تقدم لجنة ElCoM الاكاديمية

دفتر لمادة:

تحكم آلي متقدم

جزيل الشكر للطالبة:

سارة ابو سارة

Revision :

[system vs signal]

- system ≡ is amathmatical model that represent the transformation of some input $x(t)$ into output $y(t)$

$$\text{input } x(t) \longrightarrow \boxed{\text{system}} \longrightarrow \text{output } y(t)$$

ex : continuos time system

$$y(t) = 3 - 3e^{-4t} \quad , \quad t \in [0, \infty]$$

discrete time system

$$y[k] = \sin[wk] \quad , \quad k : \text{integar no. "sample index"}$$

- signals ≡ they are functions of time that carries information.

No.

[Linear System] → ch.2 موضوع موجود بالكتاب

\# to check that the system is Linear or not by satisfying the 2 conditions :

1. scaling , $y(Ax) = A\,y(x)$
2. superposition , $y(x_1 + x_2) = y(x_1) + y(x_2)$

وبشكل عام، يعتبر النظام Linear لما يكون عبارة عن

$$\boxed{\text{output} = \underset{\text{gain}}{k} * \text{input}}$$

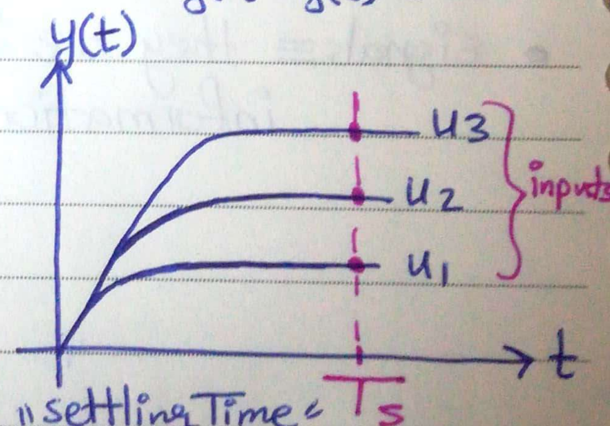
Ex : $y = au + b$, is y a linear system?

تعتبر علاقة خطية لكن non linear sys.

No , because of b

[not zero initial conditions $y(0) = \dot{y}(0) = b$]

Ex : لحتى أحدد ال Linearity بثبّت الزمن وبشوف العلاقة بين ال (u) input ← عند ال T_s أفضل شيء
وال (y) output



$y(u_1) =$

$y(u_2) =$

$y(u_3) =$

هاد المنحنى بيمثل ال
(output response with time)
ما بحدد ال Linearity للنظام من خلاله ... مباشرة

FIVE APPLE

Ex Use scaling and superposition conditions, to check if the following system equations linear or not?

① $y = x$

scaling $\rightarrow$ $y(\alpha x_1 + \beta x_2) = y(\alpha x_1) + y(\beta x_2)$

assume that $\Rightarrow$ $x_1 = 1$ $\quad \alpha = 2$

$x_2 = 3$ $\quad \beta = 4$

$y(2+12) \overset{?}{=} y(2) + y(12)$

$14 \overset{?}{=} 2 + 12$ ✓

superposition $\rightarrow$ $y(x_1 + x_2) = y(x_1) + y(x_2)$

$y(4) \overset{?}{=} y(1) + y(3)$

$4 \overset{?}{=} 1 + 3$ ✓

linear equa. + linear system

linear equation for system modeling ...

② $y = 3x + 1$

Scaling: $y(2+12) \overset{?}{=} y(2) + y(12)$

$[3(14)+1] \overset{?}{=} [3(2)+1] + [3(12)+1]$

$43 \overset{?}{=} 7 + 37$ ✓

superposition: $y(1+3) \overset{?}{=} y(1) + y(3)$

$13 \overset{?}{=} 4 + 10$

X

non linear system

③ $y = e^{-x}$

Scaling: $y(2+12) \overset{?}{=} y(2) + y(12)$

$e^{-14} \overset{?}{=} e^{-2} + e^{-12}$

X

Superposition: $y(4) \overset{?}{=} y(1) + y(3)$

$e^{-4} \overset{?}{=} e^{-1} + e^{-3}$

X

non linear system
non linear equation

No.

⇒ To make the system Linear :

Taylor Series Expansion method

tool to design a microcontroller for non linear system if that system works at small intervals.

[بأفترات زمنية متقاربة يكون عندها ال output خطي ..]

initial value

$$y(x) = y(x_0) + \frac{dy}{dx}\Big|_{x=x_0}(x-x_0) + \frac{d^2y}{dx^2}\Big|_{x=x_0}\frac{(x-x_0)^2}{2!} + \cdots$$

Higher order Term will be neglected because its values are too much small, closer to the 0

Linearization ≡ method used to find mathmatical model to find good controller, but the controller works in non linearity when we simulated it.

لازم يكون النظام Linear ⇐

حتى أقدر آخذ ال Laplace

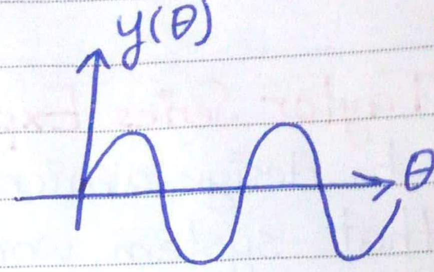
لل differential Equation model تبعه

No.

①

examples

$\theta_0 \downarrow$ $y(0) = 0$



1] $y(\theta) = \sin\theta$

$y(\theta) = y(0) + [\cos\theta]\Big|_{\theta=0} (\theta - 0)$

$= 0 + \theta$

$$\boxed{y(\theta) = \theta}$$

2] $\underline{\sin y} + ky + b\dot{y} = m\ddot{y}$

non linear term

بعمل Linearization
للحدّ غير ال Linear

$\rightarrow \quad y + ky + b\dot{y} = m\ddot{y}$

3] $y \underline{\sin y} + ky + b\dot{y} = m\ddot{y}$

non Linear term

$\underline{y^2} + ky + b\dot{y} = m\ddot{y}$, still non linear because of (y^2)

taylor series for zero's initial values we will get

$\emptyset + ky + b\dot{y} = m\ddot{y}$

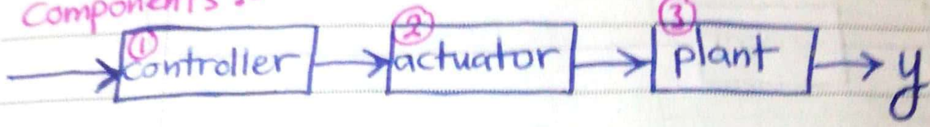
No.

CH.1 : Introduction to control systems

[Open Loop vs Closed loop system]

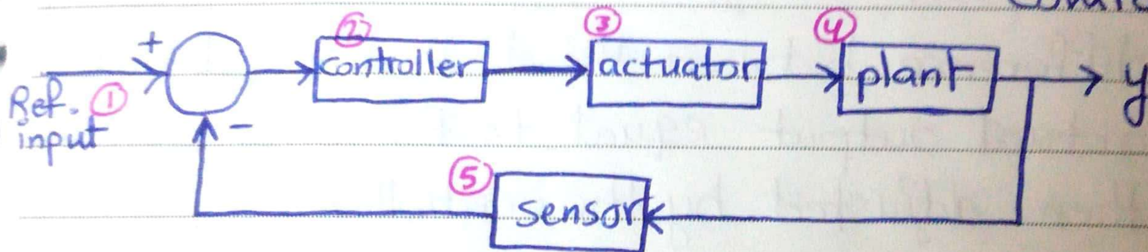
- Open Loop System →

Components :

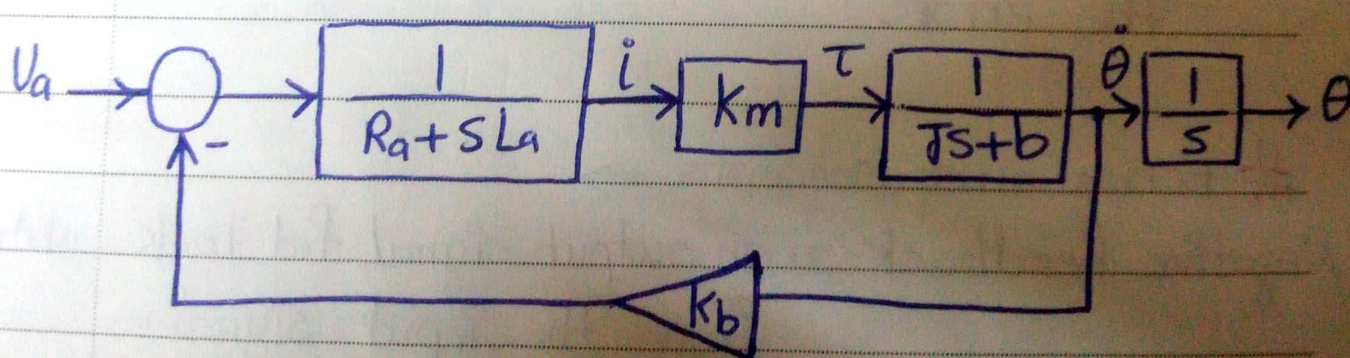


- Closed Loop system → لما القيمة المتغيرة اللي موصل عليها مع ال sensor ـــ تتغير معها ال controller

Components :



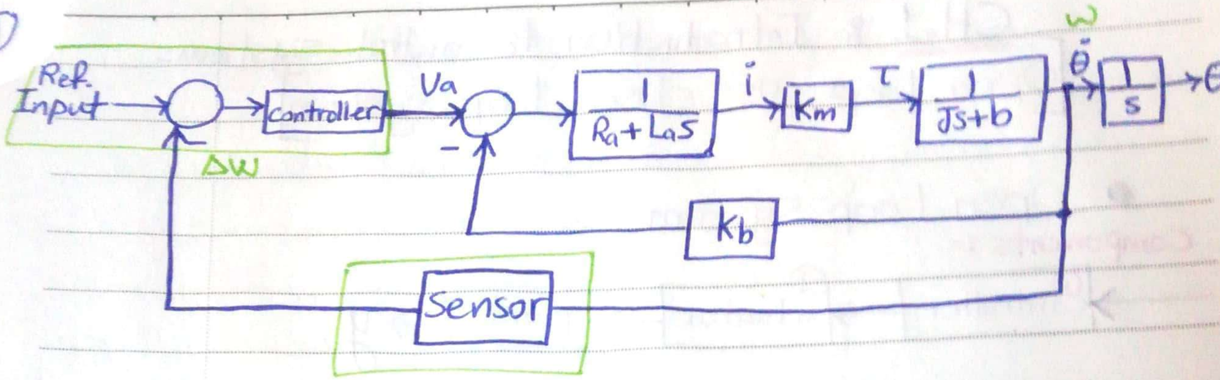
Ex : Armature Controlled DC motor



[open loop Sys.] معتبره

V_a : not the desired output "reference input"

①



this is a closed loop system ..

أهمية الـ (feedback) بالـ [Closed loop system] ⇐

∴ the difference between the desired output and the actual output equal to the error, which is then adjusted by the controller.

the output of the controller causes the actuator to modulate the process to reduce the error.

terms and concepts ⇒

- Negative feedback: an output signal fed back, subtracts from the input signal.
- Optimization: the adjustment of the parameters to acheive the most favorable or advantageous design
- productivity: the ratio of physical output to physical input of an industrial process.

No.

Ch.2 : Mathmatical Models of systems

Describe the relation between input and output by set of differential equations.

result not a requirements

*Linear Ordinary equation →

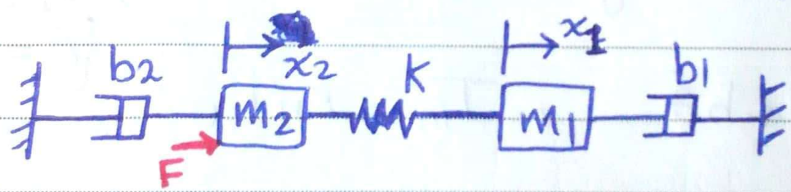
1. No product of variable $(x_1 x_2)$
2. No power more than one (x^2)
3. No non linear function $(e^x, \sin x)$

this will lead to linear system under zero initial condition only.

$$\begin{pmatrix} y(0)=0 \\ \dot{y}(0)=0 \end{pmatrix}$$

قوة التغير عند بداية التشغيل

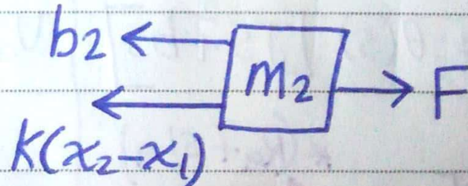
Ex:



2DOF , 2 independent motions

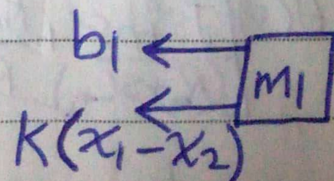
نعمل FBD لل (moving objects)

$$\boxed{\Sigma F = m\ddot{x}}$$



$F - F_s - F_d = m_2\ddot{x}_2$

$$\begin{pmatrix} F = m_2\ddot{x}_2 + b_2\dot{x}_2 + k(x_2-x_1) \\ 0 = m_1\ddot{x}_1 + b_1\dot{x}_1 + k(x_1-x_2) \end{pmatrix}$$

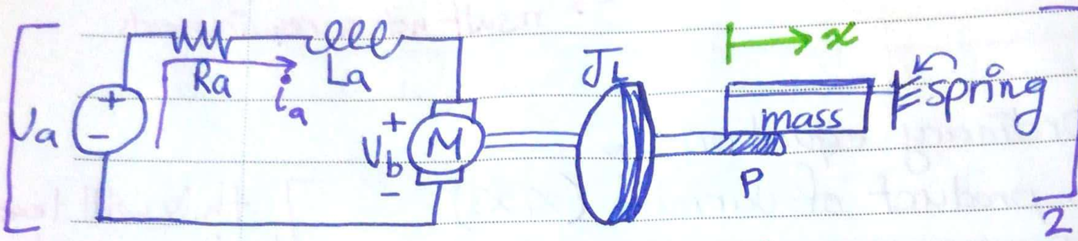


It's ordinary linear DE

FIVE APPLE

① **Ex:** Find the relation between the mass displacement and the input voltage? "for the electromechanical system shown below"

$\dfrac{x}{V_a}$?

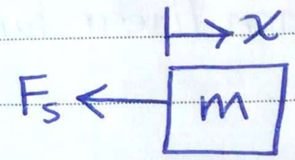


1 DOF system

لأنه مشي

2 dep. motion

$V_a \rightarrow i_a \rightarrow \tau \rightarrow \omega \rightarrow \theta \rightarrow x$



FBD for the mass

$m\ddot{x} = -kx$

$x = P\theta$

$\boxed{\theta = \dfrac{x}{P}}$

\# Armature controlled DC motor modelling ⇒

$* \ \Sigma T = J\ddot{\theta} + b\dot{\theta} \quad , \quad T_{in} = k_m i_a$

$* \ V_a - V_b = R_a i_a + L_a \dfrac{di_a}{dt}$

$V_a - \boxed{K_b \dot{\theta}} = R_a i_a + L_a \dfrac{di_a}{dt}$

$I_a(s) = \dfrac{V_a(s) - K_b s\theta(s)}{R_a + sL_a}$

$k_m i_a = J\ddot{\theta} + b\dot{\theta}$

$k_m \left[\dfrac{V_a(s) - k_b s\theta(s)}{R_a + sL_a}\right] = \theta(s)\left[Js^2 + bs\right]$

$\dfrac{K_m V_a(s)}{\cancel{R_a + sL_a}} = \theta(s)\left[\dfrac{Js^2 + bs}{(R_a + sL_a)} * (R_a + sL_a) + \dfrac{K_m K_b s}{\cancel{R_a + sL_a}}\right]$

$\dfrac{\theta(s)}{V_a(s)} = \dfrac{K_m}{(R_a + sL_a)(Js^2 + bs) + sK_m K_b}$

No.

$\Rightarrow$ while $x = P\theta$:

$$\theta(s) = \frac{1}{P} x(s)$$

$$P * \frac{\cancel{\frac{1}{P}} x(s)}{V_a(s)} = \frac{K_m * P}{(R_a + sL_a)(Js^2 + bs) + sK_m K_b}$$

$$\therefore \frac{x(s)}{V_a(s)} = \frac{K_m P}{(R_a + sL_a)(Js^2 + bs) + sK_m K_b}$$

No.

①

Differential Equation Solution →

[the output (variable of interest) behaviour with time] بدي أشوف من خلالها

by : Laplace and Laplace Inverse

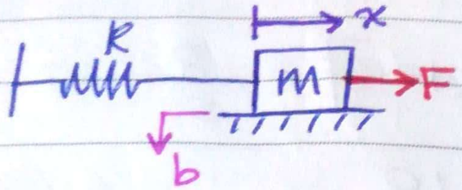
Revision

$f(t)$		$F(s)$
$u(t)$	$\longrightarrow$	$\frac{1}{s}$
e^{-at}	$\longrightarrow$	$\frac{1}{s+a}$
t^n	$\longrightarrow$	$\frac{n!}{s^{n+1}}$
$\sin(wt)$	$\longrightarrow$	$\frac{w}{s^2+w^2}$
$\cos(wt)$	$\longrightarrow$	$\frac{s}{s^2+w^2}$
$\delta(t)$	$\longrightarrow$	1
$e^{-at}\sin wt$	$\longrightarrow$	$\frac{w}{(s+a)^2+w^2}$
$e^{-at}\cos wt$	$\longrightarrow$	$\frac{s+a}{(s+a)^2+w^2}$
$t^n e^{at}$	$\longrightarrow$	$\frac{n!}{(s-a)^{n+1}}$

FIVE APPLE

No.

Examples

I) 

1DOF system

$F = m\ddot{x} + b\dot{x} + kx$, 2nd order so we need to know

الإزاحة الابتدائية ⇐ $x(0) = ?$

السرعة الابتدائية ⇐ $\dot{x}(0) = ?$

* to become a Linear system?

$x(0) = \dot{x}(0) = 0$

* about the system oscillation (its response)?

اذا زدت قيمة ال (damping constant b)
تقل ال oscillation بسبب زيادة هذا العامل لامتصاص
القوة بالتالي سيقل تأثيرها على الكتلة

لكن ، زيادة قيمة ال (spring constant k)
سيزداد ال oscillation بسبب زيادة هذا العامل في توليد
قوة معاكسة لل F المطبقة على ال mass

If we know that : $[F = 10N , k = 6 , b = 5 , m = 1]$

$\mathcal{L}[10] = \mathcal{L}[\ddot{x} + 5\dot{x} + 6x]$

→

No.

①
$$\frac{10}{s} = s^2 x(s) + 5s\,x(s) + 6x(s)$$

$$x(s)\left[s^2 + 5s + 6\right] = \frac{10}{s}$$

$$x(s) = \frac{10}{s(s^2+5s+6)}$$

by taking laplace and then its inverse, we will know the roots (s) that indicate the system response

$$\frac{10}{s(s^2+5s+6)} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{s+2}$$

$$\left[B = \frac{10}{3},\ C = -5,\ A = \frac{5}{3}\right]$$

$$x(t) = \frac{5}{3} + \frac{10}{3}e^{-3t} - 5e^{-2t}$$

system response relation

لنتأكد نتحقق

$x(0) = 0$ ⇐ "initial Value" $x(0)$ بال

$$x(0) = \frac{5}{3} + \frac{10}{3} - 5$$

$$= \frac{15}{3} - 5 = 5 - 5 = 0 \ \checkmark$$

* system response curve : يعني تمثيل العلاقة لل (output) بالنسبة ⇒ للزمن (time) نحصل عليها من خلال

(mathmatical model) لل $L^{-1} \leftarrow L$
(partial fraction)

No.

- Under what values, the system have oscillations?

$\boxed{b^2 - 4km < 0}$ ← complex حسب الـ roots اذا كانت
هيكون في oscillation

$\ddot{x} + 5\dot{x} + 6x = 10$

$5^2 - 4(6)(1) = 1 > 0$ ← في المثال السابق

ما في oscillation

for example :

If $\quad 10 = 3x + 2\dot{x} + \ddot{x}$

$m = 1$
$k = 3$
$b = 2$
$\Downarrow$
$\boxed{4 - 3(1)(4)} < 0$
في oscillation

$$x(s) = \frac{10}{s(s^2 + 2s + 3)}$$

$$\frac{10}{s(s^2+2s+3)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+3}$$

$$\frac{10}{s(s^2+2s+3)} = \frac{A}{s} + \frac{Bs+C}{(s+1)^2+2}$$

$$10 = A(s^2+2s+3) + (Bs+C)s$$

$$10 = As^2 + Bs^2 + 2As + Cs + 3A$$

نجمع الحدود المتشابهة ثم نقارن المعاملات بين الطرفين

$A = \frac{10}{3}$, $\quad As^2 + Bs^2 = 0$

$B = \frac{-10}{3}$

$2A + C = 0$

$C = \frac{-20}{3}$

→

No.

①
$$x(s) = \frac{10/3}{s} + \frac{-(10/3)\, s+1-1}{(s+1)^2+2} - \frac{(20/3)}{(s+1)^2+2}$$

$$L^{-1}\left[x(s) = \frac{10}{3s} - \frac{\frac{10}{3}(s+1)}{(s+1)^2+\underbrace{2}_{\omega^2}} + \frac{\frac{10}{3} * \sqrt{2}}{[(s+1)^2+2] * \sqrt{2}} - \frac{\frac{20}{3} * \sqrt{2}}{[(s+1)^2+2]\sqrt{2}}\right]$$

$$x(t) = \frac{10}{3} - \frac{10}{3}(e^{-t}\cos\sqrt{2}t) + \frac{10}{3\sqrt{2}}(e^{-t}\sin\sqrt{2}\,t)$$
$$- \frac{20}{3\sqrt{2}}(e^{-t}\sin\sqrt{2}\,t)$$

$$\therefore\ x(t) = \frac{10}{3} - \frac{10}{3}(e^{-t}\cos\sqrt{2}t) - \frac{10}{3\sqrt{2}}(e^{-t}\sin\sqrt{2}\,t)$$

∴ منحدد
ازاي oscillation من خلال ال (differential equ.) ⇐ $\left(\left[b^2-4km\right] < 0\right)$ ✓

* الهدف من اللابلاس :
تحويل المعادلة التفاضلية إلى معادلة جبرية
كيف أقدر ألاقي نسبة بين ال input وال output
(ايجاد علاقة بينهم تربطهم سوا)

[Transfer Function] ⟹

No.

Ex For $Y(s) = \frac{2}{(s+1)(s+2)^2}$

$$\frac{2}{(s+1)(s+2)^2} = \frac{A}{s+1} + \frac{B}{(s+2)} + \frac{C}{(s+2)^2}$$

for repeated roots case, add another constant

$$2 = A(s+2)^2 + B(s+1)(s+2) + C(s+1)$$

$\vdots$

No.

Transfer Function ≡ the ratio between the input and the output in s domain, under zero initial conditions to get a linear system.

for example →

$$H(s) = \frac{1}{ms^2 + bs + K}$$

numerator $R(s)$

$q(s)$ denominator

s : the root of $q(s)$

if $ms^2 + bs + K = 0$, we call it a characteristic equation

for certain values of s

لأنه من خلالها نقدر أعرف
مواصفات كثيرة عن سلوك
النظام من ناحية

Roots → stable or not

if it's stable → oscillatory or not

it's form → error or not

⋮

No.

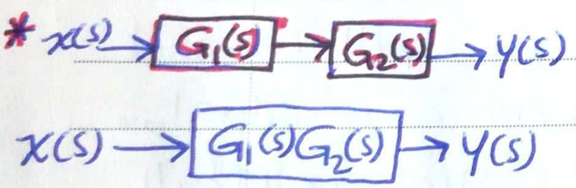
Ways to find the transfer function :

1- from mathmatical modelling [by the laplace]
2- Block diagram eliminating
3- signal flow graph [by mason's rule]

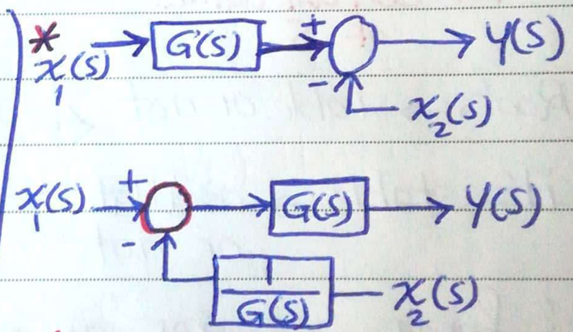
that we can apply it on the block diagram of asystem also without convert it to asignal flow graph

Revision

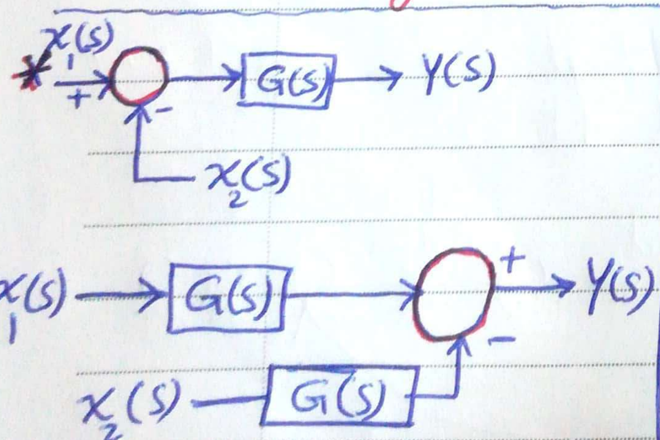
Block diagram eliminating (reduction)



Cascading



moving asumming pt. ahead of ablock



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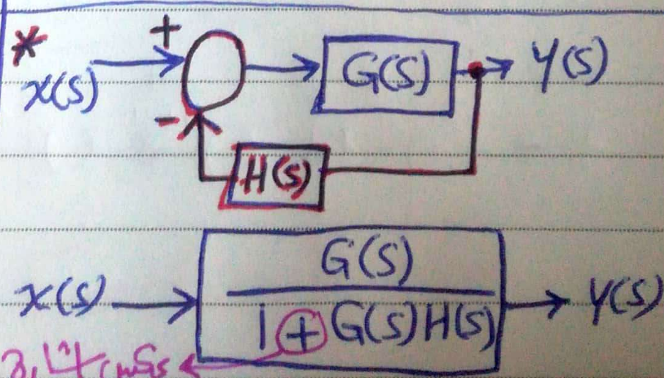

عكس اشارة feedback!

eliminating feedback loop

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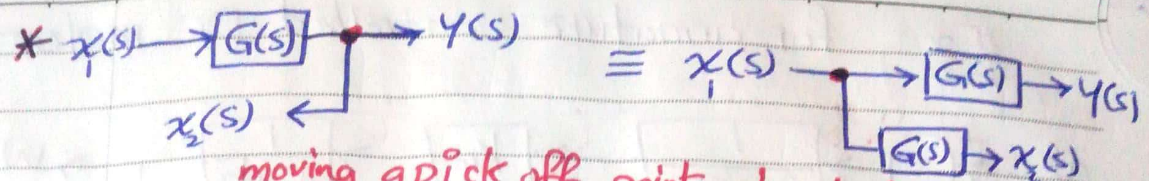
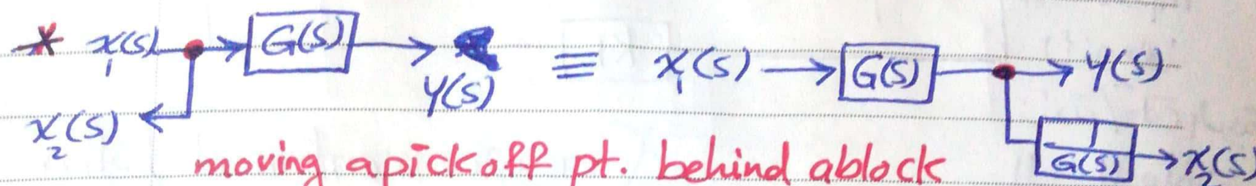


moving apick off point ahead of ablock



moving apickoff pt. behind ablock

⇒ Mason's Rule :

a way to find the ratio between the independet input in the system and the dependent signal.

$$\boxed{T_{ij} = \frac{\sum P_k \Delta k}{\Delta}}$$

P : forward Path

كل path إلها Δk

Δ : determinant

Δk : cofactor , eliminate the loops that touches the path.

