EICoM
ELECTRICAL COMPUTER MECHATRONICS

دوائر كهربائية 1

د.وفاء الشرع

للطالبة المبدعة

عنود جراندوقة

إرادة - ثقة - تغيير

chapter 2:- 12/7/2023

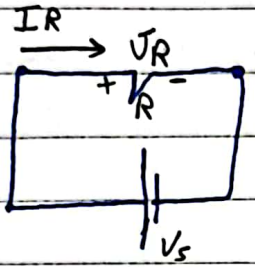
Electrical ccts components

resistans R (Ω)



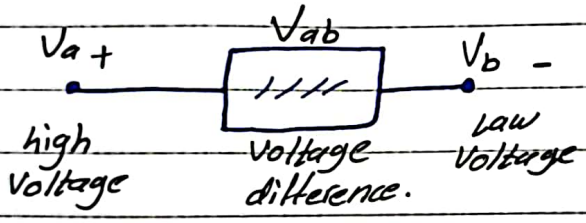
KΩ = 10^3 Ω

MΩ = 10^6 Ω



V_R :- voltage across R

I_R :- current flowing through R

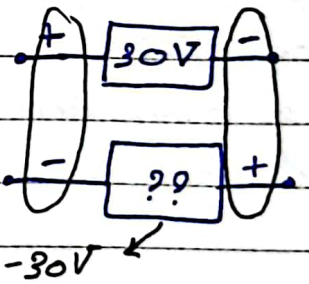


$V_{ab} = V_a - V_b$

DC AC

Voltage (V, v)

Volt → V



current (I, i) A

mA = 10^{-3} A

μA = 10^{-6} A

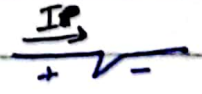
nA = 10^{-9} A

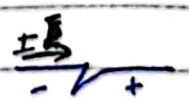
PA = 10^{-12} A

ohm's law (resistors)

$V_R = I_R R$

$V = IR$

$V = \pm (IR)$ $V = +(IR)$ 

$V = -(IR)$ 

Five Apple

Resistors

$0 < R < \infty$



use ohm's law to find
V given I, R
I " V, R
R " I, V

short ccts :- (SC)



R = 0
Vaa = Va − Va = zero.
Don't use ohm's law
to find Isc .

open ccts :- (OC)

R = ∞



$V_{oc} = V_{ab} = V_a - V_b$
Ioc = zero
Don't use ohm's law
to find Voc

* Power (P, p) watt
KW = 10^3 W
MW = 10^6 W
mW = 10^{-3} W
MW = 10^{-6} W

$P \overset{+}{=} (I\,V)$ → ·بلا إشارة

absorbed. ·توصيل

delivered by ·توليد generated

$$P_R = I^2R = \frac{V^2}{R}$$

18/7/2023

$P_{abs} = I^2R = \frac{V^2}{R}$ for R find generated power $= -P_{abs}$

In any cct
$\Sigma P_{gen} = zero$
$\Sigma P_{abs} = \Sigma P_{gen}$
$\Sigma P_{abs} = zero.$

$V_R \propto I_R$
$P_{gen} = -P_{abs}$

$P_{abs} = \pm IV = +I^2R = +\frac{V^2}{R}$ دايما قيمة موجبة
$P_{gen} = -IV = -I^2R = -\frac{V^2}{R}$ دايما قيمة سالبة

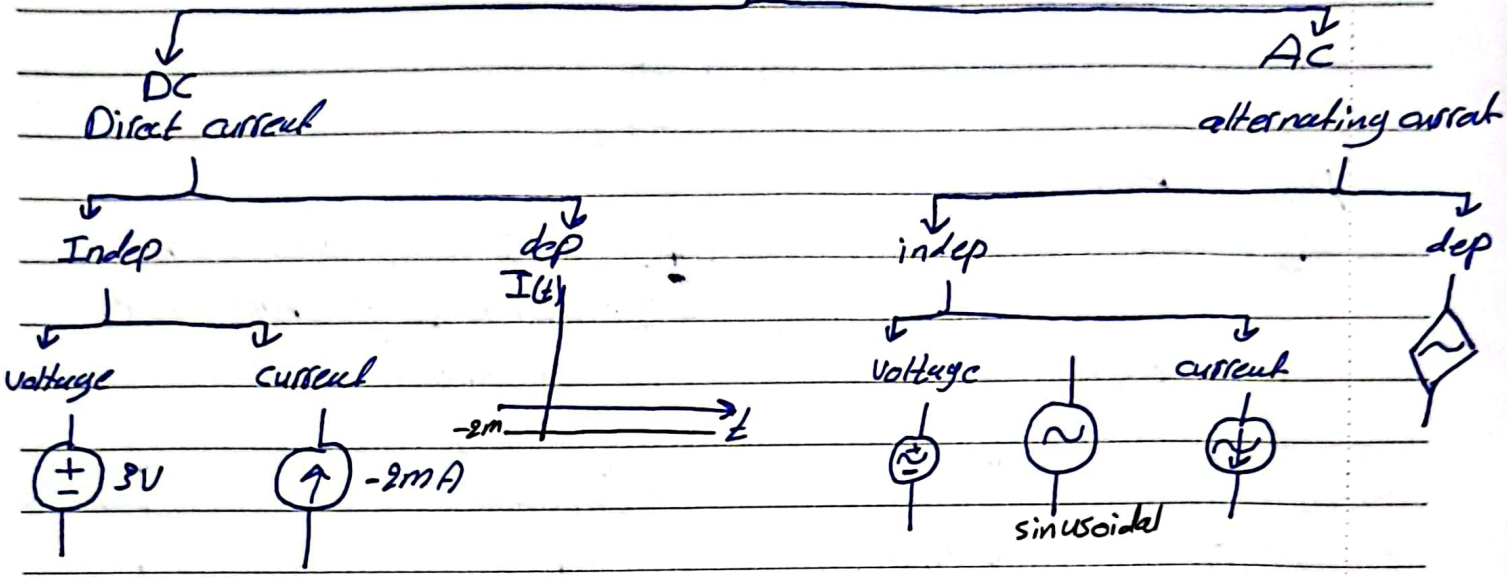
* sources *
- Voltage: ± Value (V)
- current: ↓ Value (A)

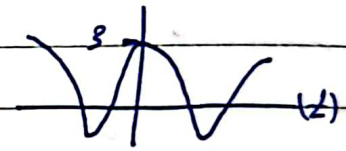
* sources *
- Independant
- dependent

* Independent *
- Voltage: 5V
- current: -3mA

* dependent *
- Voltage source
  - Voltage controlled: V_x, $3V_x$ (V)
  - current controlled: I_y, I_y (V)
- current source
  - Voltage controlled: V_z, V_z (A)
  - current controlled: I_z, $0.5I_z$ (A)

Sources





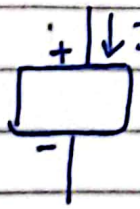
Power

absorbed

generated.

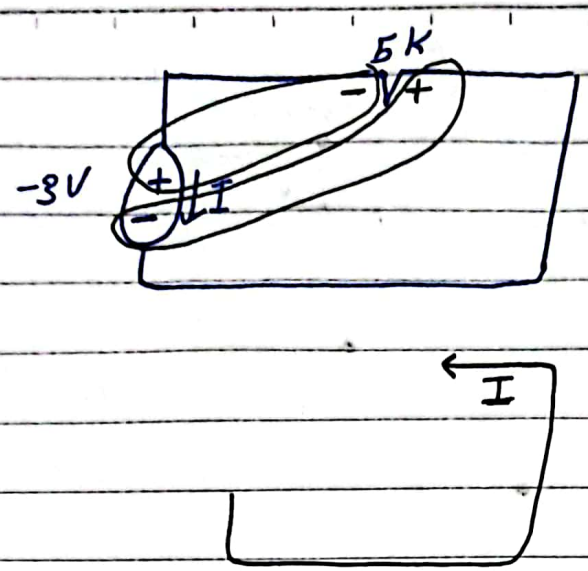
$P_{absorbed} = \pm(IV)$

Passive sign convention method



$P = +(IV)$

Ex:-



Find:- 1. I

2. P_{abs} by 5K, -3V

3. P_{gen} by 5K, -3V

Solu:-

1. $I = \frac{V}{R} = \frac{3}{5K} = 0.6mA$

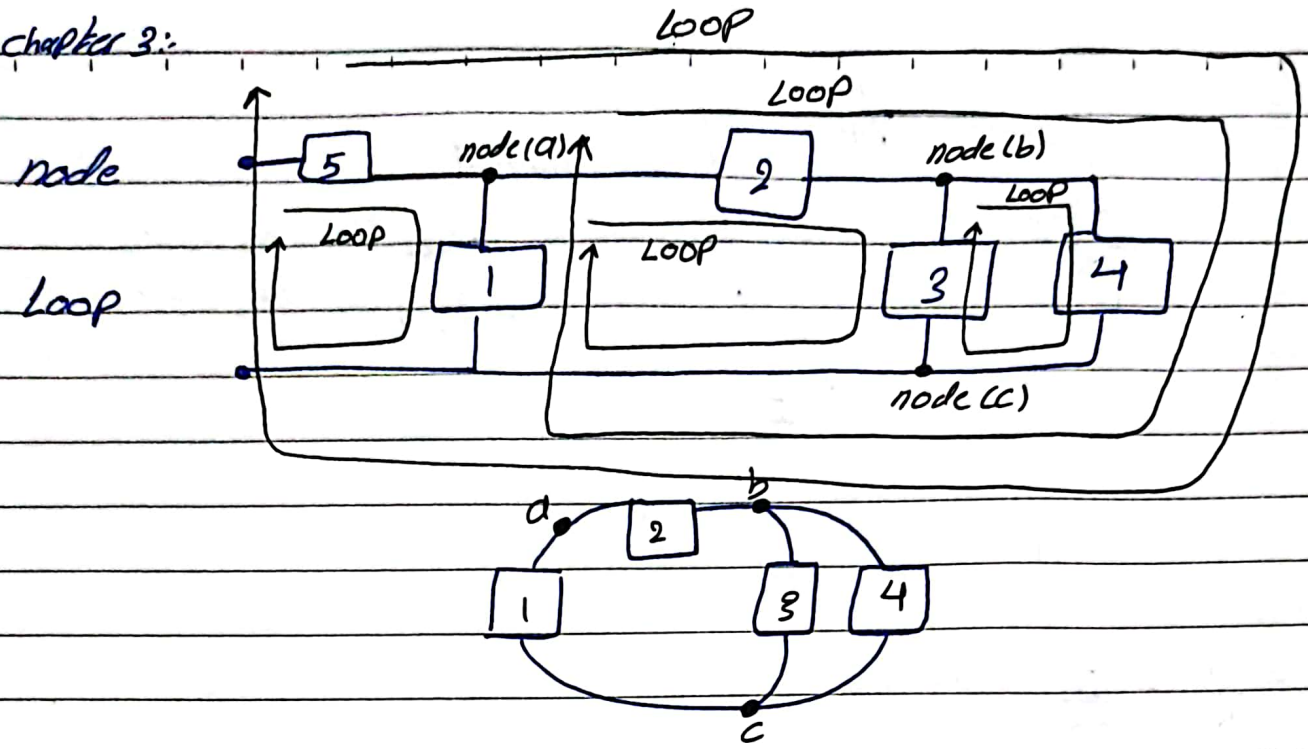
2. $P_{abs\ by\ 5K} = \frac{V^2}{R} = I^2R$ $\quad P_{abs}$ by $-3V = -1.8m$

$= \frac{(3)^2}{5K}$ ~~$(0.6m)^2$~~ $(0.6m)^2 (5K)$

$= \pm 1.8m$ watt

3. $P_{gen\ by\ 5K} = -1.8m$ watt $\quad P_{gen}$ by $-3V = +1.8m$

chapter 3:-



Kirchoff's laws.

Kirchoff's current law

KCL / node

$$\sum_{node} I_{in} = \sum_{node} I_{out}$$

1. to find unknown current in/out of a node.
2. to find unknown voltage for a resistor connected with any node.

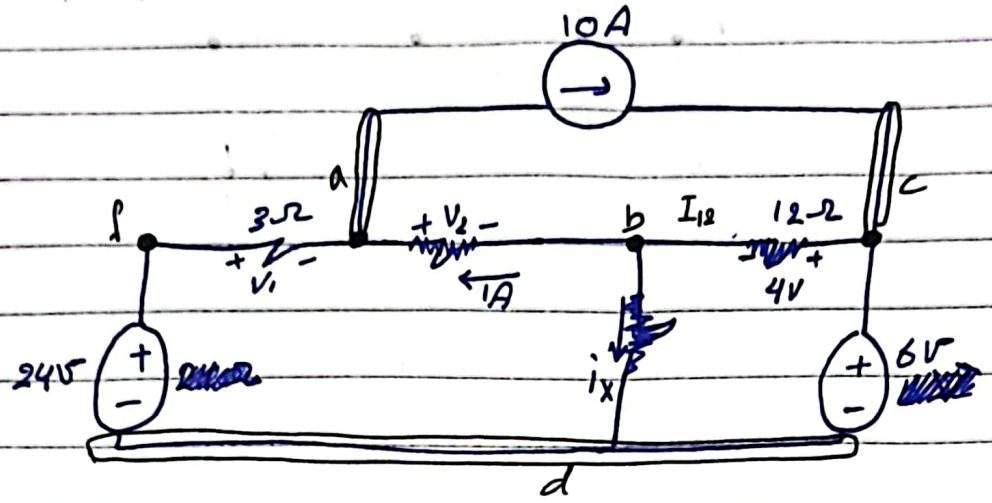
Kirchoff's voltage law

KVL / Loop

$$\sum_{Loop} V = 0$$

1. to find an unknown voltage over a loop.
2. to find an unknown current flowing through a resistor in any loop.

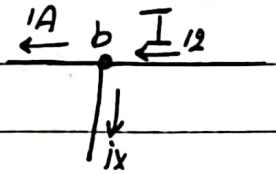
23/F/2023



Find: 1. V_1 2. V_2 3. i_x 4. P_{abs} by 10A, 6V, 12Ω

5. P_{gen} by 24V, 3Ω

to find i_x



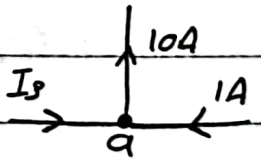
KCL/b

$\Sigma I_{in_b} = \Sigma I_{out_b}$

$i_{12} = i_x + 1$

$\frac{4}{12} = i_x + 1$

$\therefore I_x = \frac{-2}{3} A$



to find V_1

find i_3

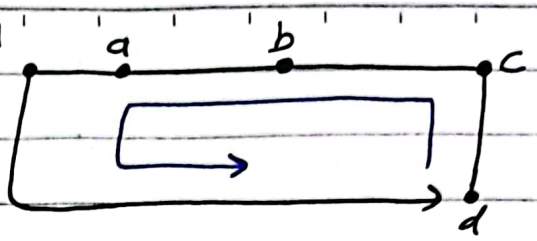
$\Sigma I_{in_a} = \Sigma I_{out_a}$

$I_3 + 1 = 10A$

$I_3 = 9A$

$\therefore V_1 = R.I = 3 \times 9 = 27 V$

to find V_2



KVL/Loop

$\Sigma V_{loop} = 0$

$-6 + 4 - V_2 - V_1 + 24 = 0$

$-6 + 4 - V_2 - 27 + 24 = 0$

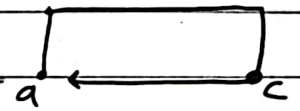
$* V_2 = -5V$

$P = \pm(IV)$

4.

$* P_{abs\ 10A} = 10 * V_{10}$

KVL/Loop



$4 - V_2 + V_{10} = 0$

$4 - -5 + V_{10} = 0$

$4 + 5 + V_{10} = 0$

$V_{10} = -9$

$\rightarrow P_{abs\ 10A} = -90W$

$* P_{abs\ 6\Omega} = 6(i_6)$

$* P_{abs\ 12\Omega} = \frac{4^2}{12} = \frac{V^2}{R}$

5. $P_{abs\ 24V} = -i_3(24)$ $\rightarrow$ سالب عكس حالة غير (-)

$* P_{gen\ 24V} = -(-i_3(24))$



$* P_{gen\ 3\Omega} = -(i_3^2 R)$

Series / Parallel elements:

Series elements:

- One connection point between series element.
- Series elements have same current.
- each of series elements has own voltage.
- you can switch between series element

Parallel elements:

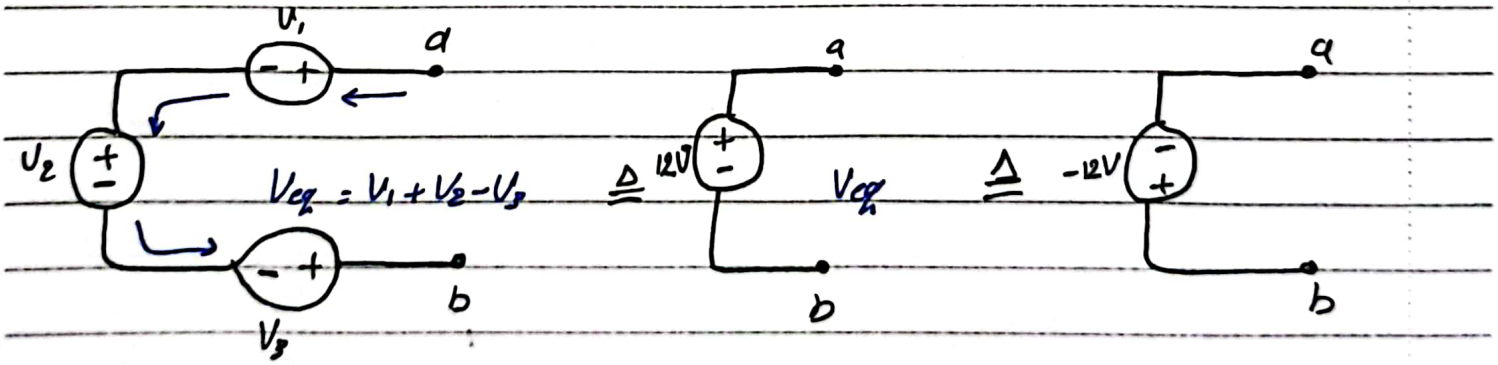
- Parallel elements connected with 2 connection point
- Parallel elements have same voltage.
- different current for Parallel elements.
- you can switch between parallel elements.

24/7/2023

Equivalent Elements:

equivalent sources, (only for indep sources

Series sources
(Voltage sources)



Case 1:-

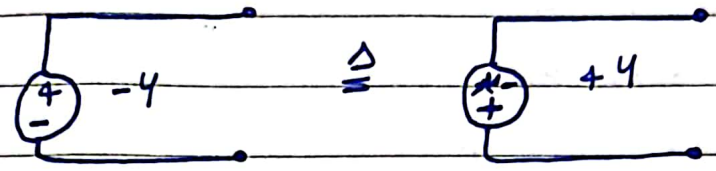
$V_1 = 10 \quad V_2 = 5 \quad V_3 = 3$

$V_{eq} = V_1 + V_2 + V_3 = +12V = -(-12)V$

Case 2:-

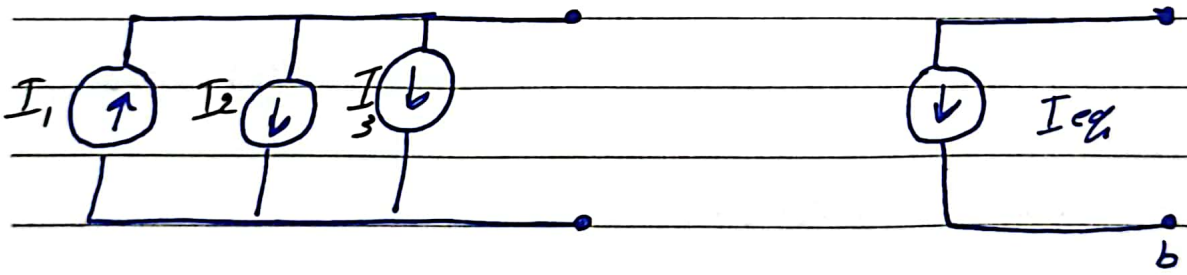
$V_1 = 3 \quad V_2 = 2 \quad V_3 = 8$

$V_{eq} = V_1 + V_2 - V_3 = (-4)$



* no equivalent sources for series indep current sources.
* no equivalent sources for parallel indep voltage sources.

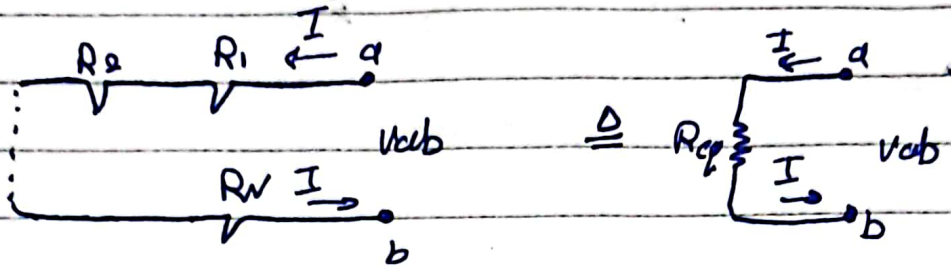
equivalent current sources for indep current sources.



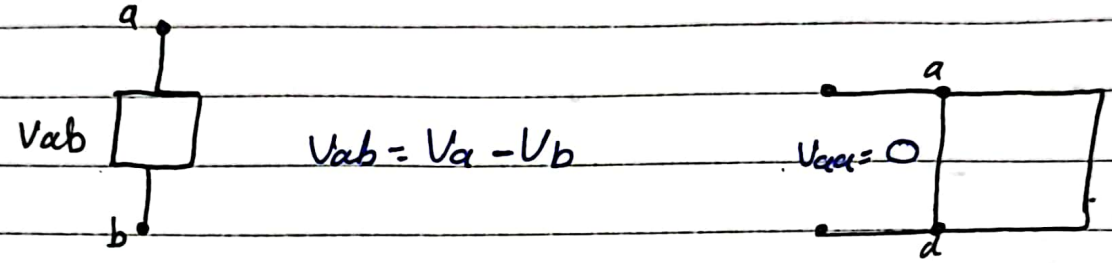
$$I_{eq} = I_2 + I_3 - I_1$$

Series and parallel Resistors :-

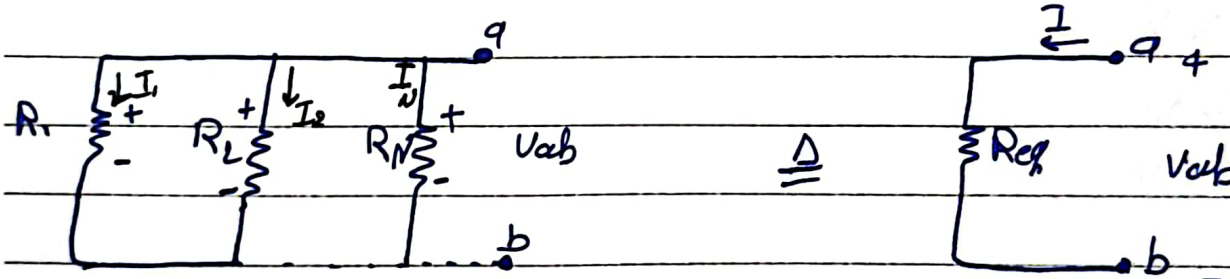
series Resistors :-



$V_{ab} = I(R_1 + R_2 --- + R_N)$ $\qquad$ $V_{ab} = I R_{eq}$.



Parallel Resistors :-

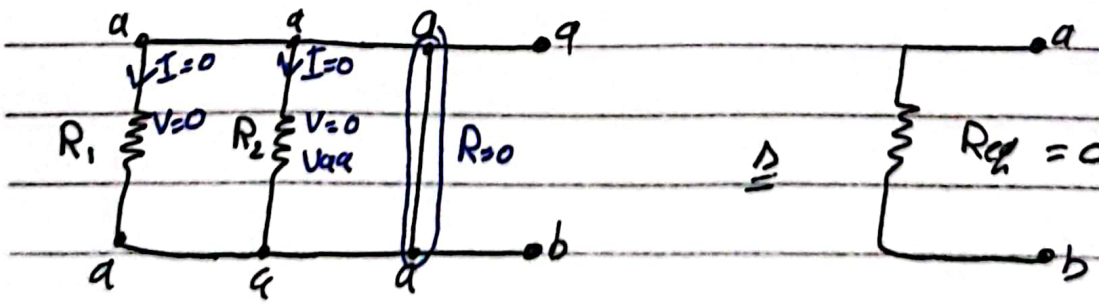


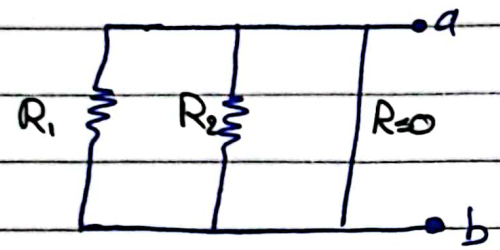
$I = I_1 + I_2 + --- + I_N$

$I = V_{ab}\left(\frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_N}\right)$ $\qquad$ $I = \frac{V_{ab}}{R_{eq}}$.

