ElCoM
ELECTRICAL COMPUTER MECHATRONICS

تقدم لجنة ElCoM الاكاديمية

دفتر لمادة:

تفاضل و تكامل (2)

من شرح:

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جزيل الشكر للطالبة:

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Integration by parts :-

$\int u \cdot dv = u \cdot v - \int v \cdot du$

$$\begin{array}{ccccc} \text{نشتق} & u & \searrow^{x} & dv & \text{نكامل} \\ \downarrow & du & \xleftarrow{-} & v & \downarrow \end{array} \qquad = u \cdot v - \int v \cdot du$$

Ex :-

الاسهل دائماً
اشتق كثير الحدود

1. $\int x e^x \, dx$

$$\begin{array}{ccc} x & \searrow^{x} & e^x \\ 1 & \leftarrow & e^x \end{array}$$

$= xe^x - \int e^x$

$= e^{\cancel{x}} - e^x + C$

2. $\int \ln x \, dx$

$$\begin{array}{ccc} \ln x & \searrow^{x} & 1 \\ \frac{1}{x} & \xleftarrow{-} & x \end{array}$$

$= x\ln(x) - \int \frac{1}{\cancel{x}} * \cancel{x} \qquad = x\ln(x) - x + C$

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3. $\int_0^{\frac{1}{2}} x \cos(\pi x) \, dx$

$$\begin{array}{ccc} x & \searrow & \cos(\pi x) \\ 1 & \leftarrow & \frac{\sin(\pi x)}{\pi} \end{array}$$

$\frac{x\sin(\pi x)}{\pi} - \int_0^{\frac{1}{2}} \frac{\sin(\pi x)}{\pi}$

$= \frac{x\sin(\pi x)}{\pi} + \frac{1}{\pi} \frac{\cos(\pi x)}{\pi} \Big|_0^{\frac{1}{2}}$

$\frac{\frac{1}{2}}{\pi} - \frac{1}{\cancel{\pi^2 * \cos \frac{\pi}{2}}} - (0 - \frac{1}{\pi^2} \cos(0)$

$\frac{1}{2\pi} + \frac{1}{\pi^2}$

4. $\int x \ln(x) \, dx$

$$\begin{array}{ccc} \ln x & \searrow & x \\ \frac{1}{x} & \leftarrow & \frac{x^2}{2} \end{array}$$

$= \frac{x^2 \ln x}{2} - \int \frac{x}{2}$

$= \frac{x^2 \ln(x)}{2} - \frac{x^2}{4} + C$

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طريقة ثانية :

$\int x^2 e^x \ dx$

$x^2 \quad e^x$

$2x \quad e^x$

$= e^x x^2 - \int 2xe^x$

$2x \quad e^x$

$2 \quad e^x$

$x^2e^x - 2xe^x - \int 2e^x$

$x^2e^x - 2xe^x - 2e^x + C$

x^2	$+$	e^x
$2x$	$-$	e^x
2	$+$	e^x
zero		e^x

(H.w)

1) $\int (2x^2+7x) \ \cos(x+1) \ dx$

~~[illegible]~~

2) $\int (x^3-7x^2) \ e^{2x-1} \ dx$

$\int x^5 \sin x \ dx$

x^5	$+$	$\sin x$
$5x^4$	$-$	$-\cos x$
$20x^3$	$+$	$-\sin x$
$60x^2$	$-$	$\cos x$
$120x$	$+$	$+\sin x$
120	$-$	$-\cos x$
0		$-\sin x$

لو في هاي الحالة لو ارجع ارجع السهم ←
لما اضرب بجرفي ما راح استفيد.
عشان هيك بحط صفر احا اذا كان
عنا اي اشي غير كثير حدود بقدر
احل نفس الاشي وباي لحظة برجع
السهم (ـــ) وبتعادل بطلع الحل

$\int e^x \ \sin x \ dx$

فيه الاقترانات الدورية .

دوري عكسيش منه بيرجع
اشتق ومنه اتكامل
نفس الاشي

$e^x \quad \sin x$

$e^x \quad -\cos x$

$-e^x\cos x + \int e^x \cos x \ dx$

$e^x \quad \cos x$

$e^x \quad \sin x$

$\int e^x \sin x = -e^x \cos x + e^x \sin(x) - \int e^x \sin x$

$\frac{2}{2} \int e^x \sin x = (\frac{1}{2})(-e^x \cos(x) + e^x \sin(x)) + C$

وممكن احلها ع طريقة كجدول خارج 2 تنتهي
بس اول مرتين اجزاء وبعد هيك سترجع نفس المسألة.
وبخلص

(H.W)

$\int e^{-2x} \sin(3x) \, dx$

$\int x^4 (\ln(x))^2 \, dx$

$y = \ln(x) \qquad dy = \frac{1}{x} \qquad dx = x\,dy$

$\int x^4 (y)^2 \qquad e^y = e^{\ln x}$

$\int (e^y)^5 \, y^2 \, dy$

$\int y^2 \, e^{5y} \, dy$

y^2	e^{5y}
$2y$	$\frac{e^{5y}}{5}$
2	$\frac{e^{5y}}{25}$
zero	$\frac{e^{5y}}{125}$

$$= \frac{y^2e^{5y}}{5} - \frac{2y\,e^{5y}}{25} + \frac{2e^{5y}}{125} + C$$

← ارجع مكان (y) قيمتها

$$\frac{\ln x^2 \; x^5}{5} - \frac{2x^5 \ln x}{25} + \frac{2x^5}{125} + C.$$

$y = \ln(x)$

$e^{5y} = e^{\boxed{5}\ln(x)}$

$\cancel{e^{(\ln x)}}^5$

$\int \frac{x^2e^x}{(x+2)^2} \, dx$

في حالة كان في مقام بالدرجة

مستحيل احطها بالاشتقاق لانها راح تكبرها

انه الدرجة بالمقام

x^2e^x	$\frac{1}{(x+2)^2}$
$x^2e^x + 2xe^x$	$\frac{-1}{(x+2)}$

$$\frac{-x^2e^x}{(x+2)} + \int xe^x \frac{\cancel{(x+2)}}{\cancel{x+2}} = \frac{-x^2e^x}{(x+2)} + xe^x - e^x + C$$

Trigonometric Integrals :-

$\boxed{1}$ $\int \sin^{n}(x)\, dx$, $\int \cos^{n} x\ dx$

* odd power :-

$\int \sin^{\textcircled{n}} x\ dx$

$\int \sin^{n-1}(x)\ \sin(x)\ dx$

$\int (\sin^{2} x)^{\frac{n-1}{2}}\ \sin(x)\ dx$

$\int (1-\cos^{2}(x))^{\frac{n-1}{2}}\ \sin(x)\ dx$

نبدل $y = \cos(x)$

Ex:- $\int \cos^{5}(x)\ dx$

$\int \cos^{4}(x)\ \cos(x)\ dx$

$\int (\cos^{2} x)^{2}\ \cos x\ dx$

$\int (1-\sin^{2} x)^{2}\ \cos x\ dx$

$y = \sin(x)$

$dx = \dfrac{dy}{\cos(x)}$

$\int (1-y^{2})^{2} \cancel{\cos x}\ \dfrac{dy}{\cancel{\cos x}}$

$\int 1 - 2y^{2} + y^{4}\ dy$

$= y - \frac{2}{3} y^{3} + \frac{y^{5}}{5} = \sin(x) - \frac{2}{3}(\sin x)^{3} + \frac{(\sin x)^{5}}{5}$

Note:-

$(a+b)^{3} = a^{3} + 3ab^{2} + 3a^{2}b + b^{3}$

H.W:- $\int \sin^{7}(x)\ dx$

w $\boxed{2}$

$\boxed{2}$ $\int \sin^{7}(x)\ dx$

$\int \sin^6(x)\ \sin x\ dx$

${}^2x)^3$

$\int (1-\cos^2(x))^3\ \sin x\ dx$

$y = \cos(x)$

$dy = -\sin x\ dx$

$dx = \frac{-dy}{\sin x}$

$\int (1-y^2)^3\ \cancel{\sin x}\ \frac{dy}{\cancel{-\sin x}}$

$\int y^6 + 3y^4 - 3y^2 + 1\quad dy$

$= \frac{y^7}{7} + \frac{3}{5}y^5 - \frac{3}{3}y^3 + y + C$

$\int \cos^6(x)\ dx$

$\int (\cos^2(x))^3 = \int \frac{1}{2}(1+\cos 2x)^3\ dx$

$\frac{1}{8}\int (1+\cos 2x)^3$

$= \frac{1}{8}\int 1 + 3\cos^2(2x) + 3\cos(2x) + \cos(2x)^3\ dx$

$= \frac{1}{8}\left(x + \frac{3}{2}\sin 2x + 3\int \left(\frac{1}{2} + \cos 2x\right) + \int (\cos 2x)^2 \cos 2x\, dx\right.$

$= \frac{1}{8}\left(x + \frac{3}{2}\sin 2x\right) + 3\left(\frac{1}{2}x + \frac{\sin 2x}{2}\right) + \int (1-\sin^2 x)^2 \cos 2x\ dx$

ده يتحل زي هنعمله اللي فوق

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* even power :-

$\cos^2(x) = \frac{1}{2}(1+\cos(2x))$

$\sin^2(x) = \frac{1}{2}(1-\cos(2x))$

Ex :- $\int \sin^4(x) \; dx$

$$\int (\sin^2 x)^2 = \int (\frac{1}{2} (1 - \cos 2x)^2$$

$$= \frac{1}{4} \int 1 - 2\cos(2x) + \cos(2x)^2$$

$$= \frac{1}{4} (x - \frac{2\sin(2x)}{2} + \int \cos(2x)^2$$

$$\frac{1}{2} \int (1 + \cos 4x)$$

$$\frac{1}{2} \int x + \frac{\sin 4x}{4}$$

$$= \frac{1}{4} x - \frac{1}{4} \sin 2x + \frac{1}{8} x + \frac{1}{32} \sin(4x) + C$$

$$= \frac{3}{8} x - \frac{1}{4} \sin 2x + \frac{1}{32} \sin(4x) + C$$

H.W :- $\int \cos^6(x) \; dx$

قانون لتسهيل الحل :-

$$\int \sin^n(x) \; dx = \frac{-1}{n} \sin^{n-1}(x) \cos x + \frac{n-1}{n} \int \sin^{n-2}(x) \; dx$$

$$\int \cos^n x \; dx = \frac{1}{n} \cos^{n-1} x \sin x + \frac{n-1}{n} \int \cos^{n-2}(x) \; dx$$

Ex :- $\int \sin^4(x) \; dx =$

$$\frac{-1}{4} \sin^3(x) \cos x + \frac{3}{4} \int \sin^2 x \; dx$$

وهكذا

$\boxed{2}$ $\int \sin^{n}(x) \cos^{m}(x) \, dx$

a) n or m odd

Ex:- $\int \sin^{3}(x) \cos^{106}(x) \, dx$

$\int \sin^{(x)} \sin^{2}(x) \cos^{106}(x) \, dx$

$\int \sin(x) (1-\cos^{2}x) \cos^{106}(x) \, dx$ $\quad y = \cos(x)$

$\int \cancel{\sin(x)} (1-y^{2}) y^{106} \frac{dy}{-\cancel{\sin(x)}}$ $\quad dx = \frac{dy}{-\sin x}$

$-\int y^{106} - y^{108}$

$= \frac{y^{109}}{109} - \frac{y^{107}}{107} + C$

$\frac{(\cos x)^{109}}{109} - \frac{\cos(x)^{107}}{107} + C$

$\int \cos^{5}(x) \sin^{11}(x) \, dx$

$\int \cos (\cos^{2})^{2} \sin^{11}(x) \, dx$

$\int \cos (1-\sin^{2}x)^{2} \sin^{11}(x) \, dx$

$y = \sin$

$d\cancel{x} = \frac{dy}{\cos(x)}$

$\int \cancel{\cos} (1-y^{2}) y^{11} \frac{dy}{\cancel{\cos x}}$

$\int (1-2y^{2}+y^{4}) y^{11} \, dy$

$\int y^{11} - 2y^{13} + y^{15} \, dy$

$= \frac{y^{12}}{12} - \frac{2y^{14}}{14} + \frac{y^{16}}{16} + C$

وبرجع تاتا (y) قيمتها .

(H.W) :-

① $\int \sin^5(x) \quad \cos^{17}(x) \quad dx$

② $\int \sin^7(x) \quad \cos^{20}(x) \quad dx$

هنا اذا كانت القوتين زوجيات
لازم اعملهم بدلالة متغير واحد اما
(sin) / (cos) ونفضل الشي بالقوة الاصغر.

$\int \sin^4(x) \quad \cos^6(x) \quad dx$

(طريقة واحد)

$\int (\sin^2 x)^2 \quad \cos^6(x) \quad dx$

$\int (1-\cos^2 x)^2 \cos^6 x \quad dx$

$\int (1-2\cos^2 x+\cos^4 x) \cos^6(x) \quad dx$

$\int (\cos^6(x) - 2\cos^8(x) + \cos^{10}(x))$

نكمل الحل

(طريقة ثانيه)

$\int \sin^4(x) \quad \cos^6(x) \quad dx$

$= \int (\sin^2 x)^2 \quad (\cos^2(x))^3 \quad dx$

$= (\frac{1}{2}(1-\cos 2x)^2 \quad (\frac{1}{2}(1+\cos 2x))^3$

نكمل الحل

(طريقة ثالثه)

$\int \sin^4(x) \quad \cos^6(x) \quad dx$

$\int \underline{\sin^4(x) \quad \cos^4(x)} \cos^2(x) \quad dx$

$\int (\sin x \quad \cos x)^4 \quad \cos^2(x)$

$\int (\frac{\sin 2x}{2})^4 \quad (\frac{1}{2}(1+\cos 2x))$

H.W :-

1) $\int \sin^2 x \quad \cos^2(x) \quad dx$

2) $\int \sin^2 x \quad \cos^4(x) \quad dx$

$\frac{1}{32} \int \sin^4 2x \quad dx \quad + \int \sin^4 2x \quad \cos(2x) \quad dx$ بالتعويض في اس (1)

$\int \sin \alpha x \ \cos \beta(x) \ dx$

$\sin(a) \cos(b) = \frac{1}{2}(\sin(a-b) + \sin(a+b))$

$\sin(a) \sin(b) = \frac{1}{2}(\cos(a-b) - \cos(a+b))$

$\cos(a) \cos(b) = \frac{1}{2}(\cos(a-b) + \cos(a+b))$

H.W :-

1. $\int \cos(10x+3) \ \sin(2x-5)$
2. $\int \sin 2x \ \sin(7x) \ dx$
3. $\int \cos(2x-1) \ \cos(3x+5) \ dx$

مهم

Ex : ① $\int \tan(x) \ dx = \ln|\sec(x)| + C \qquad = -\ln|\cos x|$

② $\int \sec(x) \ dx = \int \sec x \left[\frac{\sec(x) \ \tan(x)}{\sec(x) \ \tan(x)} \right.$

$= \ln|\sec(x) + \tan(x)| + C$

③ $\int \sec^2 x \ dx = \tan(x) + C$

④ $\int \tan^2(x) \ dx = \int \sec^2(x) - 1 \ dx$

$= \tan(x) - x + C$

$\int \tan^3(x) = \int \tan(x) \ \tan^2(x)$

$\int (\sec^2 - 1) \ \tan(x)$

$\int (\tan x \ \sec^2(x)) - \int \tan x$

تعويض

$\int \sec^3(x)\ dx$ أجزاء //

$$\begin{tikzcd}
\sec(x) \arrow[dr] & \sec^2(x) \\
\sec(x)\tan(x) & \tan(x) \arrow[l]
\end{tikzcd}$$

$= \sec(x)\tan(x) - \int \sec(x) \underbrace{\tan^2(x)}_{(\sec^2 x - 1)}$

$\int \sec^3 = \sec(x)\tan(x) - \int \sec^3 x + \int \sec x\ dx$

$\int \sec^3(x) = \frac{1}{2} (\sec x \tan(x) + \ln |\sec(x) + \tan(x)$

$\int \tan^n(x)\ \sec^m(x)$

- m even
- n odd
- m odd and n even مستثناة

Ex:-

$\int \tan^6(x)\ \sec^4(x)\ dx$ $\qquad \sec^2(x) = \tan^2(x) + 1$

$\int \tan^6(x)\ \sec^2(x)\ \sec^2(x)\ dx$

$\int y^6\ (y^2+1)\ \cancel{\sec^2}\ \frac{dy}{\cancel{\sec^2 x}}$ $\qquad y = \tan x$

$dy = \frac{dx}{\sec^2 x}$

$\int y^8 + y^6\ dx$

$= \frac{y^9}{9} + \frac{y^7}{7} + C$

$\boxed{2}$ $\int \tan^{10}(x) \quad \sec^{6}(x) \, dx$ $\qquad y = \tan(x)$

$\int \tan^{13}(x) \quad \sec^{6}(x) \quad dx$

في حال اذا كالوا كوشن بنرجع بس
بشتغل على القوة الأقل .

$\int \tan^5(x) \quad \sec^7(x) \quad dx$

$\int \tan^4(x) \quad \sec^6(x) \quad (\tan x \sec x) \quad dx$

$\int (\tan^2)^2 \quad \sec^6(x) \quad (\tan x \sec x) \quad dx$ $\qquad y = \cancel{\tan}\ \sec(x)$

$dx = \dfrac{dy}{\tan x \sec x}$

$\int ((\sec^2 - 1)^2)^2 \quad y^6 (\cancel{\tan x \sec x}) \dfrac{dy}{(\cancel{\sec x \tan x})}$

$\int (y^2 - 1)^2 \quad y^6 \quad dy$

$\int (y^4 - 2y^2 + 1) \, y^6$

$\int y^{10} - 2y^8 + y^6 \quad dy \qquad = \dfrac{y^{11}}{11} - \dfrac{2y^9}{9} + \dfrac{y^7}{7} + C$

H.W

$\boxed{1}$ $\int \tan^7(x) \quad \sec^{20}(x) \quad dx$

$\boxed{2}$ $\int \tan^3(x) \quad \sec^7(x) \quad dx$

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$\int \tan^4(x) \quad \sec^7(x) \quad dx$ (الحالة الثالثة)

في هاي الحالة بنجيبهم كلهم بدلالة متغير واحد

$\int (\tan^2(x))^2 \quad \sec^7$

$\int (\sec^2 x - 1) \sec^7(x)\, dx$

$\int (\sec^4 - 2\sec^2(x) + 1) \sec^7(x)\, dx$

$\int (\sec^{11}(x) - 2\sec^9(x) + \sec^7(x) \quad dx$

وبنحل ↓ ع القانون

* قوانين مهمة :-

$$\int \tan^n(x)\, dx = \frac{\tan^{n-1}(x)}{n-1} - \int \tan^{n-2}(x)\, dx$$

$$\int \sec^n(x)\, dx = \frac{\sec^{n-2}(x)\tan(x)}{n-1} + \frac{n-2}{n-1}\int \sec^{n-2}(x)\, d$$

H.W

$\int \sec^4(3x) \quad \tan^{10}(3x)$

$\int \frac{\tan^3 x}{\sqrt{\sec^5(x)}}\, dx$

$\int \frac{\sec^4(x)}{\sqrt[3]{\tan^3 x}}\, dx$

تعويض (12. 15. 19. 34. 45, 46 . 47. 48) ومن اسئلة الكتاب :-
(P476 - P 477)

ومن (P 468 - P 469) اي سؤال منهم بين الارقام (3 - 42)

اجزاء (12 . 20. 30 , 29. 34)

Trigonometric subistitutions :-

لما تكون المسألة بالغالب

$\sqrt{a^2 - x^2}$ او $\sqrt{x^2 - a^2}$ او $\sqrt{x^2 + a^2} \rightarrow$

فبهاي الحالة بنفرض $= a \sin\theta$

$\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}]$

هون بنعوض

$x = a \sec\theta$

$\theta \in [0, \frac{\pi}{2})$ or $[\pi, \frac{3\pi}{2}]$

يعني الاول والثالث .

