ElCoM
ELECTRICAL COMPUTER MECHATRONICS

تقدم لجنة ElCoM الاكاديمية

دفتر لمادة:

كهرومغناطيسية (2)

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CH. 7 :- Plane Wave Propegation

Time varing electric field $\Leftrightarrow$ Time varing magnatic field

[3 - 30] MHz

1) free space 2) guided

* Propegation medium

1) loss less medium (ex. air) ($\sigma = 0$)

2) lossy medium ($\sigma > 0$, $\sigma \neq 0$)

7-1 :- Time harmonic fields

* Time varying $\vec{E}, \vec{H}, \vec{D}, \vec{B}, \rho_v, \vec{J}$ depends on (x, y, z) and time

$$E(x,y,z,t) = Re[\vec{E}(x,y,z)\, e^{j\omega t}]$$

$$\downarrow$$

$$x^{\wedge}E_x + y^{\wedge}E_y + z^{\wedge}E_z$$

$$\vec{\nabla} \cdot D = \rho_v \qquad \qquad \nabla \cdot \vec{B} = 0$$

$$\nabla \times \vec{E} = \frac{-dB}{dt} = j\omega M \vec{H} \quad , \quad \nabla \cdot \vec{E} = \frac{\rho_v}{\epsilon_o}$$

$$\nabla \cdot \vec{H} = 0 \qquad \nabla \times \vec{H} = \vec{J} + j\omega \epsilon \vec{E} = \vec{J} + \frac{d\vec{D}}{dt}$$

Complex parmativity

$$\nabla \times \vec{H} = \sigma \vec{E} + j\omega \epsilon_o \vec{E} = \vec{E}(\sigma + j\omega \epsilon_o) = j\omega\left(\frac{\sigma}{j\omega} + \epsilon_o\right)\vec{E}$$

$$= j\omega\left(\epsilon_o - \frac{j\sigma}{\omega}\right)\vec{E}$$

For lossless medium with $\sigma = 0$

$$\nabla \times \vec{H} = j\omega \epsilon \vec{E}$$

$\epsilon_c = \epsilon - j\frac{\sigma}{\omega} = \epsilon' - j\epsilon''$

$\nabla \times \vec{H} = j\omega\epsilon_c\vec{E}$, $\epsilon' = \epsilon_o$, $\epsilon'' = \frac{\sigma}{\omega}$

For loss less medium $\sigma = 0$, $\epsilon_c = \epsilon' = \epsilon_o$

* wave equation

$\nabla \times (\nabla \times \vec{E}) = -j\omega\mu(\nabla \times \vec{H}) = -j\omega\mu(j\omega\epsilon_c\vec{E}) = \omega^2\mu\epsilon_c\vec{E}$

$\nabla \times (\nabla \times \vec{E}) = \nabla(\nabla \cdot \vec{E}) - \nabla^2\vec{E}$

$\nabla^2\vec{E} = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)\vec{E}$

$\nabla^2\vec{E} - \omega^2\mu\epsilon_c\vec{E} = 0$

γ^3 :- propegation constant.

$\gamma^2 = -\omega^2\mu\epsilon_c$

$\nabla^2\vec{E} - \gamma^2\vec{E} = 0$ (wave equation of $\vec{E}$)

$\nabla^2\vec{H} - \gamma^2\vec{H} = 0$ (wave equation of $\vec{H}$)

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$$\boxed{\nabla^2\vec{E} - \gamma^2\vec{E} = 0} \quad ; \nabla^2\vec{H} - \gamma^2\vec{H} = 0$$

$\gamma^2 \triangleq -w^2\mu\varepsilon_r$

$\varepsilon_r = \epsilon' - j\epsilon'' = \epsilon - j\frac{\sigma}{\omega}$

$$\nabla\cdot\vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$\vec{E} = \hat{x}E_x + \hat{y}E_y + \hat{z}E_z$

$\nabla^2 T = \nabla\cdot(\nabla T) \quad ; \nabla T = \hat{x}\frac{\partial T}{\partial x} + \hat{y}\frac{\partial T}{\partial y} + \hat{z}\frac{\partial T}{\partial z}$

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2}$$

$$\nabla^2\vec{E} = \hat{x}\nabla^2 E_x + \hat{y}\nabla^2 E_y + \hat{z}\nabla^2 E_z$$

$$= \hat{x}\left[\frac{\partial E_x}{\partial x} + \frac{\partial E_x}{\partial y} + \frac{\partial E_x}{\partial z}\right] + \hat{y}\left[\frac{\partial E_y}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_y}{\partial z}\right] + \hat{z}\left[\frac{\partial E_z}{\partial x} + \frac{\partial E_z}{\partial y} + \frac{\partial E_z}{\partial z}\right]$$

*Plane wave propagation in a "lossless mediam"

Propagation properity
- phase voltage (u_p)
- wave length (λ)

goverend by = $\left[\begin{array}{l} \omega \ ; \text{angular frequency} \\ \epsilon \ ; \mu \ ; \sigma \end{array}\right]$ depends on

$\# \sigma = 0 \rightarrow \varepsilon_r = \epsilon - j\cancel{\frac{\sigma}{w}} = \epsilon$

Wave number $\Rightarrow k^2 = -\gamma^2 = -(-w^2\mu\varepsilon) = w^2\mu\epsilon$

$k = w\sqrt{\mu\epsilon} \quad ; \nabla^2\vec{E} + k^2\vec{E} = 0$

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$$\nabla^2 \vec{E} + k^2 \vec{E} = 0$$

$$\Rightarrow \hat{x}\left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right) + \hat{y}\left(\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2}\right) + \hat{z}\left(\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial^2 E_z}{\partial y^2} + \frac{\partial^2 E_z}{\partial z^2}\right) + k^2(\hat{x}E_x + \hat{y}E_y + \hat{z}E_z) = 0$$

$$\rightarrow \hat{x}\left(\frac{\partial^2 E_x}{\partial x^2} + \frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right) + \hat{x} E_x k^2 = 0$$

$$\rightarrow \hat{y}\left(\frac{\partial^2 E_y}{\partial x^2} + \frac{\partial^2 E_y}{\partial y^2} + \frac{\partial^2 E_y}{\partial z^2}\right) + \hat{y} k^2 E_y = 0$$

$$\boxed{k = \omega\sqrt{\mu\epsilon}}$$

$$\rightarrow \hat{z}\left(\frac{\partial^2 E_z}{\partial x^2} + \frac{\partial E_z}{\partial y^2} + \frac{\partial E_z}{\partial z}\right) + \hat{z} k^2 E_z = 0$$

∴ for uniform plane wave $\vec{E}$ and $\vec{H}$ have uniform properties over cell points of an infinite plane ($x-y$ plane)

$$\frac{\partial E_x}{\partial x} = 0 = \frac{\partial E_x}{\partial y} = \frac{\partial E_x}{\partial z} = \frac{\partial E_y}{\partial x} = \frac{\partial E_y}{\partial y}$$

$$= \frac{\partial H_x}{\partial x} = \frac{\partial H_x}{\partial y} = \frac{\partial H_y}{\partial x} = \frac{\partial H_y}{\partial y} = 0$$

$$E_z = H_z = 0$$

$$\nabla \times \vec{H} = j\omega\epsilon\vec{E}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$\nabla \times \vec{E} = -j\omega\mu\vec{H}$$

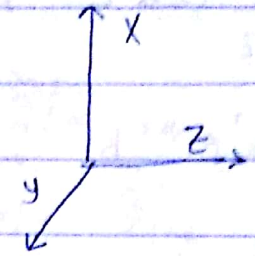
$$\hat{z}\frac{\partial \hat{z}}{\partial x} - \frac{\partial z}{\partial y} = -j\omega\mu\hat{z}H_z$$

*wher are have similar 3-equations

\# consider $\hat{z}$-componant

$$\hat{z}\left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right) = \hat{z} j\omega\epsilon E_z$$

$$\frac{d^2 E_x}{\partial z^2} + k E_x = 0$$



$$\frac{d^2 E_x}{\partial z^2} + k E_y = 0$$

$$\frac{\partial^2 H_x}{\partial z^2} + k^2 H_x = 0$$

$$\frac{d^2 H_y}{\partial z^2} + k^2 H_y = 0$$

assume
we have only E_x ($E_y = 0$) ; we have wave in the direction only

$$E(z) = \hat{x} E(x) = \hat{x} E_{x_0}^{+} e^{-jkz} \quad , \quad E_y = E_z = 0$$

$$\therefore Re\left[\hat{x} E_{x_0}^{+} e^{-jkz} e^{j\omega t}\right]$$

$$E(z) = \hat{x} E_{x_0}^{+} e^{-jkz}$$

$$k^2 = \omega^2 \mu \epsilon$$

$$Re\left[\vec{E}(z) e^{j\omega t}\right]$$ كيف انتقل لـ time domain

$$kz^{\lambda} = 2\pi \rightarrow \lambda = \frac{2\pi}{k}$$

H Field كيف اوجد

$$\nabla \times \vec{E} = j\omega\mu \vec{H}$$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ \vec{E_x} & 0 & 0 \end{vmatrix} = -j\omega\mu(\hat{x} H_x + \hat{y} H_y + \hat{z} H_z)$$

$$\hat{x}(0) + \hat{y}\frac{d\vec{E_x}}{dz} + \hat{z}\underbrace{\frac{d\vec{E_x}}{dy}}_{\text{zero}}$$

$$\Rightarrow \hat{y}\frac{d\vec{E_x}}{dz} = -j\omega\mu H_y \hat{y}$$

$$H_y = \frac{1}{-j\omega\mu}\frac{\partial}{\partial z}\left(E_{x_0}^{+} e^{-jkz}\right) = \frac{\cancel{-}1}{\cancel{j}\omega\mu} E_{x_0}^{+} e^{-jkz}(\cancel{-}\cancel{j}k)$$

$$= \frac{1}{\omega\mu} E_{x_0}^{+} e^{-jkz} k$$

$$H_y = \underbrace{\frac{k}{\omega\mu} E_{x_0}^{+}}_{H_{y_0}^{+}} e^{-jkz}$$

$$\eta \text{ (intrinsic impedance)} = \frac{\omega\mu}{k} = \frac{\omega\mu}{\omega\sqrt{\mu\varepsilon}} = \sqrt{\frac{\mu}{\varepsilon}} \qquad \eta : \text{eata}$$

$$\vec{\tilde{E}}(z) = \hat{x} E_{x_0}^{+} e^{-jkz}$$

$$\vec{\tilde{H}}(z) = \frac{\hat{y} E_{x_0}^{+}}{\eta} e^{-jkz} = \hat{y} H_{y_0}^{+} e^{-jkz}$$

In general $E_{x_0}^{+} = |E_{x_0}^{+}| e^{j\phi}$

$$\boxed{\eta = \frac{\omega\mu}{k} = \sqrt{\frac{\mu}{\varepsilon}}}$$

Instantaneuse (time domain) $\vec{E}$ and $\vec{H}$

$$\vec{E}(z,t) = \mathrm{Re}\left[\vec{\tilde{E}}(z) e^{j\omega t}\right]$$

$$= \mathrm{Re}\left[\hat{x} |E_{x_0}^{+}| e^{j\phi} e^{-jkz} e^{+j\omega t}\right]$$

$$= \hat{x} |E_{x_0}^{+}| \cos(\omega t - kz + \phi) \quad V/m$$

$$\vec{H}(z,t) = \mathrm{Re}\left[\hat{y} \vec{\tilde{H}}(z) e^{j\omega t}\right] = \hat{y} \frac{|E_{x_0}^{+}|}{\eta} \cos(\omega t - kz + \phi) \quad A/m$$

Note :- $\vec{E}$ and $\vec{H}$ are in-phase for loss less medium (since η is real)

$$\omega t - kz = \text{constant} \Rightarrow z = \frac{\omega t - \text{cons}}{k} \Rightarrow \boxed{\frac{\partial z}{\partial t} = \frac{\omega}{k}} = \frac{\omega}{\omega\sqrt{\mu\varepsilon}} = \frac{1}{\sqrt{\mu\varepsilon}}$$

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$$U_p = \frac{w}{k} = \frac{1}{\sqrt{\mu \epsilon_o}} = \frac{2\pi f}{k}$$ $\quad f = \frac{2\pi}{k}$

$$\boxed{\lambda = \frac{2\pi}{k} = \frac{U_p}{f}} \Rightarrow k = \frac{2\pi}{\lambda}$$ $\quad U_p = \frac{w}{k} \quad \lambda = \frac{U_p}{f}$

In free space $\mu = \mu_o = 4\pi * 10^{-7}$; $\epsilon = \epsilon_o = \frac{10^{-9}}{36\pi}$

$$C = \frac{1}{\sqrt{\mu_o \epsilon_o}} \approx 3 * 10^8 \ m/s$$

$$\eta_o = \sqrt{\frac{\mu_o}{\epsilon_o}} = 377\Omega = 120\pi$$

example:- Plane wave ($1MHz$) traveling in the $(+z)$ direction, the electric field points along the peak value of $E = 1.2\pi$ mV/m ; E has a maximum at $(t=0, z=50m)$; find the time domain expression $\vec{E}(z,t)$, $\vec{H}(z,t)$, let $E_x^{\wedge} = x^{\wedge} |E_{xo}^{+}| e^{j\phi} e^{-jkz}$

(ضوئية)

$C = f\lambda \Rightarrow 3*10^8 = 10^6 \lambda \Rightarrow \lambda = 300 \ m$

$k = \frac{2\pi}{\lambda} = \frac{2\pi}{300}$ $\quad Re[x^{\wedge} 1.2\pi e^{j\phi} e^{-jkz} e^{jwt}] = x^{\wedge} 1.2 \cos$

$$\vec{E}(z,t) = x^{\wedge} 1.2\pi * 10^{-3} \cos(2\pi * 10^6 t - \frac{2\pi}{300} z + \phi)$$

Peak at $t=0, z=50m \Rightarrow -\frac{2\pi}{300} * 50 + \phi = 0 \Rightarrow \boxed{\phi = \frac{\pi}{3}}$

$$\vec{E}(z,t) = x^{\wedge} 1.2\pi * 10^{-3} \cos(2\pi * 10^6 t - \frac{2\pi}{300} z + \frac{\pi}{3})$$

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$$\vec{H}(z,t) = \hat{y}\, 1.2\pi * 10^{-3} \cos\left(2\pi * 10^{6} t - \frac{2\pi}{300} + \frac{\pi}{3}\right)$$

~~Example:-~~ General relation between $\vec{E}$ and $\vec{H}$

$$\tilde{\vec{H}} = \frac{1}{\eta} \hat{k} \times \tilde{\vec{E}}$$ uniform plane wave in a direction of unit vector $\hat{k}$.

$$\tilde{\vec{E}} = -\eta\, \hat{k} \times \tilde{\vec{H}}$$

If $\vec{E} = \hat{x} E_x(z) = \hat{x} E_{x0}^{+} e^{-jkz}$ $\quad \hat{k} = \hat{z}$

$$\vec{H} = \frac{1}{\eta} \hat{z} \times \hat{x} E_{x0}^{+} e^{-jkz} = \hat{y} \frac{E_{x0}^{+}}{\eta} e^{-jkz}$$

If the wave is travilling in the $-ve$ (z) direction $\vec{E} = \hat{x} E_{x0}^{+} e^{jkz}$

$\hat{k} = -\hat{z}$

$$\vec{H} = \frac{1}{\eta} -\hat{k} \times \vec{E} = \frac{1}{\eta} -\hat{k} \times \hat{x} E_{x0}^{+} e^{jkz}$$

$$\vec{H} = -\hat{y} \frac{1}{\eta} E_{x0}^{+} e^{jkz}$$

Ingeneral :-

$$\vec{E} = \hat{x} E_x(z) + \hat{y} E_y(z) \qquad \hat{k} = \hat{z}$$

$$\vec{H} = \frac{1}{\eta} \hat{z} \times [\hat{x} E_x(z) + \hat{y} E_y(z)] = \hat{y} \frac{E_x(z)}{\eta} - \hat{x} \frac{E_y(z)}{\eta}$$

$$H_x(z) = \frac{-E_y(z)}{\eta}, \qquad H_y(z) = \frac{E_x(z)}{\eta}$$

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example :- 10 MHZ , plane wave travelling in anon-magnatic material and

$\varepsilon_r = 9$

$\boxed{a}$ phase velocity $\boxed{b}$ wave number (k) $\boxed{c}$ wave length in the medium.

$\boxed{d}$ Interinsic impedant.