$R_{eq} = (R_1^{-1} + R_2^{-1} \ldots + R_N^{-1})$

special cases (Parallel Resistors)

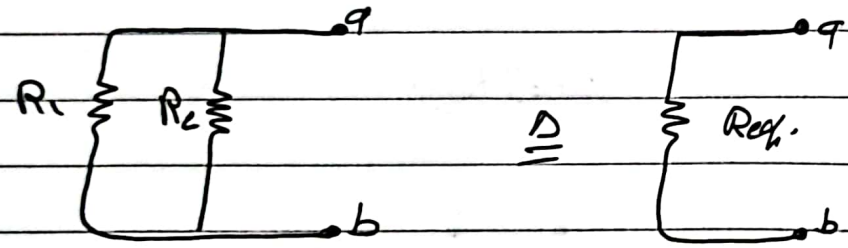




$$R_{equ} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{0}\right)^{-1}$$

$$= \frac{1}{\infty} = \text{zero}$$

*

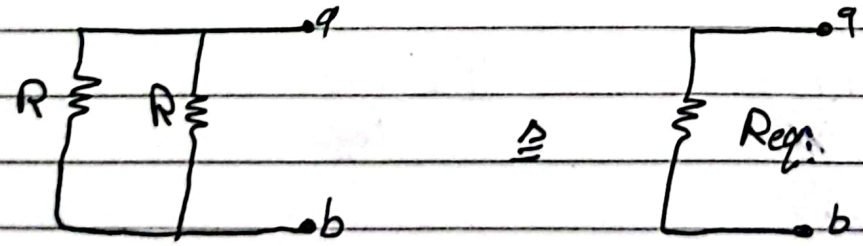


$$R_{eq} = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)^{-1}$$

$$= \left(\frac{R_1 + R_2}{R_1 R_2}\right)^{-1}$$

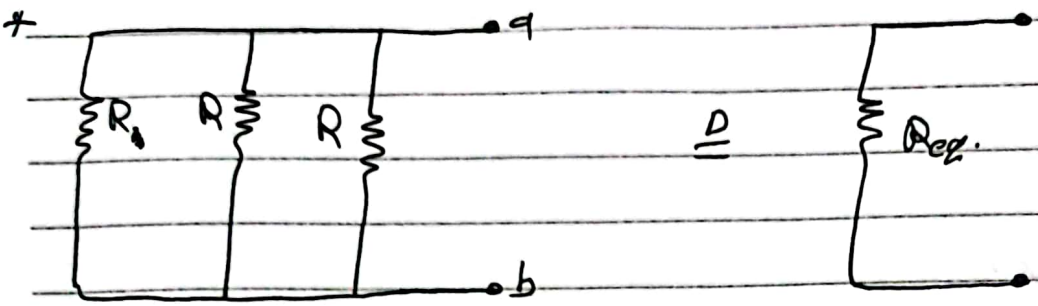
$$R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

*



$$R_{eq} = \frac{RR}{R+R}$$

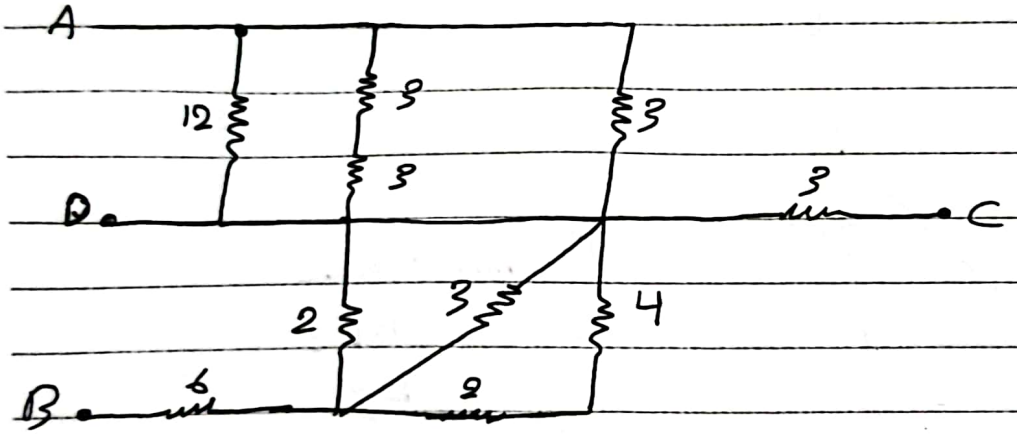
$$= \frac{R^2}{2R} = \frac{R}{2}$$



$$R_{eq} = \left(\frac{1}{R} + \frac{1}{R} + \frac{1}{R}\right)^{-1} = \left(\frac{3}{R}\right)^{-1} = \frac{R}{3}$$

* for N1 parallel Resistors with same value:-

$$R_{eq} = \frac{R}{N}$$

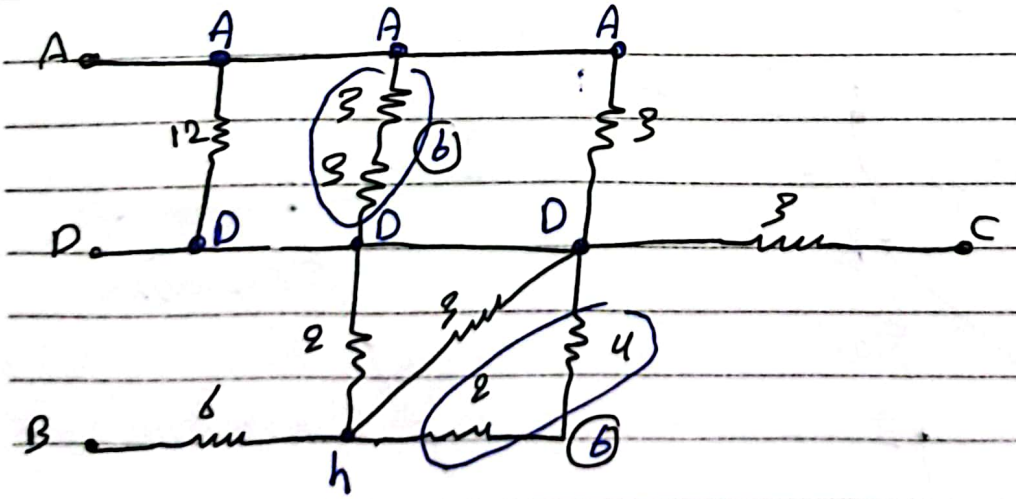


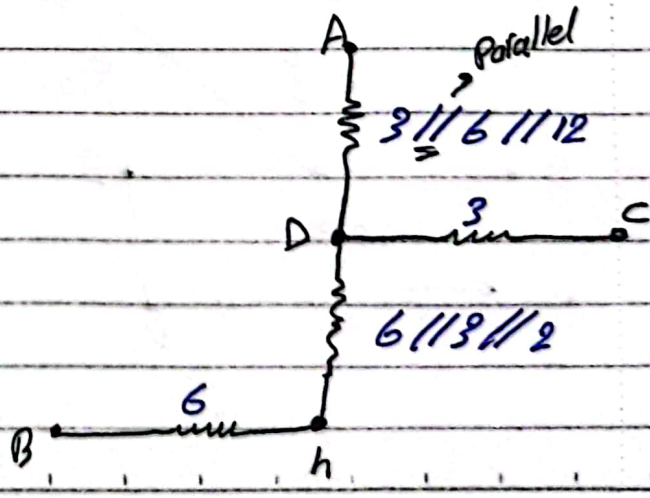
find R_{eq} between.

$AC = R_{AC} = R_{CA}$

$BC = R_{BC} = R_{BC}$

$AB = R_{AB} = R_{BA}$





$R_{AC} = (3 // 6 // 12) + 3$

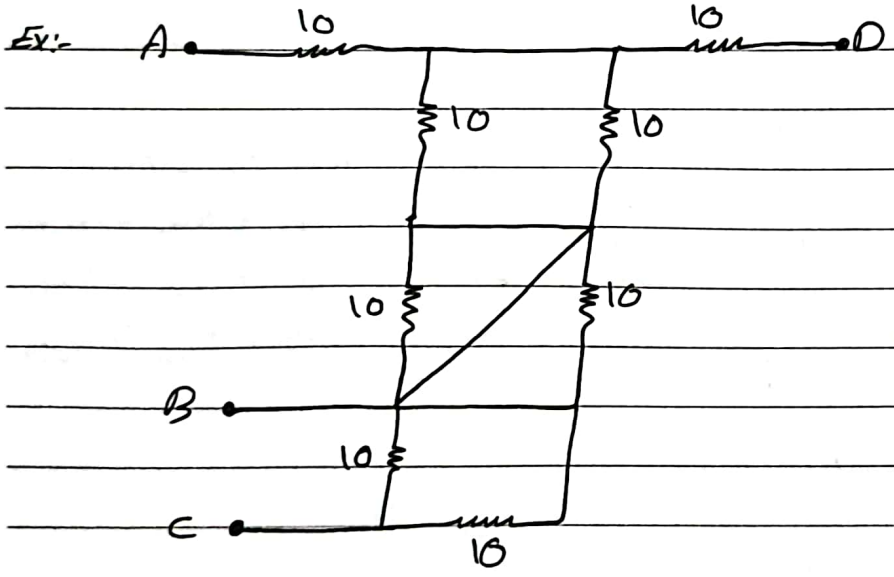
$\frac{24}{14} + 3$

$R_{CB} = 6 + (6 // 3 // 12) + 3$

$= 10$

$R_{AB} = 6 + (6 // 3 // 12) + (3 // 6 // 12)$

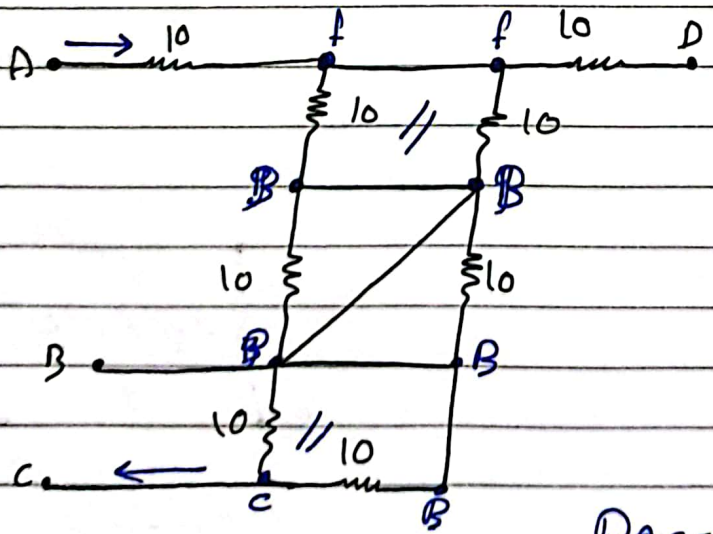
Ex:-

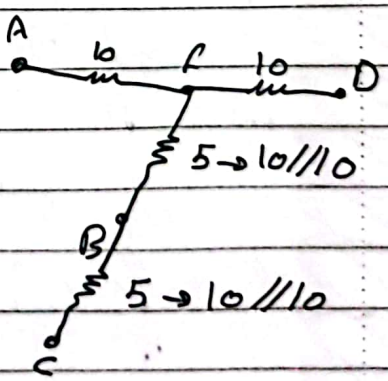


find :- R_{AC}

R_{DB}

R_{BC}





$R_{AC} = 10 + 5 + 5 = 20$

R_{DB}

R_{BC}

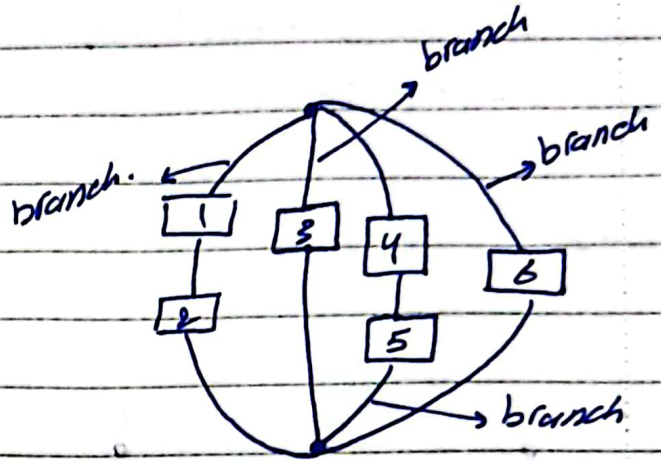
Five Apple

25/7/2023

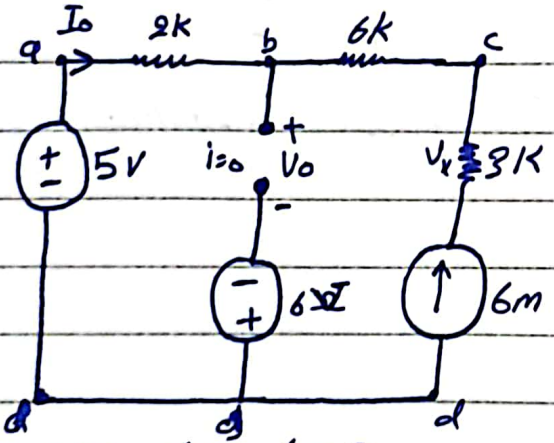
* Special cases of ccts:-

1. Single Loop ccts (closed)
   - consists of series element (have same current) KVL / loop.

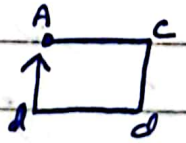
2. single node pair ccts
   - consists of parallel branches (have same voltage) use KCL / node.



Ex:-



one close loop



find: I_o, V_o, V_x

P_{gen} by 6m, 3K

P_{abs} by 5V

P_{abs} by 2K

- $I_o = -6mA$ ↓ (±) (↑)

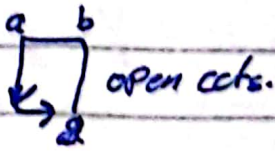
  series (have same current).

- $V_x = -(6m) \times (3K) = -18\ V$

- V_o : to find V_o use KVL/loop

  ~~[illegible]~~

  $6 + (2K \times 6m) + 5 - V_o = 0.$

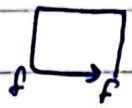


open ccts.

to find $P_{gen\ 6mA}$

find V_{source}

KVL / close loop



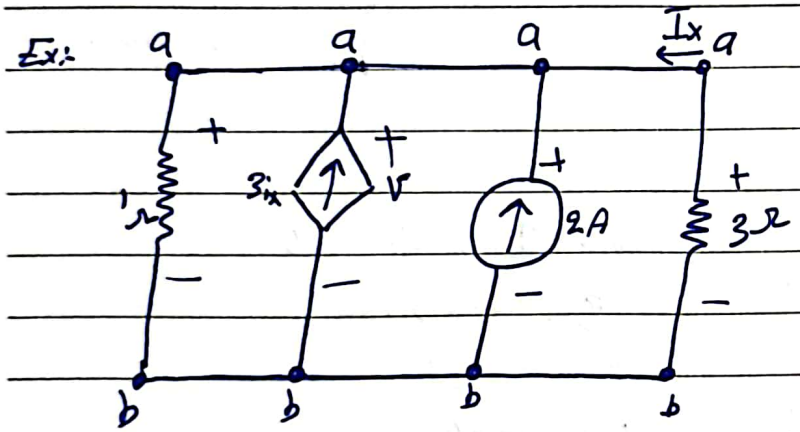
$V_{source} + 6m(2+3+6)K + 5 = 0$

$P_{abs} = (I\ V_{source}) = 6m\ V_{source}$

$P_{gen\ 6mA} = -(6m\ V_{source})$

~~$P_{gen\ 2A}$~~

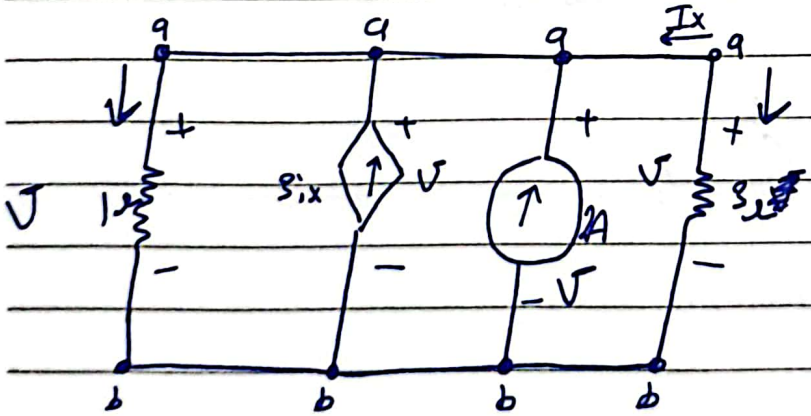
Ex:



find P_{gen} by dep source.

i_x , V

P_{abs} by ind source.



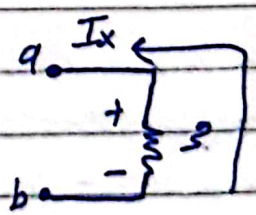
have same voltage.

بنزل تعويض عند

$\Sigma in_a = \Sigma out_a$ to find V

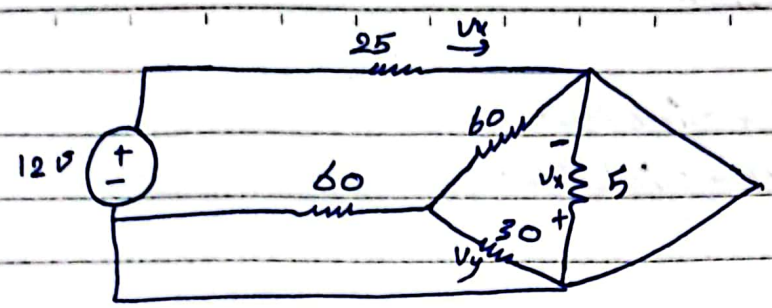
$3i_x + 2 = \frac{V}{1} + \frac{V}{3}$ ... ①

$-3i_x = V$ ... ②



$V_s = -3i_x$

Ex:



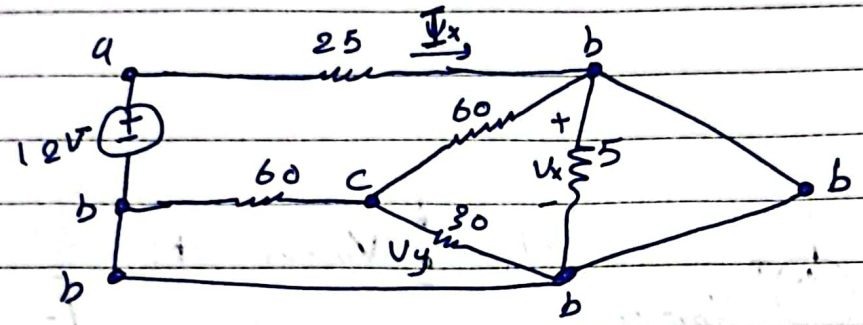
Find V_x

i_x

V_y

Pabs by 30Ω

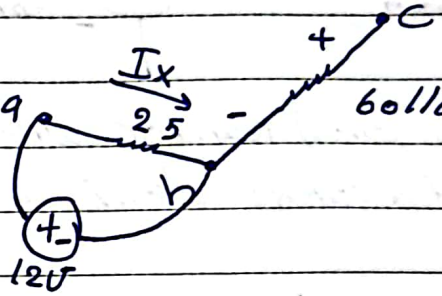
by 12V



$V_x = V_{bb} = 0$

$V_y = 0$ به خاطر اتصال کوتاه

$I_x = \frac{12}{25}$



Pabs 30Ω = zero

Pabs 12V : $-I_x(12)$.

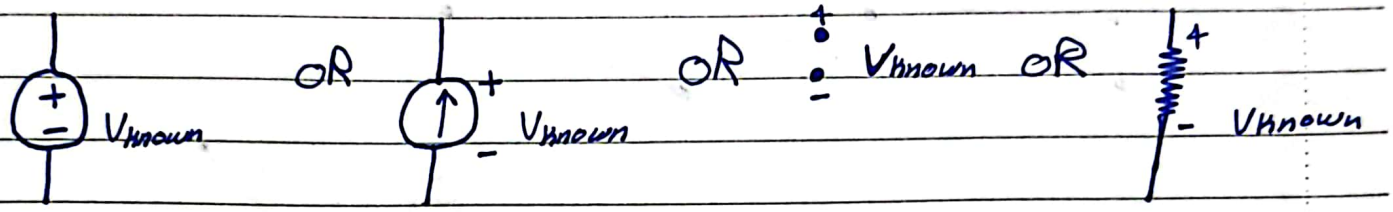
26/7/2023.

current and voltage division.

* current division → KCL + ohm's law
* voltage division → KVL + ohm's law

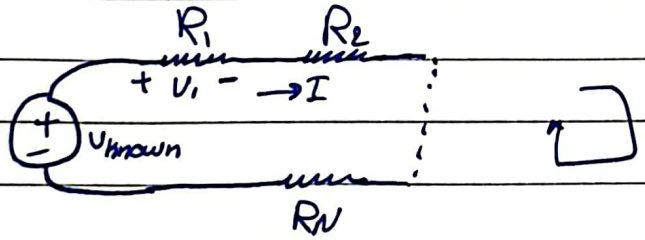
* Voltage div

known voltage.



series (only) resistor with in a loop

to find a voltage across any of series resistor.



find V_1

* $V_1 = IR_1$

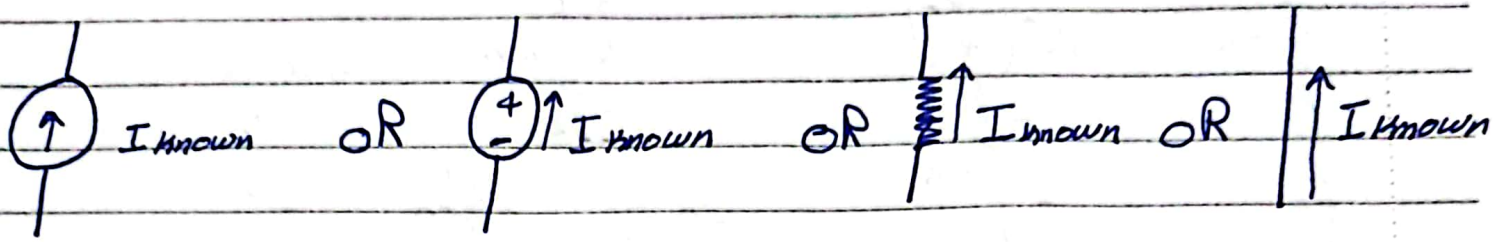
to find I (KVL) $-V_{known} + I(R_1 + R_2 + \cdots + R_N) = 0$

$$I = \frac{V_{known}}{R_1 + R_2 + \cdots + R_N}$$

$$V_K = \frac{\pm V_{known}\, R_K}{R_1 + R_2 + \cdots + R_N}$$

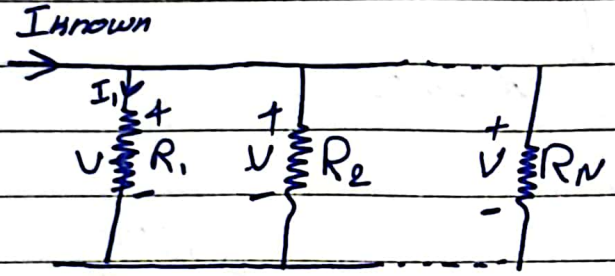
+Current div

Known current



~~Parallel Resistors only across a node~~

Parallel Resistors (only) across a node.



Find I_1

$I = \frac{V}{R_1}$ to find V → KCL

$I_{known} = I_1 + I_2 + \cdots + I_N$

$= \frac{V}{R_1} + \frac{V}{R_2} + \cdots + \frac{V}{R_N}$

$I_{known} = V\left[\frac{1}{R_1} + \frac{1}{R_2} + \cdots \frac{1}{R_N}\right]$

$= V[G_1 + G_2 + \cdots G_N]$

G: conductor

G

$G = \frac{1}{R}$

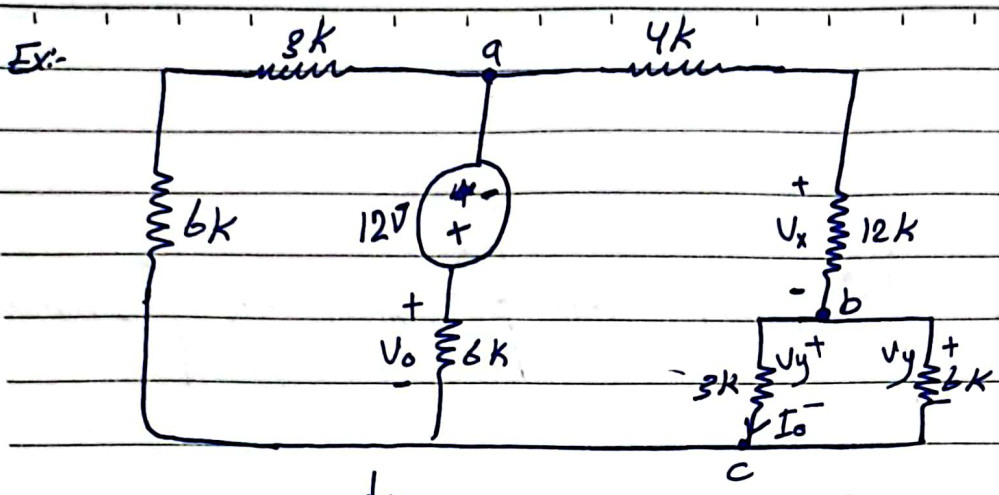
$R = \frac{1}{G}$

$I_K = \pm \frac{I_{known}\, G_K}{G_1 + G_2 + \cdots + G_N}$

+: different direction.

