Chapter 2 :- "Signals and signal space"

* Classification of Signals:-

① Energy Vs. Power Signals:-

- $x(t)$ is energy signal if

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt < \infty$$

and • $x(t)$ is power signal if:

$$P_x = \left\{ \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt < \infty \right\} > 0$$

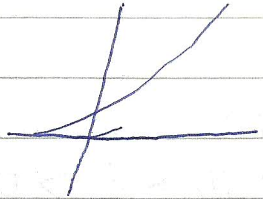
- $$E(t) = \int_{-\infty} p(t)\, dt \Rightarrow p(t) = \frac{dE(t)}{dt}$$ E is the Area Under power...

* It's neccesary that $x(t)$ be either power or energy signal.
* most of the time well behaved Aperiodic signals are energy signals.
* most of the time " " periodic " " power signals
* special case of power signals:
if $x(t)$ is periodic power signal

$$\Rightarrow P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x(t)|^2 dt$$

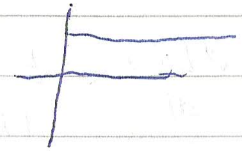
$*$ $R.M.S = \sqrt{P_x} = \sqrt{\lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt}$

Ex. $x(t) = e^{\alpha t}$, $\alpha > 0$?



$E = \infty$ ∴ not energy signal.

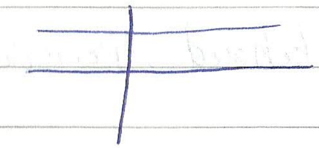
$x(t) = u(t)$?



$E_x = \infty$ ∴ Not energy signal

$P_x = 1$ power signal.

$x(t) = K$?

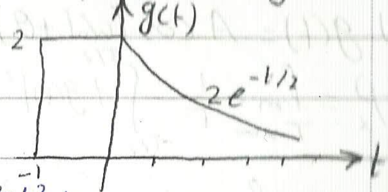


$E_x = \infty$ ∴ Not energy

$P_x = K^2$ ∴ power signal

مع انها ليست periodic

Ex. is this signal is Energy or power signal ?!?



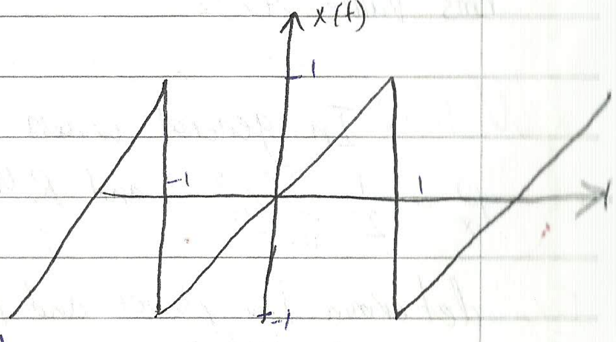
$$E_x = \int_{-\infty}^{\infty} |g(t)|^2 \, dt$$

$$= \int_{-\infty}^{-1} 0 \, dt + \int_{-1}^{0} |2|^2 dt + \int_{0}^{\infty} |2e^{-t/2}|^2 \, dt$$

$$= 0 + 4t\Big|_{-1}^{0} + -4e^{-t}\Big|_{0}^{\infty} = 4 + 4 = 8$$

So it is an Energy signal.

Ex. is this signal is Energy or power signal ?



$T = 2$ (period)

$x(t) = t$

$$P_x = \frac{1}{2} \int_{-1}^{1} |x(t)|^2 \, dt$$

$$= \frac{1}{2} \int_{-1}^{1} t^2 \, dt = \frac{1}{3}$$

So it is a power signal...

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Ex. determin the power and rms. of :-

(a) $g(t) = A\cos(\omega_0 t + \theta)$.

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |g(t)|^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} A^2 \cos^2(\omega_0 t + \theta)$$

$$= \quad ,, \quad ,, \quad \left[\frac{1}{2} + \frac{1}{2}\cos(2\omega_0 t + 2\theta)\right] dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} \frac{A^2}{2}\, dt + \underbrace{\lim_{T\to\infty} \frac{1}{\cancel{T}}\int_{-T/2}^{T/2} \frac{A^2}{2}\cancel{\cos(2\omega_0 t + 2\theta)}\, dt}_{\text{zero}}$$

$$= \lim_{T\to\infty} \frac{1}{T}\left(\frac{A^2}{2}\left(\frac{T}{2} + \frac{T}{2}\right)\right) = \lim_{T\to\infty} \frac{1}{\cancel{T}}\, \frac{A^2}{2}(\cancel{T})$$

$$= A^2/2$$

rms value $= A/\sqrt{2}$

Note:- In general when $x(t) = \sum_{n=1}^{N} C_n \cos(\omega_n t + \theta_n)$.

$P_x = \frac{1}{2}\sum_{n=1}^{N} C_n^2$ and R.M.S value $= \sqrt{\frac{1}{2}\sum_{n=1}^{N} C_n^2}$

Ex. determin the power and R.M.S of $x(t) = D e^{j\omega_0 t}$?

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |D e^{j\omega_0 t}|^2\, dt$$

$$= \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |D|^2\, dt$$

$|D e^{j\omega_0 t}| = |D(\cos(\omega_0 t) + j\sin(\omega_0 t))|$

$= |D|\,|\cos(\omega_0 t) + j\sin(\omega_0 t)|$

$= |D|\sqrt{\cos^2(\omega_0 t) + \sin^2(\omega_0 t)}$

$= |D|\sqrt{1} = |D|$

② Causal and anti causal and non causal signals:-

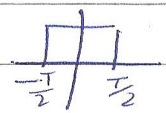
if $x(t) = 0 \;\forall\; t<0 \Rightarrow$ causal

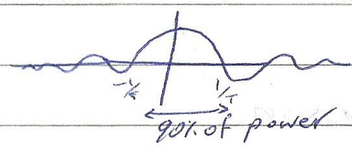
if $x(t) = 0 \;\forall\; t>0 \Rightarrow$ anti causal

if $x(t) \neq 0 \;\forall\; t \Rightarrow$ non causal.

③ Time limitted Vs. band limitted.

finite support — has finite B.W. (essential)





here the Theoretical B.W $= \infty$

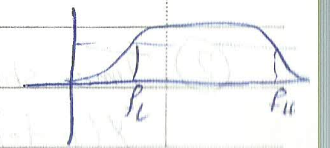
but essential B.W $= \frac{2}{T}$ (90% of power signal)

④ base band Vs. band pass.

low freq. — high freq.

⑤ Wide band Vs. narrow band signal.

$\alpha = \frac{f_H}{f_L} \gg 1$ order of (10)



by modulation (wide band $\to$ narrow band)

* operation ON signals :-

1) shift in time ($t_o > 0$)

$x(t-t_o)$ shift Right (delay)

$x(t+t_o)$ " left (advance)

2) Time scalling :-

$x(at)$: $|a| > 1 \Rightarrow$ compression

$|a| < 1 \Rightarrow$ expanssion.

3) Time inverssion :

$x(-t) \Rightarrow$ mirror Image of $x(t)$ around y-axiss.

* Classification of signals :-

① liner Vs. non linear :-

if the signal is homogenous + superposition.

$$F(\alpha_1 x_1(t) + \alpha_2 x_2(t)) = \alpha_1 y_1(t) + \alpha_2 y_2(t)$$

② Time shift invariant :-

$x(t-t_o) \Rightarrow y(t-t_o)$

③ Causal Vs. non Causal :-

if o/p depends on the present + previous values of i/p

$y(t_o) = F(x(t))$, $t < t_o \Rightarrow$ Causal ...

* Auto Corrolation and Cross Corrolation fn's :-

① Auto Corrolation of Energy Signal :-

If $x(t)$ is energy signal, then

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} x(t)\, x^*(t-\tau)\, dt = x(t) * \cancel{x(t)} \; x^*(-t)$$

↑ Complex Signal

* $R_{xx}(\tau)$ يفيد to measure the similarity bet. the signal $x(t)$ and it's delay version $x(t-\tau)$

$$R_{xx}(0) = \int_{-\infty}^{\infty} x(t)\, x^*(t)\, dt = \int_{-\infty}^{\infty} |x(t)|^2 dt = E_x$$

* ~~Corrolation coefficent~~ proparelies :-

① $|R_{xx}(\tau)| \leq R_{xx}(0)$

② $R_{xx}(\tau) \rightleftharpoons \psi_x(f) = X(f) \cdot X^*(f) = |X(f)|^2 \equiv$ Energy spec density (J/

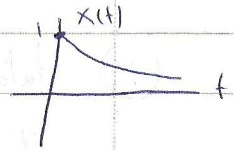
$$\therefore E_x = \int_{-\infty}^{\infty} \psi_x(f)\, df = \int_{-\infty}^{\infty} |x(t)|^2\, dt$$

③ $R_{xx}(-\tau) = R_{xx}^*(\tau)$, $Re[R_{xx}(\tau)]$ is even

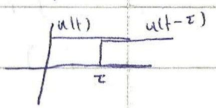
$Im[R_{xx}(\tau)]$ is odd

* If $x(t)$ is real $\Rightarrow R_{xx}(\tau)$ is Real and even.

Ex. Find $R_{xx}(\tau)$, $\psi_x(f)$, E_x for $x(t) = e^{-\alpha t}u(t)$?



$$R_{xx}(\tau) = \int_{-\infty}^{\infty} e^{-\alpha t} u(t)\, e^{-\alpha(t-\tau)} u(t-\tau)\, dt$$



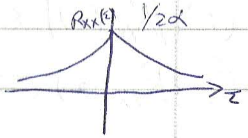
$$= \int_{\tau}^{\infty} e^{-\alpha t} e^{-\alpha(t-\tau)} dt$$

$$= e^{\alpha\tau}\int_{\tau}^{\infty} e^{-2\alpha t} dt = e^{\alpha\tau}\left[\frac{-1}{2\alpha} e^{-2\alpha t}\right]_{\tau}^{\infty}$$

$$= e^{\alpha\tau}\left(\frac{e^{-2\alpha\tau}}{2\alpha}\right) = \frac{1}{2\alpha} e^{-\alpha\tau} u(\tau)$$

$\therefore$ $x(t)$ is Real $\Rightarrow R_{xx}(\tau)$ is real and even.

$$R_{xx}(\tau) = R_{xx}(-\tau) \Rightarrow R_{xx} = \frac{1}{2\alpha} e^{-\alpha|\tau|} \text{ for all } \tau$$

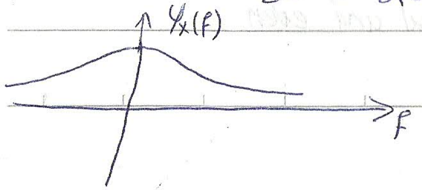


- $E_x = R_{xx}(0) = \frac{1}{2\alpha}$
- $\psi_x(f) = \int_{-\infty}^{\infty} R_{xx}(\tau) e^{-j2\pi f\tau} d\tau$

$$= \int_{-\infty}^{0} \frac{1}{2\alpha} e^{\alpha\tau} e^{-j2\pi f\tau} d\tau + \int_{0}^{\infty} \frac{1}{2\alpha} e^{-\alpha\tau} e^{-j2\pi f\tau} d\tau$$

$$= \frac{1}{2\alpha}\left[\int_{-\infty}^{0} e^{\tau(\alpha - j2\pi f)} d\tau + \int_{0}^{\infty} e^{-\tau(\alpha + j2\pi f)} d\tau\right]$$

$$= \frac{1}{2\alpha}\left[\frac{1}{\alpha - j2\pi f} + \frac{1}{\alpha + j2\pi f}\right] = \frac{1}{\alpha^2 + (2\pi f)^2}$$



- ممكن أجيب $\psi_x(f)$ من $X(f)$

اللي هو ال Forier لـ $x(t)$ وبربعه بيطلع $\psi_x(f)$

(2) Aouto corrolation of power signals :-

if $x(t)$ is complex power signal, then the auto corr. fn. is

$$R_{xx}(\tau) = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, x^*(t-\tau)\, dt$$

- For periodic power signal, $R_{xx}(\tau) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, x^*(t-\tau)\, dt$

- $R_{xx}(\tau) = R_{xx}(\tau + nT_0)$ "is periodic with som periodic as"
- $R_{xx}(\tau) \rightleftharpoons S_x(f)$ "power spectral density"

$$= |X(f)|^2 \text{ for periodic } x(t) \text{ with } T_0$$

- $P_x = R_{xx}(0) = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt = \int_{-\infty}^{\infty} S_x(f) df = a_0^2 + \sum \frac{a_n^2}{2} + \frac{b_n^2}{2}$

$$= C_0^2 + \sum \frac{C_n^2}{2}$$

$$= \sum |D_n|^2$$

- ~~$S_x(f) = \sum |C_n|^2 \delta(f - nf_0)$~~

- ~~$P_x = \int S_x(f) df = \sum |C_n|^2$~~

- $|R_{xx}(\tau)| \leq R_{xx}(0)$

~~(3) Cross correlation of ...~~

- $S_x(f) = a_0^2 \delta(f) + \sum_{n=1}^{\infty} \frac{a_n^2}{2} \delta(f - nf_0) + \sum_{n=1}^{\infty} \frac{b_n^2}{2} \delta(f - nf_0)$

$$= C_0^2 \delta(f) + \sum \frac{C_n^2}{2} \delta(f - nf_0) = \sum_{n=-\infty}^{\infty} |D_n|^2 \delta(f - nf_0)$$

Ex. $x(t) = A \sin 2\pi f_c t$?

$R_{xx}(\tau) = \frac{A^2}{2} \cos 2\pi f_c \tau$, $P = R_{xx}(0) = \frac{A^2}{2}$ or $P = \int S_x(f) df = \frac{A^2}{2}$

- $S_x(f) = F(R_{xx}(\tau)) = \frac{A^2}{4}[\delta(f - f_c) + \delta(f + f_c)]$

$$\sin\alpha \sin\beta = \frac{1}{2}[\cos(\alpha - \beta) - \cos(\alpha + \beta)]$$

~~$*$ If $R_{xy}(0)=0 \Rightarrow x \perp y = \int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt \triangleq \langle x(t), y(t)\rangle$ "linear product"~~

II Cross correlation of energy signal :-

If $x(t), y(t)$ are complex energy signal, then there cross-correlation fn.

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} x(t)\, y^*(t-\tau)\, dt \neq$$

$$R_{yx}(\tau) = \int_{-\infty}^{\infty} y(t)\, x^*(t-\tau)\, dt$$

$*$ If $R_{xy}(0) = 0 \Rightarrow x \perp y = \int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt \triangleq \langle x(t), y(t)\rangle$ "linear product"

- $R_{yx}(\tau) = R^*_{xy}(-\tau)$
- $R_{xy}(\tau) \rightleftharpoons \Psi_{xy}(f) = X(f)\, Y^*(f)$ (CESD)

  Cross ESD
- $E_{xy} = \int_{-\infty}^{\infty} \Psi_{xy}(f)\, df$

Ex: $x(t) = e^{-\alpha t} u(t)$, $y(t) = e^{\alpha t} u(-t)$ find $R_{xy}(\tau)$, $\Psi_{xy}(f)$ and plot ?

Corrolation coeffecint

- $-1 \leq \rho = \dfrac{R_{xy}(\tau)}{\sqrt{E_x E_y}} \leq 1$

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2 Cross product of power signals :-

If $x(t), y(t)$ are complex-valued power signals then thear cross corl. :-

$$R_{xy}(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x(t)\, y^*(t-\tau)\, dt$$

- If $x(t)$ and $y(t)$ are periodic with T_0

$$R_{xy}(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} x(t)\, y^*(t-\tau)\, dt$$

- $R_{yx}(\tau) = R^*_{xy}(-\tau)$ but $R_{xy}(\tau) \neq R_{yx}(\tau)$
- $|R_{xy}(\tau)| \not\leq R_{xy}(0) = \int_{-\infty}^{\infty} x(t)\, y^*(t)\, dt = \langle x(t), y(t)\rangle$
- If $x(t) \perp y(t)$ "orthogonal" $\Longleftrightarrow R_{xy}(\tau) = 0$
- $S_{xy}(f) = \sum_{n=-\infty} |C_n|^2 \delta(f - nf_0) \cdot \sum_{K=-\infty} |C_K|^2 \delta(f - Kf_0)$

  $= \sum_n \sum_K |C_n|^2 |C_K|^2 \delta(f - nf_0)\, \delta(f - Kf_0)$

- $R_{xy}(\tau) \rightleftharpoons S_{xy}(f) = S_x(f) \cdot S_y^*(f)$ (cross psd)

- $P_{xy} = \int_{-\infty}^{\infty} S_{xy}(f)\, df$

Ex. $x(t) = A\sin 2\pi f_c t$, $y(t) = A\cos 2\pi f_c t$, $R_{xy}(\tau)$, $\Psi_{xy}(f)$, P_{xy}?

$P_{xy} = \frac{A^4}{8}$

* Frequancy domain representation:-

F.S is periodic power Signal.

$x(t)$ is periodic $\Rightarrow$

$$x(t) = a_0 + a_1\cos\omega_0 t + a_2\cos 2\omega_0 t + \dots$$
$$+ b_1\sin\omega_0 t + b_2\sin 2\omega_0 t + \dots$$
$$= a_0 + \sum_{n=1}^{\infty} a_n\cos 2\pi f_0 n t + \sum_{n=1}^{\infty} b_n \sin 2\pi f_0 n t$$

- $a_0 = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\, dt \triangleq$ DC. Component.

