_y = 0;\qquad \dfrac{4 - y_D}{\sqrt{(4 + y_D)^2 + 144}} T_{DE} - \dfrac{14 - y_D}{\sqrt{(14 - y_D)^2 + 225}} T_{CD} - 250 = 0$

$$\frac{-108 + 27y_D}{\sqrt{(14 - y_D)^2 + 225}} T_{CD} = 3000 \qquad \textbf{(4)}$$

Combining Eqs. (1) & (2)

$$79y_B + 20y_D = 826$$

Combining Eqs. (3) & (4)

$$45y_B + 276y_D = 2334$$

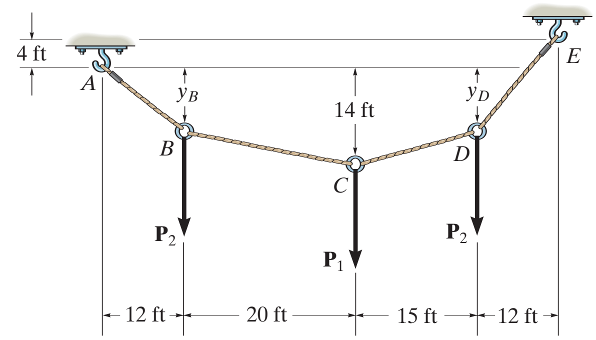
$$y_B = 8.67 \text{ ft} \qquad \textbf{Ans.}$$

$$y_D = 7.04 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$y_B = 8.67$ ft
$y_D = 7.04$ ft

***7–100.**

The cable supports the three loads shown. Determine the magnitude of $\mathbf{P}_1$ if $P_2 = 300$ lb and $y_B = 8$ ft. Also find the sag y_D.

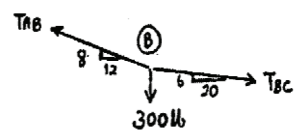


SOLUTION

At B

$$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{20}{\sqrt{436}} T_{BC} - \frac{12}{\sqrt{208}} T_{AB} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{-6}{\sqrt{436}} T_{BC} + \frac{8}{\sqrt{208}} T_{AB} - 300 = 0$$



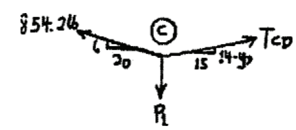
$$T_{AB} = 983.3 \text{ lb}$$

$$T_{BC} = 854.2 \text{ lb}$$

At C

$$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{-20}{\sqrt{436}}(854.2) + \frac{15}{\sqrt{(14 - y_D)^2 + 225}} T_{CD} = 0 \qquad \textbf{(1)}$$

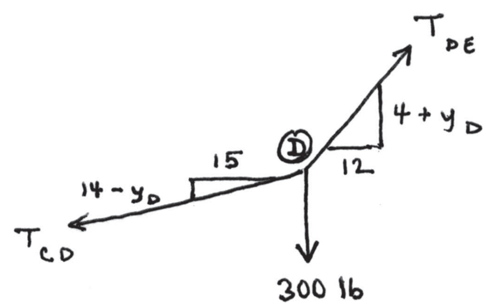
$$+\uparrow \Sigma F_y = 0; \qquad \frac{6}{\sqrt{436}}(854.2) + \frac{14 - y_D}{\sqrt{(14 - y_D)^2 + 225}} T_{CD} - P_1 = 0 \qquad \textbf{(2)}$$



At D

$$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{12}{\sqrt{(4 + y_D)^2 + 144}} T_{DE} - \frac{15}{\sqrt{(14 - y_D)^2 + 225}} T_{CD} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{4 + y_D}{\sqrt{(4 + y_D)^2 + 144}} T_{DE} - \frac{14 - y_D}{\sqrt{(14 - y_D)^2 + 225}} T_{CD} - 300 = 0 \qquad \textbf{(4)}$$



From Eq. 1,

$$\frac{T_{CD}}{\sqrt{(14 - y_D)^2 + 225}} = 54.545 \text{ lb}$$

and then from Eq. 3,

$$\frac{T_{DE}}{\sqrt{(4 + y_D)^2 + 144}} = 68.182 \text{ lb}$$

Now Eq. 2 simplifies to

$$P_1 + 54.545 y_D = 1009.09 \qquad \textbf{(5)}$$

and Eq. 4 simplifies to

$$68.182(4 + y_D) - 54.545(14 - y_D) = 300$$

$y_D = 6.444 = 6.44$ ft **Ans.**

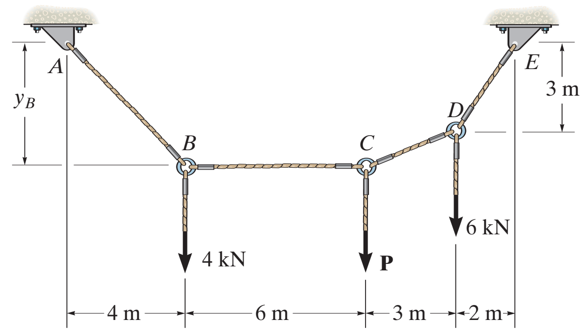
Substituting into Eq. 5,

$P_1 = 658$ lb **Ans.**

Ans:
$y_D = 6.44$ ft
$P_1 = 658$ lb

7–101.

Determine the force P needed to hold the cable in the position shown, i.e., so segment BC remains horizontal. Also, compute the sag y_B and the maximum tension in the cable.



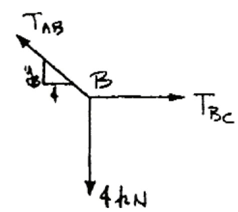
SOLUTION

Joint B:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad T_{BC} - \frac{4}{\sqrt{y_B^2 + 16}} T_{AB} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{y_B}{\sqrt{y_B^2 + 16}} T_{AB} - 4 = 0$$

$$y_B T_{BC} = 16 \qquad \textbf{(1)}$$

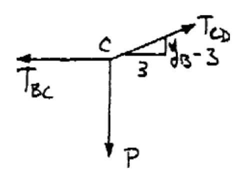


Joint C:

$$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} - T_{BC} = 0 \qquad \textbf{(2)}$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{y_B - 3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} - P = 0$$

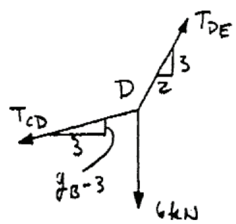
$$(y_B - 3)T_{BC} = 3P \qquad \textbf{(3)}$$



Combining Eqs. (1) and (2):

$$\frac{3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} = \frac{16}{y_B} \qquad \textbf{(4)}$$

Joint D:



$$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{2}{\sqrt{13}} T_{DE} - \frac{3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{3}{\sqrt{13}} T_{DE} - \frac{y_B - 3}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} - 6 = 0$$

$$\frac{15 - 2\,y_B}{\sqrt{(y_B - 3)^2 + 9}} T_{CD} = 12 \qquad \textbf{(5)}$$

From Eqs. (1) and (3): $\quad 3\,y_B P - 16\,y_B + 48 = 0$

From Eqs. (4) and (5): $\quad y_B = 3.53 \text{ m}$ **Ans.**

$P = 0.8 \text{ kN}$ **Ans.**

$T_{AB} = 6.05 \text{ kN}$

$T_{BC} = 4.53 \text{ kN}$

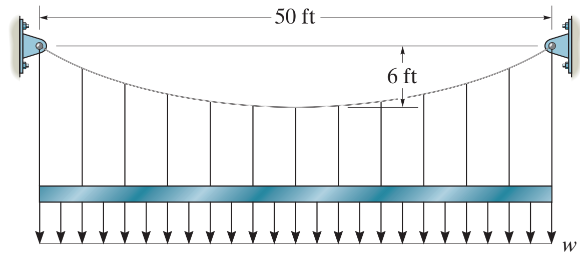
$T_{CD} = 4.60 \text{ kN}$

$T_{max} = T_{DE} = 8.17 \text{ kN}$ **Ans.**

Ans:
$y_B = 3.53 \text{ m}$
$P = 0.8 \text{ kN}$
$T_{\max} = T_{DE} = 8.17 \text{ kN}$

7–102.

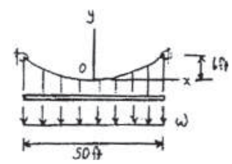
Determine the maximum uniform loading w, measured in lb/ft, that the cable can support if it is capable of sustaining a maximum tension of 3000 lb before it will break.



SOLUTION

$$y = \frac{1}{F_H}\int\left(\int w dx\right)dx$$

At $x = 0, \frac{dy}{dx} = 0$



At $x = 0, \ y = 0$

$$C_1 = C_2 = 0$$

$$y = \frac{w}{2F_H}x^2$$

At $x = 25$ ft, $y = 6$ ft $\qquad F_H = 52.08\, w$

$$\left.\frac{dy}{dx}\right|_{max} = \tan\theta_{max} = \left.\frac{w}{F_H}x\right|_{x=25 \text{ ft}}$$

$$\theta_{max} = \tan^{-1}(0.48) = 25.64^\circ$$

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = 3000$$

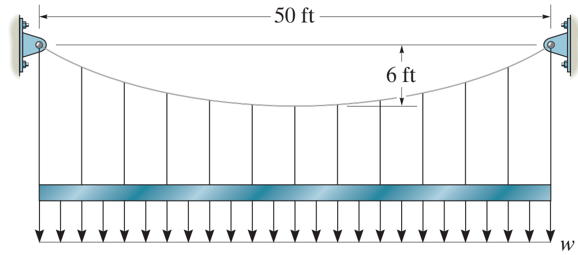
$$F_H = 2705 \text{ lb}$$

$$w = 51.9 \text{ lb/ft}$$ **Ans.**

Ans:
$w = 51.9$ lb/ft

7–103.

The cable is subjected to a uniform loading of $w = 250$ lb/ft. Determine the maximum and minimum tension in the cable.



SOLUTION

From Example 7–12:

$$F_H = \frac{w_0 L^2}{8h} = \frac{250\,(50)^2}{8\,(6)} = 13\,021 \text{ lb}$$

$$\theta_{max} = \tan^{-1}\left(\frac{w_0 L}{2F_H}\right) = \tan^{-1}\left(\frac{250\,(50)}{2(13\,021)}\right) = 25.64°$$

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{13\,021}{\cos 25.64°} = 14.4 \text{ kip}$$ **Ans.**

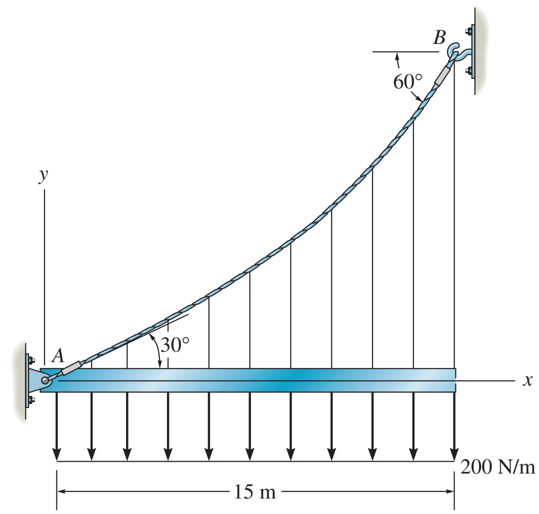
The minimum tension occurs at $\theta = 0°$.

$$T_{min} = F_H = 13.0 \text{ kip}$$ **Ans.**

Ans:
$T_{\text{max}} = 14.4$ kip
$T_{\text{min}} = 13.0$ kip

***7–104.**

The cable AB is subjected to a uniform loading of 200 N/m. If the weight of the cable is neglected and the slope angles at points A and B are 30° and 60°, respectively, determine the curve that defines the cable shape and the maximum tension developed in the cable.



SOLUTION

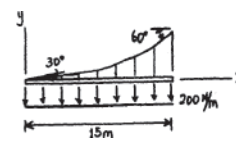
$$y = \frac{1}{F_H}\int\left(\int 200\,dx\right)dx$$

$$y = \frac{1}{F_H}(100x^2 + C_1x + C_2)$$

$$\frac{dy}{dx} = \frac{1}{F_H}(200x + C_1)$$

At $x = 0$, $\quad y = 0; \quad C_2 = 0$

At $x = 0$, $\quad \frac{dy}{dx} = \tan 30°; \quad C_1 = F_H \tan 30°$



$$y = \frac{1}{F_H}(100x^2 + F_H \tan 30° x)$$

At $x = 15$ m, $\quad \frac{dy}{dx} = \tan 60°; \quad F_H = 2598\text{ N}$

$$y = (38.5x^2 + 577x)(10^{-3})\text{ m} \qquad \textbf{Ans.}$$

$$\theta_{max} = 60°$$

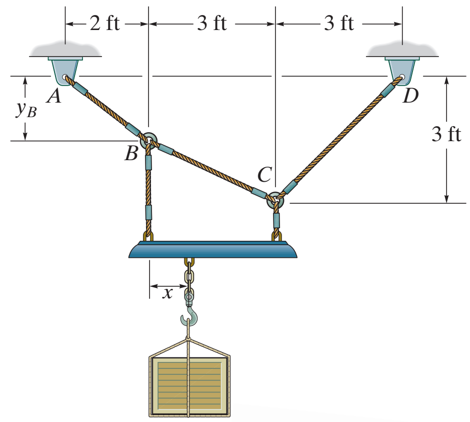
$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{2598}{\cos 60°} = 5196\text{ N}$$

$$T_{max} = 5.20\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$\theta_{max} = 60°$
$T_{max} = 5.20$ kN

7–105.

If $x = 2$ ft and the crate weighs 300 lb, which cable segment AB, BC, or CD has the greatest tension? What is this force and what is the sag y_B?



SOLUTION

The forces $\mathbf{F}_B$ and $\mathbf{F}_C$ exerted on joints B and C will be obtained by considering the equilibrium on the free-body diagram, Fig. *a*.

$+\Sigma M_E = 0;\qquad F_C(3) - 300(2) = 0\qquad F_C = 200 \text{ lb}$

$+\Sigma M_F = 0;\qquad 300(1) - F_B(3) = 0\qquad F_B = 200 \text{ lb}$

Referring to Fig. *b*, we have

$+\Sigma M_A = 0;\qquad T_{CD}\,\sin 45°(8) - 200(5) - 100(2) = 0$

$T_{CD} = 212.13 \text{ lb} = 212 \text{ lb (max)}$

Using these results and analyzing the equilibrium of joint C, Fig. *c*, we obtain

$\xrightarrow{+} \Sigma F_x = 0;\qquad 212.13\,\cos 45° - T_{BC}\cos\theta = 0$

$+\uparrow \Sigma F_y = 0;\qquad T_{BC}\sin\theta + 212.13\,\sin 45° - 200 = 0$

$T_{AB} = T_{CD} = 212 \text{ lb (max)}$ **Ans.**

Solving,

$T_{BC} = 158.11 \text{ lb}\qquad \theta = 18.43°$

Using these results to analyze the equilibrium of joint B, Fig. *d*, we have

$\xrightarrow{+} \Sigma F_x = 0;\qquad 158.11\cos 18.43° - T_{AB}\cos\phi = 0$

$+\uparrow \Sigma F_y = 0;\qquad T_{AB}\sin\phi - 100 - 158.11\sin 18.43° = 0$

Solving,

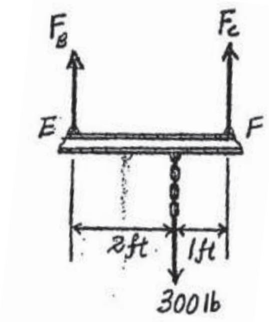
$\phi = 45°$

$T_{AB} = 212.13 \text{ lb} = 212 \text{ lb (max)}$

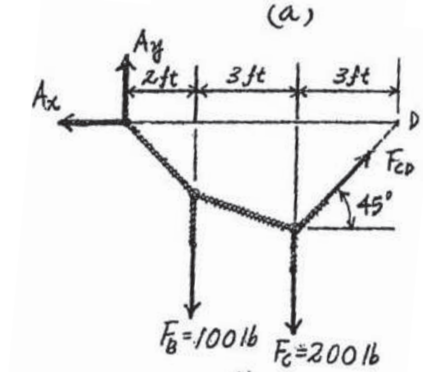
Thus, both cables AB and CD are subjected to maximum tension. The sag y_B is given by

$$\frac{y_B}{2} = \tan\phi = \tan 45°$$

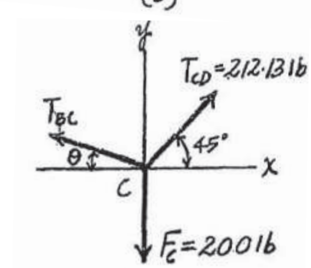
$y_B = 2 \text{ ft}$ **Ans.**



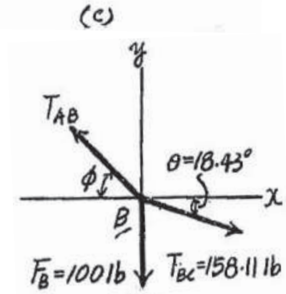
(a)



(b)



(c)

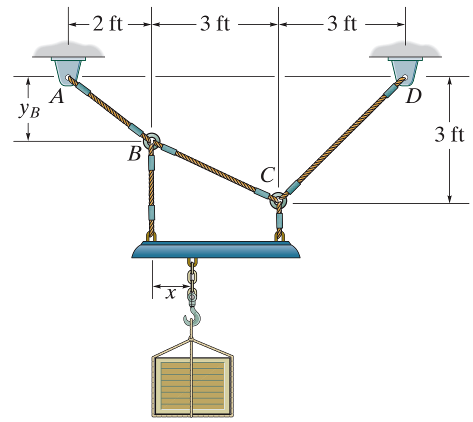


(d)

Ans:
$T_{AB} = T_{CD} = 212$ lb (max), $y_B = 2$ ft

7–106.

If $y_B = 1.5$ ft, determine the largest weight of the crate and its placement x so that neither cable segment AB, BC, or CD is subjected to a tension that exceeds 200 lb.

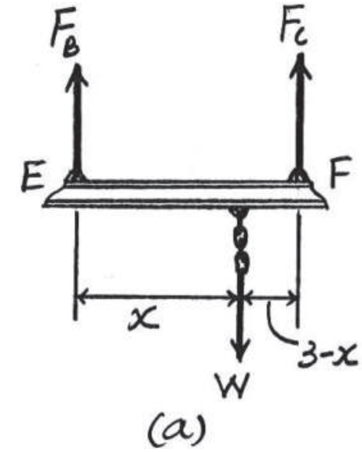


SOLUTION

The forces $\mathbf{F}_B$ and $\mathbf{F}_C$ exerted on joints B and C will be obtained by considering the equilibrium on the free-body diagram, Fig. *a*.

$$\zeta + \Sigma M_E = 0; \qquad F_C(3) - w(x) = 0 \qquad F_C = \frac{wx}{3}$$

$$\zeta + \Sigma M_F = 0; \qquad w(3 - x) - F_B(3) = 0 \qquad F_B = \frac{w}{3}(3 - x)$$



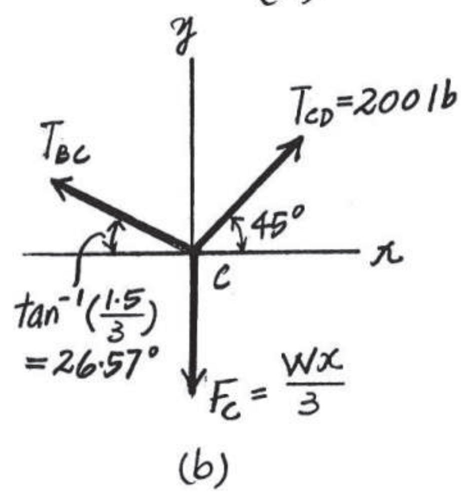
Since the horizontal component of tensile force developed in each cable is constant, cable CD, which has the greatest angle with the horizontal, will be subjected to the greatest tension. Thus, we will set $T_{CD} = 200$ lb.

First, we will analyze the equilibrium of joint C, Fig. *b*.

$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 200 \cos 45° - T_{BC} \cos 26.57° = 0 \qquad T_{BC} = 158.11 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad 200 \sin 45° + 158.11 \sin 26.57° - \frac{wx}{3} = 0$$

$$\frac{wx}{3} = 212.13 \qquad \textbf{(1)}$$



Using the result of T_{BC} to analyze the equilibrium of joint B, Fig. *c*, we have

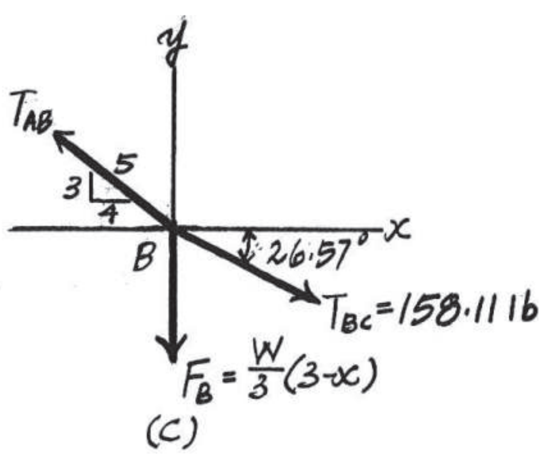
$$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 158.11 \cos 26.57° - T_{AB}\left(\frac{4}{5}\right) = 0 \qquad T_{AB} = 176.78 \text{ lb}$$

$$+\uparrow \Sigma F_y = 0; \qquad 176.78\left(\frac{3}{5}\right) - 158.11 \sin 26.57° - \frac{w}{3}(3 - x) = 0$$

$$\frac{w}{3}(3 - x) = 35.36 \qquad \textbf{(2)}$$

Solving Eqs. (1) and (2)

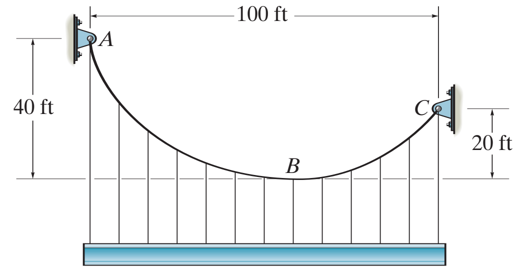
$$x = 2.57 \text{ ft} \qquad W = 247 \text{ lb} \qquad \textbf{Ans.}$$



Ans:
$x = 2.57$ ft
$W = 247$ lb

7–107.

The cable supports a girder which weighs 850 lb/ft. Determine the tension in the cable at points *A*, *B*, and *C*.

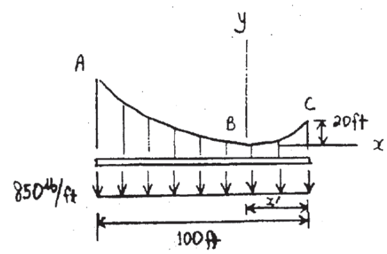


SOLUTION

$$y = \frac{1}{F_H}\int\left(\int w_0\,dx\right)dx$$

$$y = \frac{1}{F_H}(425x^2 + C_1x + C_2)$$

$$\frac{dy}{dx} = \frac{850}{F_H}x + \frac{C_1}{F_H}$$



At $x = 0,\ \dfrac{dy}{dx} = 0\ \ C_1 = 0$

At $x = 0,\ y = 0\quad C_2 = 0$

$$y = \frac{425}{F_H}x^2$$

At $y = 20\text{ ft},\ x = x'$

$$20 = \frac{425(x')^2}{F_H}$$

At $y = 40\text{ ft},\ x = (100-x')$

$$40 = \frac{425(100-x')^2}{F_H}$$

$$2(x')^2 = (x')^2 - 200x' + 100^2$$

$$(x')^2 + 200x' - 100^2 = 0$$

$$x' = \frac{-200 \mp \sqrt{200^2 + 4(100)^2}}{2} = 41.42\text{ ft}$$

$$F_H = 36\,459\text{ lb}$$

At *A*,

$$\frac{dy}{dx} = \tan\theta_A = \left.\frac{2(425)x}{F_H}\right|_{x=-58.58\text{ ft}} = 1.366$$

$$\theta_A = 53.79^\circ$$

$$T_A = \frac{F_H}{\cos\theta_A} = \frac{36\,459}{\cos 53.79^\circ} = 61\,714\text{ lb}$$

$$T_A = 61.7\text{ kip}\qquad\textbf{Ans.}$$

7–107. Continued

At B,

$$T_B = F_H = 36.5 \text{ kip}$$ **Ans.**

At C,

$$\frac{dy}{dx} = \tan\theta_C = \left.\frac{2(425)x}{F_H}\right|_{x=41.42 \text{ ft}} = 0.9657$$

$$\theta_C = 44.0°$$

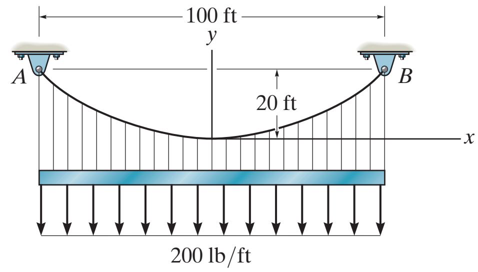
$$T_C = \frac{F_H}{\cos\theta_C} = \frac{36\,459}{\cos 44.0°} = 50\,683 \text{ lb}$$

$$T_C = 50.7 \text{ kip}$$ **Ans.**

Ans:
$T_A = 61.7$ kip
$T_B = 36.5$ kip
$T_C = 50.7$ kip

***7–108.**

The cable is subjected to a uniform loading of $w = 200$ lb/ft. Determine the maximum and minimum tension in the cable.



SOLUTION

The Equation of The Cable.

$$y = \frac{1}{F_H}\int\left(\int w(x)dx\right)dx$$

$$y = \frac{1}{F_H}\left(\frac{w_o}{2}x^2 + C_1x + C_2\right) \qquad \textbf{(1)}$$

$$\frac{dy}{dx} = \frac{1}{F_H}(w_ox + C_1) \qquad \textbf{(2)}$$

Boundary Conditions. At $x = 0$, $y = 0$. Then Eq (1) gives

$$0 = \frac{1}{F_H}(0 + 0 + C_2) \qquad C_2 = 0$$

At $x = 0, \dfrac{dy}{dx} = 0$. Then Eq (2) gives

$$0 = \frac{1}{F_H}(0 + C_1) \qquad C_1 = 0$$

Thus, the equation of the cable becomes

$$y = \frac{w_o}{2F_H}x^2 \qquad \textbf{(3)}$$

and the slope of the cable is

$$\frac{dy}{dx} = \frac{w_o}{F_H}x \qquad \textbf{(4)}$$

Here, $w_o = 200$ lb/ft. Also, at $x = 50$ ft, $y = 20$ ft. Then using Eq (3),

$$20 = \frac{200}{2F_H}(50^2) \qquad F_H = 12{,}500 \text{ lb} = 12.5 \text{ kip}$$

Thus,

$$T_{\min} = F_H = 12.5 \text{ kip} \qquad \textbf{Ans.}$$

$\theta_{\max}$ occurs at $x = 50$ ft. Using Eq. 4

$$\tan\theta_{\max} = \left.\frac{dy}{dx}\right|_{x = 50\text{ ft}} = \left(\frac{200}{12{,}500}\right)(50) \qquad \theta_{\max} = 38.66^\circ$$

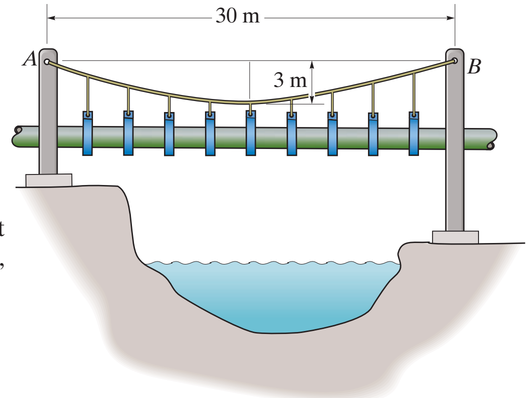
Thus,

$$T_{\max} = \frac{F_H}{\cos\theta_{\max}} = \frac{12.5}{\cos 38.66^\circ} = 16.0 \text{ kip} \qquad \textbf{Ans.}$$

Ans:
$T_{\min} = 12.5$ kip
$T_{\max} = 16.0$ kip

7–109.

If the pipe has a mass per unit length of 1500 kg/m, determine the maximum tension developed in the cable.



SOLUTION

As shown in Fig. *a*, the origin of the x, y coordinate system is set at the lowest point of the cable. Here, $w(x) = w_0 = 1500(9.81) = 14.715(10^3)$ N/m. Using Eq. 7–12, we can write

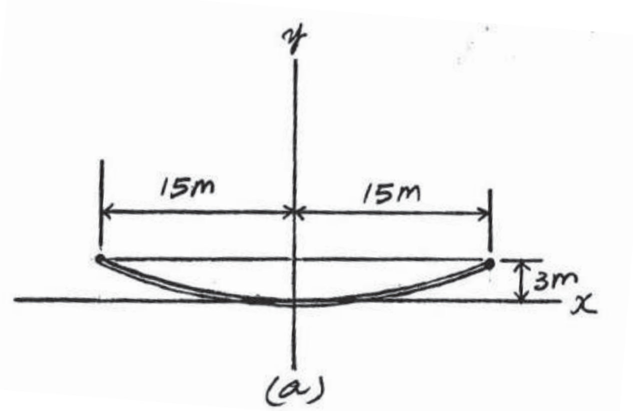
$$y = \frac{1}{F_H}\int\left(\int w_0 dx\right)dx$$

$$= \frac{1}{F_H}\left(\frac{14.715(10^3)}{2}x^2 + c_1 x + c_2\right)$$

Applying the boundary condition $\dfrac{dy}{dx} = 0$ at $x = 0$, results in $c_1 = 0$.

Applying the boundary condition $y = 0$ at $x = 0$ results in $c_2 = 0$. Thus,

$$y = \frac{7.3575(10^3)}{F_H}x^2$$



Applying the boundary condition $y = 3$ m at $x = 15$ m, we have

$$3 = \frac{7.3575(10^3)}{F_H}(15)^2 \qquad F_H = 551.81(10^3)\text{ N}$$

Substituting this result into Eq. (1), we have

$$\frac{dy}{dx} = 0.02667x$$

The maximum tension occurs at either points at A or B where the cable has the greatest angle with the horizontal. Here,

$$\theta_{\max} = \tan^{-1}\left(\left.\frac{dy}{dx}\right|_{15\text{ m}}\right) = \tan^{-1}[0.02667(15)] = 21.80^\circ$$

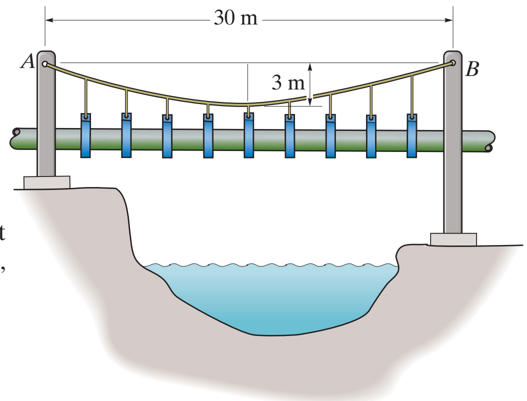
Thus,

$$T_{\max} = \frac{F_H}{\cos\theta_{\max}} = \frac{551.8(10^3)}{\cos 21.80^\circ} = 594.32(10^3)\text{ N} = 594\text{ kN}$$

Ans:
$T_{\max} = 594$ kN

7–110.

If the pipe has a mass per unit length of 1500 kg/m, determine the minimum tension developed in the cable.



SOLUTION

As shown in Fig. *a*, the origin of the x, y coordinate system is set at the lowest point of the cable. Here, $w(x) = w_0 = 1500(9.81) = 14.715(10^3)$ N/m. Using Eq. 7–12, we can write

$$y = \frac{1}{F_H}\int\left(\int w_0 dx\right)dx$$

$$= \frac{1}{F_H}\left(\frac{14.715(10^3)}{2}x^2 + c_1x + c_2\right)$$

Applying the boundary condition $\frac{dy}{dx} = 0$ at $x = 0$, results in $c_1 = 0$.
Applying the boundary condition $y = 0$ at $x = 0$ results in $c_2 = 0$. Thus,

$$y = \frac{7.3575(10^3)}{F_H}x^2$$

Applying the boundary condition $y = 3$ m at $x = 15$ m, we have

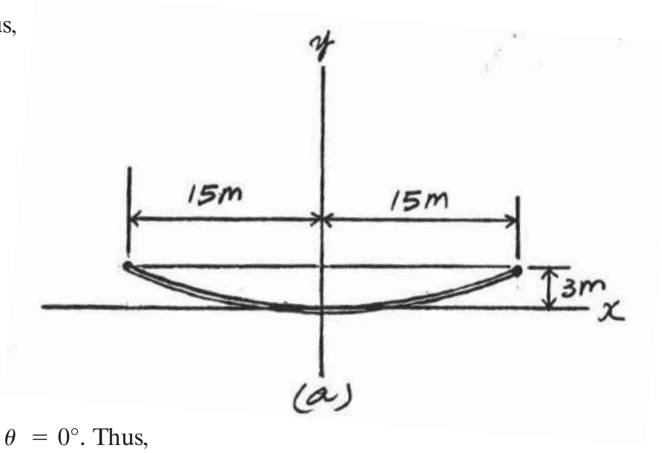
$$3 = \frac{7.3575(10^3)}{F_H}(15)^2 \qquad F_H = 551.81(10^3)\text{ N}$$

Substituting this result into Eq. (1), we have

$$\frac{dy}{dx} = 0.02667x$$

The minimum tension occurs at the lowest point of the cable, where $\theta = 0°$. Thus,

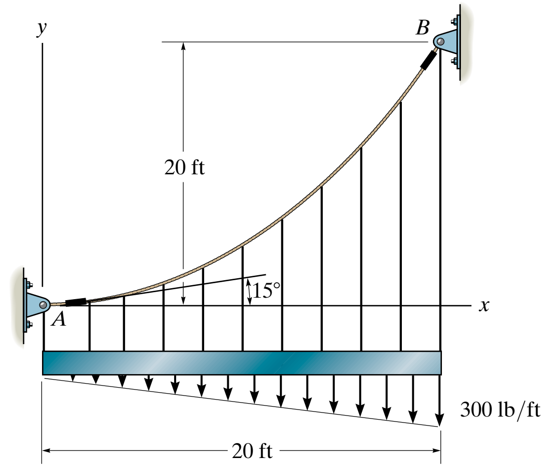
$$T_{\min} = F_H = 551.81(10^3)\text{ N} = 552\text{ kN}$$



Ans:
$T_{\min} = 552$ kN

7–111.

Determine the maximum tension developed in the cable if it is subjected to the triangular distributed load.



SOLUTION

The Equation of The Cable. Here, $w(x) = 15x$.

$$y = \frac{1}{F_H}\int\left(\int w(x)dx\right)dx$$

$$y = \frac{1}{F_H}\int\left(\int 15x\,dx\right)dx$$

$$y = \frac{1}{F_H}\int\left(\frac{15}{2}x^2 + C_1\right)dx$$

$$y = \frac{1}{F_H}\int\left(\frac{5}{2}x^3 + C_1x + C_2\right) \qquad \textbf{(1)}$$

$$\frac{dy}{dx} = \frac{1}{F_H}\left(\frac{15}{2}x^2 + C_1\right) \qquad \textbf{(2)}$$

Boundary Conditions. At $x = 0$, $y = 0$. Then Eq (1) gives

$$0 = \frac{1}{F_H}(0 + 0 + C_2) \qquad C_2 = 0$$

Also, at $x = 0, \dfrac{dy}{dx} = \tan 15°$. Then Eq (2) gives

$$\tan 15° = \frac{1}{F_H}(0 + C_1) \qquad C_1 = F_H \tan 15°$$

Thus, the equation of the cable becomes

$$y = \frac{1}{F_H}\left(\frac{5}{2}x^3 + F_H \tan 15° x\right)$$

$$y = \frac{5}{2F_H}x^3 + \tan 15°\,x \qquad \textbf{(3)}$$

And the slope of the cable is

$$\frac{dy}{dx} = \frac{15}{2F_H}x^2 + \tan 15° \qquad \textbf{(4)}$$

Also, at $x = 20$ ft, $y = 20$ ft. Then using Eq. 3,

$$20 = \left(\frac{5}{2F_H}\right)(20^3) + \tan 15°(20)$$

$$F_H = 1366.03 \text{ lb}$$

θ_{max} occurs at $x = 20$ ft. Then Eq (4) gives

$$\tan\theta_{max} = \left.\frac{dy}{dx}\right|_{x = 20\text{ ft}} = \left[\frac{15}{2(1366.02)}\right](20^2) + \tan 15° \qquad \theta_{max} = 67.91°$$

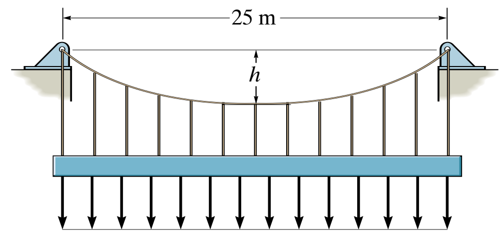
Thus

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{1366.02}{\cos 67.91°} = 3632.65 \text{ lb} = 3.63 \text{ kip} \qquad \textbf{Ans.}$$

Ans:
$T_{max} = 3.63$ kip

***7–112.**

The cable will break when the maximum tension reaches $T_{\max} = 10$ kN. Determine the minimum sag h if it supports the uniform distributed load of $w = 600$ N/m.



SOLUTION

The Equation of The Cable:

$$y = \frac{1}{F_H}\int\left(\int w(x)dx\right)dx$$

$$= \frac{1}{F_H}\left(\frac{w_0}{2}x^2 + C_1x + C_2\right) \qquad \textbf{[1]}$$

$$\frac{dy}{dx} = \frac{1}{F_H}(w_0x + C_1) \qquad \textbf{[2]}$$

Boundary Conditions:

$y = 0$ at $x = 0$, then from Eq.[1] $0 = \frac{1}{F_H}(C_2) \qquad C_2 = 0$

$\frac{dy}{dx} = 0$ at $x = 0$, then from Eq.[2] $0 = \frac{1}{F_H}(C_1) \qquad C_1 = 0$

Thus,

$$y = \frac{w_0}{2F_H}x^2 \qquad \textbf{[3]}$$

$$\frac{dy}{dx} = \frac{w_0}{F_H}x \qquad \textbf{[4]}$$

$y = h$, at $x = 12.5$m, then from Eq.[3] $h = \frac{w_0}{2F_H}(12.5^2) \qquad F_H = \frac{78.125}{h}w_0$

$\theta = \theta_{max}$ at $x = 12.5$ m and the maximum tension occurs when $\theta = \theta_{max}$. From Eq.[4] $\tan\theta_{max} = \left.\frac{dy}{dx}\right|_{x-12.5\text{m}} = \frac{w_0}{\frac{18.125}{h}w_0}x = 0.0128h(12.5) = 0.160h$

Thus, $\cos\theta_{max} = \frac{1}{\sqrt{0.0256h^2 + 1}}$

The maximum tension in the cable is

$$T_{max} = \frac{F_H}{\cos\theta_{max}}$$

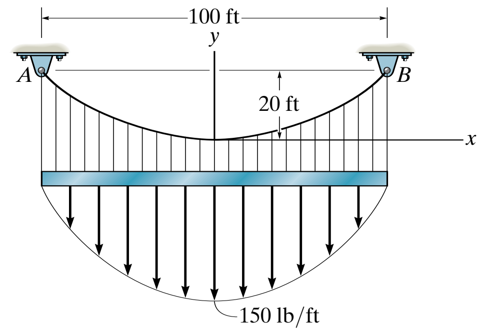
$$10 = \frac{\frac{18.125}{h}(0.6)}{\frac{1}{\sqrt{0.0256h^2 + 1}}}$$

$$h = 7.09 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$h = 7.09$ m

7–113.

The cable is subjected to the parabolic loading $w = 150(1 - (x/50)^2)$ lb/ft, where x is in ft. Determine the equation $y = f(x)$ which defines the cable shape AB and the maximum tension in the cable.



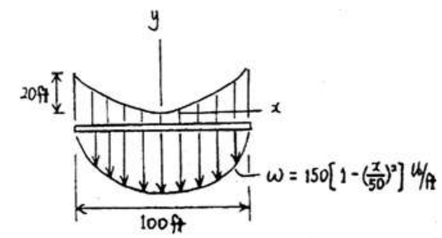
SOLUTION

$$y = \frac{1}{F_H}\int\left(\int w(x)dx\right)dx$$

$$y = \frac{1}{F_H}\int [150(x - \frac{x^3}{3(50)^2}) + C_1]dx$$

$$y = \frac{1}{F_H}(75x^2 - \frac{x^4}{200} + C_1x + C_2)$$

$$\frac{dy}{dx} = \frac{150x}{F_H} - \frac{1}{50F_H}x^3 + \frac{C_1}{F_H}$$



At $x = 0, \dfrac{dy}{dx} = 0 \quad C_1 = 0$

At $x = 0, \ y = 0 \quad C_2 = 0$

$$y = \frac{1}{F_H}\left(75x^2 - \frac{x^2}{200}\right)$$

At $x = 50$ ft, $y = 20$ ft $\qquad F_H = 7813$ lb

$$y = \frac{x^2}{7813}\left(75 - \frac{x^2}{200}\right) \text{ ft}$$ **Ans.**

$$\frac{dy}{dx} = \frac{1}{7813}\left(150x - \frac{4x^3}{200}\right)\bigg|_{x = 50\text{ ft}} = \tan\theta_{max}$$

$$\theta_{max} = 32.62°$$

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{7813}{\cos 32.62°} = 9275.9 \text{ lb}$$

$$T_{max} = 9.28 \text{ kip}$$ **Ans.**

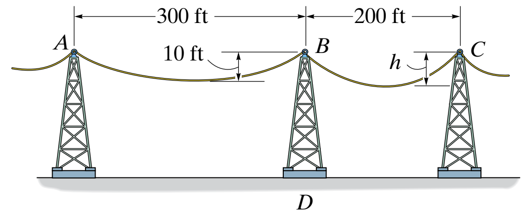
Ans:

$y = \dfrac{x^2}{7813}\left(75 - \dfrac{x^2}{200}\right)$

$T_{\max} = 9.28$ kip

7–114.

The power transmission cable weighs 10 lb/ft. If the resultant horizontal force on tower BD is required to be zero, determine the sag h of cable BC.



SOLUTION

The origin of the x, y coordinate system is set at the lowest point of the cables. Here, $w_0 = 10$ lb/ft. Using Eq. 4 of Example 7–13,

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

$$y = \frac{F_H}{10}\left[\cosh\left(\frac{10}{F_H}x\right) - 1\right] \text{ ft}$$

Applying the boundary condition of cable AB, $y = 10$ ft at $x = 150$ ft,

$$10 = \frac{(F_H)_{AB}}{10}\left[\cosh\left(\frac{10(150)}{(F_H)_{AB}}\right) - 1\right]$$

Solving by trial and error yields

$$(F_H)_{AB} = 11266.63 \text{ lb}$$

Since the resultant horizontal force at B is required to be zero, $(F_H)_{BC} = (F_H)_{AB} = 11266.62$ lb. Applying the boundary condition of cable BC $y = h$ at $x = -100$ ft to Eq. (1), we obtain

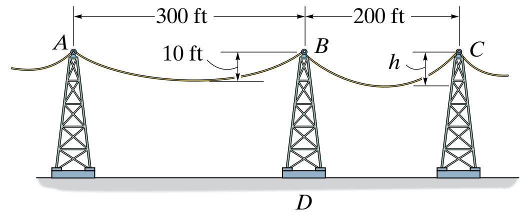
$$h = \frac{11266.62}{10}\left\{\cosh\left[\frac{10(-100)}{11266.62}\right] - 1\right\}$$

$$= 4.44 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$h = 4.44$ ft

7–115.

The power transmission cable weighs 10 lb/ft. If $h = 10$ ft, determine the resultant horizontal and vertical forces the cables exert on tower BD.



SOLUTION

The origin of the x, y coordinate system is set at the lowest point of the cables. Here, $w_0 = 10$ lb/ft. Using Eq. 4 of Example 7–13,

$$y = \frac{F_H}{w0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

$$y = \frac{F_H}{10}\left[\cosh\left(\frac{10}{F_H}x\right) - 1\right]\text{ft}$$

Applying the boundary condition of cable AB, $y = 10$ ft at $x = 150$ ft,

$$10 = \frac{(F_H)_{AB}}{10}\left[\cosh\left(\frac{10(150)}{(F_H)_{AB}}\right) - 1\right]$$

Solving by trial and error yields

$$(F_H)_{AB} = 11266.63 \text{ lb}$$

Applying the boundary condition of cable BC, $y = 10$ ft at $x = -100$ ft to Eq. (2), we have

$$10 = \frac{(F_H)_{BC}}{10}\left[\cosh\left(\frac{10(100)}{(F_H)_{BC}}\right) - 1\right]$$

Solving by trial and error yields

$$(F_H)_{BC} = 5016.58 \text{ lb}$$

Thus, the resultant horizontal force at B is

$(F_H)_R = (F_H)_{AB} - (F_H)_{BC} = 11266.63 - 5016.58 = 6250 \text{ lb} = 6.25 \text{ kip}$ **Ans.**

Using Eq. (1), $\tan(\theta_B)_{AB} = \sin\text{h}\left[\frac{10(150)}{11266.63}\right] = 0.13353$ and $\tan(\theta_B)_{BC} = \sin\text{h}\left[\frac{10(-100)}{5016.58}\right] = 0.20066$. Thus, the vertical force of cables AB and BC acting on point B are

$$(F_v)_{AB} = (F_H)_{AB}\tan(\theta_B)_{AB} = 11266.63(0.13353) = 1504.44 \text{ lb}$$

$$(F_v)_{BC} = (F_H)_{BC}\tan(\theta_B)_{BC} = 5016.58(0.20066) = 1006.64 \text{ lb}$$

The resultant vertical force at B is therefore

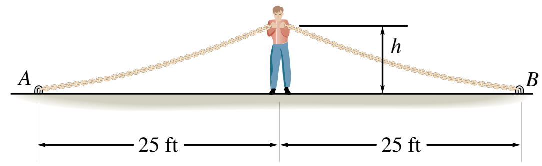
$$(F_v)_R = (F_v)_{AB} + (F_v)_{BC} = 1504.44 + 1006.64$$

$= 2511.07 \text{ lb} = 2.51 \text{ kip}$ **Ans.**

Ans:
$(F_h)_R = 6.25$ kip
$(F_v)_R = 2.51$ kip

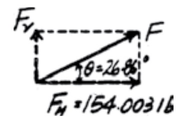
***7–116.**

The man picks up the 52-ft chain and holds it just high enough so it is completely off the ground. The chain has points of attachment A and B that are 50 ft apart. If the chain has a weight of 3 lb/ft, and the man weighs 150 lb, determine the force he exerts on the ground. Also, how high h must he lift the chain? *Hint*: The slopes at A and B are zero.



SOLUTION

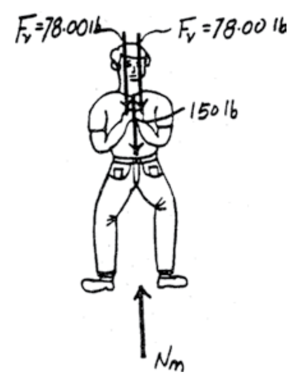
Deflection Curve of The Cable:



$$x = \int \frac{ds}{\left[1 + (1/F_H^2)\left(\int w_0\, ds\right)^2\right]^{\frac{1}{2}}} \qquad \text{where } w_0 = 3 \text{ lb/ft}$$

Performing the integration yields

$$x = \frac{F_H}{3}\left\{\sinh^{-1}\left[\frac{1}{F_H}(3s + C_1)\right] + C_2\right\} \tag{1}$$



From Eq. 7–14

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_0\, ds = \frac{1}{F_H}(3s + C_1) \tag{2}$$

Boundary Conditions:

$\dfrac{dy}{dx} = 0$ at $s = 0$. From Eq. (2) $\qquad 0 = \dfrac{1}{F_H}(0 + C_1) \qquad C_1 = 0$

Then, Eq. (2) becomes

$$\frac{dy}{dx} = \tan\theta = \frac{3s}{F_H} \tag{3}$$

$s = 0$ at $x = 0$ and use the result $C_1 = 0$. From Eq. (1)

$$x = \frac{F_H}{3}\left\{\sinh^{-1}\left[\frac{1}{F_H}(0 + 0)\right] + C_2\right\} \qquad C_2 = 0$$

Rearranging Eq. (1), we have

$$s = \frac{F_H}{3}\sinh\left(\frac{3}{F_H}x\right) \tag{4}$$

Substituting Eq. (4) into (3) yields

$$\frac{dy}{dx} = \sinh\left(\frac{3}{F_H}x\right)$$

Performing the integration

$$y = \frac{F_H}{3}\cosh\left(\frac{3}{F_H}x\right) + C_3 \tag{5}$$

$y = 0$ at $x = 0$. From Eq. (5) $0 = \dfrac{F_H}{3}\cosh 0 + C_3$, thus, $C_3 = -\dfrac{F_H}{3}$

***7–116. Continued**

Then, Eq. (5) becomes

$$y = \frac{F_H}{3}\left[\cosh\left(\frac{3}{F_H}x\right) - 1\right] \qquad \textbf{(6)}$$

$s = 26$ ft at $x = 25$ ft. From Eq. (4)

$$26 = \frac{F_H}{3}\sinh\left[\frac{3}{F_H}(25)\right]$$

$$F_H = 154.003 \text{ lb}$$

By trial and error

$y = h$ at $x = 25$ ft. From Eq. (6)

$$h = \frac{154.003}{3}\left\{\cosh\left[\frac{3}{154.003}(25)\right] - 1\right\} = 6.21 \text{ ft} \qquad \textbf{Ans.}$$

From Eq. (3)

$$\left.\frac{dy}{dx}\right|_{s=26\text{ ft}} = \tan\theta = \frac{3(26)}{154.003} = 0.5065 \qquad \theta = 26.86^\circ$$

The vertical force F_V that each chain exerts on the man is

$$F_V = F_H \tan\theta = 154.003 \tan 26.86^\circ = 78.00 \text{ lb}$$

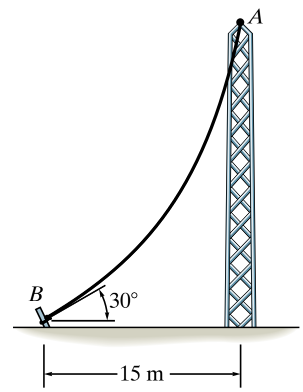
Equation of Equilibrium: By considering the equilibrium of the man,

$$+\uparrow \Sigma F_y = 0; \quad N_m - 150 - 2(78.00) = 0 \qquad N_m = 306 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$h = 6.21$ ft
$N_m = 306$ lb

7–117.

The cable has a mass of 0.5 kg/m, and is 25 m long. Determine the vertical and horizontal components of force it exerts on the top of the tower.

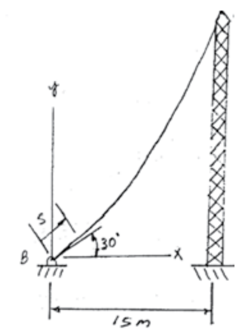


SOLUTION

$$x = \int \frac{ds}{\left\{1 + \frac{1}{F_H^2}(w_0\, ds)^2\right\}^{\frac{1}{2}}}$$

Performing the integration yields:

$$x = \frac{F_H}{4.905}\left\{\sinh^{-1}\left[\frac{1}{F_H}(4.905s + C_1)\right] + C_2\right\} \qquad \textbf{(1)}$$



rom Eq. 7-13

$$\frac{dy}{dx} = \frac{1}{F_H}\int w_0\, ds$$

$$\frac{dy}{dx} = \frac{1}{F_H}(4.905s + C_1)$$

At $s = 0$; $\frac{dy}{dx} = \tan 30°$. Hence $C_1 = F_H \tan 30°$

$$\frac{dy}{dx} = \frac{4.905s}{F_H} + \tan 30° \qquad \textbf{(2)}$$

Applying boundary conditions at $x = 0$; $s = 0$ to Eq.(1) and using the result $C_1 = F_H \tan 30°$ yields $C_2 = \; -\sinh^{-1}(\tan 30°)$. Hence

$$x = \frac{F_H}{4.905}\left\{\sinh^{-1}\left[\frac{1}{F_H}(4.905s + F_H \tan 30°)\right] - \sinh^{-1}(\tan 30°)\right\} \qquad \textbf{(3)}$$

At $x = 15$ m; $s = 25$ m. From Eq.(3)

$$15 = \frac{F_H}{4.905}\left\{\sinh^{-1}\left[\frac{1}{F_H}(4.905(25) + \mathrm{F_H} \tan 30°)\right] - \sinh^{-1}(\tan 30°)\right\}$$

By trial and error $F_H = 73.94$ N

At point A, $s = 25$ m From Eq.(2)

$$\tan \theta_A = \left.\frac{dy}{dx}\right|_{s=25\text{ m}} = \frac{4.905(25)}{73.94} + \tan 30° \qquad \theta_A = 65.90°$$

$$(F_v)_A = F_H \tan \theta_{\mathrm{A}} = 73.94 \tan 65.90° = 165 \text{ N} \qquad \textbf{Ans.}$$

$$(F_H)_A = F_H = 73.9 \text{ N} \qquad \textbf{Ans.}$$

Ans:
$(F_v)_A = 165$ N
$(F_h)_A = 73.9$ N

7–118.

A 50-ft cable is suspended between two points a distance of 15 ft apart and at the same elevation. If the minimum tension in the cable is 200 lb, determine the total weight of the cable and the maximum tension developed in the cable.

SOLUTION

$T_{min} = F_H = 200 \text{ lb}$

From Example 7–13:

$$s = \frac{F_H}{w_0}\sinh\left(\frac{w_0\,x}{F_H}\right)$$

$$\frac{50}{2} = \frac{200}{w_0}\sinh\left(\frac{w_0}{200}\left(\frac{15}{2}\right)\right)$$

Solving,

$$w_0 = 79.9 \text{ lb/ft}$$

$$\text{Total weight} = w_0\,l = 79.9\,(50) = 4.00 \text{ kip} \qquad \textbf{Ans.}$$

$$\left.\frac{dy}{dx}\right|_{max} = \tan\theta_{max} = \frac{w_0\,s}{F_H}$$

$$\theta_{max} = \tan^{-1}\left[\frac{79.9\,(25)}{200}\right] = 84.3°$$

Then,

$$T_{max} = \frac{F_H}{\cos\theta_{max}} = \frac{200}{\cos 84.3°} = 2.01 \text{ kip} \qquad \textbf{Ans.}$$

Ans:
$W = 4.00 \text{ kip}$
$T_{\max} = 2.01 \text{ kip}$

7–119.

Show that the deflection curve of the cable discussed in Example 7–13 reduces to Eq. 4 in Example 7–12 when the *hyperbolic cosine function* is expanded in terms of a series and only the first two terms are retained. (The answer indicates that the *catenary* may be replaced by a parabola in the analysis of problems in which the sag is small. In this case, the cable weight is assumed to be uniformly distributed along the horizontal.)

SOLUTION

$$\cosh x = 1 + \frac{x^2}{21} + \cdots$$

Substituting into

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

$$= \frac{F_H}{w_0}\left[1 + \frac{w_0^2 x^2}{2F_H^2} + \cdots - 1\right]$$

$$= \frac{w_0 x^2}{2F_H}$$

Using Eq. (3) in Example 7–12,

$$F_H = \frac{w_0 L^2}{8h}$$

We get $y = \dfrac{4h}{L^2}x^2$ **QED**

***7–120.**

A telephone line (cable) stretches between two points which are 150 ft apart and at the same elevation. The line sags 5 ft and the cable has a weight of 0.3 lb/ft. Determine the length of the cable and the maximum tension in the cable.

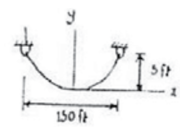
SOLUTION

$$w = 0.3 \text{ lb/ft}$$

From Example 7–13,

$$s = \frac{F_H}{w}\sinh\left(\frac{w}{F_H}x\right)$$

$$y = \frac{F_H}{w}\left[\cosh\left(\frac{w}{F_H}x\right) - 1\right]$$



At $x = 75$ ft, $y = 5$ ft, $w = 0.3$ lb/ft

$$5 = \frac{F_H}{w}\left[\cosh\left(\frac{75w}{F_H}\right) - 1\right]$$

$$F_H = 169.0 \text{ lb}$$

$$\left.\frac{dy}{dx}\right|_{\max} = \tan\theta_{\max} = \left.\sinh\left(\frac{w}{F_H}x\right)\right|_{x=75\text{ ft}}$$

$$\theta_{\max} = \tan^{-1}\left[\sinh\left(\frac{75(0.3)}{169}\right)\right] = 7.606°$$

$$T_{\max} = \frac{F_H}{\cos\theta_{\max}} = \frac{169}{\cos 7.606°} = 170 \text{ lb}$$ **Ans.**

$$s = \frac{169.0}{0.3}\sinh\left[\frac{0.3}{169.0}(75)\right] = 75.22$$

$$L = 2s = 150 \text{ ft}$$ **Ans.**

Ans:
$T_{\max} = 170$ lb
$L = 150$ ft

7–121.

A cable has a weight of 2 lb/ft. If it can span 100 ft and has a sag of 12 ft, determine the length of the cable. The ends of the cable are supported from the same elevation.

SOLUTION

From Eq. (5) of Example 7–13:

$$h = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0 L}{2F_H}\right) - 1\right]$$

$$12 = \frac{F_H}{2}\left[\cosh\left(\frac{2(100)}{2F_H}\right) - 1\right]$$

$$24 = F_H\left[\cosh\left(\frac{100}{F_H}\right) - 1\right]$$

$$F_H = 212.2 \text{ lb}$$

From Eq. (3) of Example 7–13:

$$s = \frac{F_H}{w_0}\sinh\left(\frac{w_0}{F_H}x\right)$$

$$\frac{l}{2} = \frac{212.2}{2}\sinh\left(\frac{2(50)}{212.2}\right)$$

$$l = 104 \text{ ft}$$ **Ans.**

Ans:
$l = 104$ ft

7–122.

A cable has a weight of 3 lb/ft and is supported at points that are 500 ft apart and at the same elevation. If it has a length of 600 ft, determine the sag.

SOLUTION

$w_0 = 3 \text{ lb/ft}$

From Example 7–15,

$$s = \frac{F_H}{w_0} \sinh\left(\frac{w_0}{F_H} x\right)$$

At $x = 250$ ft, $\qquad s = 300$ ft

$$300 = \frac{F_H}{3} \sinh\left(\frac{3(250)}{F_H}\right)$$

$F_H = 704.3 \text{ lb}$

$$y = \frac{F_H}{w_0}\left[\cosh \frac{w_0}{F_H} x - 1\right]$$

$$h = \frac{704.3}{3}\left[\cosh\left(\frac{3(250)}{704.3}\right) - 1\right]$$

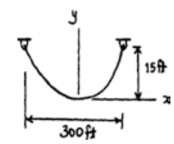
$h = 146 \text{ ft}$ **Ans.**

Ans:
$h = 146$ ft

7–123.

A cable has a weight of 5 lb/ft. If it can span 300 ft and has a sag of 15 ft, determine the length of the cable. The ends of the cable are supported at the same elevation.

SOLUTION



$$w_0 = 5 \text{ lb/ft}$$

From Example 7–15,

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

At $x = 150$ ft, $y = 15$ ft

$$\frac{15w_0}{F_H} = \cosh\left(\frac{150w_0}{F_H}\right) - 1$$

$$F_H = 3762 \text{ lb}$$

$$s = \frac{F_H}{w_0}\sinh\left(\frac{w_0}{F_H}x\right)$$

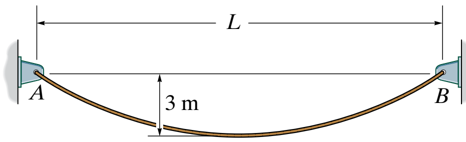
$$s = 151.0 \text{ ft}$$

$$L = 2s = 302 \text{ ft}$$ **Ans.**

Ans:
$L = 302$ ft

***7–124.**

The 10 kg m cable is suspended between the supports A and B. If the cable can sustain a maximum tension of 1.5 kN and the maximum sag is 3 m, determine the maximum distance L between the supports



SOLUTION

The origin of the x, y coordinate system is set at the lowest point of the cable. Here $w_0 = 10(9.81)\text{ N/m} = 98.1\text{ N/m}$. Using Eq. (4) of Example 7–13,

$$y = \frac{F_H}{w_0}\left[\cosh\left(\frac{w_0}{F_H}x\right) - 1\right]$$

$$y = \frac{F_H}{98.1}\left[\cosh\left(\frac{98.1x}{F_H}\right) - 1\right]$$

Applying the boundary equation $y = 3$ m at $x = \dfrac{L}{2}$, we have

$$3 = \frac{F_H}{98.1}\left[\cosh\left(\frac{49.05L}{F_H}\right) - 1\right]$$

The maximum tension occurs at either points A or B where the cable makes the greatest angle with the horizontal. From Eq. (1),

$$\tan\theta_{\max} = \sinh\left(\frac{49.05L}{F_H}\right)$$

By referring to the geometry shown in Fig. b, we have

$$\cos\theta_{\max} = \frac{1}{\sqrt{1 + \sinh^2\left(\dfrac{49.05L}{F_H}\right)}} = \frac{1}{\cosh\left(\dfrac{49.05L}{F_H}\right)}$$

hus,

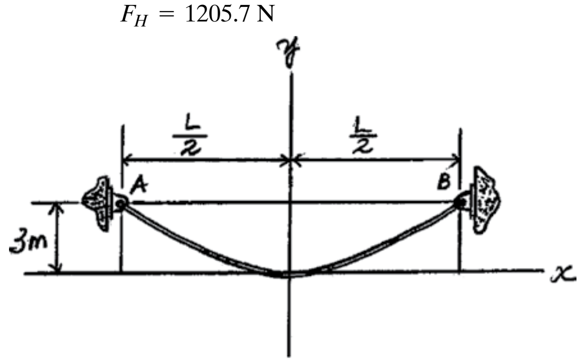
$$T_{\max} = \frac{F_H}{\cos\theta_{\max}}$$

$$1500 = F_H\cosh\left(\frac{49.05L}{F_H}\right) \qquad \textbf{(3)}$$

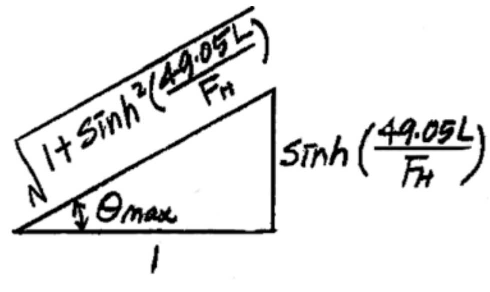
Solving Eqs. (2) and (3) yields

$L = 16.8\text{ m}$ **Ans.**

$F_H = 1205.7\text{ N}$



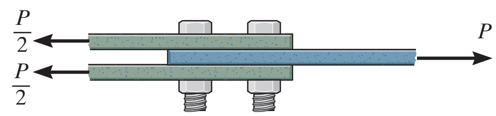
(a)



(b)

8–1.

Determine the maximum force P the connection can support so that no slipping occurs between the plates. There are four bolts used for the connection and each is tightened so that it is subjected to a tension of 4 kN. The coefficient of static friction between the plates is $\mu_s = 0.4$.

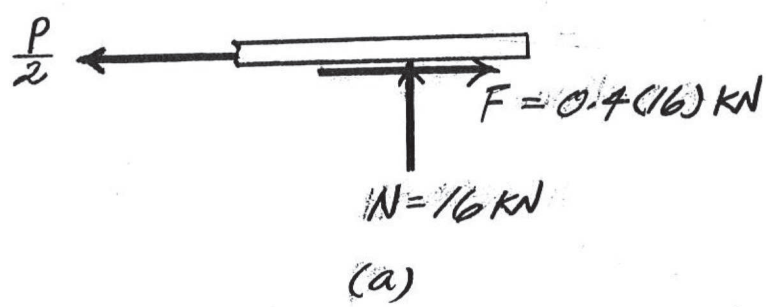


SOLUTION

Free-Body Diagram: The normal reaction acting on the contacting surface is equal to the sum total tension of the bolts. Thus, $N = 4(4)\text{ kN} = 16\text{ kN}$. When the plate is on the verge of slipping, the magnitude of the friction force acting on each contact surface can be computed using the friction formula $F = \mu_s N = 0.4(16)\text{ kN}$. As indicated on the free-body diagram of the upper plate, **F** acts to the right since the plate has a tendency to move to the left.

Equations of Equilibrium:

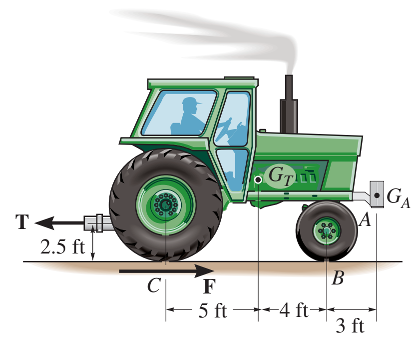
$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.4(16) - \dfrac{P}{2} = 0 \qquad p = 12.8\text{ kN}$ **Ans.**



Ans:
$P = 12.8\text{ kN}$

8–2.

The tractor exerts a towing force $T = 400$ lb. Determine the normal reactions at each of the two front and two rear tires and the tractive frictional force **F** on each rear tire needed to pull the load forward at constant velocity. The tractor has a weight of 7500 lb and a center of gravity located at G_T. An additional weight of 600 lb is added to its front having a center of gravity at G_A. Take $\mu_s = 0.4$. The front wheels are free to roll.



SOLUTION

Equations of Equilibrium:

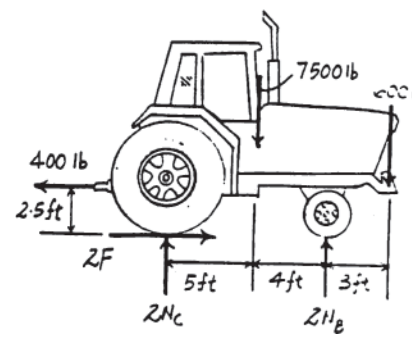
$\circlearrowleft + \Sigma M_C = 0 \qquad 2N_B(9) + 400(2.5) - 7500(5) - 600(12) = 0$

$N_B = 2427.78 \text{ lb} = 2.43 \text{ kip}$ **Ans.**

$+\uparrow \Sigma F_y = 0; \qquad 2N_C + 2(2427.78) - 7500 - 600 = 0$

$N_C = 1622.22 \text{ lb} = 1.62 \text{ kip}$ **Ans.**

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 2F - 400 = 0 \qquad F = 200 \text{ lb}$ **Ans.**

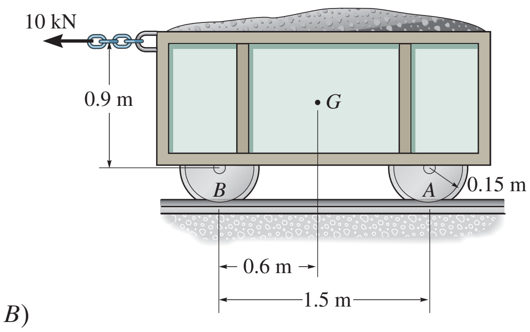


Friction: The maximum friction force that can be developed between each of the rear tires and the ground is $F_{\max} = \mu_s N_C = 0.4(1622.22) = 648.89$ lb. Since $F_{\max} > F = 200$ lb, the rear tires will not slip. Hence the tractor is capable of towing the 400 lb load.

Ans:
$N_B = 2.43$ kip
$N_C = 1.62$ kip
$F = 200$ lb

8–3.

The mine car and its contents have a total mass of 6 Mg and a center of gravity at G. If the coefficient of static friction between the wheels and the tracks is $\mu_s = 0.4$ when the wheels are locked, find the normal force acting on the front wheels at B and the rear wheels at A when the brakes at both A and B are locked. Does the car move?



SOLUTION

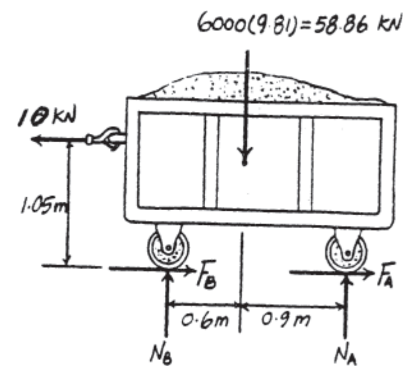
Equations of Equilibrium: The normal reactions acting on the wheels at (A and B) are independent as to whether the wheels are locked or not. Hence, the normal reactions acting on the wheels are the same for both cases.

$$\zeta + \Sigma M_B = 0; \qquad N_A(1.5) + 10(1.05) - 58.86(0.6) = 0$$

$$N_A = 16.544 \text{ kN} = 16.5 \text{ kN} \qquad \textbf{Ans.}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B + 16.544 - 58.86 = 0$$

$$N_B = 42.316 \text{ kN} = 42.3 \text{ kN} \qquad \textbf{Ans.}$$

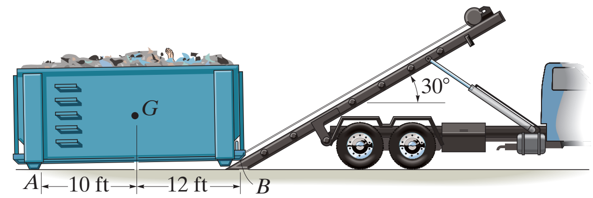


When both wheels at A and B are locked, then $(F_A)_{\max} = \mu_s N_A = 0.4(16.544) = 6.6176$ kN and $(F_B)_{\max} = \mu_s N_B = 0.4(42.316) = 16.9264$ kN. Since $(F_A)_{\max} + (F_B)_{\max} = 23.544 \text{ kN} > 10$ kN, the wheels do not slip. Thus, **the mine car does not move**. **Ans.**

Ans:
$N_A = 16.5$ kN
$N_B = 42.3$ kN
It does not move.

***8–4.**

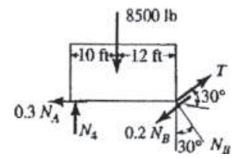
The winch on the truck is used to hoist the garbage bin onto the bed of the truck. If the loaded bin has a weight of 8500 lb and center of gravity at G, determine the force in the cable needed to begin the lift. The coefficients of static friction at A and B are $\mu_A = 0.3$ and $\mu_B = 0.2$, respectively. Neglect the height of the support at A.



SOLUTION

$\zeta + \Sigma M_B = 0; \quad 8500(12) - N_A(22) = 0$

$N_A = 4636.364 \text{ lb}$



$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad T \cos 30°$

$- 0.2N_B \cos 30° - N_B \sin 30° - 0.3(4636.364) = 0$

$T(0.86603) - 0.67321\, N_B = 1390.91$

$+\uparrow \Sigma F_y = 0; \quad 4636.364 - 8500 + T \sin 30° + N_B \cos 30°$

$- 0.2N_B \sin 30° = 0$

$T(0.5) + 0.766025\, N_B = 3863.636$

Solving:

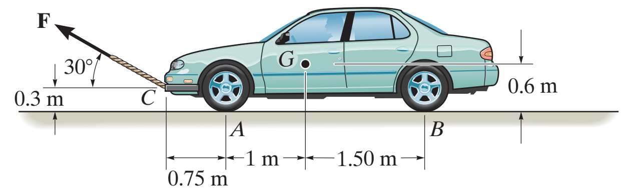
$T = 3666.5 \text{ lb} = 3.67 \text{ kip}$ **Ans.**

$N_B = 2650.6 \text{ lb}$

Ans:
$T = 3.67 \text{ kip}$

8–5.

The automobile has a mass of 2 Mg and center of mass at G. Determine the towing force **F** required to move the car if the back brakes are locked, and the front wheels are free to roll. Take $\mu_s = 0.3$.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the car shown in Fig. *a*,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_B - F\cos 30^\circ = 0 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \qquad N_A + N_B + F\sin 30^\circ - 2000(9.81) = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft +\Sigma M_A = 0; \qquad F\cos 30^\circ(0.3) - F\sin 30^\circ(0.75) + N_B(2.5) - 2000(9.81)(1) = 0 \qquad \textbf{(3)}$$

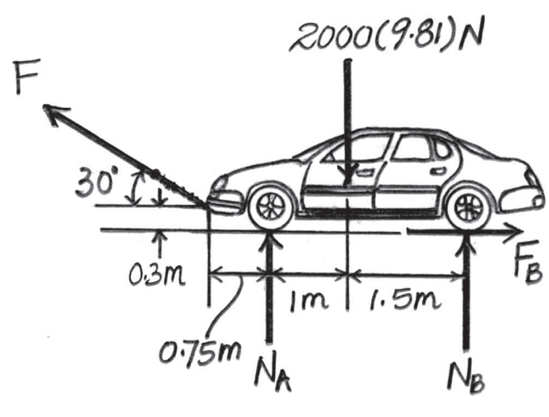
Friction. It is required that the rear wheels are on the verge to slip. Thus

$$F_B = \mu_s N_B = 0.3\,N_B \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4),

$$F = 2{,}762.72\text{ N} = 2.76\text{ kN} \qquad \textbf{Ans.}$$

$$N_B = 7975.30\text{ N} \quad N_A = 10{,}263.34\text{ N} \quad F_B = 2392.59\text{ N}$$

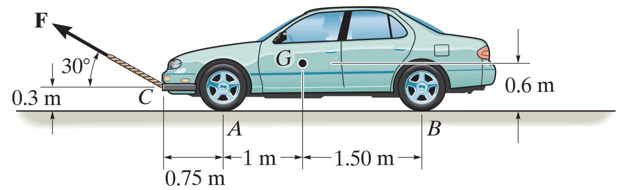


(a)

Ans:
$F = 2.76$ kN

8–6.

The automobile has a mass of 2 Mg and center of mass at G. Determine the towing force **F** required to move the car. Both the front and rear brakes are locked. Take $\mu_s = 0.3$.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the car shown in Fig. *a*,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad F_A + F_B - F\cos 30° = 0 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \qquad F\sin 30° + N_A + N_B - 2000(9.81) = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft +\Sigma M_A = 0; \qquad F\cos 30°(0.3) - F\sin 30°(0.75) + N_B(2.5) - 200(9.81)(1) = 0 \qquad \textbf{(3)}$$

Friction. It is required that both the front and rear wheels are on the verge to slip. Thus

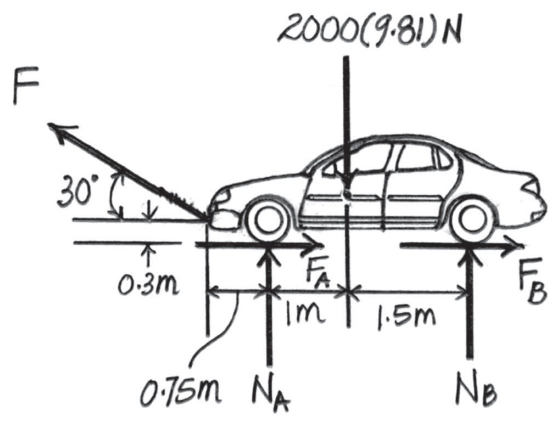
$$F_A = \mu_s N_A = 0.3\,N_A \qquad \textbf{(4)}$$

$$F_B = \mu_s N_B = 0.3\,N_B \qquad \textbf{(5)}$$

Solving Eqs. (1) to (5),

$$F = 5793.16\text{ N} = 5.79\text{ kN} \qquad \textbf{Ans.}$$

$N_B = 8114.93$ N $\quad N_A = 8608.49$ N $\quad F_A = 2582.55$ N $\quad F_B = 2434.48$ N

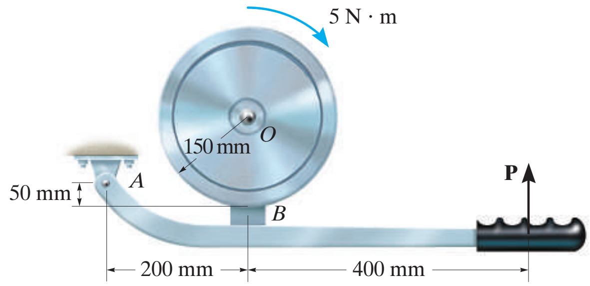


(a)

Ans:
$F = 5.79$ kN

8–7.

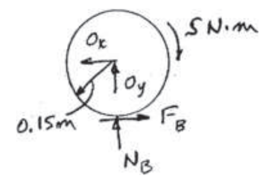
The block brake consists of a pin-connected lever and friction block at B. The coefficient of static friction between the wheel and the lever is $\mu_s = 0.3$, and a torque of $5\ \mathrm{N \cdot m}$ is applied to the wheel. Determine if the brake can hold the wheel stationary when the force applied to the lever is (a) $P = 30$ N, (b) $P = 70$ N.



SOLUTION

To hold lever:

$\circlearrowleft + \Sigma M_O = 0; \qquad F_B(0.15) - 5 = 0; \qquad F_B = 33.333\ \mathrm{N}$

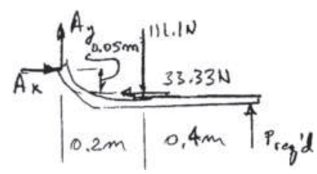


Require

$$N_B = \frac{33.333\ \mathrm{N}}{0.3} = 111.1\ \mathrm{N}$$

Lever,

$\circlearrowleft + \Sigma M_A = 0; \qquad P_{\text{Reqd.}}(0.6) - 111.1(0.2) - 33.333(0.05) = 0$



$P_{\text{Reqd.}} = 39.8\ \mathrm{N}$

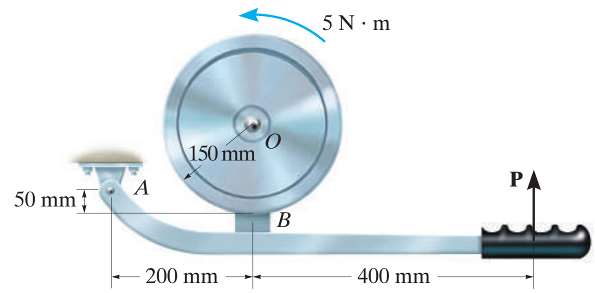
a) $P = 30\ \mathrm{N} < 39.8\ \mathrm{N}$ No **Ans.**

b) $P = 70\ \mathrm{N} > 39.8\ \mathrm{N}$ Yes **Ans.**

Ans:
No
Yes

***8–8.**

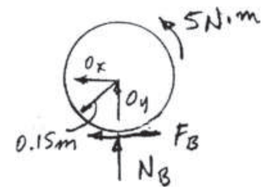
The block brake consists of a pin-connected lever and friction block at B. The coefficient of static friction between the wheel and the lever is $\mu_s = 0.3$, and a torque of $5\ \mathrm{N \cdot m}$ is applied to the wheel. Determine if the brake can hold the wheel stationary when the force applied to the lever is (a) $P = 30\ \mathrm{N}$, (b) $P = 70\ \mathrm{N}$.



SOLUTION

To hold lever:

$\circlearrowleft + \Sigma M_O = 0; \qquad -F_B(0.15) + 5 = 0; \qquad F_B = 33.333\ \mathrm{N}$



Require

$$N_B = \frac{33.333\ \mathrm{N}}{0.3} = 111.1\ \mathrm{N}$$

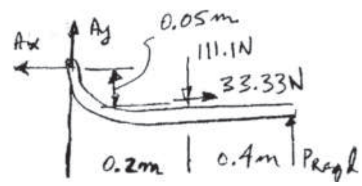
Lever,

$\circlearrowleft + \Sigma M_A = 0; \qquad P_{\text{Reqd.}}(0.6) - 111.1(0.2) + 33.333(0.05) = 0$

$P_{\text{Reqd.}} = 34.26\ \mathrm{N}$

a) $P = 30\ \mathrm{N} < 34.26\ \mathrm{N}$ No **Ans.**

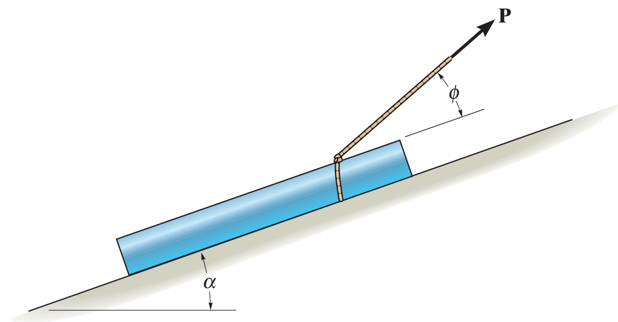
b) $P = 70\ \mathrm{N} > 34.26\ \mathrm{N}$ Yes **Ans.**



Ans:
No
Yes

8–9.

The pipe of weight W is to be pulled up the inclined plane of slope α using a force $\mathbf{P}$. If $\mathbf{P}$ acts at an angle ϕ, show that for slipping $P = W\sin(\alpha + \theta)/\cos(\phi - \theta)$, where θ is the angle of static friction; $\theta = \tan^{-1}\mu_s$.