Five Apple

Ex:-



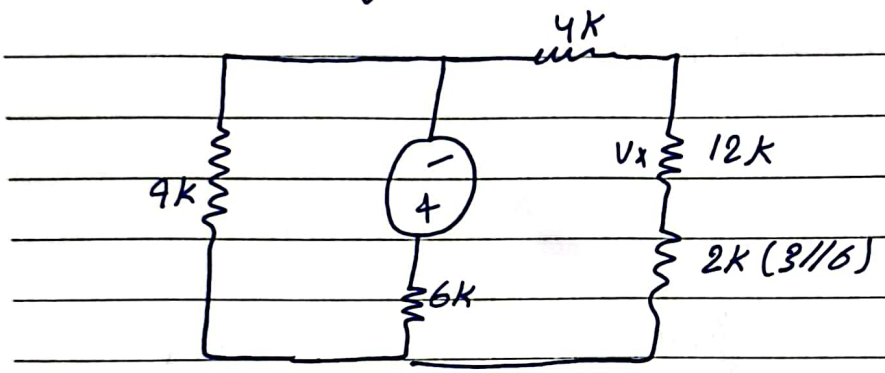
Find Vx

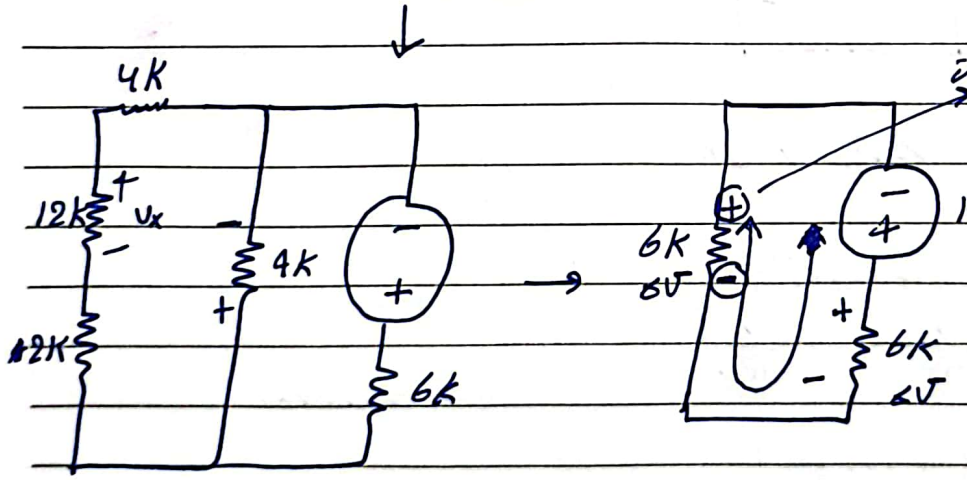
Vo

Io

Pabs by 12k

Psrc by 12V



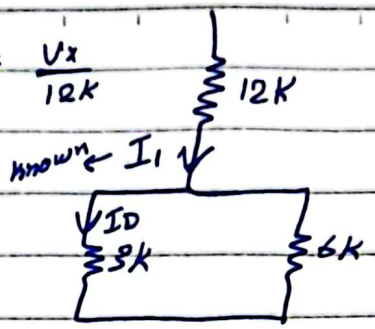


$$* V_o = \frac{+6k * 12}{(6+6)k} = +6\,V$$

$$* V_x = \frac{-6 * 12}{4+12+2}$$

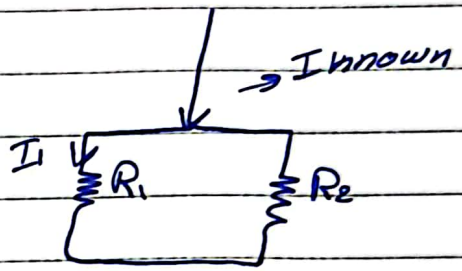
$$* V_y = \frac{2k * 6}{12+4+12} \longrightarrow I_o = \frac{V_y}{3k}$$

$I_1 = \frac{V_x}{12k}$



I_o کی طرح تقسیم کی

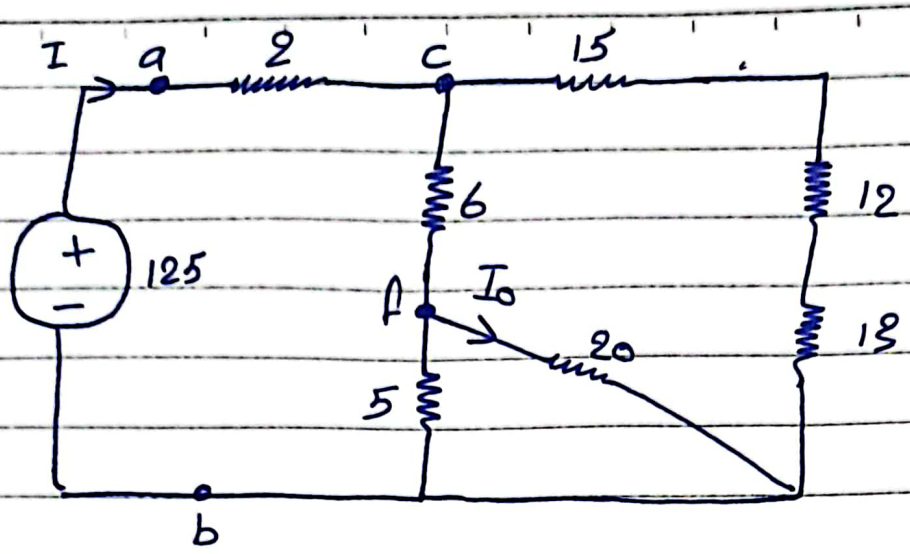
$$I_o = +I_{known} \times \frac{\frac{1}{3}}{\frac{1}{3}+\frac{1}{6}}$$



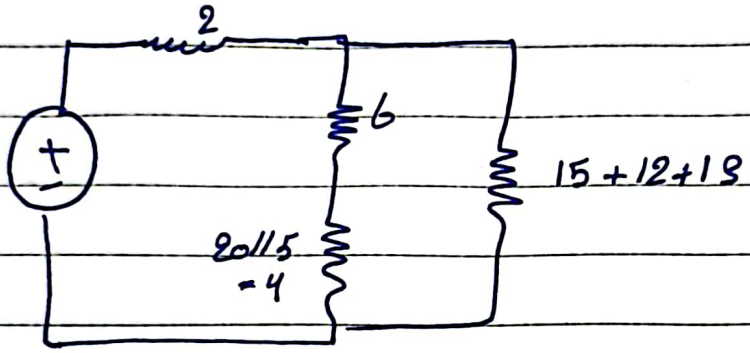
$$I_1 = \frac{I_{known} \cancel{R_1}}{\frac{R_1+R_2}{\cancel{R_1} R_2}}$$

$$I_1 = \frac{I_{known} R_2}{R_1+R_2}$$

Ex:

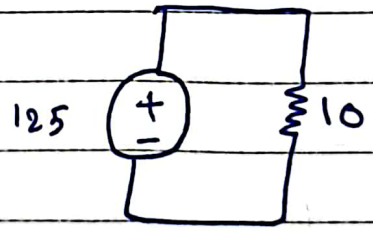


find Req ab.
Io , I



$R_{eq} = ((15+12+18) \,//\, (6+4)) + 2 = 10\,\Omega$

$$I = \frac{125}{10}$$



$$I_o = \frac{+I_b \times 5}{20+5}$$

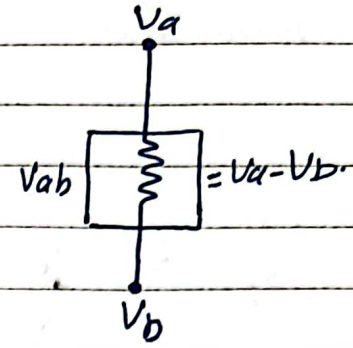
1/8/2023

chapter 4:- Nodal analysis and mesh analysis:-
are used to reduse the no. of eqns.

Nodal analysis (Voltage node analysis)

KCL /node

$\sum I_{in} = \sum I_{out}$



* method

1. Label the nodes

node a → Va

b → Vb

c → Vc

⋮ ⋮

n → Vn

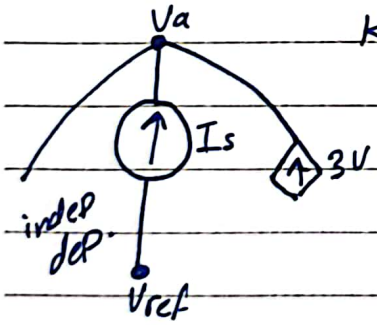
2. choose a refrance node ⏚

V_{ref} = zero

eqns = # node − 1

Cases:-

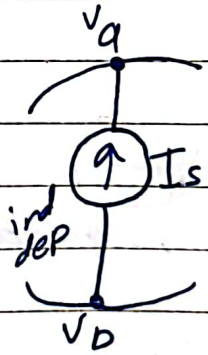
1.



KCL/a

$\sum I_{in} = \sum I_{out}$

$I_s + \dots = \dots$



KCL/a

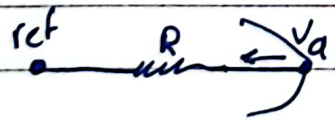
$\sum I_{in} = \sum I_{out}$

$I_s + \dots = \dots$

KCL/b

$\sum I_{in} = \sum I_{out}$

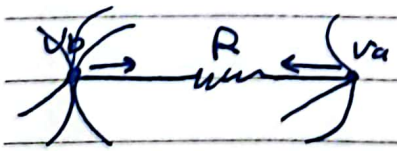
$\dots = I_s + \dots$

2. 

KCL/a

$\sum I_{in} = \sum I_{out}$

$\dots = \frac{V_a}{R} + \dots$



KCL/d

$\sum I_{in} = \sum I_{out}$

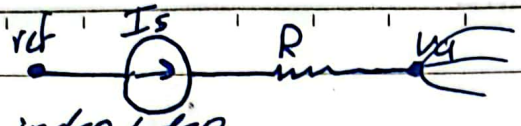
$\dots = \frac{V_a - V_b}{R} + \dots$

KCL/b

$\sum I_{in} = \sum I_{out}$

$\dots = \frac{V_b - V_a}{R} + \dots$

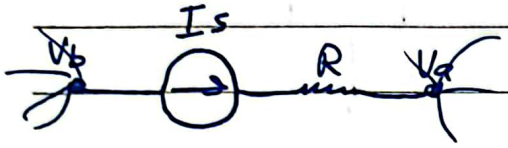
CamScanner

3. 

KCL / a

$\Sigma I_{in} = \Sigma I_{out}$

$I_{s+\cdots} = \cdots$



KCL / a

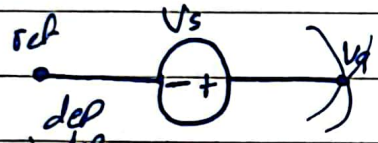
$\Sigma I_{in} = \Sigma I_{out}$

$I_{s+\cdots} = \cdots$

KCL / b

$\Sigma I_{in} = \Sigma I_{out}$

$\cdots = I_{s+\cdots}$

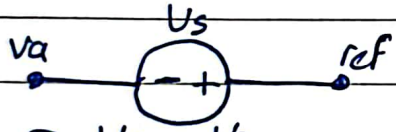
4. 

KCL / a

$\Sigma I_{in} = \Sigma I_{out}$

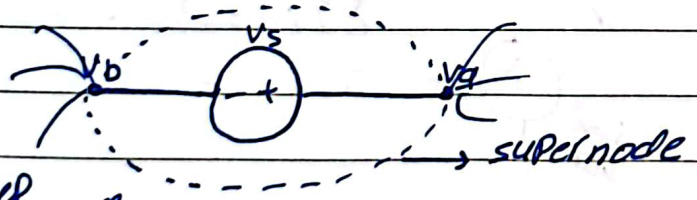
✗ don't use KCL / a to find voltage.

$V_a = V_s$ → $V_{a-0} = V_s$



$0 - V_a = V_s$

$V_a = -V_s$

5. 

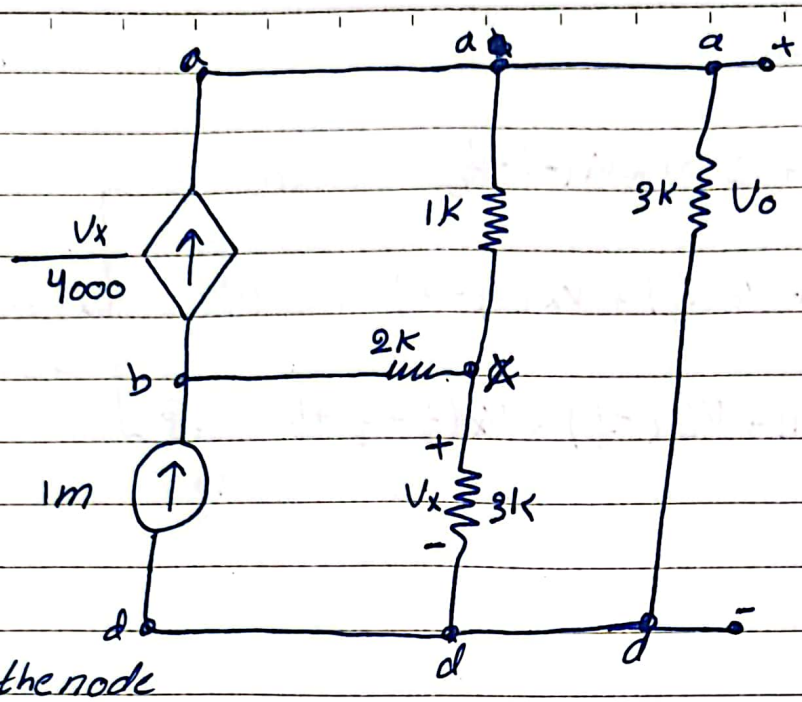
Indep dep voltage source between 2 node:

KCL / supernode:

$\Sigma I_{in} = \Sigma I_{out}$ ---- ①

$V_a - V_b = V_s$ ---- ②

Ex:-



Find Vx

Pgen by Im, $\frac{Vx}{4000}$, 3K

using nodal analysis.

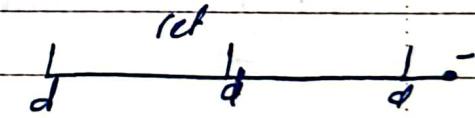
Label the node

choose a ref

cct has indep/dep current source + resistor.

choose any node as a ref OR choose the node with maximum no. of branches as a ref

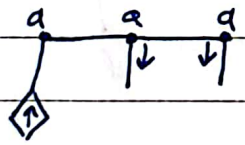
as a ref ← node ... 3 branches ... node ... ref



KCL /a

$\Sigma I_{in} = \Sigma I_{out}$

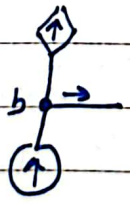
$\frac{Vx}{4k} = \frac{Va - Vx}{k} + \frac{Vo}{3K}$



KCL /b

$\Sigma I_{in} = \Sigma I_{out}$

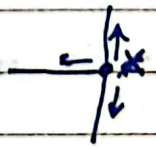
$Im = \frac{Vx}{4k} + \frac{Vb - Vx}{2k}$



KCL /c

$\Sigma I_{in} = \Sigma I_{out}$

$zero = \frac{Vx - Vb}{2k} + \frac{Vx}{3k} + \frac{Vx - Vg}{K}$



$$\text{zero} = V_a\left(1+\frac{1}{5}\right)+V_b(0)+V_x\left(-1-\frac{1}{4}\right) \quad \text{---} (1)$$

$$1 = V_a(0)+V_b\left(\frac{1}{2}\right)+V_x\left(\frac{1}{4}-\frac{1}{2}\right) \quad \text{---} (2)$$

$$\text{zero} = V_a\left(-1\right)+V_b\left(-\frac{1}{2}\right)+V_x\left(\frac{1}{2}+\frac{1}{5}+1\right) \quad \text{---} (3)$$

→ Using calculator.

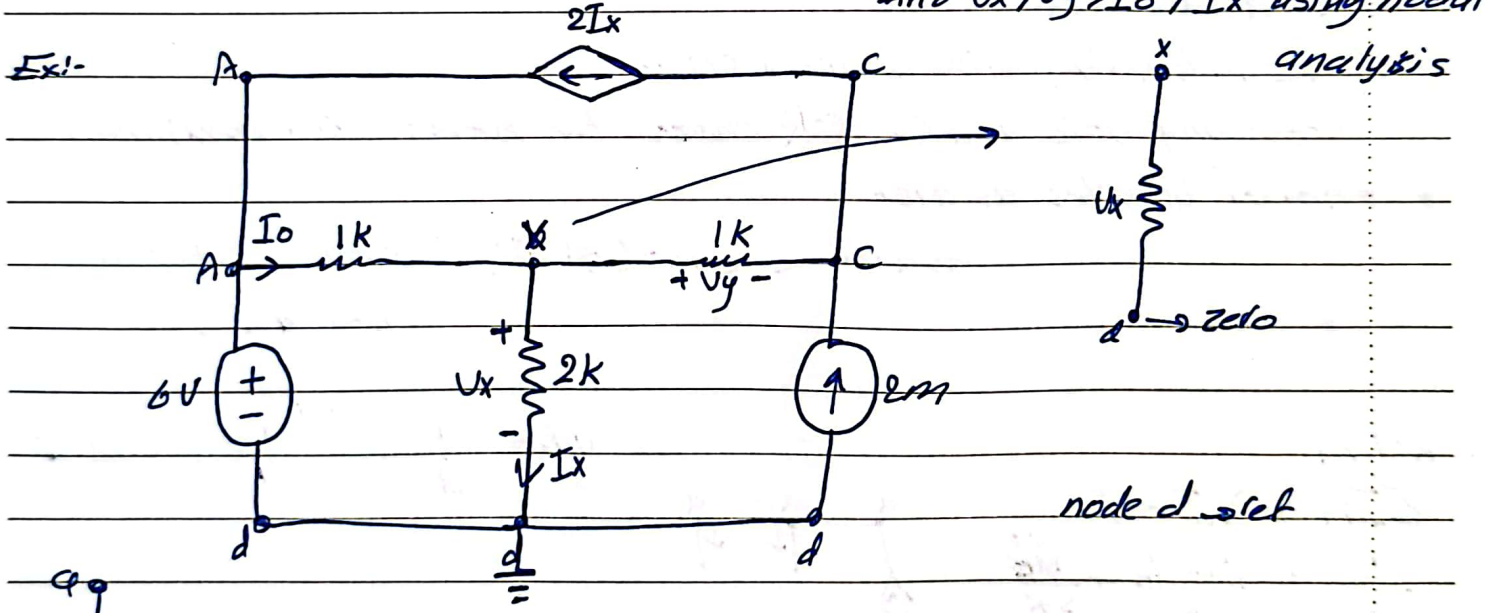
$V_a = 1.45$

$V_b = 2.75$

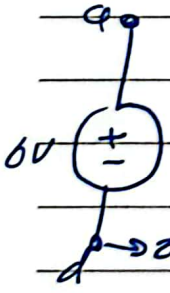
$V_x = 1.54$

Find V_x, V_y, I_o, I_x using nodal analysis

Ex:-



node d → ref



$V_a = 6V$

KCL / b

$$\Sigma I_{in} = \Sigma I_{out}$$

$$0 = \frac{V_x - V_a}{K} + \frac{V_x}{2K} + \frac{V_x - V_c}{K}$$

KCL / c

$$\Sigma I_{in} = \Sigma I_{out}$$

$$2m = \frac{V_c - V_x}{1K} + 2I_x$$

$$V_x = 2KI_x \rightarrow I_x = \frac{V_x}{2K}$$

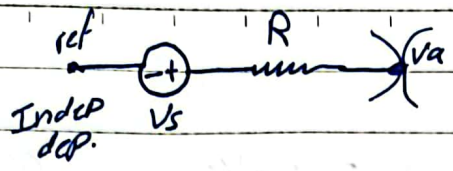
$$V_y = V_x - V_c$$

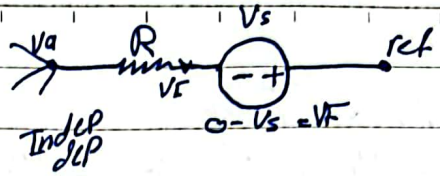
$$I_o = \frac{V_a - V_x}{K}$$

CamScanner

7/8/2023

6.





Kcl / a

$\Sigma I_{in} = \Sigma I_{out}$

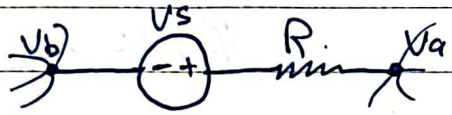
$\dots = \frac{Va - Vs}{R}$

Kcl / a

$\Sigma I_{in} = \Sigma I_{out}$

$\dots = \frac{Va - -Vs}{R}$ a...

7.



KCL / a

$\Sigma I_{in} = \Sigma I_{out}$

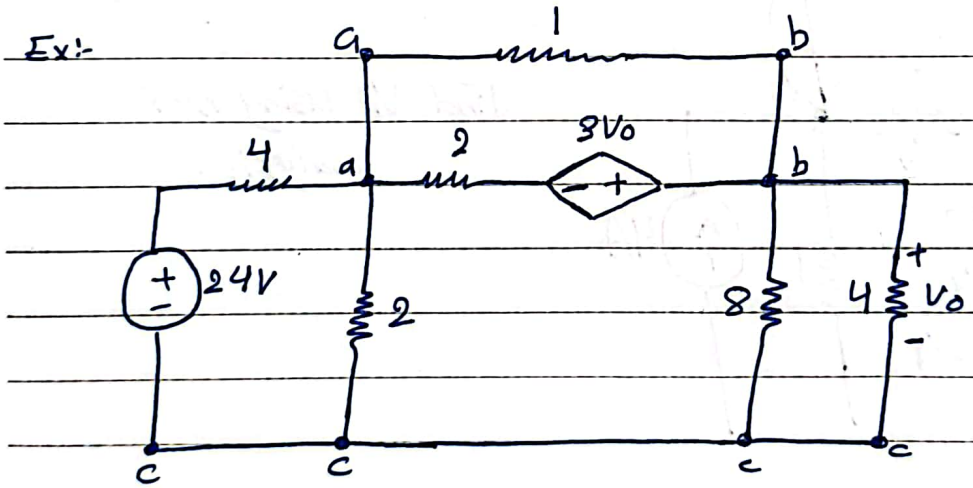
$\dots = \frac{Va - Vs - Vb}{R}$

KCL / b

$\Sigma I_{in} = \Sigma I_{out}$

$\dots = \frac{Vb - Vs - Va}{R}$

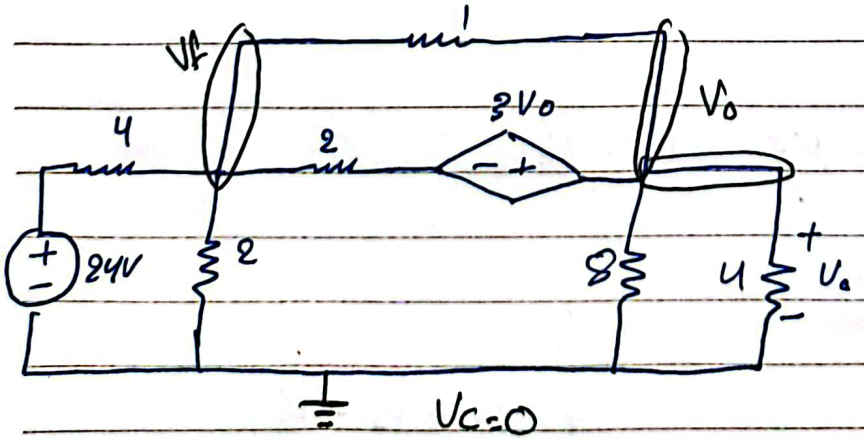
Ex:-



Find :- Vo using nodal analysis

Pabs by 24V

3Vo



CamScanner

KCL / f

$\Sigma I_{in} = \Sigma I_{out}$

$$0 = \frac{V_f}{2} + \frac{V_f - V_o}{1} + \frac{V_f - 24}{4} + \frac{V_f - 3V_o - V_o}{2} \quad \text{---} (1)$$

KCL / node o

$\Sigma I_{in} = \Sigma I_{out}$

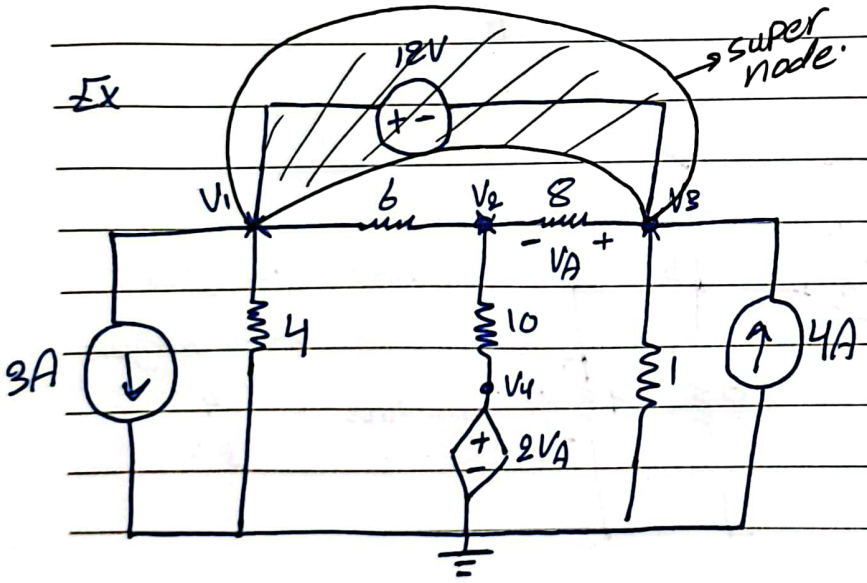
$$0 = \frac{V_o}{4} + \frac{V_o}{8} + \frac{V_o - V_f}{1} + \frac{V_o - 3V_o - V_f}{2} \quad \text{---} (2)$$

V_f, V_o ??

$$P_{abs\ 24V} = 24\left(\frac{V_f - 24}{4}\right)$$

$$P_{abs\ 3V_o} = 3V_o\left(\frac{V_o - 3V_o - V_f}{2}\right)$$

Ex



Find VA using nodal analysis.

KCL / super node:

$\Sigma I_{in} = \Sigma I_{out}$

$$4 = 3 + \frac{V_1}{4} + \frac{V_1 - V_2}{6} + \frac{V_3}{1} + \frac{V_3 - V_2}{8} \quad \text{---} (1)$$

$$V_1 - V_3 = 12 \quad \text{---} (2)$$

KCL / node 2

$\Sigma I_{in} = \Sigma I_{out}$

$$0 = \frac{V_2 - V_3}{8} + \frac{V_2 - V_1}{6} + \frac{V_2 - 2V_A}{10} \quad \text{---} (3)$$

$V_A = V_3 - V_2$

CamScanner

Mesh analysis.

mesh → loop

KVL / mesh.

mesh. عبارة عن loop / loop عبارة عن mesh

mesh 1 → I_1

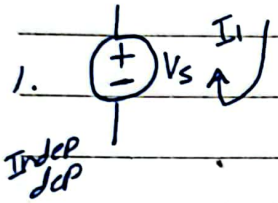
mesh 2 → I_2

mesh N → I_N

#unknowns = #meshes = # eqns.

cases:

1.

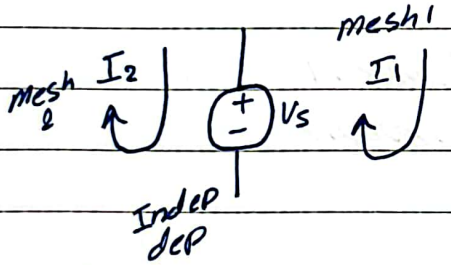


KVL / mesh 1

$\Sigma V = 0$

$-V_s + \cdots = 0$

2.



