ElCoM
ELECTRICAL COMPUTER MECHATRONICS

تقدم لجنة ElCoM الاكاديمية

تلخيص لمادة:

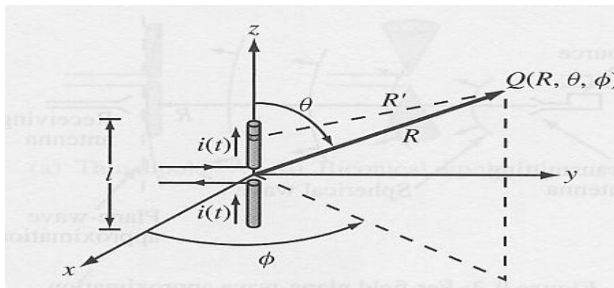
الهوائيات وانتشار الموجات

Antenna Summary
Chapter 9

For Reactive Near Field Region $R = .62\sqrt{L^3/\lambda}$ *& For Radiating Near Field Region* $R = 2L^2/\lambda$

9-1 The Short Dipole

For A short dipole, also called a *Hertzian dipole*: l should not exceed $\lambda/50$.



Short Dipole

$$\widetilde{\mathbf{A}}(\mathbf{R}) = \frac{\mu_0}{4\pi}\int_{\mathcal{V}'} \frac{\widetilde{\mathbf{J}}e^{-jkR'}}{R'}\,d\mathcal{V}'$$

and

$$\widetilde{\mathbf{H}} = \frac{1}{\mu_0}\nabla \times \widetilde{\mathbf{A}},$$
$$\widetilde{\mathbf{E}} = \frac{1}{j\omega\varepsilon_0}\nabla \times \widetilde{\mathbf{H}},$$

For Short Dipole:

$$\widetilde{A}_R = \frac{\mu_0 I_0 l}{4\pi}\cos\theta\left(\frac{e^{-jkR}}{R}\right),$$
$$\widetilde{A}_\theta = -\frac{\mu_0 I_0 l}{4\pi}\sin\theta\left(\frac{e^{-jkR}}{R}\right),$$
$$\widetilde{A}_\phi = 0.$$

>>>

$$\widetilde{H}_\phi = \frac{I_0 l k^2}{4\pi}e^{-jkR}\left[\frac{j}{kR} + \frac{1}{(kR)^2}\right]\sin\theta \qquad (9.8a)$$
$$\widetilde{E}_R = \frac{2I_0 l k^2}{4\pi}\eta_0 e^{-jkR}\left[\frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right]\cos\theta \qquad (9.8b)$$
$$\widetilde{E}_\theta = \frac{I_0 l k^2}{4\pi}\eta_0 e^{-jkR}\left[\frac{j}{kR} + \frac{1}{(kR)^2} - \frac{j}{(kR)^3}\right]\sin\theta, \qquad (9.8c)$$

where $\eta_0 = \sqrt{\mu_0/\varepsilon_0} \cong 120\pi$ (Ω) is the intrinsic impedance of free space. The remaining components ($\widetilde{H}_R$, $\widetilde{H}_\theta$, and $\widetilde{E}_\phi$) are everywhere zero.

And $k = \omega/c = 2\pi/\lambda$

9-1.1 Far-Field Approximation

For Far Field Approximation: $R \gg \lambda$ or, equivalently, $kR = 2\pi R/\lambda \gg 1$

This condition allows us to neglect the terms varying as $1/(kR)^2$ and $1/(kR)^3$ in Eqs. (9.8a) to (9.8c) in favor of the terms varying as $1/kR$, which yields the far-field expressions :

$$\widetilde{E}_\theta = \frac{jI_0 lk\eta_0}{4\pi}\left(\frac{e^{-jkR}}{R}\right)\sin\theta \quad \text{(V/m)}, \qquad (9.9a)$$
$$\widetilde{H}_\phi = \frac{\widetilde{E}_\theta}{\eta_0} \quad \text{(A/m)}, \qquad (9.9b)$$

And $\tilde{E}_R$ is negligible.

9-1.2 Power Density :

Given $\widetilde{\mathbf{E}}$ and $\widetilde{\mathbf{H}}$ in phasor form, the *time-average Poynting vector* of the radiated wave, which is also called the *power density*. :

$$\mathbf{S}_{av} = \frac{1}{2}\Re\mathfrak{e}\left(\widetilde{\mathbf{E}} \times \widetilde{\mathbf{H}}^*\right) \qquad (\text{W/m}^2).$$

For the short dipole, use of Eqs. (9.9a) and (9.9b) gives

$$\mathbf{S}_{av} = \hat{\mathbf{R}}\, S(R, \theta), \qquad (9.11)$$

With

$$S(R, \theta) = \left(\frac{\eta_0 k^2 I_0^2 l^2}{32\pi^2 R^2}\right) \sin^2\theta = S_0 \sin^2\theta \qquad (\text{W/m}^2). \qquad (9.12)$$

the *normalized radiation intensity* $F(\theta, \phi)$, :

$$F(\theta, \phi) = \frac{S(R, \theta, \phi)}{S_{max}} \qquad \text{(dimensionless)}.$$

For Hertzian dipole :

$$S_{max} = S_0 = \frac{\eta_0 k^2 I_0^2 l^2}{32\pi^2 R^2} = \frac{15\pi I_0^2}{R^2}\left(\frac{l}{\lambda}\right)^2$$

\>>>> $F(\theta, \phi) = F(\theta) = \sin^2\theta.$

9-2 Antenna Radiation Characteristics :

$$dP_{rad} = \mathbf{S}_{av} \cdot d\mathbf{A} = \mathbf{S}_{av} \cdot \hat{\mathbf{R}}\, dA = S\, dA$$

And in the spherical coordinate system : $dA = R^2 \sin\theta\, d\theta\, d\phi$, .

and the *solid angle* $d\Omega$ associated with dA, defined as the subtended area divided by R^2, is given by

$$d\Omega = \frac{dA}{R^2} = \sin\theta\, d\theta\, d\phi \qquad (\text{sr}). \qquad (9.18)$$

\>>>> $dP_{rad} = R^2 S(R, \theta, \phi)\, d\Omega.$

$$\begin{aligned} P_{rad} &= R^2 \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} S(R, \theta, \phi) \sin\theta\, d\theta\, d\phi \\ &= R^2 S_{max} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} F(\theta, \phi) \sin\theta\, d\theta\, d\phi \\ &= R^2 S_{max} \iint_{4\pi} F(\theta, \phi)\, d\Omega \qquad (\text{W}), \qquad (9.20) \end{aligned}$$

(Total Radiated Power)

9-2.1 Antenna Pattern:

$F\ (\text{dB}) = 10 \log F.$

pattern solid *angle* Ω_p : $\Omega_p = \iint_{4\pi} F(\theta, \phi)\, d\Omega \quad (\text{sr}).$

For an isotropic antenna with $F(\theta, \phi) = 1$ in all directions, $\Omega_p = 4\pi$ (sr).

The *half-power beamwidth*, simply the beamwidth β, $\beta = \theta_2 - \theta_1$,

9-2.3 Antenna Directivity:

$$D = \frac{F_{max}}{F_{av}} = \frac{1}{\dfrac{1}{4\pi}\iint_{4\pi} F(\theta,\phi)\, d\Omega} = \frac{4\pi}{\Omega_p} \quad \text{(dimensionless)}, \qquad (9.23)$$

By using Eq. (9.20) in Eq. (9.23), D can be expressed as

$$D = \frac{4\pi R^2 S_{max}}{P_{rad}} = \frac{S_{max}}{S_{av}}, \qquad (9.24)$$

where $S_{av} = P_{rad}/(4\pi R^2)$ *And* $D\ (\text{dB}) = 10 \log D.$

For an antenna with a single main lobe pointing in the z-direction as shown in Fig. 9-12, Ω_p may be approximated as the product of the half-power beamwidths β_{xz} and β_{yz} (in radians):

$$\Omega_p \simeq \beta_{xz}\beta_{yz}, \qquad (9.25)$$

and therefore

$$D = \frac{4\pi}{\Omega_p} \simeq \frac{4\pi}{\beta_{xz}\beta_{yz}}. \qquad (9.26)$$

Directivity of a Hertzian Dipole:

$D = 1.5$ or, equivalently, 1.76 dB.

9-2.4 Antenna Gain

The *radiation efficiency* ξ : $\xi = \dfrac{P_{rad}}{P_t}$ (dimensionless).

The *gain* of an antenna is defined as $G = \dfrac{4\pi R^2 S_{max}}{P_t}$ >> $G = \xi D$ (dimensionless).