مجموع حاصل ضرب كل

$$\Delta = 1 - (L_1 + L_2 + L_3 + ..) + \sum(\text{every two nontouching Loops})$$

$$- \sum(\text{every three non touching loops})$$

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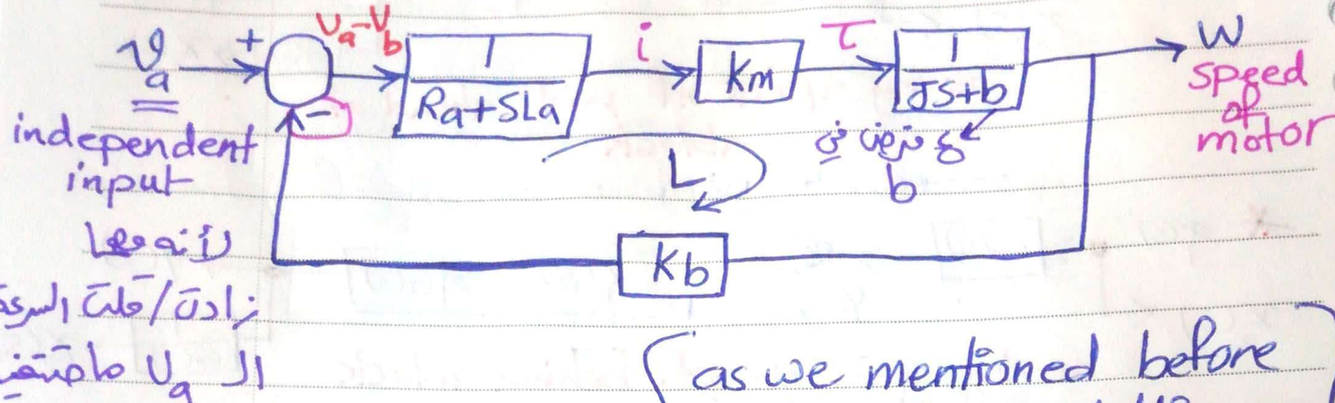
$$\Delta k_{ij} = 1 - \sum(\text{loops that nontouching a certain Path})$$

No.

Ex: for armature controlled DC motor

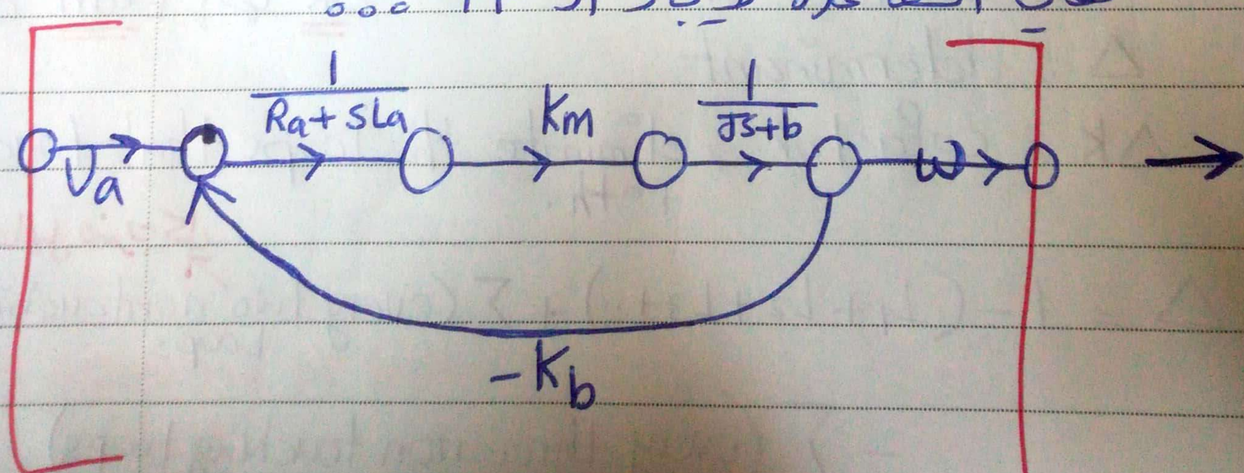


as we mentioned before that this modelling indicate to an open loop system.

ما بقدر أطلع ال TF باستخدام ال (mason's rule)
بين ال τ أو i مع $v_a - V_b$

لأنه ال $(v_a - V_b) \Leftarrow$ dependent input

لكن فقط بين v_a وأي output فأنا بقدر أستخدم
هاي القاعدة لإيجاد ال TF ...



In signal flow graph representation

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Scanned by CamScanner

$$T_{U_a,W} = \frac{P_1 \Delta_1}{\Delta}$$, because i have just one forward path

$$P_1 = \frac{k_m}{(Js+b)(R_a+sL_a)}$$

$$\Delta = 1 - L = 1 + \frac{k_b k_m}{(Js+b)(R_a+sL_a)}$$

Just one loop

because of (-) from (1-L) with the (-) from the feedback signal, together becomes (+)

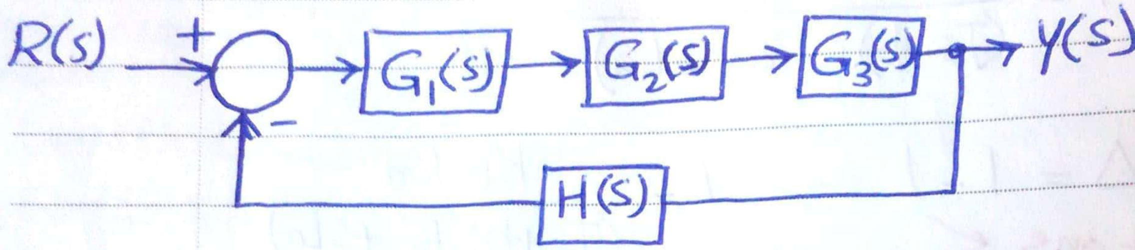
$$\Delta_1 = 1 - 0 = 1$$
(for P_1)

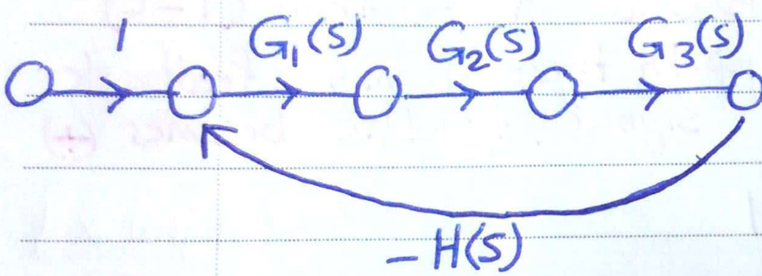
$$\therefore \quad T_{U_a,W} = \frac{k_m}{(Js+b)(R_a+sL_a) + k_b k_m}$$

No.

Ex : how to converts from a block diagram to a signal flow graph?

[مثال توضيحي]





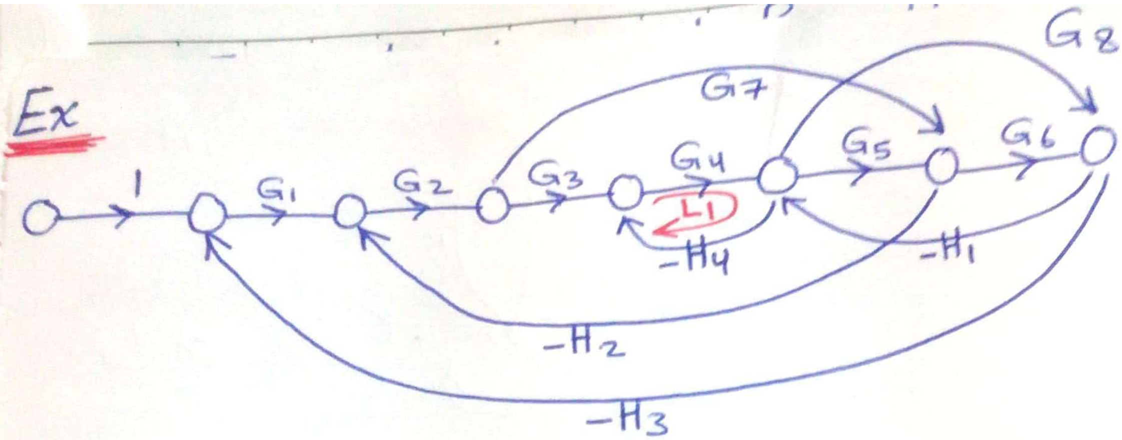
بين كل block
خط 2 nodes
وحدة قبلها و
وحدة بعدها

1 path
1 loop

$$\frac{Y(s)}{R(s)} \ldots\ldots$$

بعبّر عن ال block
بـ branch قبله وبعده o
summing point

Ex



$P_1 = G_1 G_2 G_3 G_4 G_5 G_6$

$P_2 = G_1 G_2 G_7 G_6$

$P_3 = G_1 G_2 G_3 G_4 G_8$

$$TF = \frac{P_1 \Delta_1 + P_2 \Delta_2 + P_3 \Delta_3}{\Delta}$$

$L_1 = -G_4 H_4$

$L_2 = -G_5 G_6 H_1$

$L_3 = -G_2 G_3 G_4 G_5 H_2$

$L_4 = -G_1 G_2 G_3 G_4 G_5 G_6 H_3$

$L_5 = -G_1 G_2 G_7 G_6 H_3$

$L_6 = -G_1 G_2 G_3 G_4 G_8 H_3$

$L_7 = -G_2 G_7 H_2$

$L_8 = -G_8 H_1$

$\Delta_1 = 1 - 0 = 1$

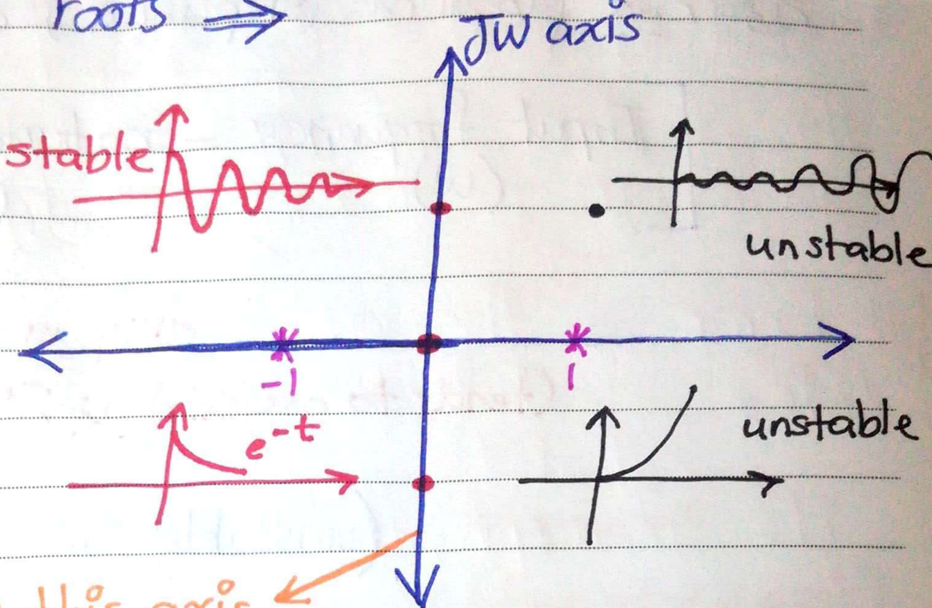
$\Delta_2 = 1 - L_1$

$\Delta_3 = 1 - 0 = 1$

مجموع حاصل ضرب كل (2 non touching loops)

$$\Delta = 1 - \sum_{i=1}^{8} L_i + [L_1 L_7 + L_1 L_5 + L_7 L_8]$$

System stability determination by its roots ⟹



roots on this axis
(marginally stable)

فقط بيكون stable عند قيمة واحدة

$\boxed{\text{practical examples}}$

* لو طبقنا فولتية (step input)
على موتور هيتحرك الموتور
بزاوية تتغير مع الزمن بالزيادة
وهنا يعني (unbounded output for bounded input)

بالتالي ∴ Unstable system

* عند عكس قطبية الـ armature بالموتور هتراوح قيمة التغير بين
two values (bounded output)
for bounded input

stable system ⟸

∴ عند تطبيق (step input) على النظام بحيث تكون

$$\left[\ \underset{(\omega)}{\text{Input frequency}} = \underset{(\omega_n)}{\text{natural frequency}}\ \right]$$

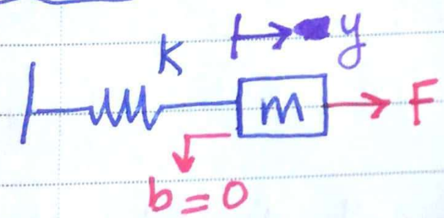
⇓

هي قيمة ال ω اللي النظام عندها يميل للتذبذب (tends to oscillate)

بيكون النظام كاملا (unstable)

Examples

1



$$H(s) = \frac{1}{ms^2+k}$$

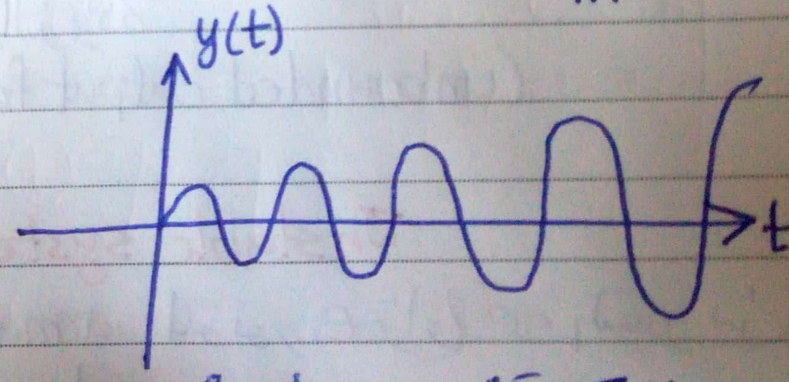
معامل s^2 لازم يساوي 1

$$= \frac{\frac{1}{m}}{s^2 + \frac{k}{m}}$$

$$\omega = \omega_n = \sqrt{\frac{k}{m}}$$

$$f(t) = \cos\sqrt{\frac{k}{m}}\,t$$

$$f(t) = \sin\sqrt{\frac{k}{m}}\,t$$



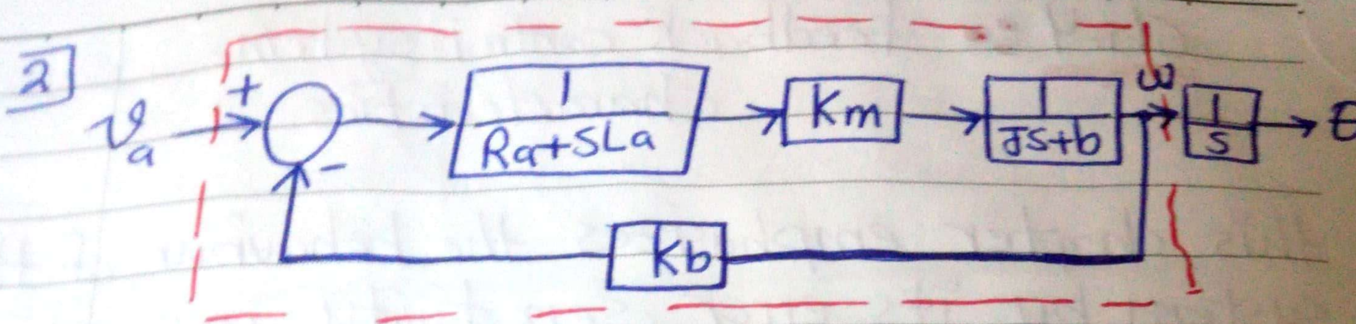
y تزيد مع الزمن

unstable system

لكن عمليا ال spring بالنهاية بينقطع بعد فترة طويلة نسبيا من الزمن

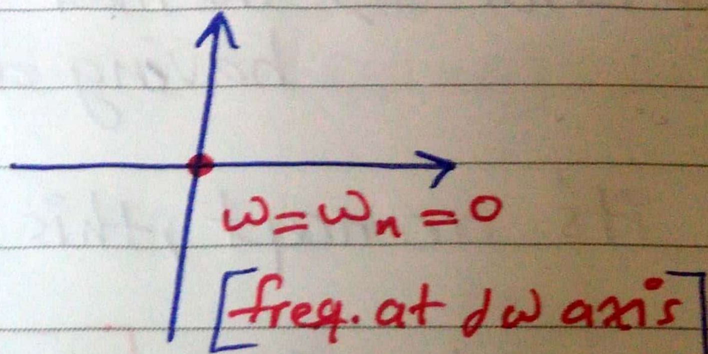
No.

2



$$\frac{K_m}{G(s)}$$

$$H(s) = \frac{K_m}{S\,G(s)}$$



No.

Ch.4 : feedback control system characteristics

this chapter emphasizes the behaviour of the system by its error, sensitivity, response, ...

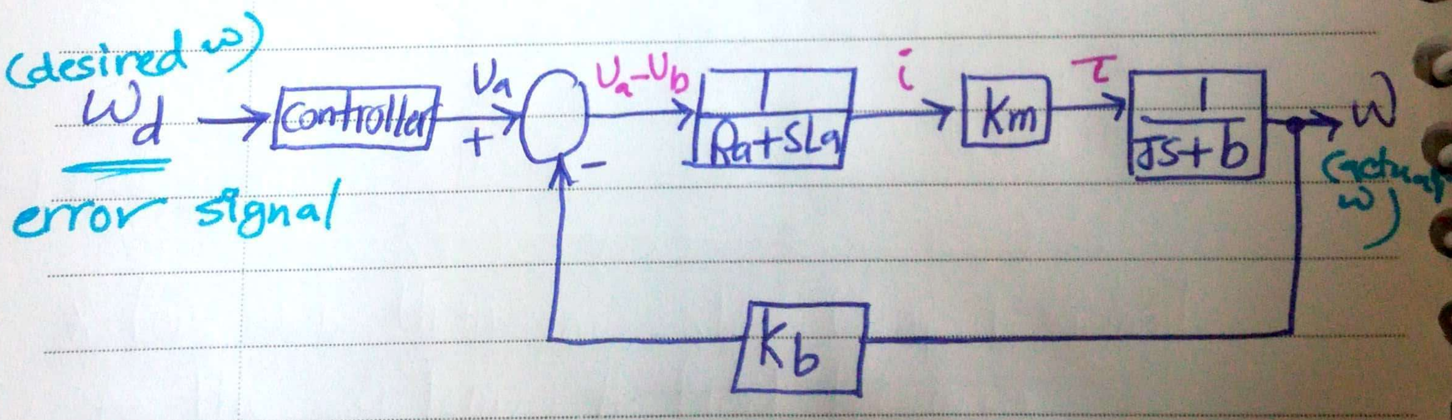
1# Error ⇒ the most desired system, which is having a zero error.

it's a concept, this is relating in time domain

In s domain, $\boxed{E(s) = R(s) - Y(s)}$

$R(s)$: desired output (Reference input) "not any input"

$Y(s)$: actual output



ال error موجودة لأنه عندك step input (V_a)

لأ تتغير تتغير ال ω (speed) ففي عندي error

No.

note → we don't care about the time that we calculate the error occured at it. (transient time)

- Steady State Error (final value theorem) ⇒

in time domain function (إلها نهاية محدودة)

$$e_{ss} = y(\infty) = \lim_{s \to 0} s\,E(s)$$

$$= \lim_{t \to \infty} E(t)$$

(ess) حساب قيمة الخطأ يتم عند استقرار النظام

Ex ⇒ ① $y(t) = \sin(\omega t)$

where y(t) describe E(t)

$$y(s) = \frac{\omega}{s^2 + \omega^2}$$

$$e_{ss} = \lim_{s \to 0} s \cdot \frac{\omega}{s^2 + \omega^2} = 0$$

مشكلة لأنه دوار
time domain function
ما إلها نهاية
[there's no final limit]

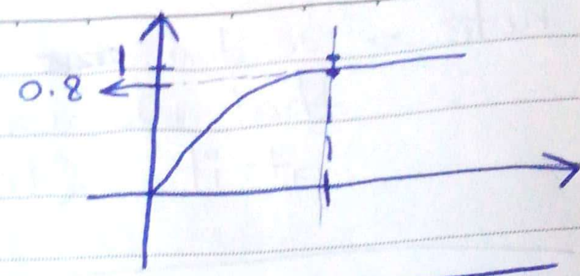
② $y(t) = e^{-t}$

$$y(\infty) = \lim_{t \to \infty} e^{-t} = 0$$

③ $y(t) = e^{t}$

$$y(\infty) = \lim_{t \to \infty} e^{t} = \infty$$

Error = 0.2
(ess)



Note System Order ≡ highest power of the TF at its denominator.

يعني معرفته بتجيب (System response) ← mainly transient response

System type ≡ indicates to how much does the integrator used in the system.

يعني بال (steady state error)

لما يكون عندي error ومدى أشيلها برفع ال system type

بإضافة $\left[\frac{1}{s}\right]$ ...

Note

* for more than one external input signal to the system, to find the error → i calculate it from each one of these signals because the error is an accumulative value

→ this process is called [superposition]

Ex for $E(s) = \frac{1}{s(s+1)}$, find the steady state error?

$$e_{ss} = \lim_{t \to \infty} [1 - e^{-t}] = 1 - e^{-\infty} = 1 - 0 = \underline{1}$$

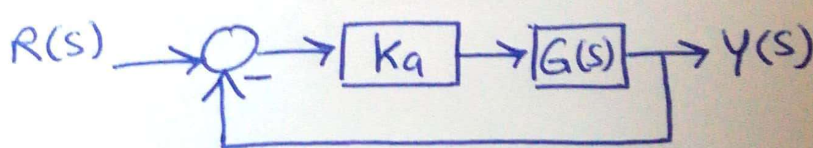
$$= \lim_{s \to 0} \cancel{s} \cdot \frac{1}{\cancel{s}(s+1)} = \frac{1}{0+1} = \underline{1}$$

$$\boxed{e^{\infty} = \infty}$$

$$\boxed{e^{-\infty} = 0}$$

Ex

If $G(s) = \frac{10}{s(0.001s+1)}$, for a unit step input, find the steady state error?



$$E(s) = R(s) - Y(s)$$

$$\boxed{Y(s) = \frac{K_a G(s) \cdot R(s)}{1 + K_a G(s)}} \rightarrow E(s) = \underbrace{R(s)}_{\substack{\text{unit step} \\ \frac{1}{s}}} \left[1 - \frac{K_a G(s)}{1 + K_a G(s)} \right]$$

توحيد مقامات

$$\therefore E(s) = \frac{R(s)}{1 + K_a G(s)}$$

$$e_{ss} = \lim_{s \to 0} s\, E(s) = \cancel{s} * \frac{\frac{1}{\cancel{s}}}{1 + \underbrace{\left[K_a \frac{10}{\underbrace{s(0.001s+1)}_{0}} \right]}_{\infty}}$$

$$e_{ss} = \frac{1}{\infty} = 0$$

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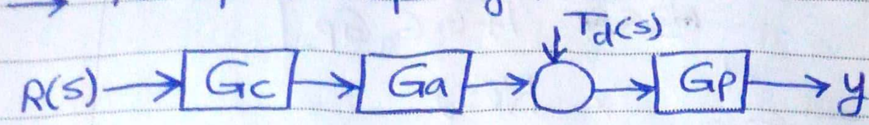
No.

→ For open loop system :-



$$y(s) = G_c G_a G_p R(s) + G_p T_d(s)$$

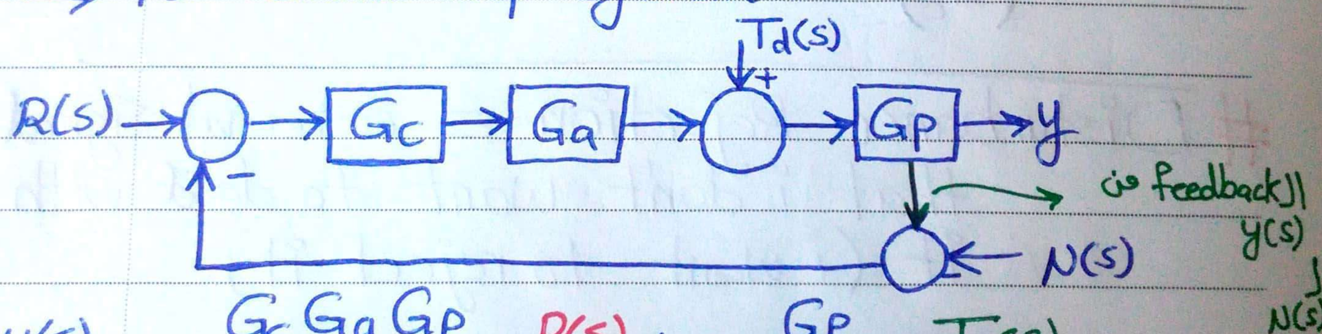
$T_d(s)$: disturbance
$N(s)$: Noise
$R(s)$: desired output

- the error as aresult of the desired output $R(s)$ $[T_d(s) = 0]$

$$E(s) = R(s) - Y(s)$$
$$= \left(1 - G_c G_a G_p\right) R(s)$$

Error في

→ For closed loop system :-



$$y(s) = \frac{G_c G_a G_p}{1 + G_c G_a G_p} R(s) + \frac{G_p}{1 + G_c G_a G_p} T_d(s)$$

$$- \frac{G_c G_a G_p}{1 + G_c G_a G_p} N(s)$$

- the error as aresult of the desired output $R(s)$

$[T_d(s) = N(s) = \phi]$

$$E(s) = R(s) - Y(s)$$
$$= R(s) - \frac{G_c G_a G_p}{1 + G_c G_a G_p} R(s)$$

FIVE APPLE

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$$E(s) = R(s)\left[\frac{1+\cancel{*}1}{1+G_cG_aG_p} - \frac{G_cG_aG_p}{1+G_cG_aG_p}\right]$$

$$= \frac{1}{1+G_cG_aG_p} * R(s)$$

ازا زدنا من ال
(G_c) controller Gain
مكبر التحكم (خفقل ال error)

Result ⇒ Closed loop better than Open loop, because i can reduce the error (controlling it) by choosing large gain (G_c)

Disturbance Rejection ≡ external signal that i don't want to deal with it (i want to reject it)

* Error as aresult of disturbance :

$$E(s) = R(s) - Y(s) \quad , \quad R(s) = 0 = N(s)$$

$$= 0 - Y(s)$$

$$= -Y(s)$$

- for open loop system given before →

$$E(s) = -G_p T_d(s)$$

Nothing will reduce/remove the error resulting from the disturbance in the open loop system....

- for closed loop system given before →

$$E(s) = -Y(s) \quad , \quad \boxed{R(s) = N(s) = 0}$$

$$= \frac{-G_P}{1+G_c G_a G_p} T_d(s)$$

→ I can reduce the error in the closed loop that the disturbance affect it, by increasing G_c

[Closed loop > open loop]
better than

Noise Rejection ≡ external signal i don't want to deal with, occurs during the measurement process

Disturbance	Noise
High power	Low power
low frequency	High frequency

- Closed loop → $E(s) = -Y(s) \quad , \quad \boxed{R(s) = T_d(s) = 0}$

$$= -\left(\frac{-G_c G_a G_p}{1+G_c G_a G_p}\right) N(s)$$

ما بقدر ألعب على قيمة G_c لأنها مأثرة بسط ومقام، فالحل للتخفيف عن ال $N(s)$ أستخدم [Low Pass Filter]

بشيلك على الـ Frequency اذا كبيرة بيقللها
(low gain)

FIVE APPLE

∴ Low Pass filter → High freq. ، بيقللها ، بيصغّر ال Gain

low freq. ، بيمررها ، بيكبر ال Gain

2# Sensetivity ⇒ the ratio of the change in the system transfer function for a small incremental change to a certain parameter changes.