C ⇒ سرعة الضوء

$u_p = \frac{w}{k}$

$$\boxed{a}\ U_P = \frac{1}{\sqrt{\mu \varepsilon}} = \frac{1}{\sqrt{\mu \varepsilon_0 \varepsilon_r}} = \ldots$$

$$= \frac{\cancel{w}}{\cancel{w}\sqrt{\mu_0 \varepsilon_0 \varepsilon_r}}$$

$$u_p = \frac{1}{\sqrt{\mu \varepsilon}}$$

$$U_P = \frac{C}{\sqrt{\varepsilon_r}} \Rightarrow \varepsilon_r = \left(\frac{C}{U_P}\right)^2$$

$$u_p = \frac{1}{\sqrt{\mu \varepsilon_0 \varepsilon_r}}$$

$$U_P = \frac{3 * 10^8}{\sqrt{9}} \Rightarrow U_P = 1 * 10^8 \ m/sec$$

$$U_P = \frac{w}{k} \Rightarrow k = \frac{w}{U_P} \Rightarrow K = \frac{2\pi * 10^7}{10^8} = 0.2\pi$$

$$k = w\sqrt{\mu \varepsilon} =$$

$$\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2\pi} = 10m$$

$$\eta = \sqrt{\frac{\mu}{\varepsilon}} = \sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}} = \sqrt{\frac{\mu_0}{\varepsilon_0}}\ \varepsilon_r = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{9}} = 127.67\ \Omega$$

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example:- Givin $\vec{\tilde{E}} = \hat{z}\, 10\, e^{-j4\pi y}$ (mV/m) wave is travilling in a loss less medium with $\eta = 188.5\ \Omega$, Find

[a] magnatic field phasor [3] Instantusly $\vec{E}(y,t)$ if the medium is non magnatic.

$\hat{k} = \hat{y}$

$$\vec{H} = \frac{1}{\eta}\, \hat{k} \times \vec{\tilde{E}} = \frac{1}{1885}\, \hat{y} \times \hat{z}\, 10\, e^{-j4\pi y} = \hat{x}\, 53 e^{-j4/\pi y}$$

$$k = w\sqrt{\mu \epsilon} \Rightarrow w_o = \frac{k}{\sqrt{\mu_o \epsilon_o}\sqrt{\epsilon_r}} = \frac{kc}{\sqrt{\epsilon}}$$

example:- $\vec{H}(z,t) = \hat{z}\ 30\cos(10^8 t - 0.5y)$ mA/m ; In a non magnartic direction of the wave propapulity.

[b] Phase velocity [c] λ [ ] ϵ_r

[e] phasor.

[a] $\hat{k} = \hat{y}$ [B] $UP = \frac{1}{\sqrt{\mu \epsilon}} = \frac{w}{k}$ $k = 0.5$

$$UP = \frac{105}{0.5} = 2 * 10^{8}\ m/sec$$

[c] $UP = f\lambda \Rightarrow \lambda = \frac{2 * 10^{+8}}{0.5} \Rightarrow \lambda = 4\pi$

[d] $\epsilon_r = \left(\frac{C}{UP}\right)^2$

[e] $\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_o}{\epsilon_o \cdot \epsilon_r}} = \frac{\eta_o}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{2.25}} = 251.3\ \Omega$

$$\vec{E} = -\hat{x}\ 25.3 \times 30\cos(10^8 t - 0.5y)$$

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example :- Non-magnatic ($\mu = \mu_o$) ; $\vec{E}(z,t) = \hat{y}\, 3\sin(\pi * 10^7 t - 0.2\pi x)$
$+ \hat{z}\, 4\cos(\pi * 10^7 t - 0.2\pi x)$ V/m

Find : $\boxed{a}$ λ $\boxed{b}$ ϵ_r $\boxed{c}$ $\vec{H}$ time domain

$\boxed{a}$ $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.2\pi} = \boxed{10\,m = \lambda}$

$\boxed{b}$ $k = \omega\sqrt{\mu\epsilon_o} \Rightarrow \epsilon_r = \left(\frac{C}{up}\right)^2 = \left(\frac{3*10^8}{5*10^7}\right) \Rightarrow \boxed{\epsilon_r = 36}$

$\mu = \mu_o \rightarrow$ lose less

$up = \frac{1}{\sqrt{\mu\epsilon}} = \frac{\omega}{k} = \frac{\pi * 10^7}{0.2\pi} = 5 * 10^7$ m/s

$\boxed{c}$ $\vec{H}$ time domain

$\eta = \frac{\eta_o}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{36}} = 62.83\,\Omega$

$\vec{H} = \frac{1}{\eta}\hat{k} \times \vec{E}$; $\hat{k} = \hat{x}$

$= \frac{1}{62.83}\hat{x} \times [\hat{y}\, 3\sin(\pi * 10^7 t - 0.2\pi x) + \hat{z}\, 4\cos(\pi * 10^7 t - 0.2\pi x)$

$= \hat{z}\, 47.7 \sin(\pi * 10^7 t - 0.2\pi x) - \hat{y}\, 63.66 \cos(\pi * 10^7 t - 0.2\pi x)$

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Wave polarization

The locus of the tip of the $\vec{E}$-vector (in the plane orthogonal to direction of propegation) at agivin point in space in space as a function of time.

* General polarization → elliptical

linear circular

$\vec{E} = \hat{x} E_x(z) + \hat{y} E_y(z)$

* Polarization depends on :-

① phase of E_{y0} relative to E_{x0}.

② Absolute value of E_{x0}, E_{y0}.

$E_x(z) = E_{x0}\, e^{-jkz}$

$E_y(z) = E_{y0}\, e^{-jkz}$

$E_{x0} = a_x$ ← reference (phase 0)

$E_{y0} = a_y e^{j\delta}$ $\delta = \delta_y - \delta_x$

$E_{x0} = a_x e^{-jkz}$

$\cancel{E_x(z) = E_{x0} e}$

$\overrightarrow{E(z)} = (\hat{x} a_x + \hat{y} a_y e^{+j\delta})\, e^{-kz}$

$E_{y0} = a_y e^{-jkz} e^{+j\delta}$

* Instautary

$E(z,t) = Re[\overrightarrow{E(x)}\, e^{j\omega t}]$

$= \hat{x} a_x \cos(\omega t - kz) + \hat{y} a_y \cos(\omega t - kz + \delta)$

Modles of $E(z,t)$

① $E(z,t) = [[E_x(z,t)] + E_x^2(z,t)]$

$= E_y(z,t)^{1/2}$

$\psi(z,t) \triangleq \tan^{-1}\left(\dfrac{E_y(z,t)}{E_x(z,t)} \right)$

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linear polarization

Close $Z = 0$

$\delta = 0$ (in phase) , $\delta = -\pi$ (out of phase)

1) $\delta = 0 \Rightarrow E(0,t) = (\hat{x} a_x + \hat{y} a_y) \cos(wt - kz)$

2) $\delta = \pi \Rightarrow \vec{E} = (\hat{x} a_x - \hat{y} a_y) \cos(wt - kz)$

$$|\vec{E}(0,t)| = \sqrt{a_x^2 + a_y^2} \cos(wt - kz)$$

$$\psi(t) = \tan^{-1}\left(\frac{E_y(t)}{E_x(t)}\right) =$$

inphase $\delta \Rightarrow \psi(t) = \tan^{-1}\left(\frac{a_y}{a_x}\right)$

out phase $\Rightarrow \psi(t) = \tan^{-1}\left(\frac{-a_y}{a_x}\right)$

if $a_y = 0 \Rightarrow \psi = 0$ or $180° \Rightarrow$ x-polarized

if $a_x = 0 \Rightarrow \psi = 90°$ or $270 \rightarrow$ y-polarized

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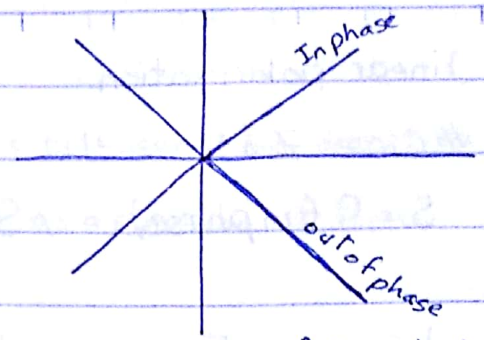
$\hat{x} a_x + \hat{y} a_y e^{j\delta}$, $\delta = 0$, $\delta = \pi$

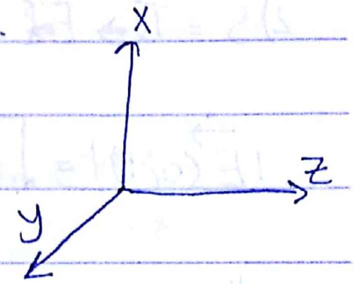
$|E| = \sqrt{a_x^2 + a_y^2} \cos wt$



example :- the electric field is $\vec{E}(z,t) = \hat{x}\, 3\cos(wt - kz) + \hat{y}\, 4\cos(wt - kz)$

Find :- ① Polarization state. ② Modulas of $\vec{E}$.

③ Inclination angle.



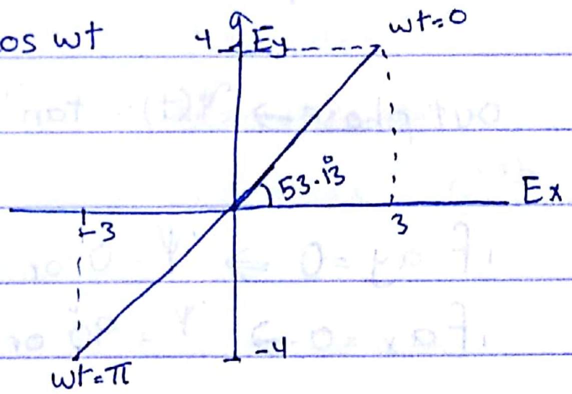
① Polarization state

$\delta = \delta_y - \delta_x = 0 - 0 = 0$

linear polarization (in-phase)

② Modulas of $\vec{E}$

$|\vec{E}| = \sqrt{a_x^2 + a_y^2} \cos wt = \sqrt{3^2 + 4^2} \cos wt = 5 \cos wt$



③ Inclination angle

$\psi = \tan^{-1}\left(\frac{4}{3}\right) = 53.13°$

Circular Polarization

1) $a_y = a_x$ 2) $\delta = \pm \pi/2$ $\begin{cases} -\pi/2 \rightarrow \text{right hand circular (RHC)} \\ \pi/2 \rightarrow \text{left hand circular (LHC)} \end{cases}$

LHC polarization $(a_y = a_x,\ \delta = \pi/2) = \delta_y - \delta_x\ (+z \rightarrow)$

$\vec{E}(z) = (\hat{x} a_x + \hat{y} a_y e^{j\delta})\, e^{-jk\delta}$

$\vec{E}(z,t) = R\,[E_z\, e^{jwt}]$

$= \hat{x} a_x \cos(wt - kz) + \hat{y} a_y \cos(wt - kz + \frac{\pi}{2})$

$\overrightarrow{E}(z,t) = \hat{x}\, a_x \cos(\omega t - kz) + \hat{y}\, a_y \cos(\frac{\pi}{2} - (-\omega t + kz)) \Rightarrow \hat{y}\, a_y \sin(-\omega t + kz)$

$\Rightarrow -\hat{y}\, a_y \sin(\omega t - kz)$

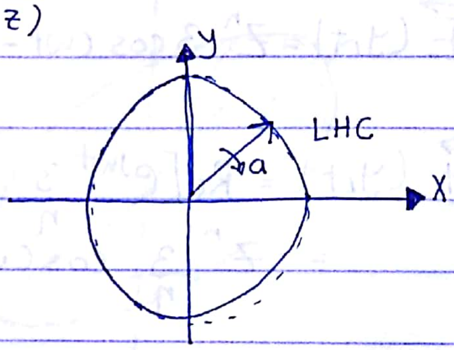
$\overrightarrow{E}(z,t) = \hat{x}\, a_x \cos(\omega t - kz) - \hat{y}\, a_y \sin(\omega t - kz)$

$|E| = \sqrt{E_x^2 + E_y^2} = \sqrt{a_x^2 \cos^2(\omega t - kz) + a_y \sin^2(\omega t - kz)}$ let $a_x = a_y = a$

$= \sqrt{a^2 * 1} = a$ independent on t or z

$\psi(z,t) = \tan^{-1}\left(\frac{E_y(z,t)}{E_x(z,t)}\right) = \tan^{-1}\left(\frac{-a \sin(\omega t - kz)}{a \cos(\omega t - kz)}\right) = \tan^{-1}(-\tan(\omega t - kz))$

$= -(\omega t - kz) = \boxed{kz - \omega t}$

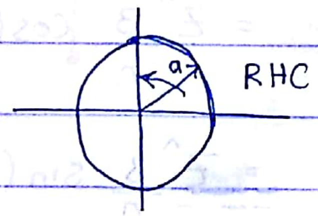


RHC polarization ($a_x = a_y = a$ }, $\delta = -\pi/2$)

$E(z,t) = \hat{x}\, a \cos(\omega t - kz) + \hat{y}\, a \sin(\omega t - kz)$

$|\overrightarrow{E}| = a$

$\psi(z,t) = \omega t - kz$



example :- Find the expresion is (RHC, polarization) $E(y,t)$; $H(y,t)$

modulas = 3 mv/m , medium is non magnatic ($\mu = \mu_0$) ; ($\epsilon = 4\epsilon_0$) ;

$\sigma = 0$, propegation direction (+y direction

$a_x = a_y = 3\ mV/m$, $\delta = -\pi/2 = \delta_x - \delta_z$ $\quad \delta_z = 0$

$\boxed{\delta = -\pi/2 = \delta_x}$

$E(y) = 3(\hat{z} + \hat{x} e^{-j\frac{\pi}{2}}) e^{-jky}$

$H(y) = \frac{1}{\eta} \hat{k} \times \overrightarrow{E} = \frac{1}{\eta} \hat{y} \times 3(\hat{z} + \hat{x} e^{-j\pi/2}) e^{-jky} = \frac{3}{\eta} (\hat{x} \pm \hat{z} e^{-j\pi/2}) e^{-jkz}$

$= \frac{3}{\eta} (\hat{x} + \hat{z} e^{j\frac{\pi}{2}}) e^{-jky}$

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$$k = \omega\sqrt{\mu\epsilon} = \omega\sqrt{\mu_0\epsilon_0}\sqrt{\epsilon_r} = \frac{\omega}{c}\sqrt{\epsilon_r}$$

$$= \frac{2\pi * 10^8}{3 * 10^8}\sqrt{4} = \frac{4\pi}{3}$$

$$\eta = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{4}} = 60\pi$$

$$\vec{E}(y,t) = Re[\vec{E}(y)e^{j\omega t}] = 3[\hat{z}\cos(\omega t - ky) + \hat{x}\cos(\omega t - k\cancel{z}y - \pi/2)]$$
$$+ \hat{x}\sin(\omega t - ky)$$

$$\vec{H}(y,t) = \frac{1}{\eta} * [E y,t] = \frac{3}{60\pi}[\hat{x}\cos(\omega t - ky) - \hat{z}\sin(\omega t - ky)] \ mA/m$$

$$\vec{E}(y,t) = \hat{z}\,3\cos(\omega t - ky) + \hat{x}\,3\sin(\omega t - ky) \ mV/m$$

$$\vec{H}(y,t) = Re[e^{j\omega t}\frac{3}{\eta}(\hat{z}e^{j\frac{\pi}{2}} + \hat{x})e^{-jky}$$

$$= \hat{z}\frac{3}{\eta}\cos(\omega t - ky + \pi/2) + \hat{x}\frac{3}{\eta}\cos(\omega t - ky)$$

$$= \hat{z}\frac{3}{\eta}\cos(\pi/2 - (-\omega t + ky)) + \hat{x}\frac{3}{\eta}\cos(\omega t - ky)$$

$$= \hat{z}\frac{3}{\eta}\sin(-\omega t + ky) + \hat{x}\frac{3}{\eta}\cos(\omega t - ky)$$

$$= -\hat{z}\frac{\cancel{3}1}{20\pi}\sin(\omega t - ky) + \hat{x}\frac{1}{20\pi}\cos(\omega t - ky)$$

example :- if the electric field phasor is givin by $\vec{E}(x) = (y^\wedge - z j) e^{-jkx}$

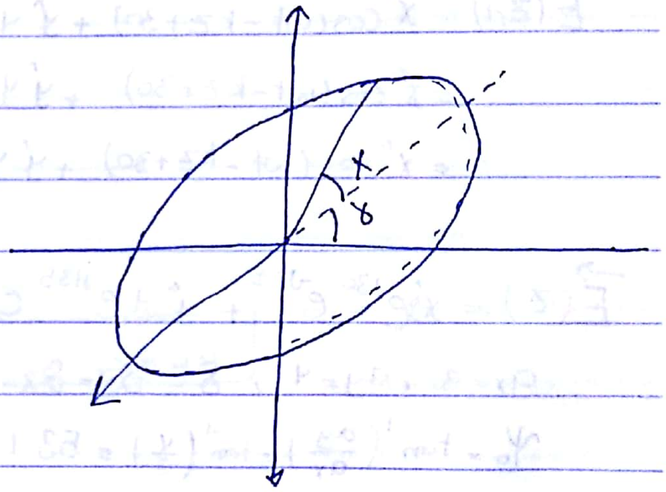
find the polarization state.