- $a_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\cos 2\pi n f_0 t\, dt$

- $b_n = \frac{2}{T_0}\int_{-T_0/2}^{T_0/2} x(t)\sin 2\pi n f_0 t\, dt$

Ac Component $= \sum a_n + \sum b_n$

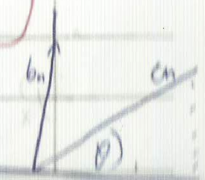
- DC power $= a_0^2$, Ac power $= \sum \frac{a_n^2}{2} + \sum \frac{b_n^2}{2}$

$\Rightarrow$ Total power $= dc_{power} + Ac_{power}$
$$= a_0^2 + \sum \frac{a_n^2}{2} + \sum \frac{b_n^2}{2}.$$

- Compact Triganometric F.S:-

$$x(t) = C_0 + 2\sum_{n=1}^{\infty} C_n\cos(n\omega_0 t + \theta_n)$$

- $C_0 = a_0 = \frac{1}{T_0}\int_{T_0} x(t)\, dt$, $C_n = \sqrt{a_n^2 + b_n^2}$



- $\theta_n = \tan^{-1}\left(\frac{-b_n}{\ }\right)$

★ C_n Vs. f $\triangleq$ Amplitude spectrum. "plot"

★ θ_n Vs. f $\triangleq$ phase spectrum. "plot"

- $P_{dc} = C_0^2$, $P_{AC} = \sum_{n=1}^{\infty} \frac{C_n^2}{2}$, $\Rightarrow P_x = C_0^2 + \sum_{1}^{\infty} \frac{C_n^2}{2}$

* Complex Exponential F.S :-

$$X(t) = \sum_{-\infty}^{\infty} D_n e^{j2\pi f_0 t}$$

$$D_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} X(t) e^{-j2\pi n f_0 t} dt$$

- $D_n = |D_n| e^{\pm j\theta_n}$, $\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right)$

- $|D_n| = \frac{C_n}{2} = \frac{\sqrt{b_n^2 + a_n^2}}{2}$ $\Rightarrow$ $a_n = 2Re[D_n]$, $b_n = -2\,Im[D_n]$

★ D_n Vs. f "plot"

θ_n Vs. f "

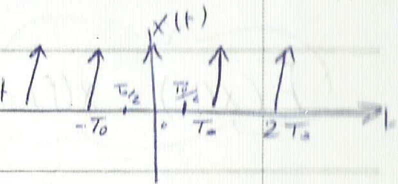
- $S_x(f) = X(f) \cdot X^*(f) = |X(f)|^2$

$= |\sum D_n \, \delta(f - nf_0)|^2$

$= \sum |D_n|^2 \delta(f - nf_0)$

- $P_x = \int_{-\infty}^{\infty} S_x(f)\, df = \sum |D_n|^2$

Ex. $X(t) = \sum_{K=-\infty}^{\infty} \delta(t - KT_0)$ find F.S. of $x(t)$?



$X(t) = a_0 + \sum a_n \cos 2\pi n f_0 t + \sum b_n \sin 2\pi n f_0 t$

- $a_0 = \frac{1}{T_0} \int_{T_0} X(t)\, dt = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t)\, dt = \frac{1}{T_0}$

$a_n = \frac{2}{T_0} \int_{T_0} x(t) \cos 2\pi n f_0 t\, dt = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t) \cos 2\pi n f_0 t\, dt = \frac{2}{T_0}$

$b_n = \frac{2}{T_0} \int_{T_0} x(t) \sin 2\pi n f_0 t\, dt = \frac{2}{T_0} \int_{-T_0/2}^{T_0/2} \delta(t) \sin 2\pi n f_0 t\, dt = 0$

$\Rightarrow X(t) = \frac{1}{T_0} + \sum \frac{2}{T_0} \cos 2\pi n f_0 t$, $\theta_n = \tan^{-1}\left(\frac{b_n}{a_n}\right) = 0$

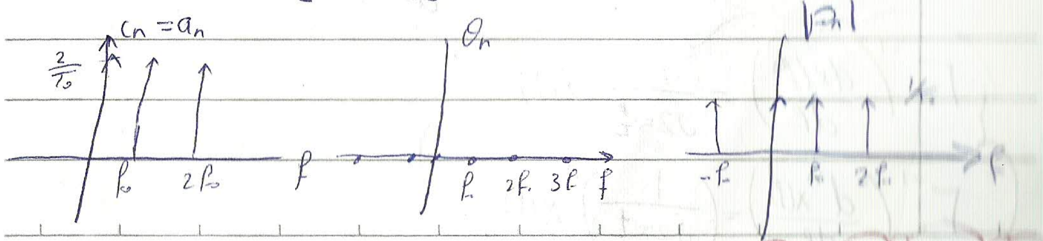
- $X(t) = C_0 + \sum C_n \cos(2\pi n f_0 t - \theta_n)$

$C_0 = a_0 = \frac{1}{T_0}$, $C_n = \sqrt{a_n^2 + b_n^2} = a_n = \frac{2}{T_0}$, $\theta_n = 0$

$\Rightarrow X(t) = \frac{1}{T_0} + \frac{2}{T_0} \sum_{n=1}^{\infty} \cos 2\pi n f_0 t$

- $X(t) = \sum_{-\infty}^{\infty} D_n e^{j2\pi n f_0 t}$,

$D_n = \frac{C_n}{2} = \frac{1}{T_0}$, $\theta_n = 0$



* F.T. of the signals:-

$$F(x(t)) = X(f) = \int_{-\infty}^{\infty} x(t)\, e^{-j2\pi f t}\, dt$$

and $x(t) = F^{-1}[X(f)] = \int_{-\infty}^{\infty} X(f) e^{j2\pi f t}\, dt$

• In general $X(f)$ is Complex fn.

• $X(f) = |X(f)|\, e^{j\theta(f)}$, where $|X(f)|$ Vs. f. is Amp. Spectra (even)

$\theta(f)$ Vs. f. is phase " (odd)

* For real signal $X(f) = X^*(f)$

* propareties of F.T:-

1. linearity: $F[X_1(t) + X_2(t)] = X_1(f) + X_2(f)$
2. Congogation:- if $x^*(t) \rightleftharpoons X^*(-f)$
   $x(-t) \rightleftharpoons X^*(f)$ [For Real Signal only]
3. Diffrantial in time:- $F\left(\frac{dx(t)}{dt}\right) = j2\pi f\, X(f)$
4. Diff in Frequancy: $F\left(\frac{d^n x(t)}{dt}\right) = (j2\pi f)^n X(f)$

$$F^{-1}\left(\frac{dX(f)}{df}\right) = \frac{1}{j2\pi t}\, x(t)$$

$$F^{-1}\left(\frac{d^n X(f)}{df}\right) = \left(\frac{1}{j2\pi t}\right)^n x(t)$$

5. Time shifting: $F(x(t \mp t_0)) = X(f)\, e^{\mp j2\pi f t_0}$
6. Time Scaling: $F[x(at)] = \frac{1}{|a|} X\left(\frac{f}{a}\right)$
7. Freq. shifting: $F(x(t)\, e^{\pm j2\pi f_c t}) = X(f \mp f_c)$
8. duality: if $x(t) \rightleftharpoons X(f)$ then $X(t) \rightleftharpoons x(-f)$
9. integration in Time:- $F\left[\int_0^t x(\tau)\, d\tau\right] = \frac{X(f)}{j2\pi f} + \frac{X(0)}{2}\delta(f)$
10. " in Freq. $F^{-1}\left[\int_{j2\pi f}^{\infty} X(f)\, df\right] = j2\pi t\, x(t)$
11. Convolution in Time:- $F[x(t) * y(t)] = X(f) . Y(f)$
12. Multiplication in time:- $F[x(t) . y(t)] = X(f) * Y(f)$

* Most proparetics used:-

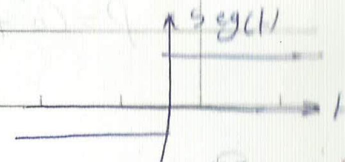
• $F(x(t) \cos 2\pi f_c t) = \frac{1}{2}[X(f-f_c) + X(f+f_c)]$

• $F(x(t) \sin 2\pi f t) = \frac{1}{2j}[X(f-f_c) + X(f+f_c)]$

• $\delta(t) \rightleftharpoons 1$

• $u(t) \rightleftharpoons \frac{1}{j2\pi f} + \frac{1}{2}\delta(f)$ • $seg(t) \rightleftharpoons \frac{1}{j\pi f}$

• $e^{\pm j2\pi f_c t} \rightleftharpoons \delta(f \mp f_c)$



$seg(t) = u(t) - u(-t)$

* Forier Transform of periodic Signals :-

If $x(t)$ is periodic power signal with period T_0

- $X_p(t) = \sum D_n e^{j2\pi f_0 t}$
- $D_n = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} X_p(t) e^{-j2\pi n f_0 t} dt$

$= \frac{1}{T_0} X(nf_0)$ where $x_p(t) = \begin{cases} X_p(t) & -\frac{T_0}{2} < t < \frac{T_0}{2} \\ 0 & \text{else} \end{cases}$

- $F(X_p(t)) = \frac{1}{T_0} \sum X(nf_0)\, \delta(f - nf_0)$

* $S_{xp}(f) = F(X_p(t)) \cdot [F(X_p(t))]^*$

power spectral dencity

$= \left(\frac{1}{T_0}\right)^2 \sum_n |X(nf_0)|^2 \delta(f - nf_0)$

$= \sum |D_n|^2 \delta(f - nf_0)$

* $P_{xp} = \int_{-\infty}^{\infty} S_{xp}(f) df = \sum |D_n|^2 = \left(\frac{1}{T_0}\right)^2 \sum |X(nf_0)|^2$

- $x_p(t) = C_0 + \sum C_n \cos(2\pi f_0 n + \theta_n)$
- $S(x_p(t)) = \cos(f) + \sum \frac{C_n}{2} [\delta(f - nf_0) e^{j\theta_n} + \delta(f + nf_0) e^{-j\theta_n}]$

$= \cos^2(f) + \sum \frac{C_n^2}{4} (\delta(f - nf_0) + \delta(f + nf_0))$

- $P = C_0^2 + \sum \frac{C_n^2}{2}$

* Energy spectral density of Modulate Signal :-

$X(t)$ is energy signal.

$X(t) = m(t) \cos 2\pi f_c t$.

$\Rightarrow X(f) = \frac{1}{2} [M(f - f_c) + M(f + f_c)]$

$\Psi_X(f) = X(f) \cdot X^*(f) = \frac{1}{4} [M^2(f - f_c) + M^2(f + f_c)]$

- $E_X = \int_{-\infty}^{\infty} \Psi_X(f) df = \frac{1}{4} \int_{-\infty} [M^2(f - f_c) + M^2(f + f_c)] df$

$= \frac{1}{4} \int_{-\infty}^{\infty} \Psi_m(f) + \Psi_m(f)\, df = \frac{1}{4} [E_m + E_m] = \frac{1}{2} E_m$

Similarity for power signal $X_p(t) \Rightarrow P_X = \frac{P_m}{2}$

وهاي مشتركة ال
ال energy وال power
بنفس طعم

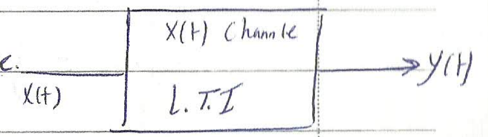
* Transformation of signal through liner sys. :-

i/p and o/p relation ship of L.T.I charactarized either time or freq.

① Time domain ; for L.T.I sys. I/o relation ship is

$y(t) = x(t) * h(t)$

where $h(t)$ is impulse responce.



$$\boxed{h(t) = y(t)\Big|_{x(t)=\delta(t)}}$$

Now if the input is $u(t)$:-

$$y(t) = u(t) * h(t)$$

$$\frac{dy(t)}{dt} = \frac{du(t)}{dt} * h(t)$$

$$= \delta(t) * h(t) = h(t)$$

لأنه لا نستطيع ادخال $\delta(t)$
عالـ signal فبندخل $u(t)$
بعدين بنشتقه بيطلع $\delta(t)$

② Frequancy domain :- I/O relation ship is:

$Y(f) = X(f) \cdot H(f)$ where $H(f)$ is L.T.I Transfere fn.

$$H(f) = \frac{y(t)}{x(t)}\Big|_{x(t)=e^{j2\pi ft}}$$

• proof :- $y(t) = h(t) * x(t)$, if $x(t) = e^{j2\pi ft}$

$$\Rightarrow y(t) = \int_{-\infty}^{\infty} h(\tau) x(t-\tau) d\tau$$

$$= \int_{-\infty}^{\infty} h(\tau) e^{j2\pi f(t-\tau)} d\tau$$

$$= e^{j2\pi ft} \int_{-\infty}^{\infty} h(\tau) e^{-j2\pi f\tau} d\tau$$

$$\Rightarrow \boxed{y(t) = e^{j2\pi ft} H(f)}$$

$* \; Y(f) = X(f) \cdot H(f) \Rightarrow |Y(f)| = |X(f)| \cdot |H(f)|$

$$\angle Y(f) = \angle X(f) + \angle H(f) = e^{j(\theta_f + \beta_f)}$$

$$* \; H(f) = \underbrace{|H(f)|}_{\text{Amp. Vs. } f} \; e^{j\beta f} \quad \angle \beta f \rightarrow \theta \text{ Vs. } f$$

* If $x_p(t)$ is periodic :

$$y(t) = x_p(t) * h(t)$$

$$Y(f) = X_p(f) \cdot H(f) = \left(\frac{1}{T_0} \sum X(nf_0)\, \delta(f - nf_0) \cdot H(f)\right)$$

$$= \frac{1}{T_0} X(nf_0) \sum \int_{-\infty}^{\infty} H(f)\, \delta(f - nf_0)\, e^{j2\pi ft} df$$

$$y(t) = F^{-1}(\;) = \frac{1}{T_0} X(nf_0) \sum H(nf_0)\, e^{j2\pi f_0 t}$$

$$= \sum \frac{1}{T_0} H(nf_0) X(nf_0)\, e^{j2\pi n f_0 t}$$

$$= \sum \alpha(n)\, e^{j2\pi f_0 t}$$

$\alpha(n)$ → F.S Coeff. of $y(t)$

* Parsevals (Energy)(power) Theorem ;-

• $E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt = \int_{-\infty}^{\infty} |X(f)|^2 df = R_{xx}(0)$

$|X(f)|^2$ → $\psi_x(f)$

• $P_x = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} |x_p(t)|^2 dt = \int \underbrace{|X_p(f)|^2}_{S_{xp}(f)} df = R_{xx}(0)$

* distortionless Transmittion:-

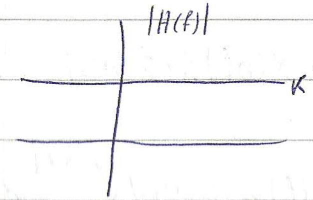
Common Chandle is Called distortionless if the I/o relation ship is given by:-

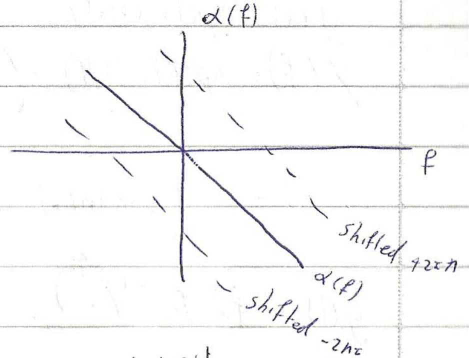
$y(t) = K\,x(t-t_d)$, K: attenuation Factor

t_d: time delay.

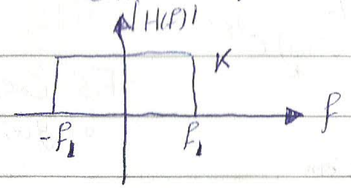
$\Rightarrow Y(f) = K\,X(f)\,e^{-j2\pi f t_d}$, $H(f) = \frac{Y(f)}{X(f)} = K\,e^{-j2\pi f t_d}$

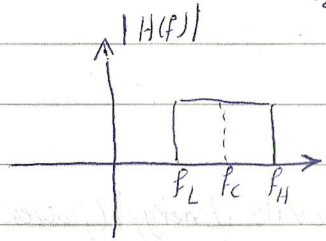
$\therefore |H(f)| = K$, $\alpha(f) = -2\pi f t_d \pm 2n\pi$

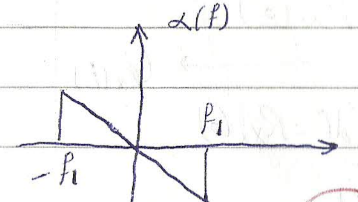


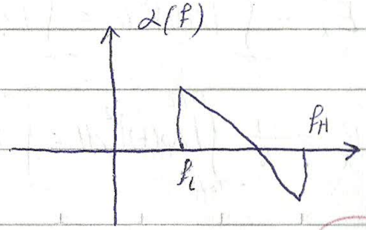


For Ideal LPF







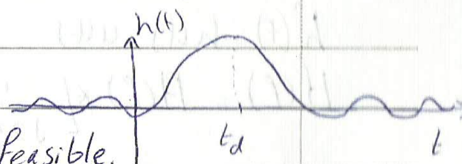


- For ideal LPF:-

$H(f) = K\,\mathrm{rect}\left(\frac{f}{2f_1}\right)e^{-j2\pi f t_d}$

كلاهما not vesible

$h(t) = F^{-1}[H(f)] = 2f_1 K\,\mathrm{sinc}\,2f_1(t-t_d)$



Since Ideal channle is not ~~B~~ feasible.

$\Rightarrow$ all Common. channle are dispersive wich responce in two types of distortion:-

① Amplitude distortion:-

Caused by $|H(f)|$ is not Constant for $\forall f$ $\Rightarrow$ fadding channle

* There is no ~~distortion~~ distortionless channles

② phase distortion:-

Caused by $\alpha(f)$ is not linear with freq.