9-2.5 Radiation Resistance:

$$P_{\rm rad} = \frac{1}{2} I_0^2 R_{\rm rad},$$

$$P_{\rm loss} = \frac{1}{2} I_0^2 R_{\rm loss},$$

where I_0 is the amplitude of the sinusoidal current exciting the antenna.

$$\xi = \frac{P_{\rm rad}}{P_{\rm t}} = \frac{P_{\rm rad}}{P_{\rm rad} + P_{\rm loss}} = \frac{R_{\rm rad}}{R_{\rm rad} + R_{\rm loss}} \cdot \quad (9.31)$$

Resistance or Circler wire of length l and radius a $R_{\rm loss} = \frac{l}{2\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}}$,

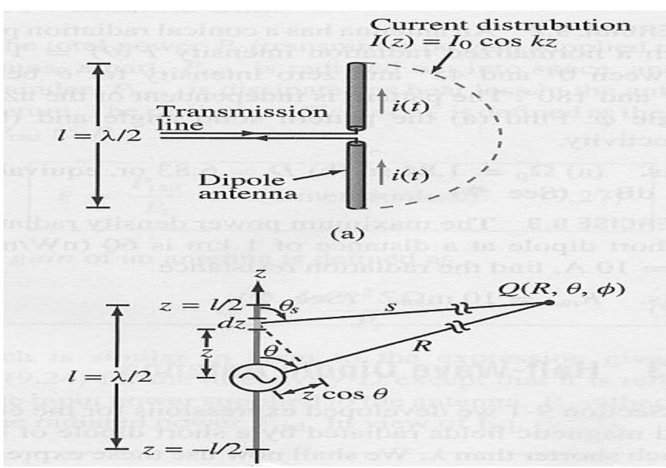
For Hertzian dipole:

$$P_{\rm rad} = \frac{4\pi R^2}{1.5} \times \frac{15\pi I_0^2}{R^2}\left(\frac{l}{\lambda}\right)^2 = 40\pi^2 I_0^2 \left(\frac{l}{\lambda}\right)^2 .$$

$R_{\rm rad} = 80\pi^2 (l/\lambda)^2$ *(See derivation in Example 9.3)*

9-3 Half-Wave Dipole Antenna:

For Half Wave Dipole: $i(t) = I_0 \cos\omega t \cos kz = \Re\mathrm{e}\left[I_0 \cos kz\, e^{j\omega t}\right]$, $\widetilde{I}(z) = I_0 \cos kz, \quad \frac{-\lambda}{4} \le z \le \frac{\lambda}{4}$,



$$d\widetilde{E}_\theta(z) = \frac{jk\eta_0}{4\pi}\widetilde{I}(z)\, dz \left(\frac{e^{-jks}}{s}\right)\sin\theta_s,$$

Two Approximations:

1. $1/s \simeq 1/R$ *Also* $\theta_s \simeq \theta$ *(Magnitude Approximation)*
2. $s \simeq R - z\cos\theta$. *(Phase Approximation)*

$$d\widetilde{E}_\theta = \frac{jk\eta_0}{4\pi}\,\widetilde{I}(z)\,dz\left(\frac{e^{-jkR}}{R}\right)\sin\theta\, e^{jkz\cos\theta}.$$ >>> $$\widetilde{E}_\theta = \int_{z=-\lambda/4}^{\lambda/4} d\widetilde{E}_\theta.$$

carrying out the integration, the following expressions are obtained:

$$\widetilde{E}_\theta = j\,60 I_0\left[\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right]\left(\frac{e^{-jkR}}{R}\right), \quad (9.42a)$$

$$\widetilde{H}_\phi = \frac{\widetilde{E}_\theta}{\eta_0}, \quad (9.42b)$$

and the corresponding time-average power density is

$$S(R,\theta) = \frac{|\widetilde{E}_\theta|^2}{2\eta_0} = \frac{15 I_0^2}{\pi R^2}\left[\frac{\cos^2[(\pi/2)\cos\theta]}{\sin^2\theta}\right] = S_0\left[\frac{\cos^2[(\pi/2)\cos\theta]}{\sin^2\theta}\right] \quad (\text{W/m}^2). \quad (9.43)$$ …

Examination of Eq. (9.43) reveals that $S(R,\theta)$ is maximum at $\theta = \pi/2$, and its value is

$$S_{\max} = S_0 = \frac{15 I_0^2}{(\pi R^2)}.$$

Hence, the normalized radiation intensity is

$$F(\theta) = \frac{S(R,\theta)}{S_0} = \left[\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right]^2. \quad (9.44)$$

9-3.1 Directivity of $\lambda/2$ Dipole :

$$P_{\text{rad}} = R^2 \iint_{4\pi} S(R,\theta)\, d\Omega = \frac{15 I_0^2}{\pi}\int_0^{2\pi}\int_0^{\pi}\left[\frac{\cos[(\pi/2)\cos\theta]}{\sin\theta}\right]^2 \sin\theta\, d\theta\, d\phi. \quad (9.45)$$ >>> $$P_{\text{rad}} = 36.6\, I_0^2$$

$$D = \frac{4\pi R^2 S_{\max}}{P_{\text{rad}}} = \frac{4\pi R^2}{36.6 I_0^2}\left(\frac{15 I_0^2}{\pi R^2}\right) = 1.64$$ or, equivalently, 2.15 dB.

9-3.2 Radiation Resistance of $\lambda/2$ Dipole

$$R_{\text{rad}} = \frac{2P_{\text{rad}}}{I_0^2} = \frac{2\times 36.6 I_0^2}{I_0^2} \simeq 73\ \Omega.$$

X_{in} = 42 Ω at $l/\lambda = 0.5$ *And* down to zero at $l/\lambda = 0.48$

$$Z_{\text{in}} = R_{\text{in}} + jX_{\text{in}}$$ *Where* $$R_{\text{in}} = R_{\text{rad}} + R_{\text{loss}}.$$

For the half-wave dipole, R_{loss} is much smaller than R_{rad} and can be ignored. Hence,

$$Z_{\text{in}} \cong R_{\text{rad}} + jX_{\text{in}}.$$

9-3.3 Quarter-Wave Monopole Antenna :

for a $\lambda/4$ monopole, $P_{rad} = 18.3 I_0^2$ and its radiation resistance is $R_{rad} = 36.5\ \Omega$. *Also Xin = 21.25*

9-4 Dipole of Arbitrary Length :

the current phasor $\widetilde{I}(z)$ can be expressed as a sine function with an argument that goes to zero at $z = \pm l/2$:

$$\widetilde{I}(z) = \begin{cases} I_0 \sin[k(l/2 - z)], & \text{for } 0 \le z \le l/2, \\ I_0 \sin[k(l/2 + z)], & \text{for } -l/2 \le z < 0. \end{cases} \quad (9.52)$$

$$d\widetilde{E}_\theta = \frac{jk\eta_0 I_0}{4\pi}\left(\frac{e^{-jkR}}{R}\right)\sin\theta\, e^{jkz\cos\theta}\, dz \times \begin{cases} \sin[k(l/2 - z)], & \text{for } 0 \le z \le l/2, \\ \sin[k(l/2 + z)], & \text{for } -l/2 \le z < 0. \end{cases} \quad (9.53)$$

>> $$\widetilde{E}_\theta = \int_{-l/2}^{l/2} d\widetilde{E}_\theta$$ >>

$$\widetilde{E}_\theta = j60 I_0 \left(\frac{e^{-jkR}}{R}\right) \times \left[\frac{\cos\left(\frac{kl}{2}\cos\theta\right) - \cos\left(\frac{kl}{2}\right)}{\sin\theta}\right]. \quad (9.55)$$

$$S(\theta) = \frac{|\widetilde{E}_\theta|^2}{2\eta_0} = \frac{15 I_0^2}{\pi R^2}\left[\frac{\cos\left(\frac{\pi l}{\lambda}\cos\theta\right) - \cos\left(\frac{\pi l}{\lambda}\right)}{\sin\theta}\right]^2 \quad (9.56)$$

For *Wave length-long dipole* : $S_{max} = 60 I_0^2/(\pi R^2)$

9-5 Effective Area of a Receiving Antenna :

effective area A_e: $A_e = \frac{P_{int}}{S_i}$ (m²). (9.57)

$$Z_{in} = R_{rad} + jX_{in},$$
$$Z_L = R_L + jX_L,$$

for maximum power transfer, the load impedance must be chosen such that $Z_L = Z_{in}^*$, >> a source $\widetilde{V}_{oc}$ connected across a resistance equal to $2R_{rad}$.

$$P_L = \frac{1}{2}|\widetilde{I}_L|^2 R_{rad} = \frac{1}{2}\left[\frac{|\widetilde{V}_{oc}|}{2R_{rad}}\right]^2 R_{rad} = \frac{|\widetilde{V}_{oc}|^2}{8R_{rad}}, \quad (9.59)$$

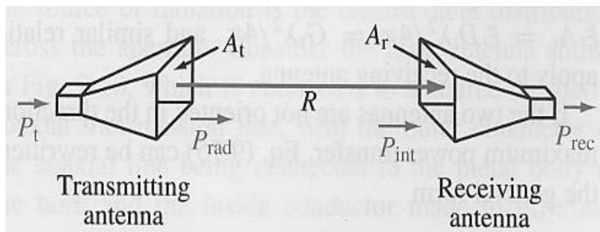
>>> $$P_{int} = P_L = \frac{|\widetilde{V}_{oc}|^2}{8R_{rad}}.$$

$$S_i = \frac{|\widetilde{E}_i|^2}{2\eta_0} = \frac{|\widetilde{E}_i|^2}{240\pi}.$$ >>>> $$A_e = \frac{P_{int}}{S_i} = \frac{30\pi |\widetilde{V}_{oc}|^2}{R_{rad} |\widetilde{E}_i|^2}.$$

For short Dipole: $A_e = \frac{3\lambda^2}{8\pi}$ (m²) (short dipole). (9.63) *For ay antenna:* $A_e = \frac{\lambda^2 D}{4\pi}$ (m²) (any antenna). (9.64)

See example 9.4 for derivation of maximum power transfer.

9-6 Friis Transmission Formula:



$$S_{iso} = \frac{P_t}{4\pi R^2} \quad S_r = G_t S_{iso} = \xi_t D_t S_{iso} = \frac{\xi_t D_t P_t}{4\pi R^2}, \quad = \frac{\xi_t A_t P_t}{\lambda^2 R^2}$$

$$P_{int} = S_r A_r = \frac{\xi_t A_t A_r P_t}{\lambda^2 R^2} \quad P_{rec} = \xi_r P_{int}. \quad \frac{P_{rec}}{P_t} = \frac{\xi_t \xi_r A_t A_r}{\lambda^2 R^2} = G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2. \quad (9.75)$$ *(Power transfer ratio)*

We used $\xi_t A_t = \xi_t D_t \lambda^2/4\pi = G_t \lambda^2/4\pi$, and similar relations to the receiving antenna.

