ElCoM
ELECTRICAL COMPUTER MECHATRONICS

تقدم لجنة ElCoM الاكاديمية

دفتر لمادة:

دوائر كهربائية 2

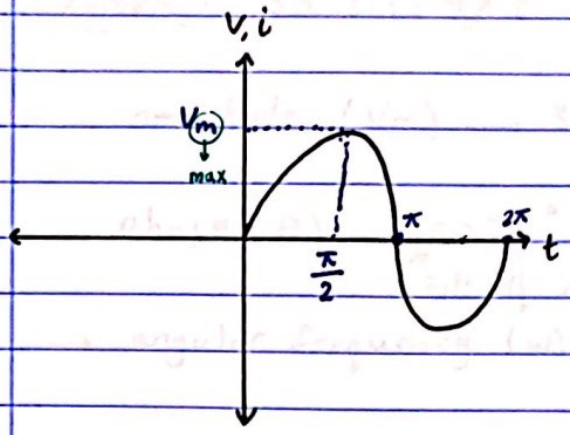
من شرح:

م.زهرة غانم

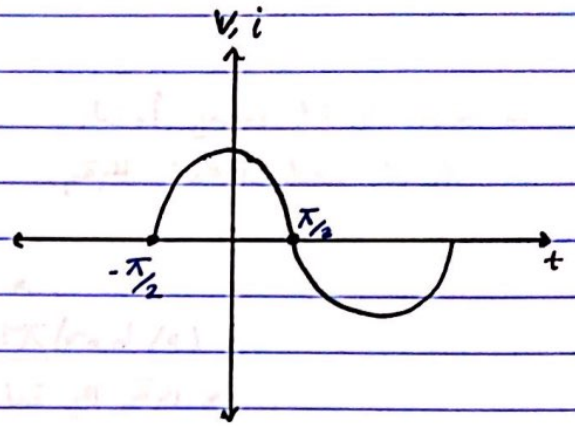
جزيل الشكر للطالب:

محمد زغول

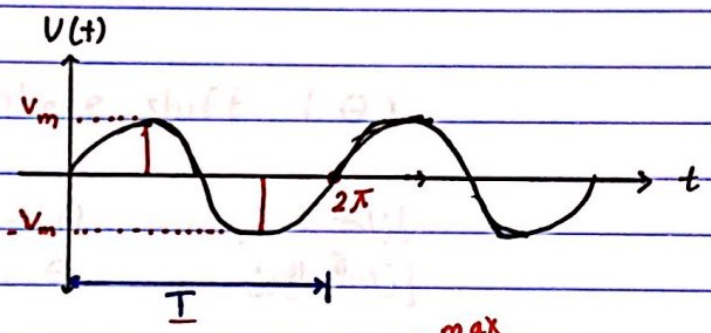
CH 10: Sinusoidal steady state analysis.



$\sin(wt)$



$\cos(wt)$



- Amplitude (V_m) (peak value). ← max
- V peak to peak (V_{p-p}) = $2V_m$ ← القيمة من القاع للقاع.
- frequency (f) (Hz). ← (كم مرة [illegible] رح تعيد حالها)
- angular frequency (w) (rad/s).
- period (T) (s).
- phase shift (θ).

$$w = 2\pi f = \frac{2\pi}{T}$$

$$f = \frac{1}{T} \rightarrow Hz = \frac{1}{s}$$

* T is constant.

$$\Rightarrow v(t) = V_m \sin(wt \pm \theta)$$

OR

$$v(t) = V_m \cos(wt \pm \theta)$$

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Example: $30 \sin(4\pi t - 75^\circ)$

→ amplitude (V_m) ⟶ 30 * ما حدد لي اذا فولتيج أو تيار فما بنحط وحدة للرقم.

→ phase (θ) ⟶ $\ominus 75^\circ$ * مهم آخذ الإشارة

→ angular frequency (w) ⟶ 4π (rad/s) * الرقم اللي قبل الـ (t)

→ period (T) ⟶ $\frac{1}{f} = \frac{1}{2}$ (s)

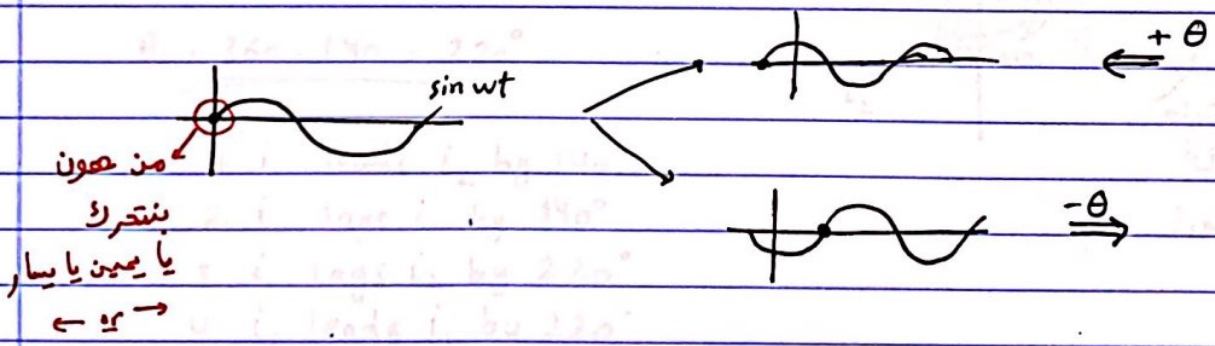
$w = 2\pi f$

$f = \frac{w}{2\pi} \Rightarrow \frac{4\pi}{2\pi} = 2$ Hz

phase shift (θ)

$+\theta$ ⟶ إزاحة لليسار

$-\theta$ ⟶ إزاحة لليمين



$V_1(t) = V_{m_1} \sin(wt + \theta_1)$

$V_2(t) = V_{m_2} \sin(wt + \theta_2)$

If $\theta_1 = \theta_2$ ⟶ V_1, V_2 In phase.

$\theta_1 \neq \theta_2$ ⟶ V_1, V_2 out of phase, phase shift (leading and lagging).

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* conditions of comparing between 2 signals :-

1. have the same frequency.
2. Both should be sin OR cos * اغلب الوقت بنحول sin لـ cos.

$$\begin{cases} \sin wt = \cos(wt-90) \\ \cos wt = \sin(wt+90) \end{cases}$$

3. Both should be positive OR negative signals.

$$\begin{cases} -\sin wt = \sin(wt \pm 180^\circ) \\ -\cos wt = \cos(wt \pm 180^\circ) \end{cases}$$

بختار اللي بتسهل الزاوية

Ex: $i_1(t) = 5\cos(3t+30^\circ)$

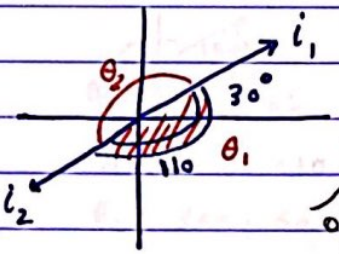
$i_2(t) = 2\sin(3t-20^\circ)$

$\hookrightarrow 2\cos(5t-20^\circ-90^\circ) \rightarrow 2\cos(5t \ominus 110^\circ)$

يدل على عكس عقارب الساعة.

$\theta_1 = 30+110 = 140^\circ$

$\theta_2 = 360-140 = 220^\circ$



1. i_1 leads i_2 by 140°
2. i_2 lags i_1 by 140°
3. i_1 lags i_2 by 220°
4. i_2 leads i_1 by 220°

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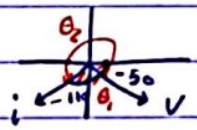
Ex$_2$: $v(t) = 2\cos(\pi t - 50^\circ)$

$i(t) = 10\sin(\pi t - 20^\circ)$

$\hookrightarrow 10\sin(\pi t - 110)$

$\theta_1 = 110^\circ - 50^\circ = 60^\circ$

$\theta_2 = 360^\circ - 60^\circ = 300^\circ$



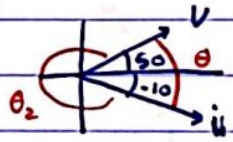
1. V lead i by 60°
2. i lag V by 60° OR -300°
3. V lag i by 300°
4. i ~~lag~~ lead V by 300° OR -60°

E$_3$: $v(t) = -3\sin(t - 40^\circ) \longrightarrow 3\cos(t - 40^\circ - 90 + 180^\circ)$

$i(t) = 10\cos(t - 10^\circ)$ $\quad 3\cos(t + 50^\circ)$

V(t) lags i(t) by ...300°...

OR(-60°)



$\theta_1 = 50 + 10 = 60^\circ$

$\theta_2 = 360 - 60 = 300^\circ$

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CH 10.2

« Forced response to sin fnction »

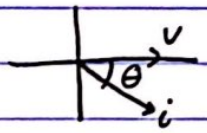
For RL :

* $V(t) = V_m \cos \omega t$

* $i(t) = \frac{V_m}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \underbrace{\tan^{-1}\frac{\omega L}{R}}_{\theta}\right)$ ← i

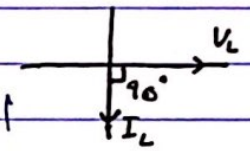
▣ w : excitative frequency

* i_L lag V by $\tan^{-1}\frac{\omega L}{R}$ for RL



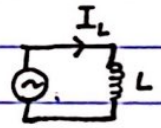
$\varphi = \tan^{-1}\infty = 90^\circ$ → but for $\boxed{R=0}$

$i(t) = \cos(\omega t - 90)$



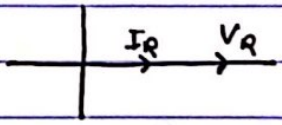
* i lag V by 90° for pure Inductor.

(if it's not pure Inductor, i still lag V but by θ ($\tan^{-1}\frac{\omega L}{R}$) and not by 90°).



$\varphi = \tan^{-1} 0 = 0$ → for $\boxed{L=0}$

$i(t) = \frac{V_m}{R} \cos \omega t$



* i_R, V_R In phase. (for pure Resistive).

→ for $\boxed{\omega = 0}$

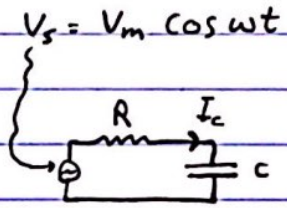
$i_L = \frac{V}{R}$ (DC source).

* DC

L → S.C

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For RC :-



* $V_s = V_m \cos \omega t$

* $i_c(t) = \frac{V_m}{\sqrt{1 + \omega^2 R^2 C^2}} \cos\left(\omega t + \tan^{-1} \frac{1}{\omega C R}\right)$

* $i_c(t)$ lead V by $\tan^{-1} \frac{1}{\omega R C}$

→ for **pure capacitive** (R = 0).
i lead V by 90°.

$\phi = 90°$ for pure C.

- I in phase V → pure R
- I Lag V by 90° → pure L
- I lead V by 90° → pure C

- I lag V by θ → RL
- I lead V by θ → RC

* for RL
i Lag V by θ $\tan^{-1} \frac{\omega L}{R}$

* for RC
i lead V by θ $\tan^{-1} \frac{1}{\omega R C}$

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CH 10.4

≪ The complex forcing function ≫

* $x + jy$ ⟶ Rectangular form.
* $A e^{j\theta}$ ⟶ polar form.

$$A = \sqrt{x^2 + y^2} \qquad \theta = \tan^{-1}\left(\frac{y}{x}\right)$$

$$x = A\cos\theta \qquad y = A\sin\theta$$

$$\#\{ x + jy = A\cos\theta + j A\sin\theta \}\#$$

$A\angle\theta$ ⟶ phasor form

A: amplitude θ: phase

لما نحول لـ phasor form لازم الـ frequency (ω) نفسو.

$$\Rightarrow V = V_m e^{j\theta}$$
$$= V_m\angle\theta \quad \leftarrow \text{phasor form}$$
$$v(t) = Re[V_m e^{j\omega t} e^{j\theta}]$$

$$\Rightarrow I = I_m e^{j\phi}$$
$$= I_m\angle\phi \quad \leftarrow \text{phasor form}$$
$$i(t) = I_m[\cos(\omega t + \phi)]$$
OR $$i(t) = Re[I_m(e^{j\omega t} e^{j\phi})]$$

θ ← ثيتا ← V للفولتية.
Φ ← فاي ← I للتيار.

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✳ Ex₁: Find the phasor form then convert it to time domain.

1. $i = -3 + j4\ A$

$A\angle\theta \rightarrow A = \sqrt{(-3)^2 + (4)^2} = 5$

$\theta = \tan^{-1}\left(\frac{4}{-3}\right) = 126.869$

$\rightarrow 5\angle 126.869^\circ$

$\rightarrow 5\cos(wt + 126.87^\circ)$

2. $V = j8e^{-j20}\ V$

$1\angle 90^\circ * 8\angle -20 = 8\angle 70^\circ$

$\rightarrow 8\cos(wt + 70^\circ)$

* الزاويتين بنجمعهم والـ amplitude بنضربه.

✳ Ex₂: Find Instantanus value of (I) at $t = 1ms$

$w = 2000\ rad/s$.

1. $i = j10\ A$

$10\angle 90^\circ \rightarrow 10\cos(wt + 90^\circ)$ ← $i(t)$

$i(1m)\ 10\cos(2000 * 1m + 90^\circ)$

$= -9\ A$

نحول Rad إلى degree

$\frac{② * 180}{\pi}$

jy ضنا x = صفر & يوجد معي

$-j5 = 5\angle -90^\circ$ $j = 90$

$+j5 = 5\angle 90^\circ$

2. $i = 20 + j10A$

$= 22.36\angle 26.56^\circ$

$i(t) = 22.36\cos(wt + 26.56^\circ)$

$= -17.4\ A$

مهم نحول فالنهاية إلى Time Domain.

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CH 10.5 « Impedance and Admittance »

ممانعة

Impedance $(Z) = \frac{\text{phasor Voltage}}{\text{phasor current}}$ (Ω)

(R) بحاصل معاملة المقاومة.

Phasors for R, L, C :-

* for R:

المقاومة هي نفسها دائماً * بالـ Time أو بالـ frequency domain

$\Rightarrow Z_R = R = \frac{V}{I}$

Time Domain ⟶ (phasor) frequency Domain

$v(t) = i(t)\,R \longrightarrow V = IR$

L ≡ Inductor

* for L:

* ممانعة (L) تعتمد على (L) او frequency كلما زاد (L) يزيد (Z_L)

$\Rightarrow \frac{V}{I} = jwL = Z_L$

Time Domain ⟶ (phasor) frequency Domain

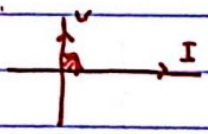
$V_L = L\frac{di}{dt} \longrightarrow V = jwLI$

Inductive Reactance ⟶ $X_L = wL$ (بدون j)

$Z_L = jwL = jX_L$

1. $w = 0$ DC ⟶ $Z_L = 0$ S.C
2. $w = \infty \longrightarrow Z_L = \infty$ O.C

I_L Lag V_L by 90°



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C ≡ capacitor * for C:

$$\Rightarrow Z_c = \frac{1}{jwC}$$

Time Domain ⟶ (phasor) (frequency Domain)

$$i_c = C\frac{dv}{dt} \longrightarrow I = jwCV$$

$$Z_c = \frac{V}{I} = \frac{1}{jwC} = \ominus\frac{j}{wC}$$

Capacitive Reactance:

$$X_c = \frac{1}{wc} \ (\Omega) \qquad \text{(بدون ذ)}$$

- $w = 0 \xrightarrow{DC} Z_c = \infty$ O.C
- $w = \infty \longrightarrow Z_c = 0$ S.C

I lead V by 90°
for pure capacitive

سعاد حية

Admittance:

$$Y = \frac{1}{Z} = \frac{I}{V} \qquad \left(\frac{1}{\Omega} \rightarrow S \text{ (siemens)}\right)$$

C	L	pure
$Z = \ominus j2$	$Z = j2$	$Z = 2$

pure Inductive Imped.. نهائياً بتذكري !

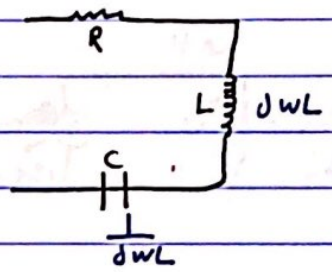
- If R + jX ⟶ Inductive.
- If R - jX ⟶ capacitive.
- If R ⟶ pure Resistive.
- If +jX ⟶ pure Inductive.
- If -jX ⟶ pure capacitance.

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* for R, C, L:

$$Z_{eq} = R + j\omega L + \frac{1}{j\omega L}$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$



$$\Rightarrow Z_{eq} = R + j\,(X_L - X_C)$$

$A\cos\theta$ = Real part (R_{eq}) — Im. part (X_{eq}) = $A\sin\theta$

$$Z_{eq} = A\angle\theta = A(\cos\theta + j\sin\theta)$$

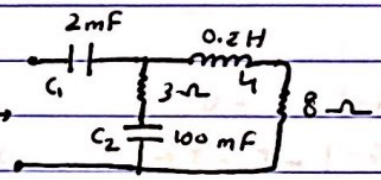
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* **Example,** Find Z_{eq} { $w = 50$ rad/s }

$*Z_{C1} = \frac{-j}{\omega C} = -j10\ \Omega$

$*Z_{C2} = \frac{-j}{50 * 100m} = -j0.2\ \Omega$

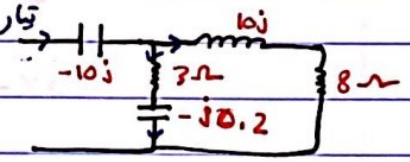
$*Z_L = j\omega L = j10\ \Omega$



بنحول كلشي Impedence

$$Z_{eq} = \{(8 + j10) \,//\, (3 - j0.2)\} - j10$$

// مكرونيم عك مجموعهم



$$Z_{eq} = 10.1\angle -75.05^\circ$$

$R_{eq} = A\cos\theta = 2.6$

$X_{eq} = A\sin\theta = -9.76$

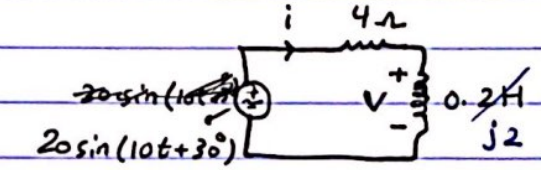
الزاوية سالبة يعني
capacitive Impedence.

* لو كان في مصدر جهد
بكون i lead V
لأن Z_{eq} capacitive

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* **Ex_2:** Determine $v(t)$ and $i(t)$

$w = 10$ Rad/s



① source: $20\angle 30^\circ$

② $Z_R = 4\Omega$

$Z_L = jwL = j * 10 * 0.2 = j2\ (\Omega)$

$$i = \frac{V}{Z} \rightarrow \frac{20\angle 30^\circ}{4+j2} = 4.47\angle 3.435^\circ$$

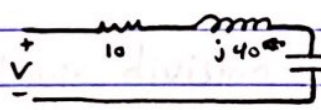
$$\Rightarrow \underline{i(t)} = 4.47 \sin(10t + 3.435)$$

$$V = I * Z \rightarrow 4.472\angle 3.435 * j2 = 8.94\angle 93.435^\circ$$

$$\Rightarrow \underline{v(t)} = 8.944 \sin(10t + 93.435^\circ)$$

---

* **Ex_3:**



$\rightarrow V = V_m\angle -25$

$\rightarrow I = I_m\angle -70$

find X_c??

sol: $Z_{eq} = 10 + j40 - jxc = \frac{V}{I}$

$\angle -25 - \angle -70$ $\quad \frac{V_m\angle -25}{I_m\angle -70} = 10 + j(40 - xc)$

$$\frac{V_m}{I_m}\angle 45^\circ = 10 + j(40 - xc) \rightarrow \begin{cases} \frac{V_m}{I_m}\cos(45^\circ) = 10 \\ \frac{V_m}{I_m}\sin(45^\circ) = 40 - xc \end{cases}$$

$\times$ ← فيها مجهولين (ما بستفيد منها) $\quad \sqrt{10^2 + (40-xc)^2} = \frac{V_m}{I_m}$ $\quad / \quad \tan^{-1}\frac{40-x_c}{10} = 45^\circ$ ← فيها مجهول واحد

$\rightarrow$ ~~tan~~$^{-1}$ $\frac{40 - xc}{10} = 1 \rightsquigarrow \boxed{xc = 30\ \Omega}$

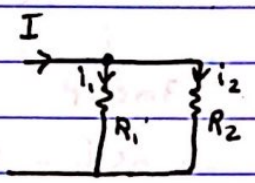
← ركبنا tan للطرفين.

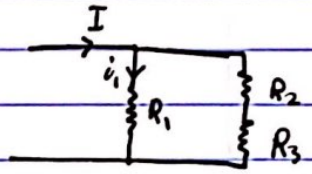
* لو طلب (c) يجب أن يعطينا w ← $xc = \frac{1}{wc} = 30$

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≪ Reminder on current and Voltage division ≫

current division:



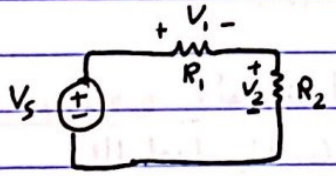


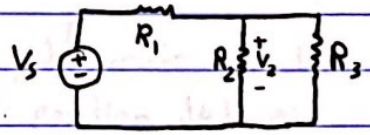
$$i_{②} = \frac{R_{①}}{R_1 + R_2} \cdot I$$

$$i_1 = \frac{R_2 + R_3}{R_1 + R_2 + R_3} \cdot I$$

$$i_1 = \frac{R_2}{R_1 + R_2} \cdot I$$

Voltage division:





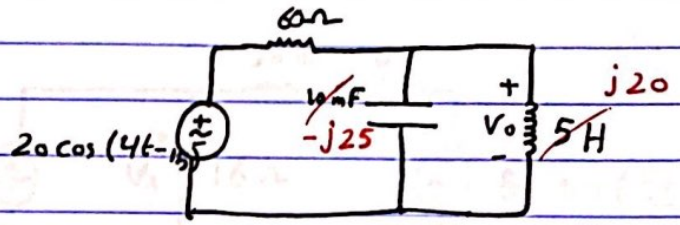
$$V_{①} = \frac{R_{①}}{R_1 + R_2} \cdot V_S$$

$$V_2 = \frac{R_2 // R_3}{R_1 + (R_2 // R_3)} \cdot V_S$$

$$V_2 = \frac{R_2}{R_1 + R_2} \cdot V_S$$

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* Ex.: Find $V_o(t)$



① $20\angle -15$

② $Z_R = 60\ \Omega$

$$Z_c = \frac{-j}{\omega C} = \frac{-j}{4 * 10mF} = -j25\ \Omega$$

$$Z_L = j\omega L = j20\ \Omega$$

⇒ Using voltage Division

$$V_o = \frac{(j20 \,//\, -j25)}{(j20 \,//\, -j25) + 60} * 20\angle -15 = 17.27\angle 16$$

$$\boxed{V_o(t) = 17.27 \cos(4t - 16^\circ)}$$

<< superposition >>

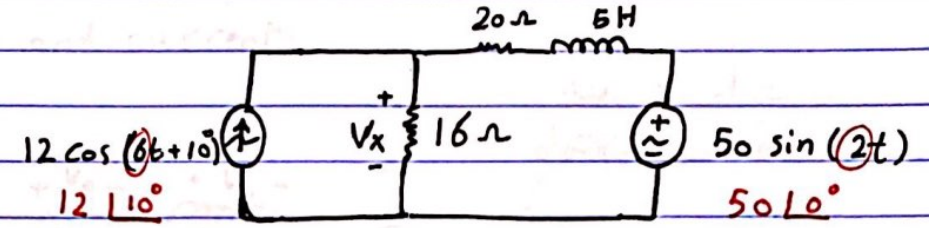
* لما يكون عندي أكثر من source بالمادة حاي وعندي w مختلفة لازم اعمل superposition من خلال إني اعمل Kill لل sources واخلي واحد وأشوف تأثيرو على السيركيت

← لما اعمل Kill لـ Voltage source (V=0) ← S.C (سلك فاضي).

← لما اعمل Kill لـ current source (I=0) ← O.C (سلك مقطوع).

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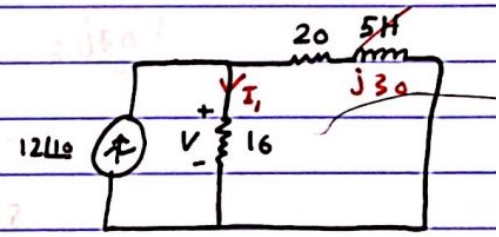
* Ex: For the circuit shown, Find the value $V_{(x)}(t)$.



[1] Voltage source → off → S.C

الشغال على source الـ (w) اللي بجيب منها

$Z_L = jwL \rightarrow j * 6 * 5 = j30$

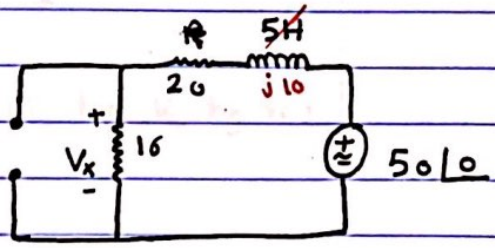


Voltage division $\rightarrow$ $I_1 = 12\angle 10 * \frac{20 + j30}{20 + j30 + 16} = 9.23\angle 26.5°$

$V_1 = i Z_R = 9.23\angle 26.5° * 16 = 147.7\angle 26.5°$

$V_1(t) = 147.7 \cos(6t + 26.5°)$

[2] Current source → off → O.C



$Z_L = jwL \rightarrow j * 2 * 5 = j10$

$V_2 = 50\angle 0 * \frac{16}{16 + 20 + j10}$

$V_2 = 21.14\angle -15.5 \rightarrow V_2(t) = 21.14 \sin(2t - 15.5°)$

$V_{(x)}(t) = V_1(t) + V_2(t)$

$= 147.7 \cos(6t + 26.5°) + 21.14 \sin(2t - 15.5°)$

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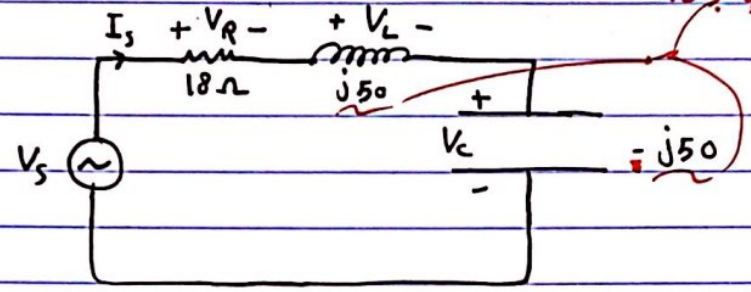
CH 10.10

<< phasor diagram >>

⇒ sketch in a complex plane for all Voltages and currents.

* متساويتين بالمقدار متعاكستين بالإتجاه.

Ex:



construct the phasor diagram?

Soli ▪ choose (I_s) as a referance. (the angle = 0).

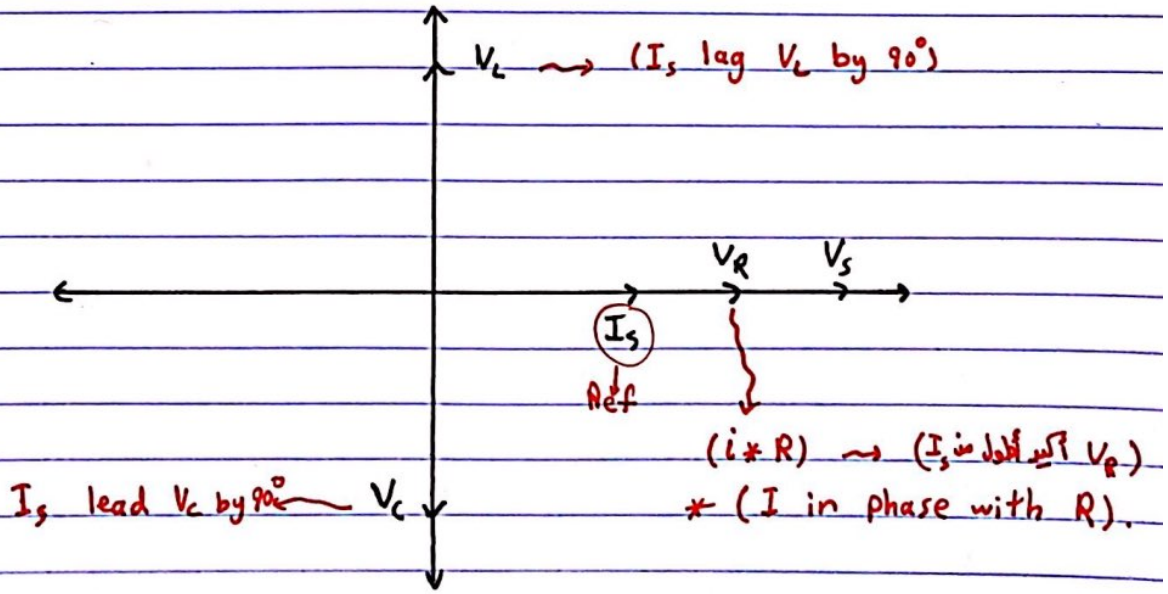
(I_s ↓ التوصيل توالي).

(لو كان توازي باخذ Voltage كـ Referance).

$I_s = 1\angle 0$

* لو الفولتيج Ref ← $V_s = 1\angle 0$

* نقارن كل عناصر الدارة بـ (I_s).



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$I_s = 1\angle 0$

$V_L = I_s Z_L = (1\angle 0)(j50) = j50\ V = 50\angle 90^\circ\ V$

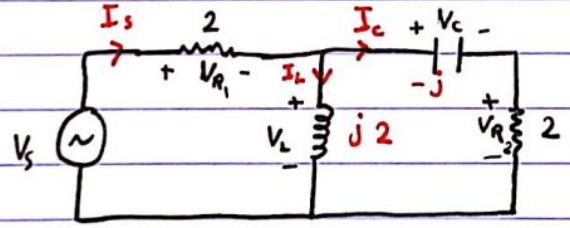
$V_R = I_s R = (1\angle 0)(10) = 10\ V$

$V_c = I_s Z_c = (1\angle 0)(-j50) = -j50\ V = 50\angle -90^\circ\ V$

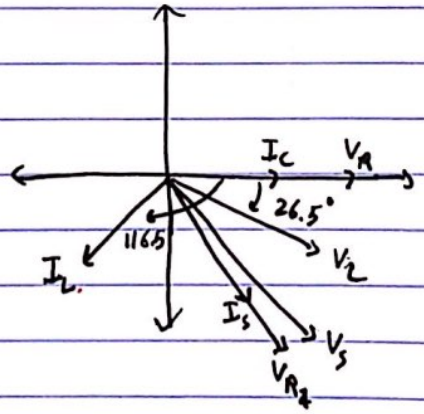
$V_s = V_R + V_c + V_L = V_R = 10\ V$ #

* Ex: choose I_c as a Reference

$I_c = 1\angle 0$



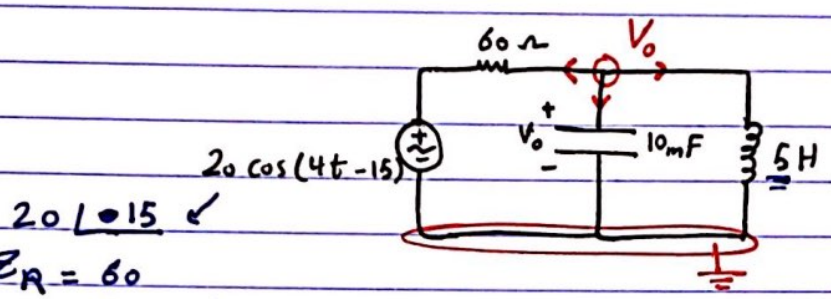
1. $I_c = 1\angle 0$
2. $V_c = I_c Z_c = (1\angle 0)(-j) = -j = 1\angle -90^\circ$
3. $V_{R_2} = I_c * 2 = 2\ V$
4. $V_L = V_c + V_R = 2.2\angle -26.5^\circ$
5. $I_L = V_L / Z_L = 1.1\angle -116.5^\circ$
6. $I_s = I_c + I_L = 1.1\angle -62.3^\circ$ ← KCL
7. $V_{R_1} = I_s * 2 = 2.2\angle -62.65^\circ$
8. $V_s = V_{R_1} + V_L$



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Extra Examples

Example: Find $V_o(t)$ using nodal analysis.