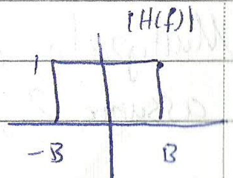
i.e. propegation velocity $V_p = \frac{c}{nf}$

$\Rightarrow$ suffer diff. delays.

$\Rightarrow$ diff. arival time $\Rightarrow$ dispertion $\Leftarrow$ يزيد الـ width الـ signal

* Ideal and practical filters:-

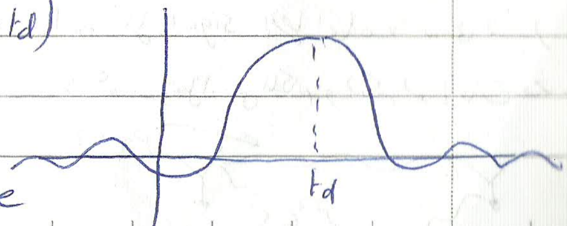
$H(f) = \begin{cases} e^{-j2\pi f t_d} & |f| \le B \\ 0 & \text{else} \end{cases}$



where B is message B.W

$h(t) = 2B\,\mathrm{Sinc}\,(t-t_d)$

. it's non Causal signal $\Rightarrow$ unrealizable



to make it realizable we Multiply it by u(t)...

$$\hat{h}(t) = h(t)\cdot u(t)$$

$$H(f) = H(f) \times \left[\frac{1}{j2\pi f} + \frac{1}{2}\delta(f)\right]$$

* Signals distortion over Comm. channles:-

① NoN Ideal Ampl. or phase responce. of the channle ⇒ linear distortion.

② NoN linear characteristics of the channle (distortionless fading)

$$x(t) \longrightarrow \boxed{n.L} \longrightarrow y(t)$$

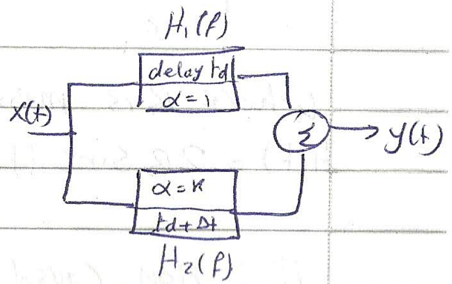
$$y(t) = a_0 + a_1 x(t) + a_2 x^2(t) \cdots + a_n x^n(t)$$

$$Y(f) = a_0\,\delta(f) + a_1 X(f) + a_2 \underbrace{X(f) * X(f)}_{\text{expand}} + \cdots$$

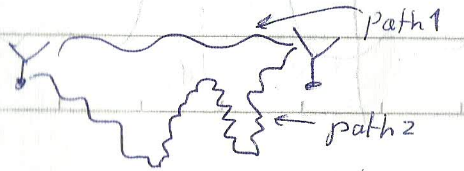
expand ⇒ distortion

③ Multipath distortion :-

assume 2 path sys.



أنه كل signal الها اتجاه معين في المسار
ولكن بتوصل لنفس المكان و لكن بأوقات مختلفة



$$X(f) \longrightarrow \boxed{H(f) = H_1(f) + H_2(f)} \longrightarrow Y(f)$$

$$H_1(f) = e^{-j2\pi f t_d} \qquad H_2(f) = K e^{-2j\pi f (t_d + \Delta t)}$$

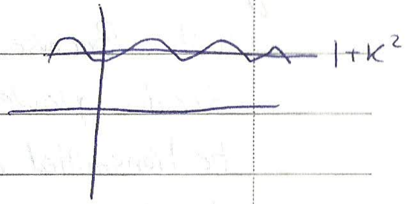
$$\Rightarrow H(f) = e^{-j2\pi f t_d} + K e^{-j2\pi f (t_d + \Delta t)}$$

$$= e^{-j2\pi f t_d}\left[1 + K e^{-j2\pi f \Delta t}\right]$$

$$|H(f)| = \left|e^{-j2\pi f t_d}\right| \left|1 + K e^{-j2\pi f \Delta t}\right|$$

$$= 1 \cdot \sqrt{(1 + K\cos 2\pi f \Delta t)^2 + (K \sin 2\pi f \Delta t)^2}$$

$$= \sqrt{1 + K^2 + 2K\cos 2\pi f \Delta t}$$



r 4: Amplitude Modulations :-

* Modulation: is aprocess that causes ashift in the range of frequancies of the signal.

① Base band ~~Command~~ communication :-
is acommunation where the base band signals are transmitted without modulation, that is without any shift in the range of frequancies of the Signal
. Note: Because the baseband signals have sizable power at low frequancies, they can't be transmitted over aradio link, but it is suitable for transmission a pair of wires, Coaxial cable, or obtical fibers).

② Carrier Communication :-
is a communaication that uses Modulation to shift the frequancy spectrum of a signal.
• Types of Modulation :-
① Amplitude Modulation (AM)
② Frequancy Modulation (FM) } Angle Modulation
③ phase Modulation (PM)

* Modulation is used to to transmit :-
① Analog baseband signals.
② Digital baseband signals.

. Note: length of antenna $\propto \lambda$ (Wave length) where :

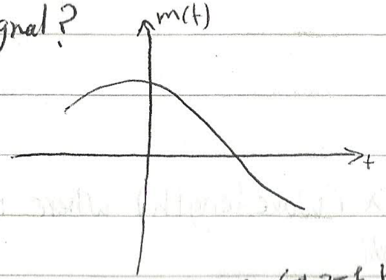
$$\lambda = \frac{C \leftarrow \text{speed of light}}{F \leftarrow \text{frequancy}}$$

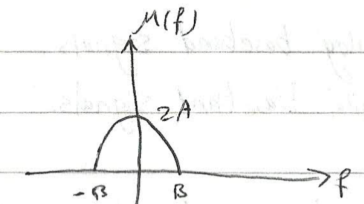
DSB - SC

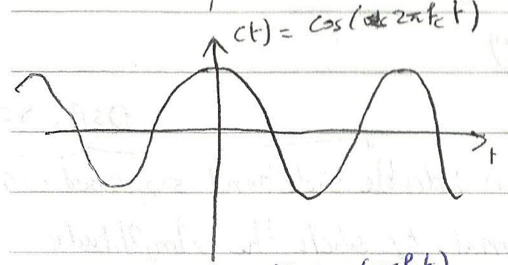
* 4.2: Amplitude modulation : double side band suppressed carrier:
. an AM signal is a signal by wich the Amplitude on analog sinosoidal carrier $c(t) = A_c \cos(2\pi f_c t + \theta_c)$ is varied ~~according~~ in proportion to the baseband (Massage) signal $m(t)$ "the modulating signal".
(The frequancy f_c and θ_c are constants)...

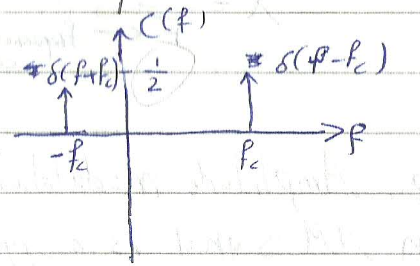
. Modulation parameters:-
1) Carrier $\rightsquigarrow A_c \cos(f_c 2\pi t + \theta_c)$
2) Modulating signal $\rightsquigarrow m(t)$
3) Modulated signal $\rightsquigarrow m(t) \cos(2\pi f_c t)$

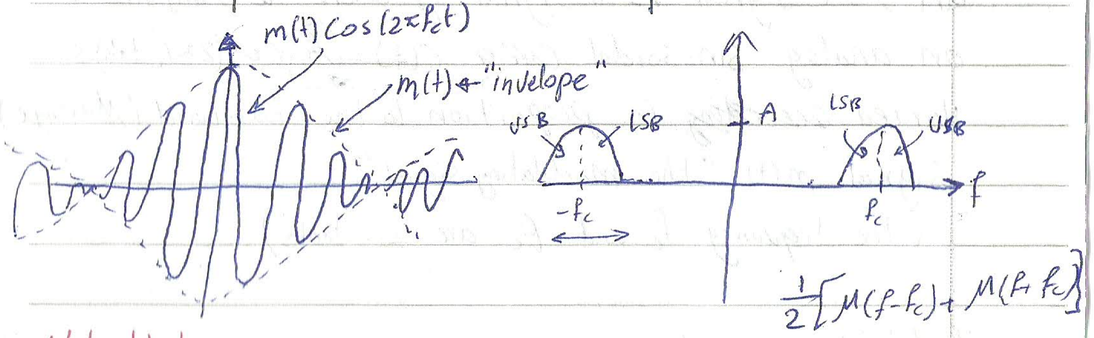
Ex. assum $m(t)$ shoun and it's F.T:- draw the Modulated Signal?







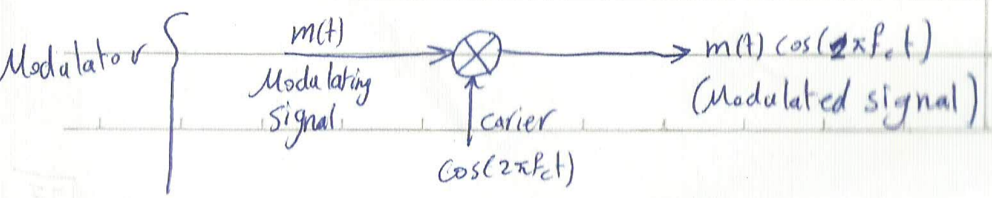




Note that :-

① if the B.W of $m(t)$ is B, then the B.W of the modulated signal is $2B$. (لانو صار عنا Conv. مشتقة)

② allways we assum that $f_c >> B$ to avoid the overlap of the spectra centered at (f_c) and $(-f_c)$...

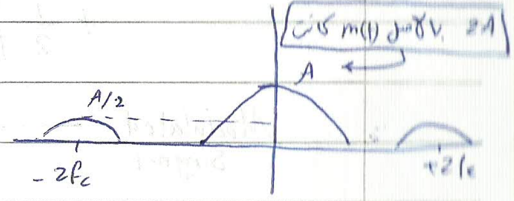


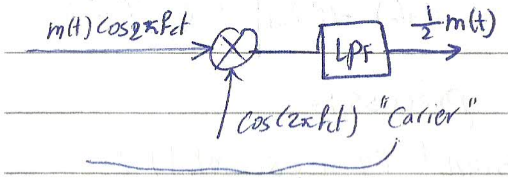
* **Demodulation:-** The process of recovering the signal from the modulated signal. (demodulation oR detection)

- Demodulation Consist of :-
  1) Multiplication of the modulated signal by $[\cos(2\pi f_c t)]$
  2) A low pass filter.

$$\Rightarrow e(t) = m(t)\cos(2\pi f_c t) \cdot \cos(2\pi f_c t) = \frac{1}{2}[m(t) + m(t)\cos(4\pi f_c t)$$

$$E(f) = \frac{1}{2}M(f) + \frac{1}{4}[M(f-2f_c) + M(f+2f_c)]$$





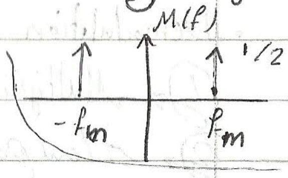
demodulator

Note: we need to generate a local carier at the reciver have the same frequancy and phase as the modulator Carrier.

Ex. $m(t) = \cos(2\pi f_m t)$ "

$\cancel{M(f)} \; M(f) = \frac{1}{2}[\delta(f-f_m) + \delta(f+f_m)]$

modulating Signal



• $DSB\_SC(t) = m(t)\cos 2\pi f_c t$

$= \cos 2\pi f_c t . \cos 2\pi f_m t$

$= \frac{1}{2}[\cos(2\pi f_c + 2\pi f_m)t \; \cancel{\cos}$

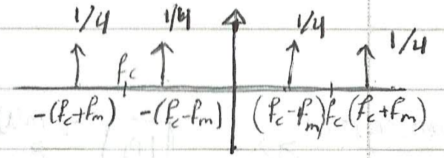
$+ \cos(2\pi f_c - 2\pi f_m)t]$

~~$DSB\_SC(f) = \frac{1}{2}[\frac{1}{2}f\,\delta(f_c+f_m$~~

$\therefore DSB\_SC(f) = \frac{1}{2}\left[\frac{1}{2}(\delta(f+f_c+f_m)) + \frac{1}{2}\delta(f-(f_c+f_m))\right]$

$+ \frac{1}{2}\left[\frac{1}{2}\delta(f-(f_c-f_m)) + \frac{1}{2}\delta(f+(f_c-f_m))\right]$

Modulated Signal $\longrightarrow$



• Now in demodulation process:-

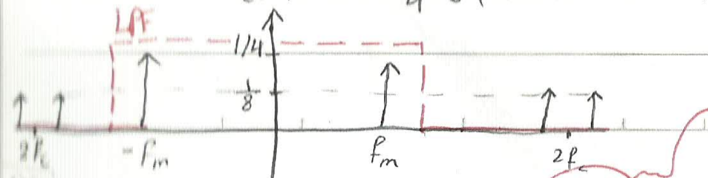
Multiply the modulated signal by $\cos(2\pi f_c t)$:-

$e(t) = \cos 2\pi f_m t \; \cos^2 2\pi f_c t = \frac{1}{2}\cos 2\pi f_m t(1 + \cos 4\pi f_c t)$

$\Rightarrow e(f) = \frac{1}{2}\left[\frac{1}{2}(\delta(f+f_m) + \delta(f-f_m)) + \left[\frac{1}{4}\delta(f+(2f_c+f_m))\right.\right.$

$+ \frac{1}{4}\delta(f-(2f_c+f_m)) + \frac{1}{4}\delta(f+(-2f_c+f_m))$

$\left.\left. + \frac{1}{4}\delta(f-(-2f_c+f_m))\right]\right]$



هلا بنضرب e(t) ب LPF
بيطيروا ال δ's ع 2fc
وبضل ال δ's ع ال fm اللي هي
نصف m(t) الأصلية وهيك نرجع ال m(t) Signal

Note: $f_c > f_m$ to avoid the overlap of the spectra centered at f_c and $-f_c$ ...

* **Demodulators:- (DSB_SC)**

The different between the Modulator and the Demodulator is the output filter.

- Modulators $\rightsquigarrow$ BPF tuned to f_c
- demodulators $\longrightarrow$ LPF

• This type of demodulation is called "coherent" or "syncronous" because:

The carrier and the reciver have the same phase and the same frequancy...

4.3: **Amplitud Modulation; double side band-large carrier [DSB-LC]**

* **Dissadvantages of DSB_SC:-**

The ~~Received Signal~~ R_x must generate acarrier at the same frequancy and phase as T_x ...

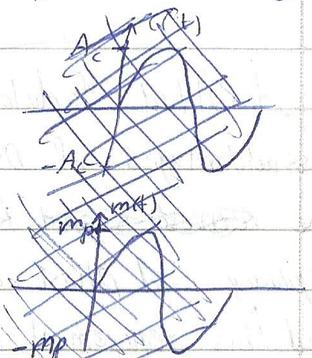
and this requirement is not easy to achieve in practice, because the traveled signal (Modulated signal) may have travilled hundreds of miles and could suffer from some unknown freq. shift.

and The solution is to make T_x send the carrier together with the modulated signal, So that there is no need to generate a carrier at the R_x...
and This method is called DSB_LC

* DSB_LC dissadvantages :-
In this case the transmeter requires higher power...

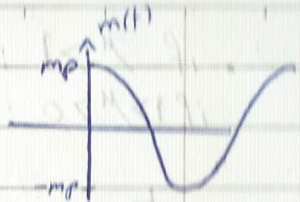
* DSB_LC :-
$c(t) = A_c \cos 2\pi f_c t$, f_c is RF range.

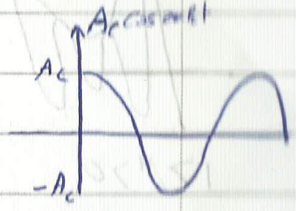
- AM signal $= A_c \cos 2\pi f_c t + m(t) \cos 2\pi f_c t$
$S_{AM}(t) = [A_c + m(t)] \cos 2\pi f_c t$ (Modulated signal)

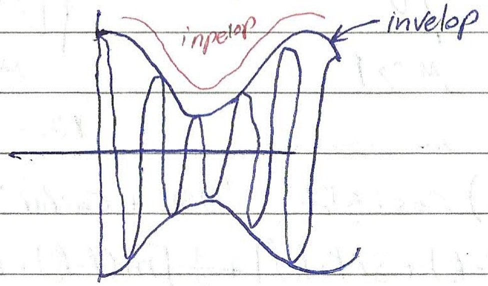


- We have two cases :-
① $A_c + m(t) \geq 0$
- the envelope has the ~~sam~~ same shape as m(t)
- we can detect the desired signal m(t) by deticting the invelope.

② $A_c + m(t) < 0$:
the invelope don't have the Same shape as m(t)
$\Rightarrow$ we can't detect the desiered signal m(t) by invelope detector.

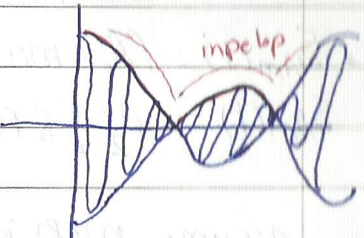






$A_c + m(t) > 0$
- we can use invelope detector.

