Fundamentals of Applied Electromagnetics 6e

by

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Solved Problems

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Chapter 1 Solved Problems

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Problem 1.4 A wave traveling along a string is given by

$$y(x,t) = 2\sin(4\pi t + 10\pi x) \quad \text{(cm)},$$

where x is the distance along the string in meters and y is the vertical displacement. Determine: (a) the direction of wave travel, (b) the reference phase ϕ_0, (c) the frequency, (d) the wavelength, and (e) the phase velocity.

Solution:

(a) We start by converting the given expression into a cosine function of the form given by (1.17):

$$y(x,t) = 2\cos\left(4\pi t + 10\pi x - \frac{\pi}{2}\right) \quad \text{(cm)}.$$

Since the coefficients of t and x both have the same sign, the wave is traveling in the negative x-direction.

(b) From the cosine expression, $\phi_0 = -\pi/2$.

(c) $\omega = 2\pi f = 4\pi$,

$$f = 4\pi/2\pi = 2 \text{ Hz}.$$

(d) $2\pi/\lambda = 10\pi$,

$$\lambda = 2\pi/10\pi = 0.2 \text{ m}.$$

(e) $u_\text{p} = f\lambda = 2 \times 0.2 = 0.4$ (m/s).

Problem 1.7 A wave traveling along a string in the $+x$-direction is given by

$$y_1(x,t) = A\cos(\omega t - \beta x),$$

where $x = 0$ is the end of the string, which is tied rigidly to a wall, as shown in Fig. (P1.7). When wave $y_1(x,t)$ arrives at the wall, a reflected wave $y_2(x,t)$ is generated. Hence, at any location on the string, the vertical displacement y_s will be the sum of the incident and reflected waves:

$$y_s(x,t) = y_1(x,t) + y_2(x,t).$$

(a) Write down an expression for $y_2(x,t)$, keeping in mind its direction of travel and the fact that the end of the string cannot move.

(b) Generate plots of $y_1(x,t)$, $y_2(x,t)$ and $y_s(x,t)$ versus x over the range $-2\lambda \le x \le 0$ at $\omega t = \pi/4$ and at $\omega t = \pi/2$.

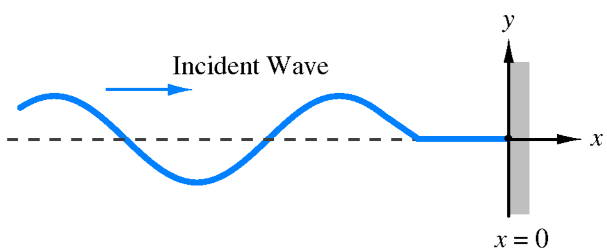


Figure P1.7: Wave on a string tied to a wall at $x = 0$ (Problem 1.7).

Solution:

(a) Since wave $y_2(x,t)$ was caused by wave $y_1(x,t)$, the two waves must have the same angular frequency ω, and since $y_2(x,t)$ is traveling on the same string as $y_1(x,t)$, the two waves must have the same phase constant β. Hence, with its direction being in the negative x-direction, $y_2(x,t)$ is given by the general form

$$y_2(x,t) = B\cos(\omega t + \beta x + \phi_0), \tag{1.1}$$

where B and ϕ_0 are yet-to-be-determined constants. The total displacement is

$$y_s(x,t) = y_1(x,t) + y_2(x,t) = A\cos(\omega t - \beta x) + B\cos(\omega t + \beta x + \phi_0).$$

Since the string cannot move at $x = 0$, the point at which it is attached to the wall, $y_s(0,t) = 0$ for all t. Thus,

$$y_s(0,t) = A\cos\omega t + B\cos(\omega t + \phi_0) = 0. \tag{1.2}$$

(i) Easy Solution: The physics of the problem suggests that a possible solution for (1.2) is $B = -A$ and $\phi_0 = 0$, in which case we have

$$y_2(x,t) = -A\cos(\omega t + \beta x). \tag{1.3}$$

(ii) Rigorous Solution: By expanding the second term in (1.2), we have

$$A\cos\omega t + B(\cos\omega t\cos\phi_0 - \sin\omega t\sin\phi_0) = 0,$$

or

$$(A + B\cos\phi_0)\cos\omega t - (B\sin\phi_0)\sin\omega t = 0. \tag{1.4}$$

This equation has to be satisfied for all values of t. At $t = 0$, it gives

$$A + B\cos\phi_0 = 0, \tag{1.5}$$

and at $\omega t = \pi/2$, (1.4) gives

$$B\sin\phi_0 = 0. \tag{1.6}$$

Equations (1.5) and (1.6) can be satisfied simultaneously only if

$$A = B = 0 \tag{1.7}$$

or

$$A = -B \quad \text{and} \quad \phi_0 = 0. \tag{1.8}$$

Clearly (1.7) is not an acceptable solution because it means that $y_1(x,t) = 0$, which is contrary to the statement of the problem. The solution given by (1.8) leads to (1.3).

(b) At $\omega t = \pi/4$,

$$y_1(x,t) = A\cos(\pi/4 - \beta x) = A\cos\left(\frac{\pi}{4} - \frac{2\pi x}{\lambda}\right),$$
$$y_2(x,t) = -A\cos(\omega t + \beta x) = -A\cos\left(\frac{\pi}{4} + \frac{2\pi x}{\lambda}\right).$$

Plots of y_1, y_2, and y_3 are shown in Fig. P1.7(b).

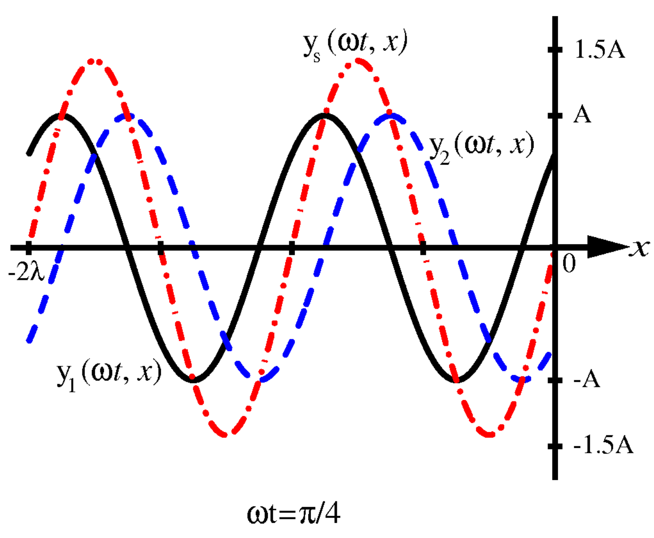


Figure P1.7: (b) Plots of y_1, y_2, and y_s versus x at $\omega t = \pi/4$.

At $\omega t = \pi/2$,

$$y_1(x,t) = A\cos(\pi/2 - \beta x) = A\sin\beta x = A\sin\frac{2\pi x}{\lambda},$$
$$y_2(x,t) = -A\cos(\pi/2 + \beta x) = A\sin\beta x = A\sin\frac{2\pi x}{\lambda}.$$

Plots of y_1, y_2, and y_3 are shown in Fig. P1.7(c).

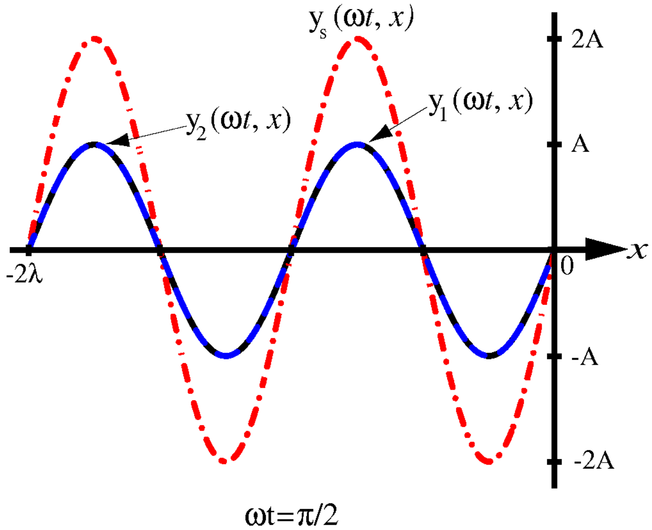


Figure P1.7: (c) Plots of y_1, y_2, and y_s versus x at $\omega t = \pi/2$.

Problem 1.15 A laser beam traveling through fog was observed to have an intensity of 1 (μW/m^2) at a distance of 2 m from the laser gun and an intensity of 0.2 (μW/m^2) at a distance of 3 m. Given that the intensity of an electromagnetic wave is proportional to the square of its electric-field amplitude, find the attenuation constant α of fog.

Solution: If the electric field is of the form

$$E(x,t) = E_0 e^{-\alpha x} \cos(\omega t - \beta x),$$

then the intensity must have a form

$$\begin{aligned} I(x,t) &\approx [E_0 e^{-\alpha x} \cos(\omega t - \beta x)]^2 \\ &\approx E_0^2 e^{-2\alpha x} \cos^2(\omega t - \beta x) \end{aligned}$$

or

$$I(x,t) = I_0 e^{-2\alpha x} \cos^2(\omega t - \beta x)$$

where we define $I_0 \approx E_0^2$. We observe that the magnitude of the intensity varies as $I_0 e^{-2\alpha x}$. Hence,

$$\begin{aligned} &\text{at } x = 2 \text{ m}, \qquad I_0 e^{-4\alpha} = 1 \times 10^{-6} \quad (\text{W/m}^2), \\ &\text{at } x = 3 \text{ m}, \qquad I_0 e^{-6\alpha} = 0.2 \times 10^{-6} \quad (\text{W/m}^2). \end{aligned}$$

$$\begin{aligned} \frac{I_0 e^{-4\alpha}}{I_0 e^{-6\alpha}} &= \frac{10^{-6}}{0.2 \times 10^{-6}} = 5 \\ e^{-4\alpha} \cdot e^{6\alpha} &= e^{2\alpha} = 5 \\ \alpha &= 0.8 \quad (\text{NP/m}). \end{aligned}$$

Problem 1.18 Complex numbers z_1 and z_2 are given by

$$z_1 = -3 + j2$$
$$z_2 = 1 - j2$$

Determine **(a)** $z_1 z_2$, **(b)** z_1/z_2^*, **(c)** z_1^2, and **(d)** $z_1 z_1^*$, all all in polar form.

Solution:

(a) We first convert z_1 and z_2 to polar form:

$$\begin{aligned} z_1 = -(3 - j2) &= -\left(\sqrt{3^2 + 2^2}\, e^{-j\tan^{-1} 2/3}\right) \\ &= -\sqrt{13}\, e^{-j33.7^\circ} \\ &= \sqrt{13}\, e^{j(180^\circ - 33.7^\circ)} \\ &= \sqrt{13}\, e^{j146.3^\circ}. \end{aligned}$$

$$\begin{aligned} z_2 = 1 - j2 &= \sqrt{1 + 4}\, e^{-j\tan^{-1} 2} \\ &= \sqrt{5}\, e^{-j63.4^\circ}. \end{aligned}$$

$$\begin{aligned} z_1 z_2 &= \sqrt{13}\, e^{j146.3^\circ} \times \sqrt{5}\, e^{-j63.4^\circ} \\ &= \sqrt{65}\, e^{j82.9^\circ}. \end{aligned}$$

(b)

$$\frac{z_1}{z_2^*} = \frac{\sqrt{13}\, e^{j146.3^\circ}}{\sqrt{5}\, e^{j63.4^\circ}} = \sqrt{\frac{13}{5}}\, e^{j82.9^\circ}.$$

(c)

$$\begin{aligned} z_1^2 = (\sqrt{13})^2 (e^{j146.3^\circ})^2 &= 13 e^{j292.6^\circ} \\ &= 13 e^{-j360^\circ} e^{j292.6^\circ} \\ &= 13 e^{-j67.4^\circ}. \end{aligned}$$

(c)

$$\begin{aligned} z_1 z_1^* &= \sqrt{13}\, e^{j146.3^\circ} \times \sqrt{13}\, e^{-j146.3^\circ} \\ &= 13. \end{aligned}$$

Problem 1.20 Find complex numbers $t = z_1 + z_2$ and $s = z_1 - z_2$, both in polar form, for each of the following pairs:

(a) $z_1 = 2 + j3,\ z_2 = 1 - j3,$

(b) $z_1 = 3,\ z_2 = -j3,$

(c) $z_1 = 3\angle 30^\circ,\ z_2 = 3\angle -30^\circ,$

(d) $z_1 = 3\angle 30^\circ,\ z_2 = 3\angle -150^\circ.$

Solution:

(d)

$$t = z_1 + z_2 = 3\angle 30^\circ + 3\angle -150^\circ = (2.6 + j1.5) + (-2.6 - j1.5) = 0,$$
$$s = z_1 - z_2 = (2.6 + j1.5) - (-2.6 - j1.5) = 5.2 + j3 = 6e^{j30^\circ}.$$

Problem 1.21 Complex numbers z_1 and z_2 are given by

$$z_1 = 5\angle{-60^\circ},$$
$$z_2 = 4\angle{45^\circ}.$$

(a) Determine the product $z_1 z_2$ in polar form.
(b) Determine the product $z_1 z_2^*$ in polar form.
(c) Determine the ratio z_1/z_2 in polar form.
(d) Determine the ratio z_1^*/z_2^* in polar form.
(e) Determine $\sqrt{z_1}$ in polar form.

Solution:

(c) $$\frac{z_1}{z_2} = \frac{5e^{-j60^\circ}}{4e^{j45^\circ}} = 1.25e^{-j105^\circ}.$$

Problem 1.24 If $z = 3e^{j\pi/6}$, find the value of e^z.

Solution:

$$\begin{aligned} z = 3e^{j\pi/6} &= 3\cos\pi/6 + j3\sin\pi/6 \\ &= 2.6 + j1.5 \end{aligned}$$

$$\begin{aligned} e^z = e^{2.6+j1.5} &= e^{2.6} \times e^{j1.5} \\ &= e^{2.6}(\cos 1.5 + j\sin 1.5) \\ &= 13.46(0.07 + j0.98) \\ &= 0.95 + j13.43. \end{aligned}$$

Problem 1.26 Find the phasors of the following time functions:

(a) $\upsilon(t) = 9\cos(\omega t - \pi/3)$ (V)

(b) $\upsilon(t) = 12\sin(\omega t + \pi/4)$ (V)

(c) $i(x,t) = 5e^{-3x}\sin(\omega t + \pi/6)$ (A)

(d) $i(t) = -2\cos(\omega t + 3\pi/4)$ (A)

(e) $i(t) = 4\sin(\omega t + \pi/3) + 3\cos(\omega t - \pi/6)$ (A)

Solution:

(d)

$$i(t) = -2\cos(\omega t + 3\pi/4),$$
$$\widetilde{I} = -2e^{j3\pi/4} = 2e^{-j\pi}e^{j3\pi/4} = 2e^{-j\pi/4} \text{ A}.$$

Problem 1.27 Find the instantaneous time sinusoidal functions corresponding to the following phasors:

(a) $\widetilde{V} = -5e^{j\pi/3}$ (V)
(b) $\widetilde{V} = j6e^{-j\pi/4}$ (V)
(c) $\widetilde{I} = (6 + j8)$ (A)
(d) $\tilde{I} = -3 + j2$ (A)
(e) $\tilde{I} = j$ (A)
(f) $\tilde{I} = 2e^{j\pi/6}$ (A)

Solution:

(d)

$$\widetilde{I} = -3 + j2 = 3.61\, e^{j146.31^\circ},$$
$$i(t) = \mathfrak{Re}\{3.61\, e^{j146.31^\circ} e^{j\omega t}\} = 3.61 \cos(\omega t + 146.31^\circ) \text{ A}.$$

Problem 1.29 The voltage source of the circuit shown in Fig. P1.29 is given by

$$v_s(t) = 25\cos(4\times 10^4 t - 45^\circ) \quad \text{(V)}.$$

Obtain an expression for $i_L(t)$, the current flowing through the inductor.

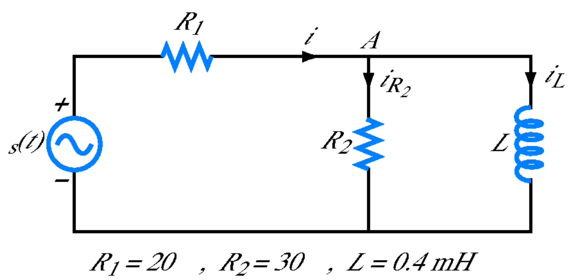


Figure P1.29: Circuit for Problem 1.29.

Solution: Based on the given voltage expression, the phasor source voltage is

$$\widetilde{V}_s = 25e^{-j45^\circ} \quad \text{(V)}. \tag{1.9}$$

The voltage equation for the left-hand side loop is

$$R_1 i + R_2 i_{R_2} = v_s \tag{1.10}$$

For the right-hand loop,

$$R_2 i_{R_2} = L\frac{di_L}{dt}\,, \tag{1.11}$$

and at node A,

$$i = i_{R_2} + i_L. \tag{1.12}$$

Next, we convert Eqs. (2)–(4) into phasor form:

$$R_1\widetilde{I} + R_2\widetilde{I}_{R_2} = \widetilde{V}_s \tag{1.13}$$

$$R_2\widetilde{I}_{R_2} = j\omega L\widetilde{I}_L \tag{1.14}$$

$$\widetilde{I} = \widetilde{I}_{R_2} + \widetilde{I}_L \tag{1.15}$$

Upon combining (6) and (7) to solve for $\widetilde{I}_{R_2}$ in terms of $\widetilde{I}$, we have:

$$\widetilde{I}_{R_2} = \frac{j\omega L}{R_2 + j\omega L} I\,. \tag{1.16}$$

Substituting (8) in (5) and then solving for $\widetilde{I}$ leads to:

$$
\begin{aligned}
R_1\widetilde{I} + \frac{jR_2\omega L}{R_2 + j\omega L}\widetilde{I} &= \widetilde{V}_\text{s} \\
\widetilde{I}\left(R_1 + \frac{jR_2\omega L}{R_2 + j\omega L}\right) &= \widetilde{V}_\text{s} \\
\widetilde{I}\left(\frac{R_1R_2 + jR_1\omega L + jR_2\omega L}{R_2 + j\omega L}\right) &= \widetilde{V}_\text{s} \\
\widetilde{I} = \left(\frac{R_2 + j\omega L}{R_1R_2 + j\omega L(R_1 + R_2)}\right)\widetilde{V}_\text{s}. & \qquad (1.17)
\end{aligned}
$$

Combining (6) and (7) to solve for $\widetilde{I}_\text{L}$ in terms of $\widetilde{I}$ gives

$$
\widetilde{I}_\text{L} = \frac{R_2}{R_2 + j\omega L}\widetilde{I}. \qquad (1.18)
$$

Combining (9) and (10) leads to

$$
\begin{aligned}
\widetilde{I}_\text{L} &= \left(\frac{R_2}{R_2 + j\omega L}\right)\left(\frac{R_2 + j\omega L}{R_1R_2 + j\omega L(R_1 + R_2)}\right)\widetilde{V}_\text{s} \\
&= \frac{R_2}{R_1R_2 + + j\omega L(R_1 + R_2)}\widetilde{V}_\text{s}.
\end{aligned}
$$

Using (1) for $\widetilde{V}_\text{s}$ and replacing R_1, R_2, L and ω with their numerical values, we have

$$
\begin{aligned}
\widetilde{I}_\text{L} &= \frac{30}{20 \times 30 + j4 \times 10^4 \times 0.4 \times 10^{-3}(20 + 30)}\, 25e^{-j45^\circ} \\
&= \frac{30 \times 25}{600 + j800}\, e^{-j45^\circ} \\
&= \frac{7.5}{6 + j8}\, e^{-j45^\circ} = \frac{7.5e^{-j45^\circ}}{10e^{j53.1^\circ}} = 0.75e^{-j98.1^\circ} \quad \text{(A)}.
\end{aligned}
$$

Finally,

$$
\begin{aligned}
i_\text{L}(t) &= \mathfrak{Re}[\widetilde{I}_\text{L}e^{j\omega t}] \\
&= 0.75\cos(4 \times 10^4 t - 98.1^\circ) \quad \text{(A)}.
\end{aligned}
$$

Chapter 2 Solved Problems

Problem 2-5

Problem 2-16

Problem 2-34

Problem 2-45

Problem 2-48

Problem 2-64

Problem 2-75

Problem 2.5 For the parallel-plate transmission line of Problem 2.4, the line parameters are given by: $R' = 1\ \Omega$/m, $L' = 167$ nH/m, $G' = 0$, and $C' = 172$ pF/m. Find α, β, u_{p}, and Z_0 at 1 GHz.