$20\angle -15$

$Z_R = 60$

$Z_C = -j/wC = -j/4*10\,mF = -j25\,\Omega$

$Z_L = jwL = j*4*5 = j20\,\Omega$

للحل على النودال بفرض تيار طالع من النود من جميع الإتجاهات.

$$\frac{V_o - 20\angle -15}{60} + \frac{V_o - Zero}{-j25} + \frac{V_o - Zero}{j20} = Zero$$

$$\frac{V_o}{60} - \frac{20\angle -15}{60} + \frac{V_o}{-j25} + \frac{V_o}{j20} = Zero$$

$$V_o\left(\frac{1}{60} + \frac{1}{-j25} + \frac{1}{j20}\right) = \frac{20\angle -15}{60}$$

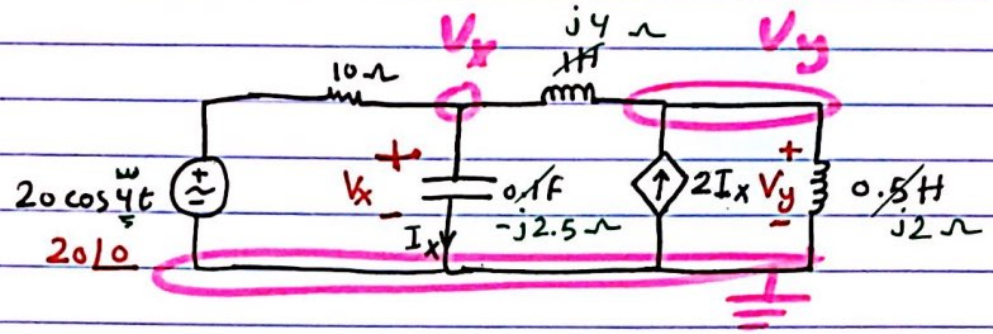
$$V_o = 17.271\angle 16$$

$$\boxed{V_o(t) = 17.271\cos(4t + 16^\circ)}$$

المطلوب.

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Example: Find $V_x(t)$, $V_y(t)$



Sol:

$\boxed{w=4}$

$$Z_{0.5H} = jwL = j*4*0.5 = j2\ \Omega$$
$$Z_{1H} = jwL = j*4*1 = j4\ \Omega$$
$$Z_c = -j/wc = -j2.5$$

KCL @ node V_x →
$$\frac{V_x - V_y}{j4} + \frac{V_x - 20\angle 0}{10} + \frac{V_x - 0}{-j2.5} = 0 \quad \dots ①$$

KCL @ node V_y →
$$\frac{V_y - V_x}{j4} + \frac{V_y - 0}{j2} = 2\underbrace{(I_x)}_{\text{داخل بالنود عشان صيك}} \rightarrow 2 * \frac{V_x - 0}{-j2.5} \quad \dots ②$$

$$V_x\left(\frac{1}{j4} + \frac{1}{10} + \frac{1}{-j2.5}\right) - \frac{V_y}{j4} = \frac{20\angle 0}{10}$$

① … $0.1803\angle 56.31^\circ\ V_x - 0.25\angle -90^\circ\ V_y = 2$
(A) (B) (E)

$$-0.25\angle -90^\circ\ V_x + 0.75\angle -90^\circ\ V_y = 0.8\angle 90\ V_x$$

② … $0.55\angle 90^\circ\ V_x - 0.75\angle -90^\circ\ V_y = 0$
(C) (D) (F)

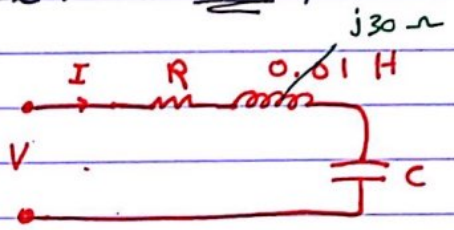
$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_x \\ V_y \end{bmatrix} = \begin{bmatrix} E \\ F \end{bmatrix}$$

$V_x = 19\angle 18.42^\circ\ V.$
$V_y = 14\angle -161.568\ V.$

$$V_x = \frac{ED - BF}{AD - BC} \qquad V_y = \frac{AF - EC}{AD - BC}$$ ← Using a calculator.

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Ex: $L = 0.01\ H$, $V(t) = 353.5\cos(3000t - 10°)$,
$i(t) = 12.5\cos(3000t - 55°)$,
for series RLC, Find C, R ?



$V = 353.5\angle -10°$

$I = 12.5\angle -55°$

$w = 3000\ rad/s$

$Z_L = jwL = j*3000*0.01 = j30\ \Omega$

$$Z_{eq} = \frac{V}{I} = \frac{353.5\angle -10°}{12.5\angle -55}$$

$$= 20 + j20$$

R $\quad$ $(X_L - X_C)$

$R = 20\ \Omega$

$X_L - X_C = 20$

$30 - X_C = 20$

$30 - 20 = X_C$

$\boxed{X_C = 10}$

Remember: $X_C = \frac{1}{wC}$

$X_L = wL$

$X_C = \frac{1}{wC} \Rightarrow C = \frac{1}{X_C w}$

$C = \frac{1}{X_C w} = \frac{1}{30000}$

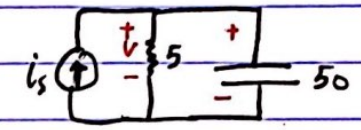
$= 3.33 * 10^{-5}\ MF$

20

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Ex: $\omega = 1000$ rad/s

sketch the phasor diagram for the following cct.



V → reference → $V = 1\angle 0$

$Z_C = -j/\omega C = -j10\ \Omega$

$I_C = V/Z \Rightarrow \frac{1\angle 0}{-j10} \Rightarrow 0.1\angle 90°$

$I_R = \frac{V_R}{Z_R} = \frac{1\angle 0}{5} = 0.2\angle 0$

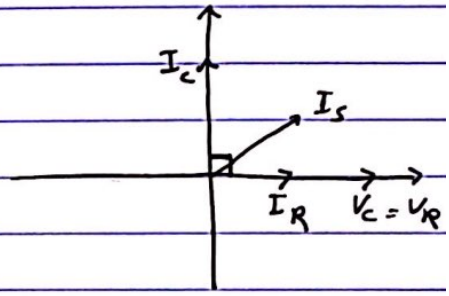
$V = I * R$

I من لطول V

$i_s = I_R + I_C$

$0.2 + 0.1\angle 90$

$I_S = 0.22\angle 26.56$ A



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CH:11

≪ AC Power Analysis ≫

→ Average and RMS values :-

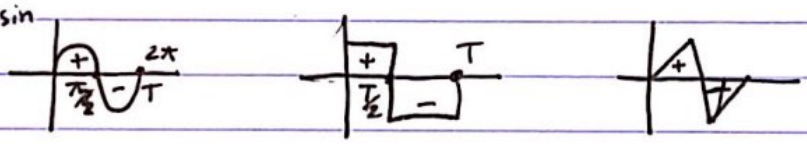
* $Y_{avg} = \frac{1}{T}\int_{0}^{T} y(t)\,.\,dt$

for sin and cos $T = 2\pi$ (فترتهم).

→ for half symmetry function :-

Y_{avg} = Zero (cos, sin مثل)

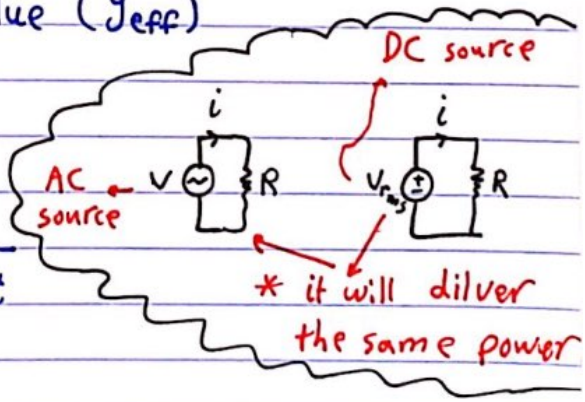
كيف يعني ؟ يعني إنو الموجب يساوي السالب فال Y_{avg} رح يصفر ، و مش شرط بس ال sin و cos.



$$\int_{0}^{T/2} V + \int_{T/2}^{T} -V = \text{zero}$$

RMS :- Root mean square valve (Y_{rms})
OR → Effective value (y_{eff})

$Y_{rms} \equiv y_{eff}$



$$Y_{rms} = Y_{eff} = \sqrt{\frac{1}{T}\int_{0}^{T} y^2(t).dt}$$

$$Y_{rms}^2 = \frac{1}{T}\int_{0}^{T} y^2(t).dt$$

اسهل استخدامو.

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Ex: $V(t) = V_m \cos(wt + \theta)$, Find:-

1. V_{avg} = ~~zero~~ (becuase it's a half symmetry function).

2. Effective Value :

$$V_{rms}^2 = \frac{1}{T} \int_0^T V(t)^2 . dt$$

2π → Period for cos functions.

$$V_{rms}^2 = \frac{1}{2\pi} \int_0^{2\pi} V_m^2 \cos^2(wt+\theta) . dt$$

$$= \frac{V_m^2}{2\pi} \int_0^{2\pi} \frac{1}{2} + \frac{1}{2} \cos(2wt + 2\theta) . dt$$

$$= \frac{V_m^2}{4\pi} \int_0^{2\pi} \frac{1}{2} \cos(2wt + 2\theta) . dt$$

$$= \frac{V_m^2}{4\pi} (T+0) \Big]_0^{2\pi}$$

$$\sqrt{V_{rms}^2} = \sqrt{\frac{V_m^2}{2}}$$

$$V_{rms} = \frac{V_m}{\sqrt{2}}$$

حفظ

هذول بس لل sin و cos ⟹

$V_{rms} = \frac{V_m}{\sqrt{2}}$, $I_{rms} = \frac{I_m}{\sqrt{2}}$, $V_{avg} = 0$

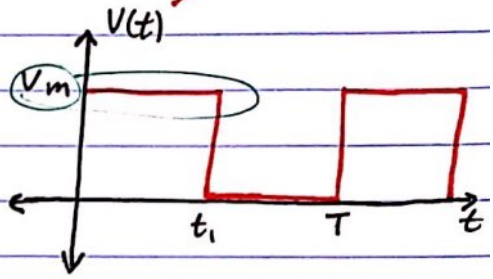
* يعني لو اعطاني اقتران غير ال sin و cos لازم اشتق مرة ثانية.

* $A\cos^2(wt) = A\left[\frac{1}{2} + \frac{1}{2}\cos(2wt)\right]$

* $A\sin^2(wt) = A\left[\frac{1}{2} - \frac{1}{2}\cos(2wt)\right]$

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ممكن تكون $I(t)$ والجواب بكون بدلالة I_m و I_{rms}



★ Ex: Find V_{avg} , V_{rms} ?

sol: ① $V_{avg} = \frac{1}{T}\int_0^T V(t).dt$

لأنو خط ثابت

$= \frac{1}{T}\left[\int_0^{t_1} V_m.dt + \int_{t_1}^{T} 0\right]$

$= \frac{1}{T}\int_0^{t_1} V_m.dt = \frac{1}{T} V_m (t_1 - 0)$

$$\boxed{V_{avg} = \frac{1}{T} * V_m * t_1}$$

② $V_{Rms}^2 = \frac{1}{T}\int_0^T V^2(t).dt$

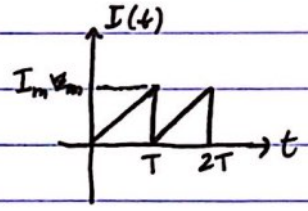
$= \frac{1}{T}\left[\int_0^{t_1} V_m^2.dt + \int_{t_1}^{T} 0.dt\right]$

$= \frac{1}{T} V_m^2 * t_1 \longrightarrow V_{rms}$ ②

ولكن هو طالب V_{rms} مش تربيع

$V_{rms} = \sqrt{\frac{V_m^2 * t_1}{T}} \Longrightarrow \boxed{V_{rms} = V_m\sqrt{\frac{t_1}{T}}}$

★ Ex: Find 1. I_{avg} , 2. I_{rms}



* عشان اطلع الاقتران $I(t)$ لازم استخدم معادلة الخط المستقيم $(y-y_0) = slope\,(x-x_0)$

$I(t) = \frac{I_m}{T} t$

① $I_{avg} = \frac{1}{T}\int_0^T I(t).dt$

$I_{avg} = \frac{1}{T}\int_0^T \frac{I_m}{T} t.dt$

$I_{avg} = \frac{I_m}{2}$

② $I_{rms}^2 = \frac{1}{T}\int_0^T \frac{I_m^2}{T^2} t^2.dt$

$I_{rms}^2 = \frac{I_m^2}{3}$

$I_{rms} = \frac{I_m}{\sqrt{3}}$

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Note:- we have 3 different ways to calculate RMS Values:-

1. By using the general equation:

$$Y_{rms} = \sqrt{\frac{1}{T}\int_0^T y^2(t)\,dt}$$

2. $Y_{rms} = \sqrt{(Y_{rms_1})^2 + (Y_{rms_2})^2 + \dots}$ ← we use this when we have different (w)

3. Same frequency → Add then Y_{rms}

Examples: Calculate V_{avg} and V_{rms}.

a) $5 + 4\cos(wt) = V(t)$

$V_{avg} = 5 + 0 = 5$

$$V_{RMS} = \sqrt{(5)^2 + \left(\frac{4}{\sqrt{2}}\right)^2} = \sqrt{25+8} = \sqrt{33}$$

b) $V(t) = 3\cos(\pi t) + 4\sin^2(\pi t)$.

$3\cos(\pi t) + 4\left(\frac{1}{2} + \frac{1}{2}\cos(2\pi t)\right)$

$3\cos(\pi t) + 2 + 2\cos(2\pi t)$ Different (w).

$V_{avg} = 2$

$$V_{rms} = \sqrt{2^2 + \left(\frac{3}{\sqrt{2}}\right)^2 + \left(\frac{2}{\sqrt{2}}\right)^2} = \text{——}$$

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c) $V(t) = 10 + 3\cos(10t) + 4\cos(10t - 120)$

↳ $10 + 3\angle 0 + 4\angle -120$

$V_{avg} = 10$ $= 10 + 3.6\angle -73.9°$

$= 10 + 3.6\cos(10t - 73.9°)$

$V_{rms} = \sqrt{(10)^2 + \left(\frac{3.6}{\sqrt{2}}\right)^2} = 10.3$

d) $6\cos(25t) + 4\sin(25t + 30)$

$6\cos(25t) + 4\cos(25t - 60°)$

$6\angle 0 + 4\angle -60° = 8.7\angle -23°$

$= 8.7\cos(25t - 23°)$

$P_{avg} = 0$

$RMS = \frac{8.7}{\sqrt{2}} = 6.152$

e) $V(t) = 2 + 3\cos(100t) + 4\cos(100t - 120)$

$3\angle 0 + 4\angle -120 = 3.6\angle -73°$

$V_{avg} = 2$

$V_{RMS} \rightarrow \sqrt{2^2 + \left(\frac{3.6}{\sqrt{2}}\right)^2} = 3.24$

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<<power calculation>>

$P = VI \longrightarrow p(t) = V(t)\, i(t) \longrightarrow$ Instantaneous power

$p(t) = V_m \cos((wt)+\theta)\ I_m \cos(wt+\varphi)$

$\cos A \cos B = \frac{1}{2}\left[\cos(A-B) + \cos(A+B)\right]$ ← متطابقة

$V(t) = V_m \cos(wt+\theta)$

$i(t) = I_m \cos(wt+\varphi)$

$P_t = P(t) = \underbrace{\frac{V_m I_m}{2}\cos(\theta-\varphi)}_{\text{constant}} + \underbrace{\frac{V_m I_m}{2}\cos(2wt+\theta+\varphi)}_{\text{Function Double (w).}\ (2w).}$

وحدتها واط (w)

Doesn't depend on time ← $P_{avg} = \frac{1}{T}\int_0^T P_t \, dt$

$P_{avg} = \frac{1}{T}\int_0^T \frac{V_m I_m}{2}\cos(\theta-\varphi) + \frac{V_m I_m}{2}\cos(2wt+\theta+\varphi)\, dt$

$P_{avg} = \frac{V_m I_m}{2}\cos(\underbrace{\theta}_{v} - \underbrace{\varphi}_{i})$ (Average power, Active power, Real Power (true))

$V_{rms} = \frac{V_m}{\sqrt{2}} \qquad I_{rms} = \frac{I_m}{\sqrt{2}}$

$V_m = \sqrt{2}\, V_{rms} \qquad I_m = \sqrt{2}\, I_{rms}$

$= \frac{(V_{rms}\sqrt{2}) * (I_{rms}\sqrt{2})}{2}\cos(\theta-\varphi)$

$P_{avg} = V_{rms} I_{rms} \cos(\theta-\varphi)$ ← صورة ثانية للقانون وبعل حسب المعطيات

وحدة الـ P_{avg} ← watt (w)

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Ex: $p(t) = 2 + 10\cos(50t)$

P_{avg} ? → 2 watt — الجزء الثابت من $p(t)$ ← P_{avg}

w = ? → $50 = 2w$

$\boxed{w = 25}$ rad/s

* Apperant power :-

$$P_{app} = S = |\underset{\text{Complex power}}{S}| = \frac{V_m I_m}{2} = V_{rms} I_{rms} \quad (VA)$$

* وحدتها فولت امبير للتمييز عن الـ (P) السابق.

* Power factor :-

$P_{avg} = P_{app} * P_F$

$P_F \rightarrow = \cos(\theta - \varphi)$

$$Z = \frac{V\angle\theta}{I\angle\varphi} = \frac{V}{I}\underbrace{\angle\theta - \varphi}_{\theta_z}$$

* هي زاوية الـ Impedance وتحدد نوعه بالدارة.

$$0 \leq PF = \cos(\theta - \varphi) \leq 1$$

- $\cos(\pm 90) = 0$
  - $\cos(90)$ → pure Inductive
  - $\cos(-90)$ → pure capacitive
- → pure Resistive
  - $\cos(0) = 1$
  - $\theta = \varphi$ → In phase.

- $(\theta - \varphi)$ → ⊕ Inductive → lagging PF
- $(\theta - \varphi)$ → ⊖ capacitive → leading PF

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* **for pure R**

$\cos(\theta - \Phi) = 1 = pF$ $\quad V = iR$

$P_{avg} = \frac{V_m I_m}{2} = P_{app}$

$\hookleftarrow \frac{V_m^2}{2R} = \frac{I_m^2 * R}{2}$ → Peak value

$= V_{rms} I_{rms} = P_{app}$

$\hookleftarrow \frac{V_{rms}^2}{R} = I_{rms}^2 * R$ → RMS

* **for Pure Reactance** ← pure C, pure L

$(\theta - \Phi) = \pm 90$ (L, C)

* P_{avg} (capacitor, inductor) = zero w ← دايمًا

$PF = 0$

$P_{app} = V_{rms} I_{rms} = \frac{V_m I_m}{2}$

* (L, C) ← يخزنوا الطاقة (ما بستهلكوها).
* (R) ← بتضيع الطاقة (تعمل على تحويل الطاقة أو اضاعتها على شكل حرارة).

* Ex: if $Z = 5 + j3$, $i = 6\angle 30°$ A, Find average power.

$V = Z * i = 34.99\angle 60.97°$

$P_{avg} = \frac{VI}{2} \cos(\theta - \Phi)$

$\frac{34.99 * 6}{2} \cos(60.97 - 30) = 90.005$ w

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Ex: $V(t) = \overset{V_m}{4} \cos \frac{\pi}{6} t$ Find P_t , P_{avg}

$Z = 2\angle 60 \rightarrow 1 + j1.73$

$I = \frac{V}{Z} = 2\angle -60 \ A \rightarrow i(t) = 2\cos(\frac{\pi}{6}t - 60)$ (I_m)

$P_t = v_t \cdot i_t = 4\cos\frac{\pi}{6}t \cdot 2\cos(\frac{\pi}{6}t - 60)$

متطابقة → $\cos A \cos B = \frac{1}{2}[\cos(A+B) + \cos(A-B)]$

$\Rightarrow P_t = 4\cos(\frac{2\pi}{6}t - 60) + 4\cos(60)$

$4\cos(60)$ ↓ P_{avg} (because it's constant)

$P_{avg} = 4\cos(60) = 2\ w$

OR $P_{avg} = \frac{V_m I_m}{2}\cos(\theta - \phi)$ نفس

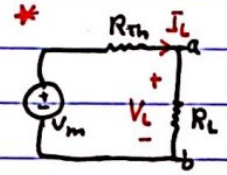
$= \frac{4*2}{2}\cos(0 - -60)$

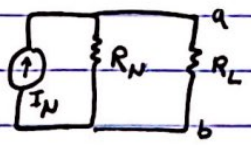
$= 4\cos(60) = 2\ w$

OR $P_{avg} = \frac{I_m^2 R}{2} = \frac{(2)^2 * 1}{2} = 2\ w$

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<< Thevenin and Norton >>



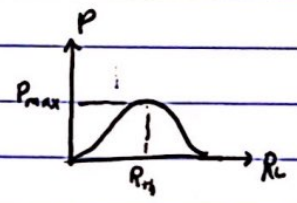


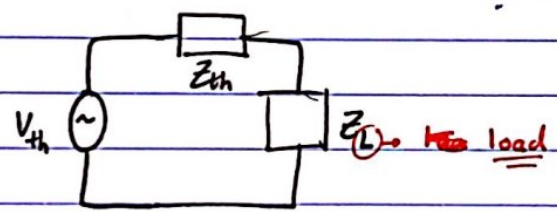
* $I_L \neq I_N$
لأنو لازم يكون في R_L بس

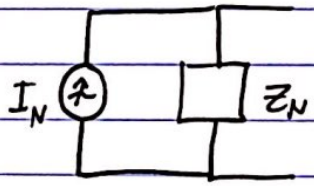
- $V_{Th} = V(o.c)$
- $I_N = I(s.c)$
- $V_{Th} = I_N R_{Th}$
- $I_N = \frac{V_{th}}{R_{th}}$
- $R_L = R_{th}$
- to get max power:

$$\rightarrow P_{max} = \frac{V_{th}^2}{4 R_{th}}$$

نحصل على أعلى طاقة عندما تكون:
$(R_{th} = R_L)$







* to get maximum power :-

$$Z_L = Z_{th}^{*}$$ → * conjegate

$Z = x + jw$
$Z^* = x - jw$

$$\Rightarrow P_{max} = \frac{|V_{th}|^2}{8 R_{th}}$$ → for peak

for the load (Z_L)

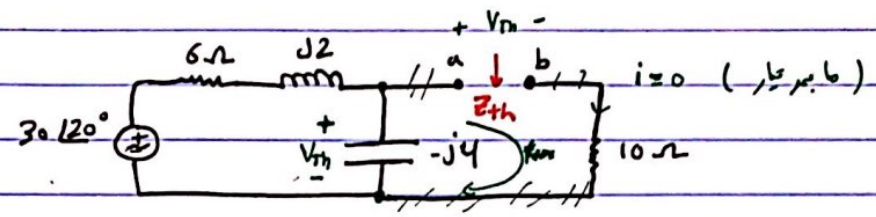
$$P_{max} = \frac{|V_{th}|_{rms}^2}{4 R_{th}}$$ → for rms (DC)

$$P_{max} = \frac{|I|^2 R_L}{2}$$

$$R_L = \sqrt{R_{th}^2 + X_{th}^2} = |Z_{th}|$$

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Ex: Find (Z_L) to get max power , Find V_{th}

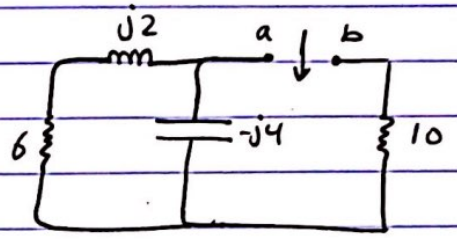


Sol: $Z_L = Z_{th}^*$

① → kill all the sources

V.s → s.c

C.s → o.c



$Z_{eq} \rightarrow ((6+j2) // -j4) + 10$

R_{Th} , X_{Th}

$Z_{Th} = Z_{eq} = 12.4 - j3.2\ \Omega = 12.8\angle -14.47$

$|Z_{Th}| = R_L$

$Z_L = Z_{eq}^* = 12.4 + j3.2\ \Omega$

$= 12.8\angle +14.47\ \Omega$

لو طالب بالسؤال Z_L بدل R_L ↓ load

$R_L \rightarrow a, b$

$R_L = |Z_{th}| = 12.8\ \Omega$

↳ load Resistor

using Voltage division:

$$V_{Th} = \frac{-j4}{-j4 + 6 + j2} \cdot 30\angle 20$$

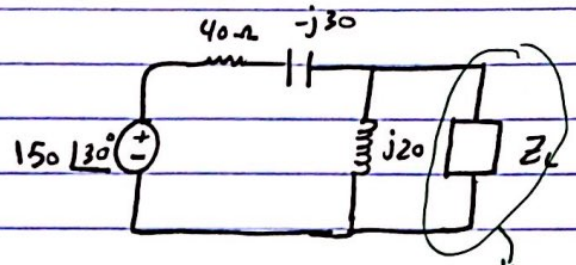
$$V_{Th} = 19\angle -51.57\ V$$

$$P_{max} = \frac{|V_{Th}|^2}{8 R_{Th}} = \frac{(19)^2}{8 * 12.4} = 3.63\ W$$

* دايمًا حط القوانين وبين شو بتعوض عشان العلامات.

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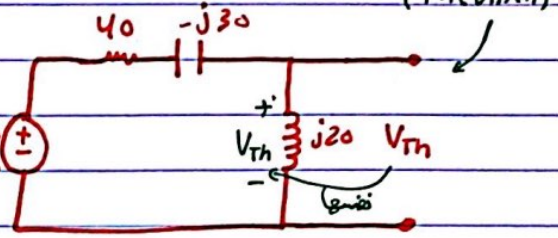
Ex: Find ① Z_L, ② power max for Z_L, ③ R_L, ④ P_{max} for R_L



Using Voltage division

$$V_{Th} = 150\angle 30 * \frac{j20}{j20+40-j30}$$

$$V_{Th} = \frac{72.76\angle 134^\circ}{|V_{th}|}$$



* to calculate the Z_{th} we kill the voltage source → ⊕ S.C

$$Z_{th} = (40-j30) // j20 = \underset{R_{th}}{9.412} + j\,22.35\ \Omega$$

① $Z_L = Z_{th}^* = 9.412 - j22.35\ \Omega$

② $P_{max_{Z_L}} = \frac{|V_{th}|^2}{8R_{th}} = \frac{(72.76)^2}{8(9.412)} = 70.3\ w$

③ $R_L = |Z_{th}| = 24.25\ \Omega$

$Z_{th} = \underset{|Z_{th}|}{24.25\angle 67^\circ}\ \Omega$

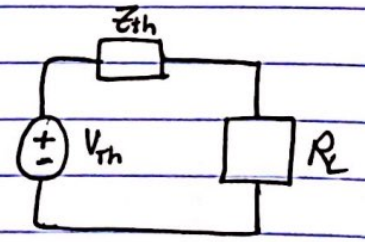
④ $P_{max_{R_L}} = \frac{1}{2}|I|^2 R_L$

$$I = \frac{V_{th}'}{Z_{th}' + R_L'} = 1.8\angle 100.5^\circ$$

هاي إلي بدي اياها لأنو القانون بدو |I|

$$= \frac{1}{2}(1.8)^2(24.25)$$

$$= 39.285\ w$$



* شكل الدارة النهائي بعد thevinen هيك بطلع منها التيار.

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CH 11.5 << complex power >>

① complex power وحدتها (VA)

$$S = V_{rms} I_{rms} \cos(\theta - \varphi) + j V_{rms} I_{rms} \sin(\theta - \varphi)$$

$V_{rms} I_{rms} \cos(\theta - \varphi)$ → P_{avg} (Real).

$V_{rms} I_{rms} \sin(\theta - \varphi)$ → Q (Imaginary).

- $P_{avg} = Re\{S\}$
- $Q = Im\{S\}$

② $S = P_{avg} + jQ$ → $Q \Rightarrow$ Reactive power, $P_{avg} \Rightarrow$ active power

③ $S = V_{rms} I_{rms} \angle \theta - \varphi$ $(S = P_{app} \angle \theta - \varphi)$

④ $S = V_{rms} I_{rms}^{*}$ ← conjugate.

(عنا ④ قوانين للـ complex power بنحل حسب المعطيات.)

$P_{app} = S = |S| = V_{rms} I_{rms}$ بدون زوايا.

Ex: $V = 5 \angle 20\ V_{rms}$, $I = 2 \angle -50\ I_{rms}$ } find the complex power?

محددين بالسؤال انو RMS Values.

I_{rms}^{*} هون قلبت الإشارة لأنو

$$S = 5 \angle 20 * 2 \angle 50$$

$$= 10 \angle 70^{\circ}\ VA$$

Volts Ampere

- $S \rightarrow VA$
- $S = |S| = P_{app} \rightarrow VA$
- $P_{avg} \rightarrow W$
- $Q \rightarrow VAR$

* اختلاف الوحدات للتمييز.

