Engineering Economy

Chapter 1: Introduction to Engineering Economy

The purpose of this book is to develop and illustrate the principles and methodology required to answer the basic economic question of any design: Do its benefits exceed its cost?

Engineering economy…

involves the systematic evaluation of the economic merits of proposed solutions to engineering problems.

Engineering economy…

Engineering economy is the dollars-and-cents side of the decisions that engineers make or recommend as they work to position a firm to be profitable in a highly competitive marketplace.

Engineering economy…

these decisions are trade-offs among different types of costs and the performance (response time, safety, weight, reliability, etc.) provided by the proposed design or problem solution.

Engineering economy…

The mission of engineering economy is to balance these trade-offs in the most economical manner.

Engineering economy…

If an engineer at Ford Motor Company invents a new transmission lubricant that increases fuel mileage by 10% and extends the life of the transmission by 30,000 miles, how much can the company afford to spend to implement this invention? Engineering economy can provide an answer.

Solutions to engineering problems must

- Promote the well-being and survival of an organization,
- Embody creative and innovative technology and ideas,
- Permit identification and scrutiny of their estimated outcomes, and
- Translate profitability to the "bottom line" through a valid and acceptable measure of merit.

Engineering economic analysis can play a role in many types of situations.

- Choosing the best design for a high-efficiency gas furnace.
- Selecting the most suitable robot for a welding operation on an automotive assembly line.
- Making a recommendation about whether jet airplanes for an overnight delivery service should be purchased or leased.
- Determining the optimal staffing plan for a computer help desk.

There are seven fundamental principles of engineering economy.

- Develop the alternatives
- Focus on the differences
- Use a consistent viewpoint
- Use a common unit of measure
- Consider all relevant criteria
- Make uncertainty explicit
- Revisit your decisions

Develop the alternatives

- Carefully define the problem! Then the choice (decision) is among alternatives. The alternatives need to be identified and then defined for subsequent analysis.

Developing and defining the alternatives for detailed evaluation is important because of the resulting impact on the quality of the decision.

Focus on the differences

- Only the differences in expected future outcomes among the alternatives are relevant to their comparison and should be considered in the decision.

only the differences in the future outcomes of the alternatives are important. Outcomes that are common to all alternatives can be disregarded in the comparison and decision.

Use a consistent viewpoint

- The prospective outcomes of the alternatives, economic and other, should be consistently developed from a defined viewpoint (perspective).

The perspective of the decision maker, which is often that of the owners of the firm, would normally be used. However, it is important that the viewpoint for the particular decision be first defined and then used consistently in the description, analysis, and comparison of the alternatives

Use a common unit of measure

- Using a common unit of measurement to enumerate as many of the prospective outcomes as possible will simplify the analysis of the alternatives.

It is desirable to make as many prospective outcomes as possible commensurable (directly comparable). For economic consequences, a monetary unit such as dollars is the common measure.

Consider all relevant criteria

- Selection of a preferred alternative (decision making) requires the use of a criterion (or several criteria). The decision process should consider both the outcomes enumerated in the monetary unit and those expressed in some other unit of measurement or made explicit in a descriptive manner.

Make uncertainty explicit

- Risk and uncertainty are inherent in estimating the future outcomes of the alternatives and should be recognized in their analysis and comparison.

Revisit your decisions

- Improved decision making results from an adaptive process; to the extent practicable, the initial projected outcomes of the selected alternative should be subsequently compared with actual results achieved.

Engineering economic analysis procedure

- Problem definition
- Development of alternatives
- Development of prospective outcomes
- Selection of a decision criterion
- Analysis and comparison of alternatives.
- Selection of the preferred alternative.
- Performance monitoring and postevaluation of results.

Electronic spreadsheets are a powerful addition to the analysis arsenal.

- Most engineering economy problems can be formulated and solved using a spreadsheet.
- Large problems can be quickly solved.
- Proper formulation allows key parameters to be changed.
- Graphical output is easily generated.

Engineering Economy

Chapter 2: Cost Concepts and Design Economics

The objective of Chapter 2 is to analyze short-term alternatives when the time value of money is not a factor.

Costs can be categorized in several different ways.

- *Fixed cost*: unaffected by changes in activity level Rent
- *Variable cost*: vary in total with the quantity of output (or similar measure of activity) Material
- *Incremental cost*: additional cost resulting from increasing output of a system by one (or more) units

EXAMPLE 2-1 Fixed and Variable Costs

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In connection with surfacing a new highway, a contractor has a choice of two sites on which to set up the asphalt-mixing plant equipment. The contractor estimates that it will cost \$2.75 per cubic yard mile (yd^3-mile) to haul the asphalt-paving material from the mixing plant to the job location. Factors relating to the two mixing sites are as follows (production costs at each site are the same):

Cost Factor	Site *A*	Site *B*
Average hauling distance	4 miles	3 miles
Monthly rental of site	\$2,000	\$7,000
Cost to set up and remove equipment	\$15,000	\$50,000
Hauling expense	\$2.75/yd^3-mile	\$2.75/yd^3-mile
Flagperson	Not required	\$150/day

The job requires 50,000 cubic yards of mixed-asphalt-paving material. It is estimated that four months (17 weeks of five working days per week) will be required for the job. Compare the two sites in terms of their fixed, variable, and total costs. Assume that the cost of the return trip is negligible. Which is the better site? For the selected site, how many cubic yards of paving material does the contractor have to deliver before starting to make a profit if paid \$12 per cubic yard delivered to the job location?

Solution

The fixed and variable costs for this job are indicated in the table shown next. Site rental, setup, and removal costs (and the cost of the flagperson at Site *B*) would be constant for the total job, but the hauling cost would vary in total amount with the distance and thus with the total output quantity of yd^3-miles (x).

Cost	Fixed	Variable	Site *A*		Site *B*	
Rent	√		=	$8,000	=	$28,000
Setup/removal	√		=	15,000	=	50,000
Flagperson	√		=	0	5(17)($150) =	12,750
Hauling		√	4(50,000)($2.75) =	550,000	3(50,000)($2.75) =	412,500
			Total:	$573,000		$503,250

Site *B*, which has the larger fixed costs, has the smaller total cost for the job. Note that the extra fixed costs of Site *B* are being "traded off" for reduced variable costs at this site.

The contractor will begin to make a profit at the point where total revenue equals total cost as a function of the cubic yards of asphalt pavement mix delivered. Based on Site *B*, we have

$$3(\$2.75) = \$8.25 \text{ in variable cost per yd}^3 \text{ delivered}$$

$$\text{Total cost} = \text{total revenue}$$

$$\$90{,}750 + \$8.25x = \$12x$$

$$x = 24{,}200 \text{ yd}^3 \text{ delivered.}$$

Therefore, by using Site *B*, the contractor will begin to make a profit on the job after delivering 24,200 cubic yards of material.

More ways to categorize costs

- *Direct*: can be measured and allocated to a specific work activity (Materials + Labor)
- *Indirect*: difficult to attribute or allocate to a specific output or work activity (also *overhead* or *burden*)
- *Standard cost*: cost per unit of output, established in advance of production or service delivery

Some useful cost terminology

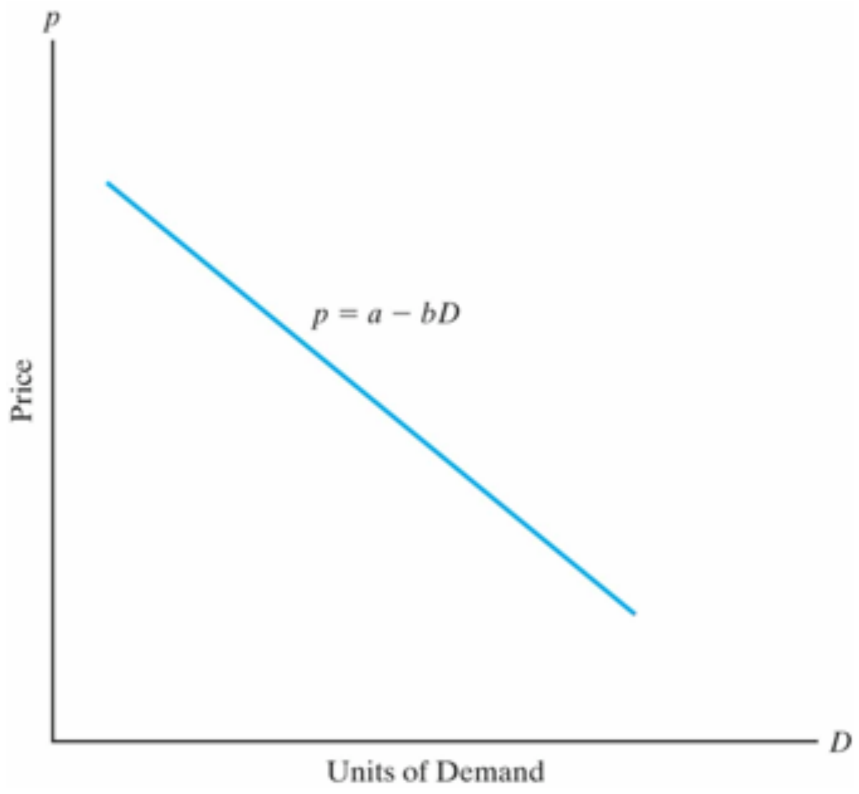
- Cash cost: a cost that involves a payment of cash.
- Book cost: a cost that does not involve a cash transaction but is reflected in the accounting system.
- Sunk cost: a cost that has occurred in the past and has no relevance to estimates of future costs and revenues related to an alternative course of action.

More useful cost terminology

- *Opportunity cost*: the monetary advantage foregone due to limited resources. The cost of the best rejected opportunity.
- *Life-cycle cost*: the summation of all costs related to a product, structure, system, or service during its life span.

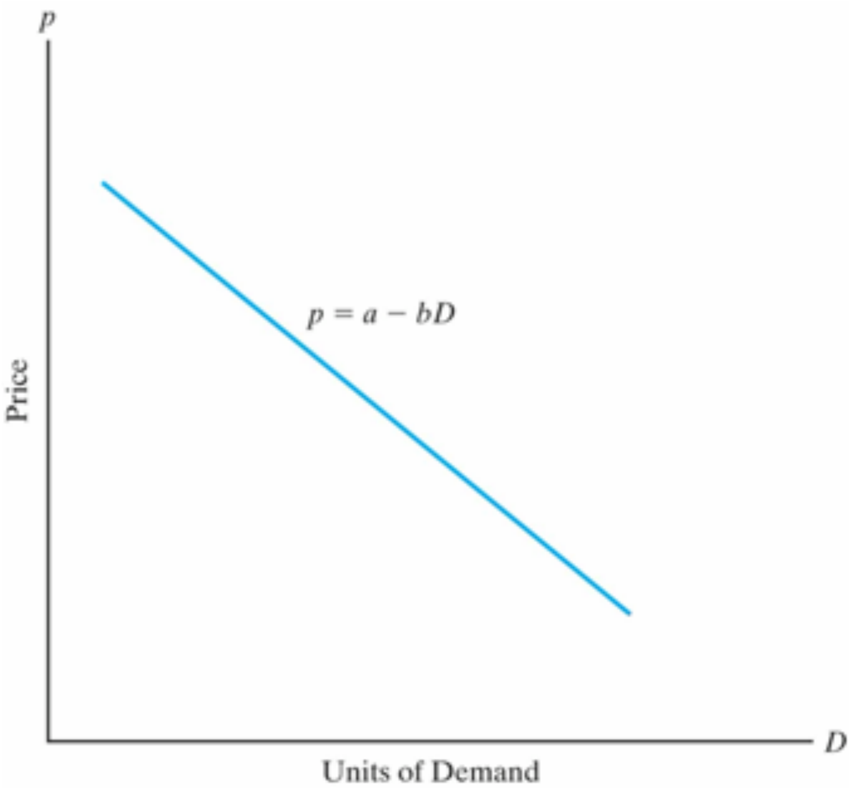
The general price-demand relationship

The demand for a product or service is directly related to its price according to p=a-bD where p is price, D is demand, and a and b are constants that depend on the particular product or service.



The general price-demand relationship

As the price increases, the demand decreases.



Total revenue depends on price and demand.

Total revenue is the product of the selling price per unit, *p*, and the number of units sold, *D*.

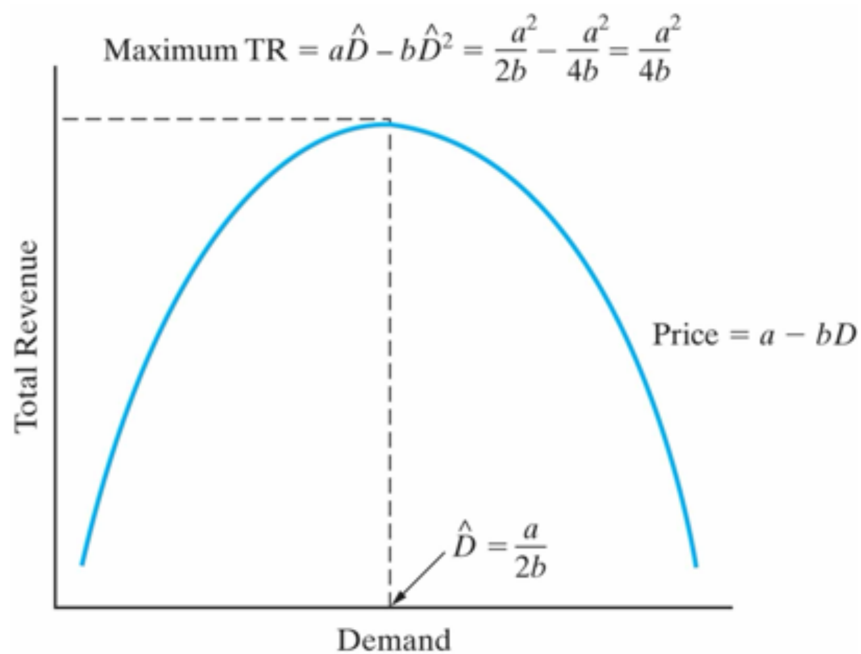
$$TR = pD = (a - bD)D = aD - bD^2$$

$$\text{for} \quad 0 \leq D \leq \frac{a}{b} \quad \text{and} \quad a > 0, \quad b > 0$$

Calculus can help determine the demand that maximizes revenue.

$$\frac{dTR}{dD} = a - 2bD = 0$$

Solving, the optimal demand is



We can also find maximum profit…

$CT = CF + CV$

where CF and CV denote fixed and variable costs, respectively.

For the linear relationship assumed here,

$CV = cv \cdot D$,

We can also find maximum profit…

Profit is revenue minus cost, so

$$\text{Profit} = -bD^2 + (a - c_v)D - C_F$$

for

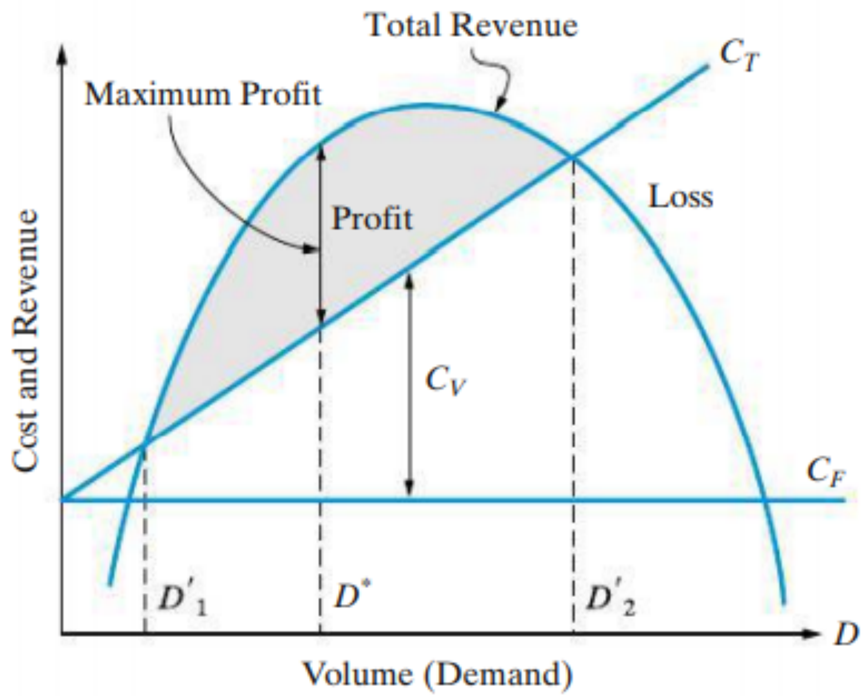
$$0 \le D \le \frac{a}{b} \quad \text{and} \quad a > 0, b > 0$$

Differentiating, we can find the value of D that maximizes profit.

$$D^* = \frac{a - c_v}{2b}$$

We can also find maximum profit…

Figure 2-4 Combined Cost and Revenue Functions, and Breakeven Points, as Functions of Volume, and Their Effect on Typical Profit (Scenario 1)



We can also find maximum profit…

Profit (loss) = total revenue − total costs

$$= (aD - bD^2) - (CF + c_vD)$$

$$= -bD^2 + (a - c_v)D - CF$$

for

$$0 \le D \le a\ b \text{ and } a > 0,\ b > 0.$$

In order for a profit to occur, based on Equation, and to achieve the typical results depicted in Figure 2-4, two conditions must be met:

We can also find maximum profit…

1. $(a - cv) > 0$; that is, the price per unit that will result in no demand has to be greater than the variable cost per unit. (This avoids negative demand.).

2. Total revenue (TR) must exceed total cost (CT) for the period involved. If these conditions are met, we can find the optimal demand at which maximum profit will occur by taking the first derivative of Equation (2-9) with respect to D and setting it equal to zero:

$d(\text{profit})/dD = a - cv - 2bD = 0.$

The optimal value of D that maximizes profit is

Dò $= (a - cv)/2b$

And we can find revenue/cost breakeven.

Breakeven is found when total revenue = total cost.
Solving, we find the demand at which this occurs.

$$D' = \frac{-(a - c_v) \pm \sqrt{(a - c_v)^2 - 4(-b)(-C_F)}}{2(-b)}$$

EXAMPLE 2-4 **Optimal Demand When Demand Is a Function of Price**

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A company produces an electronic timing switch that is used in consumer and commercial products. The fixed cost (C_F) is \$73,000 per month, and the variable cost (c_v) is \$83 per unit. The selling price per unit is $p = \$180 - 0.02(D)$, based on Equation (2-1). For this situation,

(a) determine the optimal volume for this product and confirm that a profit occurs (instead of a loss) at this demand.

(b) find the volumes at which breakeven occurs; that is, what is the range of profitable demand? Solve by hand and by spreadsheet.

Solution by Hand

(a) $D^* = \frac{a - c_v}{2b} = \frac{\$180 - \$83}{2(0.02)} = 2{,}425$ units per month [from Equation (2-10)].

Is $(a - c_v) > 0$?

$$(\$180 - \$83) = \$97, \quad \text{which is greater than 0.}$$

And is (total revenue − total cost) > 0 for $D^* = 2{,}425$ units per month?

$$[\$180(2{,}425) - 0.02(2{,}425)^2] - [\$73{,}000 + \$83(2{,}425)] = \$44{,}612$$

A demand of $D^* = 2{,}425$ units per month results in a maximum profit of \$44,612 per month. Notice that the second derivative is negative (−0.04).

(b) Total revenue = total cost (breakeven point)

$$-bD^2 + (a - c_v)D - C_F = 0 \qquad \text{[from Equation (2-11)]}$$

$$-0.02D^2 + (\$180 - \$83)D - \$73{,}000 = 0$$

$$-0.02D^2 + 97D - 73{,}000 = 0$$

And, from Equation (2-12),

$$D' = \frac{-97 \pm [(97)^2 - 4(-0.02)(-73{,}000)]^{0.5}}{2(-0.02)}$$

$$D'_1 = \frac{-97 + 59.74}{-0.04} = 932 \text{ units per month}$$

$$D'_2 = \frac{-97 - 59.74}{-0.04} = 3{,}918 \text{ units per month.}$$

Thus, the range of profitable demand is 932–3,918 units per month.

EXAMPLE 2-5 Breakeven Point When Price Is Independent of Demand

VIDEO SOLUTIONS

An engineering consulting firm measures its output in a standard service hour unit, which is a function of the personnel grade levels in the professional staff. The variable cost (c_v) is \$62 per standard service hour. The charge-out rate [i.e., selling price (p)] is \$85.56 per hour. The maximum output of the firm is 160,000 hours per year, and its fixed cost (C_F) is \$2,024,000 per year. For this firm,

(a) what is the breakeven point in standard service hours and in percentage of total capacity?

(b) what is the percentage reduction in the breakeven point (sensitivity) if fixed costs are reduced 10%; if variable cost per hour is reduced 10%; and if the selling price per unit is increased by 10%?

Solution

(a)

$$\text{Total revenue} = \text{total cost} \qquad \text{(breakeven point)}$$

$$pD' = C_F + c_v D'$$

$$D' = \frac{C_F}{(p - c_v)}, \tag{2-13}$$

and

$$D' = \frac{\$2{,}024{,}000}{(\$85.56 - \$62)} = 85{,}908 \text{ hours per year}$$

$$D' = \frac{85{,}908}{160{,}000} = 0.537,$$

or 53.7% of capacity.

(b) A 10% reduction in C_F gives

$$D' = \frac{0.9(\$2{,}024{,}000)}{(\$85.56 - \$62)} = 77{,}318 \text{ hours per year}$$

and

$$\frac{85{,}908 - 77{,}318}{85{,}908} = 0.10,$$

or a 10% reduction in D'.

A 10% reduction in c_v gives

$$D' = \frac{\$2{,}024{,}000}{[\$85.56 - 0.9(\$62)]} = 68{,}011 \text{ hours per year}$$

and

$$\frac{85{,}908 - 68{,}011}{85{,}908} = 0.208,$$

or a 20.8% reduction in D'.

A 10% increase in p gives

$$D' = \frac{\$2{,}024{,}000}{[1.1(\$85.56) - \$62]} = 63{,}021 \text{ hours per year}$$

and

$$\frac{85{,}908 - 63{,}021}{85{,}908} = 0.266,$$

or a 26.6% reduction in D'.

Thus, the breakeven point is more sensitive to a reduction in variable cost per hour than to the same percentage reduction in the fixed cost. Furthermore, notice that the breakeven point in this example is highly sensitive to the selling price per unit, p.

Two main tasks are involved in cost-driven design optimization.

Engineers must maintain a life-cycle viewpoint as they design products, processes, and services. Such a complete perspective ensures that engineers consider initial investment costs, operation and maintenance expenses and other annual expenses in later years, and environmental and social consequences over the life of their designs.

Engineers must consider cost in the design of products, processes and services.

- "Cost-driven design optimization" is critical in today's competitive business environment.
- In our brief examination we examine discrete and continuous problems that consider a single primary cost driver.
- Cost models are developed around the design variable, X.

Two main tasks are involved in cost-driven design optimization.

For cost-driven design optimization problems, the two main tasks are as follows:

1. Determine the optimal value for a certain alternative's design variable.

For example, what velocity of an aircraft minimizes the total annual costs of owning and operating the aircraft?

Two main tasks are involved in cost-driven design optimization.

2. Select the best alternative, each with its own unique value for the design variable.

For example, what insulation thickness is best for a home in Virginia: R11, R19, R30, or R38?

cost function.

In general, the cost models developed in these problems consist of three types of costs:

1. fixed cost(s)
2. cost(s) that vary directly with the design variable
3. cost(s) that vary indirectly with the design variable

Here is a simplified cost function.

$$\text{Cost} = aX + \frac{b}{X} + k$$

where,

a is a parameter that represents the directly varying cost(s),

b is a parameter that represents the indirectly varying cost(s),

k is a parameter that represents the fixed cost(s), and

X represents the design variable in question.

Optimizing a design with respect to cost is a four-step process.

- Identify the design variable that is the primary cost driver.
- Express the cost model in terms of the design variable.
- For continuous cost functions, differentiate to find the optimal value. For discrete functions, calculate cost over a range of values of the design variable.
- Solve the equation in step 3 for a continuous function. For discrete, the optimum value has the minimum cost value found in step 3.

Example

EXAMPLE 2-6 **How Fast Should the Airplane Fly?**

The cost of operating a jet-powered commercial (passenger-carrying) airplane varies as the three-halves (3/2) power of its velocity; specifically, $C_O = knv^{3/2}$, where n is the trip length in miles, k is a constant of proportionality, and v is velocity in miles per hour. It is known that at 400 miles per hour, the *average* cost of operation is $300 per mile. The company that owns the aircraft wants to minimize the cost of operation, but that cost must be balanced against the cost of the passengers' time (C_C), which has been set at $300,000 per hour.

(a) At what velocity should the trip be planned to minimize the total cost, which is the sum of the cost of operating the airplane and the cost of passengers' time?

(b) How do you know that your answer for the problem in Part (a) minimizes the total cost?

Example

Solution

(a) The equation for total cost (C_T) is

$$C_T = C_O + C_C = knv^{3/2} + (\$300{,}000 \text{ per hour})\left(\frac{n}{v}\right),$$

where n/v has time (hours) as its unit.

Now we solve for the value of k:

$$\frac{C_O}{n} = kv^{3/2}$$

$$\frac{\$300}{\text{mile}} = k\left(400\,\frac{\text{miles}}{\text{hour}}\right)^{3/2}$$

$$k = \frac{\$300/\text{mile}}{\left(400\,\frac{\text{miles}}{\text{hour}}\right)^{3/2}}$$

$$k = \frac{\$300/\text{mile}}{8000\left(\frac{\text{miles}^{3/2}}{\text{hour}^{3/2}}\right)}$$

$$k = \$0.0375\,\frac{\text{hours}^{3/2}}{\text{miles}^{5/2}}.$$

Example

$$C_T = \left(\$0.0375\frac{\text{hours}^{3/2}}{\text{miles}^{5/2}}\right)(n \text{ miles})\left(v\frac{\text{miles}}{\text{hour}}\right)^{3/2} + \left(\frac{\$300{,}000}{\text{hour}}\right)\left(\frac{n \text{ miles}}{v\frac{\text{miles}}{\text{hour}}}\right)$$

$$C_T = \$0.0375nv^{3/2} + \$300{,}000\left(\frac{n}{v}\right).$$

Next, the first derivative is taken:

$$\frac{dC_T}{dv} = \frac{3}{2}(\$0.0375)nv^{1/2} - \frac{\$300{,}000n}{v^2} = 0.$$

So,

$$0.05625v^{1/2} - \frac{300{,}000}{v^2} = 0$$

$$0.05625v^{5/2} - 300{,}000 = 0$$

$$v^{5/2} = \frac{300{,}000}{0.05625} = 5{,}333{,}333$$

$$v^* = (5{,}333{,}333)^{0.4} = 490.68 \text{ mph.}$$

Example

(b) Finally, we check the second derivative to confirm a minimum cost solution:

$$\frac{d^2C_T}{dv^2} = \frac{0.028125}{v^{1/2}} + \frac{600{,}000}{v^3} \qquad \text{for } v > 0, \text{ and therefore, } \frac{d^2C_T}{dv^2} > 0.$$

The company concludes that $v = 490.68$ mph minimizes the total cost of this particular airplane's flight.

“Present economy studies” can ignore the time value of money.

- Alternatives are being compared over one year or less.
- When revenues and other economic benefits vary among alternatives, choose the alternative that maximizes overall profitability of defect-free output.
- When revenues and other economic benefits are not present or are constant among alternatives, choose the alternative that minimizes total cost per defect-free unit.

Engineering Economy

Chapter 4: The Time Value of Money

The objective of Chapter 4 is to explain time value of money calculations and to illustrate economic equivalence.

Money has a time value.

- *Capital* refers to wealth in the form of money or property that can be used to produce more wealth.
- Engineering economy studies involve the commitment of capital for extended periods of time.
- A dollar today is worth more than a dollar one or more years from now (for several reasons).

Return to capital in the form of interest and profit is an essential ingredient of engineering economy studies.

- Interest and profit pay the providers of capital for forgoing its use during the time the capital is being used.
- Interest and profit are payments for the *risk* the investor takes in letting another use his or her capital.
- Any project or venture must provide a sufficient return to be financially attractive to the suppliers of money or property.

Simple Interest: infrequently used

When the total interest earned or charged is linearly proportional to the initial amount of the loan (principal), the interest rate, and the number of interest periods, the interest and interest rate are said to be *simple*.

Computation of simple interest

The total interest, I, earned or paid may be computed using the formula below.

$$\underline{I} = (P)(N)(i)$$

P = principal amount lent or borrowed

N = number of interest periods (e.g., years)

i = interest rate per interest period

The total amount repaid at the end of N interest periods is $P + \underline{I}$.

If $5,000 were loaned for five years at a simple interest rate of 7% per year, the interest earned would be $\underline{\mathrm{I}} = (P)(N)(i)$

$$\underline{\mathrm{I}} = \$5,000 \quad \text{x} \quad 5 \quad \text{x} \quad 0.07 = \$1,750$$

So, the total amount repaid at the end of five years would be the original amount ($5,000) plus the interest ($1,750), or $6,750.

$$\underline{I} = (P)(N)(i)$$

Compound interest reflects both the remaining principal and any accumulated interest. For $1,000 at 10%…

Period	(1) Amount owed at beginning of period	(2)=(1)x10% Interest amount for period	(3)=(1)+(2) Amount owed at end of period
1	$1,000	$100	$1,100
2	$500	$50	$550
3	$500	$50	$550

Compound interest is commonly used in personal and professional financial transactions.

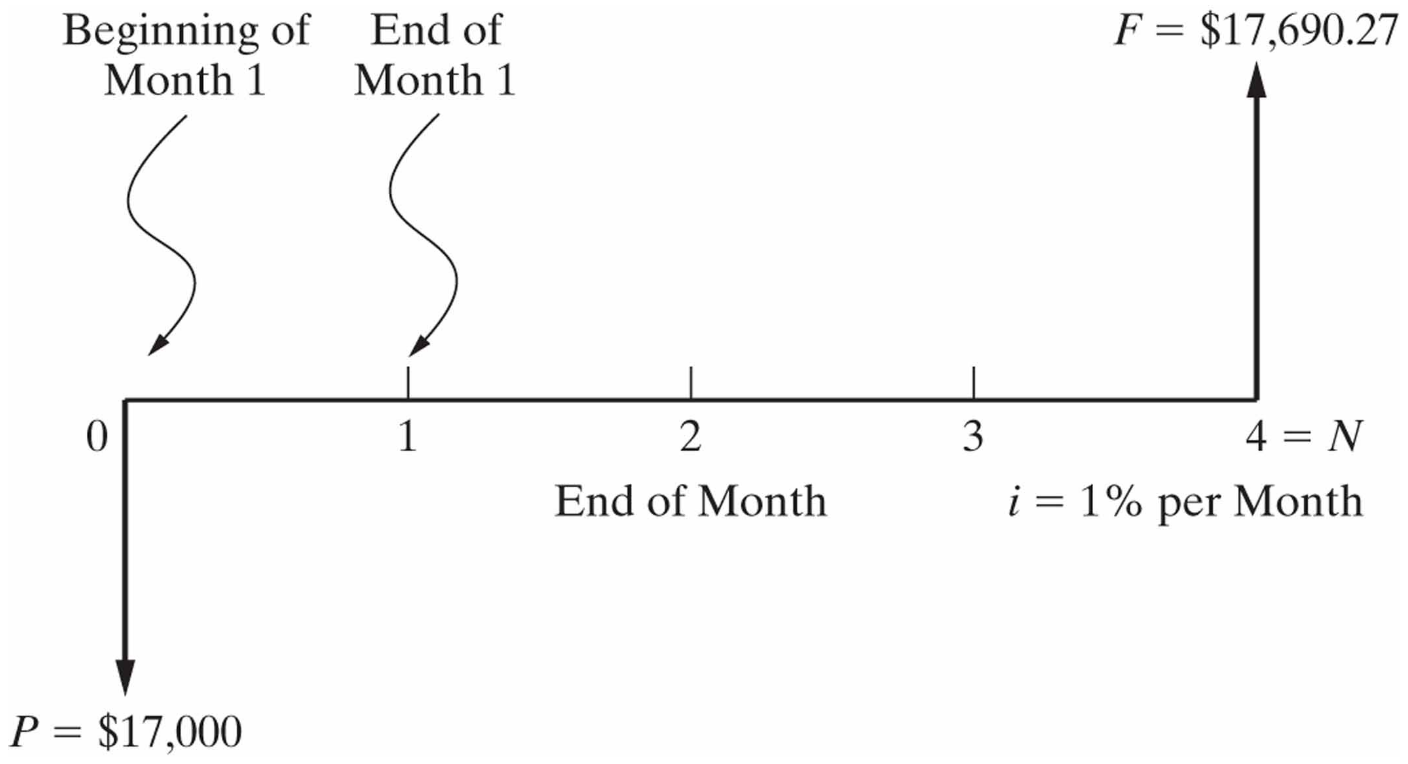
Economic equivalence allows us to compare alternatives on a common basis.

- Each alternative can be reduced to an *equivalent basis* dependent on
  - interest rate,
  - amount of money involved, and
  - timing of monetary receipts or expenses.
- Using these elements we can "move" cash flows so that we can compare them at particular points in time.

We need some tools to find economic equivalence.

- Notation used in formulas for compound interest calculations.
  - i = effective interest rate per interest period
  - N = number of compounding (interest) periods
  - P = present sum of money; *equivalent* value of one or more cash flows at a reference point in time; the present
  - F = future sum of money; *equivalent* value of one or more cash flows at a reference point in time; the future
  - A = end-of-period cash flows in a uniform series continuing for a certain number of periods, starting at the end of the first period and continuing through the last

A cash flow diagram is an indispensable tool for clarifying and visualizing a series of cash flows.



