Analog communications

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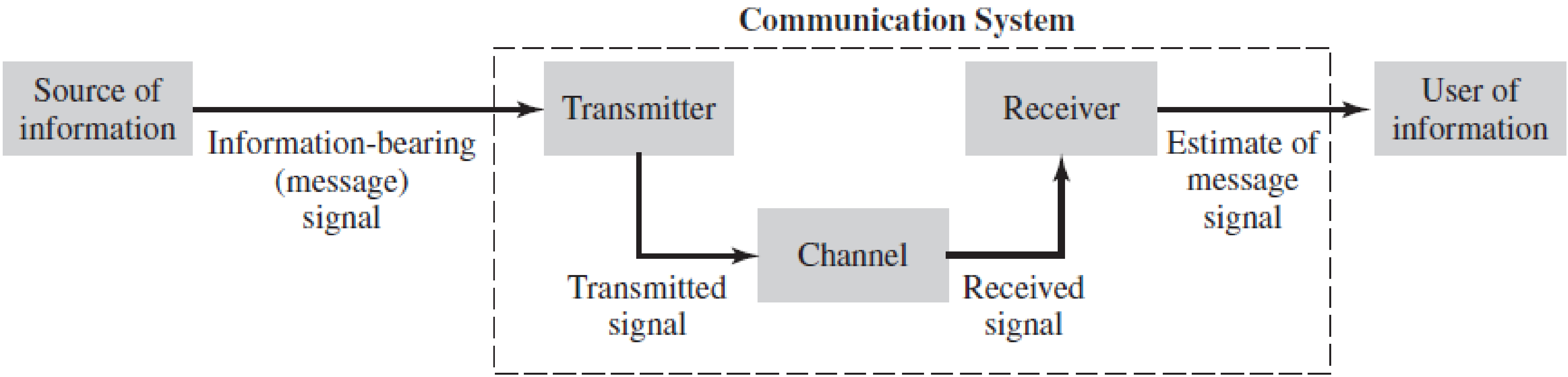
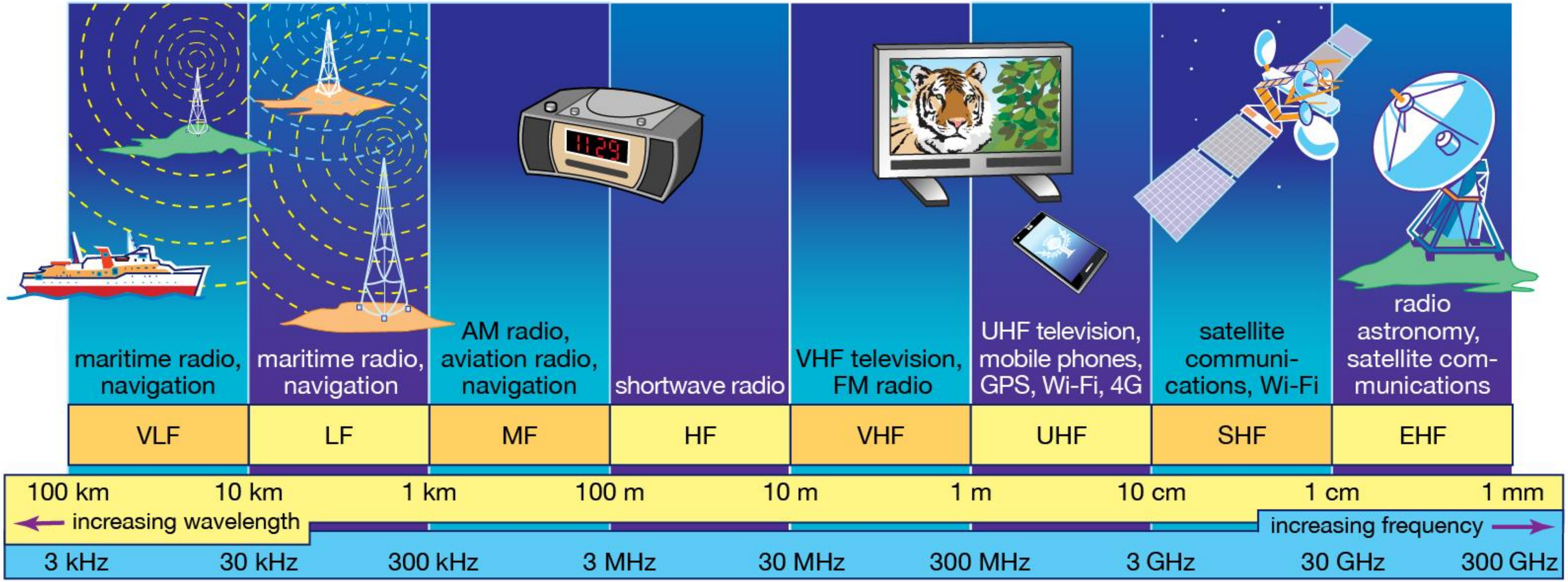
Communication system elements



$\lambda = \frac{c}{f}$, wave-length ➔ antenna length

Communication system elements

1. The source originates a message, such as a human voice, a television picture, an e-mail message, or data. If the data is nonelectric (e.g., human voice, e-mail text, a scene)
2. The transmitter transforms the input (message) signal into an appropriate form for efficient transmission. The transmitter may consist of one or more subsystems: an analog-to-digital (A/D) converter, an encoder, and a modulator.
3. The channel is a medium of choice that can convey the electric signals at the transmitter output over a distance. A typical channel can be a pair of twisted copper wires (e.g., in telephone and DSL), coaxial cable (e.g. in television and Internet), an optical fiber, or a radio cellular link.
4. The receiver reprocesses the signal received from the channel by reversing the signal transformation made at the transmitter and removing the distortions caused by the channel. The receiver output is passed to the output transducer, which converts the electric signal to its original form—the message.
5. The destination is the unit where the message transmission terminates.



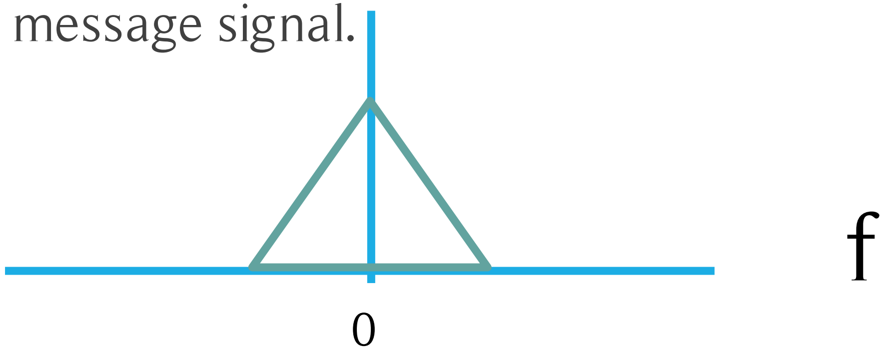
$$\mathrm{mt} = \mathrm{A}\sin(2\,\pi\, f_m\, t) \Rightarrow freq = f_m$$

$$\mathrm{mt} = \mathrm{A}\sin(100\pi t) \Rightarrow 100\pi t = 2\,\pi\, f_m\, t \Rightarrow f_m = 50\ Hz$$

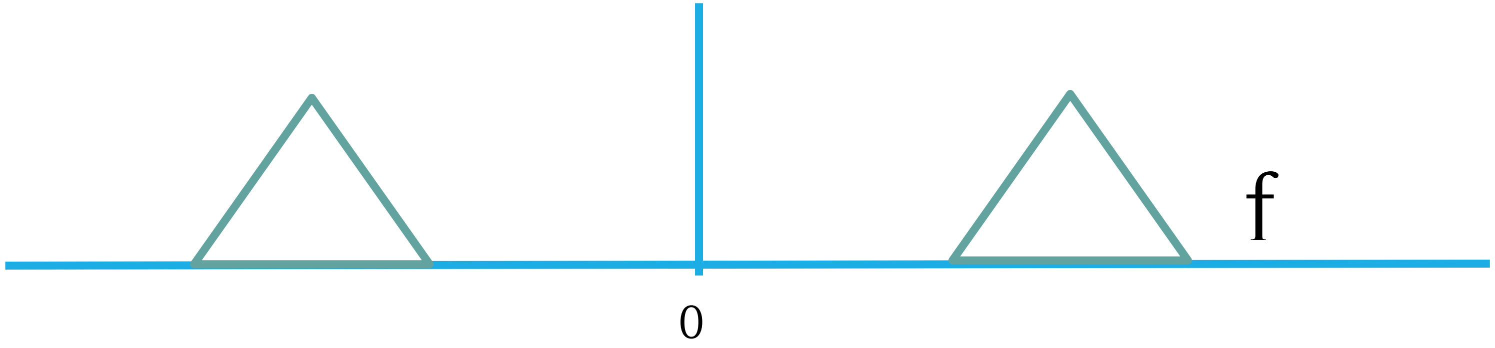
$$\mathrm{mt} = \mathrm{A}\sin(100\pi t) = \mathrm{A}\sin(2\pi\, 50\, t)$$

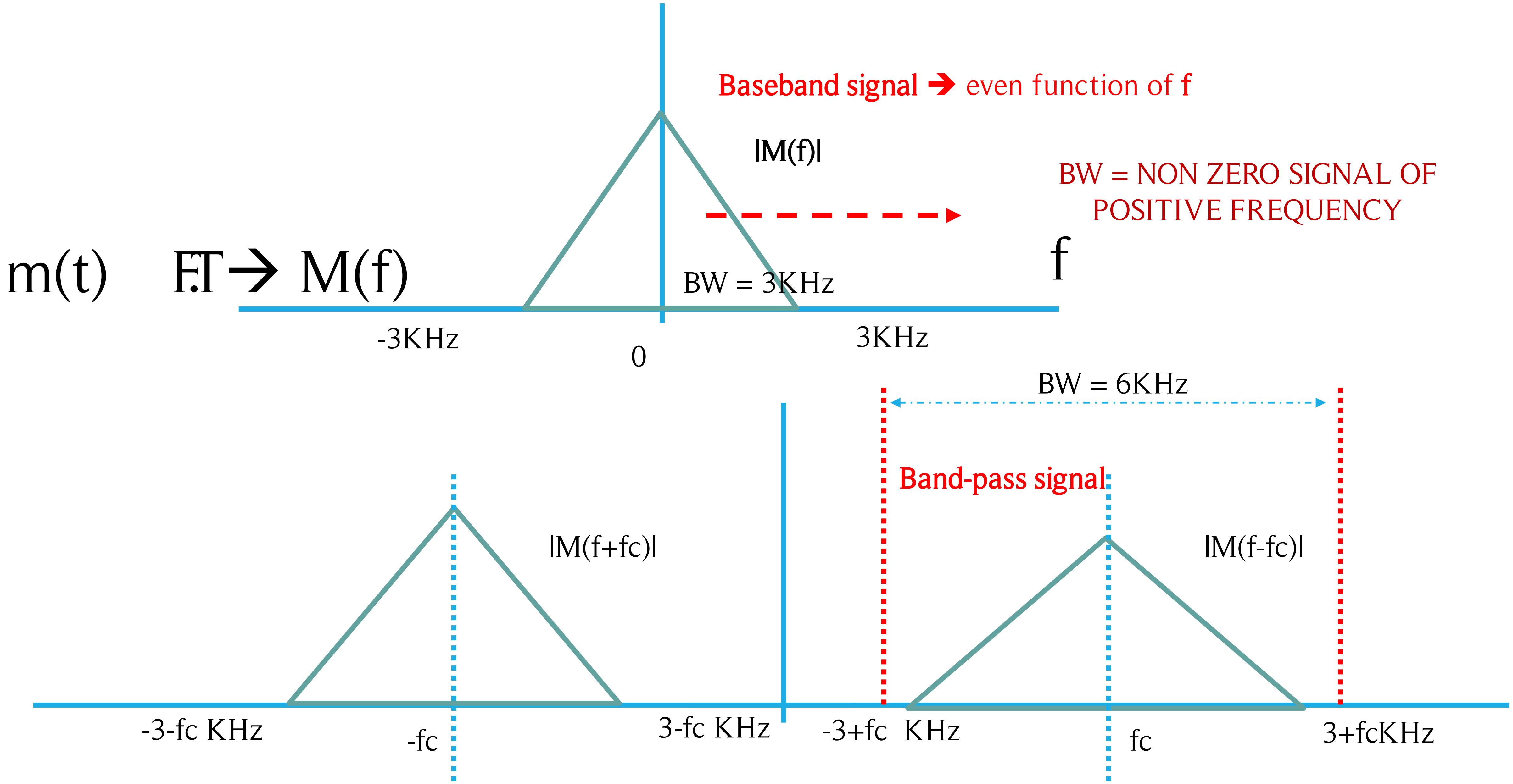
Transmission types:

Base band: the band of the frequencies supported by the channel closely matches the frequency occupied by the message signal.



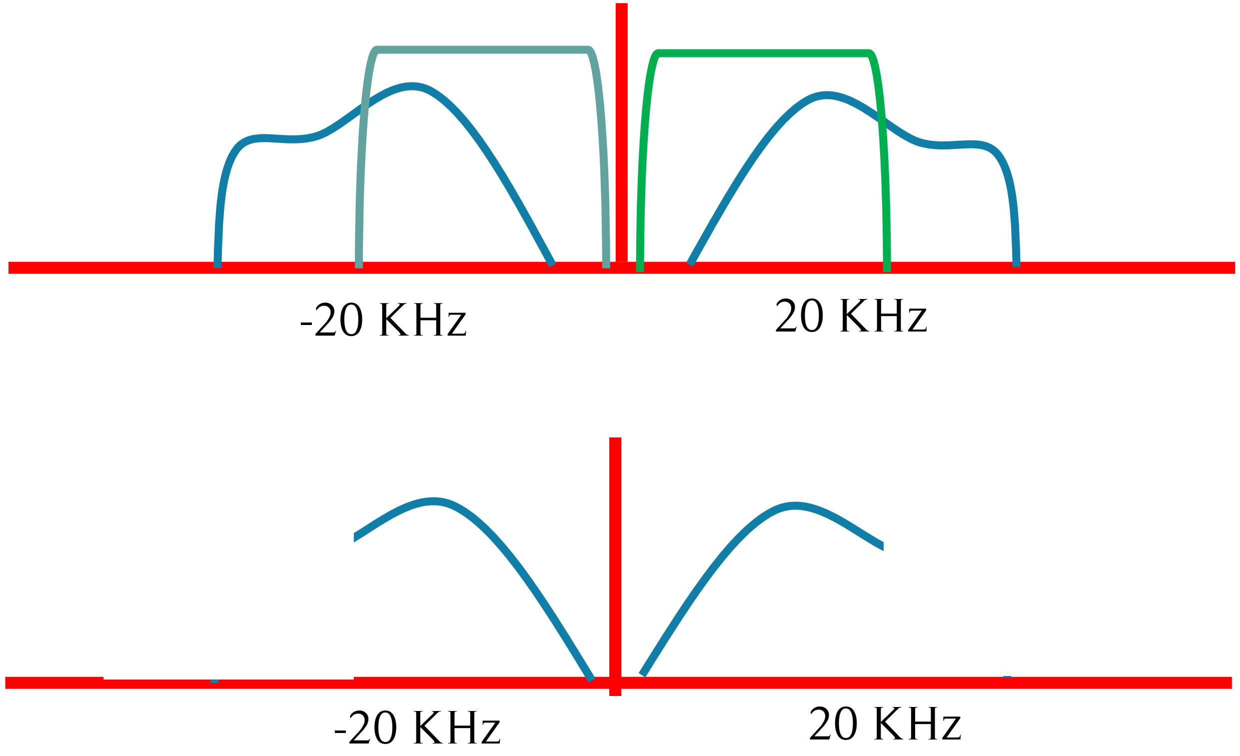
Band pass: the transmission ban of the channel is centered at a frequency much higher than the highest frequency component of the message signal.

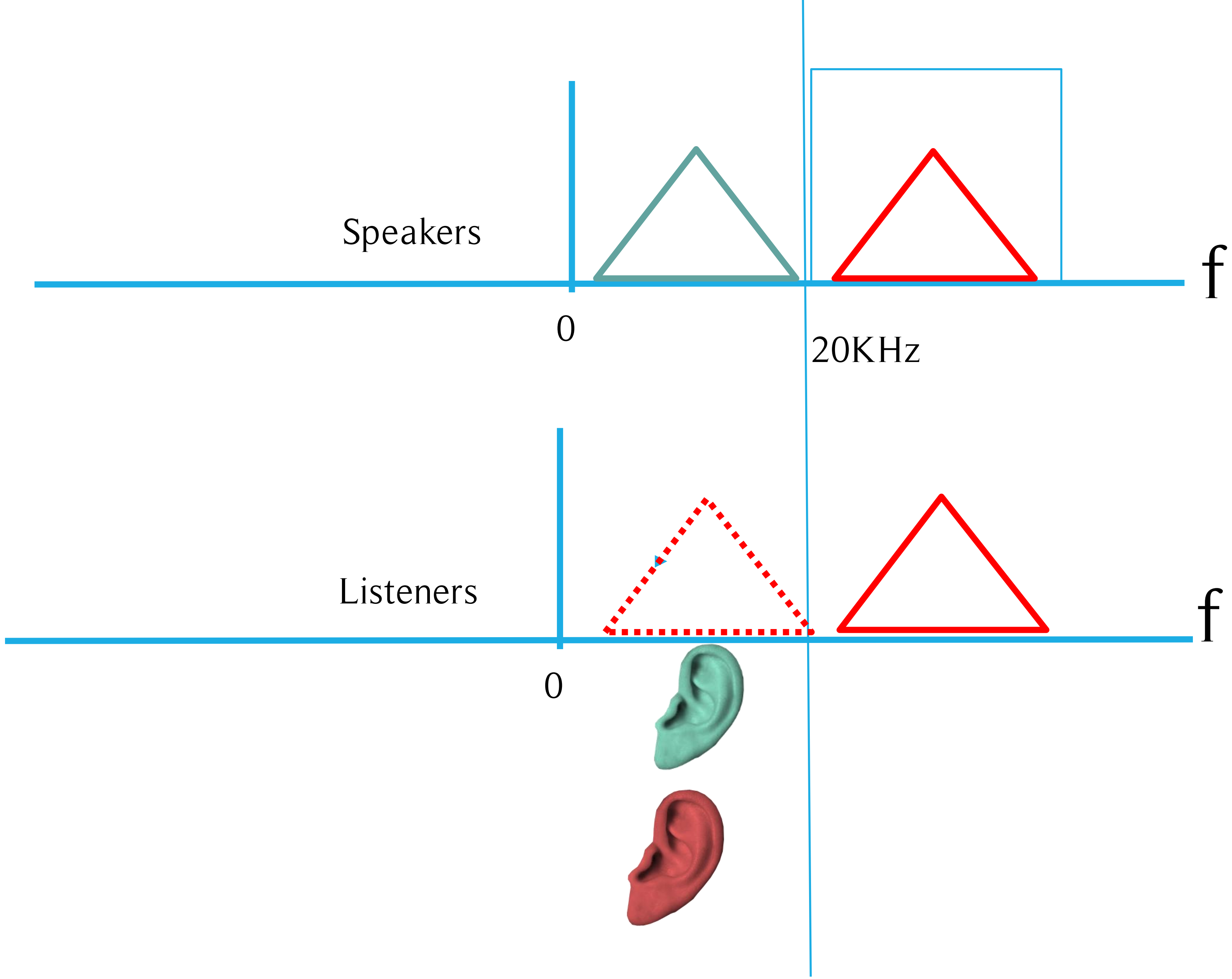


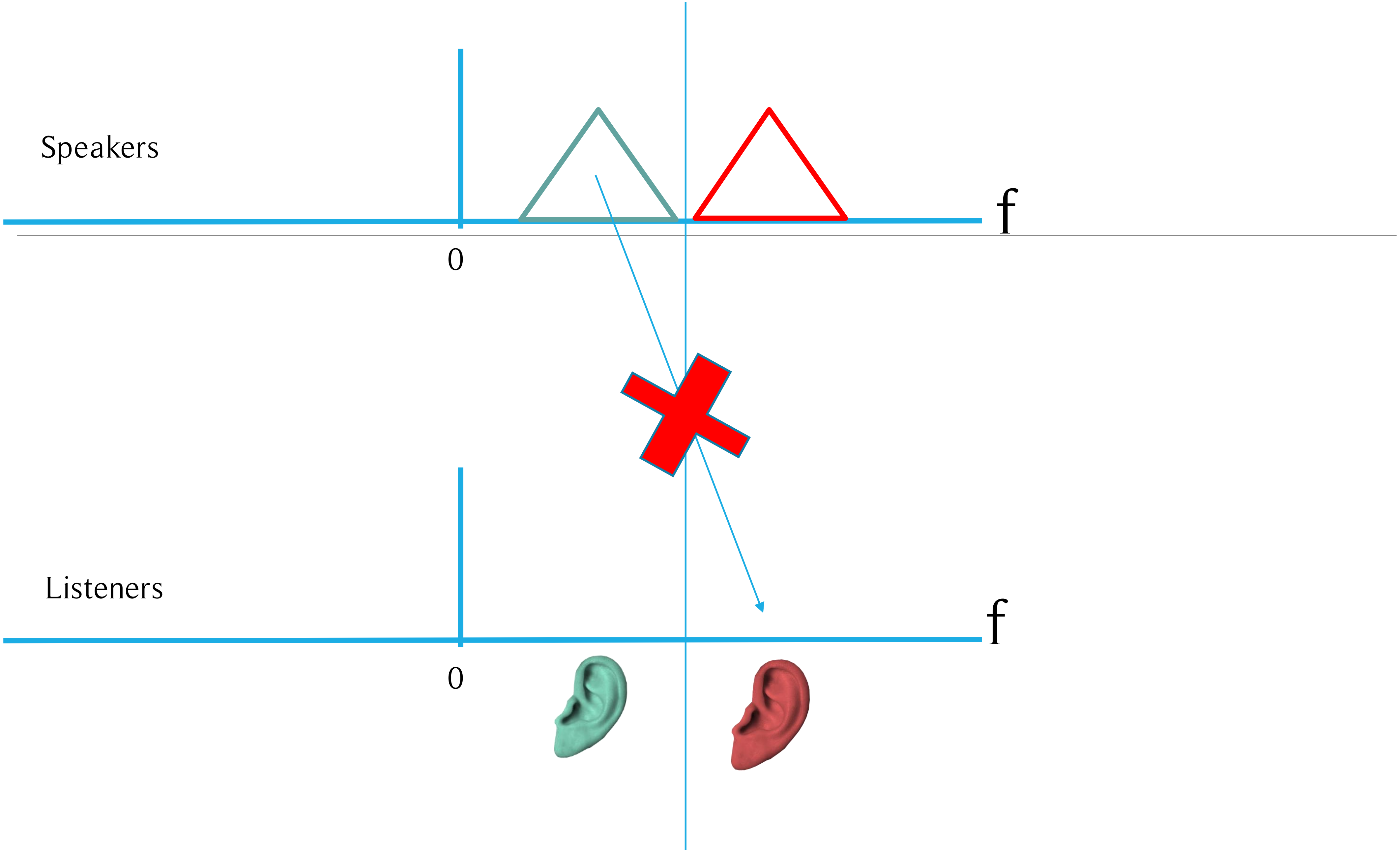


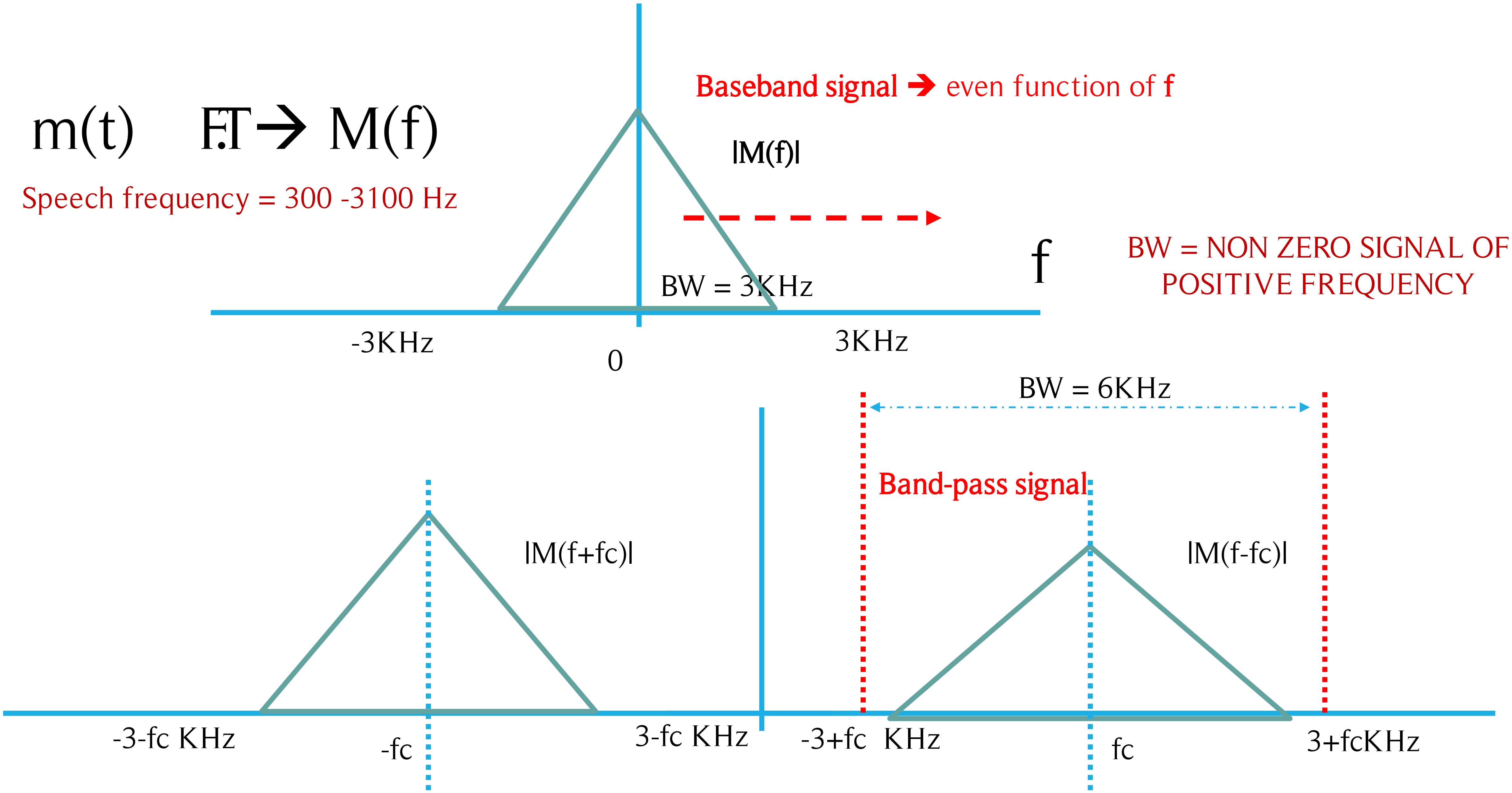
Baseband signal → even function of f
|M(f)|
BW = NON ZERO SIGNAL OF POSITIVE FREQUENCY
m(t) F.T → M(f)
f
BW = 3KHz
-3KHz
0
3KHz
BW = 6KHz
Band-pass signal
|M(f+fc)|
|M(f-fc)|
-3-fc KHz
-fc
3-fc KHz
-3+fc KHz
fc
3+fcKHz

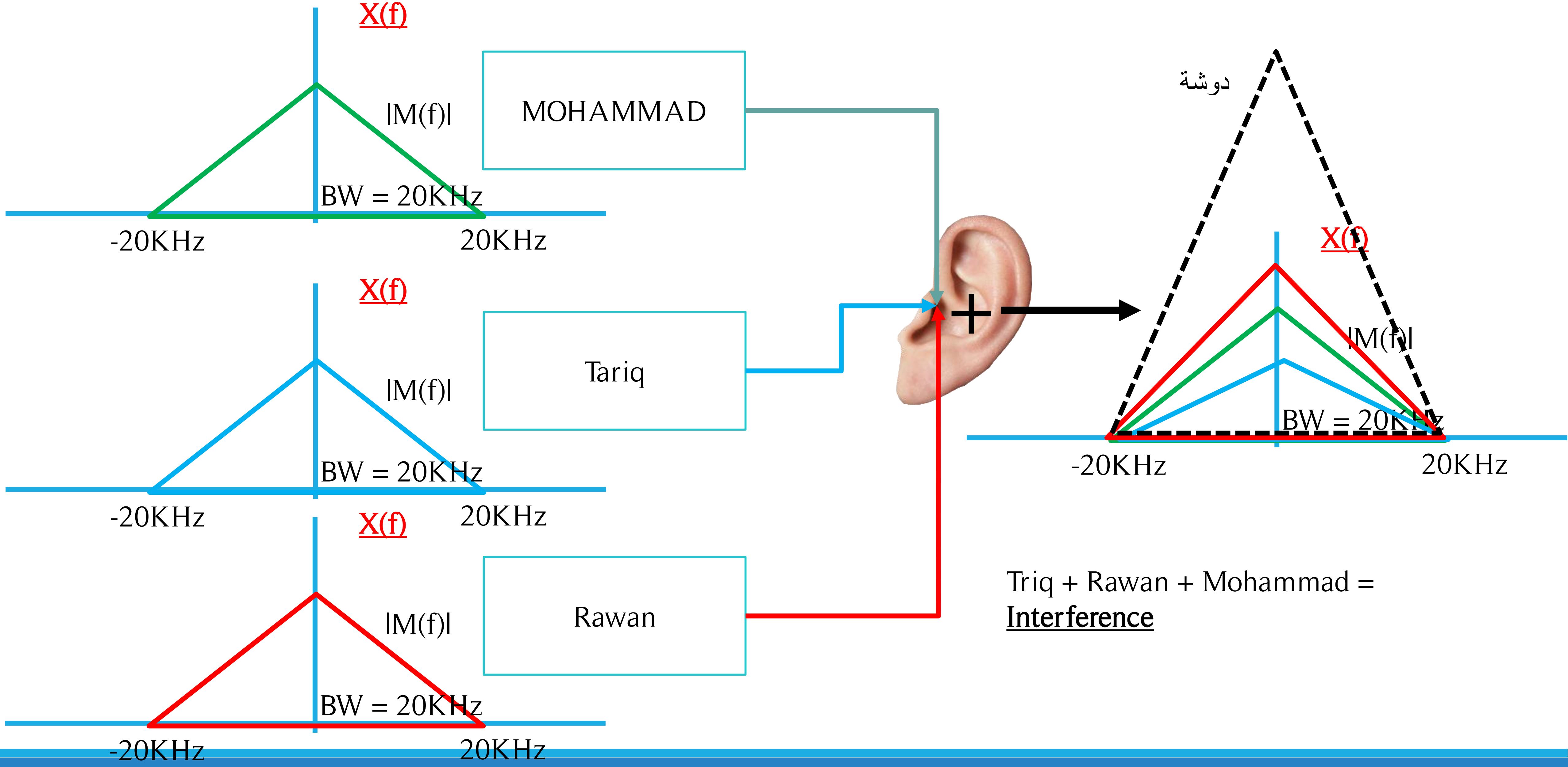
The response of human ear on frequency domain



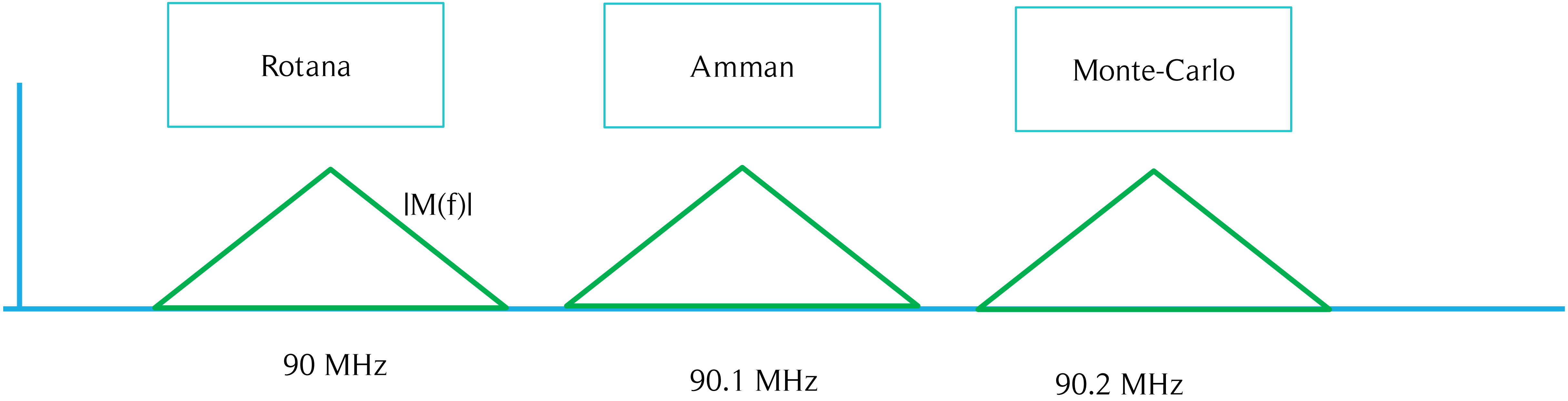

Speakers
0
20KHz
f
Listeners
0
f

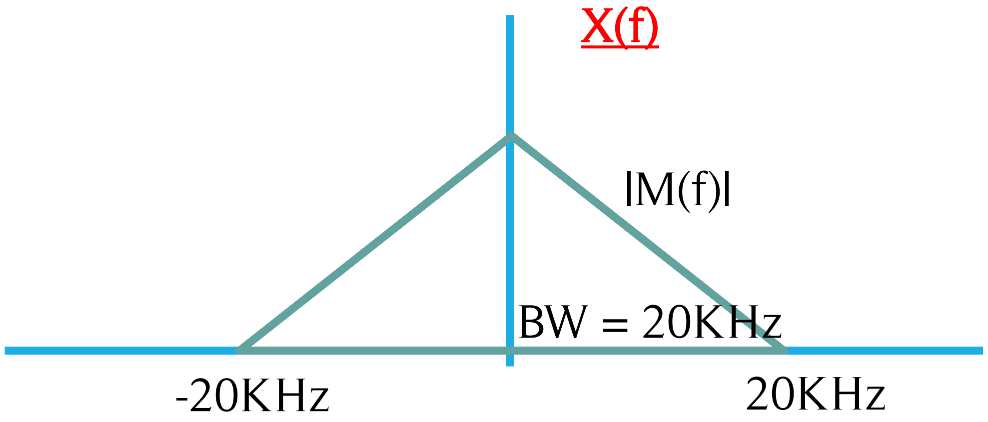

Speakers
f
0
Listeners
f
0


m(t) F.T → M(f)
Speech frequency = 300 -3100 Hz
Baseband signal → even function of f
|M(f)|
BW = 3KHz
f
BW = NON ZERO SIGNAL OF POSITIVE FREQUENCY
-3KHz
0
3KHz
BW = 6KHz
Band-pass signal
|M(f+fc)|
|M(f-fc)|
-3-fc KHz
-fc
3-fc KHz
-3+fc KHz
fc
3+fcKHz



Triq + Rawan + Mohammad =
<u>Interference</u>


Rotana
Amman
Monte-Carlo
|M(f)|
90 MHz
90.1 MHz
90.2 MHz


X(f)
|M(f)|
BW = 20KHz
-20KHz
20KHz

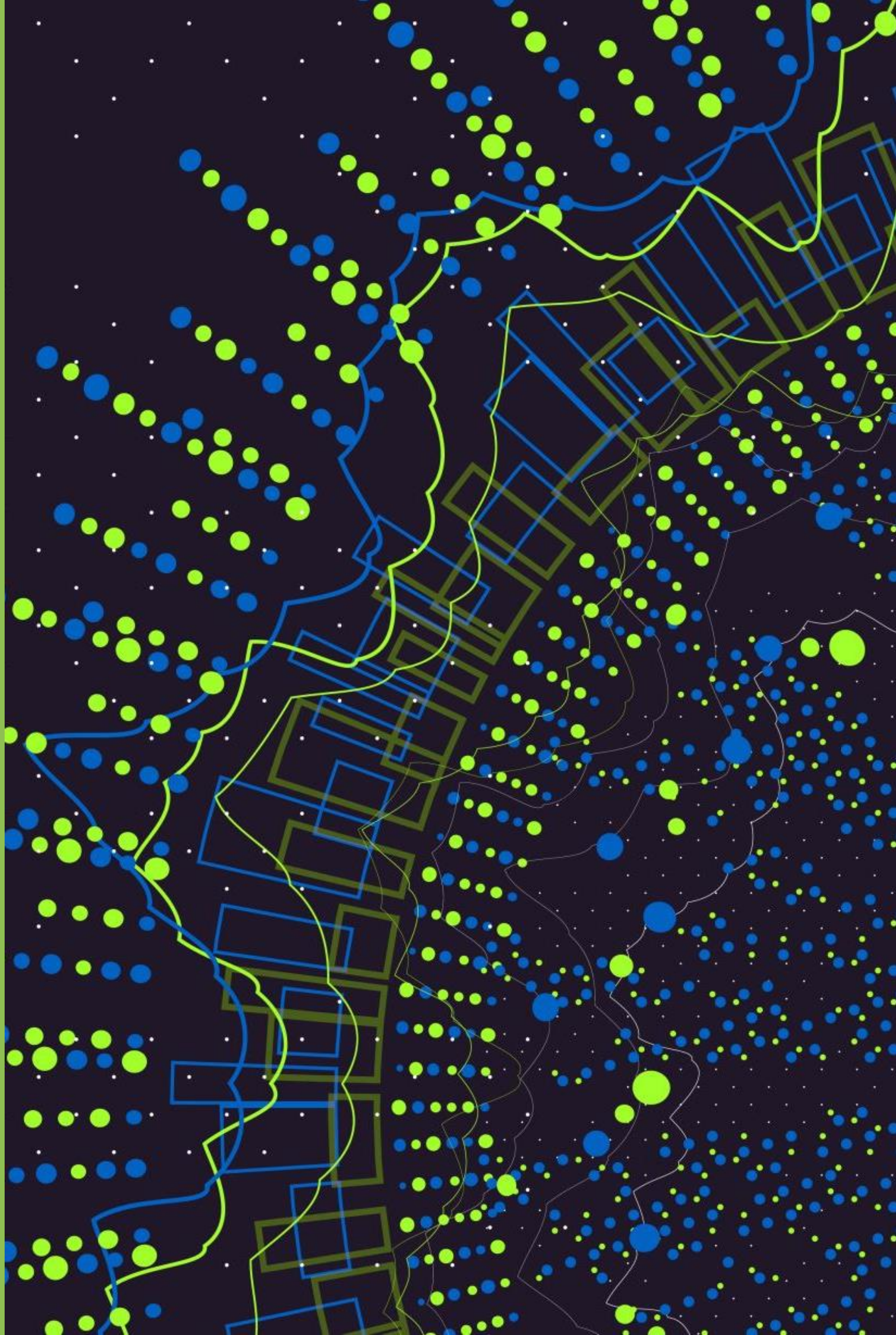

H(f)

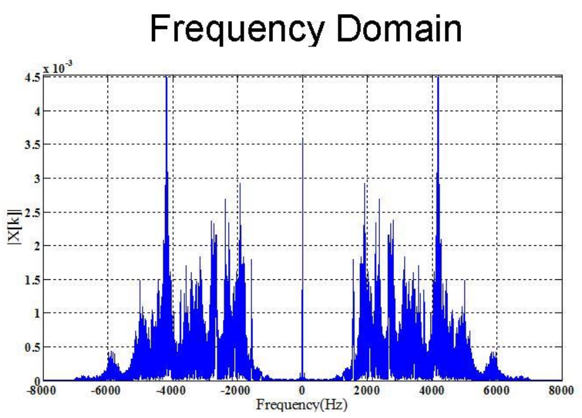
Primary Resources and Operational Requirements

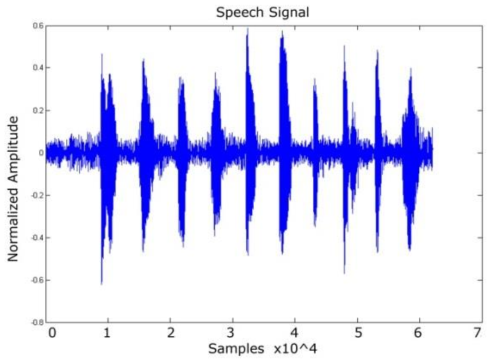
- Transmitted power, which is defined as the average power of the transmitted signal
- Channel bandwidth, which is defined by the width of the passband of the channel.

Issues in modulation theory that need to be addressed

1. Time-domain description of the modulated signal.
2. Frequency-domain description of the modulated signal.
3. Detection of the original information-bearing signal and evaluation of the effect of noise on the receiver.







FOURIER ANALYSIS

Fourier analysis provides the mathematical basis for evaluating the following issues:

- Frequency-domain description of a modulated signal, including its transmission bandwidth.
- Transmission of a signal through a linear system exemplified by a communication channel or (frequency-selective) filter.
- Correlation (i.e., similarity) between a pair of signals.


Highpass
ON
Frequency
30 Hz
Q
0.707
24dB/oct
Lowshelf
ON
100 Hz
0.0 dB
0.707
Parametrics
Active
ON ON ON ON ON ON ON ON
Freq
63.183 Hz 125.218 Hz 251.525 Hz 500 Hz 1000 Hz 2000 Hz 4000 Hz 8000 Hz
Level
1.2 dB 0.0 dB 0.0 dB 0.0 dB 0.0 dB -36.1 dB 0.0 dB 0.0 dB
Q
1 1 1 1 1 1 1 1
Highshelf
ON
5000 Hz
0.0 dB
0.707
Lowpass
ON
Frequency
18000 Hz
Q
0.707
24dB/oct
calf studio gear
File Analysis Options Settings
RTA 240 Bands
PEAK
SR: 44100 Hz
Cal: 0.0 dB
FPS: 50 (20.0 ms)
mouse position
PEAK dB fs -3.1
dB
0 -12 -24 -36 -48 -60 -72 -84 -96 108
31 63 125 250 500 1000 2000 4000 8000 16000
Search the web and Windows
9:55 AM
1/28/2017
Right Ctrl
Saturday, Jan 28, 11:55 AM

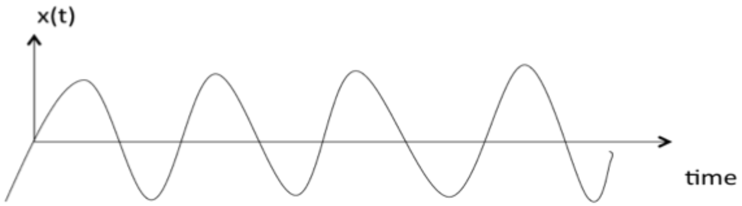
Classes of signals in communication systems

1. Periodic and non-periodic

A continuous time signal x(t) is said to be periodic if and only if:

$$x(t+T) = x(t) \text{ for } -\infty < t < \infty$$

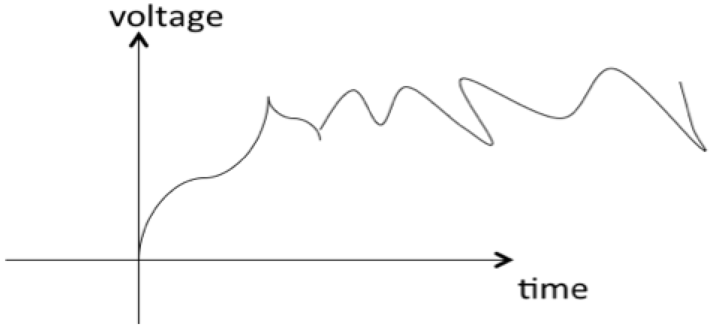
Where, T is a positive constant that represents the time period of the periodic signal.



2. Deterministic and non-deterministic signals:

A signal is said to be deterministic if there is no uncertainty with respect to its value at any instant of time. Or signals which can be defined exactly by a mathematical formula are known as deterministic signals.

A signal is said to be non-deterministic if there is uncertainty with respect to its value at some instant of time. Non-deterministic signals are random in nature hence they are called random signals. Random signals cannot be described by a mathematical equation. They are modelled in probabilistic terms.



Classes of signals in communication systems

3- Energy and Power Signals

A signal is said to be energy signal when it has finite energy.

$$\text{Energy } E = \int_{-\infty}^{\infty} x^2(t)dt$$

A signal is said to be power signal when it has finite power.

$$\text{Power } P = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} x^2(t)dt$$

NOTE: A signal cannot be both, energy and power simultaneously. Also, a signal may be neither energy nor power signal.

Power of energy signal = 0

Energy of power signal = ∞

$x(t)$ is an energy signal if $0 < E < \infty$

$x(t)$ is a power signal if $0 < \mathrm{P} < \infty$

MODULATION THEORY

Modulation is a signal-processing operation that is basic to the transmission of an information- bearing signal over a communication channel, whether in the context of digital or analog communications. The carrier wave may take one of two basic forms, depending on the application of interest:

- Sinusoidal carrier wave, whose amplitude, phase, or frequency is the parameter chosen for modification by the information-bearing signal.
- Periodic sequence of pulses, whose amplitude, width, or position is the parameter chosen for modification by the information-bearing signal.

Modulation and de-modulation

Modulation : In electronics and telecommunications, modulation is the process of varying one or more properties of a periodic waveform, called the carrier signal, with a separate signal called the modulation signal that typically contains information to be transmitted.

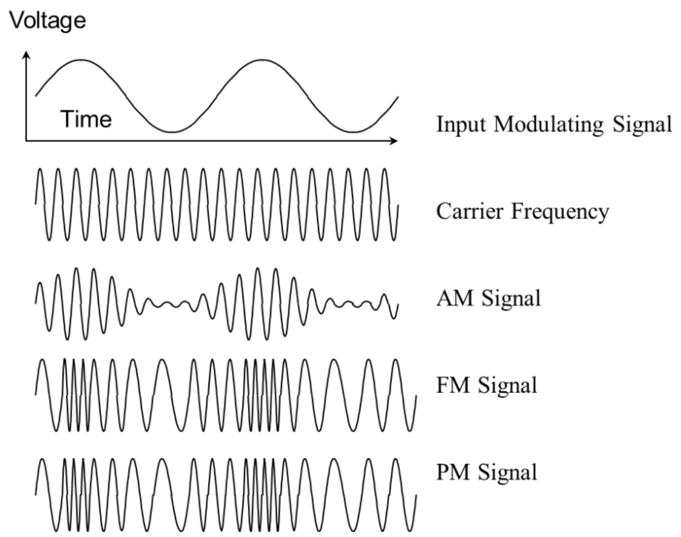
Demodulation: Demodulation is extracting the original information-bearing signal from a carrier wave.

Types of modulation:

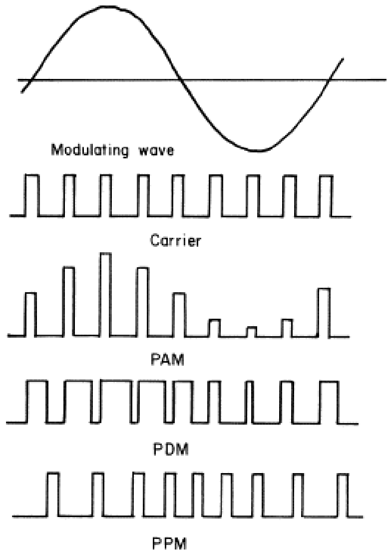
1- Continuous wave of modulation.

2- Pulse modulation.

Continuous wave of modulation.



Pulse modulation.



Types of modulation

Continuous wave of modulation:

1- Amplitude modulation

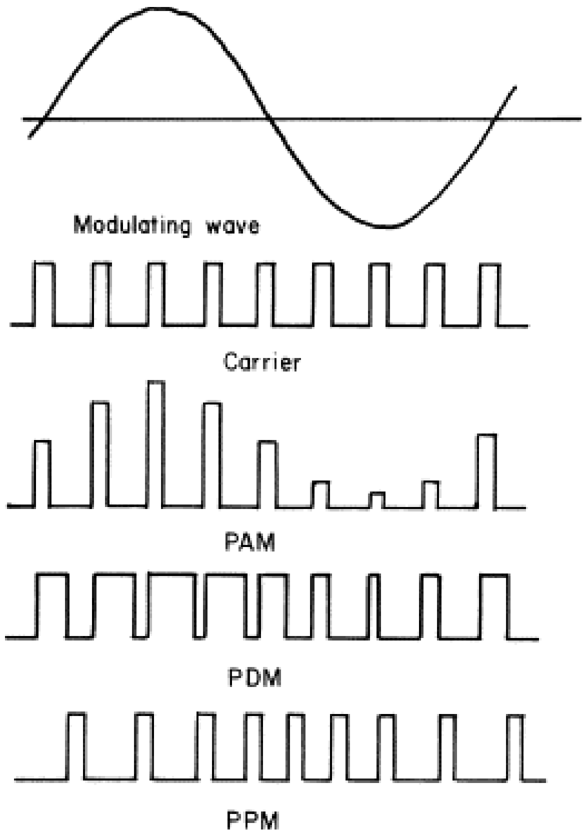
2- Angle modulation

$$c(t) = A\cos(\phi(t)); \quad \phi(t) = 2\pi f t$$

Pulse modulation

1- Analog pulse modulation: in analog pulse modulation, the amplitude (PAM), duration (PDM) or position (PPM) of the pulse is varied in accordance with the sample values of the message signal.

2- Digital pulse modulation.

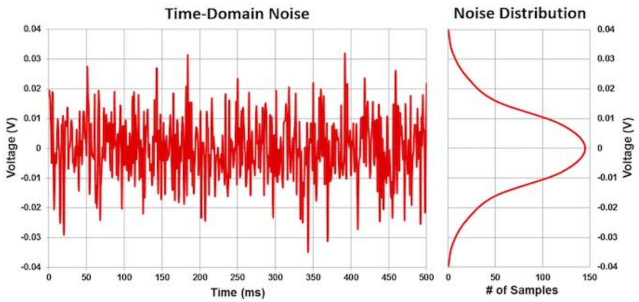


DETECTION THEORY

The signal-detection problem is complicated by two issues:

- The presence of noise.
- Factors such as the unknown phase-shift introduced into the carrier wave due to transmission of the sinusoidally modulated signal over the channel.

Shannon's channel capacity



Channel capacity, in electrical engineering, computer science, and information theory, is the tight upper bound on the rate at which information can be reliably transmitted over a communication channel.