SOLUTION

$+\nwarrow\Sigma F_{y'} = 0;\quad N + P\sin\phi - W\cos\alpha = 0 \qquad N = W\cos\alpha - P\sin\phi$

$+\nearrow\Sigma F_{x'} = 0;\quad P\cos\phi - W\sin\alpha - \tan\theta(W\cos\alpha - P\sin\phi) = 0$

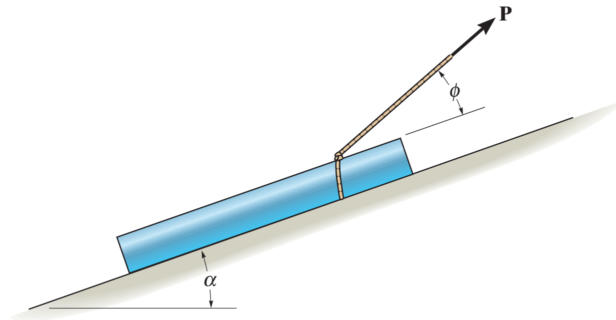
$$P = \frac{W(\sin\alpha + \tan\theta\cos\alpha)}{\cos\phi + \tan\theta\sin\phi}$$

$$= \frac{W(\cos\theta\sin\alpha + \sin\theta\cos\alpha)}{\cos\phi\cos\theta + \sin\phi\sin\theta} = \frac{W\sin(\alpha + \theta)}{\cos(\phi - \theta)}$$

Q.E.D.

8–10.

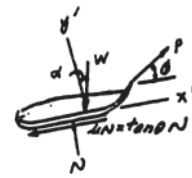
Determine the angle ϕ at which the applied force **P** should act on the pipe so that the magnitude of **P** is as small as possible for pulling the pipe up the incline. What is the corresponding value of P? The pipe weighs W and the slope α is known. Express the answer in terms of the angle of kinetic friction, $\theta = \tan^{-1} \mu_k$.



SOLUTION

$+\nwarrow \Sigma F_{y'} = 0; \qquad N + P \sin\phi - W\cos\alpha = 0 \qquad N = W\cos\alpha - P\sin\phi$

$+\nearrow \Sigma F_{x'} = 0; \qquad P\cos\phi - W\sin\alpha - \tan\theta(W\cos\alpha - P\sin\phi) = 0$



$$P = \frac{W(\sin\alpha + \tan\theta\cos\alpha)}{\cos\phi + \tan\theta\sin\phi}$$

$$= \frac{W(\cos\theta\sin\alpha + \sin\theta\cos\alpha)}{\cos\phi\cos\theta + \sin\phi\sin\theta}$$

$$= \frac{W\sin(\alpha + \theta)}{\cos(\phi - \theta)}$$

$$\frac{dP}{d\phi} = \frac{W\sin(\alpha+\theta)\sin(\phi-\theta)}{\cos^2(\phi-\theta)} = 0$$

$W\sin(\alpha+\theta)\sin(\phi-\theta) = 0 \qquad W\sin(\alpha+\theta) = 0$

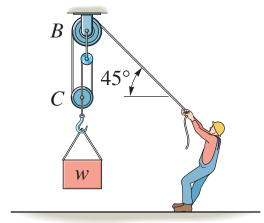
$\sin(\phi - \theta) = 0 \qquad \phi - \theta = 0 \qquad \phi = \theta$ **Ans.**

$P = \dfrac{W\sin(\alpha+\theta)}{\cos(\theta-\theta)} = W\sin(\alpha+\theta)$ **Ans.**

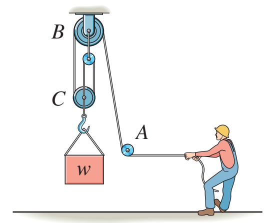
Ans:
$\phi = \theta$
$P = W\sin(\alpha + \theta)$

8–11.

Determine the maximum weight W the man can lift with constant velocity using the pulley system, without and then with the "leading block" or pulley at A. The man has a weight of 200 lb and the coefficient of static friction between his feet and the ground is $\mu_s = 0.6$.



(a)



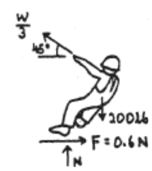
(b)

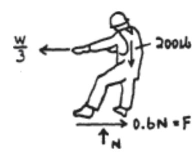
SOLUTION

a) $+\uparrow \Sigma F_y = 0; \qquad \dfrac{W}{3}\sin 45° + N - 200 = 0$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad -\dfrac{W}{3}\cos 45° + 0.6\,N = 0$

$W = 318 \text{ lb}$ **Ans.**





b) $+\uparrow \Sigma F_y = 0; \qquad N = 200 \text{ lb}$

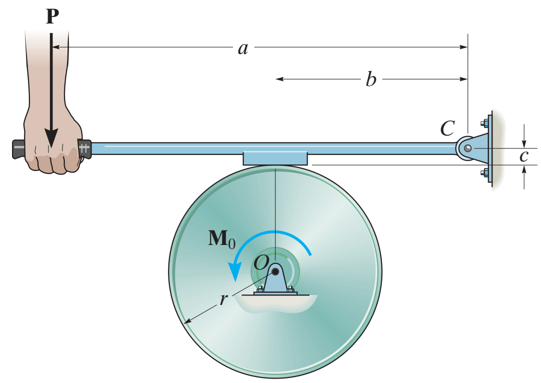
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 0.6(200) = \dfrac{W}{3}$

$W = 360 \text{ lb}$ **Ans.**

Ans:
$W = 318 \text{ lb}$
$W = 360 \text{ lb}$

***8–12.**

The block brake is used to stop the wheel from rotating when the wheel is subjected to a couple moment $\mathbf{M}_0$. If the coefficient of static friction between the wheel and the block is μ_s, determine the smallest force P that should be applied.



SOLUTION

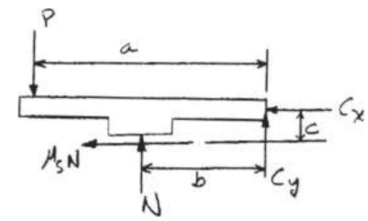
$\circlearrowleft + \Sigma M_C = 0; \qquad Pa - Nb - \mu_s Nc = 0$

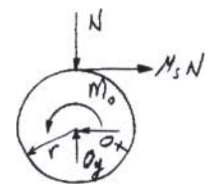
$$N = \frac{Pa}{(b + \mu_s c)}$$

$\circlearrowright + \Sigma M_O = 0; \qquad \mu_s Nr - M_0 = 0$

$$\mu_s P\left(\frac{a}{b + \mu_s c}\right)r = M_0$$

$$P = \frac{M_0}{\mu_s ra}(b + \mu_s c)$$ **Ans.**



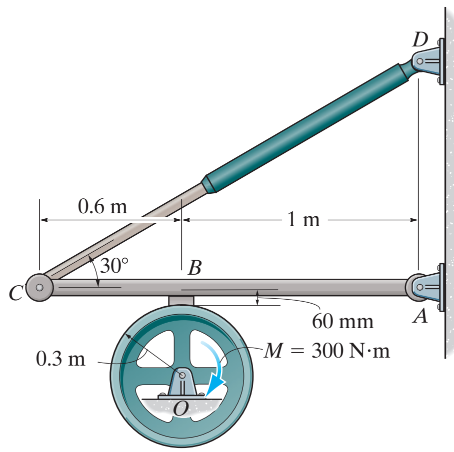


Ans:

$$P = \frac{M_0}{\mu_s ra}(b + \mu_s c)$$

8–13.

If a torque of $M = 300\ \text{N} \cdot \text{m}$ is applied to the flywheel, determine the force that must be developed in the hydraulic cylinder CD to prevent the flywheel from rotating. The coefficient of static friction between the friction pad at B and the flywheel is $\mu_s = 0.4$.



SOLUTION

Free-BodyDiagram: First we will consider the equilibrium of the flywheel using the free-body diagram shown in Fig. *a*. Here, the frictional force $\mathbf{F}_B$ must act to the left to produce the counterclockwise moment opposing the impending clockwise rotational motion caused by the $300\ \text{N} \cdot \text{m}$ couple moment. Since the wheel is required to be on the verge of slipping, then $F_B = \mu_s N_B = 0.4\ N_B$. Subsequently, the free-body diagram of member ABC shown in Fig. *b* will be used to determine $\mathbf{F}_{CD}$.

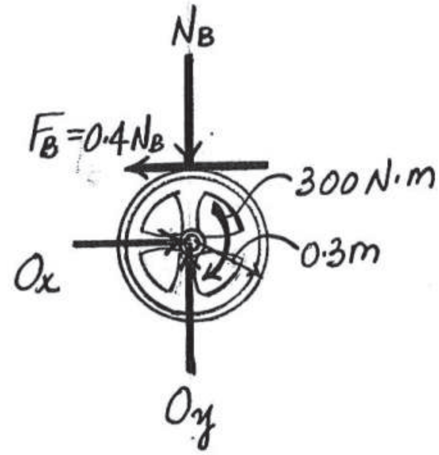
Equations of Equilibrium: We have

$$\circlearrowleft + \Sigma M_O = 0; \qquad 0.4\ N_B(0.3) - 300 = 0 \qquad N_B = 2500\ \text{N}$$

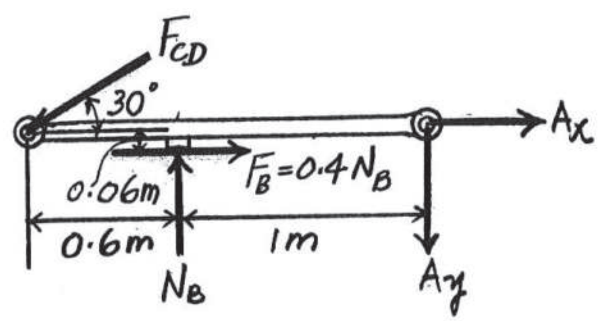
Using this result,

$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{CD} \sin 30°(1.6) + 0.4(2500)(0.06) - 2500(1) = 0$$

$$F_{CD} = 3050\ \text{N} = 3.05\ \text{kN} \qquad \textbf{Ans.}$$



(a)

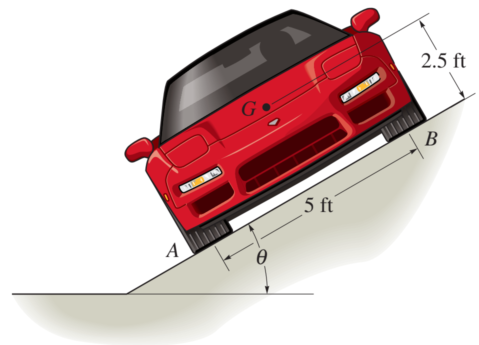


(b)

Ans:
$F_{CD} = 3.05\ \text{kN}$

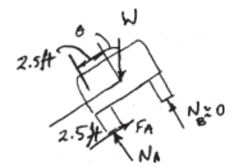
8–14.

The car has a mass of 1.6 Mg and center of mass at G. If the coefficient of static friction between the shoulder of the road and the tires is $\mu_s = 0.4$, determine the greatest slope θ the shoulder can have without causing the car to slip or tip over if the car travels along the shoulder at constant velocity.



SOLUTION

Tipping:

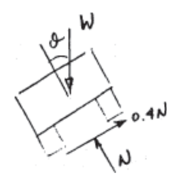


$\circlearrowleft + \Sigma M_A = 0; \qquad -W\cos\theta(2.5) + W\sin\theta(2.5) = 0$

$\tan\theta = 1$

$\theta = 45^\circ$

Slipping:



$\nearrow + \Sigma F_x = 0; \qquad 0.4\,N - W\sin\theta = 0$

$\nwarrow + \Sigma F_y = 0; \qquad N - W\cos\theta = 0$

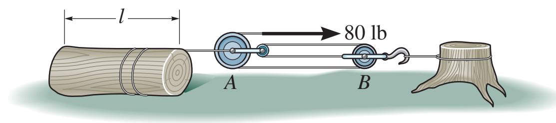
$\tan\theta = 0.4$

$\theta = 21.8^\circ$ **Ans.** (car slips before it tips)

Ans:
$\theta = 21.8^\circ$

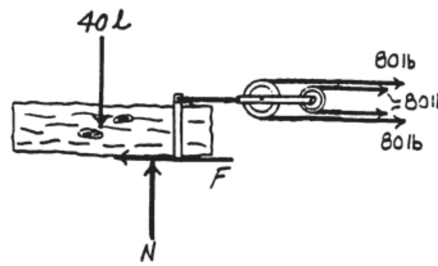
8–15.

The log has a coefficient of static friction of $\mu_s = 0.3$ with the ground and a weight of 40 lb/ft. If a man can pull on the rope with a maximum force of 80 lb, determine the greatest length l of log he can drag.



SOLUTION

Equations of Equilibrium:



$+\uparrow \Sigma F_y = 0; \qquad N - 40l = 0 \qquad N = 40l$

$\xrightarrow{+} \Sigma F_x = 0; \qquad 4(80) - F = 0 \qquad F = 320 \text{ lb}$

Friction: Since the log slides,

$$F = (F)_{max} = \mu_s N$$

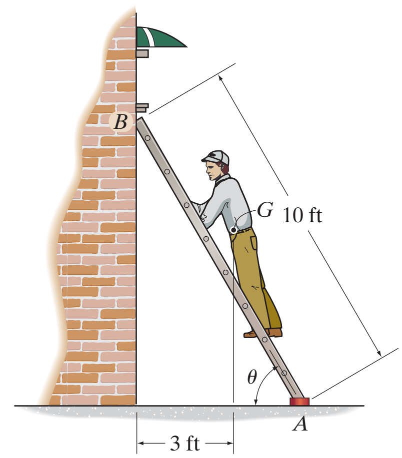
$$320 = 0.3(40l)$$

$$l = 26.7 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$l = 26.7$ ft

***8–16.**

The 180-lb man climbs up the ladder and stops at the position shown after he senses that the ladder is on the verge of slipping. Determine the inclination θ of the ladder if the coefficient of static friction between the friction pad A and the ground is $\mu_s = 0.4$. Assume the wall at B is smooth. The center of gravity for the man is at G. Neglect the weight of the ladder.



SOLUTION

Free - Body Diagram. Since the weight of the man tends to cause the friction pad A to slide to the right, the frictional force $\mathbf{F}_A$ must act to the left as indicated on the free - body diagram of the ladder, Fig. *a.* Here, the ladder is on the verge of slipping. Thus, $F_A = \mu_s N_A$.

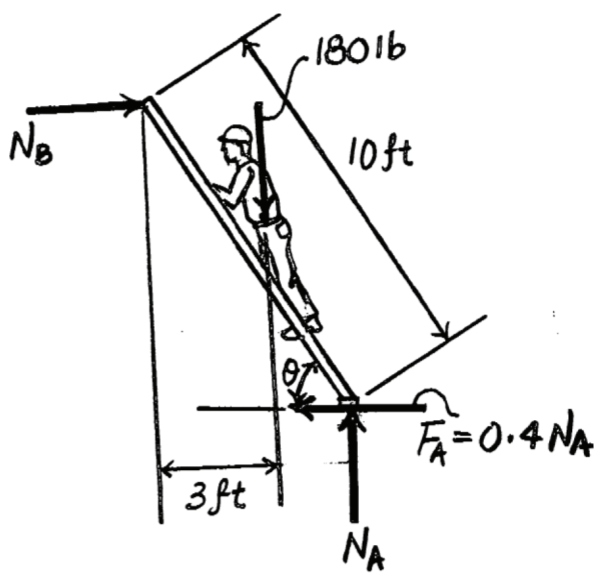
Equations of Equilibrium.

$+\uparrow \Sigma F_y = 0;$ $\quad N_A - 180 = 0 \quad\quad N_A = 180 \text{ lb}$

$\circlearrowleft + \Sigma M_B = 0;$ $\quad 180(10 \cos \theta°) - 0.4(180)(10 \sin \theta°) - 180(3) = 0$

$\cos \theta - 0.4 \sin \theta = 0.3$

$\theta = 52.0°$ **Ans.**

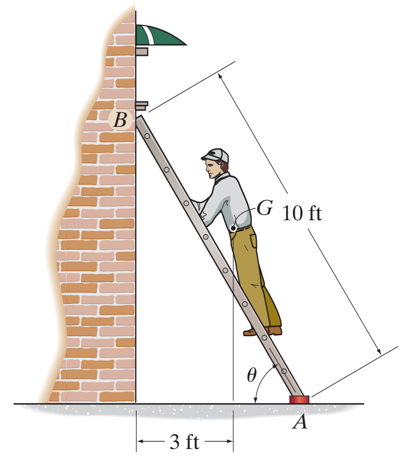


(a)

Ans:
$\theta = 52.0°$

8–17.

The 180-lb man climbs up the ladder and stops at the position shown after he senses that the ladder is on the verge of slipping. Determine the coefficient of static friction between the friction pad at A and ground if the inclination of the ladder is $\theta = 60°$ and the wall at B is smooth. The center of gravity for the man is at G. Neglect the weight of the ladder.



SOLUTION

Free - Body Diagram. Since the weight of the man tends ot cause the friction pad A to slide to the right, the frictional force $\mathbf{F}_A$ must act to the left as indicated on the free - body diagram of the ladder, Fig. *a*. Here, the ladder is on the verge of slipping. Thus, $F_A = \mu_s N_A$.

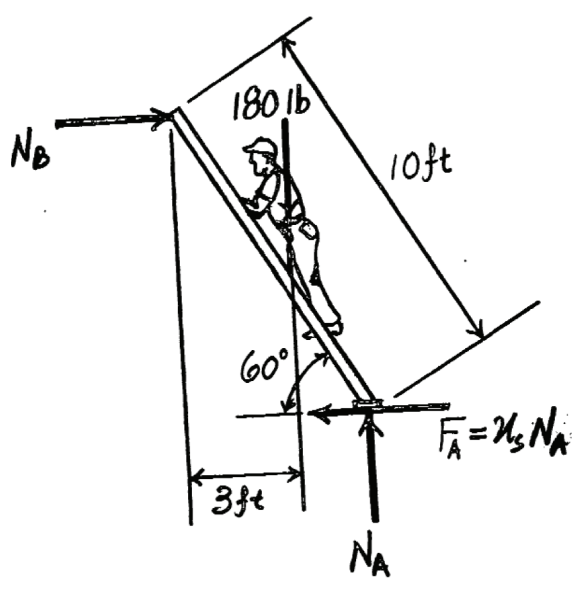
Equations of Equilibrium.

$+\uparrow \Sigma F_y = 0; \quad N_A - 180 = 0 \quad N_A = 180 \text{ lb}$

$\circlearrowleft + \Sigma M_B = 0; \quad 180(10 \cos 60°) - \mu_s(180)(10 \sin 60°) - 180(3) = 0$

$180 \cos \theta - 72 \sin \theta = 54$

$\mu_s = 0.231$ **Ans.**

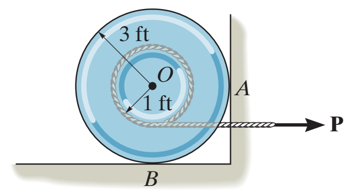


(a)

Ans:
$\mu_s = 0.231$

8–18.

The spool of wire having a weight of 300 lb rests on the ground at B and against the wall at A. Determine the force P required to begin pulling the wire horizontally off the spool. The coefficient of static friction between the spool and its points of contact is $\mu_s = 0.25$.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the spool shown in Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - N_A - F_B = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B - F_A - 300 = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad P(1) - F_B(3) - F_A(3) = 0 \qquad \textbf{(3)}$$

Frictions. It is required that slipping occurs at A and B. Thus,

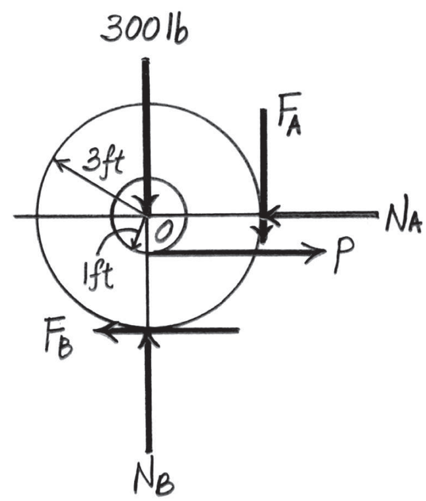
$$F_A = \mu N_A = 0.25\, N_A \qquad \textbf{(4)}$$

$$F_B = \mu N_B = 0.25\, N_B \qquad \textbf{(5)}$$

Solving Eqs. (1) to (5),

$$P = 1350 \text{ lb} \qquad \textbf{Ans.}$$

$$N_A = 1200 \text{ lb} \quad N_B = 600 \text{ lb} \quad F_A = 300 \text{ lb} \quad F_B = 150 \text{ lb}$$

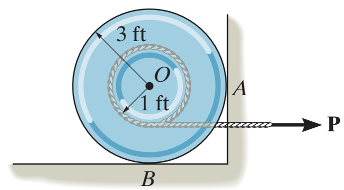


(a)

Ans:
$P = 1350$ lb

8–19.

The spool of wire having a weight of 300 lb rests on the ground at B and against the wall at A. Determine the normal force acting on the spool at A if $P = 300$ lb. The coefficient of static friction between the spool and the ground at B is $\mu_s = 0.35$. The wall at A is smooth.



SOLUTION

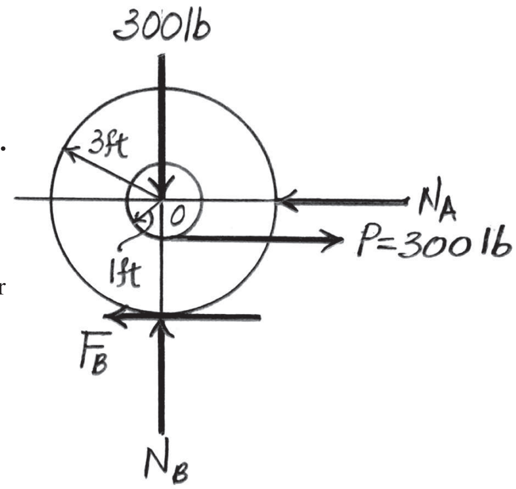
Equations of Equilibrium. Referring to the *FBD* of the spool shown in Fig. *a*,

$\zeta + \Sigma M_B = 0; \quad N_A(3) - 300(2) = 0 \quad N_A = 200 \text{ lb}$ **Ans.**

$\zeta + \Sigma M_O = 0; \quad 300(1) - F_B(3) = 0 \quad F_B = 100 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad N_B - 300 = 0 \quad N_B = 300 \text{ lb}$

Friction. Since $F_B < (F_B)_{\max} = \mu_s N_B = 0.35(300) = 105$ lb, slipping will not occur at B. Thus, the spool will remain at rest.

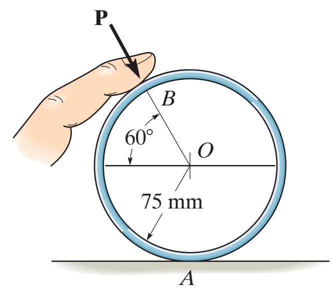


(a)

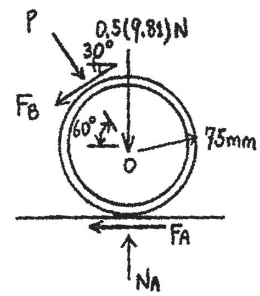
Ans:
$N_A = 200$ lb

***8–20.**

The ring has a mass of 0.5 kg and is resting on the surface of the table. In an effort to move the ring a normal force **P** from the finger is exerted on it. If this force is directed towards the ring's center O as shown, determine its magnitude when the ring is on the verge of slipping at A. The coefficient of static friction at A is $\mu_A = 0.2$ and at B, $\mu_B = 0.3$.



SOLUTION



$F_A = F_B$

$P \cos 60° - F_B \cos 30° - F_A = 0$

$N_A - 0.5(9.81) - P \sin 60° - F_B \sin 30° = 0$

$F_A = 0.2\, N_A$

$N_A = 19.34 \text{ N}$

$F_A = F_B = 3.868 \text{ N}$

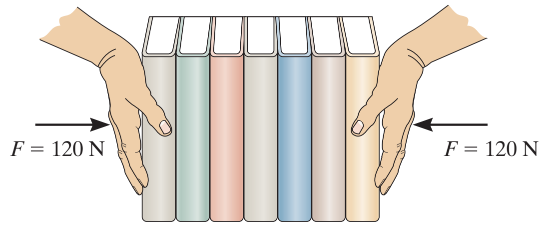
$P = 14.4 \text{ N}$ **Ans.**

$(F_B)_{max} = 0.3(14.44) = 4.33 \text{ N} > 3.868 \text{ N}$ (**O.K!**)

Ans:
$P = 14.4 \text{ N}$

8–21.

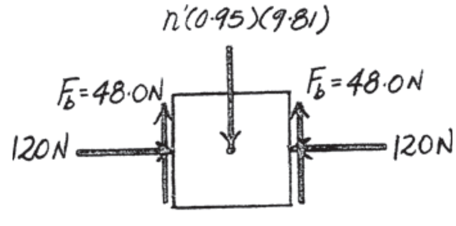
A man attempts to support a stack of books horizontally by applying a compressive force of $F = 120$ N to the ends of the stack with his hands. If each book has a mass of 0.95 kg, determine the greatest number of books that can be supported in the stack. The coefficient of static friction between the man's hands and a book is $(\mu_s)_h = 0.6$ and between any two books $(\mu_s)_b = 0.4$.



SOLUTION

Equations of Equilibrium and Friction: Let n' be the number of books that are on the verge of sliding together between the two books at the edge. Thus, $F_b = (\mu_s)_b N = 0.4(120) = 48.0$ N. From FBD (a),

$$+\uparrow \Sigma F_y = 0; \qquad 2(48.0) - n'(0.95)(9.81) = 0 \qquad n' = 10.30$$



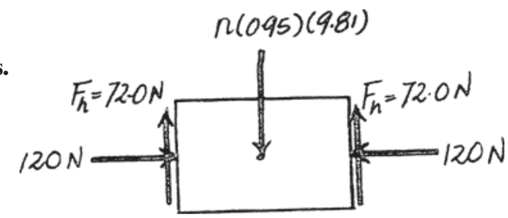
(a)

Let n be the number of books are on the verge of sliding together in the stack between the hands. Thus, $F_k = (\mu_s)_k N = 0.6(120) = 72.0$ N. From FBD (b),

$$+\uparrow \Sigma F_y = 0; \qquad 2(72.0) - n(0.95)(9.81) = 0 \qquad n = 15.45$$

Thus, the maximum number of books can be supported in stack is

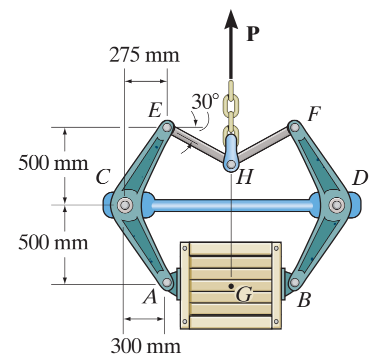
$$n = 10 + 2 = 12 \qquad \textbf{Ans.}$$



Ans:
$n = 12$

8–22.

The tongs are used to lift the 150-kg crate, whose center of mass is at G. Determine the least coefficient of static friction at the pivot blocks so that the crate can be lifted.



SOLUTION

Free - Body Diagram. Since the crate is suspended from the tongs, **P** must be equal to the weight of the crate; i.e., $P = 150(9.81)$N as indicated on the free - body diagram of joint H shown in Fig.a. Since the crate is required to be on the verge of slipping downward, $\mathbf{F}_A$ and $\mathbf{F}_B$ must act upward so that $F_A = \mu_s N_A$ and $F_B = \mu_s N_B$ as indicated on the free - body diagram of the crate shown in Fig. c.

Equations of Equilibrium. Referring to Fig. a,

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad F_{HE} \cos 30° - F_{HF} \cos 30° = 0 \qquad F_{HE} = F_{HF} = F$

$+\uparrow \Sigma F_y = 0; \qquad 150(9.81) - 2F \sin 30° = 0 \qquad F = 1471.5 \text{ N}$

Referring to Fig. b,

$\circlearrowleft + \Sigma M_C = 0; \quad 1471.5 \cos 30°(0.5) + 1471.5 \sin 30°(0.275) - N_A(0.5) - \mu_s N_A(0.3) = 0$

$$0.5N_A + 0.3\mu_s N_A = 839.51 \qquad \textbf{(1)}$$

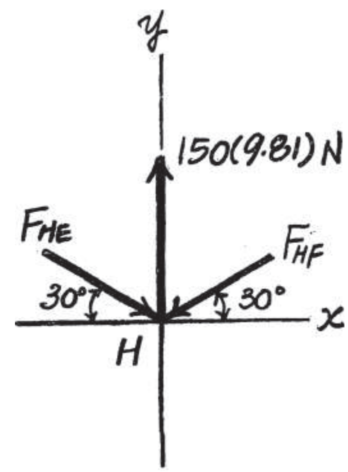
Due to the symmetry of the system and loading, $N_B = N_A$. Referring to Fig. c,

$+\uparrow \Sigma F_y = 0; \qquad 2\mu_s N_A - 150(9.81) = 0 \qquad \textbf{(2)}$

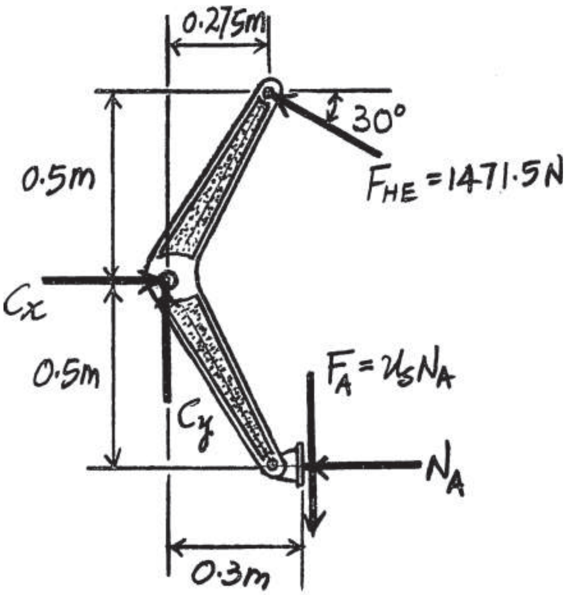
Solving Eqs. (1) and (2), yields

$$N_A = 1237.57 \text{ N}$$

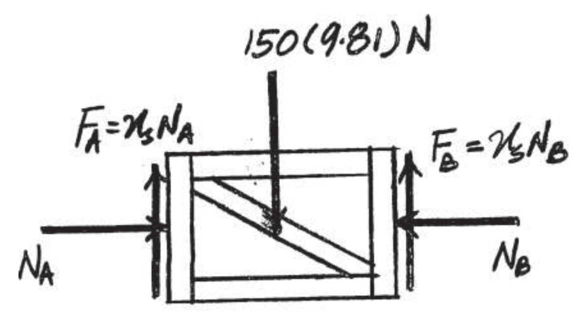
$$\mu_s = 0.595 \qquad \textbf{Ans.}$$



(a)



(b)

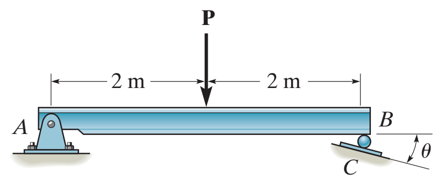


(c)

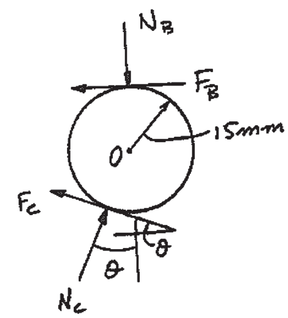
Ans:
$\mu_s = 0.595$

8–23.

The beam is supported by a pin at A and a roller at B which has negligible weight and a radius of 15 mm. If the coefficient of static friction is $\mu_B = \mu_C = 0.3$, determine the largest angle θ of the incline so that the roller does not slip for any force **P** applied to the beam.



SOLUTION



$\circlearrowleft + \Sigma M_O = 0;\qquad F_B(15) - F_C(15) = 0 \qquad$ **(1)**

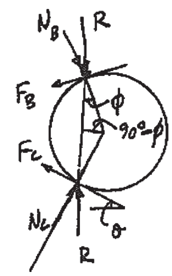
$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad -F_B - F_C\cos\theta + N_C\sin\theta = 0 \qquad$ **(2)**

$+\uparrow \Sigma F_y = 0;\qquad N_C\cos\theta + F_C\sin\theta - N_B = 0 \qquad$ **(3)**

Assume slipping at C so that

$$F_C = 0.3\,N_C$$

Then from Eqs. (1) and (2),



$$F_B = F_C$$

$$-0.3\,N_C - 0.3\,N_C\cos\theta + N_C\sin\theta = 0$$

$$(-0.3 - 0.3\cos\theta + \sin\theta)\,N_C = 0 \qquad \textbf{(4)}$$

The term in parentheses is zero when

$$\theta = 33.4^\circ \qquad \textbf{Ans.}$$

From Eq. (3), $\qquad N_C(\cos 33.4^\circ + 0.3\sin 33.4^\circ) = N_B$

$$N_C = N_B$$

Since Eq. (4) is satisfied for any value of N_C, any value of P can act on the beam. Also, the roller is a "two-force member."

$$2(90^\circ - \phi) + \theta = 180^\circ$$

$$\phi = \frac{\theta}{2}$$

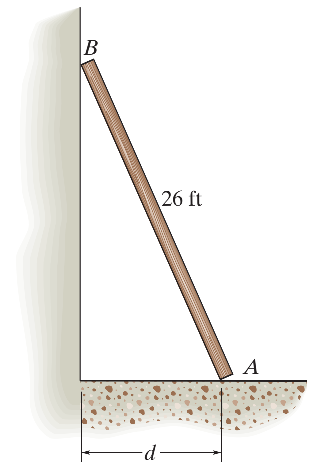
$$\phi = \tan^{-1}\left(\frac{\mu N}{N}\right) = \tan^{-1}(0.3) = 16.7^\circ$$

thus $\qquad \theta = 2(16.7^\circ) = 33.4^\circ \qquad$ **Ans.**

Ans:
$\theta = 33.4^\circ$

***8–24.**

The uniform thin pole has a weight of 30 lb and a length of 26 ft. If it is placed against the smooth wall and on the rough floor in the position $d = 10$ ft, will it remain in this position when it is released? The coefficient of static friction is $\mu_s = 0.3$.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \quad 30\,(5) - N_B\,(24) = 0$

$N_B = 6.25 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad 6.25 - F_A = 0$

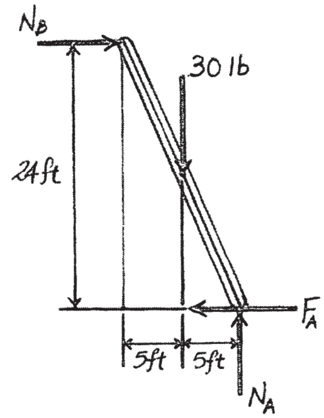
$F_A = 6.25 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad N_A - 30 = 0$

$N_A = 30 \text{ lb}$

$(F_A)_{max} = 0.3\,(30) = 9 \text{ lb} > 6.25 \text{ lb}$

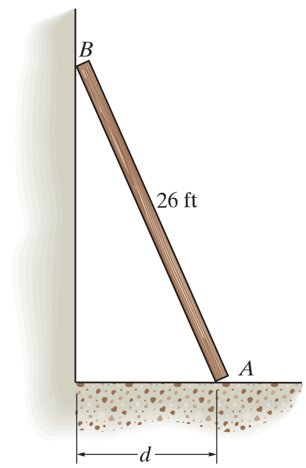
Yes, the pole will remain stationary. **Ans.**



Ans:
Yes, the pole will remain stationary.

8–25.

The uniform pole has a weight of 30 lb and a length of 26 ft. Determine the maximum distance d it can be placed from the smooth wall and not slip. The coefficient of static friction between the floor and the pole is $\mu_s = 0.3$.



SOLUTION

$+\uparrow \Sigma F_y = 0; \qquad N_A - 30 = 0$

$N_A = 30 \text{ lb}$

$F_A = (F_A)_{max} = 0.3\,(30) = 9 \text{ lb}$

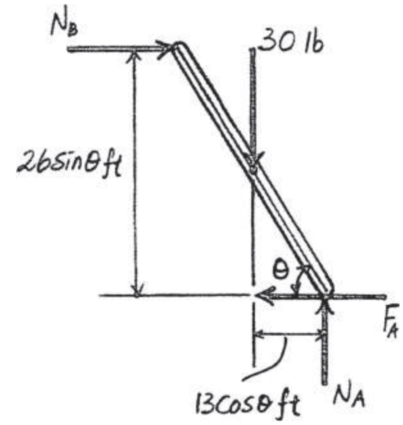
$\xrightarrow{+} \Sigma F_x = 0; \qquad N_B - 9 = 0$

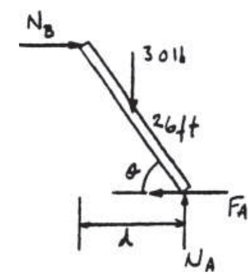
$N_B = 9 \text{ lb}$

$\zeta + \Sigma M_A = 0; \qquad 30\,(13 \cos\theta) - 9\,(26 \sin\theta) = 0$

$\theta = 59.04^\circ$

$d = 26 \cos 59.04^\circ = 13.4 \text{ ft}$ **Ans.**

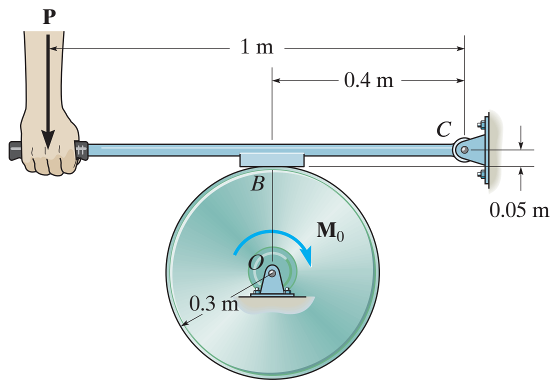




Ans:
$d = 13.4$ ft

8–26.

The block brake is used to stop the wheel from rotating when the wheel is subjected to a couple moment $M_0 = 360\ \text{N}\cdot\text{m}$. If the coefficient of static friction between the wheel and the block is $\mu_s = 0.6$, determine the smallest force P that should be applied.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the lever arm shown in Fig. *a*,

$$\circlearrowleft + \Sigma M_C = 0; \qquad P(1) + F_B(0.05) - N_B(0.4) = 0 \qquad \textbf{(1)}$$

Also, the *FBD* of the wheel, Fig. *b*,

$$\circlearrowleft + \Sigma M_O = 0; \qquad F_B(0.3) - 360 = 0 \quad F_B = 1200\ \text{N}$$

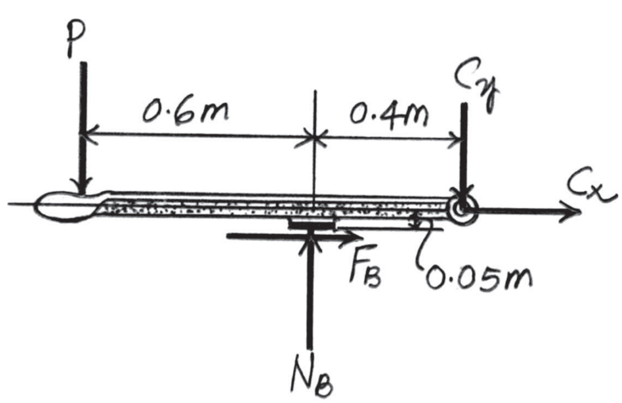
Friction. It is required that the wheel is on the verge to rotate thus slip at B. Then

$$F_B = \mu_s N_B; \quad 1200 = 0.6\, N_B \quad N_B = 2000\ \text{N}$$

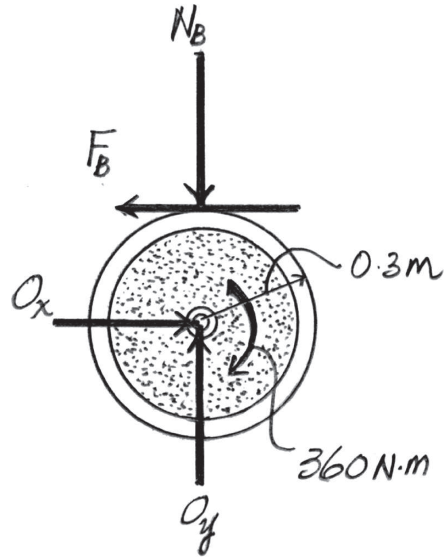
Substitute the result of F_B and N_B into Eq. (1)

$$P(1) + 1200(0.05) - 2000(0.4) = 0$$

$$P = 740\ \text{N} \qquad \textbf{Ans.}$$



(a)

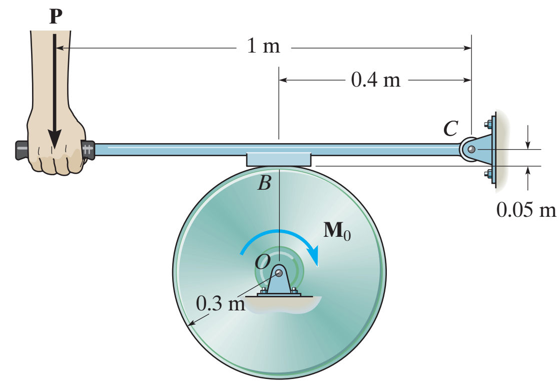


(b)

Ans:
$P = 740\ \text{N}$

8–27.

Solve Prob. 8–26 if the couple moment $\mathbf{M}_0$ is applied counterclockwise.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the lever arm shown in Fig. *a*,

$$\circlearrowleft+\Sigma M_C = 0; \qquad P(1) - F_B(0.05) - N_B(0.4) = 0 \qquad \textbf{(1)}$$

Also, the *FBD* of the wheel, Fig. *b*

$$\circlearrowleft+\Sigma M_O = 0; \qquad 360 - F_B(0.3) = 0 \quad F_B = 1200\text{ N}$$

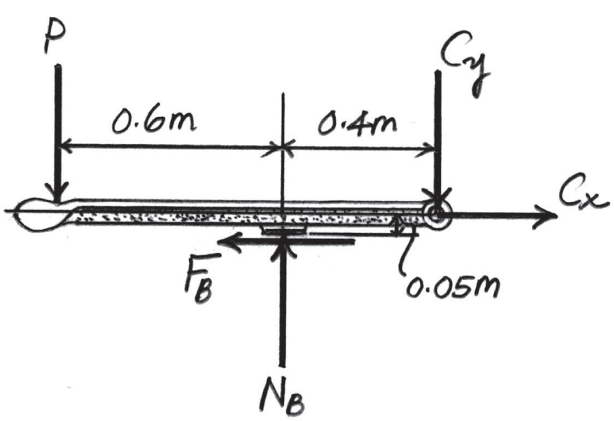
Friction. It is required that the wheel is on the verge to rotate thus slip at B. Then

$$F_B = \mu_s N_B; \quad 1200 = 0.6\, N_B \quad N_B = 2000\text{ N}$$

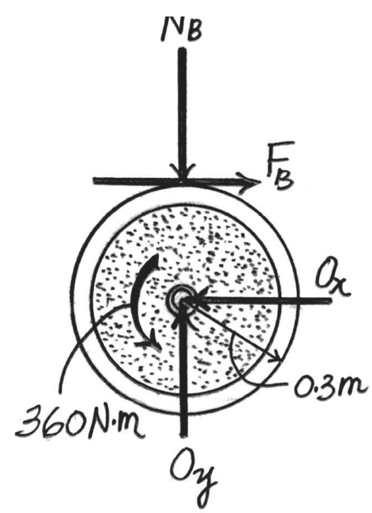
Substituting the result of $\mathbf{F}_B$ and $\mathbf{N}_B$ into Eq. (1),

$$P(1) - 1200(0.05) - 2000(0.4) = 0$$

$$P = 860\text{ N} \qquad \textbf{Ans.}$$



(a)

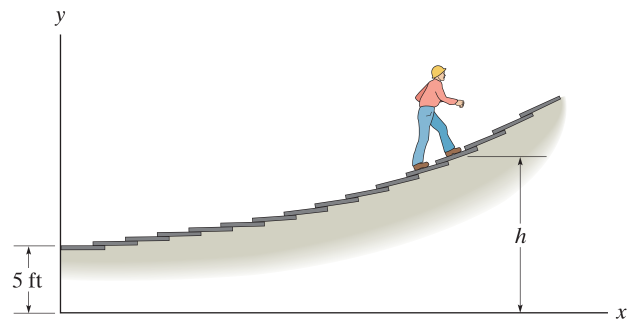


(b)

Ans:
$P = 860\text{ N}$

***8–28.**

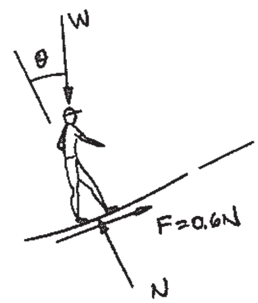
A worker walks up the sloped roof that is defined by the curve $y = (5e^{0.01x})$ ft, where x is in feet. Determine how high h he can go without slipping. The coefficient of static friction is $\mu_s = 0.6$.



SOLUTION

$\overset{+}{\nearrow}\Sigma F_x = 0; \quad 0.6\,N - W\sin\theta = 0$

$\overset{+}{\nwarrow}\Sigma F_y = 0; \quad N - W\cos\theta = 0$



$\tan\theta = 0.6$

$y = 5\,e^{0.01\,x}$

$$\frac{dy}{dx} = \tan\theta = 0.05\,e^{0.01\,x}$$

$0.6 = 0.05\,e^{0.01x}$

$\ln 12 = \ln e^{0.01x}$

$2.48 = 0.01\,x$

$x = 248.49$ ft

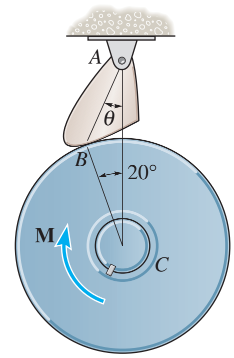
$h = 5\,e^{0.01(248.49)}$

$h = 60.0$ ft **Ans.**

Ans:
$h = 60.0$ ft

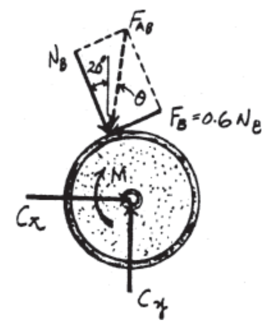
8–29.

The friction pawl is pinned at A and rests against the wheel at B. It allows freedom of movement when the wheel is rotating counterclockwise about C. Clockwise rotation is prevented due to friction of the pawl which tends to bind the wheel. If $(\mu_s)_B = 0.6$, determine the design angle θ which will prevent clockwise motion for any value of applied moment M. *Hint:* Neglect the weight of the pawl so that it becomes a two-force member.



SOLUTION

Friction: When the wheel is on the verge of rotating, slipping would have to occur. Hence, $F_B = \mu N_B = 0.6N_B$. From the force diagram (F_{AB} is the force developed in the two force member AB)



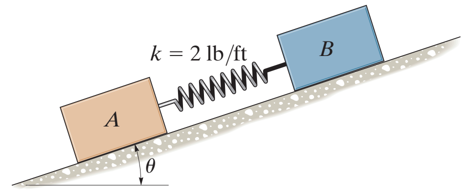
$$\tan(20° + \theta) = \frac{0.6N_B}{N_B} = 0.6$$

$$\theta = 11.0° \qquad \textbf{Ans.}$$

Ans:
$\theta = 11.0°$

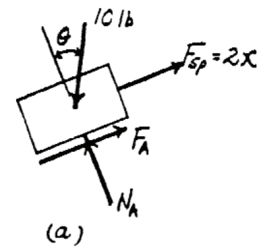
8–30.

Two blocks A and B have a weight of 10 lb and 6 lb, respectively. They are resting on the incline for which the coefficients of static friction are $\mu_A = 0.15$ and $\mu_B = 0.25$. Determine the incline angle θ for which both blocks begin to slide. Also find the required stretch or compression in the connecting spring for this to occur. The spring has a stiffness of $k = 2$ lb/ft.



SOLUTION

Equations of Equilibrium: Using the spring force formula, $F_{sp} = kx = 2x$, from FBD (a),



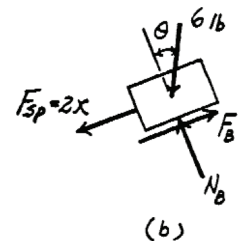
$+\nearrow \Sigma F_{x'} = 0; \qquad 2x + F_A - 10 \sin\theta = 0$ **(1)**

$\nwarrow + \Sigma F_{y'} = 0; \qquad N_A - 10\cos\theta = 0$ **(2)**

From FBD (b),

$+\nearrow \Sigma F_{x'} = 0; \qquad F_B - 2x - 6\sin\theta = 0$ **(3)**

$\nwarrow + \Sigma F_{y'} = 0; \qquad N_B - 6\cos\theta = 0$ **(4)**



Friction: If block A and B are on the verge to move, slipping would have to occur at point A and B. Hence. $F_A = \mu_{sA} N_A = 0.15 N_A$ and $F_B = \mu_{sB} N_B = 0.25 N_B$. Substituting these values into Eqs. (1), (2), (3) and (4) and solving we have

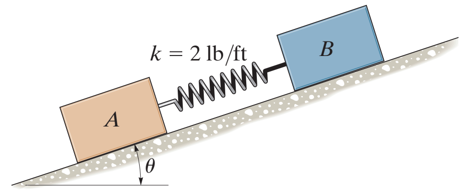
$$\theta = 10.6^\circ \qquad x = 0.184 \text{ ft}$$ **Ans.**

$$N_A = 9.829 \text{ lb} \qquad N_B = 5.897 \text{ lb}$$

Ans:
$\theta = 10.6^\circ$
$x = 0.184$ ft

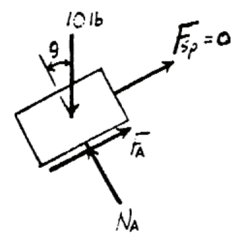
8–31.

Two blocks A and B have a weight of 10 lb and 6 lb, respectively. They are resting on the incline for which the coefficients of static friction are $\mu_A = 0.15$ and $\mu_B = 0.25$. Determine the angle θ which will cause motion of one of the blocks. What is the friction force under each of the blocks when this occurs? The spring has a stiffness of $k = 2$ lb/ft and is originally unstretched.



SOLUTION

Equations of Equilibrium: Since neither block A nor block B is moving yet, the spring force $F_{sp} = 0$. From FBD (a),



(a)

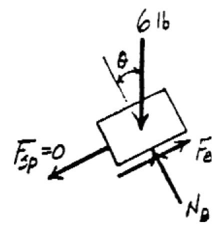
$+\nearrow\Sigma F_{x'} = 0; \qquad F_A - 10\sin\theta = 0 \qquad \textbf{(1)}$

$\nwarrow+\Sigma F_{y'} = 0; \qquad N_A - 10\cos\theta = 0 \qquad \textbf{(2)}$

From FBD (b),

$+\nearrow\Sigma F_{x'} = 0; \qquad F_B - 6\sin\theta = 0 \qquad \textbf{(3)}$

$\nwarrow+\Sigma F_{y'} = 0; \qquad N_B - 6\cos\theta = 0 \qquad \textbf{(4)}$



Friction: Assuming block A is on the verge of slipping, then

$$F_A = \mu_A N_A = 0.15 N_A \qquad \textbf{(5)}$$

Solving Eqs. (1), (2), (3), (4), and (5) yields

$$\theta = 8.531^\circ \qquad N_A = 9.889 \text{ lb} \qquad F_A = 1.483 \text{ lb}$$

$$F_B = 0.8900 \text{ lb} \qquad N_B = 5.934 \text{ lb}$$

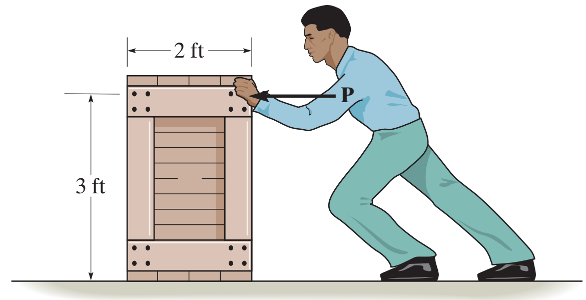
Since $(F_B)_{max} = \mu_B N_B = 0.25(5.934) = 1.483 \text{ lb} > F_B$, block B does not slip. Therefore, the above assumption is correct. Thus

$$\theta = 8.53^\circ \qquad F_A = 1.48 \text{ lb} \qquad F_B = 0.890 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$\theta = 8.53^\circ$
$F_A = 1.48$ lb
$F_B = 0.890$ lb

***8–32.**

Determine the smallest force P that must be applied in order to cause the 150-lb uniform crate to move. The coefficent of static friction between the crate and the floor is $\mu_s = 0.5$.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the crate shown in Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F - P = 0 \qquad (1)$$

$$+\uparrow \Sigma F_y = 0; \qquad N - 150 = 0 \quad N = 150 \text{ lb}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad P(3) - 150x = 0 \qquad (2)$$

Friction. Assuming that the crate slides before tipping. Thus

$$F = \mu N = 0.5(150) = 75 \text{ lb}$$

Substitute this value into Eq. (1)

$$P = 75 \text{ lb}$$

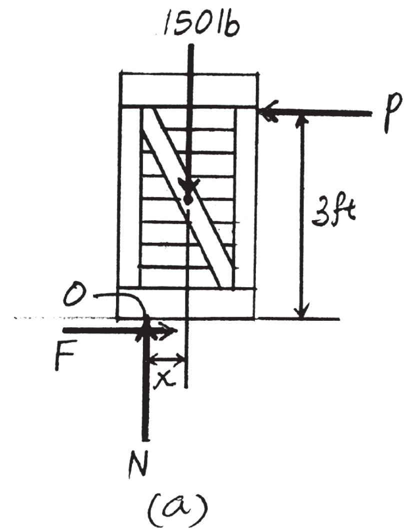
Then Eq. (2) gives

$$75(3) - 150x = 0 \quad x = 1.5 \text{ ft}$$

Since $x > 1$ ft, the crate tips before sliding. Thus, the assumption was wrong. Substitute $x = 1$ ft into Eq. (2),

$$P(3) - 150(1) = 0$$

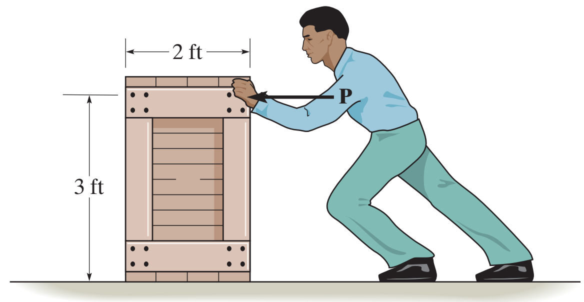
$$P = 50 \text{ lb} \qquad \textbf{Ans.}$$



Ans:
50 lb

8–33.

The man having a weight of 200 lb pushes horizontally on the crate. If the coefficient of static friction between the 450-lb crate and the floor is $\mu_s = 0.3$ and between his shoes and the floor is $\mu'_s = 0.6$, determine if he can move the crate.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the crate shown in Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_C - P = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_C - 450 = 0 \quad N_C = 450 \text{ lb}$$

$$\zeta + \Sigma M_O = 0; \qquad P(3) - 450(x) = 0 \qquad \textbf{(2)}$$

Also, from the *FBD* of the man, Fig. *b*,

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - F_m = 0 \qquad \textbf{(3)}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_m - 200 = 0 \quad N_m = 200 \text{ lb}$$

Friction. Assuming that the crate slides before tipping. Thus

$$F_C = \mu_s N_C = 0.3(450) = 135 \text{ lb}$$

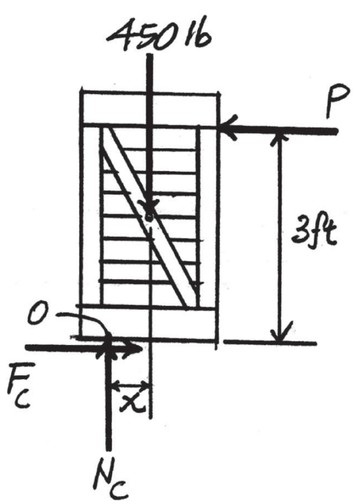
Using this result to solve Eqs. (1), (2) and (3)

$$F_m = P = 135 \text{ lb} \quad x = 0.9 \text{ ft}$$

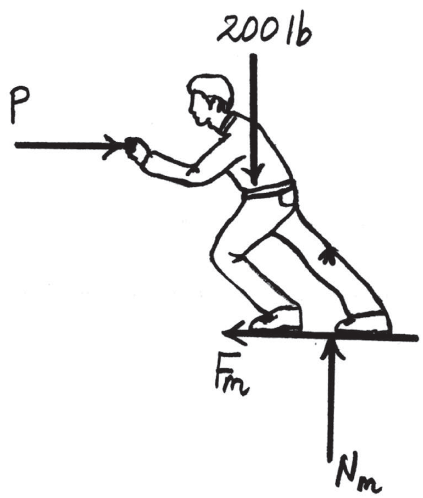
Since $x < 1$ ft, the crate indeed slides before tipping as assumed.

Also, since $F_m > (F_m)_{max} = \mu_s' N_C = 0.6(200) = 120$ lb, the man slips.

Thus **he is not able to move the crate.**



(a)

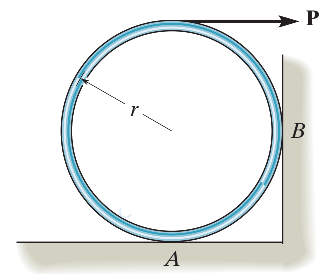


(b)

Ans:
No

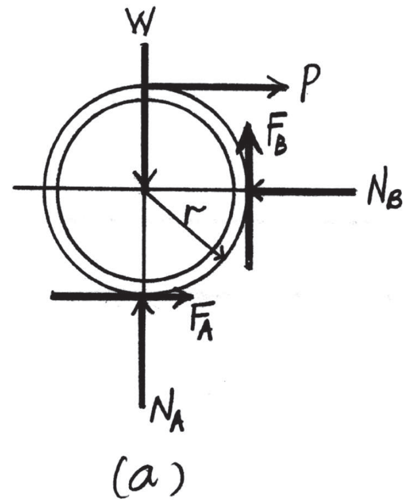
8–34.

The uniform hoop of weight W is subjected to the horizontal force P. Determine the coefficient of static friction between the hoop and the surface of A and B if the hoop is on the verge of rotating.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the hoop shown in Fig. *a*,



$\xrightarrow{+} \Sigma F_x = 0; \qquad P + F_A - N_B = 0$ **(1)**

$+\uparrow \Sigma F_y = 0; \qquad N_A + F_B - W = 0$ **(2)**

$\circlearrowleft + \Sigma M_A = 0; \qquad N_B(r) + F_B(r) - P(2r) = 0$ **(3)**

Friction. It is required that slipping occurs at point A and B. Thus

$$F_A = \mu_s N_A \quad \textbf{(4)}$$

$$F_B = \mu_s N_B \quad \textbf{(5)}$$

Substituting Eq. (5) into (3),

$$N_B r + \mu_s N_B r = 2Pr \quad N_B = \frac{2P}{1 + \mu_s} \quad \textbf{(6)}$$

Substituting Eq. (4) into (1) and Eq. (5) into (2), we obtain

$$N_B - \mu_s N_A = P \quad \textbf{(7)}$$

$$N_A + \mu_s N_B = W \quad \textbf{(8)}$$

Eliminate $\mathbf{N}_A$ from Eqs. (7) and (8),

$$N_B = \frac{P + \mu_s W}{1 + \mu_s^2} \quad \textbf{(9)}$$

Equating Eq. (6) and (9)

$$\frac{2P}{1 + \mu_s} = \frac{P + \mu_s W}{1 + \mu_s^2}$$

$$2P(1 + \mu_s^2) = (P + \mu_s W)(1 + \mu_s)$$

$$2P + 2\mu_s^2 P = P + P\mu_s + \mu_s W + \mu_s^2 W$$

$$(2P - W)\mu_s^2 - (P + W)\mu_s + P = 0$$

If $P = \frac{1}{2}W$, the quadratic term drops out, and then

$$\mu_s = \frac{P}{P + W}$$

$$= \frac{\frac{1}{2}W}{\frac{1}{2}W + W}$$

$$= \frac{1}{3} \qquad \textbf{Ans.}$$

8–34. Continued

If $P \neq \frac{1}{2}W$, then

$$\mu_s = \frac{(P + W) \pm \sqrt{[-(P + W)]^2 - 4(2P - W)P}}{2(2P - W)}$$

$$\mu_s = \frac{(P + W) \pm \sqrt{W^2 + 6PW - 7P^2}}{2(2P - W)}$$

$$\mu_s = \frac{(P + W) \pm \sqrt{(W + 7P)(W - P)}}{2(2P - W)}$$

In order to have a solution,

$$(W + 7P)(W - P) > 0$$

Since $W + 7P > 0$ then

$$W - P > 0 \quad W > P$$

Also, $P > 0$. Thus

$$0 < P < W$$

Choosing the smaller value of μ_s,

$$\mu_s = \frac{(P + W) - \sqrt{(W + 7P)(W - P)}}{2(2P - W)} \text{ for } 0 < P < W \text{ and } P \neq \frac{W}{2} \qquad \textbf{Ans.}$$

The two solutions, for $P = \frac{1}{2}W$ and $P \neq \frac{1}{2}W$, are continuous.

Note: Choosing the larger value of μ_s in the quadratic solution leads to $N_A, F_A < 0$, which is nonphysical. Also, $(\mu_s)_{\max} = 1$. For $\mu_s > 1$, the hoop will tend to climb the wall rather than rotate in place.

Ans:

If $P = \frac{1}{2}W$

$\mu_s = \frac{1}{3}$

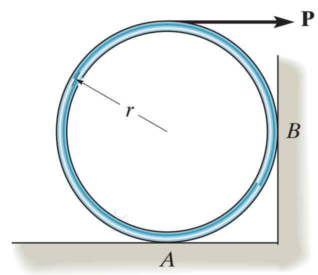
If $P \neq \frac{1}{2}W$

$$\mu_s = \frac{(P + W) - \sqrt{(W + 7P)(W - P)}}{2(2P - W)}$$

for $0 < P < W$

8–35.

Determine the maximum horizontal force **P** that can be applied to the 30-lb hoop without causing it to rotate. The coefficient of static friction between the hoop and the surfaces A and B is $\mu_s = 0.2$. Take $r = 300$ mm.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the hoop shown in Fig. *a*,

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad P + F_A - N_B = 0 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \qquad N_A + F_B - 30 = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft+\Sigma M_A = 0; \qquad F_B(0.3) + N_B(0.3) - P(0.6) = 0 \qquad \textbf{(3)}$$

Friction. Assuming that the hoop is on the verge to rotate due to the slipping occur at A and B. Then

$$F_A = \mu_s N_A = 0.2\, N_A \qquad \textbf{(4)}$$

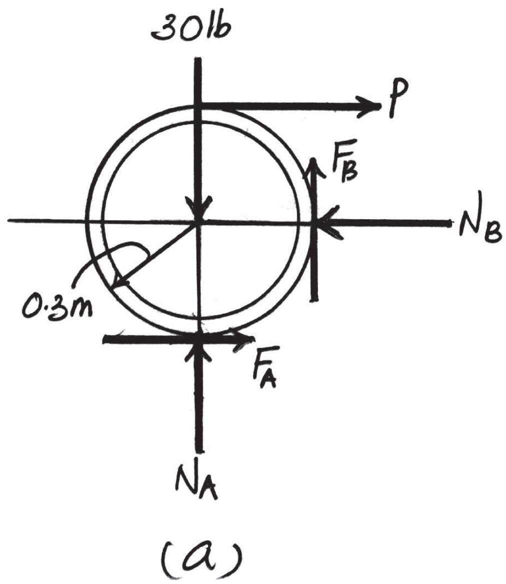
$$F_B = \mu_s N_B = 0.2\, N_B \qquad \textbf{(5)}$$

Solving Eq. (1) to (5)

$$N_A = 27.27 \text{ lb} \quad N_B = 13.64 \text{ lb} \quad F_A = 5.455 \text{ lb} \quad F_B = 2.727 \text{ lb}$$

$$P = 8.182 \text{ lb} = 8.18 \text{ lb} \qquad \textbf{Ans.}$$

Since N_A is positive, the hoop will be in contact with the floor. Thus, the assumption was correct.

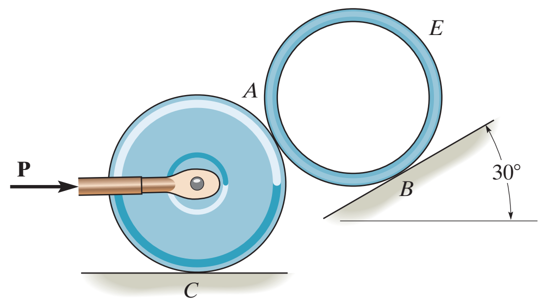


(a)

Ans:
$P = 8.18$ lb

***8–36.**

Determine the minimum force P needed to push the tube E up the incline. The force acts parallel to the plane, and the coefficients of static friction at the contacting surfaces are $\mu_A = 0.2$, $\mu_B = 0.3$, and $\mu_C = 0.4$. The 100-kg roller and 40-kg tube each have a radius of 150 mm.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the roller, Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \quad P - N_A \cos 30° - F_A \sin 30° - F_C = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \quad N_C + F_A \cos 30° - N_A \sin 30° - 100(9.81) = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_D = 0; \quad F_A(0.15) - F_C(0.15) = 0 \qquad \textbf{(3)}$$

Also, for the *FBD* of the tube, Fig. *b*,

$$+\nearrow \Sigma F_x = 0; \quad N_A - F_B - 40(9.81) \sin 30° = 0 \qquad \textbf{(4)}$$

$$+\nwarrow \Sigma F_y = 0; \quad N_B - F_A - 40(9.81) \cos 30° = 0 \qquad \textbf{(5)}$$

$$\circlearrowleft + \Sigma M_E = 0; \quad F_A(0.15) - F_B(0.15) = 0 \qquad \textbf{(6)}$$

Friction. Assuming that slipping is about to occur at A. Thus

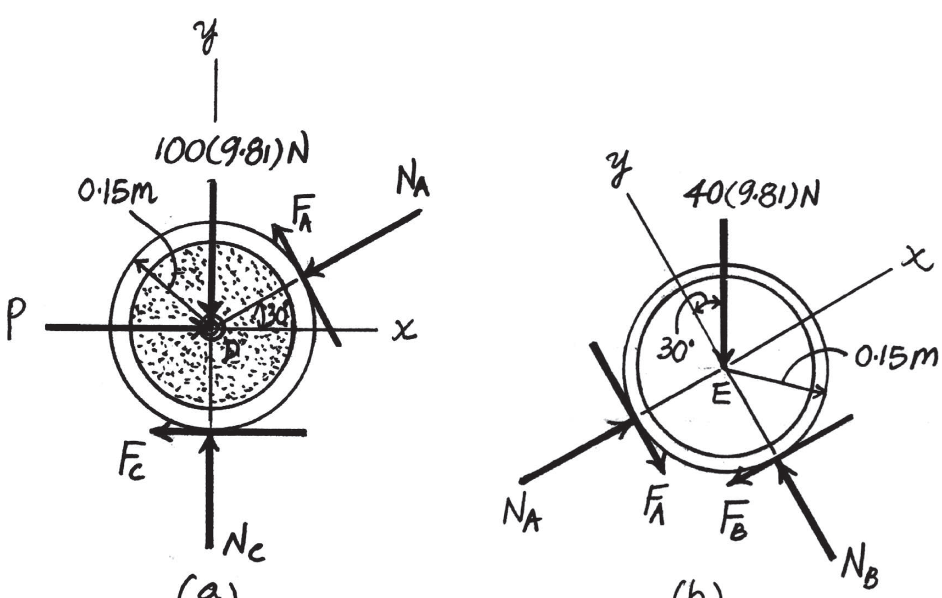
$$F_A = \mu_A N_A = 0.2\, N_A \qquad \textbf{(7)}$$

Solving Eqs. (1) to (7)

$$P = 285.97 \text{ N} = 286 \text{ N} \qquad \textbf{Ans.}$$

$N_A = 245.25$ N $\quad N_B = 388.88$ N $\quad N_C = 1061.15$ N $\quad F_A = F_B = F_C = 49.05$ N

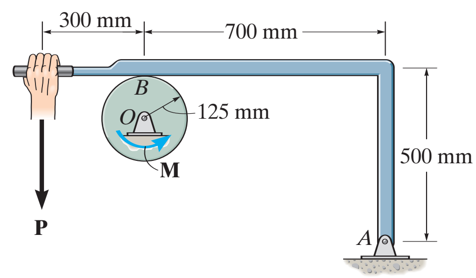
Since $F_B < (F_B)_{max} = \mu_B N_B = 0.3(388.88) = 116.66$ N and $F_C < (F_C)_{max} = \mu_C N_C = 0.4(1061.15) = 424.46$ N, slipping indeed will not occur at B and C. Thus, the assumption was correct.



Ans:
286 N

8–37.

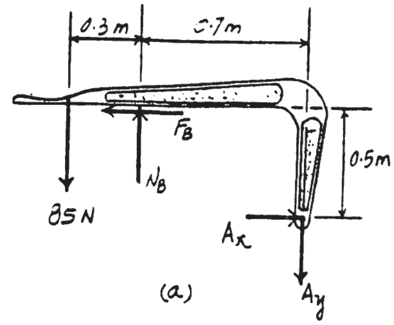
The coefficients of static and kinetic friction between the drum and brake bar are $\mu_s = 0.4$ and $\mu_k = 0.3$, respectively. If $M = 50\text{ N}\cdot\text{m}$ and $P = 85\text{ N}$ determine the horizontal and vertical components of reaction at the pin O. Neglect the weight and thickness of the brake. The drum has a mass of 25 kg.



SOLUTION

Equations of Equilibrium: From FBD (b),

$$\circlearrowleft + \Sigma M_O = 0 \qquad 50 - F_B(0.125) = 0 \qquad F_B = 400\text{ N}$$

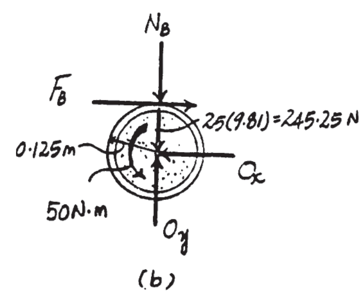


From FBD (a),

$$\circlearrowleft + \Sigma M_A = 0; \qquad 85(1.00) + 400(0.5) - N_B(0.7) = 0$$

$$N_B = 407.14\text{ N}$$

Friction: Since $F_B > (F_B)_{\max} = \mu_s N_B = 0.4(407.14) = 162.86\text{ N}$, the drum slips at point B and rotates. Therefore, the coefficient of kinetic friction should be used. Thus, $F_B = \mu_k N_B = 0.3 N_B$.



Equations of Equilibrium: From FBD (b),

$$\circlearrowleft + \Sigma M_A = 0; \qquad 85(1.00) + 0.3N_B(0.5) - N_B(0.7) = 0$$

$$N_B = 154.54\text{ N}$$

From FBD (b),

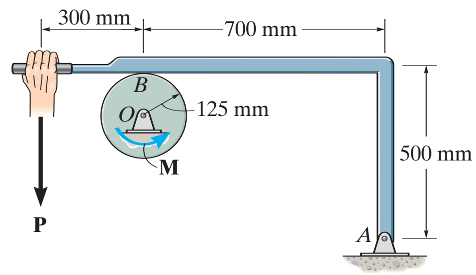
$$+\uparrow \Sigma F_y = 0; \qquad O_y - 245.25 - 154.54 = 0 \qquad O_y = 400\text{ N} \qquad \textbf{Ans.}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.3(154.54) - O_x = 0 \qquad O_x = 46.4\text{ N} \qquad \textbf{Ans.}$$

Ans:
$O_y = 400\text{ N}$
$O_x = 46.4\text{ N}$

8–38.

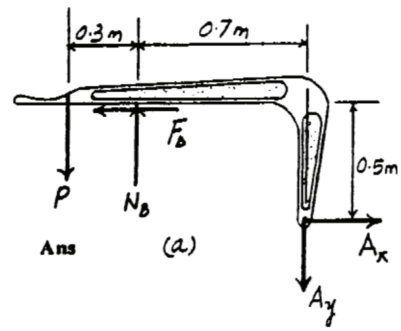
The coefficient of static friction between the drum and brake bar is $\mu_s = 0.4$. If the moment $M = 35\ \text{N}\cdot\text{m}$, determine the smallest force P that needs to be applied to the brake bar in order to prevent the drum from rotating. Also determine the corresponding horizontal and vertical components of reaction at pin O. Neglect the weight and thickness of the brake bar. The drum has a mass of 25 kg.



SOLUTION

Equations of Equilibrium: From FBD (b),

$\circlearrowleft + \Sigma M_O = 0 \qquad 35 - F_B(0.125) = 0 \qquad F_B = 280\ \text{N}$



From FBD (a),

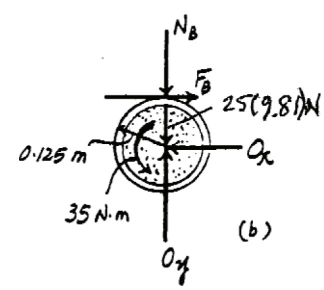
$\circlearrowleft + \Sigma M_A = 0; \qquad P(1.00) + 280(0.5) - N_B(0.7) = 0$

Friction: When the drum is on the verge of rotating,

$$F_B = \mu_s N_B$$

$$280 = 0.4 N_B$$

$$N_B = 700\ \text{N}$$



Substituting $N_B = 700$ N into Eq. [1] yields

$$P = 350\ \text{N} \qquad \textbf{Ans.}$$

Equations of Equilibrium: From FBD (b),

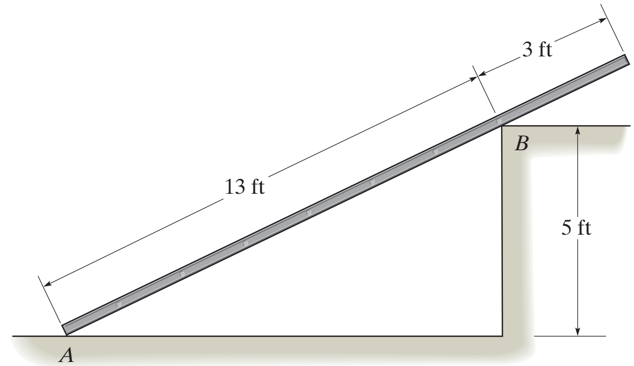
$+\uparrow \Sigma F_y = 0; \qquad O_y - 245.25 - 700 = 0 \qquad O_y = 945\ \text{N}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0; \qquad 280 - O_x = 0 \qquad O_x = 280\ \text{N}$ **Ans.**

Ans:
$P = 350$ N
$O_y = 945$ N
$O_x = 280$ N

8–39.

Determine the smallest coefficient of static friction at both A and B needed to hold the uniform 100-lb bar in equilibrium. Neglect the thickness of the bar. Take $\mu_A = \mu_B = \mu$.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the bar shown in Fig. *a*,

$$\zeta+\Sigma M_A = 0; \qquad N_B(13) - 100\left(\frac{12}{13}\right)(8) = 0 \quad N_B = 56.80 \text{ lb}$$

$$\xrightarrow{+}\Sigma F_x = 0; \qquad F_A + F_B\left(\frac{12}{13}\right) - 56.80\left(\frac{5}{13}\right) = 0 \qquad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \qquad N_A + F_B\left(\frac{5}{13}\right) + 56.80\left(\frac{12}{13}\right) - 100 = 0 \qquad \textbf{(2)}$$

Friction. It is required that slipping occurs at A and B. Thus

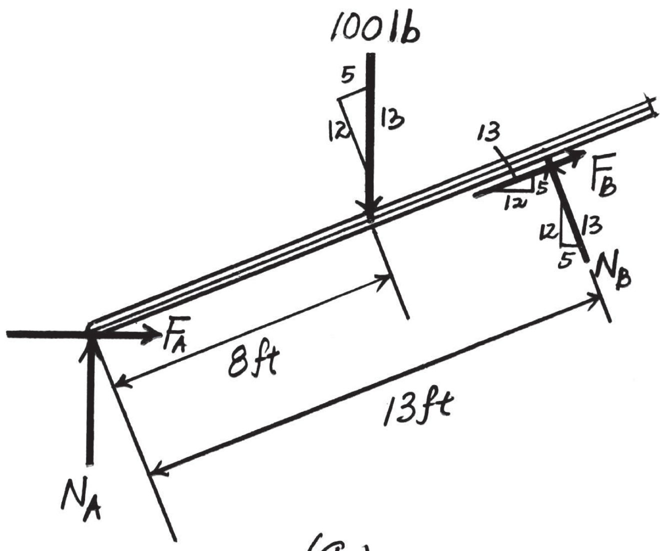
$$F_A = \mu_s N_A \qquad \textbf{(3)}$$

$$F_B = \mu_s N_B = \mu_s(56.80) \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4)

$$\mu_s = 0.230 \qquad \textbf{Ans.}$$

$$N_A = 42.54 \text{ lb} \quad F_A = 9.786 \text{ lb} \quad F_B = 13.07 \text{ lb}$$

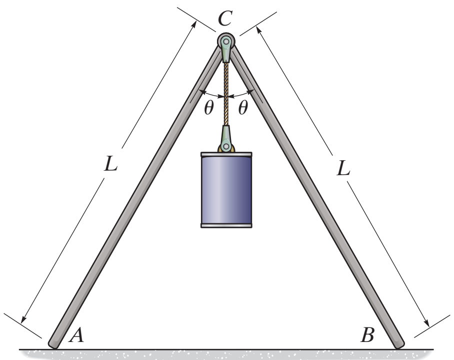


(a)

Ans:
$\mu_s = 0.230$

***8–40.**

If $\theta = 30°$ determine the minimum coefficient of static friction at A and B so that equilibrium of the supporting frame is maintained regardless of the mass of the cylinder C. Neglect the mass of the rods.



SOLUTION

Free-Body Diagram: Due to the symmetrical loading and system, ends A and B of the rod will slip simultaneously. Since end B tends to move to the right, the friction force $\mathbf{F}_B$ must act to the left as indicated on the free-body diagram shown in Fig. *a*.

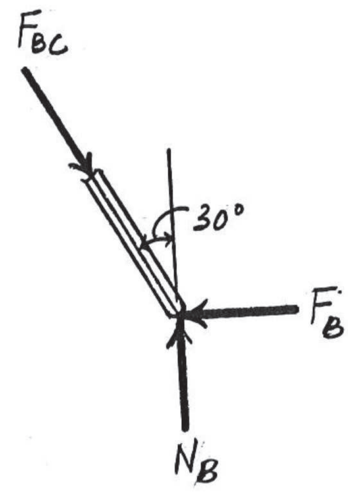
Equations of Equilibrium: We have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{BC} \sin 30° - F_B = 0 \qquad F_B = 0.5F_{BC}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B - F_{BC} \cos 30° = 0 \qquad N_B = 0.8660\, F_{BC}$$

Therefore, to prevent slipping the coefficient of static friction ends A and B must be at least

$$\mu_s = \frac{F_B}{N_B} = \frac{0.5F_{BC}}{0.8660F_{BC}} = 0.577 \qquad \textbf{Ans.}$$

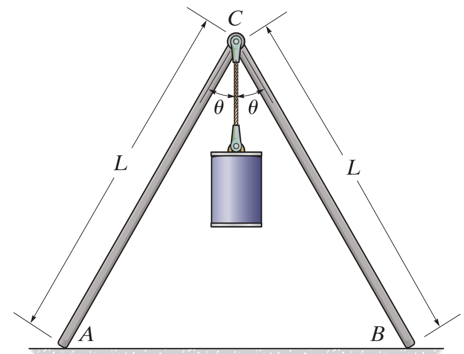


(a)

Ans:
$\mu_s = 0.577$

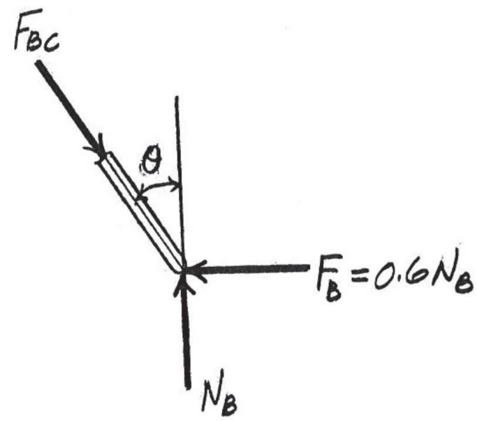
8–41.

If the coefficient of static friction at A and B is $\mu_s = 0.6$, determine the maximum angle θ so that the frame remains in equilibrium, regardless of the mass of the cylinder. Neglect the mass of the rods.



