تقدم لجنة ElCoM الاكاديمية

تلخيص لمادة

تحكم متقدم

من شرح:

د.سعد العجلوني

جزيل الشكر للطالبة:

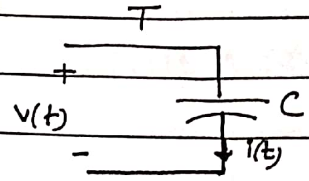
دانيا عمر

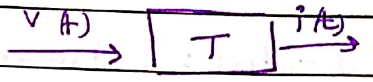
✓ Lec 1:

Review signals vs systems (LTI)

→ A signal: a function Ex (v(t))

→ A system: A transformation





⊛ The capacitor system: $T: v(t) \longrightarrow i(t) = T\{v(t)\}$.

$$i(t) = C\frac{dv(t)}{dt} = C \lim_{\Delta t \to 0} \frac{v(t+\Delta t) - v(t)}{\Delta t}$$

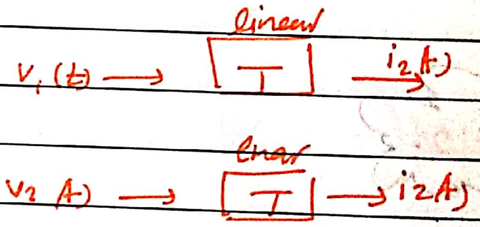
⊛ A linear system satisfies: ① scaling property. $T\{v_1(t)\} = i_1(t)$

$\longrightarrow T\{a v_1(t)\} = a\, i(t)$

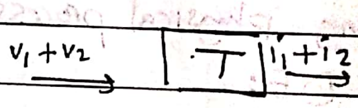
② superposition property.

$T: v_1(t) \longrightarrow i_1(t)$

$T: v_2(t) \longrightarrow i_2(t)$



linear $T: v_1(t) + v_2(t) \longrightarrow i_1(t) + i_2(t)$



Q: Is the system $T_*: x(t) \longrightarrow y(t) = 3x(t) + 1$

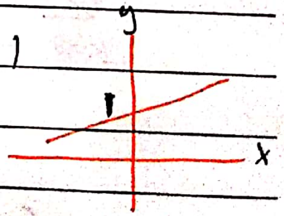
S.P → $T_*: x_1 \rightarrow 3x_1 + 1 = y_1$

$T_*: x_2 \rightarrow 3x_2 + 1 = y_2$

→ $T_*: x_1 + x_2 \overset{??}{=} y_1 + y_2$ ✗

Scaling) $x_1 \longrightarrow y_1 = 3x_1 + 1$

$5x_1 \longrightarrow 3(5x) + = 5(3x_1) + 1 \overset{?}{\neq} 5y$ ✗



TI: if we delay an Input before processing, output will be equal to output delayed after processing.

✓ TI if sys has a time dependent function that is not adirect function of time.

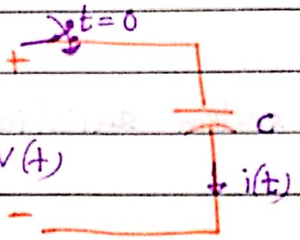
$$x(t) \longrightarrow \boxed{T} \xrightarrow{y(t)}$$

$$x(t-t_0) \longrightarrow \boxed{T} \xrightarrow{y(t-t_0)}$$

TI - test: Assume input: $x(t) \longrightarrow y(t)$

If $T\{x(t-t_0)\} = y(t-t_0)$, then system T is "TI"

$$i(t) \longrightarrow \overset{T}{\boxed{LTI}} \xrightarrow{v(t)}$$



$$v(t) = \frac{1}{C}\int_0^t i(\tau)\,d\tau$$

Lec 2:

Many physical processes (system) can be described as the following DE:

$$\sum_{k=0}^{m} C_k \frac{d^k y(t)}{dt} = \sum_{k=0}^{n} b_k \frac{d^k x(t)}{dt}$$

assume:

$$x(t) \longrightarrow \boxed{TLI} \xrightarrow{y(t)}$$

let $m > n$

where C_k, b_k are constants

② such systems are "LTI" systems.

✓

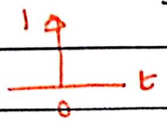
⊛ output of LTI systems due to an arbitrary Input (Convolution)

$$x(t) \xrightarrow{} \boxed{T_{LTI}} \xrightarrow{} y(t) = x(t) * h(t) = h(t) * x(t)$$

$$= \int_{-\infty}^{\infty} x(\tau)\, h(t-\tau)\, d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$$

∴ $\delta(t) \xrightarrow{} \boxed{T_{LTI}} \xrightarrow{} h(t)$: impulse response



⊛ If T_{LTI} is causal then → $h(t) = 0$ for $t < 0$.

→ $\int_{0}^{\infty} h(\tau)\, x(t-\tau)\, d\tau$

⊛ It's hard to analyze a system using convolution.

→ Solution: Use freq domain.

⊛ Definition: Frequency Response $\overset{def}{=} \mathcal{F}\{h(t)\} = H(j\omega) = \int_{-\infty}^{\infty} h(t) e^{-j\omega t} dt$

→ If causal: $H(j\omega) = \int_{0}^{\infty} h(t) e^{-j\omega t} dt.$

$x(t) \xrightarrow{} \overset{T}{\boxed{h(t)}} \xrightarrow{} y(t)$

→ let $\boxed{s = j\omega}$ ⇒ $H(s) = \int h(t)\, e^{-st} dt = \mathcal{L}\{h(t)\}\Big|_{s=j\omega}$

⊛ Freq Response $\equiv H(j\omega) \equiv \mathcal{F}\{h(t)\} = \mathcal{L}\{h(t)\}\Big|_{s=j\omega}$ ⊛

Why do analysis in freq Domain?:-

① Convolution becomes multiplication
in Time Domain ⟷ in Freq Domain

T.D F.D

$$y(t) = h(t) * x(t) \xleftrightarrow{F} Y(j\omega) = \overset{\text{Freq. Response}}{H(j\omega)} . X(j\omega)$$

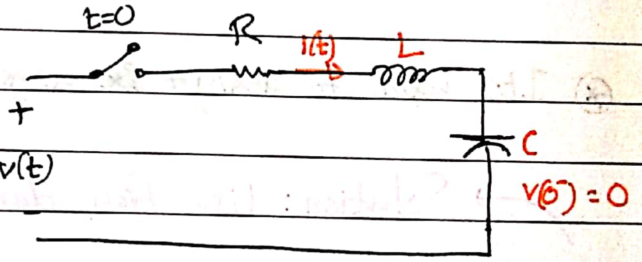
$$\xleftrightarrow{L} Y(s) = \underset{\text{T.F}}{\boxed{H(s)}} . X(s)$$

② DE's become Algebraic Equations

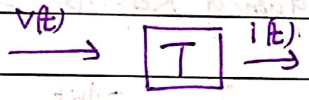
✓ ↱ Table 2.1 and 2.2 pag .36

✓ ↱ Ex 2.1 and 2.2 in text book page : 37.

Ex RLC circuit:



Find a math relationship between input v(t), and output i(t) ?



KVL $v(t) = i(t) R + L \frac{di(t)}{dt} + \frac{1}{C}\int i(t)\, dt$

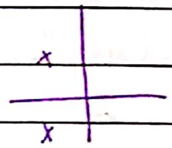
$$\mathcal{L}\left(\dot{v}(t) = \frac{di(t)}{dt} . R + L \frac{d^2 i(t)}{dt^2} + \frac{1}{C} i(t) \right)$$

This is a LTI system then find T.F for the system, i.e, $H(s) = \frac{I(s)}{V(s)}$

$$S V(S) = RS\, I(S) + S^2 L\, I(S) + \frac{1}{C} I(S)$$

$$T.F = \frac{I(S)}{V(S)} = \frac{S}{LS^2 + RS + \frac{1}{C}}$$

lec 3:



→ $S V(s) = RS\, I(s) + LS^2\, I(s) + \frac{1}{C}\, I(s)$

$$H(s) = \frac{I(s)}{V(s)} = \frac{S}{LS^2 + RS + \frac{1}{C}} = \frac{S}{(S+P_1)(S+P_2)}$$

→ P_1, P_2 are complex numbers.

$$\Downarrow$$

P.F.E

$$= \frac{K_1}{S+P_1} + \frac{K_2}{S+P_2}$$

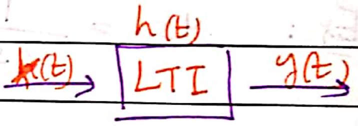
$$\mathcal{L}^{-1} \Downarrow$$

$$K_1 e^{-P_1 t} u(t).$$

⊛ Find $h(t) = \mathcal{L}^{-1}\{H(s)\}$

⊛ In general, how would you find the response (to a LTI sys) using inverse laplace transform?

$Y(s) = H(s)\ X(s)$



steps to get $y(t)$ from $Y(s)$:

① Rewrite $Y(s)$ in the form of $Y(s) = A(s) + \frac{N(s)}{D(s)}$; $A(s), N(s), D(s)$ are polynomials in S

→ * such that the degree of $N(s)$ must be strickly less that the degree of $D(s)$. *

② Conduct a partial fraction expansion (P.F.E) on $\frac{N(s)}{D(s)}$.

→ P.F.E : let $\frac{N(s)}{D(s)} = \frac{(S+z_1)(S+z_2)\cdots(S+z_n)}{(S+P_1)(S+P_2)\cdots(S+P_Q)}$, M, Q

z_i, P_j are complex.

∴ $i = 1, 2, \ldots m$

~~j = 1 Fig~~ Apple

$j = 1, 2 \cdots Q$

✓

Case 1: If all poles are distinct [each pole is of multiplicity = 1]

then → $\frac{N(s)}{D(s)} = \frac{k_1}{s+P_1} + \frac{k_2}{s+P_2} + \cdots \frac{k_Q}{s+P_Q}$

→ $K(j) \Rightarrow [j = 1, 2, \cdots Q]$ are called "residue"

where $K(j) = \left[\frac{N(s)}{D(s)} (s+P_j) \right]_{s \to -P_j}$

Case 2: If a pole, P_j, has multiplicity > 1

Ex $Y(s) = \frac{1}{(s+1)(s+3)^2}$ HW page 40. ✓

Case 1 → Ex: $Y(s) = \frac{1}{s(s+1)}$, find $y(t)$.

Sol: rewrite $Y(s) = A(s) + \frac{N(s)}{D(s)}$, $A(s) = 0$

$N(s) = 1$

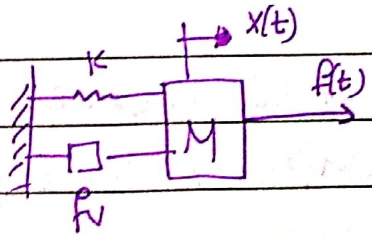
$D(s) = s(s+1)$.

→ $= (1 - e^{-t})\, u(t)$

[Section 2.5 translational mechanical sys:]

table 2.4 and Ex 2.16 ✓ page 62 - 63.

Ex 2.16



⊛ how to find EOM $\xrightarrow{\text{Draw}}$ FBD

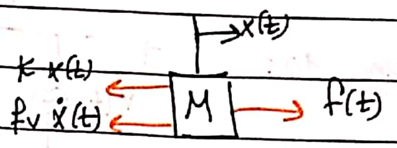
⊛ Assume the mass M is displaced in the +ve displacement direction

④ nominal point: sy'Jega هاتشتغل ال
point معين

2

أول ال steady state هو مفروض zero

FBD →



$\xrightarrow{F(s)} \boxed{TF} \xrightarrow{X(s)}$

$$\Sigma F = M \frac{d^2 x(t)}{dt} = f(t) - f_v \dot{x}(t) - k\, x(t).$$

$M\ddot{x} + f_v\dot{x} + kx = f(t)$, find the $TF = \frac{X(s)}{F(s)}$.

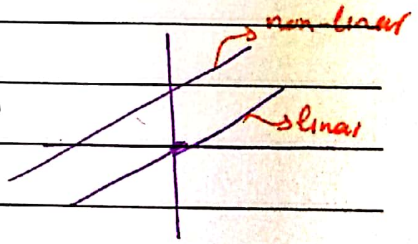
$$TF = \frac{X(s)}{F(s)} = \frac{1}{Ms^2 + f_v s + k}.$$

lec 4:

Sec 2.10: Non linear systems

$\xrightarrow{x} \boxed{} \xrightarrow{y}$

Sec 2.11: linearization:



⊛ How to linearize a non-linear function, f ?

$f: x \longrightarrow y$ $\xrightarrow{x} \boxed{f} \xrightarrow{y}$

① linearization is done about a nominal point, x_0. open page 8.

② expand f about x_0 using taylor Series expansion

→ let $x = x_0 + \Delta x$

$$f(x) = f(x_0 + \Delta x) = f(x_0) + \frac{d\,f(x)}{dx}\Big|_{x=x_0} \Delta x + \frac{d^2 f(x)}{dx}\Big|_{x=x_0} \frac{\Delta x^2}{2!} + \frac{d^3 f x}{dx}\Big|_{x=x_0} \frac{\Delta x^3}{3!} + \cdots$$

$\xrightarrow{\Delta x} \boxed{} \xrightarrow{y = f(x)}$

$$y = f(x) = f(x_0 + \Delta x) \approx f(x_0) + \frac{df(x)}{dx}\Big|_{x=x_0} \Delta x.$$

TS

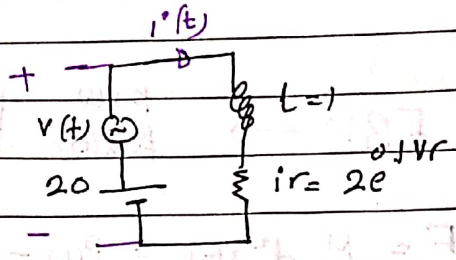
$$\text{let } f(x) - f(x_0) = \Delta f = \frac{df(x)}{dx}\Big|_{x=x_0} \Delta x = m_{x_0} \Delta x.$$

$\xrightarrow{\Delta x} \boxed{} \xrightarrow{\Delta f}$

linear

.✓

↱ Solve Ex: 2.28 page 90:



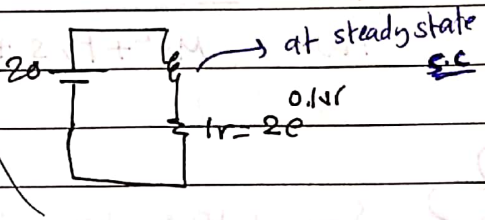
↱ find TF $\frac{V_L(S)}{V(S)}$:-

let $v_0 = 20$ $\xrightarrow{\text{using}}$ KVL:

$$20 + v(t) = \frac{di}{dt} + \underbrace{\boxed{10 \ln \frac{i}{2}}}_{}\ \overset{f(i)\ \text{non-linear}}{}$$

→ [Initial goal: linearization about i_0]

$\therefore\ i_0 = 2e^{0.1(20)} = 14.78$



$\longrightarrow 10 \ln \frac{i_0}{2} = 20$

let $i = i_0 + \Delta i$

↳ $f(i) = f(i_0 + \Delta i) = 10 \ln \frac{i}{2} = f(i_0) + \frac{df}{di}\Big|_{i=i_0} \Delta i$

$$= 10 \ln \frac{i_0}{2} + 10\left(\frac{1}{i/2} \times \frac{1}{2}\right)\Big|_{i=i_0} \Delta i$$

$$= 20 + 10 \frac{1}{10} \Delta i$$

⊛ Recall from KVL: $20 + v(t) = \frac{di}{dt} + 20 + 0.67\,\Delta i$

$$\frac{di(t)}{dt} = \frac{d(i_0 + \Delta i)}{dt} = \overset{0}{\frac{di_0}{dt}} + \frac{d\Delta i}{dt}$$

Eq ⊛ ------→ $v(t) = \frac{d\Delta i(t)}{dt} + 0.67\,\Delta i$, let $\mathcal{L}\{\Delta i(t)\} = \Delta I(s)$.

taking $\mathcal{L}$ ⟶ $V(S) = S\,\Delta I(S) + 0.67\,\Delta I(S)$

$\frac{\Delta I(S)}{V(S)} = \frac{\$}{S+0.67}$,

Goal: $v_L(t) = L\frac{di(t)}{dt}$

$v_L(t) = L\frac{d\Delta i}{dt}$

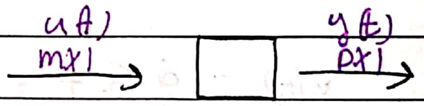
$\boxed{V_L(S) = S L\,\Delta I(S)}$,

$\hookrightarrow \frac{VL(s)}{V(s)} = \frac{s}{s+0.67}$. #

Ch 3 : Intro to modeling in the time-domain

(State-space (s-s) representation).

(HW) Study App. G (see moodle)



⊗ Given a LTI system represented by an nth order ODE; then the (S-S) representation is obtained by rewriting the ODE as follows:

[state-eq] $\underset{n\times1}{\dot{X}(t)} = \underset{n\times n}{A}\ \underset{n\times1}{X(t)} + \underset{n\times m}{B}\ \underset{m\times1}{u(t)}$

[output-eq] $\underset{p\times1}{y(t)} = \underset{p\times n}{C}\ \underset{n\times1}{X(t)} + \underset{p\times m}{D}\ \underset{m\times1}{u(t)}$

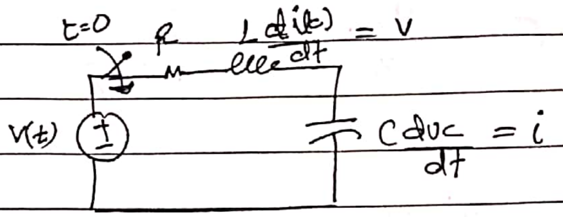
, A, B, C, D are constant matrices.

⊗ state variable : أي var تغير بتغير ال input

lec 5 :

Ex RLC circuit :

$v(t) \longrightarrow \boxed{LTI} \longrightarrow i(t)$



KVL $v(t) = i(t)R + L\frac{di(t)}{dt} + \frac{1}{C}\int_0^t i(\tau)\,d\tau$

$x(t) = i(t)$

$x_2(t) = v_c(t)$

Recall : $i(t) = \frac{dq}{dt} \Rightarrow \int_0^t i(\tau)\,d\tau = q(t)$

$$v(t) = \dot{q}R + L\ddot{q} + \frac{1}{C}q$$

let : $x_1(t) = q(t)$ ①

$x_2(t) = \dot{q}(t) = \dot{x}_1(t) \Longrightarrow \boxed{\dot{x}_1(t) = x_2}$

②

$$\boxed{\dot{x}_2(t) = \frac{-1}{LC}x_1(t) - \frac{R}{L}x_2(t) + \frac{1}{L}v(t)}$$

let $X(t) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

$$\underset{2\times1}{\dot{x} = \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix}} = \overset{A}{\underset{2\times2}{\begin{bmatrix} 0 & 1 \\ \frac{-1}{LC} & \frac{-R}{L} \end{bmatrix}}} \underset{2\times1}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} + \underset{B}{\underset{2\times1}{\begin{bmatrix} 0 \\ 1/L \end{bmatrix}}} \underset{1\times1}{v(t)}$$

$$i(t) = \underset{1\times1}{y(t)} = \underset{1\times2}{\begin{bmatrix} 0 & 1 \end{bmatrix}} \underset{2\times1}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} + \underset{1\times1}{D}\ \underset{1\times1}{v(t)}$$

→ Matricies A, B, C, D give us the S-S representation.