Cash flow tables are essential to modeling engineering economy problems in a spreadsheet

	A	B	C	D	E
1		Alternative *A*	Alternative *B*	Difference	Cumulative
2	End of Year	Net Cash Flow	Net Cash Flow	(*B*-*A*)	Difference
3	0 (now)	$ (18,000)	$ (60,000)	$ (42,000)	$ (42,000)
4	1	$ (34,400)	$ (25,000)	$ 9,400	$ (32,600)
5	2	$ (34,400)	$ (25,000)	$ 9,400	$ (23,200)
6	3	$ (34,400)	$ (25,000)	$ 9,400	$ (13,800)
7	4	$ (34,400)	$ (34,400)	$ -	$ (13,800)
8	5	$ (34,400)	$ (25,000)	$ 9,400	$ (4,400)
9	6	$ (34,400)	$ (25,000)	$ 9,400	$ 5,000
10	7	$ (34,400)	$ (25,000)	$ 9,400	$ 14,400
11	8	$ (32,400)	$ (17,000)	$ 15,400	$ 29,800
12	Total	$ (291,200)	$ (261,400)		

= – 25000 – 9400

= C3 – B3

= SUM(D$3:D3)

= – 34400 + 2000

= SUM(B3:B11)

= – 25000 + 8000

We can apply compound interest formulas to "move" cash flows along the cash flow diagram.

Using the standard notation, we find that a present amount, *P*, can grow into a future amount, *F*, in *N* time periods at interest rate *i* according to the formula below.

$$F = P(1+i)^N$$

In a similar way we can find *P* given *F* by

$$P = F(1+i)^{-N}$$

It is common to use standard notation for interest factors.

$$(1+i)^N = (F/P, i, N)$$

This is also known as the *single payment compound amount* factor. The term on the right is read "*F* given *P* at *i*% interest per period for *N* interest periods."

$$(1+i)^{-N} = (P/F, i, N)$$

is called the *single payment present worth* factor.

We can use these to find economically equivalent values at different points in time.

$2,500 at time zero is equivalent to how much after six years if the interest rate is 8% per year?

$$F = \$2,500(F/P, 8\%, 6) = \$2,500(1.5869) = \$3,967$$

$3,000 at the end of year seven is equivalent to how much today (time zero) if the interest rate is 6% per year?

$$P = \$3,000(P/F, 6\%, 7) = \$3,000(0.6651) = \$1,995$$

There are interest factors for a series of end-of-period cash flows.

$$F = A\left[\frac{(1+i)^N - 1}{i}\right] = A(F/A, i\%, N)$$

How much will you have in 40 years if you save $3,000 each year and your account earns 8% interest each year?

$$F = \$3,000(F/A, 8\%, 40) = \$3,000(259.0565) = \$777,170$$

Finding the present amount from a series of end-of-period cash flows.

$$P = A\left[\frac{(1+i)^N - 1}{i(1+i)^N}\right] = A(P/A, i\%, N)$$

How much would is needed today to provide an annual amount of $50,000 each year for 20 years, at 9% interest each year?

$$P = \$50,000(P/A, 9\%, N) = \$50,000(9.1285) = \$456,427$$

Finding A when given F.

$$A = F\left[\frac{i}{(1+i)^N - 1}\right] = F(A/F, i\%, N)$$

How much would you need to set aside each year for 25 years, at 10% interest, to have accumulated $1,000,000 at the end of the 25 years?

$$A = \$1,000,000(A/F, 10\%, 25) = \$1,000,000(0.0102) = \$10,200$$

Finding A when given P.

$$A = P\left[\frac{i(1+i)^N}{(1+i)^N - 1}\right] = P(A/P, i\%, N)$$

If you had $500,000 today in an account earning 10% each year, how much could you withdraw each year for 25 years?

$$A = \$500,000(A/P, 10\%, 25) = \$500,000(0.1102) = \$55,100$$

Symbols and Cash Flow Diagrams

- The mathematical relations used in engineering economy employ the following symbols:

NOTE

The dollar amount of F and A are considered at the end of the interest period.

P = Value of sum of money at a time denoted as the present.

F = Value or sum of money at some future time, or a single sum of money at the end of n interest period.

A = A series of periodic, equal amount of money.

n = Number of interest periods.

i = Interest rate per interest period.

Cash Flow

- Every person or company has cash receipts (income) and cash disbursement (costs).

- The results of income and costs is called cash flow.

Cash Flow = Receipts – Disbursements

- A positive cash flow indicates a net receipts in a particular interest period or year.

- A negative cash flow indicates a net disbursement in that period.

Cash Flow

- Example: If you buy a printer in 1999 for $300, maintain it for three years at a cost of $20 per year, and then sell it for $50, what are your cash flows for each year?

Year	Receipts	Disbursement	Cash Flow
1999	0	$300	- $300
2000	0	$20	- $20
2001	0	$20	- $20
2002	$50	$20	+ $30

- Its important to remember that all receipts and disbursements and thus cash flows are assumed to be end-of period amounts. Therefore, 1999 is the present (now) and 2002 is the end of year 3.

Cash Flow

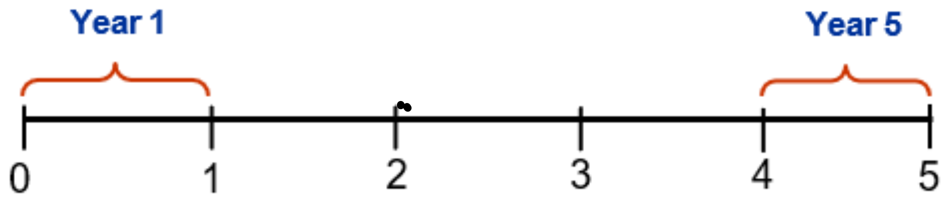
- Example: Suppose you borrowed $1,000 on May 1, 1984, and agree to repay the loan in one lump sum of $1,402.60 at the end of four years at 7%. Tabulate the cash flows?

Date	Receipts	Disbursement	Cash Flow
May 1, 1984	$1,000	0	+ $1,000
May 1, 1985	0	0	0
May 1, 1986	0	0	0
May 1, 1987	0	0	0
May 1, 1988	0	$1,402.60	- $1,402.60

Cash Flow Diagrams

- A cash flow diagram is simply a graphical representation of cash flows (in vertical direction) on a time scale (in horizontal direction). Time zero is considered to be present, and time 1 is the end of time period 1.

- This cash flow diagram is set up for five years.



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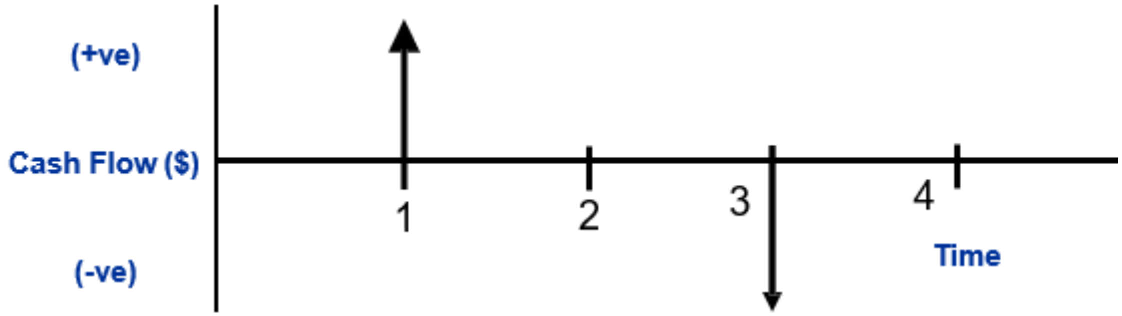
Cash Flow Diagrams

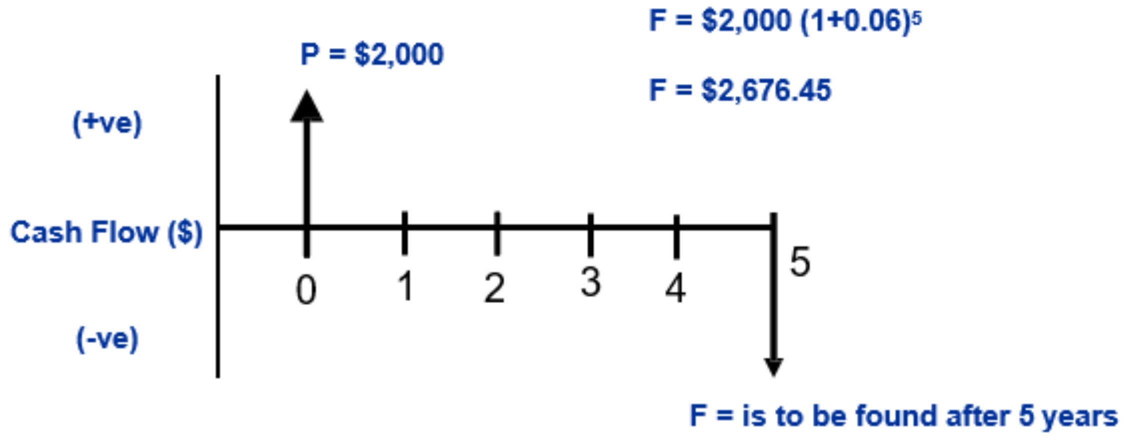
- The direction of the cash flows (income or out go) is indicated by the direction of the arrows.

- From the investor's point of view, the borrowed funds are cash flows entering the system, while the debt repayments are cash flows leaving the system.



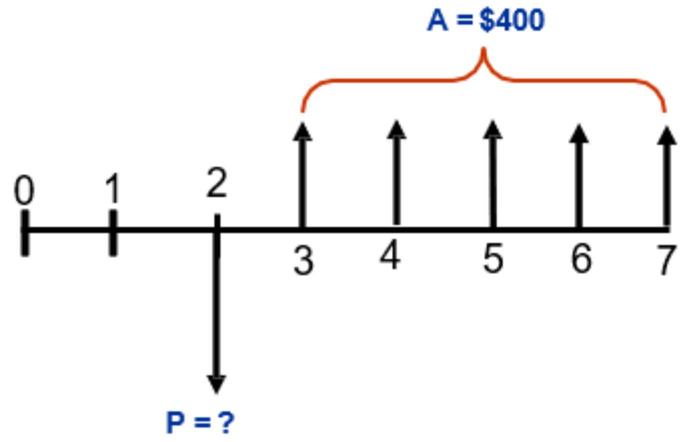
Cash Flow Diagrams

- Example: If you borrow \$2,000 now and must repay the loan plus interest (at rate of 6% per year) after five years. Draw the cash flow diagram. What is the total amount you must pay?



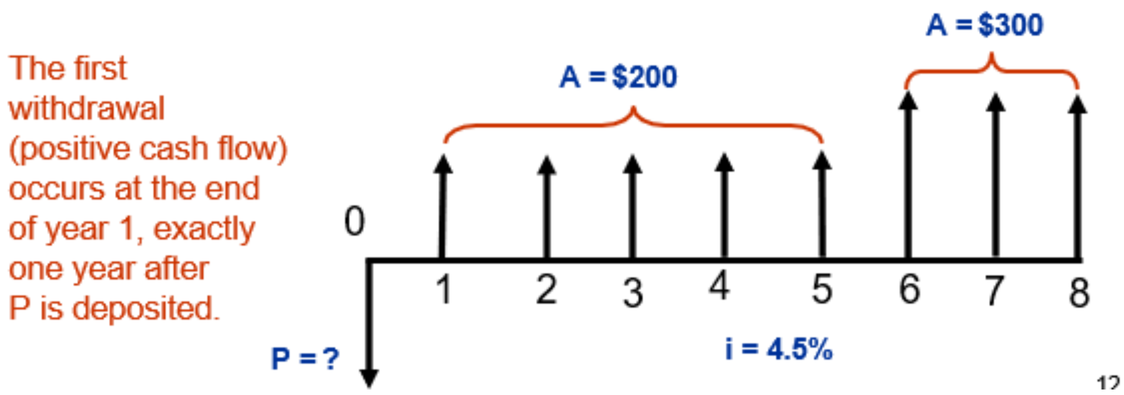
Cash Flow Diagrams

- **Example:** Assume that you want to deposit an amount (P) into an account two years from now in order to be able to withdraw $400 per year for five years starting three years from now. Assume that the interest rate is 5.5% per year. Construct the cash flow diagram.



Cash Flow Diagrams

- Example: Suppose that you want to make a deposit into your account now such that you can withdraw an equal amount (A1) of $200 per year for the first five years starting one year after your deposit and a different annual amount (A2) of $300 per year for the following three years. With an interest rate (i) of 4.5% per year, construct the cash flow diagram.



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It can be challenging to solve for N or i.

- We may know P, A, and i and want to find N.
- We may know P, A, and N and want to find i.
- These problems present special challenges that are best handled on a spreadsheet.

Finding N

Acme borrowed \$100,000 from a local bank, which charges them an interest rate of 7% per year. If Acme pays the bank \$8,000 per year, how many years will it take to pay off the loan?

$$\$100,000 = \$8,000(P/A, 7\%, N)$$

So,

$$(P/A, 7\%, N) = \frac{\$100,000}{\$8,000} = 12.5 = \frac{(1.07)^N - 1}{0.07(1.07)^N}$$

This can be solved by using the interest tables and interpolation, but we generally resort to a computer solution.

TABLE C-10 Discrete Compounding; $i = 7\%$

	Single Payment		Uniform Series				Uniform Gradient		
	Compound Amount Factor	Present Worth Factor	Compound Amount Factor	Present Worth Factor	Sinking Fund Factor	Capital Recovery Factor	Gradient Present Worth Factor	Gradient Uniform Series Factor	
	To Find *F* Given *P*	To Find *P* Given *F*	To Find *F* Given *A*	To Find *P* Given *A*	To Find *A* Given *F*	To Find *A* Given *P*	To Find *P* Given *G*	To Find *A* Given *G*	
N	*F/P*	*P/F*	*F/A*	*P/A*	*A/F*	*A/P*	*P/G*	*A/G*	*N*
1	1.0700	0.9346	1.0000	0.9346	1.0000	1.0700	0.000	0.0000	1
2	1.1449	0.8734	2.0700	1.8080	0.4831	0.5531	0.873	0.4831	2
3	1.2250	0.8163	3.2149	2.6243	0.3111	0.3811	2.506	0.9549	3
4	1.3108	0.7629	4.4399	3.3872	0.2252	0.2952	4.795	1.4155	4
5	1.4026	0.7130	5.7507	4.1002	0.1739	0.2439	7.647	1.8650	5
6	1.5007	0.6663	7.1533	4.7665	0.1398	0.2098	10.978	2.3032	6
7	1.6058	0.6227	8.6540	5.3893	0.1156	0.1856	14.715	2.7304	7
8	1.7182	0.5820	10.2598	5.9713	0.0975	0.1675	18.789	3.1465	8
9	1.8385	0.5439	11.9780	6.5152	0.0835	0.1535	23.140	3.5517	9
10	1.9672	0.5083	13.8164	7.0236	0.0724	0.1424	27.716	3.9461	10
11	2.1049	0.4751	15.7836	7.4987	0.0634	0.1334	32.467	4.3296	11
12	2.2522	0.4440	17.8885	7.9427	0.0559	0.1259	37.351	4.7025	12
13	2.4098	0.4150	20.1406	8.3577	0.0497	0.1197	42.330	5.0648	13
14	2.5785	0.3878	22.5505	8.7455	0.0443	0.1143	47.372	5.4167	14
15	2.7590	0.3624	25.1290	9.1079	0.0398	0.1098	52.446	5.7583	15
16	2.9522	0.3387	27.8881	9.4466	0.0359	0.1059	57.527	6.0897	16
17	3.1588	0.3166	30.8402	9.7632	0.0324	0.1024	62.592	6.4110	17
18	3.3799	0.2959	33.9990	10.0591	0.0294	0.0994	67.622	6.7225	18
19	3.6165	0.2765	37.3790	10.3356	0.0268	0.0968	72.599	7.0242	19
20	3.8697	0.2584	40.9955	10.5940	0.0244	0.0944	77.509	7.3163	20
21	4.1406	0.2415	44.8652	10.8355	0.0223	0.0923	82.339	7.5990	21
22	4.4304	0.2257	49.0057	11.0612	0.0204	0.0904	87.079	7.8725	22
23	4.7405	0.2109	53.4361	11.2722	0.0187	0.0887	91.720	8.1369	23
24	5.0724	0.1971	58.1767	11.4693	0.0172	0.0872	96.255	8.3923	24
25	5.4274	0.1842	63.2490	11.6536	0.0158	0.0858	100.677	8.6391	25
30	7.6123	0.1314	94.4608	12.4090	0.0106	0.0806	120.972	9.7487	30
35	10.6766	0.0937	138.2369	12.9477	0.0072	0.0772	138.135	10.6687	35
40	14.9745	0.0668	199.6351	13.3317	0.0050	0.0750	152.293	11.4233	40
45	21.0023	0.0476	285.7495	13.6055	0.0035	0.0735	163.756	12.0360	45
50	29.4570	0.0339	406.5289	13.8007	0.0025	0.0725	172.905	12.5287	50
60	57.9464	0.0173	813.5204	14.0392	0.0012	0.0712	185.768	13.2321	60
80	224.2344	0.0045	3189.0627	14.2220	0.0003	0.0703	198.075	13.9273	80
100	867.7163	0.0012	12381.6618	14.2693	0.0001	0.0701	202.200	14.1703	100
∞				14.2857		0.0700			∞

Finding *i*

Jill invested $1,000 each year for five years in a local company and sold her interest after five years for $8,000. What annual rate of return did Jill earn?

$$\$8,000 = \$1,000(F/A, i\%, 5)$$

So,

$$(F/A, i\%, 5) = \frac{\$8,000}{\$1,000} = 8 = \frac{(1+i)^5 - 1}{i}$$

Again, this can be solved using the interest tables and interpolation, but we generally resort to a computer solution.

There are specific spreadsheet functions to find *N* and *i*.

The Excel function used to solve for *N* is

NPER(*rate*, *pmt*, *pv*,*fv*), which will compute the number of payments of magnitude *pmt* required to pay off a present amount (*pv)* at a fixed interest rate (*rate*).

One Excel function used to solve for *i* is

RATE(*nper*, *pmt*, *pv*, *fv*), which returns a fixed interest rate for an annuity of *pmt* that lasts for *nper* periods to either its present value (*pv*) or future value (*fv*).

We need to be able to handle cash flows that do not occur until some time in the future.

- Deferred annuities are uniform series that do not begin until some time in the future.
- If the annuity is deferred J periods then the first payment (cash flow) begins at the end of period $J+1$.

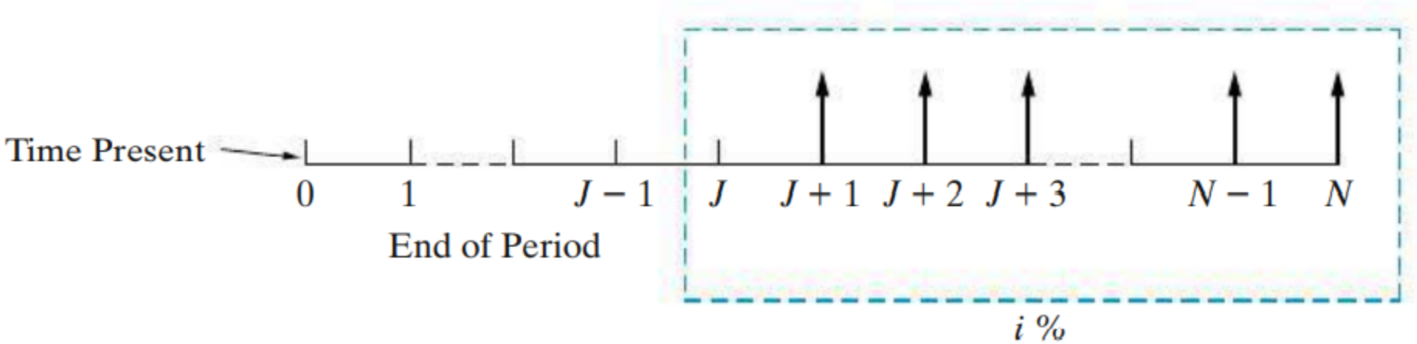

Figure 4-9 General Cash-Flow Representation of a Deferred Annuity (Uniform Series)

Finding the value at time *0* of a deferred annuity is a two-step process.

1. Use (*P/A*, *i*%, *N-J*) find the value of the deferred annuity at the end of period *J* (where there are *N-J* cash flows in the annuity).
2. Use (*P/F*, *i*%, *J*) to find the value of the deferred annuity at time zero.

$$P_0 = A(P/A, i\%, N-J)(P/F, i\%, J)$$

Example 1

EXAMPLE 4-14 Present Equivalent of a Deferred Annuity

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To illustrate the preceding discussion, suppose that a father, on the day his son is born, wishes to determine what lump amount would have to be paid into an account bearing interest of 12% per year to provide withdrawals of \$2,000 on each of the son's 18th, 19th, 20th, and 21st birthdays.

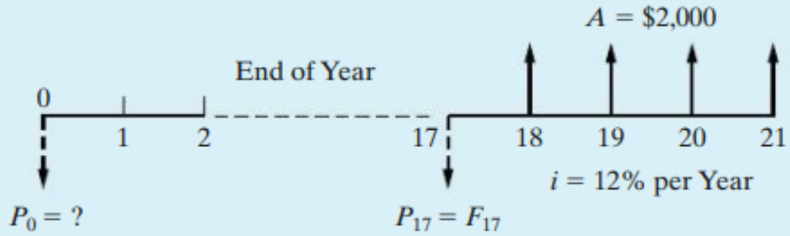
Solution

The problem is represented in the following cash-flow diagram. One should first recognize that an ordinary annuity of four withdrawals of \$2,000 each is involved and that the present equivalent of this annuity occurs at the 17th birthday when a $(P/A, i\%, N - J)$ factor is utilized. In this problem, $N = 21$ and $J = 17$. It is often helpful to use a *subscript* with P or F to denote the respective point in time.

Example 1

Hence,

$$P_{17} = A(P/A, 12\%, 4) = \$2{,}000(3.0373) = \$6{,}074.60.$$



Note the dashed arrow in the cash-flow diagram denoting P_{17}. Now that P_{17} is known, the next step is to calculate P_0. With respect to P_0, P_{17} is a future equivalent, and hence it could also be denoted F_{17}. Money at a given point in time, such as the end of period 17, is the same regardless of whether it is called a present equivalent or a future equivalent. Hence,

$$P_0 = F_{17}(P/F, 12\%, 17) = \$6{,}074.60(0.1456) = \$884.46,$$

which is the amount that the father would have to deposit on the day his son is born.

Example 2

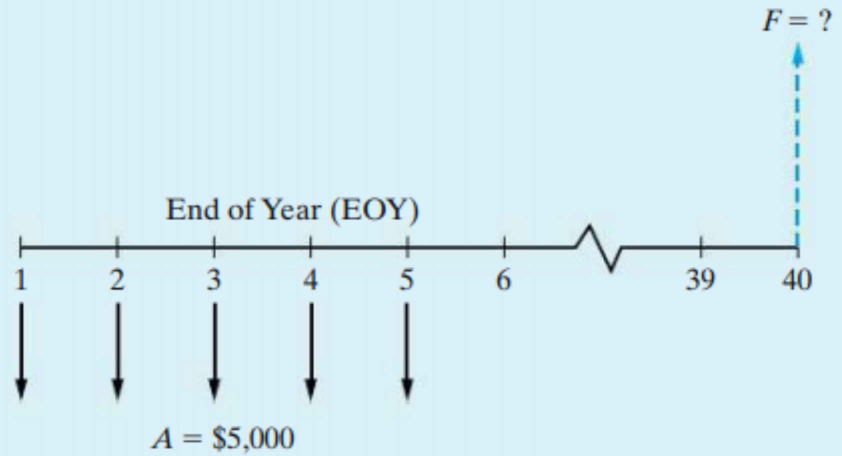
EXAMPLE 4-15 Deferred Future Value of an Annuity

When you take your first job, you decide to start saving right away for your retirement. You put $5,000 per year into the company's 401(k) plan, which averages 8% interest per year. Five years later, you move to another job and start a new 401(k) plan. You never get around to merging the funds in the two plans. If the first plan continued to earn interest at the rate of 8% per year for 35 years after you stopped making contributions, how much is the account worth?

Example 2

Solution

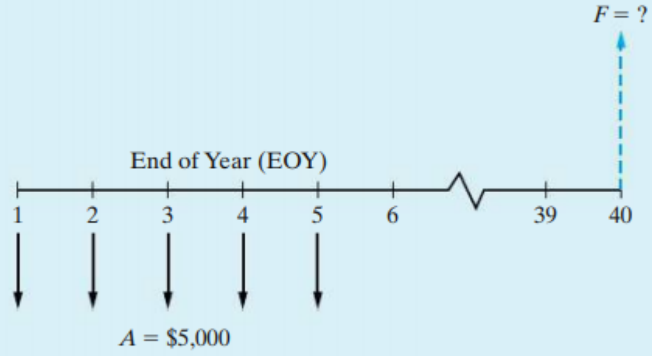
The following cash-flow diagram clarifies the timing of the cash flows for the original 401(k) investment plan.



Example 2

Solution

The following cash-flow diagram clarifies the timing of the cash flows for the original 401(k) investment plan.



The easiest way to approach this is to first find the future equivalent of the annuity as of time 5.

$$F_5 = \$5{,}000\,(F/A, 8\%, 5) = \$5{,}000\,(5.8666) = \$29{,}333.$$

To determine F_{40}, F_5 can now be denoted P_5, and

$$F_{40} = P_5\,(F/P, 8\%, 35) = \$29{,}333\,(14.7853) = \$433{,}697.$$

Example

EXAMPLE 4-16 **Calculating Equivalent *P*, *F*, and *A* Values**

VIDEO SOLUTIONS

Figure 4-10 depicts an example problem with a series of year-end cash flows extending over eight years. The amounts are \$100 for the first year, \$200 for the second year, \$500 for the third year, and \$400 for each year from the fourth through the eighth. These could represent something like the expected maintenance expenditures for a certain piece of equipment or payments into a fund. Note that the payments are shown at the end of each year, which is a standard assumption (convention) for this book and for economic analyses in general, unless we have information to the contrary. It is desired to find

(a) the present equivalent expenditure, P_0;

(b) the future equivalent expenditure, F_8;

(c) the annual equivalent expenditure, A

of these cash flows if the annual interest rate is 20%. Solve by hand and by using a spreadsheet.

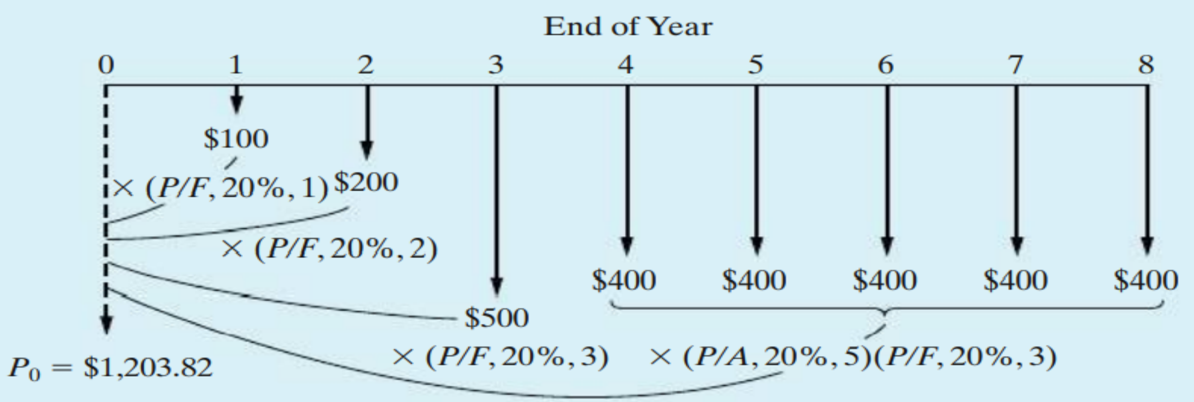
Example

Solution by Hand

(a) To find the equivalent P_0, we need to sum the equivalent values of all payments as of the beginning of the first year (time zero). The required movements of money through time are shown graphically in Figure 4-10(a).

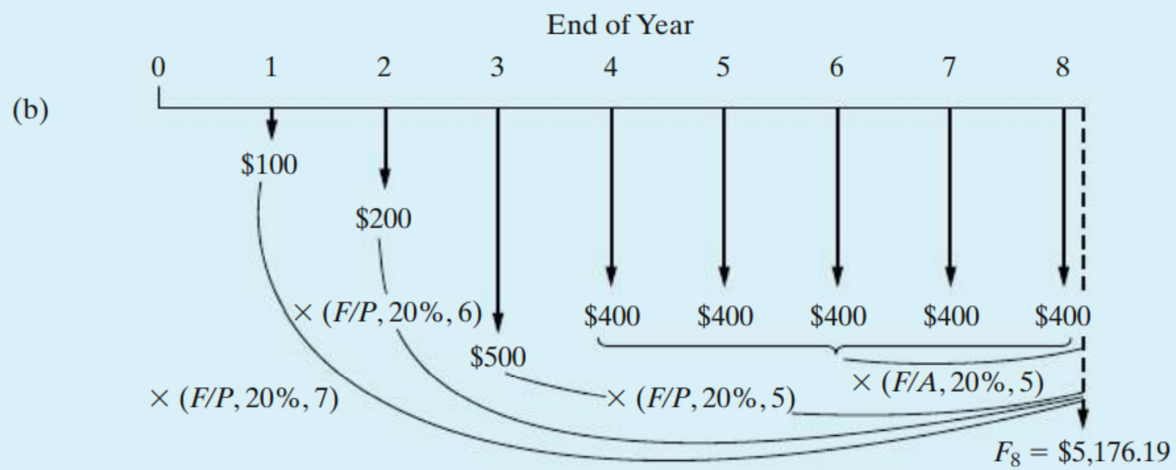
$$\begin{aligned}
P_0 = F_1(P/F, 20\%, 1) &= \$100(0.8333) &= \$83.33 \\
+F_2(P/F, 20\%, 2) &+ \$200(0.6944) &+ 138.88 \\
+F_3(P/F, 20\%, 3) &+ \$500(0.5787) &+ 289.35 \\
+A(P/A, 20\%, 5) \times (P/F, 20\%, 3) &+ \$400(2.9900) \times (0.5787) &+ 692.26 \\
& & \overline{\$1{,}203.82.}
\end{aligned}$$

(a)



Example

(b) To find the equivalent F_8, we can sum the equivalent values of all payments as of the end of the eighth year (time eight). Figure 4-10(b) indicates these

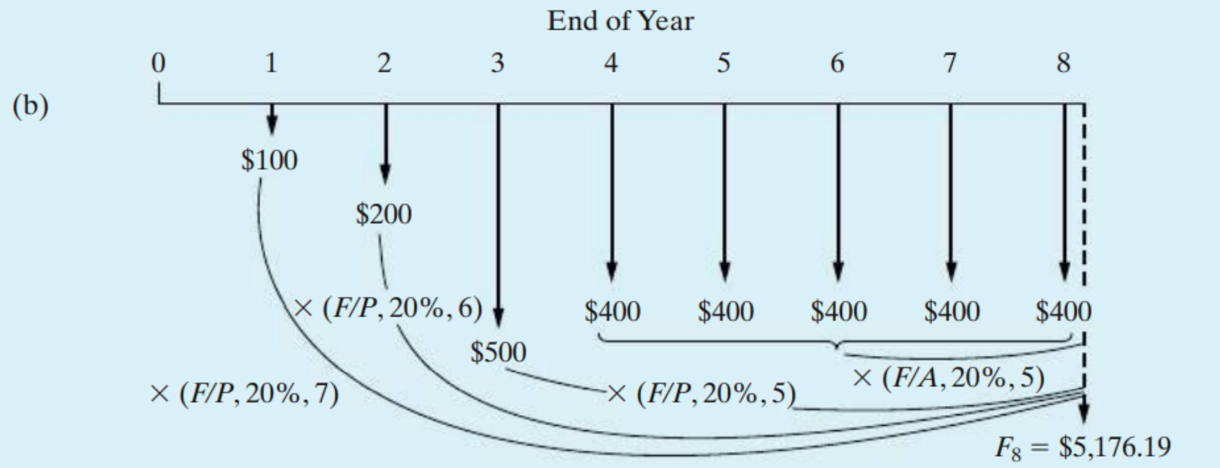


movements of money through time. However, since the equivalent P_0 is already known to be $1,203.82, we can directly calculate

$$F_8 = P_0(F/P, 20\%, 8) = \$1{,}203.82(4.2998) = \$5{,}176.19.$$

Example

(b) To find the equivalent F_8, we can sum the equivalent values of all payments as of the end of the eighth year (time eight). Figure 4-10(b) indicates these



movements of money through time. However, since the equivalent P_0 is already known to be $1,203.82, we can directly calculate

$$F_8 = P_0(F/P, 20\%, 8) = \$1{,}203.82(4.2998) = \$5{,}176.19.$$

Example

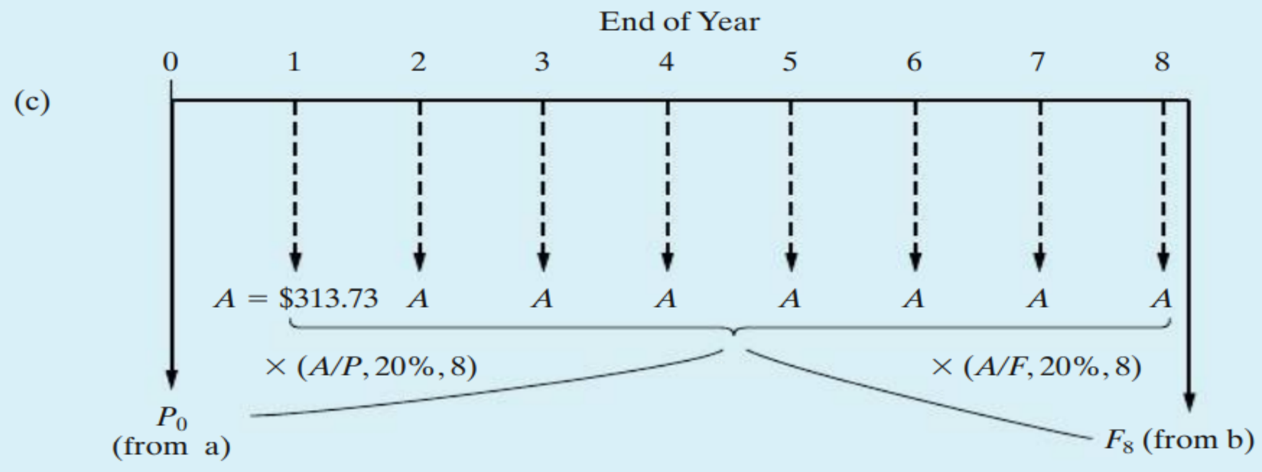
(c) The equivalent A of the irregular cash flows can be calculated directly from either P_0 or F_8 as

$$A = P_0(A/P, 20\%, 8) = \$1{,}203.82(0.2606) = \$313.73$$

or

$$A = F_8(A/F, 20\%, 8) = \$5{,}176.19(0.0606) = \$313.73.$$

The computation of A from P_0 and F_8 is shown in Figure 4-10(c). Thus, we find that the irregular series of payments shown in Figure 4-10 is equivalent to \$1,203.82 at time zero, \$5,176.19 at time eight, or a uniform series of \$313.73 at the end of each of the eight years.