Solution: At 1 GHz, $\omega = 2\pi f = 2\pi \times 10^9$ rad/s. Application of (2.22) gives:

$$\begin{aligned}
\gamma &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\
&= [(1 + j2\pi \times 10^9 \times 167 \times 10^{-9})(0 + j2\pi \times 10^9 \times 172 \times 10^{-12})]^{1/2} \\
&= [(1 + j1049)(j1.1)]^{1/2} \\
&= \left[\sqrt{1 + (1049)^2}\, e^{j\tan^{-1} 1049} \times 1.1 e^{j90^\circ}\right]^{1/2}, \qquad (j = e^{j90^\circ}) \\
&= \left[1049 e^{j89.95^\circ} \times 1.1 e^{j90^\circ}\right]^{1/2} \\
&= \left[1154 e^{j179.95^\circ}\right]^{1/2} \\
&= 34 e^{j89.97^\circ} = 34 \cos 89.97^\circ + j34 \sin 89.97^\circ = 0.016 + j34.
\end{aligned}$$

Hence,

$$\begin{aligned}
\alpha &= 0.016 \text{ Np/m}, \\
\beta &= 34 \text{ rad/m}.
\end{aligned}$$

$$\begin{aligned}
u_{\text{p}} &= \frac{\omega}{\beta} = \frac{2\pi f}{\beta} = \frac{2\pi \times 10^9}{34} = 1.85 \times 10^8 \text{ m/s}. \\
Z_0 &= \left[\frac{R' + j\omega L'}{G' + j\omega C'}\right]^{1/2} \\
&= \left[\frac{1049 e^{j89.95^\circ}}{1.1 e^{j90^\circ}}\right]^{1/2} \\
&= \left[954 e^{-j0.05^\circ}\right]^{1/2} \\
&= 31 e^{-j0.025^\circ} \simeq (31 - j0.01)\ \Omega.
\end{aligned}$$

Problem 2.16 A transmission line operating at 125 MHz has $Z_0 = 40\ \Omega$, $\alpha = 0.02$ (Np/m), and $\beta = 0.75$ rad/m. Find the line parameters R', L', G', and C'.

Solution: Given an arbitrary transmission line, $f = 125$ MHz, $Z_0 = 40\ \Omega$, $\alpha = 0.02$ Np/m, and $\beta = 0.75$ rad/m. Since Z_0 is real and $\alpha \neq 0$, the line is distortionless. From Problem 2.13, $\beta = \omega\sqrt{L'C'}$ and $Z_0 = \sqrt{L'/C'}$, therefore,

$$L' = \frac{\beta Z_0}{\omega} = \frac{0.75 \times 40}{2\pi \times 125 \times 10^6} = 38.2 \text{ nH/m}.$$

Then, from $Z_0 = \sqrt{L'/C'}$,

$$C' = \frac{L'}{Z_0^2} = \frac{38.2 \text{ nH/m}}{40^2} = 23.9 \text{ pF/m}.$$

From $\alpha = \sqrt{R'G'}$ and $R'C' = L'G'$,

$$R' = \sqrt{R'G'}\sqrt{\frac{R'}{G'}} = \sqrt{R'G'}\sqrt{\frac{L'}{C'}} = \alpha Z_0 = 0.02 \text{ Np/m} \times 40\ \Omega = 0.6\ \Omega\text{/m}$$

and

$$G' = \frac{\alpha^2}{R'} = \frac{(0.02 \text{ Np/m})^2}{0.8\ \Omega\text{/m}} = 0.5 \text{ mS/m}.$$

Problem 2.34 A 50-Ω lossless line is terminated in a load impedance $Z_L = (30 - j20)$ Ω.

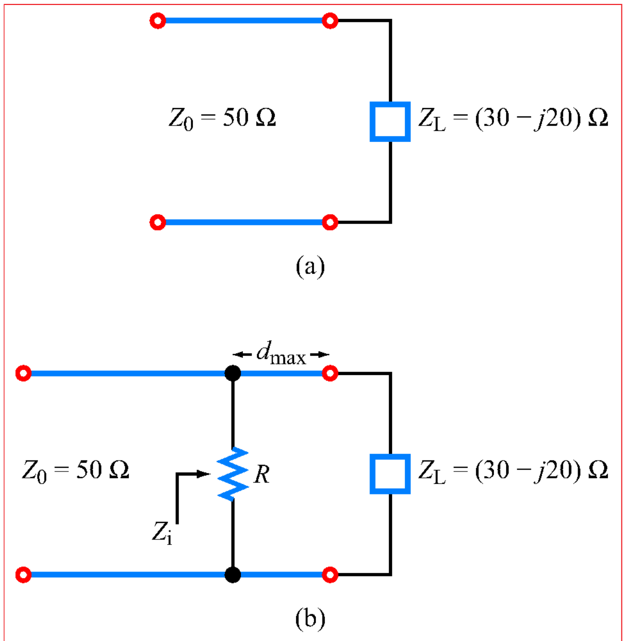


Figure P2.34: Circuit for Problem 2.34.

(a) Calculate Γ and S.

(b) It has been proposed that by placing an appropriately selected resistor across the line at a distance d_{max} from the load (as shown in Fig. P2.34(b)), where d_{max} is the distance from the load of a voltage maximum, then it is possible to render $Z_i = Z_0$, thereby eliminating reflection back to the end. Show that the proposed approach is valid and find the value of the shunt resistance.

Solution:

(a)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{30 - j20 - 50}{30 - j20 + 50} = \frac{-20 - j20}{80 - j20} = \frac{-(20 + j20)}{80 - j20} = 0.34e^{-j121^\circ}.$$

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = \frac{1 + 0.34}{1 - 0.34} = 2.$$

(b) We start by finding d_{max}, the distance of the voltage maximum nearest to the load. Using (2.70) with $n = 1$,

$$d_{max} = \frac{\theta_r \lambda}{4\pi} + \frac{\lambda}{2} = \left(\frac{-121^\circ \pi}{180^\circ}\right)\frac{\lambda}{4\pi} + \frac{\lambda}{2} = 0.33\lambda.$$

Applying (2.79) at $d = d_{max} = 0.33\lambda$, for which $\beta l = (2\pi/\lambda) \times 0.33\lambda = 2.07$ radians, the value of Z_{in} before adding the shunt resistance is:

$$
\begin{aligned}
Z_{\text{in}} &= Z_0 \left(\frac{Z_{\text{L}} + jZ_0 \tan \beta l}{Z_0 + jZ_{\text{L}} \tan \beta l} \right) \\
&= 50 \left(\frac{(30 - j20) + j50 \tan 2.07}{50 + j(30 - j20) \tan 2.07} \right) = (102 + j0)\ \Omega.
\end{aligned}
$$

Thus, at the location A (at a distance d_{max} from the load), the input impedance is purely real. If we add a shunt resistor R in parallel such that the combination is equal to Z_0, then the new Z_{in} at any point to the left of that location will be equal to Z_0.

Hence, we need to select R such that

$$
\frac{1}{R} + \frac{1}{102} = \frac{1}{50}
$$

or $R = 98\ \Omega$.

Problem 2.45 The circuit shown in Fig. P2.45 consists of a 100-Ω lossless transmission line terminated in a load with $Z_L = (50 + j100)$ Ω. If the peak value of the load voltage was measured to be $|\widetilde{V}_L| = 12$ V, determine:

(a) the time-average power dissipated in the load,

(b) the time-average power incident on the line,

(c) the time-average power reflected by the load.

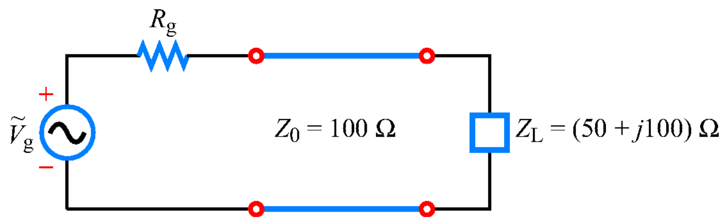


Figure P2.45: Circuit for Problem 2.45.

Solution:

(a)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 + j100 - 100}{50 + j100 + 100} = \frac{-50 + j100}{150 + j100} = 0.62e^{j82.9^\circ}.$$

The time average power dissipated in the load is:

$$\begin{aligned} P_{av} &= \frac{1}{2}|\widetilde{I}_L|^2 R_L \\ &= \frac{1}{2}\left|\frac{\widetilde{V}_L}{Z_L}\right|^2 R_L \\ &= \frac{1}{2}\frac{|\widetilde{V}_L|^2}{|Z_L|^2} R_L = \frac{1}{2} \times 12^2 \times \frac{50}{50^2 + 100^2} = 0.29 \text{ W.} \end{aligned}$$

(b)

$$P_{av} = P^i_{av}(1 - |\Gamma|^2)$$

Hence,

$$P^i_{av} = \frac{P_{av}}{1 - |\Gamma|^2} = \frac{0.29}{1 - 0.62^2} = 0.47 \text{ W.}$$

(c)

$$P^r_{av} = -|\Gamma|^2 P^i_{av} = -(0.62)^2 \times 0.47 = -0.18 \text{ W.}$$

Problem 2.48 Repeat Problem 2.47 using CD Module 2.6.

Solution:

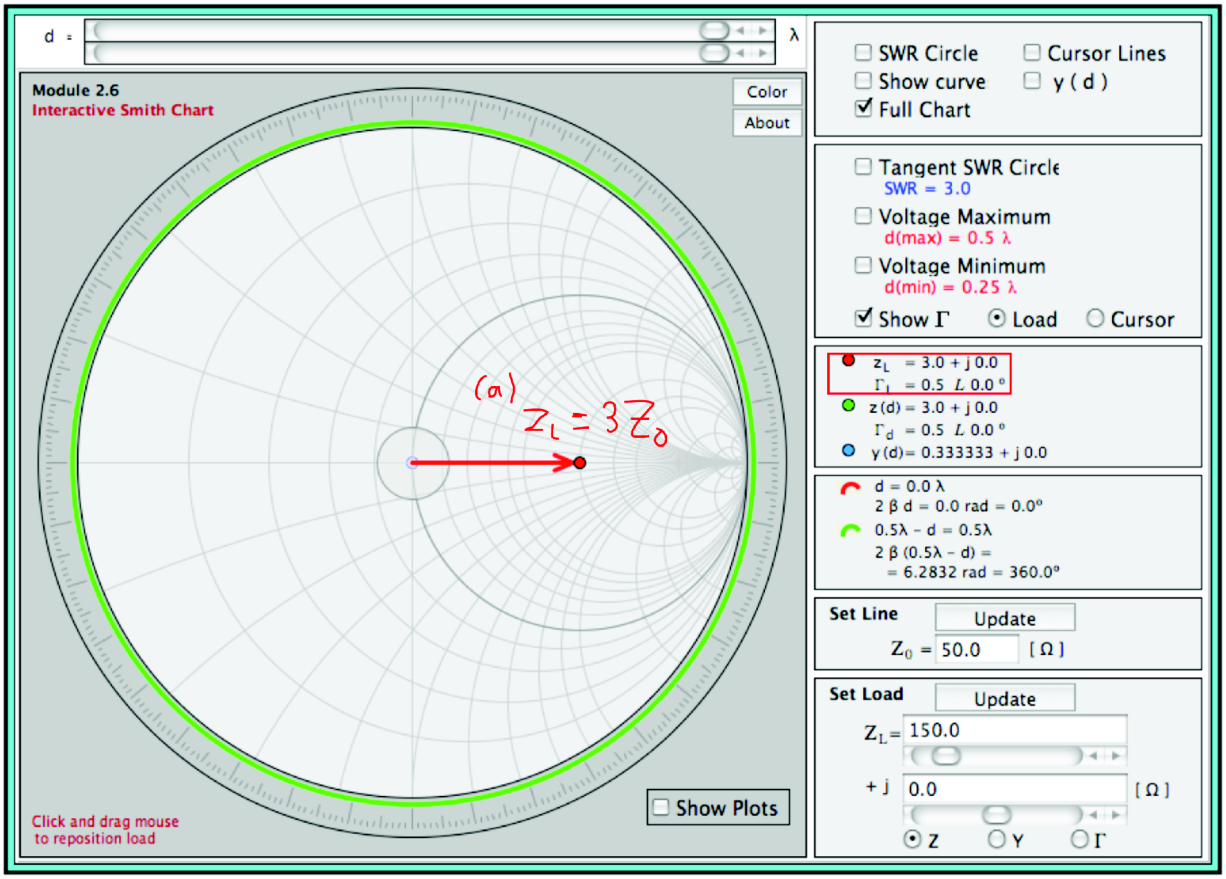


Figure P2.48(a)

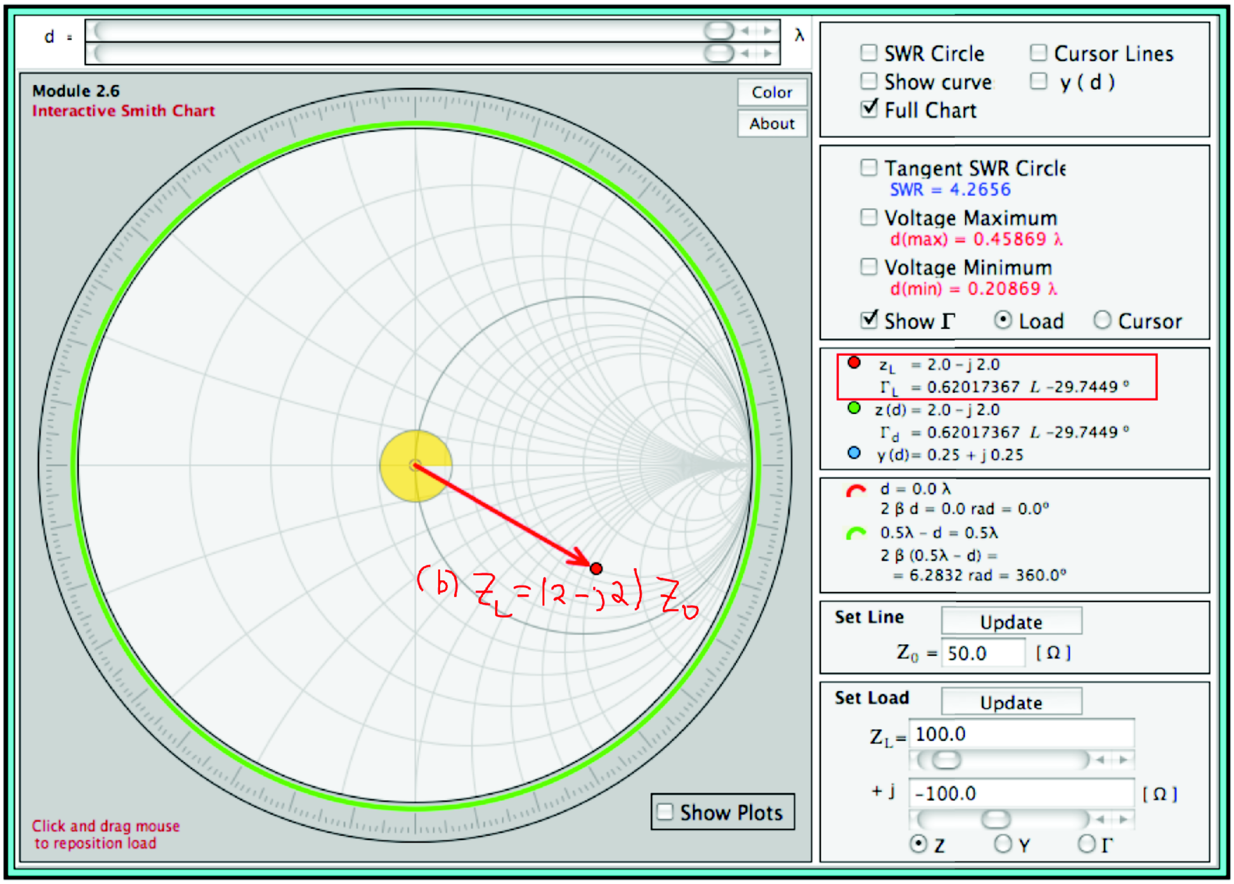


Figure P2.48(b)

