ElCoM
ELECTRICAL COMPUTER MECHATRONICS

نمذجة ومحاكاة

د.محمد القادري

للطالب المبدع
حمزة اسماعيل

إرادة - ثقة - تغيير

Modeling and Simulation :-
Modeling Process :-

* Modeling Process :-

Real World System

define the goal of the System
We can make some approximation to the System (Real World System)

Domain Knowladeg
I need it to write governing equation

We will have dynamic System governed by differenitial equation in time domain

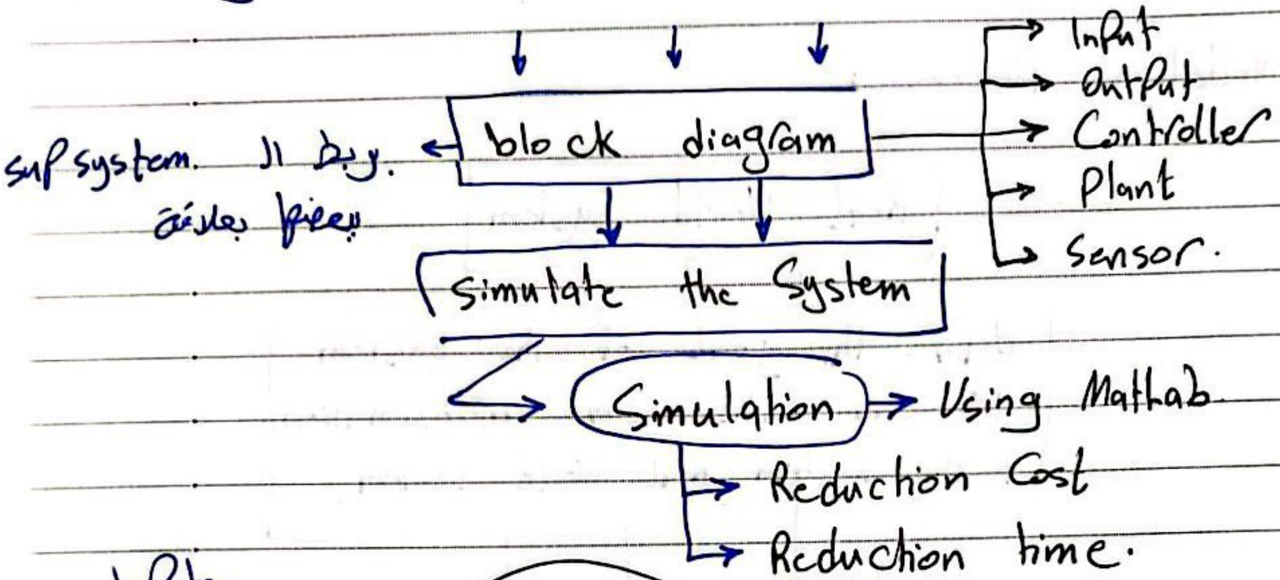
Ordinary differential equation

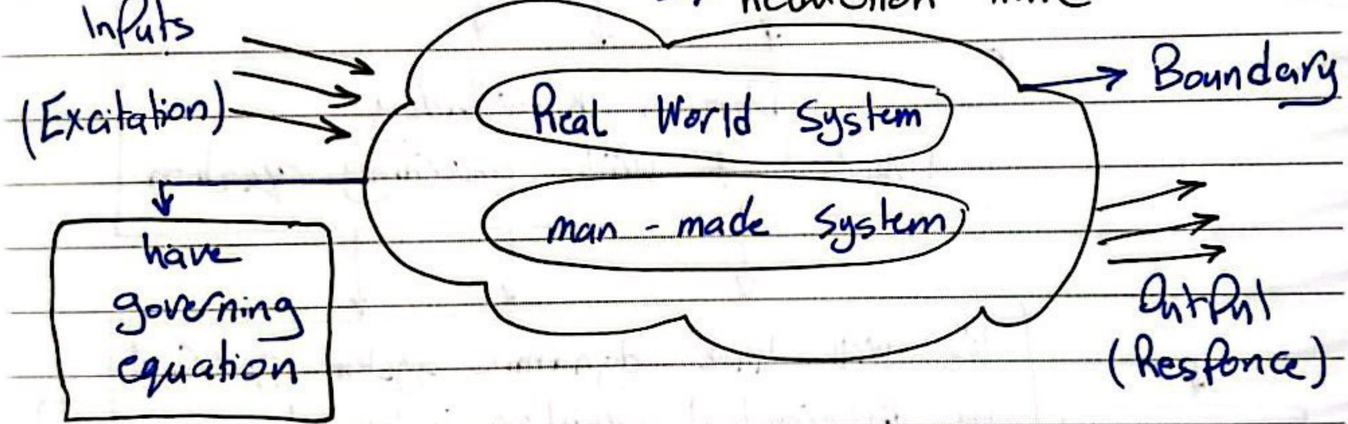
by Laplace transform. We will transform the System to S-domain (transfer function)

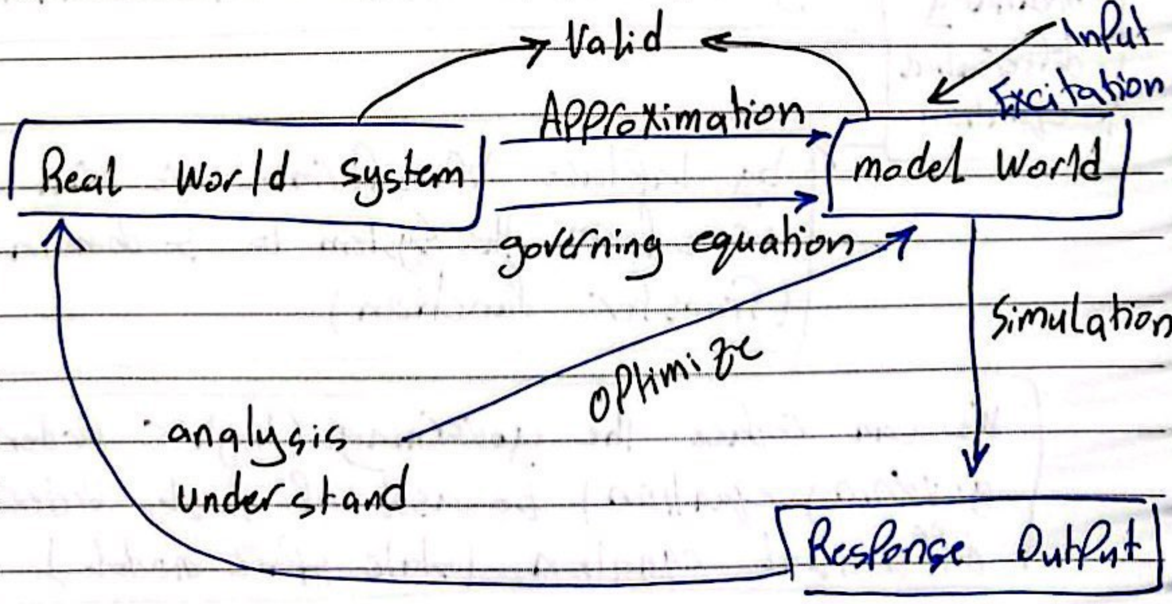
We can reduce the governing (higher Order governing equation) to a set of first Order differential equation (state space model)

Modeling and Simulation :-

Modeling Process :-







Modeling and Simulation :-

Modeling Process :-

Physical system

↓

dynamic system

↓

Consitituted equation / governing equ

- for first order system → need one equ
- for second order system and higher → need more than one equ.

↓

Transfer function

$$\frac{\text{Output}}{\text{Input}} \rightarrow \frac{Y(s)}{R(s)}$$

$$T.F = \frac{(s+2)(s-1)}{(s^2+2s+3)(s-1)}$$

→ Poles in right hand side → Unstable.

Poles / Zeros Cancelation.

state space Representation

بقدر استخرج كل system

حتى أعرف ال stability

Modeling and Simulation :-
Modeling Process :-

- System (Plant) :-
Collection of Interacting Components of Interest defined by a system boundery.

- Dynamic system :-
Rate of change of response / state Variable can not be neglected.

- state Variable :-
a set of Variables can Completely identify the ~~system~~ dynamic state of the system.

- Input :-
Excitation is (Known, UnKnown) to the system

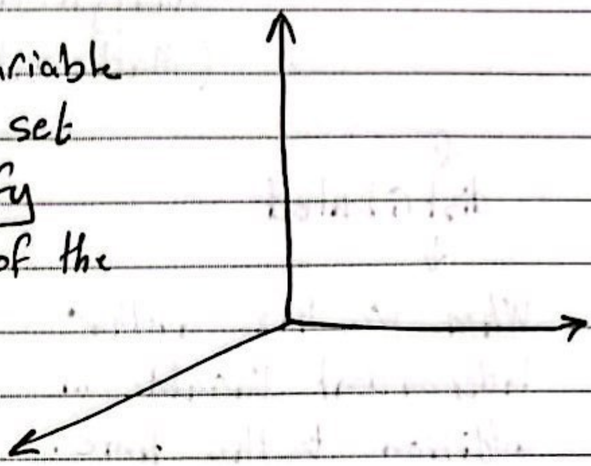
- Output :-
Response to the system. describe the dynamic behaver of the system.

Modeling and Simulation :-

Modeling Process :-

* state Variable :-

We can define state variable of the system as a min set of Variables that Identify the dynamic behavier of the system Completely.



* Model Types :-

1. Physical Model (Prototypes) :

advantage → بكون عندك فكرة عن ال design

disadvantage → expensive, high effort (time).

2. Analytical Model (Mathematical model) :-

Flexibility to change the Parameters of the System

3. Experimental Model

use Input / Output experimetal data for model Identification

4. Computer (Numerical) Model :-

Use in finite system analysis

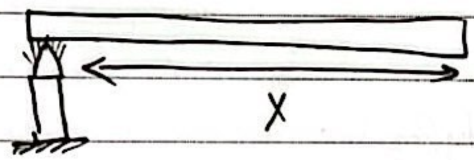
Modeling and Simulation:-

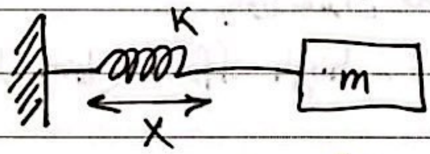
Modeling Process :-

Analytical Model
(Mathematical model)

distributed

When we have another Independent Variable in addition to the time.





lumped

Lumped System

determenstic

[Same inital Condition with same input → same output → Same dynamic behavier]

stochastic

[same initial Condition with same Input → different Output]

Modeling and Simulation :-

Modeling Process :-

Mathematical Models.

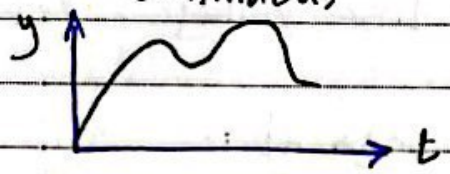
Distributed System

Lumped system

Deterministic

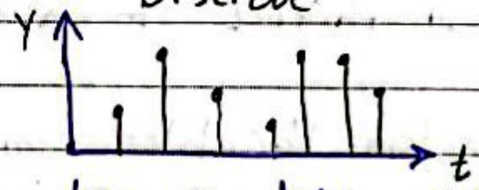
stochastic

Continuous

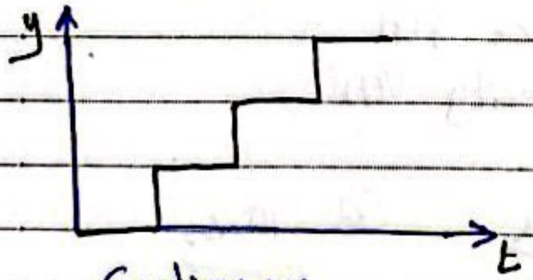


time Can take any real Value

Discrete



time can take only specific point.



Continuous

Modeling and Simulation

Modeling Process :-

* Through and Across Variable :-

- Through Variable :-

Represent a quantity that passes through an element and measured with a gauge Connected in series to an element. like Current, force.

- Across Variable :-

Whose Value is determined by a measuring a difference of element, and measured with a gauge connected in Parallel to an element. like Voltage, Velocity

* System Variables and Ideal segment :-

1- Mechanical System :-

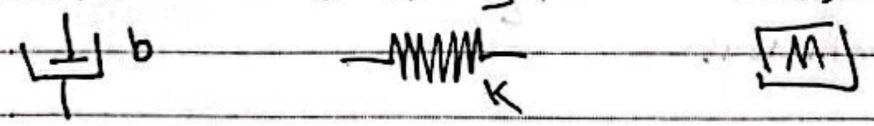
- System Variables :

Through Variable :- force $f(t)$

Across Variable :- Velocity $V(t)$

- Ideal segment :

① damper ② spring ③ Mass

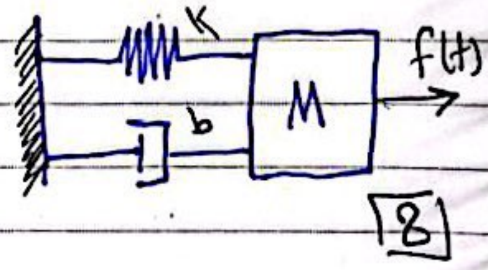


Constitutive equation :-

$F_b = b\, V(t)$

$F_s = K x = K \int V(t)\, dt$

$F = m a = m \frac{dV}{dt}$



Modeling and Simulation

Modeling Process :-

* System Variable and Ideal segments :-

1. Mechanical system :

for Rotational Mechanical System.

. Through Variable :- Torque $T(t)$

Across Variable : Angular Velocity $W(t)$

Ideal Segment :

① damper ② spring ③ Inerta mass

b K M J

Constitutive equation

$T_b = b\ W(t)$.

$T_s = K \int W(t)\ dt$.

$T = J \dfrac{dW(t)}{dt}$

2. Electrical system ::

-System Variables :-

Through Variable :- Current $I(t)$

Across Variable : Voltage $V(t)$.

Ideal segment :-

① Resistor R ② Capacitance ③ Inductive

C L

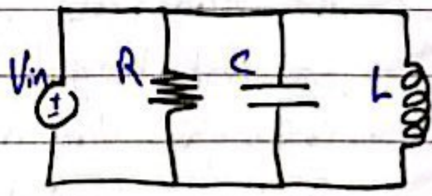
Modeling and Simulation :-

Modeling Process :-

* System Variables and Ideal segments :-

2- Electrical system :

Constitutive equation



$I_R(t) = \frac{V(t)}{R} \rightarrow V(t) = R\, I_R(t)$

$I_c(t) = C \frac{dV(t)}{dt} \rightarrow V(t) = \frac{1}{C} \int I_c(t)\, dt$

$I_L(t) = \frac{1}{L} \int V(t)\, dt \rightarrow V(t) = L \frac{dI_L(t)}{dt}$

3- Fluid System :-

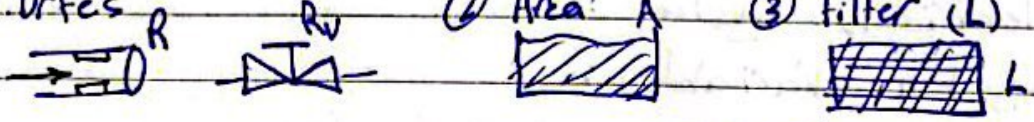
- System Variables:-

Through Variable :- Flow rate $q(t)$

Across Variable :- Level $H(t)$ / Pressure $P(t)$.

- Ideal segment.

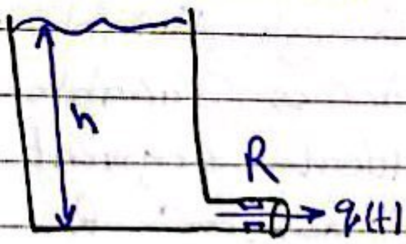
① Orfes ② Area: A ③ filter (L)

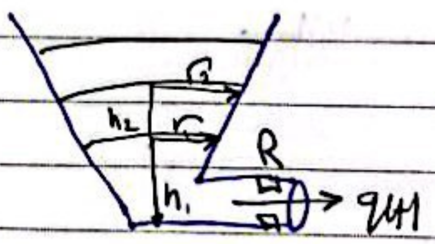


Constitutive equation

$H(t) = R * q(t)$

$q(t) = \frac{dV}{dt} = A \frac{dh(t)}{dt}$





Modeling and Simulation :-
Modeling Process :-

* System Variables and Ideal segment.
4- Thermal System :-
- System Variable :-
Through Variable : heat flow $q_h(t)$
Across Variable : Temperature $T(t)$
- Ideal segments :
① Thermal Resistance (R) ② Thermal Capacitance (C)
Constitutive equation :

$$T(t) = R\, q_h(t)$$

$$q_h(t) = C \frac{dT(t)}{dt}$$

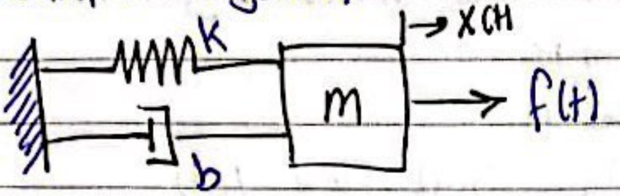
* steps to develop mathematical model :-
1. Identify the Input / Output state Variable and the purpose of the system
2. Identify system variables and initial Condition and some cases we need to determine the Input or Excitation to the system .
3- Approximation of Various element in the System.
4- draw free body diagram for the System.
5- derive the governing equation or Constitutive equation
6- Eliminate auxilary Variables from the set of Constitutive.
7- Express the derived equation as Input / Output Pairs assuming Initial Condition and boundary Condition

Scanned with CamScanner

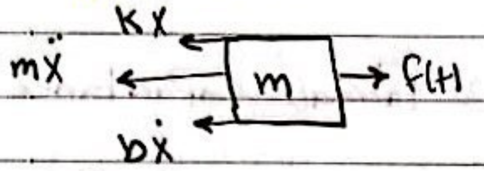
Modeling and Simulation :-

Modeling Process :-

* spring - damper system :



at equilibriam

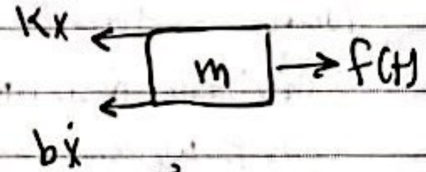


$\Sigma \vec{F} = 0.$

$f(t) - Kx - b\dot{x} - m\ddot{x} = 0.$

$m\ddot{x} + b\dot{x} + Kx = f(t)$

no equilibriam.



$\Sigma \vec{F} = m\vec{a}$

$f(t) - Kx - b\dot{x} = m\ddot{x}$

$m\ddot{x} + b\dot{x} + Kx = f(t)$

$m\ddot{x} + b\dot{x} + Kx = f(t) \xLeftrightarrow{\text{same}} m\ddot{x} + b\dot{x} + Kx = f(t)$

I'm interisted in studying the displacement of the system with different External force (input)

$\frac{X(s)}{F(s)} = \frac{\text{Output}}{\text{Input}} = T.F.$

Laplace Transform → Convert differential equation to algabric equation.

$\mathcal{L}\, \dddot{y}(t) = s^3 y(s) -$

$\mathcal{L}\, \ddot{y}(t) = s^2 y(s) - \underbrace{\cancel{s\,y(0)}}_{\text{Zero Initial condition}} - \underbrace{\cancel{\dot{y}(0)}}_{\text{Zero Initial condition}}$

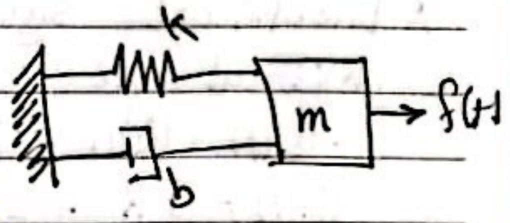
$\mathcal{L}\, \dot{y}(t) = s\,y(s) - y(0)$

$\mathcal{L}\, y(t) = y(s).$

Modeling and Simulation

Modeling Process :-

* Spring - damper system :



$\Sigma F = ma$.

$m\ddot{x} + b\dot{x} + kx = f(t)$

by taking laplace Transform and Considering zero initial Condition.

$ms^2 X(s) + bs X(s) + K X(s) = f(s)$

$$\frac{X(s)}{f(s)} = \frac{1}{ms^2 + bs + K}$$

If we interest about Velocity.

$m\dot{V} + bV + K\int V\,dt = f(t)$.

$msV(s) + bV(s) + \frac{K}{s} V(s) = f(s)$

$$\frac{V(s)}{f(s)} = \frac{s}{ms^2 + bs + k}$$

general form of second order Transfer function

$$T.f = \frac{K_P W_n^2}{s^2 + 2\xi W_n s + W_n^2}$$

K_P: proportional constant

W_n: natural frequanca

ξ: damping ratio.

$$\frac{X(s)}{F(s)} = \frac{1/m}{s^2 + \frac{b}{m}s + \frac{k}{m}}$$

$\rightarrow K_P = \frac{1}{m}$

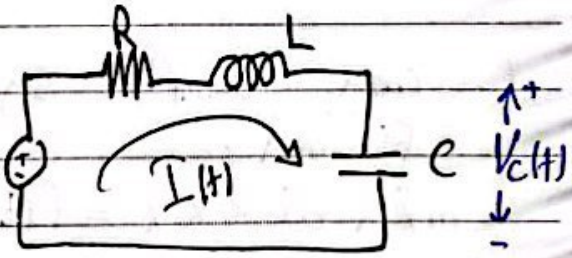
$\rightarrow W_n = \sqrt{\frac{k}{m}}$.

$\rightarrow \xi = \frac{b}{2\sqrt{Km}}$

Modeling and Simulation :-

Modeling Process :-

* Electrical System :-



KVL at the circuit.

$-V_{in}(t) + V_R(t) + V_L(t) + V_c(t) = 0$

$V_{in}(t) = IR + L\dfrac{dI(t)}{dt} + \dfrac{1}{C}\int I\,dt \qquad I_R = I_L = I_c = I$

but $I = I_c(t) = C\dfrac{dV_c(t)}{dt}$.

$V_{in}(t) = RC\dfrac{dV_c(t)}{dt} + LC\dfrac{d^2V_c(t)}{dt^2} + V_c(t)$ take LaPlace.

$V_{in}(s) = RCs\,V_c(s) + LCs^2\,V_c(s) + V_c(s)$

$$\frac{V_c(s)}{V_{in}} = \frac{1}{LCs^2 + RCs + 1}$$

$$T.f = \frac{K_P W_n^2}{s^2 + 2\xi W_n s + W_n^2} = \frac{V_c(s)}{V_{in}} = \frac{1/LC}{s^2 + \frac{R}{L}s + \frac{1}{LC}}$$

K_P : Proportional constant $\rightarrow K_P = 1$

W_n : natural frequence $\rightarrow W_n = \dfrac{1}{\sqrt{LC}}$

ξ : damping ratio

$\rightarrow \xi = \dfrac{R}{2\sqrt{\frac{L}{C}}}$

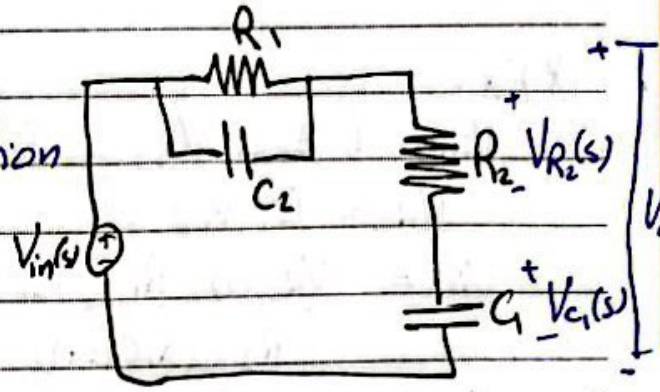
Try for $\dfrac{V_L(s)}{V_{in}(s)}$.