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Sometimes cash flows change by a constant amount each period.

We can model these situations as a *uniform gradient* of cash flows. The table below shows such a gradient.

End of Period	Cash Flows
1	0
2	G
3	$2G$
:	:
N	$(N-1)G$

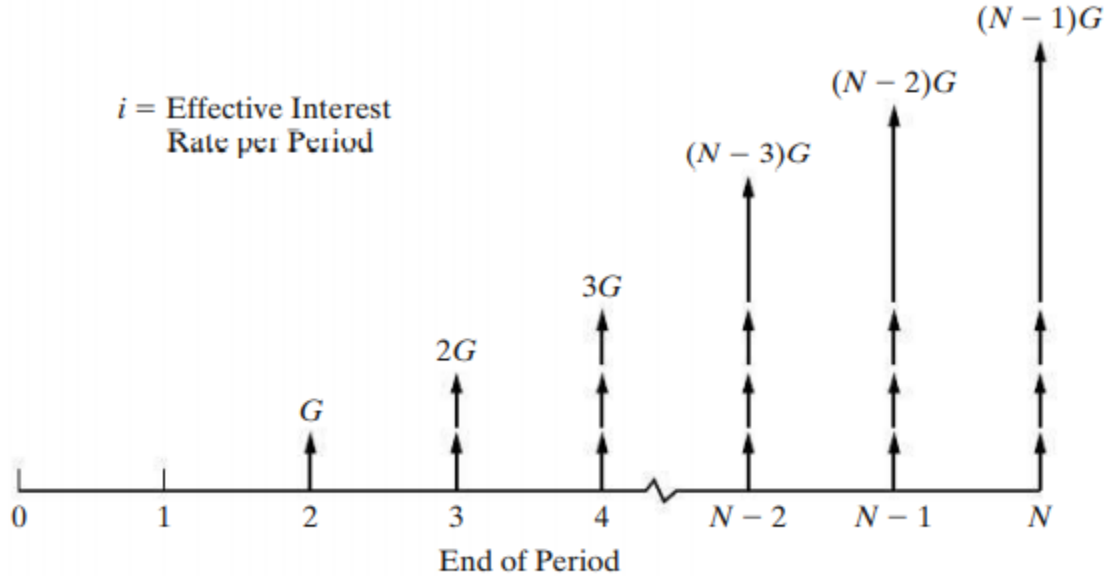


Figure 4-13 Cash-Flow Diagram for a Uniform Gradient Increasing by G Dollars per Period

It is easy to find the present value of a uniform gradient series.

Similar to the other types of cash flows, there is a formula (albeit quite complicated) we can use to find the present value, and a set of factors developed for interest tables.

$$(P/G, i\%, N) = \frac{1}{i}\left[\frac{(1+i)^N - 1}{i(1+i)^N} - \frac{N}{(1+i)^N}\right]$$

We can also find *A* or *F* equivalent to a uniform gradient series.

$$(A/G, i\%, N) = \frac{1}{i} - \frac{N}{(1+i)^N - 1}$$

$$(F/G, i\%, N) = \frac{1}{i}(F/A, i\%, N) - \frac{N}{i}$$

Example

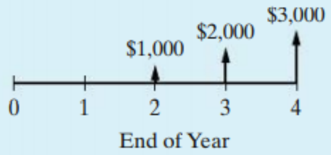
EXAMPLE 4-20 **Using the Gradient Conversion Factors to Find *P* and *A***

As an example of the straightforward use of the gradient conversion factors, suppose that certain EOY cash flows are expected to be \$1,000 for the *second* year, \$2,000 for the third year, and \$3,000 for the fourth year and that, if interest is 15% per year, it is desired to find

(a) present equivalent value at the beginning of the first year,

(b) uniform annual equivalent value at the end of each of the four years.

Solution



Observe that this schedule of cash flows fits the model of the arithmetic gradient formulas with $G = \$1{,}000$ and $N = 4$. Note that there is no cash flow at the end of the first period.

(a) The present equivalent can be calculated as

$$P_0 = G(P/G, 15\%, 4) = \$1{,}000(3.79) = \$3{,}790.$$

(b) The annual equivalent can be calculated from Equation (4-26) as

$$A = G(A/G, 15\%, 4) = \$1{,}000(1.3263) = \$1{,}326.30.$$

Of course, once P_0 is known, the value of A can be calculated as

$$A = P_0(A/P, 15\%, 4) = \$3{,}790(0.3503) = \$1{,}326.30.$$

Example

EXAMPLE 4-21 **Present Equivalent of an Increasing Arithmetic Gradient Series**

As a further example of the use of arithmetic gradient formulas, suppose that we have cash flows as follows:

End of Year	Cash Flows ($)
1	5,000
2	6,000
3	7,000
4	8,000

Also, assume that we wish to calculate their present equivalent at $i = 15\%$ per year, using gradient conversion factors.

Solution

The schedule of cash flows is depicted in the left-hand diagram of Figure 4-14. The right two diagrams of Figure 4-14 show how the original schedule can be broken into two separate sets of cash flows, an annuity series of $5,000 plus an arithmetic gradient of $1,000 that fits the general gradient model for which factors are tabled. The summed present equivalents of these two separate sets of cash flows equal the present equivalent of the original problem. Thus, using the symbols shown in Figure 4-14, we have

$$P_{0T} = P_{0A} + P_{0G}$$

$$= A(P/A, 15\%, 4) + G(P/G, 15\%, 4)$$

$$= \$5{,}000(2.8550) + \$1{,}000(3.79) = \$14{,}275 + 3{,}790 = \$18{,}065.$$

Example

The annual equivalent of the original cash flows could be calculated with the aid of Equation (4-26) as follows:

$$A_T = A + A_G$$

$$= \$5{,}000 + \$1{,}000(A/G, 15\%, 4) = \$6{,}326.30.$$

A_T is equivalent to P_{0T} because $6,326.30$(P/A, 15\%, 4)$ = $18,061, which is the same value obtained previously (subject to round-off error).

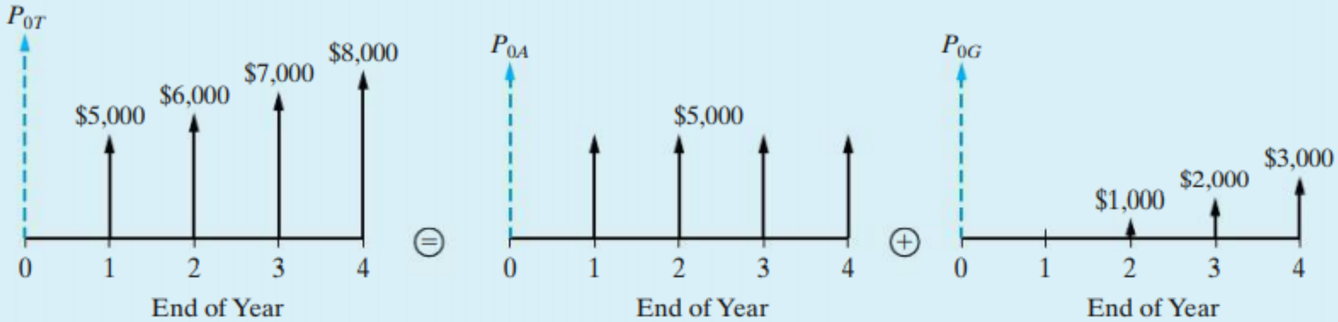
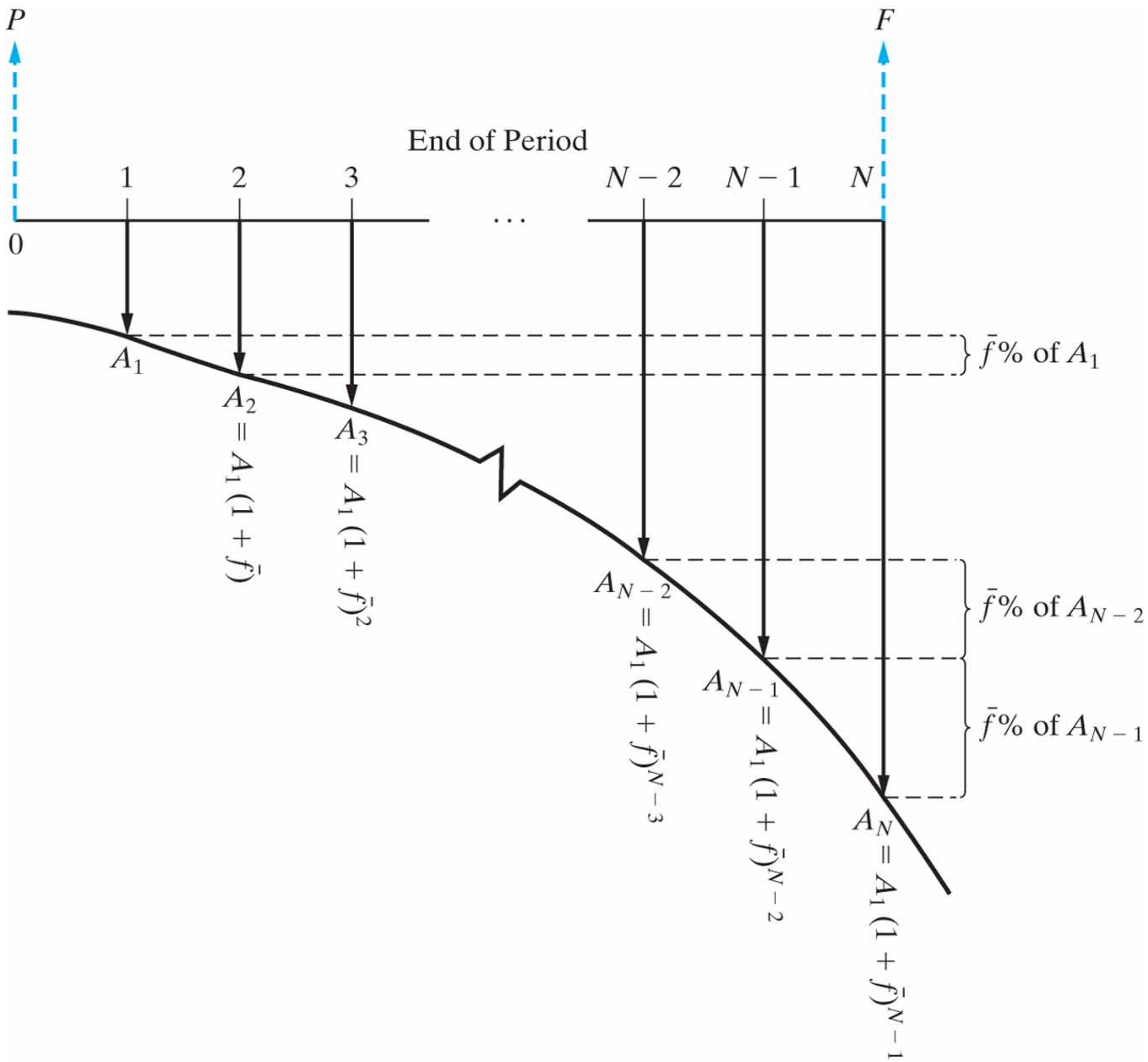


Figure 4-14 Breakdown of Cash Flows for Example 4-21

Sometimes cash flows change by a constant rate, $\bar{f}$, each period--this is a *geometric gradient series*.

This table presents a geometric gradient series. It begins at the end of year 1 and has a rate of growth, $\bar{f}$, of 20%.

End of Year	Cash Flows ($)
1	1,000
2	1,200
3	1,440
4	1,728


P
F
End of Period
1
2
3
N − 2
N − 1
N
0
. . .
A_1
$A_2 = A_1(1+\bar{f})$
$A_3 = A_1(1+\bar{f})^2$
$A_{N-2} = A_1(1+\bar{f})^{N-3}$
$A_{N-1} = A_1(1+\bar{f})^{N-2}$
$A_N = A_1(1+\bar{f})^{N-1}$
$\bar{f}$% of A_1
$\bar{f}$% of A_{N-2}
$\bar{f}$% of A_{N-1}

We can find the present value of a geometric series by using the appropriate formula below.

If $\bar{f} \neq i$

$$\frac{A_1[1 - (P/F, i\%, N)(F/P, \bar{f}\%, N]}{1 - \bar{f}}$$

If $\bar{f} = i$

$$A_1 N(P/F, i\%, 1)$$

Where A_1 is the initial cash flow in the series.

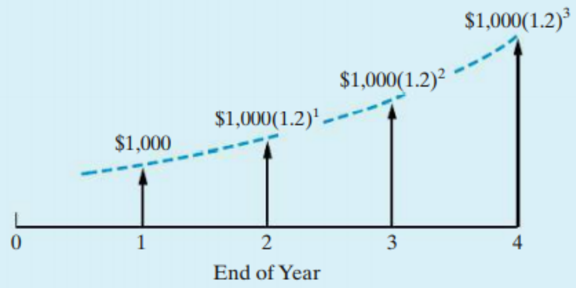
Example

EXAMPLE 4-23 **Equivalence Calculations for an Increasing Geometric Gradient Series**

VIDEO SOLUTIONS

Consider the following EOY geometric sequence of cash flows and determine the P, A, and F equivalent values. The rate of increase is 20% per year after the first year, and the interest rate is 25% per year.

Solution



$$P = \frac{\$1{,}000\left[1 - (P/F, 25\%, 4)(F/P, 20\%, 4)\right]}{0.25 - 0.20}$$

$$\begin{aligned} P &= \frac{\$1{,}000}{0.05}\left[1 - (0.4096)(2.0736)\right] \\ &= \$20{,}000(0.15065) \\ &= \$3{,}013; \\ A &= \$3{,}013(A/P, 25\%, 4) = \$1{,}275.70; \\ F &= \$3{,}013(F/P, 25\%, 4) = \$7{,}355.94. \end{aligned}$$

When interest rates vary with time different procedures are necessary.

- Interest rates often change with time (e.g., a variable rate mortgage).
- We often must resort to moving cash flows one period at a time, reflecting the interest rate for that single period.

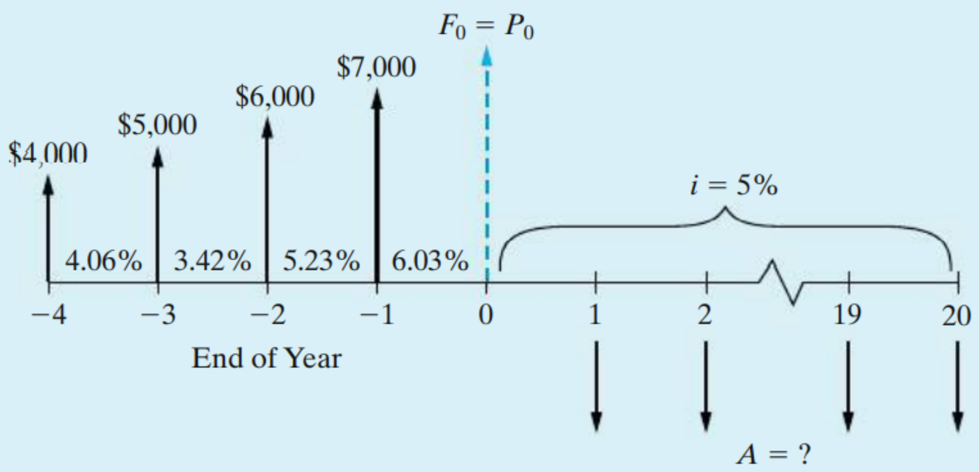
EXAMPLE 4-27 Compounding with Changing Interest Rates

VIDEO SOLUTIONS

Ashea Smith is a 22-year-old senior who used the Stafford loan program to borrow \$4,000 four years ago when the interest rate was 4.06% per year. \$5,000 was borrowed three years ago at 3.42%. Two years ago she borrowed \$6,000 at 5.23%, and last year \$7,000 was borrowed at 6.03% per year. Now she would like to consolidate her debt into a single 20-year loan with a 5% fixed annual interest rate. If Ashea makes annual payments (starting in one year) to repay her total debt, what is the amount of each payment?

Solution

The following cash-flow diagram clarifies the timing of Ashea's loans and the applicable interest rates. The diagram is drawn using Ashea's viewpoint.



Before we can find the annual repayment amount, we need to find the current (time 0) equivalent value of the four loans. This problem can be solved by compounding the amount owed at the beginning of each year by the interest rate that applies to each individual year and repeating this process over the four years to obtain the total current equivalent value.

$$F_{-3} = \$4{,}000(F/P, 4.06\%, 1) + \$5{,}000 = \$4{,}000(1.0406)$$

$$+ \$5{,}000 = \$9{,}162.40$$

$$F_{-2} = \$9{,}162.40(F/P, 3.42\%, 1) + \$6{,}000 = \$15{,}475.75$$

$$F_{-1} = \$15{,}475.75(F/P, 5.23\%, 1) + \$7{,}000 = \$23{,}285.13$$

$$F_0 = \$23{,}285.13(F/P, 6.03\%, 1) = \$24{,}689.22$$

Notice that it was a simple matter to substitute $(F/P, i\%, N) = (1 + i)^N$ for the noninteger values of i.

Now that we have the current equivalent value of the amount Ashea borrowed ($F_0 = P_0$), we can easily compute her annual repayment amount over 20 years when the interest rate is fixed at 5% per year.

$$A = \$24{,}689.22(A/P, 5\%, 20) = \$24{,}689.22(0.0802) = \$1{,}980.08 \text{ per year}$$

Comment

The total principal borrowed was \$4,000 + \$5,000 + \$6,000 + \$7,000 = \$22,000. Notice that a total of \$17,601.60 (20 × \$1,980.08 − \$22,000) in interest is repaid over the entire 20-year loan period. This interest amount is close to the amount of principal originally borrowed. *Moral:* Borrow as little as possible and repay as quickly as possible to reduce interest expense! See `www.finaid.com`.

The present equivalent of a cash flow occurring at the end of period N can be computed with the equation below, where i_k is the interest rate for the k^{th} period.

$$P = \frac{F_N}{\prod_{k=1}^{N}(1+i_k)}$$

If F_4 = \$2,500 and i_1=8%, i_2=10%, and i_3=11%, then

$$P = \$2,500(P/F, 8\%, 1)(P/F, 10\%, 1)(P/F, 11\%, 1)$$

$$P = \$2,500(0.9259)(0.9091)(0.9009) = \$1,896$$

Nominal and effective interest rates.

- More often than not, the time between successive compounding, or the interest period, is less than one year (e.g., daily, monthly, quarterly).
- The annual rate is known as a *nominal* rate.
- A *nominal* rate of 12%, compounded monthly, means an interest of 1% (12%/12) would accrue each month, and the annual rate would be *effectively* somewhat greater than 12%.
- The more frequent the compounding the greater the *effective* interest.

The effect of more frequent compounding can be easily determined.

Let r be the nominal, annual interest rate and M the number of compounding periods per year. We can find, i, the effective interest by using the formula below.

$$i = (1 + \frac{r}{M})^M - 1$$

Finding effective interest rates.

For an 18% nominal rate, compounded quarterly, the effective interest is.

$$i = (1 + \frac{0.18}{4})^4 - 1 = 19.25\%$$

For a 7% nominal rate, compounded monthly, the effective interest is.

$$i = (1 + \frac{0.07}{12})^{12} - 1 = 7.23\%$$

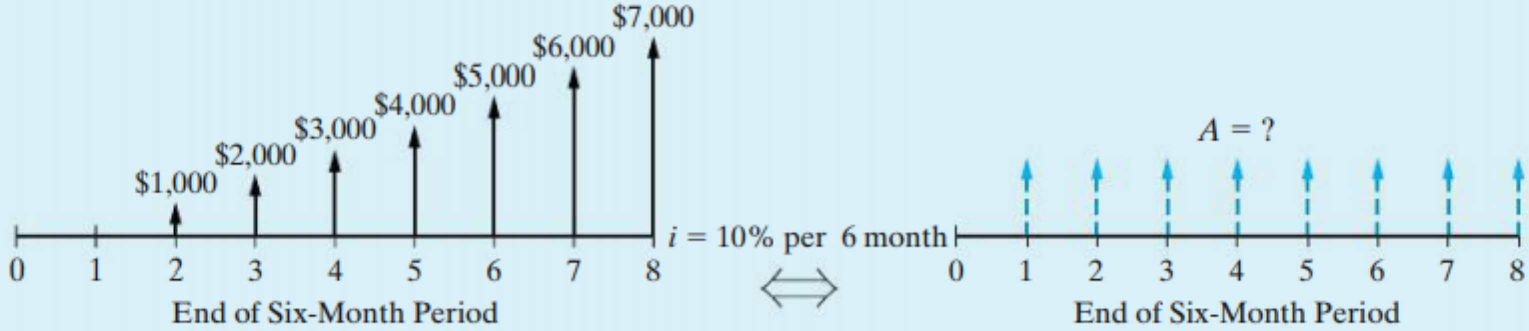
EXAMPLE 4-31 Uniform Gradient Series and Semiannual Compounding

Certain operating savings are expected to be 0 at the end of the first six months, to be $1,000 at the end of the second six months, and to increase by $1,000 at the end of each six-month period thereafter, for a total of four years. It is desired to find the equivalent uniform amount, A, at the end of each of the eight six-month periods if the nominal interest rate is 20% compounded semiannually.

Solution

A cash-flow diagram is given below, and the solution is

$$A = G(A/G, 10\%, 8) = \$1{,}000(3.0045) = \$3{,}004.50.$$



The symbol "⟺" in between the cash-flow diagrams indicates that the left-hand cash-flow diagram is *equivalent to* the right-hand cash-flow diagram when the correct value of A has been determined. In Example 4-31, the interest rate per six-month period is 10%, and cash flows occur every six months.

EXAMPLE 4-32 Finding the Interest Rate on a Loan

A loan of \$15,000 requires monthly payments of \$477 over a 36-month period of time. These payments include both principal and interest.

(a) What is the nominal interest rate (APR) for this loan?

(b) What is the effective interest rate per year?

(c) Determine the amount of unpaid loan principal after 20 months.

Solution

(a) We can set up an equivalence relationship to solve for the unknown interest rate since we know that $P = \$15{,}000$, $A = \$477$, and $N = 36$ months.

$$\$477 = \$15{,}000(A/P, i_{mo}, 36)$$

$$(A/P,\ i_{mo}, 36) = 0.0318$$

We can now look through Appendix C to find values of i that have an $(A/P,\ i,\ 36)$ value close to 0.0318. From Table C-3 ($i = 3/4\%$), we find $(A/P, 3/4\%, 36) = 0.0318$. Therefore,

$$i_{mo} = 0.75\% \text{ per month}$$

and

$$r = 12 \times 0.75\% = 9\% \text{ per year, compounded monthly.}$$

(b) Using Equation (4-32),

$$i_{eff} = \left(1 + \frac{0.09}{12}\right)^{12} - 1 = 0.0938 \text{ or } 9.38\% \text{ per year.}$$

(c) We can find the amount of the unpaid loan principal after 20 months by finding the equivalent value of the remaining 16 monthly payments as of month 20.

$$P_{20} = \$477(P/A, 3/4\%, 16) = \$477(15.0243) = \$7{,}166.59$$

After 20 payments have been made, almost half of the original principal amount remains. Notice that we used the monthly interest rate of $3/4\%$ in our calculation since the cash flows are occurring monthly.

Interest can be compounded continuously.

- Interest is typically compounded at the end of discrete periods.
- In most companies cash is always flowing, and should be immediately put to use.
- We can allow compounding to occur continuously throughout the period.
- The effect of this compared to discrete compounding is small in most cases.

We can use the effective interest formula to derive the interest factors.

$$i = (1 + \frac{r}{M})^M - 1$$

As the number of compounding periods gets larger (*M* gets larger), we find that

$$i = e^r - 1$$

Continuous compounding interest factors.

$$(P/F, \underline{\text{r}}\%, N) = e^{-rN}$$

$$(F/A, \underline{\text{r}}\%, N) = \frac{e^{rN} - 1}{e^r - 1}$$

$$(P/A, \underline{\text{r}}\%, N) = \frac{e^{rN} - 1}{e^{rN}(e^r - 1)}$$

The other factors can be found from these.

TABLE 4-5 Continuous Compounding and Discrete Cash Flows: Interest Factors and Symbols[a]

To Find:	Given:	Factor by which to Multiply "Given"	Factor Name	Factor Functional Symbol
For single cash flows:				
F	P	e^{rN}	Continuous compounding compound amount (single cash flow)	$(F/P,\underline{r}\%,N)$
P	F	e^{-rN}	Continuous compounding present equivalent (single cash flow)	$(P/F,\underline{r}\%,N)$
For uniform series (annuities):				
F	A	$\frac{e^{rN}-1}{e^{r}-1}$	Continuous compounding compound amount (uniform series)	$(F/A,\underline{r}\%,N)$
P	A	$\frac{e^{rN}-1}{e^{rN}(e^{r}-1)}$	Continuous compounding present equivalent (uniform series)	$(P/A,\underline{r}\%,N)$
A	F	$\frac{e^{r}-1}{e^{rN}-1}$	Continuous compounding sinking fund	$(A/F,\underline{r}\%,N)$
A	P	$\frac{e^{rN}(e^{r}-1)}{e^{rN}-1}$	Continuous compounding capital recovery	$(A/P,\underline{r}\%,N)$

EXAMPLE 4-33 Continuous Compounding and Single Amounts

You have $10,000 to invest for two years. Your bank offers 5% interest, compounded continuously for funds in a money market account. Assuming no additional deposits or withdrawals, how much money will be in that account at the end of two years?

Solution

$$F = \$10{,}000\ (F/P, \underline{r} = 5\%, 2) = \$10{,}000\ e^{(0.05)(2)} = \$10{,}000\ (1.1052) = \$11{,}052$$

Comment

If the interest rate was 5% compounded annually, the account would have been worth

$$F = \$10{,}000\ (F/P, 5\%, 2) = \$10{,}000\ (1.1025) = \$11{,}025.$$

EXAMPLE 4-34 Continuous Compounding and Annual Payments

Suppose that one has a present loan of \$1,000 and desires to determine what equivalent uniform EOY payments, A, could be obtained from it for 10 years if the nominal interest rate is 20% compounded continuously ($M = \infty$).

Solution

Here we utilize the formulation

$$A = P(A/P, \underline{r}\%, N).$$

Since the (A/P) factor is not tabled for continuous compounding, we substitute its inverse (P/A), which is tabled in Appendix D. Thus,

$$A = P \times \frac{1}{(P/A, \underline{20}\%, 10)} = \$1{,}000 \times \frac{1}{3.9054} = \$256.$$

Note that the answer to the same problem, with discrete annual compounding ($M = 1$), is

$$A = P(A/P, 20\%, 10)$$

$$= \$1{,}000(0.2385) = \$239.$$

EXAMPLE 4-35 Continuous Compounding and Semiannual Payments

VIDEO SOLUTIONS

An individual needs $12,000 immediately as a down payment on a new home. Suppose that he can borrow this money from his insurance company. He must repay the loan in equal payments every six months over the next eight years. The nominal interest rate being charged is 7% compounded continuously. What is the amount of each payment?

Solution

The nominal interest rate per six months is 3.5%. Thus, A each six months is $\$12{,}000(A/P, \underline{r} = 3.5\%, 16)$. By substituting terms in Equation (4-38) and then using its inverse, we determine the value of A per six months to be $997:

$$A = \$12{,}000 \left[\frac{1}{(P/A, \underline{r} = 3.5\%, 16)} \right] = \frac{\$12{,}000}{12.038} = \$997.$$

Engineering Economy

Chapter 5: Evaluating a Single Project

The objective of Chapter 5 is to discuss and critique contemporary methods for determining project profitability.

Proposed capital projects can be evaluated in several ways.

- Present worth (PW)
- Future worth (FW)
- Annual worth (AW)
- Internal rate of return (IRR)
- External rate of return (ERR)
- Payback period (generally not appropriate as a primary decision rule)

To be attractive, a capital project must provide a return that exceeds a minimum level established by the organization. This minimum level is reflected in a firm's Minimum Attractive Rate of Return (MARR).

Many elements contribute to determining the MARR.

- Amount, source, and cost of money available
- Number and purpose of good projects available
- Perceived risk of investment opportunities
- Type of organization involved (government, public utility,etc)

Objective

- the MARR should be chosen to maximize the economic well-being of an organization, subject to the types of considerations.

The most-used method is the present worth method.

- The PW method is based on the concept of equivalent worth of all cash flows relative to some base or beginning point in time called the present.
- All cash inflows and outflows are discounted to the present point in time at an interest rate that is generally the MARR.

The most-used method is the present worth method.

A positive PW for an investment project means that the project is acceptable (it satisfies the MARR).

Present Worth (PW)

$$\begin{aligned} \text{PW}(i\%) &= F_0(1+i)^0 + F_1(1+i)^{-1} + F_2(1+i)^{-2} \\ &\quad + \cdots + F_k(1+i)^{-k} + \cdots + F_N(1+i)^{-N} \\ &= \sum_{k=0}^{N} F_k(1+i)^{-k}. \end{aligned} \tag{5-1}$$

- i = effective interest rate or MARR
- K = index of each compounding period
- Fk = future cash flow at the end of period k
- N = # of compounded periods .

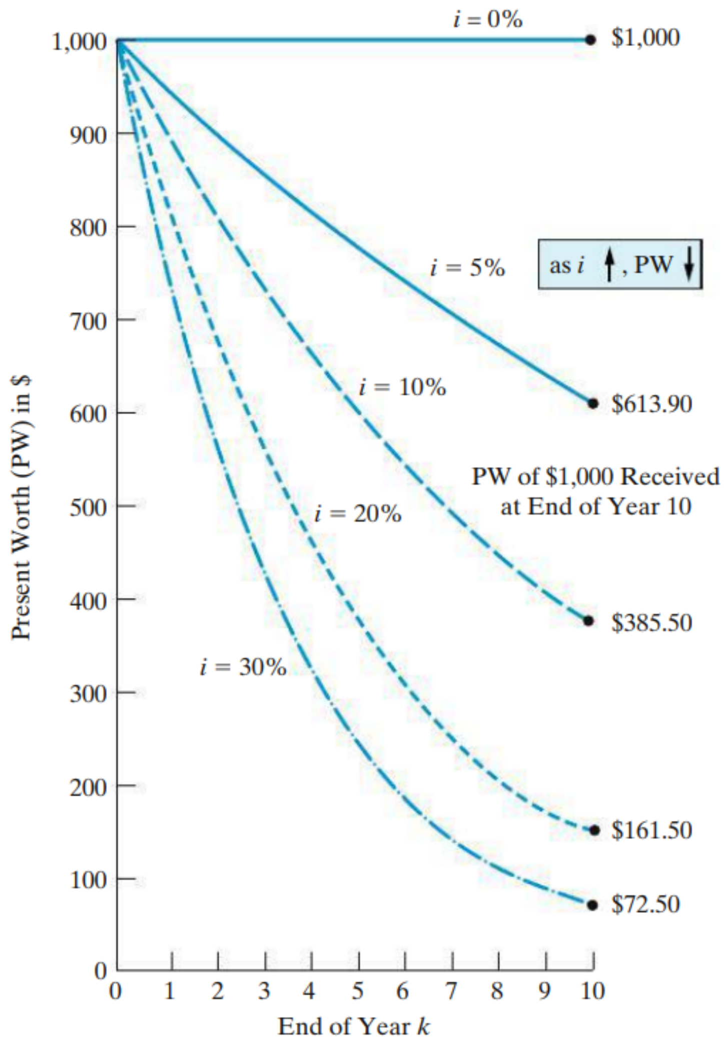
- PW Decision Rule:

If PW (i=MARR) ≥ 0 , the project is economically justified

Note

- The higher the interest rate and the further into the future a cash flow occurs , the lower its PW is
- See figure 5-2
- PW of 1,000 10 years from now i =5% is <u>$613.90</u>
- PW of 1,000 10years from now i=10% is <u>$385.5</u>

Figure 5-2 PW of $1,000 Received at the End of Year *k* at an Interest Rate of *i*% per Year



Example(PW)

Consider a project that has an initial investment of $50,000 and that returns $18,000 per year for the next four years. If the MARR is 12%, is this a good investment?