If the two antennas are not oriented in the direction of maximum power transfer, Eq. (9.75) can be rewritten in the general form

$$\frac{P_{rec}}{P_t} = G_t G_r \left(\frac{\lambda}{4\pi R}\right)^2 F_t(\theta_t, \phi_t)\, F_r(\theta_r, \phi_r), \quad (9.76)$$

Noise Level: $P_n = K T_{sys} B$ (W),

where T_{sys} [measured in kelvins (K)] is a figure of merit, called the *system noise temperature*, that characterizes the noise performance of the receiver–antenna combination, K is Boltzmann's constant [1.38×10^{-23} (J/K)], and B is the receiver bandwidth in Hz.

The *signal-to-noise ratio* S_n: $S_n = P_{rec}/P_n$

9-7 Radiation by Large-Aperture Antennas:

To satisfy the far – field condition: $R \geq 2d^2/\lambda,$ where d is the longest linear dimension of the radiating aperture.

$$\widetilde{E}(R,\theta,\phi) = \frac{j}{\lambda}\left(\frac{e^{-jkR}}{R}\right)\widetilde{h}(\theta,\phi), \qquad (9.80)$$

where

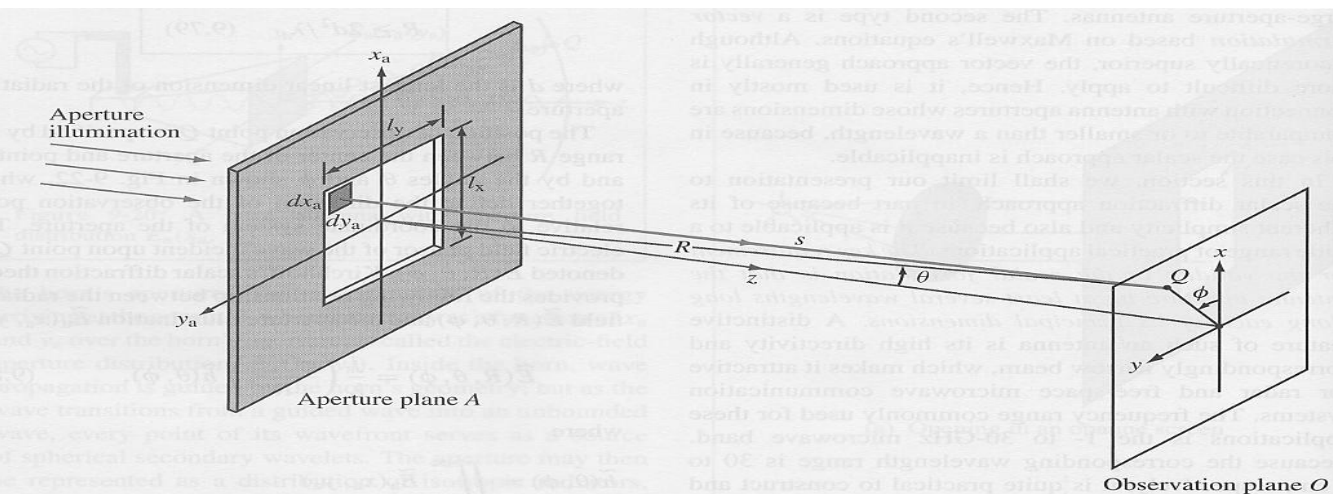
$$\widetilde{h}(\theta,\phi) = \iint_{-\infty}^{\infty} \widetilde{E}_{\rm a}(x_{\rm a}, y_{\rm a}) \cdot \exp\left[jk\sin\theta(x_{\rm a}\cos\phi + y_{\rm a}\sin\phi)\right] dx_{\rm a}\, dy_{\rm a}. \qquad (9.81)$$

We shall refer to $\widetilde{h}(\theta,\phi)$ as the *form factor* of $\widetilde{E}(R,\theta,\phi)$.

Note: Ea is the exciting field.

$$S(R,\theta,\phi) = \frac{|\widetilde{E}(R,\theta,\phi)|^2}{2\eta_0} = \frac{|\widetilde{h}(\theta,\phi)|^2}{2\eta_0\lambda^2 R^2} \qquad (9.82)$$

9-8 Rectangular Aperture with Uniform Aperture Distribution:



a uniform field distribution:

$$\widetilde{E}_{\rm a}(x_{\rm a}, y_{\rm a}) = \begin{cases} E_0, & \text{for } -l_x/2 \leq x_{\rm a} \leq l_x/2 \\ & \text{and } -l_y/2 \leq y_{\rm a} \leq l_y/2, \\ 0, & \text{otherwise.} \end{cases}$$

Assuming $\phi = 0$

$$\widetilde{h}(\theta) = \int_{y_{\rm a}=-l_y/2}^{l_y/2} \int_{x_{\rm a}=-l_x/2}^{l_x/2} E_0 \exp[jkx_{\rm a}\sin\theta]\, dx_{\rm a}\, dy_{\rm a}$$

$$\begin{aligned}\widetilde{h}(\theta) &= \frac{2E_0 l_y}{\left(\frac{2\pi}{\lambda}\sin\theta\right)} \sin(\pi l_x \sin\theta/\lambda) \\ &= E_0 l_x l_y \frac{\sin(\pi l_x \sin\theta/\lambda)}{\pi l_x \sin\theta/\lambda} \\ &= E_0 A_{\rm p}\, \mathrm{sinc}(\pi l_x \sin\theta/\lambda),\end{aligned}$$

We used $\mathrm{sinc}\, t = \frac{\sin t}{t}$

And from equation 9.82 >> $S(R,\theta) = S_0 \operatorname{sinc}^2(\pi l_x \sin\theta/\lambda)$ $(x\text{–}z \text{ plane}),$ (9.89) where $S_0 = E_0^2 A_p^2/(2\eta_0 \lambda^2 R^2)$

$$F(\theta) = \frac{S(R,\theta)}{S_{max}} = \operatorname{sinc}^2(\pi l_x \sin\theta/\lambda) = \operatorname{sinc}^2(\pi\gamma) \quad (x\text{–}z \text{ plane}),$$ *>>>* $\gamma = (l_x/\lambda)\sin\theta$

9-8.1 Beamwidth:

$\beta_{xz} = \theta_2 - \theta_1$ *And* $\theta_1 = -\theta_2$ *>>>* $\beta_{xz} = 2\theta_2$

$F(\theta_2) = \operatorname{sinc}^2(\pi l_x \sin\theta/\lambda) = 0.5$ *>>(from tables)>>* $\frac{\pi l_x}{\lambda}\sin\theta_2 = 1.39$ *>>* $\sin\theta_2 = 0.44\frac{\lambda}{l_x}$

Because $\lambda/l_x \ll 1$ … $\sin\theta_2 \simeq \theta_2$ … $\beta_{xz} = 2\theta_2 \simeq 2\sin\theta_2 = 0.88\frac{\lambda}{l_x}$

A similar solution for the y–z plane ($\phi = \pi/2$) gives

$$\beta_{yz} = 0.88\frac{\lambda}{l_y} \quad \text{(rad)}. \quad (9.94b)$$

$$\beta_{xz} = k_x \frac{\lambda}{l_x}, \quad (9.95)$$

where k_x is a constant related to the steepness of the taper. For a uniform distribution with no taper, $k_x = 0.88$, and for a highly tapered distribution, $k_x \simeq 2$. In the typical case, $k_x \simeq 1$.

For a circularly symmetric antenna pattern: $\beta \simeq \lambda/d$.

For a cylindrical reflector $\beta_{xz} \simeq \frac{\lambda}{l_x}$ *And* $\beta_{yz} \simeq \frac{\lambda}{l_y}$

9-8.2 Directivity and Effective Area:

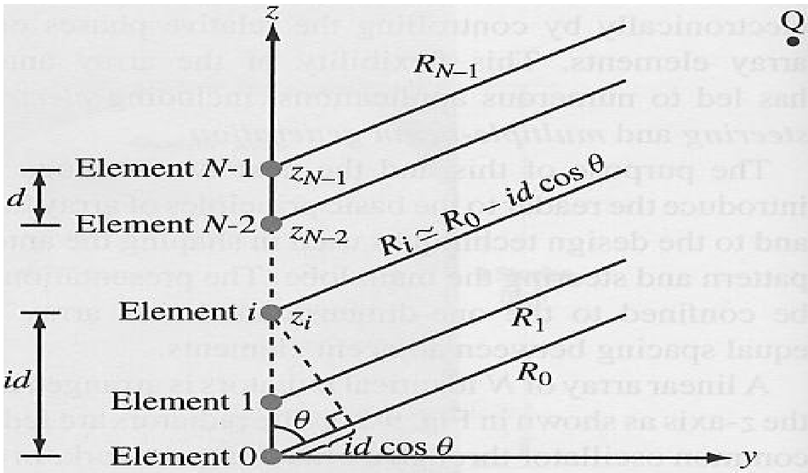
$D \simeq \frac{4\pi}{\beta_{xz}\beta_{yz}}$ $D \simeq \frac{4\pi l_x l_y}{\lambda^2} = \frac{4\pi A_p}{\lambda^2}$. *Also* $D = \frac{4\pi A_e}{\lambda^2}$

for aperture antennas, their effective apertures are approximately equal to their physical apertures; that is, $A_e \simeq A_p$.