$\vec{E} = (x^\wedge a_x + y^\wedge a_y e^{j\delta}) e^{-jkz}$

$\vec{E}(x) = (y^\wedge + z^\wedge e^{-j\pi/2}) e^{-jkx}$

$a_y = a_z = 1$

$\delta = \delta_z - \delta_y = \frac{-\pi}{2} - 0 = -\pi/2 = \delta \Rightarrow$ RHC polarization



Eliptic polarization

$a_x \neq 0, \; a_y \neq 0, \; \delta \neq 0$

γ : rotation angle.

χ : ellipticity angle.

$\vec{\tilde{E}}(z) = (x^\wedge a_x + y^\wedge a_y e^{j\delta}) e^{-jkz}$

$\vec{E}(z,t) = x^\wedge a_x \cos(\omega t - kz) + y^\wedge a_y \cos(\omega t - ky)$

Steps :-

1) $\delta = \delta_y - \delta_x$ 2) $\psi_0 = \tan^{-1}\left(\frac{a_y}{a_x}\right)$

3) Rotation angle

$\tan(2\pi) = Pout = 2\psi \cos(\delta)$

$-\pi \leq 2 \quad \leq \pi \quad \frac{-\pi}{2} \leq \gamma \leq \frac{\ldots}{2}$ $\Rightarrow \gamma > 0$ if $\cos\delta > 0$

$\Rightarrow \delta < 0$ if $\cos\delta < 0$

/ /

4) elliptically angle (χ

$\sin(2\chi) = \sin(2\psi_0)\sin\delta$ $\qquad -\frac{\pi}{4} \leq \chi \leq \frac{\pi}{4}$

$\chi > 0$ if $\sin\delta > 0$ $\qquad -\frac{\pi}{2} < 2\chi < \frac{\pi}{2}$

$\chi < 0$ if $\sin\delta < 0$

example :- Find the polarization state of a plane wave

$$E(z,t) = \hat{x}\,3\cos(\omega t - kz + 30) - \hat{y}\,4\sin(\omega t - kz + 45^\circ)\ \text{mV/m}$$

$$\vec{E}(z,t) = \hat{x}\cos(\omega t - kz + 30) + \hat{y}\,4\sin(-\omega t + kz - 45^\circ)$$

$$= \hat{x}\cos(\omega t - kz + 30) + \hat{y}\,4\cos(90 + \omega t - kz + 45^\circ)$$

$$= \hat{x}\cos(\omega t - kz + 30) + \hat{y}\,4\cos(\omega t - kz + 135^\circ)$$

$-\hat{y}\,4\cos(90 - \omega t + kz - 45)$

$-\hat{y}\,4\cos(45 - \omega t + kz)$

$-\hat{y}\,4\cos(\omega t - kz - 45)$

$-\hat{y}\,4e^{-j45}e^{-jkz}$

$$\vec{E}(z) = \hat{x}\,3e^{j30}e^{-jkz} + \hat{y}\,4e^{j135}e^{-jkz}$$

$\hat{y}\,4e^{j180}e^{-j45}e^{-jkz}$

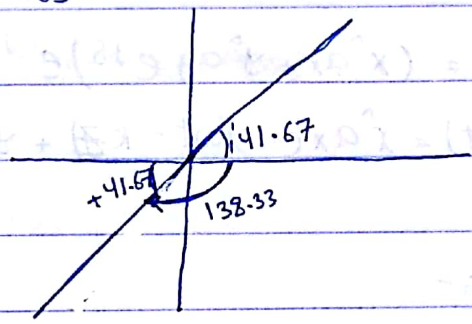
$a_x = 3$, $a_y = 4$, $\delta = \delta_y - \delta_x = 135^\circ - 30^\circ = 105^\circ$

$\psi_0 = \tan^{-1}\left(\frac{a_y}{a_x}\right) = \tan^{-1}\left(\frac{4}{3}\right) = 53.1$

$-\ \tan(2\gamma) = \tan(2\psi_0)\cos\delta \Rightarrow \tan(2\gamma) = 0.89$

$\tan^{-1}(0.89) = 41.67 = 2\gamma \rightarrow \cancel{\gamma} =$

$-(180 - 41.67) = -138.33^\circ$



$\gamma = \begin{cases} 20.83^\circ \\ -69.16^\circ \end{cases}$

$\sin(2\chi) = \sin(2\psi_0)\sin\delta$

$= 0.93$

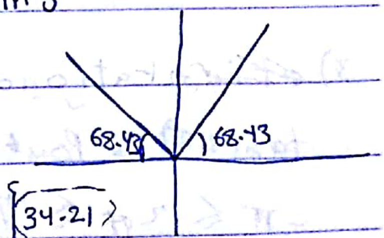
$\gamma > 0$ if $\cos\delta > 0$

$\cos(\delta) = \cos(105) = -ve$

$(0 \rightarrow 90^\circ)\ (2\chi) = 68.43$

$\boxed{\gamma = -69.16}$

$2\chi \begin{cases} 68.43 \\ 180 - 68.43 \end{cases} \Rightarrow \chi \begin{cases} 34.21 \\ \ \end{cases}$



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$$\vec{E}(z,t) = -\hat{x}\,10\sin(\omega t - kz - 60) + \hat{y}\,30\cos(\omega t - kz)$$

$$\therefore -\hat{x}\,10\sin(\omega t - kz - 60) = +\hat{x}\,10\sin(-\omega t + kz + 60) = \hat{x}\,10\cos(\omega t - kz - 60 + 90^\circ)$$

$$= \hat{x}\,10\cos(\omega t - kz + 30)$$

$$\vec{E}(z,t) = \hat{x}\,10\cos(\omega t - kz + 30) + \hat{y}\,30\cos(\omega t - kz)$$

$$\vec{E}(z) = \hat{x}\,10\,e^{j30^\circ}e^{-jkz} + \hat{y}\,30\cos(\omega t - kz)$$

$+ve \rightarrow LHR$

$\delta = \delta_y - \delta_x = 0 - 30^\circ = -30$, $a_x = 10$, $a_y = 30$ $\quad -ve \rightarrow RHR$

$\Psi_0 = \tan^{-1}\left(\frac{30}{10}\right) = 71.56^\circ$

$\tan(2\gamma) = \tan(2\Psi_0)\tan\delta = -0.649$

$(0 \rightarrow 90)$ $2\gamma = 32.98 \Rightarrow 2\gamma = \begin{cases} -32.98 \\ 147.02 \end{cases}$

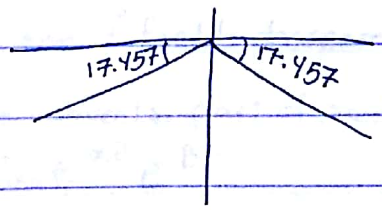
$\cos(\delta) = \cos(-30) = +ve$

$2\gamma = 147.02 \rightarrow \boxed{\gamma = 73.31}$ $\qquad \boxed{-ve \rightarrow RHR}$

$\sin(2\chi) = \sin(2\Psi_0)\sin\delta = -0.3$

$(0 \rightarrow 90^\circ)$ $2\chi = \sin^{-1}(-0.3) = -17.457$

$2\chi = \begin{cases} -17.457 & ✓ \\ -162.54 & \times \end{cases}$ $\qquad 2\chi = -17.457 \Rightarrow \boxed{\chi = -8.728}$



Plane wave propagation in Lossy Medium

$$\nabla^2\vec{E} - \gamma^2\vec{E} = 0 \quad , \gamma^2 = -\omega^2 \mu \epsilon_r = \omega^2 \mu (\epsilon' - j\epsilon'')$$

$\gamma = \alpha + j\beta$ $\qquad \epsilon_0 \qquad \frac{\sigma}{\omega}$

$$(\alpha + j\beta)^2 = \alpha^2 - \beta^2 + j2\alpha\beta = -\omega^2\mu\epsilon' - j\omega\mu\epsilon'')$$

$$\alpha^2 - \beta^2 = -\omega^2\mu\epsilon' \quad , 2\alpha\beta = \omega^2\mu\epsilon''$$

$$\alpha = \omega\left[\frac{\mu\epsilon'}{2}\left[\sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2} - 1\right]\right]^{1/2}$$

$$\beta = \omega\left[\frac{\mu\epsilon'}{2}\left[\sqrt{1+\left(\frac{\epsilon''}{\epsilon'}\right)^2} + 1\right]\right]^{1/2}$$

$$e^{-\alpha z} e^{-j\beta z} \Rightarrow z\alpha\beta - 2\omega^2\left[\left(\frac{\mu\epsilon'}{2}\right)^2\left(\frac{\epsilon''}{\epsilon}\right)^2\right]^{1/2} = 2\omega\left[\frac{\mu^2\epsilon''^2}{4}\right]^{1/2}$$

$\vec{E} = E_{x_0} e^{-\gamma z} = E_{x_0} e^{-\alpha z} e^{-j\beta z}$

$\vec{E} = \hat{x} E_{x_0} e^{-\alpha z} e^{-j\beta z}$

$\nabla \times \vec{E} = -j\omega\mu \vec{H}$

$$\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ d/dx & d/dy & d/dz \\ E_{x_0} e^{-\gamma z} & 0 & 0 \end{vmatrix} = \hat{x}(0) + \hat{y}\left(\frac{d}{dz} E_{x_0} e^{-\gamma z}\right) + \hat{z}(0)$$

$$= -j\omega\mu \left[\hat{x} H_x + \hat{y} H_y + \hat{z} H_z\right]$$

$\hat{y} E_{x_0} (-\gamma) e^{-\gamma z} = \hat{y}(-j\omega\mu H_y)$

$H_y = \frac{\gamma}{j\omega\mu} E_{x_0} e^{-\gamma z} = \frac{E_{x_0}}{\eta_c} e^{-\alpha z} e^{-j\beta z}$

$\cancel{\eta_c = \frac{\gamma}{j\omega\mu}}$ $\quad \eta_c = \frac{j\omega\mu}{\gamma} = \frac{j\omega\mu}{\sqrt{j\omega^2 \mu\epsilon_c}} = \sqrt{\frac{\mu}{\epsilon_c}}$

$\eta_c = \sqrt{\frac{\mu}{\epsilon_c}} = \sqrt{\frac{\mu}{\epsilon' - j\epsilon''}} = \sqrt{\frac{\mu}{\epsilon_r'}} \left(1 - j\frac{\epsilon''}{\epsilon'}\right)^{-1/2}$

$\epsilon' = \epsilon \; , \; \epsilon'' = \frac{\sigma}{\omega}$

\# Both $\vec{E}$ and $\vec{H}$ have no longer equal phase.

\# Both $\vec{E}$ and $\vec{H}$ Fields decrease exponantially with (z).

\# Medium converts part of power into heat.

$\vec{E} = \hat{x} E_{x_0} e^{-\alpha z} e^{-\beta z}$

∴ Skin depth δ_s

$\delta_s = \frac{1}{\alpha}$ at a distance of $z = \delta_s$ magnitude delves by e^{-1}

/ /

δ_s characterizes how well an EM wave can penetrate a medium.

1) Perfect dielectric ($\sigma = 0$)

$\delta_s = \infty$; $\epsilon'' = \sigma / \omega$

$$\alpha = \omega \left[\frac{\mu \epsilon'}{2} \left(\sqrt{1 + \left(\frac{\epsilon''}{\epsilon'}\right)^2} - 1 \right) \right]^{1/2}$$

2) Perfect conductor ($\sigma = \infty$)

$\sigma = \infty \Rightarrow \alpha = \infty \Rightarrow \delta_s = \frac{1}{\alpha} = 0$

* The factor $\frac{\epsilon''}{\epsilon'} = \frac{\sigma}{\omega \epsilon}$ plays an important role in determine how lossy the medium.

1) If $\frac{\epsilon''}{\epsilon'} < 10^{-2}$ ⇒ The medium is a low-lossy medium.

2) If $\frac{\epsilon''}{\epsilon'} > 100$ ⇒ The medium ~~have~~ is a good conductors

3) If $10^{-2} < \frac{\epsilon''}{\epsilon'} < 10^{2}$ → The medium have quasi-conductor.

Low lossy direction

$\gamma^2 = -\omega^2 \mu \epsilon_c \Rightarrow \gamma = j\omega\sqrt{\mu \epsilon_c} \Rightarrow \epsilon_c = \epsilon_r \epsilon$

$\gamma = j\omega \sqrt{\mu \epsilon'} \left(1 - j\frac{\epsilon''}{\epsilon'}\right)^{1/2}$

ϵ'' / ϵ'

$(1-x)^{\frac{1}{2}} \simeq 1 - \frac{x}{2}$

$$\gamma = jw\sqrt{\mu\epsilon'}\,\frac{\epsilon''}{2\epsilon'} + jw\sqrt{\mu\epsilon'}$$

$$= \frac{w\epsilon''}{2}\sqrt{\frac{\mu}{\epsilon'}} + jw\sqrt{\mu\epsilon'} = \alpha + j\beta$$

$$\boxed{\alpha = \frac{\sigma}{2}\sqrt{\frac{\mu}{\epsilon}}} \qquad \boxed{\beta = w\sqrt{\mu\epsilon'}}$$

$$\eta_c = \sqrt{\left[\frac{\mu}{\epsilon'}\left(\frac{1}{1 - j\frac{\epsilon''}{\epsilon'}}\right)\right]}$$

$$\eta_c = \sqrt{\frac{\mu}{\epsilon_0}} \quad \text{at low-loss medium}$$

* Good conductor $\left(\frac{\epsilon''}{\epsilon'} > 100\right\} = \frac{\sigma}{w\epsilon} > 100)$

$$\boxed{\alpha = \beta = \sqrt{\pi f \mu \sigma}}$$

إثبات $\Rightarrow \alpha = w\sqrt{\frac{\mu\epsilon''}{2}} = w\sqrt{\frac{\mu\sigma}{2w}}$

$$= \sqrt{\frac{\mu\sigma w}{2}} = \sqrt{\pi f \mu \sigma} = \beta$$

$$\eta_c = \sqrt{\frac{j\mu}{\epsilon''}} = (1+j)\sqrt{\frac{\pi f \mu}{\sigma}} = (1+j)\frac{\alpha}{\sigma}$$

* القوانين بتنشتق مش حفظ

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example :- plane wave in the +ve z direction, Sea surface $\Rightarrow Z=0$

For sea water $\Rightarrow \epsilon_r = 80 ; \mu_r = 1 ; \sigma = 4$. If the magnatic field at

$Z=0$, is givin by :- $\vec{H}(0,t) = \hat{y}\, 100 \cos(2\pi * 10^3 t + 15°)$ mA/m

$\boxed{a}$ find $\vec{E}(z,t)$, $\vec{H}(z,t)$.

$\boxed{b}$ Find the depth at which amplitude of $\vec{E}$ is 1% of the value at $Z=0$.

