تقدم لجنة ElCoM الاكاديمية

دفتر لمادة:

نمذجة ومحاكاة

جزيل الشكر للطالبة:

سارة ابو سارة

No. Modelling

Ch.1 8 Introduction

Modeling Engineering System Dynamics →

The modeling process of engineering system dynamics starts by identifying the fundamental properties of an actual system.

The minimum set of variables necessary to fully define the system configuration is formed of the degrees of freedom (DOF).

نستخدم

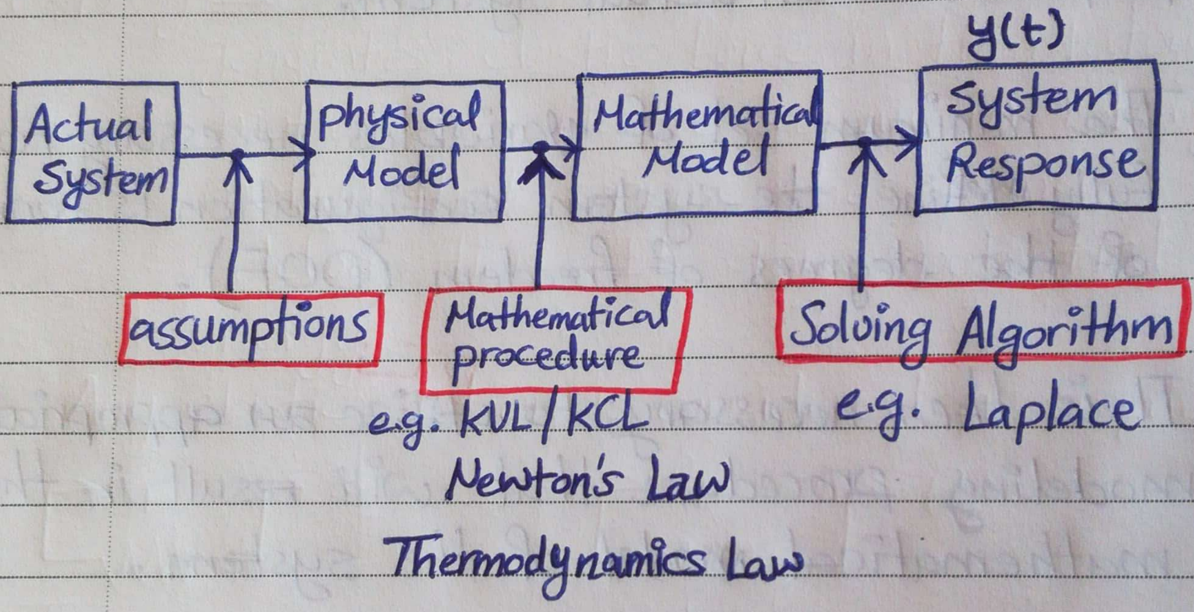
It is then necessary to utilize an appropriate modeling procedure that will result in the mathematical model of the system.

Generally ..

A mathematical model describing the dynamic behaviour of an engineering system consists of a differential equation combining parameters with known/unknown functions and derivatives.

Then, Solving the mathematical model through adequate mathematical procedures that deliver the solution.

∴ Mathematical Model → Equations that describes the relationship between the inputs and the outputs of the system.

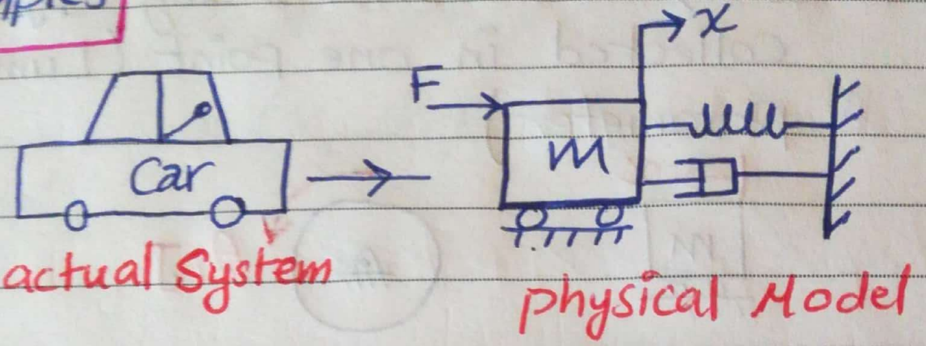


the Fig. above shows the flow in a process connecting an actual dynamic system to its response..

$\boxed{m}$ زي السيارة مثلاً نقدر أفترضها كأنها هي أنا
mass عشان أقدر أدرس حركتها ..

Examples

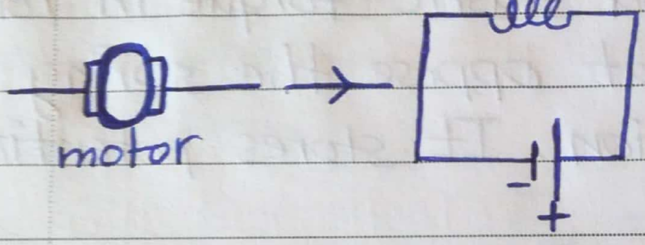
1.



$$\Sigma F = m\ddot{x} \rightarrow \begin{array}{l} x(t) = \\ \dot{x}(t) = \\ \ddot{x}(t) = \end{array}$$

2.

ملفات مطبق عليها جهد من مصدر



$$\text{KVL / KCL} \rightarrow \begin{array}{l} I(t) = \\ V(t) = \end{array}$$

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* Types of Systems ⇒

1 Mechanical System

Basic Elements : a. mass b. spring c. damper

Input : a. Force or b. Torque

Output : a. Displacement b. velocity c. acceleration

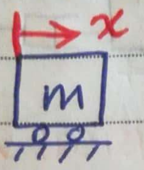
Types of Motion : a. Translational
- → mass (m)
- → Spring K
- → damper b

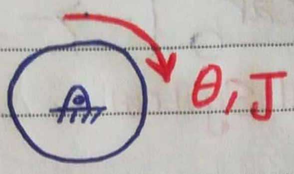
b. Rotational
- → Inertia (J)
- → Rotational spring
- → Rotational damper

No.

Ch.2 : Mechanical System I

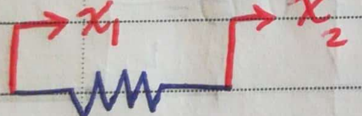
mass → all the mass of any object is Collected in one point (Lumped Parameter)



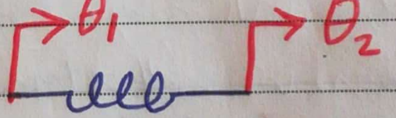


Spring → Mechanical element "elastic" that generates elastic force in translatory motion and elastic torque in rotary motion that oppose the spring deformation. It stores potential energy.

Any elastic action is considered as aspring.

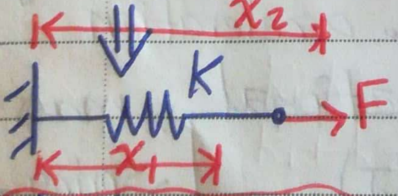


Trans. Spring



Rot. Spring

بيخزّن طاقة وضع حيث عندما
يتم افلاته من القوة التي تعرّض إليها
يعود لوضعه الطبيعي



$$\boxed{\Delta x = x_2 - x_1}$$

$$F_s = K \cdot \Delta x = k(x_2 - x_1)$$

"Spring Force"

For Rot. Spring $F_s = k . \Delta\theta = k(\theta_2 - \theta_1)$

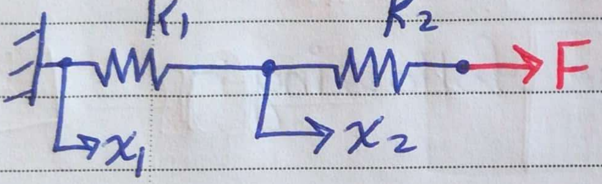
$$E_p = \frac{1}{2} k (x_2 - x_1)^2 = \frac{1}{2} k (\theta_2 - \theta_1)^2$$

"Potential Energy"

* Combination of Springs :-

$\boxed{F = kx}$

a. Series →



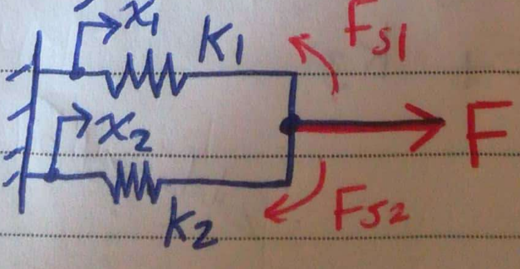
$$F_{s1} = F_{s2}$$

$$x_{eq} = x_1 + x_2$$

$$\frac{F}{k_{eq}} = \frac{F_{s1}}{k_1} + \frac{F_{s2}}{K_2} \Rightarrow \boxed{K_{eq} = \frac{k_1 k_2}{k_1 + k_2}}$$

القوة اللي بتأثر على ال K_2 هي نفسها المؤثرة ع النابض K1
فيكون مجموعهم هو ال total deflection

b. Parallel →



No.

$$x_{eq} = x_1 = x_2$$

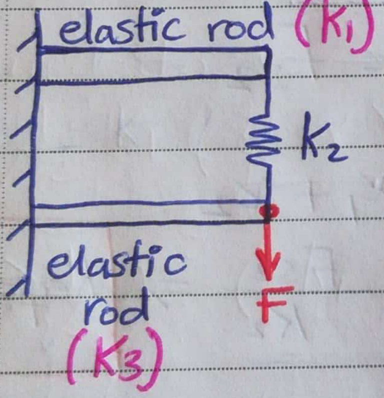
$$F = F_{S1} + F_{S2}$$

$$K_{eq}\, x_{eq} = k_1 x_1 + k_2 x_2$$

$$\boxed{K_{eq} = k_1 + k_2}$$

Ex: Find K_{eq} for the following :-

1



بعبّر عن ال Elastic rod
ب spring كونه مُصمّم
ليتمدد لفعل قوة خارجية ثم يعود
لوضعه الطبيعي بعد زوال تأثير
القوة وهذا يشبه النابض ..

K_1 in series with $K_2 \rightarrow K_{12} = \dfrac{K_1 K_2}{K_1 + K_2}$

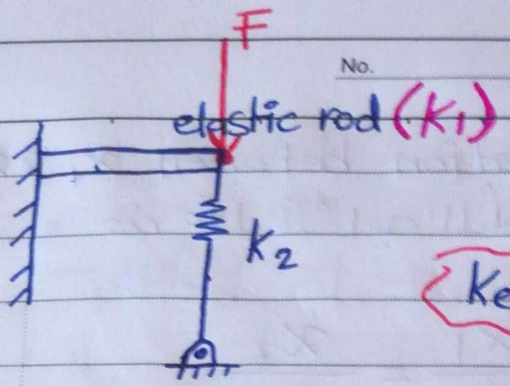
K_{12} in parallel with $K_3 \rightarrow \boxed{K_{eq} = K_{12} + K_3}$

$$K_{eq} = \frac{K_1 K_2}{K_1 + K_2} + K_3$$

توحيد مقامات

⋮

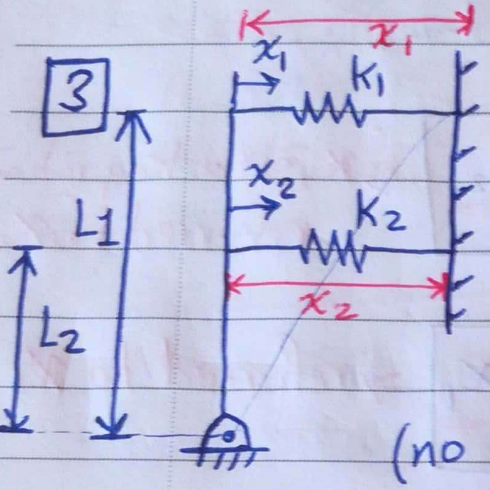
2



k_1 in parallel with k_2

$$\boxed{k_{eq} = k_1 + k_2}$$

3



الفكرة: طول ما في فورس خارجية مؤثرة عنقطة معينة بالنظام هنا منقدر نحدد الـ (k_{eq}) الآن من خلال قانون الـ [طاقة وضع/Potential Energy]

كونه ماني حركة للنابض (no Kinetic energy)

$$E_p = \frac{1}{2} k_1 (x_1)^2 + \frac{1}{2} k_2 (x_2)^2 = \frac{1}{2} k_{eq} x_{1,2}^2$$

ملاحظة: هذا النظام عبارة عن 1DOF كونه المتغيرات $(x_2$ & $x_1)$ بيعتمدوا ببعض

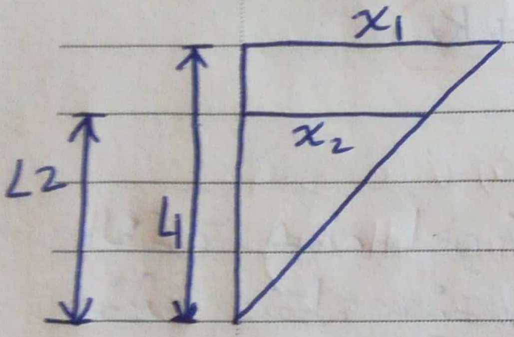
DOF ≡ No. of independent variables "motions"

$(x_2$ & $x_1)$ بيعتمدوا ببعض، يعني في علاقة بينهم فنحط وحدة بدلالة الأخرى ونعوضها بعلاقة الـ E_p لإيجاد الـ k_{eq} بدلالة x_1 أو بدلالة x_2 حسب السؤال شو بيحدد...

No.

→ to find the relation between x_1 & x_2 :

من خلال تشابه المثلثات



$$\frac{x_1}{L_1} = \frac{x_2}{L_2}$$

$$\boxed{x_1 = \frac{L_1}{L_2} x_2} \rightarrow$$ x_2 بتطلع Keq بدلالة

referred to x_2

or

$$\boxed{x_2 = \frac{L_2}{L_1} x_1} \rightarrow$$ referred to x_1

* Referred to x_2 →

$$E_p \Rightarrow \frac{1}{2} k_1 \left(\frac{L_1}{L_2} x_2\right)^2 + \frac{1}{2} k_2 x_2^2 = \frac{1}{2} k_{eq} x_2^2$$

$$\Rightarrow \cancel{\frac{1}{2}} \cancel{x_2^2} \left(k_1 \left(\frac{L_1}{L_2}\right)^2 + k_2 \right) = \cancel{\frac{1}{2}} k_{eq} \cancel{x_2^2}$$

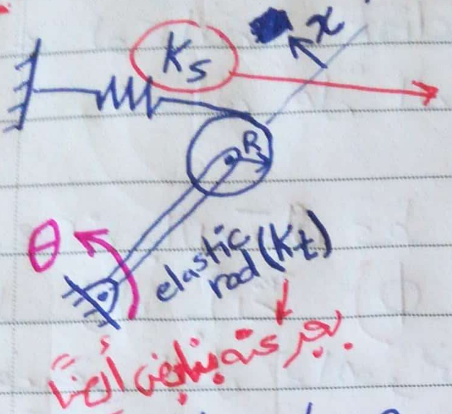
$$\therefore \boxed{k_{eq} = k_1 \left(\frac{L_1}{L_2}\right)^2 + k_2}$$

or

referred to x_1 →

$$\boxed{k_{eq} = k_1 + \left(\frac{L_2}{L_1}\right)^2 k_2}$$

Ex : Find k_{eq} for the below system :-



بيتحرك (linearly)
بالترابط مع حركة θ
اللي بتخلي ال disk
يضغط ويضغط معه النابض

Here, there's 2 variables $\rightarrow x$
$\rightarrow \theta$

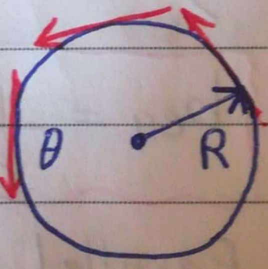
لكنهم بيعتمدوا ع بعض فإذن ⇐ 1DOF
منوجد العلاقة بين x و θ ونعوضها بال Ep
كونه ما في قوة خارجية مؤثرة ع النظام ...

$$E_p = \frac{1}{2} k_t \theta^2 + \frac{1}{2} k_s x^2$$

referred to θ :

$$E_p \Rightarrow \frac{1}{2} k_t \theta^2 + \frac{1}{2} k_s (R^2 \theta^2)$$
$$= \frac{1}{2} k_{eq} \theta^2$$

* For circular motion:



$V_{linear} = R\dot{\theta}$
$a = R\ddot{\theta}$
so $\Rightarrow \boxed{x = R\theta}$

$$\cancel{\frac{1}{2}} \cancel{\theta^2} (k_t + R^2 k_s) = \cancel{\frac{1}{2}} k_{eq} \cancel{\theta^2}$$

$$\therefore \boxed{k_{eq} = k_t + R^2 k_s}$$

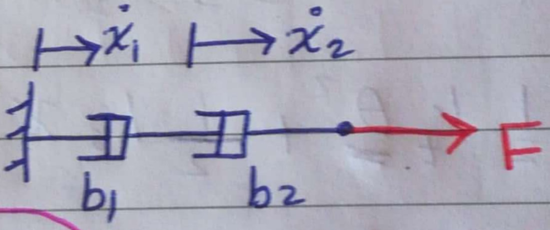
مهم ← لو كانت الـ disk تتحرك بـ ϕ مختلفة عن θ
هيختلف حل السؤال

ويمكن بالامتحان تحط θ و ϕ لكن يذكر في صيغة
السؤال بأنّ (θ the same of the ϕ)
فيبقى الحل كما هو في هذه الحالة ...

Damping element → dissipates energy in the form of heat losses .