كلما كان ال [system] ← less Sensetivity كلما كان أفضل

(أقل تأثر بالظروف المحيطة)

لكن ك [measurement sys.] ← more Sensetivity كلما كان

كلما كان أفضل لتطوير الأداء وزيادة الدقة

$$S_G^T = \frac{dT}{dG} \cdot \frac{G}{T}$$

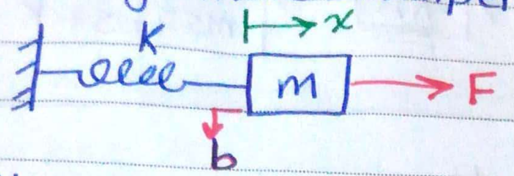
- for open loop system , $S_G^T = 1$,
  High Sensetivity (100%)

هاد بيعني إنه النظام بتأثر وبتغير بشكل كبير لأي عامل جوانية

No.

- for closed loop system,

Ex) Spring-mass-damper



* Find the system sensetivity with respect to the damping Coefficient ? certain Parameter

[لأنها أكثر معامل ممكن يتغير بعكس m و k]

$$S_b^T = \frac{dT}{db} \cdot \frac{b}{T}$$

- to find the transfer function →

$$\Sigma F = m\ddot{x}$$

$$F - kx - b\dot{x} = m\ddot{x}$$

$$\mathcal{L}[m\ddot{x} + b\dot{x} + kx = F]$$

$$ms^2x(s) + bsx(s) + kx(s) = F(s)$$

$$H(s) = \frac{\text{output}}{\text{input}} = \frac{x(s)}{F(s)} = \frac{1}{ms^2 + bs + k}$$

$$\Rightarrow S_b^T = \frac{-1 * s}{(ms^2+bs+k)^2} * \frac{b}{\frac{1}{ms^2+bs+k}}$$

[سالب الثابت * مشتقة المقام / المقام²]

$$= \frac{-s}{(ms^2+bs+k)^{\cancel{2}}} * \cancel{(ms^2+bs+k)}\, b$$

$$= \frac{-bs}{ms^2 + bs + k}$$

معناها يتأثر الـ Sys. Sensetivity بال (freq.) للنظام

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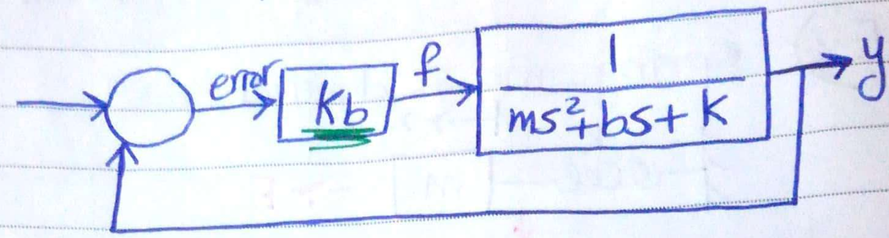
No.

* ex. follow the previous ⇒



$$S_b^T = \frac{-bs}{ms^2+bs+k+k_b}$$

:: زيادة ال gain(K_b)
للنظام مجل مدى تأثر النظام بال frequency أقل مما كانت عليه في
المثال السابق ...
لأنه المقام زادت قيمته بإضافة (k_b) وبالتالي قلت $[S_b^T]$ والمتأثرة
بالأصل من ال S [freq.]

Ex) Design Example :
English Channel boring machines
[من الكتاب صفحة 232]

* the design objective is to select the gain (K), so that the response to input angle changes is desirable while we maintain minimal error due to the disturbance.

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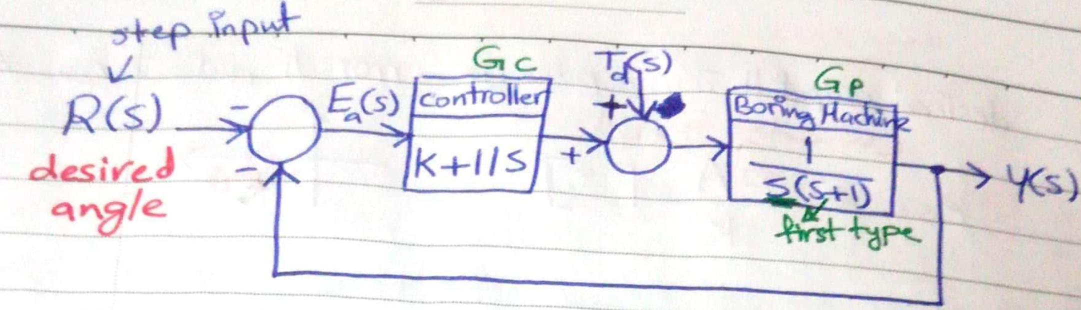
No.



$(K + 11S)$: PD controller
↓ proportional → derivative

$$y(s) = \frac{K+11S}{S(S+1)+(K+11S)} R(s) + \frac{1}{S^2+12S+K} T_d(s)$$

$$\boxed{E(s) = R(s) - Y(s)}$$

$\Rightarrow$

$$E(s) = R(s) - \left[\frac{K+11S}{S(S+1)+K+11S} R(s) + \frac{1 \cdot T_d(s)}{S^2+12S+K}\right]$$

- $T_d(s) = 0$, $E(s) = R(s)\left[1 - \frac{K+11S}{S(S+1)+K+11S}\right]$

$$e_{ss} = \lim_{S\to 0} SE(s) = \left(1 - \frac{\cancel{K}^{1}+0}{0+\cancel{K}+0}\right) * 0 = 0$$

$\therefore$ R(s) : step input

- $R(s) = 0$, $E(s) = \frac{-1}{S^2+12S+K} T_d(s)$

$$\frac{y(s)}{R(s)} = \frac{\frac{K+11S}{S(S+1)}}{\frac{S(S+1)}{S(S+1)}1 + \frac{K+11S}{S(S+1)}} = \frac{K+11S}{S(S+1)+K+11S}$$

$$\frac{y(s)}{T_d(s)} = \frac{\frac{1}{S(S+1)}}{\frac{S(S+1)}{S(S+1)}1 + \frac{K+11S}{S(S+1)}} = \frac{1}{S^2+S+K+11S} = \frac{1}{S^2+12S+K}$$

$$e_{ss} = \lim_{S\to 0} SE(s)$$

$$= S * \frac{-1}{S^2+12S+K} T_d(s) = \frac{-A}{K} \text{ [degree]}$$

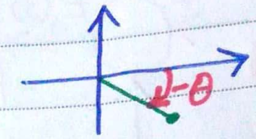
بزيد k ,بيقل ال ess

No.

* وحدة قياس الـ error من نفس وحدة الـ desired output

الزاوية معاكسة CW

$$e_{ss} = \frac{-A}{k} \, [^\circ]$$



* If $A = 1$
  $k = 100$
  $e_{ss} = \frac{-1}{100} = -0.01^\circ$, less error

* If $k = 20$, $e_{ss} = -0.05$, more error

$k=20$

$$\therefore \; y(s) = \frac{20 + 11s}{s^2 + 12s + \underline{20}} R(s) + \frac{1}{s^2 + 12s + \underline{20}} T_d(s)$$

$$\omega_n = \sqrt{20} = 4.5$$

$k=100$

$$y(s) = \frac{100 + 11s}{s^2 + 12s + \underline{100}} R(s) + \frac{1}{s^2 + 12s + \underline{100}} T_d(s)$$

$$\omega_n = 10$$

Recall : (ω_n) : قيمة ω اللي بيميل النظام عندها للتذبذب (oscillating)

[less (ω_n) $\longrightarrow$ less oscillating]

No.

حسب اللي عملناه قبل بحدّد هالمواصفة

* error shouldn't exceed ...
* max. peak shouldn't exceed ...
بشوف ال gain(k) اللي بيعطيني ال (max. peak)

for the Sensetivity →

assuming $T_d(s)$

$$S_{G_p}^{T} = \frac{dT}{dG_p} \cdot \frac{G_p}{T}$$

$$G_p = \frac{1}{s(s+1)}$$

$$G_c = k + 11s$$

$$TF = \frac{Y(s)}{R(s)} = \frac{G_c G_p}{1 + G_c G_p}$$

$$\frac{dT}{dG_p} = \frac{G_c(1 + G_c G_p) - G_c^2 G_p}{(1 + G_c G_p)^2}$$

$$S_{G_p}^{T} = \frac{G_c(1 + G_c G_p) - G_c^2 G_p}{(1 + G_c G_p)^{\cancel{2}}} * \frac{\cancel{G_p}}{\frac{G_c G_p}{(1 + G_c G_p)}} \qquad \frac{\cancel{1+G_c G_p}}{G_c \cancel{G_p}}$$

$$= \frac{\cancel{G_c}\left[(1 \cancel{+ G_c G_p}) \cancel{- G_c G_p}\right]}{1 + G_c G_p} * \frac{1}{\cancel{G_c}}$$

$$= \frac{1}{1 + G_c G_p} = \frac{1}{1 + \frac{k + 11s}{s(s+1)}}$$

FIVE APPLE

No.

$$\therefore \quad S_{G_p}^{T} = \frac{s(s+1)}{s^2+12s+K}$$

(s) in the numerator, indicates that the sensetivity changes as the frequency changes.

* If $s<1 \longrightarrow S_{G_p}^{T} \simeq \dfrac{s}{K}$ $\left[\dfrac{s(\cancel{s+1})}{\cancel{s^2}+\cancel{12s}+K} \right]$ بإهمال

for example:

$K=20, \; s=0.1$

$S_{G_p}^{T} = 0.005$

$K=100, \; s=0.1$

$S_{G_p}^{T} = 0.001$

بزيادة K بيقل مدى تأثر النظام بال frequency

عند أي قيمة

$s<1$

No.

Ch.5 : System Response [closed loop performance]

Revision

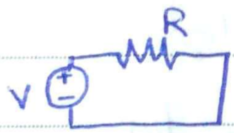
* by its order →

1. Zero Order

$V_{in} \rightarrow \boxed{K} \rightarrow V_{out}$

No S in the denominator

ex



$$\frac{I}{V} = H(s) = \frac{1}{R}$$

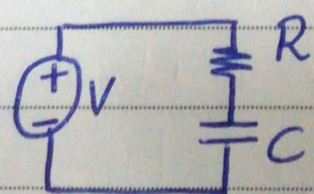
2. First Order

$$\frac{Y(s)}{R(s)} = \frac{1}{\tau s + 1}, \quad \tau : \text{time constant}$$

$$\boxed{y(\tau) = 0.63\, y_{ss}}$$

$T_s = 4\tau$ (settling time)

ex RC-CCT



$$\frac{I}{V} = \frac{1}{Z}$$

$$= \frac{1}{R + \frac{1}{CS}} = \frac{CS}{RCS + 1}$$

[redacted]

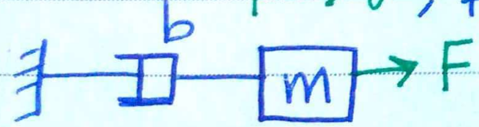
3. 2^{nd} Order

$$\frac{Y(s)}{R(s)} = \frac{b}{s^2+as+b}$$

ex
RLC
CCT

$$\frac{I}{V} = \frac{1}{Z} = \frac{1}{R+Ls+\frac{1}{Cs}} = \frac{Cs}{RCs+Ls^2C+1}$$

OR ⇒



ϑ , first order

x , second order

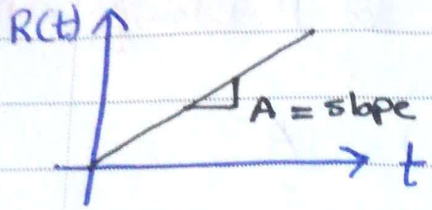
لو حطينا كمان spring بحيث كان الاوتبوت [v / السرعة]

صار (Second Order)

No.

* by its input :-

1. Ramp input , الاقتران الخطي



Steady state value بتغير الـ

input بتغير نوع الـ

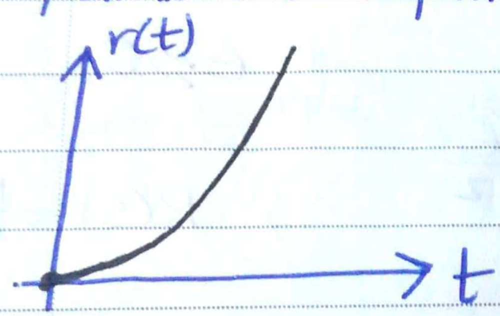
[هو دايمًا مستخدمه]

$$r(t) = \begin{cases} At & , t \geq 0 \\ 0 & , t < 0 \end{cases} \quad , \quad R(s) = \frac{A}{s^2}$$

Unbounded input

Unstable sys.

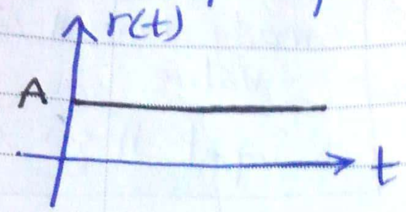
2. Parabolic input , التربيعي



$$r(t) = \begin{cases} At^2 & , t \geq 0 \\ 0 & , t < 0 \end{cases} \quad , \quad R(s) = \frac{2A}{s^3}$$

No.

3. Step input



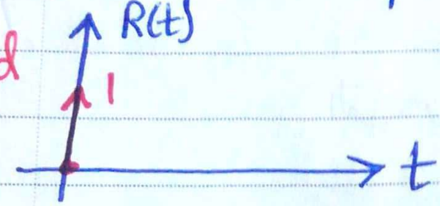
قيمة ثابتة على فترة طويلة من الزمن لكن لا تستمر فعلياً لل ∞ | bounded input

$$r(t) = \begin{cases} A & , t \geq 0 \\ 0 & , t < 0 \end{cases} \quad , R(s) = \frac{A}{s}$$

4. Unit impulse input

عبارة عن input قيمة بالنسبة للزمن كبيرة لكن مدى تأثيره قليل (لحظي)

bounded input



small value $\epsilon \approx 0$

$$R(t) = \begin{cases} \frac{1}{\epsilon} & , \frac{-\epsilon}{2} < \epsilon < \frac{\epsilon}{2} \\ 0 & , \text{otherwise} \end{cases} \quad , R(s) = 1$$

∴ Step and impulse inputs are the most important, we consider them as a testing inputs for the time response of control system.

No.

⇒ As we mentioned before,

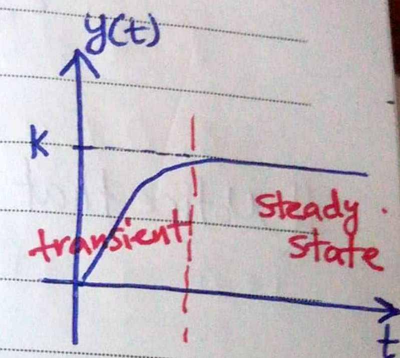
system order: يعني من خلال معرفته، تحديد استجابة النظام عبر الزمن (system Response) for stable systems ...

system Response, includes:

1. Steady State Response

(ess) يعني أعرف منها الـ

من خلال الـ system type

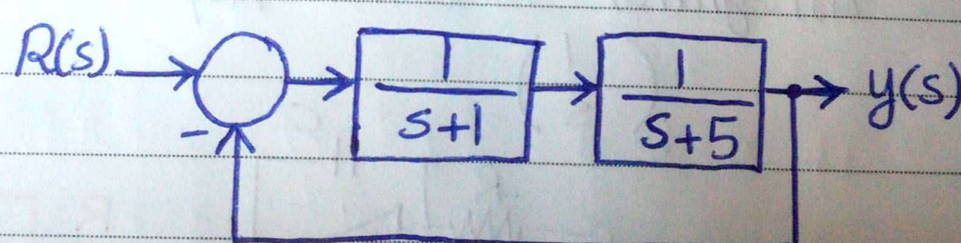


Ex: 1st Order system

system type ≡ indicates to how much does the integrator $\left(\frac{1}{s}\right)$ used in the open loop system.

for closed loop sys. ← لذلك لمعرفة الـ type

ex]



• for this closed loop sys. ⇒ نعتبره [open loop] بإهمال وجود الفيدباك ونوجد الـ transfer function

→ Zero type ←

[بالتالي في error بسبب أنّ الـ
Input type > system type]

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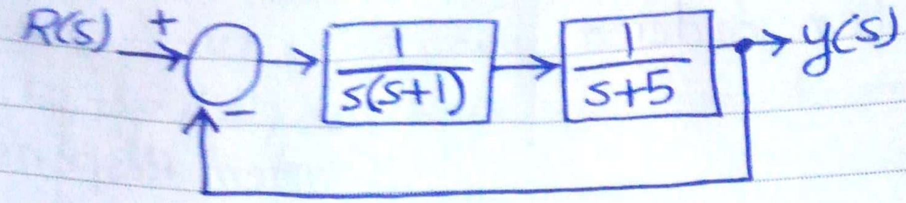
لذلك لإزالة e_{ss} نضيف integrator$\left(\frac{1}{s}\right)$ على النظام السابق ، …:



∴ system type = input type

$e_{ss} = 0$

Notice that ⇒ For closed loop system, when

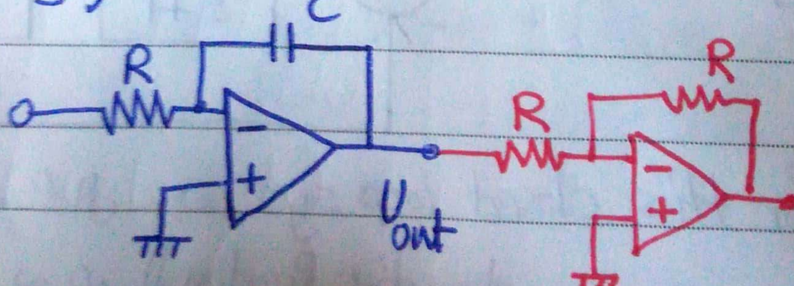
input type $\leq$ system type

the system will immediately have

<u>Zero</u> (e_{ss})

Integrator Addition Representation :

$\left(\frac{1}{s}\right)$



$V_{out} = \frac{-1}{RCS}$

للتخلص من الإشارة السالبة بـ (V_{out})

No.

2. Transient Response :

Peak ⇄ oscillatory ← يعني أعرف ازا النظام

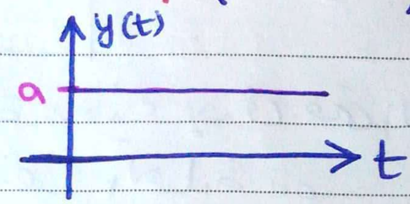
T_s or not osc.

T_s ↵

system Order ⇐ من خلال

System Order : [for unit step input]

① Zero Order : مرجع النظام لل (final value) مباشرة



no transient response

② First Order :

transfer func ← $H(s) = \frac{k}{\tau s+1}$

output ← $y(s) = \frac{k}{s(\tau s+1)}$ to find $y(t)$ [Response]

$$\frac{k}{s(\tau s+1)} = \frac{A}{s} + \frac{B}{\tau s+1}$$

$$k = A(\tau s+1) + B(s) \rightarrow k = A\tau s + Bs + A$$

$$\boxed{A = k}$$

$$A\tau + B = 0 \rightarrow \boxed{B = -\tau k}$$

$$y(s) = \frac{k}{s} + \frac{-\cancel{\tau} k}{\cancel{\tau}(s+\frac{1}{\tau})} = \frac{k}{s} - \frac{k}{s+\frac{1}{\tau}}$$

$$\therefore \boxed{y(t) = k - ke^{-t/\tau}}$$

Never has oscillation

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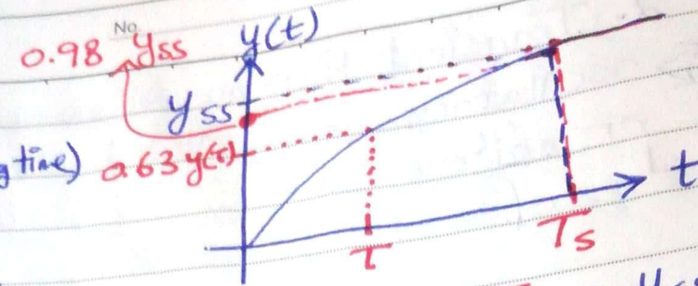
$\boxed{T_s = 4\tau}$ (settling time)



Settling time : Is the time when the system output equal :

1. $0.98\, y_{ss}$, $T_s = 4\tau$
2. $0.95\, y_{ss}$, $T_s = 3\tau$

$$\left[K = \frac{y_{ss}}{R_{ss}}\right]$$

for unit step input ($R_{ss} = 1$)

$\boxed{K = y_{ss}}$

(Ts) عبارة عن قيمة معينة بعرف منها النظام بيستقر والسرعة التي تمت فيها العملية ..

Practical example

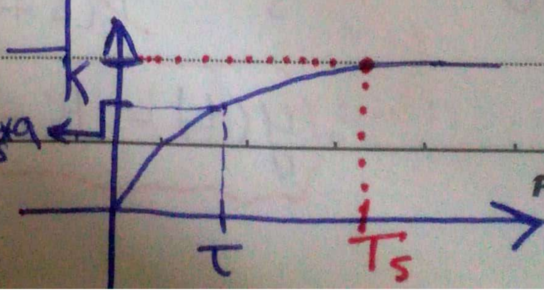
لـ (DC motor) بدي أعطيه فولتية معينة
وبدي أقيس الـ output/speed
من خلال الـ tachometer [بحول السرعة لفولتية]
ثم وصله مع الـ oscillator لرؤية الـ Response

نستطيع تقريبه من الـ [1st and 2nd orders] لإيجاد الـ TF للموتور

بحاجة لشغلتين → final value (K) , (τ)

$$\left[TF = \frac{K}{\tau s + 1}\right]$$ ومباشرة

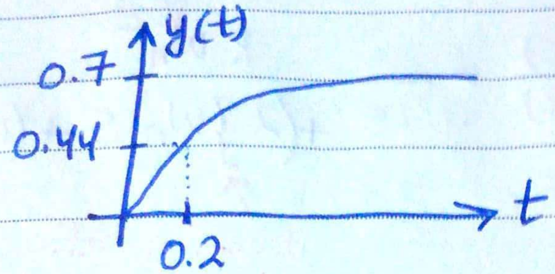
a : applied voltage



No.

ع T_s بتوقف عند [0.98 y_{ss}] or [0.95 y_{ss}] موش y_{ss} بالزبط.

ex



for a unit step input find $\frac{Y(s)}{R(s)}$, K, τ, T_s, $y(t)$?

→ $K = y_{ss} = 0.7$

to check

$$\left[\begin{array}{l} 0.63\, y_{ss} = 0.44 \\ 0.63 * 0.7 = 0.44 \ \checkmark \end{array}\right]$$

$\tau = 0.2$, $T_s = 4\tau = 0.8$ sec.

$$\frac{Y(s)}{R(s)} = \frac{0.7}{0.2s+1}$$

$$y(t) = 0.7 - 0.7\, e^{\frac{-t}{0.2}}$$

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No.

(3) Second Order :

General form : $\left[\dfrac{Y(s)}{R(s)} = \dfrac{K\,\omega_n^{2}}{s^2 + (2\zeta\omega_n)s + (\omega_n^2)} \right]$

natural freq.

damping ratio

كلما زادت قيمتها قلت الـ oscillating بالنظام

من خلال roots النظام مقيد الـ system response

$$S_{1,2} = -\zeta\omega_n \pm \sqrt{\zeta^2\omega_n^2 - \omega_n^2}$$

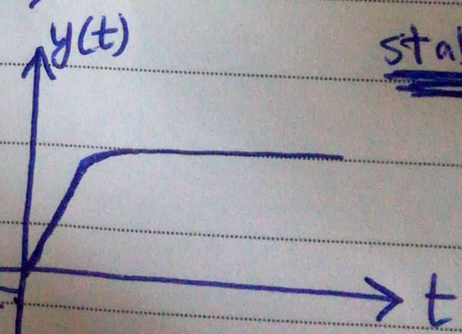
$$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$$

complex — في (oscillating) ⇓ $\zeta < 1$

Real — ما في (oscillating) ⇓ $\zeta > 1$

1. Overdamped system →

$\boxed{\zeta > 1}$



stable

- $S_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$
- $y(t) = 1 + \dfrac{\omega_n}{2\sqrt{\zeta - 1}}\left[\dfrac{e^{-s_1 t}}{s_1} - \dfrac{e^{-s_2 t}}{s_2}\right]$

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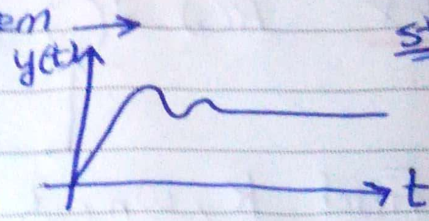
No.

2. Critically damped system → **stable**

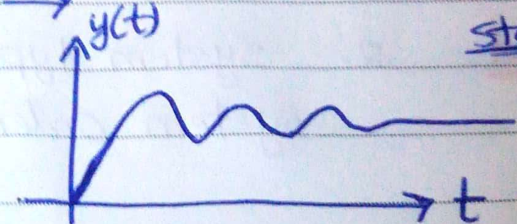
$\boxed{\zeta = 1}$

- $s_{1,2} = -\omega_n$
- $y(t) = 1 - e^{-\omega_n t}(1 + \omega_n t)$



3. Under damped system → **stable**

$\boxed{0 < \zeta < 1}$

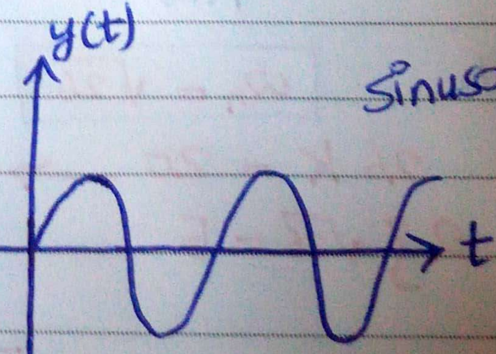


- $s_{1,2} = -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$
  $= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$
- $y(t) = 1 - \dfrac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}} \sin\left(\omega_n\sqrt{1-\zeta^2}\, t + \cos^{-1}\zeta\right)$

4. Undamped system →

$\boxed{\zeta = 0}$

Sinusoidal



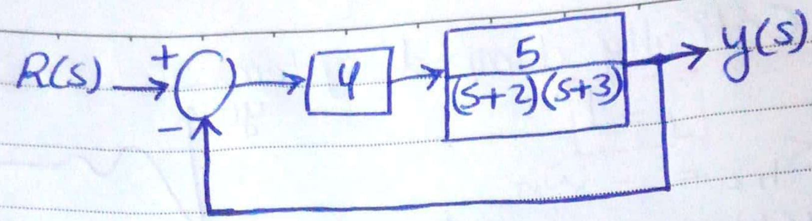
- $s_{1,2} = \pm j\omega_n$
- $y(t) = 1 - \cos(\omega_n t)$

2 poles on $j\omega$ axis

∴ marginally stable system

No.

Ex



① Find k, ω_n, ζ, $y(t)$?