KVL / mesh 1

$\Sigma V = 0$

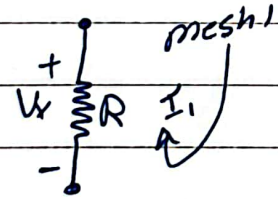
$-V_s + \cdots = 0$

KVL / mesh 2

$\Sigma V = 0$

$V_s + \cdots = 0$

3.



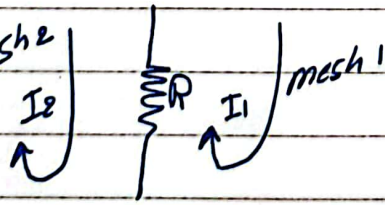
KVL / mesh 1

$\Sigma V = 0$

$-I_1 R + \cdots = 0$

$-V_x + \cdots = 0$

4.



KVL / mesh 1

$\Sigma V = 0$

$(I_1 - I_2) R + \cdots = 0$

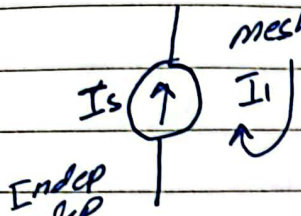
KVL / mesh 2

$\Sigma V = 0$

$(I_2 - I_1) R + \cdots = 0$

CamScanner

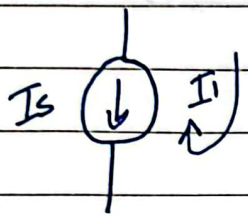
5.



KVL/mesh1

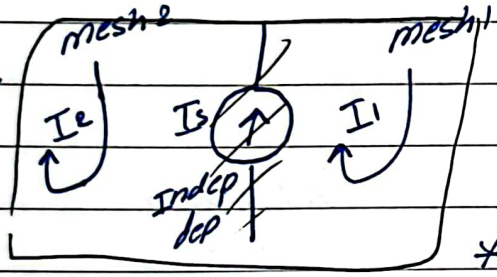
$\Sigma V = 0$ X Don't use KVL/mesh to find the current.

$\rightarrow I_1 = I_s$



$\rightarrow I_1 = -I_s$

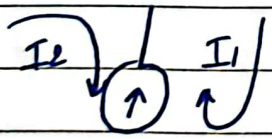
6.



supermeshes (between two meshes).

+ Don't use KVL/mesh1
KVL/mesh2

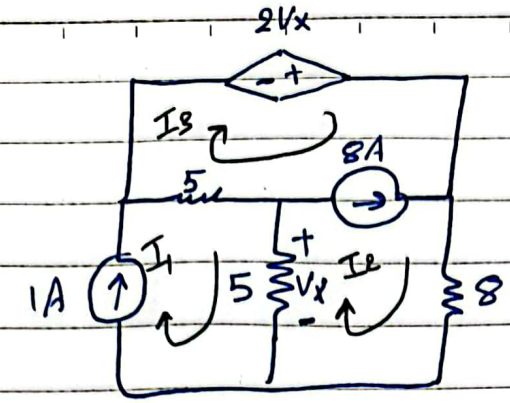
KVL/supermesh



$I_1 - I_2 = I_s$

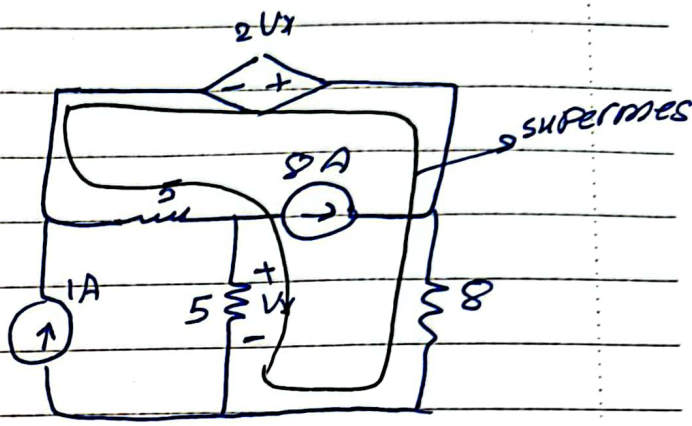
CamScanner

Ex:-



Find Vx using mesh analysis.

$I_1 = 1A$



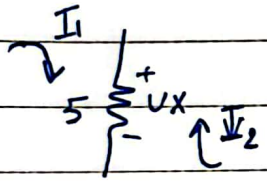
KVL / supermesh (I_2, I_3)

$\Sigma V = 0$

$-2V_x + 8I_2 + 5(I_2 - I_1) + 5(I_3 - I_1) = 0$ (1A, 1A)

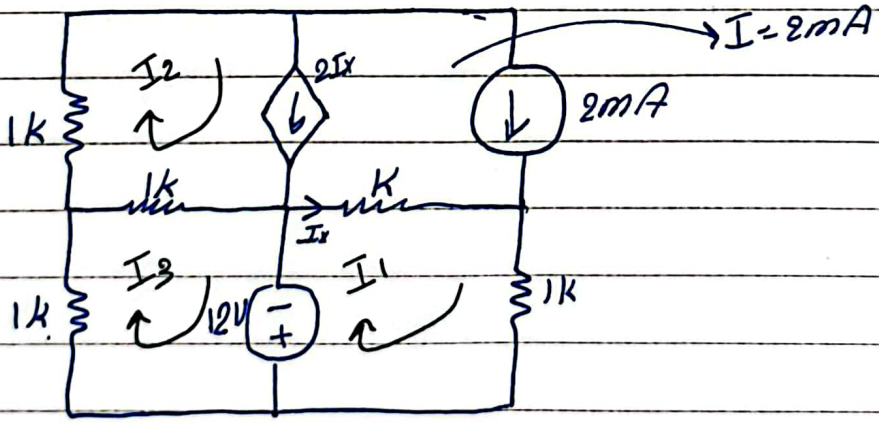
---(1)

$8 = I_2 - I_3$ ---(2)



$5(I_1 - I_2) = V_x$ ---(3)

Ex:-



$2I_x = I_2 - 2mA$ ---(1)

KVL / mesh 1

$1KI_1 + 12 + K(I_1 - 2m) = 0$ ---(2)

KVL / mesh 3

$-12 + KI_3 + K_1(I_3 - I_2) = 0$ ---(3)

$I_1 - 2m = I_x$ ---(4)

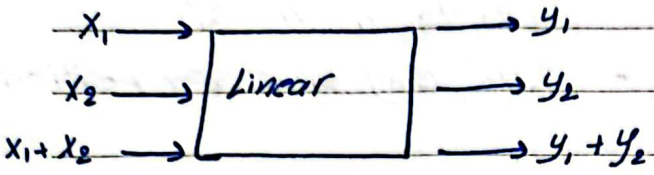
Five Apple

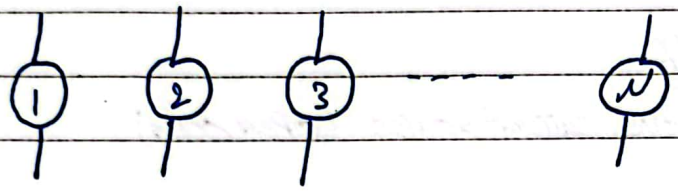
CamScanner

Chapter 5:-

Super position (Indep sources only)

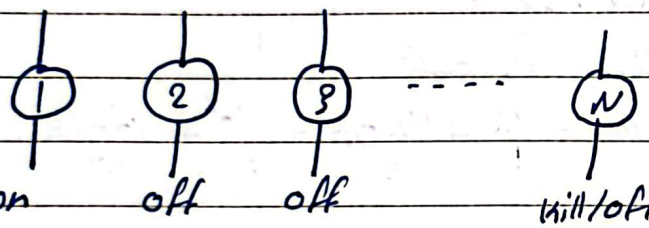
Linear ccts.





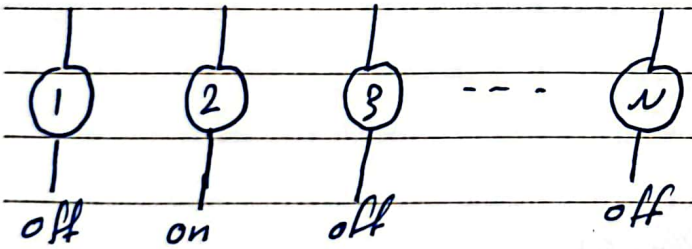
Find i, V

super position.



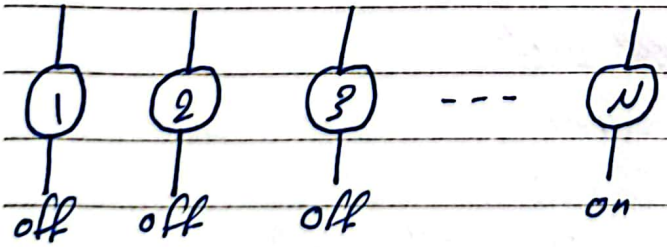
$i' = i$ to source 1

$v' = v$ " " 1



$i'' = i$ to source 2

$v'' = v$ " " 2



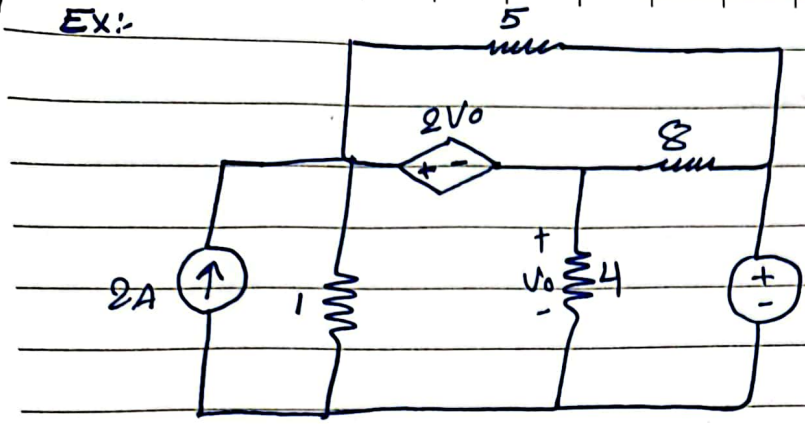
$i^N = i$ to source N

$v^n = v$ to source N

$$i = i' + i'' + \dots + i^N$$

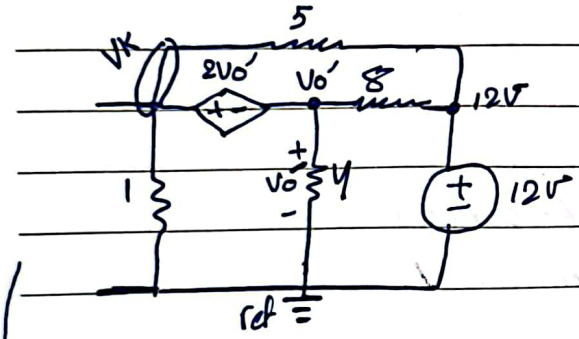
$$v = v' + v'' + \dots + v^N$$

Ex:-



1. Find Vo due to 12V
2. Vo " " 2A
3. Vo using super position.

1. V_o' : 12V/on 2A/off
↳ remove current source (open ckts)



$V_k - V_o' = 2V_o'$

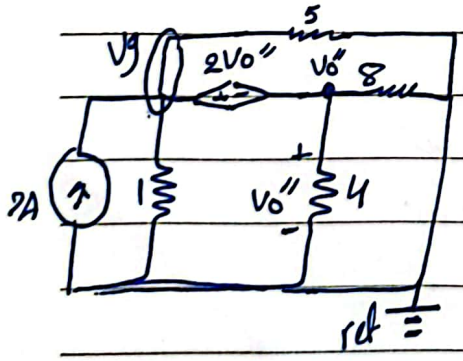
KCL / super

$\Sigma I_{in} = \Sigma I_{out}$

$$0 = \frac{V_k}{1} + \frac{V_k - 12}{5} + \frac{V_o'}{4} + \frac{V_o' - 12}{8}$$

↳ nodal analysis ohile
. (Kirchhoff's Eqn) mesh Kirchhoff

2. V_o'' : 2A/on 12V/off
↳ (short ckt)



$V_g - V_o'' = 2V_o''$

KCL / super

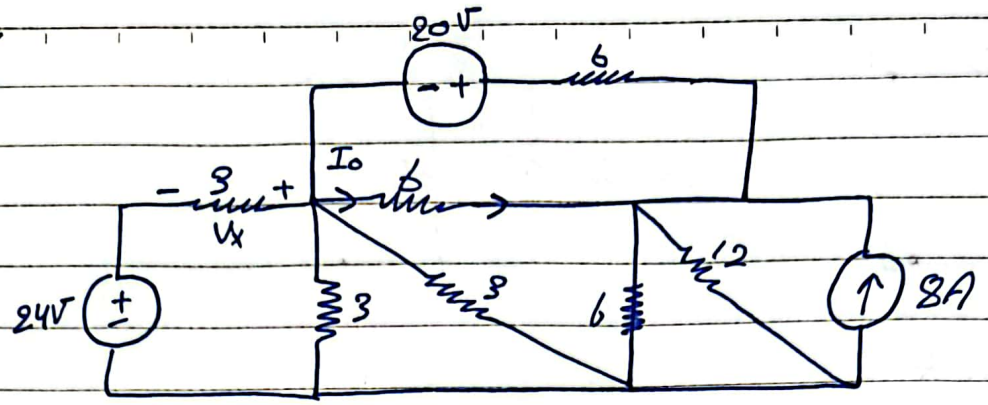
$\Sigma I_{in} = \Sigma I_{out}$

$$2 = \frac{V_g}{1} + \frac{V_g}{5} + \frac{V_o''}{4} + \frac{V_o''}{8}$$

3. $V_o = V_o' + V_o''$

CamScanner

Ex.:



Find:

i_o due to 24V = 0.5

V_x 8A = 4

i_o 20V = −1.04.

* Source transformation. 8/8/2023

Current source ⟺ Voltage source.

Indep dep — Indep dep.

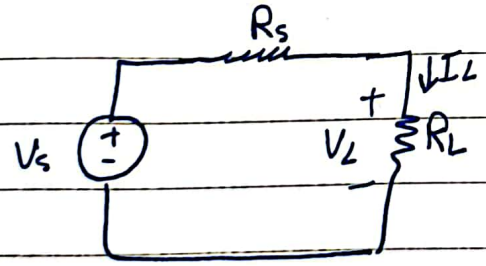


fig-1.

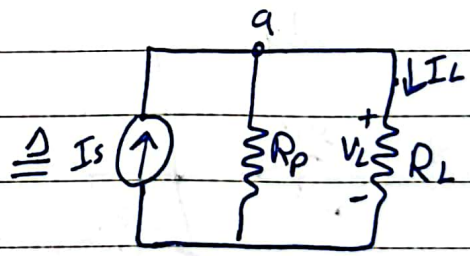


fig-2.

$$V_L = \frac{R_L}{R_L + R_S} V_S$$

$$V_L = \frac{R_L \left(\frac{R_P I_S}{R_P + R_L} \right)}{} I_L$$

$V_S = I_S R_P$

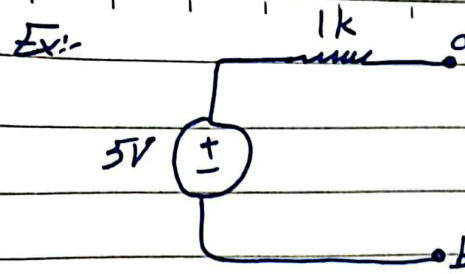
$R_L + R_S = R_P + R_L$

$R_S = R_P$

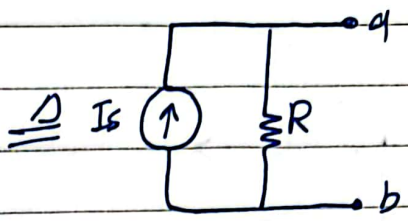
Five Apple

CamScanner

Ex:-



Δ



Find Is, R

$R = 1K\Omega$ $\qquad Is = \frac{Vs}{R} = \frac{5}{1k} = 5mA$

Ex:-

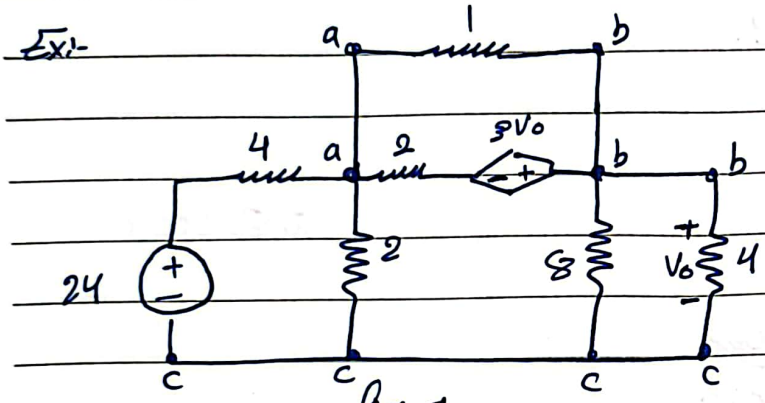


fig a

Δ

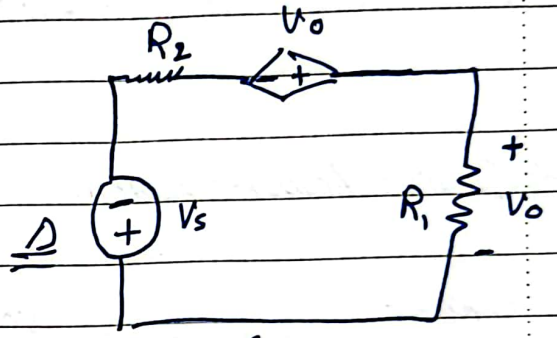
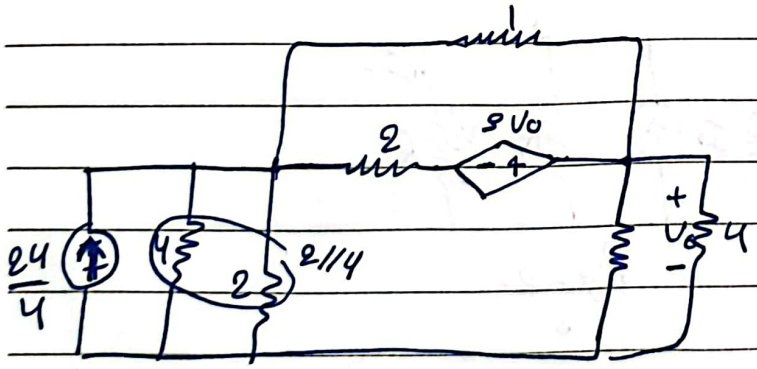
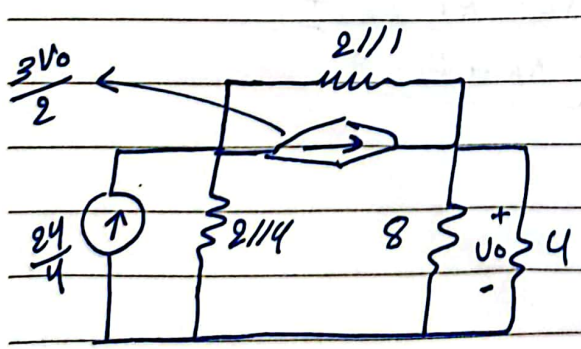
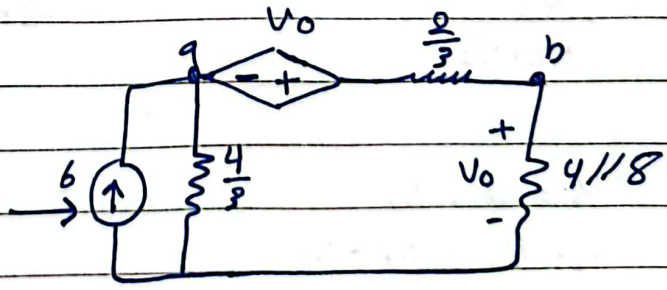


fig b

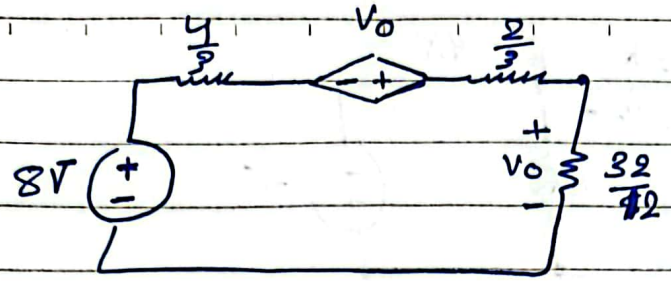
Find Re, R_1, Vs, Vo







CamScanner



$V_s = -8V \qquad R_1 = \frac{8}{9}$

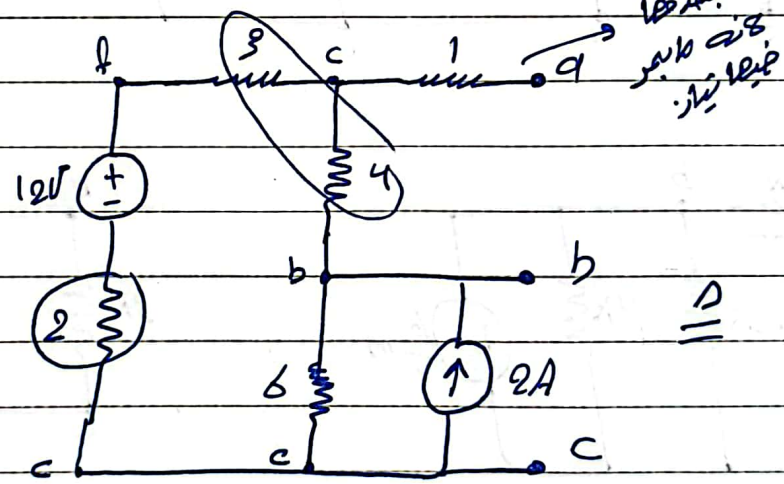
$R_2 = \frac{6}{9} = 2$

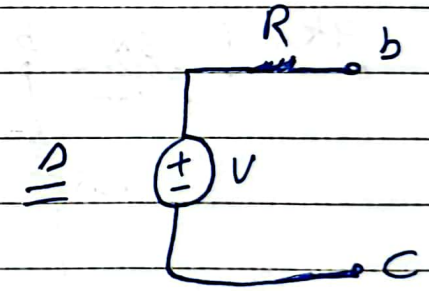
$-8 + 2I - V_6 + V_0 = 0$

$I = 4$

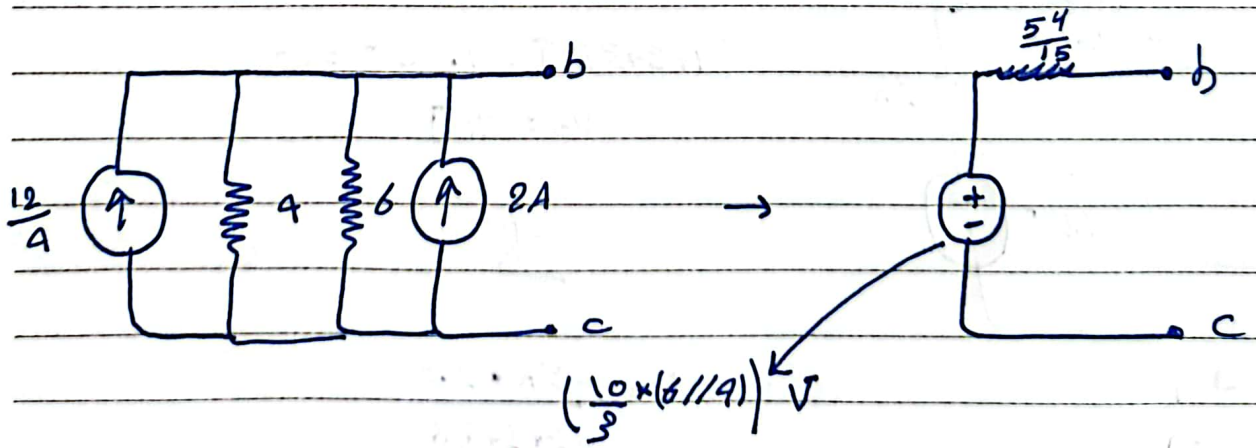
$V_o = \frac{8}{9} I = \frac{32}{9} V$

Ex.:-

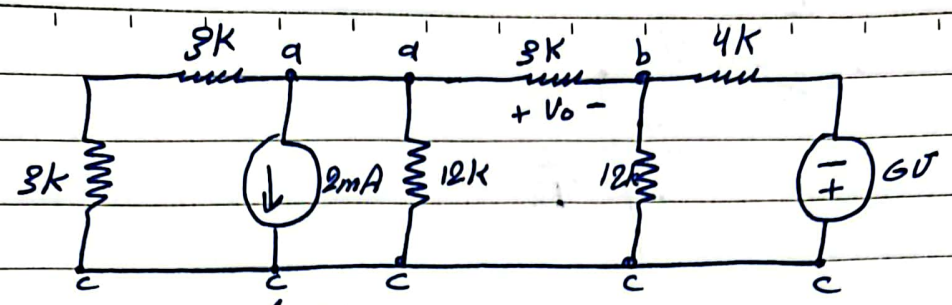




using source transformation to find V, R



Ex:-



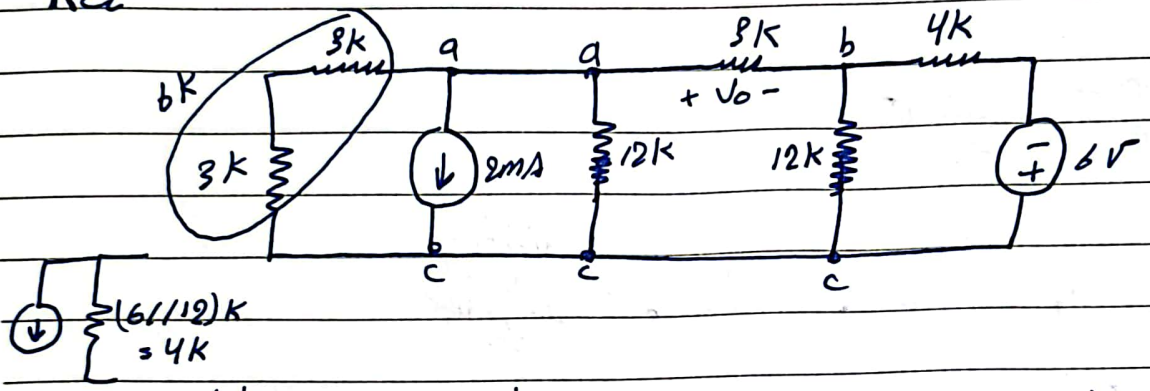
Find Vo using source tran.