An application of the channel capacity concept to an additive white Gaussian noise (AWGN) channel with B Hz bandwidth and signal-to-noise ratio SNR is the Shannon theorem:

$$C = B\log_2(1 + SNR) \quad \text{bits/hertz}$$

C: maximum channel capacity

B: channel bandwidth

SNR: Signal to noise ratio

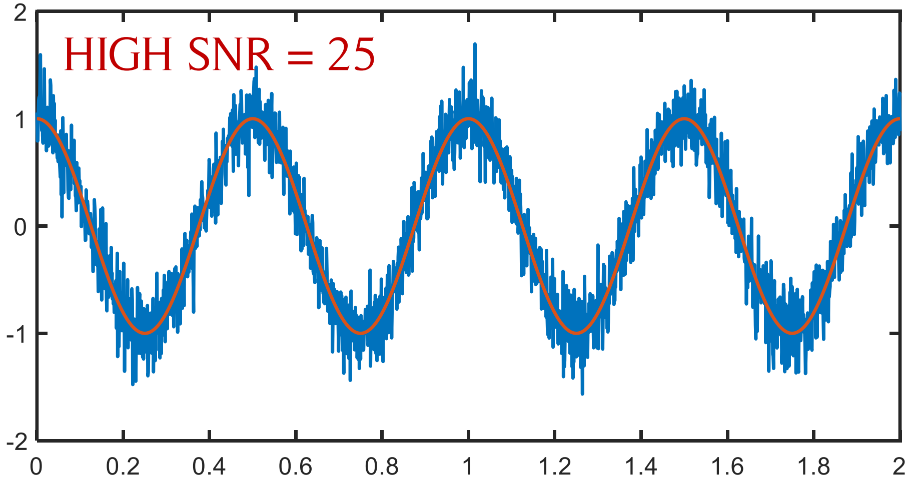
Signal Noise

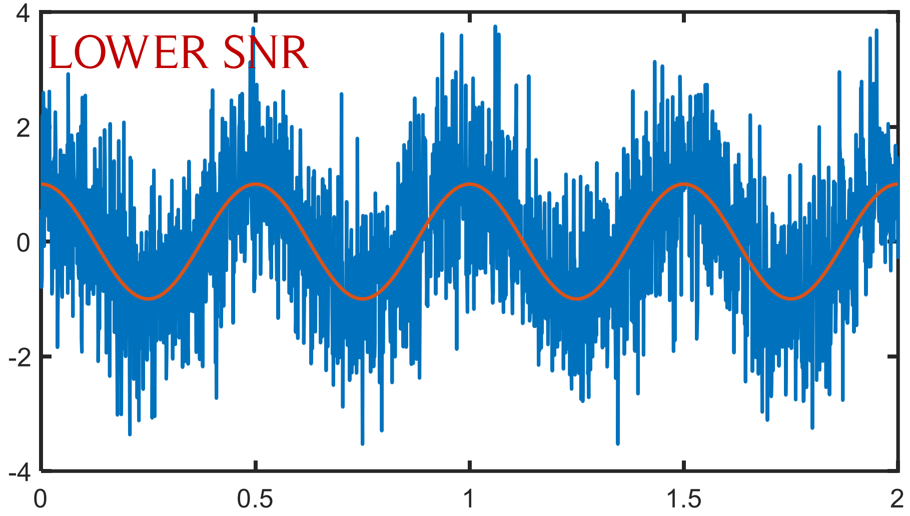
$$r(t) = s(t) + n(t)$$

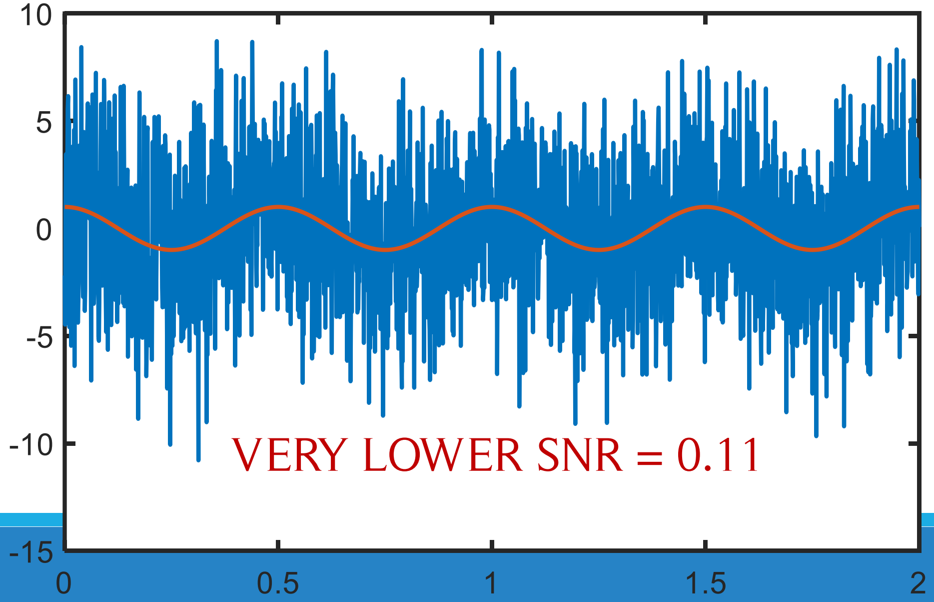
Power spectral density (PSD) frequency domain

$SNR: signal\ to\ noise\ ratio$ ← signal power / noise power

$$SNR = \frac{S}{N} = \frac{P_S}{P_N}$$



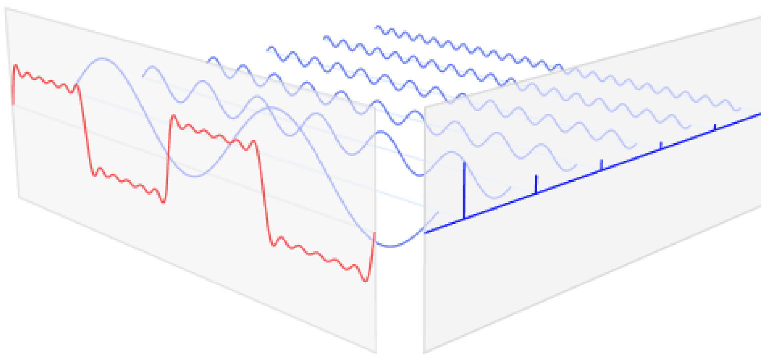


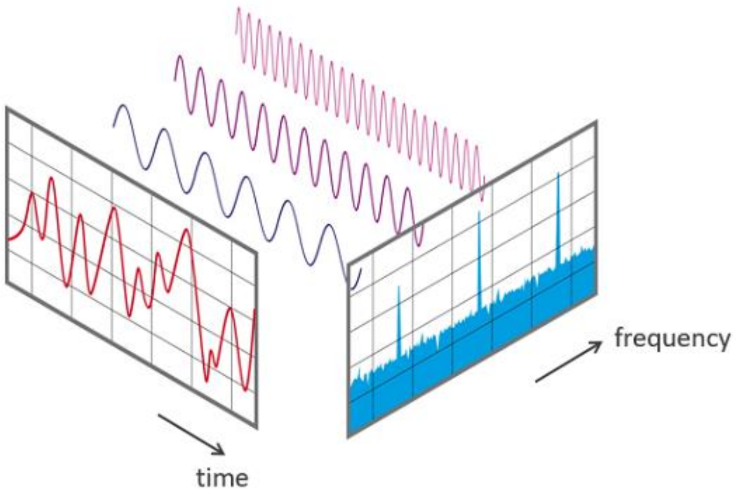


Power spectral density (PSD) frequency domain ↔ Autocorrelation function

Fourier transform

Fourier transform (FT) is a mathematical transform that decomposes functions depending on space or time into functions depending on spatial or temporal frequency, such as the expression of a musical chord in terms of the amplitudes and frequencies of its constituent notes. The term Fourier transform refers to both the frequency domain representation and the mathematical operation that associates the frequency domain representation to a function of space or time.





Fourier transform

Let $g(t)$ denote to a non periodic deterministic signal. Then:

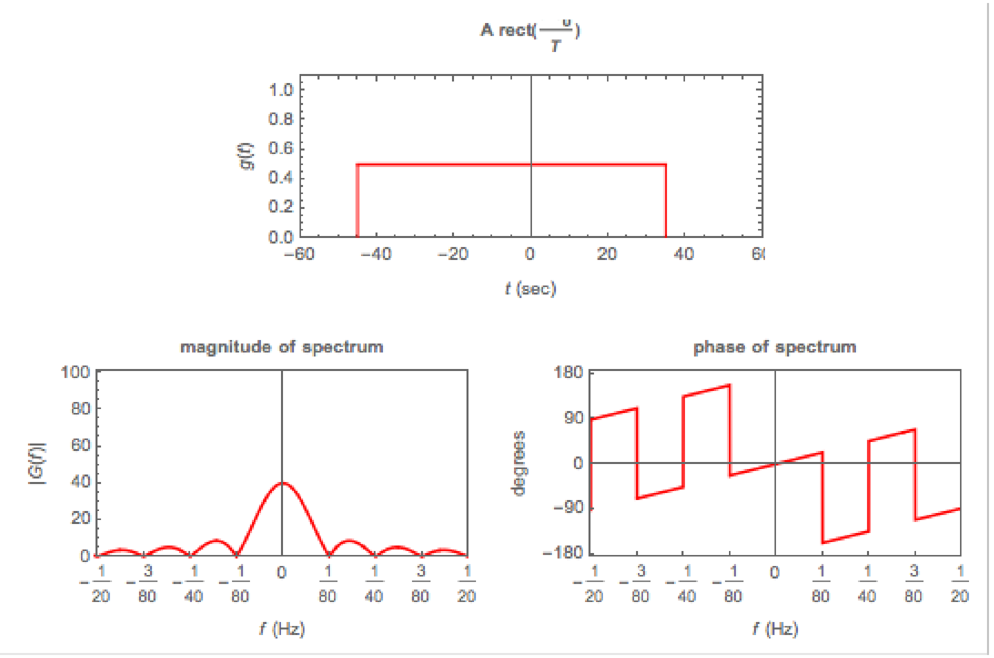
$$G(f) = \int_{-\infty}^{\infty} g(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} g(t)e^{-j2\pi f t}dt$$

$$g(t) = \int_{-\infty}^{\infty} G(f)e^{j2\pi f t}df$$

$$G(f) = |G(f)|e^{j\theta(f)}$$

$\theta(f)$: phase spectrum

$|G(f)|$: amplitude spectrum



$|G(f)|$ is even function of f (symmetric)

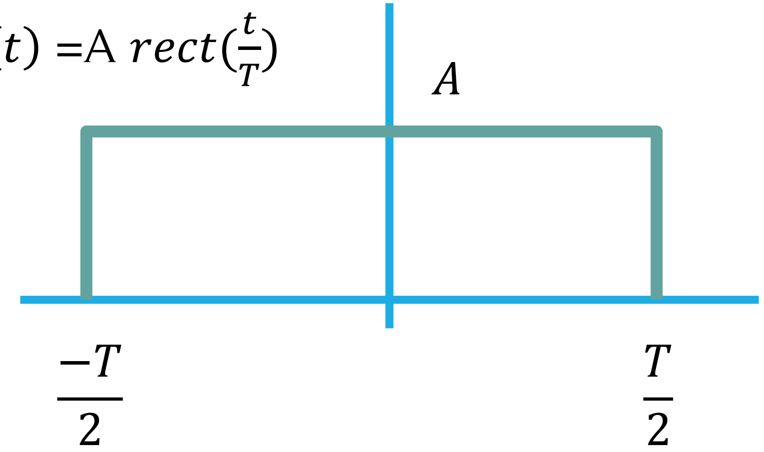
$\theta(f)$ is odd function of f (un-symmetric)

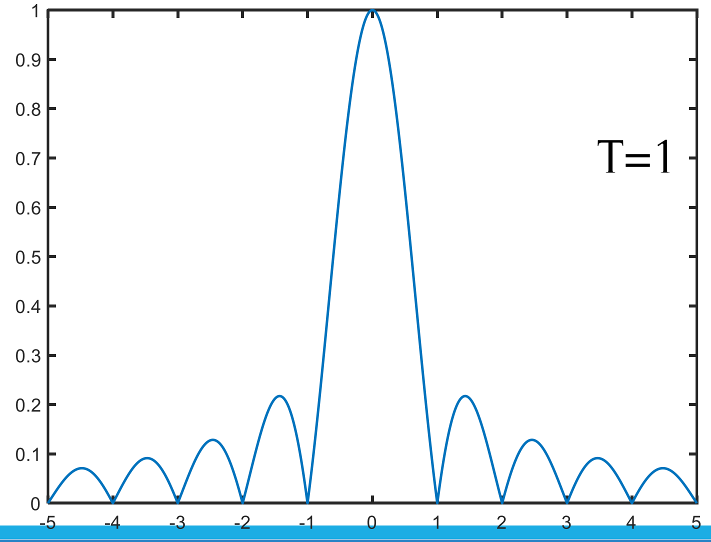
Example:

Let $g(t)$ be a rectangular pulse

$$G(f) = \int_{-\frac{T}{2}}^{\frac{T}{2}} Ae^{-j\omega t}dt = \frac{ATsin(\pi f\ T)}{\pi fT} = \text{ATsinc(f T)}$$

$g(t)$ =A $rect(\frac{t}{T})$



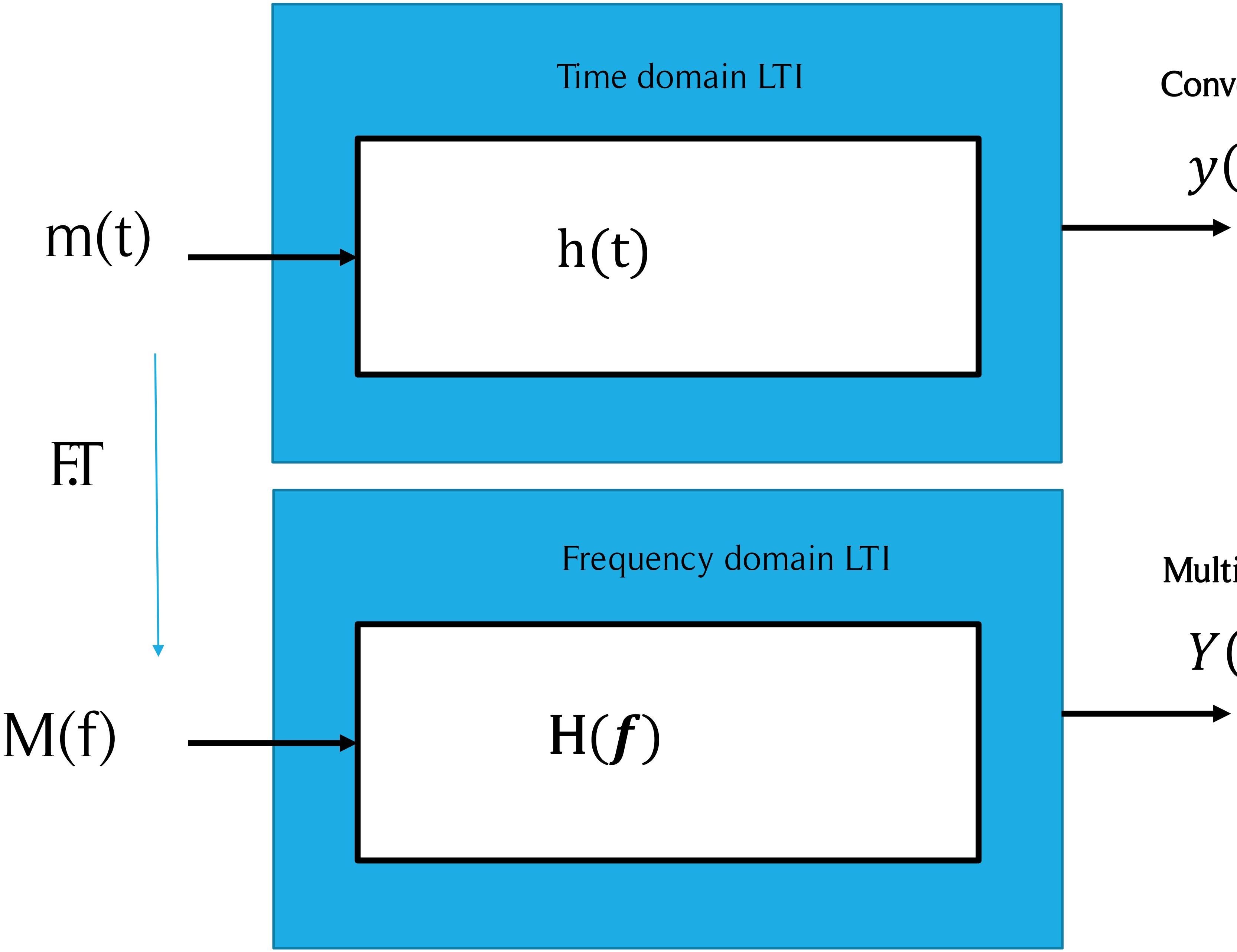


Properties of Fourier transform

Operation	FT Property: Given $\mathbf{g}(\boldsymbol{t}) \Leftrightarrow \mathbf{G}(\boldsymbol{f})$
Linearity	$ag_1(t) + bg_2(t) \Leftrightarrow aG_1(f) + bG_2(f)$
Time Shifting	$g(t - t_0) \Leftrightarrow e^{-j2\pi f t_0}\, G\,(f)$
Time Scaling	$g(at) \Leftrightarrow \frac{1}{\lvert a \rvert} G\left(\frac{\mathrm{f}}{a}\right)$
Modulation (1)	$g(t)\cos(2\pi f_o t) = \frac{g(t)}{2}\left(e^{j2\pi f_o t} + e^{-j2\pi f_o t}\right) \Leftrightarrow \frac{1}{2}[G(\mathrm{f} - f_0) + G(\mathrm{f} + f_0)]$
Modulation (2) = Frequency shifting	$g(t)\, e^{j2\pi f_o t} \Leftrightarrow G(\mathrm{f} - f_0)$
Area under G(f)	$A = g(0) = \int_{-\infty}^{\infty} G(f) e^0 df = \int_{-\infty}^{\infty} G(f) df$
Area under g(t)	$A = G(0) = \int_{-\infty}^{\infty} g(t) e^0 dt = \int_{-\infty}^{\infty} g(t) dt$

Properties of Fourier transform

Differentiation	If $x(t) = \frac{dg(t)}{dt}$, then $X(f) = j2\pi f \cdot G(f)$
Integration	If $x(t) = \int_{-\infty}^{t} g(\alpha)\, d\alpha$, then $X(f) = \frac{1}{j2\pi f} G(f)$
Convolution	$g(t) * x(t) \Leftrightarrow G(f)X(f)$, where $g(t) * x(t) \equiv \int_{-\infty}^{\infty} g(\alpha)\, x(t-\alpha) d\alpha$
Multiplication	$x(t) \cdot g(t) \Leftrightarrow X(f) * G(f) = \int_{-\infty}^{\infty} g(\alpha)\, x(f-\alpha) d\alpha$
Duality	If $g(t) \Leftrightarrow G(f)$, then $G(t) \Leftrightarrow g(-f)$
Hermitian Symmetry	If g(t) is real-valued then $G(-f) = G^*(f)$ $(\lvert G(-f)\rvert = \lvert G(f)\rvert$ and $\angle G(-f) = -\angle G(f))$
Conjugation	$g^*(t) \Leftrightarrow G^*(-f)$
Parseval's Theorem	$P_{avg} = \int_{-\infty}^{\infty} \lvert g(t)\rvert^2 dt = \int_{-\infty}^{\infty} \lvert G(2\pi f)\rvert^2 df$

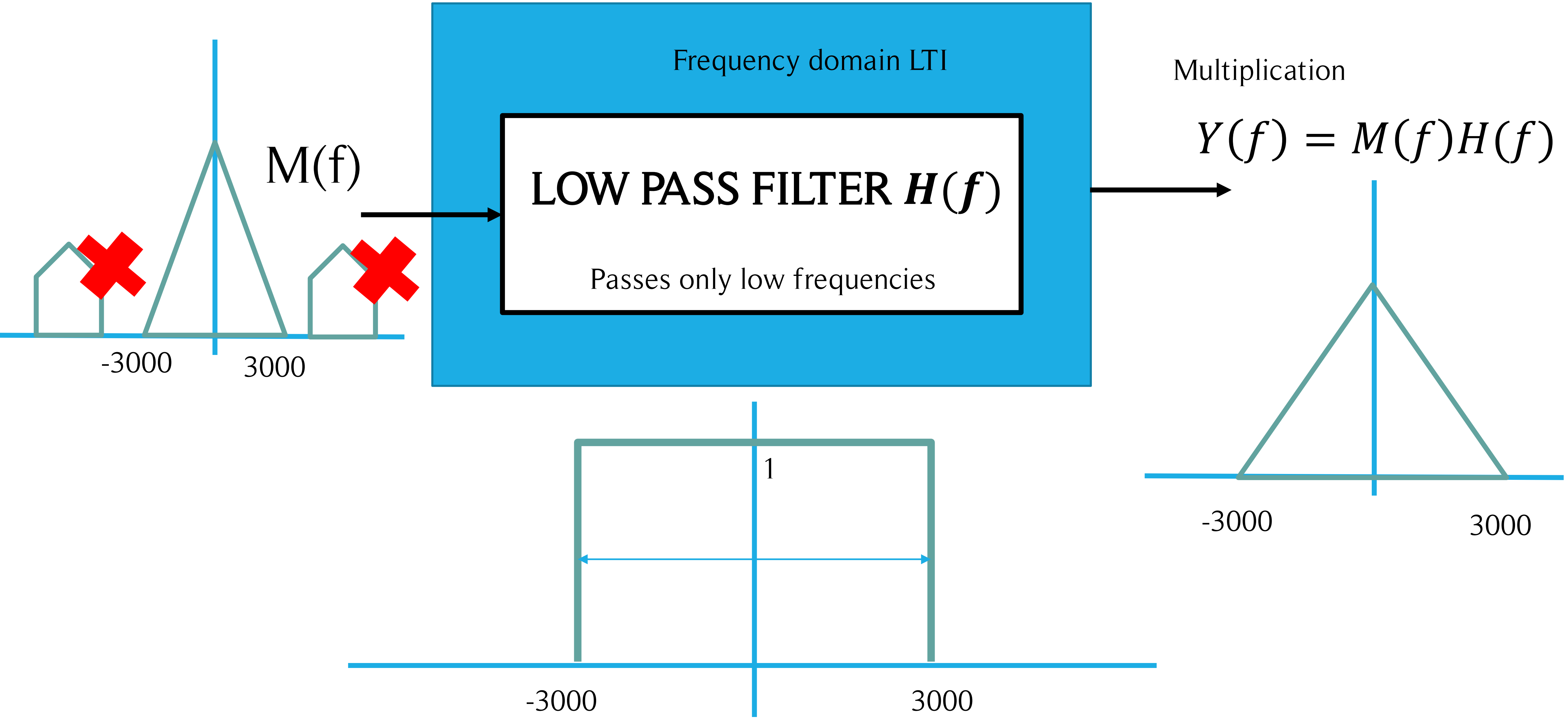


Convolution

$$y(t) = m(t) * h(t)$$

Multiplication

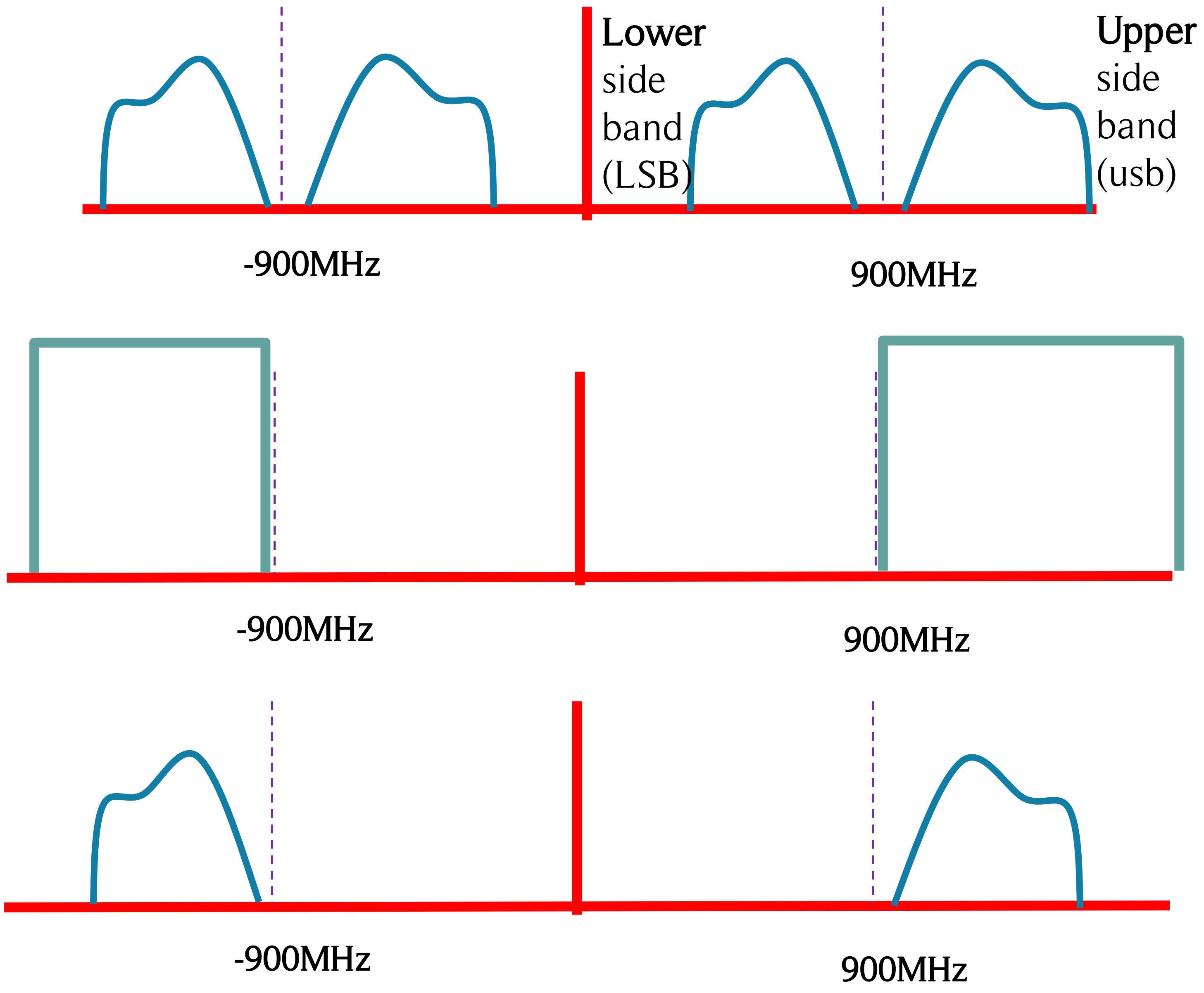
$$Y(f) = M(f)H(f)$$


M(f)
-3000
3000
Frequency domain LTI
LOW PASS FILTER $H(f)$
Passes only low frequencies
1
-3000
3000
Multiplication
$Y(f) = M(f)H(f)$
-3000
3000

U(f)

H(f)

$Y(f)=H(f)U(f)$



Passing the pass band signal through ideal filter to filter out the lower side band of the signal

Hilbert transform (90 degree *phase shift*)

Hilbert transform of a signal **g(t)** is defined as the transform in which **phase-angle** of all components of the signal is **shifted** by ±90 degrees

<u>Hilbert transform of g(t) is represented with</u>

$$\hat{g}(t) = \frac{1}{\pi t} * g(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{g(\tau)}{t-\tau}\, d\tau$$

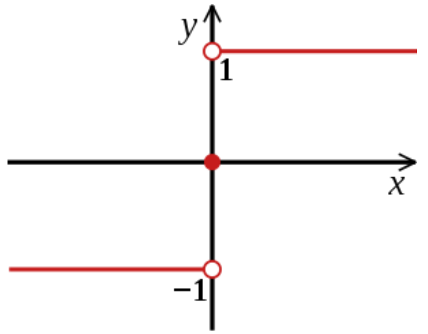
The inverse Hilbert transform is given by:

$$g(t) = \frac{1}{\pi t} * \hat{g}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{\hat{g}(t)}{t-\tau}\, d\tau$$

Frequency domain representation

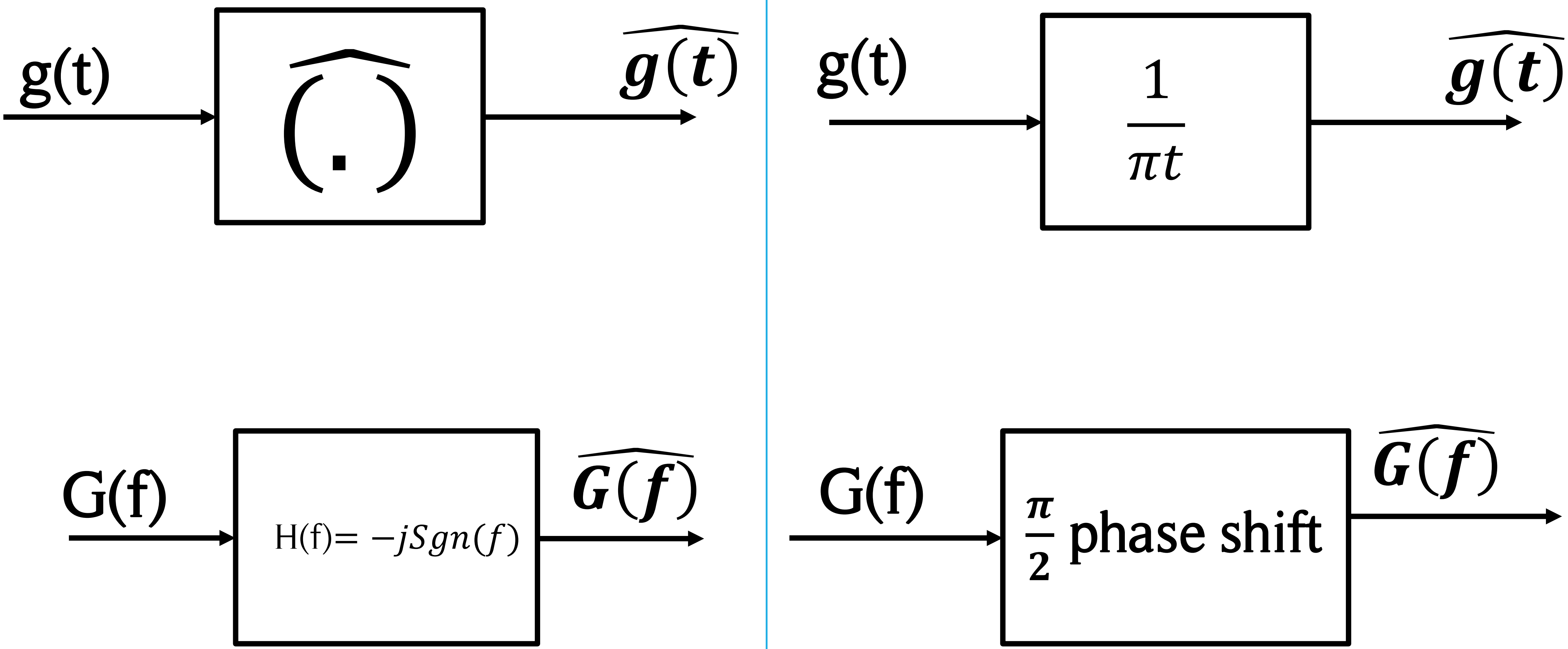
$\hat{G}(f) = G(f)[-jsgn(f)] = -jsgn(f)G(f)$

$$\mathrm{sgn}(x) = \begin{cases} 1, x > 0 \\ 0, x = 0 \\ -1, x < 0 \end{cases}$$



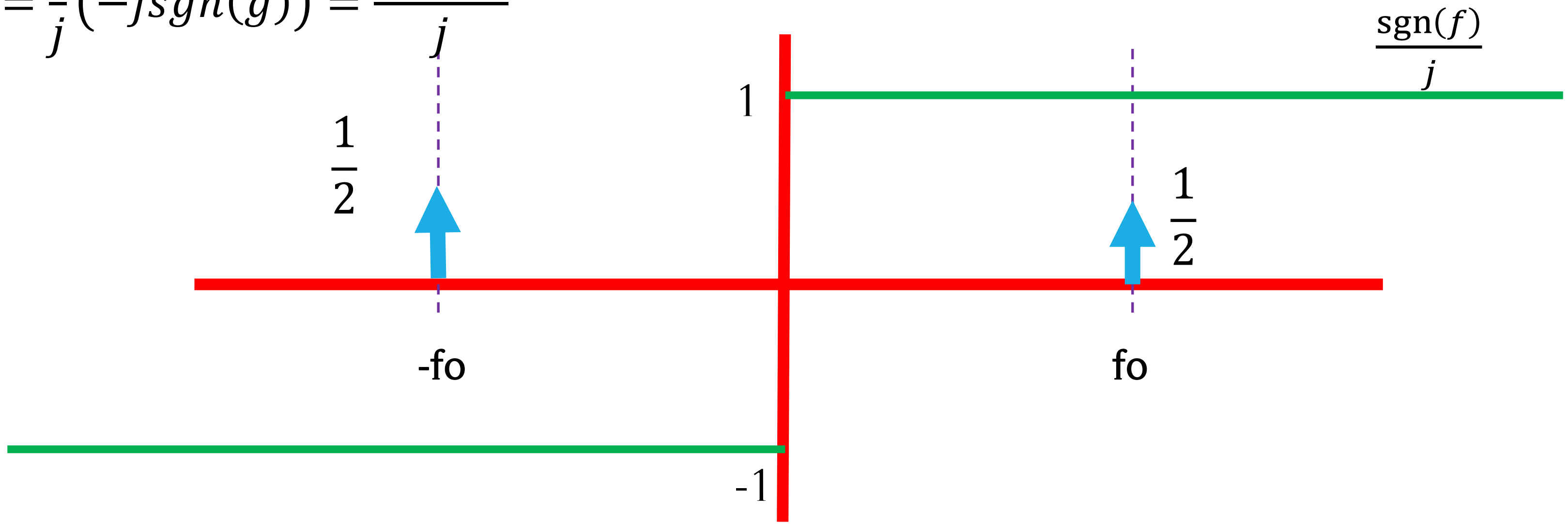
$$-\mathrm{jsgn}(f) = \begin{cases} -j, f > 0 \\ 0, f = 0 \\ j, f < 0 \end{cases}$$

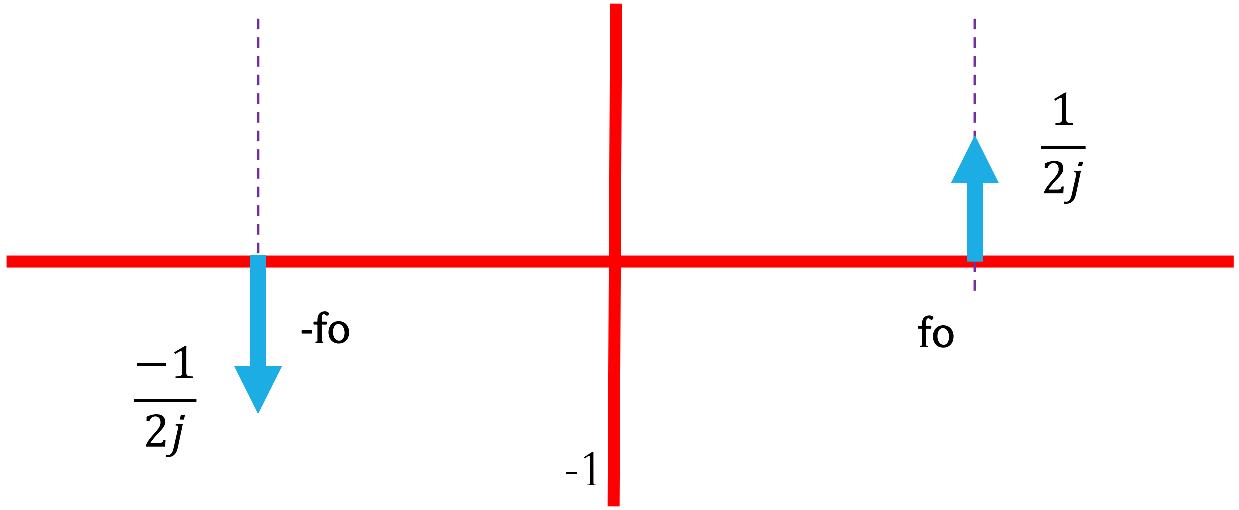
$$\frac{1}{\pi t} \ll\gg -jsgn(f)$$



Different representations of Hilbert transform

$$-jsgn(f) = \frac{j}{j}\left(-jsgn(g)\right) = \frac{\mathrm{sgn}(f)}{j}$$





Hilbert transform (Example)

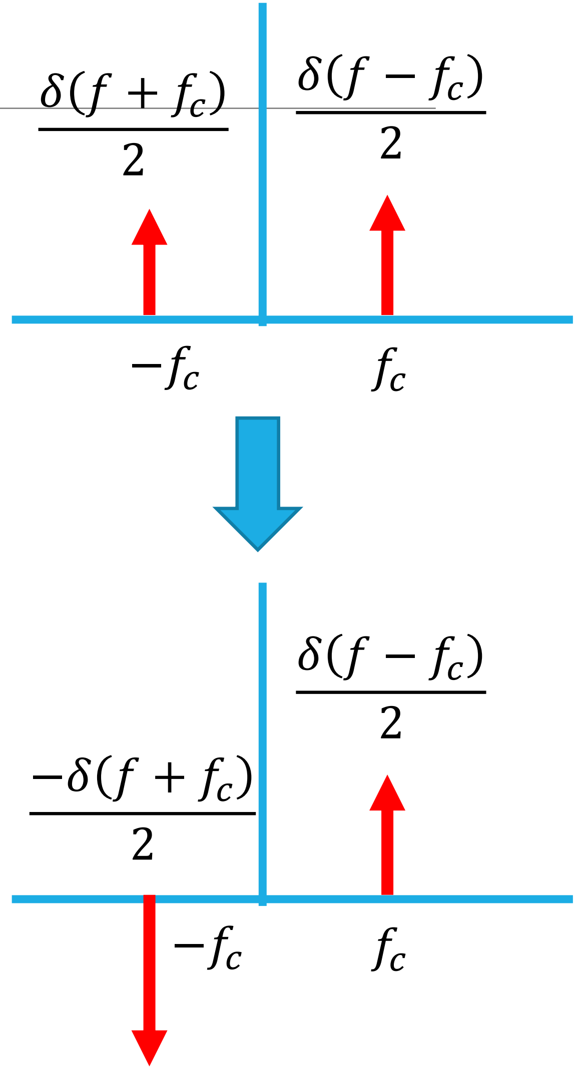
Ex: For $g(t) = \cos(2\pi f_c t)$, find $\hat{g}(t)$

$$G(f) = \frac{1}{2}[\delta(f - f_c) + \delta(f + f_c)]$$

$$\hat{G}(f) = \frac{-jsgn(f)}{2}[\delta(f - f_c) + \delta(f + f_c)] = -\mathrm{jsgn(f)G(f)}$$

$$\hat{G}(f) = \frac{-j}{2}[\delta(f - f_c) - \delta(f + f_c)] = \frac{1}{2j}[\delta(f - f_c) - \delta(f + f_c)]$$

➔ $\hat{g}(t) = \sin(2\pi f_c t)$



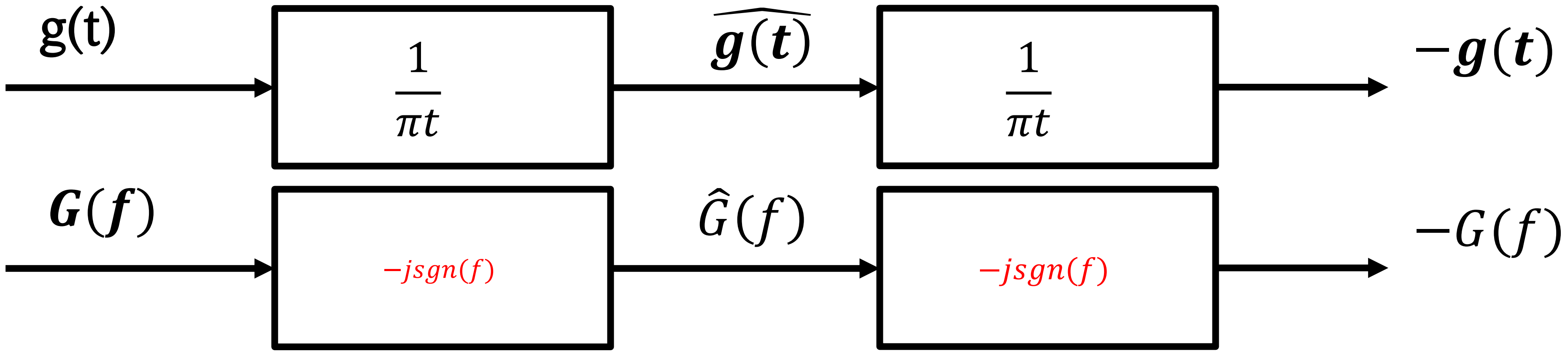
Hilbert transform properties

1- $g(t)$ *and* $\hat{g}(t)$ have the same amplitude spectrum:

$$|-jsgn(f)G(f)| = |G(f)|$$

$$|\hat{G}(f)| = |G(f)|$$

2- The Hilbert transform of $\hat{g}(t)$ is $-g(t)$, provided that $G(0) = 0$:



$$-g(t) = g(t) * \frac{1}{\pi t} * \frac{1}{\pi t} \Rightarrow G(f) = G(f)(-\mathrm{jsgn(f)})(-\mathrm{jsgn(f)})$$

Hilbert transform properties

3- A signal $g(t)$ and its Hilbert transform $\hat{g}(t)$ are orthogonal:

$$\int_{-\infty}^{\infty} g(t)\ \hat{g}(t)dt = 0$$

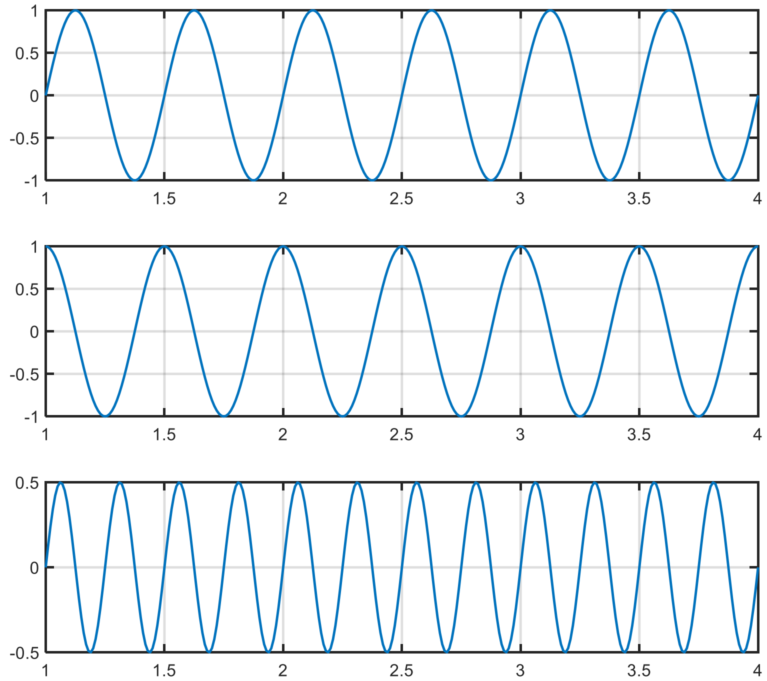
Ex: $\int_{-\infty}^{\infty} \sin(2\pi f_c t)\cos(2\pi f_c t)dt = 0$ ➔

$$\sin\alpha + \sin\beta = 2\sin\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\cos\alpha + \cos\beta = 2\cos\frac{\alpha+\beta}{2}\cos\frac{\alpha-\beta}{2}$$

$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos\alpha - \cos\beta = -2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$



Pre-envelop (Analytic signal)

Useful link

An analytic signal is a complex signal created by taking a signal and then adding in quadrature its Hilbert Transform. It is also called the pre-envelope of the real signal

$g_+(t) = g(t) + j\hat{g}(t)$: pre-envelop for positive frequency

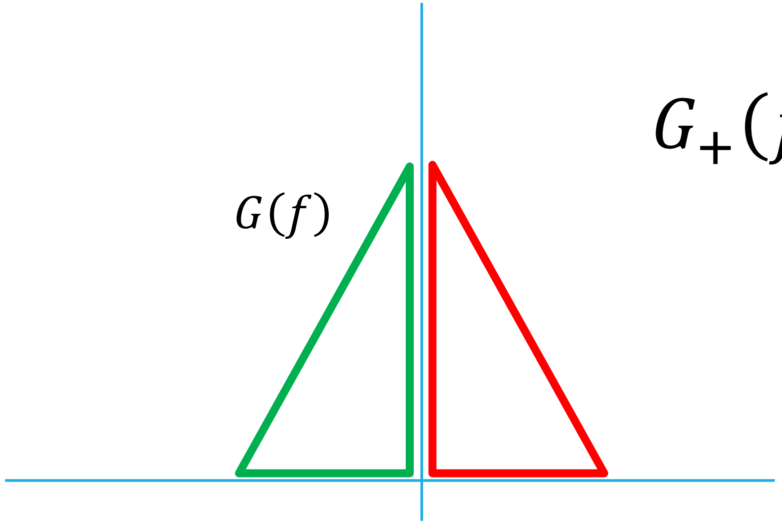
$g_-(t) = g(t) - j\hat{g}(t)$: pre-envelop for negative frequency

In frequency domain:

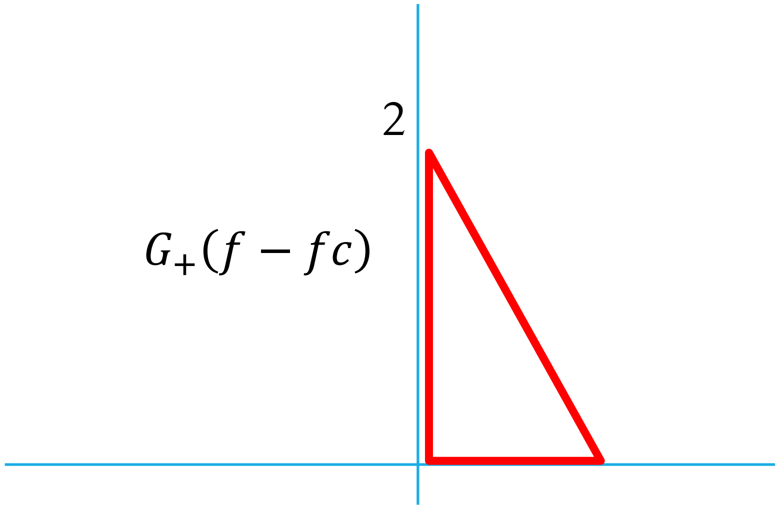
$$G_+(f) = \mathrm{G(f)} + \mathrm{j[-jsgn(f)G(f)]} = \begin{cases} 2G(f), f > 0 \\ 0, f \leq 0 \end{cases}$$: pre-envelop for positive frequency