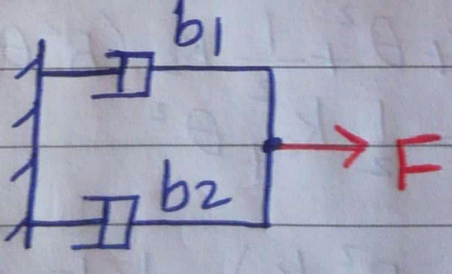
Combination :

a.) Series



$$\boxed{b_{eq} = \frac{b_1 b_2}{b_1 + b_2}}$$

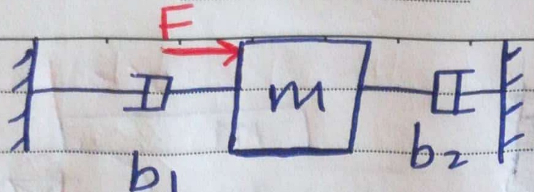
b.) Parallel



$$\boxed{b_{eq} = b_1 + b_2}$$

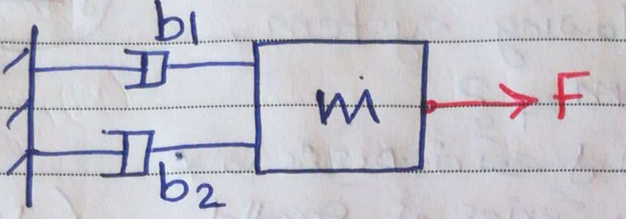
No.

Ex

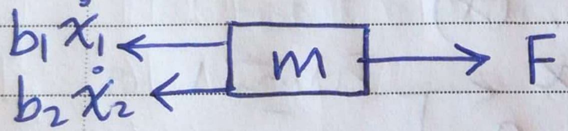


Parallel

≡



لو شكّينا بالموضوع بإمكاننا نرسم FBD ونحدّد اتجاه حركة ال (damping force)



الفورس توزعت ع ال 2dampers

$$\therefore \quad b_{eq} = b_1 + b_2$$

\# Mass / Inertia → resistance to acceleration, it stores kinetic energy.

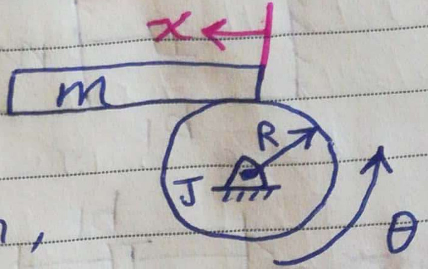
$$E_k = \frac{1}{2} m\dot{x}^2 = \frac{1}{2} J\dot{\theta}^2$$

ملاحظة :- بال spring وال damper لحتى نوجد Keq و beq لمّا يستصعب علينا تحديد نوع التوصيل بينهم اذا series أو parallel كنا نلجأ لقانون ال ,,Potential Energy,, Ep

لكن في حساب Meq ، Jeq سنلجأ لقانون ,,Kinetic Energy,, E_k

حيث تكون بدلالة متغير واحد فقط بالنهاية..

Ex)



For the following system, find meq ?

* ما في هون توصيل لل mass إذا series أو parallel ...

(بيشبه مبدأ عمل Rack & pinion)

ال (disk) بعمل rotation

بتسبب حركة ال rod بشكل خطي بمقدار (x) ∴

∴ المتغيرات x ← θ ←

بيعتمدوا ع بعض ضمن علاقة

$$\begin{pmatrix} x = R\theta \\ \dot{x} = R\dot{\theta} \\ \ddot{x} = R\ddot{\theta} \end{pmatrix}$$

∴ So the system is **1DOF**

cuz there are 2 variables but dependent with each other ..

* to find meq :-

$$E_k = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J \dot{\theta}^2$$

referred to $\dot{x}$: $E_k = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} J \frac{\dot{x}^2}{R^2}$

$$= \frac{1}{2} m_{eq} \dot{x}^2$$

$$\cancel{\frac{1}{2}}\cancel{\dot{x}^2}\left(m+\frac{J}{R^2}\right)=\cancel{\frac{1}{2}}m_{eq}\cancel{\dot{x}^2}$$

$$\therefore \boxed{m_{eq}=m+\frac{J}{R^2}}$$

<u>or</u>

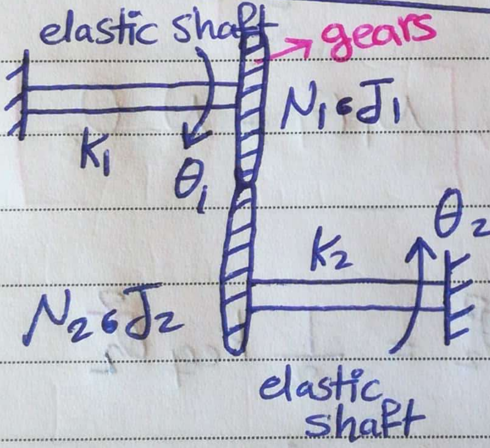
referred to $\dot{\theta}$: $\frac{1}{2}m(R\dot{\theta})^2+\frac{1}{2}J\dot{\theta}^2=\frac{1}{2}m_{eq}\dot{\theta}^2$

$$\cancel{\frac{1}{2}}\cancel{\dot{\theta}^2}\left(R^2m+J\right)=\cancel{\frac{1}{2}}m_{eq}\cancel{\dot{\theta}^2}$$

$$\therefore \boxed{m_{eq}=R^2m+J}$$

Ex)

[illegible]



we have 2 gears

each have: N, J

[N: no. of teeth

J: Inertia]

Find: ① k_{eq} ② J_{eq} ?

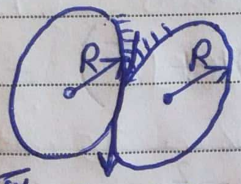
في متغيرين ← θ_1 (التفاف الـ shaft مع الـ gear الأول 1
(الموصول معه)

← θ_2 (التفاف الـ shaft الثاني مع الـ gear الثاني
(الموصول معه)

وبما إنه ال 2 gears موصولين ببعض مضاد ببعض
إنه النظام ← 1 DOF

والعلاقة بين θ_1 و θ_2 بتحدد من خلال ال Gear Ratio

* عند نقطة التقاء المسنّنات تكون سرعة ال gear الأول
متساوية مع سرعة ال gear الثاني..



⇒ Gears

$\boxed{v(\dot{x}) = R\dot{\theta}}$ ولأنه Circular motion فإنّ

For Given $\underline{\underline{R}}$ → $\left[R_1\dot{\theta}_1 = R_2\dot{\theta}_2 \right]$

But, for given $\underline{\underline{N}}$ → $\left[N_1\theta_1 = N_2\theta_2 \right]$
as in this example

$$E_p = \frac{1}{2}k_1\theta_1^2 + \frac{1}{2}k_2\theta_2^2 = \frac{1}{2}k_{eq}\theta_2^2$$

[redacted] $\frac{1}{2}k\left(\frac{N_2}{N_1}\right)^2\theta_2^2 + \frac{1}{2}k_2\theta_2$

$$= \frac{1}{2}k_{eq}\theta_2^2$$

$$\cancel{\frac{1}{2}}\cancel{\theta_2^2}\left(k_1\left(\frac{N_2}{N_1}\right)^2 + k_2\right) = \cancel{\frac{1}{2}}k_{eq}\cancel{\theta_2^2}$$

$$\therefore \left[k_{eq} = k_1\left(\frac{N_2}{N_1}\right)^2 + k_2 \right]$$

referred to θ_2 $\searrow$

$$\underline{\underline{2}} \quad E_k = \frac{1}{2}J_1\dot{\theta}_1^2 + \frac{1}{2}J_2\dot{\theta}_2^2 = \frac{1}{2}J_{eq}\dot{\theta}_2^2$$

$$\frac{1}{2}J_1\left(\frac{N_2}{N_1}\right)^2\dot{\theta}_2^2 + \frac{1}{2}J_2\dot{\theta}_2^2 = \frac{1}{2}J_{eq}\dot{\theta}_2^2$$

$$\cancel{\frac{1}{2}\dot{\theta}_2^2}\left(J_1\left(\frac{N_2}{N_1}\right)^2 + J_2\right) = \cancel{\frac{1}{2}}J_{eq}\cancel{\dot{\theta}_2^2}$$

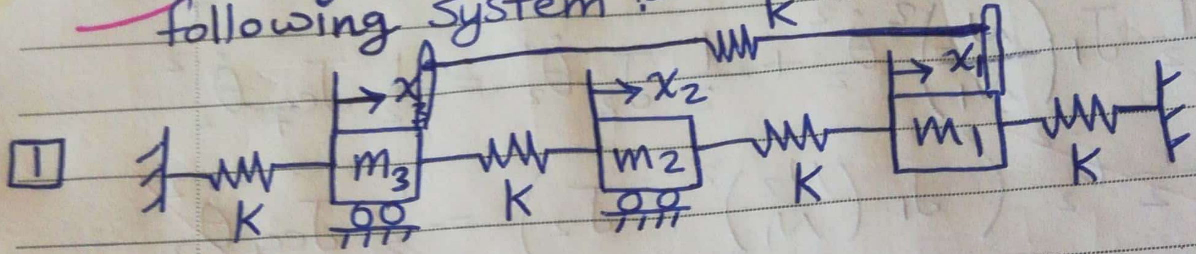
$$\therefore \boxed{J_{eq} = J_1\left(\frac{N_2}{N_1}\right)^2 + J_2}$$

Degree of Freedom (DOF) $\equiv$ The minimum number of independent coordinates that can specify the position of th system completely..

No. [redacted] Mechanical system mathematical model

Ex) Derive the equation of motion for the following system :-

1



* يوجد في هذا النظام ٣ متغيرات ← x_1 ، x_2 ، x_3

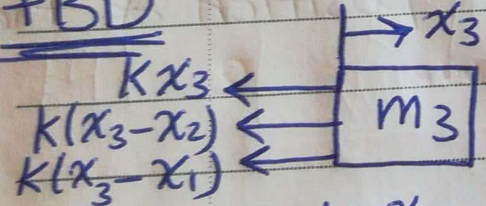
3DOF

لأنه لا يوجد علاقة تجمع بينهم

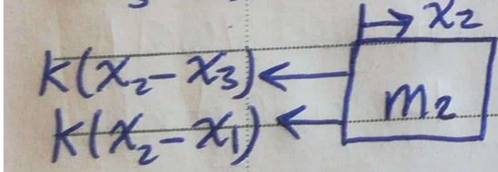
* طريقة تحليل النظام → Lagrange

→ FBD

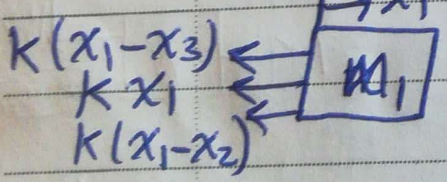
FBD



$$m_3\ddot{x}_3+kx_3+k(x_3-x_2)+k(x_3-x_1)=0$$



$$m_2\ddot{x}_2+k(x_2-x_3)+k(x_2-x_1)=0$$



$$m_1\ddot{x}_1+k(x_1-x_3)+k(x_1-x_2)+kx_1=0$$

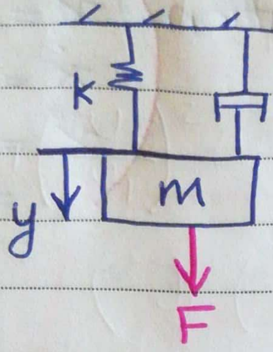
٣ معادلات بـ ٣ مجاهيل نستخدم الالاس

for given m، k

to find $x_1/x_2/x_3$

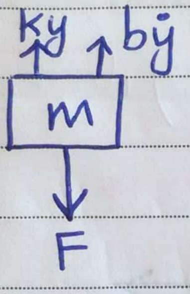
No.

2



(1DOF
المتغير y)

FBD :



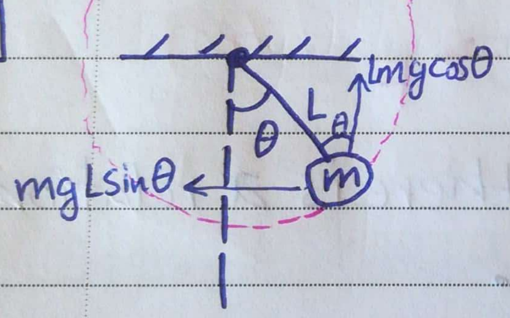
$\Sigma F_x = 0$

$\Sigma F_y = m\ddot{y} \rightarrow$

$m\ddot{y} + ky + b\dot{y} = F$

ما حطينا قوة الوزن لسبب ال Static Deformation *

3



If $J = mL^2$:

$\Sigma T = J\ddot{\theta}$

$-\cancel{m}g\cancel{L}\sin\theta = \cancel{m}L^{\cancel{2}}\ddot{\theta}$

$\frac{-g\sin\theta}{L} = \frac{\cancel{L}\ddot{\theta}}{\cancel{L}}$

$-\frac{g}{L}\sin\theta = \ddot{\theta} \rightarrow$ non-linear system

After Linearization :

$\sin\theta \approx \theta$

$-\frac{g}{L}\overset{\curvearrowright}{\theta} = \ddot{\theta}$

$\ddot{\theta} + \frac{g}{L}\theta = 0$

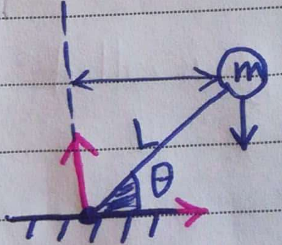
No.

Or by using the relation $\left(\begin{aligned} x &= R\theta \\ \dot{x} &= R\dot{\theta} \\ \ddot{x} &= R\ddot{\theta} \\ &= L\ddot{\theta} \end{aligned} \right)$

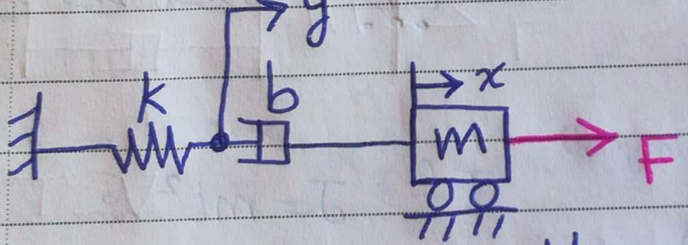
$$\Sigma T = J\ddot{\theta}$$

$$-mgL\sin\theta =$$

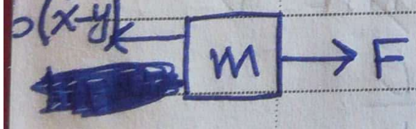
Inverted Pendulum →



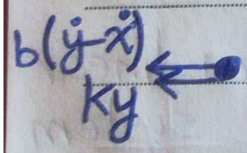
$\boxed{4}$



1DOF system because there is a relationship between $\underline{y}$ and $\underline{x}$ ...



$$\Rightarrow m\ddot{x} + b(\dot{x} - \dot{y}) = F \quad — ①$$



$$\Rightarrow -ky - b(\dot{y} - \dot{x}) = 0$$

$$b(\dot{y} - \dot{x}) = -ky$$

$$② — \boxed{b(\dot{x} - \dot{y}) = ky}$$

substitute in ①

$$\left[m\ddot{x} + ky = F \right]$$

FIVE APPLE

Scanned by CamScanner

by taking the laplace for ②, we get :

$$bs\,x(s) - bs\,y(s) = k\,y(s)$$

$$\boxed{x(s) = y(s)\left[\frac{bs+k}{bs}\right]}$$

هاي العلاقة اللي تربط بين المتغير x والمتغير y
معلة من النظام 1DOF

Deriving the differential equation using lagrange method

(بدلاً من ال FBD
اللي ما بتفيدني لما بدي أعمل موديلنج
لأنظمة أكثر تعقيداً ممّا سبق
نوعاً ما ..