هون بنفرض

$x = a \tan\theta$

$\theta \in (-\frac{\pi}{2}, \frac{\pi}{2})$

وبالغالب بكونوا في جذور

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in general

$\sqrt{a^2 - b^2x^2} \Rightarrow x = \frac{a}{b} \sin\theta$

Ex :-

$$\int \frac{dx}{x^2\sqrt{4-x^2}}$$

$x = 2\sin\theta$

$dx = 2\cos\theta \, d\theta$

$$\int \frac{2\cos\theta \quad d\theta}{4\sin^2\theta \ \sqrt{4 - 4\sin^2\theta}}$$

$\sqrt{4(1-\sin^2\theta} = 2\sqrt{\cos^2_{(x)}} = 2\cos\theta$

$$\int \frac{\cancel{2\cos\theta} \ d\theta}{4\sin^2\theta \ \cancel{2\cos\theta}}$$

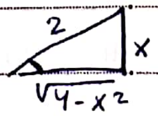
$$= \frac{1}{4}\int \frac{1}{\sin^2\theta} = \frac{1}{4}\int \csc^2\theta \ d\theta = -\frac{1}{4} \cot\theta + C$$

هون برجع للتعريف

$x = 2\sin\theta$

يعني

$\sin\theta = \frac{x}{2}$



$$\therefore = -\frac{1}{4} \ \frac{\sqrt{4-x^2}}{x} + C$$

$\int \frac{x^2}{\sqrt{5+x^2}} \, dx$

$x = \sqrt{5} \tan\theta$

$dx = \sqrt{5} \sec^2(x) \, d\theta$

$= \int \frac{5 \tan^2\theta \quad \sqrt{5} \quad \sec^2(x)}{\cancel{\sqrt{5} \sec\theta}}$

$\sqrt{5+x^2} = \sqrt{5} \sec\theta$

$\sqrt{5+5\tan^2(x)} = \sqrt{5} \sqrt{1+\tan^2(x)}$

$= \sqrt{5} \sqrt{\sec^2\theta}$

$= \sqrt{5} \sec\theta$

$\int 5 \tan^2\theta \quad \sec(\theta) \cdot d\theta$

$5 \int (\sec^2 - 1) \quad \sec\theta$

$5 \int \sec^3\theta - \sec\theta \quad d\theta$

$\int \sqrt{4-x^2} \, dx$

$x = 2\sin\theta$

$dx = 2\cos\theta$

$= \sqrt{4 - 4\sin^2\varphi}$

$= 2\sqrt{1-\sin^2\theta}$

$= 2\sqrt{\cos^2\varphi}$

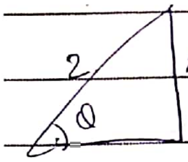
$= 2\cos\varphi$

$= \int 2\cos\varphi \; 2\cos\varphi \, d\theta$

$= 4 \int \cos^2\varphi \, d\theta$

$= 4 \times \frac{1}{2} \int 1 + \cancel{\cos 2\theta} \cos 2\varphi$

$= 2\left(\theta + \frac{\sin 2\theta}{2}\right) + C$



$= 2\theta + \frac{\cancel{2}}{\cancel{2}} \times 2 \sin\theta \cos\varphi + C$

Integrals by partial fraction :-

اذا كانت درجة لبسط
اكبر او يساوي درجة لمقام

find the partial fraction decomposition of :-

① بحلل المقام اول اشي

$$\boxed{1}\quad \frac{7X}{(X-3)(3X+7)(2X-5)}$$

بحط لانه المعام حقو

$$= \frac{A}{X+3} + \frac{B}{(3X+7)} + \frac{C}{(2X-5)}$$

$$\boxed{2}\quad \frac{2}{(X-1)^3(X+7)} = \frac{A}{(X-1)} + \frac{B}{(X-1)^2} + \frac{C}{(X+1)^3} + \frac{D}{(X+7)}$$

عشان يعني اللي جوا يكون (linear)

$$\boxed{3}\quad \frac{2X}{(X^2+7)(X-3)} = \frac{AX+B}{(X^2+7)} + \frac{C}{(X-3)}$$

تربيع كامل

$$\boxed{4}\quad \frac{2}{(X^2+X+1)^3 X^2} = \frac{cX+d}{(X^2+X+1)} + \frac{eX+f}{(X^2+X+1)^2} + \frac{gX+h}{(X^2+X+1)^3} + \frac{A}{X} + \frac{B}{X^2}$$

مو تربيع هو عبارة عن
خطي مرتين x*x

Ex :- Evalut the following integral

$$\int \frac{1}{x^2+3X-4)} = \frac{1}{(X-1)(X+4)}$$

$$\frac{1}{x^2+3X-4} = \frac{A}{(X-1)} + \frac{B}{(X+4)} = (A(X+4) + B(X-1))$$

توحيد مقامات

متساويات

$$A(X+4) + B(X-1) = 1$$

بعوض ارقام عشان تصفر مرة (A) و مرة B

$$\frac{1}{5} = A \leftarrow \boxed{X=1}$$ عندما

$$\frac{-1}{5} = B \leftarrow \boxed{X=-4}$$ عندما

هلأ برجع للتكامل

$$\int \frac{\frac{-1}{5}}{(X+4)} + \frac{\frac{1}{5}}{(X-1)} \, dX$$

$$= \frac{-1}{5} \ln|X+4| + \frac{1}{5} \ln|X-1| + C$$

$$= \frac{1}{5} (-\ln |X-4| + \ln |X-1|$$

$$= \ln \sqrt[5]{\frac{|X-1|}{|X+4|}} + C$$

متساويات بس الشكل يختلف

$$* \int \frac{dX}{X(X^2-1)} = \frac{A}{X} + \frac{B}{(X-1)} + \frac{C}{(X+1)}$$

$$= \frac{A(X-1)(X+1) + B(X)(X+1) + C(X)(X-1)}{X(X-1)(X+1)} = \frac{1}{X(X-1)(X+1)}$$

$-A = 1$ $\boxed{A = -1}$ عندما نعوض (zero)

$-2C = -1$ $\boxed{C = +\frac{1}{2}}$ عندما نعوض (-1)

$2B = 1$ $\boxed{B = \frac{1}{2}}$ عندما نعوض (1)

$$\int \frac{-1}{X} + \frac{\frac{1}{2}}{(X-1)} + \frac{+\frac{1}{2}}{(X+1)} \, dX$$

$$= -\ln|X| + \frac{1}{2} \ln|X-1| + \frac{1}{2} \ln|X+1| + C$$

$$\int \frac{dx}{x^3+x} \qquad = x(x^2+1)$$

$$\frac{A}{x} + \frac{bx+c}{(x^2+1)}$$

$$A(x^2+1) + (bx+c)x = 1$$

$\boxed{A=1}$ عندما نعوض $\boxed{x=0}$

هلأ بيك اعوض ارقام عشان تساعدني

مثلاً $\boxed{A=1}$

$2+b+c=1 \Rightarrow B+C=-1$ ← اول معادلة

لما اعوض $\boxed{x=-1}$

$2+b-c=1 \Rightarrow B-C=-1$ ← ثاني معادلة

بالحذف $2B=-2$ $\qquad B=-1$

$C = \text{zero}$

$$\int \frac{1}{x} + \frac{-x}{x^2+1} \, dx$$

$$\ln|x| - \frac{1}{2}\ln|x^2+1| + C$$

لو (C) موجود

مثلا $C=5$ بوزع المقام مرة للـ (B)

ومرة للـ (C)

$$\frac{5}{x^2+1}$$

$$5\tan^{-}(x)$$

(H.W)

$$\int \frac{x^3-2x^2-2}{x^2+1}\,dx \qquad \int \frac{1}{16x^3-4x^2+4x-1}\,dx$$

$$\int \frac{\sin x}{\cos^2(x)+\cos(x)}\,dx \qquad \int \frac{5}{\sqrt{x}-1}\,dx$$

$$\int \frac{1-x+2x^2-x^3}{x(x^2+1)^2} = \frac{A}{x} + \frac{bx+c}{(x^2+1)} + \frac{dx+e}{(x^2+1)^2}$$

مكررة حسبه

يعني هون لو ما احط عادي لأنها نفس المقام.

$$\frac{A(x^2+1)(x^2+1)^2 + bx+c\,(x)(x^2+1)^2 + dx+e\,(x)(x^2+1)}{(x)(x^2+1)(x^2+1)^2}$$

وبقدر هاد ما اكتبه بكل المرات

يعني هاي الطريقة الأسهل توحيد المقامات

$$A(x^2+1)^2 + (Bx+c)(x^2+1)^{\cancel{2}}x + (Dx+e)x = 1-x+2x^2-x^3$$

2 +2 ± 1

عندما $\boxed{x=0}$ ⇐ $\boxed{A=1}$

عندما $\boxed{x=1}$ ⇐

$4+2(b+c)+(d+e)=1$ ~ ①

$2b+2c+d+e=-3$ ~ ①

عندما $\boxed{x=-1}$

$4 \mp 2(-b+c) \mp (-d+e) = 5$

$+2b+^{-}2c+d-e=1$ ~ ②

ويكمل

- The partial fraction de composition of → هاد السؤال يعني ما بدو تكمل الحل للآخر بس تعمل اول خطوة.

$$\int \frac{e^x}{(e^x-1)^2(e^x+3)} dx \qquad y = e^x \qquad dx = \frac{dy}{e^x}$$

$$= \int \frac{1}{(y-1)^2(y+3)} dy = \frac{1}{(y+1)(y-1)(y+3)}$$

$$= \frac{A}{(y+1)} + \frac{B}{(y-1)^2} + \frac{C}{(y+3)}$$

$$\frac{A(y-1)(y+3) + B(y+3) + C(y-1)^2}{(y+3)(y-1)^2}$$

$\boxed{B = \frac{1}{4}} \Longleftarrow \boxed{y=1}$ عندما

$\boxed{C = \frac{1}{16}} \Longleftarrow \boxed{y=-3}$

$\boxed{A = \frac{-1}{16}} \Longleftarrow \boxed{y=0}$ عندما

او اي واحد غيره

$$-3A + 3\left(\frac{1}{4}\right) + \frac{1}{16} = 1$$

$$= \int \frac{\frac{-1}{16}}{(y-1)} + \frac{\frac{1}{4}}{(y-1)^2} + \frac{\frac{1}{16}}{(y+3)}$$

$$= \frac{-1}{16} \ln|y-1| + \frac{-1/4}{(y-1)} + \frac{1}{16} \ln|y+3| + C$$

* strategy of Integration

بوزع

Ex :- $\int \sqrt{x}\,(x+1)\,dx$

$\boxed{2}$ $\int \frac{\tan\theta}{\sec^2\theta}\,d\theta$ $y = \sec\theta$

$\boxed{3}$ $\int \frac{\tan^3\theta}{\cos^3\theta}\,d\theta$

هون بطلع ال cos لفوق

$\boxed{4}$ $\int \frac{1}{(1+e^x)}\,dx$ $y = e^x$

$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}$

هون راح ادور على قوة على اساس اخلص من التربيعي والتكعيبي ف راح افرض $(x = y^6)$

يعني مثلاً لو كان $\frac{\sqrt[6]{x}}{\sqrt[8]{x}+1}$ ← يعني هون بفرض $y^{24} = x$

لأنه العامل المشترك الاصغر داكل اسهل من $x = y^{48}$

$\int \frac{\sqrt{y^6}}{1+\sqrt[3]{y^6}}\, 6y^5\,dy$.

$6\int \frac{y^8}{1+y^2}\,dy$

$$\begin{array}{r|l} & y^6 - y^4 + y^2 - 1 \\ \hline 1+y^2 & y^8 \\ & y^8 + y^6 \\ \hline & -y^6 \\ & -y^6 - y^4 \\ \hline & y^4 \\ & y^4 + y^2 \\ \hline & -y^2 \\ & -y^2 - 1 \\ \hline & 1 \end{array}$$

$\int y^6 - y^4 + y^2 - 1 + \frac{1}{y^2+1}$

$\frac{y^7}{7} - \frac{y^5}{5} + \frac{y^3}{3} - y + \tan^{-1}y + C$

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$\int \frac{dx}{2+\sin(x)}$ ضرب مرافق.

$\int \frac{dx}{1-\sin x + \cos x}$

حفظ

$y = \tan \frac{x}{2}$ او $x = 2\tan^{-1} y$

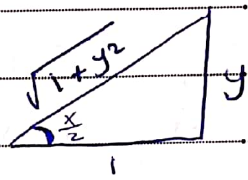
$dx = \frac{2}{1+y^2}\, dy$

$\sin 2\theta = 2\sin\theta \cos\theta$ $\boxed{\theta = \frac{x}{2}}$

$\sin x = 2\sin\frac{x}{2} \cos\frac{x}{2}$

$\cos 2x = \cos^2 x - \sin^2(x)$

$\cos x = \cos^2\frac{x}{2} - \sin^2\frac{x}{2}$



$\sin\frac{x}{2} = \frac{y}{\sqrt{1+y^2}}$ $\cos\frac{x}{y} = \frac{1}{\sqrt{1+y^2}}$

$\sin x = 2 \frac{y}{\sqrt{1+y^2}} \cdot \frac{1}{\sqrt{1+y^2}} = \boxed{\frac{2y}{1+y^2}}$ حفظ

$\cos x = \frac{1}{1+y^2} - \frac{y^2}{1+y^2} = \boxed{\frac{1-y^2}{1+y^2}}$ حفظا

* اللي حطيتهم بالمربعات والدوائر احفظهم حفظ اسهل.

$= \int \frac{\frac{2}{1+y^2}\, dy}{1 - \frac{2y}{1+y^2} + \frac{1-y^2}{1+y^2}} = \int \frac{2/\cancel{1+y^2}}{\frac{1+y^2}{\cancel{1+y^2}} - \frac{2y}{\cancel{1+y^2}} + \frac{1-y^2}{\cancel{1+y^2}}}$

$\int 2/1-2y+1 = \int \frac{2}{2-2y} = \int \frac{1}{1-y}\, dy$

$= -\ln|1-y|$

منعكس احسن ع المراف

$$\int \frac{dx}{\sin x - \cos x} = \int \frac{\frac{2}{\cancel{1+y^2}}\,dy}{\frac{2y}{\cancel{1+y^2}} - \frac{1-y^2}{\cancel{1+y^2}}} = \int \frac{2}{2y - 1 + y^2}\,dy$$

$$= \int \frac{2}{(y - \quad)(y + \quad)}$$

منكمل كسور جزئية

* Improper Integral

التكامل المعتل .

انه يكون في نقطة عدم اتصال بالفترة اللي يعطيني اياها / او يكون احد الحدود ∞ او

هلا نهاية الحالة بحل اذا كانت المشكلة بالأطراف بجزأ للتكامل واحد اقا

اذا كانت المشكلة بالنص بجزأ التكامل ل (٢)

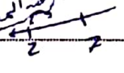
$$\int_a^{\infty} f(x) = \lim_{b \to \infty} \int_a^b f(x)\,dx$$

$$\int_{-\infty}^{b} f(x) = \lim_{t \to -\infty} \int_t^b f(x)\,dx$$

هيك بحل لو عندي ∞ او $-\infty$ ع الاطراف

$$\int_2^7 \frac{1}{x-2} = \int_c^b f(x)\,dx = \lim_{t \to c^+} \int_t^b f(x)\,dx$$

نقطة عدم اتصال



اوله من اليمين فقط

$$\int_0^3 \frac{1}{(x^2-9)} = \int_a^c f(x)\,dx = \lim_{t \to c^-} \int_a^t f(x)\,dx$$

اذا كانت المشكلة في المنتصف .

$$\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$$

$$= \lim_{t \to c^-} \int_a^t f(x) + \lim_{s \to c^+} \int_s^b f(x)\,dx.$$

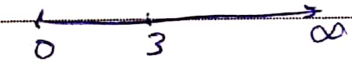
مثال تجريبة فقط :

$$\int_0^\infty \frac{1}{x(x-3)(x+7)}$$

عندي هون ٣ مشاكل .

(3 و zero و ∞)

بضطر احط ٤ تكاملات



$$\overset{\text{اللي بدي}}{}\int_0^1 + \int_1^3 + \int_3^{\overset{\text{اللي بدي اياه}}{100}} + \int_{100}^{\infty}$$

كل تكامل يكون فيه مشكله وحدة .

Ex : $\int_{-\infty}^{\infty} \frac{1}{(x-7)(x-1)(x+3)}$

$$\int_{-\infty}^{5} + \int_{5}^{-3} + \int_{-3}^{0} + \int_{0}^{1} + \int_{1}^{5} + \int_{5}^{7} + \int_{7}^{10} + \int_{10}^{\infty}$$

Improper Integral

II الصفر المقام لو يطلع صفر اذا Improper

$$\int_0^2 \cot \, dx$$

$$\int_a^\infty f(x)\,dx = \lim_{t\to\infty} \int_a^t f(x)\,dx$$

if f(x) dis cont at $x=c$

$$\int_c^b f(x)\,dx = \lim_{t\to c^+} \int_t^b f(x)\,dx \quad / \quad \int_b^c f(x)\,dx = \lim_{t\to c^-} \int_b^t f(x)\,dx$$

Ex :- $$\int_0^\infty \frac{1}{e^{3x}}\,dx = \lim_{t\to\infty} \int_0^t \frac{1}{e^{3x}}\,dx$$

$$= \lim_{t\to\infty} \int_0^t e^{-3x}\,dx = \lim_{t\to\infty} \left(\frac{e^{-3x}}{-3}\right)_0^\infty$$

$$\lim_{t\to\infty} \frac{e^{-3t}}{-3} + \frac{1}{3}$$

$$= 0 + \frac{1}{3} = \frac{1}{3}$$ converege to $\frac{1}{3}$

لو السؤال ضع دائرة ع

$$\int_0^\infty e^{-3x} = \frac{e^{-3}}{3}\Big|_0 = 0 + \frac{1}{3} = \frac{1}{3}$$

① ايركم .

$$\int_{-\infty}^{\infty} \frac{1}{1+x^2}\,dx = \int_{-\infty} \frac{1}{1+x^2}\,dx + \int_1^\infty \frac{1}{1+x^2}\,dx$$

$$\lim_{t\to-\infty} \int_t^1 \frac{1}{1+x^2} + \lim_{s\to\infty} \int_1^s \frac{1}{1+x^2}\,dx$$

$$\lim_{t\to-\infty} \tan^{-1}(x)\Big|_t^1 + \lim_{s\to\infty} \tan^{-1}x\Big|_1^s$$

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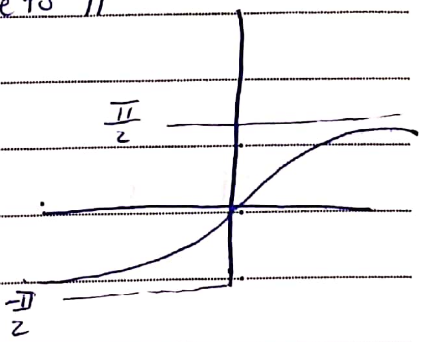
$$\lim_{t\to-\infty} \cancel{\tan^{-1}(100)} - \tan^{-1}(t) + \lim_{s\to\infty} \tan^{-1}(s) - \cancel{\tan^{-1}(100)}$$

$$= -\frac{-\pi}{2} + \frac{\pi}{2} = \frac{2\pi}{2} = \pi$$ converege to π



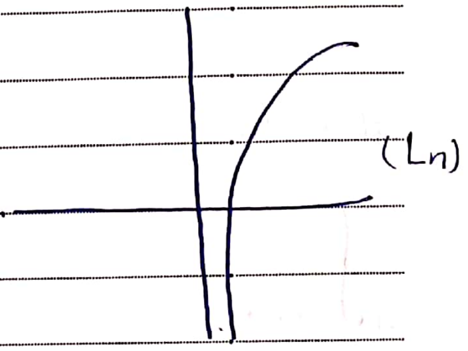
$$\int_0^1 \frac{1}{x}\,dx$$ صفر المقام (0)

$$\lim_{t\to0^+} \int_t^1 \frac{1}{x}\,dx = \lim_{t\to0^+} \ln|x| \Big|_t^1$$

$$\lim_{t\to0^+} = \ln(1) - \ln(t)$$

$$0 - -\infty = \infty$$

$$\int_0^1 \frac{1}{x}\,dx = \text{diverge}$$



$\boxed{4}$ $$\int_0^\infty \frac{2}{x^2-1}\,dx$$

$$\int_0^1 \frac{2}{x^2-1}\,dx + \int_1^7 \frac{2}{x^2-1}\,dx + \int_7^\infty \frac{2}{x^2-1}\,dx$$

$$\lim_{t\to1^-} \int_0^t \frac{2}{x^2-1}\,dx + \lim_{s\to1^+} \int_s^7 \frac{2}{x^2-1}\,dx + \lim_{w\to\infty} \int_7^w \frac{2}{x^2-1}\,dx$$

H.W $\int_0^4 \frac{dx}{(x-4)^2}$ / $\int_2^\infty \frac{1}{x\sqrt{\ln x}}\,dx$ / $\int_e^\infty \frac{1}{x(\ln x)^2}\,dx$

الاسئلة من (5_40) page 527 بالكتاب

Ex :- Find the valu (s) of p such that

$$\int_0^1 \frac{1}{x-2p}\, dx \quad \text{is improper}$$

$x = 2p = 0$

$$0 \le x \le 1$$

$$\frac{0}{2} \le \frac{\cancel{2}p}{\cancel{2}} \le \frac{1}{2}$$

$$0 \le p \le \frac{1}{2}$$

H.W :-

$$\int_{-1}^{2} \frac{dx}{2x+p} \qquad \int_{-7}^{5} \frac{dx}{\sqrt[3]{x}-7p}$$

Ex : Find the value of (P) such that $\int_1^{\infty} \frac{1}{x^P} dx$ is convergent

$p \neq 1$

$\lim_{t \to \infty} \int_1^t \frac{1}{x^P} dx = \lim_{t \to \infty} \frac{P^{-P+1}}{-P+1} \Big|_1^t$

$\frac{1}{1-P} \left(\lim_{t \to \infty} \frac{1}{t^{P-1}} - 1 \right) \to \frac{1}{\infty^0}$ converge $\to$ If power > 0 العوة

$P-1>0$

$\boxed{P>1}$

If $\boxed{P=1}$ divege

$$\boxed{\int_1^{\infty} \frac{1}{x^P} dx \quad \text{converge if } P>1}$$

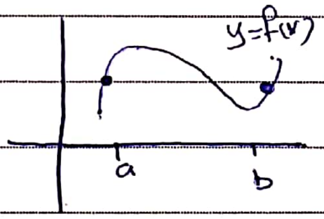
حفظ

$\int_1^{\infty} \frac{1}{\sqrt{x}} \to$ هون القوه $(\frac{1}{2})$ قبل

اذاً divarge

CH $\underline{\underline{8}}$:- Applications on Integration

Arc Length



Length

$$L = \int_a^b \sqrt{1 + f'(x)^e} \, dX$$

أغلب الاسئلة تطلب
بس اكتب التكامل
بدونه فا اعمل حل
واطلع جواب

Find the length of the following ..