$\boxed{a}$ $\vec{E}(z) = -\eta\, \underbrace{\vec{k} \times \vec{H}}_{}$ $\quad \hat{z} \times \hat{y}$ $\qquad 0.01 < \frac{\epsilon''}{\epsilon} < 100$

$= \hat{x}\, E_{x0}\, e^{-\alpha z}\, e^{-j\beta z}$

$\vec{H}(z) = \frac{1}{\eta_c}\, \hat{z} \times \hat{x}\, E_{x0}\, e^{-\alpha z}\, e^{-j\beta z} = \hat{y}\, \frac{E_{x0}}{\eta_c}\, e^{-\alpha z}\, e^{j\beta z}$

$\frac{\sigma}{\omega\epsilon} = \frac{4}{2\pi * 10^3 * \epsilon_0 * 80} = 9 * 10^5 \gg 100$

Good conductor

$\alpha = \sqrt{\pi f \mu \sigma} = \beta = \sqrt{\pi * 10^3 * 4\pi * 10^{-7} * 4}$

$\alpha = 0.126 = \beta$

$\eta_c = (1+j)\frac{\alpha}{\sigma} = (1+j)\frac{0.126}{4} = \sqrt{2}\, e^{+j\frac{\pi}{4}} * \frac{0.126}{4}$

$\eta_c = 0.044\, e^{+j\frac{\pi}{4}}$

$E_{x0} = |E_{x0}|\, e^{j\phi_0}$

$\vec{E}(z,t) = \mathrm{Re}\left[e^{j\omega t}\, \hat{x}\, |E_{x0}|\, e^{j\phi_0}\, e^{-\alpha z}\, e^{-j\beta z}\right]$

$= \hat{x}\, |E_{x0}|\, e^{-\alpha z} \cos(\omega t - \beta z + \phi_0)$

$= \hat{x}\, |E_{x0}|\, e^{-0.126 z} \cos(2\pi * 10^3 t - 0.126 z + \phi_0)$

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$$\vec{H}(z) = \hat{y}\,\frac{|E_{xo}|}{\eta_c}\, e^{-\alpha z} e^{-jBz} e^{j\phi_o}$$

$$= \hat{y}\,\underline{|E_{xo}|\,22.5}\, e^{-0.126} \cos(2\pi * 10^3 t - 0.126z - 45 + \phi_o)$$

$$\vec{H}(0,t) = \hat{y}\, 100 \cos(2\pi * 10^3 t + 15)$$

$$\vec{H}(0,t) = \hat{y}\, 22.5\,|E_{xo}| \cos(2\pi * 10^3 t - 45 + \phi_o)$$

$$15^\circ = -45 + \phi_o \Rightarrow \boxed{\phi_o = 60^\circ}$$

$$E_{xo} = \frac{100 * 10^{-3}}{22.5} = 4.44 * 10^{-3}$$

$$\vec{E}(z,t) = \hat{x}\, 4.44\, e^{-0.126z} \cos(2\pi * 10^3 - 0.126z + 60^\circ)\ mv/m$$

$$\vec{H}(z,t) = \hat{y}\, 100\, e^{-0.126z} \cos(2\pi * 10^3 - 0.126z + 15^\circ)\ mv/m$$

$\boxed{b}$ $0.01 = \dfrac{e^{-\alpha z}}{e^{-\alpha y}}$, $e^{-\alpha z} = 0.01$

$$-\alpha z = \ln^{0.1} \rightarrow z = \frac{\ln^{0.1}}{-\alpha} = \frac{\ln 0.1}{-0.126} \rightarrow \boxed{z = 36m}$$

Example :- Copper parameters , $\mu = \mu_o = 4\pi * 10^{-7}$, $\epsilon = \epsilon_o = \dfrac{10^{-9}}{36\pi}$

$\sigma = 5.8 * 10^7$, over what frequency range copper is a good conductor

$$\frac{\epsilon''}{\epsilon''} = \frac{\sigma}{w\epsilon} > 100 \Rightarrow \frac{w\epsilon}{\sigma} < 0.01 \rightarrow f < \frac{0.01\,\sigma}{2\pi\,\epsilon}$$

$$f < \frac{0.01 * 5.8 * 10^7}{2\cancel{\pi} * \frac{10^{4}}{36\cancel{\pi}}} = f < \frac{5.8 * 10^7}{\frac{200 * 10^{9}}{36}} = 1.044 * 10^{16}\ Hz$$

example :- wave is travilling in a medium with skin depth (δ_s) . find $\frac{E[3\delta_s]}{E[0]}$

$$\frac{E[3\delta_s]}{E[0]} = \frac{E_{x0}\, e^{-\alpha 3 \delta}}{E_{x0}\, e^{-0}} = e^{-\alpha 3 \frac{1}{\alpha}} = e^{-3} = 5\%$$

example :- In a medium with $\epsilon_r = 9$, $\mu_r = 1$, $\sigma = 0.1$. Find the phase angle by which $\vec{H}$ leads $\vec{E}$ at 100 MHz

$$\vec{H} = \frac{1}{\eta_c}\, \hat{k} \times \vec{E}$$

$$\frac{\sigma}{\omega\epsilon} = \frac{0.1}{2\pi \times 10^8 \times 9\epsilon_r} = 2$$

$$\eta_c = \sqrt{\frac{\mu}{\epsilon'}}\left(1 - j\frac{\epsilon''}{\epsilon'}\right)^{-1/2} \Rightarrow = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_r}}\left(1 - j\frac{\sigma}{\omega\epsilon}\right)^{-1/2}$$

$$= \frac{120\pi}{\sqrt{9}}\left(1 - j\,2\right)^{-\frac{1}{2}} = 84.04 \angle 31.72$$

$\theta_{\eta_c} = 31.72$; $\vec{H}$ leads $\vec{E}$ by $(-\theta_{\eta c})$; by $(-31.72$

Example :- Based on measurment at 1 MHz $\delta_d = 2m$, $\eta_c = 28.1 \angle 45^\circ$

$\cancel{28.1 \angle 45 = (1+j)\frac{\alpha\epsilon}{\sigma}}$

ⓐ σ ⓑ wave length in the medium ⓒ Phase volecity in the medium

45° of $\eta \rightarrow$ have good conductor

ⓐ $\sigma \rightarrow \eta_c = (1+j)\frac{\alpha\epsilon}{\sigma} \Rightarrow 28.1 \angle 45 = (1+j)\frac{\alpha\epsilon}{\sigma}$

$$28.1 \cos 45 + j\, 28.1 \sin 45 = \frac{\alpha\epsilon}{\sigma} + j\frac{\alpha\epsilon}{\sigma}$$

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$\alpha = \frac{1}{S_s} = \frac{1}{2} = \boxed{0.5 = \alpha}$

$\sigma = \frac{\alpha}{22.1/\sqrt{2}} \Rightarrow \sigma = \frac{0.5}{22.1/\sqrt{2}} = \sigma = 2.52 * 10^{-2}$

$B = \alpha = 0.5$

(b) $\lambda = \frac{2\pi}{B} = \frac{2\pi}{0.5} = 4\pi = \boxed{12.57\,m = \lambda}$

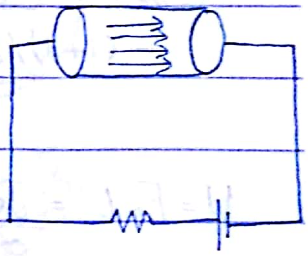
(c) $U_p = \frac{\omega}{B} = \frac{2\pi * 10^6}{0.5} = 4\pi * 10^6 = 1.256 * 10^7 \ m/sec$

$U_p = f\lambda$

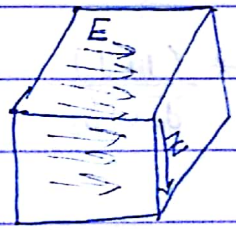
First

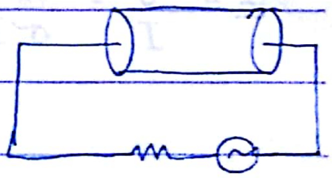
Current flow in a good conductor

→ For DC, current has uniform density over the wire cross section.



→ For time varing current density decreases exponentially towards the axis of wire.





$\vec{E}(0) = \hat{x} E_o$

$E(z) = \hat{x} E_o e^{-\alpha z} e^{-j\beta z}$

$\vec{H} = \frac{1}{\eta} \hat{z} \times \hat{x} E_o e^{-\alpha z} e^{-j\beta z} = \hat{y} \frac{E_o}{\eta_c} e^{-\alpha z} e^{-j\beta z}$

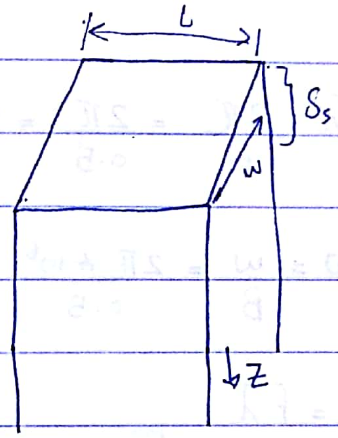
$$J_x(z) = \sigma E_0 \, e^{-\alpha z} e^{-j\beta z} \qquad J_0 = \sigma E_0$$

$$J_x(z) = J_0 \, e^{-\alpha z} e^{-j\beta z} \quad (A/m^2)$$

For good conductor $\left(\alpha = \beta = \frac{1}{\delta_s}\right)$

$$J_x(z) = J_0 \, e^{-(\alpha + j\beta)z} = J_0 \, e^{-(1+j)z/\delta_s}$$

Current flow in a good conductor



$$I = \int_{z=0}^{\infty} J_x(z) \underbrace{(w\,dz)}_{\text{Area}}$$

$$= w \int_0^{\infty} J_0 \, e^{-(1+j)z/\delta_s} dz$$

$$= \left. \frac{J_0 w \, e^{-(1+j)z/\delta_s}}{-(1+j)/\delta_s} \right]_0^{\infty} = \frac{J_0 w \delta_s}{(1+j)}$$

$$V = E_0 L = \frac{J_0}{\sigma} L$$

$$Z = \frac{V}{I} = \frac{J_0 L}{\sigma} * \frac{(1+j)}{J_0 w \delta_s} = \frac{(1+j)}{\sigma \delta_s} \frac{L}{w}$$

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Z_s (Surface impedance)

Impedance for a unit length ($l=1m$) and width ($w=1m$)

$$Z_s = \frac{1+j}{\sigma \delta_s} = R_s + jwL_s$$

surface resistant

$$\boxed{R_s = \frac{1}{\sigma \delta_s}} \Rightarrow R_s = \sqrt{\frac{\pi f \mu}{\sigma}} = \frac{\sqrt{\pi f \mu \sigma}}{\sigma}$$

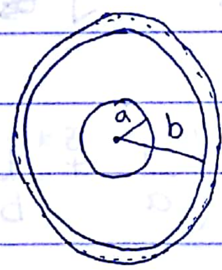
$$wL_s = \frac{1}{\sigma \delta_s} \Rightarrow L_s = \frac{1}{2\pi f \cdot \sigma \delta_s} = \frac{1}{2}\sqrt{\frac{\mu}{\pi f \sigma}}$$

$$R = R_s \frac{l}{\text{width}} = \frac{1}{\sigma \delta_s} \frac{l}{\text{width}}$$

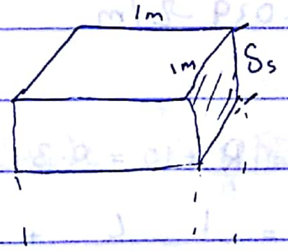
Coaxial cable reactance

For copper $\sigma = 5.8 * 10^{+7}$, Assume $f = 1MHz$

$$\delta_s = \frac{1}{\sqrt{\pi f \mu \sigma}} = 0.066 \text{ mm}$$

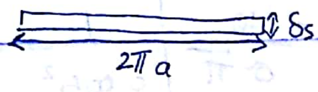


$$R' = \frac{R}{L}$$



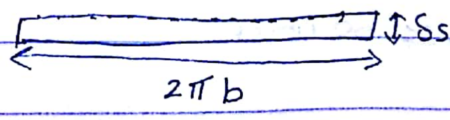
$$R = \frac{R_s \, l}{\text{Width}}$$

$$R' = \frac{R_s}{\text{width}}$$



$$R' = R_{in} + R_{out}, \quad R_{in} = \frac{R_s}{2\pi a}, \quad R_{out} = \frac{R_s}{2\pi b}$$

$$R' = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right) \Omega/m$$



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example :- Copper Coxial cable $\sigma = 5.8 * 10^{7}$, $\epsilon_r = 1$, $\mu_r = 1$,
outer thickies = 0.5mm, a = 0.5cm, b = 1cm, Find:-
① Surface resistance (R_s) ② Ac resistance at 10 MHz ③ R_{AC}/R_{DC}

$\frac{\sigma}{w\epsilon} \gg 1$ good conductor

$$\delta_s = \frac{1}{\alpha} = \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi * 10^{7} * 4\pi * 10^{-7} * 5.8 * 10^{7}}} = 0.021 \text{ mm}$$

$$\frac{d}{\delta_s} = \frac{0.5\text{mm}}{0.021} \simeq 25 \text{ (very thick)}$$

$$R_s = \frac{1}{\sigma \delta_s} = \sqrt{\frac{\pi f \mu}{\sigma}} = \frac{1}{5.8 * 10^{7} * 2.1 * 10^{-5}} = 8.4 * 10^{-4} \Omega$$

$$R' = \frac{R_s}{2\pi}\left(\frac{1}{a} + \frac{1}{b}\right) = \frac{8.4 * 10^{-4}}{2\pi}\left(\frac{1}{2} + 1\right) * 10^{+3}$$

$= 0.039 \ \Omega/m$

$R_{AC}(10m) = R' * 10 = 0.39 \ \Omega$

$$R_{DC}(10m) = \frac{1}{\sigma}\frac{L}{A_{in}} + \frac{1}{\sigma}\frac{L}{A_{out}}$$

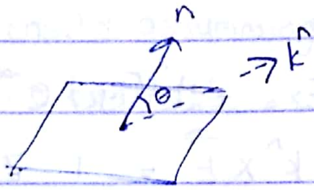
$$= \frac{10}{\sigma \pi}\left[\frac{1}{0.005^2} + \frac{1}{0.01^2 - 0.0045^2}\right] = 7.624 * 10^{-3}$$

$\frac{R_{AC}}{R_{DC}} \simeq 50$ times

Power Density

Poynting vector $\vec{S}$

$\vec{S} = \vec{E} \times \vec{H}$ (time domain) (w/m^2)



$P = \int_A \vec{S} \cdot \vec{n} \, dA \qquad = SA\cos\theta$

surface normal

Time average power density

$\vec{S}_\omega = \frac{1}{2} \mathrm{Re}[\vec{E} \times \vec{H}^* \quad (w/m^2$

For a loss less medium

$\vec{E}(z) = (\hat{x} E_{x_0} + \hat{y} E_{y_0}) e^{-jkz}$

$\vec{H}(z) = \frac{1}{\eta} k \hat{x} \vec{E} \Rightarrow \frac{1}{\eta} \hat{z} \times (\hat{x} E_{x_0} + \hat{y} E_{y_0}) e^{-jkz}$

$= \frac{1}{\eta} (\hat{y} E_{x_0} - \hat{x} E_{y_0}) e^{-jkz}$

$S = \frac{1}{2} \mathrm{Re}[\vec{E} \times \vec{H}] = \frac{1}{2} \mathrm{Re}[(\hat{x} E_{x_0} + \hat{y} E_{y_0}) e^{-jkz} \times (\frac{1}{\eta} [\hat{y} E_{x_0} - \hat{x} E_{y_0}] e^{-jkz})$

$= \frac{1}{2} \mathrm{Re}\left[\frac{\hat{z} E_{x_0}^2}{\eta} + \frac{\hat{z} E_{y_0}^2}{\eta}\right] \quad [\vec{E}^2 = E_{x_0}^2 + E_{y_0}^2]$

$= \cancel{\frac{1}{2} \mathrm{Re}[\hat{z}} = \hat{z} \frac{\vec{E}}{\eta}$

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lossy medium

$$\vec{E}(z) = (\hat{x} E_{xo} + \hat{y} E_{yo}) e^{-\alpha z} e^{-j\beta z}$$

$$\vec{H} = \frac{1}{\eta_c} \hat{k} \times \vec{E} = \frac{1}{\eta_c} \hat{z} \times (\hat{x} E_{xo} + \hat{y} E_{yo}) e^{-\alpha z} e^{-j\beta z}$$

$$= \frac{1}{\eta_c} [\hat{y} E_{xo} - \hat{x} E_{yo}] e^{-\alpha z} e^{-j\beta z}$$

$$\vec{S}_{av} = \frac{1}{2} Re[(\hat{x} E_{xo} + \hat{y} E_{yo}) e^{-\alpha z} \cancel{e^{-j\beta z}}] \times \frac{1}{\eta_c} [\hat{y} E_{xo} - \hat{x} E_{yo}] e^{-\alpha z} \cancel{e^{j\beta z}}$$

$$= \frac{1}{2} Re [(\hat{z} E_{xo}^2 + \hat{z} E_{yo}^2) e^{-2\alpha z} \qquad \eta_c = \eta_c e^{j\theta_n}$$

$$\vec{S_n} = \hat{z} \frac{|E_o|^2}{2|\eta_c|} e^{-2\alpha z} \cos(\theta_n)$$

$$\boxed{\text{problems :- } 1, 3, 4, 6 \to 11, 14, 15, 16, 19, 21, 23 \to 28}$$

$$\vec{S} = \vec{E} \times \vec{H}$$

$$\vec{S}_{av} = \frac{1}{2} Re [\vec{E} \times \vec{H}^*] = \hat{z} \frac{|E|^2}{2\eta} \quad (w/m^2) \quad \text{loss less}$$

$$= \hat{z} \frac{|E_o|^2}{2|\eta_c|} e^{-2\alpha z} \cos(\theta_n)$$

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Example :- A submarince at depth 200m uses a wire antenna to recive at 1kHz. Find the average power density assumming $|E_o| = 4.44 \, mV/m$; $\alpha = 0.126$; $\eta_c = 0.044 \angle 45^\circ$

$$\vec{S}_{av}(z) = \hat{z} \frac{|E_o|^2}{2|\eta_c|} e^{-2\alpha z} \cos(\theta_n)$$

$$= \hat{z} \frac{(4.44 * 10^{-3})^2}{2 * 0.044} e^{-0.126 z} \cos(\theta_n) = 2.1 * 10^{-26} \, w/m^2$$

Example :- A wave travelling in a non-magnatic medium with $\epsilon_r = 9$ has $\vec{E}$-Field $\vec{E}(z,t) = \hat{y} \, 3\cos(\pi * 10^7 t + kx) - \hat{z} \, 2\cos(\pi * 10^7 t + kx)$

Find :-

① directing of prop $\rightarrow$ -ve $\hat{x}$ $(-\hat{x})$.