$V_{in} = IR + V_L(t) + \dfrac{1}{C}\int I\,dt \quad \because I = I_L = \dfrac{1}{L}\int V_L(t)\,dt$

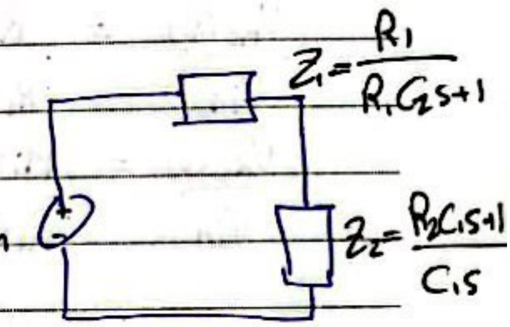
$V_{in}(s) = \dfrac{R}{Ls}V_L(s) + V_L(s) + \dfrac{1}{CLs^2}V_L(s)$

$$\frac{V_L(s)}{V_{in}(s)} = \frac{LCs^2}{LCs^2 + RCs + 1}$$

Modeling and Simulation :-
Modeling Process :-



* Ex: for the following system develope the Transfer function
$\frac{V_o(s)}{V_{in}(s)}$, $\frac{V_{R_2}(s)}{V_{in}(s)}$, $\frac{V_{C_1}(s)}{V_{in}(s)}$



$$V_{in} = I \, Z_{eq}$$

$$Z_{eq} = Z_1 + Z_2 = \left[\frac{R_1}{R_1 C_2 s + 1} + \frac{R_2 C_1 s + 1}{C_1 s} \right]$$

$$I = \frac{C_1 s (R_1 C_2 s + 1)}{R_1 C_1 s + (R_1 C_2 s + 1)(R_2 C_1 s + 1)} V_{in}$$

$$V_{in} = I Z_1 + V_o(s)$$

$$V_{in}(s) = \frac{R_1 C_1 s}{R_1 C_1 s + (R_1 C_2 s + 1)(R_2 C_1 s + 1)} V_{in}(s) + V_o(s)$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{\cancel{R_1 C_1 s} + (R_1 C_2 s + 1)(R_2 C_1 s + 1) - \cancel{R_1 C_1 s}}{R_1 C_1 s + (R_1 C_2 s + 1)(R_2 C_1 s + 1)}$$

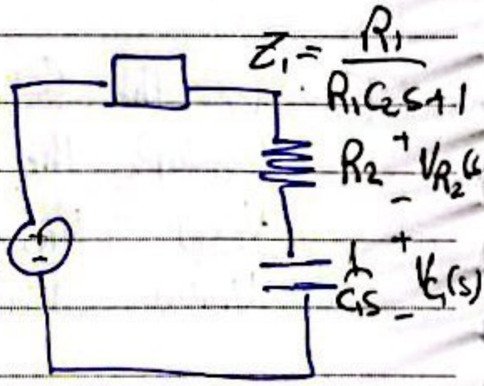
$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{(R_1 C_2 s + 1)(R_2 C_1 s + 1)}{R_1 R_2 C_1 C_2 s^2 + (R_1 C_1 + R_1 C_2 + R_2 C_1) s + 1}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{s^2 + \frac{(R_2 C_1 + R_1 C_2)}{R_1 R_2 C_1 C_2} s + \frac{1}{R_1 R_2 C_1 C_2}}{s^2 + \frac{R_1 C_1 + R_1 C_2 + R_2 C_1}{R_1 R_2 C_1 C_2} + \frac{1}{R_1 R_2 C_1 C_2}}$$

Modeling and Simulation :-

Modeling Process :-

*Ex :-



$$V_{in} = I Z_1 + V_{R_2} + I \frac{1}{C_1 s}$$

$$V_{R_2} = I \; R_2 .$$

$$V_{in} = \frac{V_{R_2}(s)}{R_2} \frac{R_1}{(R_1C_2s+1)} + V_{R_2}(s) + \frac{V_{R_2}(s)}{R_2C_1s}$$

$$\frac{V_{R_2}(s)}{V_{in}} = \frac{R_2 C_1 s (R_1C_2s+1)}{R_1C_1s + (R_1C_2s+1) + R_2C_1s(R_1C_2s+1)}$$

$$\frac{V_{R_2}(s)}{V_{in}} = \frac{R_1R_2C_1C_2s^2 + R_2C_1s}{R_1R_2C_1C_2s^2 + (R_2C_1 + R_1C_2 + R_1C_1)s + 1}$$

$$V_{in} = I Z_1 + I R_2 + V_{C_1}(s)$$

$$I = I_{C_1}(s) = C_1 s \; V_{C_1}(s)$$

$$V_{in} = \frac{C_1 s \; R_1}{R_1C_2s+1} V_{C_1}(s) + R_2C_1s \; V_{C_1}(s) + V_{C_1}(s)$$

~~Vout~~

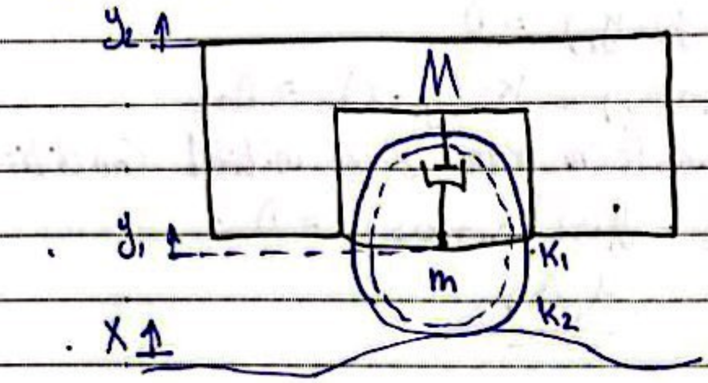
$$\frac{V_{C_1}(s)}{V_{in}(s)} = \frac{R_1C_2s+1}{R_1C_1s + R_2C_1s(R_1C_2s+1) + (R_1C_2s+1)}$$

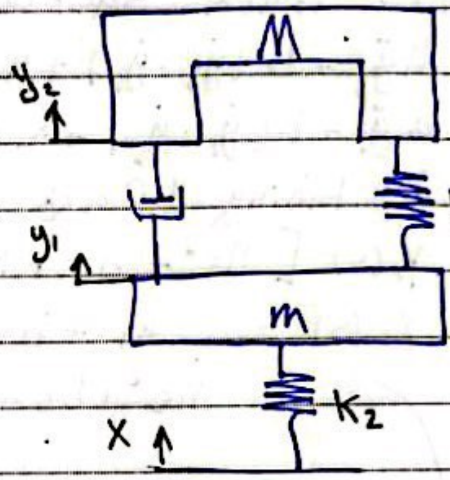
$$\frac{V_{C_1}(s)}{V_{in}(s)} = \frac{R_1C_2s+1}{R_1R_2C_1C_2s^2 + (R_1C_1 + R_2C_1 + R_1C_2)s + 1}$$

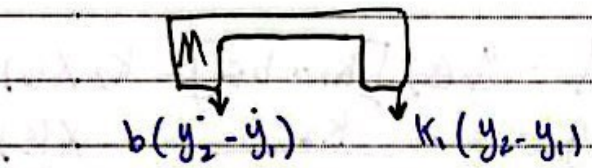
Modeling and Simulation :-

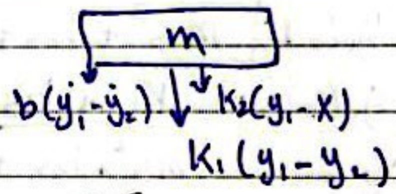
Modeling Process :-

* Ex : Truck suspension.









$\Sigma F = Ma$.

$-b(\dot{y}_2-\dot{y}_1) - K_1(y_2-y_1) = M\ddot{y}_2$

$M\ddot{y}_2 + b(\dot{y}_2-\dot{y}_1) + K_1(y_2-y_1) = 0$

$\Sigma f = ma$.

$-b(\dot{y}_1-\dot{y}_2) - K_2(y_1-X)$
$- K_1(y_1-y_2) = m\ddot{y}_1$.

$$M\ddot{y}_2 + b(\dot{y}_2-\dot{y}_1) + K_1(y_2-y_1) = 0 \quad \text{---- (1)}$$

$$M\ddot{y}_1 + b(\dot{y}_1-\dot{y}_2) + K_2(y_1-X) + K_1(y_1-y_2) = 0 \quad \text{--- (2)}$$

Excitation input ⇒ displacement.
Here. We are intersted in studying $y_2(s)$ with External input $X(s)$.

$$\frac{y_2(s)}{X(s)} = \frac{\text{Output}}{\text{Input}} = T.f.$$

Modeling and Simulation :

Modeling Process :-

* Ex :- Truck suspension :

$M\ddot{y}_2 + b(\dot{y}_2 - \dot{y}_1) + K_1(y_2 - y_1) = 0$

$m\ddot{y}_1 + b(\dot{y}_1 - \dot{y}_2) + K_1(y_1 - y_2) + K_2(y_1 - X) = 0.$

by taking laplace transform with zero initial condition

$Y_2(s)\left[Ms^2 + bs + K_1\right] - y_1(s)\left[K_1 + bs\right] = 0.$

$$Y_2(s) = \frac{K_1 + bs}{Ms^2 + bs + K_1}\, \underline{\underline{y_1(s)}}$$

$y_1(s)\left[ms^2 + bs + K_1 + K_2\right] - y_2(s)\left[K_1 + bs\right] = K_2 X(s)$

$$y_1(s) = \frac{K_1 + bs}{ms^2 + bs + K_1 + K_2}\, y_2(s) + \frac{K_2}{ms^2 + bs + K_1 + K_2}\, X(s)$$

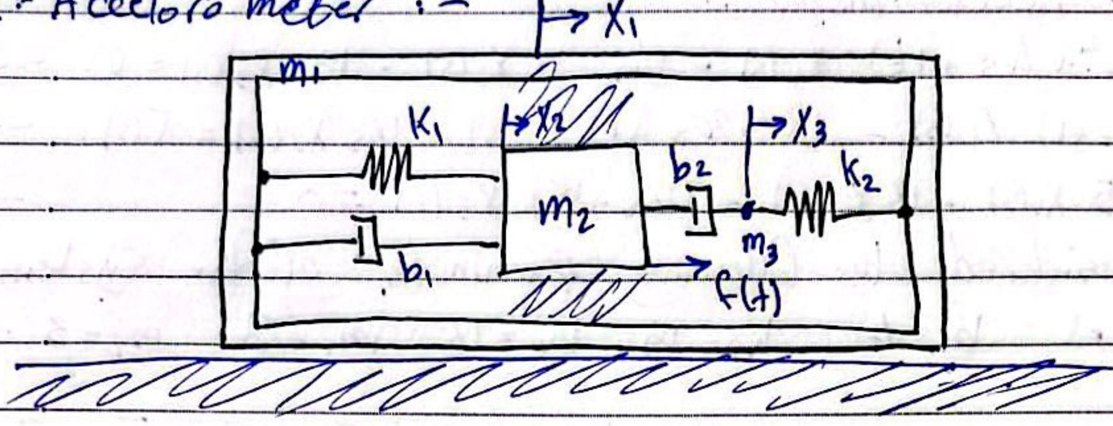
$$Y_2(s) = \frac{K_1 + bs}{Ms^2 + bs + K_1}\left[\frac{K_2 X(s) + (K_1 + bs)\, y_2(s)}{ms^2 + bs + K_1 + K_2}\right]$$

$$\frac{y_2(s)}{X(s)} = \frac{K_2(K + bs)}{(ms^2 + bs + K_1 + K_2)(Ms^2 + bs + K_1) - (K_1 + bs)^2}$$

Modeling and Simulation

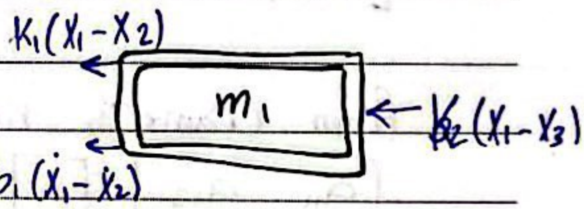
Modeling Process :-

* Ex :- Acceloro meter :-



We can study different resPonce of the system with respect to change in excitation external input $f(t)$

$$T.F = \frac{X_3(s)}{F(s)} \;,\; \frac{X_2(s)}{F(s)} = ??$$



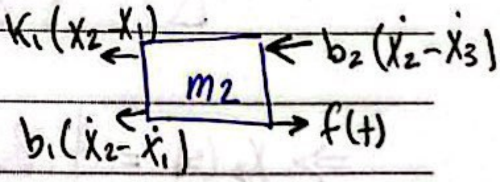
for M_1 :-

$\Sigma F = m_1 a$.

$$-K_1(X_1-X_2) - K_2(X_1-X_3) - b_1(\dot{X}_1-\dot{X}_2) = m_1 \ddot{X}_1$$

$$m_1 s^2 X_1(s) + b_1 s(X_1(s)-X_2(s)) + K_1(X_1(s)-X_2(s)) + K_2(X_1(s)-X_3(s)) = 0.$$

for M_2 :-

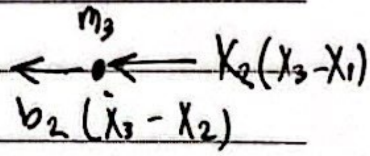


$\Sigma F = m_2 a$.

$$f(t) - b_2(\dot{X}_2-\dot{X}_3) - b_1(\dot{X}_2-\dot{X}_1) - K_1(X_2-X_1) = m_2 \ddot{X}_2$$

$$m_2 s^2 X_2(s) + b_2 s(X_2(s)-X_3(s)) + b_1 s(X_2(s)-X_1(s)) + K_1(X_2(s)-X_1(s)) = f(s)$$

for M_3 :-



$\Sigma F = m_3 a$.

$$-K_2(X_3-X_1) - b_2(\dot{X}_3-X_2) = 0$$

$$K_2(X_3(s)-X_1(s)) - b_2 s(X_3(s)-X_2(s)) = 0.$$

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Modeling and Simulation :-

Modeling Process :-

*Ex :- Accelorometer :-

$(8s^2+4s+16)\,X_1(s) - (4s-1)\,X_2(s) - 15\,X_3(s) = 0$ ... ①

$-(4s+1)\,X_1(s) + (3s^2+20s+1)\,X_2(s) - 16s\,X_3(s) = F(s)$ ... ②

$-15\,X_1(s) - 16\,X_2(s) + (16s+15)\,X_3(s) = 0$ ... ③

Considered the following parameter of the system

$K_1=1 \quad b_1=4 \quad K_2=15 \quad b_2=16 \quad m_1=8 \quad m_2=3$.

$$\begin{bmatrix} 8s^2+4s+16 & -(4s-1) & -15 \\ -(4s+1) & 3s^2+20s+1 & -16s \\ -15 & -16 & 16s+15 \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \\ X_3(s) \end{bmatrix} = \begin{bmatrix} 0 \\ F(s) \\ 0 \end{bmatrix}$$

from Cramer's rule :-

$$\begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \rightarrow X_1 = \frac{\begin{vmatrix} b_1 & a_{12} \\ b_2 & a_{22} \end{vmatrix}}{\Delta_A}$$

$$X_2 = \frac{1}{\Delta} * \begin{vmatrix} a_{11} & b_1 \\ a_{21} & b_2 \end{vmatrix}$$

$$\Rightarrow X_3(s) = \frac{\begin{vmatrix} 8s^2+4s+16 & -(4s-1) & 0 \\ -(4s+1) & 3s^2+20s+1 & F(s) \\ -15 & -16 & 0 \end{vmatrix}}{\begin{vmatrix} 8s^2+4s+16 & -(4s-1) & -15 \\ -(4s+1) & 3s^2+20s+1 & -16s \\ -15 & -16 & 16s+15 \end{vmatrix}}$$

Modeling and Simulation :-
Modeling Process :-

* Ex : Accelorometer :-

$$\Delta = \begin{vmatrix} 8s^2+4s+16 & -(4s-1) & -15 \\ -(4s+1) & 3s^2+20s+1 & -16s \\ -15 & -16 & 16s+15 \end{vmatrix}$$

$$= (8s^2+4s+16)\left[(3s^2+20s+1)(16s+15) - 16(16s)\right]$$
$$+ (4s-1)\left[-(4s+1)(16s+15) - 15(16s)\right]$$
$$- 15\left[16(4s+1) + 15(3s^2+20s+1)\right]$$

$$X_3(s) = \frac{(8s^2+4s+16)(16f(s)) + (4s-1)(15f(s))}{\Delta}$$

$$\frac{X_3(s)}{F(s)} = \frac{16(8s^2+4s+16) + 15(4s-1)}{\Delta}$$

$$X_2(s) = \frac{\begin{vmatrix} 8s^2+4s+16 & 0 & -15 \\ -(4s+1) & F(s) & -16s \\ -15 & 0 & 16s+15 \end{vmatrix}}{\Delta}$$

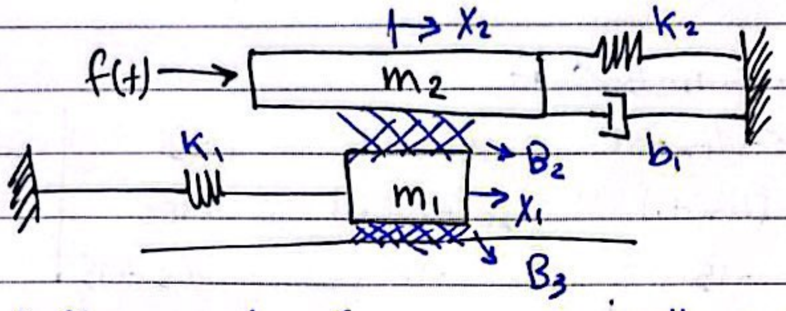
$$\frac{X_2(s)}{F(s)} = \frac{(8s^2+4s+16)(16s+15) - 15(15)}{\Delta}$$

$$\frac{X_1(s)}{F(s)} = \frac{(4s-1)(16s+15) + 15(16)}{\Delta}$$

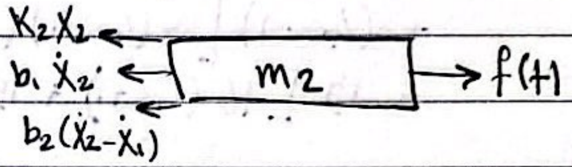
Modeling and Simulation :-

Modeling Process :-

* Ex :-



study displacement of m_1 , m_2 with respect to $F(s)$



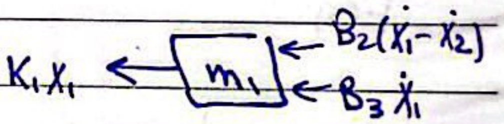
$$\Sigma F = m_2 \vec{a} .$$

$$f(t) - b_1 \dot{X}_2 - b_2(\dot{X}_2 - \dot{X}_1) - K_2 X_2 = m_2 \ddot{X}_2$$

$$m_2 \ddot{X}_2 + b_1 \dot{X}_2 + b_2(\dot{X}_2 - \dot{X}_1) + K_2 X_2 = f(t)$$

$$m_2 s^2 X_2 + b_1 s X_2 + b_2 s (X_2 - X_1) + K_2 X_2 = F(s) .$$

$$(m_2 s^2 + b_1 s + b_2 s + K_2) \; X_2(s) - b_2 s \; X_1(s) = f(s)$$



$$\Sigma F = m_1 \vec{a} .$$

$$-K_1 X_1 - b_2(\dot{X}_1 - \dot{X}_2) - b_3 \dot{X}_1 = m_1 \ddot{X}_1 .$$

$$m_1 \ddot{X}_1 + b_2(\dot{X}_1 - \dot{X}_2) + b_3 \dot{X}_1 + K_1 X_1 = 0 .$$

$$m_1 s^2 X_1 + b_2 s (X_1 - X_2) + b_3 s X_1 + K_1 X_1 = 0 .$$

$$(m_1 s^2 + b_2 s + b_3 s + K_1) \; X_1(s) + -b_2 s \; X_2(s) = 0 .$$

$$\begin{bmatrix} -b_2 s & m_2 s^2 + b_1 s + b_2 s + K_2 \\ m_1 s^2 + b_2 s + b_3 s + K_1 & -b_2 s \end{bmatrix} \begin{bmatrix} X_1(s) \\ X_2(s) \end{bmatrix} = \begin{bmatrix} F(s) \\ 0 \end{bmatrix}$$

$$\Delta = b_2^2 s^2 - (m_2 s^2 + b_1 s + b_2 s + K_2)(m_1 s^2 + b_2 s + b_3 s + K_1)$$

Modeling and Simulation :-
Modeling Process :-

* Ex :-

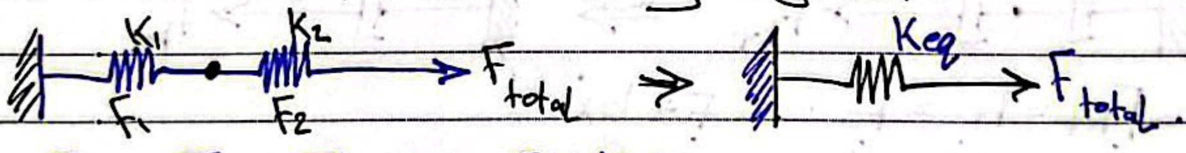
$$X_1(s) = \begin{vmatrix} F(s) & m_2s^2 + b_1s + b_2s + K_2 \\ 0 & -b_2s \end{vmatrix} * \frac{1}{\Delta}$$

$$\frac{X_1(s)}{F(s)} = \frac{-b_2 s}{b_2^2 s^2 - (m_2s^2 + b_1s + b_2s + K_2)(m_1s^2 + b_3s + b_2s + K_1)}$$

$$X_2(s) = \frac{\begin{vmatrix} -b_2s & F(s) \\ m_1s^2 + b_3s + b_2s + K_1 & 0 \end{vmatrix}}{\Delta}$$

$$\frac{X_2(s)}{F(s)} = \frac{-(m_1s^2 + b_3s + b_2s + K_1)}{b_2^2 s^2 - (m_2s^2 + b_1s + b_2s + K_2)(m_1s^2 + b_3s + b_2s + K_1)}$$

* Series and Parallel spring system :-



$F_{tot} = F_1 = F_2$. $F = KX$.

$X_{tot} = X_1 + X_2$ $X = F/K$.

$$\frac{\cancel{F_{tot}}}{K_{eq}} = \frac{\cancel{F_1}}{K_1} + \frac{\cancel{F_2}}{K_2}$$

$$\frac{1}{K_{eq}} = \frac{1}{K_1} + \frac{1}{K_2} \Rightarrow \boxed{K_{eq} = \frac{K_1 K_2}{K_1 + K_2}}$$

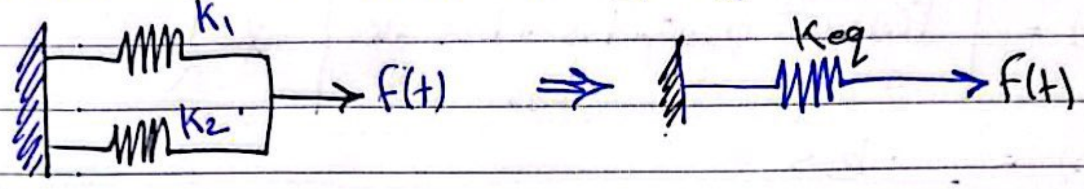
spring in series → Resistance in Parallel.

$$\boxed{\frac{1}{K_{eq}} = \sum_{i=1}^{n} \frac{1}{K_i}}$$

Modeling and Simulation :-

Modeling Process :-

*** Series and Parallel spring system :-**



$X_{tot} = X_1 = X_2$.

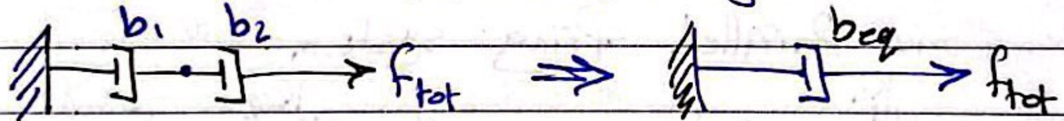
$F_{tot} = F_1 + F_2$.

$K_{eq} \cancel{X_{tot}} = K_1 \cancel{X_1} + K_2 \cancel{X_2}$.

$\boxed{K_{eq} = K_1 + K_2}$ $\boxed{K_{eq} = \sum_{i=1}^{n} K_i}$

spring in Parallel → Resistance in series.

*** Series and Parallel damper system :-**



$F_{tot} = F_1 = F_2$. $F = b\dot{x}$

$\dot{X}_{tot} = \dot{X}_1 + \dot{X}_2$ $\dot{X} = F/b$

$\frac{\cancel{F_{tot}}}{b_{eq}} = \frac{\cancel{F_1}}{b_1} + \frac{\cancel{F_2}}{b_2} \rightarrow \frac{1}{b_{eq}} = \frac{1}{b_1} + \frac{1}{b_2}$.

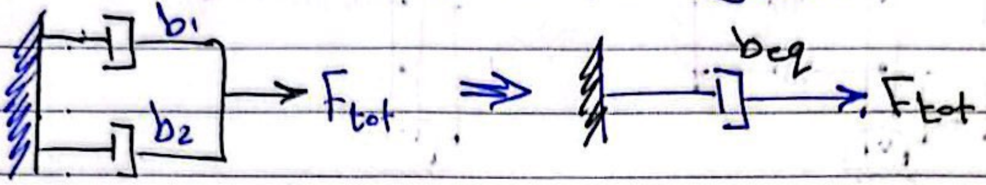
$\boxed{b_{eq} = \frac{b_1 b_2}{b_1 + b_2}}$ $\boxed{\frac{1}{b_{eq}} = \sum_{i=1}^{n} \frac{1}{b_i}}$

damper in series → Resistance in Parallel.