① $y = \sqrt{x}$ between $x=0$ and $x=3$

$y' = \frac{1}{2\sqrt{x}}$ $L = \int_0^3 \sqrt{1 + \frac{1}{4x}}$

2 $y = x^2$ betwen the points $(3,9)$, $(5,25)$

$y' = 2x$ $L = \int_3^5 \sqrt{1+4x^2}\, dx$

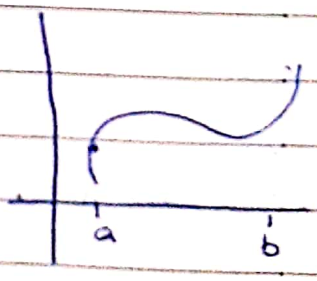
H.W ③ $f(x) = \sec x$ $(x=0$ to $x = \frac{\pi}{4})$

④ $f(x) = \ln x$ $(x=1, x=3)$

⑤ $y^3 = x^2$ from $(1,1)$ $(4,8)$

2 Area of surface

surface area by rotating $f(x)$ about x-axis



$$\text{Area} = A = \int_a^b 2\pi f(x) \sqrt{1 + (f'(x))^2}\, dx$$

$\boxed{1}$ Ex.: Find the surface area of $f(x) = \ln x$ revolved about x-axis $x \in [1, e]$

$$f'(x) = (\ln x)' = \frac{1}{x}$$

$$A = \int_1^e 2\pi \ln x \sqrt{1 + \frac{1}{x^2}}$$

$\boxed{3}$ $y = \cot(x)$
$x \in [\frac{\pi}{4}, \frac{\pi}{2}]$

$\boxed{4}$ $y = e^{5x}$ $x = 0$ to $x = 3$

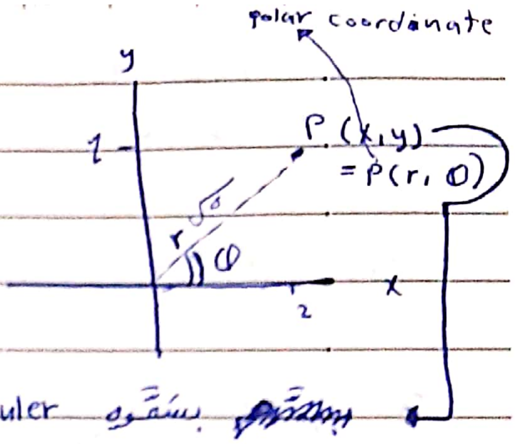
$\boxed{2}$ $y = x^2$ $\dot{y} = \cancel{3}\, 2x$

$$A = \int 2\pi x^2 \sqrt{1 + 4x^2} \, dx$$

- نهاية مادة الامتحان الاول

CH 10

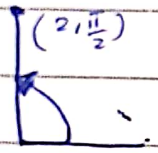
\+ polar coordinaty



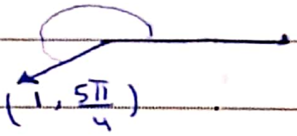
~~بسمّوه~~ بسمّوه rectatanguler
(cortisume)

EX :- plote the points whose polar coordinates are :-

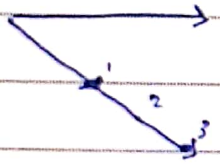
1 $(2, \frac{\pi}{2})$



2 $(1, \frac{5\pi}{4})$



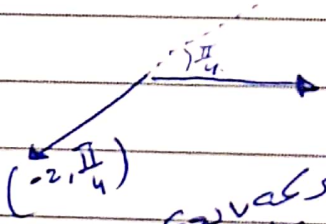
3 $(3, \frac{-\pi}{3})$



$(3, -\frac{\pi}{3})$ الزاوية سالبة بعكس عقارب الساعة

ه(r) السالب.

4 $(-2, \frac{\pi}{4})$



بركبه بالعكس لأنه سالب.

اذا ما عرفت

احدد السالب بحيث انه موجود بنزيد للزاوية (180)
وبركبه ونطلع نفس الاشي.

5 $(1, 3\pi)$ $(1, 5\pi)$, $(1, -7\pi)$

$* \; (r, \theta) = (r, \theta + 2n\pi)$

$(-r, \theta) = (r, ((\theta + \pi)) + 2n\pi)$

هاد لازم اذا بدي اغير الاشارة.

وهاي اذا بدي اجيب اسماء ثانية.

Ex:- $(1, \frac{\pi}{3}) = (1, \frac{\pi}{3} + 2n\pi)$

$(-1, (\frac{\pi}{3} + \pi) + 2n\pi)$

الصيغة العامة بشكل النقاط.

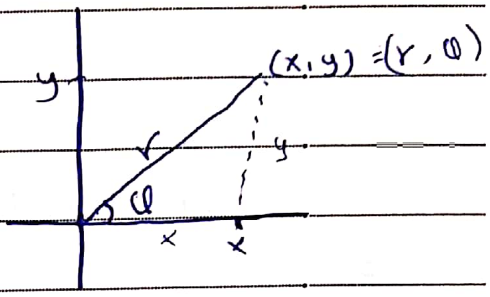
من خلال المثلث :-

$\boxed{1}$ $r^2 = x^2 + y^2$

$\boxed{2}$ $\tan\varphi = \frac{y}{x}$

or $\varphi = \tan^{-}(\frac{y}{x})$

مهم



$\boxed{3}$ $\cos\varphi = \frac{x}{r}$

$x = r\cos\varphi$

$\boxed{4}$ $\sin\varphi = \frac{y}{r}$

$y = r\sin\varphi$

Ex:- convert the following points from polar to cartisiam:

يعني بدي احول لـ (x, y)

$\boxed{1}$ $(\overset{r}{2}, \overset{\varphi}{\frac{\pi}{3}}) \Rightarrow x = r\cos\varphi = 2 \times \frac{1}{2} = 1$

$y = r\sin\varphi = \cancel{2} \times \frac{\sqrt{3}}{\cancel{2}} = \sqrt{3}$

$\therefore (\overset{r}{2}, \overset{\varphi}{\frac{\pi}{3}}) = (\overset{x}{1}, \overset{y}{\sqrt{3}})$

$\boxed{2}$ $(\overset{r}{1}, \overset{\varphi}{\pi})$

$x = r\cos\varphi = 1 \times -1 = -1$

$y = r\sin\varphi = \text{zero}$ $\therefore (-1, 0)$

34

$\boxed{3}$ $\left(2, \frac{-2\pi}{3}\right)$

هون (r) موجب مابدي اغير اشارته بس لو بدي اغير اشارة الزاوية بجمع (2nπ).

$\boxed{4}$ $\left(-2, \frac{3\pi}{4}\right)$

Ex:- Convert the following points from cartisiom to Polar.

① $(\overset{x}{\sqrt{3}}, \overset{y}{-1}) \Rightarrow r = \sqrt{x^2+y^2} = \sqrt{3+1} = 2$

$$\theta = \tan^{-1}\frac{y}{x} = \tan^{-}\frac{-1}{\sqrt{3}} =$$

اللي الها (tan) هي $\frac{1}{\sqrt{3}}$ هي $\left(\frac{\pi}{6}\right)$ وهي بالربع الرابع لأنه (x) موجب و (y) الب

$\Rightarrow 2\pi - \frac{\pi}{6} = \frac{11\pi}{6}$

$\left(2, \frac{11\pi}{6}\right)$

(Find all points) - لو كان السؤال

$\left(2, \frac{11\pi}{6} + 2n\pi\right)$ and $\left(2, \left(\frac{11\pi}{6} + \pi\right) + 2n\pi\right)$

بجمع او بنقص π
بس انا هون بالربع الرابع لو
النقص بكون احسن مشتغلح

$\left(-2, \frac{5\pi}{6} + 2n\pi\right)$

H.W:-

$(-3, 3)$

E(X) Find apolar equation for the curve represeted by the given cartisian equation.

$\boxed{1}$ $y = 1 +$ ~~[illegible]~~ $3X$

$r\sin\varphi = 1 + 3r\cos\varphi$

$$r = \frac{1}{\sin\varphi - 3\cos\varphi}$$

$\boxed{2}$ $x^2+y^2 = 4$

$r^2 = 4$ $\qquad$ $r = \pm 2$

$\boxed{3}$ $y = 2$ $\qquad$ $r\sin\varphi = 2$ $\qquad$ $r = 2\csc\varphi$

$\boxed{4}$ $xy = 4$ $\qquad$ $\boxed{5}$ $4y^2 = x$

$\boxed{6}$ $y = x$ $\qquad$ $\boxed{7}$ $(x^2+y^2)^2 = x^2 - y^2$

$\boxed{Ex}$ Identify the curve by finding acartain equation

لحل الاسئلة دي حاول اخلي

$(r\cos\varphi)$ مع بعض $(r\sin\varphi)$ مع بعض

او اخلي $(r)^2$ لأنه x^2+y^2

$\boxed{1}$ $r = 2\cos\varphi$

$r^2 = 2r\cos\varphi$ $\qquad$ بضرب الطرفين ب(r)

$x^2+y^2 = 2X$

$\boxed{2}$ $r^2 = 5$

$X^2 + y^2 = 5$

معادلة دائرة

المركز $(0,0)$

نق : $\sqrt{5}$

معادلة الدائرة :- $(x-a)^2 + (y-b)^2 = r^2$

المركز (a,b)

نصف القطر r

$\boxed{3}$ $r^2 \sin 2\theta = 1$

$2r^2 \sin\theta \cos\theta = 1$

$2(r\sin\theta)(r\cos\theta) = 1$

$2 \quad y \quad x = 1$

$\boxed{4}$ $r^2 \cos 2\theta = 1$

$r^2(\cos^2\theta - \sin^2\theta) = 1$

$r^2\cos^2\theta - r^2\sin^2\theta = 1$

$x^2 - y^2 = 1$

$\boxed{5}$ $r = \tan\theta \sec\theta$

$\boxed{6}$ $r = 4\sec \quad = \quad r\cos\theta = 4$

$x = 4$

$\boxed{7}$ $r^2 = \sin 2\theta$

$\boxed{8}$ $r = \frac{6}{(3-\sin\theta)}$

* polar curves

$\boxed{Ex}$ sckt ch the following curves

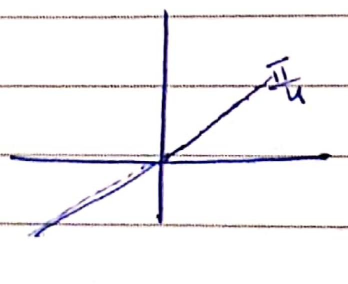
$\boxed{1}$ $\theta = \frac{\pi}{4}$

هون الزاوية ثابتة

مش لهو بتخين ال (٢)

ودائماً لما تكون الزاوية ثابتة الرسمة بتكون خط مستقيم.

مذ عند الزاوية



$y = x$

ولو ماعرفت كيف

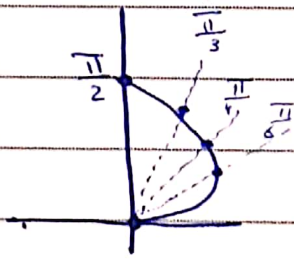
$\theta = \frac{\pi}{4}$.

$\tan\theta = \tan\frac{\pi}{4}$

$\frac{y}{x} = 1$

مثال

$r = \sin\theta$



θ	r
0	0
$\frac{\pi}{6}$	$\frac{1}{2}$
$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$
$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$
$\frac{\pi}{2}$	1

معادلة كخط مستقيم

$\boxed{2}$ $r = \frac{3}{2\cos\varphi - 3\sin\varphi}$

هاي بالرسمة صعبة خلنحولها اسهل.

$2r\cos\varphi - 3r\sin\varphi = 3$

$2x - 3y = 3$

$y = \frac{2}{3}x - 1$

لو طلب منه (slop) نحولها ونطلع

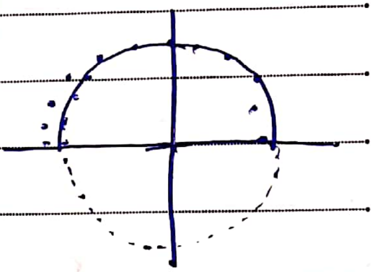
ال (slop) $\left(\frac{2}{3}\right)$

$r = \frac{c}{a\cos\varphi + b\sin\varphi}$ $\xrightarrow{\text{هي نفسها}}$ $y = \frac{c}{b} - \frac{a}{b}x$

$ax + by = c$

$\boxed{3}$ $r = \frac{7}{3\cos\varphi + 5\sin\varphi}$ $-$ slop $= \frac{3}{5}$

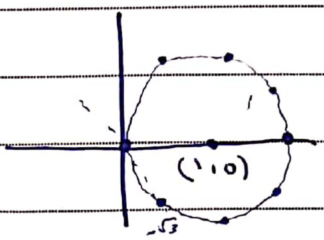
$\boxed{4}$ $r = 2$ الثابت الـ (r) واللي يتغير الزاوية



* بطلع عنا دائرة نصف قطرها (2-)

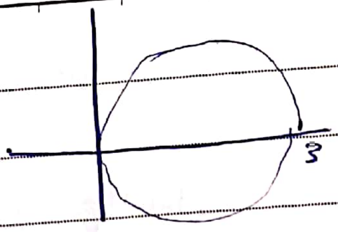
- وحتى لو كان ($r=-2$) عشان راح تكون دائرة ونصف قطرها $|-2| = 2$

$\boxed{5}$ $r = 2\cos\varphi$

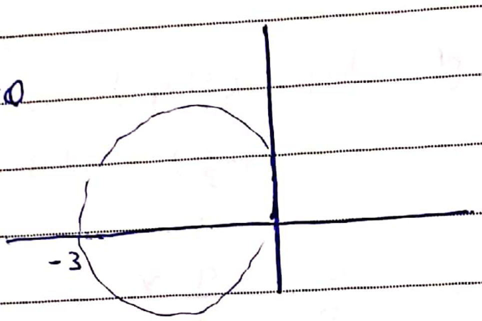


r	
0	2
$\frac{\pi}{6}$	$\sqrt{3}$
$\frac{\pi}{4}$	$\sqrt{2}$
$\frac{\pi}{3}$	1
$\frac{\pi}{2}$	0
$\frac{\pi}{6}$ بالربع الثاني	$-\sqrt{3}$ → نعين بركه بالعكس.

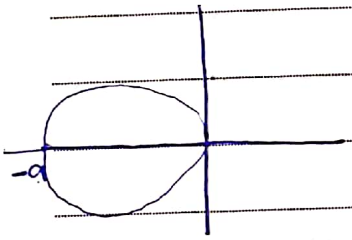
$r = 3\cos\theta$

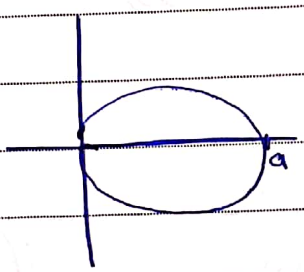


$r = -3\cos\theta$

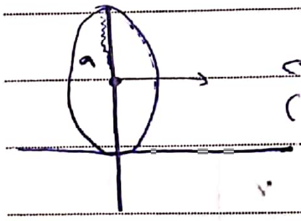


$r = 2a\cos\theta$



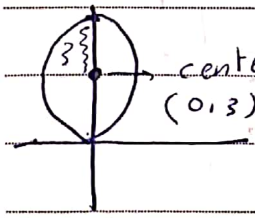


$\boxed{8}$ $a > 0$. $r = 2a\sin\theta$



centar
(0, a)

$\boxed{9}$ $r = 6\sin\theta$

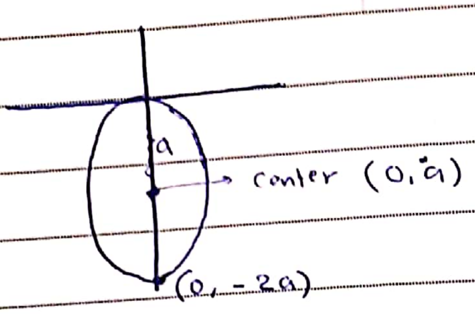


center
(0, 3)

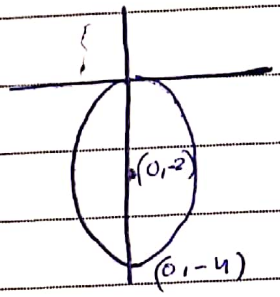
$\boxed{10}$ $r = -2a \sin\theta$

$a>0$ دكها سالب

$a<0$ اد



$\boxed{11}$ $r = -4 \sin\theta$



* In general :-

$r = 2a\cos\theta + 2b\sin\theta$

cericle with center (a, b)

and radia r

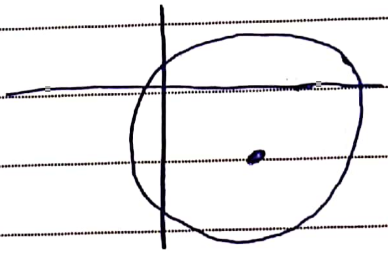
$= \sqrt{a^2 + b^2}$

Ex :- $r = 6\cos\theta - 2\sin\theta$

$\downarrow a = 3$ $\quad\downarrow b = -1$

$(3, -1)$

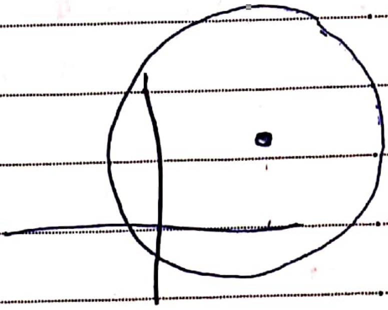
radian $= \sqrt{9+1} = \sqrt{10}$



Ex :- $r = 10\cos\theta + 8\sin\theta$

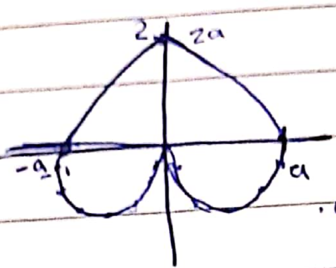
$(5, 4)$ center

$\sqrt{25+16} = \sqrt{41}$ radian



$r = 1 + \sin\phi$

ϕ	r
0	1
$\frac{\pi}{2}$	2



الرسمة حفظا اسهل.

الحالة الاولى

$a>0$

-Note :- Cardiad ♡

$\boxed{1} + \{\sin\phi$

معاملات متشابهة

موجب × موجب = موجب

$* \ r = a(1 \oplus \sin\phi)$

$\ominus a(1 \ominus \sin\phi)$

سالب × سالب = موجب

مثلا. $1 - 2\sin\phi$

المعاملات مو متشابهة اذاً مو Cardiad.

∴ اذا الجواب موجب الرسمة.

للأعلى ♡

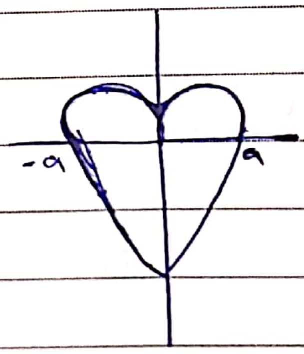
الحالة الثانية

سالب × موجب = سالب

$r = \ominus a \ (1 \ominus \sin\phi)$

$= \ominus a \ (1 \oplus \sin\phi)$

موجب × سالب = سالب



∴ اذا الجواب سالب الرسمة.

♡ للأسفل

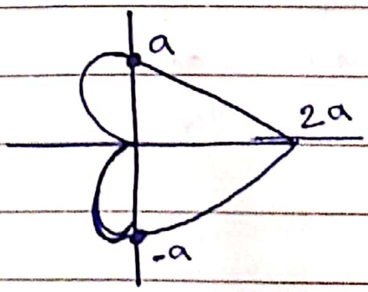
الحالة الثالثة

موجب

$r = \oplus a(1 \oplus \cos\phi)$

$= \ominus a(1 \ominus \cos\phi)$

سالب × سالب = موجب



اذا الجواب موجب الرسمة

ع اليمين

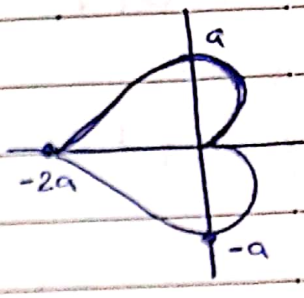
الحالة الرابعة

سالب × موجب = سالب

$r = \oplus \overbrace{a(1 \ominus}\cos\phi)$

$\ominus \underbrace{a(1 \oplus}\cos\phi)$

موجب × سالب = سالب

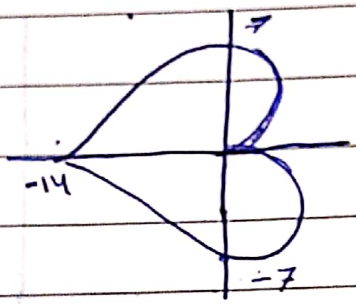


* اذا الجواب سالب الحركة لليسار

Ex:-

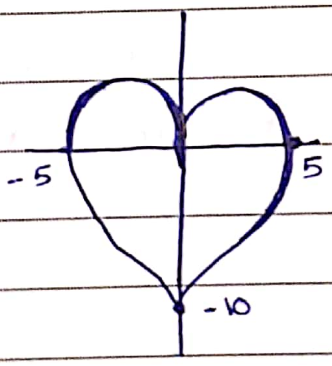
1) $r = 7 - 7\cos\phi$

$7(1-\cos\phi)$



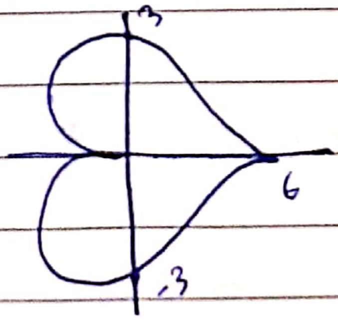
2) $r = -5 - 5\sin\phi$

$-5(1+\sin\phi)$



3) $r = 3 + 3\cos\phi$

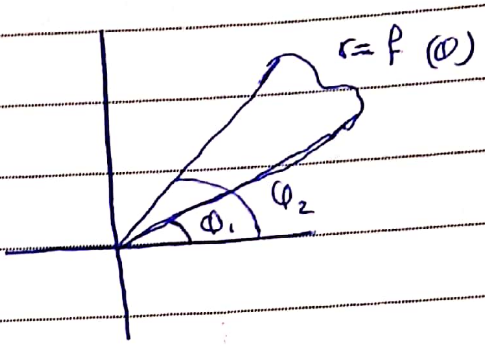
$3(1+\cos\phi)$

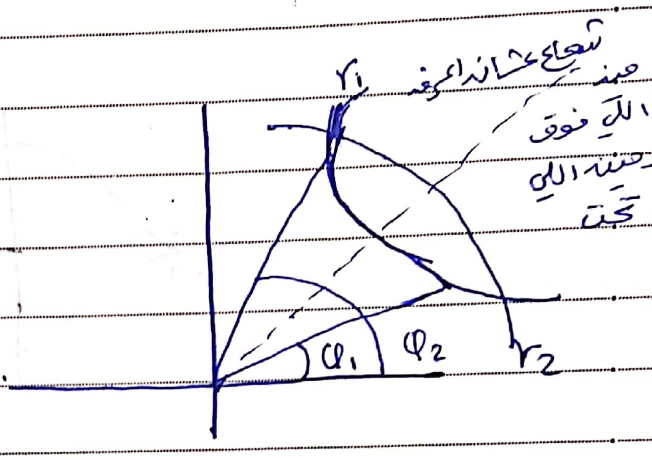


* Area in polar coordinats.

$$A = \int_{\Phi_1}^{\Phi_2} \frac{1}{2}(f(\Phi))^2 \, d\Phi$$

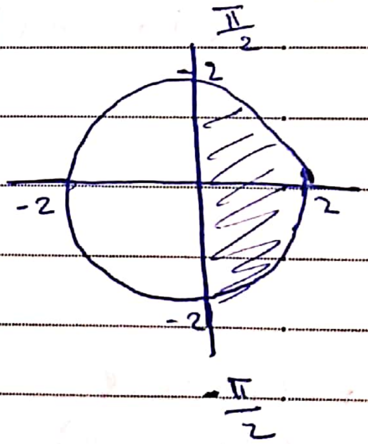
$$= \int_{\Phi_1}^{\Phi_2} \frac{1}{2} r^2 \, d\Phi$$





$$A = \frac{1}{2}\int_{\Phi_1}^{\Phi_2} r_2^2 - r_1^2 \, d\Phi$$

Ex:- Find the area of the region in the right half plane and insicle the circle $r = 2$



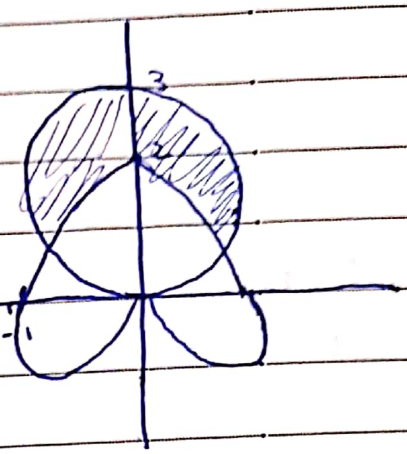
$$A = \frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \underset{\text{ثابت}}{\underbrace{r^2}} \, d\Phi$$

$$\frac{1}{2}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} (2)^2$$

$$= \frac{1}{2} \times 4 = 2 \, d\Phi = 2\pi$$

التي تقع

Find the area of the region that lies inside the circle $r = 3\sin\varphi$ and out side the cardioid $r = 1 + \sin\varphi$.