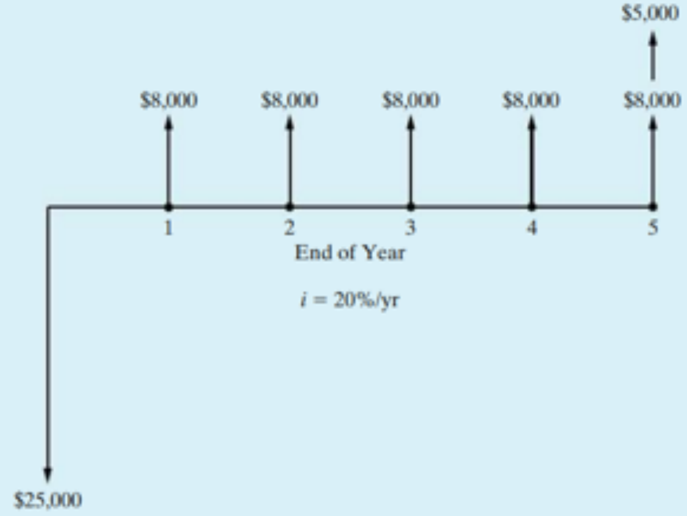
PW = -50,000 + 18,000 (P/A, 12%, 4)

PW = -50,000 + 18,000 (3.0373)

PW = $4,671.40 → This is a good investment!

EXAMPLE 5-1 Evaluation of New Equipment Purchase Using PW

A piece of new equipment has been proposed by engineers to increase the productivity of a certain manual welding operation. The investment cost is \$25,000, and the equipment will have a market value of \$5,000 at the end of a study period of five years. Increased productivity attributable to the equipment will amount to \$8,000 per year after extra operating costs have been subtracted from the revenue generated by the additional production. A cash-flow diagram for this investment opportunity is given below. If the firm's MARR is 20% per year, is this proposal a sound one? Use the PW method.



Solution

$$\text{PW} = \text{PW of cash inflows} - \text{PW of cash outflows},$$

or

$$\begin{aligned}\text{PW}(20\%) &= \$8{,}000(P/A, 20\%, 5) + \$5{,}000(P/F, 20\%, 5) - \$25{,}000 \\ &= \$934.29.\end{aligned}$$

Because $\text{PW}(20\%) \geq 0$, this equipment is economically justified.

specified for a nominal interest rate (r) of 20% per year, the PW would have been calculated by using the interest factors presented in Appendix D:

$$\begin{aligned}\text{PW}(\underline{r} = 20\%) &= -\$25{,}000 + \$8{,}000(P/A, \underline{r} = 20\%, 5) \\ &\quad + \$5{,}000(P/F, \underline{r} = 20\%, 5) \\ &= -\$25{,}000 + \$8{,}000(2.8551) + \$5{,}000(0.3679) \\ &= -\$319.60.\end{aligned}$$

EXAMPLE 5-2 Present Worth of a Space-Heating System

A retrofitted space-heating system is being considered for a small office building. The system can be purchased and installed for $110,000, and it will save an estimated 300,000 kilowatt-hours (kWh) of electric power each year over a six-year period. A kilowatt-hour of electricity costs $0.10, and the company uses a MARR of 15% per year in its economic evaluations of refurbished systems. The market value of the system will be $8,000 at the end of six years, and additional annual operating and maintenance expenses are negligible. Use the PW method to determine whether this system should be installed.

Solution

To find the PW of the proposed heating system, we need to find the present equivalent of all associated cash flows. The estimated annual savings in electrical power is worth 300,000 kWh $\times$ \$0.10/kWh = \$30,000 per year. At a MARR of 15%, we get

$$\begin{aligned} \text{PW}(15\%) &= -\$110{,}000 + \$30{,}000\ (P/A, 15\%, 6) + \$8{,}000\ (P/F, 15\%, 6) \\ &= -\$110{,}000 + \$30{,}000(3.7845) + \$8{,}000(0.4323) \\ &= \$6{,}993.40. \end{aligned}$$

Since PW(15%) ≥ 0, we conclude that the retrofitted space-heating system should be installed.

Present Worth Example

Consider a project that has an initial investment of $50,000 and that returns $18,000 per year for the next four years. If the MARR is 12%, is this a good investment?

$$PW = -50{,}000 + 18{,}000\ (P/A,\ 12\%,\ 4)$$

$$PW = -50{,}000 + 18{,}000\ (3.0373)$$

$PW = \$4{,}671.40$ → This is a good investment!

Assumptions of the PW Method

- It is assumed that we know the future with certainty (we don't live in a certain world!). For example, we presume to know with certainty future interest rates and other factors.

Assumptions of the PW Method

- It is assumed we can borrow and lend money at the same interest rate (i.e., capital markets are perfect). Regrettably, the real world has neither certainty nor perfect (frictionless, e.g., no taxes and/or commissions) capital markets.

Bond Value

A bond is an IOU where you agree to lend the bond issuer money for a specified length of time (say, 10 years). In return, you receive periodic interest payments (e.g., quarterly) from the issuer plus a promise to return the face value of the bond

Bond value is a good example of present worth.

The commercial value of a bond is the PW of all future net cash flows expected to be received--the period dividend [face value (Z) times the bond rate (r)], and the redemption price (C), all discounted to the present at the bond's yield rate, $i\%$.

$$V_N = C\,(P/F, i\%, N) + rZ\,(P/A, i\%, N)$$

Bond example

What is the value of a 6%, 10-year bond with a par (and redemption) value of $20,000 that pays dividends semi-annually, if the purchaser wishes to earn an 8% return?

$$V_N = \$20{,}000\ (P/F,\ 4\%,\ 20) + (0.03)\$20{,}000\ (P/A,\ 4\%,\ 20)$$

$$V_N = \$20{,}000\ (0.4564) + (0.03)\$20{,}000\ (13.5903)$$

$$V_N = \$17{,}282.18$$

EXAMPLE 5-3 Stan Moneymaker Wants to Buy a Bond

VIDEO SOLUTIONS

Stan Moneymaker has the opportunity to purchase a certain U.S. Treasury bond that matures in eight years and has a face value of \$10,000. This means that Stan will receive \$10,000 cash when the bond's maturity date is reached. The bond stipulates a fixed nominal interest rate of 8% per year, but interest payments are made to the bondholder every three months; therefore, each payment amounts to 2% of the face value.

Stan would like to earn 10% nominal interest (compounded quarterly) per year on his investment, because interest rates in the economy have risen since the bond was issued. How much should Stan be willing to pay for the bond?

Solution

To establish the value of this bond, in view of the stated conditions, the PW of future cash flows during the next eight years (the study period) must be evaluated. Interest payments are quarterly. Because Stan Moneymaker desires to obtain 10% *nominal interest per year* on the investment, the PW is computed at $i = 10\%/4 = 2.5\%$ per quarter for the remaining $8(4) = 32$ quarters of the bond's life:

$$V_N = \$10{,}000(P/F, 2.5\%, 32) + \$10{,}000(0.02)(P/A, 2.5\%, 32)$$

$$= \$4{,}537.71 + \$4{,}369.84 = \$8{,}907.55.$$

Thus, Stan should pay no more than \$8,907.55 when 10% nominal interest per year is desired.

Engineering Economy, Fourteenth Edition
By William G. Sullivan, Elin M. Wicks, and C. Patrick Koelling

Capitalized worth is a special variation of present worth.

- Capitalized worth is the present worth of all revenues or expenses over an infinite length of time.
- If only expenses are considered this is sometimes referred to as *capitalized cost*.
- The capitalized worth method is especially useful in problems involving endowments and public projects with indefinite lives.

The application of CW concepts.

The CW of a series of end-of-period uniform payments A, with interest at *i%* per period, is *A(P/A, i%, N)*. As *N* becomes very large (if the *A* are perpetual payments), the (P/A) term approaches *1/i*. So, *CW = A(1/i)*.

$$\text{CW}(i\%) = \text{PW}_{N\to\infty} = A(P/A, i\%, \infty) = A\left[\lim_{N\to\infty} \frac{(1+i)^N - 1}{i(1+i)^N}\right] = A\left(\frac{1}{i}\right).$$

EXAMPLE 5-5 Determining the Capitalized Worth of a Bridge

VIDEO SOLUTIONS

A new bridge across the Cumberland River is being planned near a busy highway intersection in the commercial part of a mid-western town. The construction (first) cost of the bridge is \$1,900,000 and annual upkeep is estimated to be \$25,000. In addition to annual upkeep, major maintenance work is anticipated every eight years at a cost of \$350,000 per occurrence. The town government's MARR is 8% per year.

(a) For this problem, what analysis period (N) is, practically speaking, defined as forever?

(b) If the bridge has an expected life of 50 years, what is the capitalized worth (CW) of the bridge over a 100-year study period?

Solution

(a) A practical approximation of "forever" (infinity) is dependent on the interest rate. By examining the $(A/P, i\%, N)$ factor as N increases in the Appendix C tables, we observe that this factor approaches a value of i as N becomes large. For $i = 8\%$ (Table C-11), the $(A/P, 8\%, 100)$ factor is 0.08. So $N = 100$ years is, for practical purposes, "forever" in this example.

(b) The CW is determined as follows:

$$\text{CW}(8\%) = -\$1{,}900{,}000 - \$1{,}900{,}000\,(P/F, 8\%, 50) - [\$350{,}000\,(A/F, 8\%, 8)]/0.08 - \$25{,}000/0.08.$$

The CW turns out to be −\$2,664,220 over a 100-year study period, assuming the bridge is replaced at the end of year 50 for \$1,900,000.

Future Worth (FW) method is an alternative to the PW method.

- Looking at FW is appropriate since the primary objective is to maximize the future wealth of owners of the firm.
- FW is based on the equivalent worth of all cash inflows and outflows at the end of the study period at an interest rate that is generally the MARR.
- Decisions made using FW and PW will be the same.

Future worth example.

A $45,000 investment in a new conveyor system is projected to improve throughput and increasing revenue by $14,000 per year for five years. The conveyor will have an estimated market value of $4,000 at the end of five years. Using FW and a MARR of 12%, is this a good investment?

$$FW = -\$45{,}000(F/P,\ 12\%,\ 5) + \$14{,}000(F/A,\ 12\%,\ 5) + \$4{,}000$$

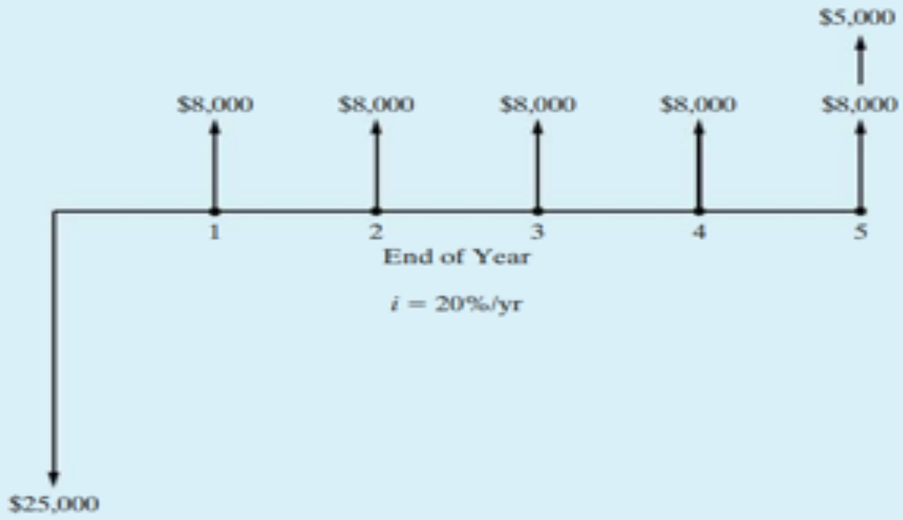
$$FW = -\$45{,}000(1.7623) + \$14{,}000(6.3528) + \$4{,}000$$

$FW = \$13{,}635.70$ → This is a good investment!

EXAMPLE 5-1 **Evaluation of New Equipment Purchase Using PW**

VIDEO SOLUTIONS

A piece of new equipment has been proposed by engineers to increase the productivity of a certain manual welding operation. The investment cost is $25,000, and the equipment will have a market value of $5,000 at the end of a study period of five years. Increased productivity attributable to the equipment will amount to $8,000 per year after extra operating costs have been subtracted from the revenue generated by the additional production. A cash-flow diagram for this investment opportunity is given below. If the firm's MARR is 20% per year, is this proposal a sound one? Use the PW method.



EXAMPLE 5-6 The Relationship between FW and PW

Evaluate the FW of the potential improvement project described in Example 5-1. Show the relationship between FW and PW for this example.

Solution

$$\begin{aligned} FW(20\%) &= -\$25{,}000(F/P, 20\%, 5) \\ &\quad + \$8{,}000(F/A, 20\%, 5) + \$5{,}000 \\ &= \$2{,}324.80. \end{aligned}$$

Again, the project is shown to be a good investment (FW $\geq$ 0). The PW is a multiple of the equivalent FW value:

$$PW(20\%) = \$2{,}324.80(P/F, 20\%, 5) = \$934.29.$$

EXAMPLE 5-7 Sensitivity Analysis Using FW (Example 5-2 Revisited)

In Example 5-2, the \$110,000 retrofitted space-heating system was projected to save \$30,000 per year in electrical power and be worth \$8,000 at the end of the six-year study period. Use the FW method to determine whether the project is still economically justified if the system has zero market value after six years. The MARR is 15% per year.

Solution

In this example, we need to find the future equivalent of the \$110,000 investment and the \$30,000 annual savings at an interest rate of 15% per year.

$$
\begin{aligned}
\text{FW}(15\%) &= -\$110{,}000\,(F/P, 15\%, 6) + \$30{,}000\,(F/A, 15\%, 6) \\
&= -\$110{,}000\,(2.3131) + \$30{,}000\,(8.7537) \\
&= \$8{,}170.
\end{aligned}
$$

The heating system is still a profitable project (FW $\geq$ 0) even if it has no market value at the end of the study period.

Annual Worth (AW) is another way to assess projects.

- Annual worth is an equal periodic series of dollar amounts that is *equivalent* to the cash inflows and outflows, at an interest rate that is generally the MARR.
- The AW of a project is annual equivalent revenue or savings (R) minus annual equivalent expenses(E), less its annual capital recovery (*CR*) amount.

$$AW(i\%) = \underline{R} - \underline{E} - CR(i\%)$$

Capital recovery reflects the capital cost of the asset.

- *CR* is the annual equivalent cost of the capital invested.
- The *CR* covers the following items.
  - Loss in value of the asset.
  - Interest on invested capital (at the MARR).
- The *CR* distributes the initial cost (*I*) and the salvage value (*S*) across the life of the asset.

$$CR(i\%) = I(A/P, i\%, N) - S(A/F, i\%, N)$$

Capital recovery

As an example, consider a device that will cost $10,000, last five years, and have a salvage (market) value of $2,000. Thus, the loss in value of this asset over five years is $8,000. Additionally, the MARR is 10% per year.

Capital recovery

It can be shown that, no matter which method of calculating an asset's loss in value over time is used, the equivalent annual CR amount is the same. For example, if a uniform loss in value is assumed ($8,000/5 years = $1,600 per year), the equivalent annual CR amount is calculated to be $2,310, as shown in Table 5-1.

Capital recovery

TABLE 5-1 Calculation of Equivalent Annual CR Amount

Year	Value of Investment at Beginning of Year[a]	Uniform Loss in Value	Interest on Beginning-of-Year Investment at $i = 10\%$	CR Amount for Year	PW of CR Amount at $i = 10\%$
1	$10,000	$1,600	$1,000	$2,600	$2,600(P/F, 10%, 1) = $2,364
2	8,400	1,600	840	2,440	$2,440(P/F, 10%, 2) = $2,016
3	6,800	1,600	680	2,280	$2,280(P/F, 10%, 3) = $1,713
4	5,200	1,600	520	2,120	$2,120(P/F, 10%, 4) = $1,448
5	3,600	1,600	360	1,960	$1,960(P/F, 10%, 5) = $1,217
					$8,758

$\text{CR}(10\%) = \$8{,}758(A/P, 10\%, 5) = \$2{,}310.$

$$\text{AW}(i\%) = \underline{R} - \underline{E} - I(A/P, i\%, N) + S(A/F, i\%, N).$$

When Equation (5-5) is applied to the example in Table 5-1, the CR cost is

$$\begin{aligned}\text{CR}(10\%) &= \$10{,}000(A/P, 10\%, 5) - \$2{,}000(A/F, 10\%, 5) \\ &= \$10{,}000(0.2638) - \$2{,}000(0.1638) = \$2{,}310.\end{aligned}$$

A project requires an initial investment of $45,000, has a salvage value of $12,000 after six years, incurs annual expenses of $6,000, and provides an annual revenue of $18,000. Using a MARR of 10%, determine the AW of this project.

$$AW(10\%) = \underline{R} - \underline{E} - CR(10\%)$$

$$CR(10\%) = 45,000(A/P, 10\%, 6) - 12,000(A/F, 10\%, 6)$$

$$CR(10\%) = 8,777$$

$$AW(10\%) = 18,000 - 6,000 - 8,777 = \$3,223$$

Since the AW is positive, it's a good investment.

EXAMPLE 5-8 **(Example 5-1 Revisited)**

By using the AW method and Equation (5-4), determine whether the equipment described in Example 5-1 should be recommended.

Solution

The AW Method applied to Example 5-1 yields the following:

$$\text{AW}(20\%) = \overbrace{\$8{,}000}^{R-E} - \overbrace{[\$25{,}000(A/P, 20\%, 5) - \$5{,}000(A/F, 20\%, 5)]}^{\text{CR amount [Equation (5-5)]}}$$

$$= \$8{,}000 - \$8{,}359.50 + \$671.90$$

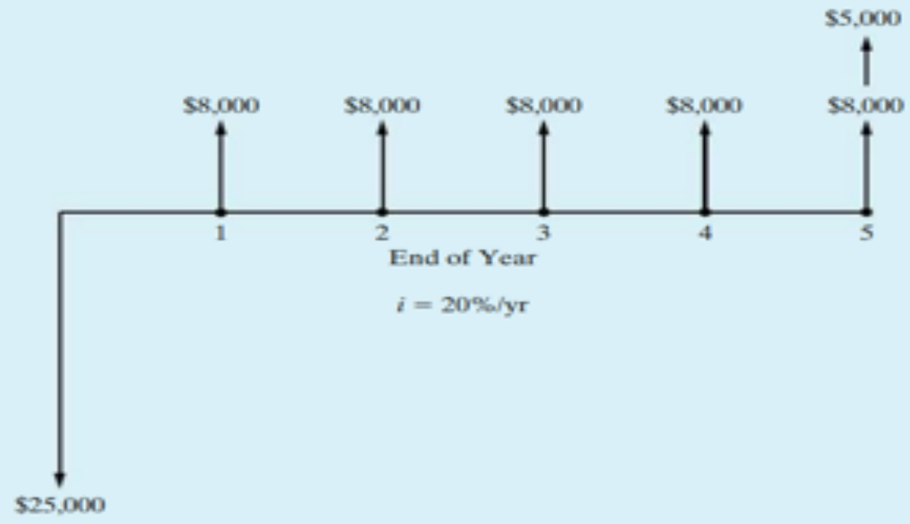
Because its AW(20%) is positive, the equipment more than pays for itself over a period of five years, while earning a 20% return per year on the unrecovered investment. In fact, the annual equivalent "surplus" is $312.40, which means that the equipment provided more than a 20% return on beginning-of-year unrecovered investment. This piece of equipment should be recommended as an attractive investment opportunity. Also, we can confirm that the AW(20%) is equivalent to PW(20%) = $934.29 in Example 5-1 and FW(20%) = $2,324.80 in Example 5-6. That is,

$$\text{AW}(20\%) = \$934.29(A/P, 20\%, 5) = \$312.40, \text{ and also}$$

$$\text{AW}(20\%) = \$2{,}324.80(A/F, 20\%, 5) = \$312.40.$$

EXAMPLE 5-1 **Evaluation of New Equipment Purchase Using PW**

A piece of new equipment has been proposed by engineers to increase the productivity of a certain manual welding operation. The investment cost is $25,000, and the equipment will have a market value of $5,000 at the end of a study period of five years. Increased productivity attributable to the equipment will amount to $8,000 per year after extra operating costs have been subtracted from the revenue generated by the additional production. A cash-flow diagram for this investment opportunity is given below. If the firm's MARR is 20% per year, is this proposal a sound one? Use the PW method.



Internal Rate of Return

- The internal rate of return (IRR) method is the most widely used rate of return method for performing engineering economic analysis.
- It is also called the *investor's method*, the *discounted cash flow* method, and the *profitability index*.
- If the IRR for a project is greater than the MARR, then the project is *acceptable*.

How the IRR works

- The IRR is the interest rate that equates the equivalent worth of an alternative's cash *inflows* (revenue, *R*) to the equivalent worth of cash *outflows* (expenses, *E*).
- The IRR is sometimes referred to as the *breakeven interest rate.*

The IRR is the interest *i'%* at which

$$\sum_{k=0}^{N} R_k(P/F, i'\%, k) = \sum_{k=0}^{N} E_k(P/F, i'\%, k)$$

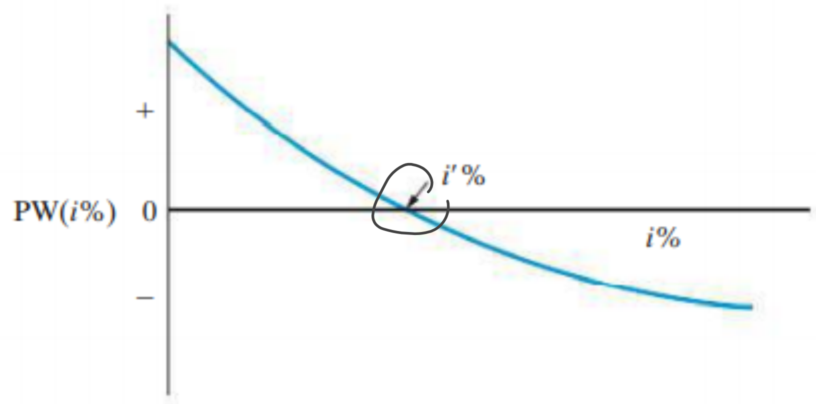
Solving for the IRR is a bit more complicated than PW, FW, or AW

- The method of solving for the *i'%* that equates revenues and expenses normally involves trial-and-error calculations, or solving numerically using mathematical software.
- The use of spreadsheet software can greatly assist in solving for the IRR. Excel uses the *IRR(range, guess)* or *RATE(nper, pmt, pv)* functions.

computing the IRR for an alternative is to determine the i ′ at which its net PW is zero. In equation form, the IRR is the

$$\text{PW} = \sum_{k=0}^{N} R_k(P/F, i'\%, k) - \sum_{k=0}^{N} E_k(P/F, i'\%, k) = 0.$$

Figure 5-3 Plot of PW versus Interest Rate



Investment-balance diagram.

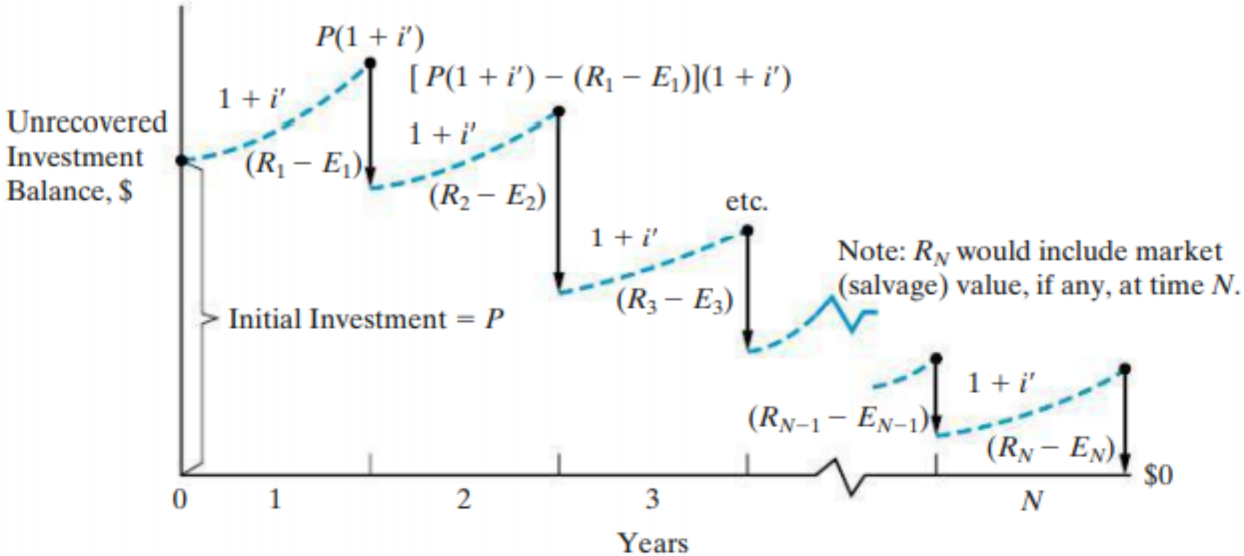


Figure 5-4 Investment-Balance Diagram Showing IRR

EXAMPLE 5-12 **Economic Desirability of a Project Using the IRR Method**

AMT, Inc., is considering the purchase of a digital camera for the maintenance of design specifications by feeding digital pictures directly into an engineering workstation where computer-aided design files can be superimposed over the digital pictures. Differences between the two images can be noted, and corrections, as appropriate, can then be made by design engineers. The capital investment requirement is $345,000 and the estimated market value of the system after a six-year study period is $115,000. Annual revenues attributable to the new camera system will be $120,000, whereas additional annual expenses will be $22,000. You have been asked by management to determine the IRR of this project and to make a recommendation. The corporation's MARR is 20% per year. Solve first by using linear interpolation and then by using a spreadsheet.

Solution by Linear Interpolation

In this example, we can easily see that the sum of positive cash flows ($835,000) exceeds the sum of negative cash flows ($455,000). Thus, it is likely that a positive-valued IRR can be determined. By writing an equation for the PW of the project's total net cash flow and setting it equal to zero, we can compute the IRR:

$$\begin{aligned} \text{PW} = 0 = &-\$345{,}000 + (\$120{,}000 - \$22{,}000)(P/A, i'\%, 6) \\ &+ \$115{,}000(P/F, i'\%, 6) \\ i'\% &= \ ? \end{aligned}$$

To use linear interpolation, we first need to try a few values for i'. A good starting point is to use the MARR.

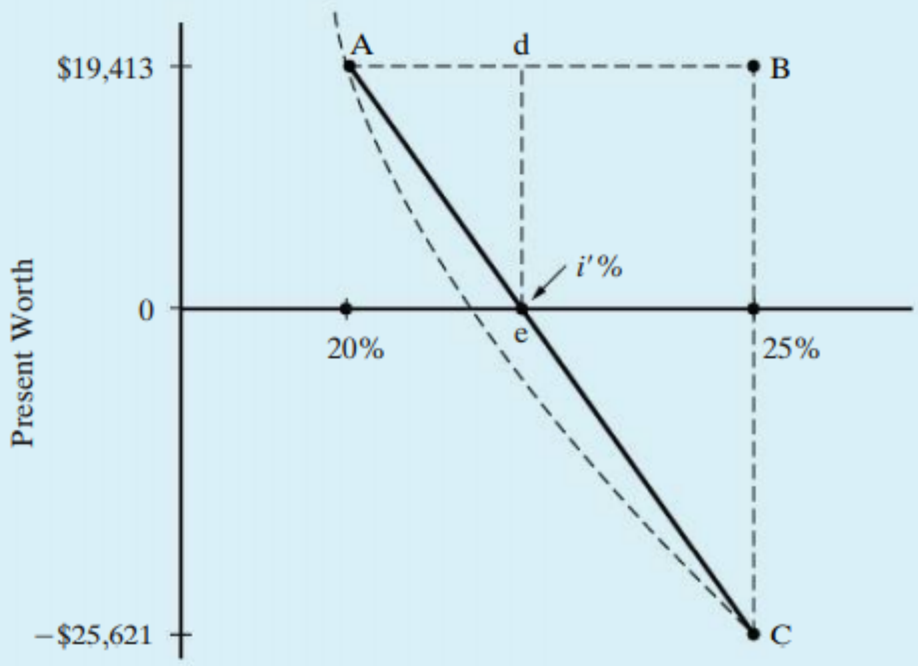
$$\text{At } i' = 20\%: \quad \text{PW} = -\$345{,}000 + \$98{,}000(3.3255) + \$115{,}000(0.3349) = +\$19{,}413$$

Since the PW is positive at 20%, we know that $i' > 20\%$.

$$\text{At } i' = 25\%: \quad \text{PW} = -\$345{,}000 + \$98{,}000(2.9514) + \$115{,}000(0.2621) = -\$25{,}621$$

Now that we have both a positive and a negative PW, the answer is bracketed ($20\% \leq i'\% \leq 25\%$). The dashed curve in Figure 5-5 is what we are linearly approximating. The answer, $i'\%$, can be determined by using the similar triangles represented by dashed lines in Figure 5-5.

Figure 5-5 Use of Linear Interpolation to Find the Approximation of IRR for Example 5-12



Now that we have both a positive and a negative PW, the answer is bracketed ($20\% \leq i'\% \leq 25\%$). The dashed curve in Figure 5-5 is what we are linearly approximating. The answer, $i'\%$, can be determined by using the similar triangles represented by dashed lines in Figure 5-5.

$$\frac{\text{line BA}}{\text{line BC}} = \frac{\text{line dA}}{\text{line de}}.$$

Here, BA is the line segment $B - A = 25\% - 20\%$. Thus,

$$\frac{25\% - 20\%}{\$19{,}413 - (-\$25{,}621)} = \frac{i'\% - 20\%}{\$19{,}413 - \$0}$$

$$i' \approx 22.16\%.$$

Because the IRR of the project (22.16%) is greater than the MARR, the project is acceptable.

EXAMPLE 5-13 Evaluation of New Equipment Purchase, Using the Internal Rate of Return Method (Example 5-1 Revisited)

A piece of new equipment has been proposed by engineers to increase the productivity of a certain manual welding operation. The investment cost is $25,000, and the equipment will have a market (salvage) value of $5,000 at the end of its expected life of five years. Increased productivity attributable to the equipment will amount to $8,000 per year after extra operating costs have been subtracted from the value of the additional production. Use a spreadsheet to evaluate the IRR of the proposed equipment. Is the investment a good one? Recall that the MARR is 20% per year.

Spreadsheet Solution

The spreadsheet solution for this problem is shown in Figure 5-7. In column E of Figure 5-7(a), the individual EOY cash flows for year five (net annual savings

	A	B	C	D	E
1	MARR =	20%			
2	Capital Investment =	$ 25,000			
3	Market Value =	$ 5,000			
4	Useful Life =	5			
5	Net Annual Savings =	$ 8,000			
6					
7	EOY	Cash Flow		EOY	Cash Flow
8	0	$ (25,000)		0	$ (25,000)
9	1	$ 8,000		1	$ 8,000
10	2	$ 8,000		2	$ 8,000
11	3	$ 8,000		3	$ 8,000
12	4	$ 8,000		4	$ 8,000
13	5	$ 8,000		5	$ 13,000
14	5	$ 5,000			
15					
16				IRR =	21.58%

= –B2

= B5

= B3

= B8

= B13 + B14

= IRR(E8:E13, B1)

(a) Direct Computation of IRR

Figure 5-7 Spreadsheet Solution, Example 5-13

Installment Financing

A rather common application of the IRR method is in so-called installment financing types of problems.

These problems are associated with financing arrangements for purchasing merchandise "on time." The total interest, or finance, charge is often paid by the borrower on the basis of what amount is owed at the beginning of the loan instead of on the unpaid loan balance.

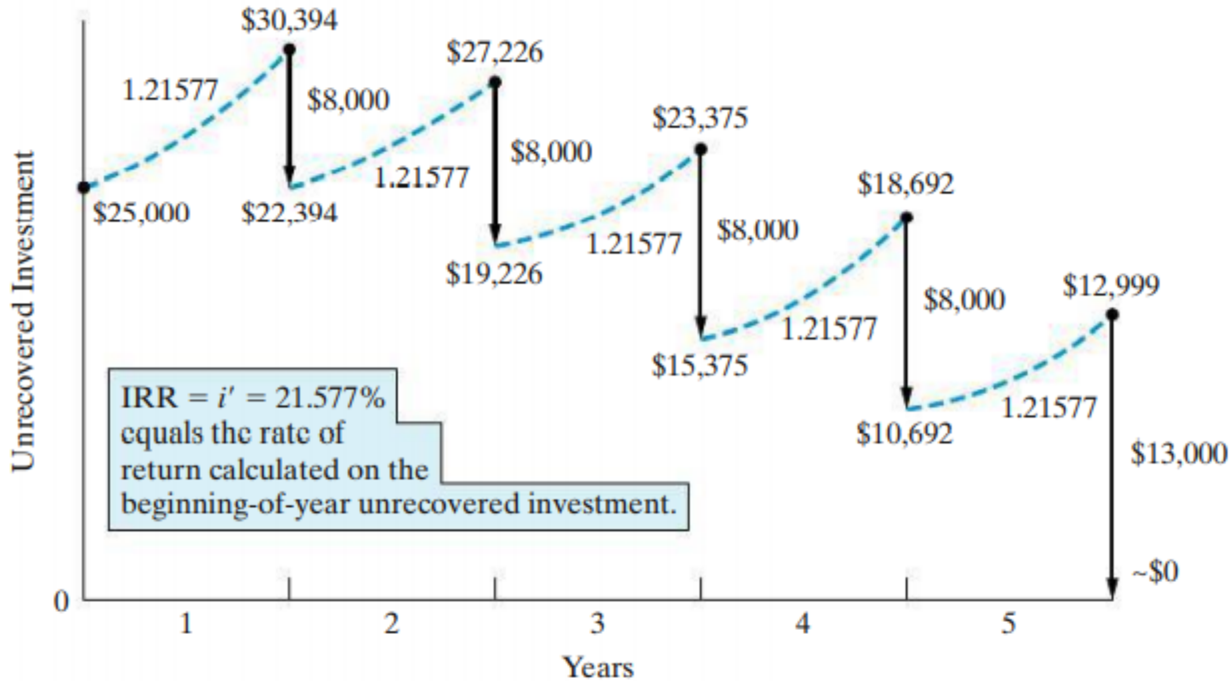


Figure 5-8 Investment-Balance Diagram for Example 5-13

EXAMPLE 5-14 Do You Know What Your Effective Interest Rate Is?