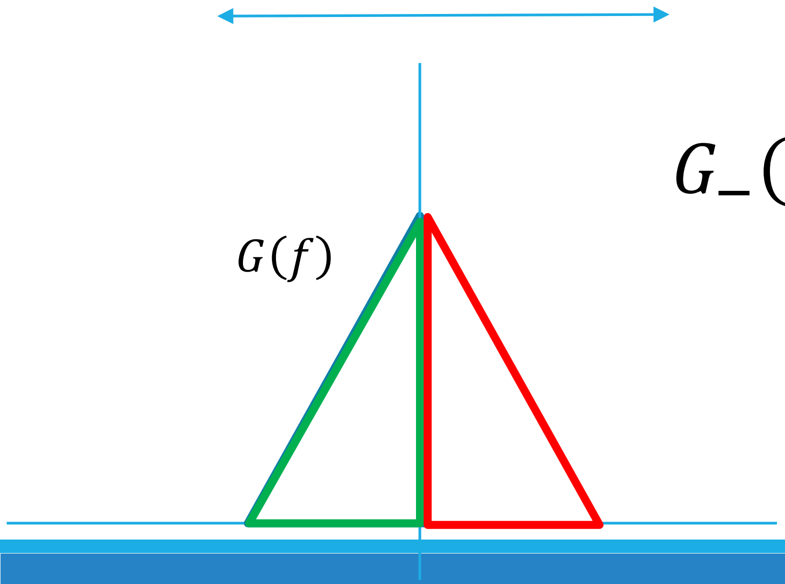
$$G_-(f) = \mathrm{G(f)} - \mathrm{j[-jsgn(f)G(f)]} = \begin{cases} 0, f \geq 0 \\ 2G(f), f < 0 \end{cases}$$: pre-envelop for negative frequency

$$g_+(t) = g(t) + j\hat{g}(t)$$

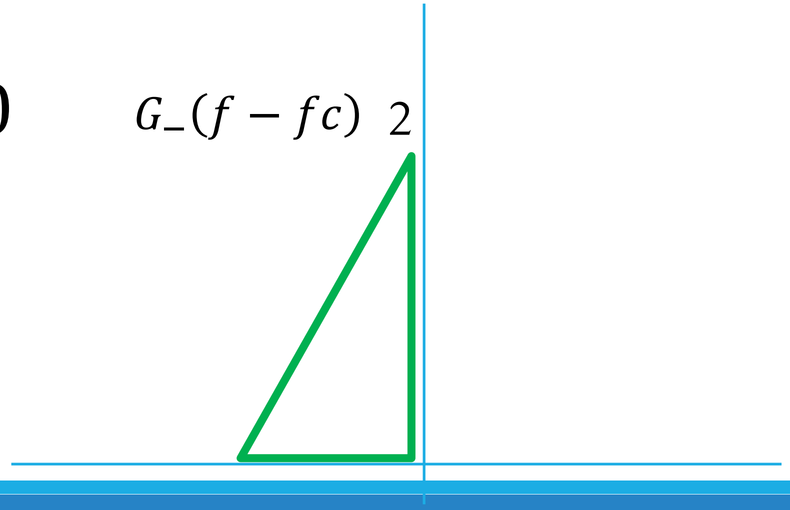


$$G_+(f) = 2G(f), \qquad f > 0$$

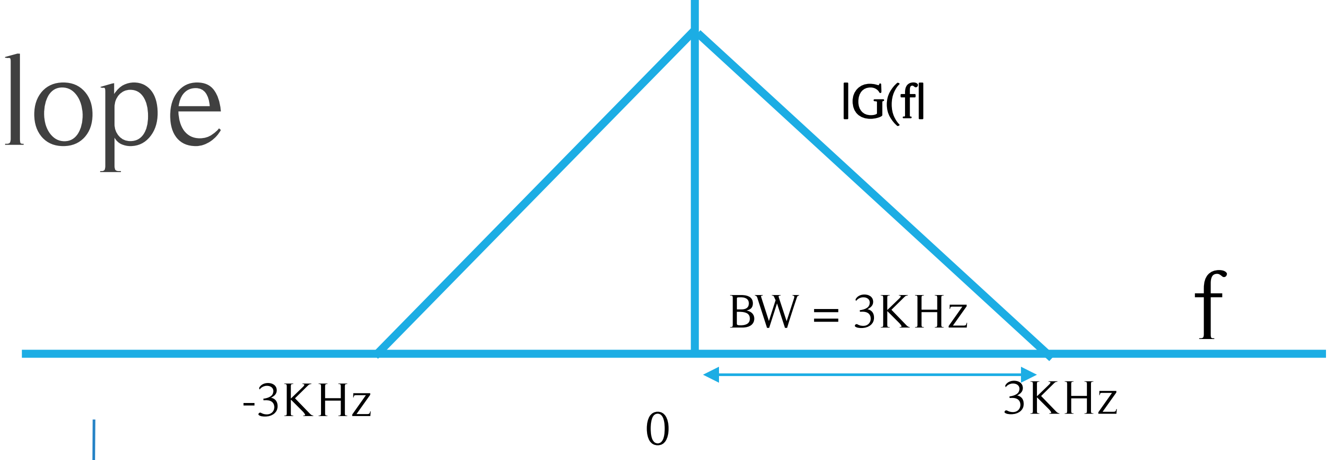


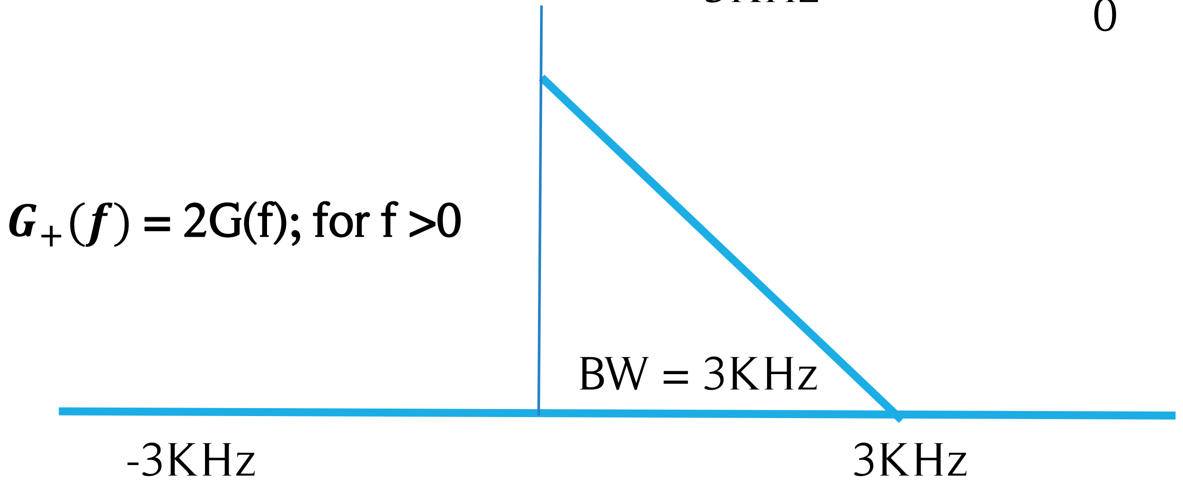


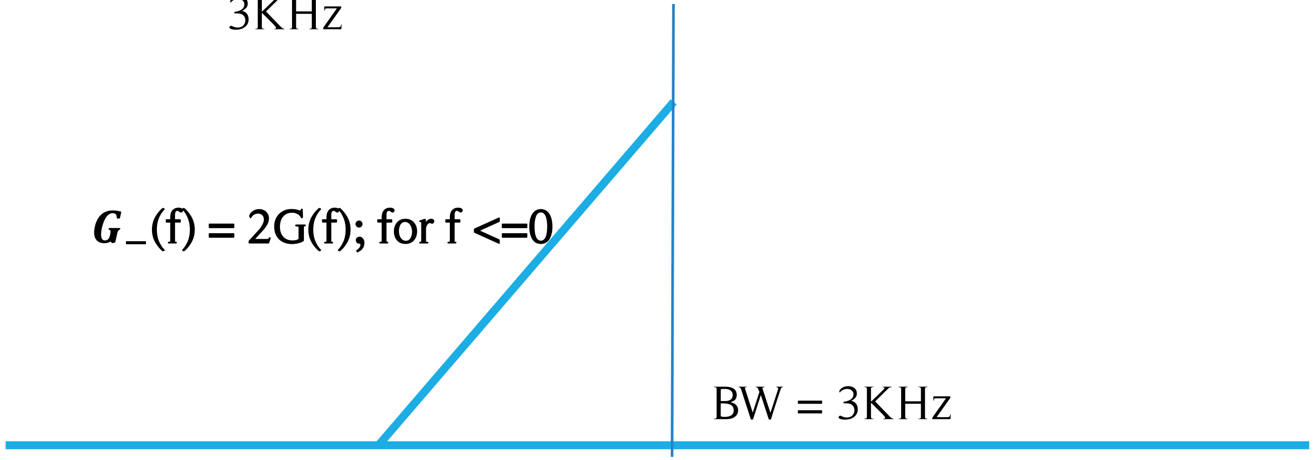
$$G_-(f) = 2G(f), \qquad f < 0$$

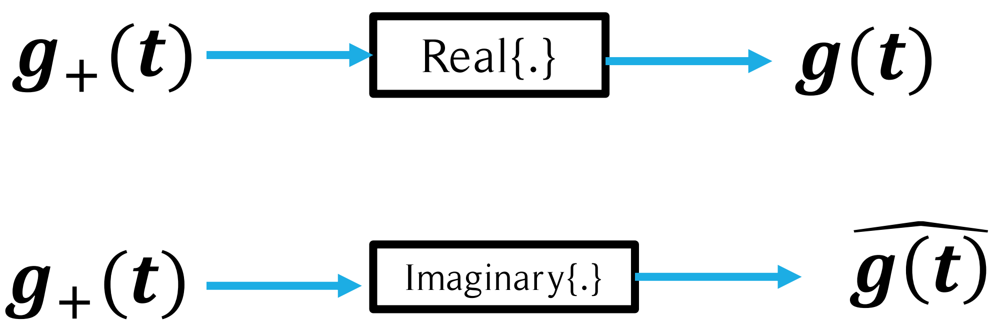


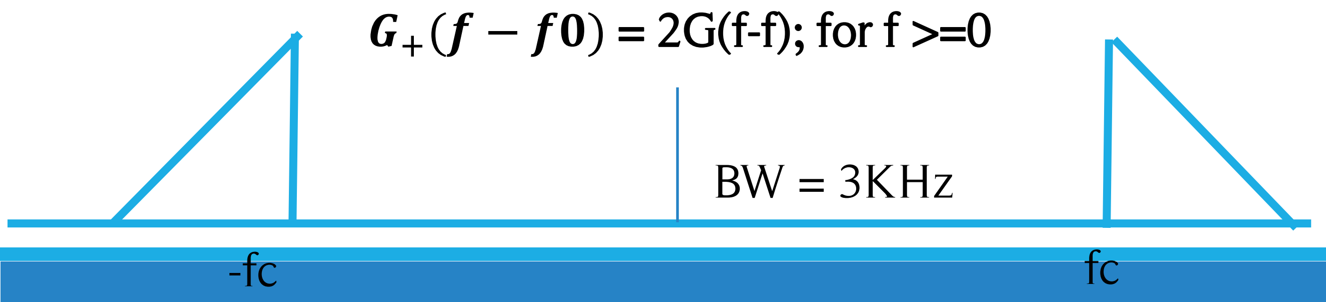
Pre envelope





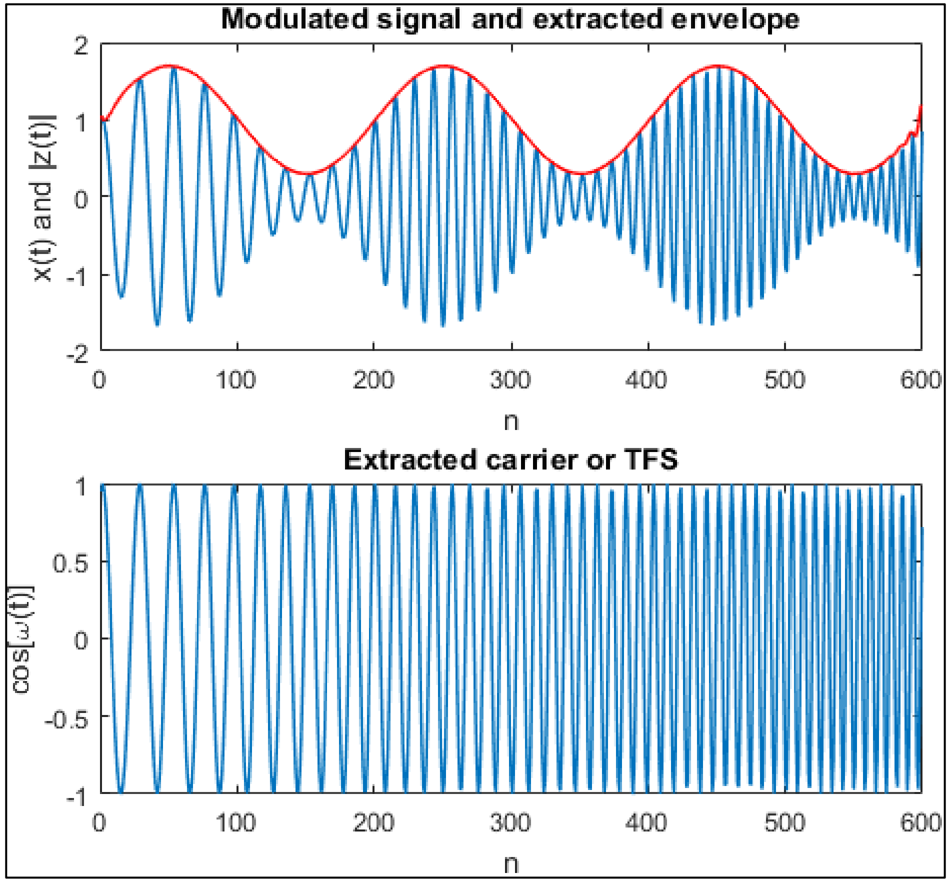






MATLAB code for envelope detection

```
fs = 600; %sampling frequency in Hz
t = 0:1/fs:1-1/fs; %time base
a_t = 1.0 + 0.7 * sin(2.0*pi*3.0*t) ; %information signal
c_t = chirp(t,20,t(end),80); %chirp carrier
x = a_t .* c_t; %modulated signal

subplot(2,1,1); plot(x);hold on; %plot the modulated signal

z = hilbert(x); %form the analytical signal
inst_amplitude = abs(z); %envelope extraction
inst_phase = unwrap(angle(z));%inst phase
inst_freq = diff(inst_phase)/(2*pi)*fs;%inst frequency

%Regenerate the carrier from the instantaneous phase
regenerated_carrier = cos(inst_phase);

plot(inst_amplitude,'r'); %overlay the extracted envelope
title('Modulated signal and extracted envelope'); xlabel('n');
ylabel('x(t) and |z(t)|');
subplot(2,1,2); plot(cos(inst_phase));
title('Extracted carrier or TFS'); xlabel('n');
ylabel('cos[\omega(t)]');
```



Representation of Band-Pass signal

Let g(t) be a narrow band signal with G(f) is its FT, then the positive pre-envelope of g(t) can be expressed as:

$$g_+(t) = \tilde{g}(t)e^{j2\pi f_c t}$$

$\tilde{g}(t)$: complex envelope, (**Read as: g Tilda of t**)

$G_+(f) = \tilde{G}(f - f_c)$

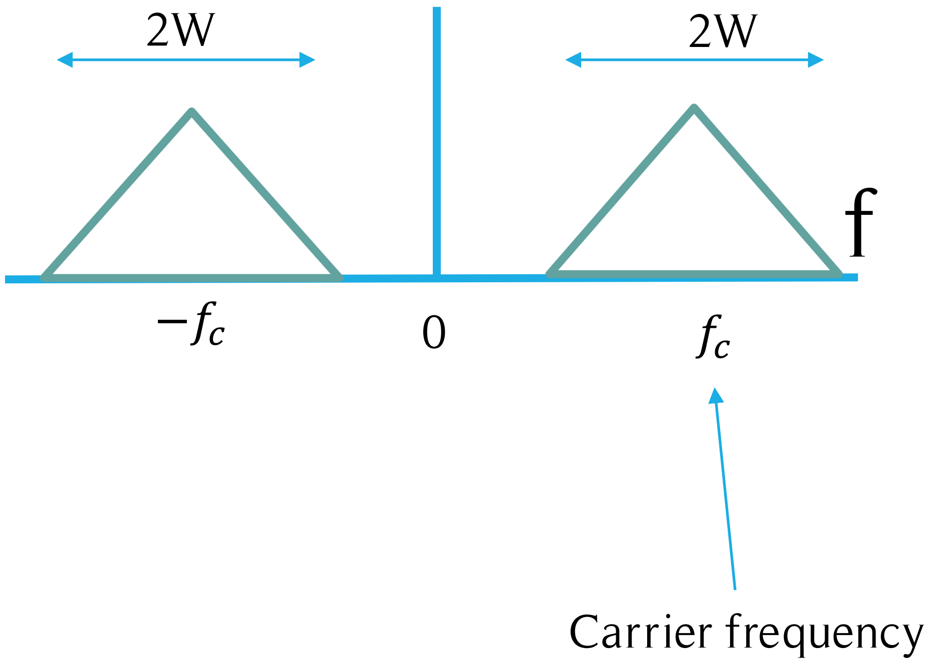
g(t) = Re$\{g_+(t)\}$= Re$\{\tilde{g}(t)e^{j2\pi f_c t}\}$

g(t) = Re$\{g_+(t)\}$= Re$\{\tilde{g}(t)[\cos(2\pi f_c t) + \mathrm{j}\sin(2\pi f_c t)]\}$

$\tilde{g}(t) = \mathrm{g}_I(t) + j g_Q(t)$

$\mathrm{g}_I(t)$: in phase component

$\mathrm{g}_Q(t)$: quadrature component

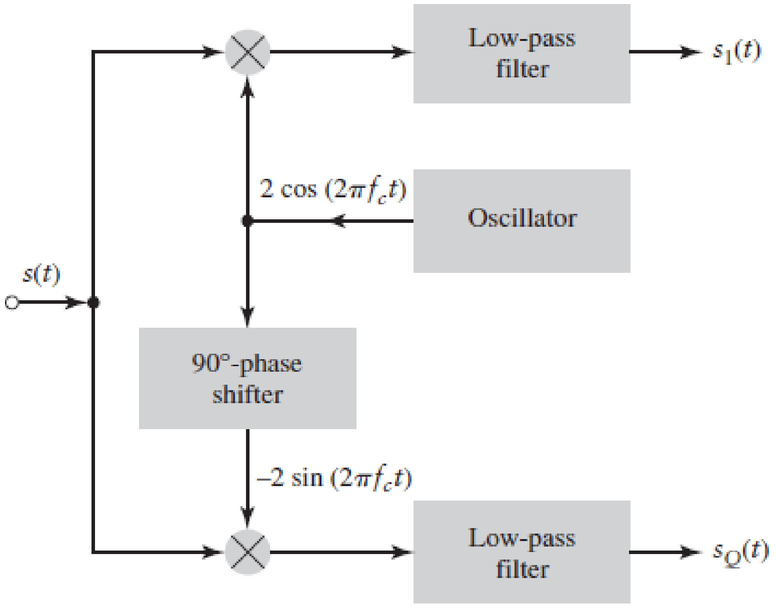


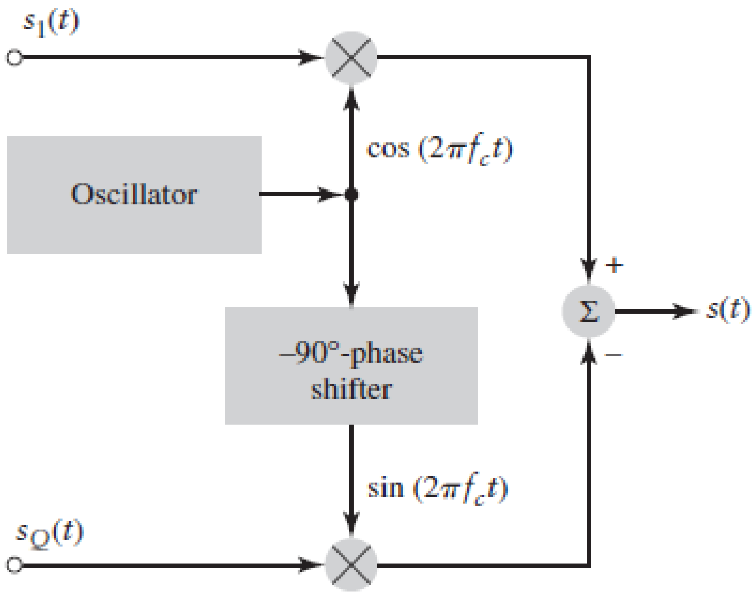
Representation of Band-Pass signal

$\tilde{g}(t) = g_I(t) + jg_Q(t)$

$g(t) = Re\{[g_I(t) + jg_Q(t)].[\cos(2\pi f_c t) + j\sin(2\pi f_c t)]\}$

$g(t) = g_I(t)\cos(2\pi f_c t) - g_Q(t)\sin(2\pi f_c t)$





Hybrid form of amplitude and angle modulation

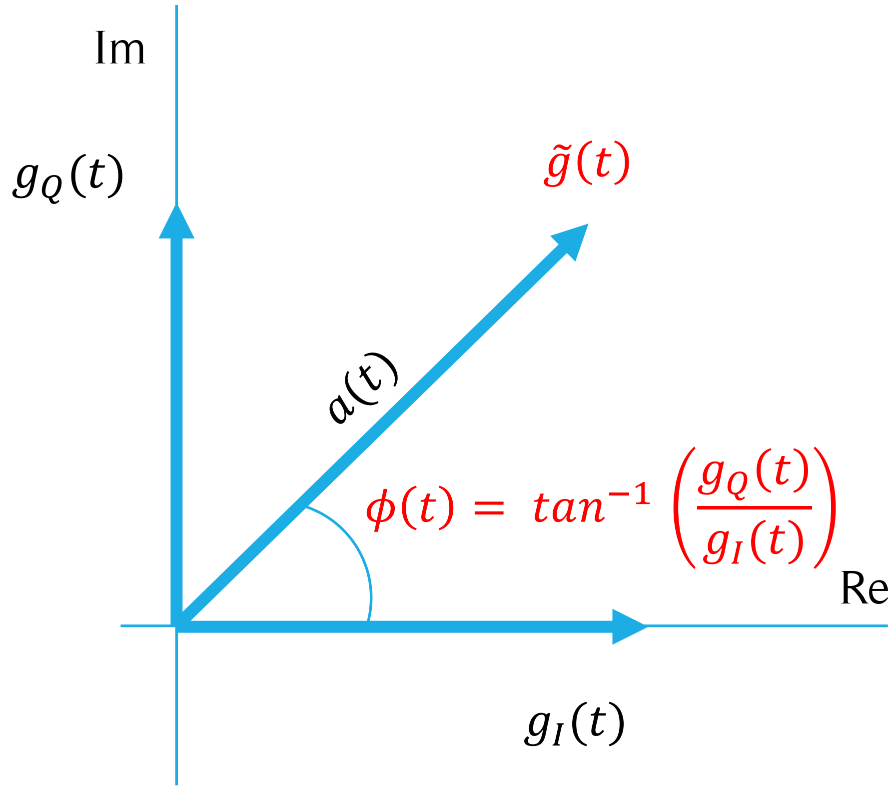
$\tilde{g}(t) = g_I(t) + jg_Q(t)$

$\tilde{g}(t) = \sqrt{g_I(t)^2 + g_Q(t)^2}\, e^{j\tan^{-1}\left(\frac{g_Q(t)}{g_I(t)}\right)}$

$a(t) = \sqrt{g_I(t)^2 + g_Q(t)^2}$, natural envelope

$\phi(t) = \tan^{-1}\left(\frac{g_Q(t)}{g_I(t)}\right)$, phase of g(t)

$\tilde{g}(t) = a(t)e^{j\phi(t)}$ } Real valued low pass signal



$g(t) = Re\{a(t)e^{j\phi(t)}e^{j2\pi f_c t}\}$

$\boldsymbol{g(t) = a(t)\cos(2\pi f_c t + \phi(t))}$

Continuous wave modulation

Carrier➔ sinusoidal signal $c(t)$

Base band signal (information signal)➔modulation wave $m(t)$

Results ➔ modulated wave s(t)

Amplitude modulation (FULL AM)

$c(t) = A_c \cos(2\pi f_c t)$

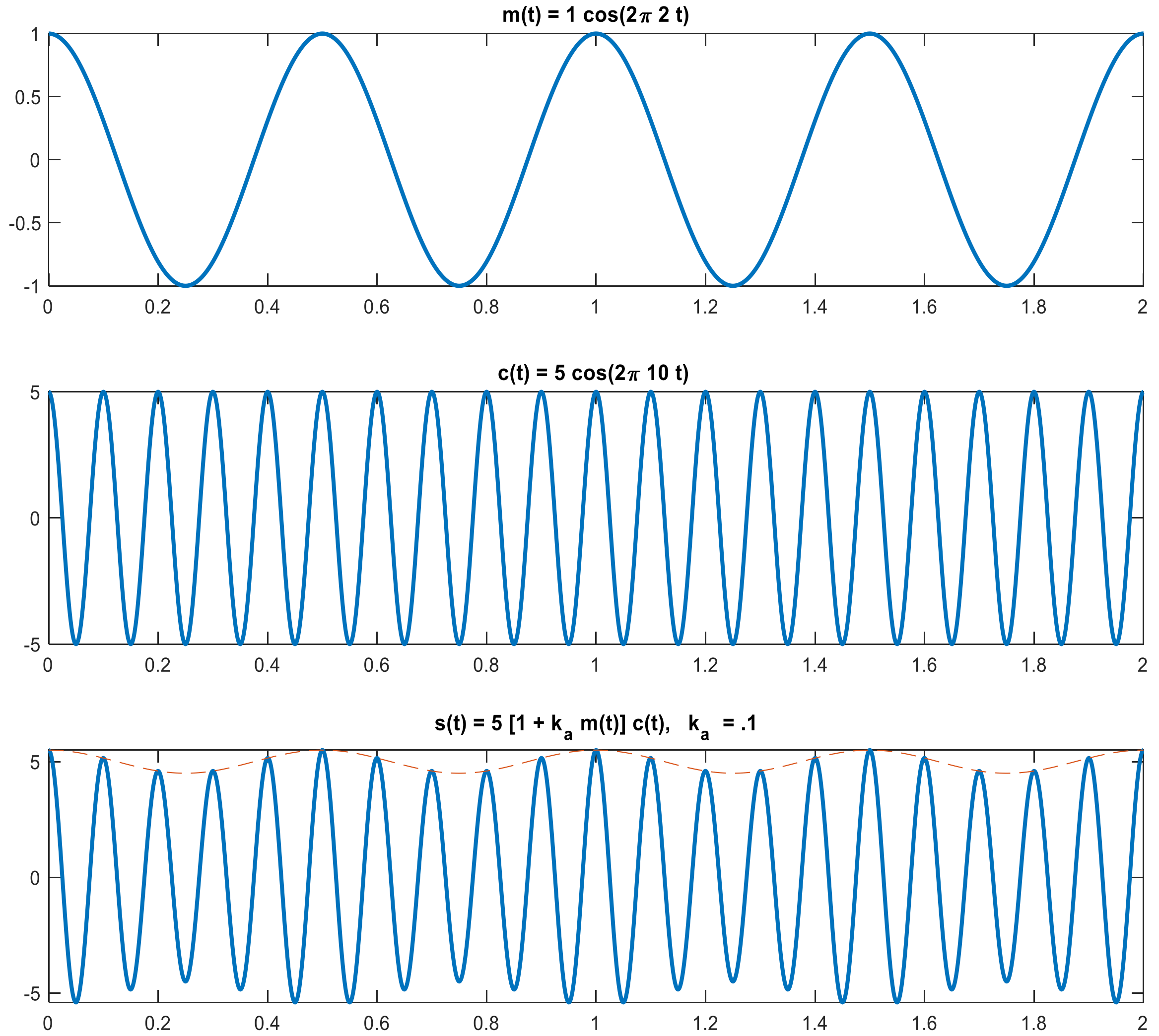
A_C: Amplitude

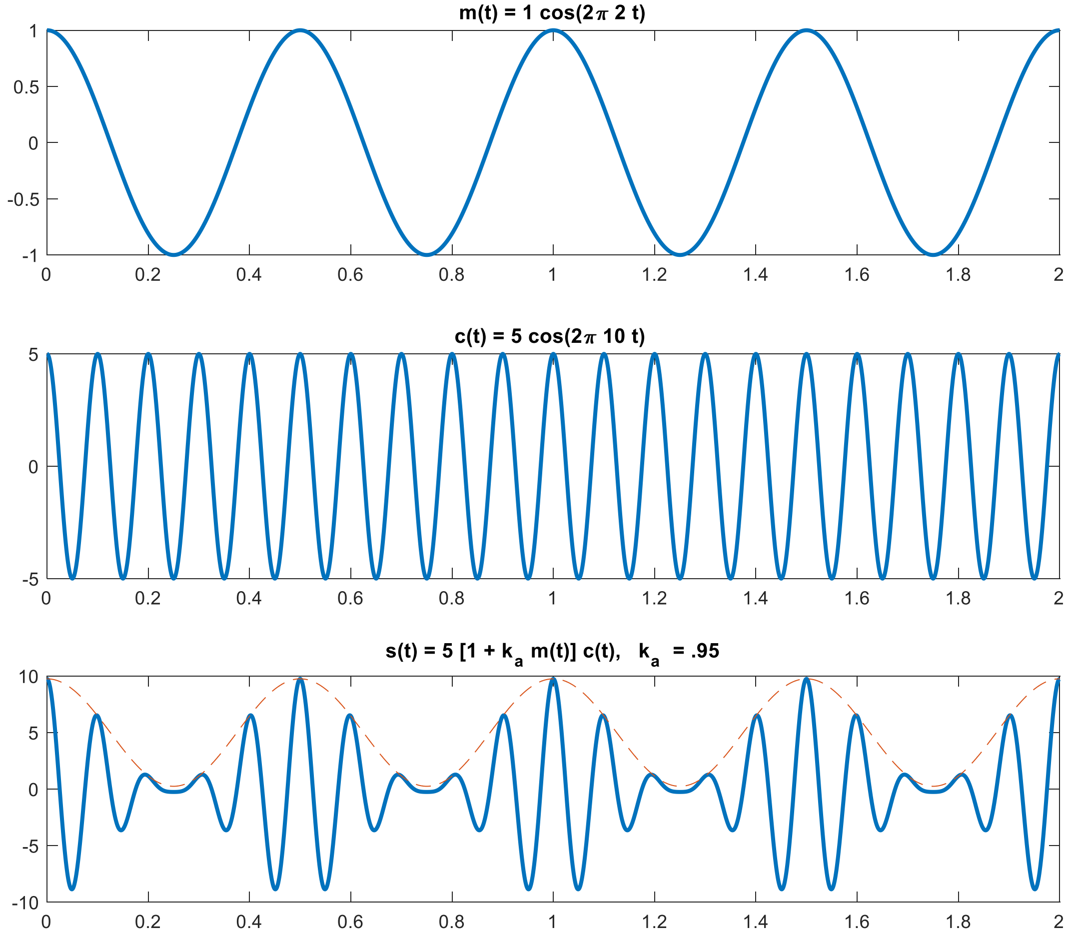
f_c: $carrier\ frequency$

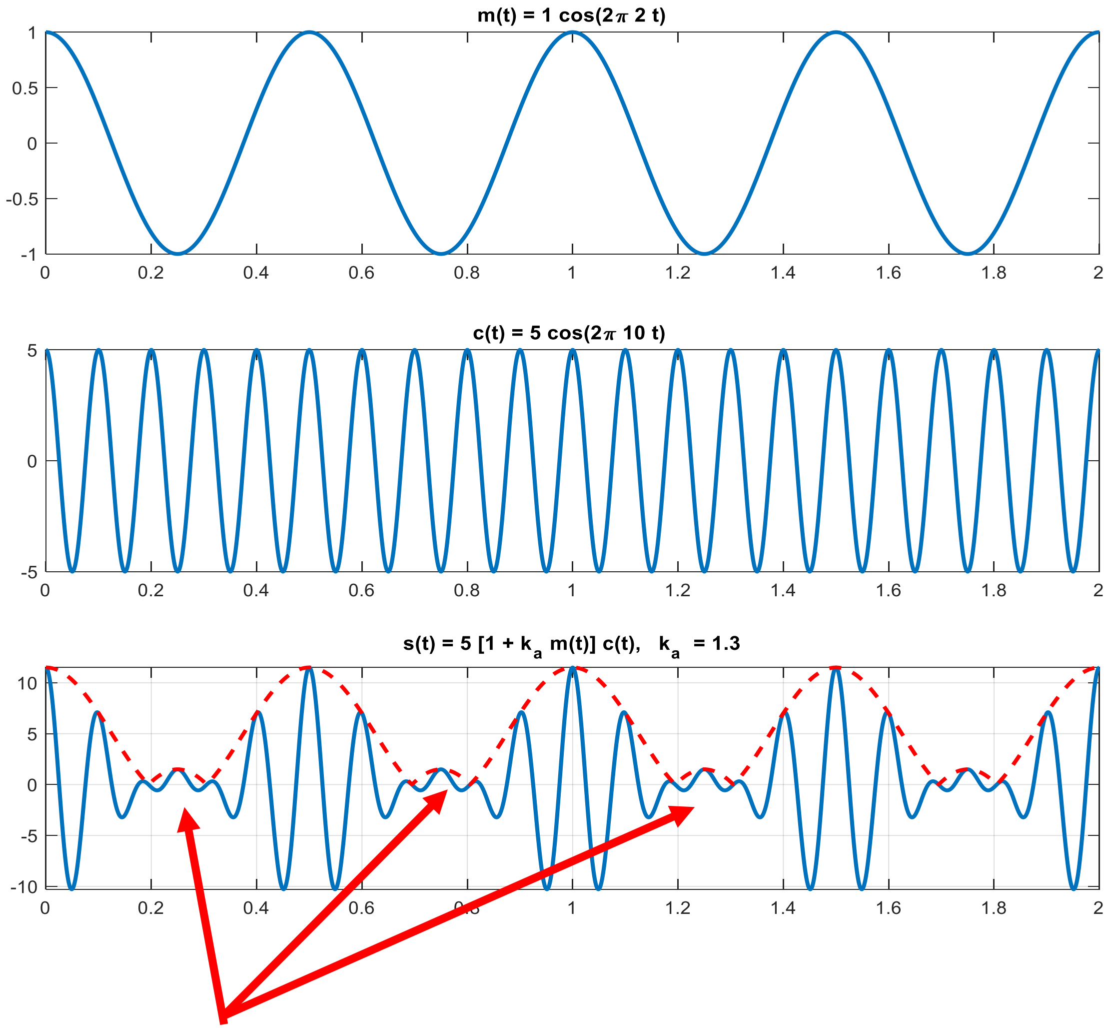
$$\mathbf{s(t) = A_c \cos(2\pi f_c t)\,[1 + k_a m(t)],\ \ |k_a m(t)| \leq 1,\ f_c \gg 2W}$$

$\boldsymbol{k_a}$: a constant called amplitude sensitivity of the modulator

```
%% Amplitude modulation with varying K_a

t = 0:.001:2;

n = 0;
ct = 5*cos(2*pi*10*t);
mt = cos(2*pi*2*t);
ka = 1.3;
st = (1 + ka.*mt).*ct;

subplot(3,1,1)
plot(t,mt,"LineWidth",2)
title("m(t) = 1 cos(2\pi 2 t)")
subplot(3,1,2)
plot(t,ct,"LineWidth",2)
title("c(t) = 5 cos(2\pi 10 t)  ")
subplot(3,1,3)
plot(t,st+ n,"LineWidth",2)
title("s(t) = 5 [1 + k_a m(t)] c(t),   k_a  = 1.3")
hold on
plot(t,abs(5*(1 + ka.*mt)),"--r","LineWidth",2)

grid on
```

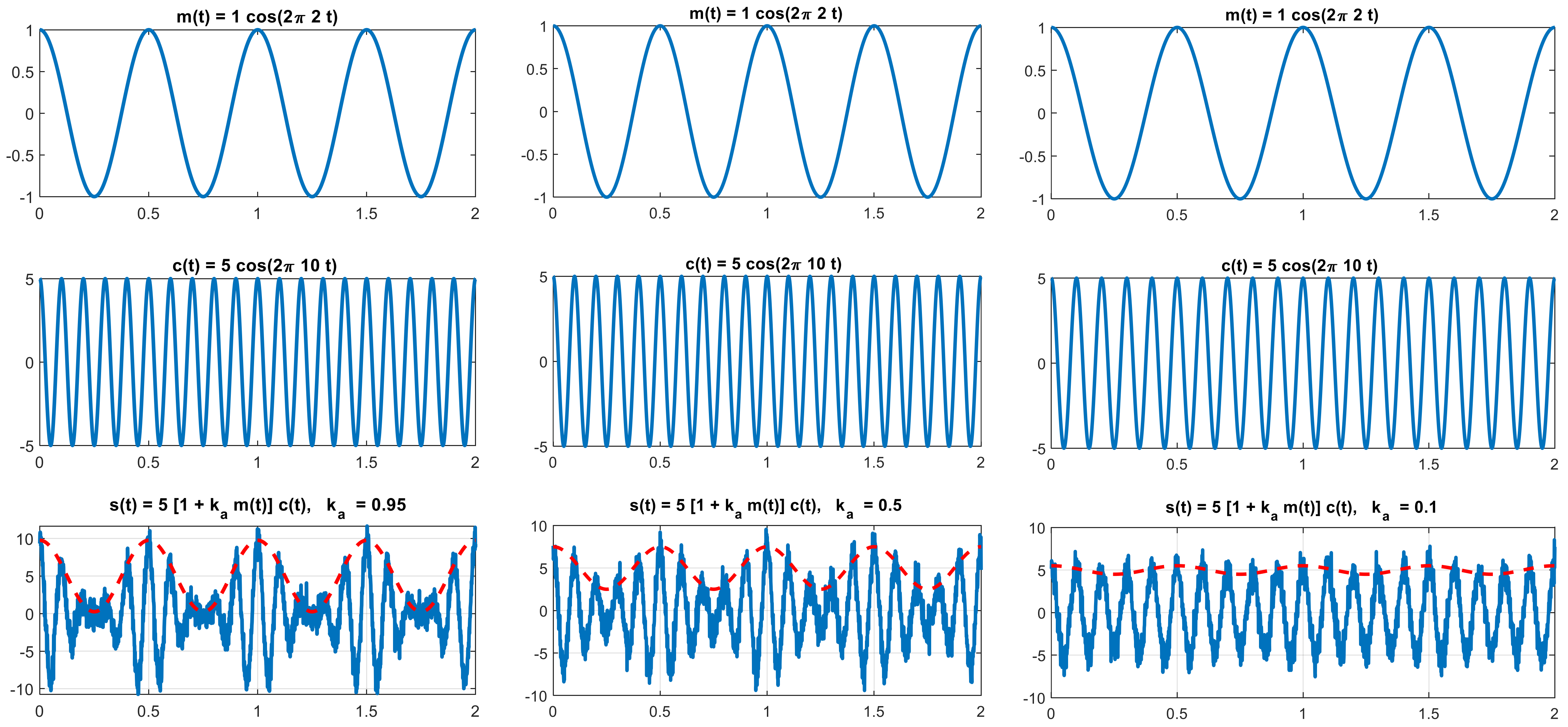


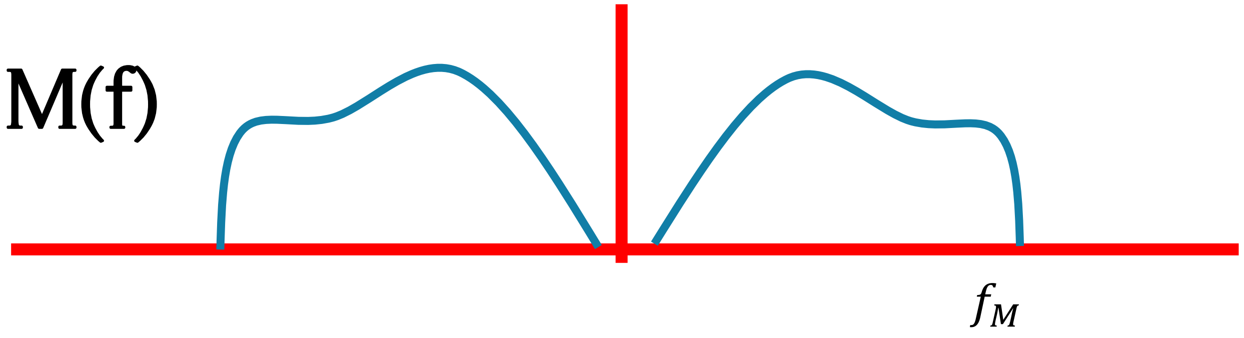


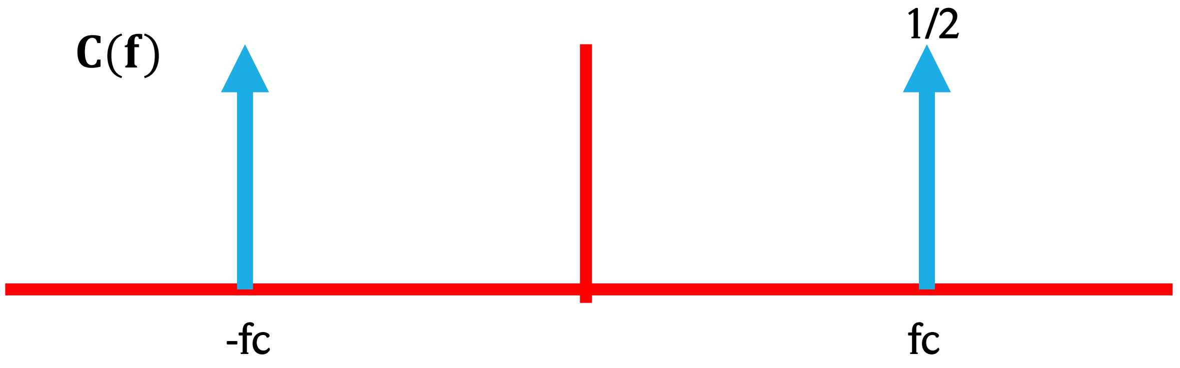


Phase reversal or envelope distortion

Noise effect on AM vs. amplitude sensitivity k_a



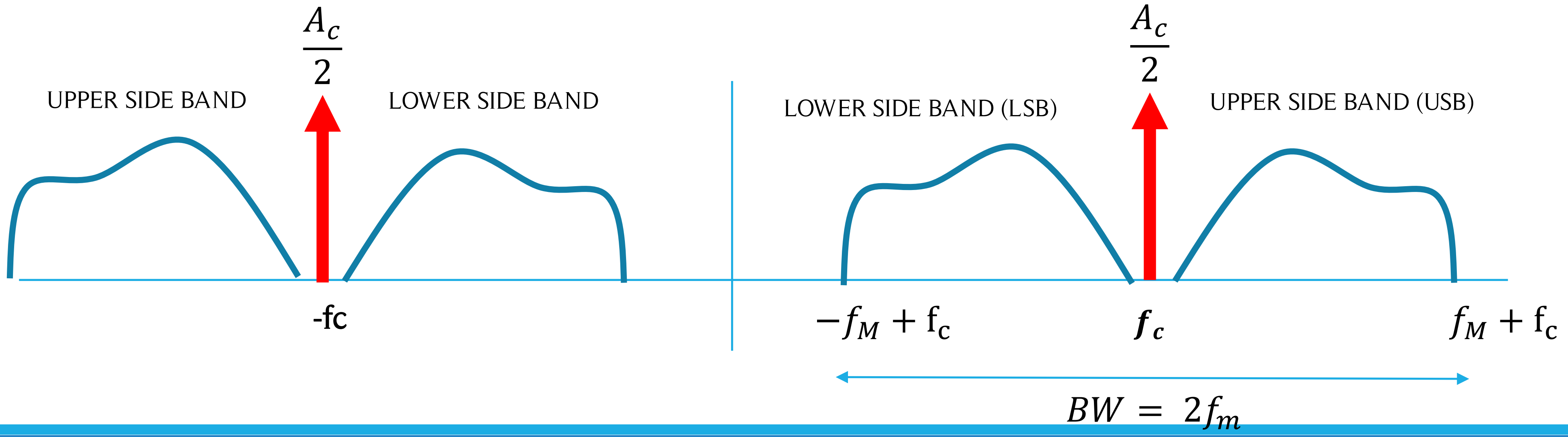




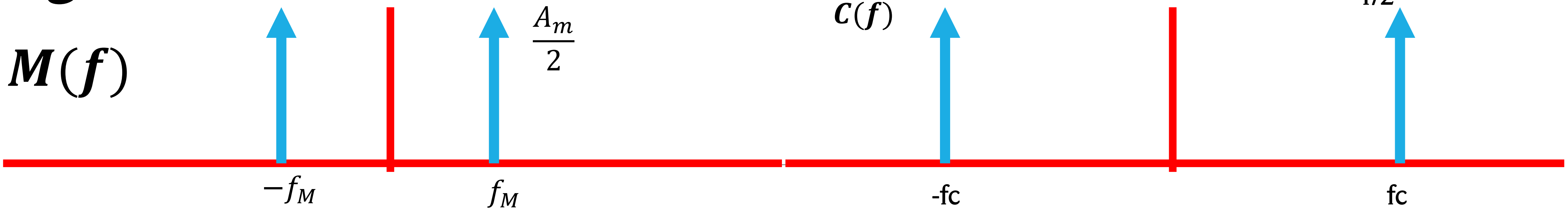
$$s(t) = A_c \cos(2\pi f_c t)\,[1 + k_a m(t)]$$

$$s(t) = A_c \cos(2\pi f_c t) + A_c k_a m(t) \cos(2\pi f_c t) = A_c \cos(2\pi f_c t) + A_c k_a m(t)\left(\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}\right)$$

$$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c k_a}{2}[M(f - f_c) + M(f + f_c)]$$



Single tone modulation



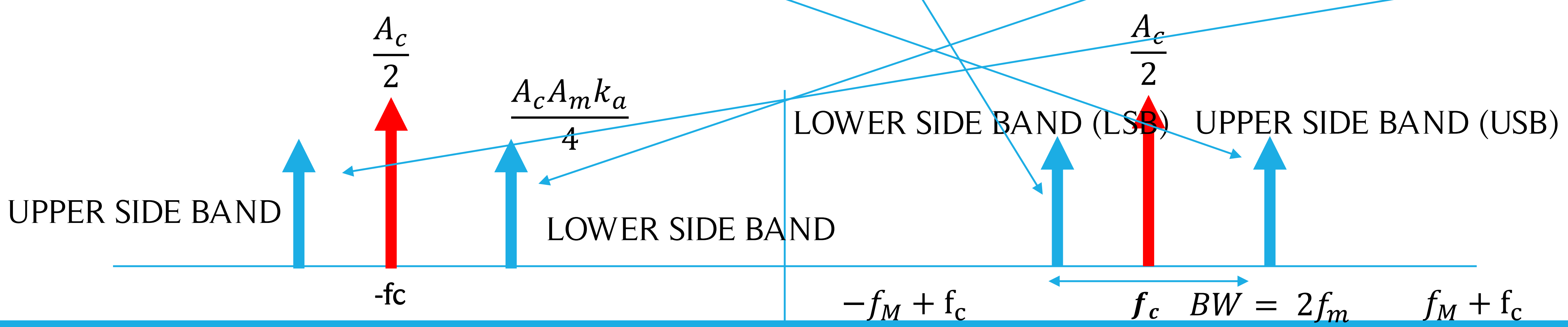
$m(t) = A_m \cos(2\pi f_m t)$

$\boldsymbol{\mu = A_m k_a}$ = *modulation factor* (index) = *percentage modulation < 100%*

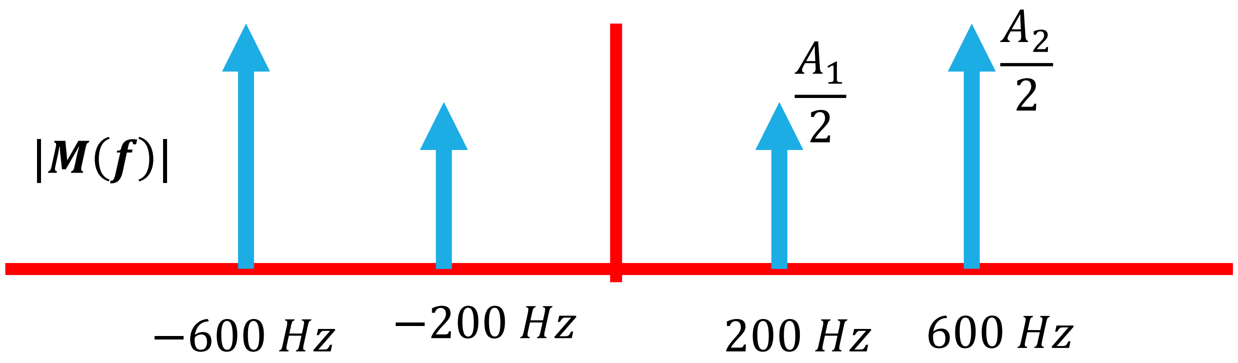
$s(t) = A_c \cos(2\pi f_c t)\,[1 + k_a m(t)]$

$s(t) = A_c \cos(2\pi f_c t) + A_c \mu \cos(2\pi f_m t) \cos(2\pi f_c t)$

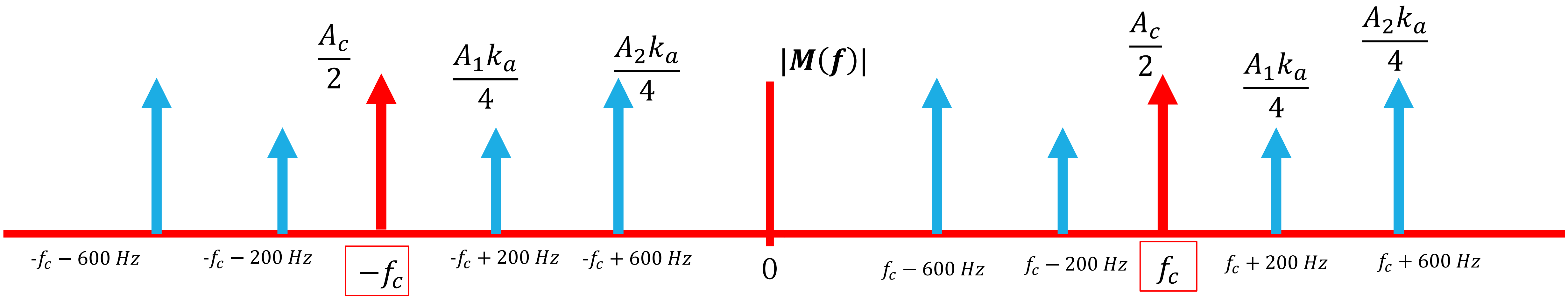
$S(f) = \frac{A_c}{2}[\delta(f - f_c) + \delta(f + f_c)] + \frac{A_c k_a}{4}[\delta(f - f_c - f_m) + \delta(f - f_c + f_m) + \delta(f + f_c - f_m) + \delta(f + f_c + f_m)]$



EX: $m(t) = A_1 \cos(2\pi 200t) + A_2 \cos(2\pi 600t)$, plot the amplitude spectrum of **s(t)** for FULL-AM



$$s(t) = A_c \cos(2\pi f_c t)\,[1 + k_a m(t)] = A_c \cos(2\pi f_c t)\,[1 + k_a[A_1 \cos(2\pi 200t) + A_2 \cos(2\pi 600t)]]$$



Power analysis of AM

Carrier power = $\frac{A_C^2}{2}$

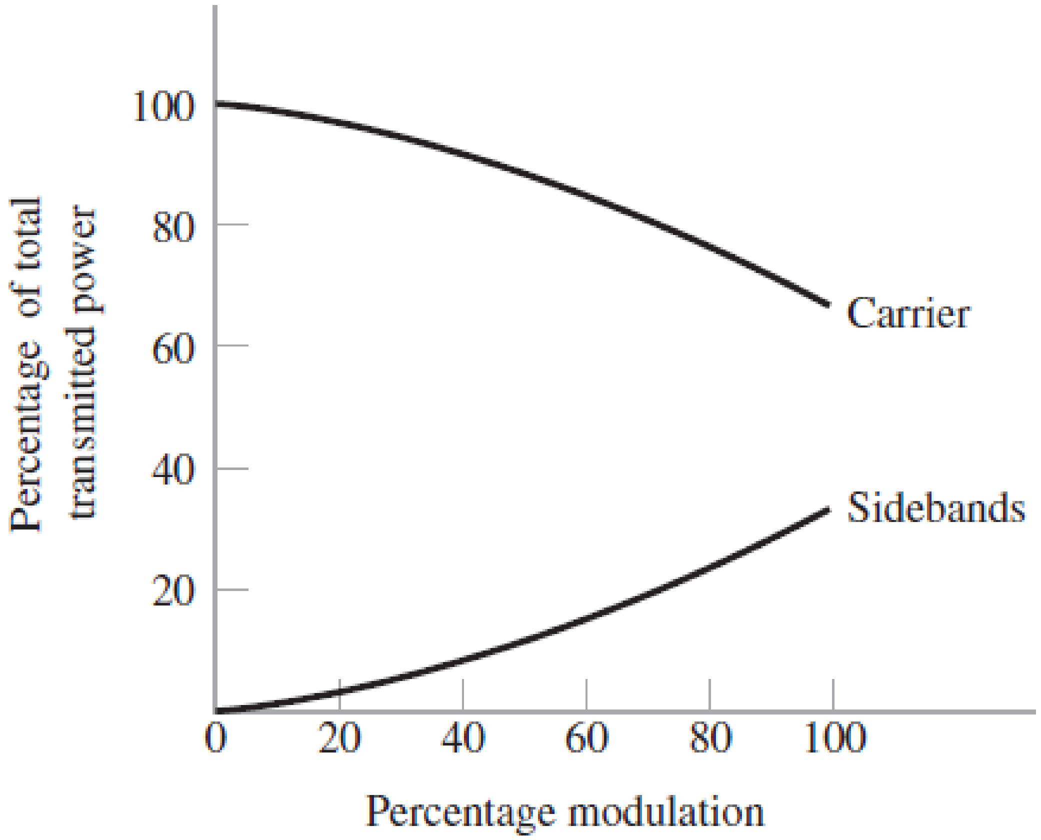
Upper side band power = $\frac{A_C^2 \mu^2}{8}$

Lower side band power = $\frac{A_C^2 \mu^2}{8}$

Power efficiency = $\boldsymbol{\eta}$

$$= \frac{\textit{total power in sidebands}}{\textit{total power of } s(t)} = \frac{\frac{A_C^2\mu^2}{4}}{\frac{A_C^2\mu^2}{4}+\frac{A_C^2}{2}}$$

$$= \frac{\mu^2}{\mu^2+2}$$



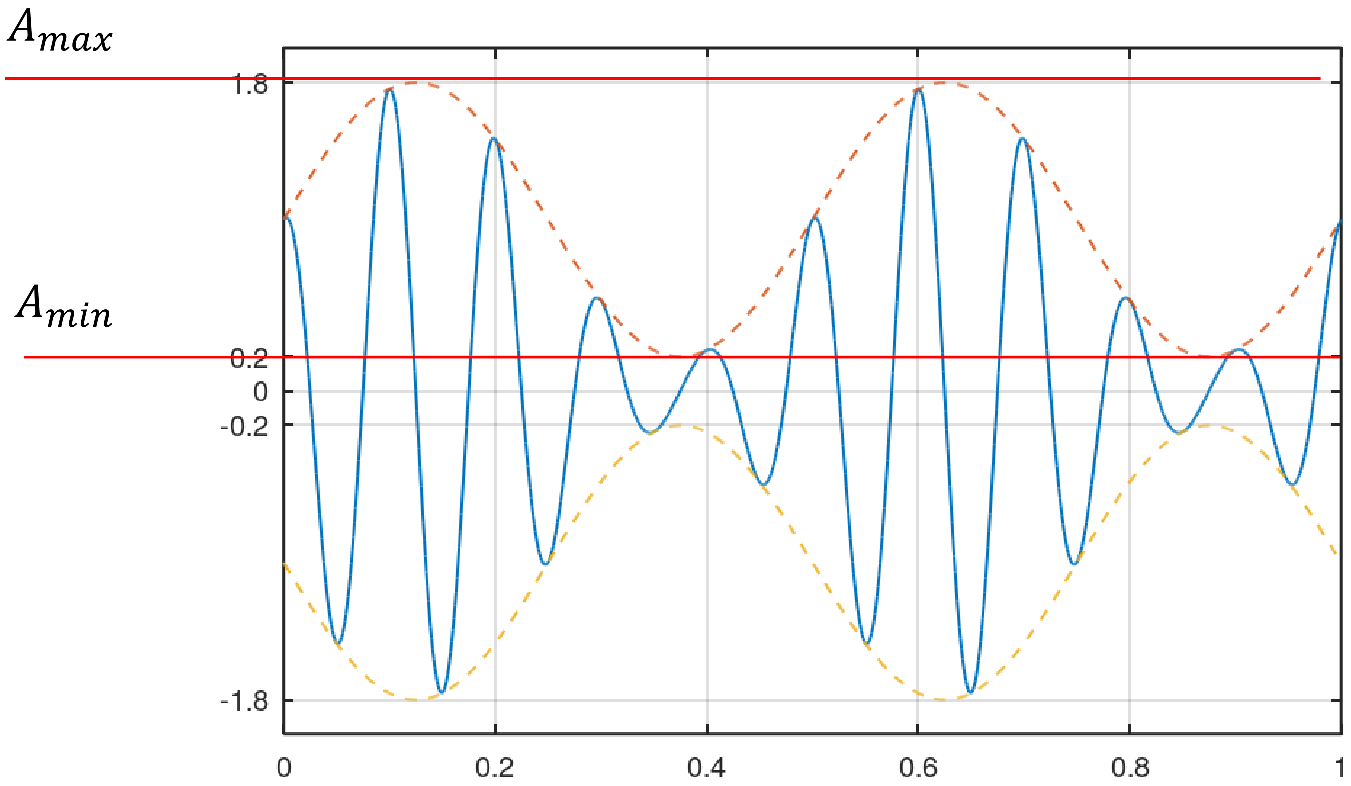
Percentage modulation

$$\mu = \frac{A_{max} - A_{min}}{A_{max} + A_{min}}$$

Ex: in the figure to the right, measure the percentage modulation.

$$\mu = \frac{1.8 - .2}{1.8 + .2} = \frac{1.6}{2} = 0.8 = 80\%$$

$$\eta = \frac{0.8^2}{0.8^2 + 2} = \frac{0.64}{2.64} = 24.24\ \%$$



$\boldsymbol{\mu = A_m k_a}$ **= *modulation factor = percentage modulation***

Switching modulator

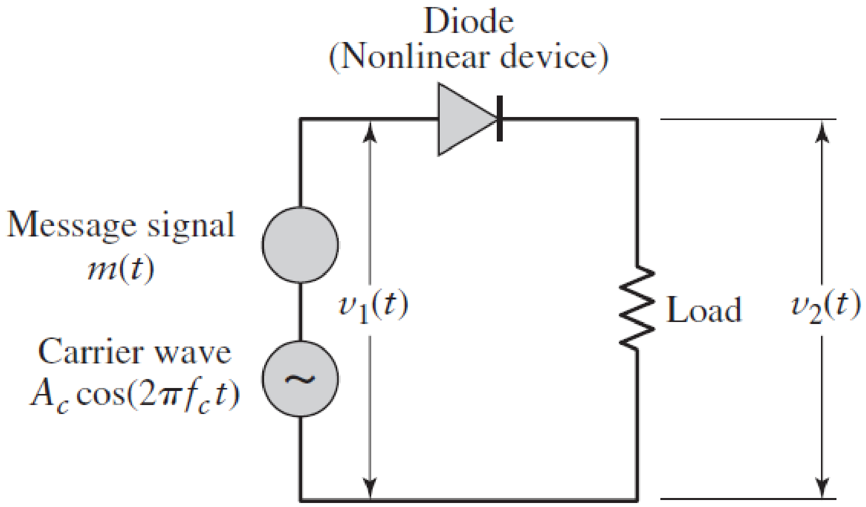
Switching modulator is a simple version of AM modulation techniques.

