ElCoM
ELECTRICAL COMPUTER MECHATRONICS

تقدم لجنة ElCoM الاكاديمية

دفتر لمادة:

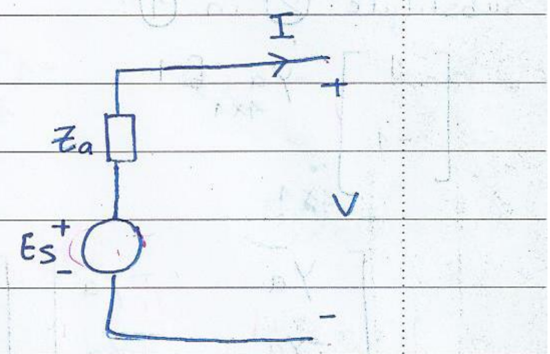
تحليل أنظمة القوى

جزيل الشكر للطالبة:

ثروة النتشة

* Admittance matrix (ybus):-

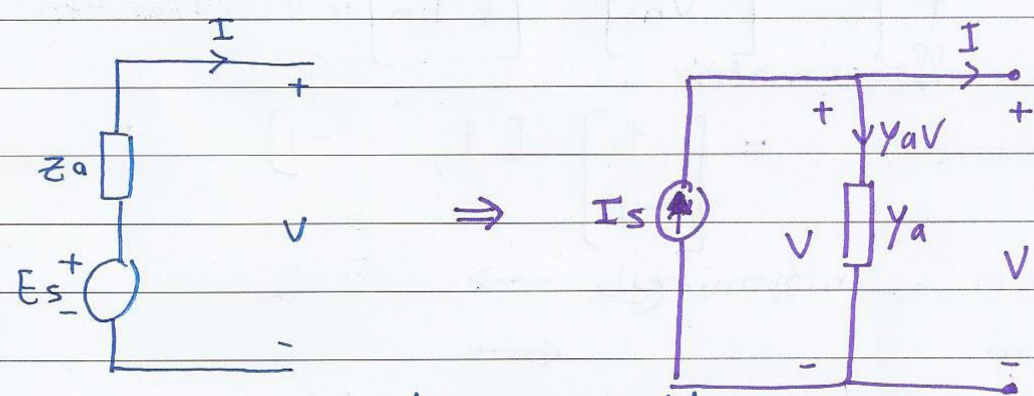
$$E_s = \underbrace{V}_{\text{terminal voltage}} + \underbrace{Z_a}_{\text{impedence}} I$$



$$\div Z_a \Rightarrow \frac{E_s}{Z_a} = \frac{V}{Z_a} + I$$

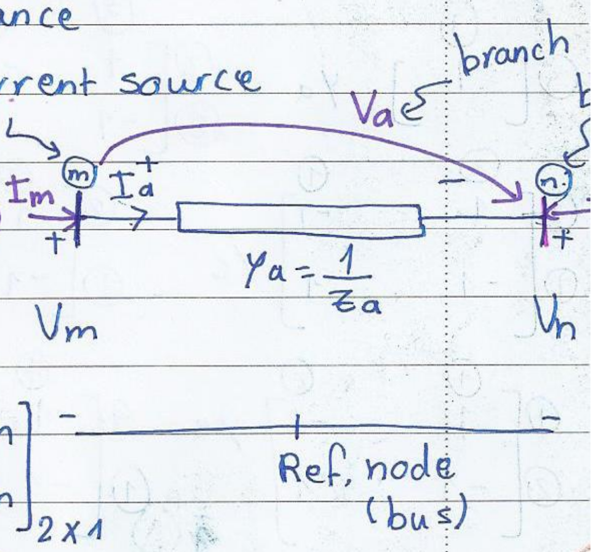
we have: $\frac{1}{Z_a} = Y_a \rightarrow$ admitt

$$I_s = Y_a V + I$$



impedence $\rightarrow$ admittance

voltage source $\rightarrow$ current source



$$I_m = I_a \qquad I_n = -I_a \Rightarrow \begin{bmatrix} I_m \\ I_n \end{bmatrix}_{2\times1} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}_{2\times1} I_{a_{1\times1}} \quad \text{--- (1)}$$

$$V_a = V_m - V_a \qquad V_{a_{1\times1}} = \begin{bmatrix} 1 & -1 \end{bmatrix}_{1\times2} \begin{bmatrix} V_m \\ V_n \end{bmatrix}_{2\times1}$$

we have: $Y_a V_a = I_a$ branch

$$Y_a \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} V_m \\ V_n \end{bmatrix} = I_a \quad \text{--- (2)}$$

substitute ② in ①:

$$\begin{bmatrix} 1 \\ -1 \end{bmatrix}_{2\times1} y_{a\,1\times1} \begin{bmatrix} 1 & -1 \end{bmatrix}_{1\times2} \begin{bmatrix} V_m \\ V_n \end{bmatrix}_{2\times1} = \begin{bmatrix} I_m \\ I_n \end{bmatrix}_{2\times1}$$

$$\begin{bmatrix} y_a & -y_a \\ -y_a & y_a \end{bmatrix} \begin{bmatrix} V_m \\ V_n \end{bmatrix} = \begin{bmatrix} I_m \\ I_n \end{bmatrix}$$ → nodal admittance equation

داخل :

طالع منه ← $$\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} y_a \begin{bmatrix} V_m \\ V_n \end{bmatrix} = \begin{bmatrix} I_m \\ I_n \end{bmatrix}$$

coefficent matrix

branch between two buses : $= \begin{bmatrix} 1 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & -1 \end{bmatrix}$

1 : m → n   طالع من m وداخل n →

-1 : n → m   ←

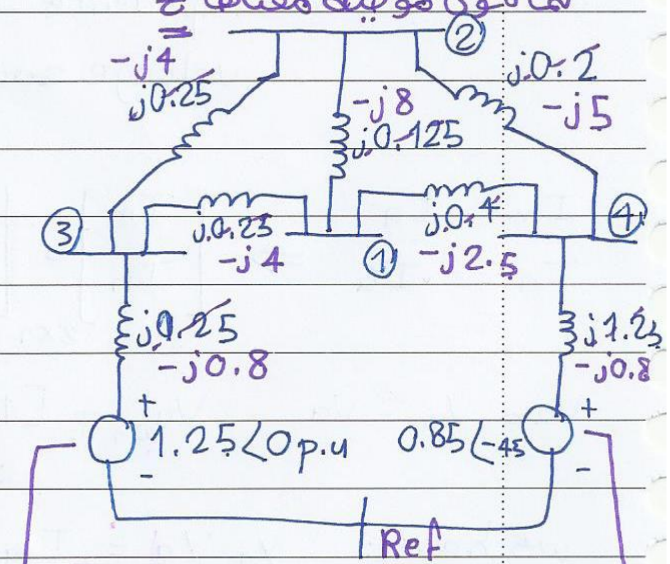
$$\begin{array}{cc} & ③ \\ ③ & \begin{bmatrix} 1 \end{bmatrix} \end{array} y_a \qquad \begin{array}{ccc} & ③ & ② \\ ③ & 1 & -1 \\ ② & -1 & 1 \end{array} y_b$$

$$\begin{array}{ccc} & ③ & ① \\ ③ & 1 & -1 \\ ① & -1 & 1 \end{array} y_c \qquad \begin{array}{ccc} & ② & ① \\ ② & 1 & -1 \\ ① & -1 & 1 \end{array} y_d$$

$$\begin{array}{ccc} & ④ & ② \\ ④ & 1 & -1 \\ ② & -1 & 1 \end{array} y_e \qquad \begin{array}{ccc} & ④ & ① \\ ④ & 1 & -1 \\ ① & -1 & 1 \end{array} y_f$$

$$\begin{array}{cc} & ④ \\ ④ & \begin{bmatrix} 1 \end{bmatrix} \end{array} y_g$$

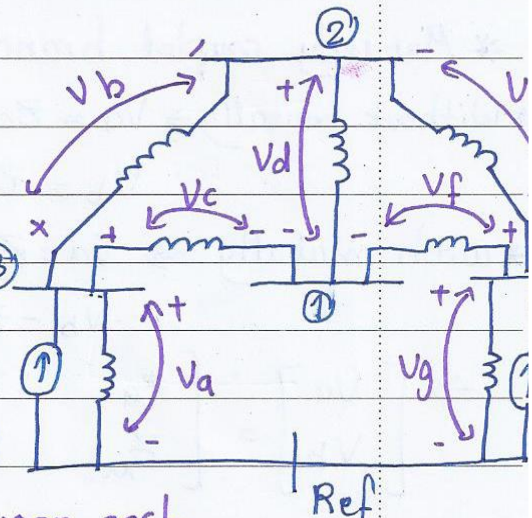
لما تكون موجبة معناها Z



$\frac{1.25}{j1.25} = 1\angle -90$ $\frac{0.85\angle -45}{j1.25}$

$$
Y_{bus} = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\
\begin{bmatrix}
y_c+y_d+y_f & -y_d & -y_c & -y_f \\
-y_d & y_b+y_d+y_e & -y_b & -y_e \\
-y_c & -y_b & y_a+y_b+y_c & 0 \\
-y_f & -y_e & 0 & y_e+y_f+y_e
\end{bmatrix}
\end{array}
$$

→ no direct connection between them



There is summtry around the digonal because its the same branch between each of two branches

$$
Y_{bus} = \begin{bmatrix}
-j14.5 & j8 & j4 & j2.5 \\
j8 & -j17 & j4 & j5 \\
j4 & j4 & -j8.8 & 0 \\
j2.5 & j5 & 0 & -j8.3
\end{bmatrix}
$$

$$I = y_a V$$

unknown

$$
\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}
$$

$$
\begin{bmatrix} 0 \\ 0 \\ 1\angle{-90} \\ 0.68\angle{-135} \end{bmatrix} = \begin{bmatrix} \quad \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}
$$

$$V = Y_{bus}^{-1} \; I$$

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$V_1 \rightarrow Z I_1 + Z_m I_2$

$V_2 \rightarrow Z I_2 + Z_m I_1$

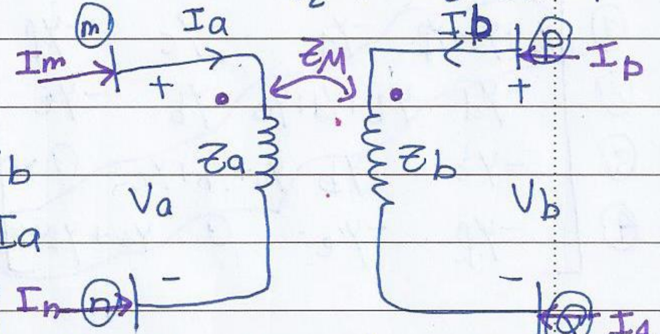
* Mutually coupled branches in ybus:

* Without mutally ⇒ $V_a = Z_a I_a$

$$V_b = Z_b I_b$$

* with mutally ⇒ $V_a = Z_a I_a + Z_M I_b$

$$V_b = Z_b I_b + Z_M I_a$$



$$\Rightarrow \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} Z_a & Z_M \\ Z_M & Z_b \end{bmatrix} \begin{bmatrix} I_a \\ I_b \end{bmatrix}$$

نفس اتجاه dot ← نفس اتجاه اللف

$$[V] = [Z]\,[I]$$

$$[Z]^{-1}\,[V] = [I]$$

$$[y_{bus}]\,[V] = [I]$$

$$\begin{bmatrix} y_a & y_m \\ y_m & y_b \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} I_a \\ I_b \end{bmatrix}$$

"branches with nodes"

nodes = branches

m, n ↓ a, b

p, Q relation

Where: $y_{bus} = [Z]^{-1} = \begin{bmatrix} Z_a & Z_M \\ Z_M & Z_b \end{bmatrix}^{-1} = \begin{bmatrix} y_a & y_m \\ y_m & y_b \end{bmatrix}$

$$= \frac{1}{Z_a Z_b - Z_M^2} \begin{bmatrix} Z_b & -Z_m \\ -Z_m & Z_a \end{bmatrix}$$

$$y_a = \frac{Z_b}{Z_a Z_b - Z_M^2}$$

$V_a = V_m - V_n$

$V_b = V_p - V_Q$

$$\Rightarrow \begin{bmatrix} V_a \\ V_b \end{bmatrix}_{2\times1} = \begin{bmatrix} V_m - V_n \\ V_p - V_Q \end{bmatrix} = \begin{array}{c} \\ a \\ b \end{array}\!\!\begin{array}{c} \begin{array}{cccc} m & n & p & Q \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix}_{2\times4} \end{array} \begin{bmatrix} V_m \\ V_n \\ V_p \\ V_Q \end{bmatrix}_{4\times1}$$

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$I_m = +I_a$
$I_n = -I_a$
$I_p = +I_b$
$I_Q = -I_b$

$$\Rightarrow \begin{bmatrix} I_m \\ I_n \\ I_p \\ I_Q \end{bmatrix}_{4\times1} = A^T \begin{bmatrix} I_a \\ I_b \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 0 \\ 0 & 1 \\ 0 & -1 \end{bmatrix}_{4\times2} \begin{bmatrix} I_a \\ I_b \end{bmatrix}_{2\times1}$$

coefficient in branches.

branches.

$$Y\,V = I$$

$$\begin{bmatrix} y_a & y_m \\ y_m & y_b \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \begin{bmatrix} I_a \\ I_b \end{bmatrix}$$

* multible two side by A^T and substitute from above

$$\underbrace{\underset{4\times2}{A^T} \begin{bmatrix} y_a & y_m \\ y_m & y_b \end{bmatrix}_{2\times2} \underset{2\times4}{A}}_{4\times4} \begin{bmatrix} V_m \\ V_n \\ V_p \\ V_q \end{bmatrix}_{4\times1} = \begin{bmatrix} I_m \\ I_n \\ I_p \\ I_q \end{bmatrix}_{4\times1}$$

nodes.

$$\begin{bmatrix} y_a & -y_a & y_m & -y_m \\ -y_a & y_a & -y_m & y_m \\ -y_m & -y_m & y_b & -y_b \\ -y_m & y_m & -y_b & y_b \end{bmatrix} \begin{bmatrix} V_m \\ V_n \\ V_p \\ V_q \end{bmatrix} = \begin{bmatrix} I_m \\ I_n \\ I_p \\ I_q \end{bmatrix}$$

coefficients in nodes

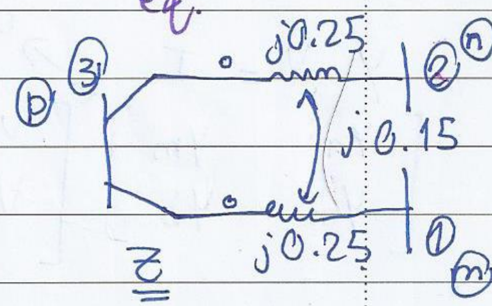
N O T E B O O K

nodal

$$\underbrace{\left[\begin{array}{c|c} \begin{array}{c|cc} & m & n \\ \hline m & 1 & -1 \\ n & -1 & 1 \end{array}\; y_a & \begin{array}{c|cc} & p & Q \\ \hline p & 1 & -1 \\ Q & -1 & 1 \end{array}\; y_m \\ \hline \begin{array}{c|cc} & p & Q \\ \hline p & 1 & -1 \\ Q & -1 & 1 \end{array}\; y_m & \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} y_b \end{array}\right]}_{\text{nodal admittance coeff.}} \begin{bmatrix} V_m \\ V_n \\ V_p \\ V_q \end{bmatrix} = \begin{bmatrix} I_m \\ I_n \\ I_p \\ I_q \end{bmatrix} \; - ✻$$

— ✻ nodal admittance eq.

Ex: $Z = \begin{bmatrix} j0.25 & j0.15 \\ j0.15 & j0.25 \end{bmatrix}$ (Z_a, Z_M, Z_M, Z_b)



$$Y = Z^{-1} = j\begin{bmatrix} 0.25 & 0.15 \\ 0.15 & 0.25 \end{bmatrix}^{-1}$$

$$= j\begin{bmatrix} -6.25 & 3.75 \\ 3.75 & -6.25 \end{bmatrix}$$ (y_a, Y_M, Y_M, Y_b)

$m = 1$

$n = 2$

$p = 3$

$Q =$ مشترك

$$\Rightarrow \left[\begin{array}{c|c} \begin{array}{c|cc} & 3 & 1 \\ \hline 3 & 1 & -1 \\ 1 & -1 & 1 \end{array}\; (-j6.25) & \begin{array}{c|cc} & 3 & 2 \\ \hline 3 & 1 & -1 \\ 2 & -1 & 1 \end{array}\; (j3.75) \\ \hline \begin{array}{c|cc} & 3 & 1 \\ \hline 3 & 1 & -1 \\ 2 & -1 & 1 \end{array}\; j3.75 & \begin{array}{c|cc} & 3 & 2 \\ \hline 3 & 1 & -1 \\ 2 & -1 & 1 \end{array}\; -j6.25 \end{array}\right]$$

4x4

$j3.75 + j...$

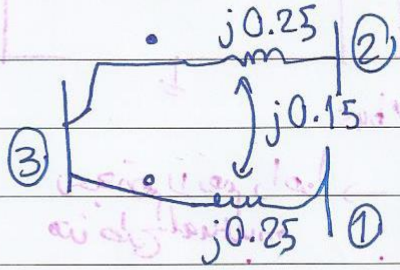
$$\begin{bmatrix} -j6.25 & j3.75 & j6.25 - j3.75 \\ j3.75 & -j6.25 & -j3.75 + j6.25 \\ j6.25 - j3.75 & -j3.75 + j6.25 & -j6.25 - j6.25 + j3.75 + j3.75 \end{bmatrix}$$

branch كانت في بين ① و② نفس قيمة Z_m بس سالبة

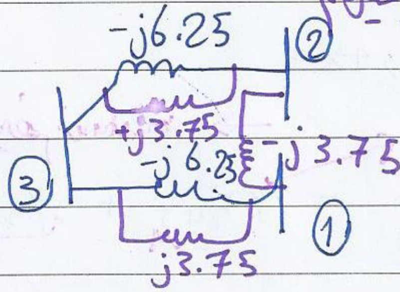
$$\begin{array}{c|c} & \begin{array}{ccc} ① & ② & ③ \end{array} \\ \hline \begin{array}{c} ① \\ ② \\ ③ \end{array} & \begin{bmatrix} -j6.25 & j3.75 & j2.5 \\ j3.75 & -j6.25 & j2.5 \\ j2.5 & j2.5 & -j6.25 \end{bmatrix} \end{array}$$

$\Rightarrow$ جمعنا 4×4 وطلعت 3×3 لانه ③ مشترك بينهم

$-j6.25 - j6.25 + 2Z_M$



$\Rightarrow$

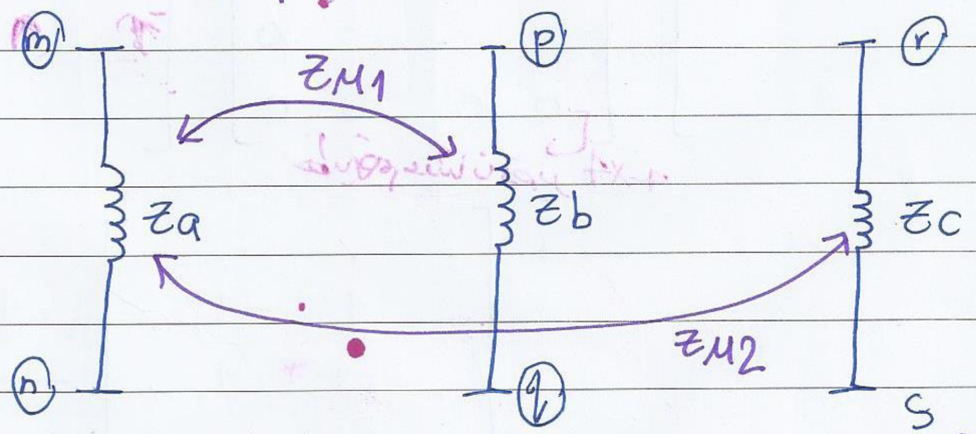


Z Y

بلغي Z_M وبصير اللي بأشر عليهم مش معهم نفس Z_M

$-Z_M$ بينهم مشترك

* For 3-coupled Branches:



$$\begin{bmatrix} Z_a & Z_{M1} & Z_{M2} \\ Z_{M1} & Z_b & 0 \\ Z_{M2} & 0 & Z_c \end{bmatrix}$$

no mutual between ② & ③

$$= \begin{bmatrix} Y_a & Y_{M1} & Y_{M3} \\ Y_{M1} & Y_b & Y_{M2} \\ Y_{M3} & Y_{M2} & Y_c \end{bmatrix}$$

مش شرط يكون 0

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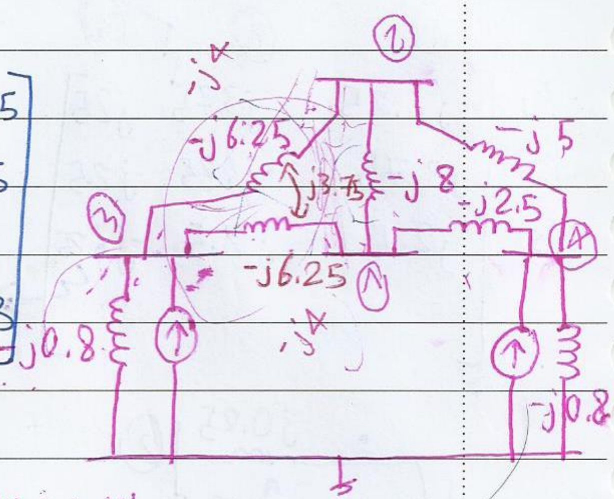
في السؤال

$$Y_{bus\ with\ M} = \begin{bmatrix} -j16.75 & j11.75 & j2.5 & j2.5 \\ j11.75 & -j19.25 & j2.5 & j5 \\ j2.5 & j2.5 & -j\cancel{\ }5.8 & 0 \\ j2.5 & j5 & 0 & -j8.3 \end{bmatrix}$$



Y_{bus} with $M \Rightarrow 4\times4$

?? Y_{bus} M 3×3 → mutual بين اللي بيضم Y_{bus}

$$Y_{bus\ without\ M} = Y_{bus\ with\ M} - Y_{bus\ M} + \underbrace{Y_{13} + Y_{23}}$$ → رجعتهم لانهم راحوا من طرح mutual

$$= \begin{bmatrix} -j16.75 & & & \\ & & & \\ & & & \\ & & & \end{bmatrix} - \begin{bmatrix} -j6.25 & & & 0 \\ & & & 0 \\ & & & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} 1 & \cdot & -1 & \cdot \\ -1 & \cdot & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}(-4j)$$

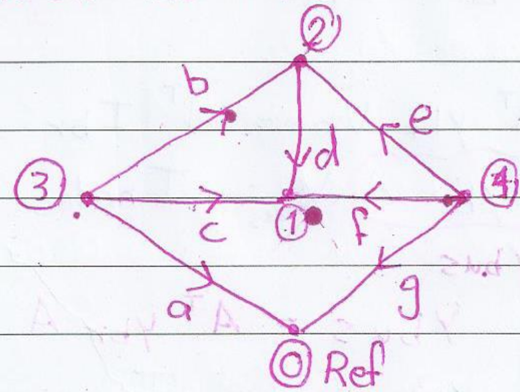
ضفتهم عشان تصير 4x4

$$+ \begin{bmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & 1 & -1 & \cdot \\ \cdot & -1 & 1 & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}(-4j)$$

$$Y_{bus\ M\ 3\times3} = \begin{bmatrix} \ \end{bmatrix} \Rightarrow$$

The network incident matrix:

convert buses to nodes



directed line segment for branche

1, 2, 3, 4 → nodes

a, b, c, d... → branch

The voltage drop cross the branches:

$V_a = V_3$

$V_b = V_3 - V_2$

$V_c = V_3 - V_1$

$V_d = V_2 - V_1$

$V_e = V_4 - V_2$

$V_f = V_4 - V_1$

$V_g = V4$

$$\underbrace{\begin{bmatrix} V_a \\ V_b \\ V_c \\ V_d \\ V_e \\ V_f \\ V_g \end{bmatrix}}_{7\times1,\ V_{branchs}} = \underbrace{\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ -1 & 0 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}_{7\times4}}_{A} \underbrace{\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix}}_{4\times1,\ V_{nodes}\ \text{buses}}$$

$V_{branchs} = A\, V_{nodes}$ **

+1 ← الي خارج

-1 ← الداخل

KCL at nodes:

$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array}\begin{array}{c} \begin{array}{ccccccc} a & b & c & d & e & f & g \end{array} \\ \begin{bmatrix} 0 & 0 & -1 & -1 & 0 & -1 & 0 \\ 0 & -1 & 0 & +1 & -1 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 1 \end{bmatrix} \end{array} \underbrace{\begin{bmatrix} I_a \\ I_b \\ I_c \\ I_d \\ I_e \\ I_f \\ I_g \end{bmatrix}}_{branchs} = \underbrace{\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}}_{buses}$$

$A^T\, I_{branchs} = I_{nodes}$

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we have: $y_{br} V_{br} = I_{br}$

$* A^T \quad A^T y_{br} V_{br} = A^T I_{br}$

$A^T y_{br} A \, V_{nodes} = I_{nodes}$ نعوض

Y_{bus}

$$Y_{bus} = A^T y_{br} A$$

$$y_{br} V_{br} = I_{br}$$

$$\begin{array}{c|c}
 & \begin{array}{ccccccc} 1 & 2 & 3 & & & & \end{array} \\
\begin{array}{c} a \\ b \\ c \\ d \\ e \\ f \\ g \end{array} &
\left[\begin{array}{ccccccc}
-j0.8 & 0 & 0 & & & & \\
0 & -j6.25 & j3.75 & & & & \\
0 & j3.75 & -j6.25 & & & & \\
0 & & & -j8 & & & \\
 & & & & -j5 & & \\
 & & & & & j1.25 & \\
 & & & & & & -j0.8
\end{array}\right]_{7\times7}
\left[\begin{array}{c} V_a \\ V_b \\ V_c \\ V_d \\ V_e \\ V_f \\ V_g \end{array}\right]_{7\times1}
=
\left[\begin{array}{c} I_a \\ I_b \\ I_c \\ I_d \\ I_e \\ I_f \\ I_g \end{array}\right]
\end{array}$$

+ mutual

+ mutual

$y_{branches}$

$$Y_{bus} V = I$$

$$V = Y_{bus}^{-1} I$$

The method of success elemination:

$$Y_{bus} V = I$$

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$$\begin{bmatrix} Y_{11} & Y_{12} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

وإذا كان صعب نوجد inv, ⇐ بنستخدم Gauss elemination

we need $\rightarrow$

$$\begin{bmatrix} 1 & \sim & \sim & \sim \\ 0 & 1 & \sim & \sim \\ 0 & 0 & 1 & \sim \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}$$

gauss elemination to find $\underline{V}$

$V_4 =$

$V_3 + V_4 =$

$V_2 + V_3 + V_4 =$

$V_1 + V_2 + V_3 + V_4 =$

If cancel bus 4 $\rightarrow$

$$\begin{bmatrix} Y_{11} & Y_{12} & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{bmatrix}$$

$$Y^{11}_{new} = Y^{11}_{old} - \frac{Y_{14}\,Y_{41}}{Y_{44}}$$

تقاطع الصف مع العمود اللي فيه Y^{11}

$$Y^{23}_{new} = Y^{23}_{old} - \frac{Y_{43}\,Y_{24}}{Y_{44}}$$

Kron reduction

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Impedence matrix Zbus:

* Direct determination of Zbus:

1. from ref (0) to new (p) → buses + 1 → Zbuses size increase 1.
   [Z_b] → new branch
2. from exicting bus (k) to new bus (p) → buses + 1
   Zbus size increase 1

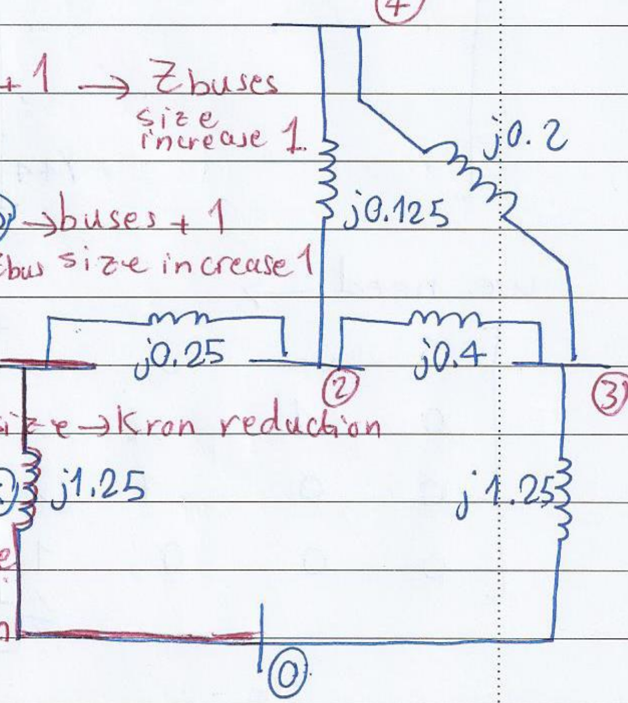
$$\begin{array}{c|c} & \begin{array}{cc} k & p \end{array} \\ \begin{array}{c} k \\ p \end{array} & \left[\begin{array}{c|c} Z_{org.} & \\ \hline & Z_{kk}+Z_b \end{array}\right] \end{array}$$

3. from exiciting bus (k) to ref (0) → same size → Kron reduction

$$\begin{array}{c|c} & \begin{array}{cc} k & 0 \end{array} \\ \begin{array}{c} k \\ 0 \end{array} & \left[\begin{array}{c|c} Z_{ORG} & \\ \hline & Z_{kk}+Z_b \end{array}\right] \end{array}$$

4. between two exiciting buses (j) & (k) → same size → Kron reduction

$$\left[\begin{array}{c|c} Z_{ORG} & col_j - col_k \\ \hline row_j - row_k & Z_{jj}+Z_{kk}-2Z_{jk}+Z_b \end{array}\right]$$



$$\begin{array}{c|c} & 1 \\ 1 & [\,j1.25\,] \end{array}$$

(1)

(2)
$$\begin{bmatrix} j1.25 & j1.25 \\ j1.25 & j1.25 + j0.5 = j1.5 \end{bmatrix}$$

$$\begin{array}{c|c} & \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{array}{c} 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} 1.25 & 1.25 & 1.25 \\ 1.25 & 1.5 & 1.5 \\ 1.25 & 1.5 & 1.5+0.4 = 1.9 \end{bmatrix} \end{array}$$

Kron reduction

③ to ①

$$\begin{bmatrix} 1.25 & 1.25 & 1.25 & 1.25 \\ 1.25 & 1.5 & 1.5 & 1.25 \\ 1.25 & 1.5 & 1.9 & 1.9 \\ 1.25 & 1.25 & 1.9 & 1.9 \end{bmatrix} \Rightarrow \text{reduction}$$

$1.9 + 1.25 = 3.15$

$$\begin{array}{cccc} & ① & ② & ③ \\ \begin{matrix} ① \\ ② \\ ③ \end{matrix} & \multicolumn{3}{c}{\begin{bmatrix} 1.25 - \frac{1.25 \cdot 1.25}{3.15} & \text{—} & \text{—} \\ & & \\ & & \end{bmatrix}} \end{array}$$

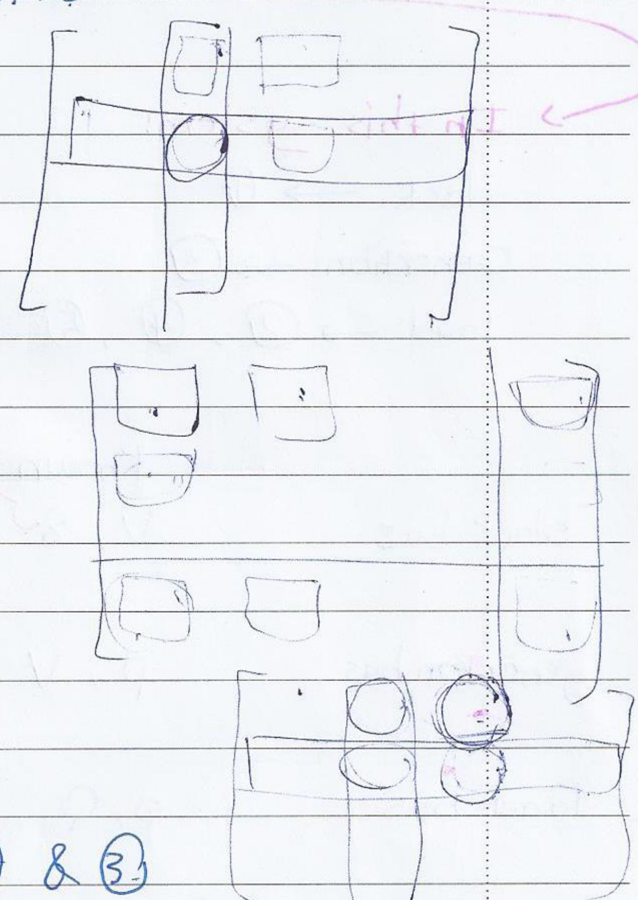
① ② ③ → colum ③

Y_{33} 0.7397

Row ③

$0.3797 + 0.2$

4x4



Between ② ~~exsit~~ exsisting ④ & ③

① ② ③ ④ ~ col.② − col④

① ② ③ ④

row② − row④

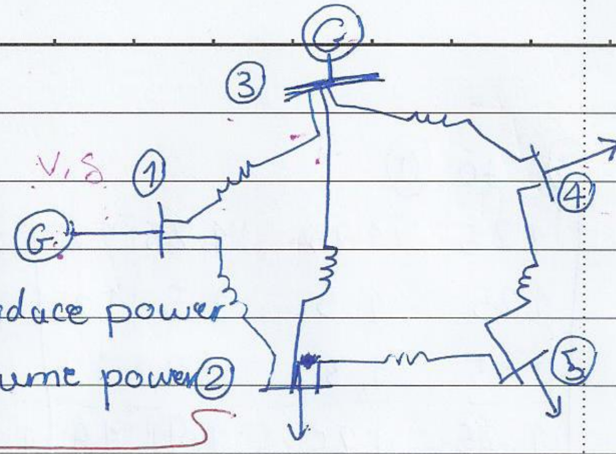
⇒ Reduction 4 x 4

$Z_{44} + Z_{22} - 2 Z_{24} + j0.15$

Subject Date No.

power flow solution:

Buses:
- → slack bus : bus #1
- → genertion bus → produce power
- → load bus → consume power



5 buses → Y_{bus}, Z_{bus} 5X5

genertion bus — load bus

In this system:

slack → ①

Genertion → ③

load → ②, ④, ⑤

V ثابت

	Known	unknown	equations
slack bus	V, δ	p, Q	
genertion bus	p, V	Q, δ	Δp
load bus	p, Q	V, δ	Δp, ΔQ

We have:

$$\bar{Y}_{ij} \angle \theta_{ij} = Y_{ij} \cos \theta_{ij} + j\, Y_{ij} \sin \theta_{ij}$$

$i \rightarrow$ raw

$j \rightarrow$ column

$$= \underbrace{G_{ij}}_{\text{conductance}} + j\, B_{ij}$$

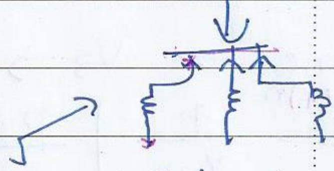
B_{ij} → susbetance

R + j Q

G + j B

Subject Date No.

$V_i = |V_i| \angle \delta_i$



I_i → net current injected

$I_i = y_{i1} V_1 + y_{i2} V_2 + y_{i3} V_3 + \cdots y_{iN} V_N$ كل الcurrent اللي داخلة n

$= \sum_{n=1}^{N} y_{iN} V_N$ او طالعة من i = bus

$\Delta p_i = p_{i,sech} - p_{ical}$

sechale المعطاة → $p_{i,sech}$; calculation المحسوبة → p_{ical}

change (mismatch) $= P_{gi} - P_{di} - P_{ical}$

Where : $p_{i,sech} = P_{gi} - P_{di}$

P_{gi} → generation ; P_{di} → demand

Same for Q ⇒

$\Delta Q_i = Q_{i,sech} - Q_{ical.}$

$= Q_{gi} - Q_{di} - Q_{ical.}$

Consider 4-bus power system

bus ① → slack

at bus ② → $S_2 = P_{2sch} + j\, Q_{2sch} = V_2 I_2^*$

$= P_{2sch} - j\, Q_{2sch} = V_2^* I_2$ → conj للطرفين

$I_2 = \sum_{j=1}^{4} Y_{2j} V_j$

$P_{2sch} - j\, Q_{sch} = V_2^* [Y_{21} V_1 + Y_{22} V_2 + \cdots$

$$V_2^{(1)n} = \frac{1}{Y_{22}} \left[\frac{P_{2sch} - j\, Q_{2sch}}{V_2^{*(0)n-1}} - \left(Y_{21} V_1 + Y_{23} V_3^{(0)n-1} + Y_{24} V_4^{(0)n-1} \right) \right]$$

Y_{21} → Y_{ij}

ما بنقدر عليها ← ما بتغير

Subject No.

لانها طلعتها قبل (اضرفيتها)

Same for $V_3 \Rightarrow$

$$V_3^{(1)n} = \frac{1}{y_{33}}\left[\frac{P_{3sch} - j\,Q_{3sch}}{V_3^{*(0)\,n-1}} - \left(y_{31}V_1 + y_{32}V_2^{(1)n} + y_{34}V_4^{(0)n-1}\right)\right]$$

at slack bus $\Rightarrow \overline{V_1} = |V_1| \angle \delta_1 \Rightarrow$ ثابت ما بتغير

يضل نفس القيمة بكل itertion

$V_2 = V_2^{(0)}$

$V_3 = V_3^{(0)}$

$V_4 = V_4^{(0)}$

$$V_i^{(K)} = \frac{1}{y_{ii}}\left[\frac{P_{isch} - j\,Q_{isch}}{V_i^{*(K-1)}} - \sum_{j=1}^{i-1} y_{ij}V_j - \sum_{j=i+1}^{N} y_{ij}V_j^{(K-1)}\right]$$

باخذ دايما اخر قيمة ← طلعت معي

power flow $\Leftrightarrow$ بالنهاية بدنا

1) V's

2) I's $\Rightarrow$ P, Q

Example:

3-buses power system with G only at bus ①

use Guass seaidal method to find the voltages at buses.

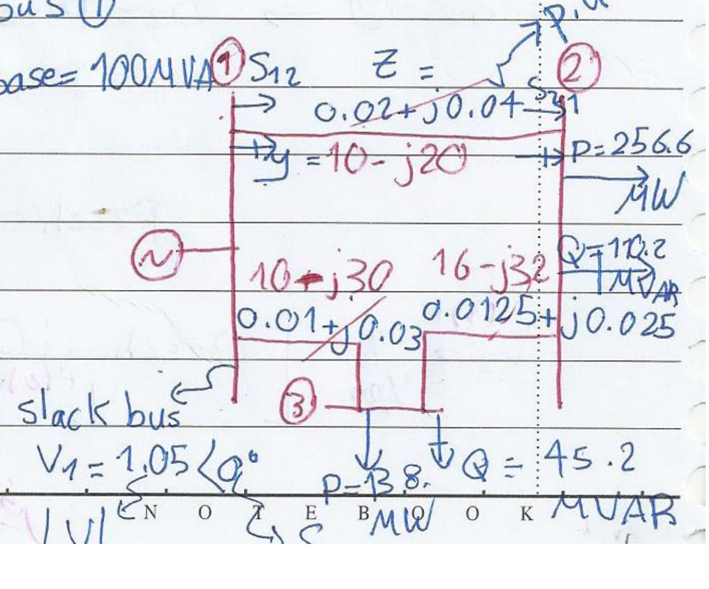
$S_{base} = 100\,MVA$

The admittances:

$y_{12} = \frac{1}{z} = 10 - j20$

$y_{13} = 10 - j30$

$y_{32} = 16 - j32$



إشارة السالب اذا كان load
انتبه اذا كانت z او y
دايماً نتعامل مع p.u

at pQ buses (load buses) $\Rightarrow$

$$S_2^{sch} = \frac{-(256.6 + j\,110.2)}{100}$$

load

$$= -2.566 - j\,1.102 \quad p.u$$

$$S_3^{sch} = \frac{-(138.6 + j\,45.2)}{100}$$

$$= -1.386 - j\,0.452 \quad p.u$$

بالقانون نحط +
عشان ما انلخبط
لانه y_{12} , y_{23} سالبة
$[\,-y_{12}\,]$

assume: $V_2^{(0)} = 1 + j0 = 1\angle 0^\circ$ p.u
$V_3^{(0)} = 1 + j0 = 1\angle 0^\circ$ p.u

أفضل شي دنفرضها 1

$Y_{bus} = [\,Y_{11}\ Y_{12}$

$$V_2^{(1)} = \frac{1}{Y_{22}}\left[\frac{P_{2sch} - j\,Q_{2sch}}{V_2^{(0)*}} - \left(y_{12}V_1 + y_{23}V_3^{(0)}\right)\right]$$

+ $\Rightarrow$ from
− $\Rightarrow$ from y

$$= \frac{1}{26 - j52}\left[\frac{-2.566 + j1.102}{1 - j0} + (10 - j\,20)(1.05 + j0) + (16 - j32)(1 + j\right.$$

$$V_2^{(1)} = 0.9825 - j\,0.0310$$

from fig.

$$V_3^{(1)} = \frac{1}{Y_{33}}\left[\frac{P_{3sch} - j\,Q_{3sch}}{V_3^{(0)*}} + y_{13}V_1 + y_{23}V_2^{(1)}\right]$$

$$= \frac{1}{26 - j62}\left[\frac{-1.386 + j\,0.452}{1 - j0} + (10 - j30)(1.05 + j0) + (16 - j32)(0.9825 - j0.0310)\right.$$

$$V_3^{(1)} = 1.0011 - j\,0.0353$$

Subject | Date | No.

Take four digits!!

* Second itteration ⇒

$V_2^{(2)} = 0.9816 - j0.0520$

$V_3^{(2)} = 1.0008 - j\,0.0459$

$\downarrow$

Stop: ⇒ $V_2^{(7)} = 0.9800 - j0.0600 = 0.98183 \angle -3.5035^\circ$ p.u

$V_3^{(7)} = 1 - j0.05 = 1.00125 \angle -2.8624^\circ$ p.u

⇒ The slack bus power: $\rightarrow S = VI^* = P + jQ$ ⇒ بس افتكر conj.

$$P_1 - jQ_1 = V_1^* \left[V_1 (y_{12} + y_{23}) - (y_{12} V_2 + y_{13} V_3) \right]$$

$$= 1.05 \left[1.05(20 - j50) - (10 - j20)(0.98 - j0.06) - (10 - j30)(1 - j0.05) \right]$$

$$= \underbrace{4.095}_{P_1} - j\,\underbrace{1.890}_{Q_1}$$

$P_1 = 4.095$ p.u $= 4.095 * 100$ MVA $= 409.5$ MW

$Q_1 = 1.890$ p.u $= 1.890 * 100$ MVA $= 1.89$ MVAR

* Line currents

$I_{12} = y_{12}(V_1 - V_2) = 1.9 - j0.8$ p.u

$I_{21} = -I_{12} = -1.9 + j0.8$ p.u.

$I_{13} = y_{13}(V_1 - V_3) = 2 - j1$

$I_{31} = -I_{13} = -2 + j1$

$I_{23} = y_{23}(V_2 - V_3) = -0.64 + j0.48$

$I_{32} = -I_{23} = 0.64 - j0.48$

The line flows:

$S_{12} = V_1 I_{12}^* = (1.05 + j0)(1.9 + j0.8) = 1.995 + j0.84$ p.u

طالعة من bus 1

$= 199.5 + j84$ MVAR

$S_{21} = V_2 I_{21}^* = -191.0$ MW $+ j67$ MVAR

$S_{13} = V_1 I_{13}^* = 210 + j105$

$S_{31} = V_3 I_{31}^* = -205 - j90$

$S_{23} = V_2 I_{23}^* = -65.6 - j43.2$

$S_{32} = V_3 I_{32}^* = 66.4 + j44.8$

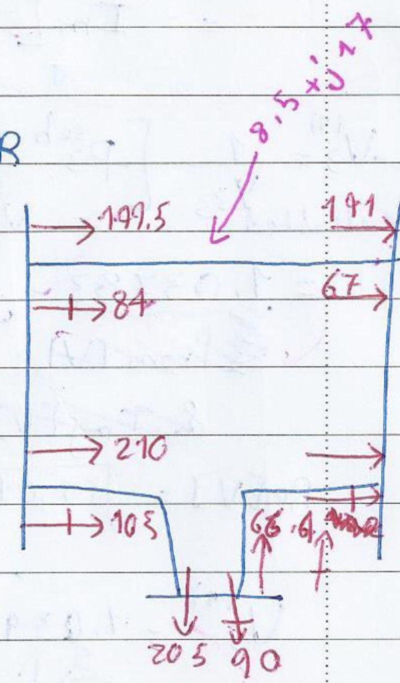
The losses:

جمع متجهات

$S_{L12} = S_{12} + S_{21} = 8.5$ MW $+ j17$ MVAR

$S_{L13} = S_{13} + S_{31} = 5 + j15$

$S_{L23} = S_{23} + S_{32} = 0.8 + j1.6$



load → S → P, Q

S → slack bus

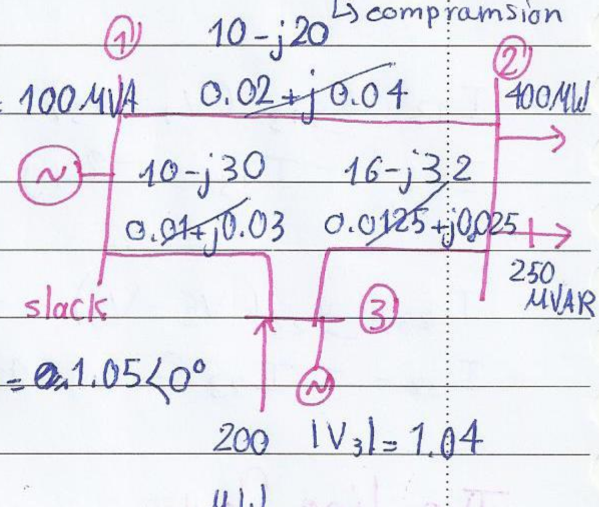
generation bus (pV) → P, V → Q

iteration $\underline{V}$

↳ compramsion

Ex:

$S_{base} = 100\,MVA$



$$S_2^{sch} = \frac{-(400 + j250)}{100} \quad \text{(load)}$$

$$= -4 - j2.5 \text{ pu.}$$

$$P_3^{sch} = +\frac{200}{100} = 2 \text{ p.u}$$

assume: $V_2^{(0)} = 1 \text{ p.u.}$

$V_3^{(0)} = 1.04 \text{ p.u.}$

$$V_2^{(1)} = \frac{1}{y_{22}} \left[\frac{P_2^{sch} - jQ_2^{sch}}{V_2^{(0)*}} + y_{12} V_1 + y_{23} V_3^{(0)} \right]$$

$$= 0.97462 - j0.042307$$

$$Q_3 = -\operatorname{Im}\left[V_3^{*(0)} \left(V_3^{(0)} y_{33} - y_{13} V_1 - y_{23} V_2^{(1)} \right) \right]$$

$$= -\operatorname{Im}[2.362 - j1.16] = 1.16$$

$$V_3^{(1)} = \frac{1}{y_{33}} \left[\frac{P_3^{sch} - jQ_3^{sch}}{V_3^{(0)*}} + y_{13} V_1 + y_{23} V_2^{(1)} \right]$$

calculted

$$= 1.03783 - j\,0.00517$$

~~from |V|~~ ↳ from calculated

~~& Img[V]~~ ↳ constant But $|V_3| = 1.04$ → const.