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In 1915, Albert Epstein allegedly borrowed \$7,000 from a large New York bank on the condition that he would repay 7% of the loan every three months, until a total of 50 payments had been made. At the time of the 50th payment, the \$7,000 loan would be completely repaid. Albert computed his annual interest rate to be $[0.07(\$7{,}000) \times 4]/\$7{,}000 = 0.28$ (28%).

(a) What true *effective* annual interest rate did Albert pay?

(b) What, if anything, was wrong with his calculation?

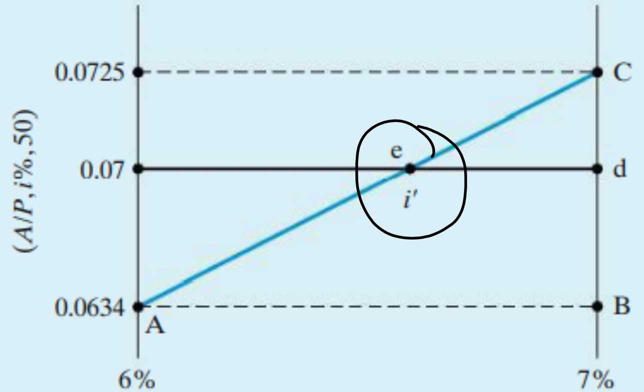
Solution

(a) The true interest rate per quarter is found by equating the equivalent value of the amount borrowed to the equivalent value of the amounts repaid. Equating the AW amounts per quarter, we find

$$\$7{,}000(A/P, i'\%/\text{quarter}, 50 \text{ quarters}) = 0.07(\$7{,}000) \text{ per quarter},$$
$$(A/P, i'\%, 50) = 0.07.$$

Linearly interpolating to find i'%/quarter by using similar triangles is the next step:

$$(A/P, 6\%, 50) = 0.0634,$$
$$(A/P, 7\%, 50) = 0.0725.$$



$$\frac{\text{line de}}{\text{line BA}} = \frac{\text{line Cd}}{\text{line CB}}$$

$$\frac{7\% - i'\%}{7\% - 6\%} = \frac{0.0725 - 0.07}{0.0725 - 0.0634},$$

$$i'\% = 7\% - 1\%\left(\frac{0.0025}{0.0091}\right),$$

$$\text{or } i'\% \simeq 6.73\% \text{ per quarter.}$$

Now we can compute the effective $i'\%$ per year that Albert was paying:

$$i'\% = [(1.0673)^4 - 1]100\%$$

$$\simeq 30\% \text{ per year.}$$

(b) Even though Albert's answer of 28% is close to the true value of 30%, his calculation is insensitive to how long his payments were made. For instance, he would get 28% for an answer when 20, 50, or 70 quarterly payments of $490 were made! For 20 quarterly payments, the true effective interest rate is 14.5% per year, and for 70 quarterly payments, it is 31% per year. As more payments are made, the true annual effective interest rate being charged by the bank will increase, but Albert's method would not reveal by how much.

Another Solution

When the initial loan amount, the payment amount, and the number of payments are known, Excel has a useful financial function, RATE (*nper*, *pmt*, *pv*), that will return the interest rate per period. For this example,

$$\text{RATE}(50, -490, 7000) = 6.73\%.$$

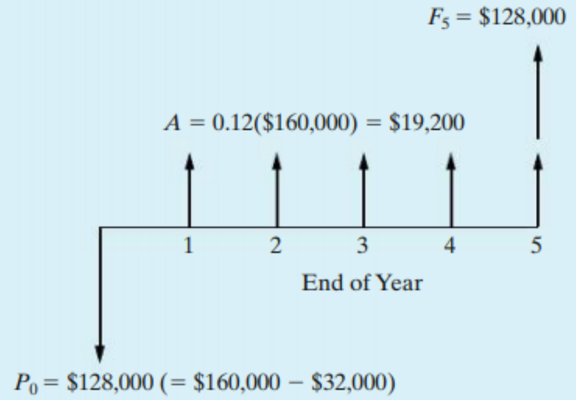
This is the same quarterly interest rate we obtained via linear interpolation in Part (a).

EXAMPLE 5-16 Effective Interest Rate for Purchase of New Aircraft Equipment

A small airline executive charter company needs to borrow $160,000 to purchase a prototype synthetic vision system for one of its business jets. The SVS is intended to improve the pilots' situational awareness when visibility is impaired. The local (and only) banker makes this statement: "We can loan you $160,000 at a very favorable rate of 12% per year for a five-year loan. However, to secure this loan, you must agree to establish a checking account (with no interest) in which the *minimum* average balance is $32,000. In addition, your interest payments are due at the end of each year, and the principal will be repaid in a lump-sum amount at the end of year five." What is the true effective annual interest rate being charged?

Solution

The cash-flow diagram from the banker's viewpoint appears below. When solving for an unknown interest rate, it is good practice to draw a *cash-flow diagram* prior to writing an equivalence relationship. Notice that $P_0 =$ \$160,000 − \$32,000 = \$128,000. Because the bank is requiring the company to open an account worth \$32,000, the bank is only \$128,000 out of pocket. This same principal applies to F_5 in that the company only needs to repay \$128,000 since the \$32,000 on deposit can be used to repay the original principal.



The interest rate (IRR) that establishes equivalence between positive and negative cash flows can now easily be computed:*

$$P_0 = F_5(P/F, i'\%, 5) + A(P/A, i'\%, 5),$$

$$\$128{,}000 = \$128{,}000(P/F, i'\%, 5) + \$19{,}200(P/A, i'\%, 5).$$

If we try $i' = 15\%$, we discover that \$128,000 = \$128,000. Therefore, the true effective interest rate is 15% per year.

Challenges in applying the IRR method.

- It is computationally difficult without proper tools.
- In rare instances multiple rates of return can be found. (See Appendix 5-A.)
- The IRR method must be carefully applied and interpreted when comparing two more mutually exclusive alternatives (e.g., do not directly compare internal rates of return).

Reinvesting revenue—the External Rate of Return (ERR)

- The IRR assumes revenues generated are reinvested at the IRR—which may not be an accurate situation.
- The ERR takes into account the interest rate, ε, external to a project at which net cash flows generated (or required) by a project over its life can be reinvested (or borrowed). This is usually the MARR.
- If the ERR happens to equal the project's IRR, then using the ERR and IRR produce identical results.

The ERR procedure

- Discount all the net cash *outflows* to time *0* at ε% per compounding period.
- Compound all the net cash *inflows* to period *N* at at ε%.
- Solve for the ERR, the interest rate that establishes equivalence between the two quantities.

ERR is the $i'\%$ at which

$$\sum_{k=0}^{N} E_k(P/F, \varepsilon\%, k)(F/P, i'\%, N) = \sum_{k=0}^{N} R_k(F/P, \varepsilon\%, N-k)$$

where

R_k = excess of receipts over expenses in period k,
E_k = excess of expenses over receipts in period k,
N = project life or number of periods, and
ε = external reinvestment rate per period.

Applying the ERR method

For the cash flows given below, find the ERR when the external reinvestment rate is $\varepsilon = 12\%$ (equal to the MARR).

Year	0	1	2	3	4
Cash Flow	-$15,000	-$7,000	$10,000	$10,000	$10,000

Expenses $15,000 + 7,000(P/F, 12\%, 1) = 21,250$

Revenue $10,000(F/A, 12\%, 3) = 33,744$

Solving, we find

$$21,250(F/P, i'\%, 4) = 33,744$$

$$i' = 16.67\% > 12\%$$

EXAMPLE 5-18 Determining the Acceptability of a Project, Using ERR

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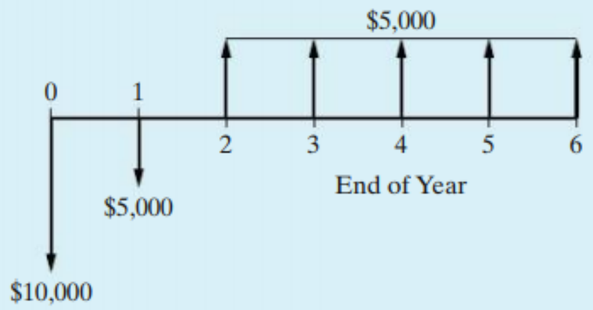
When $\epsilon = 15\%$ and MARR = 20% per year, determine whether the project (whose net cash-flow diagram appears next) is acceptable. Notice in this example that the use of an $\epsilon\%$ different from the MARR is illustrated. This might occur if, for some reason, part or all of the funds related to a project are "handled" outside the firm's normal capital structure.



Solution

$$E_0 = \$10{,}000\ (k = 0),$$

$$E_1 = \$5{,}000\ (k = 1),$$

$$R_k = \$5{,}000 \text{ for } k = 2, 3, \ldots, 6,$$

$$[\$10{,}000 + \$5{,}000(P/F, 15\%, 1)](F/P, i'\%, 6) = \$5{,}000(F/A, 15\%, 5);$$

$$i'\% = 15.3\%.$$

The $i'\%$ is less than the MARR = 20%; therefore, this project would be unacceptable according to the ERR method.

The payback period method is simple, but possibly misleading.

- The simple payback period is the number of years required for cash *inflows* to just equal cash *outflows*.
- It is a measure of *liquidity* rather than a measure of profitability.

Payback is simple to calculate.

The payback period is the *smallest* value of θ ($\theta \leq N$) for which the relationship below is satisfied.

$$\sum_{k=1}^{\theta}(R_k - E_k) - I \geq 0$$

For *discounted* payback future cash flows are discounted back to the present, so the relationship to satisfy becomes

$$\sum_{k=1}^{\theta'}(R_k - E_k)(P/F, i\%, k) - I \geq 0$$

EXAMPLE 5-19 Determining the Simple Payback Period

A public school is being renovated for \$13.5 million. The building has geothermal heating and cooling, high-efficiency windows, and a solar array that permits the school to sell electricity back to the local electric utility. The annual value of these benefits is estimated to be \$2.7 million. In addition, the residual value of the school at the end of its 40-year life is negligible. What is the simple payback period and internal rate of return for the renovated school?

Solution

The simple payback period is

$$\frac{\$13.5 \text{ million}}{\$2.7 \text{ million/year}} = 5 \text{ years.}$$

This is fairly good for a publically sponsored project. The IRR can be computed using the equation

$$0 = -\$13.5 \text{ million} + \$2.7 \text{ million } (P/A, i'\%, 40),$$

yielding $i'\%$ (the IRR) $= 20\%$ per year. The IRR indicates the project is profitable for a MARR of 20% per year or less.

Problems with the payback period method.

- It doesn't reflect any cash flows occurring after θ, or θ'.
- It doesn't indicate anything about project desirability except the speed with which the initial investment is recovered.
- Recommendation: use the payback period only as supplemental information in conjunction with one or more of the other methods in this chapter.

Finding the simple and discounted payback period for a set of cash flows.

The cumulative cash flows in the table were calculated using the formulas for simple and discounted payback.

From the calculations $\theta = 4$ years and $\theta' = 5$ years.

End of Year	Net Cash Flow	Cumulative PW at *0%*	Cumulative PW at *6%*
0	-$42,000	-$42,000	-$42,000
1	$12,000	-$30,000	-$30,679
2	$11,000	-$19,000	-$20,889
3	$10,000	-$9,000	-$12,493
4	$10,000	$1,000	-$4,572
5	$9,000		$2,153

Engineering Economy

Chapter 6: Comparison and Selection Among Alternatives

The objective of chapter 6 is to evaluate correctly capital investment alternatives when the time value of money is a key influence.

Making decisions means comparing alternatives.

- In this chapter we examine *feasible design alternatives.*
- The decisions considered are those selecting from among a set of *mutually exclusive* alternatives—when selecting one excludes the choice of any of the others.

Mutually exclusive alternatives (MEAs)

- We examine these on the basis of economic considerations alone.
- The alternatives may have different initial investments and their annual revenues and costs may vary.
- The alternatives must provide comparable "usefulness": performance, quality, etc.
- The basic methods from chapter 5 provide the basis for economic comparison of the alternatives.

Apply this rule, based on Principle 2 from Chapter 1.

The alternative that requires the minimum investment of capital and produces satisfactory functional results will be chosen unless the incremental capital associated with an alternative having a larger investment can be justified with respect to its incremental benefits. This alternative is the *base alternative*.

For alternatives that have a larger investment than the base…

If the extra benefits obtained by investing additional capital are better than those that could be obtained from investment of the same capital elsewhere in the company at the MARR, the investment should be made.

(Please note that there are some cautions when considering more than two alternatives, which will be examined later.)

There are two basic types of alternatives.

Investment Alternatives

Those with initial (or front-end) capital investment that produces positive cash flows from increased revenue, savings through reduced costs, or both.

Cost Alternatives

Those with all negative cash flows, except for a possible positive cash flow from disposal of assets at the end of the project's useful life.

Select the alternative that gives you the most money!

- For *investment alternatives* the PW of all cash flows must be positive, at the MARR, to be attractive. Select the alternative with the largest PW.
- For *cost alternatives* the PW of all cash flows will be negative. Select the alternative with the largest (smallest in absolute value) PW.

Investment alternative example

Use a MARR of 10% and useful life of 5 years to select between the investment alternatives below.

	Alternative	
	A	B
Capital investment	-$100,000	-$125,000
Annual revenues less expenses	$34,000	$41,000

$$PW_A = -100,000 + 34,000(P/A, 10\%, 5) = 28,887$$

$$PW_B = -125,000 + 41,000(P/A, 10\%, 5) = 30,423$$

Both alternatives are attractive, but Alternative B provides a greater present worth, so is better economically.

Cost alternative example

Use a MARR of 12% and useful life of 4 years to select between the cost alternatives below.

	Alternative C	Alternative D
Capital investment	-$80,000	-$60,000
Annual expenses	-$25,000	-$30,000

$$PW_C = -80,000 - 25,000(P/A, 12\%, 4) = -155,933$$

$$PW_D = -60,000 - 30,000(P/A, 12\%, 4) = -151,119$$

Alternative D costs less than Alternative C, it has a greater PW, so is better economically.

Determining the study period.

- A *study period* (or *planning horizon*) is the time period over which MEAs are compared, and it must be appropriate for the decision situation.
- MEAs can have *equal* lives (in which case the study period used is these equal lives), or they can have *unequal* lives, and at least one does not match the study period.
- The equal life case is straightforward, and was used in the previous two examples.

Unequal lives are handled in one of two ways.

- Repeatability assumption
  - The study period is either indefinitely long or equal to a common multiple of the lives of the MEAs.
  - The economic consequences expected during the MEAs' life spans will also happen in succeeding life spans (replacements).
- Coterminated assumption: uses a finite and identical study period for all MEAs. Cash flow adjustments may be made to satisfy alternative performance needs over the study period.

Comparing MEAs with equal lives.

When lives are equal adjustments to cash flows are not required. The MEAs can be compared by directly comparing their *equivalent worth* (*PW, FW, or AW*) calculated using the MARR. The decision will be the same regardless of the equivalent worth method you use. For a MARR of 12%, select from among the MEAs below.

	Alternatives			
	A	B	C	D
Capital investment	-$150,000	-$85,000	-$75,000	-$120,000
Annual revenues	$28,000	$16,000	$15,000	$22,000
Annual expenses	-$1,000	-$550	-$500	-$700
Market Value (EOL)	$20,000	$10,000	$6,000	$11,000
Life (years)	10	10	10	10

Selecting the best alternative.

Present worth analysis → select Alternative A (but C is close).

$$PW_A = -150,000 + 27,000(P/A, 12\%, 10) + 20,000(P/F, 12\%, 10) = 8,995$$

$$PW_B = -85,000 + 15,450(P/A, 12\%, 10) + 10,000(P/F, 12\%, 10) = 5,516$$

$$PW_C = -75,000 + 14,500(P/A, 12\%, 10) + 6,000(P/F, 12\%, 10) = 8,860$$

$$PW_D = -120,000 + 21,300(P/A, 12\%, 10) + 11,000(P/F, 12\%, 10) = 3,891$$

Annual worth analysis—the decision is the same.

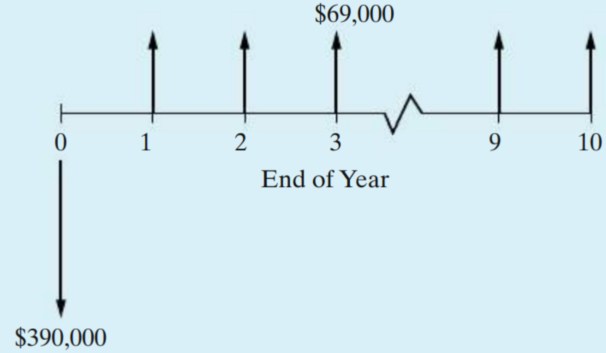
$AW_A = \$1,592$ $\quad$ $AW_C = \$1,568$

$AW_B = \$976$ $\quad$ $AW_D = \$689$

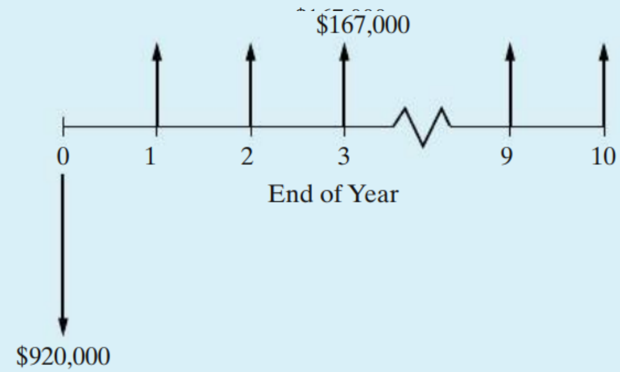
EXAMPLE 6-1 Analyzing Investment Alternatives by Using Equivalent Worth

Best Flight, Inc., is considering three mutually exclusive alternatives for implementing an automated passenger check-in counter at its hub airport. Each alternative meets the same service requirements, but differences in capital investment amounts and benefits (cost savings) exist among them. The study period is 10 years, and the useful lives of all three alternatives are also 10 years. Market values of all alternatives are assumed to be zero at the end of their useful lives. If the airline's MARR is 10% per year, which alternative should be selected in view of the cash-flow diagrams shown on page 248?

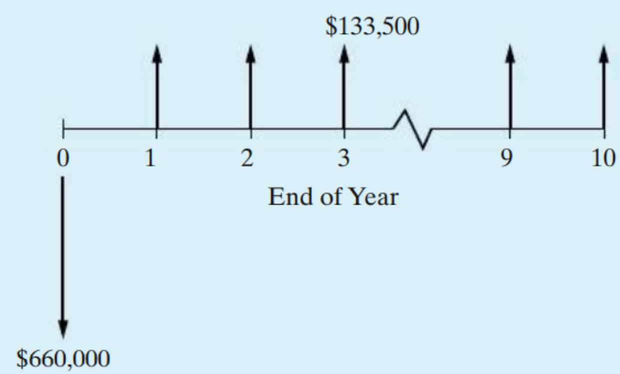
Alternative *A*:



Alternative *B*:



Alternative *C*:



Solution by the PW Method

$$PW(10\%)_A = -\$390{,}000 + \$69{,}000(P/A, 10\%, 10) = \$33{,}977,$$

$$PW(10\%)_B = -\$920{,}000 + \$167{,}000(P/A, 10\%, 10) = \$106{,}148,$$

$$PW(10\%)_C = -\$660{,}000 + \$133{,}500(P/A, 10\%, 10) = \$160{,}304.$$

Based on the PW method, Alternative C would be selected because it has the largest PW value (\$160,304). The order of preference is $C \succ B \succ A$, where $C \succ B$ means C is preferred to B.

Solution by the AW Method

$$\text{AW}(10\%)_A = -\$390{,}000(A/P, 10\%, 10) + \$69{,}000 = \$5{,}547,$$

$$\text{AW}(10\%)_B = -\$920{,}000(A/P, 10\%, 10) + \$167{,}000 = \$17{,}316,$$

$$\text{AW}(10\%)_C = -\$660{,}000(A/P, 10\%, 10) + \$133{,}500 = \$26{,}118.$$

Alternative C is again chosen because it has the largest AW value (\$26,118).

Solution by the FW Method

$$\text{FW}(10\%)_A = -\$390{,}000(F/P, 10\%, 10) + \$69{,}000(F/A, 10\%, 10) = \$88{,}138,$$

$$\text{FW}(10\%)_B = -\$920{,}000(F/P, 10\%, 10) + \$167{,}000(F/A, 10\%, 10) = \$275{,}342,$$

$$\text{FW}(10\%)_C = -\$660{,}000(F/P, 10\%, 10) + \$133{,}500(F/A, 10\%, 10) = \$415{,}801.$$

Based on the FW method, the choice is again Alternative C because it has the largest FW value (\$415,801). For all three methods (PW, AW, and FW) in this example, notice that $C \succ B \succ A$ because of the equivalency relationship among the methods. Also, notice that Rule 1 (Section 6.2.2) applies in this example, since the economic benefits (cost savings) vary among the alternatives.

EXAMPLE 6-2 Analyzing Cost-Only Alternatives, Using Equivalent Worth

VIDEO SOLUTIONS

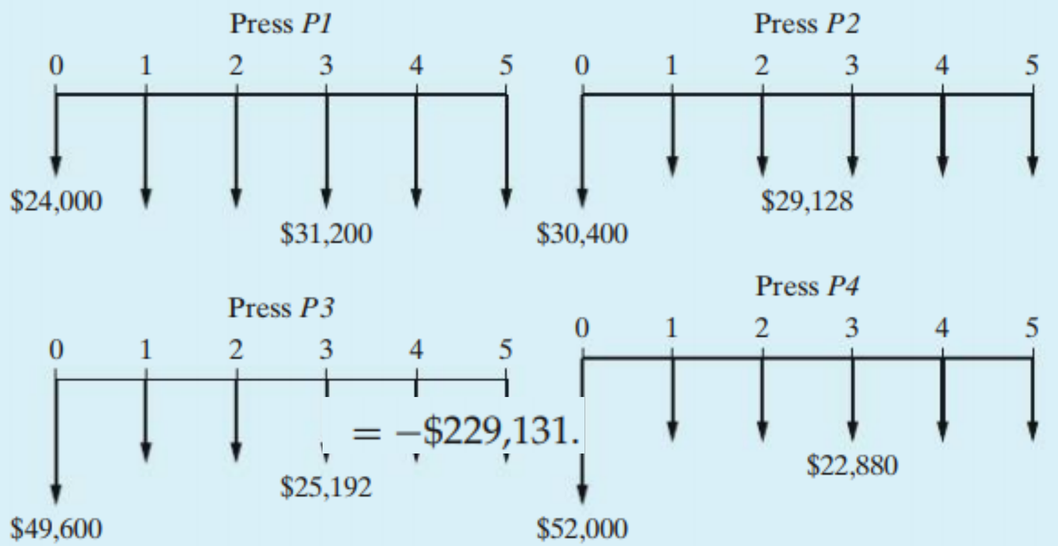
A company is planning to install a new automated plastic-molding press. Four different presses are available. The initial capital investments and annual expenses for these four mutually exclusive alternatives are as follows:

	Press			
	P1	*P2*	*P3*	*P4*
Capital investment	$24,000	$30,400	$49,600	$52,000
Useful life (years)	5	5	5	5
Annual expenses				
Power	−2,720	2,720	4,800	5,040
Labor	−26,400	24,000	16,800	14,800
Maintenance	−1,600	1,800	2,600	2,000
Property taxes and insurance	−480	608	992	1,040
Total annual expenses	$31,200	$29,128	$25,192	$22,880

Assume that each press has the same output capacity (120,000 units per year) and has no market value at the end of its useful life; the selected analysis period is five years; and any additional capital invested is expected to earn at least 10% per year. Which press should be chosen if 120,000 nondefective units per year are produced by each press and all units can be sold? The selling price is $0.375 per unit. Solve by hand and by spreadsheet.

Solution by Hand

Since the same number of nondefective units per year will be produced and sold using each press, revenue can be disregarded (Principle 2, Chapter 1). The end-of-year cash-flow diagrams of the four presses are:



The preferred alternative will minimize the equivalent worth of total costs over the five-year analysis period (Rule 2, page 245). That is, the four alternatives can be compared as cost alternatives. The PW, AW, and FW calculations for Alternative *P1* are

$$\text{PW}(10\%)_{P1} = -\$24{,}000 - \$31{,}200(P/A, 10\%, 5) = -\$142{,}273,$$

$$\text{AW}(10\%)_{P1} = -\$24{,}000(A/P, 10\%, 5) - \$31{,}200 = -\$37{,}531,$$

$$\text{FW}(10\%)_{P1} = -\$24{,}000(F/P, 10\%, 5) - \$31{,}200(F/A, 10\%, 5) = -\$229{,}131.$$

= −$229,131.

The PW, AW, and FW values for Alternatives *P2*, *P3*, and *P4* are determined with similar calculations and shown for all four presses in Table 6-1. Alternative *P4* minimizes all three equivalent-worth values of total costs and is the preferred alternative. The preference ranking (*P4* ≻ *P2* ≻ *P1* ≻ *P3*) resulting from the analysis is the same for all three methods.

TABLE 6-1 Comparison of Four Molding Presses, Using the PW, AW, and FW Methods to Minimize Total Costs

	Press (Equivalent-Worth Values)			
Method	*P1*	*P2*	*P3*	*P4*
Present worth	−$142,273	−$140,818	−$145,098	−$138,734
Annual worth	−37,531	−37,148	−38,276	−36,598
Future worth	−229,131	−226,788	−233,689	−223,431

EXAMPLE 6-3 Analyzing Alternatives with Different Reject Rates

Consider the four plastic molding presses of Example 6-2. Suppose that each press is still capable of producing 120,000 total units per year, but the estimated reject rate is different for each alternative. This means that the expected revenue will differ among the alternatives since only nondefective units can be sold. The data for the four presses are summarized below. The life of each press (and the study period) is five years.

	Press			
	P1	*P2*	*P3*	*P4*
Capital investment	\$24,000	\$30,400	\$49,600	\$52,000
Total annual expenses	\$31,200	\$29,128	\$25,192	\$22,880
Reject rate	8.4%	0.3%	2.6%	5.6%

If all nondefective units can be sold for \$0.375 per unit, which press should be chosen? Solve by hand and by spreadsheet.

Solution by Hand

In this example, each of the four alternative presses produces 120,000 units per year, but they have different estimated reject rates. Therefore, the number of nondefective output units produced and sold per year, as well as the annual revenues received by the company, varies among the alternatives. But the annual expenses are assumed to be unaffected by the reject rates. In this situation, the preferred alternative will maximize overall profitability (Rule 1, Section 6.2.2). That is, the four presses need to be compared as investment alternatives. The PW, AW, and FW calculations for Alternative *P4* are given below:

$$\begin{aligned}
PW(10\%)_{P4} &= -\$52{,}000 + [(1-0.056)(120{,}000)(\$0.375)-\$22{,}880](P/A, 10\%, 5)\\
&= \$22{,}300,\\
AW(10\%)_{P4} &= -\$52{,}000(A/P, 10\%, 5) + [(1-0.056)(120{,}000)(\$0.375) - \$22{,}880]\\
&= \$5{,}882,\\
FW(10\%)_{P4} &= -\$52{,}000(F/P, 10\%, 5)\\
&\quad + [(1-0.056)(120{,}000)(\$0.375) - \$22{,}800](F/A, 10\%, 5)\\
&= \$35{,}914.
\end{aligned}$$

The PW, AW, and FW values for Alternatives *P1*, *P2*, and *P3* are determined with similar calculations and shown for all four alternatives in Table 6-2. Alternative *P2* maximizes all three equivalent-worth measures of overall profitability and is preferred [versus *P4* in Example 6-2]. The preference ranking ($P2 \succ P4 \succ P3 \succ P1$) is the same for the three methods but is different from the ranking in Example 6-2. The different preferred alternative and preference ranking are the result of the varying capability among the presses to produce nondefective output units.

TABLE 6-2 Comparison of Four Molding Presses, Using the PW, AW, and FW Methods to Maximize Overall Profitability

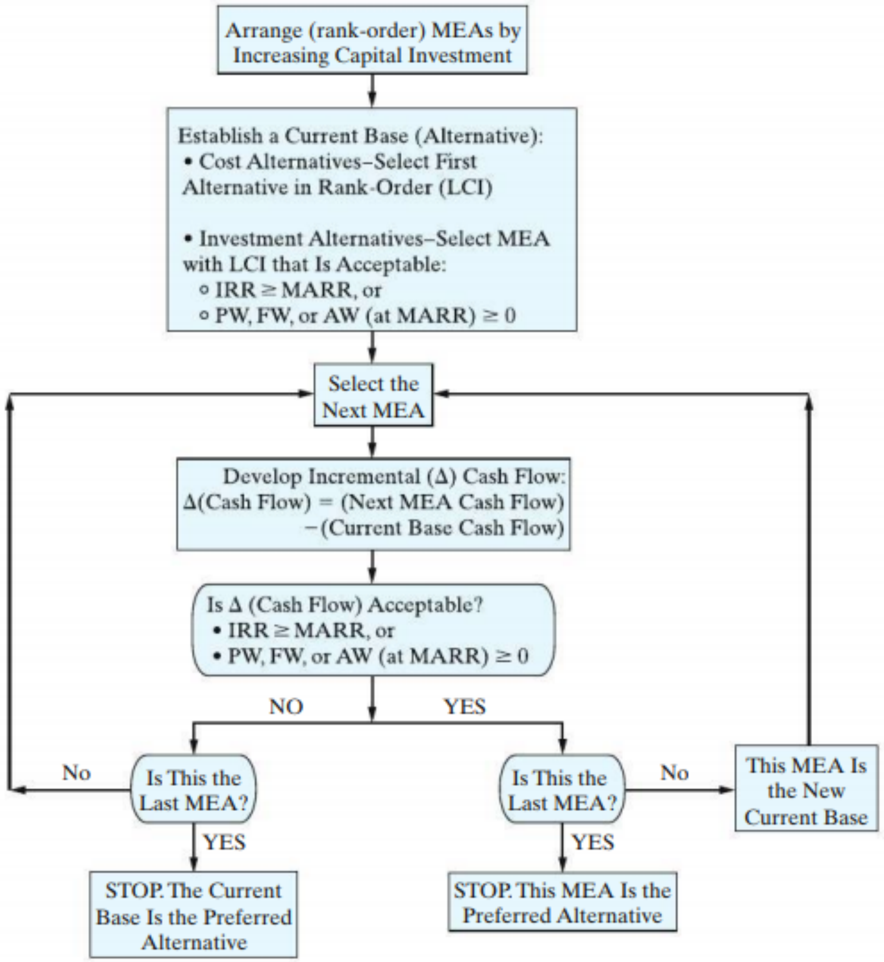
	Press (Equivalent-Worth Values)			
Method	*P1*	*P2*	*P3*	*P4*
Present worth	$13,984	$29,256	$21,053	$22,300
Annual worth	3,689	7,718	5,554	5,882
Future worth	22,521	47,117	33,906	35,914

Use the incremental investment analysis procedure.

- Arrange (rank order) the feasible alternatives based on increasing capital investment.
- Establish a base alternative.
  - Cost alternatives—the first alternative is the base.
  - Investment alternatives—the first *acceptable* alternative ($IRR > MARR$) is the base.
- Iteratively evaluate differences (incremental cash flows) between alternatives until all have been considered.

Evaluating incremental cash flows

- Work up the order of ranked alternatives smallest to largest.
- Subtract cash flows of the lower ranked alternative from the higher ranked.
- Determine if the incremental initial investment in the higher ranked alternative is *attractive* (e.g., *IRR>MARR*, *PW, FW, AW* all *>0*). If it is attractive, it is the "winner." If not, the lower ranked alternative is the "winner." The "loser" from this comparison is removed from consideration. Continue until all alternatives have been considered.
- This works for both *cost* and *investment* alternatives.



MEA: mutually exclusive alternative
LCI: least capital investment

Figure 6-6 Incremental Investment Analysis Procedure

Incremental analysis

	Alt. A	Alt. B	Alt. B-Alt. A
Initial cost	-$25,000	-$35,000	-$10,000
Net annual income	$7,500	$10,200	$3,200
IRR on total cash flow	15%	14%	11%

Which is preferred using a 5 year study period and MARR=10%?

Both alternatives A and B are acceptable—each one has a rate of return that exceeds the MARR. Choosing Alternative A because of its larger IRR would be an incorrect decision. By examining the incremental cash flows we see that the extra amount invested in Alternative B earns a return that exceeds the IRR—so B is preferred to A. Also note…

$$PW_A = -25,000 + 7,500(P/A, 10\%, 5) = 3,431$$

$$PW_B = -35,000 + 10,200(P/A, 10\%, 5) = 3,666$$

EXAMPLE 6-4 Incremental Analysis: Investment Alternatives

VIDEO SOLUTIONS

Suppose that we are analyzing the following six mutually exclusive alternatives for a small investment project, using the IRR method. The useful life of each alternative is 10 years, and the MARR is 10% per year. Also, net annual revenues less expenses vary among all alternatives, and Rule 1, Section 6.2.2, applies. If the study period is 10 years, and the market (salvage) values are zero, which alternative should be chosen? Notice that the alternatives have been *rank-ordered* from *low capital investment* to high capital investment.