9-9 . Antenna Arrays :

$$\widetilde{E}_{\mathrm{e}}(R,\theta,\phi) = \frac{e^{-jkR}}{R}\,\widetilde{f}_{\mathrm{e}}(\theta,\phi)$$
>>>
$$S_{\mathrm{e}}(R,\theta,\phi) = \frac{1}{2\eta_0}|\widetilde{E}_{\mathrm{e}}(R,\theta,\phi)|^2 = \frac{1}{2\eta_0 R^2}|\widetilde{f}_{\mathrm{e}}(\theta,\phi)|^2.$$
…
$$\widetilde{E}_i(R_i,\theta,\phi) = A_i\frac{e^{-jkR_i}}{R_i}\,\widetilde{f}_{\mathrm{e}}(\theta,\phi)$$

where $A_i = a_i e^{j\psi_i}$ is a *complex feeding coefficient*



To satisfy Far field condition $R_0 \geq \frac{2l^2}{\lambda} = \frac{2(N-1)^2 d^2}{\lambda}$ *Where* $l = (N-1)d$ *array length*

Two Approximations:

1. $R_i = R_0$ *(Magnitude Approximation)*

2. $R_i \simeq R_0 - z_i\cos\theta = R_0 - id\cos\theta$ where $z_i = id$ is the distance between the ith element and the zeroth element *(Phase Approximation)*

$$\widetilde{E}(R_0,\theta,\phi) = \widetilde{f}_{\mathrm{e}}(\theta,\phi)\left(\frac{e^{-jkR_0}}{R_0}\right) \times \left[\sum_{i=0}^{N-1} A_i e^{jikd\cos\theta}\right],$$
>>>>
$$S(R_0,\theta,\phi) = \frac{1}{2\eta_0}|\widetilde{E}(R_0,\theta,\phi)|^2 = \frac{1}{2\eta_0 R_0^2}|\widetilde{f}_{\mathrm{e}}(\theta,\phi)|^2\left|\sum_{i=0}^{N-1} A_i e^{jikd\cos\theta}\right|^2 = S_{\mathrm{e}}(R_0,\theta,\phi)\left|\sum_{i=0}^{N-1} A_i e^{jikd\cos\theta}\right|^2, \quad (9.106)$$

Denoting the array factor by
$$F_{\mathrm{a}}(\theta) = \left|\sum_{i=0}^{N-1} A_i e^{jikd\cos\theta}\right|^2,$$
>>>
$$S(R_0,\theta,\phi) = S_{\mathrm{e}}(R_0,\theta,\phi)\,F_{\mathrm{a}}(\theta). \quad (9.108)$$

$$A_i = a_i e^{j\psi_i}.$$
>>>
$$F_{\mathrm{a}}(\theta) = \left|\sum_{i=0}^{N-1} a_i e^{j\psi_i} e^{jikd\cos\theta}\right|^2. \quad (9.110)$$

9-10 *N*-Element Array with Uniform Phase Distribution

For $\psi_i = \psi_0$

$$F_a(\theta) = \left| e^{j\psi_0} \sum_{i=0}^{N-1} a_i e^{jikd\cos\theta} \right|^2$$
$$= |e^{j\psi_0}|^2 \left| \sum_{i=0}^{N-1} a_i e^{jikd\cos\theta} \right|^2$$
$$= \left| \sum_{i=0}^{N-1} a_i e^{jikd\cos\theta} \right|^2 .$$

The phase difference between the fields radiated by adjacent elements is

$$\gamma = kd\cos\theta = \frac{2\pi d}{\lambda}\cos\theta. \quad (9.112)$$

>>> In terms of γ, Eq. (9.111) takes the compact form

$$F_a(\gamma) = \left| \sum_{i=0}^{N-1} a_i e^{ji\gamma} \right|^2 \quad \text{(uniform phase)}. \quad (9.113)$$

For a uniform amplitude distribution with $a_i = 1$ >>

$$F_a(\gamma) = \frac{\sin^2(N\gamma/2)}{\sin^2(\gamma/2)}$$
(uniform amplitude and phase). (9.120)

(Derivation page 410&411)

The maximum value of $F_a(\gamma)$ can be shown to occur at $\gamma = 0$ (or $\theta = \pi/2$) and is equal to N^2 normalized array factor is given by :

$$F_{an}(\gamma) = \frac{F_a(\gamma)}{F_{a,max}} = \frac{\sin^2(N\gamma/2)}{N^2\sin^2(\gamma/2)}$$
$$= \frac{\sin^2\left[\frac{N\pi d}{\lambda}\cos\theta\right]}{N^2\sin^2\left[\frac{\pi d}{\lambda}\cos\theta\right]}.$$

9-11 Electronic Scanning of Arrays :

For Electronic steering $\psi_i = -i\delta$ >>

$$F_a(\theta) = \left| \sum_{i=0}^{N-1} a_i e^{-ji\delta} e^{jikd\cos\theta} \right|^2$$
$$= \left| \sum_{i=0}^{N-1} a_i e^{ji(kd\cos\theta - \delta)} \right|^2$$
$$= \left| \sum_{i=0}^{N-1} a_i e^{ji\gamma'} \right|^2 \triangleq F_a(\gamma'),$$

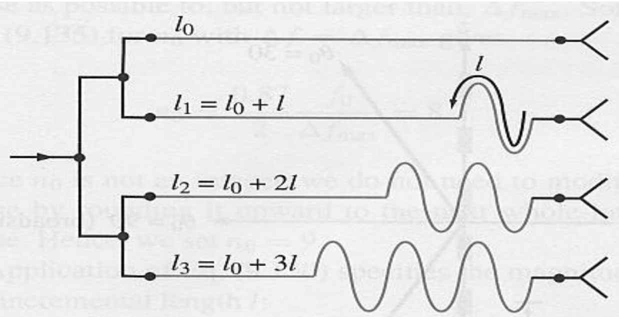
where

$$\gamma' = kd\cos\theta - \delta.$$ *And* $$\delta = kd\cos\theta_0. \quad (9.125)$$ >> $$\gamma' = kd(\cos\theta - \cos\theta_0). \quad (9.126)$$

9-11.1 Uniform-Amplitude Excitation:

$$F_{\text{an}}(\gamma') = \frac{\sin^2(N\gamma'/2)}{N^2\sin^2(\gamma'/2)}$$

9-11.2 Array Feeding:



For Frequency Scanning; $l_i = il + l_0$

where l_0 is the path length of the zeroth element. Wave propagation at a frequency f on a transmission line of length l_i is characterized by a phase factor $e^{-j\beta l_i}$, where $\beta = 2\pi f/u_p$ is the phase constant of the line and u_p is its propagation velocity >>

$$\psi_i(f) = -\beta(l_i - l_0) = -\frac{2\pi}{u_p}f(l_i - l_0)$$
$$= -\frac{2\pi i}{u_p}fl. \quad (9.129)$$

let $$l = \frac{n_0 u_p}{f_0}$$

>>
$$\begin{aligned}\psi_1(f_0 + \Delta f) &= -\frac{2\pi}{u_p}(f_0 + \Delta f)l \\ &= -\frac{2\pi f_0 l}{u_p} - \left(\frac{2\pi l}{u_p}\right)\Delta f \\ &= -2n_0\pi - 2n_0\pi\left(\frac{\Delta f}{f_0}\right) \\ &= -2n_0\pi - \delta,\end{aligned}$$
with $$\delta = 2n_0\pi\left(\frac{\Delta f}{f_0}\right).$$

Similarly, $\psi_2(f_0 + \Delta f) = 2\psi_1$ and $\psi_3(f_0 + \Delta f) = 3\psi_1$. *…Ignore 2π and its multiples (no influence of relative phases).*

$$\cos\theta_0 = \frac{2n_0\pi}{kd}\left(\frac{\Delta f}{f_0}\right).$$ *(Using equation 125)*

Additional Formulas You Might Need:

Voltage Reflection coefficient: $\Gamma = \dfrac{R_{rad} - Z_0}{R_{rad} + Z_0}$ *standing wave ratio:* $S = \dfrac{1 + |\Gamma|}{1 - |\Gamma|}$ *(See problem 12)*

For Direct and Reflected Waves from antenna:

$S = \dfrac{|E|^2}{2\eta_0}$ *Direct:* $E_d = \sqrt{2\eta_0 S_i}\, e^{-jkR}$ *…Reflected* $E_r = \left(\sqrt{\dfrac{30\eta_0}{\pi}}\,\dfrac{I_0}{R'}\, e^{-jkR'}\right)\Gamma.$

Table 7-1: Expressions for α, β, η_c, u_p, and λ for various types of media.

	Any Medium	Lossless Medium ($\sigma = 0$)	Low-loss Medium ($\varepsilon''/\varepsilon' \ll 1$)	Good Conductor ($\varepsilon''/\varepsilon' \gg 1$)	Units
$\alpha =$	$\omega\left[\dfrac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\dfrac{\varepsilon''}{\varepsilon'}\right)^2}-1\right]\right]^{1/2}$	0	$\dfrac{\sigma}{2}\sqrt{\dfrac{\mu}{\varepsilon}}$	$\sqrt{\pi f\mu\sigma}$	(Np/m)
$\beta =$	$\omega\left[\dfrac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\dfrac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]\right]^{1/2}$	$\omega\sqrt{\mu\varepsilon}$	$\omega\sqrt{\mu\varepsilon}$	$\sqrt{\pi f\mu\sigma}$	(rad/m)
$\eta_c =$	$\sqrt{\dfrac{\mu}{\varepsilon'}}\left(1 - j\dfrac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$	$\sqrt{\dfrac{\mu}{\varepsilon}}$	$\sqrt{\dfrac{\mu}{\varepsilon}}$	$(1+j)\dfrac{\alpha}{\sigma}$	(Ω)
$u_p =$	ω/β	$1/\sqrt{\mu\varepsilon}$	$1/\sqrt{\mu\varepsilon}$	$\sqrt{4\pi f/\mu\sigma}$	(m/s)
$\lambda =$	$2\pi/\beta = u_p/f$	u_p/f	u_p/f	u_p/f	(m)

Notes: $\varepsilon' = \varepsilon$; $\varepsilon'' = \sigma/\omega$; in free space, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$; in practice, a material is considered a low-loss medium if $\varepsilon''/\varepsilon' = \sigma/\omega\varepsilon < 0.01$ and a good conducting medium if $\varepsilon''/\varepsilon' > 100$.