Modeling and simulation :-

Modeling Process :-

* Series and Parallel damper system :-



$\dot{X}_{tot} = \dot{X}_1 = \dot{X}_2$

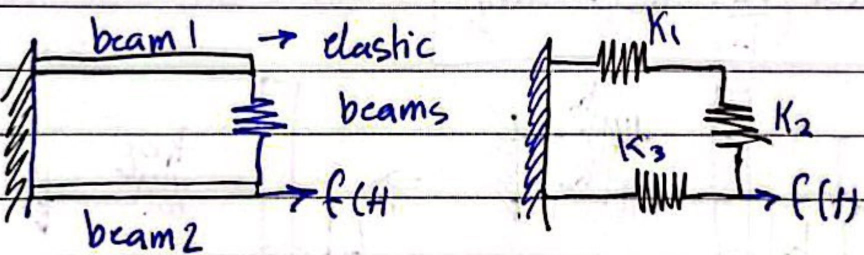
$F_{tot} = F_1 + F_2$

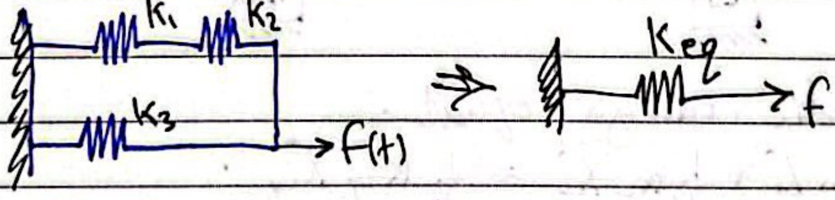
$b_{eq} \cancel{\dot{X}_{tot}} = b_1 \cancel{\dot{X}_1} + b_2 \cancel{\dot{X}_2}$

$\boxed{b_{eq} = b_1 + b_2} \Rightarrow \boxed{b_{eq} = \sum_{i=1}^{n} b_i}$

damper in Parallel → Resistance in series .

* Ex :-





K_1 , K_2 in series .

$$K_{eq} = \frac{K_1 K_2}{K_1 + K_2}$$

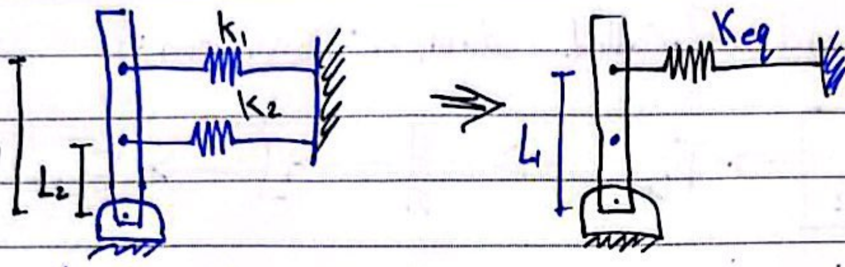
K_{eq} , K_3 in Parallel .

$$K_{eq_{tot}} = K_3 + K_{eq} .$$

$$K_{eq_{tot}} = \frac{K_3 (K_1 + K_2) + K_1 K_2}{(K_1 + K_2)}$$

Modeling and Simulation :-
Modeling Process :-

* Ex :-



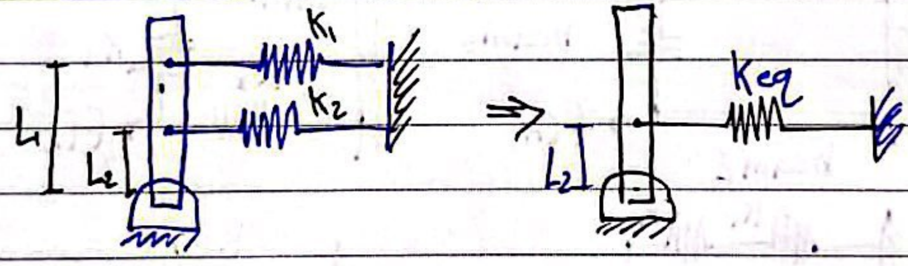
Kinetic energy equal.

$$\frac{1}{2}k_1 X_1^2 + \frac{1}{2}K_2 X_2^2 = \frac{1}{2}K_{eq} X_{eq}^2$$

من تشابه المثلثات $\frac{L_1}{L_2} = \frac{X_1}{X_2}$ $\boxed{X_1 = \frac{L_1}{L_2} X_2}$ $\boxed{X_2 = \frac{L_2}{L_1} X_1}$

$$\cancel{\frac{1}{2}} K_1 \cancel{X_1^2} + \cancel{\frac{1}{2}} K_2 \left(\frac{L_2}{L_1} \cancel{X_1}\right)^2 = \cancel{\frac{1}{2}} K_{eq} \cancel{X_1^2}.$$

$$\boxed{K_1 + \left(\frac{L_2}{L_1}\right)^2 K_2 = K_{eq}}$$



Kinetic energy equal.

$$\frac{1}{2}k_1 X_1^2 + \frac{1}{2}k_2 X_2^2 = \frac{1}{2}K_{eq} X_{eq}^2$$

$$\boxed{X_1 = \frac{L_1}{L_2} X_2}$$

$$\cancel{\frac{1}{2}} k_1 \left(\frac{L_1}{L_2}\right)^2 \cancel{X_2^2} + \cancel{\frac{1}{2}} K_2 \cancel{X_2^2} = \cancel{\frac{1}{2}} K_{eq} \cancel{X_2^2}.$$

$$\boxed{K_1 \left(\frac{L_1}{L_2}\right)^2 + K_2 = K_{eq}}$$

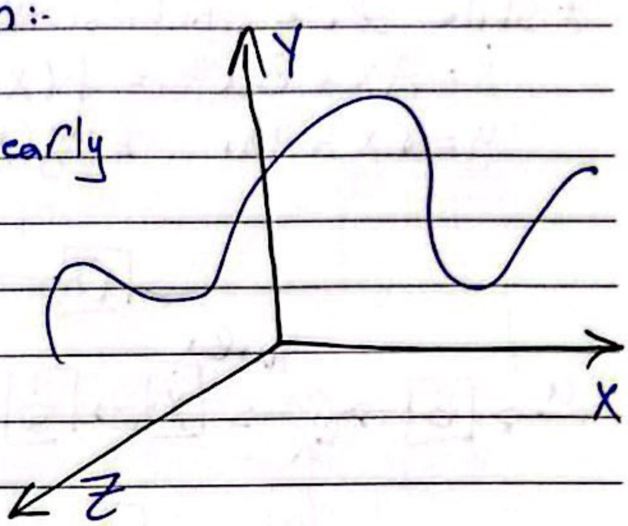
Modeling and Simulation :-

Modeling Process :-

* State Space Representation:-

State Variables :-

are the Smallest set of linearly Independent Variable that Completely describe the dynamic behaviour of the System at any given time.



* The linear form of state space Representation Can be expressed as :-

$\dot{X} = AX + Bu.$

$y = CX + Du.$

Where :- X :- state space Variable (state Vector).

u :- Input Vector.

y :- Output Vector.

A :- state matrix $\Rightarrow n * n \rightarrow$ # of state Variable

B :- Input matrix $\Rightarrow n * r \rightarrow$ # of Inputs

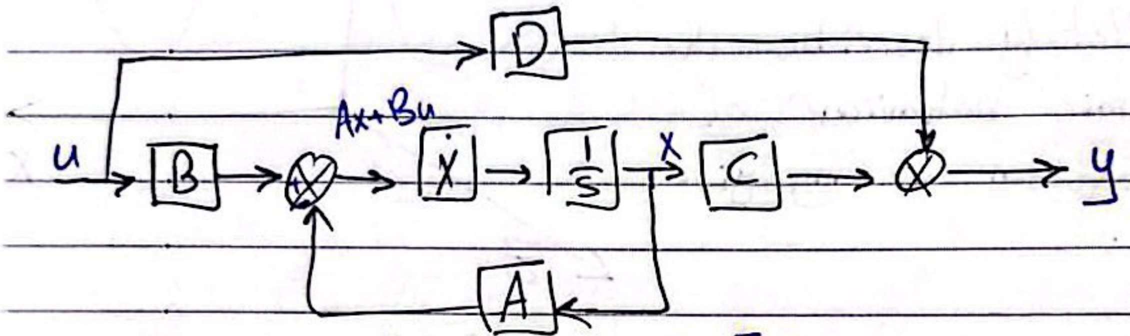
C :- Output matrix $\Rightarrow m * n \rightarrow$ # of Outputs.

D :- Feedback matrix $\Rightarrow m * r$

Modeling and Simulation
Modeling Process :-

* State Space Representation :-

$\dot{X} = AX + Bu = f(X, u, t) \rightarrow$ linear form
$y = CX + Du = g(X, u, t) \rightarrow$ linear form.



X :- is state Vector $X = [X_1, X_2, X_3, \ldots, X_n]^T$.
u :- is Input Vector $u = [u_1, u_2, u_3, \ldots, u_r]^T$.
y :- is Output Vector $Y = [y_1, y_2, y_3, \ldots, y_m]^T$.

* general Representation of state space.

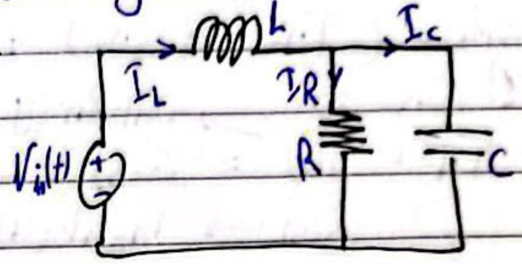
$\dot{X} = F(X, u, t)$ → either linear
$y = H(X, u, t)$ → non linear.

Modeling and Simulation :-

Modeling Process :-

* Development of state space system.

given that we are interested in studying (V_c) with varying (V_{in}) Assuming that (V_c), (I_L) Our Choice for state Variables.



by taking KCL.

Assume $x_1 = V_c$, $x_2 = I_L$.

$V_{in} = u$, $y = V_c$

$$I_L = I_R + I_c$$

$$I_L = \frac{V_R}{R} + C\frac{dV_c}{dt} \Rightarrow I_L = \frac{V_c}{R} + C\frac{dV_c}{dt}.$$

$$x_2 = \frac{x_1}{R} + C\dot{x}_1 \Rightarrow \boxed{\dot{x}_1 = -\frac{1}{RC}x_1 + \frac{1}{C}x_2}$$

by taking KVL :-

$$V_{in} = L\frac{dI_L}{dt} + V_c \Rightarrow \boxed{\dot{x}_2 = -\frac{1}{L}x_1 + \frac{u}{L}}$$

$$\dot{x} = Ax + Bu.$$

$$y = Cx + Du.$$

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix} = \begin{bmatrix}-\frac{1}{RC} & \frac{1}{C}\\ -\frac{1}{L} & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix} + \begin{bmatrix}0\\ \frac{1}{L}\end{bmatrix}u$$

$$y = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1\\ x_2\end{bmatrix}$$

Modeling and Simulation
Modeling Process :-

* Conversion between Transfer function and state space (T.F → S.S) :-

① Case 1 :-

$$T.F = \frac{y(s)}{R(s)} = G(s)$$

$y(s) \rightarrow$ Output (y)

$R(s) \rightarrow$ Input (u)

$$\frac{y(s)}{R(s)} = \frac{24}{s^3 + 9s^2 + 26s + 24}$$

$$(s^3 + 9s^2 + 26s + 24)\, y(s) = 24\, R(s)$$

by taking laplace Inverse .

$$\mathcal{L}^{-1}(s^3 + 9s^2 + 26s + 24)\, y(s) = \mathcal{L}^{-1}\, 24\, R(s)$$

$$\Rightarrow \dddot{y}(t) + 9\ddot{y}(t) + 26\dot{y}(t) + 24y(t) = 24\, u(t).$$

Z_1, Z_2, Z_3 are my state Variable .

$Z_1 = y \rightarrow \dot{Z}_1 = \dot{y} \Rightarrow \dot{Z}_1 = Z_2$.

$Z_2 = \dot{y} \rightarrow \dot{Z}_2 = \ddot{y} \Rightarrow \dot{Z}_2 = Z_3$.

$Z_3 = \ddot{y} \rightarrow \dot{Z}_3 = \dddot{y} \Rightarrow \dot{Z}_3 = \dddot{y}$

$\dot{Z}_3 = \dddot{y}(t) = 24u - 9Z_3 - 26Z_2 + 24Z_1$.

$$\begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \\ \dot{Z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -24 & -26 & -9 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 24 \end{bmatrix} u(t) .$$

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} Z_1 \\ Z_2 \\ Z_3 \end{bmatrix}$$

Modeling and Simulation :-

Modeling Process :-

*Conversion between Transfer function and state space

② Case 2 :-

$$T.F = \frac{y(s)}{R(s)} = G(s)$$

$$T.F = \frac{Y(s)}{U(s)} = \frac{3+20s}{s^3+9s^2+3s+2}$$

$$(s^3+9s^2+3s+2)\,Y(s) = (3+20s)\,U(s)$$

take laplace Inverse.

$$\dddot{y}(t) + 9\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = 3U(t) + 20\dot{U}(t).$$

let $z_1 = y(t) + V_1.$ $\quad V_1 = U(t)$

$z_2 = \dot{y}(t)$. $\quad V_2 = \dot{U}(t)$.

$z_3 = \ddot{y}(t)$

$\dot{z}_1 = \dot{y}(t) + \dot{V}_1 \rightarrow \dot{z}_1 = z_2 + V_2.$

$\dot{z}_2 = \ddot{y}(t) \rightarrow \dot{z}_2 = z_3.$

$\dot{z}_3 = 3V_1 + 20V_2 - 9z_3 - 3z_2 - 2(z_1 - V_1)$

$$\begin{bmatrix}\dot{z}_1\\ \dot{z}_2\\ \dot{z}_3\end{bmatrix} = \begin{bmatrix}0 & 1 & 0\\ 0 & 0 & 1\\ -2 & -3 & -9\end{bmatrix}\begin{bmatrix}z_1\\ z_2\\ z_3\end{bmatrix} + \begin{bmatrix}0 & 1\\ 0 & 0\\ 5 & 20\end{bmatrix}\begin{bmatrix}V_1\\ V_2\end{bmatrix}$$

$$y = \begin{bmatrix}1 & 0 & 0\end{bmatrix}\begin{bmatrix}z_1\\ z_2\\ z_3\end{bmatrix} + \begin{bmatrix}-1 & 0\end{bmatrix}\begin{bmatrix}V_1\\ V_2\end{bmatrix}$$

Modeling and Simulation

Modeling Process :-

* Conversion from state space Representation to Transfer function: (S.S → T.F) :-

$\dot{X}(t) = A X(t) + B U(t)$.

$Y(t) = C X(t) + D U(t)$.

by taking laplace Transform.

$s X(s) = A X(s) + B U(s)$ ... ①

$Y(s) = C X(s) + D U(s)$ ... ②

from eq ① $(sI - A) X(s) = B U(s)$.

$X(s) = (sI - A)^{-1} B U(s)$

from eq ② $Y(s) = C (sI - A)^{-1} B U(s) + D \cdot U(s)$

$T.F = \dfrac{Y(s)}{U(s)} = C (sI - A)^{-1} B + D$.

$$(sI - A)^{-1} = \frac{adj(sI - A)}{det(sI - A)}.$$

if $(sI - A) = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}$

$adj(sI - A) = \begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix}$

$det(sI - A) = s^2$.

$(sI - A)^{-1} = \dfrac{1}{s^2} \begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix}$

if $(sI - A) = \begin{bmatrix} 0 & s & 0 \\ -2s & 0 & s \\ 1 & 2 & 3s \end{bmatrix}$

$\begin{bmatrix} 0 & -2s & 1 \\ s & 0 & 2 \\ 0 & s & 3s \end{bmatrix} \rightarrow adj = \begin{bmatrix} -2s & -3s^2 & s^2 \\ 6s^2+s & 0 & 0 \\ -2s & s & 2s^2 \end{bmatrix}$

$det = 0 - s(-6s^2 - s) + 0$.

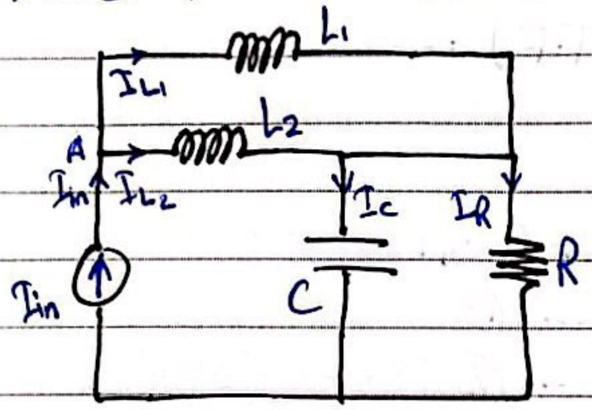
$= 6s^3 + s^2$.

$$(sI - A)^{-1} = \frac{adj(sI - A)}{det(sI - A)}.$$

Modeling and Simulation :-

Modeling Process :-

* State space of Electrical System :-



* Note :
When you chosse your state Variable you must check that all your chossen are linearly Independent.

$I_{in} = I_{L2} + I_{L1}$
$I_{L2} + I_{L1} = I_C + I_R$ → choose $X_1 = I_{L1}$.

$V_{L1} = V_{L2}$.
$V_C = V_R$ → choose $X_2 = V_C$

$I_{in} = I_{L2} + I_{L1} = I_C + I_R$.

$I_{in} = I_C + I_R$.

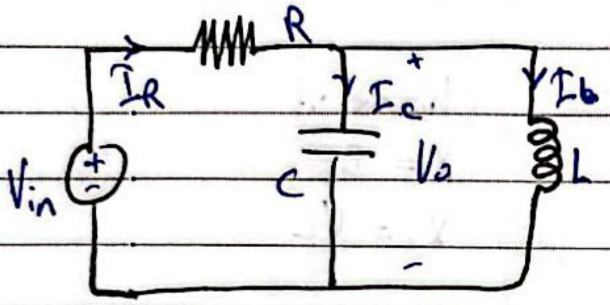
$I_{in} = C\frac{dV_C}{dt} + \frac{V_R}{R}$, $V_R = V_C$

$I_{in} = C\dot{X}_2 + \frac{X_2}{R} \Rightarrow \boxed{\dot{X}_2 = \frac{-1}{RC}X_2 + \frac{1}{C}I_{in}}$.

Modeling and Simulation :-

Modeling Process :-

* State space of Electrical System :-



Choose :-

$X_1 = V_c$

$X_2 = I_L$

$V_{in} = V_R + V_c \quad , V_c = V_L$

$V_{in} = V_R + V_L$

$I_R = I_c + I_L$

$\rightarrow V_{in} = V_R + V_c . \qquad , V_R = I_R R$

$V_{in} = I_R R + V_c \qquad , I_R = I_c + I_L$

$V_{in} = (I_c + I_L) R + V_c . \qquad , I_c = C \dfrac{dV_c}{dt} .$

$V_{in} = RC \dfrac{dV_c}{dt} + R I_L + V_c .$

$$\boxed{\dot{X}_1 = \frac{1}{RC} \left(V_{in} - R X_2 - X_1 \right)} .$$

$$V_c = V_L \rightarrow V_c = L \frac{dI_L}{dt} \rightarrow \boxed{\dot{X}_2 = \frac{1}{L} X_1}$$

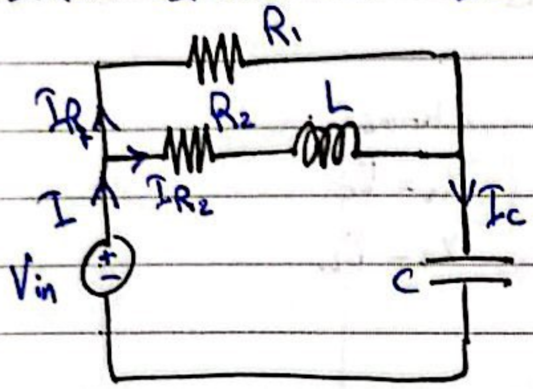
$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \end{bmatrix} = \begin{bmatrix} \frac{-1}{RC} & \frac{-1}{C} \\ \frac{1}{L} & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{RC} \\ 0 \end{bmatrix} V_{in} .$$

$$y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}$$

Modeling and Simulation :-

Modeling Process :-

*State space of Electrical System :-



chosse :.

$X_1 = I_L$.

$X_2 = V_c$.

$V_{in} = V_{R_2} + V_L + V_c$.

$V_{in} = V_{R_1} + V_c$.

$V_{R_1} = V_{R_2} + V_L$

$I = I_{R_1} + I_{R_2}$, $I_{R_2} = I_L$.

$I_c = I_{R_1} + I_{R_2}$

$V_{in} = V_{R_2} + V_L + V_c$

$V_{in} = I_{R_2} R_2 + L \dfrac{dI_L}{dt} + V_c \qquad I_{R_2} = I_L$

$$\boxed{\dot{X}_1 = \frac{1}{L}\left(V_{in} - R_2 X_1 - X_2\right)}.$$

$V_{in} = V_{R_1} + V_c \rightarrow V_{in} = I_{R_1} R_1 + V_c$.

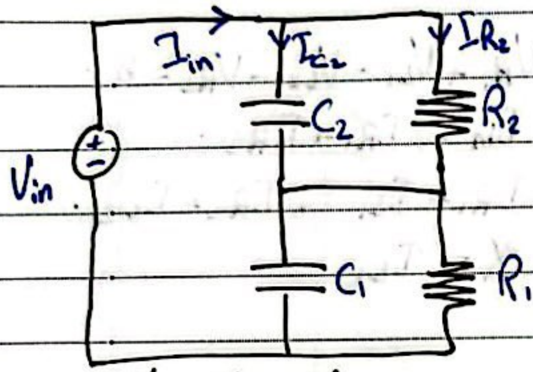
$I_c = I_L + I_{R_1} \rightarrow c \dfrac{dV_c}{dt} = I_L + I_{R_1}$.

$V_{in} = R_1 C \dfrac{dV_c}{dt} - R_1 I_L + V_c$

$$\boxed{\dot{X}_2 = \frac{1}{R_1 C}\left(V_{in} + R_1 X_1 - X_2\right)}$$

Modeling and Simulation:
Modeling Process :-

* State Variable of Electrical System :-



$V_{C_2} = V_{R_2}$, $V_{C_1} = V_{R_1}$.

$V_{in} = V_{C_2} + V_{C_1}$

$V_{in} = V_{R_2} + V_{R_1}$.

$V_{C_2} + V_{C_1} - V_{R_2} - V_{R_1} = 0$.

$I_{in} = I_{C_2} + I_{R_2}$

$I_{in} = I_{C_1} + I_{R_1}$

let $X = V_{C_1}$

$-V_{in} + V_{C_2} + V_{C_1} = 0$

$I_{C_2} + I_{R_2} = I_{C_1} + I_{R_1}$

$$C_2 \frac{dV_{C_2}}{dt} + \frac{V_{C_2}}{R_2} = C_1 \frac{dV_{C_1}}{dt} + \frac{V_{C_1}}{R_1}$$

$$C_2 \left(\frac{dV_{in}}{dt} - \frac{dV_{C_1}}{dt} \right) + \frac{V_{in} - V_{C_1}}{R_2} = C_1 \frac{dV_{C_1}}{dt} + \frac{V_{C_1}}{R_1} .$$

let $V_1 = V_{in}$, $V_2 = \dot{V}_{in}$

$$C_2 (V_2 - \dot{X}) + \frac{V_1 - X}{R_2} = C_1 \dot{X} + \frac{X}{R_1}$$

$$\dot{X} (C_1 + C_2) = X \left(\frac{-1}{R_2} - \frac{1}{R_1} \right) + \frac{1}{R_2} V_1 + C_2 V_2 .$$

$$\dot{X} = \left[\frac{-(R_1 + R_2)}{R_1 R_2 (C_1 + C_2)} \right] X + \left[\frac{1}{R_2 (C_1 + C_2)} \quad \frac{C_2}{C_1 + C_2} \right] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

$y = V_{C_1} = X$.