② determine the system order and its type?

sol

2. System type : Zero type

system order : $\dfrac{y(s)}{R(s)} = \dfrac{\dfrac{20}{\cancel{(s+2)(s+3)}}}{\dfrac{\ldots 1 +}{\cancel{(s+2)(s+3)}} \dfrac{20}{\cancel{(s+2)(s+3)}}}$

$$\frac{y(s)}{R(s)} = \frac{20}{(s+2)(s+3)+20}$$

$$H(s) = \frac{y(s)}{R(s)} = \frac{20}{s^2+5s+26}$$

1. $\boxed{\omega_n = \sqrt{26}}$

$26k = 20 \longrightarrow \boxed{k = \frac{20}{26}}$

$2\zeta\sqrt{26} = 5 \longrightarrow \boxed{\zeta = 0.5}$ underdamped response

$$y(t) = 1 - \frac{e^{-0.5\sqrt{26}\,t}}{\sqrt{1-(0.5^2)}} \sin\left(\sqrt{26}\sqrt{(1-0.5^2)}\,t + \cos^{-1} 0.5\right)$$

#

Ch.6 System stability

for high order system

Characteristic Equation

$$\left[\Delta(s) = a_0 s^N + a_1 s^{N-1} + a_2 s^{N-2} + a_3 s^{N-3} + \dots + a_N s^0\right]$$

2 conditions should be satisfied →

① no zero coefficient for Δ

لأنه لو كان مقام الـ (TF) ← $s^3 + s + 1$

هيكون السيستم مباشرة unstable

② no sign change

لو كان في معامل لـ s بمقام TF سالب هيكون

unstable system

* We can determine whether the system stable or not by [Routh_Herwitz criteria] →

s^N	a_0	a_2	a_4	.. a_6
s^{N-1}	$\boxed{a_1}$	a_3	a_5	.. a_7
s^{N-2}	b_1	b_2	b_3	
s^{N-3}	c_1	c_2	c_3	
⋮				
s^0				

$$b_1 = \frac{a_1 a_0 - a_0 a_3}{a_1}, \quad b_2 = \frac{a_1 a_4 - a_0 a_5}{a_1}, \quad b_3 = \frac{a_1 a_6 - a_0 a_7}{a_1}$$

$$c_1 = \frac{b_1 a_3 - a_1 b_2}{b_1} , \quad c_2 = \frac{b_1 a_5 - a_1 b_3}{b_1}$$

$$c_3 = \frac{b_1 a_7 - a_1 b_4}{b_1}$$

Case (1) :

Ex $\Delta(s) = s^3 + 2s^2 + 4s + K$

s^3	1	4
s^2	2	K
s^1	$\frac{8-k}{2}$	0
s^0	K	

If $k \Rightarrow 0 < k < 8$ [the system is stable, no sign change]

$K = 2 \rightarrow \frac{8-k}{2} = 3$

$K = 2$

if $K = -1 \rightarrow$ 1, 2, $9/2$, -1 [unstable system, sign change]

* المفروض يكون حتى النظام stable : العمود الأول كله +ve أو كله -ve أي تغير على الاشارة بين قيمة لقيمة يعتبر (unstable system)

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Case(2) : If the first element only in the row = 0

Ex $\Delta(s) = s^5 + 2s^4 + 2s^3 + 4s^2 + 11s + 10$

s^5	1	2	11
s^4	2	4	10
s^3	$\cancel{0}\,\epsilon$	6	0
s^2	$\cancel{0}\ \frac{4\epsilon-12}{\epsilon}$	10	0
s^1	$\boxed{6}$	0	
s^0	10		

→ أولما بيطلع معنا zero عأول الصف منشيله بـ ϵ وبنكمل الحسابات التانية على أساسه...

$$\frac{4\epsilon-12}{\epsilon} = 4 - \frac{12}{\epsilon} \cong \frac{-12}{\epsilon}$$

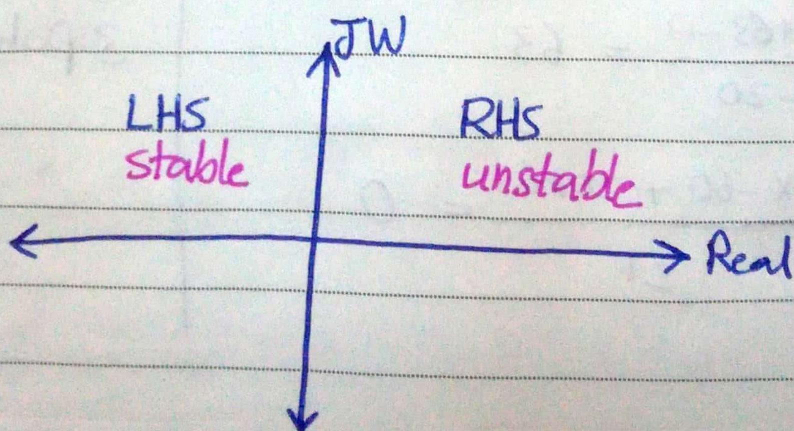
$$\frac{\cancel{\frac{-12}{\epsilon}} * 6 - \cancel{10\epsilon}}{\cancel{\frac{-12}{\epsilon}}} = 6$$

∴ unstable system , 2 times sign change

2 poles on the RHS

3 poles on the LHS

(note) The no. of poles on the RHS = The no. of sign change



No.

Case(3) : If all row elements = 0

In this case, ① poles on $J\omega$ axis
② poles on $S = \pm \alpha$: [α : real no.]

So the system then either unstable or marginally stable.

$$\Delta(S) = S^5 + S^4 + 4S^3 + 24S^2 + 3S + 63$$

S^5	1	4	3
S^4	1	24	63
S^3	-20	-60	0
S^2	21	63	0
S^1	$\cancel{0}$ 42	0	0
S^0	63		

$\rightarrow 21S^2 + 63$ / auxiliary polynomial

$S_{1,2} = \pm j\sqrt{3}$

$42S$

$$\frac{-20 * 24 + 60}{-20} = 24 - 3 = 21$$

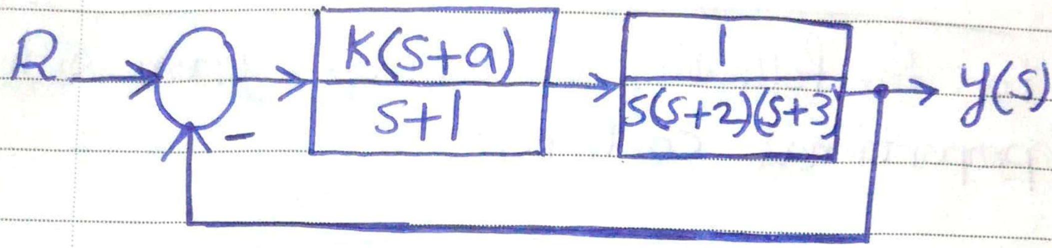
$$\frac{-20 * 63 - 0}{-20} = 63$$

$$\frac{21 * -60 + 20 * 63}{21} = 0$$

unstable system
2 poles RHS
3 poles LHS

Design Examples →

① Determine the range of (k) and (a) for which the system is stable [using routh-herwitz] ?



$$TF = \frac{k(s+a)}{k(s+a)+s^4+6s^3+11s^2+6s}$$

$$\Delta(s) = R(s) = s^4 + 6s^3 + 11s^2 + (6+k)s + ka$$

$6+k > 0$

$k > -6$

s^4	1	11	ka
s^3	6	6+k	0
s^2	$\frac{60-k}{6}$	ka	0
s^1	c_1		
s^0	ka		

$-6 < k < 60$

* $\frac{60-k}{6} > 0 \rightarrow \boxed{k < 60}$ $\quad \boxed{ka > 0}$

* $c_1 > 0 \rightarrow \frac{60k+360-k^2-6k-36ka}{60-k} > 0$

No.

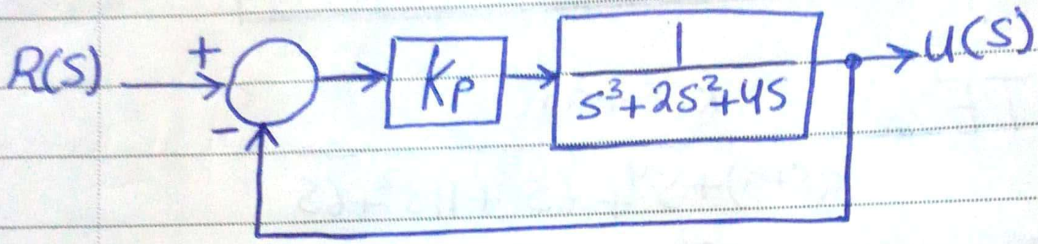
$$\boxed{54K - K^2 + 360 - 36\,Ka > 0} * -$$

$$K^2 - 54K + 36Ka - 360 < 0$$

$\vdots$

---

(2) For the following closed loop system with a proportional controller



$$TF = \frac{k_p}{s(s^2+2s+4)}$$ for the open loop sys. make it marginally stable

$$TF = \frac{k_p}{s^3+2s^2+4s+k_p}$$ $\leftarrow$ closed Loop

applying RH Criteria $\rightarrow$

s^3	1	4
s^2	2	k_p
s^1	$\frac{8-k_p}{2}$	0
s^0	k_p	

$$k_p > 0$$

$$\frac{8-k_p}{2} > 0$$

$$0 < k_p < 8$$

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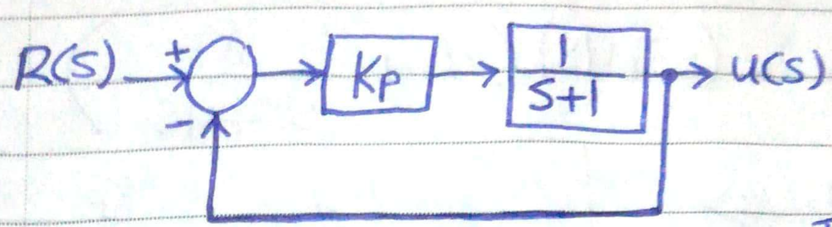
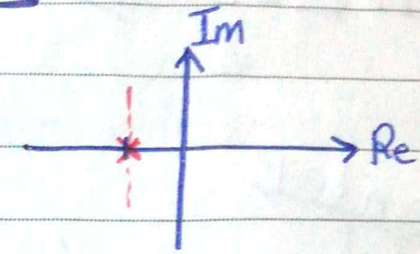
No.

Ch.7 : The Root Locus Method

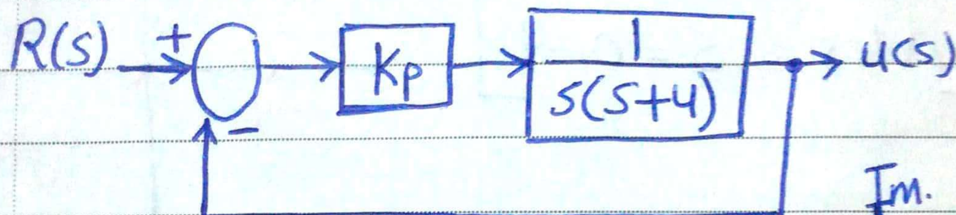
#1



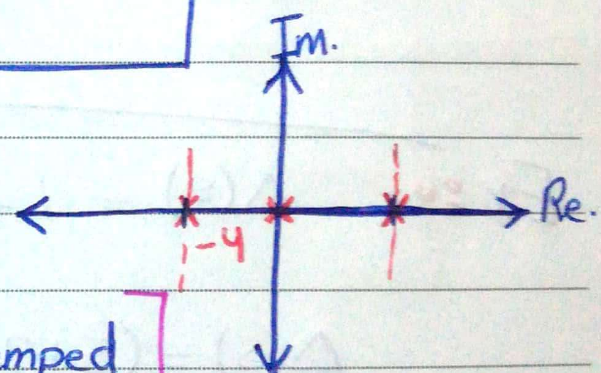
$$H(S) = \frac{K_p}{S+1+K_p}$$



#2



$$H(S) = \frac{K_p}{S^2+4S+K_p}$$



∴ [Real Roots mean → Overdamped system]

$$H(S) = \frac{y(S)}{R(S)} \begin{array}{l} \leftarrow \text{Zeros} \text{ أصفار البسط} \\ \leftarrow \text{poles} \text{ أصفار المقام} \end{array}$$

$$\Delta = R(S) = 1 + K\,G(S)$$

No.

#2 → $\Delta = s^2 + 4s + K_p$

$$= (s^2+4s)\left(1 + \frac{K_p}{s^2+4s}\right)$$

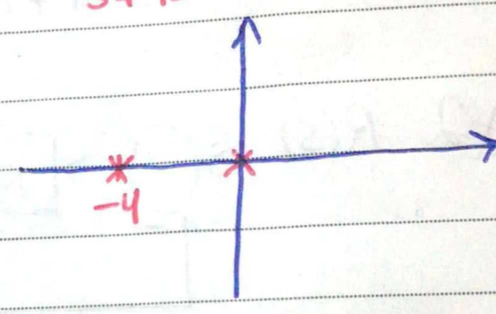
If $\Delta = 0$ ⇒ $\left[\Delta = 1 + \frac{K_p}{s^2+4s}\right]$

$G(s) = \frac{1}{s^2+4s}$, open loop TF

open Loop

no Zero's

poles : $s = 0, -4$



Ex :

$$\Delta(s) = 1 + \frac{K(s+1)}{s+2}$$ ← Zeros, ↖ poles

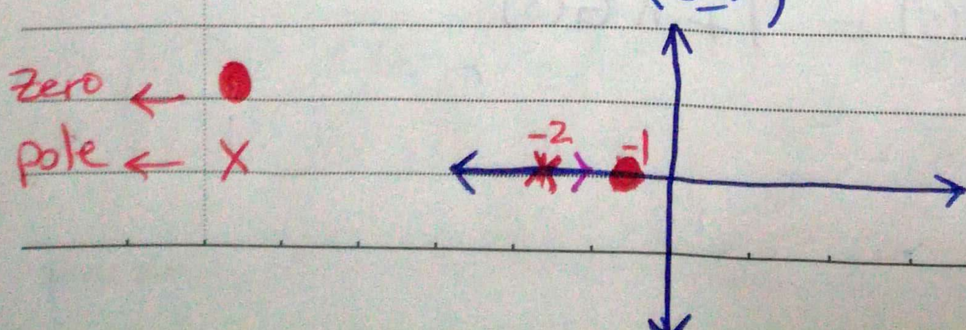
$\Delta(s) = (s+2) + K(s+1)$ ← بعد ما ساويناه بالصفر

when $k=0$, open loop poles

as $k \to \infty$, $(s+2)$ negligible

⇐ كل pole بيروح لـ open loop zero لما $[k \to \infty]$

بس اذا ما فيش (Zero موجود) بتروح (Zero ∞)



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Root Loci →

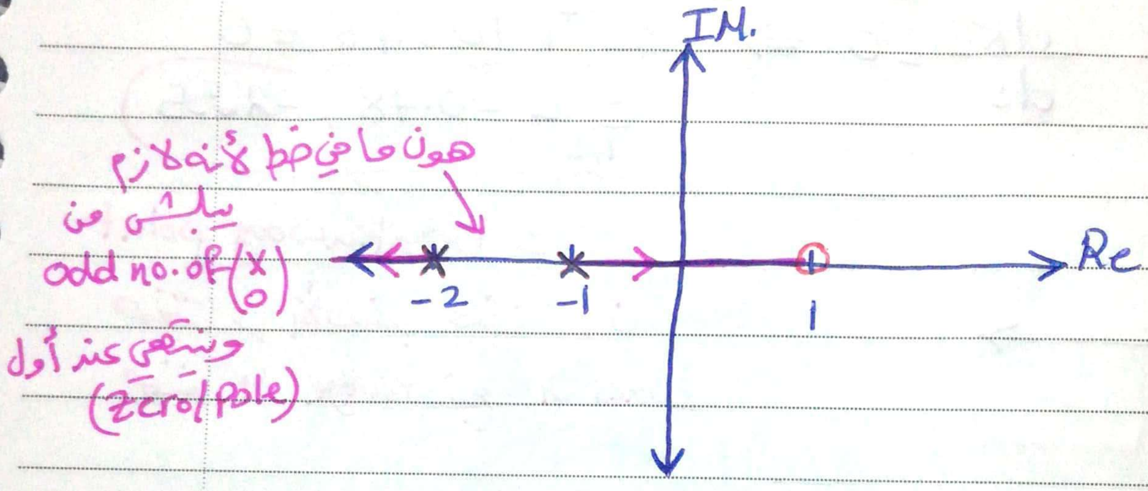
بلش من بين الـ (s-plane) بعد عدد فردي من الـ zero/pole

بحط لعند أول pole/zero

Ex

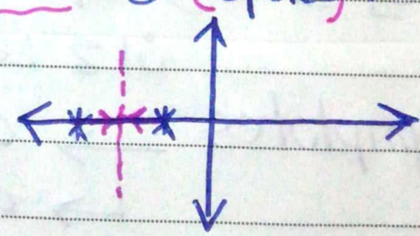
$$\Delta(s) = 1 + \frac{k(s-1)}{(s+1)(s+2)}$$

as k increase , system will be unstable more



* Breakaway point →

(Bp) اذا كان عنا (2poles) على root loci واحد

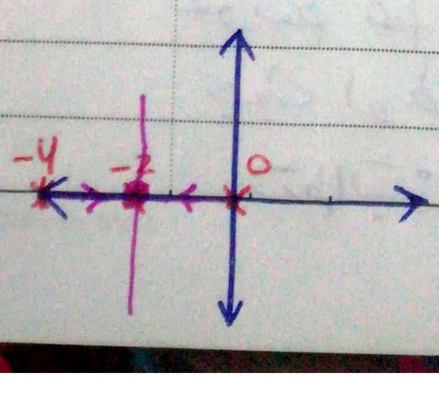


Ex

$$\Delta(s) = s^2 + 4s + kp$$

$$\Delta(s) = 1 + KG(s) \longrightarrow |KG(s)| = 1$$

$$\boxed{K = \frac{1}{|G(s)|}} = s^2 + 4s$$



$$\frac{dK}{ds} = 0 \rightarrow \frac{dK}{ds} = 2s + 4 = 0$$

$$\boxed{s = -2} \leftarrow B_p$$

No.

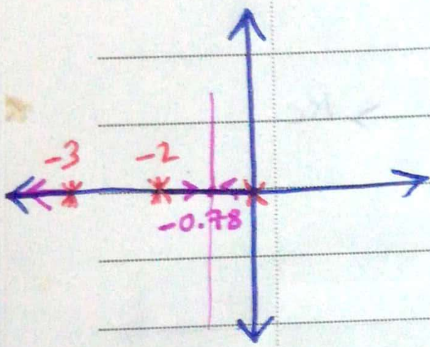
Find the breakaway point :

$$G(s) = \frac{1}{s(s^2+5s+6)}$$

$$K = \frac{1}{|G(s)|} = s(s^2+5s+6) = s^3+5s^2+6s$$

$$\frac{dK}{ds} = 0 \Rightarrow 3s^2+10s+6 = 0$$

$$s_{1,2} = -0.78 , -2.55$$



2 breakaway point

منختار الأنسب بحيث يكون

ضمن ال range تبع ال 2poles

كيف بمشي ال root ؟

بنمد خط وهمي من عند ال Bp بزبطه وبعرف الزاوية اللي

بده يمشي ال root عليها كمان

Assymptotes → center of assymptot
↳ angle of assymptot

مو شرط يكون هالخط عند ال (Root Loci)

هو فقط خط وهمي ال roots بتلحقه، متماثلة حوالين ال x axis

بحيث لو طبقنا اللي فوق على اللي تحت (حوالين ال x)

متطابقين بيكونوا .

No.

* Center of assymptot →

$$\sigma_A = \frac{\sum \text{Poles} - \sum \text{zeros}}{\underbrace{\text{No. of poles}}_{N} - \underbrace{\text{No. of zeros}}_{M}}$$

Ex $\Delta(s) = s^3 + 5s^2 + 6s = s(s^2 + 5s + 6)$

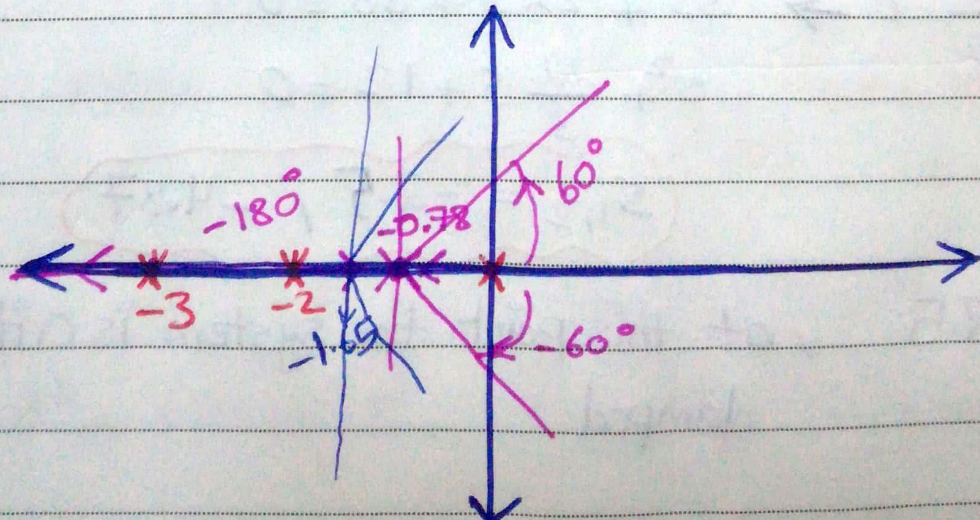
$$\frac{-2-3-0}{3-0} = \frac{-5}{3} = -1.67$$

* Angle of assymptot

$$\theta = \frac{180(2L+1)}{N-M} \qquad L = 0, 1$$

$$\theta = \frac{180}{3-0} = 60^\circ \quad ?$$

$$\theta = \frac{180(3)}{3} = 180^\circ \quad ?$$



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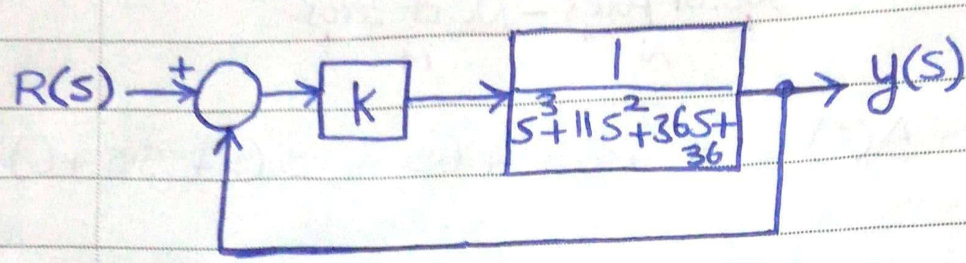
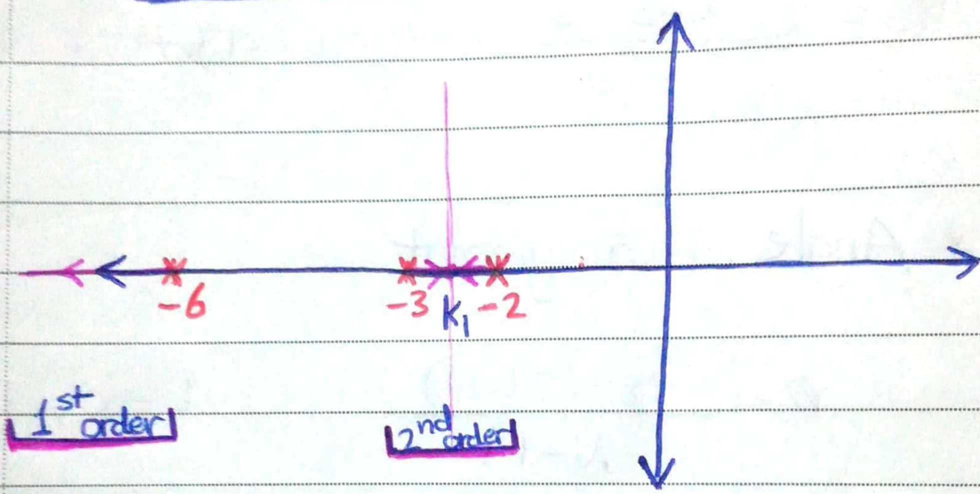
No.

Ex If $\Delta(s) = 1 + \dfrac{K}{(s+2)(s+3)(s+6)}$

third order system





* to find (k_1) [Breakaway Point] →

$G(s) = \dfrac{1}{s^3+11s^2+36s+36}$, $K = \dfrac{1}{|G(s)|} = s^3+11s^2+36s+36$

$\dfrac{dk}{ds} = 0 \rightarrow 3s^2 + 22s + 36 = 0$

$s^2 + \dfrac{22}{3}s + 12 = 0$

$s_{1,2} = -2.5 \, , \, -4.87$

$K_1 = -2.5$, at this point the system is critically damped

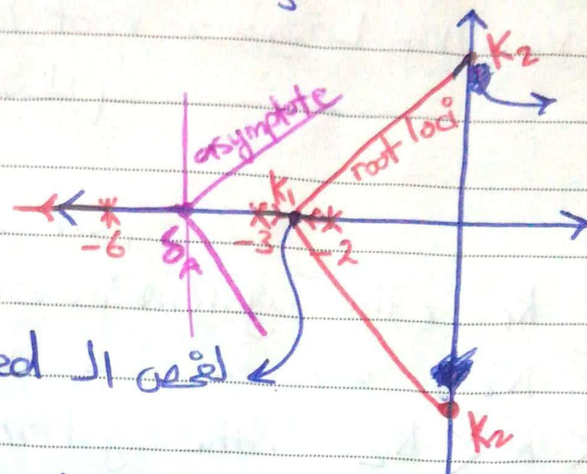
No.

Center of asymptote $(\delta_A) = \frac{11}{3} = -3.67$

$\theta_1 = 60°$

$\theta_2 = -60$

$\theta_3 = -180$



لغص ال marginally stable

لغص ال overdamped

* at k_2, the system is marginally stable.
apply Routh herwitz to get it

$\Delta(s) = s^3 + 11s^2 + 36s + (36+k)$

s^3	1	36
s^2	$\boxed{11}$	36+k
s^1	a	0
s^0	36+k	

$a = \frac{396 - 36 - k}{11}$

$a = \frac{360-k}{11}$

*ع أي أساس حطينا k_1 و k_2 بمواقع معينة ع الرسمة؟
لغص شروط النظام
* stability
* overdamped

$36 + k > 0 \rightarrow k > -36$

but practically, there's no negative controller gain (k) $0 < k < \infty$

$\therefore \boxed{k > 0}$

$11s^2 + (36+k)s^0 = 0 \longrightarrow s_{1,2} = \pm j\sqrt{\frac{396}{11}}$

تعويض

$\frac{360-k}{11} = 0 \rightarrow k = 360$

FIVE APPLE

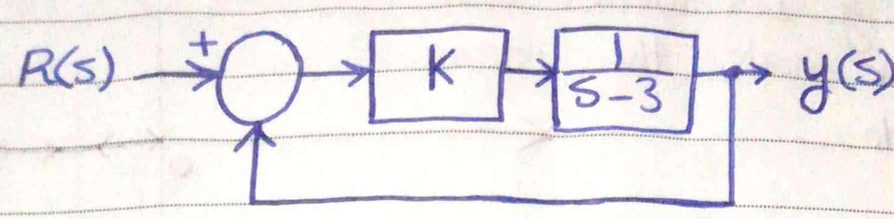
No.

* قيمة ال Controller Gain
على ال root locus وتحديدها ، إلها علاقة بحالة
استجابة النظام

ففي المثال السابق

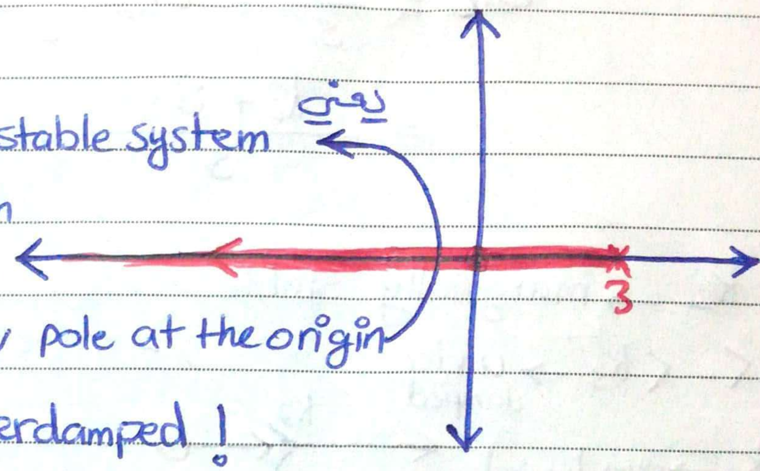
* at K_1 , the system is overdamped
* K_2 , " " " marginally stable
* $K_1 < K < K_2$, the system is underdamped

Ex



by making the system closed loop, we can solve it

$$\therefore \frac{y(s)}{R(s)} = \frac{K}{S-3+K} \rightarrow \Delta = 1 + \frac{K}{S-3} = 0$$



* at $K=3$, marginally stable system يعني

$K>3$, stable system

\# $\Delta(s) = S-3+3 = S$

$y(s)/R(s) = \frac{3}{S}$, pole at the origin

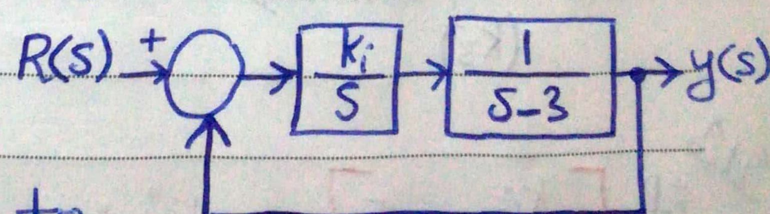
* First Order, overdamped!