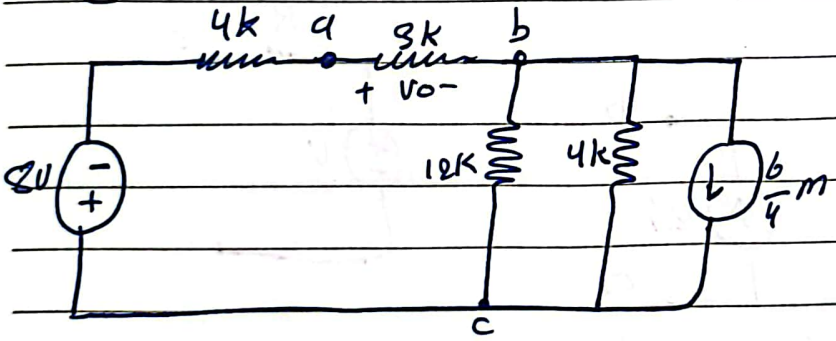
* to reduce the cct

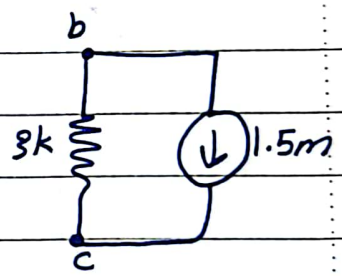
↓ single node pair cct

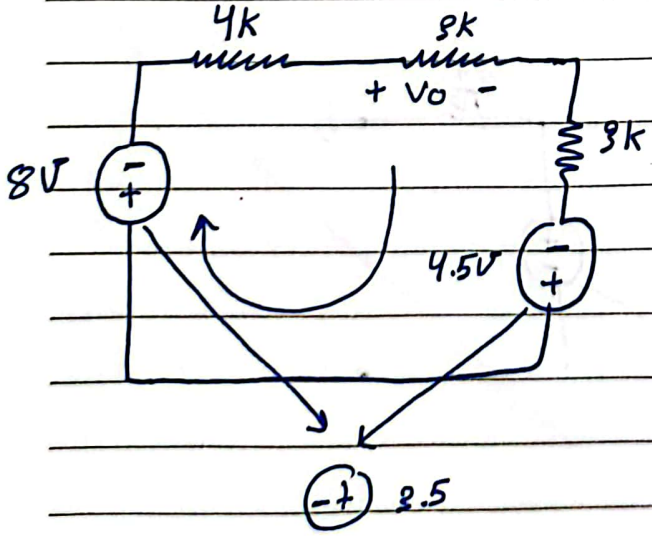
↓ single loop ccl KVL

KCL









$(4+3+3)kI - 4.5 + 8 = 0$

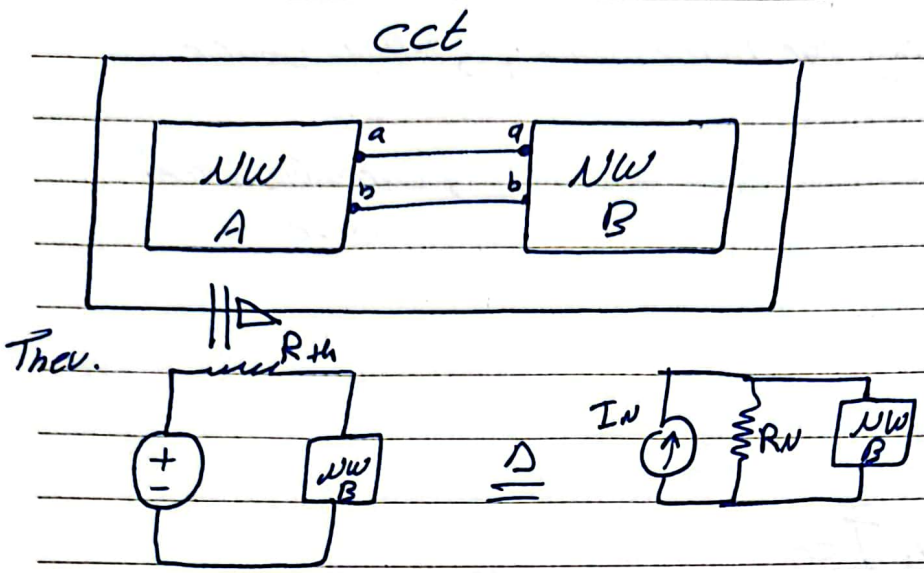
$Vo = 3k\,I$

OR

$$Vo = \frac{-3k * 3.5}{(3+3+4)k}$$

CamScanner

Thevinin / Norton equivalent ckts



Rth :- Thevinin Resistor

Vth :- Thevinin Volt.

IN :- Notron current

RN :- Notron Resistor

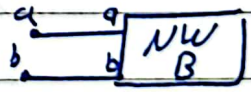
$V_{th} = I_N R_N$

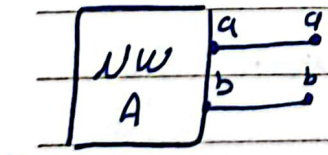
$R_N = R_{th}$

Cases NWA

case 1:

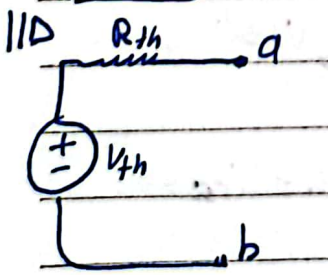
NA consists of Indep sources + resistors





O.C Find V_{th} / R_{th} / I_N

$V_{th} = V_{o.c} = V_{ab}$ terminals connected NWA with NWB

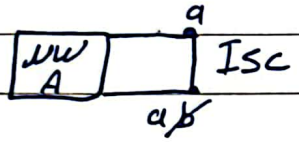


to find R_{th}

$R_{th} = R_{equ}$ → after killing the indep source: → Voltage source → S.C
→ current source: → O.C

to find $I_N = \frac{V_{th}}{R_{th}}$

OR



$I_N = I_{sc}$

case 2:-

NA → (Indep / dep) source + resistor.

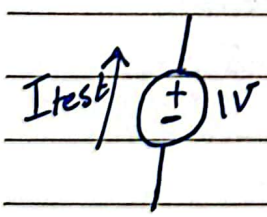
$V_{th} = V_{oc}$ = (V) between 2 node ~~connected~~ NWA with NWB

$I_N = I_{sc}$

R_{th} → $\frac{V_{th}}{I_N}$
→ kill the indep then add external indep source.

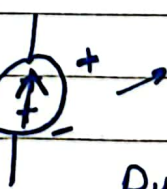
external indep source:

Voltage source



$R_{th} = \frac{1}{I_{test}}$

current source:



$R_{th} = V_{test}$

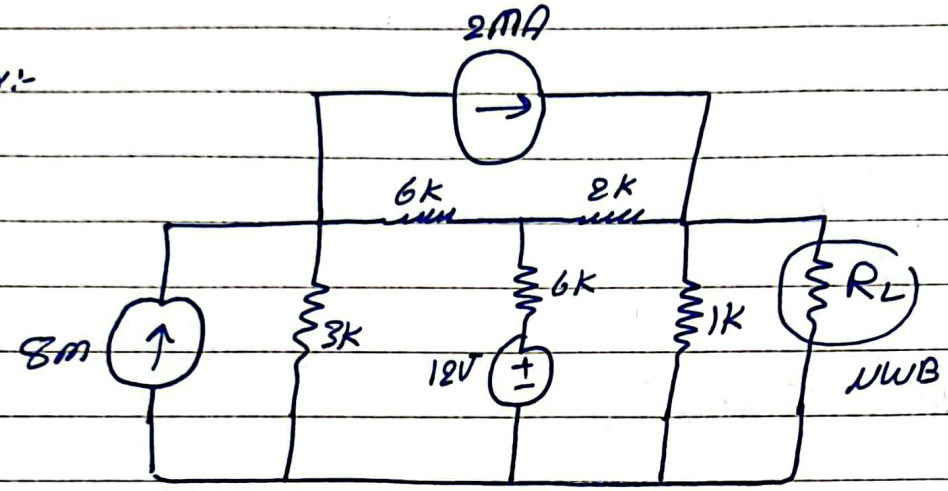
case 3:-

NA → dep source + R

$V_{th} = I_N = \text{zero}$

to find R_{th} → use external indep source.

Ex:-

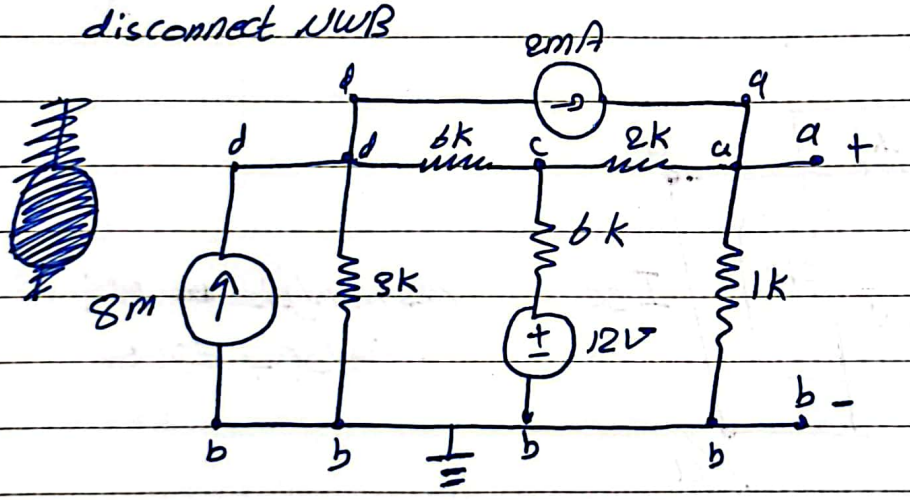


Find:-

V_{th} , R_{th} , R_N

I_N

disconnect NWB



b → ref → $V_b = 0$

to find

$V_{th} = V_{o.c} = V_{ab} = V_a - V_b = V_a$

KCL / a

$\Sigma I_{in} = \Sigma I_{out}$

$2m = \frac{V_a}{k} + \frac{V_a - V_c}{2k}$

KCL / c

$0 = \frac{V_c - V_d}{6k} + \frac{V_c - 12}{6k} + \frac{V_c - V_a}{2k}$

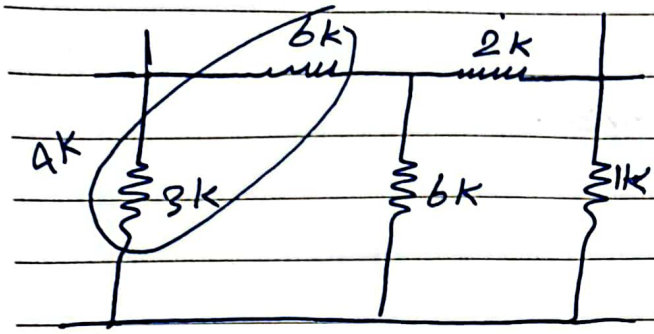
KCL / d

$8m = 2m + \frac{V_d - V_c}{6k} + \frac{V_d}{3k}$

nodal analysis · لیلہ

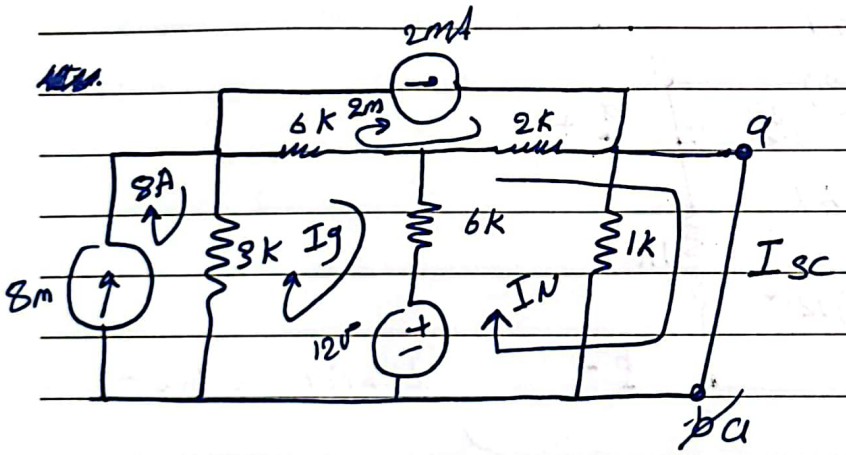
mesh analysis ڈی بی

$R_{th} = R_N$ → Kill indep source.



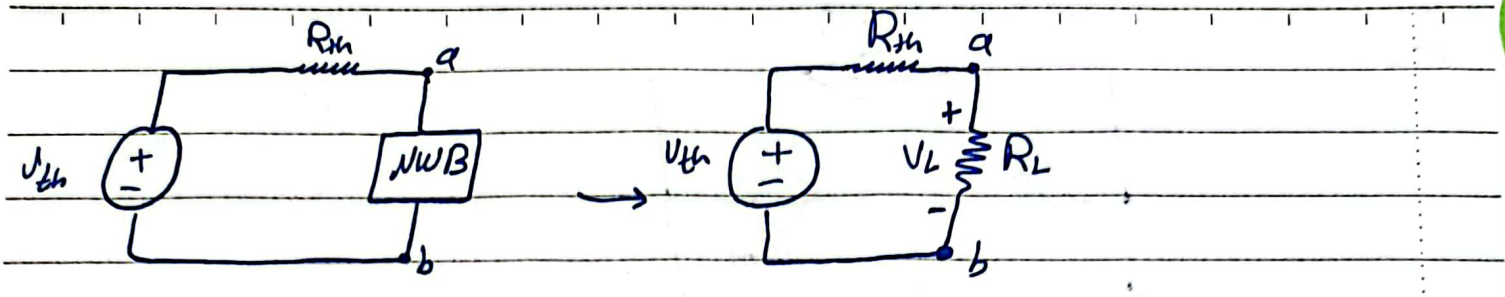
$$R_{th} = \left[[(4 // 6) + 2] // 1 \right] k$$

$$I_N = \frac{V_{th}}{R_{th}} \quad \text{OR} \quad I_N = I_{sc}$$



mesh analysis بنقدر نحلها بطريقة أسهل.

4/8/2023.



$$V_L = \frac{R_L \, V_{th}}{R_L + R_{th}}$$

$$P_L = \frac{V_L^2}{R_L} = \frac{R_L \, V_{th}^2}{(R_L + R_{th})^2}$$

to find Pmax

$$\frac{dP}{dR_L} = 0 \rightarrow 0 = \frac{(R_L + R_{th})^2 V_{th}^2 - 2R_L V_{th}^2 (R_L + R_{th})}{(R_L + R_{th})^4}$$

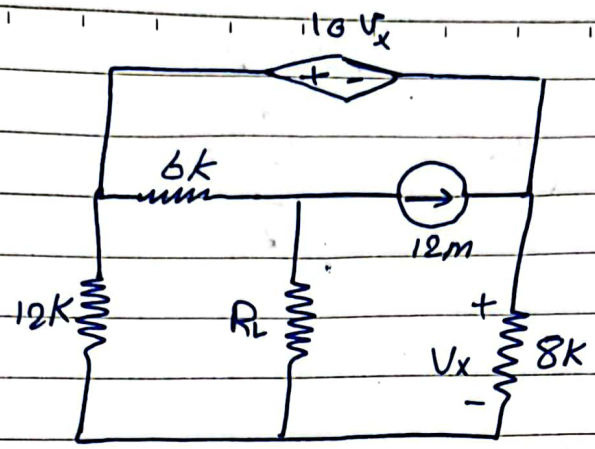
$$2R_L = R_L + R_{th}$$

$$R_L = R_{th}$$

at Pmax $R_L = R_{th}$

$$P_{max} = \frac{V_{th}^2}{4R_{th}}.$$

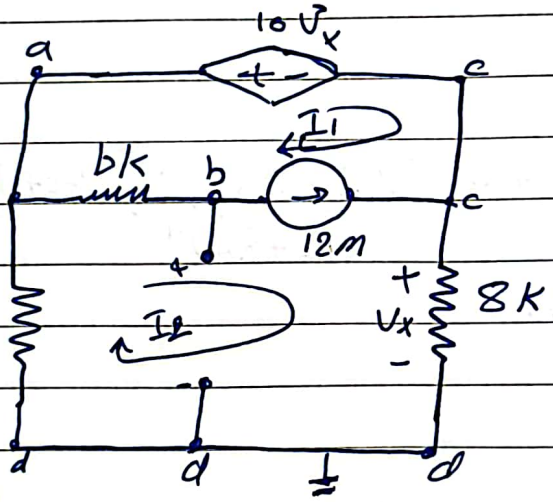
Ex:-



Disconnected NW B = RL

Determine the case of NW A

case 2



$V_x = 8K I_2$

Find I_N across R_L

R_N:

~~[illegible]~~

Pmax that could be dissipated in R_L

to find I_N → I_{sc}

OR → $\frac{V_{th}}{R_{th}}$

Firstly find V_{th}

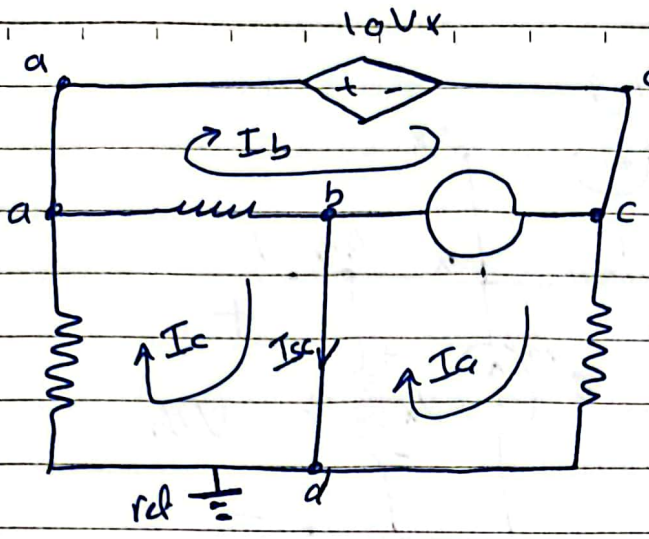
$V_{th} = V_{o.c} = V_{bd} = V_b - V_d$

$10V_x + V_x + 12I_2 = 0$

$12m = I_2 - I_1$

$6K(12m) + V_{th} + 12I_2 = 0$

$V_{th} = 72\,V$



to find $I_N \rightarrow I_{sc}$

* بس يكون في branch مشترك supernode
branch الي بناقي
reference. * ما عبر يكون عندي نقطتين

mesh → $10V_x + V_x + 6K(I_b - I_c) = 0$

$I_a - I_b = 12m$

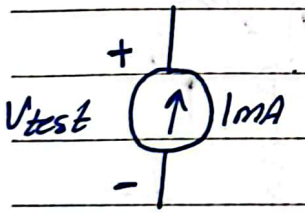
$V_x = 8K I_a$

$6K(I_c - I_b) - 12I_c = 0$

$I_N = I_c - I_a$

$R_{th} \rightarrow \frac{V_{th}}{I_N}$

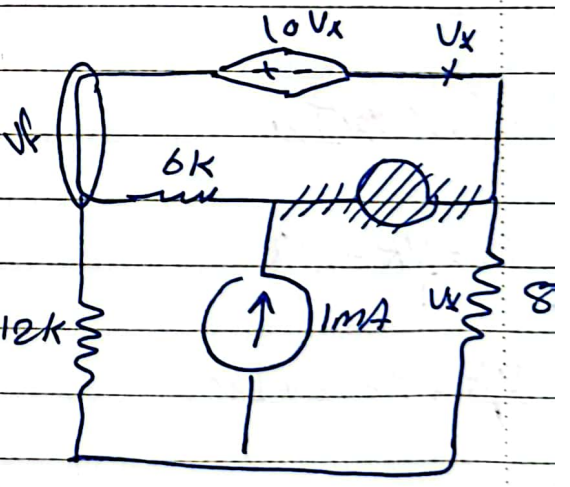
$\rightarrow$ or kill indep then add external.



$R_{th} = \frac{V_{test}}{1mA}$

KCL

$1m = \frac{V_f}{12K} + \frac{V_f - 10V_x}{8K}$



$V_f - V_x = 10V_x$

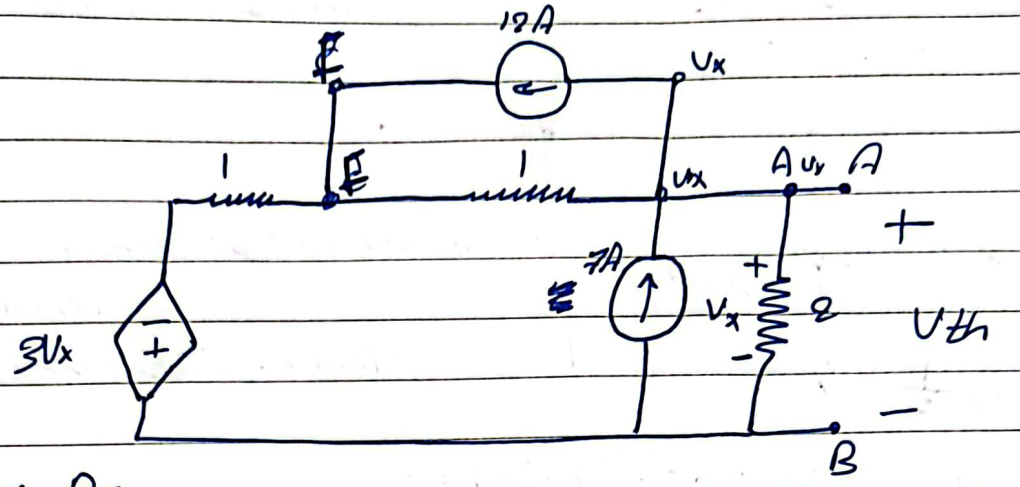
$V_{test} - V_f = 6$

$V_{test} = 6 + V_f$

OR ohm's

$V_x = 8K \times \frac{I_f - 10V_x}{8K}$

Ex:-



Find V_{th}, I_N, R_{th}

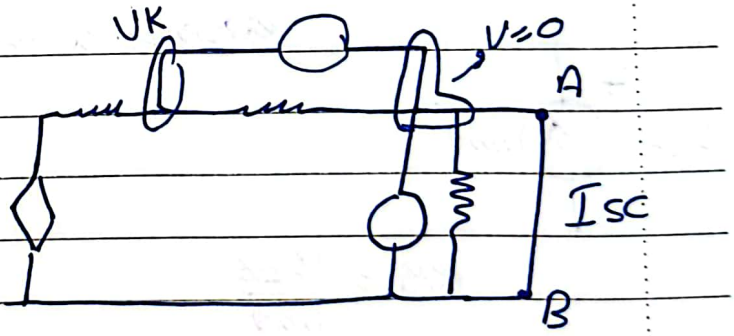
P_{abs} by $R_L = 5$ if R_L connected between A and B

to find V_{th}

$V_{th} = V_{oc} = V_{AB} = V_A - V_B = V_x$

$$7 = 12 + \frac{V_x - V_f}{1} + \frac{V_x}{2}$$

$$12 = \frac{V_f - V_x}{1} + \frac{V_f + 3V_x}{1}$$

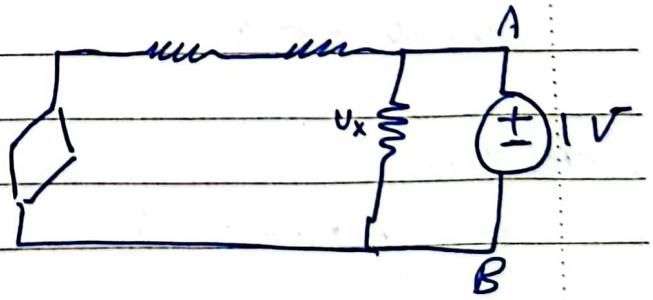


to find I_N

$$12 = \frac{V_K}{1} + \frac{V_K}{1}$$

$$\frac{V_K}{1} + 7 = 12 + I_N$$

to find R_{th} kill the indep and add ex



$$I_{test} = \frac{1}{2} + \frac{1-3}{2}$$

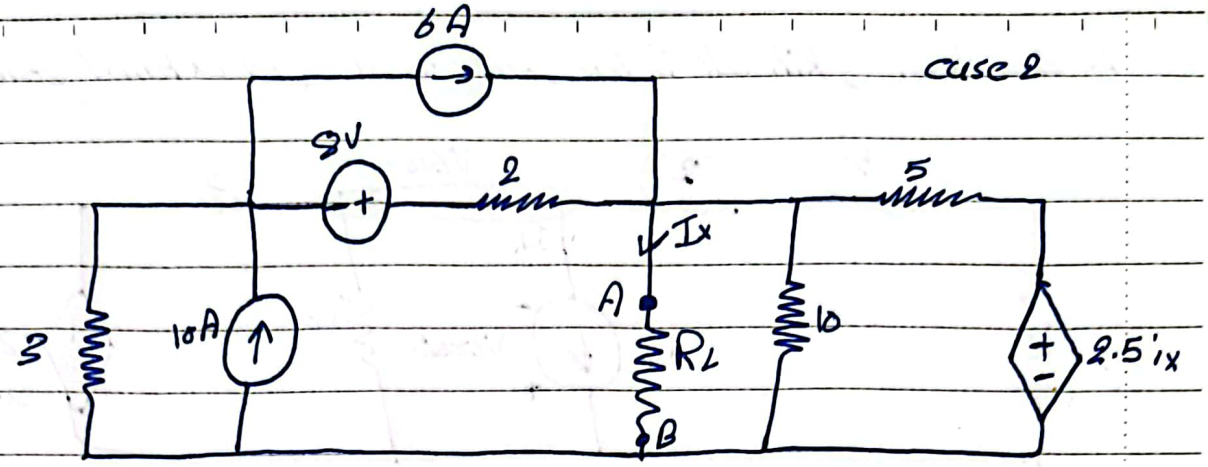
$$R_{th} = \frac{1}{I_{test}}$$

Five Apple

CamScanner

13/8/2023.