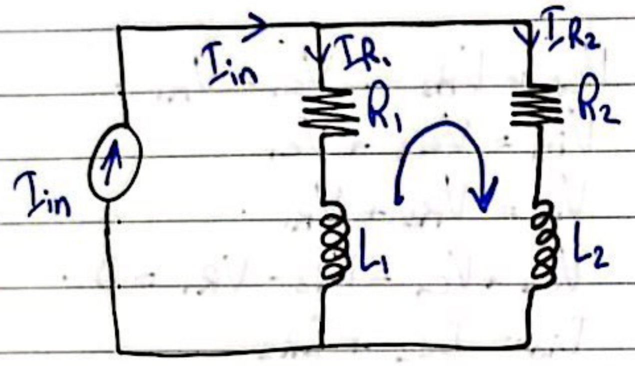
$y = [1] \; X$

Scanned with CamScanner

Modeling and Simulation :-

Modeling Process :-

* State Variable of Electrical System :-



$V_{R_1} + V_{L_1} - V_{L_2} - V_{R_2} = 0.$

$I_{in} = I_{R_1} + I_{R_2}.$

$I_{R_1} = I_{L_1} \quad , I_{R_2} = I_{L_2}.$

$X = I_{L_1}$

$$-V_{L_1} - V_{R_1} + V_{L_2} + V_{R_2} = 0.$$

$$-L_1 \frac{dI_{L_1}}{dt} - I_{L_1} R_1 + L_2 \frac{dI_{L_2}}{dt} + I_{L_2} R_2 = 0.$$

$$-L_1 \frac{dI_{L_1}}{dt} - I_{L_1} R_1 + L_2 \left(\frac{dI_{in}}{dt} - \frac{dI_{L_1}}{dt} \right) + (I_{in} - I_{L_1}) R_2 = 0.$$

$$-L_1 \dot{X} - X R_1 + L_2 \left(\frac{dI_{in}}{dt} - \dot{X} \right) + R_2 (I_{in} - X) = 0.$$

Let $V_1 = I_{in}, \quad V_2 = \frac{dI_{in}}{dt}.$

$$-L_1 \dot{X} - R_1 X + L_2 (V_2 - \dot{X}) + R_2 (V_1 - X) = 0.$$

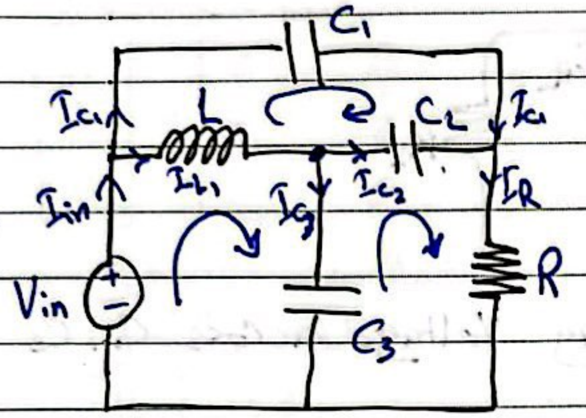
$$\dot{X} \frac{\cancel{(L_1 + L_2)}}{\cancel{(L_1 + L_2)}} = \frac{(-R_1 - R_2)}{(L_1 + L_2)} X + \frac{R_2 V_1}{(L_1 + L_2)} + \frac{L_2 V_2}{(L_1 + L_2)}.$$

$$\dot{X} = \left[\frac{-(R_1 + R_2)}{(L_1 + L_2)} \right] X + \left[\frac{R_2}{L_1 + L_2} \quad \frac{L_2}{L_1 + L_2} \right] \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Modeling and Simulation :-

Modeling Process :-

* State Variable of Electrical System :-



$V_{in} = V_L + V_{c_3}$.
$V_{in} = V_L + V_{c_2} + V_R$.
$V_{in} = V_{c_1} + V_R$.
$V_{c_2} + V_R - V_{c_3} = 0$
$V_{c_1} - V_{c_2} - V_L = 0$.
$I_{in} = I_{L_1} + I_{c_1}$.
$I_L = I_{c_3} + I_{c_2}$.
$I_{c_2} + I_{c_1} = I_R$

We can choose as state Variable.

$\boxed{X_1 = V_{c_1}}$, $\boxed{X_2 = V_{c_2}}$, $\boxed{X_3 = I_L}$

Or

$\boxed{X_1 = V_{c_1}}$, $\boxed{X_2 = V_{c_3}}$, $\boxed{X_3 = I_L}$

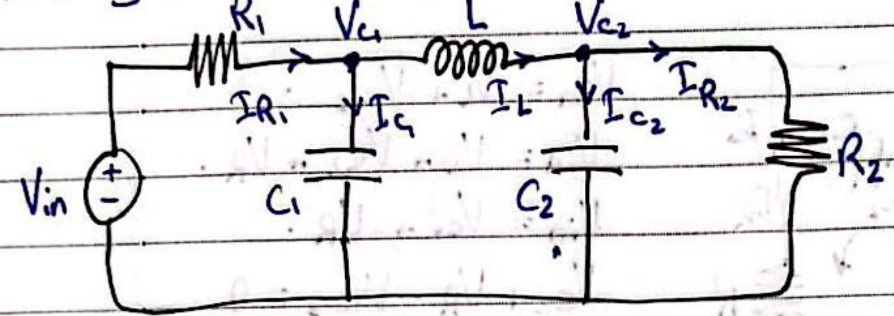
$V_{c_2} + V_R - V_{c_3} = 0$.

$-V_{c_3} + V_{c_2} + (V_{c_3} - V_{c_2})R = 0$.

Modeling and Simulation :-

Modeling Process :-

* Single Input multi Output (SIMO) system :-



We are Interested in studying Voltage across C_1, C_2 With Varying (V_{in}) :

$$\left.\begin{aligned} I_{R_1} &= I_{C_1} + I_L \\ I_L &= I_{C_2} + I_{R_2} \end{aligned}\right] \rightarrow \text{by KCL}$$

$I_{R_1} = \frac{V_{R_1}}{R_1} = \frac{V_{in} - V_{C_1}}{R_1}$, $I_{C_1} = C_1 \frac{dV_{C_1}}{dt}$.

$V_L = L\frac{dI_L}{dt} \rightarrow V_L = V_{C_2} - V_{C_1} = L\frac{dI_L}{dt}$.

Let $X_1 = I_L$, $X_2 = V_{C_1}$, $X_3 = V_{C_2}$ state Variable.

$\rightarrow I_{C_1} = I_{R_1} - I_L$.

$$C_1\frac{dV_{C_1}}{dt} = \frac{V_{in} - V_{C_1}}{R_1} - I_L \Rightarrow \boxed{\dot{X}_2 = \frac{-1}{C_1} I_L \; \frac{-1}{C_1 R_1} V_{C_1} + \frac{1}{C_1} V_{in}}$$

$I_{C_2} = I_L - I_{R_2}$.

$$C_2\frac{dV_{C_2}}{dt} = I_L - \frac{V_{C_2}}{R_2} \Rightarrow \boxed{\dot{X}_3 = \frac{1}{C_2} I_L - \frac{1}{C_2 R_2} V_{C_2}}$$

$$V_L = L\frac{dI_L}{dt} \rightarrow \frac{dI_L}{dt} = \frac{1}{L}\left(V_{C_2} - V_{C_1}\right).$$

$$\boxed{\dot{X}_1 = \frac{-1}{L} V_{C_1} + \frac{1}{L} V_{C_2}}$$

Modeling and Simulation :-
Modeling Process :-

* Single Input multi Output (SIMO) System :-

$$\dot{X} = AX + Bu.$$

$$\begin{bmatrix} \dot{I}_L \\ \dot{V}_{c_1} \\ \dot{V}_{c_2} \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{L} & \frac{1}{L} \\ \frac{-1}{C_1} & \frac{-1}{C_1R_1} & \frac{1}{C_1} \\ \frac{1}{C_2} & 0 & \frac{-1}{C_2R_2} \end{bmatrix} \begin{bmatrix} I_L \\ V_{c_1} \\ V_{c_2} \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{1}{C_1} \\ 0 \end{bmatrix} V_{in}.$$

$$y_1 = V_{c_1}, \quad y_2 = V_{c_2}.$$

$$y = CX + Du.$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} I_L \\ V_{c_1} \\ V_{c_2} \end{bmatrix}$$

- If We are Interested in studying Current throug R_1, R_2 With Varying V_{in}.

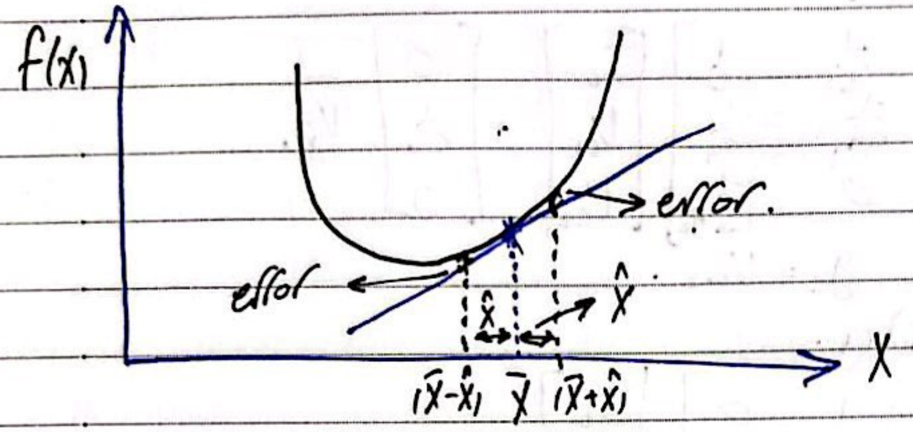
$$y_1 = I_{R_1} = \frac{V_{in} - V_{c_1}}{R_1} = \frac{-1}{R_1} V_{c_1} + \frac{1}{R_1} V_{in}.$$

$$y_2 = I_{R_2} = \frac{V_{c_2}}{R_2}.$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 0 & \frac{-1}{R_1} & 0 \\ 0 & 0 & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} I_L \\ V_{c_1} \\ V_{c_2} \end{bmatrix} + \begin{bmatrix} \frac{1}{R_1} \\ 0 \end{bmatrix} V_{in}.$$

Modeling and Simulation:-
Modeling Process:-

* Linearization of non-linear system:-



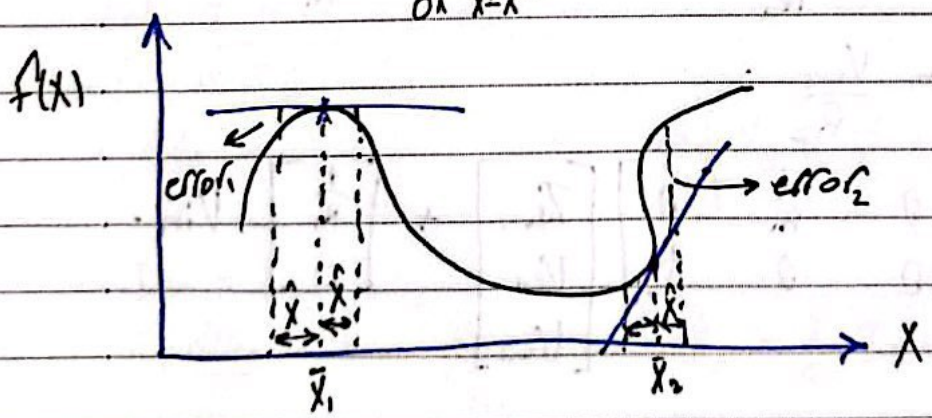
$\bar{x}$:- Operation Point.
$\hat{x}$:- Increament distance.

$y-y_0 = m(x-\cancel{x_0})$.

$f(x)-f(\bar{x}) = \frac{\partial f}{\partial x}\Big|_{x=\bar{x}} (x-\bar{x})$ $\qquad x = \bar{x}+\hat{x}$.

$\bar{x} = x-\hat{x}$, $\hat{x} = x-\bar{x}$.

$\Rightarrow f(x) = f(\bar{x}) + \frac{\partial f}{\partial x}\Big|_{x=\bar{x}} (x-\bar{x})$.



for the same system chossening, the Operating Point determine the system Valuedity of the system.

Modeling and Simulation :-

Modeling Process:-

* Linearization of non-linear system:-

- for non-linear system with one Independent Variable, the linearized Version of the system Can be expressed as following:-

$$f(x) = f(\bar{x}) + \frac{df}{\partial x}\Big|_{x=\bar{x}} (x-\bar{x}). \qquad x = \bar{x} + \hat{x}.$$

$$f(\bar{x}+\hat{x}) = f(\bar{x}) + \frac{df}{\partial x}\Big|_{x=\bar{x}} (\hat{x}) .$$

- for non-linear system with two Independent Variables, the linearized Version of the System Can be expressed as following:-

$$\dot{x} = f_1(\bar{x}_1, \bar{x}_2, \bar{u}, t) \rightarrow \text{non Linear system}.$$

$$\dot{x} = f(\bar{x}_1, \bar{x}_2, \bar{u}, t) + \frac{\partial f}{\partial x_1}\Big|_{\bar{x}_1, \bar{x}_2, u} \hat{x}_1 + \frac{\partial f}{\partial x_2}\Big|_{\bar{x}_1, \bar{x}_2, u} \hat{x}_2 .$$

$$+ \frac{\partial f}{\partial u}\Big|_{\bar{x}_1, \bar{x}_2, u} \hat{u} + \frac{1}{2}\frac{\partial^2 f}{\partial x_1^2}\Big|_{\bar{x}_1, \bar{x}_2, u} (\hat{x}_1)^2 + \frac{1}{2}\frac{\partial^2 f}{\partial x_2^2}\Big|_{\bar{x}_1, \bar{x}_2, u} (\hat{x}_2)^2$$

$$+ \frac{1}{2}\frac{\partial^2 f}{\partial x_1 \partial x_2}\Big|_{\bar{x}_1, \bar{x}_2, u} (\hat{x}_1)(\hat{x}_2) + \frac{1}{2}\frac{\partial^2 f}{\partial u^2}\Big|_{\bar{x}_1, \bar{x}_2, u} (\hat{u})^2 .$$

Modeling and Simulation :-

Modeling Process :-

* formulation of linearization for general case

if we have the non-linear system expressed as :-

$$\dot{q}_1 = \frac{dq_1}{dt} = f_1(q_1, q_2, q_3, \dots, q_n, r_1, r_2, \dots, r_m, t).$$

$$\dot{q}_2 = \frac{dq_2}{dt} = f_2(q_1, q_2, q_3, \dots, q_n, r_1, r_2, \dots, r_m, t).$$

$$\vdots$$

$$\dot{q}_n = \frac{dq_n}{dt} = f_n(q_1, q_2, q_3, \dots, q_n, r_1, r_2, \dots, r_m, t).$$

State Vector $q = [q_1, q_2, q_3, \dots, q_n]^T$.

Input Vector $r = [r_1, r_2, r_3, \dots, r_m]^T$.

$$g_1 = y_1(q_1, q_2, q_3, \dots, q_n, r_1, r_2, r_m, t).$$

$$g_2 = y_2(q_1, q_2, q_3, \dots, q_n, r_1, r_2, \dots, r_m, t).$$

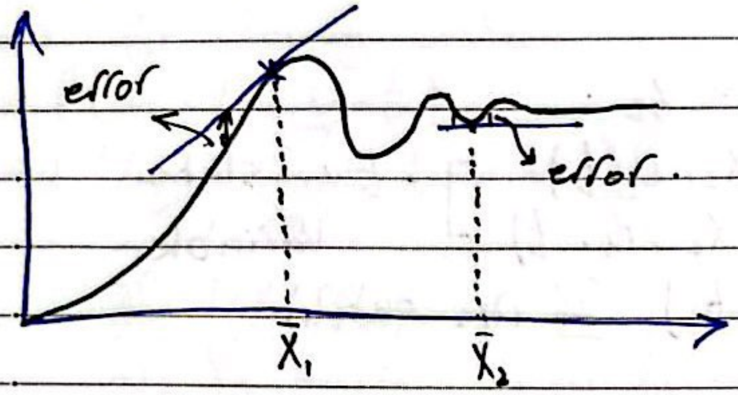
$$\vdots$$

$$g_c = y_c(q_1, q_2, q_3, \dots, q_n, r_1, r_2, \dots, r_m, t).$$

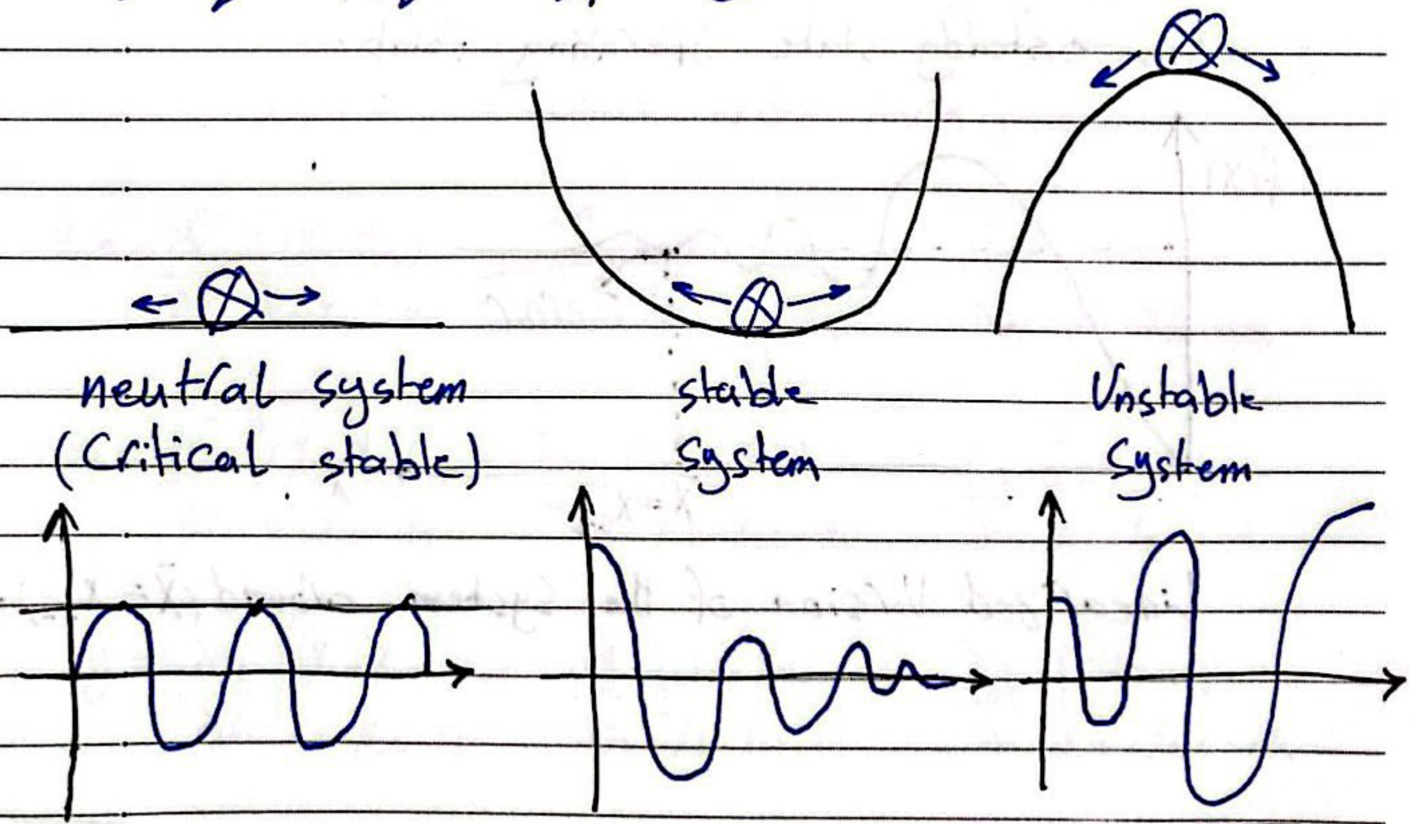
Output Vector $g = [g_1, g_2, g_3, \dots, g_c]$.

Modeling and Simulation :-
Modeling Process :-

* Operation Point ($\bar{X}$) :-



We choose the ($\bar{X}$) as steady state Condition.
Equillibrium state.
at steady state Condition Change of rate = Zero.
$\Rightarrow \dot{q} = f(q, r, t) = Zero.$



Modeling and Simulation :-
Modeling Process :-

* for a non-linear system with 2 Independent Variable (X_1, X_2) :-

$X_1 = q_1$, $X_2 = q_2$.

$\dot{q}_1 = \dot{X}_1 = f_1(X_1, X_2, u, t)$] → two state
$\dot{q}_2 = \dot{X}_2 = f_2(X_1, X_2, u, t)$] Variable.

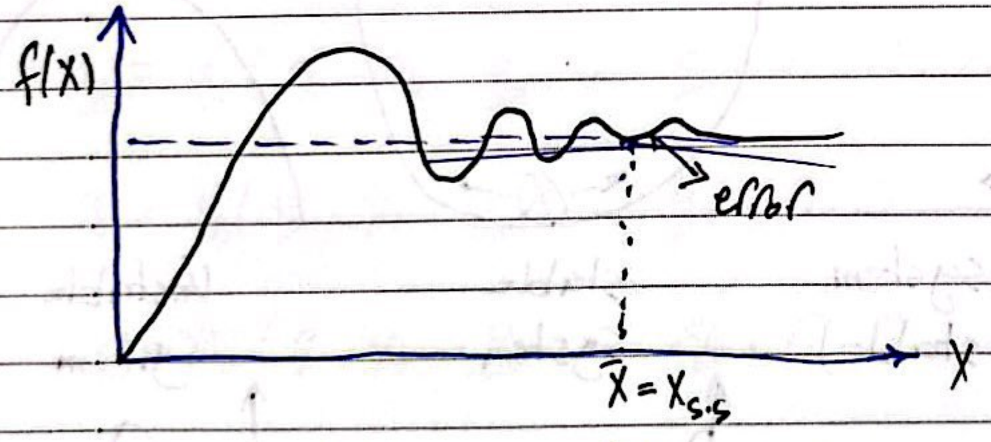
$y = g(X_1 X_2, u, t)$ → One output.

* so now we have the Operating Point at steady state Condition :-

$X = \bar{X} + \hat{X}$ if we choose ($\bar{X}$) as (X_{ss}) Steady state

$X = X_{s.s} + \hat{X}$.

$X_{s.s}$ → steady state Operating Point.



Linearized Version of the System around ($\bar{X} = X_{ss}$).

Modeling and Simulation :-
Modeling process :-

* Now we want to derive the linearized system around the steady state :-

$$\dot{X} = f(X_1, X_2; u, t)$$

$$\dot{X} = f(X_{1s}, X_{2s}, u_s, t) + \frac{\partial f}{\partial X_1}\Big|_{X_{1s}, X_{2s}, u_s}(X_1 - X_{1s})$$

zero

$$+ \frac{\partial f}{\partial X_2}\Big|_{X_{1s}, X_{2s}, u_s}(X_2 - X_{2s}) + \frac{\partial f}{\partial u}\Big|_{X_{1s}, X_{2s}, u_s}(u - u_s)$$

zero

$$+ \frac{1}{2}\Big[\frac{\partial^2 f}{\partial X_1^2}\Big|_{X_{1s}, X_{2s}, u_s}(X_1 - X_s)^2 + \frac{\partial^2 f}{\partial X_2^2}\Big|_{X_{1s}, X_{2s}, u_s}(X_2 - X_s)^2 +$$

$$+ \frac{\partial^2 f}{\partial X_1 \partial X_2}\Big|_{X_{1s}, X_{2s}, u_s}(X_1 - X_{1s})(X_2 - X_{2s}) + \frac{\partial^2 f}{\partial u^2}\Big|_{X_{1s}, X_{2s}, u_s}(u - u_s)^2\Big]$$

HOT $\rightarrow 2^{nd}$ (High order term).

$$\dot{X}_1 = \frac{\partial f_1}{\partial X_1}\Big|_{X_{1s}, X_{2s}, u_s}(X_1 - X_{1s}) + \frac{\partial f_1}{\partial X_2}\Big|_{X_{1s}, X_{2s}, u_s}(X_2 - X_{2s}) + \frac{\partial f_1}{\partial u}\Big|_{X_{1s}, X_{2s}, u_s}(u - u_s)$$

$$\dot{X}_2 = \frac{\partial f_2}{\partial X_1}\Big|_{X_{1s}, X_{2s}, u_s}(X_1 - X_{1s}) + \frac{\partial f_2}{\partial X_2}\Big|_{X_{1s}, X_{2s}, u_s}(X_2 - X_{2s}) + \frac{\partial f_2}{\partial u}\Big|_{X_{1s}, X_{2s}, u_s}(u - u_s).$$

$\vdots$

$$y = \frac{\partial g}{\partial X_1}\Big|_{X_{1s}, X_{2s}, u_s}(X_1 - X_{12}) + \frac{\partial g}{\partial X_2}\Big|_{X_{1s}, X_{2s}, u_s}(X_2 - X_{2s}) + \frac{\partial g}{\partial u}\Big|_{X_{1u}, X_{2u}, u_s}(u - u_s)$$

Modeling and Simulation :-

Modeling process :-

* for linearized System :-

$\dot{\hat{x}} = A\hat{x} + B\hat{u}$.

$\hat{y} = C\hat{x} + D\hat{u}$.

$A = [a_{ij}]$, $i = 1, \dots, n$, $j = 1, \dots, n$, $a_{ij} = \left.\dfrac{\partial f_i}{\partial x_j}\right|_{s.s}$.