SOLUTION

Free-Body Diagram: Due to the symmetrical loading and system, ends A and B of the rod will slip simultaneously. Since end B is on the verge of sliding to the right, the friction force F_B must act to the left such that $F_B = \mu_s N_B = 0.6N_B$ as indicated on the free-body diagram shown in Fig. *a*.



Equations of Equilibrium: We have

$+\uparrow \Sigma F_y = 0;\qquad N_B - F_{BC}\cos\theta = 0 \qquad N_B = F_{BC}\cos\theta$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad F_{BC}\sin\theta - 0.6(F_{BC}\cos\theta) = 0$

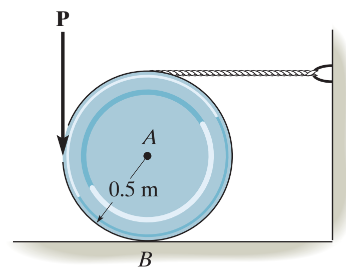
$\tan\theta = 0.6$

$\theta = 31.0°$ **Ans.**

Ans:
$\theta = 31.0°$

8–42.

The 100-kg disk rests on a surface for which $\mu_B = 0.2$. Determine the smallest vertical force **P** that can be applied tangentially to the disk which will cause motion to impend.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the disk shown in Fig. *a*,

$$+\uparrow \Sigma F_y = 0; \qquad N_B - P - 100(9.81) = 0 \tag{1}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad P(0.5) - F_B(1) = 0 \tag{2}$$

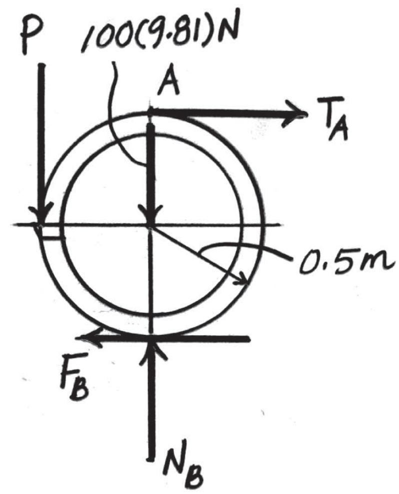
Friction. It is required that slipping impends at B.Thus,

$$F_B = \mu_B N_B = 0.2\, N_B \tag{3}$$

Solving Eqs. (1), (2) and (3)

$$P = 654 \text{ N} \qquad \textbf{Ans.}$$

$$N_B = 1635 \text{ N} \quad F_B = 327 \text{ N}$$

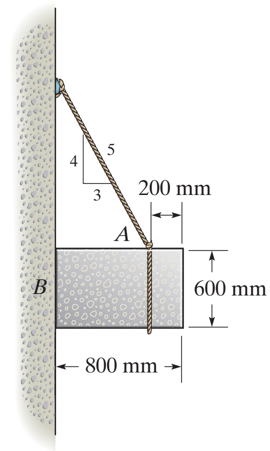


(a)

Ans:
$P = 654$ N

8–43.

Investigate whether the equilibrium can be maintained. The uniform block has a mass of 500 kg, and the coefficient of static friction is $\mu_s = 0.3$.



SOLUTION

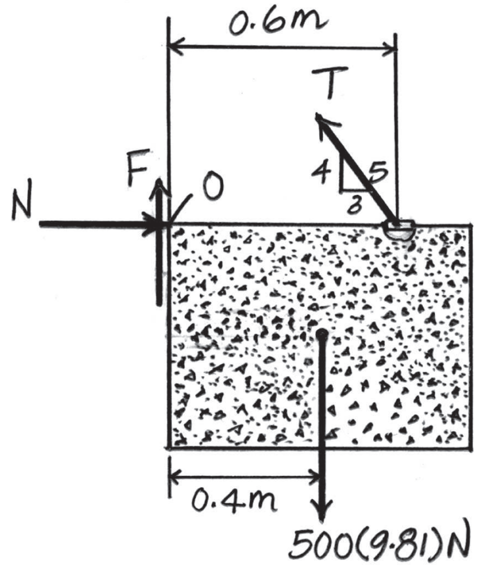
Equations of Equilibrium. The block would move only if it slips at corner O. Referring to the *FBD* of the block shown in Fig. *a*,

$$\zeta + \Sigma M_O = 0; \qquad T\left(\frac{4}{5}\right)(0.6) - 500(9.81)(0.4) = 0 \quad T = 4087.5 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N - 4087.5\left(\frac{3}{5}\right) = 0 \quad N = 2452.5 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad F + 4087.5\left(\frac{4}{5}\right) - 500(9.81) = 0 \quad F = 1635 \text{ N}$$

Friction. Since $F > (F)_{\max} = \mu_s N = 0.3(2452.5) = 735.75$ N, slipping occurs at O. Thus, **the block fails to be in equilibrium.**

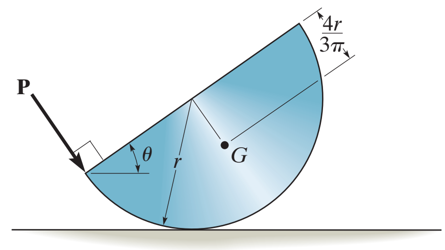


(a)

Ans:
The block fails to be in equilibrium.

***8–44.**

The homogenous semicylinder has a mass of 20 kg and mass center at G. If force **P** is applied at the edge, and $r = 300$ mm, determine the angle θ at which the semicylinder is on the verge of slipping. The coefficient of static friction between the plane and the cylinder is $\mu_s = 0.3$. Also, what is the corresponding force P for this case?



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the semicylinder shown in Fig. *a*,

$$\xrightarrow{+} \Sigma F_x = 0; \quad P \sin\theta - F = 0 \quad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \quad N - P\cos\theta - 20(9.81) = 0 \quad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_A = 0; \quad P[0.3(1 - \sin\theta)] - 20(9.81)\left[\frac{4(0.3)}{3\pi}\sin\theta\right] = 0$$

$$P = \frac{261.6}{\pi}\left(\frac{\sin\theta}{1 - \sin\theta}\right) \quad \textbf{(3)}$$

Friction. Since the semicylinder is required to be on the verge to slip at point A,

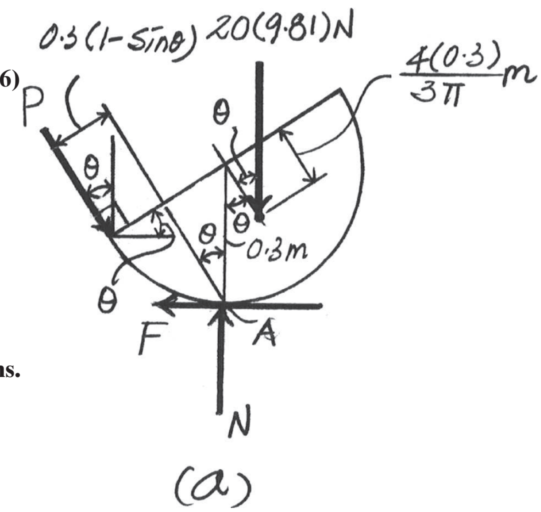
$$F = \mu_s N = 0.3\,\text{N} \quad \textbf{(4)}$$

Substitute Eq. (4) into (1),

$$P\sin\theta - 0.3\,\text{N} = 0 \quad \textbf{(5)}$$

Eliminate N from Eqs. (2) and (5), we obtain

$$P = \frac{58.86}{\sin\theta - 0.3\cos\theta} \quad \textbf{(6)}$$



(a)

Equating Eq. (3) and (6)

$$\frac{261.6}{\pi}\left(\frac{\sin\theta}{1 - \sin\theta}\right) = \frac{58.56}{\sin\theta - 0.3\cos\theta}$$

$$\sin\theta(\sin\theta - 0.3\cos\theta + 0.225\,\pi) - 0.225\,\pi = 0$$

Solving by trial and error

$$\theta = 39.50° = 39.5° \qquad \textbf{Ans.}$$

Substitute the result into Eq. 6

$$P = \frac{58.86}{\sin 39.50° - 0.3\cos 39.50°}$$

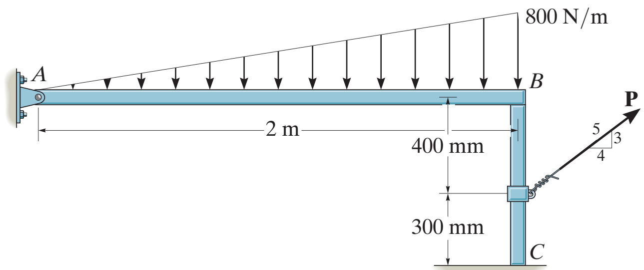
$$= 145.51\text{ N} \qquad \textbf{Ans.}$$

$$= 146\text{ N}$$

Ans:
146 N

8–45.

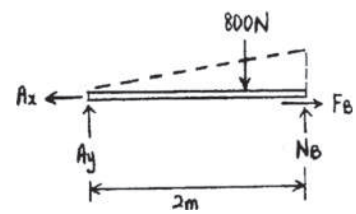
The beam AB has a negligible mass and thickness and is subjected to a triangular distributed loading. It is supported at one end by a pin and at the other end by a post having a mass of 50 kg and negligible thickness. Determine the minimum force P needed to move the post. The coefficients of static friction at B and C are $\mu_B = 0.4$ and $\mu_C = 0.2$, respectively.



SOLUTION

Member AB:

$$\circlearrowleft + \Sigma M_A = 0; \qquad -800\left(\frac{4}{3}\right) + N_B(2) = 0$$



$$N_B = 533.3 \text{ N}$$

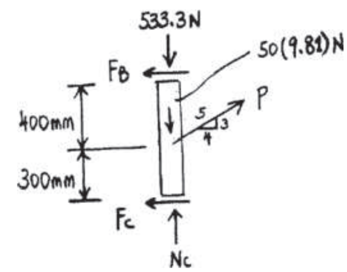
Post:

Assume slipping occurs at C; $F_C = 0.2\,N_C$

$$\circlearrowleft + \Sigma M_C = 0; \qquad -\frac{4}{5}P(0.3) + F_B(0.7) = 0$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{4}{5}P - F_B - 0.2N_C = 0$$

$$+\uparrow \Sigma F_y = 0; \qquad \frac{3}{5}P + N_C - 533.3 - 50(9.81) = 0$$



$$P = 355 \text{ N}$$ **Ans.**

$$N_C = 811.0 \text{ N}$$

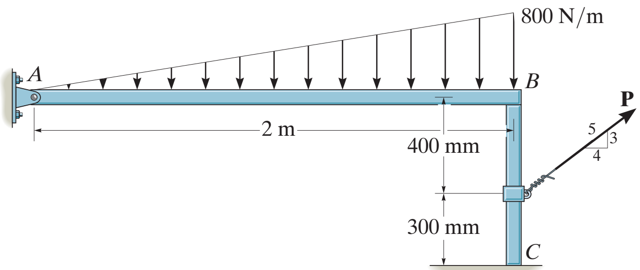
$$F_B = 121.6 \text{ N}$$

$$(F_B)_{max} = 0.4(533.3) = 213.3 \text{ N} > 121.6 \text{ N}$$ **(O.K.!)**

Ans:
$P = 335$ N

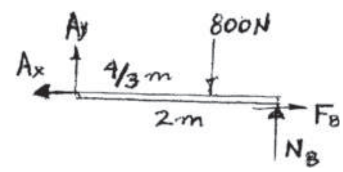
8–46.

The beam AB has a negligible mass and thickness and is subjected to a triangular distributed loading. It is supported at one end by a pin and at the other end by a post having a mass of 50 kg and negligible thickness. Determine the two coefficients of static friction at B and at C so that when the magnitude of the applied force is increased to $P = 150$ N, the post slips at both B and C simultaneously.



SOLUTION

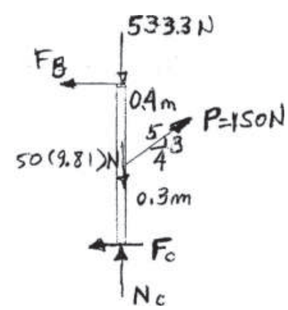
Member AB:



$\circlearrowleft + \Sigma M_A = 0; \qquad -800\left(\frac{4}{3}\right) + N_B(2) = 0$

$N_B = 533.3$ N

Post:



$+\uparrow \Sigma F_y = 0; \qquad N_C - 533.3 + 150\left(\frac{3}{5}\right) - 50(9.81) = 0$

$N_C = 933.83$ N

$\circlearrowleft + \Sigma M_C = 0; \qquad -\frac{4}{5}(150)(0.3) + F_B(0.7) = 0$

$F_B = 51.429$ N

$\xrightarrow{+} \Sigma F_x = 0; \qquad \frac{4}{5}(150) - F_C - 51.429 = 0$

$F_C = 68.571$ N

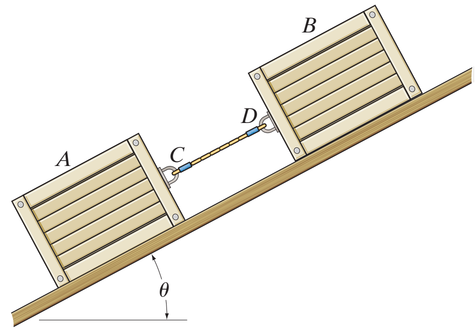
$$\mu_C = \frac{F_C}{N_C} = \frac{68.571}{933.83} = 0.0734 \qquad \textbf{Ans.}$$

$$\mu_B = \frac{F_B}{N_B} = \frac{51.429}{533.3} = 0.0964 \qquad \textbf{Ans.}$$

Ans:
$\mu_C = 0.0734$
$\mu_B = 0.0964$

8–47.

Crates A and B weigh 200 lb and 150 lb, respectively. They are connected together with a cable and placed on the inclined plane. If the angle θ is gradually increased, determine θ when the crates begin to slide. The coefficients of static friction between the crates and the plane are $\mu_A = 0.25$ and $\mu_B = 0.35$.



SOLUTION

Free - Body Diagram. Since both crates are required to be on the verge of sliding down the plane, the frictional forces $\mathbf{F}_A$ and $\mathbf{F}_B$ must act up the plane so that $F_A = \mu_A N_A = 0.25 N_A$ and $F_B = \mu_B N_B = 0.35 N_B$ as indicated on the free - body diagram of the crates shown in Figs. *a* and *b*.

Equations of Equilibrium. Referring to Fig. *a*,

$$\nwarrow + \Sigma F_{y'} = 0; \quad N_A - 200 \cos \theta = 0 \qquad N_A = 200 \cos \theta$$

$$+\nearrow \Sigma F_{x'} = 0; \quad F_{CD} + 0.25(200 \cos \theta) - 200 \sin \theta = 0 \qquad \textbf{(1)}$$

Also, by referring to Fig. *b*,

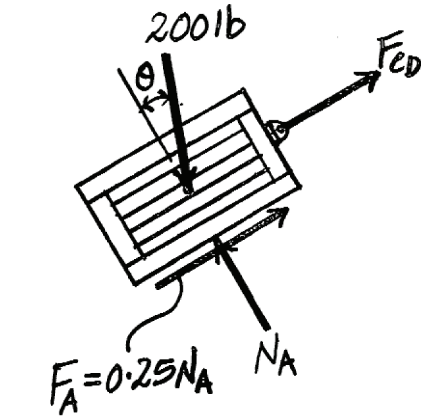
$$\nwarrow + \Sigma F_{y'} = 0; \quad N_B - 150 \cos \theta = 0 \qquad N_B = 150 \cos \theta$$

$$+\nearrow \Sigma F_{x'} = 0; \quad 0.35(150 \cos \theta) - F_{CD} - 150 \sin \theta = 0 \qquad \textbf{(2)}$$

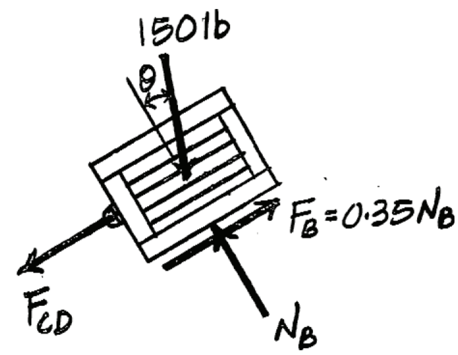
Solving Eqs. (1) and (2), yields

$$\theta = 16.3° \qquad \textbf{Ans.}$$

$$F_{CD} = 8.23 \text{ lb}$$



(a)

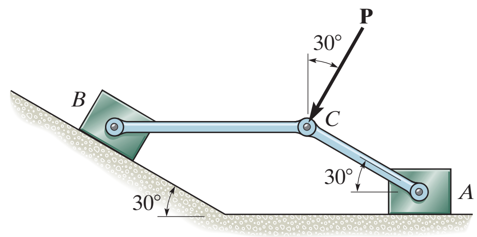


(b)

Ans:
$\theta = 16.3°$

***8–48.**

Two blocks A and B, each having a mass of 5 kg, are connected by the linkage shown. If the coefficient of static friction at the contacting surfaces is $\mu_s = 0.5$, determine the largest force P that can be applied to pin C of the linkage without causing the blocks to move. Neglect the weight of the links.



SOLUTION

Equations of Equilibrium. Analyze the equilibrium of Joint C Fig. a,

$$+\uparrow \Sigma F_y = 0; \quad F_{AC} \sin 30^\circ - P \cos 30^\circ = 0 \quad F_{AC} = \sqrt{3}\,P$$

$$\xrightarrow{+} \Sigma F_x = 0; \quad F_{BC} - P \sin 30^\circ - (\sqrt{3}P) \cos 30^\circ = 0 \quad F_{BC} = 2\,P$$

Referring to the FBD of block B, Fig. b

$$+\nwarrow \Sigma F_x = 0; \quad 2\,P \cos 30^\circ - F_B - 5(9.81) \sin 30^\circ = 0 \tag{1}$$

$$+\nearrow \Sigma F_y = 0; \quad N_B - 2\,P \sin 30^\circ - 5(9.81) \cos 30^\circ = 0 \tag{2}$$

Also, the FBD of block A, Fig. C

$$\xrightarrow{+} \Sigma F_x = 0; \quad \sqrt{3}P \cos 30^\circ - F_A = 0 \tag{3}$$

$$+\uparrow \Sigma F_y = 0; \quad N_A - \sqrt{3}P \sin 30^\circ - 5(9.81) = 0 \tag{4}$$

Friction. Assuming that block A slides first. Then

$$F_A = \mu_s N_A = 0.5\,N_A \tag{5}$$

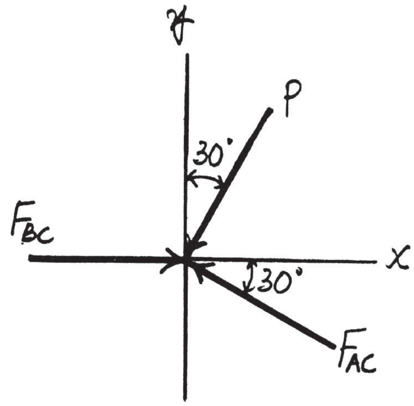
Solving Eqs. (1) to (5)

$$P = 22.99 \text{ N} = 23.0 \text{ N} \qquad \textbf{Ans.}$$

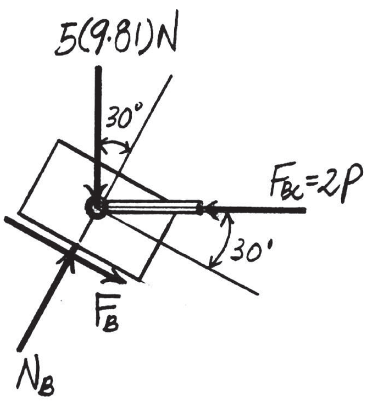
$$N_A = 68.96 \text{ N} \quad F_A = 34.48 \text{ N} \quad F_B = 15.29 \text{ N} \quad N_B = 65.46 \text{ N}$$

Since $F_B < (F_B)_{\max} = \mu_s N_B = 0.5(65.46) = 32.73$ N, Block B will not slide.

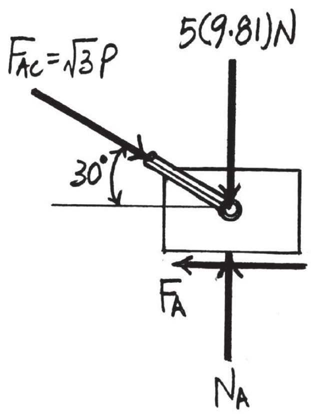
Thus, the assumption was correct.



(a)



(b)

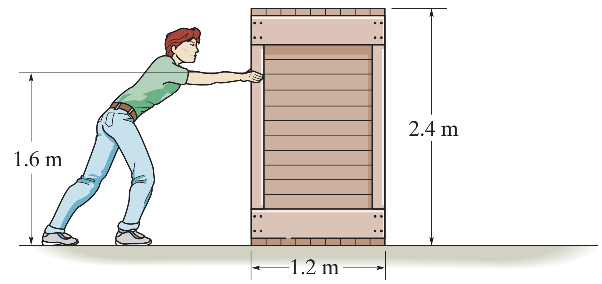


(c)

Ans:
23.0 N

8–49.

The uniform crate has a mass of 150 kg. If the coefficient of static friction between the crate and the floor is $\mu_s = 0.2$, determine whether the 85-kg man can move the crate. The coefficient of static friction between his shoes and the floor is $\mu'_s = 0.4$. Assume the man only exerts a horizontal force on the crate.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the crate shown in Fig. *a*,

$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad P - F_C = 0$ **(1)**

$+\uparrow\Sigma F_y = 0; \quad N_C - 150(9.81) = 0 \quad N_C = 1471.5\text{ N}$

$\circlearrowleft + \Sigma M_O = 0; \quad 150(9.81)x - P(1.6) = 0$ **(2)**

Also, from the *FBD* of the man, Fig. *b*,

$+\uparrow\Sigma F_y = 0; \quad N_m - 85(9.81) = 0 \quad N_m = 833.85\text{ N}$

$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad F_m - P = 0$ **(3)**

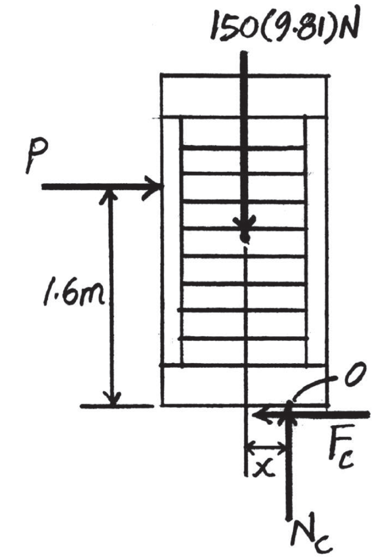
Friction. Assuming that the crate slips before tipping. Then

$$F_C = \mu_s N_C = 0.2(1471.5) = 294.3\text{ N}$$

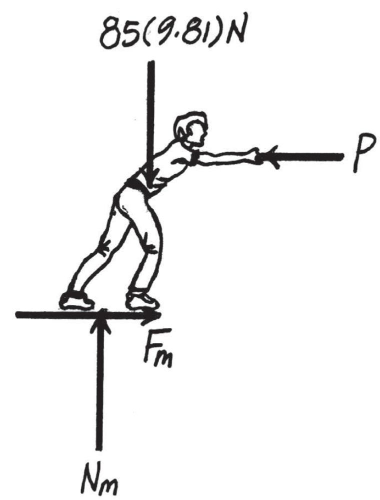
Solving Eqs. (1) to (3) using this result,

$$F_m = P = 294.3\text{ N} \quad x = 0.32\text{ m}$$

Since $x < 0.6$ m, the crate indeed slips before tipping as assumed. Also since $F_m < (F_m)_{\max} = \mu_s' N_m = 0.4(833.85) = 333.54$ N, the man will not slip. Therefore, **he is able to move the crate.**



(a)

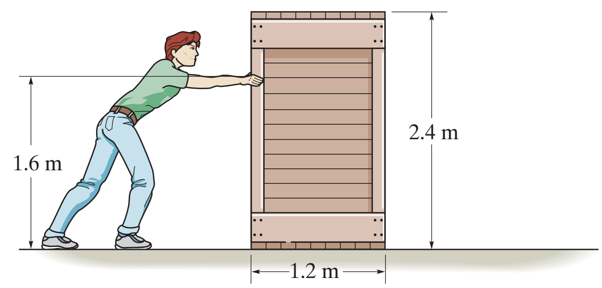


(b)

Ans:
He is able to move the crate.

8–50.

The uniform crate has a mass of 150 kg. If the coefficient of static friction between the crate and the floor is $\mu_s = 0.2$, determine the smallest mass of the man so he can move the crate. The coefficient of static friction between his shoes and the floor is $\mu'_s = 0.45$. Assume the man exerts only a horizontal force on the crate.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the crate shown in Fig. *a*,

$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad P - F_C = 0$ **(1)**

$+\uparrow\Sigma F_y = 0; \quad N_C - 150(9.81) = 0 \quad N_C = 1471.5\text{ N}$

$\circlearrowleft +\Sigma M_O = 0; \quad 150(9.81)x - P(1.6) = 0$ **(2)**

Also, from the *FBD* of the man, Fig. *b*,

$+\uparrow\Sigma F_y = 0; \quad N_m - m(9.81) = 0 \quad N_m = 9.81\text{ m}$ **(3)**

$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad F_m - P = 0$ **(4)**

Friction. Assuming that the crate slips before tipping. Then

$$F_C = \mu_s N_C = 0.2(1471.5) = 294.3\text{ N}$$

Also, it is required that the man is on the verge of slipping. Then

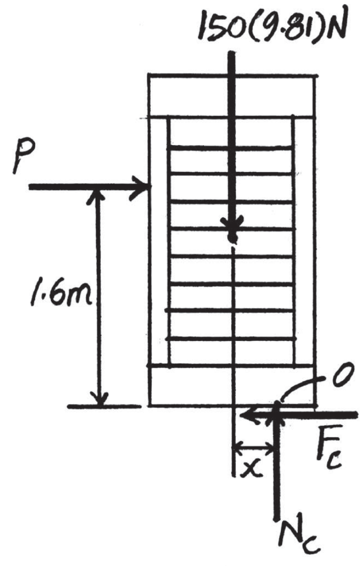
$$F_m = \mu_s' N_m = 0.45\, N_m \quad \textbf{(5)}$$

Solving Eqs. (1) to (5) using the result of $\mathbf{F}_C$,

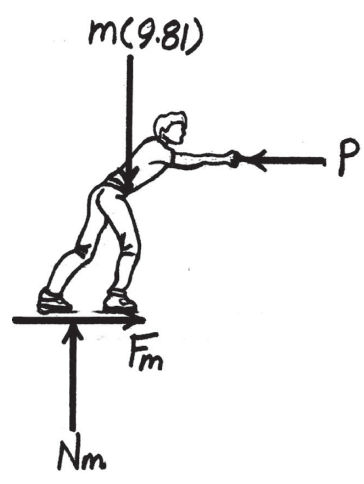
$$F_m = P = 294.3\text{ N} \quad x = 0.32\text{ m} \quad N_m = 654\text{ N}$$

$$m = 66.667\text{ kg} = 66.7\text{ kg} \qquad \textbf{Ans.}$$

Since $x < 0.6$ m, the crate indeed slips before tipping as assumed.



(a)

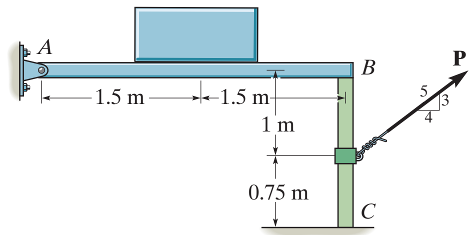


(b)

Ans:
$m = 66.7$ kg

8–51.

Beam AB has a negligible mass and thickness, and supports the 200-kg uniform block. It is pinned at A and rests on the top of a post, having a mass of 20 kg and negligible thickness. Determine the minimum force P needed to move the post. The coefficients of static friction at B and C are $\mu_B = 0.4$ and $\mu_C = 0.2$, respectively.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of member AB shown in Fig. a,

$$\circlearrowleft + \Sigma M_A = 0; \quad N_B(3) - 200(9.81)(1.5) = 0 \quad N_B = 981 \text{ N}$$

Then consider the *FBD* of member BC shown in Fig. b,

$$+\uparrow \Sigma F_y = 0; \quad N_C + P\left(\frac{3}{5}\right) - 981 - 20(9.81) = 0 \qquad \textbf{(1)}$$

$$\circlearrowleft + \Sigma M_C = 0; \quad F_B(1.75) - P\left(\frac{4}{5}\right)(0.75) = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_B = 0; \quad P\left(\frac{4}{5}\right)(1) - F_C(1.75) = 0 \qquad \textbf{(3)}$$

Friction. Assuming that slipping occurs at C. Then

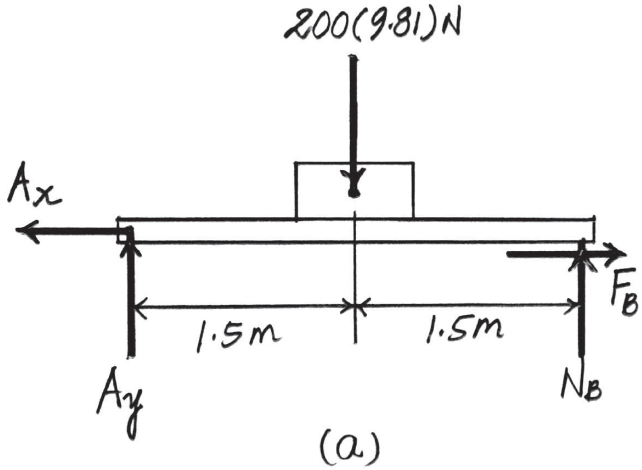
$$F_C = \mu_C N_C = 0.2\, N_C \qquad \textbf{(4)}$$

Solving Eqs. (1) to (4)

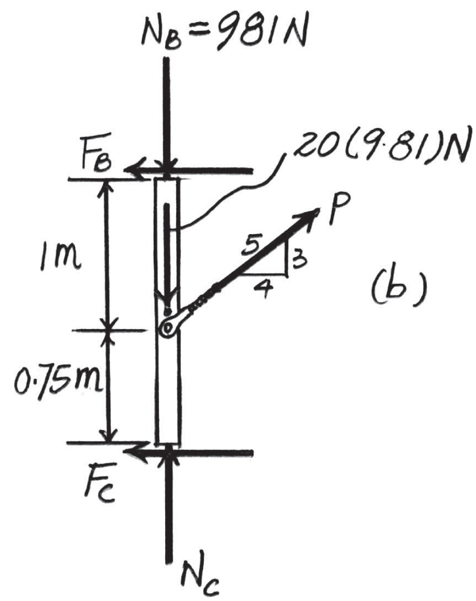
$$P = 407.94 \text{ N} = 408 \text{ N} \qquad \textbf{Ans.}$$

$$N_C = 932.44 \text{ N} \quad F_C = 186.49 \text{ N} \quad F_B = 139.87$$

Since $F_B < (F_B)_{\max} = \mu_B N_B = 0.4(981) \text{ N} = 392.4$. Indeed slipping will not occur at B. Thus, the assumption is correct.



(a)

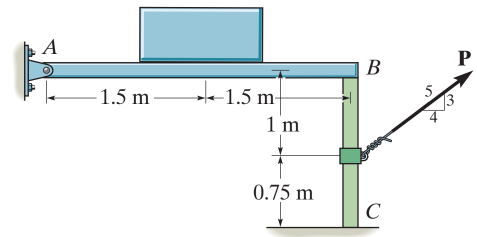


(b)

Ans:
$P = 408$ N

***8–52.**

Beam AB has a negligible mass and thickness, and supports the 200-kg uniform block. It is pinned at A and rests on the top of a post, having a mass of 20 kg and negligible thickness. Determine the two coefficients of static friction at B and at C so that when the magnitude of the applied force is increased to $P = 300$ N, the post slips at both B and C simultaneously.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of member AB shown in Fig. a,

$$\zeta+\Sigma M_A = 0; \qquad N_B(3) - 200(9.81)(1.5) = 0 \quad N_B = 981 \text{ N}$$

Then consider the *FBD* of member BC shown in Fig. b,

$$+\uparrow \Sigma F_y = 0; \qquad N_C + 300\left(\frac{3}{5}\right) - 981 - 20(9.81) = 0 \quad N_C = 997.2 \text{ N}$$

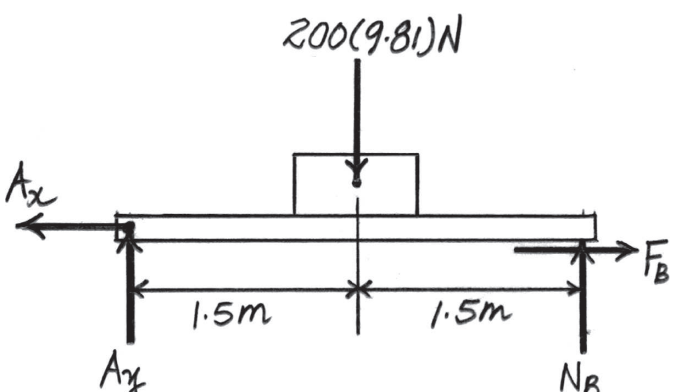
$$\zeta+\Sigma M_C = 0; \qquad F_B(1.75) - 300\left(\frac{4}{5}\right)(0.75) = 0 \qquad F_B = 102.86 \text{ N}$$

$$\zeta+\Sigma M_B = 0; \qquad 300\left(\frac{4}{5}\right)(1) - F_C(1.75) = 0 \qquad F_C = 137.14 \text{ N}$$

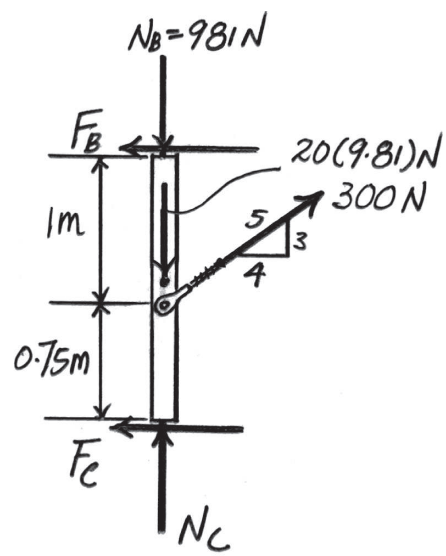
Friction. It is required that slipping occurs at B and simultaneously. Then

$$F_B = \mu_B N_B; \qquad 102.86 = \mu_B(981) \qquad \mu_B = 0.1048 = 0.105 \qquad \textbf{Ans.}$$

$$F_C = \mu_C N_C; \qquad 137.14 = \mu_C(997.2) \quad \mu_C = 0.1375 = 0.138 \qquad \textbf{Ans.}$$



(a)

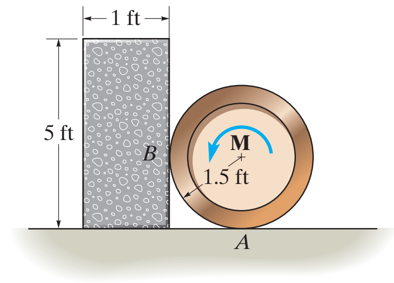


(b)

Ans:
$\mu_B = 0.105$
$\mu_C = 0.138$

8–53.

Determine the smallest couple moment that can be applied to the 150-lb wheel that will cause impending motion. The uniform concrete block has a weight of 300 lb. The coefficients of static friction are $\mu_A = 0.2$, $\mu_B = 0.3$, and between the concrete block and the floor, $\mu = 0.4$.



SOLUTION

Equations of Equilibrium. Referring to the *FBD* of the concrete block, Fig. *a*.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_C - N_B = 0 \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_C - F_B - 300 = 0 \qquad \textbf{(2)}$$

$$\circlearrowleft + \Sigma M_O = 0; \qquad N_B(1.5) - 300x - F_B(0.5 + x) = 0 \qquad \textbf{(3)}$$

Also, from the *FBD* of the wheel, Fig. *b*.

$$\xrightarrow{+} \Sigma F_x = 0; \qquad N_B - F_A = 0 \qquad \textbf{(4)}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_A - F_B - 150 = 0 \qquad \textbf{(5)}$$

$$\circlearrowleft + \Sigma M_A = 0; \qquad M - N_B(1.5) - F_B(1.5) = 0 \qquad \textbf{(6)}$$

Friction. Assuming that the impending motion is caused by the rotation of wheel due to the slipping at A and B. Thus,

$$F_A = \mu_A N_A = 0.2 N_A \qquad \textbf{(7)}$$

$$F_B = \mu_B N_B = 0.3 N_B \qquad \textbf{(8)}$$

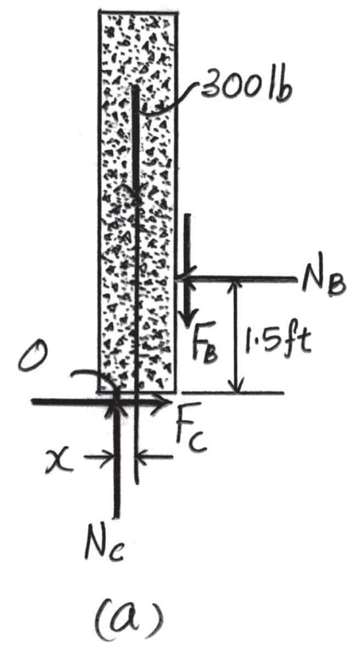
Solving Eqs. (1) to (8),

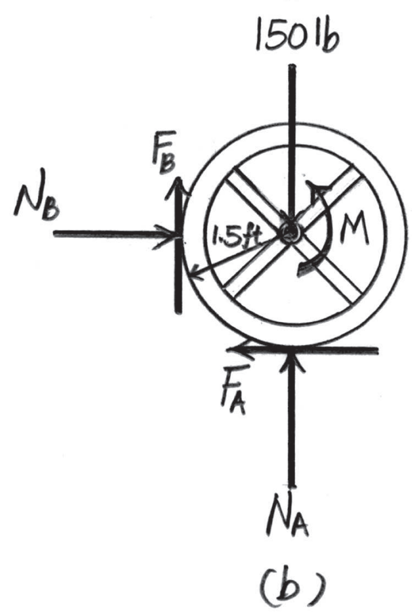
$$N_A = 141.51\text{ lb} \quad F_A = 28.30\text{ lb} \quad N_B = 28.30\text{ lb} \quad F_B = 8.491\text{ lb}$$

$$N_C = 308.49\text{ lb} \quad F_C = 28.30\text{ lb} \quad x = 0.1239\text{ ft}$$

$$M = 55.19\text{ lb}\cdot\text{ft} = 55.2\text{ lb}\cdot\text{ft} \qquad \textbf{Ans.}$$

Since $F_C < (F_C)_{\max} = \mu_C N_C = 0.4(308.49) = 123.40$ lb, and $x < 0.5$ ft, the concrete block will not slide or tip. Also, N_A is positive, so the wheel will be in contact with the floor. Thus, the assumption was correct.

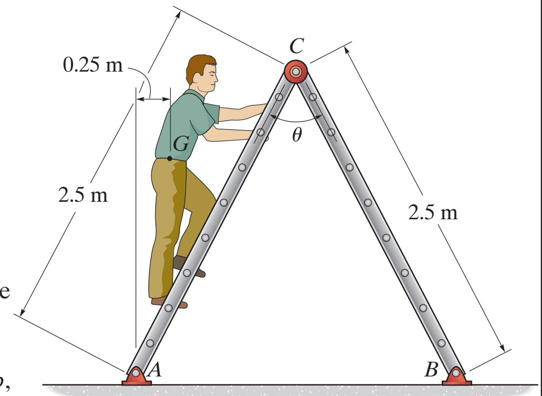




Ans:
$M = 55.2$ lb · ft

8–54.

Determine the greatest angle u so that the ladder does not slip when it supports the 75-kg man in the position shown. The surface is rather slippery, where the coefficient of static friction at A and B is $\mu_s = 0.3$.



SOLUTION

Free-Body Diagram: The slipping could occur at either end A or B of the ladder. We will assume that slipping occurs at end B. Thus, $F_B = \mu_s N_B = 0.3N_B$.

Equations of Equilibrium: Referring to the free-body diagram shown in Fig. *b*, we have

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_{BC} \sin \theta/2 - 0.3N_B = 0$$

$$F_{BC} \sin \theta/2 = 0.3N_B \qquad \textbf{(1)}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_B - F_{BC} \cos \theta/2 = 0$$

$$F_{BC} \cos \theta/2 = N_B \textbf{(2)}$$

Dividing Eq. (1) by Eq. (2) yields

$$\tan \theta/2 = 0.3$$

$$\theta = 33.40° = 33.4° \qquad \textbf{Ans.}$$

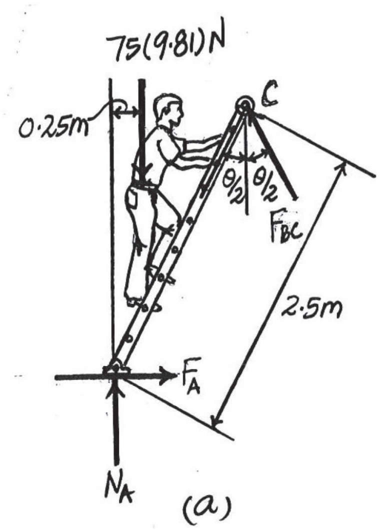
Using this result and referring to the free-body diagram of member AC shown in Fig. *a*, we have

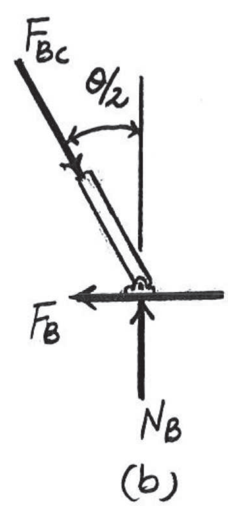
$$\circlearrowleft + \Sigma M_A = 0; \qquad F_{BC} \sin 33.40°(2.5) - 75(9.81)(0.25) = 0 \qquad F_{BC} = 133.66 \text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad F_A - 133.66 \sin\left(\frac{33.40°}{2}\right) = 0 \qquad F_A = 38.40 \text{ N}$$

$$+\uparrow \Sigma F_y = 0; \qquad N_A + 133.66 \cos\left(\frac{33.40°}{2}\right) - 75(9.81) = 0 \qquad N_A = 607.73 \text{ N}$$

Since $F_A < (F_A)_{\max} = \mu_s N_A = 0.3(607.73) = 182.32$ N, end A will not slip. Thus, the above assumption is correct.

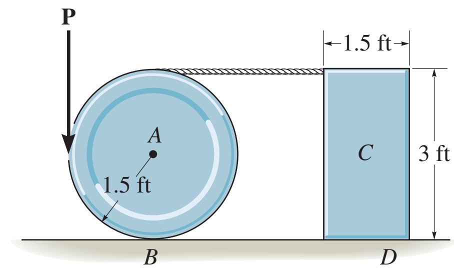




Ans:
$\theta = 33.4°$

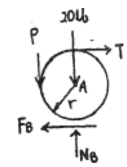
8–55.

The wheel weighs 20 lb and rests on a surface for which $\mu_B = 0.2$. A cord wrapped around it is attached to the top of the 30-lb homogeneous block. If the coefficient of static friction at D is $\mu_D = 0.3$, determine the smallest vertical force that can be applied tangentially to the wheel which will cause motion to impend.



SOLUTION

Cylinder A:



Assume slipping at B, $\quad F_B = 0.2N_B$

$\circlearrowleft + \Sigma M_A = 0; \quad F_B + T = P$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_B = T$

$+\uparrow \Sigma F_y = 0; \quad N_B = 20 + P$

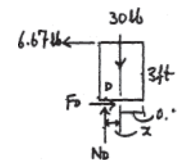
$N_B = 20 + 2(0.2N_B)$

$N_B = 33.33 \text{ lb}$

$F_B = 6.67 \text{ lb}$

$T = 6.67 \text{ lb}$

$P = 13.3 \text{ lb}$ **Ans.**



$\overset{+}{\rightarrow} \Sigma F_x = 0; \quad F_D = 6.67 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \quad N_D = 30 \text{ lb}$

$(F_D)_{max} = 0.3(30) = 9 \text{ lb} > 6.67 \text{ lb}$ **O.K.**

(No slipping occurs)

$\circlearrowleft + \Sigma M_D = 0; \quad -30(x) + 6.67(3) = 0$

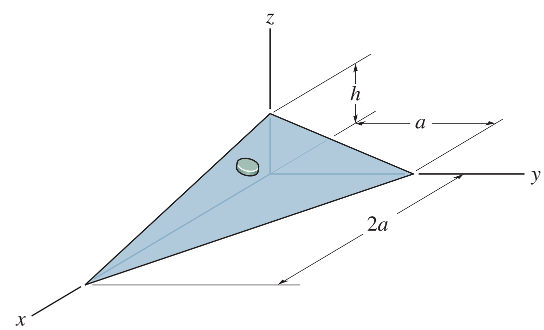
$x = 0.667 \text{ ft} < \dfrac{1.5}{2} = 0.75 \text{ ft}$ **O.K.**

(No tipping occurs)

Ans:
$P = 13.3 \text{ lb}$

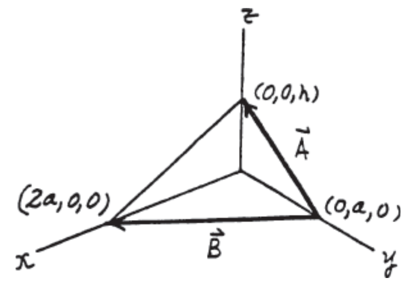
***8–56.**

The disk has a weight W and lies on a plane which has a coefficient of static friction μ. Determine the maximum height h to which the plane can be lifted without causing the disk to slip.



SOLUTION

Unit Vector: The unit vector perpendicular to the inclined plane can be determined using cross product.



$$\mathbf{A} = (0-0)\mathbf{i} + (0-a)\mathbf{j} + (h-0)\mathbf{k} = -a\mathbf{j} + h\mathbf{k}$$

$$\mathbf{B} = (2a-0)\mathbf{i} + (0-a)\mathbf{j} + (0-0)\mathbf{k} = 2a\mathbf{i} - a\mathbf{j}$$

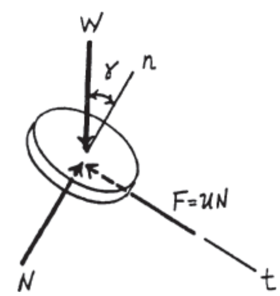
Then

$$\mathbf{N} = \mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & -a & h \\ 2a & -a & 0 \end{vmatrix} = ah\mathbf{i} + 2ah\mathbf{j} + 2a^2\mathbf{k}$$

$$n = \frac{\mathbf{N}}{N} = \frac{ah\mathbf{i} + 2ah\mathbf{j} + 2a^2\mathbf{k}}{a\sqrt{5h^2 + 4a^2}}$$

Thus

$$\cos\gamma = \frac{2a}{\sqrt{5h^2 + 4a^2}} \qquad \text{hence} \qquad \sin\gamma = \frac{\sqrt{5}h}{\sqrt{5h^2 + 4a^2}}$$



Equations of Equilibrium and Friction: When the disk is on the verge of sliding down the plane, $F = \mu N$.

$$\Sigma F_n = 0; \qquad N - W\cos\gamma = 0 \qquad N = W\cos\gamma \qquad \textbf{(1)}$$

$$\Sigma F_t = 0; \qquad W\sin\gamma - \mu N = 0 \qquad N = \frac{W\sin\gamma}{\mu} \qquad \textbf{(2)}$$

Divide Eq. (2) by (1) yields

$$\frac{\sin\gamma}{\mu\cos\gamma} = 1$$

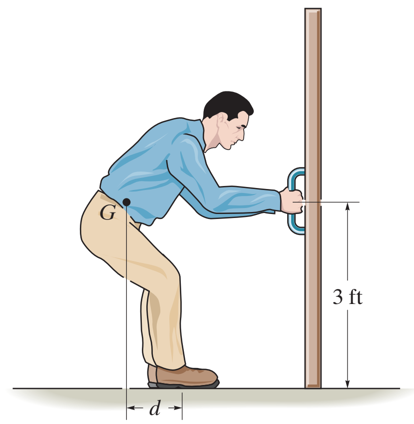
$$\frac{\frac{\sqrt{5}h}{\sqrt{5h^2 + 4a^2}}}{\mu\left(\frac{2a}{\sqrt{5h^2 + 4a^2}}\right)} = 1$$

$$h = \frac{2}{\sqrt{5}}a\mu \qquad \textbf{Ans.}$$

Ans:
$h = \dfrac{2}{\sqrt{5}}a\mu$

8–57.

The man has a weight of 200 lb, and the coefficient of static friction between his shoes and the floor is $\mu_s = 0.5$. Determine where he should position his center of gravity G at d in order to exert the maximum horizontal force on the door. What is this force?



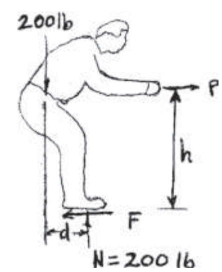
SOLUTION

$F_{max} = 0.5\,N = 0.5(200) = 100$ lb

$\xrightarrow{+} \Sigma F_x = 0; \qquad P - 100 = 0; \qquad P = 100$ lb **Ans.**

$\circlearrowleft + \Sigma M_O = 0; \qquad 200(d) - 100(3) = 0$

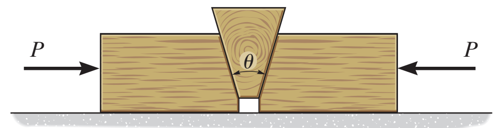
$d = 1.50$ ft **Ans.**



Ans:
$P = 100$ N
$d = 1.50$ ft

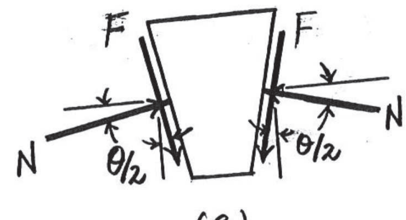
8–58.

Determine the largest angle u that will cause the wedge to be self-locking regardless of the magnitude of horizontal force P applied to the blocks. The coefficient of static friction between the wedge and the blocks is $\mu_s = 0.3$. Neglect the weight of the wedge.



SOLUTION

Free-Body Diagram: For the wedge to be self-locking, the frictional force F indicated on the free-body diagram of the wedge shown in Fig. *a* must act downward and its magnitude must be $F \leq \mu_s N = 0.3N$.



(a)

Equations of Equilibrium: Referring to Fig. *a,* we have

$$+\uparrow \Sigma F_y = 0; \qquad 2N\sin\theta/2 - 2F\cos\theta/2 = 0$$

$$F = N\tan\theta/2$$

Using the requirement $F \leq 0.3N$, we obtain

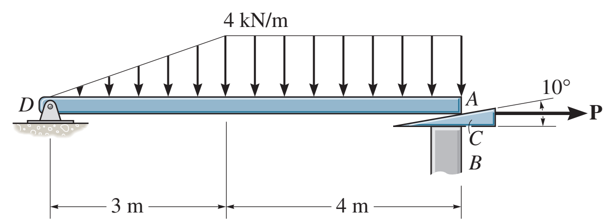
$$N\tan\theta/2 \leq 0.3N$$

$$\theta = 33.4° \qquad \textbf{Ans.}$$

Ans:
$\theta = 33.4°$

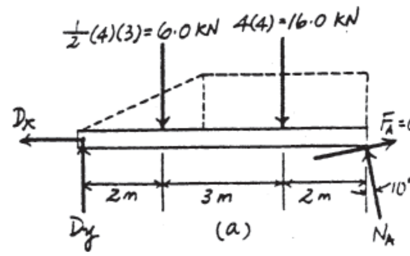
8–59.

If the beam AD is loaded as shown, determine the horizontal force P which must be applied to the wedge in order to remove it from under the beam. The coefficients of static friction at the wedge's top and bottom surfaces are $\mu_{CA} = 0.25$ and $\mu_{CB} = 0.35$, respectively. If $P = 0$, is the wedge self-locking? Neglect the weight and size of the wedge and the thickness of the beam.



SOLUTION

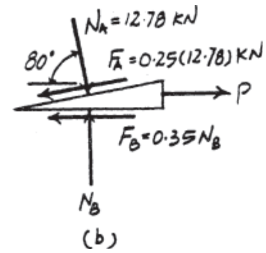
Equations of Equilibrium and Friction: If the wedge is on the verge of moving to the right, then slipping will have to occur at both contact surfaces. Thus, $F_A = \mu_{s\,A}\, N_A = 0.25N_A$ and $F_B = \mu_{s\,B}\, N_B = 0.35N_B$. From FBD (a),



$$\circlearrowleft + \Sigma M_D = 0; \qquad N_A \cos 10°(7) + 0.25N_A \sin 10°(7) - 6.00(2) - 16.0(5) = 0$$

$$N_A = 12.78 \text{ kN}$$

From FBD (b),



$$+\uparrow \Sigma F_y = 0; \qquad N_B - 12.78 \sin 80° - 0.25(12.78) \sin 10° = 0$$

$$N_B = 13.14 \text{ kN}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P + 12.78 \cos 80° - 0.25(12.78) \cos 10° - 0.35(13.14) = 0$$

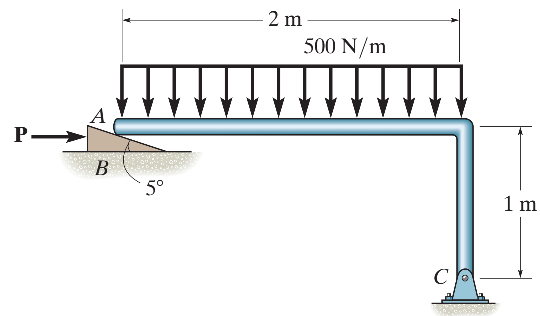
$$P = 5.53 \text{ kN} \qquad \textbf{Ans.}$$

Since a force P (> 0) is required to pull out the wedge, **the wedge will be self-locking when** $P = 0$. **Ans.**

Ans:
$P = 5.53$ kN
yes

***8–60.**

The wedge is used to level the member. Determine the horizontal force **P** that must be applied to begin to push the wedge forward. The coefficient of static friction between the wedge and the two surfaces of contact is $\mu_s = 0.2$. Neglect the weight of the wedge.



SOLUTION

Equations of Equilibrium and Friction. Since the wedge is required to be on the verge to slide to the right, then slipping will have to occur at both of its contact surfaces. Thus, $F_A = \mu_s N_A = 0.2\,N_A$ and $F_B = \mu_s N_B$. Referring to the *FBD* diagram of member *AC* shown in Fig. *a*

$$\circlearrowleft + \Sigma M_C = 0; \qquad 500(2)(1) - N_A \cos 5°(2) - N_A \sin 5°(1)$$

$$-0.2\,N_A \cos 5°(1) + 0.2\,N_A \sin 5°(2) = 0$$

$$N_A = 445.65\text{ N}$$

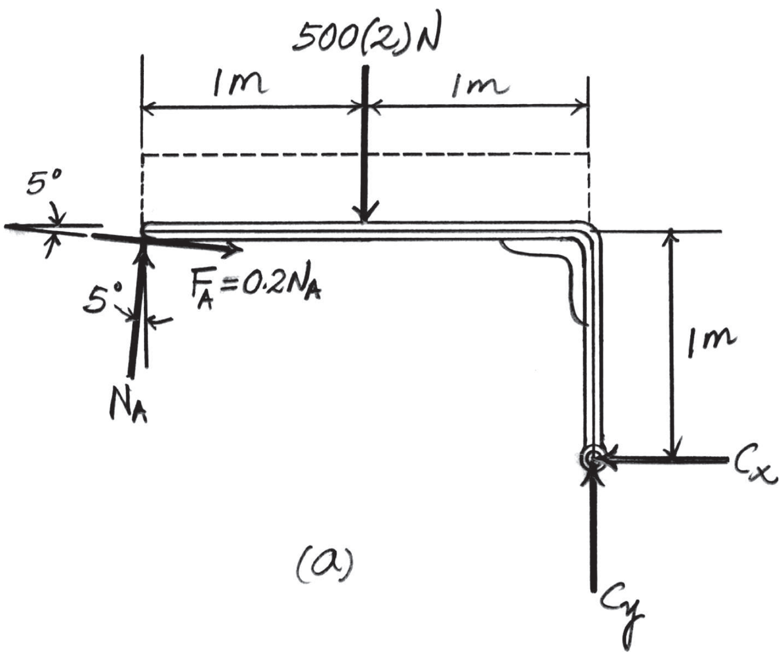
Using this result and the *FBD* of the wedge, Fig. *b*,

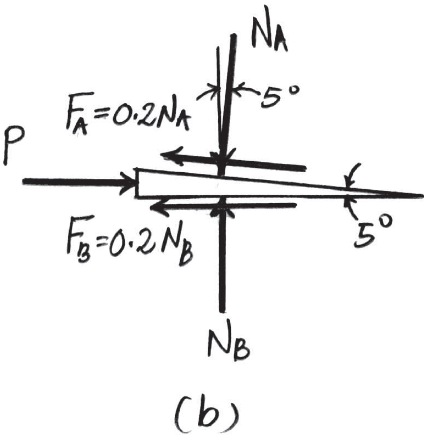
$$+\uparrow \Sigma F_y = 0; \qquad N_B - 445.65 \cos 5° + 0.2(445.65) \sin 5° = 0$$

$$N_B = 436.18\text{ N}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad P - 0.2(445.65) \cos 5° - 445.65 \sin 5° - 0.2(436.18) = 0$$

$$P = 214.87\text{ N} = 215\text{ N} \qquad \textbf{Ans.}$$

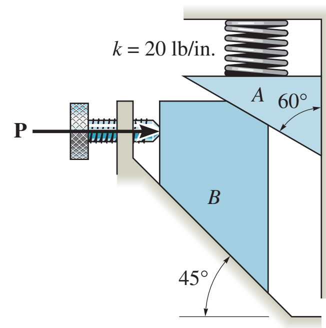




Ans:
215 N

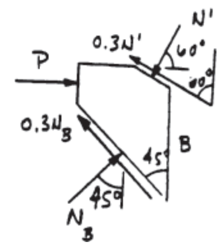
8–61.

The two blocks used in a measuring device have negligible weight. If the spring is compressed 5 in. when in the position shown, determine the smallest axial force P which the adjustment screw must exert on B in order to start the movement of B downward. The end of the screw is *smooth* and the coefficient of static friction at all other points of contact is $\mu_s = 0.3$.



SOLUTION

Note that when block B moves downward, block A will also come downward.



Block A:

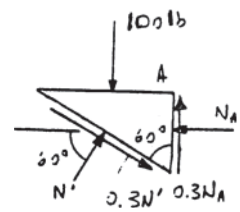
$\xrightarrow{+} \Sigma F_x = 0;\qquad N' \cos 60° + 0.3N' \sin 60° - N_A = 0$

$+\uparrow \Sigma F_y = 0;\qquad 0.3\,N_A - 0.3\,N' \cos 60° + N' \sin 60° - 100 = 0$

Block B:

$\xrightarrow{+} \Sigma F_x = 0;\qquad N_B \sin 45° - N_B \sin 45° + P - 0.3N' \sin 60° - N' \cos 60° = 0$

$+\uparrow \Sigma F_y = 0;\qquad N_B \cos 45° + 0.3\,N_B \cos 45° + 0.3\,N' \cos 60° - N' \sin 60° = 0$



Solving,

$$N' = 105.9 \text{ lb}$$

$$N_B = 82.5 \text{ lb}$$

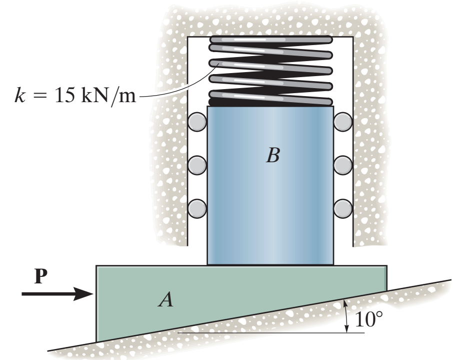
$$N_A = 80.5 \text{ lb}$$

$$P = 39.6 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$P = 39.6$ lb

8–62.

If $P = 250$ N, determine the required minimum compression in the spring so that the wedge will not move to the right. Neglect the weight of A and B. The coefficient of static friction for all contacting surfaces is $\mu_s = 0.35$. Neglect friction at the rollers.



SOLUTION

Free-Body Diagram: The spring force acting on the cylinder is $F_{sp} = kx = 15(10^3)x$. Since it is required that the wedge is on the verge to slide to the right, the frictional force must act to the left on the top and bottom surfaces of the wedge and their magnitude can be determined using friction formula.

$(F_f)_1 = \mu N_1 = 0.35 N_1 \qquad (F_f)_2 = 0.35 N_2$

Equations of Equilibrium: Referring to the FBD of the cylinder, Fig. *a*,

$+\uparrow \Sigma F_y = 0; \qquad N_1 - 15(10^3)x = 0 \qquad N_1 = 15(10^3)x$

Thus, $(F_f)_1 = 0.35[15(10^3)x] = 5.25(10^3)x$

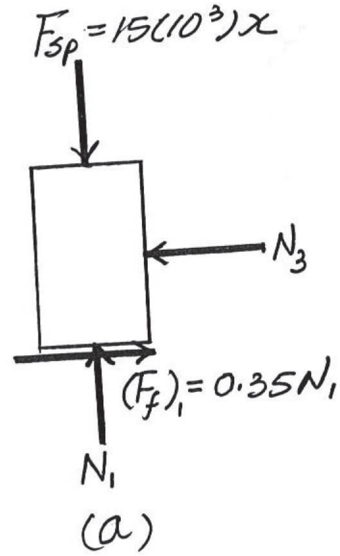
Referring to the FBD of the wedge shown in Fig. *b*,

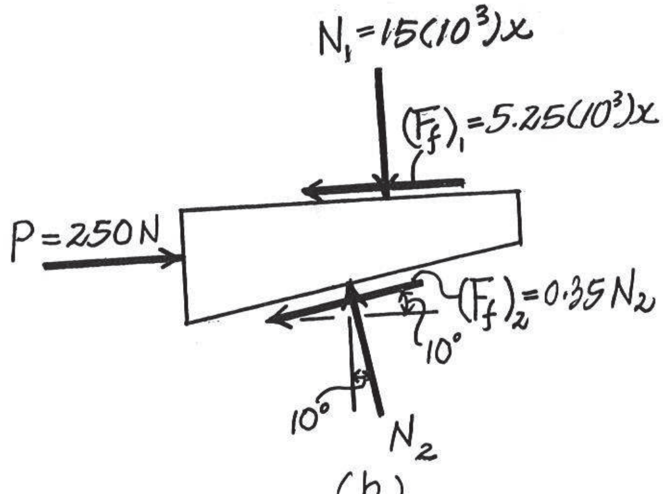
$+\uparrow \Sigma F_y = 0; \qquad N_2 \cos 10^\circ - 0.35 N_2 \sin 10^\circ - 15(10^3)x = 0$

$N_2 = 16.233(10^3)x$

$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 250 - 5.25(10^3)x - 0.35[16.233(10^3)x]\cos 10^\circ - [16.233(10^3)x]\sin 10^\circ = 0$

$x = 0.01830\,m = 18.3\,mm$ **Ans.**

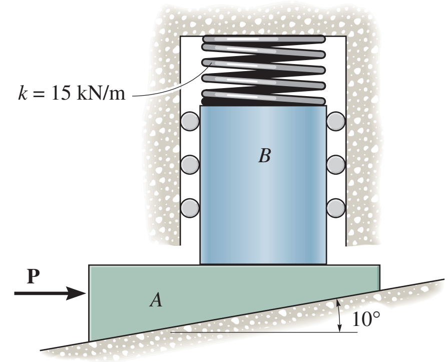




Ans:
$x = 18.3$ mm

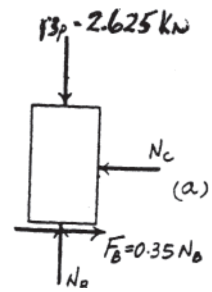
8–63.

Determine the minimum applied force **P** required to move wedge A to the right. The spring is compressed a distance of 175 mm. Neglect the weight of A and B. The coefficient of static friction for all contacting surfaces is $\mu_s = 0.35$. Neglect friction at the rollers.



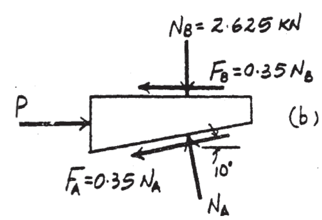
SOLUTION

Equations of Equilibrium and Friction: Using the spring formula, $F_{sp} = kx = 15(0.175) = 2.625$ kN. If the wedge is on the verge of moving to the right, then slipping will have to occur at both contact surfaces. Thus, $F_A = \mu_s N_A = 0.35N_A$ and $F_B = \mu_s N_B = 0.35N_B$. From FBD (a),



$+\uparrow \Sigma F_y = 0; \qquad N_B - 2.625 = 0 \qquad N_B = 2.625 \text{ kN}$

From FBD (b),



$+\uparrow \Sigma F_y = 0; \qquad N_A \cos 10^\circ - 0.35N_A \sin 10^\circ - 2.625 = 0$

$$N_A = 2.841 \text{ kN}$$

$\xrightarrow{+} \Sigma F_x = 0; \qquad P - 0.35(2.625) - 0.35(2.841) \cos 10^\circ$

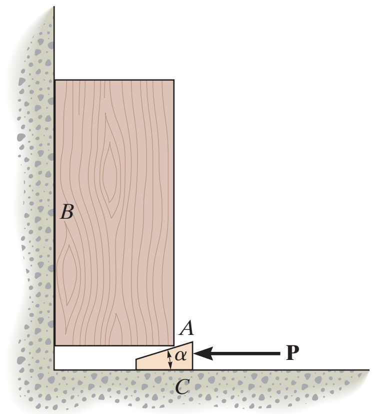
$$- 2.841 \sin 10^\circ = 0$$

$$P = 2.39 \text{ kN} \qquad \textbf{Ans.}$$

Ans:
$P = 2.39$ kN

***8–64.**

If the coefficient of static friction between all the surfaces of contact is μ_s, determine the force **P** that must be applied to the wedge in order to lift the block having a weight W.



SOLUTION

Equations of Equilibrium and Friction. Since the wedge is required to be on the verge sliding to the left, then slipping will have to occur at both of its contact surfaces. Thus, $F_A = \mu_s N_A$, $F_B = \mu_s N_B$ and $F_C = \mu_s N_C$. Referring to the *FBD* of the wedge shown in Fig. *a*.

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad \mu_s N_C + \mu_s N_A \cos\alpha + N_A \sin\alpha - P = 0 \quad \textbf{(1)}$$

$$+\uparrow\Sigma F_y = 0; \quad N_C + \mu_s N_A \sin\alpha - N_A \cos\alpha = 0 \quad \textbf{(2)}$$

Also, from the *FBD* of the block, Fig. *b*

$$\overset{+}{\rightarrow}\Sigma F_x = 0; \quad N_B - N_A \sin\alpha - \mu_s N_A \cos\alpha = 0 \quad \textbf{(3)}$$

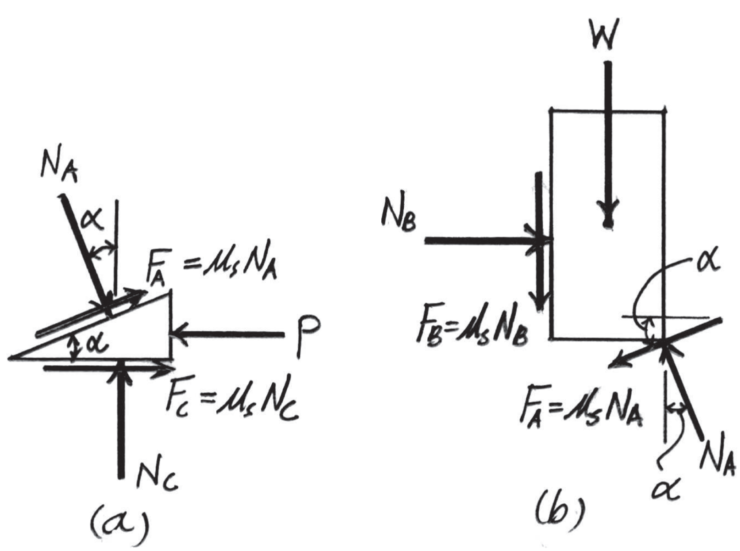
$$+\uparrow\Sigma F_y = 0; \quad N_A \cos\alpha - \mu_s N_A \sin\alpha - \mu_s N_B - W = 0 \quad \textbf{(4)}$$

Solving Eqs. (1) to (4)

$$N_A = \frac{W}{\cos\alpha(1-\mu_s^2) - 2\mu_s \sin\alpha} \qquad N_B = \left[\frac{\sin\alpha + \mu_s\cos\alpha}{\cos\alpha\,(1-\mu_s^2) - 2\mu_s\sin\alpha}\right]W$$

$$N_C = \left[\frac{\cos\alpha + \mu_s\sin\alpha}{\cos\alpha(1-\mu_s^2) - 2\mu_s\sin\alpha}\right]W$$

$$P = \left[\frac{2\mu_s\cos\alpha + \sin\alpha(1-\mu_s^2)}{\cos\alpha(1-\mu_s^2) - 2\mu_s\sin\alpha}\right]W \qquad \textbf{Ans.}$$

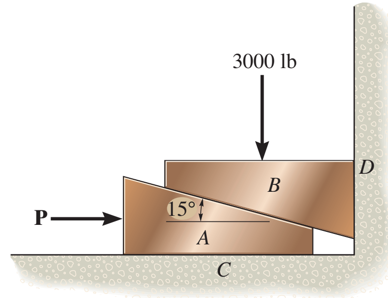


Ans:

$$P = \left[\frac{2\mu_s\cos\alpha + \sin\alpha(1-\mu_s^2)}{\cos\alpha(1-\mu_s^2) - 2\mu_s\sin\alpha}\right]W$$

8–65.

Determine the smallest force P needed to lift the 3000-lb load. The coefficient of static friction between A and C and between B and D is $\mu_s = 0.3$, and between A and B $\mu_{s'} = 0.4$. Neglect the weight of each wedge.



SOLUTION

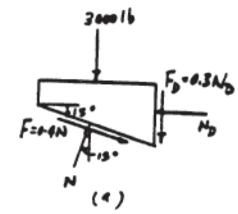
From FBD (a):

$\xrightarrow{+} \Sigma F_x = 0; \quad 0.4N \cos 15° + N \sin 15° - N_D = 0$ **(1)**

$+\uparrow \Sigma F_y = 0; \quad N \cos 15° - 0.4N \sin 15° - 0.3N_D - 3000 = 0$ **(2)**

Solving Eqs. (1) and (2) yields:

$$N = 4485.4 \text{ lb} \qquad N_D = 2893.9 \text{ lb}$$

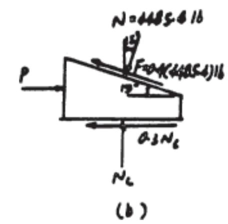


From FBD (b):

$+\uparrow \Sigma F_y = 0; \quad N_C + 0.4(4485.4) \sin 15° - 4485.4 \cos 15° = 0 \qquad N_C = 3868.2 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad P - 0.3(3868.2) - 4485.4 \sin 15° - 1794.1 \cos 15° = 0$

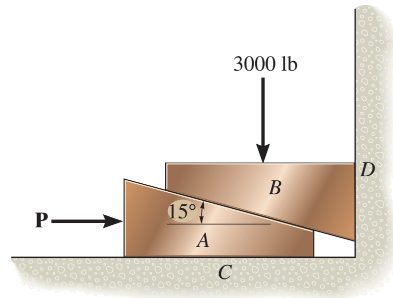
$$P = 4054 \text{ lb} = 4.05 \text{ kip}$$ **Ans.**



Ans:
$P = 4.05$ kip

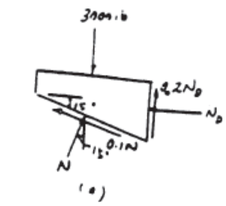
8–66.

Determine the reversed horizontal force $-\mathbf{P}$ needed to pull out wedge A. The coefficient of static friction between A and C and between B and D is $\mu_s = 0.2$, and between A and B $\mu_{s'} = 0.1$. Neglect the weight of each wedge.



SOLUTION

From FBD (a):



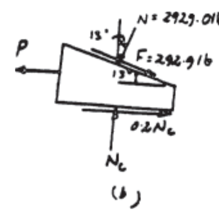
$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad N \sin 15^\circ - 0.1N \cos 15^\circ - N_D = 0$ **(1)**

$+\uparrow\Sigma F_y = 0;\qquad N \cos 15^\circ + 0.1N \sin 15^\circ + 0.2N_D - 3000 = 0$ **(2)**

Solving Eqs. (1) and (2) yields:

$$N = 2929.0 \text{ lb} \qquad N_D = 475.2 \text{ lb}$$

From FBD (b):



$+\uparrow\Sigma F_y = 0;\qquad N_C - 292.9 \sin 15^\circ - 2929.0 \cos 15^\circ = 0 \qquad N_C = 2905.0 \text{ lb}$

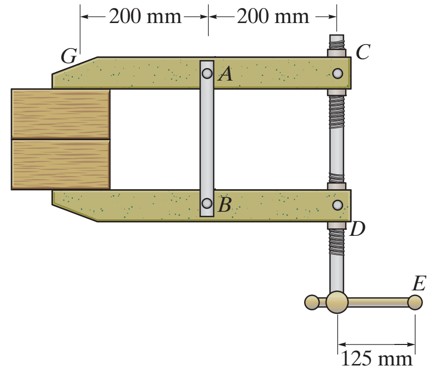
$\overset{+}{\rightarrow}\Sigma F_x = 0;\qquad 0.2(2905.0) + 292.9 \cos 15^\circ - 2929.0 \sin 15^\circ - P = 0$

$$P = 106 \text{ lb}$$ **Ans.**

Ans:
$P = 106$ lb

8–67.

If the clamping force at G is 900 N, determine the horizontal force **F** that must be applied perpendicular to the handle of the lever at E. The mean diameter and lead of both single square-threaded screws at C and D are 25 mm and 5 mm, respectively. The coefficient of static friction is $\mu_s = 0.3$.



SOLUTION

Referring to the free-body diagram of member GAC shown in Fig. a, we have $\Sigma M_A = 0; F_{CD}(0.2) - 900(0.2) = 0 \qquad F_{CD} = 900\,\text{N}$

Since the screw is being tightened, Eq. 8–3 should be used. Here, $\theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left[\frac{5}{2\pi(12.5)}\right] = 3.643°$;

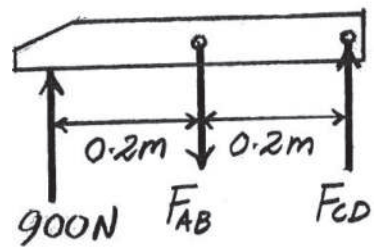
$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$; and $M = F(0.125)$. Since **M** must overcome the friction of two screws,

$$M = 2[Wr\tan(\phi_s + \theta)]$$

$$F(0.125) = 2\,[900(0.0125)\tan(16.699° + 3.643°)]$$

$$F = 66.7\,\text{N}$$ **Ans.**

Note: Since $\phi_s > \theta$, the screw is self-locking.

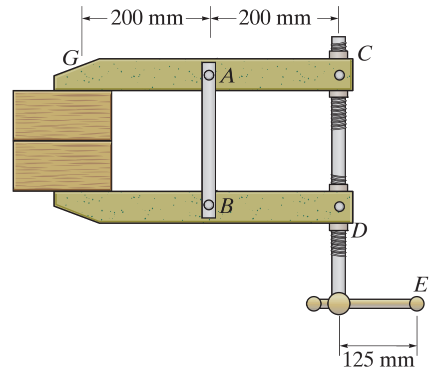


(a)

Ans:
$F = 66.7\text{ N}$

***8–68.**

If a horizontal force of $F = 50$ N is applied perpendicular to the handle of the lever at E, determine the clamping force developed at G. The mean diameter and lead of the single square-threaded screw at C and D are 25 mm and 5 mm, respectively. The coefficient of static friction is $\mu_s = 0.3$.



SOLUTION

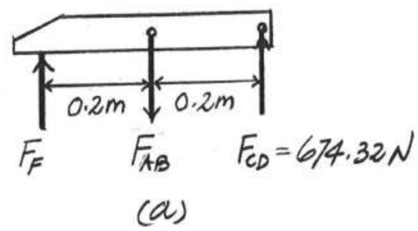
Since the screw is being tightened, Eq. 8–3 should be used. Here, $\theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left[\frac{5}{2\pi(12.5)}\right] = 3.643°$;

$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$; and $M = 50(0.125)$. Since **M** must overcome the friction of two screws,

$$M = 2[Wr\tan(\phi_s + \theta)]$$

$$50(0.125) = 2[F_{CD}(0.0125)\tan(16.699° + 3.643°)]$$

$$F_{CD} = 674.32\,\text{N}$$ **Ans.**



Using the result of F_{CD} and referring to the free-body diagram of member GAC shown in Fig. a, we have

$$\Sigma M_A = 0;\ 674.32(0.2) - F_G(0.2) = 0$$

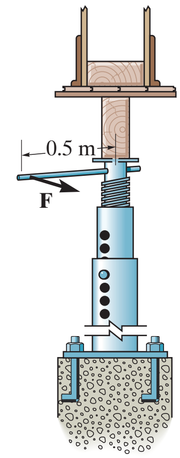
$$F_G = 674\,\text{N}$$ **Ans.**

Note: Since $\phi_s > \theta$, the screws are self-locking.

Ans:
$F_{CD} = 674.32$ N
$F_G = 674$ N

8–69.

The column is used to support the upper floor. If a force $F = 80$ N is applied perpendicular to the handle to tighten the screw, determine the compressive force in the column. The square-threaded screw on the jack has a coefficient of static friction of $\mu_s = 0.4$, mean diameter of 25 mm, and a lead of 3 mm.



SOLUTION

$$M = W(r)\tan(\phi_s + \theta_p)$$

$$\phi_s = \tan^{-1}(0.4) = 21.80^\circ$$

$$\theta_p = \tan^{-1}\left[\frac{3}{2\pi(12.5)}\right] = 2.188^\circ$$

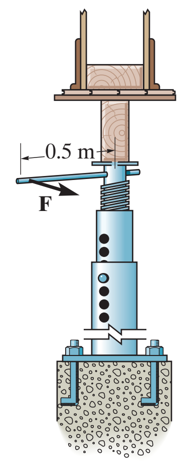
$$80(0.5) = W(0.0125)\tan(21.80^\circ + 2.188^\circ)$$

$$W = 7.19 \text{ kN}$$ **Ans.**

Ans:
$W = 7.19$ kN

8–70.

If the force **F** is removed from the handle of the jack in Prob. 8–69, determine if the screw is self-locking.



SOLUTION

$$\phi_s = \tan^{-1}(0.4) = 21.80^\circ$$

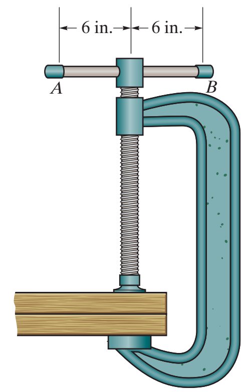
$$\theta_p = \tan^{-1}\left[\frac{3}{2\pi(12.5)}\right] = 2.188^\circ$$

Since $\phi_s > \theta_p$, the screw is self locking. **Ans.**

Ans:
The screw is self-locking.

8–71.

If couple forces of $F = 10$ lb are applied perpendicular to the lever of the clamp at A and B, determine the clamping force on the boards. The single square-threaded screw of the clamp has a mean diameter of 1 in. and a lead of 0.25 in. The coefficient of static friction is $\mu_s = 0.3$.



SOLUTION

Since the screw is being tightened, Eq. 8–3 should be used. Here,

$$M = 10(12) = 120 \text{ lb} \cdot \text{in};\ \theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left[\frac{0.25}{2\pi(0.5)}\right] = 4.550°;$$

$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$. Thus

$$M = Wr\tan(\phi_s + \theta)$$

$$120 = P(0.5)\tan(16.699° + 4.550°)$$

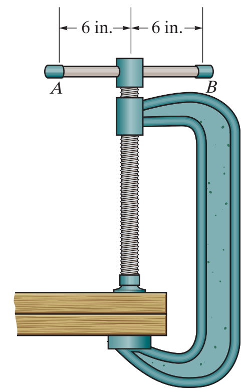
$$P = 617 \text{ lb}$$ **Ans.**

Note: Since $\phi_s > \theta$, the screw is self-locking.

Ans:
$P = 617$ lb

***8–72.**

If the clamping force on the boards is 600 lb, determine the required magnitude of the couple forces that must be applied perpendicular to the lever AB of the clamp at A and B in order to loosen the screw. The single square-threaded screw has a mean diameter of 1 in. and a lead of 0.25 in. The coefficient of static friction is $\mu_s = 0.3$.



