Question:

Explain Faraday's law and the implication of Lenz's law.

Answer:

Step 1

When a galvanometer detects the flow of current through the coil, it means that a voltage has been induced across the galvanometer terminals. This voltage is called the electromotive force V_{emf} and the process is called electromagnetic induction. The emf induced in a closed conducting loop of N is given by,

$$V_{emf} = -N\frac{d\Phi}{dt}$$
$$= -N\frac{d}{dt}\int_S \mathbf{B}\cdot d\mathbf{s}$$

This is known as Faraday's law.

Step 2

The polarity of transformer emf V_{emf}^{tr} and hence the direction of current I is governed by Lenz's law, which states the current in the loop is always in a direction that opposes the change of magnetic flux $\Phi(t)$ that produced I.

Therefore, the explanation to the faradays law's and the implication of Lenz's law has being discussed and explained above.

Question:

Under what circumstances is the net voltage around a closed loop equal to zero?

Answer:

Step 1

The relation between voltage and magnetic flux density $\mathbf{B}$ is,

$$V = -N\int_S \frac{\partial \mathbf{B}}{\partial t} \cdot d\mathbf{s}$$

Here,

N is number of turns.

Step 2

Under dc conditions, magnetic field that is not varying with time $\frac{\partial \mathbf{B}}{\partial t}$ is zero.

The voltage is,

$$\begin{aligned} V &= -N\int_S (0) \cdot d\mathbf{s} \\ &= 0 \text{ V} \end{aligned}$$

Therefore, the net voltage around a closed loop is equal to zero under dc conditions as the magnetic field **B** is static.

Question:

Suppose the magnetic flux density linking the loop of Fig (Example) is given by B = −z ̂0.3e-1 (T). What would the direction of the current be, relative to that shown in Fig. for t ≥ 0?

Example: Lenz's Law

Determine voltages V1 and V2 acros; the 2-Ω and 4-Ω resistors shown,in Fig. The loop is located in the x-y plane, its area is 4 m2. the magnetic flux density is B=−z ̂0.3t (T). and the internal resistance of the wire may be ignored.

Solution:

The flux flowing through the loop is

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s} = \int_S (-\hat{z}0.3t) \cdot \hat{z}\, ds$$

= −0.3t × 4 = −1.2t (Wb),

and the corresponding transformer emf is

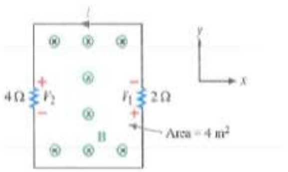
$$V_{emf}^{tr} = -\frac{d\Phi}{dt} = 1.2$$

Since the magnetic flux through the loop is along the −z-direction (into the page) and increases in magnitude with time t. Lenz's law states that the induced current I should he in a direction such that the magnetic flux density Bind it induces counteracts the direction of change Φ Hence. I has to be in the direction shown in the circuit because the corresponding Bind is along the + z-direction in the region inside the loop area. This, in turn, means that V1 and V2 are positive voltages. The total voltage ot 1.2 V is distributed across two resistors in series. Consequently.

$$I = \frac{V_{emf}^{tr}}{R_1 + R_2}$$
$$= \frac{1.2}{2+4} = 0.2 \text{ A}$$

W, − I R1 = 0.2 × 2 = 0.4 V.V2 = I R2 = 0.2 × 4 = 0.8 V.

Figure



Answer:

Step 1

Consider the loop located in x-y plane in Figure 6-4. Refer to the Figure 6-4 in the textbook. The magnetic flux density $\mathbf{B}$ linking the loop is $-\hat{\mathbf{z}}\,0.3e^{-t}$ (T). The direction of the magnetic flux through the loop is along the negative z- direction and its magnitude is decreasing with time for $t \geq 0$.