* كيف حددنا قيمة K=3 ؟

\# make the previous example an underdamped system: using [PID, PD, I, ...] controller type →

I] using I (integral) controller

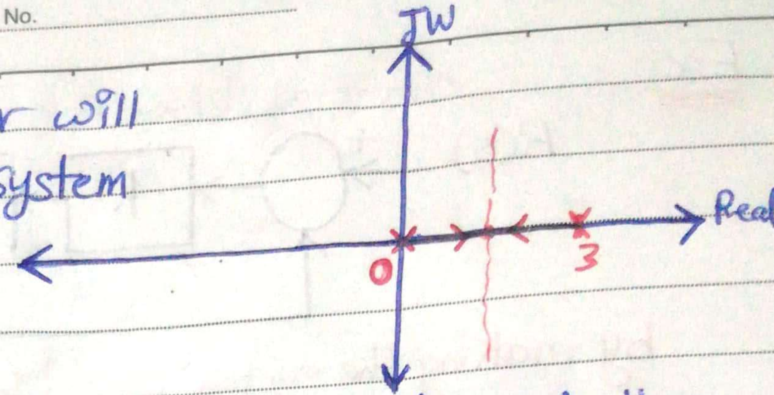
$$\boxed{G_c = \frac{1}{S}}$$



raising system type to First type

No.

adding I Controller will never make the system stable...



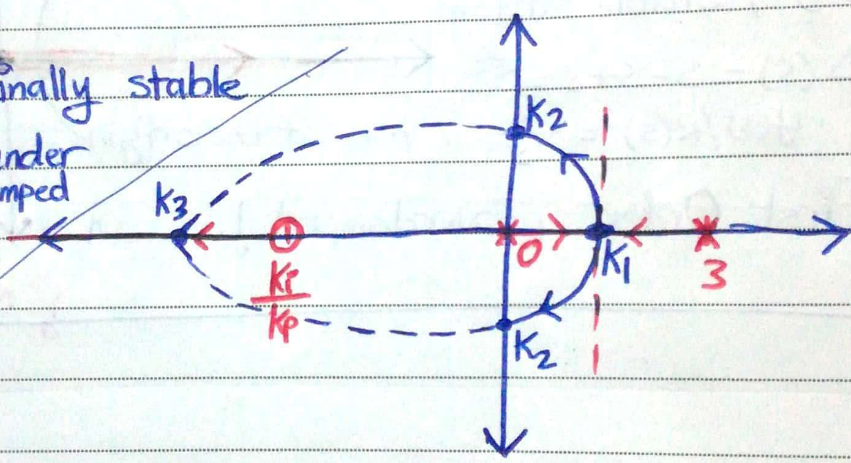
sol

2 using PI (proportional integral) controller

$$G_c = \frac{k_i}{s} + (k_p) * \frac{s}{s} \quad \text{توحيد مقامات}$$

$$= \frac{k_i + s k_p}{s} = \frac{k_p\left(\frac{k_i}{k_p} + s\right)}{s}$$

* at k_2 → marginally stable
* $k_2 < \dots < k_3$ → under damped
* $\dots > k_3$ → overdamped



* Break away point → (k_1)

*كيف نوجد قيمة الـ break-in point ؟

* Break-in point → (k_3)

* على أي أساس وضعنا $\left[\frac{k_i}{k_p} = -5\right]$ ؟ فرضًا ..

مثلاً

if $\left[\frac{k_i}{k_p} = -5\right]$ → نثبت قيمة (k_p) بعدين نختار (k_i) التانية

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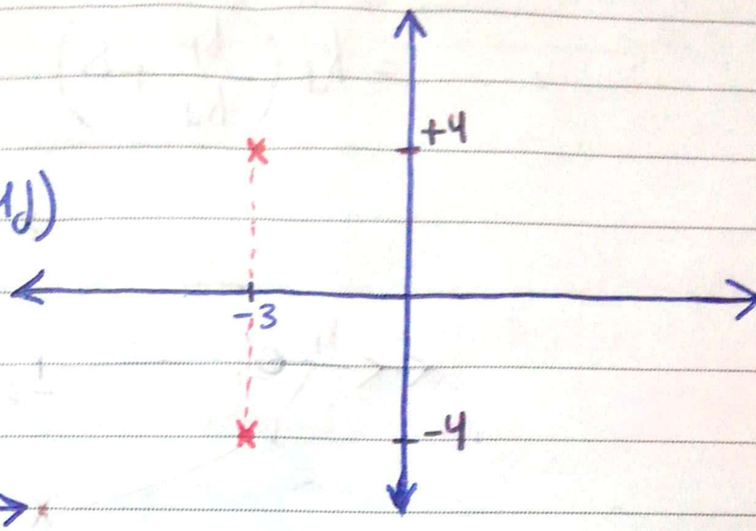
No.

Ex : Find the open loop transfer function from the below S-plane figure →

$S_{1,2} = -3 \pm 4j$



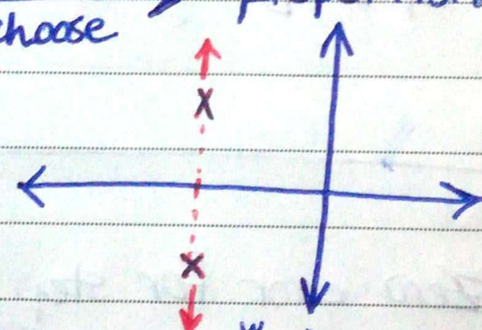
$\Delta(s) = (s-3-4j)(s-3+4j)$

$= s^2+6s+25$

$\longrightarrow \boxed{\dfrac{1}{s^2+6s+25}} \longrightarrow$

اختيار نوع الـ (controller) يتم بناءً على مواصفات محددة ← ①

* more oscillation $\xrightarrow[\text{choose}]{\text{we}}$ proportional controller



لكن لحتى أخلّي النظام stable ، بدي أجيبهم على الـ (x-axis)

∴ Zero adding, will make $(-3 \pm 4j)$ follow that zero on the x-axis to have an underdamped system

طلعت عندي (root Loci)

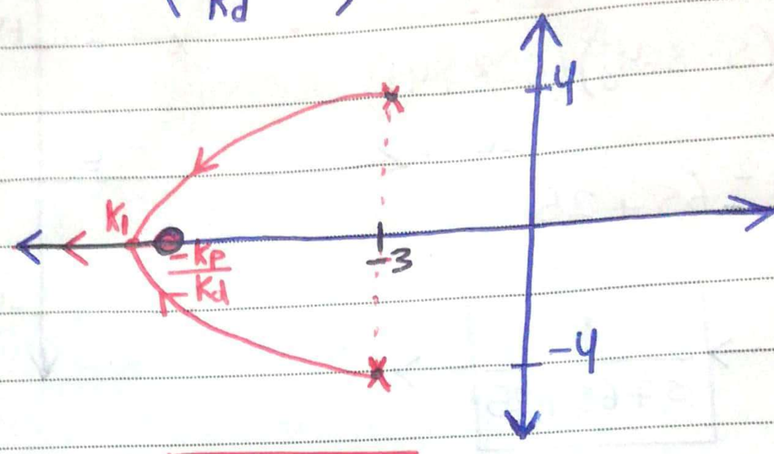
FIVE APPLE

No.

② * PD controller for adding zero

$$G_c = K_p + K_d S$$

$$= K_d\left(\frac{K_p}{K_d} + S\right)$$



مثال

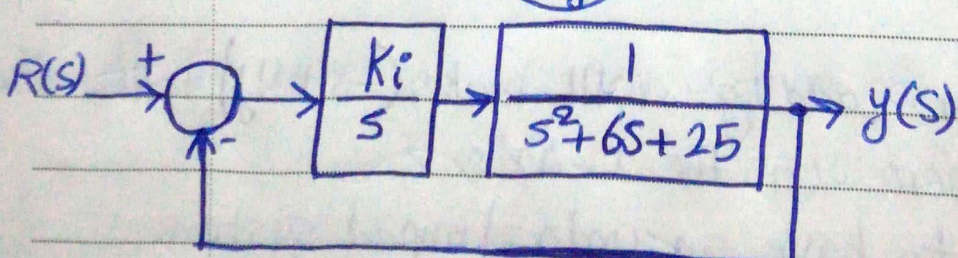
If $\frac{K_p}{K_d} = 10 \rightarrow \boxed{K_p = 10 K_d}$

$K_1 = K_D$! ?

③

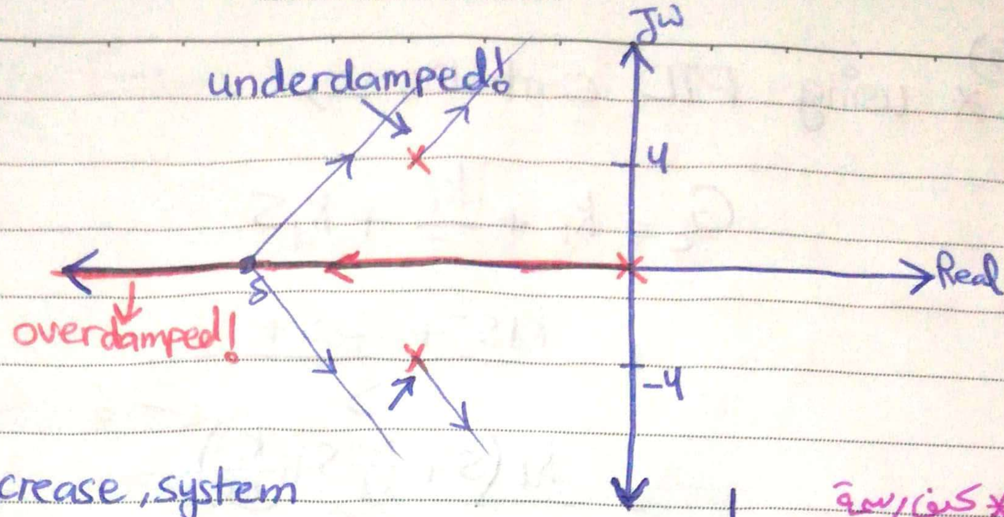
* to get zero error for step input → [I controller]

[increase system type by adding integrator]



third order

No.

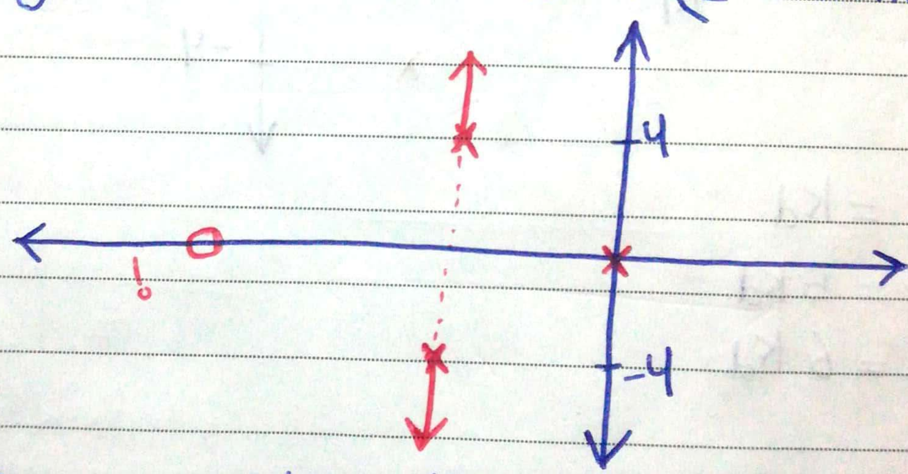


* as k increase, system being unstable !

i'll never get an overdamped system !

* كيف رسمة
Root loci بالزبط
تكون ؟
فقط على الـ x-axis
نغض النظر عن الـ
complex Roots

④
* adding PI controller instead of (I controller) →



* as k increase, we get a stable system but i will not get an overdamped system also

[فلازم أجبرهم يمثلوا على الـ root Loci
بإني أضيف كمان **Zero**
to get underdamped system ...]
(by using PID Controller type.) →

FIVE APPLE

No.

(5)

* using PID controller →

$$G_c = k_p + \frac{k_i}{s} + k_d s$$

$$= \frac{k_d s^2 + k_p s + k_i}{s}$$

$$= \frac{k_d \left(s^2 + \frac{k_p}{k_d} s + \frac{k_i}{k_d}\right)}{s} \rightarrow s^2 + 5s + 6$$

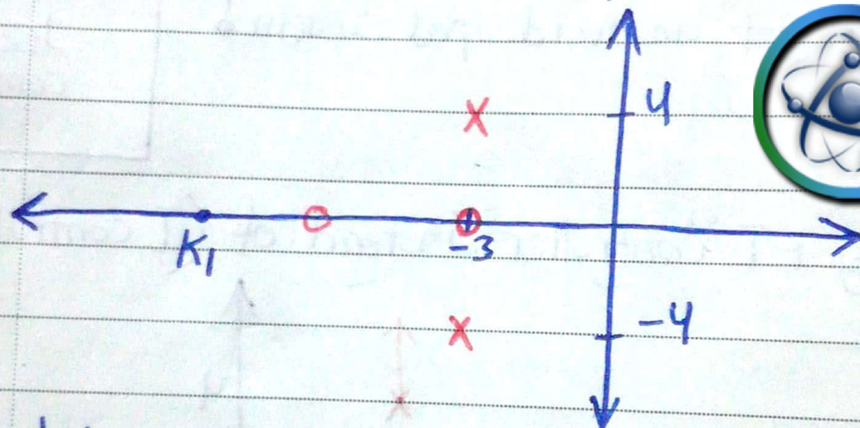
assuming

6

5

$(s+3)(s+2)$

بثبت قيمة 2 وبغير عالتالت (kd)



$k_1 = k_d$

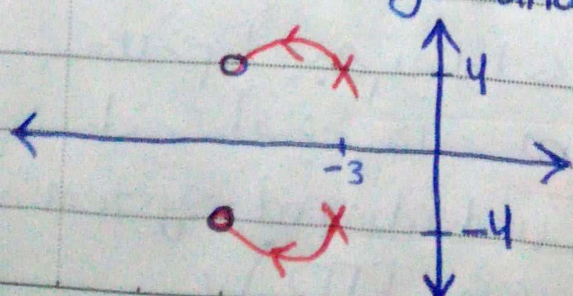
$k_p = 5 k_d$

$k_i = 6 k_d$

(6)

\# to have always underdamped system ::

بيضيف ال (Zero's)



No.

Design Example :

Ex(7.9) $G_m = \dfrac{1}{(\tau_1 s+1)(\tau_2 s+1)\, s}$

$\tau_1 = 0.1$ sec.

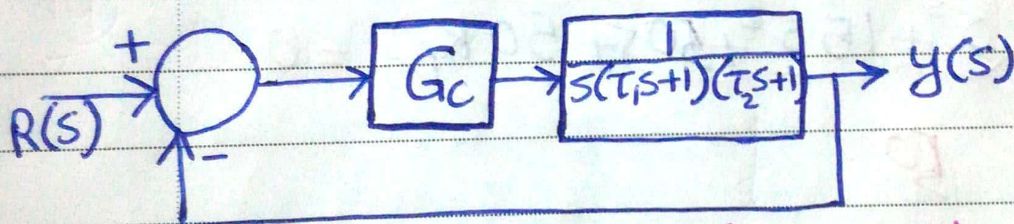
$\tau_2 = 0.2$ sec.

* $e_{ss} \leqslant 0.1$ mm , to have a stable sys.

$1 [mm/s^2]$

For ramp input, $r(t) = At \longrightarrow R(s) = \dfrac{A}{s^2}$

Input type: Second
system type : One



For open loop system (G_m alone) → marginally stable

$e_{ss} \cong \infty$

* this system must satisfied the following conditions :

① to have a stable system

② to get $e_{ss} \leqslant 0.1$

③ $T_s = 7.8s$

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No.

* For stable system :
We want to choose certain values of
k (controller gain) to reach this condition.
and by using specific method for stability checking (Routh – herwitz)

$$H(s) = \frac{K_p}{s(0.1s+1)(0.2s+1)+K_p}$$

$$= \frac{K_p}{0.02s^3+0.3s^2+0.2+K_p} = \frac{K_p}{0.02(s^3+15s^2+\underset{50s}{\downarrow})+K_p}$$

$$H(s) = \frac{50\,K_p}{s^3+15s^2+50s+50K_p} = 0$$

s^3	1	50
s^2	15	$50K_p$
s^1	$\frac{15\times50-50K_p}{15}$	0
s^0	$50K_p$	

$$50K_p > 0$$

$$K_p > 0$$

$$\rightarrow \frac{15\times50-50K_p}{15} > 0$$

$$50K_p < 15\times50$$

$$50K_p < 750$$

$$K_p < 15$$

$$\boxed{0 < K_p < 15}$$ For stable system

No.

* for $e_{ss} \leq 0.1\,mm$:

$$E(s) = R(s) - y(s)$$

$$e_{ss} = \lim_{s\to 0} s * \left[\frac{1}{s^2} - \frac{1}{s^2} * \frac{50K_p}{s^3+15s^2+50K_p+50s}\right]$$

$$= \lim_{s\to 0} \frac{1}{s} * \left[1 - \frac{50K_p}{s^3+15s^2+50K_p+50s}\right]$$

$$= \lim_{s\to 0} \frac{1}{s} * \left[\frac{s^3+15s^2+50s+\cancel{50K_p}-\cancel{50K_p}}{s^3+15s^2+50s+50K_p}\right]$$

$$= \lim_{s\to 0} \frac{1}{\cancel{s}} * \frac{\cancel{s}(s^2+15s+50)}{s^3+15s^2+50s+50K_p}$$

$$= \frac{0+0+50}{0+0+0+50K_p}$$

$$= \frac{1}{K_p} \quad \# \quad \leq 0.1 \rightarrow K_p \geq 10$$

$\therefore$ $10 \leq K_p < 15$ for stable system with an error less than or equal 0.1 mm

→

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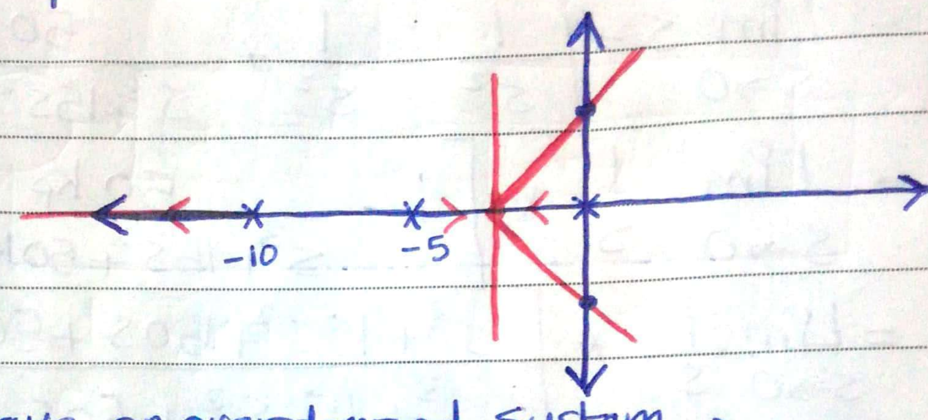
No.

Root locus for it :

* For proportional controller (K_p) :



to have an overdamped system →

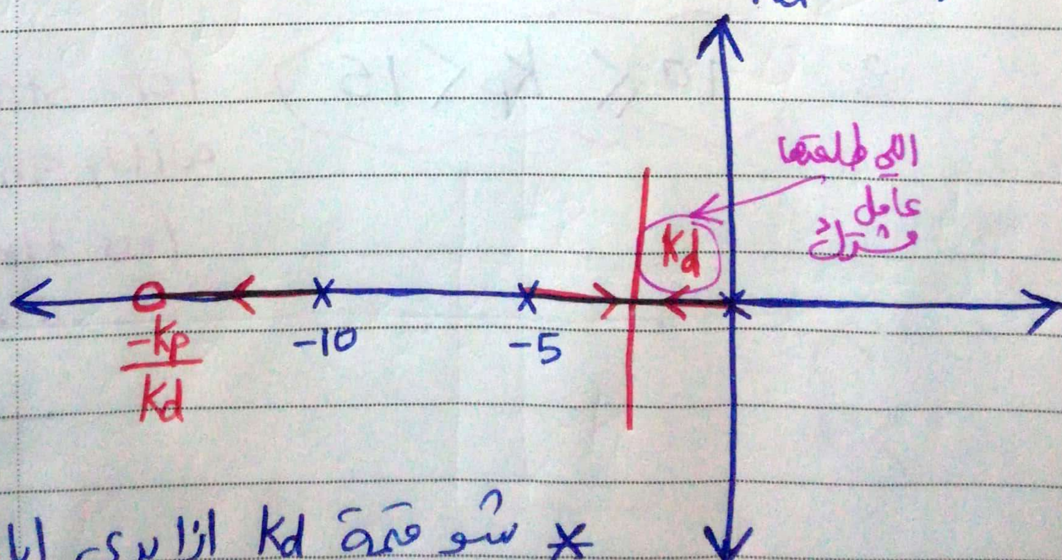
$$\boxed{0 < K_p \leq 3}$$

which contributes the previous condition

$$10 \leq K_p < 15$$

so, if we choose a PD controller :

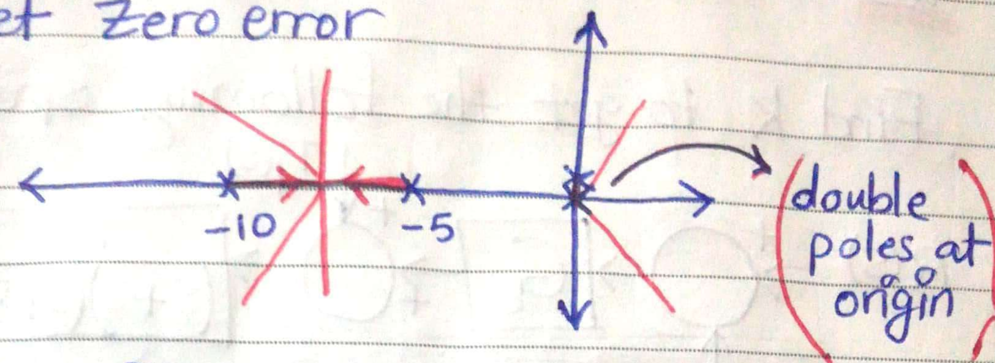
$$G_c = K_p + K_d S = K_d \left(\frac{K_p}{K_d} + S \right)$$



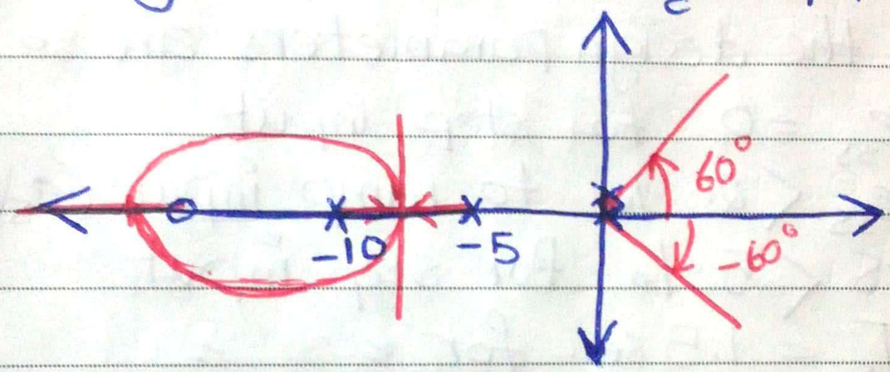
* شو قيمة K_d ازا بدي اياه overdamped

No.

* If i use an integrator (k_I) :
to get zero error



* by adding PI controller $G_c = k_p + \frac{k_I}{s}$

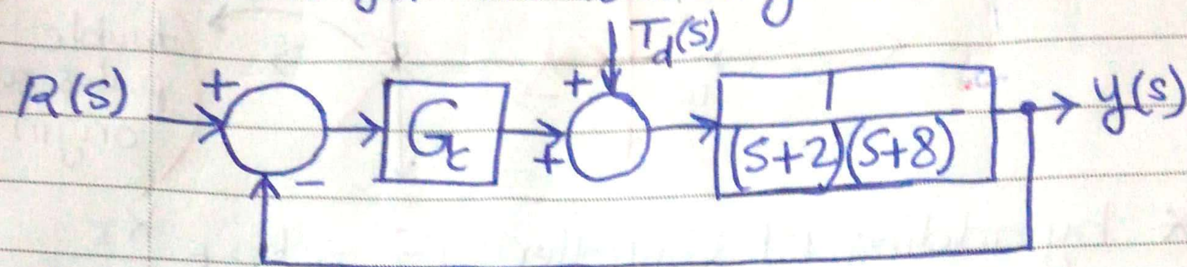


the system will be always marginally stable if i need zero error.

No.

Ex Automatic Velocity Control

Find K to get the following specifications



where the design parameters are so

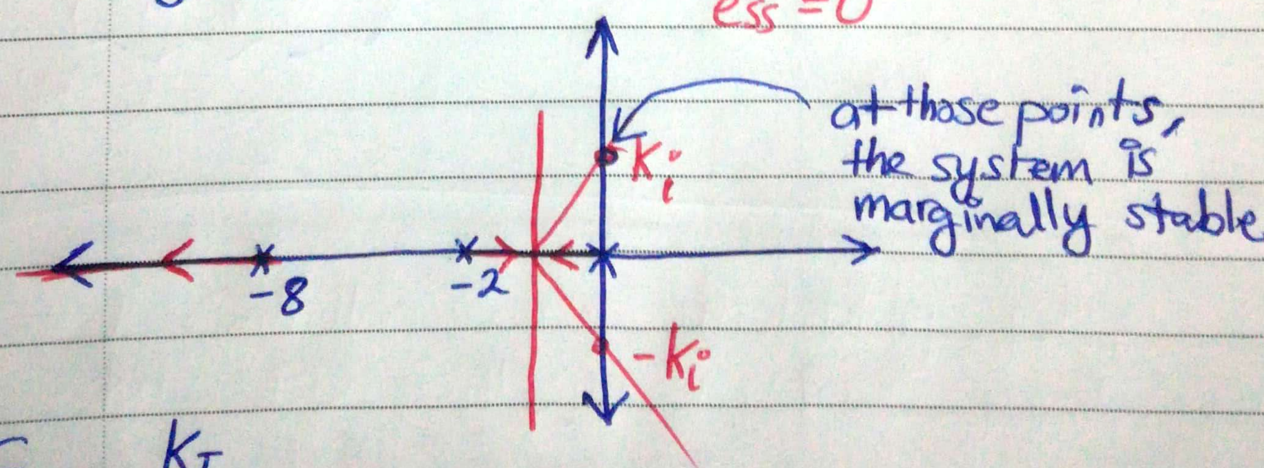
1- $e_{ss} = 0$ to step input
2- $e_{ss} < 25\%$ to ramp input ($e_{ss} < 25\% R(s)$)
3- O.S $< 5\%$ for step input
4- $T_s = 1.5$ sec. for " "

---

① $R(S) = \frac{1}{S}$

[first input type
zero system "] to get $e_{ss} = 0$

by adding (K_I)



$G_c = \frac{K_I}{S}$

No.

$$H(s) = \frac{k_i}{s^3+10s^2+16s+k_i}$$

s^3	1	16
s^2	10	k_i
s^1	$\frac{160-k_i}{10}$	0
s^0	k_i	

$\rightarrow \frac{160-k_i}{10} > 0$

$k_i < 160$

$k_i > 0$

$$0 < K_I < 160$$

(2) For $R(s) = \frac{1}{s^2}$

$$E(s) = R(s) - Y(s) = \frac{1}{s^2}\left[1 - \frac{k_i}{s^3+10s^2+16s+k_i}\right]$$

$$e_{ss} = \lim_{s \to 0} \cancel{s} * \frac{1}{s^{\cancel{2}}}\left[\frac{s^3+10s^2+16s}{s^3+10s^2+16s+k_i}\right]$$

$$= \lim_{s \to 0} \frac{1}{\cancel{s}} * \frac{\cancel{s}(s^2+10s+16)}{s^3+10s^2+16s+k_i}$$

$$= \frac{16}{k_i} \leq 0.25t$$

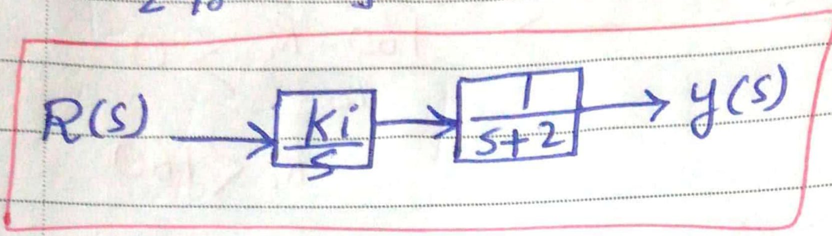
$$k_i \geq \frac{64}{t}$$

FIVE APPLE

No.