1.04

$$Re[V] = \sqrt{(1.04)^2 - (0.00517)^2} = 1.039987$$

بيثبت Real ويغير Img

بحيث ال اضلاع ثابتة

$$V_3^{(1)} = \underbrace{1.039987}_{\text{from } |V| \text{ \& Img}[V]} - j\,\underbrace{0.00517}_{\text{constant}}$$

$V_2^{(2)}$

$Q_3^{(2)}$

$V_{3cal.}^{(2)} \longrightarrow V_3^{(2)}$

ببشت Img. ⇐ لازم Q بسبب وجود Img

When Vs became constant ⇒ we find I ⇒ S ⇒ losses ⇒ power flow

Ch3:→

symtrical: 3-phase with ground → أخطر نوع

unsymtrical: single phase

line to line

line to line to ground

transient , steady state → before fault

subtransient triensient — AC only

AC → +ve , -ve

DC → one direction

very large. الماعلى I

small than sub. (أقل)

normal condit ش

upnormal condition → subtransient , transient , steady state

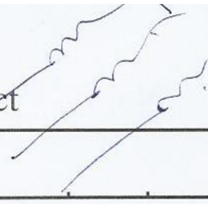
intial dc ≃ $\frac{1}{2}$ AC → $\underline{\underline{0}}$ بعدين بصير

periodic have $\underline{\underline{f}}$

subtransient → T'' → I'' → جزء من الثانية $I'' = 10I$

transient → T' → I' $I' = 5I$

steady state → T → I $T' > T''$



Ex: 1 ~~16~~ X 15, 14 X

I(t) rms $\longrightarrow$ I'', I', I_{ss} T → cycle

$I_{total} = I_{D.C} + I_{A.C}$ Ea → base قسمنا

$\frac{1}{2}$ A.C → 1 p.u

subtransient

one cycle $T = \frac{1}{f} = \frac{1}{60}$

two $\Rightarrow \frac{2}{60} = \frac{1}{30}$

Ex: 2

Base : 100 MVA → transformer

115 KV actual at high voltage side

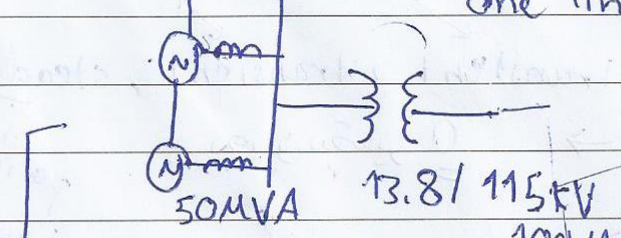
$X_1'' = X_2'' = \underbrace{0.2 * \frac{13.8^2}{50}}_{\text{actual at G}} * \left(\frac{115}{13.8}\right)^2 \div \frac{115^2}{100}$ → base at high voltage side

0.2 → p.u at G; 50 → G base

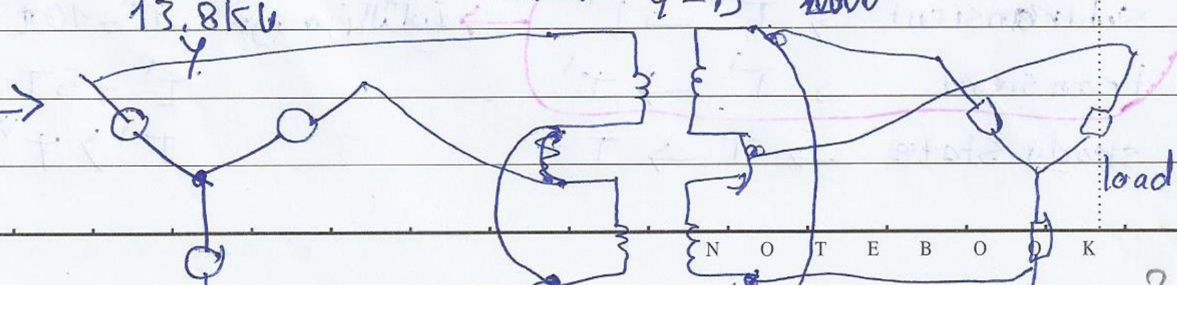
الطرفين الخاصين بالـ transformer

$= 0.4 \Rightarrow$ p.u

one line digram



بس هو بالاصل 3-ϕ



Subject Date No.

3-phase fault for network: intiailly unloaded ⇒ $I=0$, subtransient voltage $=0$

on bus 2 with unloaded ~~bus~~ ⇒

voltage before fault $I=0$, $V=V_2=V_f$

voltage the same between any two buses

2) short cct ⇒ bus two $V=0$ V_f, $-V_f$

how to make bus short cct?

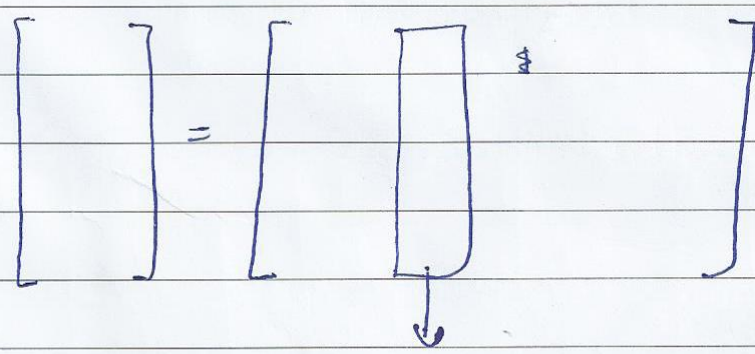
additional voltage with opposite polarit

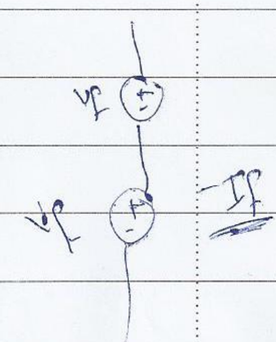
Ybus the same

change in bus 2 $0-\underline{V_f}=-\underline{V_f}$ باقي زي ماهو ⇐ ΔV

$\underbrace{-I_f''}$ ⇒ change in current

$-V_f$ ground لل polarity اتجاه





العمود الثاني راح يتغير

$I_f = \dfrac{V_f}{Z_{nn}}$ → before fault voltage

Z_{nn} → bus ⇒ اللي صار فيها fault

↳ Z_{22}

$0-V_f$

Δ ⇒ other buses ⇒ ratio $\dfrac{Z_{nm}}{Z_{nn}}$ bus fault

change

$V_{new} = V_{old} + \Delta V$

N O T E B O O K

per unit → Z_{bus} $\quad$ $Y \Rightarrow$ branch نفصلهن

Y_{bus} $\quad$ Z بس هلا مش نفصلهن

$Z \Rightarrow$ بانفصلهن ناخذ $^{-1}$

I_f $\quad$ prefault $V_f = 1\angle 0$

$V_f \rightarrow \frac{V_f}{Z_{nn}}$

$\Rightarrow V$ buses $\quad$ $V_n = 0$

I branches $\quad$ That's it

Power System Analysis
"symmetrical faults" Dr. Majdi

3.72

Preliminaries

High currents due to a fault must be detected by protective circuitry and the circuit breakers on the affected transmission line should automatically open for a brief period (about 1/3 second). This will allow ionized air to deionize. If the fault was transient, normal operation should be restored after reclosing the breaker. Therefore, many transient faults are cleared automatically. Otherwise, the circuit breaker should open again isolating the transmission line.

Selecting an appropriate circuit breaker (type, size, etc.) is important…

Type of fault	Abbreviation	Type
1 Single line-to-ground	SLG	Unsymmetrical
2 Line-to-line	LL	Unsymmetrical
3 Double line-to-ground	LLG	Unsymmetrical
4 Symmetrical three-phase	3P	Symmetrical

DC component
Transient

S.S

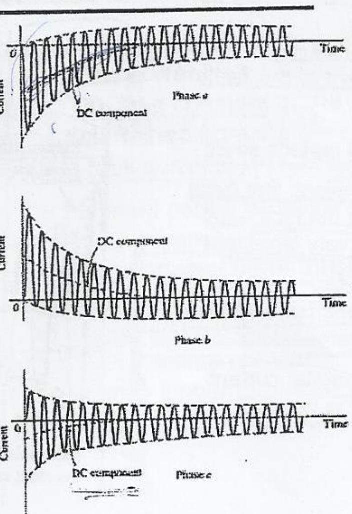
Fault current transients in machines

When a symmetrical 3-phase fault occurs at the terminals of a synchronous generator, the resulting current flow in the phases of the generator can appear as shown.

The current can be represented as a transient DC component added on top of a symmetrical AC component.

~~AC added to DC~~

Therefore, while before the fault, only AC voltages and currents were present within the generator, immediately after the fault, both AC and DC currents are present.



25 Transient | steady state

AC + DC =

before AC, after AC + DC

transient

Fault current transients in machines

When the fault occurs, the AC component of current jumps to a very large value, but the total current cannot change instantly: series inductance of the machine will prevent this. The transient DC component of current is just large enough that the sum of the AC and DC components just ***after*** the fault equals the AC current just ***before*** the fault. Since the instantaneous values of current at the moment of the fault are different in each phase, the magnitude of DC components will be different in different phases.

large after

DC Jump

→ AC before = AC + DC after.

These DC components decay fairly quickly, but they initially average about 50 - 60% of the AC current flow the instant after the fault occurs. The total initial current is therefore typically 1.5 or 1.6 times the AC component alone.

??

AC + DC = 1.5 AC's alone

DC 0.6 AC
0.5 AC

→ (0.6 + 1) A 1.6 AC

(0.5 + 1) AC 1.5 AC

Fault current transients in machines

Periods

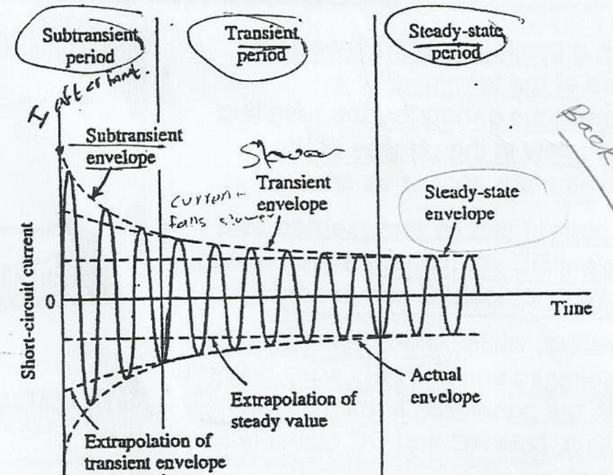
Symmetrical AC component of the fault current:

Roughly 3 periods seen:

1) Subtransient: first cycle or so after the fault – AC current is very large and falls rapidly;

2) Transient: current falls at a slower rate;

3) Steady-state: current gets back to normal.



after fault.

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Fault current transients in machines

Current

$I'' = 10 I$

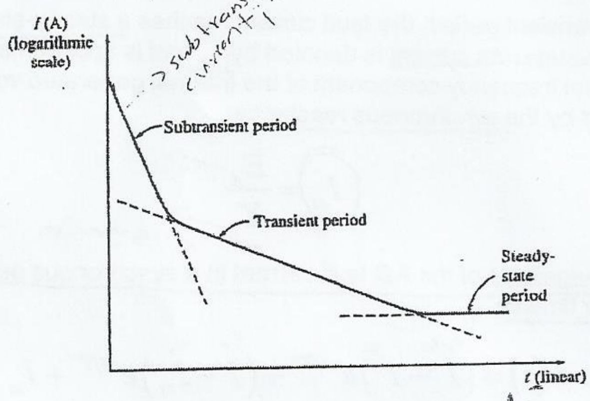
$I' = 5 I$

$T' > T''$

It is possible to observe the three periods of fault current if the rms magnitude of the AC component current is plotted as a function of time on a semilogarithmic scale.

AC semi log ←

It is possible to determine the time constants for the three periods…



Subtransient $\Rightarrow T'' \Rightarrow I''$

Transient $\Rightarrow T' \Rightarrow I'$

steady state $\Rightarrow T \Rightarrow I$

I'' → sub transient current → damper winding → T'' (slope)

I' → transient → DC component of current in field cct → T'

Fault current transients in machines

The AC current flowing in the generator during the subtransient period is called the subtransient current and is denoted by I''. This current is caused by the damper windings of synchronous machines. The time constant of the subtransient current is denoted by T'' and it can be determined from the slope. This current may be 10 times the steady-state fault current. $[I''(t) = 10\, I_{ss}(t)]$ →

fault

The AC current flowing in the generator during the transient period is called the transient current and is denoted by I'. It is caused by a transient DC component of current induced in the field circuit of a synchronous generator at the time of fault. This transient field current increases the internal generated voltage of a machine and, therefore, an increased fault current. The time constant of a field circuit T' is much larger than the time constant of the damper winding, therefore, the transient period lasts longer than the subtransient. This current is often as much as 5 times the steady-state fault current. $\boxed{I'(t) = 5\, I_{steady}(t)}$

→ $(I_F \uparrow, E_A \uparrow, I_F \uparrow)$

→ increase $E_A \uparrow, I_F \uparrow$

$T' > T''$

sub.

I'' → damper winding

I' → field cct

$\uparrow E_a \uparrow I_F$

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Fault current transients in machines

After the transient period, the fault current reaches a steady-state condition. the steady-state rms current is denoted by I_{ss} and is approximated by the fundamental frequency component of the internal generated voltage normalized by the synchronous reactance:

$$I_{ss} = \frac{E_A}{X_s} \quad \text{synch} \tag{12.11.1}$$

The rms magnitude of the AC fault current in a synchronous generator varies over time as

$$I(t) = \left(I'' - I'\right)e^{-t/T''} + \left(I' - I_{ss}\right)e^{-t/T'} + I_{ss} \tag{12.11.2}$$

AC fault current

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Fault current transients in machines

Usually, subtransient and transient reactances are defined for convenience.

The subtransient reactance is the ratio of the fundamental component of the internal generated voltage to the subtransient component of current at the beginning of the fault.

after the fault occurs

$$\text{Subtransient reactance} \leftarrow X'' = \frac{E_A}{I''} \quad E_A \rightarrow \text{internal generated Voltage} \tag{12.12.1}$$

Similarly, the transient reactance is the ratio of the fundamental component of the internal generated voltage to the transient component of current at the beginning of the fault. This value of current is found by extrapolating the transient region back to time zero

$$\text{transient} \leftarrow X' = \frac{E_A}{I'} \tag{12.12.2}$$

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Fault current transients in machines

For the purposes of sizing protective equipment, the subtransient current is often assumed to be

$$I'' = \frac{E_A}{X''} \tag{12.13.1}$$

to find the currents

and the transient current is

$$I' = \frac{E_A}{X'} \tag{12.13.2}$$

which are the maximum values of the respective currents.

Note: we assume that all three phases are shorted simultaneously. If the fault does not affect all three phases equally, the analysis below is not applicable.

unsymmetrical

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Fault transients in synchronous motors

When a short circuit develops in a system containing a synchronous motor, the motor starts acting as a generator converting its mechanical energy (stored in the inertia of its rotor) into electrical power, which is supplied to the power system.

Since a synchronous motor is physically the same machine as a synchronous generator, it also has a subtransient reactance and transient reactance that must be considered in determining the total fault current in the power system.

motor → synch → acts like a generator. (sub transient + transient) reac

→ induc → acts just considered sub transient

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Fault transients in induction motors

An induction motor is an AC machine that has only damper windings on its rotor. Since damper windings are major source of current during the subtransient period, the induction motors in a power system should be considered during calculations of subtransient currents flowing in faults.

Since currents in a damper winding are of little importance during the transient and steady-state periods of faults, induction motors may be ignored in fault current analysis after the subtransient period. → not important (transient + steady

just subtransient.

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Fault current transients

Example 12-1: A 100 MVA, 13.8 kV, Y-connected, 3 phase 60 Hz synchronous generator is operating at the rated voltage and no load when a 3 phase fault occurs at its terminals. Its reactances per unit to the machine's own base are

① I'', I', I_{ss}

$$X_s = 1.00 \qquad X' = 0.25 \qquad X'' = 0.12$$

and the time constants are

$$T' = 1.10\,s \qquad T'' = 0.04\,s$$

The initial DC component in this machine averages 50 percent of the initial AC component. $I_{DC} = 0.5\,I''$ → I''

a) What is the AC component of current in this generator the instant after the fault? $I'' = ??$

b) What is the total current (AC + DC) in the generator right after the fault occurs? $\boxed{I_{Tot} = I'' 1.5}$

c) What will the AC component of the current be after 2 cycles? After 5 s?

$\sqrt{3}$

30

17

Fault current transients

The base current of the generator can be computed as

$$I_{L,base}=\frac{S_{base}}{\sqrt{3}V_{L,base}}=\frac{100,000,000}{\sqrt{3}\cdot 13,800}=4,184\ A$$

The subtransient, transient, and steady-state currents are (per-unit and Amps)

$[I_{actual} = I_{P.u} \cdot I_{Base}]$

$$I''=\frac{E_A}{X''}=\frac{1.0}{0.12}=8.333\ pu=34,900\ A$$

$$I'=\frac{E_A}{X'}=\frac{1.0}{0.25}=4\ pu=16,700\ A$$

$$I_{ss}=\frac{E_A}{X_s}=\frac{1.0}{1.0}=1\ pu=4,184\ A$$

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Fault current transients

a) The initial AC component of current is I'' = 34,900 A.

b) The total current (AC and DC) at the beginning of the fault is

$$I_{tot}=1.5I''=52,350\ A$$

c) The AC component of current as a function of time is

$$I(t)=(I''-I')e^{-\frac{t}{T''}}+(I'-I_{ss})e^{-\frac{t}{T'}}+I_{ss}=18,200\cdot e^{-\frac{t}{0.04}}+12,516\cdot e^{-\frac{t}{1.1}}+4,184\ A$$

actual

After 2 cycles t = 1/30 s and the total current is

$T = \frac{1}{f} = \frac{1}{60}$

$$I\left(\frac{1}{30}\right)=7,910+12,142+4,184=24,236\ A$$

$T = \frac{1}{30}$

which has the largest contribution from the transient current component – transient period. At 5 s, the current reduces to

$$I(5)=0+133+4,184=4,317\ A$$

which is in a steady-state period.

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$S = \sqrt{3}\, V_L I_L$

Base = 100 MVA

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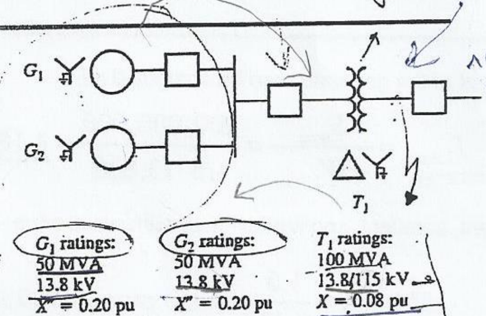
Fault current transients

Example 12-2: Two generators are connected in parallel to the low-voltage side of a transformer. Generators G_1 and G_2 are each rated at 50 MVA, 13.8 kV, with a subtransient resistance of 0.2 pu. Transformer T_1 is rated at 100 MVA, 13.8/115 kV with a series reactance of 0.08 pu and negligible resistance.



Assume that initially the voltage on the high side of the transformer is 120 kV, that the transformer is unloaded, and that there are no circulating currents between the generators.

Calculate the subtransient fault current that will flow if a 3 phase fault occurs at the high-voltage side of transformer. → $I'' = \frac{E_a}{X_s''}$

actual

$$p.u = \frac{\boxed{\text{actual}} * a^2}{\text{Base} = \frac{V^2}{S} = \frac{115^2}{100}} \qquad 0.2\ \text{p.u} * \frac{V^2}{S}\left(\frac{115}{13.8}\right)^2$$

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Fault current transients

Let choose the per-unit base values for this power system to be 100 MVA and 115 kV at the high-voltage side and 13.8 kV at the low-voltage side of the transformer.

The subtransient reactance of the two generators to the system base is

$$Z_{new} = Z_{given}\left(\frac{V_{given}}{V_{new}}\right)^2\left(\frac{S_{new}}{S_{given}}\right)$$

a^2

a^2

Therefore:

$$X_1'' = X_2'' = 0.2\cdot\left(\frac{13{,}800}{13{,}800}\right)^2\left(\frac{100{,}000}{50{,}000}\right) = 0.4\ pu$$

$$\frac{\underbrace{\underbrace{0.2}_{Z_{p.u}} * \underbrace{\frac{(13.8k)^2}{50}}_{Z_{base}}}_{\text{actual}} * \underbrace{\left(\frac{115}{13.8}\right)^2}_{\text{refering}}}{(115)^2/100}$$

refering

The reactance of the transformer is already given on the system base, it will not change

act straight fwd! $X_T = 0.08\ pu$

$$X_1'' = X_2'' = \frac{\left[\overbrace{0.2 * \frac{13.8^2}{50}}^{\text{Pu at gene}} * \left(\frac{115}{13.8}\right)\right]^2}{\frac{115^2}{100}} = 0.4$$

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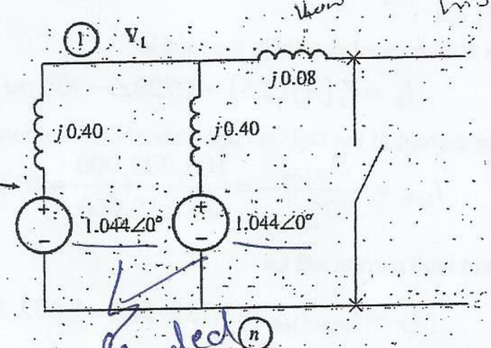
Fault current transients

The per-unit voltage on the high-voltage side of the transformer is

$$V_{pu} = \frac{actual\ value}{base\ value} = \frac{120,000}{115,000} = 1.044\ pu$$

Since there is no load on the system, the voltage at the terminals of each generator, and the internal generated voltage of each generator must also be 1.044 pu. The per-phase per-unit equivalent circuit of the system is



We observe that the phases of internal generated voltages are arbitrarily chosen as 0^0. The phase angles of both voltages must be the same since the generators were working in parallel.