② average power carried by wave.

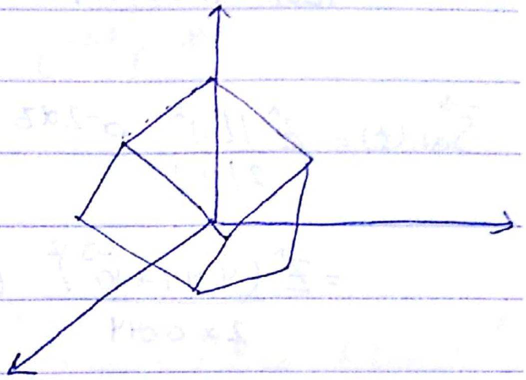
$$\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_o}{\epsilon_o \epsilon_r}} = \frac{\eta_o}{\sqrt{\epsilon_r}} = \frac{120\pi}{3} = 40\pi$$

$$\vec{S}_{av} = \hat{k} \frac{|E_o|^2}{2\eta} = -\hat{x} \frac{(3^2 + 2^2)}{2 * 40\pi} = -\hat{x} \, 0.05 \left(\frac{W}{m^2}\right)$$

Example:- plane wave $\vec{E} = \hat{x}\, 100\, e^{-20y} \cos(2\pi * 10 t - 40 y)$

$\vec{H} = -\hat{z}\, 0.64\, e^{-20y} \cos(2\pi * 10 t - 40 y - 36.85^\circ)$

$$\vec{H} = \frac{1}{\eta_c} k\, \vec{E}$$



ⓐ Find the power density of the wave.

ⓑ Find the avarge power entiring the box.

$$\vec{S} = \vec{E} \times \vec{H}$$

$$= \hat{x}(100\, e^{-20y} \cos(wt - 40y)) \times (-\hat{z}\, 0.64 e^{-20y} \cos(wt - 40y - 36.85))$$

$$= \hat{y}\, 64\, e^{-40y}\, \frac{1}{2}\left[\cos(2wt - 80y - 36.85) + \cos(36.85)\right]$$

Power entrey $= a_c\,[\vec{S}_{av}|_{y=0} - S_{av}|_{y=b}]$

$$\vec{S}_{av} = \hat{y}\, \frac{E_o^2}{2\eta_c}\, e^{-2\alpha y} \cos(\theta_n) = a_c \times \frac{E_o^2}{2\eta_c}\,(1 - e^{-2\alpha b}) \cos(\theta_n)$$

$$\vec{H} = \frac{1}{\eta_c}\, \hat{k}\, \vec{E} \Rightarrow -\hat{z}\, 0.64\, \cancel{e^{-20y}}\, e^{-j(40y + 36.85^\circ)} \; \cancel{\cos 2\pi *} = \frac{1}{\eta_c}\, \hat{y} \times \hat{x}\, 100\, \cancel{e^{-20y}}\, e^{-j40y}$$

$$\eta_c = \frac{100}{0.64}\, e^{+j36.85}$$

$$a_c\, \frac{100^2}{2\, \frac{100}{0.64}} \cos(36.85^\circ)\,(1 - e^{-40b}) \text{ watt}$$

$$\vec{S}_{av} = \frac{1}{2} Re\left[\hat{x}(100\, \cancel{e^{-20y}}\, \cancel{e^{-j40y}}) \times \hat{z}(0.64\, e^{-j36.85}\, \cancel{e^{-20y}}\, \cancel{e^{-j40y}})\right)$$

$$= \frac{1}{2}\, \hat{y}\, 64 \cos(36.85^\circ)\, e^{-40y}$$

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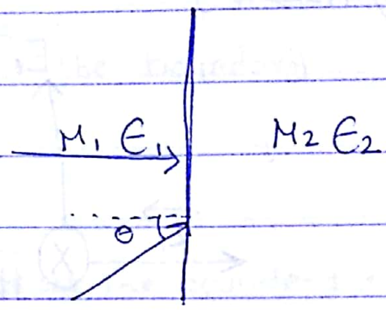
CH.8 :- Wave Reflection and Transmition

* Normal incidence (lossless medium)

$\sigma = 0$



The bondary surface is $(Z=0)$

medium - 1 : $Z < 0$

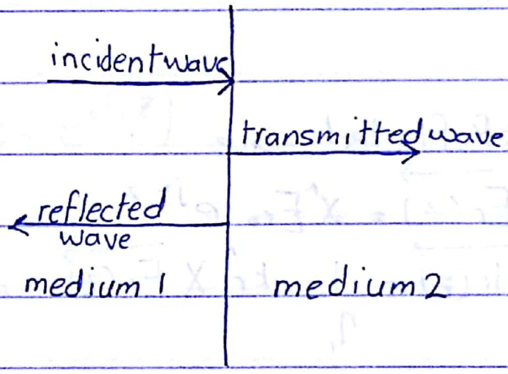
medium - 2 : $Z \geq 0$

In medium

we have x-polarized wave.

$(\vec{E_i}, \vec{H_i})$ i: incident

k_i : direction of incident wave $= \hat{Z}$

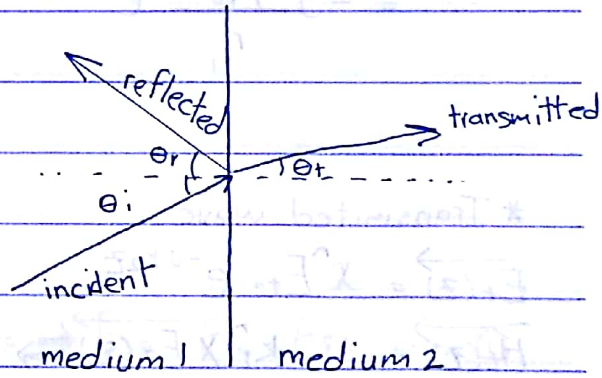


Reflected wave

$(\vec{E_r}, \vec{H_r})$ $\hat{k_r} = -\hat{Z}$

Transmitied wave

$(\vec{E_t}, \vec{H_t})$ $\hat{k_t} = \hat{Z}$

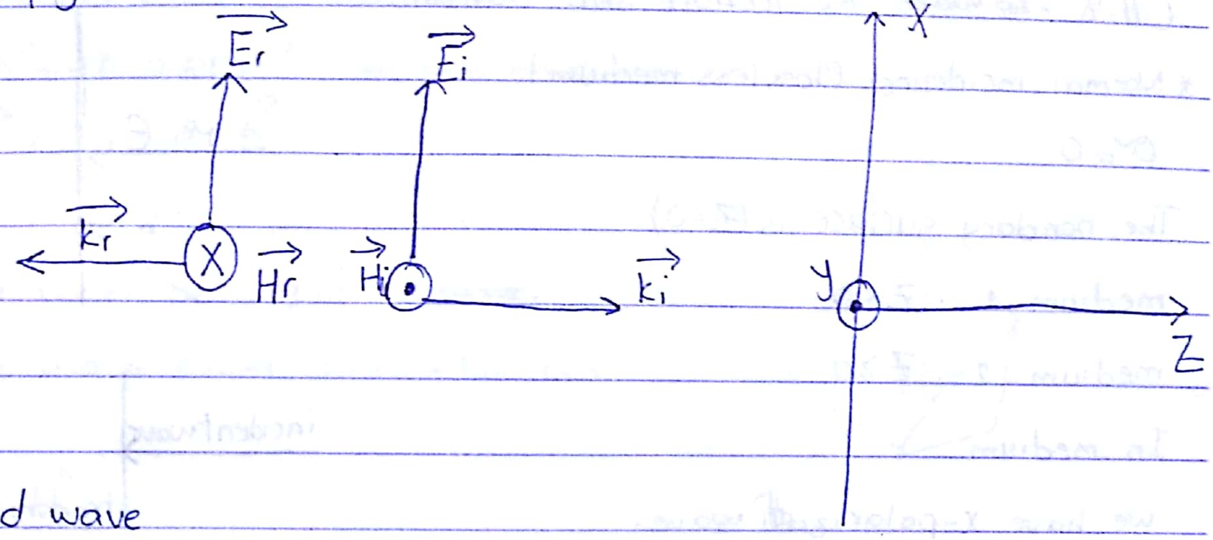


Incident wave

$\vec{E_i}(z) = \hat{x} E_{io} \, e^{-jk_1 z}$

$H_i(z) = \frac{1}{\eta_1} \hat{k_i} \times \vec{E_i} = \frac{1}{\eta} \hat{Z} \times \hat{x} E_{io} \, e^{-jk_1 z}$

$= \frac{1}{\eta} \hat{y} E_{io} \, e^{-jk_1 z}$



$\odot H \Rightarrow +\hat{y}$

*Reflected wave

$\overrightarrow{E_r(z)} = \hat{x} E_{r0}\, e^{jk_1 z}$

$\overrightarrow{H_r(z)} = \frac{1}{\eta_1} \hat{k_r} \times \overrightarrow{E_r(z)} = \frac{1}{\eta_1} (-\hat{z}) \times \hat{x} E_{r0}\, e^{jk_1 z}$

$= -\hat{y} \frac{E_{r0}}{\eta_1} e^{jk_1 z}$

μ_1	μ_2
ϵ_1	ϵ_2
$k_1 = w\sqrt{\mu_1 \epsilon_1}$	$k_2 = w\sqrt{\mu_2 \epsilon_2}$
$\eta_1 = \sqrt{\frac{\mu_1}{\epsilon_1}}$	$\eta_2 = \sqrt{\frac{\mu_2}{\epsilon_2}}$

*Transmited wave

$\overrightarrow{E_t(z)} = \hat{x} E_{t0}\, e^{-jk_2 z}$

$\overrightarrow{H_t(z)} = \frac{1}{\eta_2} \hat{k_t} \times \overrightarrow{E_t(z)} \Rightarrow \frac{1}{\eta_2} \hat{z} \times \hat{x} E_{t0}\, e^{-jk_2 z}$

$\Rightarrow \frac{1}{\eta_2} \hat{y} E_{t0}\, e^{-jk_2 z}$

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Relating E_{ro} and E_{to} to E_{io}

All wave have E H fields tanyontional to the boundary.

Boundary condition :-

Tanyential $\vec{E}$ and $\vec{H}$ fields are continous accross the boundary.

Medium - 1

$$\vec{E_1(z)} = \vec{E_i(z)} + \vec{E_r(z)} = \hat{x}\left(E_{io}\, e^{-jk_1 z} + E_{ro}\, e^{jk_1 z}\right)$$

$$\vec{H_1}(z) = \hat{y}\left(\frac{E_{io}}{\eta_1} e^{-jk_1 z} - \frac{E_{ro}\, e^{jk_1 z}}{\eta_1}\right)$$

Medium - 2

$$\vec{E_2(z)} = \hat{x}\, E_{to}\, e^{-jk_2 z}$$

$$\vec{H_2(z)} = \hat{y}\, \frac{E_{to}}{\eta_2}\, e^{-jk_2 z}$$

* Bondary condition

$\boxed{A}$ $E_1(z=0) = E_2(z=0) \Rightarrow \hat{x}(E_{io} \mp E_{ro}) = E_{To} \rightarrow E_{io} - E_{ro} = E_{To}$ .... (1)

$\cancel{\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{To}}{\eta_2}}$

$\boxed{B}$ $\vec{H_1(0)} = \vec{H_2(0)} \Rightarrow \hat{y}\left(\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1}\right) = \hat{y}\,\frac{E_{To}}{\eta_2}$ ... (2)

$$\frac{E_{io}}{\eta_1} - \frac{E_{ro}}{\eta_1} = \frac{E_{io}}{\eta_2} - \frac{E_{ro}}{\eta_2} \Rightarrow \frac{E_{io}}{\eta_1} - \frac{E_{io}}{\eta_2} = \frac{E_{ro}}{\eta_1} + \frac{E_{ro}}{\eta_2}$$

$$E_{io}(n_2 - n_1) = E_{ro}(n_2 + n_1)$$

$$\boxed{\frac{E_{io}}{E_{ro}} = \frac{n_2 + n_1}{n_2 - n_1}} = \Gamma \text{ (reflection coeffitiant)}$$

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$$\frac{E_{io}}{\eta_1} - \left(\frac{E_{To} - E_{io}}{\eta_1}\right) = \frac{E_{To}}{\eta_2}$$

$$E_{io}\left(\frac{2}{\eta_1}\right) = E_{To}\left(\frac{1}{\eta_2} + \frac{1}{\eta_1}\right)$$

$$E_{io}\left(\frac{2}{\cancel{\eta_1}}\right) = E_{To}\left(\frac{\eta_1 + \eta_2}{\cancel{\eta_1}\,\eta_2}\right) \Rightarrow \frac{E_{To}}{E_{io}} = \frac{2\eta_2}{\eta_1 + \eta_2}$$

τ (transmition coficient) $= \dfrac{2\eta_2}{\eta_1 + \eta_2}$

$$\tau = 1 + \Gamma$$

* For non magnatic medies $\mu_1 = \mu_2 = \mu_0$

$$\eta = \sqrt{\frac{\mu}{\epsilon}} \quad ; \quad \eta_1 = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_{r_1}}} \quad , \quad \eta_2 = \sqrt{\frac{\mu_0}{\epsilon_0 \epsilon_{r_2}}}$$

$$\Gamma = \frac{\sqrt{\epsilon_{r_1}} - \sqrt{\epsilon_{r_2}}}{\sqrt{\epsilon_{r_1}} + \sqrt{\epsilon_{r_2}}}$$ non-~~mag~~magnetic

Power flows in Loss less medim

$$\vec{S}_{av_1} = \frac{1}{2} \, Re\left[\vec{E_1} \times \vec{H_1^*}\right]$$

$$\vec{E_1} = \hat{x}\left[E_{io} e^{-jk_1 z} + \underbrace{\Gamma E_{io}}_{E_{ro}} e^{+jk_1 z}\right]$$

$$\vec{H} = \hat{y}\left[\frac{E_{io}}{\eta_1} e^{-jk_1 z} + \frac{\Gamma E_{io}}{\eta_1} e^{jk_1 z}\right]$$

$$\vec{H_1^*} = \hat{y}\left[\frac{E_{io}^*}{\eta_1} e^{jk_1 z} + \Gamma^* \frac{E_{io}^*}{\eta_1} e^{-jk_1 z}\right]$$

$$S_{av_1} = \frac{1}{2}\left[\hat{z}\left[\frac{|E_{io}|^2}{\eta_1}\left(1 \cancel{+} |\Gamma|^2 - \Gamma^* e^{-j2k_1 z} + \Gamma e^{j2k_1 z}\right)\right]\right]$$

$$\rightarrow |\Gamma| e^{j\theta_r} e^{j2k_1z} - |\Gamma| e^{-j\theta_r} e^{-j2k_1z} \quad * \frac{2j}{2j}$$

$$= \frac{1}{2} Re\left[\hat{z} \left[\frac{|E_{io}|^2}{\eta_1} \left(1 - |\Gamma|^2 + 2j\sin(2k_1z + \theta_r) \right) \right] \right]$$

$$\vec{S}_{av_1} = \hat{z} \frac{|E_{io}|^2}{2\eta} (1 - \Gamma)^2$$

$$\vec{S}_{av_i} = \hat{z} \frac{|E_{io}|^2}{2\eta_1} \quad , \quad \vec{S}_{av_r} = -\hat{z} |\Gamma^2| \frac{|E_{io}|^2}{2\eta_1} = -|\Gamma^2| \vec{S}_{av_i}$$

$$\vec{S}_{av_r} = -|\Gamma^2| \vec{S}_{av_i}$$

$$\vec{S}_{av_2} = \frac{1}{2} Re\left[\vec{E}_t \times \vec{H}_t^* \right] = \frac{1}{2} Re\left[\hat{x} \tau E_{io} e^{-jk_2z} \times \hat{y} \tau^* \frac{E_{io}^*}{\eta_2} e^{jk_2z} \right]$$

$$= \frac{1}{2} Re\left[\hat{z} |\tau|^2 \frac{|E_{io}|^2}{\eta_2} \right]$$

$$\vec{S}_{av_2} = \hat{z} |\tau|^2 \frac{|E_{io}|^2}{2\eta_2}$$

$$\vec{S}_{av_1} = \vec{S}_{av_2} \rightarrow \boxed{\frac{|\tau|^2}{\eta_2} = \frac{1 - |\Gamma|^2}{\eta_1}} \quad \text{Loss less}$$