لأنه هون شكل الـ invelope نفس شكل الـ m(t)



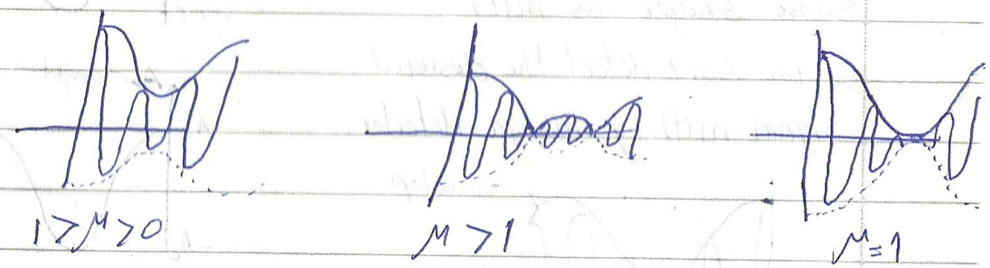
$A_c + m(t) < 0$
- we can't use invelope ditector

لأنه شكل الـ invelope صار غير شكل الـ m(t)

∴ The condition for envelope detection of an Am;
$$\underbrace{A_c + m(t) > 0 \text{ for all } t}$$

- another forms of the previous conditions :-
$$\underbrace{\mu = \frac{m_p}{A_c}}$$, ~~is the~~ μ: the Modulation index

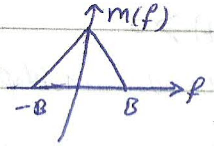
- if $\mu > 1$: "over Modulation" $(m_p > A_c)$
- if $1 > \mu > 0$: "envelop detector" $(m_p < A_c)$



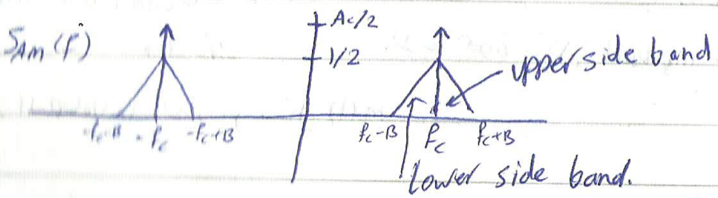
- $S_{Am}(t) = (A_c + m(t)) \cos 2\pi f_c t$ "DSB + Carrier"

$$S_{Am}(f) = \frac{A_c}{2}\left[\delta(f-f_c) + \delta(f+f_c)\right] + \frac{1}{2}\left[m(f-f_c) + m(f+f_c)\right]$$

- assume $m(f)$ is given as



$\Rightarrow S_{Am}(f)$



في شرط مهم $B \ll f_c$ موجود

× Side band & carrier power :-

$$P_{AM} = \frac{A_c^2}{2} + \frac{1}{2} P_m$$

power in carrier

$P_m = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} m^2(t)$ "power of side band"

The Power efficiency :-

$$\eta\% = \frac{\text{useful power}}{\text{total power}} = \frac{P_{sideband}}{P_{carrier} + P_{sideband}} = \frac{\frac{1}{2}P_m}{\frac{A_c^2}{2} + \frac{1}{2}P_m} \times 100\%$$

- For the special case of tone modulation (single tone)

$$\eta\% = \frac{P_m}{A_c^2 + P_m} \times 100$$

$m(t) = \mu A_c \cos 2\pi f_c t \Rightarrow P_m = \frac{(\mu A_c)^2}{2}$

$\mu A_c = m_p$

$m(t) = \underbrace{A_{m1} \cos 2\pi f_1 t}_{\text{single tone}} + A_{m2} \cos \ldots$ Multi tone

$$\eta\% = \frac{\mu^2}{\mu^2 + 2} \times 100\%$$

- For 100% Modulation $\Rightarrow \boxed{\mu = 1} \Rightarrow \eta_{max} = \frac{1}{3} = \boxed{33.3\%}$
- In AM, broadcast sys's عملياً $\boxed{\eta < 25\%}$ $\Rightarrow$ 66% waste of power

* draw backs of AM :-
  ① 50% BW wast. ② 66% power wast.

Ex let $m(t) = A_m \cos 2\pi f_m t$, $c(t) = A_c \cos 2\pi f_c t$
assume $f_m = 100\ Hz$, $f_c = 10\ KHz$ plot $S_{AM}(t)$, $S_{AM}(f)$ for $\mu = 0.5$, $\mu = 1$, $\mu = 1.5$?

Sol.

① $\mu = \frac{mp}{A_c} = 0.5 \Rightarrow mp = \boxed{0.5A_c = A_m}$

$S_{Am}(t) = [A_c + m(t)] \cos 2\pi f_c t$

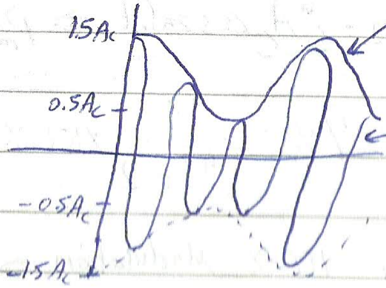
$= [A_c + A_m \cos 2\pi f_m t] \cos 2\pi f_c t$

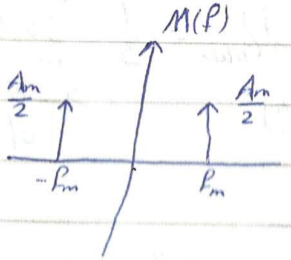
$= A_c[1 + 0.5 \cos 2\pi f_m t] \cos 2\pi f_c t$

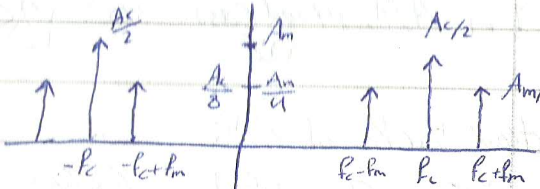
$= A(t) \cos 2\pi f_c t$

$A(t)|_{max} = 1+0.5 = 1.5A_c$

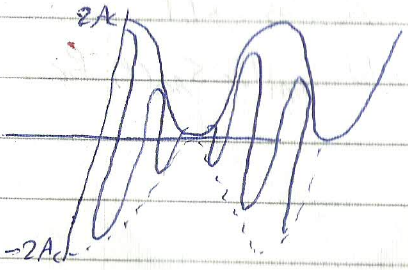
$A(t)|_{min} = 1-0.5 = 0.5A_c$

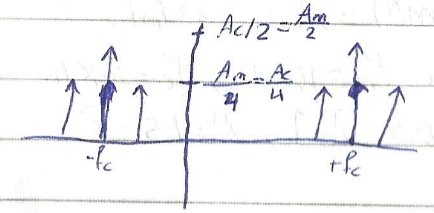




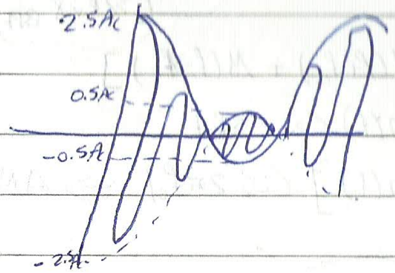


② $\mu = 1$





③ $\mu = 1.5$



مهم صار SC ولسه DC

if $\mu = \infty \Rightarrow A_c = 0$
$\Rightarrow$ There is no modulation

DC

باستخدام demodulator
coherent detector (synchronous)

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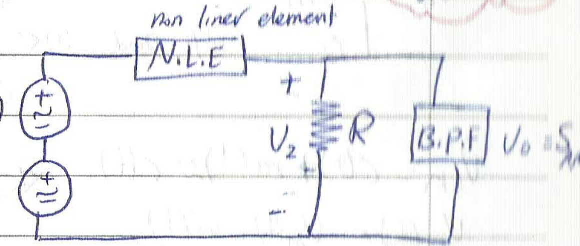
* AM Modulators:-

① square law Modulator:-

ممكن تكون DC في non linear region

I/p and o/p of NLE:



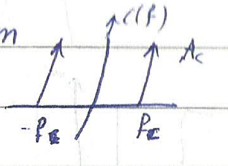
$V_2(t) = a_1 V_1(t) + a_2 V_1^2(t)$

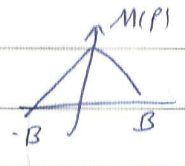
$= a_1(m(t) + c(t)) + a_2(m(t) + c(t))^2$

$= a_1 m(t) + a_1 c(t) + a_2 m^2(t) + a_2 c^2(t) + a_2 2m(t)c(t)$

$\Rightarrow V_2(f) = a_1 M(f) + a_1 C(f) + a_2 M(f) * M(f) + a_2 C(f) * C(f) + a_2 2M(f) *$

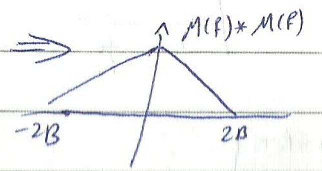
• assum

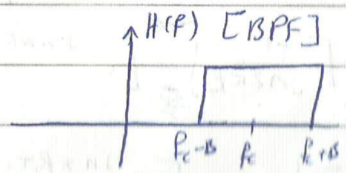




$(B \ll f_c)$

$\Rightarrow$





$\Rightarrow V_o$ of BPF $[M(f),\ M(f)*M(f),\ C(f)*C(f)]$

$f_c \gg B$ BPF

$\Rightarrow V_o(f) = a\,C(f) + A_c a_2 [M(f+f_c) + M(f-f_c)]$

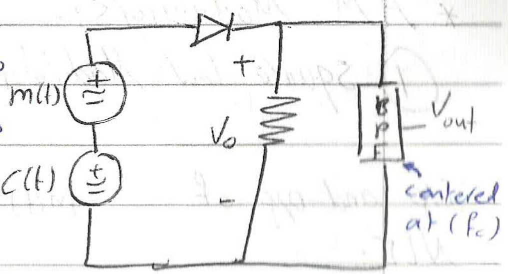
$\Rightarrow V_o(t) = a_1 c(t) + A_c a_2 m(t) \cos 2\pi f_c t$

$= [a_1 A_c + a_2 A_c m(t)] \cos 2\pi f_c t \Rightarrow$ AM signal

② Switching modulator :-

assum Ideal diode;

$$V_o = \begin{cases} V_{in} & \text{ON Diode, } C > 0 \\ 0 & \text{off Diode } C(t) < 0 \end{cases}$$

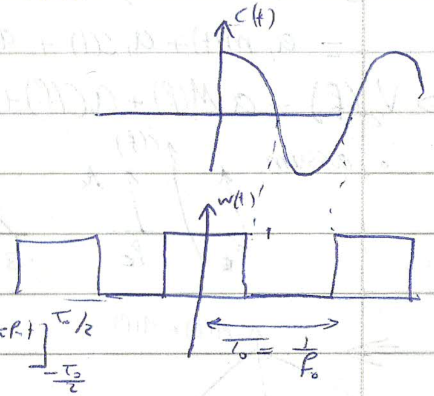


$V_{in} = c(t) + m(t) \simeq c(t)$ — assum $|m(t)| \ll A_c$, $m_p \ll A_c$

$V_o(t) = V_{in}(t)\, w(t)$,

where $w(t)$ is periodic pulse train

With period $T_0 = \frac{1}{f_c}$



$w(t) = \sum D_n e^{j2\pi n f_0 t}$

$$D_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} 1 \cdot e^{-j2\pi f_0 t}\, dt$$

$$= \frac{1}{T_0}\left[\frac{-1}{jn2\pi f_0}\right] \left[e^{-j2n\pi f_0 t}\right]_{-T_0/2}^{T_0/2}$$

$$= \frac{-1}{T_0 jn2\pi f_0}\left[e^{-jn\pi f_0 T_0} - e^{jn\pi f_0 T_0}\right]$$

$$= \frac{1}{T_0 n\pi f_0}\left[\frac{e^{jn\pi f_0 T_0} - e^{-jn\pi f_0 T_0}}{2j}\right]$$

$\therefore w(t) = C_0 + \sum C_n \cos 2\pi n f_0 t = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1} \cos 2\pi(2n-1) f_c t$

$= \frac{1}{2} + \frac{2}{\pi}\left[\cos 2\pi f_c t - \frac{1}{3}\cos 2\pi 3 f_c t + \frac{1}{5}\cos 2\pi 5 f_c t + \dots\right]$

$V_o(t) = [c(t) + m(t)]\left[\frac{1}{2} + \frac{2}{\pi}(\cos 2\pi f_c t - \dots)\right]$

$V_o(t) \rightarrow$ BPF $\rightarrow$ only

$= \left[\frac{A_c}{2}\cos 2\pi f_c t + \frac{2A_c}{\pi}\cos^2 2\pi f_c t + \frac{1}{2} m(t) + \frac{2m(t)}{\pi}\cos 2\pi f_c t\right]$

~~$V_{out}(t)$~~

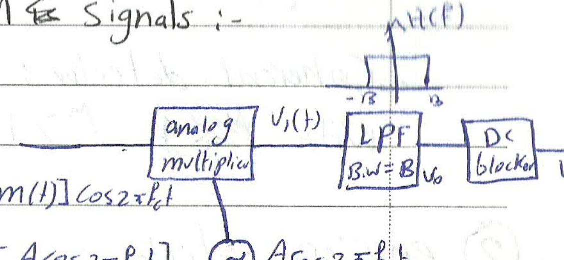
$\therefore V_{out}(t) = \left[\frac{A_c}{2} + \frac{2m(t)}{\pi}\right] \cos 2\pi f_c t$

low frequency ... BPF ... Will not pass...

Δ AM Signal

* DeModulation of AM Signals :-

① Coherent detector :-



$s(t) = [A_c + m(t)] \cos 2\pi f_c t$

$V_1(t) = [A_c + m(t)] \cos 2\pi f_c t\, [A \cos 2\pi f_c t]$

$= A A_c \cos^2 2\pi f_c t + A m(t) \cos^2 2\pi f_c t$

$= \frac{A A_c}{2}[1 + \cos 4\pi f_c t] + A m(t)[\cos 4\pi f_c t + 1]$

$V_1(f) = \frac{A A_c}{2}\delta(f) + \frac{A A_c}{4}[\delta(f - f_c) + \delta(f + f_c)] + \frac{A}{4}[M(f + f_c) + M(f - f_c)] + \frac{A}{2} M(f)$

$\therefore V_o(f) = \frac{A A_c}{2}\delta(f) + \frac{A}{2} M(f)$ "pass from LPF"

$\Rightarrow V_o(t) = \frac{A A_c}{2} + \frac{A}{2} m(t)$

$\Rightarrow V_o^*(t) = \frac{A}{2} m(t)$ "pass from DC. blocker"

* If there is a phase drift in L.osc. ;

$L.osc = A\cos(2\pi f_c t + \theta)$

$\Rightarrow V_o(t) = \frac{A}{2} m(t)\cos\theta = K m(t)$

if θ is Constant } is valid

if $\theta \neq \frac{\pi}{2}$

but if $\cos\theta$ is fn. of time $\Rightarrow$ it is a Modulated signal

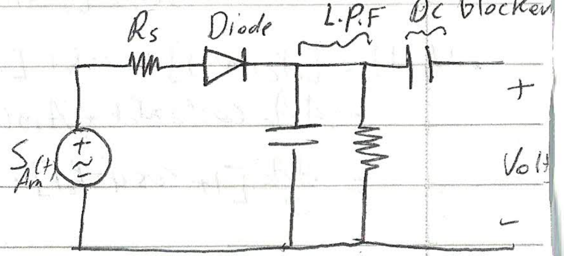
* If there is a ~~phase~~ drift in frequancy :-

$V_i(t) = A\cos 2\pi t(f_c + \Delta f) = A\cos(2\pi f_c t + 2\pi \Delta f t)$

V_o ? find V_o with your self.

. Coherent detector ; varly Used for AM Signals with ~~[illegible]~~ $\mu \geq 1$

② envilope detector :-



. Used for AM with ($\mu \leq 1$).

. $V_o(t) = K m(t) + \text{ripple}$

. in +ve H.C :

D is ON $\Rightarrow$ the c is charging

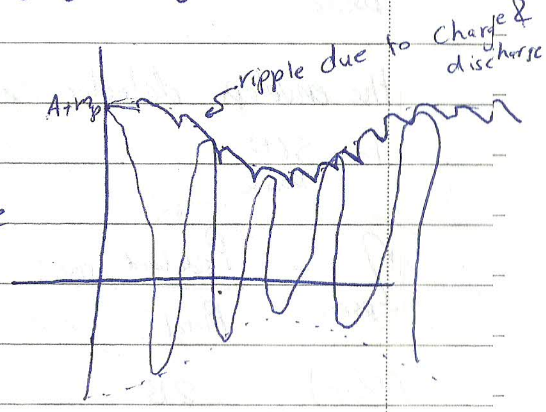
$\tau_{ch} = R_s C \ll \frac{1}{f_c}$ to have very high speed of charging...

. In −ve H.C :

"D" is off , C is discharging through R

$\tau_{dsch.} = RC$

$\frac{1}{B} \gg R_C \gg \frac{1}{f_c}$

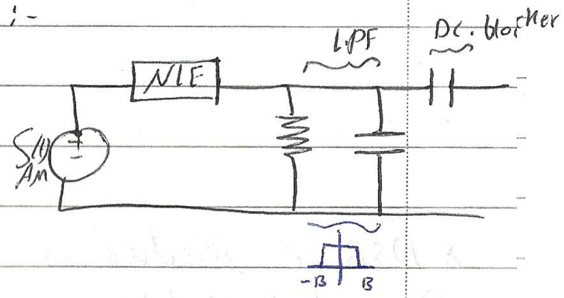


. to Make charg & discharge synchronous with the signal :

$\tau_{ds} = RC \ll \frac{\sqrt{1-\mu^2}}{2\pi B \mu}$

. if $\mu = 1 \Rightarrow RC = 0 \Rightarrow \tau_{ds} = 0$ "impossible"

③ Squaring loop detector :-



$V_1(t) = a_1 V_{in}(t) + a_2 V_{in}^2(t)$

$\Rightarrow V_o(t) = \frac{a_2}{2}[A_c + m(t)]^2$

$= \underbrace{\frac{a_2}{2} A_c^2}_{\text{تتشال بالـ dc. blocker}} + \underbrace{a_2 A_c m(t) + \frac{a_2}{2} m^2(t)}_{\text{بيعدوا من الـ LPF}}$

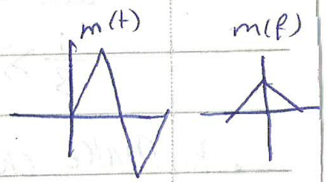
$\rho = \frac{\frac{a_2}{2} m^2(t)}{\frac{a_2}{2} 2 A_c m(t)} \Big| \rightarrow$ fn. of μ

minimize

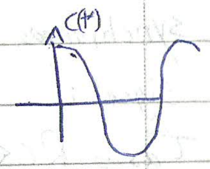
* DSBSC :

$S_{DSBSC}(t) = A m(t) \cos 2\pi f_c t$

the envelope detector can't be used to $S_{DSBSC}(t)$.