	Alternative					
	A	*B*	*C*	*D*	*E*	*F*
Capital investment	$900	$1,500	$2,500	$4,000	$5,000	$7,000
Annual revenues less expenses	150	276	400	925	1,125	1,425

Solution

For each of the feasible alternatives, the IRR on the total cash flow can be computed by determining the interest rate at which the PW, FW, or AW equals zero (use of AW is illustrated for Alternative A):*

$$0 = -\$900(A/P, i'_A\%, 10) + \$150; \quad i'\% = ?$$

By trial and error, we determine that $i'_A\% = 10.6\%$. In the same manner, the IRRs of all the alternatives are computed and summarized:

	A	*B*	*C*	*D*	*E*	*F*
IRR on total cash flow	10.6%	13.0%	9.6%	19.1%	18.3%	15.6%

At this point, *only Alternative C is unacceptable* and can be eliminated from the comparison because its IRR is less than MARR of 10% per year. Also, A is the base alternative from which to begin the incremental investment analysis procedure, because it is the mutually exclusive alternative with the lowest capital investment whose IRR (10.6%) is equal to or greater than MARR (10%).

TABLE 6-3 Comparison of Five Acceptable Investment Alternatives Using the IRR Method (Example 6-4)

Increment Considered	A	$\Delta(B - A)$	$\Delta(D - B)$	$\Delta(E - D)$	$\Delta(F - E)$
Δ Capital investment	\$900	\$600	\$2,500	\$1,000	\$2,000
Δ Annual revenues less expenses	\$150	\$126	\$649	\$200	\$300
IRR_Δ	10.6%	16.4%	22.6%	15.1%	8.1%
Is increment justified?	Yes	Yes	Yes	Yes	No

This pre-analysis of the feasibility of each alternative is not required by the incremental analysis procedure. It is useful, however, when analyzing a larger set of mutually exclusive alternatives. You can immediately eliminate nonfeasible (nonprofitable) alternatives, as well as easily identify the base alternative.

As discussed in Section 6.4.2.1, it is not necessarily correct to select the alternative that maximizes the IRR on total cash flow. That is to say, Alternative D may not be the best choice, since *maximization of IRR does not guarantee maximization of equivalent worth on total investment at the MARR*. Therefore, to make the correct choice, we must examine each increment of capital investment to see if it will pay its own way. Table 6-3 provides the analysis of the five remaining alternatives, and the IRRs on incremental cash flows are again computed by setting $AW_\Delta(i') = 0$ for cash-flow differences between alternatives.

From Table 6-3, it is apparent that Alternative E will be chosen (not D) because it requires the largest investment for which the last increment of capital investment is justified. That is, we desire to invest additional increments of the \$7,000 presumably available for this project as long as each avoidable increment of investment can earn 10% per year or better.

EXAMPLE 6-5 Incremental Analysis: Cost-Only Alternatives

The estimated capital investment and the annual expenses (based on 1,500 hours of operation per year) for four alternative designs of a diesel-powered air compressor are shown, as well as the estimated market value for each design at the end of the common five-year useful life. The perspective (Principle 3, Chapter 1) of these cost estimates is that of the typical user (construction company, plant facilities department, government highway department, and so on). The study period is five years, and the MARR is 20% per year. One of the designs must be selected for the compressor, and each design provides the same level of service. On the basis of this information,

(a) determine the preferred design alternative, using the IRR method

(b) show that the PW method (i = MARR), using the incremental analysis procedure, results in the same decision.

Solve by hand and by spreadsheet.

	Design Alternative			
	D1	*D2*	*D3*	*D4*
Capital investment	$100,000	$140,600	$148,200	$122,000
Annual expenses	29,000	16,900	14,800	22,100
Useful life (years)	5	5	5	5
Market value	10,000	14,000	25,600	14,000

Observe that this example is a *cost-type situation with four mutually exclusive cost alternatives*. The following solution demonstrates the use of the incremental analysis procedure to compare cost alternatives and applies Rule 2 in Section 6.2.2.

Solution by Hand

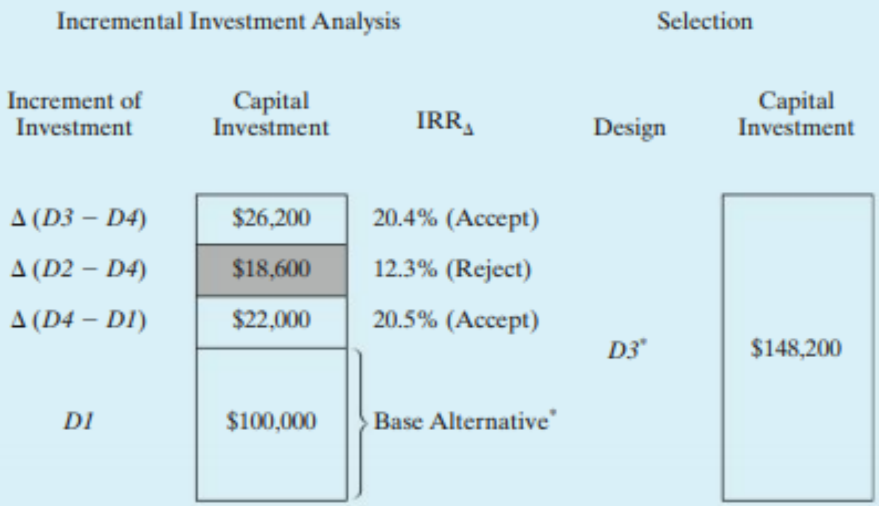
The first step is to arrange (rank-order) the four mutually exclusive cost alternatives on the basis of their increasing capital investment costs. Therefore, the order of the alternatives for incremental analysis is *D1*, *D4*, *D2*, and *D3*.

Since these are cost alternatives, the one with the least capital investment, *D1*, is the base alternative. Therefore, the base alternative will be preferred unless additional increments of capital investment can produce cost savings (benefits) that lead to a return equal to or greater than the MARR.

The first incremental cash flow to be analyzed is that between designs *D1* and *D4*, $\Delta(D4 - D1)$. The results of this analysis, and of subsequent differences between the cost alternatives, are summarized in Table 6-4, and the incremental investment analysis for the IRR method is illustrated in Figure 6-7. These results show the following:

TABLE 6-4 Comparison of Four Cost (Design) Alternatives Using the IRR and PW Methods with Incremental Analysis (Example 6-5)

Increment Considered	Δ(D4 – D1)	Δ(D2 – D4)	Δ(D3 – D4)
Δ Capital investment	\$22,000	\$18,600	\$26,200
Δ Annual expense (savings)	6,900	5,200	7,300
Δ Market value	4,000	0	11,600
Useful life (years)	5	5	5
IRR_{Δ}	20.5%	12.3%	20.4%
Is increment justified?	Yes	No	Yes
$PW_{\Delta}(20\%)$	\$243	–\$3,049	\$293
Is increment justified?	Yes	No	Yes



* Since these are cost alternatives, the IRR of *D3* cannot be determined.

Figure 6-7 Representation of Capital Investment Increments and IRR on Increments Considered in Selecting Design 3 (*D3*) in Example 6-5

1. The incremental cash flows between the cost alternatives are, in fact, investment alternatives.
2. The first increment, $\Delta(D4 - D1)$, is justified ($IRR_\Delta = 20.5\%$ is greater than MARR = 20%, and $PW_\Delta(20\%) = \$243 > 0$); the increment $\Delta(D2 - D4)$ is not justified; and the last increment, $\Delta(D3 - D4)$—not $\Delta(D3 - D2)$, because Design *D2* has already been shown to be unacceptable—is justified, resulting in the selection of Design *D3* for the air compressor. It is the highest investment for which each increment of investment capital is justified from the user's perspective.
3. The same capital investment decision results from the IRR method and the PW method, using the incremental analysis procedure, because *when the equivalent worth of an investment at i = MARR is greater than zero, its IRR is greater than MARR* (from the definition of the IRR; Chapter 5).

EXAMPLE 6-6 Incremental Analysis Using ERR

In an automotive parts plant, an engineering team is analyzing an improvement project to increase the productivity of a flexible manufacturing center. The estimated net cash flows for the three feasible alternatives being compared are shown in Table 6-5. The analysis period is six years, and MARR for capital investments at the plant is 20% per year. Using the ERR method, which alternative should be selected? ($\epsilon =$ MARR.)

Solution

The procedure for using the ERR method to compare mutually exclusive alternatives is the same as for the IRR method. The only difference is in the calculation methodology.

Table 6-5 provides a tabulation of the calculation and acceptability of each increment of capital investment considered. Since these three feasible alternatives are a mutually exclusive set of investment alternatives, the base alternative is the one with the least capital investment cost that is economically justified. For Alternative *A*, the PW of the negative cash-flow amounts (at $i = \epsilon\%$)

TABLE 6-5 Comparison of Three Mutually Exclusive Alternatives Using the ERR Method (Example 6-6)

	Alternative Cash Flows			Incremental Analysis of Alternatives		
End of Period	*A*	*B*	*C*	*A*[a]	$\Delta(B-A)$	$\Delta(C-A)$
0	−$640,000	−$680,000	−$755,000	−$640,000	−$40,000	−$115,000
1	262,000	−40,000	205,000	262,000	−302,000	−57,000
2	290,000	392,000	406,000	290,000	102,000	116,000
3	302,000	380,000	400,000	302,000	78,000	98,000
4	310,000	380,000	390,000	310,000	70,000	80,000
5	310,000	380,000	390,000	310,000	70,000	80,000
6	260,000	380,000	324,000	260,000	120,000	64,000
			Incremental analysis:			
			Δ PW of negative cash-flow amounts	640,000	291,657	162,498
			Δ FW of positive cash-flow amounts	2,853,535	651,091	685,082
			ERR	28.3%	14.3%	27.1%
			Is increment justified?	Yes	No	Yes

[a] The net cash flow for Alternative *A*, which is the incremental cash flow between making no change ($0) and implementing Alternative *A*.

is just the \$640,000 investment cost. Therefore, the ERR for Alternative A is the following:

$$\$640{,}000(F/P, i'\%, 6) = \$262{,}000(F/P, 20\%, 5) + \cdots + \$260{,}000$$
$$= \$2{,}853{,}535$$
$$(F/P, i'\%, 6) = (1 + i')^6 = \$2{,}853{,}535/\$640{,}000 = 4.4586$$
$$(1 + i') = (4.4586)^{1/6} = 1.2829$$
$$i' = 0.2829, \text{ or ERR} = 28.3\%.$$

Using a MARR = 20% per year, this capital investment is justified, and Alternative A is an acceptable base alternative. By using similar calculations, the increment $\Delta(B - A)$, earning 14.3%, is not justified and the increment $\Delta(C - A)$, earning 27.1%, is justified. Therefore, Alternative C is the preferred alternative for the improvement project. Note in this example that revenues varied among the alternatives and that Rule 1, Section 6.2.2, was applied.

Using rates of return is another way to compare alternatives.

- The return on investment (rate of return) is a popular measure of investment performance.
- Selecting the alternative with the largest rate of return can lead to incorrect decisions—do not compare the IRR of one alternative to the IRR of another alternative. The only legitimate comparison is the IRR to the MARR.
- Remember, the *base alternative* must be attractive (rate of return greater than the MARR), and the *additional* investment in other alternatives must itself make a satisfactory rate of return on that increment.

Comparing MEAs with unequal lives.

- The repeatability assumption, when applicable, simplified comparison of alternatives.
- If repeatability cannot be used, an appropriate study period must be selected (the coterminated assumption). This is most often used in engineering practice because product life cycles are becoming shorter.

The useful life of an alternative is less than the study period.

- Cost alternatives
  - Contracting or leasing for remaining years may be appropriate
  - Repeat part of the useful life and use an estimated market value to truncate
- Investment alternatives
  - Cash flows reinvested at the MARR at the end of the study period
  - Replace with another asset, with possibly different cash flows, after the study period

The useful life of an alternative is greater than the study period.

- Truncate the alternative at the end of the study period, using an estimated market value.
- The underlying principle in all such analysis is to compare the MEAs in a decision situation over the same study (analysis) period.

Equivalent worth methods can be used for MEAs with unequal lives.

- If repeatability can be assumed, the MEAs are most easily compared by finding the annual worth (AW) of each alternative over its own useful life, and recommending the one having the most economical value.
- For cotermination, use any equivalent worth method using the cash flows available for the study period.

EXAMPLE 6-7 Useful Lives ≠ Study Period: The Repeatability Assumption

VIDEO SOLUTIONS

The following data have been estimated for two mutually exclusive investment alternatives, *A* and *B*, associated with a small engineering project for which revenues as well as expenses are involved. They have useful lives of four and six years, respectively. If MARR = 10% per year, show which alternative is more desirable by using equivalent-worth methods (computed by hand and by spreadsheet). Use the repeatability assumption.

	A	*B*
Capital investment	\$3,500	\$5,000
Annual cash flow	1,255	1,480
Useful life (years)	4	6
Market value at end of useful life	0	0

Solution by the PW Method

The PW (or FW) solution must be based on the total study period (12 years). The PW of the initial useful life cycle will be different than the PW of subsequent replacement cycles:

$$\begin{aligned} \text{PW}(10\%)_A &= -\$3{,}500 - \$3{,}500[(P/F, 10\%, 4) + (P/F, 10\%, 8)] \\ &\quad + (\$1{,}255)(P/A, 10\%, 12) \\ &= \$1{,}028, \end{aligned}$$

$$\begin{aligned} \text{PW}(10\%)_B &= -\$5{,}000 - \$5{,}000(P/F, 10\%, 6) \\ &\quad + (\$1{,}480)(P/A, 10\%, 12) \\ &= \$2{,}262. \end{aligned}$$

Based on the PW method, we would select Alternative *B*.

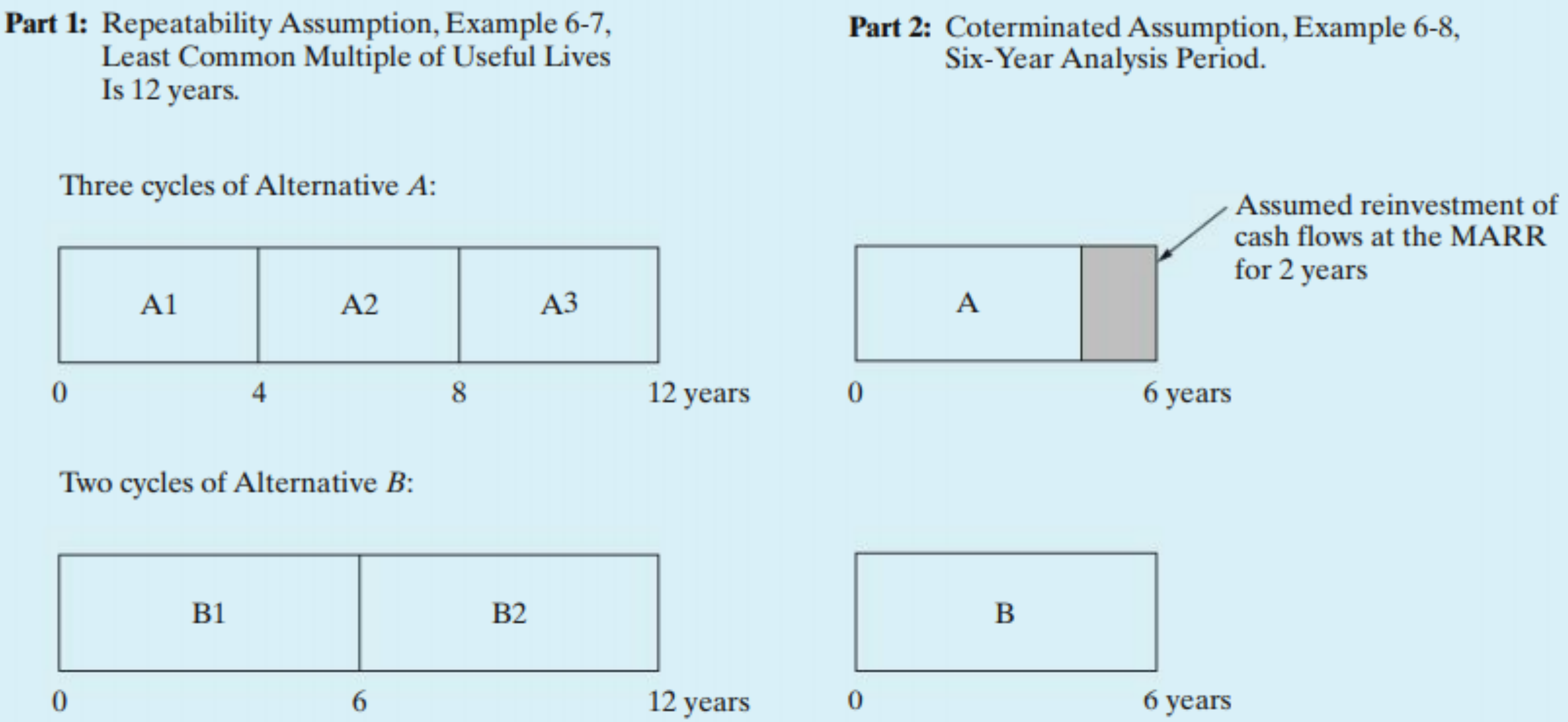


Figure 6-9 Illustration of Repeatability Assumption (Example 6-7) and Coterminated Assumption (Example 6-8)

Solution by the AW Method

The like replacement of assets assumes that the economic estimates for the initial useful life cycle will be repeated in each subsequent replacement cycle. Consequently, the AW will have the same value for each cycle and for the study period (12 years). This is demonstrated in the next AW solution by calculating (1) the AW of each alternative over the 12-year analysis period based on the previous PW values and (2) determining the AW of each alternative over one useful life cycle. Based on the previously calculated PW values, the AW values are

$$\text{AW}(10\%)_A = \$1{,}028(A/P, 10\%, 12) = \$151,$$

$$\text{AW}(10\%)_B = \$2{,}262(A/P, 10\%, 12) = \$332.$$

Next, the AW of each alternative is calculated over one useful life cycle:

$$\text{AW}(10\%)_A = -\$3{,}500(A/P, 10\%, 4) + (\$1{,}255) = \$151,$$

$$\text{AW}(10\%)_B = -\$5{,}000(A/P, 10\%, 6) + (\$1{,}480) = \$332.$$

This confirms that both calculations for each alternative result in the same AW value, and we would again select Alternative *B* because it has the larger value ($332).

EXAMPLE 6-8 Useful Lives ≠ Study Period: The Coterminated Assumption

Suppose that Example 6-7 is modified such that an analysis period of six years is used (coterminated assumption) instead of 12 years, which was based on repeatability and the least common multiple of the useful lives. Perhaps the responsible manager did not agree with the repeatability assumption and wanted a six-year analysis period because it is the planning horizon used in the company for small investment projects.

Solution

An assumption used for an investment alternative (when useful life is less than the study period) is that all cash flows will be reinvested by the firm at the MARR until the end of the study period. This assumption applies to Alternative *A*, which has a four-year useful life (two years less than the study period), and it is illustrated in Part 2 of Figure 6-9. We use the FW method to analyze this situation:

$$\text{FW}(10\%)_A = [-\$3{,}500(F/P, 10\%, 4) + (\$1{,}255)(F/A, 10\%, 4)](F/P, 10\%, 2)$$

$$= \$847,$$

$$\text{FW}(10\%)_B = -\$5{,}000(F/P, 10\%, 6) + (\$1{,}480)(F/A, 10\%, 6)$$

$$= \$2{,}561.$$

Based on the FW of each alternative at the end of the six-year study period, we would select Alternative *B* because it has the larger value ($2,561).

EXAMPLE 6-10 AW and Repeatability: Perfect Together!

VIDEO SOLUTIONS

Three products will be manufactured in a new facility at the Apex Manufacturing Company. They each require an identical manufacturing operation, but different production times, on a broaching machine. Two alternative types of broaching machines (*M1* and *M2*) are being considered for purchase. One machine type must be selected.

For the same level of annual demand for the three products, *annual* production requirements (machine hours) and annual operating expenses (per machine) are listed next. Which machine should be selected if the MARR is 20% per year? Solve by hand and by spreadsheet. Show all work to support your recommendation. (Use Rule 2 on page 245 to make your recommendation.)

Product	Machine *M1*	Machine *M2*
ABC	1,500 hr	900 hr
MNQ	1,750 hr	1,000 hr
STV	2,600 hr	2,300 hr
	5,850 hr	4,200 hr
Capital investment	$15,000 per machine	$22,000 per machine
Expected life	five years	eight years
Annual expenses	$4,000 per machine	$6,000 per machine

Assumptions: The facility will operate 2,000 hours per year. Machine availability is 90% for Machine *M1* and 80% for Machine *M2*. The yield of Machine *M1* is 95%, and the yield of Machine *M2* is 90%. Annual operating expenses are based on an assumed operation of 2,000 hours per year, and workers are paid during any idle time of Machine *M1* or Machine *M2*. Market values of both machines are negligible.

Solution by Hand

The company will need 5,850 hours/[2,000 hours (0.90)(0.95)] = 3.42 (four machines of type *M1*) or 4,200 hours/[2,000 hours (0.80)(0.90)] = 2.92 (three machines of type *M2*). The maximum operation time of 2,000 hours per year in the denominator must be multiplied by the availability of each machine and the yield of each machine, as indicated.

The annual cost of ownership, assuming a MARR = 20% per year, is $15,000(4)(*A*/*P*, 20%, 5) = $20,064 for Machine *M1* and $22,000(3)(*A*/*P*, 20%, 8) = $17,200 for Machine *M2*.

There is an excess capacity when four Machine *M1*s and three Machine *M2*s are used to provide the machine-hours (5,850 and 4,200, respectively) just given. If we assume that the operator is paid for idle time he or she may experience on *M1* or *M2*, the annual expense for the operation of four M1s is 4 machines × $4,000 per machine = $16,000. For three *M2*s, the annual expense is 3 machines × $6,000 per machine = $18,000.

The total equivalent annual cost for four Machine *M1*s is $20,064 + $16,000 = $36,064. Similarly, the total equivalent annual expense for three Machine *M2*s is $17,200 + $18,000 = $35,200. By a slim margin, Machine *M2* is the preferred choice to minimize equivalent annual costs with the repeatability assumption.

EXAMPLE 6-11 Modeling Estimated Expenses as Arithmetic Gradients

You are a member of an engineering project team that is designing a new processing facility. Your present design task involves the portion of the catalytic system that requires pumping a hydrocarbon slurry that is corrosive and contains abrasive particles. For final analysis and comparison, you have selected two fully lined slurry pump units, of equal output capacity, from different manufacturers. Each unit has a large-diameter impeller required and an integrated electric motor with solid-state controls. Both units will provide the same level of service (support) to the catalytic system but have different useful lives and costs.

	Pump Model	
	SP240	HEPS9
Capital investment	$33,200	$47,600
Annual expenses:		
Electrical energy	$2,165	$1,720
Maintenance	$1,100 in year 1, and increasing $500/yr thereafter	$500 in year 4, and increasing $100/yr thereafter
Useful life (years)	5	9
Market value (end of useful life)	0	5,000

The new processing facility is needed by your firm at least as far into the future as the strategic plan forecasts operating requirements. The MARR is 20% per year. Based on this information, which slurry pump should you select?

Solution

Notice that the estimates for maintenance expenses involve an arithmetic gradient series (Chapter 4). A cash-flow diagram is very useful in this situation to help keep track of the various cash-flow series. The cash-flow diagrams for pump models SP240 and HEPS9 are shown in Figure 6-12.

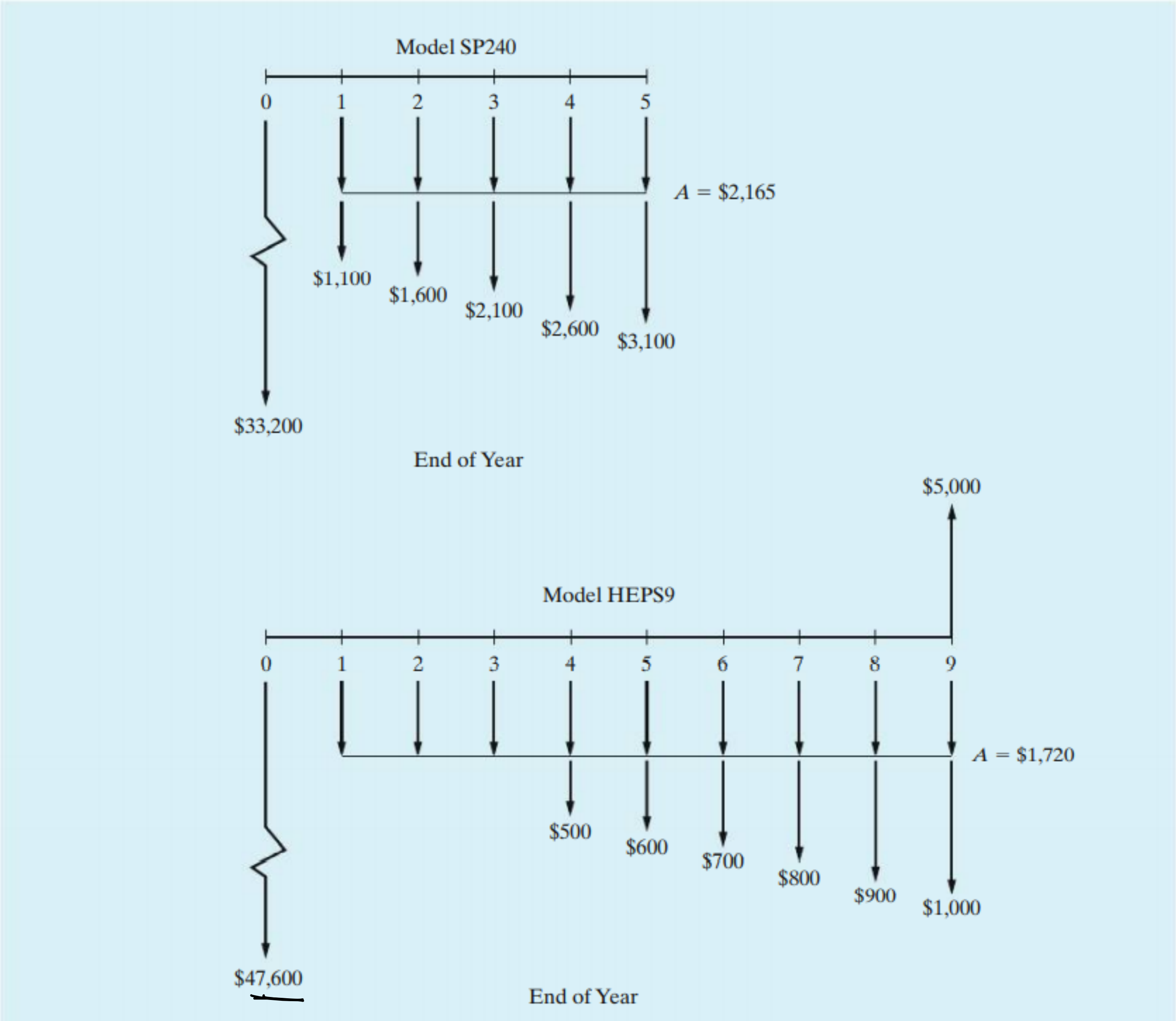
The repeatability assumption is a logical choice for this analysis, and a study period of either infinite or 45 years (least common multiple of the useful lives) in length can be used. With repeatability, the AW over the initial *useful life* of each alternative is the same as its AW over the length of either study period:

$$\begin{aligned} \text{AW}(20\%)_{\text{SP240}} &= -\$33{,}200(A/P, 20\%, 5) - \$2{,}165 \\ &\quad -[\$1{,}100 + \$500(A/G, 20\%, 5)] \\ &= -\$15{,}187, \\ \text{AW}(20\%)_{\text{HEPS9}} &= -\$47{,}600(A/P, 20\%, 9) + \$5{,}000(A/F, 20\%, 9) \\ &\quad -\$1{,}720 - [\$500(P/A, 20\%, 6) \\ &\quad +\$100(P/G, 20\%, 6)] \times (P/F, 20\%, 3) \times (A/P, 20\%, 9) \\ &= -\$13{,}622. \end{aligned}$$

Based on Rule 2 (Section 6.2.2), you should select pump model HEPS9, since the AW over its useful life (nine years) has the smaller negative value (–$13,622).

As additional information, the following two points support in choosing the repeatability assumption in Example 6-11:

1. The repeatability assumption is commensurate with the long planning horizon for the new processing facility and with the design and operating requirements of the catalytic system.


Model SP240
0
1
2
3
4
5
A = $2,165
$1,100
$1,600
$2,100
$2,600
$3,100
$33,200
End of Year
$5,000
Model HEPS9
0
1
2
3
4
5
6
7
8
9
A = $1,720
$500
$600
$700
$800
$900
$1,000
$47,600
End of Year

2. If the initial estimated costs change for future pump-replacement cycles, a logical assumption is that the ratio of the AW values for the two alternatives will remain approximately the same. Competition between the two manufacturers should cause this to happen. Hence, the pump selected (model HEPS9) should continue to be the preferred alternative.

If the existing model is redesigned or new models of slurry pumps become available, however, another study to analyze and compare all feasible alternatives is required before a replacement of the selected pump occurs.

EXAMPLE 6-9 Comparing Alternatives Using CW

We now revisit the problem posed at the beginning of the chapter involving two containment alternatives for coal combustion by-products. Because an indefinitely long study period is specified, we use the CW method to compare the two storage methods. First we compute the AW of each system over its useful

Alternatives for Waste Storage

A large electric utility company is considering two methods for containing and storing its coal combustion by-products (fly ash). One method is wet slurry storage, and the second method is dry storage of the fly ash. The company will adopt one of these methods for all 28 fly ash impoundments at its seven coal-fired power plants. Wet storage has an initial capital investment of $2 billion, followed by annual maintenance expenses of $300 million over the 10-year life of the method. Dry storage has a $2.5 billion capital investment and $150 million per year annual upkeep expenditures over its 7-year life. If the utility's MARR is 10% per year, which method of fly ash storage should be selected assuming an indefinitely long study period? In life, and then we determine the capitalized worth (refer to Section 5.3.3) over a very long study period.

$$\text{Wet:}\quad AW(10\%) = -\$2{,}000{,}000{,}000\,(A/P,\ 10\%,\ 10) - \$300{,}000{,}000$$
$$= -\$625{,}400{,}000$$
$$CW(10\%) = AW(10\%)/0.10 = -\$6{,}254{,}000{,}000$$

$$\text{Dry:}\quad AW(10\%) = -\$2{,}500{,}000{,}000\,(A/P,\ 10\%,\ 7) - \$150{,}000{,}000$$
$$= -\$663{,}500{,}000$$
$$CW(10\%) = AW(10\%)/0.10 = -\$6{,}635{,}000{,}000$$

We recommend the wet slurry storage method because it has the lesser negative (greater) CW.

We can use incremental rate of return analysis on MEAs with unequal lives.

Equate the MEAs annual worths (AW) over their respective lives.

	A	B
Capital Investment	\$3,500	\$5,000
Annual Cash Flow	\$1,255	\$1,480
Useful Live (years)	4	6

$$-\$3,500(A/P, i^{*}\%, 4) + \$1,255 = -\$5,000(A/P, i^{*}\%, 6) + \$1,480$$

Solving, we find *i**=26%, so Alt B is preferred.

Rate-of-Return Analysis

- Up until this point, we have solved problems with unequal lives with the use of the equivalent-worth methods (AW being the most convenient).
- The analysis of alternatives having unequal lives can also be accomplished by using rate-of-return methods.
- When the co-termination method is used, the incremental analysis procedure can be applied directly.

Rate-of-Return Analysis

- When the study period is either indefinitely long or equal to a common multiple of the useful lives, however, computing the incremental cash flows can be quite cumbersome.
- For example, the implicit study period in Example 6-11 is 45 years! In this instance, a more direct approach is useful.
- In general, the IRR of an increment of capital is the interest rate, i ò, that equates the equivalent worth of the higher capital investment cost alternative to the equivalent worth of the lower capital investment cost alternative

Rate-of-Return Analysis

- The decision rule using this approach is that, if i ò ≥ MARR, the increment is justified and the alternative with the higher capital investment cost is preferred.
- When repeatability applies, we simply need to develop the AW equation for each alternative over its own useful life and find the interest rate that makes them equal.

- To demonstrate, consider the alternatives that were analyzed in Example 6-8 by using a MARR of 10% per year and a study period of 12 years (repeatability assumption).

	A	*B*
Capital investment	\$3,500	\$5,000
Annual cash flow	1,255	1,480
Useful life (years)	4	6

Equating the AW of the alternatives over their respective lives, we get

$$\text{AW}_A(i^*\%) = \text{AW}_B(i^*\%)$$

$$-\$3{,}500(A/P, i^*\%, 4) + \$1{,}255 = -\$5{,}000(A/P, i^*\%, 6) + \$1{,}480.$$

By trial and error, the IRR of the extra capital needed to repeatedly invest in Alternative B (instead of Alternative A) over the study period is i ò = 26%. Since this value is greater than the MARR, the increment is justified and Alternative B is preferred. This is the same decision arrived at in Example 6-7.

The Imputed Market Value Technique

- Obtaining a current estimate from the marketplace for a piece of equipment or another type of asset is the preferred procedure in engineering practice when a market value at time T < (useful life) is required.