Fault current transients

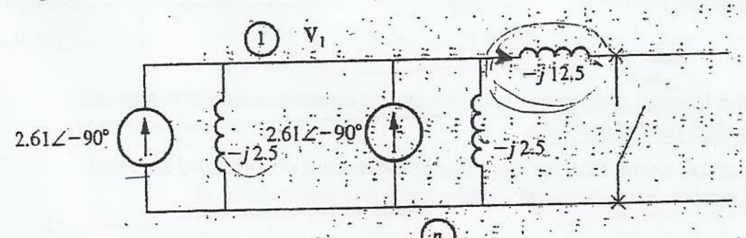
To find the subtransient fault current, we need to solve for the voltage at the bus 1 of the system. To find this voltage, we must convert first the per-unit impedances to admittances, and the voltage sources to equivalent current sources. The Thevenin impedance of each generator is $Z_{Th} = j0.4$, so the short-circuit current of each generator is

Fault

$$I_{sc} = \frac{V_{oc}}{Z_{th}} = \frac{1.044\angle 0°}{j0.4} = 2.61\angle -90°$$

p.u

The equivalent circuit



Fault current transients

Then the node equation for voltage V_1

$$V_1(-j2.5)+V_1(-j2.5)+V_1(-j12.5)=2.61\angle-90°+2.61\angle-90°$$

$$V_1=\frac{5.22\angle 90°}{-j17.5}=0.298\angle 0°$$

Therefore, the subtransient current in the fault is

$$I_F=V_1(-j12.5)=3.729\angle-90°\ pu$$

Since the base current at the high-voltage side of the transformer is

$$I_{base}=\frac{S_{3\phi,base}}{\sqrt{3}V_{LL,base}}=\frac{100,000,000}{\sqrt{3}\cdot 115,000}=502\ A$$

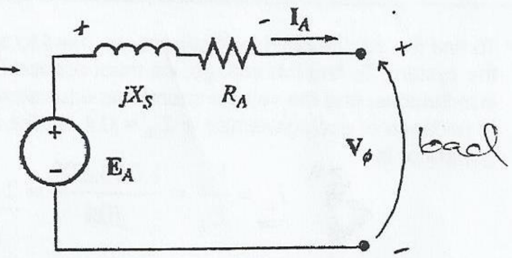
the subtransient fault current will be

$$I_F=I_{F,pu}I_{base}=3.729\cdot 502=1,872\ A$$

EA

Internal generated voltages of loaded machines under transient conditions

The per-phase equivalent circuit of a synchronous generator is shown.



The internal generated voltage is found from the Kirchhoff's voltage law as

$$E_A=V_\phi+jX_sI_A+R_AI_A \qquad (12.24.1)$$

If the series resistance can be ignored, the internal generated voltage is

load

$$E_A=V_\phi+jX_sI_A \qquad (12.24.2)$$

RA=0

Therefore, the internal generated voltage within a synchronous generator will change with load on the machine.

A similar equation exists relating the internal generated voltage and terminal voltage of a synchronous generator under subtransient conditions.

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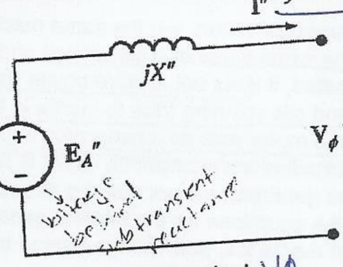
Internal generated voltages of loaded machines under transient conditions



The per-phase equivalent circuit of a synchronous generator under subtransient conditions is shown.

The internal generated voltage found from the Kirchhoff's voltage law is

$$E_A'' = V_\phi + jX''I_A \qquad (12.25.1)$$

E_A'' → before the fault

Therefore, the internal voltage E_A'' under subtransient conditions can be calculated if the load current and the terminal V_ϕ voltage are known just before the fault. The voltage determined by (12.25.1) is the voltage driving the subtransient fault current flow from the generator and is called sometimes the voltage behind subtransient reactance.

Since this voltage varies as a function of the load on the generator before the fault occurs, the subtransient current flow in a fault will depend on the prefault load conditions of the power system. This variation is typically less than 10 % for different load conditions. The voltage behind subtransient reactance can be approximated as the prefault phase voltage of the generator.

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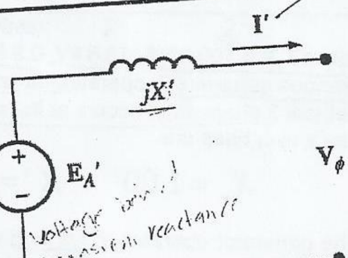
Internal generated voltages of loaded machines under transient conditions



The per-phase equivalent circuit of a synchronous generator under transient conditions is shown.

The internal generated voltage found from the Kirchhoff's voltage law is

$$E_A' = V_\phi + jX'I_A \qquad (12.26.1)$$

Therefore, the internal voltage E_A' under transient conditions can be calculated if the load current and the terminal voltage are known just before the fault. The voltage determined by (12.26.1) is the voltage driving the transient fault current flow from the generator and is called sometimes the voltage behind transient reactance.

E_A'

Since this voltage varies as a function of the load (V_ϕ) on the generator before the fault occurs, the transient current flow in a fault will depend on the prefault load conditions of the power system. This variation is also typically less than 10 % for different load conditions. The voltage behind transient reactance can be approximated as the prefault phase voltage of the generator.

V_ϕ fault

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Internal generated voltages of synchronous motors

Since synchronous motors are the same machines as synchronous generators, they have the same types of subtransient and transient reactances. When the motor is short-circuited, it does not receive power from the line but its field circuit is still energized and still spinning (due to inertia in the machine and in its load). Therefore, the motor acts as a generator, supplying power to the fault. The equivalent circuit of a synchronous motor is the same as the one of the synchronous generator except that the direction of the current flow is reversed. Therefore, the equations for the internal generated voltage, voltage behind the subtransient reactance, and voltage behind transient reactance become

transient and sub-Tran

to the motor = the gene

$$E_A = V_\phi - jX_s I_A \qquad (12.27.1)$$

$$E''_A = V_\phi + jX'' I_A \qquad (12.27.2)$$

$$E'_A = V_\phi + jX' I_A \qquad (12.27.3)$$

di + تتغير ثابت ⊖ IA

These voltages can be used in subtransient and transient fault current analyses similarly to the analysis of synchronous generators.

when we have a fault Motor acts as generator

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Internal generated voltages of loaded machines under transient conditions

Example 12-3: A 100 MVA (S), 13.8 kV (V_L), 0.9 PF ($\cos\theta$) lagging, Y-connected, 3 phase, 60 Hz synchronous generator is operating at rated voltage and $\boxed{\text{full load}}$ when a symmetrical 3 phase fault occurs at its terminals. The reactances in per-unit to the machine's own base are

$$X_s = 1.00 \qquad X' = 0.25 \qquad X'' = 0.12$$

a) If the generator operates at full load when the fault develops, what is the subtransient fault current produced by this generator?

b) If the generator operates at no load and rated voltage when the fault develops, what is the subtransient fault current produced by this generator? Observe that this calculation is equivalent to ignoring the effects of prefault load on fault currents.

c) How much difference does calculating the voltage behind subtransient reactance make in the fault current calculations?

36

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Internal generated voltages of loaded machines under transient conditions

The base current of the generator can be computed as

$S = \sqrt{3}\, V_L I_L$

I_{base}

$$I_{L,base} = \frac{S_{base}}{\sqrt{3}V_{L,base}} = \frac{100{,}000{,}000}{\sqrt{3}\cdot 13{,}800} = 4{,}184\ A \longrightarrow \boxed{\text{rated}}$$

a) Before the fault, the generator was working at rated conditions and the per-unit current was:

Power factor = P< urren

$$I_A = 1.0\angle -25.84°\ pu$$

The voltage behind subtransient reactance is

$$E_A'' = V_\phi + jX'' I_A = 1\angle 0° + (j0.12)\cdot 1\angle -25.84° = 1.058\angle 5.86°$$

Therefore, the per-unit fault current when terminals are shorted is

$$I_F = \frac{1.058\angle 5.86°}{j0.12} = 8.815\angle -84.1°\ pu = 36{,}880\ A$$

x

No load ⇒ Currents Zero

30

Internal generated voltages of loaded machines under transient conditions

b) Before the fault, the generator was assumed to be at no-load conditions and the per-unit current was:

$$I_A = 0.0\angle 0°\ pu$$

internal reactance

The voltage behind subtransient reactance is

$$E_A'' = V_\phi + jX'' I_A = 1\angle 0° + (j0.12)\cdot 0\angle 0° = 1.0\angle 0°$$

Therefore, the per-unit fault current when the terminals are shorted is

$$I_F = \frac{1.0\angle 0°}{j0.12} = 8.333\angle -90°\ pu = 34{,}870\ A$$

c) The difference in fault current between these two cases is

$$Difference = \frac{36{,}880 - 34{,}870}{34{,}870}\cdot 100\% = 5.76\%$$

The difference in the fault current when the voltage behind subtransient reactance is considered and when it is ignored is small and usually systems are assumed unloaded.

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Fault current calculations using the impedance matrix

So far, we were considering simple circuits. To determine the fault current in a system:

1) Create a per-phase per-unit equivalent circuit of the power system using either subtransient reactances (if subtransient currents are needed) or transient reactances (if transient currents are needed).

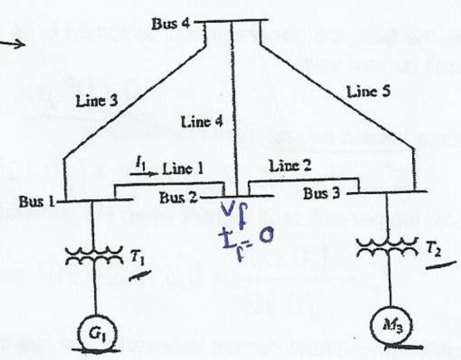
2) Add a short circuit between one node of the equivalent circuit and the neutral and calculate the current flow through that short by standard analysis.

This approach always works but can get complex while dealing with complex systems. Therefore, a nodal analysis technique will be used.

We introduce a new voltage source in the system to represent the effects of a fault at a bus. By solving for the currents introduced by this additional voltage source, we will find fault currents automatically…

Fault current calculations using the impedance matrix

Let us consider a power system shown. →



Assuming that we need to find the subtransient fault current at some node in the system, we need to create a per-phase, per-unit equivalent circuit using subtransient reactances X". Additionally, we assume that the system is initially unloaded, making the voltages behind subtransient reactances

$$\begin{cases} E''_{A_1} = 1\angle 0° \ pu \\ E''_{A_2} = 1\angle 0° \ pu \end{cases}$$

معناه
fault قبل $V = V_f$

Fault current calculations using the impedance matrix

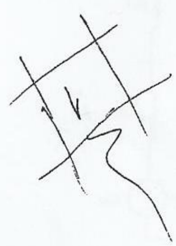
The resulting equivalent circuit is shown.

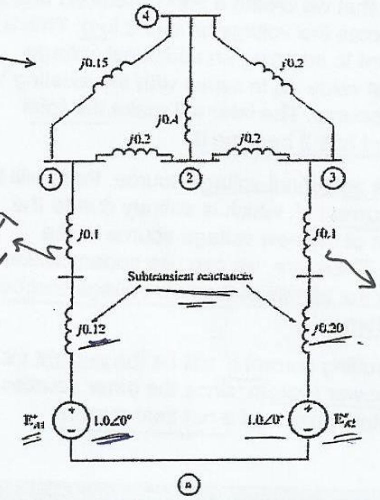
Suppose that we need to determine the subtransient fault current at $\boxed{\text{bus 2}}$ when a symmetrical 3 phase fault occurs on that bus.

intially unloaded

$I = 0$

$V_2 = V_f$

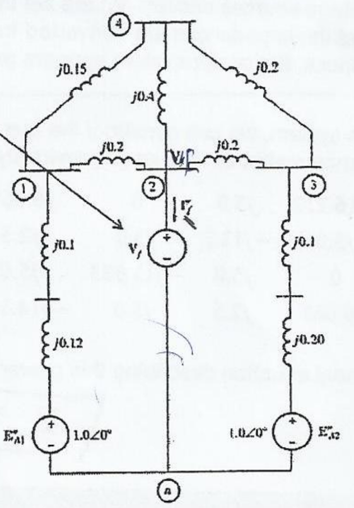




Fault current calculations using the impedance matrix

Before the fault, the voltage on bus 2 was V_f. If we introduce a voltage source of value V_f between bus 2 and the neutral, nothing will change in the system.

Since the system operates normally before the fault, there will be no current I_f'' through that source.



V_2

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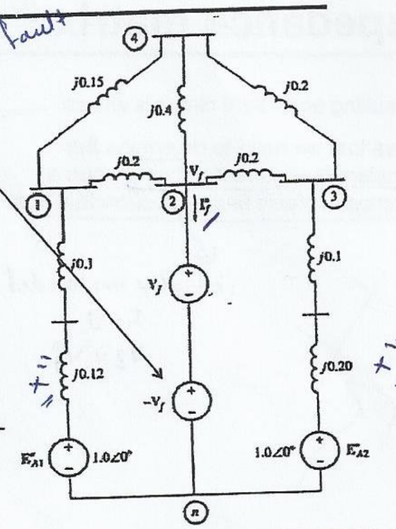
Fault current calculations using the impedance matrix

Assume that we create a short circuit on bus 2, → fault which forces the voltage on bus 2 to 0. This is equivalent to inserting an additional voltage source of value $-V_f$ in series with the existing voltage source. The later will make the total voltage at bus 2 become 0.

With this additional voltage source, there will be a fault current I_f'', which is entirely due to the insertion of the new voltage source to the system. Therefore, we can use superposition to analyze the effects of the new voltage source on the system.

The resulting current I_f'' will be the current for the entire power system, since the other sources in the system produced a net zero current.



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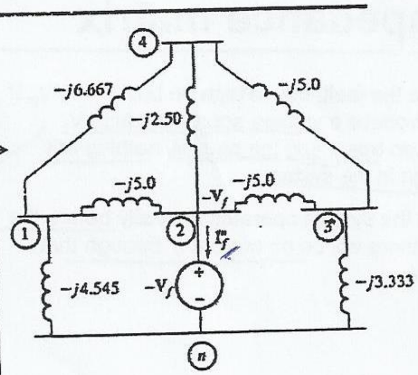
Fault current calculations using the impedance matrix

If all voltage sources except $-V_f''$ are set to zero and the impedances are converted to admittances, the power system appears as shown.



For this system, we can construct the bus admittance matrix as discussed previously:

$$Y_{bus} = \begin{bmatrix} -j16.212 & j5.0 & 0 & j6.667 \\ j5.0 & -j12.5 & j5.0 & j2.5 \\ 0 & j5.0 & -j13.333 & j5.0 \\ j6.667 & j2.5 & j5.0 & -j14.167 \end{bmatrix}$$

The nodal equation describing this power system is

$$\boxed{\mathbf{Y_{bus} V = I}} \qquad (12.36.1)$$

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Fault current calculations using the impedance matrix

With all other voltage sources set to zero, the voltage at bus 2 is $-V_f$, and the current entering the bus 2 is $-I_f''$. Therefore, the nodal equation becomes

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ -V_f \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -I_f'' \\ 0 \\ 0 \end{bmatrix} \qquad (12.37.1)$$

where ΔV_1, ΔV_3, and ΔV_4 are the changes in the voltages at those busses due to the current $-I_f''$ injected at bus 2 by the fault.

The solution to (12.37.1) is found as

$$\mathbf{V} = \mathbf{Y}_{\mathbf{bus}}^{-1}\mathbf{I} = \mathbf{Z}_{\mathbf{bus}}\mathbf{I} \qquad (12.37.2)$$

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Fault current calculations using the impedance matrix

colum of bus (fault)

Which, in the case considered, is

$$\begin{bmatrix} \Delta V_1 \\ -V_f \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} 0 \\ -I_f'' \\ 0 \\ 0 \end{bmatrix} \qquad (12.38.1)$$

4x4 4x1

where $Z_{bus} = Y_{bus}^{-1}$. Since only bus 2 has current injected at it, the system (12.38.1) reduces to

$$\begin{aligned} \Delta V_1 &= -Z_{12} I_f'' \\ -V_f &= -Z_{22} I_f'' \\ \Delta V_3 &= -Z_{32} I_f'' \\ \Delta V_4 &= -Z_{42} I_f'' \end{aligned} \qquad (12.38.2)$$

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Fault current calculations using the impedance matrix

Therefore, the fault current at bus 2 is just the prefault voltage V_f at bus 2 divided by Z_{22}, the driving point impedance at bus 2.

$$I_f'' = \frac{V_f}{Z_{22}} \qquad (12.39.1)$$

The voltage differences at each of the nodes due to the fault current can be calculated by substituting (12.39.1) into (12.38.2):

$$\begin{aligned}
\Delta V_1 &= -Z_{12} I_f'' = -\frac{Z_{12}}{Z_{22}} V_f \\
\Delta V_2 &= -V_f = -V_f \\
\Delta V_3 &= -Z_{32} I_f'' = -\frac{Z_{32}}{Z_{22}} V_f \\
\Delta V_4 &= -Z_{42} I_f'' = -\frac{Z_{42}}{Z_{22}} V_f
\end{aligned} \qquad (12.39.2)$$

Fault current calculations using the impedance matrix

Assuming that the power system was running at no load conditions before the fault, it is easy to calculate the voltages at every bus during the fault. At no load, the voltage will be the same on every bus in the power system, so the voltage on every bus in the system is V_f. The change in voltage on every bus caused by the fault current $-I_f''$ is specified by (12.39.2), so the total voltage during the fault is

during fault

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \underbrace{\begin{bmatrix} V_f \\ V_f \\ V_f \\ V_f \end{bmatrix}}_{\text{pre-fault voltage}} + \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} V_f \\ V_f \\ V_f \\ V_f \end{bmatrix} + \begin{bmatrix} -\frac{Z_{12}}{Z_{22}} V_f \\ -V_f \\ -\frac{Z_{32}}{Z_{22}} V_f \\ -\frac{Z_{42}}{Z_{22}} V_f \end{bmatrix} = \begin{bmatrix} 1-\frac{Z_{12}}{Z_{22}} \\ 0 \\ 1-\frac{Z_{32}}{Z_{22}} \\ 1-\frac{Z_{42}}{Z_{22}} \end{bmatrix} V_f \qquad (12.40.1)$$

shortcct

Therefore, we can calculate the voltage at every bus in the power system during the fault from a knowledge of the pre-fault voltage at the faulted bus and the bus impedance matrix!

اعرف V قبل fault ~~Also~~ Zbus

42

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Fault current calculations using the impedance matrix

Once these bus voltages are known, we can calculate the fault current flowing in the transmission line using bus voltages and the bus admittance matrix.

The general procedure for finding the bus voltages and line currents during a symmetrical 3 phase fault is as follows:

1. Create a per-unit equivalent circuit of the power system. Include subtransient reactances of each synchronous and induction machine when looking for subtransient fault currents; include transient reactances of each synchronous machine when looking for transient fault currents. Y, Z
2. Calculate the bus admittance matrix Y_{bus}. Include the admittances of all transmission lines, transformers, etc. between busses including the admittances of the loads or generators themselves at each bus.
3. Calculate the bus impedance matrix Z_{bus} as inverse of the bus admittance matrix. why we need it? → because they are in laws.
4. Assume that the power system is at no load and determine the voltage at every bus, which will be the same for every bus and the same as the internal voltage of the generators in the system. This is the pre-fault voltage V_f. Volt. قبل

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Fault current calculations using the impedance matrix

5. Calculate the current at the faulted bus *i* as

$$I''_{f,i} = \frac{V_f}{Z_{ii}} \tag{12.42.1}$$

6. Calculate the voltages at each bus during the fault. The voltage at bus *j* during a symmetrical 3 phase fault at the bus *i* is found as

$$V_j = \left(1 - \frac{Z_{ji}}{Z_{ii}}\right) V_f \tag{12.42.2}$$

7. Calculate the currents in any desired transmission line during the fault. The current through a line between bus *i* and bus *j* is found as

$$\boxed{I_{ij} = -Y_{ij}\left(V_i - V_j\right)} \tag{12.42.3}$$

fault

matrix من

4/3

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Fault current calculations using the impedance matrix

Example 12-4: The power system depicted in slide 32 is working at no load when a symmetrical 3 phase fault is developed on bus 2.

Y_{bus} Z_{bus}

a) Calculate the per-unit subtransient fault current I_f'' at bus 2.

b) Calculate the per-unit voltage at every bus in the system during the subtransient period. $\Delta V_i = -Z_{ij} * I_f$ $V_f + \Delta V$

$I'' = \frac{V_f}{Z_{ii}}$ 22

c) Calculate the per-unit current I_1 flowing in line 1 during the subtransient period of the fault.

$V_f = 1\angle 0$ p.u

refer to it's self

Following the 7 steps discussed before, we write:

1. The per-phase per-unit equivalent circuit is shown in slide 33.
2. The bus admittance matrix was previously calculated as

$$Y_{bus} = \begin{bmatrix} -j16.212 & j5.0 & 0 & j6.667 \\ j5.0 & -j12.5 & j5.0 & j2.5 \\ 0 & j5.0 & -j13.333 & j5.0 \\ j6.667 & j2.5 & j5.0 & -j14.167 \end{bmatrix}$$

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Fault current calculations using the impedance matrix

3. The bus impedance matrix calculated using Matlab as the inverse of Y_{bus} is

$$Z_{bus} = \begin{bmatrix} 0+j0.1515 & 0+j0.1232 & 0+j0.0934 & 0+j0.1260 \\ 0+j0.1232 & 0+j0.2104 & 0+j0.1321 & 0+j0.1417 \\ 0+j0.0934 & 0+j0.1321 & 0+j0.1726 & 0+j0.1282 \\ 0+j0.1260 & 0+j0.1417 & 0+j0.1282 & 0+j0.2001 \end{bmatrix}$$

4. For the given power system, the no-load voltage at every bus is equal to the pre-fault voltage at the bus that is

$$V_f = 1.00\angle 0° \ pu$$

5. The current at the faulted bus is computed as

$$I''_{f,2} = \frac{V_f}{Z_{22}} = \frac{1.00\angle 0°}{j0.2104} = 4.753\angle -90° \ pu$$

$-j4.753$

$\frac{V_i - V_j}{Z_{ij}} \Rightarrow -Y_{ij}(V_i - V_j)$

from matrix

44

120 120 120 symtrical

7 5 10

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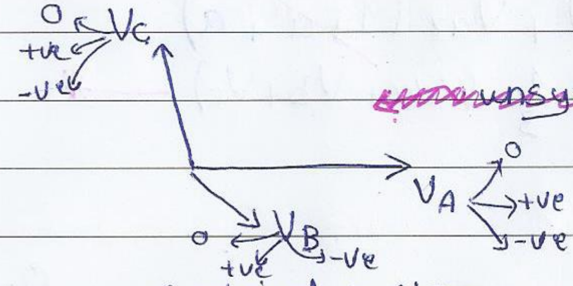
unsymtrical: Amplitude or phase or both

-ve comp
+ve comp
zero

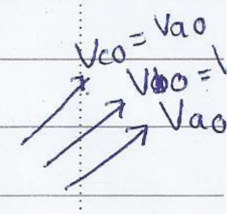
Ch: symetrical components:



symtrical system

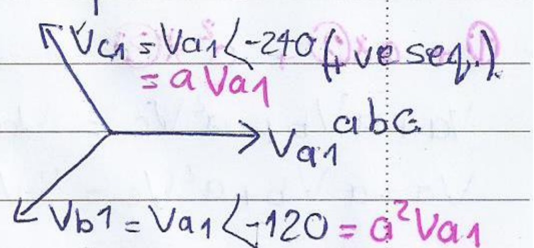
$\Longrightarrow$ can be resolved into 3-balanced ~~unsymtrical system~~ system:

1) Zero seqence



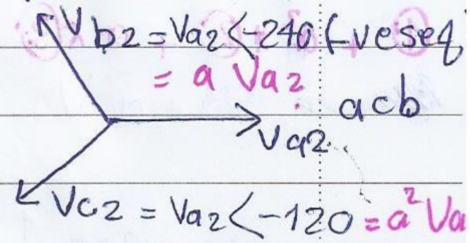
+

2)



+

3)



* Zero sequence: three components with same magnitude and phase

$$V_A = V_{ao} + V_{a1} + V_{a2} = V_{ao} + V_{a1} + V_{a2} \quad (1)$$
$$V_B = V_{bo} + V_{b1} + V_{b2} = V_{ao} + a^2 V_{a1} + a V_{a2} \quad (2)$$
$$V_C = V_{co} + V_{c1} + V_{c2} = V_{ao} + a V_{a1} + a^2 V_{a2} \quad (3)$$