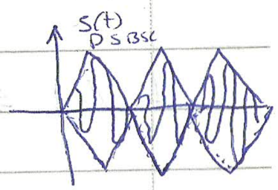


$$\eta_{DSBSC} = \frac{P_{side\,band}}{P_{total}} \times 100\%$$



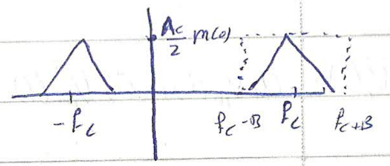
$(B_T)_{DSBSC} = 2B$

$P_s(t) = \frac{A_c^2}{2} P_m = P_{s(t)\,total} = P_{sideband}$



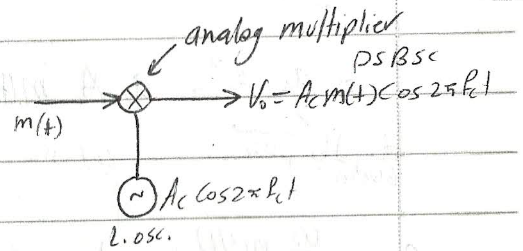
$\Rightarrow \eta_{DSBSC} = 100\%$

invelop $\neq m(t)$



* DSBSC Modulators:-

① product Modulator :-

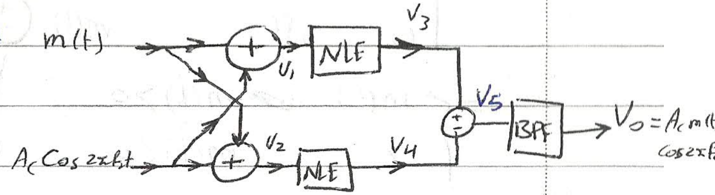


② Square low Modulator (Signal balance modulator):-

$V_1(t) = m(t) + A_c \cos 2\pi f_c t$

~~$V_2(t) = b_1 V_2^2 + b_2 V_2^2$~~

$V_2(t) = A_c \cos 2\pi f_c t - m(t)$



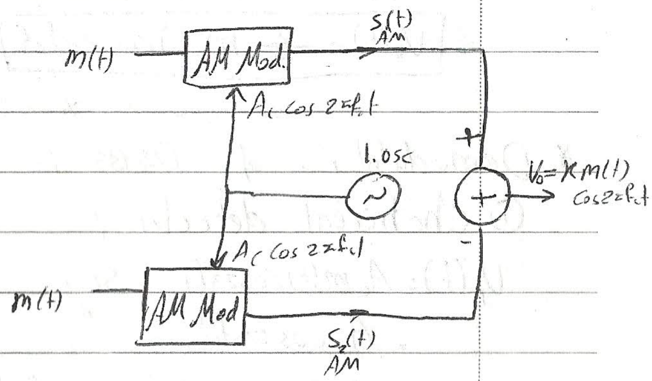
$V_3 = a_1 V_1^2 + a_2 V_1^2$

$V_4 = b_1 V_2^2 + b_2 V_2^2$

$V_5(t) = V_3 - V_4 = 2 A_c a_1 m(t) + 4 A_c b_1 m(t) \cos 2\pi f_c t$

$V_0(t) = 4 A_c b_1 m(t) \cos 2\pi f_c t = \boxed{K m(t) \cos 2\pi f_c t} = S(t)_{DSBSC}$

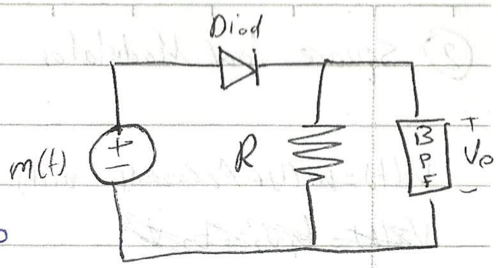
③ balanced Modulator :-

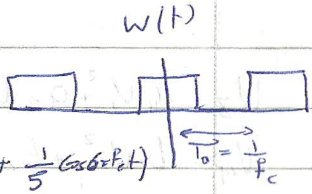


(14) Switching Modulator :

$$V_1(t) = \begin{cases} V_{in} & ON \\ 0 & off \end{cases}$$

$$= \begin{cases} m(t) & m(t) > 0 \\ 0 & m(t) < 0 \end{cases}$$





$$w(t) = \sum C_n \cos 2\pi f_c t$$

$$= \frac{1}{2} + \frac{2}{\pi}\left(\cos 2\pi f_c t - \frac{1}{3}\cos 6\pi f_c t + \frac{1}{5}\cos 10\pi f_c t\right)$$

$$V_2(t) = W(t) . V_{in} \cancel{= \Sigma m(t) + A_c \cos 2\pi f_c t}$$

$$V_2(t) = \frac{1}{2} m(t) + \frac{2}{\pi} m(t) \cos 2\pi f_c t$$

$$V_2(t) \rightarrow BPF \text{ (at } f_c) \rightarrow V_o(t)$$

$$\therefore \boxed{V_o(t) = \frac{2}{\pi} m(t) \cos 2\pi f_c t}$$

*

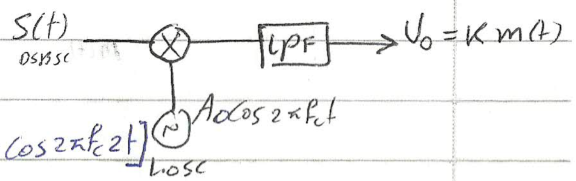
* Demodulators of DSBSC :-

① ChohERENT detector ;

$$V_1(t) = A_c m(t) \cos 2\pi f_c t$$

$$. A_o \cos 2\pi f_c t$$

$$= \frac{A_o A_c}{2} m(t) [1 + \cos 2\pi f_c 2t]$$



$V_o(t) = \frac{A_o A_c}{2} m(t)$, provide the L.osc. is in Complete ChohERENT with T osc.

---

lack of Choherance :

- ① phase drift $A_o \cos[t 2\pi f_c + \theta]$
- ② Frequancy drift $A_o \cos[2\pi(f_c + \Delta f) t]$

معناه بدل ال L.osc ح يطلع تشوه ع ح يصير بـ $V_o(t)$

* to assure coherancy :-

① Very stable quartz osc.
② pilot carrier (low power)
③ phase locked loop C_K : PPL :-

$$f_i(t) = f_c + K V_o(t)$$

$$\cancel{V_1(t) = AA^*}$$

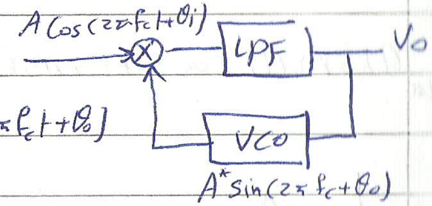
$$V_1(t) = AA^* \cos(2\pi f_c t + \theta_i) \sin(2\pi f_c t + \theta_o)$$

$$= \frac{AA^*}{2}\left[\sin 2\pi f_c t + \theta_i + \theta_o\right.$$

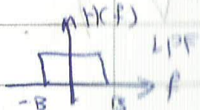
ما راح يمر

$$\left. + \sin(\theta_i - \theta_o)\right]$$

راح يمر



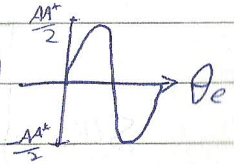
VCO : Voltage controlled osc



$$V_1(t) \rightarrow LPF \rightarrow V_o(t)$$

$$V_o(t) = \frac{AA^*}{2} \sin(\theta_i - \theta_o)$$

$$= \frac{AA^*}{2} \sin \theta_e$$



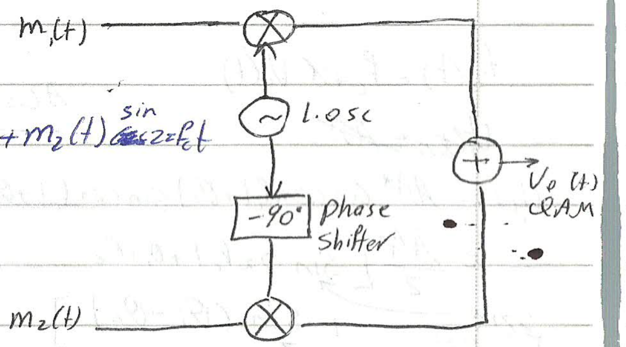
$$. \text{if } \theta_i = \theta_o \Rightarrow \theta_e = 0 \Rightarrow V_o = 0 \Rightarrow f_i(t) = f_c$$

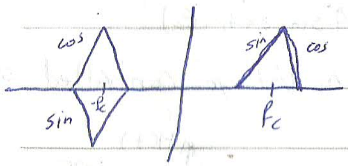
* Two reciveres imbeded in the pll ck's :
  ① Squaring loop reciever.
  ② ~~\$~~ Costas reciever.

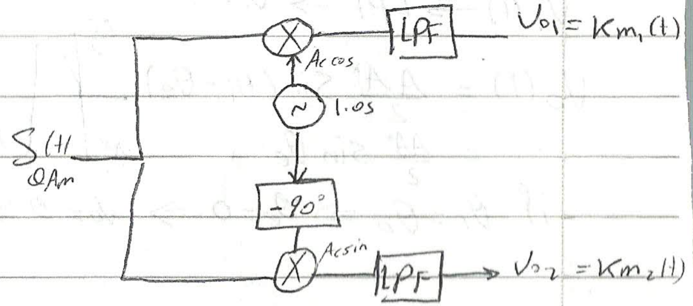
4.4) Quadrature Amplitude Modulation (QAM) :-

The DSB signals occupy twice the bandwidth ~~requered~~ required for the base band, This disadvantage Can be overcome by transmitting two DSB signals Using carriers of the same frequancy but in phase quadrature.

$$S_{QAM}(t) = m_1(t)\cos 2\pi f_c t + m_2(t)\sin 2\pi f_c t$$

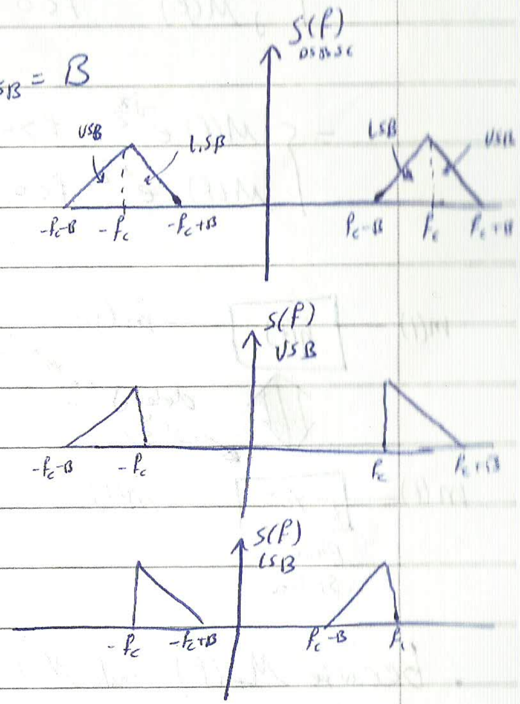






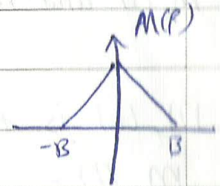
* Single Side band Modulation (SSB) :-

$$\boxed{\eta_{SSB} = 100\%} \qquad (B_T)_{SSB} = B$$



* The hilpart of m(t) is;

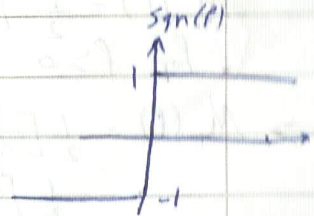
$$\hat{m}(t) = m(t) * \frac{1}{\pi t}$$



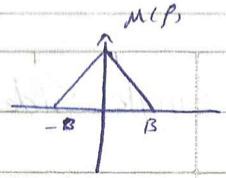
$$m(t) \longrightarrow \boxed{h(t) = \frac{1}{\pi t}} \longrightarrow \hat{m}(t)$$

$$M(f) \longrightarrow \boxed{H(f) = -j\,\mathrm{sgn}(f)} \longrightarrow \hat{M}(f)$$

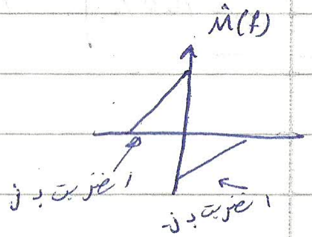
$$\boxed{\hat{M}(f) = M(f) \cdot -j\,\mathrm{sgn}(f)}$$

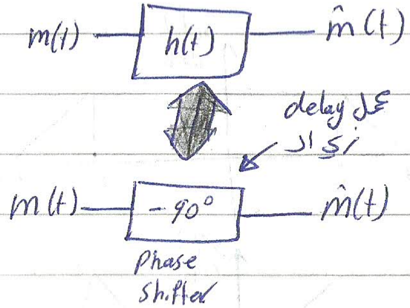


$$\hat{M}(f) = \begin{cases} -j\,M(f) & f>0 \\ j\,M(f) & f<0 \end{cases}$$



$$= \begin{cases} M(f)\, e^{-j\frac{\pi}{2}} & f>0 \\ M(f)\cdot e^{j\frac{\pi}{2}} & f<0 \end{cases}$$





- because $M_+(f)$ and $M_-(f)$ are not even fn. $\Rightarrow$ $m_+(t)$ and $m_-(t)$ Can't be real (they are complex).

- $m_+(t)$ is +ve Complex pre invelop of $m(t)$:-

$$m_+(t) = \frac{1}{2}[m(t) + j\hat{m}(t)]$$

$$m_-(t) = \frac{1}{2}[m(t) - j\hat{m}(t)]$$

$m_+(t) + m_-(t) = m(t)$

- for $f>0$

$$\Rightarrow M_+(f) = \frac{1}{2}[M(f) + j\hat{M}(f)]$$

$$= \frac{1}{2}[M(f) + j(-jM(f)\,\mathrm{sgn}(f))]$$

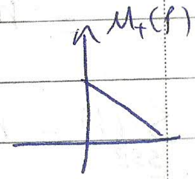
$$= \frac{1}{2}[M(f) + M(f)\,\mathrm{sgn}(f)] = \frac{1}{2}[M(f) + M(f)] = M(f)$$

- for $f<0$

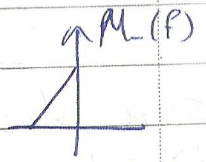
$$M_+(f) = \frac{1}{2}[M(f) + j(-jM(f)\,\mathrm{sgn}(f))]$$

$$= \frac{1}{2}[M(f) + M(f)\underbrace{\mathrm{sgn}(f)}_{-1}] = 0$$

$$\Rightarrow M_+(f) = \begin{cases} M(f) & f>0 \\ 0 & f<0 \end{cases}$$



- $$M_-(f) = \begin{cases} 0 & f>0 \\ M(f) & f<0 \end{cases}$$



* هلا بدي ارجع ل M(f) ← مجموع ال 2 signals

- $$S_{USB}(f) = [M_+(f-f_c) + M_-(f+f_c)]$$

$$S_{USB}(t) = m_+(t)e^{j2\pi f_c t} + m_-(t)e^{-j2\pi f_c t}$$

$$= [\tfrac{1}{2}m(t) + j\hat{m}(t)]e^{j2\pi f_c t} + [\tfrac{1}{2}m(t) - j\hat{m}(t)]e^{-j2\pi f_c t}$$

$$= m(t)\left[\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}\right] + j\hat{m}(t)\left[\frac{e^{j2\pi f_c t} - e^{-j2\pi f_c t}}{2}\right]$$

$$= \boxed{m(t)\cos 2\pi f_c t - \hat{m}(t)\sin 2\pi f_c t}$$

[illegible]

$$S_{LSB} = \Big\}$$

$$= m(t)\cos 2\pi f_c t + \hat{m}(t)\sin 2\pi f_c t$$

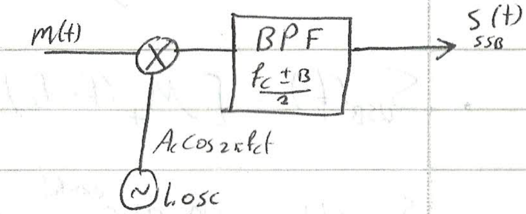
LSB

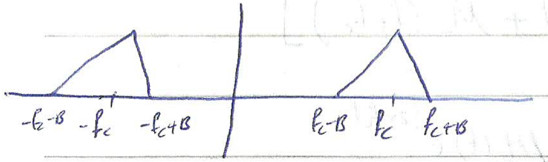
$$\therefore \boxed{S_{SSB}(t) = m(t)\cos 2\pi f_c t \pm \hat{m}(t)\sin 2\pi f_c t}$$