③ — على قانون الـ 0.5% للنظام 2^{nd} order أدناه

④ $T_s\big|_{2\%} = \dfrac{4}{\zeta\omega_n}$, for the 2^{nd} order



$\Delta(s) = s^2 + \underline{\underline{2}}s + k_i$

$2 = 2\zeta\omega_n \longrightarrow \boxed{\zeta\omega_n = 1}$

منطلع قيمة ζ عند حساب الـ (0.5) في الفرع الثالث

نعوض ω_n

$\omega_n^2 = k_i \longleftarrow$ وبعدها منعرف قيمة k_i

$k_i = \sqrt{\omega_n^2}$

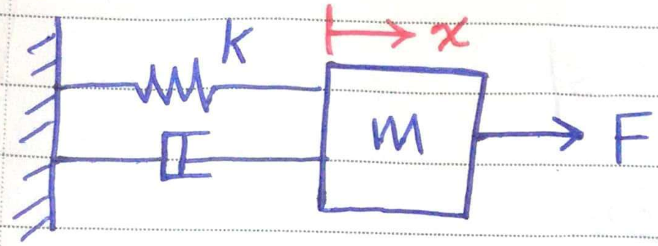
∴ I controller causes unstabilitiy system while PD will give us stable sys. always

No.

Ch.7

How to select parameters using root locus

Ex) spring-mass-damper



$m = 2\,kg$

$b = 3\ \frac{N.S}{m}$

I should choose k when the system is overdamped

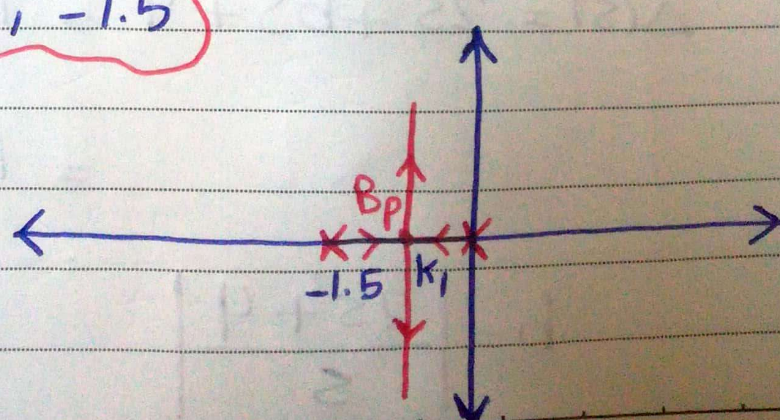
* find k ?

$$H(s) = \frac{1}{ms^2 + bs + k}$$

$$= \frac{1}{2s^2 + 3s + k}$$

$$\Delta(s) = 1 + \frac{k}{2s^2 + 3s} \rightarrow s(2s+3) = 0$$

$S_{1,2} = 0, -1.5$



FIVE APPLE

No.

$$k = \left| \frac{1}{G(s)} \right| = |2s^2 + 3s|$$

$$\frac{dk}{ds} = 4s + 3 = 0 \rightarrow \boxed{s = -0.75}$$

$$k = |2s^2 + 3s|_{s=-0.75}$$

$$\boxed{K = 1.125}$$

∴ k must be $0 < k < 1.125$

when $k = 1.125$, the system will be critically damped.

---

* the same question but assume that $k = 4$ N.m and b is unknown?

→ $H(s) = \dfrac{1}{2s^2 + bs + 4}$

$$\Delta(s) = 2s^2 + bs + 4 = 1 + bG(s)$$

?

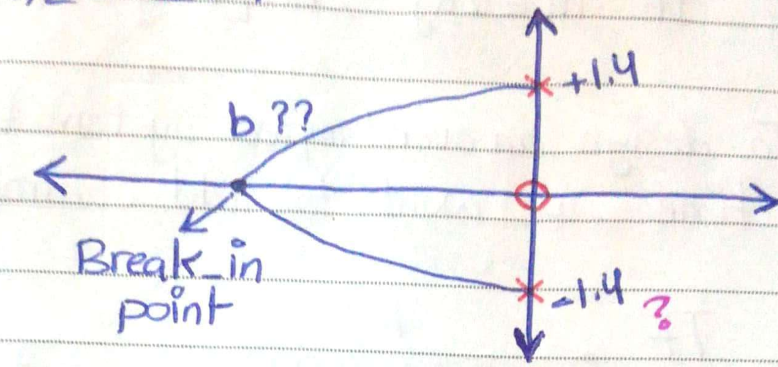
$$= 1 + \frac{bs}{2s^2 + 4}$$

$$b = \left| \frac{2s^2 + 4}{s} \right|$$

$$\frac{db}{ds} = \frac{4s(s) - (2s^2+4)}{s^2} = 0$$

$$2s^2 - 4 = 0$$

$$s_{1,2} = \pm 1.4$$



- at $s = -1.4$:

  to find b , $b\Big|_{s=-1.4} = \left|\frac{2s^2+4}{s}\right| = \frac{8}{\sqrt{2}}$ [N.s/m]

  for $b \geq \frac{8}{\sqrt{2}}$ [N.s/m]

  the system will be overdamped

* The same question before , but with :

$m = 1kg$

$b = ?$ [N.s/m]

$k = 4$ [N.m]

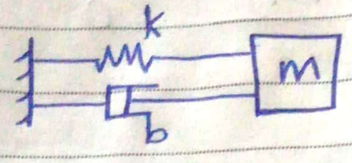
$T_s = \frac{4}{\zeta \omega_n}$, 2% criterion

$$T_s = \frac{4}{\zeta\omega_n} \leq 4 \text{ sec.} \rightarrow \zeta\omega_n \geq 1$$

from TF $\quad 2\zeta\omega_n = b$

$$2 * [\zeta\omega_n \geq 1] = b \rightarrow b \geq 2$$

Ex:



If $m = 2\,kg$, $b = [4-6]\ N.s/m$, $K = 6\ N/m$

* To design an overdamped system for the whole time , we want to add controller (change TF)

$$TF_{open} = \frac{1}{ms^2 + bs + K} = \frac{1}{2s^2 + bs + 6}$$

صمم الميزان ليتناسب مع ال (Worst Case scenario)
أسوأ شي ممكن تتعامل معه السيستم (min. value)
For b

$$\therefore \quad b = 4 \quad , \quad TF_{open} = \frac{1}{\frac{2s^2}{2} + \frac{4s}{2} + \frac{6}{2}}$$

$$= \frac{1/2}{s^2 + 2s + 3}$$

$2\zeta\omega_n = 2$, $\omega_n = \sqrt{3}$

$\boxed{\zeta = \frac{1}{\sqrt{3}}} < 1$, the system is underdamped

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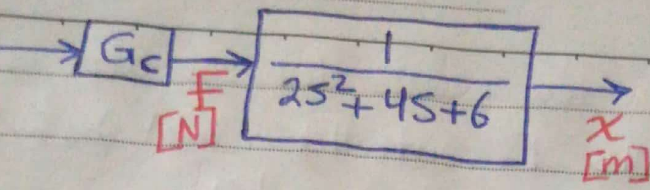


Open Loop Solution :

$$G_c = \frac{k}{\tau s+1} \quad \text{"as a filter"}$$

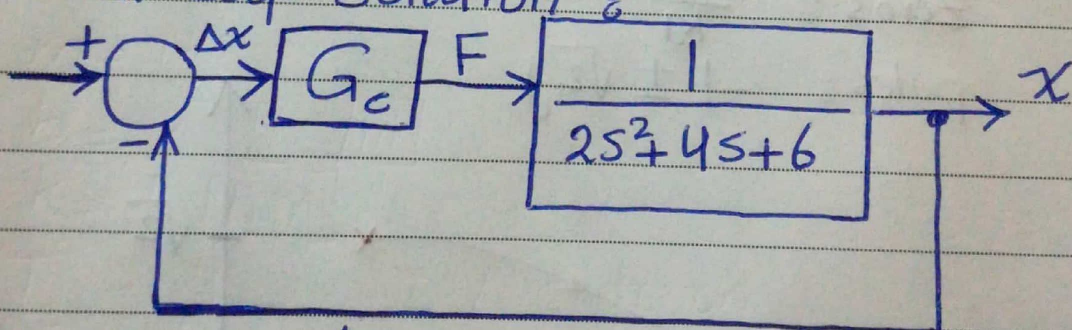
$$TF_{open} = \frac{k}{\tau s+1} * \frac{1}{2s^2+4s+6} \quad , \quad \left[R(s) = \frac{A}{s}\right] \text{ step input}$$

$$y(s) = \frac{A}{s} * \frac{k}{\tau s+1} * \frac{1}{2s^2+4s+6}$$

ازا عملناها طريقة الكسور الجزئية ما بصير أي تغيير
على استجابة النظام وضلها underdamped دامجة
استجابة النظام من الدرجة الأولى والثانية ∴

∴ No Solution

Closed Loop solution :



$$TF_{closed} = \frac{1}{2s^2+4s+6+G_c}$$

** حتى أحصل على أقل oscillation
بالباية لازم تكون معامل الاخماد (b) قيمته أكبر
من معامل النابض اللي بينتج قوة معاكسة للإزاحة
بالتالي لازم نستخدم (PD controller) ليضيفلي S عالمقام ليزيد من معامله
اللي بيمثل b

⇐ مابيزبط أصنّف D controller لحاله ..

PD controller $= K_p + sK_d$

$$= K_d\left(s + \frac{K_p}{K_d}\right) \;?$$

sol

* to find the values of K_p & K_d :
من خلال ال (root locus) لإيجاد متغير واحد فقط بعد تثبيت ال ratio ← $\frac{K_p}{K_d}$

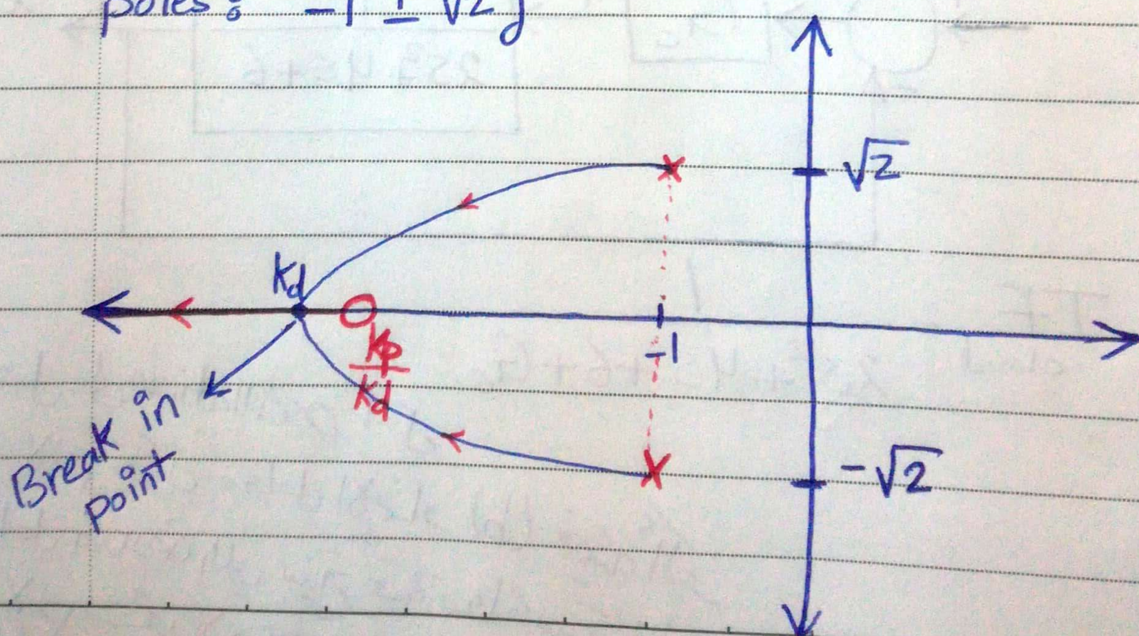
اللي بيمثل ال zero على ال root Locus

$$\Delta(s) = 1 + \frac{K_d\left(s + \frac{K_p}{K_d}\right)}{2s^2 + 4s + 6}$$

to draw Root Locus

zeros : $\frac{K_p}{K_d}$

poles : $-1 \pm \sqrt{2}\,j$



* منفرض قيمة معينة لل $\frac{k_p}{k_d}$ مثلاً (-5)

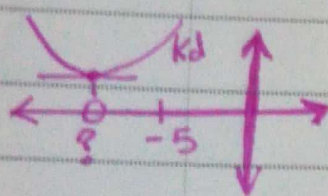
لنظام ← سريع الاستجابة نختار قيم كبيرة لل $\frac{k_p}{k_d}$ للشمال

* To find the value of (k_d) :

$$\Delta(s) = 0$$

$$1 + \frac{k_d\left(s + \frac{k_p}{k_d}\right)}{2s^2 + 4s + 6} = 0$$

$$-(2s^2 + 4s + 6) = k_d\left(s + \frac{k_p}{k_d}\right)$$



$$k_d = \left| \frac{2s^2 + 4s + 6}{s + 5} \right|$$

$$\frac{dk_d}{ds} = 0 \rightarrow \frac{(s+5)(4s+4) - (2s^2 + 4s + 6)}{(s+5)^2} = 0$$

$$2s^2 + 20s + 14 = 0$$

$$s_{1,2} = -9.24 \checkmark, \; -0.8 \times$$

① $k_d \geq 9.24$

نختار القيمة اللي ضمن ال $\left(\begin{matrix}\text{root}\\ \text{Locus}\end{matrix}\right)$

$\frac{k_p}{k_d} = 5$, when $k_d = 10$

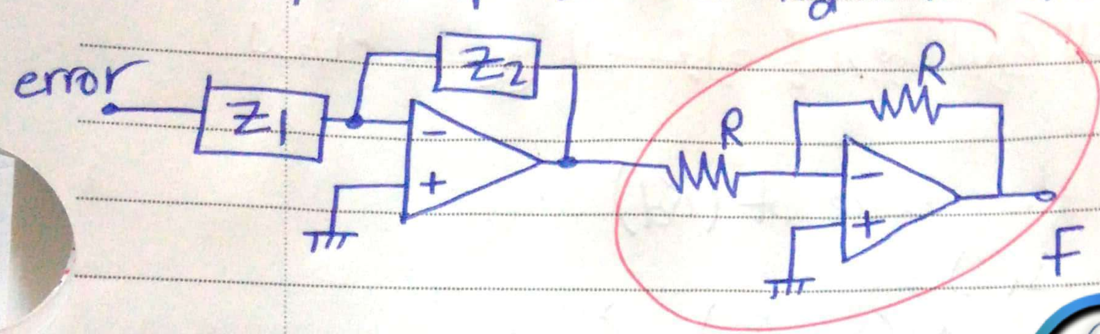
مثلاً أو أي قيمة اكبر من أو تساوي 9.24

$k_p = 50$

∴ the system will be Overdamped.

No.

Q: Design the controller circuit using Op_Amp where $k_d = 10$ and $k_p = 50$



gain = 1

Ch.3 : State Variable modelling

The system at any point in time, has a state.
this state is fully described by the state variables.

∴ كل variable
بيعبّر عن حالة
معينة للنظام

∴ الهدف من المتغيرات هاي تحوّللي النظام
(From n^{th} order DE $\xrightarrow{to}$ first order DE)

for example →
Student has lots of state describing him
hungry / Sleepy / Concentrating / ...

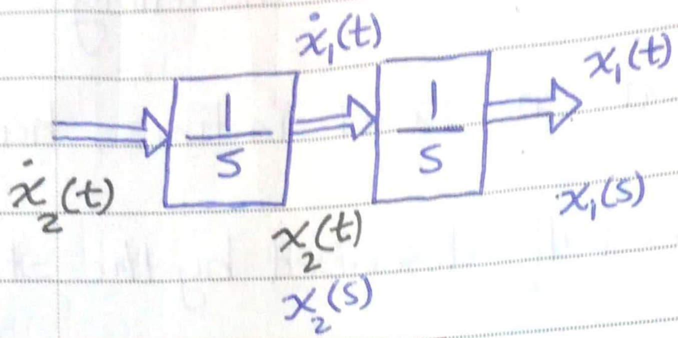
$$y(t) = C_1 x_1(t) + C_2 x_2(t) + C_3 x_3(t)$$

ما بيعتمدوا هالحالات على بعض

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

No.

الفكرة ⇐



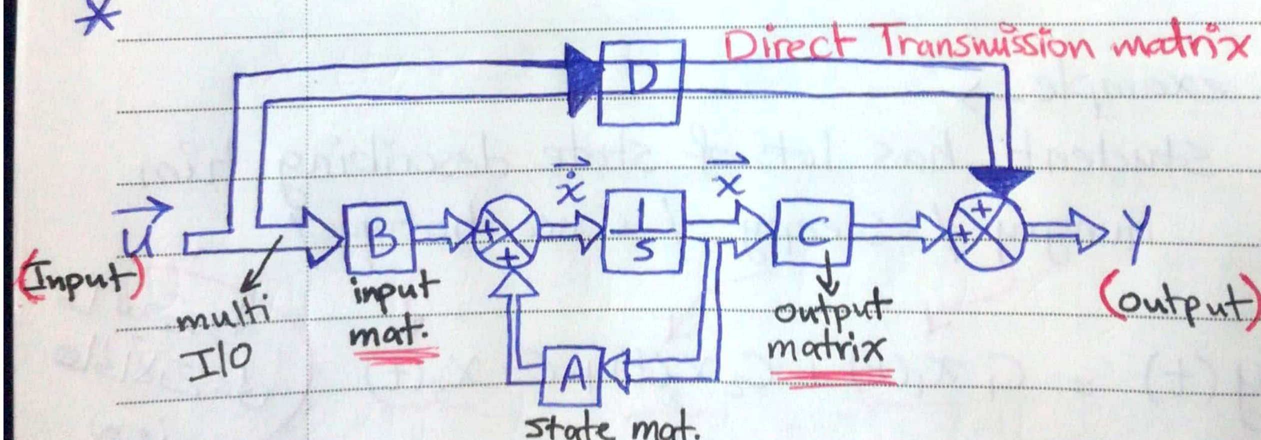
$$\boxed{x_2(s) = \dot{x}_1(s)}$$

هيك بيكون تخلصت من ال $\ddot{x}_1(t)$ ∴

يعني تخلصت من ال Second Order DE

\#

*



From the figure above,

① $\dot{x} = Ax + Bu$

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$$\underset{n\times1}{\begin{bmatrix}\dot x_1\\ \dot x_2\\ \vdots\\ \dot x_n\end{bmatrix}} = \underset{n\times n}{\begin{bmatrix} a_{11}&a_{12}&\cdots&a_{1n}\\ a_{21}&a_{22}&\cdots&a_{2n}\\ \vdots&&&\vdots\\ a_{n1}&\cdots&&a_{nn}\end{bmatrix}}\underset{n\times1}{\begin{bmatrix}x_1\\x_2\\ \vdots\\x_n\end{bmatrix}} + \underset{n\times i}{\begin{bmatrix}b_{11}&b_{12}&\cdots&b_{1i}\\ b_{21}&b_{22}&\cdots&b_{2i}\\ \vdots&&&\vdots\\ b_{n1}&\cdots&&b_{ni}\end{bmatrix}}\underset{i\times1}{\begin{bmatrix}u_1\\u_2\\ \vdots\\u_i\end{bmatrix}}$$

عدد ال inputs

② $y = Cx + Du$

$$\underset{J\times1}{\begin{bmatrix}y_1\\y_2\\ \vdots\\y_J\end{bmatrix}} = \underset{J\times n}{\begin{bmatrix} & & \\ & C & \\ & & \end{bmatrix}}\underset{n\times1}{\begin{bmatrix}x_1\\x_2\\ \vdots\\x_n\end{bmatrix}} + \underset{J\times i}{\begin{bmatrix} & & \\ & D & \\ & & \end{bmatrix}}\underset{i\times1}{\begin{bmatrix}u_1\\u_2\\ \vdots\\u_i\end{bmatrix}}$$

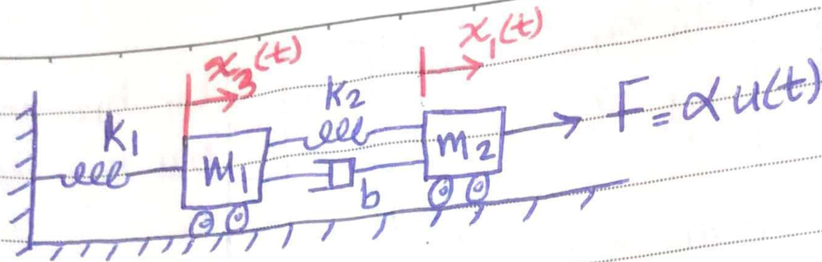
عدد ال outputs

⇐ y بتعتمد على قيم x
وأغلب الحالات
(Du) term = 0

y : Output
u : Input
x : state Variable
J : No. of outputs
i : No. of Inputs
n : No. of state Variables

No.

Ex



* Construct state space model for the above system :

- System Order = 4 , [Energy storage elements] حسب عدد ال

k_1 m_1 m_2 k_2

state Variables

- $\dot{x}_1(t) = \underline{x_2(t)}$ — ①

$\dot{x}_3(t) = \underline{x_4(t)}$ — ②

بدل من $\dot{x}_3(t)$ بدل من $\dot{x}_1(t)$

① $m_1 \dot{x}_4(t) = -k_1 x_3(t) - k_2(x_3(t) - x_1(t)) - b(x_4(t) - x_2(t))$

بدل من $\dot{x}_3(t)$

$$\dot{x}_4(t) = \frac{k_2}{m_1} x_1(t) + \frac{b}{m_1} x_2(t) - \frac{k_1+k_2}{m_1} x_3(t) - \frac{b}{m_1} x_4(t)$$ — ③

بدل من $\dot{x}_1(t)$ $\dot{x}_3(t)$

② $m_2 \dot{x}_2(t) = -k_2(x_1(t) - x_3(t)) - b(x_2(t) - x_4(t)) + \underbrace{\alpha u(t)}_{\text{دلالة ال F}}$

$$\dot{x}_2(t) = \frac{-k_2}{m_2} x_1(t) - \frac{b}{m_2} x_2(t) + \frac{k_2}{m_2} x_3(t) + \frac{b}{m_2} x_4(t) + \frac{\alpha}{m_2} u(t)$$ — ④

No.

$$A = \begin{array}{c} \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{-k_2}{m_2} & \frac{-b}{m_2} & \frac{k_2}{m_2} & \frac{b}{m_2} \\ 0 & 0 & 0 & 1 \\ \frac{k_2}{m_1} & \frac{b}{m_1} & \frac{-(k_1+k_2)}{m_1} & 0 \end{bmatrix}_{4\times4} \end{array} \rightarrow \frac{-b}{m_1}$$

معاملات x_1، x_2، x_3، x_4 معادلة $\dot{x}_1$

معاملات (x_1، x_2، x_3، x_4) معادلة $\dot{x}_2$

معاملات x_1 x_2 x_3 x_4 معادلة $\dot{x}_3$

المعاملات في معادلة $\dot{x}_4$

$$B = \begin{bmatrix} 0 \\ \frac{\alpha}{m_2} \\ 0 \\ 0 \end{bmatrix}_{4\times1}$$

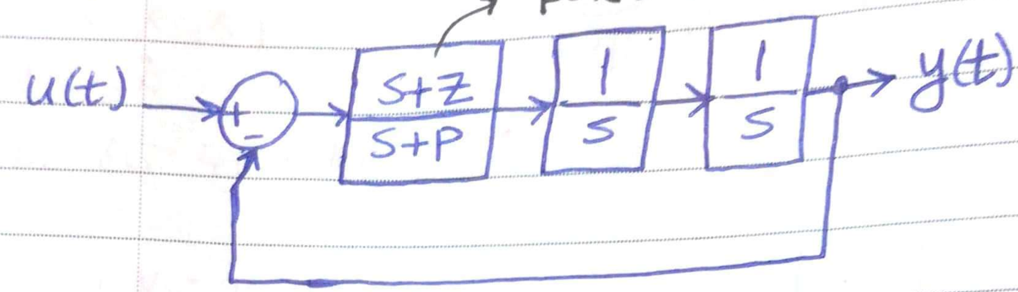
معاملات الـ input ($\alpha u(t)$) في كلاً من $\dot{x}_1(t)$ $\dot{x}_2(t)$ $\dot{x}_3(t)$ $\dot{x}_4(t)$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}_{2\times4}$$

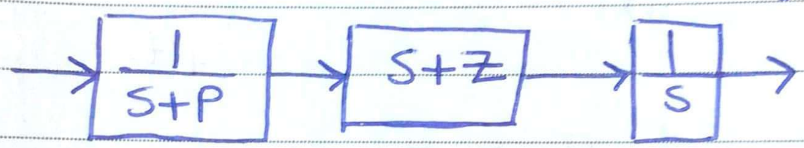
$$D = \begin{bmatrix} 0 \\ 0 \end{bmatrix}_{2\times1}$$

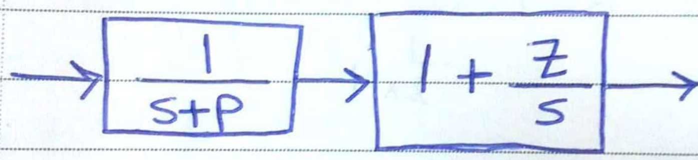
Ex From the block diagram, find the state space model :-

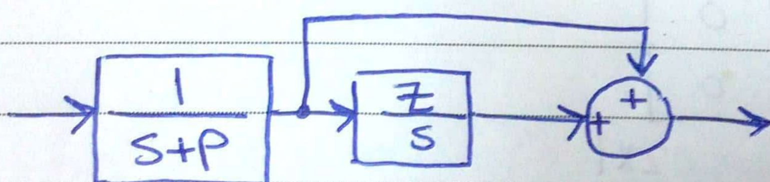
Zero's : $-Z$ [rad/s]
poles : $-P$ [rad/s]



Solution :- بغيّر على شكل البلوك دياغرام لطريقة تسمح لتمثيل وبناء ال (state space model)

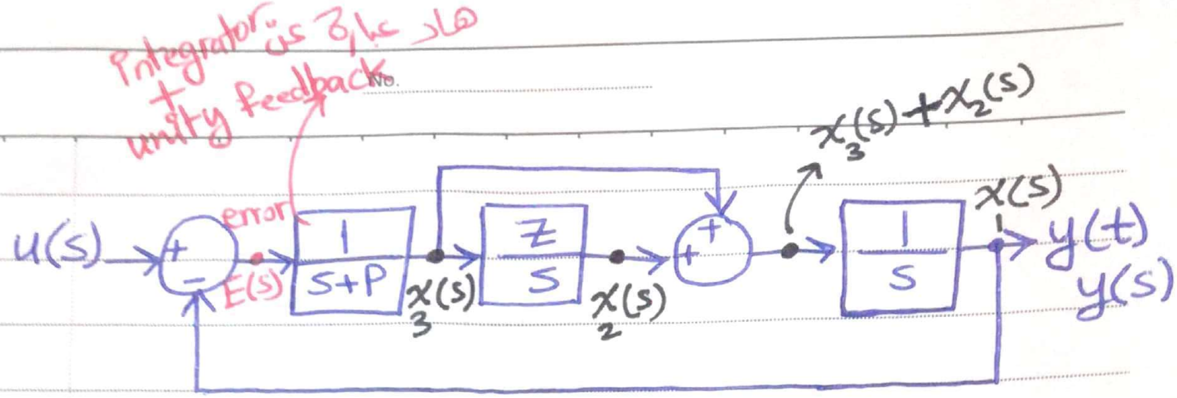






* الفكرة بالتعديل على Block Diagram ⇐ [جعل كل بلوك بيعبّر عن Integrator]

ونحط على ال (output) تبع كل integrator " state space variable " ويجهز المعادلات



① $x_1(s) = \frac{1}{s}\left(x_2(s) + x_3(s)\right) \rightarrow s\,x_1(s) = x_2(s) + x_3(s)$

② $x_2(s) = \frac{Z}{s}\left(x_3(s)\right) \rightarrow s\,x_2(s) = Z\,x_3(s)$

③ $E(s) = \left(\frac{1}{s+P}\right)\left[u(s) - x_1(s)\right]$

منشتق المعادلات الي فوق :

$\dot{x}_1(t) = \emptyset\, x_1(t) + x_2(t) + x_3(t) + \emptyset\, u(t)$

$\dot{x}_2(t) = \emptyset\, x_1(t) + \emptyset\, x_2(t) + Z\,x_3(t) + \emptyset\, u(t)$

$\dot{x}_3(t) = x_1(t) + 0\,x_2(t) - P\,x_3(t) + u(t)$

$y(t) = x_1(t) + \emptyset\, x_2(t) + \emptyset\, x_3(t)$

$A = \underset{3\times3}{\begin{bmatrix} 0 & 1 & 1 \\ 0 & 0 & Z \\ 1 & 0 & -P \end{bmatrix}}$, $B = \underset{3\times1}{\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}}$, $C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}_{1\times3}$

$D = \begin{bmatrix} 0 \end{bmatrix}_{1\times1}$

No.