$\theta_i = \theta_r$ (Snell's law of reflection), (8.28a)

$\dfrac{\sin\theta_t}{\sin\theta_i} = \dfrac{u_{p_2}}{u_{p_1}} = \sqrt{\dfrac{\mu_1\varepsilon_1}{\mu_2\varepsilon_2}}$ (Snell's law of refraction). (8.28b)

>> $n = \dfrac{c}{u_p} = \sqrt{\dfrac{\mu\varepsilon}{\mu_0\varepsilon_0}} = \sqrt{\mu_r\varepsilon_r}\,.$ (8.29)

$\dfrac{\sin\theta_t}{\sin\theta_i} = \dfrac{n_1}{n_2} = \sqrt{\dfrac{\mu_{r_1}\varepsilon_{r_1}}{\mu_{r_2}\varepsilon_{r_2}}}\,.$ *And:* $\dfrac{\sin\theta_t}{\sin\theta_i} = \dfrac{n_1}{n_2} = \sqrt{\dfrac{\varepsilon_{r_1}}{\varepsilon_{r_2}}} = \dfrac{\eta_2}{\eta_1}$ (for $\mu_1 = \mu_2$), (8.31) *Where* where $\eta = \sqrt{\mu/\varepsilon}$ is the intrinsic impedance

Table 8-2: Expressions for Γ, τ, R, and T for wave incidence from a medium with intrinsic impedance η_1 onto a medium with intrinsic impedance η_2. Angles θ_i and θ_t are the angles of incidence and transmission, respectively.

Property	Normal Incidence $\theta_i = \theta_t = 0$	Perpendicular Polarization	Parallel Polarization
Reflection coefficient	$\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$	$\Gamma_\perp = \dfrac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$	$\Gamma_\parallel = \dfrac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$
Transmission coefficient	$\tau = \dfrac{2\eta_2}{\eta_2 + \eta_1}$	$\tau_\perp = \dfrac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$	$\tau_\parallel = \dfrac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$
Relation of Γ to τ	$\tau = 1 + \Gamma$	$\tau_\perp = 1 + \Gamma_\perp$	$\tau_\parallel = (1 + \Gamma_\parallel)\dfrac{\cos\theta_i}{\cos\theta_t}$
Reflectivity	$R = \|\Gamma\|^2$	$R_\perp = \|\Gamma_\perp\|^2$	$R_\parallel = \|\Gamma_\parallel\|^2$
Transmissivity	$T = \|\tau\|^2\left(\dfrac{\eta_1}{\eta_2}\right)$	$T_\perp = \|\tau_\perp\|^2\dfrac{\eta_1\cos\theta_t}{\eta_2\cos\theta_i}$	$T_\parallel = \|\tau_\parallel\|^2\dfrac{\eta_1\cos\theta_t}{\eta_2\cos\theta_i}$
Relation of R to T	$T = 1 - R$	$T_\perp = 1 - R_\perp$	$T_\parallel = 1 - R_\parallel$

Notes: (1) $\sin\theta_t = \sqrt{\mu_1\varepsilon_1/\mu_2\varepsilon_2}\,\sin\theta_i$; (2) $\eta_1 = \sqrt{\mu_1/\varepsilon_1}$; (3) $\eta_2 = \sqrt{\mu_2/\varepsilon_2}$; (4) for nonmagnetic media, $\eta_2/\eta_1 = n_1/n_2$.

(See problem 38)

Tables:

Table B-2: CONDUCTIVITY σ OF SOME COMMON MATERIALS[a]

Material	Conductivity, σ (S/m)	Material	Conductivity, σ (S/m)
Conductors		*Semiconductors*	
Silver	6.2×10^7	Pure germanium	2.2
Copper	5.8×10^7	Pure silicon	4.4×10^{-4}
Gold	4.1×10^7	*Insulators*	
Aluminum	3.5×10^7	Wet soil	$\sim 10^{-2}$
Tungsten	1.8×10^7	Fresh water	$\sim 10^{-3}$
Zinc	1.7×10^7	Distilled water	$\sim 10^{-4}$
Brass	1.5×10^7	Dry soil	$\sim 10^{-4}$
Iron	10^7	Glass	10^{-12}
Bronze	10^7	Hard rubber	10^{-15}
Tin	9×10^6	Paraffin	10^{-15}
Lead	5×10^6	Mica	10^{-15}
Mercury	10^6	Fused quartz	10^{-17}
Carbon	3×10^4	Wax	10^{-17}
Seawater	4		
Animal body (average)	0.3 (poor cond.)		

[a]These are low-frequency values at room temperature (20° C).

Table B-3: RELATIVE PERMEABILITY μ_r OF SOME COMMON MATERIALS[a]

$\mu = \mu_r \mu_0$ and $\mu_0 = 4\pi \times 10^{-7}$ H/m.

Material	Relative Permeability, μ_r
Diamagnetic	
Bismuth	$0.99983 \simeq 1$
Gold	$0.99996 \simeq 1$
Mercury	$0.99997 \simeq 1$
Silver	$0.99998 \simeq 1$
Copper	$0.99999 \simeq 1$
Water	$0.99999 \simeq 1$
Paramagnetic	
Air	$1.000004 \simeq 1$
Aluminum	$1.00002 \simeq 1$
Tungsten	$1.00008 \simeq 1$
Titanium	$1.0002 \simeq 1$
Platinum	$1.0003 \simeq 1$

CARTESIAN (RECTANGULAR)

$$\nabla \times \mathbf{A} = \begin{vmatrix} \hat{\mathbf{x}} & \hat{\mathbf{y}} & \hat{\mathbf{z}} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} = \hat{\mathbf{x}}\left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y}\right)$$

CYLINDRICAL COORDINATES (r, ϕ, z)

$$\nabla \times \mathbf{A} = \frac{1}{r}\begin{vmatrix} \hat{\mathbf{r}} & \hat{\boldsymbol{\phi}} r & \hat{\mathbf{z}} \\ \frac{\partial}{\partial r} & \frac{\partial}{\partial \phi} & \frac{\partial}{\partial z} \\ A_r & rA_\phi & A_z \end{vmatrix} = \hat{\mathbf{r}}\left(\frac{1}{r}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z}\right) + \hat{\boldsymbol{\phi}}\left(\frac{\partial A_r}{\partial z} - \frac{\partial A_z}{\partial r}\right) + \hat{\mathbf{z}}\frac{1}{r}\left[\frac{\partial}{\partial r}(rA_\phi) - \frac{\partial A_r}{\partial \phi}\right]$$

SPHERICAL COORDINATES (R, θ, ϕ)

$$\nabla \times \mathbf{A} = \frac{1}{R^2 \sin\theta}\begin{vmatrix} \hat{\mathbf{R}} & \hat{\boldsymbol{\theta}} R & \hat{\boldsymbol{\phi}} R\sin\theta \\ \frac{\partial}{\partial R} & \frac{\partial}{\partial \theta} & \frac{\partial}{\partial \phi} \\ A_R & RA_\theta & (R\sin\theta)A_\phi \end{vmatrix}$$

$$= \hat{\mathbf{R}}\frac{1}{R\sin\theta}\left[\frac{\partial}{\partial\theta}(A_\phi \sin\theta) - \frac{\partial A_\theta}{\partial\phi}\right] + \hat{\boldsymbol{\theta}}\frac{1}{R}\left[\frac{1}{\sin\theta}\frac{\partial A_R}{\partial\phi} - \frac{\partial}{\partial R}(RA_\phi)\right] + \hat{\boldsymbol{\phi}}\frac{1}{R}\left[\frac{\partial}{\partial R}(RA_\theta) - \frac{\partial A_R}{\partial\theta}\right]$$