SOLUTION

Since the screw is being loosened, Eq. 8–5 should be used. Here,

$M = F(12)$; $\theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left[\frac{0.25}{2\pi(0.5)}\right] = 4.550°$;

$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$; and $W = 600$ lb. Thus

$$M = Wr\tan(\phi_s - \theta)$$

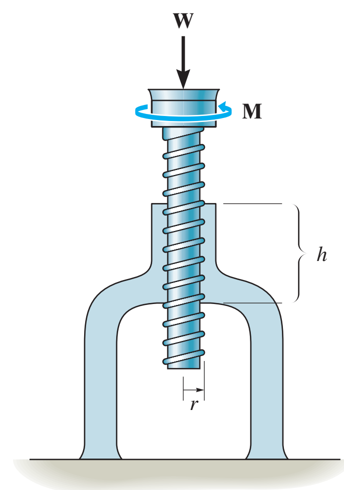
$$F(12) = 600(0.5)\tan(16.699° - 4.550°)$$

$$F = 5.38 \text{ lb}$$ **Ans.**

Ans:
$F = 5.38$ lb

8–73.

Prove that the lead l must be less than $2\pi r\mu_s$ for the jack screw shown in Fig. 8–15 to be "self-locking."



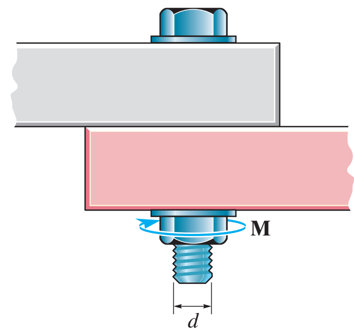
SOLUTION

For self–locking, $\phi_s > \theta_P$ or $\tan\phi_s > \tan\theta_p$;

$$\mu_s > \frac{l}{2\pi r}; \qquad l < 2\pi r\mu_s$$ **Q.E.D.**

8–74.

The square-threaded bolt is used to join two plates together. If the bolt has a mean diameter of $d = 20$ mm and a lead of $l = 3$ mm, determine the smallest torque M required to loosen the bolt if the tension in the bolt is $T = 40$ kN. The coefficient of static friction between the threads and the bolt is $\mu_s = 0.15$.



SOLUTION

$$\phi = \tan^{-1} 0.15 = 8.531^\circ$$

$$\theta = \tan^{-1} \frac{3}{2\pi(10)} = 2.734^\circ$$

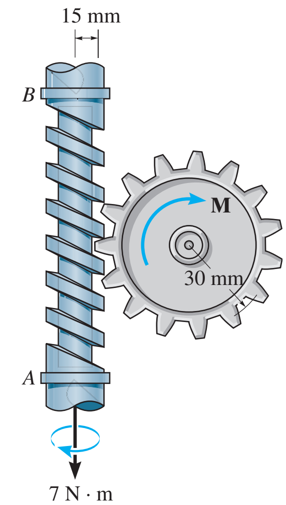
$$M = r\,W \tan(\phi - \theta) = (0.01)(40\,000) \tan(8.531^\circ - 2.734^\circ)$$

$$M = 40.6 \text{ N} \cdot \text{m}$$ **Ans.**

Ans:
$M = 40.6 \text{ N} \cdot \text{m}$

8–75.

The shaft has a square-threaded screw with a lead of 8 mm and a mean radius of 15 mm. If it is in contact with a plate gear having a mean radius of 30 mm, determine the resisting torque **M** on the plate gear which can be overcome if a torque of 7 N · m is applied to the shaft. The coefficient of static friction at the screw is $\mu_B = 0.2$. Neglect friction of the bearings located at A and B.



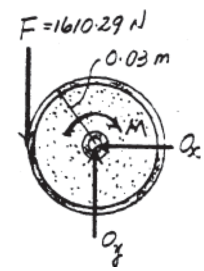
SOLUTION

Frictional Forces on Screw: Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{8}{2\pi(15)}\right] = 4.852°$, $W = F$, $M = 7\text{ N}\cdot\text{m}$ and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.2) = 11.310°$. Applying Eq. 8–3, we have

$$M = Wr\tan(\theta + \phi)$$

$$7 = F(0.015)\tan(4.852° + 11.310°)$$

$$F = 1610.29\text{ N}$$



Note: Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if force **F** is removed.

Equations of Equilibrium:

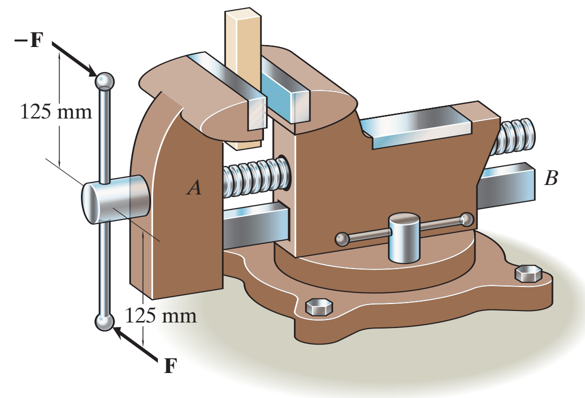
$\circlearrowleft + \Sigma M_O = 0; \qquad 1610.29(0.03) - M = 0$

$$M = 48.3\text{ N}\cdot\text{m}$$ **Ans.**

Ans:
$M = 48.3\text{ N}\cdot\text{m}$

***8–76.**

If couple forces of $F = 35$ N are applied to the handle of the machinist's vise, determine the compressive force developed in the block. Neglect friction at the bearing A. The guide at B is smooth. The single square-threaded screw has a mean radius of 6 mm and a lead of 8 mm, and the coefficient of static friction is $\mu_s = 0.27$.



SOLUTION

$\phi = \tan^{-1}(0.27) = 15.11°$

$\theta = \tan^{-1}\left(\dfrac{8}{2\pi(6)}\right) = 11.98°$

$M = Wr\tan(\theta + \phi)$

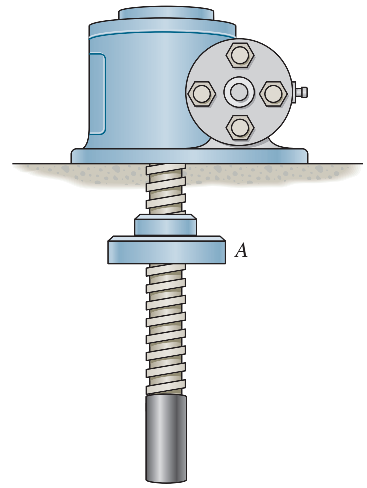
$35\ (0.250) = P(0.006)\tan(11.98° + 15.11°)$

$P = 2851\text{ N} = 2.85\text{ kN}$ **Ans.**

Ans:
$P = 2.85$ kN

8–77.

The square-threaded screw has a mean diameter of 20 mm and a lead of 4 mm. If the weight of the plate A is 5 lb, determine the smallest coefficient of static friction between the screw and the plate so that the plate does not travel down the screw when the plate is suspended as shown.



SOLUTION

Frictional Forces on Screw: This requires a "self-locking" screw where $\phi_s \geq \theta$. Here, $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{4}{2\pi(10)}\right] = 3.643°$.

$$\phi_s = \tan^{-1}\mu_s$$

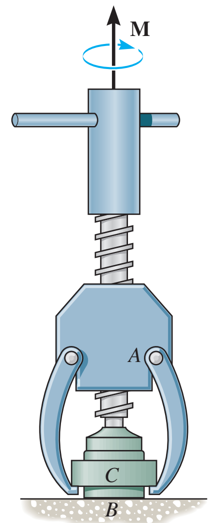
$$\mu_s = \tan\phi_s \qquad \text{where } \phi_s = \theta = 3.643°$$

$$= 0.0637 \qquad \textbf{Ans.}$$

Ans:
$\mu_s = 0.0637$

8–78.

The device is used to pull the battery cable terminal C from the post of a battery. If the required pulling force is 85 lb, determine the torque **M** that must be applied to the handle on the screw to tighten it. The screw has square threads, a mean diameter of 0.2 in., a lead of 0.08 in., and the coefficient of static friction is $\mu_s = 0.5$.



SOLUTION

Frictional Forces on Screw: Here, $\theta = \tan^{-1}\left(\dfrac{l}{2\pi r}\right) = \tan^{-1}\left[\dfrac{0.08}{2\pi(0.1)}\right] = 7.256°$, $W = 85$ lb and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.5) = 26.565°$. Applying Eq. 8–3, we have

$$M = Wr\tan(\theta + \phi)$$

$$= 85(0.1)\tan(7.256° + 26.565°)$$

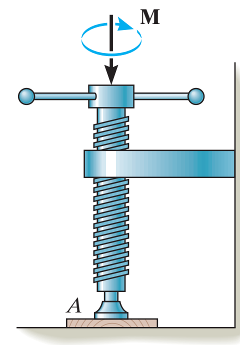
$$= 5.69 \text{ lb}\cdot\text{in} \qquad \textbf{Ans.}$$

Note: Since $\phi_s > \theta$, the screw is self-locking. It will not unscrew even if the moment is removed.

Ans:
$M = 5.69 \text{ lb}\cdot\text{in.}$

8–79.

Determine the clamping force on the board A if the screw is tightened with a torque of $M = 8\ \text{N}\cdot\text{m}$. The square-threaded screw has a mean radius of 10 mm and a lead of 3 mm, and the coefficient of static friction is $\mu_s = 0.35$.



SOLUTION

Frictional Forces on Screw. Here $\theta = \tan^{-1}\left(\dfrac{l}{2\pi r}\right) = \tan^{-1}\left[\dfrac{3}{2\pi\,(10)}\right] = 2.7336°$,

$W = F$ and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.35) = 19.2900°$.

$$M = Wr\tan\,(\theta + \phi_s)$$

$$8 = F(0.01)\tan\,(2.7336° + 19.2900°)$$

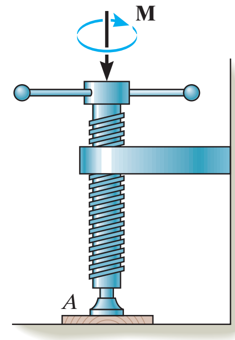
$$F = 1977.72\text{ N} = 1.98\text{ kN}$$ **Ans.**

Note: Since $\phi_s > \theta$, the screw is "self-locking". It will not unscrew even if the torque is removed.

Ans:
$F = 1.98$ kN

***8–80.**

If the required clamping force at the board A is to be 2 kN, determine the torque M that must be applied to the screw to tighten it down. The square-threaded screw has a mean radius of 10 mm and a lead of 3 mm, and the coefficient of static friction is $\mu_s = 0.35$.



SOLUTION

Frictional Forces on Screw. Here $\theta = \tan^{-1}\left(\frac{l}{2\pi r}\right) = \tan^{-1}\left[\frac{3}{2\pi\,(10)}\right] = 2.7336°$,

$W = 2000$ N and $\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.35) = 19.2900°$.

$$M = Wr\tan(\theta + \phi_s)$$

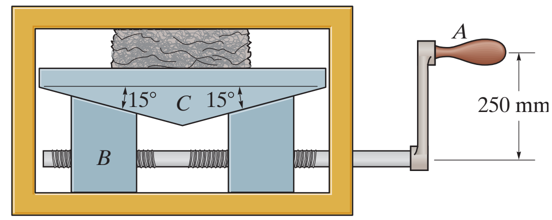
$$= 2000\,(0.01)\tan(2.7336° + 19.2900°)$$

$$= 8.09\ \text{N}\cdot\text{m}$$ **Ans.**

Ans:
8.09 N · m

8–81.

If a horizontal force of $P = 100$ N is applied perpendicular to the handle of the lever at A, determine the compressive force $\mathbf{F}$ exerted on the material. Each single square-threaded screw has a mean diameter of 25 mm and a lead of 7.5 mm. The coefficient of static friction at all contacting surfaces of the wedges is $\mu_s = 0.2$, and the coefficient of static friction at the screw is $\mu_s' = 0.15$.



SOLUTION

Since the screws are being tightened, Eq. 8–3 should be used. Here,

$$\theta = \tan^{-1}\left(\frac{L}{2\pi r}\right) = \tan^{-1}\left[\frac{7.5}{2\pi(12.5)}\right] = 5.455°;$$

$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.15) = 8.531°$; $M = 100(0.25) = 25\text{ N}\cdot\text{m}$; and $W = T$, where T is the tension in the screw shank. Since $\mathbf{M}$ must overcome the friction of two screws,

$$M = 2[Wr, \tan(\phi_s + \theta)]$$
$$25 = 2[T(0.0125)\tan(8.531° + 5.455°)]$$
$$T = 4015.09\text{ N} = 4.02\text{ kN} \qquad \textbf{Ans.}$$

Referring to the free-body diagram of wedge B shown in Fig. a using the result of T, we have

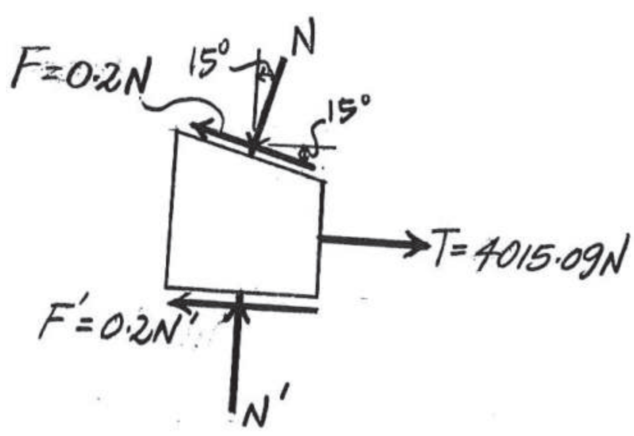
$$\overset{+}{\rightarrow}\Sigma F_x = 0; \qquad 4015.09 - 0.2N' - 0.2N\cos 15° - N\sin 15° = 0 \qquad \textbf{(1)}$$
$$+\uparrow\Sigma F_y = 0; \qquad N' + 0.2N\sin 15° - N\cos 15° = 0 \qquad \textbf{(2)}$$

Solving,

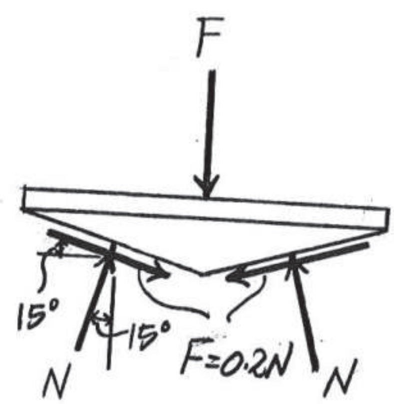
$$N = 6324.60\text{ N} \qquad N' = 5781.71\text{ N}$$

Using the result of N and referring to the free-body diagram of wedge C shown in Fig. b, we have

$$+\uparrow\Sigma F_y = 0; \qquad 2(6324.60)\cos 15° - 2[0.2(6324.60)\sin 15°] - F = 0$$
$$F = 11563.42\text{ N} = 11.6\text{ kN} \qquad \textbf{Ans.}$$



(a)

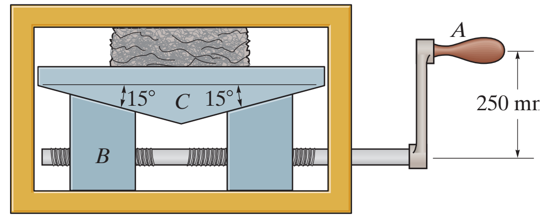


(b)

Ans:
$T = 4.02$ kN
$F = 11.6$ kN

8–82.

Determine the horizontal force **P** that must be applied perpendicular to the handle of the lever at A in order to develop a compressive force of 12 kN on the material. Each single square-threaded screw has a mean diameter of 25 mm and a lead of 7.5 mm. The coefficient of static friction at all contacting surfaces of the wedges is $\mu_s = 0.2$, and the coefficient of static friction at the screw is $\mu_s' = 0.15$.



SOLUTION

Referring to the free-body diagram of wedge C shown in Fig. a, we have

$+\uparrow\Sigma F_y = 0;\qquad 2N\cos 15^\circ - 2[0.2N\sin 15^\circ] - 12000 = 0$

$N = 6563.39\text{ N}$

Using the result of N and referring to the free-body diagram of wedge B shown in Fig. b, we have

$+\uparrow\Sigma F_y = 0;\qquad N' - 6563.39\cos 15^\circ + 0.2(6563.39)\sin 15^\circ = 0$

$N' = 6000\text{ N}$

$\xrightarrow{+}\Sigma F_x = 0;\qquad T - 6563.39\sin 15^\circ - 0.2(6563.39)\cos 15^\circ - 0.2(6000) = 0$

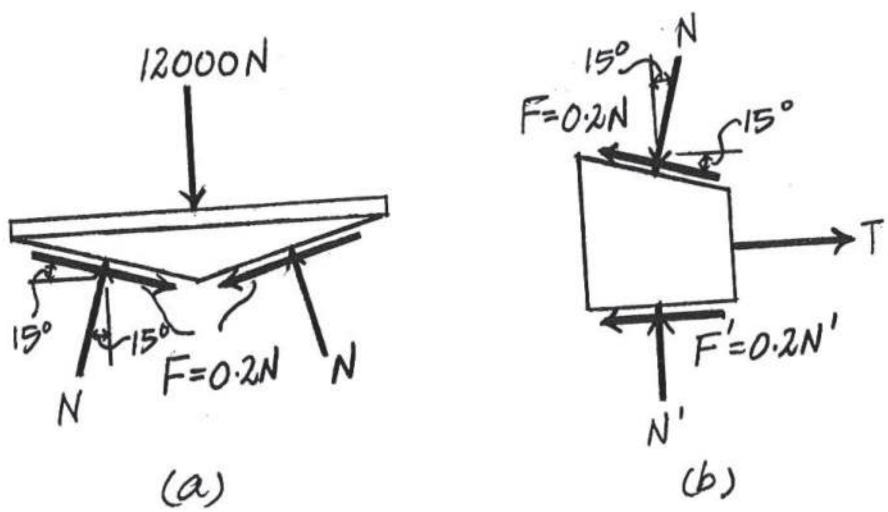
$T = 4166.68\text{ N}$

Since the screw is being tightened, Eq. 8–3 should be used. Here,

$$\theta = \tan^{-1}\left[\frac{L}{2\pi r}\right] = \tan^{-1}\left[\frac{7.5}{2\pi(12.5)}\right] = 5.455^\circ;$$

$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.15) = 8.531^\circ$; $M = P(0.25)$; and $W = T = 4166.68\text{N}$. Since **M** must overcome the friction of two screws,

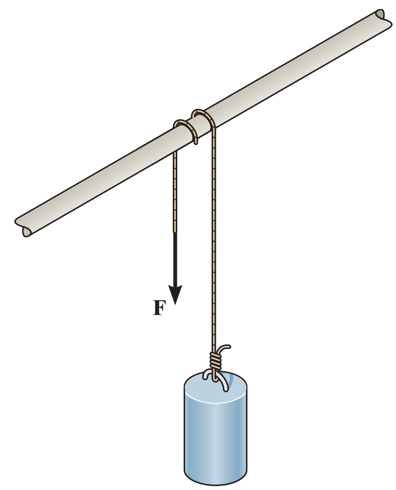
$$M = 2[Wr\tan(\phi_s + \theta)]$$
$$P(0.25) = 2[4166.68(0.0125)\tan(8.531^\circ + 5.455^\circ)]$$
$$P = 104\text{ N}$$ **Ans.**



Ans:
$P = 104\text{ N}$

8–83.

A cylinder having a mass of 250 kg is to be supported by the cord which wraps over the pipe. Determine the smallest vertical force F needed to support the load if the cord passes (a) once over the pipe, $\beta = 180°$, and (b) two times over the pipe, $\beta = 540°$. Take $\mu_s = 0.2$.



SOLUTION

Frictional Force on Flat Belt: Here, $T_1 = F$ and $T_2 = 250(9.81) = 2452.5$ N. Applying Eq. 8–6, we have

a) If $\beta = 180° = \pi$ rad

$$T_2 = T_1 e^{\mu\beta}$$

$$2452.5 = Fe^{0.2\pi}$$

$$F = 1308.38 \text{ N} = 1.31 \text{ kN}$$ **Ans.**

b) If $\beta = 540° = 3\pi$ rad

$$T_2 = T_1 e^{\mu\beta}$$

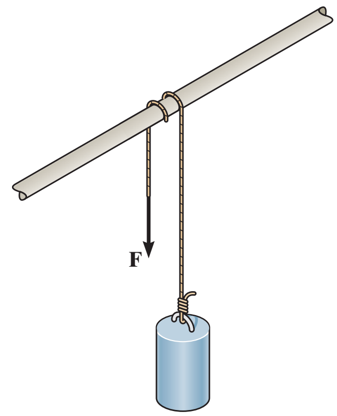
$$2452.5 = Fe^{0.2(3\pi)}$$

$$F = 372.38 \text{ N} = 372 \text{ N}$$ **Ans.**

Ans:
$F = 1.31$ kN
$F = 372$ N

***8–84.**

A cylinder having a mass of 250 kg is to be supported by the cord which wraps over the pipe. Determine the largest vertical force F that can be applied to the cord without moving the cylinder. The cord passes (a) once over the pipe, $\beta = 180°$, and (b) two times over the pipe, $\beta = 540°$. Take $\mu_s = 0.2$.



SOLUTION

Frictional Force on Flat Belt: Here, $T_1 = 250(9.81) = 2452.5\text{ N}$ and $T_2 = F$. Applying Eq. 8–6, we have

a) If $\beta = 180° = \pi$ rad

$$T_2 = T_1\, e^{\mu\beta}$$

$$F = 2452.5e^{0.2\,\pi}$$

$$F = 4597.10\text{ N} = 4.60\text{ kN} \qquad \textbf{Ans.}$$

b) If $\beta = 540° = 3\,\pi$ rad

$$T_2 = T_1 e^{\mu\beta}$$

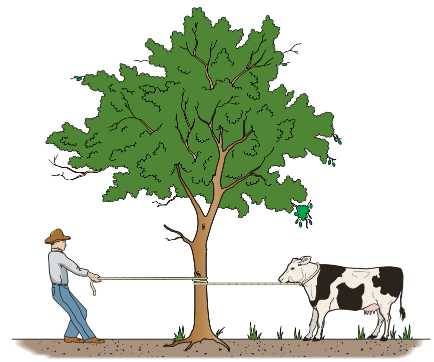
$$F = 2452.5e^{0.2(3\,\pi)}$$

$$F = 16\,152.32\text{ N} = 16.2\text{ kN} \qquad \textbf{Ans.}$$

Ans:
$F = 4.60$ kN
$F = 16.2$ kN

8–85.

A 180-lb farmer tries to restrain the cow from escaping by wrapping the rope two turns around the tree trunk as shown. If the cow exerts a force of 250 lb on the rope, determine if the farmer can successfully restrain the cow. The coefficient of static friction between the rope and the tree trunk is $\mu_s = 0.15$, and between the farmer's shoes and the ground $\mu_s' = 0.3$.



SOLUTION

Since the cow is on the verge of moving, the force it exerts on the rope is $T_2 = 250$ lb and the force exerted by the man on the rope is T_1. Here, $\beta = 2(2\pi) = 4\pi$ rad. Thus,

$$T_2 = T_1 e^{\mu_s \beta}$$

$$250 = T_1 e^{0.15(4\pi)}$$

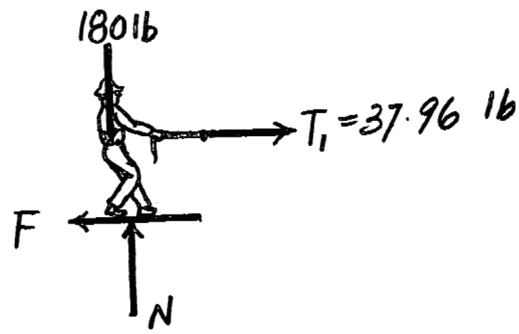
$$T_1 = 37.96 \text{ lb}$$

Using this result and referring to the free - body diagram of the man shown in Fig. *a*,

$+\uparrow \Sigma F_y = 0;$ $\quad N - 180 = 0 \quad N = 180$ lb

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad 37.96 - F = 0 \quad F = 37.96$ lb

Since $F < F_{max} = \mu_s' N = 0.3(180) = 54$ lb, the man will not slip, and he will successfully restrain the cow.

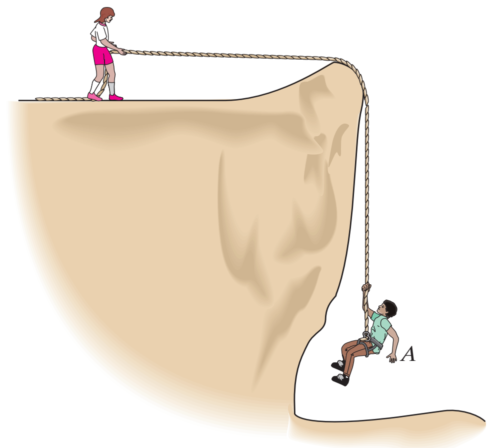


(a)

Ans:
He will successfully restrain the cow.

8–86.

The 100-lb boy at A is suspended from the cable that passes over the quarter circular cliff rock. Determine if it is possible for the 185-lb woman to hoist him up; and if this is possible, what smallest force must she exert on the horizontal cable? The coefficient of static friction between the cable and the rock is $\mu_s = 0.2$, and between the shoes of the woman and the ground $\mu_s' = 0.8$.



SOLUTION

$$\beta = \frac{\pi}{2}$$

$$T_2 = T_1 e^{\mu\beta} = 100\, e^{0.2\frac{\pi}{2}} = 136.9 \text{ lb}$$

$+\uparrow \Sigma F_y = 0;\qquad N - 185 = 0$

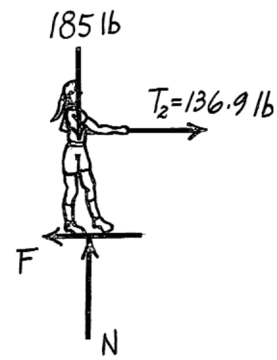
$$N = 185 \text{ lb}$$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 136.9 - F = 0$

$$F = 136.9 \text{ lb}$$

$$F_{max} = 0.8\,(185) = 148 \text{ lb} > 136.9 \text{ lb}$$

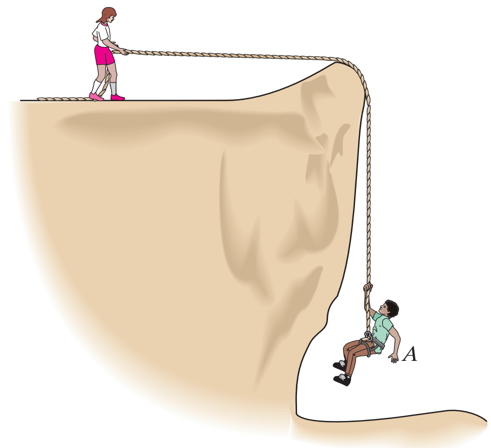
Yes, just barely. **Ans.**



Ans:
Yes, it is possible.
$F = 137$ lb

8–87.

The 100-lb boy at A is suspended from the cable that passes over the quarter circular cliff rock. What horizontal force must the woman at A exert on the cable in order to let the boy descend at constant velocity? The coefficients of static and kinetic friction between the cable and the rock are $\mu_s = 0.4$ and $\mu_k = 0.35$, respectively.



SOLUTION

$$\beta = \frac{\pi}{2}$$

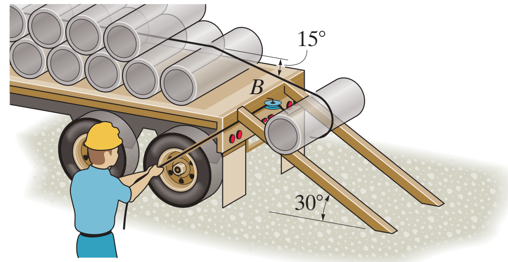
$$T_2 = T_1\, e^{\mu\beta}; \qquad 100 = T_1\, e^{0.35\frac{\pi}{2}}$$

$$T_1 = 57.7 \text{ lb}$$ **Ans.**

Ans:
$T_1 = 57.7$ lb

***8–88.**

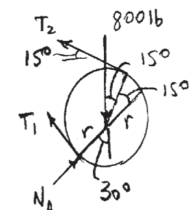
The uniform concrete pipe has a weight of 800 lb and is unloaded slowly from the truck bed using the rope and skids shown. If the coefficient of kinetic friction between the rope and pipe is $\mu_k = 0.3$, determine the force the worker must exert on the rope to lower the pipe at constant speed. There is a pulley at B, and the pipe does not slip on the skids. The lower portion of the rope is parallel to the skids.



SOLUTION

$\circlearrowleft + \Sigma M_A = 0; \quad -800(r \sin 30°) + T_2 \cos 15°(r \cos 15° + r \cos 30°) + T_2 \sin 15°(r \sin 15° + r \sin 15°) = 0$

$T_2 = 203.466$ lb



$\beta = 180° + 15° = 195°$

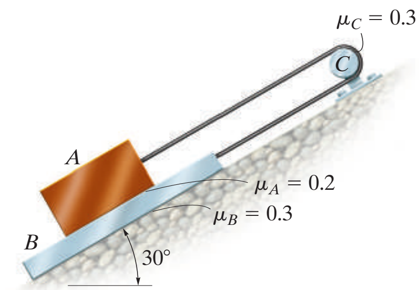
$T_2 = T_1 e^{\mu\beta}, \qquad 203.466 = T_1 e^{(0.3)(\frac{195°}{180°})(\pi)}$

$T_1 = 73.3$ lb **Ans.**

Ans:
$T_1 = 73.3$ lb

8–89.

A cable is attached to the 20-kg plate B, passes over a fixed peg at C, and is attached to the block at A. Using the coefficients of static friction shown, determine the smallest mass of block A so that it will prevent sliding motion of B down the plane.



SOLUTION

Block A:

$$+\nearrow\Sigma F_x = 0; \qquad T_1 - 0.2\,N_A - W_A \sin 30^\circ = 0 \qquad \textbf{(1)}$$

$$+\nwarrow\Sigma F_y = 0; \qquad N_A - W_A \cos 30^\circ = 0 \qquad \textbf{(2)}$$

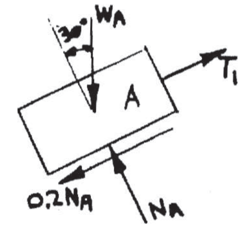


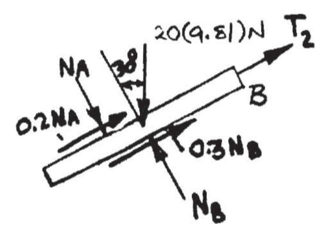
Plate B:

$$+\nearrow\Sigma F_x = 0; \qquad T_2 - 20(9.81)\sin 30^\circ + 0.3\,N_B + 0.2\,N_A = 0 \qquad \textbf{(3)}$$

$$+\nwarrow\Sigma F_y = 0; \qquad N_B - N_A - 20(9.81)\cos 30^\circ = 0 \qquad \textbf{(4)}$$

Peg C:

$$T_2 = T_1 e^{\mu\beta}; \qquad T_2 = T_1\, e^{0.3\pi} \qquad \textbf{(5)}$$



Solving Eqs. (1)–(5) yields

$T_1 = 14.68$ N; $\quad T_2 = 37.68$ N; $\quad N_A = 18.89$ N; $\quad N_B = 188.8$ N; $\quad W_A = 21.81$ N

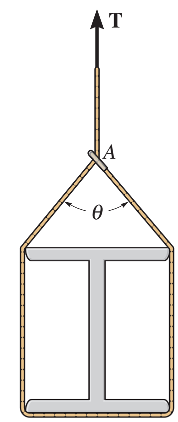
Thus,

$$m_A = \frac{21.81}{9.81} = 2.22 \text{ kg} \qquad \textbf{Ans.}$$

Ans:
$m_A = 2.22$ kg

8–90.

The smooth beam is being hoisted using a rope which is wrapped around the beam and passes through a ring at A as shown. If the end of the rope is subjected to a tension **T** and the coefficient of static friction between the rope and ring is $\mu_s = 0.3$, determine the angle of θ for equilibrium.



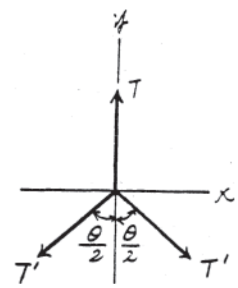
SOLUTION

Equation of Equilibrium:

$$+\uparrow \Sigma F_x = 0; \qquad T - 2T' \cos\frac{\theta}{2} = 0 \qquad T = 2T' \cos\frac{\theta}{2} \qquad \textbf{(1)}$$

Frictional Force on Flat Belt: Here, $\beta = \dfrac{\theta}{2}$, $T_2 = T$ and $T_1 = T'$. Applying Eq. 8–6 $T_2 = T_1 e^{\mu\beta}$, we have

$$T = T' e^{0.3(\theta/2)} = T' e^{0.15\,\theta} \qquad \textbf{(2)}$$



Substituting Eq. (1) into (2) yields

$$2T' \cos\frac{\theta}{2} = T' e^{0.15\,\theta}$$

$$e^{0.15\,\theta} = 2\cos\frac{\theta}{2}$$

Solving by trial and error

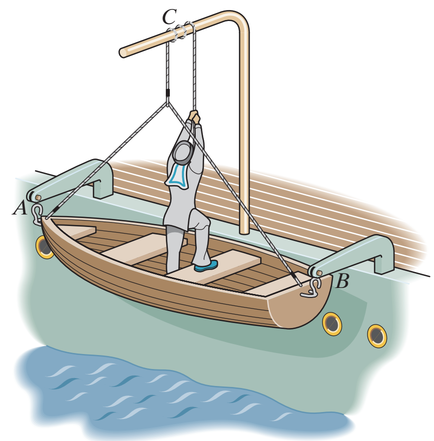
$$\theta = 1.73104 \text{ rad} = 99.2^\circ \qquad \textbf{Ans.}$$

The other solution, which starts with $T' = Te^{0.3(0/2)}$ based on cinching the ring tight, is 2.4326 rad $= 139°$. Any angle from 99.2° to 139° is equilibrium.

Ans:
$\theta = 92.2^\circ$

8–91.

The boat has a weight of 500 lb and is held in position off the side of a ship by the spars at A and B. A man having a weight of 130 lb gets in the boat, wraps a rope around an overhead boom at C, and ties it to the end of the boat as shown. If the boat is disconnected from the spars, determine the *minimum number of half turns* the rope must make around the boom so that the boat can be safely lowered into the water at constant velocity. Also, what is the normal force between the boat and the man? The coefficient of kinetic friction between the rope and the boom is $\mu_s = 0.15$. *Hint*: The problem requires that the normal force between the man's feet and the boat be as small as possible.



SOLUTION

Frictional Force on Flat Belt: If the normal force between the man and the boat is equal to zero, then, $T_1 = 130$ lb and $T_2 = 500$ lb. Applying Eq. 8–6, we have

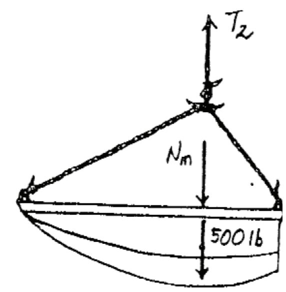
$$T_2 = T_1 e^{\mu\beta}$$

$$500 = 130e^{0.15\beta}$$

$$\beta = 8.980 \text{ rad}$$

The least number of half turns of the rope required is $\dfrac{8.980}{\pi} = 2.86$ turns. Thus

Use $n = 3$ half turns **Ans.**



(a)

Equations of Equilibrium: From FBD (a),

$+\uparrow\Sigma F_y = 0; \qquad T_2 - N_m - 500 = 0 \qquad T_2 = N_m + 500$

From FBD (b),

$+\uparrow\Sigma F_y = 0; \qquad T_1 + N_m - 130 = 0 \qquad T_1 = 130 - N_m$

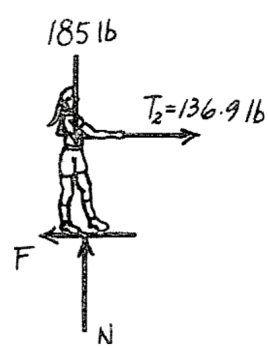
(b)

Frictional Force on Flat Belts: Here, $\beta = 3\,\pi$ rad. Applying Eq. 8–6, we have

$$T_2 = T_1 e^{\mu\beta}$$

$$N_m + 500 = (130 - N_m)\, e^{0.15\,(3\,\pi)}$$

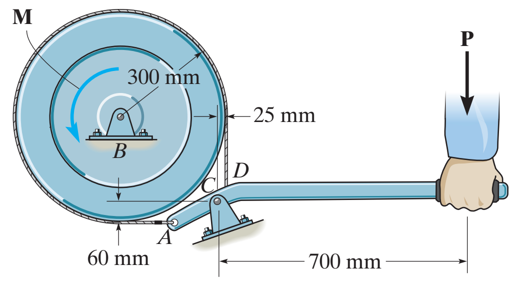
$$N_m = 6.74 \text{ lb}$$ **Ans.**



Ans:
$n = 3$ half turns
$N_m = 6.74$ lb

***8–92.**

Determine the force P that must be applied to the handle of the lever so that B the wheel is on the verge of turning if $M = 300\ \text{N}\cdot\text{m}$. The coefficient of static friction between the belt and the wheel is $\mu_s = 0.3$.



SOLUTION

Frictional Force on Flat Belt. Here $\beta = 270° = \dfrac{3\pi}{2}$ rad.

$$T_D = T_A e^{\mu_s \beta}$$

$$T_D = T_A e^{0.3\left(\frac{3\pi}{2}\right)}$$

$$T_D = 4.1112\ T_A \qquad \textbf{(1)}$$

Equations of Equilibrium. Referring to the *FBD* of the wheel shown in Fig. *a*,

$\circlearrowleft + \Sigma M_B = 0;\quad 300 + T_A(0.3) - T_D(0.3) = 0 \qquad$ **(2)**

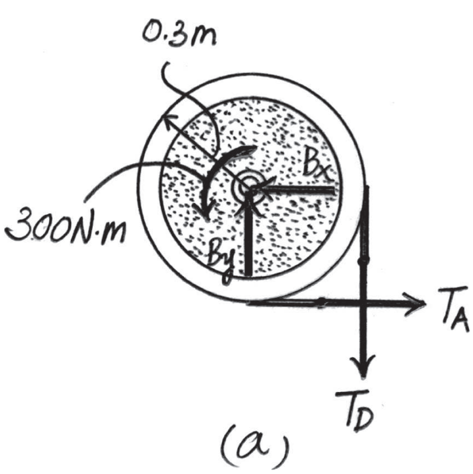
Solving Eqs. (1) and (2),

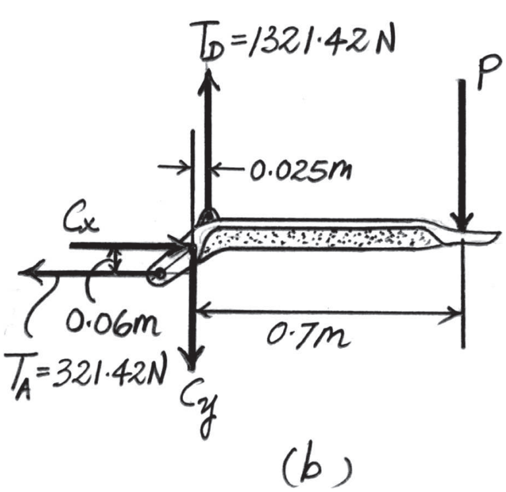
$$T_A = 321.42\ \text{N} \quad T_D = 1321.42\ \text{N}$$

Subsequently, from the *FBD* of the lever, Fig. *b*

$\circlearrowleft + \Sigma M_C = 0;\quad 1321.42(0.025) - 321.42(0.06) - P(0.7) = 0$

$P = 19.64\ \text{N} = 19.6\ \text{N}$ **Ans.**

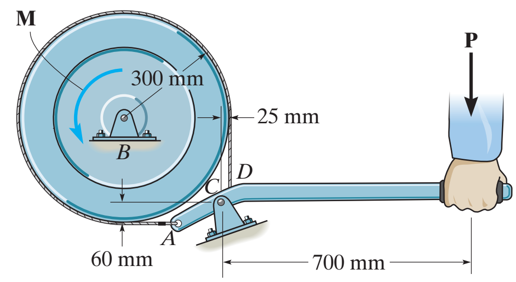




Ans:
19.6 N

8–93.

If a force of $P = 30$ N is applied to the handle of the lever, determine the largest couple moment **M** that can be resisted so that the wheel does not turn. The coefficient of static friction between the belt and the wheel is $\mu_s = 0.3$.



SOLUTION

Frictional Force on Flat Belt. Here $\beta = 270° = \dfrac{3\pi}{2}$ rad.

$$T_D = T_A e^{\mu\beta}$$

$$T_D = T_A e^{0.3\left(\frac{3\pi}{2}\right)}$$

$$T_D = 4.1112\, T_A \qquad \textbf{(1)}$$

Equations of Equilibrium. Referring to the *FBD* of the wheel shown in Fig. *a*,

$\circlearrowleft + \Sigma M_B = 0;\qquad M + T_A(0.3) - T_D(0.3) = 0 \qquad$ **(2)**

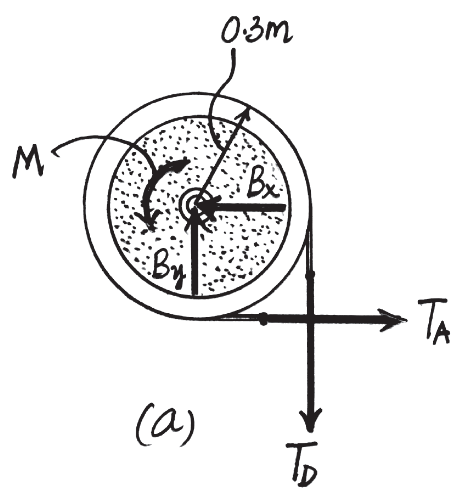
Solving Eqs. (1) and (2)

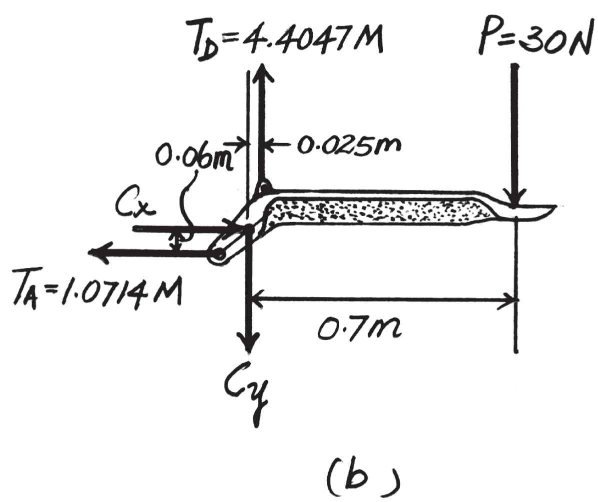
$$T_A = 1.0714\text{ m} \qquad T_D = 4.4047\text{ m}$$

Subsequently, from the *FBD* of the lever, Fig. *b*

$\circlearrowleft + \Sigma M_C = 0;\qquad 4.4047\, M(0.025) - 1.0714\, M(0.06) - 30(0.7) = 0$

$$M = 458.17\text{ N}\cdot\text{m} = 458\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

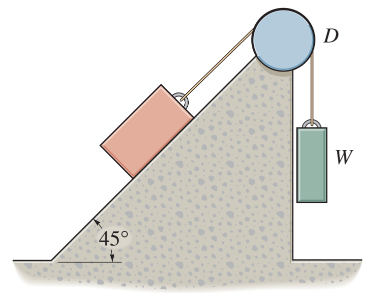




Ans:
$M = 458\text{ N}\cdot\text{m}$

***8–96.**

Determine the maximum and the minimum values of weight W which may be applied without causing the 50-lb block to slip. The coefficient of static friction between the block and the plane is $\mu_s = 0.2$, and between the rope and the drum D $\mu_s' = 0.3$.

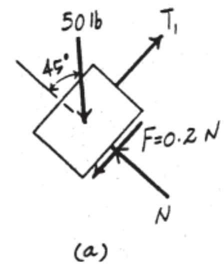


SOLUTION

Equations of Equilibrium and Friction: Since the block is on the verge of sliding up or down the plane, then, $F = \mu_s N = 0.2N$. If the block is on the verge of sliding up the plane [FBD (a)],

$\nwarrow + \Sigma F_{y'} = 0; \qquad N - 50\cos 45° = 0 \qquad N = 35.36 \text{ lb}$

$\nearrow + \Sigma F_{x'} = 0; \qquad T_1 - 0.2(35.36) - 50\sin 45° = 0 \qquad T_1 = 42.43 \text{ lb}$

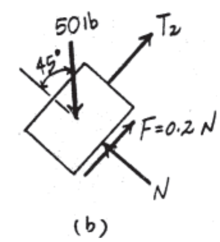


If the block is on the verge of sliding down the plane [FBD (b)],

$\nwarrow + \Sigma F_{y'} = 0; \qquad N - 50\cos 45° = 0 \qquad N = 35.36 \text{ lb}$

$\nearrow + \Sigma F_{x'} = 0; \qquad T_2 + 0.2(35.36) - 50\sin 45° = 0 \qquad T_2 = 28.28 \text{ lb}$

Frictional Force on Flat Belt: Here, $\beta = 45° + 90° = 135° = \dfrac{3\pi}{4}$ rad. If the block is on the verge of sliding up the plane, $T_1 = 42.43$ lb and $T_2 = W$.

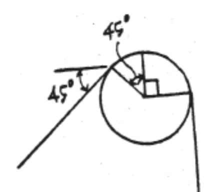


$$T_2 = T_1 e^{\mu\beta}$$

$$W = 42.43e^{0.3\left(\frac{3\pi}{4}\right)}$$

$$= 86.02 \text{ lb} = 86.0 \text{ lb}$$ **Ans.**

If the block is on the verge of sliding down the plane, $T_1 = W$ and $T_2 = 28.28$ lb.



$$T_2 = T_1 e^{\mu\beta}$$

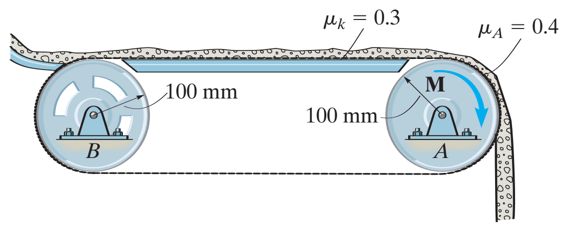
$$28.28 = We^{0.3\left(\frac{3\pi}{4}\right)}$$

$$W = 13.95 \text{ lb} = 13.9 \text{ lb}$$ **Ans.**

Ans:
$W = 86.0$ lb
$W = 13.9$ lb

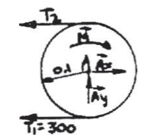
8–97.

Granular material, having a density of $1.5\ \mathrm{Mg/m^3}$, is transported on a conveyor belt that slides over the fixed surface, having a coefficient of kinetic friction of $\mu_k = 0.3$. Operation of the belt is provided by a motor that supplies a torque **M** to wheel A. The wheel at B is free to turn, and the coefficient of static friction between the wheel at A and the belt is $\mu_A = 0.4$. If the belt is subjected to a pretension of 300 N when no load is on the belt, determine the greatest volume V of material that is permitted on the belt at any time without allowing the belt to stop. What is the torque **M** required to drive the belt when it is subjected to this maximum load?



SOLUTION

Wheel A:

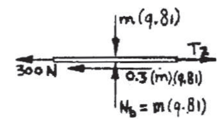


$\circlearrowleft + \Sigma M_A = 0; \qquad -M - 300\,(0.1) + T_2(0.1) = 0$

$T_2 = T_1 e^{\mu\beta}; \qquad T_2 = 300e^{0.4(\pi)} = 1054.1\ \text{N}$

Thus, $M = 75.4\ \text{N}\cdot\text{m}$ **Ans.**

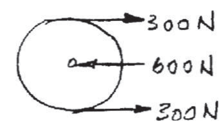
Belt,



$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 1054.1 - 0.3\,(m)\,(9.81) - 300 = 0$

$m = 256.2\ \text{kg}$

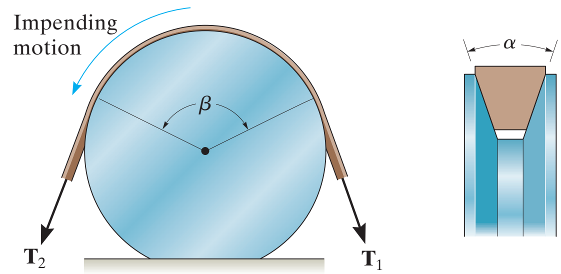
$V = \dfrac{m}{p} = \dfrac{256.2}{1500} = 0.171\ \text{m}^3$ **Ans.**



Ans:
$M = 75.4\ \text{N}\cdot\text{m}$
$V = 0.171\ \text{m}^3$

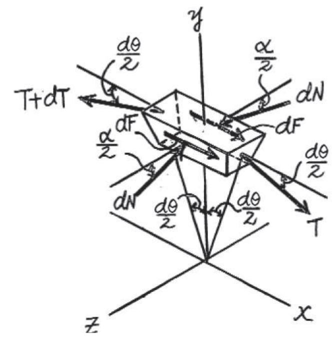
8–98.

Show that the frictional relationship between the belt tensions, the coefficient of friction μ, and the angular contacts α and β for the V-belt is $T_2 = T_1 e^{\mu\beta/\sin(\alpha/2)}$.



SOLUTION

FBD of a section of the belt is shown.



Proceeding in the general manner:

$\Sigma F_x = 0;$ $\qquad -(T + dT)\cos\frac{d\theta}{2} + T\cos\frac{d\theta}{2} + 2\,dF = 0$

$\Sigma F_y = 0;$ $\qquad -(T + dT)\sin\frac{d\theta}{2} - T\sin\frac{d\theta}{2} + 2\,dN\sin\frac{\alpha}{2} = 0$

Replace $\qquad \sin\frac{d\theta}{2}$ by $\frac{d\theta}{2}$,

$\cos\frac{d\theta}{2}$ by 1,

$dF = \mu\,dN$

Using this and $(dT)(d\theta) \rightarrow 0$, the above relations become

$$dT = 2\mu\,dN$$

$$T\,d\theta = 2\left(dN\sin\frac{\alpha}{2}\right)$$

Combine

$$\frac{dT}{T} = \mu\frac{d\theta}{\sin\frac{\alpha}{2}}$$

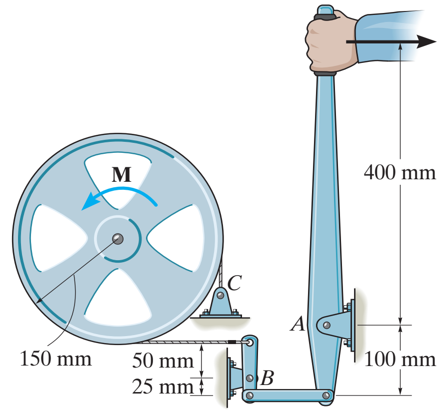
Integrate from $\theta = 0,\ T = T_1$
to $\theta = \beta, T = T_2$

we get,

$$T_2 = T_1\,e^{\left(\frac{\mu\beta}{\sin\frac{\alpha}{2}}\right)} \qquad \textbf{Q.E.D}$$

8–99.

The wheel is subjected to a torque of $M = 50\ \text{N}\cdot\text{m}$. If the coefficient of kinetic friction between the band brake and the rim of the wheel is $\mu_k = 0.3$, determine the smallest horizontal force P that must be applied to the lever to stop the wheel.



SOLUTION

Wheel:

$\zeta + \Sigma M_O = 0; \qquad -T_2(0.150) + T_1(0.150) + 50 = 0$

$T_2 = T_1 e^{\mu\beta}; \qquad T_2 = T_1 e^{0.3\left(\frac{3\pi}{2}\right)}$

$T_1 = 107.14\ \text{N}$

Link:

$\zeta + \Sigma M_B = 0; \qquad 107.14\ (0.05) - F\ (0.025) = 0$

$F = 214.28\ \text{N}$

Lever:

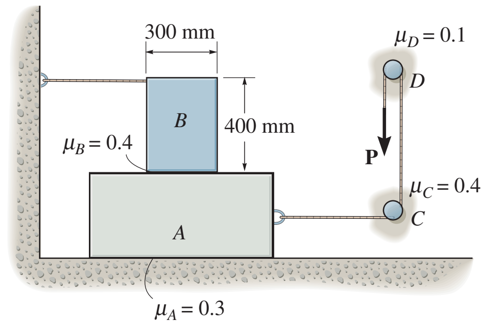
$\zeta + \Sigma M_A = 0; \qquad -P\ (0.4) + 214.28\ (0.1) = 0$

$P = 53.6\ \text{N}$ **Ans.**

Ans:
$P = 53.6\ \text{N}$

***8–100.**

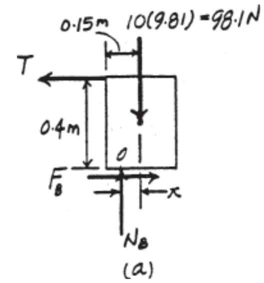
Blocks A and B have a mass of 7 kg and 10 kg, respectively. Using the coefficients of static friction indicated, determine the largest vertical force P which can be applied to the cord without causing motion.



SOLUTION

Frictional Forces on Flat Belts: When the cord pass over peg D, $\beta = 180° = \pi$ rad and $T_2 = P$. Applying Eq. 8–6, $T_2 = T_1 e^{\mu\beta}$, we have

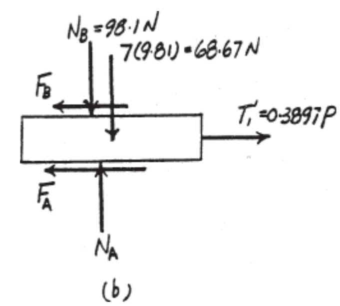
$$P = T_1 e^{0.1\,\pi} \qquad T_1 = 0.7304P$$



When the cord pass over peg C, $\beta = 90° = \dfrac{\pi}{2}$ rad and $T_2{}' = T_1 = 0.7304P$. Applying Eq. 8–6, $T_2{}' = T_1{}' e^{\mu\beta}$, we have

$$0.7304P = T_1{}' e^{0.4(\pi/2)} \qquad T_1{}' = 0.3897P$$

Equations of Equilibrium: From FDB (b),



$+\uparrow \Sigma F_y = 0; \qquad N_B - 98.1 = 0 \qquad N_B = 98.1\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad F_B - T = 0$ **(1)**

$\circlearrowleft + \Sigma M_O = 0; \qquad T(0.4) - 98.1(x) = 0$ **(2)**

From FDB (b),

$+\uparrow \Sigma F_y = 0; \qquad N_A - 98.1 - 68.67 = 0 \qquad N_A = 166.77\text{ N}$

$\xrightarrow{+} \Sigma F_x = 0; \qquad 0.3897P - F_B - F_A = 0$ **(3)**

Friction: Assuming the block B is on the verge of tipping, then $x = 0.15$ m. A1 for motion to occur, block A will have slip. Hence, $F_A = (\mu_s)_A N_A = 0.3(166.77) = 50.031$ N. Substituting these values into Eqs. (1), (2) and (3) and solving yields

$$P = 222.81\text{ N} = 223\text{ N} \qquad \textbf{Ans.}$$

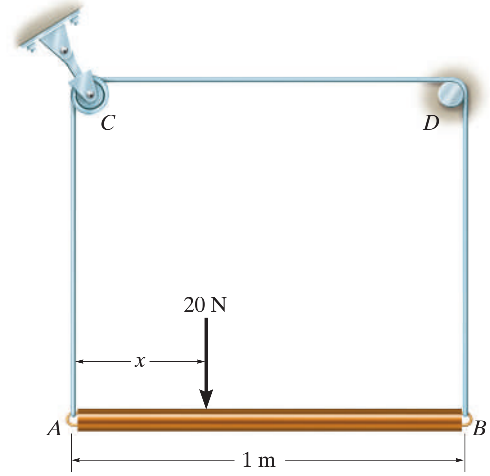
$$F_B = T = 36.79\text{ N}$$

Since $(F_B)_{max} = (\mu_s)_B N_B = 0.4(98.1) = 39.24\text{ N} > F_B$, block B does not slip but tips. Therefore, the above assumption is correct.

Ans:
$P = 223$ N

8–101.

The uniform bar AB is supported by a rope that passes over a frictionless pulley at C and a fixed peg at D. If the coefficient of static friction between the rope and the peg is $\mu_D = 0.3$, determine the smallest distance x from the end of the bar at which a 20-N force may be placed and not cause the bar to move.



SOLUTION

$\zeta + \Sigma M_A = 0; \qquad -20\,(x) + T_B\,(1) = 0$

$+\uparrow \Sigma F_y = 0; \qquad T_A + T_B - 20 = 0$

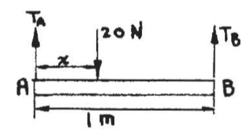
$T_2 = T_1 e^{\mu\beta}; \qquad T_A = T_B e^{0.3(\frac{\pi}{2})} = 1.602\,T_B$

Solving,

$$T_A = 12.3\text{ N}$$

$$T_B = 7.69\text{ N}$$

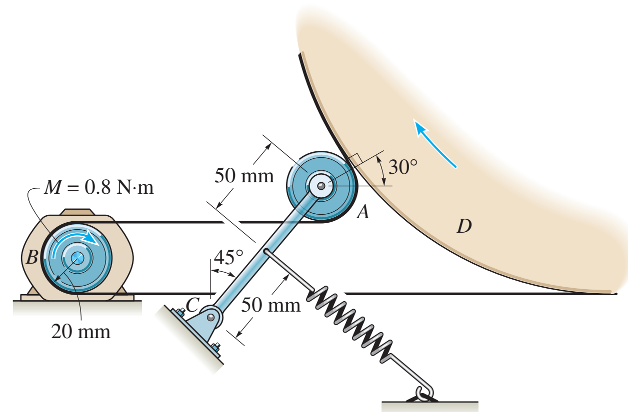
$$x = 0.384\text{ m}$$ **Ans.**



Ans:
$x = 0.384$ m

8–102.

The belt on the portable dryer wraps around the drum D, idler pulley A, and motor pulley B. If the motor can develop a maximum torque of $M = 0.80\text{ N}\cdot\text{m}$, determine the smallest spring tension required to hold the belt from slipping. The coefficient of static friction between the belt and the drum and motor pulley is $\mu_s = 0.3$.



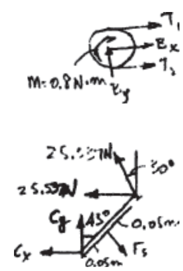
SOLUTION

$\zeta + \Sigma M_B = 0; \qquad -T_1\,(0.02) + T_2\,(0.02) - 0.8 = 0$

$T_2 = T_1\,e^{\mu\beta}; \qquad T_2 = T_1\,e^{(0.3)(\pi)} = 2.5663T_1$

$T_1 = 25.537\text{ N}$

$T_2 = 65.53\text{ N}$



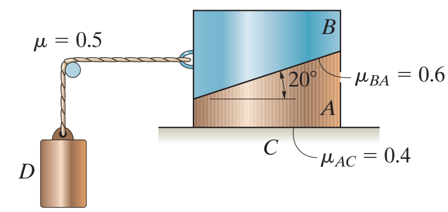
$\zeta + \Sigma M_C = 0; \qquad -F_s(0.05) + (25.537 + 25.537\sin 30°)(0.1\cos 45°) + 25.537\cos 30°(0.1\sin 45°) = 0$

$F_s = 85.4\text{ N}$ **Ans.**

Ans:
$F_s = 85.4\text{ N}$

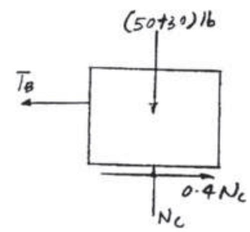
8–103.

Blocks A and B weigh 50 lb and 30 lb, respectively. Using the coefficients of static friction indicated, determine the greatest weight of block D without causing motion.



SOLUTION

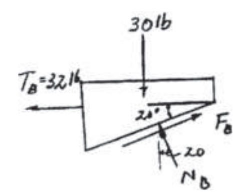
For block A and B: Assuming block B does not slip



$+\uparrow \Sigma F_y = 0;\qquad N_C - (50 + 30) = 0 \qquad N_C = 80\text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 0.4(80) - T_B = 0 \qquad T_B = 32\text{ lb}$

For block B:



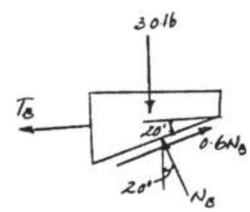
$+\uparrow \Sigma F_y = 0;\qquad N_B \cos 20° + F_B \sin 20° - 30 = 0 \qquad$ **(1)**

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad F_B \cos 20° - N_B \sin 20° - 32 = 0 \qquad$ **(2)**

Solving Eqs. (1) and (2) yields:

$F_B = 40.32\text{ lb} \qquad N_B = 17.25\text{ lb}$

Since $F_B = 40.32\text{ lb} > \mu N_B = 0.6(17.25) = 10.35\text{ lb}$, slipping does occur between A and B. Therefore, the assumption is no good.



Since slipping occurs, $F_B = 0.6\,N_B$.

$+\uparrow \Sigma F_y = 0;\qquad N_B \cos 20° + 0.6N_B \sin 20° - 30 = 0 \qquad N_B = 26.20\text{ lb}$

$\overset{+}{\rightarrow} \Sigma F_x = 0;\qquad 0.6(26.20)\cos 20° - 26.20 \sin 20° - T_B = 0 \qquad T_B = 5.812\text{ lb}$

$T_2 = T_1 e^{\mu\beta} \qquad$ Where $\qquad T_2 = W_D,\ T_1 = T_B = 5.812\text{ lb},\ \beta = 0.5\pi\text{ rad}$

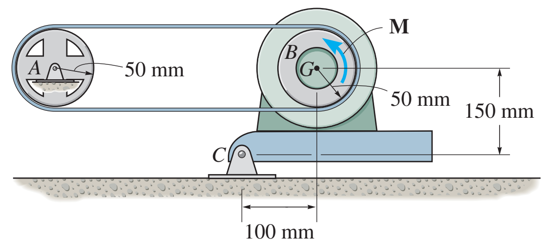
$W_D = 5.812e^{0.5(0.5\pi)}$

$\quad = 12.7\text{ lb}$ **Ans.**

Ans:
$W_D = 12.7\text{ lb}$

***8–104.**

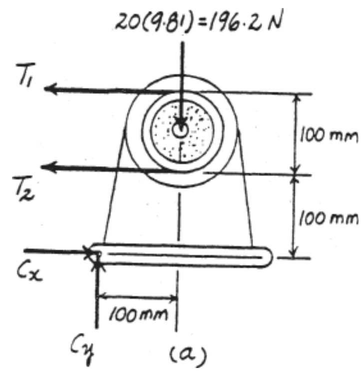
The 20-kg motor has a center of gravity at G and is pin-connected at C to maintain a tension in the drive belt. Determine the smallest counterclockwise twist or torque **M** that must be supplied by the motor to turn the disk B if wheel A locks and causes the belt to slip over the disk. No slipping occurs at A. The coefficient of static friction between the belt and the disk is $\mu_s = 0.3$.



SOLUTION

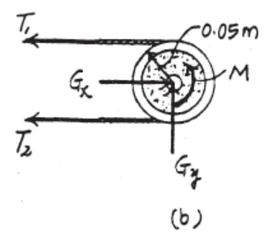
Equations of Equilibrium: From FBD (a),

$$\circlearrowleft+\Sigma M_C = 0; \quad T_2\,(100) + T_1\,(200) - 196.2(100) = 0 \qquad \textbf{(1)}$$



From FBD (b),

$$\circlearrowleft+\Sigma M_O = 0; \qquad M + T_1\,(0.05) - T_2\,(0.05) = 0 \qquad \textbf{(2)}$$



Frictional Force on Flat Belt: Here, $\beta = 180° = \pi$ rad. Applying Eq. 8–6, $T_2 = T_1\,e^{\mu\beta}$, we have

$$T_2 = T_1\,e^{0.3\pi} = 2.566T_1 \qquad \textbf{(3)}$$

Solving Eqs. (1), (2), and (3) yields

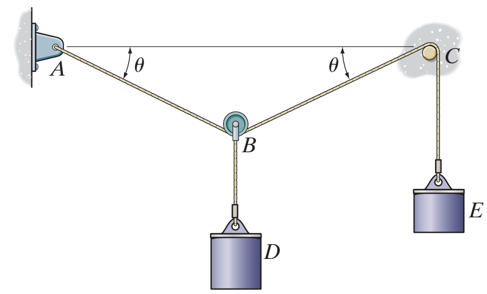
$$M = 3.37\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

$$T_1 = 42.97\ \text{N} \qquad T_2 = 110.27\ \text{N}$$

Ans:
$M = 3.37\ \text{N}\cdot\text{m}$

8–105.

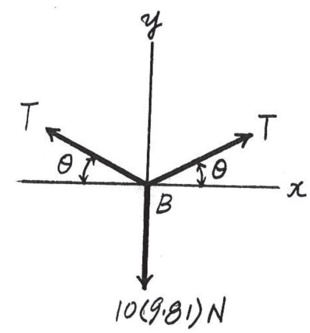
A 10-kg cylinder D, which is attached to a small pulley B, is placed on the cord as shown. Determine the largest angle u so that the cord does not slip over the peg at C. The cylinder at E also has a mass of 10 kg, and the coefficient of static friction between the cord and the peg is $\mu_s = 0.1$.



SOLUTION

Since pully B is smooth, the tension in the cord between pegs A and C remains constant. Referring to the free-body diagram of the joint B shown in Fig. a, we have

$$+\uparrow \Sigma F_y = 0; \qquad 2T \sin\theta - 10(9.81) = 0 \qquad T = \frac{49.05}{\sin\theta}$$



(a)

In the case where cylinder E is on the verge of ascending, $T_2 = T = \dfrac{49.05}{\sin\theta}$ and $T_1 = 10(9.81)\,\text{N}$. Here, $\dfrac{\pi}{2} + \theta$, Fig. b. Thus,

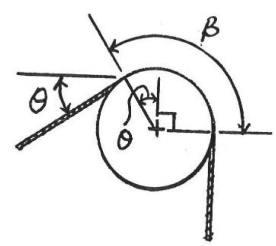
$$T_2 = T_1 e^{\mu_s \beta}$$

$$\frac{49.05}{\sin\theta} = 10(9.81)\, e^{0.1\left(\frac{\pi}{2} + \theta\right)}$$

$$\ln \frac{0.5}{\sin\theta} = 0.1\left(\frac{\pi}{2} + \theta\right)$$

Solving by trial and error, yields

$$\theta = 0.4221 \text{ rad} = 24.2^\circ$$



(b)

In the case where cylinder E is on the verge of descending, $T_2 = 10(9.81)$ N and $T_1 = \dfrac{49.05}{\sin\theta}$. Here, $\dfrac{\pi}{2} + \theta$. Thus,

$$T_2 = T_1 e^{\mu_s \beta}$$

$$10(9.81) = \frac{49.05}{\sin\theta}\, e^{0.1\left(\frac{\pi}{2} + \theta\right)}$$

$$\ln\,(2 \sin\theta) = 0.1\left(\frac{\pi}{2} + \theta\right)$$

Solving by trial and error, yields

$$\theta = 0.6764 \text{ rad} = 38.8^\circ$$

Thus, the range of θ at which the wire does not slip over peg C is

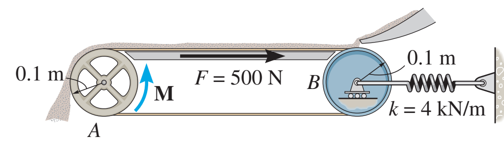
$$24.2^\circ < \theta < 38.8^\circ$$

$$\theta_{\max} = 38.8^\circ \qquad \textbf{Ans.}$$

Ans:
$\theta_{\max} = 38.8^\circ$

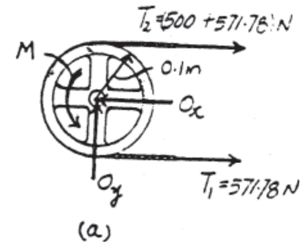
8–106.

A conveyer belt is used to transfer granular material and the frictional resistance on the top of the belt is $F = 500$ N. Determine the smallest stretch of the spring attached to the moveable axle of the idle pulley B so that the belt does not slip at the drive pulley A when the torque **M** is applied. What minimum torque **M** is required to keep the belt moving? The coefficient of static friction between the belt and the wheel at A is $\mu_s = 0.2$.



SOLUTION

Frictional Force on Flat Belt: Here, $\beta = 180° = \pi$ rad and $T_2 = 500 + T$ and $T_1 = T$. Applying Eq. 8–6, we have



$$T_2 = T_1 e^{\mu\beta}$$

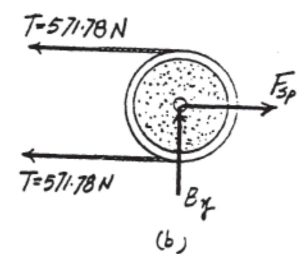
$$500 + T = Te^{0.2\pi}$$

$$T = 571.78 \text{ N}$$

Equations of Equilibrium: From FBD (a),

$\circlearrowleft + \Sigma M_O = 0;\qquad M + 571.78(0.1) - (500 + 578.1)(0.1) = 0$

$$M = 50.0 \text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$



From FBD (b),

$\overset{+}{\rightarrow} \Sigma F_x = 0;\quad F_{sp} - 2(578.71) = 0 \qquad F_{sp} = 1143.57 \text{ N}$

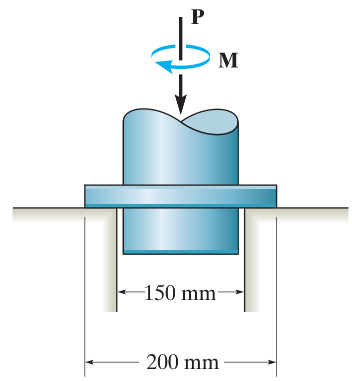
Thus, the spring stretch is

$$x = \frac{F_{sp}}{k} = \frac{1143.57}{4000} = 0.2859 \text{ m} = 286 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$M = 50.0 \text{ N}\cdot\text{m}$
$x = 286$ mm

8–107.

The collar bearing uniformly supports an axial force of $P = 5$ kN. If the coefficient of static friction is $\mu_s = 0.3$, determine the smallest torque M required to overcome friction.



SOLUTION

Bearing Friction. With $R_2 = 0.1$ m, $R_1 = 0.075$ m, $P = 5(10^3)$ N, and $\mu_s = 0.3$,

$$M = \frac{2}{3}\mu_s P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

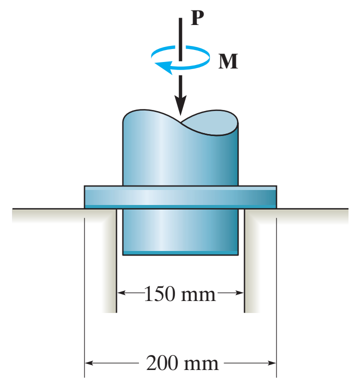
$$= \frac{2}{3}(0.3)\left[5(10^3)\right]\left(\frac{0.1^3 - 0.075^3}{0.1^2 - 0.075^2}\right)$$

$$= 132\ \text{N}\cdot\text{m}$$

Ans:
$M = 132\ \text{N}\cdot\text{m}$

***8–108.**

The collar bearing uniformly supports an axial force of $P = 8$ kN. If a torque of $M = 200$ N·m is applied to the shaft and causes it to rotate at constant velocity, determine the coefficient of kinetic friction at the surface of contact.



SOLUTION

Bearing Friction. With $R_2 = 0.1$ m, $R_1 = 0.075$ m, $M = 300$ N·m, and $P = 8(10^3)$ N,

$$M = \frac{2}{3}\mu_k P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

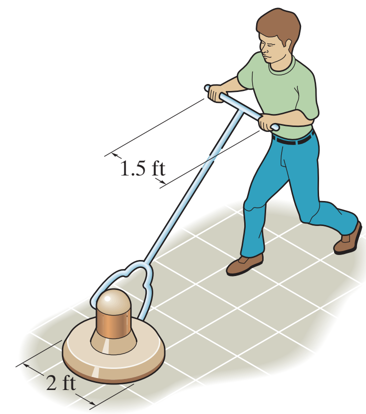
$$200 = \frac{2}{3}\mu_k\left[8(10^3)\right]\left(\frac{0.1^3 - 0.075^3}{0.1^2 - 0.075^2}\right)$$

$$\mu_k = 0.284$$ **Ans.**

Ans:
$\mu_k = 0.284$

8–109.

The floor-polishing machine rotates at a constant angular velocity. If it has a weight of 80 lb. determine the couple forces F the operator must apply to the handles to hold the machine stationary. The coefficient of kinetic friction between the floor and brush is $\mu_k = 0.3$. Assume the brush exerts a uniform pressure on the floor.



SOLUTION

$$M = \frac{2}{3}\mu\, P\, R$$

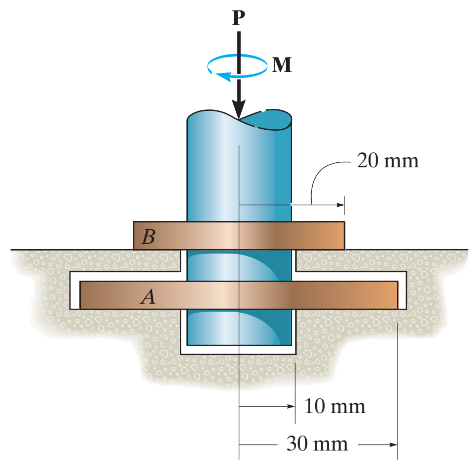
$$F(1.5) = \frac{2}{3}(0.3)\,(80)(1)$$

$F = 10.7$ lb **Ans.**

Ans:
$F = 10.7$ lb

8–110.

The *double-collar bearing* is subjected to an axial force $P = 4\text{ kN}$. Assuming that collar A supports $0.75P$ and collar B supports $0.25P$, both with a uniform distribution of pressure, determine the maximum frictional moment M that may be resisted by the bearing. Take $\mu_s = 0.2$ for both collars.



SOLUTION

$$M = \frac{2}{3}\mu_s\, P\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

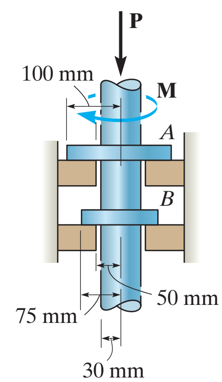
$$M = \frac{2}{5}(0.2)\left(\frac{(0.03)^3 - (0.01)^3}{(0.03)^2 - (0.01)^2}(0.75)\,(4000) + \frac{(0.02)^3 - (0.01)^3}{(0.02)^2 - (0.01)^2}(0.25)\,(4000)\right)$$

$$= 16.1\ \text{N}\cdot\text{m}$$ **Ans.**

Ans:
$M = 16.1\ \text{N}\cdot\text{m}$

8–111.

The *double-collar bearing* is subjected to an axial force $P = 16$ kN. Assuming that collar A supports $0.75P$ and collar B supports $0.25P$, both with a uniform distribution of pressure, determine the smallest torque **M** that must be applied to overcome friction. Take $\mu_s = 0.2$ for both collars.



SOLUTION

Bearing Friction. Here $(R_A)_2 = 0.1$ m, $(R_A)_1 = 0.05$ m, $P_A = 0.75\left[16\left(10^3\right)\text{ N}\right] = 12\left(10^3\right)$ N, $(R_B)_2 = 0.075$ m, $(R_B)_1 = 0.05$ m and $P_B = 0.25\left[16\left(10^3\right)\text{ N}\right] = 4\left(10^3\right)$ N.

$$\mu = \frac{2}{3}\mu_s P_A\left[\frac{(R_A)_2^3 - (R_A)_1^3}{(R_A)_2^2 - (R_A)_1^2}\right] + \frac{2}{3}\mu_s P_B\left[\frac{(R_B)_2^3 - (R_B)_1^3}{(R_B)_2^2 - (R_B)_1^2}\right]$$

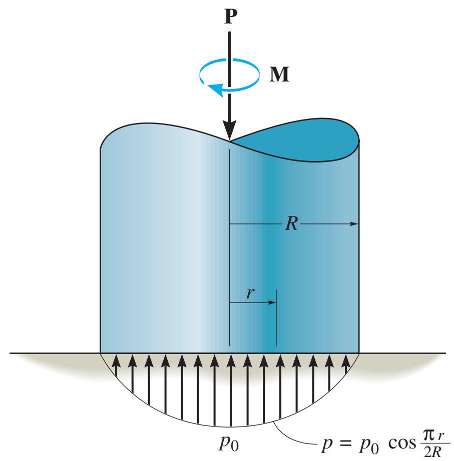
$$= \frac{2}{3}(0.2)\left[12\left(10^3\right)\right]\left(\frac{0.1^3 - 0.05^3}{0.1^2 - 0.05^2}\right) + \frac{2}{3}(0.2)\left[4\left(10^3\right)\right]\left(\frac{0.075^3 - 0.05^3}{0.075^2 - 0.05^2}\right)$$

$$= 237.33\text{ N}\cdot\text{m} = 237\text{ N}\cdot\text{m} \qquad \textbf{Ans.}$$

Ans:
$M = 237$ N·m

***8–112.**

The pivot bearing is subjected to a pressure distribution at its surface of contact which varies as shown. If the coefficient of static friction is μ, determine the torque M required to overcome friction if the shaft supports an axial force **P.**



SOLUTION

$$dF = \mu\, dN = \mu\, p_0 \cos\left(\frac{\pi r}{2R}\right) dA$$

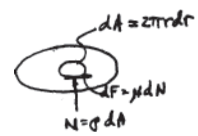
$$M = \int_A r\mu\, p_0 \cos\left(\frac{\pi r}{2R}\right) r\, dr\, d\theta$$

$$= \mu\, p_0 \int_0^R \left(r^2 \cos\left(\frac{\pi r}{2R}\right) dr\right) \int_0^{2\pi} d\theta$$

$$= \mu\, p_0 \left[\frac{2r}{\left(\frac{\pi}{2R}\right)^2} \cos\left(\frac{\pi r}{2R}\right) + \frac{\left(\frac{\pi}{2R}\right)^2 r^2 - 2}{\left(\frac{\pi}{2R}\right)^3} \sin\left(\frac{\pi r}{2R}\right)\right]_0^R (2\pi)$$

$$= \mu p_0 \left(\frac{16R^3}{\pi^2}\right)\left[\left(\frac{\pi}{2}\right)^2 - 2\right]$$

$$= 0.7577\mu\, p_0\, R^3$$



$$P = \int_A dN = \int_0^R p_0 \left(\cos\left(\frac{\pi r}{2R}\right) r\,dr\right) \int_0^{2\pi} d\theta$$

$$= p_0 \left[\frac{1}{\left(\frac{\pi}{2R}\right)^2} \cos\left(\frac{\pi r}{2R}\right) + \frac{r}{\left(\frac{\pi}{2R}\right)} \sin\left(\frac{\pi r}{2R}\right)\right]_0^R (2\pi)$$

$$= 4p_0\, R^2 \left(1 - \frac{2}{\pi}\right)$$

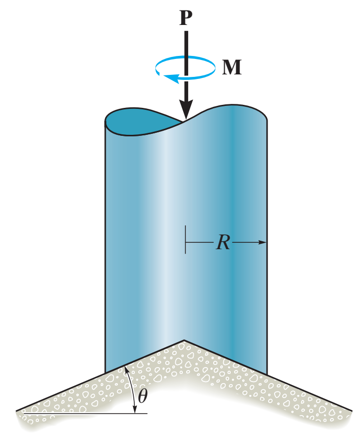
$$= 1.454 p_0\, R^2$$

Thus, $\quad M = 0.521\, P\mu R \quad$ **Ans.**

Ans:
$M = 0.521\, P\mu R$

8–113.

The conical bearing is subjected to a constant pressure distribution at its surface of contact. If the coefficient of static friction is μ_s, determine the torque M required to overcome friction if the shaft supports an axial force $\mathbf{P}$.



SOLUTION

The differential area (shaded) $dA = 2\pi r\left(\dfrac{dr}{\cos\theta}\right) = \dfrac{2\pi r dr}{\cos\theta}$

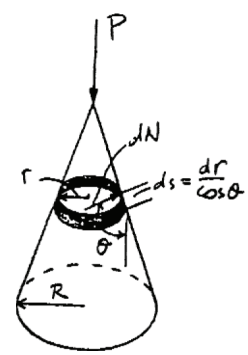
$$P = \int p\cos\theta\, dA = \int p\cos\theta\left(\frac{2\pi r dr}{\cos\theta}\right) = 2\pi p\int_0^R r dr$$

$$P = \pi p R^2 \qquad p = \frac{P}{\pi R^2}$$

$$dN = p dA = \frac{P}{\pi R^2}\left(\frac{2\pi r dr}{\cos\theta}\right) = \frac{2P}{R^2\cos\theta} r dr$$

$$M = \int r dF = \int \mu_s r dN = \frac{2\mu_s P}{R^2\cos\theta}\int_0^R r^2\, dr$$

$$= \frac{2\mu_s P}{R^2\cos\theta}\frac{R^3}{3} = \frac{2\mu_s PR}{3\cos\theta} \qquad \textbf{Ans.}$$

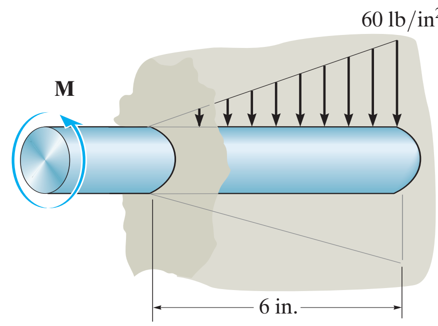


Ans:

$M = \dfrac{2\mu_s PR}{3\cos\theta}$

8–114.

