Question:

What is a uniform plane wave? Describe its properties, both physically and mathematically. Under what conditions is it appropriate to treat a spherical wave as a plane wave?

Answer:

Step 1

A uniform plane wave is a wave with wavefronts of the shape of planes, so it's uniform in any plane vertical to the direction of propagation. In fact, in order to be uniform across a plane, it also has to be infinite.

Step 2

Physical Properties: It has constant frequency; therefore its wave equation can be expressed in sinusoid form. It's a TEM (transverse electromagnetic mode) wave, which means, it electric field and magnetic field perpendicular to each other, both transverse to the direction of propagation. The amplitudes of the electric field and magnetic field of a uniform plane wave have a constant ratio, which is called the intrinsic impedance of the medium.

$$|H| = \frac{|E|}{\eta}$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}}$$

Step 3

Mathematical Properties: The solution of the wave equation can be written in the form of

$$\mathbf{E} = \hat{x}E_x + \hat{y}E_y$$

With

$$E_x = E_{0x}\cos(\omega t - \beta z + \phi_{0x})$$
$$E_y = E_{0y}\cos(\omega t - \beta z + \phi_{0y})$$

Assuming z is the direction of propagation.

Step 4

When observed from very far away such that the wavefronts of a spherical wave seem to be planar, the spherical wave can be treated as plane wave effectively.

Question:

Since $\tilde{\mathbf{E}}$ and $\tilde{\mathbf{H}}$ are governed by wave equations of the same form, does it follow that $\tilde{\mathbf{E}} = \tilde{\mathbf{H}}$? Explain.

Answer:

Step 1

Knowing that the electric field and magnetic field has wave equations of the same form, we still can't conclude that they have the same solution.

Step 2

$$\nabla^2\tilde{\mathbf{H}} - \gamma^2\tilde{\mathbf{H}} = 0$$
$$\nabla^2\tilde{\mathbf{E}} - \gamma^2\tilde{\mathbf{E}} = 0$$

Step 3

Since there can be various solutions to the equation

$$\nabla^2 V - \gamma^2 V = 0$$

And they are not necessarily the same. In fact, if a sinusoid function satisfies this equation, a constant number multiply by this function is also a solution to the equation.

Example: Suppose V_1 is such a solution, so we have

$$\nabla^2 V_1 - \gamma^2 V_1 = 0$$

$$\frac{\partial^2 V_1}{\partial x^2} - \frac{\partial^2 V_1}{\partial y^2} - \frac{\partial^2 V_1}{\partial z^2} = \gamma^2 V_1$$

Step 4

Now we multiply it by a constant number c, and

$$\frac{\partial^2 (cV_1)}{\partial x^2} - \frac{\partial^2 (cV_1)}{\partial y^2} - \frac{\partial^2 (cV_1)}{\partial z^2} = \gamma^2 (cV_1)$$

Still holds. So we know that cV_1 is also a solution to the equation.

Step 5

So we can only infer that the solutions to the wave equations of electrical field and magnetic field have the same form, but we can't say they are the same.

Question:

Answer:

Step 1

A TEM wave traveling in the y-direction means, by the definition of TEM (transverse electromagnetic) wave, its electric field and magnetic field are perpendicular to y, the direction of propagation. Therefore, neither of its electric field nor its magnetic field can have components along y. But it's fine to have components along x or z, because that doesn't conflict with the definition of TEM waves.

Question:

An elliptically polarized wave is characterized by amplitudes ax and ay and by the phase difference δ. 11 ax and ay are both nonzero, what should δ be in order for the polarization state to reduce to linear polarization?

Answer:

Step 1

Linear polarization happens only when the x-component and y-component of the field are in-phase or out-of-phase. This means, for a wave to be linearly polarized, it must have the phase difference $\boxed{\delta = 0 \text{ or } \delta = \pi}$.

Question:

Which of the following two descriptions defines an RHC polarized wave: A wave incident upon an observer is RHC polarized if its electric field appears to the observer to rotate in a counterclockwise direction

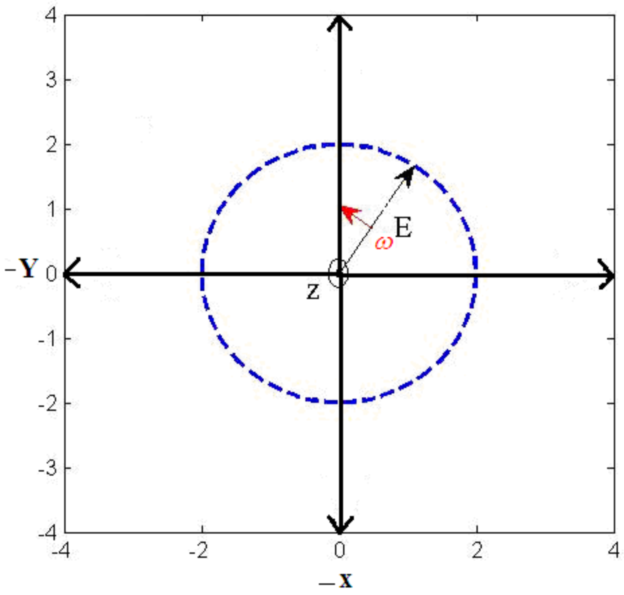
(a) as a function of time in a fixed plane perpendicular to the direction of wave travel or