عشان اوجد نقاط التقاطع بساوي الاقترانين ببعض

$3\sin\varphi = 1 + \sin\varphi$

$2\sin\varphi = 1$

$\sin\varphi = \frac{1}{2}$ $\quad \varphi = \frac{\pi}{6}, \frac{5\pi}{6}$

$$A = \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (3\sin\varphi)^2 - (1 + \sin\varphi)^2$$

وبتكمل

والجواب $= \pi$

1 - Find the crea of the region inside $r = 2\cos\varphi$ outside $r = 1$

2 inside $r = 2\sin\varphi$ out side $r = 3\sin\varphi$

3 inside $r = \sqrt{3}\cos\varphi$ inside $r = \sin\varphi$ } نفس السؤال بس التعبير مختلف

(common area مشتركة او , inclosed مغلقة)

4 inside $r = 1 + \cos\varphi$ inside $r = 1 - \cos\varphi$

44

* common to the carves $r = 3\cos\theta$ and $r = 3\sin\theta$

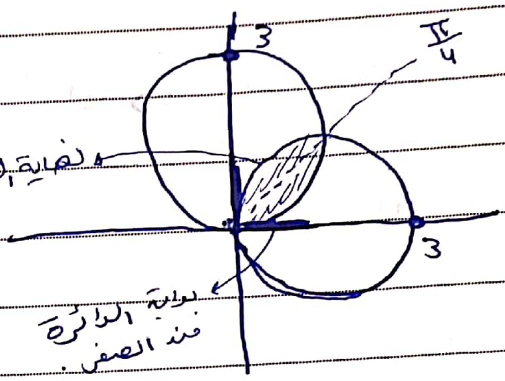
هون مو حاكي انهم دوائر بس من السؤال بعرف انهم دوائر

نقاط التقاطع ..

$3\cos\theta = 3\sin\theta$

$\tan\theta = 1$

$\theta = \frac{\pi}{4}$



$$A = \int_0^{\pi/4} \frac{1}{2}(3\sin\theta)^2 + \int_{\pi/4}^{\pi/2} \frac{1}{2}(3\cos\theta)^2 \, d\theta$$

- هون ما في واحد فوق الثاني لما اطلع شعاع بتنقسم المنطقة لنصين وكل نص بيدي على اقتران واحد فقط.

* parametric equation

curves defined by parametric equation

$y = f(x)$

dependental variable

↳ independent variable.

$y = f(x)$

$y = x^2 - 1$

x	y
0	-1
1	0
2	3

عشوائي .

يعتمد على قيمة (x)

suppose that x and y are both given as afunctions of thired variable t

dependent variable

$$\left.\begin{aligned} x &= f(t) \\ y &= g(t) \end{aligned}\right\}$$ parametric equation.

such that

$a \leqslant t \leqslant b$

independent variable

parameter

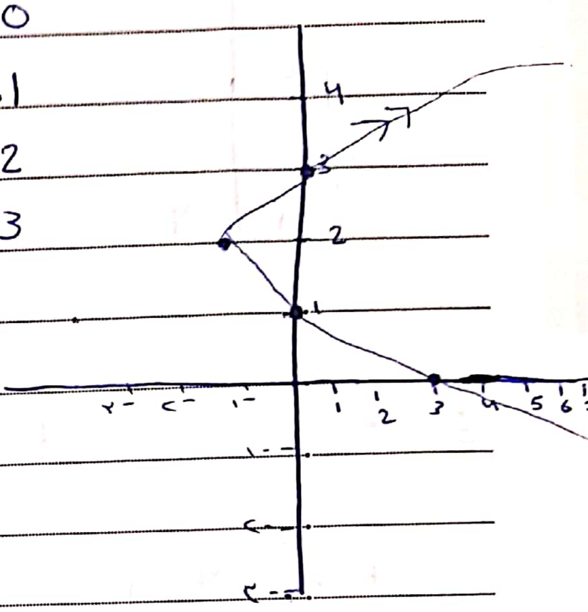
Ex :- sketch and identify the curve defined by p.E

$x = t^2 - 2t$

$y = t + 1$

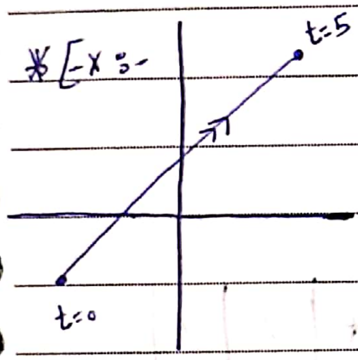
$\infty - < t < \infty$

t	x	y
-2	8	-1
-1	3	0
0	0	1
1	-1	2
2	0	3

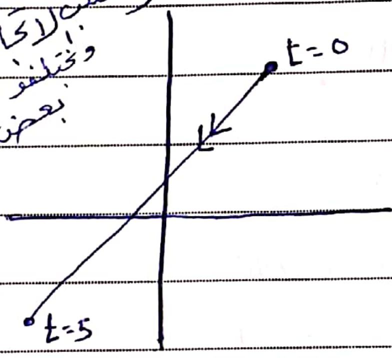


$0 \leq t \leq 5$

*Ex :-



حسب الاتجاه ويختلفو عن بعض



$X \quad Y = t^2 - 2t$ عوّض $y = t + 1$

$\boxed{t = y - 1}$

$x = (y-1)^2 - 2(y-1)$

$x = y^2 - 2y + 1 - 2y + 2$

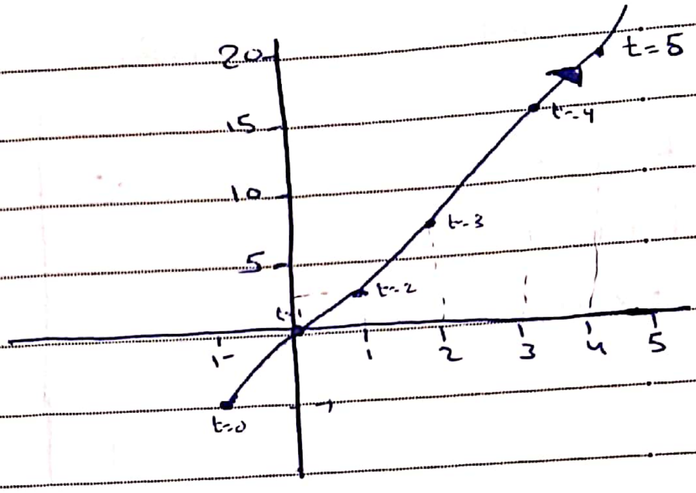
$\boxed{x = y^2 - 4y + 3}$

Ex :- sketch and indentify the curve defined by p.E

$x = t - 1 \qquad y = t^2 - 1 \qquad 0 \leq t \leq 5$

$t = x + 1 \qquad y = (x+1)^2 - 1$

t	x	y
0	-1	-1
1	0	0
2	1	3
3	2	8
4	3	15
5	4	24



Ex :- $x = \cos t$ $y = \sin t$

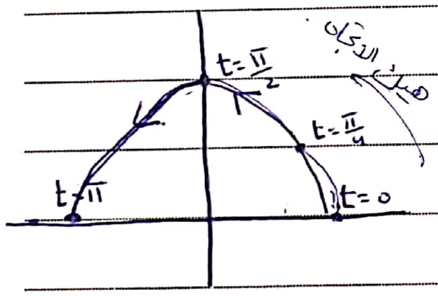
$0 \le t \le \pi$ → يعني نص دايرة لو $0 \le t \le 2t$ كانت دايرة كاملة.

$\sin^2 t + \cos^2 t = 1$

$y^2 + x^2 = 1$

$x^2 + y^2 = 1$ ⟹ Circle Center $(0,0)$

radian $= 1$



t	x	y
0	1	0
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{2}$	0	1
π	-1	0

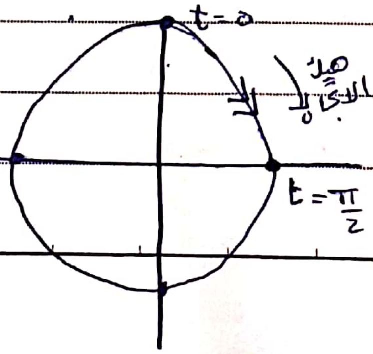
* Ex :-

$x = \sin t$, $y = \cos t$ $0 \le t \le 2\pi$

$\sin^2 t + \cos^2 t = 1$

$x^2 + y^2 = 1$

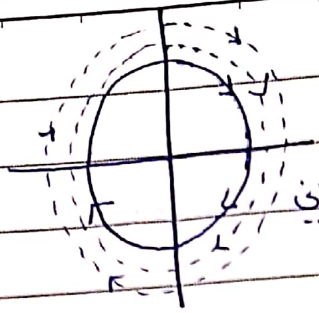
$(0,0)$ $r = 1$



t	x	5
0	0	1
$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$
$\frac{\pi}{2}$	1	0
π	0	-1
$\frac{3\pi}{2}$	-1	0
2π	0	1

لو كانت

$0 \leq t \leq 4\pi$



دعشان ابيّن انه رسمت دائرتين .

برسم دائرتين منقطات او بلون ثاني

[H.W] $x = \sin 2t$ $\quad y = \cos 2t$ $\quad 0 \leq t \leq 2\pi$

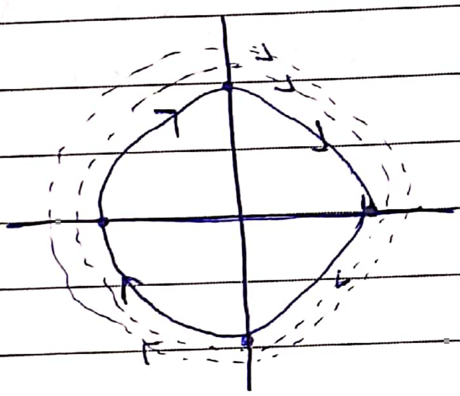
$\sin^2(2t) + \cos^2(2t) = 1$

center(0,0)

r=1

$x^2 + y^2 = 1$

t	x	y
0	0	1
$\frac{\pi}{4}$	1	0
$\frac{\pi}{2}$	0	-1
$\frac{3\pi}{4}$	-1	0
π	0	1
$\frac{5\pi}{4}$	1	0
$\frac{3\pi}{2}$	0	-1



Ex:- ① $x = t^2 + t$

$y = t^2 - t$

$-2 \leq t \leq 2$

[2] $x = t^2$

$y = t3 - 4t$

$-3 \leq t \leq 3$

[3] $x = \cos^2 t$

$y = 1 - \sin t$

$0 \leq t \leq \frac{\pi}{2}$

[4] $x = e^{-t} + t$

$y = e^{t} - t$

$-2 \leq t \leq 2$

[5] $x = e^{6} - 1$

$y = e^{2t}$

[6] $x = \frac{1}{2} \cos \theta$

$y = 2 \sin \theta$

$x = a + r \cos t$

$y = b + r \sin t$

circle with center (a,b) and raduis r (directin c.c.w)

Ex :-

$\cos t = \frac{x-3}{2}$

$\sin t = \frac{y+1}{2}$

$\frac{(x-3)^2}{4} + \frac{(y+1)^2}{4} = 1$

$x = 3 + 2\cos t$

$y = -1 + 2\sin t$

$0 \le t \le 2\pi$

circl center $(3,-1)$

radian $r = 2$

The p.E for the line pass thought (x_1, y), (x_2, y_2) is

$x = x_1 + (x_2 - x_1)t$

$y = y_1 + (y_2 - y_1)t$

بين $\boxed{0 \le t \le 1}$

Ex :- $(2,1)$, $(3,4)$

$x = 2 + t$

$y = 1 + 3t$

$0 \le t \le 1$

* Calculus with parametric curves.

$x = f(t)$ $\qquad$ $y = g(t)$

$\frac{dy}{dt} \quad \frac{dt}{dx}$

$$\boxed{\frac{dy}{dx} = \frac{dy/dt}{dx/dt}} = \frac{g'(t)}{f'(t)}$$

$$\frac{d^2y}{d^2x} = \frac{d}{dx}\left(\frac{dy}{dx}\right)$$

$$\boxed{\frac{\frac{d}{dt}(dy/dx)}{\frac{dx}{dt}}}$$

Ex :- $X = t^2 - 1$ Find :- $\frac{dy}{dX}$, $\frac{dy^2}{dX^2}$

$y = t + 3$

$\boxed{1}$ $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{1}{2t}$

$\boxed{2}$ $\frac{d^2y}{dx^2} = \frac{d/dt\left(\frac{1}{2t}\right)}{2t} = \frac{\frac{-1}{2t^2}}{2t} = \frac{-1}{4t}$

1) slope at the point $t = t_0$ $\Rightarrow \left.\frac{dy}{dx}\right|_{t_0}$

2) horizantel tangent :- $\Rightarrow \frac{dy}{dt} = 0$ $\frac{dx}{dt} \neq 0$

Ex :- consider the curve C , defined by the P.E

$x = t^2$, $y = t^3 - 3t$

1) show that C has two tangent at $(\overset{x}{3}, \overset{y}{0})$ and find there equation.

$\boxed{y=0}$ $t^3 - 3t = 0$

$t(t^2 - 3) = 0$ $t = 0$ / $t = \sqrt{3}$ / $t = -\sqrt{3}$

$\boxed{x = t^2}$ when $t = 0$ $x = 0$ / $t = \sqrt{3} \Rightarrow x = 3$ / $t = -\sqrt{3} \Rightarrow X = 3$

$(\overset{x}{0}, \overset{y}{0})$ $(\overset{x}{3}, \overset{y}{0})$ $(\overset{x}{3}, \overset{y}{0})$

$\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{3t^2 - 3}{2t}$

slope at $t = \sqrt{3}$ $\left.\frac{dy}{dx}\right|_{t=\sqrt{3}} = \frac{3(3) - 3}{2\sqrt{3}} = \sqrt{3}$

slope at $t = -\sqrt{3}$ $\left.\frac{dy}{dx}\right|_{t=-\sqrt{3}} \frac{3(3) - 3}{2(-\sqrt{3})} = -\sqrt{3}$

$t = \sqrt{3}$ $(\overset{x_0}{3}, \overset{y_0}{0}) \Rightarrow y - 0 = \sqrt{3}(x - 3) \Rightarrow y = \sqrt{3}x - 3\sqrt{3}$

$t = -\sqrt{3}$ $(\overset{x_0}{3}, \overset{y_0}{0}) \Rightarrow y - 0 = -\sqrt{3}(x - 3) \Rightarrow y = -\sqrt{3}x + 3\sqrt{3}$

$x = t^2$ $\qquad$ $y = t^3 - 3t$

b) Find the points on C where the tangent is horizantal $\Rightarrow \frac{dy}{dx} \neq 0$

$\frac{dy}{dx} = 0$

$3t^2 - 3 = 0$

$t^2 = 1 \qquad \boxed{t = \pm 1}$

$t = 1 \qquad x = 1 \qquad y = 1 - 3 = -2 \qquad (1, -2)$

$t = -1 \qquad x = 1 \qquad y = -1 - -3 = 2 \qquad (1, 2)$

* horizantal tangent at the points $(1, 2) / (1, -2)$

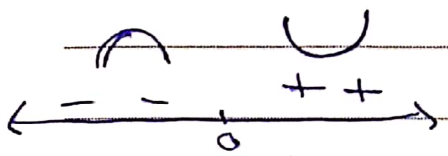
$\boxed{c}$ Find the points on C where the tangent is vertical

$\frac{dx}{dt} = 0 \qquad \frac{dy}{dt} \neq 0$

$\frac{dy}{dx} = \frac{3t^2 - 3}{2t}$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt} = \left(\frac{dy}{dx}\right)}{\frac{dx}{dt}} = \frac{\frac{(2t)(6t) - (2(3t^2 - 3)}{4t^2}}{2t}$$

$$= \frac{6t^2 + 6}{8t^3} = \frac{3t^2 + 3}{4t^3}$$



concave down $\quad t < 0$

concave up $\quad t > 0$

$3t^2 + 3 = 0$

$t^2 = -1$ x

$8t^3 = 0 \qquad t = 0$

Arc Length (on P.E)

$x = f(t)$ $y = g(t)$ $a \le t \le b$

then :-

$$L = \int_a^b \sqrt{(\underset{\frac{dx}{dt}}{f'(t)})^2 + (\underset{\frac{dy}{dt}}{g'(t)})^2}\ dt$$

Arc length

$$= \int_a^b \sqrt{(x')^2 + (y')^2}\ dt$$

Ex:- Find the exact Length of

$\boxed{1}$ $x = \cos t$ $y = \sin t$

$0 \le t \le \pi$ $\frac{dx}{dt} = -\sin t$ / $\frac{dy}{dt} = \cos t$

$$= \int_0^\pi \sqrt{(-\sin t)^2 + (\cos t)^2}\ dt$$

$$\int_0^\pi 1\, dt = \pi$$

$\boxed{2}$ $x = 1 + 3t^2$ $y = 4 + 2t^3$ $0 \le t \le 1$

$\frac{dx}{dt} = 6t$ $\frac{dy}{dt} = 6t^2$

$L = \int_0^1 \sqrt{36t^2 + 36t^4}$ عامل مشترك $(36t^2)$ مش رح اكملها بسبب ونفرض قلش بعدها.

(H.w)

$\boxed{1}$ $x = e^t + e^{-t}$; $y = 5 - 2t$ $0 \le t \le 3$

$\boxed{2}$ $x = t \sin t$; $y = t \cos t$ $0 \le t \le 1$

$\boxed{3}$ $x = 3\cos t - \cos 3t$; $y = 3\sin t - \sin 3t$ $0 \le t \le \pi$

52

* Area of surface :-

$x = f(t)$ $y = g(t)$ $a \le t \le b$

$$A = \int_a^b 2\pi \; g(t) \sqrt{(f'(t))^2 + (g' t)^2} \; dt$$

Area of surface.

$$= \int_a^b 2\pi \, y \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \; dt$$

Ex :- find the area of the surface of

$x = \cos t$ $y = \sin t$ $0 \le t \le \pi$

$$A = \int_0^\pi 2\pi \; \sin t \sqrt{(\sin t)^2 + (\cos t)^2} \; dt$$

P. 651 ص من اسئلة الكتاب *

(3 - 20) / (37 - 40) / (57 - 60) / (61 - 63)

[CH 11] sequences المتتاليات

↘ Infinite sequences

$1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots\ldots, \frac{1}{n}$ ↷ general term الحد العام

First term الحد الاول — second term

- لو افرض ان المتتالية اسمها (a)
فالحد الاول يكون (a_1) والحد الثاني (a_2) ....

- اغلب المتتاليات تكون على شكل نمط واني بعرف اكمل المتتالية.

* بدل ما اكتب ال sequence بالشكل ؟ بكتبها بدلالة الحد العام

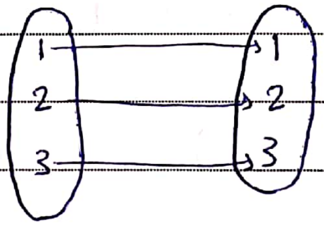
$\{a_n\} = \{\frac{1}{n}\}_{\square}^{\square}$ بكتب بداية قيم (n) ونهايتها $= \{\frac{1}{n}\}_{1}^{\infty} = \{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5} \ldots\}$

a_n هي عبارة عن الحد العام.