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example :- $\epsilon_{r1} = 9$, normal incedunt, $\epsilon_{r2} = 4$, both media are non magnatic givin $H_i(y,t) = \hat{z}\, 2\cos(2\pi * 10^9 t - ky)$ (A/m)

Find :-

[a] time domain expression for E, H in each of the two media.

$$\vec{E}(y,t) = -\eta_1 \vec{k_i} \times \vec{H_i}$$

$$\eta_1 = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{120\pi}{3} = 40\pi\ \Omega$$

$$k_1 = \frac{\omega}{C}\sqrt{\epsilon_r} = \frac{2\pi * 10^9}{3 * 10^8}\sqrt{9} = 20\pi$$

$$\vec{E}(y,t) = -\hat{x}\, 251.34 \cos(2\pi * 10^9 t - 20\pi y)$$

$$k_2 = \frac{\omega\sqrt{\epsilon_{r2}}}{C} = \frac{40\pi}{3}$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\epsilon_{r2}}} = \frac{120\pi}{2} = 60\pi\ \Omega$$

$$\Gamma = \frac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \frac{20\pi}{100\pi} = \frac{1}{5}$$

$$\tau = 1 + \Gamma \Rightarrow \boxed{\tau = 1.2}$$

$$\vec{E_r}(t,y) = \Gamma E_i(y,t) = -\hat{x}(50.27 \cos(2\pi * 10^9 t + 20\pi y)$$

$$E_1(y,t) = E_i(y,t) + E_r(y,t)$$

$$= -\hat{x}(251.34 \cos(2\pi * 10^9 t - 20\pi y) + \hat{x}(50.27 \cos(2\pi * 10^9 t + 20\pi y)$$

$\eta_2 = 60\pi$

$E_t(y) = -\hat{x}(\tau\, 40\pi) e^{-jk_2 y}$.

$k_2 = 41.917$

$$\vec{E_t}(y,t) = -\hat{x}(96\pi \cos(2\pi * 10^9 t - k_2 y))$$

$$= -\hat{x}\boxed{301.593 \cos(2\pi * 10^9 t - 41.92y)}$$

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$$\vec{H_r}(y,t) = \frac{1}{\eta_1} \hat{k_r} \times \vec{E_r} = \frac{1}{40\pi}\left(-\hat{y} \times -\hat{x}\ \overset{16\pi}{\cancel{80.27}} \cos(\omega t + 20\pi y)\right)$$

$$= \frac{16}{40\pi}\left(-\hat{z}\cos(\omega t + 20\pi y)\right) = -\hat{z}\,0.4 \cos(\omega t + 20\pi y)\ (A/m)$$

$$\vec{H_t}(y,t) = \frac{1}{\eta_2} \hat{k_t} \times \vec{E_t} = \frac{1}{60\pi}\left(\hat{y} \times -\hat{x}\ 96\pi \cos\left(\omega t \mp \frac{40\pi}{3} y\right)\right)$$

$$= \hat{z}\,1.6 \cos\left(\omega t - \frac{40\pi}{3} y\right) (A/m)$$

$\boxed{2}$ $$S_{avi} = \frac{\hat{y}\,|E_{io}|^2}{2\eta_1} = \frac{\hat{y}(80\pi)^2}{2 * 40\pi} = \hat{y}\,80\pi \ \ w/m^2$$

$$\vec{S_{avr}} = -\hat{y}\,\Gamma^2 \vec{S_{avi}} = -\hat{y}\,(0.04)\,80\pi = -\hat{y}\,10.05 \ \ w/m^2$$

$$\vec{S_{avt}} = \hat{y}\,\tau^2 \frac{E_i^2}{2\eta_2} = \hat{y}\ 1.44 \frac{(80\pi)^2}{2 * 60\pi} = \hat{y}\,24.127 \ \ w/m^2$$

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Example :- A beam of yellow light with wave length 0.6 μm is normally incident in air upon a glass surface. Assume the glass is sufficiently thick as to ignore its back surface. if the surface is situated in the plane $z = 0$ and the relative permittivity of glass is 2.25, determine :-

① the location of the electric field maxima in medium !

② the standing wave ratio

③ the fraction of incident power transmitted into the glass medium.

① $\eta_1 = \dfrac{\eta_0}{\sqrt{\epsilon_{r_1}}} = 120\pi$; $\eta_2 = \dfrac{120\pi}{\sqrt{2.25}} = 251.327 = 80\pi$

$\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1} = \dfrac{-40\pi}{200\pi} = -0.2$

$\tau = 1 + \Gamma = 0.8$

$S = \dfrac{1 + |\Gamma|}{1 - |\Gamma|} = \dfrac{1.2}{0.8} = 1.5$

③ $\dfrac{S_{av_t}}{S_{av_i}} = \dfrac{\tau^2 \frac{E_{io}^2}{\eta_2}}{\frac{E_{io}^2}{\eta_1}} = \dfrac{\eta_1}{\eta_2} \tau^2 \Rightarrow \dfrac{120\pi}{80\pi} \, 0.64 = 0.96 = 96\%$

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ex:- A 1 GHz $\hat{x}$ polarization plane wave in Air travilling in the (+z) direction is insident on the (x-y) plane (z=0) with E_{io} = 12mV on amaterial surface with ($\mu_r = 1$, $\epsilon = 1$, $\sigma = 5.8 * 10^7$ S/m) .obtain time domain expression for $\vec{E}$, $\vec{H}$ in the meadium

$$\vec{E_i(z)} = \hat{x} E_{io} e^{-jk_1 z}$$

$$C = \frac{w}{k} = up^2$$

$$k_1 = \frac{w}{C}$$

$$k_1 = \frac{2\pi}{\lambda}$$

$$n_1 = n_o = 120\pi = 377$$

$$\frac{\sigma}{w\epsilon} = \frac{5.8 * 10^7}{2\pi * 10^9 * \frac{1}{36\pi} * 10^{-4}} = 10^4 \gg 100$$

good Conductor

$$n_{c2} = (1+j)\sqrt{\frac{\pi f \mu}{\sigma}} = (1+j)\, 8.25 * 10^{-3} \Omega$$

$$\Gamma = \frac{n_{c2} - n_{c1}}{n_{c2} + n_{c1}} = \frac{(1+j)8.25 * 10^{-3} - 377}{(1+j)8.25 * 10^{-3} + 377}$$

$$= -1$$

$$\tau = 1 + \Gamma = 0$$

$$\vec{E_1(z)} = \hat{x} E_{io} \left(e^{-jk_1 z} - e^{jk_1 z}\right) \frac{2j}{2j} = -\hat{x} E_{io}(2j) \sin k_1 z$$

$$\vec{E_1(z,t)} = -\hat{x}\, 24 \cos\left(wt - \frac{\pi}{2}\right) \sin k_1 z$$

$$= \hat{x}\, 24 \sin\left(\frac{20\pi}{3} z\right) \sin(2\pi * 10^9 t) \text{ mV/m}$$

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$$\tilde{\vec{H}}_1(z,t) = \frac{1}{\eta} \overbrace{\hat{z} \times \hat{x} E_{io}}^{\frac{1}{\eta} k \times \vec{E}} e^{-jk_1 z} - \frac{1}{\eta} (-\hat{z}) \times \hat{x} E_{io} e^{jk_1 z}$$

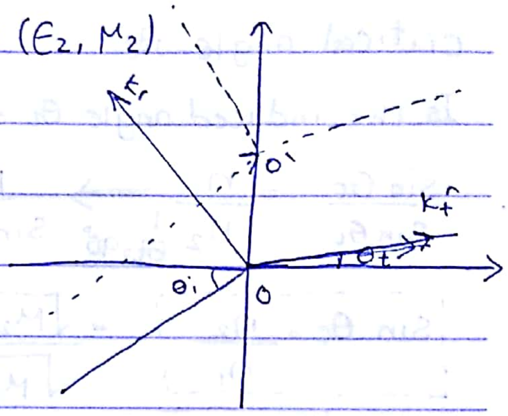
$$= \frac{\cancel{8}}{\underset{377}{\eta_1}} \hat{y} \overset{12}{E_{io}} e^{-jk_1 z} + \frac{\hat{y} E_{io}}{\eta_1} e^{jk_1 z}$$

$$= \hat{y}\ 64 \cos\left(\frac{20\pi}{3} z\right) \cos\left(2\pi * 10^4 t\right) (MA/m)$$

* if $\Gamma = -1$ the boundary between dielectric and perfect conductor.

Snell's Low

Bondary between two dielectric media (ϵ_1, μ_1) ; (ϵ_2, μ_2)



$$\frac{A_i O'}{up_1} = \frac{A_r o}{up_1} = \frac{O A_t}{up_2}$$

$A_i o' = OO' \sin\theta_i$

$A_r o = OO' \sin\theta_r$

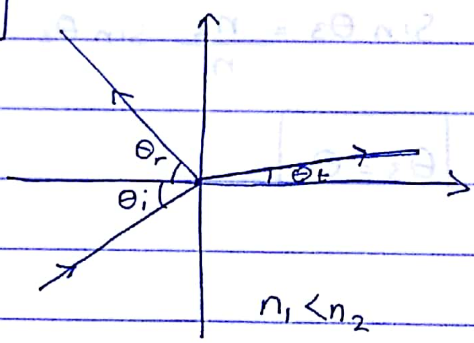
$O A_t = OO' \sin\theta_t$

$$\frac{OO' \sin\theta_i}{up_1} = \frac{OO' \sin\theta_r}{up_1} \Rightarrow \boxed{\sin\theta_i = \sin\theta_r}$$

$$\frac{OO' \sin\theta_i}{up_1} = \frac{OO' \sin\theta_t}{up_2} \Rightarrow \boxed{\frac{\theta_t}{\theta_i} = \frac{up_2}{up_1}}$$ Snell's Low for refaction

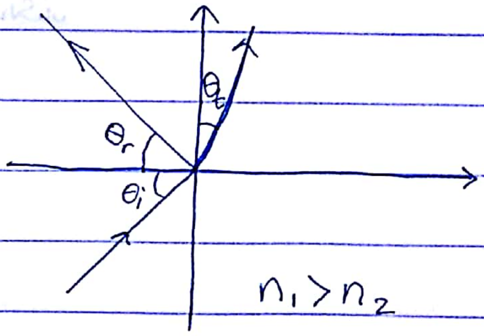
$$= \frac{\sqrt{\mu_1 \epsilon_1}}{\sqrt{\mu_2 \epsilon_2}}$$

$$\boxed{\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2}}$$



Index of Refraction

$$n = \frac{c}{up} = \sqrt{\frac{\cancel{\mu_0} \mu_r \cancel{\epsilon_0} \epsilon_r}{\cancel{\mu_0} \cancel{\epsilon_0}}} = \sqrt{\mu_r \epsilon_r}$$



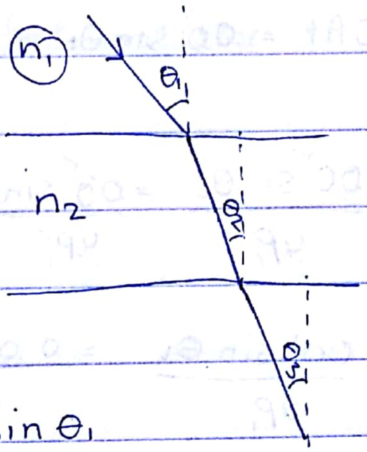
critical angle θ_c

Is the induced angle θ_i at which $\theta_t = \frac{\pi}{2}$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{n_1}{n_2} \xrightarrow[\theta_t = 90^\circ]{} \frac{1}{\sin\theta_c} = \frac{n_1}{n_2}$$

$$\sin\theta_c = \frac{n_2}{n_1} = \frac{\sqrt{\mu_{2r}\,\varepsilon_{2r}}}{\sqrt{\mu_{1r}\,\varepsilon_{1r}}} \quad (\mu = \mu_0)$$

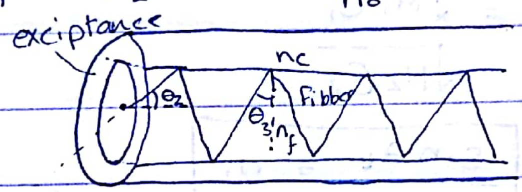
$$= \frac{\sqrt{\varepsilon_{2r}}}{\sqrt{\varepsilon_{1r}}} = \sqrt{\frac{\varepsilon_{2r}}{\varepsilon_{1r}}} \quad \text{(non-magnatic)}$$

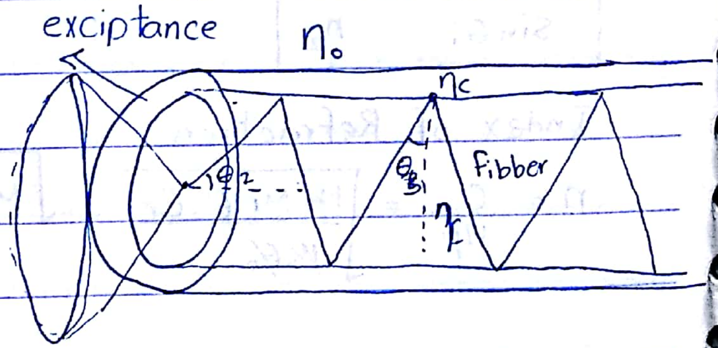


$$\frac{\sin\theta_2}{\sin\theta_1} = \frac{n_1}{n_2}, \quad \frac{\sin\theta_3}{\sin\theta_2} = \frac{n_2}{n_1}$$

$$\sin\theta_3 = \frac{n_2}{n_1}\sin\theta_2 \Rightarrow \sin\theta_3 = \frac{n_2}{n_1} * \frac{n_1}{n_2}\sin\theta_1$$

$$\boxed{\theta_3 = \theta_1}$$





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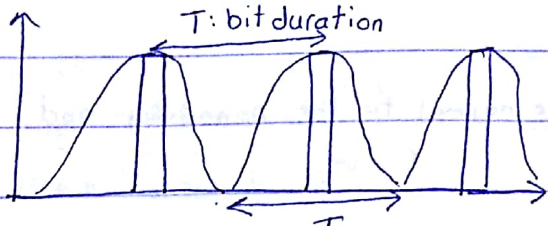
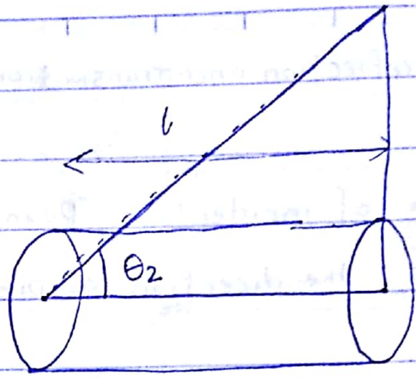
Max bit rate





$T \geq 2\tau$, $f = \frac{1}{T}$

$t_{min} = \frac{L}{u_p} = \frac{L}{c/n_f} = \frac{L n_f}{c}$ $\left(\frac{c}{u_p} = n_f\right)$

$L_{max} = \frac{1}{\cos\theta_2} = L\frac{n_f}{n_c} = \frac{L n_f}{c}\left(\frac{n_t}{n_c} - 1\right)$

$t_{max} = \frac{L_{max}}{u_p} = \frac{L n_f / n_c}{c/n_f} = \frac{L n_f^2}{c n_c}$

$f_{max} = \frac{1}{2\tau} = \frac{c\, n_c}{2 L n_f (n_f - n_c)}$

example :- 1 km opt. fiber $n_f = 1.52$, $n_c = 1.49$

$\theta_a = \sin^{-1}\left(\frac{1}{n_0}\sqrt{n_f^2 - n_c^2}\right) = \sin^{-1}\left(\frac{1}{120\pi}\sqrt{1.52^2 - 1.49^2}\right)$

$= \sin^{-1}\left(\frac{1}{120\pi} * 0.3\right)$

$f_p = \frac{c\, n_c}{2L\, n_f (n_f - n_c)} = 4.9013 * 10^6$ HZ b/sec

Wave reflection and transmision of oblique incident

*plane of incident :- Plane containing the normal to the boundary and the direction of propagation

Best way to solvies reflection / transmission problem.