- This approach, however, may not be feasible in some cases. For example, a type of asset may have low turnover in the market place, and information for recent transactions is not available.

The Imputed Market Value Technique

- It is sometimes necessary to estimate the market value for an asset without current and representative historical data.

- The imputed market value technique, which is sometimes called the implied market value, can be used for this purpose as well as for comparison with marketplace values when current data are available.

The Imputed Market Value Technique

The estimating procedure used in the technique is based on logical assumptions about the value of the remaining useful life for an asset. If an imputed market value is needed for a piece of equipment, say, at the end of year T < (useful life), the estimate is calculated on the basis of the sum of two parts, as follows:

$$\text{MV}_T = [\text{PW at EOY } T \text{ of remaining capital recovery (CR) amounts}]$$
$$+[\text{PW at EOY } T \text{ of original market value at end of useful life}],$$

where PW is computed at $i = \text{MARR}$.

EXAMPLE 6-12 Estimating a New Market Value when Useful Life > Study Period

Use the imputed market value technique to develop an estimated market value for pump model HEPS9 (Example 6-11) at EOY five. The MARR remains 20% per year.

Solution

The original information from Example 6-11 will be used in the solution: capital investment = \$47,600, useful life = nine years, and market value = \$5,000 at the end of useful life.

First, compute the PW at EOY five of the remaining CR amounts [Equation (5-5)]:

$$\begin{aligned} PW(20\%)_{CR} &= [\$47{,}600(A/P, 20\%, 9) - \$5{,}000(A/F, 20\%, 9)] \times (P/A, 20\%, 4) \\ &= \$29{,}949. \end{aligned}$$

Next, compute the PW at EOY five of the original MV at the end of useful life (nine years):

$$PW(20\%)_{MV} = \$5{,}000(P/F, 20\%, 4) = \$2{,}412.$$

Then, the estimated market value at EOY five ($T = 5$) is as follows:

$$\begin{aligned} MV_5 &= PW_{CR} + PW_{MV} \\ &= \$29{,}949 + \$2{,}412 = \$32{,}361. \end{aligned}$$

Personal Finances

- Sound financial planning is all about making wise choices for your particular circumstances (e.g., your amount of personal savings, your job security, your attitude toward risk).
- Thus far in Chapter 6, we have focused on facilitating good decision making from the perspective of a corporation.
- Now we apply these same principles to several problems you are likely to face soon in your personal decision making. Two of the largest investments you'll ever make involve houses and automobiles.

Personal Finances

- It turns out that people who use credit cards tend to spend more money than others who pay cash or write checks. An enlightening exercise to see how addicted you are to credit cards is to go cold turkey for two months.
- A fundamental lesson underlying this section is to save now rather than spending on luxury purchases.
- By choosing to save now, we are making an attempt to minimize the risk of making poor decisions later on.

EXAMPLE 6-13 Automobile Financing Options

You have decided to purchase a new automobile with a hybrid-fueled engine and a six-speed transmission. After the trade-in of your present car, the purchase price of the new automobile is \$30,000. This balance can be financed by an auto dealer at 2.9% APR with payments over 48 months. Alternatively, you can get a \$2,000 discount on the purchase price if you finance the loan balance at an APR of 8.9% over 48 months. Should you accept the 2.9% financing plan or accept the dealer's offer of a \$2,000 rebate with 8.9% financing? Both APRs are compounded monthly.

Solution

In this example, we assume that your objective is to minimize your monthly car payment.

2.9% financing monthly payment:

$$\$30{,}000\,(A/P,\ 2.9\%/12,\ 48\text{ months}) = \$30{,}000(0.0221) = \$663.00 \text{ per month}$$

8.9% financing monthly payment:

$$\$28{,}000\,(A/P,\ 8.9\%/12,\ 48\text{ months}) = \$28{,}000(0.0248) = \$694.90 \text{ per month}$$

Therefore, to minimize your monthly payment, you should select the 2.9% financing option.

What If Questions

When shopping for an automobile you'll find that there are many financing options like the ones in this example available to you. You may find it useful to ask yourself questions such as "how high would the rebate have to be for me to prefer the rebate option," or "how low would the APR have to be for me to select the rebate option?" The answers to these questions can be found through simple equivalence calculations.

(a) How much would the rebate have to be?

Let X = rebate amount. Using the monthly payment of \$663 from the 2.9% financing option, we can solve for the rebate amount that would yield the same monthly payment.

$$(\$30{,}000 - X)(A/P,\ 8.9\%/12 \text{ months},\ 48 \text{ months}) = \$663$$

$$(\$30{,}000 - X)(0.0248) = \$663$$

$$X = \$3{,}266$$

(b) How low would the interest rate have to be?

Now we want to find the interest rate that equates borrowing \$28,000 to 48 monthly payments of \$663. This question is easily solved using a spreadsheet package.

$$\text{RATE}\ (48, -663,\ 28000) = 0.535\% \text{ per month}$$

$$\text{APR} = 0.535\% \times 12 = 6.42\%$$

EXAMPLE 6-14 Mortgage Financing Options

A general rule of thumb is that your monthly mortgage payment should not exceed 28% of your household's gross monthly income. Consider the situation of Jerry and Tracy, who just committed to a \$300,000 mortgage on their dream home. They have reduced their financing choices to a 30-year conventional mortgage at 6% APR, or a 30-year interest-only mortgage at 6% APR.

(a) Which mortgage, if either, do they qualify for if their combined gross annual income is \$70,000?

(b) What is the disadvantage in an interest-only mortgage compared to the conventional mortgage?

Solution

(a) Using the general rule of thumb, Jerry and Tracy can afford a monthly mortgage payment of $(0.28)(\$70{,}000/12) = \$1{,}633$. The monthly payment for the conventional mortgage is

$$\$300{,}000\,(A/P,\ 0.5\%,\ 360) = \$1{,}800.$$

For the interest-only mortgage, the monthly payment is

$$(0.005)(\$300{,}000) = \$1{,}500.$$

Thus, the conventional mortgage payment is larger than what the guideline suggests is affordable. This type of loan is marginal because it stretches their budget too much. They easily qualify for the interest-only mortgage because the \$1,500 payment is less than \$1,633.

(b) If home prices fall in the next several years, Jerry and Tracy may have "negative equity" in their home because no principal has been repaid in their monthly interest-only payments. They will not have any buffer to fall back on should they have to sell their house for less than they purchased it for.

It is important to note that interest-only loans don't remain interest-only for the entire loan period. The length of time that interest-only payments may be made is defined in the mortgage contract and can be as short as 5 years or as long as 15 years. After the interest-only period is over, the monthly payment adjusts to include principal and interest. It is calculated to repay the entire loan by the end of the loan period. Suppose Jerry and Tracy's interest-only period was five years. After this time, the monthly payment would become

$$\$300{,}000(A/P, 0.5\%, 300) = \$1{,}932.90.$$

Before Jerry and Tracy accept this type of loan, they should be confident that they will be able to afford the $1,932.90 monthly payment in five years.

EXAMPLE 6-15 Comparison of Two Savings Plans

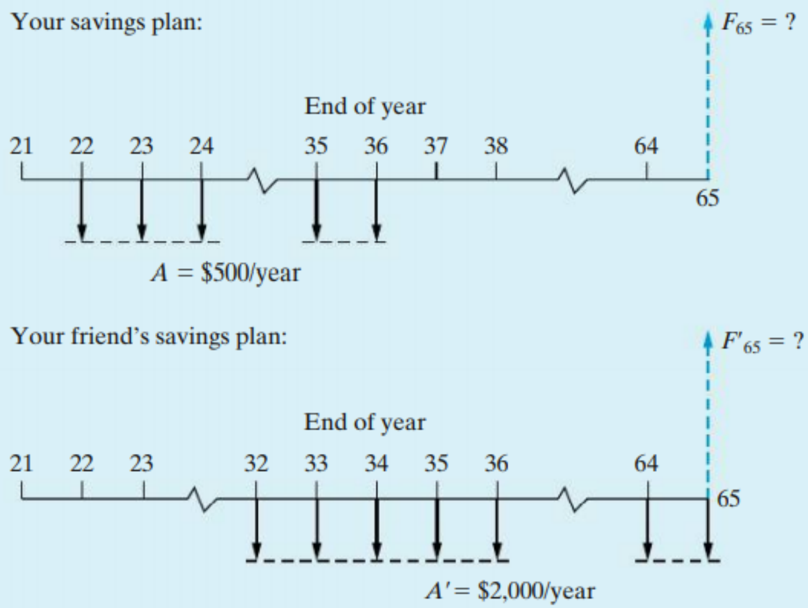
Suppose you start a savings plan in which you save \$500 each year for 15 years. You make your first payment at age 22 and then leave the accumulated sum in the savings plan (and make no more annual payments) until you reach age 65, at which time you withdraw the total accumulated amount. The average annual interest rate you'll earn on this savings plan is 10%.

A friend of yours (exactly your age) from Minnesota State University waits 10 years to start her savings plan. (That is, she is 32 years old.) She decides to save \$2,000 each year in an account earning interest at the rate of 10% per year. She will make these annual payments until she is 65 years old, at which time she will withdraw the total accumulated amount.

How old will you be when your friend's *accumulated* savings amount (including interest) exceeds yours? State any assumptions you think are necessary.

Solution

Creating cash-flow diagrams for Example 6-15 is an important first step in solving for the unknown number of years, N, until the future equivalent values of both savings plans are equal. The two diagrams are shown below. The future equivalent (F) of your plan is $\$500(F/A, 10\%, 15)(F/P, 10\%, N - 36)$ and that of your friend is $F' = \$2{,}000(F/A, 10\%, N - 31)$. It is clear that N, the age at which $F = F'$, is greater than 32. Assuming that the interest rate remains constant at 10% per year, the value of N can be determined by trial and error:



N	Your Plan's F	Friend's F′
36	$15,886	$12,210
38	$19,222	$18,974
39	$21,145	$22,872
40	$23,259	$27,159

By the time you reach age 39, your friend's accumulated savings will exceed yours. (If you had deposited $1,000 instead of $500, you would be over 76 years when your friend's plan surpassed yours. Moral: Start saving early!)

EXAMPLE 6-16 Credit Card Offers

Randy just cancelled his credit card with a large bank. A week later, a representative of the bank called Randy with an offer of a "better" credit card that will advance Randy $2,000 when he accepts it. Randy could not refuse the offer and several days later receives a check for $2,000 from the bank. With this money, Randy decides to buy a new computer.

At the next billing cycle (a month later), the $2,000 advance appears as a charge against Randy's account, and the APR is stated to be 21% (compounded monthly). At this point in time, Randy elects to pay the minimum monthly payment of $40 and cuts up the credit card so that he cannot make any additional purchases.

(a) Over what period of time does this payment extend in order to repay the $2,000 principal?

(b) If Randy decides to repay all remaining principal after having made 15 monthly payments, how much will he repay?

Solution

(a) You may be surprised to know that the majority of credit card companies determine the minimum monthly payment based on a repayment period of 10 years. The monthly interest rate being charged for Randy's card is $21\%/12 = 1.75\%$. We can solve the following equivalence relationship to determine the number of $40 monthly payments required to pay off a loan principal of $2,000.

$$\$2{,}000 = \$40(P/A, 1.75\%, N)$$

This is easily solved using the NPER (rate, payment, principal) function in Excel.

$$\text{NPER}(1.75\%, -40, 2000) = 119.86$$

Sure enough, it will take Randy 120 months to pay off this debt.

(b) After having made 15 payments, Randy has 105 payments remaining. To find the single sum payoff for this loan, we simply have to determine the present worth of the remaining payments.

$$\text{Payoff} - \$40(P/A, 1.75\%, 105) - \$1{,}915.96$$

This payoff amount assumes no penalty for early repayment of the loan (which is typically the case when it comes to credit cards). Notice how very little principal ($84.06) was repaid in the early part of the loan.

Engineering Economy

Chapter 7: Depreciation and Income Taxes

The objective of Chapter 7 is to explain how depreciation affects income taxes and how income taxes affect economic decision making.

Income taxes usually represent a significant cash outflow. Depreciation is an important element in finding after-tax cash flows

Depreciation is the decrease in value of physical properties with the passage of time.

- It is an accounting concept, a *non-cash* cost, that establishes an annual deduction against before-tax income.
- It is used to reflect effect of time and use on asset's value in firm's financial statements.
- It is intended to approximate the yearly fraction of an asset's value used in the production of income.

Depreciation

- Definition: Loss of value for a fixed asset
- Example: You purchased a car worth $15,000 at the beginning of year 2000.

End of Year	Market Value	Loss of Value
0	$15,000	
1	10,000	$5,000
2	8,000	2,000
3	6,000	2,000
4	5,000	1,000
5	4,000	1,000

Depreciation

Why Do We Need to Consider Depreciation?

Business Expense: Depreciation is viewed as part of business expenses that reduce taxable income.

Gross Income -Expenses:
(Cost of goods sold) (Depreciation)
(operating expenses)

Taxable Income

- Income taxes

Net income (profit)

Factors to Consider in Asset Depreciation

- Depreciable life (how long?)
- Salvage value (disposal value)
- Cost basis (depreciation basis)
- Method of depreciation (how?)

Property is depreciable if

- it must be used in business or held to produce income.
- it has a determinable useful life and longer than one year.
- it is something that wears out, decays, gets used up, becomes obsolete, or loses value from natural causes (ex: land is not depreciable).
- it is not inventory, stock in trade, or investment property..

Depreciable property is

- TANGIBLE - can be seen or touched

  <u>personal property</u> - includes assets such as machinery, vehicles, equipment, furniture, etc...

  <u>real property</u> - anything growing on, or attached to land

  (Since land does not have a determinable life itself, it is not depreciable)

- Or it is INTANGIBLE - personal property, such as copyright, patent or franchise

- Properties are depreciated according to a depreciation schedule.

WHEN DEPRECIATION STARTS AND STOPS

- Depreciation <u>starts</u> when property is placed in service for use in business or for production of income.
- Property is considered in service when it is ready and available for specific use, even if not actually used yet.
- Depreciation <u>stops</u> when it is retired from service.

Some Definitions

- Adjusted (cost) basis: The original cost basis of the asset, adjusted by allowable increases or decreases (used to calculate depreciation deductions)

- Basis (or cost basis): The initial cost of acquiring an asset (purchase price plus any sales tax), including transportation expenses and other normal costs of making the asset serviceable for its intended use. (also called unadjusted cost basis)

Some Definitions

- Book Value (BV): The worth of a depreciable property shown on accounting records of a company.
  - It is the original adjusted cost basis less all allowable depreciation deductions.
  - It represents the amount of capital that remains invested in the property and must be recovered in the future through accounting process.

$$BV_k = \text{Adjusted cost basis} - \sum_{j=1}^{k} d_j$$

$$BV_k = \text{Book value at the end of year } k$$

$$d_j = \text{Depreciation deduction for year } j$$

Some Definitions

- Market Value (MV): The amount that will be paid by a willing buyer to willing seller for a property.

- Market Value approximates present value of what will be received through ownership of property, including time-value of money (or profit).

Some Definitions

- Recovery period: The number of years over which the basis of a property is recovered through the accounting process.
- Normally recovery period is the useful life of the property for classical methods.
- Recovery Rate is the percentage for each year of recovery period used to calculate an annual depreciation deduction.

Some Definitions

- Salvage Value (SV): The estimated value of a property at the end of its useful life.

-- it is the expected selling price of property when asset can no longer be used productively.

-- net salvage value is used when expenses incurred in disposing of property; cash outflows must be deducted from cash inflows for final net salvage value.

-- with classical methods of depreciation, estimated salvage value is established and used.

-- with new methods of depreciation (MACRS), the salvage value of depreciable property is defined to be zero

Some Definitions

- Useful life: The expected (or estimated) period of time that a property will be used in a trade or business to produce income.

- Sometimes it is referred to as depreciable life.

Cost Basis

Cost of new hole-punching machine (Invoice price)	$62,500
+ Freight	725
+ Installation labor	2,150
+ Site preparation	3,500
Cost basis to use in depreciation calculation	$68,875

Cost Basis with Trade-In Allowance

Old hole-punching machine (book value)	$4,000
Less: Trade-in allowance	5,000
Unrecognized gains	$1,000
Cost of new hole-punching machine	$62,500
Less: Unrecognized gains	(1,000)
Freight	725
Installation labor	2,150
Site preparation	3,500
Cost of machine (cost basis)	$67,875

DEPRECIATION CONCEPTS

The following terms are used in the classical (historical) depreciation method equations:

N = depreciable life of the asset in years

B = cost basis, including allowable adjustments

d_k = annual depreciation deduction in year k ($1 \leq k \leq N$)

d_{k^*} = cummulative depreciation through year k

BV_k = book value at the end of year k

BV_N = book value at the end of the depreciable (useful) life SV_N = salvage value at the end of year N

R = the ratio of depreciation in any one year to the BV at the beginning of the year

Classical Depreciation Methods

- Types of Classical Depreciation Methods:
  - Straight-Line Method
  - Declining Balance Method
  - Sum of the Years' Digits Method
  - Unit Production Method

Straight line (SL): constant amount of depreciation each year over the depreciable life of the asset.

$$d_k = \frac{B - SV_N}{N}$$

$$d_k^* = kd_k \text{ for } 1 \le k \le N$$

$$BV_k = B - d_k^*$$

- N = depreciable life
- B = cost basis
- d_k = depreciaton in *k*
- $d_k{}^*$=*cumulative depreciation through year k*
- BV_k = book value at end of *k*
- SV_N = salvage value

This method requires an estimate of the final SV at the end of year N.

Example for Straight Line Method of Depreciation

Example: B = \$7000 (purchase price); SV=\$1000; N=5 years

d_k (Constant/Yr.)= (7000-1000)/5= 1,200

d_3^*= Accumulated depreciation at the end of Yr.3 =

3(1200) = 3600

BV(3) = Book value at the beginning of Yr.4 =

Purchase price - Sum(of depreciation charges) =

7000 - 3(1200)= \$3,400

EXAMPLE 7-1 SL Depreciation

A laser surgical tool has a cost basis of $200,000 and a five-year depreciable life. The estimated SV of the laser is $20,000 at the end of five years. Determine the annual depreciation amounts using the SL method. Tabulate the annual depreciation amounts and the book value of the laser at the end of each year.

Solution

The depreciation amount, cumulative depreciation, and BV for each year are obtained by applying Equations (7-2), (7-3), and (7-4). Sample calculations for year three are as follows:

$$d_3 = \frac{\$200{,}000 - \$20{,}000}{5} = \$36{,}000$$

$$d_3^* = 3\left(\frac{\$200{,}000 - \$20{,}000}{5}\right) = \$108{,}000$$

$$BV_3 = \$200{,}000 - \$108{,}000 = \$92{,}000$$

The depreciation and BV amounts for each year are shown in the following table.

EOY, k	d_k	BV_k
0	—	$200,000
1	$36,000	$164,000
2	$36,000	$128,000
3	$36,000	$92,000
4	$36,000	$56,000
5	$36,000	$20,000

Note that the BV at the end of the depreciable life is equal to the SV used to calculate the yearly depreciation amount.

Declining-balance (DB): a constant-percentage of the remaining BV is depreciated each year.

$$d_k = B(1 - R)^{k-1}(R)$$

$$BV_k = B(1 - R)^k$$

The constant percentage is determined by R, where

$R = 2/N$ when 200% declining balance is used,

$R = 1.5/N$ when 150% declining balance is used.

Declining Balance

$$d_k^* = B\left[1 - (1 - R)^k\right]$$

d^*_k = cummulative depreciation through year k

B: Cost basis (e.g. Purchase price)

DB is sometimes called constant percentage method.

DB method assumes annual cost of depreciation is a fixed percentage of BV at beginning of year.

$d_1 = B(R)$ annual depreciation in year 1

$d_k = B(1 - R)^{k-1}(R)$ annual depreciation in year k

$d_{k*} = B[1 - (1 - R)^k]$ cummulative depreciation through year k

$BV_k = B(1 - R)^k$ book value at the end of year k

$BV_N = B(1 - R)^N$ book value at the end of the depreciable life

EXAMPLE 7-2 DB Depreciation

A new electric saw for cutting small pieces of lumber in a furniture manufacturing plant has a cost basis of $4,000 and a 10-year depreciable life. The estimated SV of the saw is zero at the end of 10 years. Use the DB method to calculate the annual depreciation amounts when

(a) $R = 2/N$ (200% DB method)

(b) $R = 1.5/N$ (150% DB method).

Tabulate the annual depreciation amount and BV for each year.

Solution

Annual depreciation, cumulative depreciation, and BV are determined by using Equations (7-6), (7-7), and (7-8), respectively. Sample calculations for year six are as follows:

(a)

$$R = 2/10 = 0.2,$$

$$d_6 = \$4{,}000(1 - 0.2)^5(0.2) = \$262.14,$$

$$d_6^* = \$4{,}000[1 - (1 - 0.2)^6] = \$2{,}951.42,$$

$$BV_6 = \$4{,}000(1 - 0.2)^6 = \$1{,}048.58.$$

(b)

$$R = 1.5/10 = 0.15,$$

$$d_6 = \$4{,}000(1 - 0.15)^5(0.15) = \$266.22,$$

EXAMPLE 7-2 DB Depreciation

$$d_6^* = \$4{,}000[1 - (1 - 0.15)^6] = \$2{,}491.40,$$

$$BV_6 = \$4{,}000(1 - 0.15)^6 = \$1{,}508.60.$$

The depreciation and BV amounts for each year, when $R = 2/N = 0.2$, are shown in the following table:

200% DB Method Only		
EOY, k	d_k	BV_k
0	—	\$4,000
1	\$800	3,200
2	640	2,560
3	512	2,048
4	409.60	1,638.40
5	327.68	1,310.72
6	262.14	1,048.58
7	209.72	838.86
8	167.77	671.09
9	134.22	536.87
10	107.37	429.50

DB with Switchover to SL

- When DB method is used, BV never reaches 0.

- In order for BV to reach 0, we allow switching from DB to SL in the year in which a larger depreciation amount is obtained from the SL method.

TABLE 7-1 The 200% DB Method with Switchover to the SL Method (Example 7-1)

		Depreciation Method		
Year, k	(1) Beginning-of-Year BV[a]	(2) 200% DB Method[b]	(3) SL Method[c]	(4) Depreciation Amount Selected[d]
1	$4,000.00	$800.00	>$400.00	$800.00
2	3,200.00	640.00	>355.56	640.00
3	2,560.00	512.00	>320.00	512.00
4	2,048.00	409.60	>292.57	409.60
5	1,638.40	327.68	>273.07	327.68
6	1,310.72	262.14	=262.14	262.14 (switch)
7	1,048.58	209.72	<262.14	262.14
8	786.44	167.77	<262.14	262.14
9	524.30	134.22	<262.14	262.14
10	262.16	107.37	<262.14	262.14
		$3,570.50		$4,000.00

[a]Column 1 for year k less column 4 for year k equals the entry in column 1 for year $k+1$.

[b]200% (= 2/10) of column 1.

[c]Column 1 minus estimated SV_N divided by the remaining years from the beginning of the year through the 10th year.

[d]Select the larger amount in column 2 or column 3.

Figure: 07-T01

In the previous example;

When using Declining Balance (DB) method, for example: d_3 =

annual depreciation deduction in year 3

BV_3 = book value at the end of year 3 (at the beginning of year 4)

$$d_k = B(1-R)^{k-1}(R)$$

$$d_3 = 4000(1-0{,}2)^{3-1}(0{,}2) = 512$$

$$BV_k = B(1-R)^k$$

$$BV_3 = 4000(1-0{,}2)^3 = 2048$$

BV_3 is the book value at the end of year 3 (and also at the beginning of year 4) see year 4 at the table.

Half-Year Rule

If the half year rule is applied, **during year 1**, for specified assets, **one-half of the normally allowable depreciation can be claimed**.

Example: **Straight Line Method**

Depreciation rate = 50%

Half-year exempt (not applied).

Purchase price = $10,000 in 2001 depreciation in 2001= 10,000x0.5= $5000 depreciation in 2002= $5000

Sum of depreciation = $10,000

Now assume that for this class, half-year rule is applicable; Depreciation=50% depreciation in 2001 = ($10,000)(0.5)(**1/2**) = $**2,500**

depreciation in 2002 = ($10,000)(0.5) = $5,000 depreciation in 2003 = ($10,000-($2500+$5000))= $2,500 Sum of depreciation = $10,000

Sum-of-Years' Digits (SOYD) Method

- Principle

Depreciation concept similar to DB but with decreasing depreciation rate.

Charges a larger fraction of the cost as an expense of the early years than of the later years.

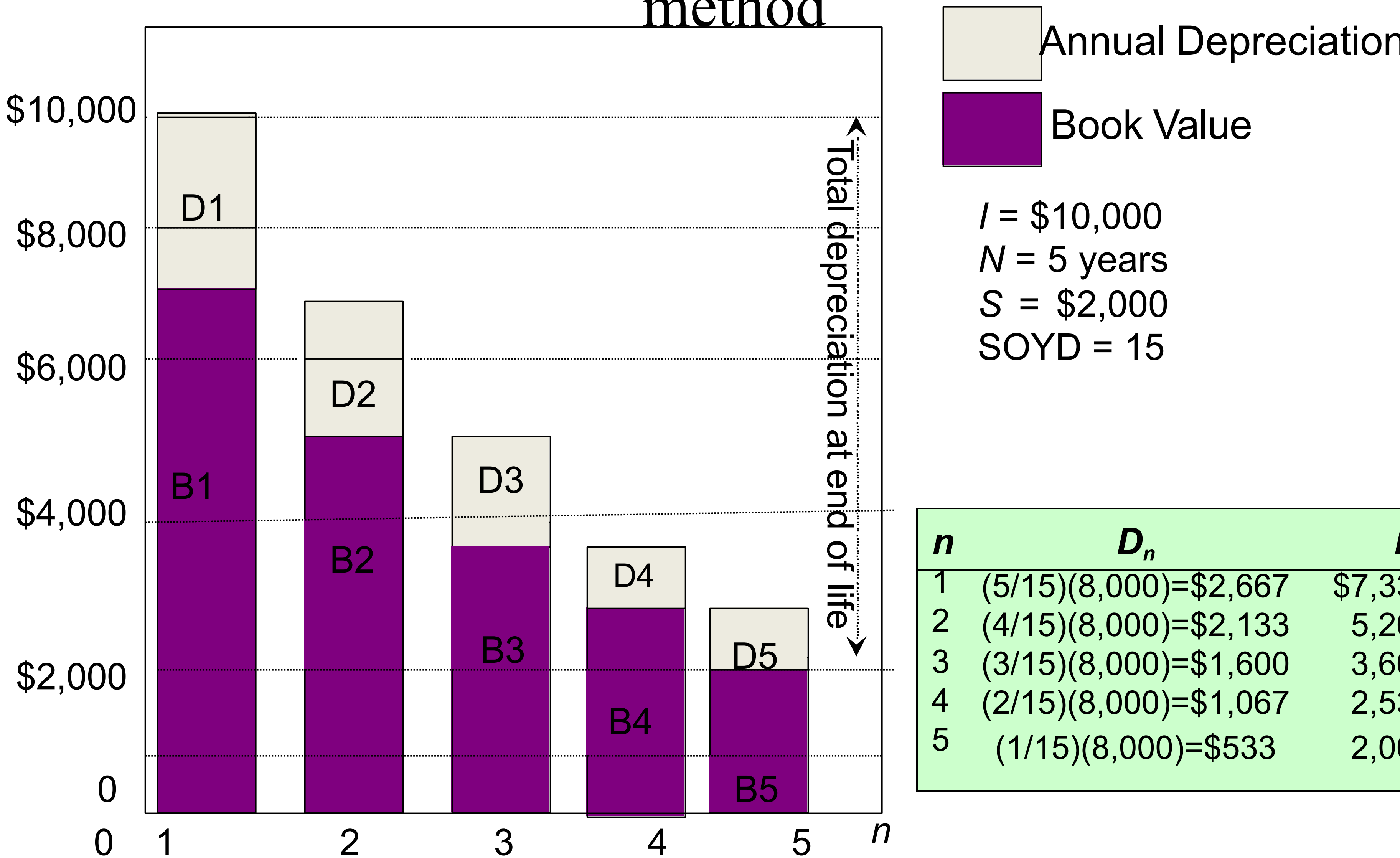
- Formula
  - •Annual Depreciation
  - •Book Value

$$D_n = (I - S)(N - n + 1) / SOYD$$

$$B_n = I - \sum_{j=1}^{n} D_j$$

where $SOYD = N(N+1)/2$

Example – Sum-of-years' digits method



I = $10,000
N = 5 years
S = $2,000
SOYD = 15

n	D_n	B_n
1	(5/15)(8,000)=$2,667	$7,333
2	(4/15)(8,000)=$2,133	5,200
3	(3/15)(8,000)=$1,600	3,600
4	(2/15)(8,000)=$1,067	2,533
5	(1/15)(8,000)=$533	2,000

The *units-of-production* method can be used when the decrease in value of the asset is mostly a function of use, instead of time. The cost basis is allocated equally over the number of units produced over the asset's life. The depreciation per unit of production is found from the formula below.

$$\frac{B - SV_N}{\text{(Estimated lifetime production units)}}$$

EXAMPLE 7-3 Depreciation Based on Activity

A piece of equipment used in a business has a basis of \$50,000 and is expected to have a \$10,000 SV when replaced after 30,000 hours of use. Find its depreciation rate per hour of use, and find its BV after 10,000 hours of operation.

Solution

$$\text{Depreciation per unit of production} = \frac{\$50{,}000 - \$10{,}000}{30{,}000 \text{ hours}} = \$1.33 \text{ per hour.}$$

$$\text{After 10,000 hours, BV} = \$50{,}000 - \frac{\$1.33}{\text{hour}}(10{,}000 \text{ hours}), \text{ or BV} = \$36{,}700.$$

ACCELERATED COST RECOVERY SYSTEM (ACRS)

- ACRS recognizes an asset as belonging to one of four (tangible) property classes
- The Internal Revenue Service (**IRS)** is the U.S. government agency responsible for tax collection and tax law enforcement.
- IRS prescribes the specific series of depreciation per property class.
- Rates are based on 150% Declining Balance depreciation, later switching to Straight-Line.

TABLE 7-2 MACRS Class Lives and Recovery Periods[a]

Asset Class	Description of Assets	Class Life	Recovery Period GDS[b]	Recovery Period ADS
00.11	Office furniture and equipment	10	7	10
00.12	Information systems, including computers	6	5	5
00.22	Automobiles, taxis	3	5	5
00.23	Buses	9	5	9
00.241	Light general purpose trucks	4	5	5
00.242	Heavy general purpose trucks	6	5	6
00.26	Tractor units for use over the road	4	3	4
01.1	Agriculture	10	7	10
10.0	Mining	10	7	10
13.2	Production of petroleum and natural gas	14	7	14
13.3	Petroleum refining	16	10	16
15.0	Construction	6	5	6
22.3	Manufacture of carpets	9	5	9
24.4	Manufacture of wood products and furniture	10	7	10
28.0	Manufacture of chemicals and allied products	9.5	5	9.5
30.1	Manufacture of rubber products	14	7	14
32.2	Manufacture of cement	20	15	20
34.0	Manufacture of fabricated metal products	12	7	12
36.0	Manufacture of electronic components, products, and systems	6	5	6
37.11	Manufacture of motor vehicles	12	7	12
37.2	Manufacture of aerospace products	10	7	10
48.12	Telephone central office equipment	18	10	18
49.13	Electric utility steam production plant	28	20	28
49.21	Gas utility distribution facilities	35	20	35
79.0	Recreation	10	7	10

MODIFIED ACCELERATED COST RECOVERY SYSTEM (MACRS)

- Used in USA.
- It allows faster depreciation.
- MACRS is the principal method for computing depreciation deductions in USA after 1986.
- SV_N is defined to be 0 ; useful life estimates are not used directly in calculating depreciation amounts.

EXAMPLE 7-4 MACRS Depreciation with GDS

A firm purchased and placed in service a new piece of semiconductor manufacturing equipment. The cost basis for the equipment is \$100,000. Determine

(a) the depreciation charge permissible in the fourth year,

(b) the BV at the end of the fourth year,

(c) the cumulative depreciation through the third year,

(d) the BV at the end of the fifth year if the equipment is disposed of at that time.

Solution

From Table 7-2, it may be seen that the semiconductor (electronic) manufacturing equipment has a class life of six years and a GDS recovery period of five years. The recovery rates that apply are given in Table 7-3.

(a) The depreciation deduction, or cost-recovery allowance, that is allowable in year four (d_4) is 0.1152 (\$100,000) = \$11,520.

(b) The BV at the end of year four (BV_4) is the cost basis less depreciation charges in years one through four:

$$BV_4 = \$100,000 - \$100,000(0.20 + 0.32 + 0.192 + 0.1152)$$
$$= \$17,280.$$

(c) Accumulated depreciation through year three, d_3^*, is the sum of depreciation amounts in years one through three:

$$d_3^* = d_1 + d_2 + d_3$$
$$= \$100,000(0.20 + 0.32 + 0.192)$$
$$= \$71,200.$$

(d) The depreciation deduction in year five can only be (0.5)(0.1152)(\$100,000) = \$5,760 when the equipment is disposed of prior to year six. Thus, the BV at the end of year five is BV_4 − \$5,760 = \$11,520.