Lagrange Method : $\boxed{L = T - V}$
(L)

$T \equiv$ Kinetic energy (translation + Rotation)

$V \equiv$ Potential energy (spring + gravity)

لما بيكون الجسم بعيد عن خط ال ref. بمسافة معينة

من العلاقة (L = T−V) يتم تطبيق أدناه اللي تمثل ال DE
المطلوبة ←

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \frac{\partial L}{\partial q_i} = F_{nc}$$

مشتقة علاقة ال L
بالنسبة لل $\dot{q}_i$

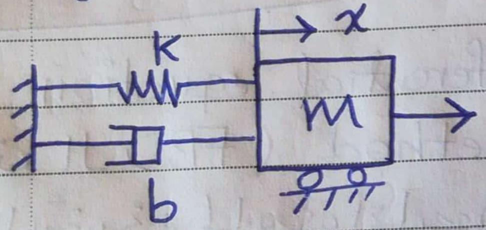
where $F_{nc} \equiv$ Non conservative force (damping, friction, external force)

القوة اللي منضيعها وما منقدر نستعيدها

$q_i \equiv$ system variable

Ex) Derive the DE for the following systems using Lagrange method :-

$\boxed{1}$



ما تم ذكر انه في friction عند الجسم (m) ∴ friction less

$L = T - V$

$T = \frac{1}{2} m v^2 = \frac{1}{2} m \dot{x}^2$ → mass للـ

$V = \frac{1}{2} k x^2$ → للنابض

$\therefore \quad L = \frac{1}{2} m \dot{x}^2 - \frac{1}{2} k x^2$

$\frac{\partial L}{\partial \dot{x}} = m\dot{x}$ → أي رمز غير $\dot{x}$ يعتبر ثابت بالنسبة إله

$\frac{\partial (m\dot{x})}{\partial t} = m\ddot{x}$ → we talk about $m\dot{x}(t) \xrightarrow[\text{بدلالة الزمن}]{\text{مشتقها}} m\ddot{x}(t)$

$\frac{\partial L}{\partial x} = -kx$

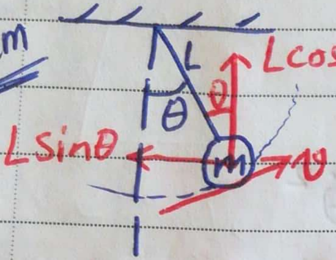
external force

$$m\ddot{x} + kx = F - b\dot{x}$$

damper

$$\therefore \boxed{m\ddot{x} + kx + b\dot{x} = F}$$

$\boxed{2}$ pendulum



→ 1 variable (θ) / 1 DOF

$$L = T - V$$

$$T = \frac{1}{2} m v^2 = \frac{1}{2} m (L\dot{\theta})^2$$

$v = \dot{x}$

$\dot{x} = L\dot{\theta}$

gravity

$$V = -mgL\cos\theta$$

$$L = \frac{1}{2} mL^2\dot{\theta}^2 + mgL\cos\theta$$

$$\frac{\partial L}{\partial \dot{\theta}} = mL^2\dot{\theta}$$

$$\frac{\partial (mL^2\dot{\theta})}{\partial t} = mL^2\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = -mgL\sin\theta \cong -mgL\theta$$

Taylor Series

[for small motion $\sin\theta \cong \theta$]

$$\therefore mL^2\ddot{\theta} + mgL\theta = 0$$

$$L\ddot{\theta} + g\theta = 0 \longrightarrow \boxed{\ddot{\theta} + \frac{g}{L}\theta = 0}$$

$\boxed{3}$

Inverted pendulum

$\vartheta = L\dot{\theta}$

1 variable (θ) / 1 DOF

$$L = T - V$$

$$T = \frac{1}{2} m \vartheta^2 = \frac{1}{2} m L^2 \dot{\theta}^2$$

$$V = \overset{\text{above the Ref.}}{\oplus} \underbrace{mgL\cos\theta}_{\text{gravity}} + \underbrace{\frac{1}{2} k_t \theta^2}_{\text{Rotational spring}}$$

$$L = \frac{1}{2} m L^2 \dot{\theta}^2 - mgL\underline{\underline{\cos\theta}} - \frac{1}{2} k_t \theta^2$$

لاحظي ما استبدلناها ب θ
إلّا آخر خطوة

$$\frac{\partial L}{\partial \dot{\theta}} = mL^2\dot{\theta}$$

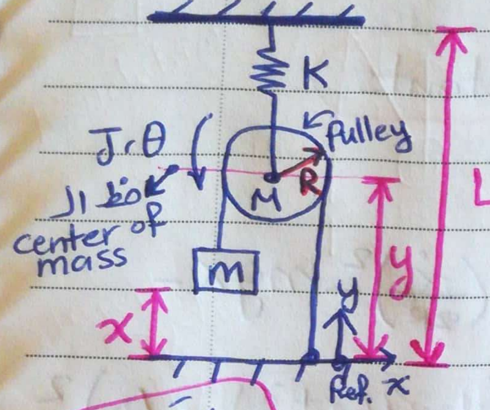
$$\frac{\partial (mL^2\dot{\theta})}{\partial t} = mL^2\ddot{\theta}$$

$$\frac{\partial L}{\partial \theta} = mgL\sin\theta - \frac{1}{2}k_t\theta = mgL\theta - \frac{1}{2}k_t\theta$$

$$\boxed{mL^2\ddot{\theta} - mgL\theta + \frac{1}{2}k_t\theta = \phi}$$

Ex Derive the DE of the following system using lagrange method :-

$\boxed{1}$ Spring Free Length (Δ)



كتلة البكرة M تختلف عن ال J تبعها

we have here 3 variables but all in 1DOF

x , y , θ

$$\boxed{y = R\theta} \rightarrow \dot{y} = R\dot{\theta} \quad \text{—} ①$$

طول البكرة كاملة

$$L_o = y + (y - x) = 2y - x \quad \text{—} ②$$

$$\boxed{L = T - V}$$

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} M \dot{y}^2 + \frac{1}{2} J \dot{\theta}^2$$

طلّع كل وحدة بدلالة متغيرها بعدين بنعوض وحدة محل التانية

For gravity

$$V = mg\,x + Mgy + \frac{1}{2} k\left((L-y)-\Delta\right)^2$$

الوزن

البعد عن Ref.

where → $\dot{y}$: translational speed

$\dot{\theta}$: Rotational speed

من العلاقات ① و ② أعلاه اذا بدنا نحل ال Lagrange equ. بدلالة $\dot{x}$

From ② → $y = \frac{L_o + x}{2}$, $\dot{y} = \frac{\dot{x}}{2}$

فنعوضها بـ ①

$$\frac{\dot{x}}{2} = R\dot{\theta} \rightarrow \boxed{\dot{\theta} = \frac{\dot{x}}{2R}}$$

$$T = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} M \left(\frac{\dot{x}}{2}\right)^2 + \frac{1}{2} J \left(\frac{\dot{x}}{2R}\right)^2$$

$$V = mg\,x + Mg\left(\frac{L_0 + x}{2}\right) + \frac{1}{2} k\left(L - \left(\frac{L_0 + x}{2}\right) - \Delta\right)^2$$

$$\boxed{L \,, \underset{R}{L_0} \,, \Delta \Rightarrow \text{constants}}$$

$$\Rightarrow \quad L = T - V$$

$$= \frac{1}{2} m \dot{x}^2 + \frac{1}{2} M \left(\frac{\dot{x}}{2}\right)^2 + \frac{1}{2} J \left(\frac{\dot{x}}{2R}\right)^2 - mgx -$$

$$Mg\left(\frac{L_0 + x}{2}\right) - \frac{1}{2} k\left(L - \left(\frac{L_0 + x}{2}\right) - \Delta\right)^2$$

$$\frac{\partial L}{\partial \dot{x}} = m\dot{x} + M\frac{\dot{x}}{2 \times 2} + J\frac{\dot{x}}{2R \times 2R} - 0$$

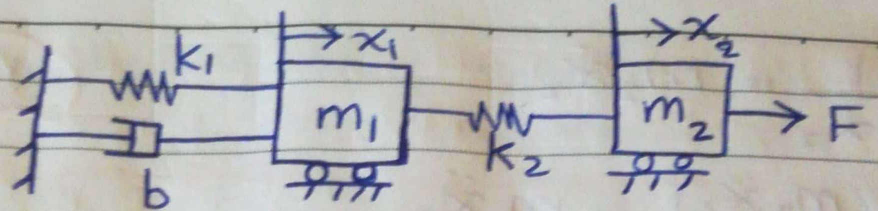
$$\frac{\partial L}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right) = m\ddot{x} + \frac{M}{4}\ddot{x} + J\frac{\ddot{x}}{4R^2}$$

$$\frac{\partial L}{\partial x} = -(mg) - \left(Mg \times \frac{1}{2}\right) + \frac{K}{2}\left(L - \frac{L_0 + x}{2} - \Delta\right)$$

$$\therefore 0 = m\ddot{x} + \frac{M}{4}\ddot{x} + J\frac{\ddot{x}}{4R^2} + mg + \frac{M}{2}g - \frac{1}{2}k\left(L - \left(\frac{L_0 + x}{2}\right)\Delta\right)$$

$$\boxed{0 = \ddot{x}\left(m + \frac{M}{4} + \frac{J}{4R^2}\right) + g\left(m + \frac{M}{2}\right) - \frac{1}{2}k\left(L - \left(\frac{L_0 + K}{2}\right) - \Delta\right)}$$

$\boxed{2}$



In this system, there's 2 variables / 2DOF cuz they're independent variables (x_1 & x_2)

هيطلع عندي معادلتين باشتقاق ال Lagrange بالنسبة ل x_1 و x_2

$$T = \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2$$

$$V = \frac{1}{2} k_1 x_1^2 + \frac{1}{2} k_2 (x_1 - x_2)^2$$

$$\therefore \quad L = T - V$$

$$= \frac{1}{2} m_1 \dot{x}_1^2 + \frac{1}{2} m_2 \dot{x}_2^2 - \frac{1}{2} k_1 x_1^2 - \frac{1}{2} k_2 (x_1 - x_2)^2$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial x_1}\right) - \frac{\partial L}{\partial x_1} = -b\dot{x}_1$$

حركة m_1

$$\frac{\partial L}{\partial x_1} = -k_1 x_1 - k_2 (x_1 - x_2)$$

$$\frac{\partial L}{\partial \dot{x}_1} = m_1 \dot{x}_1 \longrightarrow \frac{\partial (m_1 \dot{x}_1)}{\partial t} = m_1 \ddot{x}_1$$

$$m_1 \ddot{x}_1 - (-k_1 x_1 - k_2 (x_1 - x_2)) = -b\dot{x}_1$$

$$\boxed{m_1 \ddot{x}_1 + k_1 x_1 + k_2 (x_1 - x_2) + b\dot{x}_1 = 0}$$

No.

حركة m_2

$$\frac{\partial L}{\partial \dot{x}_2} = m_2\dot{x}_2$$

$$\frac{\partial(m_2\dot{x}_2)}{\partial t} = m_2\ddot{x}_2$$

$$\frac{\partial L}{\partial x_2} = +k_2(x_1 - x_2)$$

$$\boxed{m_2\ddot{x}_2 - k_2(x_1 - x_2) = F}$$

$\Rightarrow$ If we say that :

$$L = \frac{1}{2} m_1\dot{x}_1^2 + \frac{1}{2} m_2\dot{x}_2^2 - \frac{1}{2} k_1 x_1^2 - \frac{1}{2} k_2 \underline{(x_2 - x_1)}^2$$

عكس السابقة

For m1 : $m_1\ddot{x}_1 = \frac{\partial}{\partial t}(m_1\dot{x}_1)$

$$\frac{\partial L}{\partial x_1} = -k_1 x_1 + k_2(x_2 - x_1)$$

$$m_1\ddot{x}_1 + k_1 x_1 - k_2(x_2 - x_1) = -b\dot{x}_1$$

the same as before # $\leftarrow$ $$\boxed{m_1\ddot{x}_1 + b\dot{x}_1 + k_1 x_1 + k_2(x_1 - x_2) = 0}$$

For m_2 : $\frac{\partial}{\partial t}(m_2\dot{x}_2) = m_2\ddot{x}_2$

$$\frac{\partial L}{\partial x_2} = -k_2(x_2 - x_1)$$

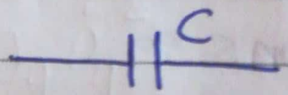
$$\boxed{m_2\ddot{x}_2 + k_2(x_2 - x_1) = F}$$ $\rightarrow$ also, the same as before #

Ch.4 : Electrical System

- Basic Elements → Resistor, Capacitor, Inductor
- Describing Variables → charge, current, voltage

---

1 Capacitor :-



بيخزن قيمة معينة من ال Energy

$$U(t) = \frac{1}{c}\int I.dt = \frac{Q \rightarrow \text{charge}}{c}$$

[redacted]

$$I = \frac{dQ}{dt}$$

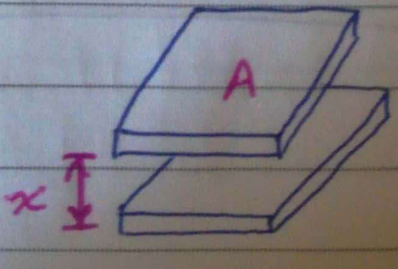
$$E_c = \frac{1}{2} c \vartheta^2 = \frac{1}{2}\frac{Q^2}{c}$$

$$I(t) = c\,\frac{dV_c}{dt}$$

ex 1 Capacitor for two parallel plates

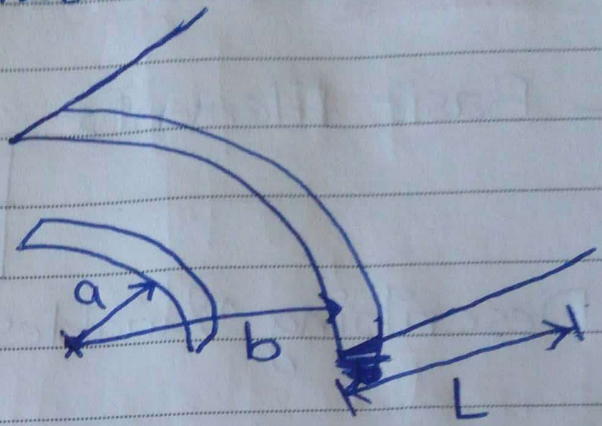
$$C = \frac{\epsilon A}{x}$$

ϵ : dielectric constant



ex2| Capacitor for two concentric Cylinedar

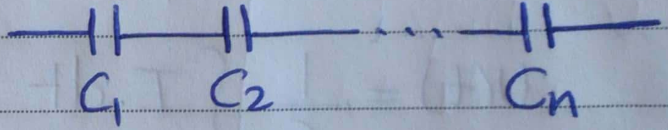
$$C = \frac{2\pi \epsilon L}{\ln\left(\frac{b}{a}\right)}$$



where $(b \gg a)$

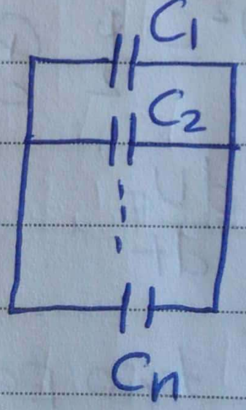
Combinations :

a. series →



$$\frac{1}{C_{eq}} = \sum_{i=1}^{n} \frac{1}{C_i}$$

b. Parallel →



$$C_{eq} = \sum_{i=1}^{n} C_i$$

2 Resistor :-

$$v(t) = IR$$

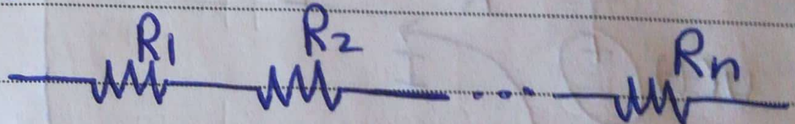
$$R = \frac{\rho L}{A}$$

ρ : electric resistivity

power dissipated $= IV = \frac{V^2}{R} = I^2R$

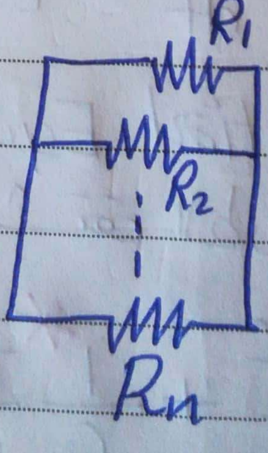
Combinations :

a. Series →



$$R_{eq} = \sum_{i=1}^{n} R_i$$

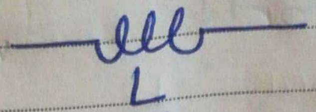
b. Parallel



$$\frac{1}{R_{eq}} = \sum_{i=1}^{n} \frac{1}{R_i}$$

[redacted]

[3] Inductor :-

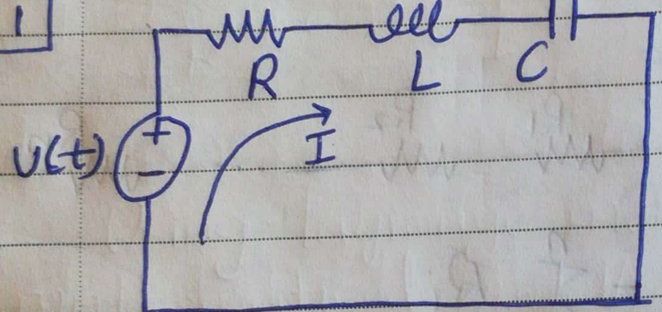


$$v(t) = L\frac{dI}{dF} = L\ddot{Q}(t)$$

$$E_L = \frac{1}{2} L I^2$$

Ex

[1]



Output : I

Input : V

$I = I_R = I_C = I_L$

a. $v(t) = V_R + V_L + V_c$

$$V = IR + L\frac{dI}{dt} + \frac{1}{c}\int I\,dt$$

Differential Equ. ← لتكون

نشتق الطرفين

$$\mathcal{L}\left[\dot{v} = \dot{I}R + L\ddot{I} + \overset{*}{\frac{1}{c}} I\right]$$

$$s\,v(s) = sI(s)R + s^2LI(s) + \frac{1}{c}I(s)$$

$$s\,v(s) = I(s)\left[Rs + s^2L + \frac{1}{c}\right]$$

$$\frac{I(s)}{V(s)} = \frac{s}{Rs + Ls^2 + \frac{1}{c}}$$

b. using lagrange method →

$\boxed{L = T - V}$, T : inductors

V : Capacitors

non-conservative force : (V/I) sources + Resistor voltage IR

$$L = \frac{1}{2} L \dot{Q}^2 - \frac{1}{2C} Q^2$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{Q}}\right) - \frac{\partial L}{\partial Q} = F_{nc} \Rightarrow$$

مشتقة الشحنة ≡ التيار /

$$\frac{\partial L}{\partial \dot{Q}} = L\dot{Q}$$

$$\frac{\partial (L\dot{Q})}{\partial t} = L\ddot{Q}$$

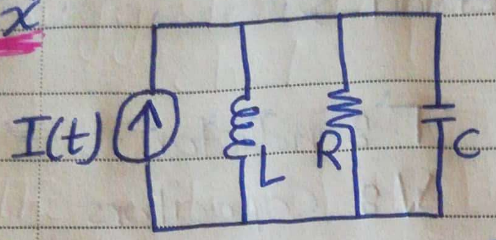
$$\frac{\partial L}{\partial Q} = \frac{-1}{C} Q$$

$$\therefore \quad L\dot{Q} - \left(\frac{-Q}{C}\right) = v(t) - \underbrace{R\dot{Q}}_{\rightarrow RI}$$

$$\boxed{L\ddot{Q} + R\dot{Q} + \frac{1}{C} Q = v(t)}$$

ملاحظة - تم التركيز عاللوحات العديدة في الامتحان
هو مثل هاد المثال

Ex



I : input

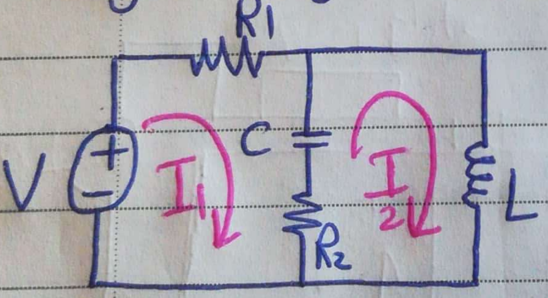
v : output

$$I(t) = I_R + I_c + I_L$$

$$= \frac{v}{R} + C\frac{dv_c}{dt} + \frac{1}{L}\int v(t).dt$$

$$\boxed{\dot{I} = \frac{\dot{v}}{R} + C\ddot{v} + \frac{1}{L}v}$$

Ex Using lagrange method, derive the mathematical modelling for the following system ?