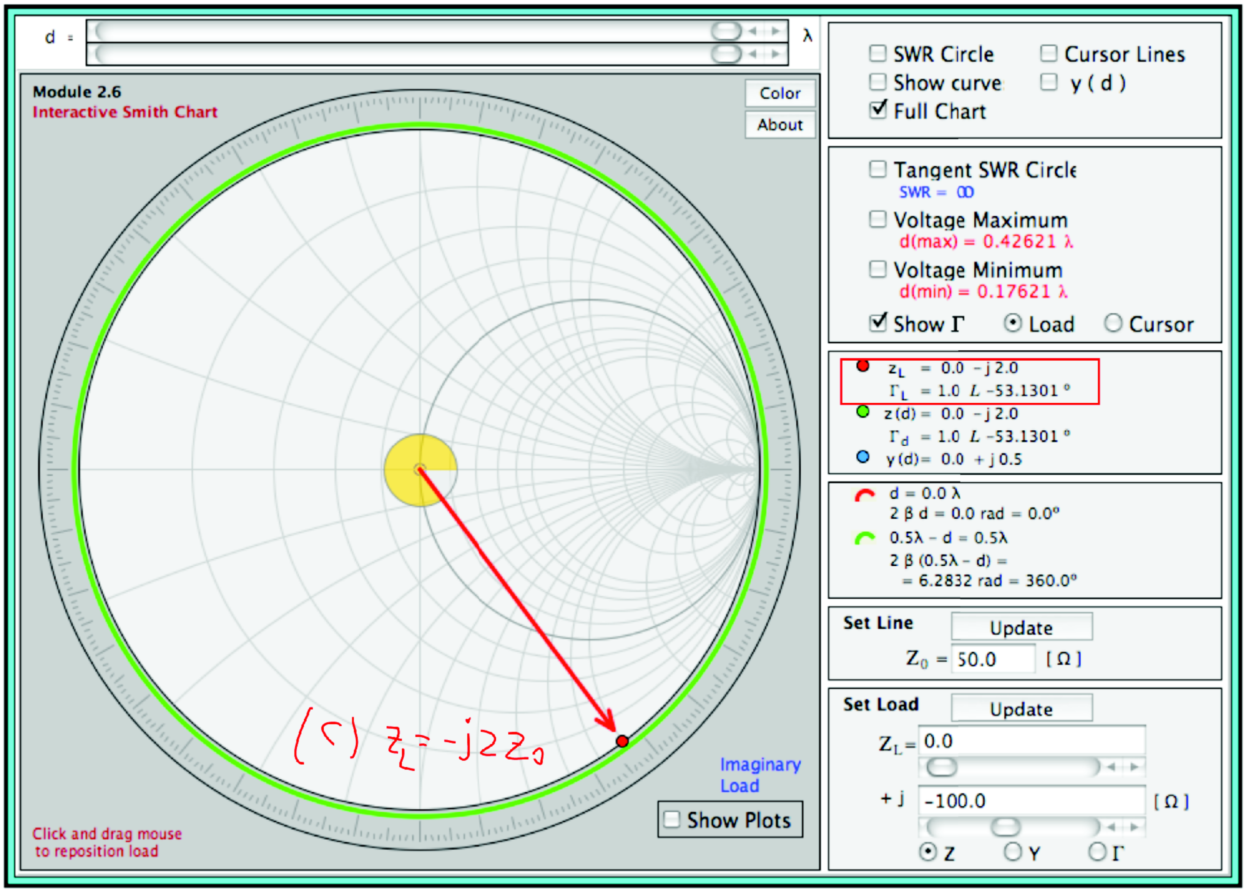


Figure P2.48(c)

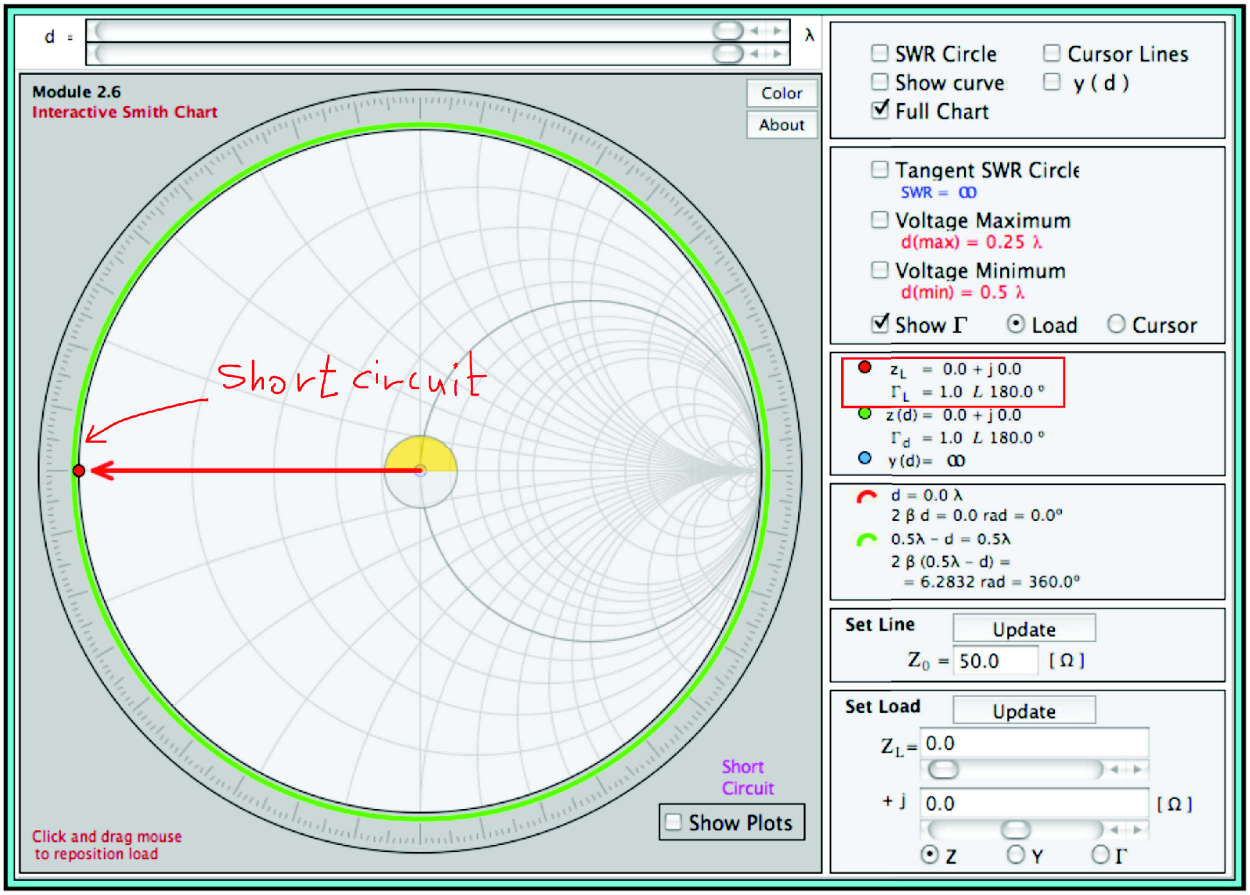


Figure P2.48(d)

Problem 2.64 Use CD Module 2.7 to design a quarter-wavelength transformer to match a load with $Z_L = (100 - j200)$ Ω to a 50-Ω line.

Solution: Figure P2.64(a) displays the first solution of Module 2.7 where a $\lambda/4$ section of $Z_{02} = 15.5015$ Ω is inserted at distance $d_1 = 0.21829\lambda$ from the load.

Figure P2.64(b) displays a summary of the two possible solutions for matching the load to the feedline with a $\lambda/4$ transformer.

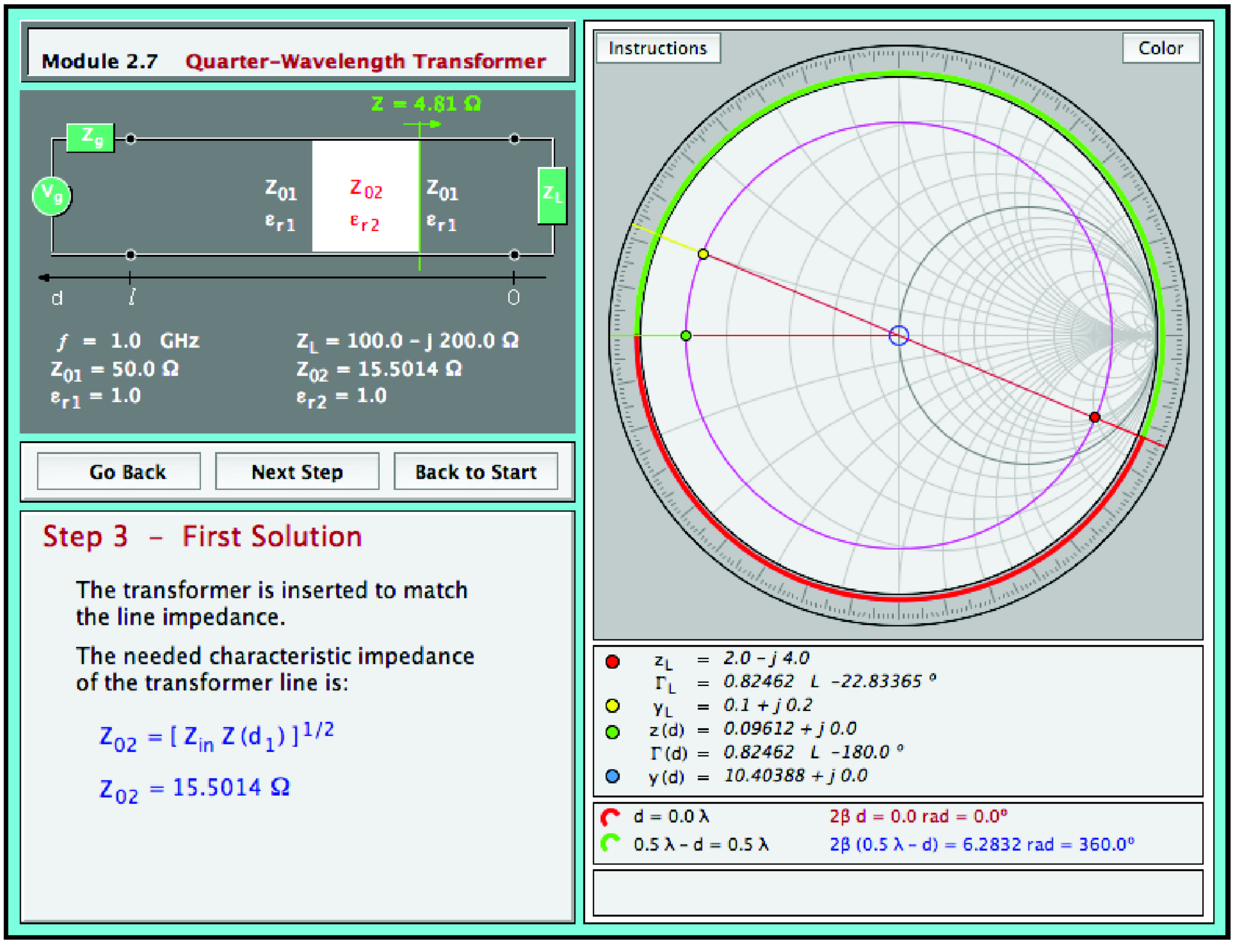


Figure P2.64(a)

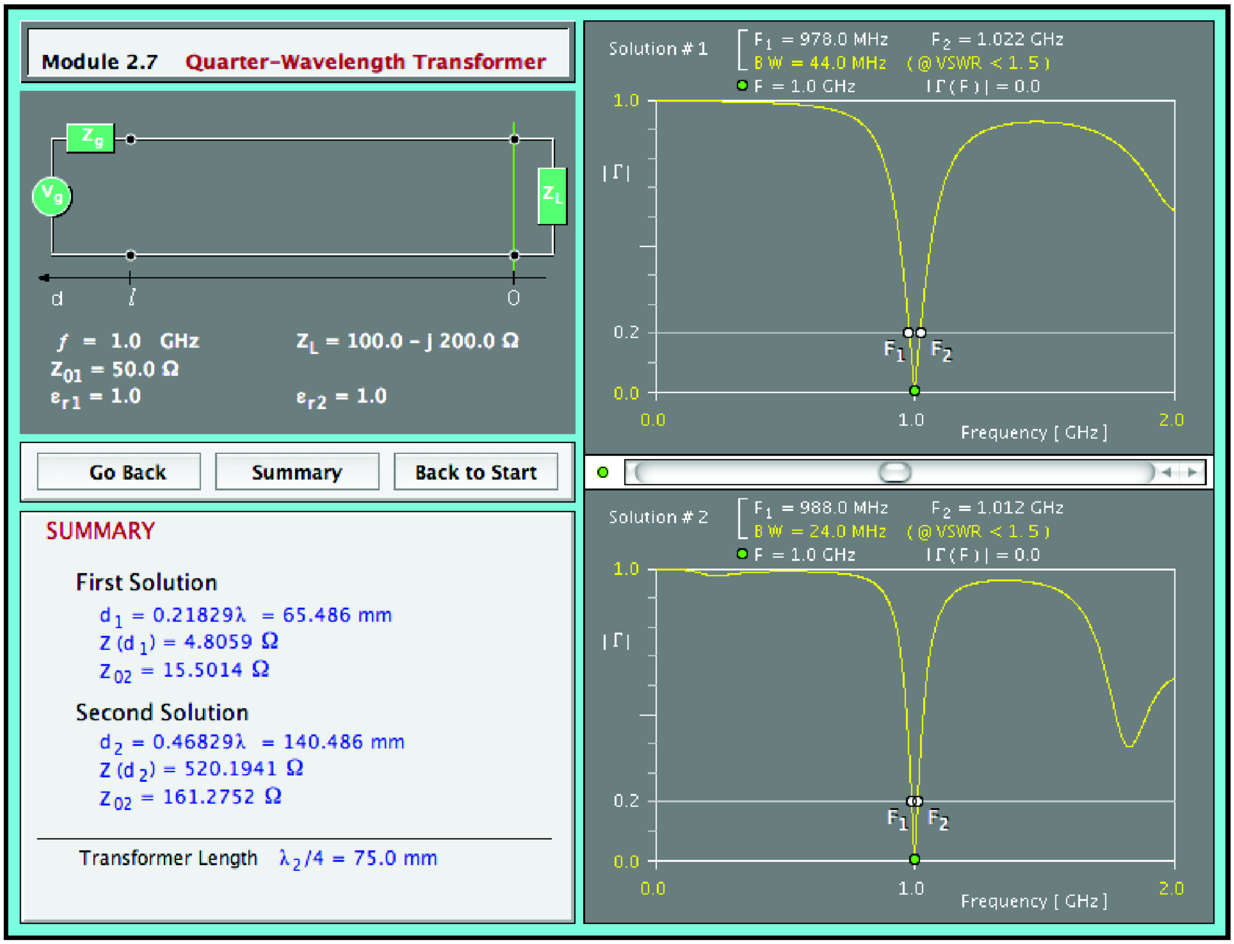


Figure P2.64(b)

Problem 2.75 Generate a bounce diagram for the voltage $V(z,t)$ for a 1-m–long lossless line characterized by $Z_0 = 50\ \Omega$ and $u_p = 2c/3$ (where c is the velocity of light) if the line is fed by a step voltage applied at $t = 0$ by a generator circuit with $V_g = 60$ V and $R_g = 100\ \Omega$. The line is terminated in a load $R_L = 25\ \Omega$. Use the bounce diagram to plot $V(t)$ at a point midway along the length of the line from $t = 0$ to $t = 25$ ns.

Solution:

$$\Gamma_g = \frac{R_g - Z_0}{R_g + Z_0} = \frac{100-50}{100+50} = \frac{50}{150} = \frac{1}{3},$$
$$\Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{25-50}{25+50} = \frac{-25}{75} = \frac{-1}{3}.$$

From Eq. (2.149b),

$$V_1^+ = \frac{V_g Z_0}{R_g + Z_0} = \frac{60 \times 50}{100+50} = 20 \text{ V}.$$

Also,

$$T = \frac{l}{u_p} = \frac{l}{2c/3} = \frac{3}{2 \times 3 \times 10^8} = 5 \text{ ns}.$$

The bounce diagram is shown in Fig. P2.75(a) and the plot of $V(t)$ in Fig. P2.75(b).

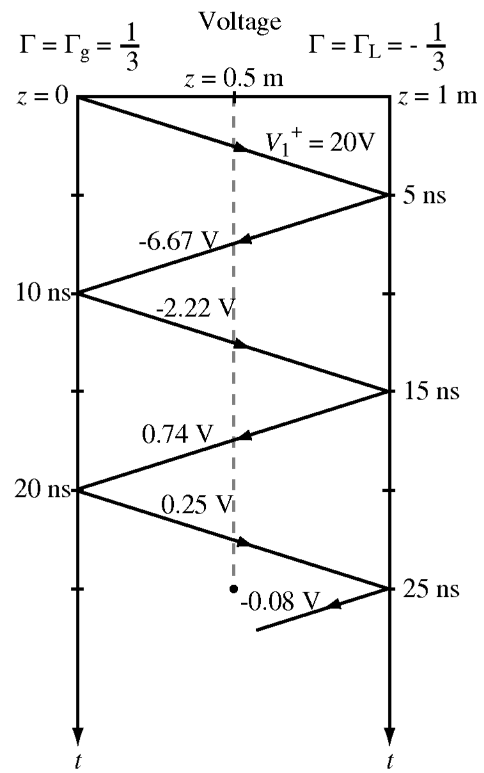


Figure P2.75: (a) Bounce diagram for Problem 2.75.

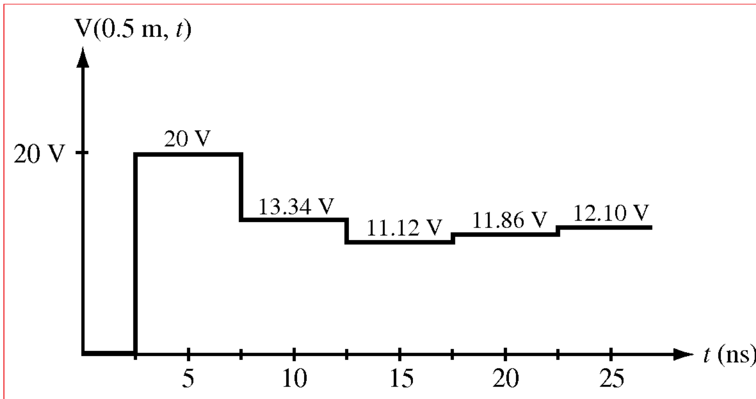


Figure P2.75: (b) Time response of voltage.

Chapter 3 Solved Problems

Problem 3-9

Problem 3-17

Problem 3-19

Problem 3-20

Problem 3-22

Problem 3-23

Problem 3-25

Problem 3-33

Problem 3-35

Problem 3-36

Problem 3-41

Problem 3-50

Problem 3-55

Problem 3-57

Problem 3.9 Find an expression for the unit vector directed toward the origin from an arbitrary point on the line described by $x = 1$ and $z = 2$.