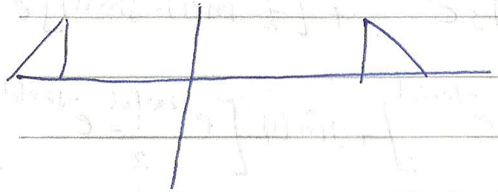
USB

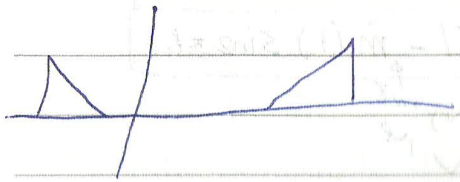
* SSB Modulators :-

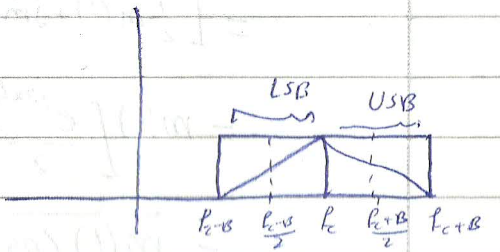
1) frequancy discriminator :-



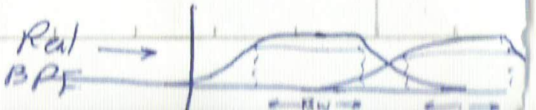




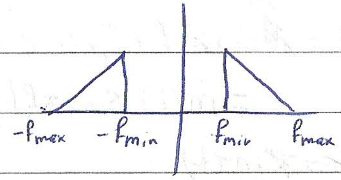


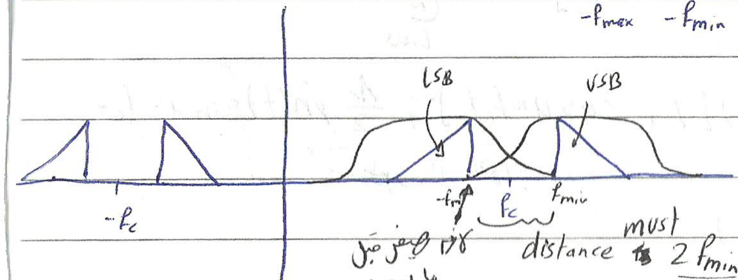


but this BPF is not possible



. So, it only possible with M(f) doesn't start from (f = 0)

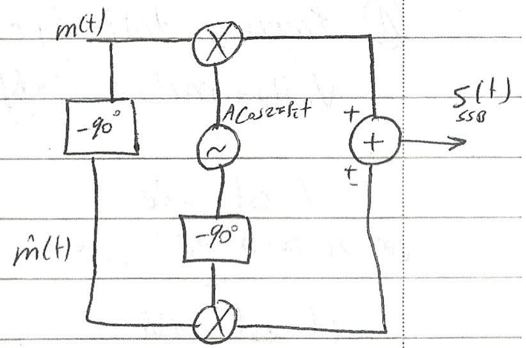




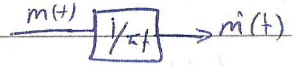
لأ يقص من ما يوصل ال LSB

distance must $2f_{min}$ between 2 filters

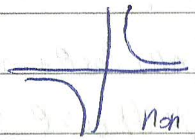
② phase discreminator



فعليا ما بقدر اعمل $\frac{1}{\pi t}$ عشان اطلع $\hat{m}(t)$



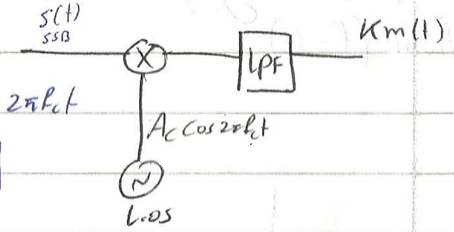
لأن $\frac{1}{\pi t}$



Non Causal

* SSB Demodulators:-

1) Choherent detector:-



$V_1(t) = A_c \cos 2\pi f_c t \,[m(t) \cos 2\pi f_c t \mp \hat{m}(t) \sin 2\pi f_c t]$

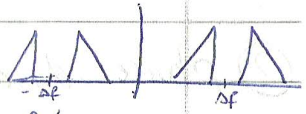
~~$V_o(t) = Km(t)$~~

$= \frac{A_c}{2} m(t)[1 + \underbrace{\cos 4\pi f_c t] \mp \frac{A_c}{2} \hat{m}(t) \sin 4\pi f_c t}_{\text{بيطير بال LPF}}$

$= \frac{A_c}{2} m(t)$

if there is :

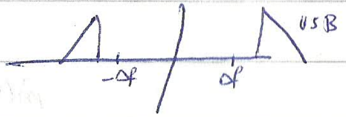
(A) frequany drift by Δf :



$V_o(t) = m(t) \cos 2\pi \Delta f t \mp \hat{m}(t) \sin 2\pi \Delta f t$

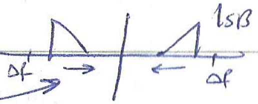
if Δf -ve

احيطلع بعطني ال USB



if Δf +ve

احيطلع بعطني ال LSB



(B) phase drift :

$V_o(t) = K\,[m(t) \cos\theta \pm \hat{m}(t) \sin\theta]$

$V_o(f) = K[M(f) \cos\theta \pm (-j\,\mathrm{sgn}(f) M(f) \sin\theta)$

$= \begin{cases} M(f)(\cos\theta \pm j\sin\theta) & f > 0 \\ M(f)(\cos\theta \mp j\sin\theta) & f < 0 \end{cases}$

$= K M(f)\, e^{\pm j\theta}$ ← if θ fn. of time the $V_o(t)$ will be distorted signal

2) envelope detector:-

$S_{SSB+C}(t) = [m(t) \cos 2\pi f_c t \mp \hat{m}(t) \sin 2\pi f_c t] + A_c \cos 2\pi f_c t$

$= [A_c + m(t)] \cos 2\pi f_c t \mp \hat{m}(t) \sin 2\pi f_c t$

$= A(t) \underbrace{\cos(2\pi f_c t + \theta)}_{\text{by Forier sieries.}}$, wher $A(t) = \sqrt{(A_c + m(t))^2 + \hat{m}^2(t)}$

$= \sqrt{A_c^2 + 2A_c m(t) + m^2(t) + \hat{m}^2(t)}$

$= A_c \sqrt{1 + \frac{2m(t)}{A_c}}$, if $A_c \gg m(t)$

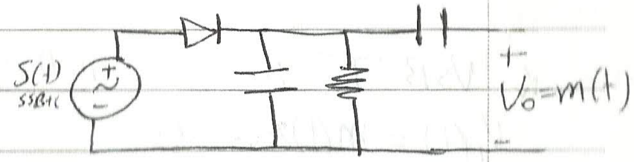
$\theta = \tan^{-1}\left(\frac{\hat{m}(t)}{A_c + m(t)}\right)$

• $\boxed{\mu \ll 1}$ $\boxed{\eta \ll 1}$

$(1+x)^n = 1 + nx$ for $x \ll 1$

$\Rightarrow \boxed{A(t) = A_c + m(t)}$

∴ envelope detector can be used to demodulate SSB+C ...



* Note that :

DSB-LC $A_c \geq m(t)$

SSB+C $A_c \gg m(t)$

4.5: Vestigal Sideband (VSB) :-

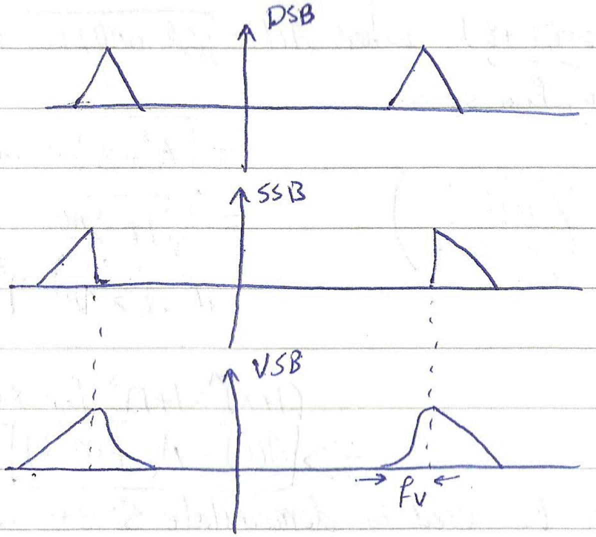
1) less B.W than DSB $(B_T)_{VSB} \leq B + 0.25B$

$\leq \boxed{1.25B}$

2) $\eta_{VSB} = 100\%$ carrier ما في

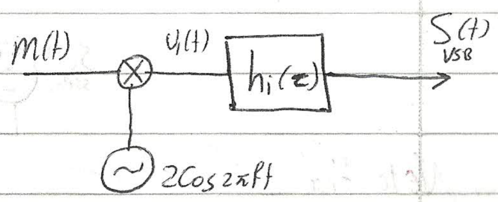
3) easy to generate (less complex than SSB)

قلل الـ BW باستخدام أقل real filter أعقد



$\boxed{f_V = 0.25B}$

VSB Tx :-



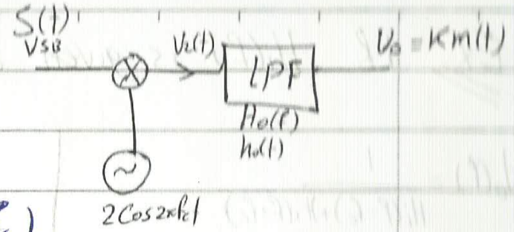
$V_1(t) = m(t) 2\cos 2\pi f_c t$

$S_{VSB}(t) = V_1(t) * h_i(t)$

$S_{VSB}(f) = V_1(f) \cdot H_i(f)$

$= [M(f+f_c) + M(f-f_c)] \cdot H_i(f) \quad \text{---} (1)$

VSB Rx :-



$V_2(t) = S_{VSB}(t) \cdot 2\cos 2\pi f_c t$

$V_2(f) = S_{VSB}(f+f_c) + S_{VSB}(f-f_c)$

$V_o(t) = V_2(t) * h_o(t)$

$V_o(f) = V_2(f) \cdot H_o(f)$

$= [S_{VSB}(f-f_c) + S_{VSB}(f+f_c)] \cdot H_o(f) \quad \text{---} (2)$

sub (1) in (2) :

$V_o(f) = \Big([M(f-2f_c) + M(f)] H_i(f-f_c) + [M(f) + M(f+2f_c)] H_i(f+f_c)\Big) \underset{LPF}{H_o(f)} = K M(f)$

$\Rightarrow M(f)[H_i(f-f_c) + H_i(f+f_c)] H_o(f) = K M(f)$

assume $K = 1$

$\Rightarrow \boxed{H_o(f) = \dfrac{1}{H_i(f-f_c) + H_i(f+f_c)}}$

$|f| \leq B$

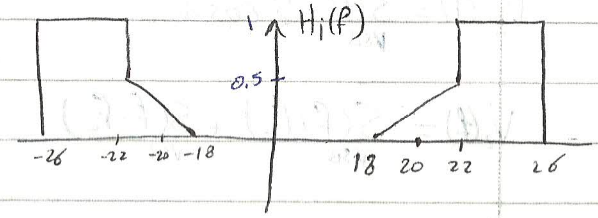
$\boxed{-B \leq f \leq B}$

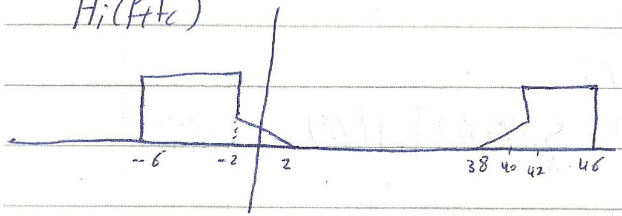
يعني بغض النظر عن نوع الـ filter بالـ Tx بقدر أرجع الـ signal باستخدام الـ filter الـ Rx بحيث انه يعمل للـ filter الـ Tx shift يعني بجمعه لـ shift بالعكس وبطلع المقلوب بيكون هو الـ filter المطلوب بالـ Rx

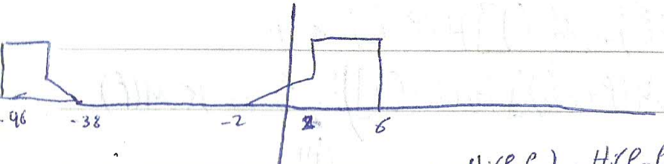
$\Rightarrow H_i(f)$ must given..

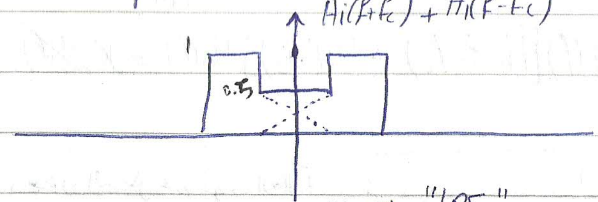
Ex. if $H_i(f)$ is given and $\boxed{f_c = 20KHz}$ find $H_o(f)$?

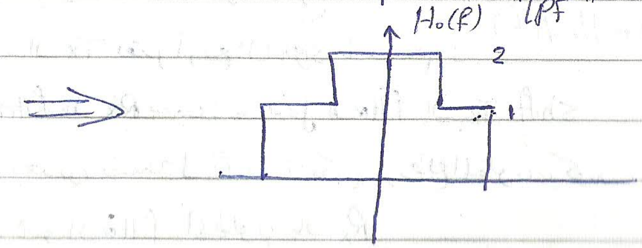
$$H_o(f) = \frac{1}{H_i(f-f_c)+H_i(f+f_c)}$$







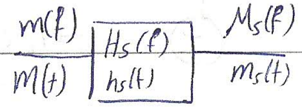




* Detection of VSB signals:-

$$S_{VSB}(t) = m(t)\cos 2\pi f_c t \mp m_s(t)\sin 2\pi f_c t$$; - USB + LSB Vistige

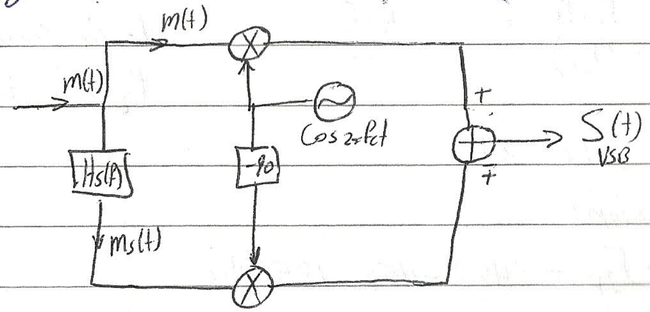
+ USB + LSB Vistige



• detiction of VSB is that of SSB;

1) coherent detector.

2) envelope detector. VSB+C

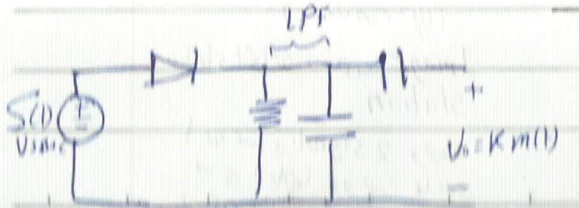


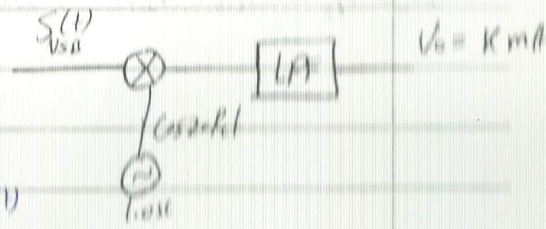
$$S_{VSB+C}(t) = [m(t)+A_c]\cos 2\pi f_c t \mp m_s(t)\sin 2\pi f_c t$$

$$= A(t)\cos(2\pi f_c t+\theta), \quad A(t) = \sqrt{(m(t)+A_c)^2+m_s^2(t)}$$

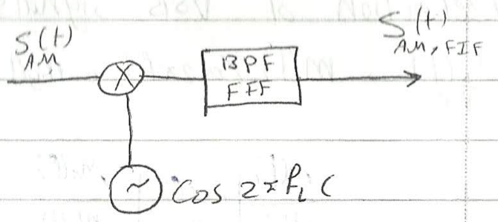
$$\theta = \tan^{-1}\left(\frac{m(t)}{m(t)+A_c}\right)$$

must $A_c \gg |m(t)|$

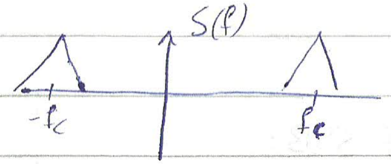


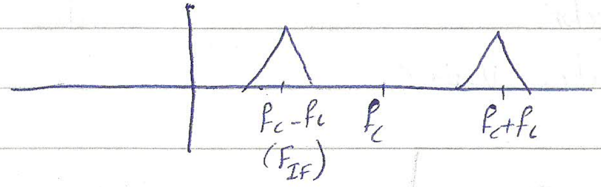


* Frequancy mixing:-



- $F_{IF} = 455\ KHz$
- $f_c = 520 - 1600\ KHz$





- Up Conversion
  $f_L = f_c + F_{IF}$
- down Conversion
  $f_L = f_c - F_{IF}$

- assum $f_c = 1400\ KHz$.
- Up conversion;
  - $f_L = f_c + F_{IF} = 1400 + 455 = 1855\ KHZ$
- down conv.
  $f_L = f_c - F_{IF} = 945\ KHz$
- if I have another signal with $f_{c_2} = \underset{f_{c_1}}{1400} + 2(455)$
  $= 2310$

$\Rightarrow f_{L_2} = f_{c_2} - F_{IF} = 2310 - 455 = 1855\ KHz$!!;

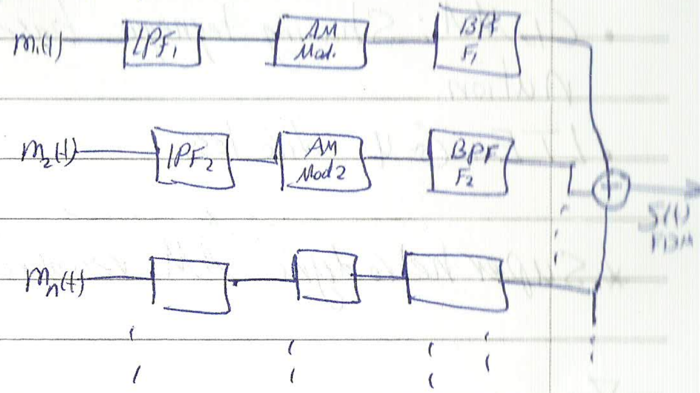
رح يصير ع نفس تردد f_{L_1}
(up conver.)