* sequence ⇒ عبارة عن function

Dom = $\mathbb{N}$

Range = $\mathbb{R}$



* $\{a_n\}_{n=1}^{\infty}$

convergent — divergent

$\lim_{n\to\infty} a_n = \underset{\text{اي عدد}}{\overset{\text{exist}}{L}}$

$\lim_{n\to\infty} a_n = d.n.e$ الجواب او $\infty, -\infty$

54

Ex :- $1, \frac{1}{2}, \frac{1}{3}, \ldots\ldots\ldots, \frac{1}{n}$

$$\lim_{n \to \infty} \frac{1}{n} = \text{zero}$$

$$\left\{\frac{1}{n}\right\}_{n=1}^{\infty} \xrightarrow{\text{converge}} 0$$

Ex :- $\underset{a_1}{1}, \underset{a_2}{3}, \underset{a_3}{5}, \underset{a_4}{7}, \ldots\ldots, \underset{a_n}{2n-1}$

العلاقة بنضرب ن(٢) وبنقص (١)

$$\lim_{n \to \infty} 2n-1 = \infty \qquad \therefore \{2n-1\}_{n=1}^{\infty} \text{ divergent}$$

If $\{a_n\}$ and $\{b_n\}$ are convergent seq and constant:

$\boxed{1}$ $\lim_{n \to \infty} (a_n \pm b_n) = \lim_{n \to \infty} a_n + \lim_{n \to \infty} b_n$

$\boxed{2}$ $\lim_{n \to \infty} c\,a_n = c \lim_{n \to \infty} a_n$

$\boxed{3}$ $\lim_{n \to \infty} (a_n\, b_n) = \lim_{n \to \infty} a_n \cdot \lim_{n \to \infty} b_n$

$\boxed{4}$ $\lim_{n \to \infty} c = c$

$\boxed{5}$ $\lim_{n \to \infty} \frac{a_n}{b_n} = \frac{\lim_{n \to \infty} a_n}{\lim_{n \to \infty} b_n} \qquad \lim_{n \to \infty} b_n \neq 0$

$\boxed{6}$ $\lim_{n \to \infty} (a_n)^p = \left(\lim_{n \to \infty} a_n\right)^p , \quad p > 0$

$a_n > 0$

Ex :- Find a formula for the general term a_n :-

$\boxed{1}$ $\left\{ 1, \frac{1}{3}, \frac{1}{5}, \frac{1}{7}, \ldots\ldots \right\}$

$\underset{a_1}{1}, \underset{a_2}{\frac{1}{3}}, \underset{a_3}{\frac{1}{5}}, \underset{a_4}{\frac{1}{7}}, \ldots, a_n$

$\therefore a_n = \frac{1}{2n-1}$

∴ المقام عبارة عن العدد ضرب (٢) وبطرح (١)

$\boxed{2}$ $\underset{a_1}{\frac{1}{2}}, \underset{a_2}{\frac{1}{4}}, \underset{a_3}{\frac{1}{8}}, \frac{1}{16}, \frac{1}{32}, \ldots\ldots$

الحد العام $\left[a_n = \frac{1}{2^n} \right.$

$\boxed{3}$ $\underset{a_1}{1}, \underset{a_2}{\frac{-1}{3}}, \underset{a_3}{\frac{1}{9}}, \underset{a_4}{\frac{-1}{27}}, \underset{a_5}{\frac{1}{81}}$

* هون المقام مبين عندي انه

$\ldots\ldots, \underset{n=4}{(3)^3}, \underset{n=3}{(3)^2}, \underset{n=2}{(3)^1}$

$a_n = (-1)^{n+1} \frac{1}{3^{n-1}}$

هيك بحل مشكلة الإشارة

- الزوجيات اشارتهم سالبة والفرديات موجبة.

* هون الإشارة عشان احلها بستخدم

$(-1)^n$ ← بستخدمها اذا كانت الحدود الفردية السالبة

$(-1)^{n+1}$ ← بستخدمها اذا كانت الحدود الزوجية السالبة [وهاي اللي راح استخدمها]

او $(-1)^{n-1}$

نفس الشي

$\boxed{4}$ $\left\{\underset{a_1}{-3},\ \underset{a_2}{2},\ \underset{a_3}{\frac{-4 \nearrow 2^2}{3}},\ \underset{a_4}{\frac{8 \nearrow 2^3}{9}},\ \underset{a_5}{\frac{-16 \nearrow 2^4}{27}},\ \dots\right\}$

اول شي الحدود الفردية هي السالب
بستخدم قاعدة $(-1)^n$
- بلبش منه الحد الثالث عشان اعرف القاعدة

$$a_n = (-1)^n \frac{2^{n-1}}{3^{n-2}}$$

[H.W]

$\boxed{5}$ $\{5, 8, 11, 14, 17 \sim\}$

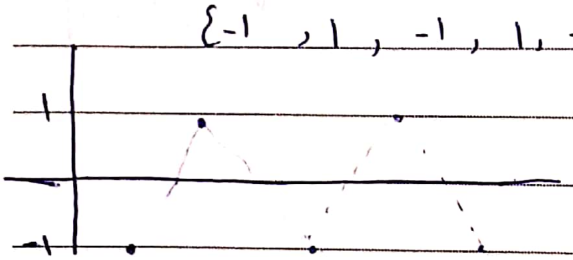
$\boxed{6}$ $\left\{\frac{1}{2}, \frac{-4}{3}, \frac{9}{4}, \frac{16}{5}, \frac{25}{6}\right\}$

$\boxed{7}$ $\{1, 0, -1, 0, 1, 0, -1, 0\}$

EX:- Determine whether the following seq convergen or divergen & Find the limit.

$\boxed{1}$ $a_n = (-1)^n \implies \therefore$ diverge

$\{-1, 1, -1, 1, -1, \dots\}$



$$\therefore \lim_{n\to\infty} (-1)^n = d.n.e$$

$\boxed{\sin}$ بتشبه دالة $(\sin)$ لأنو بتضل تطلع وتنزل ما الها نهاية d.n.e

57

$\boxed{2}$ $a_n = 1 - (0.2)^n$

$\lim\limits_{n\to\infty} 1 - \underset{\text{الأساس}}{(0.2)^n}$

مثل كالك (١) اذا كان الأساس
من ٥ و ١ فإن $\underset{\text{للأساس}}{\lim}$ صفر
واذا كان الكبر منه (١) بكونه (div)

$= \lim\limits_{n\to\infty} 1 - \lim\limits_{n\to\infty} (0.2)^n$

$= 1 - 0 = 1$

$\therefore \{a_n\}$ converge to 1

$\boxed{3}$ $a_n = \dfrac{n^3}{n^3+1}$ $\dfrac{\text{Poly}}{\text{Poly}} \Rightarrow$ مثل كالك (١) $\dfrac{\text{اعلى حد}}{\text{اعلى حد}}$

$\lim\limits_{n\to\infty} \dfrac{n^3}{n^3} = \dfrac{1}{1} = 1$

$\therefore \{a_n\}$ converge to 1

$\boxed{4}$ $a_n = \dfrac{3 + 5n^2}{n + n^2}$

$\lim\limits_{n\to\infty} \dfrac{3+5n^2}{n+n^2} = \dfrac{5}{1} = 5$

$\therefore \{a_n\}$ converge to 5

Thm If $\lim\limits_{n\to\infty} \overset{\text{seq تحقق}}{|a_n|} = 0$ $\overset{\text{فإن}}{\text{then}}$ $\lim\limits_{n\to\infty} a_n = 0$

سنستخدمها لما يكون عندي (seq) سالبة

$\boxed{5}$ $a_n = \dfrac{(-1)^n}{n}$ $\Rightarrow |a_n| = \left|\dfrac{-1^n}{n}\right| = \dfrac{1}{n}$

$\lim\limits_{n\to\infty} |a_n| = \lim\limits_{n\to\infty} \dfrac{1}{n} = \text{zero}$

$\therefore \lim \dfrac{(-1)^n}{n} = \text{zero}$

متتالية فيها حد موجب وحد سالب

* Alterneting seq

بالغالب هيك شكلها

$\{(-1)^n a_n\}$ or $\{(-1)^{n+1} a_n\}$

* If $\lim\limits_{n\to\infty} a_n = L$ (convergent)

$\therefore \lim\limits_{n\to\infty} \underset{\text{الحدود الزوجية}}{a_{2n}} = \boxed{\text{or}} \lim\limits_{n\to\infty} \underset{\text{الحدود الفردية}}{a_{2n+1}} = L$

يعني اذا كان (con) فإن اي جزء منها يكون النهاية اله (con) والعكس غير صحيح

$\lim\limits_{n\to\infty} a_{2n} \neq \lim\limits_{n\to\infty} a_{2n+1}$

↳ then $\{a_n\}$ divergent *

$\boxed{6}$ $a_n = \left\{(-1)^n \dfrac{3n^3 - 7n - 1}{7n - n^3}\right\}_{n=1}^{\infty}$

* لو بدي استخدم نظرية المطلق راح يكون الجواب $\frac{3}{-1} = |-3| = 3$ اذاً مو صفر
يعني ما بقدر استخدمو

* - بجيب اوجد الحدود الزوجية . سالب بقوة زوجي بروح

$\lim\limits_{n\to\infty} a_{2n} = \lim\limits_{n\to\infty} \cancel{(-1)^{2n}} \dfrac{3(2n)^3 - 7 * 2n - 1}{7 * 2n - (2n)^3}$ $\frac{\text{poly}}{\text{poly}}$

$= \lim\limits_{n\to\infty} \dfrac{3 * 24 n^3}{-24 n^3} = -3$

* - هلأ بوجد النهاية للحدود الفردية

سالب قوة فردي = سالب

$\lim\limits_{n\to\infty} a_{2n+1} = \lim\limits_{n\to\infty} (-1)^{2n+1} \; 3 \dfrac{(2n+1)^3 - 7(2n+1) - 1}{7(2n+1) - (2n+1)^3}$

$= \lim\limits_{n\to\infty} (-1) \dfrac{3(2n+1)^3}{(2n+1)^3} = 3 \dfrac{24 n^3}{-24 n^3} = +3$

بعد ما اول بوحد اكبر قوة ع اكبر قوة.

$\lim\limits_{n\to\infty} a_{2n} \neq \lim\limits_{n\to\infty} a_{2n+1}$

$= \{(-1)^n \frac{3n^3 - 7n - 1}{}$? divergent

$\boxed{7}$ $a_n = \begin{cases} 2 + \frac{1}{n} & n \text{ even} \\ \frac{3n-1}{n^3} & n \text{ odd} \end{cases}$

$\lim_{n\to\infty} \overset{\text{زوجي}}{a_{2n}} = \lim 2 + \frac{1}{n} = 2$

$\lim_{n\to\infty} a_{2n+1} = \lim_{n\to\infty} \frac{3n-1}{n^3} = 0$

$\therefore \{a_n\}$ divergent

$\boxed{8}$ $\left\{ \left(1 - \frac{3}{7n}\right)^{2n} \right\}_{n=1}^{\infty}$ بالتعويض بطلع $(1)^{\infty}$ وهاي احدى حالات لوبيتال بركبلها (Ln) وبكمل ⇒

أو عن طريق قاعدة $\left[\lim_{n\to\infty} \left(1 + \frac{a}{n}\right)^{bn} = e^{ab} \right]$ [illegible]

$\therefore \lim_{n\to\infty} \left(1 - \frac{3}{7n}\right)^{2n} = e^{\frac{-3}{7} \times 2} = e^{-\frac{6}{7}}$ $\therefore$ convergent to $e^{-\frac{6}{7}}$

$\boxed{9}$ $a_n = \left(\frac{n+3}{n-5}\right)^{7n}$ بالتعويض $(1)^{\infty}$ ⇐ لوبيتال وبركب (ln)

ع القاعدة. $\therefore \lim_{n\to\infty} \left(\frac{n+3}{n-5}\right)^{7n} = \lim \frac{\cancel{n}\left(1 + \frac{3}{n}\right)^{7n}}{\cancel{n}\left(1 - \frac{5}{n}\right)^{7n}}$

$= \frac{\lim_{n\to\infty} \left(1 + \frac{3}{n}\right)^{7n}}{\lim_{n\to\infty} \left(1 - \frac{5}{n}\right)^{7n}} = \frac{e^{21}}{e^{-35}} = e^{56}$

هل con ولا div

* $\{a^n\}_{n=1}^{\infty}$

هون منعتمد على الحد اللي تحت مثلاً لو كان (1)

$(1)^\infty = 1$ ولو كان $(-1) \Leftarrow (-1)^\infty = \text{div}$

لو كان $(\frac{1}{2}) \Leftarrow (\frac{1}{2})^n = \frac{1}{2^n} = \frac{1}{\infty} = \text{zero}$

لو كانت $(7) \Leftarrow (7)^\infty = \infty \Leftarrow \text{div}$

↳ convergent if $-1 < a \leq 1$

↳ divergent $a \leq -1$ or $a > 1$

Ex:-

[10] $a_n = \frac{1}{5^n} \Rightarrow \lim\limits_{n\to\infty} = (\frac{1}{5})^n$ ∴ converge to zero

العددية (-1,1)

ودائماً كل الكسور بتكون converge to $\underline{\underline{0}}$ وحالة خاصة عند (1) بتكون converg to $\underline{\underline{1}}$

$\{(1)^n\}$

[11] $\lim (1)^n$ = converge to $\underline{\underline{1}}$

[12] $\{(-1)^n\}$ divergent.

[13] $\left\{\frac{3^n}{e^n}\right\}_{n=1}^{\infty}$ conv or div ??

$$\lim \frac{3^n}{e^n} = \lim_{n\to\infty} \left(\frac{3}{e}\right)^n = \infty \quad \therefore \text{divergent.}$$

الجواب أكبر من (1)

لو العكس $(\frac{e}{3})^n$ بتكون convergent

[14] $\left\{\frac{7^{n+1}}{10^n}\right\}_{n=1}^{\infty} \Rightarrow \frac{7^n \cdot 7}{10^n} = 7\left(\frac{7}{10}\right)^n$

$$\lim \frac{7^{n+1}}{10^n} = \lim_{n\to\infty} 7 \cdot \left(\frac{7}{10}\right)^n = 7 \lim \left(\frac{7}{10}\right)^n = 7 * 0 = 0$$

∴ converge to zero.

$\boxed{15}$ $a_n = \frac{(-2)^n}{\pi^n} = \left(\frac{-2}{\pi}\right)^2 = \lim_{n\to\infty} \left(\frac{-2}{\pi}\right)^n = 0$

لو العدد بين (1,-1)

$\therefore$ converg to zero.

$\boxed{16}$ $\left\{(5^n + 7^n)^{\frac{2}{n}}\right\}_{n=1}^{\infty}$

بقدر استخدم لوبيتال.

بطلع العدد الأكبر عامل مشترك

$$\left(7^n\left(\frac{5^n}{7^n} + 1\right)\right)^{\frac{2}{n}} = 7^{n \times \frac{2}{n}} * \left(\left(\frac{5^n}{7}\right) + 1\right)^{\frac{2}{n}}$$

$$= 49 * \left(\left(\frac{5}{7}\right)^n + 1\right)^{\frac{2}{n}}$$

$$49 \cdot \lim_{n\to\infty} \left(\left(\frac{5}{9}\right)^n + 1\right)^{\frac{2}{n}} = 49\,(0+1)^0 = 49 \times 1^0 = 49$$

$\therefore$ converge to $\underline{\underline{49}}$

$\boxed{17}$ $\left\{\sin n\right\}_{n=1}^{\infty}$ $\lim_{n\to\infty} \sin n = \text{d.n.e.}$ $\therefore$ divergent.

$\boxed{18}$ $a_n = \frac{\cos 2n}{n+1}$

صورة لما تكون عندي $\frac{\sin \text{ أو } / \cos}{\text{Poly}}$ نستخدم

squeezing Thm

$$b_n \le a_n \le c_n$$

b_n ⇓ Conveng to (L)

a_n ↓ فأكيد ستكون Converg to (L)

c_n ↓ Converg to (L)

بعوض lim

$$\frac{-1}{n+1} \le \frac{\cos 2n}{n+1} \le \frac{1}{n+1}$$

$$\lim_{n\to\infty} \frac{-1}{n+1} = \text{zero} \;/\; \lim_{n\to\infty} \frac{1}{n+1} = \text{zero}$$

$\therefore \lim \frac{\cos 2n}{n+1} = \underline{\underline{\text{zero}}}$ $\therefore$ converg to zero.

$\boxed{19}$ $\left\{\frac{n(\cos^2 n + n)}{n^2+1}\right\}_{n=1}^{\infty}$

squezzing ← $= \frac{n\cos^2 n}{n^2+1} + \frac{n^2}{n^2+1}$ → $\frac{\text{poly}}{\text{poly}} = \frac{1}{1} = 1$

$$0 \le \cos^2 n \le 1$$

$$0 \le n\cos^2 n \le n \Rightarrow \underset{\downarrow\, 0}{0} \le \frac{n\cos^2 n}{n^2+1} \le \underset{\downarrow\, 0}{\frac{n}{n^2+1}}$$

لأنه درجة البسط أقل من المقام

$\therefore \lim_{n\to\infty} \frac{\cos 2n}{n^2+1} = \text{zero}$

$\therefore \lim \frac{n\cos^2 n}{n^2+1} + \frac{n^2}{n^2+1} = 0 + 1 = 0$

$\therefore$ converg to 1

$\boxed{20}$ $a_n = \cos n\pi$

في حالة كان في (π) مع cos او sin
السهل اني افكها.

$a_1 = \cos \pi = -1$

$a_2 = \cos 2\pi = 1$

$a_3 = \cos 3\pi = -1$

∴ اذا المتتالية $(-1, 1, -1, 1)$

اذاً هي نفسها $(-1)^n$

∴ divergent

$\boxed{21}$ $a_n = \sin n\pi$

$a_1 = 0$ $a_3 = 0$

$a_2 = 0$ $a_4 = 0$

∴ $\lim\limits_{n \to \infty} 0 = 0$

∴ converge to zero.

[H.W] page 700

16 / 17 / 18 / 27 / 31 / 36 / 38 / 39 / 42 / 43 / 48 / 51

اختياري (3 - 56)

$\boxed{22}$ Find the value of b such that the seq $\left\{ \frac{b^n}{7^n} \right\}_{n=1}^{\infty}$ is convergent

Sol: $\left(\frac{b}{7}\right)^n$ Convergent $-1 < \frac{b}{7} \leq 1$

$b \in (-7, 7]$ $-7 < b \leq 7$

$\boxed{23}$ $\boxed{22}$ نفس سؤال

H.W $\left\{ \frac{2b^{n+1}}{\pi^n} \right\}_{n=1}^{\infty}$

[recursive sequences] صورة كل حد يعتمد على قيمة الحد اللي قبله

Ex :- $a_1 = 3$ $\xrightarrow{\text{الحد اللي بعد}}$ $a_{n+1} = \dfrac{1}{a_n - 1}$

$\therefore\ a_1 = 3$ / $a_2 = \dfrac{1}{3-1} = \dfrac{1}{2}$ / $a_3 = \dfrac{1}{\frac{1}{2}-1} = \dfrac{1}{-\frac{1}{2}} = -2$

$a_4 = \dfrac{1}{-2-1} = \dfrac{-1}{3}$

$* \lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} \underbrace{a_{n+1}}_{\text{اداي اشي}}$] هاي القاعدة اللي راح نعتمد عليها بالحل

Ex :- $\boxed{1}$ let $a_1 = \sqrt{6}$ $a_{n+1} = \sqrt{6 + a_n}$ and ~~determine~~. find

find the first $\underline{\underline{5}}$ terms

$\lim\limits_{n\to\infty} a_n$

$\therefore\ a_1 = \sqrt{6}$ / $a_2 = \sqrt{6+\sqrt{6}}$ / $a_3 = \sqrt{6+\sqrt{6+\sqrt{6}}}$

$a_4 = \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}$ $a_5 = \sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6+\sqrt{6}}}}}$

$\lim\limits_{n\to\infty} a_n = L = \lim\limits_{n\to\infty} a_{n+1}$

$\lim\limits_{n\to\infty} a_{n+1} = \lim\limits_{n\to\infty} \sqrt{6 + a_n}$

$L = \sqrt{6 + L} \Rightarrow L^2 = 6 + L$

$L^2 - L - 6 = 0$

$(L-3)\ (L+2)$

$\therefore$ converge to $\underline{\underline{3}}$ $L = 3$ $\boxed{L = -2}$ X

راح ارفضه لأن كل القيم اللي معي بالسؤال موجبة

$\boxed{2}$ $a_1 = 1$ $\qquad a_{n+1} = \frac{1}{1+a_n}$

$a_1 = 1$ / $a_2 = \frac{1}{2}$ / $a_3 = \frac{1}{1+\frac{1}{2}} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$

$a_4 = \frac{1}{1+\frac{2}{3}} = \frac{1}{\frac{5}{3}} = \frac{3}{5}$ / $a_5 = \frac{1}{1+\frac{3}{5}} = \frac{1}{\frac{8}{5}} = \frac{5}{8}$

$$\lim_{n\to\infty} a_{n+1} = \lim_{n\to\infty} \frac{1}{1+a_n}$$

$$L \cancel{=} \frac{1}{1+L} \Rightarrow L + L^2 = 1$$

$$L^2 + L - 1 = 0$$

$$L = \frac{-1 \pm \sqrt{1 - 4 \times 1 \times (-1)}}{2 \times 1}$$

Converge to $\frac{-1+\sqrt{5}}{2}$

$$L = \frac{-1 \pm \sqrt{5}}{2}$$

القيم كلها موجبة اذاً نهمل السالب

H.W $\boxed{1}$ $a_1 = 1$ $\qquad a_{n+1} = 3 - \frac{1}{a_n}$ $\qquad$ * أول (5) حدود.

$\boxed{2}$ $a_1 = 1$ / $a_2 = 5$ $\qquad a_n = \frac{a_{n-1}}{3 + a_{n-2}}$

$$a_3 = \frac{a_{3-1}}{3 + a_{3-2}} = \frac{a_2}{3+a_1} = \frac{5}{3+1} = \frac{5}{4}$$

$$a_4 = \frac{a_3}{3+a_2} = \frac{\frac{5}{4}}{3+5} = \frac{5}{32}$$

$$\lim_{n\to\infty} a_n = \lim \frac{a_{n-1}}{3 + a_{n-2}}$$

$$L = \frac{L}{3+L} \Rightarrow 3L + L^2 = L$$

$$2L + L^2 = 0$$

$$\underset{\checkmark}{L=0} \;/\; \underbrace{L = -2}_{\times}$$

65

Monotone sequence :-

Def A seq $\{a_n\}_{n=1}^{\infty}$ is called

1) increasing if $a_1 \leq a_2 \leq a_3 \leq \cdots \leq a_n \leq \cdots$

2) decreasing if $a_1 \geq a_2 \geq a_3 \geq \cdots \geq a_n \geq \cdots$

3) a seq that either increasing or decreasing is said to be Monotone

Ex :- $\boxed{1}$ $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots\ldots$ القانون $\Rightarrow \frac{n}{n+1}$

$\therefore$ increasing

$\boxed{2}$ $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots, \frac{1}{n}$

$\therefore$ decreasing

$\boxed{3}$ $1,1,1, 2,2,2, 3,3,3, \ldots$

$\therefore$ increasing.

$\boxed{4}$ $1,1,1,1,1,1, \ldots$

$\therefore$ increasing and decreasing.

$\boxed{5}$ $1, \frac{-1}{2}, \frac{1}{3}, \frac{-1}{4}, \frac{1}{5}, \frac{-1}{6}$

$\therefore$ neither increasing nor decreasing

$\therefore$ is not Monoton.