Solution: An arbitrary point on the given line is $(1, y, 2)$. The vector from this point to $(0, 0, 0)$ is:

$$\begin{aligned}
\mathbf{A} &= \hat{\mathbf{x}}(0-1) + \hat{\mathbf{y}}(0-y) + \hat{\mathbf{z}}(0-2) = -\hat{\mathbf{x}} - \hat{\mathbf{y}}y - 2\hat{\mathbf{z}}, \\
|\mathbf{A}| &= \sqrt{1 + y^2 + 4} = \sqrt{5 + y^2}, \\
\hat{\mathbf{a}} &= \frac{\mathbf{A}}{|\mathbf{A}|} = \frac{-\hat{\mathbf{x}} - \hat{\mathbf{y}}y - \hat{\mathbf{z}}2}{\sqrt{5 + y^2}}.
\end{aligned}$$

Problem 3.17 Find a vector **G** whose magnitude is 4 and whose direction is perpendicular to both vectors **E** and **F**, where $\mathbf{E} = \hat{\mathbf{x}} + \hat{\mathbf{y}}\,2 - \hat{\mathbf{z}}\,2$ and $\mathbf{F} = \hat{\mathbf{y}}\,3 - \hat{\mathbf{z}}\,6$.

Solution: The cross product of two vectors produces a third vector which is perpendicular to both of the original vectors. Two vectors exist that satisfy the stated conditions, one along $\mathbf{E} \times \mathbf{F}$ and another along the opposite direction. Hence,

$$\begin{aligned}
\mathbf{G} = \pm 4 \frac{\mathbf{E} \times \mathbf{F}}{|\mathbf{E} \times \mathbf{F}|} &= \pm 4 \frac{(\hat{\mathbf{x}} + \hat{\mathbf{y}}\,2 - \hat{\mathbf{z}}\,2) \times (\hat{\mathbf{y}}\,3 - \hat{\mathbf{z}}\,6)}{|(\hat{\mathbf{x}} + \hat{\mathbf{y}}\,2 - \hat{\mathbf{z}}\,2) \times (\hat{\mathbf{y}}\,3 - \hat{\mathbf{z}}\,6)|} \\
&= \pm 4 \frac{(-\hat{\mathbf{x}}\,6 + \hat{\mathbf{y}}\,6 + \hat{\mathbf{z}}\,3)}{\sqrt{36 + 36 + 9}} \\
&= \pm \frac{4}{9} (-\hat{\mathbf{x}}\,6 + \hat{\mathbf{y}}\,6 + \hat{\mathbf{z}}\,3) = \pm \left(-\hat{\mathbf{x}}\,\frac{8}{3} + \hat{\mathbf{y}}\,\frac{8}{3} + \hat{\mathbf{z}}\,\frac{4}{3} \right).
\end{aligned}$$

Problem 3.19 Vector field $\mathbf{E}$ is given by

$$\mathbf{E} = \hat{\mathbf{R}}\,5R\cos\theta - \hat{\boldsymbol{\theta}}\frac{12}{R}\sin\theta\cos\phi + \hat{\boldsymbol{\phi}}3\sin\phi.$$

Determine the component of $\mathbf{E}$ tangential to the spherical surface $R = 2$ at point $P = (2, 30^\circ, 60^\circ)$.

Solution: At P, $\mathbf{E}$ is given by

$$\begin{aligned}\mathbf{E} &= \hat{\mathbf{R}}\,5\times 2\cos 30^\circ - \hat{\boldsymbol{\theta}}\frac{12}{2}\sin 30^\circ\cos 60^\circ + \hat{\boldsymbol{\phi}}3\sin 60^\circ \\ &= \hat{\mathbf{R}}\,8.67 - \hat{\boldsymbol{\theta}}1.5 + \hat{\boldsymbol{\phi}}2.6.\end{aligned}$$

The $\hat{\mathbf{R}}$ component is normal to the spherical surface while the other two are tangential. Hence,

$$\mathbf{E}_{\mathrm{t}} = -\hat{\boldsymbol{\theta}}1.5 + \hat{\boldsymbol{\phi}}2.6.$$

Problem 3.20 When sketching or demonstrating the spatial variation of a vector field, we often use arrows, as in Fig. P3.20, wherein the length of the arrow is made to be proportional to the strength of the field and the direction of the arrow is the same as that of the field's. The sketch shown in Fig. P3.20, which represents the vector field $\mathbf{E} = \hat{\mathbf{r}}r$, consists of arrows pointing radially away from the origin and their lengths increase linearly in proportion to their distance away from the origin. Using this arrow representation, sketch each of the following vector fields:

(a) $\mathbf{E}_1 = -\hat{\mathbf{x}}y$,

(b) $\mathbf{E}_2 = \hat{\mathbf{y}}x$,

(c) $\mathbf{E}_3 = \hat{\mathbf{x}}x + \hat{\mathbf{y}}y$,

(d) $\mathbf{E}_4 = \hat{\mathbf{x}}x + \hat{\mathbf{y}}2y$,

(e) $\mathbf{E}_5 = \hat{\boldsymbol{\phi}}r$,

(f) $\mathbf{E}_6 = \hat{\mathbf{r}}\sin\phi$.

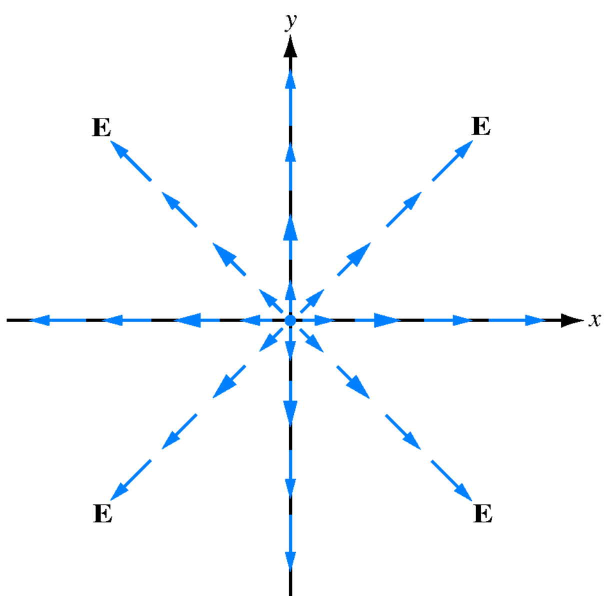


Figure P3.20: Arrow representation for vector field $\mathbf{E} = \hat{\mathbf{r}}r$ (Problem 3.20).

Solution:

(b)

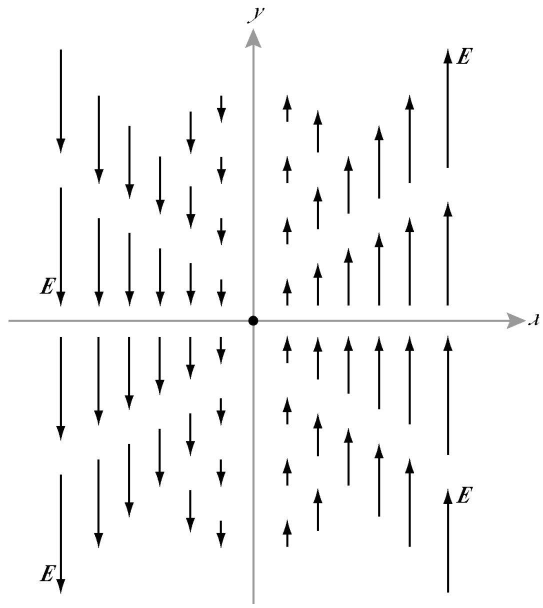


Figure P3.20(b): $\mathbf{E}_2 = -\hat{\mathbf{y}}\, x$

(e)

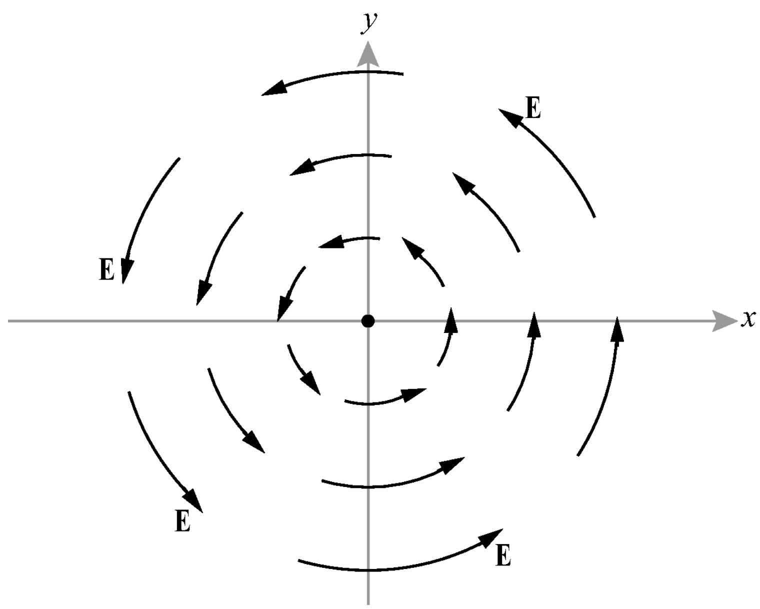


Figure P3.20(e): $\mathbf{E}_5 = \hat{\boldsymbol{\phi}}\, r$

Problem 3.22 Convert the coordinates of the following points from Cartesian to cylindrical and spherical coordinates:

(a) $P_1 = (1, 2, 0)$,
(b) $P_2 = (0, 0, 2)$,
(c) $P_3 = (1, 1, 3)$,
(d) $P_4 = (-2, 2, -2)$.

Solution: Use the "coordinate variables" column in Table 3-2.

(a) In the cylindrical coordinate system,

$$P_1 = (\sqrt{1^2 + 2^2}, \tan^{-1}(2/1), 0) = (\sqrt{5}, 1.107 \text{ rad}, 0) \approx (2.24, 63.4^\circ, 0).$$

In the spherical coordinate system,

$$\begin{aligned} P_1 &= (\sqrt{1^2 + 2^2 + 0^2}, \tan^{-1}(\sqrt{1^2 + 2^2}/0), \tan^{-1}(2/1)) \\ &= (\sqrt{5}, \pi/2 \text{ rad}, 1.107 \text{ rad}) \approx (2.24, 90.0^\circ, 63.4^\circ). \end{aligned}$$

Note that in both the cylindrical and spherical coordinates, ϕ is in Quadrant I.

Problem 3.23 Convert the coordinates of the following points from cylindrical to Cartesian coordinates:

(a) $P_1 = (2, \pi/4, -3)$,

(b) $P_2 = (3, 0, -2)$,

(c) $P_3 = (4, \pi, 5)$.

Solution: (b) $P_2 = (x, y, z) = P_2 = (3\cos 0, 3\sin 0, -2) = P_2 = (3, 0, -2)$.

Problem 3.25 Use the appropriate expression for the differential surface area $d\mathbf{s}$ to determine the area of each of the following surfaces:

(a) $r = 3;\ 0 \le \phi \le \pi/3;\ -2 \le z \le 2,$

(b) $2 \le r \le 5;\ \pi/2 \le \phi \le \pi;\ z = 0,$

(c) $2 \le r \le 5;\ \phi = \pi/4;\ -2 \le z \le 2,$

(d) $R = 2;\ 0 \le \theta \le \pi/3;\ 0 \le \phi \le \pi,$

(e) $0 \le R \le 5;\ \theta = \pi/3;\ 0 \le \phi \le 2\pi.$

Also sketch the outlines of each of the surfaces.

Solution:

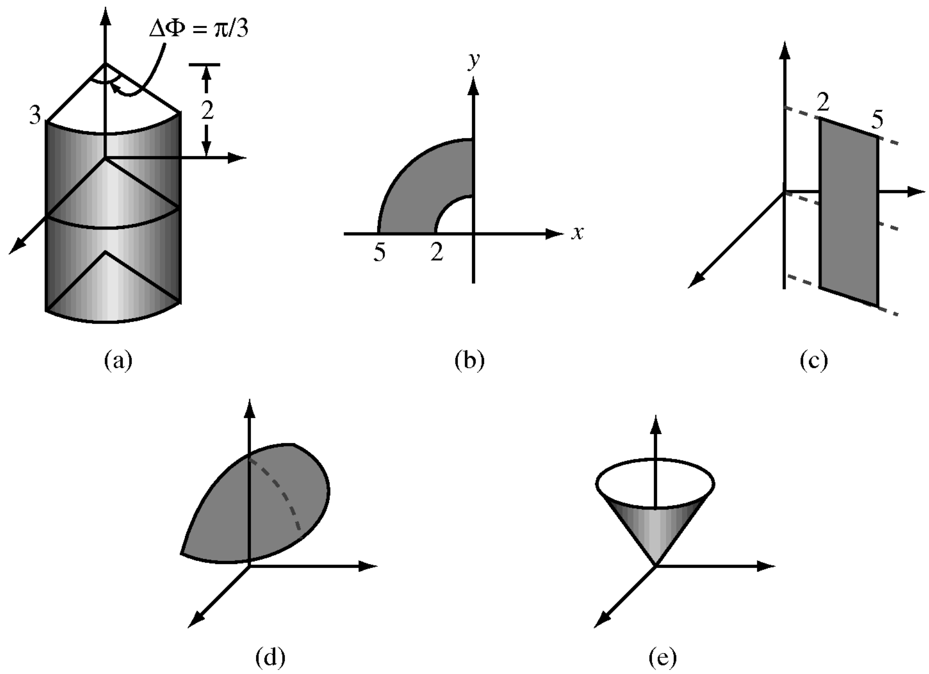


Figure P3.25: Surfaces described by Problem 3.25.

(d) Using Eq. (3.50b),

$$A = \int_{\theta=0}^{\pi/3} \int_{\phi=0}^{\pi} \left(R^2 \sin\theta\right)\Big|_{R=2} d\phi\, d\theta = \left(\left(-4\phi\cos\theta\right)\Big|_{\theta=0}^{\pi/3}\right)\Big|_{\phi=0}^{\pi} = 2\pi.$$

Problem 3.33 Transform the vector

$$\mathbf{A} = \hat{\mathbf{R}} \sin^2\theta \cos\phi + \hat{\boldsymbol{\theta}} \cos^2\phi - \hat{\boldsymbol{\phi}} \sin\phi$$

into cylindrical coordinates and then evaluate it at $P = (2, \pi/2, \pi/2)$.

Solution: From Table 3-2,

$$\begin{aligned}
\mathbf{A} &= (\hat{\mathbf{r}} \sin\theta + \hat{\mathbf{z}} \cos\theta) \sin^2\theta \cos\phi + (\hat{\mathbf{r}} \cos\theta - \hat{\mathbf{z}} \sin\theta) \cos^2\phi - \hat{\boldsymbol{\phi}} \sin\phi \\
&= \hat{\mathbf{r}} (\sin^3\theta \cos\phi + \cos\theta \cos^2\phi) - \hat{\boldsymbol{\phi}} \sin\phi + \hat{\mathbf{z}} (\cos\theta \sin^2\theta \cos\phi - \sin\theta \cos^2\phi)
\end{aligned}$$

At $P = (2, \pi/2, \pi/2)$,

$$\mathbf{A} = -\hat{\boldsymbol{\phi}}.$$

Problem 3.35 Transform the following vectors into spherical coordinates and then evaluate them at the indicated points:

(a) $\mathbf{A} = \hat{\mathbf{x}}y^2 + \hat{\mathbf{y}}xz + \hat{\mathbf{z}}4$ at $P_1 = (1,-1,2)$,

(b) $\mathbf{B} = \hat{\mathbf{y}}(x^2+y^2+z^2) - \hat{\mathbf{z}}(x^2+y^2)$ at $P_2 = (-1,0,2)$,

(c) $\mathbf{C} = \hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi + \hat{\mathbf{z}}\cos\phi\sin\phi$ at $P_3 = (2,\pi/4,2)$, and

(d) $\mathbf{D} = \hat{\mathbf{x}}y^2/(x^2+y^2) - \hat{\mathbf{y}}x^2/(x^2+y^2) + \hat{\mathbf{z}}4$ at $P_4 = (1,-1,2)$.

Solution: From Table 3-2:

(c)

$$\begin{aligned}
\mathbf{C} &= (\hat{\mathbf{R}}\sin\theta + \hat{\boldsymbol{\theta}}\cos\theta)\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi + (\hat{\mathbf{R}}\cos\theta - \hat{\boldsymbol{\theta}}\sin\theta)\cos\phi\sin\phi \\
&= \hat{\mathbf{R}}\cos\phi(\sin\theta + \cos\theta\sin\phi) + \hat{\boldsymbol{\theta}}\cos\phi(\cos\theta - \sin\theta\sin\phi) - \hat{\boldsymbol{\phi}}\sin\phi, \\
P_3 &= \left(\sqrt{2^2+2^2}, \tan^{-1}(2/2), \pi/4\right) = (2\sqrt{2}, 45^\circ, 45^\circ), \\
\mathbf{C}(P_3) &\approx \hat{\mathbf{R}}0.854 + \hat{\boldsymbol{\theta}}0.146 - \hat{\boldsymbol{\phi}}0.707.
\end{aligned}$$

Problem 3.36 Find the gradient of the following scalar functions:

(a) $T = 3/(x^2 + z^2)$,

(b) $V = xy^2z^4$,

(c) $U = z\cos\phi/(1 + r^2)$,

(d) $W = e^{-R}\sin\theta$,

(e) $S = 4x^2e^{-z} + y^3$,

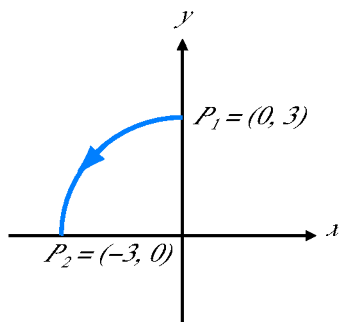
(f) $N = r^2\cos^2\phi$,

(g) $M = R\cos\theta\sin\phi$.

Solution: **(d)** From Eq. (3.83),

$$\nabla W = -\hat{\mathbf{R}}e^{-R}\sin\theta + \hat{\boldsymbol{\theta}}(e^{-R}/R)\cos\theta.$$

Problem 3.41 Evaluate the line integral of $\mathbf{E} = \hat{\mathbf{x}}x - \hat{\mathbf{y}}y$ along the segment P_1 to P_2 of the circular path shown in the figure.