To determine the current in the circuit, first evaluate the magnetic flux Φ flowing through the surface of the loop, using the following formula:

$$\Phi = \int_S \mathbf{B} \cdot d\mathbf{s}$$

Substitute $\mathbf{B} = -\hat{\mathbf{z}}\,0.3e^{-t}$ (T) in this expression for the magnetic flux.

$$\begin{aligned}\Phi &= \int_S \mathbf{B} \cdot d\mathbf{s} \\ &= \int_S \left(-\hat{\mathbf{z}}\,0.3e^{-t}\right) \cdot \hat{\mathbf{z}}\,ds \\ &= \int_S \left(-0.3e^{-t}\right) \cdot ds \\ &= \left(-0.3e^{-t}\right) \int_S ds\end{aligned}$$

The integral over the surface of the loop $\int_S ds$ yields the area of the loop. The area of the loop is 4 m^2 .Hence, substitute the value 4 m^2 for $\int_S ds$ in the expression for magnetic flux and simplify it further.

$$\begin{aligned}\Phi &= \left(-0.3e^{-t}\right)(4) \\ &= -1.2e^{-t} \quad \text{(Wb)}\end{aligned}$$

Step 2

Here, the loop is stationary , so the electromotive force induced in it is called transformer electro motive force (emf).Determine the transformer electro motive force (emf) induced in the closed conducting loop having N number of turns, using the following formula:

$$V_{\text{emf}}^{\text{tr}} = -N \frac{d\Phi}{dt}$$

Substitute 1 for and N $-1.2e^{-t}$ for Φ in this expression for the magnetic flux.

$$\begin{aligned}V_{\text{emf}} &= -N \frac{d}{dt}\left(-1.2e^{-t}\right) \\ &= -(1)(-1.2)\frac{d}{dt}\left(e^{-t}\right) \\ &= 1.2\left(-e^{-t}\right) \\ &= -1.2e^{-t} \quad \text{(V)}\end{aligned}$$

Step 3

Determine the current I flowing through the circuit using the following formula:

$$I = \frac{V_{emf}^{tr}}{R_1 + R_2}$$

Substitute $-1.2e^{-t}$ for V_{emf}, $R_1 = 2\,\Omega$ and $R_2 = 4\,\Omega$ in the expression for current.

$$\begin{aligned} I &= \frac{-1.2e^{-t}\ (\mathrm{V})}{2\,\Omega + 4\,\Omega} \\ &= \frac{-1.2e^{-t}\ (\mathrm{V})}{6\,\Omega} \\ &= -0.2e^{-t}\ (\mathrm{A}) \end{aligned}$$

Obtain the value of current for $t \geq 0$. Substitute $t = 0$ in the expression and simplify it further.

$$\begin{aligned} I &= -0.2e^{-(0)}\ (\mathrm{A}) \\ &= -0.2\ (\mathrm{A}) \end{aligned}$$

Step 4

The polarity of transformer emf V_{emf}^{tr} and hence the direction of current I is governed by Lenz's law, which states the current in the loop is always in a direction that opposes the change of magnetic flux $\Phi(t)$ that produced I.

The value of current I is $-0.2\ (\mathrm{A})$. For $t \geq 0$, the transformer emf, $V_{emf} < 0$, so according to Lenz's law the direction of current needs to be in a direction that can oppose the change in Φ that is $\frac{d\Phi}{dt}$. Hence, in this case the current flows in reverse direction relative to the direction of current in Figure 6-4.

The direction of the induced field $\mathbf{B}_{ind}$ is in the same direction as that of time varying field $\mathbf{B}(t)$ but the change in Φ that is $\frac{d\Phi}{dt}$ is negative to decrease the magnitude of the $\mathbf{B}(t)$.

Hence, the direction of current relative to the direction of the current flowing in the loop in Figure 6-4 is determined. The **current flows in the reverse direction** to that shown in Figure 6-4.

Question:

Suppose that no friction is involved in sliding the conducting bar of Fig. and that the horizontal arms of the circuit are very long. Hence, if the bar is given an initial push, it should continue moving at a constant velocity, and its movement generates electrical energy in the form of an induced emf, indefinitely. Is this a valid argument? If not, why not? How can we generate electrical energy without having to supply an equal amount of energy by other means?