Assume: $1\angle 120^\circ = a$

$+120$: $1\angle 240^\circ = a * a = a^2$

$+120$: $1\angle 360 = 1\angle 0 = a^3$

$\Rightarrow a^4 = a^3\, a = a$

$a^{23} = a^2 (a^3)^7 = a^2$

$1 = a^{n3}$

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{ao} \\ V_{a1} \\ V_{a2} \end{bmatrix}$$

$$[V_{abc}] = [A]\,[V_{a012}]$$

symtrical

unsymtrical

$a = 1\angle 120^\circ$

$$\Rightarrow [V_{012}] = [A^{-1}]\,[V_{abc}]$$

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①+②+③

$$V_a + V_b + V_c = 3V_{a0} + V_{a1}\cancel{(1+a+a^2)}^{0} + V_{a2}\cancel{(1+a^2+a)}^{0}$$

$$= 3V_{a0} \Rightarrow V_{a0} = \frac{1}{3}(V_a + V_b + V_c) \longrightarrow *$$

① + a*② + a^2*③

$$V_a + aV_b + a^2V_c = V_{a0}\cancel{(1+a+a^2)}^{0} + V_{a1}(1+a^3+a^3) + \cancel{(1+a^3+a^2)}^{0}V_{a2}$$

$$V_a + aV_b + a^2V_c = 3V_{a1} \Rightarrow V_{a1} = \frac{1}{3}(V_a + aV_b + a^2V_c) \longrightarrow *$$

① + a^2*② + a*③

$$V_a + a^2V_b + aV_c = V_{a0}\cancel{(1+a^2+a)}^{0} + V_{a1}\cancel{(1+a+a^2)}^{0} + V_{a2}(1+a^3+a^3)$$

$$= 3V_{a2}$$

$$\Rightarrow V_{a2} = \frac{1}{3}(V_a + a^2V_b + aV_c) \longrightarrow *$$

$$\Rightarrow \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}$$

$\longrightarrow A^{-1}$

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \Rightarrow A^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^3 \\ 1 & a^2 & a \end{bmatrix}$$

$$A^{-1} = \frac{1}{3}A$$

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$\Rightarrow \quad V_{abc} = A \, V_{a012}$

$V_{a012} = \frac{1}{3} A^{-1} V_{abc}$

Where: $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$

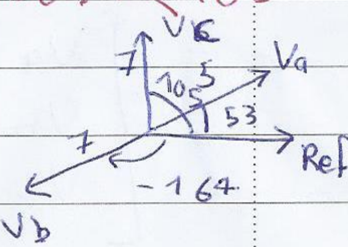
$$A^{-1} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}$$

Same for currents:

$[I_{abc}] = [A]\,[I_{012}]$

$[I_{012}] = A^{-1}\,[I_{abc}]$

Ex: Given that $V_a = 5\angle 53 \quad V_b = 7\angle -164 \quad V_c = 7\angle 105$

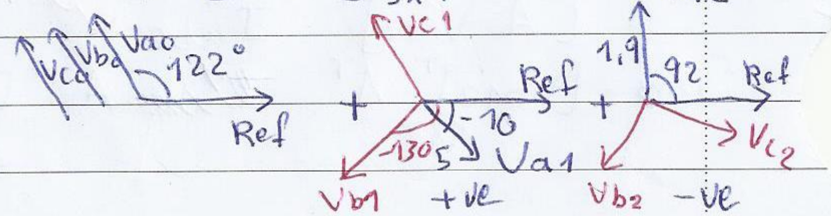


$V_{a012} = A^{-1} V_{abc}$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 5\angle 53 \\ 7\angle -164 \\ 7\angle 105 \end{bmatrix}$$

3x1

$$= \begin{bmatrix} 3.5\angle 122 \\ 5\angle -10 \\ 1.9\angle 92 \end{bmatrix}$$



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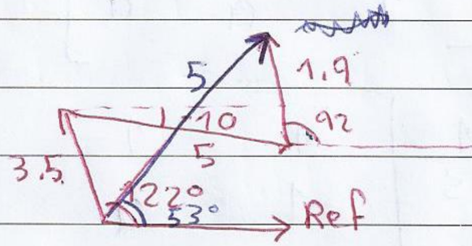
$\Rightarrow$ $V_{b0} = 3.5\angle 122^\circ$
$V_{b1} = 5\angle -130$
$V_{b2} = 1.9\angle -148$

$\Rightarrow$ $V_{c0} = 3.5\angle 122^\circ$
$V_{c1} = 5\angle 110$
$V_{c2} = 1.9\angle -28$

Note that:

$$3.5\angle 122^\circ + 5\angle -10 + 1.9\angle 92 = ?? = 5\angle 53 = V_a$$



abc ⇐ اللي على الواقع

012 ⇐ تحليل للـabc

$$S_{3\text{-}\phi} = P_{3\text{-}\phi} + jQ_{3\text{-}\phi}$$
$$= S_a + S_b + S_c$$
$$= V_a I_a^* + V_b I_b^* + V_c I_c^*$$
$$= [V_a \quad V_b \quad V_c]\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$
$$= \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix}^T \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}^*$$
$$= V_{abc}^T \; I_{abc}^*$$
$$= [A\,V_{012}]^T \; I_{abc}^* \rightarrow [A\,I_{012}]^*$$
$$S_{3\text{-}\phi} = \underbrace{[V_{012}]^T}_{1\times3} A^T [A]^* [I_{012}]^*$$

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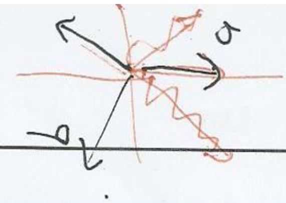


$A^* = 3A^{-1}$, $A^T = A$

$$S_{3\phi} = [V_{012}]^T \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} I_{012}^*$$

$$= [V_{012}]^T \cdot 3\, I_{3\times3}\, [I_{012}]^*$$

$$= 3\, [V_{012}]^T\, [I_{012}]^*$$

$$= 3 \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix}^T \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

$$= 3\, [V_{a0} \quad V_{a1} \quad V_{a2}] \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}^*$$

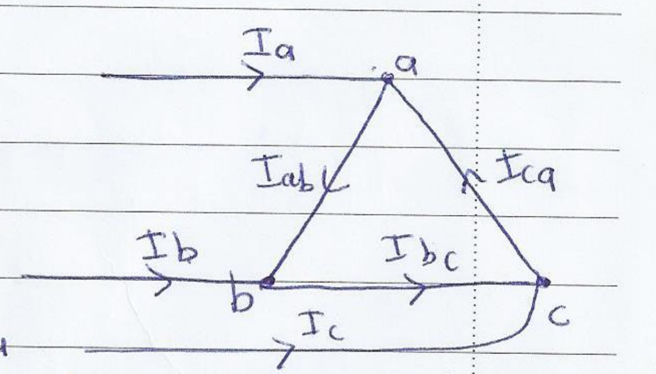
$$S_{3\phi} = 3\,(V_{a0} I_{a0}^* + V_{a1} I_{a2}^* + V_{a2} I_{a2}^*)$$ ⟹ بجزها من وحدة
مرهم بس Symtrical

Case 1:

$I_a = I_{ab} - I_{ca}$

$I_b = I_{bc} - I_{ab}$

$I_c = I_{ca} - I_{bc}$



$$\Rightarrow \underbrace{I_a + I_b + I_c}_{3 I_0 = 0} = I_{ab} + I_{bc} + I_{ca}$$

(0 1 2 0 1 2 0 1 2)

$-(I_{ca} + I_{ab} + I_{bc}) = 0$

$3 I_0 = 0$

$I_{a0} = 0 \Rightarrow I_{a0} = I_{b0} = I_{c0} = 0$ (No zero seq.) for any Δ-connection

$\Rightarrow I_a = I_{a1} + I_{a2}$

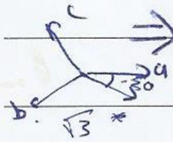
$I_b = I_{b1} + I_{b2}$

$I_c = I_{c1} + I_{c2}$

We have: $I_a = \underbrace{I_{ab}}_{0\ 1\ 2} - \underbrace{I_{ca}}_{0\ 1\ 2} = I_{a1} + I_{a2}$

$(\cancel{I_{ab0}} - I_{ca0}) + (I_{ab1} - I_{ca1}) + (I_{ab2} - I_{ca2})$

$= I_{a1} + I_{a2}$



$\Rightarrow I_{a1} = I_{ab1} - \overset{= a I_{ab1}}{I_{ca1}} = I_{ab1}\sqrt{3}\angle -30$

$I_{a2} = I_{ab2} - \overset{= a^2 I_{ab2}}{I_{ca2}} = \sqrt{3}\, I_{ab2}\angle 30$

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grounded without impedance

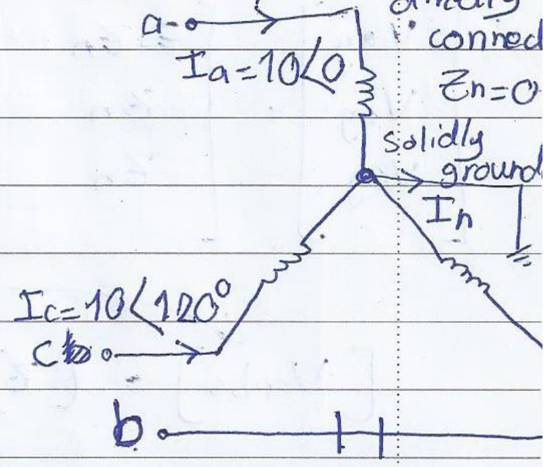
directly connected $Z_n = 0$

solidly ground

* 3-ϕ line feed a balance y-load with one phase open

Find: I_{012} & I_n. ~~Balanced~~

balanced ~~but with open~~ unblanced



$$I_{a012} = A^{-1}\, I_{abc}$$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 10\angle 0 \\ a \\ 10\angle 12a \end{bmatrix}$$

$$= \begin{bmatrix} 3.333\angle 60 \\ 0 \\ 3.333\angle -60 \end{bmatrix} \rightsquigarrow 20???$$

unblanced $\Leftarrow I_b = 0$. open

$I_b = \cancel{10}\angle 240$

$$I_n = I_a + I_b + I_c$$

0 1 2 0 1 2 0 1 2 $\rightsquigarrow$ components

$$= 3 I_0$$

$$= 3 \cdot 3.333\angle 60$$

$$= 10\angle 60.$$

OR: diredly from phase.

$$I_n = 10\angle 0 + 0 + 10\angle 120$$

$$= 10\angle 60$$

***

$I_n = I_a + I_b + I_c$ a

$$V_{ag} = I_a Z + I_n Z_n = (Z + Z_n) I_a + Z_n I_b + Z_n I_c$$

$$V_{bg} = I_b Z + I_n Z_n = Z_n I_a + (Z + Z_n) I_b + Z_n I_c$$

$$V_{cg} = I_c Z + I_n Z_n = Z_n I_a + Z_n I_b + (Z + Z_n) I_c$$

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grounded without impedance

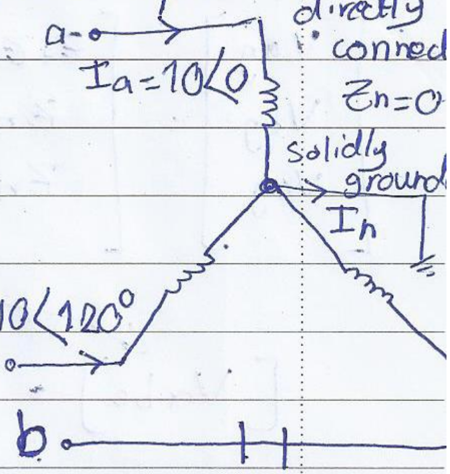
directly connected $Z_n = 0$

solidly grounded

* 3-ϕ line feed a balance y-load with one phase open

Find: I_{012} & I_n ~~Unbalanced~~

balanced ~~but with open~~ unblanced



$I_{a012} = A^{-1}\, I_{abc}$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 10\angle 0 \\ a \\ 10\angle 120 \end{bmatrix}$$

$$= \begin{bmatrix} 3.333\angle 60 \\ 0 \\ 3.333\angle -60 \end{bmatrix} \rightsquigarrow 20\,???$$

unblanced $\leftarrow I_b = 0$

$I_b = \cancel{10}\angle 240$

$I_n = I_a + I_b + I_c$

0 1 2 0 1 2 0 1 2 $\rightsquigarrow$ components

$= 3 I_0$

$= 3 \cdot 3.333\angle 60$

$= 10\angle 60.$

OR: diredly from phase.

$I_n = 10\angle 0 + 0 + 10\angle 120$

$= 10\angle 60$

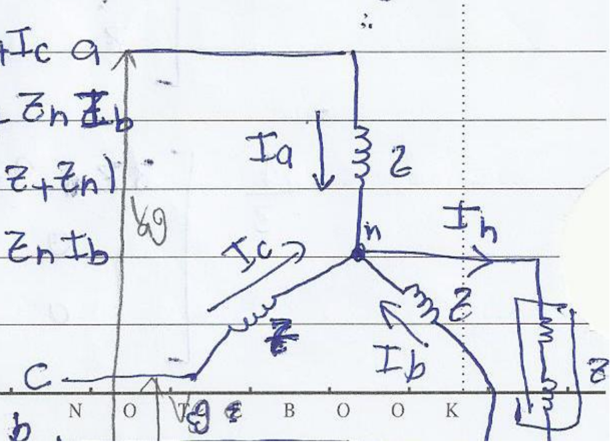
**

$I_n = I_a + I_b + I_c$

$V_{ag} = I_a Z + I_n Z_n = (Z + Z_n) I_a + Z_n I_b$

$V_{bg} = I_b Z + I_n Z_n = Z_n I_a + (Z + Z_n)$

$V_{cg} = I_c Z + I_n Z_n = Z_n I_a + Z_n I_b$



$$\underbrace{\begin{bmatrix} V_{ag} \\ V_{bg} \\ V_{cg} \end{bmatrix}}_{V_{abc}} = \underbrace{\begin{bmatrix} Z+Z_n & Z_n & Z_n \\ Z_n & Z+Z_n & Z_n \\ Z_n & Z_n & Z+Z_n \end{bmatrix}}_{Z_{abc}} \underbrace{\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}}_{I_{abc}}$$

$$[V_{abc}] = [Z]_{abc} [I_{abc}]$$

$$[A][V_{012}] = [Z_{abc}] \cdot [A][I_{012}]$$

$$[V_{012}] = \underbrace{[A^{-1}][Z_{abc}][A]}_{Z_{012}}[I_{012}]$$

we have:

$$[Z_{012}] = [A]^{-1} [Z_{abc}] [A]$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z+Z_n & Z_n & Z_n \\ Z_n & Z+Z_n & Z_n \\ Z_n & Z_n & Z+Z_n \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} Z+3Z_n & Z+3Z_n & Z+3Z_n \\ Z & aZ & a^2 Z_n \\ Z & a^2 Z & aZ \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$= \frac{1}{3} \begin{bmatrix} 3Z+9Z_n & 0 & 0 \\ 0 & 3Z & 0 \\ 0 & 0 & 3Z \end{bmatrix}$$

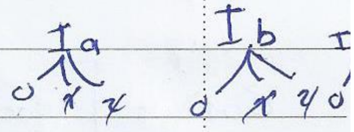
$$Z_{012} = \begin{bmatrix} Z+3Z_n & 0 & 0 \\ 0 & Z & 0 \\ 0 & 0 & Z \end{bmatrix}$$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} Z+3Z_n & 0 & 0 \\ 0 & Z & 0 \\ 0 & 0 & Z \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

$V_0 = (Z + 3Z_n) I_0$

$V_1 = Z I_1$

$V_2 = Z I_2$

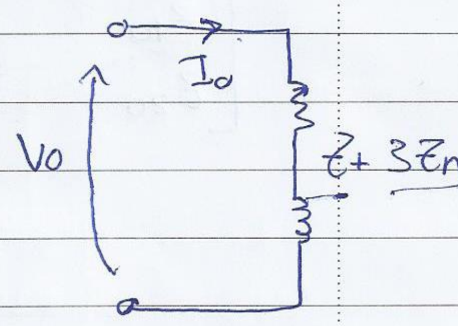


n بس بمر ب ⇐ $3I_{ao}$ ⇐

$3I_{ao} \Leftarrow I_{co}, I_{bo}, I_{ao} \Rightarrow$ I_n بمر

$I_2, I_1 \Rightarrow$ I_n ما بمر

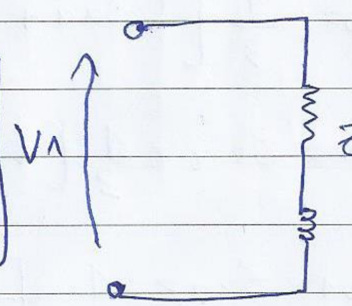
لانهم بلغوا بعض



If Δ-connection:

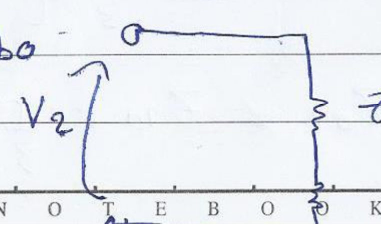
$$\Rightarrow \begin{bmatrix} V_{ab_0} \\ V_{bc} \\ V_{ca} \end{bmatrix} = \begin{bmatrix} Z_\Delta & 0 & 0 \\ 0 & Z_\Delta & 0 \\ 0 & 0 & Z_\Delta \end{bmatrix} \begin{bmatrix} I_{ab} \\ I_{bc} \\ I_{ca} \end{bmatrix}$$

بدل Z في Y



$V_{ab} + V_{bc} + V_{ca} = 3V_{ab0} = 3Z_\Delta I_{ab0}$

in Δ-connection, we have $I_{abc} = 0$



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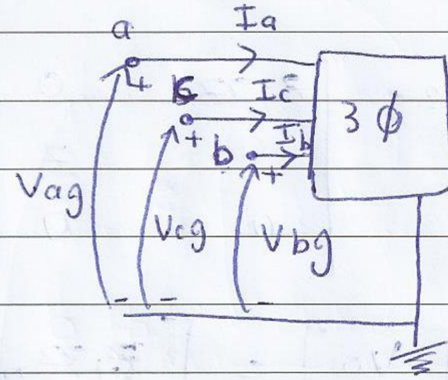
$* 3-\phi$



$$V_{abc} = Z_{abc} I_{abc}$$

self نفسهم mutual

$$Z_{abc} = \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix}$$

$$A\, V_{012} = Z_{012}\, A\, I_{012}$$

$$V_{012} = \underbrace{A^{-1} Z_{012} A}_{Z_{012}}\, I_{012}$$

مش نفسهم

$$Z_{012} = \begin{bmatrix} Z_{00} & Z_{01} & Z_{02} \\ Z_{10} & Z_{11} & Z_{12} \\ Z_{20} & Z_{21} & Z_{22} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

$$\Rightarrow Z_{00} = \frac{1}{3}\left[(Z_{aa} + Z_{bb} + Z_{cc}) + 2(Z_{ab} + Z_{bc} + Z_{ca}) \right]$$

$$Z_{11} = Z_{22} = \frac{1}{3}\left[(Z_{aa} + Z_{bb} + Z_{cc}) - (Z_{ab} + Z_{bc} + Z_{ca}) \right]$$

$$Z_{01} = Z_{20} = \frac{1}{3}\left[(Z_{aa} + a^2 Z_{bb} + a Z_{cc}) - (Z_{bc} + a^2 Z_{ab} + a Z_{ca}) \right]$$

$$Z_{02} = Z_{10} = \frac{1}{3}\left[(Z_{aa} + a Z_{bb} + a^2 Z_{cc}) - (Z_{bc} + a^2 Z_{ab} + a Z_{ca}) \right]$$

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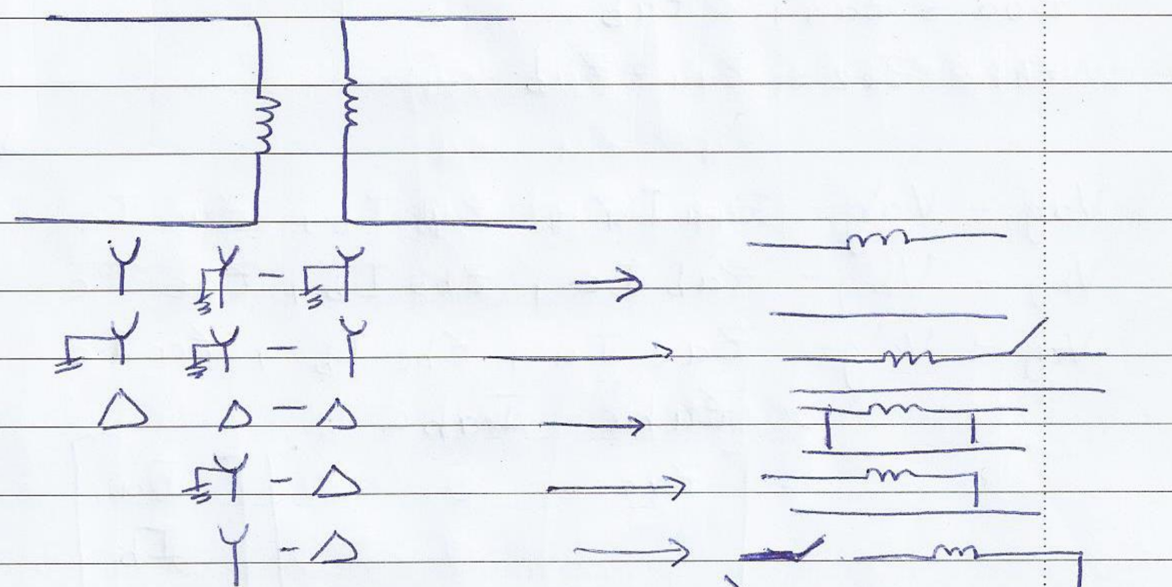
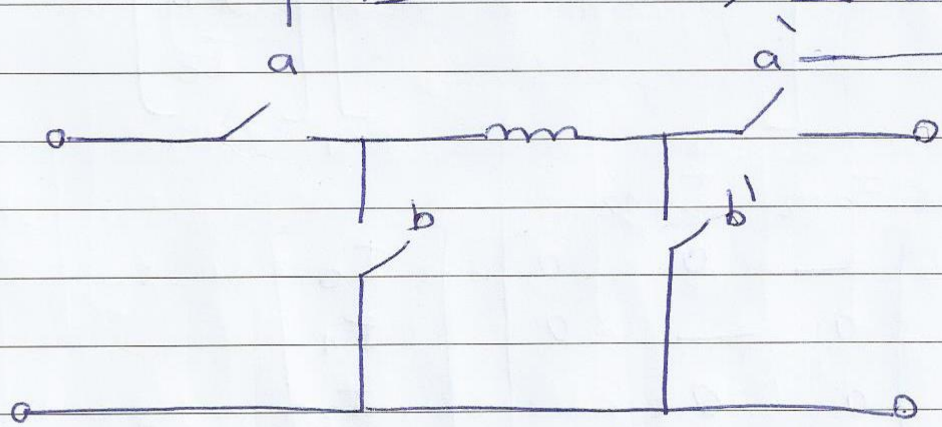
$$Z_{12} = \frac{1}{3}\left[\left(Z_{aa} + a^2 Z_{bb} + a Z_{cc}\right) + 2\left(Z_{bc} + a Z_{ab} + a^2 Z_{ca}\right)\right]$$

$$Z_{21} = \left[\left(Z_{aa} + a Z_{bb} + a^2 Z_{cc}\right) + 2\left(Z_{bc} + a^2 Z_{ab} + a Z_{ca}\right)\right]$$