Solution: We need to calculate:

$$\int_{P_1}^{P_2} \mathbf{E} \cdot d\boldsymbol{\ell}.$$

Since the path is along the perimeter of a circle, it is best to use cylindrical coordinates, which requires expressing both $\mathbf{E}$ and $d\boldsymbol{\ell}$ in cylindrical coordinates. Using Table 3-2,

$$\begin{aligned}\mathbf{E} = \hat{\mathbf{x}}x - \hat{\mathbf{y}}y &= (\hat{\mathbf{r}}\cos\phi - \hat{\boldsymbol{\phi}}\sin\phi)r\cos\phi - (\hat{\mathbf{r}}\sin\phi + \hat{\boldsymbol{\phi}}\cos\phi)r\sin\phi \\ &= \hat{\mathbf{r}}\, r(\cos^2\phi - \sin^2\phi) - \hat{\boldsymbol{\phi}}2r\sin\phi\cos\phi\end{aligned}$$

The designated path is along the ϕ-direction at a constant $r = 3$. From Table 3-1, the applicable component of $d\boldsymbol{\ell}$ is:

$$d\boldsymbol{\ell} = \hat{\boldsymbol{\phi}}\, r\, d\phi.$$

Hence,

$$\begin{aligned}\int_{P_1}^{P_2} \mathbf{E} \cdot d\boldsymbol{\ell} &= \int_{\phi=90^\circ}^{\phi=180^\circ} \left[\hat{\mathbf{r}}\, r(\cos^2\phi - \sin^2\phi) - \hat{\boldsymbol{\phi}}\, 2r\sin\phi\cos\phi\right] \cdot \hat{\boldsymbol{\phi}}\, r\, d\phi \Big|_{r=3} \\ &= \int_{90^\circ}^{180^\circ} -2r^2 \sin\phi\cos\phi\, d\phi \Big|_{r=3} \\ &= -2r^2 \left. \left. \frac{\sin^2\phi}{2} \right|_{\phi=90^\circ}^{180^\circ} \right|_{r=3} = 9.\end{aligned}$$

Problem 3.50 For the vector field $\mathbf{E} = \hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2 + 2y^2)$, calculate

(a) $\oint_C \mathbf{E} \cdot d\mathbf{l}$ around the triangular contour shown in Fig. P3.50(a), and

(b) $\int_S (\nabla \times \mathbf{E}) \cdot d\mathbf{s}$ over the area of the triangle.

Solution: In addition to the independent condition that $z = 0$, the three lines of the triangle are represented by the equations $y = 0$, $x = 1$, and $y = x$, respectively.

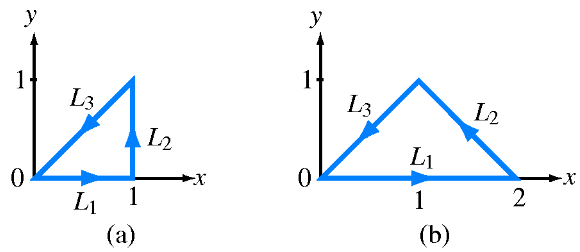


Figure P3.50: Contours for (a) Problem 3.50 and (b) Problem 3.51.

(a)

$$\oint \mathbf{E} \cdot d\mathbf{l} = L_1 + L_2 + L_3,$$

$$L_1 = \int (\hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2 + 2y^2)) \cdot (\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz)$$

$$= \int_{x=0}^{1} (xy)|_{y=0,z=0}\,dx - \int_{y=0}^{0} \left(x^2 + 2y^2\right)\big|_{z=0}\,dy + \int_{z=0}^{0} (0)|_{y=0}\,dz = 0,$$

$$L_2 = \int (\hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2 + 2y^2)) \cdot (\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz)$$

$$= \int_{x=1}^{1} (xy)|_{z=0}\,dx - \int_{y=0}^{1} \left(x^2 + 2y^2\right)\big|_{x=1,z=0}\,dy + \int_{z=0}^{0} (0)|_{x=1}\,dz$$

$$= 0 - \left(y + \frac{2y^3}{3}\right)\bigg|_{y=0}^{1} + 0 = \frac{-5}{3},$$

$$L_3 = \int (\hat{\mathbf{x}}xy - \hat{\mathbf{y}}(x^2 + 2y^2)) \cdot (\hat{\mathbf{x}}\,dx + \hat{\mathbf{y}}\,dy + \hat{\mathbf{z}}\,dz)$$

$$= \int_{x=1}^{0} (xy)|_{y=x,\ z=0}\,dx - \int_{y=1}^{0} \left(x^2 + 2y^2\right)\big|_{x=y,\ z=0}\,dy + \int_{z=0}^{0} (0)|_{y=x}\,dz$$

$$= \left(\frac{x^3}{3}\right)\bigg|_{x=1}^{0} - \left(y^3\right)\big|_{y=1}^{0} + 0 = \frac{2}{3}.$$

Therefore,

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0 - \frac{5}{3} + \frac{2}{3} = -1.$$

(b) From Eq. (3.105), $\nabla\times\mathbf{E} = -\hat{\mathbf{z}}3x$, so that

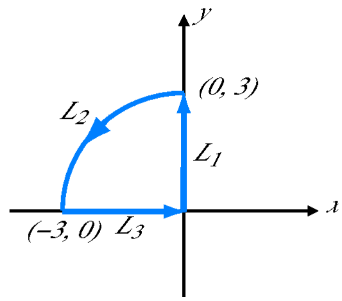
$$\iint \nabla\times\mathbf{E}\cdot d\mathbf{s} = \int_{x=0}^{1}\int_{y=0}^{x} \left((-\hat{\mathbf{z}}3x)\cdot(\hat{\mathbf{z}}\,dy\,dx)\right)\Big|_{z=0}$$

$$= -\int_{x=0}^{1}\int_{y=0}^{x} 3x\,dy\,dx = -\int_{x=0}^{1} 3x(x-0)\,dx = -\left(x^3\right)\Big|_0^1 = -1.$$

Problem 3.55 Verify Stokes's theorem for the vector field $\mathbf{B} = (\hat{\mathbf{r}} \cos\phi + \hat{\boldsymbol{\phi}} \sin\phi)$ by evaluating:

(a) $\oint_C \mathbf{B} \cdot d\boldsymbol{\ell}$ over the path comprising a quarter section of a circle, as shown in Fig. P3.55, and

(b) $\int_S (\nabla \times \mathbf{B}) \cdot d\mathbf{s}$ over the surface of the quarter section.



Solution:

(a)

$$\oint_C \mathbf{B} \cdot d\boldsymbol{\ell} = \int_{L_1} \mathbf{B} \cdot d\boldsymbol{\ell} + \int_{L_2} \mathbf{B} \cdot d\boldsymbol{\ell} + \int_{L_3} \mathbf{B} \cdot d\boldsymbol{\ell}$$

Given the shape of the path, it is best to use cylindrical coordinates. $\mathbf{B}$ is already expressed in cylindrical coordinates, and we need to choose $d\boldsymbol{\ell}$ in cylindrical coordinates:

$$d\boldsymbol{\ell} = \hat{\mathbf{r}}\; dr + \hat{\boldsymbol{\phi}} r\, d\phi + \hat{\mathbf{z}}\, dz.$$

Along path L_1, $d\phi = 0$ and $dz = 0$. Hence, $d\boldsymbol{\ell} = \hat{\mathbf{r}}\; dr$ and

$$\begin{aligned}\int_{L_1} \mathbf{B} \cdot d\boldsymbol{\ell} &= \int_{r=0}^{r=3} (\hat{\mathbf{r}} \cos\phi + \hat{\boldsymbol{\phi}} \sin\phi) \cdot \hat{\mathbf{r}}\; dr \Big|_{\phi=90^\circ} \\ &= \int_{r=0}^{3} \cos\phi\; dr \Big|_{\phi=90^\circ} = r\cos\phi|_{r=0}^{3}\Big|_{\phi=90^\circ} = 0.\end{aligned}$$

Along L_2, $dr = dz = 0$. Hence, $d\boldsymbol{\ell} = \hat{\boldsymbol{\phi}} r\, d\phi$ and

$$\begin{aligned}\int_{L_2} \mathbf{B} \cdot d\boldsymbol{\ell} &= \int_{\phi=90^\circ}^{180^\circ} (\hat{\mathbf{r}} \cos\phi + \hat{\boldsymbol{\phi}} \sin\phi) \cdot \hat{\boldsymbol{\phi}} r\, d\phi \Big|_{r=3} \\ &= -3\cos\phi|_{90^\circ}^{180^\circ} = 3.\end{aligned}$$

Along L_3, $dz = 0$ and $d\phi = 0$. Hence, $d\boldsymbol{\ell} = \hat{\mathbf{r}}\; dr$ and

$$\begin{aligned}\int_{L_3} \mathbf{B} \cdot d\boldsymbol{\ell} &= \int_{r=3}^{0} (\hat{\mathbf{r}} \cos\phi + \hat{\boldsymbol{\phi}} \sin\phi) \cdot \hat{\mathbf{r}}\; dr \Big|_{\phi=180^\circ} \\ &= \int_{r=3}^{0} \cos\phi\; dr|_{\phi=180^\circ} = -r|_3^0 = 3.\end{aligned}$$

Hence,

$$\oint_C \mathbf{B} \cdot d\boldsymbol{\ell} = 0 + 3 + 3 = 6.$$

(b)

$$\begin{aligned}
\nabla \times \mathbf{B} &= \hat{\mathbf{z}} \frac{1}{r} \left(\frac{\partial}{\partial r} \left(r B_\phi - \frac{\partial B_r}{\partial \phi} \right) \right) \\
&= \hat{\mathbf{z}} \frac{1}{r} \left(\frac{\partial}{\partial r} (r \sin\phi) - \frac{\partial}{\partial \phi} (\cos\phi) \right) \\
&= \hat{\mathbf{z}} \frac{1}{r} (\sin\phi + \sin\phi) = \hat{\mathbf{z}} \frac{2}{r} \sin\phi . \\
\int_S (\nabla \times \mathbf{B}) \cdot d\mathbf{s} &= \int_{r=0}^{3} \int_{\phi=90^\circ}^{180^\circ} \left(\hat{\mathbf{z}} \frac{2}{r} \sin\phi \right) \cdot \hat{\mathbf{z}}\, r \, dr \, d\phi \\
&= \ -2r\big|_{r=0}^{3} \cos\phi \Big|_{\phi=90^\circ}^{180^\circ} = 6.
\end{aligned}$$

Hence, Stokes's theorem is verified.

Problem 3.57 Find the Laplacian of the following scalar functions:

(a) $V = 4xy^2z^3$,

(b) $V = xy + yz + zx$,

(c) $V = 3/(x^2 + y^2)$,

(d) $V = 5e^{-r}\cos\phi$,

(e) $V = 10e^{-R}\sin\theta$.

Solution:

(a) From Eq. (3.110), $\nabla^2(4xy^2z^3) = 8xz^3 + 24xy^2z$.

(b) $\nabla^2(xy + yz + zx) = 0$.

(c) From the inside back cover of the book,

$$\nabla^2\left(\frac{3}{x^2+y^2}\right) = \nabla^2(3r^{-2}) = 12r^{-4} = \frac{12}{(x^2+y^2)^2}.$$

(d)

$$\nabla^2(5e^{-r}\cos\phi) = 5e^{-r}\cos\phi\left(1 - \frac{1}{r} - \frac{1}{r^2}\right).$$

(e)

$$\nabla^2(10e^{-R}\sin\theta) = 10e^{-R}\left(\sin\theta\left(1 - \frac{2}{R}\right) + \frac{\cos^2\theta - \sin^2\theta}{R^2\sin\theta}\right).$$

Chapter 4 Solved Problems

Problem 4-5

Problem 4-9

Problem 4-12

Problem 4-29

Problem 4-37

Problem 4-47

Problem 4-57

Problem 4-60

Problem 4-62

Problem 4.5 Find the total charge on a circular disk defined by $r \leq a$ and $z = 0$ if:

(a) $\rho_s = \rho_{s0} \cos\phi$ (C/m^2)

(b) $\rho_s = \rho_{s0} \sin^2\phi$ (C/m^2)

(c) $\rho_s = \rho_{s0} e^{-r}$ (C/m^2)

(d) $\rho_s = \rho_{s0} e^{-r} \sin^2\phi$ (C/m^2)

where ρ_{s0} is a constant.

Solution:

(c)

$$
\begin{aligned}
Q = \int_{r=0}^{a} \int_{\phi=0}^{2\pi} \rho_{s0} e^{-r} r \, dr \, d\phi &= 2\pi\rho_{s0} \int_{0}^{a} r e^{-r} \, dr \\
&= 2\pi\rho_{s0} \left[-re^{-r} - e^{-r} \right]_0^a \\
&= 2\pi\rho_{s0} [1 - e^{-a}(1+a)].
\end{aligned}
$$

Problem 4.9 A circular beam of charge of radius a consists of electrons moving with a constant speed u along the $+z$-direction. The beam's axis is coincident with the z-axis and the electron charge density is given by

$$\rho_{\rm v} = -cr^2 \quad \text{(c/m}^3\text{)}$$

where c is a constant and r is the radial distance from the axis of the beam.

(a) Determine the charge density per unit length.

(b) Determine the current crossing the z-plane.

Solution:

(a)

$$\begin{aligned} \rho_l &= \int \rho_{\rm v}\, ds \\ &= \int_{r=0}^{a} \int_{\phi=0}^{2\pi} -cr^2 \cdot r\, dr\, d\phi = -2\pi c \left. \frac{r^4}{4} \right|_0^a = -\frac{\pi c a^4}{2} \quad \text{(C/m)}. \end{aligned}$$

(b)

$$\begin{aligned} \mathbf{J} &= \rho_{\rm v} \mathbf{u} = -\hat{\mathbf{z}} c r^2 u \quad \text{(A/m}^2\text{)} \\ I &= \int \mathbf{J} \cdot d\mathbf{s} \\ &= \int_{r=0}^{a} \int_{\phi=0}^{2\pi} (-\hat{\mathbf{z}} c u r^2) \cdot \hat{\mathbf{z}}\, r\, dr\, d\phi \\ &= -2\pi c u \int_0^a r^3\, dr = -\frac{\pi c u a^4}{2} = \rho_l u. \quad \text{(A)}. \end{aligned}$$

Problem 4.12 Three point charges, each with $q = 3$ nC, are located at the corners of a triangle in the x–y plane, with one corner at the origin, another at $(2\text{ cm}, 0, 0)$, and the third at $(0, 2\text{ cm}, 0)$. Find the force acting on the charge located at the origin.

Solution: Use Eq. (4.19) to determine the electric field at the origin due to the other two point charges [Fig. P4.12]:

$$\mathbf{E} = \frac{1}{4\pi\varepsilon}\left[\frac{3\text{ nC}\,(-\hat{\mathbf{x}}0.02)}{(0.02)^3}\right] + \frac{3\text{ nC}\,(-\hat{\mathbf{y}}0.02)}{(0.02)^3} = -67.4(\hat{\mathbf{x}}+\hat{\mathbf{y}})\text{ (kV/m) at }\mathbf{R}=0.$$

Employ Eq. (4.14) to find the force $\mathbf{F} = q\mathbf{E} = -202.2(\hat{\mathbf{x}}+\hat{\mathbf{y}})$ (μN).

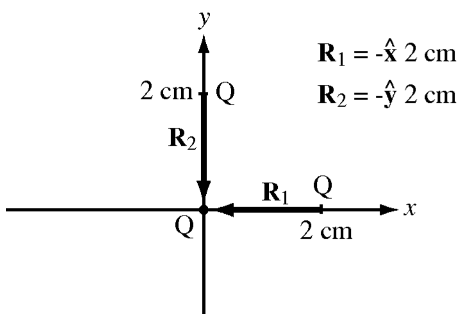


Figure P4.12: Locations of charges in Problem 4.12.

Problem 4.29 A spherical shell with outer radius b surrounds a charge-free cavity of radius $a < b$ (Fig. P4.29). If the shell contains a charge density given by

$$\rho_v = -\frac{\rho_{v0}}{R^2}, \qquad a \le R \le b,$$

where ρ_{v0} is a positive constant, determine $\mathbf{D}$ in all regions.

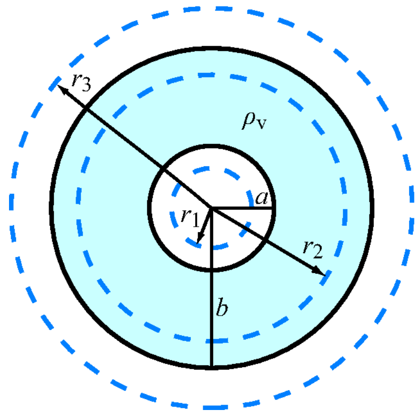


Figure P4.29: Problem 4.29.

Solution: Symmetry dictates that $\mathbf{D}$ is radially oriented. Thus,

$$\mathbf{D} = \hat{\mathbf{R}} D_R.$$

At any R, Gauss's law gives

$$\oint_S \mathbf{D} \cdot d\mathbf{s} = Q$$
$$\int_S \hat{\mathbf{R}} D_R \cdot \hat{\mathbf{R}}\, ds = Q$$
$$4\pi R^2 D_R = Q$$
$$D_R = \frac{Q}{4\pi R^2}.$$

(a) For $R < a$, no charge is contained in the cavity. Hence, $Q = 0$, and

$$D_R = 0, \qquad R \le a.$$

(b) For $a \le R \le b$,

$$Q = \int_{R=a}^{R} \rho_v \, dV = \int_{R=a}^{R} -\frac{\rho_{v0}}{R^2} \cdot 4\pi R^2 \, dR$$
$$= -4\pi\rho_{v0}(R - a).$$

Hence,

$$D_R = -\frac{\rho_{v0}(R-a)}{R^2}\,, \qquad a \le R \le b.$$

(c) For $R \ge b$,

$$Q = \int_{R=a}^{b} \rho_v \, dV = -4\pi\rho_{v0}(b-a)$$

$$D_R = -\frac{\rho_{v0}(b-a)}{R^2}\,, \qquad R \ge b.$$

Problem 4.37 Two infinite lines of charge, both parallel to the z-axis, lie in the x–z plane, one with density ρ_ℓ and located at $x = a$ and the other with density $-\rho_\ell$ and located at $x = -a$. Obtain an expression for the electric potential $V(x,y)$ at a point $P = (x,y)$ relative to the potential at the origin.

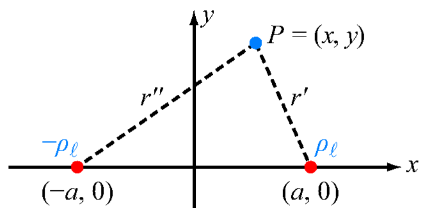


Figure P4.37: Problem 4.37.

