Question:

What boundary conditions were used in the derivations of the expressions for Γ and τ?

Answer:

Step 1

Write the expression for reflection coefficient, Γ.

$$\Gamma = \frac{E_0^r}{E_0^i}$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$$

Write the expression for transmission coefficient, τ.

$$\tau = \frac{E_0^t}{E_0^i}$$

$$\tau = \frac{2\eta_2}{\eta_2 + \eta_1}$$

Here,

η_1 and η_2 are the intrinsic impedances of medium 1 and medium 2 respectively.

E_0^r, E_0^t and E_0^i are the amplitudes of the reflected, transmitted and incident electric fields respectively.

Step 2

In deriving the reflection and transmission coefficients, use the boundary conditions for the total electric field, that the tangential components of electric fields $\widetilde{\mathbf{E}}_1(0)$ and $\widetilde{\mathbf{E}}_2(0)$ are always continuous across the boundary if the two media are continuous.

$$\widetilde{\mathbf{E}}_1(0)=\widetilde{\mathbf{E}}_2(0)$$

or

$$E_0^i+E_0^r=E_0^t$$

In deriving the reflection and transmission coefficients, use the boundary conditions for the total magnetic fields that the tangential components of the magnetic fields $\widetilde{\mathbf{H}}_1(0)$ and $\widetilde{\mathbf{H}}_2(0)$ are continuous across the boundary, provided there is no current source passing through the boundary.

$$\widetilde{\mathbf{H}}_1(0)=\widetilde{\mathbf{H}}_2(0)$$

or

$$\frac{E_0^i}{\eta_1}-\frac{E_0^r}{\eta_1}=\frac{E_0^t}{\eta_2}$$

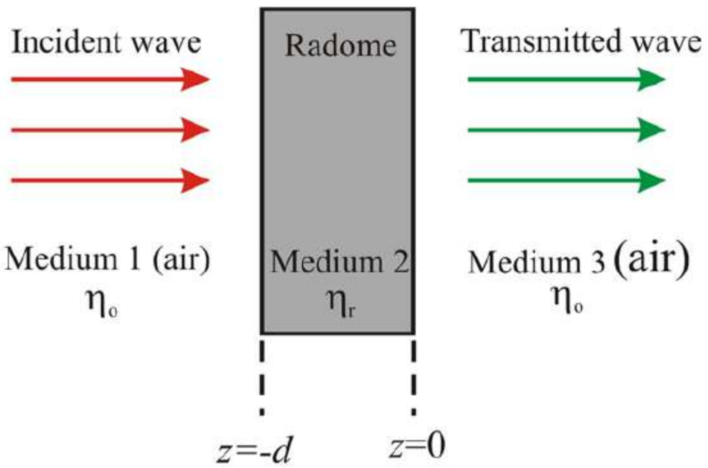
Question:

In the radar radome design of Example 8-1, all the incident energy in medium 1 ends up getting transmitted into medium 3, and vice versa. Does this imply that no reflections take place within medium 2? Explain.

Answer:

Step 1

Given figure is



Step 2

For the radome to "appear" transparent to the incident wave, the reflection coefficient must be zero at $z=-d$, thereby guaranteeing total transmission of the incident power into medium 3. Since $Z_L=\eta_0$, no reflection will take place at $z=-d$. That is no reflections take place within medium 2. Hence all the incident energy in medium 1 ends up getting transmitted into medium 3, and vice versa.

Question:

Explain on the basis of boundary conditions why it is necessary that $\Gamma = -1$ at the boundary between a dielectric and a perfect conductor.

Answer:

Step 1

At the boundary between a dielectric and a perfect conductor, the boundary conditions require that the tangential components of electric fields are continuous across the boundary, namely

$E_{1t} = E_{2t}$ ...... (1)

Step 2

But we know that in a perfect conductor, no electric field should exist.

Hence $E_{2t} = 0$

Now equation (1) becomes

$E_{1t} = 0$

Step 3

To make the total tangential component of electric field in medium 1 disappear, it is required that

$$E_{1t} = E_{it} + E_{rt}$$
$$E_{1t} = 0$$

Hence it must be the case that the reflection coefficient

$\boxed{\Gamma = -1}$

Question:

Can total internal reflection take place for a wave incident from medium 1 (with n1) onto medium 2 (with n2) when n2 > n1?