$B = [b_{ij}]$, $i = 1, \dots, n$ $j = 1, \dots, m$ $b_{ij} = \left.\dfrac{\partial f_i}{\partial u_j}\right|_{s.s}$

$C = [C_{ij}]$, $i = 1, \dots, r$ $j = 1, \dots, n$ $C_{ij} = \left.\dfrac{\partial g_i}{\partial x_j}\right|_{s.s}$

$D = [D_{ij}]$, $i = 1, \dots, r$, $j = 1, \dots, m$ $D_{ij} = \left.\dfrac{\partial g_i}{\partial u_j}\right|_{s.s}$

$$\dot{\hat{x}} = \begin{bmatrix} \dot{\hat{x}}_1 \\ \dot{\hat{x}}_2 \end{bmatrix} = \begin{bmatrix} \left.\dfrac{\partial f_1}{\partial x_1}\right|_{s.s} & \left.\dfrac{\partial f_1}{\partial x_2}\right|_{s.s} \\ \left.\dfrac{\partial f_2}{\partial x_1}\right|_{s.s} & \left.\dfrac{\partial f_2}{\partial x_2}\right|_{s.s} \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} \left.\dfrac{\partial f_1}{\partial u}\right|_{s.s} \\ \left.\dfrac{\partial f_2}{\partial u}\right|_{s.s} \end{bmatrix} [\hat{u}]$$

$$\hat{y} = \begin{bmatrix} \left.\dfrac{\partial g_1}{\partial x_1}\right|_{s.s} & \left.\dfrac{\partial g_1}{\partial x_2}\right|_{s.s} \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} + \begin{bmatrix} \left.\dfrac{\partial g_1}{\partial u}\right|_{s.s} \end{bmatrix} \hat{u} .$$

Modeling and Simulation :-

Modeling Process :-

* Linearization steps of steady state Condition :-

$\dot{X}_1 = f_1(X_1, X_2, \dots, u_1, u_2, t)$.

$\dot{X}_2 = f_2(X_1, X_2, \dots, u_1, u_2, t)$.

$y = g(X_1, X_2, \dots, u_1, u_2, t)$.

1 Set all state Variables equations ($\dot{X}$) to Zero

$\dot{X}_1 = f_1 = 0 \quad , \quad \dot{X}_2 = f_2 = 0$.

2- Solve the resulting algebric equations and find steady state Condition.

$\dot{X}_1 = f_1 = 0 \rightarrow X_{ss} = a \dots$,

$\dot{X}_2 = f_2 = 0 \rightarrow X_{ss} = b \dots$.

3- Compute the drivative of all non linear term with respecte all Independent Variables at steady state operating Point (state Variable), (Input Vector).

4- Represent the system in state space matrix form

$\dot{\hat{X}} = A\hat{X} + B\hat{u}$.

$\hat{y} = C\hat{X} + D\hat{u}$.

Modeling and Simulation :-

Modeling Process :-

* Ex:- for a given non-linear state equations. Find the linearized Version of the System at steady state Condition

$$f_1 = \dot{x} = 14x - \frac{1}{2}x^2 - xy$$
$$f_2 = \dot{y} = 16y - \frac{1}{2}y^2 - xy$$

→ state equation

$\dot{x}, \dot{y}$ → state Variable.

① Find the Operating Point in this Case is steady state → $f_1 = 0$, $f_2 = 0$.

$$\dot{Z} = \begin{bmatrix} \dot{Z}_1 \\ \dot{Z}_2 \end{bmatrix} = \begin{bmatrix} \dot{x} \\ \dot{y} \end{bmatrix}$$

$f_1 = 0 \rightarrow 14x - \frac{1}{2}x^2 - xy = 0$

$f_2 = 0 \rightarrow 16y - \frac{1}{2}y^2 - xy = 0$

$14x - \frac{1}{2}x^2 - xy = x(14 - \frac{1}{2}x - y) = 0 \rightarrow x = 0.$

$16y - \frac{1}{2}y^2 - xy = y(16 - \frac{1}{2}y - x) = 0 \rightarrow y = 0.$

first Pair → (0, 0).

second Pair → $(0, 16 - \frac{1}{2}y - x = 0) \rightarrow (0, 32)$.

thierd Pair → $(14 - \frac{1}{2}x - y = 0, 0) \rightarrow (28, 0)$.

fourth Pair → $(14 - \frac{1}{2}x - y = 0, 16 - \frac{1}{2}y - x = 0)$.

→ (12, 8).

$$a_{ij} = \frac{\partial f_i}{\partial x_j} \Rightarrow A = \begin{bmatrix} \left.\frac{\partial f_1}{\partial x}\right|_{s.s} & \left.\frac{\partial f_1}{\partial y}\right|_{s.s} \\ \left.\frac{\partial f_2}{\partial x}\right|_{s.s} & \left.\frac{\partial f_2}{\partial y}\right|_{s.s} \end{bmatrix}$$

$$A = \begin{bmatrix} 14 - x - y & -x \\ -y & 16 - y - x \end{bmatrix}$$

Modeling and Simulation :-

Modeling Process :-

* Ex :-

$$A = \begin{bmatrix} 14-x-y & -x \\ -y & 16-y-x \end{bmatrix}$$

steady state Point.
(0,0), (0,32), (28,0), (12,8)

$$A\Big|_{(0,0)} = \begin{bmatrix} 14 & 0 \\ 0 & 16 \end{bmatrix}$$

$$A\Big|_{(0,32)} = \begin{bmatrix} -18 & 0 \\ -32 & -16 \end{bmatrix}$$

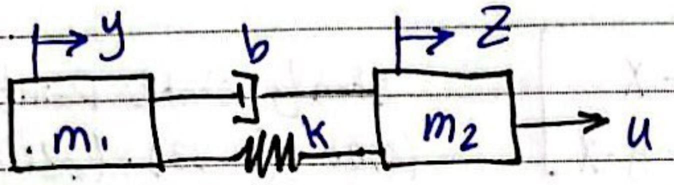
$$A\Big|_{(28,0)} = \begin{bmatrix} -14 & -28 \\ 0 & -12 \end{bmatrix}$$

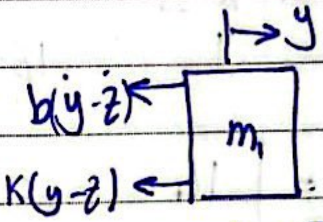
$$A\Big|_{(12,8)} = \begin{bmatrix} -6 & -12 \\ -8 & -4 \end{bmatrix}$$

Modeling and Semulations :-

Modeling Process :-

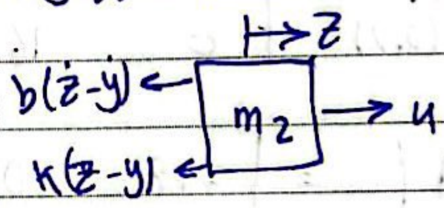
* Selection of state Variable :-





$\sum F = m_1 a$.

$m_1 \ddot{y} + b(\dot{y} - \dot{z}) + K(y - z) = 0$



$\sum F = m_2 a$

$m_2 \ddot{z} + b(\dot{z} - \dot{y}) + K(z - y) = u(t)$

if we choose

$x_1 = y \;\rightarrow\; \dot{x}_1 = \dot{y} \;\rightarrow\; \dot{x}_1 = x_2$

$x_2 = \dot{y} \;\rightarrow\; \dot{x}_2 = \ddot{y} \qquad \dot{x}_2 = \frac{1}{m_1}\left(-b(x_2 - x_4) - K(x_1 - x_3)\right)$

$x_3 = z \;\rightarrow\; \dot{x}_3 = \dot{z} \;\rightarrow\; \dot{x}_3 = x_4$

$x_4 = \dot{z} \;\rightarrow\; \dot{x}_4 = \ddot{z} \qquad \dot{x}_4 = \frac{1}{m_2}\left(u(t) - b(x_4 - x_2) - K(x_3 - x_1)\right)$

$\dot{x} = Ax + Bu$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k}{m_1} & \frac{-b}{m_1} & \frac{k}{m_1} & \frac{b}{m_1} \\ 0 & 0 & 0 & 1 \\ \frac{k}{m_2} & \frac{b}{m_2} & \frac{-k}{m_2} & \frac{-b}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{1}{m_2} \end{bmatrix} u(t).$$

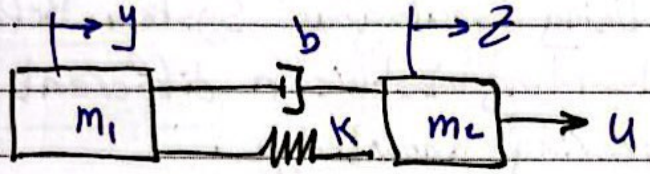
$y_1 = x_1 \quad , \quad y_2 = x_3$.

$y = Cx + Du$.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Modeling and Simulations :-
Modeling Process :-

* Selection of state Variable :-



$m_1\ddot{y} + b(\dot{y}-\dot{z}) + K(y-z) = 0$.
$m_2\ddot{z} + b(\dot{z}-\dot{y}) + K(z-y) = u(t)$.
if we chosse : (We are Intrested in studying y and z)
$x_1 = y - z$, $x_2 = \dot{y}$. $x_3 = \dot{z}$
We can develop state Variable representation of the system Using the above state Variables if we are Intrested in studying only relation displacement between mass (1) and mass (2).

$x_1 = y - z \longrightarrow \dot{x}_1 = \dot{y} - \dot{z} \longrightarrow \dot{x}_1 = x_2 - x_3$
$\dot{x}_2 = \dot{y} \longrightarrow \dot{x}_2 = \ddot{y} \longrightarrow \dot{x}_2 = \frac{-1}{m_1}(b(x_2 - x_3) + K(x_1))$
$x_3 = \dot{z} \longrightarrow \dot{x}_3 = \ddot{z} \longrightarrow \dot{x}_3 = \frac{-1}{m_2}(-U(t) + b(x_3 - x_2) + K(-x_1)$

$\dot{x} = Ax + Bu$.

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -1 \\ \frac{-K}{m_1} & \frac{-b}{m_1} & \frac{b}{m_1} \\ \frac{K}{m_2} & \frac{b}{m_2} & \frac{-b}{m_2} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{m_2} \end{bmatrix} u(t).$$

$y = x_1$
$y = Cx + Du$.

$$y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Modeling and Simulations :-

Modeling Process :-

* Analogous System Representation :-

*The advantages of Using analogous System Representation

1- Conveying understanding between different there of engineering Knowledge.

2- to use the anolytical tools available in different themes of engineering.

3- Unifying on interesting system (mixed system) we can use anologus system representation to unify the system in one domain.

*We will deal with two different analogus system Representation as following:-

1- force - Current analogus Representation (Parallel analogus Representation).

2- force - Voltage analogus Representation. (series analogus Representation).

*System Variables and Ideal segment :-

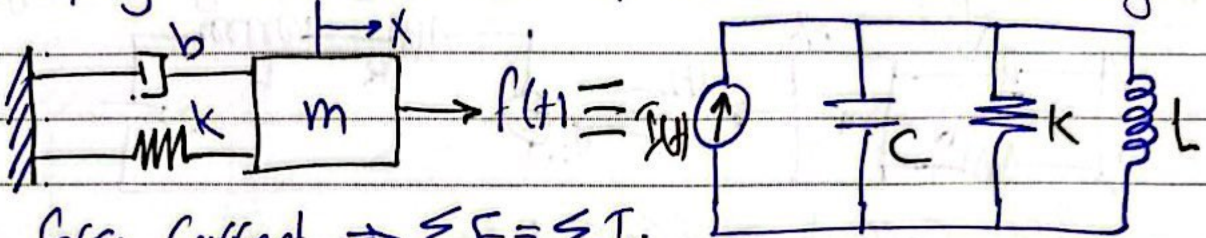
1- Mechanical System
- Through Variable : $f(t)$
- Across Variable : $V(t)$
- Ideal segment :
  - b, K, m.

2- Electrical system
- Through Variable :- $I(t)$
- Across Variable :- $V(t)$
- Ideal Segment :
  - R, C, L.

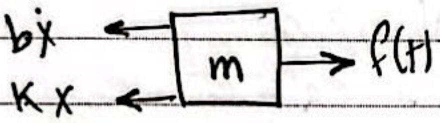
Modeling and Simulations :-

Modeling Process :-

* developing Conversion table of force Current analogus :-



force Current $\Rightarrow \Sigma F = \Sigma I$.



$\Sigma I_{in} = \Sigma I_{out}$.

$I_{in}(t) = I_C + I_R + I_L$.

$I_{in} = C\dfrac{de_c}{dt} + \dfrac{e_R}{R} + \dfrac{1}{L}\int e_L dt$

$\Sigma F = ma$.

$f(t) = m\ddot{x} + b\dot{x} + kx$

$f(s) = (ms^2 + bs + k)X(s)$

$I_{in}(s) = (Cs + \frac{1}{R} + \frac{1}{Ls})\, E(s)$

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2 + bs + k} \qquad \frac{E(s)}{I(s)} = \frac{1}{Cs + \frac{1}{R} + \frac{1}{Ls}}$$

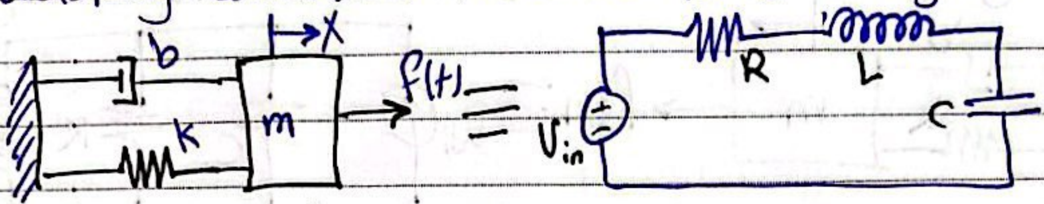
$$\frac{V(s)}{F(s)} = \frac{s}{ms^2 + bs + k} \qquad \frac{E(s)}{I(s)} = \frac{s}{Cs^2 + \frac{1}{R}s + \frac{1}{L}}$$

* Conversion table of force-Current analogus.

Mechanical	Electrical
force (f) ⟷	Current (I)
Velocity (V) ⟷	Voltage (e)
mass (m) ⟷	Capacitor (C)
spring (k) ⟷	$(\frac{1}{L})$
damper (b) ⟷	$(\frac{1}{R})$

Modeling and Simulations :-
Modeling Process :-

* developing Conversion table of force - Voltage Analogus :-



force Current $\Rightarrow \sum f = \sum V$

$\sum f = ma.$

$m\ddot{x} + b\dot{x} + kx = f(t)$

$(ms^2 + bs + k)\,X(s) = f(s)$

$$\frac{X(s)}{f(s)} = \frac{1}{ms^2 + bs + k}$$

$$\frac{V(s)}{f(s)} = \frac{s}{ms^2 + bs + k}$$

$\sum V = \text{Zero}.$

$e_{in} = e_R + e_L + e_C.$

$e_{in} = I_R R + L\frac{dI_L}{dt} + \frac{1}{C}\int I_C dt$

$e_{in}(s) = (R + Ls + \frac{1}{Cs})\,I(s)$

$$\frac{I(s)}{E(s)} = \frac{1}{R + Ls + \frac{1}{Cs}}$$

$$\frac{I(s)}{E(s)} = \frac{s}{Ls^2 + Rs + \frac{1}{C}}.$$

* Conversion table of force - Voltage analogus :-

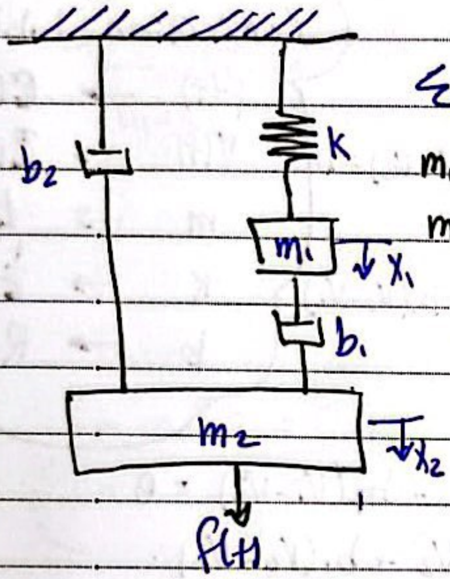
Mechanical	Electrical
force $(f(t))$ $\leftrightarrow$	Voltage (e)
Velocity $V(t)$ $\leftrightarrow$	Current $I(t)$
mass (m) $\leftrightarrow$	Inductor (L)
spring (K) $\leftrightarrow$	$(\frac{1}{C})$
damper (b) $\leftrightarrow$	(R)

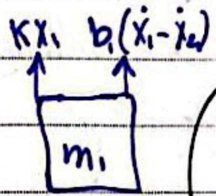
Modeling and Simulations :-

Modeling Process :-

* Example of Mechanical System :-

Using force Current Representation

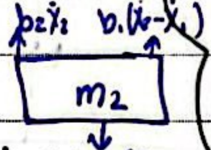




$\Sigma F = m_1 a$

$m_1\ddot{x}_1 + b_1(\dot{x}_1 - \dot{x}_2) + k(x_1) = 0.$

$m_1\dot{V}_1 + b_1(V_1 - V_2) + k\int V_1\,dt = 0.$



$\Sigma F = m_2 a.$

$f(t) = m_2\ddot{x}_2 + b_2\dot{x}_2 + b_1(\dot{x}_2 - \dot{x}_1)$

$f(t) = m_2\dot{V}_2 + b_2 V_2 + b_1(V_2 - V_1).$

Conversion table.

$f(t)$	$\rightarrow I$
$V(t)$	$\rightarrow e$
m	$\rightarrow C$
k	$\rightarrow \frac{1}{L}$
b	$\rightarrow \frac{1}{R}$.

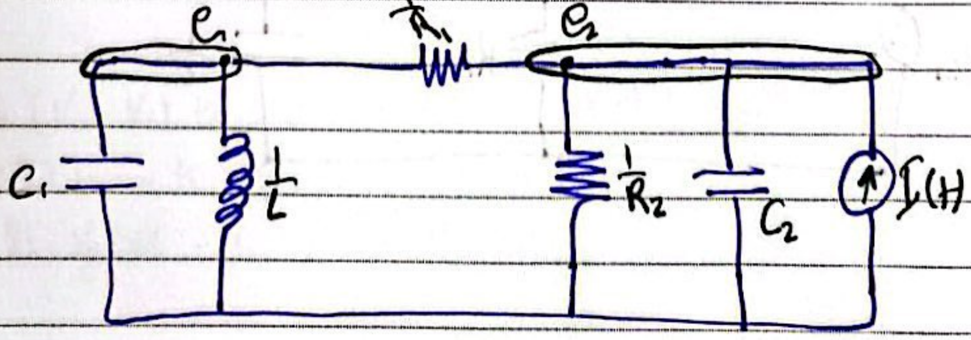
$m_1\dot{V}_1 + b_1(V_1 - V_2) + k\int V_1\,dt = 0.$

$f(t) = m_2\dot{V}_2 + b_2 V_2 + b_1(V_2 - V_1).$

Replace Velocity Variable, to Voltage Variable and Replace Mechanical Component to Electrical Component

$C_2\frac{de_2}{dt} + \frac{1}{R_2}e_2 + \frac{1}{R_1}(e_2 - e_1) = I(t)$

$C_1\frac{de_1}{dt} + \frac{1}{R_1}(e_1 - e_2) + \frac{1}{L}\int e_1\,dt = 0.$

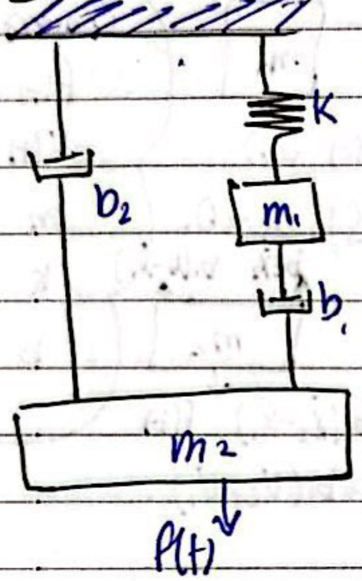


Modeling and Simulations :-

Modeling Process :-

* Example of Mechanical System :-

Using force - Voltage Representation:



$\Sigma F = m_1 a.$

$m_1\dot{V}_1 + K\int V_1 dt + b_1(V_1 - V_2) = 0$

$\Sigma f = m_2 a$

$F(t) = m_2\dot{V}_2 + b_2 V_2 + b_1(V_2 - V_1)$

Conversion table

$f(t)$	$\rightarrow$	$e(t)$
$V(t)$	$\rightarrow$	$I(t)$
m	$\rightarrow$	L
K	$\rightarrow$	$\frac{1}{C}$
b	$\rightarrow$	R

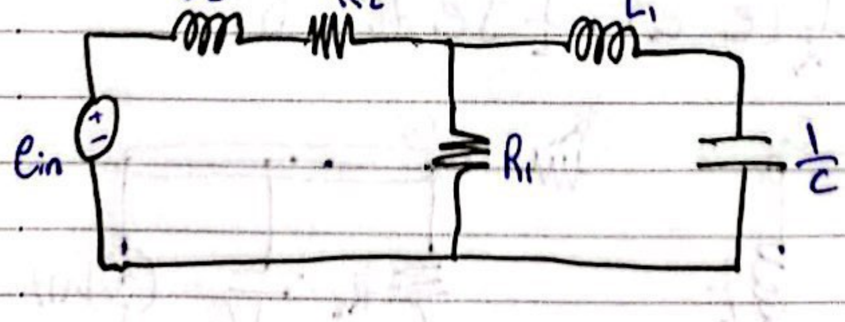
$m_1\dot{V}_1 + K\int V_1 dt + b_1(V_1 - V_2) = 0.$

$f(t) = m_2\dot{V}_2 + b_2 V_2 + b_1(V_2 - V_1).$

Replace Velocity Variable by Current Variable and Replace Mechanical Component to Electrical Component.

$L_1 \frac{d I_1(t)}{dt} + \frac{1}{C}\int I_1(t) + R_1(I_1 - I_2) = 0.$

$e(t) = L_2 \frac{d I_2(t)}{dt} + R_2 . I_2(t) + R_1(I_2 - I_1).$

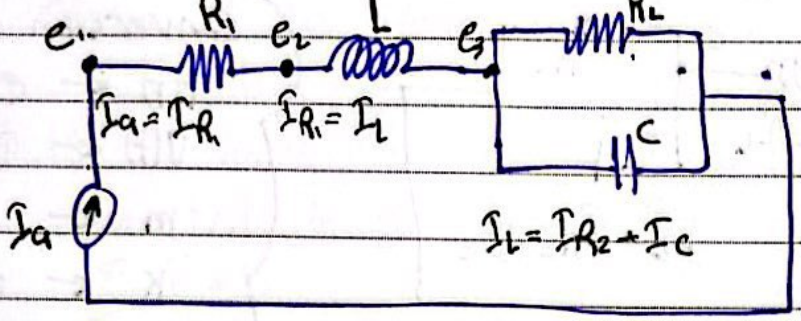


Modeling and Simulations :-

Modeling Process :-

* Example of Electrical System :-

Using force - Current Representation.



Conversion	table
$f(t)$ ←	$I(t)$
$V(t)$ ←	$e(t)$
m ←	C
K ←	$\frac{1}{L}$
b ←	$\frac{1}{R}$

Based on node (1) (e_1) :-

$$I_a + I_{R_1} = 0.$$

$$I_a + \frac{e_2 - e_1}{R_1} = 0$$

Based on node (2) (e_2)

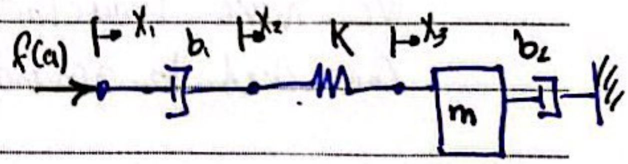
$$I_{R_1} + I_L = 0.$$

$$\frac{e_1 - e_2}{R_1} + \frac{1}{L}\int (e_3 - e_2)\,dt = 0$$

Based on node (3) (e_3) :-

$$I_L - I_{R_2} - I_C = 0$$

$$\frac{1}{L}\int (e_2 - e_3)\,dt - \frac{e_3}{R_2} - C\frac{de_3}{dt} = 0$$



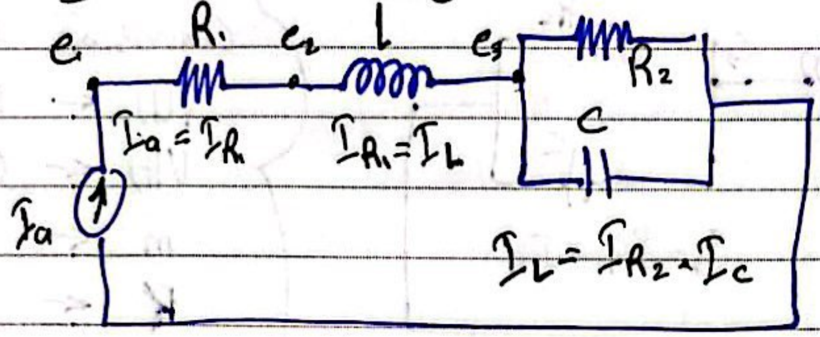
$$f_a + (V_2 - V_1)\,b_1 = 0.$$

$$(V_1 - V_2)\,b_1 + K\int ((V_3 - V_2)\,dt = 0$$

$$K\int (V_2 - V_3)\,dt - b_2 V_3 - m\dot{V}_3 = 0$$

Modeling and Simulations :-
Modeling Process :-

*** Example of Electrical System :-**
Using force - Voltage Representation.



Conversion table

$f(t)$	←	$e(t)$
$v(t)$	←	$I(t)$
m	←	L
K	←	$\frac{1}{C}$
b	←	R

$$I_a + \frac{e_2 - e_1}{R_1} = 0$$

$$\frac{e_1 - e_2}{R_1} + \frac{1}{L}\int (e_3 - e_2)\,dt = 0$$

$$\frac{1}{L}\int (e_2 - e_3)\,dt - \frac{e_3}{R_2} - C\frac{de_3}{dt} = 0$$

$$V_a + \frac{f_2 - f_1}{b_1} = 0$$

$$\frac{f_1 - f_2}{b_1} + \frac{1}{m}\int (f_3 - f_2)\,dt = 0$$

$$\frac{1}{m}\int (f_2 - f_3)\,dt - \frac{f_3}{b_2} - \frac{1}{K}\frac{d f_3(t)}{dt} = 0$$

We have Inductors, One terminals of it not Connected to ground → force - Voltage failded.