Table 11.17 p450 ⇒





In Zero seq. Y → a , a` if $\underline{\perp}$Y → close

Δ → b, b`

N O T E B O O K

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* symtrical transimission line:

x condition for symtrical load:

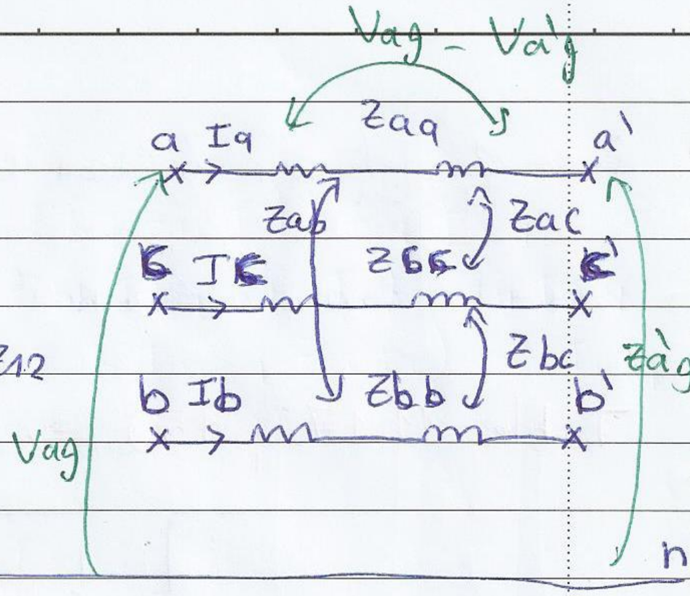
$$Z_{aa} = Z_{bb} = Z_{cc}$$

$$Z_{ab} = Z_{bc} = Z_{ca}$$

$$\Rightarrow Z_{01} = Z_{10} = Z_{02} = Z_{20} = Z_{12} = Z_{21} = 0$$

$$Z_{00} = Z_{aa} + 2Z_{ab}$$

$$Z_{11} = Z_{22} = Z_{aa} - Z_{ab}$$



$$V_{ag} - V_{a'g} = Z_{aa} I_a + Z_{ab} I_b + Z_{ac} I_c$$

$$V_{bg} - V_{b'g} = Z_{ab} I_a + Z_{bb} I_b + Z_{ac} I_c$$

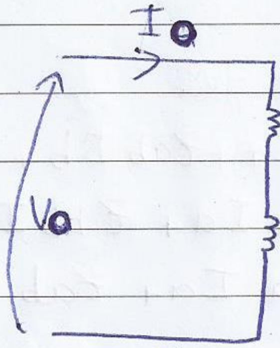
$$V_{cg} - V_{c'g} = Z_{ac} I_a + Z_{bc} I_b + Z_{cc} I_c$$

$$= Z_{abc} \; I_{abc}$$

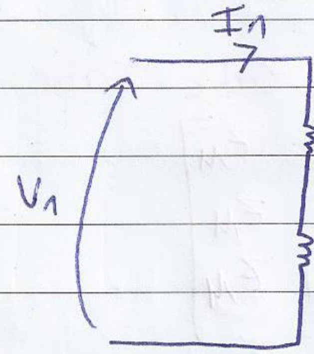
$$= \begin{bmatrix} Z_{aa} & - & \\ \vdots & - & - \\ \vdots & \vdots & - \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix}$$

$$V_{012} = Z_{012} \; I_{012}$$

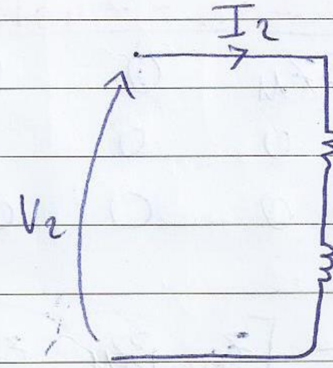
$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} \text{—} & 0 & 0 \\ 0 & \text{—} & 0 \\ 0 & 0 & \text{—} \end{bmatrix} \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$



$Z_{00} = Z_{aa} + 2Z_{ab}$



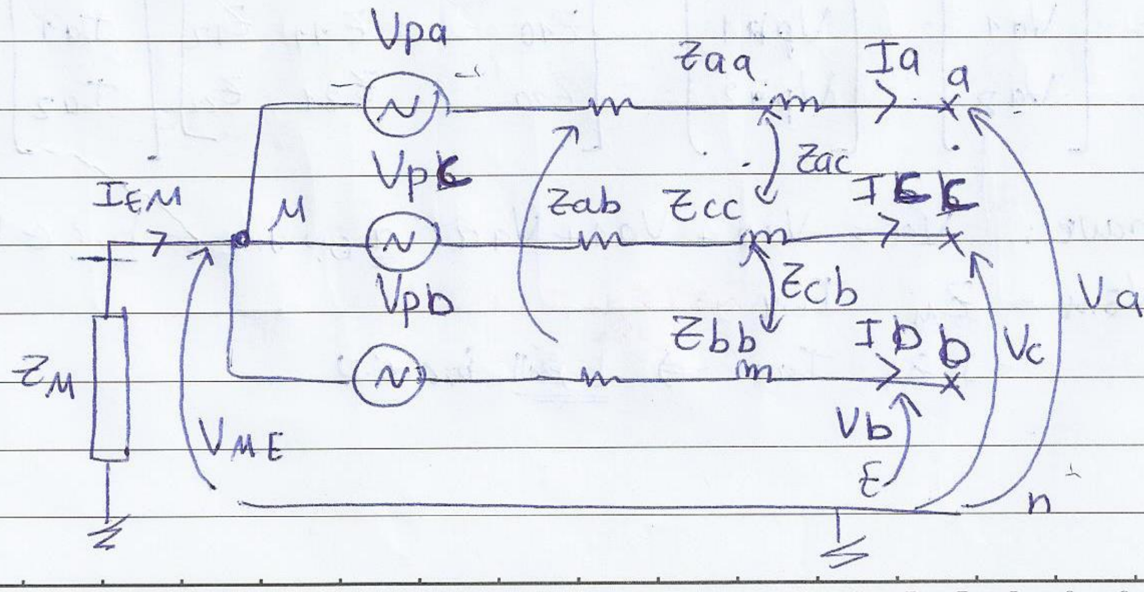
$Z_{11} = Z_{aa} - Z_{ab}$



$Z_{22} = Z_{aa} - Z_{ab}$

$V_a = U_{pa} - (Z_{aa} I_a \ldots)$

$- I_{EM} Z_M$



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KCL $\Rightarrow$ $I_{EM} = I_a + I_b + I_c$

KVL $\Rightarrow$ $V_a = V_{pa} - (Z_{aa} I_a + Z_{ab} I_b + Z_{ac} I_c) - Z_M I_{EM}$

$V_b = V_{pb} - (Z_{ab} I_a + Z_{bb} I_b + Z_{bc} I_c) - Z_M I_{EM}$

$V_c = V_{pc} - (Z_{ca} I_a + Z_{ab} I_b + Z_{cc} I_c) - Z_M I_{EM}$

$\Rightarrow$ $V_{abc} = V_{p\,abc} - Z_{abc} I_{abc} - Z_M I_{abc}$

Where $\Rightarrow$ $Z_M = \begin{bmatrix} Z_M & Z_M & Z_M \\ Z_M & Z_M & Z_M \\ Z_M & Z_M & Z_M \end{bmatrix}$

$V_{abc} = Z_{abc} I_{abc}$

$V_{012} = Z_{012} I_{012}$

$\Rightarrow$ $V_{012} = V_{p012} - Z_{012} I_{012} - Z_{M012} I_{012}$,

Where: $Z_{M012} = \begin{bmatrix} 3Z_M & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \begin{bmatrix} V_{pa0} \\ V_{pa1} \\ V_{pa2} \end{bmatrix} - \begin{bmatrix} Z_{00}+3Z_M & Z_{01} & Z_{02} \\ Z_{10} & Z_{11} & Z_{12} \\ Z_{20} & Z_{21} & Z_{22} \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix}$$

we have: $V_a = V_{a0} + V_{a1} + V_{a2}$ a بفضل بين ∈ b,c ∈ لبقوا

$V_{EM} = Z_M \; I_{EM} \rightsquigarrow (I_a + I_b + I_c)$

$= 3 Z_M \; I_{a0}$ $\Rightarrow$ لانضل b,c النقوا

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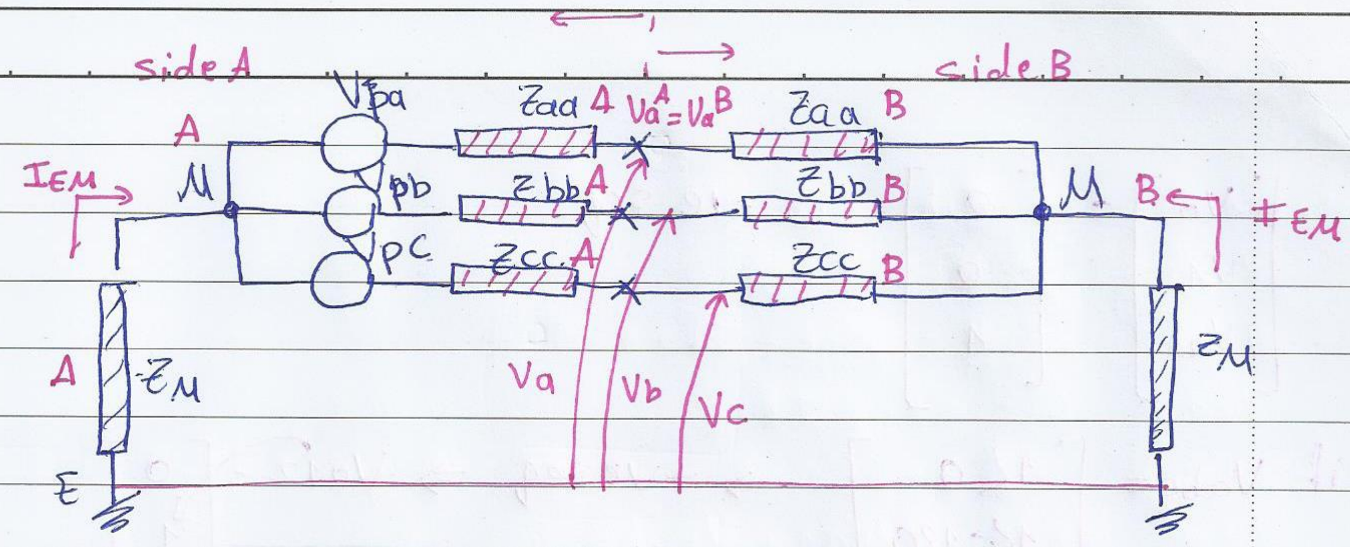


$$V^A_{012} = V_{p012} - Z^A_{012}\, I_{012} - Z^A_{M012}\, I_{012}$$

$||$

$$V^B_{012} = Z^B_{012}\, I_{012} + Z^B_{M012}\, I_{012}$$

$$\Rightarrow V_{p012} = \left(Z^A_{012} + Z^A_{M012} + Z^B_{012} + Z^B_{M012} \right) I_{012}$$

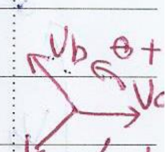
$$\Rightarrow Z_{00} = Z^A_{00} + 3Z^A_M + Z^B_{00} + 3Z^B_M$$

$$Z_{10} = Z^A_{10} + Z^B_{10}$$

$$Z_{11} = Z^A_{11} + Z^B_{11}$$

Ex:

If we have -ve sequence $\Rightarrow V_{abc} = \begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \begin{bmatrix} 1\angle 0 \\ 1\angle 120 \\ 1\angle -120 \end{bmatrix}$



acbseq

-ve seq

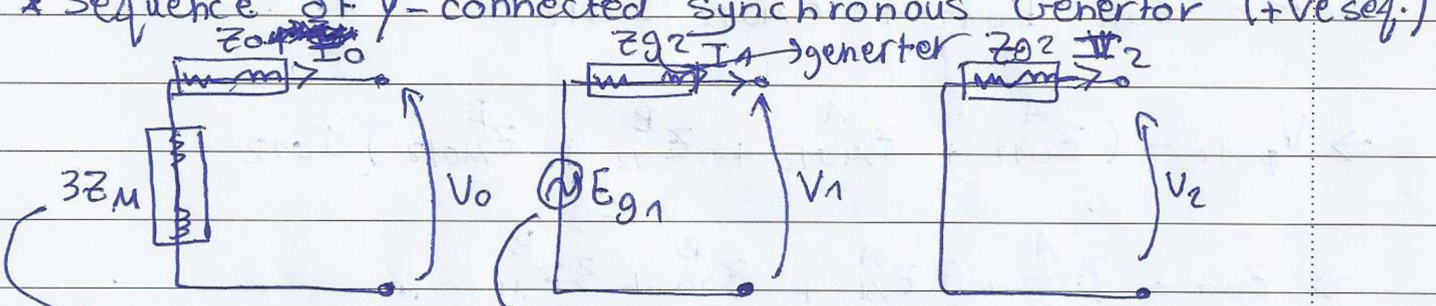
$$V_{012} = A^{-1}\, V_{abc}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 1\angle 0 \\ 1\angle 120 \\ 1\angle -120 \end{bmatrix}$$

$$\begin{bmatrix} V_0 \\ V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \Rightarrow -ve\ seq.$$

$$\text{if } V_{abc} = \begin{bmatrix} 1\angle 0 \\ 1\angle -120 \\ 1\angle 120 \end{bmatrix} \rightarrow +ve\ seq \Rightarrow V_{012} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

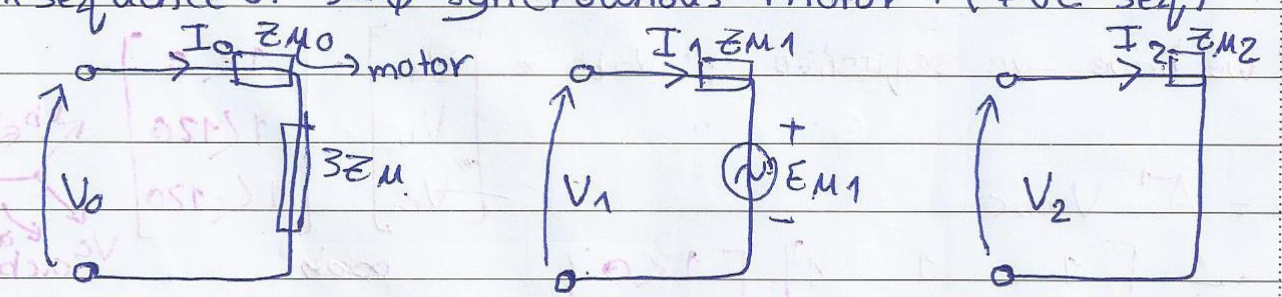
* sequence of y-connected synchronous Genertor (+ve seq.)



0 ماتطلعش غير

مصدر بس
+ve seq.

* sequence of 3-ϕ syncrounous motor: (+ve seq)



zero-seq. +ve seq -ve seq.

* If the motor induction ⇒ There is No EMf ⇒ short
* If y-connected synch. G (unsymtrical) ⇒ E_g will apear in all sequences

ch: unsymtrical faults:
- → Single line to ground fault (1LG fault)
- → Line to Line fault (LL fault)
- → Double line to ground fault (2LG fault)

* assumptions:
- The power system is blanaced before the fault.
- pre-fault +ve sequence voltages are the same at all buses & at the fault location.
- $R=0$ & $C=0$ ⇒ L (inductive)

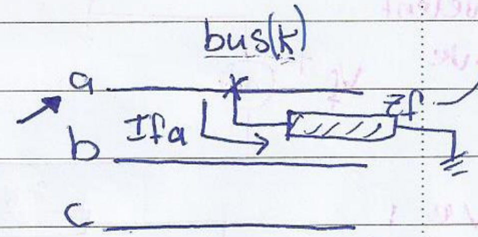
$Z_f = 0 \leftarrow$ solidly ارثنا

1) 1LG fault: at phase a

* unloaded before fault.

⇒ $I_{fb} = I_{fc} = 0$

I_{fa} ✓



I_{fa}: f: during fault.

$$I_{fa_{012}} = A^{-1} \cdot I_{fabc} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{fa} \\ 0 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_{fa0} \\ I_{fa1} \\ I_{fa2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} I_{fa} \\ I_{fa} \\ I_{fa} \end{bmatrix}$$

$\Rightarrow I_{fa0} = I_{fa1} = I_{fa2} = \frac{1}{3} I_{fa}$

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at bus k : $\Rightarrow Z_{th} = Z_{kk}$

$V_{ka0} = -Z_{kk0} I_{fa0}$

bus ← → phase

$V_{ka1} = V_f - Z_{kk1} I_{fa1}$ are equal

$V_{ka2} = -Z_{kk2} I_{fa2}$

$V_{ka} = V_{ka0} + V_{ka1} + V_{ka2}$

$= V_f - (Z_{kk0} + Z_{kk1} + Z_{kk2}) I_{fa0}$

We have : $V_{ka} = Z_f I_{fa}$

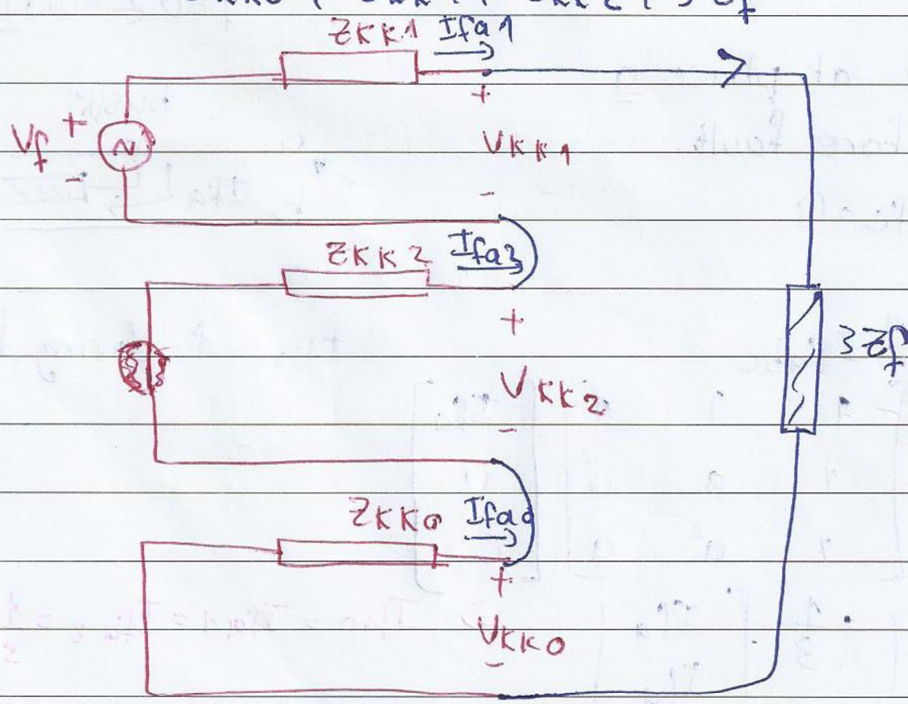
$= Z_f (I_{fa0} + I_{fa1} + I_{fa2})$

$= 3 Z_f I_{fa}$

$$\Rightarrow I_{fa0} = \frac{V_f}{Z_{kk0} + Z_{kk1} + Z_{kk2} + 3Z_f}$$

equivelent for +ve

-ve :



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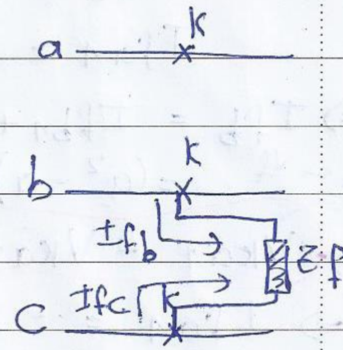
2) LL fault: b to c fault at bus $\underline{k}$

$I_{fa} = 0$

$I_{fb} = -I_{fc}$



$$\Rightarrow I_{fa012} = A^{-1} I_{fabc}$$

$$= \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ I_{fb} \\ -I_{fb} \end{bmatrix}$$

$$I_{fa012} = \frac{1}{3} \begin{bmatrix} 0 \\ (a-a^2) I_{fb} \\ (a^2-a) I_{fb} \end{bmatrix}$$

$\Rightarrow I_{fa0} = 0 \longrightarrow$ No zero sequence

$I_{fa1} = -I_{fa2}$

~~I_{fa2}~~

We have: $V_{kb} - V_{kc} = Z_f I_{fb}$

$V_{kb} \to 0, 1, 2$; $V_{kc} \to 0, 1, 2$

نفس القيمة وطرح

$$= (V_{kb1} - V_{kc1}) + (V_{kb2} - V_{kc2})$$

$$= (a^2 - a) V_{ka1} + (a - a^2) V_{ka2}$$

$$= (a^2 - a)(V_{ka1} - V_{ka2})$$

We have: $I_{fa0} = I_{fb0} = 0$

$I_{fa1} = -I_{fa2}$

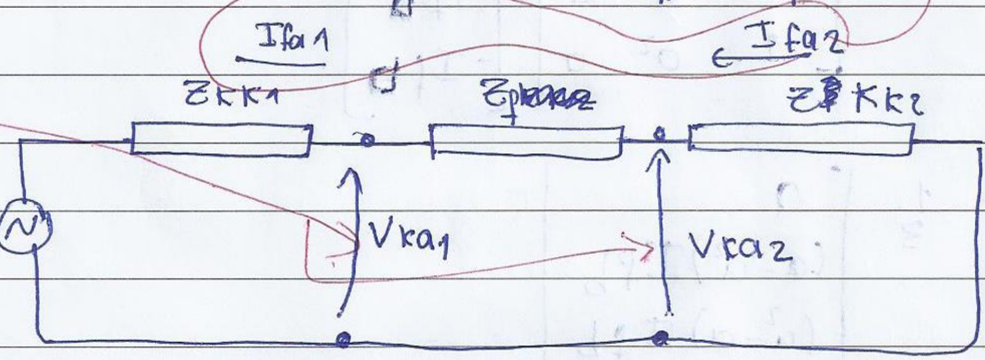
$\Rightarrow I_{fb} = I_{fb1} + I_{fb2}$

$= (a^2 - a)\, I_{fa1}$

$-Z_{kk1} I_{fa1} + V_f$ $\quad$ $-Z_{kk2}$ I_{fa2}

$\Rightarrow V_{ka1} - V_{ka2} = Z_f\, I_{fa1} \Rightarrow$ من مساواة I_{fb} نعوض

$\Rightarrow I_{fa1} = -I_{fa2} = \dfrac{V_f}{Z_{kk1} + Z_{kk2} + Z_f}$



3) Double line to ground fault: 2LG fault:

* fault ~~...~~ occured at node k.

phases b-to-C through impedence Z_f to ground.