We assume the following for a proper switching diode circuit operation:

1- it assumed that the diode is ideal ($r_d = 0$ 🡐 forward bias, $r_d = \infty$ 🡐 reverse bias)

2- $c(t)$ is large in amplitude.

3- $m(t)$ is weak if compared to $c(t)$



$$v_1(t) = A_c \cos(2\pi f_c t) + m(t)$$

$$v_o = v_2(t) = \begin{cases} v_1(t), c(t) > 0 \\ 0, c(t) < 0 \end{cases}$$

$$v_o = [A_c \cos(2\pi f_c t) + m(t)] g_{T_o}(t)$$

Switching modulator

$$g_{T_o}(t) = square\ (f_c) = \frac{1}{2} + \frac{2}{\pi}\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}\cos(2\pi f_c t(2n-1))$$

The output $v_o(t)$ can be seen as two components:

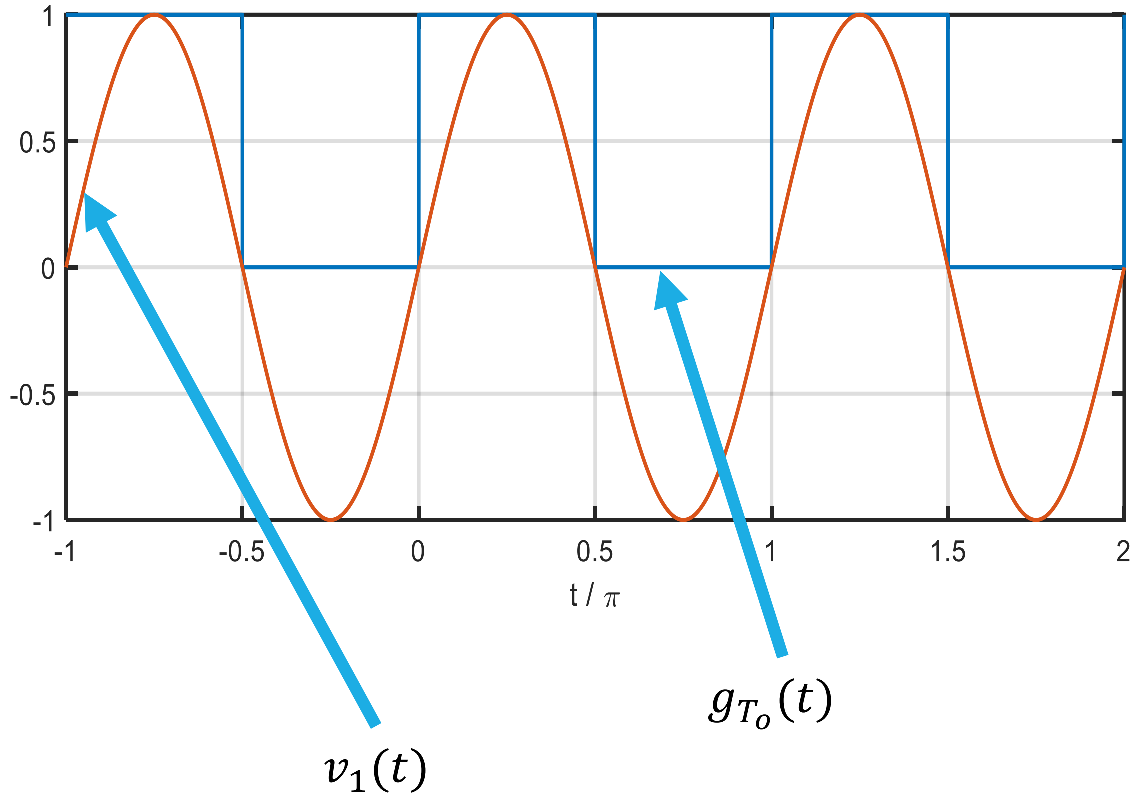
1- The wanted component:

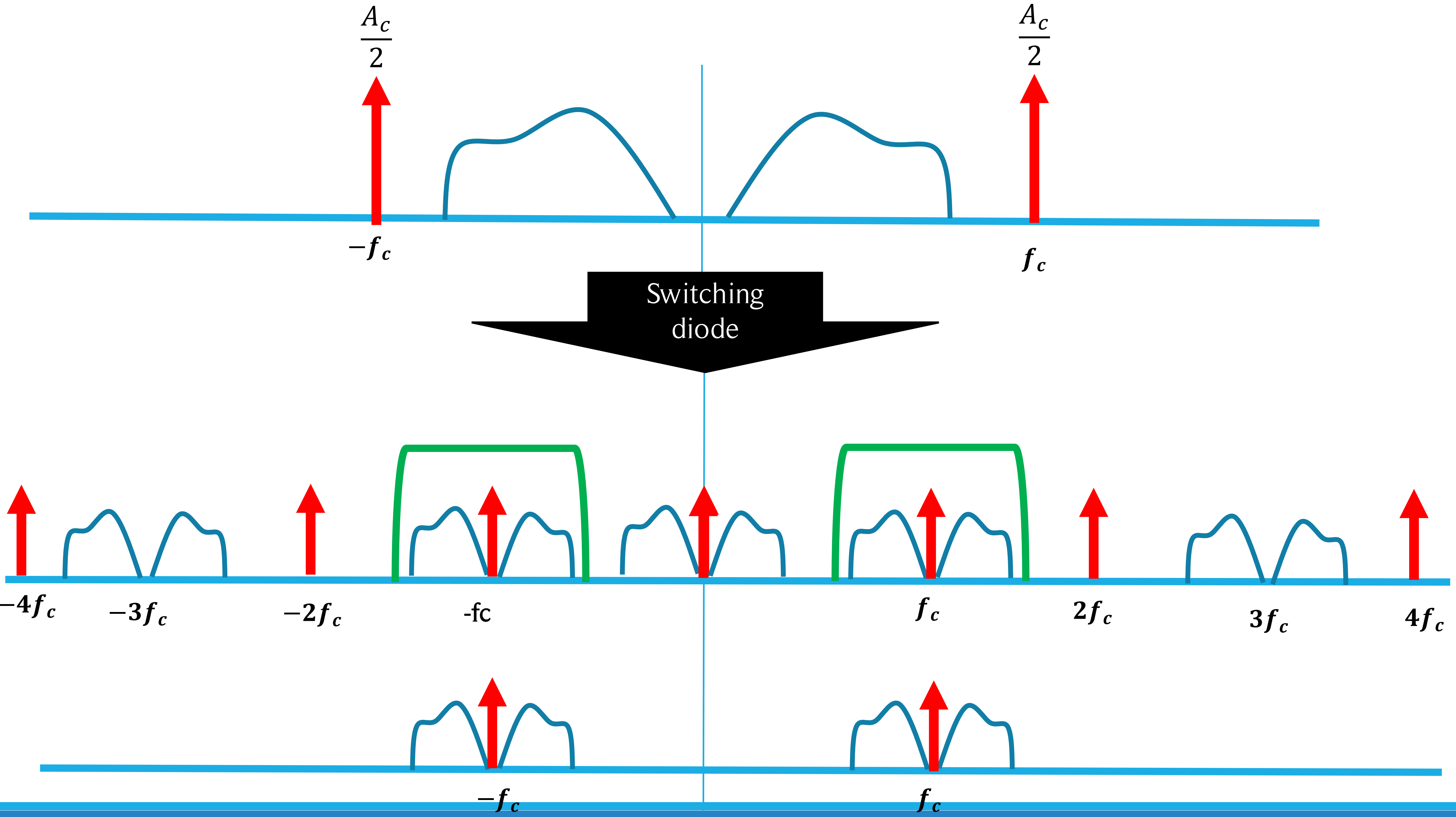
$$\frac{\boldsymbol{A_C}}{\mathbf{2}}\left[\mathbf{1} + \frac{\mathbf{2}}{\boldsymbol{\pi A_c}}\boldsymbol{m(t)}\right]\mathbf{cos(2\pi f_c t)} = \frac{A_C}{2}[1 + k_a m(t)]\cos(2\pi f_c t)$$

2- Unwanted components:

a- deltas at $\pm 2f_c, \pm 4f_c, \pm 6f_c, \ldots.$

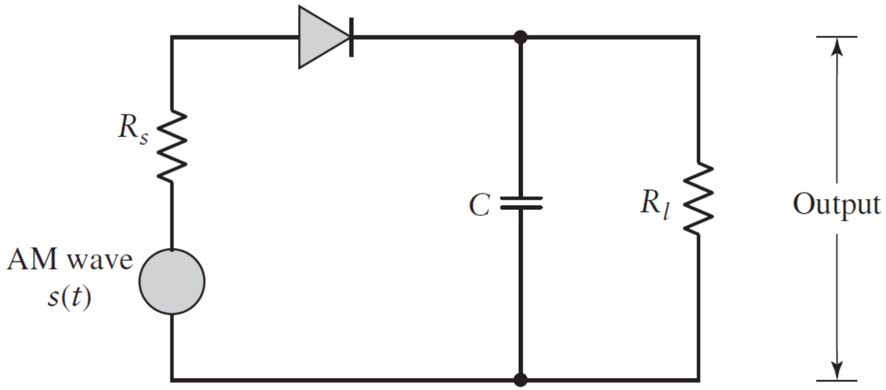
b- versions of the message signal at $\pm 1f_c, \pm 3f_c, \pm 5f_c, \ldots.$





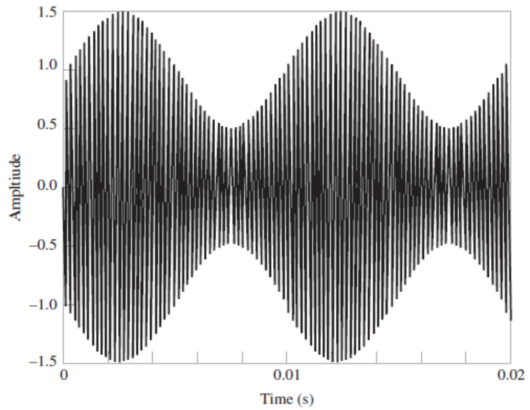
$\frac{A_c}{2}$
$\frac{A_c}{2}$
$-f_c$
f_c
Switching diode
$-4f_c$
$-3f_c$
$-2f_c$
-fc
f_c
$2f_c$
$3f_c$
$4f_c$
$-f_c$
f_c

Non- Coherent detection Envelope detector - (FULL) AM RECIVER



Demodulation of an AM wave can be accomplished by means of a simple and yet hi effective circuit called the envelope detector, provided two practical conditions are satisfied:

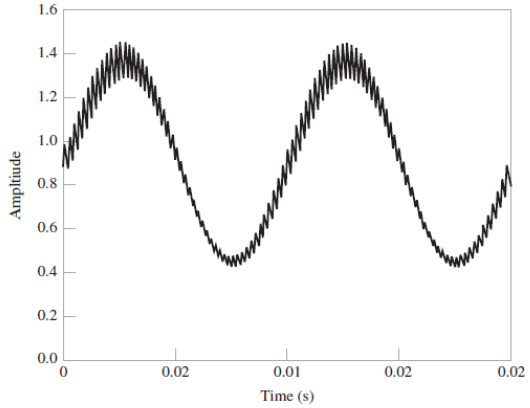
1. The AM wave is narrowband, which means that the carrier frequency is large compared to the message bandwidth ($f_c \gg W$).
2. The percentage modulation in the AM wave is less than 100 percent.



(b)

Also, we need to ensure the following conditions:

1. The charging time constant $(r_f + R_s)C$ must be short compared with the carrier period $\frac{1}{f_c}$ ➔ $\boldsymbol{(r_f + R_s)C >> \frac{1}{f_c}}$.
2. **The discharging time constant $R_l C$ must be long enough to ensure that the capacitor discharges slowly through the load resistor R_l between positive peaks of the carrier wave, but not so long that the capacitor voltage will not discharge at the maximum rate of change of the modulating wave** ➔ $\boldsymbol{\frac{1}{f_c} \ll R_l C \ll \frac{1}{W}}$



Full AM limitations

- It is wasteful of bandwidth: it requires 2W while W is enough.
- It is wasteful of power: $\frac{2}{3}$ of the power is for the carrier.
- AM detectors are very sensitive to noise.

Modified forms of AM

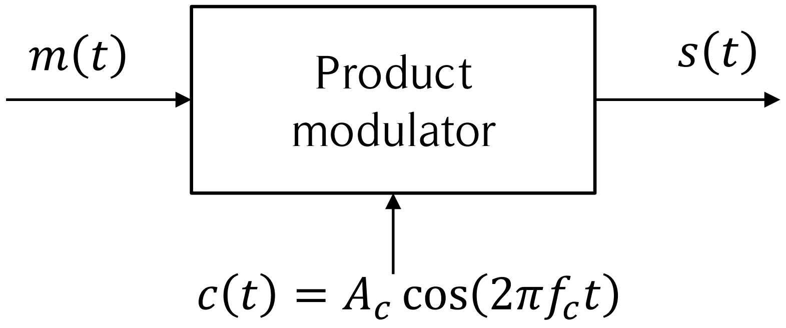
- Double side band suppressed carrier (DSB-SC)
  - We only transmit the side bands without the carrier.
- Single side band (SSB) modulation
  - Only one side band is transmitted.
- Vestigial side band modulation (VSB)
  - One side band is transmitted with trace of the other

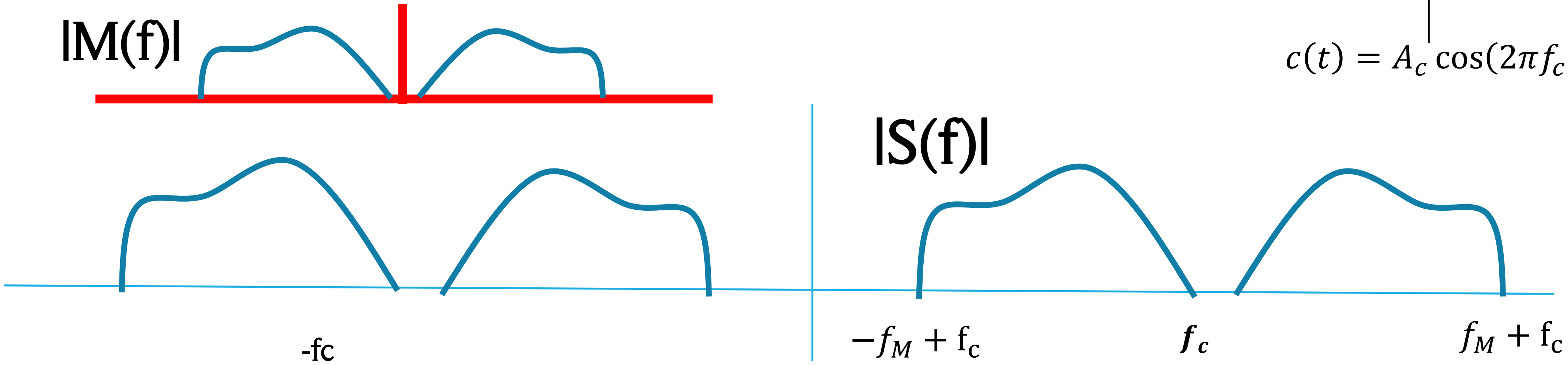
Double side band suppressed carrier (DSB-SC) modulation

Basically, double sideband-suppressed carrier (DSB-SC) modulation consists of the product of the message signal and the carrier wave:

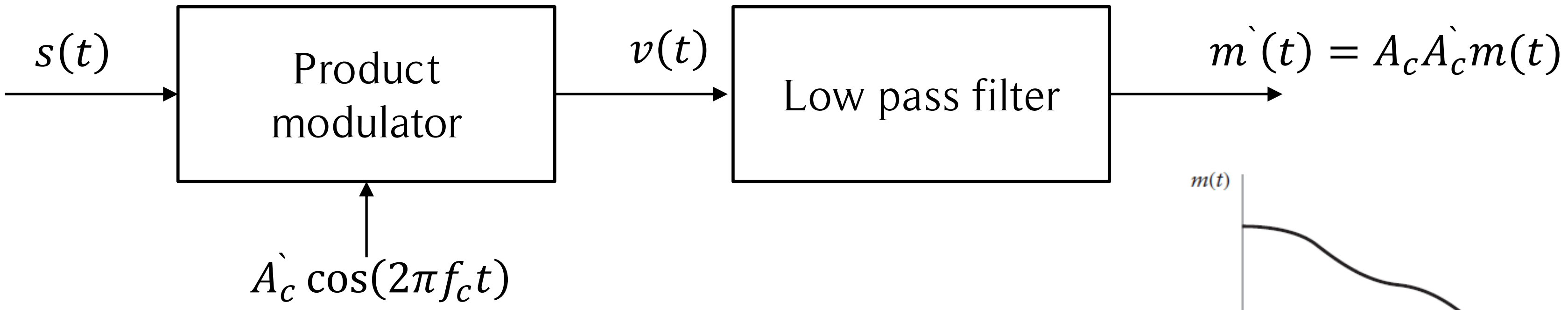
$$s(t) = m(t)c(t) = A_c \cos(2\pi f_c t)\, m(t)$$

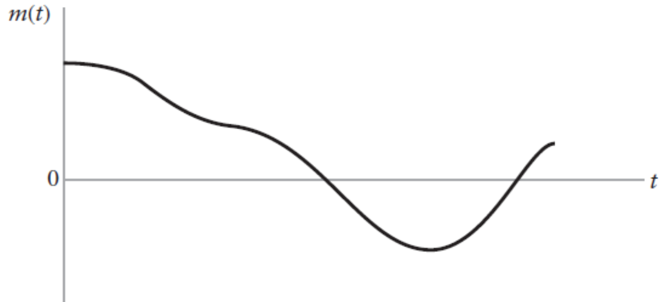
$$s(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c)]$$



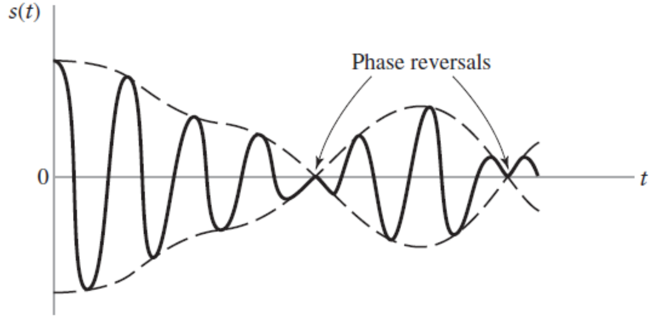


Double side band suppressed carrier (DSB-SC) De-modulation





(a)



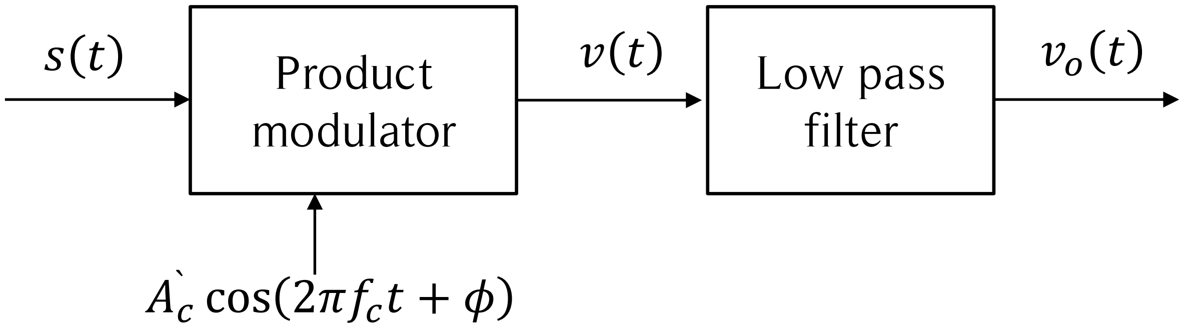
Modulated signal undergoes a *phase reversal* whenever the message signal crosses zero. The envelope of a DSB-SC modulated signal is therefore different from the message signal, which means that simple demodulation using an envelope detection is not a viable option for DSB-SC modulation.

Coherent detection of DSB-SC

$$v(t) = \grave{A}_c \cos(2\pi f_c t + \phi)\, s(t)$$

$$v(t) = \grave{A}_c A_c \cos(2\pi f_c t + \phi) \cos(2\pi f_c t)\, m(t)$$

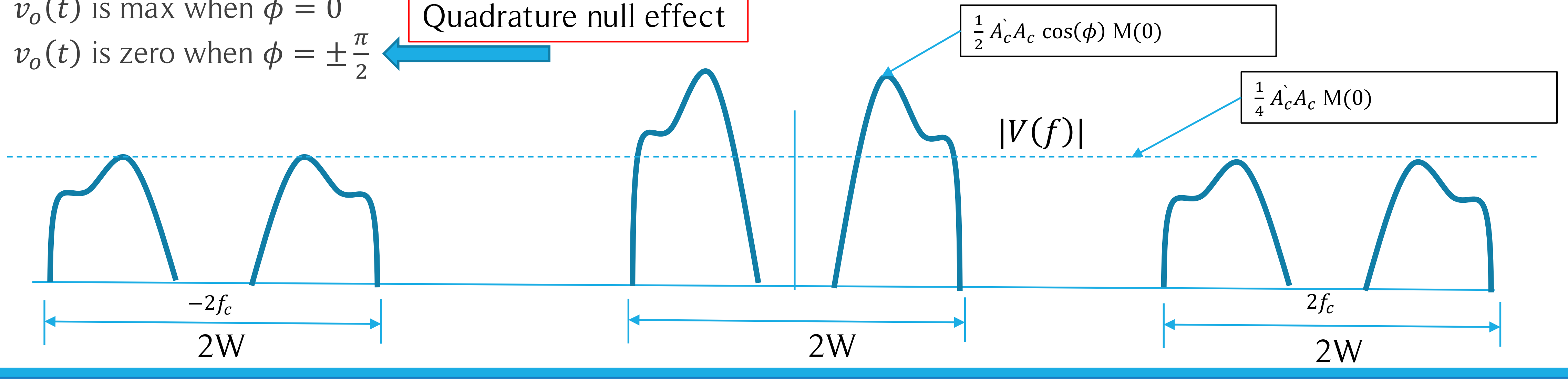
$$v(t) = \frac{\grave{A}_c A_c}{2} \cos(4\pi f_c t + \phi)\, m(t) + \frac{\grave{A}_c A_c}{2} \cos(\phi)\, m(t)$$



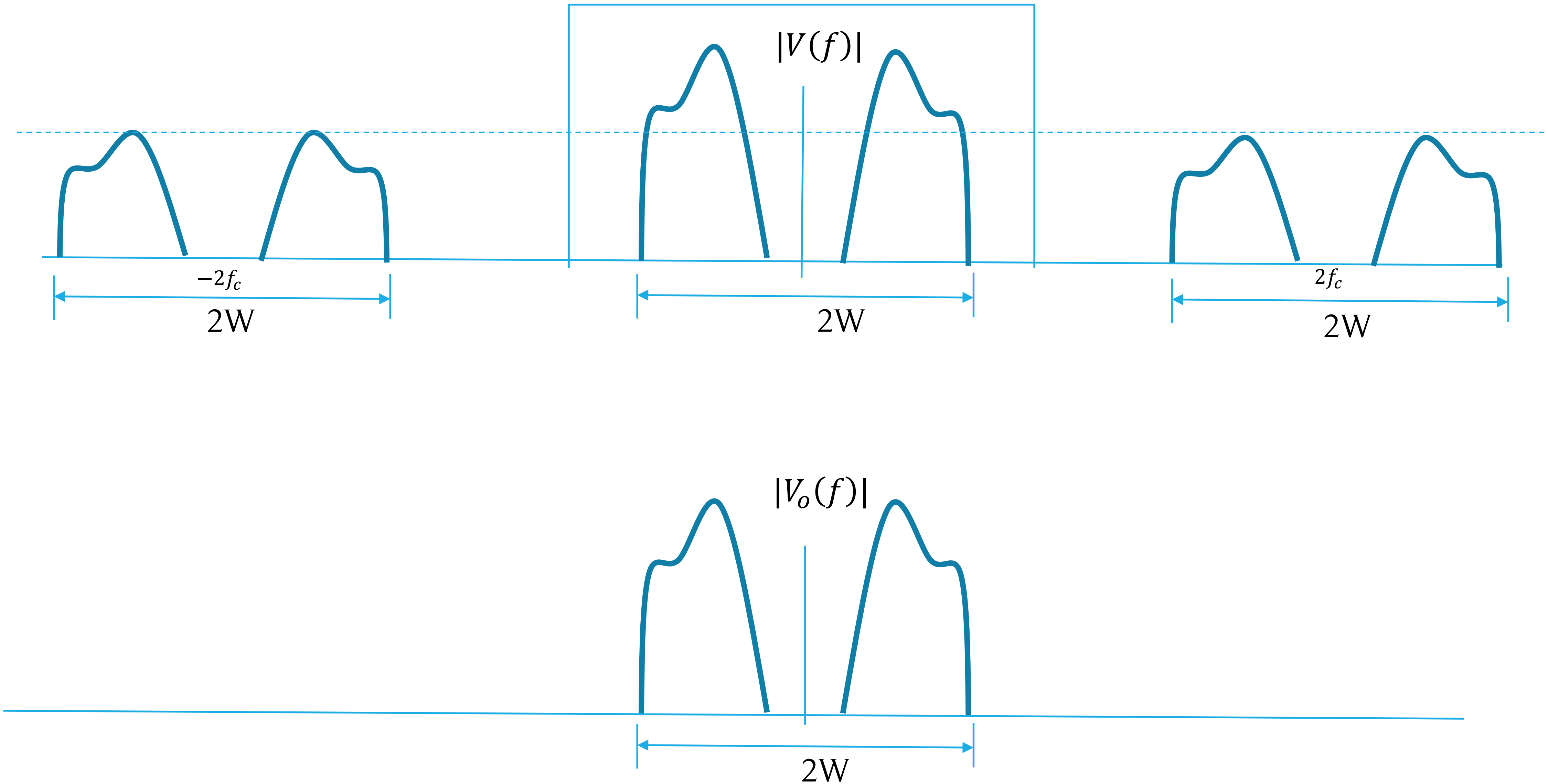
➔ $v_o(t) = \frac{\grave{A}_c A_c}{2} \cos(\phi)\, m(t)$

- $v_o(t)$ is max when $\phi = 0$
- $v_o(t)$ is zero when $\phi = \pm\frac{\pi}{2}$

Quadrature null effect

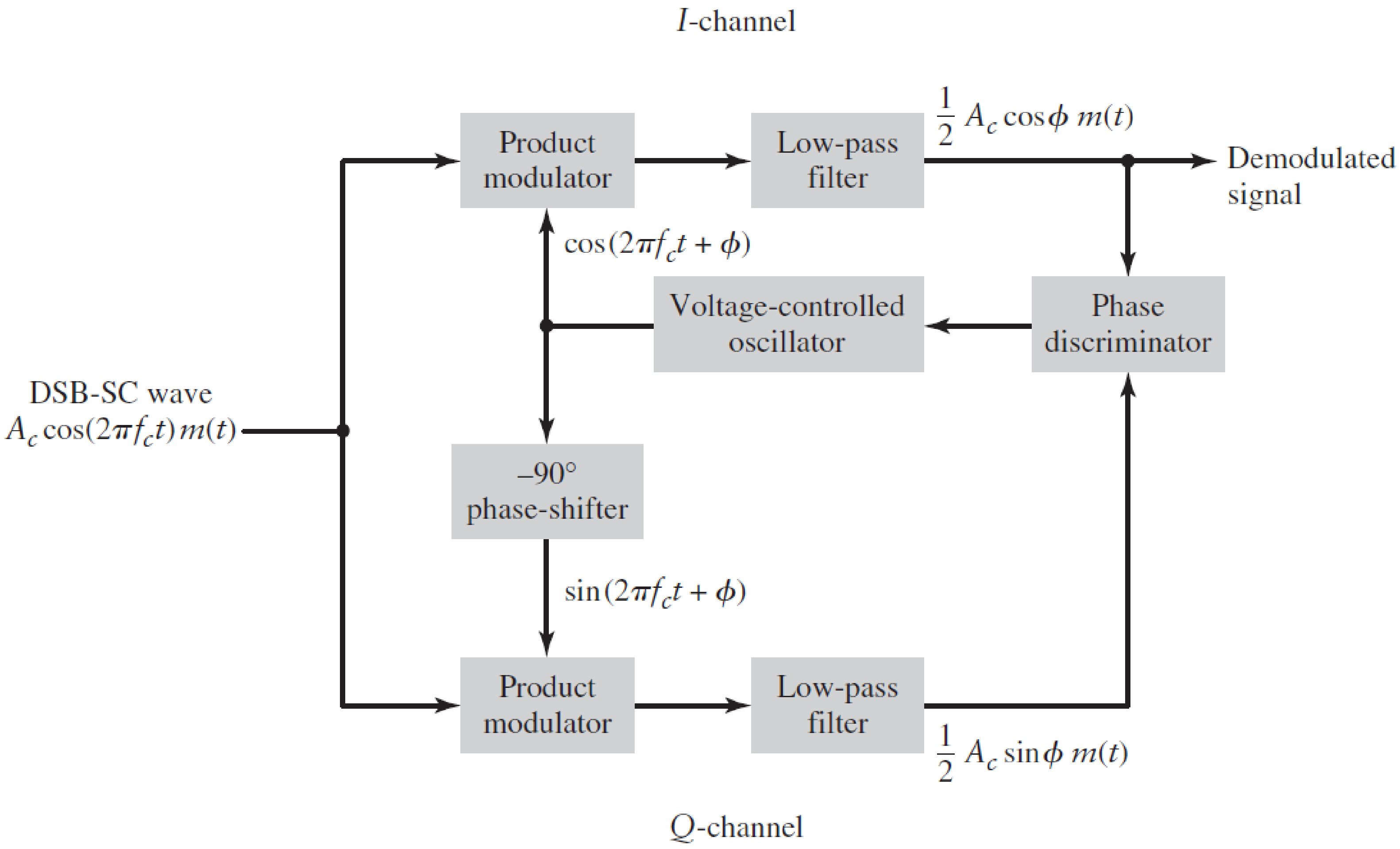


DSB-SC DEMODULATION



Costas Receiver (Coherent detection)

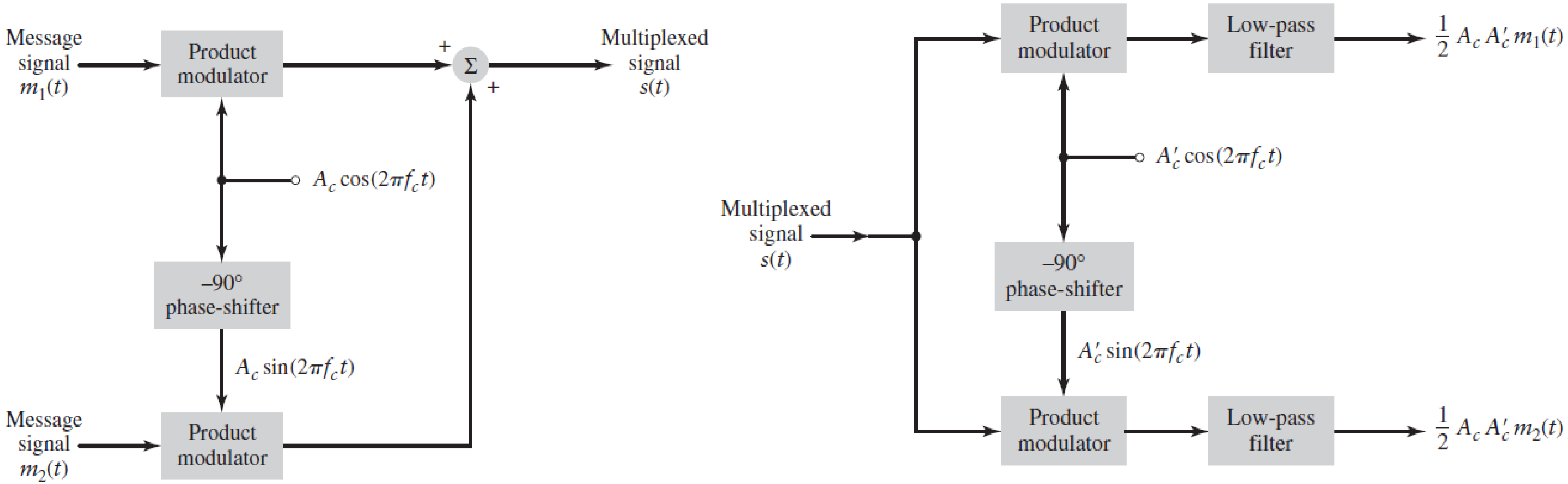
- Coherent detection of a DSB-SC modulated wave requires that the locally generated carrier in the receiver be synchronous in both frequency and phase with the oscillator responsible for generating the carrier in the transmitter.
- One method of satisfying this requirement is to use the Costas receiver



Quadrature carrier multiplexing

The quadrature null effect of the coherent detector may also be put to good use in the construction of the so-called quadrature-carrier multiplexing or quadrature-amplitude modulation (QAM).

$$s(t) = A_c m_1(t)\cos(2\pi f_C t) + A_c m_2(t)\sin(2\pi f_C t)$$



$$\sin a \cos b = \frac{1}{2}[\sin(a+b) + \sin(a-b)]$$

$$\cos a \sin b = \frac{1}{2}[\sin(a+b) - \sin(a-b)]$$

$$\cos a \cos b = \frac{1}{2}[\cos(a+b) + \cos(a-b)]$$

$$\sin a \sin b = -\frac{1}{2}[\cos(a+b) - \cos(a-b)]$$

First branch

$$[\boldsymbol{m_1}(t)\cos(2\pi f_c t) + m_2(\mathrm{t})\sin(2\pi f_c t)]\cos(2\pi f_c t) =$$
$$[\boldsymbol{m_1}(t)cos(4\pi f_c t) + \boldsymbol{m_1}(\boldsymbol{t})cos(0)] + [m_2(t)\,sin(4\pi f_c t)$$
$$+ [m_2\,(t)\,sin(0)]$$

$$\rightarrow \text{LPF} \rightarrow m_1(t)\cos(0) = m_1(t)$$

Second branch

$$[m_1(t)\cos(2\pi f_c t) + m_2(\mathrm{t})\sin(2\pi f_c t)]\sin(2\pi f_c t) =$$
$$[m_1(t)sin(4\pi f_c t) - m_1(t)\sin(0)] + [m_2(t)\,cos(0)$$
$$- m_2(t)\cos(4\pi f_c t)]$$

$$\rightarrow \text{LPF} \rightarrow m_2(t)\cos(0) = m_2(t)$$

Single side band modulation (SSB)

- DSB-SC takes care of the major disadvantage of the full AM and saves the wasted power in the carrier
- To achieve saving the bandwidth, we need to suppress one of the two sidebands in the DSB-SC modulated wave.
- SSB modulation relies solely on the lower sideband or upper sideband to transmit the message signal across a communication channel. Depending on which sideband is transmitted, we speak of lower SSB or upper SSB modulation.

The SSB formula is as follows:

$$s(t) = A_c m(t)\cos(2\pi f_c t) \pm A_c\,\hat{m}(t)\sin(2\pi f_c t)$$

In the frequency domain we can right for the upper side band:

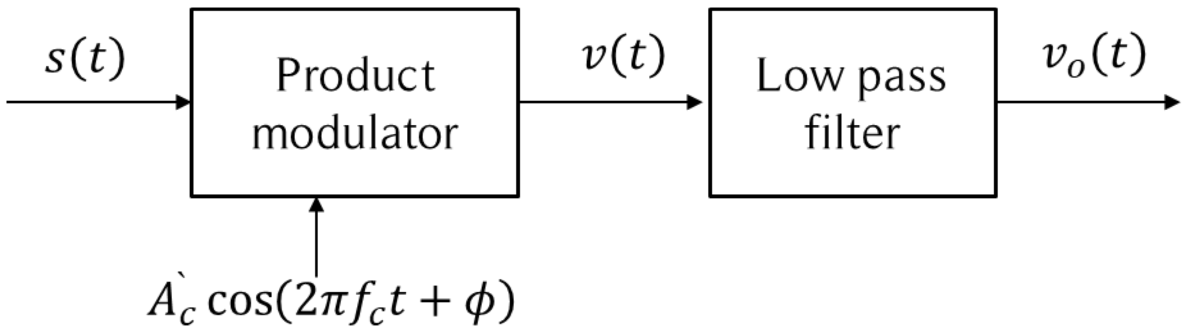
$$\boldsymbol{s(f) = \begin{cases} \frac{A_c}{2} M(f - f_c)\ , f \geq f_c \\ 0, 0 < f \leq f_c \end{cases}}$$

And for the lower side band

$$\boldsymbol{s(f) = \begin{cases} 0\ , f \geq f_c \\ \frac{A_c}{2} M(f - f_c), 0 < f \leq f_c \end{cases}}$$

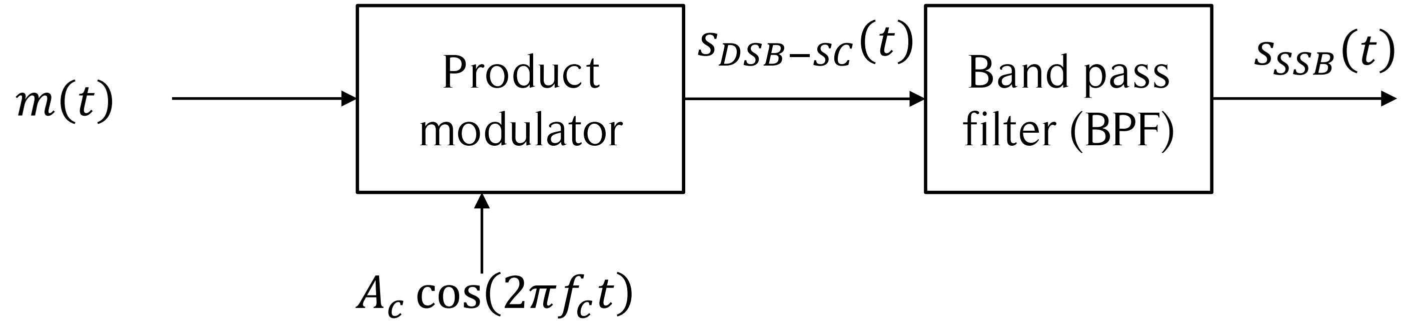
COHERENT DETECTION OF SSB

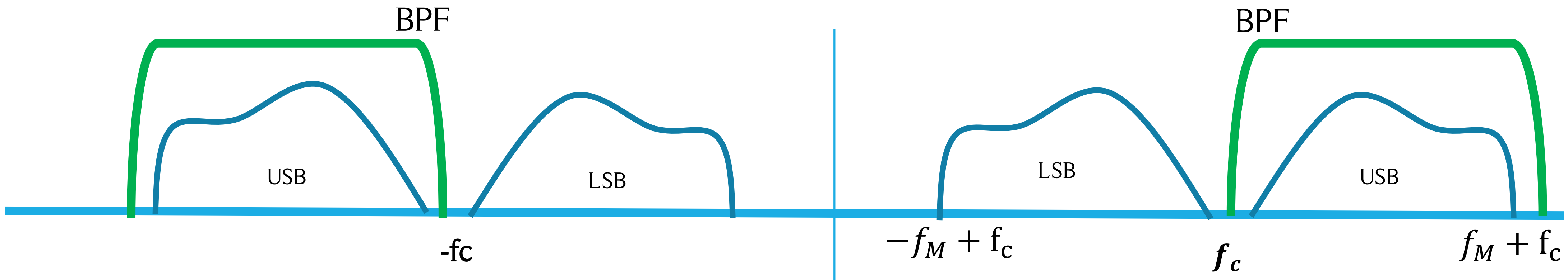
- The demodulation of DSB-SC is complicated by the suppression of the carrier in the transmitted signal.
- To make up for the absence of the carrier in the received signal, the receiver resorts to the use of coherent detection, which requires synchronization of a local oscillator in the receiver with the oscillator responsible for generating the carrier in the transmitter.
- The synchronization requirement has to be in both phase and frequency.
- However, the demodulation of SSB is further complicated by the additional suppression of the upper or lower sideband.
- The coherent detector of the DSB-SC applies equally well to the demodulation SSB; the only difference between these two applications is how the modulated wave $S(t)$ is defined



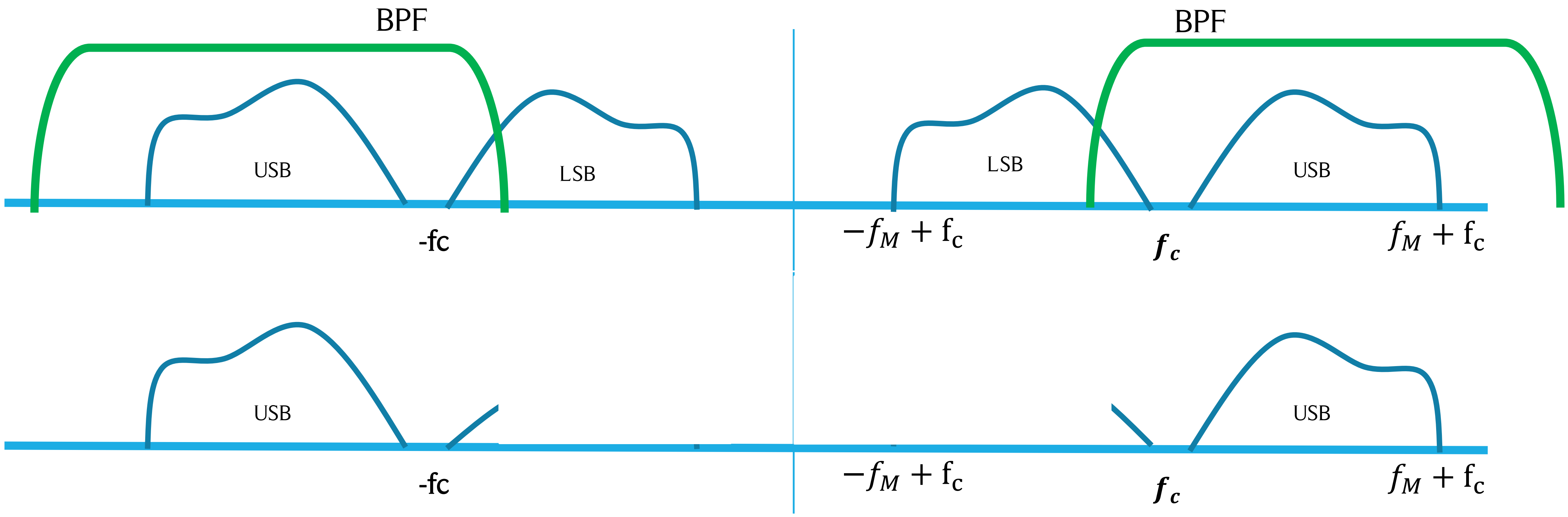
SSB modulators:

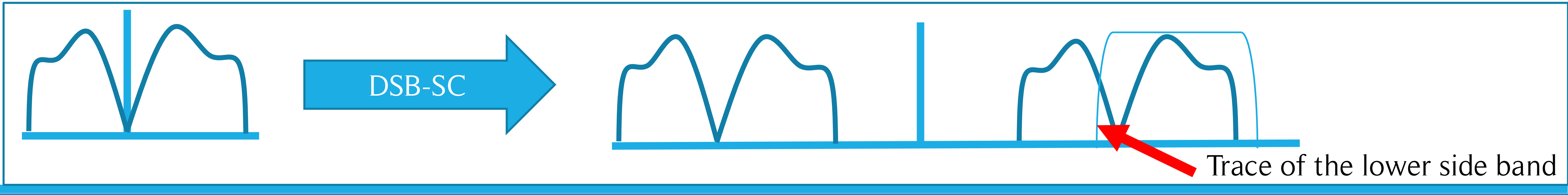
- 1) **Frequency Discrimination Method**: consists of two components: product modulator followed by band-pass filter. The product modulator produces a DSB-SC modulated wave with an upper sideband and a lower sideband. The band-pass filter is designed to transmit one of these two sidebands, depending on whether the upper SSB or lower SSB is the desired modulation.