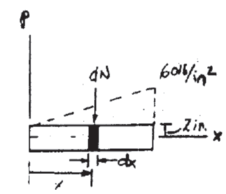
The 4-in.-diameter shaft is held in the hole such that the normal pressure acting around the shaft varies linearly with its depth as shown. Determine the frictional torque that must be overcome to rotate the shaft. Take $\mu_s = 0.2$.



SOLUTION

Express the pressure p as the function of x:

$$P = \frac{60}{6}x = 10x$$



The differential area (shaded) $dA = 2\pi(2)dx = 4\pi dx$

$$dN = p\,dA = 10x(4\pi dx) = 40\pi x dx$$

$$T = 2\int dF = 2\int \mu dN = 80\pi\mu \int_0^6 x dx$$

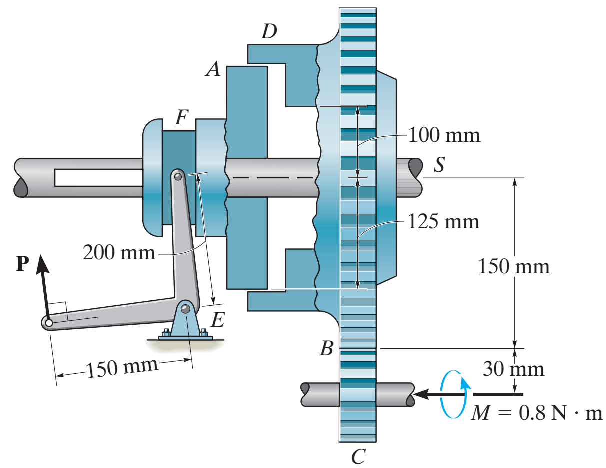
$$= 1440\pi\mu \text{ lb} \cdot \text{in.}$$

$$= 1440\pi(0.2) = 905 \text{ lb} \cdot \text{in.}$$ **Ans.**

Ans:
$T = 905 \text{ lb} \cdot \text{in.}$

8–115.

The plate clutch consists of a flat plate A that slides over the rotating shaft S. The shaft is fixed to the driving plate gear B. If the gear C, which is in mesh with B, is subjected to a torque of $M = 0.8\text{N}\cdot\text{m}$, determine the smallest force P, that must be applied via the control arm, to stop the rotation. The coefficient of static friction between the plates A and D is $\mu_s = 0.4$. Assume the bearing pressure between A and D to be uniform.



SOLUTION

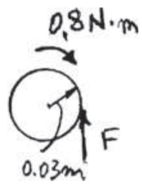
$$F = \frac{0.8}{0.03} = 26.667 \text{ N}$$

$$M = 26.667(0.150) = 4.00 \text{ N}\cdot\text{m}$$

$$M = \frac{2}{3}\mu P'\left(\frac{R_2^3 - R_1^3}{R_2^2 - R_1^2}\right)$$

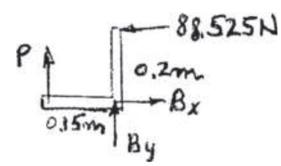
$$4.00 = \frac{2}{3}(0.4)(P')\left(\frac{(0.125)^3 - (0.1)^3}{(0.125)^2 - (0.1)^2}\right)$$

$$P' = 88.525 \text{ N}$$



$$\zeta + \Sigma M_F = 0; \qquad 88.525(0.2) - P(0.15) = 0$$

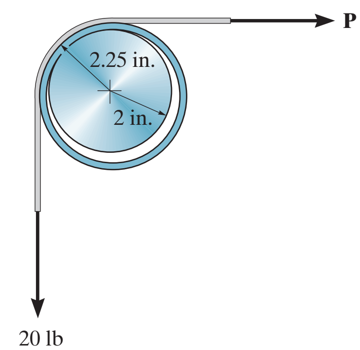
$$P = 118 \text{ N}$$ **Ans.**



Ans:
$P = 118$ N

***8–116.**

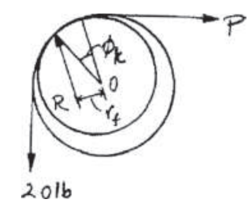
The collar fits *loosely* around a fixed shaft that has a radius of 2 in. If the coefficient of kinetic friction between the shaft and the collar is $\mu_k = 0.3$, determine the force P on the horizontal segment of the belt so that the collar rotates *counterclockwise* with a constant angular velocity. Assume that the belt does not slip on the collar; rather, the collar slips on the shaft. Neglect the weight and thickness of the belt and collar. The radius, measured from the center of the collar to the mean thickness of the belt, is 2.25 in.



SOLUTION

$\phi_k = \tan^{-1}\mu_k = \tan^{-1}0.3 = 16.699°$

$r_f = 2\sin 16.699° = 0.5747$ in.



Equilibrium:

$+\uparrow \Sigma F_y = 0; \quad R_y - 20 = 0 \quad R_y = 20$ lb

$\xrightarrow{+} \Sigma F_x = 0; \quad P - R_x = 0 \quad R_x = P$

Hence $R = \sqrt{R_x^2 + R_y^2} = \sqrt{P^2 + 20^2}$

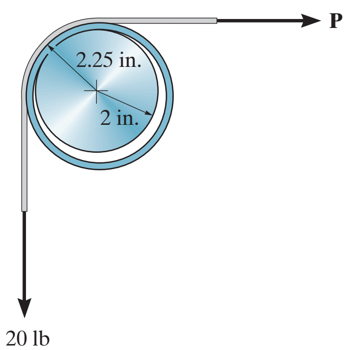
$\circlearrowleft + \Sigma M_O = 0; \quad -\left(\sqrt{P^2 + 20^2}\right)(0.5747) + 20(2.25) - P(2.25) = 0$

$P = 13.8$ lb **Ans.**

Ans:
$P = 13.8$ lb

8–117.

The collar fits *loosely* around a fixed shaft that has a radius of 2 in. If the coefficient of kinetic friction between the shaft and the collar is $\mu_k = 0.3$, determine the force P on the horizontal segment of the belt so that the collar rotates *clockwise* with a constant angular velocity. Assume that the belt does not slip on the collar; rather, the collar slips on the shaft. Neglect the weight and thickness of the belt and collar. The radius, measured from the center of the collar to the mean thickness of the belt, is 2.25 in.



SOLUTION

$\phi_k = \tan^{-1}\mu_k = \tan^{-1} 0.3 = 16.699°$

$r_f = 2 \sin 16.699° = 0.5747$ in.

Equilibrium:

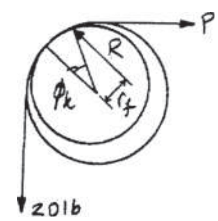
$+\uparrow \Sigma F_y = 0; \quad R_y - 20 = 0 \quad R_y = 20 \text{ lb}$

$\xrightarrow{+} \Sigma F_x = 0; \quad P - R_x = 0 \quad R_x = P$

Hence $R = \sqrt{R_x^2 + R_y^2} = \sqrt{P^2 + 20^2}$

$\circlearrowleft + \Sigma M_O = 0; \quad \left(\sqrt{P^2 + 20^2}\right)(0.5747) + 20(2.25) - P(2.25) = 0$

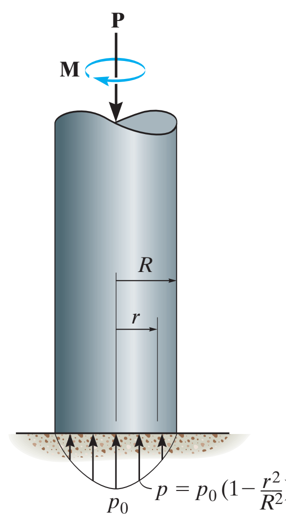
$P = 29.0 \text{ lb}$ **Ans.**



Ans:
$P = 29.0$ lb

8–118.

The pivot bearing is subjected to a parabolic pressure distribution at its surface of contact. If the coefficient of static friction is μ_s, determine the torque M required to overcome friction and turn the shaft if it supports an axial force **P**.



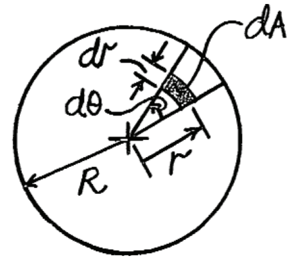
SOLUTION

The differential are $dA = (rd\theta)(dr)$

$$P = \int p\,dA = \int p_0\left(1 - \frac{r^2}{R^2}\right)(rd\theta)(dr) = p_0\int_0^{2\pi} d\theta \int_0^R r\left(1 - \frac{r^2}{R^2}\right)dr$$

$$P = \frac{\pi R^2\, p_0}{2} \qquad p_0 = \frac{2P}{\pi R^2}$$

$$dN = p\,dA = \frac{2P}{\pi R^2}\left(1 - \frac{r^2}{R^2}\right)(rd\theta)(dr)$$



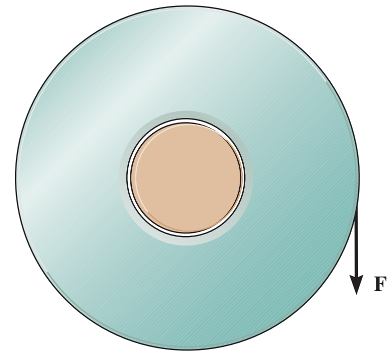
$$M = \int r\,dF = \int \mu_s\, r\,dN = \frac{2\mu_s P}{\pi R^2}\int_0^{2\pi} d\theta \int_0^R r^2\left(1 - \frac{r^2}{R^2}\right)dr$$

$$= \frac{8}{15}\mu_s PR \qquad \textbf{Ans.}$$

Ans:
$M = \frac{8}{15}\mu_s PR$

8–119.

A disk having an outer diameter of 120 mm fits loosely over a fixed shaft having a diameter of 30 mm. If the coefficient of static friction between the disk and the shaft is $\mu_s = 0.15$ and the disk has a mass of 50 kg, determine the smallest vertical force **F** acting on the rim which must be applied to the disk to cause it to slip over the shaft.



SOLUTION

Frictional Force on Journal Bearing: Here, $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.15 = 8.531°$. Then the radius of friction circle is

$$r_f = r \sin\phi_s = 0.015 \sin 8.531° = 2.225(10^{-3})\text{ m}$$

Equation of Equilibrium:

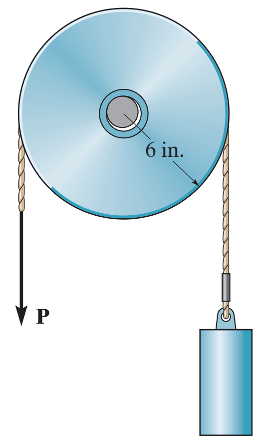
$$\circlearrowleft + \Sigma M_P = 0; \qquad 490.5(2.225)(10^{-3}) - F[0.06 - (2.225)(10^{-3})] = 0$$

$$F = 18.9\text{ N}$$ **Ans.**

Ans:
$F = 18.9$ N

***8–120.**

The 4-lb pulley has a diameter of 1 ft and the axle has a diameter of 1 in. If the coefficient of kinetic friction between the axle and the pulley is $\mu_k = 0.20$, determine the vertical force P on the rope required to lift the 20-lb block at constant velocity.



SOLUTION

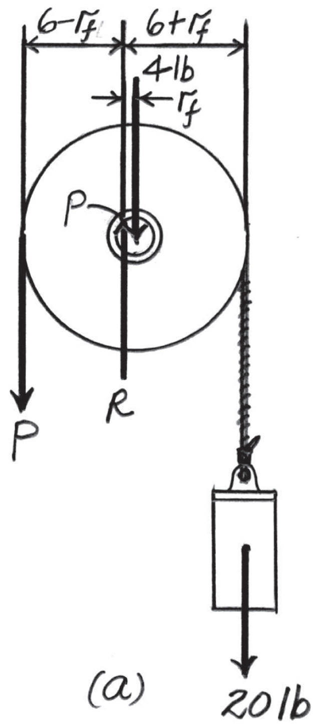
Frictional Force on Journal Bearing. Here $\phi_k = \tan^{-1}\mu_k = \tan^{-1} 0.2 = 11.3099°$. Then the radius of the friction circle is

$$r_f = r \sin\phi_k = 0.5 \sin 11.3099° = 0.09806 \text{ in.}$$

Equations of Equilibrium. Referring to the *FBD* of the pulley shown in Fig. *a*,

$\circlearrowleft + \Sigma M_P = 0;\qquad P(6 - 0.09806) - 4(0.09806) - 20(6 + 0.09806) = 0$

$$P = 20.73 = 20.7 \text{ lb}$$ **Ans.**

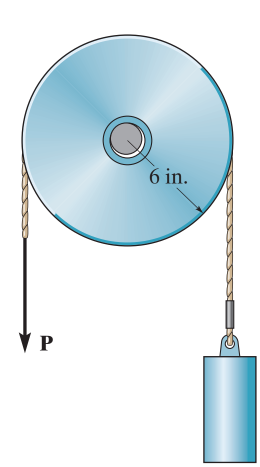


(a)

Ans:
$P = 20.7$ lb

8–121.

Solve Prob. 8–120 if the force **P** is applied horizontally to the left.



SOLUTION

Frictional Force on Journal Bearing. Here $\phi_k = \tan^{-1}\mu_k = \tan^{-1} 0.2 = 11.3099°$. Then the radius of the friction circle is

$$r_f = r \sin \phi_k = 0.5 \sin 11.3099° = 0.09806 \text{ in.}$$

Equations of Equilibrium. Referring to the *FBD* of the pulley shown in Fig. *a*.

$$\circlearrowleft + \Sigma M_O = 0; \qquad P(6) - 20(6) - R(0.09806) = 0$$

$$R = 61.1882\, P - 1223.76 \qquad \textbf{(1)}$$

$$\xrightarrow{+} \Sigma F_x = 0; \qquad R_x - P = 0 \qquad R_x = P$$

$$+\uparrow \Sigma F_y = 0; \qquad R_y - 4 - 20 = 0 \quad R_y = 24 \text{ lb}$$

Thus, the magnitude of **R** is

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{P^2 + 24^2} \qquad \textbf{(2)}$$

Equating Eqs. (1) and (2)

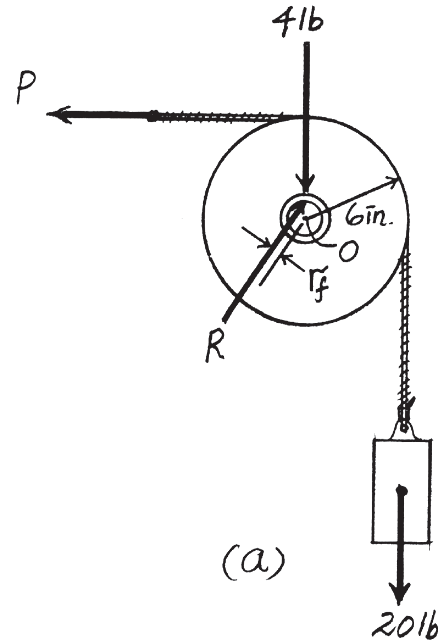
$$61.1882\, P - 1223.76 = \sqrt{P^2 + 24^2}$$

$$3743.00\, P^2 - 149{,}760.00\, P + 1{,}497{,}024.00 = 0$$

$$P^2 - 40.01\, P + 399.95 = 0$$

chose the root $P > 20$ lb,

$$P = 20.52 \text{ lb} = 20.5 \text{ lb} \qquad \textbf{Ans.}$$

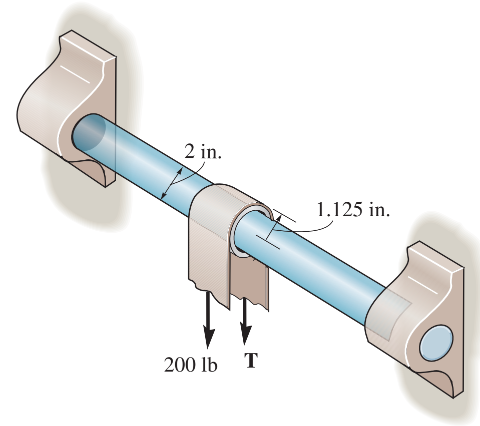


(a)

Ans:
$P = 20.5$ lb

8–122.

Determine the tension **T** in the belt needed to overcome the tension of 200 lb created on the other side. Also, what are the normal and frictional components of force developed on the collar bushing? The coefficient of static friction is $\mu_s = 0.21$.

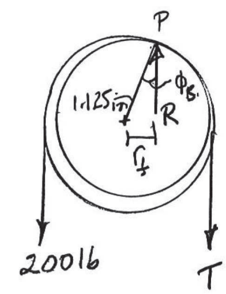


SOLUTION

Frictional Force on Journal Bearing: Here, $\phi_s = \tan^{-1}\mu_s = \tan^{-1}0.21 = 11.86°$. Then the radius of friction circle is

$$r_f = r \sin \phi_k = 1 \sin 11.86° = 0.2055 \text{ in.}$$

Equations of Equilibrium:



$\circlearrowleft + \Sigma M_P = 0; \quad 200(1.125 + 0.2055) - T(1.125 - 0.2055) = 0$

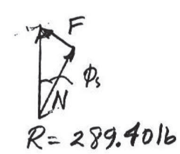
$$T = 289.41 \text{ lb} = 289 \text{ lb}$$ **Ans.**

$+\uparrow F_y = 0; \qquad R - 200 - 289.41 = 0 \qquad R = 489.41 \text{ lb}$

Thus, the normal and friction force are

$$N = R \cos \phi_s = 489.41 \cos 11.86° = 479 \text{ lb}$$ **Ans.**

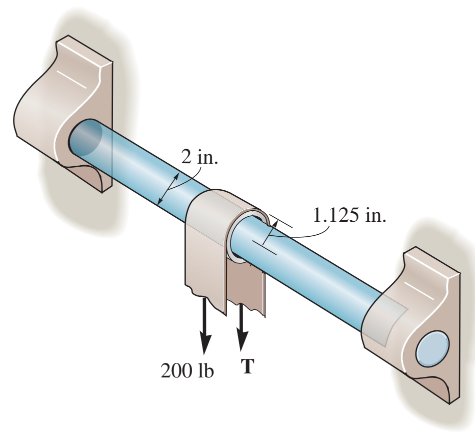
$$F = R \sin \phi_s = 489.41 \sin 11.86° = 101 \text{ lb}$$ **Ans.**



Ans:
$T = 289$ lb
$N = 479$ lb
$F = 101$ lb

8–123.

If a tension force $T = 215$ lb is required to pull the 200-lb force around the collar bushing, determine the coefficient of static friction at the contacting surface. The belt does not slip on the collar.



SOLUTION

Equation of Equilibrium:

$$\zeta + \Sigma M_P = 0; \quad 200(1.125 + r_f) - 215(1.125 - r_f) = 0$$

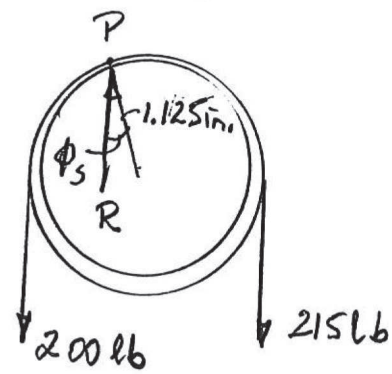
$$r_f = 0.04066 \text{ in.}$$

Frictional Force on Journal Bearing: The radius of friction circle is

$$r_f = r \sin \phi_k$$

$$0.04066 = 1 \sin \phi_k$$

$$\phi_k = 2.330°$$



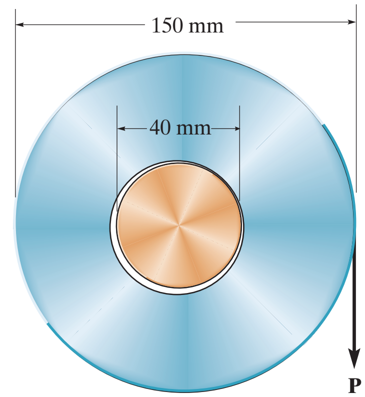
and the coefficient of static friction is

$$\mu_s = \tan \phi_s = \tan 2.330° = 0.0407 \qquad \textbf{Ans.}$$

Ans:
$\mu_s = 0.0407$

***8–124.**

The uniform disk fits loosely over a fixed shaft having a diameter of 40 mm. If the coefficient of static friction between the disk and the shaft is $\mu_s = 0.15$, determine the smallest vertical force P, acting on the rim, which must be applied to the disk to cause it to slip on the shaft. The disk has a mass of 20 kg.



SOLUTION

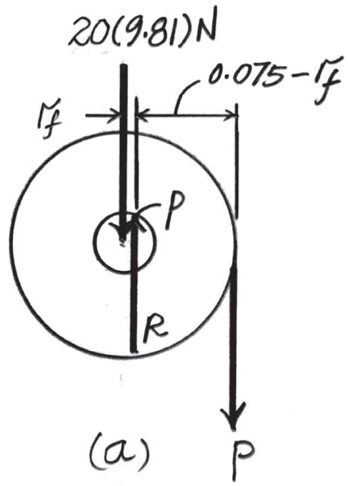
Frictional Force on Journal Bearing. Here, $\phi_k = \tan^{-1}\mu_s = \tan^{-1}0.15 = 8.5308°$. Then the radius of the friction circle is

$$r_f = r\sin\phi_s = 0.02\sin 8.5308° = 2.9668\left(10^{-3}\right)\text{ m}$$

Equations of Equilibrium. Referring to the *FBD* of the disk shown in Fig. *a*,

$$\circlearrowleft +\Sigma M_P = 0; \qquad 20(9.81)\left[2.9668\left(10^{-3}\right)\right] - P\left[0.075 - 2.9668\left(10^{-3}\right)\right] = 0$$

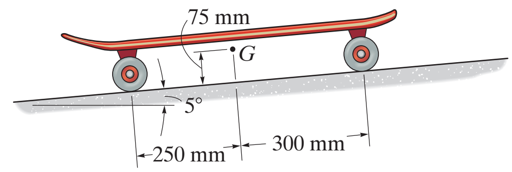
$$P = 8.08\text{ N} \qquad \textbf{Ans.}$$



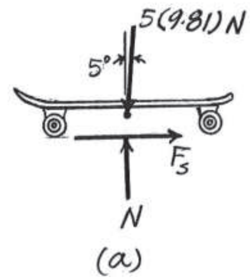
Ans:
8.08 N

8–125.

The 5-kg skateboard rolls down the 5° slope at constant speed. If the coefficient of kinetic friction between the 12.5-mm diameter axles and the wheels is $\mu_k = 0.3$, determine the radius of the wheels. Neglect rolling resistance of the wheels on the surface. The center of mass for the skateboard is at G.



SOLUTION

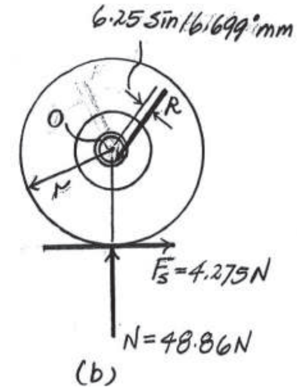


Referring to the free-body diagram of the skateboard shown in Fig. *a*, we have

$\Sigma F_{x'} = 0; \quad F_s - 5(9.81)\sin 5° = 0 \quad F_s = 4.275 \text{ N}$

$\Sigma F_{y'} = 0; \quad N - 5(9.81)\cos 5° = 0 \quad N = 48.86 \text{ N}$

The effect of the forces acting on the wheels can be represented as if these forces are acting on a single wheel as indicated on the free-body diagram shown in Fig. *b*. We have



$\Sigma F_{x'} = 0; \quad R_{x'} - 4.275 = 0 \quad R_{x'} = 4.275 \text{ N}$

$\Sigma F_{y'} = 0; \quad 48.86 - R_{y'} = 0 \quad R_{y'} = 48.86 \text{ N}$

Thus, the magnitude of **R** is

$R = \sqrt{R_{x'}{}^2 + R_{y'}{}^2} = \sqrt{4.275^2 + 48.86^2} = 49.05 \text{ N}$

$\phi_s = \tan^{-1}\mu_s = \tan^{-1}(0.3) = 16.699°$. Thus, the moment arm of **R** from point O is $(6.25 \sin 16.699°)$ mm. Using these results and writing the moment equation about point O, Fig. *b*, we have

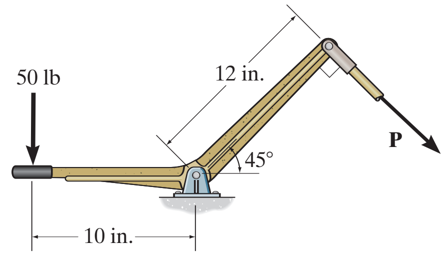
$\circlearrowleft + \Sigma M_O = 0; \quad 4.275(r) - 49.05(6.25 \sin 16.699° = 0)$

$r = 20.6 \text{ mm}$ **Ans.**

Ans:
$r = 20.6 \text{ mm}$

8–126.

The bell crank fits loosely into a 0.5-in-diameter pin. Determine the required force P which is just sufficient to rotate the bell crank clockwise. The coefficient of static friction between the pin and the bell crank is $\mu_s = 0.3$.



SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \quad P \cos 45° - R_x = 0 \quad\quad R_x = 0.7071P$

$+\uparrow \Sigma F_y = 0; \quad R_y - P \sin 45° - 50 = 0 \quad\quad R_y = 0.7071P + 50$

Thus, the magnitude of **R** is

$$R = \sqrt{R_x^{\,2} + R_y^{\,2}} = \sqrt{(0.7071P)^2 + (0.7071P + 50)^2}$$

$$= \sqrt{P^2 + 70.71P + 2500}$$

We find that $\phi_s = \tan^{-1} \mu_s = \tan^{-1}(0.3) = 16.699°$. Thus, the moment arm of **R** from point O is $(0.25 \sin 16.699°)$ mm. Using these results and writing the moment equation about point O, Fig. a,

$\zeta + \Sigma M_O = 0; \quad 50(10) + \sqrt{P^2 + 70.71P + 2500}(0.25 \sin 16.699°) - P(12) = 0$

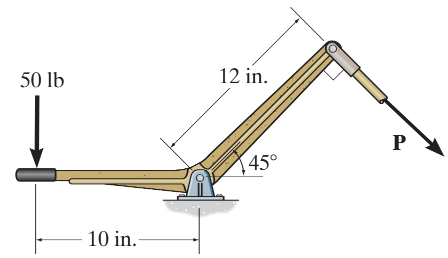
Choosing the larger root,

$P = 42.2$ lb **Ans.**

Ans:
$P = 42.2$ lb

8–127.

The bell crank fits loosely into a 0.5-in-diameter pin. If $P = 41$ lb, the bell crank is then on the verge of rotating counterclockwise. Determine the coefficient of static friction between the pin and the bell crank.



SOLUTION

$\xrightarrow{+} \Sigma F_x = 0; \qquad 41 \cos 45° - R_x = 0 \qquad R_x = 28.991 \text{ lb}$

$+\uparrow \Sigma F_y = 0; \qquad R_y - 41 \sin 45° - 50 = 0 \qquad R_y = 78.991 \text{ lb}$

Thus, the magnitude of **R** is

$$R = \sqrt{R_x^{\,2} + R_y^{\,2}} = \sqrt{28.991^2 + 78.991^2} = 84.144 \text{ lb}$$

We find that the moment arm of **R** from point O is $0.25 \sin \phi_s$. Using these results and writing the moment equation about point O, Fig. a,

$\circlearrowleft + \Sigma M_O = 0; \qquad 50(10) - 41(12) - 84.144(0.25 \sin \phi_s) = 0$

$$\phi_s = 22.35°$$

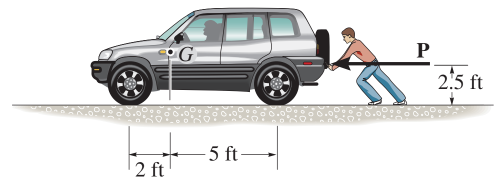
Thus,

$$\mu_s = \tan \phi_s = \tan 22.35° = 0.411 \qquad \textbf{Ans.}$$

Ans:
$\mu_s = 0.411$

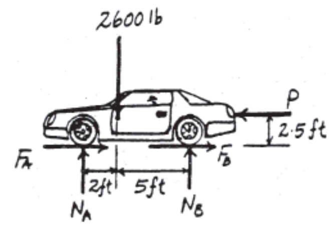
***8–128.**

The vehicle has a weight of 2600 lb and center of gravity at G. Determine the horizontal force **P** that must be applied to overcome the rolling resistance of the wheels. The coefficient of rolling resistance is 0.5 in. The tires have a diameter of 2.75 ft.



SOLUTION

Rolling Resistance: Here, $W = N_A + N_B = \dfrac{5200 - 2.5P}{7} + \dfrac{13000 + 2.5P}{7}$ $= 2600$ lb, $a = 0.5$ in. and $r = \left(\dfrac{2.75}{2}\right)(12) = 16.5$ in. Applying Eq. 8–11, we have



$$P \approx \frac{Wa}{r}$$

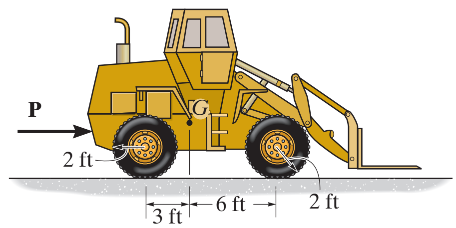
$$\approx \frac{2600(0.5)}{16.5}$$

$$\approx 78.8 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
≈ 78.8 lb

8–129.

The tractor has a weight of 16 000 lb and the coefficient of rolling resistance is $a = 2$ in. Determine the force **P** needed to overcome rolling resistance at all four wheels and push it forward.



SOLUTION

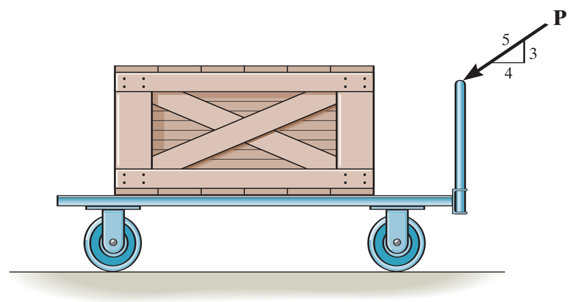
Applying Eq. 8–11 with $W = 16\,000$ lb, $a = \left(\frac{2}{12}\right)$ ft and $r = 2$ ft, we have

$$P \approx \frac{Wa}{r} = \frac{16000\left(\frac{2}{12}\right)}{2} = 1333 \text{ lb}$$ **Ans.**

Ans:
$P = 1333$ lb

8–130.

The handcart has wheels with a diameter of 6 in. If a crate having a weight of 1500 lb is placed on the cart, determine the force P that must be applied to the handle to overcome the rolling resistance. The coefficient of rolling resistance is 0.04 in. Neglect the weight of the cart.



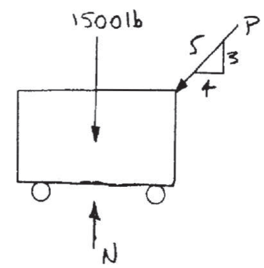
SOLUTION

$$+\uparrow \Sigma F_y = 0; \qquad N - 1500 - P\left(\frac{3}{5}\right) = 0$$

$$P = \frac{Wa}{r}, \qquad \frac{4}{5}P = \frac{\left[1500 + P\left(\frac{3}{5}\right)\right](0.04)}{3}$$

$$2.4\,P = 60 + 0.024\,P$$

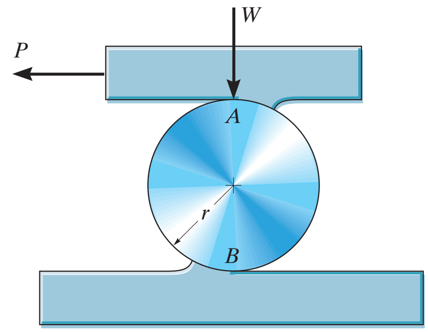
$$P = 25.3 \text{ lb} \qquad \textbf{Ans.}$$



Ans:
$P = 25.3$ lb

8–131.

The cylinder is subjected to a load that has a weight W. If the coefficients of rolling resistance for the cylinder's top and bottom surfaces are a_A and a_B, respectively, show that a horizontal force having a magnitude of $P = [W(a_A + a_B)]/2r$ is required to move the load and thereby roll the cylinder forward. Neglect the weight of the cylinder.



SOLUTION

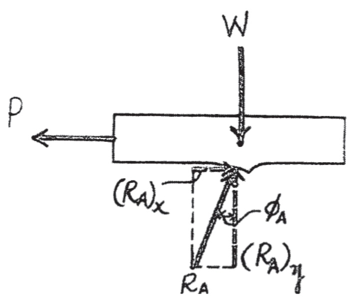
$\xrightarrow{+} \Sigma F_x = 0; \qquad (R_A)_x - P = 0 \qquad (R_A)_x = P$

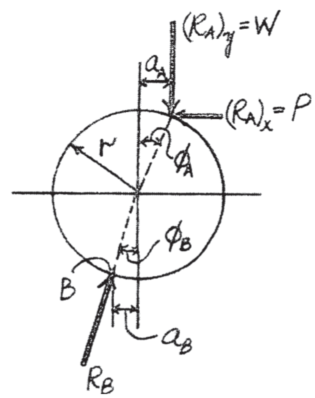
$+\uparrow \Sigma F_y = 0; \qquad (R_A)_y - W = 0 \qquad (R_A)_y = W$

$\circlearrowleft + \Sigma M_B = 0; \qquad P(r \cos \phi_A + r \cos \phi_B) - W(a_A + a_B) = 0 \qquad \textbf{(1)}$

Since ϕ_A and ϕ_B are very small, $\cos \phi_A - \cos \phi_B = 1$. Hence, from Eq. (1)

$$P = \frac{W(a_A + a_B)}{2r} \qquad \textbf{(QED)}$$





***8–132.**

The 1.4-Mg machine is to be moved over a level surface using a series of rollers for which the coefficient of rolling resistance is 0.5 mm at the ground and 0.2 mm at the bottom surface of the machine. Determine the appropriate diameter of the rollers so that the machine can be pushed forward with a horizontal force of P = 250 N. *Hint:* Use the result of Prob. 8–131.

SOLUTION

$$P = \frac{W(a_A + a_B)}{2r}$$

$$250 = \frac{1400\,(9.81)\,(0.2 + 0.5)}{2r}$$

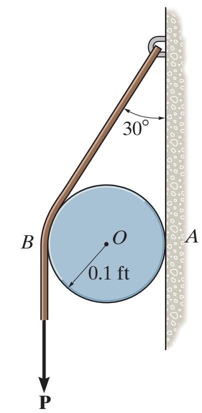
$$r = 19.2 \text{ mm}$$

$$d = 38.5 \text{ mm}$$ **Ans.**

Ans:
$d = 38.5$ mm

8–94.

A minimum force of $P = 50$ lb is required to hold the cylinder from slipping against the belt and the wall. Determine the weight of the cylinder if the coefficient of friction between the belt and cylinder is $\mu_s = 0.3$ and slipping does not occur at the wall.



SOLUTION

Equations of Equilibrium: Write the moment equation of equilibrium about point A by referring to the FBD of the cylinder shown in Fig. *a*,

$$\zeta + \Sigma M_A = 0; \quad 50(0.2) + W(0.1) - T_2 \cos 30°(0.1 + 0.1 \cos 30°) - T_2 \sin 30°(0.1 \sin 30°) = 0 \qquad \textbf{(1)}$$

Frictional Force on Flat Belt: Here, $T_1 = 50$ lb,

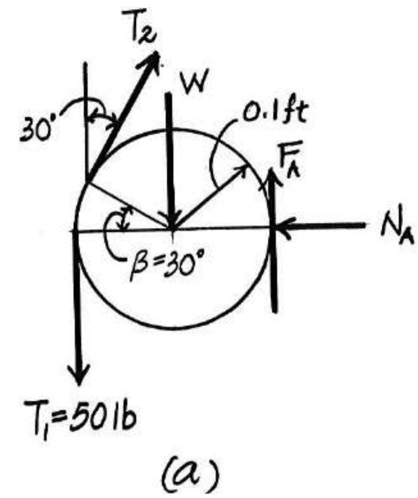
$\beta = \left(\dfrac{30°}{180°}\right)\pi = \dfrac{\pi}{6}$ rad. Applying Eq. 8–6

$$T_2 = T_1 e^{\mu\beta}$$

$$= 50e^{0.3\left(\frac{\pi}{6}\right)} = 58.50 \text{ lb}$$

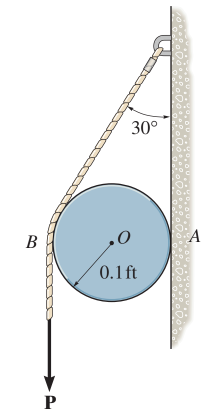
Substitute this result into Eq. (1),

$$W = 9.17 \text{ lb} \qquad \textbf{Ans.}$$



8–95.

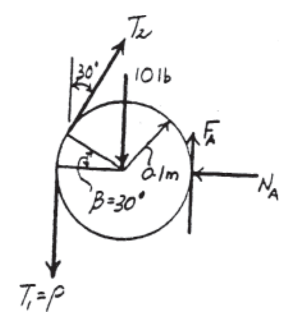
The cylinder weighs 10 lb and is held in equilibrium by the belt and wall. If slipping does not occur at the wall, determine the minimum vertical force P which must be applied to the belt for equilibrium. The coefficient of static friction between the belt and the cylinder is $\mu_s = 0.25$.



SOLUTION

Equations of Equilibrium:

$$\circlearrowleft + \Sigma M_A = 0; \qquad P(0.2) + 10(0.1) - T_2 \cos 30°(0.1 + 0.1 \cos 30°)$$
$$- T_2 \sin 30°(0.1 \sin 30°) = 0 \qquad \textbf{(1)}$$



Frictional Force on Flat Belt: Here, $\beta = 30° = \dfrac{\pi}{6}$ rad and $T_1 = P$. Applying Eq. 8–6, $T_2 = T_1\, e^{\mu\beta}$, we have

$$T_2 = Pe^{0.25(\pi/6)} = 1.140P \qquad \textbf{(2)}$$

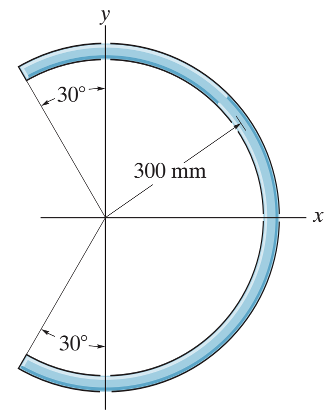
Solving Eqs. (1) and (2) yields

$$P = 78.7 \text{ lb} \qquad \textbf{Ans.}$$

$$T_2 = 89.76 \text{ lb}$$

9–1.

Locate the center of mass of the homogeneous rod bent into the shape of a circular arc.



SOLUTION

$$dL = 300\, d\,\theta$$

$$\widetilde{x} = 300 \cos\theta$$

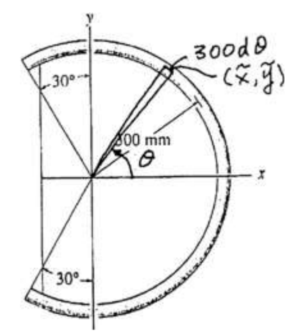
$$\widetilde{y} = 300 \sin\theta$$

$$\bar{x} = \frac{\int \widetilde{x}\, dL}{\int dL} = \frac{\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} 300 \cos\theta\,(300 d\theta)}{\int_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}} 300 d\,\theta}$$

$$= \frac{(300)^2 \left[\sin\theta\right]_{-\frac{2\pi}{3}}^{\frac{2\pi}{3}}}{300\left(\frac{4}{3}\pi\right)}$$

$$= 124 \text{ mm} \qquad \textbf{Ans.}$$

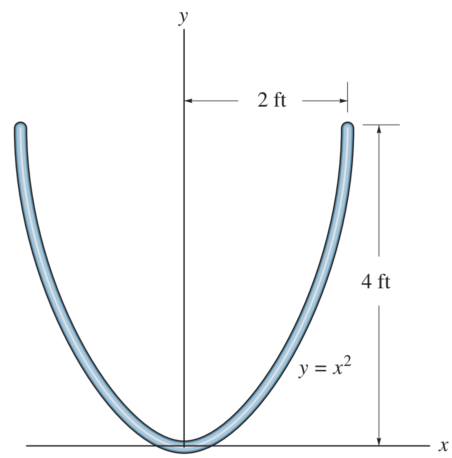
$$\bar{y} = 0 \qquad \text{(By symmetry)} \qquad \textbf{Ans.}$$



Ans:
$\bar{x} = 124$ mm
$\bar{y} = 0$

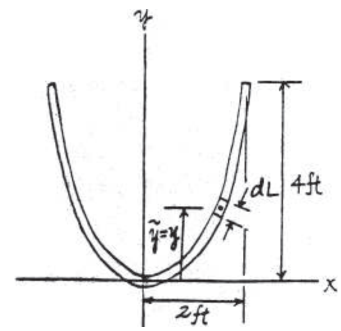
9–2.

Determine the location $(\bar{x}, \bar{y})$ of the centroid of the wire.



SOLUTION

Length and Moment Arm: The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right)dx$ and its centroid is $\tilde{y} = y = x^2$. Here, $\frac{dy}{dx} = 2x$.



Centroid: Due to symmetry

$$\tilde{x} = 0$$ **Ans.**

Applying Eq. 9–7 and performing the integration, we have

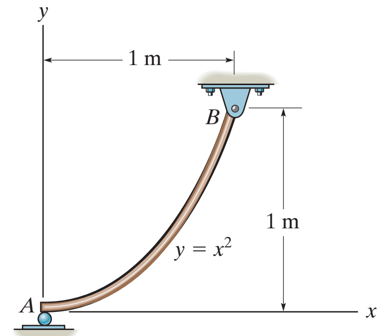
$$\tilde{y} = \frac{\int_L \tilde{y}\,dL}{\int_L dL} = \frac{\int_{-2\text{ ft}}^{2\text{ ft}} x^2\sqrt{1 + 4x^2}\,dx}{\int_{-2\text{ ft}}^{2\text{ ft}} \sqrt{1 + 4x^2}\,dx}$$

$$= \frac{16.9423}{9.2936} = 1.82 \text{ ft}$$ **Ans.**

Ans:
$\bar{x} = 0$
$\bar{y} = 1.82$ ft

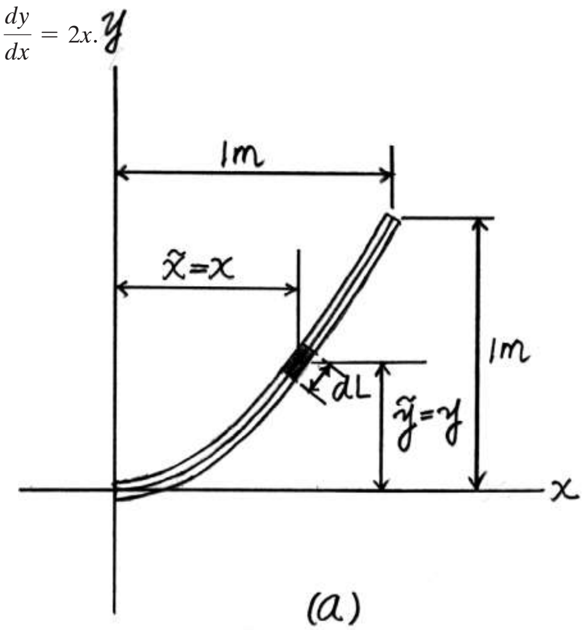
9–3.

Locate the center of gravity $\bar{x}$ of the homogeneous rod. If the rod has a weight per unit length of 100 N/m, determine the vertical reaction at A and the x and y components of reaction at the pin B.



SOLUTION

Length And Moment Arm. The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left[\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right]dx$ and its centroid is $\tilde{x} = x$. Here $\frac{dy}{dx} = 2x$.



Perform the integration

$$L = \int_L dL = \int_0^{1\text{ m}} \sqrt{1 + 4x^2}\, dx$$

$$= 2\int_0^{1\text{ m}} \sqrt{x^2 + \frac{1}{4}}\, dx$$

$$= \left[x\sqrt{x^2 + \frac{1}{4}} + \frac{1}{4}\ln\left(x + \sqrt{x^2 + \frac{1}{4}}\right)\right]_0^{1\text{ m}}$$

$$= 1.4789 \text{ m}$$

$$\int_L \tilde{x}\, dL = \int_0^{1\text{ m}} x\sqrt{1 + 4x^2}\, dx$$

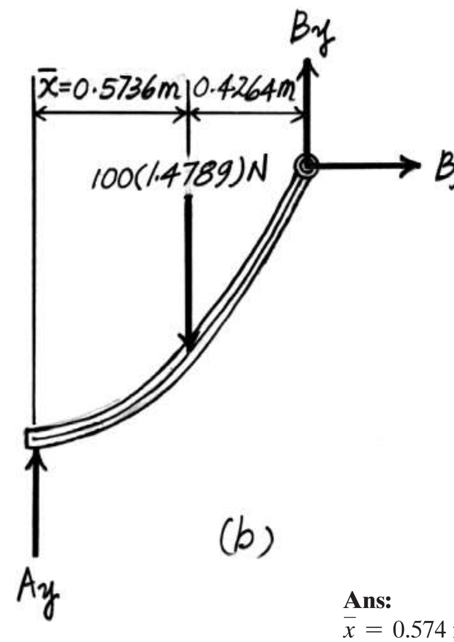
$$= 2\int_0^{1\text{ m}} x\sqrt{x^2 + \frac{1}{4}}\, dx$$

$$= \left[\frac{2}{3}\left(x^2 + \frac{1}{4}\right)^{3/2}\right]_0^{1\text{ m}}$$

$$= 0.8484 \text{ m}^2$$

Centroid.

$$\bar{x} = \frac{\int_L \tilde{x}\, dL}{\int_L dL} = \frac{0.8484 \text{ m}^2}{1.4789 \text{ m}} = 0.5736 \text{ m} = 0.574 \text{ m} \qquad \textbf{Ans.}$$



Equations of Equilibrium. Refering to the *FBD* of the rod shown in Fig. *a*

$\xrightarrow{+}\Sigma F_x = 0; \qquad B_x = 0$ **Ans.**

$\circlearrowleft + \Sigma M_B = 0; \quad 100(1.4789)\,(0.4264) - A_y(1) = 0$

$A_y = 63.06 \text{ N} = 63.1 \text{ N}$ **Ans.**

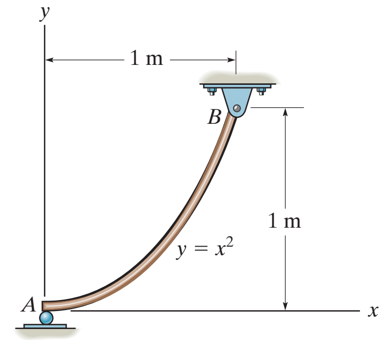
$\circlearrowleft + \Sigma M_A = 0; \quad B_y(1) - 100(1.4789)\,(0.5736) = 0$

$B_y = 84.84 \text{ N} = 84.8 \text{ N}$ **Ans.**

Ans:
$\bar{x} = 0.574$ m
$B_x = 0$
$A_y = 63.1$ N
$B_y = 84.8$ N

***9–4.**

Locate the center of gravity $\bar{y}$ of the homogeneous rod.



SOLUTION

Length And Moment Arm. The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left[\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right]dx$ and its centroid is $\tilde{y} = y$. Here $\frac{dy}{dx} = 2x$.

Perform the integration,

$$L = \int_L dL = \int_0^{1\text{ m}} \sqrt{1 + 4x^2}\, dx$$

$$= 2\int_0^{1\text{ m}} \sqrt{x^2 + \frac{1}{4}}\, dx$$

$$= \left[x\sqrt{x^2 + \frac{1}{4}} + \frac{1}{4}\ln\left(x + \sqrt{x^2 + \frac{1}{4}}\right)\right]_0^{1\text{ m}}$$

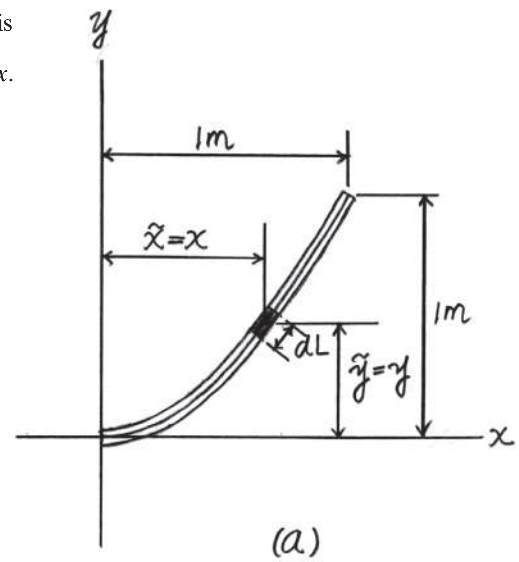
$$= 1.4789 \text{ m}$$

$$\int_L \tilde{y}\, dL = \int_0^{1\text{ m}} x^2\sqrt{1 + 4x^2}\, dx$$

$$= 2\int_0^{1\text{ m}} x^2\sqrt{x^2 + \frac{1}{4}}\, dx$$

$$= 2\left[\frac{x}{4}\sqrt{\left(x^2 + \frac{1}{4}\right)^3} - \frac{1}{32}x\sqrt{x^2 + \frac{1}{4}} - \frac{1}{128}\ln\left(x + \sqrt{x^2 + \frac{1}{4}}\right)\right]_0^{1\text{ m}}$$

$$= 0.6063 \text{ m}^2$$



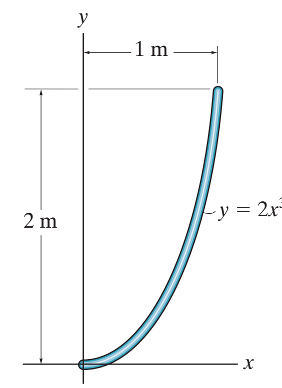
Centroid.

$$\bar{y} = \frac{\int_L \tilde{y}\, dL}{\int_L dL} = \frac{0.6063 \text{ m}^2}{1.4789 \text{ m}} = 0.40998 \text{ m} = 0.410 \text{ m}$$ **Ans.**

Ans:
$\bar{y} = 0.410$ m

9–5.

Determine the distance $\bar{y}$ to the center of gravity of the homogeneous rod.



SOLUTION

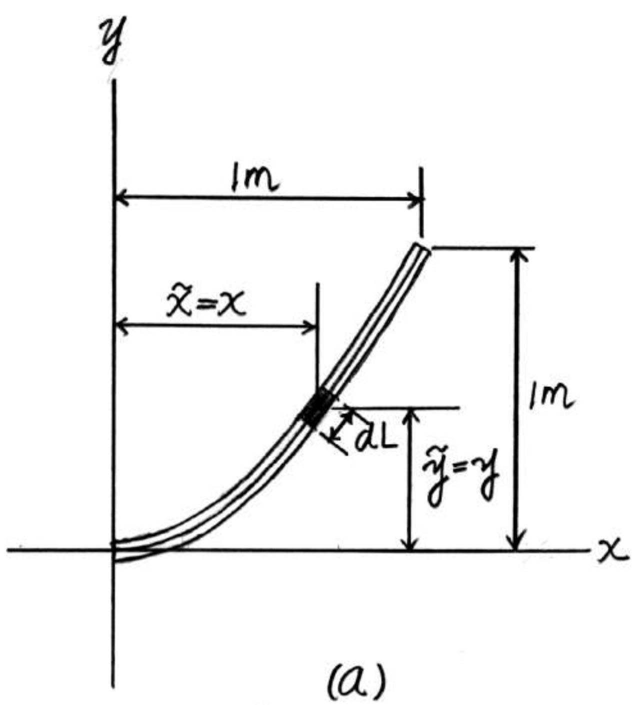
Length And Moment Arm. The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right) dx$ and its centroid is at $\tilde{y} = y$. Here $\frac{dy}{dx} = 6x^2$. Evaluate the integral numerically,

$$L = \int_L dL = \int_0^{1\text{ m}} \sqrt{1 + 36x^4}\, dx = 2.4214 \text{ m}$$

$$\int_L \tilde{y}\, dL = \int_0^{1\text{ m}} 2x^3\sqrt{1 + 36x^4}\, dx = 2.0747 \text{ m}^2$$

Centroid. Applying Eq. 9–7,

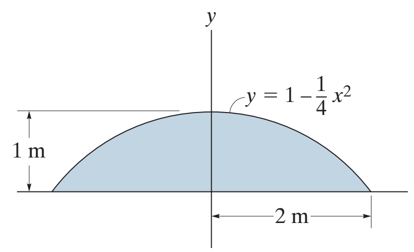
$$\bar{y} = \frac{\int_L \tilde{y} dL}{\int_L dL} = \frac{2.0747 \text{ m}^2}{2.4214 \text{ m}} = 0.8568 = 0.857 \text{ m} \qquad \textbf{Ans.}$$



Ans:
$\bar{y} = 0.857$ m

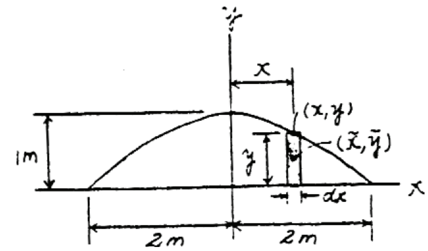
9–6.

Locate the centroid $\overline{y}$ of the area.



SOLUTION

Area and Moment Arm: The area of the differential element is $dA = ydx = \left(1 - \frac{1}{4}x^2\right)dx$ and its centroid is $\widetilde{y} = \frac{y}{2} = \frac{1}{2}\left(1 - \frac{1}{4}x^2\right)$.



Centroid: Due to symmetry

$$\overline{x} = 0 \qquad \textbf{Ans.}$$

Applying Eq. 9–4 and performing the integration, we have

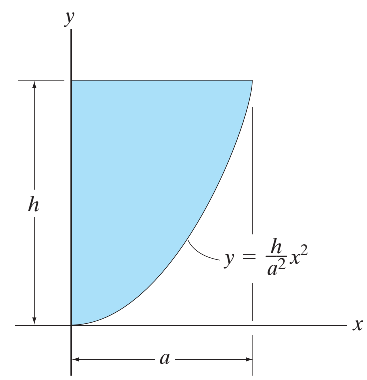
$$\overline{y} = \frac{\int_A \widetilde{y}\,dA}{\int_A dA} = \frac{\int_{-2\text{m}}^{2\text{m}} \frac{1}{2}\left(1 - \frac{1}{4}x^2\right)\left(1 - \frac{1}{4}x^2\right)dx}{\int_{-2\text{m}}^{2\text{m}}\left(1 - \frac{1}{4}x^2\right)dx}$$

$$= \frac{\left(\frac{x}{2} - \frac{x^3}{12} + \frac{x^5}{160}\right)\Big|_{-2\text{m}}^{2\text{m}}}{\left(x - \frac{x^3}{12}\right)\Big|_{-2\text{m}}^{2\text{m}}} = \frac{2}{5}\text{ m} \qquad \textbf{Ans.}$$

Ans:
$\overline{y} = \frac{2}{5}\text{ m}$

9–7.

Determine the area and the centroid $\bar{x}$ of the parabolic area.



SOLUTION

Differential Element: The area element parallel to the x axis shown shaded in Fig. a will be considered. The area of the element is

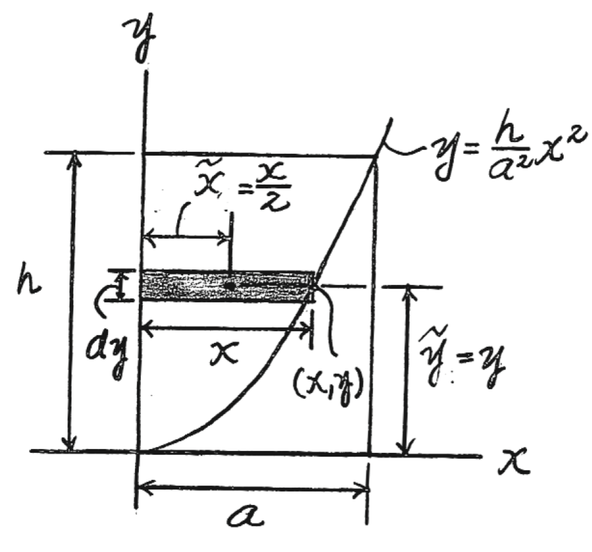
$$dA = x\,dy = \frac{a}{h^{1/2}}\,y^{1/2}\,dy$$

Centroid: The centroid of the element is located at $\tilde{x} = \frac{x}{2} = \frac{a}{2h^{1/2}}\,y^{1/2}$ and $\tilde{y} = y$.

Area: Integrating,

$$A = \int_A dA = \int_0^h \frac{a}{h^{1/2}}\,y^{1/2}\,dy = \frac{2a}{3h^{1/2}}\left(y^{3/2}\right)\Big|_0^h = \frac{2}{3}ah \qquad \textbf{Ans.}$$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^h\left(\frac{a}{2h^{1/2}}y^{1/2}\right)\left(\frac{a}{h^{1/2}}\,y^{1/2}\,dy\right)}{\frac{2}{3}ah} = \frac{\int_0^h \frac{a^2}{2h}\,y\,dy}{\frac{2}{3}ah} = \frac{\frac{a^2}{2h}\left(\frac{y^2}{2}\right)\Big|_0^h}{\frac{2}{3}ah} = \frac{3}{8}a \qquad \textbf{Ans.}$$



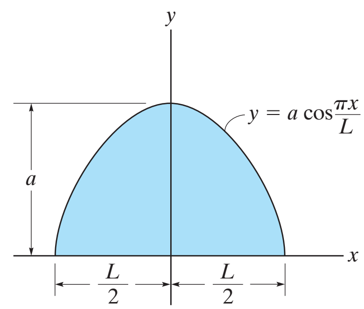
(a)

Ans:

$\bar{x} = \frac{3}{8}a$

***9–8.**

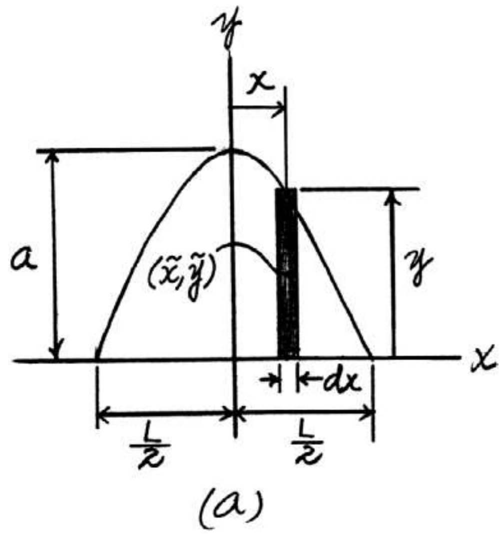
Locate the centroid of the shaded area.



SOLUTION

Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = ydx = a\cos\frac{\pi}{L}x\,dx$ and its centroid is at $\tilde{y} = \frac{y}{2} = \frac{a}{2}\cos\frac{\pi}{2}x$.

Centroid. Perform the integration



$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_{-L/2}^{L/2}\left(\frac{a}{2}\cos\frac{\pi}{L}x\right)\left(a\cos\frac{\pi}{L}x\,dx\right)}{\int_{-L/2}^{L/2} a\cos\frac{\pi}{L}x\,dx}$$

$$= \frac{\int_{-L/2}^{L/2}\frac{a^2}{4}\left(\cos\frac{2\pi}{L}x + 1\right)dx}{\int_{-L/2}^{L/2} a\cos\frac{\pi}{L}x\,dx}$$

$$= \frac{\frac{a^2}{4}\left(\frac{L}{2\pi}\sin\frac{2\pi}{L}x + x\right)\Big|_{-L/2}^{L/2}}{\left(\frac{aL}{\pi}\sin\frac{\pi}{L}x\right)\Big|_{-L/2}^{L/2}}$$

$$= \frac{a^2L/4}{2aL/\pi} = \frac{\pi}{8}a \qquad \textbf{Ans.}$$

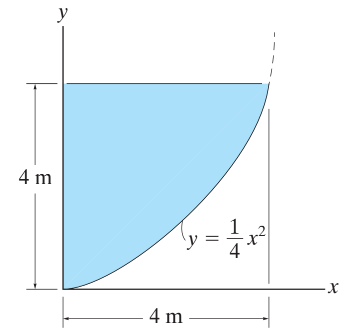
Due to Symmetry,

$$\bar{x} = 0 \qquad \textbf{Ans.}$$

Ans:
$\bar{y} = \frac{\pi}{8}a$
$\bar{x} = 0$

9–9.

Locate the centroid $\bar{x}$ of the shaded area.



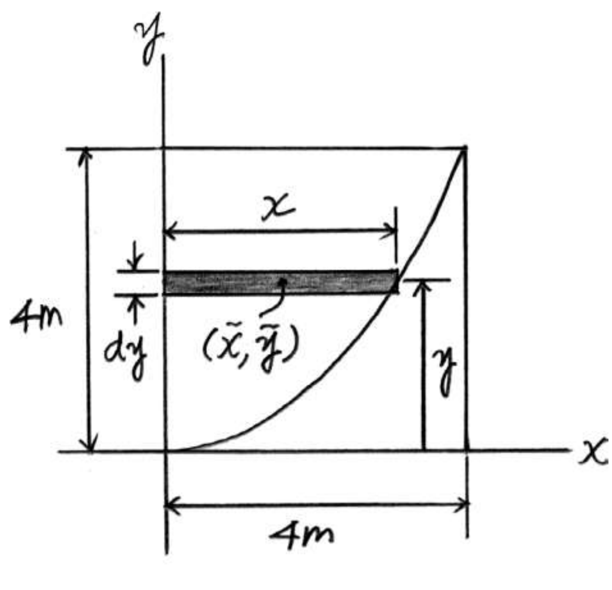
SOLUTION

Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = x\,dy$ and its centroid is at $\tilde{x} = \frac{1}{2}x$. Here, $x = 2y^{1/2}$

Centroid. Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ m}} \frac{1}{2}\left(2y^{1/2}\right)\left(2y^{1/2}\,dy\right)}{\int_0^{4\text{ m}} 2y^{1/2}\,dy}$$

$$= \frac{3}{2}\text{ m} \qquad \textbf{Ans.}$$

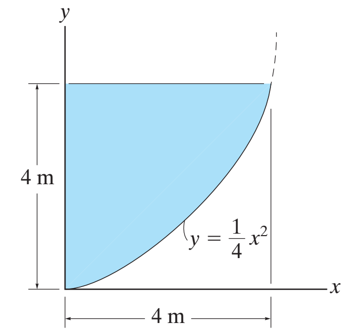


(a)

Ans:
$\bar{x} = \frac{3}{2}$ m

9–10.

Locate the centroid $\bar{y}$ of the shaded area.



SOLUTION

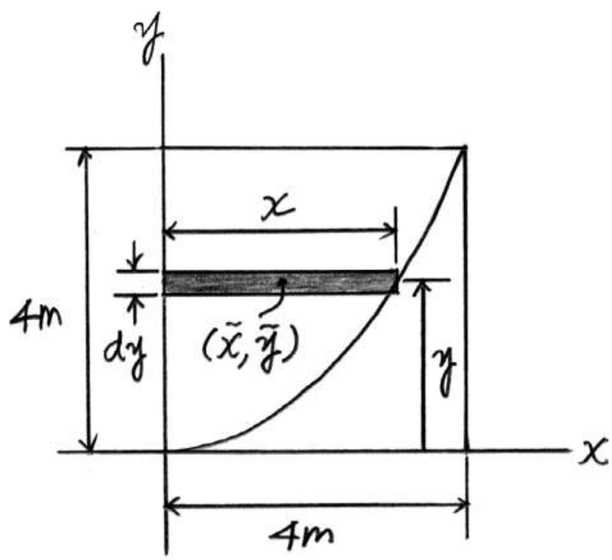
Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = x\,dy$ and its centroid is at $\tilde{y} = y$. Here, $x = 2y^{1/2}$.

Centroid. Perform the integration

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ m}} y\left(2y^{1/2}\,dy\right)}{\int_0^{4\text{ m}} 2y^{1/2}\,dy}$$

$$= \frac{\left(\frac{4}{5}y^{5/2}\right)\Big|_0^{4\text{ m}}}{\left(\frac{4}{3}y^{3/2}\right)\Big|_0^{4\text{ m}}}$$

$$= \frac{12}{5}\text{ m}$$ **Ans.**

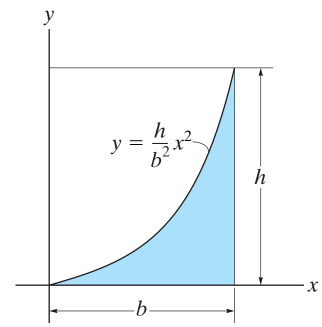


(a)

Ans:
$\bar{y} = \frac{12}{5}$ m

9–11.

Locate the centroid $\bar{x}$ of the area.

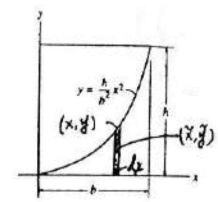


SOLUTION

$dA = y\,dx$

$\tilde{x} = x$

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^b \frac{h}{b^2}x^3\,dx}{\int_0^b \frac{h}{b^2}x^2\,dx} = \frac{\left[\frac{h}{4b^2}x^4\right]_0^b}{\left[\frac{h}{3b^2}x^3\right]_0^b} = \frac{3}{4}b$$ **Ans.**

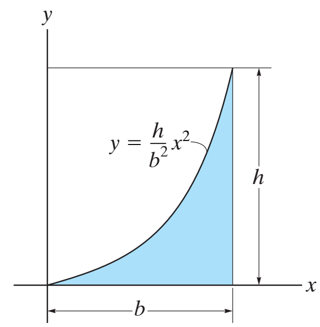


Ans:

$\bar{x} = \frac{3}{4}b$

***9–12.**

Locate the centroid $\bar{y}$ of the shaded area.



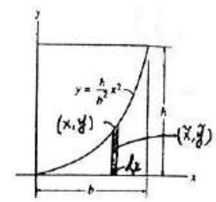
SOLUTION

$$dA = y\,dx$$

$$\tilde{y} = \frac{y}{2}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^b \frac{h^2}{2b^4}x^4\,dx}{\int_0^b \frac{h}{b^2}x^2\,dx} = \frac{\left[\frac{h^2}{10b^4}x^5\right]_0^b}{\left[\frac{h}{3b^2}x^3\right]_0^b} = \frac{3}{10}h$$

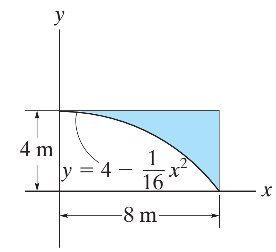
Ans.



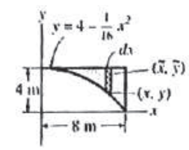
Ans:
$\bar{y} = \frac{3}{10}h$

9–13.

Locate the centroid $\overline{x}$ of the shaded area.



SOLUTION



$$dA = (4 - y)dx = \left(\frac{1}{16}x^2\right)dx$$

$$\widetilde{x} = x$$

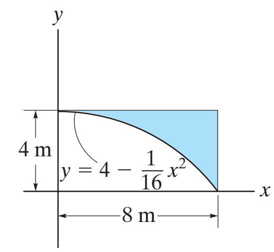
$$\overline{x} = \frac{\int_A \widetilde{x}dA}{\int_A dA} = \frac{\int_0^8 x\left(\frac{x^2}{16}\right)dx}{\int_0^8 \left(\frac{1}{16}x^2\right)dx}$$

$$\overline{x} = 6 \text{ m}$$ **Ans.**

Ans:
$\overline{x} = 6$ m

9–14.

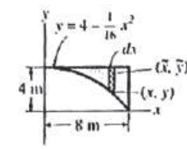
Locate the centroid $\overline{y}$ of the shaded area.



SOLUTION

$$dA = (4 - y)dx = \left(\frac{1}{16}x^2\right)dx$$

$$\overline{y} = \frac{4 + y}{2}$$



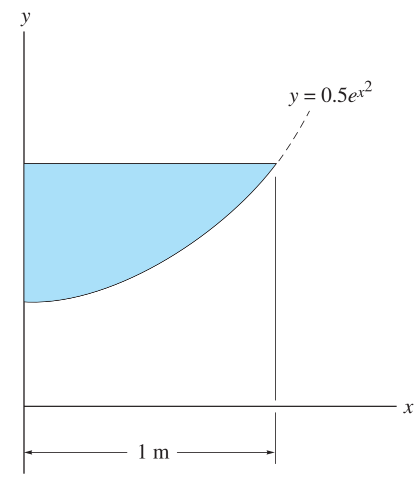
$$\overline{y} = \frac{\int_A \widetilde{y}\, dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^8 \left(8 - \frac{x^2}{16}\right)\left(\frac{x^2}{16}\right) dx}{\int_0^8 \left(\frac{1}{16}x^2\right) dx}$$

$\overline{y} = 2.8 \text{ m}$ **Ans.**

Ans:
$\overline{y} = 2.8 \text{ m}$

9–15.

Locate the centroid $\overline{x}$ of the shaded area. Solve the problem by evaluating the integrals using Simpson's rule.



SOLUTION

At $x = 1$ m

$$y = 0.5e^{1^2} = 1.359 \text{ m}$$

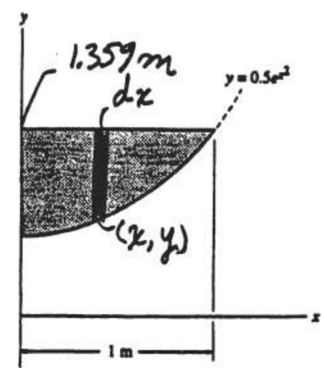
$$\int_A dA = \int_0^1 (1.359 - y)\,dx = \int_0^1 \left(1.359 = 0.5\,e^{x^2}\right) dx = 0.6278 \text{ m}^2$$

$$\overline{x} = x$$

$$\int_A \overline{x}\,dA = \int_0^1 x\left(1.359 - 0.5\,e^{x^2}\right) dx$$

$$= 0.25 \text{ m}^3$$

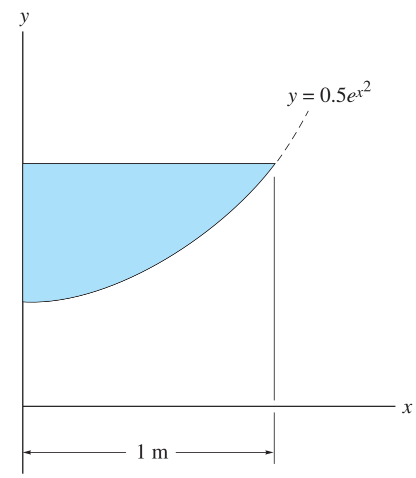
$$\overline{x} = \frac{\int_A \overline{x}\,dA}{\int_A dA} = \frac{0.25}{0.6278} = 0.398 \text{ m}$$ **Ans.**



Ans:
$\overline{x} = 0.398$ m

***9–16.**

Locate the centroid $\overline{y}$ of the shaded area. Solve the problem by evaluating the integrals using Simpson's rule.



SOLUTION

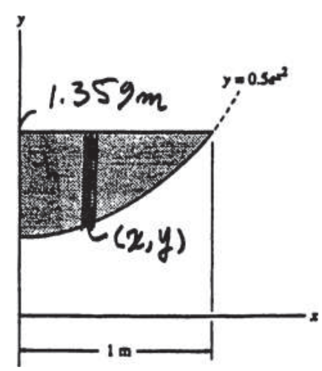
$$\int_A dA = \int_0^1 (1.359 - y)\,dx = \int_0^1 \left(1.359 - 0.5e^{x^2}\right) dx = 0.6278 \text{ m}^2$$

$$\tilde{y} = \frac{1.359 + y}{2}$$

$$\int_A \tilde{y}\,dA = \int_0^1 \left(\frac{1.359 + 0.5\,e^{x^2}}{2}\right)\left(1.359 - 0.5\,e^{x^2}\right) dx$$

$$= \frac{1}{2}\int_0^1 \left(1.847 - 0.25\,e^{2x^2}\right) dx = 0.6278 \text{ m}^3$$

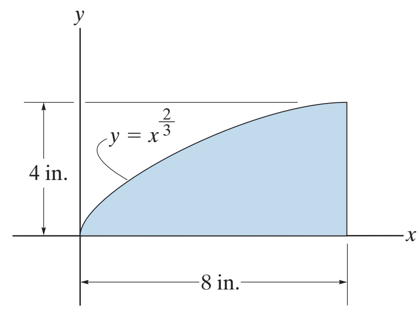
$$\overline{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{0.6278}{0.6278} = 1.00 \text{ m} \qquad \textbf{Ans.}$$



Ans:
$\overline{y} = 1.00$ m

9–17.

Locate the centroid $\bar{y}$ of the area.



SOLUTION

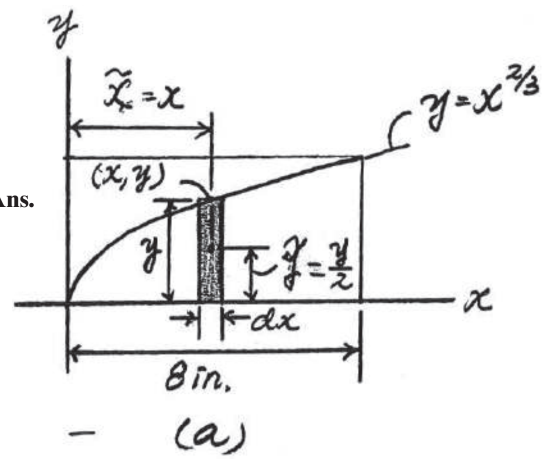
Area: Integrating the area of the differential element gives

$$A = \int_A dA = \int_0^{8\text{ in.}} x^{2/3}\,dx = \left[\frac{3}{5}x^{5/3}\right]\Bigg|_0^{8\text{ in.}} = 19.2\text{ in.}^2$$

Centroid: The centroid of the element is located at $\tilde{y} = y/2 = \dfrac{1}{2}x^{2/3}$. Applying Eq. 9–4, we have

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{8\text{ in.}} \frac{1}{2}x^{2/3}\left(x^{2/3}\right)dx}{19.2} = \frac{\int_0^{8\text{ in.}} \frac{1}{2}x^{4/3}dx}{19.2}$$

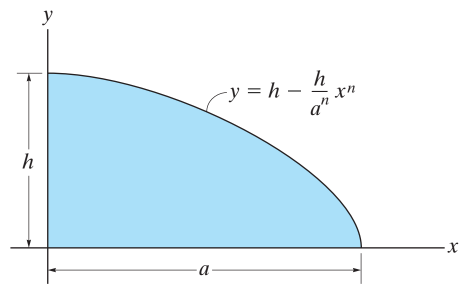
$$= \frac{\left[\frac{3}{14}x^{7/3}\right]\Bigg|_0^{8\text{ in.}}}{19.2} = 1.43\text{ in.}$$ **Ans.**



Ans:
$\bar{y} = 1.43$ in.

9–18.

Locate the centroid $\overline{x}$ of the area.

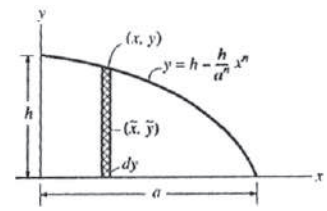


SOLUTION

$$dA = y\,dx$$

$$\widetilde{x} = x$$

$$\overline{x} = \frac{\int_A \widetilde{x}\,dA}{\int_A dA} = \frac{\int_0^a \left(hx - \frac{h}{a^n}x^{n+1}\right)dx}{\int_0^a \left(h - \frac{h}{a^n}x^n\right)dx}$$



$$= \frac{\left[\frac{h}{2}x^2 - \frac{h(x^{n+2})}{a^n(n+2)}\right]_0^a}{\left[hx - \frac{h(x^{n+1})}{a^n(n+1)}\right]_0^a}$$

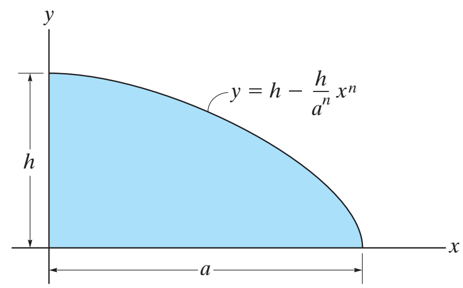
$$\overline{x} = \frac{\left(\frac{h}{2} - \frac{h}{n+2}\right)a^2}{\left(h - \frac{h}{n+1}\right)a} = \frac{a(1+n)}{2(2+n)}$$ **Ans.**

Ans:

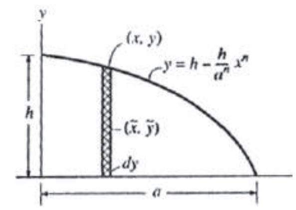
$$\overline{x} = \frac{a(1+n)}{2(2+n)}$$

9–19.

Locate the centroid $\overline{y}$ of the area.



SOLUTION



$$dA = y\,dx$$

$$\tilde{y} = \frac{y}{2}$$

$$\overline{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^a \left(h^2 - 2\frac{h^2}{a^n}x^n + \frac{h^2}{a^{2n}}x^{2n}\right)dx}{\int_0^a \left(h - \frac{h}{a^n}x^n\right)dx}$$

$$= \frac{\frac{1}{2}\left[h^2x - \frac{2h^2(x^{n+1})}{a^n(n+1)} + \frac{h^2(x^{2n+1})}{a^{2n}(2n+1)}\right]_0^a}{\left[hx - \frac{h(x^{n+1})}{a^n(n+1)}\right]_0^a}$$

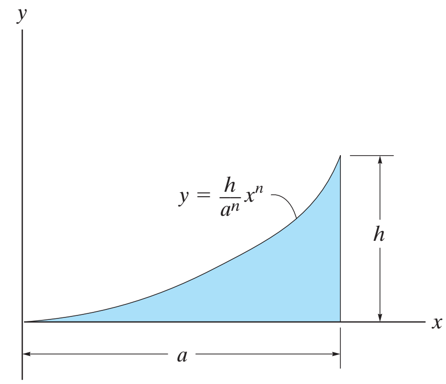
$$\overline{y} = \frac{\frac{2n^2}{2(n+1)(2n+1)}h}{\frac{n}{n+1}} = \frac{hn}{2n+1} \qquad \textbf{Ans.}$$

Ans:

$$\overline{y} = \frac{hn}{2n+1}$$

***9–20.**

Locate the centroid $\overline{y}$ of the shaded area.

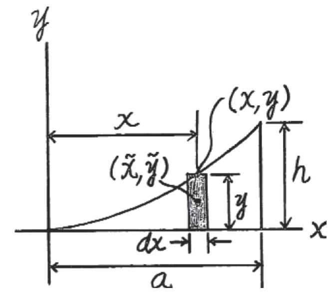


SOLUTION

$$dA = y\,dx$$

$$\overline{y} = \frac{y}{2}$$

$$\overline{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\frac{1}{2}\int_0^a \frac{h^2}{a^{2n}}\,x^{2n}\,dx}{\int_0^a \frac{h}{a^n}\,x^n\,dx} = \frac{\frac{h^2(a^{2n+1})}{2a^{2n}(2n+1)}}{\frac{h(a^{n+1})}{a^n(n+1)}} = \frac{hn+1}{2(2n+1)}$$ **Ans.**

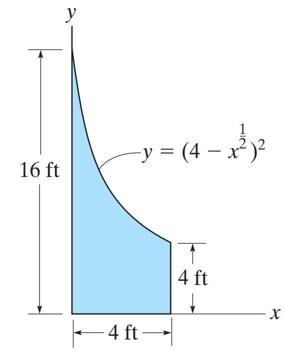


Ans:

$$\overline{y} = \frac{hn+1}{2(2n+1)}$$

9–21.

Locate the centroid $\bar{x}$ of the shaded area.