⊗ how to choose state variables:

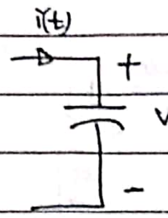
① must be a system variable (any signal that responds to input signal stimulous)

② must be linearly independent (each variable cannot be expressed as a linear combination of the other variable)

↳ جمعناها زي ال super position.

Ex let us say you picked x_1, x_2, x_3 as state variables, If $x_1 = 3x_2 + 2x_3$. variables are not linearly independent.

Ex $x_1 = 2x_2$ linearly dependent



$x_1 = i$

$x_2 = \frac{dv_c}{dt}$

$i(t) = C\frac{dv_c}{dt} \Rightarrow x_1(t) = Cx_2(t)$

linearly dependent.

Note: A set of objects $x_i ; (i = 1, 2 \dots m)$ are said to be linearly independent If:

$$\sum_{i=1}^{m} c_i x_i = 0 \quad , \text{ only if } c_i = 0 \text{ for all values of } \underline{i}.$$

ⓧ يعني اذا ما في linear combination مع بعض ال obj وضربناها بال constants وجمعناهم كلهم لـ variab ال i ضربه constant وجمعناه بيصير أصل ما يطلعه $c_i=0$ يعني لما تكون ال constants = 0 بيكون L.indep

Note c_i ⇒ are constants $(i = 1, 2, \dots m)$

③ A min # of state variables must be selected.

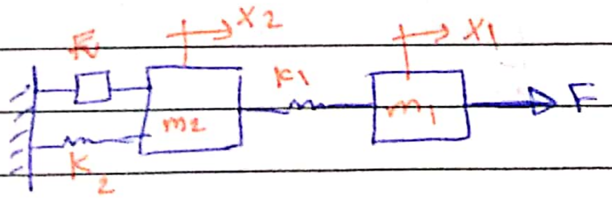
Rules: For a simple linear circuit:

\# of state variables = # of energy storage elements (C, L)

For a translational mechanical system:

\# of state variables = # of DOF × 2

\# of DOFs = # of directions that independent masses move in.



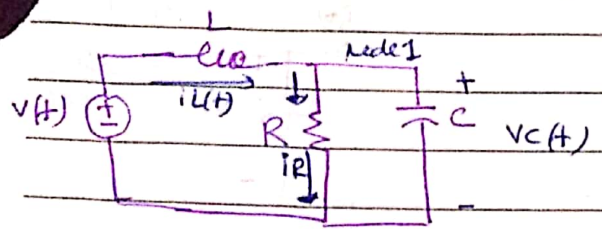
How to choose state variable for:

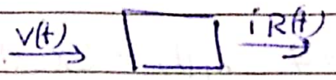
→ 1) circuit: choose the differential variables of energy storage element.

→ 2) Tran. mechanical system:- choose state variables to be the position [displacement] and velocity of each point of linearly independent motion [DOF].

- lec 6

(Ex 3.1 page 124)





Q: find S-S representation:

⊛ state variables

let: $x_1 = v_c$

$x_2 = i_L$

Sol: $L\frac{di}{dt} = V_L = v - v_c \rightarrow \frac{di_L}{dt} = \frac{-v_c}{L} + \frac{v}{L}$

$$\boxed{\dot{x}_2 = \frac{-x_1}{L} + \frac{v}{L}} \; ⊛$$

$C\frac{dv_c}{dt} = i_c$

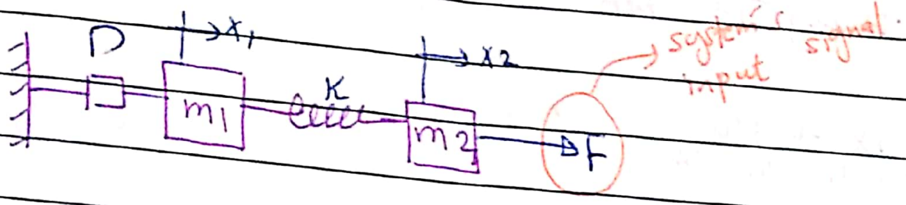
$\dot{x}_1 = \frac{i_c}{c} = \frac{i_L - i_R}{c} = \frac{i_L}{c} - \frac{v_c}{RC}$

$$\boxed{\dot{x}_1 = \frac{-x_1}{Rc} + \frac{x_2}{c}} \; ⊛$$

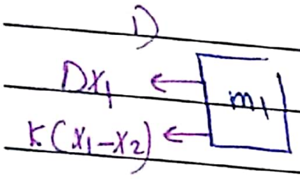
$$\therefore \underbrace{\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix}}_{2X1} = \underbrace{\begin{bmatrix} \frac{-1}{RC} & 1/c \\ \frac{-1}{L} & 0 \end{bmatrix}}_{2X2} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{2X1} + \underbrace{\begin{bmatrix} 0 \\ 1/L \end{bmatrix}}_{2X1} \underbrace{v(t)}_{1X1}$$

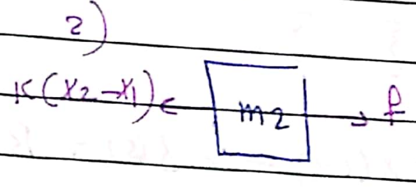
$$\therefore \boxed{\underbrace{i_R}_{1X1} = \frac{v_c}{R}} = \underbrace{\begin{bmatrix} \frac{1}{R} & 0 \end{bmatrix}}_{1X2} \underbrace{\begin{bmatrix} x_1 = v_c \\ x_2 = i_L \end{bmatrix}}_{2X1} + \underbrace{\begin{bmatrix} 0 \end{bmatrix}}_{1X1} \underbrace{v(t)}_{1X1}$$

Ex 3.3 P: 128 →



Sol: draw a FBD:





⊛ we expect the state eq. to have 4 state variables.

M1 ↱ $\sum F_{x_1} = m_1\ddot{x}_1 = -K(x_1-x_2) - D\dot{x}_1$.

M2 ↱ $\sum F_{x_{m2}} = m_2\ddot{x}_2 = f - K(x_2-x_1)$

→ state vector :

$$= \begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix} = X = \begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix}$$

$$\dot{X} = A\,X(t) + B\,f(t)$$

[Back to Book page 128]

$$y = C\,X(t) + D\,f(t) = \underset{2\times1}{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}} = \underset{2\times4}{\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}} \underset{4\times1}{\begin{bmatrix} x_1 \\ \dot{x}_1 \\ x_2 \\ \dot{x}_2 \end{bmatrix}} + \underset{2\times1}{\begin{bmatrix} 0 \\ 0 \end{bmatrix}} \underset{1\times1}{f(t)}$$

$$\underset{4\times1}{\begin{bmatrix} \dot{x}_1 \\ \dot{v}_1 \\ \dot{x}_2 \\ \dot{v}_2 \end{bmatrix}} = \underset{4\times4}{\begin{bmatrix} 0 & 1 & 0 & 0 \\ -K/m & -D/M & K/m & 0 \\ 0 & 0 & 0 & 1 \\ K/m_2 & \cancel{\ }\ 0 & \cancel{0}\ -\frac{K}{m_2} & 0 \end{bmatrix}} \underset{4\times1}{\begin{bmatrix} x_1 \\ v_1 \\ x_2 \\ v_2 \end{bmatrix}} + \underset{4\times1}{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1/m_2 \end{bmatrix}} \underset{1\times1}{\begin{bmatrix} f_1 \end{bmatrix}}$$

$$\boxed{\begin{aligned} \dot{x}_1 &= v_1 \\ \dot{x}_2 &= v_2 \end{aligned}}$$

Sec 3.6 finding T.F from s-s representation.

Given: $\underset{n\times1}{\dot{x}} = \underset{n\times n}{A}\underset{n\times1}{x} + \underset{n\times m}{B}\underset{m\times1}{u} \longrightarrow (1)$

$\underset{p\times1}{y} = \underset{p\times n}{C}\underset{n\times1}{x} + \underset{p\times m}{D}\underset{m\times1}{u} \longrightarrow (2)$

Taking $\mathcal{L}\{\quad\}$

$s X(S) - x(t=0) = A X(s) + B U(s)$

$(SI - A) X(s) = x_0 + B U(s)$

$X(s) = \underbrace{(SI-A)^{-1} x(s) + (SI-A)^{-1} B U(s)}$

Take:

$\mathcal{L}\{Eq(2)\}$ $Y(s) = C(X(s)) + D U(s)$

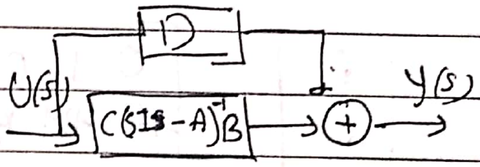
$y(s) = C[(SI-A)^{-1}] x_0 + C[SI-A]^{-1} B U(s) + D(s)$

$= C(SI-A)^{-1} x(0) + [C(SI-A)^{-1} B + D] U(s)$

④ to find T.F, let $x(0)=0$:

$Y(s) = \underbrace{[C(SI-A)^{-1} B + D]}_{\left(\underset{p\times m}{H}\right)} U(s)$

→ system T.F



① let $D=0$ and let $m=p=1$ (SISO)

$Y(s) = C(SI-A)^{-1} B U(s)$

$$\frac{Y(s)}{U(s)} = \frac{C\,\overbrace{Adj\,(SI-A)}^{\text{polynomail in } S}\,B}{Det\,(SI-A)}$$

② في حالة ال MIMO
لا نحكي قسمة $\frac{Y(s)}{U(s)}$

② $m \neq 1$, $P \neq 1$ (MIMO)

$$\underset{P\times 1}{Y(s)} = \boxed{\Bigg[\frac{\underset{P\times n}{C}\ adj\,[SI-A]\ \underset{n\times m}{B}}{\underset{P\times n}{\ }\ \underbrace{det\,[SI-A]}_{n\times n}} + \underset{P\times m}{D}\Bigg]}\ \underset{m\times 1}{U(s)}$$

only in ZOD is special case

$\Downarrow$ $\boxed{P\times m}$

$H_{P\times m}$ is called the system's [Matrix tranfer function.]

$$\therefore \underset{P\times m}{H} = \begin{bmatrix} \frac{N_{11}(s)}{Q(s)} & \frac{N_{12}(s)}{Q(s)} & \cdots & \frac{N_{1m}}{Q(s)} \\ \frac{N_{21}(s)}{Qs} & \frac{N_{22}(s)}{Qs} & \cdots & \frac{N_{2m}}{Qs} \\ \frac{N_{P1}(s)}{Q(s)} & \cdots & \cdots & \frac{N_{Pm}(s)}{Q(s)} \end{bmatrix}$$

T.F مقامها
بعطين ال poles
وبسطها بعطي
ال zero

; $Q(s) = det\,[SI-A]$ ↳ by setting $det\,[SI-A]=0$ we can find Poles.

$$\underset{m\times 1}{W} = \begin{bmatrix} u_1 \\ u_2 \\ \vdots \\ u_m \end{bmatrix}$$

$$y_{P\times 1} = \begin{bmatrix} y \\ \vdots \\ y_P \end{bmatrix}$$

lec 7 ✓ HW Ex 3.6 page 137:

Sec 3.5 : Converting a DE to S.S representation ; and systems implementation using integration.

Ex Given the DE $\ddot{y}(t) + 2\dot{y}(t) + 3y(t) = 7u(t)$.

Sol: let: $x_1 = y$

let: $x_2 = \dot{y} = \dot{x}_1 \longrightarrow \boxed{\dot{x}_1 = x_2}$

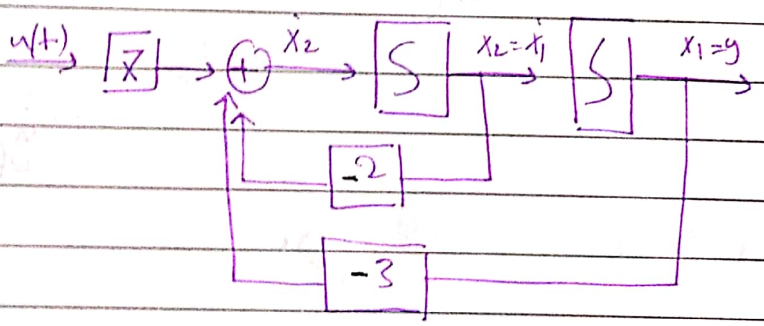
$\dot{x}_2 = \ddot{y} = -3y - 2\dot{y} + 7u$, let: $x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$; $\dot{x} = \begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix}$

$$\boxed{\dot{x}_2 = -3x_1 - 2x_2 + 7u.}$$

$$\dot{x} = \underbrace{\begin{bmatrix} \dot{x}_1 \\ x_2 \end{bmatrix}}_{2\times1} = \underbrace{\begin{bmatrix} 0 & 1 \\ -3 & -2 \end{bmatrix}}_{2\times2} \underbrace{\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}}_{2\times1} + \underbrace{\begin{bmatrix} 0 \\ 7 \end{bmatrix}}_{2\times1} \underbrace{u}_{1\times1}$$

$$y = x_1 = [1 \ 0]\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [0]\, u.$$

In general, given DE: $y(t) + a_{n-1} y^{n-1}(t) + \cdots a_1 \dot{y}(t) + a_0 y(t) = b_0 u(t)$



④ let: $x = \begin{bmatrix} ~~x~~ y \\ ~~x~~ \dot{y} \\ \vdots \\ y^{n-1} \end{bmatrix}$ $\curvearrowright$ S–S represntation will be:

$$\dot{x} = \underbrace{\begin{bmatrix} \underset{n-1\times1}{0} & \vdots & I \\ -a_0 & a_1 & \cdots & a_{n-1} \end{bmatrix}}_{n\times n} x + \underbrace{\begin{bmatrix} 0 \\ 0 \\ \vdots \\ b_0 \end{bmatrix}}_{n\times1} u(t)$$

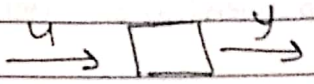
controllable canonical form.

OR

phase-var form.

$$y = [1\ 0\ 0 \ldots 0]\, X + [0]\, U$$

Ex Given $\ddot{y}+2\dot{y}+3y=\dot{u}+u$ ...⊛

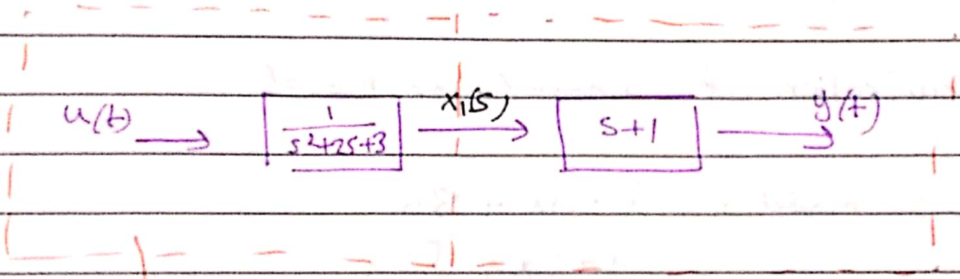


assuming zero I.Cs: $\mathcal{L}\{⊛\}=$

$$[s^2+2s+3]Y(s)=[s+1]U(s)$$

$$\frac{Y(s)}{U(s)}=\frac{s+1}{s^2+3\cancel{s}+2s}=\frac{1}{s^2+2s+3}\cdot s+1$$

$$=\frac{X_1(s)}{U(s)}\cdot\frac{Y(s)}{X(s)}$$



$$\frac{X(s)}{U(s)}=\frac{1}{s^2+2s+3}$$

$\mathcal{L}^{-1}$: $\ddot{x}_1+2\dot{x}_1+3x=u.$

$\dot{x}_2=-2x_2-3x_1+u$

let $x_2=\dot{x}_1\Rightarrow$ state equation: $x=\begin{bmatrix}x_1\\x_2\end{bmatrix}$

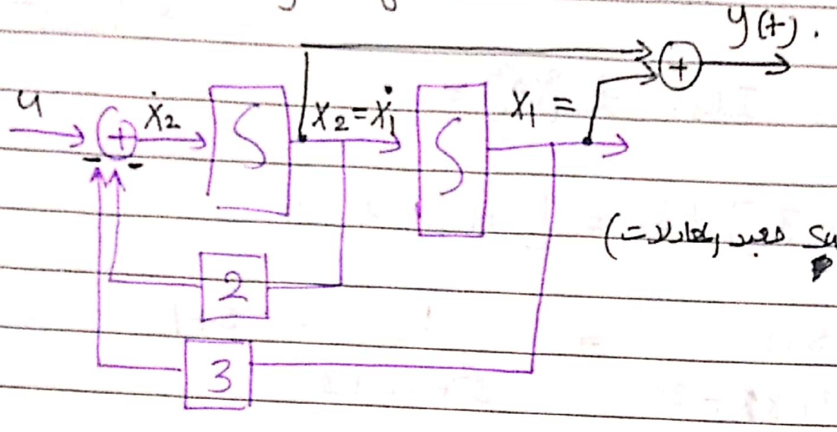
$$\rightarrow\ \dot{x}=\begin{bmatrix}\dot{x}_1\\\dot{x}_2\end{bmatrix}=\begin{bmatrix}0&1\\-3&-2\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}u\ \text{... S-eq}$$ ✓

But $Y(s)=[s+1]X_1(s)\xRightarrow{\mathcal{L}^{-1}}y(t)=\dot{x}_1(t)+x_1(t)$

$=x_2+x_1$

$$\therefore\ y(t)=\begin{bmatrix}1&1\end{bmatrix}\begin{bmatrix}x_1\\x_2\end{bmatrix}\ \text{... output-eq}$$ ✓

Implementation using integration:



(كد ال Summing معبر بالمعادلات)

H.W Ex 3.4 & Ex3.5 in T.B.

lec 8

linearization of a non-linear model:

⊛ linear S-S model : $\dot{x} = Ax + Bu$

$y = Cx + Du$

→ Given the sys of equations:

$$\underset{n\times1}{\dot{x}}(t) = \underbrace{\underset{n\times1}{f}(\underset{n\times1}{x(t)}, \underset{m\times1}{u(t)})}_{\text{non-linear}} \neq A\,x(t) + B\,u(t)$$

and $y = (\underbrace{h(x(t), u(t))}_{\text{non-linear}})$

[around a point of equi.]