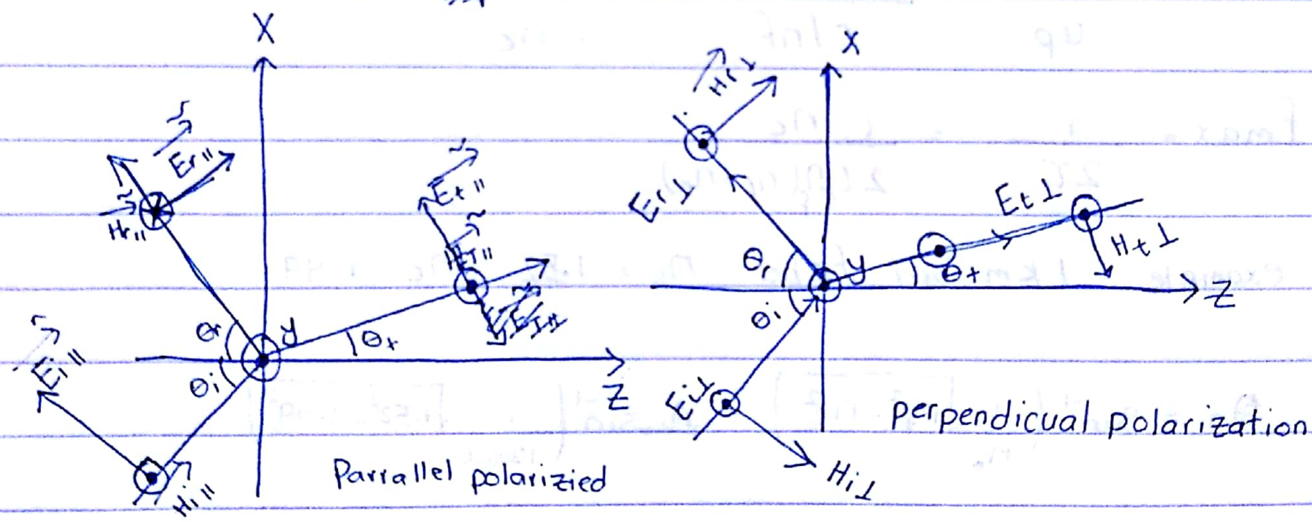
1) Dec the incidant wave ($\vec{\tilde{E_i}}$, $\vec{\tilde{H_i}}$) into

a) perpendicular polarized component. ($\vec{\tilde{E_i}}\perp$, $\vec{\tilde{H_i}}\perp$)

b) parrallel polarized component ($\vec{\tilde{E_i}}_{\parallel}$, $\vec{\tilde{H_i}}_{\parallel}$)

2) Reflected wave → ($\vec{\tilde{E_r}}\perp$, $\vec{\tilde{H_r}}\perp$)

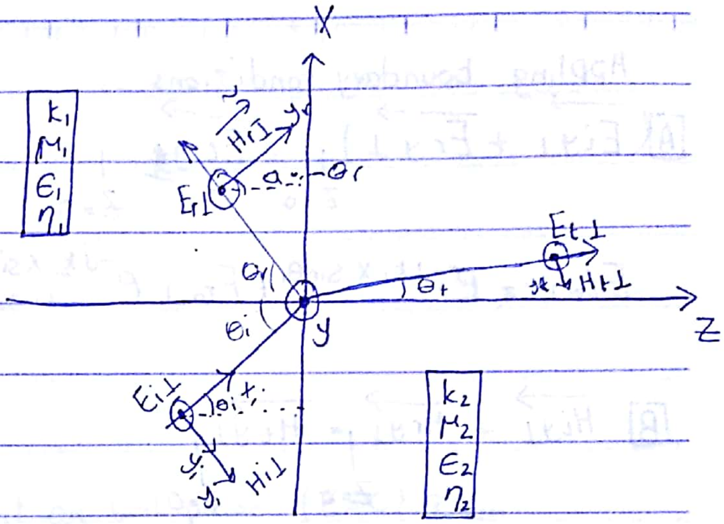
→ ($\vec{\tilde{E_r}}_{\parallel}$, $\vec{\tilde{H_r}}_{\parallel}$)



Parrallel polarizied

perpendicual polarization

perrpendicular polarization



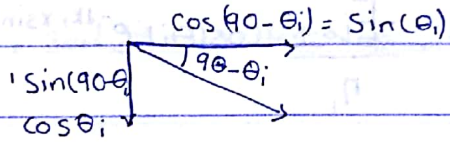
$$\vec{E}_{i\perp} = \hat{y}\, E_{io\perp}\, e^{-jk_1 x_i}$$

$$\vec{H}_{i\perp} = \hat{y}\, \frac{E_{io\perp}}{\eta_1}\, e^{-jk_1 x_i}$$

$\Delta Z \rightarrow \Delta Z \cos\theta_i$

$\Delta X \rightarrow \Delta X \sin\theta_i$

$x_i = X\sin\theta_i + Z\cos\theta_i$



$$E_{i\perp} = \hat{y}_i\, E_{io\perp}\, e^{-jk_1(x\sin\theta_i + z\cos\theta_i)}$$

$$\vec{H}_{i\perp} = (-\hat{x}\cos\theta_i + \hat{z}\sin\theta_i)\,\frac{E_{io\perp}}{\eta_i}\, e^{-jk_1(x\sin\theta_i + z\cos\theta_i)}$$

$x_i = -\hat{x}\cos\theta_i + \hat{z}\sin\theta_i$

Reflected wave :-

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$$\vec{E}_{r\perp} = +\hat{y}\, E_{ro\perp}\, e^{-jk_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\vec{H}_{r\perp} = (\hat{x}\cos\theta_r + \hat{z}\sin\theta_r)\,\frac{E_{ro\perp}}{\eta_1}\, e^{-jk_1(x\sin\theta_r - z\cos\theta_r)}$$

$$\vec{E}_{t\perp} = \hat{y}\, E_{to\perp}\, e^{-jk_2(z\cos\theta_t + x\sin\theta_t)}$$

$$\vec{H}_{t\perp} = (-\hat{x}\cos\theta_t + \hat{z}\sin\theta_t)\,\frac{E_{to\perp}}{\eta_2}\, e^{-jk_2(x\sin\theta_t + z\cos\theta_t)}$$

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Appling boundary conditions

$\boxed{A}$ $(\vec{E_{iy\perp}} + \vec{E_{ry\perp}})\Big|_{z=0} = \vec{E_{ty}}\Big|_{z=0}$

$$E_{io\perp}\, e^{-jk_1 x\sin\theta_i} + E_{ro\perp}\, e^{-jk_1 x\sin\theta_r} = E_{to\perp}\, e^{-jk_2 x\sin\theta_t}$$

$\boxed{B}$ $\vec{H_{iy\perp}} + \vec{H_{ry\perp}}\Big|_{z=0} = \vec{H_{ty\perp}}\Big|_{z=0}$

$$\frac{-E_{io\perp}}{\eta_1}\cos(\theta_i)\, e^{-jk_1 x\sin\theta_i} + \frac{E_{ro\perp}}{\eta_1}\cos(\theta_r)\, e^{-jk_1 x\sin\theta_r} = \frac{-E_{to\perp}}{\eta_2}\cos(\theta_t)\, e^{-jk_2 x\sin\theta_t}$$

1) $E_{io\perp} + E_{ro\perp} = E_{to\perp}$

2) $\frac{\cos(\theta_i)}{\eta_1}\left[-E_{io\perp} + E_{ro\perp}\right] = -\frac{\cos(\theta_t)}{\eta_2} E_{to\perp}$

$$\Gamma_\perp = \frac{E_{ro\perp}}{E_{io\perp}} = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} \qquad \boxed{\tau_\perp = 1 + \Gamma_\perp}$$

$$\tau_\perp = \frac{E_{to\perp}}{E_{io\perp}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_i + \eta_1\cos\theta_t}$$

$\therefore$ if medium 2 is perfect conductor $\sigma = \infty$

$\Rightarrow \eta_2 = 0$

$\Gamma_\perp = -1 , \quad \tau_\perp = 0$

$\therefore$ For non-magnatic conductor $(\mu_1 = \mu_2 = \mu_0)$

$$\Gamma_\perp = \frac{\cos\theta_i - \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_i}}{\cos\theta_i + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_i}}$$

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$$\Gamma_{\perp} = \frac{\cos\theta_i - \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_i}}{\cos\theta_i + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_i}}$$

we have used $\frac{\epsilon_2}{\epsilon_1} = \left(\frac{n_2}{n_1}\right)^2$ "SNLS Low"

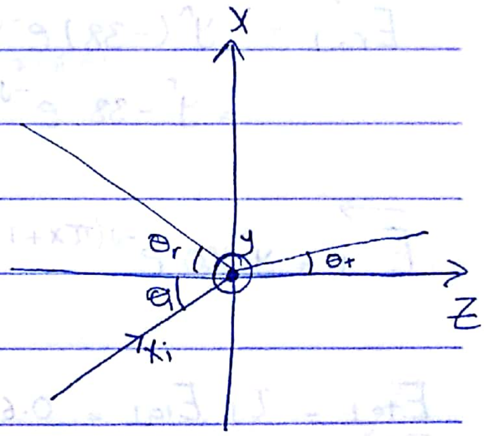
example :- plane wave air is incident on soil plane ($Z=0$) Given :-

$E_{i\perp} = \hat{y}\, 100 \cos(\omega t - \pi x - 1.73\pi Z)$ V/m, soil is loss less, ~~wth~~ with ($\epsilon_r = 4$), non magnatic. Find :-

[a] Find k_1, k_2, θ_i [b] Find all fields (E, H) in both medium

X-Z plane is the incident plane prop ($+\hat{z}$)($+\hat{x}$) directions.

$\vec{E}_{i\perp} = \hat{y}\, 100\, e^{-j\pi x - j1.73\pi Z} = \hat{y}\, 100\, e$



$k_1(Z\cos\theta_i + X\sin\theta_i) = k_1 Z\cos\theta_i + k_1 X\sin\theta_i$

$k_1 Z\cos\theta_i + k_1 X\sin\theta_i = \pi X + 1.73\pi Z$

$k_1 \sin\theta_i = \pi$, $k_1\cos\theta_i = 1.73\pi$

$k_1^2 \sin\theta_i^2 + k_1^2\cos\theta_i^2 = \pi^2 + 1.73^2\pi^2$

$k_1^2(\sin^2\theta_i + \cos^2\theta_i^3) = \cancel{1.73\pi^2}\ \pi^2(1 + 1.73^2)$

k_1

$$\frac{k_1 \sin\theta_i}{k_1 \cos\theta_i} = \frac{\cancel{\pi}}{1.73\cancel{\pi}} \Rightarrow \tan\theta_i = \frac{1}{1.73} \Rightarrow \boxed{\theta_i = 30^\circ}$$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{k_1}{k_2} \Rightarrow \sin\theta_t = \sin\theta_i \, \frac{2\pi}{4\pi} = 0.25$$

$$\theta_t = \sin^{-1}(0.25) = 14.5^\circ$$

$$\eta_2 = \frac{\eta_0}{\sqrt{\epsilon_r}} = \frac{120\pi}{\sqrt{4}} = 60\pi \qquad \eta_1 = 120\pi = 377$$

$$E_{ro\perp} = \Gamma E_{io\perp} = -0.38 * 100 = -38$$

$$\vec{E}_{ro\perp} = \hat{y}(-38)\, e^{-jX_r k_1} = \hat{y}\,{-38}\, e^{-jk_1(-Z\cos\theta_r + X\sin\theta_r)}$$
$$= \hat{y}\,{-38}\, e^{-j(\pi X - 1.73\pi Z)}$$

$$\vec{E}_{\perp} = \hat{y}\,100\, e^{-j(\pi X + 1.73\pi Z)} - \hat{y}\,38\, e^{-j(\pi X + 1.73\pi Z)}$$

$$E_{to\perp} = \tau_\perp E_{io\perp} = 0.62 * 100 = 62$$

$$\vec{E}_{i\perp} = \vec{E}_{t\perp} = \hat{y}\,62\, e^{-jk_2 X_t}$$
$$= \hat{y}\,62\, e^{-j2\pi(Z\cos 14.5 + X\sin 14.5)} = \hat{y}\,62\, e^{-j(3.87\pi Z + \pi X)}$$

$$E_{i\perp,} = \hat{y}\left[100\cos(\omega t - \pi X - 1.73\pi Z) - 38\cos(\omega t - \pi X + 1.73\pi Z)\right]$$

$$E_{2\perp} = \hat{y}\,62\cos(\omega t - \pi X - 3.87\pi Z)$$

$$H_{i\perp} = \frac{1}{\eta_1} \cancel{\cos\theta}\, k_i \times \overrightarrow{E_{i\perp}}$$

$$\hat{k_i} = \cos\theta_i\, \hat{z} + \sin\theta_i\, \hat{x} = \hat{z}\, 0.866 + \hat{x}\, 0.5$$

$$\overrightarrow{H_{i\perp}} = \frac{1}{\eta_1} (\hat{z}\, 0.866 + \hat{x}\, 0.5) \times (\hat{y}\, 100\, e^{-j(\pi x + 1.73\pi z)})$$

$$= \frac{1}{377} (-\hat{x}\, 86.6 + \hat{z}\, 50)\, e^{-j(\pi x + 1.73\pi z)}$$

$$= (-\hat{x}\, 0.2297 + \hat{z}\, 0.1326) \cos(wt - \pi x - 1.73\pi z)$$

$$\hat{k_r} = \hat{z}(-\cos\theta_r) + \hat{x}(\sin\theta_r) = -\hat{z}\, 0.866 + \hat{x}\, 0.5$$

$$\overrightarrow{H_{r\perp}} = \frac{1}{377} (-\hat{z}\, 0.866 + \hat{x}\, 0.5) \times (-\hat{y}\, 38\, e^{-j(\pi x - 1.73\pi z)}$$

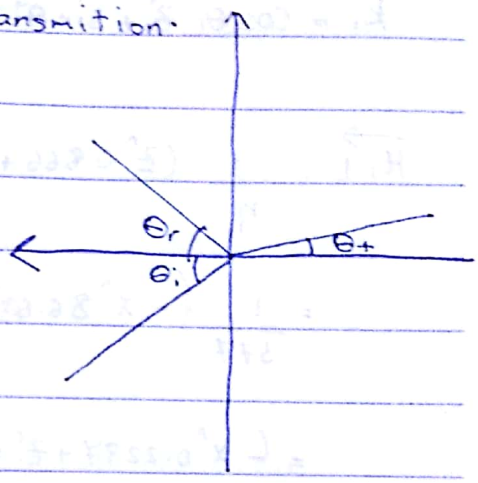
Example :- Uniform plane wave in air having $E_i = -\hat{y}8\cos(\omega t - 4x - 3z)$ V/m is incident on a dielectric slab $z \geq 0$ with ($\mu_r = 1$, $\epsilon_r = 2.5$, $\sigma = 0$)

Find :-

① angle of incident. ② angle of transmition.

③ refflected of transmited $\vec{E}$-Fields.

④ " of transmitted $\vec{H}$-Fields.