$I_1 \equiv Q_1$

$I_2 \equiv Q_2$

Input (v)

Output (Q_1 / Q_2)

„indep. Variables„ مافي علاقة بينهم ↙

$$L = T - V$$

$$= \frac{1}{2}L\dot{Q}_2^2 - \frac{1}{2C}(Q_1 - Q_2)^2$$

No.

$$\rightarrow \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{Q}_1}\right) - \frac{\partial L}{\partial Q_1} = v(t) - R_1\dot{Q}_1 - R_2(\dot{Q}_1 - \dot{Q}_2)$$

$$0 - \left(\frac{-1}{C}(Q_1 - Q_2)\right) = v(t) - R_1\dot{Q}_1 - R_2(\dot{Q}_1 - \dot{Q}_2)$$

$$\left[\frac{Q_1 - Q_2}{C} = v(t) - R_1\dot{Q}_1 - R_2(\dot{Q}_1 - \dot{Q}_2)\right]$$

$$\rightarrow \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{Q}_2}\right) - \frac{\partial L}{\partial Q_2} = -R_2(\dot{Q}_2 - \dot{Q}_1)$$

$$L\ddot{Q}_2 - \left(\frac{1}{C}(Q_1 - Q_2)\right) = -R_2(\dot{Q}_2 - \dot{Q}_1)$$

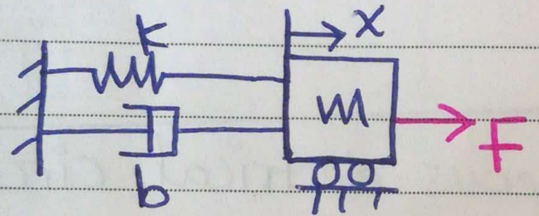
$$\left[L\ddot{Q}_2 - \frac{1}{C}(Q_1 - Q_2) = -R_2(\dot{Q}_2 - \dot{Q}_1)\right]$$

Ch.10 Analogy between mechanical and electrical system

1. Force_voltage analogy
↓
Force = V
m = L
b = R
k = $\frac{1}{c}$

2. Force_current analogy
↓
Force = I
m = C
b = $\frac{1}{R}$
k = $\frac{1}{L}$

Ex)

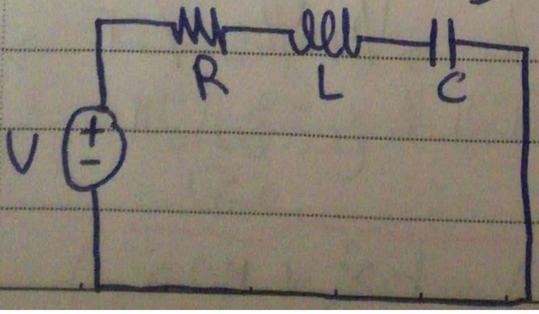


a) Using Force voltage analogy ?

$$\underbrace{m\ddot{x}}_{L} + \underbrace{b\dot{x}}_{R} + \underbrace{kx}_{\frac{1}{c}} = \underbrace{f(t)}_{V}$$

→ لعمل مودِلينغ للنظام الميكانيكي لفهم آلية مكافئة بالنظام الكهربائي المقابل

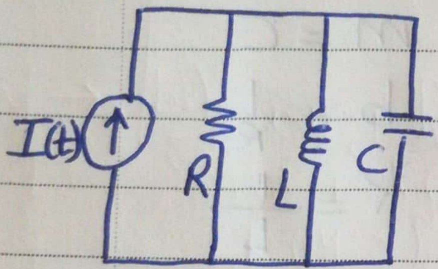
* الفولتية تتوزع وهذا يكون بتوصيل مكونات الدارة الكهربائية ع التوالي *



b.) Using force-current analogy?

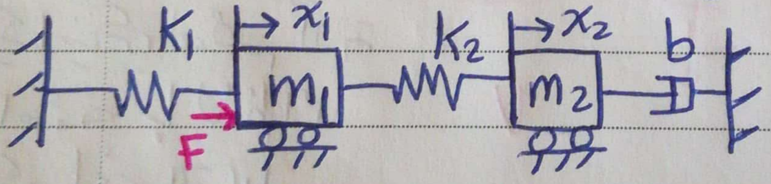
$$\underset{C}{m\ddot{x}} + \underset{\frac{1}{R}}{b\dot{x}} + \underset{\frac{1}{L}}{kx} = \underset{I}{f}$$

* التيار يتوزع ع التوازي *



Ex Draw the analogeous electrical circuit using : a- Force_current

b- Force_voltage



$m_1\ddot{x}_1 + k_1x_1 + k_2(x_1 - x_2) = F$ —— ①

$m_2\ddot{x}_2 + b\dot{x}_2 + k_2(x_2 - x_1) = 0$ —— ②

a) $m_1 ، m_2 \rightarrow C_1 ، C_2$ / $k_1 ، k_2 \rightarrow 1/L_1 ، 1/L_2$ / $F \rightarrow I(t)$

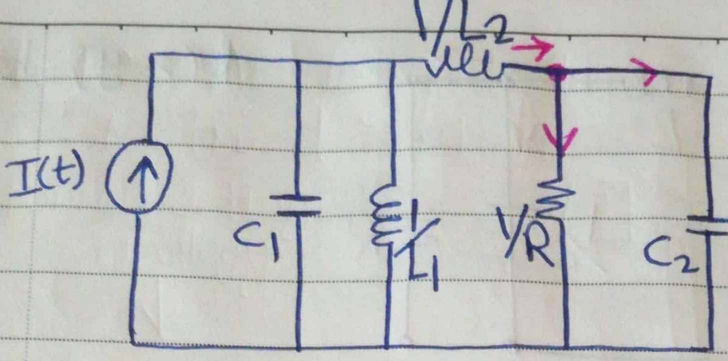
From ① ⇒ التيار يتوزع على العناصر

لكن

(لما يكون في coupling) From ②

$$k_2(x_1 - x_2) = b\dot{x}_2 + m_2\ddot{x}_2$$

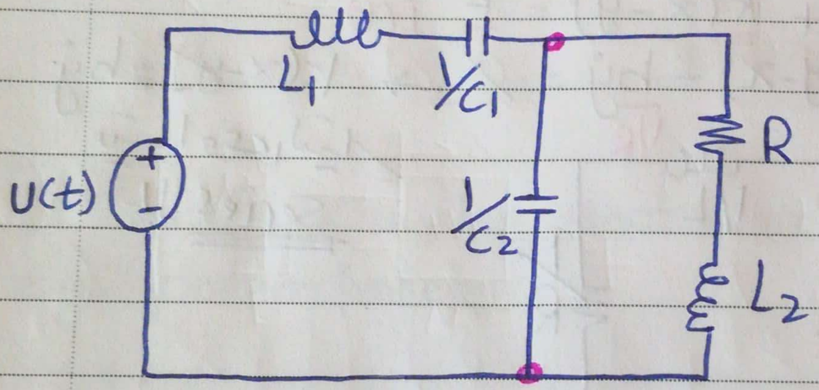
No.



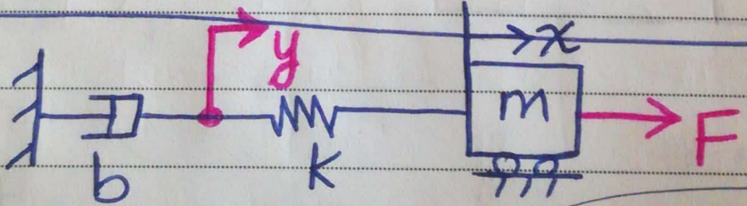
في حالة الـ
coupling
بتركها لآخر شي

b) $m_1, m_2 \rightarrow L_1, L_2$ / $k_1, k_2 \rightarrow \frac{1}{C_1}, \frac{1}{C_2}$

$b \Rightarrow R$ / $F \rightarrow V(t)$

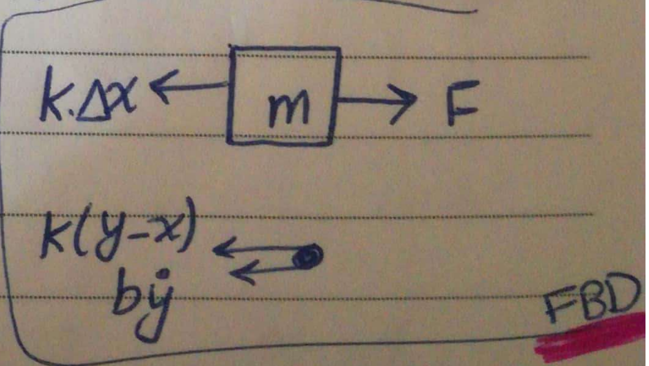


Ex)



$F = m\ddot{x} + k(x-y)$ — ①

$-k(y-x) - b\dot{y} = 0$ — ②

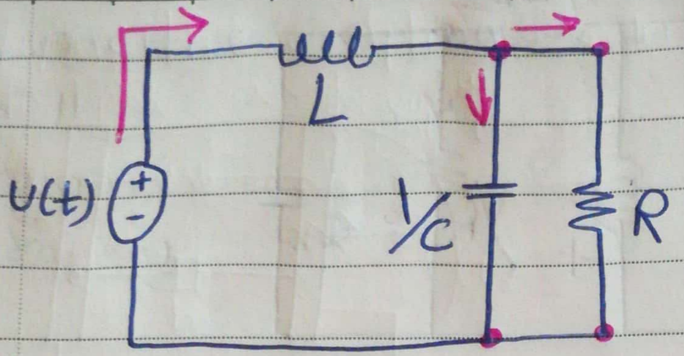


a.)

#using force-voltage :

$F \rightarrow V$ / $m \rightarrow L$ / $k \rightarrow \frac{1}{c}$ / $b \rightarrow R$

No.



$k(x-y) = b\dot{y}$

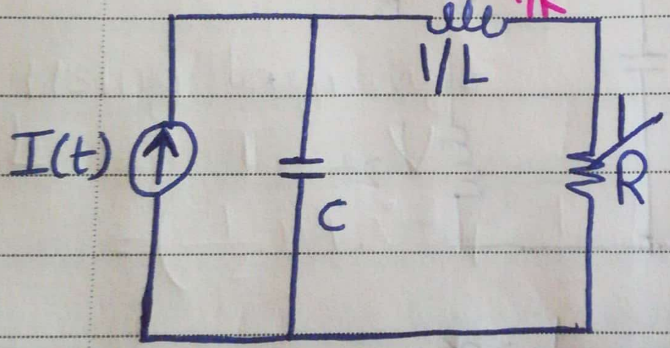
Ex → b.) using force current :

$c\ \underline{m}\ddot{x} + \overset{1/L}{\underline{k}}(x-y) = \underline{F}\ I(t)$

$-k(y-x) - \underset{1/R}{\underline{b}}\dot{y} = 0 \rightarrow k(x-y) = b\dot{y}$

بيساوي التيار

Series ال

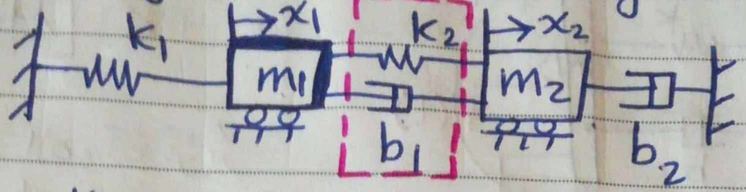


First Exam

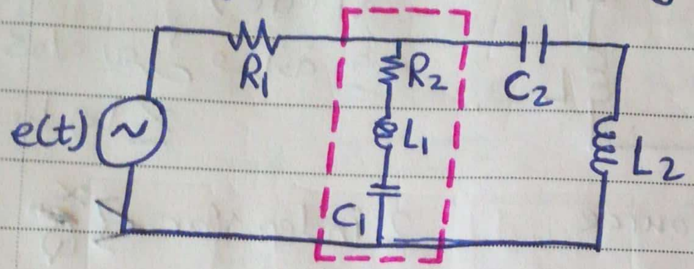
No.

Coupling and combined systems →

1] Coupling in mechanical system



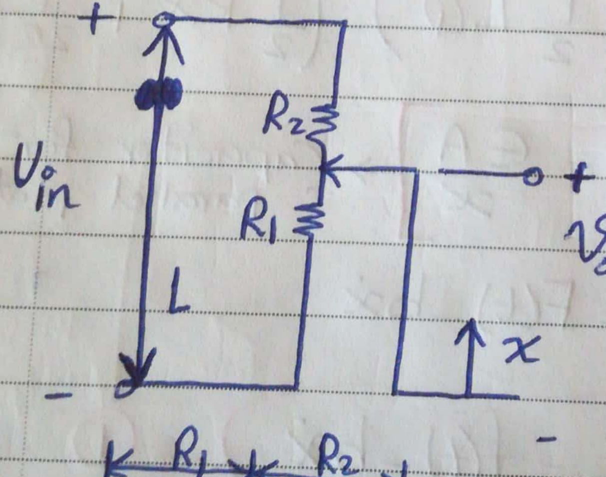
2] Coupling in electrical system



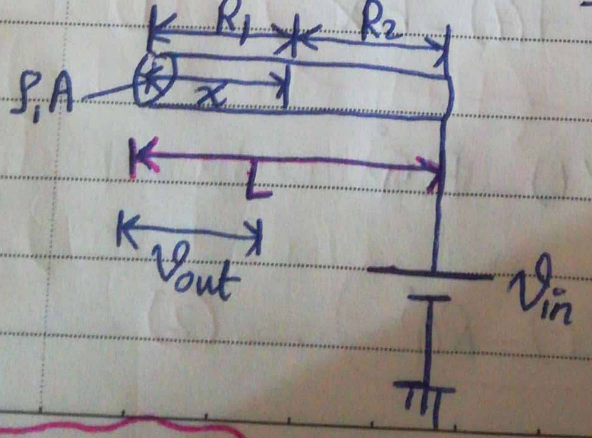
بيكون ال coupling بين indep. var.

3] Resistive coupling electromechanical system

e.g. potentiometer



* $$V_{out} = \frac{R_1}{R_1+R_2} V_{in}$$



* $$V_{out} = \frac{x}{L} V_{in}$$

$$R_1 = \frac{\rho x}{A}$$

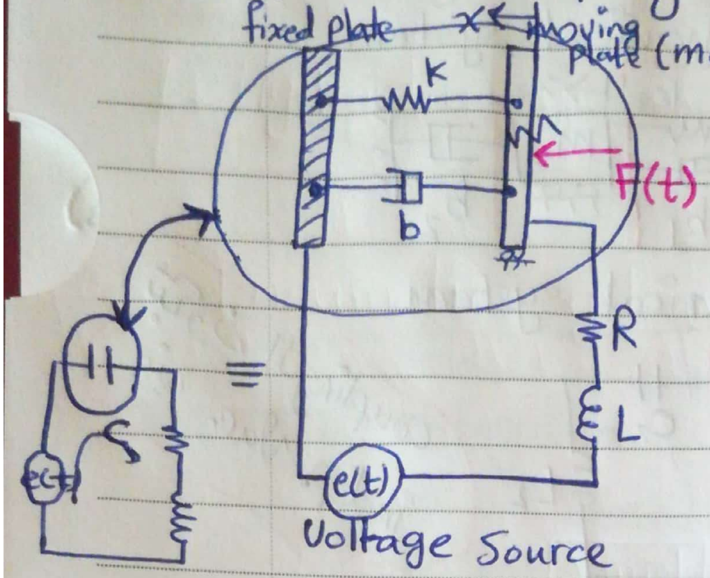
$$\boxed{R = \frac{\rho L}{A}}$$

$$R_2 = \frac{\rho(L-x)}{A}$$

Electromechanical Sys. → Resistive, Capacitive, Inductive

No.