For Stability :-

asymptote stability ≡ The initial value will go to zero in case of unforce "no external force".

the root of $\det[SI - A] \xrightarrow{\text{called}}$ Eigen Value

that should be on LHS

(ex) For $A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}_{2\times2}$

$$= \det \; S\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$$

$$= \det \left| \begin{bmatrix} S & 0 \\ 0 & S \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \right| = \det \begin{bmatrix} S & -1 \\ +2 & S+3 \end{bmatrix}$$

$$\det[SI - A] = S(S+3) + 2$$
$$= S^2 + 3S + 2$$

Eigen Values are $[-2, -1]$

∴ all poles are eigen values

But, not all the eigen values are the poles.

(ex) For $H(S) = \dfrac{\cancel{(S-1)}}{\cancel{(S-1)}(S+2)(S+3)}$

pole zero cancellation here

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② Controllability: Record

اذا قدرت أنقل value لـ قيمة ثانية خلال infinite time. فهذا الـ...

نحسب $\mathcal{V}_c = \begin{bmatrix} B & AB & \dots & A^{n-1}B \end{bmatrix}_{n\times n}$ Controllability matrix.

if V_c is full rank "of rank n" ∴ sys controllable.
(عادة للتعريف)

it should have n independent rows ; $\det \neq 0$.

$\text{rank } 1 \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ not full.

Ex:

Record "مع جواب" لبس لم نحلها.

$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$V_c = \begin{bmatrix} 0 & 1 \\ 1 & -3 \end{bmatrix}$

③ Observability: مراقبة

اذا أعرف الـ o/p وعندي معلومات عن x أقدر أعرف الـ initial cond.

$V_o = \begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}_{n\times n}$ observability matrix.
if V_o is full rank the sys. is oberuable.

Ex ①

$V_o = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ 2×2.

$\dot{x} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}$

Ex 2: غير مهم

$y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$ c

$V_o = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{b}{m} \end{bmatrix}$ observable

Ch.8 : Frequency Response

استجابة النظام ل $(\omega's)$ مختلفة

8.1

$$r(t) \longrightarrow \boxed{G(s)} \longrightarrow y(t)$$

It's the steady state response, when the input signal r(t) is a sinusoidal wave

$$\boxed{r(t) = a \sin(\omega t)}$$

Where, y(t) will be a sinusoidal signal also, but with different amplitude, according to the TF., and phase shift, according to the poles.,

$$\boxed{y(t) = a\,|T(j\omega)|\,\sin(\omega t + \theta)}$$

θ : phase shift $\angle T(j\omega)$

→ there is no info. about the transient response unless we convert it to the laplace ..

No.

Remember

* $Z_R = R$ | $Z_L = JWL$

  $Z_C = \dfrac{1}{JWC}$

* $\omega\,[HZ] = 2\pi f\ \ [rad/s]$

اللي مهم نتعامل معها

To find the magnitude and the phase shift for the system TF :

① $T(s) = \dfrac{1}{S+2}$, $r(t) = 3\sin(2t)$

$\omega = 2\ \frac{rad}{s}$

$T(JW) = \dfrac{1}{2J+2}$

$2\ \frac{rad}{s}$

$\angle T(2J) = \angle \dfrac{1}{2J+2} = -45^\circ$ / من الآلة الحاسبة مباشرة

OR : $\angle T(2J) = \angle 1 - \angle 2J+2$
$= \phi - 45^\circ \quad = -45^\circ$

$|T(2J)| = \dfrac{1}{\sqrt{(2)^2+(2)^2}} = \dfrac{1}{\sqrt{8}}$

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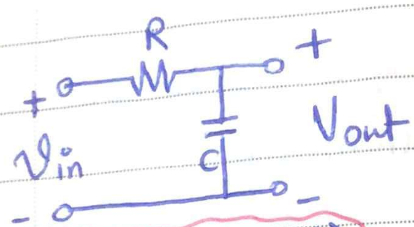
No.

$$\therefore\ y_{ss}(t) = \frac{3}{\sqrt{8}} \sin\left(2t - \frac{\pi}{4}\right)$$

② 

Recall that: this is a low-pass filter

if $v_{in} = a \sin(\omega t)$ →

$$H(j\omega) = \frac{V_{out}}{V_{in}} = \frac{Z_c}{Z_R + Z_c}$$

$$= \frac{1/j\omega C}{R + 1/j\omega C}$$

$$= \frac{1}{1 + RC(j\omega)}$$

* if we check the system response for different ω's:

- $\omega = 0$ [DC signal] → $|T(j\omega)| = 1$

$$y_{ss}(t) = a$$

- $\omega = \frac{1}{RC}$ → $|T(j\omega)| = \frac{1}{\sqrt{1^2 + (j\frac{RC}{RC})^2}} = \frac{1}{\sqrt{2}}$

$$\theta = \angle \frac{1}{1+j} = -45^\circ$$

$$y_{ss}(t) = \frac{a}{\sqrt{2}} \sin\left(\frac{1}{RC} t - \frac{\pi}{4}\right)$$

- $\omega = \infty$ → $|T(\infty)| = 0$ $\quad \boxed{\frac{1}{\infty} = \phi}$

$$\angle T(j\omega) = \frac{-\pi}{2} \text{ rad}$$

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No.

عم بدخل عالسركت اشارات بترددات مختلفة

$$\left[0 \rightarrow \frac{1}{RC} \rightarrow \infty \right]$$

ω قليلة $\leftarrow$ $|T(j\omega)|$ عم تقل مع بزيادة ω

بيحصل attenuation للسيجنال

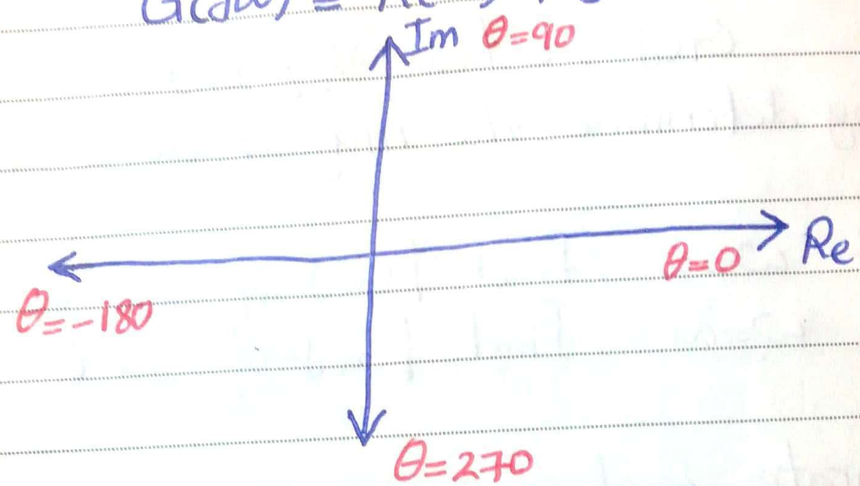
بتمرر الاشارات ذات التردد القليل (زي ماهي من غير تغيير) وتضعف ذات التردد العالي $\Leftarrow$ Low Pass filter

No.

8.2 Frequency Response Plots (Polar Plot)

- The transfer function of a system $G(s)$ can be described in the frequency domain by:

$$G(j\omega) = R(\omega) + jX(\omega)$$



- $G(s)$ represented by its magnitude $|G(j\omega)|$ and a phase $\theta(j\omega)$, as:

$$G(j\omega) = |G(j\omega)|\, e^{j\theta(\omega)} = |G(j\omega)| \angle \theta(\omega)$$

where,

$$\left[\begin{array}{l} |G(j\omega)| = \sqrt{R^2(\omega) + X^2(\omega)} \\ \theta(\omega) = \tan^{-1} \dfrac{X(\omega)}{R(\omega)} \end{array}\right]$$

No.

* steps for sketching the polar plot :

① Determine the system transfer function $G(s)$

② put $\boxed{s = j\omega}$, and write the sys. equation in polar form

$$G(j\omega) = |G(j\omega)| \angle G(j\omega)$$

by determine the magnitude & phase

③ at $\omega = 0$, find $|G(j\omega)|$ and θ
at $\omega = \infty$, find $|G(j\omega)|$ and θ

④ Separate Real and Imaginary part

Real part $= 0$	Img. $= 0$
بطلّع ω	بطلع ω
ونعوضها بـ $G(j\omega)$	ونعوضها في $G(j\omega)$
لإيجاد (I_m)	لإيجاد (R_e)

$$G(j\omega) = R(\omega) + jX(\omega)$$

No.

Ex : If $G(s) = \dfrac{100}{(s+2)(s+4)(s+8)}$

① $G(j\omega) = \dfrac{100}{(2+j\omega)(4+j\omega)(8+j\omega)}$

② $|G(j\omega)| = \dfrac{100}{\sqrt{4+\omega^2}\sqrt{16+\omega^2}\sqrt{64+\omega^2}}$, $\theta = -\tan^{-1}\dfrac{\omega}{2} - \tan^{-1}\dfrac{\omega}{4} - \tan^{-1}\dfrac{\omega}{8}$

③ at $\omega=0 \rightarrow |G(j\omega)| = \dfrac{100}{\sqrt{4}\,\sqrt{16}\,\sqrt{64}} = 1.56$

$\theta = -\tan^{-1}0 - \tan^{-1}0 - \tan^{-1}0 = 0°$

at $\omega=\infty \rightarrow |G(j\omega)| = \dfrac{100}{\infty} = 0$

$\theta = -\tan^{-1}\infty - \tan^{-1}\infty - \tan^{-1}\infty = -270$

$\tan^{-1}\infty = 90°$

$(8-6j\omega-\omega^2)$

④ $G(j\omega) = \dfrac{100}{(2+j\omega)(4+j\omega)(8+j\omega)} \times \dfrac{(2-j\omega)(4-j\omega)(8-j\omega)}{(2-j\omega)(4-j\omega)(8-j\omega)}$

$= \dfrac{100\,[64-8j\omega-48j\omega-6\omega^2-8\omega^2+j\omega^3]}{(4+\omega^2)(16+\omega^2)(64+\omega^2)}$

$= \underbrace{\dfrac{6400-1400\omega^2}{(4+\omega^2)(16+\omega^2)(64+\omega^2)}}_{Re} - j\,\underbrace{\dfrac{5600\omega-100\omega^3}{(4+\omega^2)(16+\omega^2)(64+\omega^2)}}_{Im}$

No.

* $Re = \phi \longrightarrow$

$6400 = 1400\,\omega^2$

$\boxed{\omega = 2.13 \text{ rad/s}}$

$G(j2.13) = 0.91$

$\theta = -90$

تقاطع

التقاطع

$Im = \phi \longrightarrow$

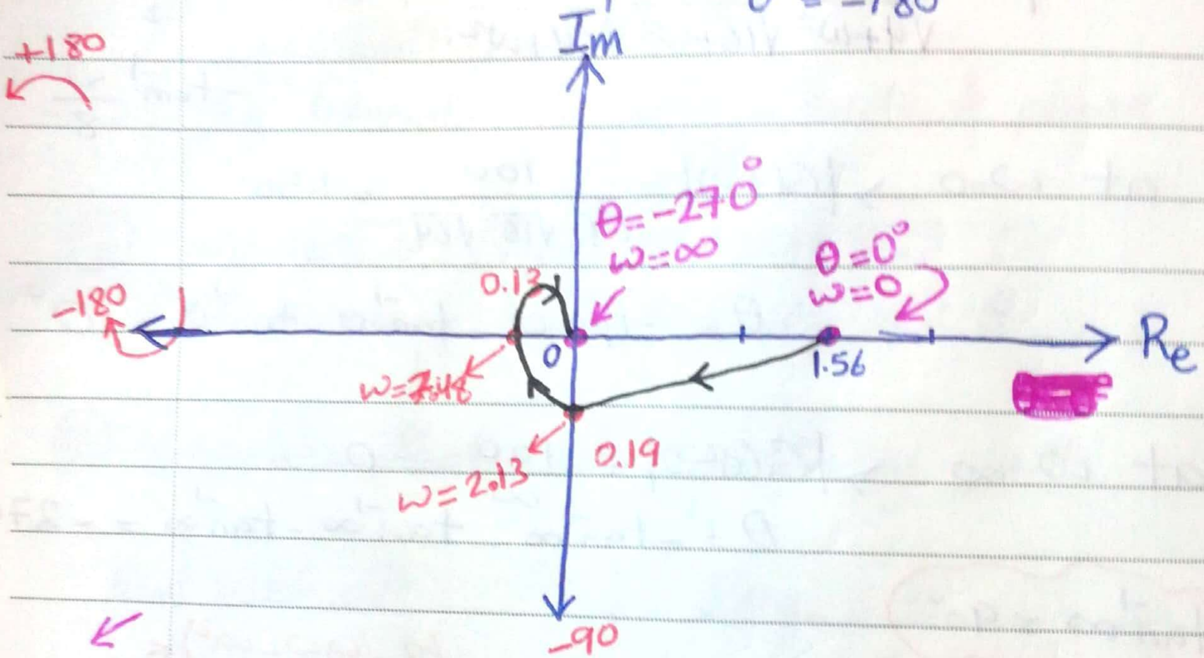
$5600\,\omega - 100\,\omega^3 = 0$

$5600 = 100\,\omega^2$

$\boxed{\omega = 7.48 \text{ rad/s}}$

$G(j7.48) = 0.13$

$\theta = -180$



هاي الرسمة
تمثل $G(j\omega)$
مختصين عليها

$|G(j\omega)|$ & $\theta \longrightarrow$ at certain ω

∴ بلّش بالرسم عند $\omega=0$ ، بمرّ بنقاط التقاطع اللي أوجدناهم
بالخطوة السابقة ، وينتهي عند $\omega=\infty$

Ex : $G(s) = \dfrac{10}{(s+2)(s+4)}$

① $G(j\omega) = \dfrac{10}{(2+j\omega)(4+j\omega)}$

② $|G(j\omega)| = \dfrac{10}{\sqrt{4+\omega^2}\sqrt{16+\omega^2}}$, $\theta = -\tan^{-1}\left(\dfrac{\omega}{2}\right) - \tan^{-1}\left(\dfrac{\omega}{4}\right)$

③ at $\omega = 0 \rightarrow |G(j\omega)| = 1.25$, $\theta = 0^\circ$

$\omega = \infty \rightarrow |G(j\omega)| = 0$, $\theta = -90 - 90 = -180^\circ$

④ $\dfrac{10}{(2+j\omega)(4+j\omega)} * \dfrac{(2-j\omega)(4-j\omega)}{(2-j\omega)(4-j\omega)}$

$G(j\omega) = \dfrac{10(8-6j\omega-\omega^2)}{(4+\omega^2)(16+\omega^2)}$

$G(j\omega) = \dfrac{80-10\omega^2}{(4+\omega^2)(16+\omega^2)} - j\dfrac{60\omega}{(4+\omega^2)(16+\omega^2)}$

* $Re = 0 \rightarrow$

$\omega = \sqrt{8} = 2\sqrt{2}$ rad/s

$G(j\omega) = 0.589$

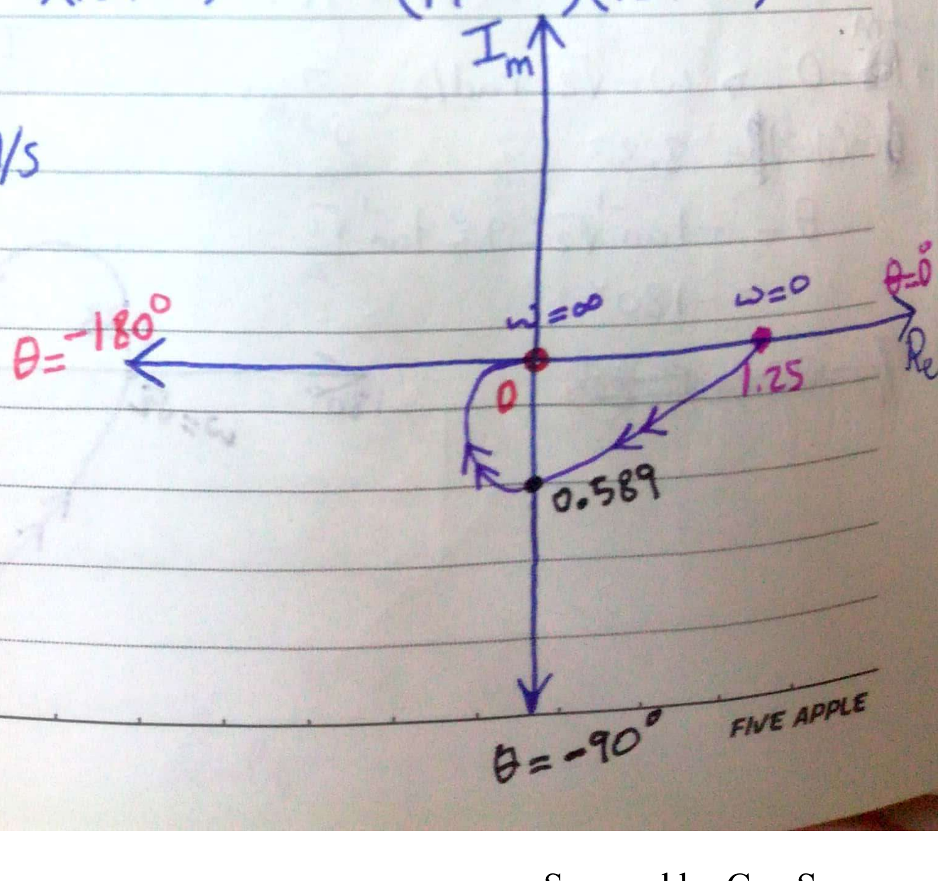
$\theta = -90^\circ$

* $I_m = 0 \rightarrow$

$\omega = 0$

$G(j\omega) = 1.25$

$\theta = 0^\circ$



FIVE APPLE

Scanned by CamScanner

* Bode Plot → لوصف ال (System Freq.)
"Logarithmic Plot"

- Sketching the phase and the gain (magnitude) of $G(s)$ as a function of ω [rad/s]

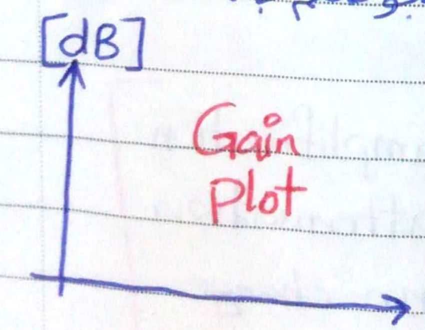
- It simplifies the determination of the graphical portrayal of the freq. response.

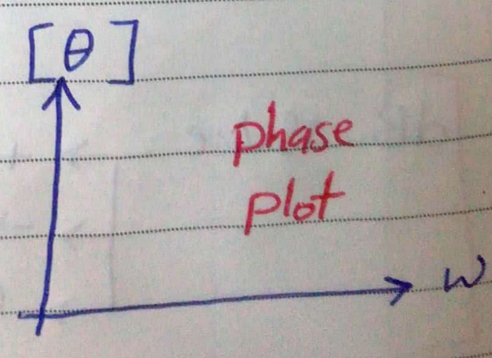
$$\boxed{\text{Logarithm gain}\,[dB] = 20 \log |G(j\omega)|}$$

- **Magnitude Plot:**
  - find $(|G(j\omega)|)$
  - $G_{dB} = 20 \log |G(j\omega)|$

  عند قيم معينة لـ ω نعوضهم بالعلاقة أعلاه

- **Phase Plot:**
  - find (θ)
  - عند قيم ω معينة بوجد الزوايا

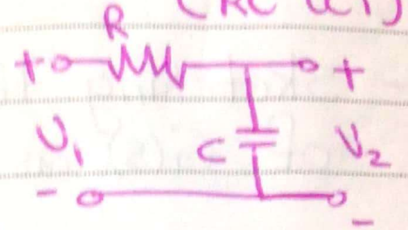




إلهم نفس ال (Frequency scale)

No.

Ex) For $G(s) = \dfrac{1}{RCS+1}$ → It's a low Pass Filter CCT (RC CCT)



→ $G(j\omega) = \dfrac{1}{1+j\left(\frac{\omega}{\omega_0}\right)}$

where, $\left[\omega_0 = \dfrac{1}{RC}\right]$ → Corner Freq.

→ $|G(j\omega)| = \dfrac{1}{\sqrt{1+\left(\frac{\omega}{\omega_0}\right)^2}}$ ، $\theta = \phi - \tan^{-1}\left(\frac{\omega}{\omega_0}\right)$

بحدد قيم معينة ω بوجد عندهم $|G(j\omega)|$ & θ ....
ونرسم ال Bode Plot ع أساسهم

* at $(\omega = 0)$ → $|G(j\omega)| = 1$ ، $\theta = -\tan^{-1}(0) = \phi$

$[dB] = 20 \log 1$
$= 0$ dB

* at $(\omega = \omega_0)$ → $[dB] = 20 \log\left(\frac{1}{\sqrt{2}}\right)$ ، $\theta = -\tan^{-1}(1) = -45^\circ$
$= -3$dB

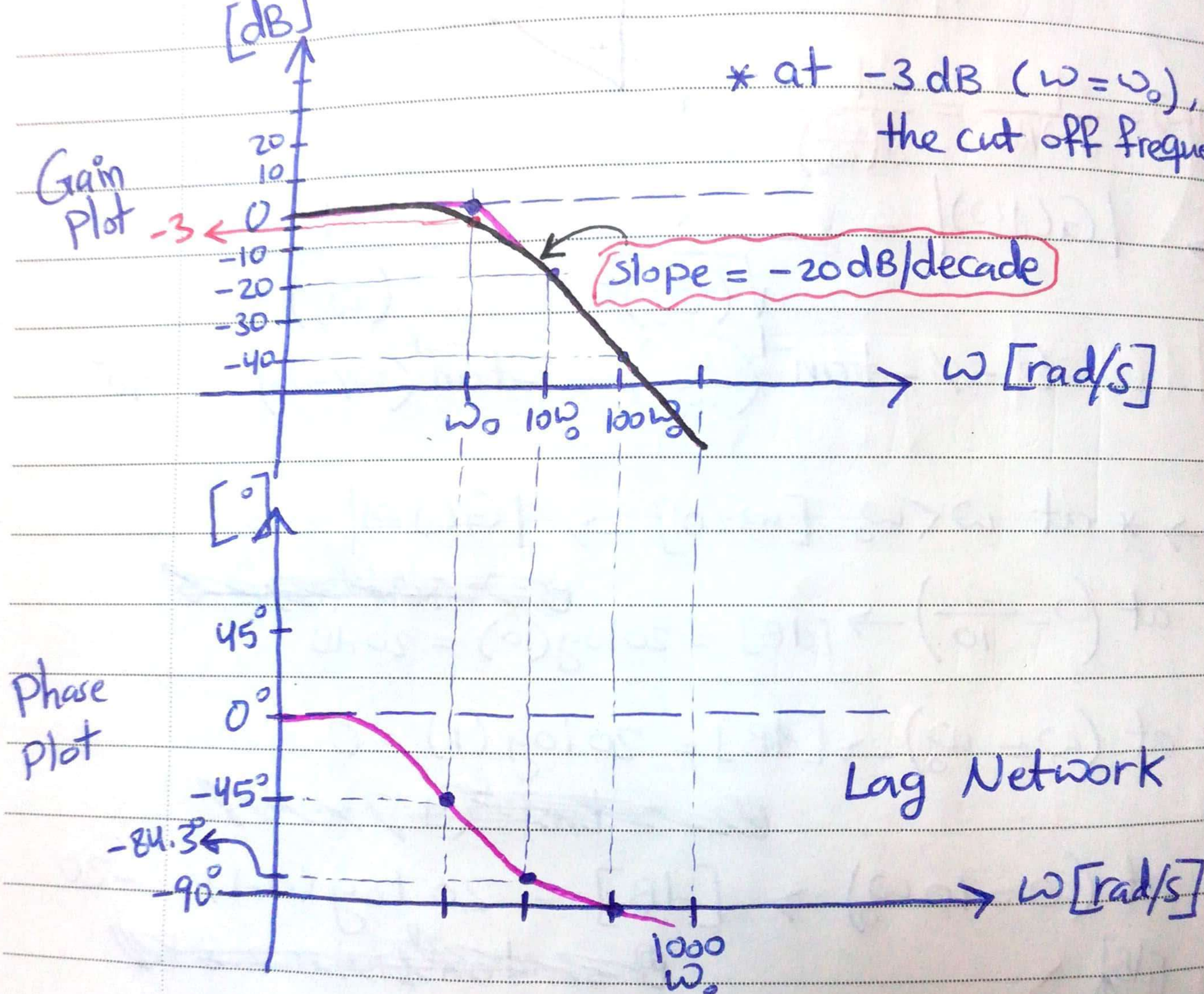
* at $(\omega = 10\,\omega_0)$ → $[dB] = 20 \log(0.1)$ ، $\theta = -\tan^{-1}(10)$
$= -20$ dB $\qquad = -84.3^\circ$

∴ dB Value
- → +ve ، amplification
- → -ve ، attenuation
- → 0 ، no change

FIVE APPLE

* فنستنتج مما سبق أن النظام بعمل Signal Attenuating
عند قيم الـ Freq. العالية ابتداءً من (ω_0)
وعلى قيم الـ Freq. القليلة بمررها كما هي
لذا هو $\Leftarrow$ [Low Pass Filter]

* at -3 dB $(\omega = \omega_0)$, called the cut off frequency

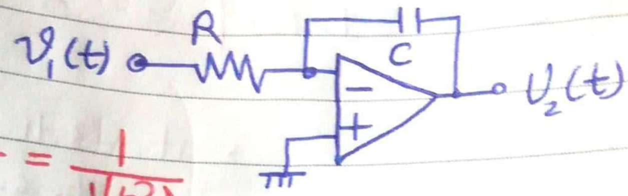


[decade = 10x]

No.

Ex) Integrator, $G(s) = \dfrac{1}{s}$

Sketch the Bode Plot?