(b) as a function of travel distance at a fixed time

Answer:

Step 1

By the definition of RHC polarized wave, when the direction of propagation is upon the observer, the wave must be rotating counterclockwise in a fixed plane orthogonal to the direction of propagation as a function of time. As illustrated in Figure 1 below.



RHC polarized wave

Question:

How does β of a low- loss dielectric medium compare to that of a lossless medium?

Answer:

Step 1

The phase constant β of a lossless medium and of a low-loss medium can be treated as the same effectively. They both have the form

$\beta = \omega\sqrt{\mu\varepsilon}$ and

That's because for low-loss medium, we know that

$$\sqrt{\frac{\varepsilon''}{\varepsilon'}} \;\square\; 1$$

Step 2

Hence we can apply the approximation

$$\sqrt{1+x^2} \approx 1+\frac{x}{2}$$ and

We can calculate β as

$$\beta = \omega\left\{\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]\right\}^{1/2}$$

$$\approx \omega\left\{\frac{\mu\varepsilon'}{2}\left[1+\frac{\frac{\varepsilon''}{\varepsilon'}}{2}+1\right]\right\}^{1/2}$$

$$= \omega\left\{\mu\varepsilon' + \frac{\mu\varepsilon''}{4}\right\}^{1/2}$$

And know that $\varepsilon'' \ll \varepsilon'$, we have

$$\beta \approx \omega\sqrt{\mu\varepsilon'} = \omega\sqrt{\mu\varepsilon}$$

Phase constant (β) of low-loss dielectric medium is same as the phase constant of lossless medium

Question:

In a good conductor, does the phase of H lead or lag that of E and by how much?

Answer:

Step 1

From the expressions of electric field and magnetic field in a good conductor

$$E(z) = \left|E_0 e^{-\alpha z} e^{-j\beta z}\right|$$

$$H(z) = \left|\frac{E_0}{\eta_c} e^{-\alpha z} e^{-j\beta z}\right|$$

Step 2

And we know for good conductor,

$$\eta_c = (1+j)\frac{\alpha}{\sigma}$$
$$= \left(\sqrt{2}e^{j\pi/4}\right)\frac{\alpha}{\sigma}$$

Step 3

Therefore

$$H(z) = \left|\frac{E_0}{\eta_c} e^{-\alpha z} e^{-j\beta z}\right|$$
$$= \left|\frac{E_0}{\sqrt{2}} e^{-j\pi/4} e^{-\alpha z} e^{-j\beta z}\right|$$
$$= \left|e^{-j\pi/4} E(z)\right|$$

Hence we conclude that the magnetic field lags electric field by $\frac{\pi}{4}$

Question:

Attenuation means that a wave loses energy as it propagates in a lossy medium. What happens to the lost energy?

Answer:

Step 1

When a wave propagates in a lossy medium, it loses energy which converted to other form of energies such as heat, light and etc.

Question:

Is a conducting medium dispersive or dispersionless? Explain.

Answer:

Step 1

Write the formula for phase constant for good conductor.

$$\beta = \sqrt{\pi f \mu \sigma}$$

Step 2

A conducting medium is **dispersive** because the phase constant is frequency-dependent.

The lights of different frequencies will have different phase constant and therefore they will be dispersed.

Therefore, the conducting medium is **dispersive**.

Question:

Compare the flow of current through a wire in the dc and ac cases. Compare the corresponding dc and ac resistances of the wire.

Answer:

Step 1

For a wire of length l, radius r, conductivity σ, we have the dc resistance

$$R_{dc} = \frac{l}{\sigma A}$$

We know the cross-sectional is $A = \pi r^2$, gives

$$\boxed{R_{dc} = \frac{l}{\sigma \pi r^2}}$$

And given permeability μ, we have the ac resistance

$$R_{ac} = R_s \frac{l}{2\pi r}$$
$$= \frac{l}{2\pi r}\sqrt{\frac{\pi f \mu}{\sigma}}$$

$$\boxed{R_{ac} = \frac{l}{2r}\sqrt{\frac{f\mu}{\sigma\pi}}}$$

And this is a frequency-dependent function. (Assuming the skin depth is much smaller than the radius of the wire)