Answer:

Step 1

No. When $n_2 > n_1$, the critical angle would be

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

This equation has no solution for θ_c when $\frac{n_2}{n_1} > 1$. Hence total internal reflection is impossible in this case.

Question:

What is the difference between the boundary conditions applied in Section 8-1-1 for normal incidence and those applied in Section 8-4.1 for oblique incidence with perpendicular polarization?

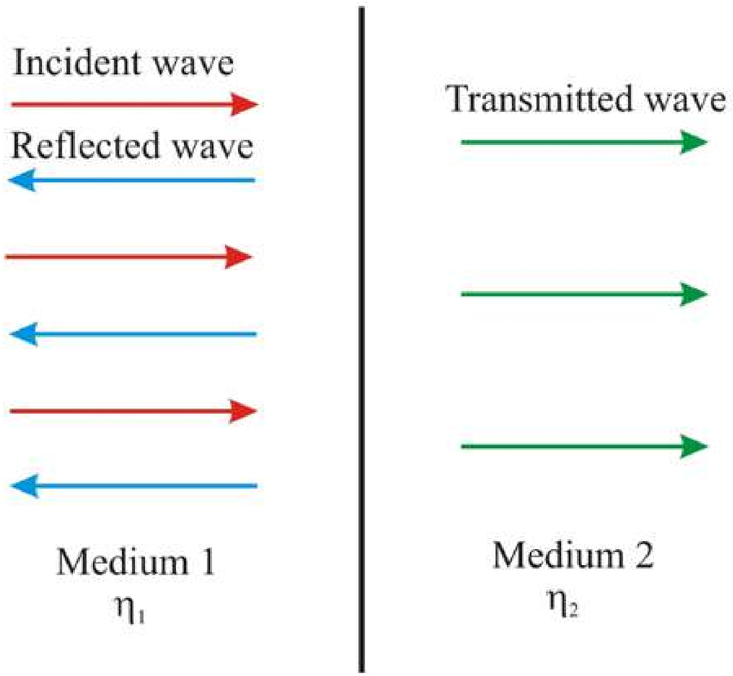
Answer:

Step 1

Boundary conditions require that the tangential components of both fields are always continuous across the boundary if the two media are contiguous and when there's no current source at the boundary.

Step 2

Ray representation of wave reflection and transmission at normal incidence is shown in below figure:

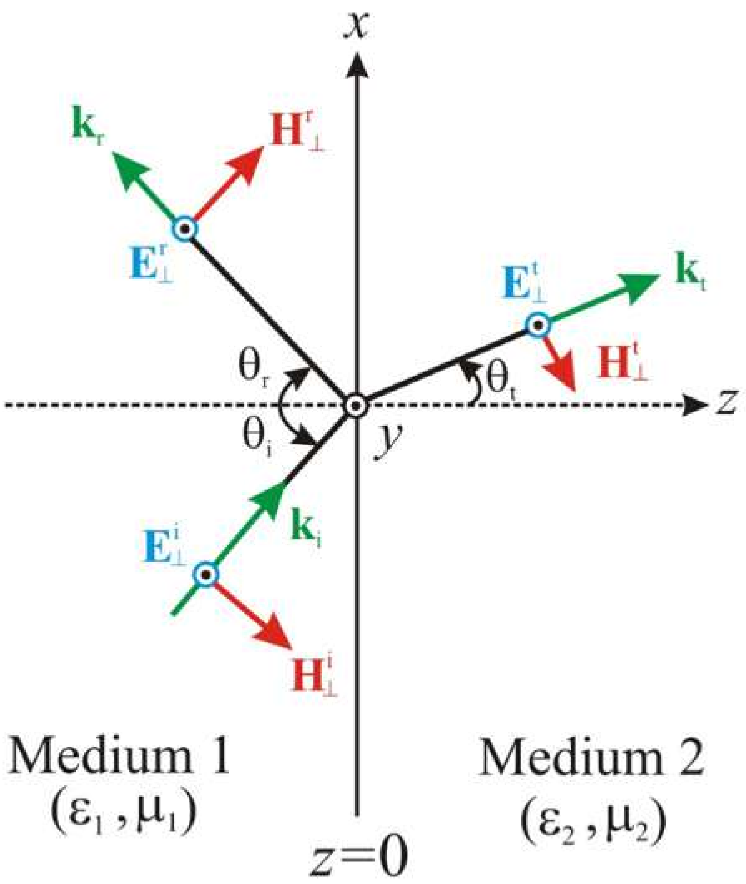


Step 3

For normal incidence, the electric fields and magnetic fields of all the waves, including incident, reflected and transmitted waves, are all tangential to the boundary.