∴ we obtain a linear S-S model by linearing about an equilibrium point (x_0, u_0), such that $f(x_0, u_0) = 0$

→ let $x(t) = x_0 + \Delta x(t)$; $u(t) = u_0 + \Delta u(t)$

$$\rightarrow \underset{n\times 1}{\dot{x}(t)} \approx \underset{n\times 1}{f(x_0, u_0)} + \underset{\substack{n\times n\\ \text{matrix}}}{\left.\frac{df}{dx}\right|_{\substack{x(t)=x_0\\ u(t)=u_0}}} \underset{n\times 1}{\Delta x(t)} + \underset{n\times m}{\left.\frac{\partial f}{\partial u}\right|_{\substack{x(t)=x_0\\ u(t)=u_0}}} \underset{m\times 1}{\Delta u(t)}$$

where $\frac{\partial f}{\partial x}$ is a matrix let : $\underset{n\times n}{\frac{\partial f}{\partial x}} = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_1}{\partial x_2} & \cdots & \frac{\partial f_1}{\partial x_n} \\ \frac{\partial f_2}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots & \frac{\partial f_2}{\partial x_n} \\ & & \cdots & \\ \frac{\partial f_n}{\partial x_1} & & & \frac{\partial f_n}{\partial x_n} \end{bmatrix}$

$$\rightarrow \underset{n\times 1}{f(x,u)} = \begin{bmatrix} f_1(x,u) \\ f_2(x,u) \\ \vdots \\ f_n(x,u) \end{bmatrix} ; \quad \underset{n\times 1}{x(t)} = \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_n(t) \end{bmatrix} ; \quad \underset{n\times 1}{x_0} = \begin{bmatrix} x_{0}1 \\ x_0 2 \\ \vdots \\ x_0 n \end{bmatrix}$$

$$\rightarrow \underset{m\times 1}{u(t)} = \begin{bmatrix} u_1(t) \\ \vdots \\ u_m(t) \end{bmatrix}$$

$$\rightarrow \Delta x(t) = x(t) - x_0$$

$$\frac{\partial f}{\partial u} = \begin{bmatrix} \frac{\partial f_1}{\partial u_1} & \cdots & \frac{\partial f_1}{\partial u_m} \\ \vdots & \ddots & \\ \frac{\partial f_n}{\partial u_1} & \cdots & \frac{\partial f_n}{\partial u_m} \end{bmatrix} , \quad B = \left.\frac{\partial f}{\partial u}\right|_{\substack{x=x_0\\ u=u_0}} ; \quad A = \left.\frac{\partial f}{\partial x}\right|_{\substack{x=x_0\\ u=u_0}}$$

$$\dot{x}(t) = \cancelto{0}{f(x_0, u_0)} + A\Delta x(t) + B\Delta u$$

$$\rightarrow \dot{x}(t) = A\Delta x + B\Delta u \quad ; \quad \text{But } \dot{x}(t) = \Delta \dot{x}(t)$$

$$\boxed{\Delta \dot{x}(t) = A\Delta x(t) + B\,\delta u(t)} \rightarrow$$ we can solve for Δx

but $x(t) = x_0 + \Delta x(t)$

Q What would you do to linearize a non-linear equation?

$y(t) = h(x(t), u(t))$

$$y(t) \approx h(x_0, u_0) + \underbrace{\frac{\partial h}{\partial x}\Big|_{\substack{x=x_0\\u=u_0}}}_{C} \Delta x + \underbrace{\frac{\partial h}{\partial u}\Big|_{\substack{x=x_0\\u=u_0}}}_{D} \Delta u$$

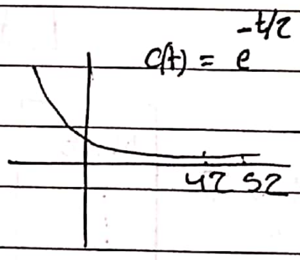
$\Delta y = y(t) - h(x_0, u_0)$

$\Delta y = C\Delta x(t) + D\Delta u(t)$

HW Ex 3.2 ✓
page 139.

Chapter 4 :>

$c(t) = e^{-st} = e^{\frac{-t}{1/5}}$



what is time constant ?!

$e^{-t/\tau} \rightarrow$ time constant ; settling time $= 4\tau = \frac{4}{a}$

for 2nd order system:-

$$T(s) = \frac{K}{s^2 + 2\zeta\omega_n s + \omega_n^2} \Longrightarrow T_s\Big|_{\substack{\text{2nd}\\\text{order}}} = \frac{4}{\zeta\omega_n}, \tau\ \frac{1}{\zeta\omega_n}$$

step response: $c_{\substack{\text{2nd}\\\text{order}}}(t) = \text{S-S value} + \underbrace{e^{-\zeta\omega_n t}\cos(\omega_d + \phi)}_{\text{transient}}$

1st order : stepresponse $= 1 - e^{-at}$

$T_r = \frac{2.2}{a} = 2.2\tau$

$T_s = 4\tau = \frac{4}{a}$

• Lec 9 :

$$R(s)=\frac{1}{s} \longrightarrow \boxed{T(s)=\frac{a}{s+a}} \longrightarrow C(s)$$

$$C(s) = T(s)\,R(s) = \frac{a}{s(s+a)}$$

$$C(t) = \mathcal{L}^{-1}\left\{\frac{a}{s(s+a)}\right\} = \mathcal{L}^{-1}\left\{\frac{1}{s} + \frac{-1}{s+a}\right\} = 1 + -e^{-at}$$

$$= 1 - e^{-t/\tau} \quad ; \tau = \frac{1}{a}$$

after 4τ, $C(t)$ will reach steady state $= \underbrace{C_f(t)}_{\text{forced}} + \underbrace{C_n(t)}_{\text{natural response (transient)}}$

∴ after for $t > 4\tau$, $C(t) \approx C_f(t)$

⊛ for the 1st order system, steady-state step response equals 1. $=$ DC gain

$$\lim_{t\to\infty} C(t) = \lim_{s\to0} sC(s) = \lim_{s\to0} S\,R(s)\,T(s) = \lim S\frac{1}{S}\,T(s)$$

$$\hookrightarrow \lim_{s\to0} T(s) = \lim_{s\to0} \frac{a}{s+0} = \frac{a}{a} = 1.$$

⊛ Steady state response $\equiv$ DC- gain of system. #

↳ Recall freq Response $= T(s)\Big|_{s\to j\omega} = T(j\omega)$

$|T(j\omega)|$ = system gain for an input frequency $\omega = \omega_0$.

DC Gain $= |T(j\omega)=0| = |T(s)=0|$.

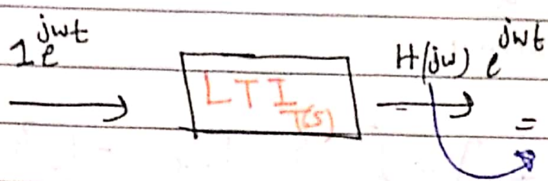
⊛ ال DC gain :

نعوض بال TF

s=0 ونطلع ال DC gain

$C(t) = DC-\text{gain} * \text{scaling of input.}$

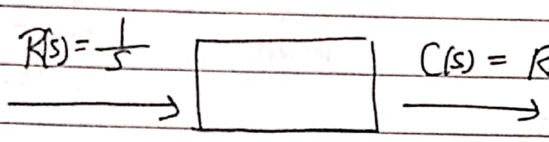
$C(t) = 3(2\,u(t) = 6\;u(t)$



$= |H(j\omega)|\, e^{j\measuredangle H(j\omega)}$

$= |H(j\omega)|\, e^{j(\omega t + \measuredangle H(j\omega))}$

↳ 2nd order underdamped system: $0 < \zeta < 1$



$C(s) = R(s)\;T(s) = \frac{\omega_n^2}{s(s^2+2\zeta\omega_n s+\omega_n^2)} = \frac{\omega_n^2}{s(s+p)(s+p_*)}$

$= \frac{K_1}{s} + \frac{K_2}{s-p} + \frac{K_3}{s+p^*}$

$P = -\zeta\omega_n + j\omega_d$

$= -\zeta\omega_n + j\omega_n\sqrt{1-\zeta^2}$

↳ $s^2+2\zeta\omega_n s+\omega_n^2 = (s-p)\,(s-p)^*$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$\rightarrow (s-1)(s-2) = \text{roots} = (1,2)$

s^2-3s+2

$\mathcal{L}^{-1}\left\{\frac{k_1}{s} + \frac{k_2}{s-p} + \frac{k_3}{s-p^*}\right\}$

$= \left(K_1 + K_2 e^{pt} + K_3 e^{p^* t}\right) u(t).$

⊛ Try an example, to find that $K_3 = K_2^*$

step response $c(t) = \underbrace{k_1\,u(t)}_{c_f} + \underbrace{\left[K_2 e^{pt} + K_3 e^{p^* t}\right]}_{*}$

has a phase shift

★ $[k_2 e^{pt} + k_3 e^{p^*t}]$: C_t (transient response) let $k_2 = Ae^{j\theta}$

$= [A e^{j\theta} e^{-\zeta\omega_n t} e^{j\omega_d t} + A e^{-j\theta} e^{-\zeta\omega_n t} e^{-j\omega_d t}]$

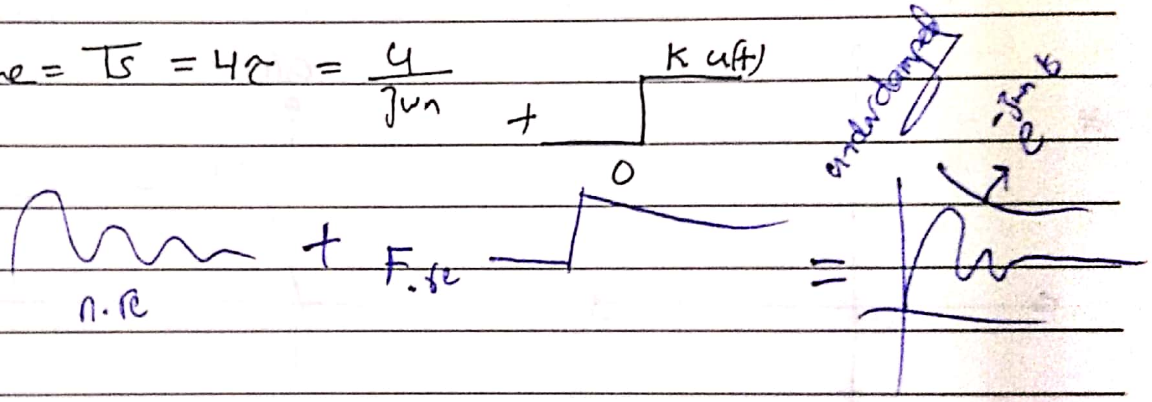
$= A e^{-\zeta\omega_n t} (e^{j(\omega_d t+\theta)} + e^{-j(\omega_d t+\theta)}]$ page 172

$= A e^{-\zeta\omega_n t} \, 2\cos(\omega_d t+\theta)$

$= A e^{-t/\tau} \, 2\cos(\omega_d t+\theta)$, where $\tau = \frac{1}{\zeta\omega_n}$.

→ Settling time $= T_s = 4\tau = \frac{4}{\zeta\omega_n}$

c(t) =



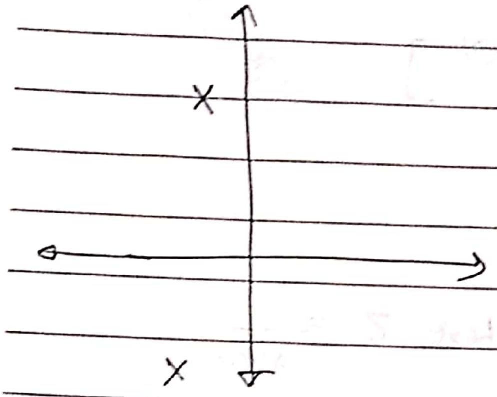
Lec 10: (HW) Ex 4.5 page 178.

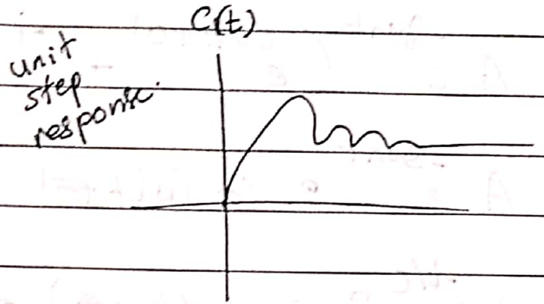
$T(s) = \frac{k\,\omega_n^2}{s^2+2\zeta\omega_n s+\omega_n^2}$; roots of characterstics equation = system poles.

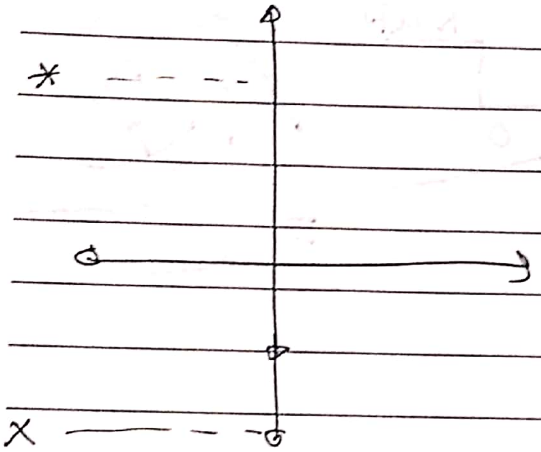
[underdamped system] $0 < \zeta < 1$, $p_1 = -\zeta\omega_n + j\omega_d$
$p_2 = -\zeta\omega_n + -j\omega_d$.

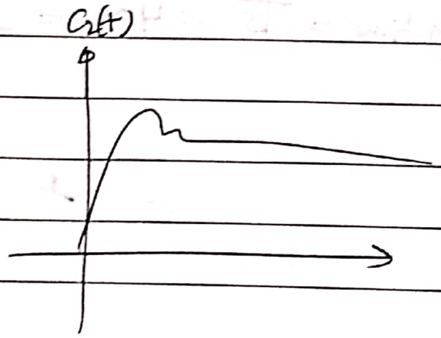
(HW) Ex 4.6 page 180.

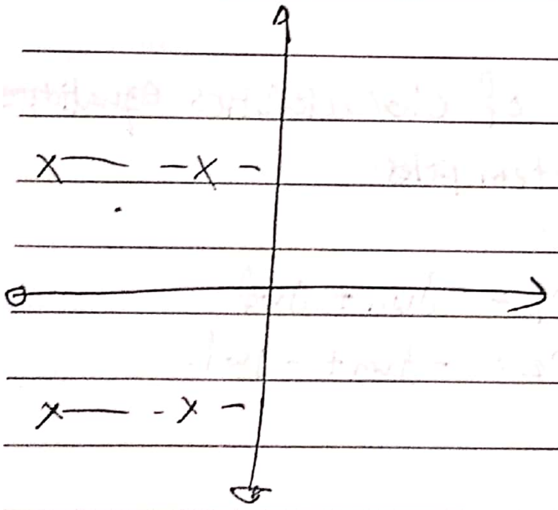
Sec 4.7: Approximating the transient response of a system having more than 2 poles.









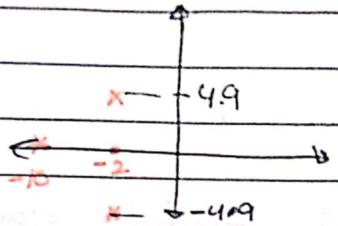


$$T(s)=\frac{K}{\underbrace{(s^2+2\zeta\omega_n s+\omega_{n_1})}\underbrace{(s^2+2\zeta_2\omega_{n_2}s+\omega_{n_2}^2)}}$$

Find step response: c(t):

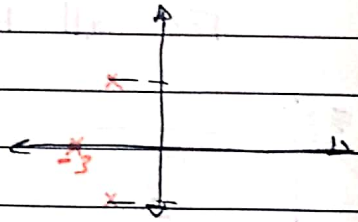
$$C(s)=\underbrace{\frac{K_1}{s}}_{dc}+\left[\frac{K_2}{}+\frac{K_3}{}\right]$$

→ $T_1(s) = \dfrac{24.542}{s^2+4s+24.542}$



$T_2(s) = \dfrac{245.42}{(s+10)(s^2+4s+24.542)}$

$T_3(s) = \dfrac{73.626}{(s+3)(s^2+4s+24.542)}$



→ find 2nd order approximation for T_2 and T_3 :

$T(s) \approx \dfrac{24.542}{s^2+4s+24.542}$ ⟷ $T_2 \approx T_1$ approx $\underline{\underline{A}}$

$T_3(3) \approx T_1$ approx $\underline{\underline{B}}$ ∴ A is Better than B.

Lec 11 : Check lec 11 PDF.

Second:

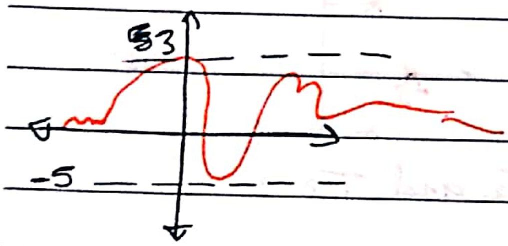
- Lec12:

Ch6:

Defintions:

+ve real number.

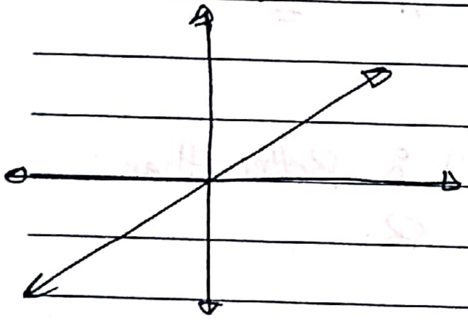
① A bounded signal, $r(t)$, is such that $|r(t)| < B$ For all time, t,.



for all time

$|r(t)| < 5$

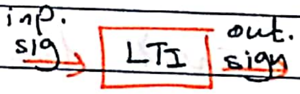
$r_2(t) = (t)$



we can't find a +ve real number, B, that satisfies $|r_2(t)| < B$.