4 Capacitive Coupling → in electromechanical sys.



Voltage Source

مبدأ عمل [Microphone]

الموجات الصوتية تؤثر بقوة على ال (moving plate) تجعله يتحرك مسافة معينة مقدارها x

2 indep. var → x, Q

using lagrange :

T : mass / Inductor
V : spring / Capacitor

$$L = T - V$$

$$= \left(\frac{1}{2} m \dot{x}^2 + \frac{1}{2} L \dot{Q}^2\right) - \left(\frac{1}{2} k x^2 + \frac{1}{2C} Q^2\right)$$

where $\left[C = \frac{\epsilon A}{x}\right]$ → Capacitor for 2 parallel plates

$$* \ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = F(t) - b\dot{x}$$

$$\boxed{m\ddot{x} + kx + \frac{Q^2}{2\epsilon A} = f(t) - b\dot{x}} \quad — ①$$

$$* \ \frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{q}}\right) - \frac{\partial L}{\partial q} = e(t) - R\dot{Q}$$

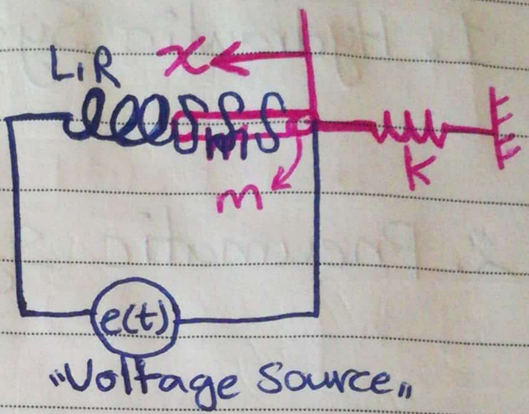
$$\boxed{L\ddot{Q} + \left(\frac{x}{\epsilon A}\right) Q = e(t) - R\dot{Q}} \quad — ②$$

هون ال coupling انه ~~الواسطة اللي بالنظام الكهربائي~~ عبارة عن نظام ميكانيكي

FIVE APPLE

5] Inductive coupling

Ex : Solenoid



متغيرات النظام :

x and $\dot{Q}$

because of displacement but it's not dependent with Q → 2DOF

Coil ≡ L+R

$$L = T - V$$

$$= \left[\frac{1}{2} m\dot{x}^2 + \frac{1}{2} L\dot{Q}^2\right]_T - \left[\frac{1}{2} k x^2\right]_V$$

$$\frac{d}{\partial t}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = \phi$$

$$\boxed{m\ddot{x} \pm kx = \phi} \quad ..①$$

For "Mechanical"

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \dot{Q}}\right) - \frac{\partial L}{\partial Q} = e(t) - R\dot{Q}$$

For "Electrical"

$$L\ddot{Q} + \phi = e(t) - R\dot{Q}$$

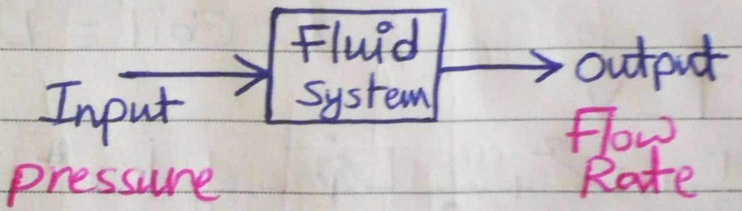
$$\boxed{L\ddot{Q} + R\dot{Q} = e(t)} \quad ..②$$

No.

Ch.5 : Fluid System

1. Hydraulic System → Liquid (incompressable) لا ينضغط
ρ : constant

2. Pneumatic system → gases (compressable)
ρ : variable



- Mass Flow Rate (pneumatic) "variable ρ"
- Volume Flow Rate (Hydraulic) "constant ρ"

Inputs

mec. elec.
Pressure ≡ Force ≡ Voltage
q ≡ Flow Rate ≡ Velocity ≡ current
$(q, \dot{m})$ mec. elec.
$\dot{V}$

for analogy purposes

\# Fundamental Laws :- (Hydraulic System)
① Conservation of mass (Continuity Equation)
② Hydrostatic Pressure Law (Pascal Law)

[redacted]

* # Conservation of mass :

$$\frac{\partial m}{\partial t} = \dot{m}_{in} - \dot{m}_{out} = 0 \rightarrow \boxed{\dot{m}_{in} = \dot{m}_{out}}$$

Flow Rate

mass flow rate

$$\left[\text{mass} = \underset{\rho}{\text{density}} * \underset{\forall}{\text{Volume}} \right]$$

$$\rightarrow \boxed{\cancel{\rho}\,\dot{\forall}_{in} = \cancel{\rho}\,\dot{\forall}_{out}}$$

→ volume flow Rate

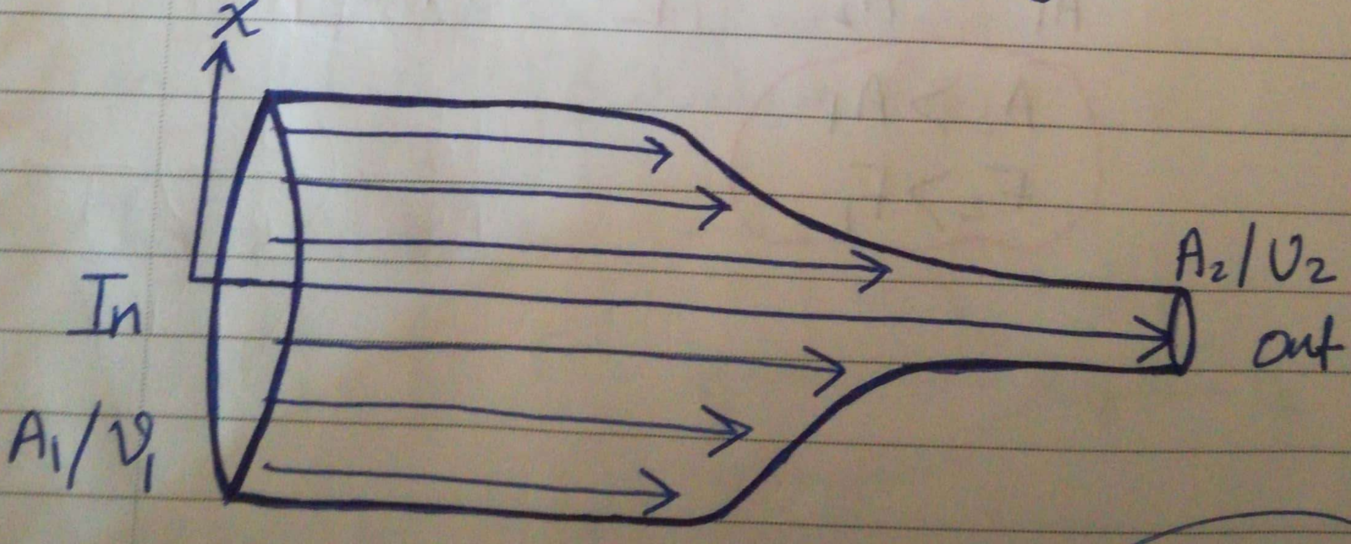
$$\left[\text{Volume} = \text{Area} * \underset{x}{\text{Height}} \right]$$

$$A_1 \frac{\partial x_1}{\partial t} = A_2 \frac{\partial x_2}{\partial t}$$

→ الارتفاع بتغير مع الزمن بثبات المساحة (A)

$$\boxed{A_1 v_1 = A_2 v_2}$$

→ velocity



$\forall$ = volume

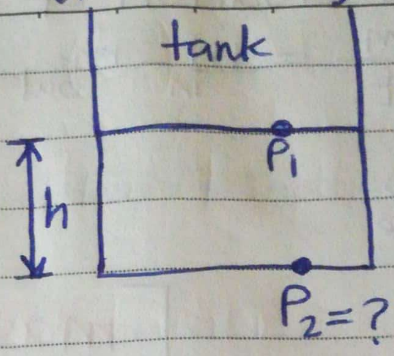
v = velocity

Hydrostatic Pressure Law (Pascal Law) →

* $P_1 = P_{atm}$

$$\boxed{P_2 = P_{atm} + \rho g h}$$

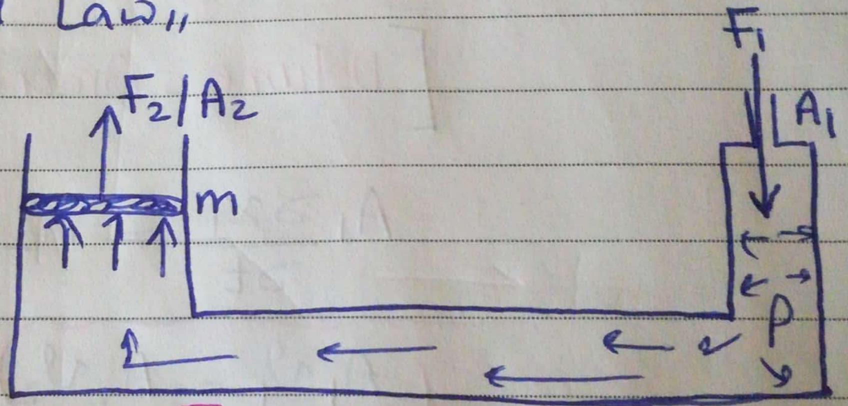
اذا كان الخزان مفتوح



$\boxed{P_2 = \rho g h}$ ⇐ بينما لو كان مغلق من الأعلى

"Pascal Law"

* $P = \frac{F}{A}$



$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \rightarrow \boxed{F_2 = \frac{A_2}{A_1} F_1}$$

$A_2 > A_1$

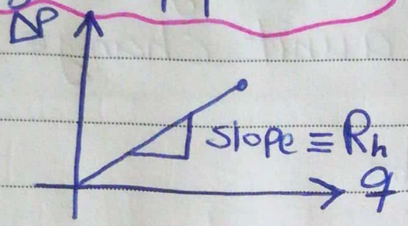
$F_2 > F_1$

The Basic Element modelling :

for hydraulic system

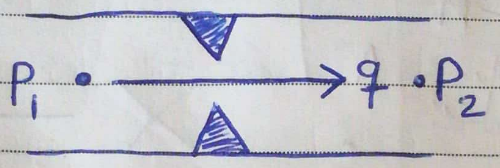
① Hydraulic Resistance (R_h) →

the resistance to flow because of a valve or change in pipe diameter. $\Delta P = R_h q$



[damper
بالنظام الميكانيكي]

Orifice,



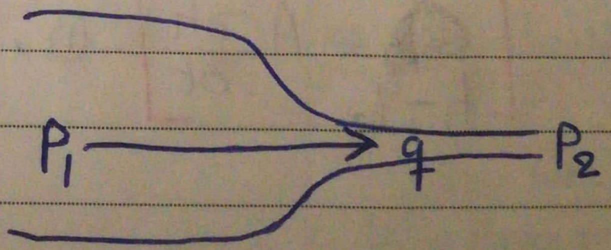
Flow Rate

- يختلف الضغط على نفس المكان

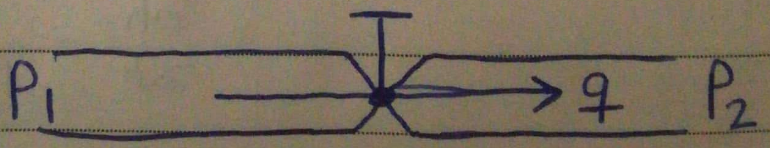
- بنفس مبدأ الـ Valve بحيث بعمل تضيق فيها، بالتالي تختلف المساحة

[Δ Area]

Change in diameter,



Value,



$$P_1 \neq P_2$$

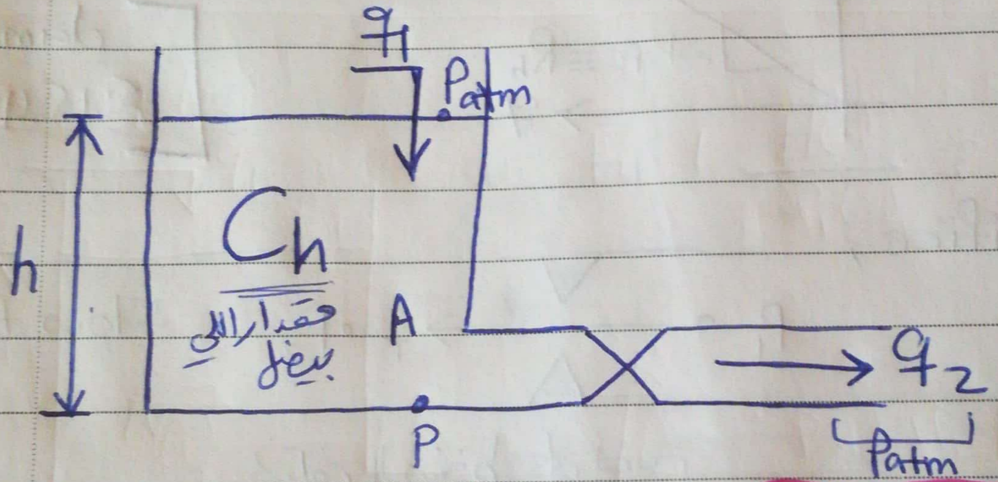
$$R_h = \frac{P_2 - P_1}{q}$$

No.

② Hydraulic Capacitance (C_h) →

Used to describe energy storage with a liquid, where it's stored in the form of potential energy.

A height of liquid container is one form of storage, is the change in volume stored to cause a unit change in h



volume = Ah

$\forall$: Volume

$$q_1 - q_2 = \frac{\partial \forall}{\partial t}$$

الارتفاع بيتغير

$$q_1 - q_2 = \frac{\partial h}{\partial t} * A$$

$$\left[q_1 - q_2 = A\frac{\partial h}{\partial t} \right] \ldots ①,$$

$$P = \rho g h \ldots ②$$

$$\frac{\partial P}{\partial t} = \rho g \frac{\partial h}{\partial t}$$

$$\frac{\partial h}{\partial t} = \frac{1}{\rho g} \frac{\partial P}{\partial t}$$ sub in ①

$$[q_1 - q_2] = \overset{②}{\frac{A}{\rho g}} \frac{\partial P}{\partial t}$$

C_h → تعتبر متغيرة في النظام
لأن المساحة تختلف حسب
الشكل والارتفاع كذلك **Ah**

$$\left[E_c = \frac{1}{2C_h} v^2\right]$$

③ Hydraulic Inertance → زي الـInductor

It's equivalent to inertia/mass in mechanical system, and to the inductor in electrical system.

Long pipe

$$F_1 - F_2 = P_1 A - P_2 A \quad .. ①$$

$$F_1 - F_2 = m a \rightarrow F_1 - F_2 = m \frac{\partial v}{\partial t} \quad .. ②$$

Velocity

$$\underbrace{\cancel{A}(P_1 - P_2)}_{①} = \underbrace{\cancel{A} L \rho \frac{\partial v}{\partial t}}_{②} \quad .. ③$$

$$P_1 - P_2 = \frac{L\rho}{A} \frac{\partial q}{\partial t}$$

$\frac{L\rho}{A}$ → I_h

$$\left[E_I = \frac{1}{2} L v^2\right]$$

→ تخزن (Kinetic Energy)

mass = volume * ρ

m = Area * height * density

$$\boxed{m = AL\rho}$$

flow rate

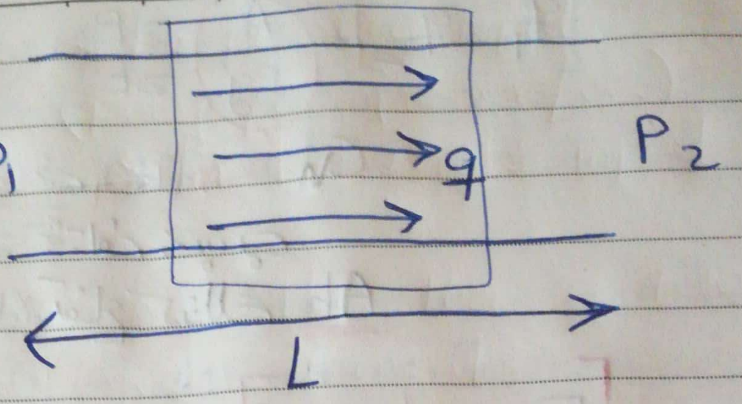
q = Area * velocity

$$q = Av$$

$$\frac{\partial q}{\partial t} = A \frac{\partial v}{\partial t} \rightarrow \boxed{\frac{\partial v}{\partial t} = \frac{1}{A} \frac{\partial q}{\partial t}}$$

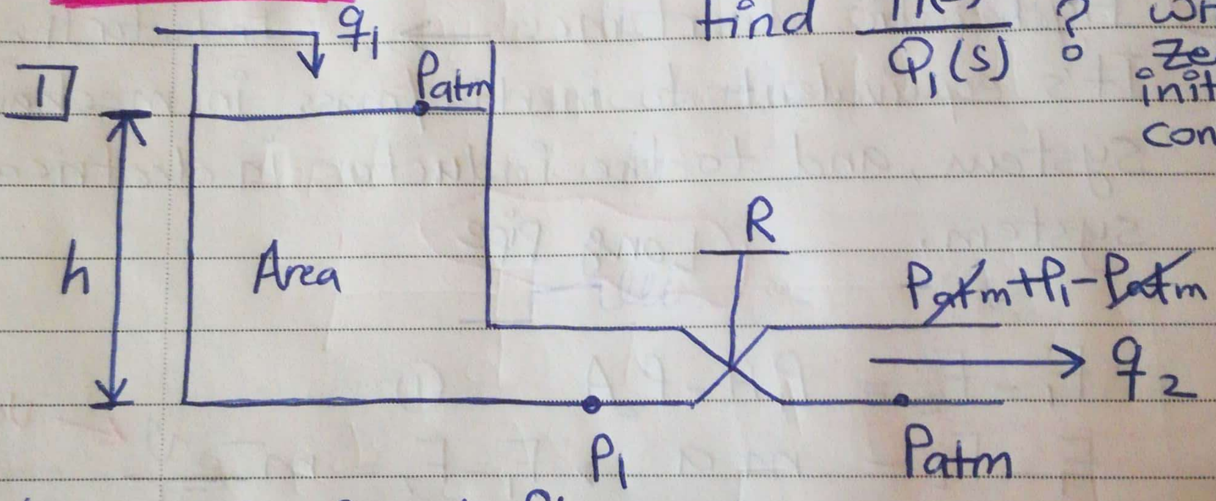
sub in

For Long pipe



Examples

Find $\frac{H(s)}{Q_1(s)}$? with zero initial conditions



where q_1 : input flow rate
q_2 : Output " "

C_h ← مين عندي بيأ؟ ع النظام
change in Area ←

$$P_1 - P_2 = R q_2$$

Rh/ $$\boxed{\rho g h(t) = R q_2}$$ ← معادلة الـ R !