* لحتى اعرف اذا increasing or decreasing

		increasing	decreasing
	$\boxed{1}$	$a_1 \leq a_2 \leq a_3$	$a_1 \geq a_2 \geq a_3 \geq \cdots \geq a_n \geq a_{n+1}$
طرح	$\boxed{2}$	$a_{n+1} \ominus a_n \overset{\text{موجب}}{\geq} 0$	$a_{n+1} \ominus a_n \overset{\text{سالب}}{\leq} 0$
قسمة (في اسس) طريقة	$\boxed{3}$	$\frac{a_{n+1}}{a_n} \geq 1$	$\frac{a_{n+1}}{a_n} \leq 1$
عن طريق المشتقة (دوال)	$\boxed{4}$	$a_n = f(n)$ $f'(n) > 0$	$f'(n) < 0$

Ex :- $\boxed{1}$ $\{n+(-1)^n\}_{n=1}^{\infty}$ لو بدي احسب الحد الاول واعوض (1) = صفر

$\overset{a_1}{0}, \overset{a_2}{3}, \overset{a_3}{2}, \overset{a_4}{5}, \overset{a_5}{4}$ $\therefore$ not Monotonic.

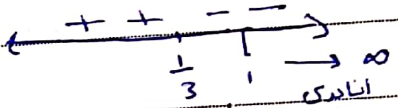
عن طريقة الاشتقاقة $\boxed{2}$ $\{ne^{-3n}\}_{n=1}^{\infty}$

$f(x) = xe^{-3x}$

$f'(x) = -3xe^{-3x} + e^{-3x}$

$e^{-3x}(-3x+1) = 0$

$x = +\frac{1}{3}$



انا بدي من (1) لل ∞ وهي سالبة

$\therefore \{ne^{-3n}\}_{n=1}^{\infty}$ is decreasing.

$\boxed{3}$ $\left\{\frac{n}{2n-1}\right\}_{n=1}^{\infty}$ تنحل عن طريق بالقسمة.

الحل عنه طريق القسمة ⇐ يعني مكان كل (n) نحط n+1

$$\frac{a_{n+1}}{a_n} = \frac{\frac{n+1}{2(n+1)-1}}{\frac{n}{2n-1}} = \frac{\frac{n+1}{2n+1}}{\frac{n}{2n-1}} = \frac{n+1}{2n+1} \times \frac{2n-1}{n}$$

$$= \frac{2n^2+2n-2n-1}{2n^2+n}$$

$$= \frac{2n^2+n-1}{2n^2+n}$$

البسط اقل من المقام لواحد يعني العدد اقل من (1)

$\therefore$ decreasing

$\boxed{4}$ $\left\{\frac{n^n}{n!}\right\}_{n=1}^{\infty}$

قسمة المضروب بعضه فنشتغل على القسمة

$a_{n+1} = \frac{(n+1)^{n+1}}{(n+1)!}$

بدل كل (n) نحط n+1

$$\frac{a_{n+1}}{a_n} = \frac{\frac{(n+1)^{n+1}}{(n+1)!}}{\frac{n^n}{n!}} = \frac{(n+1)^{n+1}}{(n+1)\cancel{n!}} \cdot \frac{\cancel{n!}}{n^n}$$

$$= \frac{(n+1)^n}{n^n} = \left(\frac{n+1}{n}\right)^n$$

but $\frac{n+1}{n} > 1$

$(n+1)! \Rightarrow (n+1)\,n!$

يعني بتعمل زي ما بدي وبحط آخر شي (!)

$\therefore$ increasing

$\boxed{5}$ $\left\{ \tan^{-1} n \right\}_{n=1}^{\infty}$

ما بتزبط فشتقها بحل ع المشتقة

$f(x) = \tan^{-1} x$

$f'(x) = \dfrac{1}{1+x^2}$ $\quad \longleftrightarrow{+ \; + \; + \; + \; +}$

$\therefore$ increasing . seq

[H.W]

$\boxed{6}$ $\left\{ \dfrac{\ln (n+n)}{(n+n)} \right\}_{n=1}^{\infty}$ باستخدام المشتقة.

$\boxed{7}$ $\left\{ \dfrac{10}{n!} \right\}_{n=1}^{\infty}$ باستخدام القسمة

$\boxed{8}$ $\left\{ (n-8)^2 \right\}_{n=1}^{\infty}$

* page 701
(72 - 78)

نهاية مادة السكند

11.2 Infinite series.

$a_1 , a_2 , a_3 , \dots\dots , a_n$

$a_1 + a_2 + a_3 + \dots\dots + a_n + \dots = \sum_{i=1}^{\infty} a_i$

المجموع يُسمى ← infinite Series.

$\sum_{i=1}^{\infty} a_i = a_1 + a_2 + \dots$

$$\underbrace{\text{convergent} \qquad\qquad \text{divergent}}$$

convergent — اذا كان المجموع موجود.

divergent — اذا كان ∞ أو -∞

Ex $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{L}{5} + \dots$

$= \sum_{n=1}^{\infty} \frac{1}{n}$ تُسمى harmonic series

$\therefore$ divergent.

* seq of partial sem

$\sum_{n=1}^{\infty} an = a_1 + a_2 + a_3 + \dots + a_n + \dots$

عنصر اول $S_1 = a_1$. عنصر ثاني $S_2 = a_1 + a_2 = S_1 + a_2$

$S_3 = a_1 + a_2 + a_3 = a_3 + S_2$ $\qquad a_n = a_1 + a_2 + \dots + a_n = \sum_{n=1} a_i$

وهذا صار عندي seq

$S_1 , S_2 , S_3 , \dots\dots$ $\qquad \lim_{n \to \infty} S_n = \lim_{n \to \infty} \sum_{i=1}^{n} a_i$

Thm If the seq of partial sem $\{S_n\}$ is convergent to (L) then $\sum_{i=1}^{\infty} a_i = \underline{\underline{L}}$ (converg)

- If the seq $\{S_n\}$ is divergent then the series $\sum_{i=1}^{\infty} a_i$ divergent

69

Ex $\sum_{i=1}^{\infty} (-1)^i = -1 + 1 - 1 + 1 - 1 + 1 - 1 + 1$

$S_1 = -1$ $S_2 = -1 + 1 = 0$ $S_3 = -1 + 1 - 1 = -1$

مهم جدا المثال

$S_4 = -1 + 1 - 1 + 1 = 0$

متغير seq $= -1, 0, -1, 0, \ldots$

$\lim_{n \to \infty} S_n = \text{d.n.e} \Rightarrow \sum_{i=1}^{\infty} (-1)^i$ is divergent

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Ex If the partiel sum of the series $\sum_{K=1} a_K$ is

$S_k = \frac{9 + 4K}{3 + 2K}$ find $\sum_{K=1}^{\infty} a_K$

اذا ال seq الها جواب ال (Σ) الها جواب بتوجد

$\sum_{k=1}^{\infty} a_k = \lim_{K \to \infty} \frac{9 + 4K}{3 + 2K} \frac{\text{Poly}}{\text{poly}} = \frac{4}{2} = 2$

Ex $\sum_{K=1}^{\infty} \frac{1}{K(K+1)}$ determine whether the series converg or div

$\frac{1}{K(K+1)} = \frac{A}{K} + \frac{B}{K+1} \Rightarrow A(K+1) + B(K) = 1$

$A = 1$ مثلاً العوض صفر

$B = -1$ (1) //

$\sum_{K=1} \frac{1}{K(K+1)} = \sum_{K=1}^{\infty} \frac{1}{K} - \frac{1}{K+1}$

$S_1 = a_1 = \boxed{1 - \frac{1}{2}}$ $S_2 = a_1 + a_2 = \underbrace{1 - \cancel{\frac{1}{2}}}_{a_1} + \cancel{\frac{1}{2}} - \frac{1}{3} = \boxed{1 - \frac{1}{3}}$

$S_3 = S_2 + a_3 = \underbrace{1 - \cancel{\frac{1}{3}}}_{S_2} + \cancel{\frac{1}{3}} - \frac{1}{4}$ $S_4 = S_3 + a_4 = 1 - \cancel{\frac{1}{4}} + \cancel{\frac{1}{4}} - \frac{1}{5}$ $\boxed{1 - \frac{1}{5}}$

$S_n = 1 - \frac{1}{n+1}$

$\therefore \sum_{K=1}^{\infty} \frac{1}{K(K+1)} = \lim_{K \to \infty} S_K = \lim_{K \to \infty} 1 - \frac{1}{K+1} = 1$

$\therefore \sum_{K=1}^{\infty} \frac{1}{K(K+1)} = 1$ $\therefore$ converg to 1

~~Ex~~ H.W

$$\sum_{x=3}^{\infty} \frac{1}{k-2} - \frac{1}{k+2}$$

الجواب: $= \frac{25}{48}$

* Geometric series :-

$$\sum_{\boxed{k=0}}^{\infty} (\overset{\text{عدد}}{r})^{\boxed{k}} = 1 + r + r^2 + r^3 + r^4$$

لازم يبلش من الصفر

القوة لازم تكون k فقط

Ex
$1 + 2 + 4 + ~~8~~ + 8 + 16$

$1 + \frac{1}{2} + \frac{1}{8} + \frac{1}{16}$

$$* \quad \sum_{k=1}^{\infty} (r)^k \begin{cases} \text{converg} & \frac{1}{1-r} \qquad -1 < r < 1 \\ \text{(divergent)} & r \geq 1 \quad \text{or} \quad r \leq -1 \end{cases}$$

Ex [1]
$$\sum_{k=0}^{\infty} \left(\frac{1}{3}\right)^k = \frac{1}{1-r} = \frac{1}{1-\frac{1}{3}} = \frac{3}{2}$$

بين (-1,1)

[2]
$$\sum_{\boxed{k=3}}^{\infty} a^k \longrightarrow \sum_{n=0}^{\infty} a^{n+3} \qquad n = k-3$$

لازم يكون صفر

[3]
$$\sum_{0}^{\infty} a^{k+3} = a^3 \sum_{0}^{\infty} a^k$$

$$* \quad a^{m+n} = a^m \cdot a^n$$
$$a^{m-2} = \frac{a^m}{a^n}$$

القوة هو (k) لحال

[4]
$$\frac{\boxed{3^k \cdot 7^k}}{5^k} \Rightarrow a^n b^n = (ab)^n$$

لازم يكون عندي عدد واحد فقط

$$\left(\frac{21}{5}\right)^k$$

(Thm) If $\sum_{k=i}^{\infty} a_k = L$ and $\sum_{k=i}^{\infty} b_k = m$ and c constant then

$$\sum a_k + b_k = \sum a_k + \sum b_k$$

$$\sum c a_k = c \sum a_k$$

Ex $\boxed{1}$ $\sum_{K=0}^{\infty} 5 \frac{1}{4^K} = 5 \sum_{K=0}^{\infty} \underbrace{\left(\frac{1}{4}\right)^K}_{\text{conv}} = 5\left(\frac{1}{1-\frac{1}{4}}\right) = \frac{20}{3}$

$\boxed{2}$ $\sum_{K=1}^{\infty} 3^{2K} \cdot 5^{1-K}$ $\qquad 1+n=K$

لازم ترجع لصفر

$\sum_{n=0}^{\infty} 3^{2(n+1)} \cdot 5^{\cancel{1}-n-\cancel{1}} \Rightarrow \sum_{n=0}^{\infty} \frac{3^{2n}\,\cdot 3^2}{5^{n}}$

$9 \sum_{n=0}^{\infty} \frac{(3^2)^n}{5^n} = \underbrace{\left(\frac{9}{5}\right)^n}_{\text{اكبر من 1}}$

$\therefore$ divergent

$\boxed{3}$ $\sum_{K=2}^{\infty} \frac{2^K \cdot 3^{K-2}}{7^{K+1}}$ $\qquad \boxed{K=n+2}$

$\sum_{n=0}^{\infty} \frac{2^{n+2} \cdot 3^n}{7^{n+3}} \Rightarrow \sum_{n=0} \frac{3^n \cdot 2^2 \cdot 2^n}{7^n \cdot 7^3} = \frac{2^2}{7^3} \cdot \sum_{n=0}^{\infty} \left(\frac{3 \cdot 2}{7}\right)^n$

$\frac{4}{7^3} \sum_{n=0}^{\infty} \underbrace{\left(\frac{6}{7}\right)^n}_{\text{أقل من واحد}} \Rightarrow \frac{4}{7^3}\left(\frac{1}{1-\frac{6}{7}}\right) = \frac{4}{7^{\cancel{3}\,2}} \times \cancel{7}$

$= \frac{4}{49}$

$\boxed{4}$ $\sum_{n=1}^{\infty} \frac{7^{n+1}}{10^n}$ $\qquad (t+1=n)$

$\sum_{t=0}^{\infty} \frac{7^{t+2}}{10^{t+1}} = \sum_{t=0} \frac{7^t \cdot 7^2}{10^t \cdot 10} = \frac{49}{10} \sum_{t=0}^{\infty} \underbrace{\left(\frac{7}{10}\right)^n}_{\text{أقل من واحد}}$

اذًا بنطبق القاعدة

$\frac{49}{10}\left(\frac{1}{1-\frac{7}{10}}\right) = \frac{49}{10}\left(\frac{1}{\frac{3}{10}}\right) = \frac{49}{\cancel{10}} \times \frac{\cancel{10}}{3} = \frac{4}{3}$

(H.W)

$\boxed{5}$ $\sum_{n=0}^{\infty} \frac{1+2^n}{3^n} \Rightarrow \sum_{n=0}^{\infty} \frac{1}{3^n} + \sum_{n=0}^{\infty} \frac{2^n}{3^n}$

$\boxed{6}$ $\sum_{n=1}^{\infty} (0.8)^{n-1} - (0.3)^n$ $\qquad \boxed{7}$ $\sum_{K=0}^{\infty} \left(\frac{\pi}{3}\right)^K$

$\boxed{8}$ $\sum_{K=1}^{\infty} \frac{1}{e^n} + \frac{1}{n(n+1)}$ $\qquad \boxed{9}$ $\sum_{K=1}^{\infty} \frac{1}{\sqrt{K}} - \frac{1}{\sqrt{K+1}}$ $\qquad$ 72

$\boxed{10}$ $\sum_{K=1}^{\infty} \frac{1}{3K} - \frac{1}{3^{n+1}}$

* $\boxed{\text{Thm}}$:- $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent → general (harmonic) sereis.

بشكل عام
in general $\sum \frac{\text{عدد}}{[illegible]}$ ⇒ divergent.

$\boxed{11}$ $\sum_{n=3}^{\infty} \frac{7}{2n+1}$ ⇒ divergent

* divergent test

$\boxed{\text{Thm}}$ if $\sum_{n=1}^{\infty} a_n$ is convergent then $\lim_{n\to\infty} a_n = 0$

اذا كان (conv) اكيد ال (lim) صفر ، واذا (lim) مو صفر فأكيد (div).

$\boxed{\text{Thm}}$ If $\lim_{n\to\infty} a_n$ d.n.e or $\lim_{n\to\infty} a_n \neq 0$ then $\sum_{n=1}^{\infty} a_n$ is divergent.

ما بدقق ع [illegible]

عادي شو ما تكون

Ex $\boxed{13}$ $\sum_{n=1}^{\infty} \frac{n^2+1}{3n^2-2}$ $\therefore \lim_{n\to\infty} \frac{n^2+1}{3n^2-2} = \frac{1}{3} \neq 0$

→ divergent.

$\boxed{14}$ $\sum \frac{1+3^n}{2^n}$ $\xrightarrow{\text{Geometric لو احلل}}$ $\underset{\text{con.}}{\frac{1}{2^n}} + \underset{\text{div}}{\frac{3^n}{2^n}}$ ✗

$\lim_{n\to\infty} \frac{1}{2^n} + \left(\frac{3}{2}\right)^n$
$= 0 + \infty = \infty$ $\therefore$ divergent.
ما بساوي صفر.

$\boxed{15}$ $\sum_{n=1}^{\infty} \frac{1}{n} - \frac{1}{n+1}$ → هون التنتين (harmonic) يعني اتنين بس بدي اشوف طرحهم (Converg)

$S_1 = 1-\frac{1}{2}$ $S_2 = 1-\frac{1}{3}$ $S_n = 1-\frac{1}{n}$ → $\underline{1}$ ليس صفر.

$\boxed{16}$ $\sum_{n=1}^{\infty} \frac{e^n}{n^2}$ $\lim_{n\to\infty} \frac{e^n}{n^2} = \infty$ (div)

[illegible]
x^x
$x!$
e^x
poly
Log

اذا كانت العلاقة بهاد الاتجاه الجواب $\boxed{\infty}$ ↓

↑ واذا بهاد الجواب $\boxed{0}$

$\boxed{17}$ $\sum_{n=1}^{\infty} \sqrt[n]{2}$ $\lim_{n\to\infty} \sqrt[n]{2} = \lim_{n\to\infty} 2^{\frac{1}{n}}$

$\ln y = \lim_{n\to\infty} \frac{1}{n} \ln 2 = \text{zero}$

$y = e^0 = 1$ $\therefore$ divergent

قاعدة
$\lim_{x\to\infty} \frac{\ln x}{x} = 0$
$\lim_{x\to\infty} \frac{x}{\ln x} = \infty$

[H.W] $\boxed{18}$ $\sum_{n=1}^{\infty} \ln\left(\frac{n^2+1}{2n^2+1}\right)$ $\boxed{19}$ $\sum_{n=1}^{\infty} \tan^{-1}(n)$

* Integral test :-

$\sum_{k=1}^{\infty} a_k$

$a_k = f(x)$

$$\left[\begin{array}{l} 1)\ f(x) > 0 \\ 2)\ f(x)\ \text{decreasing} \\ 3)\ f(x)\ \text{cont} \end{array}\right]$$

الشروط عشان اطبق ال test

test $\int_1^{\infty} f(x)\,dx$ is convergent if and only if $\sum_{n=1}^{\infty} a_n$ converg

Ex $\boxed{20}$ Show that $\sum_{n=1}^{\infty} \frac{1}{n}$ is divergent

بستخدم (Integral test) لا اشوف انه الاقتران قابل للتكامل

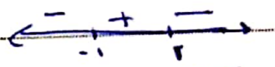
$f_x = \frac{1}{x}$

$\int_1^{\infty} \frac{1}{x}\,dx =$ [illegible]

$\lim_{b\to\infty} \int_1^{b} \frac{1}{x}\,dx$

1) $f(x)$ cont on $[1,\infty)$

2) $f'(x) = \frac{-1}{x^2}$ عشان اكون متناقص



3) $f(x) > 0$

$x \geq 1 \overset{\text{اذا}}{\Longrightarrow} \frac{1}{x} > 0$

$\lim_{b\to\infty} \ln x |_1^{b} = \lim_{b\to\infty} \ln b = \infty \therefore$ divergent

$\boxed{21}$ $\sum_{k=1}^{\infty} \frac{1}{k^2} =$ منطبق الشروط وبطلع كلهم صح

$\lim_{b\to K} \int_1^{b} \frac{1}{x^2}\,dx$

مهمة

* $\boxed{\text{Thm}}$ (p-series)

$$\sum_{k=1}^{\infty} \frac{1}{k^p} \Rightarrow \begin{cases} \text{convergent} & p > 1 \\ \text{divergent} & p \leq 1 \end{cases}$$

Ex $\boxed{22}$ $\sum_{k=1}^{\infty} \frac{1}{\sqrt[3]{k}} \Rightarrow$ (p-series) $p = \frac{1}{3}$

$\therefore$ divergent.

$\boxed{23}$ $\sum_{k=1}^{\infty} \frac{1}{\sqrt[5]{k^7}}$ (p-series) $p = \frac{7}{5}$

$\therefore$ convergent

$\boxed{24}$ $\sum_{k=1}^{\infty} \frac{7}{k^3}$ (p-series) $p = 3$

$\therefore$ convergent.

لازم اكتبها تاكد منهم

$\boxed{25}$ $\sum_{n=1}^{\infty} \frac{1}{n^2+1} \Rightarrow f(x) = \frac{1}{x^2+1}$

* check 1) decreasing
2) $f_x > 0$
3) cont

$$\int_1^{\infty} \frac{1}{x^2+1}\,dx \qquad \lim_{b\to\infty} \int_1^{b} \frac{1}{x^2+1}\,dx = \lim_{b\to\infty} \tan^{-1}x \Big|_1^b$$

$$= \lim_{b\to\infty} \tan^{-1}b - \tan^{-1}1 = \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4} \qquad \therefore \underline{\underline{\text{Conv}}}$$

$\therefore \sum_{n=1}^{\infty} \frac{1}{n^2+1}$ $\underline{\underline{\text{convergent}}}$ → بس ما بحكي (conv to $\frac{\pi}{4}$) ما بعرف الا انها (conv) زي النهاية اما ($\frac{\pi}{4}$) ما دخلها

الشروط

note :-

1) $\sum_{n=b}^{\infty} a_n$ → بيصير من (b) ∞ N
→ decreasing
→ $f(x) > 0$

واذا $\therefore \int_b^{\infty} \text{conv} \overset{\text{اكيد}}{\Longrightarrow} \underline{\underline{\sum \text{conv}}}$

2) $\sum_{n=1}^{\infty}$ conv $\longleftrightarrow$ $\sum_{n=k} a_n$

يعني اذا كانت (∑) اولها موجبة متزايد او متناقص بوخذ من اي مكان لاي فترة ونشتغل عليها واذا كانت ما في الفترة (con) اذا اكيد الاولى conv

يعني اذا الشروط ما تحققت من اول شي بعدين بلشت تتحقق بوخذ ما بين الفترة ونشتغل عليها.

26 $\sum_{n=1}^{\infty} \frac{\ln n}{n} \Rightarrow \int_1^{\infty} \frac{\ln x}{x}$

check
f_x : cont
decreasing
$f_x > 0$

$$\lim_{b\to\infty} \int_1^{b} \frac{\ln x}{x} \qquad y = \ln x$$

$$\int_1^{b} y\,dy$$

$$\lim_{b\to\infty} \frac{y^2}{2}\Big|_{\ln 2}^{\ln b} = \lim_{b\to\infty} \frac{\ln(b)^2}{2} - \frac{\ln 2}{2} = \infty \quad \therefore \text{divergent}$$

$\therefore \sum_{n=1}^{\infty} \frac{\ln n}{n}$ divergent

لازم اشيك ع الشروط .
(check).