SOLUTION

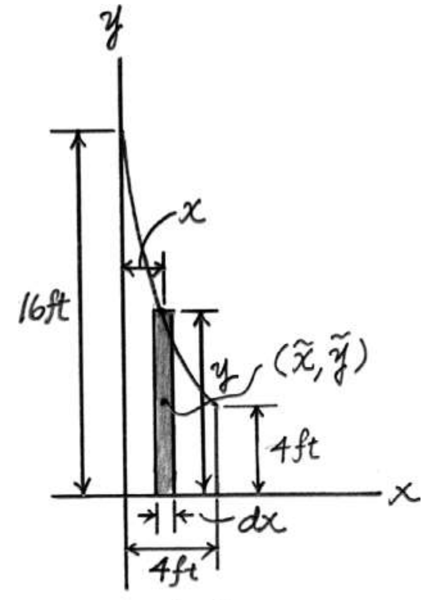
Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = (4 - x^{1/2})^2\,dx = (x - 8x^{1/2} + 16)dx$ and its centroid is at $\tilde{x} = x$.

Centroid. Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\displaystyle\int_0^{4\text{ ft}} x(x - 8x^{1/2} + 16)dx}{\displaystyle\int_0^{4\text{ ft}} (x - 8x^{1/2} + 16)\,dx}$$

$$= \frac{\left(\dfrac{x^3}{3} - \dfrac{16}{5}x^{5/2} + 8x^2\right)\Big|_0^{4\text{ ft}}}{\left(\dfrac{x^2}{2} - \dfrac{16}{3}x^{3/2} + 16x\right)\Big|_0^{4\text{ ft}}}$$

$$= 1\frac{3}{5}\text{ ft}$$ **Ans.**

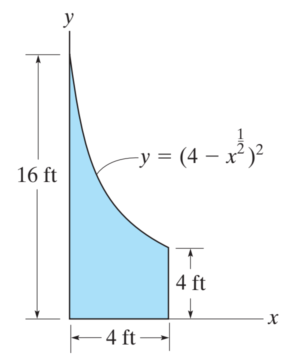


(a)

Ans:
$\bar{x} = 1\frac{3}{5}$ ft

9–22.

Locate the centroid $\bar{y}$ of the shaded area.



SOLUTION

Area And Moment Arm. The area of the differential element shown shaded in Fig. a is $dA = y\,dx = \left(4 - x^{\frac{1}{2}}\right)^2 dx = (x - 8x^{1/2} + 16)dx$ and its centroid is at $\tilde{y} = \dfrac{y}{2} = \dfrac{1}{2}\left(4 - x^{1/2}\right)^2$.

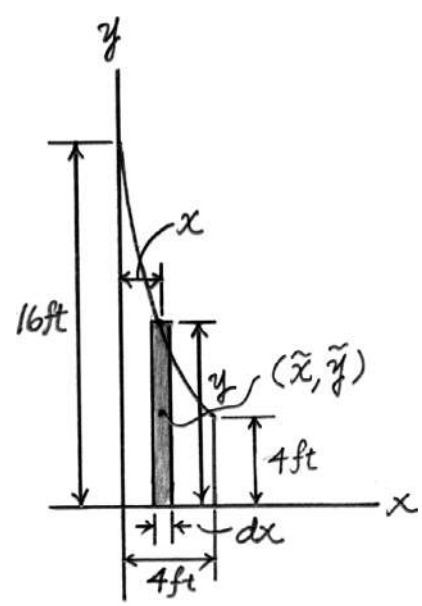
Centroid. Perform the integration,

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ ft}} \frac{1}{2}\left(4 - x^{1/2}\right)^2\left(x - 8x^{1/2} + 16\right)dx}{\int_0^{4\text{ ft}} (x - 8x^{1/2} + 16)dx}$$

$$= \frac{\int_0^{4\text{ ft}} \left(\frac{1}{2}x^2 - 8x^{3/2} + 48x - 128x^{1/2} + 128\right)dx}{\int_0^{4\text{ ft}} \left(x - 8x^{1/2} + 16\right)dx}$$

$$= \frac{\left(\frac{x^3}{6} - \frac{16}{5}x^{5/2} + 24x^2 - \frac{256}{3}x^{3/2} + 128x\right)\Big|_0^{4\text{ ft}}}{\left(\frac{x^2}{2} - \frac{16}{3}x^{3/2} + 16x\right)\Big|_0^{4\text{ ft}}}$$

$$= 4\,\frac{8}{55}\text{ ft}$$ **Ans.**

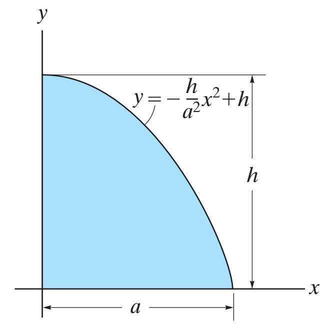


(a)

Ans:
$\bar{y} = 4\,\frac{8}{55}$ ft

9–23.

Locate the centroid $\bar{x}$ of the shaded area.



SOLUTION

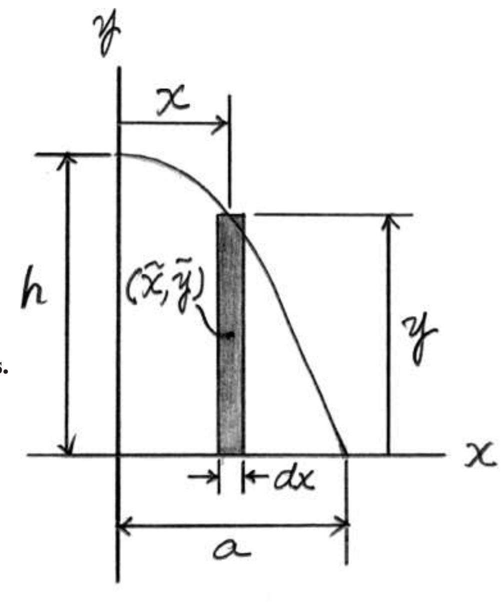
Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \left(-\frac{h}{a^2}x^2 + h\right)dx$ and its centroid is at $\tilde{x} = x$.

Centroid. Perform the integration,

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^a x\left(-\frac{h}{a^2}x^2 + h\right)dx}{\int_0^a \left(-\frac{h}{a^2}x^2 + h\right)dx}$$

$$= \frac{\left(-\frac{h}{4a^2}x^4 + \frac{h}{2}x^2\right)\Big|_0^a}{\left(-\frac{h}{3a^2}x^3 + hx\right)\Big|_0^a}$$

$$= \frac{3}{8}a$$ **Ans.**



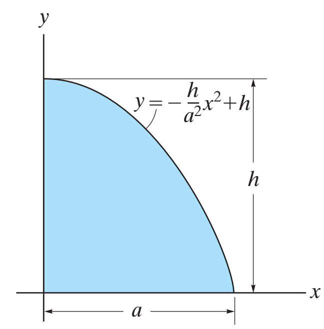
(a)

Ans:

$\bar{x} = \frac{3}{8}a$

***9–24.**

Locate the centroid $\bar{y}$ of the shaded area.



SOLUTION

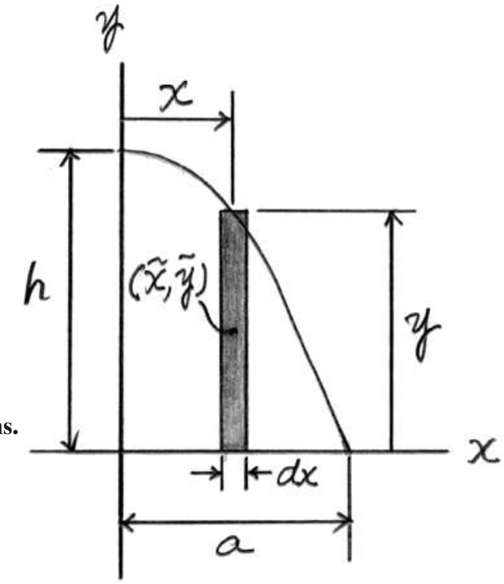
Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \left(-\frac{h}{a^2}x^2 + h\right)dx$ and its centroid is at $\tilde{y} = \frac{y}{2} = \frac{1}{2}\left(-\frac{h^2}{a}x^2 + h\right)$.

Centroid. Perform the integration,

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^a \frac{1}{2}\left(-\frac{h}{a^2}x^2 + h\right)\left(-\frac{h}{a^2}x^2 + h\right)dx}{\int_0^a \left(-\frac{h}{a^2}x^2 + h\right)dx}$$

$$= \frac{\frac{1}{2}\left(\frac{h^2}{5a^4}x^5 - \frac{2h^2}{3a^2}x^3 + h^2x\right)\Big|_0^a}{\left(-\frac{h}{3a^2}x^3 + hx\right)\Big|_0^a}$$

$$= \frac{2}{5}h \qquad \textbf{Ans.}$$

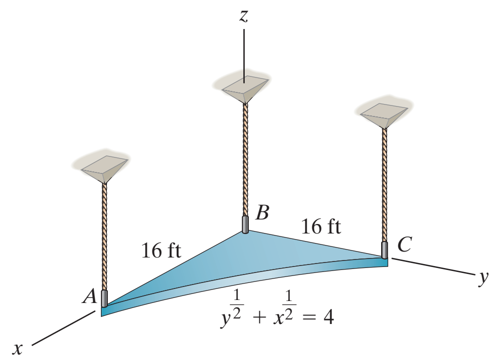


(a)

Ans:
$\bar{y} = \frac{2}{5}h$

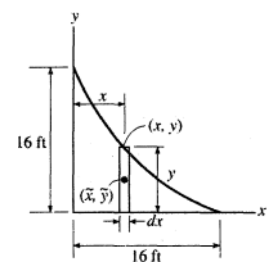
9–25.

The plate has a thickness of 0.25 ft and a specific weight of $\gamma = 180\ \text{lb/ft}^3$. Determine the location of its center of gravity. Also, find the tension in each of the cords used to support it.



SOLUTION

Area and Moment Arm: Here, $y = x - 8x^{\frac{1}{2}} + 16$. The area of the differential element is $dA = y\,dx = (x - 8x^{\frac{1}{2}} + 16)dx$ and its centroid is $\tilde{x} = x$ and $\tilde{y} = \frac{1}{2}(x - 8x^{\frac{1}{2}} + 16)$. Evaluating the integrals, we have



$$A = \int_A dA = \int_0^{16\text{ ft}} (x - 8x^{\frac{1}{2}} + 16)dx$$

$$= \left(\frac{1}{2}x^2 - \frac{16}{3}x^{\frac{3}{2}} + 16x\right)\Big|_0^{16\text{ ft}} = 42.67\ \text{ft}^2$$

$$\int_A \tilde{x}\,dA = \int_0^{16\text{ ft}} x[(x - 8x^{\frac{1}{2}} + 16)dx]$$

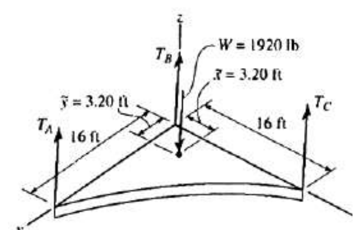
$$= \left(\frac{1}{3}x^3 - \frac{16}{5}x^{\frac{5}{2}} + 8x^2\right)\Big|_0^{16\text{ ft}} = 136.53\ \text{ft}^3$$

$$\int_A \tilde{y}\,dA = \int_0^{16\text{ ft}} \frac{1}{2}(x - 8x^{\frac{1}{2}} + 16)[(x - 8x^{\frac{1}{2}} + 16)dx]$$

$$= \frac{1}{2}\left(\frac{1}{3}x^3 - \frac{32}{5}x^{\frac{5}{2}} + 48x^2 - \frac{512}{3}x^{\frac{3}{2}} + 256x\right)\Big|_0^{16\text{ ft}}$$

$$= 136.53\ \text{ft}^3$$

Centroid: Applying Eq. 9–6, we have



$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{136.53}{42.67} = 3.20\ \text{ft} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{136.53}{42.67} = 3.20\ \text{ft} \qquad \textbf{Ans.}$$

Equations of Equilibrium: The weight of the plate is $W = 42.67(0.25)(180) = 1920$ lb.

$\Sigma M_x = 0;\quad 1920(3.20) - T_A(16) = 0 \quad T_A = 384\ \text{lb}$ **Ans.**

$\Sigma M_y = 0;\quad T_C(16) - 1920(3.20) = 0 \quad T_C = 384\ \text{lb}$ **Ans.**

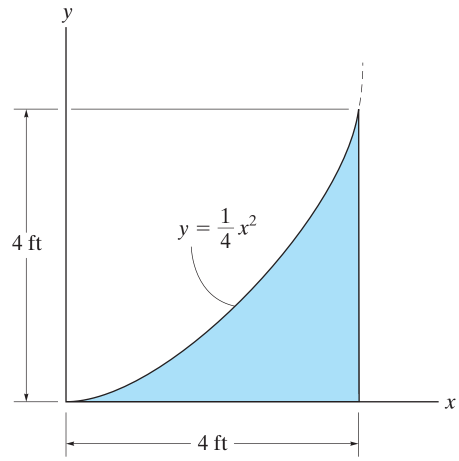
$\Sigma F_z = 0;\quad T_B + 384 + 384 - 1920 = 0$

$T_B = 1152\ \text{lb} = 1.15\ \text{kip}$ **Ans.**

Ans:
$\bar{x} = 3.20$ ft
$\bar{y} = 3.20$ ft
$T_A = 384$ lb
$T_C = 384$ lb
$T_B = 1.15$ kip

9–26.

Locate the centroid $\bar{x}$ of the shaded area.



SOLUTION

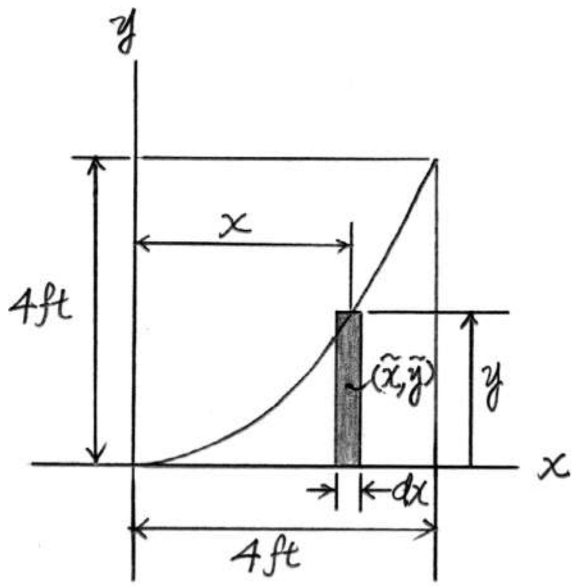
Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \frac{1}{4}x^2\,dx$ and its centroid is at $\tilde{x} = x$.

Centroid. Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ ft}} x\left(\frac{1}{4}x^2\,dx\right)}{\int_0^{4\text{ ft}} \frac{1}{4}x^2 dx}$$

$$= \frac{\left(\frac{1}{16}x^4\right)\Big|_0^{4\text{ ft}}}{\left(\frac{1}{12}x^3\right)\Big|_0^{4\text{ ft}}}$$

$$= 3\text{ ft}$$ **Ans.**

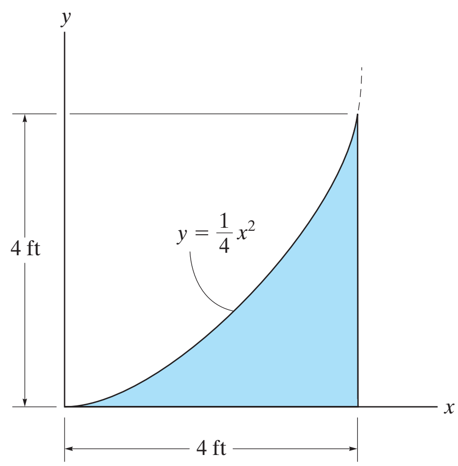


(a)

Ans:
$\bar{x} = 3$ ft

9–27.

Locate the centroid $\bar{y}$ of the shaded area.



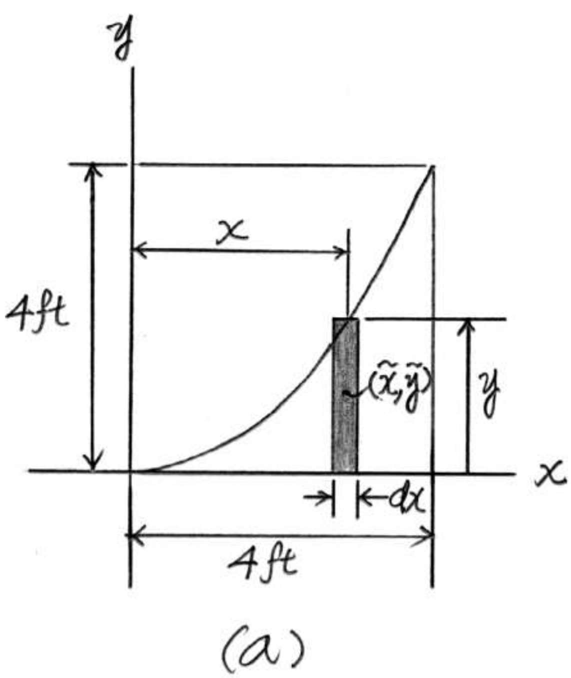
SOLUTION

Area And Moment Arm. The area of the differential element shown shaded in Fig. *a* is $dA = y\,dx = \frac{1}{4}x^2\,dx$ and its centroid is located at $\tilde{y} = \frac{y}{2} = \frac{1}{2}\left(\frac{1}{4}x^2\right) = \frac{1}{8}x^2$.

Centroid. Perform the integration,

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^{4\text{ ft}} \frac{1}{8}x^2\left(\frac{1}{4}x^2\,dx\right)}{\int_0^{4\text{ ft}} \frac{1}{4}x^2 dx}$$

$$= \frac{6}{5}\text{ ft}$$ **Ans.**

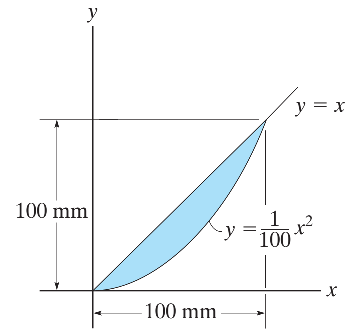


(a)

Ans:
$\bar{y} = \frac{6}{5}$ ft

***9–28.**

Locate the centroid $\bar{x}$ of the shaded area.



SOLUTION

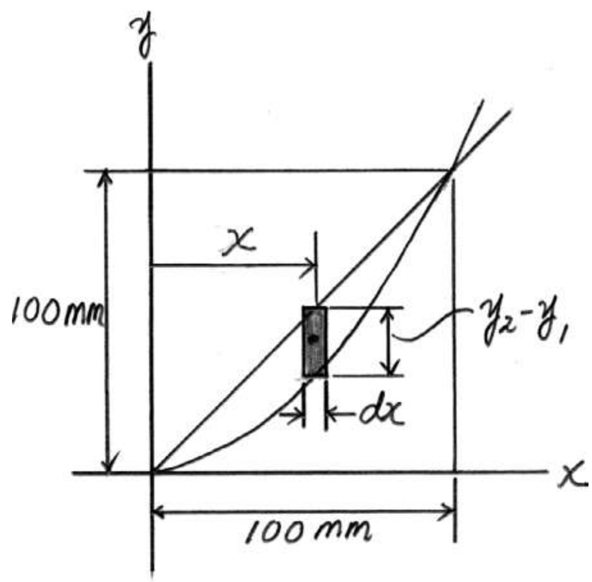
Area And Moment Arm. Here, $y_2 = x$ and $y_1 = \dfrac{1}{100}x^2$. Thus the area of the differential element shown shaded in Fig. *a* is $dA = (y_2 - y_1)\,dx = \left(x - \dfrac{1}{100}x^2\right)dx$ and its centroid is at $\tilde{x} = x$.

Centroid. Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\displaystyle\int_0^{100\text{ mm}} x\left(x - \frac{1}{100}x^2\right)dx}{\displaystyle\int_0^{100\text{ mm}} \left(x - \frac{1}{100}x^2\right)dx}$$

$$= \frac{\left(\dfrac{x^3}{3} - \dfrac{1}{400}x^4\right)\Bigg|_0^{100\text{ mm}}}{\left(\dfrac{x^2}{2} - \dfrac{1}{300}x^3\right)\Bigg|_0^{100\text{ mm}}}$$

$$= 50.0\text{ mm}$$ **Ans.**

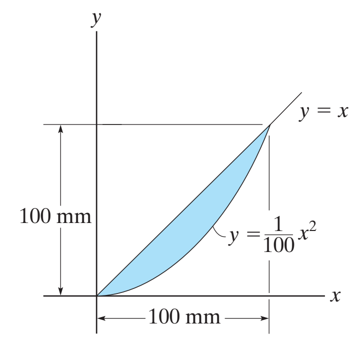


(a)

Ans:
$\bar{x} = 50.0$ mm

9–29.

Locate the centroid $\bar{y}$ of the shaded area.



SOLUTION

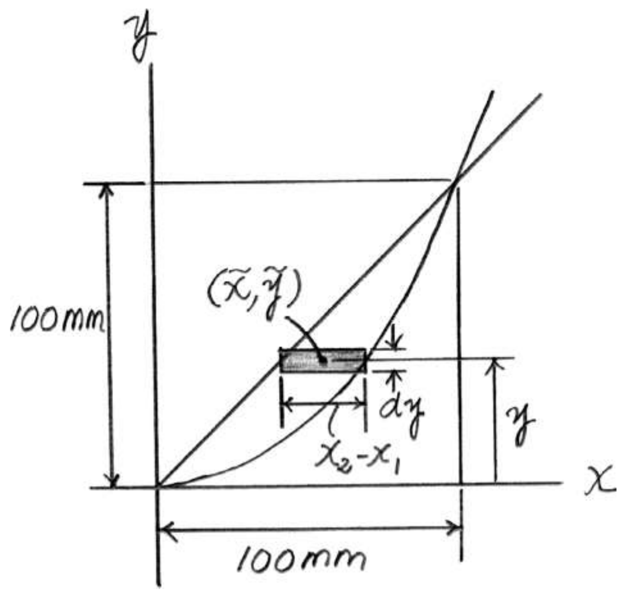
Area And Moment Arm. Here, $x_2 = 10y^{1/2}$ and $x_1 = y$. Thus, the area of the differential element shown shaded in Fig. *a* is $dA = (x_2 - x_1)\,dy = \left(10y^{1/2} - y\right)dy$ and its centroid is at $\tilde{y} = y$.

Centroid. Perform the integration,

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\displaystyle\int_0^{100\text{ mm}} y\left(10y^{1/2} - y\right)dy}{\displaystyle\int_0^{100\text{ mm}}\left(10y^{1/2} - y\right)dy}$$

$$= \frac{\left(4y^{5/2} - \dfrac{y^3}{3}\right)\Bigg|_0^{100\text{ mm}}}{\left(\dfrac{20}{3}y^{3/2} - \dfrac{y^2}{2}\right)\Bigg|_0^{100\text{ mm}}}$$

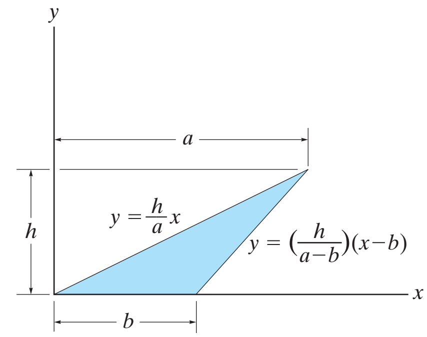
$$= 40.0\text{ mm} \qquad \textbf{Ans.}$$



Ans:
$\bar{y} = 40.0$ mm

9–30.

Locate the centroid $\bar{x}$ of the shaded area.



SOLUTION

Area And Moment Arm. Here $x_1 = \frac{a}{h}y$ and $x_2 = \left(\frac{a-b}{h}\right)y + b$. Thus the area of the differential element is $dA = (x_2 - x_1)\,dy = \left[\left(\frac{a-b}{h}\right)y + b - \frac{a}{h}y\right]dy = \left(b - \frac{b}{h}y\right)dy$ and its centroid is at $\tilde{x} = x_1 + \frac{x_2 - x_1}{2} = \frac{1}{2}(x_2 + x_1) = \frac{a}{h}y - \frac{b}{2h}y + \frac{b}{2}$.

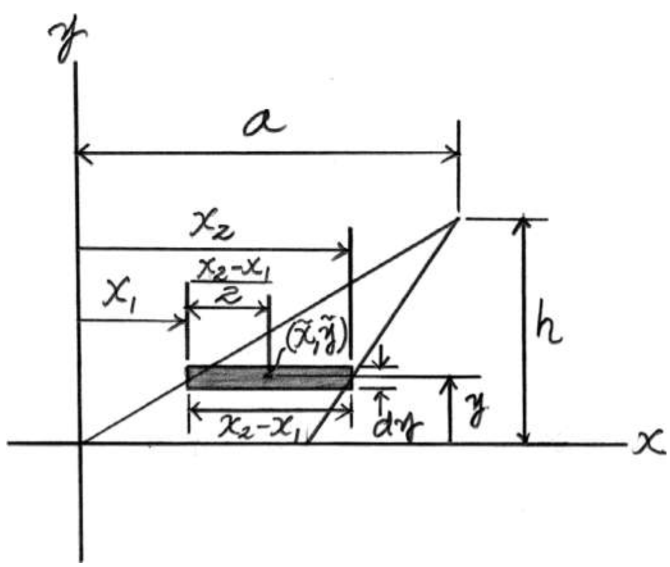
Centroid. Perform the integration,

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^h \left(\frac{a}{h}y - \frac{b}{2h}y + \frac{b}{2}\right)\left[\left(b - \frac{b}{h}y\right)dy\right]}{\int_0^h \left(b - \frac{b}{h}y\right)dy}$$

$$= \frac{\left[\frac{b}{2h}(a-b)y^2 + \frac{b}{6h^2}(b-2a)y^3 + \frac{b^2}{2}y\right]\Big|_0^h}{\left(by - \frac{b}{2h}y^2\right)\Big|_0^h}$$

$$= \frac{\frac{bh}{6}(a+b)}{\frac{bh}{2}}$$

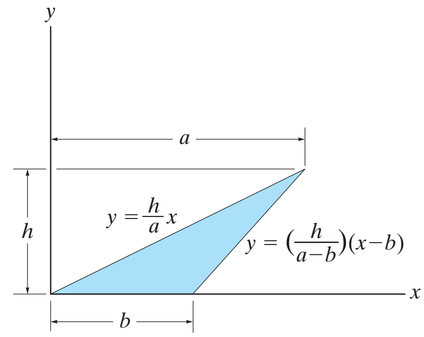
$$= \frac{1}{3}(a+b) \qquad \textbf{Ans.}$$



Ans:
$\bar{x} = \frac{1}{3}(a+b)$

9–31.

Locate the centroid $\bar{y}$ of the shaded area.



SOLUTION

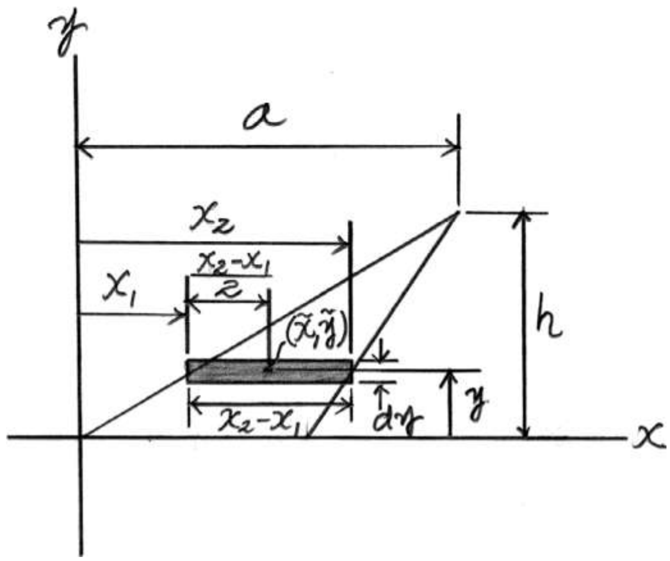
Area And Moment Arm. Here, $x_1 = \dfrac{a}{h}y$ and $x_2 = \left(\dfrac{a-b}{h}\right)y + b$. Thus the area of the differential element is $dA = \left(x_2 - x_1\right)dy = \left[\left(\dfrac{a-b}{h}\right)y + b - \dfrac{a}{h}y\right]dy = \left(b - \dfrac{b}{h}y\right)dy$ and its centroid is at $\tilde{y} = y$.

Centroid. Perform the integration,

$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\displaystyle\int_0^h y\left(b - \frac{b}{h}y\right)dy}{\displaystyle\int_0^h \left(b - \frac{b}{h}y\right)dy}$$

$$= \frac{\left.\left(\dfrac{b}{2}y^2 - \dfrac{b}{3h}y^3\right)\right|_0^h}{\left.\left(by - \dfrac{b}{2h}y^2\right)\right|_0^h}$$

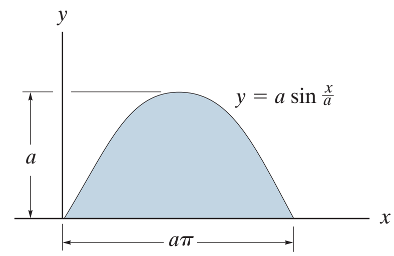
$$= \frac{\dfrac{1}{6}bh^2}{\dfrac{1}{2}bh} = \frac{h}{3}$$ **Ans.**



Ans:
$\bar{y} = \dfrac{h}{3}$

***9–32.**

Locate the centroid $\overline{x}$ of the shaded area.



SOLUTION

Area and Moment Arm: The area of the differential element is $dA = y\,dx = a \sin \frac{x}{a} dx$ and its centroid are $\overline{x} = x$

$$\overline{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_0^{\pi a} x\left(a \sin \frac{x}{a} dx\right)}{\int_0^{\pi a} a \sin \frac{x}{a} dx}$$

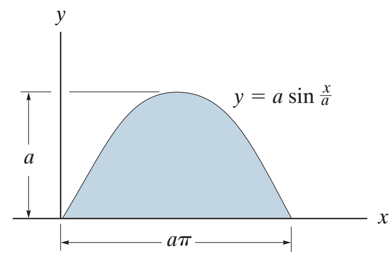
$$= \frac{\left[a^3 \sin \frac{x}{a} - x\left(a^2 \cos \frac{x}{a}\right)\right]\Big|_0^{\pi a}}{\left(-a^2 \cos \frac{x}{a}\right)\Big|_0^{\pi a}}$$

$$= \frac{\pi}{2} a \qquad \textbf{Ans.}$$

Ans:
$\overline{x} = \frac{\pi}{2} a$

9–33.

Locate the centroid $\bar{y}$ of the shaded area.



SOLUTION

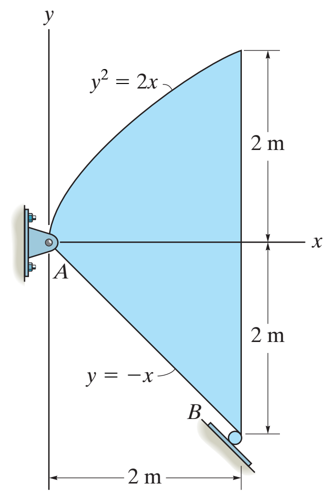
Area and Moment Arm: The area of the differential element is $dA = y dx = a \sin \frac{x}{a} dx$ and its centroid are $\bar{y} = \frac{y}{2} = \frac{a}{2} \sin \frac{x}{a}$.

$$\bar{y} = \frac{\int_A \bar{y} dA}{\int_A dA} = \frac{\int_0^{\pi a} \frac{a}{2} \sin \frac{x}{a}\left(a \sin \frac{x}{a} dx\right)}{\int_0^{\pi a} a \sin \frac{x}{a} dx} = \frac{\left[\frac{1}{4}a^2\left(x - \frac{1}{2}a \sin \frac{2x}{a}\right)\right]\Big|_0^{\pi a}}{\left(-a^2 \cos \frac{x}{a}\right)\Big|_0^{\pi a}} = \frac{\pi a}{8} \qquad \textbf{Ans.}$$

Ans:
$\bar{y} = \frac{\pi a}{8}$

9–34.

The steel plate is 0.3 m thick and has a density of 7850 kg/m^3. Determine the location of its center of mass. Also compute the reactions at the pin and roller support.



SOLUTION

$$y_1 = -x_1$$

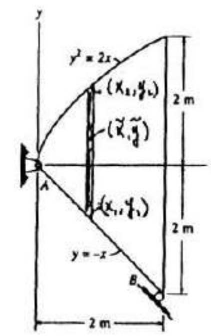
$$y_2^2 = 2x_2$$

$$dA = (y_2 - y_1)\,dx = \left(\sqrt{2x} + x\right)dx$$

$$\widetilde{x} = x$$

$$\widetilde{y} = \frac{y_2 + y_1}{2} = \frac{\sqrt{2x} - x}{2}$$

$$\overline{x} = \frac{\int_A \widetilde{x}\,dA}{\int_A dA} = \frac{\int_0^2 x\left(\sqrt{2x} + x\right)dx}{\int_0^2 \left(\sqrt{2x} + x\right)dx} = \frac{\left[\frac{2\sqrt{2}}{5}x^{5/2} + \frac{1}{3}x^3\right]_0^2}{\left[\frac{2\sqrt{2}}{3}x^{3/2} + \frac{1}{2}x^2\right]_0^2} = 1.2571 = 1.26 \text{ m} \quad \textbf{Ans.}$$



$$\overline{y} = \frac{\int_A \widetilde{y}\,dA}{\int_A dA} = \frac{\int_0^2 \frac{\sqrt{2x} - x}{2}\left(\sqrt{2x} + x\right)dx}{\int_0^2 \left(\sqrt{2x} + x\right)dx} = \frac{\left[\frac{x^2}{2} - \frac{1}{6}x^3\right]_0^2}{\left[\frac{2\sqrt{2}}{3}x^{3/2} + \frac{1}{2}x^2\right]_0^2} = 0.143 \text{ m} \quad \textbf{Ans.}$$

$$A = 4.667 \text{ m}^2$$

$$W = 7850(9.81)(4.667)(0.3) = 107.81 \text{ kN}$$

$\circlearrowleft + \Sigma M_A = 0;$ $\quad -1.2571(107.81) + N_B\left(2\sqrt{2}\right) = 0$

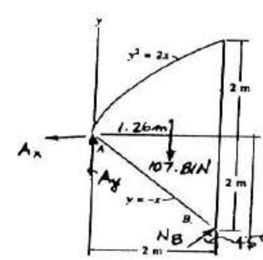
$N_B = 47.92 = 47.9 \text{ kN}$ **Ans.**

$\xrightarrow{+} \Sigma F_x = 0;$ $\quad -A_x + 47.92 \sin 45° = 0$

$A_x = 33.9 \text{ kN}$ **Ans.**

$+\uparrow \Sigma F_y = 0;$ $\quad A_y + 47.92 \cos 45° - 107.81 = 0$

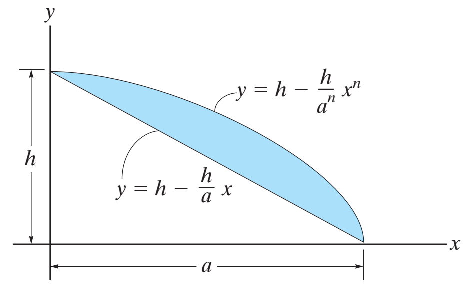
$A_y = 73.9 \text{ kN}$ **Ans.**



Ans:
$\overline{x} = 1.26$ m
$\overline{y} = 0.143$ m
$N_B = 47.9$ kN
$A_x = 33.9$ kN
$A_y = 73.9$ kN

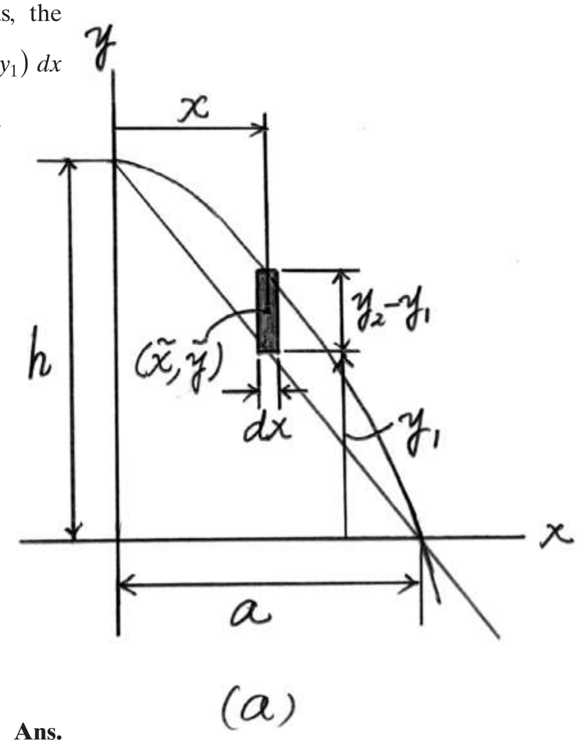
9–35.

Locate the centroid $\bar{x}$ of the shaded area.



SOLUTION

Area And Moment Arm. Here, $y_2 = h - \dfrac{h}{a^n}x^n$ and $y_1 = h - \dfrac{h}{a}x$. Thus, the area of the differential element shown shaded in Fig. a is $dA = (y_2 - y_1)\,dx$ $= \left[h - \dfrac{h}{a^n}x^n - \left(h - \dfrac{h}{a}x\right)\right]dx = \left(\dfrac{h}{a}x - \dfrac{h}{a^n}x^n\right)dx$ and its centroid is $\tilde{x} = x$.



Centroid. Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\displaystyle\int_0^a x\left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx}{\displaystyle\int_0^a\left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx}$$

$$= \frac{\left[\dfrac{h}{3a}x^3 - \dfrac{h}{a^n(n+2)}x^{n+2}\right]\Bigg|_0^a}{\left[\dfrac{h}{2a}x^2 - \dfrac{h}{a^n(n+1)}x^{n+1}\right)\Bigg|_0^a}$$

$$= \frac{\dfrac{ha^2(n-1)}{3(n+2)}}{\dfrac{ha(n-1)}{2(n+1)}}$$

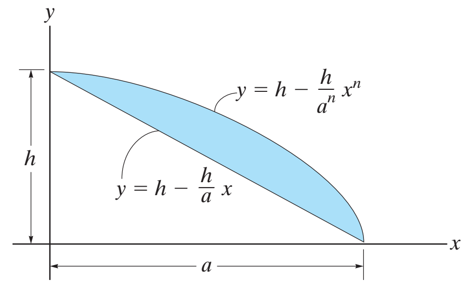
$$= \left[\frac{2(n+1)}{3(n+2)}\right]a \qquad \textbf{Ans.}$$

Ans:

$$\bar{x} = \left[\frac{2(n+1)}{3(n+2)}\right]a$$

***9–36.**

Locate the centroid $\bar{y}$ of the shaded area.



SOLUTION

Area And Moment Arm. Here, $y_2 = h - \frac{h}{a^n}x^n$ and $y_1 = h - \frac{h}{a}x$. Thus, the area of the differential element shown shaded in Fig. *a* is $dA = (y_2 - y_1)\,dx = \left[h - \frac{h}{a^n}x^n - \left(h - \frac{h}{a}x\right)\right]dx = \left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx$ and its centroid is at

$$\tilde{y} = y_1 + \left(\frac{y_2 - y_1}{2}\right) = \frac{1}{2}(y_2 + y_1) = \frac{1}{2}\left(h - \frac{h}{a^n}x^n + h - \frac{h}{a}x\right) = \frac{1}{2}\left(2h - \frac{h}{a^n}x^n - \frac{h}{a}x\right).$$

Centroid. Perform the integration

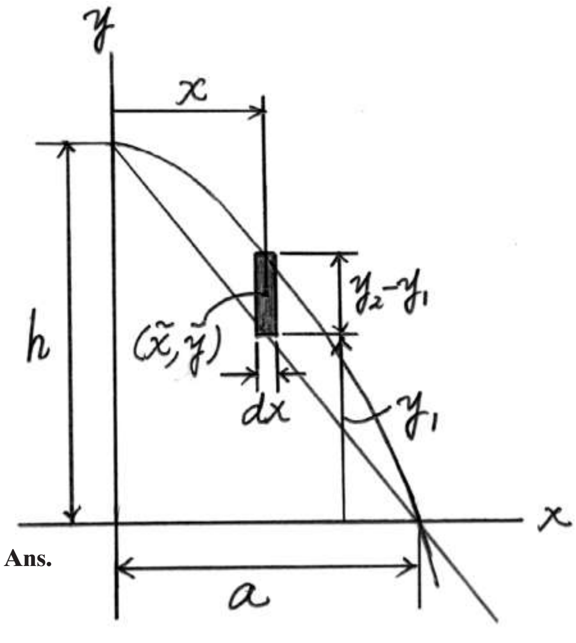
$$\bar{y} = \frac{\int_A \tilde{y}\,dA}{\int_A dA} = \frac{\int_0^a \frac{1}{2}\left(2h - \frac{h}{a^n}x^n - \frac{h}{a}x\right)\left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx}{\int_0^a \left(\frac{h}{a}x - \frac{h}{a^n}x^n\right)dx}$$

$$= \frac{\frac{1}{2}\left[\frac{h^2}{a}x^2 - \frac{h^2}{3a^2}x^3 - \frac{2h^2}{a^n(n+1)}x^{n+1} + \frac{h^2}{a^{2n}(2n+1)}x^{2n+1}\right]\Bigg|_0^a}{\left[\frac{h}{2a}x^2 - \frac{h}{a^n(n+1)}x^{n+1}\right)\Bigg|_0^a}$$

$$= \frac{h^2a\left[\frac{(4n+1)(n-1)}{6(n+1)(2n+1)}\right]}{ha\left[\frac{n-1}{2(n+1)}\right]}$$

$$= \left[\frac{(4n+1)}{3(2n+1)}\right]h$$

Ans.



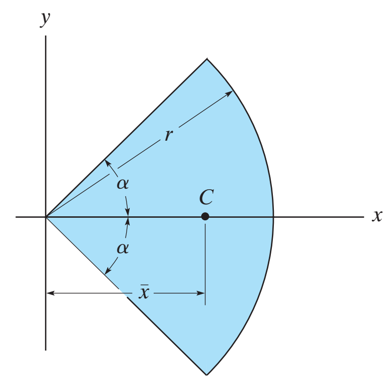
(a)

Ans:

$\bar{y} = \left[\frac{(4n+1)}{3(2n+1)}\right]h$

9–37.

Locate the centroid $\bar{x}$ of the circular sector.



SOLUTION

Area And Moment Arm. The area of the differential element shown in Fig. *a* is $dA = \frac{1}{2}r^2\,d\theta$ and its centroid is at $\tilde{x} = \frac{2}{3}r\cos\theta$.

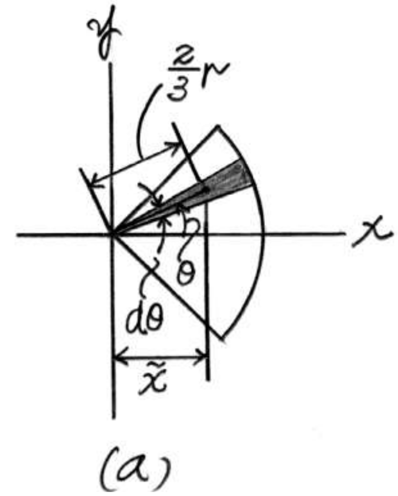
Centroid. Perform the integration

$$\bar{x} = \frac{\int_A \tilde{x}\,dA}{\int_A dA} = \frac{\int_{-\alpha}^{\alpha}\left(\frac{2}{3}r\cos\theta\right)\left(\frac{1}{2}r^2\,d\theta\right)}{\int_{-\alpha}^{\alpha}\frac{1}{2}r^2\,d\theta}$$

$$= \frac{\left(\frac{1}{3}r^3\sin\theta\right)\Big|_{-\alpha}^{\alpha}}{\left(\frac{1}{2}r^2\theta\right)\Big|_{-\alpha}^{\alpha}}$$

$$= \frac{\frac{2}{3}r^3\sin\alpha}{r^2\alpha}$$

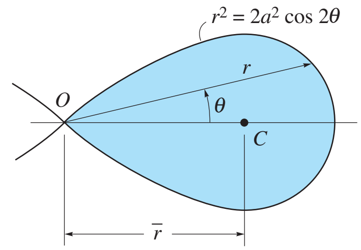
$$= \frac{2}{3}\left(\frac{r\sin\alpha}{\alpha}\right) \qquad \textbf{Ans.}$$



Ans:
$\bar{x} = \frac{2}{3}\left(\frac{r\sin\alpha}{\alpha}\right)$

9–38.

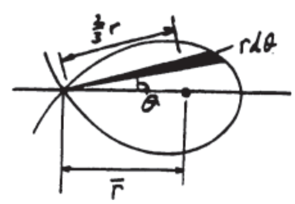
Determine the location $\bar{r}$ of the centroid C for the loop of the lemniscate, $r^2 = 2a^2 \cos 2\theta, (-45° \le \theta \le 45°)$.



SOLUTION

$$dA = \frac{1}{2}(r)\, r\, d\theta = \frac{1}{2} r^2\, d\theta$$

$$A = 2\int_0^{45°} \frac{1}{2}(2a^2 \cos 2\theta) d\theta = a^2\Big[\sin 2\theta\Big]_0^{45°} = a^2$$



$$\bar{x} = \frac{\int_A \bar{x}\, dA}{\int_A dA} = \frac{2\int_0^{45°} \left(\frac{2}{3} r \cos \theta\right)\left(\frac{1}{2} r^2 d\theta\right)}{a^2} = \frac{\frac{2}{3}\int_0^{45°} r^3 \cos\theta\, d\theta}{a^2}$$

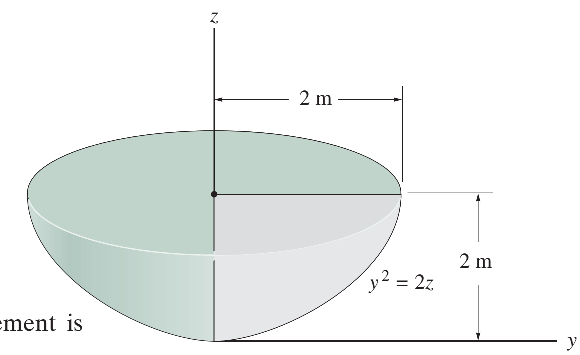
$$\int_A \bar{x}\, dA = \frac{2}{3}\int_0^{45°} r^3 \cos\theta\, d\theta = \frac{2}{3}\int_0^{45°} \left(2a^2\right)^{3/2} \cos\theta\, (\cos 2\theta)^{3/2} d\theta = 0.7854\, a^3$$

$$\bar{x} = \frac{0.7854\, a^3}{a^2} = 0.785\, a$$ **Ans.**

Ans:
$\bar{x} = 0.785\, a$

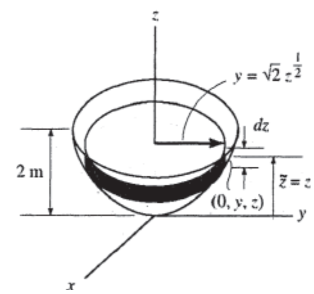
9–39.

Locate the center of gravity of the volume. The material is homogeneous.



SOLUTION

Volume and Moment Arm: The volume of the thin disk differential element is $dV = \pi y^2 dz = \pi(2z)dz = 2\pi z dz$ and its centroid $\widetilde{z} = z$.



Centroid: Due to symmetry about z axis

$$\bar{x} = \bar{y} = 0 \qquad \textbf{Ans.}$$

Applying Eq. 9–3 and performing the integration, we have

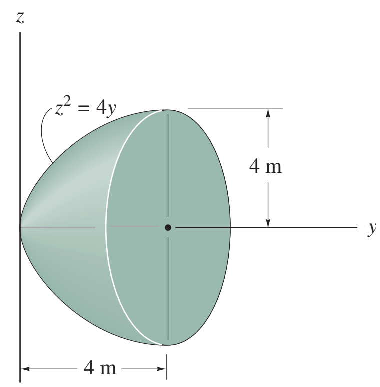
$$\bar{z} = \frac{\int_v \widetilde{z}\,dV}{\int_v dV} = \frac{\int_0^{2\text{ m}} z(2\pi z dz)}{\int_0^{2\text{ m}} 2\pi z dz}$$

$$= \frac{2\pi\left(\frac{z^3}{3}\right)\Big|_0^{2\text{ m}}}{2\pi\left(\frac{z^2}{2}\right)\Big|_0^{2\text{ m}}} = \frac{4}{3}\text{ m} \qquad \textbf{Ans.}$$

Ans:
$\bar{x} = \bar{y} = 0$
$\bar{z} = \frac{4}{3}$ m

***9–40.**

Locate the centroid $\overline{y}$ of the paraboloid.



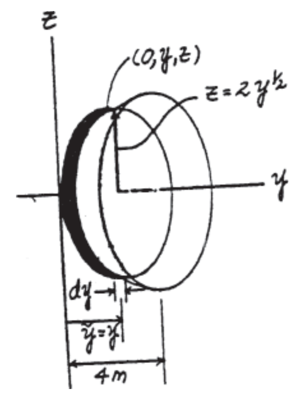
SOLUTION

Volume and Moment Arm: The volume of the thin disk differential element is $dV = \pi z^2 dy = \pi(4y)dy$ and its centroid $\widetilde{y} = y$.

Centroid: Applying Eq. 9–3 and performing the integration, we have

$$\overline{y} = \frac{\int_V \widetilde{y}\,dV}{\int_V dV} = \frac{\int_0^{4\text{ m}} y[\pi(4y)dy]}{\int_0^{4\text{ m}} \pi(4y)dy}$$

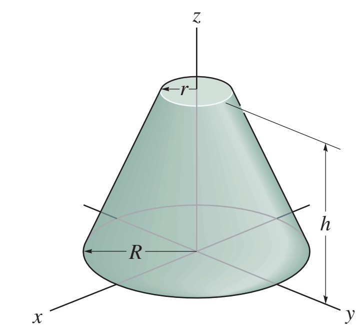
$$= \frac{4\pi\left(\dfrac{y^3}{3}\right)\Bigg|_0^{4\text{ m}}}{4\pi\left(\dfrac{y^2}{2}\right)\Bigg|_0^{4\text{ m}}} = 2.67\text{ m} \qquad \textbf{Ans.}$$



Ans:
$\overline{y} = 2.67$ m

9–41.

Locate the centroid $\bar{z}$ of the frustum of the right-circular cone.



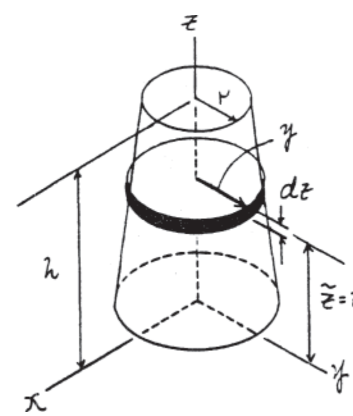
SOLUTION

Volume and Moment Arm: From the geometry, $\dfrac{y-r}{R-r} = \dfrac{h-z}{h}$, $y = \dfrac{(r-R)z + Rh}{h}$. The volume of the thin disk differential element is

$$dV = \pi y^2 dz = \pi\left[\left(\frac{(r-R)z + Rh}{h}\right)^2\right]dz$$

$$= \frac{\pi}{h^2}\left[(r-R)^2z^2 + 2Rh(r-R)z + R^2h^2\right]dz$$

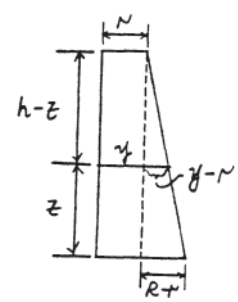
and its centroid $\tilde{z} = z$.



Centroid: Applying Eq. 9–5 and performing the integration, we have

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^h z\left\{\frac{\pi}{h^2}[(r-R)^2z^2 + 2Rh(r-R)z + R^2h^2]dz\right\}}{\int_0^h \frac{\pi}{h^2}[r-R)^2z^2 + 2Rh(r-R)z + R^2h^2]dz}$$

$$= \frac{\frac{\pi}{h^2}\left[(r-R)^2\left(\frac{z^4}{4}\right) + 2Rh(r-R)\left(\frac{z^3}{3}\right) + R^2h^2\left(\frac{z^2}{2}\right)\right]\Bigg|_0^h}{\frac{\pi}{h^2}\left[(r-R)^2\left(\frac{z^3}{3}\right) + 2Rh(r-R)\left(\frac{z^2}{2}\right) + R^2h^2(z)\right]\Bigg|_0^h}$$



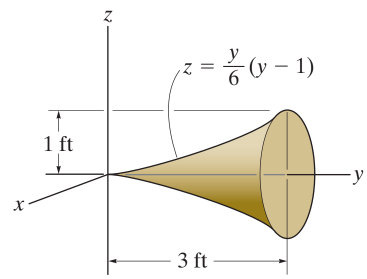
$$= \frac{R^2 + 3r^2 + 2rR}{4(R^2 + r^2 + rR)}h \qquad \textbf{Ans.}$$

Ans:

$$\bar{z} = \frac{R^2 + 3r^2 + 2rR}{4(R^2 + r^2 + rR)}h$$

9–42.

Determine the centroid $\overline{y}$ of the solid.



SOLUTION

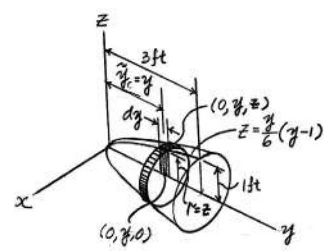
Differential Element: The thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is

$$dV = \pi z^2\,dy = \pi\left[\frac{y}{6}(y - 1)\right]^2 dy = \frac{\pi}{36}(y^4 - 2y^3 + y^2)dy$$

Centroid: The centroid of the element is located at $y_c = y$. We have

$$\overline{y} = \frac{\int_V \tilde{y}\,dV}{\int_V dV} = \frac{\int_0^{3\text{ ft}} y\left[\frac{\pi}{36}\left(y^4 - 2y^3 + y^2\right)dy\right]}{\int_0^{3\text{ ft}} \frac{\pi}{36}\left(y^4 - 2y^3 + y^2\right)dy} = \frac{\int_0^{3\text{ ft}} \frac{\pi}{36}\left(y^5 - 2y^4 + y^3\right)dy}{\int_0^{3\text{ ft}} \frac{\pi}{36}\left(y^4 - 2y^3 + y^2\right)dy} = \frac{\frac{\pi}{36}\left[\frac{y^6}{6} - \frac{2}{5}y^5 + \frac{y^4}{4}\right]\Bigg|_0^{3\text{ ft}}}{\frac{\pi}{36}\left[\frac{y^5}{5} - \frac{y^4}{2} + \frac{y^3}{3}\right]\Bigg|_0^{3\text{ ft}}}$$

$$= 2.61\text{ ft}$$ **Ans.**

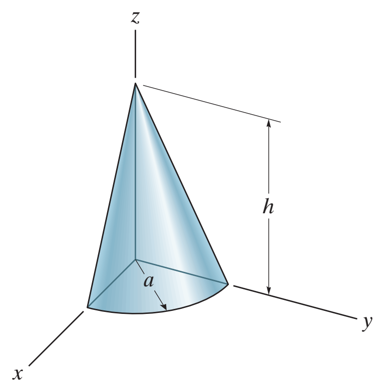


(a)

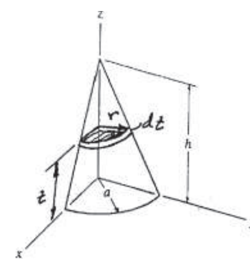
Ans:
$\overline{y} = 2.61$ ft

9–43.

Locate the centroid of the quarter-cone.



SOLUTION



$$\tilde{z} = z$$

$$r = \frac{a}{h}(h - z)$$

$$dV = \frac{\pi}{4} r^2\, dz = \frac{\pi\, a^2}{4\, h^2}(h - z)^2\, dz$$

$$\int dV = \frac{\pi\, a^2}{4\, h^2}\int_0^h (h^2 - 2hz + z^2)\, dz = \frac{\pi\, a^2}{4\, h^2}\left[h^2 z - hz^2 + \frac{z^3}{3}\right]_0^h$$

$$= \frac{\pi\, a^2}{4\, h^2}\left(\frac{h^3}{3}\right) = \frac{\pi a^2\, h}{12}$$

$$\int \tilde{z} dV = \frac{\pi\, a^2}{4\, h^2}\int_0^h (h^2 - 2hz + z^2)\, z\, dz = \frac{\pi\, a^2}{4\, h^2}\left[h^2\frac{z^2}{2} - 2h\frac{z^3}{3} + \frac{z^4}{4}\right]_0^h$$

$$= \frac{\pi\, a^2}{4\, h^2}\left(\frac{h^4}{12}\right) = \frac{\pi a^2 h^2}{48}$$

$$\bar{z} = \frac{\int \tilde{z} dV}{\int dV} = \frac{\frac{\pi\, a^2\, h^2}{48}}{\frac{\pi\, a^2 h}{12}} = \frac{h}{4} \qquad \textbf{Ans.}$$

$$\int \tilde{x} dV = \frac{\pi a^2}{4\, h^2}\int_0^h \frac{4\, r}{3\pi}(h - z)^2\, dz = \frac{\pi a^2}{4\, h^2}\int_0^h \frac{4a}{3\, \pi\, h}(h^3 - 3h^2\, z + 3hz^2 - z^3)\, dz$$

$$= \frac{\pi\, a^2}{4\, h^2}\,\frac{4a}{3\pi h}\left(h^4 - \frac{3h^4}{2} + h^4 - \frac{h^4}{4}\right)$$

$$= \frac{\pi\, a^2}{4\, h^2}\left(\frac{a\, h^3}{3\pi}\right) = \frac{a^3\, h}{12}$$

$$\bar{x} = \bar{y} = \frac{\int \tilde{x} dV}{\int dV} = \frac{\frac{a^3\, h}{12}}{\frac{\pi\, a^2 h}{12}} = \frac{a}{\pi} \qquad \textbf{Ans.}$$

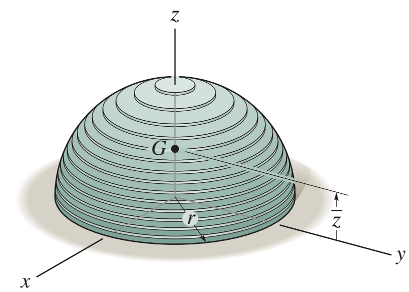
Ans:

$\bar{z} = \dfrac{h}{4}$

$\bar{x} = \bar{y} = \dfrac{a}{\pi}$

***9–44.**

The hemisphere of radius r is made from a stack of very thin plates such that the density varies with height $\rho = kz$, where k is a constant. Determine its mass and the distance to the center of mass G.



SOLUTION

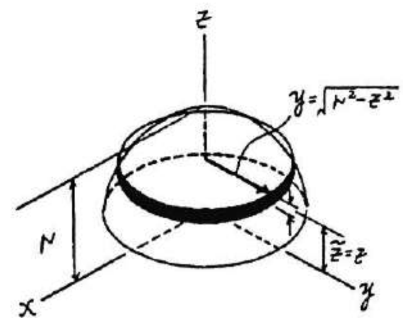
Mass and Moment Arm: The density of the material is $\rho = kz$. The mass of the thin disk differential element is $dm = \rho dV = \rho\pi y^2 dz = kz[\pi(r^2 - z^2)\,dz]$ and its centroid $\tilde{z} = z$. Evaluating the integrals, we have

$$m = \int_m dm = \int_0^r kz[\pi(r^2 - z^2)\,dz]$$

$$= \pi k\left(\frac{r^2z^2}{2} - \frac{z^4}{4}\right)\Bigg|_0^r = \frac{\pi k r^4}{4} \qquad \textbf{Ans.}$$

$$\int_m \tilde{z}\,dm = \int_0^r z\{kz[\pi(r^2 - z^2)\,dz]\}$$

$$= \pi k\left(\frac{r^2z^3}{3} - \frac{z^5}{5}\right)\Bigg|_0^r = \frac{2\pi k r^5}{15}$$



Centroid: Applying Eq. 9–3, we have

$$\bar{z} = \frac{\int_m \tilde{z}\,dm}{\int_m dm} = \frac{2\pi k r^5/15}{\pi k r^4/4} = \frac{8}{15}r \qquad \textbf{Ans.}$$

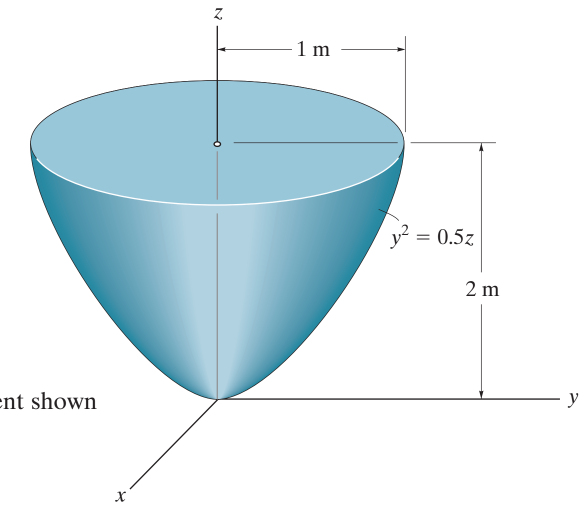
Ans:

$m = \dfrac{\pi k r^4}{4}$

$\bar{z} = \dfrac{8}{15}r$

9–45.

Locate the centroid $\bar{z}$ of the volume.



SOLUTION

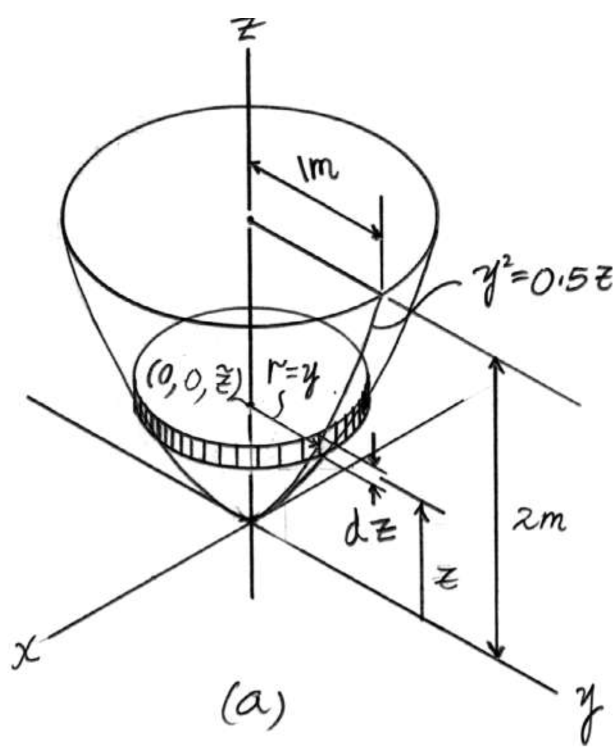
Volume And Moment Arm. The volume of the thin disk differential element shown shaded in Fig. *a* is $dV = \pi y^2\,dz = \pi(0.5z)dz$ and its centroid is at $\tilde{z} = z$.

Centroid. Perform the integration

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\int_0^{2\text{ m}} z[\pi(0.5z)dz]}{\int_0^{2\text{ m}} \pi(0.5z)dz}$$

$$= \frac{\frac{0.57}{3}z^3\Big|_0^{2\text{ m}}}{\frac{0.5\pi}{2}z^2\Big|_0^{2\text{ m}}}$$

$$= \frac{4}{3}\text{ m} \qquad \textbf{Ans.}$$

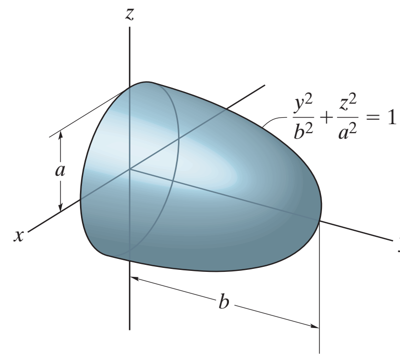


(a)

Ans:
$\bar{z} = \frac{4}{3}$ m

9–46.

Locate the centroid of the ellipsoid of revolution.

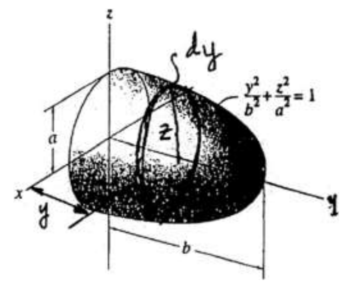


SOLUTION

$$dV = \pi\, z^2\, dy$$

$$\int dV = \int_0^b \pi\, a^2 \left(1 - \frac{y^2}{b^2}\right) dy = \pi\, a^2 \left[y - \frac{y^3}{3b^2}\right]_0^b = \frac{2\pi a^2 b}{3}$$

$$\int \tilde{y} dV = \int_0^b \pi\, a^2 y \left(1 - \frac{y^2}{b^2}\right) dy = \pi\, a^2 \left[\frac{y^2}{2} - \frac{y^4}{4b^2}\right]_0^b = \frac{\pi\, a^2 b^2}{4}$$



$$\bar{y} = \frac{\int_V \tilde{y} dV}{\int_V dV} = \frac{\frac{\pi a^2 b^2}{4}}{\frac{2\pi a^2 b}{3}} = \frac{3}{8} b \qquad \textbf{Ans.}$$

$$\bar{x} = \bar{z} = 0 \qquad \text{(By symmetry)} \qquad \textbf{Ans.}$$

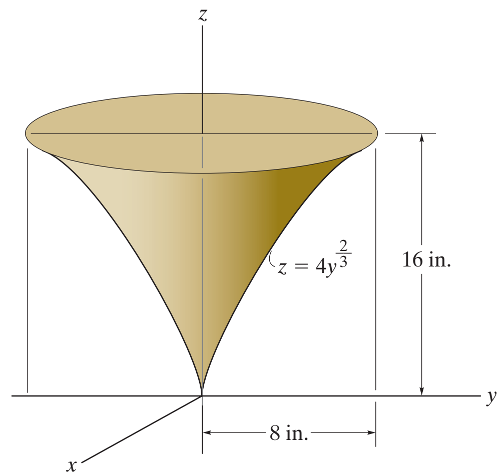
Ans:

$\bar{y} = \frac{3}{8} b$

$\bar{x} = \bar{z} = 0$

9–47.

Locate the center of gravity $\bar{z}$ of the solid.



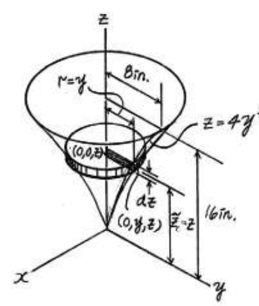
SOLUTION

Differential Element: The thin disk element shown shaded in Fig. *a* will be considered. The volume of the element is

$$dV = \pi y^2\, dz = \pi\left[\frac{1}{8} z^{3/2}\right]^2 dz = \frac{\pi}{64} z^3\, dz$$

Centroid: The centroid of the element is located at $z_c = z$. We have

$$\bar{z} = \frac{\int_V \tilde{z}\, dV}{\int_V dV} = \frac{\int_0^{16\text{ in.}} z\left[\frac{\pi}{64} z^3\, dz\right]}{\int_0^{16\text{ in.}} \frac{\pi}{64} z^3\, dz} = \frac{\int_0^{16\text{ in.}} \frac{\pi}{64} z^4\, dz}{\int_0^{16\text{ in.}} \frac{\pi}{64} z^3\, dz} = \frac{\frac{\pi}{64}\left(\frac{z^5}{5}\right)\Big|_0^{16\text{ in.}}}{\frac{\pi}{64}\left(\frac{z^4}{4}\right)\Big|_0^{16\text{ in.}}} = 12.8\text{ in.} \quad \textbf{Ans.}$$

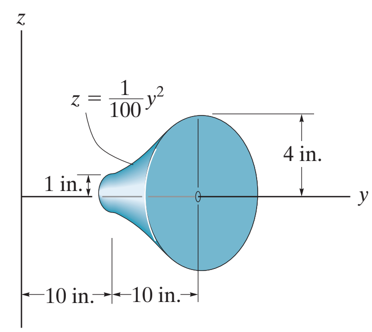


(a)

Ans:
$\bar{z} = 12.8$ in.

***9–48.**

Locate the center of gravity $\bar{y}$ of the volume. The material is homogeneous.



SOLUTION

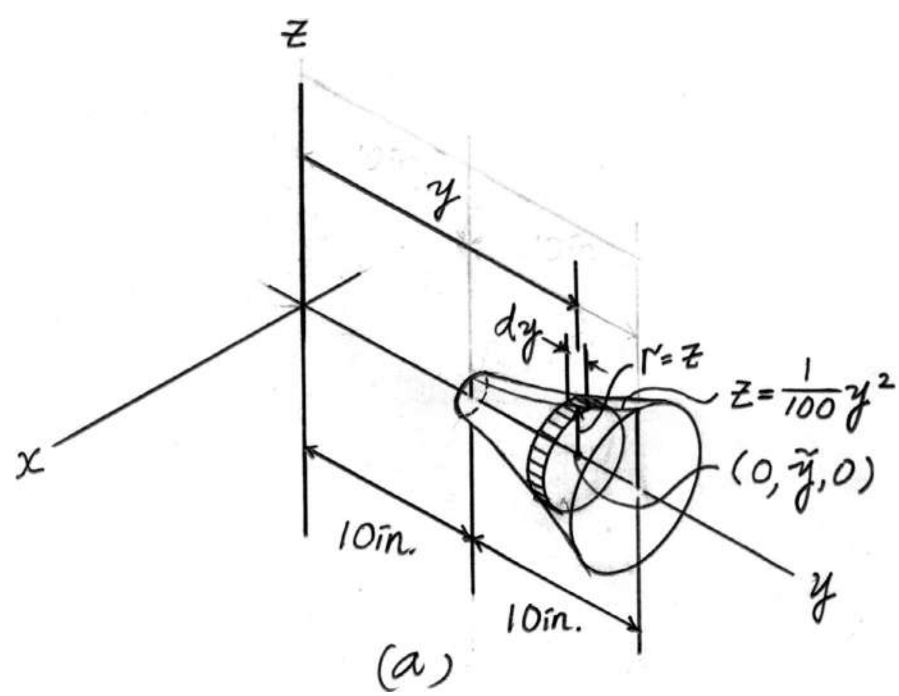
Volume And Moment Arm. The volume of the thin disk differential element shown shaded in Fig. *a* is $dV = \pi z^2\,dy = \pi\left(\frac{1}{100}y^2\right)^2 dy = \frac{\pi}{10000}y^4\,dy$ and its centroid is at $\tilde{y} = y$.

Centroid. Perform the integration

$$\bar{y} = \frac{\int_V \tilde{y}\,dV}{\int_V dV} = \frac{\int_{10\text{ in.}}^{20\text{ in.}} y\left(\frac{\pi}{10000}y^4 dy\right)}{\int_{10\text{ in.}}^{20\text{ in.}} \frac{\pi}{10000}y^4\,dy}$$

$$= \frac{\left(\frac{\pi}{60000}y^6\right)\Big|_{10\text{ in.}}^{20\text{ in.}}}{\left(\frac{\pi}{50000}y^5\right)\Big|_{10\text{ in.}}^{20\text{ in.}}}$$

$$= 16.94\text{ in.} = 16.9\text{ in.}$$ **Ans.**

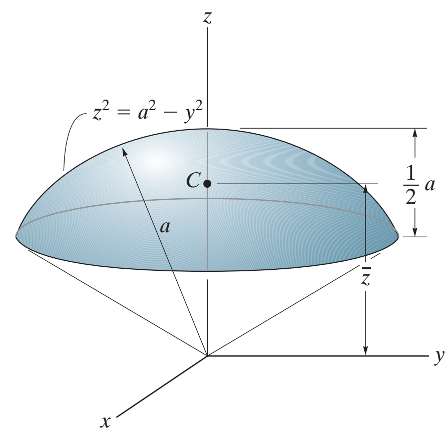


(a)

Ans:
$\bar{y} = 16.9$ in.

9–49.

Locate the centroid $\bar{z}$ of the spherical segment.



SOLUTION

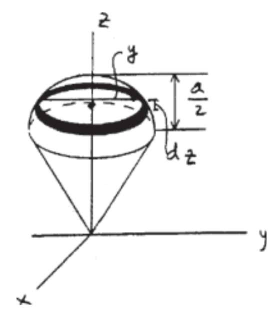
$$dV = \pi y^2 dz = \pi(a^2 - z^2)\,dz$$

$$\tilde{z} = z$$

$$\bar{z} = \frac{\int_V \tilde{z}\,dV}{\int_V dV} = \frac{\pi \int_{\frac{a}{2}}^{a} z\,(a^2 - z^2)dz}{\pi \int_{\frac{a}{2}}^{a} (a^2 - z^2)dz}$$

$$= \frac{\pi\left[a^2\left(\frac{z^2}{2}\right) - \left(\frac{z^4}{4}\right)\right]_{\frac{a}{2}}^{a}}{\pi\left[a^2(z) - \left(\frac{z^3}{3}\right)\right]_{\frac{a}{2}}^{a}} = \frac{\pi\left[\frac{a^4}{2} - \frac{a^4}{4} - \frac{a^4}{8} + \frac{a^4}{64}\right]}{\pi\left[a^3 - \frac{a^3}{3} - \frac{a^3}{2} + \frac{a^3}{24}\right]} = \frac{\pi\left[\frac{9a^4}{64}\right]}{\pi\left[\frac{5a^3}{24}\right]}$$

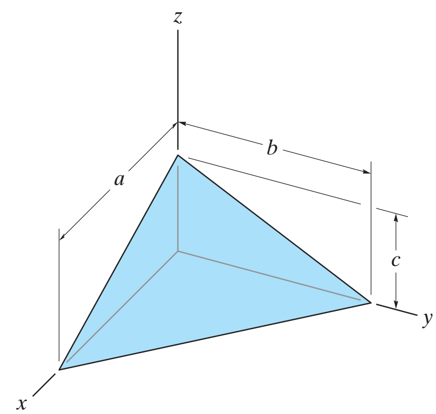
$\bar{z} = 0.675\,a$ **Ans.**



Ans:
$\bar{z} = 0.675a$

9–50.

Determine the location $\bar{z}$ of the centroid for the tetrahedron. *Hint:* Use a triangular "plate" element parallel to the x–y plane and of thickness dz.

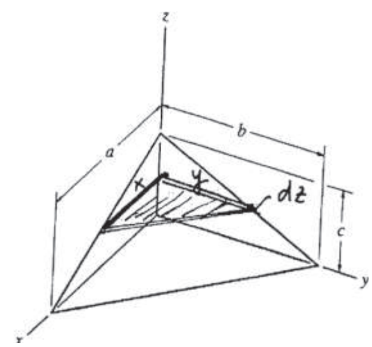


SOLUTION

$$z = c\left(1 - \frac{1}{b}y\right) = c\left(1 - \frac{1}{a}x\right)$$

$$\int dV = \int_0^c \frac{1}{2}(x)(y)dz = \frac{1}{2}\int_0^c a\left(1 - \frac{z}{c}\right)b\left(1 - \frac{z}{c}\right) dz = \frac{abc}{6}$$

$$\int \tilde{z} dV = \frac{1}{2}\int_0^c z\, a\left(1 - \frac{z}{c}\right)b\left(1 - \frac{z}{c}\right)dz = \frac{abc^2}{24}$$



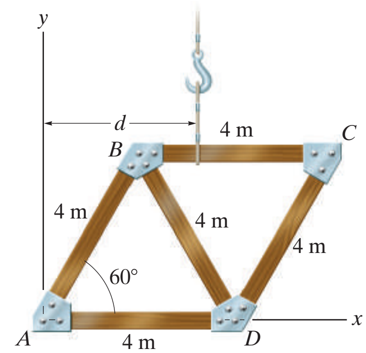
$$\bar{z} = \frac{\int \tilde{z} dV}{\int dV} = \frac{\frac{abc^2}{24}}{\frac{abc}{6}} = \frac{c}{4}$$ **Ans.**

Ans:

$\bar{z} = \dfrac{c}{4}$

9–51.

The truss is made from five members, each having a length of 4 m and a mass of 7 kg/m. If the mass of the gusset plates at the joints and the thickness of the members can be neglected, determine the distance d to where the hoisting cable must be attached, so that the truss does not tip (rotate) when it is lifted.



SOLUTION

$$\Sigma \tilde{x} M = 4(7)(1+4+2+3+5) = 420 \text{ kg} \cdot \text{m}$$

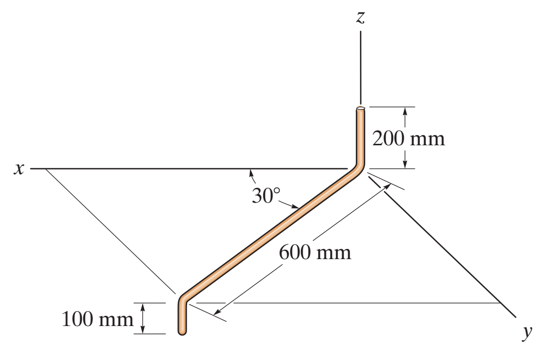
$$\Sigma M = 4(7)(5) = 140 \text{ kg}$$

$$d = \bar{x} = \frac{\Sigma \tilde{x} M}{\Sigma M} = \frac{420}{140} = 3 \text{ m}$$ **Ans.**

Ans:
$d = 3$ m

***9–52.**

Determine the location $(\bar{x}, \bar{y}, \bar{z})$ of the centroid of the homogeneous rod.



SOLUTION

Centroid. Referring to Fig. *a*, the length of the segments and the locations of their respective centroids are tabulated below

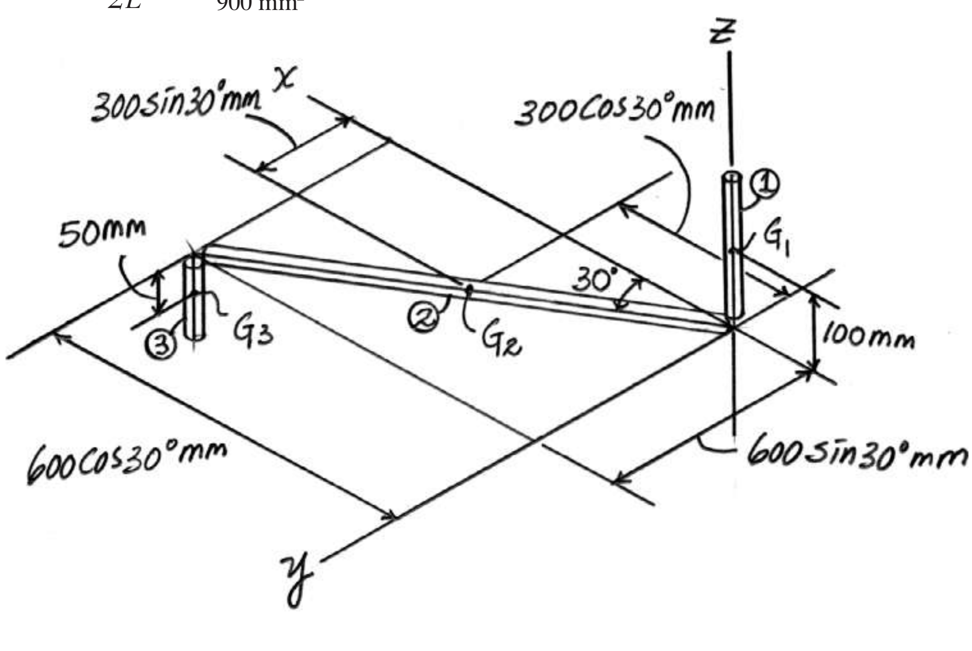
Segment	L(mm)	$\tilde{x}$(mm)	$\tilde{y}$(mm)	$\tilde{z}$(mm)	$\tilde{x}L$(mm^2)	$\tilde{y}L$(mm^2)	$\tilde{z}L$(mm^2)
1	200	0	0	100	0	0	$20.0(10^3)$
2	600	300 cos 30°	300 sin 30°	0	$155.88(10^3)$	$90.0(10^3)$	0
3	100	600 cos 30°	600 sin 30°	–50	$51.96(10^3)$	$30.0(10^3)$	$-5.0(10^3)$
Σ	900				$207.85(10^3)$	$120.0(10^3)$	$15.0(10^3)$

Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}L}{\Sigma L} = \frac{207.85(10^3)\text{mm}^2}{900 \text{ mm}} = 230.94 \text{ mm} = 231 \text{ mm} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}L}{\Sigma L} = \frac{120.0(10^3)\text{mm}^2}{900 \text{ mm}} = 133.33 \text{ mm} = 133 \text{ mm} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma\tilde{z}L}{\Sigma L} = \frac{15.0(10^3)\text{mm}^2}{900 \text{ mm}^2} = 16.67 \text{ mm} = 16.7 \text{ mm} \qquad \textbf{Ans.}$$

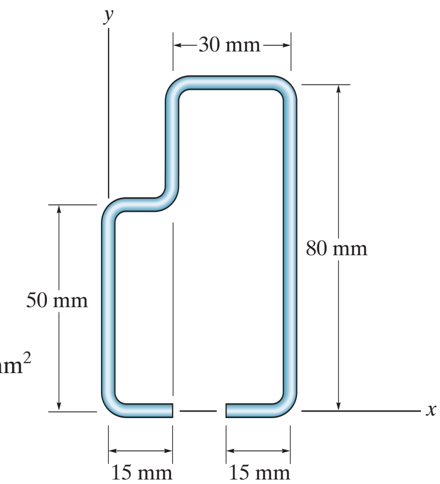


(a)

Ans:
$\bar{x} = 231$ mm
$\bar{y} = 133$ mm
$\bar{z} = 16.7$ mm

9–53.