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Reactive Power

Q ← طاقة بترجع للـ source (معلومة للفهم)

Reactive power: $Q = V_{rms} I_{rms} \sin(\theta - \varphi)$ → VAR وحدتها x Volts Amps Reactive

لو كانت* → $Q = \frac{V_m I_m}{2} \sin(\theta - \varphi)$ (rms value)

→ Q for pure R:-

$\theta = \varphi \rightarrow \theta - \varphi = 0$

$\sin(0) = 0$

$S = P_{avg} + jQ = P_{avg} = V_{rms} I_{rms} = P_{app}$

→ Q for pure C, L:-

$\theta - \varphi = \pm 90$

$\sin(\theta - \varphi) = \sin(\pm 90) = \pm 1$ (+1 pure L, −1 pure C)

$Q_L = V_{rms} I_{rms} \sin(90) = V_{rms} I_{rms}$

$Q_C = V_{rms} I_{rms} \sin(-90) = -V_{rms} I_{rms}$

$S_L = 0 + jV_{rms} I_{rms}$

$S_C = 0 - jV_{rms} I_{rms}$

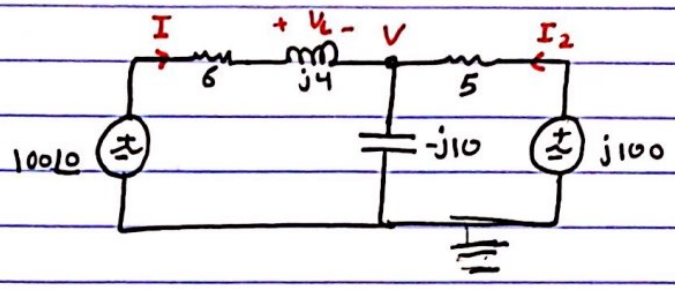
$S = V_{rms} I_{rms}^* = V_{rms} \left(\frac{V_{rms}}{Z}\right)^* = \frac{|V|^2}{Z^*}$

$I = \frac{V}{Z}$

$= |I|^2 Z$

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* Example : Find complex power for all elements :-



→ Using nodal analysis :

~~KCL~~ KCL @ nodeA:

$$\frac{V - 100\angle 0}{6 + j4} + \frac{V}{-j10} + \frac{V - j100}{5} = 53.4\angle 42.66 \quad V = ?$$

$$I_1 = \frac{100 - V}{6 + j4} = 9.8\angle -64.44 \ A$$

$$I_2 = \frac{j100 - V}{5} = 15\angle 121.6 \ A$$

$$\Rightarrow P_5 = \frac{|I_2|^2 R}{2} = \frac{(15)^2(5)}{2} = 562.5 \ w$$

$$S_5 = 562.5 + j0 \ VA$$

التيار المار بالـ Capacitor ← $I_c = I_1 + I_2 = 5.35\angle 132.7 \ A$

$$S_{-j10} = \frac{V_m I_m^*}{2} = \frac{53.4\angle 42.66 * 5.35\angle 132.7}{2}$$

$$= 0 - j142.6 \ VA$$

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$$S_6 = \frac{1}{2}|I_1|^2 R = \frac{1}{2}(9.8)^2(6) = 288.5 \text{ VA}$$

$$S_{j4} = \frac{VI^*}{2} = j192.3 \text{ VA}$$

$$S_{100\angle 0} = \frac{VI^*}{2} = \frac{(100\angle 0)(9.8\angle 64.44)}{2}$$

$$S_{100} = 211.5 + j442.3 \text{ VA} \quad \text{generated (source)}$$

$$S_{j100} = \frac{(100\angle 90)(15\angle -121.6)}{2}$$

$$S_{j100} = 638.4 - j392.3 \text{ VA} \quad \text{generated}$$

$$\Sigma S_{generated} = \Sigma S_{disipated}$$

$$\Sigma P_{avg_g} = \Sigma P_{avg_d}$$

$$\Sigma Q_g = \Sigma Q_d$$

* We can't sum up P_{app}

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Example:- A series RLC cct with a source value of $V(t) = 100 \sin(20t - 120°)$ V and $i(t) = 5 \sin(20t - 40°)$ A, if $X_L = 1.3$

Find:

1. The value of the Resistance
2. The value of the capacitive Reactance. (X_C)

$V = 100 \angle -120°$

$I = 5 \angle -40°$

$$Z_{eq} = \frac{V}{I} = 20 \angle -80°$$

$$20 \angle -80° = R + j(X_L - X_C)$$

$$\downarrow$$

$$3.473 - j19.7 = R + j(1.3 - X_C)$$

① ... $\boxed{R = 3.473\ \Omega}$

$$-19.7 = 1.3 - X_C$$

② ... $\boxed{X_C = 21}$

طالب بالسؤال Capacitive Reactance, ممكن يطلب الـ Capacitance وقتها بنستخدم $X_C = \frac{1}{\omega C}$

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Q1 Sinusoidal source supplies 10 K VAR power to $Z = 250 \angle 75^\circ\ \Omega$.

A. The complex power is ....

B. The peak Voltage is .... → V_m بدي

$$Q = V_{rms} I_{rms} \sin(\theta - \phi) \quad (\theta = \theta_z)$$

$$10k = V_{rms} I_{rms} \sin(75)$$

$$V_{rms} I_{rms} = 10.35\ K = P_{app}$$

$$P_{avg} = P_{app} * P_f \quad (P_f = \cos\theta_z)$$

$$= 10.35\ K * \cos(75) = 2678.7\ w$$

A: complex power $S = 2678.7 + j10K\ VA$

$$Z = 64.7 + j241.48 \quad (R)$$

$$P_{avg} = I^2 R \rightarrow I = 9.1$$

Peak Value طلب جزء (2)

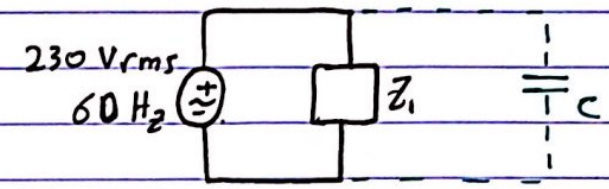
$$S = |S| = \frac{V_m I_m}{2}$$

$$10352.56 = \frac{V_m (9.1)}{2} \rightarrow V_m = 2.275\ KV$$

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Ex: Load Z_1 operating at 50 kw, 0.8 lagging pf (Pavg, L → +) find the value of parallel capacitor to raise pf to 0.95.

بضيف على التوازي capacitor زي ما هو طالب بالسؤال.



source ← $S_s = S_z + S_c$

$\Rightarrow S = \frac{VI}{P_{avg}/PF} \angle \cos^{-1} PF$ ← $\theta - \Phi$ (لو كانت leading بحط سالب)

(P_{avg}/PF ← يعني P_{app} هاي)

PF for the system after we add the parallel capacitor (0.95)

$$\frac{50k}{0.95} \angle \cos^{-1} 0.95 = \frac{50k}{0.8} \angle \cos^{-1} 0.8 + S_c$$

* The power average for the impedence and the source is the same because we use $EP_g = EP_d$

$$50 + j16.43k = 50 + j37.5k + S_c$$
$$S_c = j16.43k - j37.5k = -j21.07 \text{ KVA}$$

* we calculated the complex power for (C) so we can calculate the current that passes through it then we can caculate the value of C by using $Z_c = \frac{V}{I}$.

$$S_c = VI_c^* = -j21.07k$$
$$I_c^* = \frac{-j21.07k}{230} = -j91.6A = 91.6\angle -90$$
$$I_c = 91.6\angle 90^\circ \text{ A}$$

$$Z_c = \frac{V_c}{I_c} = \frac{230}{91.6\angle 90} = \frac{1}{j\omega C} = -j\omega c$$

$$X_c = 2.51 = \frac{1}{\omega c} \Rightarrow C = 1056 \text{ MF}$$

* if we want to raise PF we usually add a parallel capacitor ⊣⊢ to the circuit.

* if we want to decrease the PF we either add a parallel inductor ⏦ or a series capacitor. ⊣⊢

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Resonant frequency (ω_o):

$X_{eq} = X_L - X_C$

$X_L = X_C \Rightarrow X_{eq} = 0$

* The resonant frequency cancel the imaginary part (C and L) (X_{eq}).

* PF = 1 ⟶ pure resistance.

$Z_{eq} = R_{eq} + j X_{eq}$ ($X_{eq} = 0$)

$Z_{eq} = R_{eq}$, $X_{eq} = 0$

لح بس ثوابت لأنه C و L

رح يلغو بعض فالمحصلة تكون Real

* $f = f_o$

* $\omega = \omega_o$

Ex: for R, L, C → (series).

OR

Find the value of resonant frequency ?

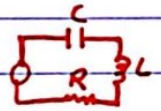
Find the value of (ω) to provide unity PF ?

هيك الأسئلة على هاد الموضوع بيجو.

@ $f = f_o$ → ① Z_{eq} = Real

② $X = 0$

③ PF = 1



$$Z_{eq} = R + j\omega L + \frac{1}{j\omega C}$$

(R: Real, $j\omega L - \frac{1}{j\omega C}$: Imaginary)

$$= R + j\left[\omega L - \frac{1}{\omega C}\right]$$

⇒ to provid PF=1

Imaginary = Zero

$\omega = \omega_o$

$\omega L - \frac{1}{\omega C} = 0$

* $\theta - \phi = \tan^{-1}\frac{\omega L - \frac{1}{\omega C}}{R}$

$PF = \cos(\theta - \phi) = \ldots$

حسب المطلوب بتطلع موضوع للقانون بدون (ω) ⟶

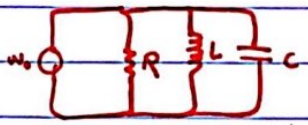
$$\omega_o = \frac{1}{\sqrt{LC}}$$

→ Resonant frequency

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Ex: for R, L, C (Parallel).

* عشان احسبهم بقلبهم بخليهم (Y) (admittance) نجمعهم وبقلب الناتج



$$Z_{eq} = R \,//\, jwL \,//\, \frac{1}{jwC}$$

$$Z_{eq} = (Real) + j(Imaginary).$$

$PF = 1$

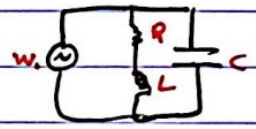
$Imaginary = Zero$

$$w_o = \frac{1}{\sqrt{LC}}$$

نفس إلاشي

Ex: Find Resonant frequency ?

$$Z_{eq} = (R + jwL) \,//\, \frac{1}{jwC}$$



$PF = 1$

$IMaginary = Zero$

$$Y_{eq} = \frac{1}{1/jwC} + \frac{1}{jwL + R}$$

لما عملت هيك صارت اسهل وانو بدل ما يكون عندي parallel ولازم اطلع المقلوب على المجموع هيك بقدر اجمع عادي طول.

$$Y_{eq} = \frac{R}{R^2 + w^2L^2} + j\left(wC - \frac{wL}{R^2 + w^2L^2}\right)$$

$\rightarrow = 0$

$w \rightarrow w_o \qquad Imaginary \rightarrow Zero$

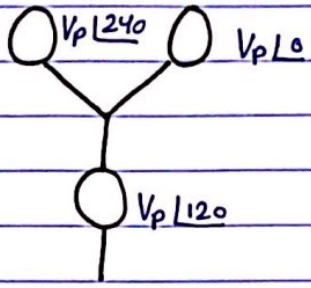
$$\# \quad w_o = \sqrt{\frac{L - R^2C}{L^2C}}$$

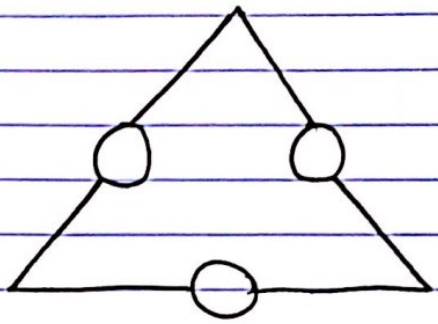
$$Y_{eq} = \frac{R}{R^2 + w^2L^2}$$

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CH:12 << Poly phase >>

3-phase:

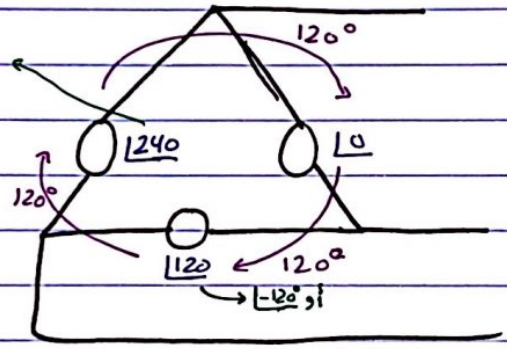




لو كانو Z بدل O بكون امبيدنس يعني Y-load. or Δ-load.

Sources → Y

Sources → Δ (Delta)

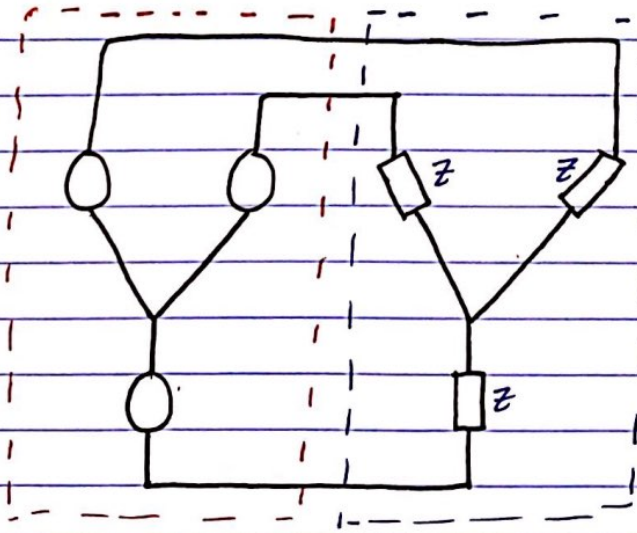


أو ممكن تكون ∠-240°

← Δ source

* دايمًا 120° بيناتهم يعني مش شرط نشوف 0° و 120° ممكن 50° و 170° +120°

(ماخذين دورة كاملة 360°).



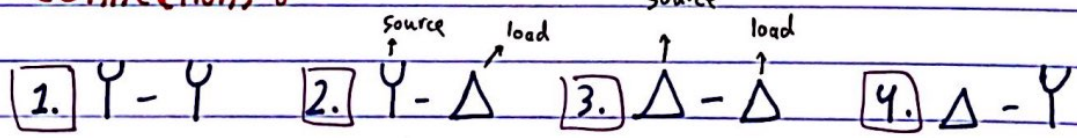
* اذا كانو Z متساويين بكون Balanced system.

source part.

Y - Y connection.

load part.

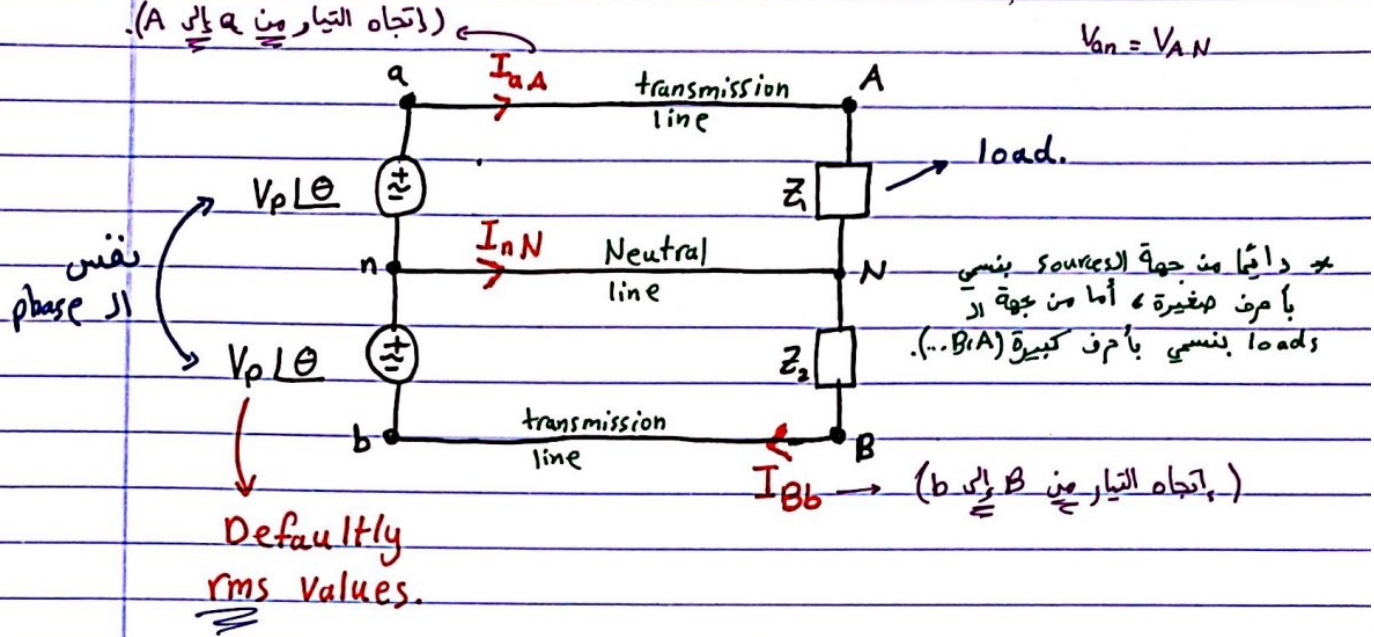
Connections :-



43

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Single phase - 3 wires system.



$V_{an} = V_{AN}$

* دائمًا من جهة الـ sources بنسمي بأحرف صغيرة، أما من جهة الـ loads بنسمي بأحرف كبيرة (A,B...).

طبعًا هون الترتيب مهم كتير

$V_{an} \neq V_{na}$

$V_{an} = -V_{na}$

$V_{an} = V_{AN} = V_p \angle\theta$

$V_{bn} = -V_{nb} = -V_p \angle\theta$

$V_{nb} = V_{NB} = V_p \angle\theta$

$V_{ab} = V_{AB} = V_{an} + V_{nb} = 2V_p \angle\theta$

$I_{aA} = \frac{V_{an}}{Z_1}$

$I_{Bb} = \frac{V_{nb}}{Z_2}$

$I_{nN} = \longrightarrow I_{Bb} - I_{aA}$

Balanced system.

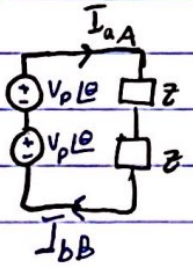
→ we call the system balanced when all the impedances are equal.

in this ex. → $(Z_1 = Z_2)$.

→ in a balanced system the $I_{nN} = \text{zero}$. (Always).

* in this example → $I_{Bb} = I_{aA}$

Balanced → (we remove the neutral line).



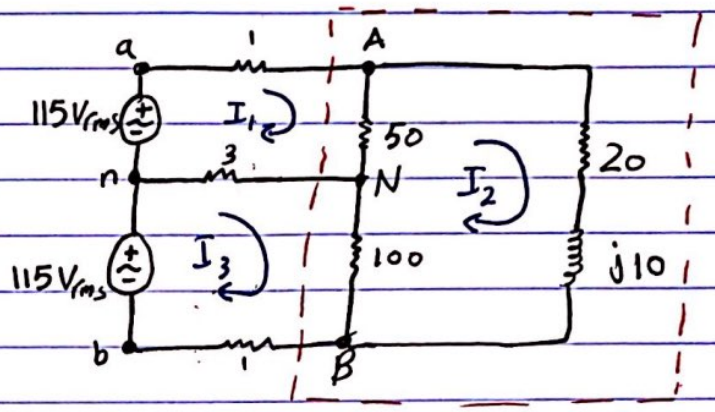
* التيار الجديد إلي بمر بالبارة كاملة.

$I = \frac{2V_p \angle\theta}{2Z} = \frac{V_p \angle\theta}{Z}$

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Example: Find total power delivered to loads power loss, input power (generating) and efficiency.

$I_{nN} = I_3 - I_1$



① power Delivered to the load:

Using mesh analysis:

↳ loads.

$$1I_1 + 50(I_1 - I_2) + 3(I_1 - I_3) = 115\angle 0 \quad \ldots(1)$$
$$50(I_2 - I_1) + (20 + j10)I_2 + 100(I_2 - I_3) = 0 \quad \ldots(2)$$
$$3(I_3 - I_1) + 100(I_3 - I_2) + 1I_3 = 115\angle 0 \quad \ldots(3)$$

* طبعًا بالامتحان ما بيجي 3 معادلات و 3 مجاهيل لأنو حلهم بدهم وقت طويل جدًا

$$\Rightarrow I_1 = 11.24\angle -19.83 \text{ A}_{rms}$$
$$I_2 = 9.39\angle -24.47 \text{ A}_{rms}$$
$$I_3 = 10.37\angle -21.8 \text{ A}_{rms}$$

$P_{j10} = 0$

انتبه إنو هون بنطرح phasor يعني $A\angle\theta - B\angle\phi$ مش A - B لحال والجواب باخز ال magnitude إلو.

$P_{50} = |I_1 - I_2|^2 R$

$P_{50} = 206\,w$

$P_{100} = |I_3 - I_2|^2 R$

$= 117\,w$

$P_{20} = |I_2|^2 (20)$

$= 1763\,w$

② power loss

$P_1 = |I_1|^2 (1)$

$= 126\,w$

$P_3 = |I_{nN}|^2 (3)$

$= 3w$

$P_1 = |I_3|^2 (1)$

$= 108\,w$

$P_{loss} = 237w$

* المقاومات على الـ Transmission and neutral lines بسميها wire Resistances

$P_{load} = P_{output}$

$P_o = 2086\,w$

Scanned with CamScanner

③ Input power

$P_{in} = P_o + P_{loss}$

$= 2086 + 237$

$P_{in} = 2323w$

* الطريقة الثانية احسن
و أضمن لأنو ما بعتمد
على إجابات الأسئلة اللي قبلو.

OR: $P_{in} = P_{s1} + P_{s2} = \underline{2323w}$

$P_{s1} = (115)(11.24)\cos(0 - 19.83) = 1216w$

$P_{s2} = (115)(10.37)\cos(0 - -21.8) = 1107w$

④ efficiency (η)

$$\eta = \frac{P_o}{P_{in}} \times 100\%$$

$$= \frac{P_{load}}{P_{load} + P_{loss}} \times 100\%$$

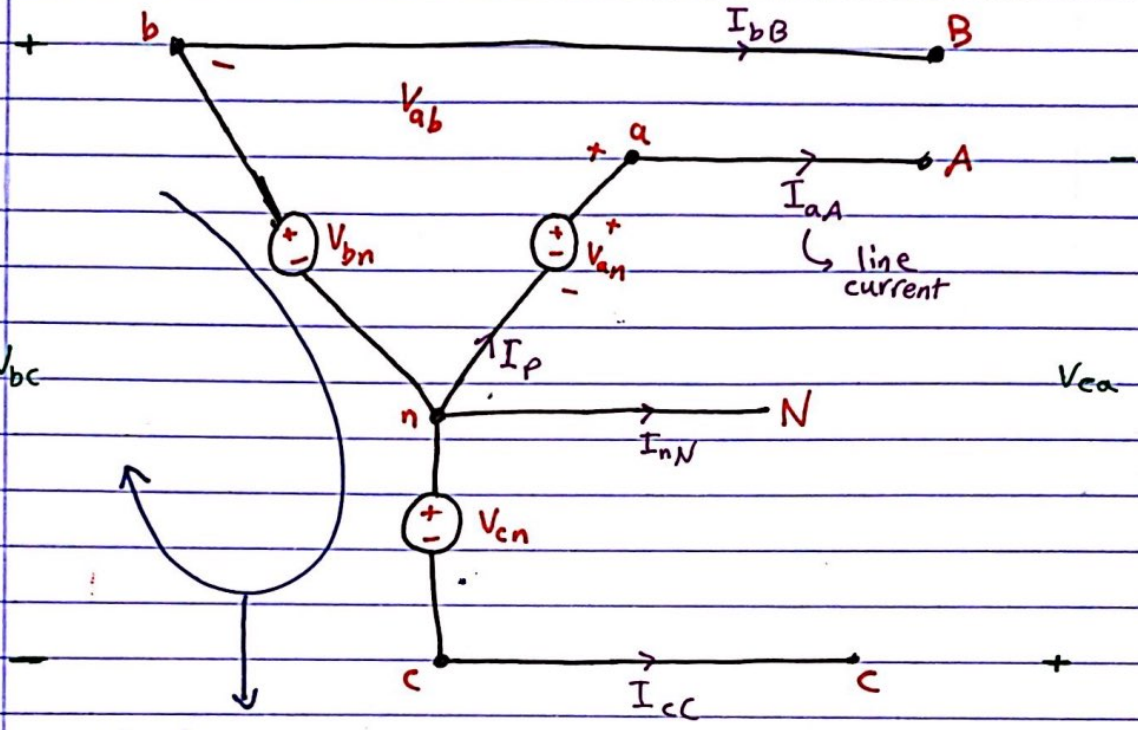
$$\eta = \frac{2086}{2086 + 237} * 100\%$$

$= 89.8\%$

* بعرف إنو حلي صح بالمنطق
لأنو الكفاءة لازم تكون دايمًا عالية
أو أكبر من 50% أو 60%
⇐ يعني مش منطق أصمم سيركيت
بتخليني أحصر طاقة كبيرة
وكفاءتها واطية.

Scanned with CamScanner

3 phase y-connection
(3 Ø wye connection). ← عالأغلب هيك بيجي بالإمتحان



V_{bc} عشان الـ loop

phase Voltages :- Voltage between line and neutral
↳ V_{an} , V_{bn} , V_{cn}

⊕ sequence
(abc sequence)

$V_{an} = V_p \angle 0$
-120°
$V_{bn} = V_p \angle -120$
-120°
$V_{cn} = V_p \angle -240$

⊖ sequence
(cba sequence)

$V_{an} = V_p \angle 0$
$+120^\circ$
$V_{bn} = V_p \angle 120$
$+120^\circ$
$V_{cn} = V_p \angle 240$

$|V_{an}| = |V_{bn}| = |V_{cn}| = V_p$ ← Balanced إذا كان

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line Voltages :- Voltage between two lines

L، V_{ab} , V_{bc} , V_{ca}

⇒ for ⊕ sequence

① $V_{ab} = V_{an} - V_{bn}$

$= V_p \angle 0 - V_p \angle -120$

فكيتوم ٤٥ بعض من خلال إني حولتوم على صيغة Acos θ ±isin θ وجمعتوم مع بعض.

$V_{ab} = \sqrt{3}\, V_p \angle 30$

-120° ② $V_{bc} = V_{bn} - V_{cn}$

$= V_p \angle -120 - V_p \angle -240$

$V_{bc} = \sqrt{3}\, V_p \angle -90$ → OR $\angle 270$ $(+360°)$

③ $V_{ca} = V_{cn} - V_{an}$

$= V_p \angle -240 - V_p \angle 0$

$V_{ca} = \sqrt{3}\, V_p \angle -210$ → OR $\angle 150$

$|V_{ab}| = |V_{bc}| = |V_{ca}| = \sqrt{3}\, V_p = V_L$ ← line

$V_L = \sqrt{3}\, V_p$

$V_L > V_p$

↑ أكبر

* line Voltage lead phase Voltage by 30°.

↓

for ⊕ sequence. ← $\theta_p + 30 = \theta_L$

Scanned with CamScanner

⇒ for ⊖ sequence

$V_{an} = V_p \angle 0$) 120°
$V_{bn} = V_p \angle 120$) 120°
$V_{cn} = V_p \angle 240$

phase Voltages

$V_{ab} = \sqrt{3}\ V_p \angle -30$) 120
$V_{bc} = \sqrt{3}\ V_p \angle 90$) 120
$V_{ca} = \sqrt{3}\ V_p \angle 210$

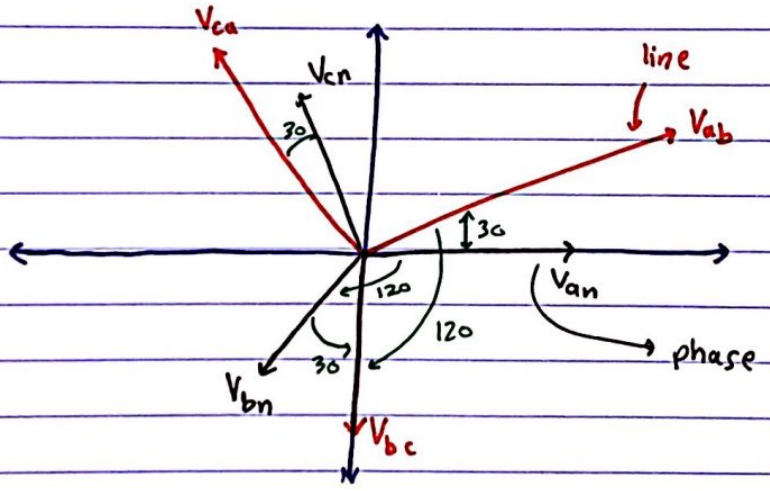
line Voltages

-30

$\theta_p - 30 = \theta_L$ ★ line lag phase by 30°

⊖ sequence

★ $I_L = I_p$ in Y- connection



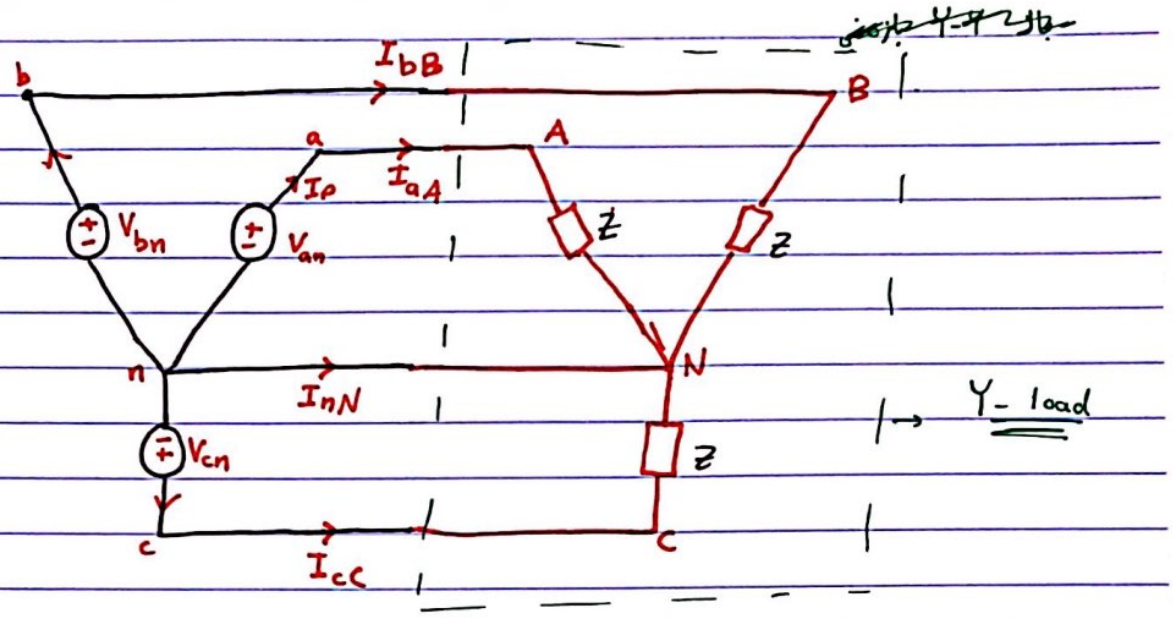
phasor Diagram
for ⊕ sequence

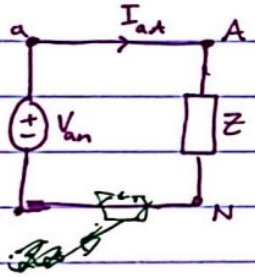
⊖ sequence will have a different phasor diagram.