$\boxed{27}$ $\sum_{n=3}^{\infty} ne^{-n}$

$$\int_{3}^{\infty} x e^{-x}\, dx = \lim_{b\to\infty} \int_{3}^{b} x e^{-x}$$

x	e^{-x}
1	$-e^{-x}$
0	e^{-x}

$$\lim_{b\to\infty} \left(-xe^{-x} - e^{-x}\right)\Big|_{3}^{b}$$

$$= \lim_{b\to\infty} \left(-be^{-b} - e^{-b}\right) - \left(-3e^{-3} - e^{-3}\right)$$

$$= \frac{-b}{e^{b}} - \frac{1}{e^{b}} + \frac{3}{e^{3}} + \frac{1}{e^{3}} \qquad = 3e^{-3} + e^{-3} \quad \therefore \text{conv}$$

$$= 0 - 0$$

$$\Longrightarrow \sum_{n=3}^{\infty} ne^{-n} \text{ is convergent}$$

H.W

$\boxed{28}$ $\sum_{K=1}^{\infty} \frac{K}{K^2+1}$ البسط مشتقة المقام .

$\boxed{29}$ $\sum_{n=2}^{\infty} \frac{\text{Ln } n}{n^2}$

$\boxed{30}$ $\sum_{n=3}^{\infty} \frac{1}{n(\text{Ln}n)(\text{Ln}(\text{Ln}n))}$

$\boxed{31}$ Find the values of (p) for which the series is convergent $\sum_{K=2}^{\infty} \frac{1}{K(\text{Ln}K)^{p}}$

Sol ~~31~~ 31 :-

$$= \lim_{K\to\infty} \int_{2}^{\infty} \frac{1}{x(\text{Ln}x)^{p}} \qquad y = \ln x \qquad dy\ \frac{1}{x}\, dx$$

$$\lim_{w\to\infty} \int_{2}^{w} \frac{1}{x(\text{Ln}x)^{p}} \Longrightarrow \lim_{w\to\infty} \int_{\ln 2}^{\text{Ln}w} \frac{1}{y^{p}}\, dy$$

$$\lim_{w\to\infty} \int_{\text{Ln}2}^{\text{Ln}w} y^{-p}\, dy \Longrightarrow \lim_{w\to\infty} \frac{y^{-p+1}}{-p+1}\Big|_{\text{Ln}2}^{\text{Ln}w}$$

$$\lim_{w\to\infty} \frac{(\ln w)^{1-p}}{1-p} - \frac{(\text{Ln}\,2)^{1-p}}{1-p}$$

هاي صعودها (∞) وطول ما هي بالبسط راح يضل div اذا صارت بالمقام بصير conv

$$\therefore (1-p) < 0 \qquad \therefore p > 1 \to (1, \infty)$$

للمصدر العوة تكون سالبة لتنزل للمقام .

$$1 < p$$

(H.W)

$\boxed{32}$ $\sum_{k=1}^{\infty} k(1+k^2)^p$ نفس سؤال $\underline{31}$

$\boxed{33}$ $\sum_{k=1}^{\infty} \frac{5-2\sqrt{k}}{k^3}$

---

* Limit comparson test

لما يكون $\frac{\text{poly}}{\text{poly}}$ او عادي اذا جذر بحل عليها ← بستخدمها.

suppose that $\sum_{k=1}^{\infty} a_k$ and $\sum b_k$ be series with positive terms.

لو عندك كم حد سالب عادي بيبتش من الحد الموجب ل آخرها.

$\lim_{k\to\infty} \frac{a_k}{b_k} = C < \infty$ and $C \neq 0$ اذا كان الجواب ثابت

واذا طلع (0) او (∞) اذًا انا اخترت اختيار خاطئ لأنه السؤال بيجيلي وحده دالتانيه انا بألفها

اذا تحققت الشروط then either both series convergent or divergent.

Ex $\boxed{34}$ $\sum_{k=1}^{\infty} \underbrace{\frac{1}{2k^2+k}}_{a_k \text{ بسميها}}$

دائمًا نوجد $\frac{\text{اكبر قوة}}{\text{اكبر قوة}}$

$b_k = \frac{1}{2k^2}$ ↳ p-series $\left(\frac{1}{k^p}\right)$

p>1 converg

$\lim_{k\to\infty} \frac{\frac{1}{2k^2+k}}{b_k} \Rightarrow \frac{1}{2k^2+k} * 2k^2$ $\left(\frac{\text{poly}}{\text{poly}}\right)$

$\lim_{k\to\infty} = 1$ والواحد $< \infty$, $\neq 0$

$\therefore \sum \frac{1}{2k^2+k}$ and $\sum \frac{1}{2k^2}$ اللي انا جبتها

convergent الاول ⇒ convergent. (p-series). p = 2

$\boxed{35}$ $\sum_{k=1}^{\infty} \underbrace{\frac{1}{\sqrt[3]{8k^2-3k}}}_{a_k}$ $b_k = \frac{1}{(8k^2)^{\frac{1}{3}}} = \frac{1}{2k^{\boxed{\frac{2}{3}}}}$ → p-series | $p<1 \therefore$ diverge

$\lim_{k\to\infty} \frac{a_k}{b_k} = \frac{\frac{1}{\sqrt[3]{8k^2-3k}}}{\frac{1}{2k^{2/3}}} = \frac{1}{\sqrt[3]{8k^2-3k}} * 2k^{\frac{2}{3}}$

$\frac{\text{اكبر قوة}}{\text{اكبر قوة}}$ ←

$\lim_{k\to\infty} \frac{2k^{2/3}}{2k^{2/3}} = 1$ ✓

بما انها مش (0) ولا (∞) اذًا الاتنين ال series الهم نفس الحكم يعني ال (c) conv او (c) div حسب اللي انا جبتها.

$\therefore \sum \frac{1}{\sqrt[3]{8k^2-3k}} \Rightarrow$ divergent.

77

H.W $\boxed{36}$ $\sum_{k=1}^{\infty} \frac{1}{(2k+3)^{17}}$

$\boxed{37}$ $\sum_{k=3}^{\infty} \underbrace{\frac{1+2^k}{1+3^k}}_{a_k}$

$b_k = \frac{2^k}{3^k}$ $\lim_{k\to\infty} \frac{a_k}{b_k}$

$= \frac{1+2^k}{1+3^k} * \frac{3^k}{2^k}$ $= \lim_{k\to\infty} \left(\frac{3^k+6^k}{2^k+6^k}\right) \cdot \frac{6^{-k}}{6^{-n}}$

$= \lim_{k\to\infty} \frac{\frac{1}{2^k}+1}{\frac{1}{3^k}+1} = \frac{0+1}{0+1} = \boxed{1}$ ✓

برجع للاصل اساسية لحتى اشوف conv/div ولا :.

$\sum b_k = \sum \frac{2^k}{3^k} = \sum \left(\frac{2}{3}\right)^k \rightarrow -1 < \frac{2}{3} < 1$ $\therefore \sum_{k=1}^{\infty} \left(\frac{2}{3}\right)^k$ Conv

(H.W) $\boxed{38}$ $\sum_{k=1}^{\infty} \frac{1}{2^k-1}$ $\boxed{39}$ $\sum_{k=1}^{\infty} \frac{2k^2+3k}{\sqrt{5+k^5}}$

$\boxed{40}$ $\sum_{k=0}^{\infty} \frac{k+2}{(k+3)^3}$

$\boxed{41}$ $\sum_{k=1}^{\infty} \underbrace{\frac{1}{k^{1+\frac{1}{k}}}}_{a_k}$ $b_k = \frac{1}{k}$

$\lim_{k\to\infty} \frac{1}{k^{1+\frac{1}{k}}} * \frac{k}{1} \Rightarrow \lim_{k\to\infty} \frac{1}{k^{\frac{1}{k}}} = \lim_{k\to\infty} k^{-\frac{1}{k}} = y$

$\ln y = \ln_{k\to\infty} \ln k^{\frac{-1}{k}} = \ln_{k\to\infty} \frac{\ln k}{-k} \rightarrow \lim_{k\to\infty} \ln \frac{\frac{1}{k}}{-1} = 0$

$y = \lim_{k\to\infty} \frac{1}{k^k} = e^0 = 1$ ✓ but $\sum_{k=1}^{\infty} \frac{1}{k}$ harmonic series $\therefore$ divergent.

$\Rightarrow \sum_{k=1}^{\infty} \frac{1}{k^{1+\frac{1}{k}}}$ is divergent.

$\boxed{42}$ $\sum_{n=1}^{\infty} \sin\left(\frac{1}{k}\right)$ $b_k = \frac{1}{k}$

$\lim_{K\to\infty} \frac{\sin\frac{1}{k}}{\frac{1}{k}} =$ $n = \frac{1}{k}$

$\lim_{n\to 0} \frac{\sin n}{n} = 1$ ✓

$\sum_{n=1}^{\infty} \frac{1}{k}$ harmonic (divergant)

$\Rightarrow \sum_{n=1}^{\infty} \sin\frac{1}{k}$ divergent

(H.W) $\boxed{43}$ $\sum_{n=1}^{\infty} \frac{n}{3n^3+1}$ $\boxed{45}$ $\sum_{n=1}^{\infty} \frac{\sqrt{n}}{n-1}$

$\boxed{44}$ $\sum_{n=1}^{\infty} \frac{9^n}{3+10^n}$ $\boxed{46}$ $\sum_{n=1}^{\infty} \left(1+\frac{1}{n}\right)^2 e^{-n}$

* Ratio test

e.g :- $\sum \frac{3^k}{2k}$ $a_k = \frac{a^k}{2k}$, $a_3 = \frac{a^3}{2(3)}$

$a_n = \frac{a^n}{2n}$ $a_{(k+1)} = \frac{3^{k+1}}{2(k+1)}$

* Ratio test

~~We~~ Let $\sum a_k$ be series with positive terms $(a_k > 0)$

$$p = \lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \begin{cases} p<1 & \text{converg} \\ p>1 & \text{diverge} \\ p=1 & \text{test failed.} \end{cases}$$

Ex $\boxed{47}$ $\sum_{k=1}^{\infty} \frac{1}{k!}$ $a_k = \frac{1}{k!}$ $a_{(k+1)} = \frac{1}{(k+1)!}$

$$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{1}{(k+1)!} \cdot \frac{k!}{1}$$

$= \lim_{k\to\infty} \frac{1}{(k+1)\cancel{k!}} \cdot \frac{\cancel{k!}}{1} \qquad = \lim_{k\to\infty} \frac{1}{k+1} = 0 < 1$

$\sum_{k=1}^{\infty} \frac{1}{k!}$ convergent.

$\boxed{48}$ $\sum_{k=1}^{\infty} \frac{(2k)!}{4^k}$ $\qquad a_k = \frac{(2k)!}{4^k}$. $a_{k+1} = \frac{(2(k+1))!}{4^{k+1}}$

$= \frac{(2k+2)!}{4^{k+1}}$

$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \lim_{k\to\infty} \frac{(2k+2)!}{4^{k+1}} \cdot \frac{4^k}{2k!}$

$\lim_{k\to\infty} \frac{(2k+2)(2k+1)\cancel{(2k)!}}{\cancel{4^k}\cdot 4} \cdot \frac{\cancel{4^k}}{\cancel{(2k)!}} = \lim_{k\to\infty} \frac{(2k+2)(2k+1)}{4} = \infty$

$\therefore \sum_{k=1}^{\infty} \frac{(2k)!}{4^k}$ divergent.

$\boxed{49}$ $\sum_{k=1}^{\infty} \frac{k^k}{k!}$ $\qquad a_k = \frac{k^k}{k!}$ $\qquad a_{k+1} = \frac{(k+1)^{k+1}}{(k+1)!}$

$\lim_{k\to\infty} \frac{(k+1)^{k+1}}{(k+1)!} \cdot \frac{k!}{k^k} \Rightarrow \lim_{k\to\infty} \frac{(k+1)^k \cdot \cancel{(k+1)}}{\cancel{(k+1)}\,\cancel{k!}} \cdot \frac{\cancel{k!}}{k^k}$

$= \lim_{k\to\infty} \left(\frac{k+1}{k}\right)^k = \lim_{k\to\infty} \left(1+\frac{1}{k}\right)^k = e = e > 1$

$\lim_{n\to\infty} \left(1+\frac{a}{k}\right)^{bk} = e^{ab}$

$\Rightarrow \sum_{k=1}^{\infty} \frac{k^k}{k!}$ divergent.

$\boxed{50}$ $\sum_{k=1}^{\infty} \frac{k}{2k}$ $\qquad \lim_{k\to\infty} \frac{k+1}{2^{k+1}} \cdot \frac{2k}{k}$

$= \lim_{k\to\infty} \frac{k+1}{2k\cdot 2} \cdot \frac{2^k}{k} = \lim_{k\to\infty} \frac{k+1}{2k} = \frac{1}{2} < 1$ convergent.

[H.W] $\boxed{51}$ $\sum_{k=1}^{\infty} \frac{2^k (k!)^2}{(2k+2)!}$ $\qquad \boxed{52}$ $\sum_{n=1}^{\infty} \frac{k!}{e^{k^2}}$

$\boxed{53}$ $\sum_{n=1}^{\infty} k\left(\frac{1}{3}\right)^k$ $\qquad \boxed{54}$ $\sum_{n=1}^{\infty} \frac{k!}{k}$

* Root test.

Let Σa_K be series with positive terms

$$\rho = \lim_{k \to \infty} \sqrt[K]{a_K} = \begin{cases} \rho < 1 & \text{converge} \\ \rho > 1 & \text{diverge.} \\ \rho = 1 & \text{test failed.} \end{cases}$$

$\boxed{Ex}$:- $\boxed{55}$ $\sum_{K=3}^{\infty} \left(\frac{4K-5}{2K+1}\right)^K$

$$\lim_{K \to \infty} \sqrt[K]{\left(\frac{4K-3}{2K+1}\right)^K} = \lim_{K \to \infty} \frac{4K-5}{2K+1} = \frac{4}{2} = 2 > 1$$

∴ divergent

$\boxed{56}$ $\sum_{K=1}^{\infty} \frac{K}{3^K} = \lim_{K \to \infty} \sqrt[K]{\frac{K}{3^K}} = \lim_{K \to \infty} \frac{\sqrt[K]{K}}{\sqrt[K]{3^K}}$

$$= \lim_{K \to \infty} \frac{K^{\frac{1}{K}}}{3} = \frac{1}{3} \underbrace{\lim_{K \to \infty} K^{\frac{1}{K}}}_{\checkmark}$$

$y = \lim_{K \to \infty} K^{\frac{1}{K}}$

$\ln y = \lim \frac{1}{K} \ln K$

$\lim_{K \to \infty} \frac{\frac{1}{K}}{1} = 0$

$y = e^0 = 1$

$\frac{1}{3} * 1 = \frac{1}{3} < 1$

∴ convergent

[H.W]

$\boxed{57}$ $\sum_{n=1}^{\infty} \left(\frac{k}{k+1}\right)^{k^2}$

$\boxed{58}$ $\sum_{K=1}^{\infty} \frac{(3K+2)^K}{(2K-1)^K}$

$\boxed{59}$ $\sum_{K=1}^{\infty} \left(\frac{K}{100}\right)^K$

* Alternating Series :-

$\sum_{n=1}^{\infty} (-1)^n a_n$, $\sum_{n=1}^{\infty} (-1)^{n-1} a_n$

$\sum_{n=1}^{\infty} (-1)^{n+1} a_n$ $\quad a_n > 0$

$\boxed{Thm}$ (Alternating series test)

$\sum_{n=1}^{\infty} (-1)^n a_n$ $\quad a_n > 0$

1) $a_{n+1} < a_n$ for all n (decreasing)

2) $\lim_{n \to \infty} a_n = 0$ ∴ then the series is convergent.

Ex $\boxed{60}$ $\sum_{k=1}^{\infty} \frac{(-1)^k}{k} = \sum_{k=1}^{\infty} (-1)^k \cdot \frac{1}{k}$

$a_k = \frac{1}{k}$ $\quad f x = \frac{1}{x} \Rightarrow f'(x) = \frac{-1}{x^2}$ $\quad \overset{- \; -}{\longleftrightarrow}$

∴ decreasing.

$\lim_{k \to \infty} \frac{1}{k} = 0$ ✓

$\Rightarrow \sum_{k=1}^{\infty} \frac{(-1)^k}{k}$ is convergent.

$\boxed{61}$ $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k}{3^k}$ $\qquad a_k = \frac{k}{3^k}$

$\frac{a_{k+1}}{a_k} = \frac{k+1}{3^{k+1}} \cdot \frac{3^k}{k} \Rightarrow \frac{k+1}{3k} < 1$ $\quad$ ∴ a_k decreasing

$\lim_{k \to \infty} \frac{k}{3^k} = \frac{\infty}{\infty} \xrightarrow{\text{لوبيتال}} \lim \frac{1}{3^k \ln 3} = 0$

$\Rightarrow \sum_{k=1}^{\infty} (-1)^{k+1} \frac{k}{3^k}$ is convergent.

[HW] $\boxed{62}$ $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+3}{k(k+1)}$ $\qquad \boxed{63}$ $\sum_{k=1}^{\infty} (-1)^{k+1} e^{-k}$

$\boxed{64}$ $\sum_{k=1}^{\infty} (-1)^k \frac{\ln k}{k}$ $\qquad \boxed{65}$ $\sum_{k=0}^{\infty} (-1)^{k+1} \frac{k+1}{\sqrt{k}+1}$

* Absolute convergent :-

$\boxed{\text{Def}}$ A series $\sum a_k$ is said to be converge absolutly if $\sum |a_k|$ converge.

E.g :- $\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \rightarrow \left| \frac{(-1)^n}{n^2} \right| = \frac{1}{n^2} \rightarrow \sum_{n=1}^{\infty} \frac{1}{n^2}$ convergent (P-series P = 2)

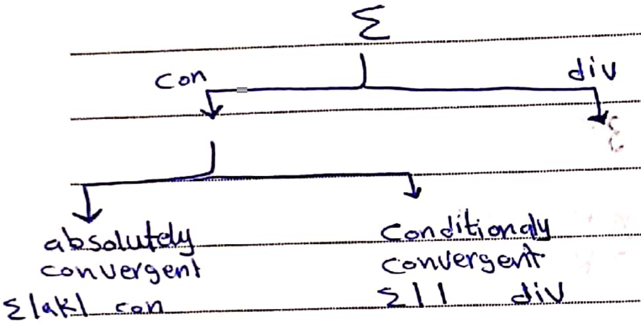
$\Rightarrow \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}$ absolutely convergent.

Thm if $\sum a_k$ is convergant then it's convergent

eg: $\sum_{k=1}^{\infty} \frac{(-1)^k}{k} \rightarrow$ convergent

↳ but $\sum_{k=1}^{\infty} \left| \frac{(-1)^k}{k} \right| = \sum_{k=1}^{\infty} \frac{1}{k}$ → harmonic series → div

Def :- A series $\sum a_k$ is called conditionally convergent if it is convergent but not absolutely convergent.



EX 66 $1-\frac{1}{2}-\frac{1}{2^2}+\frac{1}{2^3}+\frac{1}{2^4}-\frac{1}{2^5}-\frac{1}{2^6}+ \cdots$

take absolute value

$1+\frac{1}{2}+\frac{1}{2^2}+\frac{1}{2^3}+\cdots \Rightarrow \sum_{k=0}^{\infty} \frac{1}{2^k}$

$= \sum_{k=0}^{\infty} \left(\frac{1}{2}\right)^k$ geometric $r=\frac{1}{2}$ Convergent

$\Rightarrow 1-\frac{1}{2}-\frac{1}{2^2}+\frac{1}{2^3}$ absolutely Conv → conv.

67 $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{k+2}{k^3+k} \Rightarrow \left| (-1)^{k+1} \frac{k+2}{k^3+k} \right|$

$= \sum_{k=1}^{\infty} \underset{a_k}{\boxed{\frac{k+2}{k^3+k}}}$, $\sum_{k=1}^{\infty} \frac{\cancel{k}}{k^{\cancel{3}2}} = \sum_{k=1}^{\infty} \frac{1}{k^2}$

$\lim_{k\to\infty} \frac{a_k}{b_k} = \lim_{k\to\infty} \frac{k+2}{k^3+k} \cdot k^2 = 1 < \infty$

→ both convergent or both divergent.

but $\sum \frac{1}{k^2}$ converent (p - sereis p = 2)

$\Rightarrow \sum \frac{k+2}{k^3+k}$ is convergent $\Rightarrow \sum (-1)^{k+1} \frac{k+2}{k^3+k}$ is

absolutely conv $\rightarrow$ convergent

* Ratio test for absolute convergent.

$\sum_{n=1}^{\infty} a_n \rightarrow \lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \rho$ if $\rho < 1 \rightarrow \sum a_n$ abs con $\searrow$ con

if $\rho > 1 \rightarrow \sum a_n$ div ~~(test failed)~~.

$\rho = 1$ (test failed)

* Root test for abs conv.

$\sum_{n=1}^{\infty} a_n$ $\qquad \rho = \lim_{n\to\infty} \sqrt[n]{|a_n|}$

$\rho < 1 \rightarrow \sum a_n$ is abs conv $\rightarrow$ converge.

$\rho > 1 \rightarrow \sum a_n$ divergent.

$\rho = 1 \rightarrow$ test failed.

EX $\boxed{68}$ $\sum_{n=1}^{\infty} \frac{n^3}{(-3)^n} \rightarrow \left|\frac{n^3}{(-3)^n}\right| \rightarrow \underbrace{\frac{n^3}{3^n}}_{a_n}$

$a_{n+1} = \frac{(n+1)^3}{3^{n+1}}$

$\therefore \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{(n+1)^3}{3^{n+1}} \cdot \frac{3^n}{n^3} = \lim_{n\to\infty} \frac{1}{3}\left(\frac{n+1}{n}\right)^3$

$= \frac{1}{3}(1)^3 = \frac{1}{3} < 1$ $\rightarrow$ abs converg

$\therefore$ convergent.

$\boxed{69}$ $\sum_{n=1}^{\infty} (-1)^n \left(\frac{3n+2}{2n+1}\right)^n \longrightarrow \left|(-1)^n \left(\frac{3n+2}{2n+1}\right)^n\right|$

$= \sum_{n=1}^{\infty} \left(\frac{3n+2}{2n+1}\right)^n$ $\rho = \lim_{n\to\infty} \sqrt[n]{\left(\frac{3n+2}{2n+1}\right)^n}$

$= \lim_{n\to\infty} \frac{3n+2}{2n+1} = \frac{3}{2} > 1$

$\sum^{\infty} (-1)^n \left(\frac{3n+2}{2n+1}\right)^n$ is divergent.