$\therefore$ $r_2(t)$ is unbounded.

as t goes to ∞, $r_2(t)$ goes to ∞



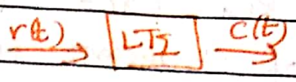
⊛ Stability definitions:

Ⓘ a system is stable if for any bounded input, the output signal will be bounded. [BIBO], That is system is BIBO stable.

Ⓘⓘ A system is marginally stable, if its output is bounded for some choie of input signal, but output is unbounded for other choices of input signals.

ⒾⒾⒾ A system is unstable, if the output is unbounded for any choice of bounded input.

⊛ Stability for LTI systems:



→ Recall that:

$$c(t) = \overset{\text{force}}{c_f(t)} + \overset{\text{transient}}{c_n(t)}$$

↪ A LTI system is stable if transient response ($c_n(t)$) goes → 0 as $t \to \infty$

Ex $T(s) = \dfrac{1}{s^2+2\zeta\omega_n s+\omega_n^2}$; poles of system

$s_{1,2} = -\zeta\omega_n \mp \omega_n\sqrt{\zeta^2-1}$ (general form)

for underdamped case: $= \dfrac{1}{(s+p)(s+p^*)}$

↪ for underdamped system:

$s_{1,2} = -\zeta\omega_n \mp j\sqrt{1-\zeta^2}$

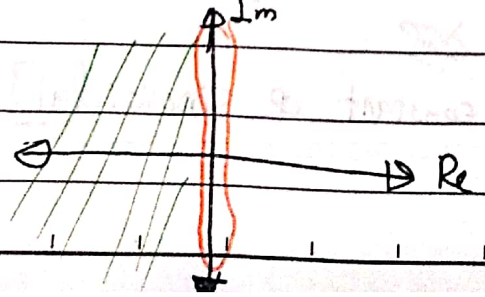
Ex ; $T(s) = \dfrac{1}{s^2+s+1}$

$Re\{1,2\} = -\dfrac{1}{2}$ → stable : $e^{-\frac{1}{2}t}$: expon. decaying.

Ex $T_2(s) = \dfrac{1}{s^2-s+1}$

$Re\{1,2\} = \dfrac{1}{2}$ → unstable : $e^{\frac{1}{2}t}$: expon. growing.

→ A LTI sys is stable if all sys poles have <u>-ve</u> real part. That is, all poles are in the open LHP.



⊗ poles have real Part Less than 0 are stable.
<0

in a stable system

⊗ Real Part of the pole tells us about the exponential decay of (transient response).

transient response

⊗ A LTI system is unstable $c_n(t) \rightarrow \infty$ as $t \rightarrow \infty$
(exponentially grows)

⊗ A LTI Sys is marginally stable if $c_n(t)$ (transient) neither grows, nor decay with time, rather transient remains costant or ossillates as $t \rightarrow \infty$

Note Pole multiplicity:

$$TF = T(s) = \frac{1}{(s+P_1)^{z_1}(s+P_2)^{z_2}}$$

z_2: is the multiplicity of P_2.

expo growing | RHP

→ Unstable systems: have poles with +ve real part and/OR imaginary-axis poles with multiplicity $z \geq 2$.

MS

at least

→ marginally stable systems: have poles on the $j\omega$-axis with multipicity of one, and/OR poles with -ve real part

expo. decaying [LHP].

Note Imaginary-axis Poles with multiplicity z $(z=1,2,\ldots)$ produce a transient response of the form:

$$c_n(t) = \sum_{k=1}^{z} A_k t^{k-1} \cos(\omega_k t + \phi_k)$$

If $z=1$, then system is M.S. [constant or sinosoidal]

{Note} ~~Ana~~ An impulse input $\left[\xrightarrow{r(t)=\delta(t)} \boxed{} \xrightarrow{h(t)}\right]$ ↷ impulse response.

↳ is a special-case theoretical input.

δ(t) is unbounded → ∞

but absolutely integrable.

→ $\int_{-\infty}^{\infty} |\delta(t)|\, dt = 1$, Impulse response of LTI systems explain the transient behaviour of the system.

LTI system is stable if $\int_{-\infty}^{\infty} | h(t) |\, dt < \infty$ (Integrable).

↳ is absolutely integrable.

Ex given $\xrightarrow{R(s)} \boxed{T(s)} \xrightarrow{C(s)}$,

$T(s) = \frac{1}{s^2(s+2)}$, find impulse response.

Sol let R(s) = 1 (r(t) = δ(t))

Impulse response = R(s) T(s) = $T(s) = \frac{1}{s^2(s+2)}$

$$= \frac{0.25}{s+2} + \frac{-0.25}{s} + \frac{0.5}{s^2}$$

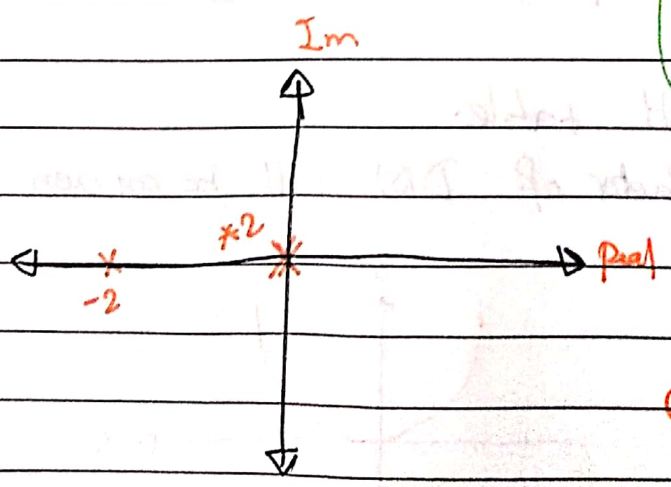
transient due to pole at -2

transient due to double (z=2) poles at origin

$\mathcal{L}^{-1}\{\ \} = (-0.25 + 0.5t)u(t)$.

↳ so: as t → ∞ transient will → ∞

∴ ustable system.



@ unstable due to (z=2) for pole: 0 at origin without any pole at the RHP

- Lec 13:

⊛ Routh-Hurwitz method : (R-H)

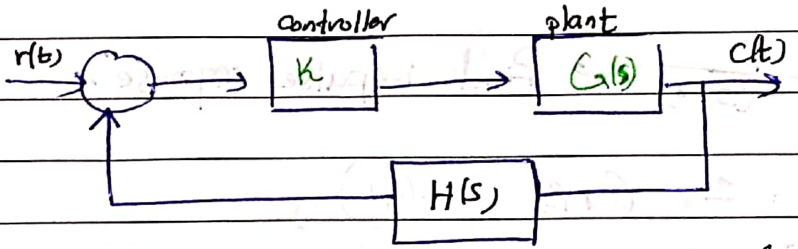
⊛ Determines stability of a system by analyzing the systems characteristic polynomial D(s). مقام ال T.F

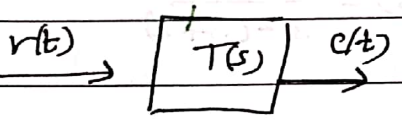
⊛ See table 6.2 page 304. ✓

⊛ R-H method finds No. of poles in open-LHP [Complex-plane], open RHP, and No of poles on jw-axis.

⊛ The strength of R-H method is in the design of stable systems, rather than simply determining poles Location of D(s).

⊛ See ex 6.1 ✓ 304





$\therefore T(s) = \frac{N(s)}{D(s,K)}$

⊛ اذا بدي بس أعرف شو ال poles نلزم المقلاب.

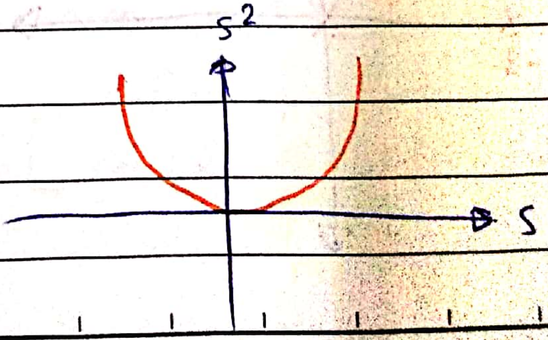
⊛ اذا زي هاد المثال بدنا أعرف قيم K بنلزم R-H

⊛ Special cases of Routh-table:-

1] zero in first column of R-H table.

✓ Ex 6.2 & Ex 6.3 page [306-307].

2] Entire row of zeros in R-H table. means that we have that a factor of D(s) will be an even polynomial.



⊛ من بعد ال row of zeros ا ما نشتغل analysing ال even polynomial فقط

6

⊛ إذا ما صار فيه sign change بعد ال row of zeros معناها انه ال poles على ال jw-axis

Ex1 S^2 is ~~a~~ even polynomial, → even polynomials include even powers of S.

Ex2 S^4+3S^2+2, is even in S.

→ Roots of even polynomials have symmetry about the origin of S-plane. [See Figure 6.5 P: 309] ✓

→ To solve Problem in R-H table, previous row differentiation.

Ex 6.5 ✓ ⊛ No row of zeros in R-H table → No roots on jw-axis (poles)

⊛ Row of zeros in R-H table ← poles on jw-axis.

$\cancel{\rightarrow}$ ⊛ إذا فيه sign change بعد ال Row of zeros معناها انه مش على ال jw-axis

→ Design Example Ex 6.9 (page 316):

③ Given S-S model, how would you build R-H table?

① we find the characteristic polynomial $D(S) = \det(SI-A)$

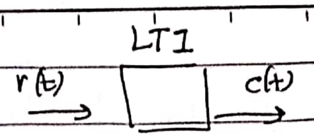
END of # Ch6 T.F. ارجعي

?? Dania check SA 6.2 P: 311.

E(s): -ve real part ⊛
أو pole واحد بال origin
تكون حتى E(s) < ∞
finite.

- Lec 14:

Ch#7:



↳ Definition : error $\overset{def}{=} e(t) = r(t) - c(t)$

define $e_{ss} = \lim_{t \to \infty} e(t)$ [steady state error]

It's desirable that $e_{ss} = 0 \Leftrightarrow \lim_{t \to \infty} c(t) = r(t)$.

↳ final value theorem: (FVT):-

let: $E(s) = R(s) - C(s) \xleftrightarrow{\mathcal{L}} e(t) = r(t) - c(t)$.

$$e_{ss} = \lim_{t \to \infty} e(t) = \lim_{s \to 0} s\,E(s)$$

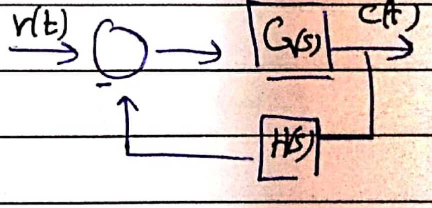
⊛ بيجمع اذا في poles بال LHP

⊛ اذا في poles على ال imaginary غير ال origin [with imaginary part] ف ال theory ما بتصير not valid

↳ FVT: is valid only if E(s) has Poles with -ve real part, and/or poles on the origin of the s-planes $\overset{=1}{}$. If E(s) has more than one poles at origin, then ess will be infinite. ∞

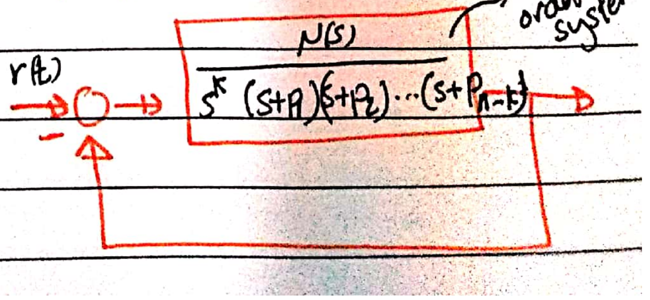
Recall:

$$T(s) = \frac{C(s)}{R(s)} = \frac{G(s)}{1 + G(s)H(s)}$$

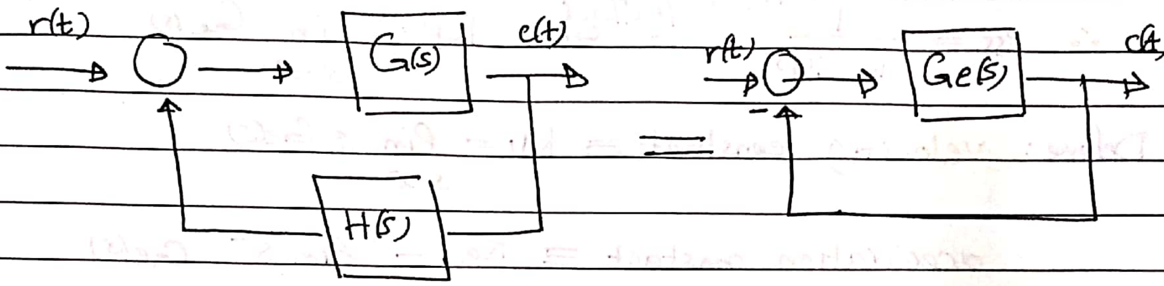


↳ System type: is Number of Pure integrators in the forward path of a unity-feedback system. [power of S]

K is the system type.



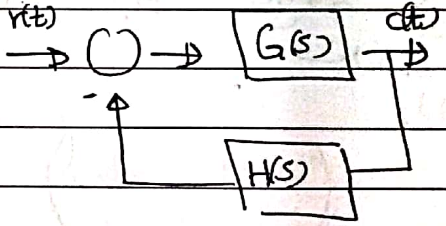
(*) ③ How to determine system type with a non-unity feed back.



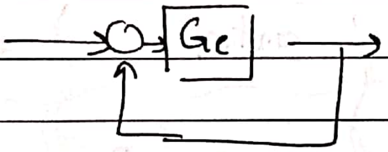
$$T(s) = \frac{G(s)}{1+G(s)H(s)} \Longrightarrow T(s) = \frac{Ge}{1+Ge} = \frac{G}{1+GH}$$

$$\boxed{\therefore Ge = \frac{G}{1+GH-G}} \#\#$$

Ex 7.8 (P354) system type, and ess for a unity-step input.



↳ Sol: to find system type,
Find $Ge = \frac{G}{1+GH-G}$

then 

↳ Given $R(s) = \frac{1}{s}$ (unity-step input)

$$e_{ss} = \lim_{t\to\infty} e(t) = \lim_{s\to 0} s\,E(s)$$

$$= \lim_{s\to 0} s\,[R(s) - C(s)]$$

$$= \lim_{s\to 0} s\,[R(s) - R(s)\,T(s)$$

$$= \lim_{s\to 0} s\,R(s)\,[1-T$$

But $R(s) = \frac{1}{s}$:

$$e_{ss} = \lim_{s\to 0} [1-T(s)]$$

✱ اذا السؤال أعطاني finite $\left[\begin{matrix} K_v \\ K_a \\ K_p \end{matrix}\right.$ قيمة ال sys stable

$$= \lim \left[1 - \frac{G_e}{1+G_e}\right] = \lim_{s\to 0} \frac{1}{G_e+1\,m} = \frac{1}{1+\lim_{s\to 0} G_e}$$

unit step ~~constant~~ input / unity feedback لازم

$$\therefore e_{ss} = \frac{1}{1+K_p}, \quad \text{where } K_p = \lim_{s\to 0} G_e(s)$$

→ position constant (static error constant)

✱ Define: velocitiy constant ≡ $K_v = \lim_{s\to 0} s\,G_e(s)$

acceleration constant ≡ $K_a = \lim_{s\to 0} s^2\,G_e(s)$

✱ table (7.2) (page 348) table assume a unity-feed back

(HW) Skill Assesment Exercise 7.2 P (348)

Ex 7.5 P 349 ✓

بطلع فيه K ونجيب ← Ex 7.6 P 350 ✓ + 7.3 ✓

Range تبع K ال نشيل

stability ال

8 لو

END of ch#7

Input	steady-state error formula	Type 0		Type 1		Type 2	
		static error constant	Error	s-e constant	error	s-e cons	error
step, $u(t)$	$\frac{1}{1+K_p}$	K_p = constant	$\frac{1}{1+K_p}$	$K_p=\infty$	0	$K_p=\infty$	0
$\frac{1}{s^2}$ ← Ramp, $t\,u(t)$	$\frac{1}{K_v}$	$K_v=0$	∞	K_v = constant	$\frac{1}{K_v}$	$K_v=\infty$	0
Parabola, $\frac{t^2}{2}u(t)$	$\frac{1}{K_a}$	$K_a=0$	∞	$K_a=0$	∞	K_a = const	$\frac{1}{K_a}$

✪ ال Poles بال splane بيغيروا موقعهم عن طريق تغير ال static gain variable $0 < K < \infty$
[closed Loop poles]

• lec 15 z

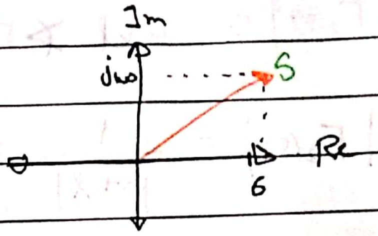
✪ يعني ال gain بيتغير من 0 وبيوصل ل ∞ $0 < K < \infty$. ما بتاع ال Poles لل sys معين.

Ch # 8: Root locus:

✪ لما نغير مكان ال C.L poles بالمسار، ال K تتغير من نقطة معينة وبيوصل لنقطة ثانية معينة

Intro: vector representation of complex number.

Given $s = 6 + j\omega_0$



✪ Mapping a value of s through a function F(s). تعويض قيمة معينة لل s
OR
. ما ندخل ال F(s) [function ال s]

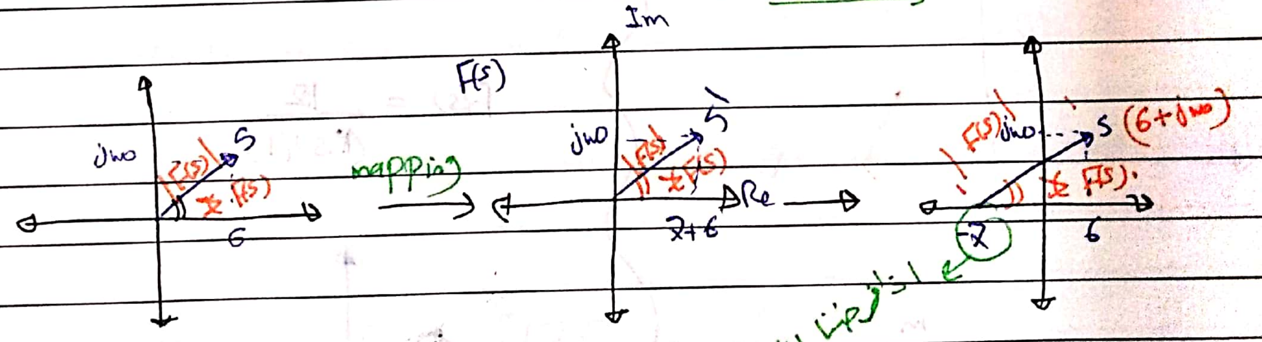
Representation of |F(s)| (magnitude / gain), and $\measuredangle$ F(s) (phase shift), That is:
(gain + phase)

$$F(s) = |F(s)| e^{j \measuredangle F(s)}$$

Ex Given F(s) = s + X, where $s = 6 + j\omega_0$

$$3 = F(s)\Big|_{s = 6 + j\omega_0} = (6 + X) + j\omega_0$$

translation with X units



ال ✪ اذا كان F(s) = TF و ال zero هو -X ✪ وموقع ال poles عن s

✪ حساب الأفقي ال mag + phase
ال function F(s) ← باستخدام أي part
عطلة / بنرسمها عن
ال zero [ال k] لحديث
TF.
التفضية إلي بنرسمها ؟
(6+jw)

Ex 385 + S.A 8.1 : two ways to solve:
① Directly substituting the point in F(s).
② Calculate the result using vectors.