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Y - Y

ex





باد Y-Y بلزمني neutral
أما باد Δ ما بلزمني line

positive sequence

$$I_{aA} = \frac{V_p \angle 0}{Z}$$

-120

$$I_{bB} = \frac{V_p \angle -120}{Z}$$

-120

$$I_{cC} = \frac{V_p \angle -240}{Z}$$

$$I_{nN} = I_{aA} + I_{bB} + I_{cC}$$

↳ for balance

$$I_{nN} = 0$$

P_{loss} for neutral = 0

بطلع تيار واحد وبتعمل
(-120) بمقدار phase shift
(+) sequence واذا كان

Scanned with CamScanner

ex: find phase, line Voltages and currents, and calculate total power dissipated by the loads for a balanced 3Φ (Y-Y) system if

$V_{an} = 200 \angle 0^\circ \; V_{rms}$

$Z_L = 100 \angle 60^\circ \; \Omega$

$\omega = 120\pi$ ⊕ sequence

Sol:

* $V_p = |V_{an}| = 200$

phase Voltages

$V_{an} = 200 \angle 0^\circ \; V_{rms}$

$V_{bn} = 200 \angle -120^\circ \; V_{rms}$ (-120°)

$V_{cn} = 200 \angle -240^\circ \; V_{rms}$ (-120°)

* $V_L = \sqrt{3} V_p = \sqrt{3} * 200$ (+30°)

* line lead phase by 30 for ⊕ sequence.

line Voltages

$V_{ab} = 200\sqrt{3} \angle 30^\circ$

$V_{bc} = 200\sqrt{3} \angle -90^\circ$ (-120°)

$V_{ca} = 200\sqrt{3} \angle -210^\circ$ (-120°)

* because it's a y-y connection

$I_L = I_p$

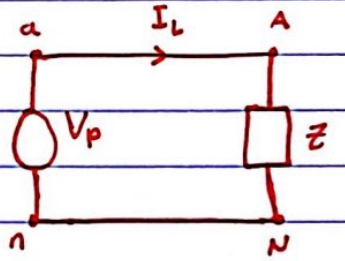
$I_{aA} = \frac{V_{an}}{Z} = \frac{200\angle 0}{100\angle 60}$

$I_{aA} = 2 \angle -60^\circ$ (-120°)

$I_{bB} = 2 \angle -180^\circ$ (-120°)

$I_{cC} = 2 \angle -300^\circ$

* $I_{nN} = 0$ (balanced system.)



$P_{perphas} = |V||I| \, PF$

$= (200)(2)\cos(60)$

$P_{phase} = 200 \; w$

total power بس هو المطلوب من per phase

$P_{total} = 3 P_{phase} = \underline{600 \; w.}$

$P_{phase} = |V_{an}||I_{aA}| \, PF$

$P_{phase} = V_p I_p \, PF$

$P_{total} = 3 P_{phase} = 3 V_p I_p PF$

$P_{phase} = \frac{V_L}{\sqrt{3}} I_L \, PF$

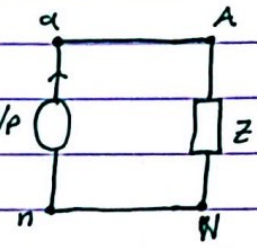
$P_{total} = \frac{3}{\sqrt{3}} V_L I_L \, PF$

$= \sqrt{3} \, V_L I_L \, P_F$

Scanned with CamScanner

Example: Balanced 3Φ Y-Y, $V_L = 300\ V$ supplying total load with 1200 w at 0.8 leading PF find I_L, per phase Impedance.

I_{aA} ⊕ sequence.



$I_p = I_L$

$V_L = \sqrt{3}\, V_p \angle 30$ ⊕ seq

⇒ TOTAL Pavg is given

① $P_{phase} = \frac{P_t}{3} = \frac{1200}{3} = 400\ w$

$V_p = \frac{V_L}{\sqrt{3}} = \frac{300}{\sqrt{3}}$

$P_{phase} = 400 = \frac{300}{\sqrt{3}} * I_p * (0.8)$ *

magnitude هاي ال فلازم اطلع الزاوية

$\Rightarrow I_p = I_L = 2.9A$

* $P_{phase} = V_p I_p PF$

OR: $P_{total} = 3 V_p I_p PF$

$1200 = 3 * \frac{300}{\sqrt{3}} * I_p * 0.8 \Rightarrow I_p = I_L = 2.9\ A$

* $PF = 0.8$

$0.8 = \cos(\theta - \phi)$ (phase Voltage.)

(من عندي) معهم احطها كانو بالسؤال حاكي leading

$-36.87 = \theta - \phi$

$V_L = 300\ V$ θ_L

$V_{ab} = 300 \angle 0\ V$

$V_{an} = \frac{300}{\sqrt{3}} \angle -30$ ϕ

$\theta_p + 30 = \theta_L$

$I_{aA} = 2.9 \angle 6.87$

$I_{bB} = 2.9 \angle ___$ -120

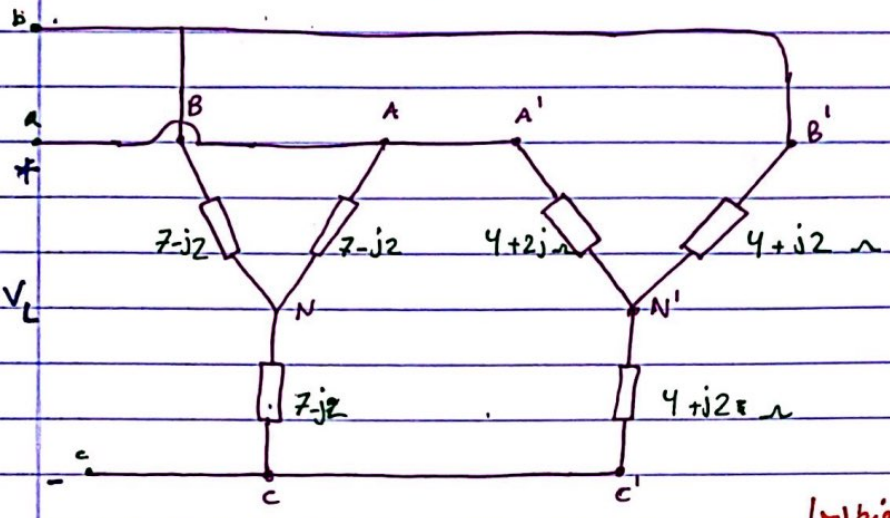
$I_{cC} = 2.9 \angle ___$ -120

② $Z = \frac{V_p}{I_p} \angle -\cos^{-1} 0.8 = \frac{300/\sqrt{3}}{2.9} \angle -\cos^{-1} 0.8$ (leading)

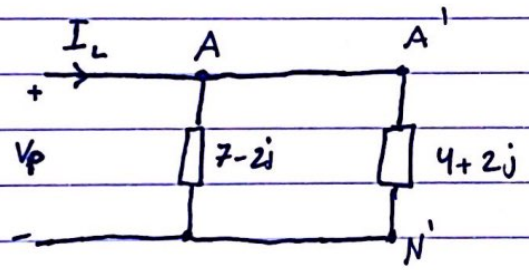
$= 60 \angle -36.87$

Scanned with CamScanner

ex: $V_L = 500\ V$, 2 balanced Y-Parallel connected loads to 3 Q system, one of the loads is capacitive $7 - j2\ \Omega$/phase, the other is inductive with $4 + j2\ \Omega$/phase per.
find V_p, I_L, total power, pf



* دايماً بلزمني V_p فبطلعها حتى لو مش طالبها بالسؤال.



(1) $V_p = \frac{V_L}{\sqrt{3}} = \frac{500}{\sqrt{3}}\ V \angle -30$

$\boxed{I_p = I_L}$

(2) $I_L = \frac{V_p}{Z_{eq}} = 97.5 \angle -40.65\ A$

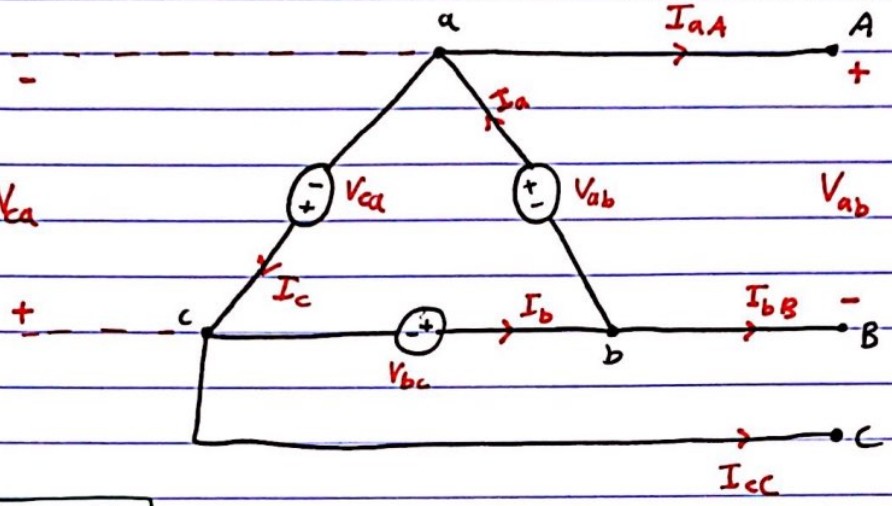
$Z_{eq} = (7 - j2) // (4 + j2)$
$= 2.9 \angle 10.62\ \Omega$

(3) $P_t = 3 P_{phase}$
$= 3 V_p I_p PF$
$= 3 \left(\frac{500}{\sqrt{3}}\right)(97.5)\cos(-30 + 40.65)$ (θ_p)
$= 83\ kw$

(4) $PF = \cos(10.65)$
$= 0.983$ lagging.

Scanned with CamScanner

Δ connection



$\boxed{V_p = V_L}$

I_a, I_b, I_c → phase currents

I_{aA}, I_{bB}, I_{cc} → line currents

for ⊕ sequence

line currents

$$I_{aA} = I_a - I_c$$

phase currents ←

$$I_a = I_p \angle 0 \quad (-120) \quad I_b = I_p \angle -120 \quad (-120) \quad I_c = I_p \angle -240$$

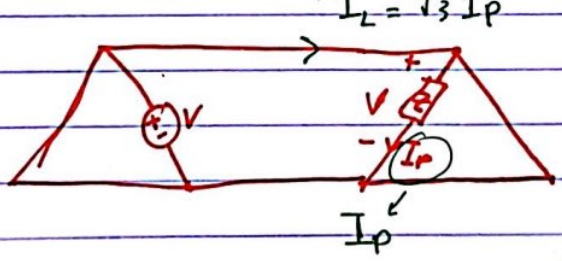
$$I_{aA} = \sqrt{3}\, I_p \angle -30^\circ \quad (-120) \quad I_{bB} = \sqrt{3}\, I_p \angle -150^\circ \quad (-120) \quad I_{cc} = \sqrt{3}\, I_p \angle -270^\circ$$

θ_p θ_L

$I_L = \sqrt{3}\, I_p$

I_L lag I_p by 30° for positive sequence.

$\theta_p - 30 = \theta_L$



Scanned with CamScanner

Example: find I_L in 3Φ system with $V_L = 300\,V$, total load power 1200 w to Δ connected load of 0.8 lagging PF. and find Z. ⊖ sequence.

$V_L = V_P$

$I_L = \sqrt{3}\, I_P \angle +30$ → -ve sequence.

$P_{total} = 3 * V_P * I_P * PF$ OR: $P_{phase} = \frac{P_t}{3} = 400\,w$

$1200 = 3 * 300 * I_P * 0.8$

$I_P = 1.67A$

$P_{phase} = V_P\, I_P\, PF$

$400 = 300\, I_P\, 0.8$

$I_P = 1.67\,A$

$\cos^{-1}(0.8) = \theta_z$

$\theta_z = \theta - \phi = 36.67°$

$\phi = -36.67°$

$I_a \leftarrow I_P = 1.67 \angle -36.67$ → phase current

$I_{aA} = 1.67 * \sqrt{3} \angle +36.67 - 30$ → line current

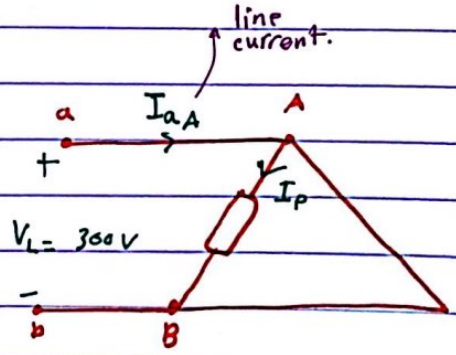
$= 2.9 \angle -66.87°$

$I_{bB} = 2.9 \angle$ (+120)

$I_{cC} = 2.9 \angle$ (+120)

$Z = \frac{V_P}{I_P} = \frac{300}{1.67} \angle +\cos^{-1} PF$ (lagging)

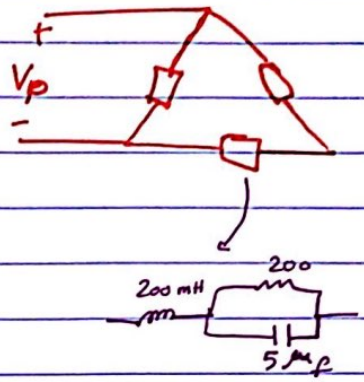
$Z = 180 \angle +36.9°\ \Omega$



$V_{ab} = V_{AB}$

Scanned with CamScanner

Example: each phase of Δ connected load consist of 200 mH (inductor) in series with parallel 5 μF (capacitor) and 200 Ω (Resistor)
$V_p = 200\ V$, $w = 400\ rad/s$
find I_p, I_L, Total power



Sol:

$V_p = V_L = 200\ V$

$Z_p = jwL + \left(\frac{1}{jwC} \parallel R\right)$

$L \rightarrow jwL = j * 400 * 200 * 10^{-3}$
$= j80$

$C \rightarrow \frac{1}{jwC} = \frac{-j}{400 * 10^{-6} * 5} = -j500$

$Z_p = 172.76 \angle 3.66^\circ\ \Omega$

$\Rightarrow I_p = \frac{V_p}{Z} = 1.158 \angle -3.66^\circ$

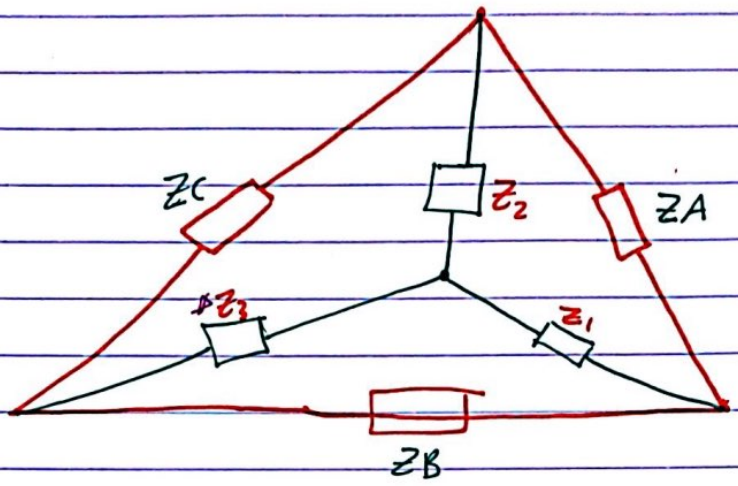
$I_L = \sqrt{3}\, I_p \angle -30$
$= \sqrt{3} * 1.158 * \angle -33.66^\circ$
$= 2 \angle -33.66^\circ\ A$

$\theta_p - 30 = \theta_L$
↳ positive sequence.

$P_t = 3 V_p I_p pf$
$= 3(200)(1.158)\cos(3.66)$
$= 693\ w$

Scanned with CamScanner

Y Δ conversion



Y → Δ

$$Z_A = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_3 \rightarrow \text{ZA ic}}$$

$$Z_B = \frac{Z_1Z_2 + Z_2Z_3 + Z_1Z_3}{Z_2}$$

$$Z_C = \frac{Z_1Z_2 + Z_1Z_3 + Z_2Z_3}{Z_1}$$

for Balanced systems

$Z_1 = Z_2 = Z_3$

$$Z_\Delta = 3Z_Y$$

Δ → Y

$$Z_1 = \frac{Z_AZ_B}{Z_A+Z_B+Z_C}$$

$$Z_2 = \frac{Z_AZ_C}{Z_A+Z_B+Z_C}$$

$$Z_3 = \frac{Z_BZ_C}{Z_A+Z_B+Z_C}$$

for Balanced systems

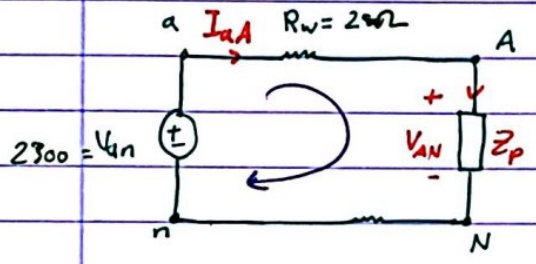
$Z_1 = Z_2 = Z_3$

$$Z_Y = \frac{Z_\Delta}{3}$$

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Example: Q16 if $V_{an} = 2300\angle 0$ V_{rms} in Balanced system (Y-Y) and $R_w = 2\Omega$, ⊕ sequence, total power supplied by the source $100 + j30$ KVA → complex power
find I_{aA}, V_{AN}, Z_p, η ↳ Real part: $P_{avg} = 100$

V_{AN}: Voltage of the load end



$$S_{source} = V_{an} * I_{aA}^{*}$$ (نسالك)

$$\frac{100 + j30\,K}{3} = 2300\angle 0^{\circ} * I_{aA}^{*}$$

① $I_{aA} = 15.13\angle -16.7$ A_{rms}

نتوقع مقاومة على السلك يعني الجهد بتوزع. $V_{an} \neq V_{AN}$

② $V_{an} = I_{aA} R_w + V_{AN}$
$= 2271\angle 0.219^{\circ}$ V

③ $Z_p = \frac{V_{AN}}{I_{aA}} \rightarrow Z_p = 143.3 + j44.3\,\Omega = 150\angle 17^{\circ}$
↓ R_{load}

④ $\eta = \frac{P_o}{P_{in}} \times 100\%$ (P_o: P_{avg} output)

$\eta = \frac{32.87\,K}{100/3\,K} \times 100\% = 98.6\%$

per phase ← $P_{in} = \frac{100}{3}\,K$ // $P_o = P_{in} - P_{loss}$

لأنو معطيني بالسؤال بس انا بحلي على per phase.

OR: $P_o = |I_{aA}|^2 (R_{load}) = (15.13)^2 (143.3) = 32.87\,Kw$

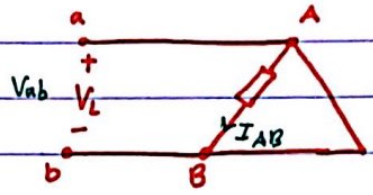
per phase ينوخذ الـ Real part من Z_p لأنو اصلًا الـ P_{avg} للـ capacitors & inductors = صفر.

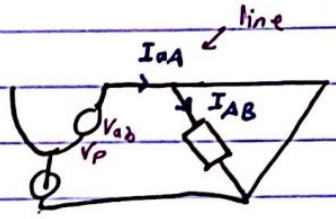
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Q Book 25: Balanced 3 wire, 3 Q Y-Δ, $R_w = 0$

$V_{an} = 200 \angle 60$ V, power per phase → 2 − j KVA

⊕ sequence, find V_{bc} ①, Z_p ②, I_{aA} ③





$V_{an} = 200 \angle 60$ → phase

$V_{ab} = 200\sqrt{3} \angle 60 + 30$ (+30)

$V_{bc} = 200\sqrt{3} \angle 90 - 120$ (−120) → line voltage

$V_{ca} = 200\sqrt{3} \angle ___$ (−120)

① $V_{bc} = 200\sqrt{3} \angle -30^\circ$

$\boxed{V_{AB} = V_{ab}}$ $\quad I_{AB} = \frac{V_{AB}}{Z}$

$S = V_{AB} I_{AB}^*$

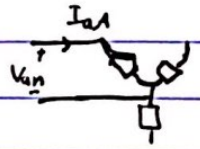
$2000 - j1000 = (200\sqrt{3} \angle 30) I_{AB}^*$

$I_{AB} = 6.467 \angle 116.565$ A → phase current

② $Z_p = \frac{V_{AB}}{I_{AB}} = 53.67 \angle -26.57 = 48 - j24$

$I_{aA} = \sqrt{3} I_{AB} \angle -30$

line → $I_{aA} = 11.2 \angle 86.57$



OR: Δ → Y

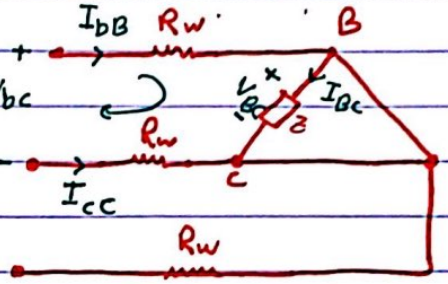
$Z_Y = \frac{Z_\Delta}{3} = \frac{53.67 \angle -26.57}{3}$

$I_{aA} = \frac{V_{an}}{Z_Y} = 11.2 \angle 86.57$

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مهم جداً.

Q 26: (book) Δ load require 15 KVA (Total P_{app}) at 0.8 lagging PF, ⊕ sequence $V_{BC} = 180\angle 30$ Vrms, $R_w = 0.75\,\Omega$, find V_{bc}, total complex power generated by the source.



① KVL: $V_{bc} = I_{bB}R_w + V_{BC} - I_{cC}R_w$ (OR: $I_{BC}\cdot Z$ بس انا مبي قيمة V_{BC} فبعوض زي ما هي.)

(I_{cC}: line current)

$$S_{load} = \frac{15K}{3}\angle +\cos^{-1}PF = V_{BC}\,I_{BC}^{*}$$

(P_{app} ↑, lagging)

OR: $P_{app} = |V_{BC}||I_{BC}|$

$V_{bc} \neq V_{BC}$ لأنو في عنا wire Resistance.

$I_{BC} = 27.78\angle -6.87$ A (phase current)

-30

line currents:
$I_{bB} = \sqrt{3} * 27.78\angle -36.87$ (line current)
$I_{cC} = \sqrt{3} * 27.78\angle -156.87$ -120

I_{bB} بتوزع → $I_{bB} \neq I_{BC}$

تابع

$$V_{bc} = (48\angle -36.87)(0.75) + 180\angle 30 - 48\angle -156.87\,(0.75)$$
$$= 233\angle 20.74\ V$$

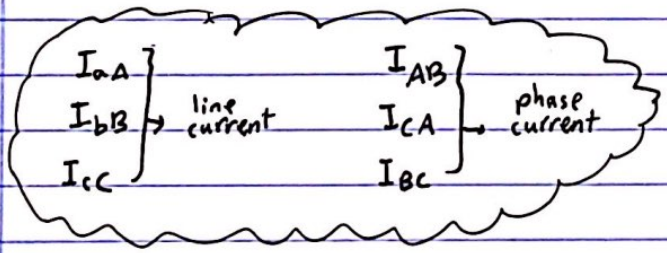
② $S_{source} = S_{load} + S_{line}$ (Total)

ضربنا بـ 3 لأنو بدنا كل الكوتال

$$= 15K\angle +\cos^{-1}PF + 3|I_L^2|R_w$$
$$= 15K\angle +\cos^{-1}PF + 3(48)^2(0.75)$$

ما قسمنا على 3 لأنو بدنا كل الكوتال.

* Pure R → $S = P_{avg} = P_{app} = V_{rms}I_{rms} = |I_{rms}^2|R$ يعني بنوخذ الـ magnitude

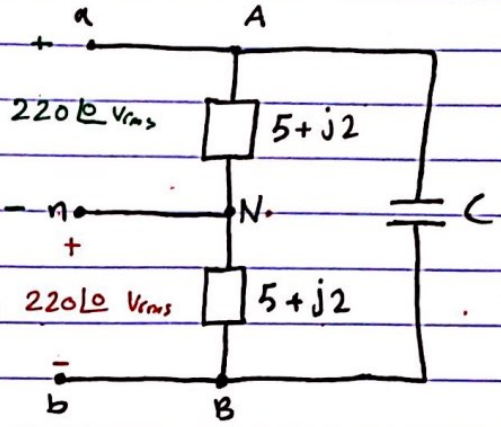


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Q book 13: Balanced 3-wire single phase, $V_{AN} = 220\ V_{rms}$ 60 Hz

1. Find the value of C providing unity PF
2. How many KVA should C handle? → means PF = 1

$V_{AN} = V_{an} = 220\ V_{rms}$ because we don't have wire Resistances.



$V_{an} = V_{nb}$ لأنو 3-wire single phase من خصائص ← $220\angle 0\ V_{rms}$

① $I_{nN} = Zero$ (Balanced system).

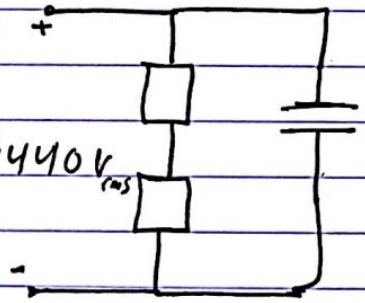
$$Z_{eq} = (10+j4) \,//\, Z_c$$
$$= (10+j4) \,//\, \frac{1}{j\omega C}$$

Admittance بشكل عشان اسهل الحل على و أحول // لـ (+)

$$Y_{eq} = j\omega C + \frac{1}{(10+j4)} * \left(\frac{10-j4}{10-j4}\right)$$

conjugate

عشان اختلص من ز الى بالمقام.



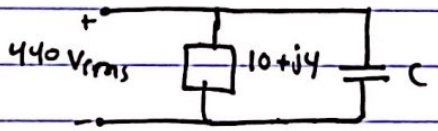
$$Y_{eq} = j\omega C + \frac{10}{116} - \frac{j4}{116}$$

عشان اطلع الـ unity PF ← I_{imag} Part لازم يكون Zero

$$\frac{-j4}{116} + j\omega C = Zero$$

$\omega \Downarrow 2\pi f$

$$C = 91.5\ \mu f$$



② $P_{app} \Rightarrow V_c * I_c$

$= 440 * 15.22$

$= 6.698\ KVA$

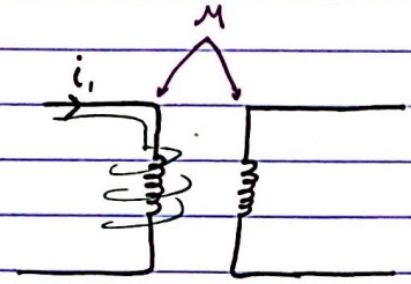
$I_c = \frac{V_c}{Z_c}$

$= 440 * 15.22\ \angle 90$ A

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CH: 13

<<magnatically coupled >> circuits

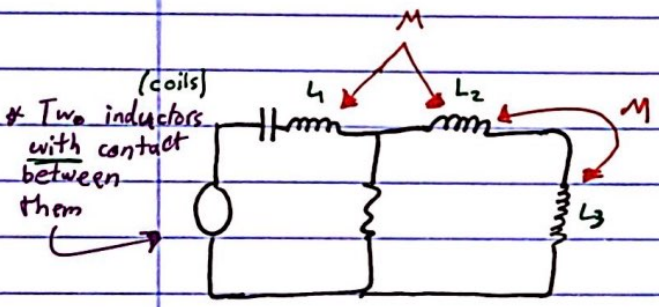


* Two inductors (coils) without contact between them

* التيار i_1 عندما يمر بالـ inductor يولد مجال مغناطيسي الذي يؤثر على الـ Inductor الثاني يؤدي إلى توليد تيار كهربائي بالرادة الثانية وبالتالي يصير عندها e_{ind}.

■ M: Mutal Inductance

* when two loops with or without contacts between them affect each other through the magnetic field generated by one of them, they are said to be magnetically coupled.

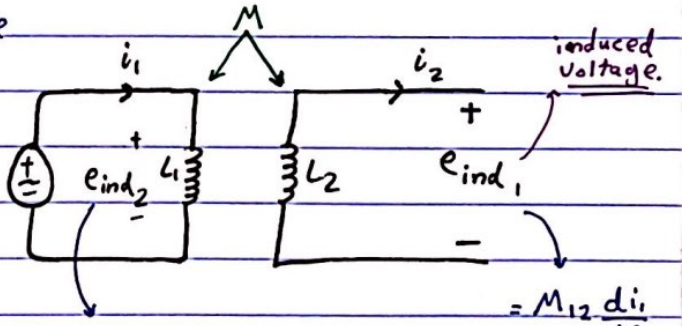


* Two inductors (coils) with contact between them

* L_1, L_2 have effects on eachother as well as L_2, L_3.
whereas L_1, L_3 don't affect eachother. because they're far from eachother
Therefor the magnatic field is far and weak to affect the other.

■ Mutual Inductance:- to describe the voltage induced on a coil from the magnetic field of the other coil.

تقاس بوحدة هنري (H) (نسبة الـ inductance)



M_{21} ≡ Mutual inductance generated from L_2 OR L_1.

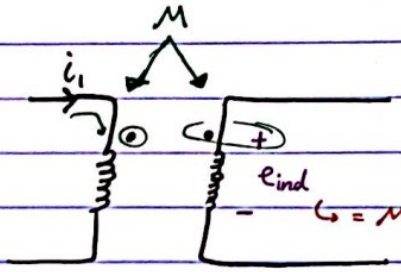
$$e_{ind_2} = M_{21} \frac{di_2}{dt}$$

$$M_{12} = M_{21} = M$$

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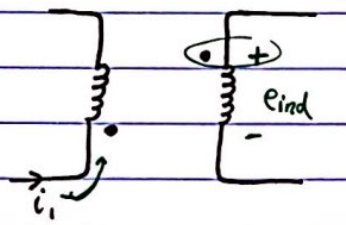
Dot connection

- if the current i_1 enters the dotted end
  ⇒ e_{ind} ⊕ at dotted end of the second coil.

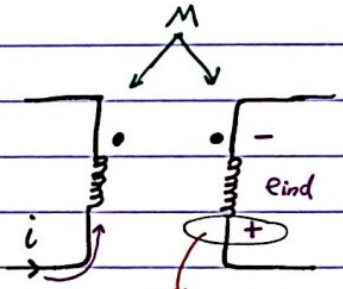


$\hookrightarrow = M \frac{di_1}{dt}$

- because the current entered the dotted end we put the positive pole of the e_{ind} where ever the dotted end is placed on the othe circuit.



- if the current enters the undotted end of the coil,
  ⇒ e_{ind} ⊕ at undotted end of the other coil.



(Undotted end).

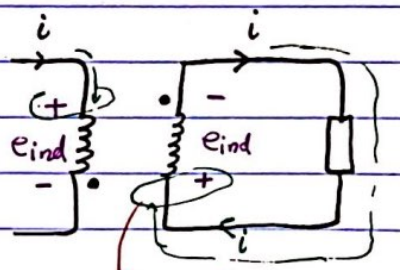
Time Domain ← $e_{ind} = M \frac{di}{dt}$

frequency Domain. ↔ $e_{ind} = JWMI$

أكثر إشي بنستعملها بهاي المادة.
للفائدة.