Modeling and Simulations:-

Modeling Process:-

* Notes:-

1. If we have Capacitor. One termial of it not Connected to ground → force-Current failued

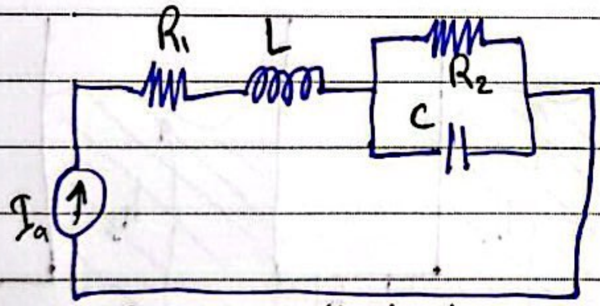
$\Sigma F = 0$, $\Sigma I = 0$. $\Sigma F = \Sigma I$.

2. If we have Inductor, One terminal of it not Connected to ground → force-Voltage failued.

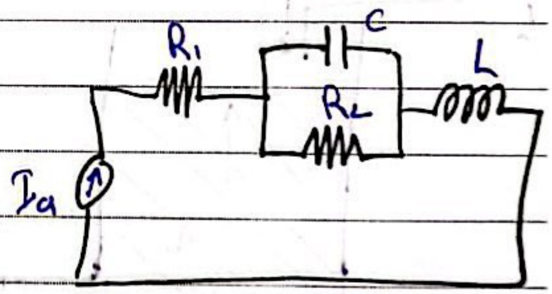
$\Sigma F = 0$, $\Sigma V = 0$, $\Sigma F = \Sigma V$.

3. Inertia: two terminal Ideal segment.

if both Capacitor and Inductors Connected to ground.



(C) Connected to ground.
→ force-Current ✓
(L) Not Connected to ground.
→ force-Voltage failued.



(L) Connected to ground
→ force-Voltage ✓
(C) Not Connected to ground
→ force-Current failued.

Modeling and Simulations :-

Modeling Process :-

* for the following fluid system Find the analogus Electrical System Representation Using force - Current analogus :-

Through Variable :- flow rate $q(t)$

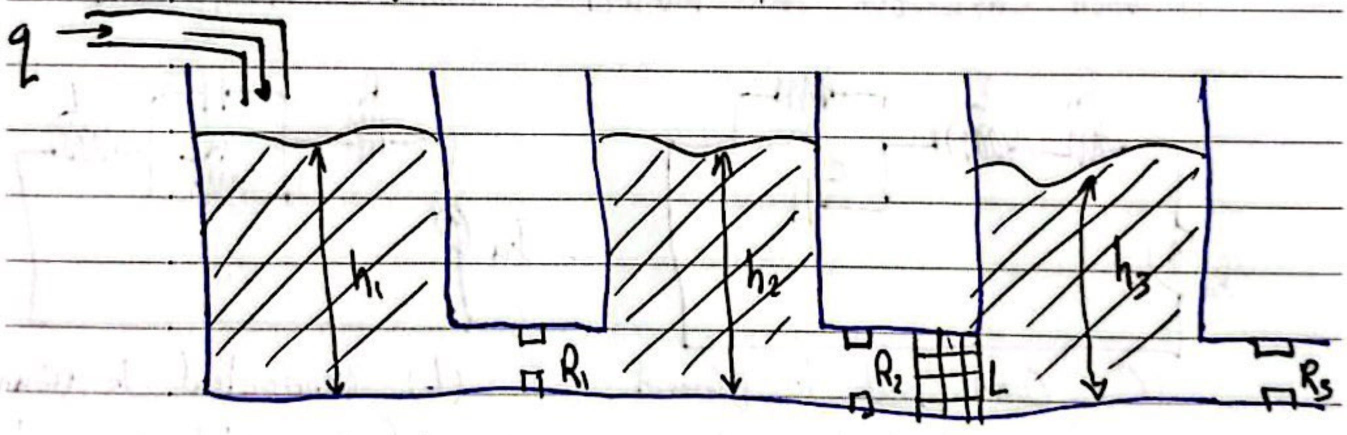
Across Variable :- level $H(t)$ / Pressure $P(t)$.

Ideal segments :- Orfes (R), Area (A), filter (L)

Constitutive equation

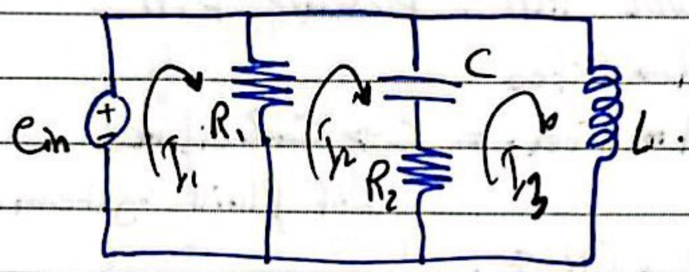
$$H(t) = R * q(t)$$

$$q(t) = A \cdot \frac{d\,h(t)}{dt}.$$



Modeling and Simulation :-
Modeling Process:-

* Example of Electrical system :-
Using force-Voltage Representation



Conversion table.
$f(t) \leftarrow e(t)$
$V(t) \leftarrow I(t)$
$m \leftarrow L$
$K \leftarrow \frac{1}{C}$
$b \leftarrow R$

$\Sigma f = 0, \quad \Sigma V = 0.$

$e_{in} = e_{R_1} = I_{R_1} R_1 = (I_1 - I_2) R_1$

$e_{in} + R(I_2 - I_1) = 0 \quad \dots (1)$

$-e_{R_1} + e_c + e_{R_2} = 0.$

$R_1(I_1 - I_2) + \frac{1}{C}\int(I_2 - I_3)\,dt + R_2(I_2 - I_3) = 0 \quad \dots (2)$

$e_c + e_{R_2} - e_L = 0.$

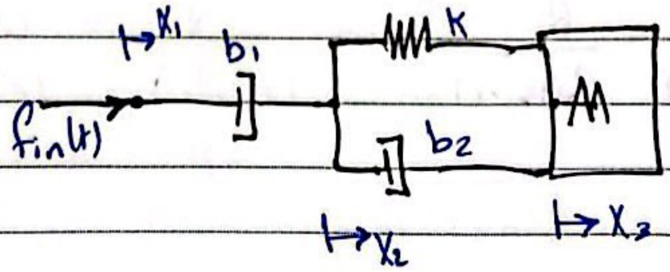
$\frac{1}{C}\int(I_2 - I_3)\,dt + R_2(I_2 - I_3) - L\frac{dI_3}{dt} = 0 \quad \dots (3)$

Replace Current Variable by Velocity Variable and Replace Electrical element to Mechanical element

$f_{in}(t) + b(V_2 - V_1) = 0.$

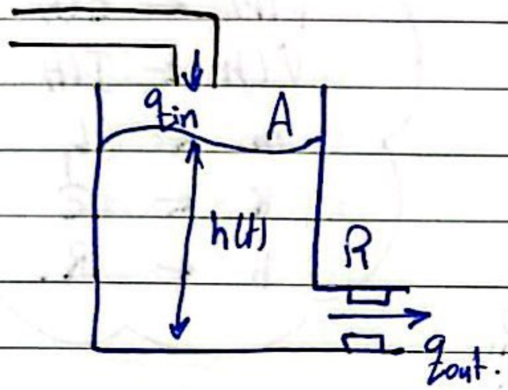
$b_1(V_1 - V_2) + K\int(V_2 - V_3)\,dt + b_2(V_2 - V_3) = 0$

$K\int(V_2 - V_3)\,dt + b_2(V_2 - V_3) - m\dot{V}_3 = 0.$



Modeling and Simulations :-
Modeling Process :-

* Fluid system :-
- Through Variable :- flow rate ($q(t)$).
- Across Variable : level $H(t)$, Pressure $P(t)$.



R → Orfes
A → Area. = C → Capacitance of fluid system.
L → filters.

Hard Approximation

$\Rightarrow q_{out} = \frac{h}{R}$ $\qquad H(t) = q(t) * R(t)$.
↳ Can be non linear.

$$q_{in} - q_{out} = \frac{dV}{dt} = A\frac{dh}{dt}.$$

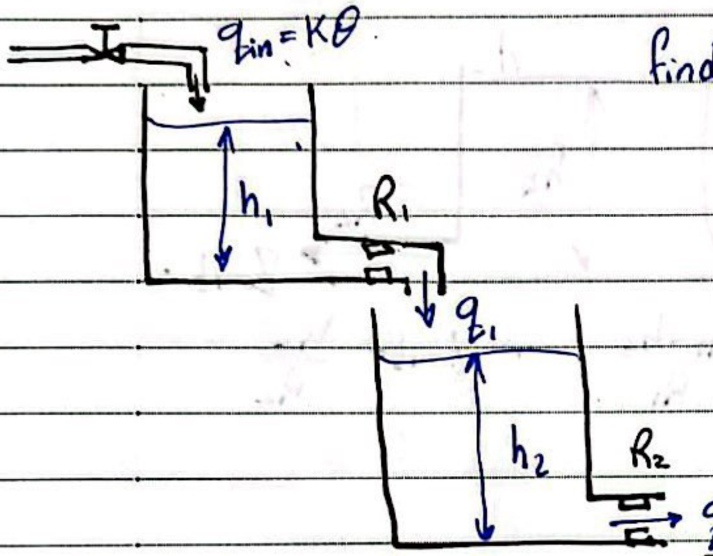
$$q_{in} - \frac{h}{R} = A\frac{dh}{dt}$$ by taking laplace transform

$$Q_{in}(s) - \frac{h(s)}{R} = As\,h(s) \Rightarrow \frac{H(s)}{Q_{in}(s)} = \frac{1}{As + \frac{1}{R}}.$$

$$\Rightarrow \frac{H(s)}{Q_{in}(s)} = \frac{R}{RAs+1}$$

Modeling and Simulation ::
Modeling Process :-

*Fluid System :-
Hard approximation and Weak Coupling :-



find $\frac{H_2(s)}{Q(s)}$

$$q_{in} - q_1 = A_1 \frac{dh_1}{dt}$$

$$K\theta - \frac{h_1}{R_1} = A_1 \frac{dh_1}{dt}$$

by taking laPlace

$$K\theta(s) - \frac{h_1(s)}{R} = A_1 s\, h_1(s) \Rightarrow \boxed{\frac{H_1(s)}{\theta(s)} = \frac{KR_1}{A_1R_1 s+1}}$$

$$q_1 - q_{out} = A_2 \frac{dh_2}{dt} \Rightarrow \frac{h_1}{R_1} - \frac{h_2}{R_2} = A_2 \frac{dh_2}{dt}$$

$$\frac{h_1(s)}{R_1} - \frac{h_2(s)}{R_2} = A_2 s\, h_2(s)$$

but $\boxed{H_1(s) = \frac{KR_1}{A_1R_1 s+1}\,\theta(s)}$

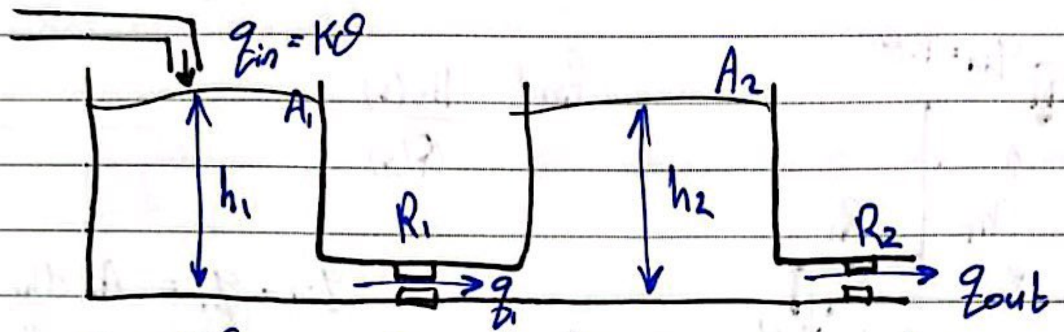
$$\frac{K\,\theta(s)}{A_1R_1 s+1} = \left(\frac{1}{R_2} + A_2 s\right) H_2(s)$$

$$\Rightarrow \frac{H_2(s)}{\theta(s)} = \frac{K}{A_1R_1 s+1} * \frac{R_2}{R_2A_2 s+1}$$

$$\frac{H_2(s)}{Q_{in}(s)} = \frac{R_2}{(A_1R_1 s+1)(R_2A_2 s+1)}$$

Modeling and Simulations:-
Modeling Process:-

* Fluid system:-
- Hard approximation and strong Coupling:-



$q_{in} = K_1\theta$, $\quad q_1 = \frac{h_1 - h_2}{R_1}$, $\quad q_{out} = \frac{h_2}{R_2}$

$$q_{in} - q_1 = A_1 \frac{dh_1}{dt}$$

$$K\theta_1 - \frac{h_1 - h_2}{R_1} = A_1 \frac{dh_1}{dt}$$ by taking laplace transform

$$K\theta(s) - \frac{h_1(s) - h_2(s)}{R_1} = A_1 s\, h_1(s)$$

$$q_1 - q_{out} = A_2 \frac{dh_2}{dt} \Rightarrow \frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2} = A_2 \frac{dh_2}{dt}$$

$$\frac{h_1(s) - h_2(s)}{R_1} - \frac{h_2(s)}{R_2} = A_2 s\, h_2(s)$$

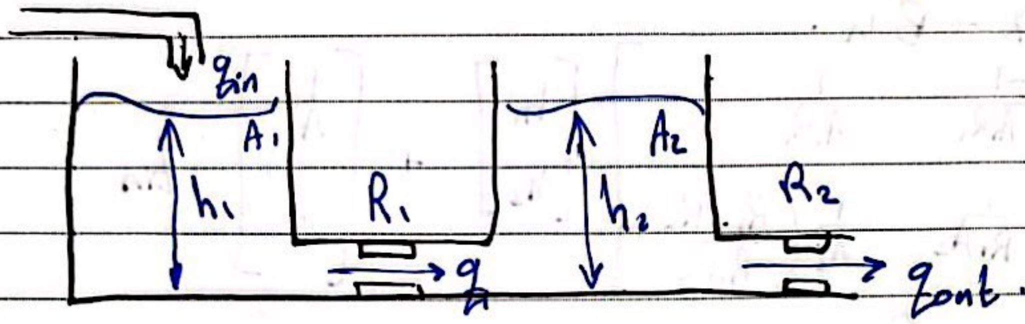
$$\frac{H_2(s)}{\theta(s)} = \frac{K R_2}{(A_1 R_1 s + 1)(A_2 R_2 s + 1) + A_1 R_2 s}$$

$$\frac{H_2(s)}{q_{in}(s)} = \frac{R_2}{(A_1 R_1 s + 1)(A_2 R_2 s + 1) + A_1 R_2 s}$$

Modeling and Simulations :-

Modeling Process :-

* state space Representation of Fluid system :-



$$q_{in} - q_1 = A_1 \frac{dh_1}{dt} \quad , \quad q_1 = \frac{h_1 - h_2}{R_1}$$

$$q_1 - q_{out} = A_2 \frac{dh_2}{dt} \quad , \quad q_{out} = \frac{h_2}{R_2}$$

$$\Rightarrow q_{in} - \frac{h_1 - h_2}{R_1} = A_1 \frac{dh_1}{dt}.$$

$$\frac{h_1 - h_2}{R_1} - \frac{h_2}{R_2} = A_2 \frac{dh_2}{dt}.$$

my state Variables are h_1, h_2.

$$X_1 = h_1 \rightarrow \dot{X}_1 = \frac{dh_1}{dt} = \frac{q_{in}}{A_1} - \frac{1}{R_1A_1} h_1 + \frac{1}{R_1A_1} h_2.$$

$$X_2 = h_2 \rightarrow \dot{X}_2 = \frac{dh_2}{dt} = \frac{1}{R_1A_2} h_1 - \frac{1}{A_2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) h_2$$

and I am interested in studying h_1 and h_2

$y_1 = h_1 = X_1$

$y_2 = h_2 = X_2$

Modeling and simulations :-

Modeling Process :-

* state space Representation of Fluid system :-

$$\dot{X} = AX + Bu.$$

$$\dot{X} = \begin{bmatrix} \frac{-1}{A_1R_1} & \frac{1}{A_1R_1} \\ \frac{1}{R_1A_2} & \frac{1}{A_2}\left(\frac{R_1+R_2}{R_1R_2}\right) \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix} q_{in}$$

$$y = CX + Du.$$

$$y = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix} + [0]u$$

if Iam Interested in studying (q_{out}) as output

$$y = q_{out} = \frac{h_2}{R_2}.$$

$$y = \begin{bmatrix} 0 & \frac{1}{R_2} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}.$$

if Iam Interested in studying (q_1) as Output.

$$y = q_1 = \frac{h_1 - h_2}{R_1} = \frac{h_1}{R_1} - \frac{h_2}{R_1}.$$

$$y = \begin{bmatrix} \frac{1}{R_1} & \frac{-1}{R_1} \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}.$$

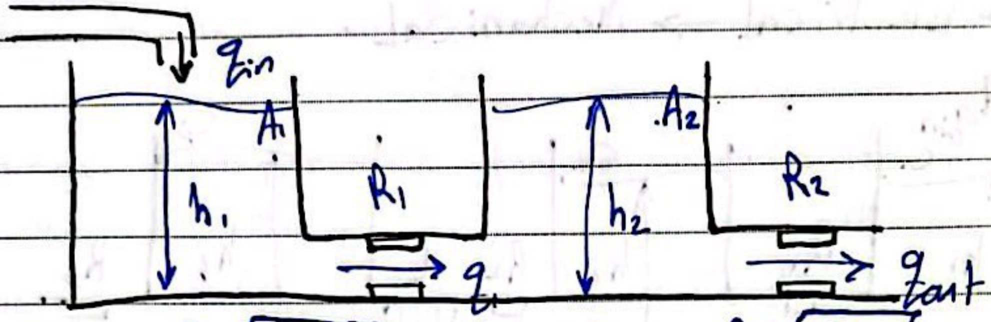
if Iam Interested in studying change of level $h_1 - h_2$

$$y = h_1 - h_2 = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} h_1 \\ h_2 \end{bmatrix}$$

Modeling and Simulations :-

Modeling Process :-

*Non linear Fluid tank System :-



$q_1 = R_1\sqrt{h_1 - h_2}$ $\qquad q_{out} = R_2\sqrt{h_2}$

$$q_{in} - q_1 = A_1\frac{dh_1}{dt} \Rightarrow \boxed{\frac{dh_1}{dt} = \frac{q_{in}}{A_1} - \frac{R_1}{A_1}\sqrt{h_1 - h_2}} = f_1$$

$$q_1 - q_{out} = A_2\frac{dh_2}{dt} \Rightarrow \boxed{\frac{dh_2}{dt} = \frac{R_1}{A_2}\sqrt{h_1 - h_2} - \frac{R_2}{A_2}\sqrt{h_2}} = f_2$$

if $y = q = \sqrt{h_2^3} - h_1$

$$A = \begin{bmatrix} \frac{\partial f_1}{\partial h_1} & \frac{\partial f_1}{\partial h_2} \\ \frac{\partial f_2}{\partial h_1} & \frac{\partial f_2}{\partial h_2} \end{bmatrix} \Bigg|_{h_{1s},\, h_{2s},\, u_s} \text{steady state} \qquad B = \begin{bmatrix} \frac{\partial f_1}{\partial q_{in}} \\ \frac{\partial f_2}{\partial q_{in}} \end{bmatrix} \Bigg|_{h_{1s},\, h_{2s},\, u_s}$$

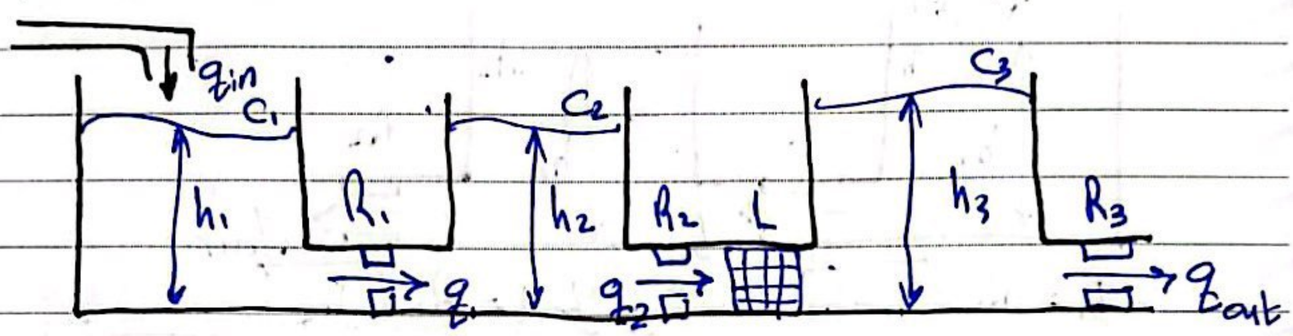
$$A = \begin{bmatrix} \frac{-R_1}{2A_1\sqrt{h_1 - h_2}} & \frac{R_1}{2A_1\sqrt{h_1 - h_2}} \\ \frac{R_1}{2A_1\sqrt{h_1 - h_2}} & \frac{-R_1}{2A_2\sqrt{h_1 - h_2}} - \frac{R_2}{2A_2\sqrt{h_2}} \end{bmatrix} \qquad B = \begin{bmatrix} \frac{1}{A_1} \\ 0 \end{bmatrix}$$

Modeling and Simulations:-

Modeling Process :-

* Analogous System Representation of Fluid system :—

Fluid → Electrical → Mechanical.