Ch/ $q_1 - q_2 = A\frac{\partial h}{\partial t} \longrightarrow q_1 = A\frac{\partial h}{\partial t} + \underline{\underline{q_2}}$

$$q_1 = A\frac{\partial h}{\partial t} + \underset{\text{ثابتة}}{\boxed{\frac{\rho g}{R}}}\, h(t)$$

$$Q_1(S) = AS\,H(S) + \overset{0}{\cancel{H(0)}} + \frac{\rho g}{R} H(s)$$

$$Q_1(s) = H(s)\left[\frac{\cancel{R*}\,AS}{R} + \frac{\rho g}{R}\right]$$

$$\frac{H(S)}{Q_1(S)} = \frac{R}{\frac{(RA)}{\rho g}S + \frac{\rho g}{\rho g}} = \frac{R/\rho g}{(R\underset{\downarrow}{C})S+1}$$

$$C_h \equiv \frac{A}{\rho g}$$

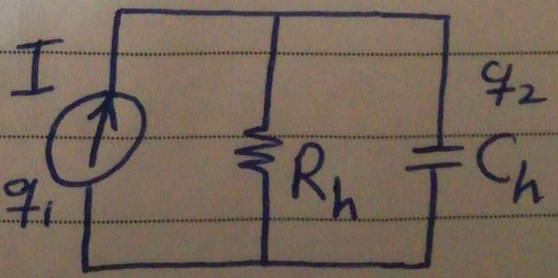
first Order system

→ the equivalent electrical system :

$Q_1(s)$: $I(s)$

Area : Resistor

q_2 : Capacitor



No.

Voltage source → for pressure input
(pump / compressor / ...)

Current source → for flow rate input

$\uparrow q_2$

(C_h) جزء بيمثل الخزان

(ΔA) جزء بيشتغل عالـ valve

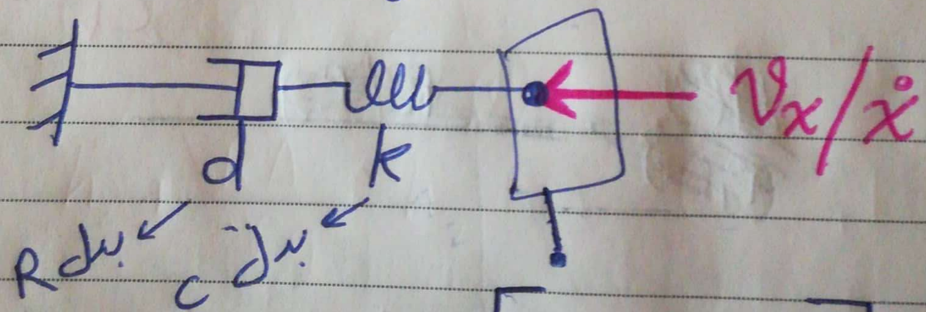
Here, there are two elements

→ the equivalent mechanical system :

حوّل مباشرةً من الـ hydraulic

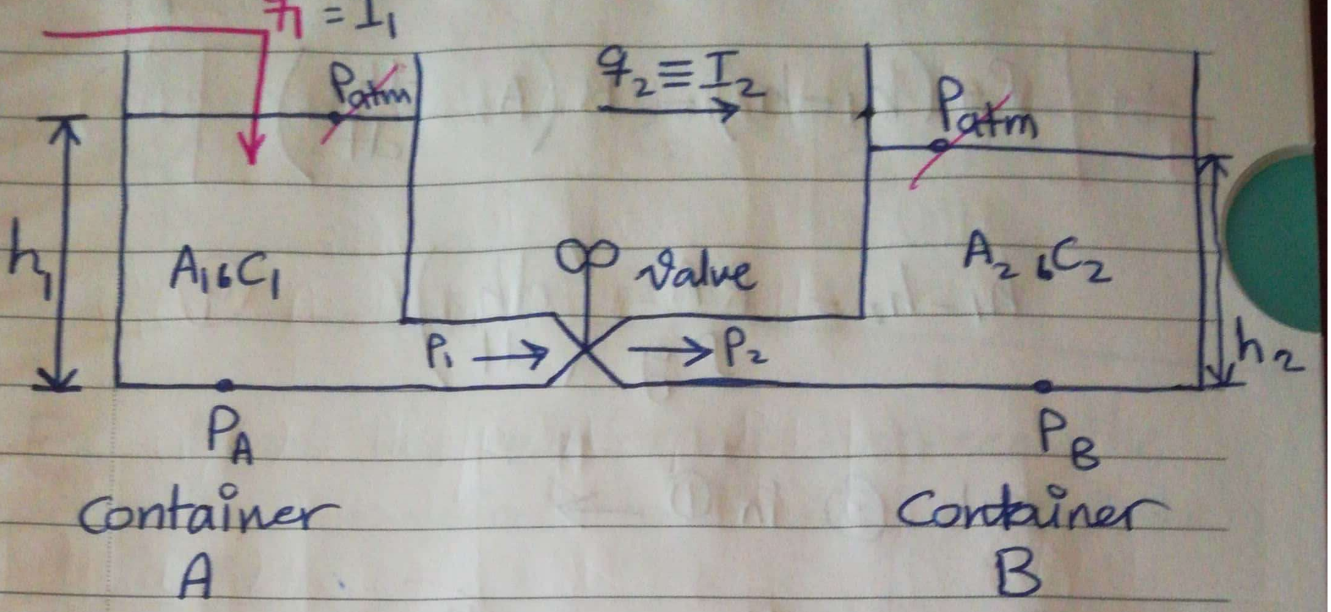
إلى الـ mechanical

Input ≡ velocity



$$\boxed{\begin{array}{l}\text{No } I_h\\ \text{so no mass}\end{array}}$$

Ex:



Find the differential equation that relates h_2 to q_1 ?

$$\left[q_1 - q_2 = A_1 \frac{dh_1}{dt} \right] \text{——} ①$$

$$P_A - P_B = Rq_2$$

$$\rho g \, (h_1 - h_2) = Rq_2 \text{——} ②$$

$$\left[q_2 - \emptyset = A_2 \frac{dh_2}{dt} \right] \text{——} ③$$

الداخل ع B

الخارج من B

ولو انه مخرج مناخده بالاعتبار

* انتبه انه التيار اللي
فايت ع R نفسه
ع C_2

Sub ③ in ② →

$$\rho g \div \left[\rho g (h_1 - h_2) = R \left(A_2 \frac{dh_2}{dt} \right) \right]$$

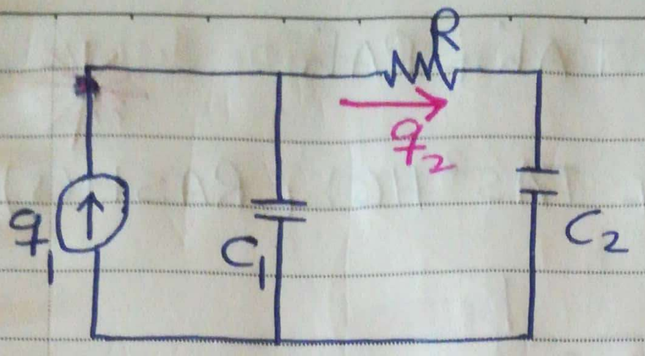
$$h_1 - h_2 = \frac{R A_2}{\rho g} \left(\frac{dh_2}{dt} \right) \quad \text{——} ④$$

Sub ③ in ① →

$$q_1 = A_2 \frac{dh_2}{dt} + A_1 \underset{\text{اشتقاق}}{\boxed{\frac{dh_1}{dt}}}$$

$$\boxed{\dot{h}_1 = \dot{h}_2 + \frac{R A_2}{\rho g} \ddot{h}_2}^{④} \quad \text{——} ⑤$$

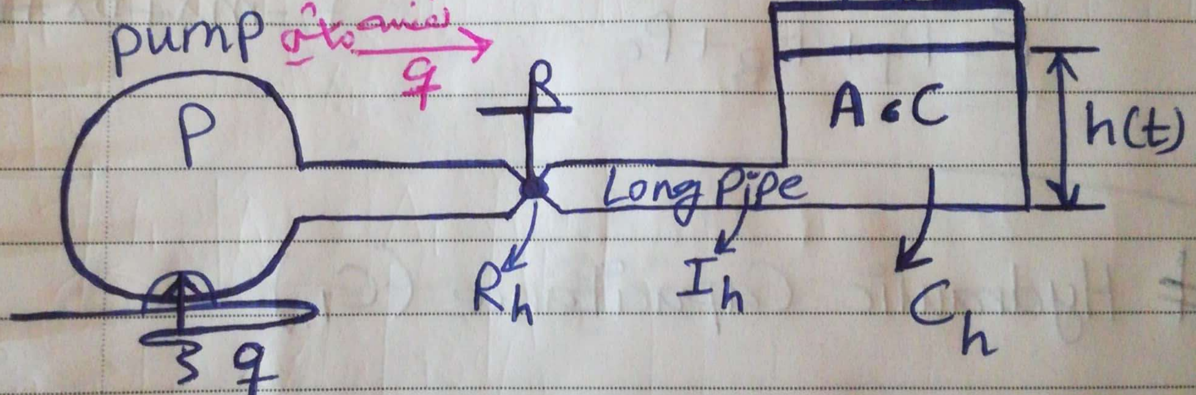
$$q_1 = A_2 \frac{dh_2}{dt} + A_1 \left[\dot{h}_2 + \frac{R A_2}{\rho g} \ddot{h}_2 \right]$$



الـ q_2 اللي
فالـ R
نفسها عـ C_2

Ex Find the relationship between the input P required to fill the tank & the Liquid.

height $h(t)$: $\dfrac{H(s)}{P(s)}$



$$q = q_c = q_I = q_c = A\dot{h}$$

$$P = P_{\textcircled{c}} + P_{\textcircled{I}} + P_{\textcircled{R}}$$

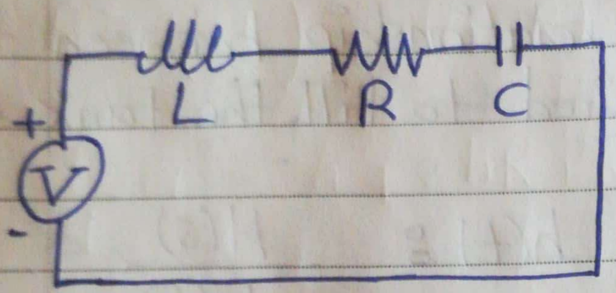
$$P = \left(\frac{1}{c}\int q\right) + I\dot{q} + Rq.$$

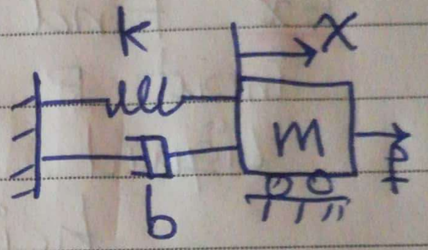
$$= \left(\frac{1}{c}\int A\dot{h}.dt\right) + IA\ddot{h} + RA\dot{h}$$

$$P = \frac{A}{c} h + IA\ddot{h} + RA\dot{h}$$

$$P(s) = \frac{A}{c} H(s) + IAS^2 H(s) + RASH(s)$$

$$\therefore \frac{H(s)}{P(s)} = \frac{1}{IAS^2 + RAS + \frac{A}{c}}$$



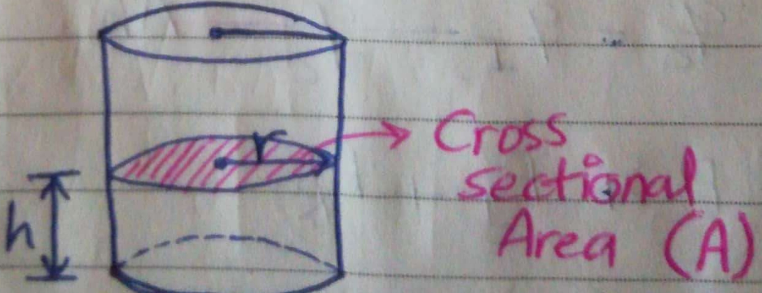


$$F = m\ddot{x} + b\dot{x} + kx$$

$$P = P_I + P_R + P_c$$

Hydraulic Capacitance (C_h)

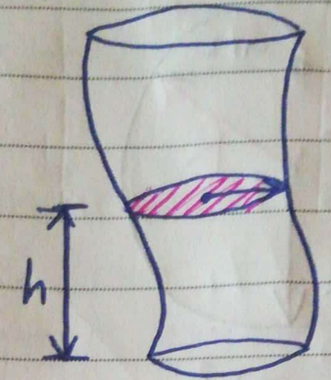
a. Vessel with constant Area



مو شرط circular

$$C_h = \frac{A}{\rho g}$$

b. Vessel with varying area

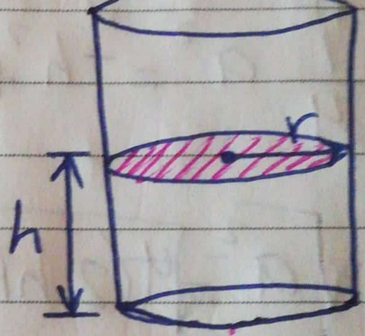


$$C_h = \frac{A(h)}{\rho g}$$

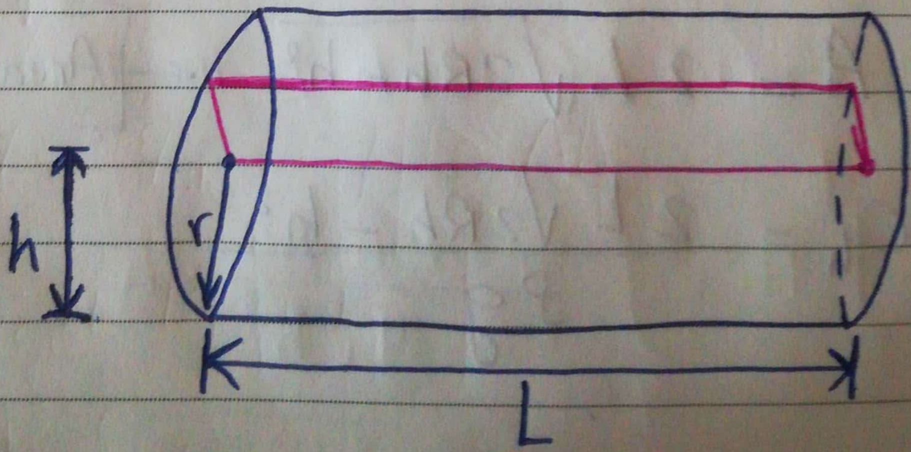
C. Vertical Cylinedar

$$C_h = \frac{A}{\rho g}$$

$$= \frac{\pi r^2}{\rho g}$$



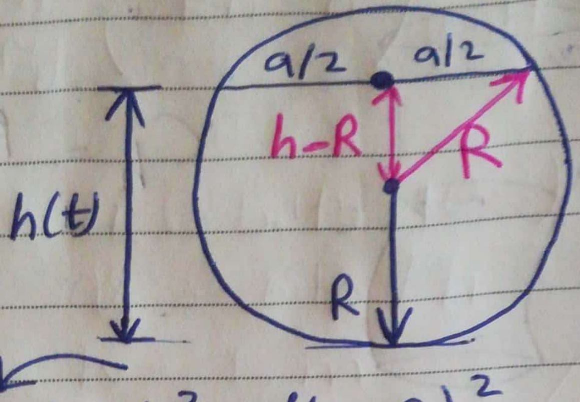
d. Horizontal Cylinedar



$$C_h = \frac{A(h)}{\rho g}$$

No.

for the horizontal Cylinedar:



$$R^2 = \left(\frac{a}{2}\right)^2 + (h-R)^2$$

$$\left[\frac{1}{4}a^2 = -h^2 + 2hR \cancel{+R^2} \cancel{-R^2}\right] * 4$$

$$\sqrt{a^2} = \sqrt{4(2hR - h^2)}$$

$$\boxed{a = 2\sqrt{2hR-h^2}}$$

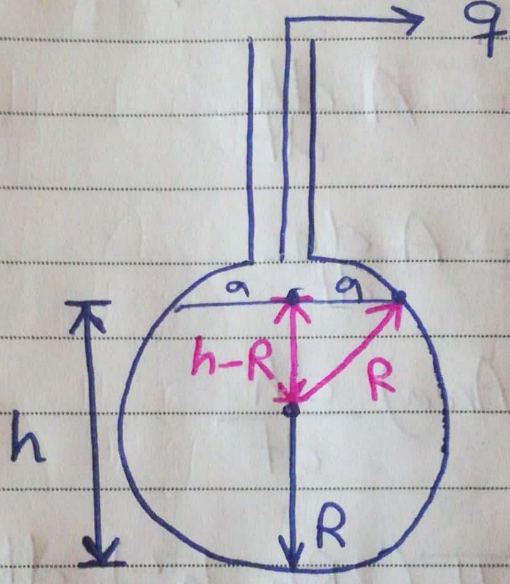
$$A = 2L\sqrt{2Rh-h^2} \leftarrow \left[\text{Area} = L*a\right]$$

$$\therefore C_h = \frac{2L\sqrt{2Rh-h^2}}{\rho g}$$

Ex: For the spherical tank shown below:

1. prove that $C_h = \frac{\pi}{\rho g}(2Rh - h^2)$

2. Determine how long it will be taken for the tank to empty, if the initial height is 9 m?



$$C_h = \frac{Area}{\rho g}$$

$$= \frac{\pi a^2(h)}{\rho g}$$

[redacted]

$$a^2 = R^2 - (h-R)^2$$

$$a^2 = \cancel{R^2} - h^2 + 2hR - \cancel{R^2}$$

$$a^2 = 2Rh - h^2$$

$$a = \sqrt{2Rh - h^2}$$

$$C_h = \frac{\pi}{\rho g}(2Rh - h^2)$$ #

2) If $a_o = 5.45(10^{-3})\ m^2$

$v_o = 6.13\sqrt{h}$

$R = 5\ m$

/ Given

$$q_{in} - q_{out} = A\frac{dh}{dt}$$

$$0 - q_{out} = A\frac{dh}{dt}$$

$$-A_o v_o = (Area)\frac{dh}{dt}$$

$$-A_o v_o = \pi(2Rh - h^2)\frac{dh}{dt}$$

$$-5.45(10^{-3})(6.13\sqrt{h}) = \pi(10h - h^2)\frac{dh}{dt}$$

$$\int_0^t dt = \int_{9m}^{0m} \frac{\pi(10h - h^2)}{-0.0334\sqrt{h}}\, dh$$

$$t = \frac{-\pi}{0.0334}\left[\frac{2}{3}(10h^{3/2}) - \frac{2}{5}h^{5/2}\right]_9^0$$

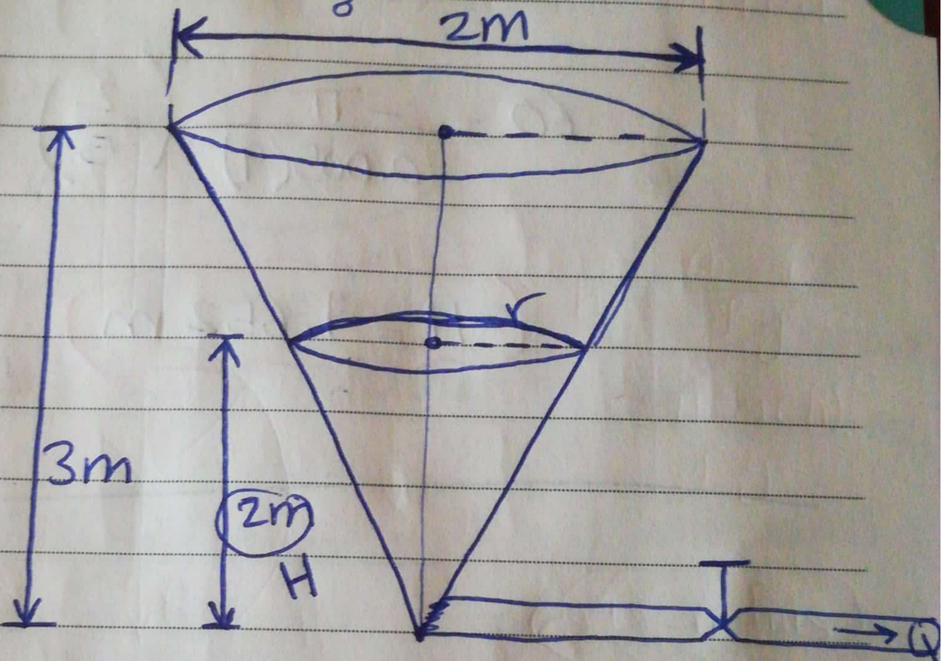
$$\boxed{t = 41\ min}$$ #

No. Sunday ..

Ex Consider the conical water tank shown below:

$$Q = 0.005\sqrt{H} \rightarrow \text{flow rate through the Valve}$$

the head is 2m at $t=0$, what will be the head at $t=60$ s?



area is a function of H

* نفس مبدأ ال (Spherical Tank)

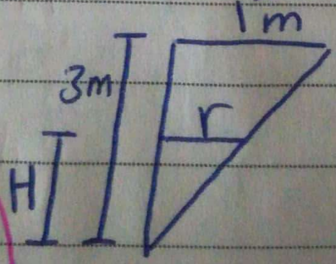
$$q_{in} - q_{out} = A\frac{dh}{dt}$$

$$0 - 0.005\sqrt{H} = A\frac{dH}{dt}$$

كم ذكرنا عن C_h اللي بيعبر عن التغيير في الحجم الناتج عن التغيير في الارتفاع

$$* \; A = \pi r^2$$

$$= \frac{\pi}{9} H^2$$



$$\frac{3}{H} = \frac{1}{r}$$

$$\boxed{r = \frac{H}{3}}$$

$$\frac{dt}{-0.005\sqrt{H}}\left(-0.005\sqrt{H} = \frac{\pi}{9} H^{2} \frac{dH}{dt}\right)$$

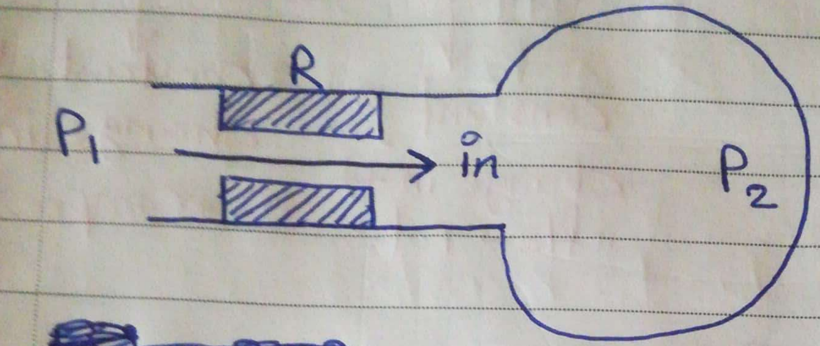
$$\int_{0}^{60} dt = \frac{-\pi}{9(0.005)} \int_{2m}^{H} H^{2} * H^{-1/2} \cdot dt$$

$$60 = \frac{-\pi}{0.005(9)} \left(\frac{2}{5}\right) H^{5/2}\Big]_{2m}^{H}$$

$$\boxed{H = 1.652\,m}$$

Pneumatic System →

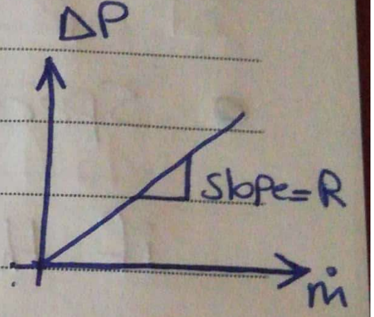
Change in Δp required to cause a unit change in q mass flow rate ($\dot{m}$)



[redacted]

P ≡ Voltage

$R\dot{m} = P_1 - P_2 \Rightarrow \dot{m} \equiv$ current



①

* Pneumatic Capacitance: the change in the gas stored in the mass (kg) required to cause a unit change in the gas pressure.

$$C_p = \frac{\partial m}{\partial P}$$

$$\dot{m} = \frac{\partial m}{\partial t} = \frac{\partial(\rho \forall)}{\partial t}$$

2 Variables here...

$$\left(\dot{m} = \rho \frac{\partial \forall}{\partial t} + \forall \frac{\partial \rho}{\partial t}\right) \frac{\partial \rho}{\partial P}$$

$$\frac{\partial m}{\partial t} = \frac{\partial P}{\partial t}\left(\rho \frac{\partial \forall}{\partial P} + \forall \frac{\partial \rho}{\partial P}\right)$$

$$\boxed{C_P = \frac{\partial m}{\partial P} = \left[\underbrace{\rho \frac{\partial \forall}{\partial P}}_{C_1} + \underbrace{\forall \frac{\partial \rho}{\partial P}}_{C_2}\right]}$$

C_1 ↓ Constant ρ
change in $\forall$
"flow rate"

C_2 ↓ Constant $\forall$
change in ρ
"compressibility"

- Special Cases :

if the chamber is a fixed volume

$$\left[C_P = \forall \frac{\partial \rho}{\partial P}\right]$$

for ideal gas and isothermal process

ما بتكسب ولا بتفقد
حرارة
نظام معزول (الحرارة فيه ثابتة)

$$P = \rho R T$$

$$\frac{\partial P}{\partial \rho} = R T$$

$$\frac{\partial \rho}{\partial P} = \frac{1}{RT} \Rightarrow \left[C_P = \rho \frac{\partial t}{\partial P} + \frac{\forall}{RT}\right]$$

② pneumatic inertance →

[redacted]

pressure = $\frac{\text{force}}{\text{area}}$

$q = A\vartheta$ and $\dot{m} = \rho q$

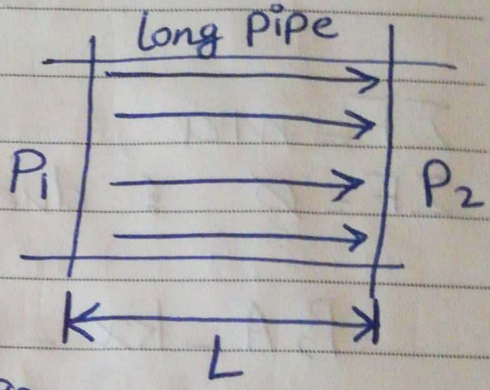
ϑ : velocity

$\Sigma F = ma \rightarrow$

$\boxed{\dot{m} = \rho A \vartheta}$

$(P_1 - P_2)A = m\frac{\partial \vartheta}{\partial t}$

$$\left[\Delta P = \underbrace{\frac{L}{A}}_{I_p}\dot{m}\right]!$$



Ex) Find the I/O relationship

I = input = P_1

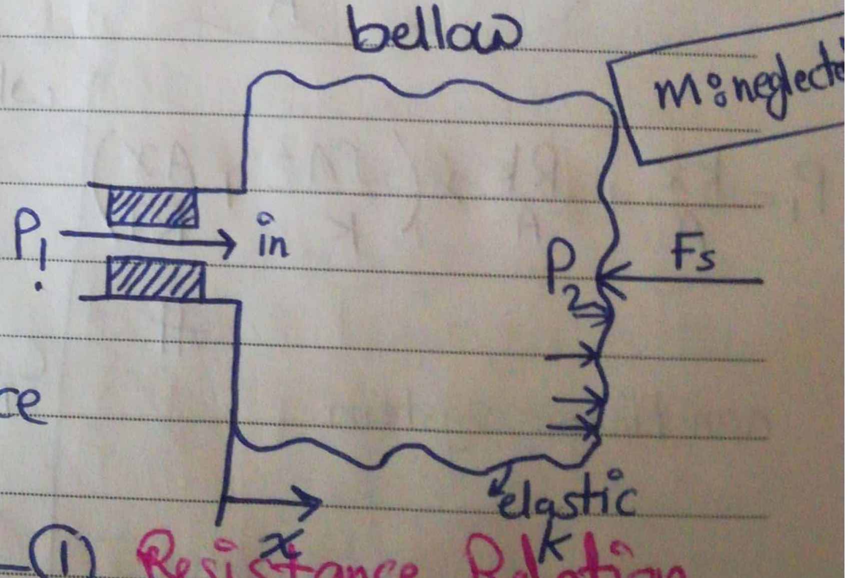
O = output = x

(Pneumatic system transfer func.)

كل الطاقة الخاصة بال elements في النظام ففي هذا المثال

→ Resistance

→ Capacitance



$P_1 - P_2 = R\dot{m}$ ——— ① Resistance Relation

$\dot{m} = C_p\dot{P}_2$ ——— ② Capacitance Relation

$\dot{P}_2$ → pressure inside the bellow

$$\boxed{P_1 = P_2 + R\dot{m}}$$

$$P_1 = P_2 + RC_p\dot{P}_2 \text{ ——— } ③$$

بدي أشوف علاقة لل R مع x

P_1 مع x

P_2 مع x

$\Sigma F = ma$

$\Sigma F = \varnothing$: due to massless

$$P_2A - kx = \varnothing \rightarrow \left[P_2 = \frac{k}{A}x\right] \text{ sub in } ③$$

$$\dot{P}_2 = \frac{k}{A}\dot{x}$$

$$\Rightarrow \boxed{P_1 = \frac{k}{A}x + RC_p\frac{k}{A}\dot{x}}$$

where, $C_p = \rho\frac{\partial\forall}{\partial P_2} + \forall\frac{\partial\rho}{\partial P_2}$

$$\boxed{\forall = Ax}$$

$$C_p = \rho\frac{\partial(Ax)}{\partial P_2} + Ax\frac{\partial\rho}{\partial P_2}$$

$$= \rho A\frac{\partial x}{\partial P_2} + Ax\frac{\partial\rho}{\partial P_2}$$

for ideal gas & isothermal Process

$$\left[C_p = \rho\frac{A^2}{k} + \frac{Ax}{RT}\right]$$

!

$$P_1 = \frac{kx}{A} + \frac{Rk}{A}\dot{x}\left(\frac{\rho A^2}{k} + \frac{Ax}{RT}\right)$$

#

non Linear system

لو ما كان (ideal gas)

بدي أطلع العلاقة بين ال ρ وال P_2

* بالفرض ← بطلع العلاقة

* No info. ← بعتبر C_p ثابتة أو بضلني على العلاقة الأخيرة اللي جبت عليها

* معطى بالسؤال

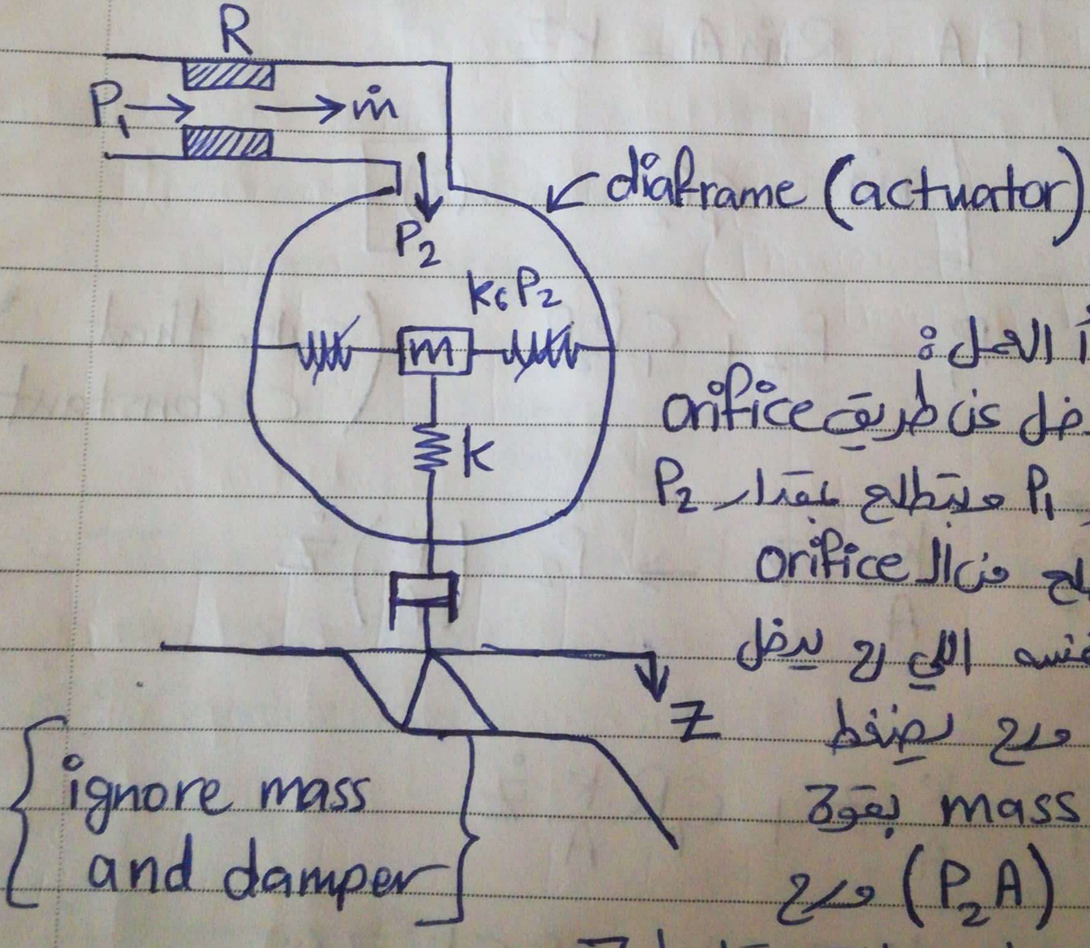
الوحيد اللي بتغير هو (x)

where $\left[P_2 = \frac{kx}{A}\right]$

بشتق وبقسم على $1/\partial P_2$

$$\frac{A}{K}\frac{\cancel{\partial P_2}}{\cancel{\partial P_2}} = \frac{k}{A}\left(\frac{\partial x}{\partial P_2}\right)$$

Ex Find $\frac{Z(s)}{P_1(s)}$?