$\frac{d}{dt} = Jw$
$= s$

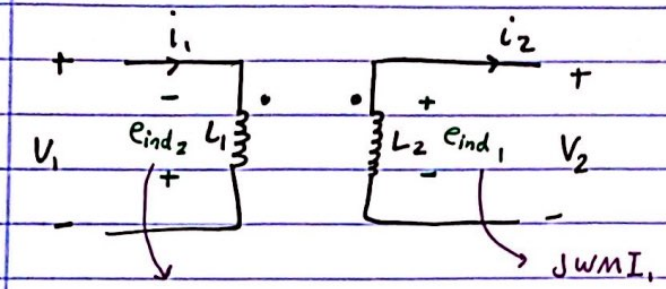
أكثر إشي موجود بالكونترول (مادة التحكم الآلي)

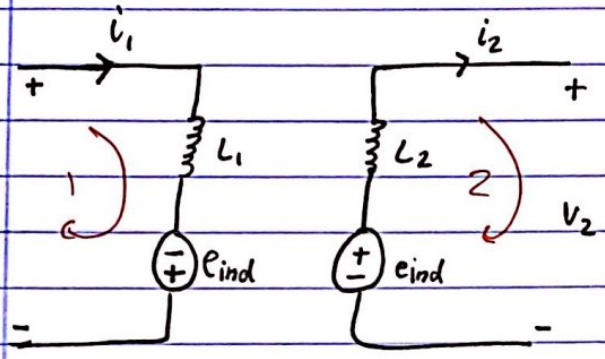


(undotted end).

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ex:





ملاحظة :

* وانتبه إنو صون $V_2 \neq e_{ind}$ لأنو في إشي إسمو self inductance من خلال KVL بتوضح.

فولتية ناتجة من مرور تيار في (L) *Voltage*

فولتية ناتجة من مرور i_2 بـ (L_1)

* زي ماحكينا بالملاحظة انو انا عندي self و Mutual ← Inductance voltages.

① $V_1(t) = L_1 \frac{di_1}{dt} - M \frac{di_2}{dt}$

$V_1 = JWL_1 I_1 - JWMI_2$

self inductance voltage — Mutual Inductance voltage

② $V_2 - e_{ind} + L_2 \frac{di_2}{dt} = 0$

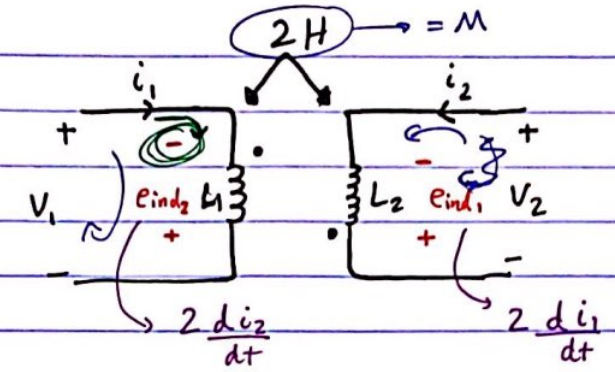
time Domain ← $V_2(t) = M \frac{di_1}{dt} - L_2 \frac{di_2}{dt}$

S-Domain ← $V_2 = JWMI_1 - JWL_2 I_2$ → phasor form.

⇒ we call that combined mutual and self-inductance.

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Example:



★ find V_1 if $i_2 = 5\sin 4t$ A
$i_1 = 0$ A

$$V_1(t) = L_1\frac{di_1}{dt} - 2\frac{di_2}{dt}$$

$$V_1(t) = -2\frac{di_2}{dt} = -40\cos 4t \text{ V}$$

■ V_1 is in the "first circuit" and when we say $i_1 = 0$ that means that there will be no self induced inductance, only mutual from the "second cct".

★ لو طلب عندي عند ثانية معينة او وقت معين بس بخوف معل t.

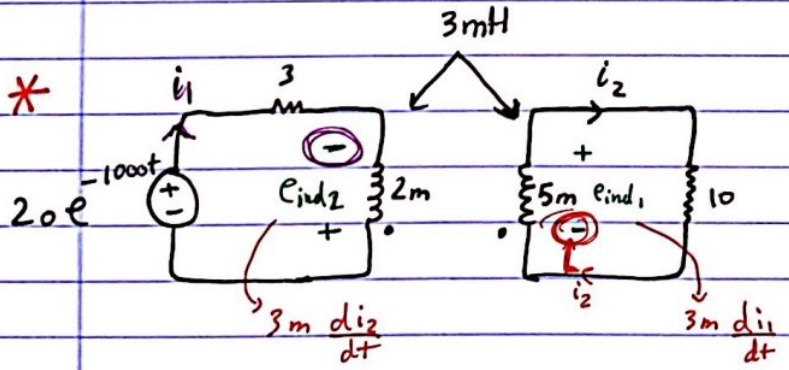
★ find $V_2(t)$ if $i_1 = -8e^{-t}$
$i_2 = 0$

$$V_2 = L_2\frac{di_2}{dt} - 2\frac{di_1}{dt}$$

$$V_2 = -16e^{-t}$$

★ it's best to take KVL with the direction of the current (less confusion).

★



$$20e^{-1000t} = 3i_1 + 2m\frac{di_1}{dt} - 3m\frac{di_2}{dt} \quad \ldots (1)$$

$$10i_2 + 5m\frac{di_2}{dt} - 3m\frac{di_1}{dt}$$

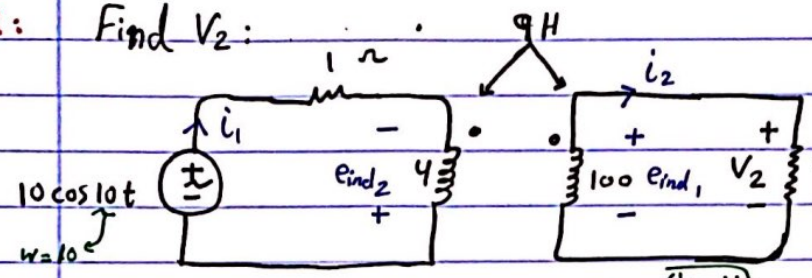
★ i_1 is not given, but i can choose the current direction based on the source
(التيار خارج من القطب الموجب).

★ بما أنو ما عندي Voltage source بالسركيت الثانية بتغير الـ e_{ind_1} في الـ source

★ إنتبه !! هون ما بقدر احول لـ phasor لأنو مش sinusoidal source.

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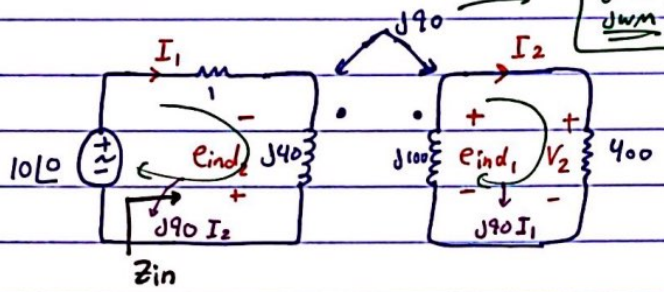
EX: Find V_2:



i_2 طلعناها من خلال إنو طلعنا i_1 بعدين طلعنا e_{ind_1} وبعتبرو الـ source حسب ما تعلمنا سابقًا، والتيار بطلع من الموجب.

نفسها $j\omega m$

Sol:



KVL → $I_1 + j40\,I_1 - j90\,I_2 - 10\angle 0 = 0$ ..... (1)

$400\,I_2 + j100\,I_2 - j90\,I_1 = 0$ ..... (2)

($400\,I_2$ = V_2)

$(400 + j100)\,I_2 = j90\,I_1$

$I_1 = ?$ $I_2 = ?$

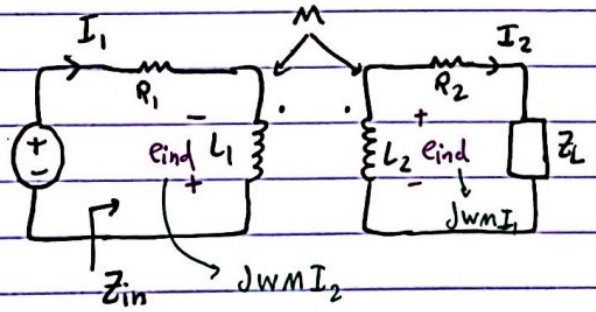
$V_2 = 400\,I_2 = \ldots\ldots\ V$

→ Z_{in}

$Z_{in} = \frac{V_s}{I_1} = 1 + j40$ if $M = 0$ (no coupling, no Mutual impedance)

if $M \neq 0$ $Z_{in} = \frac{V_s}{I_1}$

phasor form



⇒

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... continue:-

$$V_s = R_1 I_1 + j\omega L_1 I_1 - j\omega M I_2 \quad \text{.... (1)}$$

$$R_2 I_2 + Z_L I_2 + j\omega L_2 I_2 - j\omega M I_1 = 0 \quad \text{... (2)}$$

$$(R_2 + j\omega L_2 + Z_L) I_2 = j\omega M I_1$$

$$V_s = (R_1 + j\omega L_1) I_1 - j\omega M \left(\frac{j\omega M}{R_2 + j\omega L_2 + Z_L} I_1 \right)$$

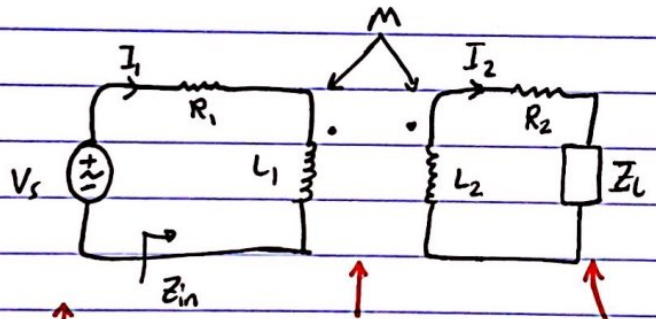
يلزم موجب.

$$Z_{in} = \frac{V_s}{I_1} = \underbrace{(R_1 + j\omega L_1)}_{\text{primary}} + \frac{\omega^2 M^2}{R_2 + j\omega L_2 + Z_L}$$

→ Reflected Impedance

Reflected from secondary

if $M = 0$ ⟶ coupling ⇒ $Z_{in} = R_1 + j\omega L_1$



Transfomer

Primary side (source side)

secondary side (load side)

Impedance from the ~~second~~ first circuit

$$Z_{in} = Z_{11} + \frac{M^2 \omega^2}{Z_{22}}$$

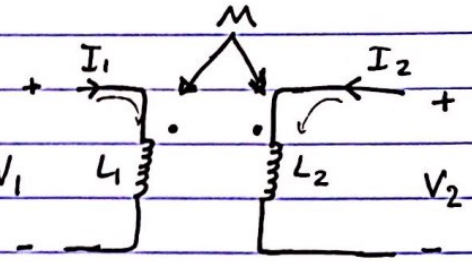
أسهل للحفظ

إشارة موجب.

↳ Impedance from the second circuit.

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Stored energy



الطاقة المختزنة في (L).
$W_L = \frac{1}{2} L I^2$

$W_C = \frac{1}{2} C V^2$

in Time Domain equation

Total stored energy (J)

$I_1, I_2 \rightarrow$ enter

$\Rightarrow W_L(t) = \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M I_1 I_2$

⊕ اذا كانوا متشابهين ـ واو داخلين او طالعين تكون

⊖ اذا كانوا مختلفين يعني واحد داخل او واحد خارج او العكس

⊕ i_1, i_2 → enter or leave dot

⊖ i_1 enter dot and i_2 leaves dot

OR i_1 leaves dot and i_2 enter dot.

$W(t) \geq 0$

$(\ )(\ ) \geq 0$

$M = K \sqrt{L_1 L_2}$ → coupling coeffecients

$K = \frac{M}{\sqrt{L_1 L_2}}$

$0 \leq K \leq 1$

$m=0$ ← no coupling.

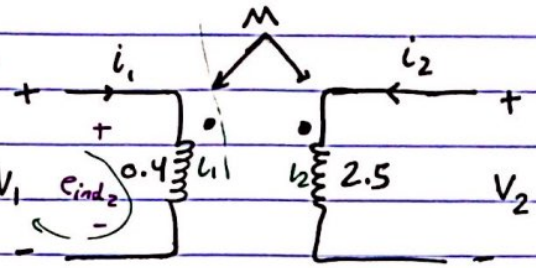
→ Tightly coupled.

* كل ما كان الـ K أقرب لـ 1
الـ inductors بكونو كثير مقربين
من بعض والعكس صحيح.

* وحدة الـ total stored energy هي جول (J).

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Example:



$K = 0.6$

$i_1 = 4i_2 = 20\cos(500t - 20)\ mA.$

Find i_2, V_1, total stored energy at $t=0$

★ من خلال عاي المعلومة بعرف إنو لازم احل على الـ time Domain مش على الـ phasor (عشان اقدر اعوض t).

Sol: 1. $i_2 = \frac{i_1}{4} = \frac{20}{4}\cos(500t - 20)\ mA$

$i_2(0) = 4.698\ m\ (A)$ ← (عوضنا 0 بدل t)

2. $V_1 = L_1\frac{di_1}{dt} + M\frac{di_2}{dt}$ ★

$M = K\sqrt{L_1 L_2}$

$= 0.6\sqrt{0.4 * 2.5}$

$= 0.6H$

$V_1 = 0.4 * (-20 * 500 * \sin(500t - 20)) * 10^{-3}$

$- 0.6 * (5 * 500 * \sin(500t - 20)) * 10^{-3} =$

★ مهم جدًا التيار إلي معي بالميلي أمبير مهم اعوضو عشان يطلع الجواب صح.

$V_1|_{t=0} = 1.88\ V$

3. Total stored energy:

$W_L = \frac{1}{2}L_1 i_1^2 + \frac{1}{2}L_2 i_2^2 \pm M * i_1 * i_2$

★ انتبه مش دايمًا بتكون موجبة ممكن سالب بس هو موجب لأنو التيارين داخلين بال Dot

$= \frac{1}{2} * 0.4 * (20\cos(500t - 20) * 10^{-3})^2 + \frac{1}{2} * 2.5 * (5\cos(500t - 20) * 10^{-3})^2$

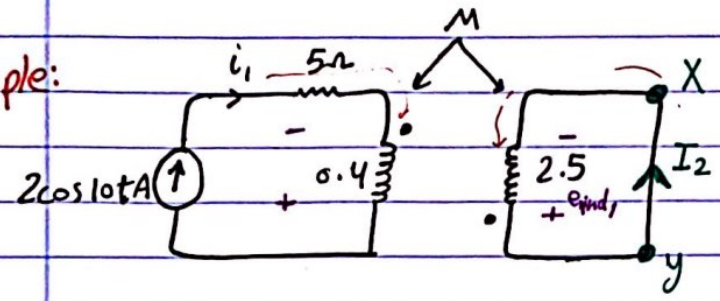
$+ 0.6\,(20\cos(500t - 20) * 10^{-3})\,(5\cos(500t - 20) * 10^{-3})$

$W_L(0) = 70.64 * 10^{-6} + 27.6 * 10^{-6} + 53 * 10^{-6}$

$W_L(0) = 151.24\ \mu J$

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Example:



$k = 0.6$

Find total stored energy if:
① X-y open ② x-y short
when $t=0$

Sol : ① X-y open
↳ $I_2 = 0$

$M = k\sqrt{L_1 L_2}$
$= \underline{0.6\,H}$

$W(t) = \frac{1}{2} * 0.4 * I_1^2 + \frac{1}{2} * 2.5 * I_2^2 \rightarrow M I_1 I_2$

"when t=0" ←

$I_1 = 2\cos 10t$

$W(0) = \frac{1}{2} * 0.4 * (2\cos(0))^2 = 0.8\,J$

② X-y short
↳ $I_2 = ??$

$e_{ind_1} = M\frac{di_1}{dt} = 0.6 * -20 * \sin 10t$
$= -12\sin 10t$

* بهاي الطريقة بطلع الـ e_{ind_1} إلي هي نفسها الـ source بال سيركيت الثانية بما إنو ما عندي مصدر ثاني

* Voltage on an inductor $= L\frac{di}{dt}$ * هي نفسها e_{ind_1}

بستخدمو عشان اطلع I_2 ← فيني ← $I_2 = \frac{1}{L_2}\int V\,dt$

لاحظ كيف صون سالب لأنو التيارين عكس اتجاه بعض

$I_2 = 0.48\cos 10t\ A$

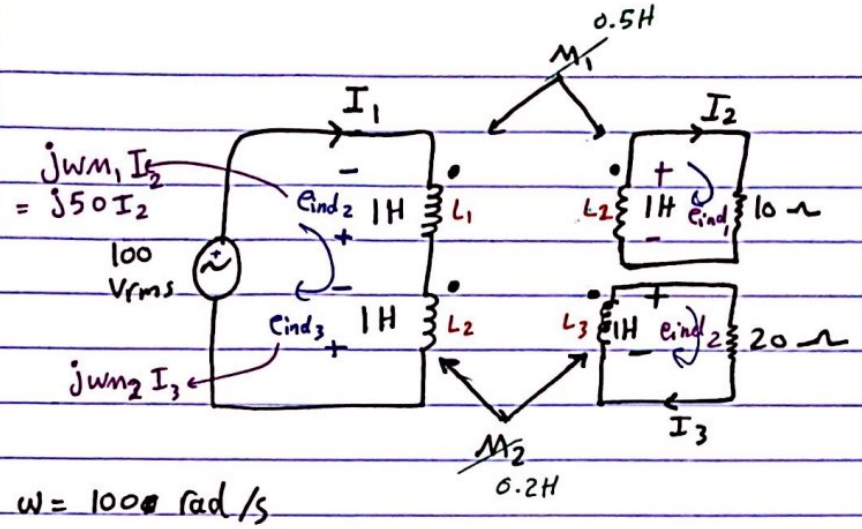
$W(0) = \frac{1}{2} * 0.4 * 2^2 + \frac{1}{2} * 2.5 * (0.48)^2 - M I_1 I_2$

$0.6 * 2 * 0.48 = 0.576$

$W(0) = 0.512\,J$

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Example:



$Z_L = j\omega L$

$Z_L = j100$

(الكلهم لأنو
إلهم نفس القيمة)

$\omega = 1000$ rad/s

$K_1 = 0.5$, $K_2 = 0.2$

Find $P_{20\Omega}$, $P_{10\Omega}$, P_{source} ← Power average of course

Sol:

$M_1 = K_1 \sqrt{L_1 * L_2} = 0.5\ H$

$M_2 = K_2 \sqrt{L_3 * L_4} = 0.2\ H$

$$-100 + j100 I_1 - j50 I_2 + j100 I_1 - j20 I_3 = 0 \quad \dots ①$$

$$I_2 10 + j100 I_2 - j50 I_1 = 0 \quad \dots ②$$

$$20 I_3 + j100 I_3 - j20 I_1 = 0 \quad \dots ③$$

مش أكيد حل حذف وتعويض وتأكد.

$I_1 = 1.328 \angle -80.4\ A$

$I_2 = 0.66 \angle -74.68\ A$

$I_3 = 0.132 \angle -74.68\ A$

$\Rightarrow P_{20} = I_3^2 (20) = 0.39\ w$

$P_{10} = I_2^2 (10) = 4.356\ w$

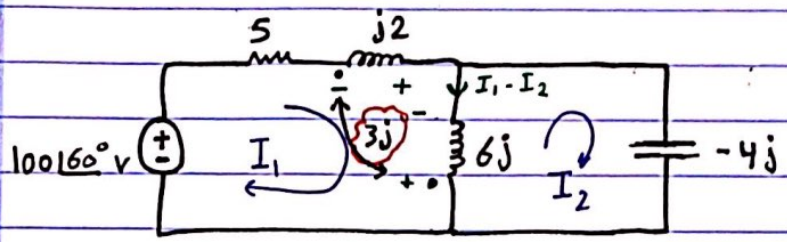
$P_{source} = V_{rms} I_1 \cos(\theta - \phi)$

$= 100 * 1.328 * \cos(80.4)$

$= 22.15\ w$

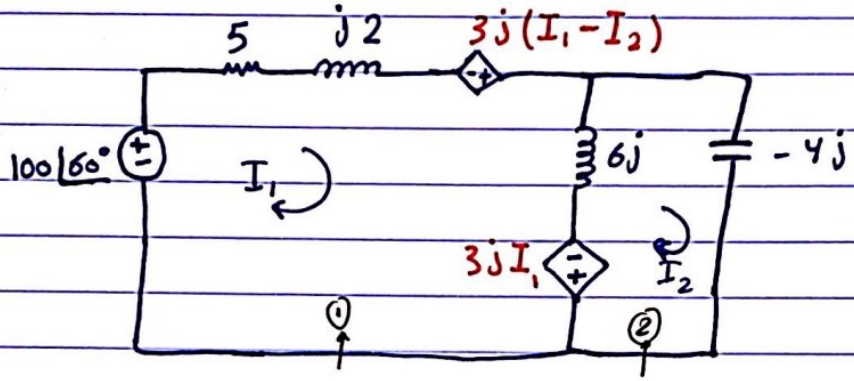
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Practice problem 13.2



$M = 3j$

Find I_1, I_2



① $100\angle 60° = 5I_1 + j2I_1 - 3j(I_1 - I_2) + 6j(I_1 - I_2) - 3jI_1$

$100\angle 60° = I_1(5 + 2j - 3j + 6j - 3j) + I_2(3j - 6j)$

$100\angle 60° = (5 + 2j)I_1 - 3jI_2 \quad \ldots ①$

② $+ 3jI_2 + 6j(I_2 - I_1) - 4jI_2 = 0$

$I_1(3j - 6j) + I_2(6j - 4j) = 0$

$-3jI_1 + 2jI_2 = 0 \quad \ldots ②$

$\vdots$

$I_1 = 17.889\angle 86.57°$ A

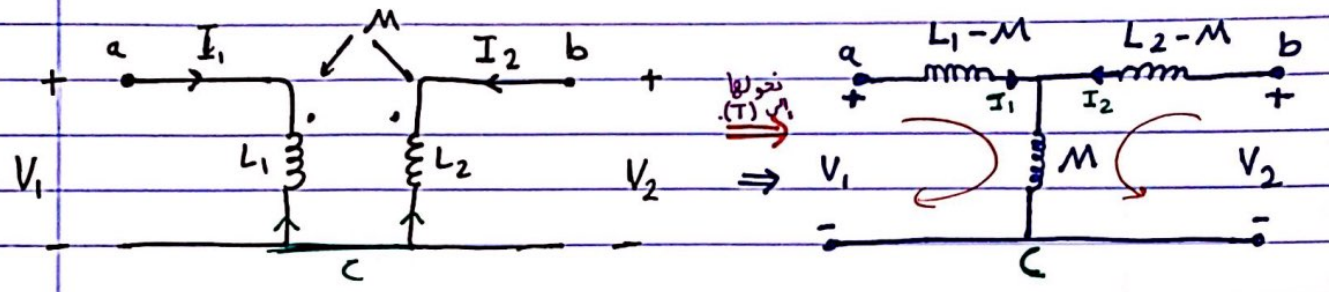
$I_2 = 26.83\angle 86.57$ A

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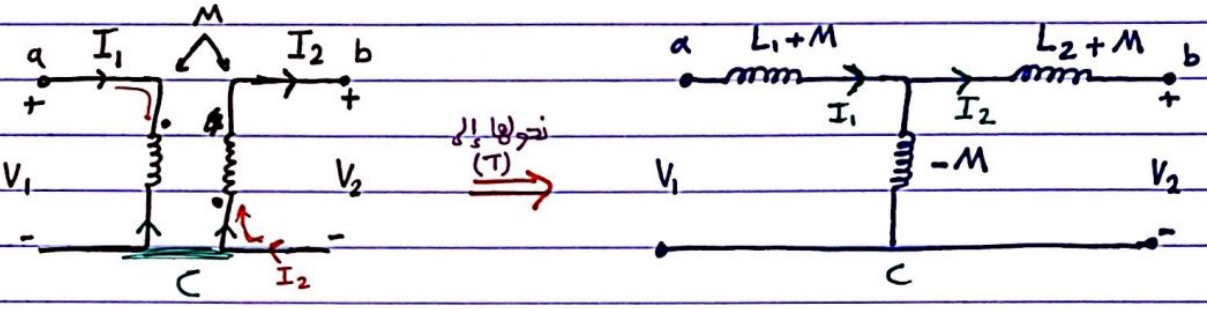
T-equivalent

This is the (T) equivalent.

[1]



[2]



* في حالة [1] اذا كانوا الـ (dot) في نفس الإتجاه (داخلين أو خارجين) ← (+M) ونطرح (L-M).

[2] اذا كانوا الـ (dot) في عكس الإتجاه (وحدة خارج وحدة داخل أو العكس) ← (-M) ونجمع (L+M).

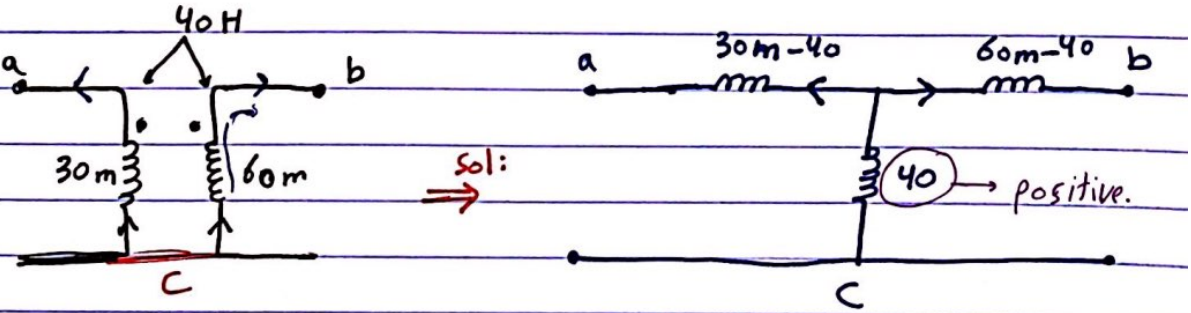
Always be careful of the current direction because

[1] ≠ [2]

in the first T-equivalent both currents enter the dot, but in the second T-equivalent one current enters the dot and one leaves the dot. (from point c).

⇒ To prove they're equivalent we just do KVL on both of them and the resulting equations should be the same.

ex:



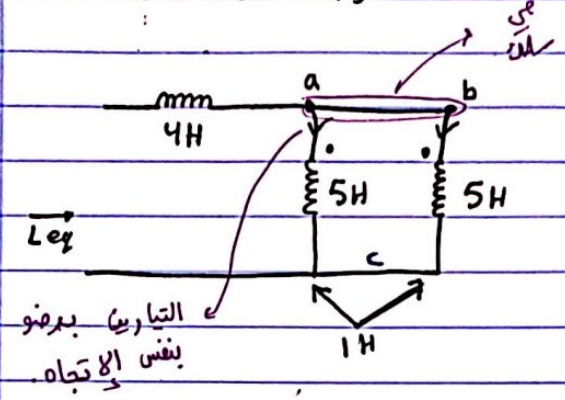
Find T equiv cct.

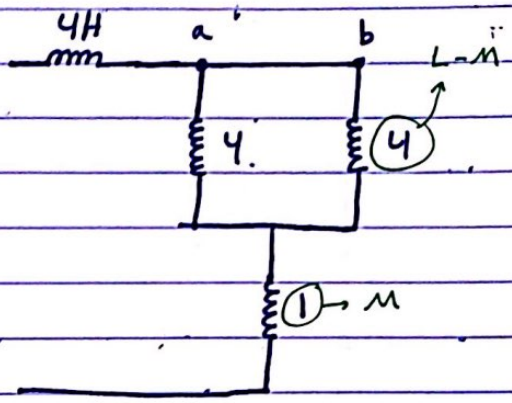
هون التيارين بنفس الإتجاه.

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EX: Find L_{eq} , Z_{in}

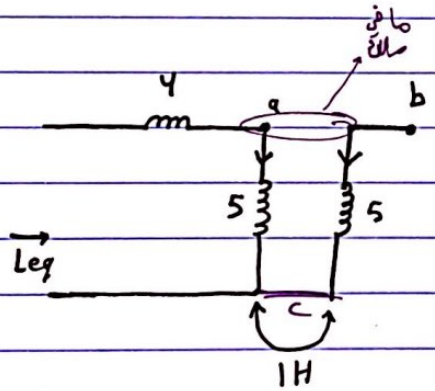
$w = 200$ rad/s

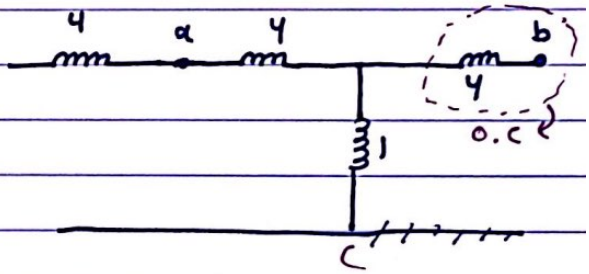




$L_{eq} = 4 + (4 // 4) + 1 = 4 + 2 + 1 = 7H$

$Z_{in} = jwL \rightarrow j * 200 * 7 = j1400\ \Omega$



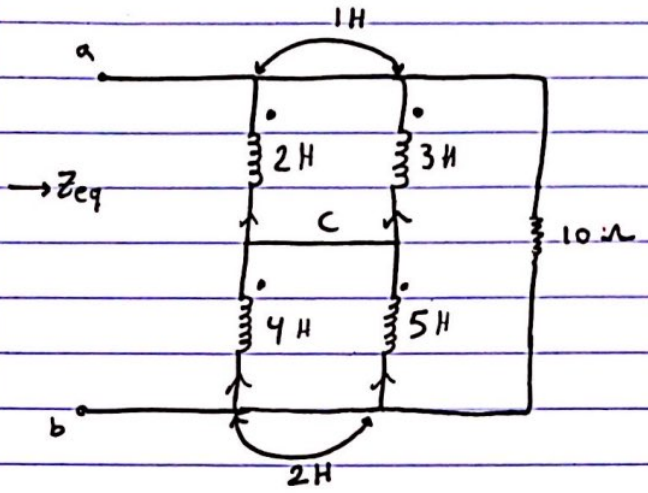


$L_{eq} = 4 + 4 + 1 = 5H$

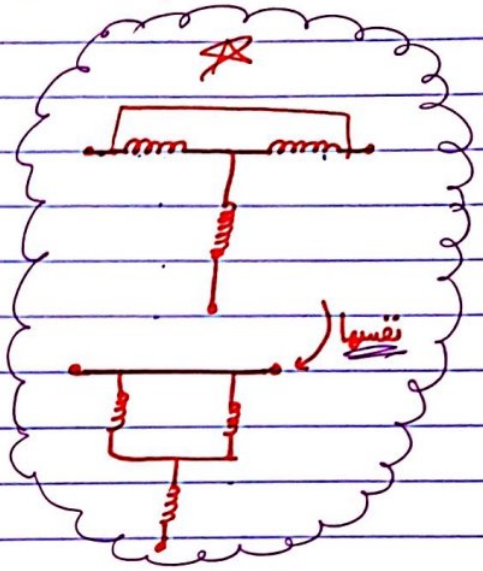
وكيز على شكله circuit الـ

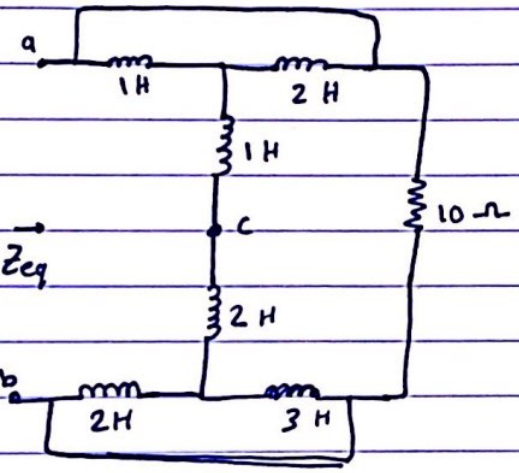
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EX: Find Z_{in}



NOTES-





باخذ Inductors المحال.