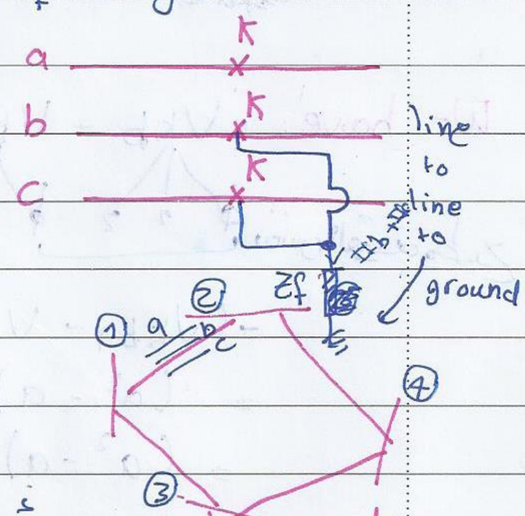
* Unloaded before $\Rightarrow 0 = I_{abc}$ fault قبل

$I_{fa} = 0 \rightarrow$ during fault and before fault

$I_{fa012} = A^{-1} I_{fabc}$

during fault

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_{fb} \\ I_{fc} \end{bmatrix}$$



3-φ مثل ما هو ← single line digram

$I_{fa0} = \frac{1}{3}(I_{fb} + I_{fc}) \Rightarrow 3 I_{fa0} = I_{fb} + I_{fc}$

$V_{kb} = V_{kc} \Rightarrow$ (c , b بين short cct)

$= Z_f (I_{fb} + I_{fc})$ $Z_f \leftarrow$ تمر $I_{fb} + I_{fc}$

$= 3 Z_f I_{fa0}$

$\Rightarrow V_{ka012} = A^{-1} V_{kabc}$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_{ka} \\ V_{kb} \\ V_{kc} \end{bmatrix}$$

$$= \frac{1}{3}\begin{bmatrix} V_{ka} + 2V_{kb} \\ V_{ka} + (a+a^2)V_{kb} \\ V_{ka} + (a+a^2)V_{kb} \end{bmatrix} \rightarrow V_{kb} = V_{kc}$$

$\Rightarrow V_{ka1} = V_{ka2}$

$\Rightarrow 3V_{ka0} = V_{ka} + 2V_{kb}$

$= V_{ka0} + V_{ka1} + V_{ka2} + 2V_{kb}$

sub $\Rightarrow V_{ka1} = V_{ka2} = V_{ka0} = 3 Z_f I_{fa0}$

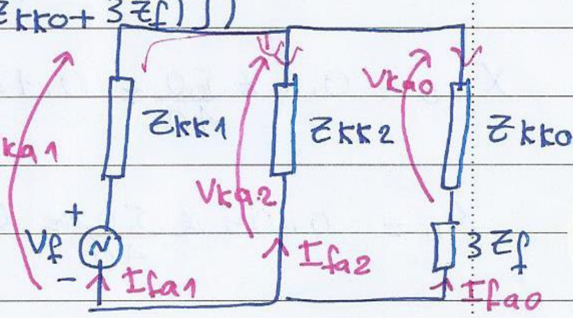
Since $\Rightarrow I_{fa} \Rightarrow I_{fa0} + I_{fa1} + I_{fa2} = 0.$

$I_{fa1} = V_f / (Z_{kk1} + [Z_{kk2} \,//\, (Z_{kk0} + 3Z_f)])$

$I_{fa0} = - I_{fa1} * \frac{Z_{kk2}}{Z_{ka2} + Z_{kk0} + 3Z_f}$

$I_{fa2} = - I_{fa1} * \frac{Z_{kk0} + 3Z_f}{Z_{kk2} + Z_{kk0} + 3Z_f}$

current divider rule



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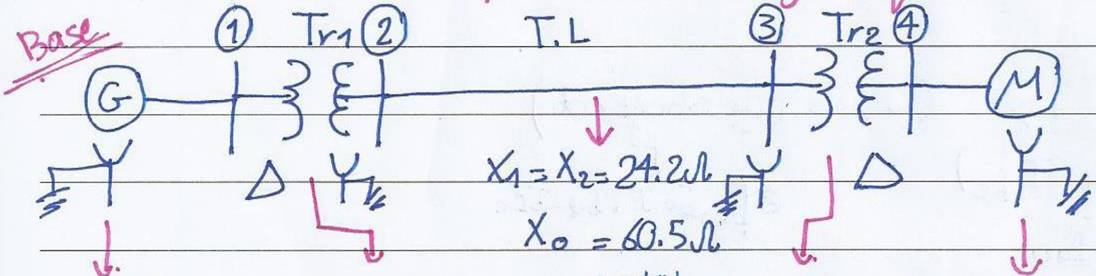
*fault current compatution using sequence Networks:



50MVA   50MVA, 20KV   110KV / 50 MVA   50MVA   40 MVA

20KV $X = 10\%$ 110KVY / 20KVΔ   20KV

$X'' = X_1 = X_2 = 20\%$ $X = 10\%$ $X'' = X_1 = X_2 = 20\%$

$X_0 = 7.5\%$ $X_0 = 10\%$ $X_n = 50\%$

Solidly grounded

* find the fault current for ⇒ 2LG fault
  LL fault
  1LG fault

$X_{G1} = X_{G2} = 0.2$

$X_{G0} = 0.075$

$X_{Tr1} = X_{Tr2} = 0.1$

$X_{M1} = X_{M2} = 0.2 * \frac{50}{40} = 0.25$

$X_{M0} = 0.1 * \frac{50}{40} = 0.125$

$X_n = 0.05 * \frac{50}{40} = 0.0625$

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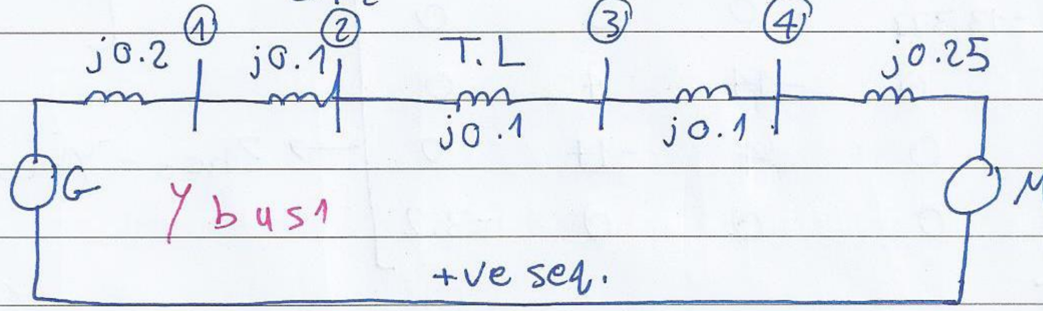
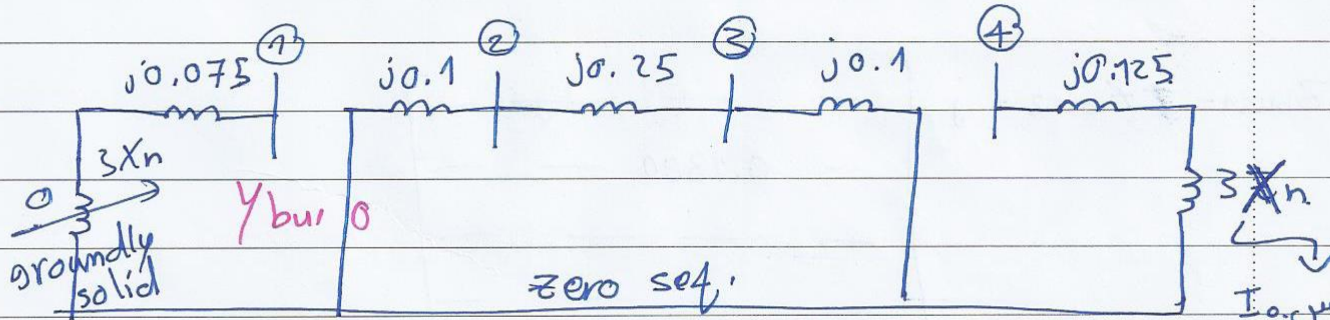
for T.L $\Rightarrow$ $Z_{\substack{base \\ at\ T.L}} = \frac{110^2}{50} = 242$

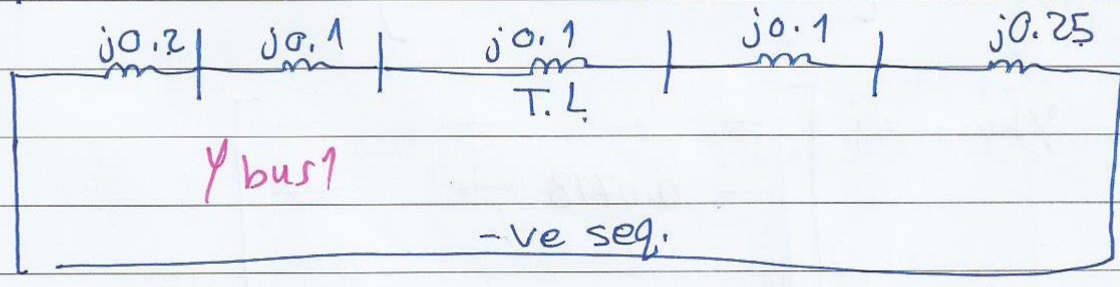
$\Rightarrow X_{T.L1} = X_{T.L2} = \frac{24.2}{242} = 0.1$

$X_{T.L0} = \frac{60.5}{242} = 0.25$





بس I_o
تمشي فيها



$$Y_{bus1} = Y_{bus2} = j\begin{bmatrix} -15 & 10 & 0 & 0 \\ 10 & -20 & 10 & 0 \\ 0 & 10 & -20 & 10 \\ 0 & 0 & 10 & -14 \end{bmatrix} \rightarrow Z_{bus1} = Z_{bus2} = Y_{bus1}^{-1} = \begin{bmatrix} \quad \end{bmatrix}$$

$$Y_{bus0} = j\begin{bmatrix} -13.333 & 0 & 0 & 0 \\ 0 & -14 & 4 & 0 \\ 0 & 4 & -14 & 0 \\ 0 & 0 & 0 & -3.2 \end{bmatrix} \rightarrow Z_{bus0} = Y_{bus0}^{-1}$$

$$Z_{bus1} = Z_{bus2} = j\begin{bmatrix} - & - & - & - \\ - & 0.1800 & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

$$Z_{bus0} = Y_{bus0}^{-1} = j\begin{bmatrix} - & - & - & - \\ - & 0.0778 & - & - \\ - & - & - & - \\ - & - & - & - \end{bmatrix}$$

$$\left.\begin{aligned} Z_{22,1} &= j0.18 \\ Z_{22,2} &= j0.13 \\ Z_{22,0} &= j0.0778 \end{aligned}\right\} \text{from } Z_{bus}$$

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0 →
1 → source
2 → ولا اشي

Thevenin: ⇒

OR $Z_{22,1} = j0.3 \,//\, j0.45 = j0.1800$

$Z_{22,2} = j0.18$

$Z_{22,0} = j0.1 \,//\, j0.35 = j0.0778$

1) 1LG at bus ②:- bulted or solidly ⇒ $Z_f =$

unloaded before fault ⇒ Bus voltages $1\angle 0$ p.u.

$$\Rightarrow I_{f0} = I_{fa1} = I_{fa2} = \frac{1}{j(0.18 + 0.18 + 0.0778)} \leftarrow Z_0 + Z_1 + Z_2 + 3Z$$

$$= -j2.2841 \text{ p.u.}$$

$I_{fa} = 3 I_{fa0} = -j6.8524$

we have: $I_{fb} = I_{fc} = 0$

$V_{2a0} = -j0.0778 I_{fa0} = -0.1777$

at bus 2

$V_{2a1} = 0.5889$

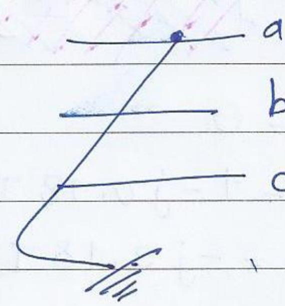
$V_{2a2} = -0.4111$

$V_{abc} = A V_{012}$

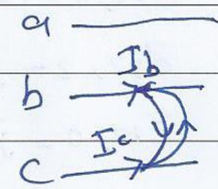
fault (groundlly)

يعني موصلة

$$= \begin{bmatrix} 0 \\ 0.9061 \angle -107.11 \\ 0.9061 \angle +107.11 \end{bmatrix}$$



2) Line to line fault L-L fault at bus 2



$$I_{fa1} = -I_{fa2} = \frac{1}{j0.18 + j0.12}$$

$$= -j2.7778$$

$$I_{fabc} = A\, I_{f012}$$

$$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ -j2.7778 \\ +j2.7778 \end{bmatrix} = \begin{bmatrix} 0 \\ -4.8113 \\ 4.8113 \end{bmatrix}$$

$\cancel{V_{2a,0} = -j\,0.0715\, I_{fa0} = 0.2318}$

$\cancel{V_{2a,1} = 1 - j0.18\, I_{fa1} = 0.2318}$

$\cancel{V_{2a,2} = -j\,0.18\, I_{fa2} = 0.2318}$

$\cancel{V_{abc} = A\, V_{2a,012}}$

$\cancel{= \begin{bmatrix} 0.6954 \\ 0 \\ 0 \end{bmatrix}}$

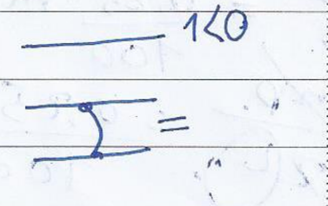
$V_{2a,0} = 0$

$V_{2a,1} = 1 - j\,0.18\, I_{fa1} = 0.5$

$V_{2a,2} = -j\,0.18\, I_{fa2} = 0.5$

$$V_{abc} = A \; V_{012}$$

$$= A \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix} = \begin{bmatrix} 1 \\ -0.5 \\ -0.5 \end{bmatrix}.$$



3) 2LG fault at bus 2

$$I_{fa1} = \frac{1}{Z_1 + [Z_2 \,//\, (Z_0 + 3Z_f)]} = \frac{1}{j0.18 + (j0.18 \,//\, j0.0798)}$$

$$= -j4.2676$$

$$I_{fa0} = -I_{fa1} * \ldots = j2.9797$$

$$I_{fa2} = -I_{fa1} * \ldots = j1.2879$$

$$I_{abc} = A \; I_{012}$$

$$= \begin{bmatrix} 0 \\ 6.657 \angle 137.11 \\ 6.657 \angle 42.89^\circ \end{bmatrix}.$$

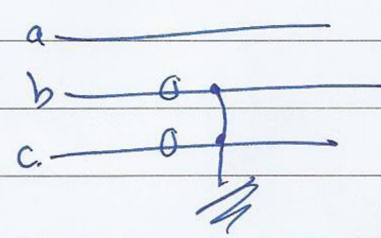
$$V_{2a,0} = -j0.07715 \; I_{fa0} = 0.2318$$

$$V_{2a,1} = 1 - j0.18 \; I_{fa1} = 0.2318$$

$$V_{2a,2} = -j0.18 \; I_{fa2} = 0.2318$$

$$\Rightarrow V_{abc} = A \; V_{2a,012}$$

$$= \begin{bmatrix} 0.6954 \\ 0 \\ 0 \end{bmatrix}$$



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$$\frac{\Delta}{2} = \frac{0.25}{100} mp$$

$$\frac{2 mp}{2L} = \frac{0.25}{100} \cancel{mp}$$

$$\frac{3L^2}{(\ln(1+M))^2}$$

$\underline{\underline{D}}$

$$\frac{\Delta}{2} = \frac{0.25}{100} mp$$

$$B_T = \frac{R}{2} \rightarrow nNf_s$$

$$\frac{2mp}{2L} = \frac{0.25}{100} \cancel{mp}$$

$\underline{M}$

$R = 2f_s N$

$\cancel{f_s = 2B}$

$47 = 10 \log SNR$

$$\frac{3L^2}{2}$$

SNR

$\alpha +$

$$\frac{3L^2 P_m}{m_p^2}$$

$\frac{1}{2}$

SNR = 3.41

$\boxed{3/6 \quad 22.77}$

$$SNR = 3L^2 \frac{P_m}{m_p^2}$$

$4.7 = 10 \log SNR$

$$6n + d \quad \frac{3L^2}{(\ln(1+M))^2} \leftarrow \text{linear}$$

$$10 \log c \quad 10 \log L^2 + \frac{10 \log 3}{(\ln(1+M))^2}$$

$$dB = 6n + d$$

$10 \log c$

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Final Analysis

17/12/2014 $\boxed{30}$

analysis

CHAPTER 8

UNSYMMETRICAL FAULTS – Finding fault current

The sequence circuits and the sequence networks developed in the previous chapter will now be used for finding out fault current during unsymmetrical faults. Specifically we are going to discuss the following three types of faults:

- Single-line-to-ground (1LG) fault
- Line-to-line (LL) fault
- Double-line-to-ground (2LG) fault

For the calculation of fault currents, we shall make the following assumptions:

1. • The power system is balanced before the fault occurs, such that of the three sequence networks only the positive sequence network is active. Also as the fault occurs, the sequence networks are connected only through the fault location.
2. • The fault current is negligible such that the pre-fault positive sequence voltages are same at all nodes and at the fault location.
3. • All the network resistances and line charging capacitances are negligible.
4. • All loads are passive except the rotating loads which are represented by synchronous machines.

Based on the assumptions stated above, the faulted network will be as shown in Fig. 8.1 where the voltage at the faulted point will be denoted by V_f and current in the three faulted phases are I_{fa}, I_{fb} and I_{fc}. We shall now discuss how the three sequence networks are connected when the three types of faults discussed above occur.

the voltage at faulted point

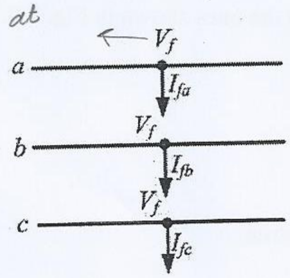

Fig. 8.1 Representation of a faulted segment.

8.1 SINGLE-LINE-TO-GROUND FAULT

Let a 1LG fault has occurred at node k of a network. The faulted segment is then as shown in Fig. 8.2 where it is assumed that phase-a has touched the ground through an impedance Z_f. Since the system is unloaded before the occurrence of the fault we have

$$I_{fb} = I_{fc} = 0 \tag{8.1}$$

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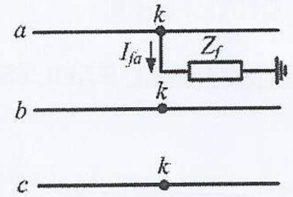

Fig. 8.2 Representation of 1LG fault.

Also the phase-a voltage at the fault point is given by

$$V_{ka} = Z_f I_{fa} \tag{8.2}$$

From (8.1) we can write

$$I_{fa012} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_{fa} \\ 0 \\ 0 \end{bmatrix} \tag{8.3}$$

Solving (8.3) we get

$$I_{fa0} = I_{fa1} = I_{fa2} = \frac{I_{fa}}{3} \tag{8.4}$$

This implies that the three sequence currents are in series for the 1LG fault. Let us denote the zero, positive and negative sequence Thevenin impedance at the faulted point as Z_{kk0}, Z_{kk1} and Z_{kk2} respectively. Also since the Thevenin voltage at the faulted phase is V_f we get three sequence circuits that are similar to the ones shown in Fig. 7.7. We can then write

$$\begin{aligned} V_{ka0} &= -Z_{kk0} I_{fa0} \\ V_{ka1} &= V_f - Z_{kk1} I_{fa1} \\ V_{ka2} &= -Z_{kk2} I_{fa2} \end{aligned} \tag{8.5}$$

Then from (8.4) and (8.5) we can write

$$\begin{aligned} V_{ka} &= V_{ka0} + V_{ka1} + V_{ka2} \\ &= V_f - \left(Z_{kk0} + Z_{kk1} + Z_{kk2}\right) I_{fa0} \end{aligned} \tag{8.6}$$

Again since

$$V_{ka} = Z_f I_{fa} = Z_f \left(I_{fa0} + I_{fa1} + I_{fa2}\right) = 3 Z_f I_{fa0}$$

we get from (8.6)

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pania - ao

$$I_{fa0} = \frac{V_f}{Z_{kk0} + Z_{kk1} + Z_{kk2} + 3Z_f} \tag{8.7}$$

The Thevenin equivalent of the sequence network is shown in Fig. 8.3.

thevenin impedance at faulted point

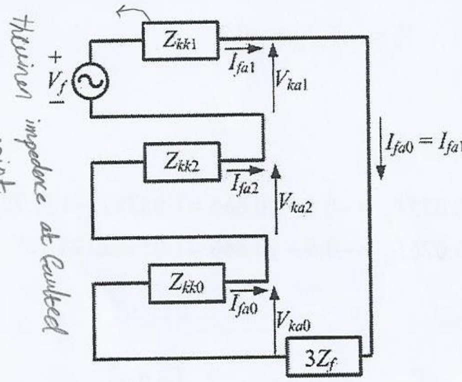
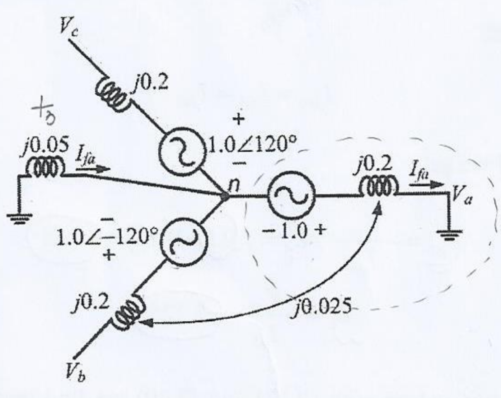


Fig. 8.3 Thevenin equivalent of a 1LG fault.

Example 8.1: A three-phase Y-connected synchronous generator is running unloaded with rated voltage when a 1LG fault occurs at its terminals. The generator is rated 20 kV, 220 MVA, with subsynchronous reactance of 0.2 per unit. Assume that the subtransient mutual reactance between the windings is 0.025 per unit. The neutral of the generator is grounded through a 0.05 per unit reactance. The equivalent circuit of the generator is shown in Fig. 8.4. We have to find out the negative and zero sequence reactances.



V = 20 kV
S = 220 MVA

Fig. 8.4 Unloaded generator of Example 8.1.

Since the generator is unloaded the internal emfs are

$$E_{an} = 1.0 \quad E_{bn} = 1.0\angle -120° \quad E_{cn} = 1.0\angle 120°$$ +ve sequence

Since no current flows in phases b and c, once the fault occurs, we have from Fig. 8.4

$$I_{fa} = \frac{1}{j(0.2+0.05)} = -j4.0$$

Then we also have

$$V_n = -X_n I_{fa} = -0.2$$

From Fig. 8.4 and (7.34) we get

$$\begin{aligned} V_a &= 0 \\ V_b &= E_{bn} + V_n + j0.025 I_{fa} = -0.6 - j0.866 = 1.0536\angle -124.72^\circ \\ V_c &= E_{cn} + V_n + j0.025 I_{fa} = -0.6 + j0.866 = 1.0536\angle 124.72^\circ \end{aligned}$$

Therefore

$$V_{a012} = \overset{A^{-1}}{C}\begin{bmatrix} 0 \\ 1.0536\angle -124.72^\circ \\ 1.0536\angle 124.72^\circ \end{bmatrix} = \begin{bmatrix} -0.4 \\ 0.7 \\ -0.3 \end{bmatrix}$$

From (7.38) we can write $Z_1 = j\omega(L_s + M_s) = j0.225$. Then from Fig. 7.7 we have

$$I_{fa1} = \frac{E_{an} - V_{a1}}{Z_1} = \frac{1-0.7}{j0.225} = -j1.3333$$

thevenin.