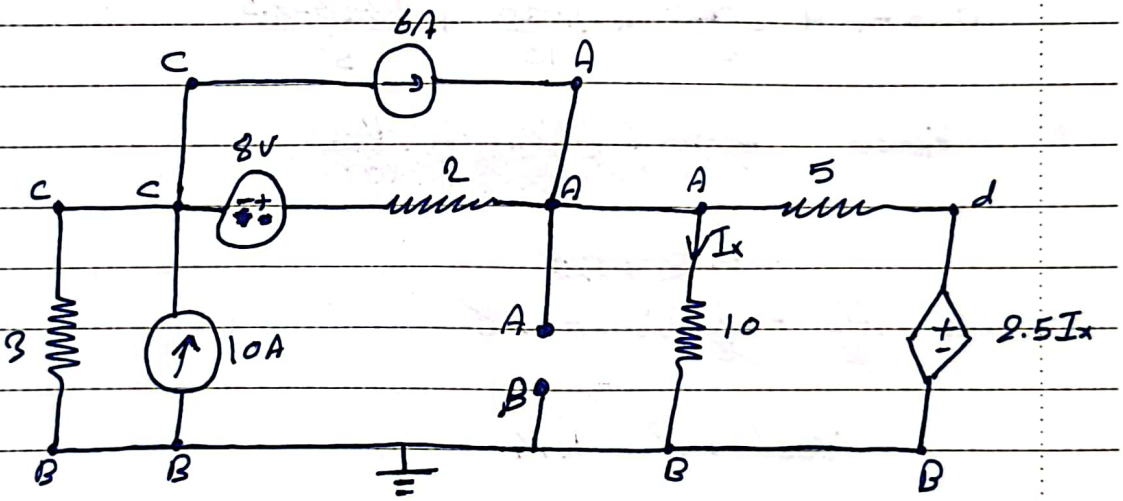
case 2



* Find V_{th}, R_{th}, I_N

P_{max} that could be dissipated in R_L

* disconnected ww B → R_L



$V_{th} = V_{o-s} = V_{ab} = V_a$

$V_{th} = V_a$.

$I_x = \frac{V_a}{10}$ → $V_d = 0.25 V_a$

KCL / a

$\Sigma I_{in} = \Sigma I_{out}$

$6 = \frac{V_a - V_c - 8}{2} + \frac{V_a}{10} + \frac{V_A - V_d}{5}$

$10 = V_a\left(\frac{1}{2} + \frac{1}{10} + \frac{1}{5}\right) + V_c\left(\frac{-1}{2}\right) + V_d\left(\frac{-1}{5}\right)$ ... (1)

KCL / c $\Sigma I_{in} = \Sigma I_{out}$

$10 = 6 + \frac{V_c - V_a + 8}{2} + \frac{V_c}{5}$

$0 = V_a\left(\frac{-1}{2}\right) + V_c\left(\frac{1}{2} + \frac{1}{5}\right) + V_d(0)$ ... (2)

KCL / d $\Sigma I_{in} = \Sigma I_{out}$

$0 = \frac{V_d - V_a}{5} + \frac{V_a}{4}$

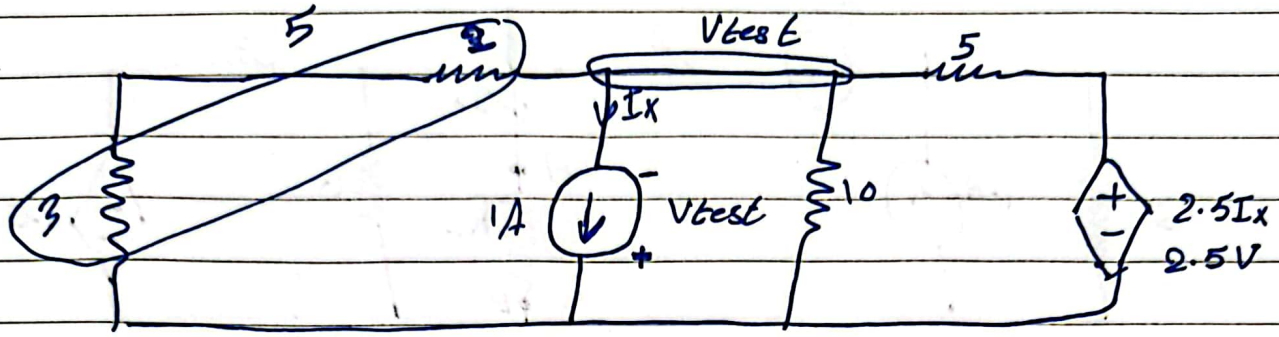
$0 = V_a\left(\frac{1}{4} - \frac{1}{5}\right) + V_c(0) + V_d\left(\frac{1}{5}\right)$ ... (3)

$V_a = V_{th} = 18.18$

$V_c = 10.9$

Five Apple

to find R_{th} → kill all indep source and add external source.



$I_x = 1A$

nodal

$V_{test} = R_{th}$.

$\Sigma I_{in} = \Sigma I_{out}$

$$0 = -1 + \frac{V_{test}}{5} + \frac{V_{test}}{10} + \frac{V_{test}-2.5}{5}$$

~~V_{test} $\frac{2V_{test}}{5} + \frac{V_{test}}{10}$~~

~~$\rightarrow$~~

$$2/x \quad \frac{3}{2} = \frac{1}{2} V_{test} \cdot x 2$$

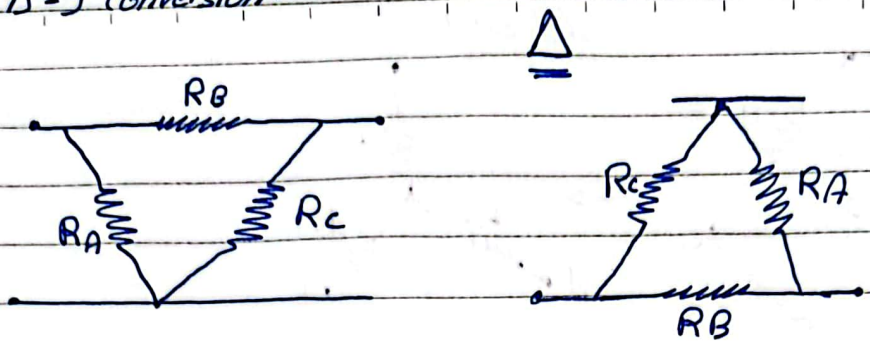
$$V_{test} = 3 = R_{th}$$

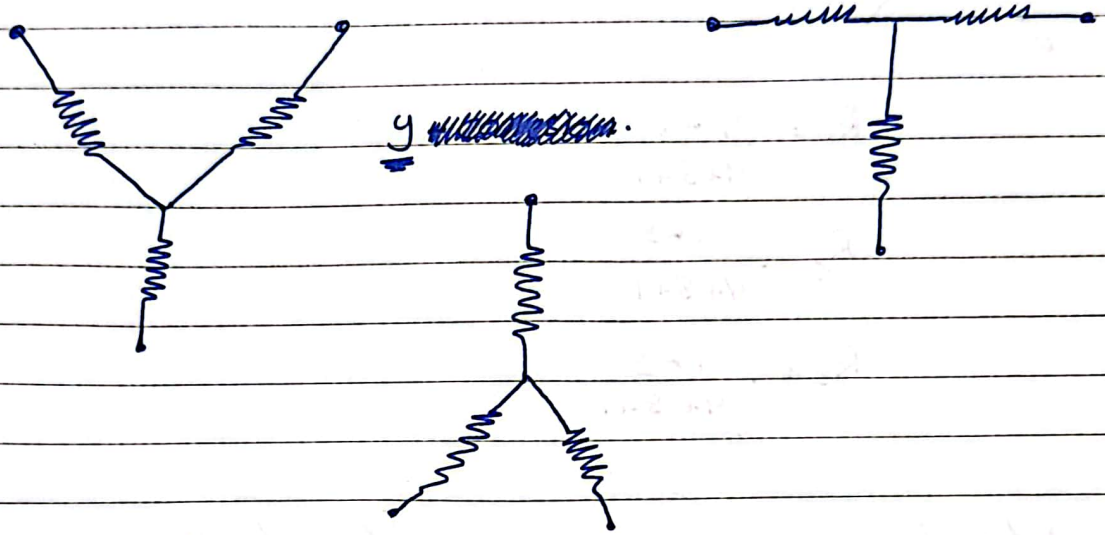
$$I_N = \frac{V_{th}}{R_{th}} = \frac{18.18}{3} = 6A$$

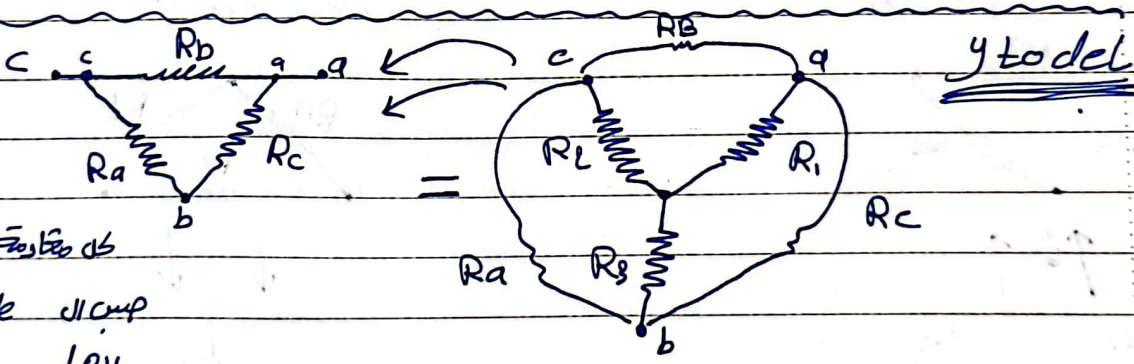
$$P_{max} = \frac{(V_{th})^2}{4R_{th}} = 27.54$$

CamScanner

Δ - y conversion.



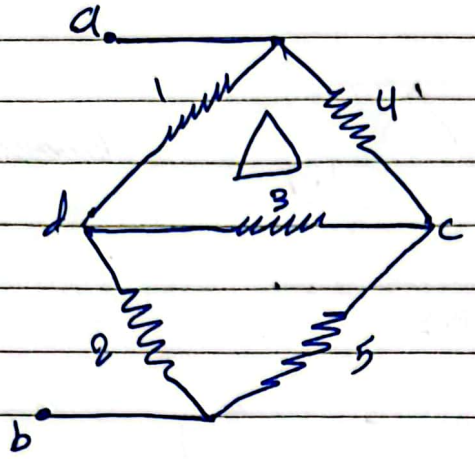




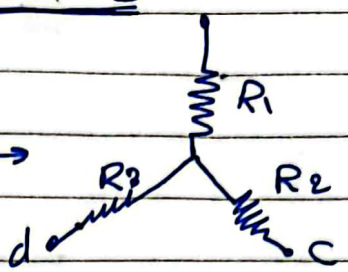
y to del

$$R_a = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_3}$$

Ex:-



del to y



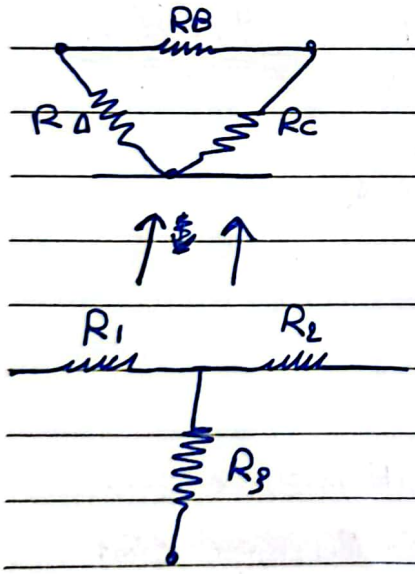
$$R_1 = \frac{4 \times 1}{4+3+1}$$

$$R_2 = \frac{4 \times 3}{4+3+1}$$

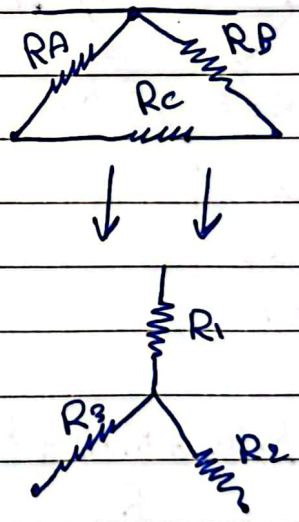
$$R_3 = \frac{1 \times 3}{4+3+1}$$

---

y to del



del to y



$$R_A = \frac{R_1R_2 + R_1R_3 + R_2R_3}{R_2}$$

$$R_B = \frac{R_1R_2 + R_1R_2 + R_2R_3}{R_3}$$

$$R_C = \frac{R_1R_2 + R_1R_3 + R_1R_3}{R_1}$$

$$R_1 = \frac{R_B R_A}{R_A + R_B + R_C}$$

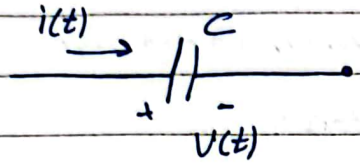
$$R_2 = \frac{R_B R_C}{R_A + R_B + R_C}$$

$$R_3 = \frac{R_A R_C}{R_A + R_B + R_C}$$

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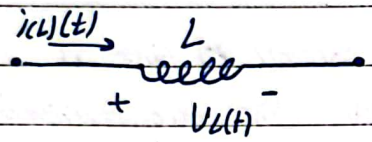
chapter 7:- Capacitor and inductors:-

capacitor :- storing energy element
stores voltage.



C :- capacitance (F)

inductors :- storing energy element
stores current



L :- inductance (H)

* Capacitor *	* inductor *
$i = C \frac{dv}{dt}$	$v = L \frac{di}{dt}$
$v(t) = \frac{1}{C} \int_{t_0}^{t} i(t')\,dt' + v(t_0)$	$i(t) = \frac{1}{L} \int_{t_0}^{t} v\,dt' + i(t_0)$

t_0 = initial time.

$P = vi = Cv \frac{dv}{dt}$ $p(t)$	$P = vi = Li \frac{di}{dt}$

The energy stored $w(t)$, $E(t) \rightarrow J$

$\int_{t_0}^{t} p\,dt' = C \int_{t_0}^{t} v \frac{dv}{dt'} dt' = C \int_{v(t_0)}^{v(t)} v'\,dv'$	$\int_{t_0}^{t} p\,dt' = L \int_{t_0}^{t} i \frac{di}{dt'} dt' = L \int_{i(t_0)}^{i(t)} i'\,di'$
$w_C(t) - w_C(t_0) = \frac{1}{2} C [v(t)^2 - v(t_0)^2]$	$w_L(t) - w_L(t_0) = \frac{1}{2} L [i(t)^2 - i(t_0)^2]$
* $w_C(t) = \frac{1}{2} C v^2$	$w_L(t) = \frac{1}{2} L i^2$
$v(t_0)$ = initial voltage.	$i(t_0)$ initial current.

Five Apple

* Series *

inductors:-

$L_{eq} = L_1 + L_2 + \cdots + L_N$

capacitors:-

$C_{eq} = \frac{1}{1/C_1 + 1/C_2 + \cdots + 1/C_N}$

* Parallel *

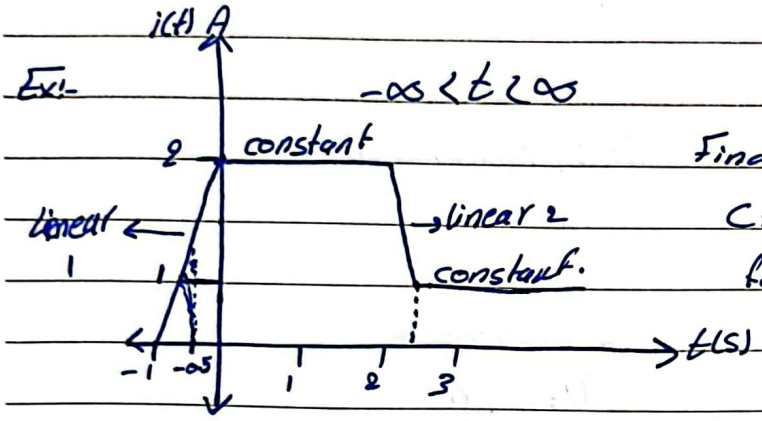
inductors:-

$L_{eq} = \frac{1}{1/L_1 + 1/L_2 + \cdots + 1/L_N}$

capacitors:-

$C_{eq} = C_1 + C_2 + \cdots + C_N$

Ex:- $-\infty < t < \infty$



Find $V_c(t)$ for all time if $C = 2$ F given the current following through the capacitor.

$i_C(t) = C \frac{dV(t)}{dt}$

$V_c(t) = \frac{1}{C} \int_{t_0}^{t} i(t')\,dt' + V_c(t_0)$

area.

$V_c(t) = 0 \quad -\infty < t < -1$

$V_c(t) = \frac{1}{C} \int_{t_0}^{t} 0\,dt' + V_c(t_0)$ ($V_c(-\infty)$)

$V_c(t) = \frac{1}{C} \int_{-1}^{t} 2(t'+1)\,dt' + V_c(-1) \quad -1 < t < 0$

Linear 1

$i(t) = \frac{m(t+1)}{t-1} + i(-1)$

$V_c(t) = \frac{1}{2} \int_{-1}^{t} 2(t'+1)\,dt' + V_c(-1)$ (0)

$i(t) = 2(t+1)$

$\frac{1}{2}\left(\frac{t'^2}{2} + t' \Big|_{-1}^{t}\right)$

m = slope

$i(-1) = 0$

$\frac{1}{2}\left((t^2+2t) - (1-2)\right) = \frac{1}{2}(t^2+2t+1)$

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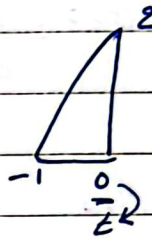
Find $v_c(-0.5)$ $\rightarrow v_c(-0.5) = \frac{0.5^2}{2} = 0.125\,V$

Find $v_c(-0.5)$ using Area method

$$v_c(-0.5) = \frac{1}{2}\left(\frac{1}{2}(0.5+1)1\right) + \overbrace{v_c(-1)}^{\text{zero}} = 0.125\,V$$

↳ قانون مساحة المثلث

Find $v_c(t)$ for $-1 < t < 0$ using area method.

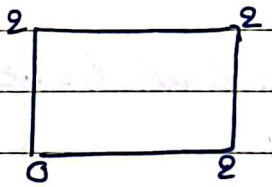


$$v_c(t) = \frac{1}{2}(0.5 \times (t+1) \times 2(t+1)) + v_c(-1)$$

$$v_c(t) = \frac{(t+1)(t+1)}{2} = \frac{t^2+2t+1}{2}$$

$v_c(t)$ $\quad 0 < t < 2$

$$v_c(t) = \frac{1}{2}\int_0^t 2\,dt' + v_c(0)$$



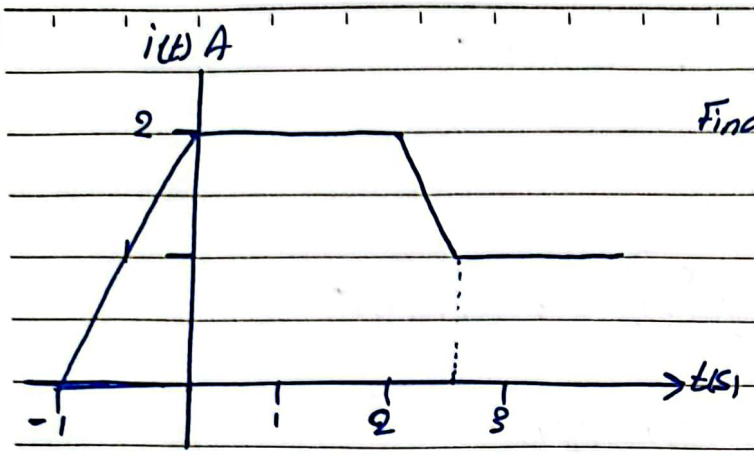
$v_c(0) = 0.5$

$v_c(t) = t + 0.5 \quad 0 < t < 2$

using area method.

$$v_c(t) = \frac{2 \times (t-0)}{2} + v_c(0)$$

$$v_c(t) = t + 0.5$$



Find $V_L(t)$ for all time if $L = 1H$

$$V_L(t) = L \frac{di(t)}{dt}$$

$V_L(t) =$

0	$-\infty < t < -1$
0.5	$-1 < t < 0$
0	$0 < t < 2$
$\frac{-1}{0.5}$	$2 < t < 2.5$
0	$2.5 < t < \infty$ → $t > 2.5$

Find the energy stored in L at $t = 1.5s$

$$w = 0.5 L i^2$$

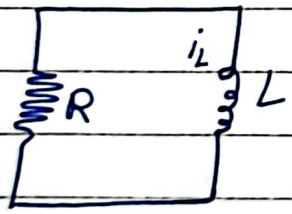
chapter 8 : RL and RC ccts

RL → cct has inductors, resistors and may contain dep source.

RC → ccts has capacitors and resistors and may contain dep source.

- **RL**
  - source free for $t > t_0$ — cct has no indep source or has no effect on RL cct
  - driven for $t > t_0$ — cct has indep source effected RL ccts
- **RC**
  - source free for $t > t_0$ — cct has no indep source has no effect on RC cct
  - driven. for $t > t_0$ — cct has indep source effected RC cct

source free RL cct



Find $i_L(t)$ for $t > t_0$ given

$i_L(t_0) = I_0$

I_0 initial current

$$V_L(t) + V_R(t) = 0$$

$$L\frac{di_L(t)}{dt} + Ri_L(t) = 0$$

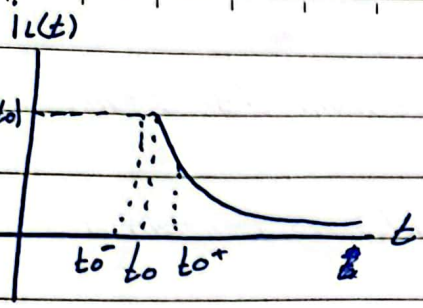
$$\int \frac{di(t)}{i(t)} = \int -\frac{R}{L}\,dt$$

$$i_L(t) = i_L(t_0)\, e^{-\left(\frac{t-t_0}{\tau}\right)} \qquad t \geq t_0$$

τ : time constant (s)

$$\tau = \frac{L_{eq}}{R_{eq}} \qquad \tau = \frac{L_{eq}}{R_{TH}} \rightarrow \text{if cct has dep source (case 3).}$$

Five Apple



if the Inductor is connected with DC indep source for a Long time it acts as a S.C

$$i_L(t_o^-) = i_L(t_o^+) = i_L(t_o)$$

Find $V_L(t)$

$$V_L(t) = L \frac{d i_L(t)}{dt}$$

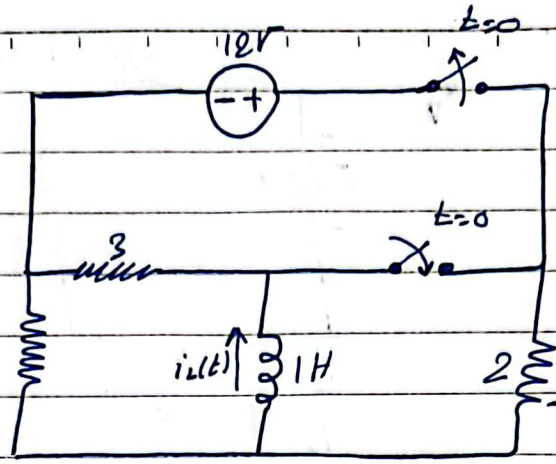
OR

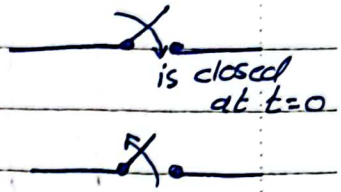
$$V(t) = V_L(t_o^+) e^{-\left(\frac{t - t_o}{\tau}\right)} \qquad t > t_o$$

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Ex:





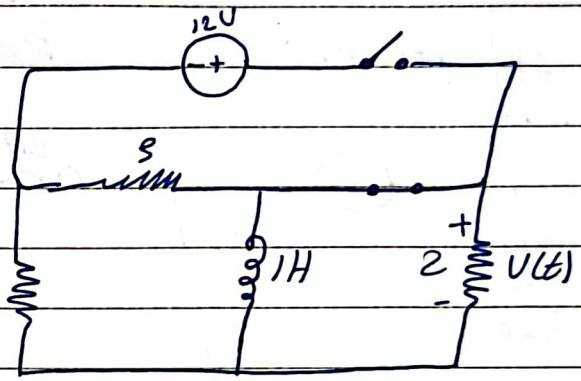
Find $i_L(t)$ for $t>0$

$V(t)$

Sourc free RL

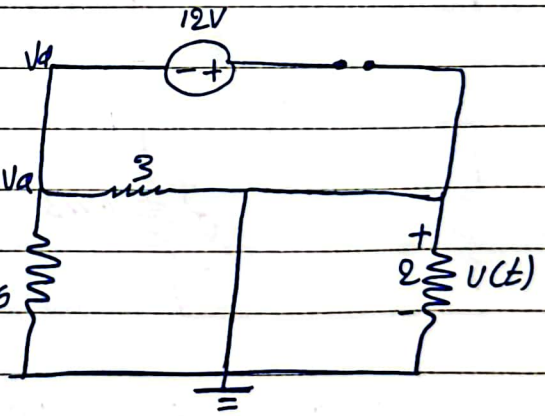
$i_L(t) = i_L(0^+)\, e^{-t/\tau}$ $t>0$

$V(t) = V(0^+)\, e^{-t/\tau}$ $t>0$



$i_L(0^+) = i_L(0^-)$

ccts $t<0$ L → s.c



$\frac{Va}{6} + \frac{Va}{3} + \frac{Va+12}{2} = 0$ (nodal)

$i_L(0^-) = \frac{-Va}{3}$

$\tau = \frac{L_{eq}}{R_{eq}} = \frac{1}{9\|2}$

$i_L(t) = \frac{-2Va}{3}\, e^{-t/\frac{1}{9\|2}}$ $t>0$

$V_2(t) = V_L(t) = L\frac{di_L(t)}{dt}$

OR

$V_L(t) = V_L(0^+)\, e^{-t/\tau}$

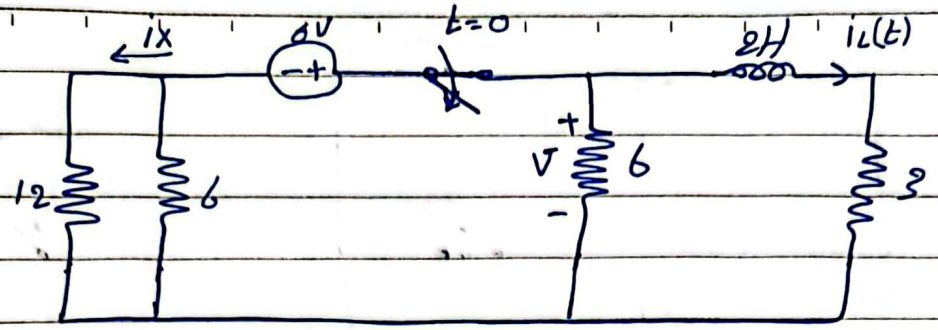
$V_2(0^+) = V_L(0^+) = 2\, i_2(0^+)$

$i_2(0^+) = \frac{9}{2+9}\, i_L(0^+)$

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Ex:-



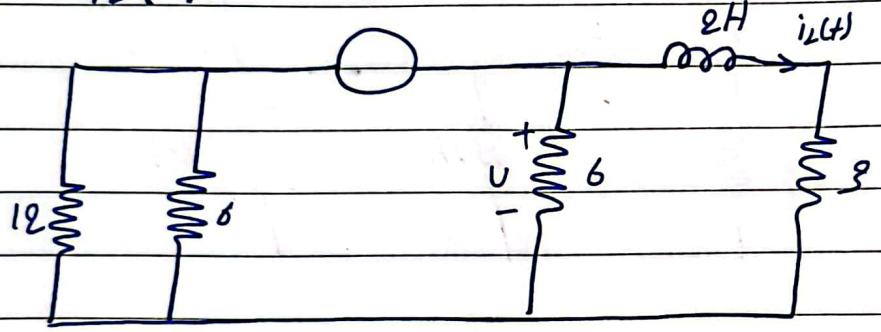
Find V, ix, iL at 0^+

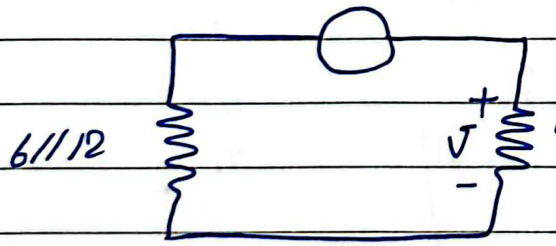
V, ix at 0^-

V, ix(t), iL(t) for $t>0$

$i_L(t=2ms)$

$-\infty < t < 0$





$$V(0^-) = \frac{+3//6 \cdot 6}{(3//6)+(6//12)}$$

$$i_L(0^+) = i_L(0^-) = \frac{V(0^-)}{3}$$

$$I = \frac{V(0^-)}{3//6} \qquad i_x(0^-) = \frac{-6}{6+12} I$$

* $i_L(0^+) = i_L(0^-)$

* $V(0^+) = [illegible] = 3\, i_L(0^+) \cdot -6$

$V(0^+) + i_L(0^+)[3+6) = 0$

* $i_x(0^+) = 0$

$i_L(t) = i_L(0^+)\, e^{-t/\tau}$ $\qquad \tau = \frac{L}{Req} = \frac{2}{3+6} = \frac{2}{9}$

$V(t) = V(0^+)\, e^{-t/\tau}$

$i_x(t) = zero$

$i_L(2m) = i_L(0^+)\, e^{\frac{-2m}{2/9}}$

L → S.C (at t_0^- DC)

L → indep.