Step 4

Ray representation of wave reflection and transmission for oblique incidence with perpendicular polarization is shown in below figure:



Step 5

For oblique incidence with perpendicular polarization, the electric fields of all the waves are tangential to the plane of incidence. However, the magnetic fields are not, they are parallel to the plane of incidence.

Question:

Why is the Brewster angle also called the polarizing angle?

Answer:

Step 1

The Brewster angle θ_B is defined as the incidence angle θ_i at which the Fresnel reflection coefficient $\Gamma = 0$.

The Brewster angle is also called the polarizing angle. This is because; it can be used to filter out one of the polarized components of a wave of arbitrary polarization. Since a wave of any polarization can be decomposed into parallel polarization and perpendicular polarization, we can incident a wave at the Brewster angle onto a nonmagnetic medium. By the definition of the Brewster angle, we know that the parallel polarized component will be totally transmitted into the medium and will not be reflected, while part of the perpendicular component still gets reflected. So the reflected wave will consist only of perpendicular polarized wave.

Question:

At the boundary, the vector sum of the tangential components of the incident and reflected electric fields has to equal the tangential component of the transmitted electric field. For εr1 = 1 and εr2 = 16, determine the Brewster angle and then verify the validity of the preceding statement by sketching to scale the tangential components of the three electric fields at the Brewster angle.

Answer:

Step 1

Brewster angle is the angle at which total transmission occurs; it exists only for parallel polarization.

The permittivity of the medium 1 is 1, and the permittivity of medium 2 is 16.

$\varepsilon_{r1} = 1, \varepsilon_{r2} = 16$

Step 2

Write the expression for the Brewster angle.

$$\theta_{B\parallel} = \tan^{-1}\left(\sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\right)$$
$$= \tan^{-1}\left(\sqrt{\frac{\varepsilon_{r2}}{\varepsilon_{r1}}}\right)$$
$$= \tan^{-1}\left(\sqrt{\frac{16}{1}}\right)$$
$$= \tan^{-1}(4)$$

$$\theta_{B\parallel} = 75.96°$$

Therefore, Brewster angle is,

$$\boxed{\theta_{B\parallel} = 75.96°}$$

Step 3

Consider a p-polarized wave the electric field lies in the plane of incidence.

The p-polarized wave has no perpendicular electric field component to the plane of incidence.

At the Brewster angle the electric field component that is perpendicular to the plane of incidence is reflected and the component that is parallel to the plane of incidence is totally transmitted.

In parallel polarization the normal component of reflected electric undergoes 180° phase change as demonstrated in the Figure 1.

Step 4

Write the expression for the boundary conditions of the tangential electric field.

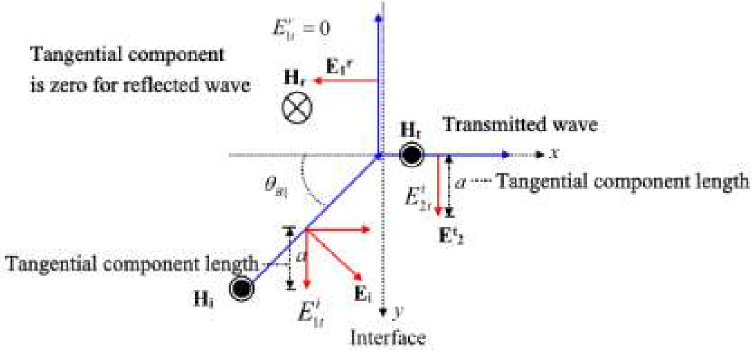
$$E_{1t} = E_{2t}$$
$$E_{1t}^{i} + E_{1t}^{r} = E_{2t}^{t}$$

The reflected component does not have the tangential component due to the transmission at the Brewster angle.

$$E_{1t}^{i} + 0 = E_{2t}^{t}$$
$$E_{1t}^{i} = E_{2t}^{t}$$

Irrespective of the permittivity of the medium the tangential components are equal by boundary conditions.