Answer:

Step 1

Refer to Figure 6-8 for a conducting bar in a magnetic field. The horizontal arms of the circuit are very long and there is no friction in sliding of the conducting bar. If an initial push is given to the bar then an emf is induced in the conductor circuit, because of that initial velocity. This induced emf causes current to flow through the closed circuit. Now, the conductor has become a current carrying conductor and it is in a magnetic field.

When a current carrying conductor is placed in a magnetic field, it experiences a force equal to the sum of the magnetic forces acting on the charged particles moving within it.

Hence, the conducting bar moves and hence an emf is induced in it. This induced emf again causes more current to flow through the conductor, and hence experiences a mechanical force. This mechanical force causes emf to be induced in the conducting bar. The process repeats and the bar continues to move with a constant velocity and hence generates electrical energy.

Thus, the argument is valid.

Step 2

There are two ways to generate electrical energy without supplying an equal amount of energy by other means.

(1) When a conductor moves in a magnetic field, an emf is induced in the conductor circuit. The magnitude of the induced emf is proportional to the strength of the magnetic field and the velocity of the moving conductor. In this process no energy is supplied to the circuit to generate electrical energy.

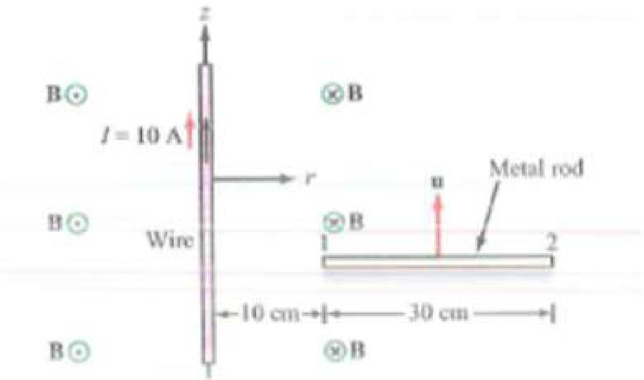
(2) When a stationary conductor is placed in a time varying magnetic field, an emf is induced in the conductor circuit, and hence electrical energy is generated without supplying energy to the circuit.

Thus, the two ways of generating electrical energy without supplying an equal amount of energy by other means are listed.

Question:

Is the current flowing in the rod of Fig. a steady current? Examine the force on a charge q at ends 1 and 2 and compare.

Figure



Answer:

Step 1

Refer to Figure 6-10 in the text book. It consists of a wire carrying current and a metal rod that moves at a constant velocity.

The current, $\mathbf{I}$ carried by the rod induces a magnetic field, $\mathbf{B}$ given by,

$$\mathbf{B} = \hat{\boldsymbol{\phi}}\frac{\mu_0 \mathbf{I}}{2\pi r}$$

Here, $\hat{\boldsymbol{\phi}}$ is a unit vector in the direction of radial distance, r from the wire into the page at the metal rod side of the wire.

Determine the cross product, $\mathbf{u}\times\mathbf{B}$ at a distance, r from the wire.

$$\mathbf{u}\times\mathbf{B} = \hat{\mathbf{z}}5\times\hat{\boldsymbol{\phi}}\frac{\mu_0 I}{2\pi r}$$

Determine the magnetic force, $\mathbf{F}_1$ on charge q at end 1, which is at a distance of 10 cm from the wire.

$$\begin{aligned}
\mathbf{F}_1 &= q(\mathbf{u}\times\mathbf{B}) \\
&= q\left(\hat{\mathbf{z}}5\times\hat{\boldsymbol{\phi}}\frac{\mu_0 I}{2\pi 10\times 10^{-2}}\right) \\
&= q\left(\hat{\mathbf{z}}5\times\hat{\boldsymbol{\phi}}\frac{10\mu_0 I}{2\pi}\right) \\
&= \frac{25\mu_0 I q}{\pi}\left(\hat{\mathbf{z}}\times\hat{\boldsymbol{\phi}}\right) \\
&= \frac{25\mu_0 I q}{\pi}\hat{\mathbf{r}}
\end{aligned}$$

The electric field at this distance is,

$$\begin{aligned}
\mathbf{E}_1 &= \frac{\mathbf{F}_1}{q} \\
&= \frac{25\mu_0 I}{\pi}\hat{\mathbf{r}}
\end{aligned}$$

Observe that the magnitude of the electric field is constant.