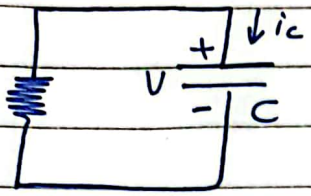
→ Long time.

$V(t_0^-) = zero.$

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source free RC cct.



Find $V_c(t)$ $t > t_0$

given $V_c(t_0) = V_0$

$$i_c(t) + i_R(t) = 0$$

$$C\frac{dV_c(t)}{dt} + \frac{V_c(t)}{R} = 0 \rightarrow \text{first order diff eqn.}$$

$$V_c(t) = V_c(t_0)\, e^{-\left(\frac{t-t_0}{\tau}\right)} \qquad t > t_0$$

L	C
stores current	stores voltage.
$V_L = \frac{L\, di_L(t)}{dt}$	$i_c(t) = \frac{C\, dV_c(t)}{dt}$
L acts as a S.C at t_0^- (DC indep sourc for a long time)	C acts as an o.c at t_0^- if ----

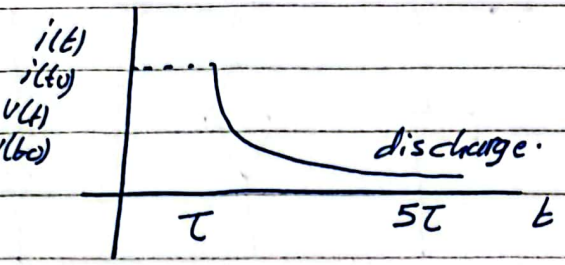
$$i(t) = i(t_0^+)\, e^{-\left(\frac{t-t_0}{\tau}\right)} \qquad t > t_0$$

$$V(t) = V(t_0^+)\, e^{-\left(\frac{t-t_0}{\tau}\right)} \qquad t > t_0$$

source free RL	source free RC
$\tau = \frac{l_{eq}}{R_{th}}$	$\tau = R_{th} C_{eq}$
R_{th} is act has dep. as seen from L	as seen from C
$i_L(t_0^+) = i_L(t_0^-)$ $t < t_0$	$V_c(t_0^-) = V_c(t_0^+)$. $t < t_0$

(5τ to charge or Dis charge.)

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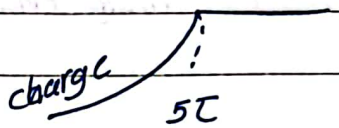


i_L $i_C(t=\tau) = V_C(0^+)e^{-1}$ $t=0$

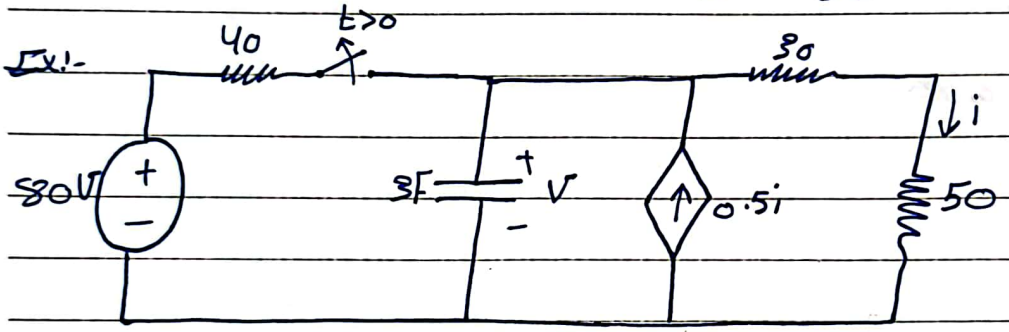
$t=2\tau = V_C(0^+)e^{-2}$

t ⋮

$t=5\tau = V_C(0^+)e^{-5}$



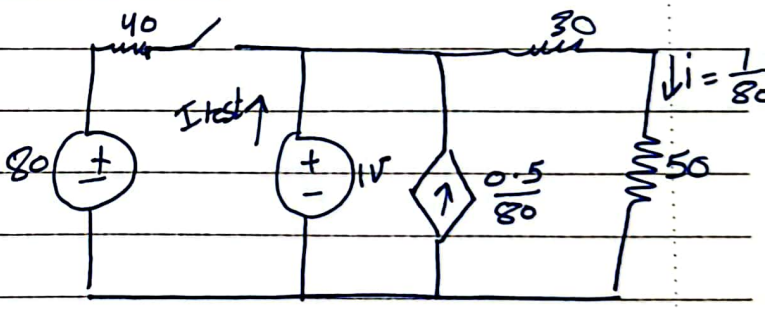
Ex:-



$i(t) = i(0^+)e^{-t/\tau}$ $t>0$

$\tau = R_{th} \cdot C$

to find R_{th} add external



$I_{test} + 0.5i = i$

$I_{test} = 0.5i$

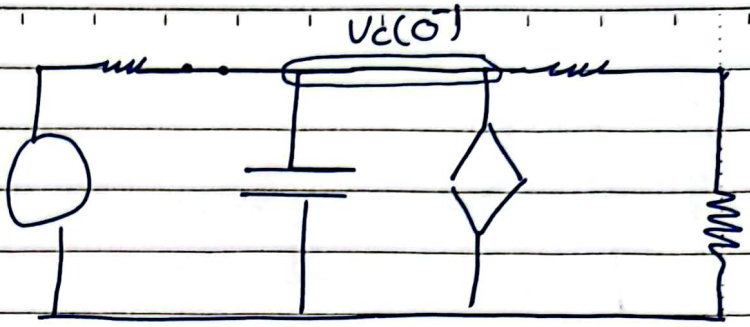
$I_{test} = \frac{0.5}{80}$

$R_{th} = \frac{1}{I_{test}} = \frac{80}{0.5} = 160$

$\tau = 160 \cdot 3 = 480s.$

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$V_c(0^-) = V_c(0^+)$



$$0.5i + i + \frac{V_c(0^-) - 80}{40}$$

$$-0.5 \times \frac{V_c(0^-)}{80} = \frac{V_c(0^-) - 80}{40} \rightarrow V_c(0^-) = 64V = V_c(0^+)$$

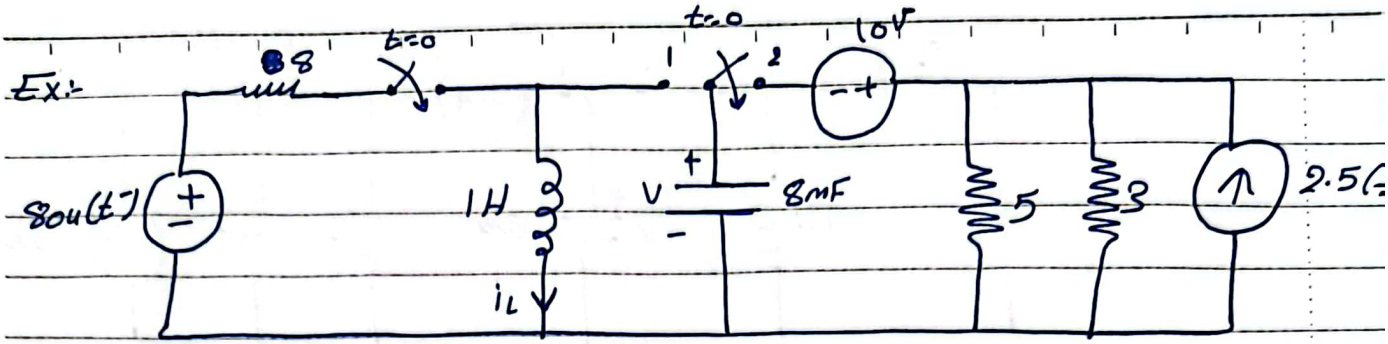
to find value at 0^+ firstly find

$R_L \rightarrow i_L(0^+) = i_L(0^-)$

$R_C \rightarrow V_c(0^+) = V_c(0^-)$

$$i(t) = \frac{64}{80} e^{-t/480}$$

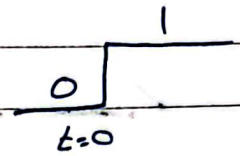
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Find $v_c(t)$ $t>0$

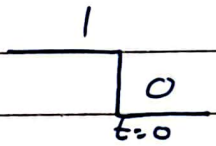
$i_L(t)$

Unit step fun $u(t)$



$$u(t-t_0) = \begin{cases} 1 & t > t_0 \\ 0 & t < t_0 \end{cases}$$

$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$

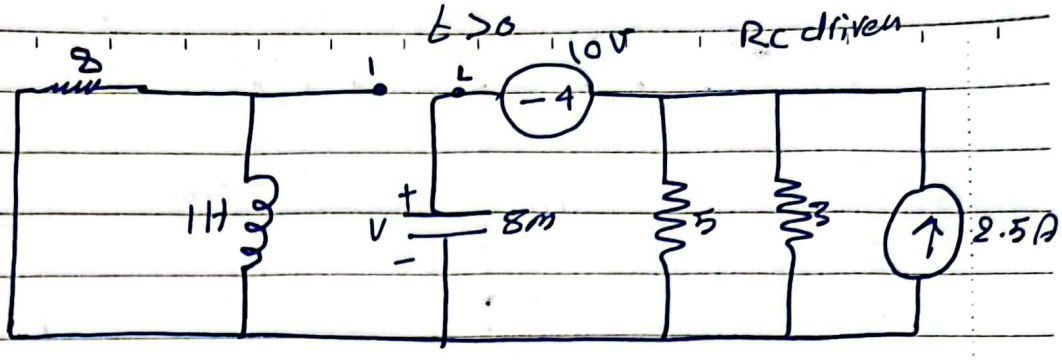


$$u_2(-t) = \begin{cases} 1 & t \geq 0 \\ 0 & t > 0 \end{cases}$$

$$80u(t) = \begin{cases} 80 & t < 0 \\ 0 & t > 0 \end{cases}$$

S.C voltage source O.S current source.

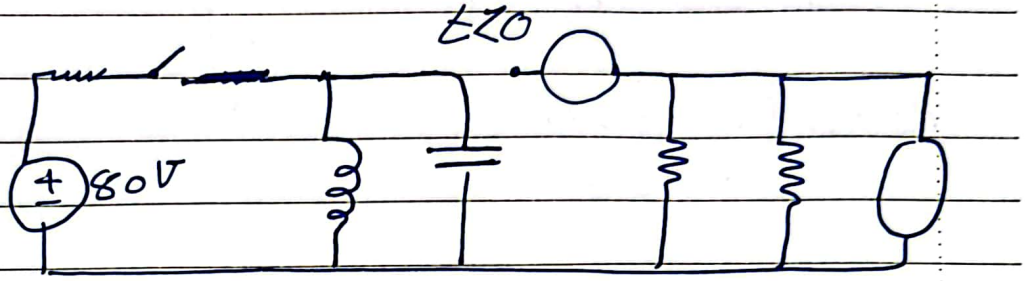
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$i_L(t) = i_L(0^+)\, e^{-t/\tau}$

ckt $t>0$

$\tau = \frac{L}{R} = \frac{1}{8}$



Driven

RL , RC

forced driven.

$i(t) = i_f + (i_{in} - i_f)\, e^{\left\{\frac{t-t_0}{\tau}\right\}}$ $t>0$

$V(t) = V_f + (V_{in} - V_f)\, e^{\left\{\frac{t-t_0}{\tau}\right\}}$ $t>0$

final values	initial values
$i_f = i(\infty)$	$i_{in} = i(t_0^-)$
$V_f = V(\infty)$	$V_{in} = V(t_0^+)$

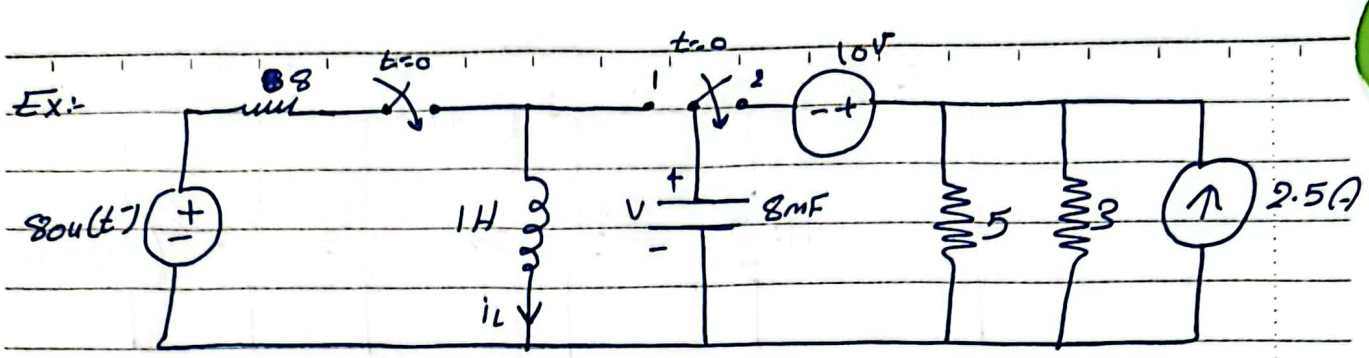
Driven

RL		RC	
$\tau = \frac{L_{eq}}{R_{th}}$	$V_L(t_0^-) = \text{zero}$	$\tau = R_{th} C_{eq}$	$i_C(t_0) = \text{zero}$
	$V_L(\infty) = \text{zero}$	Cat $\infty \to$ O.S	$i_C(\infty) = \text{zero}$.
Cat $\infty \to$ S.C	$i_L(t_0^-) = i_L(t_0^+)$		$V_C(t_0^-) = V_C(t_0^+)$

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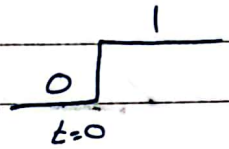
Ex:-



Find $v_c(t)$ $t>0$

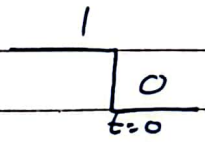
$i_L(t)$

Unit step fun $u(t)$



$$u(t-t_0) = \begin{cases} 1 & t > t_0 \\ 0 & t < t_0 \end{cases}$$

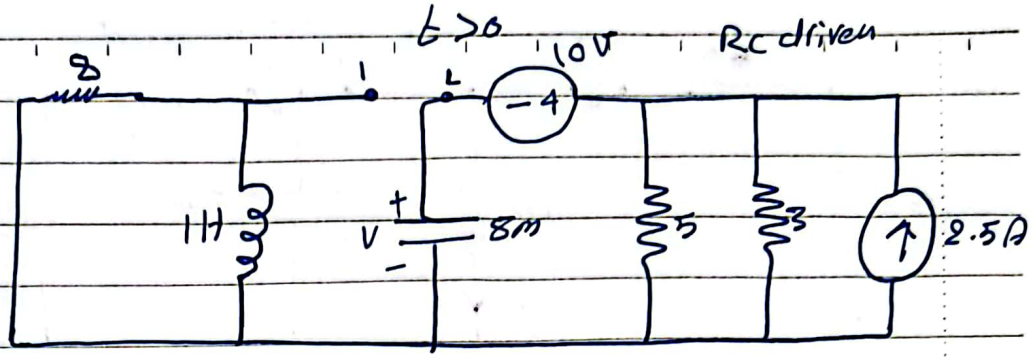
$$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$$



$$u(-t) = \begin{cases} 1 & t < 0 \\ 0 & t > 0 \end{cases}$$

$$80u(t) = \begin{cases} 80 & t < 0 \\ 0 & t > 0 \end{cases}$$

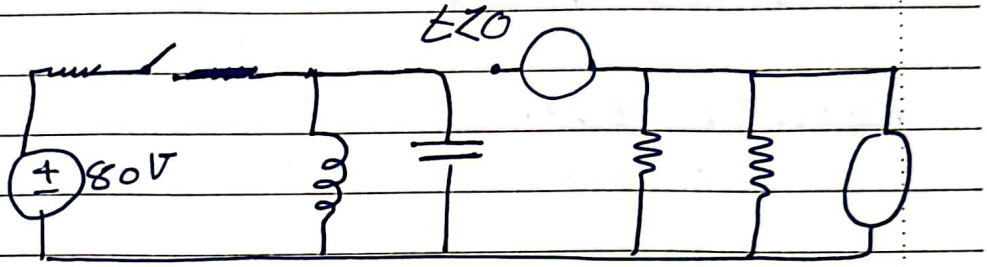
S.C voltage source O.S current source.



$i_L(t) = i_L(0^+)\, e^{-t/\tau}$

ckt $t>0$

$\tau = \frac{L}{R} = \frac{1}{8}$



Driven

R_{th} , R_C

forced driven.

$i(t) = i_f + (i_{in} - i_f)\, e^{\left\{\frac{t-t_0}{\tau}\right\}}$ $t>0$

$V(t) = V_f + (V_{in} - V_f)\, e^{\left\{\frac{t-t_0}{\tau}\right\}}$ $t>0$

final values	initial values
$i_f = i(\infty)$	$i_{in} = i(t_0^+)$
$V_f = V(\infty)$	$V_{in} = V(t_0^+)$

Driven

RL		Rc	
$\tau = \frac{L_{eq}}{R_{th}}$	$V_L(t_0^-) =$ zero	$\tau = R_{th} C_{eq}$	$i_C(t_0) =$ zero
Cal t_∞^- → S.C	$V_L(\infty) =$ zero	Cal t_∞^- → O.S	$i_C(\infty) =$ zero.
Cal t_∞^- → S.C	$i_L(t_0^-) = i_L(t_0^+)$		$V_C(t_0^-) = V_C(t_0^+)$

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$V_c(t) = V_f + (V_{in} + V_f)\, e^{-t/\tau}$ حل السؤال :-

$V_c(0^-) = V_c(0^+) = \text{zero} \rightarrow V_{in} = \text{zero}.$

$\frac{V}{5} + \frac{V}{3} = 2.5 \rightarrow V = ?$

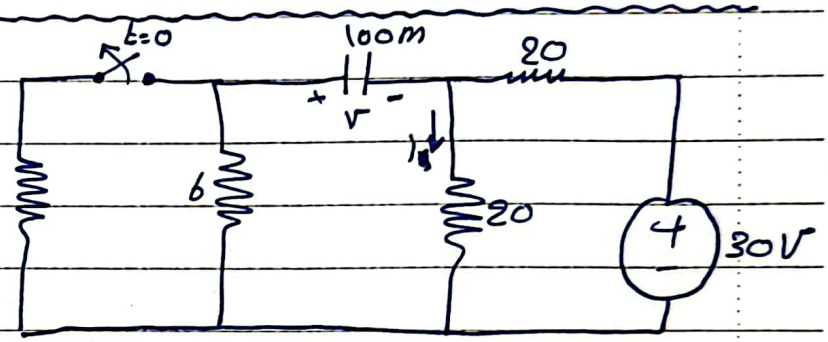
$-V_c(\infty) - 10 + V = 0 \rightarrow V_L$

$\tau = R_{th} C$

$(5 // 3) \cdot 8$

* $V_c(0^+)$ * $i_L(0^+)$ * $i_L(\infty)$ * τ

* energy stored in C at $t = \infty$



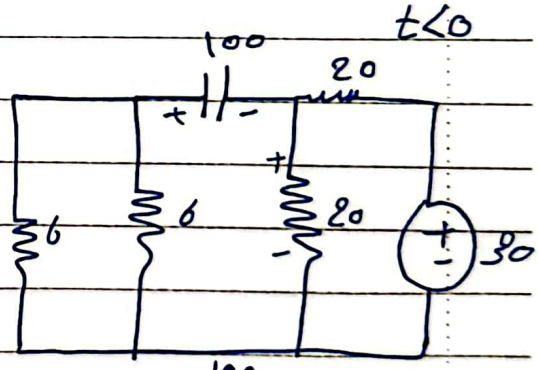
to find any value at 0^+

firstly find

$i_L(0^+) = i_L(0^-) / R_L$

$V_c(0^+) = V_c(0^-) / R_C$

$V_c(0^+) = V_c(0^-) = -15$



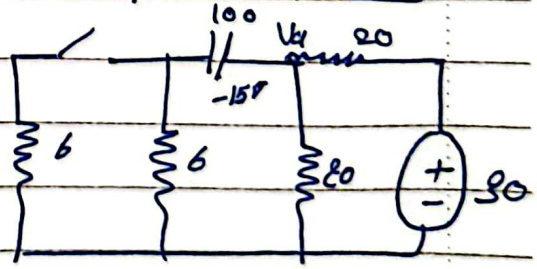
to find $i(0^+)$

$\frac{V_a}{20} + \frac{V_a - 30}{20} + \frac{V_a - 15}{6} = 0$

$i(0^+) = \frac{V_a}{20} = 0.75\,mA$

$E = \frac{1}{2} C V_{100}^2$

$= \frac{1}{2} \times 100m\,(-15)^2$



$i_c(\infty) = \text{zero}.$

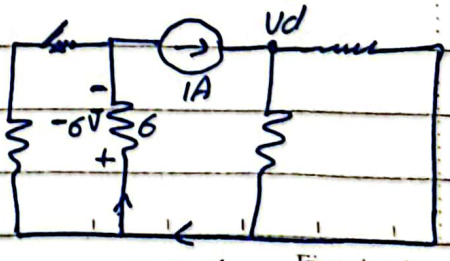
$\tau = R_{th} C$

$16 \times 100m$

$V_d = 20 \times 0.5$

$V_d + 6 = V_{test}$

$16 = V_{test} = R_{th}$



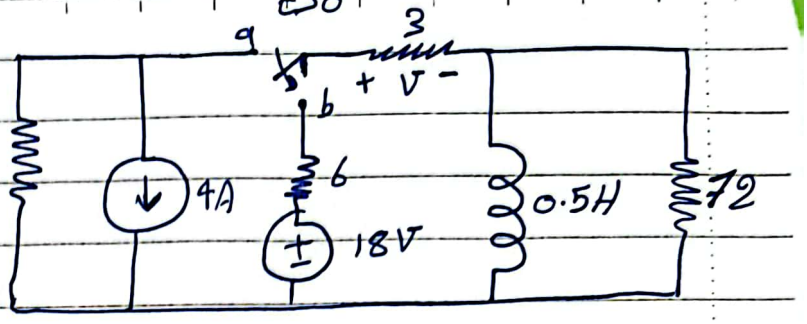
To find R_{th}.