Ex 2 $F_2(s) = \frac{1}{s+X} = \frac{1}{F(s)}$

$|F_2(s)|$?? and $\measuredangle F_2(s)$ given $\boxed{s = 6 + j\nu_0}$

مقلوبه

$|F_2(s)| = \frac{1}{|s+X|} = \frac{1}{|F(s)|}$, $\boxed{\measuredangle F_2(s) = -\measuredangle F_2(s)}$

In general If $F(s) = \frac{\prod_{i=1}^{m}(s+z_i)}{\prod_{j=1}^{n}(s+p_j)}$ then:

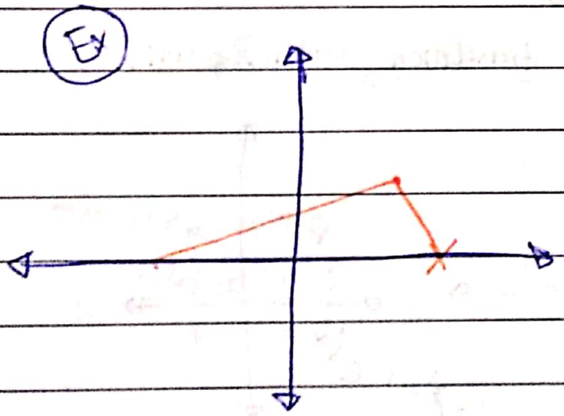
⊛ نرسم للنقطة $\underline{\underline{S}}$ نعرف كل مواقع ال zeros و poles ونرسم خط منهم للنقطة $\underline{\underline{S}}$ → طول الخط هو ال magnitude.

$$|F(s)| = \frac{\prod_{i=1}^{m}|s+z_i|}{\prod_{j=1}^{n}|s+p_j|}$$

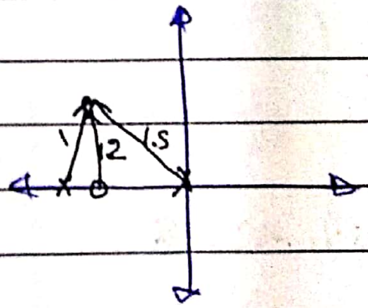
$|s+z_i|$ → [length of $\overrightarrow{V_{z_i}}$ drawn from ~~0~~ at $-z_i$ ~~to~~ to the point $\underline{S}$]

$|s+p_j|$ → [length of $\overrightarrow{V_{p_j}}$ drawn from pole at $-p_j$ to the point $\underline{S}$.]

Ex



$F(s) = \frac{1.2}{(1.5)(1)}$



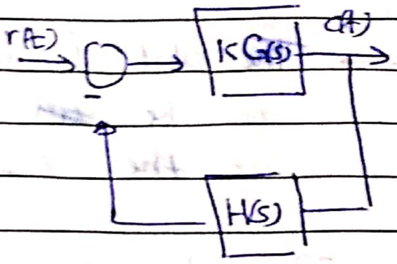
Also $\measuredangle F(s) = \sum_{i=1}^{m}\measuredangle(s+z_i) - \sum_{j=1}^{n}\measuredangle(s+p_j)$

$\boxed{\text{Ex 8.1 page 385}}$ ✓

$0 < K < \infty$

F.W ال مجرى لـه
path

Given C.L $T(s) = \dfrac{K\,G(s)}{1+K\,\underbrace{G(s)H(s)}_{\text{open loop transfer function O.L TF}}}$



O.L system

Root Locus: is the path each C.L pole will $\overset{\text{path, تعبر}}{\text{traverse}}$ as the value of gain $\underline{\underline{K}}$ is increased.

Let $G(s) = \dfrac{N_G(s)}{D_G(s)}$, $H(s) = \dfrac{N_H(s)}{D_H(s)}$

TF ال zeros : هي ال zeros ال G و ال poles ال H

$$\therefore T(s) = \frac{K\,G(s)}{1+K\,G(s)\,H(s)} = \frac{K\,N_G(s)\,D_H(s)}{D_G(s)\,D_H(s) + K\,N_G(s)\,N_H(s)}$$

$$\text{as } K \to 0 ,\quad T(s) \to \frac{\varepsilon z}{D_G(s)D_H(s) + \underbrace{\varepsilon \approx 0}_{\text{very small.}}}$$

TF ال Pole ال

poles of C.L system as $(K \to 0)$ = roots of $D_G(s)D_H(s)$
= poles of $G(s)H(s)$

ⓧ بتبدأ الحركة من عند ال open L. poles = poles of O.L systems.

$$\text{as } K \to \infty ,\quad T(s) = \frac{K\,N_G(s)\,D_H(s)}{\varepsilon + K\,N_G(s)\,N_H(s)}$$

poles of C.L system as $K \to \infty$ = roots of $N_G(s)\,N_H(s)$
= zeros of O.L system $G(s)H(s)$

ⓧ الحركة منتهية صفر

"The life in Corona."

⊛ مثال عيني لربط كلام فيه (S+3) الـ zero عندما نعمل أصفار S=+3 الـ O.L TF دي عرفنا لها نفس التأثير. 13

- Lec 16:-

↳ The plot of root locus starts (when $K \to 0$) at the ~~[illegible]~~ O.L poles, and ends as ($K \to \infty$) at the O.L zeros.

⊛ If O.L system (TF) $G(s)H(s) = \frac{1}{s}$, then:
O.L system has:

① 1 finite pole at $s=0$
② 1 infinite zero at $s=\infty$ [$\lim_{s \to \infty} \frac{1}{s} = 0$ ⊛ same effect on O.L TF as finite zeros in O.L TF.]

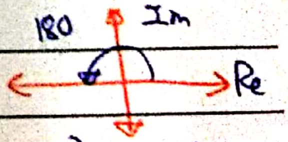
Ex If O.L T.F $G(s)H(s) = \frac{s+1}{(s+2)(s+3)(s+7)}$ [illegible]

Then: $G(s)H(s)$ has:
① 3 finite poles at $-2, -3, -7$.
② 1 finite zero at -1, and 2 zeros at $s=\infty$.
(عدد الزيرو (degree) دا لكل الدالة لازم ناخد فرق الدرجة بين العالي والسفلي)

⊛ Points in S-plane that lie on root Locus:

$$C.L\ TF = \frac{K\,G(s)}{1 + K\,G(s)\,H(s)}$$

poles of C.L system = roots of $[1 + K\,G(s)H(s)]$
= Values of s that satisfy $1 + K\,G(s)\,H(s) = 0$



$\boxed{K\,G(s)\,H(s) = -1}$:

مثلاً k=0

$\Longleftrightarrow |K\,G(s)\,H(s)| = 1$, and $\measuredangle(K\,G(s)\,H(s)) = (2k+1)\,180^\circ$
$k = 0, \pm1, \pm2, \ldots$
$= \measuredangle\, G(s)\,H(s).$

is K دايماً a positive constant

SA: 8.2 ✓

14

⊛ Read 8.5 and study picture 8.13 page 396.
Ex 8.4

∴ ~~Paths~~ points that lie on root locus are values of ⓢ that satisfy:

$$\left[\measuredangle G(s)H(s) = (2k+1)\,180^\circ \;;\; k=0,\pm1,\pm2,\pm\cdots\right]$$

where value of gain K at each point root locus is:

$$K = \frac{1}{|G(s)\,H(s)|} = \frac{1}{|F(s)|} = \frac{\prod_{i=1}^{n} |\vec{VP_i}|}{\prod_{i=1}^{m} |\vec{V_{zi}}|}$$

HW Review [study] Sec 8.4, and (Sec 8.5.) → مسار R.L

⊛ list 1-5 pages 391-392.

pm ⑤

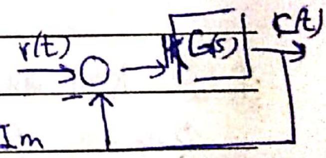
② راجعي الرسم [illegible] ②.②

lec 18:

Ex let $G(s) = \frac{(s+3)(s+4)}{(s+1)(s+2)}$, $H(s) = 1$ → unity feed-back

page 392

$\hookrightarrow$ C.L TF ≡ $T(s) = \frac{K\,G(s)}{1+K\,\boxed{G(s)}}$ → OLTF



⊛ sketching the root locus:

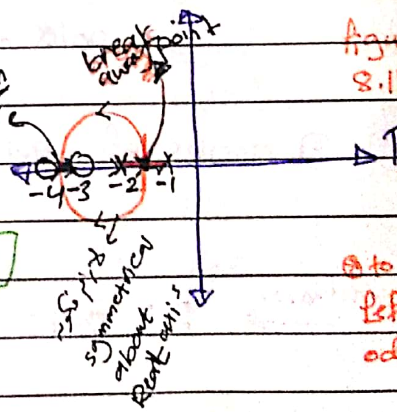


Figure 8.10 P392

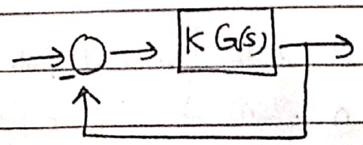
⊛ to the left of odd #s

⊛ Behaviour at Infinity: Solve Ex 8.2 Page 393

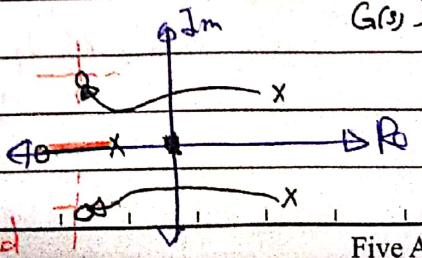
σ_a = نقطة التقاء: real axis intersept ① لازم تذكر ل
② نطلع θ قانون برضو [illegible]

⊛ Angels of arrival / departure

(⊛⊛) Ex



O.L zeros and Poles (zeros and poles of G(s))



R.L to the left of finite odd # of real axis Pole and zero.

• [illegible]

Sec 8.4: Sketching the Root Locus:-

Rules to draw R-L sketch with minimal calculations:
[بتجيبلي الـ Rules رسمة تقريبية وبأقل معادلات وحسابات ممكنة]

① Number of branches: ⓐ each C.L pole moves as the gain K varies ⓐ

ⓐ is the path of that the pole traverse.

WH

Rule: # of ~~poles~~ Branches of R-L sketch = # of CL poles

② Symmetry:

Rule: the R.L is symmetrical about the Real axis

③ Real axis segments:

Rules: ① angular contribution of apair of open Loop complex poles or zeros is zero. [بيلغوا تأثير بعض]

② ⓐ angles of real-axis pole or zeros is [0 OR 180]
→ Right ← left

conclusion → R-L exists to the left of an odd Real-axis Finite pole OR zero

④ Starting and ending points:

الـ C.L pole بيبدأ لما الـ K>0 ~~180~~ من الـ O.L poles
وينتهي عند الـ O.L zero لما الـ K>∞

عشان أحدد ال R-L شو بيصير فيه عند ال ∞
لازم ارسم asymptotes ونطلع ال θ & σ

⑤ Behavior at Infinity: ⊛ If the function approches ∞ as s → ∞ then It has a pole at ∞.

⊛ If the function approches 0 as s → ∞ then It has a zero at ∞

If the open Loop poles are more than open loop zeros then there must be zeros at ∞.
← ⊛ لازم يكون عدد ال poles زي عدد ال zero
عن طريق إضافة بعيدة.

Steps when sketching R-L: go to example 8.2 and S.A 8.3

① Calculate the asymptotes using:

$$\sigma_a = \frac{\sum \text{finite poles} - \sum \text{finite zeros}}{\#\text{finite poles} - \#\text{finite zeros}}$$

② Calculate the angles of the lines of interseet at $\boxed{\sigma_a}$ using:

$$\theta_a = \frac{(2K+1)\pi}{\#\text{finite poles} - \#\text{finite zeros}}$$

→ For different values of K: eg K = 0, 1, 2 ..

← = عدد مرات تعويض K

Note: # of lines = Δ[finite poles − finite zeros].

⊛ to have a better picture of R-L: ① we can find angles of arrival and angles of departure.
→ only for O.L complex conjegate.
for complex zeros. / for complex poles

~~real-axis break-in~~ ~~real-axis break away~~

عشان ② and calculate jω-axis crossings
احدد stability

③ real-axis break away and break in points. → For real-axis poles or zero

⊛ ممكن الـ segments تكون – كلها على الـ Real axis أو جزء منه عليه والجزء بالـ splane

⊛ الـ segments of R-L تكون دائما to the left of an odd Real axis pole OR zero

Lec 18: $\boxed{17}$

Ex 8.7 R-L using matlab page 405:-

note

rlocus(sys)

$sys = \frac{n(s)}{d(s)}$

rlocus(sys) will plot values of s that satisfy $d(s)+K\,n(s)=0$, $0<K<\infty$

$$O.L\ TF = \frac{K(s+15)}{\cancel{s(s+1)(s+10)}} = \frac{K(s+15)}{s^3+11s^2+10}$$ Ex 8.8

→ write using matlab :-

```
fx >> numgh = [1 -4 20]
   >> dengh = [1 6 8]
   >> dengh = poly([-2 -4])
   >> G=tf(numgh, dengh)
   >> rLocus(G)
   >> doc rlocus
```

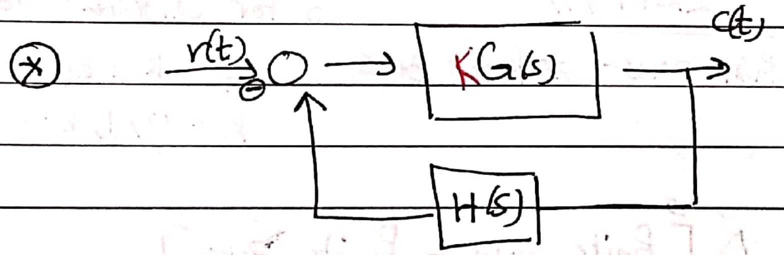
should give same result

نلاحظ عكس الإشارة.

TF بعرف الـ → G=tf(numgh, dengh) → بعطيني جواب مش مربوط بـ gain K

rLocus(G) → RL بيعطيني رسمة الـ function G

doc rlocus → بيعطيني شرح الـ Command



$$C.L\ TF = \frac{KG(s)}{1+K\,G(s)H(s)} \rightarrow ①$$

$G(s)H(s)$: O.L TF

From Lec 15:

⊛ let $G(s) = \frac{N(G(s))}{D_G(s)}$, $H(s) = \frac{N_H}{D_H(s)}$

substute ⊛ in eq ①

$$\rightarrow TF = \frac{K N_G(s) D_H(s)}{D_G(s) D_H(s) + K N_H N_G}$$

$$O.L\ TF = sys = \frac{N_G N_H}{D_G D_H} = \frac{n(s)}{d(s)}$$

$$\rightarrow C.L\ TF = \frac{K N_G D_H}{d(s) + K\,n(s)}$$

⊛ Back to Ch# 4 page 176: علاقة ζ والـ OS% مع T_c $\cos\theta = \zeta$

Back to 179

$\cos\theta \propto \zeta \propto \frac{1}{\theta}$

sgrid(Z, wn)

بيرسملي خطوط الـ ζ damping + الدوائر اللي تمثل الـ wn يكون ثابت عندها.

rlocfind

يعطي الـ gain والـ poles لأي نقطة تحددها.

بيكون غير معرف
Z = 0.45
wn = 0

Back to matLab to solve Ex 8.7 :

ⓐ شرحناه

```
>> doc sgrid
>> sgrid (0.45, 0)
```

يعني بيرسم الخط اللي عنده ζ = 0.45

لو يعني بيمسح الرسم للدائرة بتعتي

```
>> sgrid (0.45, [0.1 : 1])
```

ⓑ [K, P] = rlocfind (G) :

select a point in the plot → بعدها الـ point بيدلي → بعدها يختار الـ K أو P ويجيب لي الـ poles والـ gain

ⓒ command بيرسم breakaway بطلع الـ → صعب لازم يعني تبقى الـ poles اللي اختار p الـ rlocfind كما رأينا.

Advance

→ ⊛ See 8.7: transient Response Design via Gain Adgustment.

⊛ Assuming CL Loop sys is approximated to a 2^{nd} order sys.

→ See figure 8.20 (P.408): لأنه بـ figure (b) الـ pole ابعد عن الـ Im-axis

✓ (b) has better approximation than (a): → من P_3 بـ (a) ممكن كان بعيد تكوين → تأثير أكبر

✓ (d) has better = = (c): → لأنه بعد ما تزيد الـ gain بـ P3 أقرب للـ zero من $P_{3(c)}$

check page 407 and 408

- Lec 19:

How to use MATLAB to draw Root-Locus :

→ Ex 8.8 (p 408):-

→ Open Loop $F(s) = \frac{k(1.5+s)}{s(s+1)(s+10)} = \frac{k(1.5+s)}{s^3+11s^2+10s}$.

1>> numgh = [1 1.5] // عشان ~ أدخل الـج بنظام على شكل معاملات

2>> dengh = poly([0 -1 -10]) // عشان ~ أدخل المقام بهذه الطريقة بنظام

OR 2>> dengh= [1 11 10 0] ع شكل roots

3>> G= TF (numgh dengh) // create a TF (G) and display it.

4>> rLocus (G)// Draw RL of function G.

5>> Sgrid ($\underset{\zeta}{0.8}$ $\underset{\omega_n}{0}$); // To draw a line of specific value of ζ and/or ω_n, here $\omega_n = 0$ 'Coz I don't want to draw it. [ازا بدي ω_n أطلعها مخفية]

6>> [Ki , Pi] = rLocFind (G); // to determine the gain (k) an C.L poles (p) for a selected point on the R-L.