Non ideal BPF for SSB modulation

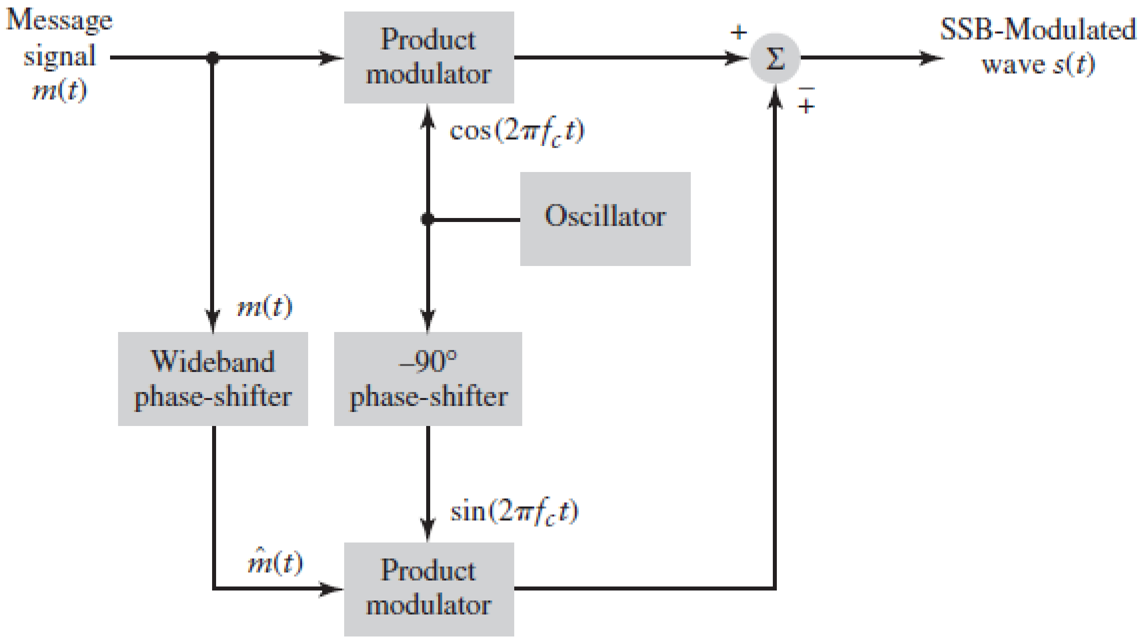




SSB modulators:

2) Phase Discrimination Method: Its implementation follows from the time-domain description of SSB waves and consists of two parallel paths, one called the in-phase path and the other called the quadrature path. Each path involves a product modulator:

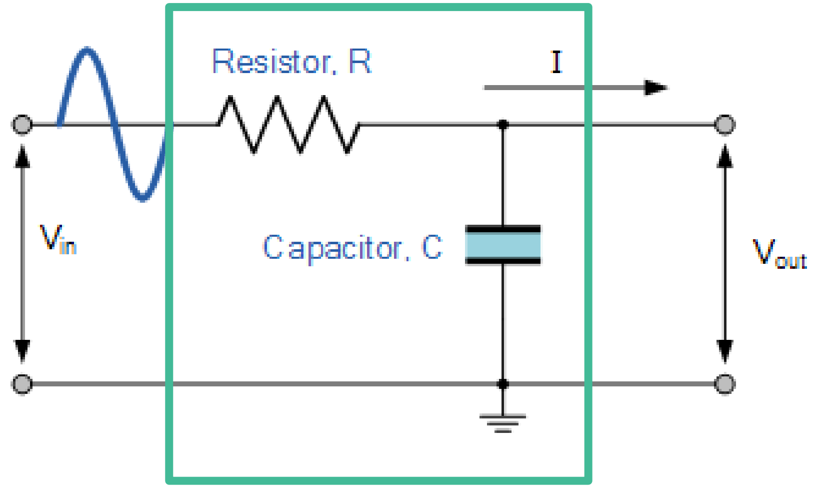
$$s(t) = A_c m(t)\cos(2\pi f_c t) \pm A_c \hat{m}(t)\sin(2\pi f_c t)$$



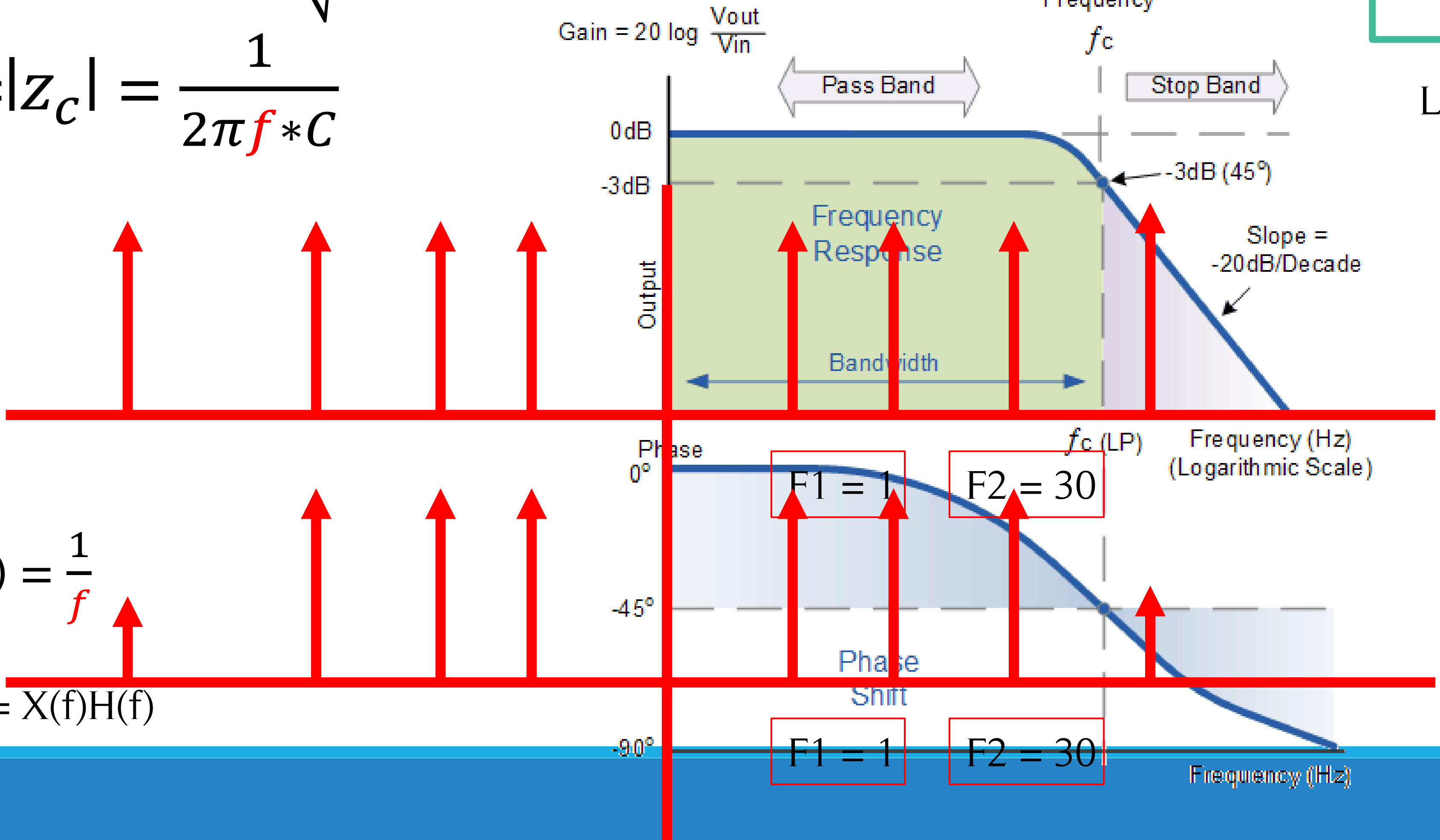
PASSIVE RC LPF

$$h(t) = \frac{V_o(t)}{V_{in}(t)} = \frac{X_c}{\sqrt{X_C^2 + R^2}}$$

$$X_c = |z_c| = \frac{1}{2\pi f * C}$$



LPF with $h(t) = \frac{V_o(t)}{V_{in}(t)}$



$$H(f) = \frac{1}{f}$$

$$Y(f) = X(f)H(f)$$

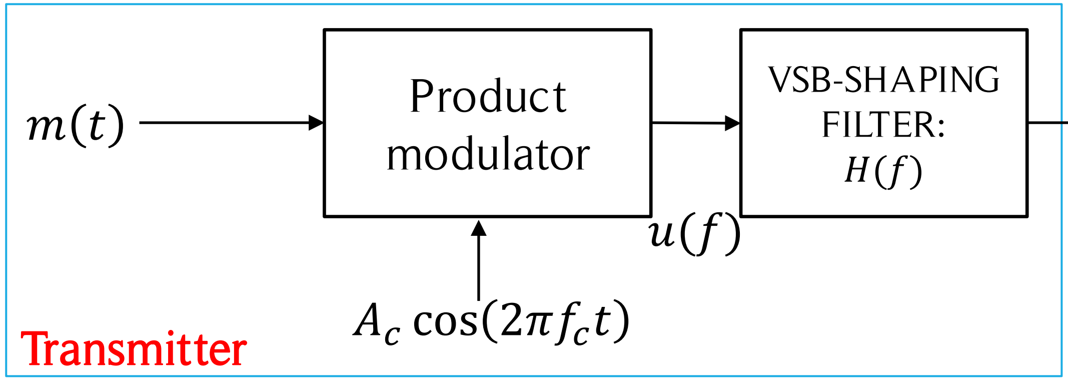
Vestigial Sideband Modulation

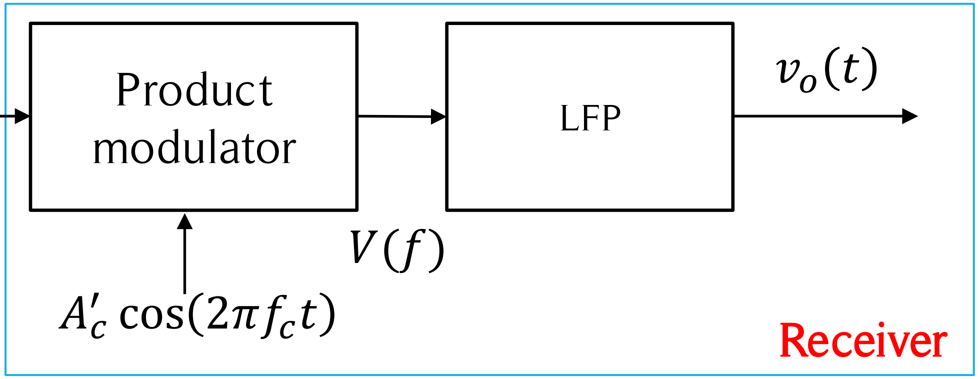
- Single-sideband modulation works satisfactorily for an information-bearing signal (e.g., speech signal) with an energy gap centered around zero frequency.
- Typically, the spectra of wideband signals (exemplified by television video signals and computer data) contain significant low frequencies, which make it impractical to use SSB modulation.
- Vestigial sideband (VSB) modulation distinguishes itself from SSB modulation in two practical respects:
  1. Instead of completely removing a sideband, a trace or vestige of that sideband is transmitted; hence, the name “vestigial sideband”.
  2. Instead of transmitting the other sideband in full, almost the whole of this second band is also transmitted. Accordingly, the transmission bandwidth of a VSB modulated signal is defined by:

$$BW = f_v + W$$

where, f_v : vestige bandwidth, W : massage bandwidth

Vestigial Sideband Modulation





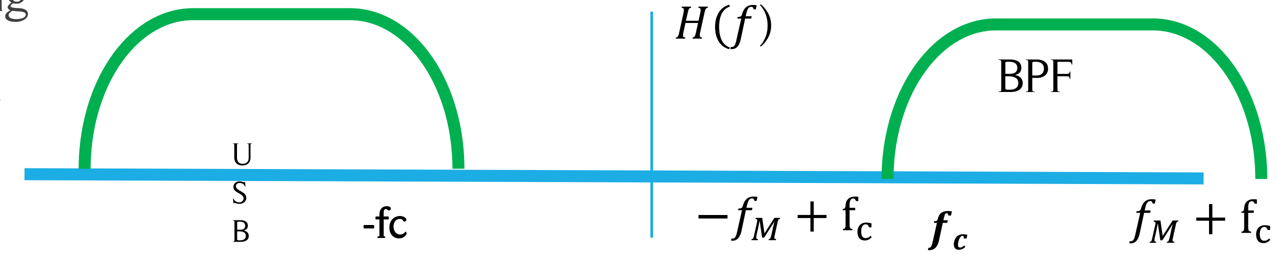
Sideband shaping filter must itself satisfy the following condition:

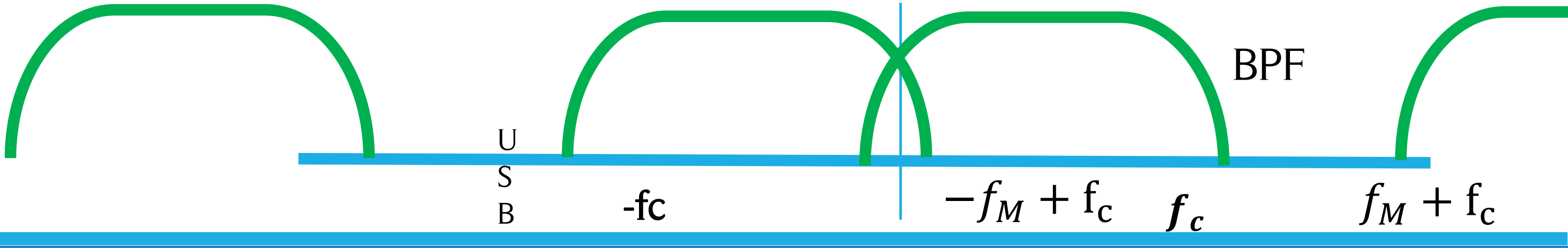
$$H(f+f_c) + H(f-f_c) = 1, \text{ for } -W \le f \le W$$

$$u(f) = \frac{A_c}{2}[M(f-f_c) + M(f+f_c]$$

$$S(f) = \frac{A_c}{2}[M(f-f_c) + M(f+f_c]H(f)$$

$$V(f) = \frac{A_c'}{2}[S(f-f_c) + S(f+f_c)]$$





$$u(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c]$$

$$S(f) = \frac{A_c}{2}[M(f - f_c) + M(f + f_c]H(f)$$

Coherent detection of VSB

$$V(f) = \frac{A_c' A_c}{4}\,[M(f - 2f_c)H(f - f_c) + M(f)H(f - f_c)] + M(f)H(f + f_c) + M(f + 2f_c)H(f + f_c)]$$

$$V(f) = \frac{A_c' A_c}{4}\,[M(f - 2f_c)H(f - f_c) + M(f + 2f_c)H(f + f_c)] + \frac{A_c' A_c}{4}\,M(f)\,[\underbrace{H(f - f_c) + H(f + f_c)}_{1}]$$

$$V_o(f) = \frac{A_c' A_c}{4} M(f)$$

$$s(t) = S_I(t)\cos 2\pi f_c t + S_Q(t)\sin 2\pi f_c t$$

$$S_I(f) = \begin{cases} S(f - f_c) + S(f + f_c), -W \le f \le W \\ 0, \quad elsewhere \end{cases}$$

$$S_I(f) = \frac{A_c}{2}\,M(f)[H(f - f_c) + H(f + f_c)]$$

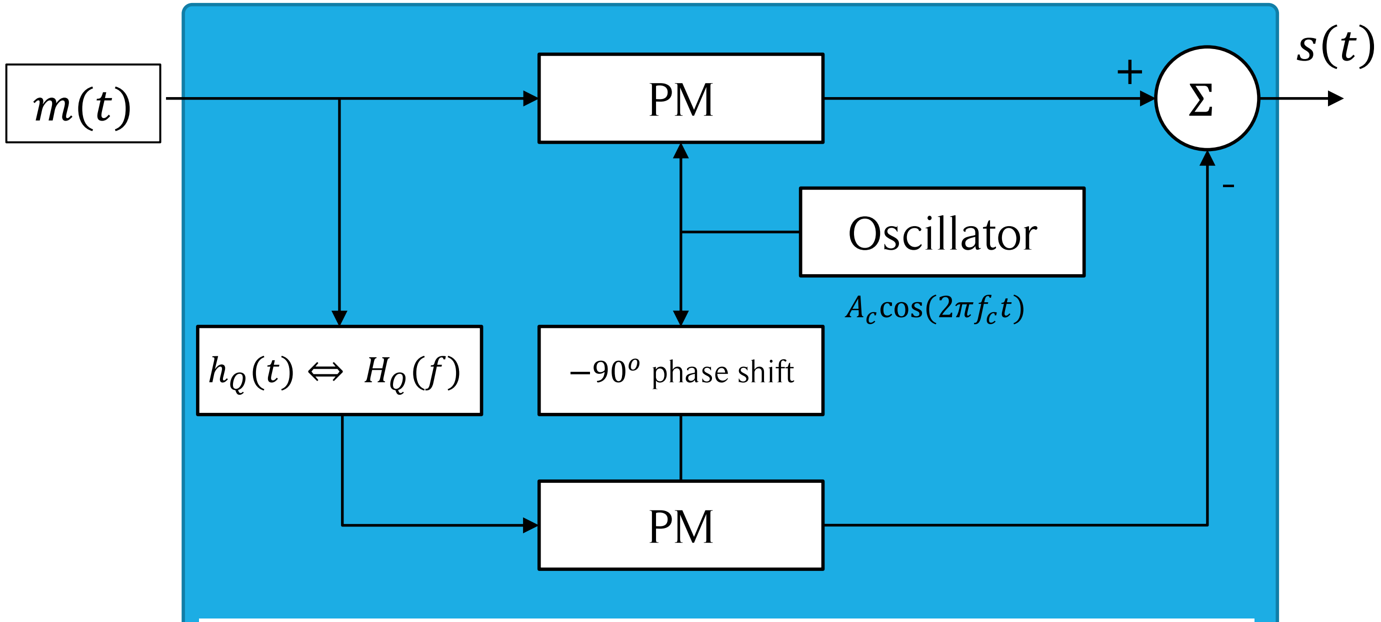
$$S_I(f) = \frac{A_c}{2}\,M(f) \Rightarrow S_I(t) = \frac{A_c}{2}\,m(t)$$

$$S_Q(f) = \begin{cases} j[S(f - f_c) - S(f + f_c)], -W \le f \le W \\ 0, \quad elsewhere \end{cases}$$

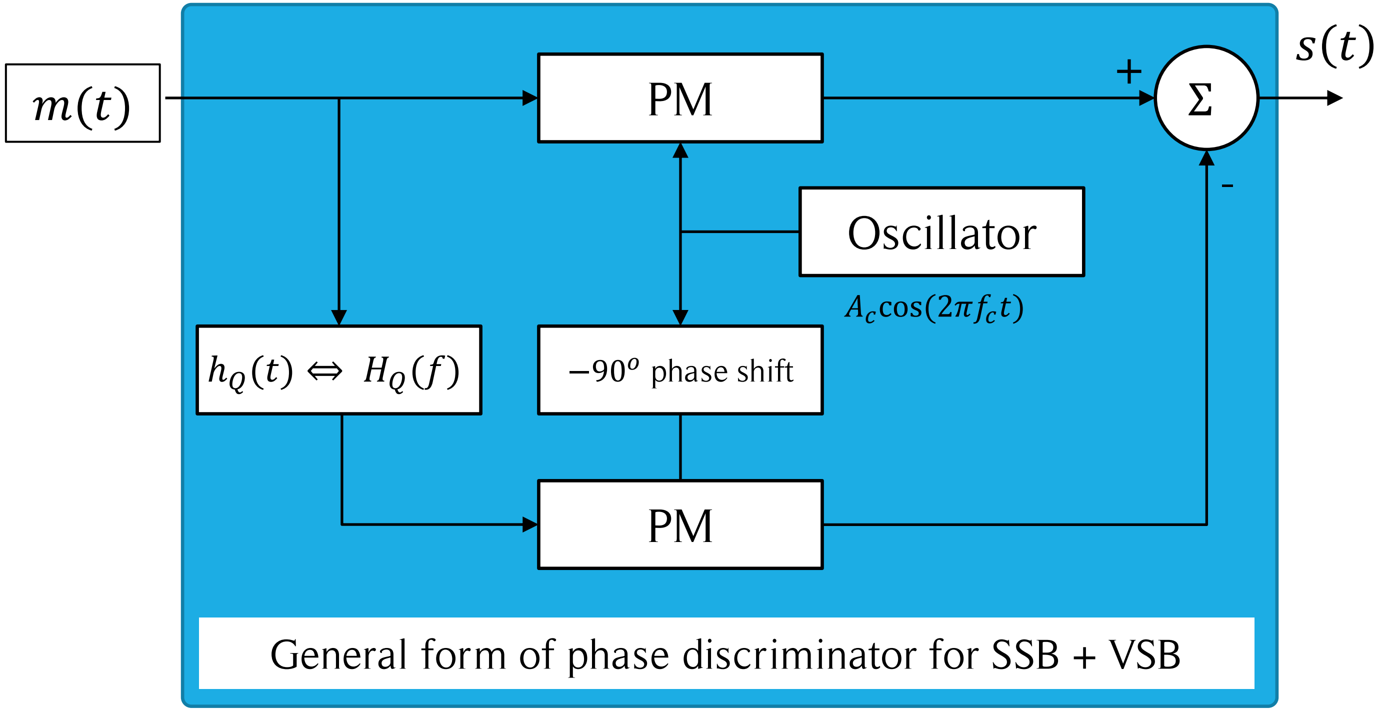
$$S_Q(f) = \frac{A_c}{2}\,M(f)j[H(f - f_c) - H(f + f_c)]$$

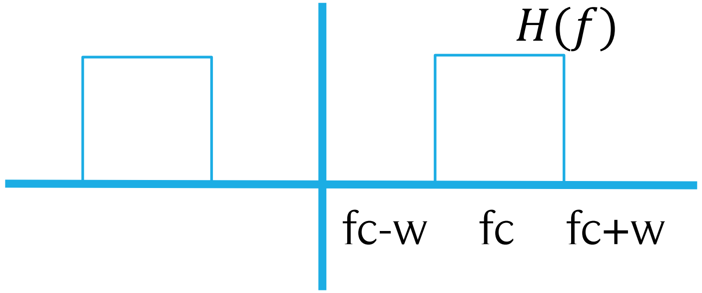
$$H_Q(f) = j[H(f - f_c) - H(f + f_c)]$$

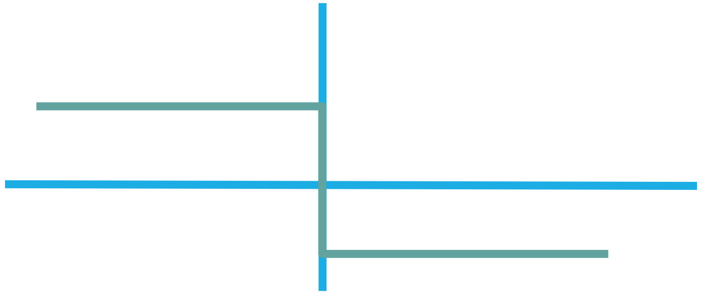
$$S_Q(f) = \frac{A_c}{2}\,M(f)H_Q(f) \Rightarrow S_Q(t) = = F^{-1}[\frac{A_c}{2}\,M(f)H_Q(f)]$$

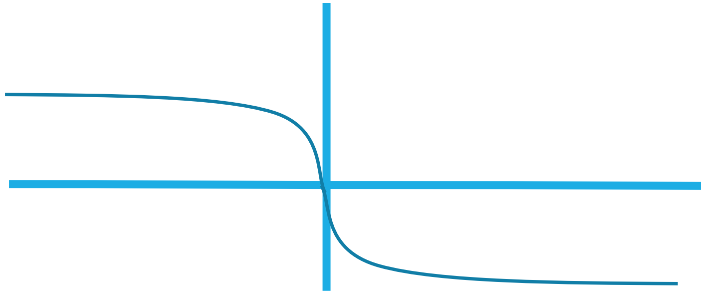


General form of phase discriminator for SSB + VSB









$$H_Q(f) = j[H(f - f_c) - H(f + f_c)]$$
$$|H_Q(f)| = |H(f - f_c) - H(f + f_c)|$$
$$|H_Q(f) = -j\,\mathrm{sgn}(f)\, H_I(f)$$

$$s(t) = A_c m(t)\cos(2\pi f_c t) - A_c\,\hat{m}(t)\sin(2\pi f_c t)$$

Or

$$s(t) = A_c m(t)\cos(2\pi f_c t) - A_c\,m'(t)\sin(2\pi f_c t)$$

EXAMPLE 3.3 Sinusoidal VSB

Consider the simple example of sinusoidal VSB modulation produced by the sinusoidal modulating wave

$$m(t) = A_m \cos(2\pi f_m t)$$

and carrier wave

$$c(t) = A_c \cos(2\pi f_c t)$$

Let the upper side-frequency at $f_c + f_m$ as well as its image at $-(f_c + f_m)$ be attenuated by the factor k. To satisfy the condition of Eq. (3.26), the lower side-frequency at $f_c - f_m$ and its image $-(f_c - f_m)$ must be attenuated by the factor $(1 - k)$. The VSB spectrum is therefore

$$S(f) = \frac{1}{4}kA_cA_m[\delta(f - (f_c + f_m)) + \delta(f + (f_c + f_m))] + \frac{1}{4}(1 - k)A_cA_m[\delta(f - (f_c - f_m)) + \delta(f + (f_c - f_m))]$$

Correspondingly, the sinusoidal VSB modulated wave is defined by

$$\begin{aligned} s(t) &= \frac{1}{4}kA_cA_m[\exp(j2\pi(f_c + f_m)t) + \exp(-j2\pi(f_c + f_m)t)] \\ &\quad + \frac{1}{4}(1 - k)A_cA_m[\exp(j2\pi(f_c - f_m)t) + \exp(-j2\pi(f_c - f_m)t)] \\ &= \frac{1}{2}kA_cA_m \cos(2\pi(f_c + f_m)t) + \frac{1}{2}(1 - k)A_cA_m \cos(2\pi(f_c - f_m)t) \end{aligned} \tag{3.30}$$

Using well-known trigonometric identities to expand the cosine terms $\cos(2\pi(f_c + f_m)t)$ and $\cos(2\pi(f_c - f_m)t)$, we may reformulate Eq. (3.30) as the linear combination of two sinusoidal DSB-SC modulated waves.

$$s(t) = \frac{1}{2}A_cA_m \cos(2\pi f_c t)\cos(2\pi f_m t) + \frac{1}{2}A_cA_m(1 - 2k)\sin((2\pi f_c t)\sin(2\pi f_m t)) \tag{3.31}$$

where the first term on the right-hand side is the in-phase component of $s(t)$ and the second term is the quadrature component.

1. $k = \frac{1}{2}$, DSB-SC
2. $k = 0$, LOWER SSB
3. $K = 1$, UPPER SSB
4. $0 < k < \frac{1}{2}$, VSB + attenuated version of USB
5. $\frac{1}{2} < k < 1$, VSB + attenuated version of LSB

The coherent detection of VSB requires synchronism of the receiver to the transmitter, which increases system complexity. To simplify the demodulation process, we may purposely add the carrier to the VSB signal (scaled by the factor k_a) prior to transmission and then use envelope detection in the receiver.

The coherent detection of VSB requires synchronism of the receiver to the transmitter, which increases system complexity. To simplify the demodulation process, we may purposely add the carrier to the VSB signal (scaled by the factor $\boldsymbol{k_a}$) prior to transmission and then use envelope detection in the receiver.

EXAMPLE 3.5 Envelope detection of VSB plus carrier

The coherent detection of VSB requires synchronism of the receiver to the transmitter, which increases system complexity. To simplify the demodulation process, we may purposely add the carrier to the VSB signal (scaled by the factor k_a) prior to transmission and then use envelope detection in the receiver.[3] Assuming sinusoidal modulation, the "VSB-plus-carrier" signal is defined [see Eq. (3.31) of Example 3.3) as

$$\begin{aligned} s_{\text{VSB+C}}(t) &= A_c\cos(2\pi f_c t) + k_a s(t), \qquad k_a = \text{amplitude sensitivity factor} \\ &= A_c\cos(2\pi f_c t) + \frac{k_a}{2}A_c A_m\cos(2\pi f_m t)\cos(2\pi f_c t) \\ &\quad + \frac{k_a}{2}A_c A_m(1-2k)\sin(2\pi f_m t)\sin(2\pi f_c t) \\ &= A_c\left[1 + \frac{k_a}{2}A_m\cos(2\pi f_m t)\right]\cos(2\pi f_c t) \\ &\quad + \frac{k_a}{2}A_c A_m(1-2k)\sin(2\pi f_m t)\sin(2\pi f_c t) \end{aligned}$$

The envelope of $s_{\text{VSB+C}}(t)$ is therefore

$$\begin{aligned} a(t) &= \left\{A_c^2\left[1 + \frac{k_a}{2}A_m\cos(2\pi f_m t)\right]^2 + A_c^2\left[\frac{k_a}{2}A_m(1-2k)\sin(2\pi f_m t)\right]^2\right\}^{1/2} \\ &= A_c\left[1 + \frac{k_a}{2}A_m\cos(2\pi f_m t)\right]\left\{1 + \left[\frac{\frac{k_a}{2}A_m(1-2k)\sin(2\pi f_m t)}{1 + \frac{k_a}{2}A_m\cos(2\pi f_m t)}\right]^2\right\}^{1/2} \end{aligned} \qquad (3.38)$$

SSB VS VSB

A. VSB modulation is a tradeoff between DSB modulation and SSB modulation.

B. Use more bandwidth than SSB but simplifies the system.
   I. The VSB filter is much easier to implement than the SSB filter, which requires near ideal frequency response at '0' or f_c .
   II. The VSB filter can be implemented at the receiver instead of at the transmitter due to power constraints.

C. C. Envelope detection is also possible for VSB:

D. Used in Television Transmission system.

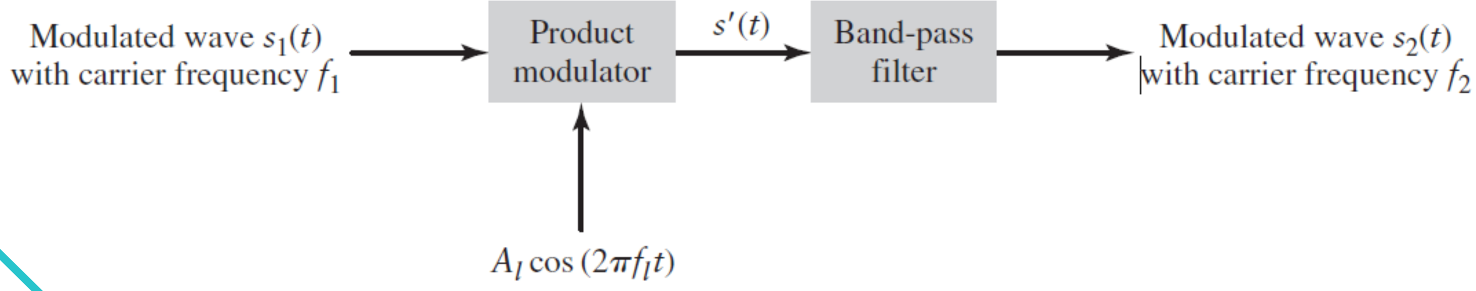
Summary of amplitude modulation techniques

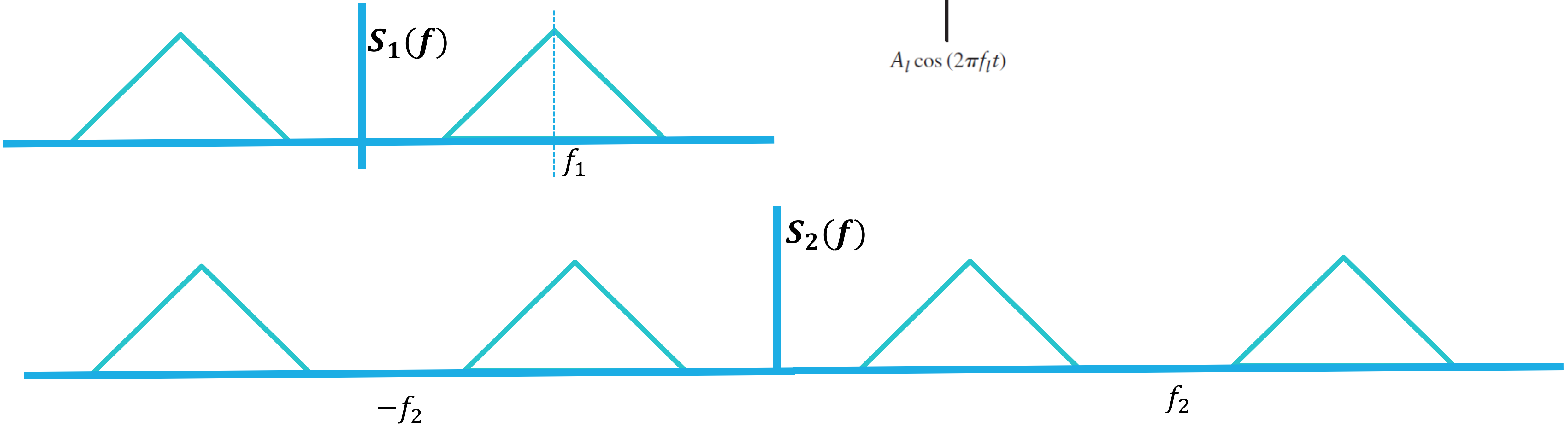
TABLE 3.1 *Different Forms of Linear Modulation as Special Cases of Eq. (3.39), assuming unit carrier amplitude*

Type of modulation	*In-phase component* $s_I(t)$	*Quadrature component* $s_Q(t)$	*Comments*
AM	$1 + k_a m(t)$	0	k_a = amplitude sensitivity $m(t)$ = message signal
DSB-SC	$m(t)$	0	
SSB:			
(a) Upper sideband transmitted	$\frac{1}{2}m(t)$	$\frac{1}{2}\hat{m}(t)$	$\hat{m}(t)$ = Hilbert transform of $m(t)$ (see part (i) of footnote 4)[4]
(b) Lower sideband transmitted	$\frac{1}{2}m(t)$	$-\frac{1}{2}\hat{m}(t)$	
VSB:			
(a) Vestige of lower sideband transmitted	$\frac{1}{2}m(t)$	$\frac{1}{2}m'(t)$	$m'(t)$ = response of filter with transfer function $H_Q(f)$ due to message signal $m(t)$. The $H_Q(f)$ is defined by the formula (see part (ii) of footnote 4)
(b) Vestige of upper sideband transmitted	$\frac{1}{2}m(t)$	$-\frac{1}{2}m'(t)$	$H_Q(f) = -j[H(f + f_c) - H(f - f_c)]$ where $H(f)$ is the transfer function of the VSB sideband shaping filter.

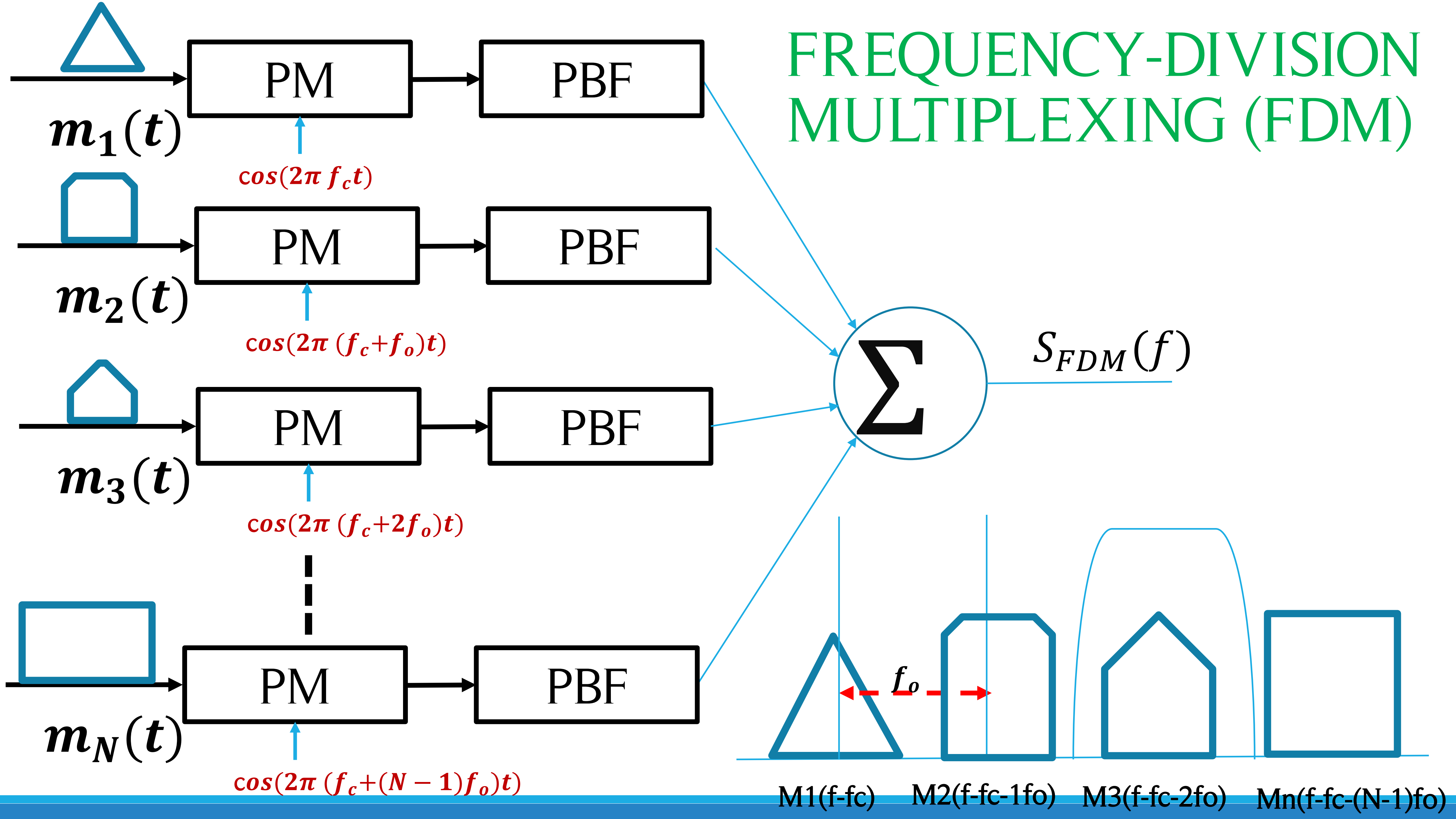
FREQUENCY TRANSLATION

Suppose that we have a modulated wave $s_1(t)$ whose spectrum is centered on a carrier frequency f_1 and the requirement is to translate it upward or downward in frequency, such that the carrier frequency is changed from f_1 to a new value f_2. This requirement is accomplished by using a ***mixer***.



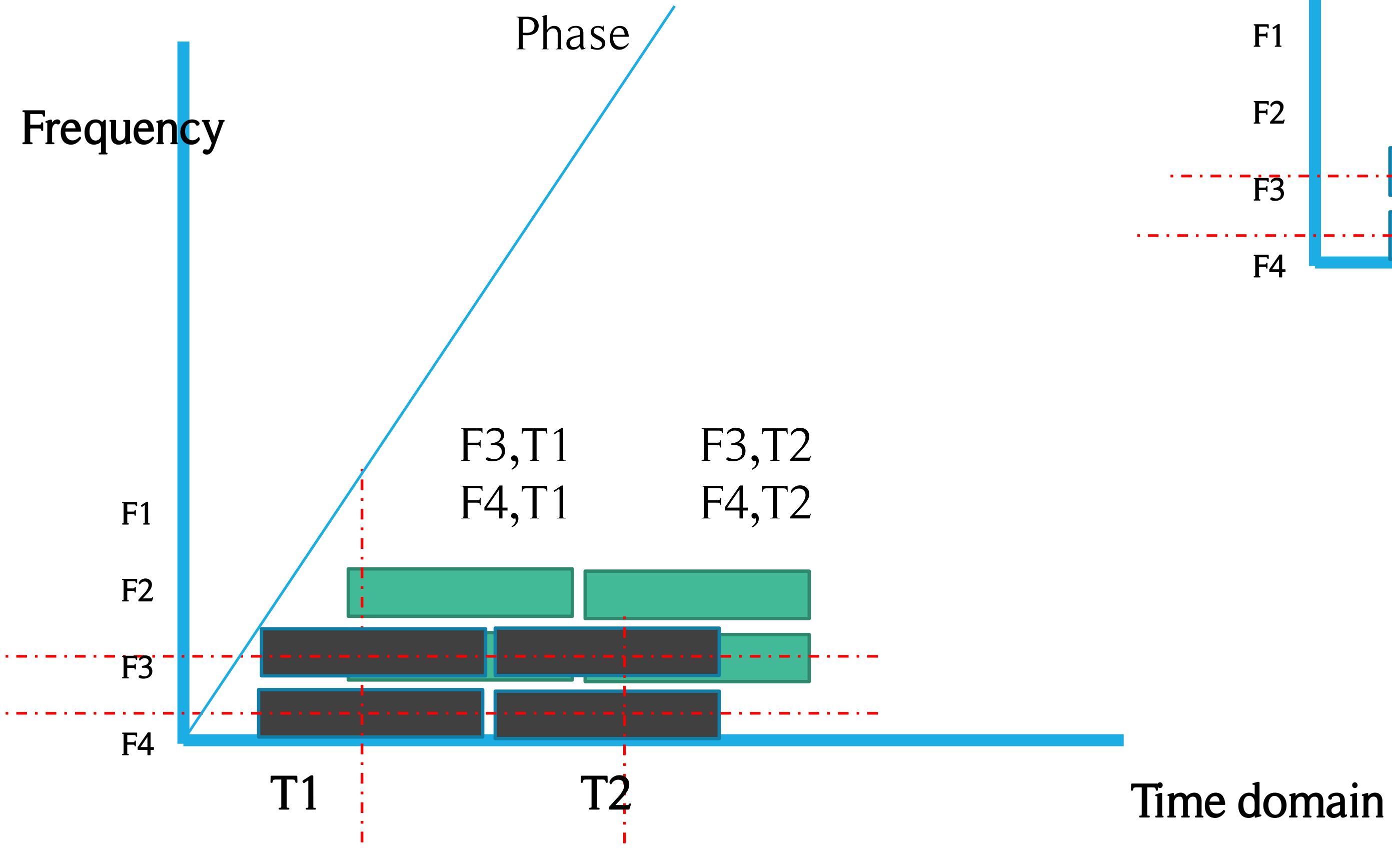


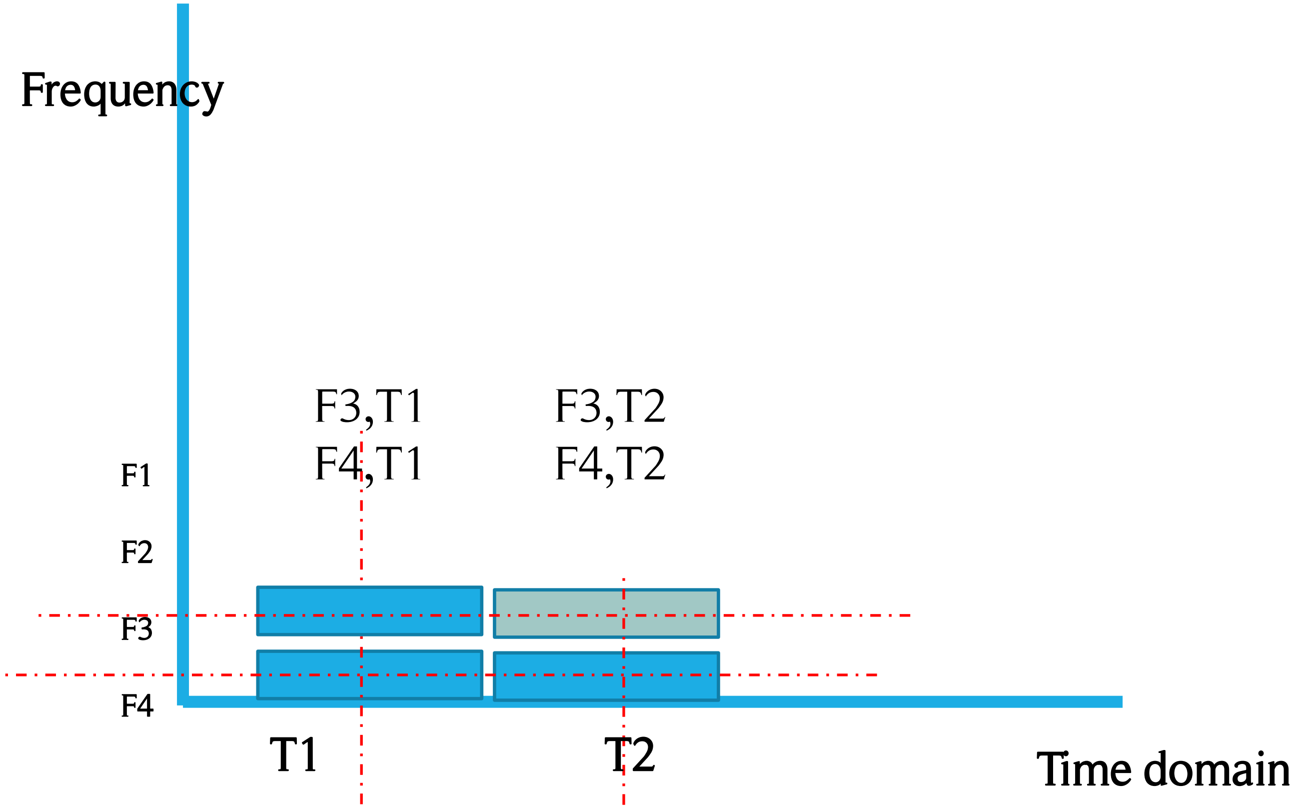
FREQUENCY-DIVISION MULTIPLEXING (FDM)



Domain = Resource

- Frequency
- Time
- Location (spatial)
- Phase
- Power





Angle Modulation

Angle Modulation

- Another way of modulating a sinusoidal carrier wave— namely, angle modulation, in which the angle of the carrier wave is varied according to the information-bearing signal.
- Angle modulation can provide better discrimination against noise and interference than amplitude modulation.
- Angle modulation is a nonlinear process, which testifies to its sophisticated nature. In the context of analog communications, this distinctive property of angle modulation has two implications:
  - The spectral analysis of angle modulation is complicated.
  - the implementation of angle modulation is demanding.
- The transmission bandwidth of an angle-modulated wave may assume an infinite extent, at least in theory.
- Additive noise would affect the performance of angle modulation to a lesser extent than amplitude modulation

Angle Modulation

Let $\theta_i(t)$ denote the angle of a modulated sinusoidal carrier at time t; it is assumed to be a function of the information-bearing signal or message signal. We express the resulting angle-modulated wave as:

$$s(t) = A_c \cos(\theta_i(t))$$

If $\theta_i(t)$ increases monotonically with time, then the average frequency in hertz, over a small interval:

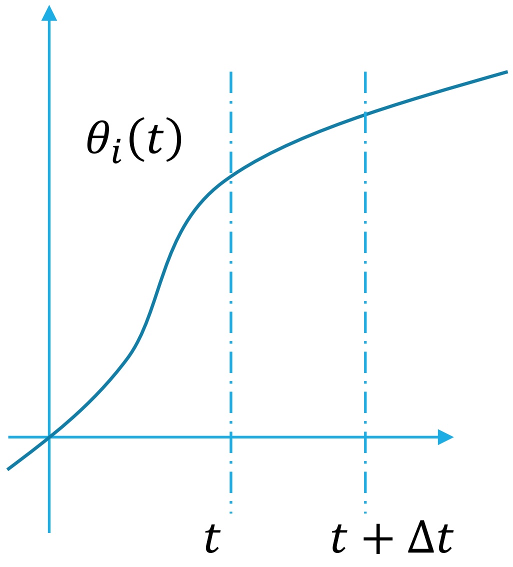
$$f_{\Delta t}(t) = \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi\Delta t}$$

If Δt ➔ 0; then the **instantaneous frequency** is defined as:

$$f_i(t) = \lim_{\Delta t \to 0} f_{\Delta t}(t)$$

$$f_i(t) = \lim_{\Delta t \to 0} \frac{\theta_i(t+\Delta t) - \theta_i(t)}{2\pi\Delta t}$$

$$= \frac{1}{2\pi}\frac{d\,\theta_i(t)}{dt}$$



Before performing any modulation:

$$\boldsymbol{\theta_i(t) = 2\pi f_c t + \phi_c}$$

There are an infinite number of ways in which the angle may be varied in some manner with the message signal. However, we shall consider only two commonly used methods, phase modulation and frequency modulation,