Using hard approximation :-

$q_1 = \frac{h_1 - h_2}{R_1}$, $q_2 = \frac{h_2 - h_3}{R_2} + \frac{1}{L}\int (h_2 - h_3)dt$, $q_{out} = \frac{h_3}{R_3}$

$q_{in} - q_1 = A_1 \frac{dh_1}{dt} \Rightarrow \frac{dh_1}{dt} = \frac{q_{in}}{A_1} - \frac{h_1 - h_2}{A_1 R_1}$

$q_1 - q_2 = A_2 \frac{dh_2}{dt} \Rightarrow \frac{dh_2}{dt} = \frac{h_1 - h_2}{A_2 R_1} - \frac{h_2 - h_3}{A_2 R_2} + \frac{1}{L A_2}\int (h_2 - h_3)dt$

$q_2 - q_{out} = A_3 \frac{dh_3}{dt} \Rightarrow \frac{dh_2}{dt} = \frac{h_2 - h_3}{C_3 R_2} - \frac{1}{C_3 L}\int (h_2 - h_3)dt - \frac{h_3}{C_3 R_3}$

$q_{in} - \frac{h_1 - h_2}{R_1} = C_1 \frac{dh_1}{dt}$

$\frac{h_1 - h_2}{R_1} - \frac{h_2 - h_3}{R_2} - \frac{1}{L}\int (h_2 - h_3)dt = C_2 \frac{dh_2}{dt}$

$\frac{h_2 - h_3}{R_2} + \frac{1}{L}\int (h_2 - h_3)dt - \frac{h_3}{R_3} = C_3 \frac{dh_3}{dt}$

Modeling and Simulations :-

Modeling Process :-

* Analogous system Representation of fluid systems :

$$q_{in} - \frac{h_1 - h_2}{R_1} = C_1 \frac{dh_1}{dt}$$

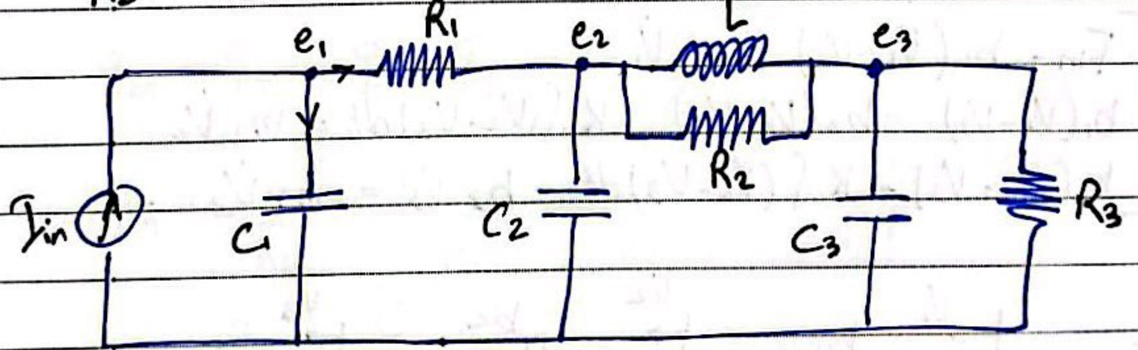
$$\frac{h_1 - h_2}{R_1} - \frac{h_2 - h_3}{R_2} - \frac{1}{L}\int (h_2 - h_3)\,dt = C_2 \frac{dh_2}{dt}$$

$$\frac{h_2 - h_3}{R_2} + \frac{1}{L}\int (h_2 - h_3)\,dt - \frac{h_3}{R_3} = C_3 \frac{dh_3}{dt}$$

$$I_{in} - \frac{e_1 - e_2}{R_1} = C_1 \frac{de_1}{dt}$$

$$\frac{e_1 - e_2}{R_1} - \frac{e_2 - e_3}{R_2} - \frac{1}{L}\int (e_2 - e_3)\,dt = C_2 \frac{de_2}{dt}$$

$$\frac{e_2 - e_3}{R_2} + \frac{1}{L}\int (e_2 - e_3)\,dt - \frac{e_3}{R_3} = C_3 \frac{de_3}{dt}$$



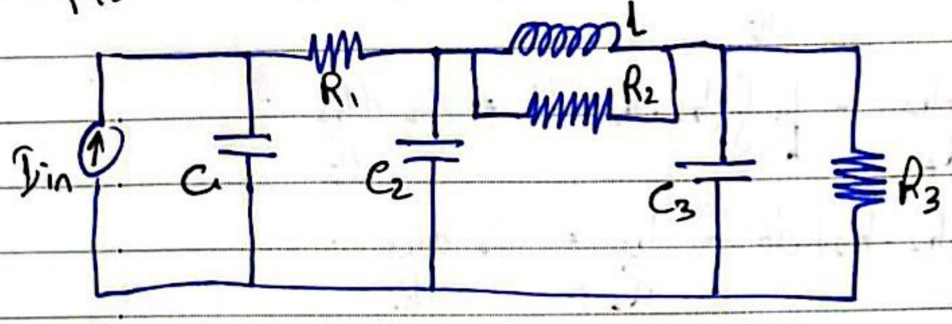
then Convert the electrical system to mechanical system Using Analogous system Representation.

Modeling and Simulations :–

Modeling Process :–

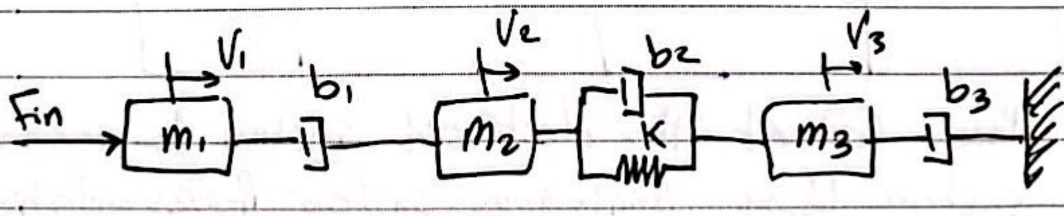
* Analogous system Representation of Fluid system:–

Fluid → Electrical → Mechanical.



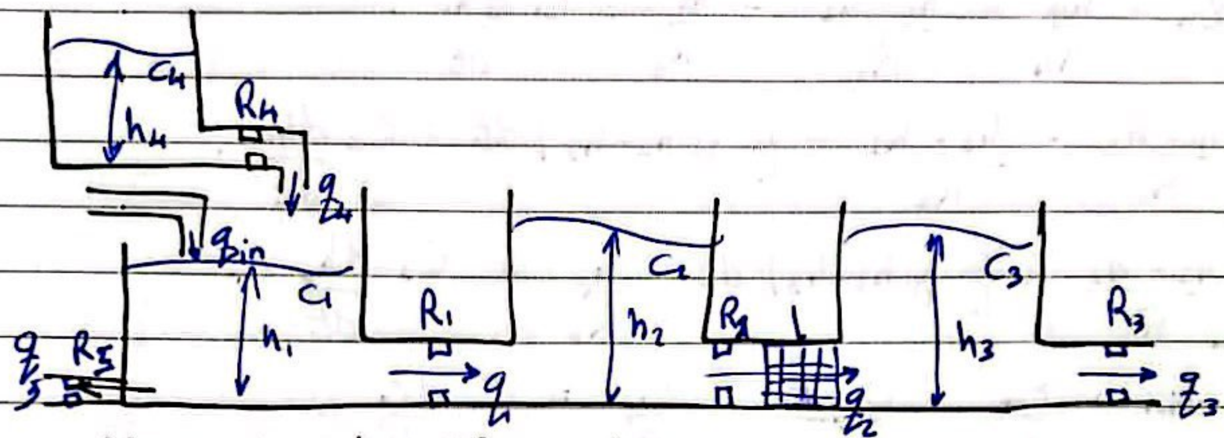
$$\left\{\begin{aligned}
& I_{in} - \frac{e_1-e_2}{R_1} = C_1 \frac{de_1}{dt} \\
& \frac{e_1-e_2}{R_1} - \frac{e_2-e_3}{R_2} + \frac{1}{L}\int(e_2-e_3)\,dt = C_2 \frac{de_2}{dt} \\
& \frac{e_2-e_3}{R_2} + \frac{1}{L}\int(e_2-e_3)\,dt - \frac{e_3}{R_3} = C_3 \frac{de_3}{dt}
\end{aligned}\right.$$

$$\rightarrow \left\{\begin{aligned}
& F_{in} - b_1(V_1-V_2) = m_1 \dot{V}_1 \\
& b_1(V_1-V_2) - b_2(V_2-V_3) - K\int(V_2-V_3)\,dt = m_2 \dot{V}_2 \\
& b_2(V_2-V_3) + K\int(V_2-V_3)\,dt - b_3 V_3 = m_3 \dot{V}_3
\end{aligned}\right.$$



Modeling and Simulations :-
Modeling process :-

*Analaguos System Representation of fluid system :-



Using hard approximation

$$q_1 = \frac{h_1 - h_2}{R_1} \quad , \quad q_2 = \frac{h_2 - h_3}{R_2} + \frac{1}{L}\int (h_2 - h_3)\,dt$$

$$q_3 = \frac{h_3}{R_3} \quad , \quad q_4 = \frac{h_4}{R_4} \quad , \quad q_5 = \frac{h_1}{R_5} .$$

$$q_{in} + q_4 - q_1 - q_5 = C_1 \frac{dh_1}{dt} .$$

$$q_{in} + \frac{h_4}{R_4} - \frac{h_1 - h_2}{R_1} - \frac{h_1}{R_5} = C_1 \frac{dh_1}{dt} \quad \text{---- } 1$$

$$q_1 - q_2 = C_2 \frac{dh_2}{dt} .$$

$$\frac{h_1 - h_2}{R_1} - \frac{h_2 - h_3}{R_2} - \frac{1}{L}\int (h_2 - h_3)\,dt = C_2 \frac{dh_2}{dt} \quad \text{---- } 2$$

$$q_2 - q_3 = C_3 \, dh_3/dt .$$

$$\frac{h_2 - h_3}{R_2} + \frac{1}{L}\int (h_2 - h_3)\,dt - \frac{h_3}{R_3} = C_3 \frac{dh_3}{dt} . \quad \text{---- } 3$$

Modeling and Simulations :-

Modeling process :-

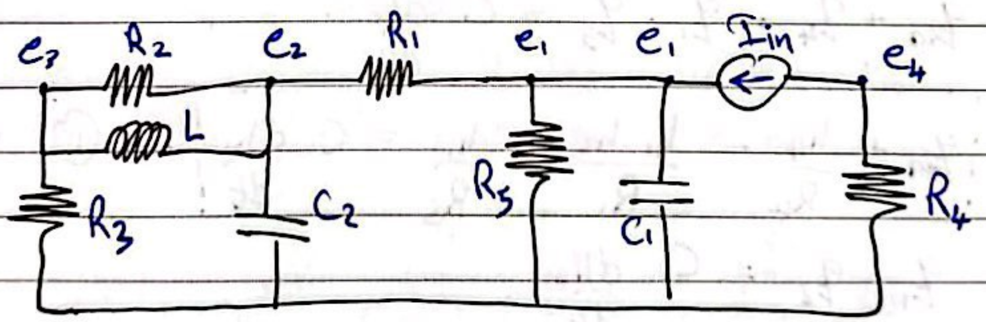
*** Analogous System Representation of fluid system**

$$\begin{cases} q_{in} + \dfrac{h_4}{R_4} - \dfrac{h_1 - h_2}{R_1} - \dfrac{h_1}{R_5} = C_1 \dfrac{dh_1}{dt} \\[2ex] \dfrac{h_1 - h_2}{R_1} - \dfrac{h_2 - h_3}{R_2} - \dfrac{1}{L}\int (h_2 - h_3)\,dt = C_2 \dfrac{dh_2}{dt} \\[2ex] \dfrac{h_2 - h_3}{R_2} + \dfrac{1}{L}\int (h_2 - h_3)\,dt - \dfrac{h_3}{R_3} = C_3 \dfrac{dh_3}{dt} \end{cases}$$

$$I_{in} + \frac{e_4}{R_4} - \frac{e_1 - e_2}{R_1} - \frac{e_1}{R_5} = C_1 \frac{de_1}{dt}$$

$$\frac{e_1 - e_2}{R_1} - \frac{e_2 - e_3}{R_2} - \frac{1}{L}\int (e_2 - e_3)\,dt = C_2 \frac{de_2}{dt}$$

$$\frac{e_2 - e_3}{R_2} + \frac{1}{L}\int (e_2 - e_3)\,dt - \frac{e_3}{R_3} = C_3 \frac{de_3}{dt}$$

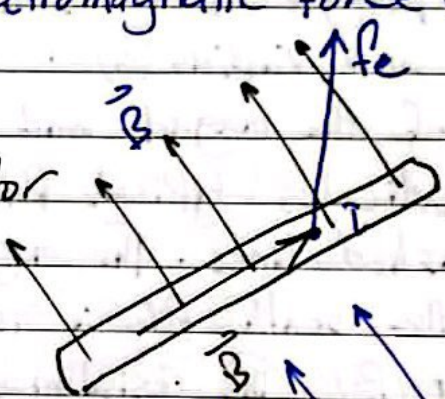


Modeling and Simulations :-
Modeling Process :-

*. Mixed System :-
We Will study electromechanical system as example of mixed system.

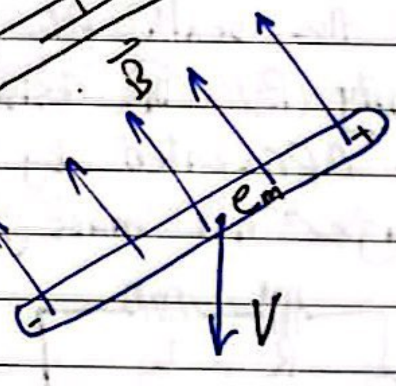
* for Carrying current wire subject to electromagnatic field. this Will Create electromagnatic force (f_e) exerted on Conductor

$\boxed{f_e = i L B}$.



i:- Current through Conductor
L: Length of Conductor
B: magnatic flux

$\boxed{e_m = V B L}$.



e_m: Induced Voltage
V: Velocity of Conductor
B: magnatic flux
L: Length of Conductor.

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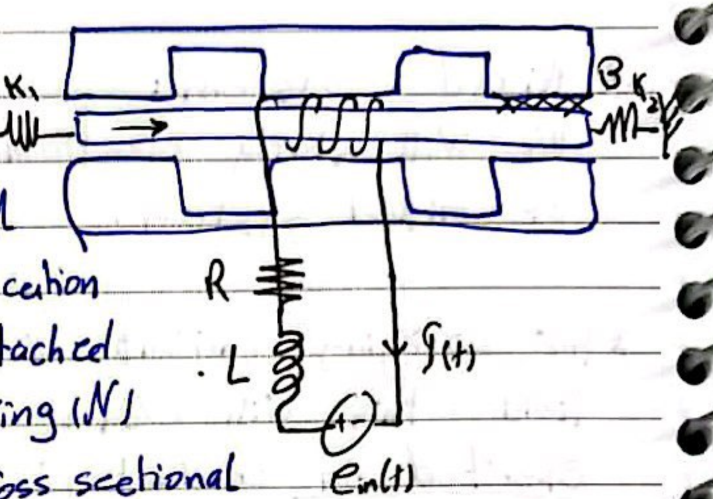
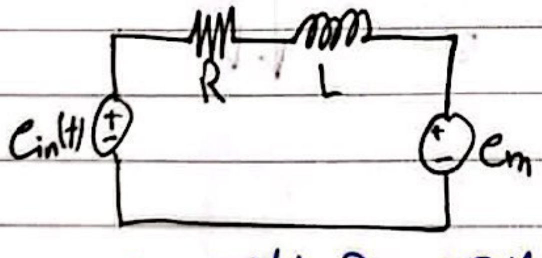
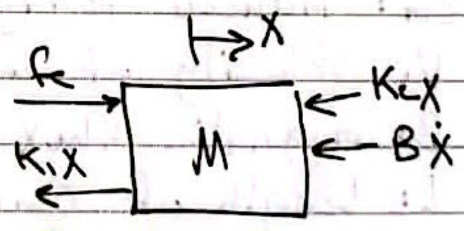
Modeling and Simulations :-

Modeling Process:-



* Ex :- A plunger is made to mov horizantally through the Center of a fixed Cylinderical Permanent magnet by the application of a Voltage source $e_{in}(t)$ attached to the plunger is a Coil having (N) turns and radius (a) · Cross scetional view of the magnet and plunger.
and indicates typical paths for the magnatic flux by dashed lines. The magnatic field between the plunger and the south pole is assumed to have a constant flux density (B) · The resistance and inductance of the Coil are represented by the lumped elements R, L and the plunger has mass (M) ·





$f_e = i\,L\,B$ ·
$= 2\pi N a\, i\, B$ ·

$e_m = VLB = 2\pi N a V B$

by taking KVL.

$-e_{in}(t) + V_R + V_L + e_m = 0$ ·

$-e_{in}(t) + iR + L\dfrac{di(t)}{dt} + 2\pi N a V B = 0$

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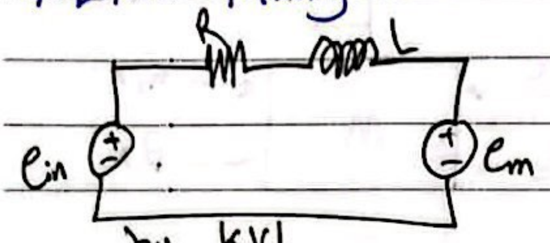
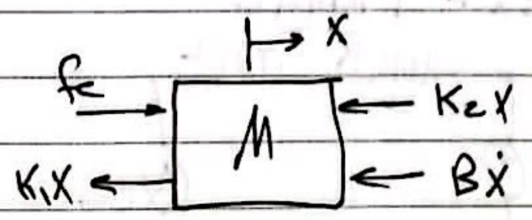
Modeling and Simulations :-
Modeling Process :-

*Ex:- Plunger.





by KVL

$-e_{in} + V_R + V_L + e_m = 0.$

$-e_{in} + iR + L\frac{di}{dt} + 2\pi N a V B = 0 \quad \ldots (1)$

$\Sigma f = Ma.$

$fe - K_1x - K_2x - B\dot{x} = M\ddot{x}$

$M\ddot{x} + B\dot{x} + K_1 x + K_2 x = 2\pi N a\, i B \quad \ldots (2)$

if we select x, V, i as state Variable.

$z_1 = x \rightarrow \dot{z}_1 = \dot{x} = V \rightarrow \dot{z}_1 = z_2.$

$z_2 = V \rightarrow \dot{z}_2 = \ddot{x} = \dot{V} = \frac{1}{M}\left(2\pi N a B z_3 - B z_2 - K_1 z_1 - K_2 z_1\right)$

$z_3 = i \rightarrow \dot{z}_3 = \frac{di}{dt} = \frac{1}{L}\left(e_{in} - R z_3 - 2\pi N a z_2 B\right).$

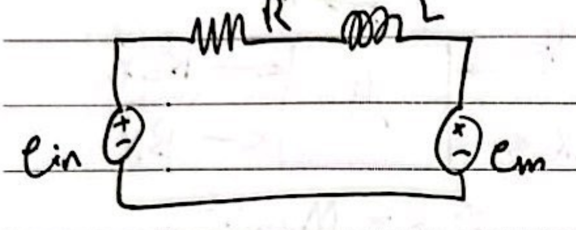
$\dot{z} = Az + Bu.$

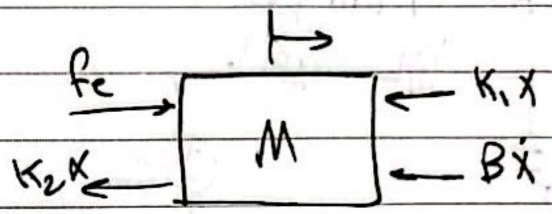
$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{-(K_1+K_2)}{M} & \frac{-B}{M} & \frac{2\pi N a B}{M} \\ 0 & \frac{-2\pi N a B}{L} & \frac{-R}{L} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} \cdot u$$

Modeling and Simulations :-

Modeling Process :-

*Ex :- Plunger





$$-e_{in} + iR + L\frac{di}{dt} + 2\pi N a V B = 0 \qquad m\ddot{X} + (K_1 + K_2)X + B\dot{X} = 2\pi N a B i$$

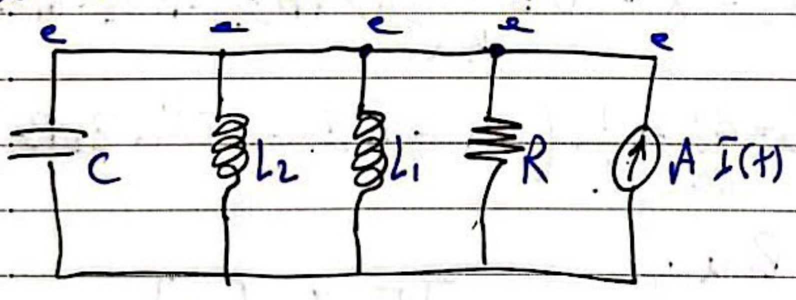
$$m\dot{V} + K_1\int V dt + K_2 \int V dt + BV = 2\pi N a B i.$$

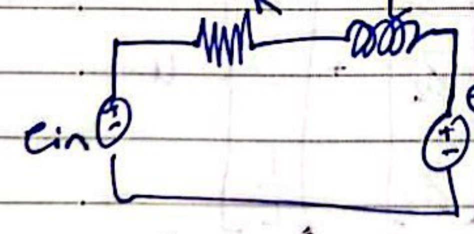
Using force Current analogous system Representation

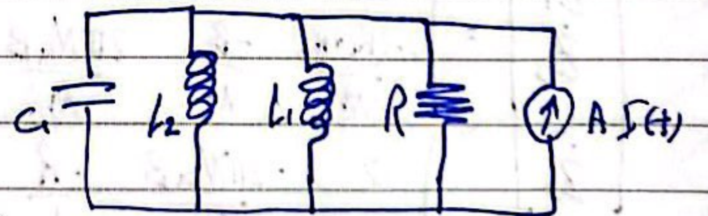
$$m\dot{V} + K_1\int V + K_2 \int V + BV = 2\pi N a B i.$$

$$C\frac{de}{dt} + \frac{1}{L_1}\int e dt + \frac{1}{L_2}\int e dt + \frac{1}{R} e = A I(t).$$

Conversion table
- $F \rightarrow I$
- $V \rightarrow e$
- $m \rightarrow C$
- $K \rightarrow \frac{1}{L}$
- $b \rightarrow \frac{1}{R}$





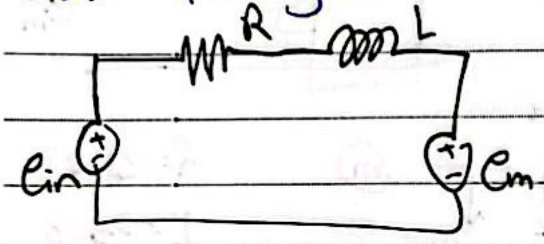


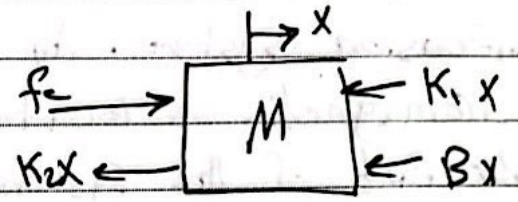
We Unify the system in electrical domain

Modeling and Simulations :-

Modeling Process :-

*Ex :- Plunger





$$-e_{in} + iR + L\frac{di}{dt} + 2\pi N a V B = 0.$$

Using force Voltag analogous

$$-e_{in} + iR + L\dot{I} + 2\pi N a B\ V = 0.$$

$$-F_{in} + b_1 V + m\dot{V} + b_2 V = 0.$$

$$m\dot{V} + b_1 V + b_2 V = f_{in}.$$

$$m\ddot{x} + b_1\dot{x} + b_2\dot{x} = f_{in}$$

Conversion table

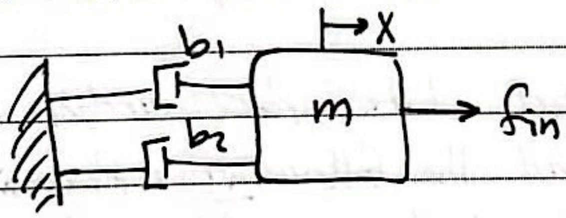
$F \to e$

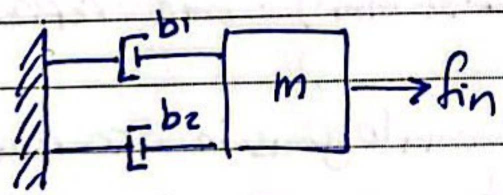
$V \to I$

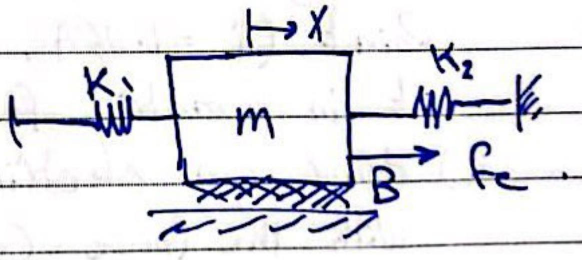
$m \to L$

$K \to \frac{1}{c}$

$b \to R$



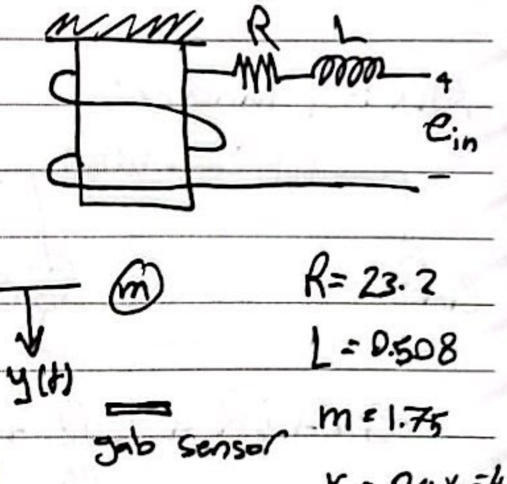




We Unify the System in mechanical domain

Modeling and Simulations :-

Modeling Process :-



$R = 23.2$
$L = 0.508$
$m = 1.75$
$K = 2.4 \times 10^{-4}$

* Ex :- Consider the electromagnetic suspension system. An electromagnetic is located at the upper part of the system. Using the electromagnatic force $f_m = K \frac{I^2}{y^2}$ We want to suspend the steel ball. A gab sensor is Used to measure the distance y(t). Assume that the state Variable are selected as $X_1 = y$, $X_2 = \dot{y}$, $X_3 = I$, also assume Input e_{in}, Output $y(t)$

a) Derive the non linear state equations for the system.