بعد الcommand ↓

اختار نقطة معينة على ال R-L من ال graphic window → بيعطيني selected point X + yi

K // عشان ~ أجيب قيمة ال gain للنقطة ال selected

K = 2.36 ويكتب k

p // " " ال pole " " " p

p = -0.82 + 0.66j

= -0.82 + 0.66j

= 9.25 + 0.00j

كل نقطة رح تعطيني 3poles → 1-C.L pole ; 2 C.L poles (Complex conjugate).

⊛ بكرر العملية حتى ... لما أطلع نقاط التقاطع

⊛ بما أنها ويطلعلي نقطة 3poles تقرب أي وحدة من هاي النقاط أعمل بينها (أو بينها) ال approximation.

Case	Third C.L pole
1	- 9.25
2	- 8.61
3	- 1.8

→ Cancellation بيصير إلها الوحيدة الحالة اللي ← هو ⊗
zero هي لأنه لأنها الوحيدة القريبة عن ال
-1.5 اللي هو

قريبة

7>> Ti = feedback (Ki * G , 1) // This command is used to find C.L TF.

>> T1 = feedback (K * G , 1)

$K_1 = 7.38$ ↳ unity feedback

8>> step (Ti) // Generate closed - Loop response for point select on root Locus.

step (T1) بجرب

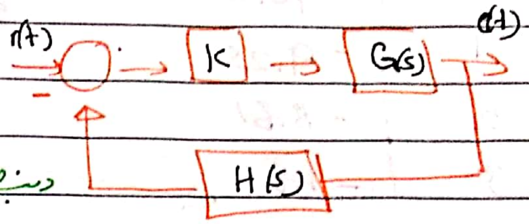
→ generate this step response for the 3points.

Note: This example gives us the overshoot value (1.52%). we can find ζ from the equation $\% OS = e^{-(\zeta\pi/\sqrt{1-\zeta^2})} * 100\%$

OR from figure 4.15 (page 186).

Lec 20 :

Sec 8.8 :- Generlized Root-Locus (see Figure p 412)



$\rightarrow$ C.L TF = $\dfrac{K\, G(s)}{1+K\, G(s)\, H(s)}$ → هاي الصورة اللي احنا متعودين عليها

رح نشوف تغيير غير الـ K شو تأثيره على ال parameter

→ If the function $K\,(G(s)\,H(s))$ is formed as : (for example) :-

$$\underbrace{K\,(G(s))\,H(s)}_{\text{function of } s} = \frac{10}{(s+2)(s+P)}$$

→ let the C.L $\overset{TF}{?}$ as this configuration ↲

$$C.L\ Tf = \frac{\frac{10}{s^2+2s+10}}{1+\underbrace{\frac{P(s+2)}{s^2+2s+10}}_{\to \text{ function of } s}}$$

→ Now; $K(G(s)\,H(s)) \equiv P_1 \ast \dfrac{s+2}{s^2+2s+10}$ ∴ now they have same configuration ⇓

Sec 8.9 Root Locus positive - feedback systems :

- ve feedback $\xrightarrow{sys}$ TF = $\dfrac{K\,G(s)}{1+K\,G(s)\,H(s)}$ → $K\,G(s)\,H(s) = -1 = (2k+1)18^{\circ}$

+ ve = $\xrightarrow{sys}$ TF = $\dfrac{K\,G(s)}{1-K\,G(s)\,H(s)}$ ⇒ $K\,G(s)\,H(s) = 1 = k(360)^{\circ}$

, $k = 0, \pm 1, \pm 2, \cdots$

Ex 8.9 p (414)

skip sec 8.10

End of #8

تكملة
Lec 20:

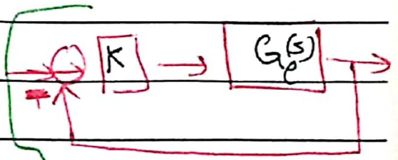
#9: Design via Root-Locus:

In general, increasing the value of K will reduce steady-state error, but overshoot will increase [assuming that C.L system is still stable].

وال } متقبل

Ex step response error $e(\infty) = \frac{1}{1+K_p}$

position error constant



→ as K increases ↑, Kp will increase ↑ → as a result $e(\infty)$ will decrease ↓

$K_p = \lim_{s \to 0} K G_e(s)$

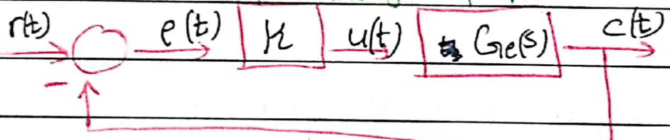
$= K \lim G_e(s)$

constant

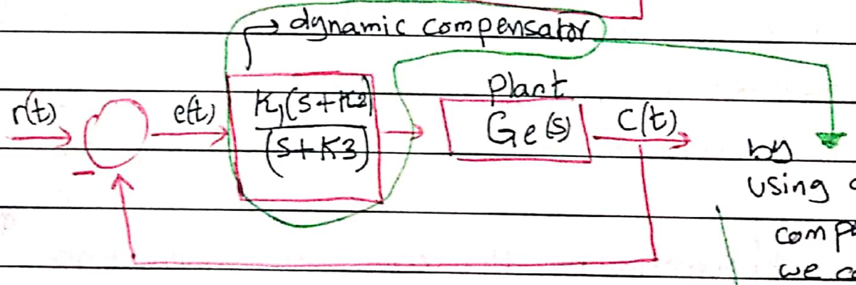
ex = 1

Instead of using a static gain controller (proportional ~~feedback~~ forward controller), we will use feed forward dynamic compensators (might contain zeros and/or poles in their TF).

→ static gain [proportional controller].



$\boxed{e(t) = r(t) - c(t).}$



by using dynamic compensators we can improve both transient and SS error at the same time.

we use dynamic compensators in order to reach the desirable overshoot, SSerror etc values as much as possible.

لـ حطي ده أقرب حد القيم المطلوبة.

مساوئ K:

تغيرها للحصول على قيمة معينة لل (SS.e) لو أثر على قيمة ال overshoot والعكس

dyn comp ⊛ ال overshoot هي parameter بـ ال Transient Response ، عند اضافة أو استخدام
تقدر نتحكم بها independtly عن ال SS. error

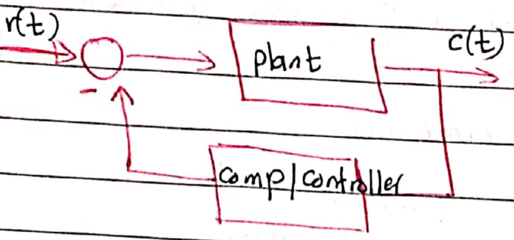
Lec 21:

⊛ Controller $\overset{OR}{\approx}$ compensator

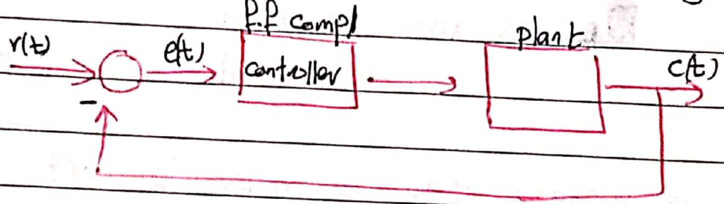
⊛ We have two types of compensator/controller configurations:

1) feedback compensator/controller:

لـ له وجوده ال Feedback.



2) feedforward compesators/controller: takes error signal as an input



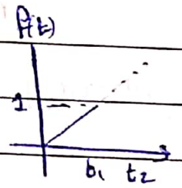
e(t)= r(t)- c(t)

⊛ we will study feed forward compensator design [Configuration 2]

⊛ What effect would you expect on transient response, when error signal is differentiated in the compensator?

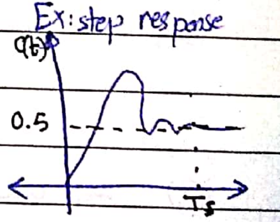
↳ Differentiating the error is expected to reduce settling time and reduce the overshoot in transient response.

[that is; it will improve transient response (يسرّع)]



r(t)=1

Ex: step response



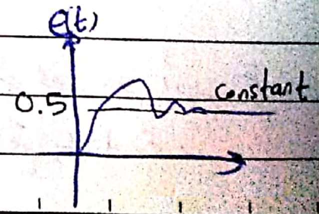
⊛ Note that, differentiating the error signal will not affect (improve) the SS. error (But) integrating the error will improve the steady-state error.

طالع ال error ← constant

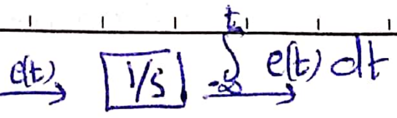
e(∞) = 1-0.5 = 0.5

Conclusion :-

For Feedforward compesators, differentiating e(t) will improve transient response ; however integrating e(t) will improve (reduce) S.S error.



/



[unity feed back ← sys لازم يكون الـ feed.for comp design قبل لا أعمل ⊛]

Lec 22:

(Integral comp مع استخدام SS es↓)

Sec 9.2: Improving SS error using Integral compensation:-

→ fig 9.3 page 454: figure 9.3 :-

⊛ في (b) تغير الـ R-L لتغير الـ TF لاستخدام $\frac{K}{s}$

In (a): استخدمنا (K) فقط وحصلنا على الـ T.R المطلوب ولكن بدي أقلل من الـ error

⇓

In (b): بدنا أقلل من الـ error استخدمنا $\frac{K}{s}$ فقلل الـ error ولكن الـ T.R تغير

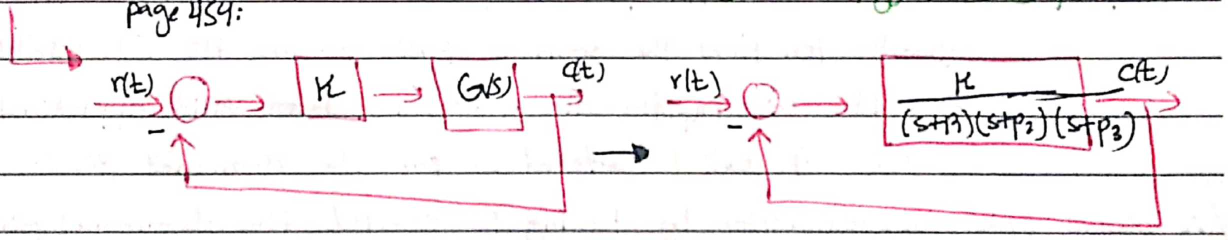
← لا مضافة pole بالـ compen → الـ T.F Open Loop تغير لذلك تغير الـ T.R وبالتالي رسم الـ R-L تغيرت.

⇓

In (c): بدنا أقلل من الـ error وحافظ على الـ T.R أو أقرب شكل إله استخدمنا $\frac{K(s+a)}{s}$ (ضفنا zero)

← الفكرة إنه الـ open loop poles ما رح يتغير → pole إضافي عن الـ origin وضفنا zero بس إنه قريب أكثر بحيث حافظنا على الـ T.R السابق ونقدر نقلل الـ SS-error.

Fig. page 454:

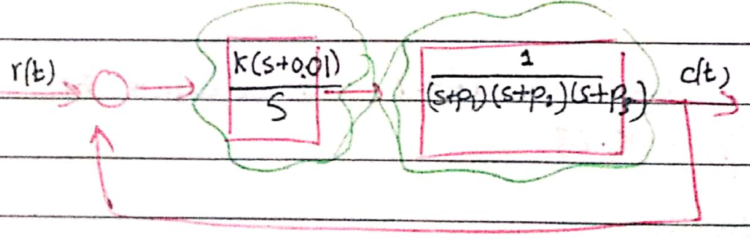


system type is Type 0

K_p is finite, $e_{ss} = \frac{1}{1+K_p}$

→ to get rid of S.S error, we increase system type through cascade compensation [Integral compensation].

System is Type 1



$$K_p = \lim_{s\to 0} G_e(s)$$

$$K_p = \lim_{s\to 0} \frac{K(s+0.01)}{s} \frac{1}{(s+p_1)(s+p_2)(s+p_3)}$$

$K_p = \infty$, $e_{ss} = \frac{1}{\infty} = 0$

(*) If the system is type 0 and kp is finite ~~so~~ to eliminate e(∞) we make the system type 1 by adding a pole integrater.

(*) If the system = type 1 = kv = = → = = = we make the system type 2 by adding = = = . 25

وهكذا لل type 2 بعدها type3

Example 9.1 (p 455):

(*) بدنا ال sys يكون عند قيمة $\zeta = 0.174$ بالتالي بده ال overshoot تكون قيمة معينة

→ use matlab command sgrid to get the line with constant damping ratio.

(*) مطلوب مني احكم بال gain k بحيث يجبروا ال closed loop poles موجودين على نقطة تقاطع رسمة ال root-loc مع ال ζ Line

go to Ch 4 p: 176 (*) ارجع

For $\zeta = .174$ ⟶ overshoot ≃ 60% ← Fig. 4.15

when Fig 9.5(a) ارجع

↳ $k = 0$: C.L poles are the O.L poles

↳ $K = 164.6$: ($-.694 \pm j3.926$ معبر عنها ب 2 complex conjugate poles.) → بصيروا هم ال dominant poles

لـ K تبع ال dominant poles تكونوا منطبقين على ال ζ Line.

Note dominant poles at $P = -1, -2$ and pole at $p = -10$: The third pole is almost ten times the dominant poles so the ~~the~~ T.R results from it will rapidly die. So, the ~~real~~ valid approximation / real T.R will be affected by only the dominant poles.

[i.e second order approximation by taking for granted the dominant poles and neglect the third one at $p = -10$.]

Q: Is it a good approximation? according to Rule of thumb
If the real part of the Dominant poles is 5 times Less than the third pole, then the approximation is a good one

↳ to find S.S error when $K = 164.6$: ① Calculate kp

$$kp = \lim_{s \to 0} K G(s) = 8.23$$

② $$e(\infty) = \frac{1}{1+kp} = \frac{1}{1+8.23} = \boxed{0.108}$$ → فهنا عشان نطفي ال error لازم نزيد نوع ال syst type او نضيف zero ← figure (b)

Figure (a): uncompensated

Figure (b): compensated

→ after Calculating the error for figure (a), go to figure (b) to elemindte the error and keep the T.R.

↓ add apure Integ ↓ add a zero

⊛ after changing the O.L T.F to reach the same result at $\zeta = 0.174$ the gain changed from 164.6 to 158.2 and the 2 complex Conj changed from $-0.694 \pm j3.926$ to $-0.688 \pm j3.832$ with the same Root-Locus (نفسها).

9.6 (b) شرح

Note that at $k = 158.2$, the added pole (fourth pole) is at point $p = -0.0902$

[very close to the added zero at $z = 0.1$ [لذلك لغت تأثير بعض] وصار كأننا ما ضافة ال Zero القريب.

⊛ Now It's a fourth order system but can be approximated easilly

the third Plose at $p = -10$ [بعيد وأثره صغير نفس قبل]

→ the fourth pole [أثره الغى] زي ما ذكرنا فوق

بضل بنحكي ال 2poles complex هم ال dominant poles.

⊛ Go to Fig 9.2: step respone

مزنة ال response لل compensated و ال uncompensated

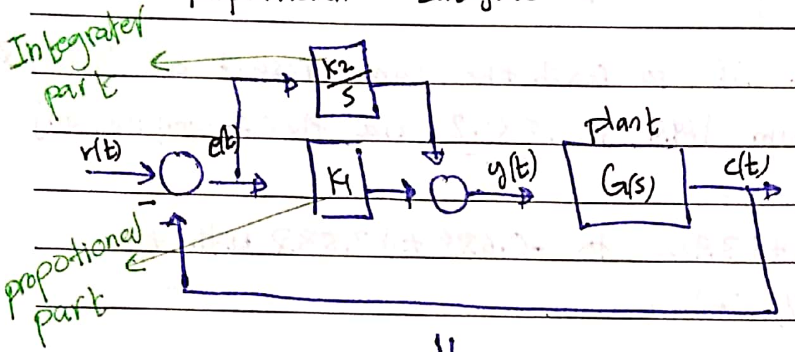
ال step input ← ال desired input = 1 ∴ لل uncomp ← $e(\infty) r(t) - c(t) = 0.108$

∴ لل compe ← $e(\infty) = 0 \rightarrow r(t) = c(t) = 1$

⊛ Both have same overshoot and Similar settling time

(*) Ideal Integral cascade compensator $\left(\frac{K(s+a)}{s}\right)$, is also called a PI-Controller

proportional   Integral.

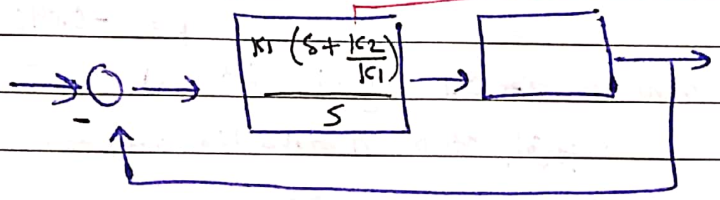


$$Y(s) = E(s)\left[K_1 + \frac{K_2}{s}\right]$$

$$\frac{Y(s)}{E(s)} = \left[K_1 + \frac{K_2}{s}\right]$$

$$= \frac{K_1\left(s + \frac{K_2}{sK_1}\right)}{S}$$

Ideal P.I controller



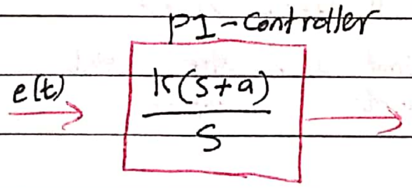
من الكتاب مثال

$$\left[\frac{\overset{\sim 15}{158}(s+0.1)}{s}\right]$$

Lec 23:- Ideal PI. compensator includes pure integration. (s in the denominator of compensator T.F)

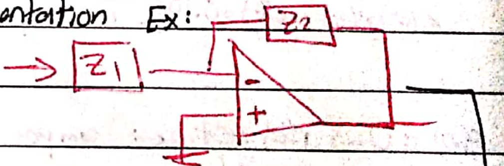
(*) To implement (realize) a PI Compensator, an active element is required such as an OP-Amp. (PI-controller)

PI-controller



[(*) التطبيق هنا ل function تحتاج لل cct معينة]

→ implementation Ex:



لما بدي cct تطلب وجود operational Amplifier (active element)

requires external power

External P.S power source ← تزويد مصدر

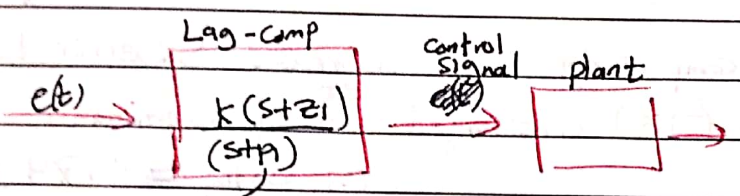
⊛ احيانا ما نقدر استخدم external power source عشان اطبق الcontroller زي كدي

وقتها نستخدم نوع من الAlternative

⊛ Alternative to using OP-Amp (active element) in realizing a PI-Controller is to use something called a Lag compensator

[non-ideal PI-controller]

لا تغيّر Freq. response
بيكون ~ في phase delay
+
لا يملك pure integrator



$p_1 > 0$

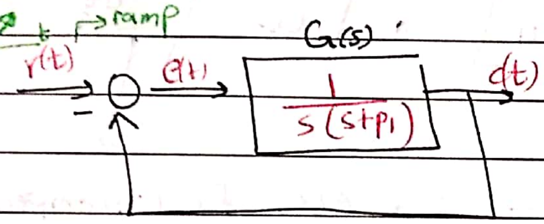
[does not have pure integrator]

∴ It doesn't increase system type.