1 - Phase modulation

$$\boldsymbol{s(t) = A_c cos\,[\theta_i(t)]}$$

$$\boldsymbol{\theta_i(t) = 2\pi f_c t + k_p m(t)}$$

k_p: phase sensitivity of the modulator [rad/volt]

$$\boldsymbol{s(t) = A_c cos\,[2\pi f_c t + k_p m(t)]}$$

2 - Frequency modulation

$$\boldsymbol{s(t) = A_c cos\,[\theta_i(t)]}$$

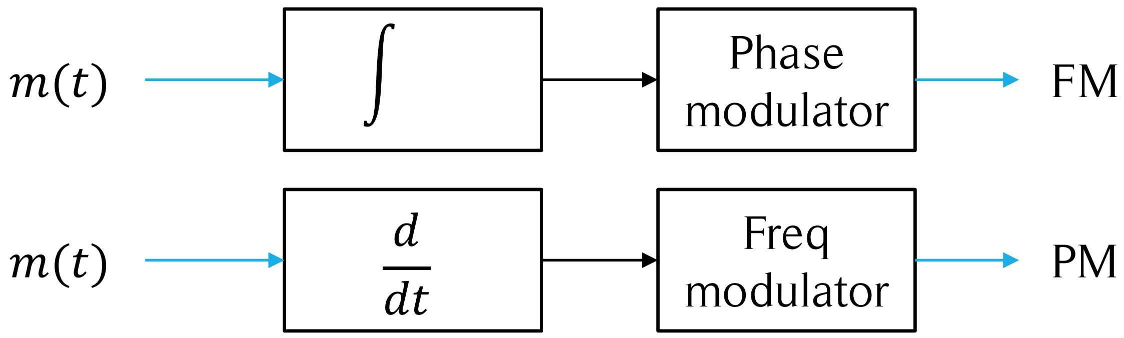
$$\boldsymbol{f_i(t) = f_c + k_f m(t)}$$

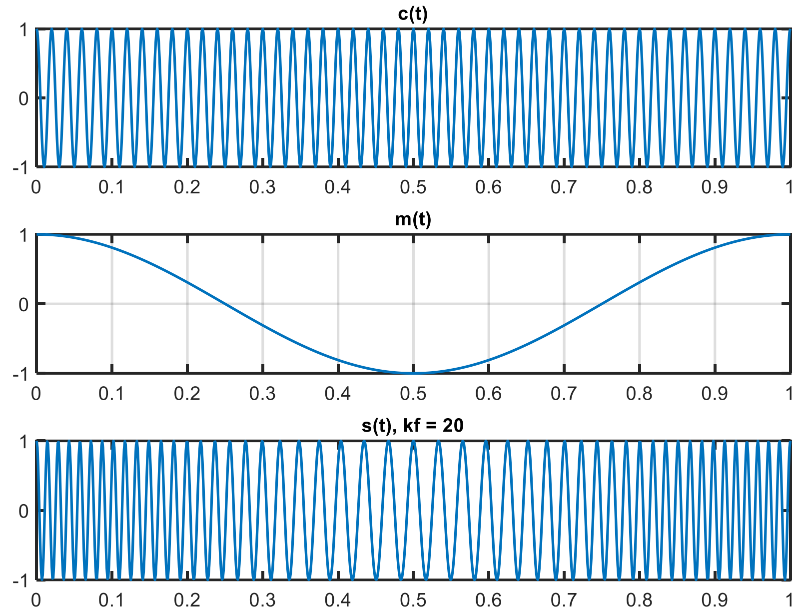
k_f: frequency sensitivity of the modulator [hertz/volt]

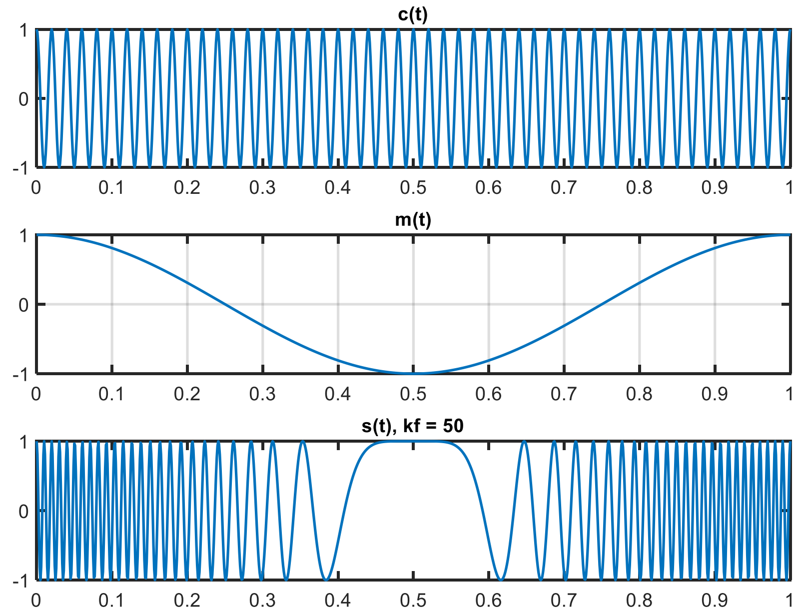
$$\boldsymbol{f_i(t) = \frac{1}{2\pi}\frac{d\,\theta_i(t)}{dt} \Rightarrow \theta_i(t) = 2\pi \int_0^t f_i(t)\,dt}$$

$$\boldsymbol{\theta_i(t) = 2\pi \int_0^t [f_c + k_f m(t)]\,dt = 2\pi f_c t + 2\pi k_f \int_0^t m(t)dt}$$

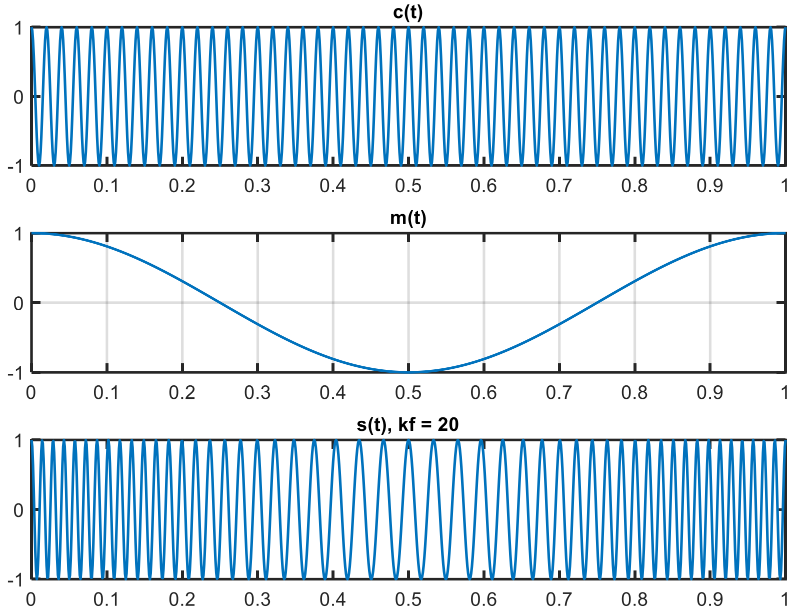
$$\boldsymbol{s(t) = A_c cos\,[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt]}$$

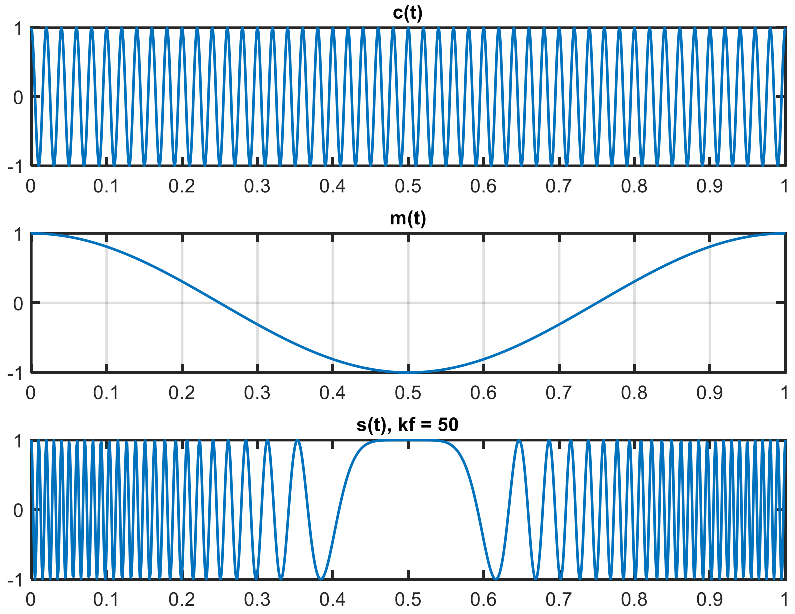






```
%% Matlab code
t = 0:.001:.1;
fm = 10;
ct = cos(2*pi*50*t);
mt = cos(2*pi*fm*t);
kf = 50;
st_FM = cos(2*pi*50*t +
2*pi*kf*sin(2*pi*fm*t)/2/fm/pi) ;

subplot(3,1,1)
plot(t,ct)
title("c(t)")
subplot(3,1,2)
grid on
plot(t,mt)
title("m(t), "+ "f_m = " + fm )
grid on
subplot(3,1,3)
plot(t,st_FM )
title("s(t), "+ "kf = " + kf )
```





Frequency modulation

$$\boldsymbol{s(t) = A_c cos\,[\theta_i(t)] \ = A_c cos\,[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt]}$$

Cases of FM modulation:

1. Single tone m(t) which produces narrow band FM (NBFM)
2. Single tone m(t) which produces wide band FM (WBFM)

$$f_i(t) = f_c + k_f m(t)$$

$$f_i(t) = f_c + \boldsymbol{k_f A_m} cos(2\pi f_m t)$$

$\boldsymbol{\Delta f = k_f A_m}$, frequency deviation

$$\theta_i(t) = 2\pi f_c t + \frac{2\pi k_f A_m \sin(2\pi f_m t)}{2\pi f_m}$$

$$\theta_i(t) = 2\pi f_c t + \frac{\Delta f}{f_m} \sin(2\pi f_m t)$$

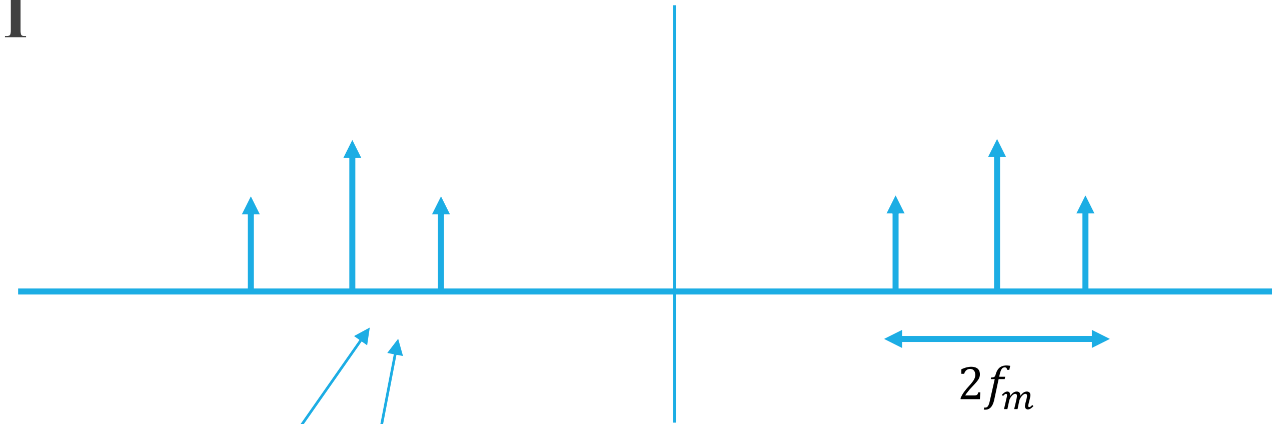
$$\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t), \ \boldsymbol{\beta : modulation\ index}, \ \beta = \frac{k_f A_m}{f_m} = \frac{\boldsymbol{\Delta f}}{\boldsymbol{f_m}}$$

$$\boldsymbol{s(t) = A_c cos\,[2\pi f_c t + \beta \ \sin(2\pi f_m t)]}$$

Types of single tone FM

$\theta_i(t) = 2\pi f_c t + \beta \sin(2\pi f_m t)$

1. If β is small compared to 1 radian ➔ NBFM
2. If β is large compared to 1 radian ➔ WBFM



❑ Narrow band FM

$\boldsymbol{s}(\boldsymbol{t}) = \boldsymbol{A_c}\boldsymbol{cos}\,[2\pi f_c t + \beta\ \sin(2\pi f_m t)]$

$\boldsymbol{s}(\boldsymbol{t}) = \boldsymbol{A_c}\boldsymbol{cos}\,[2\pi f_c t]\ \boldsymbol{cos}\,[\beta\ \sin(2\pi f_m t)]$ - $\mathbf{A_c}\mathbf{sin}[2\pi f_c t]\ \boldsymbol{sin}\,[\beta\ \sin(2\pi f_m t)]$

If $\boldsymbol{\beta}$ is small compared to 1 radian:

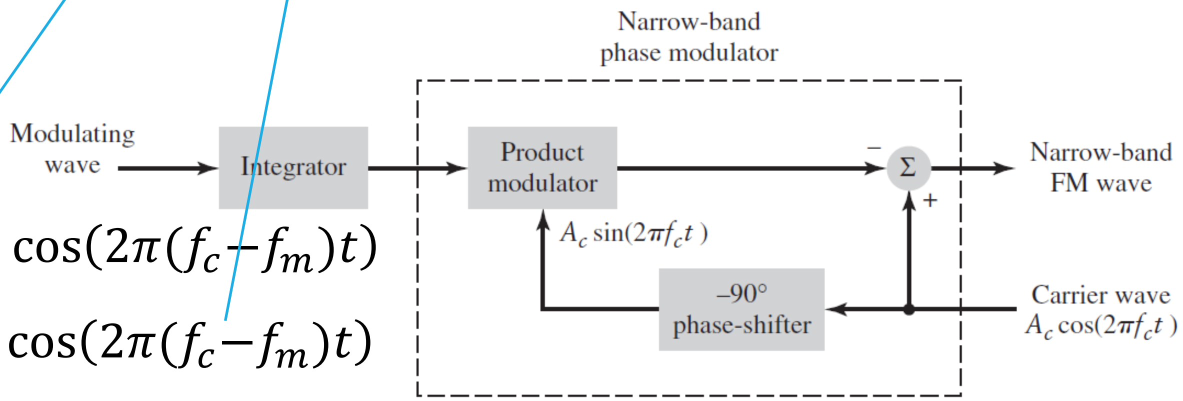
$\boldsymbol{cos}\,[\beta\ \sin(2\pi f_m t)] = 1$

$\boldsymbol{sin}\,[\beta\ \sin(2\pi f_m t)] = \beta\ \sin(2\pi f_m t)$

$\boldsymbol{s}(\boldsymbol{t}) = \boldsymbol{A_c}\boldsymbol{cos}\,[2\pi f_c t] - \mathbf{A_c}\boldsymbol{\beta}\ \mathbf{sin}[2\pi f_c t]\ \sin(2\pi f_m t)$

$\boldsymbol{s_{NBFM}}(\boldsymbol{t}) = \boldsymbol{A_c}\boldsymbol{cos}\,[2\pi f_c t] + \frac{\mathbf{A_c}}{\mathbf{2}}\boldsymbol{\beta}\ \mathbf{cos}[2\pi(f_c+f_m)t] - \frac{\mathbf{A_c}}{\mathbf{2}}\boldsymbol{\beta}\ \cos(2\pi(f_c-f_m)t)$

$\boldsymbol{s_{AM}}(\boldsymbol{t}) = \boldsymbol{A_c}\boldsymbol{cos}\,[2\pi f_c t] + \frac{\mathbf{A_c}}{\mathbf{2}}\boldsymbol{\mu}\ \mathbf{cos}[2\pi(f_c+f_m)t] + \frac{\mathbf{A_c}}{\mathbf{2}}\mu\ \cos(2\pi(f_c-f_m)t)$



❑Wide band FM (WBFM)

$m(t) = A_m \cos 2\pi f_c t$

$\boldsymbol{s(t) = A_c cos}\ [2\pi f_c t + \beta \sin(2\pi f_m t)]$

$s(t) = Re\{A_c e^{j(2\pi f_c t + \beta \sin(2\pi f_m t))}\}$

$s(t) = Re\{A_c e^{j(2\pi f_c t)} e^{j(\beta \sin(2\pi f_m t))}\}$

$\tilde{s}(t) = A_c e^{j(\beta \sin(2\pi f_m t))}$ 🡐 periodic function with fundamental frequency = f_m

Using Fourier series:

$\tilde{s}(t) = \sum_{n=-\infty}^{\infty} C_n e^{j2\pi n f_m t}, \quad T = \frac{1}{f_m}$

$$C_n = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \tilde{s}(t) e^{-j2\pi n f_m t} dt = A_c f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} e^{j(\beta \sin(2\pi f_m t) - j2\pi n f_m t)} dt$$

Let $x = 2\pi f_m t$:

$$C_n = \frac{A_c f_m}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin(x) - nx)} dx = J_n(\beta) A_c \quad \leftarrow J_n(\beta) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j(\beta \sin(x) - nx)} dx$$

$J_n(\beta)$: nth order Bessel function of first kind with argument of β.

$$\tilde{s}(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}$$

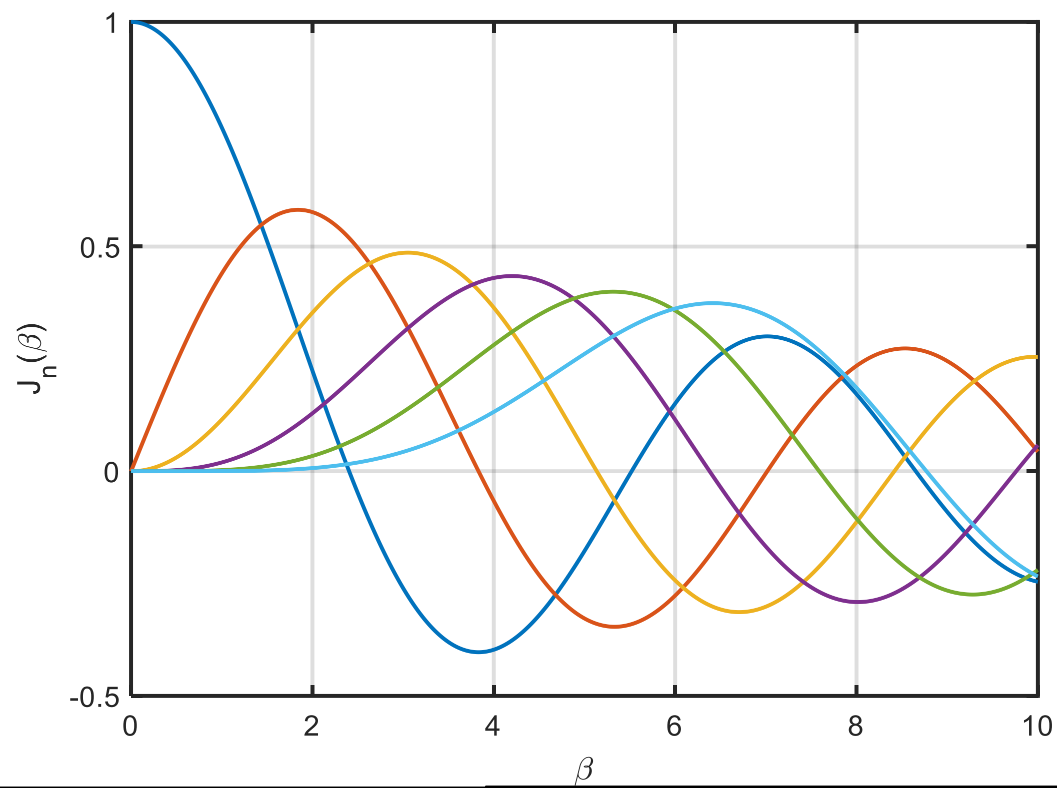
$$s(t) = Re\{\tilde{s}(t) e^{j2\pi f_c t}\}$$

$$s(t) = Re\{A_c\, e^{j2\pi f_c t} \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t}\}$$

$$s(t) = Re\{A_c \sum_{n=-\infty}^{\infty} J_n(\beta) e^{j2\pi n f_m t + j2\pi f_c t}\}$$

$$s(t) = A_c \sum_{n=-\infty}^{\infty} J_n(\beta) \cos(2\pi(f_c + n f_m)t)$$

$$S(f) = \frac{A_c}{2} \sum_{n=-\infty}^{\infty} J_n(\beta) \left[\, \delta(f + f_c + n f_m) + \delta(f - f_c - n f_m)\right]$$




```
%% matlab code ^^
for n = 0:5
beta = 0:.001:10;
y = besselj(n,beta);
plot(beta,y,"LineWidth",2);
hold on
end
grid on
xlabel("\beta")
ylabel("J_n(\beta)")
```

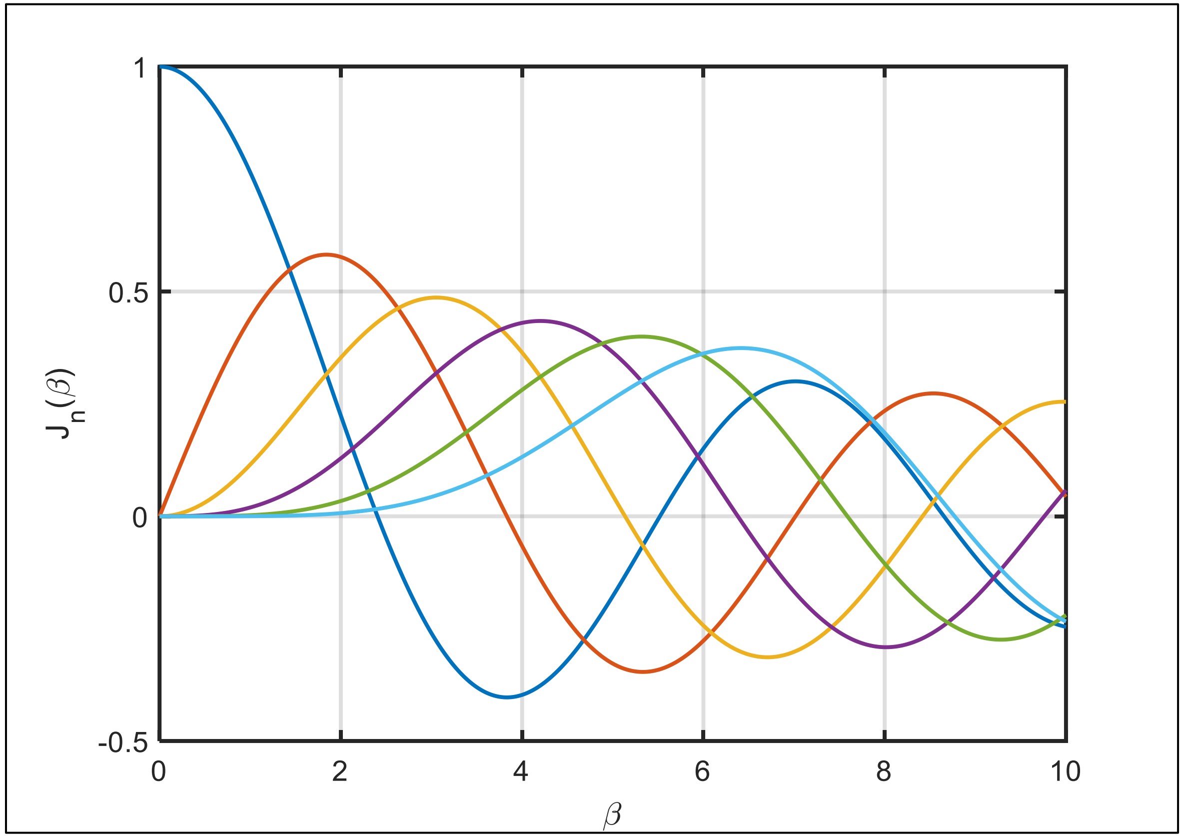
Some properties of Bessel function:

1- For n = even , $J_n(\beta) = J_{-n}(\beta)$

For n = odd , $J_n(\beta) = -J_{-n}(\beta)$,,,,,, $J_n(\beta) = (-1)^n J_{-n}(\beta)$

2- For small values of $\beta, J_0(\beta) = 1, J_1(\beta) = \frac{\beta}{2}, \; J_n(\beta) \cong 0 \; for \; n > 2$

3-$\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$



$$\boldsymbol{s(t) = A_c cos}\,[2\pi f_c t + \beta \sin(2\pi f_m t)]$$

Power of FM modulated signal:

$$P_{FM} = \frac{A_c^2}{2}$$

Power of the carrier before modulation:

$$P_c = \frac{A_c^2}{2}$$

Power of the carrier after modulation:

$$P_c = \frac{\left(A_c J_o(\beta)\right)^2}{2}$$

Side band power after modulation:

$$P_s = \frac{A_c^2}{2} - \frac{\left(A_c J_o(\beta)\right)^2}{2}$$

Transmission Bandwidth of FM Waves

FM wave contains an infinite number of side-frequencies so that the bandwidth required to transmit such a modulated wave is similarly infinite in extent. In practice, however, we find that the FM wave is effectively limited to a finite number of significant side-frequencies compatible with a specified amount of distortion

- CARSON'S RULE

$$BW = 2\Delta f + 2f_m = 2\Delta f\left(1 + \frac{1}{\beta}\right)$$

- UNIVERSAL CURVE FOR FM TRANSMISSION BANDWIDTH

  -Carson's rule is simple to use, but, unfortunately, it does not always provide a good estimate of the bandwidth requirements of communication systems using wideband frequency modulation. For a more accurate assessment of FM bandwidth, we may use a definition based on retaining the maximum number of significant side frequencies whose amplitudes are all greater than some selected value.

  -We may thus define the transmission bandwidth of an FM wave as the separation between the two frequencies beyond which none of the side frequencies is greater than one percent of the carrier amplitude obtained when the modulation is removed.

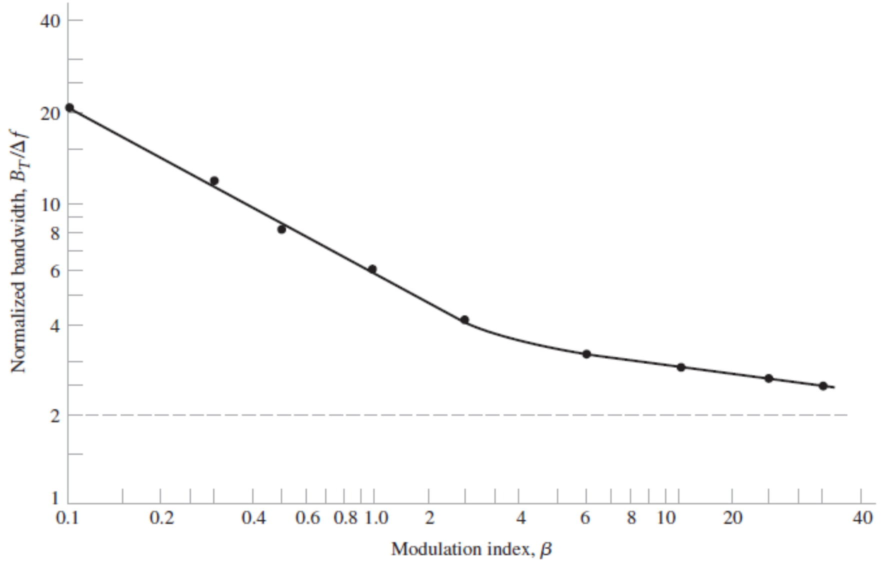
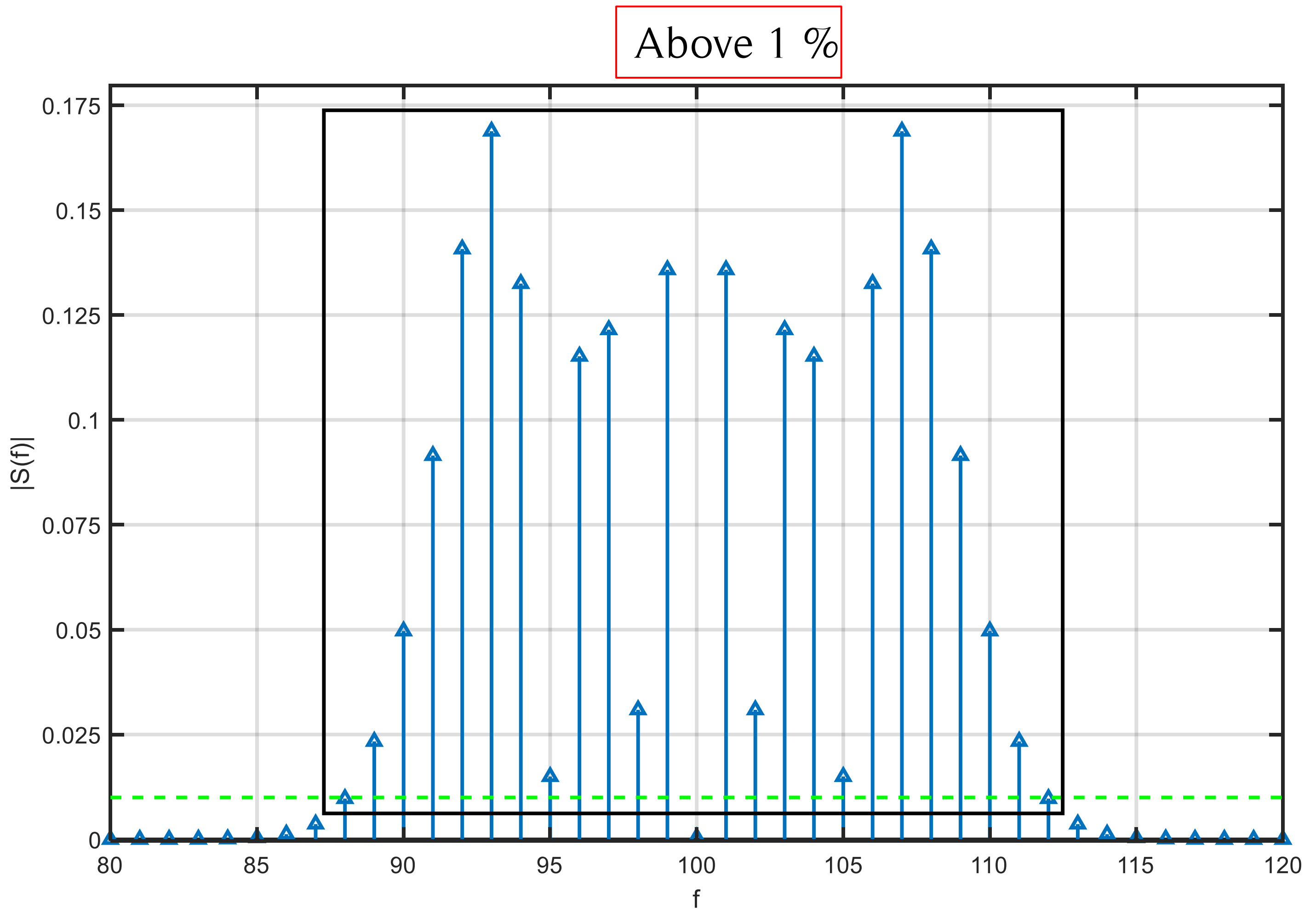


FIGURE 4.9 Universal curve for evaluating the one percent bandwidth of an FM wave.

TABLE 4.2 *Number of Significant Side-Frequencies of a Wide-Band FM Signal for Varying Modulation Index*

Modulation Index β	*Number of Significant Side-Frequencies* $2n_{max}$
0.1	2
0.3	4
0.5	4
1.0	6
2.0	8
5.0	16
10.0	28
20.0	50
30.0	70

```
clear

t = 0:.001:1;
fm = 1;
fc = 100;
beta = 8.65;

mt = 1*cos(2*pi*fm*t);

st = cos(2*pi*fc*t+beta*sin(2*pi*fm*t));

st2 = 0;
N=20;
N = -N:1:N;

for i=1:length(N)
dt(i) = besselj(N(i), beta)/2
end


fss = fc+N*fm;

stem(fss,abs(dt),"^","LineWidth",2)
set(gca,"FontSize",13)
grid on
yticks([0:0.025:1])

xlabel("f")
ylabel("|S(f)|")

hold on
plot( [80,120], [0.01 .01],"g--")
```



ARBITRARY MODULATING WAVE (General Case)

If $m(t)$ has a BW of W, then we define the deviation ratio as:

$$D = \frac{\Delta f}{W}$$

The deviation ratio D plays the same role for no sinusoidal modulation that the modulation index β plays for the case of sinusoidal modulation.

Hence, replacing β by D and replacing f_m with W, we may generalize:

$$BW = 2(\Delta f + W)$$

Ex: Commercial FM Broadcasting

In North America, the maximum value of frequency deviation Δf is fixed at 75 kHz for commercial FM broadcasting by radio. If we take the modulation frequency $W = 15$ kHz which is typically the "maximum" audio frequency of interest in FM transmission, we find that the corresponding value of the deviation ratio is

$$D = \frac{75}{15} = 5 \;\Rightarrow\; BW = 2(75 + 15) = 180 \text{ KHz}$$

On the other hand, use of the universal curve gives the transmission bandwidth of the FM signal to be:

$$BW = 3.2\,\Delta f = 3.2 \times 75 = 240 \text{ KHz}$$

In this example, Carson's rule underestimates the transmission bandwidth by 25 percent compared with the result of using the universal curve

Generation of FM Waves

To design of a frequency modulator, we need a device that produces an output signal whose instantaneous frequency is sensitive to *variations in the amplitude of an input signal in a linear manner*. There are two basic methods of generating frequency-modulated waves, one direct and the other indirect.

Direct method:

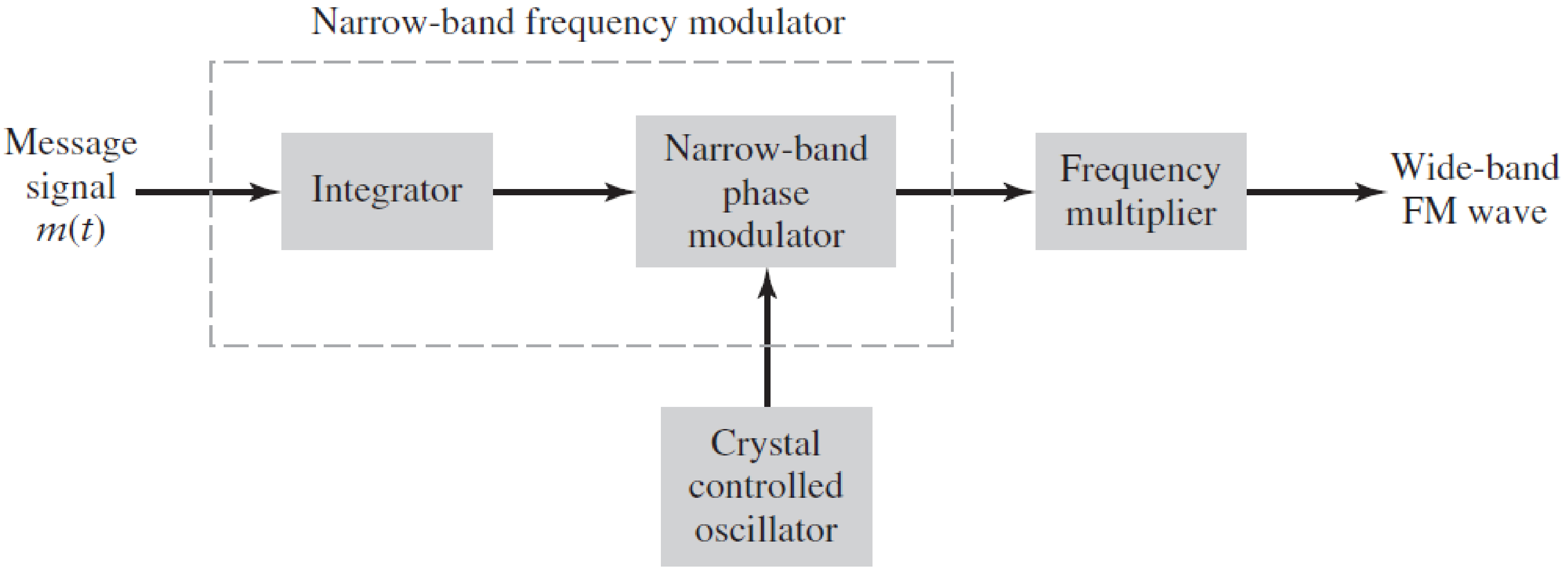
-Direct method uses a sinusoidal oscillator, with one of the reactive elements (e.g., capacitive element) in the tank circuit of the oscillator being directly controllable by the message signal.

-A serious limitation of the direct method is the tendency for the carrier frequency to drift, which is usually unacceptable for commercial radio applications.

INDIRECT METHOD: ARMSTRONG MODULATOR

The message signal is first used to produce a narrow-band FM, which is followed by frequency multiplication to increase the frequency deviation to the desired level.

The carrier-frequency stability problem is alleviated by using a highly stable oscillator (e.g., crystal oscillator) in the narrowband FM generation; this modulation scheme is called the Armstrong wide-band frequency modulator.



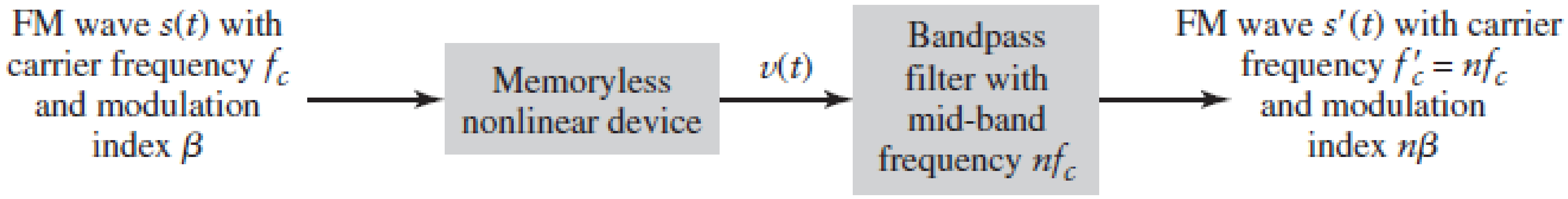


FIGURE 4.11 Block diagram of frequency multiplier.

Voltage controlled oscillator

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)\, c(t)}}$$

For a sinusoidal $m(t)$ modulating signal, with frequency f_m:

$$c(t) = C_o + \Delta C \cos(2\pi f_m t)$$

$$f_i(t) = \frac{1}{2\pi\sqrt{(L_1 + L_2)\,(C_o + k_c m(t))}} = \frac{1}{2\pi\sqrt{(L_1 + L_2)\,(C_o + A_c k_c \cos(2\pi f_m t))}}$$

C_o: total capacitance in absence of the $m(t)$.

ΔC: maximum change in the capacitance.

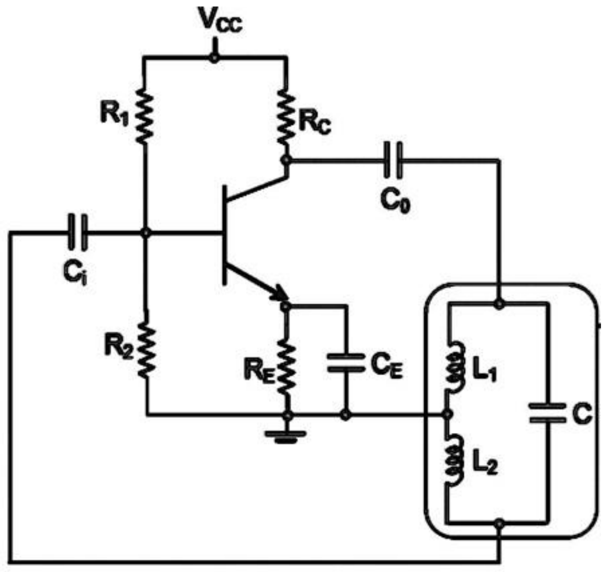
$$f_i(t) = f_o\left(1 + \frac{\Delta C}{C_o}\cos(2\pi f_m t)\right)^{-\frac{1}{2}}$$

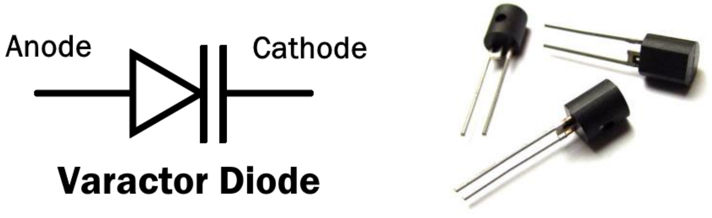
If ΔC is small compared to C_o:

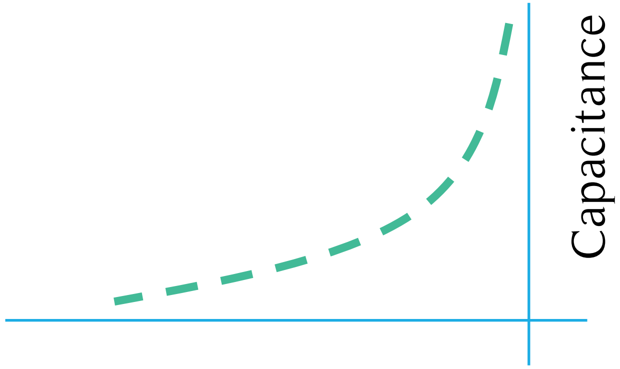
$$f_i(t) = f_o(1 - \frac{\Delta C}{2C_o}\cos(2\pi f_m t))$$

$$f_i(t) = f_o + \Delta f cos(2\pi f_m t) \;\; ; \;\; \Delta f = -\frac{\Delta C f_o}{2C_o}$$

The capacitance could be a varactor diode. The varactor diode is used in a place where the variable capacitance is required, and that capacitance is controlled with the help of the voltage

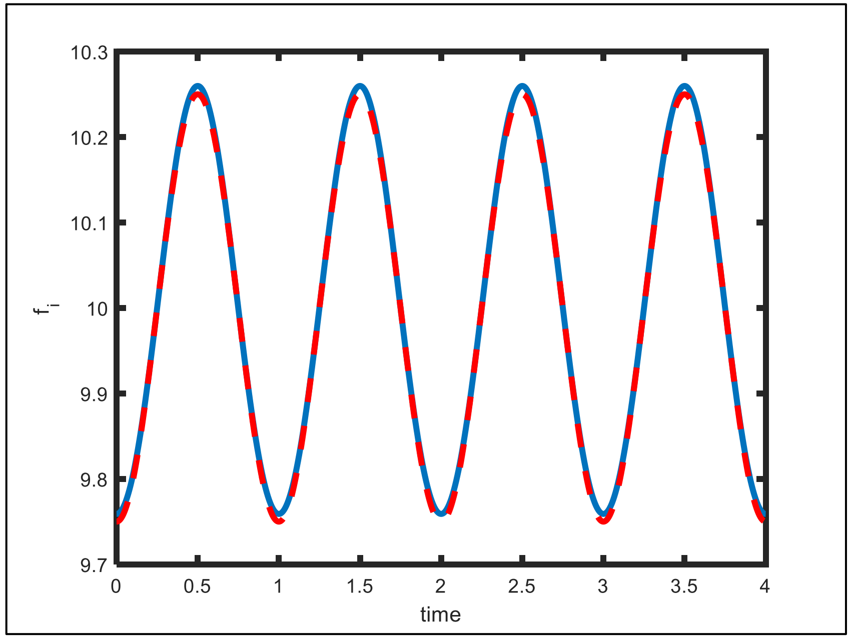


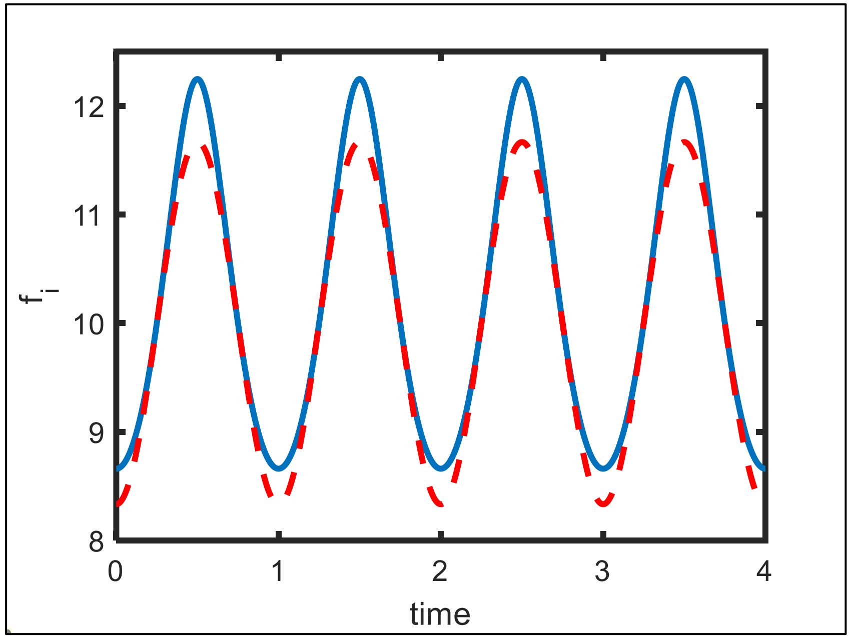




```
f0 = 10;
delta_c = 1;
c0 = 20;
fm = 1;
t = 0:.001:4;
fi1 = f0*(1+delta_c./c0.*cos(2*pi*fm*t)).^(-.5);
delta_f = -delta_c*f0/2/c0
fi_approx = f0+delta_f.*cos(2*pi*fm*t);
plot(t,fi1,t,fi_approx,"--r")
hold on
xlabel("time")
ylabel("f_i")
```

```
f0 = 10;
delta_c = 1;
c0 = 3;
fm = 1;
t = 0:.001:4;
fi1 = f0*(1+delta_c./c0.*cos(2*pi*fm*t)).^(-.5);
delta_f = -delta_c*f0/2/c0
fi_approx = f0+delta_f.*cos(2*pi*fm*t);
plot(t,fi1,t,fi_approx,"--r")
hold on
xlabel("time")
ylabel("f_i")
```





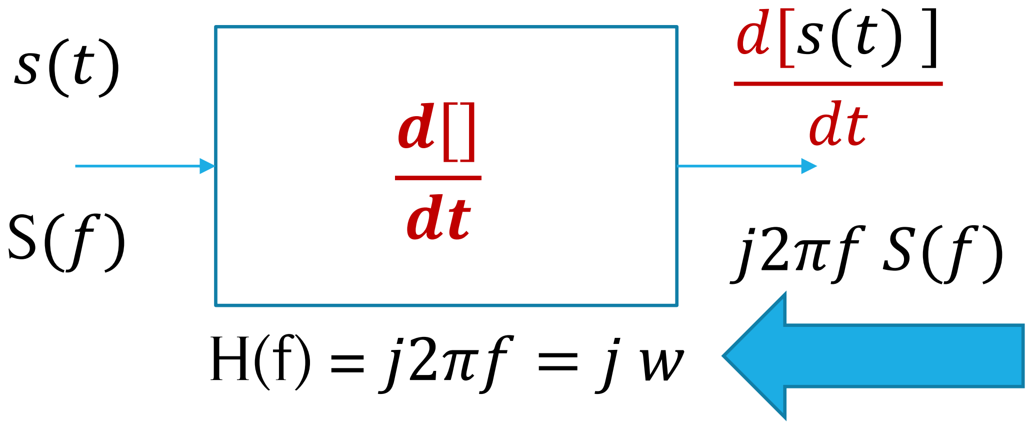
Demodulation of FM signal

With the frequency modulator being a device that produces an output signal whose instantaneous frequency varies linearly with the amplitude of the input message signal, it follows that for frequency demodulation we need a device whose output amplitude is sensitive to variations in the instantaneous frequency of the input FM wave in a linear manner too.

We will discuss two types of FM demodulation:

1- Frequency discriminator

2- Phase-locked loop (PLL)



Frequency discriminator

The frequency discriminator consists of two parts –

1- SLOPE CIRCUIT (diff)

2- ENVELOP DETECTOR

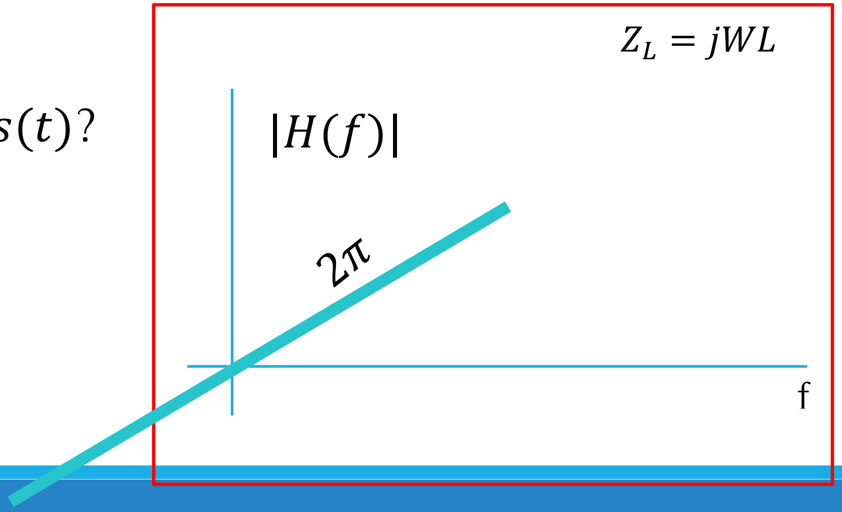
Remember: $\boldsymbol{s(t) = A_c cos\,[2\pi f_c t + 2\pi k_f \int_0^t m(t)dt]}$

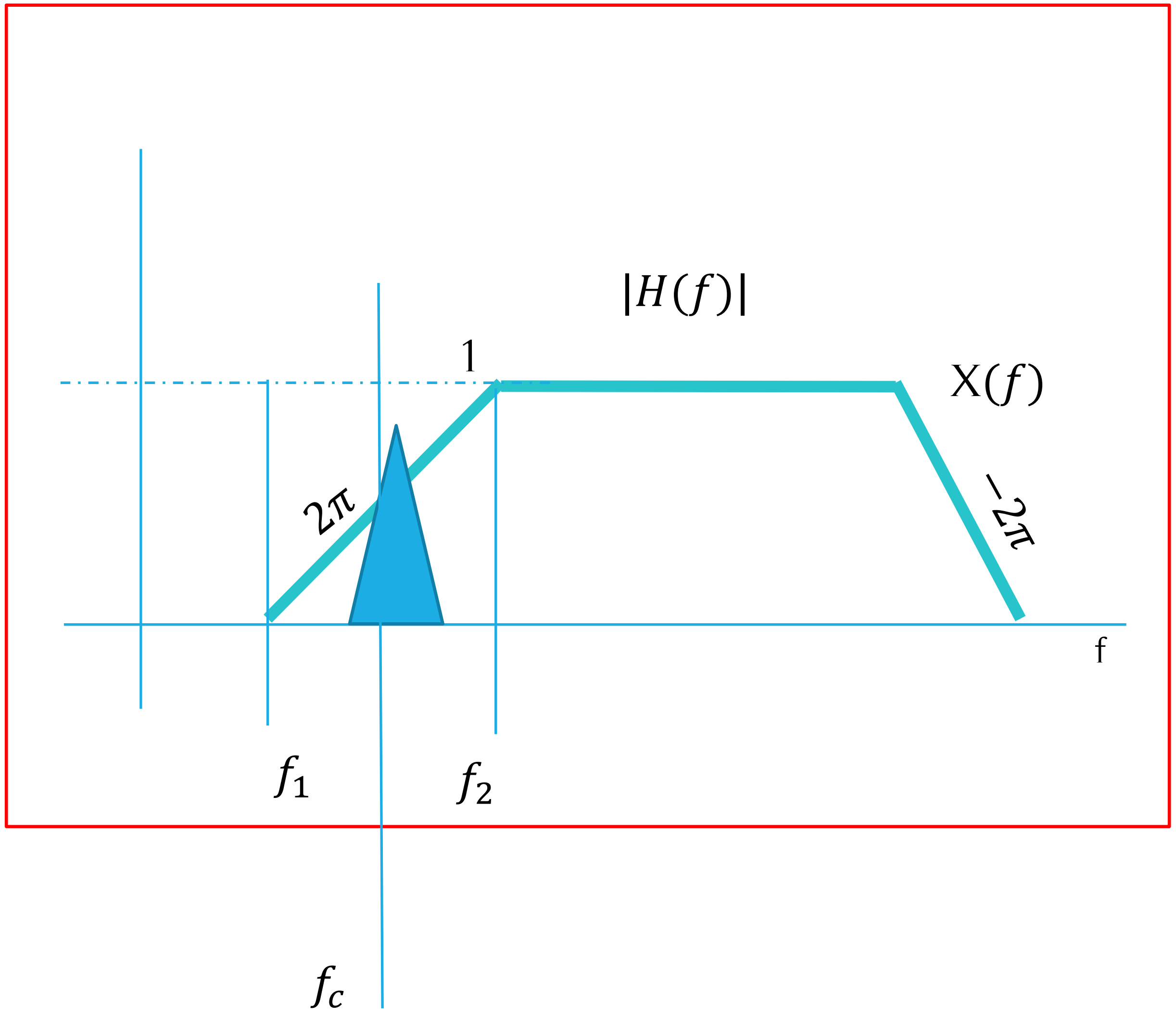
How do we recover the message signal $m(t)$ from the modulated signal $s(t)$?