Transformation	Coordinate Variables	Unit Vectors	Vector Components
Cartesian to cylindrical	$r = \sqrt[+]{x^2+y^2}$ $\phi = \tan^{-1}(y/x)$ $z = z$	$\hat{\mathbf{r}} = \hat{\mathbf{x}}\cos\phi + \hat{\mathbf{y}}\sin\phi$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_r = A_x\cos\phi + A_y\sin\phi$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$ $A_z = A_z$
Cylindrical to Cartesian	$x = r\cos\phi$ $y = r\sin\phi$ $z = z$	$\hat{\mathbf{x}} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{z}}$	$A_x = A_r\cos\phi - A_\phi\sin\phi$ $A_y = A_r\sin\phi + A_\phi\cos\phi$ $A_z = A_z$
Cartesian to spherical	$R = \sqrt[+]{x^2+y^2+z^2}$ $\theta = \tan^{-1}[\sqrt[+]{x^2+y^2}/z]$ $\phi = \tan^{-1}(y/x)$	$\hat{\mathbf{R}} = \hat{\mathbf{x}}\sin\theta\cos\phi + \hat{\mathbf{y}}\sin\theta\sin\phi + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{x}}\cos\theta\cos\phi + \hat{\mathbf{y}}\cos\theta\sin\phi - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}\sin\phi + \hat{\mathbf{y}}\cos\phi$	$A_R = A_x\sin\theta\cos\phi + A_y\sin\theta\sin\phi + A_z\cos\theta$ $A_\theta = A_x\cos\theta\cos\phi + A_y\cos\theta\sin\phi - A_z\sin\theta$ $A_\phi = -A_x\sin\phi + A_y\cos\phi$
Spherical to Cartesian	$x = R\sin\theta\cos\phi$ $y = R\sin\theta\sin\phi$ $z = R\cos\theta$	$\hat{\mathbf{x}} = \hat{\mathbf{R}}\sin\theta\cos\phi + \hat{\boldsymbol{\theta}}\cos\theta\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi$ $\hat{\mathbf{y}} = \hat{\mathbf{R}}\sin\theta\sin\phi + \hat{\boldsymbol{\theta}}\cos\theta\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_x = A_R\sin\theta\cos\phi + A_\theta\cos\theta\cos\phi - A_\phi\sin\phi$ $A_y = A_R\sin\theta\sin\phi + A_\theta\cos\theta\sin\phi + A_\phi\cos\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$
Cylindrical to spherical	$R = \sqrt[+]{r^2+z^2}$ $\theta = \tan^{-1}(r/z)$ $\phi = \phi$	$\hat{\mathbf{R}} = \hat{\mathbf{r}}\sin\theta + \hat{\mathbf{z}}\cos\theta$ $\hat{\boldsymbol{\theta}} = \hat{\mathbf{r}}\cos\theta - \hat{\mathbf{z}}\sin\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$	$A_R = A_r\sin\theta + A_z\cos\theta$ $A_\theta = A_r\cos\theta - A_z\sin\theta$ $A_\phi = A_\phi$
Spherical to cylindrical	$r = R\sin\theta$ $\phi = \phi$ $z = R\cos\theta$	$\hat{\mathbf{r}} = \hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta$ $\hat{\boldsymbol{\phi}} = \hat{\boldsymbol{\phi}}$ $\hat{\mathbf{z}} = \hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta$	$A_r = A_R\sin\theta + A_\theta\cos\theta$ $A_\phi = A_\phi$ $A_z = A_R\cos\theta - A_\theta\sin\theta$

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Helical Antennas

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Completed November 8, 2011

EXECUTIVE SUMMURY:

Helical antennas have long been popular in applications from VHF to microwaves requiring circular polarization, since they have the unique property of naturally providing circularly polarized radiation.

One area that takes advantage of this property is satellite communications. Where more gain is required than can be provided by a helical antenna alone, a helical antenna can also be used as a feed for a parabolic dish for higher gains. As we shall see, the helical antenna can be an excellent feed for a dish, with the advantage of circular polarization.

One limitation is that the usefulness of the circular polarization is limited since it cannot be easily reversed to the other sense, left-handed to right-handed or vice-versa. [1]

TABLE OF CONTENTS

List of Figures

Overview:

A helical antenna is an antenna consisting of a conducting wire wound in the form of a helix. In most cases, helical antennas are mounted over a ground plane. The feed line is connected between the bottom of the helix and the ground plane. Helical antennas can operate in one of two principal modes: normal mode or axial mode.

In the *normal mode* or *broadside* helix, the dimensions of the helix (the diameter and the pitch) are small compared with the wavelength. The antenna acts similarly to an electrically short dipole or monopole, and the radiation pattern, similar to these antennas is omnidirectional, with maximum radiation at right angles to the helix axis. The radiation is linearly polarized parallel to the helix axis.

In the *axial mode* or *end-fire* helix, the dimensions of the helix are comparable to a wavelength. The antenna functions as a directional antenna radiating a beam off the ends of the helix, along the antenna's axis. It radiates circularly polarized radio waves. [2]

Geometry and Operation:

A sketch of a typical helical antenna is shown in Figure 1. The radiating element is a helix of wire, driven at one end and radiating along the axis of the helix. A ground plane at the driven end makes the radiation unidirectional from the far (open) end.

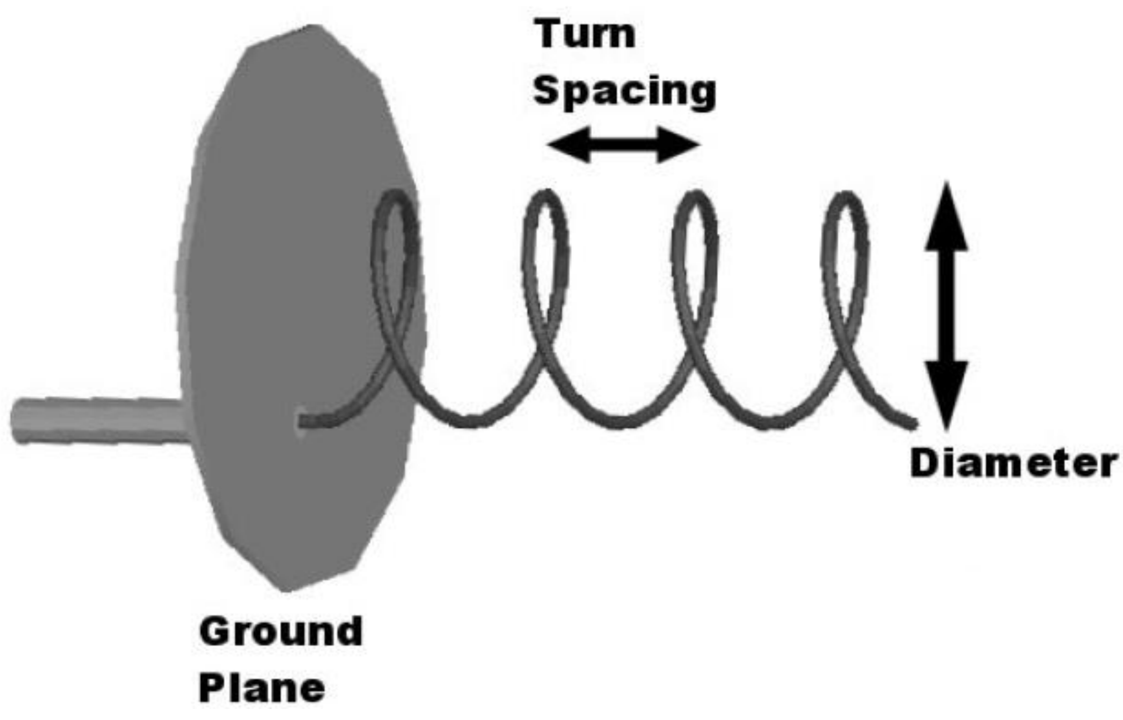

Figure 1: A sketch of a typical helical antenna [1]

The benefits of this helix antenna *axial mode* or *end-fire* helix are it has a wide bandwidth, is easily constructed, and has real input impedance.

The parameters of the helix antenna shown in figure 2 are defined Next Page.

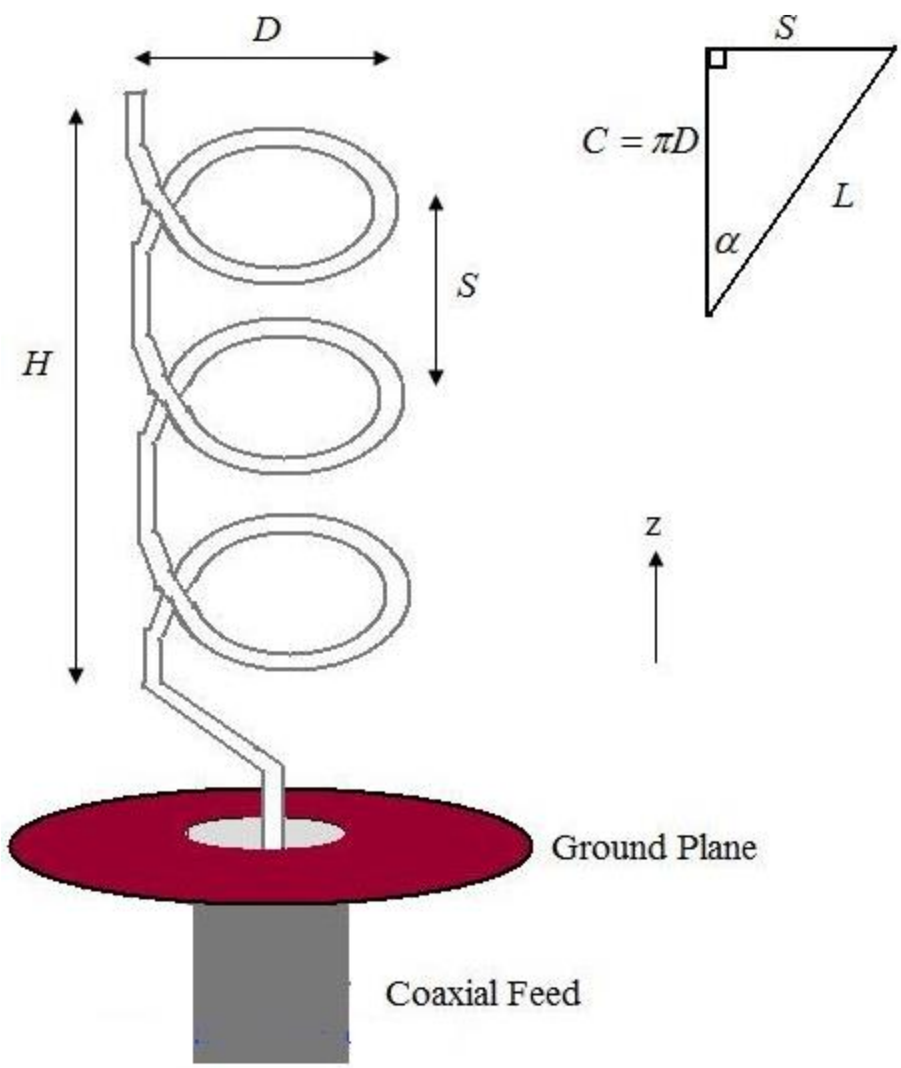

Figure 2: Geometry of Helical Antenna. [3]

- D - Diameter of a turn on the helix antenna.