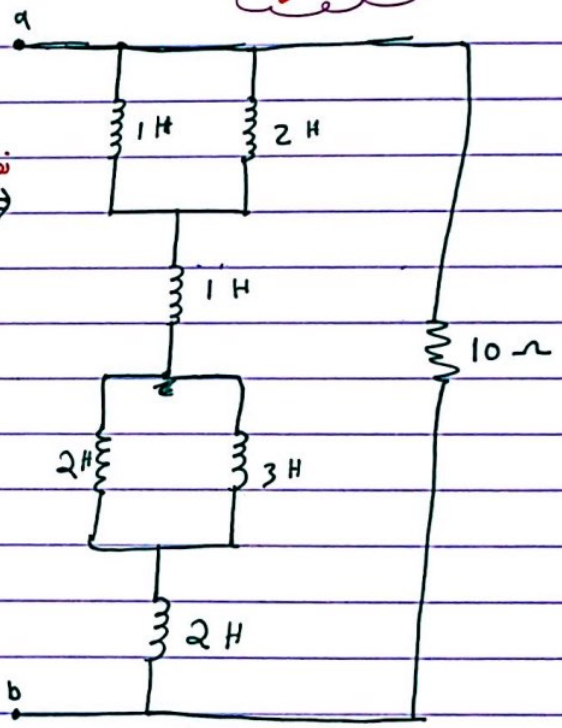
$L_{eq} = (1 // 2) + 1 + 2 + (2 // 3)$

$= 4.86\ H$

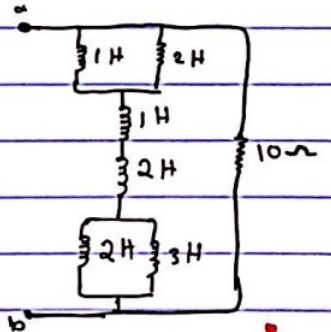
$Z_{eq} = jwL_{eq} // 10$

$= (jw * 4.86) // 10$

لو كان معطينا w بنعوض وبنكمل الحل.



أو ممكن هاذ الشكل (نقسم الأخير).

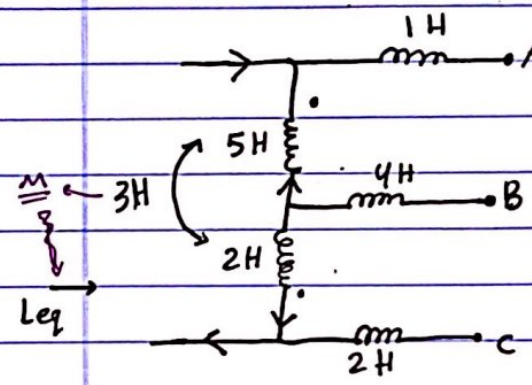


احفظ هاذ الشكل

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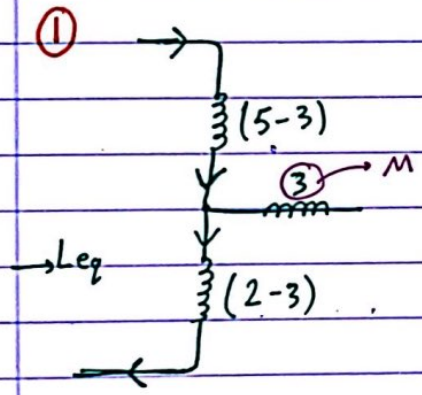
Example: Find L_{eq}, ① when there is no connection between a and b and c.
② when AB is shorted.
③ when AC is shorted.

Q35 7th Book.



* هون لاحظ إنو مش معطيني coupling عند كل الـ inductors.

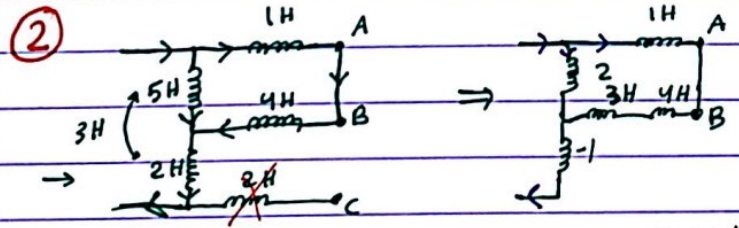
* shorted means we connect an empty wire between two points.

① 

طبعًا الـ 3 ولا زي أكنها موجودة لأنو ما بمر فيها تيار

* الـ inductor بالعادة بدرم بطلع بالموجب وخاصة لما نتعامل معاه بالابات بس هون عادي لو طلع سالب لا يتخاف.

$\rightarrow L_{eq} = 2 + -1 = 1H$

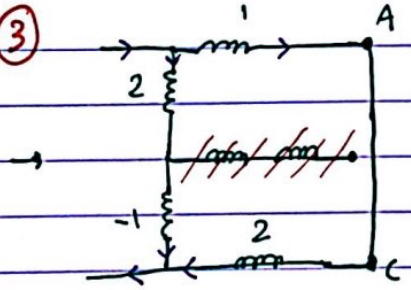
② 

توالي لأنو بمر فيهم نفس التيار.

$\Rightarrow L_{eq} = [(3+4+1)\,//\,2] - 1$

$= 0.6\,H$

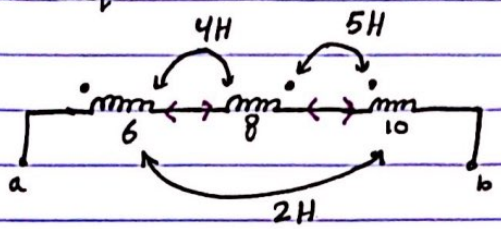
* عشان اعرف التوالي والتوازي بشوف من وين طالب مني احسب الـ equivalent وبحط تيار وبشوف متى بتمزج ومتى لا وهكذا.

③ 

$L_{eq} = (1+2)\,//\,(2-1) = 0.75H$

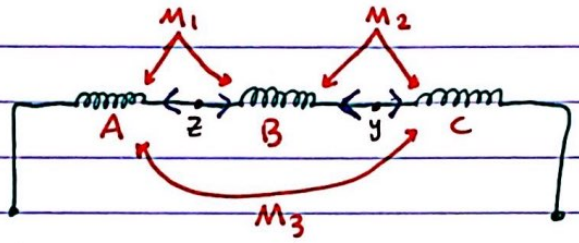
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EX: Find Leq



Leq =

$$Leq = (6-4+2) + (8-4-5) + (10-5+2)$$
$$= 10\ H.$$



Leq ⟶ each Inductor alone :

$\boxed{L_A} \Rightarrow A \pm M_1 \pm M_3$

$\boxed{L_B} \Rightarrow B \pm M_1 \pm M_2$

$\boxed{L_C} \Rightarrow C \pm M_2 \pm M_3$

$\therefore L_{eq} = L_A + L_B + L_C$

☆ لتحديد الإشارة بنشوف ويا التيار الطالع من النقاط و و ج (احنا فرضنا هيك) و بشوف اذا كان داخل بنفس الحل بحط (-) اذا كان داخل بنقطة من جهة ومن فراغي من جهة ثانية (عكس بعض) بحط (+).

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CH 14: # complex frequency

$V(t) = V_m \cos(\omega t + \theta)$

$V(t) = Re\{V_m e^{j(\omega t + \theta)}\}$

$= Re\{V_m e^{j\omega t} e^{j\theta}\}$

* الـ sinusoidal حالة خاصة من complex frequency حيث أن الـ σ فيها = صفر.
(حالة 3) ($s = j\omega$).

A+jB على صيغة ← $S = \sigma + j\omega$ → Total frequency

σ: Real Part — ω: Imaginary part

- S: complex frequency, (s^{-1}) / (seconds)$^{-1}$. → $\frac{1}{s}$
- σ: neper frequency ; (neper / sec.).
- ω: (Radian / Angular) frequency, (rad / s). ← $2\pi f$
- f: frequency, (Hz) or $(1/s)$.

★ General form for any function :-

» $f(t) = Ke^{st}$

K :- constant.

S :- complex frequency ($s = \sigma + j\omega$).

★ There are four cases for (S) :-

* قيمة ثابتة ما بتتغير بتغير الزمن.

σ = 0 and ω = 0 ← 1) $S = 0$

$f(t) = Ke^{0t} \rightarrow f(t) = K$ constant (DC source).

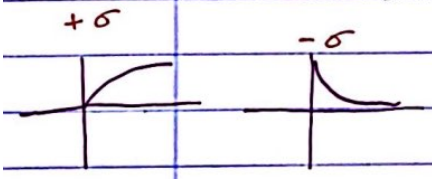
ω = 0 ← 2) if $S = \sigma$

$f(t) = Ke^{\sigma t}$ (exponential function.)

$+\sigma \rightarrow$ increasing.

$-\sigma \rightarrow$ decreasing.

which means the function's graph can be increasing or decreasing.



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$\sigma = 0$ ← [3] if $s = jw$

$$f(t) = Ke^{jwt}$$
$$= K\cos(wt) + jK\sin(wt).$$

$f(t) \rightarrow$ sinusoidal function.

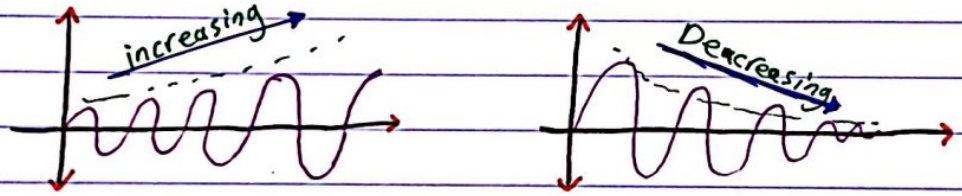
[4] if $s = \sigma + jw$

$$f(t) = Ke^{(\sigma+jw)t} = Ke^{\sigma t} \cdot e^{jwt}$$

$f(t) \rightarrow$ damped sinusoidal function.

$$f(t) = Ke^{\sigma t}[\cos(wt) + j\sin(wt)]$$

∴ Note :-



---

* if $v(t) = V_m \cos(wt + \theta)$

$$v(t) = V_m \left[\frac{e^{j(wt+\theta)} + e^{-j(wt+\theta)}}{2}\right]$$

$$v(t) = \underbrace{\frac{V_m}{2} e^{j\theta}}_{K_1} \underbrace{e^{jwt}}_{e^{s_1 t}} + \underbrace{\frac{V_m}{2} e^{-j\theta}}_{K_2} \underbrace{e^{-jwt}}_{e^{s_2 t}}$$

اذا كان الـ function sin ما بختلف عندي اشي غير الإشارة بالبسط بتصير ⊖ وبالمقام بضير عندي jz بدل 2 لحالها.

متطابقة

* Two frequencies in $v(t)$

$s_1 = +jw \quad s_2 = -jw$

* The same applies whether we have a cos or sin function → $\boxed{s = \pm jw}$

↳ Magnitude for s is (w) (The frequency for sinusoidals) (whether it's $+$ or $-$).
But now this is the complex frequency.

$K_1 = \frac{V_m}{2} e^{j\theta}$

$K_2 = \frac{V_m}{2} e^{-j\theta}$

$K_1 = K_2^*$

$s_1 = s_2^*$

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بس يعطينا sin أو cos وداخلو رقم بدلالة t عطول بعرف إنو هاذ الرقم (w) وعطول بحكي $s = \pm jw$ ولو في زاوية ماررح تهمني.

Ex: Find complex frequency

[1] $v(t) = 2\cos(5t + 30)$

$s_1 = +5j\ s^{-1}$. $s_2 = -5j\ s^{-1}$.

[2] $v(t) = 5\sin(10t - 50)$

$s = \pm j10\ s^{-1}$.

* احنا كنا قبل نحول sin ← cos وبتتغير الزاوية (θ) بس هون ماررح يفرق معي الحل.

للتوضيح ↓

$$\begin{bmatrix} v(t) = 5\cos(10t - 50 - 90) \\ v(t) = 5\cos(10t - 140) \\ s = \pm j10 \end{bmatrix}$$

(w) ← فقط بهمني

[3] $v(t) = (2e^{-100t} + e^{-200t})\cos 1000t$.

* لو كان sin 1000t رح يكون نفس الحل.

$$v(t) = 2e^{-100t}\cos 1000t + e^{-200t}\cos 1000t$$

($\pm j1000$ (w)

الحل مباشرة ← $\{s_1 = -100 + j1000\ (s^{-1}),\ s_3 = -200 + j1000\ (s^{-1})\}$, $\{s_2 = -100 - j1000\ (s^{-1}),\ s_4 = -200 - j1000\ (s^{-1})\}$

توضيح كيف طلعنا s_2 و s_1 ← $2e^{-100t}\left[\frac{e^{j1000t} + e^{-j1000t}}{2}\right]$ نعوض ← $\left(e^{-100t + j1000t} + e^{-100t - j1000t}\right)$ ($\sigma t + jwt$)

[4] $v(t) = (2 - e^{-10t}).\sin(4t + \theta)$

* إنتبه ممكن يعطينا $\cos^2$ أو $\sin^2$ لازم افكها.

$$v(t) = 2\sin(4t + \theta) - e^{-10t}\sin(4t + \theta)$$

$s_1 = j4\ (s^{-1})$ $s_3 = -10 + j4\ (s^{-1})$

$s_2 = -j4\ (s^{-1})$ $s_4 = -10 - j4\ (s^{-1})$

extra:

$v(t) = \sin(10t + 60) \rightarrow s = \pm j10\ s^{-1}$ (60 ← phase)

$v(t) = \cos(5t + 50) \rightarrow s = \pm j5\ s^{-1}$

$v(t) = e^{-10t}\cos(5t + 50) \rightarrow s = -10 \pm j5\ s^{-1}$

(5 ← Frequency. 50 ← phase)

* e^2 → constant
* e^{j2} → 2 → phase (θ)
* e^{j2t} → frequency (w) = 2
* e^{2t} → frequency σ = 2

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EX: construct a function with following frequencies :-

★ ① $S = 0, 10, -10\ s^{-1}$ ⟶ $0 \rightarrow DC$, $10 \rightarrow \sigma$, $-10 \rightarrow \sigma$

$$v(t) = A + Ke^{10t} + Be^{-10t}$$

∴ Note :-

★ العوامل الموجودة بين الأرقام تدل على أن كل رقم يمثل حد.

★ ② $S = \overset{\sigma}{-5}, \overset{w}{j8}, \overset{\sigma}{-5} \overset{w}{-j8}$

$$v(t) = Ae^{-5t} + B\cos(8t) + Ce^{-5t}\cos(8t).$$

OR $v(t) = Ae^{-5t} + B\sin(8t) + Ce^{-5t}\sin(8t)$

OR $v(t) = Ae^{-5t} + B\sin(8t+\theta) + Ce^{-5t}\cos(8t+\theta)$

EX: if $V = 12 \angle \underset{\theta}{35}\ V$ find $v(t)$ if

(k أو الثابت)

① $S = 0 \rightarrow w = 0, \sigma = 0$

إحنا لازم نكتب $V(t)$ على صورة:

$$V(t) = Ke^{\sigma t}\cos(wt+\theta)$$

$$v(t) = 12e^{0t}\cos(0t+35)$$

$$= 12\cos(35) = 9.83\ V \leftarrow (DC)$$

* Voltage source always takes the Real part only

② $S = -20\ s^{-1} \rightarrow w = 0, \sigma = -20$

$$v(t) = 12e^{-20t}\cos(0t+35)$$

$$v(t) = 9.83e^{-20t}\ V \rightarrow \text{exponential}$$

③ $S = j5\ s^{-1} \rightarrow w = 5, \sigma = 0$

$$v(t) = 12e^{0t}\cos(5t+35)$$

$$v(t) = 12\cos(5t+35) \rightarrow \text{sinusoidal}$$

④ $S = -20 + j5$

$$v(t) = 12e^{-20t}\cos(5t+35) \rightarrow \text{Damped sinusoidal.}$$

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Ex: Find complex frequency

1. $e^{-10t} \cdot \cos 10t \cdot \sin 40t$

★ لما يكون في ضرب، صدود الـ complex frequency رح يكون بيناتهم جمع.

$s = -10 \pm j10 \pm j40$

$s_1 = -10 + j10 + j40 = -10 + j50\ s^{-1}$

$s_2 = -10 + j10 - j40 = -10 - j30\ s^{-1}$

$s_3 = -10 - j10 + j40 = -10 + j30\ s^{-1}$

$s_4 = -10 - j10 - j40 = -10 - j50\ s^{-1}$

2. $12e^{-2t} + e^{-5t}\cos(50t - 210)$

$s_1 = -2\ s^{-1}$

$s_2 = -5 \pm j50\ s^{-1}$

CH 14.2 # The Damped sinusoidal forcing function :-

$V(t) \rightarrow V_m \angle\theta \longrightarrow V_m e^{j\theta}$ } phasor form

$I(t) \rightarrow I_m \angle\theta \longrightarrow I_m e^{j\theta}$ }

$V(t) = Ve^{st} \longrightarrow V = V_m e^{j\theta} / V_m \angle\theta$ بدلالة الـ complex frequency.

$I(t) = Ie^{st} \longrightarrow I = I_m e^{j\theta} / I_m \angle\theta$

Ex: 1. $24 \sin(90t + 60)$

$24\cos(90t + 60° - 90°) \rightarrow 24\cos(90t - 30°)$

phasor $\rightarrow 24\angle -30°$ ← cos عشان اطلع الـ phasor لازم اخلي الـ

$s = \pm j90\ s^{-1}$ ← أما إذا بدي اطلع الـ complex frequency ما بتفرق

2. $24e^{-10t}\cos(90t + 60)$

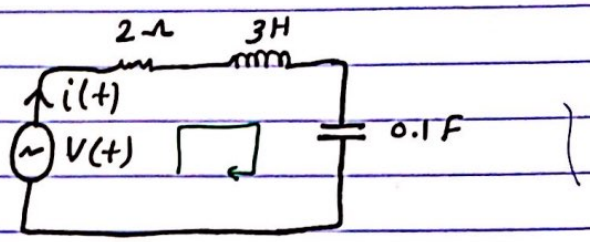
$\rightarrow 24\angle 60 \cdot e^{-10t}$

$s \Rightarrow -10 \pm j90$

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Example: $V(t) = 60\, e^{-2t} \cos(4t + 10)$, Find $i(t)$

$V_L = L \frac{di}{dt}$	$Z_L = j\omega L$
	$Z_L = sL$
$V_C = \frac{1}{C} \int i\, dt$	$Z_C = \frac{1}{j\omega C}$
	$Z_C = \frac{1}{sC}$
$s = \sigma + j\omega$	



KVL → $60\, e^{-2t} \cos(4t + 10) = 2I + 3\frac{di}{dt} + \frac{1}{0.1} \int i\, dt$

★ $V(t) = 60 \angle 10^\circ\, e^{-2t} \longrightarrow s = -2 \pm j4$

$I(t) = I_m \angle \theta^\circ\, e^{st} \longrightarrow s = \sigma \pm j\omega$

phasor → $60 \angle 10^\circ = 2 I_m + 3s I_m + \frac{I_m}{0.1s}$

$$I_m = \frac{60 \angle 10^\circ}{2 + 3s + \frac{1}{0.1s}} \Bigg|_{s = -2 \pm j4}$$

$$I_m = \frac{60 \angle 10^\circ}{2 + 3(-2 + j4) + \frac{1}{0.1(-2 + j4)}}$$

$I_m = \underbrace{5.37}_{\text{Magnitude}} \angle \underbrace{-106.56}_{\theta}$ A

$\Rightarrow i(t) = 5.37 \cos(4t - 106.6^\circ)\, e^{st}$

$= 5.37 \cos(4t - 106.6)\, e^{-2t}$ //

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CH 14.3 : # Laplace Transform

» The convertion from time domain to complex frequency domain [S].
↳ (S-Domain).

- $\mathcal{L}\{f(t)\} = F(s) = \int_{-\infty}^{\infty} e^{-st} f(t)\,dt$.

- $\mathcal{L}^{-1}\{F(s)\} = f(t)$

* بقدر أحول أي نوع من انواع الـ sources لـ S-Domain من خلال الـ laplace transform سواء كان sinusoidal, DC or any other function.

$f(t) \Longleftrightarrow F(s)$

لـ يدل على أن العملية تبديلية
يعني بقدر أحول من [time Domain] إلى [frequency Domain] و بقدر العكس.

$\int_{-\infty}^{\infty} = \int_{-\infty}^{0} + \int_{0}^{\infty}$

↓ أما هاي فهي لـ initial Value

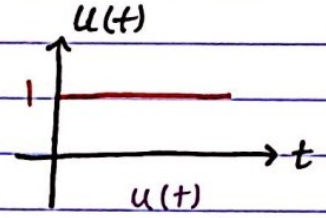
↳ رح نستخدم هاي

take laplace → one side. - $\mathcal{L}[f(t)u(t)] = F(s) = \int_{0}^{\infty} F(t)e^{-st}dt$.

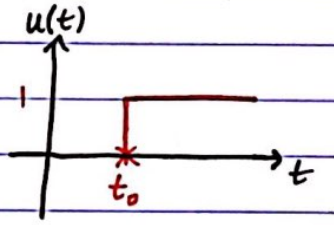
* بنا نوخذ one side لأنو بحسبي من لما تبلش التجربة (من عند 0) اما القيم الأقل من هيك بعتبرها initial value ولهذا السبب ضربنا بـ u(t)

* $u(t)$ → unit step function,

$u(t) = \begin{cases} 1 & t > 0 \\ 0 & t < 0 \end{cases}$ ↳ it's like a switch in the circuit.



* $\delta(t)$ → Impulse function.



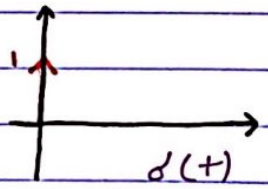
$u(t-t_o)$ إزاحة ↳

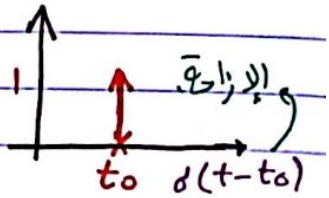
* دايمًا بس أحل و أخلص حل لازم ارجع لـ time Domain, ما بصير اضل بـ S-Domain.

- $\mathcal{L}\left(\frac{dv}{dt}\right) \rightarrow sV(s) - V(0)$ ← initial value

- $\mathcal{L}\left(\frac{d^2v}{dt^2}\right) \rightarrow s^2V(s) - sV(0) - V'(0)$

- $\mathcal{L}\left(\frac{d^3v}{dt^3}\right) \rightarrow s^3V(s) - s^2V(0) - sV'(0) - V''(0)$





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Time Domain	Frequency Domain
$A\,u(t)$	$\frac{A}{s}$
$A\,u(t-t_0)$	$\frac{A}{s}\,e^{-t_0 s}$
$A e^{-at}.u(t)$	$\frac{A}{(s+a)}$
$A e^{at}.u(t)$	$\frac{A}{(s-a)}$
$A t.u(t)$	$\frac{A}{s^2}$
$A t e^{-at} u(t)$	$\frac{A}{(s+a)^2}$
$A\delta(t)$	A
$A\,\delta(t-t_0)$	$A e^{-t_0 s}$
$f(t)\,\delta(t_0-t_0)$	$F(t_0)$
$A \sin wt\ u(t)$	$\frac{Aw}{s^2+w^2}$
$A \cos wt\ u(t)$	$\frac{As}{s^2+w^2}$

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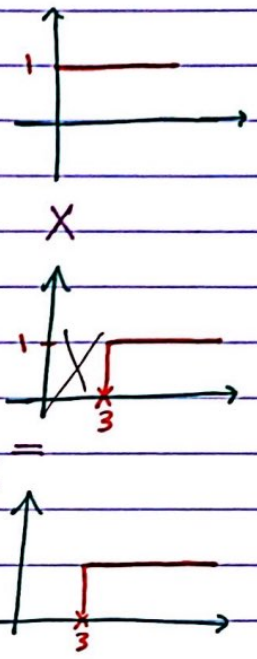
Examples: Find $V(s)$ or convert to frequency domain :-

1. $V(t) = 4\delta(t) - 3u(t)$
   $\hookrightarrow V(s) = 4 - \frac{3}{s}$.

2. $V(t) = 4\delta(t-2) - 3tu(t)$
   $\hookrightarrow V(s) = 4e^{-2s} - \frac{3}{s^2}$.

3. $V(t) = u(t) \cdot u(t-3)$
   $= u(t-3)$
   $\hookrightarrow$ then $\Rightarrow V(s) = \frac{e^{-3s}}{s}$.

* اصلاً أي اشي بضربو بـ $u(t)$ بعطيني الاشي إلي جمبو.



4. $V(t) = 10tu(t)$
   $\hookrightarrow V(s) = \frac{10}{s^2}$.

Example: Find $V(t)$ or convert to time domain:-

1. $V(s) = 10 \longrightarrow V(t) = 10\delta(t)$.
2. $V(s) = \frac{10}{s} \longrightarrow V(t) = 10u(t)$.
3. $V(s) = \frac{10}{s^2} \longrightarrow V(t) = 10tu(t)$.

* ممكن أحط $u(t)$ او ما احط، بحالة الـ impulse عادي.

* اذا طلب التحويل إلى [Time domain] وكان الـ function المعطى على صورة $\left(\frac{\text{بسط}}{\text{مقام}}\right)$ ولا يمكن حله من الجدول السابق فهنالك طريقتين للحل :-

ex: $V(s) = \frac{10s}{s+10}$

$$H(s) = \frac{N(s)}{D(s)}$$

$$\begin{array}{r|l} & 10 \\ \hline s+10 & 10s \\ & -10s \mp 100 \\ \hline & -100 \end{array}$$

$$V(s) = 10 - \frac{100}{s+10}$$

$$V(t) = 10\delta(t) - 100e^{-10t}u(t)$$

- $N < D \rightarrow$ partial fraction.
- $N \geq D \rightarrow$ long division.

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ex: convert to time domain

* $V(s) = \frac{10}{s(s+10)}$

$V(s) = \frac{A}{s} + \frac{B}{s+10} = \frac{10}{s(s+10)}$

$A(s+10) + Bs = 10$

$As + 10A + Bs = 10 + 0s$

$10A = 10 \rightarrow A = 1$

$A + B = 0 \rightarrow 1 + B = 0 \rightarrow B = -1$

$V(s) = \frac{1}{s} + \frac{-1}{s+10} \rightarrow V(t) = u(t) - e^{-10t} u(t)$

* $F(s) = \frac{2s+4}{s} \Rightarrow F(s) = 2 + \frac{4}{s}$

$F(t) = 2\delta(t) + 4u(t)$

* $F(s) = \frac{7s+5}{s(s+1)} \rightarrow$ P.F

$F(s) = \frac{A}{s} + \frac{B}{s+1} = \frac{7s+5}{s(s+1)} \rightarrow A(s+1) + Bs = 7s+5$

$As + A + Bs = 7s + 5$

$\boxed{A = 5}$

$A + B = 7 \rightarrow \boxed{B = 2}$

$F(s) = \frac{5}{s} + \frac{2}{s+1}$

$f(t) = 5u(t) + 2e^{-1t} u(t)$

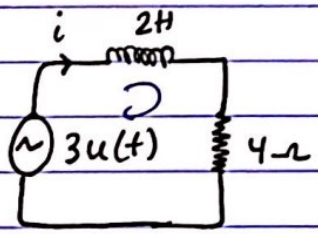
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* Integration Theory

$$\mathcal{L}\left(\int_{-\infty}^{t} f(t).dt\right) = \mathcal{L}\left(\underbrace{\int_{-\infty}^{0} f(t)}_{\text{حلينا قبل أنها constant و initial value}} + \int_{0}^{t} f(t).dt\right)$$

$$= \frac{F(0)}{s} + \frac{F(s)}{s}$$

ex: if $i(0^-) = 5A$
find $i(t)$



write diff equation (KVL)

$$3u(t) = 2\frac{di}{dt} + 4i \quad \text{— ①}$$

Take laplace to ①

$$\mathcal{L}\left(3u(t) = 2\frac{di}{dt} + 4i\right)$$

$$\frac{3}{s} = 2sI(s) - \underbrace{2I(0^-)}_{2*5=10} + 4I(s)$$

$$I(s) = \frac{3/s + 10}{2s+4} \longrightarrow \frac{3+10s}{s(2s+4)}$$

★ وحدت مقامات وبسطات.

↘ slove partial

$$\frac{3+10s}{s(2s+4)} \rightarrow \frac{A}{2s} + \frac{B}{s+2} \Longrightarrow A(s+2) + 2Bs = 3+10s$$

$$As + 2A + 2Bs = 3 + 10s$$

$$2A = 3 \rightarrow A = 1.5$$

$$A + 2B = 10 \rightarrow B = 4.25$$

$$I(s) = \frac{1.5}{2s} + \frac{4.25}{s+2}$$

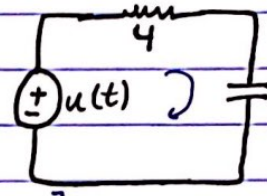
$$i(t) = 0.75\,u(t) + 4.25e^{-2t}u(t).$$

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EX: $v(0^-)$ [initial value] = 9 v, Find $i(t)$

write KVL equation

$$\mathcal{L}\left(u(t) = 4i(t) + \frac{1}{1/16}\int_{-\infty}^{t} i(t)\,dt\right)$$



$$\frac{1}{s} = 4I(s) + 16\,\mathcal{L}\left[\int_{-\infty}^{0} i(t) + \int_{0}^{t} i(t)\right]$$

ثابت عبارت 16 ← initial value

$$\frac{1}{s} = 4I(s) + \frac{V(0)}{s} + 16\frac{I(s)}{s}$$

$\mathcal{L}$ integ

$$\mathcal{L}\,16\int_{-\infty}^{0} i(t) = V(0)$$

$$\frac{1}{s} - \frac{9}{s} = \left(4 + \frac{16}{s}\right) I(s)$$

$$I(s) = \frac{-8}{4s+16} = \frac{-2}{s+4}$$

$$i(t) = -2e^{-4t}u(t)\ A$$

CH 14.7 # Initial value and final value

→ Initial value $f(0) = \lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$.

→ Final value $f(\infty) = \lim_{t\to\infty} f(t) = \lim_{s\to 0} sF(s)$.

ex: $f(t) = (1 - e^{-at})\,u(t)$ find $f(0)$, $f(\infty)$.