Thus, the length of the tangential components in the below figure are equal.



Both tangential components have the same length.

Figure 1

Question:

What are the primary limitations of coaxial cables at frequencies higher than 30 GHz?

Answer:

Step 1

At frequencies higher than 30 GHz, the coaxial cable has the following limitations:

(1) It needs to meet the size-to-wavelength requirement, so the higher the frequency, the smaller it gets. But small sizes are usually difficult to fabricate.

(2) At high frequencies, its cross section will be very small, which limits its power-handling capacity.

(3) The higher the frequency, the more significant the attenuation due to dielectric losses.

Question:

Can a TE mode have a zero magnetic field along the direction of propagation?

Answer:

Step 1

No. By the definition, a mode is called a transverse electric (TE) mode when the electric field $\mathbf{E}$ is transverse to the direction of propagation but the magnetic field $\mathbf{H}$ is not. If the magnetic field has zero component along the direction of propagation, then $\mathbf{H}$ is either transverse to the direction of propagation, or equal to zero. Neither satisfy our condition of TE mode.

Question:

What is the rationale for choosing a solution for ẽz that involves sine and cosine functions?

Answer:

Step 1

The original equation to solve is

$$\tilde{E}_z(x,y,z) = \tilde{e}_z(x,y)e^{-j\beta z} \quad \ldots\ldots (1)$$

Where $\tilde{e}_z(x,y)$ describes the dependence of $\tilde{E}_z(x,y,z)$ on (x,y) only.

Step 2

The above equation can be reduced to

$$\frac{\partial^2 \tilde{e}_z}{\partial x^2} + \frac{\partial^2 \tilde{e}_z}{\partial y^2} + k_c^2 \tilde{e}_z = 0 \quad \ldots\ldots (2)$$

The form of the partial differential equation allows us to assume a product solution of the form

$$\tilde{e}_z(x,y) = X(x)Y(y) \quad \ldots\ldots (3)$$

Step 3

By substituting equation (3) into equation (2), followed with dividing all terms by $X(x)Y(y)$, leads to

$$\frac{1}{X}\frac{d^2X}{dx^2} + \frac{1}{Y}\frac{d^2Y}{dx^2} + k_c^2 = 0 \quad \ldots\ldots (4)$$

Step 4

To satisfy above equation (4), each of the first two terms has to equal a constant. Hence, we define separation constants k_x and k_y such that

$$\frac{d^2X}{dx^2}+k_x^2X=0$$
$$\frac{d^2Y}{dy^2}+k_y^2Y=0$$

And

$$k_c^2=k_x^2+k_y^2$$

Step 5

Since $\mathbf{E}=0$ in the conducting walls, the boundary conditions require $\tilde{E}_z$ in the waveguide cavity to go to zero as x approaches 0 and a, and as y approaches 0 and b.

To satisfy these boundary conditions, sinusoidal solutions are chosen for $X(x)$ and $Y(y)$ as follows:

$$\tilde{e}_z(x,y)=X(x)Y(y)$$
$$\tilde{e}_z(x,y)=(A\cos k_x x+B\sin k_x x)(C\cos k_y y+D\sin k_y y)$$

Step 6

The boundary conditions require $\tilde{E}_z$ to equal to when x equals 0 and a, and when y equals 0 and b. So we know that $\tilde{e}_z(x,y)=X(x)Y(y)$ has at least 4 zeros:

$x=0$
$x=a$
$y=0$
$y=b$

$X(x)$ and $Y(y)$ each has at least 2 zeros. This can be satisfied by choosing appropriate sinusoidal functions.

Question:

What is an evanescent wave?

Answer:

Step 1

The evanescent waves are a kind of wave characterized by rapidly decaying amplitudes as they propagate. Their attenuation functions have the form $e^{-\alpha z}$.