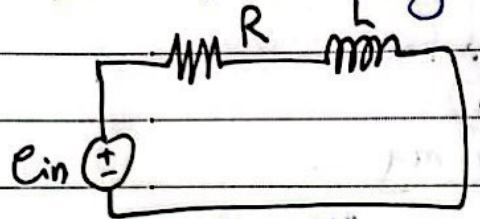
b). Find the linearized state space model of the system around the following operating Point ($i = 1.06A$, $y = 4.36\,mm$). and represent it in a matrix form.

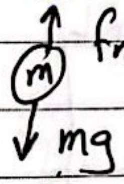
c) draw an electrical analogous in accordance With the force - Current

d) draw a mechanical analogous in accordance With the force - Voltage

Modeling and Simulations :-
Modeling Process :-

*Ex:- electromagnatic suspension system.





$f_m = \frac{K I^2}{y^2}$.

$-e_{in} + V_R + V_L = 0$

$-e_{in} + iR + L\frac{di}{dt} = 0$

$\Sigma F = ma$.

$mg - f_m = m\ddot{y}$

$mg - \frac{K I^2}{y^2} = m\ddot{y}$

$X_1 = y \rightarrow \dot{X}_1 = \dot{y} = X_2$

$X_2 = \dot{y} \rightarrow \dot{X}_2 = \ddot{y} = g - \frac{K X_3^2}{m X_1^2}$

~~$X_3 = I$~~

$X_3 = I \rightarrow \dot{X}_3 = \frac{di}{dt} = \frac{1}{L}(e_{in} - R X_3)$.

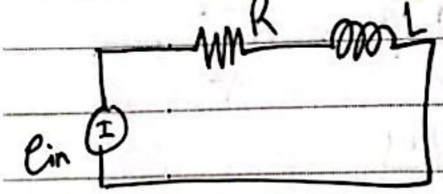
$$A = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \frac{\partial f_1}{\partial x_3} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \frac{\partial f_2}{\partial x_3} \\ \frac{\partial f_3}{\partial x_1} & \frac{\partial f_3}{\partial x_2} & \frac{\partial f_3}{\partial x_3} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ \frac{2K}{m}\frac{X_3^2}{X_1^3} & 0 & \frac{-2K}{m}\frac{X_3}{X_1^2} \\ 0 & 0 & \frac{-R}{L} \end{bmatrix}$$

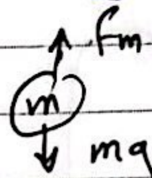
$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{0.027K}{m} & 0 & \frac{-0.111K}{m} \\ 0 & 0 & \frac{-R}{L} \end{bmatrix} , \quad B = \begin{bmatrix} \frac{\partial f_1}{\partial u} \\ \frac{\partial f_2}{\partial u} \\ \frac{\partial f_3}{\partial u} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}$$

Modeling and Simulations :-

Modeling Process :-

*Ex:- Electromagnatic suspension system :-





$$-e_{in} + iR + L\frac{di}{dt} = 0 \qquad mg - \frac{KI^2}{y^2} = m\ddot{y}$$

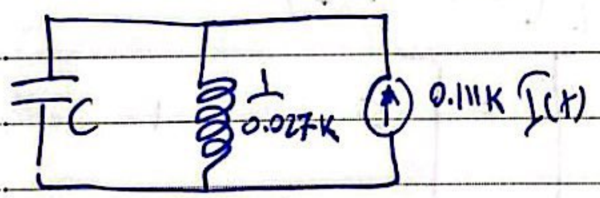
Using force Current. $\quad m\ddot{y} = 0.027Ky - 0.111KI$

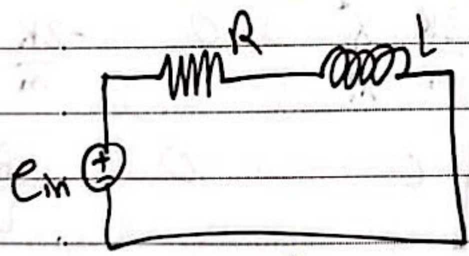
$$m\ddot{y} - 0.027K\,y + 0.111K\,I = 0.$$

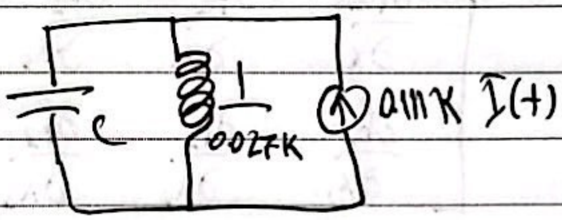
$$m\dot{V} - 0.027K\int V\,dt + 0.111K\,I = 0$$

$$C\frac{de}{dt} - \underbrace{0.027K}\int e\,dt + \underbrace{0.111K}_{A}\,\cancel{I}\,I = 0.$$

Conversion table
- $F \to I$
- $V \to e$
- $m \to C$
- $K \to \frac{1}{L}$
- $b \to \frac{1}{R}$.





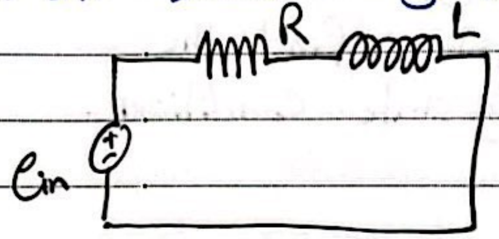


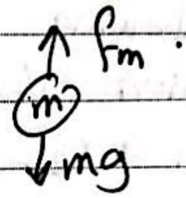
We unify the System in electrical domain

Modeling and Simulations :-

Modeling Process :-

* Ex :- Electromagnatic suspension system





$$-e_{in} + iR + L\frac{di}{dt} = 0$$

$$m\ddot{y} = 0.027Ky - 0.111KI$$

Using force - Voltage

Conversion table

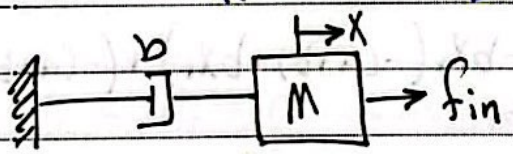
$$-e_{in} + iR + L\dot{i} = 0$$

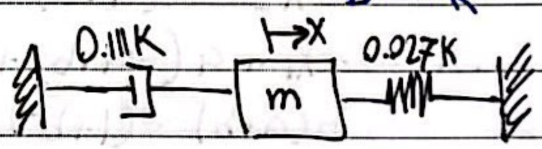
$$-F_{in} + bV + m\dot{V} = 0$$

$$m\ddot{y} - 0.027Ky + 0.111KI = 0$$

$$m\ddot{y} - \underbrace{0.027K}_{K}y + \underbrace{0.111K}_{b}V = 0$$

$F \rightarrow e$	
$V \rightarrow I$	
$m \rightarrow L$	
$K \rightarrow \frac{1}{C}$	
$b \rightarrow R$	





We Unify the System in mechanical domain

Modeling and Simulations :-
Modeling Process :-

* linearization of non linear system :-

for the following non linear system find the linearization Version at steady state Condition.

$\dot{X}_1 = -X_1 + aX_2 - bX_1X_2 + X_2^2$.

$\dot{X}_2 = -(a+b)X_1 + bX_1^2 - X_1X_2$

$\dot{X}_2 = X_1(-(a+b) + bX_1 - X_2) = 0$

$\Rightarrow \boxed{X_1 = 0}$ or $-(a+b) + bX_1 - X_2 = 0$.

$\dot{X}_1 = -X_1 + aX_2 - bX_1X_2 + X_2^2 = 0$

$X_1 = 0 \rightarrow X_2(X_2 + a) = 0$

$\Rightarrow \boxed{X_2 = 0}$, $\boxed{X_2 = -a}$

$\Rightarrow (0,0)$, $(0,-a)$.

$X_1(-(a+b) + bX_1 - X_2) = 0$.

$\rightarrow X_2 = -(a+b) + bX_1$

$0 = -X_1 + a(-(a+b) + bX_1) - bX_1(-(a+b) + bX_1) + (-(a+b) + bX_1)^2$

$b(a+b) - (1+b^2)X_1 = 0$.

$X_1 = \dfrac{b(a+b)}{(1+b^2)}$, $X_2 = \dfrac{(a+b)}{(1+b^2)}$

$$A = \begin{bmatrix} \dfrac{\partial f_1}{\partial x_1} & \dfrac{\partial f_1}{\partial x_2} \\ \dfrac{\partial f_2}{\partial x_1} & \dfrac{\partial f_2}{\partial x_2} \end{bmatrix} = \begin{bmatrix} -1 - bX_2 & a - bX_1 + 2X_2 \\ -(a+b) + 2bX_1 - X_2 & -X_1 \end{bmatrix}$$

$$A\Big|_{(0,0)} = \begin{bmatrix} -1 & a \\ -(a+b) & 0 \end{bmatrix} , \quad A\Big|_{(0,-a)} = \begin{bmatrix} -1+ab & -a \\ -b & 0 \end{bmatrix}$$

Modeling and Simulations:-
Modeling Process:-

*Thermal Systems :-
- System Variable:-
Through Variable:- heat flow $q(t)$.
Across Variable:- Temprature $T(t)$.
- Ideal segments:-
1. Thermal Resistance (R) . 2- Thermal Capacitance (C)
Constitutive equation
$$T(t) = R\, q(t)$$
$$q(t) = C \frac{dT(t)}{dt}$$

- Heat Transfer equation:-
$$\Sigma Q = \rho V c \frac{dT}{dt}.$$
$\rho V c = m c \equiv C \Rightarrow$ Thermal Capacitance.

1. Conduction:-
The net rate of transfer by conduction across the boundaries of a unit Volume is equal to the rate of heat accumulation within the unit Volume.
$$Q = -KA \frac{\partial T}{\partial t} = \frac{KA}{L} (T_2 - T_1)$$

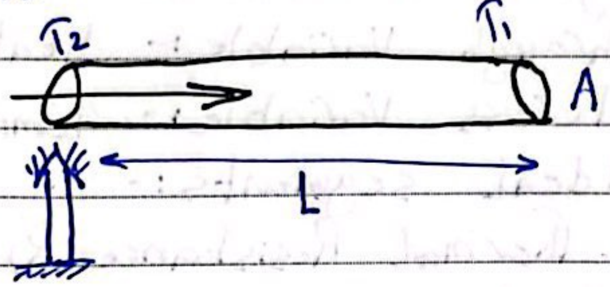
Modeling and Simulations :-

Modeling Process :-

* Thermal Systems :-

- Heat Transfer equations :-

1- Conduction



$$Q = -KA \frac{dT}{dx}$$

$$Q = \frac{KA}{L} (T_2 - T_1).$$

if $R_T = \frac{L}{KA} \Rightarrow Q = \frac{1}{R_T} (T_2 - T_1)$

R_T = Thermal Resistance.

2- Convection :-

Convection heat transfer is associated with the transfer of mass in a boundary layer of a fluid over a fixed wall

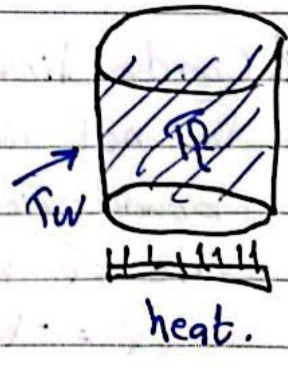
$$Q = h_c A (T_w - T_f).$$

Where :- A :- Cross section Area :

h_c :- Convection heat transfer

T_w : Wall Temprature.

T_f :- Fluid Temprature.



Modeling and Simulations :-
Modeling process :-

* Thermal Systems :-
- Heat Transfer equations :-
3. Radiation :-
The rate of heat transfer by radiation between two separated bodies having temprature T_1 and T_2 is determined by the stefan-Boltzmann Law.
$$Q = \sigma F_A F_E (T_1^4 - T_2^4).$$
σ :- stefan-boltzman Constant $\sigma = 5.667 * 10^{-8}$
F_A :- shape factor Constant
F_E : effective emissivity constant.

* Conservation of energy :-
$$\begin{pmatrix}\text{rate of}\\ \text{energy}\\ \text{stored}\\ \text{within}\\ \text{system}\end{pmatrix} = \begin{pmatrix}\text{heat}\\ \text{flow}\\ \text{rate}\\ \text{into}\\ \text{system}\end{pmatrix} + \begin{pmatrix}\text{rate of}\\ \text{heat}\\ \text{generated}\\ \text{within}\\ \text{system}\end{pmatrix} + \begin{pmatrix}\text{rate of}\\ \text{work}\\ \text{done}\\ \text{on}\\ \text{system}\end{pmatrix} - \begin{pmatrix}\text{heat}\\ \text{flow}\\ \text{rate}\\ \text{out of}\\ \text{system}\end{pmatrix}$$

$$\boxed{\underbrace{\dot{E}_{in} - \dot{E}_{out}}_{} = \frac{\partial E}{\partial t}_{system}}$$

$\dot{E}_{in}$:- the net of rate of energy of heat, work mass.

$\frac{\partial E}{\partial t}$:- rate of energy Internal Kinetic, Potential total fluid energy.

Modeling and Simulations :-
Modeling Process :-

* Conservation of energy :-

$$\overset{*}{E}_{in} - \overset{*}{E}_{out} = \frac{\partial E}{\partial t}_{system}$$

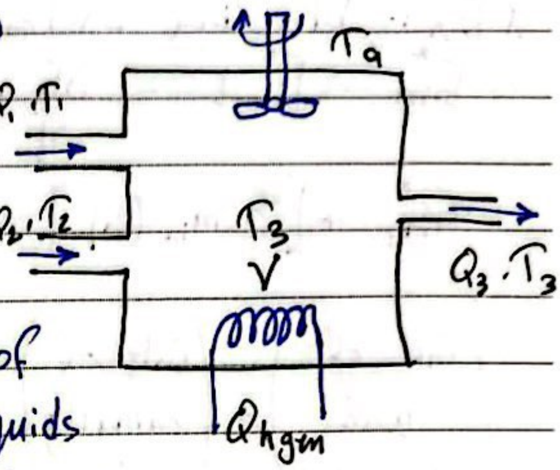
$$\left[\overset{*}{Q}_{in} + \overset{*}{W}_{in} + \Sigma \overset{*}{m}_{in}\left(h + \frac{V^2}{2} + gZ\right)\right] - \left[\overset{*}{Q}_{out} + \overset{*}{W}_{out} + \Sigma \overset{*}{m}_{out}\left(h + \frac{V^2}{2} + gZ\right)\right] = \frac{\partial E}{\partial t}_{system}$$

$\overset{*}{Q}_{in}$:- heat flow rate into system
$\overset{*}{Q}_{out}$:- heat flow rate out of system
$\overset{*}{W}_{in}$:- rate of work done on system
$\overset{*}{W}_{out}$:- rate of work ~~out~~ done out of system
$\Sigma m^*_{in}\left(h + \frac{V^2}{2} + gZ\right)$:- rate of heat generated within system
$\Delta h = C_P \Delta T \Rightarrow \Delta h = C_P (T_f - T_{ref})$,
h :- enthalbe , C_P :- Specific heat.
$V^2/2$: Kinetic energy.
gZ :- Potential energy.
$m^* = \rho * Q$ = density * flow rate.
m^* :- mass Changing not constant

*Enthalbe :- total energy

Modeling and Simulations :-

Modeling Process :-



*Ex: Consider the blending system shown. Two identical liquids of different tempretures T_1 and T_2 flowing with different flow rate Q_1 and Q_2 are perfectly mixed in a blender of Volume V. The mixture of liquids is also heated in the blender by an electrical heater supplying heat at a constant rate (Q_{hgen}). there are heat losses in the system and the Cofficient of heat transfer between the blender and the ambient air of temperature T_a is h_c. Altough the mixing in the tank is assumed to be perfect. the Work done by the system (mixer) is negligible and the Kinetic energies of the flows Q_1, Q_2 Q_3 are very small. Derive a mathematical model of the blending process.

$$\dot{E}_{in} - \dot{E}_{out} = \frac{\partial E}{\partial t}_{system}$$

$$\left[\dot{Q}_{in} + \cancel{\dot{W}_{in}} + \sum \dot{m}_{in}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)\right] - \left[\dot{Q}_{out} + \cancel{\dot{W}_{out}} + \right.$$

$$\left.\sum \dot{m}_{out}\left(h + \cancel{\frac{V^2}{2}} + \cancel{gz}\right)\right] = \frac{\partial E}{\partial t}_{system} + Q_{loss}$$

Modeling and Simulations :-

Modeling Process :-

* Ex :: blending system.

$$E^*_{in} - E^*_{out} = \frac{\partial E}{\partial t}_{system} + E_{loss}$$

$$Q^*_{in_1} + \left[\Sigma m_{in} (h)\right] = \left[\Sigma m_{out}(h)\right] + Q_{loss} + \frac{\partial E}{\partial t}_{system}$$

$$\begin{pmatrix}\text{rate of}\\ \text{heat}\\ \text{generated}\end{pmatrix} + \begin{pmatrix}\text{rate of}\\ \text{enthalpy}\\ Q_1\end{pmatrix} + \begin{pmatrix}\text{rate of}\\ \text{enthalpy}\\ Q_2\end{pmatrix} = \begin{pmatrix}\text{rate of}\\ \text{enthalpy}\\ Q_3\end{pmatrix} + \begin{pmatrix}\text{rate of}\\ \text{heat}\\ \text{loss}\end{pmatrix} + \begin{pmatrix}\text{rate of}\\ \text{change}\\ \text{of energy}\end{pmatrix}$$

rate of heat generated $= Q_{hgen}$.

rate of enthalpy of $Q_1 = Q_{h1} = \rho C_p Q_1 (T_1 - T_a)$.

rate of enthalpy of $Q_2 = Q_{h2} = \rho C_p Q_2 (T_2 - T_a)$.

rate of enthalpy of $Q_3 = Q_{h3} = \rho C_p Q_3 (T_3 - T_a)$.

but $Q_3 = Q_1 + Q_2 \Rightarrow Q_{h3} = \rho C_p (Q_1 + Q_2)(T_3 - T_a)$.

rate of heat loss $= Q_{hloss} = h_c A (T_3 - T_a)$.

rate of change of energy of system $= \frac{\partial E}{\partial t} = \rho C_p V \frac{dT_3}{dt}$

Conservation of energy become :-

$$Q_{hgen} + Q_{h1} + Q_{h2} = Q_{h3} + Q_{hloss} + \frac{\partial E}{\partial t}_{system}.$$

$$Q_{hgen} + \rho C_p Q_1 (T_1 - T_a) + \rho C_p Q_2 (T_2 - T_a) = h_c A (T_3 - T_a) + \rho C_p (Q_1 + Q_2)(T_3 - T_a) + \rho C_p V \frac{dT_3}{dt}.$$

Modeling and Simulations :-

Modeling Process :-

*Ex:- blending System :-

$$\frac{dT_3}{dt} = \frac{1}{\rho C_p V}\left(Q_{hgen} + \rho C_p Q_1 (T_1 - T_a) + \rho C_p Q_2 (T_2 - T_a) \right)$$

$$- \frac{1}{\rho C_p V}\left(\rho C_p \cdot (Q_1 + Q_2)(T_3 - T_a) - h_c A (T_3 - T_a) \right)$$

$$\frac{dT_3}{dt} = \frac{-1}{V}\left(\frac{h_c A}{\rho C_p} + Q_1 + Q_2 \right) T_3 + \frac{Q_1}{V} T_1 + \frac{Q_2}{V} T_2$$

$$+ \frac{1}{\rho C_p V} Q_{hgen} + \frac{h_c A}{\rho C_p V} T_a .$$

if the $Q_{hgen} = 0$, $h_c = 0$ then

$$\frac{dT_3}{dt} = \frac{-1}{C}\left(\frac{1}{R_1} + \frac{1}{R_2} \right) T_3 + \left(\frac{1}{R_1 C} \right) T_1 + \left(\frac{1}{R_2 C} \right) T_2$$

where $C = \rho C_p V$.

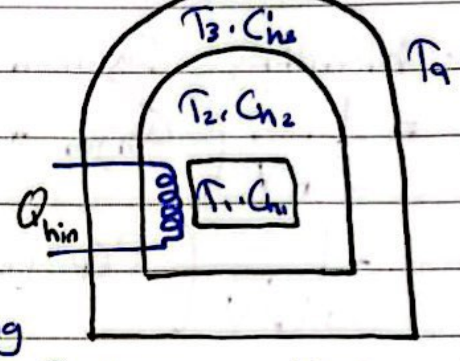
$$R_1 = \frac{1}{\rho C_p Q_1}$$

$$R_2 = \frac{1}{\rho C_p Q_2}$$

and time Constant (τ)

$$\tau = \frac{\rho C_p V}{h_c A + \rho C_p Q_3}$$

Modeling and Simulations :-
Modeling Process :-



* Problem 8.8 :- in figure shown a simple model of an industrial furnace. a packing of temprature T_1 is being heated in the furnace by an electrical heater supplying heat at the rate $Q_{hi}(t)$. the temperature inside the furnace is T_2, the walls are at temperature T_3. and the ambient temprature is T_a. the thermal capacitances of the packing. the air inside the furnace and the furnace walls are C_{h_1}, C_{h_2} and C_{h_3} respectively. Derive state variable equations for this system assuming that heat is transferred by convection only. with the convective heat transfer coefficients h_{c_1} (air - packing), h_{c_2} (air - inside walls) and h_{c_3} (outside walls - ambient air).

1. air - packing $\Rightarrow$ packing $\rightarrow$ air around packing. $(T_1 \rightarrow T_2)$

$$E^*_{in} - \cancel{E^*_{out}} = \frac{\partial E}{\partial t} \text{ system.}$$

$$Q_1 = \frac{\partial E_1}{\partial t} \Rightarrow h_{c_1} A_1 (T_2 - T_1) = m_1 C_1 \frac{dT_1}{dt}.$$

2. air - inside walls $\Rightarrow$ air around packing $\rightarrow$ walls in $(T_2 \rightarrow T_3)$

$$E^*_{in} - E^*_{out} = \frac{\partial E}{\partial t} \text{ system} \Rightarrow Q_{hin} - Q_1 - Q_2 = \frac{\partial E_2}{dt}.$$

$$Q_{hin} - h_{c_1} A_1 (T_2 - T_1) - h_{c_2} A_2 (T_2 - T_3) = m_2 C_2 \frac{dT_2}{dt}.$$

Modeling and Simulations :-
Modeling PROCESS :-

* Problem 8.8 :- Industrial furnace.

3- Outside Walls - ambient air $\Rightarrow$ Wall in - Wall out $(T_a \to T_3)$

$$\dot{E}_{in} - \dot{E}_{out} = \frac{\partial E}{\partial t}\text{ system} \Rightarrow Q_2 - Q_3 = \frac{\partial E_3}{\partial t}.$$

$$h_{c2} A_2 (T_2 - T_3) - h_{c3} A_3 (T_3 - T_a) = m_3 C_3 \frac{dT_3}{dt}.$$

state Variable.

$X_1 = T_1$, $X_2 = T_2$, $X_3 = T_3$, $y = Q_{hin} = Q_1 - Q_2$.

$$\dot{X}_1 = \frac{dT_1}{dt} = \frac{h_{c1} A_1}{m_1 C_1}(T_2 - T_1) = \frac{h_{c1} A_1}{m_1 C_1}(X_2 - X_1).$$

$$\dot{X}_2 = \frac{dT_2}{dt} = \frac{1}{m_2 C_2} Q_{hin} - \frac{h_{c2} A_2}{m_2 C_2}(T_2 - T_3) - \frac{h_{c1} A_1}{m_1 C_1}(T_2 - T_1)$$

$$\dot{X}_3 = \frac{dT_3}{dt} = \frac{h_{c2} A_2}{m_3 C_3}(T_2 - T_3) - \frac{h_{c3} A_3}{m_3 C_3}(T_3 - T_a).$$

$\dot{X} = AX + Bu.$

$$\begin{bmatrix} \dot{X}_1 \\ \dot{X}_2 \\ \dot{X}_3 \end{bmatrix} = \begin{bmatrix} \frac{-h_{c1} A_1}{m_1 C_1} & \frac{h_{c1} A_1}{m_1 C_1} & 0 \\ \frac{h_{c1} A_1}{m_1 C_1} & \left(\frac{-h_{c2} A_2}{m_2 C_2} - \frac{h_{c1} A_1}{m_1 C_1}\right) & \frac{h_{c2} A_2}{m_2 C_2} \\ 0 & \frac{h_{c2} A_2}{m_3 C_3} & \left(-\frac{h_{c2} A_2}{m_3 C_3} - \frac{h_{c3} A_3}{m_3 C_3}\right) \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \end{bmatrix}$$

$$+ \begin{bmatrix} 0 \\ \frac{1}{m_2 C_2} \\ 0 \end{bmatrix} Q_{hin}.$$

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