⊛ go to book page 458, see figure 9.9

ramp

Ex



∴ K_v is finite (non-zero value)

S.Se = e∞ for a ramp input

$$e(\infty) = \frac{1}{K_v} = \text{finite (non-zero value)}$$

↳ $K_v = \lim_{s\to 0} s\,G(s)$, In general increasing the value of any type error constant (K_p, K_v, K_a), deecreases $e(\infty)$.

⊛ الفكرة من استخدام الLag.Comp
هو انه أثر على قيمة الerror constant
الي تتناسب بشكل عكسي
مع e(∞)
زيادة الgain (K) بيقلل من الerror
وبالتالي لا نحتاج لتغيير الtype
للتخلص من الerror
مثل عند استخدام الideal.comp

from page 459:

with Lag-comp ← → without

$$K_{v_{new}} = K_v \frac{z_c}{p_c} > K \quad , \quad e(\infty) \propto \frac{1}{K_v}$$

then: $e(\infty)_{new} < e(\infty)$

See figure 9.10 p: 459:

نفس الشي ~ نحافظ على ال T.R أو رسمة ال
R-L المطلوبة بتضبيط ال zero و ال pole في حالة ال
Lag.comp بـ ابعاد قصيرة لاماكنه من ال poles الأصليين
بحيث ما يأثروا على ال SS.Rep بس بنفس الوقت
تأثر في e(∞)

⊛ Note: for the Lag Compensator the added zero and pole $\boxed{z > p}$

in this ex $\boxed{z = 0.111 > p = 0.01} \to$ Lag

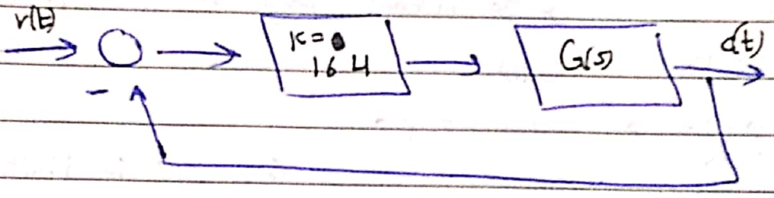
Ex 9.2 Lag Compinsator design for type 0 system:-

Notes: For type zero system increasing the static error cons. (Kp) will decrease SS error.

Goal: is to design a Lag compensator that decreases S.S error by a factor of 10 but maintains transient Response (T.S) unchanged [ζ of dominant C.L poles = 0.174].

We started with:

⊛ Uncompensated system with C.L poles having ζ = 0.174



⊛ for type 0 sys.: $e(\infty) = \frac{1}{1+K_p}$ → e(∞) For Uncompensated sys

$= \frac{1}{1+K_{P_{old}}} = \frac{1}{1+8.2} = 0.108$

$\to K_{P_{old}} = \lim_{s\to 0} KG(s) = \lim_{s\to 0} \frac{164 * 1}{(s+1)(s+2)(s+10)}$

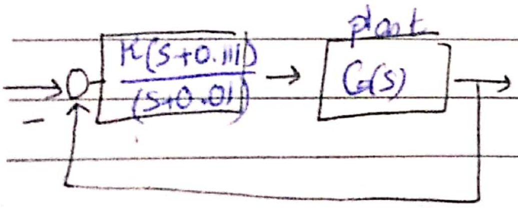
$= 8.23$

→ now to reduce the error by (10) times $e(\infty)_{new} = 0.0108$

⊛ After using Lag Compensator, we want $e(\infty)_{new} = 0.0108$

$e(\infty) = 0.0108 = \frac{1}{1+K_{P_{New}}}$ then, $\boxed{K_{P_{new}} = 91.6}$

compensated system using lag comp:



⊛ نختار قيمة ما للـ pole بحيث تكون قريبة من ال Im-axis، مثلاً صفر

then ← $P_c = 0.01$

وبناءً عليها اخترنا قيمة ال zero نحسب:

$\frac{Z_c}{P_c} = \frac{K_{P_{new}}}{K_{P_{old}}} = \frac{916}{8.23} = 11.13$

P_c ← أنا اختارها

⊛ check Figure 9.13 page 461

⊗ for Lead compensators $\boxed{P > Z}$

Lec 24:

Check 9.6 section, page 495 + 496 (op-Amp), $\hat{G}_c$, eq) + 498 → table 9.11 (Lag .Comp)
↳ table 9.10

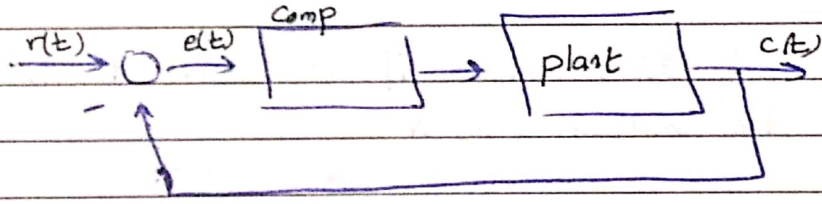
⊗ Note from table 9.11 T.F for the Lag Comp:

$$\overset{K}{\frac{R_2}{R_1+R_2}}\;\frac{s+\overset{Z_1}{\frac{1}{R_2C}}}{s+\underset{P_1}{\frac{1}{(R_1+R_2)C}}}$$

Similar

⊗ From previous Lec Lag Comp was:
$$K\,\frac{(S+Z_1)}{(S+P_1)}$$

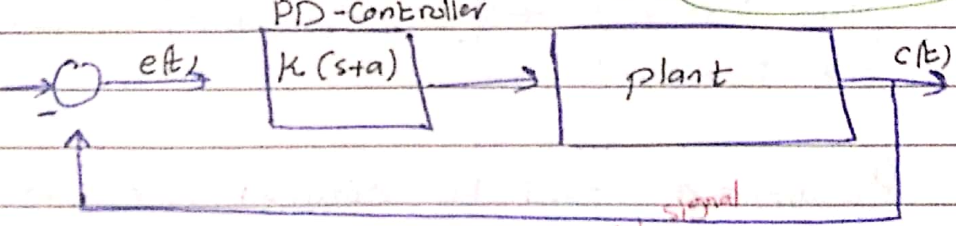
Sec 9.3: Improving Transient Response: Via Cascade Comp.

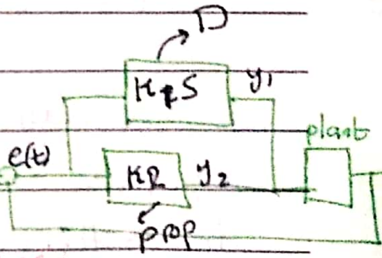


⊗ Improving the T.R usually means decreasing settling time $T(s)$ and decreasing rise time $T(r)$, while maintaining a small overshoot.

⊗ Ideal derivative compensator (PD controller) → gain + zero
proportional, derivative

PD-Controller





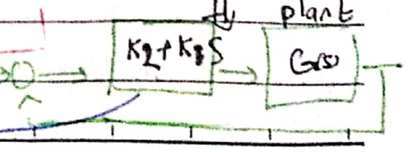
signal
true physical

⊗ Note: if the error $e(t)$ = 1 $\sin(t)$ + 0.1 $\sin(~~300~~\ 27.50\ t)$
→ noise term.

↪ $e(t) \approx \sin(t) + 0.1 \sin 314t$

↪ $\frac{de(t)}{dt} \simeq \cos(t) + 31.4 \cos 314t$

$$= K_1\left[S+\frac{K_2}{K_1}\right]$$

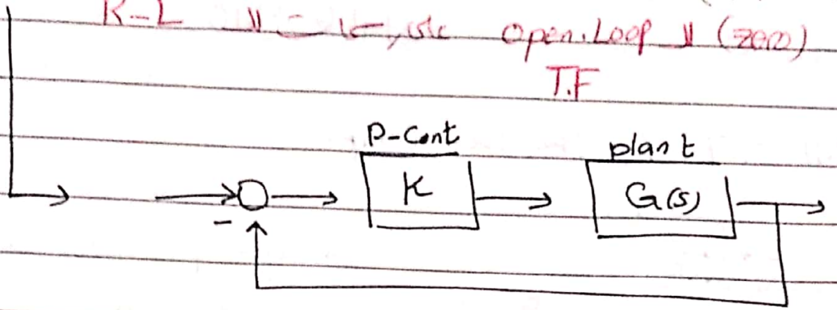


∴ differentian can Lead to amplifying the noise signal (high freq component)

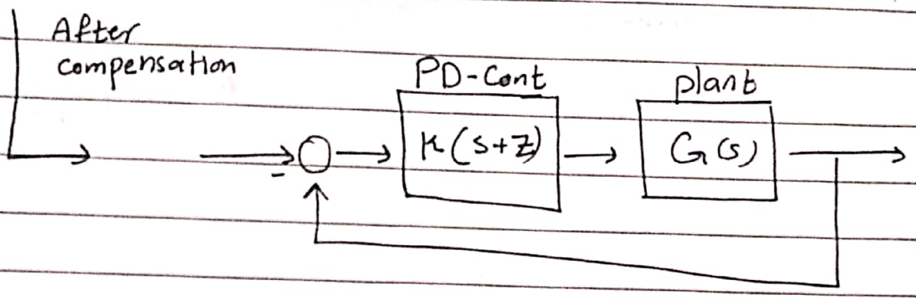
⊛ Before using a PD-Controller we have to check it first; Because sometimes with simulation it ~~may~~ may give us a good response but when using it for real the response may be very different due to nois appearance [PD-Con Amplify the noise].

⊛ Check page 464, figure 9.15 (أ.م رح توصلنا الفكرة البعد)

[أثر إضافة (zero) لل open.Loop على رسمة ال R-L
T.F



[uncompensated]
↳ only gain K



[compensated]
↳ has a zero or pole or both

Lec 25: ⊛ check (25)# video if you want to find the matlab files ⊛

↳ Check page 464 figure 9.15

⊛ [الفكرة صوره عندي برسمة (a) [uncompensated] sys يعني فقط بس [gain K]

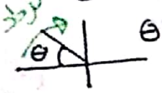
بعدها بفرع b,c,d مضيف ثلاث (zeros) عنا قيم مختلفة (م مختلفة)

ونشوف تأثيرها على رسمة] + عمل هنا الأثر بالأرقام ← Table 9.2
حيث يكون (ζ) ثابتة = 0.4

For (a) and (b) → ① adding a zero = -2 to be rusults: increasing the magnitude of the real part from |0.939| to |3| which causes the setting to decrease ② overshoot is fixed for all figures.
③ K increased → from K = 23.82 to 51.25
④ wd increased so the peak time decreased
⑤ since ζ = 0.4 is fixed and wn increased then Tr decreased

⊛ $\cos\theta = \zeta$

$\theta\uparrow \;\alpha\; \zeta\downarrow$

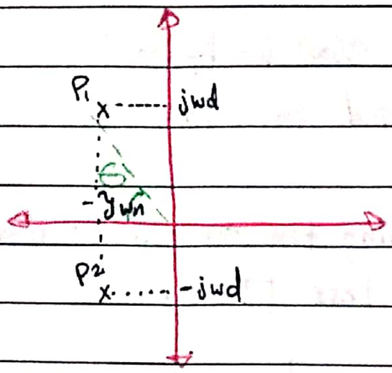


⊛ Ch4, figure 4.15
page 126
مهم

⊛ أثر الـ Dominant poles على الـ T.R parameters

$\boxed{32}$

Recall / ~~For~~ For an underdamped 2nd order system
→ that
→ $0 < \zeta < 1$



$$T(s) = \frac{1}{(s-P_1)(s+P_2)}$$

$$(s-P_1)(s-P_2) = 0$$

roots $S_{1,2} = P_1, P_2$ → $P_1 = -\zeta w_n + j w_d$, $P_2 = -\zeta w_n - j w_d$

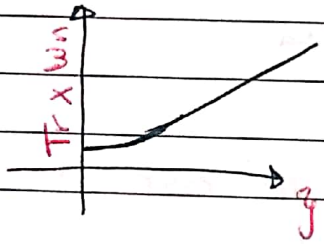
where $w_d = w_n\sqrt{1-\zeta^2}$

$$T(s) = \frac{4}{\zeta w_n} = 4\tau \overset{\text{عكسية}}{\Longrightarrow} \downarrow T_s \;\alpha\; \frac{1}{\zeta w_n}\uparrow$$

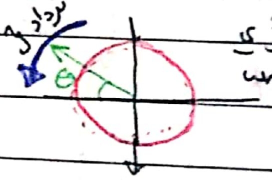
$$\overset{\text{عكسية}}{\Longrightarrow} \uparrow OS\% \;\alpha\; \frac{1}{\zeta}\downarrow$$

→ $T_p = \dfrac{\pi}{w_d}$ $\overset{\text{عكسية}}{\rightarrow}$ $T_p \;\alpha\; \dfrac{1}{w_d} \Rightarrow T_p \;\alpha\; \dfrac{1}{Im\{P_1, P_2\}}$

→ Tr ⇒ Check Ch4, page 177, figure 4.16 (مهم)



← اذا ثبتنا الـ wn وزدنا الـ ζ
بيزيد الـ Tr
← كلما زادت ζ بيزيد بقل الـ θ وبثبات wn
بيزداد الـ Tr



مسار دائري لثبات wn

← ثبات الـ ζ ← كلما زادت wn تقل الـ Tr للحفاظ ع ratio ثابت
أي لأنه ما حاصل ضرب $w_n \times Tr$

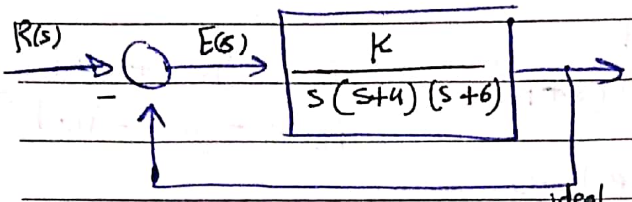
Conclusion: ① If ζ is fixed, increasing wn will decrease Tr.
② If wn is fixed, increasing ζ will increase Tr.

hint
Note: If Tp decreased, the Tr decreased [as an Indication]

⊛ For overshoot = 0.16 ⇝ $\zeta = 0.504$

Lec 26:

Ex 9.3 : PD-Controller design, Page 466 :



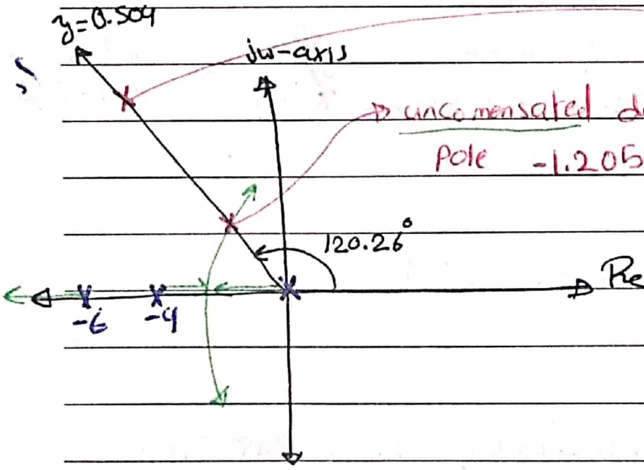
⇝ type 1 ; $e(\infty)$ for $k_p = 0$

⇝ uncompensated system.

Goal of Q: design a ideal PD-Controller, remaining the overshoot value of 0.16 and at the same time get a 3-times Less (T_s).

$\hookrightarrow T_{s_{New}} = \frac{1}{3} T_{s_{old}}$ ∴ $\left[T(s) \propto \frac{1}{\zeta \omega_n} \quad T(s) \propto \frac{1}{Re[Pole]} \right]$

Ex If uncompensated system has $(\zeta \omega_n)_{old} = 1$, then the required compensated system has to have dominant poles with real part $Re\{Pole\} = -\zeta\omega_n = -3$ ∴ $(\zeta\omega_n)_{new} = 3(\zeta\omega_n)_{old}$



uncompensated dominant pole $-1.205 + j2064$

desired compensated dominant pole $\boxed{-3.613 + j6.193}$

↳ We have to use PD-Con with a zero value that will give us the desired pole value. will keeping a fixed value of ζ.

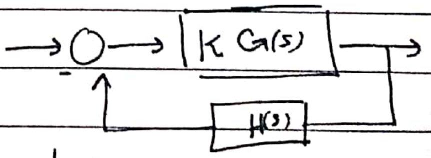
TF of:

Now Recall : $O.L = G(s)\,H(s)$

$$CL = \frac{KG}{1 + KGH}$$

↪ CL poles = roots of $1 + KG(s)H(s) = 0$

$\rightarrow \boxed{KG(s)H(s) = -1}$

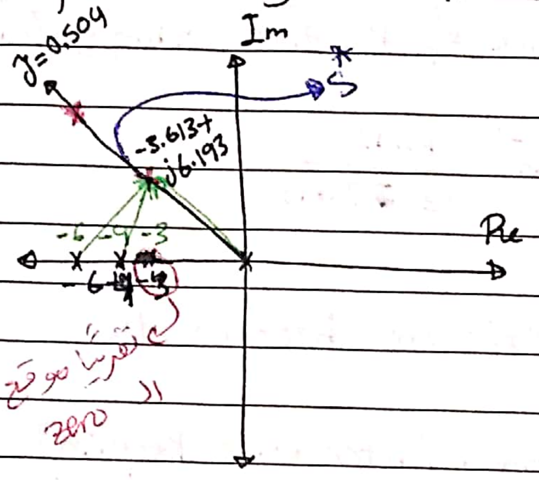


~~Ag~~ Any point(s) that satisfy the phase condition: → will Lie on R-L.