A rack is made from roll-formed sheet steel and has the cross section shown. Determine the location $(\bar{x}, \bar{y})$ of the centroid of the cross section. The dimensions are indicated at the center thickness of each segment.



SOLUTION

$$\Sigma L = 15 + 50 + 15 + 30 + 30 + 80 + 15 = 235 \text{ mm}$$

$$\Sigma \tilde{x}L = 7.5(15) + 0(50) + 7.5(15) + 15(30) + 30(30) + 45(80) + 37.5(15) = 5737.50 \text{ mm}^2$$

$$\Sigma \tilde{y}L = 0(15) + 25(50) + 50(15) + 65(30) + 80(30) + 40(80) + 0(15) = 9550 \text{ mm}^2$$

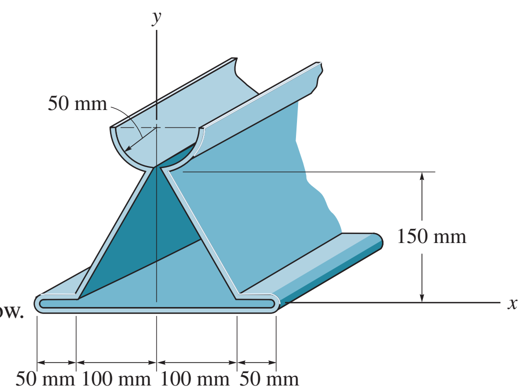
$$\bar{x} = \frac{\Sigma \tilde{x}L}{\Sigma L} = \frac{5737.50}{235} = 24.4 \text{ mm}$$ **Ans.**

$$\bar{y} = \frac{\Sigma \tilde{y}L}{\Sigma L} = \frac{9550}{235} = 40.6 \text{ mm}$$ **Ans.**

Ans:
$\bar{x} = 24.4$ mm
$\bar{y} = 40.6$ mm

9–54.

Locate the centroid $(\bar{x}, \bar{y})$ of the metal cross section. Neglect the thickness of the material and slight bends at the corners.



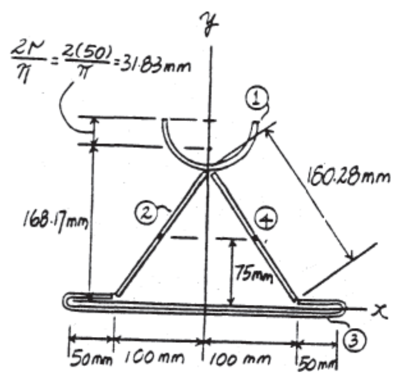
SOLUTION

Centroid: The length of each segment and its respective centroid are tabulated below.

Segment	L **(mm)**	$\tilde{y}$ **(mm)**	$\tilde{y}L$ **(mm²)**
1	50π	168.17	26415.93
2	180.28	75	13520.82
3	400	0	0
4	180.28	75	13520.82
Σ	917.63		53457.56

Due to symmetry about y axis, $\bar{x} = 0$ **Ans.**

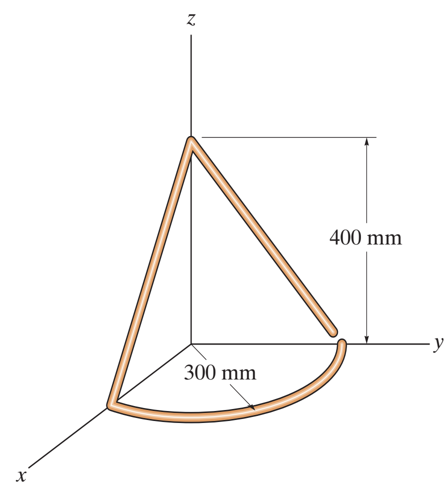
$$\bar{y} = \frac{\Sigma \tilde{y} L}{\Sigma L} = \frac{53457.56}{917.63} = 58.26 \text{ mm} = 58.3 \text{ mm}$$ **Ans.**



Ans:
$\bar{x} = 0$
$\bar{y} = 58.3$ mm

9–55.

Locate the center of gravity $(\bar{x}, \bar{y}, \bar{z})$ of the homogeneous wire.



SOLUTION

$$\Sigma \widetilde{x} L = 150(500) + 0(500) + \frac{2\,(300)}{\pi}\left(\frac{\pi}{2}\right)(300) = 165\,000 \text{ mm}^2$$

$$\Sigma L = 500 + 500 + \left(\frac{\pi}{2}\right)(300) = 1471.24 \text{ mm}$$

$$\bar{x} = \frac{\Sigma \widetilde{x} L}{\Sigma L} = \frac{165\,000}{1471.24} = 112 \text{ mm} \qquad \textbf{Ans.}$$

Due to symmetry,

$$\bar{y} = 112 \text{ mm} \qquad \textbf{Ans.}$$

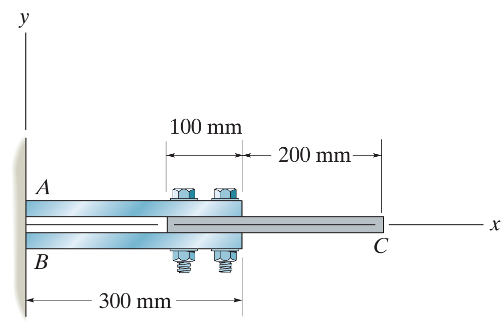
$$\Sigma \widetilde{z} L = 200(500) + 200(500) + 0\left(\frac{\pi}{2}\right)(300) = 200\,000 \text{ mm}^2$$

$$\bar{z} = \frac{\Sigma \widetilde{z} L}{\Sigma L} = \frac{200\,000}{1471.24} = 136 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$\bar{x} = 112$ mm
$\bar{y} = 112$ mm
$\bar{z} = 136$ mm

***9–56.**

The steel and aluminum plate assembly is bolted together and fastened to the wall. Each plate has a constant width in the z direction of 200 mm and thickness of 20 mm. If the density of A and B is $\rho_s = 7.85\ \text{Mg/m}^3$, and for C, $\rho_{al} = 2.71\ \text{Mg/m}^3$, determine the location x of the center of mass. Neglect the size of the bolts.



SOLUTION

$$\Sigma m = 2\left[7.85(10)^3(0.3)(0.2)(0.02)\right] + 2.71(10)^3(0.3)(0.2)(0.02) = 22.092\ \text{kg}$$

$$\Sigma \tilde{x} m = 150\{2\left[7.85(10)^3(0.3)(0.2)(0.02)\right]\} + 350\left[2.71(10)^3(0.3)(0.2)(0.02)\right]$$

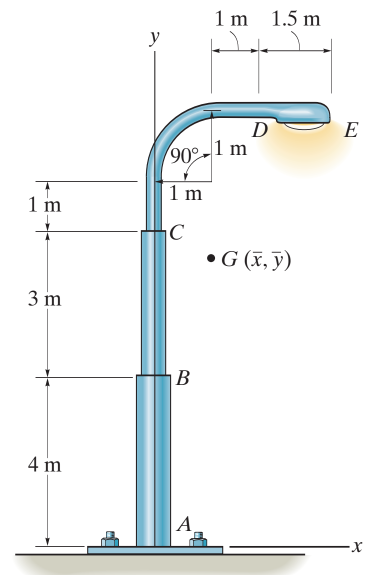
$$= 3964.2\ \text{kg.mm}$$

$$\bar{x} = \frac{\Sigma \tilde{x} m}{\Sigma m} = \frac{3964.2}{22.092} = 179\ \text{mm} \qquad \textbf{Ans.}$$

Ans:
$\bar{x} = 179$ mm

9–57.

Locate the center of gravity $G(\bar{x}, \bar{y})$ of the streetlight. Neglect the thickness of each segment. The mass per unit length of each segment is as follows: $\rho_{AB} = 12$ kg/m, $\rho_{BC} = 8$ kg/m, $\rho_{CD} = 5$ kg/m, and $\rho_{DE} = 2$ kg/m.



SOLUTION

$$\Sigma \tilde{x} m = 0(4)(12) + 0(3)(8) + 0(1)(5) + \left(1 - \frac{2(1)}{\pi}\right)\left(\frac{\pi}{2}\right)(5)$$

$$+ 1.5\,(1)\,(5) + 2.75\,(1.5)\,(2) = 18.604 \text{ kg} \cdot \text{m}$$

$$\Sigma m = 4\,(12) + 3\,(8) + 1(5) + \frac{\pi}{2}(5) + 1(5) + 1.5\,(2) = 92.854 \text{ kg}$$

$$\bar{x} = \frac{\Sigma \tilde{x} m}{\Sigma m} = \frac{18.604}{92.854} = 0.200 \text{ m}$$ **Ans.**

$$\Sigma \tilde{y} m = 2\,(4)\,(12) + 5.5\,(3)(8) + 7.5(1)\,(5) + \left(8 + \frac{2(1)}{\pi}\right)\left(\frac{\pi}{2}\right)(5)$$

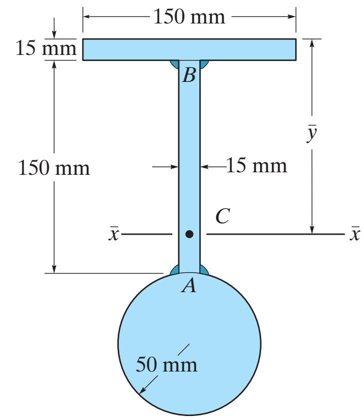
$$+ 9\,(1)\,(5) + 9(1.5)\,(2) = 405.332 \text{ kg} \cdot \text{m}$$

$$\bar{y} = \frac{\Sigma \tilde{y} m}{\Sigma m} = \frac{405.332}{92.854} = 4.37 \text{ m}$$ **Ans.**

Ans:
$\bar{x} = 0.200$ m
$\bar{y} = 4.37$ m

9–58.

Determine the location $\bar{y}$ of the centroidal axis $\bar{x}-\bar{x}$ of the beam's cross-sectional area. Neglect the size of the corner welds at A and B for the calculation.



SOLUTION

$$\Sigma \tilde{y}A = 7.5(15)(150) + 90(150)(15) + 215(\pi)(50)^2$$

$$= 1\,907\,981.05 \text{ mm}^2$$

$$\Sigma A = 15(150) + 150(15) + \pi(50)^2$$

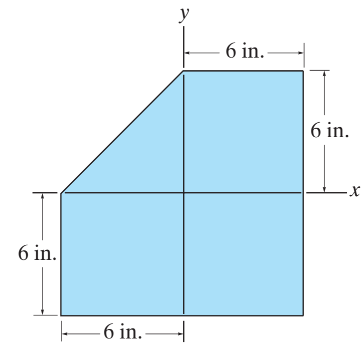
$$= 12\,353.98 \text{ mm}^2$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{1\,907\,981.05}{12\,353.98} = 154 \text{ mm}$$ **Ans.**

Ans:
$\bar{y} = 154$ mm

9–59.

Locate the centroid $(\bar{x}, \bar{y})$ of the shaded area.



SOLUTION

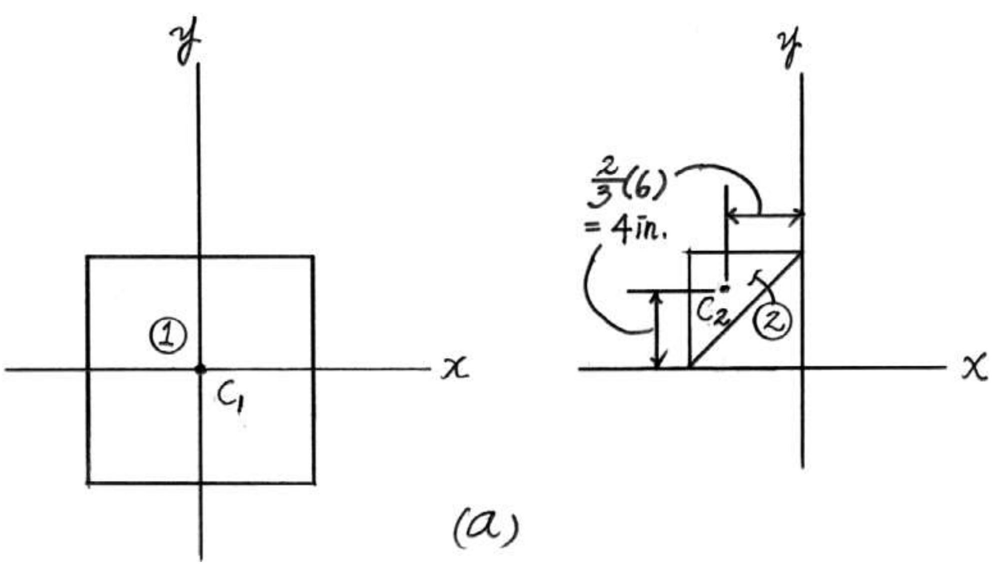
Centroid. Referring to Fiq. *a*, the areas of the segments and the locations of their respective centroids are tabulated below

Segment	A(in.^{2})	$\tilde{x}$(in.)	$\tilde{y}$(in.)	$\tilde{x}A$(in.^{3})	$\tilde{y}A$(in.^{3})
1	12(12)	0	0	0	0
2	$-\frac{1}{2}(6)(6)$	-4	4	72.0	-72.0
Σ	126			72.0	-72.0

Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{72.0 \text{ in.}^3}{126 \text{ in.}^2} = 0.5714 \text{ in.} = 0.571 \text{ in.}$$ **Ans.**

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{-72.0 \text{ in.}^3}{126 \text{ in.}^2} = -0.5714 \text{ in.} = -0.571 \text{ in.}$$ **Ans.**

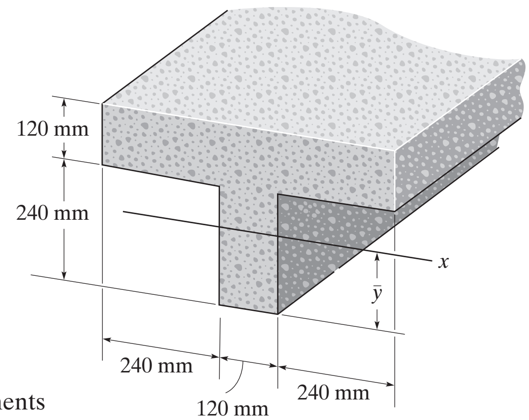


(a)

Ans:
$\bar{x} = 0.571$ in.
$\bar{y} = -0.571$ in.

***9–60.**

Locate the centroid $\bar{y}$ for the beam's cross-sectional area.

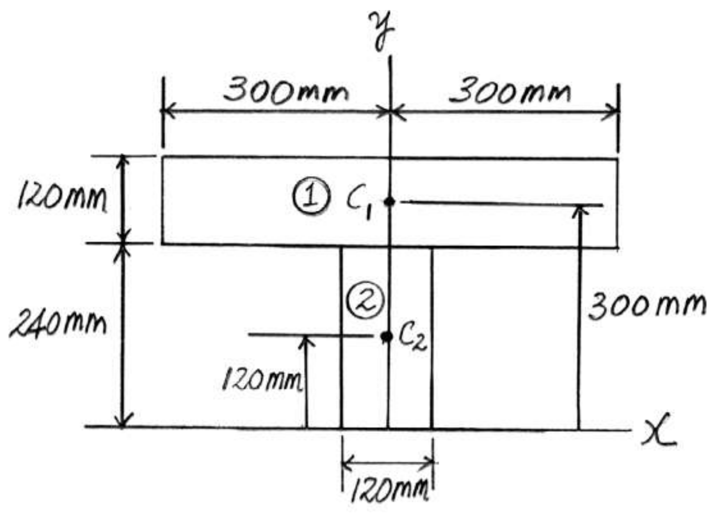


SOLUTION

Centroid. The locations of the centroids measuring from the x axis for segments ① and ② are indicated in Fig. a. Thus

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{300(120)(600) + 120(240)(120)}{120(600) + 240(120)}$$

$$= 248.57 \text{ mm} = 249 \text{ mm}$$

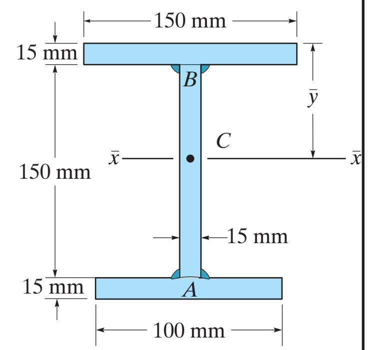


(a)

Ans:
$\bar{y} = 249$ mm

9–61.

Determine the location $\bar{y}$ of the centroid C of the beam having the cross-sectional area shown.

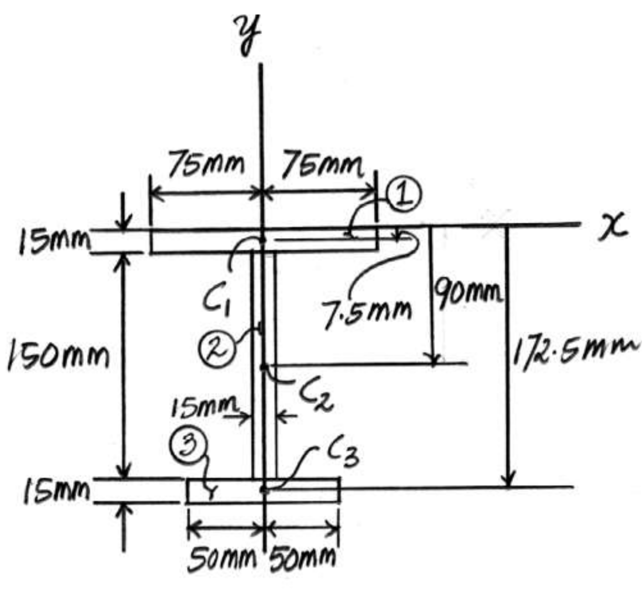


SOLUTION

Centroid. The locations of the centroids measuring from the x axis for segments ①, ② and ③ are indicated in Fig. a. Thus

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{7.5(15)(150) + 90(150)(15) + 172.5(15)(100)}{15(150) + 150(15) + 15(100)}$$

$$= 79.6875 \text{ mm} = 79.7 \text{ mm} \qquad \textbf{Ans.}$$

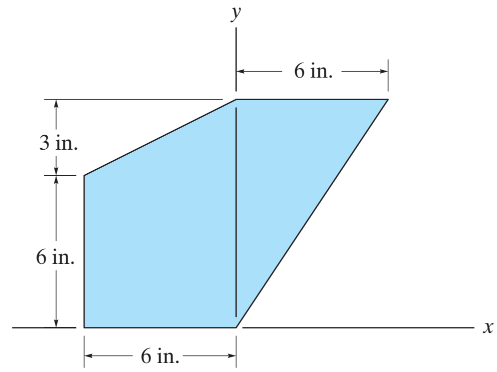


(a)

Ans:
$\bar{y} = 79.7$ mm

9–62.

Locate the centroid $(\bar{x}, \bar{y})$ of the shaded area.



SOLUTION

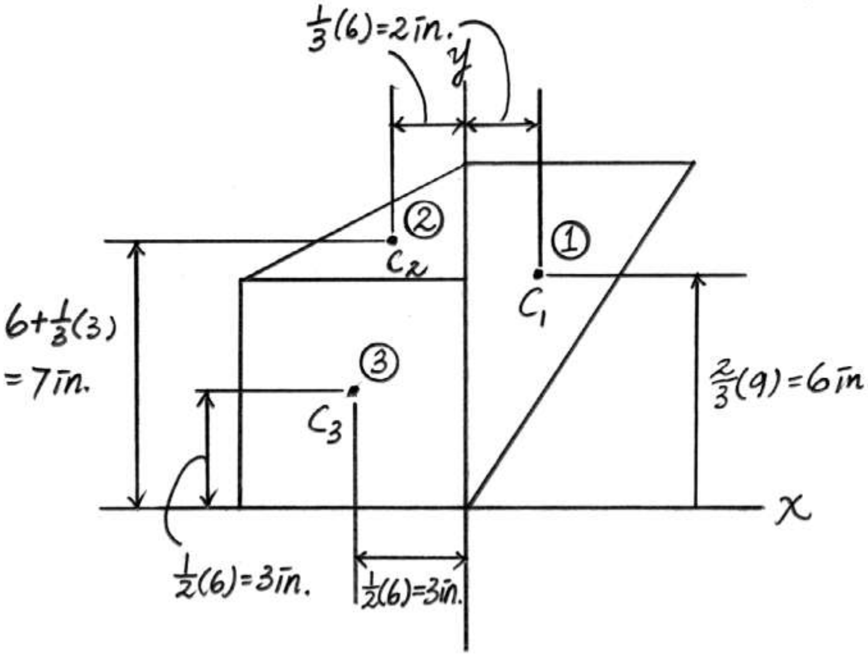
Centroid. Referring to Fig. *a*, the areas of the segments and the locations of their respective centroids are tabulated below

Segment	$A(\text{in.}^2)$	$\tilde{x}$ (in.)	$\tilde{y}$ (in.)	$\tilde{x}A(\text{in.}^3)$	$\tilde{y}A(\text{in.}^3)$
1	$\frac{1}{2}(6)(9)$	2	6	54.0	162.0
2	$\frac{1}{2}(6)(3)$	−2	7	−18.0	63.0
3	6(6)	−3	3	−108.0	108.00
Σ	72.0			−72.0	333.0

Thus,

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{-72.0\ \text{in.}^3}{72.0\ \text{in.}^2} = -1.00\ \text{in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{333.0\ \text{in.}^3}{72.0\ \text{in.}^2} = 4.625\ \text{in.} \qquad \textbf{Ans.}$$

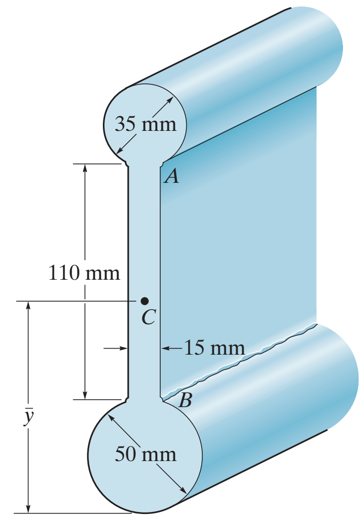


(a)

Ans:
$\bar{x} = -1.00$ in.
$\bar{y} = 4.625$ in.

9–63.

Determine the location $\bar{y}$ of the centroid of the beam's cross-sectional area. Neglect the size of the corner welds at A and B for the calculation.



SOLUTION

$$\Sigma\tilde{y}A = \pi(25)^2(25) + 15(110)(50 + 55) + \pi\left(\frac{35}{2}\right)^2\left(50 + 110 + \frac{35}{2}\right) = 393\,112\ \text{mm}^3$$

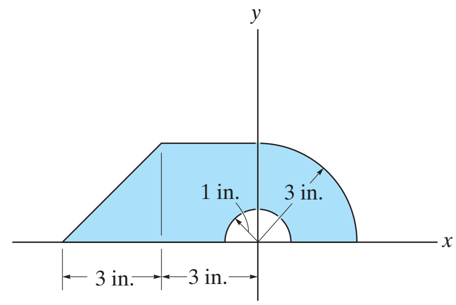
$$\Sigma A = \pi(25)^2 + 15(110) + \pi\left(\frac{35}{2}\right)^2 = 4575.6\ \text{mm}^2$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{393\,112}{4575.6} = 85.9\ \text{mm}$$ **Ans.**

Ans:
$\bar{y} = 85.9$ mm

***9–64.**

Locate the centroid $(\bar{x}, \bar{y})$ of the shaded area.



SOLUTION

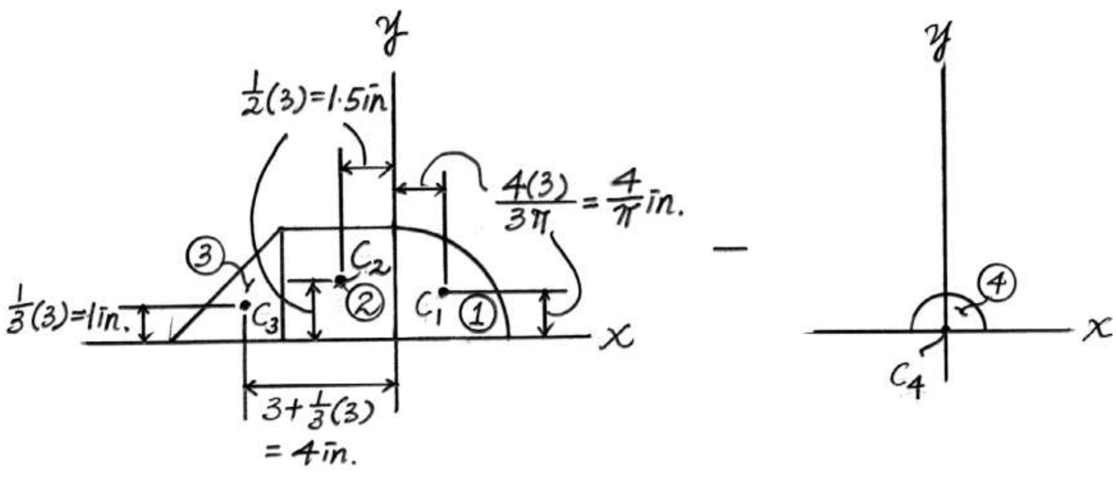
Centroid. Referring to Fig. *a*, the areas of the segments and the locations of their respective centroids are tabulated below

Segment	A(in.^{2})	$\tilde{x}$ (in.)	$\tilde{y}$ (in.)	$\tilde{x}A$(in.^{3})	$\tilde{y}A$(in.^{3})
1	$\frac{\pi}{4}(3^2)$	$\frac{4}{\pi}$	$\frac{4}{\pi}$	9.00	9.00
2	3(3)	−1.5	1.5	−13.50	13.50
3	$\frac{1}{2}(3)(3)$	−4	1	−18.00	4.50
4	$-\frac{\pi}{2}(1^2)$	0	$\frac{4}{3\pi}$	0	−0.67
Σ	18.9978			−22.50	26.33

Thus,

$$\bar{x} = \frac{\Sigma\tilde{x}A}{\Sigma A} = \frac{-22.50 \text{ in.}^3}{18.9978 \text{ in.}^2} = -1.1843 \text{ in.} = -1.18 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{26.33 \text{ in.}^3}{18.9978 \text{ in.}^2} = 1.3861 \text{ in.} = 1.39 \text{ in.} \qquad \textbf{Ans.}$$

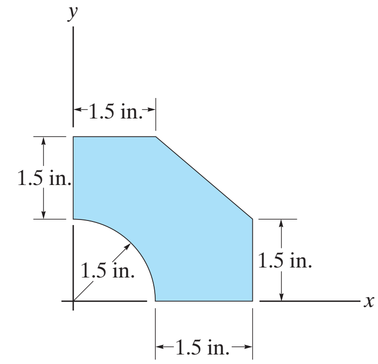


(a)

Ans:
$\bar{x} = -1.18$ in.
$\bar{y} = 1.39$ in.

9–65.

Determine the location $(\bar{x}, \bar{y})$ of the centroid C of the area.



SOLUTION

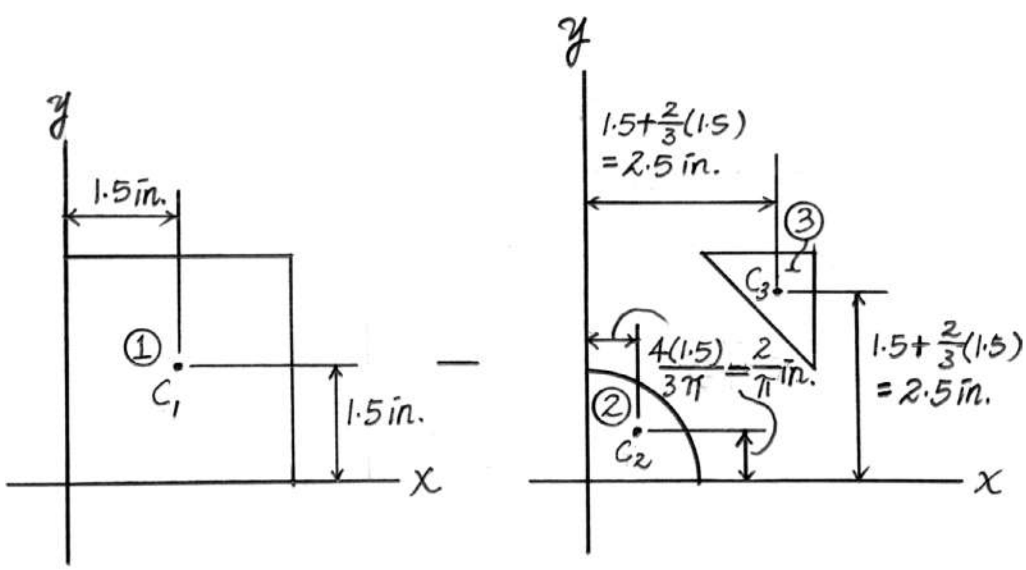
Centroid. Referring to Fig. *a*, the areas of the segments and the locations of their respective centroids are tabulated below.

Segment	A(in.^{2})	$\tilde{x}$ (in.)	$\tilde{y}$ (in.)	$\tilde{x}A$(in.^{3})	$\tilde{y}A$(in.^{3})
1	3(3)	1.5	1.5	13.5	13.5
2	$-\frac{\pi}{4}(1.5^2)$	$\frac{2}{\pi}$	$\frac{2}{\pi}$	-1.125	-1.125
3	$-\frac{1}{2}(1.5)(1.5)$	2.5	2.5	-2.8125	-2.8125
Σ	6.1079			9.5625	9.5625

Thus

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{9.5625 \text{ in.}^3}{6.1079 \text{ in.}^2} = 1.5656 \text{ in.} = 1.57 \text{ in.} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{9.5625 \text{ in.}^3}{6.1079 \text{ in.}^2} = 1.5656 \text{ in.} = 1.57 \text{ in.} \qquad \textbf{Ans.}$$

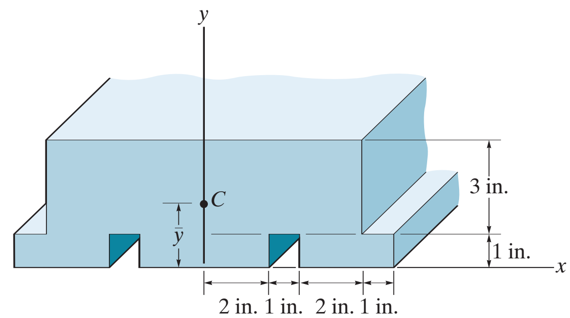


(a)

Ans:
$\bar{x} = 1.57$ in.
$\bar{y} = 1.57$ in.

9–66.

Determine the location $\bar{y}$ of the centroid C for a beam having the cross-sectional area shown. The beam is symmetric with respect to the y axis.



SOLUTION

$$\Sigma \tilde{y} A = 6(4)(2) - 1(1)(0.5) - 3(1)(2.5) = 40 \text{ in}^3$$

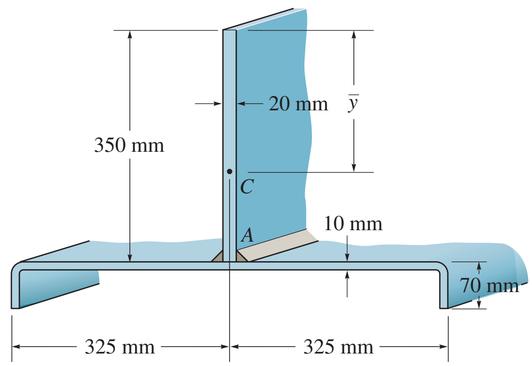
$$\Sigma A = 6(4) - 1(1) - 3(1) = 20 \text{ in}^2$$

$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{40}{20} = 2 \text{ in.}$$ **Ans.**

Ans:
$\bar{y} = 2$ in.

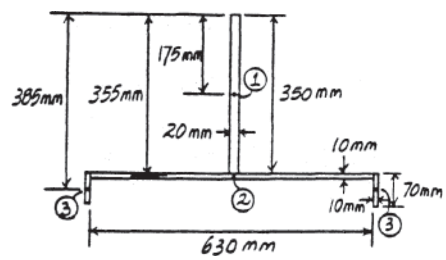
9–67.

Locate the centroid $\bar{y}$ of the cross-sectional area of the beam constructed from a channel and a plate. Assume all corners are square and neglect the size of the weld at A.



SOLUTION

Centroid: The area of each segment and its respective centroid are tabulated below.



Segment	A (mm^2)	$\tilde{y}$ (mm)	$\tilde{y}A$ (mm^3)
1	350(20)	175	1 225 000
2	630(10)	355	2 236 500
3	70(20)	385	539 000
Σ	14 700		4 000 500

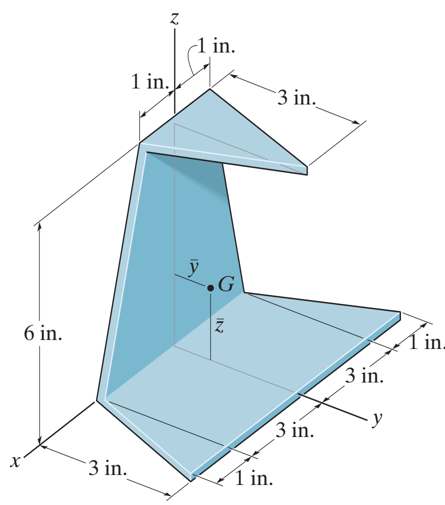
Thus,

$$\tilde{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{4\,000\,500}{14\,700} = 272.14 \text{ mm} = 272 \text{ mm}$$ **Ans.**

Ans:
$\bar{y} = 272 \text{ mm}$

***9–68.**

A triangular plate made of homogeneous material has a constant thickness which is very small. If it is folded over as shown, determine the location $\overline{y}$ of the plate's center of gravity G.



SOLUTION

$$\Sigma A = \frac{1}{2}(8)(12) = 48 \text{ in}^2$$

$$\Sigma \widetilde{y} A = 2(1)\left(\frac{1}{2}\right)(1)(3) + 1.5(6)(3) + 2(2)\left(\frac{1}{2}\right)(1)(3)$$

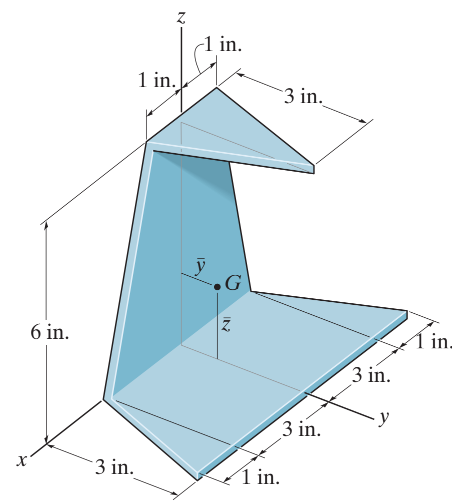
$$= 36 \text{ in}^3$$

$$\overline{y} = \frac{\Sigma \widetilde{y} A}{\Sigma A} = \frac{36}{48} = 0.75 \text{ in.}$$ **Ans.**

Ans:
$\overline{y} = 0.75$ in.

9–69.

A triangular plate made of homogeneous material has a constant thickness which is very small. If it is folded over as shown, determine the location $\bar{z}$ of the plate's center of gravity G.



SOLUTION

$$\Sigma A = \frac{1}{2}(8)(12) = 48 \text{ in}^2$$

$$\Sigma \tilde{z} A = 2(2)\left(\frac{1}{2}\right)(2)(6) + 3(6)(2) + 6\left(\frac{1}{2}\right)(2)(3)$$

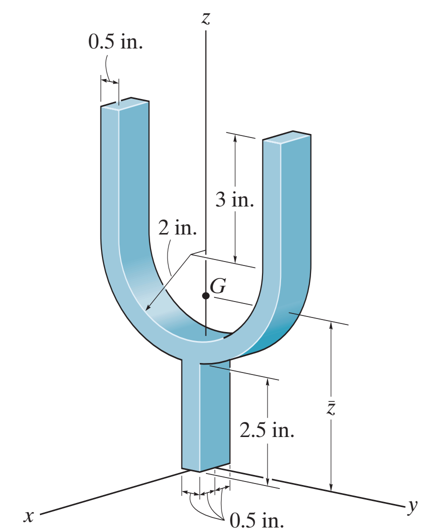
$$= 78 \text{ in}^3$$

$$\bar{z} = \frac{\Sigma \tilde{z} A}{\Sigma A} = \frac{78}{48} = 1.625 \text{ in.} \qquad \textbf{Ans.}$$

Ans:
$\bar{z} = 1.625$ in.

9–70.

Locate the center of mass $\bar{z}$ of the forked lever, which is made from a homogeneous material and has the dimensions shown.



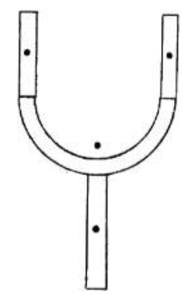
SOLUTION

$$\Sigma A = 2.5(0.5) + \left[\frac{1}{2}\pi(2.5)^2 - \frac{1}{2}\pi(2)^2\right] + 2[(3)(0.5)] = 7.7843 \text{ in}^2$$

$$\Sigma \tilde{z} A = \frac{2.5}{2}(2.5)(0.5) + \left(5 - \frac{4(2.5)}{3\pi}\right)\left(\frac{1}{2}\pi(2.5)^2\right)$$

$$-\left(5 - \frac{4(2)}{3\pi}\right)\left(\frac{1}{2}\pi(2)^2\right) + 6.5(2)(3)(0.5) = 33.651 \text{ in}^3$$

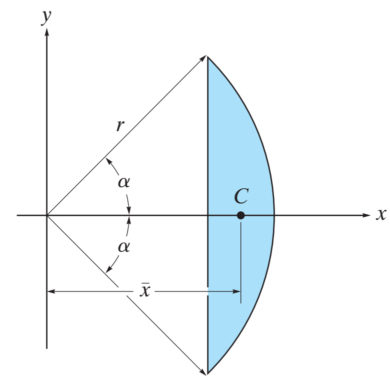
$$\bar{z} = \frac{\Sigma \tilde{z} A}{\Sigma A} = \frac{33.651}{7.7843} = 4.32 \text{ in.}$$ **Ans.**



Ans:
$\bar{z} = 4.32$ in.

9–71.

Determine the location $\bar{x}$ of the centroid C of the shaded area which is part of a circle having a radius r.



SOLUTION

Using symmetry, to simplify, consider just the top half:

$$\Sigma \tilde{x} A = \frac{1}{2} r^2 \alpha \left(\frac{2r}{3\alpha} \sin \alpha \right) - \frac{1}{2} (r \sin \alpha)(r \cos \alpha) \left(\frac{2}{3} r \cos \alpha \right)$$

$$= \frac{r^3}{3} \sin \alpha - \frac{r^3}{3} \sin \alpha \cos^2 \alpha$$

$$= \frac{r^3}{3} \sin^3 \alpha$$

$$\Sigma A = \frac{1}{2} r^2 \alpha - \frac{1}{2} (r \sin \alpha)(r \cos \alpha)$$

$$= \frac{1}{2} r^2 \left(\alpha - \frac{\sin 2\alpha}{2} \right)$$

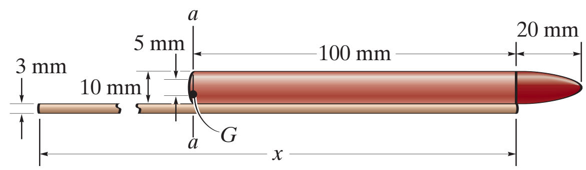
$$\bar{x} = \frac{\Sigma \tilde{x} A}{\Sigma A} = \frac{\frac{r^3}{3} \sin^3 \alpha}{\frac{1}{2} r^2 \left(\alpha - \frac{\sin 2\alpha}{2} \right)} = \frac{\frac{2}{3} r \sin^3 \alpha}{\alpha - \frac{\sin 2\alpha}{2}}$$ **Ans.**

Ans:

$$\bar{x} = \frac{\frac{2}{3} r \sin^3 \alpha}{\alpha - \frac{\sin 2\alpha}{2}}$$

***9–72.**

A toy skyrocket consists of a solid conical top, $\rho_t = 600\ \text{kg/m}^3$, a hollow cylinder, $\rho_c = 400\ \text{kg/m}^3$, and a stick having a circular cross section, $\rho_s = 300\ \text{kg/m}^3$. Determine the length of the stick, x, so that the center of gravity G of the skyrocket is located along line aa.



SOLUTION

$$\Sigma\widetilde{x}m = \left(\frac{20}{4}\right)\left[\left(\frac{1}{3}\right)\pi\,(5)^2(20)\right](600) - 50\left[\pi\left(5^2 - 2.5^2\right)(100)\right](400) - \frac{x}{2}\left[(x)\,\pi\,(1.5)^2\right](300)$$

$$= -116.24\left(10^6\right) - x^2(1060.29)\ \text{kg}\cdot\text{mm}^4/\text{m}^3$$

$$\Sigma m = \left[\frac{1}{3}\pi\,(5)^2\,(20)\right](600) + \pi\left(5^2 - 2.5^2\right)(100)(400) + \left[x\pi\,(1.5)^2\right](300)$$

$$= 2.670\left(10^6\right) + 2120.58x\ \text{kg}\cdot\text{mm}^3/\text{m}^3$$

$$\bar{x} = \frac{\Sigma\widetilde{x}m}{\Sigma m} = \frac{-116.24(10^6) - x^2(1060.29)}{2.670(10^6) + 2120.58x} = -100$$

$$-116.24\left(10^6\right) - x^2(1060.29) = -267.0\left(10^6\right) - 212.058\left(10^3\right)x$$

$$1060.29x^2 - 212.058\left(10^3\right)x - 150.80\left(10^6\right) = 0$$

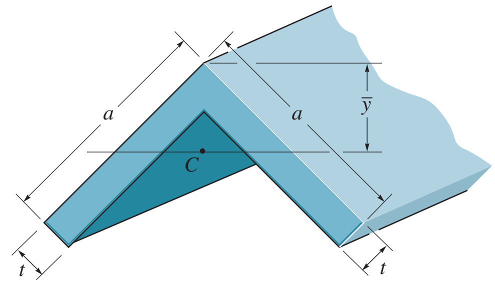
Solving for the positive root gives

$$x = 490\ \text{mm}$$ **Ans.**

Ans:
$x = 490$ mm

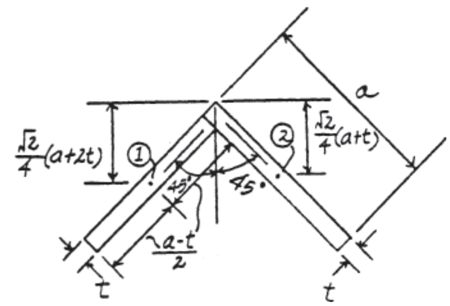
9–73.

Locate the centroid $\bar{y}$ for the cross-sectional area of the angle.



SOLUTION

Centroid : The area and the centroid for segments 1 and 2 are



$$A_1 = t(a - t)$$

$$\tilde{y}_1 = \left(\frac{a - t}{2} + \frac{t}{2}\right)\cos 45° + \frac{t}{2\cos 45°} = \frac{\sqrt{2}}{4}(a + 2t)$$

$$A_2 = at$$

$$\tilde{y}_2 = \left(\frac{a}{2} - \frac{t}{2}\right)\cos 45° + \frac{t}{2\cos 45°} = \frac{\sqrt{2}}{4}(a + t)$$

Listed in a tabular form, we have

Segment	***A***	$\tilde{y}$	$\tilde{y}A$
1	$t(a - t)$	$\frac{\sqrt{2}}{4}(a + 2t)$	$\frac{\sqrt{2}t}{4}(a^2 + at - 2t^2)$
2	at	$\frac{\sqrt{2}}{4}(a + t)$	$\frac{\sqrt{2}t}{4}(a^2 + at)$
Σ	$t(2a - t)$		$\frac{\sqrt{2}t}{2}(a^2 + at - t^2)$

Thus,

$$\bar{y} = \frac{\Sigma\tilde{y}A}{\Sigma A} = \frac{\frac{\sqrt{2}t}{2}(a^2 + at - t^2)}{t(2a - t)}$$

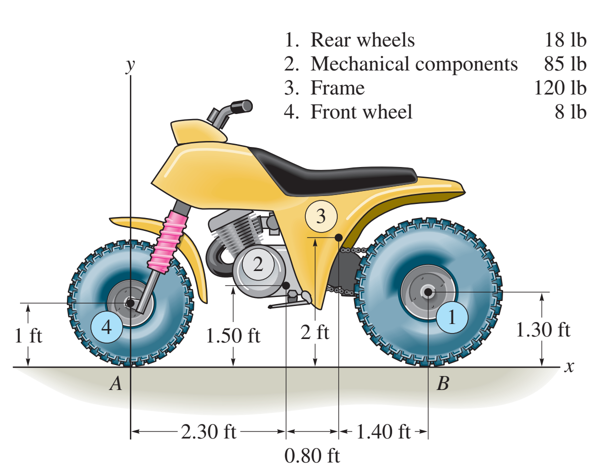
$$= \frac{\sqrt{2}(a^2 + at - t^2)}{2(2a - t)} \qquad \textbf{Ans.}$$

Ans:

$$\bar{y} = \frac{\sqrt{2}(a^2 + at - t^2)}{2(2a - t)}$$

9–74.

Determine the location $(\bar{x}, \bar{y})$ of the center of gravity of the three-wheeler. The location of the center of gravity of each component and its weight are tabulated in the figure. If the three-wheeler is symmetrical with respect to the x–y plane, determine the normal reaction each of its wheels exerts on the ground.



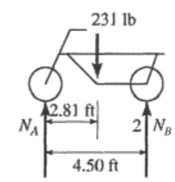
SOLUTION

$$\Sigma \tilde{x} W = 4.5(18) + 2.3(85) + 3.1(120)$$

$$= 648.5 \text{ lb} \cdot \text{ft}$$

$$\Sigma W = 18 + 85 + 120 + 8 = 231 \text{ lb}$$

$$\bar{x} = \frac{\Sigma \tilde{x} W}{\Sigma W} = \frac{648.5}{231} = 2.81 \text{ ft}$$ **Ans.**



$$\Sigma \tilde{y} W = 1.30(18) + 1.5(85) + 2(120) + 1(8)$$

$$= 398.9 \text{ lb} \cdot \text{ft}$$

$$\bar{y} = \frac{\Sigma \tilde{y} W}{\Sigma W} = \frac{398.9}{231} = 1.73 \text{ ft}$$ **Ans.**

$\circlearrowleft + \Sigma M_A = 0;$ $\quad 2(N_B)(4.5) - 231(2.81) = 0$

$$N_B = 72.1 \text{ lb}$$ **Ans.**

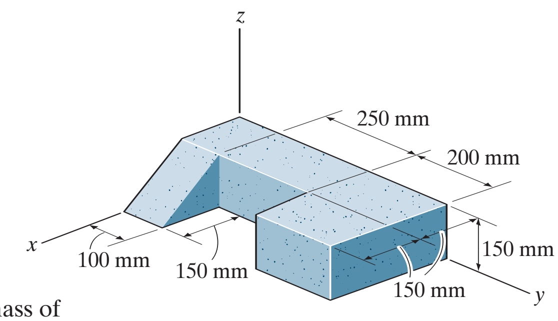
$+\uparrow \Sigma F_y = 0;$ $\quad N_A + 2(72.1) - 231 = 0$

$$N_A = 86.9 \text{ lb}$$ **Ans.**

Ans:
$\bar{x} = 2.81$ ft
$\bar{y} = 1.73$ ft
$N_B = 72.1$ lb
$N_A = 86.9$ lb

9–75.

Locate the center of mass $(\bar{x}, \bar{y}, \bar{z})$ of the homogeneous block assembly.



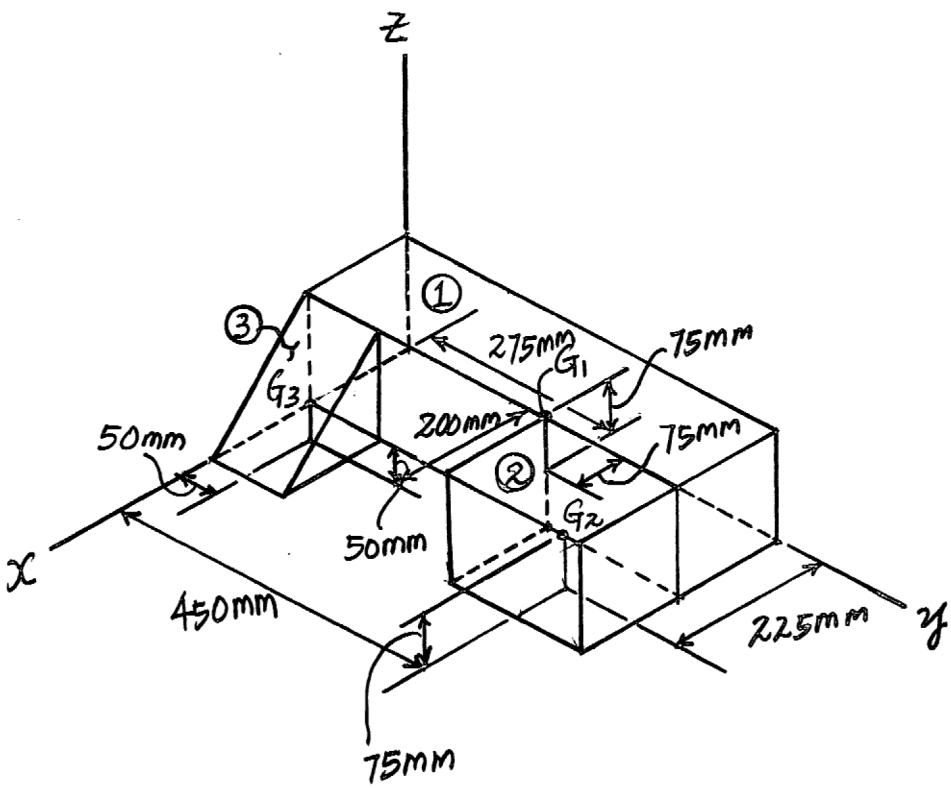
SOLUTION

Centroid: Since the block is made of a homogeneous material, the center of mass of the block coincides with the centroid of its volume. The centroid of each composite segment is shown in Fig. *a*.

$$\bar{x} = \frac{\Sigma \tilde{x}V}{\Sigma V} = \frac{(75)(150)(150)(550) + (225)(150)(150)(200) + (200)\left(\frac{1}{2}\right)(150)(150)(100)}{(150)(150)(550) + (150)(150)(200) + \frac{1}{2}(150)(150)(100)} = \frac{2.165625(10^9)}{18(10^6)} = 120 \text{ mm} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma \tilde{y}V}{\Sigma V} = \frac{(275)(150)(150)(550) + (450)(150)(150)(200) + (50)\left(\frac{1}{2}\right)(150)(150)(100)}{(150)(150)(550) + (150)(150)(200) + \frac{1}{2}(150)(150)(100)} = \frac{5.484375(10^9)}{18(10^6)} = 305 \text{ mm} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma \tilde{z}V}{\Sigma V} = \frac{(75)(150)(150)(550) + (75)(150)(150)(200) + (50)\left(\frac{1}{2}\right)(150)(150)(100)}{(150)(150)(550) + (150)(150)(200) + \frac{1}{2}(150)(150)(100)} = \frac{1.321875(10^9)}{18(10^6)} = 73.4 \text{ mm} \qquad \textbf{Ans.}$$

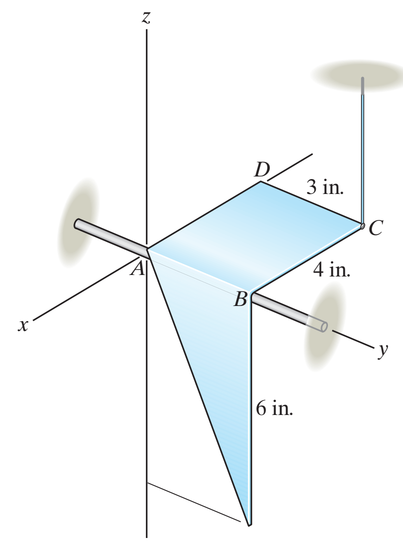


(a)

Ans:
$\bar{x} = 120$ mm
$\bar{y} = 305$ mm
$\bar{z} = 73.4$ mm

***9–76.**

The sheet metal part has the dimensions shown. Determine the location $(\bar{x}, \bar{y}, \bar{z})$ of its centroid.



SOLUTION

$$\Sigma A = 4(3) + \frac{1}{2}(3)(6) = 21 \text{ in}^2$$

$$\Sigma \tilde{x}A = -2(4)(3) + 0\left(\frac{1}{2}\right)(3)(6) = -24 \text{ in}^3$$

$$\Sigma \tilde{y}A = 1.5(4)(3) + \frac{2}{3}(3)\left(\frac{1}{2}\right)(3)(6) = 36 \text{ in}^3$$

$$\Sigma \tilde{z}A = 0(4)(3) - \frac{1}{3}(6)\left(\frac{1}{2}\right)(3)(6) = -18 \text{ in}^3$$

$$\bar{x} = \frac{\Sigma \tilde{x}A}{\Sigma A} = \frac{-24}{21} = -1.14 \text{ in.}$$ **Ans.**

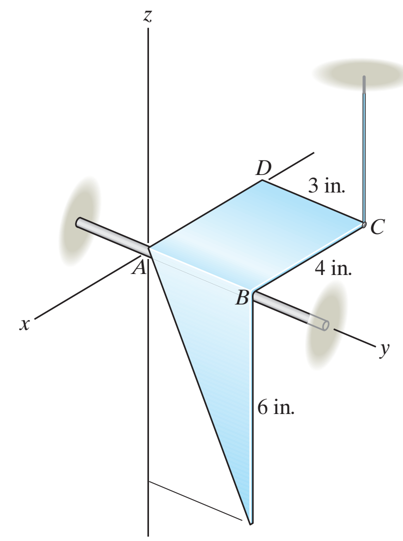
$$\bar{y} = \frac{\Sigma \tilde{y}A}{\Sigma A} = \frac{36}{21} = 1.71 \text{ in.}$$ **Ans.**

$$\bar{z} = \frac{\Sigma \tilde{z}A}{\Sigma A} = \frac{-18}{21} = -0.857 \text{ in.}$$ **Ans.**

Ans:
$\bar{x} = -1.14$ in.
$\bar{y} = 1.71$ in.
$\bar{z} = -0.857$ in.

9–77.

The sheet metal part has a weight per unit area of 2 lb/ft^2 and is supported by the smooth rod and at C. If the cord is cut, the part will rotate about the y axis until it reaches equilibrium. Determine the equilibrium angle of tilt, measured downward from the negative x axis, that AD makes with the $-x$ axis.

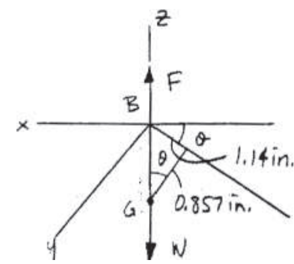


SOLUTION

Since the material is homogeneous, the center of gravity coincides with the centroid.

See solution to Prob. 9–74.

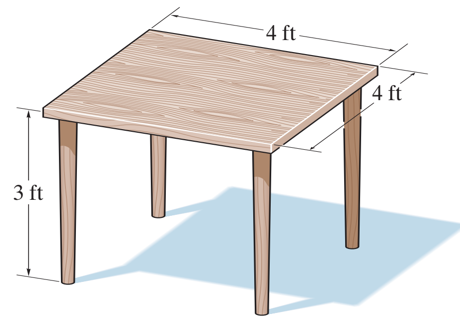
$$\theta = \tan^{-1}\left(\frac{1.14}{0.857}\right) = 53.1^\circ \qquad \textbf{Ans.}$$



Ans:
$\theta = 53.1^\circ$

9–78.

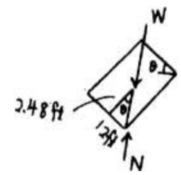
The wooden table is made from a square board having a weight of 15 lb. Each of the legs weighs 2 lb and is 3 ft long. Determine how high its center of gravity is from the floor. Also, what is the angle, measured from the horizontal, through which its top surface can be tilted on two of its legs before it begins to overturn? Neglect the thickness of each leg.



SOLUTION

$$\bar{z} = \frac{\Sigma \tilde{z} W}{\Sigma W} = \frac{15(3) + 4(2)(1.5)}{15 + 4(2)} = 2.48 \text{ ft}$$ **Ans.**

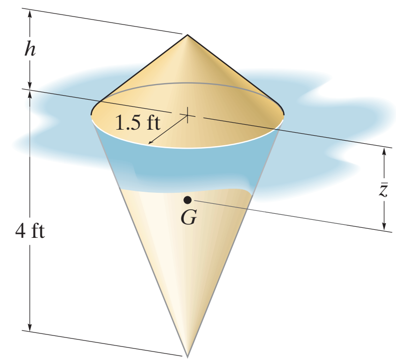
$$\theta = \tan^{-1}\left(\frac{2}{2.48}\right) = 38.9^\circ$$ **Ans.**



Ans:
$\bar{z} = 2.48$ ft
$\theta = 38.9^\circ$

9–79.

The buoy is made from two homogeneous cones each having a radius of 1.5 ft. If $h = 1.2$ ft, find the distance $\bar{z}$ to the buoy's center of gravity G.



SOLUTION

$$\Sigma \tilde{z}\, V = \frac{1}{3}\pi\,(1.5)^2(1.2)\left(-\frac{1.2}{4}\right) + \frac{1}{3}\pi(1.5)^2(4)\left(\frac{4}{4}\right)$$

$$= 8.577 \text{ ft}^4$$

$$\Sigma V = \frac{1}{3}\pi\,(1.5)^2(1.2) + \frac{1}{3}\pi(1.5)^2(4)$$

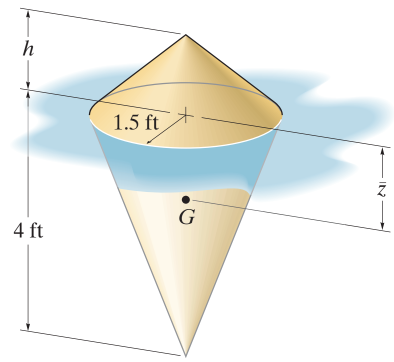
$$= 12.25 \text{ ft}^3$$

$$\bar{z} = \frac{\Sigma \tilde{z} V}{\Sigma V} = \frac{8.577}{12.25} = 0.70 \text{ ft}$$ **Ans.**

Ans:
$\bar{z} = 0.70$ ft

***9–80.**

The buoy is made from two homogeneous cones each having a radius of 1.5 ft. If it is required that the buoy's center of gravity G be located at $\bar{z} = 0.5$ ft, determine the height h of the top cone.



SOLUTION

$$\Sigma\tilde{z}V = \frac{1}{3}\pi(1.5)^2(h)\left(-\frac{h}{4}\right) + \frac{1}{3}\pi(1.5)^2(4)\left(\frac{4}{4}\right)$$

$$= -0.5890\,h^2 + 9.4248$$

$$\Sigma V = \frac{1}{3}\pi(1.5)^2(h) + \frac{1}{3}\pi(1.5)^2(4)$$

$$= 2.3562\,h + 9.4248$$

$$\tilde{z} = \frac{\Sigma\tilde{z}V}{\Sigma V} = \frac{-0.5890\,h^2 + 9.4248}{2.3562\,h + 9.4248} = 0.5$$

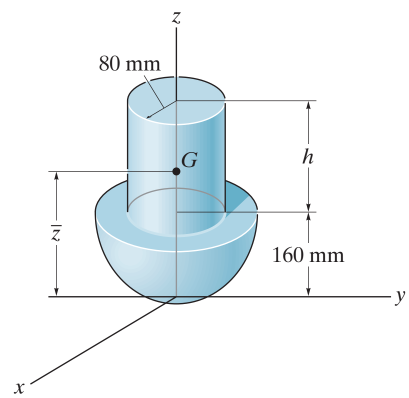
$$-0.5890\,h^2 + 9.4248 = 1.1781\,h + 4.7124$$

$$h = 2.00 \text{ ft}$$ **Ans.**

Ans:
$h = 2.00$ ft

9–81.

The assembly is made from a steel hemisphere, $\rho_{st} = 7.80\ \text{Mg/m}^3$, and an aluminum cylinder, $\rho_{al} = 2.70\ \text{Mg/m}^3$. Determine the mass center of the assembly if the height of the cylinder is $h = 200$ mm.



SOLUTION

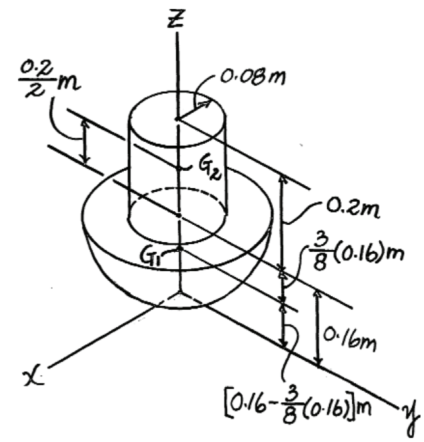
$$\Sigma\,\bar{z}m = \left[0.160 - \tfrac{3}{8}(0.160)\right]\left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \left(0.160 + \tfrac{0.2}{2}\right)\pi(0.2)(0.08)^2(2.70)$$

$$= 9.51425(10^{-3})\ \text{Mg}\cdot\text{m}$$

$$\Sigma m = \left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \pi(0.2)(0.08)^2(2.70)$$

$$= 77.7706(10^{-3})\ \text{Mg}$$

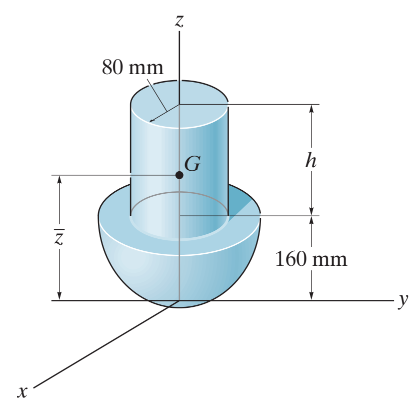
$$\bar{z} = \frac{\Sigma\,\bar{z}m}{\Sigma m} = \frac{9.51425(10^{-3})}{77.7706(10^{-3})} = 0.122\ \text{m} = 122\ \text{mm}$$ **Ans.**



Ans:
$\bar{z} = 122$ mm

9–82.

The assembly is made from a steel hemisphere, $\rho_{st} = 7.80\ \text{Mg/m}^3$, and an aluminum cylinder, $\rho_{al} = 2.70\ \text{Mg/m}^3$. Determine the height h of the cylinder so that the mass center of the assembly is located at $\bar{z} = 160$ mm.



SOLUTION

$$\Sigma\,\bar{z}m = \left[0.160 - \tfrac{3}{8}(0.160)\right]\left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \left(0.160 + \tfrac{h}{2}\right)\pi(h)(0.08)^2(2.70)$$

$$= 6.691(10^{-3}) + 8.686(10^{-3})\,h + 27.143(10^{-3})\,h^2$$

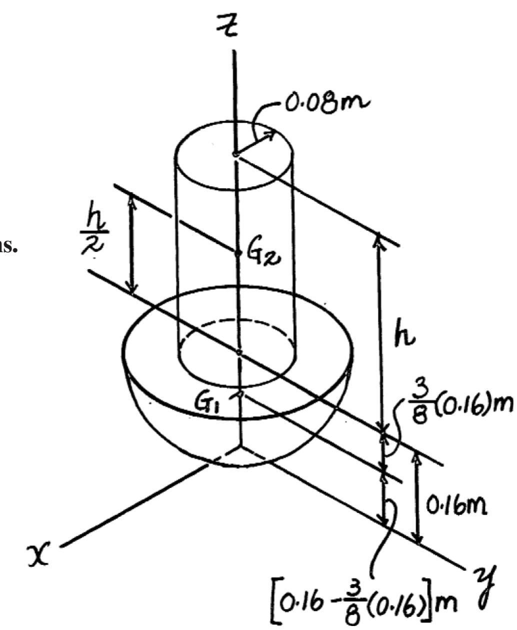
$$\Sigma m = \left(\tfrac{2}{3}\right)\pi(0.160)^3(7.80) + \pi(h)(0.08)^2(2.70)$$

$$= 66.91(10^{-3}) + 54.29(10^{-3})\,h$$

$$\bar{z} = \frac{\Sigma\,\bar{z}m}{\Sigma m} = \frac{6.691(10^{-3}) + 8.686(10^{-3})\,h + 27.143(10^{-3})\,h^2}{66.91(10^{-3}) + 54.29(10^{-3})\,h} = 0.160$$

Solving

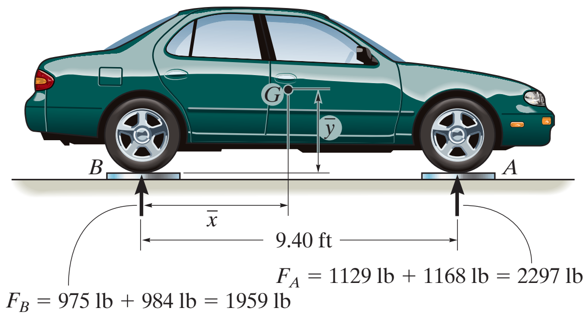
$$h = 0.385\ \text{m} = 385\ \text{mm}$$ **Ans.**

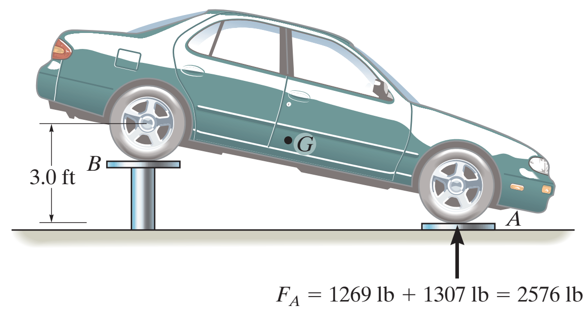


Ans:
$h = 385$ mm

9–83.

The car rests on four scales and in this position the scale readings of both the front and rear tires are shown by F_A and F_B. When the rear wheels are elevated to a height of 3 ft above the front scales, the new readings of the front wheels are also recorded. Use this data to compute the location $\bar{x}$ and $\bar{y}$ to the center of gravity G of the car. The tires each have a diameter of 1.98 ft.





SOLUTION

In horizontal position

$$W = 1959 + 2297 = 4256 \text{ lb}$$

$$\zeta + \Sigma M_B = 0; \qquad 2297(9.40) - 4256\bar{x} = 0$$

$$\bar{x} = 5.0733 = 5.07 \text{ ft} \qquad \textbf{Ans.}$$

$$\theta = \sin^{-1}\left(\frac{3 - 0.990}{9.40}\right) = 12.347°$$

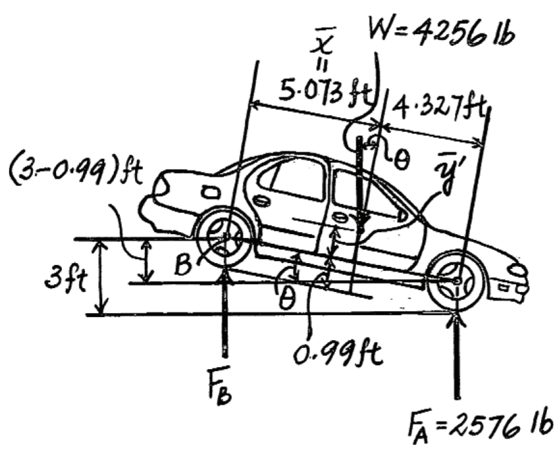
With rear whells elevated

$$\zeta + \Sigma M_B = 0; \qquad 2576(9.40 \cos 12.347°) - 4256 \cos 12.347°(5.0733)$$

$$- 4256 \sin 12.347° \, \bar{y}' = 0$$

$$\bar{y}' = 2.86 \text{ ft}$$

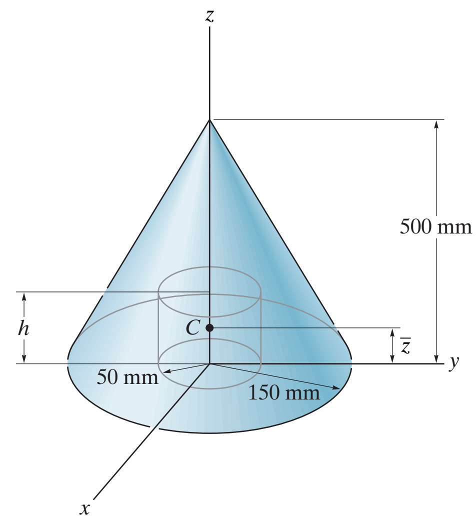
$$\bar{y} = 2.815 + 0.990 = 3.80 \text{ ft} \qquad \textbf{Ans.}$$



Ans:
$\bar{x} = 5.07$ ft
$\bar{y} = 3.80$ ft

***9–84.**

Determine the distance h to which a 100-mm diameter hole must be bored into the base of the cone so that the center of mass of the resulting shape is located at $\bar{z} = 115$ mm. The material has a density of 8 Mg/m^3.



$$\frac{\frac{1}{3}\pi(0.15)^2(0.5)\left(\frac{0.5}{4}\right) - \pi(0.05)^2(h)\left(\frac{h}{2}\right)}{\frac{1}{3}\pi(0.15)^2(0.5) - \pi(0.05)^2(h)} = 0.115$$

$$0.4313 - 0.2875\,h = 0.4688 - 1.25\,h^2$$

$$h^2 - 0.230\,h - 0.0300 = 0$$

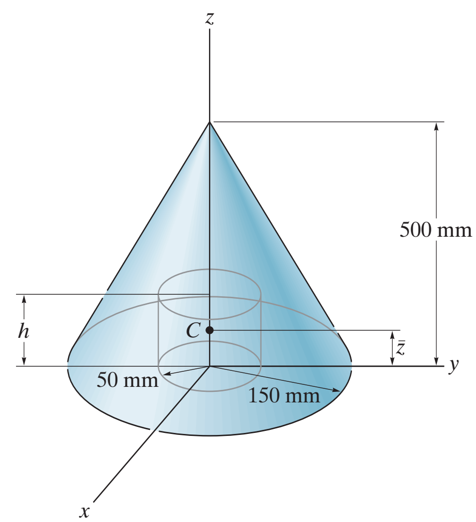
Choosing the positive root,

$$h = 323 \text{ mm}$$ **Ans.**

Ans:
$h = 323$ mm

9–85.

Determine the distance $\bar{z}$ to the centroid of the shape which consists of a cone with a hole of height $h = 50$ mm bored into its base.



SOLUTION

$$\Sigma \tilde{z} V = \frac{1}{3}\pi\,(0.15)^2(0.5)\left(\frac{0.5}{4}\right) - \pi\,(0.05)^2(0.05)\left(\frac{0.05}{2}\right)$$

$$= 1.463(10^{-3})\text{ m}^4$$

$$\Sigma V = \frac{1}{3}\pi\,(0.15)^2(0.5) - \pi\,(0.05)^2(0.05)$$

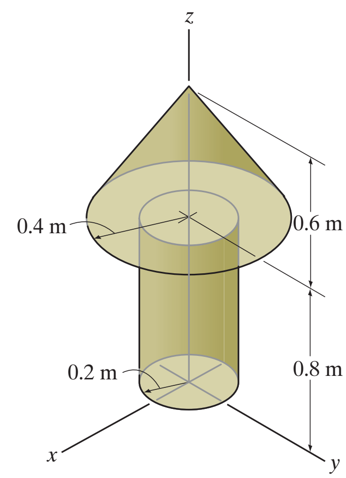
$$= 0.01139\text{ m}^3$$

$$\bar{z} = \frac{\Sigma \tilde{z}\,V}{\Sigma V} = \frac{1.463\,(10^{-3})}{0.01139} = 0.12845\text{ m} = 128\text{ mm}$$ **Ans.**

Ans:
$\bar{z} = 128$ mm

9–86.

Locate the center of mass $\bar{z}$ of the assembly. The cylinder and the cone are made from materials having densities of 5 Mg/m^3 and 9 Mg/m^3, respectively.

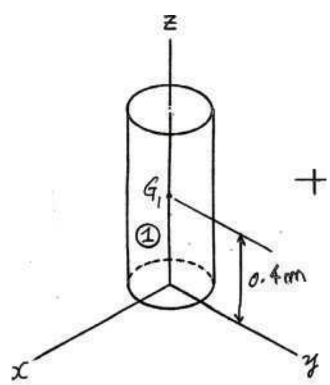


SOLUTION

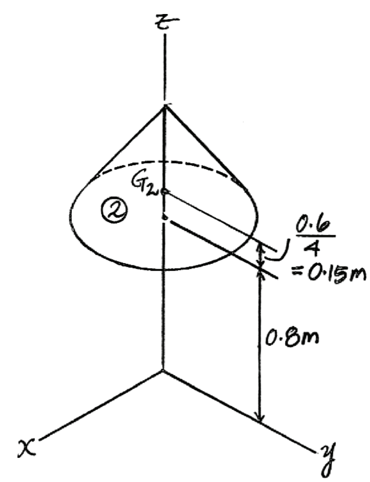
Center of mass: The assembly is broken into two composite segments, as shown in Figs. *a* and *b*.

$$\bar{z} = \frac{\Sigma \tilde{z} m}{\Sigma m} = \frac{5000(0.4)\left[\pi(0.2^2)(0.8)\right] + 9000(0.8 + 0.15)\left[\frac{1}{3}\pi(0.4^2)(0.6)\right]}{5000\left[\pi(0.2^2)(0.8)\right] + 9000\left[\frac{1}{3}\pi(0.4^2)(0.6)\right]}$$

$$= \frac{1060.60}{1407.4} = 0.754 \text{ m} = 754 \text{ mm} \qquad \textbf{Ans.}$$



(a)

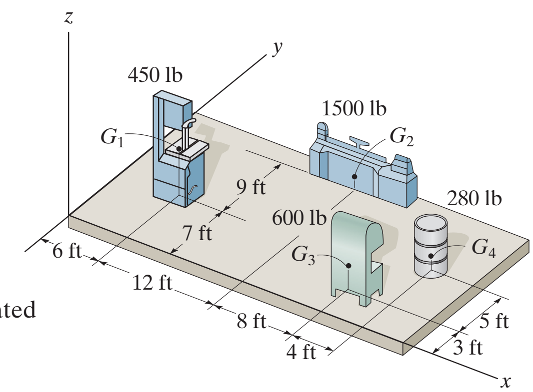


(b)

Ans:
$\bar{z} = 754$ mm

9–87.

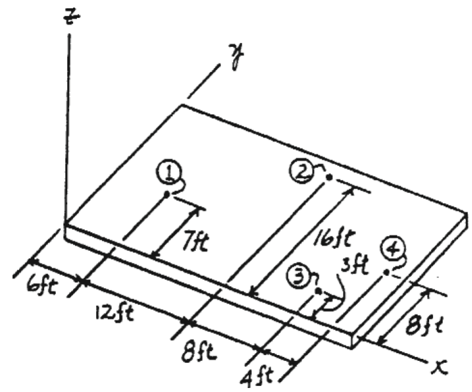
Major floor loadings in a shop are caused by the weights of the objects shown. Each force acts through its respective center of gravity G. Locate the center of gravity $(\bar{x}, \bar{y})$ of all these components.



SOLUTION

Centroid: The floor loadings on the floor and its respective centroid are tabulated below.

Loading	***W*** **(lb)**	$\bar{x}$ **(ft)**	$\bar{y}$ **(ft)**	$\bar{x}W$ **(lb · ft)**	$\bar{y}W$ **(lb · ft)**
1	450	6	7	2700	3150
2	1500	18	16	27000	24000
3	600	26	3	15600	1800
4	280	30	8	8400	2240
Σ	2830			53700	31190



Thus,

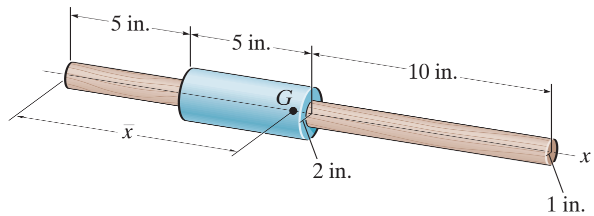
$$\bar{x} = \frac{\Sigma\bar{x}W}{\Sigma W} = \frac{53700}{2830} = 18.98 \text{ ft} = 19.0 \text{ ft} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\Sigma\bar{y}W}{\Sigma W} = \frac{31190}{2830} = 11.02 \text{ ft} = 11.0 \text{ ft} \qquad \textbf{Ans.}$$

Ans:
$\bar{x} = 19.0$ ft
$\bar{y} = 11.0$ ft

***9–88.**

The assembly consists of a 20-in. wooden dowel rod and a tight-fitting steel collar. Determine the distance $\bar{x}$ to its center of gravity if the specific weights of the materials are $\gamma_w = 150\ \text{lb/ft}^3$ and $\gamma_{st} = 490\ \text{lb/ft}^3$. The radii of the dowel and collar are shown.



SOLUTION

$$\Sigma \bar{x} W = \left\{10\pi(1)^2(20)(150) + 7.5\pi(5)(2^2 - 1^2)(490)\right\}\frac{1}{(12)^3}$$

$$= 154.8\ \text{lb}\cdot\text{in.}$$

$$\Sigma W = \left\{\pi(1)^2(20)(150) + \pi(5)(2^2 - 1^2)(490)\right\}\frac{1}{(12)^3}$$

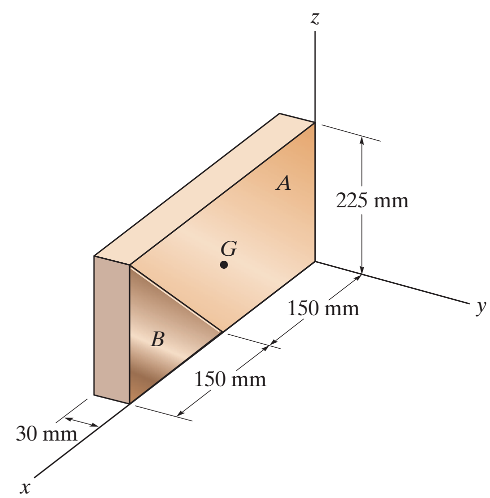
$$= 18.82\ \text{lb}$$

$$\bar{x} = \frac{\Sigma \bar{x} W}{\Sigma W} = \frac{154.8}{18.82} = 8.22\ \text{in.}$$ **Ans.**

Ans:
$\bar{x} = 8.22$ in.

9–89.

The composite plate is made from both steel (A) and brass (B) segments. Determine the mass and location $(\bar{x}, \bar{y}, \bar{z})$ of its mass center G. Take $\rho_{st} = 7.85\ \text{Mg/m}^3$ and $\rho_{br} = 8.74\ \text{Mg/m}^3$.



SOLUTION

$$\Sigma m = \Sigma \rho V = \left[8.74\left(\frac{1}{2}(0.15)(0.225)(0.03)\right)\right] + \left[7.85\left(\frac{1}{2}(0.15)(0.225)(0.03)\right)\right]$$

$$+ [7.85(0.15)(0.225)(0.03)]$$

$$= \left[4.4246\left(10^{-3}\right)\right] + \left[3.9741\left(10^{-3}\right)\right] + \left[7.9481\left(10^{-3}\right)\right]$$

$$= 16.347\left(10^{-3}\right) = 16.4 \text{ kg} \qquad \textbf{Ans.}$$

$$\Sigma \bar{x} m = \left(0.150 + \frac{2}{3}(0.150)\right)(4.4246)\left(10^{-3}\right) + \left(0.150 + \frac{1}{3}(0.150)\right)(3.9741)\left(10^{-3}\right)$$

$$+ \frac{1}{2}(0.150)(7.9481)(10^{-3}) = 2.4971(10^{-3}) \text{ kg}\cdot\text{m}$$

$$\Sigma \bar{z} m = \left(\frac{1}{3}(0.225)\right)(4.4246)\left(10^{-3}\right) + \left(\frac{2}{3}(0.225)\right)(3.9471)\left(10^{-3}\right) + \left(\frac{0.225}{2}\right)(7.9481)\left(10^{-3}\right)$$

$$= 1.8221\left(10^{-3}\right)\text{kg}\cdot\text{ m}$$

$$\bar{x} = \frac{\Sigma \bar{x} m}{\Sigma m} = \frac{2.4971(10^{-3})}{16.347(10^{-3})} = 0.153 \text{ m} = 153 \text{ mm} \qquad \textbf{Ans.}$$

Due to symmetry:

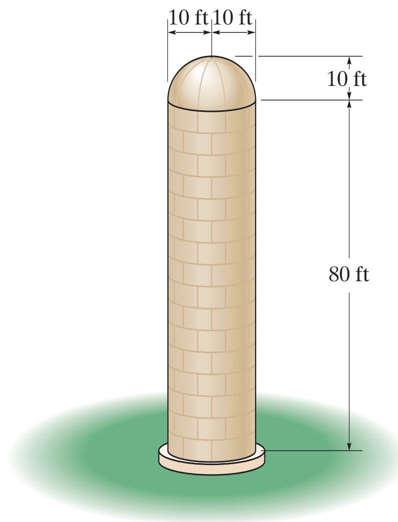
$$\bar{y} = -15 \text{ mm} \qquad \textbf{Ans.}$$

$$\bar{z} = \frac{\Sigma \bar{z} m}{\Sigma m} = \frac{1.8221(10^{-3})}{16.347(10^{-3})} = 0.1115 \text{ m} = 111 \text{ mm} \qquad \textbf{Ans.}$$

Ans:
$\Sigma m = 16.4$ kg
$\bar{x} = 153$ mm
$\bar{y} = -15$ mm
$\bar{z} = 111$ mm

9–90.