Step 2

Determine the magnetic force, $\mathbf{F}_2$ on charge q at end 2, which is at a distance of 30 cm from the wire.

$$\begin{aligned}
\mathbf{F}_2 &= q(\mathbf{u}\times\mathbf{B}) \\
&= q\left(\hat{\mathbf{z}}5\times\hat{\boldsymbol{\phi}}\frac{\mu_0 I}{2\pi 30\times 10^{-2}}\right) \\
&= q\left(\hat{\mathbf{z}}5\times\hat{\boldsymbol{\phi}}\frac{10\mu_0 I}{3\times 2\pi}\right) \\
&= \frac{25\mu_0 Iq}{3\pi}\left(\hat{\mathbf{z}}\times\hat{\boldsymbol{\phi}}\right) \\
&= \frac{25\mu_0 Iq}{3\pi}\hat{\mathbf{r}}
\end{aligned}$$

The electric field at this distance is,

$$\begin{aligned}
\mathbf{E}_2 &= \frac{\mathbf{F}_2}{q} \\
&= \frac{25\mu_0 I}{3\pi}\hat{\mathbf{r}}
\end{aligned}$$

Observe that the magnitude of the electric field is constant.

Step 3

The induced emf, $V_{12} = \int_2^1 \mathbf{E}_m \cdot d\mathbf{l}$.

As the magnitude of the electric field is constant at the both ends of the rod, the magnitude of induced emf is also constant and is different at the both ends. This difference in induced emf causes steady current to flow in the rod.

Hence, the current flowing in the rod is a steady current.

Question:

Contrast the operation of an ac motor with that of an ac generator.

Answer:

Step 1

The electromagnetic generator is the converse of the electromagnetic motor. In a motor electrical energy is supplied by a voltage source; that is, converted into mechanical energy in the form of rotating loop.

Step 2

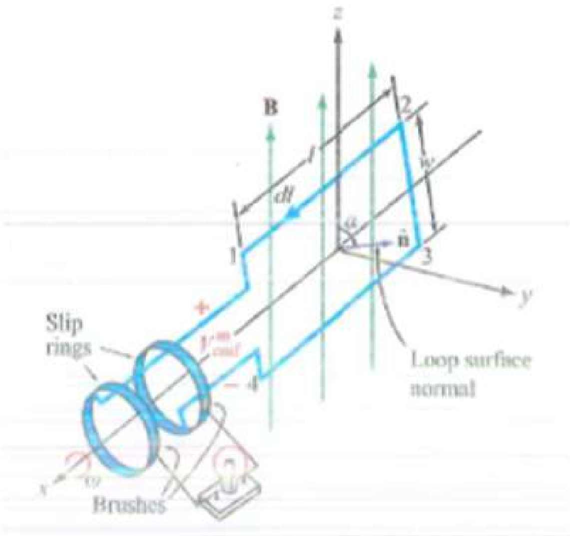
Now, instead of passing current through the loop to make it turn, the loop is made to rotate by an external force and the movement of the loop in the magnetic field produces a motional emf. Now, the motor has converted to a generator as the mechanical energy is being converted to electrical energy.

Thus, the contrast of operation of an ac motor with that of an ac generator is explained.

Question:

The rotating loop of Fig. 6-12 had a single turn. What would be the emf generated by a loop with 10 turns?

Figure



Answer:

Step 1

Consider the loop, rotating in the magnetic field in Figure 6-12. Refer to the Figure 6-12 in the textbook.

The magnetic flux Φ flowing through the surface of the loop is $B_0 A\cos(\omega t + C_0)$.