- C - Circumference of a turn on the helix antenna (C=pi*D).

- S - Vertical separation between turns for helical antenna.

- α - pitch angle, which controls how far the helix antenna grows in the z-direction per turn, and is given by

$$\alpha = \tan^{-1}\frac{S}{C}$$

- N - Number of turns on the helix antenna.

- H - Total height of helix antenna, $H=NS$.

The antenna in Figure 2 is a left handed helix antenna, because if you curl your fingers on your left hand around the helix your thumb would point up (also, the waves emitted from this helix antenna are Left Hand Circularly Polarized). If the helix antenna was wound the other way, it would be a right handed helical antenna.

The radiation pattern will be maximum in the +z direction (along the helical axis in Figure 1). The design of helical antennas is primarily based on empirical results, and the fundamental equations will be presented here.

Helix antennas of at least 3 turns will have close to circular polarization in the +z direction when the circumference C is close to a wavelength:

$$\frac{3\lambda}{4} \leq C \leq \frac{4\lambda}{3}$$

Once the circumference C is chosen, the inequalities above roughly determine the operating bandwidth of the helix antenna. For instance, if C=0.5 meters, then the highest frequency of operation will be given by the smallest wavelength that fits into the above equation, or =0.75C=0.375 meters, which corresponds to a frequency of 800 MHz. The lowest frequency of operation will be given by the largest wavelength that fits into the above equation, or =1.333C=0.667 meters, which corresponds to a frequency of 450 MHz. Hence, the *fractional BW** is 56%, which is true of axial helical antennas in general.

The fractional bandwidth of an antenna is a measure of how wideband the antenna is. If the antenna operates at center frequency fc between lower frequency f1 and upper frequency f2 (where fc=(f1+f2)/2), then the fractional bandwidth FBW is given by:

$$FBW = \frac{f_2 - f_1}{f_c}$$

The fractional bandwidth varies between 0 and 2, and is often quoted as a percentage (between 0% and 200%). The higher the percentage, the wider the bandwidth.

Wideband antennas typically have a Fractional Bandwidth of 20% or more. Antennas with a FBW of greater than 50% are referred to as ultra-wideband antennas; this means that helical antenna is an ultra-wideband antenna.

Impedance, Gain, and Radiation Pattern:

The helix antenna is a travelling wave antenna, which means the current travels along the antenna and the phase varies continuously. In addition, the input impedance is primarily real and can be approximated in Ohms by:

$$Z_{in} = 140\frac{C}{\lambda}$$

The helix antenna functions well for pitch angles (α) between 12 and 14 degrees. Typically, the pitch angle is taken as 13 degrees.

The normalized radiation pattern for the E-field components is given by:

$$E_\theta \propto E_\phi \propto \sin\frac{\pi}{2N}\cos\theta\frac{\sin\frac{N\Omega}{2}}{\sin(\Omega/2)}$$

$$\Omega = kS(\cos\theta - 1) - \pi(2 + 1/N)$$

For circular polarization, the orthogonal components of the E-field must be 90 degrees out of phase. This occurs in directions near the axis (z-axis in Figure 1) of the helix. The axial ratio for helix antennas decreases as the number of loops N is added, and can be approximated by:

$$AR = \frac{2N+1}{2N}$$

The axial ratio is the ratio of orthogonal components of an E-field. A circularly polarized field is made up of two orthogonal E-field components of equal amplitude (and 90 degrees out of phase). Because the components are equal magnitude, the axial ratio is 1 (or 0 dB).

The axial ratio for an ellipse is larger than 1 (>0 dB). The axial ratio for pure linear polarization is infinite, because one of the orthogonal components of the field is zero.

Axial ratios are often quoted for antennas in which the desired polarization is circular. The ideal value of the axial ratio for circularly polarized fields is 0 dB. In addition, the axial ratio tends to degrade away from the mainbeam of an antenna, so the axial ratio may be indicated in a spec sheet (data sheet) for an antenna as follows: "Axial Ratio: <3 dB for +-30 degrees from mainbeam". This indicates that the deviation from circular polarization is less than 3 dB over the specified angular range.

The gain of the helix antenna can be approximated by:

$$G = \frac{6.2C^2NS}{\lambda^3} = \frac{6.2C^2NSf^3}{c^3}$$

In the above, c is the speed of light. Note that for a given helix geometry (specified in terms of C, S, N), the gain increases with frequency. For an N=10 turn helix, that has a 0.5 meter circumference as above, and pitch angle of 13 degrees (giving S=0.13 meters), the gain is 8.3 (9.2 dB).

For the same example helix antenna, the pattern is shown in Figure 3:

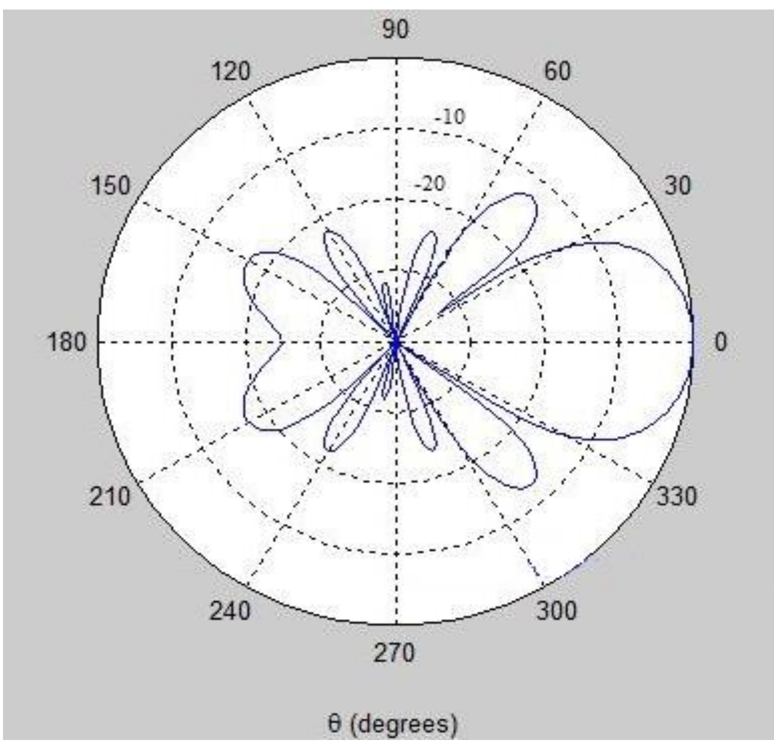

Figure 3: Radiation Pattern Helical Antenna. [3]

The Half-Power Beamwidth for helical antennas can be approximated (in degrees) by:

$$HPBW = \frac{65\lambda}{C\sqrt{\frac{NS}{\lambda}}}$$

The Beamwidth between nulls is approximately:

$$FNBW(degrees) \simeq \frac{115\lambda^{3/2}}{C\sqrt{NS}}$$

CONCLUTIONS:

- Helical antennas have long been popular in applications from VHF to microwaves requiring circular polarization.

- A helical antenna is an antenna consisting of a conducting wire wound in the form of a helix.

- In the *normal mode* or *broadside* helix, the dimensions of the helix (the diameter and the pitch) are small compared with the wavelength. The antenna acts similarly to an electrically short dipole or monopole.

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- The benefits of this helix antenna *axial mode* or *end-fire* helix are it has a wide bandwidth, is easily constructed, and has real input impedance.

- The helix antenna is a travelling wave antenna, which means the current travels along the antenna and the phase varies continuously.

- The *fractional Bandwidth* of helical antennas is 56%.

- One limitation of helical antennas is that the usefulness of the circular polarization is limited since it cannot be easily reversed to the other sense, left-handed to right-handed or vice-versa

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Helical Antennas

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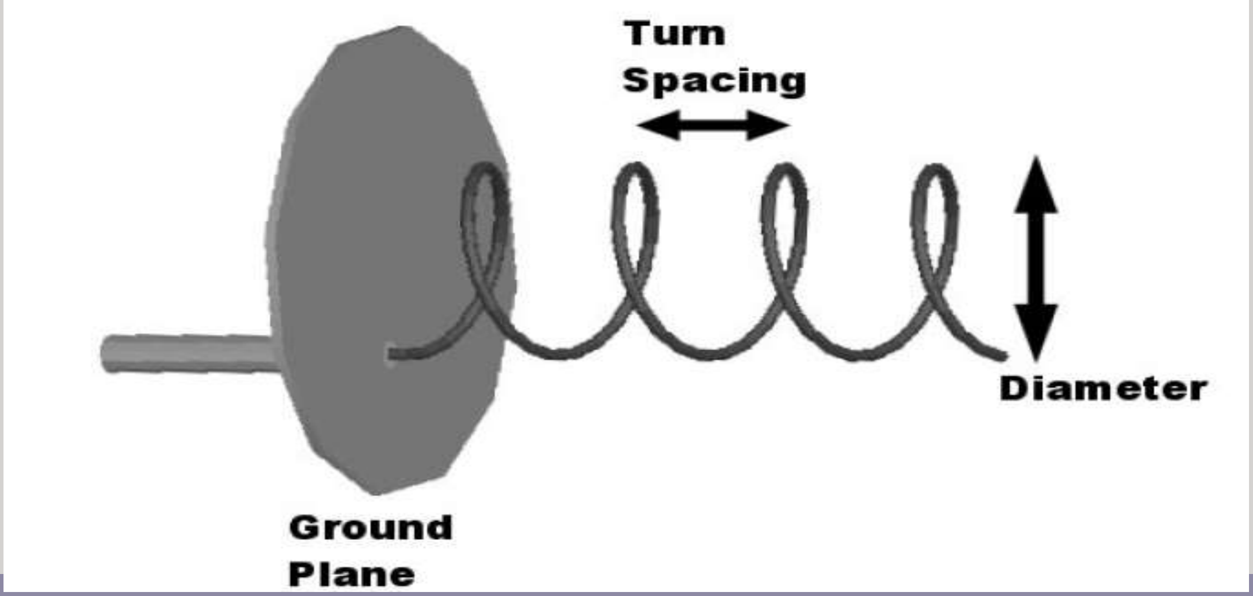
Contents

- Overview of Helical Antenna
- Geometry and Operation of Helical Antenna
- Impedance, Gain, and Radiation Pattern
- Result of Calculating the Gain and Plot of Radiation Pattern .