Solution: According to the result of Problem 4.33, the electric potential difference between a point at a distance r_1 and another at a distance r_2 from a line charge of density ρ_l is

$$V = \frac{\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{r_2}{r_1}\right).$$

Applying this result to the line charge at $x = a$, which is at a distance a from the origin:

$$\begin{aligned} V' &= \frac{\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{a}{r'}\right) \qquad (r_2 = a \text{ and } r_1 = r') \\ &= \frac{\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{a}{\sqrt{(x-a)^2 + y^2}}\right). \end{aligned}$$

Similarly, for the negative line charge at $x = -a$,

$$\begin{aligned} V'' &= \frac{-\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{a}{r''}\right) \qquad (r_2 = a \text{ and } r_1 = r') \\ &= \frac{-\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{a}{\sqrt{(x+a)^2 + y^2}}\right). \end{aligned}$$

The potential due to both lines is

$$V = V' + V'' = \frac{\rho_l}{2\pi\varepsilon_0} \left[\ln\left(\frac{a}{\sqrt{(x-a)^2 + y^2}}\right) - \ln\left(\frac{a}{\sqrt{(x+a)^2 + y^2}}\right) \right].$$

At the origin, $V = 0$, as it should be since the origin is the reference point. The potential is also zero along all points on the y-axis ($x = 0$).

Problem 4.47 A cylinder-shaped carbon resistor is 8 cm in length and its circular cross section has a diameter $d = 1$ mm.

(a) Determine the resistance R.

(b) To reduce its resistance by 40%, the carbon resistor is coated with a layer of copper of thickness t. Use the result of Problem 4.44 to determine t.

Solution: According to the result of Problem 4.33, the electric potential difference between a point at a distance r_1 and another at a distance r_2 from a line charge of density ρ_l is

$$V = \frac{\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{r_2}{r_1}\right).$$

Applying this result to the line charge at $x = a$, which is at a distance a from the origin:

$$V' = \frac{\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{a}{r'}\right) \qquad (r_2 = a \text{ and } r_1 = r')$$
$$= \frac{\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{a}{\sqrt{(x-a)^2 + y^2}}\right).$$

Similarly, for the negative line charge at $x = -a$,

$$V'' = \frac{-\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{a}{r''}\right) \qquad (r_2 = a \text{ and } r_1 = r')$$
$$= \frac{-\rho_l}{2\pi\varepsilon_0} \ln\left(\frac{a}{\sqrt{(x+a)^2 + y^2}}\right).$$

The potential due to both lines is

$$V = V' + V'' = \frac{\rho_l}{2\pi\varepsilon_0}\left[\ln\left(\frac{a}{\sqrt{(x-a)^2 + y^2}}\right) - \ln\left(\frac{a}{\sqrt{(x+a)^2 + y^2}}\right)\right].$$

At the origin, $V = 0$, as it should be since the origin is the reference point. The potential is also zero along all points on the y-axis ($x = 0$).

Problem 4.57 Use the result of Problem 4.56 to determine the capacitance for each of the following configurations:

(a) Conducting plates are on top and bottom faces of the rectangular structure in Fig. P4.57(a).

(b) Conducting plates are on front and back faces of the structure in Fig. P4.57(a).

(c) Conducting plates are on top and bottom faces of the cylindrical structure in Fig. P4.57(b).

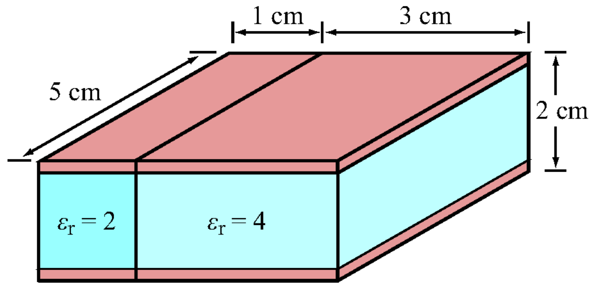
Solution:

(a) The two capacitors share the same voltage; hence they are in parallel.

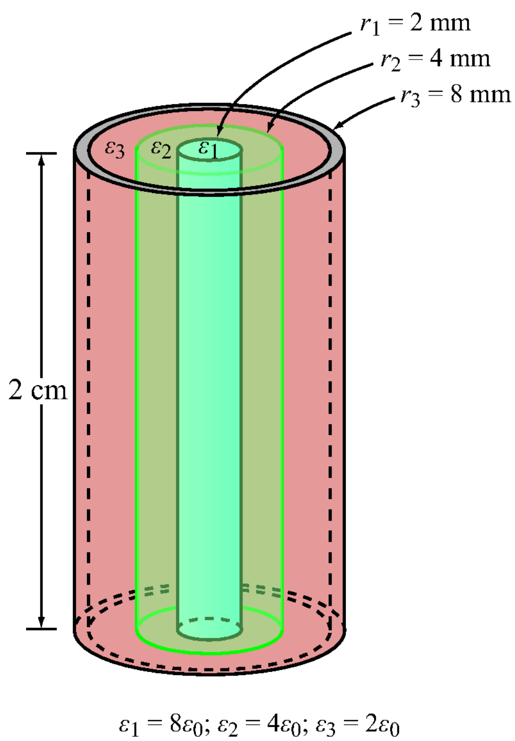
$$C_1 = \varepsilon_1 \frac{A_1}{d} = 2\varepsilon_0 \frac{(5\times 1)\times 10^{-4}}{2\times 10^{-2}} = 5\varepsilon_0 \times 10^{-2},$$

$$C_2 = \varepsilon_2 \frac{A_2}{d} = 4\varepsilon_0 \frac{(5\times 3)\times 10^{-4}}{2\times 10^{-2}} = 30\varepsilon_0 \times 10^{-2},$$

$$C = C_1 + C_2 = (5\varepsilon_0 + 30\varepsilon_0)\times 10^{-2} = 0.35\varepsilon_0 = 3.1\times 10^{-12} \text{ F}.$$



(a)



$\varepsilon_1 = 8\varepsilon_0$; $\varepsilon_2 = 4\varepsilon_0$; $\varepsilon_3 = 2\varepsilon_0$

(b)

Figure P4.57: Dielectric sections for Problems 4.57 and 4.59.

Problem 4.60 A coaxial capacitor consists of two concentric, conducting, cylindrical surfaces, one of radius a and another of radius b, as shown in Fig. P4.60. The insulating layer separating the two conducting surfaces is divided equally into two semi-cylindrical sections, one filled with dielectric ε_1 and the other filled with dielectric ε_2.

(a) Develop an expression for C in terms of the length l and the given quantities.

(b) Evaluate the value of C for $a = 2$ mm, $b = 6$ mm, $\varepsilon_{r_1} = 2$, $\varepsilon_{r_2} = 4$, and $l = 4$ cm.

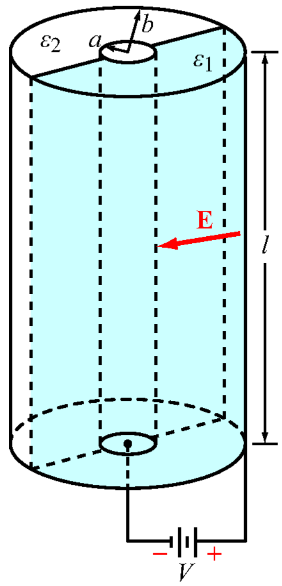


Figure P4.60: Problem 4.60.

Solution:

(a) For the indicated voltage polarity, the **E** field inside the capacitor exists in only the dielectric materials and points radially inward. Let $\mathbf{E}_1$ be the field in dielectric ε_1 and $\mathbf{E}_2$ be the field in dielectric ε_2. At the interface between the two dielectric sections, $\mathbf{E}_1$ is parallel to $\mathbf{E}_2$ and both are tangential to the interface. Since boundary conditions require that the tangential components of $\mathbf{E}_1$ and $\mathbf{E}_2$ be the same, it follows that:

$$\mathbf{E}_1 = \mathbf{E}_2 = -\hat{\mathbf{r}}E.$$

At $r = a$ (surface of inner conductor), in medium 1, the boundary condition on **D**, as stated by (4.101), leads to

$$\mathbf{D}_1 = \varepsilon_1 \mathbf{E}_1 = \hat{\mathbf{n}}\rho_{s1}$$
$$-\hat{\mathbf{r}}\varepsilon_1 E = \hat{\mathbf{r}}\rho_{s1}$$

or

$$\rho_{s1} = -\varepsilon_1 E.$$

Similarly, in medium 2

$$\rho_{s2} = -\varepsilon_2 E.$$

Thus, the **E** fields will be the same in the two dielectrics, but the charge densities will be different along the two sides of the inner conducting cylinder.

Since the same voltage applies for the two sections of the capacitor, we can treat them as two capacitors in parallel. For the capacitor half that includes dielectric ε_1, we can apply the results of Eqs. (4.114)–(4.116), but we have to keep in mind that Q is now the charge on only one half of the inner cylinder. Hence,

$$C_1 = \frac{\pi\varepsilon_1 l}{\ln(b/a)} \ .$$

Similarly,

$$C_2 = \frac{\pi\varepsilon_2 l}{\ln(b/a)} \ ,$$

and

$$C = C_1 + C_2 = \frac{\pi l(\varepsilon_1 + \varepsilon_2)}{\ln(b/a)} \ .$$

(b)

$$C = \frac{\pi \times 4 \times 10^{-2}(2+4) \times 8.85 \times 10^{-12}}{\ln(6/2)}$$
$$= 6.07 \text{ pF.}$$

Problem 4.62 Conducting wires above a conducting plane carry currents I_1 and I_2 in the directions shown in Fig. P4.62. Keeping in mind that the direction of a current is defined in terms of the movement of positive charges, what are the directions of the image currents corresponding to I_1 and I_2?

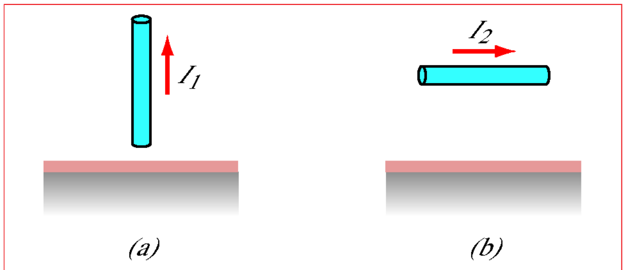


Figure P4.62: Currents above a conducting plane (Problem 4.62).

Solution:

(a) In the image current, movement of negative charges downward = movement of positive charges upward. Hence, image of I_1 is same as I_1.

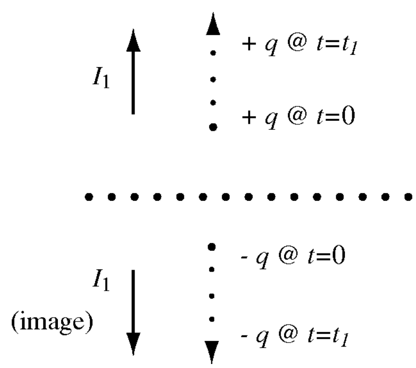


Figure P4.62: (a) Solution for part (a).

(b) In the image current, movement of negative charges to right = movement of positive charges to left.

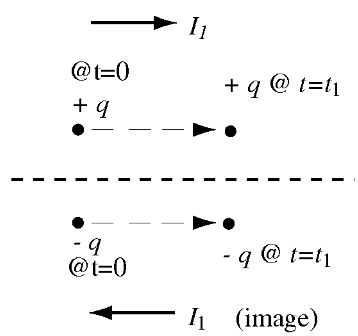


Figure P4.62: (b) Solution for part (b).

Chapter 5 Solved Problems

Problem 5-5

Problem 5-35

Problem 5.5 In a cylindrical coordinate system, a 2-m-long straight wire carrying a current of 5 A in the positive z-direction is located at $r = 4$ cm, $\phi = \pi/2$, and -1 m $\leq z \leq 1$ m.

(a) If $\mathbf{B} = \hat{\mathbf{r}}\, 0.2\cos\phi$ (T), what is the magnetic force acting on the wire?

(b) How much work is required to rotate the wire once about the z-axis in the negative ϕ-direction (while maintaining $r = 4$ cm)?

(c) At what angle ϕ is the force a maximum?

Solution:

(a)

$$\begin{aligned}\mathbf{F} &= I\boldsymbol{\ell} \times \mathbf{B} \\ &= 5\hat{\mathbf{z}}\, 2 \times [\hat{\mathbf{r}}\, 0.2\cos\phi] \\ &= \hat{\boldsymbol{\phi}}\, 2\cos\phi.\end{aligned}$$

At $\phi = \pi/2$, $\hat{\boldsymbol{\phi}} = -\hat{\mathbf{x}}$. Hence,

$$\mathbf{F} = -\hat{\mathbf{x}}\, 2\cos(\pi/2) = 0.$$

(b)

$$\begin{aligned}W &= \int_{\phi=0}^{2\pi} \mathbf{F} \cdot d\mathbf{l} = \int_0^{2\pi} \hat{\boldsymbol{\phi}}\, [2\cos\phi] \cdot (-\hat{\boldsymbol{\phi}})\, r\, d\phi \bigg|_{r=4 \text{ cm}} \\ &= -2r \int_0^{2\pi} \cos\phi\, d\phi \bigg|_{r=4 \text{ cm}} = -8 \times 10^{-2}\, [\sin\phi]_0^{2\pi} = 0.\end{aligned}$$

The force is in the $+\hat{\boldsymbol{\phi}}$-direction, which means that rotating it in the $-\hat{\boldsymbol{\phi}}$-direction would require work. However, the force varies as $\cos\phi$, which means it is positive when $-\pi/2 \leq \phi \leq \pi/2$ and negative over the second half of the circle. Thus, work is provided by the force between $\phi = \pi/2$ and $\phi = -\pi/2$ (when rotated in the $-\hat{\boldsymbol{\phi}}$-direction), and work is supplied for the second half of the rotation, resulting in a net work of zero.

(c) The force $\mathbf{F}$ is maximum when $\cos\phi = 1$, or $\phi = 0$.

Problem 5.35 The plane boundary defined by $z = 0$ separates air from a block of iron. If $\mathbf{B}_1 = \hat{\mathbf{x}}4 - \hat{\mathbf{y}}6 + \hat{\mathbf{z}}8$ in air ($z \geq 0$), find $\mathbf{B}_2$ in iron ($z \leq 0$), given that $\mu = 5000\mu_0$ for iron.

Solution: From Eq. (5.2),

$$\mathbf{H}_1 = \frac{\mathbf{B}_1}{\mu_1} = \frac{1}{\mu_1}(\hat{\mathbf{x}}4 - \hat{\mathbf{y}}6 + \hat{\mathbf{z}}8).$$

The z component is the normal component to the boundary at $z = 0$. Therefore, from Eq. (5.79), $B_{2z} = B_{1z} = 8$ while, from Eq. (5.85),

$$H_{2x} = H_{1x} = \frac{1}{\mu_1}4, \qquad H_{2y} = H_{1y} = -\frac{1}{\mu_1}6,$$

or

$$B_{2x} = \mu_2 H_{2x} = \frac{\mu_2}{\mu_1}4, \qquad B_{2y} = \mu_2 H_{2y} = -\frac{\mu_2}{\mu_1}6,$$

where $\mu_2/\mu_1 = \mu_r = 5000$. Therefore,

$$\mathbf{B}_2 = \hat{\mathbf{x}}20000 - \hat{\mathbf{y}}30000 + \hat{\mathbf{z}}8.$$

Chapter 6 Solved Problems

Problem 6-8

Problem 6.8 The transformer shown in Fig. P6.8 consists of a long wire coincident with the z-axis carrying a current $I = I_0 \cos \omega t$, coupling magnetic energy to a toroidal coil situated in the x–y plane and centered at the origin. The toroidal core uses iron material with relative permeability μ_r, around which 100 turns of a tightly wound coil serves to induce a voltage V_{emf}, as shown in the figure.

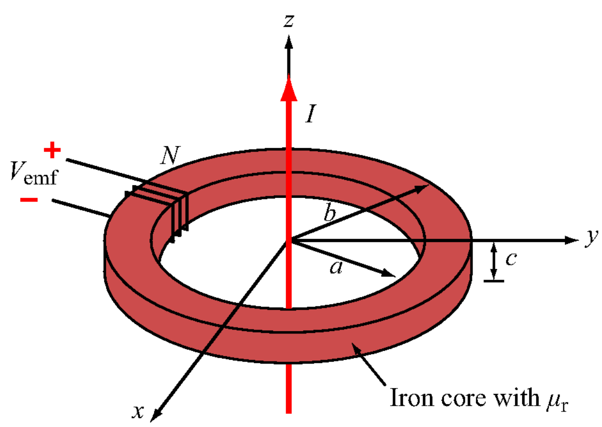


Figure P6.8: Problem 6.8.

(a) Develop an expression for V_{emf}.

(b) Calculate V_{emf} for $f = 60$ Hz, $\mu_r = 4000$, $a = 5$ cm, $b = 6$ cm, $c = 2$ cm, and $I_0 = 50$ A.

Solution:

(a) We start by calculating the magnetic flux through the coil, noting that r, the distance from the wire varies from a to b

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} = \int_a^b \hat{\mathbf{x}} \frac{\mu I}{2\pi r} \cdot \hat{\mathbf{x}} c \, dr = \frac{\mu c I}{2\pi} \ln\left(\frac{b}{a}\right)$$

$$V_{emf} = -N \frac{d\Phi}{dt} = -\frac{\mu c N}{2\pi} \ln\left(\frac{b}{a}\right) \frac{dI}{dt}$$

$$= \frac{\mu c N \omega I_0}{2\pi} \ln\left(\frac{b}{a}\right) \sin \omega t \quad \text{(V)}.$$

(b)

$$V_{emf} = \frac{4000 \times 4\pi \times 10^{-7} \times 2 \times 10^{-2} \times 100 \times 2\pi \times 60 \times 50 \ln(6/5)}{2\pi} \sin 377t$$

$$= 5.5 \sin 377t \quad \text{(V)}.$$

Chapter 7 Solved Problems

Problem 7-7

Problem 7-23

Problem 7-28

Problem 7-33

Problem 7-36

Problem 7.7 A 60-MHz plane wave traveling in the $-x$-direction in dry soil with relative permittivity $\varepsilon_r = 4$ has an electric field polarized along the z-direction. Assuming dry soil to be approximately lossless, and given that the magnetic field has a peak value of 10 (mA/m) and that its value was measured to be 7 (mA/m) at $t = 0$ and $x = -0.75$ m, develop complete expressions for the wave's electric and magnetic fields.