$G(j\omega) = \dfrac{1}{j\omega RC} = \dfrac{1}{j\left(\frac{\omega}{\omega_0}\right)}$

$\rightarrow \quad |G(j\omega)| = \dfrac{1}{\sqrt{\left(\frac{\omega}{\omega_0}\right)^2}} = \dfrac{1}{\left(\frac{\omega}{\omega_0}\right)}$

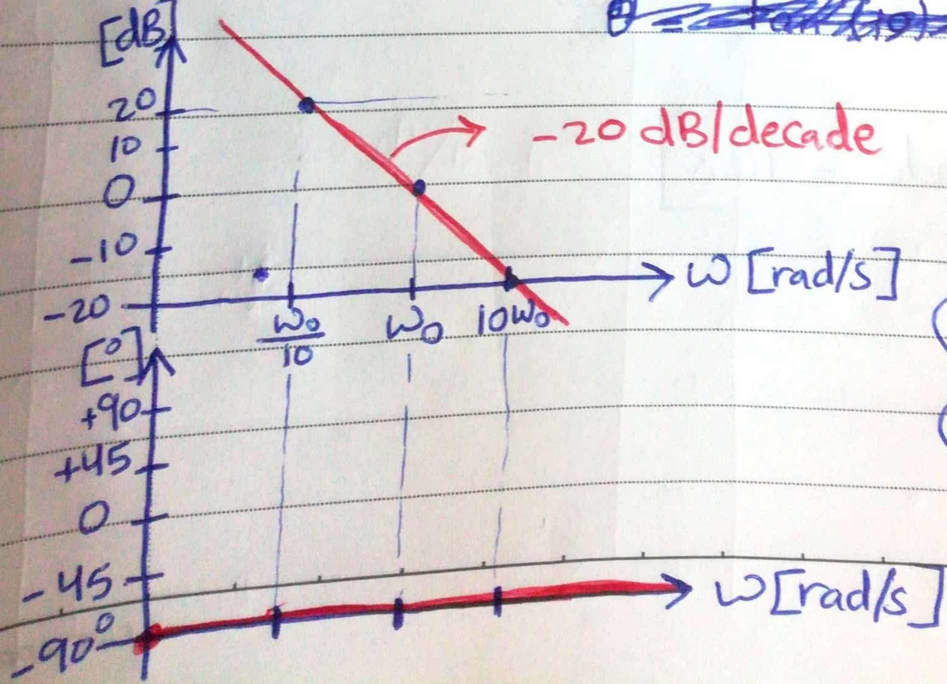
$\theta = \phi - \tan^{-1}\left(\frac{\omega}{\omega_0}\right) = -\tan^{-1}(\omega/\omega_0) = -90^\circ$

$\rightarrow$ * at $\omega < \omega_0$ $[\omega = 0] \rightarrow |G(j\omega)| = \infty$

at $\left(\omega = \frac{\omega_0}{10}\right) \rightarrow [dB] = 20\log(10) = 20\,dB$

* at $(\omega = \omega_0) \rightarrow [dB] = 20\log(1) = 0$

* at $(\omega = 10\omega_0) \rightarrow [dB] = 20\log(0.1) = -20$



$\angle 1 = 0^\circ$

$\angle j\omega = 90^\circ$

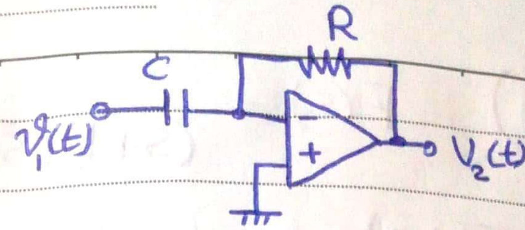
$\angle j = 90^\circ$

$\angle j^{-1} = -90^\circ$

FIVE APPLE

No.

Ex) Differentiator



$$G(s) = RCS \rightarrow G(j\omega) = j\omega RC = j\left(\frac{\omega}{\omega_0}\right)$$

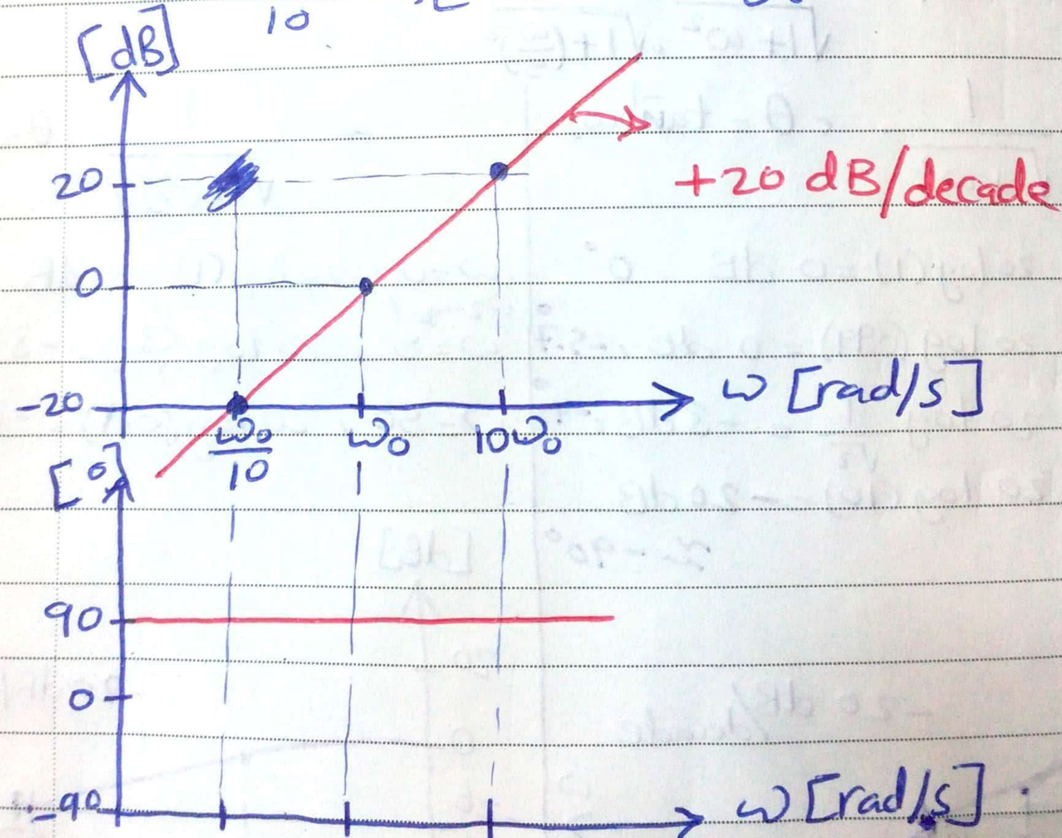
$$\rightarrow |G(j\omega)| = \sqrt{\left(\frac{\omega}{\omega_0}\right)^2} = \frac{\omega}{\omega_0} \quad , \quad \theta = +\tan^{-1}\left(\frac{\omega}{\omega_0}\right) = 90^\circ$$

* at $\omega = 0 \rightarrow [dB] = 20\log(0) = \infty$

$\omega = \omega_0 \rightarrow [dB] = 20\log(1) = 0$

$\omega = 10\omega_0 \rightarrow [dB] = 20\log(10) = 20$

$\omega = \frac{\omega_0}{10} \rightarrow [dB] = 20\log(0.1) = -20$



Recall that : $\omega_0 \equiv$ Corner Frequency

No.

Ex)

$$T(S) = \frac{5(0.1S+1)}{S(0.5S+1)\left(\left(\frac{S}{50}\right)^2 + 0.6\frac{S}{50} + 1\right)}$$

1. Constant (5)
2. Poles at origin
3. Poles at (−2)
4. Zero at (−10)
5. Complex poles, where $\omega_n = 50$

No.

Process of plotting abode Plot :-

1- Write transfer function in standard form.

$$G(s) = \frac{(s+a)(s+b)}{(s+p)(s+q)} = \frac{ab}{pq} \cdot \frac{(1+\frac{s}{a})(1+\frac{s}{b})}{(1+\frac{s}{p})(1+\frac{s}{q})}$$

$$= K \frac{(1+\frac{s}{a})(1+\frac{s}{b})}{(1+\frac{s}{p})(1+\frac{s}{q})}$$

for type 0 → slope of the 1st line is 0dB/dec.
for type 1 → " " " " " " -20 dB/dec.

2- Identity gain and slope of 1st line

3- Arrange all Freq. in asending order and define slope to each line.

$\omega = a$
$\omega = b$
$\omega = p$
$\omega = q$

4- Write Equation for phase plot and make a table of $(\theta - \omega)$ →

$$\theta = \tan^{-1}\frac{\omega}{a} + \tan^{-1}\frac{\omega}{b} - \tan^{-1}\frac{\omega}{p} - \tan^{-1}\frac{\omega}{q}$$

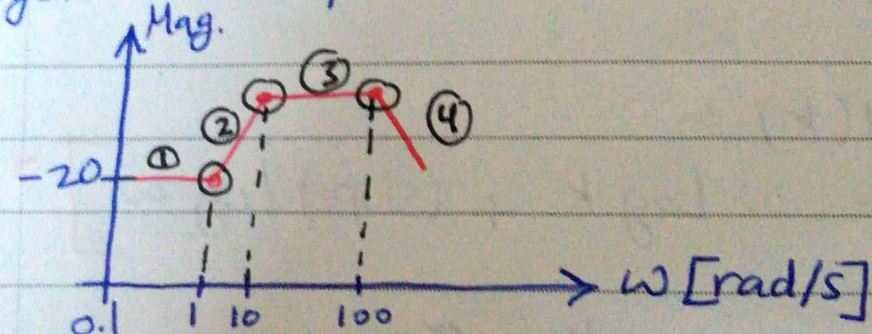
FIVE APPLE

ω	θ
0.1	
1	
10	
100	
⋮	

* Bode Plot advantages :
  - Both low & high freq.'s characteristics by one plot.
  - It can indicate the stability of the system.

Ex Find the transfer function from each system bode plot from below :

(1)



$\omega = 1$

$\omega = 10$

$\omega = 100$

* يحدد ال Corner Freq.'s من الشكل :

* at 1 → 0 slope (1st line) $\xrightarrow{so}$ type 0 system
* at 2 → +20 dB/dec.
* at 3 → 0 slope
* at 4 → -20 dB/dec.

No.

* If slope INC. by 20dB/dec, then there's **Zero**
* " " DEC. " 20dB/dec. " " " **Pole**
* Zero " at the 1^{st} line, means that there's no root at origin ≡ type 0 system.

So, $\omega = 1 \rightarrow$ Zero
$\omega = 10 \rightarrow$ pole
$\omega = 100 \rightarrow$ pole

$$\rightarrow G(s) = k \frac{(s+1)}{\left(\frac{s}{10}+1\right)\left(\frac{s}{100}+1\right)}$$

standard Equation form

$$= k \frac{(s+1)}{\frac{1}{1000}(s+10)(s+100)} = 1000k \frac{(s+1)}{(s+10)(s+100)}$$

→ to find (k):

$$\left[\text{Gain} = 20 \operatorname{Log} k + (\text{Slope}) \operatorname{Log} \omega\right]$$

→ for the first line ONLY

$$-20 = 20 \operatorname{Log} k + \varnothing$$

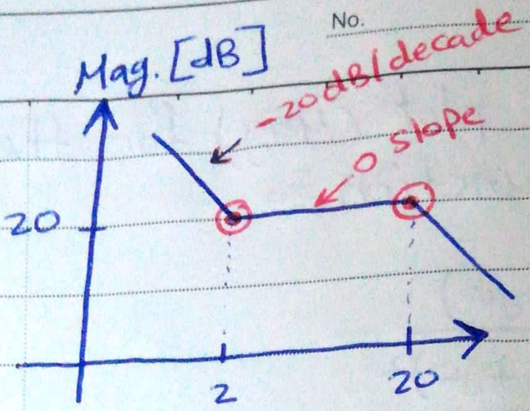
$$k = 0.1$$

$$\therefore G(s) = \frac{100(s+1)}{(s+10)(s+100)} \quad \#$$

FIVE APPLE

No.

②



sol : $\omega = 2$ rad/s $\rightarrow$ zero

$\omega = 20$ rad/s $\rightarrow$ pole

(type 1 sys.)

$\rightarrow$ 1^{st} line slope = -20 dB/dec. $\rightarrow$ there's a pole at origin

لو كان (-40dB/dec.) سيكون في [2 poles at origin]

$$\rightarrow \quad G(s) = K \frac{\left(\frac{s}{2}+1\right)}{s\left(\frac{s}{20}+1\right)}$$

$$\text{gain} = 20 \log K + (\text{slope}) \log(\omega)$$

$$20 = 20 \log K + (-20) \log(2)$$

$$20 \log K = 26$$

$$K = 10^{1.3} = 20$$

$$\therefore \quad G(s) = \frac{20\left(\frac{s}{2}+1\right)}{s\left(\frac{s}{20}+1\right)}$$

#

Ex Draw the bode plot (Mag.) for the below transfer function :

$$G(j\omega) = \frac{10^4(1+j\omega)}{(10+j\omega)(100+j\omega)^2}$$

Sol :

$$G(s) = \frac{10^4(1+s)}{(10+s)(100+s)^2}$$

$$G(s) = \frac{\overset{0.1}{10^4}(1+s)}{10 * 10^4 \left(1+\frac{s}{10}\right)\left(1+\frac{s}{100}\right)^2}$$

Zero type → No poles at origin (0 dB/dec For the 1st line)

* gain = 20 log (0.1) = −20 dB

* Corner Freq. :

$\omega = 1$ rad/s → Zero → +20dB/dec

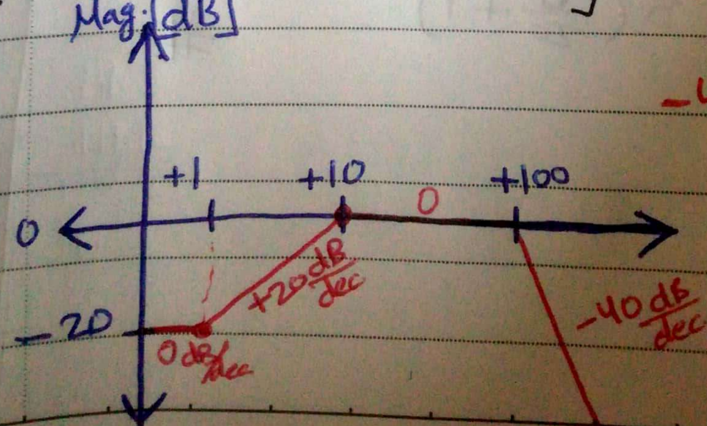
$\omega = 10$ rad/s → Pole → 0 dB/dec

$\omega = 100$ rad/s → pole → ~~−20 dB/dec~~

But : 2 poles

−40dB/dec مرفوض

عوض G(s) المعطى



No.

Ex: Determine k for the given TF below, and draw the bode plot for:

$$G(s) = \frac{k}{s(s+2)(s+4)} \quad ?$$

Sol

→ ① $G(s) = \dfrac{k}{s(8)(1+\frac{s}{2})(1+\frac{s}{4})} = \dfrac{k_1}{s(1+\frac{s}{2})(1+\frac{s}{4})}$

where, $\boxed{k_1 = \frac{k}{8}}$

→ ② If $k_1 = 1 \rightarrow$ mag. at $\omega = 1$

Gain $= 20 \log(1) = 0\,dB$

there's one pole at origin, so slope of first line is -20 dB/dec.

③ $\left[\begin{array}{l} \omega = 2 \text{ rad/s} \rightarrow \text{pole} \\ \omega = 4 \text{ rad/s} \rightarrow \text{pole} \end{array}\right]$ $\begin{array}{l} -20-20 = -40 \text{ dB/dec.} \\ -40-20 = -60 \text{ dB/dec.} \end{array}$

من TF معطى

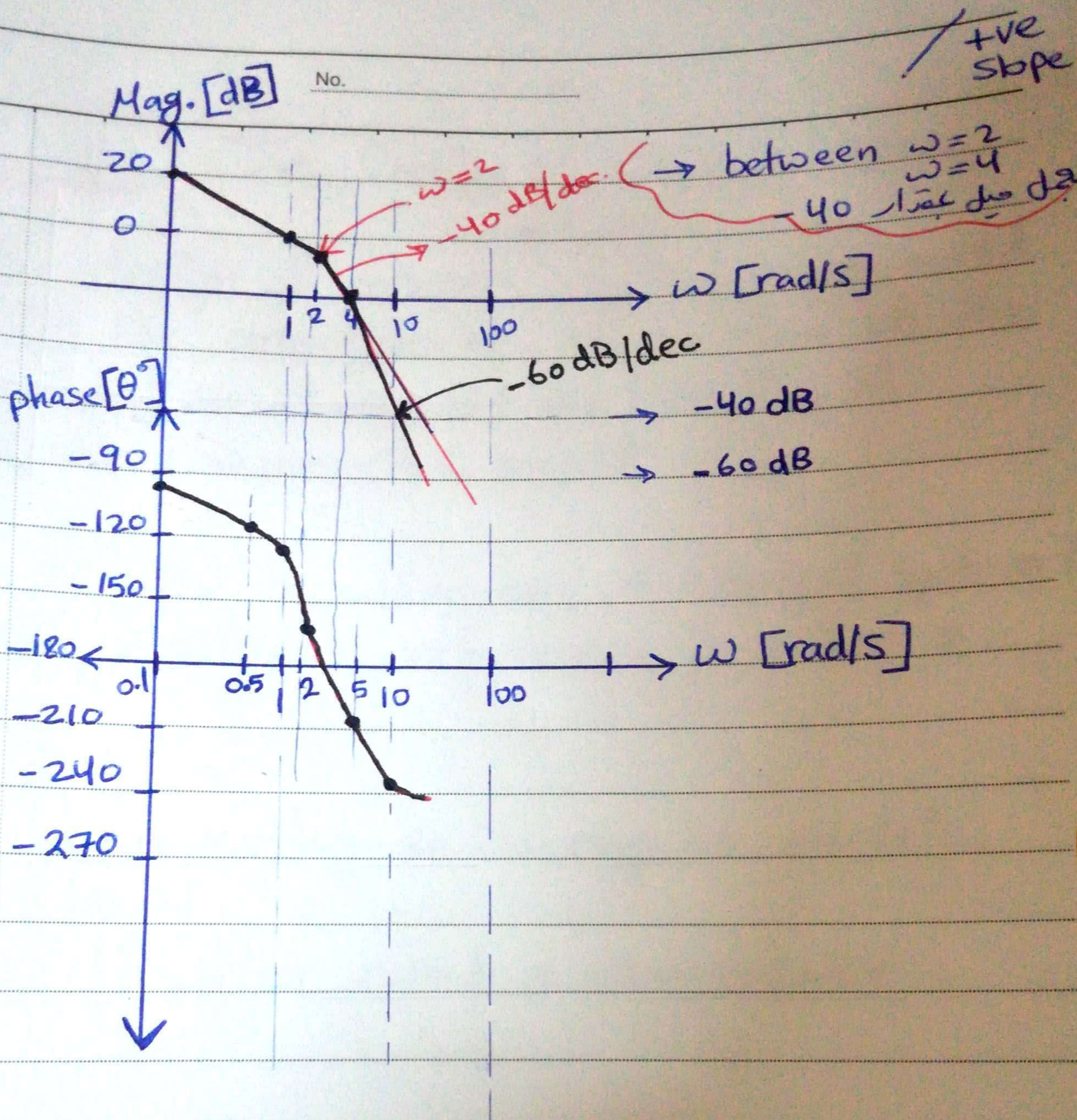
[كلما صادف pole
الميل مقدار $-20\frac{dB}{dec.}$
في كل مرة]

④

ω	θ
0.1	-94.3
0.5	-111.16
1	-130.66
2	-161.5
5	-209.5
10	-236.9
50	-263.12
100	-266.5
1000	-269.6

$$\theta = -90 - \tan^{-1}\frac{\omega}{2} - \tan^{-1}\frac{\omega}{4}$$

FIVE APPLE


+ve slope
No.
Mag. [dB]
20
0
ω=2
-40 dB/dec
→ between ω=2 ω=4
نجعل ميل بمقدار -40
ω [rad/s]
1 2 4 10 100
-60 dB/dec
phase [θ°]
→ -40 dB
→ -60 dB
-90
-120
-150
-180
-210
-240
-270
ω [rad/s]
0.1 0.5 1 2 5 10 100

No.

Ch.10 The Design of feedback control system

- The design of a control system is concerned with the arrangement, or the plan, of the system structure and the selection of suitable components & parameters.

- The alteration or adjustment of a control system to provide a suitable performance is called a Compensation.

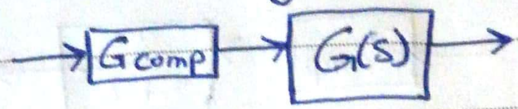
- The compensating device may be electric, mechanical, ...
  and is often called a Compensator.

∴ Compensator ≡ It's an additional component or circuit inserted into a control sys. to compensate for a deficient performance.

types ∴ ① Lag Compensator
② Lead 〃
③ Lead - Lag 〃

Ex: ① If $G(s) = \dfrac{1}{s+2}$

make the system a zero order?

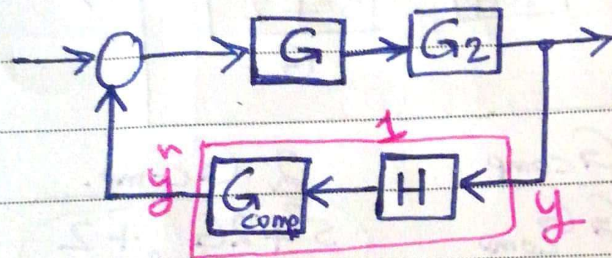


طلع s

$G_{comp} = k(s+2)$

$\therefore G(s) = k$ → the whole TF Here

②



$H(s) = \dfrac{3}{s+9}$, make $y^n = y$?

→ $G_{comp} = \dfrac{s+9}{3}$

③ If $T(s) = \dfrac{2}{s^2+2}$ → $\zeta = 0$ (Undamped Sys.)

Design a compensator that do the following tasks

1* to have $\zeta \geq 0.45$ → (at least get an underdamped)

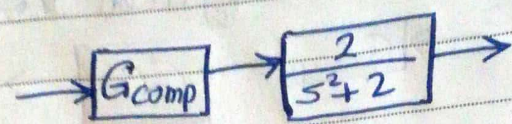
2* $T_s \leq 4$ sec.

No

(1)

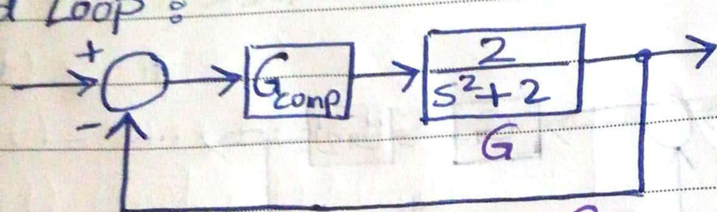
* بدنا نضيف [ξ term] على $\Delta(s)$ $\leftarrow$ $\Delta(s) = s^2 + 2\xi\omega_n s + \omega_n^2$

Open Loop :



$\times$ مارح ينفع لأنو بدي أضيف على المقام
مو مسألة ضرب وحذف أمر معين ..

Closed Loop :



$$T(s) = \frac{G\,G_{comp}}{1+G\,G_{comp}} = \frac{2\,G_{comp.}}{s^2+2G_{comp}+2}$$

$$s^2 + 2\xi\omega_n s + \omega_n^2$$

$G_{comp} = (\xi\omega_n)s$, $\omega_n^2 = 2$

$\omega_n = \sqrt{2}$

choose $\xi \geq 0.45$

ضع 0.5

$\therefore \boxed{G_{comp} = 0.7\,s}$

(2) $T_s = \dfrac{4}{\xi\omega_n} \leq 4$

$\dfrac{1}{\xi\omega_n} \leq 1 \longrightarrow \xi\sqrt{2} \geq 1$

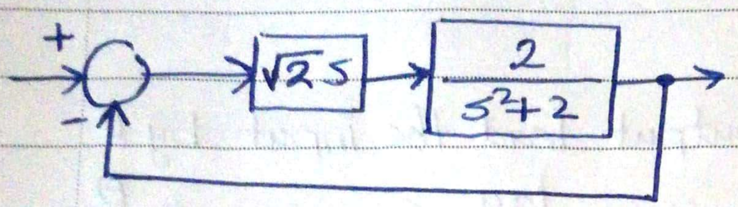
$\xi \geq \dfrac{1}{\sqrt{2}}$

$\boxed{\xi \geq 0.7}$

No.

→ so, make $\boxed{\zeta = 1}$, we'll get :

$$G_{comp} = 1 * \sqrt{2} * S = \sqrt{2}\, s$$



$$T(s) = \frac{2\sqrt{2}\, S}{s^2 + 2\sqrt{2}\, s + 2}$$

$\zeta = 1 \geq 0.45$ ✓

$T_s = 2.3 \leq 4$ ✓

No.

* Lead, Lag Compensator →

they're usually used in a system with sinusoidal output signal.

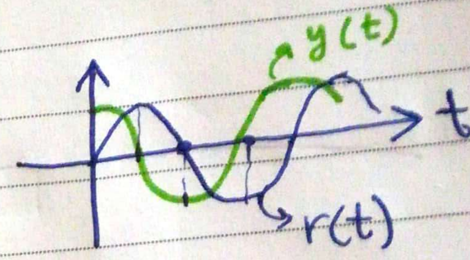
Lead : the output lead the input by θ

Lag : 〃 〃 lag 〃 〃 〃 θ

Ex) ① $r(t) = \sin(5t)$

$y(t) = \sin(5t + \frac{\pi}{2})$

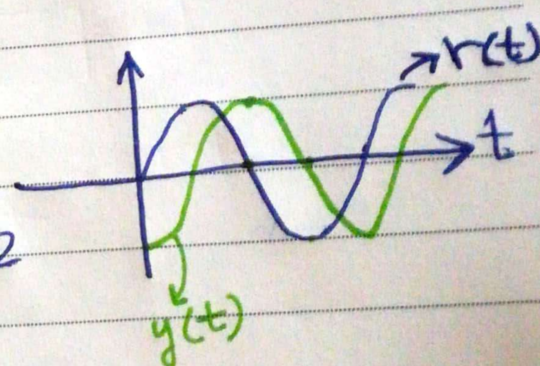
→ $y(t)$ lead $r(t)$ by $\frac{\pi}{2}$



② $r(t) = \sin(5t)$

$y(t) = \sin(5t - \frac{\pi}{2})$

→ $y(t)$ lag $r(t)$ by $\pi/2$

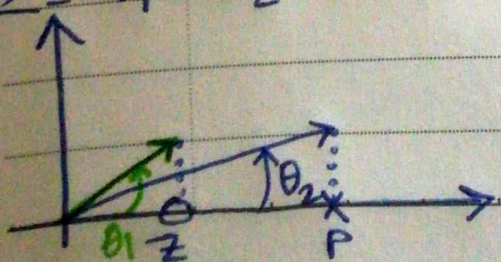


Lead :

$$\left[G_{com} = \frac{s+Z}{s+P} \right]$$

$Z \ll P$

$\underline{G(j\omega)} = \theta_1 - \theta_2$ (+ve)



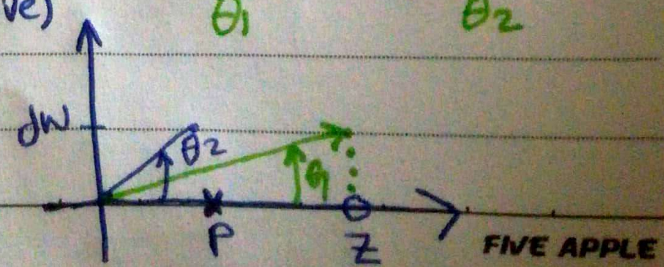
$\theta_1 > \theta_2$

Lag :

$$\left[G_{com} = \frac{s+Z}{s+P} \right]$$

$Z \gg P$

$$\underset{(-ve)}{\angle G(j\omega)} = \underset{\theta_1}{\angle j\omega + Z} - \underset{\theta_2}{\angle j\omega + P}$$



$\theta_2 > \theta_1$

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No.

Ex: Design a compensator at $\omega = 2$ rad/s where the output lead the input by $\frac{\pi}{4}$?

$$G(2j) = \frac{2j + Z}{2j + P} \qquad \text{where } Z < P$$

$$\theta_1 > \theta_2$$

$$\frac{\pi}{4} = \theta_1 - \theta_2$$

$$= \underbrace{\frac{\pi}{2}}_{Z} - \underbrace{\frac{\pi}{4}}_{P} \Rightarrow \therefore \; Z = 0, \quad P = 2$$

$$\therefore \; G_{comp} = \frac{2j}{2j + 2}$$

At any $\omega \Rightarrow \; = \frac{j\omega}{j\omega + 2} \Rightarrow G_{comp}(s) = \frac{s}{s+2}$