Differentiation?

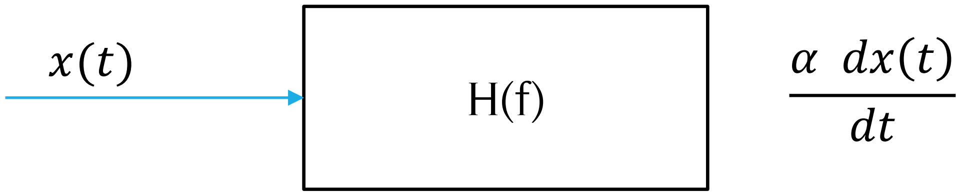
$$\frac{ds(t)}{dt} = -(2\pi f_c + 2\pi k_f m(t))A_c \sin(2\pi f_c t + 2\pi k_f \int^t m(t)dt)$$

$$\boldsymbol{H_1(f) = j2\pi\left[f - \left(f_c - \frac{B_T}{2}\right)\right], f_c - \frac{B_T}{2} \le |f| \le f_c + \frac{B_T}{2}}$$

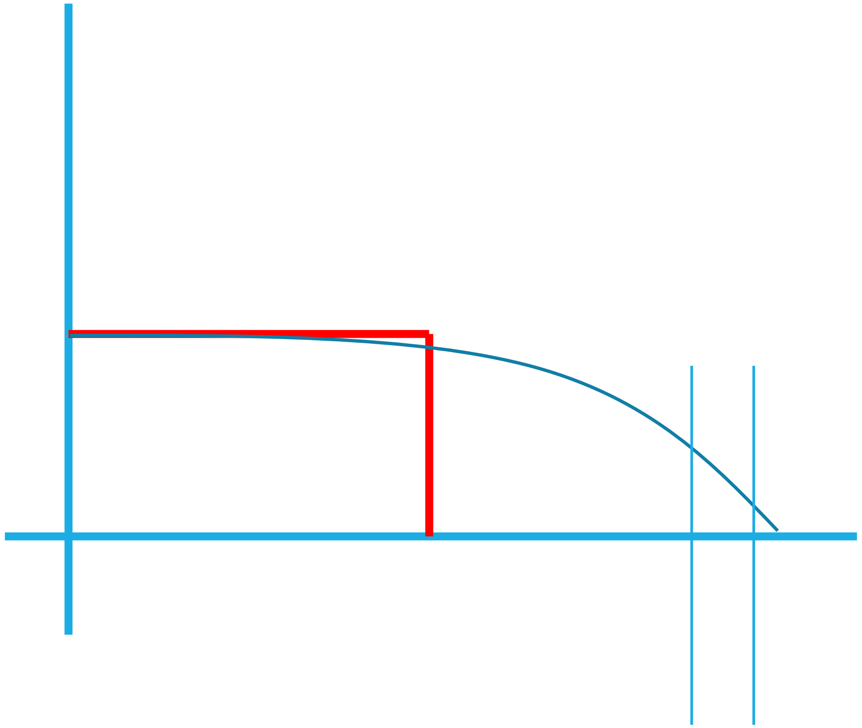




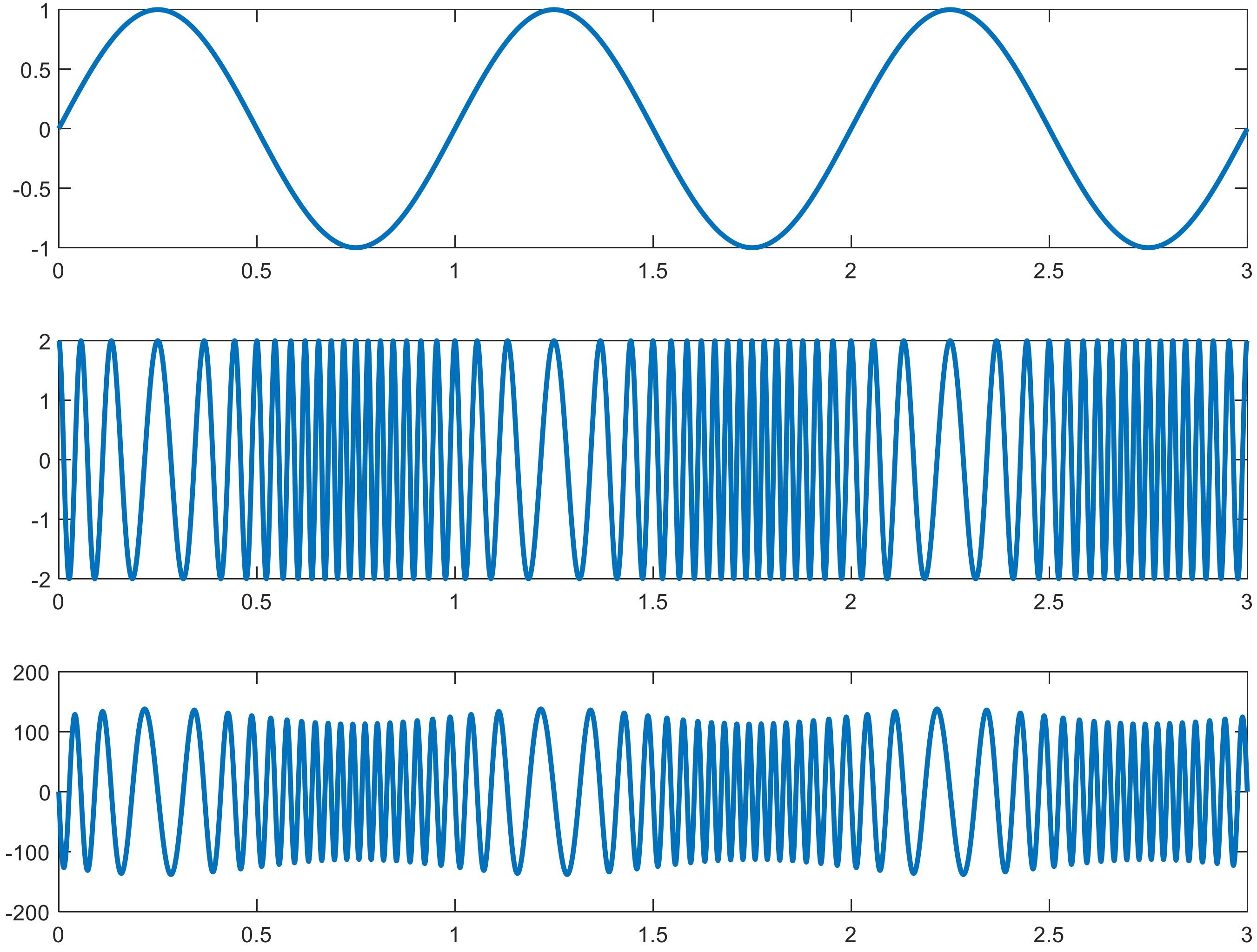
$|H(f)|$
1
$X(f)$
2π
-2π
f
f_1
f_2
f_c



$x(t)$
H(f)
$\frac{\alpha \ dx(t)}{dt}$



```
fm = 1;
fc = 20;
t = 0:.001:3;
Ac = 2;
kf= 2;

mt = sin(2*pi*fm*t)

subplot(3,1,1)
plot(t,mt,"LineWidth",2)
st = Ac*cos(2*pi*fc*t+
2*pi*kf*cos(2*pi*fm*t));

subplot(3,1,2)
plot(t,st,"LineWidth",2)
subplot(3,1,3)

dsdt = -(2*pi*fc +
2*pi*kf*sin(2*pi*fm*t))...
     .*sin(2*pi*fc*t+
2*pi*kf*cos(2*pi*fm*t));
plot(t,dsdt,"LineWidth",2)
```



It is simplest to proceed with a complex baseband representation of the signal processing performed by the discriminator

$$\tilde{s}(t) = A_c \exp(j2\pi k_f \int^t m(t)dt)$$

we may express the complex baseband filter (i.e., slope circuit) as:

$$\widetilde{H_1}(f) = \begin{cases} j2\pi\left[f + \frac{B_T}{2}\right], & -\frac{B_T}{2} \le |f| \le \frac{B_T}{2} \\ 0, & ow \end{cases}$$

$$\tilde{S}_1(f) = \frac{1}{2}\widetilde{H_1}(f)\,\tilde{S}(f) = \begin{cases} j2\pi\left[f + \frac{B_T}{2}\right]\tilde{S}(f), & -\frac{B_T}{2} \le |f| \le \frac{B_T}{2} \\ 0, & ow \end{cases}$$

$$\tilde{s}_1(t) = \frac{1}{2}\frac{d}{dt}\tilde{s}(t) + \frac{1}{2}j\pi B_T \tilde{s}(t)$$

$$\tilde{s}_1(t) = \frac{1}{2}j\pi A_c B_T\left[1 + \left(\frac{2k_f}{B_T}\right)m(t)\right]\exp(j2\pi k_f \int^t m(t)dt)$$

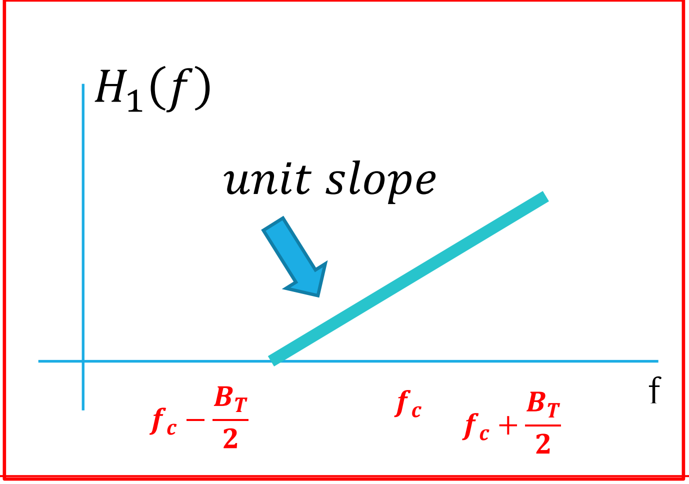
$$s_1(t) = \boldsymbol{Re}[\tilde{s}_1(t)\exp(j2\pi f_c t)]$$

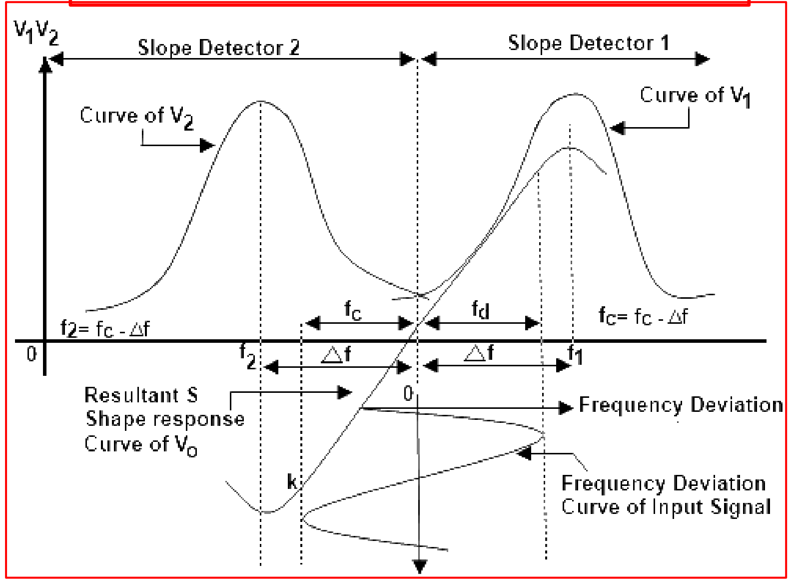
$$= \frac{1}{2}j\pi A_c B_T\left[1 + \left(\frac{2k_f}{B_T}\right)m(t)\right]\cos(2\pi f_c t + j2\pi k_f \int^t m(t)dt + \frac{\pi}{2})$$

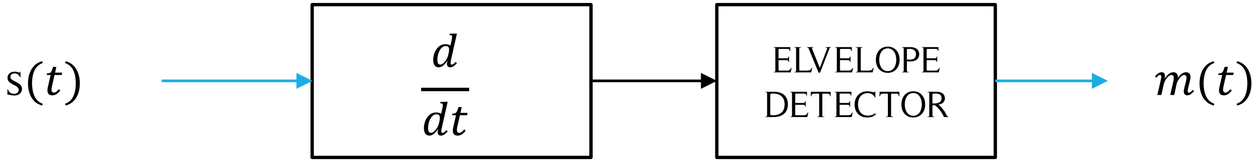
➔ $v_1(t) = \frac{1}{2}\pi\, A_c B_T\, [1 + \left(\frac{2k_f}{B_T}\right)m(t)]$

➔ The bias in is defined by the constant term

➔ To remove the bias, we may use a second slope circuit followed by an envelope detector of its own.





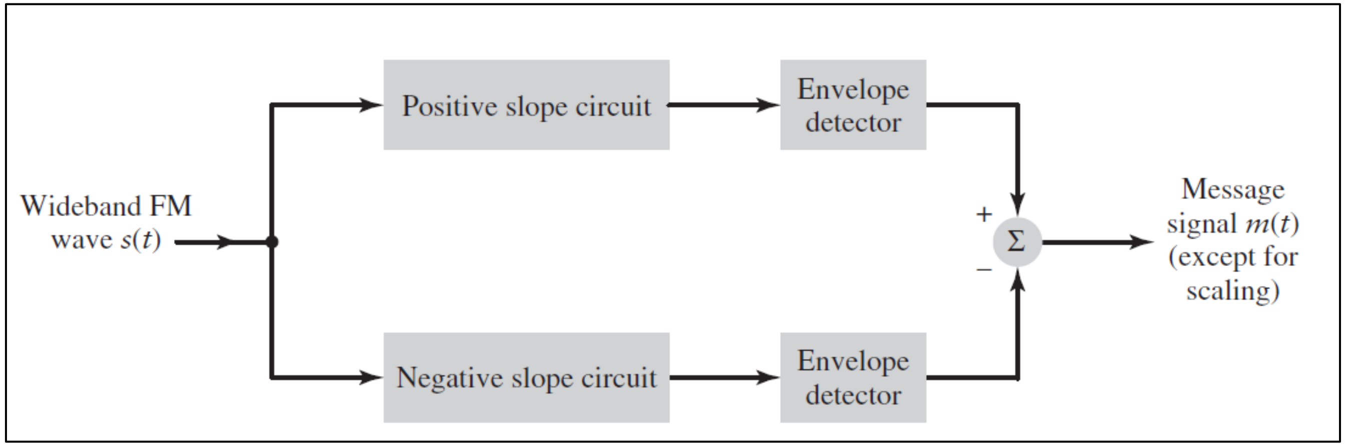


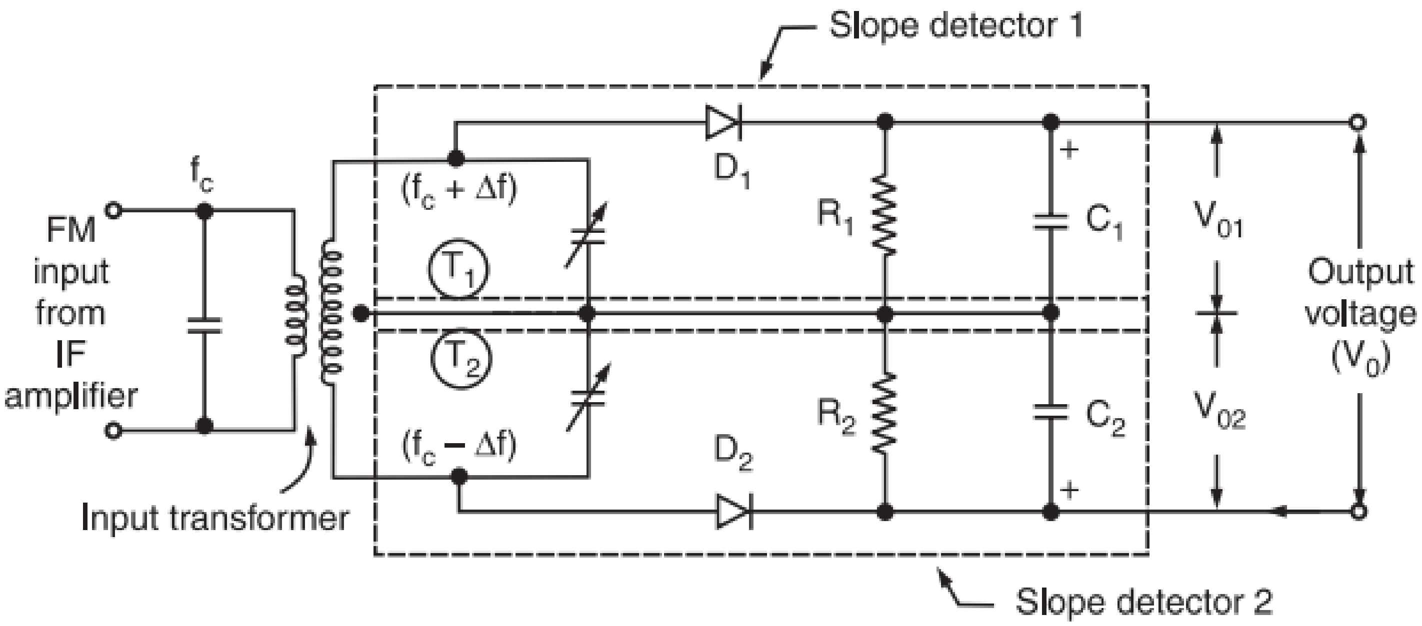
To remove the bias, we may use a second slope circuit followed by an envelope detector of its own

$$\Rightarrow v_1(t) = \frac{1}{2}\pi\, A_c B_T\, [1 + \left(\frac{2k_f}{B_T}\right) m(t)]$$

$$v_2(t) = \frac{1}{2}\pi\, A_c B_T\, [1 - \left(\frac{2k_f}{B_T}\right) m(t)]$$

$$v(t) = v_1(t) - v_2(t) = c\, m\,(t)$$

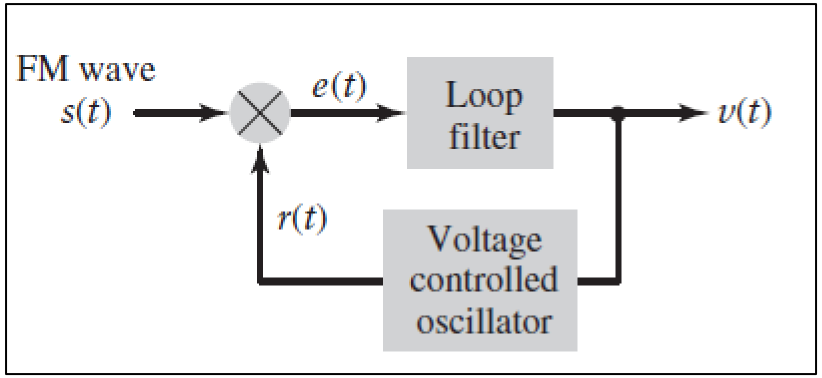




PHASE-LOCKED LOOP (PLL)

The phase-locked loop is a feedback system whose operation is closely linked to frequency modulation. It is commonly used for carrier synchronization, and indirect frequency demodulation. the phase-locked loop consists of three major components:

- *Voltage-controlled oscillator (VCO), which performs frequency modulation on its own control signal.*
- *Multiplier, which multiplies an incoming FM wave by the output of the voltage-controlled oscillator.*
- *Loop filter of a low-pass kind, the function of which is to remove the high-frequency components contained in the multiplier's output signal and thereby shape the overall frequency response of the system.*



- If $v(t)$ (control voltage) is zero, then the VCO is adjusted to satisfy the following conditions:
  1. The output frequency of the VCO is f_c
  2. The output voltage of the VCO has 90^o phase shift with respect to the carrier

$$c(t) = A_c \cos(2\pi f_c t)$$

$$r(t) = A_v \cos(2\pi f_c t - \frac{\pi}{2})$$

$$s(t) = A_c \cos\big(2\pi f_c t + \phi_1(t)\big) = A_c \cos(2\pi f_c t + 2\pi k_f \int^t m(t)dt)$$

$$r(t) = A_v \sin\big(2\pi f_c t + \phi_2(t)\big) = A_v \sin(2\pi f_c t + 2\pi k_v \int^t v(t)dt)$$

$e(t)$ will be of two terms:

- A high-frequency component, which is defined by the double-frequency term:

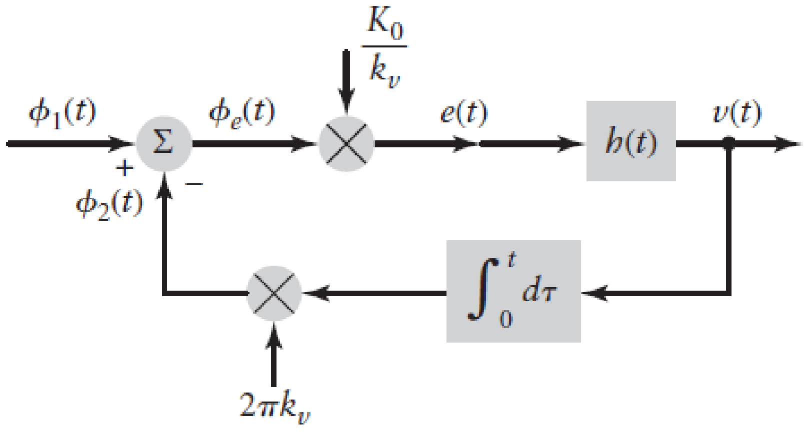
$$k_m A_c A_v \sin(4\pi f_c t + \phi_1(t) + \phi_2(t))$$

- A low-frequency component, which is defined by the difference-frequency term:

$$k_m A_c A_v \sin(\phi_1(t) - \phi_2(t))$$

The loop-filter is designed to suppress the high-frequency components in the multiplier's output:

$$e(t) = k_m A_c A_v \sin(\phi_1(t) - \phi_2(t)) = k_m A_c A_v \sin(\phi_e(t))$$



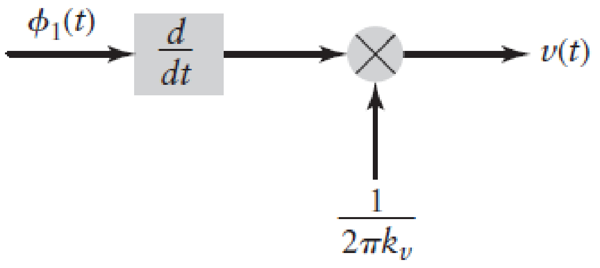
(a)

$\phi_e(t) = \phi_1(t)$ - $2\pi k_v \int^t v(t)dt$

When the phase error $\phi_e(t)$ is zero the phase-locked loop is said to be in phase-lock. It is said to be near-phase-lock when the phase error is small compared with one radian, under which condition we may use the approximation:

$$\sin(\phi_e(t)) \approx \phi_e(t)$$

$$\Rightarrow e(t) = k_m A_c A_v\ \phi_e(t) = \frac{K_0}{K_v}\ \phi_e(t)$$



(b)

loop-gain parameter of the phase-lock loop : $K_0 = k_m k_v A_c A_v$

➔ The error signal $e(t)$ acts on the loop filter to produce the overall output $v(t)$

$v(t)$ can be measured as:

$$v(t) = \int_{-\infty}^{\infty} e(\tau)h(t-\tau)d\tau$$

From linear feedback theory, we recall the following important theorem

When the open-loop transfer function of a linear feedback system has a large magnitude compared with unity for all frequencies, the closed-loop transfer function of the system is effectively determined by the inverse of the transfer function of the feedback path.

Stated in another way, the closed-loop transfer function of the feedback system becomes essentially independent of the forward path.

- The inverse of this feedback path is described in the time domain by the scaled differentiator:

$$v(t) = \frac{1}{2\pi k_v}\left(\frac{d\phi_2(t)}{dt}\right)$$

- The closed-loop time-domain behavior of the phase-locked loop is described by the overall output $v(t)$ produced in response of he angle $\phi_1(t)$
- The magnitude of the open-loop transfer function of the phase-locked loop is controlled by the loop-gain parameter K_o

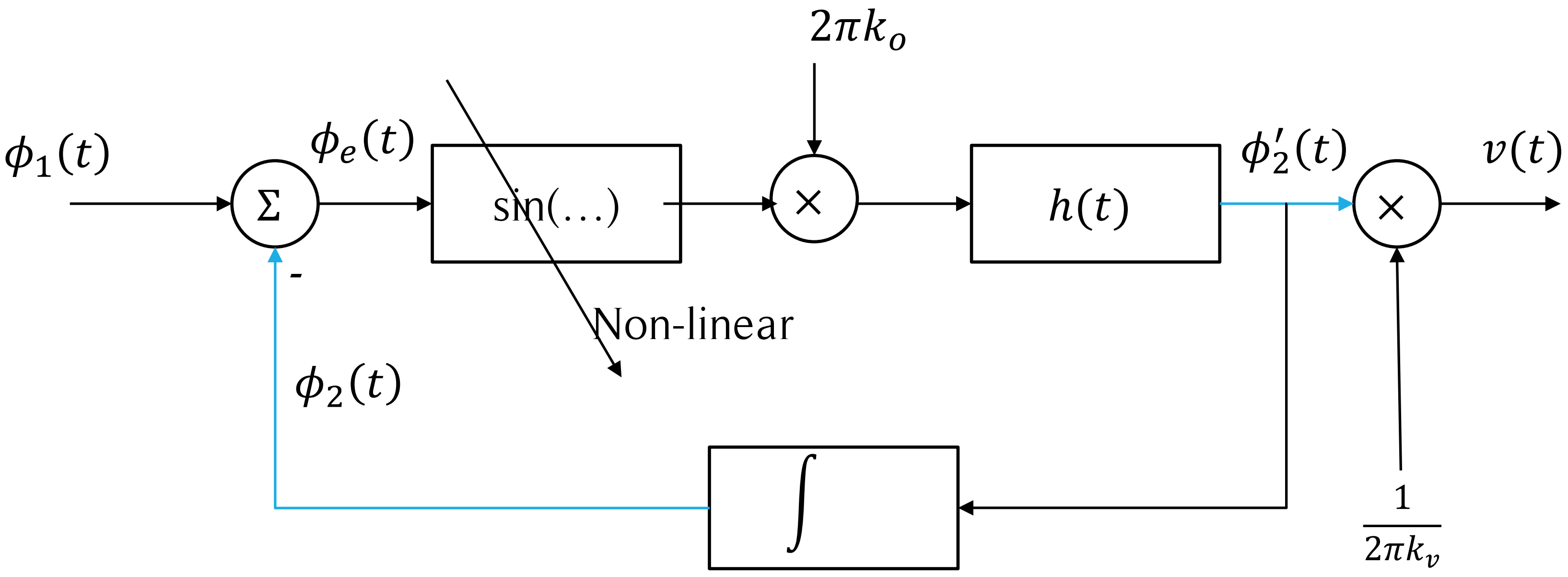
Assuming that the loop-gain parameter is large compared with unity, application of the linear feedback theorem to the model:

$$v(t) \approx \frac{1}{2\pi k_v}\left(\frac{d\phi_1(t)}{dt}\right) = \frac{1}{2\pi k_v}\frac{d}{dt}\left(2\pi k_f \int^t m(t)dt\right) = \frac{k_f}{k_v}\, m(t)$$

$$\phi_2(t) = 2\pi k_v \int^t v(t)dt \;\Rightarrow\; v(t) = \frac{1}{2\pi\, k_f}\frac{d\phi_2(t)}{dt}$$

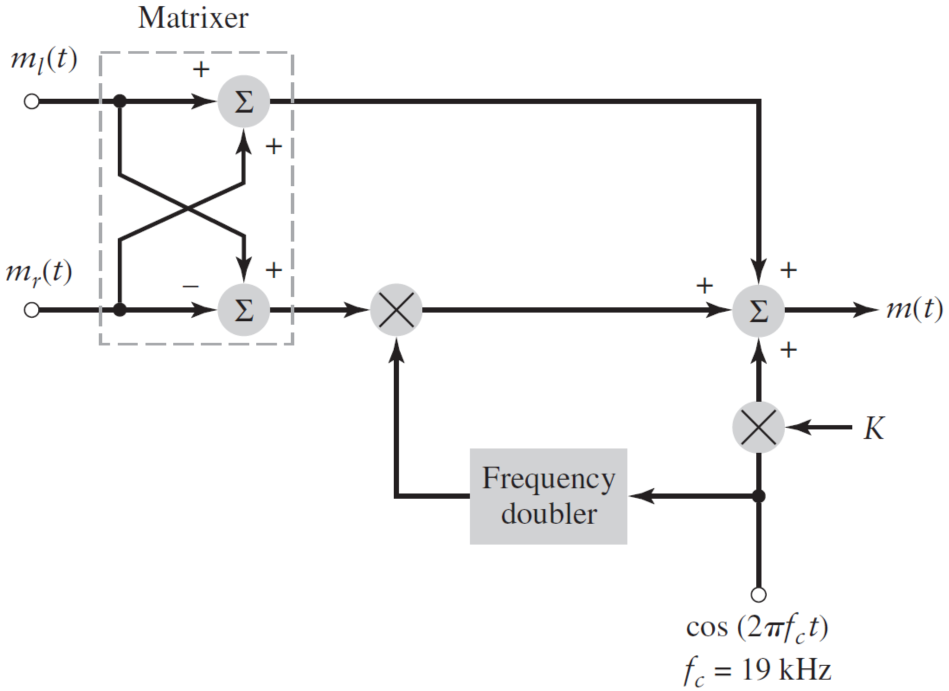
$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi k_v \int_{-\infty}^{\infty} e(\tau)h(t-\tau)d\tau$$

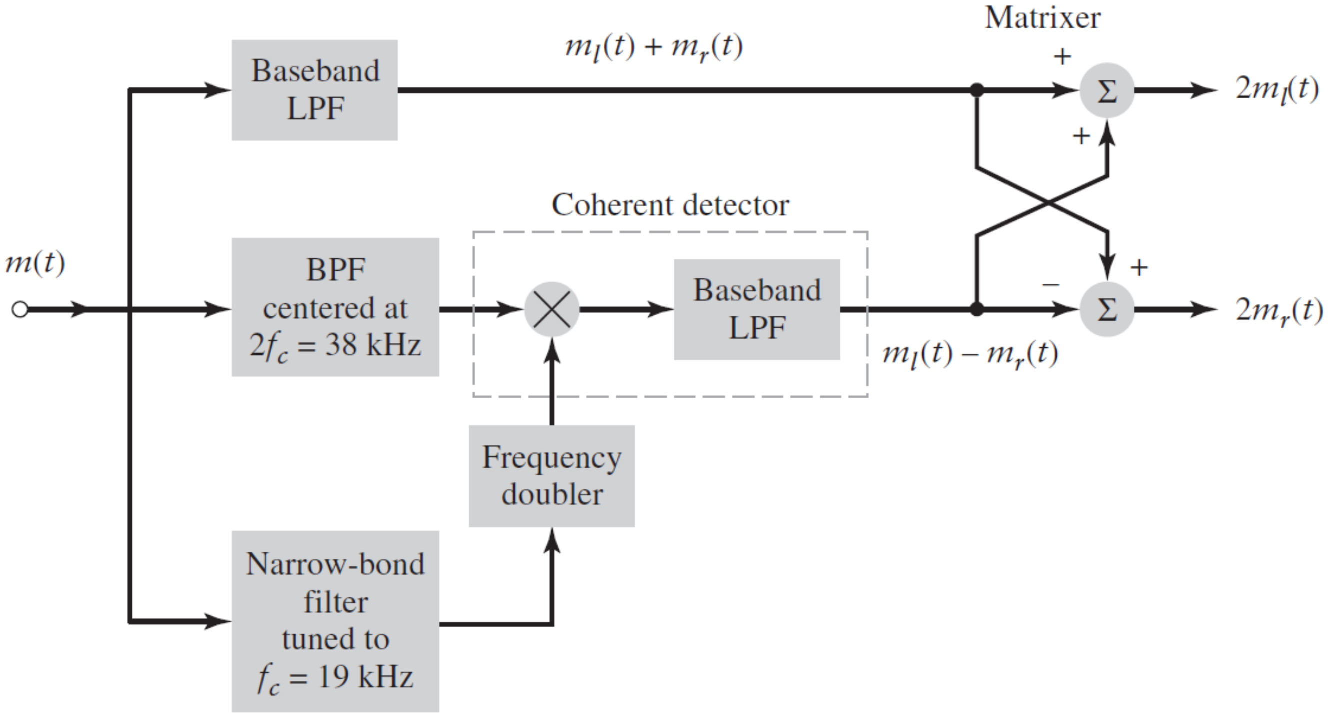
$$\frac{d\phi_e(t)}{dt} = \frac{d\phi_1(t)}{dt} - 2\pi k_v \int_{-\infty}^{\infty} \sin(\phi_e)h(t-\tau)d\tau$$



FM STEREO

Stereo multiplexing is a form of frequency-division multiplexing (FDM) designed to transmit two separate signals via the same carrier. It is widely used in FM radio broadcasting to send two different elements of a program



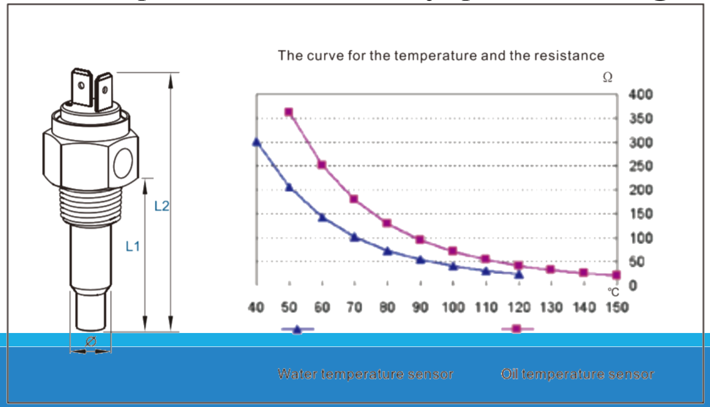


PULSE MODULATION: TRANSITION FROM ANALOG TO DIGITAL COMMUNICATIONS

In continuous-wave (CW) modulation, which we studied in AM and FM, some parameter of a sinusoidal carrier wave is varied continuously in accordance with the message signal.
In pulse modulation, some parameter of a pulse train is varied in accordance with the message signal. In this context, we may distinguish two families of pulse modulation:

1. **Analog pulse modulation:** periodic pulse train is used as the carrier wave, and some characteristic feature of each pulse (e.g., amplitude, duration, or position) is varied in a continuous manner in accordance with the corresponding sample value of the message signal.

2. **Digital pulse modulation:** the message signal is represented in a form that is discrete in both time and amplitude, thereby permitting its transmission in digital form as a sequence of coded pulses.



$50^{o}\ C$ (DECIMAL) ➔$[110010]_2$
50.00001300434450 ➔
$[100011100001101111001100001010110001110001110000010010]_2$

Sampling Process

In signal processing, sampling is the reduction of a continuous-time signal to a discrete-time signal. A common example is the conversion of a sound wave to a sequence of "samples".

The sampling is being made with a sampling period of T_s which is taken to be $\leq \frac{T_f}{2}$, where T_f is the maximum frequency in the message signal $g(t)$.

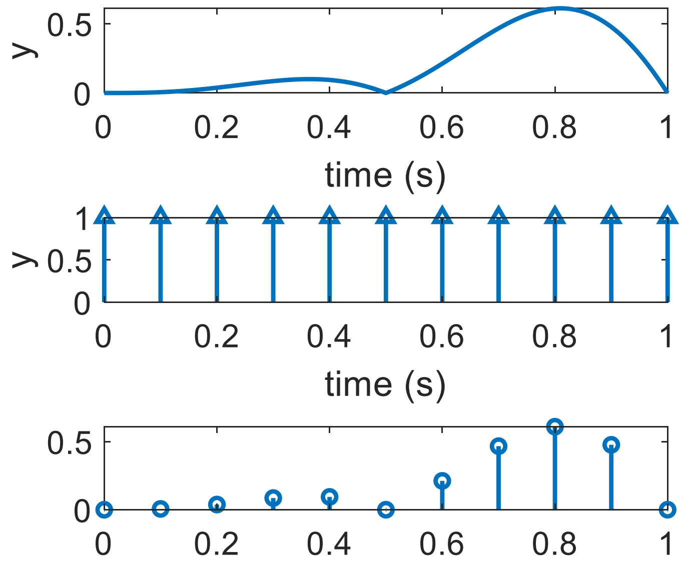
$$T_s = \frac{1}{f_s}$$

This ideal form of sampling is called instantaneous sampling.

The samples version of $g(t)$ can be writes as:

$$g_s(t) = \sum_{n=-\infty}^{\infty} g(nT_s)\delta(t - nT_s)$$

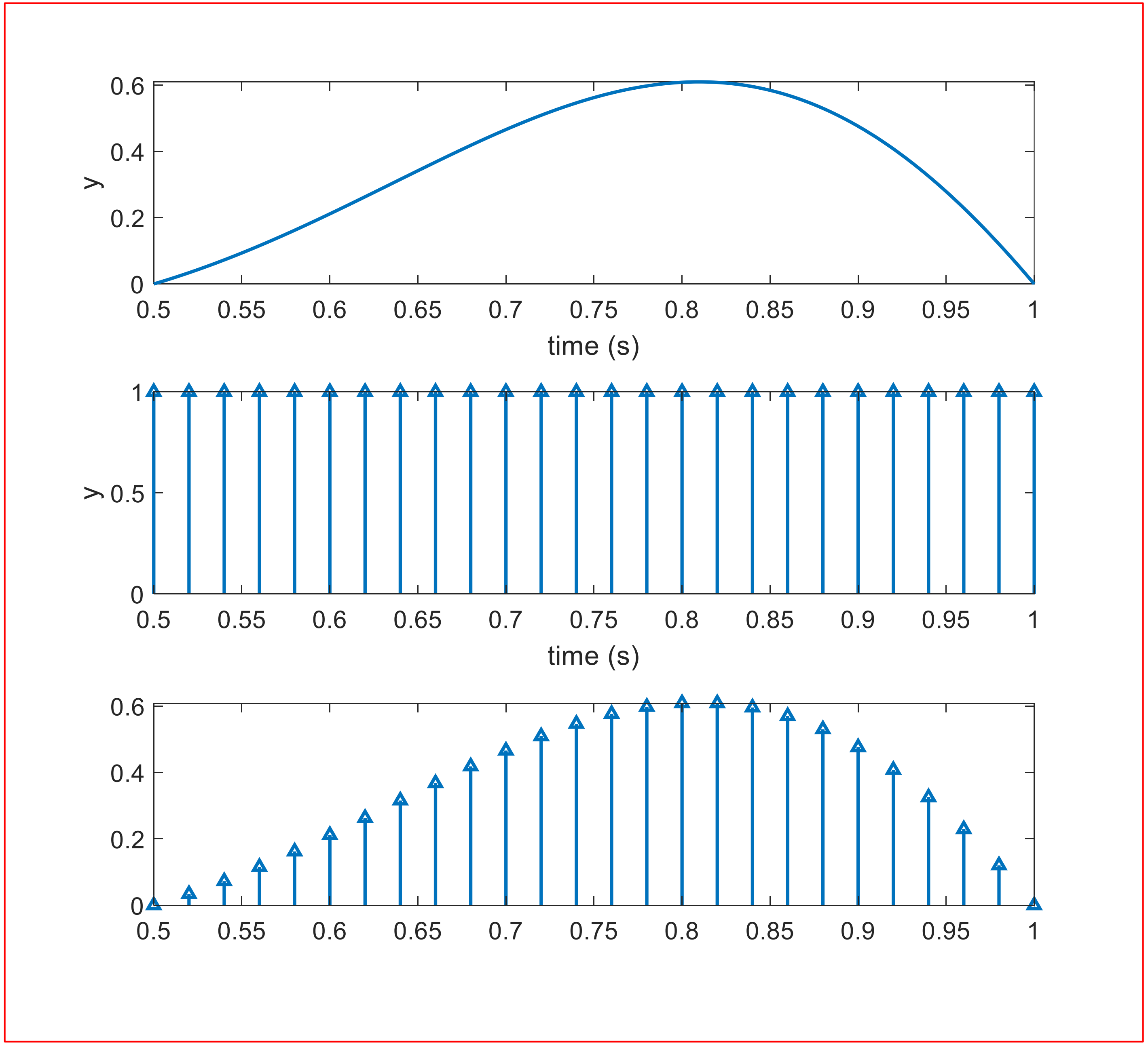
$$\{g(nT_s)\} = \{\dots, g(-T_s), g(0) \dots . g(1T_s), g(2T_s), \dots .\}$$



$f_s = 10\ samples/second$

Sampling Process

```
clear
clf
ts = .02;

t1 = .5:ts:1;
t2 = .5:.001:1;

stream = ones(1,length(t1));
y1 = t1.^2.*sin(2*pi*1*t1) ;
y2 = t2.^2.*sin(2*pi*1*t2) ;

subplot(3,1,1)
plot(t2,abs(y2),'LineWidth',2)
xlabel("time (s)")
ylabel("y")
set(gca,'FontSize',15)
subplot(3,1,2)
stem(t1,stream,'^','LineWidth',2)
xlabel("time (s)")
ylabel("y")
set(gca,'FontSize',15)

subplot(3,1,3)
xlabel("time (s)")
ylabel("y")

stem(t1,abs(y1),'^','LineWidth',2)
set(gca,'FontSize',15)
```



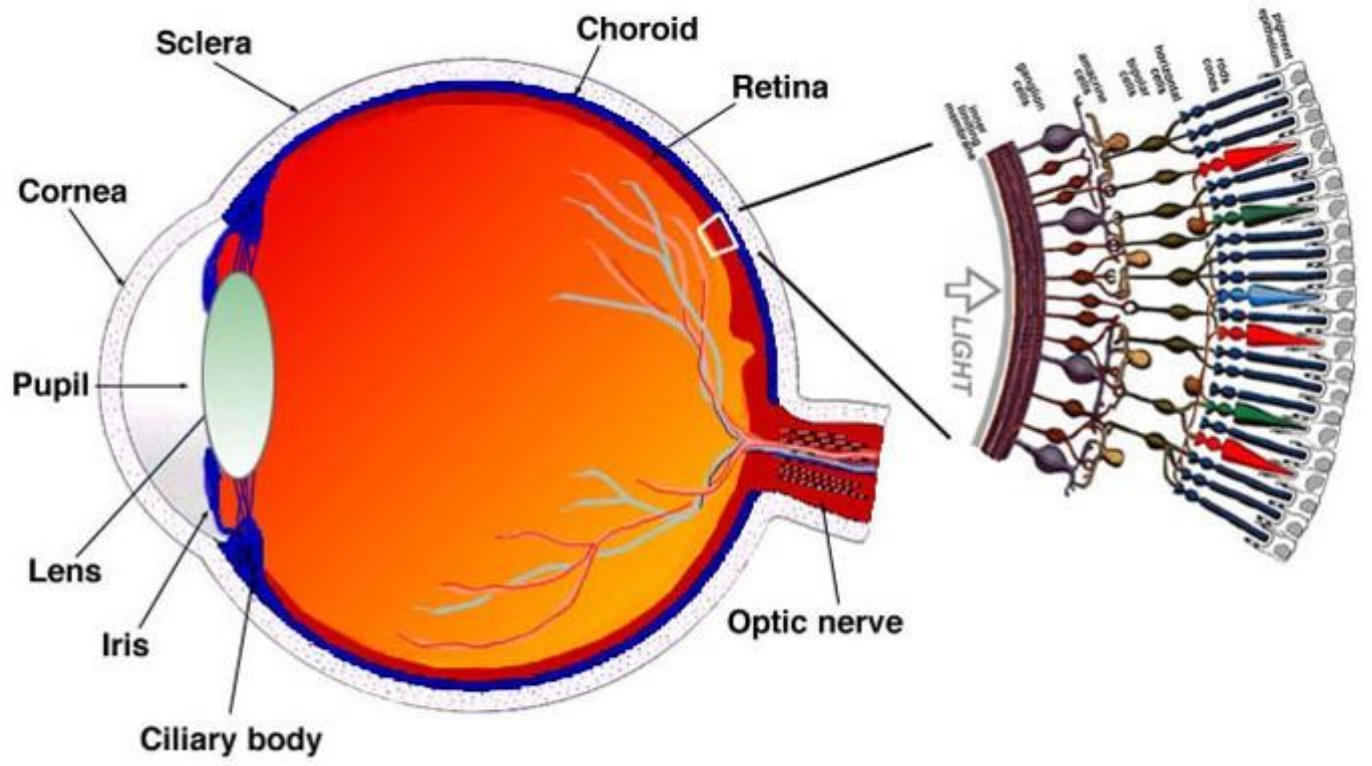


Fig. 1.1. A drawing of a section through the human eye with a schematic enlargement of the retina.

A human retina is less than a centimeter square and a half-millimeter thick. It has about 100 million neurons, of five distinct kinds.