Initial value → $f(0) = 0$

Final value → $f(\infty) = 1$

ex: $F(s) = \frac{1}{s} - \frac{1}{s+a}$

Initial value → $f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty}\left(1 - \frac{s}{s+a}\right) = \lim_{s\to\infty}\left(1 - \frac{1}{1+a/s}\right) = 0$

final value → $f(\infty) = \lim_{s\to 0} sF(s) = \lim_{s\to 0}\left(1 - \frac{s}{s+a}\right) = 1$

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ex: $F(s) = \dfrac{5s^2+10}{2s(s^2+3s+5)}$

$$f(0) = \lim_{s\to\infty} sF(s) = \lim_{s\to\infty} \frac{5s^2+10}{2(s^2+3s+5)}$$

$$= \lim_{s\to\infty} \frac{5 + {}^{10}/_{s^2}}{2\left(1+\frac{3}{s}+\frac{5}{s^2}\right)} = F(0) = \frac{5}{2}$$

$$F(\infty) = \lim_{s\to 0} sF(s) = \frac{10}{10} = 1$$

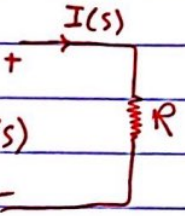
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chapter (15): circuit Analysis in S-Domain

* Sinusoidal signals is a special case of S-Domain where $\sigma = 0$ $s = \pm j\omega$.

* Models for R, L, C in s-Domain *

▶▶ For R:



$$\mathcal{L}\{v(t) = R\,i(t)\} \rightarrow \boxed{V(s) = R\,I(s)}$$

Impedance in s-Domain → $R = \dfrac{V(s)}{I(s)} \Rightarrow Z$

▶▶ For L:

initial value.

$$\mathcal{L}\{v_L(t) = L\frac{di(t)}{dt}\} \rightarrow \boxed{V_L(s) = L\left[s\,I_L(s) - I(0^-)\right]}$$

$$V_L(s) = L\,s\,I(s) - L\,I(0)$$

* we need to find a model for the inductor (a relation between the voltage and the current) so for now we will suppose $\boxed{I(0) = 0}$.

* Remember that $s = j\omega$. (FYI).

$$V_L(s) = L\,s\,I(s)$$

$$\boxed{\frac{V_L(s)}{I(s)} = Ls = Z_L}$$

($j\omega L$)

Impedance for Inductor in s-Domain.

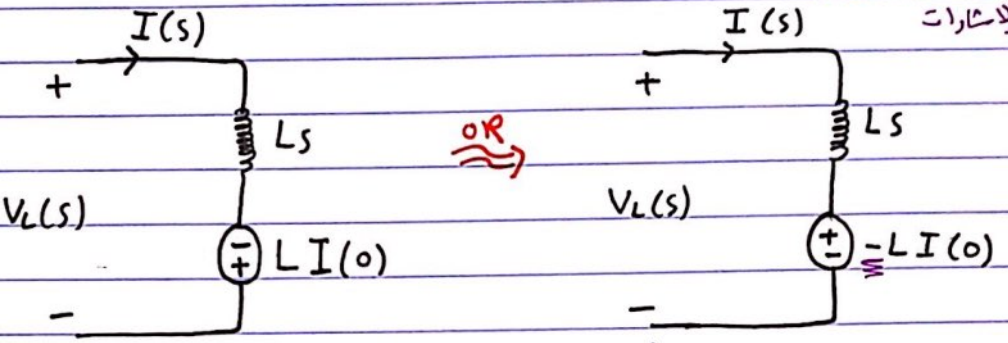
→ when i have an initial value & we need to treat it as a source and put it in our voltage or current models.

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now $V_L(s) = \underbrace{L s\, I(s)}_{\text{Voltage}} - \underbrace{L I(0)}_{\text{Voltage}}$ ... ①

← متى بجمع فولتية؟ ← لما يكونو على التوالي لأنو بتوزع.

★ الهدف إنو لما امشي بالسيركيت تطلع معي نفس المعادلة الأصلية فمركز ★ عالإشارات



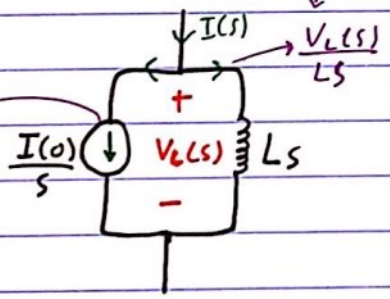
This i's called
#Voltage-Model#

---

or $\underbrace{I(s)}_{\text{current}} = \underbrace{\frac{V_L(s)}{Ls}}_{\text{current}} + \underbrace{\frac{I(0)}{s}}_{\text{current}}$

← متى بجمع تيارات؟ ← لما يكونو على نود (توازي) وعلى هاد الأساس بعمل الـ model.

* تيار عبارة عن تيار + تيار مش تيار زائد خولتيج يعني انتبه هون $\frac{I(0)}{s}$ هو تيار.



اتجاه التيار هيك لأنو لازم يطابق المعادلة وفيها ⊕ يعني لازم تكو بنفس اتجاه التيار.

This is called
#Current-model#

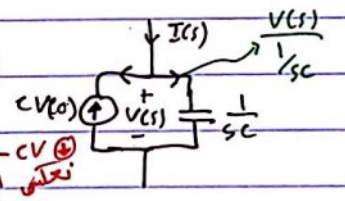
▶▶ For C:

$$\int i_c = \int c \frac{dv_c}{dt} \qquad I_c(s) = CsV_c(s) - CV_c(0^-)$$

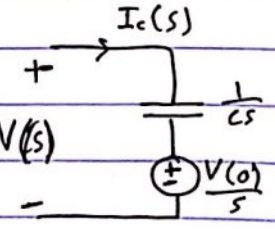
* If $V(0^-) = 0 \rightarrow I_c(s) = CsV(s)$

$$\frac{V(s)}{I(s)} = \frac{1}{sC}$$

$\frac{1}{j\omega C}$



current model



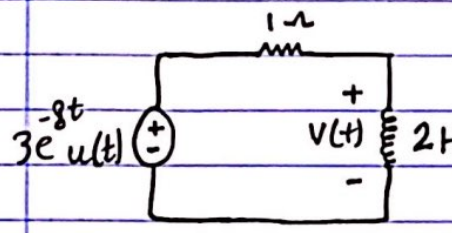
voltage model

عشان اطلع الـ voltage model

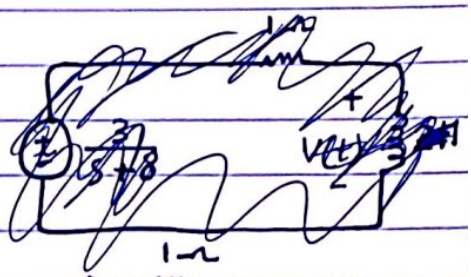
$$V(s) = \frac{I(s)}{Cs} + \frac{CV(0^-)}{Cs}$$

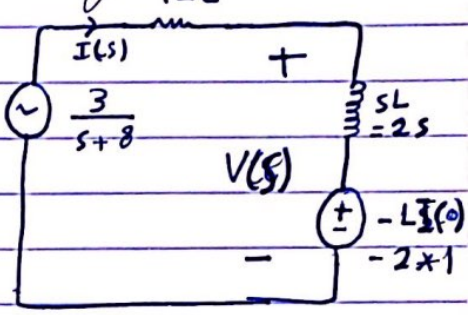
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Example: Find $V(t)$; $i(0) = 1A$



first thing to do since the source has that signal is switch to the s-Domain.





Voltage model

$$\frac{3}{s+8} = 2sI(s) + I(s) - 2$$

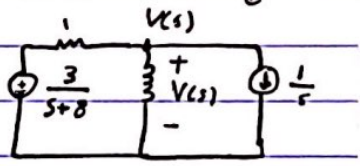
$$I(s) = \frac{\left(\frac{3}{s+8}\right)+2}{2s+1} = \frac{3+2(s+8)}{(2s+1)(s+8)}$$

$$V(s) = 2sI(s) - 2$$

$$V(s) = \frac{2s-8}{(s+8)(s+0.5)} = \frac{A}{s+8} + \frac{B}{s+0.5}$$

$$V(t) = (3.2e^{-8t} - 1.2e^{-0.5t})\,u(t)$$

OR



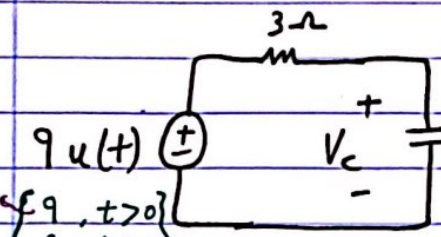
current model

$$\frac{V-\frac{3}{s+8}}{1} + \frac{V}{2s} + \frac{1}{s} = 0$$

← وبكمل الحل بطلع نفس الناتج

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Example: find $V_c(t)$ if $V_c(0^-) = -2V$



عند ∞ $\begin{cases} 9, & t>0 \\ 0, & t<0 \end{cases}$

دائمًا لازم يكون في تغير على source عشان نطلع initial value مثلًا
يعني صون من V.S → S.C

$V_c(0^-) = V_c(0) = V_c(0^+)$
$I_L(0^-) = I_L(0) = I_L(0^+)$

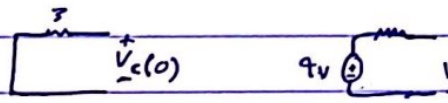
if we didn't have initial value I have to calculate it ⇒

time
$t < 0 \Rightarrow t = 0^-$
طبعًا ما في زمن اقل من صفر (بالسالب) ف (0^-) يعني اقل من الصفر بشوية.

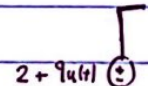
initial value → $t<0$: L → S.C, C → O.C

final value → $t=\infty$: L → S.C, C → O.C

~~final~~ → Usually The source at $t=0^-$ = 0, unless we have a different source that is not a function of t OR in this example if we had $9u(t)+1$ in that case the source = 0+1=1

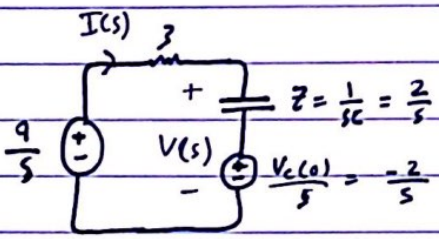


$V_c(0) = 0$ initial value
$V_c(\infty) = 9V$ final value

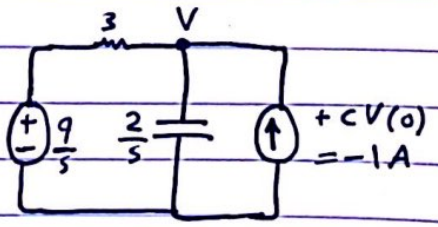


final value → 11
initial value → 2

حل:



* عالأغلب طاي اسهل لأنو بتطالع معي معادلة وحدة



$V(s) = V_c + \frac{-2}{s}$

$V(s) = \frac{2\,I(s)}{s} + \frac{2}{s}$

$I(s) = \frac{\frac{9}{s} - (-\frac{2}{s})}{3 + \frac{2}{s}}$

$V(s) = \frac{9}{s} - \frac{11}{s+2/3}$

$V(t) = 9u(t) - 11e^{-\frac{2}{3}t}u(t)$

$= (9 - 11e^{-\frac{2}{3}t})u(t)$

$V(\infty) = 9 \quad V(0) = 9 - 11 = -2$

لو اعوض ∞ بالمعادلة إلي طلعت معي بطلع $V(\infty) = 9$ وصيك بعرف إنو حلي صح.

$\frac{V - 9/s}{3} + \frac{V}{2/s} - (-1) = 0$

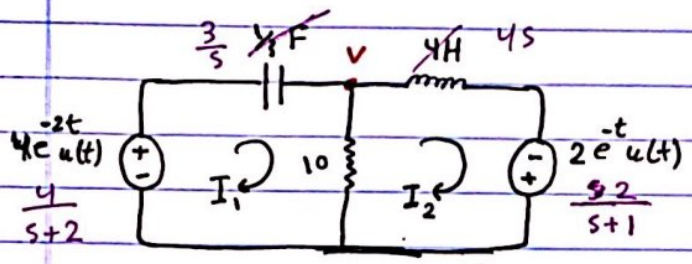
$V = ???$

Voltage model.

current model.

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ex:



find I_1 if there is (no stored energy.)
initial value يعني ما في

$I_L(0) = 0 \qquad Z_c = \frac{1}{sc}$

$V_c(0) = 0 \qquad Z_L = sL$

Mesh → $\frac{4}{s+2} = \frac{3}{s} I_1 + 10(I_1 - I_2) = 0 \quad \ldots ①$

$4s I_2 - \frac{2}{s+1} + 10(I_2 - I_1) = 0 \quad \ldots ②$

$$I_1 = \frac{2s(4s^2 + 19s + 20)}{20s^4 + 66s^3 + 37s^2 + 57s + 30}$$

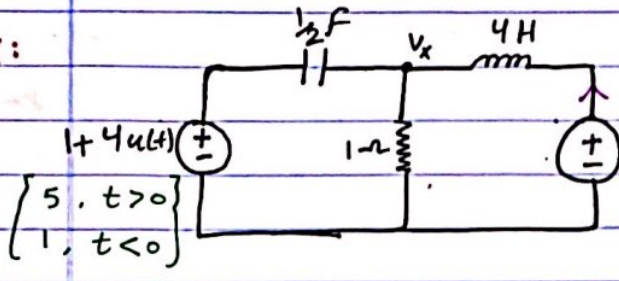
* طبعًا ما بيجي هيك لأنو الحل بدو وقت كتير.

Nodal → $\frac{V - \frac{4}{s+2}}{3/s} + \frac{V}{10} + \frac{V + \frac{2}{s+1}}{4s} = 0$

$$I_1 = \frac{\frac{4}{s+2} - V}{3/s}$$

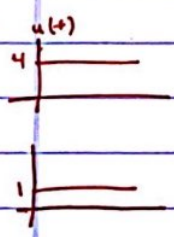
Scanned with CamScanner

ex:



▶ find $V_x(t)$

* السؤال ما ذكر وانو ماني initial value أو انو no stored energy يعني لازم احسبها.

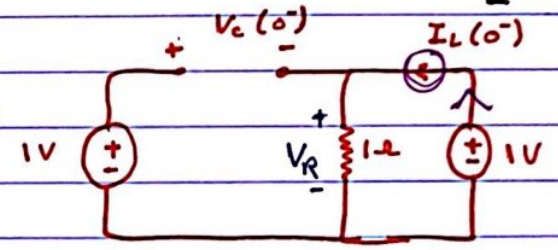


$t<0 \rightarrow \boxed{t=0^-}$

$V_c(0^-) = V_c(0) = V_c(0^+)$

$I_L(0^-) = I_L(0) = I_L(0^+)$

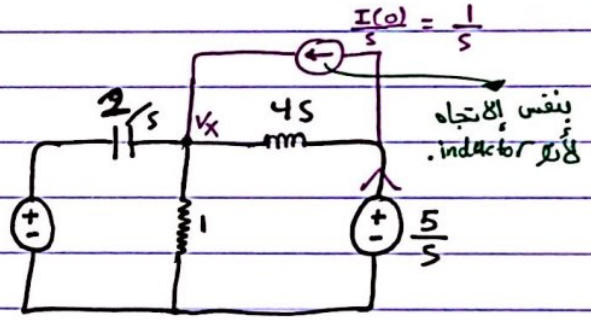
C: O.C
L: S.C



* if i did KVL → $V_c(0^-) = 1 - 1 = 0V$

$(-1 + V_c + V_R = 0)$

$I_L(0^-) = \frac{1}{1} = 1A$



$$\frac{V_x - \frac{5}{s}}{\frac{2}{s}} + \frac{V_x}{1} + \frac{V_x - \frac{5}{s}}{4s} - \frac{1}{s} = 0$$

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CH 15.4 # Transfer function (H(s)), Zeros and poles.

→ The ratio between Input and output ~~voltage~~ in S-Domain.

$$H(s) = \frac{\text{output}(s)}{\text{input}(s)}$$

- $H(s) = \frac{V_o(s)}{V_{in}(s)}$
- $H(s) = \frac{V_o(s)}{I_{in}(s)}$
- $H(s) = \frac{I_o(s)}{V_{in}(s)}$
- $H(s) = \frac{I_o(s)}{I_{in}(s)}$

⏩ H(s) has Zeros at $s = s_o$ if $H(s_o) = 0$ — اصفار البسط

⏩ H(s) has poles at $s = s_o$ if $H(s_o) = \infty$ — اصفار المقام

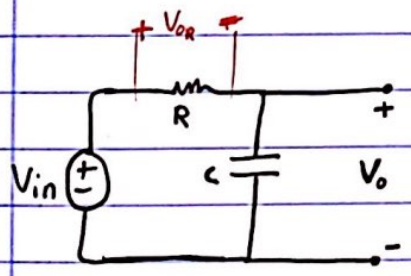
$\frac{\text{عدد}}{\text{صفر}} = \infty$

ex: * $H(s) = \frac{7}{s(3s^2 - 9s + 6)}$, find poles and Zeros.

$$H(s) = \frac{7}{s(3s-3)(s-2)} \Rightarrow \text{Zeros} \rightarrow s = \infty$$

poles → $s = 0$, $3s - 3 = 0 \rightarrow s = 1$, $s - 2 = 0 \rightarrow s = 2$

ex:



▶ find $H(s) = \frac{V_o(s)}{V_{in}(s)}$

★ بالسؤال لازم يحدد الـ input والـ output. حسب شو طالب.

using voltage division → $V_o(s) = \frac{Z_c}{Z_c + R} V_{in}$

$$V_{o_c}(s) = \frac{\frac{1}{sc}}{\frac{1}{sc} + R} V_{in}$$

$$V_{o_R}(s) = \frac{R}{R + \frac{1}{sc}} V_{in}(s)$$

$$H(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{1}{1 + RCs}$$

Zeros → $H(s) = 0 \rightarrow s = \infty$ ← zero.

poles → $H(s) = \infty \rightarrow 1 + RCs = 0$

pole → $s = -\frac{1}{RC}$

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CH 16.6 : # Bode plot ⇒ sketch of the phase and magnatude of a given transfer function as a function of frequency in jw plane.

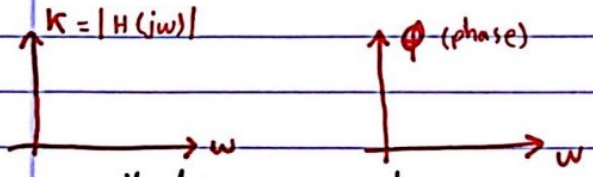
$H(s) \longrightarrow H(jw)$

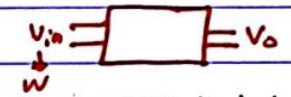
$H(jw) = A + jB$

$H(jw) = \frac{V_o(jw)}{V_{in}(jw)} = K \angle \phi$ (θ, K مجهول بدلالة frequency الـ (w))

$K = |H(jw)| = \sqrt{A^2 + B^2}$

$\phi\theta = \tan^{-1}\frac{B}{A}$





* كل ما غيرنا الـ frequency (w) بتغير الـ Impedance وبتغير الحل.

transfer function بطلع الـ وال w مجهولة فبرسم بدلالتو.

مثلاً ممكن يطلع: $H(jw) = \frac{w}{2+w} \angle \tan^{-1} w$, $|H(jw)| = \frac{w}{2+w}$, $HdB = 20 \log \frac{w}{2+w}$

$|H(jw)| \longrightarrow$ dB (decibels).

★ $HdB = 20 \log_{10} |H(jw)|$

- $\log_{10} 1 = 0$
- $\log_{10} 10 = 1$
- $\log_{10} 100 = 2$
- $\log_{10} 0.1 = -1$

ex: $H(s) = K$ plot magnitude and phase Bode plot.

$H(jw) = K \longrightarrow$ constant

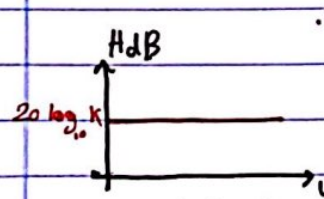
$H(jw) = K \angle 0$

$|H(jw)| = K$

$\phi = 0 = \tan^{-1}\frac{0}{K}$ ماني w.

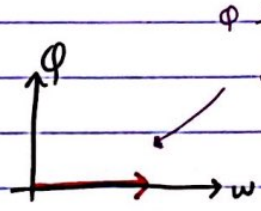
ϕ : is the phase for the transfer function $(\frac{V_o}{V_{in}})$ not for the Inductor or capacitor alone.

$HdB = 20 \log_{10} K \longrightarrow$ constant. (ما طلع بدلالة (w) و ماإلو علاقة بـ (w)).



Magnitude BP constant with (w).

* شو ما غيرت w الـ φ ما بتتغير بتظل 0.



phase Bode plot.

* لو أعطاني قيمة لـ K بس بعوضها وبحسبها.

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ex: $H(s) = s$, plot magnitude and phase Bode plot ?

$H(jw) = jw \Rightarrow H(jw) = w \angle 90^\circ$

$|H(jw)| = w \qquad \varphi = 90^\circ$

$HdB = 20 \log_{10} w$

بيلش من عند الـ 1 عشان يصفر الـ dB عشان ابلش الرسم من عند 0.

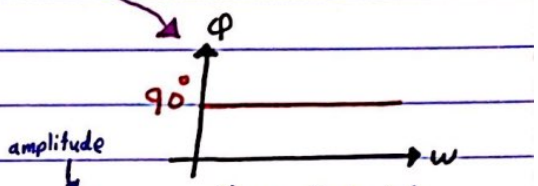
بعدين باخز مضاعفات لـ 10 (بضرب أو بقسم على 10) وكأنو بهاي الطريقة بكون عنا scale واحد وبسهل علينا الرسم.

- $w = 1 \rightarrow HdB = 0\,dB$
- $w = 10 \rightarrow HdB = 20 \log 10 = 20\,dB$ (x10)
- $w = 100 \rightarrow HdB = 20 \log 100 = 40\,dB$ (x10)
- $w = 0.1 \rightarrow HdB = 20 \log 0.1 = -20\,dB$
- $w = 0.01 \rightarrow HdB = 20 \log 0.01 = -40\,dB$

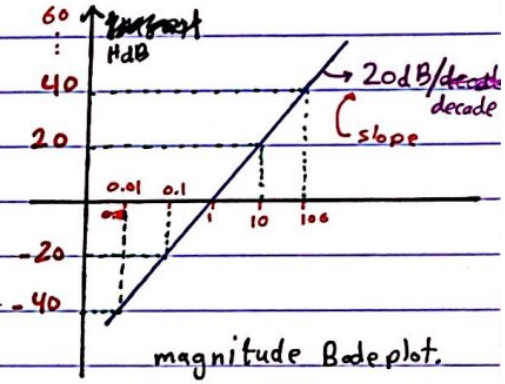
$\varphi = \tan^{-1} \frac{w}{0} = \infty \rightarrow \varphi = \frac{\pi}{2} = 90^\circ$

Slope $\Rightarrow$ 20 dB/decade.

↳ 10 (مضاعفات الـ 10).



phase Bode plot.



magnitude Bode plot.

ex: $H(s) = s^2$

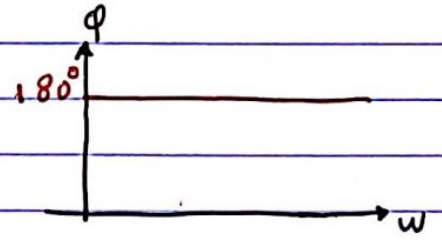
$H(jw) = (jw)^2 = -w^2$

$H(jw) = w^2 \angle 180^\circ$

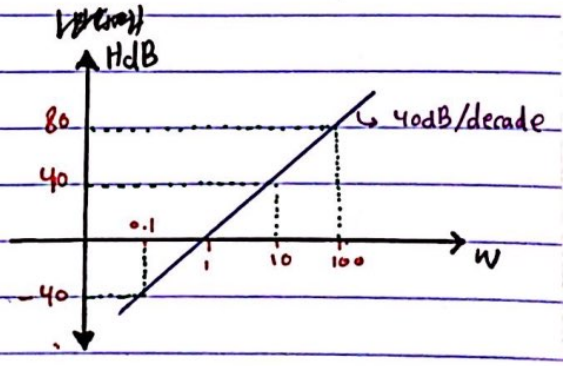
$|H(jw)| = w^2 \qquad \varphi = 180^\circ$

$HdB = 20 \log w^2$

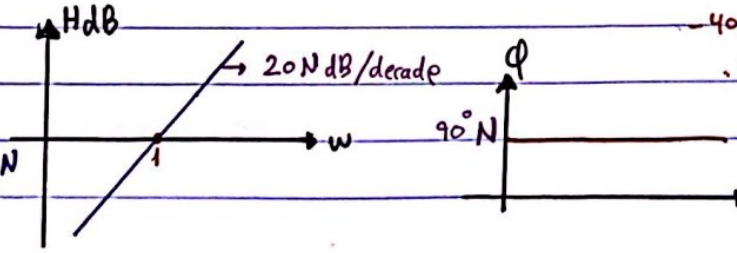
$\Rightarrow HdB = 40 \log w$



- $w = 0.1 \rightarrow HdB = 40 \log 0.1 = -40\,dB$
- $w = 1 \rightarrow HdB = 40 \log 1 = 0\,dB$
- $w = 10 \rightarrow HdB = 40 \log 10 = 40\,dB$
- $w = 100 \rightarrow HdB = 40 \log 100 = 80\,dB$



In general $\Rightarrow$ for $H(s) = s^N$

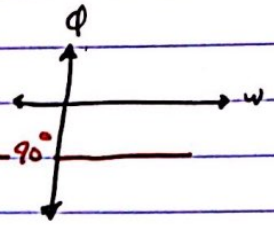


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ex: $H(s) = \frac{1}{s}$

$H(jw) = \frac{1}{jw}$ $\quad H(jw) = \frac{1}{w} \angle -90^\circ$

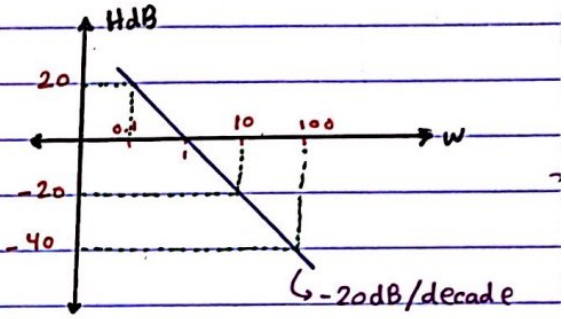
$|H(jw)| = \frac{1}{w}$ $\quad \phi = -90^\circ$



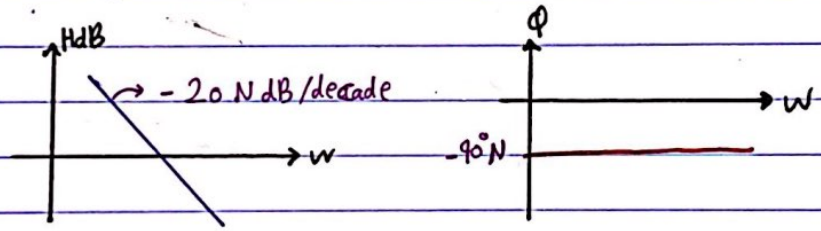
$HdB = 20 \log \frac{1}{w} \rightarrow HdB = 20 \log 1 - 20 \log w = -20 \log w$

↳ رح يطلع الـ slope متناقص.

- $w = 0.1 \rightarrow HdB = 20$ dB
- $w = 1 \rightarrow HdB = 0$ dB
- $w = 10 \rightarrow HdB = -20$ dB
- $w = 100 \rightarrow HdB = -40$ dB



In general → $H(s) = \frac{1}{s^N}$



---

ex: $H(s) = 1 + \frac{s}{a}$

Rectangular form. ← $H(jw) = 1 + j\frac{w}{a}$ // polar form → $H(jw) = \sqrt{1 + \frac{w^2}{a^2}} \angle \tan^{-1}\frac{w}{a}$ $\quad |H(jw)| = \sqrt{1 + \frac{w^2}{a^2}}$

$\phi = \tan^{-1}\frac{w}{a}$

$HdB = 20 \log \sqrt{1 + \frac{w^2}{a^2}}$

* انتبه اناصون بقارن w مع a^2 مش a.

- $w = a \rightarrow HdB = 20 \log \sqrt{2} \approx 3$dB
- $w \ll a \rightarrow HdB = 20 \log \sqrt{1} = 0$dB
- $w \gg a \rightarrow HdB = 20 \log \frac{w}{a}$

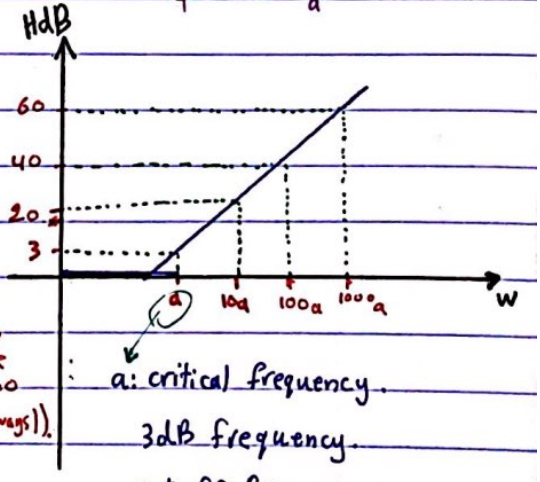
↳ $w = 10a \rightarrow HdB = 20 \log 10 = 20$dB

$w = 100a \rightarrow HdB = 20 \log 100 = 40$dB

$w = 1000a \rightarrow HdB = 20 \log 1000 = 60$dB

frequencyic بحكي كبار يعني الـ w بالنسبة لـ a بكتير.

The Base ↳ (in this example at w=0 (not always)).



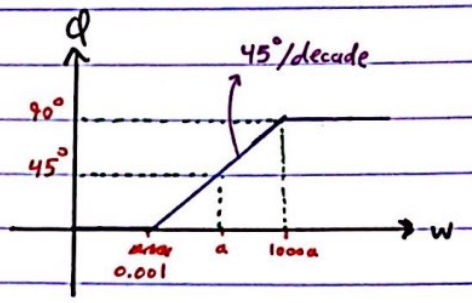
a: critical frequency.

3dB frequency.

cut off frequency.