وهاي الحالة تسمى Image station

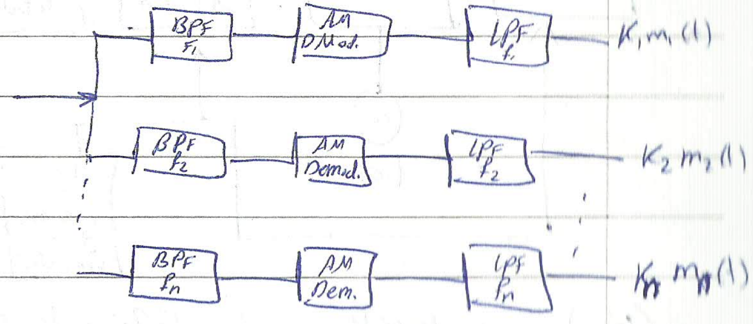
انه عندي 2 signal وحدة
بدياها ووحدة ما بدي

* FDM: Sharing legneeqes based on channle B.W...

- FDM Tx



- FDM Rx

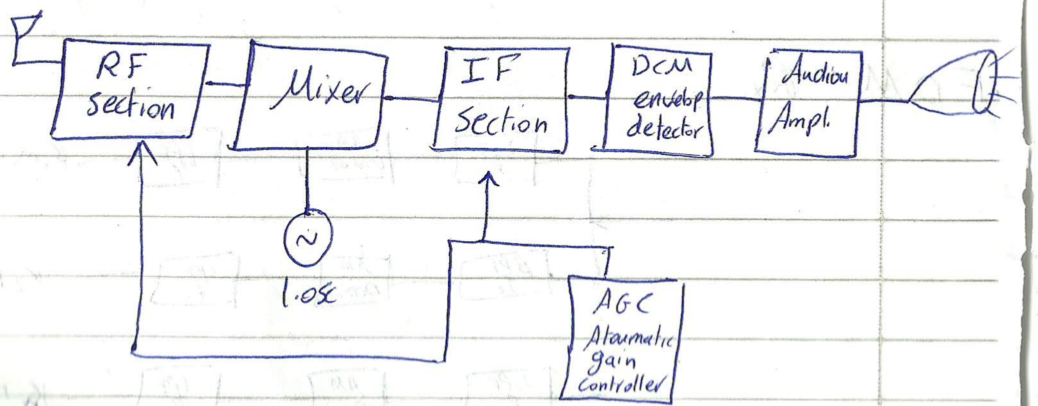


- dissadvantages: Waste of B.W if User in idle state.

* TDM; Sharing the Transmition time rates
  - Waiting time for the tocken is long as No. of users.
  - if users data are not symm. $\Rightarrow$ Zero stuffing.

- STDM: over come zero stuffing in TDM.
- CDMA: Sharing tegneque based on spectrum nution.
- LTE: G 4 Wire less.

* Super hetrodyne AM recievers:



$(B_T)_{hetrodyne} = 10 KHz$, $f_c = 520 - 1600$ KHz

Chapter 5: Angle Modulation (FM, PM)

Angle Mod. is "NON linear operation."

$$\boxed{S_{EM}(t) = A_c \cos(2\pi f_c t + \phi_i(t))}$$ "The general form"

$\phi_i(t)$ → instantanous phase

$$= A_c \cos \theta_i(t)$$

- $$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}$$ "instantanous frequancy"

$$\Rightarrow \boxed{\theta_i(t) = 2\pi \int f_i(t)\, dt}$$

(1) either $\phi(t) \propto m(t)$; "Phase Modulation"

$$\boxed{S_{PM}(t) = A_c \cos(2\pi f_c t + K_p m(t))}$$

K_p → phase modulation sensitivities

- $$f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = \frac{1}{2\pi}[2\pi f_c + K_p m'(t)]$$

$$\therefore\ f_i(t) = f_c + \frac{K_p}{2\pi} m'(t)$$ ⇒ f_i depend on the derivative of m ($m'(t)$)

OR

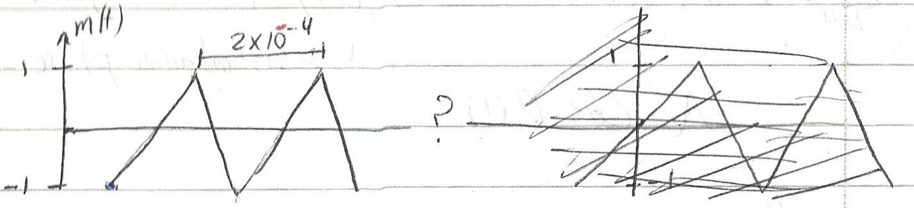
(2) $f_i(t) \propto m(t)$. "Frequancy modulation"

$\cancel{f_i(t) = K_f m(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt}}$ ⇒ f_i depend on $m(t)$

$$\Rightarrow \boxed{S_{FM}(t) = A_c \cos\left(2\pi f_c t + K_f \int m(t)\, dt\right)}$$

$$\therefore\ f_i(t) = \frac{1}{2\pi}\frac{d\theta_i(t)}{dt} = f_c + \frac{K_f}{2\pi} m(t)$$

Ex Sketch FM and PM waves for modulating Signal $m(t)$, $K_f = 2\pi \times 10^5$, $K_p = 10\pi$, $f_c = 100\,MHz$?



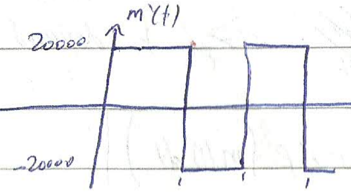
- to draw $S_{FM}(t)$ we must find $f_i(t)$ min & max.

$$\max f_i(t) = f_c + \frac{K_f}{2\pi} m(t)\Big|_{max}$$
$$= 10^8 + 10^5 (1) = 100.1\,MHz$$

$$\min f_i(t) = f_c + \frac{K_f}{2\pi} m(t)\Big|_{min} = 10^8 - 10^5 = 99.9\,MHz$$

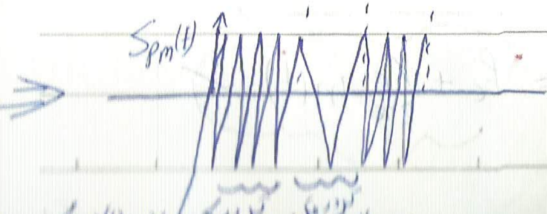
بال FM كل ما كبرت m(t) بتكبر fi(t)
أما بال PM
كل ما كبرت مشتقة m(t) بتكبر fi(t) والعكس

- Now to draw $S_{PM}(t)$ we must drwa $m'(t)$;



$$m'(t)\Big|_{max} = \frac{2}{10^{-4}} = 20000$$

$$m'(t)\Big|_{min} = -20000$$



$$f_i(t)\Big|_{max} = f_c + \frac{K_p}{2\pi} m'(t)\Big|_{max}$$
$$= 100.1\,MHz$$

$$f_i(t)\Big|_{min} = f_c + \frac{K_p}{2\pi} m'(t)\Big|_{min} = 99.9\,MHz$$

---

* $m(t) \rightarrow \boxed{h(t)} \rightarrow \left.\begin{matrix} \phi(t) = K_p m(t) \\ \phi(t) = K_f \int m(t) \end{matrix}\right\} = m(t) * h(t)$

$$h(t) = K_p\,\delta(t) \Rightarrow \phi(t) = K_p m(t)$$
$$h(t) = K_f\,u(t) \Rightarrow \phi(t) = K_f \int m(t)\,dt$$

* **Bandwidth of angle modulation waves:**

1) FM Signals:

$$S_{FM}(t) = A_c \cos\left[2\pi f_c t + K_f \int m(t)\,dt\right]$$

- $\cos\theta = Re[e^{j\theta}]$

$$\Rightarrow S_{FM}(t) = Re\left[A_c e^{j(2\pi f_c t + K_f \int m(t)\,dt)}\right]$$
$$= Re\left[A_c\, e^{j2\pi f_c t}\, \underbrace{e^{jK_f \int m(t)\,dt}}_{\cos + j\sin}\right]$$

* $$e^{jK_f \int m(t)\,dt} = \sum_{n=0}^{\infty} \frac{(jK_f \int m(t)\,dt)^n}{n!}$$

$\boxed{\beta(t) = \int m(t)}$

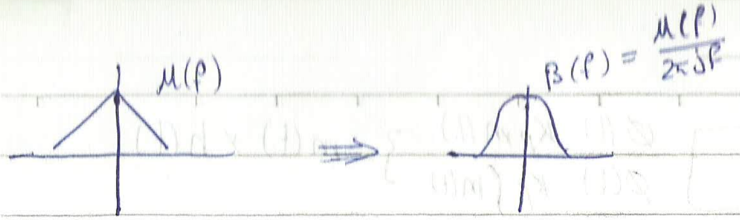
$$= \sum_{n=0}^{\infty} \frac{(jK_f \beta(t))^n}{n!}$$

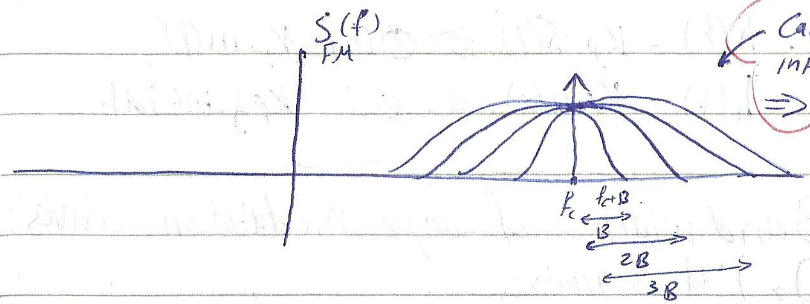
$$= 1 + \frac{jK_f \beta(t)}{2} - \frac{K_f^2 \beta^2(t)}{3!} + \frac{K_f^4 \beta^4(t)}{4!} + \cdots$$

$$\Rightarrow S_{FM}(t) = Re\left[A_c \overset{\cos + j\sin}{e^{j2\pi f_c t}} \left[1 + jK_f\beta(t) - \frac{K_f^2 \beta^2(t)}{2!} - \frac{K_f^3 \beta^3(t)}{3!} + \cdots\right]\right]$$
$$= A_c\left[\cos 2\pi f_c t - K_f \beta(t) \sin 2\pi f_c t - \frac{K_f^3 \beta^3(t)}{3!} \sin 2\pi f_c t + \cdots\right]$$

$$\Rightarrow S_{FM}(f) = A_c\Big[\tfrac{1}{2}(\delta(f-f_c)+\delta(f+f_c)) - \frac{K_f}{2j}[\gamma(f-f_c)+\gamma(f+f_c)]$$
$$- \frac{K_f^2}{4}[\gamma^2(f-f_c)+\gamma^2(f+f_c)] + \frac{K_f^3}{j3!}[\gamma^3(f-f_c)+\gamma^3(f+f_c)]\Big]$$

assume $\boxed{\beta(t) \xrightarrow{F} \gamma(f)}$





Carrier + infinit DSB's
⇒ B.W = ∞

$\Rightarrow (B_T)_{FM} = \infty$
and to minimize the B.W there are Two types of FM:-

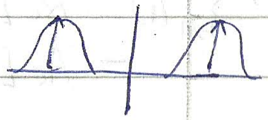
① NBFM:
assuming $|K_f m(t)| \ll 1$

$\Rightarrow \frac{K_f^2}{2} \lll 1 \Rightarrow \frac{K_f^3}{3!} \lll 1$

∴ I Can Ignore them from tylor series $(K_f^2 + \cdots)$

$\Rightarrow S_{NB} = A_c \cos 2\pi f_c t - K_f \int m(t)\,dt \,\sin 2\pi f_c t$

∴ $(B_T)_{NBFM} = 2B$ بدل ما كانت (∞)



② WBFM:
assuming $\boxed{K_f m'(t) \gg 1}$

frequancy diviation

$\Rightarrow (B_T)_{WBFM} = 2(\Delta f(t) + B) \simeq 2\Delta f(t)$ if $\Delta f(t) \gg$

wher $\Delta f(t) = \frac{K_f}{2\pi} m(t)$ or $\frac{K_p}{2\pi} m'(t)$

freq Mod. (FM) — phase Mod. (PM)

* power effeciency = 100 %

β "Modulation index" $\beta = \frac{\Delta f}{B}$

Δf: frequancy diviation
B: B.W for massage signal

بشكل عام
- if $\beta < 0.2 \Rightarrow \Delta f < B$
  $\Rightarrow (B_T) = 2B$
- if $\beta \gg \Rightarrow \Delta f \gg B$
  $(B_T) = 2\Delta f$

- $B = f_o = \frac{1}{T_o}$

• اذا كانت m(t)
low frequancy fn.
الافضل استخدام
FM
لان $\downarrow \Delta f = \frac{K_f m(t)}{2\pi}$
اما لو استخدمنا PM
$\downarrow \Delta f = \frac{K_p m'(t)}{2\pi}$
رح يقل Δf وبالتالي B.W↓
لانه Δf يعتمد بال PM ع
مشتقة ال m(t)
وال low
freq مشتقته صغيرة

* Single Tune (FM) Signal:-

let $m(t) = A_m \cos 2\pi f_m t$

$$S_{FM}(t) = A_c \cos[2\pi f_c t + k_f \int m(t)\,dt]$$

$$S_{SFM}(t) = A_c \cos[2\pi f_c t + \beta \sin 2\pi f_m t]$$

$$= Re[A_c e^{j2\pi f_c t} e^{j\beta \sin 2\pi f_m t}]$$

$$e^{j\beta \sin 2\pi f_m t} = \sum C_n e^{+j2\pi f_m t}$$

$$C_n = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} e^{j\beta \sin 2\pi f_m t} \cdot e^{-j2\pi f_m t}\,dt$$

$$x = 2\pi f_m t \Rightarrow dx = 2\pi f_m\,dt$$

$$\Rightarrow C_n = f_m \int_{-1/2f_m}^{1/2f_m} e^{j\beta \sin x} \cdot e^{-jnx} \frac{dx}{2\pi f_m}$$

$$= \frac{1}{2\pi}\int_{-1/2f_m}^{\frac{1}{2f_m}} e^{j(\beta \sin x - nx)}\,dx$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j(\beta \sin x - nx)}\,dx \doteq J_n(\beta)$$

n^{th} order Bessel function of first Kind.

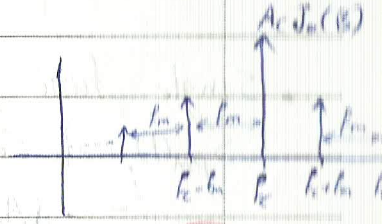
$$\Rightarrow e^{j\beta \sin 2\pi f_m t} = \sum J_n(\beta)\, e^{j2\pi f_m t}$$

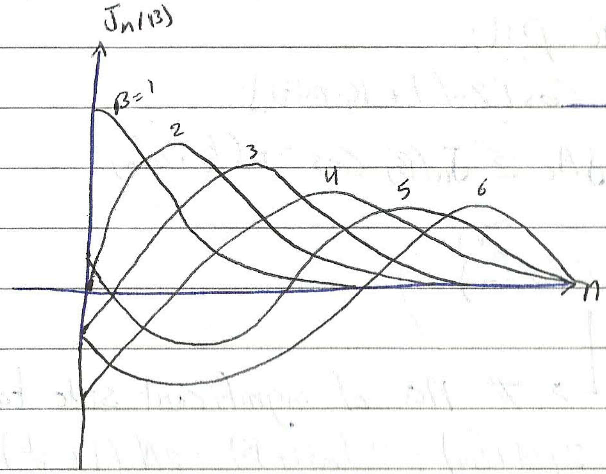
$$\Rightarrow S_{SFM}(t) = Re[A_c \sum J_n(\beta)\, e^{j2\pi n f_m t} e^{j2\pi f_c t}]$$

$$= \sum_{-\infty}^{\infty} A_c J_n(\beta) \cos(2\pi(f_c + n f_m)t)$$

$$S_{SFM}(f) = \sum \frac{A_c}{2} J_n(\beta)\, (\delta(f-(f_c+nf_m)) + \delta(f+$$

n^{th} order





* properties of $J_n(\beta)$:-

1) $J_{-n}(\beta) = (-1)^n J_n(\beta)$

2) $\sum_{n=-\infty}^{\infty} J_n^2(\beta) = 1$

3) for $\beta \ll 1$:-

$$J_0(\beta) = 1 \,,\quad J_1(\beta) = \frac{\beta}{2} \,,\quad J_n(\beta) = 0 \text{ for } n \geq 1$$

4) for $\beta \gg 1$:-

$$J_n(\beta) = 0$$

5) for $n \gg 1 \Rightarrow n \approx \beta + 1$

$$(B_T) = 2 n f_m$$

- Single tune PM:-

$$S_{PM}(t) = A_c \cos[2\pi f_c t + K_p m(t)]$$

$$= -j A_c \sum J_n(\beta) \cos 2\pi (f_c + n f_m) t$$

$$(B_T)_{FM} = n(2 f_m)$$

↳ # no. of significant side bands

- $n(2f_m) = 2(\Delta f + f_m) = 2 f_m(1 + \beta) = 2\Delta f \left(1 + \frac{1}{\beta}\right)$

$\Rightarrow$ for NBFM, NBPM $\} n = 1 \Rightarrow \boxed{B_T = 2 f_m}$, $\beta \ll 1$

$\Rightarrow$ for WBFM $\Rightarrow n = \beta + 1 \Rightarrow B_T = 2 n f_m$

$\approx \boxed{2\Delta f}$, $\beta \gg 1$

- extenstion of S_{FM}, S_{PM} is applicable to any periodic $m(t)$

* For double tune $m(t)$:-

$$S_{FM}(t) = \sum_n \sum_m J_n(\beta_1) J_m(\beta_2) \cos 2\pi (f_c + n f_{m_1} + m f_{m_2}) t$$

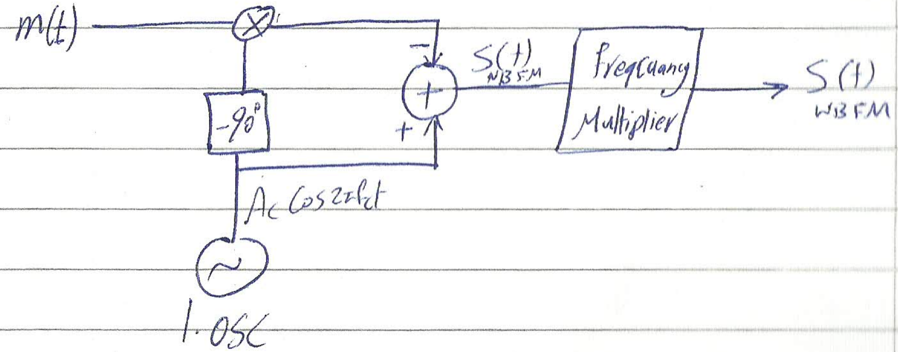
- ادرس امثلة الكتاب لانها مهمة على ذمة الدكتور.
- اذا طلب منك ترسم يكفي انك ترسم الـ positive frequancies

* FM Modulators:-

- 1) Indirect method:-

first generate NBFM, NB PM, then increase of Using frequancy Multiplier to get WBFM & WBPM.

$$S_{NBFM}(t) = A_c[\cos 2\pi f_c t - K_f m(t) \sin 2\pi f_c t]$$



* super hytrodyne reciver :-

$B_T = 200\,KHz$

② direct Method :- "Varacter Modulator" Voltage يعني

Controlled osc.