Sol

$$\vec{E_i} = \hat{y} E_{io\perp} e^{-jk_1(x\sin\theta_i + z\cos\theta_i)}$$

$E_{io\perp} = -8$

$k_1 \sin\theta_i = 4$ $\Rightarrow$ $\tan\theta_i = \frac{4}{3} \Rightarrow \boxed{\theta_i = 53.13}$

$k_1 \cos\theta_i = 3$ $\quad$ $k_1 \sin\theta_i = 4 \Rightarrow \boxed{k_1 = 5}$

$$\frac{k_1}{k_2} = \frac{\cancel{\omega}\sqrt{\mu_o\epsilon_o}}{\cancel{\omega}\sqrt{\cancel{\mu_o}\cancel{\epsilon_o}\epsilon_r}} = \frac{1}{\sqrt{2.25}} = \frac{5}{k_2} \Rightarrow k_2 = \cancel{1.5} * 5\sqrt{2.5}$$

$\boxed{k_2 = 7.9}$

$\eta_2 = 75.895\pi$

$\eta_1 = 120\pi$

$$\frac{\sin\theta_t}{\sin\theta_i} = \frac{5}{7.91} \Rightarrow \sin\theta_t = \overset{0.51}{\cancel{0.8}} \Rightarrow \boxed{\theta_t = 30.395^\circ}$$

$$\Gamma_\perp = \frac{\eta_2\cos\theta_i - \eta_1\cos\theta_t}{\eta_2\cos\theta_i + \eta_1\cos\theta_t} = \frac{\cancel{75.895}\ 45.537\pi - 103.51\pi}{45.537\pi + 103.51\pi}$$

$= -0.38896$

$\boxed{\tau_1 = 0.611}$

$E_{r0\perp} = (-0.389)(-8) = 3.112$

$$\vec{E}_{r\perp} = \hat{y}\,3.112\,e^{-jk_1(x\sin\theta_r - z\cos\theta_r)} = \hat{y}\,3.112\,e^{-j(-3z+4x)}$$

$$= \hat{y}\,3.112\cos(\omega t + 3z - 4x)$$

$$\vec{E}_{t\perp} = \hat{y}\,{-4.888}\,e^{-jk_2(+x\cos\theta_t + z\sin\theta_t)}$$

$\boxed{\theta_i = 53.13^\circ = \theta_r}$

$$= \hat{y}\,{-4.888}\,e^{-j(+6.82x+4z)} =$$

$$= \hat{y}\,{-4.888}\cos(\omega t - 4x - 6.82z)$$

$\boxed{\theta_t = 30.39^\circ}$

$$\hat{k}_i = \hat{z}\sin\theta_i - \hat{x}\cos\theta_i = 0.8\hat{z} - 0.6\hat{x}$$

$$\hat{k}_r = \hat{z}\sin\theta_r + \hat{x}\cos\theta_r = 0.8\hat{z} + 0.6\hat{x}$$

$$\hat{k}_t = -\hat{x}\cos\theta_t + \hat{z}\sin\theta_t = 0.51\hat{z} - 0.863\hat{x}$$

$$\vec{H}_{i\perp} = \frac{1}{\eta_1}\,\hat{k}_i \times \vec{E}_{i\perp} = \frac{1}{120\pi}(0.8\hat{z} - 0.6\hat{x}) \times \hat{y}\,(-8\cos(\omega t - 4x - 3z))$$

$$= (0.8\hat{x} + 0.6\hat{z})\,\frac{8}{\cancel{120\pi}\;15}\cos(\omega t - 4x - 3z)$$

$$\vec{H}_{r\perp} = \frac{1}{\eta_1}\,\hat{k}_r \times \hat{E}_{r\perp} = \frac{1}{120\pi}(0.8\hat{z} + 0.6\hat{x}) \times \hat{y}\,3.112\cos(\omega t + 3z - 4x)$$

$$= (-0.8\hat{x} + 0.6\hat{z})\,\frac{3.112}{120\pi}\cos(\omega t + 3z - 4x)$$

$$= (6.6\hat{x} + 4.953\hat{z})\cos(\omega t + 3z - 4x)\ m$$

$$H_{t\perp} = \frac{1}{75.895}(0.51\hat{z} - 0.863\hat{x}) \times \hat{y}\,(-4.888\cos(\omega t - 4x - 6.82z))$$

$$= (0.51\hat{x} + 0.863\hat{z})\,64.4\,(\cos(\omega t - 4x - 6.82z))\ m$$

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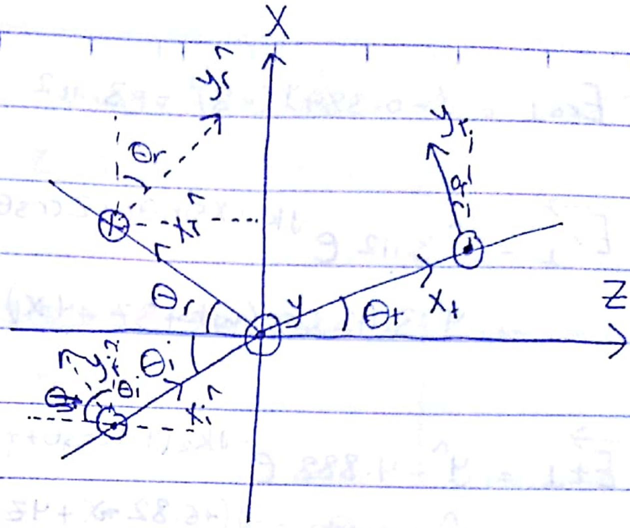

$\hat{x_i} = x\sin\theta_i + z\cos\theta_i$

$\hat{y_i} = \hat{x}\cos\theta_i - \hat{z}\sin\theta_i$

$\hat{x_r} = x\sin\theta_r - z\cos\theta_r$

$\hat{y_r} = \hat{x}\cos\theta_r - \hat{z}\sin\theta_r$

$x_t = x\sin\theta_t + z\cos\theta_t$

$y_t = \hat{x}\cos\theta_t - \hat{z}\sin\theta_t$

$$\Gamma_{\parallel} = \frac{E_{ro\parallel}}{E_{io\parallel}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\tau_{\parallel} = \frac{E_{to\parallel}}{E_{io\parallel}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \qquad \tau_{\parallel} = (1+\Gamma_{\parallel})\frac{\cos\theta_i}{\cos\theta_t}$$

if the secound medium is perfect conductor

$$\eta_c = \sqrt{\frac{\mu}{\epsilon(1 + j\frac{\sigma}{\omega t})}} \qquad \sigma = \infty \Rightarrow \eta_c = 0 \Rightarrow \Gamma_{\parallel} = -1 \Rightarrow \tau_{\parallel} = 0$$

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Example :- A plane wave in air with $\vec{E_i} = (-10\hat{y} + 5\hat{z})\cos(\omega t - 2y + 4z)$

$(\sigma = 0,\ \mu = \mu_o,\ \varepsilon_r = 4)$

① θ_i, θ_r and θ_t

② $\Gamma_{\parallel}$ and $\tau_{\parallel}$

③ total $\vec{E}$ and $\vec{H}$ fields

④ Average incident power vector

① $k_1 \sin\theta_i = \cancel{3}\ 2$ $\quad \tan\theta_i = 2 \Rightarrow \theta_i = \cancel{63.435}$ 26.565 $\quad \boxed{k_1 = 4.472}$

$k_1 \cos\theta_i = \cancel{5}\ 4$

$\frac{k_2}{k_1} = \sqrt{\frac{\mu_{r2}}{\mu_{r1}}} \Rightarrow k_2 = 2k_1 \Rightarrow \cancel{k_2 = 22.361}$ $\quad \boxed{k_2 = 8.9443}$

$\frac{\sin\theta_t}{\sin\theta_i} = \frac{k_1}{k_2} \Rightarrow \cancel{\theta_t = 26.57}$ $\quad \boxed{\theta_t = 12.921}$ $\quad \eta_1 = 120\pi \quad \eta_2 = 60\pi$

$\Gamma_{\parallel} = \frac{58.481 - 107.33}{58.481 + 107.33} = -0.295 \qquad \tau_{\parallel} = 0.705$

$E_r = (+2.95\hat{y} + 1.475\hat{z})\, e^{-j(2y - 4z)}$

$E_t = (-7.05\hat{y} + 3.525\hat{z})\, e^{-j(2y + 4z)}$

$H_i = \frac{1}{120\pi}(\hat{y}\,0.89 - \hat{z}\,0.447) \times (-10\hat{y} + 5\hat{z})\, e^{-j(2y+4z)}$

$= \frac{1}{120\pi}(-\hat{x} \quad)\, e^{-j(2y+4z)}$

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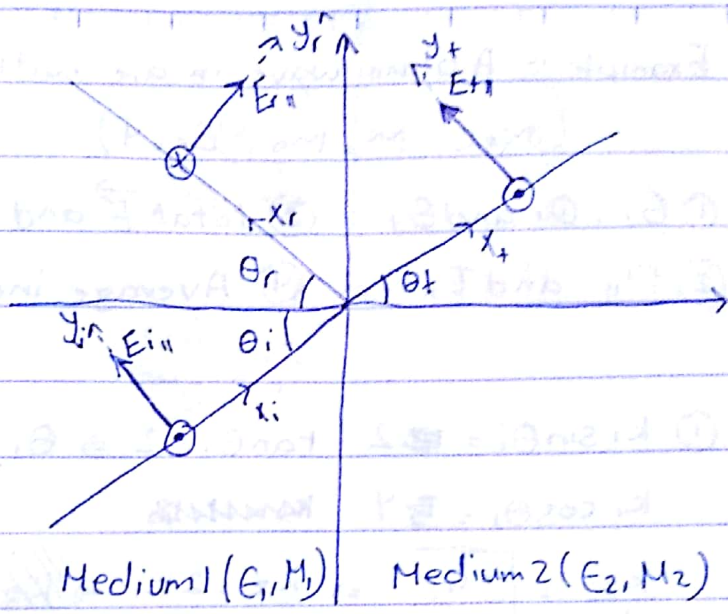
Parrallel polarization



$x_{i\parallel} = x \sin\theta_i + z\cos\theta_i$

$x_{r\parallel} = x \sin\theta_r - z\cos\theta_r$

$x_{t\parallel} = x \sin\theta_t + z\cos\theta_t$

$\hat{y}_{i\parallel} = \hat{x}\cos\theta_i - \hat{z}\sin\theta_i$

$\hat{y}_{r\parallel} = \hat{x}\cos\theta_r + \hat{z}\sin\theta_r$

$\hat{y}_{t\parallel} = \hat{x}\cos\theta_t - \hat{z}\sin\theta_t$

$$\Gamma_\parallel = \frac{E_{r\parallel}}{E_{i\parallel}} = \frac{\eta_2\cos\theta_t - \eta_1\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i} \Rightarrow \text{if } \eta_2 = 0 \Rightarrow \boxed{\Gamma_\parallel = -1}$$

$$\tau_\parallel = \frac{E_{t\parallel}}{E_{i\parallel}} = \frac{2\eta_2\cos\theta_i}{\eta_2\cos\theta_t + \eta_1\cos\theta_i}$$

$$\tau_\parallel = (1+\Gamma_\parallel)\frac{\cos\theta_i}{\cos\theta_t}$$

For non magnatic :- $\Gamma_\parallel = \dfrac{-(\epsilon_2/\epsilon_1)\cos\theta_i + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_i}}{(\epsilon_2/\epsilon_1)\cos\theta_i + \sqrt{(\epsilon_2/\epsilon_1) - \sin^2\theta_i}}$ $(\mu_1 = \mu_2)$

The vector of avarage incident power density

$$\vec{S}_{avi} = \frac{|E_i|^2}{2\eta_1} \hat{k}_i = \frac{10^2+5^2}{2*377} (\hat{z}\,0.894 + \hat{y}\,0.447)$$

$$= \hat{y}\,0.074 + \hat{z}\,0.118 \ (w/m^2)$$

or

$$\vec{S}_{avi} = \frac{1}{2} Re(\vec{E} \times \vec{H}^*) = \frac{1}{2} Re\left((-10\hat{y} + 5\hat{z}) e^{-j(2y+4z)} \times (\hat{x}\,24.638) e^{j(2y+4z)}\right)$$

#Browster angle (θ_B)

Is the incident angle at which ($\Gamma = 0$)

1) perpendicular polarization

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = 0 \Rightarrow \eta_2^2 \cos^2\theta_i = \eta_1^2 \cos^2\theta_t$$

$$\Rightarrow \eta_2^2 (1 - \sin^2\theta_i) = \eta_1^2 (1 - \sin^2\theta_t) \Rightarrow \left\{ \frac{\sin\theta_t}{\sin\theta_i} = \frac{u_{p2}}{u_{p1}} = \sqrt{\frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}} \right.$$

$$\sin^2\theta_t = \frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2} \sin^2\theta_i$$

$$\sin^2\theta_i \left(-\eta_2^2 + \eta_1^2 \frac{\mu_1 \epsilon_1}{\mu_2 \epsilon_2}\right) = \eta_1^2 - \eta_2^2$$

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$$\sin\theta_{iB\perp}\left(-\frac{\mu_2}{\epsilon_2}+\frac{\mu_1}{\epsilon_1}\;\frac{\mu_1\epsilon_1}{\mu_2\epsilon_2}\right)=\frac{\mu_1}{\epsilon_1}-\frac{\mu_2}{\epsilon_2}$$

$$\sin\theta_{B\perp}=\frac{\sqrt{1-\frac{\mu_1\epsilon_2}{\mu_2\epsilon_1}}}{\sqrt{1-\left(\frac{\mu_1}{\mu_2}\right)^2}}$$

if $\mu_1=\mu_2 \Rightarrow \sin\theta_i=\infty$

$\theta_{B\perp}$ dose not excist for $\mu_1=\mu_2$ or for non magnatic

2) Parrallel polarization

$\Gamma_{\parallel}=0$

$$\sin\theta_{B\parallel}=\sqrt{\frac{1-\frac{\epsilon_1\mu_2}{\epsilon_2\mu_1}}{1-(\epsilon_1/\epsilon_2)^2}}$$

For non magnatic $(\mu_1=\mu_2=\mu_0$

$$\theta_{B\parallel}=\sin^{-1}\left(\sqrt{\frac{1}{1+(\epsilon_1/\epsilon_2)^2}}\right)=\tan^{-1}\left(\sqrt{\frac{\epsilon_2}{\epsilon_1}}\right)$$

θ_B is called pollarizing angle

if awave with perpendicual, Parrallel polarization is incident on non-magnatic material at $\theta_i=\theta_{B\parallel}$ So that the $\parallel$-component $(\Gamma_{\parallel}=0)$ is totally transmitted only the $\perp$ component is reflected.

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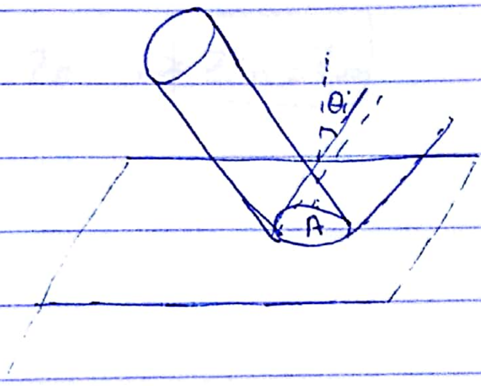
Reflected and transmitivits



$$R_\perp \text{ (reflected)} = \frac{P_{r\perp}}{P_{i\perp}}$$

$$P_{i\perp} = S_{i\perp} A_i = \frac{|E_{i\perp}|^2}{2\eta_1} A\cos\theta_i$$

$$P_{r\perp} = \frac{|E_{r\perp}|^2}{2\eta_1} A\cos\theta_r$$

$$P_{t\perp} = \frac{|E_{t\perp}|^2}{2\eta_2} A\cos\theta_t$$

$$R_\perp = \frac{|E_{\perp r}|^2}{|E_{i\perp}|^2} = |\Gamma_\perp|^2$$

$$R_{\parallel} = \frac{P_{r\parallel}}{P_{i\parallel}} = |\Gamma_{\parallel}|^2$$

$$T \text{(transmittivity)} = \frac{P_t}{P_i}$$

$$T_\perp = \frac{(|E_{t\perp}|^2/2\eta_2)\, A\cos\theta_t}{(|E_{i\perp}|^2/2\eta_1)\, A\cos\theta_i} = |\tau_\perp|^2 \frac{\eta_1\cos\theta_t}{\eta_2\cos\theta_i}$$

$$T_{\parallel} = (\tau_{\parallel})^2 \frac{\eta_1\cos\theta_t}{\eta_2\cos\theta_i}$$

$$|\Gamma_{\parallel}|^2 + |\tau_{\parallel}|^2 \frac{\eta_1\cos\theta_t}{\eta_2\cos\theta_i} = 1$$

$$P_i = P_r + P_t$$

$$|\Gamma_\perp|^2 + |\tau_\perp|^2 \frac{\eta_1\cos\theta_t}{n_2\cos\theta_i} = 1$$