$\boxed{70}$ $\sum_{n=2}^{\infty} \left(\frac{-2n}{n+1}\right)^{5n} \longrightarrow \left|\left(\frac{-2n}{n+1}\right)^{5n}\right| = \left(\frac{2n}{n+1}\right)^{5n}$

$\lim_{n\to\infty} \sqrt[n]{\left(\frac{2n}{n+1}\right)^{5\cancel{n}}} = \lim_{n\to\infty} \left(\frac{2n}{n+1}\right)^{5}$ $(2)^5 > 1$

∴ divergent

[H.W] :-

$\boxed{71}$ $\sum (-1)^n \frac{n^n}{n!}$ $\boxed{72}$ $\sum_{n=1}^{\infty} \frac{(-2)^n}{n^2}$

$\boxed{73}$ $\sum_{n=1}^{\infty} (-1)^{n-1} \frac{n}{n^2+4}$ $\boxed{74}$ $\sum_{n=0}^{\infty} \frac{(-1)^n}{5n+1}$

$\boxed{75}$ $\sum_{n=1}^{\infty} \frac{(-1)^n e^{\frac{1}{n}}}{n^3}$ $\boxed{76}$ $\sum_{n=0}^{\infty} \frac{(-3)^n}{(2n+1)!}$

$\boxed{77}$ $\sum_{n=1}^{\infty} \frac{n^{10}}{(-10)^{n+1}}$

* page 740 - 741

EX 11.7   (1 - 38)

H.W

power series :-

$\sum_{n=0}^{\infty} c_n x^n$ or $\sum_{n=0}^{\infty} c_n (x-a)^n$

↳ about 0

↳ about a

eg :- $\sum_{n=0}^{\infty} \frac{1}{n} x^n$

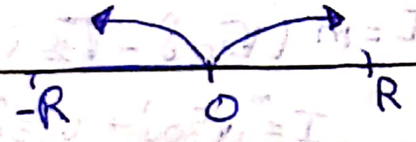
$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{n^3}$

⇒ about Zero

$\sum_{n=0}^{\infty} \frac{n!(x-3)^n}{(n+1)^n} \longrightarrow a = 3$

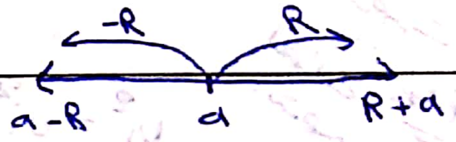
$\sum_{n=0}^{\infty} \sqrt[n]{n+1}\,(x+3)^n \longrightarrow a = -3$

about 0



R = Radus of convergent

about a



R = Radus of convergent.

about 0

Radus = $R \overset{\neq 0}{<} \infty$

$(-R, R)$

Interval of convergent

$R = 0$

$\{0\}$

Interval of convergent

$R = \infty$ متقارب في كل مكان

$(-\infty, \infty)$

Interval of convergent.

ع الاطراف نشيك على كل حده

about a

Radus $= R < \infty$

$(a-R, a+R)$

$x = a$

$R = 0$

$\{a\}$

$R = \infty$ متقارب في كل مكان

$(-\infty, \infty)$

Interval of conv + Radus طريقة ايجاد

ratio test باستخدام

for absolute convergent.

Ex Find the radius and the interval of convergent for :-

① $\sum_{n=1}^{\infty} \frac{x^n}{\underbrace{n}_{a_n}}$

$\lim \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{x^{n+1}}{n+1} \cdot \frac{n}{x^n}\right|$

$= \lim_{n\to\infty} \frac{n}{n+1} |x|$ $\qquad |x| < 1$

$\Rightarrow -1 < x < 1$

at $x = 1$

$\sum_{n=1}^{\infty} \frac{1^n}{n} = \sum_{n=1}^{\infty} \frac{1}{n}$ → harmonic.

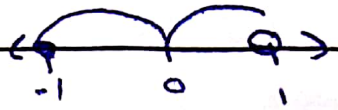
∴ divergent.

at $x = -1$

$\sum_{n=1}^{\infty} \frac{(-1)^n}{n}$ → Alternating series في مثال سابق رقم $\boxed{60}$

∴ convergent

$-1 \leq x < 1$



Interval of converge $[-1, 1)$ $\qquad R = 1$

$\underline{(1)} = \left(\frac{1 - -1}{2}\right) = \left(\frac{\text{البداية - النهاية}}{2}\right)$

(2) $\sum_{K=0}^{\infty} \frac{(-1)^K (X+1)^K}{5^K (K+2)}$ $a = -1$

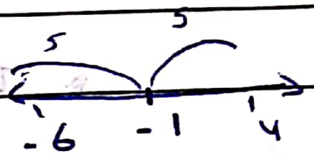
$\lim_{X\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{K\to\infty} \left| \frac{(-1)^{K+1} (X+1)^{K+1}}{5^{K+1} (K+3} \cdot \frac{5^K (K+2)}{(-1)^K (X+1)^K} \right|$

$= \lim_{K\to\infty} \left| \frac{(X+1)}{5} \cdot \frac{K+2}{K+3} \right| = \lim_{K\to\infty} \frac{K+2}{5(K+3)} |X+1|$

$= \frac{1}{5} |X+1| < 1 \Rightarrow |X+1| < 5$

$-5 < X+1 < 5$

$-6 < X < 4$



$R = \frac{4--6}{2} = 5$

$X = -6$ $\sum \frac{(-1)^K (-5)^K}{5^K (K+2)} = \sum \frac{1}{K+2}$ harmonic

divergent.

$X = 4$

$\sum_{K=0}^{\infty} \frac{(-1)^K 5^K}{5^K (K+2)} = \sum_{K=0}^{\infty} \frac{(-1)^K}{R+2}$ by Alternating series

$a_K = \frac{1}{K+2}$ $\lim_{K\to\infty} a_n = \lim \frac{1}{K+2} = 0$

$a_n' = \frac{-1}{(K+2)^2}$ $\underset{-2}{\longleftrightarrow}$ decreasing.

$\sum \frac{(-1)^K}{K+2}$ convergent.

→ Interval of converge $(-6, 4]$

③ $\sum_{n=0}^{\infty} n! \, x^n$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim_{n\to\infty} \left|\frac{(n+1)!}{n!} \cdot \frac{x^{n+1}}{x^n}\right|$

$= \lim_{n\to\infty} \left|\frac{(n+1)\cancel{n!}}{\cancel{n!}} \cdot \frac{x^{n+1}}{x^n}\right|$

$= \lim_{n\to\infty} n+1 \; |x| = \infty$

مش $\infty < 1$ ← متباعد

→ Interval of converge $= \{0\}$ $R = 0$

④ $\sum_{n=1}^{\infty} \frac{x^n}{n!}$

$\lim_{n\to\infty} \left|\frac{a_{n+1}}{a_n}\right| = \lim \left| \frac{x^{n+1}}{(n+1)!} \cdot \frac{n!}{x^n} \right|$

$\lim \frac{|x|}{n+1} = 0 < 1$ دائماً

→ Interval of convergent

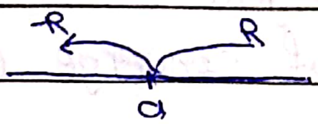
$= (-\infty, \infty)$

$R = \infty$

[طريقة سهلة] :-

$\sum c_n x^n$ or $\sum c_n (x-a)^n$

$\lim_{n\to\infty} \left|\frac{c_{n+1}}{c_n}\right| = \rho$ $R = \frac{1}{\rho}$ $\begin{cases} \frac{1}{0} \to \infty \\ \frac{1}{\infty} \to 0 \end{cases}$



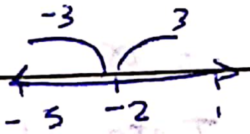
⑤ $$\sum_{n=0}^{\infty} \frac{n(x+2)^n}{3^{n+1}}$$

$$c_n = \frac{n}{3^n+1}$$

$$\lim_{n\to\infty} \left| \frac{c_{n+1}}{c_n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{n+1}{3^{n+2}} \cdot \frac{3^{n+1}}{n} \right| = \lim_{n\to\infty} \frac{1}{3} \frac{n+1}{n} = \frac{1}{3} = \rho$$

$$R = \frac{1}{\frac{1}{3}} = 3 \checkmark$$



$X = -5$

$$\sum_{n=0}^{\infty} \frac{n(-3)^n}{3^{n+1}} = \sum_{n=0}^{\infty} \frac{n(-1)^n \cancel{3^n}}{\cancel{3^n} \cdot 3} = \sum_{n=0}^{\infty} \frac{1}{3} (-1)^n n.$$

divergent test $\lim_{n\to\infty} \frac{1}{3}(-1)^n n$ = d.n.e

∴ divergent.

$X = 1$

$$\sum_{n=0}^{\infty} \frac{n(3^n)}{3^{n+1}} = \frac{1}{3} \sum_{n=0} n \qquad \lim_{n\to\infty} n = \infty$$

∴ divergent

$R = 3$

Interval of convergent $(-5, 1)$

⑥ $$\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}} \qquad R = \frac{1}{3}$$

$$\left(\frac{-1}{3}, \frac{1}{3}\right]$$

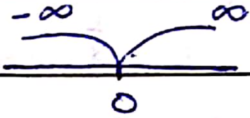
⑦ $\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2^{2n} n!}$ $\qquad C_n = \frac{(-1)^n}{2^{2n} n!}$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1}}{2^{2n+2}(n+1)!} \cdot \frac{2^{2n} n!}{(-1)^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{1}{2^2(n+1)} \right| = \lim_{n\to\infty} \frac{1}{4(n+1)} = 0 = \rho$$

$R = \frac{1}{\rho} = \infty$

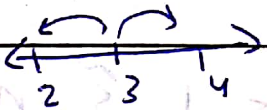
Interval $(-\infty, \infty)$



⑧ $\sum_{n=1}^{\infty} n(x-3)^n$ $\qquad C_n = n$

$$\lim \left| \frac{C_{n+1}}{C_n} \right| = \lim_{n\to\infty} \left| \frac{n+1}{n} \right| = 1 = \rho$$

$R = 1$



Interval of convergent.

i) $x=2$

$\sum_{n=1}^{\infty} n(-1)^n \rightarrow$ divergent by divergent test.

$x=4$

$\sum_{n=1}^{\infty} n \rightarrow$ divergent by diverge test

$\rightarrow$ Interval of converge

$= (2,4)$

$\boxed{19}$ $$\sum_{n=0}^{\infty} \underbrace{n!}_{c_n} (x+1)^n$$

$$\lim_{n\to\infty} \left|\frac{(n+1)!}{n!}\right| = \lim_{n\to\infty} n+1 = \infty = \rho$$

$$R = \frac{1}{\rho} = \frac{1}{\infty} = 0$$

Interval of converge = $\{-1\}$

[H.W]

$\boxed{10}$ $\sum_{n=1}^{\infty} \frac{x^n}{2n-1}$

$\boxed{11}$ $\sum_{n=0}^{\infty} \frac{x^n}{n!}$

$\boxed{12}$ $\sum_{n=1}^{\infty} \frac{(-1)^n x^n}{\sqrt[3]{n}}$

$\boxed{13}$ $\sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{\sqrt{n}}$

$\boxed{14}$ $\sum_{n=0}^{\infty} \frac{(x-2)^n}{n^2+1}$

$\boxed{15}$ $\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n \sqrt{n}}$

$\boxed{16}$ $\sum_{n=1}^{\infty} \frac{2^n}{n} (2x-4)^n$

Taylor and Maclaurin series :-

(introduction).

$$f(x) = \sum_{n=0}^{\infty} C_n x^n$$

$$= C_0 + C_1 x + C_2 x^2 + C_3 x^3 + C_4 x^4 + \cdots$$

$$f(0) = C_0$$

$$f'(x) = C_1 + 2C_2 x + 3C_3 x^2 + 4C_4 x^3 + \cdots$$

$$f'(0) = C$$

$$f''(0) = 2C_2 + (3)(2)C_3 x + 4(3)C_4 x^2 + \cdots$$

$$f''(0) = 2C_2 \Rightarrow C_2 = \frac{f''(0)}{2}$$

$$f'''(x) = 3(2)C_3 + 4(3)(2)C_4 x + \cdots$$

$$f'''(0) = 3(2)C_3 \rightarrow C_3 = \frac{f'''(0)}{3(2)}$$

$$C(4) = \frac{f^{(4)}(0)}{4(3)(2)(1)}$$

$$C_5 = \frac{f^{(5)}(0)}{5(4)(3)(2)1}$$

$$\vdots$$

$$C_n = \frac{f^{(n)}(0)}{n(n-1)(n-2)\cdots(2)(1)} = \frac{f^{(n)}(0)}{n!}$$

If f(x) has derivaties of all orders at 0 then Maclaurin series of f(x) is

$$\left[f_x = \sum_{n=0}^{\infty} \frac{f^{n}(0)}{n!} x^{n} \right] \rightarrow x \in \text{Interval of Converga}$$

* In general :-

If f(x) has derivatives for all orders at a_0 then Taylor series of f(x) about.

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(a_0)}{n!} (x - a_0)^{n}$$

Ex :- Find the Maclaurin series of the function $f(x) = e^{x}$ and its radus of convergence.

sol

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{n}(0)}{n!} x^{n}$$

$f(x) = e^{x}$

$f^{0}(0) \therefore f(0) = 1$ $\qquad$ $f'(0) = e^{x}$

$f'(0) = 1$ $\qquad$ $f''(0) = e^{x}$

$f''(0) = 1$ $\qquad$ $f^{n}(0) = e^{x}$

$\vdots$

$f^{n}(0) = 1$

$$e^{x} = \sum_{n=0}^{\infty} \frac{1}{n!} x^{n} = \left[\sum_{n=0}^{\infty} \frac{x^{n}}{n!} \right]$$

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}$$

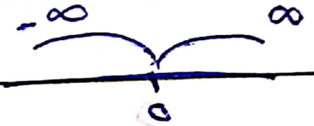
$$\lim_{n\to\infty} \left| \frac{1}{(n+1)!} \cdot \frac{n!}{1} \right|$$

$$= \lim_{n\to\infty} \left| \frac{1}{n+1} \right| = 0 < 1$$

$$R = \frac{1}{\rho} = \frac{1}{0} = \infty$$

▸ radius of converges

$(-\infty, \infty)$



eg :- $\sum_{n=0}^{\infty} \frac{7^n}{n!}$ $\qquad \sum \frac{x^n}{n!} = e^x$

① $= e^7$ convergent.

② $\sum_{n=1}^{\infty} \frac{(\frac{1}{3})^n}{(n-1)!}$ $\qquad K = n-1$

$$\sum_{K=0}^{\infty} \frac{(\frac{1}{3})^{K+1}}{K!} = \frac{1}{3} \sum_{K=0}^{\infty} \frac{(\frac{1}{3})^K}{K!}$$

$$= \frac{1}{3} e^{\frac{1}{3}}$$

eg :- $e = \sum_{n=0}^{\infty} \frac{1}{n!}$

$$e^{-1} = \sum_{n=0}^{\infty} \frac{(-1)^n}{n!}$$

احفظ $\sin x = \sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$ Interval of convergel $(-\infty, \infty)$

eg $\sin \pi = \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{(2n+1)!}$

eg $\sum_{n=0}^{\infty} \frac{(-1)^n (\frac{\pi}{2})^{2n+1}}{(2n+1)!}$

$\Rightarrow \sin \frac{\pi}{2} = 1$

eg $\sum_{n=0}^{\infty} \frac{(-1)^n (\frac{\pi}{3})^{2n}}{(2n+1)!}$

$\frac{3}{\pi} \cdot \sum_{n=0}^{\infty} \frac{(-1)^n (\frac{\pi}{3})^{2n+1}}{(2n+1)!} = \frac{3}{\pi} \sin \frac{\pi}{3}$

$= \frac{3\sqrt{3}}{2\pi}$

حفظ

f(x)	Maclaurin	I.C
e^x	$\sum_{n=0}^{\infty} \frac{x^n}{n!} = 1+x+\frac{x^2}{2}$	$(-\infty, \infty)$
$\sin x$	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{(2n+1)!}$	$f\infty, \infty)$
$\cos x$	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n}}{(2n)!}$	$(-\infty, \infty)$
$\frac{1}{1-x}$	$\sum_{n=0}^{\infty} x^n = 1+x+x^2+\dots$	$(-1, 1)$
$\ln(1+x)$	$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{x^n}{n}$	$(-1, 1]$
$\tan^{-1}(x)$	$\sum_{n=0}^{\infty} \frac{(-1)^n x^{2n+1}}{2n+1}$	$[-1, 1]$

Ex find the Maclaurin series for :-

$\boxed{1}$ $f(x) = \frac{1}{1+x}$ $\quad * \frac{1}{1+x} = \frac{1}{1-(-x)}$

$$= \sum_{n=0}^{\infty} (-x)^n = \sum_{n=0}^{\infty} \underbrace{(-1)^n x^n}_{\text{تحفظ}}$$

98

2 $\sin 2x = \sum_{n=0}^{\infty} \frac{(-1)^n \, (2x)^{2n+1}}{(2n+1)!}$

3 $\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n \, (x^2)^n$

$\underbrace{\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}}_{\text{مهم}}$

4 $\ln x = \ln(1 + (x-1)]$

$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(x-1)^n}{n}$

5 $\frac{1}{x} = \frac{1}{1-(1-x)}$

$1-y = x$

$y = 1-x$

$= \sum_{n=0}^{\infty} (1-x)^n$

6 $\ln(2+x) = \ln(1+(1+x))$

$2+x = 1+y$

$y = 1+x$

$= \sum_{n=1}^{\infty} (-1)^{n-1} \frac{(1+x)^n}{n}$

[H.W] 7 $f(x) = e^{2x}$ 8 $f(x) = \sin \frac{\pi x}{2}$

A $f(x) = \sin^2(x)$

$$\sin^2(x) = \frac{1}{2}(1 - \cos 2x)$$

$$= \frac{1}{2}\left(1 - \sum_{n=0}^{\infty} \frac{(-1)^n (2x)^{2n}}{(2n)!}\right)$$

$$= \frac{1}{2}\left(1 - (1 - \frac{(2x)^2}{2!} + \frac{(2x)^4}{4!} + \frac{(2x)^6}{6!} + \dots\right.$$

$$\frac{1}{2}\sum_{n=1}^{\infty} \frac{(-1)^{n+1} (2x)^{2n}}{(2n)!}$$

EX Find the sum of the series :-

① $$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n} (2n)!} = \sum_{n=0}^{\infty} \frac{(-1)^n (\frac{\pi}{6})^{2n}}{(2n)!}$$

$$= \cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$$

② $$\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{6^{2n} (2n)!}$$

$$\pi \sum_{n=0}^{\infty} \frac{(-1)^n (\frac{\pi}{6})^{2n}}{(2n)!} = \frac{\sqrt{3}}{2}\pi$$

③ $$\sum_{n=1}^{\infty} (-1)^{n-1} \frac{3^n}{5^n \cdot n} = \sum_{n=1}^{\infty} \frac{(-1)^{n-1} (\frac{3}{5})^n}{n}$$

$$= \ln\left(1 + \frac{3}{5}\right) = \ln\left(\frac{8}{5}\right)$$

$\boxed{4}$ $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{4^{2n+1} (2n+1)!}$

$\frac{1}{4\pi} \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n+1}}{4^{2n+1} (2n+1)!} \longrightarrow \frac{1}{4\pi} \sum_{n=0}^{\infty} \frac{(-1)^n (\frac{\pi}{4})^{2n+1}}{(2n+1)!}$

$= \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

$\boxed{5}$ $\sum_{n=1}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!}$

$= \sum_{n=1}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!} + 1 - 1$

$= \sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{(2n)!} - 1$

$= \cos(\pi) - 1 = -1 - 1 = -2$

$\boxed{6}$ $\sum_{n=1}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n-1} (2n+1)!}$

$\boxed{7}$ $\frac{1}{1 \cdot 2} - \frac{1}{3 \cdot 2^3} + \frac{1}{5 \cdot 2^5} - \frac{1}{7 \cdot 2^7} + \dots$

$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)\, 2^{2n+1}} = \sum_{n=0}^{\infty} \frac{(-1)^n (\frac{1}{2})^{2n+1}}{2n+1}$

$= \tan^{-1}\left(\frac{1}{2}\right) =$

$\boxed{8}$ $\frac{1}{1\cdot 3^{\frac{1}{2}}} - \frac{1}{3\cdot 3^{\frac{3}{2}}} + \frac{1}{5\cdot 3^{\frac{5}{2}}} - \frac{1}{7\cdot 3^{\frac{7}{2}}} + \ldots$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n}{(2n+1)(\sqrt{3})^{2n+1}}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n \left(\frac{1}{\sqrt{3}}\right)^{2n+1}}{2n+1}$$

$$= \tan^{-1}\left(\frac{1}{\sqrt{3}}\right) = \frac{\pi}{6}$$

$\boxed{9}$ $1 - \operatorname{Ln}(2) + \frac{(\operatorname{Ln}(2))^2}{2!} - \frac{\operatorname{Ln}(2)^3}{3!} + \ldots$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (\operatorname{Ln}2)^n}{n!}$$

$$= \sum_{n=0}^{\infty} \frac{(-\operatorname{Ln}2)^n}{n!} = e^{-\operatorname{Ln}2} = e^{\operatorname{Ln}\frac{1}{2}} = \frac{1}{2}$$

$\boxed{10}$ $\sum_{n=1}^{\infty} \frac{(-1)^n (\operatorname{Ln}(2))^n}{n!}$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (\operatorname{Ln}(2))^n}{n!} - 1$$

$$= 2 - 1$$

$$= 1$$

11) $1-\frac{1}{3}+\frac{1}{5}-\frac{1}{7}+\dots$

$$=\sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1} = \sum_{n=0}^{\infty}\frac{(-1)^n\,(1)^{2n+1}}{2n+1}$$

$$=\tan^{-1}(1)$$

$$=\frac{\pi}{4}$$

12) $3+\frac{9}{2!}+\frac{37}{3!}+\frac{81}{4!}+\dots$

$$\sum_{n=1}^{\infty}\frac{3^n}{n!} = \sum_{n=0}^{\infty}\frac{3^n}{n!}-1$$

$$=e^3-1$$

13) $\sum_{n=0}^{\infty}\left(\frac{-1}{3}\right)^n \frac{\pi^{2n-1}}{(2n)!}$

14) $\sum_{n=2}^{\infty}\frac{(-1)^{n-1}}{n}$