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* $i(0^+)$, * $i(\infty)$, * $i(t)$ $t>0$

* $V(0^+)$, * τ , * $V(t)$ $t>0$

$i(t=2ms)$

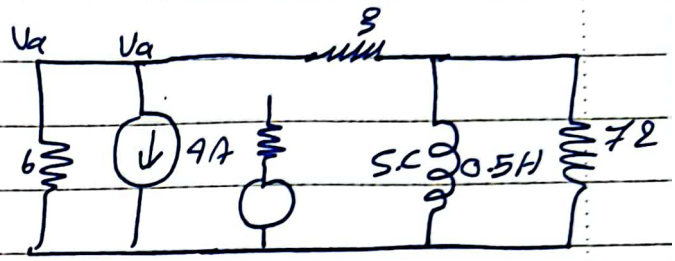


L at $\rightarrow \infty$, 0^- $\rightarrow$ S.C

$V_L(0^-) = V_L(\infty) =$ zero

$i_L(0^-) = i_L(0^+)$

$0 = 9 + \frac{Va}{6} + \frac{Va}{3}$

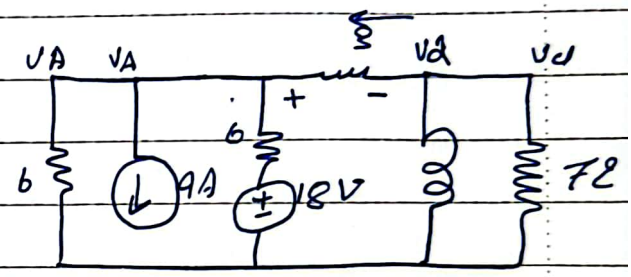


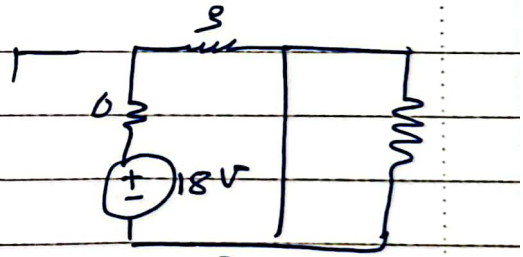
$i_L(0^-) = i_L(0^+) =$

$\frac{Vd}{72} + i_L(0^+) + \frac{Vd-18}{4}$

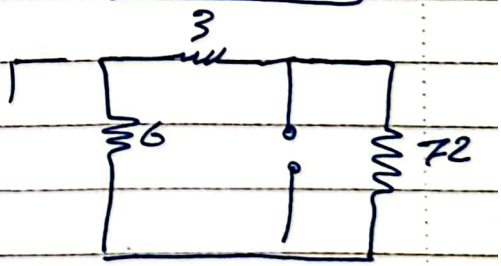
$V = -3\left(\frac{Vd-18}{9}\right)$ $\quad i_L(0^-) = \frac{Vg}{3}$

سالب كنت داخل من الجهة السابقة





$i(\infty) = \frac{18}{9}$



$\tau = \frac{L}{R_{th}} = \frac{0.5}{72//9}$

$i(t) = if + (iin - if)\, e^{-t/\tau}$ $\quad t>0$

$= i(\infty) + (i(0^+) + i(\infty))\, e^{-t/0.5//72//9}$

$i(2m)$ . . . . . $e^{\frac{-2m}{0.5//72//9}}$

$V(t) = V(\infty) + (V(0^+) - V(\infty))\, e^{-t/\tau}$

$V(\infty) = \frac{3}{3+6}\, 18 = 6\,V$

chapter 9:- RLC ckts.

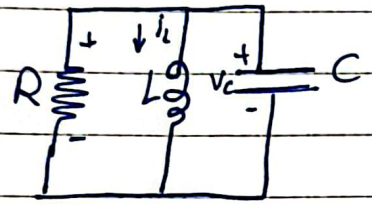
* RLC ckt has resistor, inductors, capacitor.
(series, parallel), may contain dep source.

$t>0$

source free
no indep source.
(natural response)

driven.
indep source.
(forced response).

parallel source free RLC



Find $V_c(t)$ $t>0$
$i_L(t)$ $t>0$

given
$i_L(0) = I_0 = i_L(0^-) = i_L(0^+)$
$V_c(0) = V_0 = V_c(0^-) = V_c(0^+)$

$i_C(t) + i_L(t) + i_R(t) = 0$

$$\frac{d}{dt}\left(C\frac{dv(t)}{dt} + \frac{V_c(t)}{R} + \frac{1}{L}\int V_c(t)\,dt = \text{zero} \right)$$

$$\boxed{\frac{d^2V_c(t)}{dt^2} + \frac{1}{RC}\frac{dV_c(t)}{dt} + \frac{1}{LC}V_c(t) = \text{zero}}$$ 2nd order diff eqn.

$\frac{dv}{dt} = s^1$

$\frac{d^2v}{dt^2} = s^2$

$$\boxed{s^2 + \frac{1}{RC}s + \frac{1}{LC} = \text{zero}}$$ characteristic eqn.

$V = \frac{d^0v}{dt^0} = s^0$

$\alpha = \frac{1}{2R_eqC}$ coff

$\omega_0 = \frac{1}{\sqrt{LC}}$ natural freq.

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parallel RLC

$$\alpha = \frac{1}{2R_{eq}C_{eq}}$$

↳ as seen from L, C

$$\omega_0 = \frac{1}{\sqrt{L_{eq}C_{eq}}}$$

$$s_1 = -\alpha + \sqrt{\alpha^2 - \omega_0^2}$$
$$s_2 = -\alpha - \sqrt{\alpha^2 - \omega_0^2}$$

Overdamping

$\alpha > \omega_0$

s_1, s_2 real as different

$$v_c(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t} \qquad t > 0$$
$$i_L(t) = B_1 e^{s_1 t} + B_2 e^{s_2 t}$$

to find A_1, A_2 / to find B_1, B_2

$$v_c(0^+) = v_c(0^-) = A_1 + A_2 \quad \text{---- (1)}$$
$$i_L(0^+) = i_L(0^-) = B_1 + B_2 \quad \text{... (1)}$$
$$\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C} = A_1 s_1 + A_2 s_2 \quad \text{---- (2)}$$
$$\frac{di(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{v_c(0^-)}{L} = B_1 s_1 + B_2 s_2 \quad \text{... (2)}$$

(1) and (2) → A_1, A_2

(1) and (2) → B_1, B_2

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critical damping.

$\alpha = \omega_0$

$s_1 = s_2 = \alpha$

$v_c(t) = e^{-\alpha t}(A_1 t + A_2) \quad t>0$

$i_L(t) = e^{-\alpha t}(B_1 t + B_2) \quad t>0$

* to find A_1, A_2

$v_c(0^+) = v_c(0^-) = A_2$ →

$\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C} = A_1 - \alpha A_2$

* to find B_1, B_2

$i_L(0^+) = i_L(0^-) = B_2$ →

$\frac{di_L(0^+)}{dt} = \frac{v_L(0^+)}{L} = \frac{v_c(0^+)}{L} = B_1 - \alpha B_2$

under damping.

$\alpha < \omega_0 \quad j = \sqrt{-1}$

$\omega_d = \sqrt{\omega_0^2 - \alpha^2}$

$s_1 = -\alpha + j\omega_d \quad , \quad s_2 = -\alpha - j\omega_d$

↳ complex

$v_c(t) = e^{-\alpha t}(A_1 \cos\omega_d t + A_2 \sin\omega_d t) \quad t>0$

$i_L(t) = e^{-\alpha t}(B_1 \cos\omega_d t + B_2 \sin\omega_d t) \quad t>0$

* to find A_1, A_2

$v_c(0^+) = v_c(0^-) = A_1$ →

$\frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C} = A_2\omega_d - \alpha A_1$

* to find B_1, B_2.

$i_L(0^+) = i_L(0^-) = B_1$ →

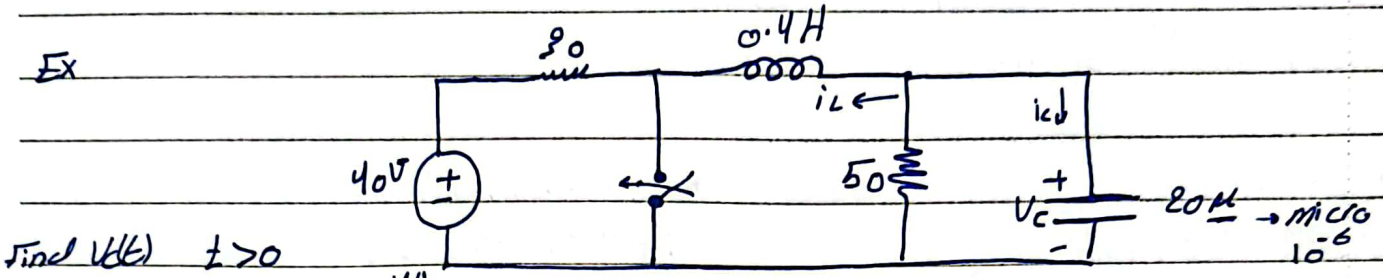
$\frac{di_L(0^+)}{dt} = \frac{v_c(0^+)}{L} + \frac{v_L(0^+)}{L} = B_2\omega_d - \alpha B_1$

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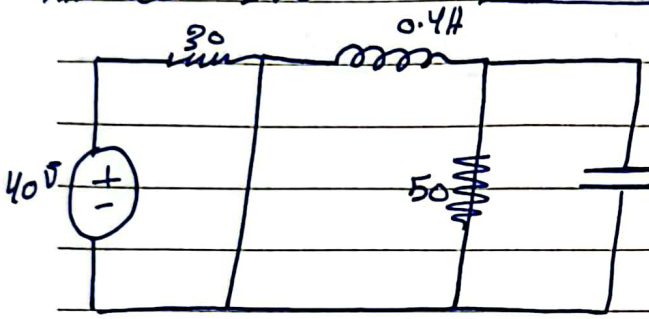
series RLC

$\alpha = \frac{R}{2L}$ $\quad \omega_0 = \frac{1}{\sqrt{L_{eq} C_{eq}}}$ Either one series or Parallel or

Ex



Find V(t) $t>0$



Parallel / source free RLC

$\alpha = \frac{1}{2R_{eq}C_{eq}} = \frac{1}{2(50)(20\mu)} = 500$

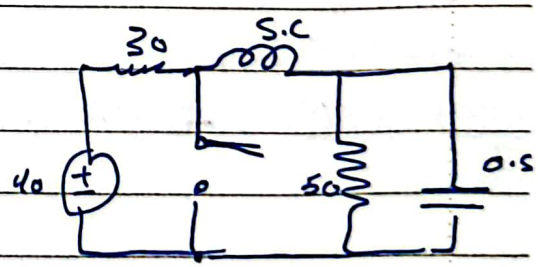
$\omega_0 = \frac{1}{\sqrt{L_{eq} C_{eq}}} = \frac{1}{\sqrt{0.4 * 20\mu}} = 353$ $\quad s_1 = -146$ $\quad s_2 =$

$\alpha > \omega_0$ overdamped.

$V(t) = A_1 e^{s_1 t} + A_2 e^{s_2 t}$ $\quad t>0$

$V_c(0^+) = V_c(0^-) = A_1 + A_2$

$25 = \frac{50}{50 * 80} \cdot 40 = A_1 + A_2$



$\frac{dV(0^+)}{dt} = \frac{i_c(0^+)}{C} = A_1 s_1 + A_2 s_2$

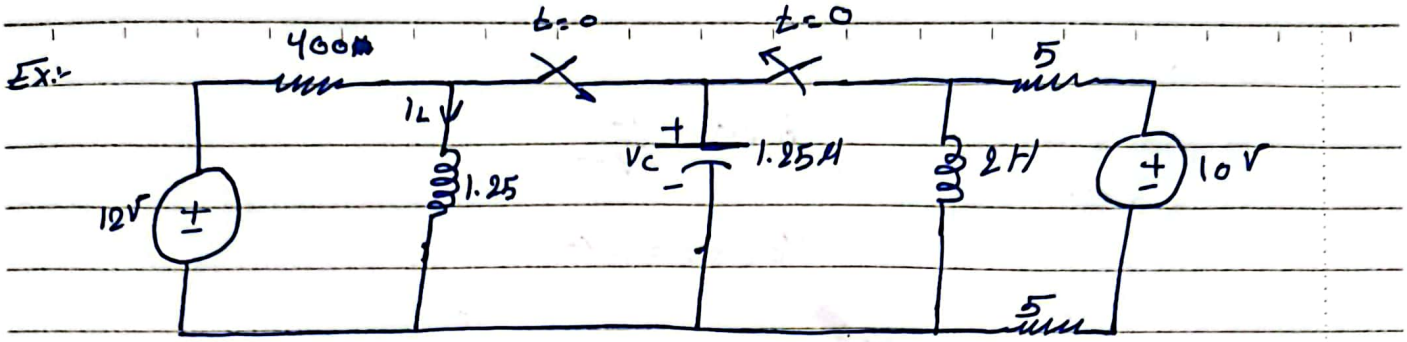
$i_c(0^+) + i_L(0^+) + i_R(0^+) = 0$

$i_L(0^+) + 0.5 + \frac{25}{50} = 0$

$i_L(0^+) = \text{zero}$ ..

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Ex:



$V_c(0^+)$

$i_L(0^+)$

$i_L(\infty)$

$V_c(\infty)$

$\frac{dv(0^+)}{dt}$

$\frac{di_L(0^+)}{dt}$

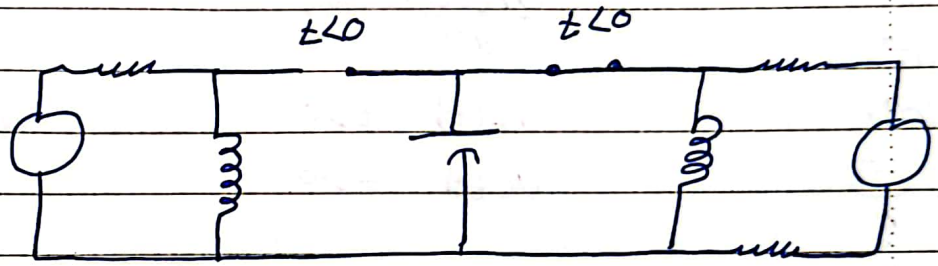
$\frac{dv(0^-)}{dt} = \frac{i_c(0^-)}{C}$

$\frac{di_L(0^-)}{dt}$

to find value of 0^+ you must find firstly

$i_L(0^-) = i_L(0^+)$

$V_c(0^-) = V_c(0^+)$

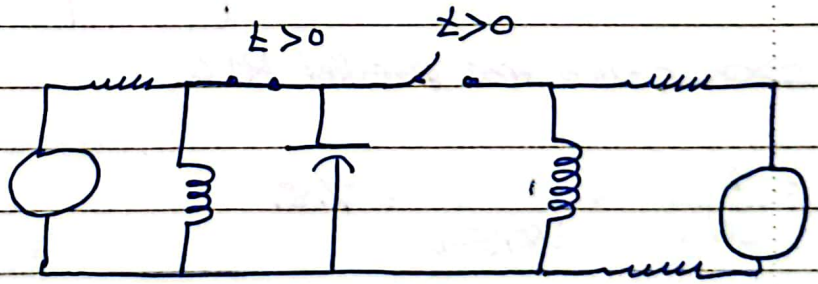


$i_L(0^-) = i_L(0^+) = \frac{12}{400}$

$V_c(0^-) = V_c(0^+) = zero$

$\frac{dv(0^-)}{dt} = \frac{i_c(0^-)}{C} = zero$ (0.C)

$\frac{di_L(0^-)}{dt} = \frac{V_L(0^-)}{L} = zero$ (S.C)



$\frac{dv_c(0^+)}{dt} = \frac{i_c(0^+)}{C}$

to find it use KCL

$i_c(0^+) + i_L(0^+) + \frac{0-12}{400} = 0$

$i_c(0^+) = zero$

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at ∞

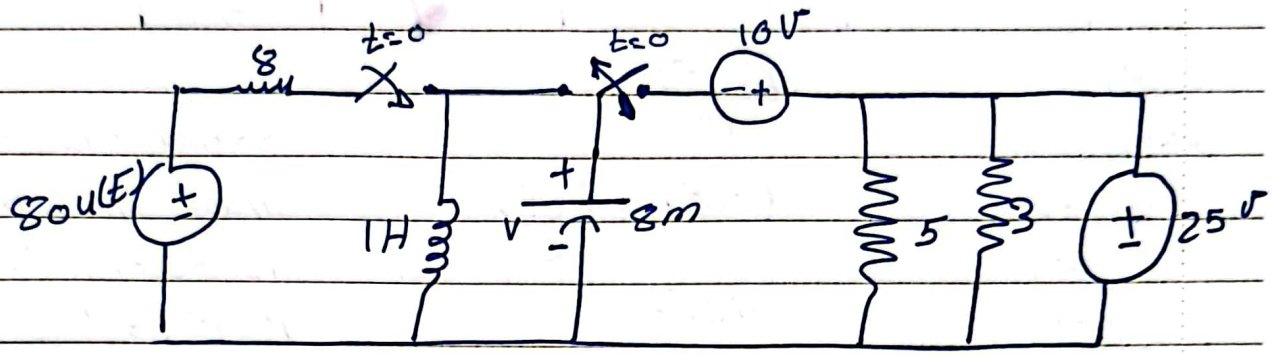
$L \rightarrow$ S.C

$C \rightarrow$ O.C

$i_L(\infty) = \frac{12}{400}$

$V_C(\infty) =$ zero.

Ex:-



α

w_0

$i_L(0^+)$

$V_C(0^+)$

$\frac{dV(0^-)}{dt}$

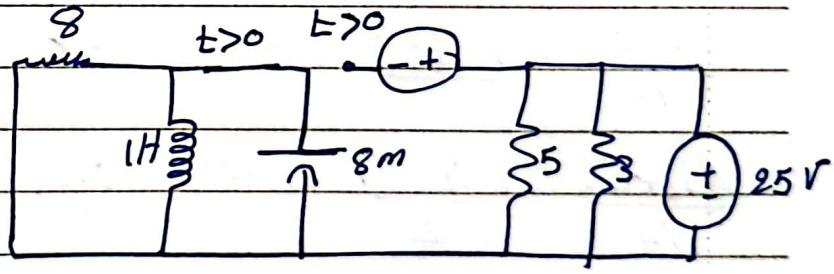
$\frac{di_L(0^-)}{dt}$

$i_L(t) \quad t>0$

$V_C(t) \quad t>0$

to find α w_0 $t>0$

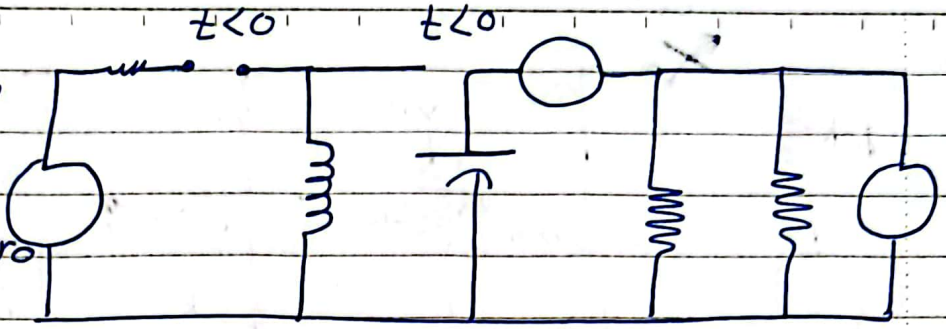
$80\,u(t^-) = \begin{cases} 80 & t \leq 0 \\ 0 & t>0 \end{cases}$



$t>0$ source free parallel RLC

$\alpha = \frac{1}{2RC} = \frac{1}{2(8)(8m)} = 7.81$

$w_0 = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{1 \times 8m}} = 11.18$



$\frac{dV_c(0^-)}{dt} = \frac{i_c(0^-)}{C} =$ zero

$\frac{di_L(0^-)}{dt} = \frac{V_L(0^-)}{L} =$ zero

$V_c(0^-) = V_c(0^+) = 15$ $\quad i_L(0^-) = i_L(0^+) =$ zero.

$-V_c(0^-) - 10 + 25 = 0$

$V_c(0^-) = 15$

to find $i_L(t)$, $V_c(t)$ firstly find α, w_o

$w_o > \alpha$

underdamped.

$i_L(t) = e^{-\alpha t}(A_1 \cos(w_d t) + A_2 \sin(w_d t)) \quad t>0 \quad w_d = \sqrt{w_o^2 - \alpha^2} \simeq 8$

$i_L(0^+) = A_1 =$ zero.

$\frac{di_L(0^+)}{dt} = \underbrace{\frac{V_L(0^+)}{dt} = \frac{V_c(0^+)}{dt}}_{\text{parallel}} = \frac{15}{1} = A_2 w_d - \alpha A_1$

$A_2 = \frac{15}{8}$

$i_L(t) = e^{-7.31t}\left(\frac{15}{8}\sin 8t\right) \quad t>0$

$V_c(t) = e^{-\alpha t}(B_1 \cos(w_d t) + B_2 \sin(w_d t))$

$V_c(0^+) = 15 = B_1$

$\frac{dV_c(0^+)}{dt} = \frac{i_c(0^+)}{C} = \frac{-15/8}{8m}$

$\frac{-15/8}{8m} = B_2 w_d - \alpha B_1$

$\frac{-15/8}{8m} = 8B_2 - (7.31)*15$

$\rightarrow B_2 = ??$

KCL to find $i_c(0^+)$

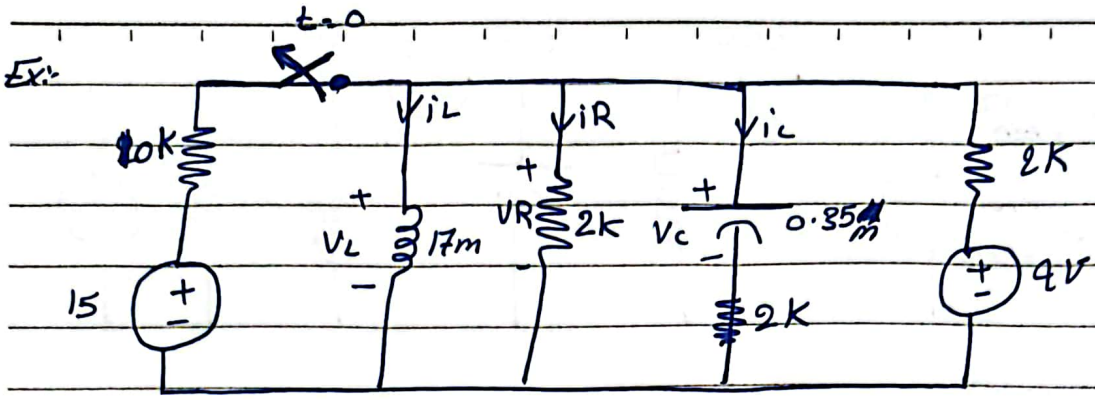
$i_c(0^+) + i_8(0^+) + i_1(0^+) = 0$

$i_c(0^+) + \frac{V_c(0^+)}{8} = 0$

$i_c(0^+) = \frac{-15}{8}$

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Ex:

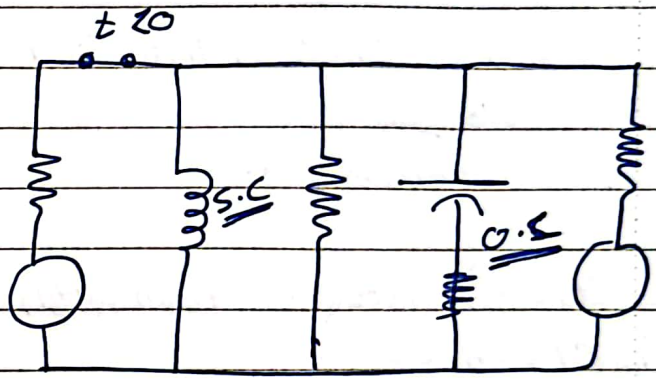


$i_L(0^+)$ $i_c(0^+)$

$V_c(0^-)$ E stored at ∞ C

$V_L(0^+)$

$i_R(0^+)$ $i_R(0^-)$



$i_c(0^-) = zero$

$i_R(0^-) = zero.$

to find any value of 0^+

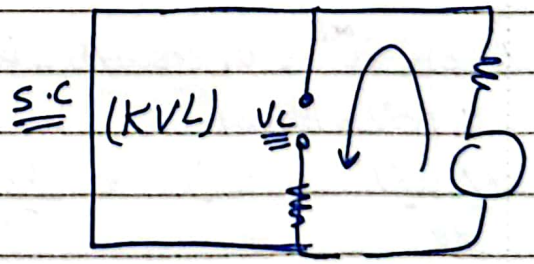
you must find

$V_c(0^-) = V_c(0^+)$

$i_L(0^-) = i_L(0^+)$

$$i_L(0^-) + \frac{0-15}{10K} + \frac{0-4}{2K} = 0 \qquad (KCL)$$

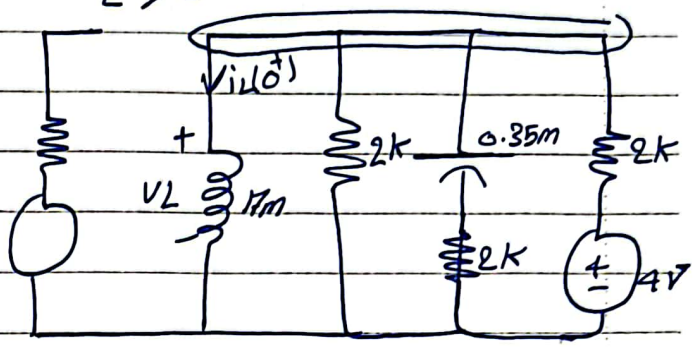
$V_c(0^-) = V_c(0^+) = zero.$ (KVL)



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t > 0



~~$i_L(0^+)$~~

$$i_L(0^+) + \frac{V_L(0^+)}{2K} + i_C(0^+) + \frac{V_L(0^+) - 9}{2K} = 0$$

$$i_L(0^+) + \frac{V_L(0^+)}{2K} + \frac{V_L(0^+)}{2K} + \frac{V_L(0^+) - 9}{2K} = 0$$

Divide ↙

$V_L(0^+) =$

E stored at $\infty = \frac{1}{2} C |V(\infty)|^2 = zero$

$C \rightarrow O.C$

$L, R \rightarrow S.C$

$$i_R(0^+) = \frac{V_L(0^+)}{2K}$$

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