← مبدأ العمل:
$\dot{m}$ تدخل عن طريق orifice
بمقدار P_1 وبتطلع بمقدار P_2
اللي بيطلع من الorifice
هو نفسه اللي رح يدخل
للخزان ورح يضغط
على ال mass بقوة
معينة (P_2A) ورح
يخليه ينزل مسافة مقدارها Z

No.

$$\Sigma F = ma$$

$$P_2 A - kZ - b\dot{Z} = ma$$

طالما السؤال انتهى

عند ال Element :

Resistance → valve

Inertance → long pipe

capacitance → $\dot{m}$

$$\boxed{P_1 - P_2 = R\dot{m}} \quad \text{—— (1)}$$

$$P_2 = P_1 - R\dot{m} \quad \ldots (2)$$

$$(P_1 - R\dot{m})A - kZ - \cancel{b\dot{Z}} = \cancel{m\ddot{Z}}$$

$$P_1 A - R\dot{m}A = kZ$$

$$\left[\dot{m} = C_p \dot{P}_2 \quad \ldots (3)\right]$$

$$P_1 = P_2 + C_p \dot{P}_2 R$$

(sub. that
C : constant)

$$\boxed{P_2 = \frac{k}{A} \cdot Z} \rightarrow \dot{P}_2 = \left(\frac{k}{A}\right)\dot{Z}$$

$$\therefore \; P_1 = \frac{k}{A} Z + C_p R \frac{k}{A} \dot{Z}$$

$$P_1(s) = \frac{k}{A} Z(s) + C_p R s \frac{k}{A} Z(s)$$

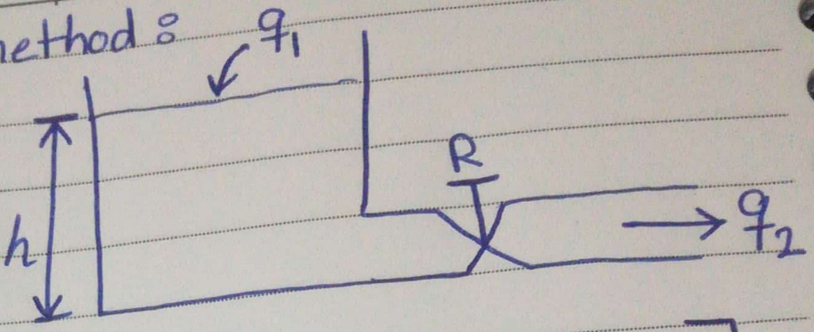
$$\therefore \; \boxed{\frac{Z(s)}{P(s)} = \frac{\frac{A}{k}}{1 + C_p R s}}$$

FIVE APPLE

Scanned by CamScanner

No.

Q: Using lagrange method: Find $\frac{H(s)}{Q_1(s)}$?



$$L = T - V$$

$$= 0 - \frac{1}{2C}\vartheta^2$$

$$\boxed{\begin{array}{c} q_1 - q_2 = A\dot{h} \\ q_2 = q_1 + A\dot{h} \end{array}}$$

$$\frac{\partial}{\partial t}\left(\frac{\partial L}{\partial \vartheta}\right) - \frac{\partial L}{\partial \vartheta} = F_{nc}$$

$$0 + \frac{\vartheta}{C} = -Rq_2 = -R(q_1 + A\dot{h})$$

لأنه عم نسحب ال tank بينا ال pump
بتضيف للنظام

$$\vartheta = Ah$$

ال ϑ تتغير هون

$$\frac{Ah}{RC} = \frac{-Rq_1}{R} - \frac{RA\dot{h}}{R}$$

$$\Rightarrow q_1 = -\left(\frac{A}{RC}h + A\dot{h}\right)$$

$$Q_1(s) = -\left[\frac{A}{RC}H(s) + ASH(s)\right]$$

بعد أخذ H(s) عامل مشترك و توحيد المقامات

$$\therefore \frac{H(s)}{Q_1(s)} = \frac{-1}{\frac{A}{RC} + \frac{RCA}{RC}s} = \frac{RC/A}{1 + (RC)s} = \frac{R/\rho g}{1 + (RC)s}$$

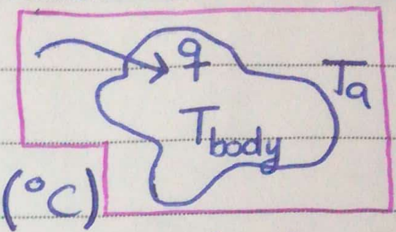
#

FIVE APPLE

No.

⇒ Thermal System

the transfer of heat from one substance to another.



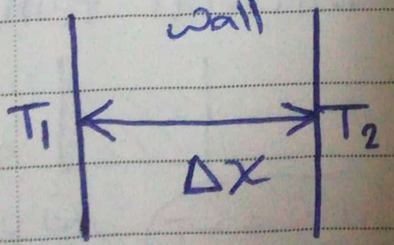
T = body temp. (°C)
T_a = ambient temp. (°C)
q = heat flow rate (k cal/sec.)

describing variables (T, q)

→ Basic Building Elements:
① Thermal Resistance (R_T)
② Thermal Capacitance (C_T) [أدي النظام بيقدر يخزن حرارة]

No Inertance in the thermal

→ Three different ways heat can flow from one substance to another:
① Conduction: $q = k \Delta T$, $k = \frac{Ab}{\Delta x}$



A // مساحة الوسط الناقل
b // thermal conductivity معامل التوصيل
Δx // wall thickness

② Convection : $q = k\Delta T$ ، $k = hA$

تتم عبر ال fluid
عن طريق تيارات الحمل

All area
h // convection
connection
يعتمد ع نوع ال fluid

③ Radiation

Basic building elements →

1] R_T : property of material that indicates its ability to transfer heat.

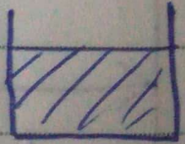
$$R_T = \frac{1}{K} = \frac{1}{Ah} \quad \text{"convection"}$$

$$R_T = \frac{1}{K} = \frac{\Delta x}{Ab} \quad \text{"Conduction"}$$

2] C_T : is the measure of heat energy required to increase the temp. of an object by acertain internal temp.

$$\rightarrow q = \overset{\downarrow}{mc}\,\frac{dT}{dt}$$

$$q = \underline{C_T}\,\frac{dT}{dt}$$

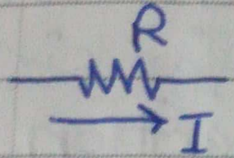


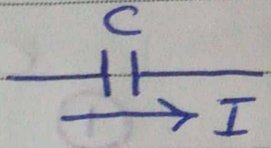
$mC = C_T$

أدو
الي بقدر
يتحمل كزن
حرارة فيه..

m // mass of the substance
c // Specific heat of substance
q // rate of heat stored

Thermal Analogy →





$$\boxed{I = \frac{U}{R}}, \quad q = \frac{\Delta T}{R_T}$$

$$\boxed{I = C\frac{dU_c}{dt}}$$

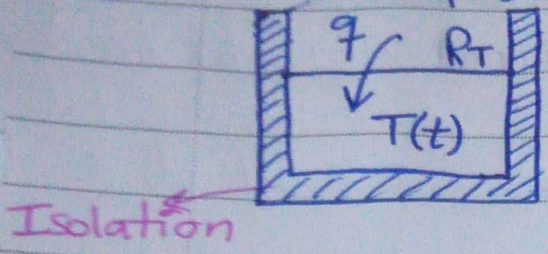
$$q = C_T \frac{\partial T}{\partial t}$$

$$\left\{\begin{array}{l} i \equiv q \\ U \equiv \Delta T \\ R \equiv R_T \\ C \equiv C_T \end{array}\right.$$

[Note : فكرة المماثلة مهمة]

Ex1 Find T_1 ?

T_1=constant



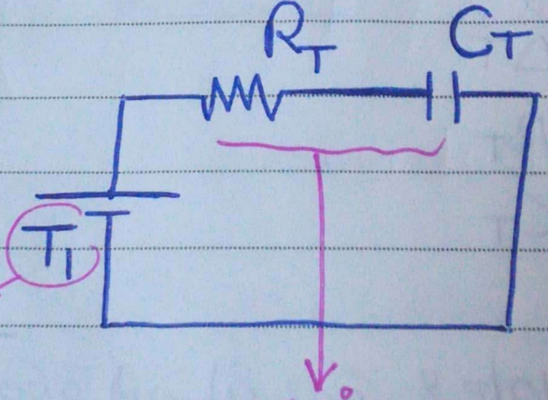
انتقال الحرارة من برّا لجوا خلال السطح المائي تُعتبر ① Conduction

أما خلال المي نفسها بين بعضها تُعتبر ② Convection

①

$$q = \frac{T_1 - T(t)}{R_T} = C_T \frac{\partial T}{\partial t} \quad ②$$

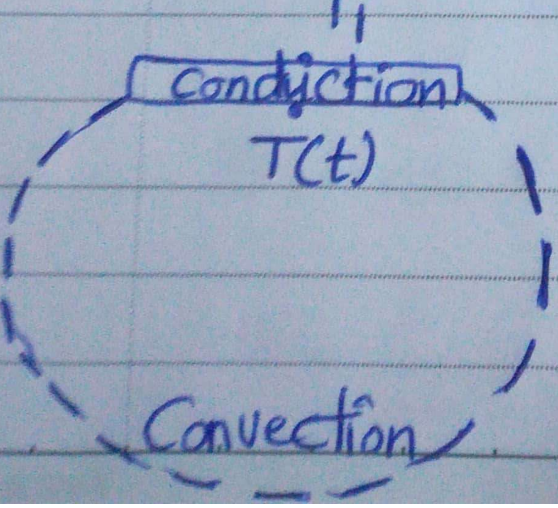
$$T_1 = T(t) + R_T C_T \frac{\partial T}{\partial t}$$



لأنو أنا ما دفعت الحرارة وسببت q

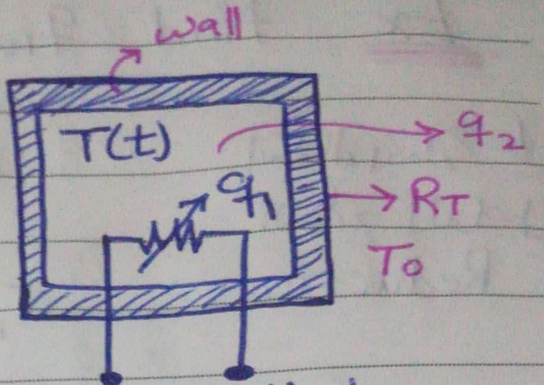
هو فرق الـ T عل تيار
اذن [V]

series
بما إنو نفس الـ q
دخلت وتوزعت عليهم
نفسها



No.

Ex.2 Find q_1 ?



Electrical Heater

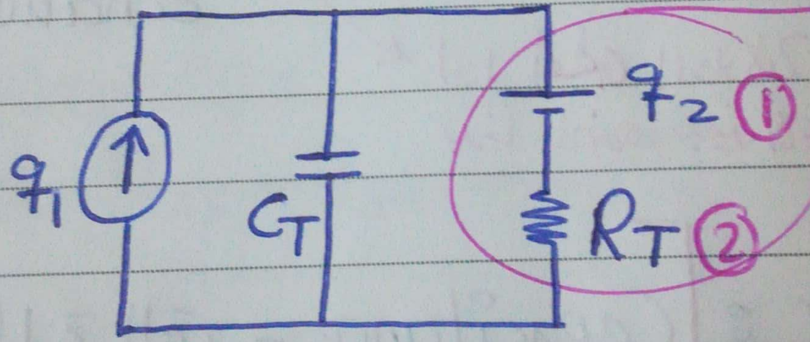
$$q_1 - q_2 = C_T \overset{\xrightarrow{\text{Avg } T}}{\frac{\partial T}{\partial t}}$$

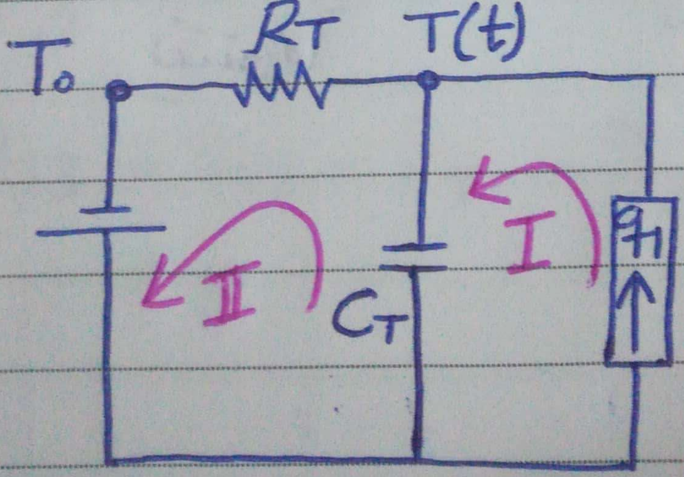
$$q_1 = q_2 + C_1 \frac{\partial T}{\partial t}$$

$$\# \quad q_2 = \frac{T(t) - T_o \;\textcircled{1}}{R_T \;\textcircled{2}}$$

∴ In terms of temp.

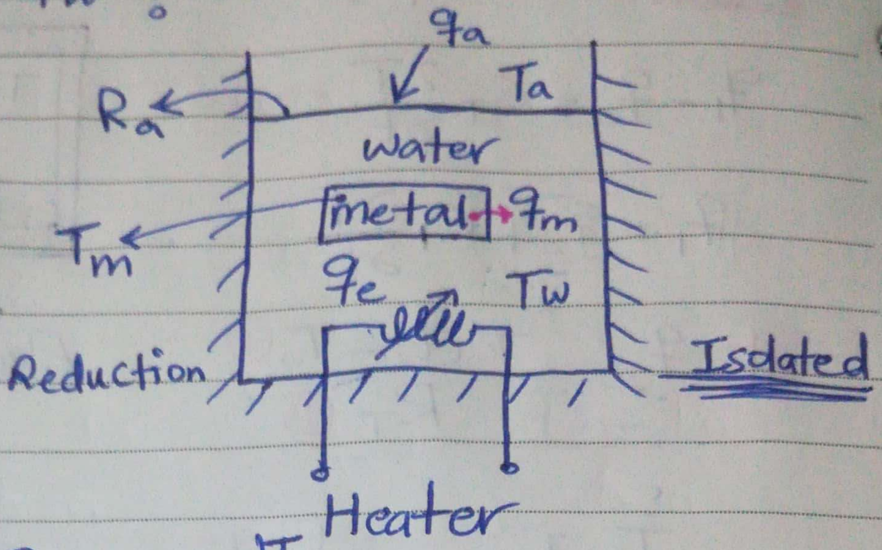
$$\left[\; \underset{\text{current}}{q_1} = \frac{\overset{\text{voltage}}{T(t) - T_o}}{R_T} + C_T \frac{\partial T}{\partial t} \;\right]$$





Ex Find q_w ?

انتقال الحرارة على السطح الفاصل
Reduction



$$q_m + q_a + q_e = C_w \frac{\partial T_w}{\partial t}$$

$$\left(\frac{T_m - T_w}{R_m}\right) + \left(\frac{T_a - T_w}{R_a}\right) + q_e = C_w \frac{\partial T_w}{\partial t}$$

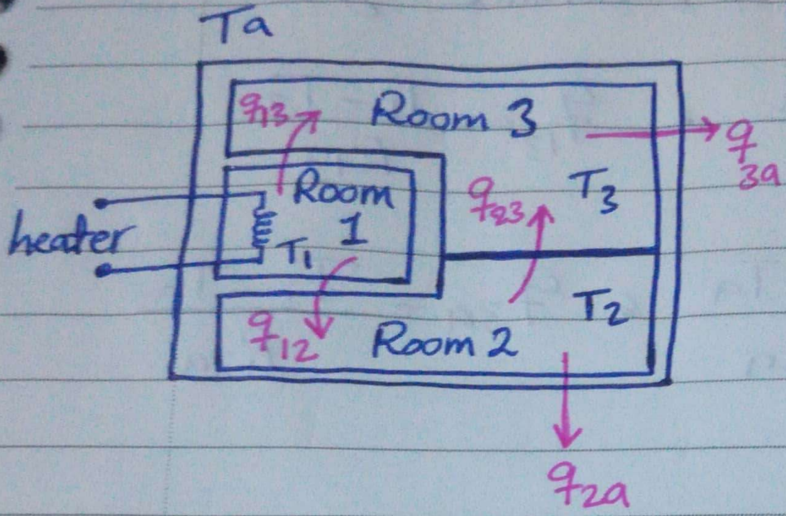
conduction

اذا أعطى المعادلات
بدنا ناخذها بعين الاعتبار

Remark : Capacitance = الحرارة التي يكتسبها - الحرارة التي يفقدها

No.

Ex For the given system
Find T_a ، T_1 ، T_2 ، and T_3 ?



بدي أوجد ال (eq.)
اللي بتربط بين
T_1 ، T_2 ، T_3

* For Room (2) :

$$q_{12} - q_{2a} - q_{23} = C_2 \dot{T}_2$$

* For Room (3) :

$$q_{13} + q_{23} - q_{3a} = C_3 \dot{T}_3$$

⇒ By adding eq_1 ، eq_2 :

$$q_{13} + q_{12} - q_{2a} - q_{3a} = C_3 \dot{T}_3 + C_2 \dot{T}_2$$

No.

عشان نعرف كيف تنتقل الحرارة ←

تنتقل "Wall Resistance"

$$q_{12} = \frac{T_1 - T_2}{R_{12}} \text{ ، } q_{13} = \frac{T_1 - T_3}{R_{13}}$$

$$q_{2a} = \frac{T_2 - T_a}{R_{2a}} \text{ ، } q_{3a} = \frac{T_3 - T_a}{R_{3a}}$$