Determine the motional electro motive force (emf) $V_{\text{emf}}^{\text{m}}$ induced in loop having N number of turns, using the following formula:

$$V_{\text{emf}}^{\text{m}} = -N\frac{d\Phi}{dt}$$

Substitute 10 for N and $B_0 A\cos(\omega t + C_0)$ for Φ in this expression for motional electro motive force.

$$\begin{aligned}
V_{\text{emf}}^{\text{m}} &= -N\frac{d}{dt}\left(B_0 A\cos(\omega t + C_0)\right) \\
&= -(10)\frac{d}{dt}\left[B_0 A\cos(\omega t + C_0)\right] \\
&= -(10)B_0 A\frac{d}{dt}\left[\cos(\omega t + C_0)\right] \\
&= -(10)B_0 A\left[-\omega\sin(\omega t + C_0)\right]
\end{aligned}$$

Simplify, the expression further.

$$V_{\text{emf}}^{\text{m}} = 10A\omega B_0\sin(\omega t + C_0)$$

Hence, the emf $V_{\text{emf}}^{\text{m}}$ generated by a loop with 10 turns is $\boxed{10A\omega B_0\sin(\omega t + C_0)}$.

Question:

The magnetic flux linking the loop shown in Fig. 6-12 is maximum when α = 0 (loop in x-y plane), and yet according to Eq. the induced emf is zero when α = 0. Conversely, when a – 90°, the flux linking the loop is zero, but Vmemf is at a maximum. Is this consistent with your expectations? Why?

Equation

Vmemf = ωlmωB0 sinα = AωB0sinα

Answer:

Step 1

Refer to Figure 6-12 in the text book for a loop rotating in a magnetic field.

As the loop is rotating in a magnetic field, the flux linking with the loop is changing. This changing flux induces emf in the loop. The induced emf is proportional to the rate of change of flux linkages.

The flux linking with the loop is,

$\Phi = B_0 A \cos\alpha$

According to Faraday's law, the induced emf is,

$$\begin{aligned} V_{emf} &= -\frac{d\Phi}{dt} \\ &= A\omega B_0 \sin\alpha \end{aligned}$$

When $\alpha = 0$, the flux is maximum $(B_0 A)$, but the rate of change of flux is zero. Hence, the induced emf is zero.

When $\alpha = 90°$, the flux is zero, but the rate of change of flux is maximum $(A\omega B_0)$. Hence, the induced emf is maximum.

Hence, the result is consistent with our expectations.

Question:

When conduction current flows through a material, a certain number of charges enter the material on one end and an equal number leave on the other end. What's the situation like for the displacement current through a perfect dielectric?

Answer:

Step 1

The conduction current, $I_c = \frac{dQ}{dt}$.

Thus, conduction current flows through a material when there is rate of change of charge, that is, when charges enter the material at one end and an equal leave at the other end.

Hence, conduction current is the rate of change of charge.

The displacement current, $I_d \propto \frac{\partial \mathbf{D}}{\partial t}$.

The displacement current is the current that flows from one end of the dielectric to the other because of time change of electric flux density, $\mathbf{D}$

Thus, displacement current does not involve any flow of charge, but it is due to the rate of change of electric field and it does not require any medium like conduction current and can flow in vacuum also.

Question:

Verify that the integral form of Ampere’s law given by Eq. leads to the boundary condition that the tangential component of H is continuous across the boundary between two dielectric media.

Answer:

Step 1

The integral form of Ampere’s law is as follows:

$$\oint_C \mathbf{H}\cdot d\mathbf{l} = I_c + \int_S \frac{\partial \mathbf{D}}{\partial t}\cdot d\mathbf{s}$$

The integral form of Ampere’s law can be also expressed as follows:

$$\oint_C \mathbf{H}\cdot d\mathbf{l} = I_c + I_d$$

$$\oint_C \mathbf{H}\cdot d\mathbf{l} = I$$

where

$I =$ total current

$I_c =$ conduction current

$I_d =$ displacement current

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