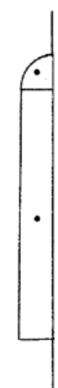
Determine the volume of the silo which consists of a cylinder and hemispherical cap. Neglect the thickness of the plates.



SOLUTION

$$V = \Sigma\theta\,\bar{r}\,A = 2\pi\left[\frac{4(10)}{3\pi}\left(\frac{1}{4}\right)\pi\,(10)^2 + 5(80)(10)\right]$$

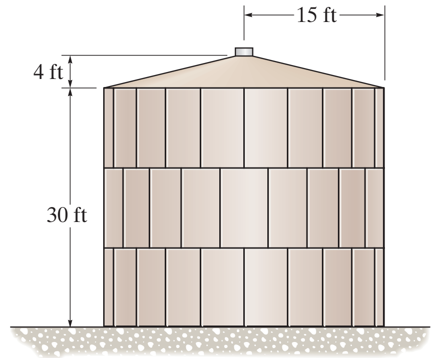
$$= 27.2\left(10^3\right)\text{ft}^3 \qquad \textbf{Ans.}$$



Ans:
$V = 27.2\left(10^3\right)\text{ft}^3$

9–91.

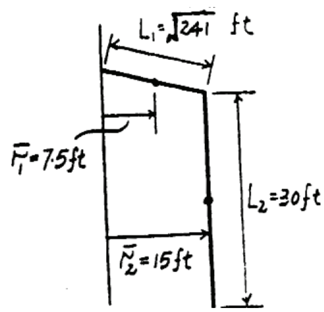
Determine the outside surface area of the storage tank.



SOLUTION

Surface Area: Applying the theorem of Pappus and Guldinus, Eq.9–7. with $\theta = 2\pi$, $L_1 = \sqrt{15^2 + 4^2} = \sqrt{241}$ ft, $L_2 = 30$ ft, $\bar{r}_1 = 7.5$ ft and $\bar{r}_2 = 15$ ft, we have

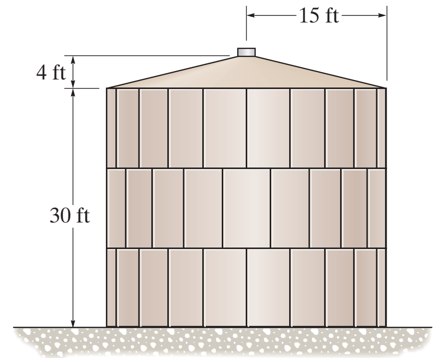
$$A = \theta \Sigma \tilde{r} L = 2\pi\left[7.5\left(\sqrt{241}\right) + 15(30)\right] = 3.56\left(10^3\right)\ \text{ft}^2$$ **Ans.**



Ans:
$A = 3.56\left(10^3\right)\ \text{ft}^2$

***9–92.**

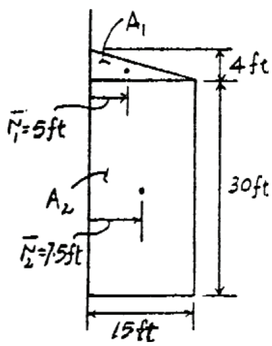
Determine the volume of the storage tank.



SOLUTION

Volume: Applying the theorem of Pappus and Guldinus, Eq. 9–8 with $\theta = 2\pi$, $\bar{r}_1 = 5\text{ ft}, \bar{r}_2 = 7.5\text{ ft}, A_1 = \frac{1}{2}(15)(4) = 30.0\text{ ft}^2$ and $A_2 = 30(15) = 450\text{ ft}^2$, we have

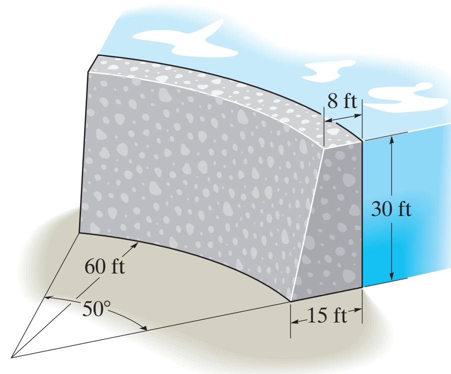
$$V = \theta \Sigma \bar{r} A = 2\pi[5(30.0) + 7.5(450)] = 22.1\left(10^3\right)\text{ ft}^3 \qquad \textbf{Ans.}$$



Ans:
$V = 22.1\left(10^3\right)\text{ ft}^3$

9–93.

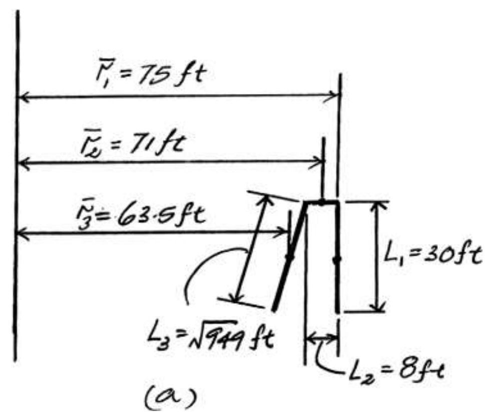
Determine the surface area of the concrete sea wall, excluding its bottom.



SOLUTION

Surface Area: Applying Theorem of Pappus and Guldinus, Eq. 9–9 with $\theta = \left(\dfrac{50}{180}\right)\pi = \dfrac{5}{18}\pi$ rad, $L_1 = 30$ ft, $L_2 = 8$ ft, $L_3 = \sqrt{7^2 + 30^2} = \sqrt{949}$ ft, $\overline{N}_1 = 75$ ft, $\overline{N}_2 = 71$ ft and $\overline{N}_3 = 63.5$ ft as indicated in Fig. *a*,

$$A_1 = \theta \Sigma \overline{N} L = \frac{5}{18}\pi\left[75(30) + 71(8) + 63.5(\sqrt{949})\right]$$
$$= 4166.25 \text{ ft}^2$$



The surface area of two sides of the wall is

$$A_2 = 2\left[\frac{1}{2}(8 + 15)(30)\right] = 690 \text{ ft}^2$$

Thus the total surface area is

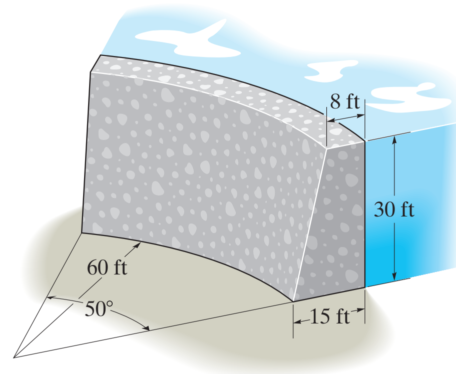
$$A = A_1 + A_2 = 4166.25 + 690$$
$$= 4856.25 \text{ ft}^2$$
$$= 4856 \text{ ft}^2$$

Ans.

Ans:
$A = 4856 \text{ ft}^2$

9–94.

A circular sea wall is made of concrete. Determine the total weight of the wall if the concrete has a specific weight of $\gamma_c = 150 \text{ lb/ft}^3$.

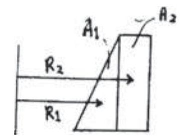


SOLUTION

$$V = \Sigma\theta\tilde{r}A = \left(\frac{50°}{180°}\right)\pi\left[\left(60 + \frac{2}{3}(7)\right)\left(\frac{1}{2}\right)(30)(7) + 71(30)(8)\right]$$

$$= 20\,795.6 \text{ ft}^3$$

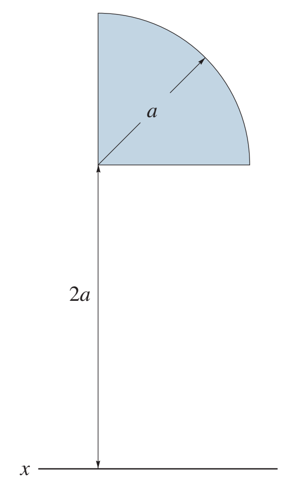
$$W = \gamma V = 150(20\,795.6) = 3.12(10^6) \text{ lb}$$ **Ans.**



Ans:
$W = 3.12(10^6)$ lb

9–95.

A ring is generated by rotating the quartercircular area about the x axis. Determine its volume.



SOLUTION

Volume: Applying the theorem of Pappus and Guldinus, Eq. 9–10, with $\theta = 2\pi$, $\bar{r} = 2a + \dfrac{4a}{3\pi} = \dfrac{6\pi + 4}{3\pi}a$ and $A = \dfrac{\pi}{4}a^2$, we have

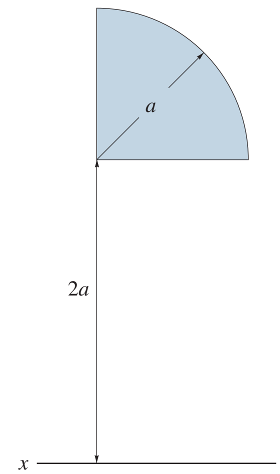
$$V = \theta \bar{r} A = 2\pi\left(\frac{6\pi + 4}{3\pi}a\right)\left(\frac{\pi}{4}a^2\right) = \frac{\pi(6\pi + 4)}{6}a^3 \qquad \textbf{Ans.}$$

Ans:

$$V = \frac{\pi(6\pi + 4)}{6}a^3$$

***9–96.**

A ring is generated by rotating the quartercircular area about the x axis. Determine its surface area.



SOLUTION

Surface Area: Applying the theorem of Pappus and Guldinus, Eq. 9–11, with $\theta = 2\pi$, $L_1 = L_3 = a, L_2 = \dfrac{\pi a}{2}, \bar{r}_1 = 2a, \bar{r}_2 = \dfrac{2(\pi + 1)}{\pi}a$ and $\bar{r}_3 = \dfrac{5}{2}a$, we have

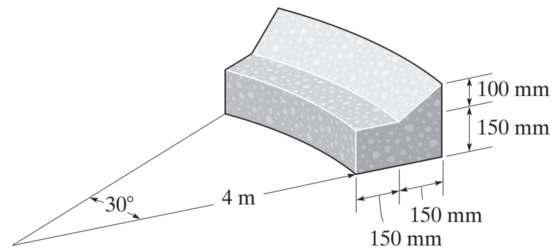
$$A = \theta\Sigma\bar{r}L = 2\pi\left[2a(a) + \left(\frac{2(\pi + 1)}{\pi}a\right)\left(\frac{\pi a}{2}\right) + \frac{5}{2}a\,(a)\right]$$

$$= \pi(2\pi + 11)a^2 \qquad \textbf{Ans.}$$

Ans:
$A = \pi(2\pi + 11)a^2$

9–97.

Determine the volume of concrete needed to construct the curb.



SOLUTION

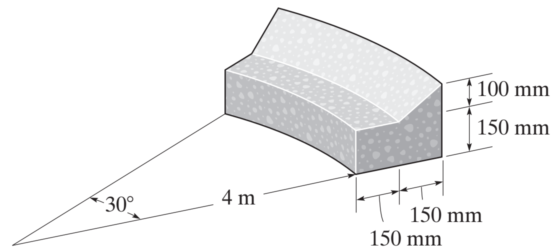
$$V = \Sigma\theta A\,\bar{r} = \left(\frac{\pi}{6}\right)[(0.15)(0.3)(4.15)] + \left(\frac{\pi}{6}\right)\left[\left(\frac{1}{2}\right)(0.15)(0.1)(4.25)\right]$$

$V = 0.114\ \text{m}^3$ **Ans.**

Ans:
$V = 0.114\ \text{m}^3$

9–98.

Determine the surface area of the curb. Do not include the area of the ends in the calculation.



SOLUTION

$$A = \Sigma\, \theta\, \bar{r}\, L = \frac{\pi}{6}\{4(0.15) + 4.075(0.15) + (4.15 + 0.075)\left(\sqrt{0.15^2 + 0.1^2}\right)$$

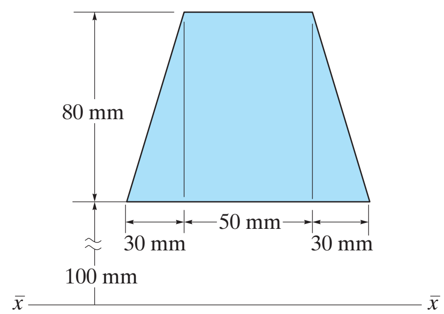
$$+ 4.3(0.25) + 4.15(0.3)\}$$

$A = 2.25\ \text{m}^2$ **Ans.**

Ans:
$A = 2.25\ \text{m}^2$

9–99.

A ring is formed by rotating the area 360° about the $\bar{x}-\bar{x}$ axes. Determine its surface area.



SOLUTION

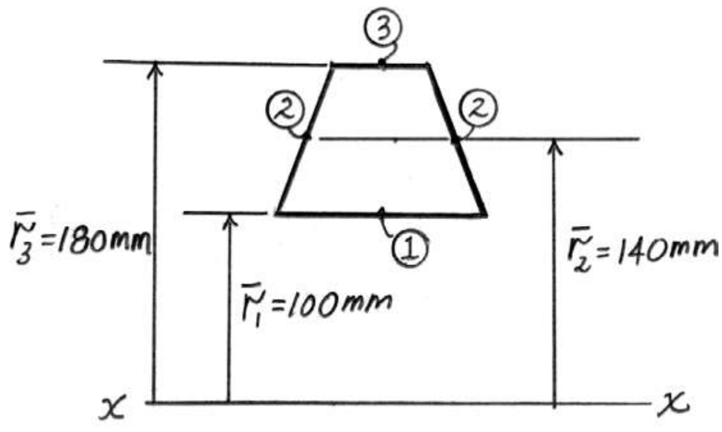
Surface Area. Referring to Fig. *a*, $L_1 = 110$ mm, $L_2 = \sqrt{30^2 + 80^2} = \sqrt{7300}$ mm $L_3 = 50$ mm, $\bar{r}_1 = 100$ mm, $\bar{r}_2 = 140$ mm and $\bar{r}_3 = 180$ mm. Applying the theorem of Pappus and Guldinus, with $\theta = 2\pi$ rad,

$$A = \theta \Sigma \bar{r} L$$

$$= 2\pi\left[100(110) + 2(140)\left(\sqrt{7300}\right) + 180(50)\right]$$

$$= 275.98\left(10^3\right)\text{ mm}^2$$

$$= 276\left(10^3\right)\text{ mm}^2 \qquad \textbf{Ans.}$$

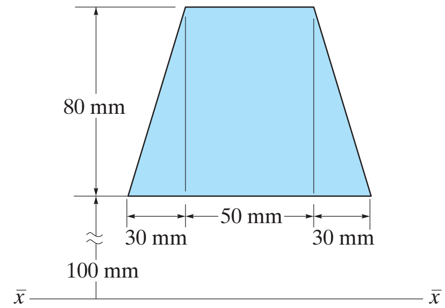


(a)

Ans:
$A = 276(10^3)$ mm^2

***9–100.**

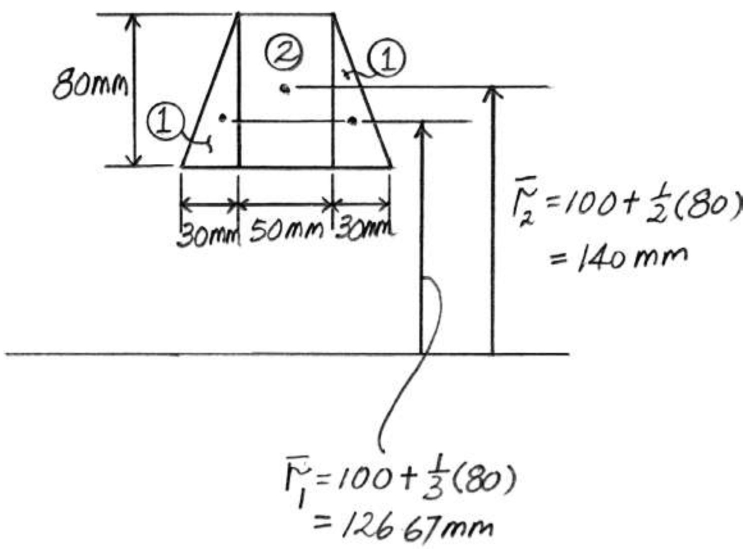
A ring is formed by rotating the area 360° about the $\bar{x}-\bar{x}$ axes. Determine its volume.



SOLUTION

Volume. Referring to Fig. *a*, $A_1 = \frac{1}{2}(60)(80) = 2400\text{ mm}^2$, $A_2 = 50(80) = 4000\text{ mm}^2$, $\bar{r}_1 = 126.67$ mm and $\bar{r}_2 = 140$ mm. Applying the theorem of Pappus and Guldinus, with $\theta = 2\pi$ rad,

$$
\begin{aligned}
V &= \theta\Sigma\bar{r}A \\
&= 2\pi\left[126.67(2400) + 140(4000)\right] \\
&= 5.429\left(10^6\right)\text{ mm}^3 \\
&= 5.43\left(10^6\right)\text{ mm}^3 \qquad \textbf{Ans.}
\end{aligned}
$$

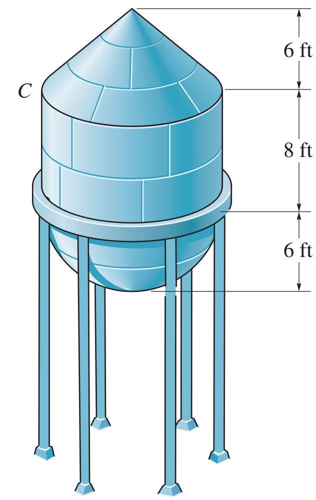


(a)

Ans:
$V = 5.43\left(10^6\right)\text{ mm}^3$

9–101.

The water-supply tank has a hemispherical bottom and cylindrical sides. Determine the weight of water in the tank when it is filled to the top at C. Take $\gamma_w = 62.4\ \text{lb/ft}^3$.

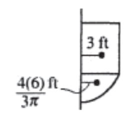


SOLUTION

$$V = \Sigma\theta\tilde{r}A = 2\pi\left\{3(8)(6) + \frac{4(6)}{3\pi}\left(\frac{1}{4}\right)(\pi)(6)^2\right\}$$

$$V = 1357.17\ \text{ft}^3$$

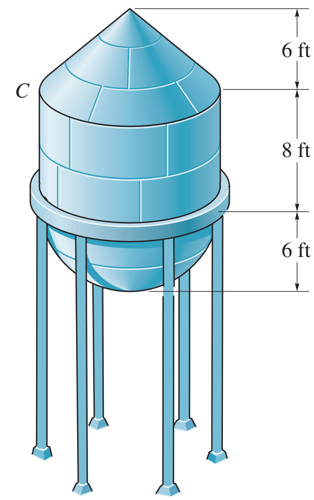
$$W = \gamma V = 62.4(1357.17) = 84.7\ \text{kip}$$ **Ans.**



Ans:
$W = 84.7\ \text{kip}$

9–102.

Determine the number of gallons of paint needed to paint the outside surface of the water-supply tank, which consists of a hemispherical bottom, cylindrical sides, and conical top. Each gallon of paint can cover 250 ft^2.

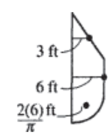


SOLUTION

$$A = \Sigma\theta\tilde{r}L = 2\pi\left\{3\left(6\sqrt{2}\right) + 6(8) + \frac{2(6)}{\pi}\left(\frac{2(6)\pi}{4}\right)\right\}$$

$$= 687.73 \text{ ft}^2$$

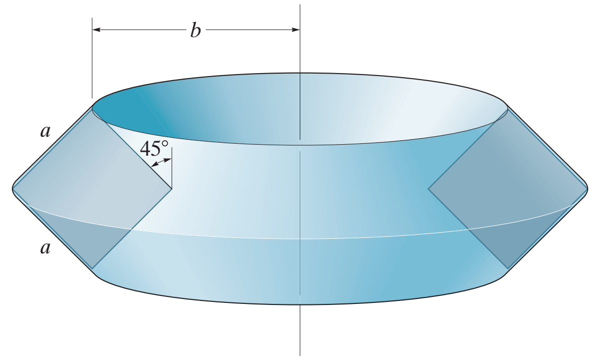
$$\text{Number of gal.} = \frac{687.73 \text{ ft}^2}{250 \text{ ft}^2/\text{gal.}} = 2.75 \text{ gal.}$$ **Ans.**



Ans:
Number of gal. $= 2.75$ gal

9–103.

Determine the surface area and the volume of the ring formed by rotating the square about the vertical axis.

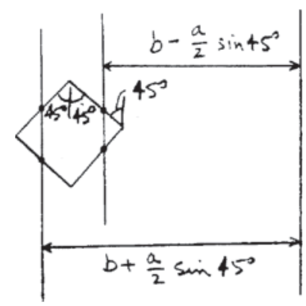


SOLUTION

$$A = \Sigma\theta\tilde{r}L = 2\left[2\pi\left(b - \frac{a}{2}\sin 45^\circ\right)(a)\right] + 2\left[2\pi\left(b + \frac{a}{2}\sin 45^\circ\right)(a)\right]$$

$$= 4\pi\left[ba - \frac{a^2}{2}\sin 45^\circ + ba + \frac{a^2}{2}\sin 45^\circ\right]$$

$$= 8\pi ba$$ **Ans.**



Also

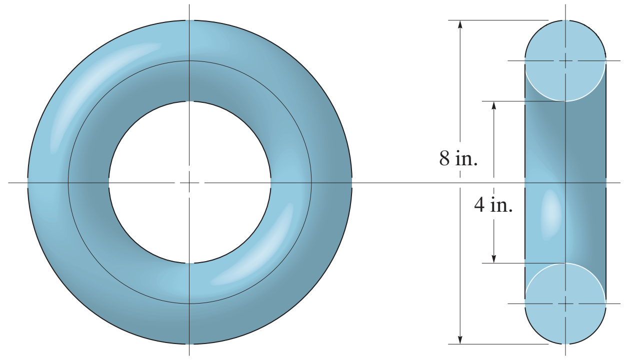
$$A = \Sigma\theta\bar{r}L = 2\pi(b)(4a) = 8\pi ba$$

$$V = \Sigma\theta\tilde{r}A = 2\pi(b)(a)^2 = 2\pi ba^2$$ **Ans.**

Ans:
$A = 8\pi ba$
$V = 2\pi ba^2$

***9–104.**

Determine the surface area of the ring. The cross section is circular as shown.



SOLUTION

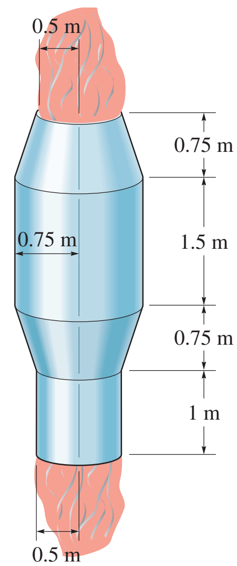
$$A = \theta \tilde{r} L = 2\pi(3)2\pi(1)$$

$$= 118 \text{ in.}^2 \qquad \textbf{Ans.}$$

Ans:
$A = 118 \text{ in.}^2$

9–105.

The heat exchanger radiates thermal energy at the rate of 2500 kJ/h for each square meter of its surface area. Determine how many joules (J) are radiated within a 5-hour period.

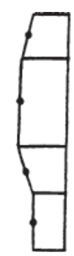


SOLUTION

$$A = \Sigma\theta\,\bar{r}\,L = (2\pi)\left[2\left(\frac{0.75 + 0.5}{2}\right)\sqrt{(0.75)^2 + (0.25)^2} + (0.75)(1.5) + (0.5)(1)\right]$$

$$= 16.419 \text{ m}^2$$

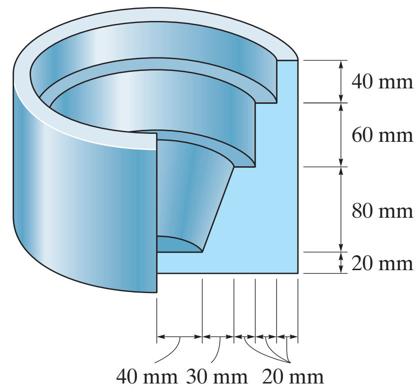
$$Q = 2500(10^3)\left(\frac{\text{J}}{\text{h}\cdot\text{m}^2}\right)(16.416 \text{ m}^2)(5 \text{ h}) = 205 \text{ MJ}$$ **Ans.**



Ans:
$Q = 205$ MJ

9–106.

Determine the interior surface area of the brake piston. It consists of a full circular part. Its cross section is shown in the figure.



SOLUTION

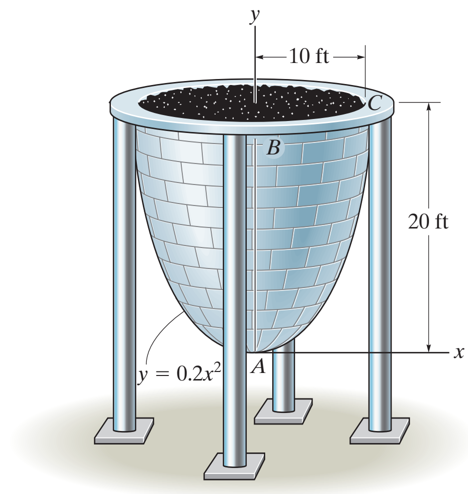
$$A = \Sigma\, \theta\, \bar{r}\, L = 2\,\pi\,[20(40) + 55\sqrt{(30)^2 + (80)^2} + 80(20) + 90(60) + 100(20) + 110(40)]$$

$A = 119(10^3)\text{ mm}^2$ **Ans.**

Ans:
$A = 119(10^3)\text{ mm}^2$

9–107.

The suspension bunker is made from plates which are curved to the natural shape which a completely flexible membrane would take if subjected to a full load of coal. This curve may be approximated by a parabola, $y = 0.2x^2$. Determine the weight of coal which the bunker would contain when completely filled. Coal has a specific weight of $\gamma = 50\ \text{lb/ft}^3$, and assume there is a 20% loss in volume due to air voids. Solve the problem by integration to determine the cross-sectional area of ABC; then use the second theorem of Pappus–Guldinus to find the volume.



SOLUTION

$$\bar{x} = \frac{x}{2}$$

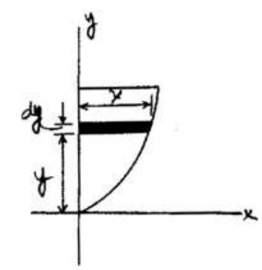
$$\tilde{y} = y$$

$$dA = x\,dy$$

$$\int_A dA = \int_0^{20} \sqrt{\frac{y}{0.2}}\,dy = \frac{2}{3\sqrt{0.2}}\,y^{\frac{3}{2}}\Big|_0^{20} = 133.3\ \text{ft}^2$$

$$\int_A \bar{x}\,dA = \int_0^{20} \frac{y}{0.4}\,dy = \frac{y^2}{0.8}\Big|_0^{20} = 500\ \text{ft}^3$$

$$\bar{x} = \frac{\int_A \bar{x}\,dA}{\int_A dA} = \frac{500}{133.3} = 3.75\ \text{ft}$$



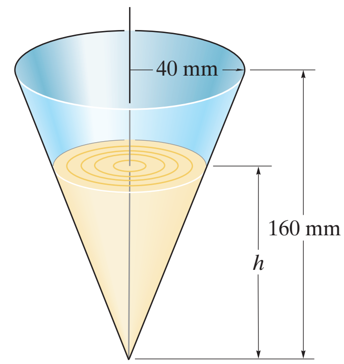
$$V = \theta\,\bar{r}\,A = 2\pi\,(3.75)\,(133.3) = 3142\ \text{ft}^3$$

$$W = 0.8\,\gamma\,V = 0.8(50)(3142) = 125\,664\ \text{lb} = 126\ \text{kip}$$ **Ans.**

Ans:
$W = 126$ kip

***9–108.**

Determine the height h to which liquid should be poured into the cup so that it contacts three-fourths the surface area on the inside of the cup. Neglect the cup's thickness for the calculation.



SOLUTION

Surface Area. From the geometry shown in Fig. a,

$$\frac{r}{h} = \frac{40}{160}; \qquad r = \frac{1}{4}h$$

Thus, $\bar{r} = \frac{1}{8}h$ and $L = \sqrt{\left(\frac{1}{4}h\right)^2 + h^2} = \frac{\sqrt{17}}{4}h$, Fig. b. Applying the theorem of Pappus and Guldinus, with $\theta = 2\pi$ rad,

$$A = \theta\,\Sigma\bar{r}\,L = 2\pi\left(\frac{1}{8}h\right)\left(\frac{\sqrt{17}}{4}h\right) = \frac{\pi\sqrt{17}}{16}h^2$$

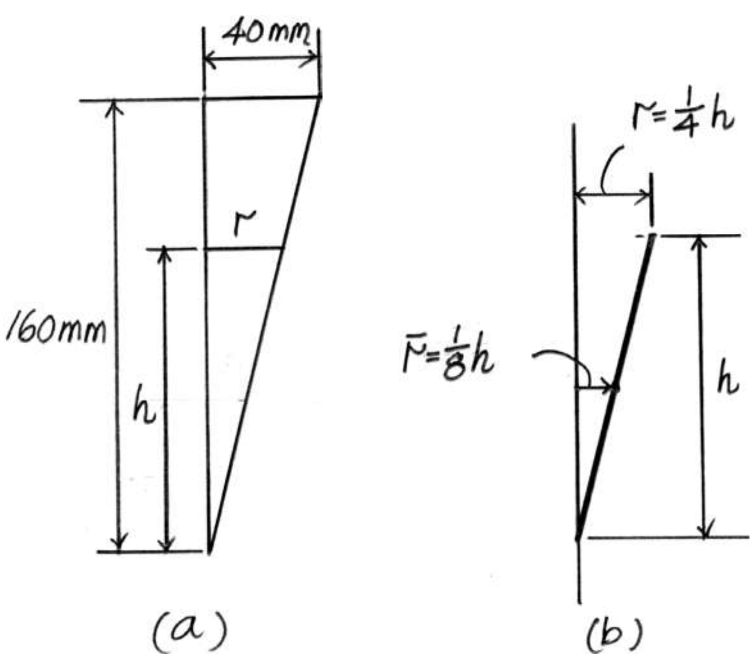
For the whole cup, $h = 160$ mm. Thus

$$A_o = \left(\frac{\pi\sqrt{17}}{16}\right)\left(160^2\right) = 1600\pi\sqrt{17}\text{ mm}^2$$

It is required that $A = \frac{3}{4}A_o = \frac{3}{4}\left(1600\pi\sqrt{17}\right) = 1200\pi\sqrt{17}\text{ mm}^2$. Thus

$$1200\pi\sqrt{17} = \frac{\pi\sqrt{17}}{16}h^2$$

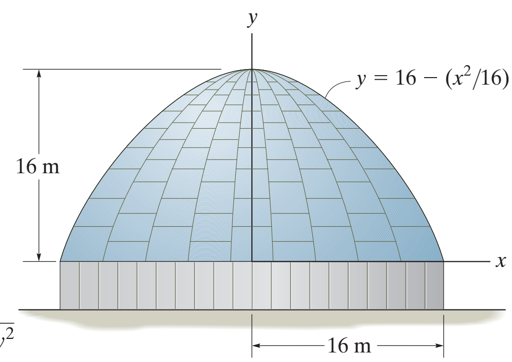
$$h = 138.56\text{ mm} = 139\text{ mm}$$ **Ans.**



Ans:
$h = 139$ mm

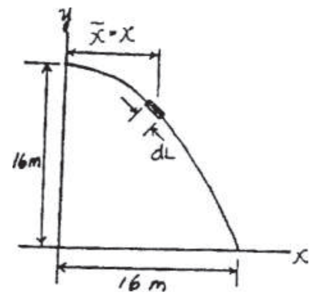
9–109.

Determine the surface area of the roof of the structure if it is formed by rotating the parabola about the y axis.



SOLUTION

Centroid: The length of the differential element is $dL = \sqrt{dx^2 + dy^2} = \left(\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\right)dx$ and its centroid is $x = x$. Here, $\frac{dy}{dx} = -\frac{x}{8}$. Evaluating the integrals, we nave



$$L = \int dL = \int_0^{16\text{ m}} \left(\sqrt{1 + \frac{x^2}{64}}\right)dx = 23.663 \text{ m}$$

$$\int_L \tilde{x}\,dL = \int_0^{16\text{ m}} \tilde{x}\left(\sqrt{1 + \frac{x^2}{64}}\right)dx = 217.181 \text{ m}^2$$

Applying Eq. 9–5, we have

$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{217.181}{23.663} = 9.178 \text{ m}$$

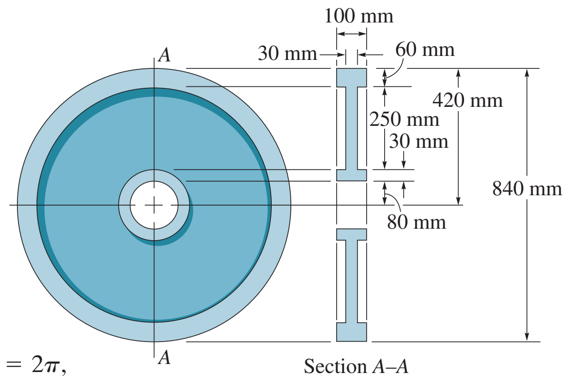
Surface Area: Applying the theorem of Pappus and Guldinus, Eq. 9–7, with $\theta = 2\pi$, $L = 23.663$ m, $\bar{r} = \bar{x} = 9.178$, we have

$$A = \theta \bar{r} L = 2\pi(9.178)\,(23.663) = 1365 \text{ m}^2$$ **Ans.**

Ans:
$A = 1365 \text{ m}^2$

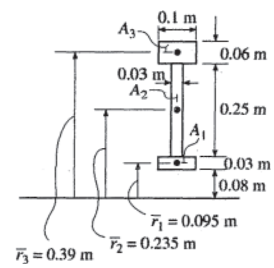
9–110.

A steel wheel has a diameter of 840 mm and a cross section as shown in the figure. Determine the total mass of the wheel if $\rho = 5\ \text{Mg/m}^3$.



SOLUTION

Volume: Applying the theorem of Pappus and Guldinus, Eq. 9–12, with $\theta = 2\pi$, $\bar{r}_1 = 0.095$ m, $\bar{r}_2 = 0.235$ m, $\bar{r}_3 = 0.39$ m, $A_1 = 0.1(0.03) = 0.003\ \text{m}^2$, $A_2 = 0.25(0.03) = 0.0075\ \text{m}^2$ and $A_3 = (0.1)(0.06) = 0.006\ \text{m}^2$, we have



$$V = \theta\Sigma\bar{r}A = 2\pi[0.095(0.003) + 0.235(0.0075) + 0.39(0.006)]$$

$$= 8.775\pi(10^{-3})\text{m}^3$$

The mass of the wheel is

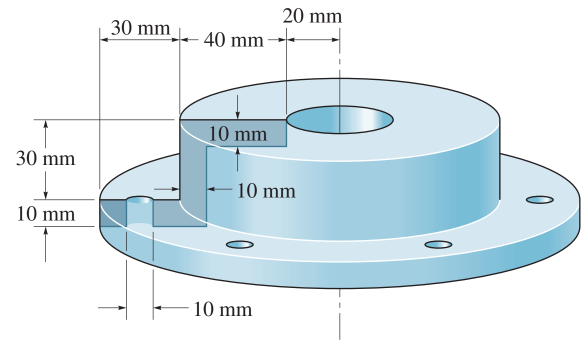
$$m = \rho V = 5(10^3)[8.775(10^{-3})\pi]$$

$$= 138\ \text{kg} \qquad \textbf{Ans.}$$

Ans:
$m = 138$ kg

9–111.

Half the cross section of the steel housing is shown in the figure. There are six 10-mm-diameter bolt holes around its rim. Determine its mass. The density of steel is 7.85 Mg/m^3. The housing is a full circular part.



SOLUTION

$$V = 2\pi[(40)(40)(10) + (55)(30)(10) + (75)(30)(10)] - 6\left[\pi\,(5)^2(10)\right] = 340.9\left(10^3\right)\text{ mm}^3$$

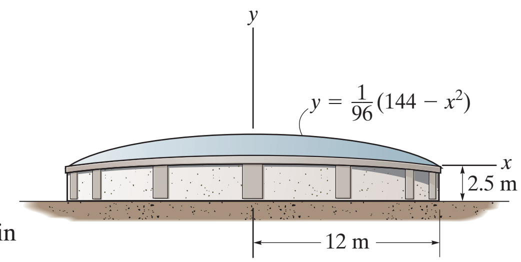
$$m = \rho V = \left(7850\,\frac{\text{kg}}{\text{m}^3}\right)(340.9)\left(10^3\right)\left(10^{-9}\right)\text{ m}^3$$

$$= 2.68\text{ kg}$$ **Ans.**

Ans:
$m = 2.68$ kg

***9–112.**

The water tank has a paraboloid-shaped roof. If one liter of paint can cover 3 m^2 of the tank, determine the number of liters required to coat the roof.



SOLUTION

Length and Centroid: The length of the differential element shown shaded in Fig. *a* is

$$dL = \sqrt{dx^2 + dy^2} = \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx$$

where $\frac{dy}{dx} = -\frac{1}{48}x$. Thus,

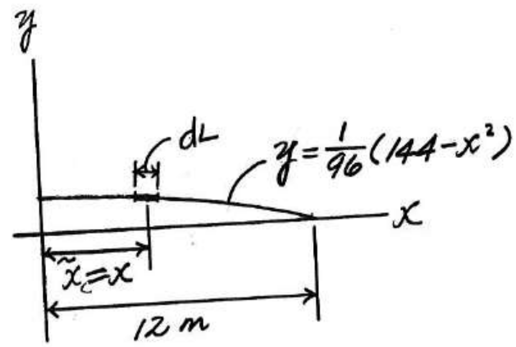
$$dL = \sqrt{1 + \left(-\frac{1}{48}x\right)^2}\,dx = \sqrt{1 + \frac{x^2}{48^2}}\,dx = \frac{1}{48}\sqrt{48^2 + x^2}\,dx$$

Integrating,

$$L = \int_L dL = \int_0^{12\text{ m}} \frac{1}{48}\sqrt{48^2 + x^2}\,dx = 12.124\text{ m}$$

The centroid $\bar{x}$ of the line can be obtained by applying Eq. 9–5 with $x_c = x$.

$$\bar{x} = \frac{\int_L \tilde{x}\,dL}{\int_L dL} = \frac{\int_0^{12\text{ m}} x\left[\frac{1}{48}\sqrt{48^2 + x^2\,dx}\right]}{12.124} = \frac{73.114}{12.124} = 6.031\text{ m}$$



(a)

Surface Area: Applying the first theorem of Pappus and Guldinus and using the results obtained above with $\bar{r} = \bar{x} = 6.031$ m, we have

$$A = 2\pi\bar{r}L = 2\pi(6.031)(12.124) = 459.39\text{ m}^2$$

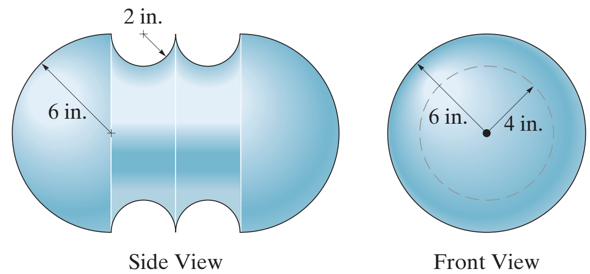
Thus, the amount of paint required is

$$\text{\# of liters} = \frac{459.39}{3} = 153\text{ liters}$$ **Ans.**

Ans:
153 liters

9–113.

Determine the volume of material needed to make the casting.



SOLUTION

$$V = \Sigma\, \theta\, A\, \bar{y}$$

$$= 2\,\pi\left[2\left(\frac{1}{4}\,\pi\right)(6)^2\left(\frac{4(6)}{3\,\pi}\right) + 2(6)(4)(3) - 2\left(\frac{1}{2}\,\pi\right)(2)^2\left(6 - \frac{4(2)}{3\pi}\right)\right]$$

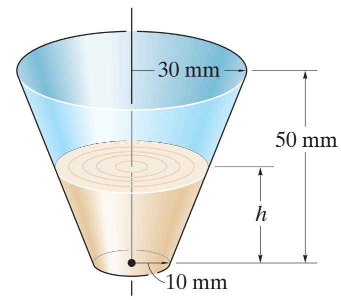
$$= 1402.8 \text{ in}^3$$

$$V = 1.40(10^3) \text{ in}^3$$ **Ans.**

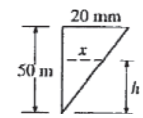
Ans:
$V = 1.40\left(10^3\right) \text{ in}^3$

9–114.

Determine the height h to which liquid should be poured into the cup so that it contacts half the surface area on the inside of the cup. Neglect the cup's thickness for the calculation.



SOLUTION



$$A = \theta \bar{z} \tilde{r} L = 2\pi\{20\sqrt{(20)^2 + (50)^2} + 5(10)\}$$

$$= 2\pi(1127.03)\ \text{mm}^2$$

$$x = \frac{20h}{50} = \frac{2h}{5}$$

$$2\pi\left\{5(10) + \left(10 + \frac{h}{5}\right)\sqrt{\left(\frac{2h}{5}\right)^2 + h^2}\right\} = \frac{1}{2}(2\pi)(1127.03)$$

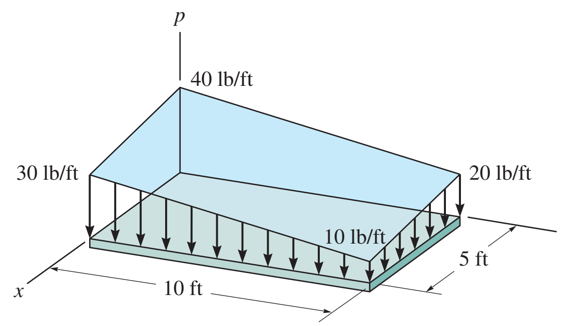
$$10.77h + 0.2154h^2 = 513.5$$

$h = 29.9$ mm **Ans.**

Ans:
$h = 29.9$ mm

9–115.

The pressure loading on the plate varies uniformly along each of its edges. Determine the magnitude of the resultant force and the coordinates $(\bar{x}, \bar{y})$ of the point where the line of action of the force intersects the plate. *Hint:* The equation defining the boundary of the load has the form $p = ax + by + c$, where the constants a, b, and c have to be determined.



SOLUTION

$p = ax + by + c$

At $x = 0, \quad y = 0; \qquad p = 40$

$40 = 0 + 0 + c; \qquad c = 40$

At $x = 5, \quad y = 0, \qquad p = 30$

$30 = a(5) + 0 + 40; \qquad a = -2$

At $x = 0; \quad y = 10, \qquad p = 20$

$20 = 0 + b(10) + 40; \qquad b = -2$

Thus,

$p = -2x - 2y + 40$

$$F_R = \int_A p(x,y)dA = \int_0^5 \int_0^{10} (-2x - 2y + 40)\, dy\, dx$$

$$= -2\left(\tfrac{1}{2}(5)^2\right)(10) - 2\left(\tfrac{1}{2}(10)^2\right)5 + 40(5)(10)$$

$$= 1250 \text{ lb}$$ **Ans.**

$$\int_A xp(x,y)\, dA = \int_0^5 \int_0^{10} (-2x^2 - 2\,yx + 40\,x)\, dy\, dx$$

$$= -2\left(\tfrac{1}{3}(5)^2\right)(10) - 2\left(\tfrac{1}{2}(10)^2\right)\left(\tfrac{1}{2}(5)^2\right) + 40\left(\tfrac{1}{2}(5)^2\right)(10)$$

$$= 2916.67 \text{ lb}\cdot\text{ft}$$

$$\bar{x} = \frac{\int_A xp(x,y)\, dA}{\int_A p(x,y)dA} = \frac{2916.67}{1250} = 2.33 \text{ ft}$$ **Ans.**

$$\int_A yp(x,y)\, dA = \int_0^5 \int_0^{10} (-2x\,y - 2y^2 + 40y)\, dy\, dx$$

$$= -2\left(\tfrac{1}{2}(5)^2\right)\left(\tfrac{1}{2}(10)^2\right) - 2\left(\tfrac{1}{3}(10)^3\right)(5) + 40(5)\left(\tfrac{1}{2}(10)^2\right)$$

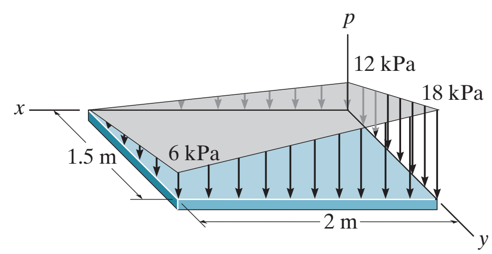
$$= 5416.67 \text{ lb}\cdot\text{ft}$$

$$\bar{y} = \frac{\int_A yp(x,y)\, dA}{\int_A p(x,y)\, dA} = \frac{5416.67}{1250} = 4.33 \text{ ft}$$ **Ans.**

Ans:
$F_R = 1250$ lb
$\bar{x} = 2.33$ ft
$\bar{y} = 4.33$ ft

***9–116.**

The load over the plate varies linearly along the sides of the plate such that $p = (12 - 6x + 4y)$ kPa. Determine the magnitude of the resultant force and the coordinates $(\bar{x}, \bar{y})$ of the point where the line of action of the force intersects the plate.



SOLUTION

Centroid. Perform the double integration.

$$F_R = \int_A p(x, y)dA = \int_0^{1.5\text{ m}} \int_0^{2\text{ m}} (12 - 6x + 4y)dxdy$$

$$= \int_0^{1.5\text{ m}} \left(12x - 3x^2 + 4xy\right)\Big|_0^{2\text{ m}} dy$$

$$= \int_0^{1.5\text{ m}} (8y + 12)dy$$

$$= (4y^2 + 12y)\Big|_0^{1.5\text{ m}}$$

$$= 27.0\text{ kN} \qquad \textbf{Ans.}$$

$$\int_A xp(x, y)dA = \int_0^{1.5\text{ m}} \int_0^{2\text{ m}} \left(12x - 6x^2 + 4xy\right)dx\,dy$$

$$= \int_0^{1.5\text{ m}} (6x^2 - 2x^3 + 2x^2y)\Big|_0^{2\text{ m}} dy$$

$$= \int_0^{1.5\text{ m}} (8y + 8)dy$$

$$= \left(4y^2 + 8y\right)\Big|_0^{1.5\text{ m}}$$

$$= 21.0\text{ kN}\cdot\text{m}$$

$$\int_A yp(x, y)dA = \int_0^{1.5\text{ m}} \int_0^{2\text{ m}} \left(12y - 6xy + 4y^2\right)dx\,dy$$

$$= \int_0^{1.5\text{ m}} \left(12xy - 3x^2y + 4xy^2\right)\Big|_0^{2\text{ m}} dy$$

$$= \int_0^{1.5\text{ m}} (8y^2 + 12y)dy$$

$$= \left(\frac{8}{3}y^3 + 6y^2\right)\Big|_0^{1.5\text{ m}}$$

$$= 22.5\text{ kN}\cdot\text{m}$$

***9–116. Continued**

Thus,

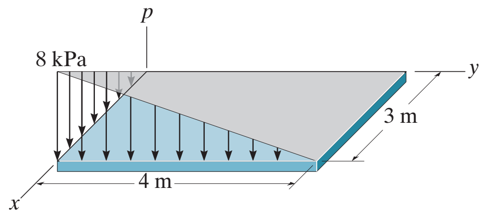
$$\bar{x} = \frac{\int_A xp(x, y)dA}{\int_A p(x, y)dA} = \frac{21.0 \text{ kN} \cdot \text{m}}{27.0 \text{ kN}} = \frac{7}{9} \text{ m} = 0.778 \text{ m} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_A yp(x, y)dA}{\int_A p(x, y)dA} = \frac{22.5 \text{ kN} \cdot \text{m}}{27.0 \text{ kN}} = 0.833 \text{ m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 27.0$ kN
$\bar{x} = 0.778$ m
$\bar{y} = 0.833$ m

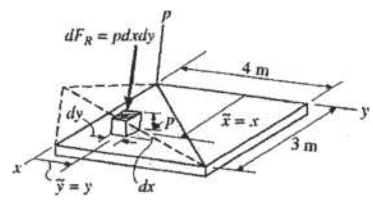
9–117.

The load over the plate varies linearly along the sides of the plate such that $p = \frac{2}{3}[x(4 - y)]$ kPa. Determine the resultant force and its position $(\bar{x}, \bar{y})$ on the plate.



SOLUTION

Resultant Force and its Location: The volume of the differential element is $dV = d\,F_R = p\,dx\,dy = \dfrac{2}{3}(x\,dx)[(4 - y)dy]$ and its centroid is at $\tilde{x} = x$ and $\tilde{y} = y$.



$$F_R = \int_{F_k} dF_R = \int_0^{3\text{ m}} \frac{2}{3}(x\,dx)\int_0^{4\text{ m}} (4 - y)dy$$

$$= \frac{2}{3}\left[\left(\frac{x^2}{2}\right)\Big|_0^{3\text{ m}}\left(4y - \frac{y^2}{2}\right)\Big|_0^{4\text{ m}}\right] = 24.0\text{ kN} \qquad \textbf{Ans.}$$

$$\int_{F_R} \tilde{x}\,dF_R = \int_0^{3\text{ m}} \frac{2}{3}(x^2dx)\int_0^{4\text{ m}} (4 - y)dy$$

$$= \frac{2}{3}\left[\left(\frac{x^3}{3}\right)\Big|_0^{3\text{ m}}\left(4y - \frac{y^2}{2}\right)\Big|_0^{4\text{m}}\right] = 48.0\text{ kN}\cdot\text{m}$$

$$\int_{F_R} \tilde{y}\,dF_R = \int_0^{3\text{ m}} \frac{2}{3}(x\,dx)\int_0^{4\text{ m}} y(4 - y)dy$$

$$= \frac{2}{3}\left[\left(\frac{x^2}{2}\right)\Big|_0^{3\text{m}}\left(2y^2 - \frac{y^3}{3}\right)\Big|_0^{4\text{ m}}\right] = 32.0\text{ kN}\cdot\text{m}$$

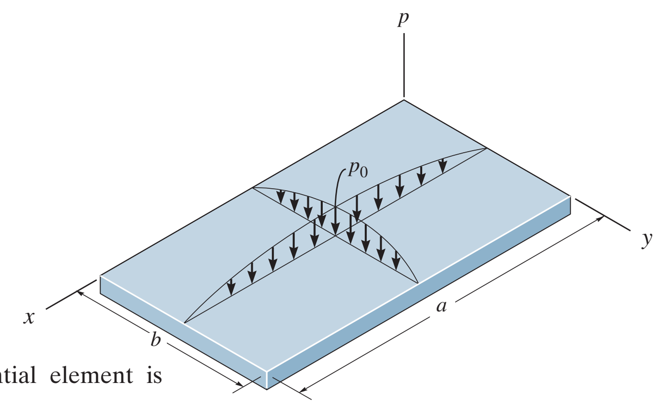
$$\bar{x} = \frac{\int_{FR} \tilde{x}\,dF_R}{\int_{FR} dF_R} = \frac{48.0}{24.0} = 2.00\text{ m} \qquad \textbf{Ans.}$$

$$\bar{y} = \frac{\int_{FR} \tilde{y}\,dF_R}{\int_{FR} dF_R} = \frac{32.0}{24.0} = 1.33\text{ m} \qquad \textbf{Ans.}$$

Ans:
$F_R = 24.0$ kN
$\bar{x} = 2.00$ m
$\bar{y} = 1.33$ m

9–118.

The rectangular plate is subjected to a distributed load over its *entire surface.* The load is defined by the expression $p = p_0 \sin(\pi x/a)\sin(\pi y/b)$, where p_0 represents the pressure acting at the center of the plate. Determine the magnitude and location of the resultant force acting on the plate.



SOLUTION

Resultant Force and its Location: The volume of the differential element is $dV = dF_R = p\,dx\,dy = p_0\left(\sin\frac{\pi x}{a}dx\right)\left(\sin\frac{\pi y}{b}dy\right)$.

$$F_R = \int_{F_R} dF_R = p_0 \int_0^a \left(\sin\frac{\pi x}{a}dx\right)\int_0^b\left(\sin\frac{\pi y}{b}dy\right)$$

$$= p_0\left[\left(-\frac{a}{\pi}\cos\frac{\pi x}{a}\right)\Big|_0^a\left(-\frac{b}{\pi}\cos\frac{\pi x}{b}\right)\Big|_0^b\right]$$

$$= \frac{4ab}{\pi^2}p_0 \qquad \textbf{Ans.}$$

Since the loading is symmetric, the location of the resultant force is at the center of the plate. Hence,

$$\bar{x} = \frac{a}{2} \qquad \bar{y} = \frac{b}{2} \qquad \textbf{Ans.}$$

Ans:

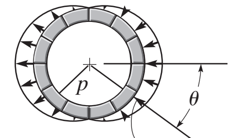
$F_R = \frac{4ab}{\pi^2}p_0$

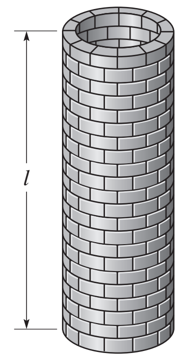
$\bar{x} = \frac{a}{2}$

$\bar{y} = \frac{b}{2}$

9–119.

A wind loading creates a positive pressure on one side of the chimney and a negative (suction) pressure on the other side, as shown. If this pressure loading acts uniformly along the chimney's length, determine the magnitude of the resultant force created by the wind.





SOLUTION

$$F_{Rx} = 2l\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (p_0 \cos\theta)\cos\theta\, r\, d\theta = 2rlp_0 \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^2\theta\, d\theta$$

$$= 2rlp_0\left(\frac{\pi}{2}\right)$$ **Ans.**

$$F_{Rx} = \pi l r p_0$$

$$F_{Ry} = 2l\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (p_0\cos\theta)\sin\theta\, r\, d\theta = 0$$

Thus,

$$F_R = \pi l r p_0$$ **Ans.**

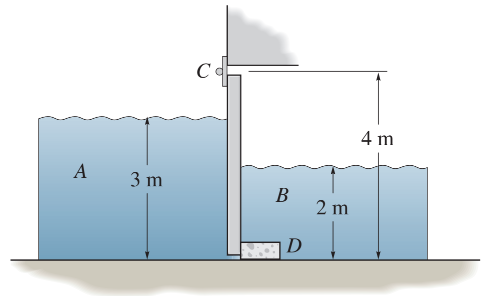
Ans:

$F_{Rx} = 2rlp_0\left(\frac{\pi}{2}\right)$

$F_R = \pi l r p_0$

***9–120.**

When the tide water A subsides, the tide gate automatically swings open to drain the marsh B. For the condition of high tide shown, determine the horizontal reactions developed at the hinge C and stop block D. The length of the gate is 6 m and its height is 4 m. $\rho_w = 1.0\ \text{Mg/m}^3$.

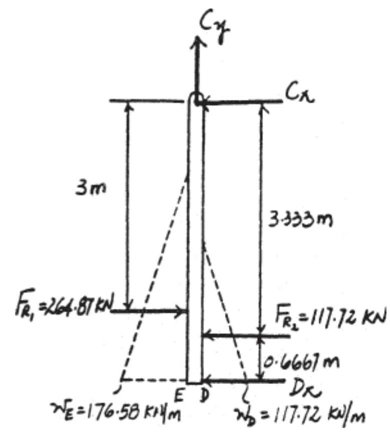


SOLUTION

Fluid Pressure: The fluid pressure at points D and E can be determined using Eq. 9–13, $p = \rho g z$.

$$p_D = 1.0(10^3)(9.81)(2) = 19\,620\ \text{N/m}^2 = 19.62\ \text{kN/m}^2$$

$$p_E = 1.0(10^3)(9.81)(3) = 29\,430\ \text{N/m}^2 = 29.43\ \text{kN/m}^2$$



Thus,

$$w_D = 19.62(6) = 117.72\ \text{kN/m}$$

$$w_E = 29.43(6) = 176.58\ \text{kN/m}$$

Resultant Forces:

$$F_{R_1} = \frac{1}{2}(176.58)(3) = 264.87\ \text{kN}$$

$$F_{R_2} = \frac{1}{2}(117.72)(2) = 117.72\ \text{kN}$$

Equations of Equilibrium:

$\zeta + \Sigma M_C = 0; \qquad 264.87(3) - 117.72(3.333) - D_x(4) = 0$

$$D_x = 100.55\ \text{kN} = 101\ \text{kN} \qquad \textbf{Ans.}$$

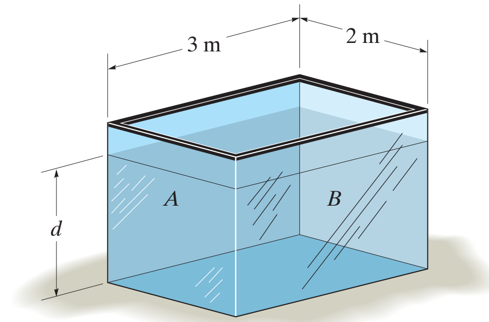
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 264.87 - 117.72 - 100.55 - C_x = 0$

$$C_x = 46.6\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$D_x = 101\ \text{kN}$
$C_x = 46.6\ \text{kN}$

9–121.

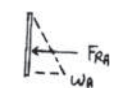
The tank is filled with water to a depth of $d = 4$ m. Determine the resultant force the water exerts on side A and side B of the tank. If oil instead of water is placed in the tank, to what depth d should it reach so that it creates the same resultant forces? $\rho_o = 900\ \text{kg/m}^3$ and $\rho_w = 1000\ \text{kg/m}^3$.

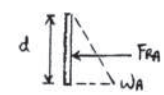


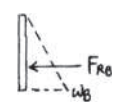
SOLUTION

For water

At side A:







$$w_A = b\,\rho_w\,g\,d$$
$$= 2(1000)(9.81)\,(4)$$
$$= 78\,480\ \text{N/m}$$

$$F_{R_A} = \frac{1}{2}(78\,480)(4) = 156\,960\ \text{N} = 157\ \text{kN}$$ **Ans.**

At side B:

$$w_B = b\,\rho_w\,g\,d$$
$$= 3(1000)(9.81)(4)$$
$$= 117\,720\ \text{N/m}$$

$$F_{R_B} = \frac{1}{2}(117\,720)(4) = 235\,440\ \text{N} = 235\ \text{kN}$$ **Ans.**

For oil

At side A:

$$w_A = b\,\rho_o\,g\,d$$
$$= 2(900)(9.81)d$$
$$= 17\,658\,d$$

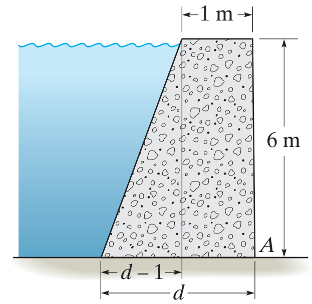
$$F_{R_A} = \frac{1}{2}(17\,658\,d)(d) = 156\,960\ \text{N}$$

$$d = 4.22\ \text{m}$$ **Ans.**

Ans:
For water: $F_{R_A} = 157$ kN
$F_{R_B} = 235$ kN
For oil: $d = 4.22$ m

9–122.

The concrete "gravity" dam is held in place by its own weight. If the density of concrete is $\rho_c = 2.5\ \text{Mg/m}^3$, and water has a density of $\rho_w = 1.0\ \text{Mg/m}^3$, determine the smallest dimension d that will prevent the dam from overturning about its end A.



SOLUTION

Loadings. The computation will be based on $b = 1$ m width of the dam. The pressure at the base of the dam is.

$$P = \rho g h = 1000(9.81)(6) = 58.86(10^3)\ pa = 58.86\ \text{kPa}$$

Thus

$$w = pb = 58.86(1) = 58.86\ \text{kN/m}$$

The forces that act on the dam and their respective points of application, shown in Fig. *a*, are

$$W_1 = 2500\left[1(6)(1)\right](9.81) = 147.15(10^3)\ \text{N} = 147.15\ \text{kN}$$

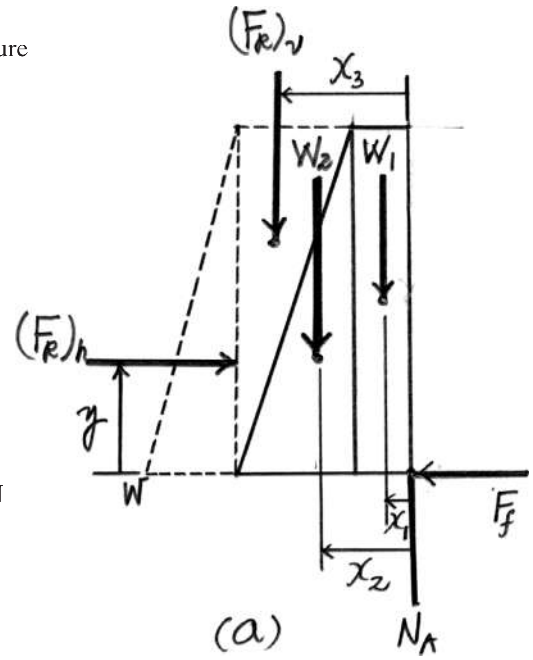
$$W_2 = 2500\left[\frac{1}{2}(d-1)(6)(1)\right](9.81) = 73.575(d-1)(10^3) = 73.575(d-1)\ \text{kN}$$

$$(F_R)_v = 1000\left[\frac{1}{2}(d-1)(6)(1)\right](9.81) = 29.43(d-1)(10^3) = 29.43(d-1)\ \text{kN}$$

$$(F_R)_h = \frac{1}{2}(58.86)(6) = 176.58\ \text{kN}$$

$$x_1 = 0.5 \quad x_2 = 1 + \frac{1}{3}(d-1) = \frac{1}{3}(d+2) \quad x_3 = 1 + \frac{2}{3}(d-1) = \frac{1}{3}(2d+1)$$

$$y = \frac{1}{3}(6) = 2\ \text{m}$$



Equation of Equilibrium. Write the moment equation of equilibrium about A by referring to the *FBD* of the dam, Fig. *a*,

$$\circlearrowleft + \Sigma M_A = 0; \quad 147.15(0.5) + [73.575(d-1)]\left[\frac{1}{3}(d+2)\right]$$

$$+ [29.43(d-1)]\left[\frac{1}{3}(2d+1)\right] - 176.58(2) = 0$$

$$44.145d^2 + 14.715d - 338.445 = 0$$

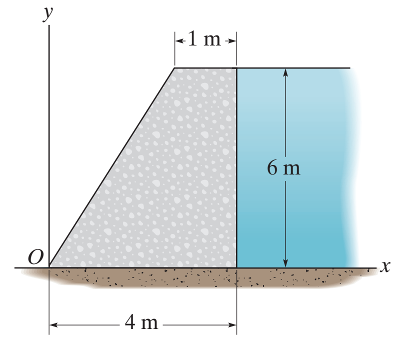
Solving and chose the positive root

$$d = 2.607\ \text{m} = 2.61\ \text{m} \qquad \textbf{Ans.}$$

Ans:
$d = 2.61$ m

9–123.

The factor of safety for tipping of the concrete dam is defined as the ratio of the stabilizing moment due to the dam's weight divided by the overturning moment about O due to the water pressure. Determine this factor if the concrete has a density of $\rho_{conc} = 2.5\ \text{Mg/m}^3$ and for water $\rho_w = 1\ \text{Mg/m}^3$.



SOLUTION

Loadings. The computation will be based on $b = 1\text{m}$ width of the dam. The pressure at the base of the dam is

$$P = p_{wgh} = 1000(9.81)(6) = 58.86(10^3)p_a = 58.86\ \text{kPa}$$

Thus,

$$w = Pb = 58.86(1) = 58.86\ \text{kN/m}$$

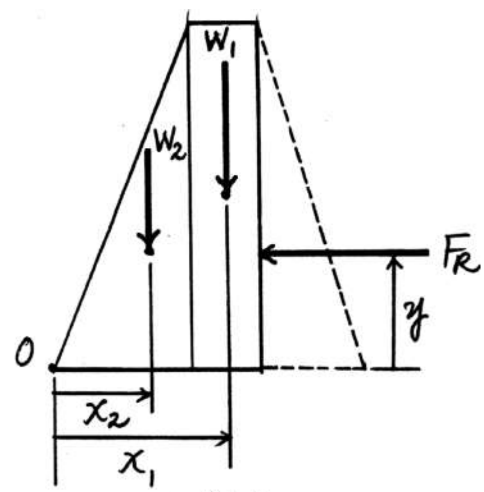
The forces that act on the dam and their respective points of application, shown in Fig. *a*, are

$$W_1 = (2500)[(1(6)(1)](9.81) = 147.15(10^3)]\ \text{N} = 147.15\ \text{kN}$$

$$W_2 = (2500)\left[\frac{1}{2}(3)(6)(1)\right](9.81) = 220.725(10^3)\ \text{N} = 220.725\ \text{kN}$$

$$F_R = \frac{1}{2}(58.86)(6) = 176.58\ \text{kN}$$

$$x_1 = 3 + \frac{1}{2}(1) = 3.5\ \text{ft} \qquad x_2 = \frac{2}{3}(3) = 2\text{m} \qquad y = \frac{1}{3}(6) = 2\ \text{m}$$



(a)

Thus, the overturning moment about O is

$$M_{OT} = 176.58(2) = 353.16\ \text{kN}\cdot\text{m}$$

And the stabilizing moment about O is

$$M_s = 147.15(3.5) + 220.725(2) = 956.475\ \text{kN}\cdot\text{m}$$

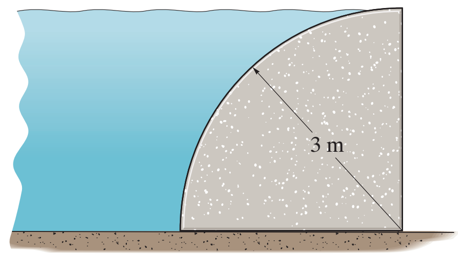
Thus, the factor of safety is

$$\text{F.S.} = \frac{M_s}{M_{OT}} = \frac{956.475}{353.16} = 2.7083 = 2.71 \qquad \textbf{Ans.}$$

Ans:
F.S. $= 2.71$

***9–124.**

The concrete dam in the shape of a quarter circle. Determine the magnitude of the resultant hydrostatic force that acts on the dam per meter of length. The density of water is $\rho_w = 1\ \text{Mg/m}^3$.



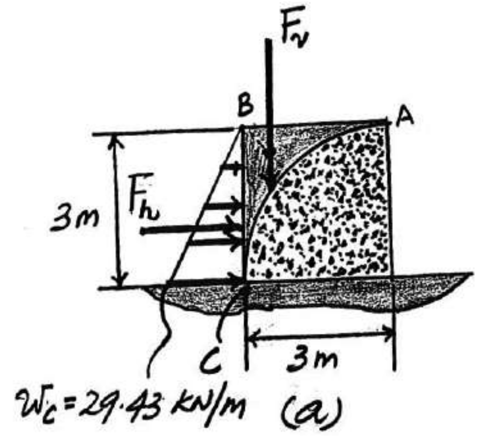
SOLUTION

Loading: The hydrostatic force acting on the circular surface of the dam consists of the vertical component $\mathbf{F}_v$ and the horizontal component $\mathbf{F}_h$ as shown in Fig. *a*.

Resultant Force Component: The vertical component $\mathbf{F}_v$ consists of the weight of water contained in the shaded area shown in Fig. *a*. For a 1-m length of dam, we have

$$F_v = \rho g A_{ABC} b = (1000)(9.81)\left[(3)(3) - \frac{\pi}{4}(3^2)\right](1) = 18947.20\ \text{N} = 18.95\ \text{kN}$$

The horizontal component $\mathbf{F}_h$ consists of the horizontal hydrostatic pressure. Since the width of the dam is constant (1 m), this loading can be represented by a triangular distributed loading with an intensity of $w_C = \rho g h_C b = 1000(9.81)(3)(1) = 29.43\ \text{kN/m}$ at point C, Fig. *a*.



$$F_h = \frac{1}{2}(29.43)(3) = 44.145\ \text{kN}$$

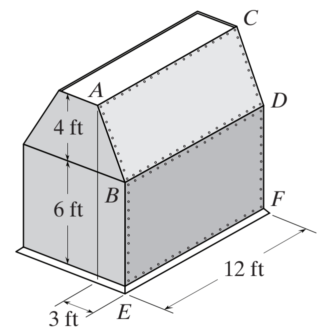
Thus, the magnitude of the resultant hydrostatic force acting on the dam is

$$F_R = \sqrt{F_h^{\,2} + F_v^{\,2}} = \sqrt{44.145^2 + 18.95^2} = 48.0\ \text{kN} \qquad \textbf{Ans.}$$

Ans:
$F_R = 48.0\ \text{kN}$

9–125.

The tank is used to store a liquid having a density of $80\ \text{lb/ft}^3$. If it is filled to the top, determine the magnitude of force the liquid exerts on each of its two sides *ABDC* and *BDFE*.



SOLUTION

$$w_1 = 80(4)(12) = 3840\ \text{lb/ft}$$

$$w_2 = 80(10)(12) = 9600\ \text{lb/ft}$$

ABDC:

$$F_1 = \frac{1}{2}(3840)(5) = 9.60\ \text{kip} \qquad \textbf{Ans.}$$

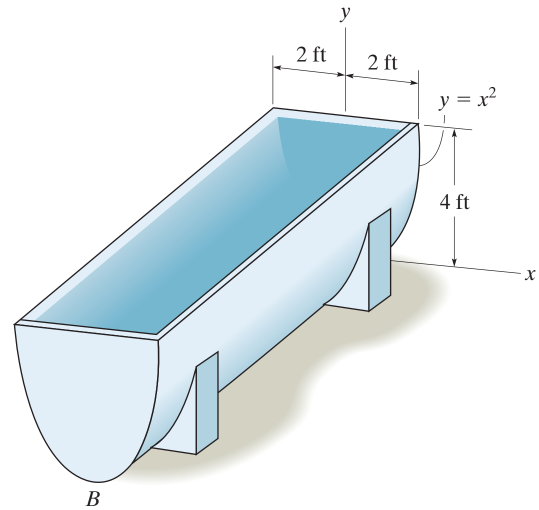
BDEF:

$$F_2 = \frac{1}{2}(9600 - 3840)(6) + 3840(6) = 40.3\ \text{kip} \qquad \textbf{Ans.}$$

Ans:
$F_1 = 9.60$ kip
$F_2 = 40.3$ kip

9–126.

The parabolic plate is subjected to a fluid pressure that varies linearly from 0 at its top to 100 lb/ft at its bottom B. Determine the magnitude of the resultant force and its location on the plate.



SOLUTION

$$F_R = \int_A p\,dA = \int_0^4 (100 - 25y)(2x\,dy)$$

$$= 2\int_0^4 (100 - 25y)\left(y^{\frac{1}{2}}\,dy\right)$$

$$= 2\left[100\left(\frac{2}{3}\right)y^{\frac{3}{2}} - 25\left(\frac{2}{5}\right)y^{\frac{5}{2}}\right]_0^4 = 426.7\text{ lb} = 427\text{ lb} \qquad \textbf{Ans.}$$

$$F_R\bar{y} = \int_A yp\,dA; \qquad 426.7\,\bar{y} = 2\int_0^4 y(100 - 25y)y^{\frac{1}{2}}\,dy$$

$$426.7\,\bar{y} = 2\left[100\left(\frac{2}{5}\right)y^{\frac{5}{2}} - 25\left(\frac{2}{7}\right)y^{\frac{7}{2}}\right]_0^4$$

$$426.7\,\bar{y} = 731.4$$

$\bar{y} = 1.71$ ft **Ans.**

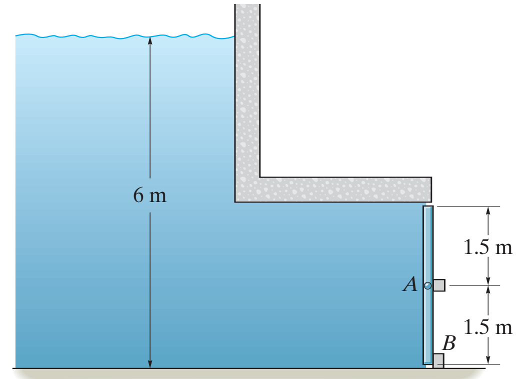
Due to symmetry,

$\bar{x} = 0$ **Ans.**

Ans:
$F_R = 427$ lb
$\bar{y} = 1.71$ ft
$\bar{x} = 0$

9–127.

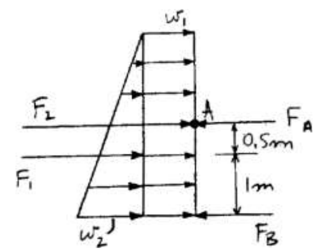
The 2-m-wide rectangular gate is pinned at its center A and is prevented from rotating by the block at B. Determine the reactions at these supports due to hydrostatic pressure. $\rho_w = 1.0\ \text{Mg/m}^3$.



SOLUTION

$w_1 = 1000(9.81)(3)(2) = 58\,860\ \text{N/m}$

$w_2 = 1000(9.81)(3)(2) = 58\,860\ \text{N/m}$



$F_1 = \frac{1}{2}(3)(58\,860) = 88\,290$

$F_2 = (58\,860)(3) = 176\,580$

$\zeta + \Sigma M_A = 0; \qquad 88\,290(0.5) - F_B\,(1.5) = 0$

$F_B = 29\,430\ \text{N} = 29.4\ \text{kN}$ **Ans.**

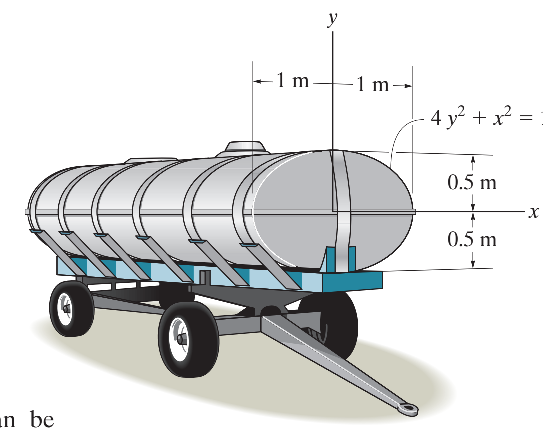
$\overset{+}{\rightarrow} \Sigma F_x = 0; \qquad 88\,290 + 176\,580 - 29\,430 - F_A = 0$

$F_A = 235\,440\,\text{N} = 235\ \text{kN}$ **Ans.**

Ans:
$F_B = 29.4\ \text{kN}$
$F_A = 235\ \text{kN}$

***9–128.**

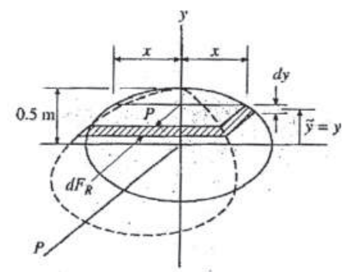
The tank is filled with a liquid which has a density of $900\ \text{kg/m}^3$. Determine the resultant force that it exerts on the elliptical end plate, and the location of the center of pressure, measured from the x axis.



SOLUTION

Fluid Pressure: The fluid pressure at an arbitrary point along y axis can be determined using Eq. 9–13, $p = \gamma(0.5 - y) = 900(9.81)(0.5 - y) = 8829(0.5 - y)$.

Resultant Force and its Location: Here, $x = \sqrt{1 - 4y^2}$. The volume of the differential element is $dV = dF_R = p(2x\,dy) = 8829(0.5 - y)[2\sqrt{1 - 4y^2}]\,dy$. Evaluating integrals using Simpson's rule, we have



$$F_R = \int_{FR} d\,F_R = 17658 \int_{-0.5\text{ m}}^{0.5\text{ m}} (0.5 - y)(\sqrt{1 - 4y^2})dy$$

$$= 6934.2\text{ N} = 6.93\text{ kN}$$ **Ans.**

$$\int_{F_R} \overline{y} d\,F_R = 17658 \int_{-0.5\text{ m}}^{0.5\text{ m}} y(0.5 - y)(\sqrt{1 - 4y^2})dy$$

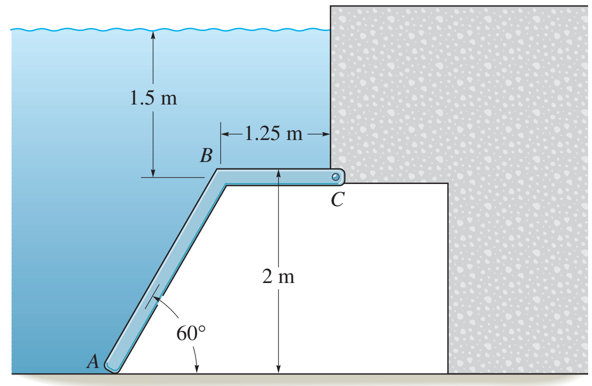
$$= -866.7\text{ N}\cdot\text{m}$$

$$\overline{y} = \frac{\int_{F_R} \widetilde{y} dF_R}{\int_{F_R} dF_R} = \frac{-866.7}{6934.2} = -0.125\text{ m}$$ **Ans.**

Ans:
$F_R = 6.93\text{ kN}$
$\overline{y} = -0.125\text{ m}$

9–129.

Determine the magnitude of the resultant force acting on the gate ABC due to hydrostatic pressure. The gate has a width of 1.5 m. $\rho_w = 1.0\ \text{Mg/m}^3$.



SOLUTION

$$w_1 = 1000(9.81)(1.5)(1.5) = 22.072\ \text{kN/m}$$

$$w_2 = 1000(9.81)(2)(1.5) = 29.43\ \text{kN/m}$$

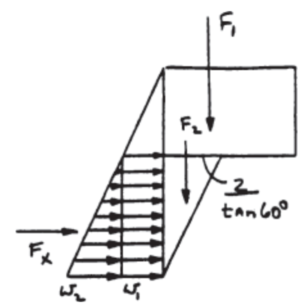
$$F_x = \frac{1}{2}(29.43)(2) + (22.0725)(2) = 73.58\ \text{kN}$$

$$F_1 = \left[(22.072)\left(1.25 + \frac{2}{\tan 60°}\right)\right] = 53.078\ \text{kN}$$

$$F_2 = \frac{1}{2}(1.5)(2)\left(\frac{2}{\tan 60°}\right)(1000)(9.81) = 16.99\ \text{kN}$$

$$F_y = F_1 + F_2 = 70.069\ \text{kN}$$

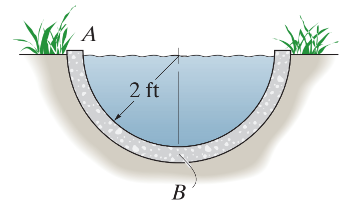
$$F = \sqrt{F_x^2 + F_y^2} = \sqrt{(73.58)^2 + (70.069)^2} = 102\ \text{kN}$$ **Ans.**



Ans:
$F = 102$ kN

9–130.

The semicircular drainage pipe is filled with water. Determine the resultant horizontal and vertical force components that the water exerts on the side AB of the pipe per foot of pipe length; $\gamma_w = 62.4 \text{ lb/ft}^3$.



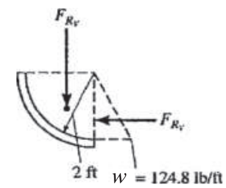
SOLUTION

Fluid Pressure: The fluid pressure at the bottom of the drain can be determined using Eq. 9–13, $p = \gamma z$.

$$p = 62.4(2) = 124.8 \text{ lb/ft}^2$$

Thus,

$$w = 124.8(1) = 124.8 \text{ lb/ft}$$



Resultant Forces: The area of the quarter circle is $A = \frac{1}{4}\pi r^2 = \frac{1}{4}\pi(2^2) = \pi \text{ ft}^2$. Then, the vertical component of the resultant force is

$$F_{R_v} = \gamma V = 62.4[\pi(1)] = 196 \text{ lb} \qquad \textbf{Ans.}$$

and the horizontal component of the resultant force is

$$F_{R_h} = \frac{1}{2}(124.8)(2) = 125 \text{ lb} \qquad \textbf{Ans.}$$

Ans:
$F_{R_v} = 196$ lb
$F_{R_h} = 125$ lb