$\measuredangle G(s)H(s) = (2K+1)180^\circ, \quad K = 0, \pm 1, \pm 2, ---$

→ Recall : $\angle F(s) = \left(\frac{s+z}{s+p}\right) = \angle(s+z) - \angle(s+p)$

→ Finding the value of the added zero for the PD-Controller.



$\angle G(s^*)H(s^*) = (2K+1)\,180^\circ$

→ to find the phase, we draw a vector from each pole and find its angle

0.L

$\therefore \underset{\text{uncompensated}}{\phi(s^*)} = \sum\theta_z - \sum\theta_p$

$= -(\theta_1 + \theta_2 + \theta_3) \approx \boxed{-275}$

p=-6 p=-4 p=0

↳ لسا مش مقبولة

↳ $-275 + \underset{\text{comp}}{\phi} = \boxed{-180}$

↳ the nearest valid point to -275

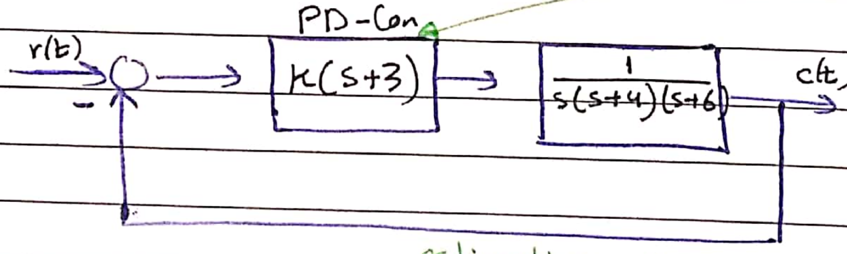
according to → $(2K+1)\,180^\circ$

غالبًا، نقطة مش بس الـ RL

$\phi_c \in [0,180]$

$\therefore \phi_c = 95^\circ$ [the added zero will have angle of 95°] and will be Located ≃ at point -3

↳ Compensated system now :



⊛ K changed from 43.35 to 47.45 (from MATLAB)

⊛ T(s) changed from 3.32 to 1.107

estimation

⊛ Check table 9.3 page 467 : the uncompensated and compensated values are taken according to the two dominant poles [2nd order approximation]

→ And the Simulation values are taken according ~~the~~ to all three poles [since it's a third order system].

(actual values)

⊛ e(∞) decreased [error for a ramp input]

type1

↳ $e(\infty)|_{Kp} = 0$

↳ $e(\infty)|_{Kv} = \boxed{}$ ✓

↳ $e(\infty)|_{Ka} = \infty$

⊛ Also, note that the Tp decreases because ωn increases when ζ is constant.

⊛ How to Calculate the exact value of the $\overset{zc}{\underline{\text{zero}}}$ using $\measuredangle \phi_c$?

Sol:

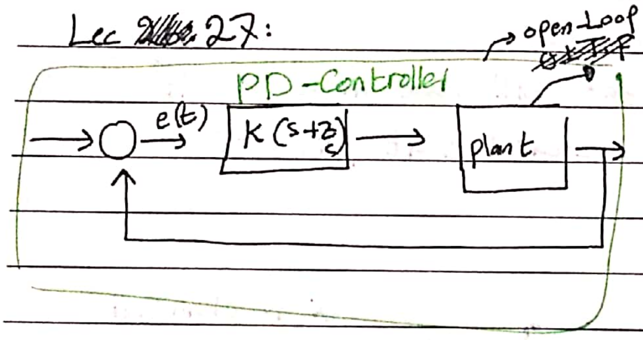
$$\measuredangle (s+z_c)\Big|_{s^*} = 95^\circ = \tan^{-1} \frac{6.193}{z_c \mp 3.613}$$

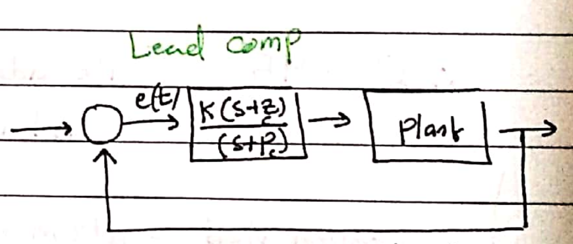
Check figure 9.22, page 469: the difference between the uncomp. and Comp. response.

[note that the comp. sys is much faster while keeping] a constant overshoot and a e(∞) of zero

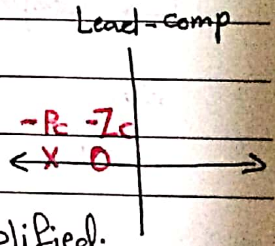
لإن step Input وال System type 1

Lec 27:





∴ |Pd| > |Zd|



→ Advantages For Lead comp:

If there was any high-freq nois, It won't be amplified.

→ disadvantages for Lead Comp:

Can't Increase the # of zeros In the O.L.T.F

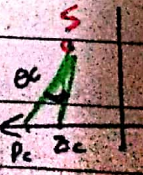
(But The PD-Controller adds a zero to the O.L.T.F)

لـ بالتالي صفر عدد الـ zero بـ الـ infinite

Check figure 9.24, page 470

⊛ بدي أعمل design لـ θc بحيث اقدر أنقل Zc و Pc بحيث يكونوا مرتبطين بعرف عن طريق قيمة θc

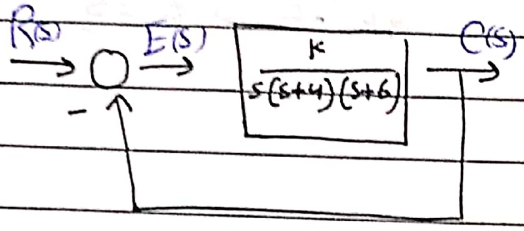
⊛ يعني بقدر أحدد أي قيمة لـ Zc وأراعي قيمة Pc بناءً على θc بحيث الاحظ انه قيمتها بسهم الا اختار مواقعهم



ⓧ لتسريع ال sys بنستخدم 1) PD-Con
2) Lead-Comp

Ex 9.4 page 471: Check Figure 9.26

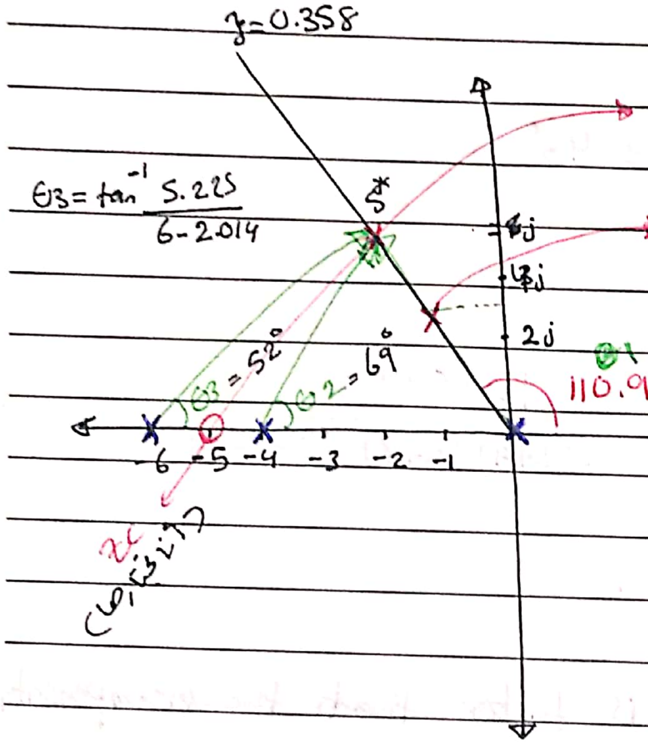
For this system ↓



Goal: Desired OS% = 30% ↓

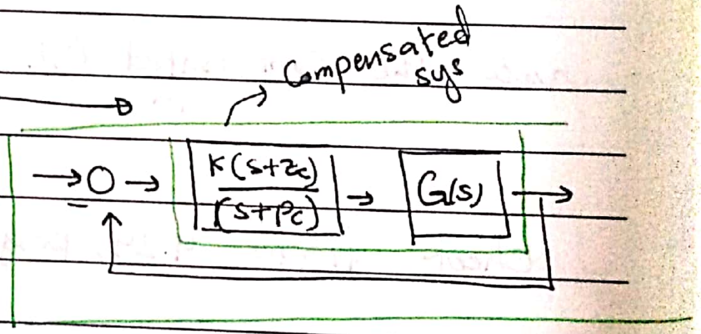
from table / Figure ($\zeta = 0.358$)

then design Lead-Com: reduce the T(s) by $\underline{\underline{2}}$ while keeping 30% overshoot



→ Desired compensated dominant Pole at $-2.014 + j5.252$

→ Uncompensated dominant Pole at $-1.007 + 2.627j$ ($K = 63.21$)



→ $Ts_{New} \propto \frac{1}{\zeta\omega_n}$ then $(\zeta\omega_n)_{New} = 2(\zeta\omega_n)_{old}$

$(\zeta\omega_n)_{New} = 2 \left| Re\left\{ \text{Dominant Poles} \right\} \right| = 2 * 1.007 = 2.014$ (لسه مش عال RL)

فلازم تطلع ال (angle) الي هتخليه عال R-L

↳ $-\theta_1 - \theta_2 - \theta_3 = -232.4°$

$-\theta_1 - \theta_2 - \theta_3 + \theta_c = (2K+1)\ 180°$ ∴ $\theta_c = \theta_{zc} - \theta_{pc}$

$\theta_c \in [0, 180]$

↳ $-232.4° + 52.4° = \boxed{-180}$ ↑ From this

ⓧ الأحسن والأسهل أحط ال zero مكان O.Loop pole زي عند (-6)
لأنه هيعمل cancellation لل zero فلما أضيف pole ماتكونش أثرت
على عدد ال poles النهائي

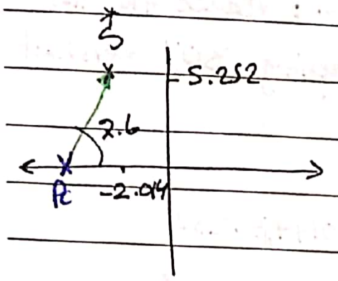
$\rightarrow \theta_c = 52.4 = \theta_{zc} - \theta_{pc}$, let compensator zero be [at $-6 \rightarrow Z_c = 6$] نفضل

But following the book at $(-5 \rightarrow Z_c = 5)$

$\therefore \theta_{zc} = \tan^{-1} \frac{5.25}{5 - 2.014} \approx 60^{\circ}$

$\rightarrow \theta_c = 52.4^{\circ} = 60^{\circ} - \theta_{Pc}$

$\theta_{Pc} = 7.6^{\circ} \longrightarrow$ find Pc



$\frac{5.252}{Pc - 2.14} = \tan 7.6$

$Pc \approx 43^{\circ}$

Check table 9.4, page 472 :

note the compensated O.L T.F s (a) $\frac{K(S+5)}{S(S+4)(S+6)(S+43)}$

Check figure 9.29, page 473 :

↳ Note that the compensated sys is faster than the uncompensated sys. (Tr and Ts decreased)

↳ Note that for system C, the overshoot is not 30% So it was a little off the sys needs, the desig were actually done for 30% OS but after simulation the actual OS% was 14.5 [this error is due to the assuming That it is 2nd order though It's not]
$\therefore$ Because of 2nd order approximation.

↳ Note that the error is reduced too.

Conclusion : Lead-compinsator affect the transient response to reach the desired one, and Also can reduce the SS. error.

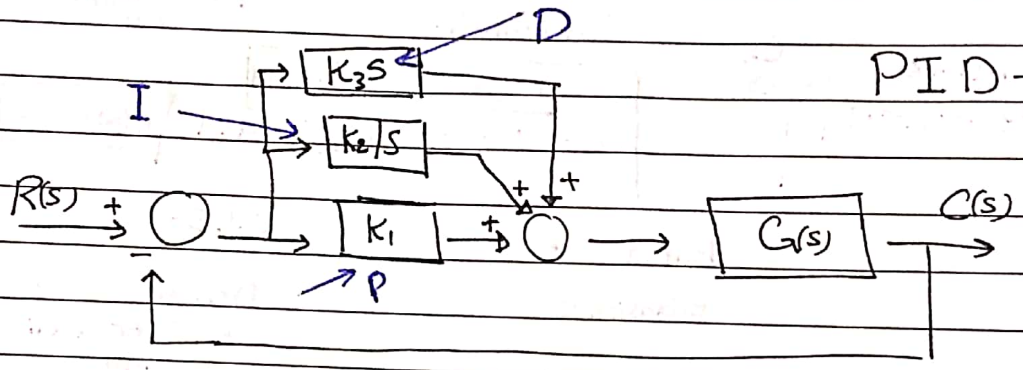
له بس مش بالضرورة نعتين

38

Lec 28:

9.4 PID and Lead-Lag Compensator :

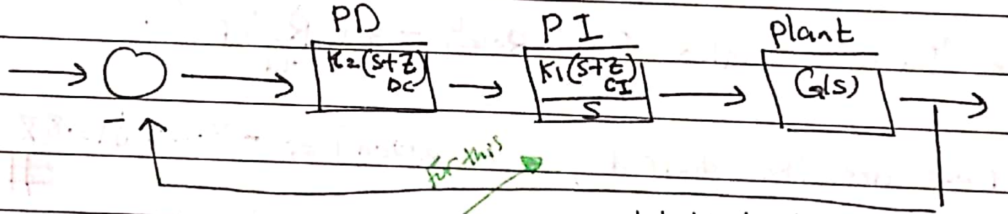
figure 9.3, Page 475 :



PID - Controller

Check T.F page 475

|| equivalent to [have PD and PI controllers in series]



$$\text{PID-Controller T.F} = \frac{K(s+z_{DC})(s+z_{IC})}{s} \quad \text{let } K = K_1 \cdot K_2$$

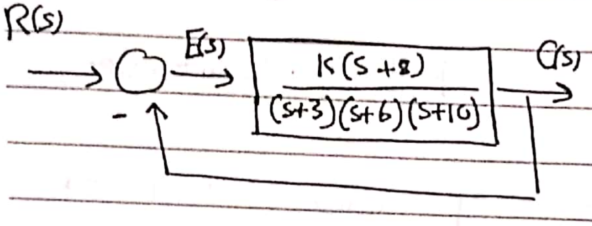
Desing Process steps : (For PID-Controller)

1) Desig PD-Controller to improve the T.R.
2) Simulation [to check that we got the desired response) T.R
3) Design PI-controller (add apole at origin) and add a zero near the pole
4) Simulation again (Check the stead-state error) good or not?

If not : re-design again, change the zeros possitions and check again.

Example 9.5, page 475

For this system:



Goal: Design a PID-Controller For 0-type sys

In which: $T_{P_{new}} = 2/3\ T_{P_{old}}$ at OS% of 20% and $e(\infty) = 0$

desired transient

$\hookrightarrow \zeta = 0.456$

Desired $e(\infty)$ for a unit step

$T_P \propto \frac{1}{\omega_d} \Rightarrow T_P \propto \frac{1}{Im\{pole\}} \Rightarrow \frac{2}{3} T_P \propto \frac{1}{3/2\ Im\{pole}$

→ to maintaine ζ value, also $Re_{New}\{Pole\} = 3/2\ Re_{old}\{pole\}$.

[those are the desired pole Location] = $-8.13 + j15.87$ # S^*

Now, the desired point is not on R-L, to make it the Root-Locus:

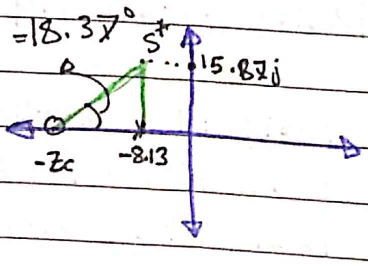
→ We add a zero In which the angle of It will make the point on the R-L.

PD

zero ⟶ three pole ⟶ For un-comp sys

1) Sol: $\theta_z - \theta_1 - \theta_2 - \theta_3 = -198.37^\circ$

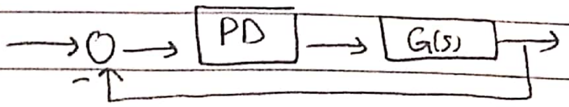
$198.37^\circ - 180 = 18.37 = \theta_{Dc}$



to find Zc: $\frac{15.87}{Zc - 8.13} = \tan 18.37^\circ$

$Zc = 55.92$

∴ PD-Controller is : $K(s + 55.92)$



Recall: PD-Controller may amplify the noise which is bad.
PD and PI require using OP-Amps (active elements) $\boxed{HO}$

PI
2) Sol: $G_{PI}(s) = \frac{K(s+0.5)}{s}$ أنا اخترتها ∴ $Z_{Ic} = 0.5$

طبعاً تكون قريب من
ال pole عند ال origin وبنفضل تكون بعيدة عن أقرب pole لل sys الأصلي

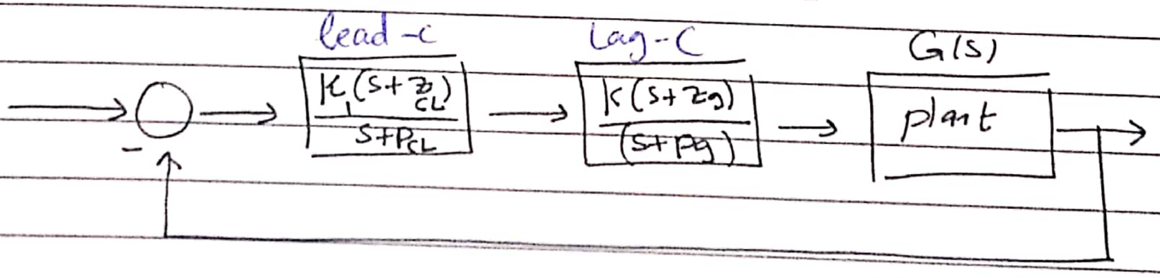
Check figure 9.35 :

K=4.6 ← For PID: faster transient response, $e(\infty) = 0$
K=121.5 ← For uncomp: slower with $e(\infty)$ = finite #
K=5.34 for PD : gives almost the desired response, but has an error

Check table 9.5, page [see the values difference]

→ In Case I don't have any active elements OR
I don't want to use PID-Controller (PI, PD),
In Stead we can use:
LAG-LEAD Comp.

⊛ LAg-Lead compensator Design: (in Series)



Improve TR
steps: 1) start with the Lead-Compensator
2) Simulation (check)
Reduce $e(\infty)$ ← 3) Lag-Com design (Note that the pole of it is not in the origin)
4) Simulation If good or not?
If not: design again (start with changing P_L and Z_L)

Ex 9.6 $\boxed{H.W}$