Sampling process and Sampling theorem

$$G_s(f) = f_s \sum_{m=-\infty}^{\infty} G(f - mf_s) \Leftrightarrow g_s(t)$$

$$G_s(f) = f_s G(f) + f_s \sum_{m \neq 0} G(f - mf_s)$$

$$\Rightarrow G(f) = \frac{G_s(f)}{f_s}, \quad -W < f < W$$

If $f_s = 2W$

$$G(f) = \frac{1}{2W} \sum_{m=-\infty}^{\infty} g\left(\frac{n}{2W}\right) e^{-\frac{j\pi n f}{W}}$$

$$g(t) = \int_{-\infty}^{\infty} G(f) e^{j2\pi f t}\, df$$

$$g(t) = \int_{-\infty}^{\infty} \left[\frac{1}{2W} \sum_{m=-\infty}^{\infty} g\left(\frac{n}{2W}\right) e^{-\frac{j\pi n f}{W}}\right] e^{j2\pi f t}\, df$$

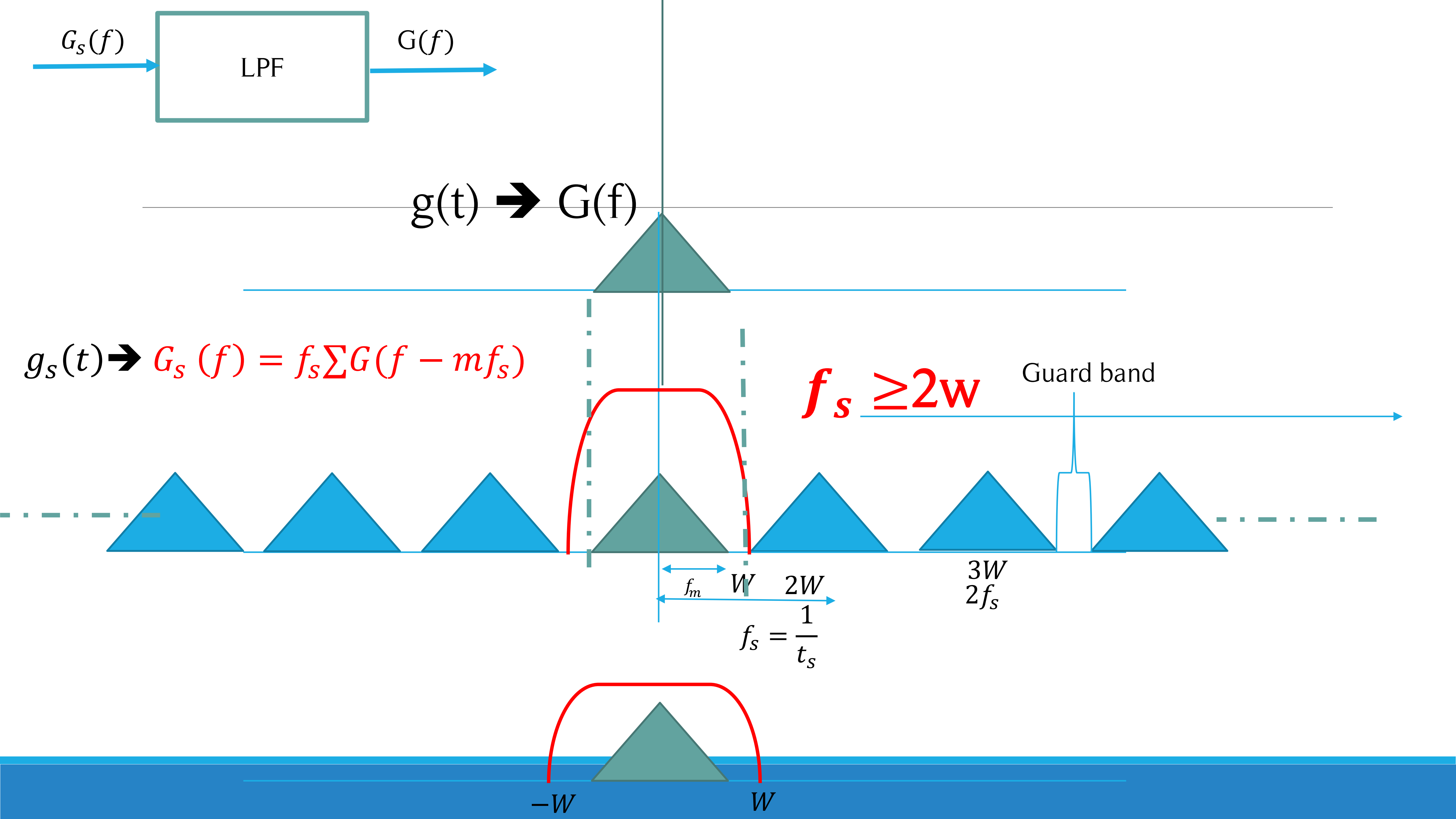
….

$$g(t) = \sum_{n=-\infty}^{\infty} g\left(\frac{n}{2W}\right) sinc(2Wt - n)$$

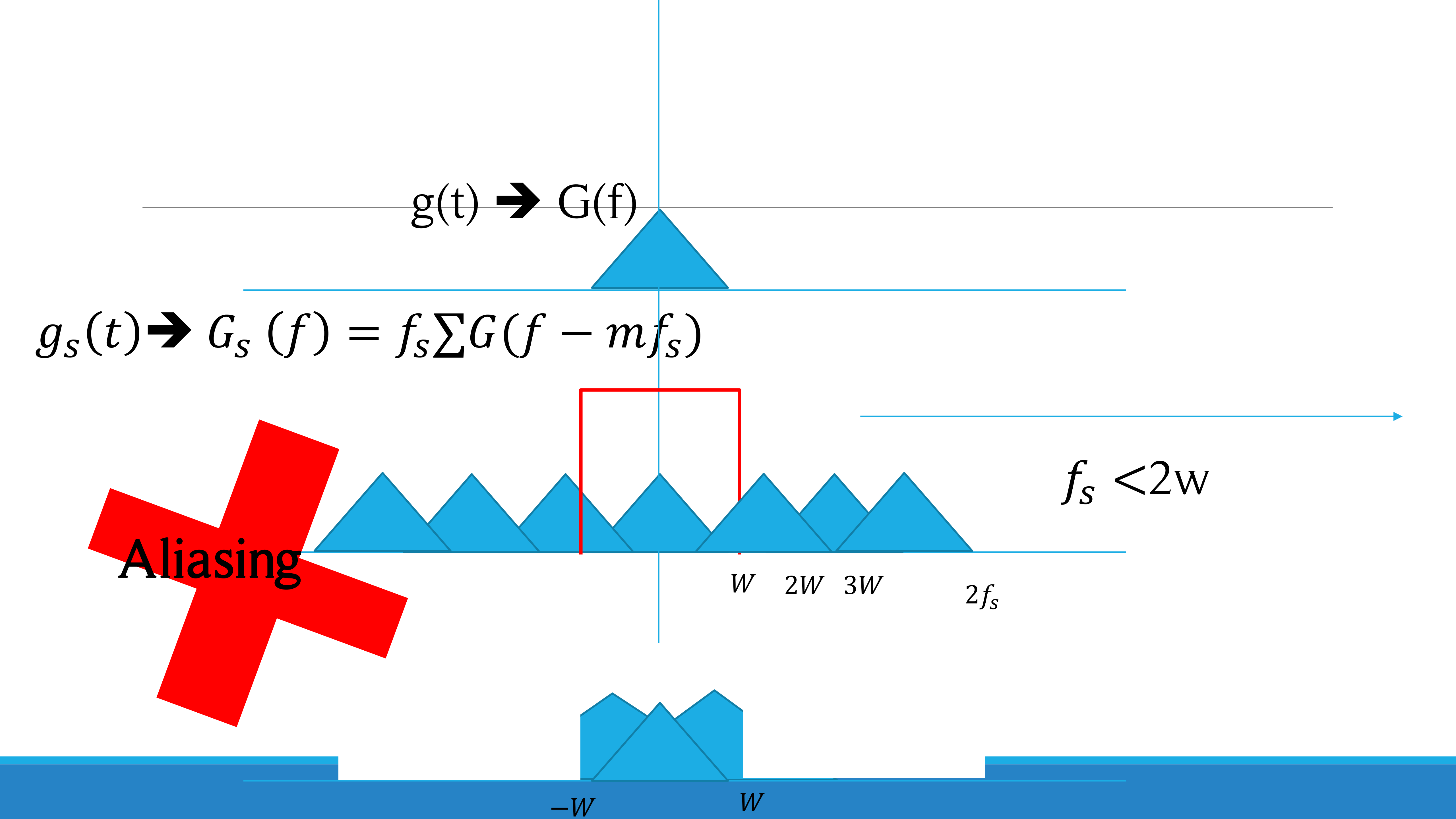
Nyquist rate

The Nyquist Sampling Theorem states that: A bandlimited continuous-time signal can be sampled and perfectly reconstructed from its samples if the waveform is sampled over twice as fast as it's highest frequency component.

$$f_s \geq 2f_m$$



$G_s(f)$
LPF
$G(f)$
g(t) ➔ G(f)
$g_s(t)$ ➔ $G_s(f) = f_s \sum G(f - m f_s)$
$f_s \geq 2w$
Guard band
f_m
W
$2W$
$3W$
$2f_s$
$f_s = \frac{1}{t_s}$
$-W$
W

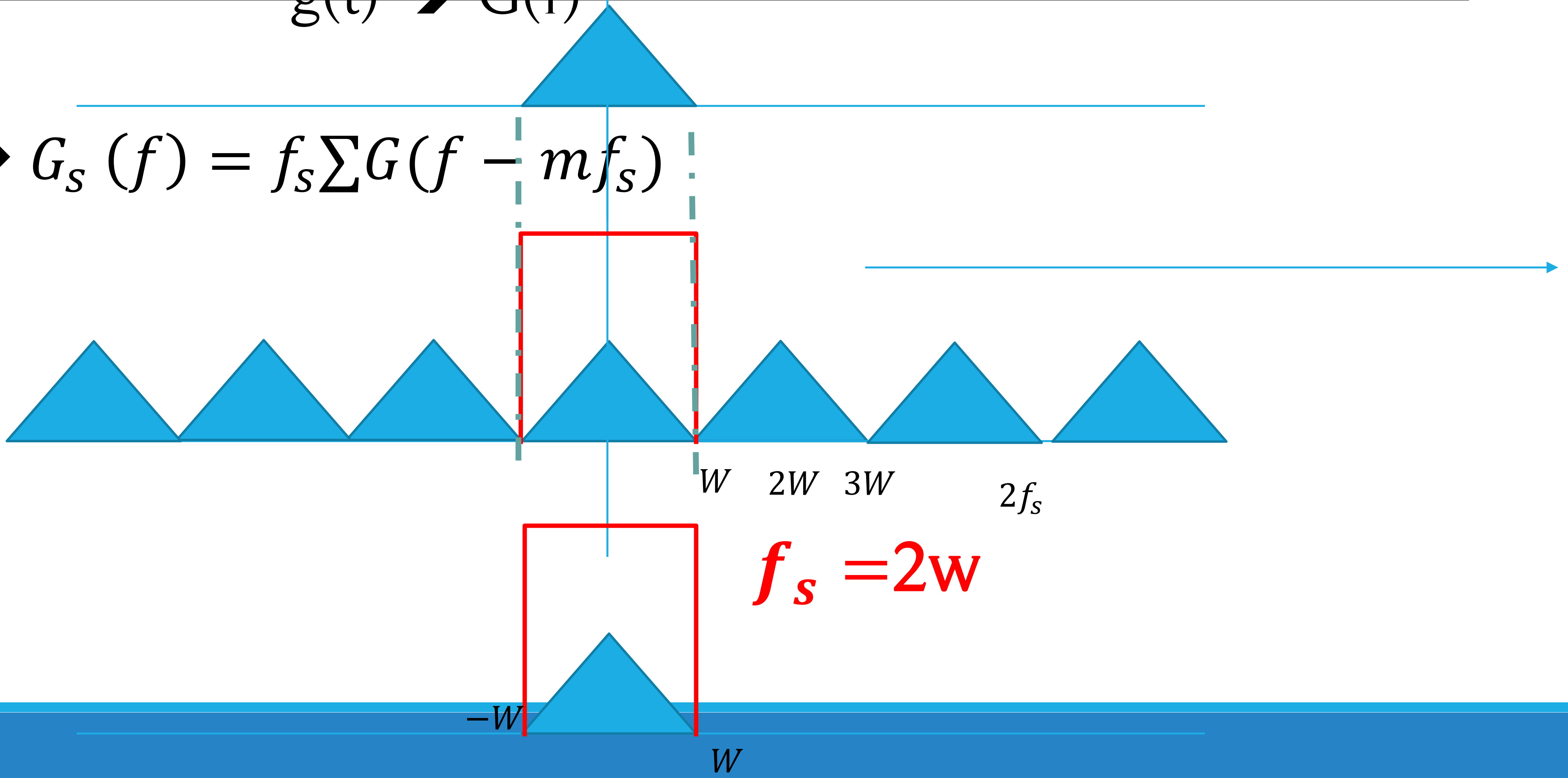


g(t) ➔ G(f)
$g_s(t)$ ➔ $G_s(f) = f_s \sum G(f - mf_s)$
f_s <2w
Aliasing
W
$2W$
$3W$
$2f_s$
$-W$
W

Nyquist rate ➔ $f_s = 2f_m = 2W$

g(t) ➔ G(f)

$g_s(t)$ ➔ $G_s\,(f) = f_s \sum G(f - mf_s)$



Pulse amplitude modulation (PAM)

$s(t) = \sum_{n=-\infty}^{\infty} m(nT_s)h(t - nT_s) = m_s(t) * h(t)$

$S(f) = M_s(f)H(f)$

$M_s(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)$

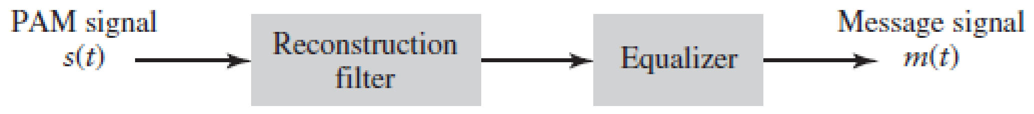
$S(f) = f_s \sum_{k=-\infty}^{\infty} M(f - kf_s)\, H(f)$

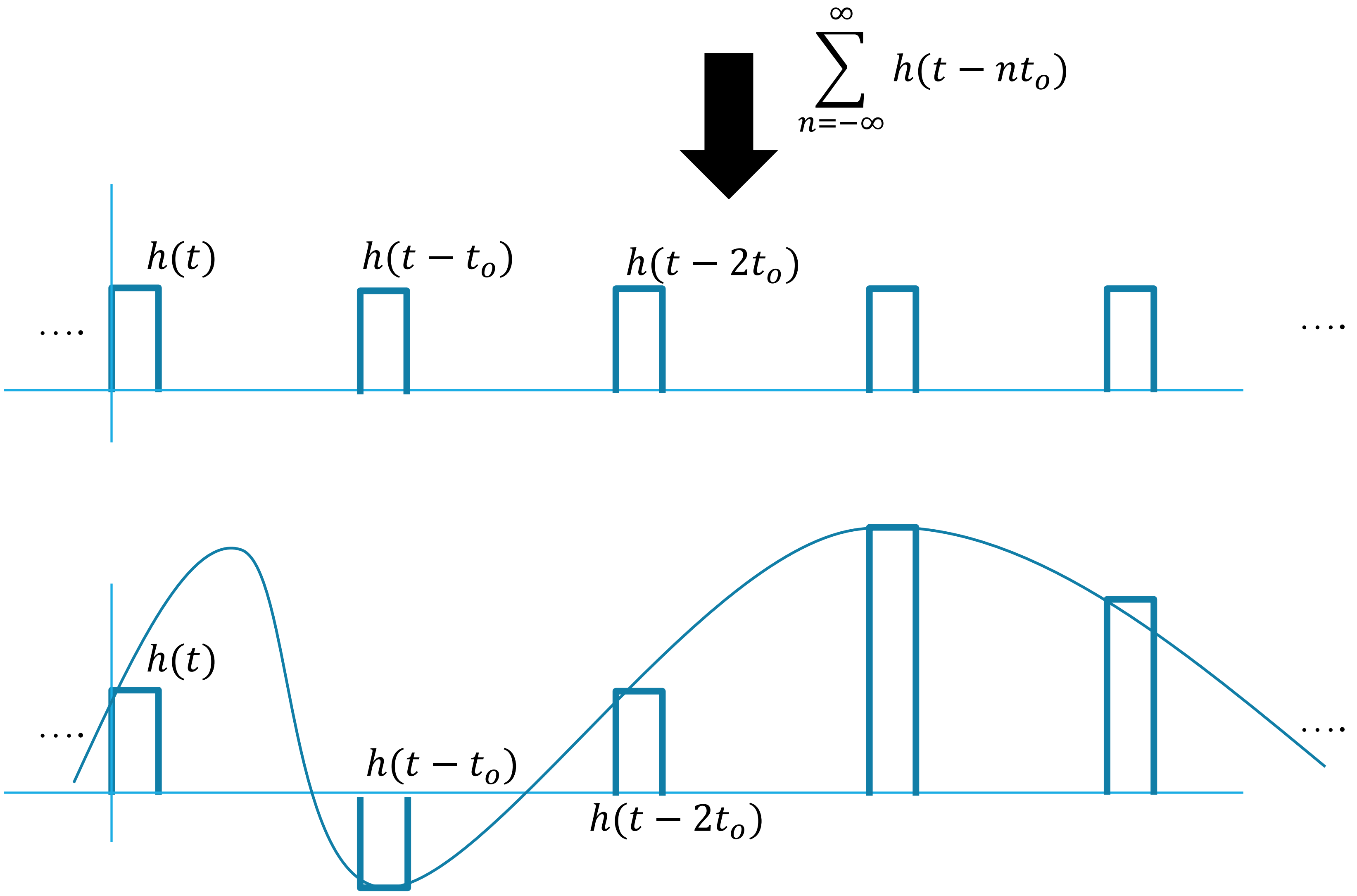
The summation terms follows therefore that the PAM signal is mathematically equivalent to the convolution of $m_s(t)$ and $h(t)$, where $h(t)$ is a standard rectangular pulse of unit amplitude and duration t.

$$h(t) = rect\left(\frac{t - \frac{T}{2}}{T}\right) = \begin{cases} 1, 0 < t < T \\ \frac{1}{2}, t = 0, t = T \\ 0, otherwise \end{cases}$$

There are two operations involved in the generation of the PAM signal:

1. Instantaneous sampling of the message signal.
2. Lengthening the duration of each sample



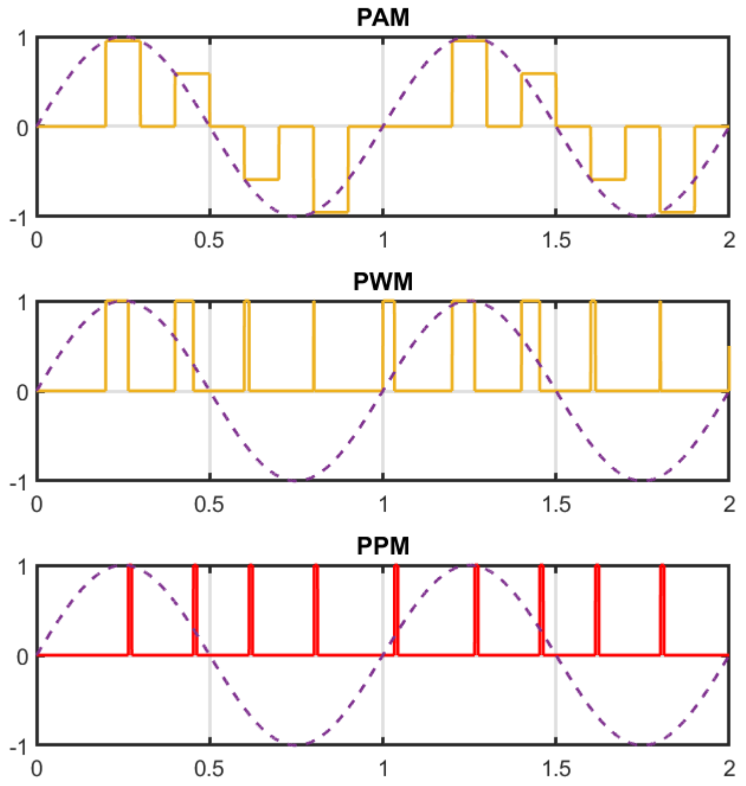


$\sum_{n=-\infty}^{\infty} h(t - nt_o)$
$h(t)$
$h(t - t_o)$
$h(t - 2t_o)$
....
....
$h(t)$
$h(t - t_o)$
$h(t - 2t_o)$
....
....

Pulse DURATION modulation (PDM) and pulse POSITION modulation (PPM)

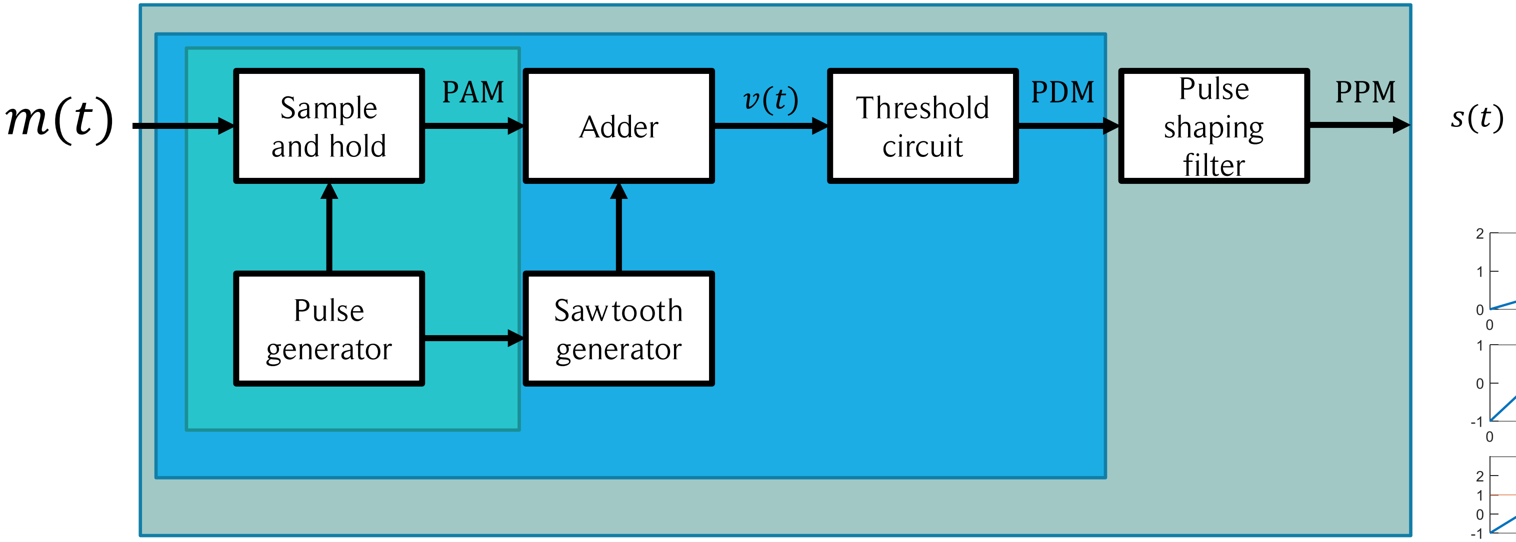
$$s(t) = \sum_{n=-\infty}^{\infty} g(t - nT_s - k_p m(nT_s))$$

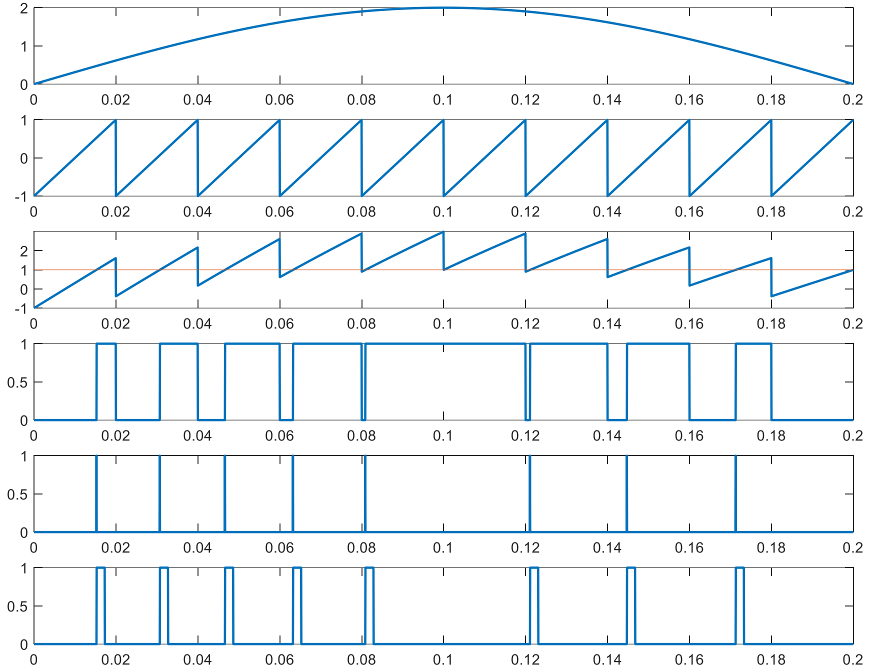
Where $g(t)$ is the standard pulse of interest, k_p is the sensitivity factor of the pulse position modulator



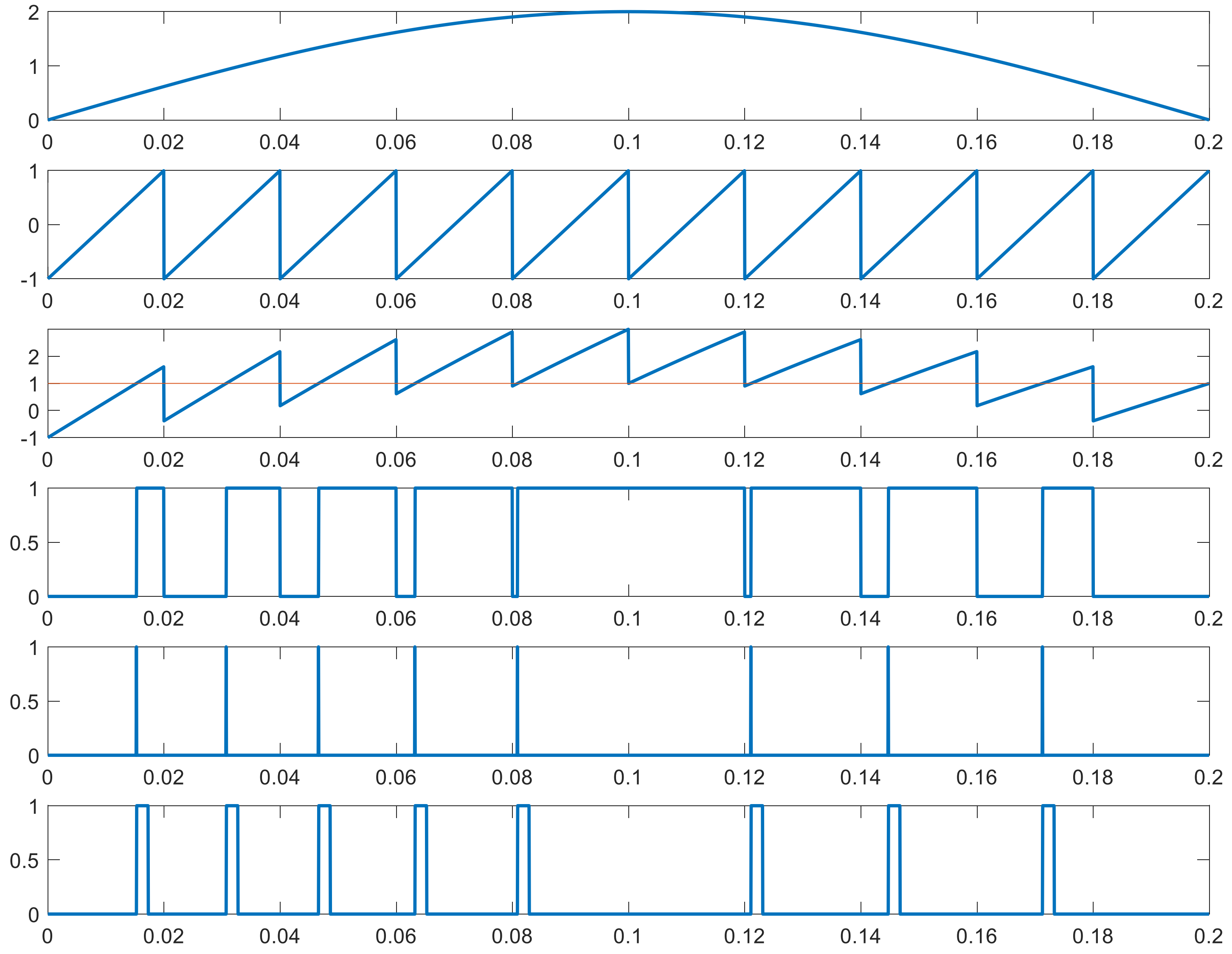

```
ts = .2;
dudtycycle = .5;
end_t = 2;
t = linspace(0,end_t,10000);
st_PWM = 0;
st_PAM = 0;
st_PPM = 0;
x_saw = sawtooth(2*pi*(1/ts)*t);
for n = 1:end_t/ts
st_PWM =  st_PWM + rct(n*ts,t,ts*(sin(2*pi*1*(n*ts))+1)/6);
st_PAM =  st_PAM + sin(2*pi*1*(n*ts))*rct(n*ts,t,ts*dudtycycle) ;
st_PPM =  st_PPM + rct(n*ts
+ts/6*(sin(2*pi*1*(n*ts))+1),t,ts*dudtycycle/10) ;
end
subplot(3,1,1)
plot(t,st_PAM )
hold on, grid on, title("PAM")
plot(t,sin(2*pi*1*(t)),"--")

subplot(3,1,2)
plot(t,st_PWM )
title("PWM"), hold on, grid on
plot(t,sin(2*pi*1*(t)),"--")

subplot(3,1,3)
plot(t,st_PPM,"-r")
hold on, grid on, title("PPM")
plot(t,sin(2*pi*1*(t)),"--")

function y = rct(q,t,w)
y  = heaviside(t-q)-heaviside(t-(q+w));
end
```

Pulse position modulation generation technique



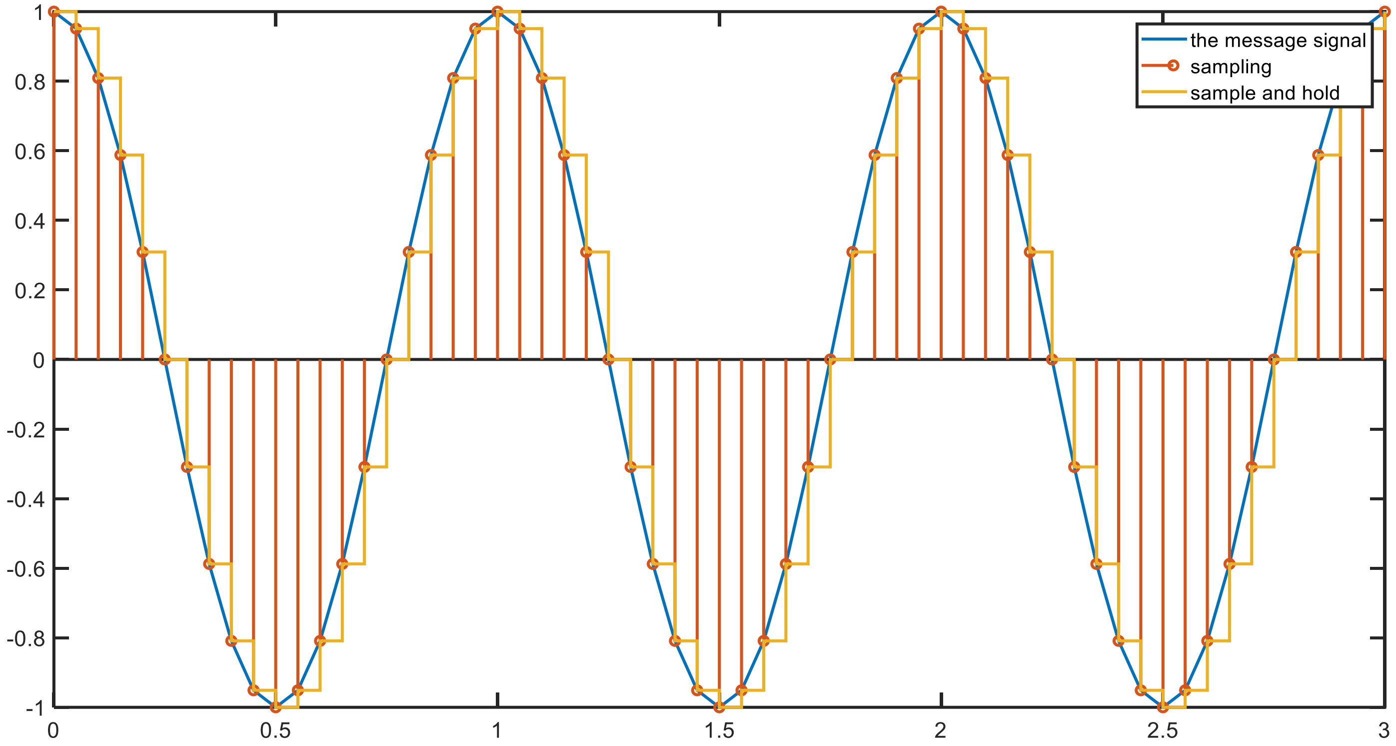


```
T = 10*(1/50);fs = 20000; t = 0:1/fs:T-1/fs;fontSIZE = 13;
subplot(6,1,1)
mt = 2*sin(2*pi*2.5*t)
plot(t,mt, 'LineWidth',2 )
set(gca,'FontSize', fontSIZE)

subplot(6,1,2)

x = sawtooth(2*pi*50*t);
plot(t,x, 'LineWidth',2 )
set(gca,'FontSize', fontSIZE)
subplot(6,1,3)
threshold = 1.0

plot(t,x+mt, 'LineWidth',2 )
thr = threshold*ones(1,length(t));
hold on

plot(t,thr)
set(gca,'FontSize', fontSIZE)

subplot(6,1,4)
plot(t,(x+mt)>=threshold, 'LineWidth',2 )
set(gca,'FontSize', fontSIZE)

subplot(6,1,5)
edges = diff([(x+mt)>=threshold, 0 ]);
plot(t,edges>0, 'LineWidth',2 )
set(gca,'FontSize', fontSIZE)

subplot(6,1,6)
PPM_OUT = ppm_shape(edges>0,40);
plot(t, PPM_OUT, 'LineWidth',2 )
set(gca,'FontSize', fontSIZE)


function [y] = ppm_shape(x,n)
y = 0;
for i = 1:n
y = y + circshift(x,i);
end
end
```



Quantization process

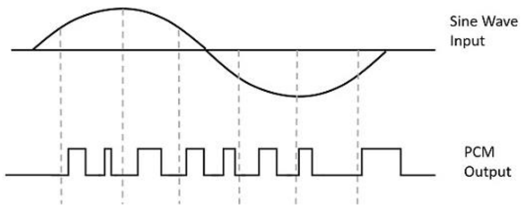
Quantization, in mathematics and digital signal processing, is the process of mapping input values from a large set (often a continuous set) to output values in a (countable) smaller set, often with a finite number of elements. Rounding and truncation are typical examples of quantization processes.



Sample	Quantization	Code
1	1	0000
1 - 0.125	2	0001
1 - 2*0.125	3	0010
---	4	0011
---	5	0100
….	…	….
X_16	16	1111

Pulse-Code Modulation (PCM)

- In pulse code modulation (PCM), a message signal is represented by a sequence of coded pulses, which is accomplished by representing the signal in discrete form in both time and amplitude.
- The basic operations performed in the transmitter of a PCM system are sampling, quantization, and encoding.
- **In other words: A signal is pulse code modulated to convert its analog information into a binary sequence, i.e., 1s and 0s. The output of a PCM will resemble a binary sequence. The following figure shows an example of PCM output with respect to instantaneous values of a given sine wave.**



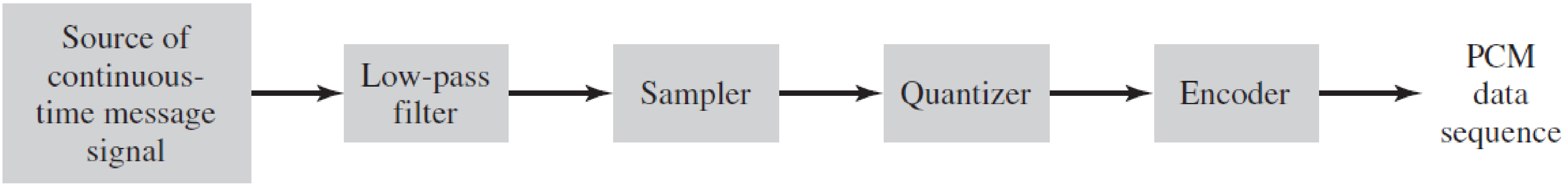
OPERATIONS IN THE TRANSMITTER

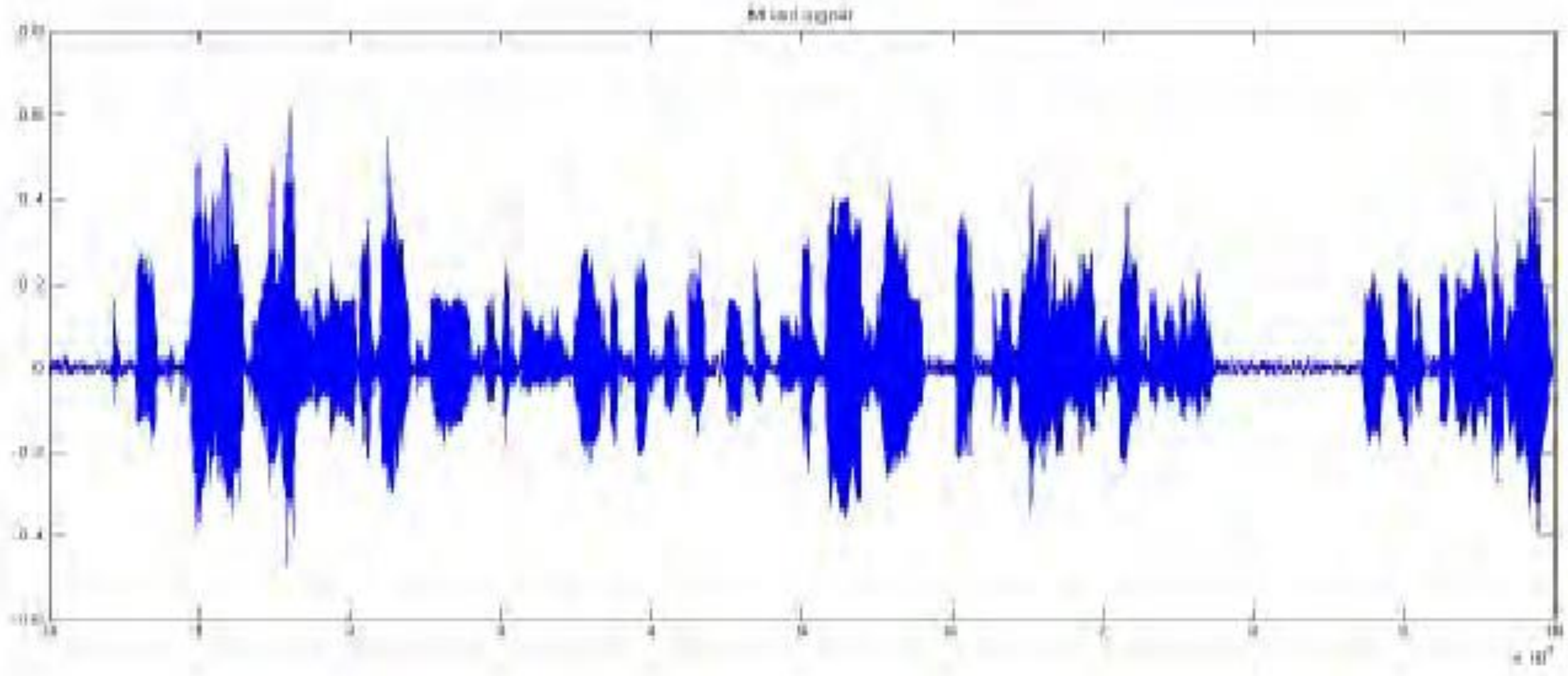
1- **Sampling.**

2- **Non uniform quantization:**

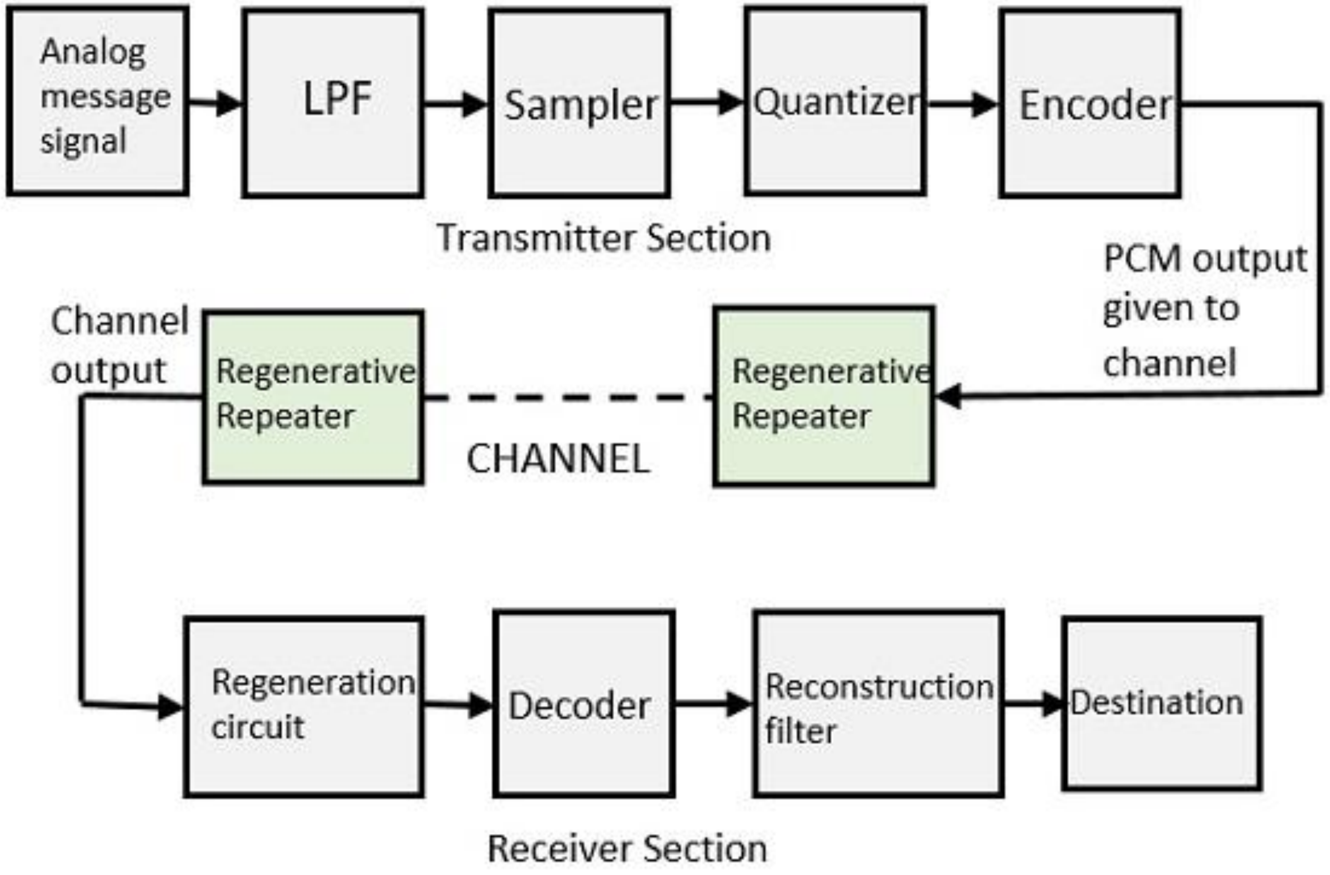
- The sampled version of the message signal is then quantized, thereby providing a new representation of the signal that is discrete in both time and amplitude. The weak passages that need more protection are favored at the expense of the loud passages.
- The use of a nonuniform quantizer is equivalent to passing the message signal through a compressor and then applying the compressed signal to a uniform quantizer.

3- **Encoding:**

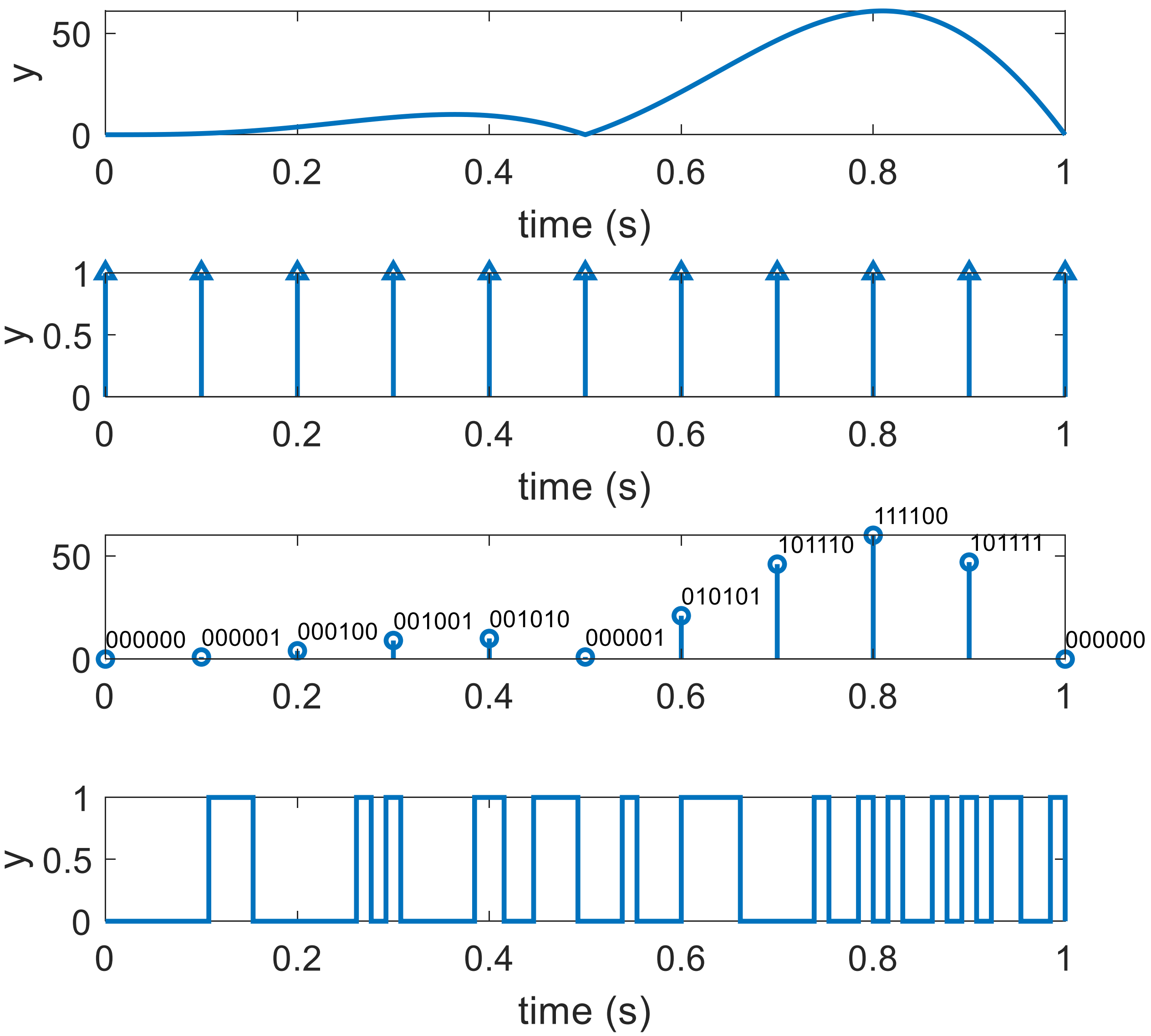




Basic Elements of PCM



```
clear
clf
Ts = .1; % SAMPLING PERIOD

t1 = 0:Ts:1;
t2 = .0:.001:1;

stream = ones(1,length(t1));
y1 = 100*t1.^2.*sin(2*pi*1*t1)  ;
y2 = 100*t2.^2.*sin(2*pi*1*t2)  ;

subplot(4,1,1)
plot(t2,abs(y2),'LineWidth',2)
xlabel("time (s)")
ylabel("y")
set(gca,'FontSize',15)
subplot(4,1,2)
stem(t1,stream,'^','LineWidth',2)
xlabel("time (s)")
ylabel("y")
set(gca,'FontSize',15)

subplot(4,1,3)
xlabel("time (s)")
ylabel("y")

stem(t1,abs(ceil(y1)),'o','LineWidth',2)
yabs = abs(ceil(y1));
set(gca,'FontSize',15)

bin_seq = dec2bin(abs(ceil(y1)))

for i=1:length(bin_seq)
text(t1(i),yabs(i)+10,bin_seq(i,:))
end

subplot(4,1,4)
xlabel("time (s)")
ylabel("y")


bin_seq = bin_seq;
bns = str2num(bin_seq(:))
t11 = linspace(0,1,length(bns)); % Time Vector
 s = stairs(t11,bns,'LineWidth',2); % Plotting3

xlabel("time (s)")
ylabel("y")
set(gca,'FontSize',15)
```



Non uniform quantization

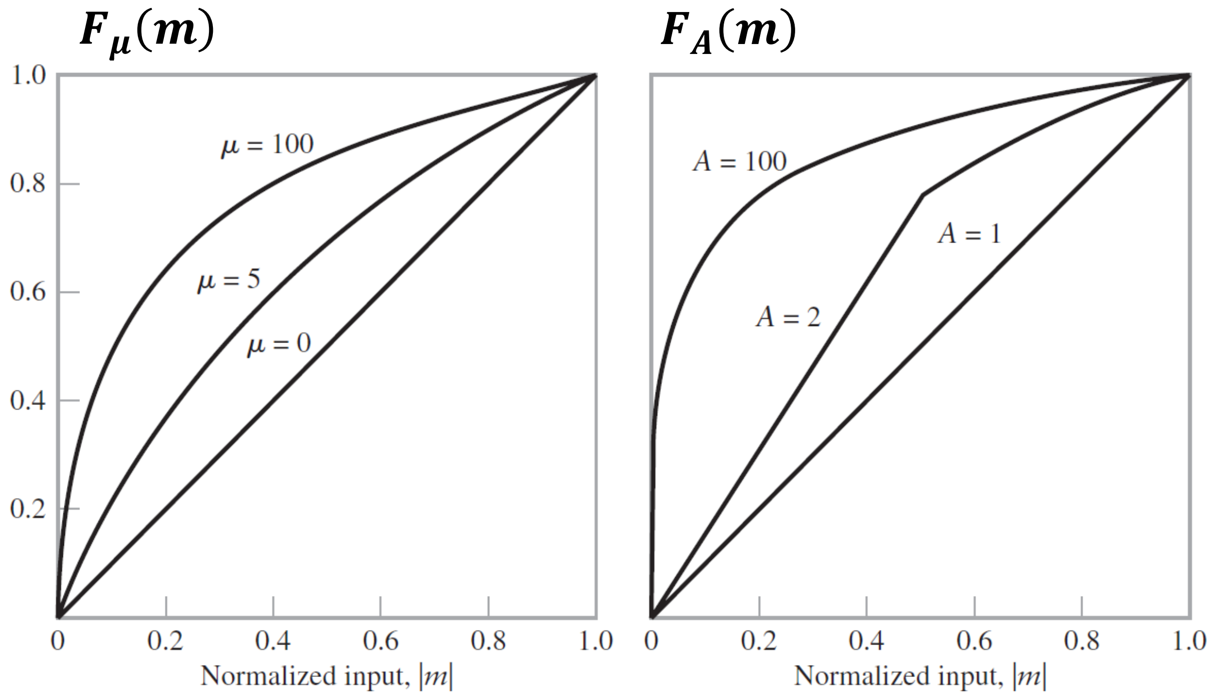
$\mu - law$: In this type of compression, the relationship between the input signal and the output signal of the compressor is as follows:

$$F_\mu(m) = \frac{log(1+\mu|m|)}{log(1+\mu)}$$

$|m|$: the normalized input signal
$F(x)$: the normalized output signal

Another compression law is called $A - law$:

$$F_A(m) = \begin{cases} \frac{A|m|}{1+log(A)}, 0 \leq |m| \leq \frac{1}{A} \\ \frac{1+log(A|m|)}{1+log(A)}, \frac{1}{A} \leq |m| \leq 1 \end{cases}$$



TIME-DIVISION MULTIPLEXING

- The sampling theorem provides the basis for transmitting the information contained in a band-limited message signal as a sequence of samples of taken uniformly at a rate that is usually slightly higher than the Nyquist rate. An important feature of the sampling process is a conservation of time. That is, the transmission of the message samples engages the communication channel for only a fraction of the sampling interval.
- We thereby obtain a time-division multiplex (TDM) system, which enables the joint utilization of a common communication channel by a plurality of independent message sources without mutual interference among them.