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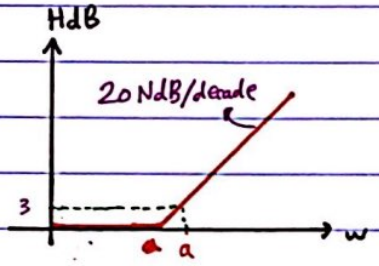
$\tan^{-1}\frac{w}{a}$

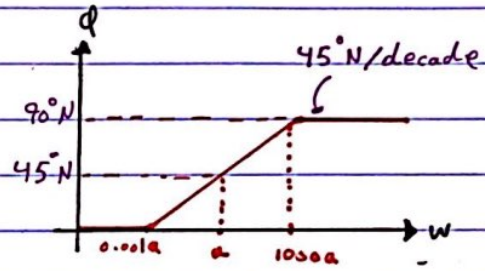


- $w \ll a \rightarrow w \leq 0.001a \rightarrow \varphi \approx \tan^{-1} 0 = 0$
- $w = a \rightarrow \tan^{-1} 1 \rightarrow \varphi = 45°$
- $w \gg a \rightarrow \tan^{-1} 1000 \rightarrow \varphi \approx 90°$

$w \geq 1000a$

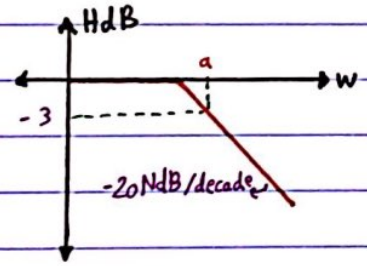
In general: $H(S) = \left(1 + \frac{S}{a}\right)^N$

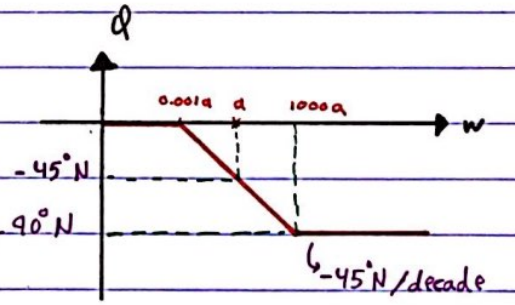




In general: $H(S) = \left(\frac{1}{1 + \frac{S}{a}}\right)^N$

$H(jw) = \left(1 + j\frac{w}{a}\right)^N = \left(\sqrt{1 + \frac{w^2}{a^2}}\right)^N \angle N \tan^{-1}\frac{w}{a}$





ex: $H(S) = 50 + S \rightarrow H(jw) = 50\left(1 + \frac{jw}{50}\right)$

$|H(jw)| = 50\sqrt{1 + \frac{w^2}{50^2}} \qquad \varphi = \tan^{-1} w/50$

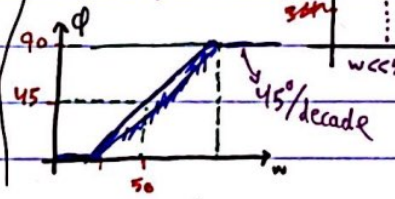
$HdB = 20 \log 50\sqrt{1 + \frac{w^2}{50^2}}$

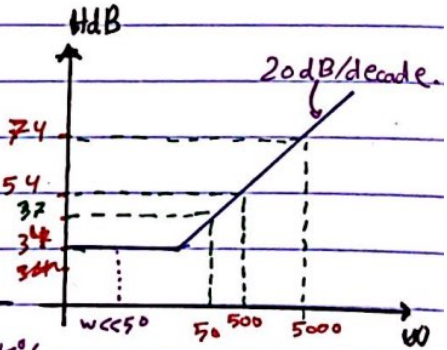
- $w \ll 50 \rightarrow HdB = 20 \log 50 = 34dB$
- $w = 50 \rightarrow HdB = 20 \log 50\sqrt{2} = 37dB$
- $w \gg 50 \rightarrow HdB = 20 \log w$ ← يقدر اعمل ١١ ١٠
  - $w = 500 \rightarrow HdB = 54dB$
  - $w = 5000 \rightarrow HdB = 74dB$

$w \ll 50 \rightarrow \varphi = 0°$

$w = 50 \rightarrow \varphi = 45°$

$w \gg 50 \rightarrow \varphi = 90°$





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ex: $H(s) = \frac{s+10}{s} \Rightarrow 1 + \frac{10}{s}$

$H(jw) = 1 + \frac{10}{jw}$ ← ابتبع لازم اطلعها فوق لأنو رح تسبب اشارة سالب

$\rightarrow H(jw) = 1 - j\frac{10}{w}$ ← السالب رح يكون مهم كثير بحساب ال phase (ϕ).

$|H(jw)| = \sqrt{1 + \frac{10^2}{w^2}}$

$HdB = 20 \log \sqrt{1 + \frac{10^2}{w^2}}$

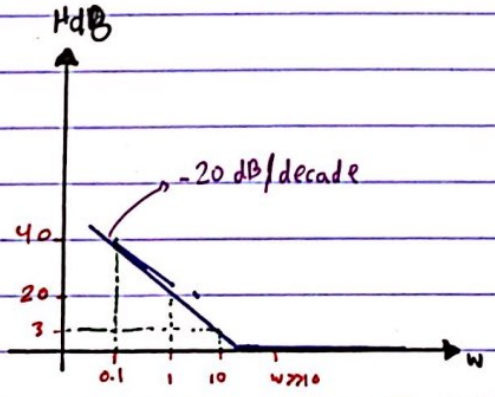
* بهاد السؤال ممكن أحلّي على طريقة الاختصار والتبسيط بحيث يطلع معي نفس الأشكال القبل وممكن أحلّي على 4 طريقة انو اخذ البسط لحال والمقام لحال يعني رسمتين للمقام و رسمتين للبسط وبطرحهم (سيتم التوضيح بالسؤال القادم).

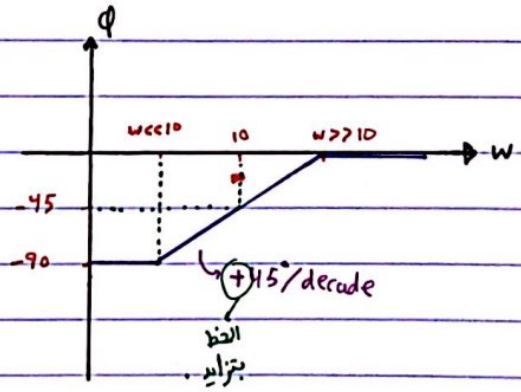
* الأفضل ما أعوض $10^2 \leftarrow 100$ عشان اعرف انو قاعد بقارن مع 10 مش مع 100.

بقدر أحلل الـ ←
- $w \gg 10 \rightarrow (w = 1000) \rightarrow HdB = 0dB$
- $w = 10 \rightarrow HdB = 20 \log \sqrt{2} = 3dB$
- $w \ll 10 \rightarrow (w = 1) \rightarrow HdB = 20dB$
- $(w = 0.1) \rightarrow HdB = 40dB$

$\phi = \tan^{-1}\left(-\frac{10}{w}\right)$

- $w = 10 \rightarrow \phi = -45^\circ$
- $w \gg 10 \rightarrow \phi \approx 0^\circ$
- $w \ll 10 \rightarrow \phi \approx -90^\circ$





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ex: $H(s) = \frac{S+10}{S+100}$

* بهاد السؤال ما بقدر اختصر اشي فلازم اطلع رسمتين البسط ورسمتين المقام واطرحهم.

$H(jw) = \frac{10+jw}{100+jw}$ ← بقدر أخليهم صيك ويمكن اخذ عامل مشترك

$$H(jw) = \frac{10\left(1+j\frac{w}{10}\right)}{100\left(1+j\frac{w}{100}\right)} = \frac{\overset{A}{10\sqrt{1+\frac{w^2}{10^2}}}\ \angle \overset{C}{\tan^{-1}\left(\frac{w}{10}\right)}}{\underset{B}{100\sqrt{1+\frac{w^2}{100^2}}}\ \angle \tan^{-1}\left(\frac{w}{100}\right)_D}$$

* بالقسمة بطرح الـphases يعني (C-D).

$HdB = 20\log\frac{A}{B} = 20\log A - 20\log B$

$$= \underbrace{20\log 10\sqrt{1+\frac{w^2}{10^2}}}_{①} - \underbrace{20\log 100\sqrt{1+\frac{w^2}{100^2}}}_{②}$$

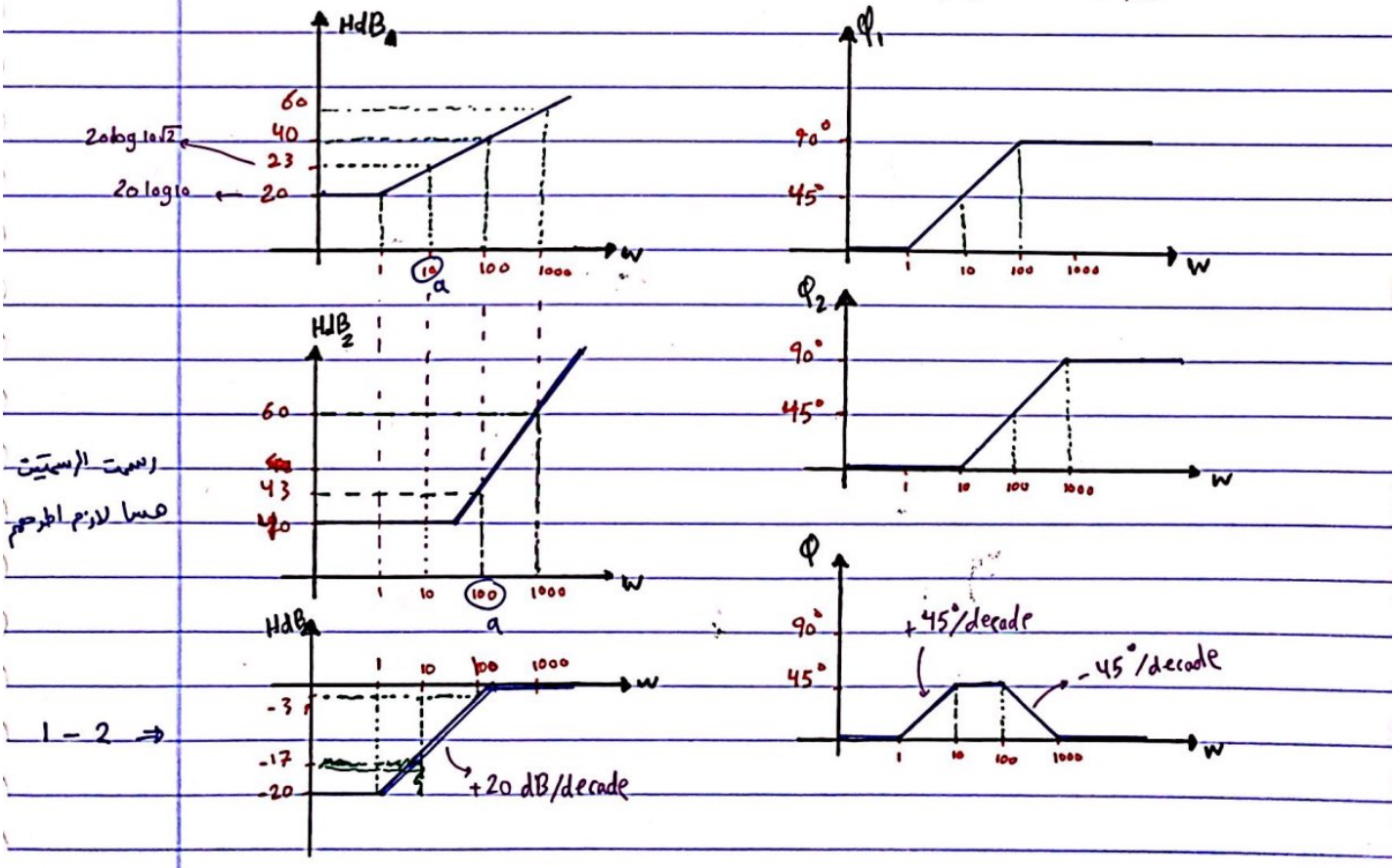
① هاي إلها رسمتها وعارفين كيف نرسمها

② وهاي نفس الإشي

* هون رح نرسم عطول للإختصار بس بالإمتحان لازم نحل بكل التفاصيل زي قبل بعدين بنرسم.

$\varphi = \frac{\tan^{-1}\frac{w}{10}}{\tan^{-1}\frac{w}{100}}$

$\varphi = \tan^{-1}_1\frac{w}{10} - \tan^{-1}_2\frac{w}{100}$



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CH 16.7 # Filters :-

* filters are used to select or reject a band of frequency



filter types according to components :-

1) passive filters (R, L, C) ← ~~دراستنا رح تكون~~ عن الـ passive فقط.

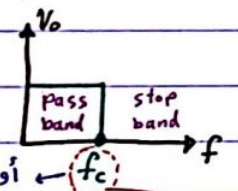
2) Active filters (R, L, C) + Op-Amp ▷ v_o

filter types according to Frequency :-

1) Low pass filter (LpF) :-

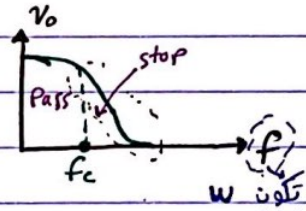
* اذا بنغير الـ source الـ frequency من 0 ← ∞ وبشوف العلاقة بينو وبين أي ~~شي~~ فولتية اذا بدي ~~اي~~ اياها.

* ما بنغير فولتية الـ source اذا بنغير الـ frequency وبالتالي بتغير الفولتيات والتيارات إلي على الـ impedance بس مش المصادر نفسو.



Ideal ← بالواقع هالإشي مش موجود.

critical frequency.



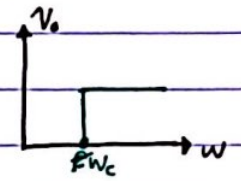
ممكن تكون w وما يختلف إشي.

practical ← ما بوصل فجأة للصفر، بوخذ ثوانٍ وقت.

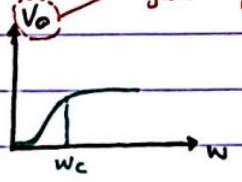
* اذا كانت الفريكونسي من صفر لقيمة معينة بتعطيني قيمة بعرف إنو هاد (LPF). ومش صفر. (output)

2) High pass filter (HPF) :-

* هاي الرسمات بتكون للـ response مش Bode plot



Ideal



practical

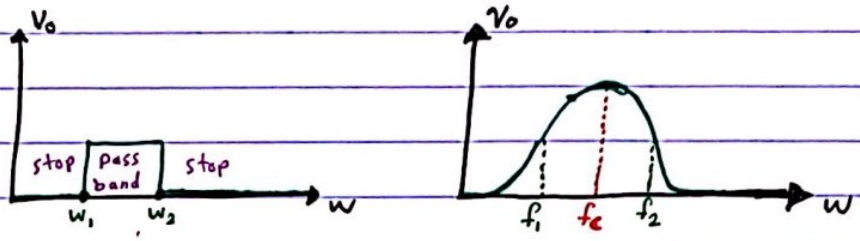
LPF passes through the low frequencies

HPF passes through the high frequencies.

* RC, RL circuits gives the (LPF) and ~~(High)~~ (HPF).

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3) Band pass filter (BpF):-

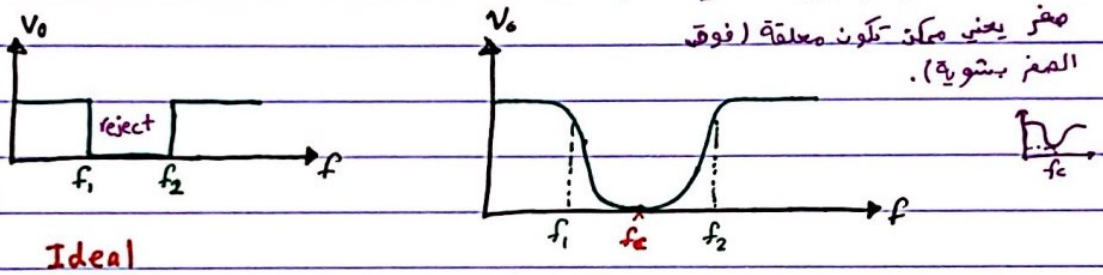


Ideal practical

* مرة تانية هاي الرسمات هي الـ Response مش الـ Bode plot بس بنقدر نرسم الـ Bode plot.

3) Band stop filter (BSF), or: Band Reject filter (BRF):-

* الـ fc مش بالضرورة تساوي صفر يعني ممكن تكون معلقة (فوق الصفر بشوية).



Ideal

Practical

* قبل f_1 في فولتية وبعد f_2 في فولتية.

* الـ (BPF) و(BSF) مكملين لبعض يعني السيركيت إلي بتعطي الـ (BPF) هي نفسها إلي بتعطي (BSF) بس من جهة تانية.
(ونفس الحكي للـ (LPF) و (HPF).

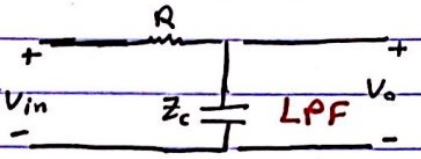
* طب كيف أعرف شو نوع الـ filter الحي؟

- بختبر الفولتية مع الـ frequency.
- بطلع الـ Transfer function وبرسم الـ Bode plot ومن خلاله بعرف

هاي الطريقة صعبة خاصة اذا كان معنا سيركيت معقدة فعشان هيك رح نستخدم الطريقة الأولى.

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RC circuit :-



$V_o = \frac{Z_c}{Z_c + R} V_{in}$ → Voltage division to find the transfer function.

$\frac{V_o}{V_{in}} = \frac{1}{1 + jwcR}$ → Transfer function

صارت على صيغة → $\frac{1}{1 + s/a}$ → $a = 1/CR$ ← 3dB

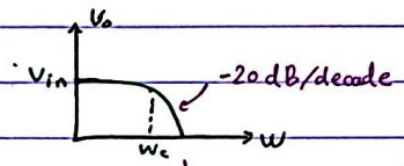
وبعدين بقدر ارسم الـ Bode plot زي ما تعلمنا قبل. بس زي ما قلنا بدنا نحل عن طريق اختبار الـ frequency :→

$* \frac{V_o}{V_{in}} = \frac{1}{1 + j\frac{R}{X_c}}$

$w = 0 \rightarrow V_o = V_{in}$

$w = \infty \rightarrow V_o = 0 \rightarrow (1/\infty)$

بالبداية كان في output وعند الـ ∞ صار 0 ، فأنا عطول بعرف وانو (LPF).



a (critical frequency).

LPF

$|1 + jwcR| = \sqrt{2}$

$\sqrt{1^2 + w^2c^2R^2} = \sqrt{2}$

عشان تتحقق المعادلة

$w_c = \frac{1}{RC}$ or $f_c = \frac{1}{2\pi RC}$

هيدا لازم اطلع الـ critical frequency وانا بعرف عي الـ 3dB يعني لازم تكون على صيغة $20\log\sqrt{2}$ بس كيفك

$\tau = RC$

$w_c = \frac{1}{\tau}$ ← هاي مش ثابت كاباستور هاي cutoff frequency.

$HdB = 20\log(\frac{1}{\sqrt{2}}) = -3\,dB = w_c$

* إشي طبيعي يكون سالب عشان الـ slope متناقص.

OR:

I can also test the impedance ~~instead~~ instead of the frequency :-

وهاي اسهل

$\Rightarrow w = 0 \rightarrow Z_c = \frac{1}{jwc} = \infty \rightarrow$ open circuit $\Rightarrow V_o = V_{in}$ ← from the ecct.

$\Rightarrow w = \infty \rightarrow Z_c = 0 \rightarrow$ short circuit $\Rightarrow V_o = 0$

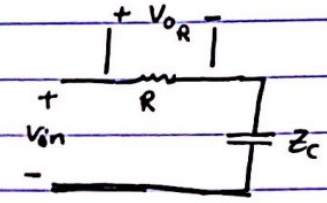
RC → LPF if V_o at the capacitor only

at $w = w_c \rightarrow X_c = R$

$X_c = \frac{1}{wc}$

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RC circuit:-



→ The same as before but the output is on the resistor instead of the capacitor.

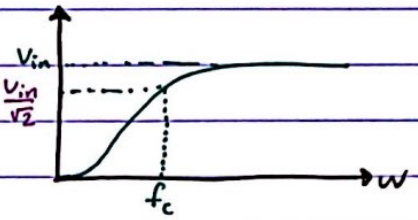
if $w = 0 \rightarrow Z_c$ open circuit → There will be no current.

$V_o = 0$

if $w = \infty \rightarrow Z_c$ short circuit

$V_o = V_{in}$

$w = w_c = \frac{1}{RC} \rightarrow V_o = \frac{1}{\sqrt{2}} V_{in}$

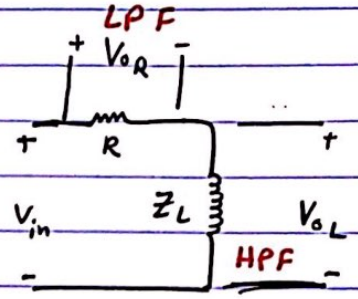


RC → HPF if the output is on the Resistor only. HPF

for RC → $f_c = \frac{1}{2\pi RC}$

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RL circuit



if $w = 0$

$Z_L = jwL = 0$ → short circuit.

$V_{oL} = 0$

$V_{oR} = V_{in}$

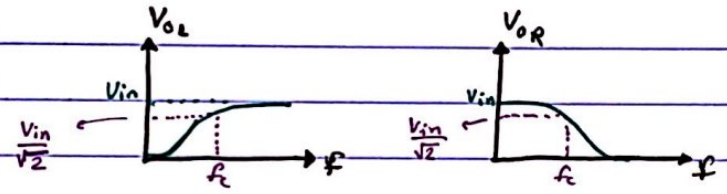
* دايمًا الفولتيج على سلك فاضي بكون صفر.

if $w = \infty$

$Z_L = jwL = \infty$ → open circuit. → (no current).

$V_{oL} = V_{in}$

$V_{oR} = 0$ ← ماشي تيار كأنو



V_{oL} HPF V_{oR} LPF

at $w = w_c$ → $V_o = \frac{1}{\sqrt{2}} V_{in}$

$w_c = \frac{R}{L} = \frac{1}{\tau}$ $\tau_L = \frac{L}{R}$

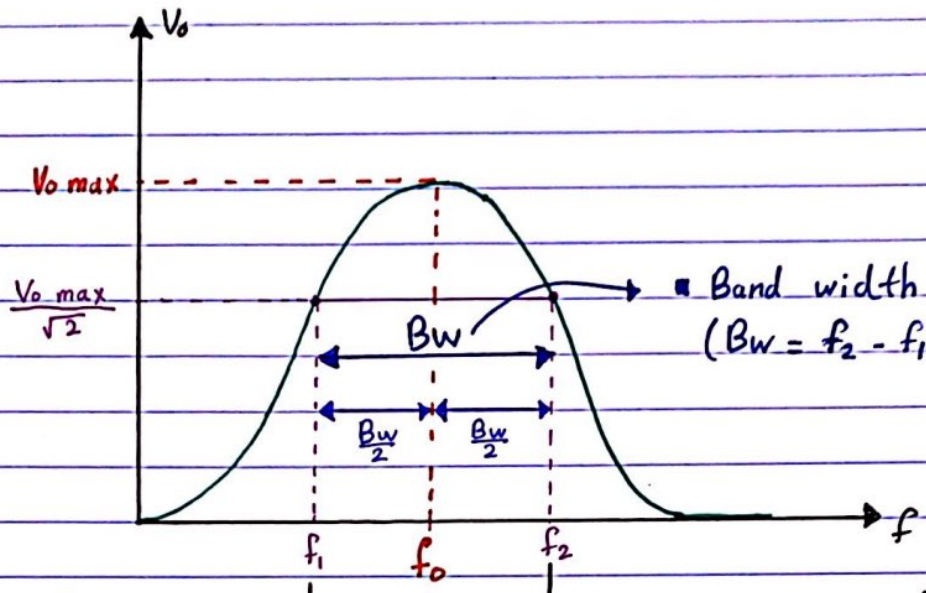
$f_c = \frac{R}{2\pi L}$

if $w = w_c$ → $X_L = R$

(RC) أفضل من (RL) لأن (L) دايمًا بأثر بمجال مغناطيسي وله مقاومة ومحجوم كبير مقارنة مع الـ (C). معه series ~~sirfies~~

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Band pass filter



- Band width ($Bw = f_2 - f_1$).

corner frequency (3dB).

center frequency (Resonant frequency).

هي الـ (f) التي تجعل الـ (Impedance) يعني [pure R] (Imag part = 0).

$$\# f_o = f_1 + \frac{Bw}{2}$$

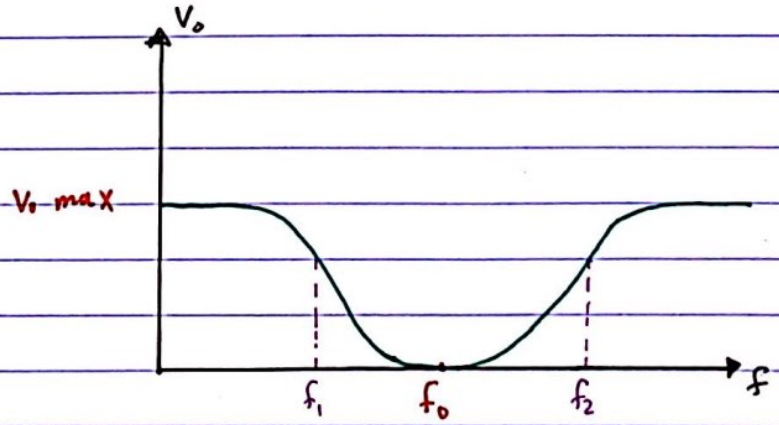
OR:

$$\# f_o = f_2 - \frac{Bw}{2}$$

$$\# f_o = \frac{f_1 + f_2}{2}$$

Band stop filter

نفس المسميات والحسابات فوق.

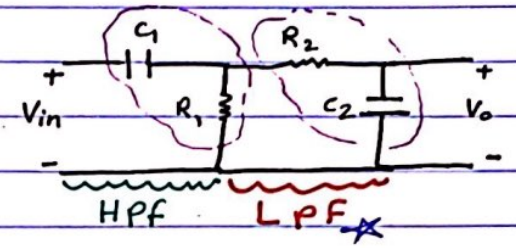


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⇒ How to get Band pass filter (BPF)

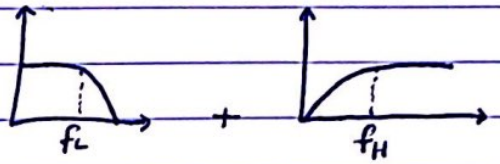
① LPF + HPF

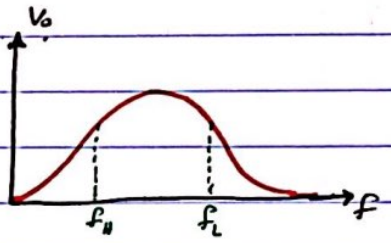
يعني بضيف LPF
و HPF ورا بعض ومش
مهم الترتيب المهم هو الشرط التالي:



$f_L > f_H$

- $f_L = \frac{1}{2\pi R_2 C_2}$
- $f_H = \frac{1}{2\pi R_1 C_1}$





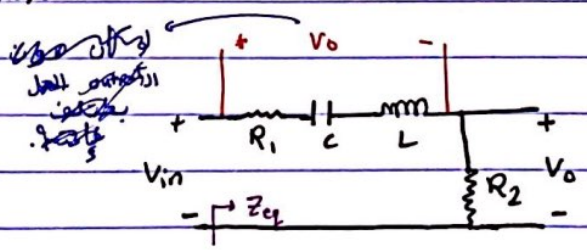
BPF

- $w = 0$ ← بعدين بسأل حالي شو الresponse إلي بيلش بصفر وبنتهي بصفر؟ إلي هو BPF.
  - $X_{C_1} = \frac{1}{wc} = \infty$ o.c
  - $X_{C_2} = \frac{1}{wc} = \infty$ o.c
  - $V_o = 0$
- $w = \infty$
  - $X_{C_1} = \frac{1}{wc} = 0$ s.c
  - $X_{C_2} = \frac{1}{wc} = 0$ s.c
  - $V_o = 0$ ← حكينا الفولتيج على سلك فاضي = 0.

★ Note: عرفت إنو Lpf مناك محلها لأنو الـ c موصولة مع الـ V_o.

② series and parallel RLC (Resonant circuits).

★ series RLC
at Resonant frequency (f_o).
Z_{eq} = Real

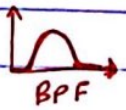


لو أخذت الـ output من عند الـ L,C
كلشي رح يطلع نفسو بس الفرق إنو عندو بهير
BSF بدل BPF وبدل ما كان $V_{o\,max}$ عند
الـ f_o بتصير $V_{o\,min}$ بالـ BSF
وممكن تكون صفر.



	$w = 0$	$w = \infty$
فتحة	L: s.c	L: o.c
نوع	c: o.c	c: s.c
filter ال	$V_o = 0$	$V_o = 0$

ما في تيار.



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$$Z_{eq} = R_1 + \frac{1}{jwC} + jwL + R_2$$

$$= R_1 + R_2 + j\underbrace{[wL - 1/wC]}_{X \rightarrow \text{imaginary part}}$$

بنعرف إنو عند الResonant ← at $f = f_o \rightarrow X = 0$

الـ $\text{imag}_{\text{part}}$ = 0. ← $L, \; w_o = \frac{1}{\sqrt{LC}} \rightarrow f_o = \frac{1}{2\pi\sqrt{LC}}$

هون بشيل الـ L و C لأنهم بلغو بعض بس مشي 0.5 إنتبه، فيضل عندي المقاومات. ← at $\boxed{f = f_o}$, V_o max

$$\boxed{V_o \max = \frac{R_2}{R_1 + R_2} V_{in}}$$

* هاي في حال كان في R_1 اذا ماكو خلص $V_o = V_{in}$. (عند الـ resonant فقط طبعًا).

Now, $\boxed{\text{To find } f_1, f_2}$ → corner frequencies.

we must find Bw.

▶▶ Quality factor (Q):- → $\frac{\text{total stored energy}}{\text{total dissipated energy}}$

$$Q = \frac{X_L}{Req} = \frac{w_o L}{R_1 + R_2}$$ ← ما إلها وحدة.

في حالة الـ series بنعتمد على تيار الإندكتور أكثر عشان هيك طلع معي القانون بدلالة L فقط.

$$Bw = \frac{w_o}{Q} = \frac{w_o}{\frac{w_o L}{R}} = \frac{R}{L} \quad (rad/s).$$ ← عبارة عن frequency.

OR:

$$Bw = \frac{f_o}{Q} = \frac{f_o}{\frac{2\pi f_o L}{R}} = \frac{R}{2\pi L} \quad (Hz).$$

■ $f_1 = f_o - \frac{Bw}{2}$ ■ $f_2 = f_o + \frac{Bw}{2}$

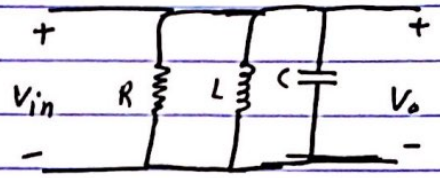
▶▶ 3dB frequency? → الأوتبوت عندهم

$$V_o|_{f_1, f_2} = \frac{V_o \max}{\sqrt{2}}$$

///

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☆ Parallel RLC



BPF

find $Z_{eq} = R // jwL // \frac{1}{jwC}$

$Z_{eq} = \underline{Real} + jx$ ← هاي مش بس R وإنتبه

at Resonant frequency → $X = 0$

$w_o = \frac{1}{\sqrt{LC}}$

$f_o = \frac{1}{2\pi\sqrt{LC}}$ ← عند هاي الفريكونسي الـ C و L بلغو بعض.

فحص نوع الـ filter.

	$w = 0$	$w = \infty$
L	S.C	O.C
C	O.C	S.C
V_o	0	0

* لو بلش بـ V_{in} وإنتهي بـ V_{in} صاف بكون Bsf.

at $f = f_o \Longrightarrow V_o = V_{o\,max}$

حسب شكل الدائرة.

▶▶ Quality factor :-

$Q = w_o R C$ ← * إني بتحكم بالسيركيت أكثر هو فولتية الكاباستور.

- $Bw = \frac{w_o}{Q} = \frac{1}{RC}$ (rad/s)
- $Bw = \frac{f_o}{Q} = \frac{1}{2\pi RC}$ (Hz)

- $f_1 = f_o - \frac{Bw}{2}$
- $f_2 = f_o + \frac{Bw}{2}$

$V_o = \frac{V_{o\,max}}{\sqrt{2}}$

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