Also note from (8.4) that

$$I_{fa0} = I_{fa1} = I_{fa2}$$

Therefore from Fig. 7.7 we get

$$\begin{aligned} Z_{g0} &= -\frac{V_{a0}}{I_{a0}} - 3Z_n = j(0.3 - 0.15) = j0.15 \\ Z_2 &= -\frac{V_{a2}}{I_{a2}} = j0.225 \end{aligned}$$

Comparing the above two values with (7.37) and (7.39) we find that Z_0 indeed is equal to $j\omega(L_s - 2M_s)$ and Z_2 is equal to $j\omega(L_s + M_s)$. Note that we can also calculate the fault current from (8.7) as

$$I_{fa0} = \frac{1}{j(\underbrace{0.225}_{Z_1} + \underbrace{0.225}_{Z_2} + \underbrace{0.15}_{Z_{g0}} + \underbrace{3\times 0.05}_{3Z_f = 3Z_n})} = -j1.3333$$

ΔΔΔ

8.2 LINE-TO-LINE FAULT

The faulted segment for an L-L fault is shown in Fig. 8.5 where it is assumed that the fault has occurred at node k of the network. In this the phases b and c got shorted through the impedance Z_f. Since the system is unloaded before the occurrence of the fault we have

$$I_{fa} = 0 \tag{8.8}$$

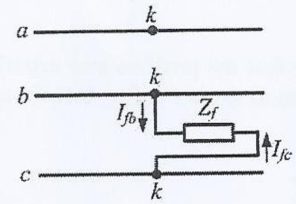


Fig. 8.5 Representation of L-L fault.

Also since phases b and c are shorted we have

$$I_{fb} = -I_{fc} \tag{8.9}$$

Therefore from (8.8) and (8.9) we have

$$I_{fa012} = C\begin{bmatrix} 0 \\ I_{fb} \\ -I_{fb} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 \\ (a-a^2)I_{fb} \\ (a^2-a)I_{fb} \end{bmatrix} \tag{8.10}$$

We can then summarize from (8.10)

$$\begin{aligned} I_{fa0} &= 0 \\ I_{fa1} &= -I_{fa2} \end{aligned} \tag{8.11}$$

Therefore no zero sequence current is injected into the network at bus k and hence the zero sequence remains a dead network for an L-L fault. The positive and negative sequence currents are negative of each other.

Now from Fig. 8.5 we get the following expression for the voltage at the faulted point

$$V_{kb} - V_{kc} = Z_f I_{fb} \tag{8.12}$$

Again

$$\begin{aligned} V_{kb} - V_{kc} &= V_{kb0} + V_{kb1} + V_{kb2} - V_{kc0} - V_{kc1} - V_{kc2} \\ &= (V_{kb1} - V_{kc1}) + (V_{kb2} - V_{kc2}) \\ &= (a^2 - a)V_{ka1} + (a - a^2)V_{ka2} \\ &= (a^2 - a)(V_{ka1} - V_{ka2}) \end{aligned} \tag{8.13}$$

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I_{fa2}

Moreover since $I_{fa0} = I_{fb0} = 0$ and $I_{fa1} = -I_{fa2}$, we can write

$$I_{fb} = I_{fb1} + I_{fb2} = a^2 I_{fa1} + a I_{fa2} = \left(a^2 - a\right) I_{fa1} \tag{8.14}$$

I_{fa2}

Therefore combining (8.12)-(8.14) we get

$$V_{ka1} - V_{ka2} = Z_f I_{fa1} \tag{8.15}$$

Equations (8.12) and (8.15) indicate that the positive and negative sequence networks are in parallel. The sequence network is then as shown in Fig. 8.6. From this network we get

$$I_{fa1} = -I_{fa2} = \frac{V_f}{Z_{kk1} + Z_{kk2} + Z_f} \tag{8.16}$$

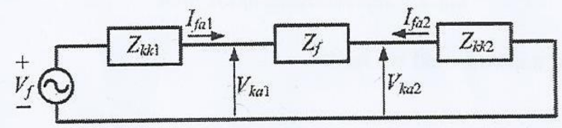

Fig. 8.6 Thevenin equivalent of an LL fault.

Example 8.2: Let us consider the same generator as given in Example 8.1. Assume that the generator is unloaded when a bolted ($Z_f = 0$) short circuit occurs between phases b and c. Then we get from (8.9) $I_{fb} = -I_{fc}$. Also since the generator is unloaded, we have $I_{fa} = 0$. Therefore from (7.34) we get

$$V_{an} = E_{an} = 1.0$$
$$V_{bn} = E_{bn} - j0.225 I_{fb} = 1\angle -120° - j0.225 I_{fb}$$
$$V_{cn} = E_{cn} - j0.225 I_{fc} = 1\angle 120° + j0.225 I_{fb}$$

Also since $V_{bn} = V_{cn}$, we can combine the above two equations to get

$$I_{fb} = -I_{fc} = \frac{1\angle -120° - 1\angle 120°}{j0.45} = -3.849$$

Then

$$I_{fa012} = C\begin{bmatrix} 0 \\ -3.849 \\ 3.849 \end{bmatrix} = \begin{bmatrix} 0 \\ -j2.2222 \\ j2.2222 \end{bmatrix}$$

We can also obtain the above equation from (8.16) as

$$I_{fa1} = -I_{fa2} = \frac{1}{j0.225 + j0.225} = -j2.2222$$

Also since the neutral current I_n is zero, we can write $V_a = 1.0$ and

$$V_b = V_c = V_{bn} = -0.5$$

Hence the sequence components of the line voltages are

$$V_{a012} = C\begin{bmatrix} 1.0 \\ -0.5 \\ -0.5 \end{bmatrix} = \begin{bmatrix} 0 \\ 0.5 \\ 0.5 \end{bmatrix}$$

Also note that

$$V_{a1} = 1.0 - j0.225 I_{fa1} = 0.5$$
$$V_{a2} = -j0.225 I_{fa2} = 0.5$$

which are the same as obtained before.

ΔΔΔ

8.3 DOUBLE-LINE-TO-GROUND FAULT

The faulted segment for a 2LG fault is shown in Fig. 8.7 where it is assumed that the fault has occurred at node k of the network. In this the phases b and c got shorted through the impedance Z_f to the ground. Since the system is unloaded before the occurrence of the fault we have the same condition as (8.8) for the phase-a current. Therefore

$$I_{fa0} = \frac{1}{3}\left(I_{fa} + I_{fb} + I_{fc}\right) = \frac{1}{3}\left(I_{fb} + I_{fc}\right) \tag{8.17}$$
$$\Rightarrow 3I_{fa0} = I_{fb} + I_{fc}$$

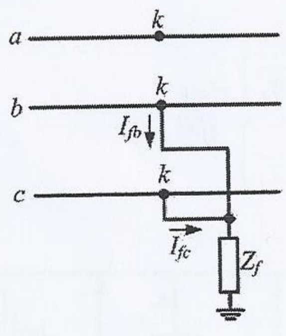

Fig. 8.7 Representation of 2LG fault.

Also the voltages of phases b and c are given by

$$V_{kb} = V_{kc} = Z_f\left(I_b + I_c\right) = 3Z_f I_{fa0} \tag{8.18}$$

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Therefore

$$V_{ka012} = C\begin{bmatrix} V_{ka} \\ V_{kb} \\ V_{kb} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} V_{ka} + 2V_{kb} \\ V_{ka} + \left(a + a^2\right)V_{kb} \\ V_{ka} + \left(a + a^2\right)V_{kb} \end{bmatrix} \tag{8.19}$$

We thus get the following two equations from (8.19)

$$V_{ka1} = V_{ka2} \tag{8.20}$$

$$3V_{ka0} = V_{ka} + 2V_{kb} = V_{ka0} + V_{ka1} + V_{ka2} + 2V_{kb} \tag{8.21}$$

Substituting (8.18) and (8.20) in (8.21) and rearranging we get

$$V_{ka1} = V_{ka2} = V_{ka0} - 3Z_f I_{fa0} \tag{8.22}$$

Also since $I_{fa} = 0$ we have

$$I_{fa0} + I_{fa1} + I_{fa2} = 0 \tag{8.23}$$

The Thevenin equivalent circuit for 2LG fault is shown in Fig. 8.8. From this figure we get

$$I_{fa1} = \frac{V_f}{Z_{kk1} + Z_{kk2} \| \left(Z_{kk0} + 3Z_f\right)} = \frac{V_f}{Z_{kk1} + \dfrac{Z_{kk2}\left(Z_{kk0} + 3Z_f\right)}{Z_{kk2} + Z_{kk0} + 3Z_f}} \tag{8.24}$$

The zero and negative sequence currents can be obtained using the current divider principle as

البعيد

$$I_{fa0} = -I_{fa1}\left(\frac{Z_{kk2}}{Z_{kk2} + Z_{kk0} + 3Z_f}\right) \tag{8.25}$$

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$$I_{fa2} = -I_{fa1}\left(\frac{Z_{kk0} + 3Z_f}{Z_{kk2} + Z_{kk0} + 3Z_f}\right) \tag{8.26}$$

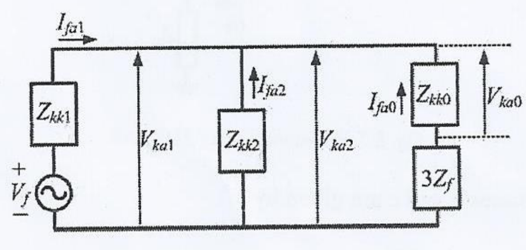

Fig. 8.8 Thevenin equivalent of a 2LG fault.

Example 8.3: Let us consider the same generator as given in Examples 8.1 and 8.2. Let us assume that the generator is operating without any load when a bolted 2LG fault occurs in phases b and c. The equivalent circuit for this fault is shown in Fig. 8.9. From this figure we can write

$$E_{bn} + V_n = 1\angle -120° + V_n = j0.2I_{fb} - j0.025I_{fc}$$
$$E_{cn} + V_n = 1\angle 120° + V_n = j0.2I_{fc} - j0.025I_{fb}$$
$$V_n = -j0.05\left(I_{fb} + I_{fc}\right)$$

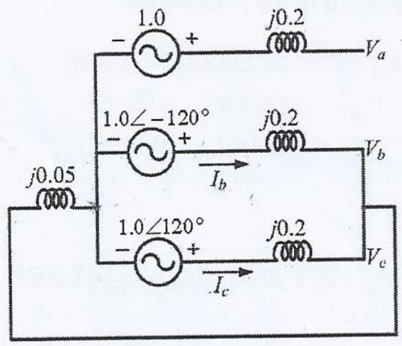

Fig. 8.9 Equivalent circuit of the generator in Fig. 8.4 for a 2LG fault in phases b and c.

Combining the above three equations we can write the following vector-matrix form

$$j\begin{bmatrix} 0.25 & 0.025 \\ 0.025 & 0.25 \end{bmatrix}\begin{bmatrix} I_{fb} \\ I_{fc} \end{bmatrix} = \begin{bmatrix} 1\angle -120° \\ 1\angle 120° \end{bmatrix}$$

Solving the above equation we get

$$I_b = -3.849 + j1.8182$$
$$I_c = 3.849 + j1.8182$$

Hence

$$i_{fa012} = C\begin{bmatrix} 0 \\ -3.849 + j1.8182 \\ -3.849 + j1.8182 \end{bmatrix} = \begin{bmatrix} j1.2121 \\ -j2.8283 \\ j1.6162 \end{bmatrix}$$

We can also obtain the above values using (8.24)-(8.26). Note from Example 8.1 that

$$Z_1 = Z_2 = j0.225,\ Z_0 = j(0.15 + 3\times 0.05) = j0.3 \text{ and } Z_f = 0$$

Then

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$$I_{fa1} = \frac{1.0}{j0.225 + \left(\dfrac{j0.225 \times j0.3}{j0.525}\right)} = -j2.8283$$

$$I_{fa2} = -I_{fa1}\frac{j0.3}{j0.525} = j1.6162$$

$$I_{fa0} = -I_{fa1}\frac{j0.225}{j0.525} = j1.2121$$

Now the sequence components of the voltages are

$$V_{a1} = 1.0 - j0.225 \times I_{fa1} = 0.3636$$
$$V_{a2} = -j0.225 \times I_{fa2} = 0.3636$$
$$V_{a0} = -j0.3 \times I_{fa0} = 0.3636$$

Also note from Fig. 8.9 that

$$V_a = E_{an} + V_n + j0.0225(I_{fb} + I_{fc}) = 1.0909$$

and $V_b = V_c = 0$. Therefore

$$V_{a012} = C\begin{bmatrix} 1.0909 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 0.3636 \\ 0.3636 \\ 0.3636 \end{bmatrix}$$

which are the same as obtained before.

ΔΔΔ

8.4 FAULT CURRENT COMPUTATION USING SEQUENCE NETWORKS

In this section we shall demonstrate the use of sequence networks in the calculation of fault currents using sequence network through some examples.

Example 8.4: Consider the network shown in Fig. 8.10. The system parameters are given below:

p.u

Generator G:	50 MVA, 20 kV, $X'' = X_1 = X_2 = 20\%$, $X_0 = 7.5\%$
Motor M:	40 MVA, 20 kV, $X'' = X_1 = X_2 = 20\%$, $X_0 = 10\%$, $X_n = 5\%$
Transformer T_1:	50 MVA, 20 kVΔ/110 kVY, $X = 10\%$
Transformer T_2:	50 MVA, 20 kVΔ/110 kVY, $X = 10\%$
Transmission line:	$X_1 = X_2 = 24.2\ \Omega$, $X_0 = 60.5\ \Omega$

because Y- is grounded.

not p.u

We shall find the fault current for when a (a) 1LG, (b) LL and (c) 2LG fault occurs at bus-2.

① find p.u after referraling.

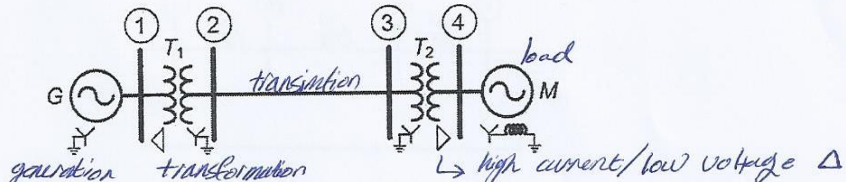


Fig. 8.10 Radial power system of Example 8.4.

Let us choose a base in the circuit of the generator. Then the per unit impedances of the generator are:

$$\boxed{X_{G1} = X_{G2} = 0.2, \quad X_{G0} = 0.075}$$

50MVA, 20KVA

The per unit impedances of the two transformers are

because they have the same S, V of the Generator.

$$X_{T1} = X_{T2} = 0.1$$

The MVA base of the motor is 40, while the base MVA of the total circuit is 50. Therefore the per unit impedances of the motor are

$$X_{M1} = X_{M2} = 0.2 \times \frac{50}{40} = 0.25, \quad X_{M0} = 0.1 \times \frac{50}{40} = 0.125, \quad X_n = 0.05 \times \frac{50}{40} = 0.0625$$

$\left(\frac{20}{20}\right)^2$ same same.

For the transmission line

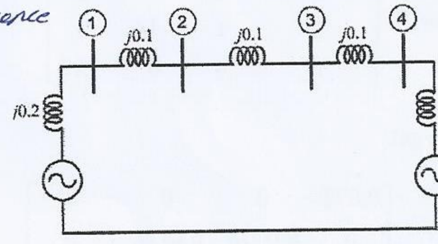
$$Z_{base} = \frac{110^2}{50} = 242\,\Omega \quad = \frac{V^2_{base}}{S_{base}}$$

Therefore

$$X_{L1} = X_{L2} = \frac{\overset{\text{actual}}{24.2}}{\underset{\text{base}}{242}} = 0.1, \quad X_{L0} = \frac{60.5}{242} = 0.25$$

Let us neglect the phase shift associated with the Y/Δ transformers. Then the positive, negative and zero sequence networks are as shown in Figs. 8.11-8.13.

system in the sequence



$X_n \rightarrow$ in sequences analysis.

Fig. 8.11 Positive sequence network of the power system of Fig. 8.10.

in the component → the sync machines appear hear.

generation

$$Z_{p.u\,old} \left(\frac{V_{new}}{V_{base}}\right)^2 \left(\frac{S_{base\,old}}{S_{new}}\right)$$

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j0.3 j0.45

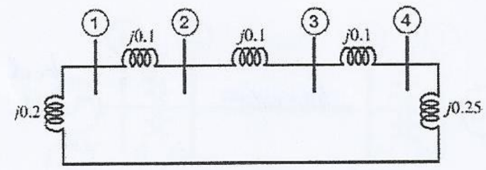

Fig. 8.12 Negative sequence network of the power system of Fig. 8.10.

taking into consideration. the zero-trans.

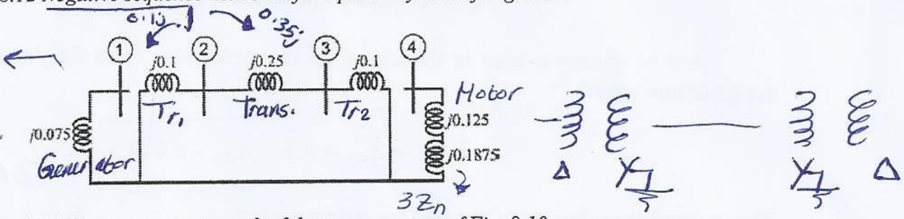

Fig. 8.13 Zero sequence network of the power system of Fig. 8.10.

trans. Connection.

From Figs. 8.11 and 8.12 we get the following Y_{bus} matrix for both positive and negative sequences

$$Y_{bus1} = Y_{bus2} = j\begin{bmatrix} -15 & 10 & 0 & 0 \\ 10 & -20 & 10 & 0 \\ 0 & 10 & -20 & 10 \\ 0 & 0 & 10 & -14 \end{bmatrix}$$

generation is shorted. in the.

Inverting the above matrix we get the following Z_{bus} matrix

$$Z_{bus1} = Z_{bus2} = j\begin{bmatrix} 0.1467 & 0.1200 & 0.0933 & 0.0667 \\ 0.1200 & \boxed{0.1800} & 0.1400 & 0.1000 \\ 0.0933 & 0.1400 & 0.1867 & 0.1333 \\ 0.0667 & 0.1000 & 0.1333 & 0.1667 \end{bmatrix}$$

Again from Fig. 8.13 we get the following Y_{bus} matrix for the zero sequence

$$Y_{bus0} = j\begin{bmatrix} -13.3333 & 0 & 0 & 0 \\ 0 & -14 & 4 & 0 \\ 0 & 4 & -14 & 0 \\ 0 & 0 & 0 & -3.2 \end{bmatrix}$$

Inverting the above matrix we get

$$Z_{bus0} = j\begin{bmatrix} 0.075 & 0 & 0 & 0 \\ 0 & 0.0778 & 0.0222 & 0 \\ 0 & 0.0222 & 0.0778 & 0 \\ 0 & 0 & 0 & 0.3125 \end{bmatrix}$$

or using inception method.

Hence for a fault in bus-2, we have the following Thevenin impedances

$$Z_1 = Z_2 = j0.18, \quad Z_0 = j0.0778$$

Z_{22} from matrix.

in the seq Z_{22}, Z_{22} in negative seq

OR Alternatively we find from Figs. 8.11 and 8.12 that

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$$\boxed{\begin{aligned} Z_1 = Z_2 &= j0.3 \,\|\, j0.45 = j0.18 \\ Z_0 &= j0.1 \,\|\, j0.35 = j0.0778 \end{aligned}}$$

(a) Single-Line-to-Ground Fault: Let a bolted 1LG fault occurs at bus-2 when the system is unloaded with bus voltages being 1.0 per unit. Then from (8.7) we get

all faults on bus 2

circuit من ال

$$I_{fa0} = I_{fa1} = I_{fa2} = \frac{1}{j(2\times 0.18 + 0.0778)} = -j2.2841 \text{ per unit}$$

$+3Z_n$ zero. Z_n = zero

Also from (8.4) we get

$$I_{fa} = 3I_{fa0} = -j6.8524 \text{ per unit}$$

Also $I_{fb} = I_{fc} = 0$. From (8.5) we get the sequence components of the voltages as

$$\begin{aligned} V_{2a0} &= -j0.0778 I_{fa0} = -0.1777 \\ V_{2a1} &= 1 - j0.18 I_{fa1} = 0.5889 \\ V_{2a2} &= -j0.18 I_{fa2} = -0.4111 \end{aligned}$$

Therefore the voltages at the faulted bus are finding abc from 012

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = C^{-1}\begin{bmatrix} V_{2a0} \\ V_{2a1} \\ V_{2a2} \end{bmatrix} = \begin{bmatrix} 0 \\ 0.9061\angle -107.11^\circ \\ 0.9061\angle 107.11^\circ \end{bmatrix}$$

(b) Line-to-Line Fault: For a bolted LL fault, we can write from (8.16)

$Z_n = 0$

$$\frac{V_f}{Z_{K1} + Z_n + Z_{K2}} = I_{fa1} = -I_{fa2} = \frac{1}{j2\times 0.18} = -j2.7778 \text{ per unit}$$

because $I_b = -I_a$.

Then the fault currents are

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = C^{-1}\begin{bmatrix} 0 \\ I_{fa1} \\ I_{fa2} \end{bmatrix} = \begin{bmatrix} 0 \\ -4.8113 \\ 4.8113 \end{bmatrix}$$

I_{fa0}

Finally the sequence components of bus-2 voltages are

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$$V_{2a0} = 0$$
$$V_{2a1} = 1 - j0.18 I_{fa1} = 0.5$$
$$V_{2a2} = -j0.18 I_{fa2} = 0.5$$

Hence faulted bus voltages are

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = \underset{= \underline{A}}{C^{-1}} \begin{bmatrix} V_{2a0} \\ V_{2a1} \\ V_{2a2} \end{bmatrix} = \begin{bmatrix} 1.0 \\ -0.5 \\ -0.5 \end{bmatrix}$$

(c)Double-Line-to-Ground Fault: Let us assumes that a bolted 2LG fault occurs at bus-2. Then

$$Z_{eq} = j0.18 \| j0.0778 = j0.0543$$

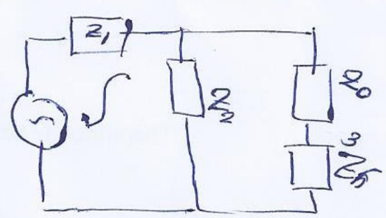
Hence from (8.24) we get the positive sequence current as

$$\frac{V_f}{Z_{11} + Z_{22} // (3Z_f + Z_{k00})} = I_{fa1} = \frac{1}{j0.18 + Z_{eq}} = -j4.2676 \text{ per unit} = \frac{V_f}{Z_1 + [Z_2 // (Z_0 + 3Z_f)]}$$

The zero and negative sequence currents are then computed from (8.25) and (8.26) as

Current devider.

$$I_{fa0} = -I_{fa1} \frac{j0.18}{j(0.18 + 0.0778)} = j2.9797 \text{ per unit}$$
$$I_{fa2} = -I_{fa1} \frac{j0.0778}{j(0.18 + 0.0778)} = j1.2879 \text{ per unit}$$



Therefore the fault currents flowing in the line are Fault ما صار فيا → short generator here.

$$\begin{bmatrix} I_{fa} \\ I_{fb} \\ I_{fc} \end{bmatrix} = C^{-1} \begin{bmatrix} I_{fa0} \\ I_{fa1} \\ I_{fa2} \end{bmatrix} = \begin{bmatrix} 0 \\ 6.657\angle 137.11° \\ 6.657\angle 42.89° \end{bmatrix}$$

مجموع هاد هو الي بمر ب Z_f

Furthermore the sequence components of bus-2 voltages are

$$V_{2a0} = -j0.0778 I_{fa0} = 0.2318$$
$$V_{2a1} = 1 - j0.18 I_{fa1} = 0.2318$$
$$V_{2a2} = -j0.18 I_{fa2} = 0.2318$$

Therefore voltages at the faulted bus are 0.2318

$$\begin{bmatrix} V_a \\ V_b \\ V_c \end{bmatrix} = C^{-1} \begin{bmatrix} V_{2a0} \\ 0.2318 \\ V_{2a1} \\ 0.2318 \\ V_{2a2} \end{bmatrix} = \begin{bmatrix} 0.6954 \\ 0 \\ 0 \end{bmatrix} ?$$