Solution: For $f = 60$ MHz $= 6 \times 10^7$ Hz, $\varepsilon_r = 4$, $\mu_r = 1$,

$$k = \frac{\omega}{c}\sqrt{\varepsilon_r} = \frac{2\pi \times 6 \times 10^7}{3 \times 10^8}\sqrt{4} = 0.8\pi \quad \text{(rad/m)}.$$

Given that $\mathbf{E}$ points along $\hat{\mathbf{z}}$ and wave travel is along $-\hat{\mathbf{x}}$, we can write

$$\mathbf{E}(x,t) = \hat{\mathbf{z}} E_0 \cos(2\pi \times 60 \times 10^6 t + 0.8\pi x + \phi_0) \quad \text{(V/m)}$$

where E_0 and ϕ_0 are unknown constants at this time. The intrinsic impedance of the medium is

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{2} = 60\pi \quad (\Omega).$$

With $\mathbf{E}$ along $\hat{\mathbf{z}}$ and $\hat{\mathbf{k}}$ along $-\hat{\mathbf{x}}$, (7.39) gives

$$\mathbf{H} = \frac{1}{\eta}\hat{\mathbf{k}} \times \mathbf{E}$$

or

$$\mathbf{H}(x,t) = \hat{\mathbf{y}}\frac{E_0}{\eta}\cos(1.2\pi \times 10^8 t + 0.8\pi x + \phi_0) \quad \text{(A/m)}.$$

Hence,

$$\frac{E_0}{\eta} = 10 \quad \text{(mA/m)}$$

$$E_0 = 10 \times 60\pi \times 10^{-3} = 0.6\pi \quad \text{(V/m)}.$$

Also,

$$H(-0.75\text{ m}, 0) = 7 \times 10^{-3} = 10\cos(-0.8\pi \times 0.75 + \phi_0) \times 10^{-3}$$

which leads to $\phi_0 = 153.6°$.

Hence,

$$\mathbf{E}(x,t) = \hat{\mathbf{z}}\, 0.6\pi \cos(1.2\pi \times 10^8 t + 0.8\pi x + 153.6°) \quad \text{(V/m)}.$$

$$\mathbf{H}(x,t) = \hat{\mathbf{y}}\, 10\cos(1.2\pi \times 10^8 t + 0.8\pi x + 153.6°) \quad \text{(mA/m)}.$$

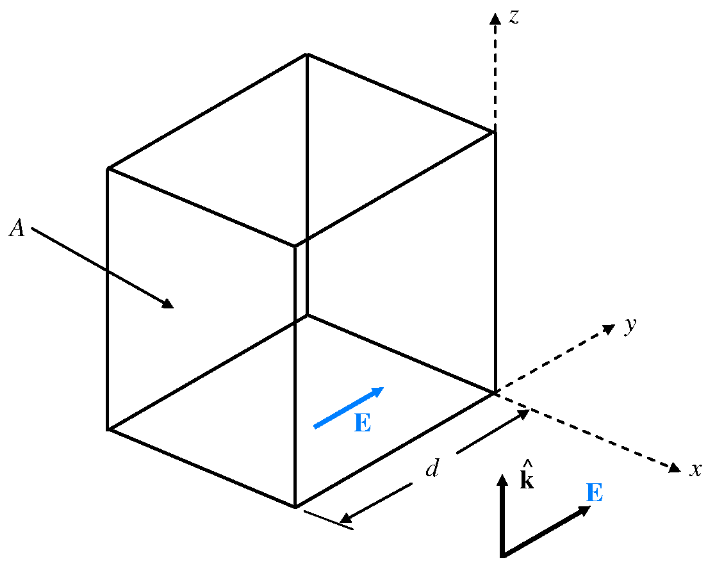
Problem 7.23 At 2 GHz, the conductivity of meat is on the order of 1 (S/m). When a material is placed inside a microwave oven and the field is activated, the presence of the electromagnetic fields in the conducting material causes energy dissipation in the material in the form of heat.

(a) Develop an expression for the time-average power per mm^3 dissipated in a material of conductivity σ if the peak electric field in the material is E_0.

(b) Evaluate the result for an electric field $E_0 = 4 \times 10^4$ (V/m).

Solution:

(a) Let us consider a small volume of the material in the shape of a box of length d and cross sectional area A. Let us assume the microwave oven creates a wave traveling along the z direction with $\mathbf{E}$ along y, as shown.



Along y, the $\mathbf{E}$ field will create a voltage difference across the length of the box V, where

$$V = Ed.$$

Conduction current through the cross sectional area A is

$$I = JA = \sigma EA.$$

Hence, the instantaneous power is

$$\begin{aligned} P = IV &= \sigma E^2 (Ad) \\ &= \sigma E^2 \mathscr{V}. \end{aligned}$$

where $\mathscr{V} = Ad$ is the small volume under consideration. The power per mm^3 is obtained by setting $\mathscr{V} = (10^{-3})^3$,

$$P' = \frac{P}{10^{-9}} = \sigma E^2 \times 10^{-9} \quad (\text{W/mm}^3).$$

As a time harmonic signal, $E = E_0 \cos\omega t$. The time average dissipated power is

$$P'_{\text{av}} = \left[\frac{1}{T}\int_0^T \sigma E_0^2 \cos^2 \omega t \, dt\right] \times 10^{-9}$$
$$= \frac{1}{2}\sigma E_0^2 \times 10^{-9} \quad (\text{W/mm}^3).$$

(b)

$$P'_{\text{av}} = \frac{1}{2} \times 1 \times (4 \times 10^4)2 \times 10^{-9} = 0.8 \quad (\text{W/mm}^3).$$

Problem 7.28 A wave traveling in a nonmagnetic medium with $\varepsilon_r = 9$ is characterized by an electric field given by

$$\mathbf{E} = [\hat{\mathbf{y}}\,3\cos(\pi \times 10^7 t + kx) - \hat{\mathbf{z}}\,2\cos(\pi \times 10^7 t + kx)] \quad \text{(V/m)}$$

Determine the direction of wave travel and average power density carried by the wave.

Solution:

$$\eta \simeq \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{9}} = 40\pi \quad (\Omega).$$

The wave is traveling in the negative x-direction.

$$\mathbf{S}_{av} = -\hat{\mathbf{x}}\frac{[3^2 + 2^2]}{2\eta} = -\hat{\mathbf{x}}\frac{13}{2 \times 40\pi} = -\hat{\mathbf{x}}0.05 \quad \text{(W/m}^2\text{)}.$$

Problem 7.33 Consider the imaginary rectangular box shown in Fig. P7.33.

(a) Determine the net power flux $P(t)$ entering the box due to a plane wave in air given by

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \cos(\omega t - ky) \quad \text{(V/m)}$$

(b) Determine the net time-average power entering the box.

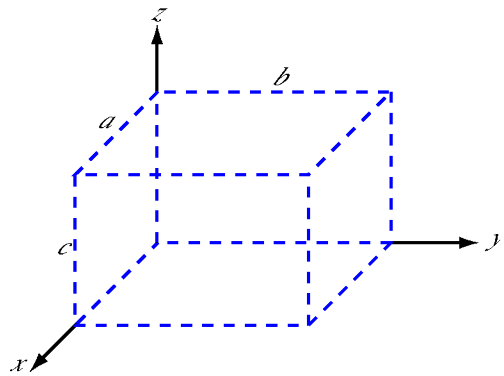


Figure P7.33: Imaginary rectangular box of Problems 7.33 and 7.34.

Solution:

(a)

$$\mathbf{E} = \hat{\mathbf{x}} E_0 \cos(\omega t - ky),$$

$$\mathbf{H} = -\hat{\mathbf{z}} \frac{E_0}{\eta_0} \cos(\omega t - ky).$$

$$\mathbf{S}(t) = \mathbf{E} \times \mathbf{H} = \hat{\mathbf{y}} \frac{E_0^2}{\eta_0} \cos^2(\omega t - ky),$$

$$P(t) = S(t)A|_{y=0} - S(t)A|_{y=b} = \frac{E_0^2}{\eta_0} ac[\cos^2 \omega t - \cos^2(\omega t - kb)].$$

(b)

$$P_{\text{av}} = \frac{1}{T} \int_0^T P(t)\, dt.$$

where $T = 2\pi/\omega$.

$$P_{\text{av}} = \frac{E_0^2 ac}{\eta_0} \left\{ \frac{\omega}{2\pi} \int_0^{2\pi/\omega} [\cos^2 \omega t - \cos^2(\omega t - kb)]\, dt \right\} = 0.$$

Net average energy entering the box is zero, which is as expected since the box is in a lossless medium (air).

Problem 7.36 A team of scientists is designing a radar as a probe for measuring the depth of the ice layer over the antarctic land mass. In order to measure a detectable echo due to the reflection by the ice-rock boundary, the thickness of the ice sheet should not exceed three skin depths. If $\varepsilon_r' = 3$ and $\varepsilon_r'' = 10^{-2}$ for ice and if the maximum anticipated ice thickness in the area under exploration is 1.2 km, what frequency range is useable with the radar?

Solution:

$$3\delta_s = 1.2 \text{ km} = 1200 \text{ m}$$
$$\delta_s = 400 \text{ m}.$$

Hence,

$$\alpha = \frac{1}{\delta_s} = \frac{1}{400} = 2.5 \times 10^{-3} \quad \text{(Np/m)}.$$

Since $\varepsilon''/\varepsilon' \ll 1$, we can use (7.75a) for α:

$$\alpha = \frac{\omega\varepsilon''}{2}\sqrt{\frac{\mu}{\varepsilon'}} = \frac{2\pi f \varepsilon_r'' \varepsilon_0}{2\sqrt{\varepsilon_r'}\sqrt{\varepsilon_0}}\sqrt{\mu_0} = \frac{\pi f \varepsilon_r''}{c\sqrt{\varepsilon_r}} = \frac{\pi f \times 10^{-2}}{3 \times 10^8 \sqrt{3}} = 6f \times 10^{-11} \text{Np/m}.$$

For $\alpha = 2.5 \times 10^{-3} = 6f \times 10^{-11}$,

$$f = 41.6 \text{ MHz}.$$

Since α increases with increasing frequency, the useable frequency range is

$$f \leq 41.6 \text{ MHz}.$$

Chapter 8 Solved Problems

Problem 8-3

Problem 8-14

Problem 8-32

Problem 8-36

Problem 8.3 A plane wave traveling in a medium with $\varepsilon_{r_1} = 9$ is normally incident upon a second medium with $\varepsilon_{r_2} = 4$. Both media are made of nonmagnetic, non-conducting materials. If the magnetic field of the incident plane wave is given by

$$\mathbf{H}^{\mathrm{i}} = \hat{\mathbf{z}}\, 2\cos(2\pi \times 10^9 t - ky) \quad \text{(A/m)}.$$

(a) Obtain time-domain expressions for the electric and magnetic fields in each of the two media.

(b) Determine the average power densities of the incident, reflected, and transmitted waves.

Solution:

(a) In medium 1,

$$\begin{aligned}
u_{\mathrm{p}} &= \frac{c}{\sqrt{\varepsilon_{r_1}}} = \frac{3 \times 10^8}{\sqrt{9}} = 1 \times 10^8 \quad \text{(m/s)},\\
k_1 &= \frac{\omega}{u_{\mathrm{p}}} = \frac{2\pi \times 10^9}{1 \times 10^8} = 20\pi \quad \text{(rad/m)},\\
\mathbf{H}^{\mathrm{i}} &= \hat{\mathbf{z}}\, 2\cos(2\pi \times 10^9 t - 20\pi y) \quad \text{(A/m)},\\
\eta_1 &= \frac{\eta_0}{\sqrt{\varepsilon_{r_1}}} = \frac{377}{3} = 125.67\ \Omega,\\
\eta_2 &= \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = \frac{377}{2} = 188.5\ \Omega,\\
\mathbf{E}^{\mathrm{i}} &= -\hat{\mathbf{x}}\, 2\eta_1 \cos(2\pi \times 10^9 t - 20\pi y)\\
&= -\hat{\mathbf{x}}\, 251.34\cos(2\pi \times 10^9 t - 20\pi y) \quad \text{(V/m)},
\end{aligned}$$

$$\begin{aligned}
\Gamma &= \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{188.5 - 125.67}{188.5 + 125.67} = 0.2,\\
\tau &= 1 + \Gamma = 1.2,\\
\mathbf{E}^{\mathrm{r}} &= -\hat{\mathbf{x}}\, 251.34 \times 0.2\cos(2\pi \times 10^9 t + 20\pi y)\\
&= -\hat{\mathbf{x}}\, 50.27\cos(2\pi \times 10^9 t + 20\pi y) \quad \text{(V/m)},\\
\mathbf{H}^{\mathrm{r}} &= -\hat{\mathbf{z}}\, \frac{50.27}{\eta_1}\cos(2\pi \times 10^9 t + 20\pi y)\\
&= -\hat{\mathbf{z}}\, 0.4\cos(2\pi \times 10^9 t + 20\pi y) \quad \text{(A/m)},
\end{aligned}$$

$$\begin{aligned}
\mathbf{E}_1 &= \mathbf{E}^{\mathrm{i}} + \mathbf{E}^{\mathrm{r}}\\
&= -\hat{\mathbf{x}}\,[25.134\cos(2\pi \times 10^9 t - 20\pi y) + 50.27\cos(2\pi \times 10^9 t + 20\pi y)] \quad \text{(V/m)},\\
\mathbf{H}_1 &= \mathbf{H}^{\mathrm{i}} + \mathbf{H}^{\mathrm{r}} = \hat{\mathbf{z}}\,[2\cos(2\pi \times 10^9 t - 20\pi y) - 0.4\cos(2\pi \times 10^9 t + 20\pi y)] \quad \text{(A/m)}.
\end{aligned}$$

In medium 2,

$$k_2 = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\, k_1 = \sqrt{\frac{4}{9}} \times 20\pi = \frac{40\pi}{3} \quad \text{(rad/m)},$$

$$
\begin{aligned}
\mathbf{E}_2 = \mathbf{E}^{\mathrm{t}} &= -\hat{\mathbf{x}}\,251.34\,\tau\cos\left(2\pi\times 10^9 t - \frac{40\pi y}{3}\right) \\
&= -\hat{\mathbf{x}}\,301.61\cos\left(2\pi\times 10^9 t - \frac{40\pi y}{3}\right) \quad \text{(V/m)}, \\
\mathbf{H}_2 = \mathbf{H}^{\mathrm{t}} &= \hat{\mathbf{z}}\,\frac{301.61}{\eta_2}\cos\left(2\pi\times 10^9 t - \frac{40\pi y}{3}\right) \\
&= \hat{\mathbf{z}}\,1.6\cos\left(2\pi\times 10^9 t - \frac{40\pi y}{3}\right) \quad \text{(A/m)}.
\end{aligned}
$$

(b)

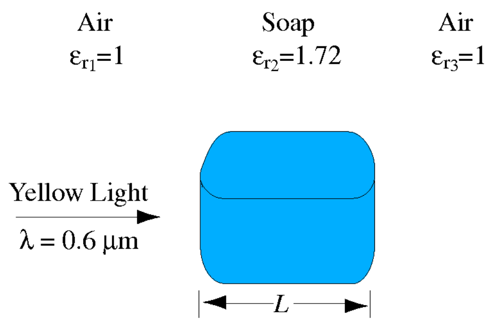
$$
\begin{aligned}
\mathbf{S}^{\mathrm{i}}_{\mathrm{av}} &= \hat{\mathbf{y}}\,\frac{|E_0|^2}{2\eta_1} = \hat{\mathbf{y}}\,\frac{(251.34)^2}{2\times 125.67} = \hat{\mathbf{y}}\,251.34 \quad (\text{W/m}^2), \\
\mathbf{S}^{\mathrm{r}}_{\mathrm{av}} &= -\hat{\mathbf{y}}\,|\Gamma|^2(251.34) = \hat{\mathbf{y}}\,10.05 \quad (\text{W/m}^2), \\
\mathbf{S}^{\mathrm{t}}_{\mathrm{av}} &= \hat{\mathbf{y}}\,(251.34 - 10.05) = \hat{\mathbf{y}}\,241.29 \quad (\text{W/m}^2).
\end{aligned}
$$

Problem 8.14 Consider a thin film of soap in air under illumination by yellow light with $\lambda = 0.6$ μm in vacuum. If the film is treated as a planar dielectric slab with $\varepsilon_r = 1.72$, surrounded on both sides by air, what film thickness would produce strong reflection of the yellow light at normal incidence?

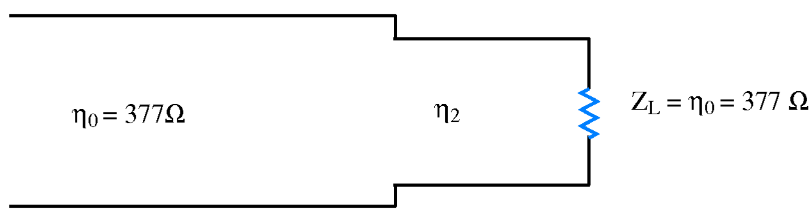
Solution: The transmission line analogue of the soap-bubble wave problem is shown in Fig. P8.14(b) where the load Z_L is equal to η_0, the impedance of the air medium on the other side of the bubble. That is,

$$\eta_0 = 377\ \Omega, \qquad \eta_1 = \frac{377}{\sqrt{1.72}} = 287.5\ \Omega.$$

The normalized load impedance is



(a) Yellow light incident on soap bubble.



(b) Transmission-line equivalent circuit

Figure P8.14: Diagrams for Problem 8.14.

$$z_L = \frac{\eta_0}{\eta_1} = 1.31.$$

For the reflection by the soap bubble to be the largest, Z_{in} needs to be the most different from η_0. This happens when z_L is transformed through a length $\lambda/4$. Hence,

$$L = \frac{\lambda}{4} = \frac{\lambda_0}{4\sqrt{\varepsilon_r}} = \frac{0.6\ \mu\text{m}}{4\sqrt{1.72}} = 0.115\ \mu\text{m},$$

where λ is the wavelength of the soap bubble material. Strong reflections will also occur if the thickness is greater than L by integer multiples of $n\lambda/2 = (0.23n)\ \mu$m.

Hence, in general

$$L = (0.115 + 0.23n)\ \mu\text{m}, \qquad n = 0, 1, 2, \ldots\ .$$

According to Section 2-7.5, transforming a load $Z_\text{L} = 377\ \Omega$ through a $\lambda/4$ section of $Z_0 = 287.5\ \Omega$ ends up presenting an input impedance of

$$Z_\text{in} = \frac{Z_0^2}{Z_\text{L}} = \frac{(287.5)^2}{377} = 219.25\ \Omega.$$

This Z_in is at the input side of the soap bubble. The reflection coefficient at that interface is

$$\Gamma = \frac{Z_\text{in} - \eta_0}{Z_\text{in} + \eta_0} = \frac{219.25 - 377}{219.25 + 377} = -0.27.$$

Any other thickness would produce a reflection coeffficient with a smaller magnitude.

Problem 8.32 A perpendicularly polarized wave in air is obliquely incident upon a planar glass–air interface at an incidence angle of 30°. The wave frequency is 600 THz (1 THz $= 10^{12}$ Hz), which corresponds to green light, and the index of refraction of the glass is 1.6. If the electric field amplitude of the incident wave is 50 V/m, determine the following:

(a) The reflection and transmission coefficients.

(b) The instantaneous expressions for **E** and **H** in the glass medium.