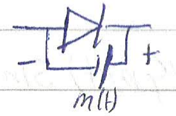
. Use VCO such that instantanous frequancy

. $f_i(t) = f_c + \frac{K_f}{2\pi} m(t)$

. $f_o = \frac{1}{2\pi\sqrt{LC}}$ ← C is function of time

$$\boxed{C(t) = C_o - K\,m(t)}$$

⇒ The variable capasitance is adiod in reverse mode



$m(t)\uparrow,\ d\uparrow,\ C\downarrow,\ f_o\uparrow$

* FM Demodulators :-

① Frequancy discriminator :-

$S_{FM}(t) = A_c \cos[2\pi f_c t + K_f \int m(t)\,dt]$

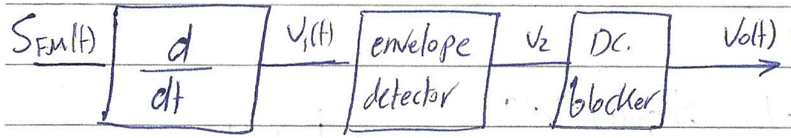
$V_1(t) = A_c[2\pi f_c + K_f m(t)] \sin[2\pi f_c t + K_f \int m(t)\,dt]$

$= A(t) \sin[2\pi f_c t + K_f \int m(t)\,dt]$

$V_2(t) = A(t)$

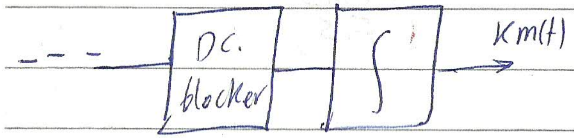
$V_o(t) = A_c K_f m(t) = K\,m(t)$

but the peack value of $S_{FM}(t)$ must be constant

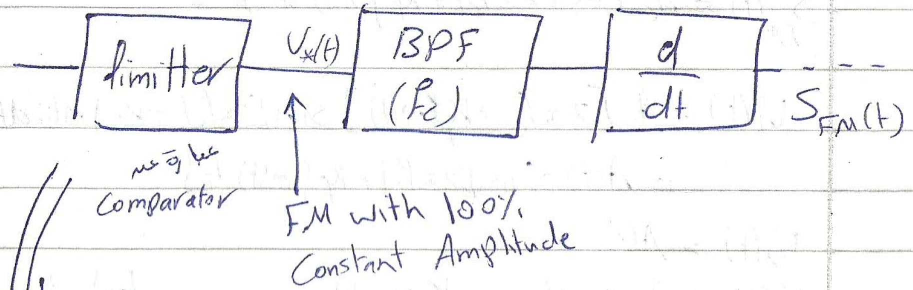


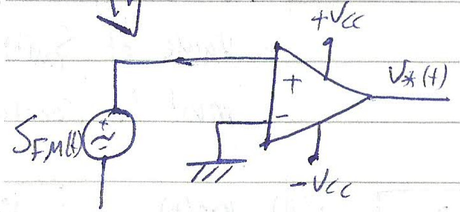
A_c Constant

. $S_{PM}(t)$: Just ~~replace~~ we put integrator after the DC. blocker.



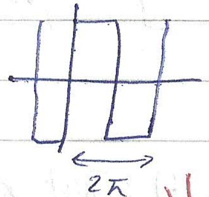
* to Assure that A_c is not fn. of time $S_{FM}(t)$ is feeded to a limitter.





$$v^*(t) = \begin{cases} +1 & S_{FM}(t) > 0 \\ -1 & S_{FM}(t) < 0 \end{cases}$$

$$= \begin{cases} +1 & \theta_i(t) > 0 \\ -1 & \theta_i(t) < 0 \end{cases}$$



it has F.S ⇐ periodic طالما

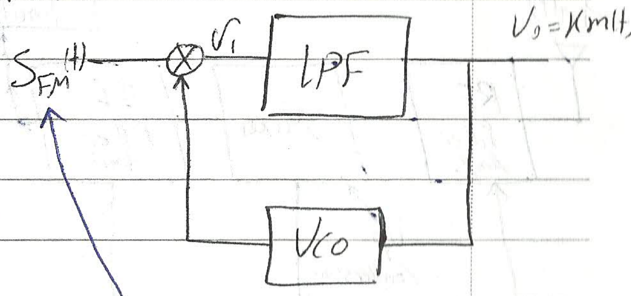
$$\Rightarrow v^*(t) = \sum C_n e^{j2\pi f_0 t}$$

$$= \frac{4}{\pi} \sum \frac{(-1)^{n-1}}{2n-1} \cos[(2n-1)\theta_i(t)]$$

$$= \frac{4}{\pi}\left[\cos\theta_i - \frac{1}{3}\cos 3\theta_i + \frac{1}{5}\cos 5\theta_i + \cdots\right]$$

$$\therefore S_{FM}(t) = \frac{4}{\pi}\cos\left[2\pi f_c t + K_f \int m(t)\,dt\right]$$

② PLL Demodulators:-



$$f_{i\,VCO} = f_c + K V_o$$

$$\cancel{f_c + 1/2\pi}$$

$$= f_c + \frac{1}{2\pi}\frac{d\theta_o(t)}{dt}$$

$$V_o(t) = \frac{1}{2\pi K}\frac{d\theta_o(t)}{dt}$$

$$S_{FM}(t) = A_c\cos[2\pi f_c t + \underbrace{K_f \int m(t)\,dt}_{\theta_i(t)}]$$

$$V_1(t) = \frac{A_c^2}{2}\left[\sin(\theta_2(t) - \theta_o(t)) + \sin[2\pi f_c t + \theta_2(t) + \theta_o(t)]\right]$$

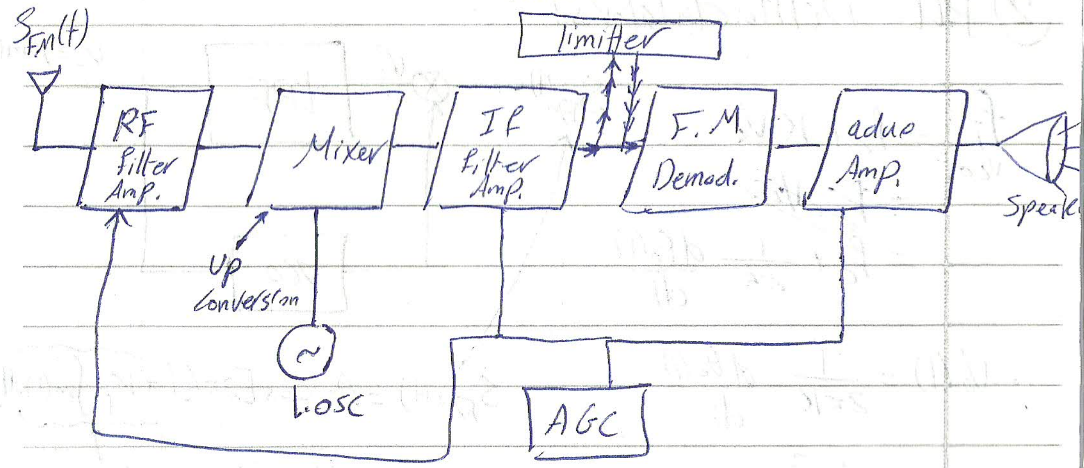
$$= \frac{A_c^2}{2}\sin\theta_e \quad ; \quad \theta_e = \theta_2 - \theta_o$$

$$= \frac{A_c^2}{2}\theta_e$$

$$\Rightarrow V_o(t) = \frac{1}{2\pi K}\frac{d\theta_o(t)}{dt} = \frac{1}{2\pi K}\frac{d}{dt}\left(K_f \int m(t)\right) = \frac{K_f}{2\pi K} m(t)$$

* FM superhytrodyne reciver:-

- $f_c : (87-108)\,MHz$, $f_{IF} = 10.7\,MHz$
- gap = 200 KHz, $(B_T)_{station} = 158\,KHz$



$\boxed{f_L = f_c + F_{IF}}$ $\boxed{f_{IMage} = f_c + 2F_{IF}}$

$\boxed{\Delta f = 75\,KHz}$

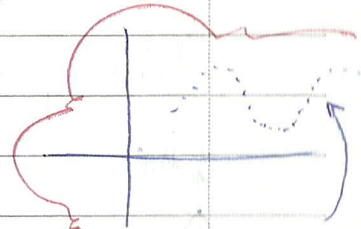
Chapter 6: Sampling and Analog to Digital Converter

6.1: Sampling Theorem:

- Nyquist Sampling theorem:
  if $m(t)$ is baseband signal limited to $(B)\,Hz$, then $m(t)$ can be reconstructed exactly from it's samples taken uniformly at rate of grater than or equal $2B$.

  $\boxed{f_s \geq 2B\,Hz}$ or equivalently at sampling interval $\boxed{T_s \leq \frac{1}{2B}}$

- The minimum allowable sampling rate $f_s = 2B$ sample/sec is called Nyquist rate.

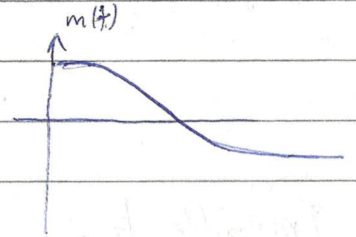


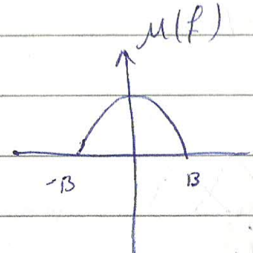
كل ما زاد عدد ال samples
قل ال error

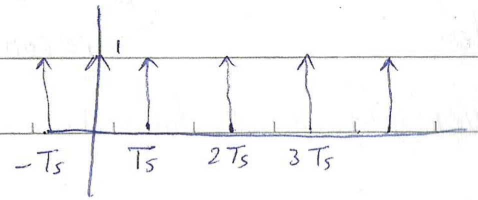
* Ideal sampling:

$\overset{*}{m}(t) = m_p(t) \cdot m(t)$; ~~where $m_p(t) = \Sigma$~~

where $m_p(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT_s)$

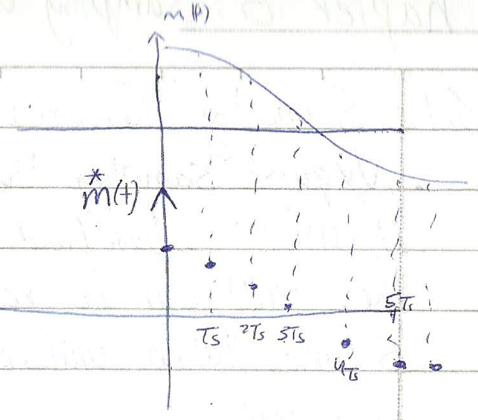






discreat time analog signal

$$\Rightarrow \overset{*}{m}(t) = m(t) \sum \delta(t - nT_s)$$
$$= \sum m(nT_s) \cdot \delta(t - nT_s)$$



• $\sum \delta(t - nT_s)$
$$= \sum C_n e^{j2\pi f_s t}$$
$$C_n = \frac{1}{T_s} \int \delta(t) e^{-j2\pi f_s t} = \frac{1}{T_s}$$

$$\Rightarrow \sum \delta(t - nT_s) = \frac{1}{T_s} \sum e^{j2\pi f_s t}$$

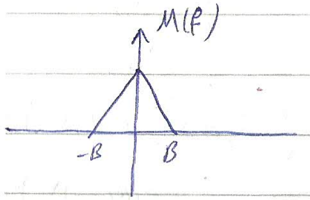
$f_s = \frac{1}{T_s}$

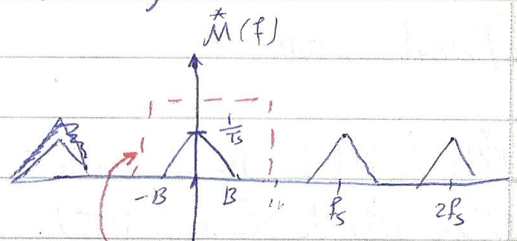
f_s: Sampling rate

T_s: Sampling time (interval)

$$\therefore \overset{*}{m}(t) = \frac{1}{T_s} \sum m(nT_s)\, e^{j2\pi f_s t}$$

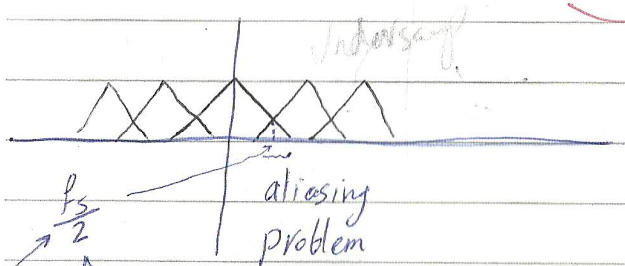
$$\Rightarrow \overset{*}{M}(f) = \frac{1}{T_s} \sum M(f - nf_s)$$





• وعشان نرجع ال signal بدنا LPF

① if $f_s < 2B$

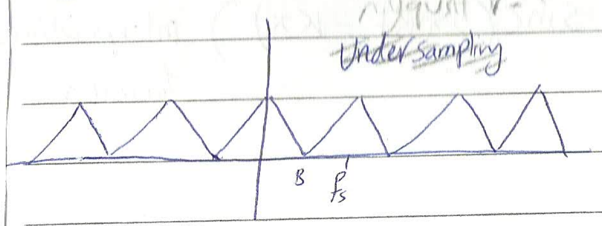


Folding frequancy if $f_s = B$

Imposible to reconstruct

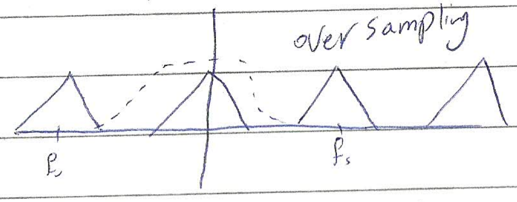
aliasing: overlaping in f-domain.

② if $f_s = 2B$:



• هاي ممكن نعملها بس بشرط انه يكون في عنا Ideal filter وفعلياً ماني عاجز

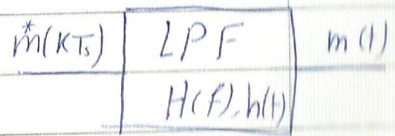
③ $f_s > 2B$:



Can be reconstructed by practical filter. (LPF)

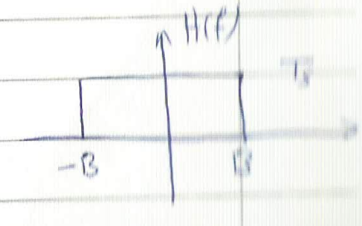
• Signal reconstruction: ($m(t)$ from $\overset{*}{m}(t)$)

* Ideal reconstruction:



$$H(f) = T_s \,\text{rect}\left(\frac{f}{2B}\right)$$

$$M(f) = \overset{*}{M}(f) \cdot H(f)$$
$$= \frac{1}{T_s} \sum M(f - nf_s) \cdot T_s \,\text{rect}\left(\frac{f}{2B}\right)$$
$$= F\left(\overset{*}{m}(kT_s) * h(t)\right)$$



$\therefore\ m(t) = \sum \overset{*}{m}(t)\ h(t - KT_s)$

$$m(t) = \sum m(KT_s)\ \text{sinc}(2\pi Bt - K\pi)$$ interpolation formula