MACRS Table

TABLE 7-3 GDS Recovery Rates (r_k) for the Six Personal Property Classes

	Depreciation Rate for Recovery Period					
Year	3-year[a]	5-year[a]	7-year[a]	10-year[a]	15-year[b]	20-year[b]
1	0.3333	0.2000	0.1429	0.1000	0.0500	0.0375
2	0.4445	0.3200	0.2449	0.1800	0.0950	0.0722
3	0.1481	0.1920	0.1749	0.1440	0.0855	0.0668
4	0.0741	0.1152	0.1249	0.1152	0.0770	0.0618
5		0.1152	0.0893	0.0922	0.0693	0.0571
6		0.0576	0.0892	0.0737	0.0623	0.0528
7			0.0893	0.0655	0.0590	0.0489
8			0.0446	0.0655	0.0590	0.0452
9				0.0656	0.0591	0.0447
10				0.0655	0.0590	0.0447
11				0.0328	0.0591	0.0446
12					0.0590	0.0446
13					0.0591	0.0446
14					0.0590	0.0446
15					0.0591	0.0446
16					0.0295	0.0446
17						0.0446
18						0.0446
19						0.0446
20						0.0446
21						0.0223

EXAMPLE 7-5 MACRS with a Trade-in

In May 2011, your company traded in a computer and peripheral equipment, used in its business, that had a BV at that time of $25,000. A new, faster computer system having a fair MV of $400,000 was acquired. Because the vendor accepted the older computer as a trade-in, a deal was agreed to whereby your company would pay $325,000 cash for the new computer system.

(a) What is the GDS property class of the new computer system?

(b) How much depreciation can be deducted each year based on this class life? (Refer to Figure 7-1.)

Solution

(a) The new computer is in Asset Class 00.12 and has a class life of six years. (See Table 7-2.) Hence, its GDS property class and recovery period are five years.

(b) The cost basis for this property is $350,000, which is the sum of the $325,000 cash price of the computer and the $25,000 BV remaining on the trade-in. (In this case, the trade-in was treated as a nontaxable transaction.)

EXAMPLE 7-5 MACRS with a Trade-in CONTINUED

MACRS (GDS) rates that apply to the $350,000 cost basis are found in Table 7-3. An allowance (half-year) is built into the year-one rate, so it does not matter that the computer was purchased in May 2011 instead of, say, November 2011. The depreciation deductions (d_k) for 2011 through 2016 are computed using Equation (7-10) and Table 7-3.

Property	Date Placed in Service	Cost Basis	Class Life	MACRS (GDS) Recovery Period
Computer System	May 2005	$350,000	6 years	5 years

Year	Depreciation Deductions
2011	0.20 × $350,000 = $70,000
2012	0.32 × 350,000 = 112,000
2013	0.192 × 350,000 = 67,200
2014	0.1152 × 350,000 = 40,320
2015	0.1152 × 350,000 = 40,320
2016	0.0576 × 350,000 = 20,160
	Total $350,000

EXAMPLE 7-7 Comparison of Depreciation Methods

The La Salle Bus Company has decided to purchase a new bus for \$85,000 with a trade-in of their old bus. The old bus has a BV of \$10,000 at the time of the trade-in. The new bus will be kept for 10 years before being sold. Its estimated SV at that time is expected to be \$5,000.

First, we must calculate the cost basis. The basis is the original purchase price of the bus plus the BV of the old bus that was traded in [Equation (7-11)]. Thus, the basis is \$85,000 + \$10,000, or \$95,000. We need to look at Table 7-2 and find buses, which are asset class 00.23. Hence, we find that buses have a nine-year class (useful) life, over which we depreciate the bus with historical methods discussed in Section 7.3, and a five-year GDS recovery period.

Solution: SL Method

For the SL method, we use the class life of 9 years, even though the bus will be kept for 10 years. By using Equations (7-2) and (7-4), we obtain the following information:

$$d_k = \frac{\$95{,}000 - \$5{,}000}{9 \text{ years}} = \$10{,}000, \quad \text{for } k = 1 \text{ to } 9.$$

SL Method		
EOY k	d_k	BV_k
0	—	\$95,000
1	\$10,000	85,000
2	10,000	75,000
3	10,000	65,000
4	10,000	55,000
5	10,000	45,000
6	10,000	35,000
7	10,000	25,000
8	10,000	15,000
9	10,000	5,000

EXAMPLE 7-7 Comparison of Depreciation Methods CONTINUED

Notice that no depreciation was taken after year nine because the class life was only nine years. Also, the final BV was the estimated SV, and the BV will remain at $5,000 until the bus is sold.

Solution: DB Method

To demonstrate this method, we will use the 200% DB equations. With Equations (7-6) and (7-8), we calculate the following:

$$R = 2/9 = 0.2222;$$

$$d_1 = \$95{,}000(0.2222) = \$21{,}111;$$

$$d_5 = \$95{,}000(1 - 0.2222)^4(0.2222) = \$7{,}726;$$

$$BV_5 = \$95{,}000(1 - 0.2222)^5 = \$27{,}040.$$

200% DB Method

EOY k	d_k	BV_k
0	—	$95,000
1	$21,111	73,889
2	16,420	57,469
3	12,771	44,698
4	9,932	34,765
5	7,726	27,040
6	6,009	21,031
7	4,674	16,357
8	3,635	12,722
9	2,827	9,895

EXAMPLE 7-7 **Comparison of Depreciation Methods** CONTINUED

Solution: DB with Switchover to SL Depreciation

To illustrate the mechanics of Table 7-1 for this example, we first specify that the bus will be depreciated by the 200% DB method ($R = 2/N$). Because DB methods never reach a zero BV, suppose that we further specify that a switchover to SL depreciation will be made to ensure a BV of $5,000 at the end of the vehicle's nine-year class life.

EOY k	Beginning-of-Year BV	200% DB Method	SL Method ($BV_9 = \$5{,}000$)	Depreciation Amount Selected
1	$95,000	$21,111	$10,000	$21,111
2	73,889	16,420	8,611	16,420
3	57,469	12,771	7,496	12,771
4	44,698	9,933	6,616	9,933
5	34,765	7,726	5,953	7,726
6	27,040	6,009	5,510	6,009
7	21,031	4,674	5,344	5,344[a]
8	15,687	3,635	5,344	5,344
9	10,344	2,827	5,344	5,344

[a] Switchover occurs in year seven.

Solution: MACRS (GDS) with Half-Year Convention

To demonstrate the GDS with the half-year convention, we will change the La Salle bus problem so that the bus is now sold in year five for Part (a) and in year six for Part (b).

(a) Selling bus in year five:

EOY k	Factor	d_k	BV_k
0	—	—	$95,000
1	0.2000	$19,000	76,000
2	0.3200	30,400	45,600
3	0.1920	18,240	27,360
4	0.1152	10,944	16,416
5	0.0576	5,472	10,944

EXAMPLE 7-7 Comparison of Depreciation Methods CONTINUED

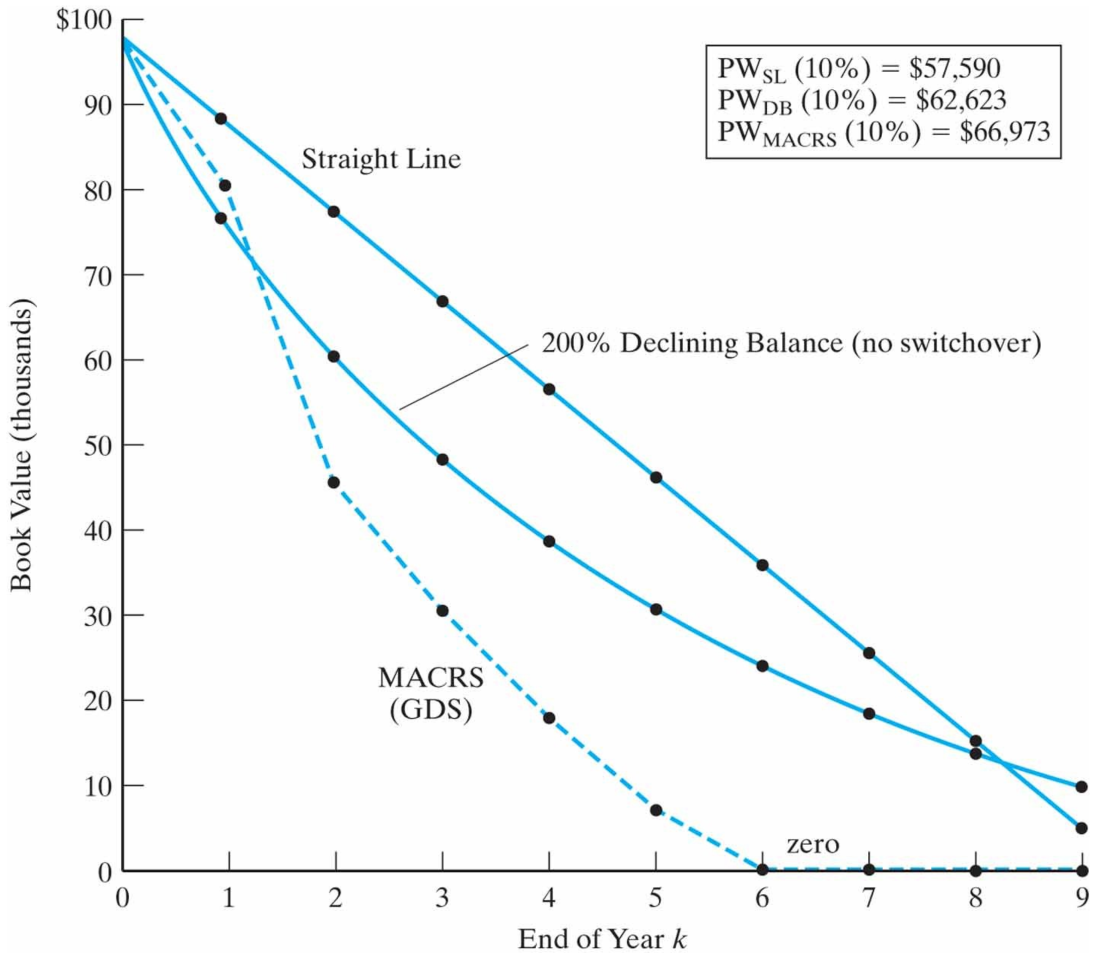
(b) Selling bus in year six:

EOY k	Factor	d_k	BV_k
0	—	—	$95,000
1	0.2000	$19,000	76,000
2	0.3200	30,400	45,600
3	0.1920	18,240	27,360
4	0.1152	10,944	16,416
5	0.1152	10,944	5,472
6	0.0576	5,472	0

Notice that, when we sold the bus in year five before the recovery period had ended, we took only half of the normal depreciation. The other years (years one through four) were not changed. When the bus was sold in year six, at the end of the recovery period, we did not divide the last year amount by two.

Selected methods of depreciation, illustrated in Example 7-7, are compared in Figure 7-2. In addition, the PW (10%) of each method is shown in Figure 7-2. Because large PWs of depreciation deductions are generally viewed as desirable, it is clear that the MACRS method is very attractive to profitable companies.

Comparison of Different Methods



Engineering Economy, Fourteenth Edition
By William G. Sullivan, Elin M. Wicks, and C. Patrick Koelling

There are many different types of taxes.

- Income taxes are assessed as a function of gross revenues minus allowable expenses.
- Property taxes are assessed as a function of the value of property owned.
- Sales taxes are assessed on the basis of purchase of goods or services. The tax amount is usually calculated by applying a percentage rate to the taxable price of a sale.
  - e.g., Value added taxes, in which tax is charged on all sales
- Excise taxes are taxes assessed as a function of the sale of certain goods or services often considered nonnecessities.
  - applied to a narrow range of products, such as gasoline or alcohol

We will focus on income taxes.

How to conduct after-tax analysis?

- MARR has to be adjusted.
- After-tax cash flows (ATCFs) should be computed.
- The same methods (i.e. Equivalent worth and rate of return methods) can be used by using the adjusted MARR and ATCFs.

How to adjust MARR for after-tax analysis?

Taking taxes into account changes our expectations of returns on projects, so our MARR (after-tax) is lower.

$$\text{Before-tax MARR} \approx \frac{\text{After-tax MARR}}{\text{1-effective income tax rate}}$$

Depreciation is not a cash flow, but it affects a corporation's taxable income, and therefore the taxes a corporation pays.

Taxable income = gross income

– all expenses except capital invest.

– depreciation deductions.

EXAMPLE 7-8 Determination of Taxable Income

A company generates $1,500,000 of gross income during its tax year and incurs operating expenses of $800,000. Property taxes on business assets amount to $48,000. The total depreciation deductions for the tax year equal $114,000. What is the taxable income of this firm?

Solution

Based on Equation (7-14), this company's taxable income for the tax year would be

$$\$1,500,000 - \$800,000 - \$48,000 - \$114,000 = \$538,000.$$

EFFECTIVE (MARGINAL) CORPORATE INCOME TAX RATE

- Personal/corporate income tax rates are based on income brackets.
- Depending on the bracket a firm's income falls within, the marginal tax rate can vary from 15% to a maximum of 40%

Effective Corporate Income Tax Rate

TABLE 7-5 Corporate Federal Income Tax Rates (2004)

If taxable income is:		The tax is:	
Over	but not over		of the amount over
0	$50,000	15%	0
$50,000	75,000	$7,500 + 25%	$50,000
75,000	100,000	13,750 + 34%	75,000
100,000	335,000	22,250 + 39%	100,000
335,000	10,000,000	113,900 + 34%	335,000
10,000,000	15,000,000	3,400,000 + 35%	10,000,000
15,000,000	18,333,333	5,150,000 + 38%	15,000,000
18,333,333	....	6,416,667 + 35%	18,333,333

Source: *Tax Information on Corporations*, IRS Publication 542, 2003.

Figure: 07-T05

From *Engineering Economy*, Thirteenth Edition, by William G. Sullivan, Elin M. Wicks, and James Luxhoj.
ISBN 0-13-186520-X.

EXAMPLE 7-9 Calculating Income Taxes

Suppose that a firm for a tax year has a gross income of $5,270,000, expenses (excluding capital) of $2,927,500, and depreciation deductions of $1,874,300. What would be its taxable income and federal income tax for the tax year, based on Equation (7-14) and Table 7-5?

Solution

Taxable income = Gross income − Expenses − Depreciation deductions

= $5,270,000 − $2,927,500 − $1,874,300

= $468,200

Income tax = 15% of first $50,000	$7,500
+ 25% of the next $25,000	6,250
+ 34% of the next $25,000	8,500
+ 39% of the next $235,000	91,650
+ 34% of the next $133,200	45,288
Total	$159,188

The total tax liability in this case is $159,188. As an added note, we could have used a flat rate of 34% in this example because the federal weighted average tax rate at taxable income = $335,000 is 34%. The remaining $133,200 of taxable income above this amount is in a 34% tax bracket (Table 7-5). So we have 0.34($468,200) = $159,188.

EXAMPLE 7-10 Project-Based Income Taxes

A small corporation is expecting an annual taxable income of $45,000 for its tax year. It is considering an additional capital investment of $100,000 in an engineering project, which is expected to create an added annual net cash flow (revenues minus expenses) of $35,000 and an added annual depreciation deduction of $20,000. What is the corporation's federal income tax liability

(a) without the added capital investment,

(b) with the added capital investment?

Solution

(a)

Income Taxes	*Rate*	*Amount*
On first $45,000	15%	$6,750
	Total	$6,750

(b)

Taxable Income		
Before added investment		$45,000
+ added net cash flow		+35,000
− depreciation deduction		−20,000
	Taxable Income	$60,000

Income Taxes on $60,000	*Rate*	*Amount*
On first $50,000	15%	$7,500
On next $10,000	25%	2,500
	Total	$10,000

The increased income tax liability from the investment is $10,000 − $6,750 = $3,250.

The disposal of a depreciable asset can result in a gain or loss based on the sale price (market value) and the current book value

$$[\text{gain (loss) on disposal}]_N = MV_N - BV_N$$

A gain is often referred to as *depreciation recapture*, and it is generally taxed as the same as ordinary income. A loss is a *capital loss*. An asset sold for more than it's cost basis results in a *capital gain*.

Capital Gain & Depreciation Recapture

- *Capital Gain*: If sale price (i.e., salvage or disposition) > original purchase price, then
- *Sale price - Purchase price = Capital gain*
- *If sale price = purchase price, no capital gain.*

- Depreciation Recapture: If sale price is higher than book value, Depreciation recapture applies.
- Depreciation *Recapture = (Sale price – book value)(tax rate)*
- *Note: if sale price > purchase price, both capital gain tax and* Depreciation *recapture apply*

Capital Gain & Depreciation Recapture: Example

- Purchase price = $50,000 five years ago.
- Sale price = $60,000
- Effective tax rate = 46% Depreciation
- rate = 20% Declining Balance method is
- used

- Capital Gain:
- Capital Gain = 60,000-50,000 = $10,000

Capital Gain & Depreciation Recapture: Example

- Assumed tax rate for capital gain = (1/2)(effective income tax rate)
- Capital gain tax = (1/2)(0.46)(10,000) = $2,300

- Depreciation Recapture:
- book value after 5 yrs of use = BV_5 = (50,000)(1- Depreciation rate of 0.2)^5 = $16,384
- Depreciation recapture = (purchase price – BV_5)(tax rate) = (50,000-16,384)(0.46) = $15,463.36

- Total tax on asset disposal = $2,300 + $15,463.36 = $17,763.36

EXAMPLE 7-11 Tax Consequences of Selling an Asset

A corporation sold a piece of equipment during the current tax year for \$78,600. The accounting records show that its cost basis, B, is \$190,000 and the accumulated depreciation is \$139,200. Assume that the effective income tax rate as a decimal is 0.40 (40%). Based on this information, what is

(a) the gain (loss) on disposal,

(b) the tax liability (or credit) resulting from this sale,

(c) the tax liability (or credit) if the accumulated depreciation was \$92,400 instead of \$139,200?

Solution

(a) The BV at the time of sale is \$190,000 − \$139,200 = \$50,800. Therefore, the gain on disposal is \$78,600 − \$50,800 = \$27,800.

(b) The tax owed on this gain is −0.40(\$27,800) = −\$11,120.

(c) With $d_k^* = \$92{,}400$, the BV at the time of sale is \$190,000 − \$92,400 = \$97,600. The loss is \$78,600 − \$97,600 = −\$19,000. The tax credit resulting from this loss on disposal is −0.40(−\$19,000) = \$7,600.

EXAMPLE 7-12 PW of MACRS Depreciation Amounts

Suppose that an asset with a cost basis of \$100,000 and an ADS recovery period of five years is being depreciated under the *Alternate Depreciation System (ADS) of MACRS*, as follows:

Year	1	2	3	4	5	6
Depreciation Deduction	\$10,000	\$20,000	\$20,000	\$20,000	\$20,000	\$10,000

If the firm's effective income tax rate remains constant at 40% during this six-year period, what is the PW of after-tax savings resulting from depreciation when MARR = 10% per year (after taxes)?

Solution

The PW of tax credits (savings) because of this depreciation schedule is

$$\text{PW}(10\%) = \sum_{k=1}^{6} 0.4d_k(1.10)^{-k} = \$4{,}000(0.9091) + \$8{,}000(0.8264) + \cdots + \$4{,}000(0.5645) = \$28{,}948.$$

After-tax economic analysis is generally the same as before-tax analysis, just using after-tax cash flows (ATCF) instead of before- tax cash flows (BTCF).

- The analysis is conducted using the after-tax MARR.

Cash flows are typically determined for each year using the notation below.

R_k = revenues (and savings) from the project during period k

E_k = cash outflows during k for deductible expenses

d_k = sum of all noncash, or book, costs during k, such as depreciation

t = effective income tax rate on ordinary income

T_k = income tax consequence during year k

$ATCF_k$ = ATCF from the project during year k

Some important cash flow formulas.

Taxable income

$$R_k - E_k - d_k$$

Ordinary income tax consequences

$$T_k = -t(R_k - E_k - d_k)$$

$$BTCF_k = R_k - E_k$$

$$ATCF_k = BTCF_k + T_k$$

$$ATCF_k = (1-t)(R_k - E_k) + td_k$$

End of Year, k	(A) Before-Tax Cash Flow (BTCF)	(B) Depreciation Deduction	$(C) = (A) - (B)$ Taxable Income (TI)	$(D) = -t(C)$ Cash Flow for Income Taxes	$(E) = (A) + (D)$ After-Tax Cash Flow (ATCF)
0	Capital investment				Capital investment
1, 2, . . . , N	Ordinary income is positive (or negative) in sign	Depreciation (positive in sign)	TI can be negative or positive in sign	$-t$ (TI) (opposite in sign from TI)	After-tax cash flows from operations
N	Market value (MV)		MV − BV†	$-t$(MV − BV)	MV − t(MV − BV)

Before-Tax IRR, PW, and so on (Computed from Col. *A*, using the before-tax MARR)

After-Tax IRR, PW, and so on (Computed from Col. *E*, using the after-tax MARR)

†BV = book value at end of year *N*

EXAMPLE 7-13 After-Tax PW of an Asset

The asset in Example 7-12 is expected to produce net cash inflows (net revenues) of $30,000 per year during the six-year period, and its terminal MV is negligible. If the effective income tax rate is 40%, how much can a firm afford to spend for this asset and still earn the MARR? What is the meaning of any excess in affordable amount over the $100,000 cost basis given in Example 7-12? [Equals the after tax PW.]

Solution

After income taxes, the PW of net revenues is $(1 - 0.4)(\$30{,}000)(P/A, 10\%, 6) = \$18{,}000(4.3553) = \$78{,}395$. After adding to this the PW of tax savings computed in Example 7-12, the affordable amount is $107,343. Because the capital investment is $100,000, the net PW equals $7,343. This same result can be obtained by using the general format (worksheet) of Figure 7-4:

EOY	(A) BTCF	(B) Depreciation Deduction	(C) = (A) − (B) Taxable Income	(D) = − 0.4(C) Income Taxes	(E) = (A) + (D) ATCF
0	−$100,000				−$100,000
1	30,000	$10,000	$20,000	−$8,000	22,000
2	30,000	20,000	10,000	−4,000	26,000
3	30,000	20,000	10,000	−4,000	26,000
4	30,000	20,000	10,000	−4,000	26,000
5	30,000	20,000	10,000	−4,000	26,000
6	30,000	10,000	20,000	−8,000	22,000
					PW(10%) of ATCF = $7,343

EXAMPLE 7-15 Computing After-Tax PW and IRR

Certain new machinery, when placed in service, is estimated to cost \$180,000. It is expected to *reduce* net annual operating expenses by \$36,000 per year for 10 years and to have a \$30,000 MV at the end of the 10th year.

(a) Develop the ATCFs and BTCFs.

(b) Calculate the before-tax and after-tax IRR. Assume that the firm is in the federal taxable income bracket of \$335,000 to \$10,000,000 and that the state income tax rate is 6%. State income taxes are deductible from federal taxable income. This machinery is in the MACRS (GDS) five-year property class.

(c) Calculate the after-tax PW when the *after-tax* MARR = 10% per year.

In this example, the study period is 10 years, but the property class of the machinery is 5 years. Solve by hand and by spreadsheet.

Solution by Hand

(a) Table 7-6 applies the format illustrated in Figure 7-4 to calculate the BTCF and ATCF for this example. In column *D*, the effective income tax rate is very close to 0.38 [from Equation (7-15)] based on the information just provided.

(b) The before-tax IRR is computed from column *A*:

$$0 = -\$180{,}000 + \$36{,}000(P/A, i'\%, 10) + \$30{,}000(P/F, i'\%, 10).$$

TABLE 7-6 ATCF Analysis of Example 7-15

TABLE 7-6 ATCF Analysis of Example 7-15

	(A)	(B)	Depreciation Deduction				(C) = (A)−(B)	(D) = −0.38(C)	(E) = (A) + (D)
End of Year, *k*	BTCF	Cost Basis	×	GDS Recovery Rate	=	Deduction	Taxable Income	Cash Flow for Income Taxes	ATCF
0	−$180,000	—		—		—			−$180,000
1	36,000	$180,000	×	0.2000	=	$36,000	0	0	36,000
2	36,000	180,000	×	0.3200	=	57,600	−21,600	+8,208	44,208
3	36,000	180,000	×	0.1920	=	34,560	1,440	−547	35,453
4	36,000	180,000	×	0.1152	=	20,736	15,264	−5,800	30,200
5	36,000	180,000	×	0.1152	=	20,736	15,264	−5,800	30,200
6	36,000	180,000	×	0.0576	=	10,368	25,632	−9,740	26,260
7–10	36,000	0				0	36,000	−13,680	22,320
10	30,000						30,000[a]	−11,400[b]	18,600
Total	$210,000							Total	$130,201
								PW (10%) =	$17,208

[a] Depreciation recapture = $MV_{10} - BV_{10}$ = \$30,000 − 0 = \$30,000 (gain on disposal).

[b] Tax on depreciation recapture = \$30,000(0.38) = \$11,400.

After-tax IRR: Set PW of column E = 0 and solve for i' in the following equation:

$$0 = -\$180{,}000 + \$36{,}000(P/F, i', 1) + \$44{,}208(P/F, i', 2) + \$35{,}453(P/F, i', 3) + \$30{,}200(P/F, i', 4) + \$30{,}200(P/F, i', 5) + \$26{,}260(P/F, i', 6) + \$22{,}320(P/A, i', 4)(P/F, i', 6) + \$18{,}600(P/F, i', 10); \quad \text{IRR} = 12.4\%.$$

EXAMPLE 7-15 Computing After-Tax PW and IRR CONTINUED

By trial and error, we find that $i' = 16.1\%$.

The entry in the last year is shown to be $30,000 because the machinery will have this estimated MV. The asset, however, was depreciated to zero with the GDS method. Therefore, when the machine is sold at the end of year 10, there will be $30,000 of *recaptured depreciation*, or gain on disposal [Equation (7-16)], which is taxed at the effective income tax rate of 38%. This tax entry is shown in column *D* (EOY 10).

By trial and error, the after-tax IRR for Example 7-15 is found to be 12.4%.

(c) When MARR = 10% per year is inserted into the PW equation at the bottom of Table 7-6, it can be determined that the after-tax PW of this investment is $17,208.

Spreadsheet Solution

Figure 7-5 displays the spreadsheet solution for Example 7-15. This spreadsheet uses the form given in Figure 7-4 to compute the ATCFs. The spreadsheet also illustrates the use of the VDB (variable declining balance) function to compute MACRS (GDS) depreciation amounts.

Cell B9 contains the cost basis, cells B10:B19 contain the BTCFs, and the year 10 MV is given in cell B20. The VDB function is used to determine the MACRS (GDS) depreciation amounts in column C. Cell B7 contains the DB percentage used for a five-year property class (see Table 7-4).

Note that in Figure 7-5, there are two row entries for the last year of the study period (year 10). The first entry accounts for expected revenues less expenses, while the second entry accounts for the disposal of the asset. To use the NPV and IRR financial functions, these values must be combined into a net cash flow for the year. This is accomplished by the adjusted ATCF and adjusted BTCF columns in the spreadsheet.

Figure 7-5 Spreadsheet Solution, Example 7-15

	A	B	C	D	E	F	G	H	I
1	After-tax MARR =	10%		Capital Investment =	$ 180,000				
2	Class Life =	5		Market Value =	$ 30,000				
3	State tax rate =	6%		Annual Savings =	$ 36,000				
4	Federal tax rate =	34%		Useful Life =	10				
5									
6	effective tax rate =	38.00%							
7	DB rate =	200%							
8	EOY	BTCF	Depreciation Deduction	Taxable Income	Cash Flow for Income Taxes	ATCF	Adjusted ATCF		Adjusted BTCF
9	0	$ (180,000)				$ (180,000)	$ (180,000)		$ (180,000)
10	1	$ 36,000	$ 36,000	$ -	$ -	$ 36,000	$ 36,000		$ 36,000
11	2	$ 36,000	$ 57,600	$ (21,600)	$ 8,208	$ 44,208	$ 44,208		$ 36,000
12	3	$ 36,000	$ 34,560	$ 1,440	$ (547)	$ 35,453	$ 35,453		$ 36,000
13	4	$ 36,000	$ 20,736	$ 15,264	$ (5,800)	$ 30,200	$ 30,200		$ 36,000
14	5	$ 36,000	$ 20,736	$ 15,264	$ (5,800)	$ 30,200	$ 30,200		$ 36,000
15	6	$ 36,000	$ 10,368	$ 25,632	$ (9,740)	$ 26,260	$ 26,260		$ 36,000
16	7	$ 36,000	$ -	$ 36,000	$ (13,680)	$ 22,320	$ 22,320		$ 36,000
17	8	$ 36,000	$ -	$ 36,000	$ (13,680)	$ 22,320	$ 22,320		$ 36,000
18	9	$ 36,000	$ -	$ 36,000	$ (13,680)	$ 22,320	$ 22,320		$ 36,000
19	10	$ 36,000	$ -	$ 36,000	$ (13,680)	$ 22,320	$ 40,920		$ 66,000
20	10	$ 30,000		$ 30,000	$ (11,400)	$ 18,600			
21									
22						PW =	$ 17,209		$ 52,771
23						AW =	$ 2,801		$ 8,588
24						IRR =	12.4%		16.1%

After-Tax Cash Flow Analysis

Example:

- For a 3 year project, the following estimates are provided:
- Purchase price=$700M. Resale (SV) =$450M. Income/yr=$500M. Expenses=$350M/yr. Depreciation=20% (half yr. rule applicable).
- Loan = $550M, to be repaid in 3 years @ 8% interest rate. Tax rate = 40% and MARR (after tax) = 9%.
- Find the Net Present Worth (after-tax)
- Drop M (million)

Calculations for the ATCF example:

- Depreciation for Yr.1 = 700(1/2)(0.2)= 70
- Depreciation for Yr. 2 = (700-70)(0.2)=126
- Depreciation for Yr.3 = (700-70-126)(0.2)=100.80
- Book Value (at the end of Yr.3) =

  700 - Sum of (70+126+100.80) = 403.20 < 450 salvage
- Since salvage > Book Value, Depreciation recapture applies.
- Depreciation recapture=[450-403.20](tax rate of 0.4)= 18.72 (a tax is a negative cash flow item).

Calculations for the ATCF example:

Loan: principal = 550; Interest = 8%

Equal annual payment A = 550 (A/P,8%,3) = 213.42

Yr.1: Interest= 550x0.08= 44.00;

Principal = 213.42-44.00 = 169.42

Yr.2: Balance of principal= 550-169.42=380.58;

Interest=380.58x0.08=30.45;

Principal=213.42-30.45 = 182.97

and so on..

Calculations for the ATCF example:

Taxable Income = income-expenses-depreciation-interest

Taxable Income for **Yr.1**= 500 – 350 – 70 – 44 = 36.00

Taxable Income for **Yr.2**= 500 – 350 – 126 – 30.45 = -6.45

And so on..

Calculations for the ATCF example:

Tax @ 40% = (Taxable Income in each year)(0.40)

For Yr. 1: 36x0.4 = 14.40

For Yr.2: -6.45x0.4 = -2.58

And so on..

After Tax Cash Flow (ATCF) = Income-Expenses-Loan repayment-Tax

After Tax Cash Flow for **Yr.1**: 500 – 350 – 213.42 – 14.40= -77.82

After Tax Cash Flow for **Yr.2**: 500 – 350 – 213.42 – (-2.58) = -60.84

And so on..

Finally
NPW

- By considering After Tax Cash Flow values:
- NPW= (-700+550) - 77.82(P/F,9%,1) - 60.84(P/F,9,2) + (-76.78+450.00-18.72)(P/F,9%,3)
- = **$1.14M** (Feasible)

Depreciation Rates for Turkey

No			Amortismana Tabi İktisadi Kıymetler	Faydalı Ömür (Yıl)	Normal Amortisman Oranı	İlgili Genel Tebliğ
1.			Binalar			333
	1.1.		Ticari, sınai, zirai ve mesleki işletmelerin idare binaları ile bunların sağlık ve sosyal hizmetlerinde kullandıkları binalar, ticarethane, muayenehane, yazıhane, banka, sigorta, pansiyon, otel, okul, hamam, banyo binaları, tiyatro ve benzeri eğlence yerleri, depo, ardiye, kapalı spor sahaları binaları, spor sahalarındaki tribünler ve benzeri hizmetlere tahsis olunan sair binalar, ikamete mahsus ev, apartman ve emsali binalar			333
	1.1.1.		Beton, kargir, demir, çelik	50	2,00%	333
	1.1.2.		Yarı kargir, yarı ahşap (Ahşap yapılar üzerine beton ve emsali püskürtmek suretiyle yapılmış binalar dahil)	33	3,03%	333
	1.1.3.		Ahşap, kerpiç	20	5,00%	333
	1.1.4.		Galip malzemesi saç, çinko, teneke olan mevsimlik faaliyette bulunan sinema, gazino ve emsali yerlerdeki cam veya ahşap tesisler ile benzeri binalar	15	6,66%	333
	1.1.5.		Galip malzemesi teneke muvakkat barakalar, inşaat şantiye binaları ve prefabrik yapılar	10	10,00%	333
	1.1.6.		Galip malzemesi cam olan binalar (Depo ve tesisler) ve benzerleri	15	6,66%	365
	1.1.7.		Sarnıç ve benzerleri	50	2,00%	333
	1.1.8.		Tanklar			333
		1.1.8.1.	İspirto ve benzeri tanklar	15	6,66%	333
		1.1.8.2.	Asit tankları, yakıt depolama tankları, reçine tankları	10	10,00%	333
		1.1.8.3.	Diğer tanklar	15	6,66%	333
	1.2.		Fabrika, atölye, istasyon, terminal, garaj, hangar, kantar dairesi, kazan dairesi, pompa dairesi, elektrik santral binaları ile emsali hizmetlere mahsus binalar			333
	1.2.1.		Beton, kargir, demir, çelik	40	2,50%	333
	1.2.2.		Yarı kargir	25	4,00%	333
	1.2.3.		Ahşap	15	6,66%	333

Source: www.gib.gov.tr (Gelir İdaresi Başkanlığı web sitesi)