Solution:

(a) For nonmagnetic materials, $(\varepsilon_2/\varepsilon_1) = (n_2/n_1)^2$. Using this relation in Eq. (8.60) gives

$$\Gamma_\perp = \frac{\cos\theta_\mathrm{i} - \sqrt{(n_2/n_1)^2 - \sin^2\theta_\mathrm{i}}}{\cos\theta_\mathrm{i} + \sqrt{(n_2/n_1)^2 - \sin^2\theta_\mathrm{i}}} = \frac{\cos 30^\circ - \sqrt{(1.6)^2 - \sin^2 30^\circ}}{\cos 30^\circ + \sqrt{(1.6)^2 - \sin^2 30^\circ}} = -0.27,$$

$$\tau_\perp = 1 + \Gamma_\perp = 1 - 0.27 = 0.73.$$

(b) In the glass medium,

$$\sin\theta_\mathrm{t} = \frac{\sin\theta_\mathrm{i}}{n_2} = \frac{\sin 30^\circ}{1.6} = 0.31,$$

or $\theta_\mathrm{t} = 18.21^\circ$.

$$\eta_2 = \sqrt{\frac{\mu_2}{\varepsilon_2}} = \frac{\eta_0}{n_2} = \frac{120\pi}{1.6} = 75\pi = 235.62 \quad (\Omega),$$

$$k_2 = \frac{\omega}{u_\mathrm{p}} = \frac{2\pi f}{c/n} = \frac{2\pi f n}{c} = \frac{2\pi \times 600 \times 10^{12} \times 1.6}{3 \times 10^8} = 6.4\pi \times 10^6 \text{ rad/m},$$

$$E_0^\mathrm{t} = \tau_\perp E_0^\mathrm{i} = 0.73 \times 50 = 36.5 \text{ V/m}.$$

From Eqs. (8.49c) and (8.49d),

$$\widetilde{\mathbf{E}}_\perp^\mathrm{t} = \hat{\mathbf{y}} E_0^\mathrm{t} e^{-jk_2(x\sin\theta_\mathrm{t} + z\cos\theta_\mathrm{t})},$$

$$\widetilde{\mathbf{H}}_\perp^\mathrm{t} = (-\hat{\mathbf{x}}\cos\theta_\mathrm{t} + \hat{\mathbf{z}}\sin\theta_\mathrm{t}) \frac{E_0^\mathrm{t}}{\eta_2} e^{-jk_2(x\sin\theta_\mathrm{t} + z\cos\theta_\mathrm{t})},$$

and the corresponding instantaneous expressions are:

$$\mathbf{E}_\perp^\mathrm{t}(x,z,t) = \hat{\mathbf{y}} 36.5 \cos(\omega t - k_2 x \sin\theta_\mathrm{t} - k_2 z \cos\theta_\mathrm{t}) \quad \text{(V/m)},$$

$$\mathbf{H}_\perp^\mathrm{t}(x,z,t) = (-\hat{\mathbf{x}}\cos\theta_\mathrm{t} - \hat{\mathbf{z}}\cos\theta_\mathrm{t}) 0.16 \cos(\omega t - k_2 x \sin\theta_\mathrm{t} - k_2 z \cos\theta_\mathrm{t}) \quad \text{(A/m)},$$

with $\omega = 2\pi \times 10^{15}$ rad/s and $k_2 = 6.4\pi \times 10^6$ rad/m.

Problem 8.36 A 50-MHz right-hand circularly polarized plane wave with an electric field modulus of 30 V/m is normally incident in air upon a dielectric medium with $\varepsilon_r = 9$ and occupying the region defined by $z \geq 0$.

(a) Write an expression for the electric field phasor of the incident wave, given that the field is a positive maximum at $z = 0$ and $t = 0$.

(b) Calculate the reflection and transmission coefficients.

(c) Write expressions for the electric field phasors of the reflected wave, the transmitted wave, and the total field in the region $z \leq 0$.

(d) Determine the percentages of the incident average power reflected by the boundary and transmitted into the second medium.

Solution:

(a)

$$k_1 = \frac{\omega}{c} = \frac{2\pi \times 50 \times 10^6}{3 \times 10^8} = \frac{\pi}{3} \text{ rad/m,}$$
$$k_2 = \frac{\omega}{c}\sqrt{\varepsilon_{r_2}} = \frac{\pi}{3}\sqrt{9} = \pi \text{ rad/m.}$$

From (7.57), RHC wave traveling in $+z$ direction:

$$\widetilde{\mathbf{E}}^{\mathrm{i}} = a_0(\hat{\mathbf{x}} + \hat{\mathbf{y}}\,e^{-j\pi/2})e^{-jk_1z} = a_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{-jk_1z}$$
$$\mathbf{E}^{\mathrm{i}}(z,t) = \mathfrak{Re}\left[\widetilde{\mathbf{E}}^{\mathrm{i}} e^{j\omega t}\right]$$
$$= \mathfrak{Re}\left[a_0(\hat{\mathbf{x}}\,e^{j(\omega t - k_1z)} + \hat{\mathbf{y}}\,e^{j(\omega t - k_1z - \pi/2)})\right]$$
$$= \hat{\mathbf{x}}\,a_0\cos(\omega t - k_1z) + \hat{\mathbf{y}}\,a_0\cos(\omega t - k_1z - \pi/2)$$
$$= \hat{\mathbf{x}}\,a_0\cos(\omega t - k_1z) + \hat{\mathbf{y}}\,a_0\sin(\omega t - k_1z)$$
$$|\mathbf{E}^{\mathrm{i}}| = \left[a_0^2\cos^2(\omega t - k_1z) + a_0^2\sin^2(\omega t - k_1z)\right]^{1/2} = a_0 = 30 \text{ V/m.}$$

Hence,

$$\widetilde{\mathbf{E}}^{\mathrm{i}} = 30(x_0 - jy_0)e^{-j\pi z/3} \quad \text{(V/m).}$$

(b)

$$\eta_1 = \eta_0 = 120\pi \quad (\Omega), \qquad \eta_2 = \frac{\eta_0}{\sqrt{\varepsilon_{r_2}}} = \frac{120\pi}{\sqrt{9}} = 40\pi \quad (\Omega).$$
$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{40\pi - 120\pi}{40\pi + 120\pi} = -0.5$$
$$\tau = 1 + \Gamma = 1 - 0.5 = 0.5.$$

(c)

$$\begin{aligned}\widetilde{\mathbf{E}}^{\mathrm{r}} &= \Gamma a_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{jk_1 z} \\ &= -0.5 \times 30(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{jk_1 z} \\ &= -15(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{j\pi z/3} \quad \text{(V/m)}. \\ \widetilde{\mathbf{E}}^{\mathrm{t}} &= \tau a_0(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{-jk_2 z} \\ &= 15(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{-j\pi z} \quad \text{(V/m)}. \\ \widetilde{\mathbf{E}}_1 &= \widetilde{\mathbf{E}}^{\mathrm{i}} + \widetilde{\mathbf{E}}^{\mathrm{r}} \\ &= 30(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{-j\pi z/3} - 15(\hat{\mathbf{x}} - j\hat{\mathbf{y}})e^{j\pi z/3} \\ &= 15(\hat{\mathbf{x}} - j\hat{\mathbf{y}})[2e^{-j\pi z/3} - e^{j\pi z/3}] \quad \text{(V/m)}.\end{aligned}$$

(d)

$$\%\text{ of reflected power} = 100 \times |\Gamma|^2 = 100 \times (0.5)^2 = 25\%$$

$$\%\text{ of transmitted power} = 100|\tau|^2 \frac{\eta_1}{\eta_2} = 100 \times (0.5)^2 \times \frac{120\pi}{40\pi} = 75\%.$$

Chapter 9 Solved Problems

Problem 9-17

Problem 9-24

Problem 9-37

Problem 9.17 Repeat parts (a)–(c) of Problem 9.15 for a dipole of length $l = \lambda$.

Solution: For $l = \lambda$, Eq. (9.56) becomes

$$S(\theta) = \frac{15I_0^2}{\pi R^2}\left[\frac{\cos(\pi\cos\theta) - \cos(\pi)}{\sin\theta}\right]^2 = \frac{15I_0^2}{\pi R^2}\left[\frac{\cos(\pi\cos\theta) + 1}{\sin\theta}\right]^2.$$

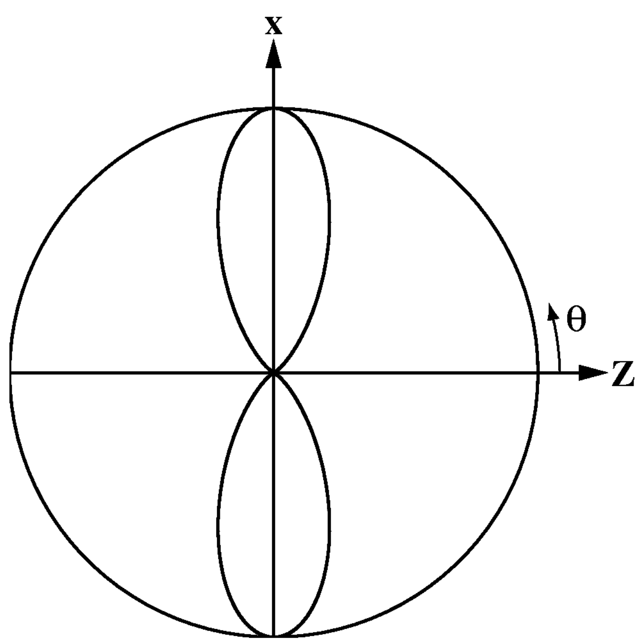


Figure P9.17: Figure P9.17: Radiation pattern of dipole of length $l = \lambda$.

Solving for the directions of maximum radiation numerically yields

$$\theta_{\max_1} = 90^\circ, \qquad \theta_{\max_2} = 270^\circ.$$

(b) From the numerical results, it was found that $S(\theta) = 15I_0^2/(\pi R^2)(4)$ at $\theta_{\max}$. Thus,

$$S_{\max} = \frac{60I_0^2}{\pi R^2}.$$

(c) The normalized radiation pattern is given by Eq. (9.13), as

$$F(\theta) = \frac{S(\theta)}{S_{\max}}.$$

Using the expression for $S(\theta)$ from part (a) with the value of $S_{\max}$ found in part (b),

$$F(\theta) = \frac{1}{4}\left[\frac{\cos(\pi\cos\theta) + 1}{\sin\theta}\right]^2.$$

The normalized radiation pattern is shown in Fig. P9.17.

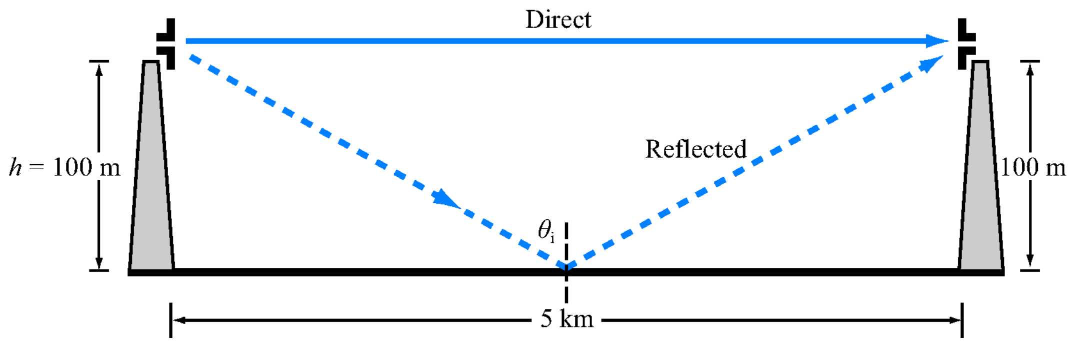


Figure P9.24: Problem 9.24.

Problem 9.24 The configuration shown in Fig. P9.24 depicts two vertically oriented half-wave dipole antennas pointed towards each other, with both positioned on 100-m–tall towers separated by a distance of 5 km. If the transit antenna is driven by a 50-MHz current with amplitude $I_0 = 2$ A, determine:

(a) The power received by the receive antenna in the absence of the surface. (Assume both antennas to be lossless.)

(b) The power received by the receive antenna after incorporating reflection by the ground surface, assuming the surface to be flat and to have $\varepsilon_r = 9$ and conductivity $\sigma = 10^{-3}$ (S/m).

Solution:

(a) Since both antennas are lossless,

$$P_{rec} = P_{int} = S_i A_{er}$$

where S_i is the incident power density and A_{er} is the effective area of the receive dipole. From Section 9-3,

$$S_i = S_0 = \frac{15 I_0^2}{\pi R^2},$$

and from (9.64) and (9.47),

$$A_{er} = \frac{\lambda^2 D}{4\pi} = \frac{\lambda^2}{4\pi} \times 1.64 = \frac{1.64\lambda^2}{4\pi}.$$

Hence,

$$P_{rec} = \frac{15 I_0^2}{\pi R^2} \times \frac{1.64\lambda^2}{4\pi} = 3.6 \times 10^{-6} \text{ W}.$$

(b) The electric field of the signal intercepted by the receive antenna now consists of a direct component, E_d, due to the directly transmitted signal, and a reflected component, E_r, due to the ground reflection. Since the power density S and the electric field E are related by

$$S = \frac{|E|^2}{2\eta_0},$$

it follows that

$$\begin{aligned} E_{\rm d} &= \sqrt{2\eta_0 S_{\rm i}}\; e^{-jkR} \\ &= \sqrt{2\eta_0 \times \frac{15 I_0^2}{\pi R^2}}\; e^{-jkR} \\ &= \sqrt{\frac{30\eta_0}{\pi}}\; \frac{I_0}{R}\; e^{-jkR} \end{aligned}$$

where the phase of the signal is measured with respect to the location of the transmit antenna, and $k = 2\pi/\lambda$. Hence,

$$E_{\rm d} = 0.024 e^{-j120^\circ} \quad \text{(V/m)}.$$

The electric field of the reflected signal is similar in form except for the fact that R should be replaced with R', where R' is the path length traveled by the reflected signal, and the electric field is modified by the reflection coefficient Γ. Thus,

$$E_{\rm r} = \left(\sqrt{\frac{30\eta_0}{\pi}}\; \frac{I_0}{R'}\; e^{-jkR'} \right) \Gamma.$$

From the problem geometry

$$R' = 2\sqrt{(2.5 \times 10^3)^2 + (100)^2} = 5004.0 \text{ m}.$$

Since the dipole is vertically oriented, the electric field is parallel polarized. To calculate Γ, we first determine

$$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega \varepsilon_0 \varepsilon_{\rm r}} = \frac{10^{-3}}{2\pi \times 50 \times 10^6 \times 8.85 \times 10^{-12} \times 9} = 0.04.$$

From Table 7-1,

$$\eta_{\rm c} \approx \eta = \sqrt{\frac{\mu}{\varepsilon}} = \frac{\eta_0}{\sqrt{\varepsilon_{\rm r}}} = \frac{\eta_0}{\sqrt{9}} = \frac{\eta_0}{3}\;.$$

From (8.66a),

$$\Gamma_\parallel = \frac{\eta_2 \cos\theta_{\rm t} - \eta_1 \cos\theta_{\rm i}}{\eta_2 \cos\theta_{\rm t} + \eta_1 \cos\theta_{\rm i}}$$

From the geometry,

$$\begin{aligned} \cos\theta_{\rm i} &= \frac{h}{(R'/2)} = \frac{100}{2502} = 0.04 \\ \theta_{\rm i} &= 87.71^\circ \\ \theta_{\rm t} &= \sin^{-1}\left(\frac{\sin\theta_{\rm i}}{\sqrt{\varepsilon_{\rm r}}}\right) = 19.46^\circ \\ \eta_1 &= \eta_0 \text{ (air)} \\ \eta_2 &= \eta = \frac{\eta_0}{3}\;. \end{aligned}$$

Hence,

$$\Gamma_{\parallel} = \frac{(\eta_0/3)\times 0.94 - \eta_0 \times 0.04}{(\eta_0/3)\times 0.94 + \eta_0 \times 0.04} = 0.77.$$

The reflected electric field is

$$\begin{aligned} E_{\rm r} &= \left(\sqrt{\frac{30\eta_0}{\pi}}\ \frac{I_0}{R'}\ e^{-jkR'} \right) \Gamma \\ &= 0.018 e^{j0.6^\circ} \quad \text{(V/m)}. \end{aligned}$$

The total electric field is

$$\begin{aligned} E &= E_{\rm d} + E_{\rm r} \\ &= 0.024 e^{-j120^\circ} + 0.018 e^{j0.6^\circ} \\ &= 0.02 e^{-j73.3^\circ} \quad \text{(V/m)}. \end{aligned}$$

The received power is

$$\begin{aligned} P_{\rm rec} &= S_{\rm i} A_{\rm er} \\ &= \frac{|E|^2}{2\eta_0} \times \frac{1.64\lambda^2}{4\pi} \\ &= 2.5 \times 10^{-6} \text{ W}. \end{aligned}$$

Problem 9.37 A five-element equally spaced linear array with $d = \lambda/2$ is excited with uniform phase and an amplitude distribution given by the binomial distribution

$$a_i = \frac{(N-1)!}{i!(N-i-1)!}, \qquad i = 0, 1, \ldots, (N-1),$$

where N is the number of elements. Develop an expression for the array factor.

Solution: Using the given formula,

$$\begin{aligned}
a_0 &= \frac{(5-1)!}{0!4!} = 1 \qquad \text{(note that } 0! = 1) \\
a_1 &= \frac{4!}{1!3!} = 4 \\
a_2 &= \frac{4!}{2!2!} = 6 \\
a_3 &= \frac{4!}{3!1!} = 4 \\
a_4 &= \frac{4!}{0!4!} = 1
\end{aligned}$$

Application of (9.113) leads to:

$$\begin{aligned}
F_a(\gamma) &= \left| \sum_{i=0}^{N-1} a_i e^{ji\gamma} \right|^2, \qquad \gamma = \frac{2\pi d}{\lambda} \cos\theta \\
&= \left| 1 + 4e^{j\gamma} + 6e^{j2\gamma} + 4e^{j3\gamma} + e^{j4\gamma} \right|^2 \\
&= \left| e^{j2\gamma} (e^{-j2\gamma} + 4e^{-j\gamma} + 6 + 4e^{j\gamma} + e^{j2\gamma}) \right|^2 \\
&= (6 + 8\cos\gamma + 2\cos 2\gamma)^2.
\end{aligned}$$

With $d = \lambda/2$, $\gamma = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} \cos\theta = \pi \cos\theta$,

$$F_a(\theta) = [6 + 8\cos(\pi\cos\theta) + 2\cos(2\pi\cos\theta)]^2.$$

Chapter 10 Solved Problems

There are no solved problems for Chapter 10