Overview:

A helical antenna is an antenna consisting of a conducting wire wound in the form of a helix. In most cases, helical antennas are mounted over a ground plane. The feed line is connected between the bottom of the helix and the ground plane. Helical antennas can operate in one of two principal modes: normal mode or axial mode.



Normal mode or Broadside helix

In the *normal mode* or *broadside* helix, the dimensions of the helix (the diameter and the pitch) are small compared with the wavelength. The antenna acts similarly to an electrically short dipole or monopole, and the radiation pattern, similar to these antennas is omnidirectional, with maximum radiation at right angles to the helix axis. The radiation is linearly polarized parallel to the helix axis.

Axial mode or End-fire helix

In the *axial mode* or *end-fire* helix, the dimensions of the helix are comparable to a wavelength. The antenna functions as a directional antenna radiating a beam off the ends of the helix, along the antenna's axis. It radiates circularly polarized radio waves.

The benefits of this helix antenna *axial mode* or *end-fire* helix are it has a wide bandwidth, is easily constructed, and has real input impedance.

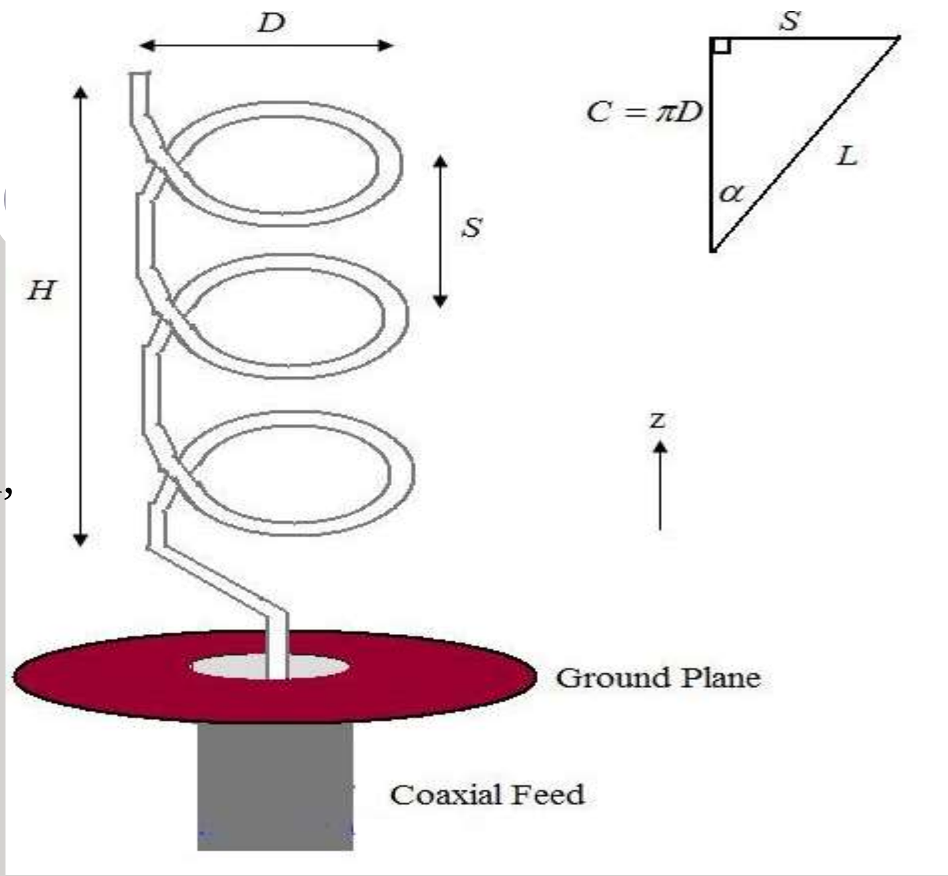
Geometry and Operation

· D - Diameter of a turn on the helix antenna.
· C - Circumference of a turn on the helix antenna (*C*=pi**D*).
· S - Vertical separation between turns for helical antenna.
· α - pitch angle, which controls how far the helix antenna grows in the z-direction per turn, and is given by

$$\alpha = \tan^{-1}\frac{S}{C}$$

· N - Number of turns on the helix antenna.

· H - Total height of helix antenna, *H*=*NS*.



Geometry and Operation

The radiation pattern will be maximum in the +z direction (along the helical axis).

The design of helical antennas is primarily based on empirical results, and the fundamental equations will be presented here.

Helix antennas of at least 3 turns will have close to circular polarization in the +z direction when the

circumference C is close to a wavelength: $\frac{3\lambda}{4} \leq C \leq \frac{4\lambda}{3}$

Impedance, Gain, and Radiation Pattern

The helix antenna is a travelling wave antenna, which means the current travels along the antenna and the phase varies continuously.

In addition, the input impedance is primarily real and can be approximated in Ohms by:

$$Z_{in} = 140 \frac{C}{\lambda}$$

Impedance, Gain, and Radiation Pattern

The helix antenna functions well for pitch angles (α) between 12 and 14 degrees.

Typically, the pitch angle is taken as 13 degrees.

The normalized radiation pattern for the E-field components is given by:

$$E_\theta \propto E_\phi \propto \sin\frac{\pi}{2N}\cos\theta\frac{\sin\frac{N\Omega}{2}}{\sin(\Omega/2)}$$

$$\Omega = kS(\cos\theta - 1) - \pi(2 + 1/N)$$

Impedance, Gain, and Radiation Pattern

For circular polarization, the orthogonal components of the E-field must be 90 degrees out of phase.

This occurs in directions near the axis (z-axis in Figure 1) of the helix. The axial ratio for helix antennas decreases as the number of loops N is added, and can be approximated by:

$$AR = \frac{2N+1}{2N}$$

Impedance, Gain, and Radiation Pattern

The axial ratio is the ratio of orthogonal components of an E-field. A circularly polarized field is made up of two orthogonal E-field components of equal amplitude (and 90 degrees out of phase).

Because the components are equal magnitude, the axial ratio is 1 (or 0 dB).

Impedance, Gain, and Radiation Pattern

The gain of the helix antenna can be approximated by:

$$G = \frac{6.2C^2NS}{\lambda^3} = \frac{6.2C^2NSf^3}{c^3}$$

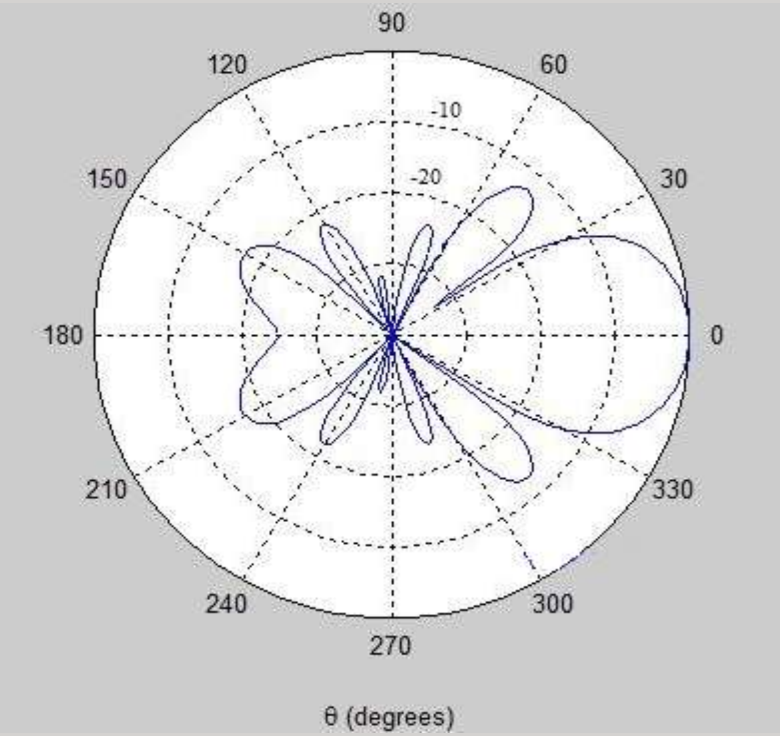
In the above, c is the speed of light.

For an N=10 turn helix, that has a 0.5 meter Circumference as above, and pitch angle of 13 degrees

(giving S=0.13 meters), the gain is 8.3 (9.2 dB).

Impedance, Gain, and Radiation Pattern

For the same example helix antenna, the pattern is shown in the figure :



Impedance, Gain, and Radiation Pattern

The Half-Power Beamwidth for helical antennas can be approximated (in degrees) by:

$$HPBW = \frac{65\lambda}{C\sqrt{\frac{NS}{\lambda}}}$$

The Beamwidth between nulls is approximately:

$$FNBW(degrees) \simeq \frac{115\lambda^{3/2}}{C\sqrt{NS}}$$

The End

Any Question ?